

Chapter 5

Transducers

5.1. Introduction

This chapter is devoted to the technology of transducers, their choice and their dynamic response.

The choice of transducer is essentially dependent on the physical functions one wishes to investigate for material characterization. Adoption of such functions must be examined carefully at the beginning of a measurement investigation.

Well-known methods used in structural dynamics are applicable here, amongst which should be mentioned:

$$\left. \begin{array}{l}
 \text{mechanical impedance defined as: } [F_i \text{ (force)}] / [\dot{x}_j \text{ (velocity)}] \\
 \text{mechanical mobility: } [\dot{x}_j \text{ (velocity)}] / [F_i \text{ (force)}] \\
 \text{dynamic stiffness: } [F_i \text{ (force)} / x_j \text{ (displacement)}] \\
 \text{dynamic receptance: } [x_j \text{ (displacement)} / F_i \text{ (force)}] \\
 \text{inertia: } [F_i \text{ (force)} / \ddot{x}_j \text{ (acceleration)}]
 \end{array} \right\} \quad [5.1]$$

In all the definitions above, force measurement is required.

Other possible methods are not currently used in structural dynamics but are of great interest for material characterization. The transfer functions presented below

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have, as input and output signals, the same nature of physical quantities: displacements x_i , x_j , velocities \dot{x}_i , \dot{x}_j , accelerations \ddot{x}_i , \ddot{x}_j , and indexes i and j , designating the localization of input and output points, respectively.

$$\left. \begin{aligned} \text{TF}_x &= \left[x_j \text{ output displacement} / x_i \text{ input displacement} \right] \\ \text{TF}_v &= \left[\dot{x}_j \text{ output velocity} / \dot{x}_i \text{ input velocity} \right] \\ \text{TF}_a &= \left[\ddot{x}_j \text{ output acceleration} / \ddot{x}_i \text{ input acceleration} \right] \end{aligned} \right\} \quad [5.2]$$

The main interest of the definitions in [5.2] resides in the fact that, the nature of the signals being the same, utilization of transducers of the same type allows transfer functions which are less dependent on each response transducer. There is a kind of auto-compensation. If the two transducers are identical in weight and frequency response, the frequency response of a sample in [5.2] takes place in the numerator and denominator as well. This transfer function is emphasized in Part II (see [CHE 10] from the same authors, Chapters 5 and 12).

In Figure 5.1, two set-ups are represented. In Figure 5.1a, the exciter and connecting part on one hand, and the sample and transducers on the other, have a similar volume. In Figure 5.1b, the volume of the exciter is much less than that of the structure being tested. The influence of the exciter in Figure 5.1b can be minimized when, as is the case in Figure 5.1a, the mechanical response of the set (exciter and the connecting part) influences the response of the sample.

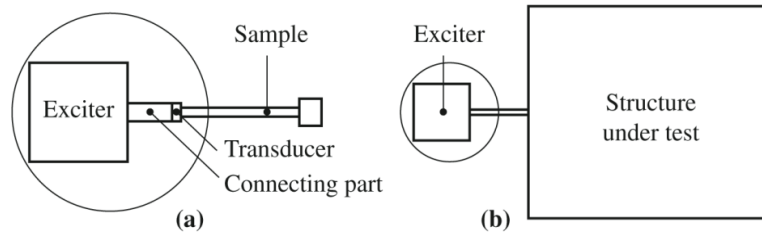


Figure 5.1. *The relative importance of a sample of (a) a set constituted by an exciter and connecting part is not the same compared to (b) a classical dynamic test on a normal structure*

5.2. Transducers and their principal performance

A transducer (or pick-up, or sensor) is a physical system which transforms shock or vibration into an optical, mechanical or electrical signal which is proportional to the motion of the parameter of interest. An electrical signal is rarely and directly usable for various reasons which will be examined later.

A signal conditioner is used (Figure 5.2). Its objective is to:

- amplify the signal and eventually convert it into another electric signal;
- to satisfy adaptation conditions, such as:
 - input electrical impedance;
 - output electrical impedance;
 - noise level;
 - working frequency range.

These specifications are an integral part of the transducer and are useful for the experimenter when choosing a transducer.



Figure 5.2. *A signal conditioner constitutes an integral part of a transducer*

5.3. The main classes of fixed reference transducers

If all the electric and electronic components were presented, there would be more than a dozen possible transducers to choose from. However, by restricting ourselves to a list of the transducers usually used for material characterization, the following types should be mentioned:

- electric condenser;
- inductance;
- mutual inductance;
- differential transformer;
- strain gauges with resistive wire; and
- piezoelectric effects.

The first four types are most popular. Figure 5.3 presents the principles of these transducers, which are commercially available.

The electric voltage obtained at the output of a signal conditioner (Figure 5.2) is rarely the same as the electric potential difference directly measured in the electric elements presented in Figure 5.3 (namely the condenser, inductance, mutual inductance).

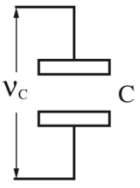
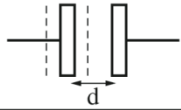

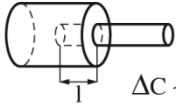
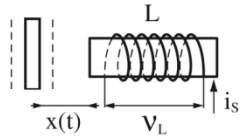
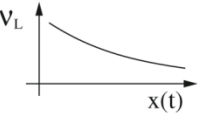
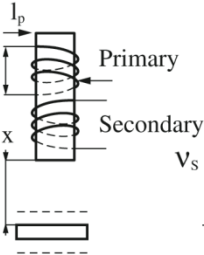
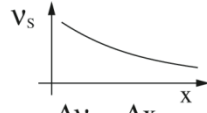
<p>Condenser</p>  <p>Contact less transducer</p>	<p>Plane electrode</p> $C = \frac{\epsilon \delta}{d}$ <p>δ : electrode surface d : distance between electrode ϵ : air dielectric constant</p>	 <p>$\Delta C \sim \frac{1}{\Delta d}$ Translation</p>  <p>$\Delta C \sim \Delta \theta$ Rotation</p>  <p>$C = \frac{2\pi\epsilon l}{\ln \frac{R_1}{R_2}}$ $\Delta C \sim \Delta l$</p>
<p>Contact less inductance transducer with permanent magnet no power supply</p>	<p>$v_L = L \frac{di_s}{dt}$</p> <p>Velocity transducer</p>	 <p>$v_L \sim \frac{1}{x(t)}$</p> 
<p>Contact less mutual inductance transducer displacement</p>	<p>$v_s = M \frac{di_p}{dt}$</p> <p>Primary winding with alternative tension</p>	 <p>v_s</p>  <p>$\Delta v_s \sim \Delta x$ Secondary</p>

Figure 5.3.a Some transducers based on classical electrical elements

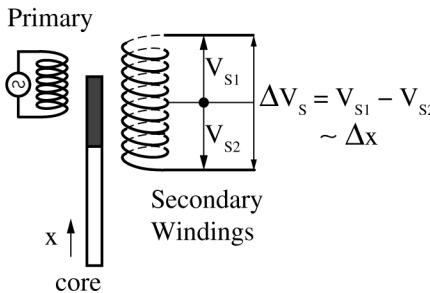
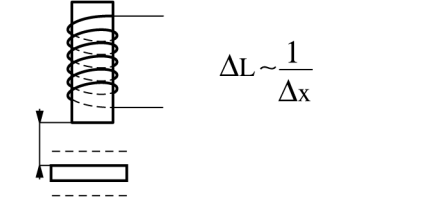
<p>Differential transformer with Moving core Displacement transducer</p>	$V_s = V_{s1} - V_{s2}$	
<p>Variable self inductance No permanent magnet Displacement transducer</p>	$v = L \frac{di}{dt}$	

Figure 5.3.b Some transducers based on classical electrical elements

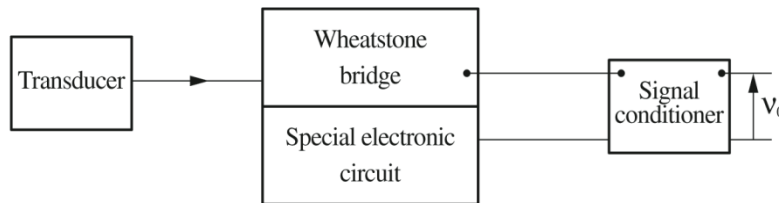


Figure 5.4. An intermediary circuit (in this case, a Wheatstone bridge; see section 5.4.2 below) is inserted between the transducer and the signal conditioner

Often an intermediary stage is inserted between transducer and signal conditioner (Figure 5.4).

Experimenters must be aware, at least, of the electrical principles governing the successive stages after the transducer. These principles allow the electrical performance and particularly the transducer limitations to be appreciated. The choice of transducer is an important matter which merits the reader's attention. Details of each type of transducer are given in the following chapters.

5.4. Condenser-type transducer

There are three types of condenser transducers: those with polarization circuits, those with a Wheatstone bridge, and those with a frequency discriminator.

5.4.1. Polarization circuit

A polarization circuit is used in a condenser microphone for acoustical applications. Figure 5.5 represents a polarization voltage V of the order of 100 Volts, which supplies the variable condenser C via a high resistance R ($\approx 10^8$ Ohms).

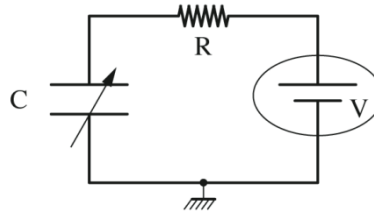


Figure 5.5. Continuous electric voltage feeding a condenser with a series resistance R

The voltage of the condenser is:

$$V_C = \frac{1}{C} \int_0^t i dt \quad [5.3]$$

with $i(t)$ current intensity function of time variable t .

