Chapter 14

Ultrasonic Benches: Characterization of Materials by Wave Propagation Techniques

14.1. Introduction

The theory of progressive wave propagation in elastic and/or viscoelastic materials has been presented elsewhere [CHE 10]. The interest of this technique, from a material characterization perspective, is discussed in this chapter. We focus attention on the easy fabrication of ultrasonic benches which use either direct contact benches or a water immersion bench. These apparatus are complementary.

14.2. Ultrasonic transducers

Technical and scientific information were gathered in Chapter 4. Remember that, when choosing an ultrasonic transducer, the main characteristics that experimenters should consider are:

– the nature of the transducer; there are two kinds of materials used in their fabrication: (a) piezoelectric disks; and (b) piezoresistive disks. The first are made from natural materials whilst the second are artificial;

– the electric voltage applied to the emitter, which depends on the nature of the material;

– the electric signal; progressive waves are used in ultrasonic benches. Consequently, the electric signals applied to the emitter are isolated impulses whose

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Mechanical Characterization of Materials and Wave Dispersion: Instrumentation and Experiment Interpretation
Edited by Yvon Chevalier and Jean Tuong Vinh

time duration must be short so as not to interfere with the received signal and to avoid wave reflections on the transducer receiver;

– the width of the pulse emitted must not be too wide in time to obtain received signals whose Fourier transforms have a restricted frequency range. This is the condition for using progressive waves;

– mechanical specifications; the commercially available transducers proposed for non destructive control are the same as for material characterization. The proposed transducers are a short cylindrical shape with a metallic housing on the lateral surface;

– frequency specification; the usual frequency range is 500 KHz to 20 MHz. The usual transducer frequencies are: 0.5, 1, 2, 3, 4, 6, 8 and 10 MHz. These frequencies are the resonance frequencies of the transducers themselves;

– the nature of the ultrasonic wave; there are two kinds of transducers: (a) extensional (or longitudinal) waves with a wave propagation direction which is collinear with the polarization direction of the wave; (b) a shear (or equivolumic) wave with wave propagation direction which is orthogonal to the polarization direction of the wave;

– whether to select a direct contact or water immersion bench; the difference lies in whether the benches are waterproof or not. Since only longitudinal waves can propagate in water, only transducers emitting or receiving these waves are used for a water immersion bench.

14.2.1. *Details of transducer technologies*

Figure 14.1 presents a transverse section of a transducer.

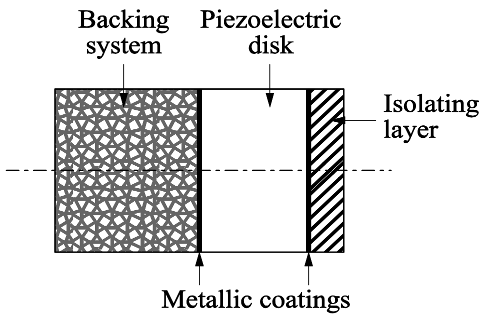

Figure 14.1. *Transverse section of an ultrasonic transducer*

Piezoelectric (or ferroelectric) disks have electrically conductive coatings on both faces. To facilitate wave propagation in one direction only (in Figure 14.1 the free face is the right-hand one), the left face is not accessible. It is glued against a special cylinder which is made of a mixture of resin and lead balls of a specified diameter. This cylinder is part of a transducer which reduces the energy of the wave propagation in the opposite direction, with respect to the right direction towards the right side in Figure 14.1. Usually, the left coating face is connected to the earth.

Between the metallic free face coating and the left face integer with the backing system a voltage is connected furnished by an impulse electric generator; see Figure 14.2.

A second coating and electrically insulated layer is painted on the metallic layer indicated above. Its role is to insulate the transducer with respect to water if a water immersion bench is adopted. By prolonged immersion in water, this coating layer might lose its insulating capacity and consequently the efficiency of the immersed transducer decreases. For direct contact transducers, the coating layer undergoes severe friction conditions in spite of special viscous coupling fluid used for the coupling between transducer and sample.

Generally, the ferroelectric transducer which is used as the emitter requires a voltage of less than that used for a quartz transducer.

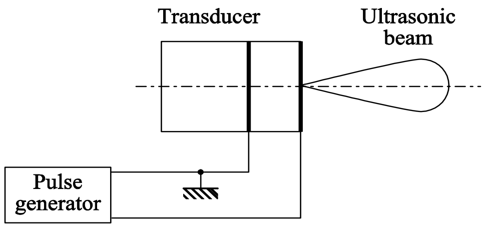

Figure 14.2. *A pulse electronic generator furnishes voltage to the transducer used as the emitter. The ultrasonic beam must not be too wide*

14.3. Pulse generator

A pulse electric generator with low output impedance ($Z_0 \cong 100\ \Omega$) and with a power of the order of 100 watts (and capable of delivering more than 100 Volts) is the best choice. This impulse generator must be capable of delivering (Figure 14.3) an adjustable impulse of time width T (several milliseconds). This impulse is repeated in time $T_S >> T$, where T_S is the sweeping period for the oscilloscope.

The width of each impulse can be adjusted so that $T \cong \frac{3}{f_r}$, where f_r is the transducer resonance frequency.

Figure 14.3(b) shows an emitted ultrasonic wave packet with decreasing oscillations. Three sinusoids are retained.[1]

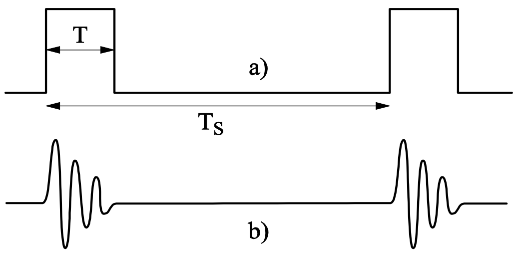

Figure 14.3. *(a) Rectangular electric pulses applied by the emitter with duration T, repeated with a period T_s are delivered by a pulse generator; (b) in contact with an emitter transducer, rectangular pulses are transformed into wave packets with the same repetition period T_S*

14.3.1. *Interest in progressive waves*

For ultrasonic waves, progressive waves are used instead of stationary waves. The main interest resides in the evaluation of the time propagation of a wave packet which is the simplest and quickest measurement compared to the measurement of a sample submitted to stationary waves. The reflected waves, after multiple reflections in the sample, exist but the adjustment of the sweeping time on the oscilloscope allows only the first transmitted wave packet to be visualized (Figure 14.4).

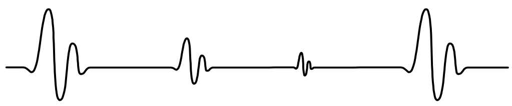

Figure 14.4. *Wave packets are used as progressive waves on the condition that the eventual multi-reflected waves decay rapidly in amplitude and do not interfere with the transmitted wave*

A second method of shaping the ultrasonic progressive wave is presented in Figure 14.5.

