Chapter 12

Derivative-Free Nonlinear Kalman Filtering for PMSG Sensorless Control

12.1. Introduction

State estimation of nonlinear electric power generators using filters is a significant topic in the area of mechatronics because it can provide improved methods for sensorless control and fault diagnosis of such electromechanical systems. In the design of nonlinear controllers for power generators, it is important to measure all state variables needed to generate a feedback control law. In case of state variables for which direct measurement is technically difficult or costly, estimation can be performed with nonlinear filtering methods [LIN 10, JIA 01, DEL 01, ANE 07, MAH 12]. This chapter proposes derivative-free nonlinear Kalman filtering for sensorless control of permanent magnet synchronous generators (PMSGs).

For nonlinear electric power generators, and under Gaussian noise, the extended Kalman filter (EKF) is frequently applied for estimating the non-measurable state variables through the processing of input and output sequences [KAM 99, BAS 93, HAR 02, NG 03]. The EKF is based on linearization of the dynamical system using Taylor series expansion [XIO 08, RIG 07, RIG 01, RIG 08, RIG 09]. Although EKF is efficient in several estimation problems, it is characterized by cumulative errors due to the

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local linearization assumption and this may affect the accuracy of the state estimation or even risk the stability of the observer-based control loop.

It is also known that we can attempt transformation of the PMSG model into the canonical (Brunovsky) form through the application of the Lie-algebra theory. By using such differential geometric methods, it is possible to arrive at a description of the system in the linear canonical form if the relative degree of the system is equal to the order of the system. After transformation to the canonical form, state estimation with the use of a linear system is possible. However, this linearization procedure requires the computation of Lie derivatives (partial derivatives on the vector fields describing the system dynamics), which can be a cumbersome computation procedure.

Aiming at finding more efficient state estimation methods for the model of the PMSG (with reference to the Lie algebra-based state estimator design), in this chapter a derivative-free approach to Kalman filtering is introduced. In the proposed derivative-free Kalman filtering method, the system is first subject to a linearization transformation that is based on the differential flatness theory and then state estimation is performed by applying the standard Kalman filter recursion to the linearized model. Unlike the Lie algebra-based estimator design method, the proposed approach provides estimates of the state vector of the PMSG without the need for derivatives and Jacobians calculation. By avoiding linearization approximations, the proposed filtering method improves the accuracy of estimation of the system state variables, and results in smooth control signal variations and in minimization of the tracking error of the associated control loop [RIG 10, RIG 11a, RIG 11b].

Differential flatness theory is currently a main topic in nonlinear dynamical systems [RUD 03, SIR 04, RIG 11c]. To find out if a dynamical system is differentially flat, the following should be examined: (1) the existence of the so-called flat output, that is a new variable which is expressed as a function of the system's state variables. The flat output and its derivatives should not be coupled in the form of an ordinary differential equation (ODE); (2) the components of the system (that is state variables and control input) should be expressed as functions of the flat output and its derivatives [LEV 11, FLI 99, LAR 07]. In certain cases, the differential flatness theory enables transformation to a linearized form (canonical Brunovsky form) for which the design of the controller becomes easier. In other cases, by showing that a system is differentially flat, we can easily design a reference trajectory as a function of the so-called flat output and can find a control law that assures tracking of this desirable trajectory [FLI 99, VIL 07].

This chapter analyzes differential flatness of the PMSG model and its resulting description in the Brunovksy (canonical) form [MAR 99]. At a second stage and with the use of the linearized model, Kalman filtering is proposed for estimating the non-directly measurable elements of the state vector of the power generator. To enable efficient operation of the power generator under variable mechanical input power (torque), the Kalman filter is extended towards a disturbances estimator. This enables us to estimate simultaneously both the state vector of the generator and the unknown mechanical input power (torque). Finally, state estimation-based control is applied to assure that the rotation speed of the generator tracks a predefined reference set point. The aggregate control input is generated by including in the state-feedback control law a term that is based on the estimation of the unknown input power and of its derivatives and which compensates for the effects of variation of this input power.

The structure of the chapter is as follows: in section 12.2, the dynamic model of the PMSG is analyzed. In section 12.3, it is shown how with the application of Lie algebra theory we can obtain a description of the generator's dynamics in the linear canonical form and based on such a linearization procedure we can design a state estimator for the system. In section 12.4, it is shown how with the use of the differential flatness theory and without the computation of partial derivatives and Jacobian matrices we can obtain again a description of a nonlinear dynamical system in the linear canonical form. In section 12.5, it is shown that the PMSG is a differentially flat system and its nonlinear model can be transformed into an equivalent linear canonical form. Through this linearization, it becomes possible to apply derivative-free nonlinear Kalman filtering for obtaining an estimate of the non-measurable elements of the PMSG's state vector. In section 12.6, methods for robust state estimation based on unknown input observers are analyzed. In section 12.7, it is shown how the derivative-free nonlinear Kalman filter can be generalized in the form of a disturbances estimator, thus enabling simultaneous estimation of both the non-measurable elements of the generator's state vector and the unknown input power (torque). In section 12.8, the proposed state estimation-based control scheme for the power generator is evaluated with the use of simulation experiments. Finally, in section 12.9, concluding remarks are stated.

12.2. Dynamic model of the permanent magnet synchronous generator

It is considered that the third-order model PMSG is connected to the power grid as shown in Figure 12.1, thus forming the model of a single machine infinite bus (SMIB) system.

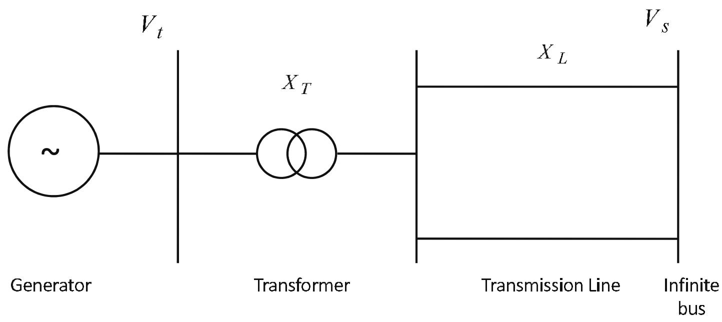


Figure 12.1. *PMSG connected to the power grid according to the single machine infinite bus (SMIB) model*

The PMSG mechanical dynamics can be represented as follows:

$$\begin{aligned} \dot{\delta} &= \omega - \omega_0 \\ \dot{\omega} &= -\frac{D}{2J}(\omega - \omega_0) + \frac{\omega_0}{2J}(P_m - P_e) \end{aligned} \qquad [12.1]$$

where δ is the turn angle of the generator's rotor, ω is the rotation speed of the rotor with respect to synchronous reference, ω_0 is the synchronous speed of the generator, J is the moment of inertia of the rotor, P_m is the mechanical input torque to the generator that is associated with the mechanical input power, D is the damping constant of the generator and T_e is the electrical torque that is associated with the generated active power. Moreover, the following variables are defined: $\Delta\delta = \delta - \delta_0$ and $\Delta\omega = \omega - \omega_0$ with ω_0 denoting the synchronous speed.

The generator's electric dynamics is described by [LIN 10, JIA 01, DEL 01, ANE 07] and [MAH 12]:

$$\dot{E}_q^{'} = \frac{1}{T_{d_o}}(E_f - E_q) \qquad [12.2]$$

where $E_q^{'}$ is the quadrature-axis transient voltage of the generator, E_q is the quadrature axis voltage of the generator, T_{d_o} is the direct axis open-circuit transient time constant of the generator and E_f is the equivalent voltage in the excitation coil.

The algebraic equations of the synchronous generator are given by

$$
\begin{aligned}
E_q &= \frac{x_{d_\Sigma}}{x'_{d_\Sigma}} E'_q - (x_d - x'_d) \frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta) \\
I_q &= \frac{V_s}{x'_{d_\Sigma}} sin(\Delta\delta) \\
I_d &= \frac{E'_q}{x'_{d_\Sigma}} - \frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta) \\
P_e &= \frac{V_s E'_q}{x'_{d_\Sigma}} sin(\Delta\delta) \\
Q_e &= \frac{V_s E'_q}{x'_{d_\Sigma}} cos(\Delta\delta) - \frac{V_s^2}{x_{d_\Sigma}} \\
V_t &= \sqrt{(E'_q - X'_d I_d)^2 + (X'_d I_q)^2}
\end{aligned}
\qquad [12.3]
$$

where $x_{d_\Sigma} = x_d + x_T + x_L$, $x'_{d_\Sigma} = x'_d + x_T + x_L$, x_d is the direct-axis synchronous reactance, x_T is the reactance of the transformer, x_L is the reactance of the transmission line, I_d and I_q are direct and quadrature axis currents of the generator, V_s is the infinite bus voltage, Q_e is the generator reactive power delivered to the infinite bus, and V_t is the terminal voltage of the generator.