The voltage V_R is given by:

$$V_R(t) = V \frac{C_1}{C_0 \sqrt{1 + \frac{1}{(RC_0\omega)^2}}} \sin(\omega t + \phi) \quad [5.4]$$

Full details of the calculation of the voltage V_R at the two sides of the resistor, R , are given in Appendix 5A.

The harmonic variation of the condenser capacity is given by:

$$C = C_0 + C_1 \sin \omega t \quad [5.5]$$

where C_1 concerns the variable part of the condenser C . C_1 is inversely proportional to the displacement $x(t)$ of one of the condenser plates, which is glued to one end of the sample, ϵ is the air dielectric, and S the condenser plate surface:

$$C(t) = \epsilon S / e(t) \tag{5.6a}$$

$$e(t) \approx x(t) \tag{5.6b}$$

Figure 5.6 shows the two amplifiers which follow the polarization circuit. Amplifier A_1 has a field effect transistor with an input impedance $Z_i \approx 100\text{ M}\Omega$.

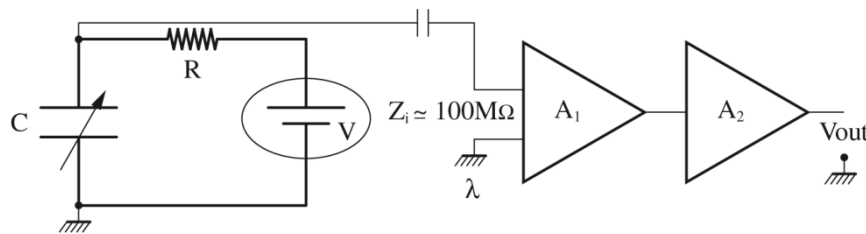


Figure 5.6. The first amplifier stage, A_1 , has a high input impedance and a voltage gain equal to 1. The last amplifier, A_2 , is a voltage amplifier with low output impedance $Z_o \approx 600\ \Omega$

5.4.2. Condenser transducer with a Wheatstone bridge

A special Wheatstone bridge with alternating potential has been used with success to obtain an electrical output voltage proportional to the condenser capacity. Figure 5.7 presents a diagram of electronic circuits. Attention is drawn to the high frequency oscillator feeding the Wheatstone bridge ($F \geq 25,000\text{ Hz}$).

It has been shown that the higher the frequency, the more sensitive the bridge, permitting measurement of displacement equal to or less than 1 micrometer.

If the condenser is made up of two parallel moving plates, as in equation [5.6.a] and [5.6b], the voltage V_R in [5.4] is proportional to variable part C_1 of the condenser capacity C . V_R is then inversely proportional to displacement $x(t)$. This non-linearity must be taken into account.

The equilibrium of the bridge at the outset is achieved by capacitive adjustment. The transducer condenser is connected to one arm of the bridge.

The main part of the conditioner circuit is the phase demodulator (see Figures 5.8 and 5.9).

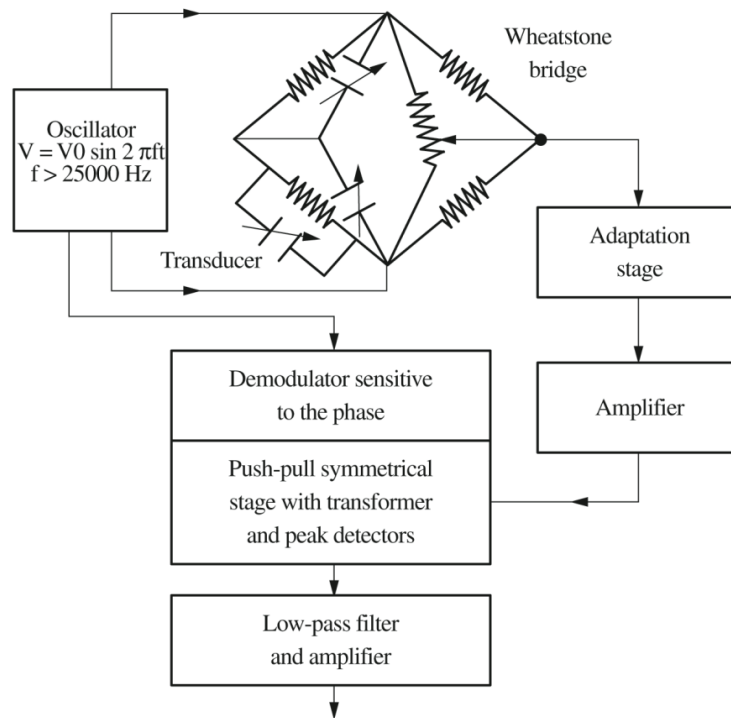


Figure 5.7. *Wheatstone bridge fed with voltage furnished by a high frequency oscillator*

There are two potentials at the output of the bridge: E_b from the bridge due to the variation of the condenser capacity, and $E_c = E_{\text{carrier}}$ the potential furnished by the oscillator (carrier wave). E_b is the vectorial result of two potentials. Two outcomes can occur during the vectorial addition: see cases (b) and (c) in Figure 5.9. The variable E_b has a positive or negative sign depending on the direction of motion of the capacity.

Thus, the demodulator is sensitive to the input signal phase. Without going into details of the electric circuit, let us say that demodulation is obtained by two special transformers with four secondary windings.

Figure 5.9a represents the output signal. The signal envelope is the desired signal which is to be extracted from the modulated signal by an envelope detector (Figure 5.9b). A low pass filter eliminates the residual variations due to the carrier signal.

To obtain an output from a Wheatstone bridge from which a capacity variation ΔC of the order of a fraction of picofarad can be detected, the frequency of the carrier oscillator must be of the order of 25,000 Hz or higher.

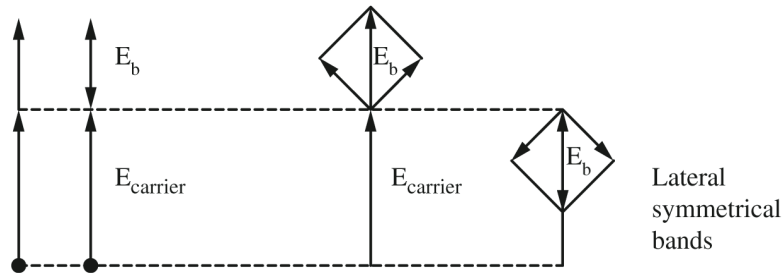


Figure 5.8. The output signal is the sum of two voltages: E_b furnished by the bridge, $E_{carrier}$ furnished by the carrier wave oscillator. E_b and E_c are always collinear, E_b has variable signs depending on the direction of E_b

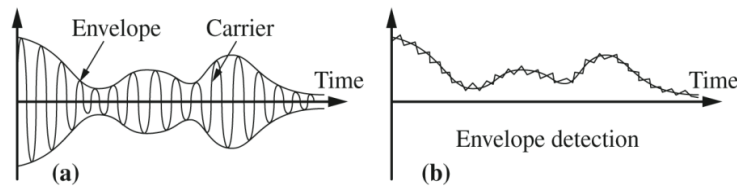


Figure 5.9. (a) Modulated signal obtained at the output of the Wheatstone bridge; (b) demodulated signal

Note that, due to the presence of a low-pass filter (Figure 5.9b), the maximum admissible highest frequency depends on the cut-off frequency of the filter (defined by the attenuation of signal measured at the 3 decibel level). In practice, the maximum usable frequency is

$$f < \frac{1}{5} \text{ or } \frac{1}{10} f_{osc}$$

with f_{osc} the oscillator frequency of the bridge.

5.4.3. Condenser transducer with a frequency discriminator

The electronic circuit used is called a frequency discriminator. It is frequently used in frequency modulation receivers. The principle is to convert a variation of frequency into a variation of electric potential. In Frequency Modulator (FM) receivers, this circuit is necessary to obtain an audible signal. With some adaptation this circuit can be used as a signal conditioner for a capacitive transducer.

5.4.3.1. Principle of a discriminator circuit

To understand the main idea of this circuit, we should examine an anti-resonant circuit, Figure 5.10. The graph of the electric response versus frequency, Figure 5.10b, is used to convert a variation of angular frequency $\Delta\omega$ into potential variation ΔV_c at the two ends of the condenser.

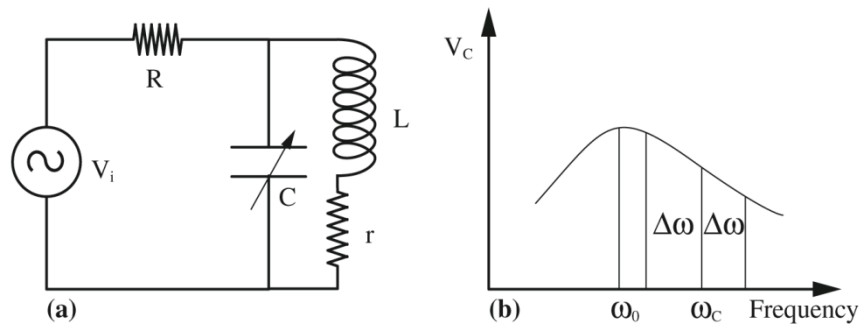


Figure 5.10. Antiresonant electric circuit used to convert circular frequency ω into voltage variation ΔV_c (a) antiresonant circuit (b) voltage V_c versus frequency

Instead of varying frequency, the capacitive variation ΔC around a center frequency f_{c0} creates a variation of voltage ΔV_c which is not linear plotted against variation in capacity.

5.4.3.2. Discriminator circuit

A discriminator circuit improves the electric circuit to obtain a linear variation of V_c (see Figure 5.11).