1. Two impulses from a tone burst generator could adjust the wave shape to obtain regularly distributed sinusoid amplitudes, and thus allow this operation.

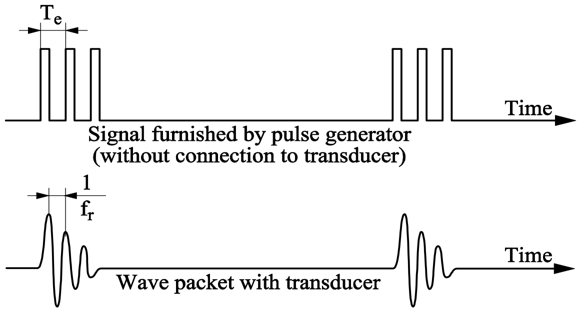

Figure 14.5. *A packet of three rectangular impulses or three sinusoids is used to strengthen the incident ultrasonic wave*

The time interval between impulses must be adjusted to the inverse of the transducer resonance frequency: $T_e = T_r = 1/f_r$.

The repetition period T_s corresponds to the sweeping period on the oscilloscope. It must not be too short for the reason that the wave packet must not be influenced by the possible multi-reflected waves (Figure 14.4).

14.3.2. *Electronic set-up*

The electronic set-up (Figure 14.6) is reduced to:

– a pulse generator with a power amplifier which delivers an electric power of 100 Watts;

– an oscilloscope with two inputs so as to observe emitted and received signals simultaneously, and which must possess systems for accurate time interval measurements;

– additional signal processing, to obtain the correlation function.

14.4. Mechanical realization of ultrasonic benches

There are two different benches to be built. The two are complementary and are both necessary for the characterization of stiffness matrix coefficients. In many circumstances it is not possible to use only one of them.

The size and the shape of samples constitute the first parameters to be taken into account for the choice of the ultrasonic bench to evaluate the set of stiffness coefficients, before any experimental campaign.

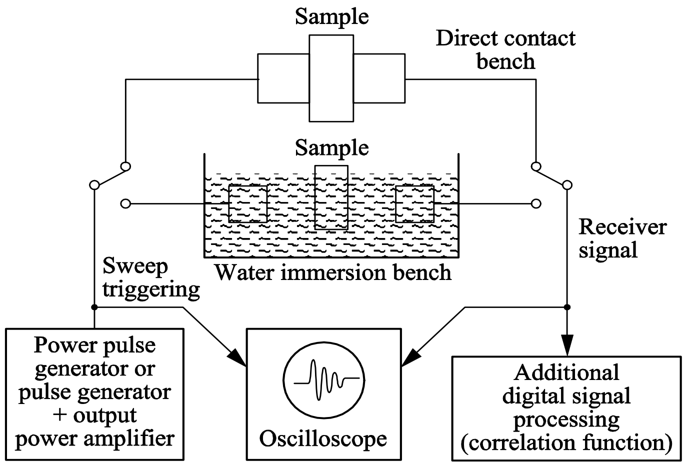

Figure 14.6. *Schematic representation of electronic set-up*

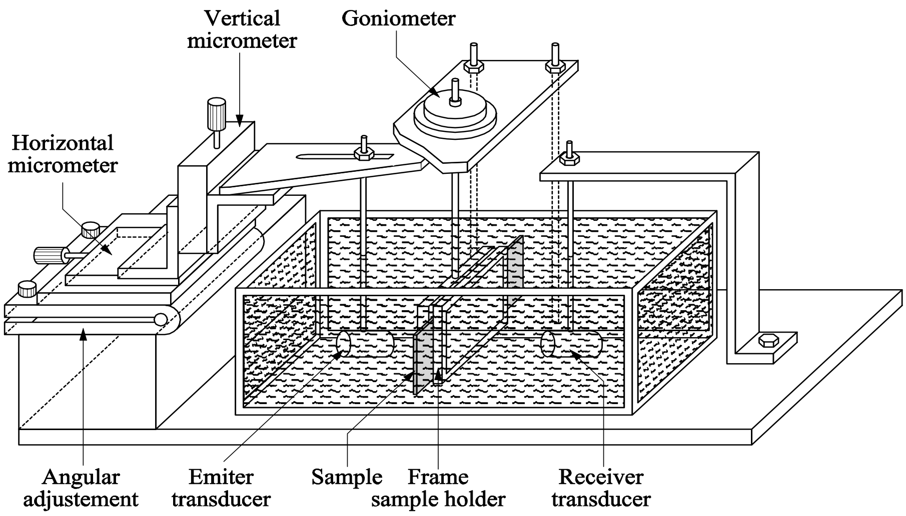

Figure 14.7. *Side view of a transmission water immersion bench. The sample can be rotated around its vertical axis by means of a goniometer which is fixed to the horizontal transverse plate. The transducer on the left moves along two axes: vertical and horizontal. The horizontal axis is collinear with the common axis of both transducers. Plates joined by a hinge pin are used for horizontal adjustment of the triangular beam supporting the left transducer*

14.4.1. *Water immersion bench*[2]

Figure 14.7 shows a water immersion test bench which is different to Markham's test bench [MAR 70].[3]

In a water tank, two identical transducers are used. Their common axis is horizontal and it is adjusted by means of two thick square plates and two translational micrometers which permit the horizontal position of the emitter to be controlled along the common axis.

To complete the adjustment of the emitted longitudinal ultrasonic beam in water[4], a system with two plates joined together by a hinge pin allows an adjustment of the angle of the emitter transducer in the range of 0 to 20°.

The sample plate is pressed between two rectangular metallic frames which are fixed along a tube and is integral with a goniometer fixed on a thick horizontal rectangular plate. The emitter transducer on the left is presented in a metallic hollow cylinder, serving as a housing, along a vertical tube. This set (emitter, housing, tube) can be adjusted in position by nuts and bolts along a rectangular slit which permits rough adjustment of the emitter position by height, with respect to the receiver on the right.

The important part of an immersion bench is the goniometer. During measurements, the operator adjusts the angle of the ultrasonic beam axis with respect to the normal direction of the plate itself. The angle must be accurate to a minute. Two micrometers (one horizontal, the other vertical) on the left serve to accurately adjust the horizontal (x, y) position of the emitter. Finally, an angular adjustment system on the left allows the horizontal position of the emitter axis to be modified.

14.4.2. *Direct contact bench*

Figure 14.8 shows two test benches: one for longitudinal wave and the other for transverse (shear) waves. As the transducers' dimensions are not the same, utilization of these two different benches avoids troublesome changes of transducer holders. Details about mechanical fabrication are indicated in Figure 14.8. For a transmitted shear wave, it is necessary to determine the polarization direction of the shear wave before measurements are taken (Figure 14.9). Remember that this direction is orthogonal to the propagation direction of the wave.