Substituting the electrical equations of the PMSG given in equation [12.3] into the equations of the electrical and mechanical dynamics of the rotor given in equations [12.1] and [12.2], respectively, the complete model of the SMIB model is obtained:

$$
\begin{aligned}
\dot{\delta} &= \omega - \omega_0 \\
\dot{\omega} &= -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E'_q}{x'_{d_\Sigma}} sin(\Delta\delta) \\
\dot{E}'_q &= -\frac{1}{T'_d} E'_q + \frac{1}{T_{d_o}} \frac{x_d - x'_d}{x'_{d_\Sigma}} V_s cos(\Delta\delta) + \frac{1}{T_{d_o}} E_f
\end{aligned}
\qquad [12.4]
$$

where $T'_d = \frac{x'_{d_\Sigma}}{x_{d_\Sigma}} T_{d_o}$ is the time constant of the field winding.

The previously analyzed SMIB model of the PMSG is described by a nonlinear state space model of the form

$$\dot{x} = f(x) + g(x)u \qquad [12.5]$$

where the state vector x is defined as

$$x = \left(\Delta\delta \ \Delta\omega \ E_q^{'}\right)^T \qquad [12.6]$$

The vector fields $f(x)$ and $g(x)$ are defined as

$$f(x) = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2H} - \omega_0 \frac{1}{2J} \frac{V_s E_q^{'}}{x_{d\sigma}^{'}} sin(\Delta\delta) \\ -\frac{1}{T_d^{'}} E_q^{'} + \frac{1}{T_{do}} \frac{x_d - x_d^{'}}{x_{d\sigma}^{'}} V_s cos(\Delta\delta) \end{pmatrix} \qquad [12.7]$$

$$g(x) = \left(0 \ 0 \ \frac{1}{T_{do}}\right)^T \qquad [12.8]$$

with control input $u = E_f$, the field voltage (equivalent voltage in the excitation coil) and measurable output the rotation speed of the rotor

$$y = h(x) = \delta - \delta_0 \qquad [12.9]$$

12.3. Lie algebra-based design of nonlinear state estimators

12.3.1. *Relative degree for nonlinear systems*

The nonlinear model of the PMSG given in equation [12.4] is in an affine in-the-input form, that is

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u \\ y(t) &= h(x) \end{aligned} \qquad [12.10]$$

where $x \in R^n$ is the state vector, $u \in R$ is the control vector, $y \in R$ is the output vector, $f(x)$ and $g(x)$ vector fields that belong in R^n and $h(x)$ is the scalar function of x. It will be shown that a state estimator for the nonlinear model of the PMSG can be derived using Lie algebra, which introduces a change of coordinates (diffeomorphism) that enables us to write a nonlinear system into an equivalent linear form. Next, the following definitions from the Lie algebra theory are used [KHA 96]:

Lie derivative: For a given differentiable scalar function $h(x)$ of $x = [x_1, x_2, x_3, \cdots x_n]^T$ and a vector field $f(x) = [f_1, f_2, f_3, \cdots, f_n]^T$, the Lie derivative of function $h(x)$ along the vector field $f(x)$ is a new scalar function defined by $L_f h(x)$ which is obtained as follows:

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \qquad [12.11]$$

Relative degree: If the Lie derivative of the function $L_f^{r-1} h(x)$ along vector field $g(x)$ is not equal to zero in a neighborhood Ω, that is

$$L_g L_f^{r-1} h(x) \neq 0 \qquad [12.12]$$

then it is said that the system has relative degree r in Ω.

The relative degree of the system is a parameter to take into account in the design of controllers or observers for nonlinear dynamical systems. If the relative degree of a system is equal to its order n, then the system is exactly linearizable. If $r < n$, then the system is partially linearizable.

12.3.2. *Nonlinear observer design for exactly linearizable systems*

Under the condition $r = n$, we have [KHA 96]:

$$\begin{aligned} &L_g L_f^{1-1} h(x) = L_g L_f^{2-1} h(x) = \cdots = L_g L_f^{n-2} h(x) = 0 \\ &L_g L_f^{n-1} h(x) \neq 0 \end{aligned} \qquad [12.13]$$

Next, a change of coordinates is performed as follows:

$$z_1 = y = h(x) = L_f^{1-1} h(x) \qquad [12.14]$$

Moreover, it holds

$$\dot{z}_1 = \frac{\partial h(x)}{\partial x} \dot{x} \qquad [12.15]$$

Substituting the system's dynamics into equation [12.15], we get

$$\begin{aligned}\dot{z}_1 &= \frac{\partial h(x)}{\partial x} f(x) + \frac{\partial h(x)}{\partial x} g(x) \cdot u \\ &= L_f^{2-1} h(x) + L_g L_f^{1-1} h(x) u\end{aligned} \quad [12.16]$$

According to the condition about the relative degree of the system, it holds $L_g L_f^{1-1} h(x) = 0$ and from equation [12.16], we get

$$\dot{z}_1 = L_f h(x) = z_2 \quad [12.17]$$

In a similar way

$$\begin{aligned}\dot{z}_2 &= L_f^2 h(x) = z_3 \\ &\cdots \\ &\cdots \\ \dot{z}_{n-1} &= L_f^{n-1} h(x) = z_n\end{aligned} \quad [12.18]$$

Again, using the property of the system's relative degree that we have $L_g L_f^{n-1} h(x) \neq 0$, we obtain

$$\dot{z}_n = L_f^n h(x) + L_g L_f^{n-1} h(x) u = \alpha(x) + b(x) u = v \quad [12.19]$$

where $\alpha(x) = L_f^n h(x)$, $b(x) = L_g L_f^{n-1} h(x)$ and $v = \alpha(x) + b(x)u$. After this change of coordinates, the system can be written as

$$\dot{z} = Az + Bu \quad [12.20]$$

where

$$z = \phi(x) = \begin{pmatrix} h(x) \\ L_f h(x) \\ \cdots \\ \cdots \\ L_f^{n-1} h(x) \end{pmatrix} \quad [12.21]$$

while matrices A,B appearing in the previous state-space equation are given by

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \qquad [12.22]$$

$$B = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \end{pmatrix}^T$$

The state observer for the transformed system of equation [12.20] is

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + Bv + K(y - h(\hat{x})) \text{ or} \\ \dot{\hat{z}} &= A\hat{z} + B[\alpha(\hat{x}) + b(\hat{x})u] + K(y - h(\hat{x})) \text{ or} \\ \dot{\hat{z}} &= A\hat{z} + B[\alpha(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u] + K(y - h(\phi^{-1}(\hat{z}))) \end{aligned} \qquad [12.23]$$

It is also possible to express the state observer using a nonlinear model, that is

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + L(y - h(\hat{x})) \qquad [12.24]$$

It will be shown that the nonlinear observer's gain L is now given by

$$L = (J_\phi(\hat{x}))^{-1}K \qquad [12.25]$$

where matrix $J_\phi(\hat{x})$ is the Jacobian of the new coordinates $z_1, \cdots, z_n$, which is obtained after the nonlinear change of coordinates, and K is the observer's gain computed for the linearized equivalent of the system of equation [12.23]. The observer gain K for the linearized system can be obtained through the Kalman filter recursion. We have that

$$d\hat{z} = \begin{pmatrix} dh(\hat{x}) \\ dL_f h(\hat{x}) \\ \cdots \\ dL_f^{n-1}h(\hat{x}) \end{pmatrix} = J_\phi(\hat{x})d\hat{x} \qquad [12.26]$$

or equivalently

$$\dot{\hat{z}} = J_\phi(\hat{x})\dot{\hat{x}} \qquad [12.27]$$

It holds that the Jacobian matrix of $\phi(\hat{x})$ with respect to $\hat{x}$ can be written as

$$J_\phi(\hat{x}) = \frac{\partial\phi(\hat{x})}{\partial\hat{x}} \qquad [12.28]$$

Using the state observer dynamics described in equation [12.24], we have

$$\frac{\partial\phi(\hat{x})}{\partial\hat{x}}\dot{\hat{x}} = \frac{\partial\phi(\hat{x})}{\partial\hat{x}}f(\hat{x}) + \frac{\partial\phi(\hat{x})}{\partial\hat{x}}g(\hat{x})u + \frac{\partial\phi(\hat{x})}{\partial\hat{x}}L(\hat{x})(y-h(\hat{x})) \qquad [12.29]$$

Considering that for the first row of the Jacobian matrix it holds $\phi(\hat{x}) = h(\hat{x})$, we have

$$\frac{\partial h(\hat{x})}{\partial\hat{x}}\dot{\hat{x}} = \frac{\partial h(\hat{x})}{\partial\hat{x}}f(\hat{x}) + \frac{\partial h(\hat{x})}{\partial\hat{x}}g(\hat{x})u + K(y-h(\hat{x})) \qquad [12.30]$$

or equivalently

$$\dot{\hat{z}}_1 = L_f h(\hat{x}) + L_g h(\hat{x})u + K_1(y-h(\hat{x})) \qquad [12.31]$$