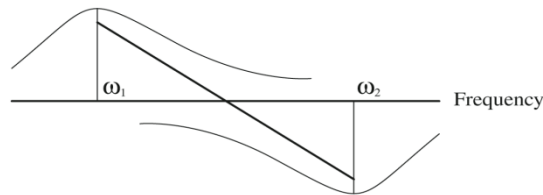


Figure 5.11. Utilization of two anti-resonant circuits at two different resonance circular frequencies ω_1, ω_2 . Between ω_1 and ω_2 the resultant voltage is linear

If we can subtract the signal from one anti-resonant circuit from that of the second circuit, we can obtain a linearized response:

$$V_{\text{output}} = V_{c1} - V_{c2} \quad [5.7]$$

This “trick” is well known in applied analog electronics. It is called a push-pull differential system. Figure 5.12 represents such a circuit, called a discriminator circuit.

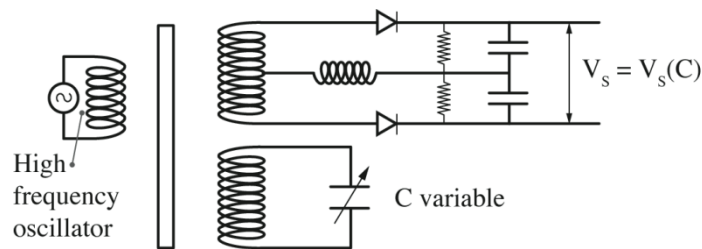


Figure 5.12. Discriminator circuit with a symmetrical secondary winding. A second secondary winding is used for a variable condenser C_V

With two diode detectors, and with resistance R and capacity C , the signal obtained at the output V_s is demodulated. The two center frequencies f_{c1} and f_{c2} must be sufficiently high (f_{c1} and $f_{c2} \approx 10^6$ Hz) so that the low pass filter gives the maximum frequency as: $f_{\text{max}} = 10^6/5 = 2 \times 10^5$ Hz.

5.4.4. Sensitivity of capacitive transducers

Condenser transducers with signal conditioners are the most sensitive transducers one can find among other industrially available transducers.

Model/make	Specification	Description
Brüel and Kjør MM004	$\Delta V_S = k\Delta x$ Sensitivity 25V/mm	Displacement transducer
Dress-Barnes 40 Type	$\Delta V_S = k\Delta x$ Sensitivity 100mV/inch or 3.9 mV/ μm	Displacement transducer
General Radio ⁽¹⁾	$\Delta C \cong 10^{-5}$ to 10^{-4} pF; $\Delta x \cong 10\mu\text{m}$	Displacement transducer with Wheatstone bridge
ONERA 20W304	$C \cong 1$ pF Sensitivity 0.24 V/ μm	Displacement transducer
CNRS Grenoble, France	10 Angstrom $< x < 100\mu\text{m}$	Dilatometer multielectrode transducer

(1) Concerns a Wheatstone bridge only

Table 5.1. Performance of some capacitive transducers with special signal conditioners

Table 5.1 shows that, with the addition of an extra specialized capacitive transducer, sensitivity goes from 20 Volts/mm to 0.2 Volt/ μm .

5.4.5. Advantages of capacitive transducers

Capacitive transducers can work at very high temperature. The dielectric constant of the condenser is the air, which does not vary with temperature (if the atmosphere is humidity free).

The second reason for their popularity is the fact that they are a contactless transducer and the simplicity of their installation. This type of transducer is much appreciated in the highest range of frequencies, going beyond megaHz at their upper frequency limit.

Figure 5.13a shows details of their construction, including mention of the brass tube which constitutes a shield for the condenser. We should mention that the useful variable capacity ΔC can be a small fraction of the maximum capacity. Figures 5.13b, c, d and e show some examples of applications. The pick-up plate in each case must be insulated.

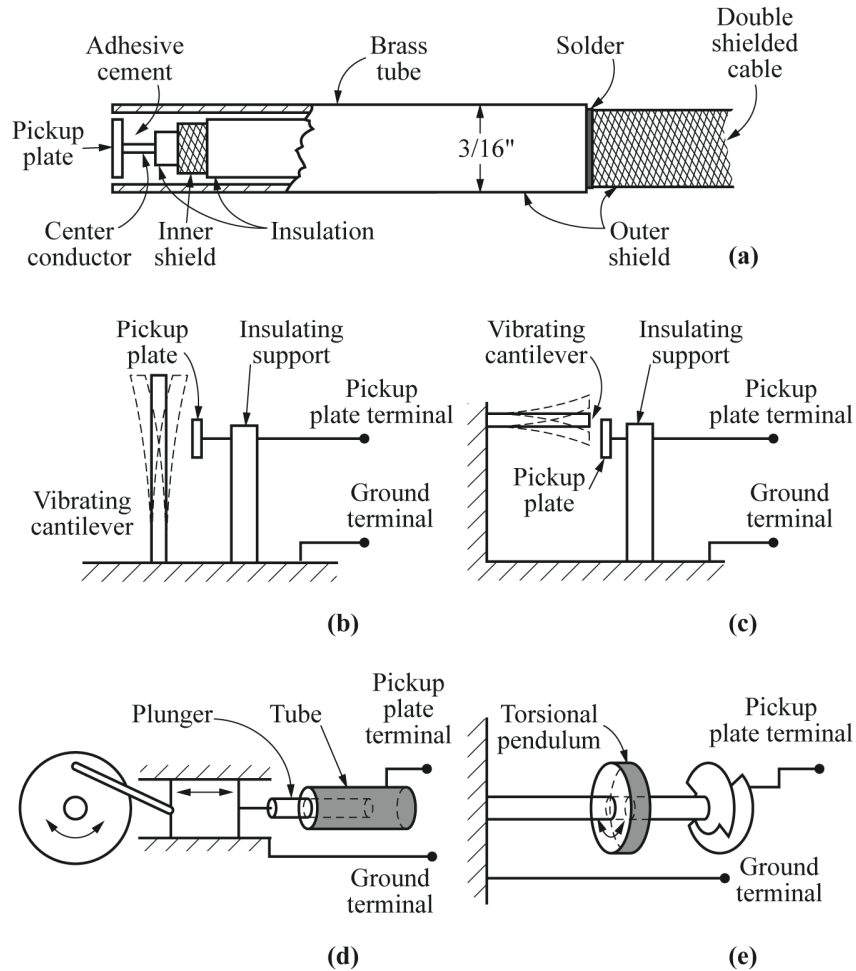


Figure 5.13. Some examples of capacitive transducer mountings

5.5. Inductance transducers

An inductance transducer is made up of a coil around a ferromagnetic core (which is not a permanent magnet). Inductance occurs in the presence of a ferromagnetic mobile target (Figure 5.14). When the transducer inductance is fed with an alternative voltage, there are magnetic force lines between the target and the transducer core. The target's motion modifies the force lines and inductance L varies. The inductance L of the transducer (Henry, unit of measure) is:

$$L = 4\pi n^2 \mu S \ell (10^{-9}) \quad [5.8]$$

where n is the number of turns, S the area of the core cross-section, ℓ the core length, and μ permeability of the core.

The variation of L versus the displacement x of the target is not linear. To transform the variation of L into an electrical signal, it is necessary to use an alternative Wheatstone bridge.

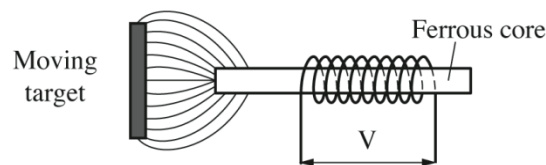


Figure 5.14. Inductance of the coil varies with the gap between the target and the transducer

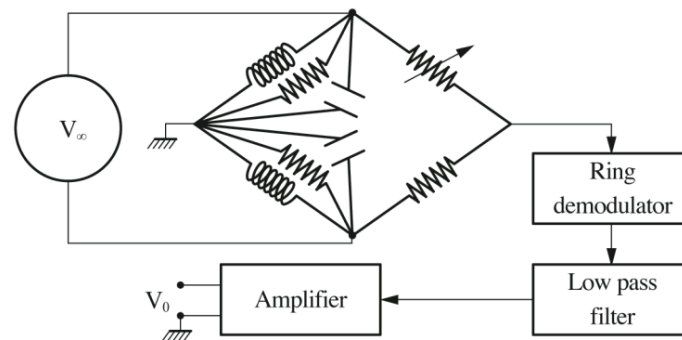


Figure 5.15. Wheatstone bridge fed by an alternative electric voltage transforming the variation of active inductance L into variable voltage

Figure 5.15 shows a single transducer, with inductance L in the arm (left side) of a Wheatstone bridge¹. To bring the bridge to equilibrium, a passive inductance L_p is used. A ring demodulator and an amplifier provide an electric signal. It is important to take into account the carrier frequency of the Wheatstone bridge, as mentioned in the previous section devoted to capacitive transducers, as this carrier frequency is

¹ A normal commercially available Wheatstone bridge is fed with an alternative voltage whose frequency is around 2,500 Hz.

not the maximum usable frequency. Generally, the low pass filter at the output signal of the bridge limits this frequency to:

$$f_{\max} \approx 1/10 \text{ to } 1/5 \text{ of the carrier frequency } f_{\text{car}} \quad [5.9]$$

In an induction-type transducer, the passive inductance (see Figure 5.16) is provided not independently but behind the active inductance. To facilitate the adjustment of the bridge equilibrium, the passive inductance has a fixed plate with a gap, with adjustable distance.

5.5.1. One active and one passive inductance

Figure 5.16 shows the disposition of one active inductance. The moving target serves for adjustment of the bridge.

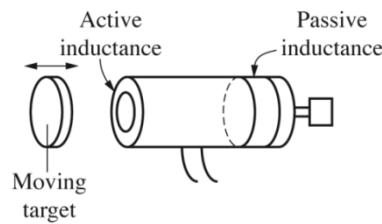


Figure 5.16. A transducer incorporating two inductances in the same housing: active inductance in front of a moving target (on the left) and passive inductance with an adjustable plate whose gap is adjusted by screw (on the right). This inductance applies only to the Wheatstone bridge equilibrium

The problem of non-linearity when using one active inductance should be examined, particularly when a moving target is subjected to large displacement. In such cases, a special electronic linearized circuit could be used to reduce the non-linearity effect, after obtaining a response from the transducer with a signal conditioner.

