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Cables and Lines

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In distribution systems the power transmission capacity is directly given by the product of the operating voltage and the maximum current that can be transmitted. The operating voltage being a fixed value, the delivery capability of the system at a given voltage depends on the conductor's capacity if carrying current.

The delivery capacity is called the ampacity of the cable system [1] and its calculation is carried out taking into consideration both steady state and transient calculations [2]. The calculations for cables in air and for buried cables are slightly different, due to the surrounding medium with which the cable has to interact. The ampacity calculations for air cables should take into account solar radiation and the amount of wind in the area in which the cable system is installed. Ampacity calculations for buried cables should consider the soil in which the cable system is installed.

Ampacity calculations require the solution of the heat transfer equations because insulation and cable size are independent parameters, inter-related by thermal considerations. Cable ampacity calculations require the determination of the temperature of the conductor for a given current loading. The ampacity rating is directly proportional to the conductor size: the larger the conductor size (lower Joule losses) the higher the ampacity. On the other hand, the insulation requirements are determined by the operating voltage and they also directly influence the ampacity value: high insulation requirements (lower heat dissipation) mean a lower ampacity. The parameters that influence the value of the ampacity are the number and types of cable, the thermal resistance of the medium surrounding the cable (soil or air), the depth of burial in the case of buried cables and the horizontal spacing between the cables of the system. The clear relationship between the conductor current and the temperature leads to a study of how the heat generated while a current is transmitted is dissipated. The resolution of the basic heat transfer equations is the first step to achieving the cable rating calculations and cable ampacity; they depend mainly on the efficiency of the dissipation process, along with the limits imposed on the insulation temperature.

Technical criteria alone are nowadays not enough to obtain the best sizing for a cable. In fact, the minimum admissible section obtained from the solution of the heat transfer equations does not take into account the cost of the losses that will be present during the cable's lifetime. The selection of the cable size should therefore take into consideration the sum of the initial cost and the cost of the losses: the cost of energy losses can be calculated by estimating load growth and the cost of energy. If the sum of the future costs of energy losses and the initial cost of purchase and installation are minimized, then the most economical size of conductor has been achieved. Using this minimization, the saving in overall cost is due to the reduction in the cost of the Joule losses compared with the increase in the cost of purchase.

2.1 Theory of Heat Transfer

An overview of the heat transfer theory is presented here because load current and conductor temperature are strictly related. The heat generated in the cable system and the rate of its dissipation must be calculated, for a given conductor material and for a given load, in order to determine the conductor temperature for a given current loading or to determine the tolerable load current for a given conductor temperature [2].

There are different mechanisms that explain how heat is transferred in different media: these three mechanisms are conduction, convection and radiation.

2.1.1 Conduction

For heat conduction, the rate equation used to express the transfer of heat between two media in contact is the equation known as Fourier's law, as in:

$$Q = -\frac{1}{\rho} A \frac{d\theta}{dx}. \quad (2.1)$$

The thermal power in the x direction is represented by Q [W] and it is directly proportional to the temperature gradient $\frac{d\theta}{dx} \left[\frac{K}{m} \right]$. This gradient represents, for a given temperature distribution, $\theta(x)$ [K], the direction and the rate at which the temperature changes. A [m²] is the area in which the thermal exchange occurs and $\rho \left[\frac{K m}{W} \right]$ is the thermal resistivity, a transport quality characteristic of the material. The minus in the equation represents the fact that heat is transferred in the direction of the decreasing temperature.

Conduction is the mechanism that acquires more importance when considering buried cables, where the conductor is in contact with other metallic parts and insulation.

2.1.2 Convection

Convection is the result of two mechanisms that work simultaneously: heat transfer conduction due to the presence of molecular motion and heat transfer due to fluid motion. The equation employed to describe convection is Newton's law:

$$Q = hA (\theta_s - \theta_{amb}). \quad (2.2)$$

Table 2.1 Range of values for the heat convection coefficient, h

Mechanism	Heat coefficient, h [W/K m ²]
Natural convection	5–25
Forced convection	
Gas	25–250
Fluids	50–20000
Boiling and condensation	2500–100 000

The convection thermal power Q [W] is proportional to the temperature difference between the surface θ_s and the ambient temperature, θ_{amb} . A [m²] is again the area of the thermal exchange surface and h $\left[\frac{W}{K m^2}\right]$ is the convection heat transfer coefficient.