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2 For theoretical considerations on ultrasonic wave propagation see [CHE 10], Chapter 10.

3 See references given at the end of this chapter for other possible benches.

4 Only longitudinal waves can travel in water.

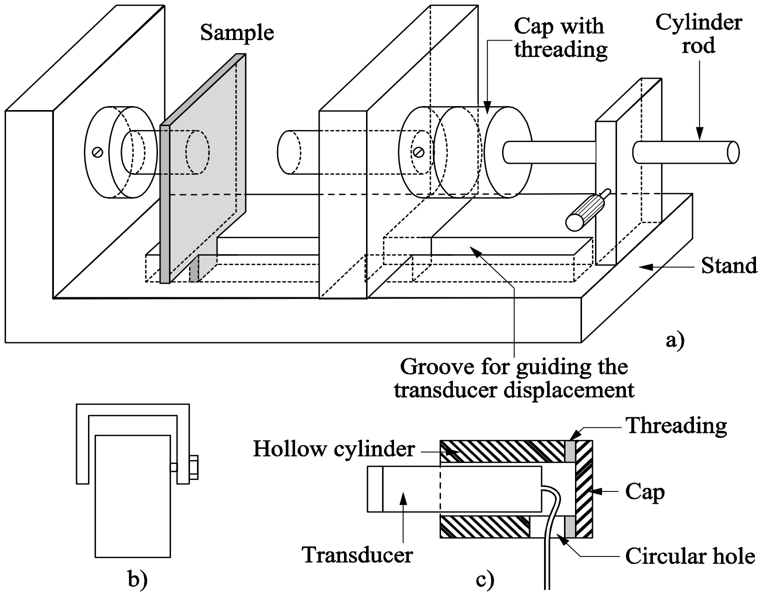

Figure 14.8. *Direct contact bench by transmission for a longitudinal wave through the thickness of the plate*

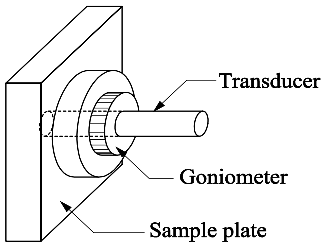

Figure 14.9. *The fabrication of a direct contact bench for a shear wave is similar to a bench for longitudinal waves presented in Figure 14.8 with the addition of a goniometer for one of the transducers. The polarization direction of the shear wave must be adjusted with accuracy on the surface of the sample plate*

An adjustment of the wave polarization direction is the first operation required with both transducers joined together. The polarization direction of both transducers must be coincident. During this operation, the angle rotation of one of the transducers is observed on the oscilloscope screen to obtain the maximum signal amplitude by means of the goniometer rotation. This direction must be well situated in the plane that is orthogonal to the transducer axis. This permits the symmetry axis of the material sample to be situated in space with respect to the polarization direction of the transducers.

The sample is then inserted between the transducers. Two thin layers of viscous liquid are laid by strokes of a soft brush onto the two opposite faces of the plate sample.

Measurements with a direct contact bench are the simplest to perform, compared to the vibration measurements using stationary waves on sample rods presented in previous chapters.

14.5. Experimental interpretation of phase velocity and group velocity

Details concerning phase velocity measurements are presented elsewhere [CHE 10]. This section provides some complementary information which might be useful for experimenters. Both phase velocity and group velocity are discussed. Stiffness matrix coefficients are examined from the point of view of experimental accuracy. We will show that non-diagonal coefficient calculations often lack accuracy. Optimization techniques are then referred to.

14.5.1. *Influence of viscoelastic dispersion*

There are two kinds of dispersions: geometrical dispersion, which is due to the boundary conditions imposed on the bound media (rod or plate), and viscoelastic dispersion, which is due to the material itself. Viscoelastic coefficients of stiffness matrixes (or compliance matrixes) depend on the nature of the material and also on its degree of symmetry. Viscoelastic dispersion, even in the ultrasonic frequency range, has a more or less pronounced influence depending on the damping properties of the material. On practical grounds, these complex stiffness matrix coefficients can be considered as asymptotic values in the so-called glassy zone in viscoelasticity.

In this section, we examine the influence of viscoelastic dispersion on velocity measurements.

14.5.1.1. *Wave packet envelope and group velocity*

As a progressive wave is favored in the ultrasonic domain, a wave packet is extensively adopted. If there is a possibility of accurately obtaining the wave envelopes of emitted and received waves, it is then possible to evaluate the group velocity.

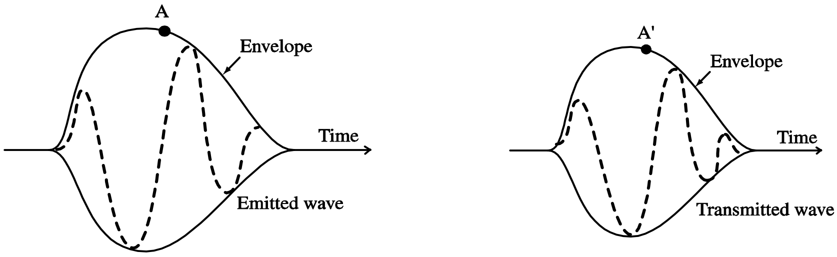

Figure 14.10. *Propagation of wave envelope; group velocity measurement*

In Figure 14.10 two such waves are represented. If the wave shape remains the same after propagation in the sample the wave shape is not modified except in its attenuation. Then two similar points on the envelopes (Figure 9.10) permit the propagation time to be estimated:

$$\Delta t_A = t_{A'} - t_A \qquad [14.1]$$

If the distance separating two points A and A' in Figure 14.10 is ΔL, then the group velocity is:

$$v_g = \frac{\Delta L}{\Delta t_A} \qquad [14.2]$$

14.5.1.2. *Phase velocity and wave inside the packet*

If the wave dispersion can be neglected, the shape of the transmitted wave packet is that of the incident wave packet but on a reduced scale due to the viscoelastic behavior of the material.

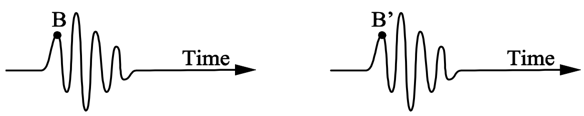

Figure 14.11. *Case of transmitted wave packet which has a similar wave shape with a reduced scale*

In Figure 14.11, if two homologous points B (B corresponding to normal incidence and concerns phase velocity) and B' (corresponding to the trajectory of the phase velocity v_p) on the two wave packets are examined and their positions measured, then the phase velocity:

$$v_p = \frac{\Delta L}{t_{B'} - t_B} = \frac{\Delta L}{\Delta t_B} \quad [14.3]$$

If the two velocities are identical:

$$v_g = v_p$$

There is no wave geometric dispersion in this case.