Moreover, it holds $L_g h(\hat{x}) = 0$, and $L_f h(\hat{x}) = \hat{z}_2$, and thus we obtain

$$\dot{\hat{z}}_1 = \hat{z}_2 + K_1(y-h(\hat{x})) \qquad [12.32]$$

In a similar manner, we have

$$\begin{aligned} &\dot{\hat{z}}_2 = \hat{z}_3 + K_2(y-h(\hat{x})) \\ &\cdots \\ &\dot{\hat{z}}_{n-1} = \hat{z}_n + K_{n-1}(y-h(\hat{x})) \end{aligned} \qquad [12.33]$$

Additionally, using that $L_g L_f^{n-1} \neq 0$, we can finally write

$$\dot{\hat{z}}_n = \alpha(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u + K_n(y-h(\hat{x})) \qquad [12.34]$$

where $\alpha(\phi^{-1}(\hat{z})) = L_f^{n-1}h(\hat{x}) = L_f^{n-1}h(\phi^{-1}(\hat{z}))$, and $b(\phi^{-1}(\hat{z})) = L_g L_f^{n-1}h(\hat{x}) = L_g L_f^{n-1}h(\phi^{-1}(\hat{z}))$. Using the previous notation, we obtain the formulation of the nonlinear estimator's gain L as a function of the observation gain K for the linearized equivalent of the system described in equation [12.23]. Thus, we finally get equation [12.25]

$$(J_\phi(\hat{x}))L = K \Rightarrow L = (J_\phi(\hat{x}))^{-1}K \qquad [12.35]$$

12.3.3. *Linearization of PMSG dynamics using Lie Algebra*

The nonlinear dynamical model of the PMSG was described in equations [12.5]–[12.9]. The linearization procedure with the use of Lie algebra gives:

$$z_1 = L_f^0 h(x) = h(x) = \Delta\delta \qquad [12.36]$$

$$z_2 = L_f^1 h(x) = \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \Rightarrow \qquad [12.37]$$

$$z_2 = x_2 = \Delta\omega$$

$$z_3 = L_f^2 h(x) = \begin{pmatrix} \frac{\partial z_2}{\partial x_1} & \frac{\partial z_2}{\partial x_2} & \frac{\partial z_2}{\partial x_3} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \Rightarrow \qquad [12.38]$$

$$z_3 = f_2 = -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \frac{\omega_0}{2J} \frac{V_s E_q^{'}}{x_{d_\Sigma}^{'}} sin(\Delta\delta)$$

Moreover, according to the previous analysis, it holds

$$\dot{z}_3 = L_f^3 h(x) + L_g L_f^2 h(x) u \qquad [12.39]$$

where

$$L_f^3 h(x) = \begin{pmatrix} \frac{\partial z_3}{\partial x_1} & \frac{\partial z_3}{\partial x_2} & \frac{\partial z_3}{\partial x_3} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \Rightarrow$$

$$L_f^3 h(x) = (-\frac{\omega_0}{2J} \frac{V_s}{x_{d\Sigma}^{'}} x_3 cos(x_1) \dot{x}_1) - \frac{D}{2J} \Bigg(\frac{-D}{2J} x_2 + \omega_0 \frac{P_m}{2J}$$
$$- \frac{\omega_0}{2J} \frac{V_s}{x_{d\Sigma}^{'}} x_3 sin(x_1) \Bigg) \Bigg(\frac{1}{T_d^{'}} x_3 + \frac{1}{T_{do}} \frac{x_d - x_d^{'}}{x_{d\Sigma}^{'}} cos(x_1) \Bigg) \Rightarrow \qquad [12.40]$$

$$L_f^3 h(x) = (\frac{D}{2J})^2 \dot{x_1} - \omega_0 \frac{D}{2J} \frac{P_m}{2J} + \frac{D}{(2J)^2} \omega_0 \frac{V_s}{x_{d\Sigma}^{'}} x_3 sin(x_1)$$
$$+ \frac{\omega_0}{2J} \frac{V_s}{x_{d\Sigma}^{'}} \frac{1}{T_d^{'}} x_3 sin(x_1) - \frac{\omega_0}{2J} \frac{V_s}{x_{d\Sigma}^{'}} \frac{1}{T_{do}} \frac{x_d - x_d^{'}}{x_{d\Sigma}^{'}} V_s cos(x_1) sin(x_1)$$
$$- \frac{\omega_0}{2H} \frac{V_s}{x_{d\Sigma}^{'}} x_3 cos(x_1) x_2$$

Finally, in an equivalent manner, we obtain

$$L_g L_f^2 h(x) = \begin{pmatrix} \frac{\partial z_3}{\partial x_1} & \frac{\partial z_3}{\partial x_2} & \frac{\partial z_3}{\partial x_3} \end{pmatrix} \cdot \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} \Rightarrow$$
$$L_g L_f^2 h(x) = -\frac{\omega_0}{2J}\frac{1}{T_{do}}\frac{V_s}{x'_{d\Sigma}} sin(x_1) \qquad [12.41]$$

Defining the control input for the linearized system $v = L_f^3 h(x) + L_g L_f^2 h(x) u$, the state-space model can be written in the following linear canonical (Brunovsky) form:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \qquad [12.42]$$

12.4. Differential flatness for nonlinear dynamical systems

12.4.1. *Definition of differentially flat systems*

Differential flatness is a structural property of a class of nonlinear systems, denoting that all system variables (such as state vector elements and control inputs) can be written in terms of a set of specific variables (the so-called flat outputs) and their derivatives. The following nonlinear system is considered:

$$\dot{x}(t) = f(x(t), u(t)) \qquad [12.43]$$

The time is $t \in R$, the state vector is $x(t) \in R^n$ with initial conditions $x(0) = x_0$ and the input is $u(t) \in R^m$. Next, the properties of differentially flat systems are given [RUD 03, SIR 04, RIG 11c, LEV 11, FLI 99, LAR 07, VIL 07]:

The finite dimensional system of equation [12.43] can be written in the general form of an ODE, that is $S_i(w, \dot{w}, \ddot{w}, \cdots, w^{(i)}),\ \ i = 1, 2, \cdots, q$. The term w is a generic notation for the system variables (these variables are, for instance, the elements of the system's state vector $x(t)$ and the elements of the control input $u(t)$) while $w^{(i)},\ i = 1, 2, \cdots, q$ are the associated derivatives. Such a system is differentially flat if there are m functions $y = (y_1, \cdots, y_m)$ of the system variables and of their time derivatives, that is $y_i = \phi(w, \dot{w}, \ddot{w}, \cdots, w^{(\alpha_i)}),\ i = 1, \cdots, m$ satisfying the following two conditions [RIG 08, FLI 99, MAR 99]:

1) There does not exist any differential relation of the form $R(y, \dot{y}, \cdots, y^{(\beta)}) = 0$ which implies that the derivatives of the flat output are not coupled in the sense of an ODE, or equivalently, it can be said that the flat output is differentially independent.

2) All system variables (that is the elements of the system's state vector w and the control input) can be expressed using only the flat output y and its time derivatives $w_i = \psi_i(y, \dot{y}, \cdots, y^{(\gamma_i)}),\ i = 1, \cdots, s.$ An equivalent definition of differentially flat systems is as follows:

DEFINITION.– *The system $\dot{x} = f(x, u)$, $x \in R^n$, $u \in R^m$ is differentially flat if there exist relations*

$$\begin{aligned} &h\colon\ R^n \times (R^m)^{r+1} \rightarrow R^m, \\ &\phi\colon\ (R^m)^r \rightarrow R^n \textit{ and} \\ &\psi\colon\ (R^m)^{r+1} \rightarrow R^m \end{aligned} \quad [12.44]$$

such that

$$\begin{aligned} &y = h(x, u, \dot{u}, \cdots, u^{(r)}), \\ &x = \phi(y, \dot{y}, \cdots, y^{(r-1)}), \textit{ and} \\ &u = \psi(y, \dot{y}, \cdots, y^{(r-1)}, y^{(r)}). \end{aligned} \quad [12.45]$$

This means that all system dynamics can be expressed as a function of the flat output and its derivatives; therefore, the state vector and the control input can be written as

$$\begin{aligned} &x(t) = \phi(y(t), \dot{y}(t), \cdots, y^{(r)}(t)), \text{and} \\ &u(t) = \psi(y(t), \dot{y}(t), \cdots, y^{(r+1)}(t)) \end{aligned} \quad [12.46]$$

It is noted that for linear systems the property of differential flatness is equivalent to that of controllability. Next, an example is given to explain the design of a differentially flat controller for finite dimensional systems of known parameters.