Convection can be classified according to the fluid motion: forced convection and free convection. The first occurs when the flow is caused by external means: wind, pumps or fans. The second arises from density differences caused by temperature variations. Convection must be strongly taken into account for cables installed in air and the heat convection coefficient, h is the most important parameter to be calculated. Table 2.1 shows the typical range of values for the h coefficient.

2.1.3 Radiation

Energy transmission by radiation is a characteristic of all matter; it does not need a medium: radiation travels by means of electromagnetic waves, which can transmit energy in a vacuum. The thermal power emitted follows Stefan–Boltzmann’s law:

$$Q = \varepsilon A \sigma_B \theta_s^{*4} \quad (2.3)$$

The thermal power Q is directly proportional to the absolute temperature θ_s^* [K] of the surface, A [m²], σ_B is Stefan–Boltzmann’s constant $\left(\sigma_B = 5.67 \cdot 10^{-8} \left[\frac{W}{K^4 m^2}\right]\right)$ and ε is the emissivity, a radiative property of the surface. Emissivity is the efficiency of a surface to emit, compared with an ideal radiator and its range of values is $0 \leq \varepsilon \leq 1$.

If radiation is incident on a surface, a portion will be absorbed according to the surface radiative property known as absorptivity, α [W/K⁴ m²], as presented in the following equation:

$$Q_{abs} = \alpha Q_{inc}, \quad (2.4)$$

where $0 \leq \alpha \leq 1$. Cables both emit and absorb radiation. Therefore to determine the net rate the following equation is employed:

$$Q = \varepsilon A_r \sigma_B (\theta_s^{*4} - \theta_{amb}^{*4}). \quad (2.5)$$

In cable systems installed in air, convection must also be taken into account, so finally the equation that should be applied is:

$$Q = h A_c (\theta_s - \theta_{amb}) + \varepsilon A_r \sigma_B (\theta_s^{*4} - \theta_{amb}^{*4}), \quad (2.6)$$

where A_c [m] is the convection surface and A_r [m] is the radiation surface.

2.2 Current Rating of Cables Installed in Free Air

The permissible current rating of cables is calculated basically using four main values: permissible temperature rise, conductor resistance, losses and thermal resistivity. However, some quantities vary with cable design and material, so one needs to rely on an international standard. Moreover, as will be explained later, the quantities relating to the operating conditions may vary from one country to another.

Considering AC cables in air, the permissible current rating is [3]:

$$I = \left[\frac{\Delta\theta - W_d [0.5 T_1 + n (T_2 + T_3 + T_4)]}{RT_1 + nR (1 + \lambda_1) T_2 + nR (1 + \lambda_1 + \lambda_2) (T_3 + T_4)} \right]^{0.5}, \quad (2.7)$$

where $\Delta\theta$ [K] is the permissible temperature rise of a conductor above ambient temperature, W_d [W/m] represents the dielectric losses per unit length per phase, n is the number of conductors in the cable, R [Ω /m] is the alternating current resistance of the conductor at its maximum operating temperature and T_i [K m/W] represents the thermal resistance, more specifically: T_1 is the thermal resistance per core between conductor and sheath, T_2 is the thermal resistance between sheath and armour, T_3 is the thermal resistance of the external serving and T_4 is the thermal resistance of the surrounding medium.

To evaluate the losses, several quantities are considered: AC resistance, dielectric losses, sheath and screen losses, armour, reinforcement and steel pipes losses. Here only AC resistance and dielectric losses are discussed, a further discussion on sheath and screen losses, armour, reinforcement and steel pipes losses can be found in the IEC 60287-1-1 [3]. Considering its maximum operating temperature, the AC resistance per unit length of the conductor is given by:

$$R = R' (1 + y_s + y_p), \quad (2.8)$$

where R [Ω /m] is the AC current resistance of conductor at maximum operating temperature, R' is the DC resistance of conductor at maximum operating temperature, y_s is the skin effect factor and y_p the proximity effect factor. The evaluation of these quantities can be done by IEC 60287-1-1 [3]. When a cable carries alternating current, the resistance is higher than when it carries direct current, mainly because of the skin effect, proximity effect, hysteresis and eddy current losses in ferromagnetic materials and the induced losses in short-circuited non-ferromagnetic materials [2]. Usually, only the skin and proximity effects are considered, except in very high voltage cables.

