Mechanical Characterization of Materials and Wave Dispersion

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Instrumentation and Experiment Interpretation

Edited by
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Table of Contents

Chapter 13. Stationary and Progressive Waves in Rings and Hollow Cylinders 335

Yvon CHEVALIER and Jean Tuong VINH

Preface

In the world of the mechanical characterization of materials, the activities of researchers and engineers can generally be classified into three main areas:

1. designing and building appropriate instruments, covering both mechanical and electronic aspects;

2. conducting experiments to obtain dynamic responses to the bounded medium constituted by the sample, using signal processing to obtain appropriate dynamic responses from the sample;

3. searching for a solution to the inverse problem in viscoelasticity. The dynamics of the sample being known, the dynamic characteristics of the material are sought so as to obtain a material response, either in the field of frequency or time.

We will examine each of these steps in turn below, enabling us to present our point of view before presenting commentaries concerning each chapter of the book.

The conception and realization of appropriate instruments

At first glance, this heading might surprise some readers, given the number of commercially available instruments in applied viscoelasticity. However, in spite of the growing number of these instruments, rarely do they cover all the needs of researchers in this domain. Most of this apparatus does not cover a large frequency range. Often, the physical phenomenon of geometric wave dispersion in the bounded medium constituted by the sample itself is ignored. Evaluation of the material's viscoelastic parameters becomes problematic and might be subject to major error.

Discussion concerning adopted practical boundary conditions is often absent and the conception of the sample holder is such that parasitic coupling between vibrations of different kinds (bending, torsional, longitudinal) occur in the sample and empirical correcting terms are proposed.

In many cases, researchers have to conceive and make the apparatus themselves, to cover a very wide range frequency range from just a few Hertz up to or beyond 100,000 Hz, in which cases special transducers are especially fabricated from ferroelectric plates.

In some situations, a large range of negative and positive temperatures are used in the experiments. In these cases, the judicious choice of boundary conditions is important, taking into account the possible creep of the sample at high temperature.

Electronic instrumentation testing is facilitated by the profusion of apparatus in the field of dynamic testing of structures. With some precautions, experimenters can use them, the special use being essentially different from that frequently adopted in the dynamics of structures. The size of the sample in mechanical characterization is often much smaller than the usual size of mechanical structures compared to the size of the usual transducers themselves.

Conducting experiments on a sample in view of the dynamic characterization of the material

There are four steps in this stage: choice of boundary conditions, search for solution of direct problem, signal processing, and, finally, the search for a solution to the inverse problem. Items one and three will be explained as follows:

The choice of sample boundary conditions

This important choice depends on a variety of parameters which govern the dynamic performances of the sample: frequency range, transducer inertias, eventual extreme ambient temperatures, nature of the vibration imposed on the sample and the sample environment.

Signal processing

This concerns a set of tests which permit the detection of hidden non-linearities in sample responses which are due to transducer responses, the excitation of signal amplitudes. Recent advances in the detection of hidden non-linearities by using the Hilbert frequency transform deserve the experimenter’s attention.

Modal analysis in the domain of structural dynamics offers a variety of computer programs available for treating either the transient time response of the structure or its frequency response.

Solving specific viscoelastic problems

There are three steps in this final stage:

– mathematical modeling of the viscoelastic sample, the nature of the vibration being known;

– research into a solution for the material viscoelastic parameters of the sample in the framework of an inverse problem, the dynamic response of the sample being known;

– research into a closed form expression of a viscoelastic modulus versus the frequency or the corresponding relaxation or creep function versus time.

Mathematical modeling of a sample

This concerns a specific problem in which a set of sample elastic coefficients are known, and in which the dynamic responses are deduced from an appropriate theory chosen by the experimenter themselves (see [CHE 10]). This step is important so as to include all the dynamic and static effects which occur in the sample. It enables us obtain the phase velocity, from which the elastic (or viscoelastic) modulus is deduced, to be evaluated correctly.