EXAMPLE.– Flatness-based control for a nonlinear system of known parameters. Consider the following model [LAR 07]:

$$\begin{aligned} &\dot{x}_1 = x_3 - x_2 u \\ &\dot{x}_2 = -x_2 + u \\ &\dot{x}_3 = x_2 - x_1 + 2x_2(u - x_2) \end{aligned} \quad [12.47]$$

The candidate flat output is $y_1 = x_1 + \frac{x_2^2}{2}$. Thus, we get:

$$\begin{aligned}
y_1 &= x_1 + \frac{x_2^2}{2} \\
y_2 &= \dot{y}_1 = (x_3 - x_2 u) + x_2(u - x_2) = x_3 - x_2^2 \\
y_3 &= \dot{y}_2 = \ddot{y}_1 = x_2 - x_1 + 2x_2(u - x_2) - 2x_2(u - x_2) = -x_1 + x_2 \\
v &= \dot{y}_3 = y_1^{(3)} = -x_3 + x_2 u - x_2 + u = -x_2 - x_3 + u(1 + x_2)
\end{aligned} \quad [12.48]$$

It can be verified that property 1 holds, that is there does not exist any differential relation of the form $R(y, \dot{y}, \cdots, y^{(\beta)}) = 0$, and this implies that the derivatives of the flat output are not coupled. Moreover, it can be shown that property 2 also holds, that is the components w of the system (elements of the system's state vector and control input) can be expressed using only the flat output y and its time derivatives $w_i = \psi_i(y, \dot{y}, \cdots, y^{(\gamma_i)}), i = i, \cdots, s$.

For instance, to calculate x_1 with respect to y_1, $\dot{y}_1$, $\ddot{y}_1$ and $y_1^{(3)}$, the relation of $\ddot{y}_1$ is used, that is:

$$x_1^2 + 2x_1(1 + \ddot{y}_1) + \ddot{y}_1^2 - 2y_1 = 0 \quad [12.49]$$

from which two possible solutions are derived, that is $x_1 = -(1 + \ddot{y}_1 - \sqrt{1 + 2(y_1 + \ddot{y}_1)})$ and $x_1 = -(1 + \ddot{y}_1 + \sqrt{1 + 2(y_1 + \ddot{y}_1)})$. Keeping the biggest of these two solutions, we obtain:

$$\begin{aligned}
x_1 &= -(1 + \ddot{y}_1) + \sqrt{1 + 2(y_1 + \ddot{y}_1)} \\
x_2 &= \ddot{y}_1 + x_1 \\
x_3 &= \dot{y}_1 + \ddot{y}_1^2 + 2x_1\ddot{y}_1 + x_1^2 \\
u &= \frac{y_1^3 + \ddot{y}_1^2 + \ddot{y}_1 + \dot{y}_1 + x_1 + 2x_1\ddot{y}_1 + x_1^2}{1 + x_1 + \ddot{y}_1}
\end{aligned} \quad [12.50]$$

The computation of the equivalent model of the system in the linear canonical form is summarized as follows: by finding the derivatives of the flat output, we get a set of equations that can be solved with respect to the state variables and the control input of the initial state-space description of the system. First, the binomial of variable x_1 given in equation [12.49] is solved providing x_1 as a function of the flat output and its derivatives. Next, using the expression for x_1 and equation [12.50], state variable x_2 is also written as a function of the flat output and its derivatives. Finally, using the expressions for both x_1 and x_2 and equation [12.50], state variable x_3 is written as a function of the flat output and its derivatives. Thus, we can finally express the state vector elements and the control input as a function of the flat output and its derivatives, which completes the proof about differential flatness of the system.

From equation [12.50], it can be concluded that the initial system of equation [12.47] is indeed differentially flat. Using the flat output and its derivatives, the system of equation [12.47] can be written in Brunovsky (canonical) form:

$$\begin{pmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{pmatrix} = \begin{pmatrix}0\ 1\ 0\\ 0\ 0\ 1\\ 0\ 0\ 0\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix} v \qquad [12.51]$$

Therefore, a transformation of the system into a linear equivalent is obtained and then it is straightforward to design a controller based on linear control theory. Thus, given the reference trajectory $[x_1^*, x_2^*, x_3^*]^T$, we can find the transformed reference trajectory $[y_1^*, \dot{y}_1^*, \ddot{y}_1^*]^T$ and select the appropriate control input v that succeeds tracking. Knowing v, the control u of the initial system can be found.

12.4.2. *Classes of differentially flat systems*

Main classes of nonlinear differentially flat systems are [MAR 99]:

1) Affine in-the-input systems: the dynamics of such systems is given by:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \qquad [12.52]$$

From equation [12.52], we can notice that the above state equation can also describe MIMO dynamical systems. Without loss of generality, it is assumed that $G = [g_1, \cdots, g_m]$ is of rank m. In a case where the flat outputs of the aforementioned system are only functions of states x, then this class of dynamical systems is called 0-flat. It has been proved that a dynamical affine system with n states and $n-1$ inputs is 0-flat if it is controllable.

2) Driftless systems: these are systems of the form

$$\dot{x} = \sum_{i=1}^{m} f_i(x)u_i \qquad [12.53]$$

For driftless systems with two inputs, that is

$$\dot{x} = f_1(x)u_1 + f_2(x)u_2 \qquad [12.54]$$

flatness holds, if and only if the rank of matrix $E_{k+1}\colon = \{E_k, [E_k, E_k]\},\ k \geq 0$ (with $E_0\colon = \{f_1, f_2\}$ and $[E_k, E_k]\colon = \{[X, Y],\ X, Y \in E_k\}$ denoting a Lie bracket) is equal to $k+2$ for $k=0,\cdots,n-2$. It has been proved that a driftless system, which is differentially flat, is also 0-flat (which means that the flat output is a function of only the elements of the state vector of the initial nonlinear system).

Moreover, for flat systems with n states and $n-2$ control inputs, that is

$$\dot{x}=\sum_{i=1}^{n-2} u_i f_i(x) \ x \in R^n \qquad [12.55]$$

the flatness property holds if controllability also holds. Furthermore, the system is 0-flat if n is even.

12.4.3. *Differential flatness and transformation into the canonical form*

The classes of systems for which the application of differential flatness theory results into a canonical form have to be defined. Transformation into the Brunovksy form can be succeeded for systems that admit static feedback linearization (that is a change of coordinates for both the system state variables and the system's control input). Single-input differentially flat systems admit static feedback linearization and therefore can be finally written in the Brunovsky form. For flat multi-input systems, necessary and sufficient conditions that allow an endogenous transformation into Brunovsky coordinates can be also stated [MAR 99, RIG 11].

In particular, for the case of a single-input differentially flat dynamical system, we have:

$$\dot{x}=f_s(x,t)+g_s(x,t)(u+\tilde{d}), \ x \in R^n, \ u \in R, \ \tilde{d} \in R \qquad [12.56]$$

where $f_s(x,t)$, $g_s(x,t)$ are nonlinear vector fields that define the system's dynamics, u denotes the control input and $\tilde{d}$ denotes additive input disturbances. Knowing that the system of equation [12.56] is differentially flat, the next step is to try to write it into a Brunovsky form. The selected flat output is again denoted by y. Then, as analyzed in section 12.4, for the state variables x_i of the system of equation [12.56], it holds

$$x_i=\phi_i(y,\dot{y},\cdots,y^{(r-1)}), i=1,\cdots,n \qquad [12.57]$$

while for the control input, it holds

$$u=\psi(y,\dot{y},\cdots,y^{(r-1)},y^{(r)}) \qquad [12.58]$$

Introducing the new state variables $y_1 = y$ and $y_i = y^{(i-1)},\ i = 2, \cdots, n$, the initial system of equation [12.56] can be written in the Brunovsky form:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \cdots \\ \cdots \\ \dot{y}_{n-1} \\ \dot{y}_n \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ \cdots \\ y_{n-1} \\ y_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ \cdots \\ 0 \\ 1 \end{pmatrix} v \qquad [12.59]$$

where $v = f(x,t) + g(x,t)(u + \tilde{d})$ is the control input for the linearized model, and $\tilde{d}$ denotes additive input disturbances.

12.5. Differential flatness of the PMSG

It will be shown that the dynamic model of the PMSG is a differentially flat one, that is it holds that all state variables and its control inputs can be written as functions of the flat output and its derivatives. Moreover, it will be shown that by expressing the elements of the state vector as functions of the flat output and its derivatives, we obtain a transformation of the PMSG model into the linear canonical (Brunovsky) form.