The dielectric losses per unit length in each phase are given by:

$$W_d = \omega C U_0^2 \tan \delta, \quad (2.9)$$

where $\omega = 2\pi f$, C [F/m] is the capacitance per unit length and U_0 [V] is the voltage to earth. Applying alternating voltage to paper and solid insulation causes charging currents to flow because the insulation acts as a large capacitor. Each time the voltage direction changes, the electrons must be realigned, expending a certain amount of work, which will produce heat and therefore a loss in real power, the dielectric loss [2].

As can be seen by its equation, the dielectric loss is voltage dependent and Table 3 of the IEC 60287-1-1 [3] gives, for the common used insulation materials, the value of U_0 . The rest of the quantities in the equation for the dielectric losses can be also be found in the same table.

Finally, the internal and the external thermal resistance of cables are discussed. The thermal resistances per unit length of a cable, T_1 , T_2 and T_3 are formulated separately. For single-core cables, the thermal resistance, T_1 , between one conductor and its sheath is:

$$T_1 = \frac{\rho_T}{2\pi} \ln \left[1 + \frac{2t_1}{d_c} \right], \quad (2.10)$$

where ρ_T [K m/W] is the thermal resistivity of insulation, d_c [mm] is the diameter of the conductor and t_1 [mm] is the thickness of the insulation between conductor and sheath.

The thermal resistivity of the materials used for insulation can be found in Table 1 of IEC 60287-2-1 [4]. In the same part of the international standard, formulations of T_1 are given for belted cables, three-core cables, oil-filled cables, and SL and SA type cables. SL and SA cables are radial field single-core metallic sheath cables with electrostatic tape acting as the insulation screen. SL and SA refer to sheathing with lead and aluminium respectively [5].

The thermal resistance between sheath and armour for single-core, two-core and three-core cables, with a common metallic sheath, is represented by T_2 and it is formulated as follows:

$$T_2 = \frac{\rho_T}{2\pi} \ln \left[1 + \frac{2t_2}{D_s} \right], \quad (2.11)$$

where D_s [mm] is the external diameter of the sheath and t_2 [mm] is the thickness of the bedding. For SL and SA type cables, the formulation is given in IEC 60287-2-1 [3].

The thermal resistance of the outer covering, T_3 , for external servings in the form of concentric layers is given by:

$$T_3 = \frac{\rho_T}{2\pi} \ln \left[1 + \frac{2t_3}{D'_a} \right], \quad (2.12)$$

where D'_a [mm] is the external diameter of the armour and t_3 [mm] is the thickness of the serving. Corrugated sheaths and pipe-type cables thermal resistances are further analysed in IEC 60287-2-1 [4].

The evaluation of external thermal resistance T_4 changes when considering cables protected from direct solar radiation or cables directly exposed to solar radiation. For the first case, the formulation is:

$$T_4 = \frac{1}{h\pi (\Delta\theta_s)^{\frac{1}{4}} D_e^*}, \quad (2.13)$$

where D_e^* [m] is the external diameter of the cable for corrugated sheaths and h is the heat dissipation coefficient (see [4]).

The international standard also presents methods of calculating $\Delta\theta_s$, the excess of cable surface temperature above ambient temperature. For groups of cables in free air, protected from solar radiation, a method for calculating reduction factors is given in IEC 60287-2-2 [6]. The method is valid when cables are mounted adjacent to each other and it is limited to:

- a maximum of nine cables in a square formation,
- a maximum of six circuits, each comprising three cables mounted in a trefoil, with up to three circuits placed side by side or two circuits placed one above the other.

Table 2.2 Ambient temperature at sea level

Climate	Ambient air temperature		Ambient ground temperature at a depth of 1 m	
	Min °C	Max °C	Min °C	Max °C
Tropical	25	55	25	40
Subtropical	10	40	15	30
Temperate	0	25	10	20

For the second case, cables directly exposed to solar radiation, the change is in the calculation of $\Delta\theta_s$ as detailed in IEC 60287-2-1 [4].