5.5.2. With two active inductances

With two active inductances, symmetrical mounting improves the linearity of the transducers. Figure 5.17 shows two possible arrangements; the passive transducer shown in Figure 5.15 is replaced by an active one. The bridge delivers two electrical signals of opposite signs. Their sum (Figure 5.17) is the linearized output voltage. This electrical system is popular in electronics and is called a “push-pull” system.

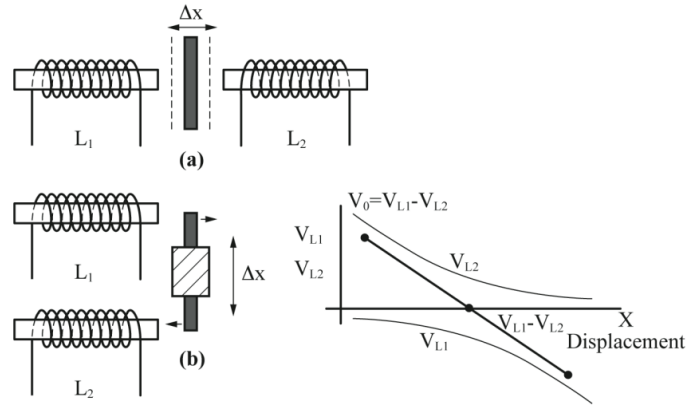


Figure 5.17. Symmetrical mounting of two inductance transducers: (a) a moving target lies between two transducers and produces variable horizontal displacement; (b) a moving target at fixed distance from the transducers. The target surface in front of the transducers changes with vertical displacement of the surface

5.6. Mutual inductance transducer

Figure 5.18 shows a transducer made from a transformer with two windings.

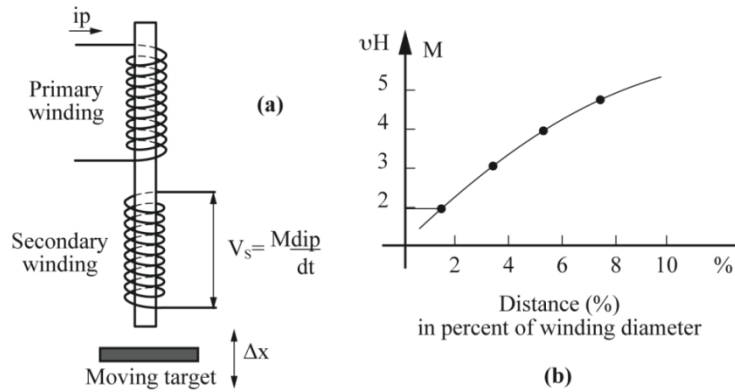


Figure 5.18. Mutual inductance transducer: (a) transformer with two windings and a moving target; (b) the non-linear response of the transducer

An alternative current is fed to the primary winding:

$$i_p = I_p \cos \omega t \quad [5.10]$$

This current produces a voltage at the secondary winding:

$$v_s \cong M[di_p/ dt] = - M\omega I_p \sin\omega t \quad [5.11]$$

The moving target in front of the second winding causes the mutual inductance to vary:

$$M = M(x) + M_0 \quad [5.12]$$

and the output signal is:

$$V_s = \{ M_0 + M [x(t)] \} \omega I_p \sin\omega t \quad [5.13]$$

A demodulator is necessary to detect the output signal envelope which varies with target displacement $x(t)$. Figure 5.18b shows that the mutual inductance variation versus the displacement is non-linear. The transducer material is not necessarily iron. An electrically conductive material (such as copper or aluminum) works quite well. The target surface in front of the transducer must be sufficiently large.

It should be noted that the frequency of the primary current in [5.10] can be as high as $f \cong 2$ MHz after signal detection, and with a low pass filter the working frequency might be of the order of $f_w = 10^5$ Hz.

A linearized electronic circuit or a push-pull mounting, similar to that presented in Figure 5.17, can be used.

5.7. Differential transformer transducer

This type of transformer was mentioned in Figure 5.3. It produces a remarkable linear response. However, one of the main disadvantages of this transducer resides in the fact that there is a connecting rod fixed to the sample. Consequently its use must be prohibited in the measurement of material damping.

5.8. Contactless inductance transducer with a permanent magnet

This type of transducer was presented in Figure 5.3. There is no polarization or feeding voltage. The core of the winding is a permanent magnet which is necessary to generate an output voltage when the target moves in front of the inductance. The voltage V_0 generated at the two terminals of the coil is:

$$V_0 = -Blv (10^{-8}) \text{ Volt} \quad [5.14]$$

where B is the magnetic density flux in Gauss, l is the length in cm, v is the velocity (cm/s) of the target.

This is a low-cost velocity transducer. If the sample material is metal, an eddy current can be produced in the sample. The main disadvantage of this transducer is the magnet which produces a permanent attractive force on the sample (if the target is ferromagnetic) which influences the damping measurement of the sample material.

5.9. Eddy current transducer

This type of transducers is very popular in structural dynamics, due to its simplicity in utilization.

5.9.1. Eddy current transducer: loss of energy in metallic targets

Figure 5.19 shows a contactless eddy current-based transducer.

The coil is fed with alternating current at high frequency. The electromagnetic field depends naturally on the permeability of the target but also on the gap between the coil and the target.

There is an energy loss in the target. This energy is taken away from the coil energy.

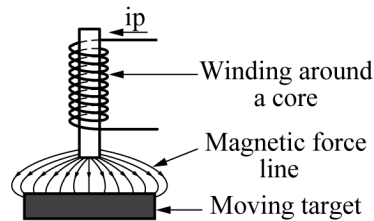


Figure 5.19. Eddy current-based transducer. Magnetic force lines are established between the winding core and the moving ferromagnetic target

If the target is too far away from the coil, the energy loss is usually low. By contrast, if the coil is close to the target, this loss increases. This energy loss gives rise to the reluctance variation of the coil.

5.9.2. Sensitivity

Figure 5.20 shows the curve sensitivity of the transducer expressed in relative voltage, obtained with various metals by comparison to aluminum resistivity. Magnetic materials are not necessarily more sensitive compared to non-magnetic materials.

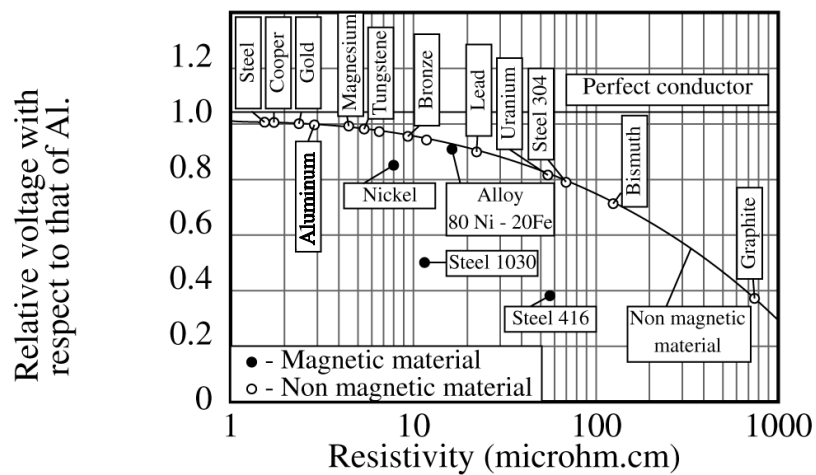


Figure 5.20. Sensitivity of metals which constitute the moving target. Output voltage depends on the metal resistivity

5.9.3. Influence of frequency

The frequency of the voltage supply plays an important role. When the frequency increases, magnetic force lines penetrate the target less deeply. This effect is similar to the Kelvin effect in a steel target. The supply frequency is generally high and of the order of megaHz. The high sensitivity of this transducer is related to the frequency.

5.9.4. Schematic diagram of an electronic circuit

The Wheatstone bridge used is similar to that used for an inductance transducer presented in Figure 5.15, except that the carrier frequency is in the range of megaHz. Two transducers are used, one for the equilibrium adjustment of the bridge, the other to act as the active transducer.

A ring demodulator, logarithmic amplifier and adjustable amplifier gain are the possible electronic circuits to follow the Wheatstone bridge (Figure 5.21).

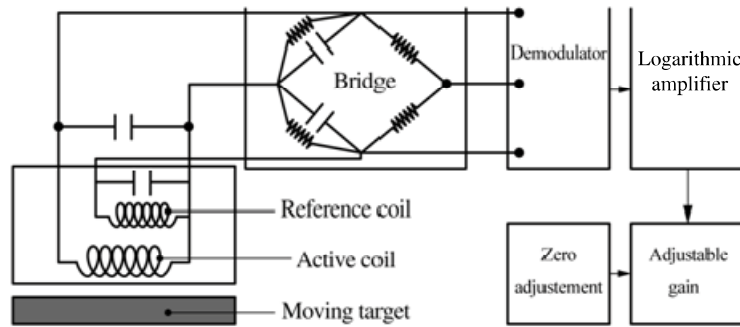


Figure 5.21. One active inductance type transducer. The other transducer serves as a reference transducer. Uses high frequency and a Wheatstone bridge

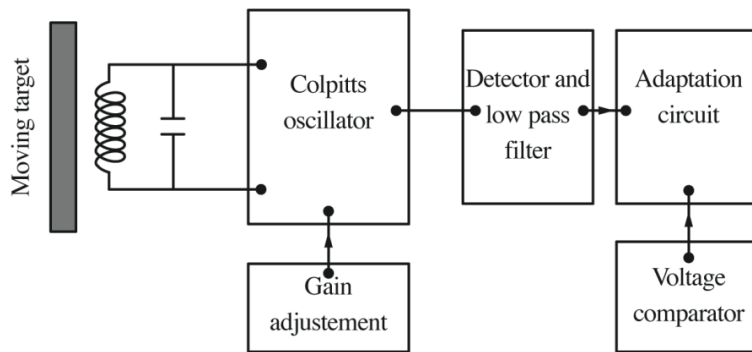


Figure 5.22. Colpitts oscillator feeding an eddy current type transducer. Inductance of the coil varies with moving target position

5.9.5. Usable frequency range

The usable frequency range is not the frequency of the oscillator which feeds the Wheatstone bridge. It is much lower: $f_{uti} = 1/10$ to $1/5$ of $f_{oscillator}$

5.9.6. Performance of some industrial eddy current transducers

Table 5.2 shows additional parameters; however, frequency response, the output voltage range, current and output impedance are not mentioned here. Moreover, thermal sensitivity should be taken into account when testing at high temperature.