Defining the state vector of the PMSG as $x = [\Delta\delta, \Delta\omega, E_q^{'}]^T$, we have that

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{D}{2J}x_2 + \omega_0\frac{P_m}{2J} - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}^{'}}x_3 sin(x_1) \\ \dot{x}_3 &= -\frac{1}{T_d^{'}}x_3 + \frac{1}{T_{do}}\frac{x_d - x_d^{'}}{x_{d\Sigma}^{'}}V_s cos(x_1) + \frac{1}{T_{do}}u \end{aligned} \qquad [12.60]$$

The flat output is chosen to be $y = x_1$. Therefore, it holds

$$x_1 = y \qquad [12.61]$$

$$x_2 = \dot{y} \qquad [12.62]$$

while from the second row of equation [12.60], we have

$$\ddot{y} = -\frac{D}{2J}\dot{y} + \omega_0\frac{P_m}{2J} - \frac{\omega_0}{2J}\frac{V_s x_3}{x_{d\Sigma}^{'}}sin(y) \qquad [12.63]$$

Thus, for $x_1 \neq \pm n\pi$ (where $n = 0, 1, 2, \cdots$), we obtain

$$x_3 = \frac{\omega_0 \frac{P_m}{2J} - \ddot{y} - \frac{D}{2J}\dot{y}}{\frac{\omega_0}{2J}\frac{V_s x_3}{x'_{d\Sigma}} sin(y)}, \text{ or} \quad [12.64]$$

$$x_3 = f_a(y, \dot{y}, \ddot{y})$$

From the third row of equation [12.60], we have

$$u = T_{do}[\dot{x}_3 + \frac{1}{T'_d} x_3 \frac{1}{T_{do}} \frac{x_d - x'_d}{x'_{d\Sigma}} V_s cos(x_1)], \text{ or} \quad [12.65]$$

$$u = f_b(y, \dot{y}, \ddot{y})$$

Therefore, all state variables and the control input of the PMSG can be written as functions of the flat output and its derivatives, and the PMSG model is a differentially flat one.

Next, the following change of variables is performed: $y_1 = y$, $y_2 = \dot{y}$, $y_3 = \ddot{y}$. It also holds

$$\dot{y}_3 = y^{(3)} = \ddot{x}_2 \Rightarrow$$

$$y^{(3)} = -\frac{D}{2J}\dot{x}_2 - \frac{\omega_0}{2H}\frac{V_s}{x_{d\Sigma}}' \dot{x}_3 sin(x_1) - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}}' x_3 cos(x_1)\dot{x}_1 \quad [12.66]$$

By substituting $\dot{x}_2$ and $\dot{x}_3$ from the second and third rows of equation [12.60], respectively, and after intermediate operations, we obtain

$$y^{(3)} = f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u \quad [12.67]$$

where

$$\begin{aligned} f_c(y, \dot{y}, \ddot{y}) = {} & (\frac{D}{2J^2})\dot{y} - \omega_0 \frac{D}{2J}\frac{P_m}{2J} + \omega_0 \frac{D}{(2J)^2}\frac{V_s}{x'_{d\Sigma}} x_3 sin(\dot{y}) \\ & + \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}\frac{1}{T'_d} x_3 sin(y) - \\ & - \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}\frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}} V_s cos(y) sin(y) - \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}} x_3 cos(y)\dot{y} \end{aligned} \quad [12.68]$$

$$g_c(y, \dot{y}, \ddot{y}) = -\frac{\omega_0}{2J}\frac{1}{T_{do}}\frac{V_s}{x'_{d\Sigma}} sin(y) \quad [12.69]$$

Thus, the system can be written in the following linear canonical (Brunovsky) form:

$$\begin{pmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{pmatrix} = \begin{pmatrix}0\ 1\ 0\\ 0\ 0\ 1\\ 0\ 0\ 0\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix} v \qquad [12.70]$$

while the new control input for the linearized system is $v = f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u$. It can be noticed that the linearized equivalent of the system that is obtained after applying differential flatness theory is the same with the one obtained after applying the Lie algebra-based method.

The controller design for the linearized system described in equation [12.70] is carried out using common pole placement methods. Since the overall system dynamics is described by the differential equation

$$y^{(3)} = v \qquad [12.71]$$

a suitable feedback control law that makes the flat output track a desirable set point $y_d(t)$ is given by

$$v = y_d^{(3)}(t) - k_1(\ddot{y}(t) - \ddot{y}_d(t)) - k_2(\dot{y}(t) - \dot{y}_d(t)) - k_3(y(t) - y_d(t))) \qquad [12.72]$$

The control input that is finally applied to the PMSG is given by

$$u(t) = g_c^{-1}(t)[v(t) - f_c(t)] \qquad [12.73]$$

The control law of equation [12.72] results in the closed-loop dynamics

$$e^{(3)}(t) + k_1\ddot{e}(t) + k_2\dot{e}(t) + k_3 e(t) = 0 \qquad [12.74]$$

By selecting the feedback control gains k_1,k_2 and k_3 such that the associated characteristic polynomial $p(s) = s^3 + k_1 s^2 + k_2 s + k_3$ is a Hurwitz one, that is it has poles in the left complex semi-plane, we have that the tracking error converges asymptotically to zero

$$lim_{t\to\infty} e(t) = 0 \Rightarrow lim_{t\to\infty} y(t) = y_d(t) \qquad [12.75]$$

Because the flat output converges to the desirable set point and because all state variables x_1,x_2 and x_3 are expressed as functions of the flat output and its derivatives, we have that the state variables also converge to the desirable set points and therefore efficient control of the PMSG is achieved.

Moreover, for the linearized equivalent of the system, we can perform state estimation with the use of the standard Kalman filter recursion and can compute also the state vector elements that are not directly measurable (i.e. ω and $E_q^{'}$).

12.6. Robust state estimation-based control of the PMSG

12.6.1. *Unknown input observers*

The previous method for PMSG controller design was based on the assumption that the mechanical input power (torque) P_m that is applied to the PMSG can be directly measured and that this input torque is a constant or piecewise constant variable. However, this is not always the case since the measurement of P_m may be technically difficult while P_m maybe a time-varying signal (e.g. mechanical excitation due to wind, steam or water flow). In the latter case, we can consider variable P_m as an external disturbance to the generator's model. The simultaneous estimation of the non-measurable elements of the PMSG state vector (that is ω and $E_q^{'}$) as well as the estimation of the disturbance term (input torque P_m) is possible if using a disturbance estimator is used [COR 05, COR 06, CHE 00, GUP 11, MIK 06].

A first type of disturbance estimator is the unknown input observer. This is applied to dynamical systems of the form

$$\begin{aligned} \dot{x} &= Ax + B(u + w_e) \\ z &= Cx \end{aligned} \qquad [12.76]$$

while the disturbance dynamics is given by

$$\begin{aligned} \dot{d} &= A_f d \\ w_e &= C_f d \end{aligned} \qquad [12.77]$$

Then, the unknown input observer provides a state estimate of the extended state vector

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{d}} \end{pmatrix} = \begin{pmatrix} A & BC_f \\ 0 & A_f \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{d} \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + K(z - C\hat{x}) \qquad [12.78]$$

In the generic case, we can assume that the disturbances vector w_e varies dynamically in time. However, in several cases, it suffices to assume a constant or piecewise constant disturbance $\dot{w}_e(z) = 0$, where $A_f = 0$ and $C_f = 1$. The observer's gain can be obtained through the standard Kalman filter recursion.

12.6.2. *Perturbation observer*

The perturbation observer is an extension of the unknown inputs observer that takes into account not only external disturbances but also parametric uncertainties. In the discrete-time form, the system dynamics is given by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w_f(k) \\ z(k) &= Cx(k) \end{aligned} \qquad [12.79]$$

while the disturbance dynamics is given by

$$\begin{aligned} d(k) &= A_f d(k-1) + B_f(B^+(\hat{x}(k) - A\hat{x}(k-1)) - u(k-1)) \\ \hat{w}_f(k) &= C_f d(k) \\ \hat{x}(k+1) &= A\hat{x}(k) + B(u(k) + \hat{w}_f(k) + L(z(k) - C\hat{x}(k)) \end{aligned} \qquad [12.80]$$

where B^+ is the Moore–Penrose pseudo-inverse of matrix B. The unknown input can represent traditional external disturbances and model uncertainties, that is $w_f = w_e + \Delta A x_k + \Delta B u_k$.

12.6.3. *Extended state observer*

The extended state observer uses a canonical form, so the unmodeled dynamics appear at the disturbance estimation part. The system's description in the canonical form is given by

$$\begin{aligned} x_1^{(n)} &= f(x, t, u, w_f) + b_m u \\ z &= x_1 \\ x &= \begin{pmatrix} x_1 \ \dot{x}_1 \ \cdots \ x_1^{(n-1)} \end{pmatrix}^T \end{aligned} \qquad [12.81]$$

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \cdots \\ \dot{\hat{x}}_{n-1} \\ \dot{\hat{x}}_n \\ \dot{\hat{f}} \end{pmatrix} = \begin{pmatrix} \hat{x}_2 \\ \cdots \\ \hat{x}_n \\ \hat{f} + b_m u \\ 0 \end{pmatrix} + K(x_1 - \hat{x}_1) \qquad [12.82]$$

The extended state observer can be also modified to take into account derivatives of the disturbance

$$\begin{aligned} x_1^{(n)} &= f(x, t, u, w_f) + b_m u \\ z &= x_1 \\ x &= \begin{pmatrix} x_1 \ \dot{x}_1 \ \cdots \ x_1^{(n-1)} \end{pmatrix}^T \\ F &= \begin{pmatrix} f \ \dot{f} \ \cdots \ f^{(h-1)} \end{pmatrix}^T \end{aligned} \qquad [12.83]$$

and now the state and disturbance observer takes the form

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \cdots \\ \dot{\hat{x}}_{n-1} \\ \dot{\hat{x}}_n \\ \dot{\hat{F}}_1 \\ \cdots \\ \dot{\hat{F}}_{h-1} \\ \dot{\hat{F}}_h \end{pmatrix} = \begin{pmatrix} \hat{x}_2 \\ \cdots \\ \hat{x}_n \\ \hat{f} + b_m u \\ \hat{F}_2 \\ \cdots \\ \hat{F}_h \\ 0 \end{pmatrix}^T + K(x_1 - \hat{x}_1) \quad [12.84]$$

The latter form of the extended state observer described in equation [12.84] enables to track various types of disturbances. For example, $h = 1$ allows the estimation of disturbance dynamics defined by its first-order derivative, and $h = 2$ allows the estimation of disturbance dynamics defined by its second-order derivative.