14.5.1.3. *Phase velocity in the case of strong viscoelastic dispersion*

In many cases, the wave is completely distorted after propagation. In Figure 14.12 the transmitted wave packet is distorted with respect to the emitted wave packet. It is then difficult to find homologous points in the two packets.

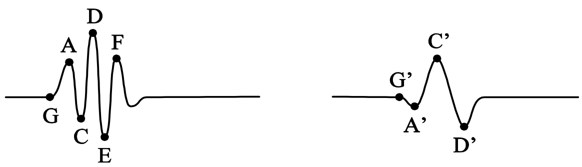

Figure 14.12. *Distorted wave packet after transmission through a sample and phase change*

In some cases there is phase inversion, illustrated by the two points C and C' homologous with a phase inversion. This difficulty happens when we are dealing with highly damped material. What we can do is to focus our attention on the beginning of the wave packet (points G and G').

However, this raises the problem of noise level. The group velocity is then evaluated instead of phase velocity. The inter-correlation function in signal processing is then referred to, permitting the group velocity to be evaluated with more accuracy.

14.5.2. *Influence of geometric dispersion*

Plates are frequently used for material characterization by ultrasonic waves. The geometric dispersion of infinite plates has been extensively examined, such bounded mediums being considered as wave guides, by Musgrave [MUS 70], Ekstein [EKS 45] and Thurston [THU 74].

With the massive introduction, from around 1970, of composite materials in industry, the extension of geometric dispersion to anisotropic materials takes into account their microstructures, their symmetry direction and their preferred fiber direction. Transverse isotropic materials have given rise to abundant scientific and technical literature, e.g. Kline [KLI 92].

In this section attention is focused on the influence of geometric dispersion on phase and group velocities.

The question is, when using ultrasonic benches by direct contact and by water immersion, can we reach a phase velocity from which stiffness coefficients can be deduced?

14.5.2.1. *Direct contact method: normal incidence and energy propagation direction*

The relationship between group velocity and phase velocity is briefly presented here. The important point to remember is that the propagation direction of energy velocity is not collinear with the propagation direction of phase velocity. This relationship is:

$$p_{\alpha} . v_e = v_p \qquad [14.3a]$$

where v_e is the energy velocity, v_p the phase velocity, and p_α the direction cosine.

Given that $v_{group} = v_e$ (the energy velocity) we can then write:

$$v_{group} . \cos\psi = v_{phase} \qquad [14.3b]$$

Figure 14.13 shows the two directions of propagation. The oblique distance is that of energy propagation, $d/\cos\psi$, where d is the distance between transducers.

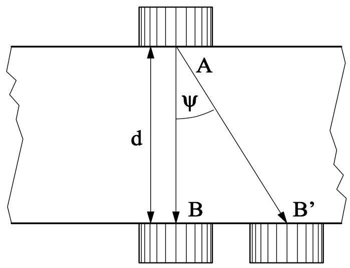

Figure 14.13. *Direct contact bench with two transducers, where d is the thickness of the sample, and ψ the angle between normal incidence and energy velocity direction*

The time delay is, using [14.3]:

$$\Delta t = (d/\cos\psi)/(v_{phase}/\cos\psi) = d/v_{phase} \qquad [14.4]$$

This is the relation adopted to evaluate v_{phase} when energy flux deviation is ignored.

It can be shown, if a pulse echo is adopted [KLI 92], that:

$$\Delta t = 2d/v_{phase} \quad [14.5]$$

14.5.2.2. *Oblique incidence and energy propagation direction*

Appendix 14A presents details of Kline's calculations for a water immersion bench. The equation obtained for the calculation of phase velocity is exactly the same as the one obtained above. From time delay measurements, the result is valid for an isotropic medium, independent of material symmetry.

14.5.3. *Signal processing*

14.5.3.1. *Correlation function*

Before the introduction of digital signal processing (1980) to improve the accuracy of time measurement, the literature concerning various electronic set-ups was abundant; Truel, Elbaum and Chick [TRU 69], however, used only analog measurement methods. Things have drastically changed since then. The advantages of digital based signal processing techniques have meant that they have replaced analog ones. The digitizer can reach a working frequency of 100 MHz to improve time resolution, and special programs are used, such as autocorrelation and cross-correlation.

However, the reader's attention should be focused on the fact that the aforementioned programs evaluate the time delay corresponding to the envelope of the wave packet. This means that this is suitable for evaluating a time delay related to group velocity. In the elastic and/or viscoelastic characterization of a material, phase velocity is needed.

The correlation functions are defined as:

$$C_{yx}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt \text{ (intercorrelation)} \quad [14.6]$$

$$\text{And } C_{yy}(\tau) = \int_{-\infty}^{+\infty} y(t) y(t+\tau)\, dt \text{ (autocorrelation)} \quad [14.7]$$

where x(t) and y(t) are input and output signals, respectively.

The main interest in these correlation functions resides in the fact that they enable the separation of a quasi-longitudinal wave from a transverse wave, when the oblique incidence of an ultrasonic wave is used. For composite materials, see Castagnede [CAS 89].

14.5.3.2. *Time Hilbert transform and analytic signal*

This transform is useful to examine the envelope as a signal. The time signal furnished by the receiver transducer is real:

$$y_r(t) = \text{Real function of time} \quad [14.8]$$

The Hilbert transform for time of y(t) is defined as:

$$y_i(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} y_r(t).\frac{1}{(t-\tau)}\,d\tau \quad [14.9]$$

which is a convolution of the signal, with the kernel of convolution product $1/\pi t$.

We then obtain a complex function, as follows:

$$y_r(t) + j\,y_i(t) = y_{complex}(t) \quad [14.10]$$

This method is used extensively when the envelope of a signal is desired [GAM 81], the envelope being related to group velocity.

14.6. Some experimental results on composite materials

The examples presented below serve to show various calculation steps.

14.6.1. *Unidirectional glass fiber-epoxy resin composite material*

Fiber volume fraction: 60%

Sample thickness: $e = 4 \times 10^{-3}$ m

Density: $\rho = 1.866 \times 10^3$ kg/m^3

Ultrasonic velocity in water: $v_0 = 1{,}500$ m/s[5]

5 Measurement of v_0 is obtained by variation of the distance separating the two transducers. The temperature of the water in the immersion bench must be accurately measured. v_o is strongly dependent on temperature.

14.6.1.1. *Longitudinal wave*

The longitudinal wave is obtained by adjusting the incidence angle i =0, and this wave is examined. The distance l_L, is taken from equations [14.B.4], [14.B.6], [14.B.7] and [14.B.8] in Appendix 14B.