Transducer target	0.5 SU non-magnetic	4S4 non-magnetic	6U1 non-magnetic	3OU1 non-magnetic
Measuring range (mm)	0.5	4	6	30
Resolution mid range (Hz)	10^{-4}	4×10^{-4}	6×10^{-4}	3×10^{-3}
Non-linearity (mm)		8×10^{-4}	1.2×10^{-3}	0.15
Sensitivity (mV/mm)	2.54×10^4	2.54×10^3	2.54×10^3	2.54×10^2

Table 5.2. Useful parameters (measuring range, resolution, non-linearity, sensitivity) to be considered when selecting an eddy current transducer.
Source: Kalman Science Corporation documents

5.9.7. Calibration procedure and linearizer circuit

This calibration procedure is also useful for other transducers. It necessitates a micro calibration system fixture. The transducer is fixed along an axis, an appropriate distance from the sample target. The accuracy of the micrometer must be equal to or greater than that required for effective measurements.

Another method consists of the use of plastic spacers inserted between the sample target and the sensor. A thickness of three spacers is necessary for calibration. The use of plastic spacers has no influence on the distribution of the lines of force.

In the case of the non-linear response of the transducer output signal, a linearizer circuit is incorporated into the whole electronic system.

5.10. Seismic transducers

This class of transducer are also called no fixed reference transducers. In the previous section, transducers with a fixed reference were presented. By contrast,

these seismic transducers have neither fixed connection nor a common reference with the moving target. They are attached to a reference which is the sample itself.

The moving reference transducers presented here are essentially the accelerometers whose basic constitutive elements are a mass, spring and damper. The electrical principle of accelerometers is not confined only to piezoelectric transducers. Piezoresistive transducers, in which acceleration produces a variation in electric resistance, also exist, as do resistive transducers, which essentially relate to the cursor of a potentiometer.

However, piezoelectric transducers are the most commonly used for their versatility, light weight and reduced size, and because they can take high frequency responses into account.

5.10.1. Functioning principle

Figure 5.23 shows a schematic diagram of an accelerometer made up of a mass, spring and damper. Let y be the displacement of mass m , and x be the displacement motion of the box's transducer, known as the carter or case. The equation of motion is:

$$m \frac{d^2(x+y)}{dt^2} + c \frac{dy}{dt} + ky = 0 \quad [5.15]$$

which can be rewritten as follows, by dividing all the terms by m :

$$m \frac{d^2y}{dt^2} + \frac{c}{m} \frac{dy}{dt} + \frac{k}{m} y = -\frac{d^2x}{dt^2} \quad [5.16]$$

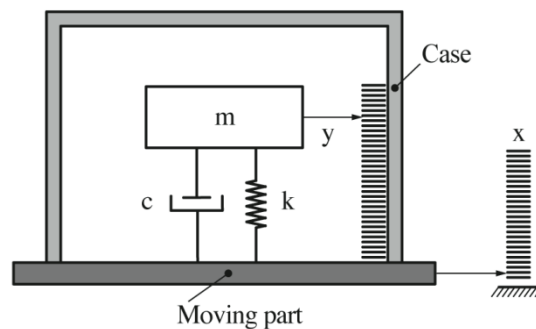


Figure 5.23. Mass, spring and dashpot constituting a seismic transducer

5.10.1.1. *Sinusoidal response*

If the case (Figure 5.23) has a motion of the form:

$$x = A \cos \omega t, \quad d^2x / dt^2 = -A \omega^2 \cos \omega t \quad [5.17]$$

Let the mass displacement y be:

$$y = B \cos (\omega t - \phi) \quad [5.18]$$

$$B = -\omega^2 A / [(\omega_0^2 - \omega^2)^2 + (c\omega / m)^2]^{0.5} \quad [5.19]$$

$$\text{with } \omega_0 = \sqrt{\frac{k}{m}} \quad \text{and } \zeta = (c/2) / \sqrt{(km)} \quad [5.20]$$

The damping factor ζ defined in [5.20] is called “the fraction of critical” damping and the critical damping coefficient c_c is defined as:

$$c_c = 2 (km)^{0.5} = 2m \omega_0$$

$$\text{so } \zeta = c / c_c \quad [5.21]$$

ω_0 is the (undamped) natural circular frequency:

$$\omega_0 = \sqrt{(k/m)} \text{ rad/s} \quad [5.22]$$

Let us recall that the damped natural circular frequency is:

$$\omega_d = \omega_0 (1 - \zeta^2)^{1/2} \text{ rad/s} \quad [5.23]$$

[5.19] is rewritten with the notation in [5.20]:

$$B/A\omega^2 = -(1 / \omega_0)^2 \{ [1 - (\omega/\omega_0)^2]^2 + (2\zeta \omega / \omega_0)^2 \}^{1/2} \quad [5.24]$$

with $d^2u / dt^2 = -\omega^2 u(t)$

[5.24] can be represented by the response of the mass m with acceleration, as an excitation. However, the right hand side of equation [5.22] shows that it is frequency dependent. The signal $y(t)$ can be considered as an acceptable transducer if the following approximation is adopted:

$$\omega / \omega_0 \ll 1 \quad [5.25]$$

5.10.1.2. Sensitivity and linearity

If the damping coefficient ζ is zero, if assertion [5.25] is valid, we have a response deduced from [5.24]

$$(B/ A\omega^2) \cong -1/ \omega_0^2 \tag{5.26}$$

which would be acceptable. However, the presence of the term $1/ \omega_0^2$ at the right hand side of [5.26] shows that we have a poor sensitivity. The product of the linear response and the undamped natural response is:

$$B [(d^2x/dt^2)]^{-1} \cdot \omega_0^2 \approx A \quad \text{if } (\omega/\omega_0) \ll 1 \tag{5.27}$$

The assumption of $\zeta = 0$ is unrealistic. Let us examine equation [5.24] and seek an optimal damping coefficient ζ . Figures 5.24 and 5.25a show the responses of a seismic system in displacement and in acceleration respectively.

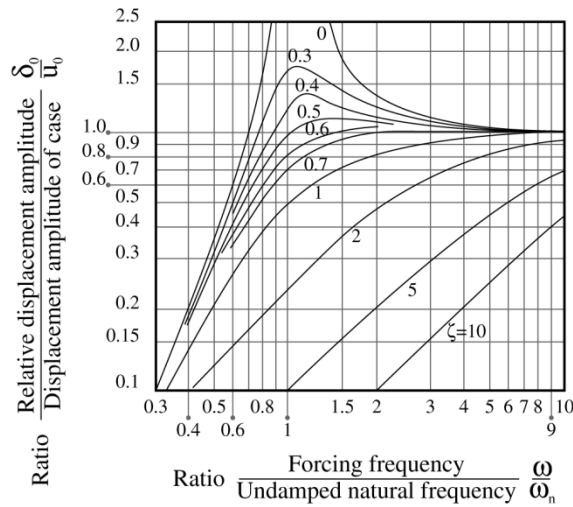


Figure 5.24. Displacement response of seismic system (mass, spring, damper) to sinusoidal displacement. Damping coefficient ζ is indicated on each curve. Note the optimal damping $\zeta = 0.7$ which gives the flattest response curve

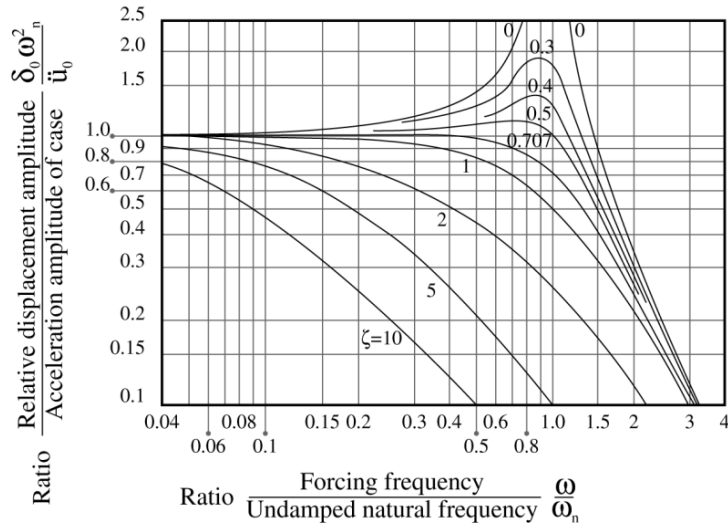


Figure 5.25a. Response of a seismic system with a sinusoidal acceleration. Damping coefficient ζ is indicated on each curve. Note the optimal damping coefficient $\zeta = 0.707$