12.7. Estimation of PMSG disturbance input with Kalman filtering

12.7.1. *State estimation with the derivative-free nonlinear Kalman filter*

It was shown that using differential flatness theory, the initial nonlinear model of the PMSG can be written in the canonical form:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \quad [12.85]$$

Thus, we have a linear model of the form

$$\begin{aligned} \dot{y}_f &= A_f y_f + B_f v \\ z_f &= C_f y_f \end{aligned} \quad [12.86]$$

where $y_f = [y_1, y_2, y_3]^T$ and matrices A_f,B_f,C_f are in the canonical form

$$A_f = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad B_f = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad C_f^T = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad [12.87]$$

where the measurable variable $y_1 = \delta$ is associated with the turn angle of the rotor. For the previous model, and after carrying out discretization of matrices A_f, B_f and C_f with common discretization methods, we can perform linear Kalman filtering using equation [12.98] and equation [12.99]. This is *derivative-free nonlinear Kalman filtering* for the model of the generator, which, unlike EKF, is performed without needing the computation of Jacobian matrices and does not introduce numerical errors due to approximative linearization with Taylor series expansion [RIG 11a, RIG 11b, RIG 12a].

12.7.2. *Kalman filter-based estimation of disturbances*

Up to now, the mechanical input torque of the generator, which has been denoted by P_m, was considered to be constant or piecewise constant while it has been also considered that it is possible to measure it. Now it is assumed that the mechanical input torque varies in time, and in that case the aggregate disturbance input exerted on the generator's model is

$$T_m = -\omega_0 \frac{D}{(2J)^2} P_m + \frac{\omega_0}{2J} \dot{P}_m \qquad [12.88]$$

It is also assumed that the dynamics of the disturbance term T_m is defined by its n-th order derivative $T_m^{(n)}$. Considering now that after expressing the system's state variables as functions of the flat outputs and their derivatives, the PMSG's dynamics is given by

$$\begin{aligned} y^{(3)} &= f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u + \frac{1}{2H}\dot{P}_m \text{ or} \\ y^{(3)} &= v - \omega_0 \frac{D}{(2J)}^2 P_m + \frac{\omega_0}{2J}\dot{P}_m \text{ or} \\ y^{(3)} &= v - T_m \end{aligned} \qquad [12.89]$$

where

$$\begin{aligned} f_c(y, \dot{y}, \ddot{y}) = &\left(\frac{D}{2J^2}\right)\dot{y} - \omega_0 \frac{D}{2J}\frac{P_m}{2J} + \frac{D}{(2J)^2}\omega_0 \frac{V_s}{x'_{d\Sigma}} x_3 sin(\dot{y}) \\ &+ \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}\frac{1}{T'_d} x_3 sin(y) \\ &- \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}\frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}} V_s cos(y) sin(y) - \frac{\omega_0}{2H}\frac{V_s}{x'_{d\Sigma}} x_3 cos(y)\dot{y} \end{aligned} \qquad [12.90]$$

$$g_c(y,\dot{y},\ddot{y}) = -\frac{D}{2M}\frac{\omega_0}{T_{do}}\frac{V_s}{x'_{d\Sigma}}sin(y) \quad [12.91]$$

$$\begin{pmatrix}\dot{y}_1\\ \dot{y}_2\\ \ddot{y}_3\end{pmatrix} = \begin{pmatrix}0\;1\;0\\ 0\;0\;1\\ 0\;0\;0\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\left(v - \omega_0\frac{D}{(2J)^2}P_m + \frac{\omega_0}{2J}\dot{P}_m\right) \quad [12.92]$$

Next, the state vector of the model of equation [12.92] is extended to include as additional state variables the disturbance input T_m and its derivatives. Then, in the new state-space description, we have $z_1 = y_1$, $z_2 = y_2$, $z_3 = y_3$, $z_4 = T_m = -\omega_0\frac{D}{(2J)^2}P_m + \frac{\omega_0}{2J}\dot{P}_m$, $z_5 = \dot{T}_m$, and $z_6 = \ddot{T}_m$. Without loss of generality, the disturbance input dynamics is assumed to be described by its third-order derivative $\dot{z}_6 = T_m^{(3)}$. Using the previous definition of state variables, we have the matrix equations

$$\dot{z} = \tilde{A}{\cdot}z + \tilde{B}{\cdot}\tilde{v} \quad [12.93]$$

where the control input is

$$\tilde{v} = \left(v\; T_m^{(3)}\right)^T \quad [12.94]$$

with

$$\tilde{A} = \begin{pmatrix}0\,1\,0\,0\,0\,0\\ 0\,0\,1\,0\,0\,0\\ 0\,0\,0\,1\,0\,0\\ 0\,0\,0\,0\,1\,0\\ 0\,0\,0\,0\,0\,1\\ 0\,0\,0\,0\,0\,0\end{pmatrix}\; \tilde{B} = \begin{pmatrix}0\,0\\ 0\,0\\ 1\,0\\ 0\,0\\ 0\,0\\ 0\,1\end{pmatrix}\; \tilde{C}^T = \begin{pmatrix}1\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix} \quad [12.95]$$

where the measurable state variable is z_1. Because the dynamics of the disturbance input are taken to be unknown in the design of the associated disturbances' estimator, we have the following dynamics:

$$\dot{z} = \tilde{A}_o{\cdot}z + \tilde{B}_o{\cdot}\tilde{v} + K(C_o z - C_o\hat{z}) \quad [12.96]$$

where $K{\in}R^{6\times 1}$ is the state estimator's gain and

$$\tilde{A}_o = \begin{pmatrix}0\,1\,0\,0\,0\,0\\ 0\,0\,1\,0\,0\,0\\ 0\,0\,0\,1\,0\,0\\ 0\,0\,0\,0\,1\,0\\ 0\,0\,0\,0\,0\,1\\ 0\,0\,0\,0\,0\,0\end{pmatrix}\; \tilde{B}_o = \begin{pmatrix}0\,0\\ 0\,0\\ 1\,0\\ 0\,0\\ 0\,0\\ 0\,0\end{pmatrix}\; \tilde{C}_o^T = \begin{pmatrix}1\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix} \quad [12.97]$$

It can be confirmed that the disturbance observer model of the PMSG defined in equation [12.97] is observable. Defining the discrete-time equivalents of matrices $\tilde{A}_o$, $\tilde{B}_o$ and $\tilde{C}_o$ as $\tilde{A}_d$, $\tilde{B}_d$, and $\tilde{C}_d$, respectively, a derivative-free nonlinear Kalman filter can be designed for the aforementioned representation of the system dynamics. The associated Kalman filter-based disturbance estimator is given by [RIG 12].

Measurement update:

$$\begin{aligned} K(k) &= P^-(k)\tilde{C}_d^T[\tilde{C}_d \cdot P^-(k)\tilde{C}_d^T + R]^{-1} \\ \hat{z}(k) &= \hat{z}^-(k) + K(k)[z(k) - \tilde{C}_d\hat{x}^-(k)] \\ P(k) &= P^-(k) - K(k)\tilde{C}_d P^-(k) \end{aligned} \qquad [12.98]$$

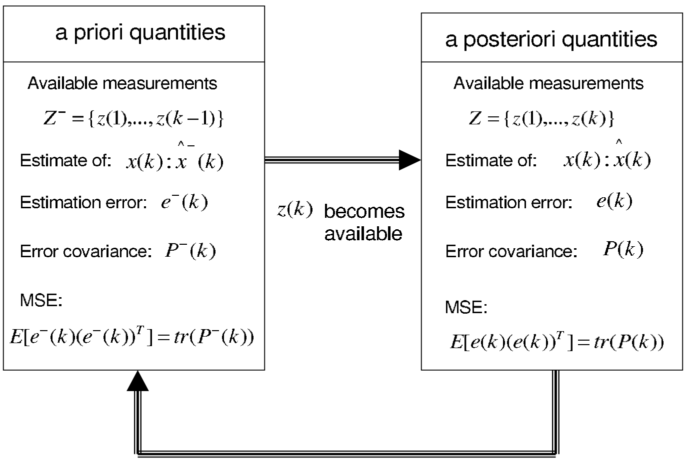


Figure 12.2. *Schematic diagram of the Kalman filter loop*

Time update:

$$\begin{aligned} P^-(k+1) &= \tilde{A}_d(k)P(k)\tilde{A}_d^T(k) + Q(k) \\ \hat{z}^-(k+1) &= \tilde{A}_d(k)\hat{z}(k) + \tilde{B}_d(k)\tilde{v}(k) \end{aligned} \qquad [12.99]$$

To compensate for the disturbance terms, it suffices to use in the control loop the modified control input, which actually removes the effects of the external disturbance variable T_m.

$$v^* = v - \hat{T}_m \quad \text{or} \quad v^* = v - \hat{z}_4 \qquad [12.100]$$

12.8. Simulation experiments

To evaluate the performance of the proposed nonlinear control scheme, which uses Kalman filtering to estimate the non-measurable state vector elements of the PMSG and the external disturbances, simulation experiments have been carried out. Different rotation speed set points had been assumed. Moreover, different input torques (mechanical input power profiles) have been considered to affect the PMSG dynamic model (SMIB model). The control loop used in the PMSG control is shown in Figure 12.3.