ℓ = -52.885 +55.200 = 2.315 = 2.32 mm.

ℓ /e = 0.58 $v_L = v_{1/1}$([6]) = v_0 / (1- ℓ /e) = 3560 m/s.

This corresponds to the longitudinal wave with:

$\vec{p}$ = (1,0,0), $\vec{d}$ = (1,0,0) , 3 is the direction of the fiber axis.

14.6.1.2. *Critical and overcritical incidence angle: $i \geq i_C$*

The measurements are effected with an incidence angle i which is beyond the critical angle and corresponds to the only wave traveling through the sample. This is a transverse wave with polarization in the incidence plane. There is consequently no longitudinal wave traveling through the sample. Choosing this incidence angle, a pure transverse wave allows the shear coefficients of stiffness matrix to be obtained via its velocity.[7]

Remember that the principle of measurement using a water immersion bench is that the sample is placed between two transducers and plunged in water. The shear wave velocity is given by equation [14.B.4] in Appendix 14B.

$$v_T = \frac{v_0}{\left[1+\frac{v_0 \delta t}{e}\left(\frac{v_0 \delta t}{e}-2\cos i\right)\right]^{\frac{1}{2}}} \qquad [14.11]$$

in which the time increment δt, corresponding to the insertion of the sample, is replaced by $v_0\delta t$, at a distance in water ℓ.

$\ell = v_0\,\delta t$, the distance increment, is adjusted by a micrometer and visualized on

ℓ= -530,86 + 55.20 = 1.34 mm

ℓ / e = 1.34 / 4 = 0.335 , e being the sample thickness.

6 1/1 in the subscript designates the direction of propagation 1 and polarization direction 1 of the wave.

7 Details of calculations can be found in [CHE 10], Chapter 10.

i = 44° 46’ (the incident angle), 2cos i = 1.419

$$v_T = v_{1/2} = \frac{1500}{[\,1+0.335(0.335-1.419)]^{1/2}} = 1{,}880 \text{ m/s.}$$

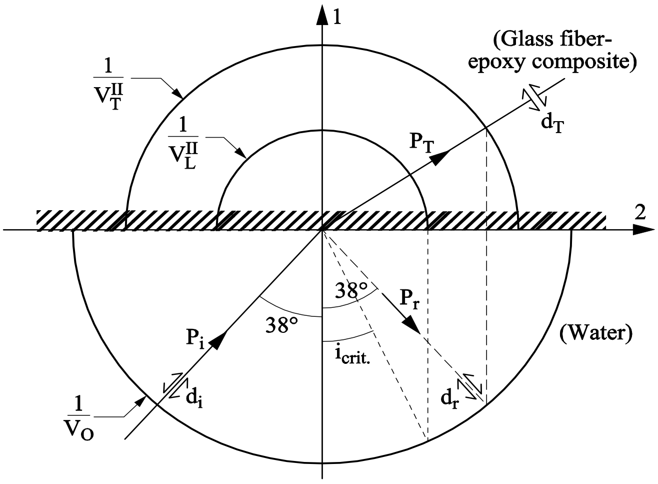

Figure 14.14. *Propagation of waves in composite material. Polarization direction $\vec{d}$ and propagation direction is in the plane (1,2) which is isotropic. Propagation of shear wave beyond critical incidence angle 38° > $i_{critical}$*

14.6.1.3. *The two stiffness coefficients of a composite material, glass fiber–epoxy resin (3 being the fiber direction of transverse isotropic material)*

$$C_{1111} = C_{11} = (v_{1/1})^2\rho = 2365 \text{ x } 10^7 \text{ Pa} = 23.65 \text{ GPa.}$$

$$C_{1212} = C_{16} = (v_{1/2})^2\rho = 660 \text{ x } 10^7 \text{ Pa} = 6.6 \text{ GPa.}$$

14.6.1.4. *Slowness surface*

The polarization unit vector $\vec{d}$ and propagation direction unit vector $\vec{p}$ are in the plane (1,2). For this plane, being an isotropic plane for a longitudinal and transverse wave, the two corresponding traces of slowness surfaces are circles. The transmitted shear wave is presented in Figure 14.14.

14.6.1.5. *Utilization of two ultrasonic benches for complete evaluation of stiffness matrix coefficients*

For transverse isotropic composite materials (see Chapter 1in [CHE 10]), five elastic stiffness coefficients are evaluated. The two elastic coefficients obtained above are completed by measurement of the three other coefficients. Figure 14.15 shows, on the two sample planes (1,2) and (1,3) in which direction the ultrasonic waves and polarization vector are represented. The wave indicated by (b) permits evaluation of the non-diagonal coefficient C_{1133} and also the phase velocity v_p at various angles, R, in the sample. This velocity gives rise to the corresponding slowness curve.

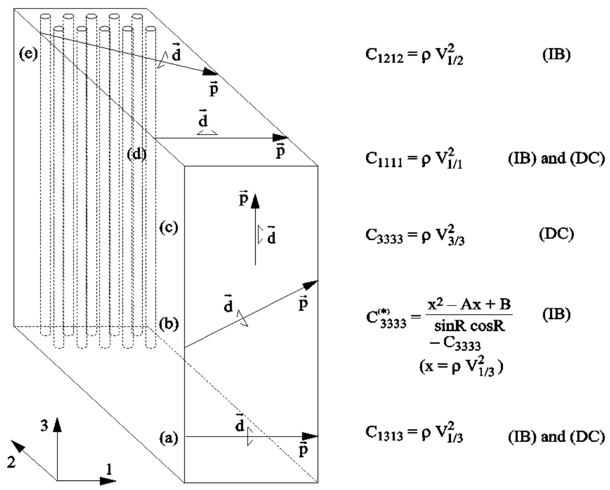

Figure 14.15. *Direction of propagation $\vec{p}$ and polarization unit vector $\vec{d}$ (represented by double arrow). IB: Immersion bench, DC: direct contact bench. Two planes (1,3) and (1,2) are explored. On each plane, wave with polarization $\vec{d}$ is indicated. Stiffness coefficient C_{ijkl} is indicated in front of the propagation direction*

14.7. Viscoelastic characterization of materials by ultrasonic waves

Viscoelastic properties of materials have been explored by many researchers. Using the principle of correspondence (Chapter 10 in [CHE 10]), let us convert the elastic equation of motion into a viscoelastic one:

$$C^*_{ijkl}\, u^*_{k,lj} = \rho\, u^*_{i,tt} \qquad [14.12]$$

The star designates a complex quantity (true only in the case of progressive plane harmonic waves).