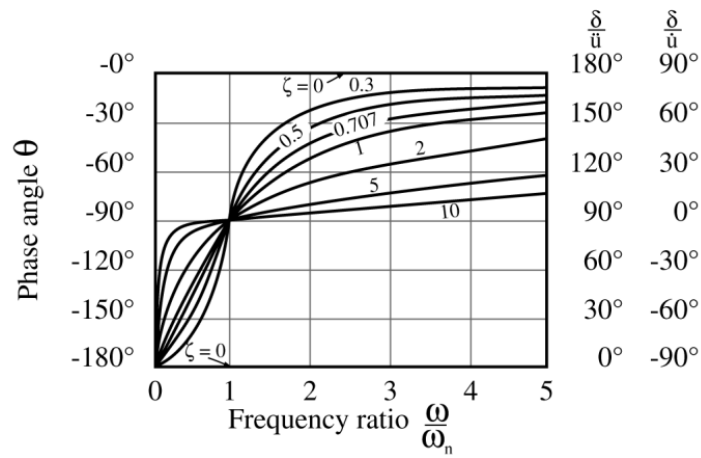


Figure 5.25b. Phase angle of seismic system. On the left, scale is related to displacement of the case. On the right, scales are related to the acceleration and velocity of the case, respectively

In these figures, low damping $\zeta < 0.6$ gives rise to high distortion of the response curve. For high damping $\zeta > 0.7$ the responses vary with frequency. The optimized damping coefficient is:

$$\zeta = \zeta_{\text{opt}} = 0.707$$

Figure 5.25a shows that this damping coefficient corresponds to the largest linear response and the proximity of natural resonance frequency²

5.10.1.3. *Practical considerations for the choice of an accelerometer*

These remarks constitute guidelines only:

- for dynamic material testing, if a large frequency range is being covered, choose an accelerometer with a low weight and high natural frequency ($f \cong 100,000$ Hz). Adopting these criteria, the experimenter will not achieve a high sensitivity for the accelerometer due to limitations with the technology;
- the accelerometer performance should always include the optimal damping coefficient ζ_{opt} which corresponds to the largest frequency range.

Other practical considerations are examined below.

5.10.2. *Technology of piezoelectric accelerometers*

Piezoelectric-type accelerometers are used widely. There are five types (see Figure 5.26) and each type differs from the other by the nature of the static preload.

Figures 5.26a-d concern compression preloads. The influence of housing distortion by mounting can influence the accelerometer performance. Figure 5.27e, a shear-type transducer, is the most rugged accelerometer and has low cross axis response. Figure 5.27 presents details of the mountings for two accelerometers of this type.

² This optimal damping coefficient is rarely exploited in electronic circuits for televisions.

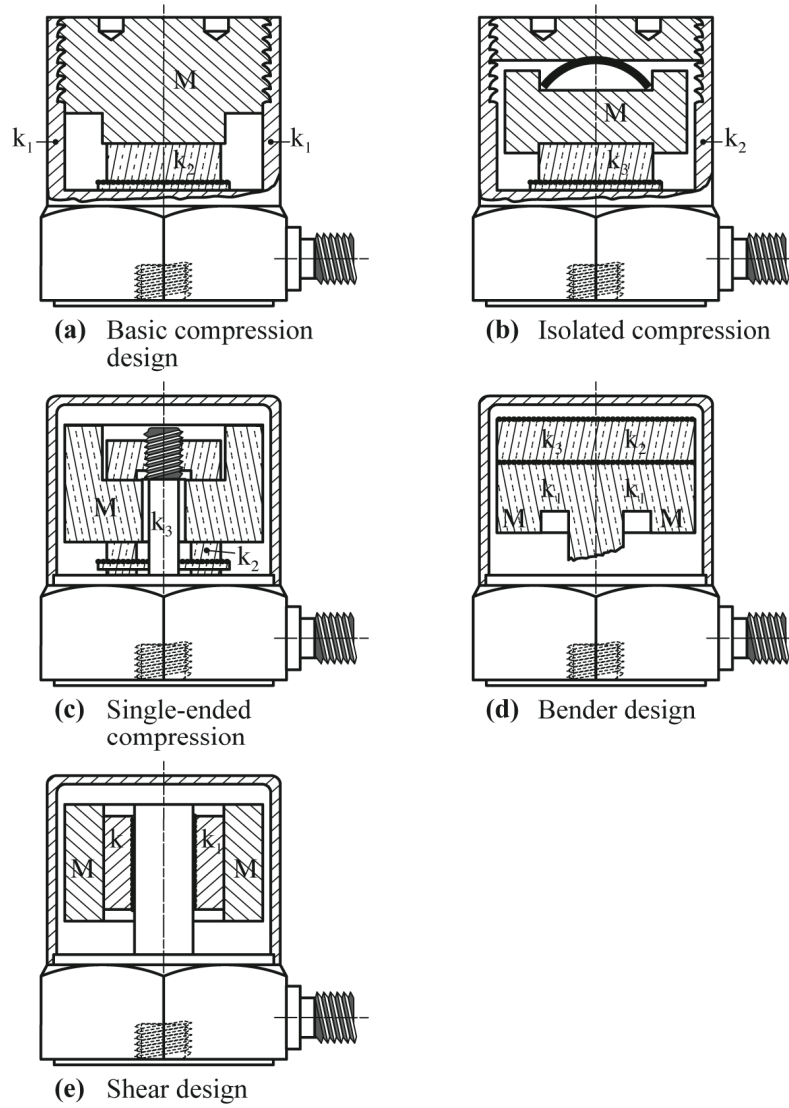


Figure 5.26. Piezoelectric accelerometers are submitted to preloads whose nature changes from one accelerometer to other (Endevco documents)

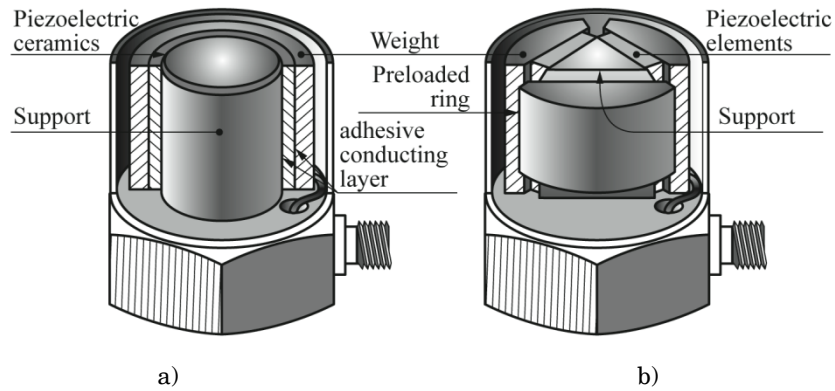


Figure 5.27. Two shear type accelerometers: (a) annular type; (b) delta type (Bruel & Kjaer document)

Characteristic	Comments
Size, dimensions, volume, surface, weight	Useful available surface for connecting depends on the active surface of the sample. Eventually transducer size also depends on the environment chamber size in which the sample is located.
Resonance frequency	The accelerometer frequency range is chosen in terms of the frequency range imposed by the sample vibrations. A high frequency range means low sensitivity of the accelerometer. Maximum working frequency of the accelerometer must include maximum frequency for the sample.
Sensitivity	Expressed in pC/g or mV/g (g = gravity acceleration, pC = picoCoulomb) High working frequency = low accelerometer sensitivity
Temperature	Normal range of temperature (-100°C to 150°C) There are accelerometers working in extreme temperature conditions (-200°C to 700°C). Accelerometer sensitivity varies with temperature.
Shock	When using an accelerometer under severe shock conditions, choose a transducer with low sensitivity. 10g is a medium shock 100g is a severe shock ($g = 9.81 \text{ m/s/s}$)
Damping	The majority of accelerometers are not adjusted with an optimal damping coefficient, $z = 0.707$
Calibration	A calibration curve with constant acceleration level and variable frequency must be consulted

Table 5.3. List of useful accelerometer characteristics

5.10.3. Acceleration performances

Table 5.3 contains the main characteristics which are necessary in the choice of this transducer for establishing an experimental set-up.

5.10.4. Transverse sensitivity of accelerometers

Transverse sensitivity is a quality factor to be taken into account when selecting an accelerometer. Figure 5.28 shows a transducer with axis oz, along which the accelerometer must be active. In a direction making an angle θ , a sensitivity:

$$e_{\theta} = e_{\max} \cos \theta \quad [5.28]$$

is obtained. Transverse sensitivity, corresponding to an excitation in the plane yz, is:

$$e_{\theta} = e_{\max} \cos (\theta - \theta_1) \quad [5.29]$$

There is a deviation θ_1 and, along the z axis, we obtain:

$$e_z = e_{\max} \cos \theta_1 \quad [5.30]$$

Along the y axis we have:

$$e_y = e_{\max} \sin \theta_1 \quad [5.31]$$

In the plane (xy) in Figure 5.28d, the sensitivity is:

$$e_{\phi} = e_{\max} \sin \theta_1 \cos \phi \quad [5.32]$$

For an ideal transducer $e_{\phi} = 0$. However, an accelerometer furnishes a response such that $e_{\phi} \neq 0$ and is independent of ϕ . So, in a general manner:

$$e_{\phi} = e_{\max} (A + B \cos \phi) \quad [5.33]$$

Sometimes A is larger than B. In that case, transverse sensitivity cannot be zero.

In practice, for $\Phi = 0^\circ$, transverse sensitivity is of the order of 0.5 to 15% of the axial sensitivity.