Solution of the inverse problem when the dynamic responses of the sample are known

Before carrying out a testing procedure, the experimenter has already chosen an appropriate theory describing the dynamic behavior of the bounded medium (correctly, constituted by the sample in a chosen frequency range). This allows an appropriate mathematical model to be obtained, the dimensions and shape of the sample, as well as the nature of the vibration, being known.

The viscoelastic modulus of the sample material versus the frequency has to be evaluated from measured dynamic responses. That constitutes the solution of the inverse problem, whose mathematical unicity is unfortunately not ensured. Additional problems concerning a numerical solution include numerical instability (depending on the type of differential equations adopted) and the mathematical instability inherent to the type of numerical method adopted (Newton–Raphson's method for example). The experimenter also has to choose an optimization criterion.

Research into a closed form expression of the complex viscoelastic modulus versus frequency[1]

In the dynamics of a structure, it is necessary to include the constitutive equations of the material in any computer program. The closed form expressions of complex modulus obtained from the experimental responses of the sample have been proposed by many researchers[2].

Editors and authors

Editors

All chapters were rewritten by the editors, J.T. Vinh and Y. Chevalier, to bring a coherent approach across the whole book. The two editors were supervisors of research undertaken by all the authors of this book[3] in the material engineering laboratory at the Institut Supérieur des Matériaux et de la Construction Mécanique (ISMCM) in France, which was directed by J.T. Vinh from 1965 to 1996; from 1996 to 2009[4], the laboratory was directed by Y. Chevalier. Each chapter can be considered as a scientific paper with large extracts of the original thesis. Additional sections written by one of the co-ordinators enable us to situate the contribution of the research author among other scientific papers.

Some chapters are extracted from the editors' lecture notes delivered at the aforementioned Institute and at oversea universities when the editors were visiting professors. These complements in mechanics and viscoelasticity might be helpful to the reader.

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[1] In many cases this stage is not necessary for chemists investigating a material. The shape of the complex modulus, particularly the damping coefficients of a material, permits them to distinguish the role of the constituents of the material itself.

[2] See references at the end of this introduction.

[3] The exception to this is Touratier's doctoral engineering thesis, which was not prepared under the editors' supervision.

[4] Since 1998, the Institute has been called the Institut Supérieur de Mécanique de Paris (ISMEP), whilst the laboratory has additional activities and has changed its name to become LISMMA (Laboratory of Engineering of Mechanical Structures and Materials)

Contributors

The contributors to the book come from various backgrounds. Many are students in mechanical engineering from engineering colleges in France who prepared written final reports on research undertaken in mechanics during a period of one year or more (equivalent to a Master's degree in English-speaking countries). Others are students who obtained mechanical engineering diplomas, or research fellows who prepared PhD theses or engineering doctoral theses over three years or more.

Presentation of the content of the book

The book is divided into two parts. Part I (Chapters 1 to 8) is devoted to mechanical and electronic instrumentations.

Chapter 1 presents guidelines for the choice of experimental set-ups.

Chapter 2 provides a short review of some of the industrial analyzers available. A sample is often assimilated to a simple model with one degree of freedom mechanical system that is constituted by a spring and dashpot whose coefficients are frequency-dependent and capable of directly describing the material's dynamic behavior. This hypothesis is compatible with the use, at low frequency, of elementary motion equations in which shear and inertia effects of the sample are neglected even for a short sample (see [CHE 10]). Geometric wave dispersion is an important phenomenon in the bounded medium constituted by the sample. If it is ignored, calculations of the viscoelastic coefficient of the material are subjected to large errors when working frequency increases. Coupling vibrations give rise to empirical corrective coefficient in the proposed formulae.

Chapter 3 presents test benches with various mechanical parts. Different sample holders are discussed, in view of adopting the compatible boundary conditions. Ideal boundary conditions transposed into applications and mechanical realizations give rise to additional constraints which have an influence on the effective length (which is different from the measured sample length). A special pseudo-clamping system is proposed so as to avoid, in some special circumstances, sample length correction due to the three-dimensional state of stress in the part of the sample submitted to compressive force. The accuracy of the material complex modulus obtained depends on the choice of the sample holder and the care with which this mechanical part is realized.