We are interested first in the waves propagating along privileged directions of the material. These correspond to the symmetry axes, for example axes (1,2,3) along which we can have a pure longitudinal wave or a pure transverse wave. This enables measurements of the stiffness elastic coefficients, such as C_{1111}, C_{2222}, C_{3333} or C_{2323}, C_{3131}, C_{1212}.

14.7.1. *Longitudinal waves*

The wave propagation equation is a star indicating complex quantities in viscoelastic regime (true only in the case of progressive plane harmonic waves):

$$C_{iiii}^{*} \cdot u_{i,ii}^{*} = \rho\, u_{i,tt}^{*} \qquad [14.13]$$

The progressive wave in [14.13] is rewritten as:

$$u_k^*(t,\vec{x}) = A_k^* \exp\left[j\omega^*(t - p_i x_i / v)\right] \quad , \quad \vec{x}=(x_1, x_2, x_3) \qquad [14.14]$$

Complex circular frequency is:

$$\omega^* = \omega' + j\omega'' \qquad [14.15]$$

This kind of notation is familiar in the dynamics of structures. The imaginary part of the circular frequency is related to damping capacity. Developing [14.14] we obtain:

$$u_k^*(t,\vec{x}) = A_k^* \exp\left[j\omega'(t - p_i x_i / v)\right]\exp\left[-\omega''(t - p_i x_i / v)\right] \quad , \quad \vec{x}=(x_1, x_2, x_3)$$

$$[14.16]$$

In the second bracket of [14.16] the exponential term is real and negative, which means that there is a decrease in amplitude when the wave is propagating in time and in space. Bringing [14.14] into [14.13], an equation of motion in a harmonic regime is obtained in which C_{iiii}^{*} is a complex stiffness coefficient, defined as:

$$C_{iiii}^* = C'_{iiii} + jC''_{iiii} \qquad \text{not summed} \qquad [14.17]$$

14.7.1.1. *Damping capacity*

The damping ratio concerning the extensional wave is:

$$\frac{C''_{iiii}}{C'_{iiii}} = \tan \delta_{C_{iiii}} \quad [14.18]$$

which is related to C_{iiii}^*, and δ_C is the damping angle.

14.7.1.2. *Evaluation of damping coefficient by logarithmic decrement*

If the wave is made up of continuous decreasing sinusoids, this coefficient is defined as:

$$\Lambda m = \frac{1}{m} \ln \frac{\theta_n}{\theta_{n+m}} \quad [14.19]$$

$$\text{and } \tan \delta_C = \Lambda\, m / \pi \quad [14.20]$$

For progressive wave propagation in a sample in an immersion bench, the wave shape is not one of decreasing sinusoids but that of a succession of wave packets with amplitude decreasing in time and space. It is not possible to evaluate the amplitudes of waves at the receiver transducer either (i) in water or (ii) in the sample.

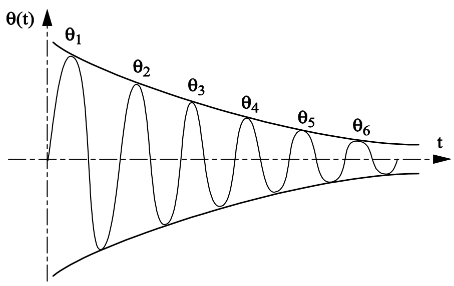

Figure 14.16. *Decay of sinusoidal oscillations. Amplitude decay is represented versus the time variable (as in Le Rolland-Sorin's pendulum) to evaluate damping*

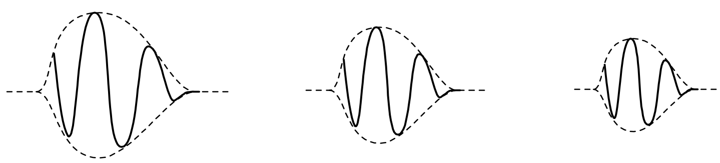

Figure 14.17. *Progressive wave packets propagating in a sample. Damping coefficients are evaluated by amplitude decay versus the space variable*

For a longitudinal wave (Figure 14.13) the distance AB' = AB = e= d for i = 0. Equation [10.69] is then adopted in which m = 1:

$$\Lambda = \ln \frac{\theta_n}{\theta_{n+m}} = \ln \frac{u_n(\text{sample})}{u_{n+m}(\text{water})} \qquad [14.21]$$

A correction for a damping wave in water can be made.

For a transverse wave, using an immersion bench, a similar equation can be written:

$$C_{ijij}^{*}\, u_{i,jj} = \rho\, u_{i,tt}^{*} \qquad [14.22]$$

The distances AB and AB' in Figure 14.13 are not the same. This difference must be taken into account in the calculation of the damping capacity of the material.

14.7.1.3. *Damping coefficients in a direct contact bench*

Measurements seem to be simpler for a direct contact bench than for an immersion bench. The difficulty resides in the evaluation of the damping capacity of the viscous liquid (or gelatin) layer which serves as a coupling fluid between the transducer and sample. For direct transmission (using two transducers) there are two coating thicknesses of coupling liquid or gelatine to be taken into account. Before measurement, one applies a compression force to reduce the thicknesses of the coupling layers to a minimum. The calculations concerning the wave damping in coupling fluid are not taken into account if we are sure that wave damping is negligible, i.e. the fluid layer thicknesses are reduced to a minimum. That explains the initial caution using a compression force.

14.7.2. *Remarks on the viscoelastic properties of materials obtained by ultrasonic methods*

The viscoelastic properties of a material are considered as asymptotic properties of the material at high frequencies.

14.7.2.1. *Frequency range*

At ultrasonic frequency, the material properties are useful to complete information obtained by vibration tests at low and medium frequencies

A few Hertz < frequency < 50,000 to 100,000 Hertz

For practical industrial applications, the results obtained at an ultrasonic frequency range should be handled with caution. The damping coefficients concerning some diagonal stiffness matrix coefficients obtained at ultrasonic frequency can be much lower than the ones measured at audio frequency. They are beyond the transition zone. Figure 14.18 shows the trend of one complex coefficient in modulus (absolute value) and phase (damping capacity).