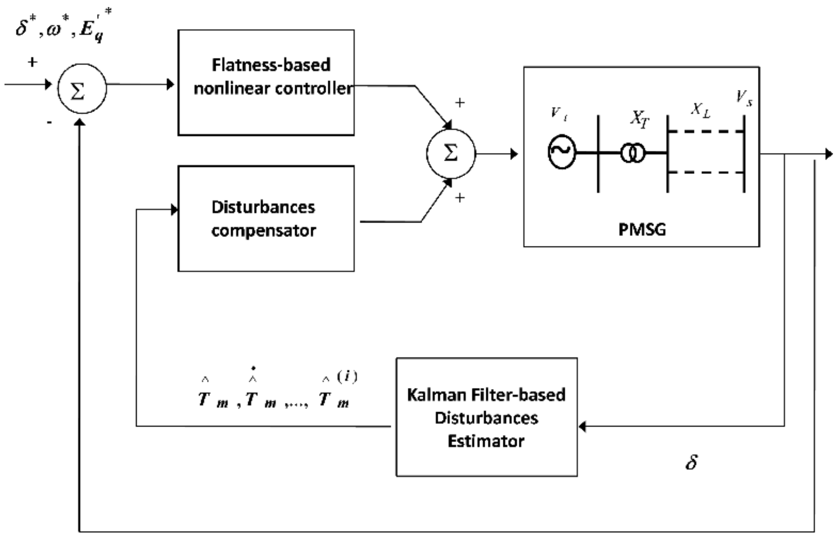
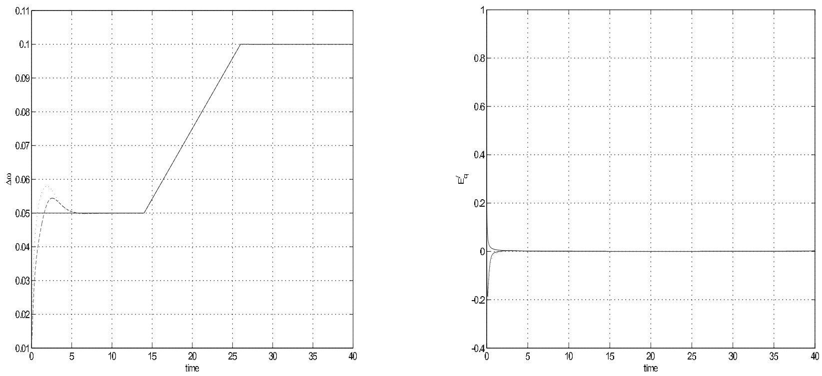


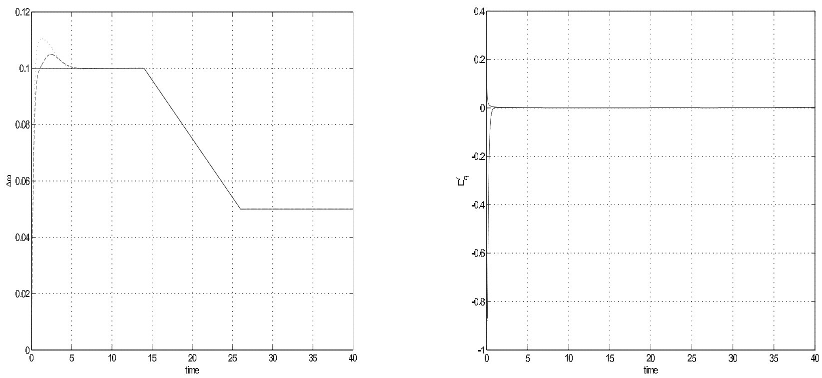
Figure 12.3. *Control loop for the PSMG comprising a flatness-based nonlinear controller and a Kalman filter-based disturbances estimator*

First, the case of measurable piecewise constant mechanical input power (torque) was examined. It was assumed that the input torque could be measured and could be directly used in the controller's design. The derivative-free nonlinear Kalman filter enabled the estimation of specific elements of the state vector, such as ω and E_q'. The associated results about the tracking performance of the control loop are shown in Figures 12.4 and 12.5. The desirable set point is denoted by a continuous line, the real value of the state variable is denoted by a dashed line and the estimated value of the state variable in shown in dotted line. The units of the PMSG state variables have been expressed in a per unit (p.u.) system. We can observe that the proposed sensorless control scheme for the PMSG succeeds fast and accurate convergence to the desirable set points.



a) **b)**

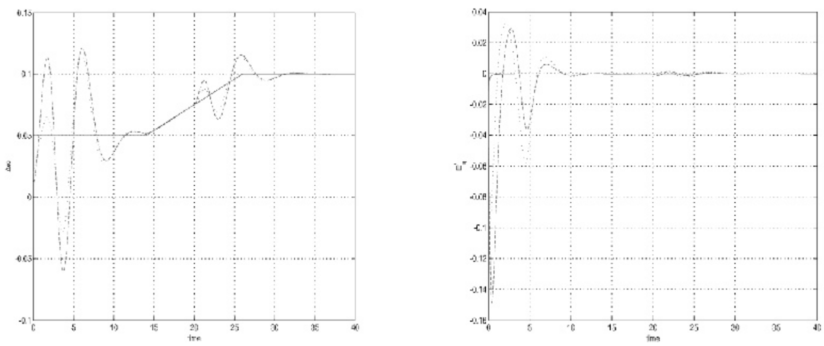
Figure 12.4. *Sensorless control of the PMSG under measurable mechanical input torque in the case of speed reference set point 1: a) convergence of the real and estimated values of the angular speed difference* $\Delta\omega$ *and b) convergence of the real and estimated values of the quadrature axis voltage* $E_q^{'}$



a) **b)**

Figure 12.5. *Sensorless control of the PMSG under measurable mechanical input torque in the case of speed reference set point 2: a) convergence of the real and estimated values of the angular speed difference* $\Delta\omega$ *and b) convergence of the real and estimated values of the quadrature axis voltage* $E_q^{'}$

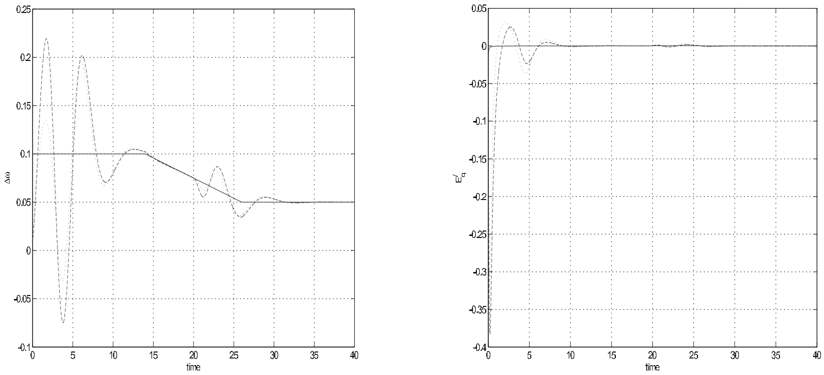
Next, the case of PMSG operation under unknown input power (torque) was examined. The input power was considered to be a disturbance input to the PMSG model and it was assumed that its change in time was defined by the third derivative of the associated variable, that is $T_m^{(3)}$, where $T_m = -\omega_0 \frac{D}{(2J)^2} P_m + \frac{\omega_0}{2J} \dot{P}_m$. The disturbance dynamics was completely unknown to the controller and its identification was performed in real time by the disturbance estimator. It is shown that the derivative-free nonlinear Kalman filter redesigned as a disturbance observer is capable of estimating simultaneously the non-measurable state variables of the generator (ω and $E_q^{'}$), as well as the unknown disturbance input T_m. A first set of results is concerned with the tracking performance of the control loop in case of piecewise constant disturbance input, as shown in Figures 12.6 and 12.7. The estimation of the piecewise constant disturbance input is shown in Figure 12.8. In addition, results about the tracking performance of the control loop in case of time-varying disturbance input are shown in Figures 12.9 and 12.10, while the estimation of the time-varying disturbance input is shown in Figure 12.11. The units of the PMSG state variables have been expressed again in the p.u. system.