Chapter 4 is devoted to electromechanical exciters as well as to piezoelectric and ferroelectric exciters. Various mechanical exciter signals (impulse, sinusoidal, white noise) are discussed. It is shown that, in many cases, we need to make special-sized transducers, or transducers working in unusual frequency ranges, by ourselves.

Chapter 5 deals with transducers in their various forms (displacement, velocity, acceleration, force, etc.). Their choice and correct mounting conditions, and the validity of final results, are reviewed.

Chapter 6 concerns electronic equipment for transducers. Practical considerations about connecting cables are discussed. The equipment devoted to digital signal processing and the useful programs derived from a fast Fourier transform (FFT) are concisely presented. Attention is focused on the transfer function and the coherence function extensively used in dynamic tests.

Chapter 7 is devoted to the Hilbert frequency transform which has recently been applied with success in experimental structural dynamics. The transform enables the detection of the presence of possible hidden non-linearities in the responses of the sample. We are convinced this is very helpful for experimenters.

In Chapter 8, various methods of damping measurements commonly adopted in structural dynamics are presented and discussed. Use of such possibilities extended to material damping evaluation, requires caution, particularly for high damping materials[5]. Making a distinction between *structural mechanical damping* and *internal material damping* avoids confusion. The problem resorts to an inverse problem and the corresponding solution is rarely directly obtained from a structural damping coefficient, except in some cases where explicit formulae are obtained by expansion into series of the eigenvalues versus structural damping coefficients. There are not one but at least two (or possibly more) material damping coefficients, depending on the symmetry of the material itself which conditions the number of coefficients. For composite non-isotropic materials, complex non-diagonal terms of compliance (or stiffness) matrixes require coupled vibrations, of two vibrations of a different nature. Evaluation of damping coefficients becomes difficult if not impossible. The energy partition is not easy to operate.

Part II (Chapters 9 to 15) is devoted to experiments and the experimental interpretation of elastic (or viscoelastic) moduli. After choosing the type of vibration (extensional, bending, and torsional) according to the modulus to be measured, appropriate equations are retained. The next step is to realize the experimental set-up including mechanical and electronic parts.

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[5] The material damping coefficient $\tan \delta$ is equal to or higher than 5×10^{-2}.

Just as Part I presents details on instrumentation, Part II presents the aforementioned set-ups concisely. Some suggested set-ups can be adopted in laboratory experiments for students and researchers, to illustrate an advanced course (or research) in elastodynamics. Special experiments can be envisaged to obtain experimental geometrical dispersion curves even at higher modes of vibrations and higher frequency.

Chapter 9 presents two different apparatuses for the torsional vibration of rods: one for lower frequency ranges and the other adapted for higher frequency exploration. The first of these uses a pseudo-clamping of the sample. The symmetry of the sample and the application of excitation of the torque in the middle of the sample permits a moderate clamping force to be adopted for the sample. Length correction is not necessary. Saint-Venant’s principle is used and the working frequency is less than 2,000 Hz.

Nugues’s theory is used and wave dispersion is discussed taking into account some parameters such as flatness (the ratio of width to thickness) and slenderness (the ratio of width to length).

Onobiono’s theory (see [CHE 10]) does not use Saint-Venant’s principle but rather Engstrom’s theory, extended to anisotropic material. This theory is more appropriate to portray warping phenomenon at higher elastodynamic modes and higher frequency. A special set-up is used for this purpose.

Chapter 10 is devoted to bending tests which are the easiest to realize. As the degree of the differential equations of the motion (Bernoulli–Euler’s or Timoshenko–Mindlin’s equations) does not exceed 4, systematic use of characteristic functions[6] is possible to find the eigenvalue equation of the vibrating rod. Wave dispersion is discussed in relation to some important parameters such as slenderness, flatness and the ratio axial Young’s modulus on shear modulus; this last parameter plays an important role in the study of composite materials.