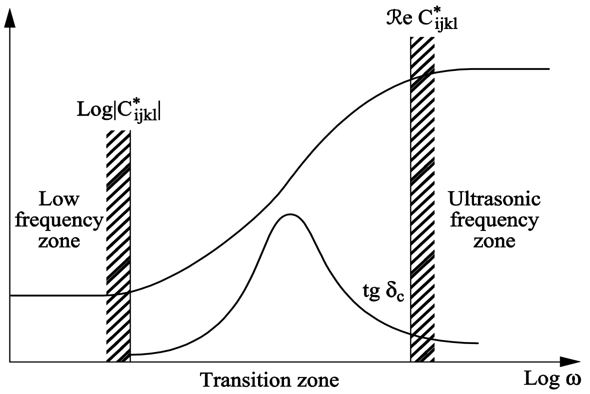

Figure 14.18. *A sketch of complex stiffness coefficient, real part and damping coefficient versus frequency*

14.7.2.2. *Relations between some stiffness coefficients and compliance coefficients*

For isotropic elastic material, the relationships are simple but they involve the measurements of a group of two stiffness coefficients. So, when going from complex stiffness coefficients to complex compliance coefficients (technical elastic moduli), this passage is possible (see the following equations):

$$C^*_{11} = C^*_{22} = C^*_{33} = \frac{E^*(1-\nu^*)}{(1+\nu^*)(1-2\nu^*)}$$

$$C^*_{44} = C^*_{55} = C^*_{66} = \frac{E^*}{2(1+\nu^*)}$$

$$C^*_{12} = C^*_{13} = C^*_{23} = \frac{E^*\nu^*}{(1-2\nu^*)}$$

where E^* and ν^* are respectively the complex Young's modulus and the complex Poisson coefficient. For anisotropic materials, depending on the degree of symmetry of the material (see Chapter 1), the group of independent stiffness coefficients is higher than two. The measurement of complex non-diagonal stiffness coefficients requires coupled waves (longitudinal and shear waves). The need for damping coefficients for evaluation raises the delicate problem of such coefficients on physical grounds.

The other difficulty is the passage from complex stiffness coefficients to the complex moduli of a compliance matrix.

14.7.2.3. *Necessity for transducers to work at the same frequency*

With the objective of material characterization, transducers of different kinds (longitudinal or extension waves, shear wave) must definitely have the same working frequencies, particularly when one wants to convert complex stiffness coefficients into complex compliance coefficients.

14.7.2.4. *Simplifying assumptions for isotropic complex moduli*

In Part II of [CHE 10] (devoted to bending, torsion and longitudinal vibration of rods), there are two elastic moduli influencing the geometric dispersion in the equations of motion. One of the simplifying hypotheses consists of admitting that the Poisson's number ν is equal to:

$$\nu = \frac{E}{2G} - 1 \qquad [14.23]$$

where E is the Young's modulus, and G the shear modulus.

This relationship would permit (if Poisson's number is constant) that, in the working frequency range, the ratio $\frac{E^*}{G^*}$ of two complex moduli was also constant and real. This assumption must be verified when one wants to convert complex stiffness coefficients into complex compliance coefficients.

14.8. Bibliography

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14.9. Appendix 14A. Oblique incidence and energy propagation direction [KLI 92]

The interest in through transmission using a water immersion bench is the ease with which material properties can be explored in any direction. Ultrasonic examination does not necessitate cutting a sample into various pieces.

The problem to be examined remains knowing how the energy flux is directed in the sample.

Descartes-Snell's law gives the relationship between the two rays as:

$$\frac{\sin\theta_i}{V_{water}} = \frac{\sin\theta_r}{V_{phase}} \qquad [14.A.1]$$

where V_{water} and V_{phase} designate the phase velocities in water and in the sample.

The relationship between phase velocity and group velocity is:

$$V_{group} \cos\phi \cos\psi = V_{phase} \qquad [14.A.2]$$

The time delay between the signal observed with and without a sample is:

$$\Delta t = t_{w/o\ sample} - t_{w\ sample} \qquad [14.A.3]$$

$$\Delta t = \frac{d_w}{V_w} - \frac{d_g}{V_g} \qquad [14.A.4]$$

Let d_g cosϕ designate the projection of a group velocity ray in the incidence plane. It forms an angle $(\theta_r + \psi)$ with the normal to the surface:

$$d_g.\cos\phi \cos(\theta r + \psi) = d \qquad [14.A.5]$$

The time delay is written as:

$$\Delta t = [d_g \cos\phi\cos(\theta_r - \theta_I + \psi)] / V_w - d_g / V_g \qquad [14.A.6]$$

Bringing [14A.5] into [14A.6] taking into account [14.3b]:

$$\Delta t = [d/\cos(\theta_r + \psi)] \{[\cos(\theta_r - \theta_I + \psi)] / V_w - [\cos\psi / V_{phase}]\} \qquad [14.A.7]$$

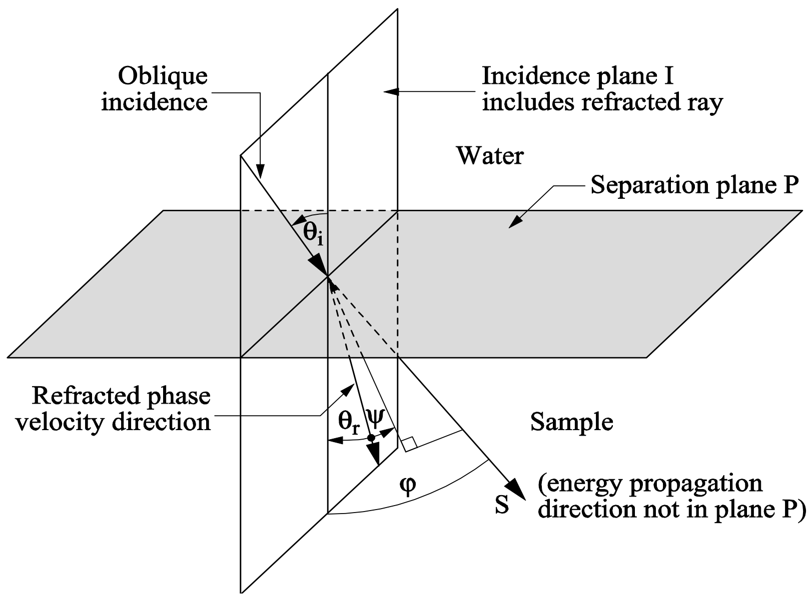

Figure 14A.1. *Plane P is the separation plane between water and the sample. The incidence plane in the water, including the normal to P and the ultrasonic incident ray in water, must contain the refracted ray in the sample (angle θ_r). S represents the propagation direction of energy; it makes an angle ϕ with the plane I. The projection of S on the plane I makes an angle Ψ with the reflected ray*

Expanding $\cos(\theta_r - \theta_I + \psi)$:

$$\Delta t = [\ d/\cos(\theta_r + \psi)]\{[\ \cos(\theta_r + \psi)\cos\theta_I / V_w] + [\sin(\theta_r + \psi)\sin\theta I /\ V_w] - \cos\psi / V_{phase}\ \}$$ [14.A.8]

Taking into account Descartes-Snell's law:

$$\Delta t = [\cos(\theta_r + \psi)]\ \{\ \cos(\ \theta_r + \psi)\cos\theta_I / Vw + [\sin(\ \theta_r + \psi)\sin\theta_r - \cos\ \psi]/V_{phase}\ ]\}$$

[14.A.9]

Expanding $\sin(\theta_r + \psi)$ and multiplying $\cos\psi$ by $(\cos^2\theta_r + \sin^2\theta_r)$, we obtain:

$$\Delta t = [d/\cos(\ \theta_r + \psi)][(\cos(\ \theta_r + \psi)\cos\theta_I) /\ V_w - (\cos(\ \theta_r\ + \psi\ )\cos\ \theta_r)/ V_{phase}]$$ [14.A.10]

$$\Delta t = d\ [\cos\theta_I / V_w - \cos\theta_r / V_{phase}\ ]$$ [14.A.11]

Letting:

$$\cos\theta_r = \sqrt{(1 - \sin\theta_r)}$$

Solving for V_{phase} yields:

$$V_{phase} = [(\frac{\Delta t}{d})^2 - \frac{2\Delta t/\boldsymbol{d}}{V_W}\cos\theta_i + (\frac{1}{V_W})^2]^{-1/2}$$ [14.A.12]

Expression [14.A.12] is exactly the same as the one obtained using phase velocity.