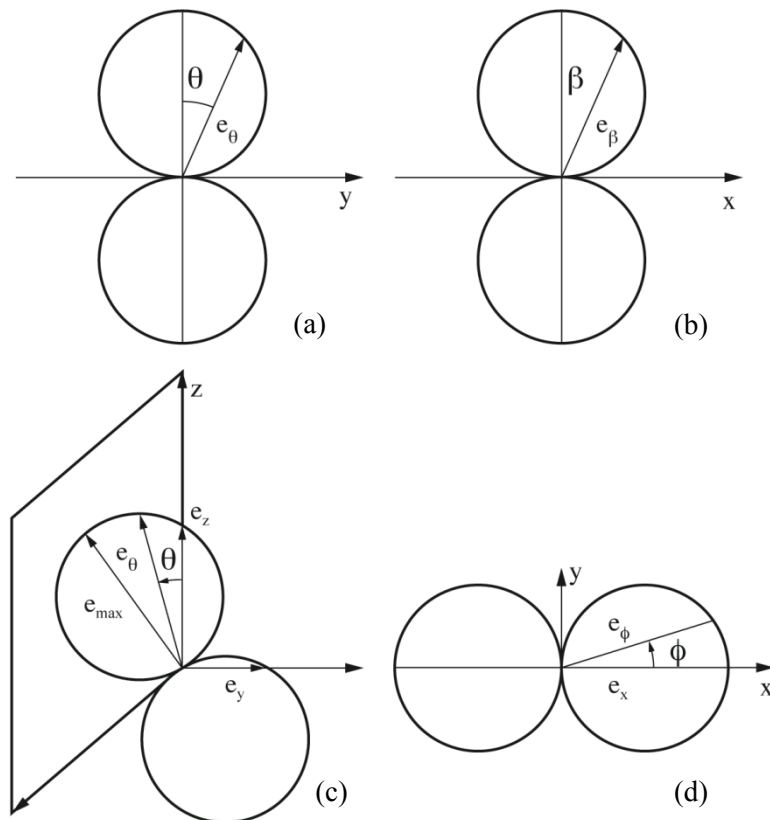


Figure 5.28. Directional sensitivity of an accelerometer

5.10.5. Accelerometer conditioners

There are two types of accelerometer conditioners: electrometer amplifiers and charge amplifiers.

5.10.5.1. Electrometer amplifier

Electrometer amplifiers have a high input impedance Z_i . An electric condenser is connected at the input (Figure 5.29). The capacity of the connecting cable C_c and the

capacity of the transducer itself must be taken into account. Q being the electric charge, the voltage between the two plots of the input Figure 5.29 is:

$$V_i = Q / (C_t + C_c + C_R) \quad [5.34]$$

The output voltage V_0 depends on the capacity of the connecting cable C_c .

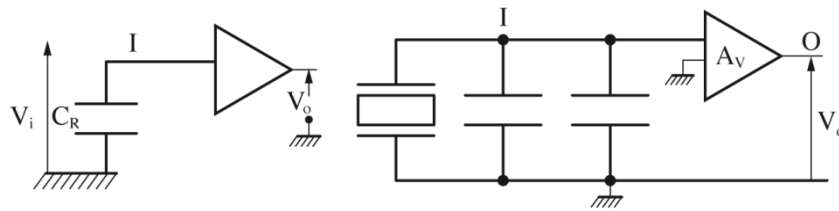


Figure 5.29. The amplifier transforms a charge variation q into voltage $V_i = q/C_R$

5.10.5.2. Improved version of an electrometer (piezotron)

Figure 5.30 shows an electrometer-type amplifier which is directly connected to a piezoelectric transducer.

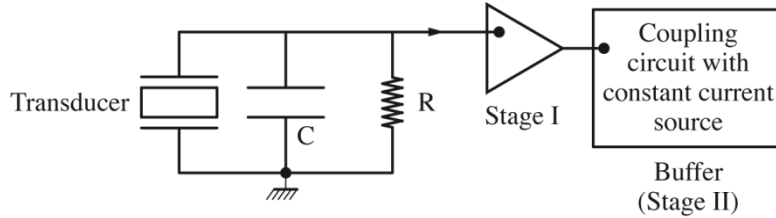


Figure 5.30. Piezotron amplifier (Kistler) with electrometer stage

A second stage is a coupler with a constant current source. The connection at the buffer output impedance being weak, the resistance R_t permits the dynamic functioning of the conditioner.

The first stage (stage I) is incorporated in the transducer case and the set is relatively immune to cable-induced noise and spurious response. Low-cost cable can be used and there is no influence on the functioning of the transducer associated with the preamplifier.

5.10.5.3. Charge amplifier

Figure 5.31 shows a special operational amplifier with two capacitive condensers, C_E at the input and C_F the feedback element. If an electric charge is applied to the input, the total capacity is the transducer connecting capacity:

$$C_E = C_T + C_R \quad [5.35]$$

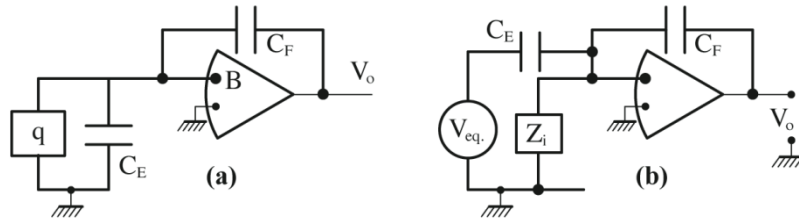


Figure 5.31. Charge amplifier using special operational amplifier with two condensers: C_E at the input and C_F which is the feedback element (a) circuit with transducer producing charge q ; (b) voltage equivalent circuit at the input

The equivalent voltage can be obtained from Figure 5.31a, by Thévenin's theorem which converts a charge source into voltage source (Figure 5.31b):

$$V_E = q / C_E \quad [5.36]$$

Applying Kirchhoff's law at the node B (amplifier input) and using the Laplace transform³

$$(v_{eqi} - v_B) C_{Ep} - (v_o - v_B) C_{Fp} - v_B / Z_i = 0 \quad [5.37]$$

Z_i is the input impedance of the transducer.

There is a second equation which binds v_B and v_o :

$$v_B = v_o / A_v \quad A_v \gg 1 \quad [5.38]$$

A_v being the voltage amplifier gain of the operational amplifier. Bringing [5.38] into [5.37]:

$$v_{eq} C_{Ep} - v_o (C_E + C_F) p / A_v - v_o / (A_v Z_i) = -v_o C_F p \quad [5.39]$$

³ p is the complex Laplace variable.

A_v being large ($A_v > 105$) in [5.37], the term v_0 / A_v can be neglected:

$$v_0 = -v_{\text{equi}} C_E / C_F \quad [5.40]$$

and from [5.36]:

$$v_0 = -q / C_F \quad [5.41]$$

[5.41] shows that the output voltage v_0 depends on the feedback condenser C_F and the charge q . Thus, the input capacity C_E does not intervene at all. Consequently this scheme permits the elimination of the influence of the capacity due to the connecting cable.

5.10.5.4. Improved version of the charge amplifier

In principle, the circuit presented in Figure 5.31a seems attractive. However, practically speaking, there is inevitably a possible long-term drift problem of the output voltage if there is no resistance in parallel with C_F . This drift tends to create a saturation. To avoid this drift, R_F is necessary. The time constant of the set (R_F, C_F) in parallel must satisfy the condition:

$$R_F C_F \omega \gg 1$$

ω being the lowest angular frequency in the working frequency range. To set v_0 to zero, an interrupter is used to eliminate the electrical charge in C_F (Figure 5.32).

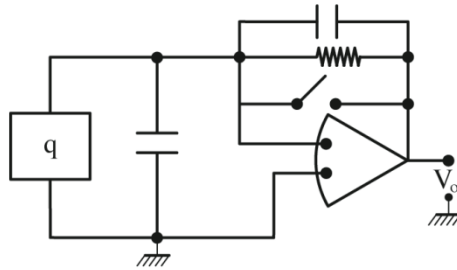


Figure 5.32. Improved charge amplifier to avoid drift of output voltage V_0

5.11. Piezoresistive accelerometer

Piezoelectric accelerometers were examined in section 5.10. Another class of accelerometers exists which are piezoresistive. That means that the transducer is not a self-generating system. The principle of piezoresistive accelerometers is that

acceleration produces a variation of resistance. This resistance can be a strain gauge resistance, or a semiconductor constituted by a single crystal of silicon. By micromachining, a seismic system with mass, stiffness and damper is obtained. The difference between these two acceleration types lies in their size. The second type can be as small as a miniature-type classical piezoelectric accelerometer. Piezoresistive transducers are interesting because:

- their lowest frequency can be as low as direct current (0 Hz) (in the measurement of a large damped material);
- a type of strain gauge resistance accelerometer can be interesting. A semiconductor resistance type accelerometer is used when maximum working frequency is chosen, with a good sensitivity;
- a wide range of acceleration from 10 G to 100 G⁴ can be obtained.

A piezoresistive transducer must be connected to a Wheatstone bridge. The carrier frequency of the bridge is chosen with caution so that the highest working frequency corresponds to the cut-off frequency of the low pass filter which inevitably follows the detector circuit at the output of the bridge⁵.

The mechanical resonance frequency of the transducer will be mentioned in the transducer manufacturer's performance list. Some useful references relating to the design of the experimental set up can be found in the bibliography at the end of this chapter.

5.12. Other transducers

Many interesting transducers are not presented in this chapter. These include:

- force transducers;
- mechanical impedance heads, incorporating force and displacement transducers. These can measure driving point impedance (force and displacement transducers at the same point);
- optical electronic transducer systems which include laser Doppler vibrometers, optical displacement systems and fiber optic reflective displacement sensors, as shown in Chu, Harris [HAR 96].

In this chapter, attention is focused on dynamic measurement of a sample using, in preference, transducers of the same type at the input and the output of the sample. We presented the reason for our choice at the beginning of the chapter.

⁴ G is the gravity acceleration.

⁵ In this respect, see section 5.9.4.

5.13. Force transducers

There are two classes of force transducers:

- piezoelectric (or ferroelectric) transducers;
- strain gauge-type transducers.

Piezoelectric-type transducers are the most commonly used in structural dynamics. Strain gauge force-type transducers are generally specially designed and manufactured by experimenters themselves, taking into account the particular experimentation conditions.