Chapter 11 concerns longitudinal vibration of rods. In classical textbooks devoted to the dynamics of structures, a second order equation of motion is referred to. Experimenters must be careful when choosing such an equation. Wave dispersion is absent from this equation and this assertion might give rise to large errors when evaluating a complex Young’s modulus. The validity of this hypothesis must be checked by evaluating the frequency range as a prerequisite condition. Elastodynamic spectrum[7] or the curve relative phase velocity versus relative wave

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[6] This function is the linear combination of trigonometric and hyperbolic functions.

[7] Represented by the curve relative phase velocity versus the relative wave number.

number presented in this chapter is appropriate in the determination of the width of frequency range.

When moving from an isotropic material to an anisotropic (composite) material, wave dispersion is much more pronounced and the experimenter’s attention is required when choosing appropriate equations of motion.

In Chapter 12, details are given on the conception and fabrication of Le Rolland-Sorin’s double pendulum which allows Young’s and shear moduli of materials to be evaluated at low frequency. This simple and artful apparatus does not require an external exciter and the only necessary measurements are of the beating period of the pendulums. With a recording of the oscillations of one of the pendulums, it is possible to evaluate the material damping coefficients.

Extension of the use of the double pendulum to measure elastic (or viscoelastic) moduli of anisotropic materials is possible.

Chapter 13 is devoted to measurements of material moduli using rings or hollow cylinders. In many circumstances, a sample is presented with an unusual shape. To fully characterize the material, it is necessary to effect measurements directly on a ring and hollow cylinder successively and also, if necessary, on a straight rod cut-off from a hollow cylinder with a curved cross-section.

In Chapter 14, ultrasonic benches are presented. A water immersion bench is devoted to measurements of two stiffness coefficients. Two direct contact benches permit the evaluation of the remaining material stiffness coefficients. Evaluation of damping coefficients is possible using a logarithmic decrement method.

Chapter 15 is not (yet) devoted to industrial applications. It concerns special devices to evaluate three kinds of waves in turn: torsional, bending and longitudinal waves using long rods and special transducers. These enable experimental evaluation of phase velocities of the three waves in an extremely wide frequency range. Theoretical wave dispersion studies are presented in parallel so as to obtain a higher approximation degree for the motion equations. A variation Hellinger–Reissner’s principle with mixed fields (of displacement and stress fields) is used for this purpose. These experiments might be interesting for researchers not only in the domain of metallic materials but also in composite anisotropic materials.

The objective of Chapter 15 is also to show that wave dispersion at high and very high frequencies belongs, in fact, for the moment to academic theoretical studies[8]. However, in the near future, promising prospective applications can be envisaged,

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[8] To our knowledge, the devices described (and the experimental higher elastodynamic spectra for composite materials at higher modes) are among the first ever presented.

including interesting applications in material characterization and in fracture mechanics.

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June 2010

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Acknowledgements

Before interpreting the dynamic responses of rods and plates, measurements taken of these vibrating bounded media seem *a priori* an easy task when we have at our disposal all the appropriate instruments especially tailored for this task. That was the belief of one of the writers of this book, 40 years ago. Often, however, we are not so lucky.

Over the years, researchers, engineers, students and also technicians have realized that they needed to do a lot of work themselves to improve instruments or invent new ones. The opinions of each researcher and student are invaluable contributions to be taken into account.

As theoretical studies concerning wave dispersions evolve over time, the range of their investigations gradually reaches new unknown frontiers. Our ambition has been to find new and appropriate devices to obtain corresponding experimental results to confirm or to invalidate theoretical results.

In all this research, we have to create appropriate devices for the new measurements ourselves. In this task, the opinions of technicians are as important as those of researchers.

Jean-Baptiste Casimir, Assistant Professor at the ISMEP, is a pioneer in the field of the continuous element method applied to mechanical structures in their various forms (rod, plates, cylinder, etc). He also got the knack of effecting complementary calculations and experiments during the last months of the university year of 2009. He supervised the writing of Chapter 12.

Professor *Fei Bin Jun*, Vice-President of the University of Aeronautics and Astronautics in Beijing, China, and our old research fellow, kindly sent us his Master of Science report (September 1985) on the Hilbert frequency transform. His pioneering work in this area is presented in Chapter 7.

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[1] His thesis was not prepared under our supervision