Although Δt corresponds to the energy propagation with group velocity, the phase velocity can be found from the time delay.

14.10. Appendix 14.B. Water immersion bench, measurement of coefficients of stiffness matrix

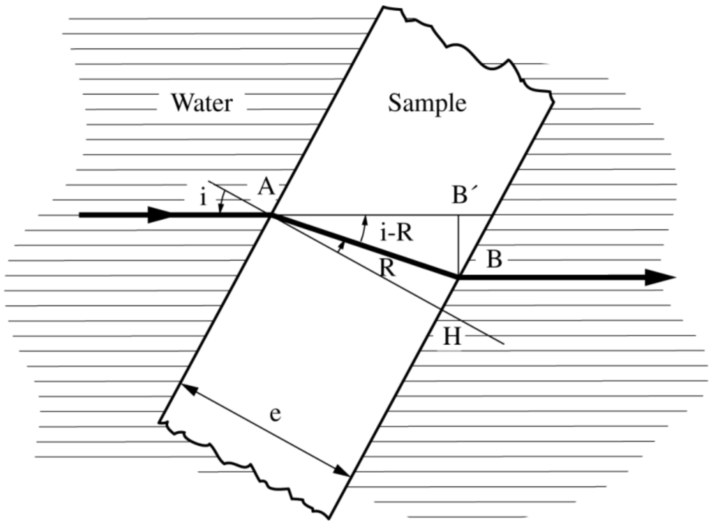

Figure 14B.1. *Ultrasonic wave transmission through a plate immersed in water*

The relationship between the incidence angle i and refraction angle R is Snell's law (see Figure 14B.1):

$$\frac{\sin i}{v_0} = \frac{\sin R}{v}$$

or, n being the refraction index between water and the sample:

$$\frac{\sin i}{\sin R} = n = \frac{v_0}{v} \qquad [14.B.1]$$

If $v > v_0$ (or $n < 1$), [14.B.1] can be written as:

$$\delta t = \frac{AB'}{v_0} - \frac{AB'}{v} = \frac{e \cos(i - R)}{v_0 \cos R} - \frac{e}{v \cos R}$$

$$\delta t = \frac{e}{v_0 \cos R} \left[\cos \left(i - R \right) - n \right] \qquad [14.B.2]$$

Developing the brackets:

$$\delta t = \frac{e}{v_0} \left(\frac{\cos i \cos R + \sin i \sin R - n}{\cos R} \right)$$

Taking into account Snell's law [14B.1] time delay can be rewritten:

$$\delta t = \frac{e}{v_0} \left(\cos i - \sqrt{n^2 - \sin^2 i} \right) \qquad [14.B.3]$$

$i \leq \pi/2$

14B.1. *Expression of phase velocity in the sample*

Equation [10.55] contains the measurable parameters i.e. δt, cos i, sin i, v_0, e. The phase velocity is contained in the refraction index.

From [14B.3]:

$$\sqrt{n^2 - \sin^2 i} = \cos i - \frac{v_0\, \delta t}{e}$$

let us extract n

$$n^2 = \sin^2 i + \left(\cos i - \frac{v_0\, \delta t}{e} \right)^2$$

we arrive at:

$$v = \frac{v_0}{\left[1 + \frac{v_0\, \delta t}{e} \left(\frac{v_0\, \delta t}{e} - 2 \cos i \right) \right]^{1/2}} \qquad [14.B.4]$$

Formula [14B.4] for velocity v can be used in two ways: with and without time measurement.

14B.2. ***Phase velocity measurement by propagation time (δt) evaluation***

Propagation of sinusoid packets in the form of tone bursts can be visualized, despite their progressive wave character, for a given position on an oscilloscope screen.

This can be obtained with repetitive wave packets and if the sweeping is synchronized to the wave emitted from the generator itself.

δt is measured by the difference of two time durations:

$$\delta t = T_{water} - T_{sample} \qquad [14.B.5]$$

where T_{water} corresponds to the position of the pulse packet at the receiver (with no sample). T_{sample} corresponds to the pulse packet after traveling through the sample.

14B.3. ***Phase velocity evaluation without time measurements***

The second method uses the oscilloscope as an electronic detector of the zero position of a wave packet. We need the possibility of adjusting the distance between the two transducers (the emitter and receiver) by mounting one of them on an adjustable micrometer platform.

14B.4.1. *Measurement of longitudinal wave velocity*

In [14B.4] the incident wave is chosen normal to the separating surface i = 0. The wave velocity is:

$$v_L = \frac{v_0}{\left(1 - \frac{v_0\ \delta t}{e}\right)} \qquad [14.B.6]$$

$$v_L = \frac{v_0}{\left(1 - \frac{\ell}{e}\right)} \qquad [14.B.7]$$

In [14B.7], l in the denominator is the distance we must vary one of the transducers so that the distance between the two transducers is equal to $\ell = v_0 \delta t$, v_0 being the velocity in water.

14B.4.2. *Measurement of transverse wave velocity*

Use is made of equation [14B.4] in which we adjust the distance ℓ between transducers, as $\ell = v_0\delta t$:

$$v_T = v_0\left[1 + \frac{l}{e}\left(\frac{l}{e} - 2\cos i\right)\right]^{-1/2} \qquad [14.B.8]$$

$$v_T = \frac{v_0}{\left[1 + \frac{\ell}{e}\left(\frac{\ell}{e} - 2\cos i\right)\right]^{1/2}}$$

ℓ can be evaluated with an accuracy of 1/100 mm.

Notice that the measurement is possible with an incident angle $i \neq 0$. We have to eliminate the longitudinal wave in the sample and retain only a transverse wave by varying the incident angle beyond the critical value so as to transform (in the sample) the longitudinal wave into a surface wave, this last wave not being permitted to travel in the sample.