5.13.1. Piezoelectric-type transducers

A force transducer using a piezoelectric (or ferroelectric) effect must not be confused with a piezoelectric accelerometer.

5.13.1.1 Force transducer fixed to a sample where there is no seismic system

However, the sample must be taken into account and contribute to modifying the amplitude of the motion in the following equation:

$$a = a_0 [m_s / (m_s + m_t)] \quad [5.42]$$

where m_s is the structure mass of the structure at the point of attachment of the force transducer, m_t is the transducer mass in the direction of force application and a is the amplitude of force with the attached transducer.

The resonance frequency of the force transducer (for the attached transducer) is

$$F_{\text{resonance}} = (1/2\pi) \sqrt{[k / (m_s + m_t)]} \quad [5.43]$$

Consequently this frequency depends on the localized sample mass, m_s .

5.13.1.2. Transducer in contact with structure during impact

The usual convenient method of excitation is to include the force transducer in a hammer head. The contact is produced only during impact, the duration of which is of the order of several milliseconds⁶.

⁶ The contact duration depends essentially on the geometry of the hammer head and the nature of the metal constituting the head, and also the stiffness of the structure itself.

In this case, the frequency range depends on the narrowness of the impulse itself. A direct Fourier transform permits the force response to be obtained. Various shaped hammer heads are used to cover various frequency ranges:

- a large frequency range can use a small curvature of the hammer head, made with hard metal;
- a reduced frequency range to obtain higher energy concentrated on a limited frequency bandwidth.

5.13.1.3. Mechanical inertance of a structure

If both a force transducer and an accelerometer are placed in the hammer itself, the driving point inertance can be measured by:

$$(\text{Acceleration at point } i) / (\text{Force at point } i)$$

5.13.2. Special strain gauge-type force transducers

Strain gauges are extensively used to fabricate all types of transducers (displacement, velocity, acceleration, pressure, force). In this section, we restrict our remarks to force transducers. The reason for this restriction can be explained by our objective of evaluating the frequency response of the sample in the largest frequency range if possible. In this respect, the adopted geometry of this type of transducer gives rise to relatively low resonance frequency (with the exception of the cylinder shaped force transducer).

Design of such force transducers for large frequency ranges is incomplete if the corresponding resonance frequency is not evaluated. For a first evaluation of the resonance frequency, the Rayleigh or Rayleigh-Ritz methods are suggested [HAR 96]. Figure 5.33 presents a simple diagram of such a transducer, with a trapezoidal or triangular plate clamped to a fixed rigid column.

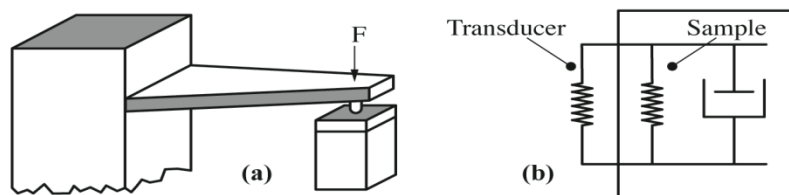


Figure 5.33. Cantilever-type force transducer.
 (a) The sample and triangular plate are rigidly mounted;
 (b) mechanical force transducer stiffness and complex sample stiffness in parallel

The plate, in the first approximation, is assimilated to a variable beam width. The force exciter, F , relates to the deformation of strain gauges. This force transducer is designed so that its stiffness k_t is not too high with respect to that of the sample k_s :

$$k_t \ll k_s \quad [5.44]$$

One of the disadvantages of this transducer is that it requires room for mounting. The second disadvantage is that its stiffness, being chosen following [5.44], combined with its own weight, gives rise to a low frequency resonance frequency (see Appendix 5B).

5.14. Bibliography

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5.15. Appendix 5A. Condenser with polarization

The equation of the elastic circuit presented in Figure 5.5 is:

$$R \cdot i(t) + = V \quad [5.A.1]$$

or:

$$RC \cdot i(t) + V(t) = V(t) \quad [5.A.2]$$

In the case of a harmonic variation of the condenser capacity:

$$C(t) = C_0 + C_1 \sin \omega t \quad [5.A.3]$$

Bringing [5.A.3] into [5.A.2]

$$V(t) = R(C_0 + C_1 \sin \omega t) i(t) + \int_0^t i(\tau) d\tau \quad [5.A.4]$$

Deriving [5.A.4.] with respect to time:

$$\dot{V}(t) = \frac{dV}{dt} = R(\omega C_1 \cos \omega t) i(t) + R(C_0 + C_1 \sin \omega t) \frac{di}{dt} + i(t) \quad [5.A.5]$$

Let $i(t)$ be

$$i(t) = I \sin(\omega t + \phi_1) \quad [5.A.6]$$

Bringing [5.A.6] into [5.A.5]:

$$\begin{aligned} \dot{V}(t) = & R\omega C_1 \cos \omega t I \sin(\omega t + \phi_1) + \\ & R(C_0 + C_1 \sin \omega t) I \omega \cos(\omega t + \phi_1) + I \sin(\omega t + \phi_1) \end{aligned} \quad [5.A.7]$$

If C_1 is such that:

$$C_1 \ll C_0 \quad [5.A.8]$$

and if $\omega RC_0 \gg 1$. [5.A.9]

[5.A.7] can then be rewritten as:

$$\begin{aligned} \dot{V}(t) = & RC_0 I \omega \cos(\omega t + \phi_1) + I \sin(\omega t + \phi_1) \\ & + R\omega C_1 \cos \omega t I \sin(\omega t + \phi_1) \end{aligned} \quad [5.A.10]$$

From [5.A.10] the current's amplitude can be deduced:

$$I = \dot{V} \left[(1 + R\omega C_1 \cos \omega t)^2 + (RC_0\omega)^2 \right]^{-1/2}$$

With the inequality [5.A.8] taken into account:

$$I = \frac{\dot{V}}{RC_0\omega} \left[1 + \frac{1}{(RC_0\omega)^2} \right]^{-1/2} \quad [5.A.11]$$

$V_R(t)$, variable voltage, is proportional to the variable part of C:

$$V_R(t) = V \frac{C_1}{C_0 \sqrt{1 + \frac{1}{(RC_0\omega)^2}}} \sin(\omega t + \phi) \quad [5.A.12]$$

$$V_R(t) \cong \frac{1}{e(t)} \cong \frac{1}{x(t)}$$

where $e(t)$ is the gap between condenser electrodes.

5.16. Appendix 5B. Eigenfrequencies of some force transducers: Rayleigh and Rayleigh-Ritz upper bound methods

In the design of transducers, equations giving strain and stress components versus applied force are given. For dynamic tests, the resonance frequency must be evaluated so as to check whether it is beyond the working maximum frequency.

Various methods of calculation are available, including:

- Rayleigh's method;
- Rayleigh-Ritz's method;
- eigenfrequency deduced from equation of motion;
- finite element method.

Attention here is focused on the first two methods used for the first evaluation of the lowest eigenfrequency.

5B.1. Rayleigh's method

For an essentially conservative elastic system, the maximum kinetic energy T is equal to the maximum of the strain energy V :

$$\text{Max}(T) = \text{max}(V) \quad [5.B.1]$$

In the first member of [17.B.1] the circular eigenfrequency is factorized and written as:

$$\omega^2 = \frac{\text{max}(T)}{\text{max}\left(\frac{V}{\omega^2}\right)} \quad [5.B.2]$$

Rayleigh's method is tractable only if an expression of displacement u versus abscissa x is adopted for a rod type: $u(x)$.

For a plate-type system, two coordinates are adopted: $u(x, y)$.

If the test function $u(x)$ or $u(x, y)$ satisfies all the boundary conditions, [5.B.2] gives rise to the exact first eigenvalue. In practice, the test function is an approximation and often the boundary conditions are partly satisfied.

It can be shown [DYM 73] in this case, that [5.B.2] constitutes an upper bound. Confidence of such a calculation is optimistic and can give rise to a false eigenvalue which is larger than the exact one. One has to adopt a more refined approximation of the test function $u(x)$ or $u(x, y)$.

The objective is to obtain an approximate eigenvalue which is as close as possible to the exact one if we arrange that the test function satisfies the maximum possible of the boundary conditions.

A second reason for interest in Rayleigh's method is that, if there are localized elastic concentrated elements and masses, these elements can be introduced in [5B.2] and Rayleigh's equation becomes:

$$\omega^2 = \frac{N}{D} \quad [5.B.3]$$

$$N = \text{max}(T) + \sum_{i=1}^n \left(\frac{1}{2}\right) k_i^2 u^2(x_i)$$

$$D = \text{max}\left(\frac{V}{\omega^2}\right) + \sum_{p=1}^m \frac{1}{2} [m_p^2 u^2(x_p)] / \omega^2$$

x_i , x_p being abscissas of the points where springs k_i and masses m_p are attached, respectively. In equation [5.B.3] the displacement $u(x)$ (or $u(x,y)$) is supposed to be the same without additional weights and stiffnesses. For details of applications, see [DYM 73], [MEI 67], [HAR 96].

5B.2. Rayleigh-Ritz's method

This method constitutes an extrapolation of Rayleigh's method. It uses a collection of test functions gathered together in a unique test function, including a set of coefficients relating the test functions.

These coefficients are considered as unknown parameters to be optimized.

Finally, the problem is transformed into a matricial eigenvalue problem. For the first eigenvalue of a force transducer, the Rayleigh-Ritz's method might give the lowest approximate eigenvalue but the calculation is lengthy.

5B.3. Preliminary experimental test on the force transducer

Before carrying dynamic tests on a sample it is advised that the force transducer be tested by an impact test. The first resonance frequency can be obtained by a Fourier transform of the time response of the strain gauge type transducer.

The experimenter has the possibility of checking the resonance frequency which must be higher than the upper bound of the frequency range.