a) b)

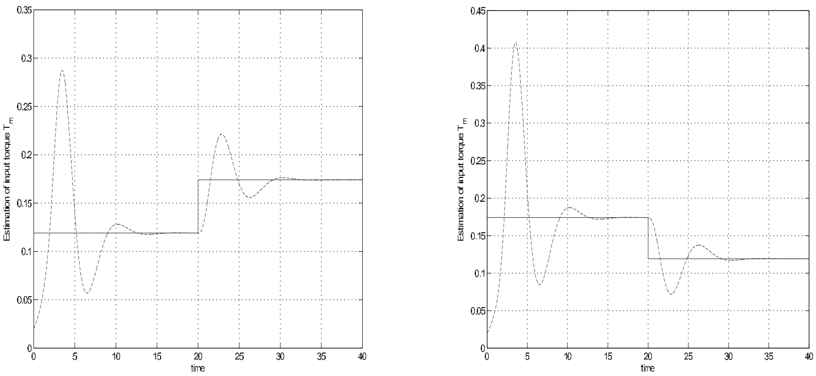
Figure 12.6. *Sensorless control of the PMSG under non-measurable (piecewise constant) mechanical input torque in the case of speed reference set point 1: a) convergence of the real and estimated values of the angular speed difference* $\Delta\omega$ *and b) convergence of the real and estimated values of the quadrature axis voltage* $E_q^{'}$

The simulation experiments have confirmed that the proposed state estimation-based control scheme not only enables implementation of PMSG control through the measurement of a small number of variables (e.g. of only the rotor's turn angle) but also improves the robustness of the PMSG control loop in the case of varying speed set points and varying mechanical input torque.



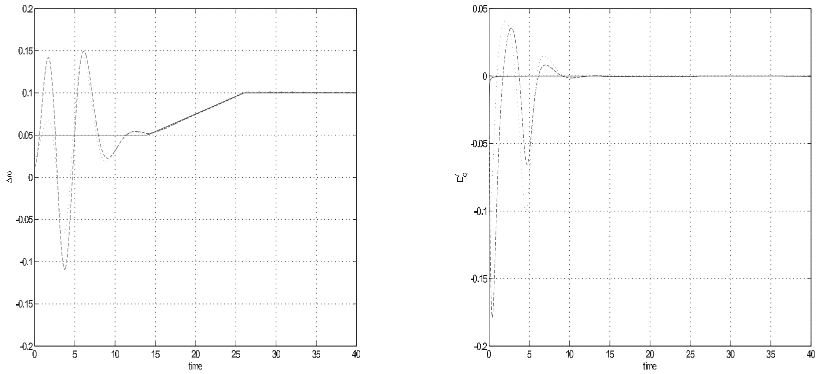
a) **b)**

Figure 12.7. *Sensorless control of the PMSG under non-measurable (piecewise constant) mechanical input torque in the case of speed reference set point 2: a) convergence of the real and estimated values of the angular speed difference* $\Delta\omega$ *and b) convergence of the real and estimated values of the quadrature axis voltage* $E_q^{'}$



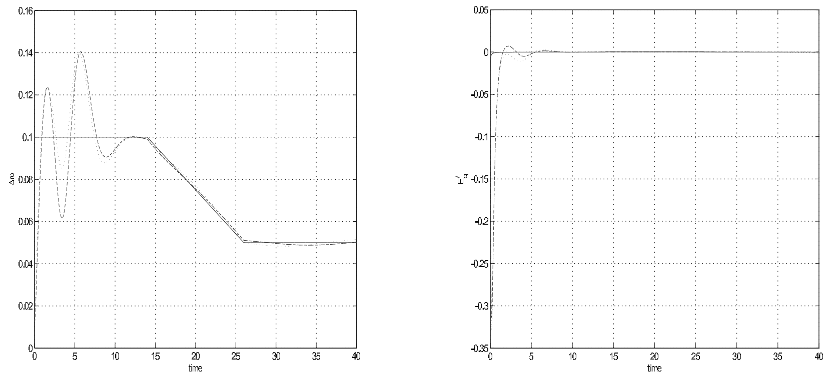
a) **b)**

Figure 12.8. *Estimation of the non-measurable (piecewise constant) input torque in sensorless control of the PMSG through the processing of rotor angle measurements: a) in the case of speed reference set point 1 and b) in the case of speed reference set point 2*



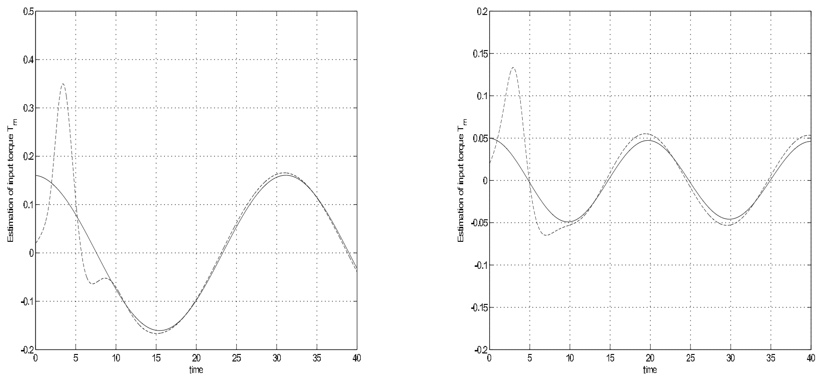
a) **b)**

Figure 12.9. *Sensorless control of the PMSG under non-measurable (time varying) mechanical input torque in the case of speed reference set point 1: a) convergence of the real and estimated values of the angular speed difference $\Delta\omega$ and b) convergence of the real and estimated values of the quadrature axis voltage $E_q^{'}$*



a) **b)**

Figure 12.10. *Sensorless control of the PMSG under non-measurable (time varying) mechanical input torque in the case of speed reference set point 2: a) convergence of the real and estimated values of the angular speed difference $\Delta\omega$ and b) convergence of the real and estimated values of the quadrature axis voltage $E_q^{'}$*



a) **b)**

Figure 12.11. *Estimation of the non-measurable (time varying) input torque in sensorless control of the PMSG: a) in the case of speed reference set point 1 and b) in the case of speed reference set point 2*

12.9. Conclusions

A new method for state estimation-based (sensorless) control of the PMSG is proposed. The method makes use of the differential flatness theory that enables us to transform the initial nonlinear model of the generator into a linear canonical (Brunovsky) form. To show that the PMSG model is a differentially flat model, the rotor's turn angle is chosen to be the flat output and it is shown that all elements of the generator's state vector and the control input can be written as functions of the flat output and its derivatives. This procedure permits us to introduce a change of state variables (diffeomorphism) and to write the initial nonlinear model of the generator into the linear canonical form. For the linearized equivalent of the system, it is possible to design a state feedback controller with the use of pole-placement methods. Unlike linearization with the use of Lie derivatives, the flatness-based linearization approach does not require the computation of partial derivatives of the elements of the state vector or the computation of Jacobian matrices.

There are two particular issues in the design of the PMSG controller: (1) there are certain elements in the generator's state vector that are not directly measurable and (2) there may be variations in the mechanical input power (torque) and it may also be impractical or costly to measure this input power. To address point 1, a new nonlinear Kalman filtering approach is introduced. The method is called derivative-free nonlinear Kalman filtering and consists of implementation of the standard Kalman filter recursion on the linearized equivalent of the PMSG model, which is obtained with the use of the differential flatness-based transformation. The fast computation features of the derivative-free nonlinear Kalman filter enable the estimation of non-measurable state vector elements in real time, while the accuracy of the provided state estimation is very satisfactory.

To address point 2, the derivative-free nonlinear Kalman filter is redesigned in the form of a disturbance estimator. The state vector of the disturbance estimator contains both the state vector elements of the linearized equivalent of the PMSG and new state vector elements that stand for the unknown mechanical input (torque) and its derivatives. Using the aforementioned disturbance observer, we can obtain estimates of both the non-measurable elements of the state vector (e.g. rotation speed and quadrature-axis transient voltage of the rotor) and estimates of the unknown disturbance input (which is due to external mechanical torque) and of its time derivatives.

To maintain the operating characteristics of the generator (e.g. rotation speed) unchanged despite variations in the input power, the initial state feedback controller of the PMSG is complemented by additional elements based on the disturbance input estimates that actually compensate for the unknown input effects. The considered disturbance estimator also makes it possible to compensate for the effects of parametric changes and modeling uncertainties in the PMSG dynamics. The efficiency of the proposed state estimation-based (sensorless control) of the PMSG is evaluated through simulation experiments.

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