The specific operating conditions of cables vary depending on the country: reference ambient temperature and thermal resistivities of soil may therefore present different values for each country.

IEC 60287-3-1 [7] presents standard operating conditions when values are not provided by national tables.

National values are available for Australia, Austria, Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Poland, Sweden, Switzerland, the United Kingdom and the United States of America.

Where national values are not given, the tables have to be used (Tables 2.2 and 2.3).

To complete this part, the purchaser is given a detailed list of information required by the cable manufacturer to select the appropriate type of cable. Information should be given on operating conditions and installation data.

Considering operation conditions, the information necessary to enable the selection of the appropriate type of cable is as follows:

- a) the nominal voltage of the system, U ;
- b) the highest voltage of the three-phase system, U_m ;
- c) lightning overvoltage;
- d) the system frequency;
- e) the type of earthing;
- f) environmental conditions should be provided, for example:

- the altitude above the sea level, if above 1000 m,
- indoor or outdoor installations,

Table 2.3 Thermal resistivity of soil

Thermal resistivity [K m/W]	Soil conditions	Weather conditions
0.7	Very moist	Continuously moist
1.0	Moist	Regular rainfall
2.0	Dry	Seldom rains
3.0	Very dry	Little or no rain

- excessive atmospheric pollution,
 - termination in SF₆ switchgear;
- g) the maximum rated current: for continuous operation, for cyclic operation and for emergency or overload operation, if any exists;
- h) the expected symmetrical and asymmetrical short-circuit currents that may flow in the case of short circuits, both between phases and to earth;
- j) the maximum time for which short-circuit currents may flow.

The installation data that are required can be divided into general data, underground cables and cables in air. Details on underground data can be found in [7]. For general data, the necessary information is:

- a) the length and profile of route;
- b) laying arrangements and how the metallic coverings are connected to each other and to earth;
- c) special laying conditions, for example cables in water. Individual installations require special consideration.

For cables in air the requirements are as follows:

- a) minimum, maximum and average ambient air temperature;
- b) type of installation;
- c) details of ventilation;
- d) whether exposed to direct sunlight;
- e) special conditions, for example fire risk.

2.3 Economic Aspects

Usually, the cost of losses during the lifetime of a cable is not taken into account when selecting a cable size. In fact, the selection leads to the minimum admissible cross-sectional area, minimizing the initial investment cost of the cable, but not taking into consideration its whole life-cycle. Therefore, the initial cost and the cost of the losses should be minimized (see IEC 60287-3-2 [8]) by estimating load growth and the cost of energy in order to provide the correct minimization of the sum of the future cost of energy losses and the initial cost of purchase and installation.

Purchase and installation costs are combined with the costs of energy losses and all these costs are expressed in comparable economic values, henceforth expressed as 'cu'. The date of purchase of the installation is therefore considered as the 'present' and future costs associated with costs of energy losses must be converted to their equivalent 'present values'. To do this, the 'discounting' process is applied so that the discounting rate is linked to the cost of borrowing money. The conditions and financial constraints of an individual installation influence the calculation of the present value of the cost of the losses. To obtain this value, it is indeed necessary to choose appropriate values for the future development of the load,

annual increases in kWh price and annual discounting rates over the economic lifetime of the cable.

There are two different ways to calculate the economic sizing of a cable. In the first, for a particular installation, for each of the conductor sizes, the range of economic currents has to be calculated. Afterwards, it is necessary to select the size whose economic range contains the required value of the load. The second method is more suitable for a single installation: the optimum cross-sectional area for the required load is calculated and then the closest standard conductor size is selected. Economic aspects are not the only aspects that should be considered when finding the optimum size for a conductor. There are that should be considered in this order:

1. Calculate the economic cross-sectional area.
2. Check that the calculated size can carry the maximum load expected to occur and the temperature limits are respected.
3. Check short-circuit and earth fault currents.
4. Check voltage drop limits.
5. Consider other criteria that may affect the installation.

Finally, the supply quality of the installation should also be considered.

2.4 Calculation of the Current Rating: Total Costs

The first step in finding the optimum cable size is to express the total cost of installing and operating a cable during its economic life. All costs should be expressed in present values and the equation that represents the total cost (CT):

$$CT = CI + CJ, \quad (2.14)$$

where CI is the cost of the installed length of cable and CJ is the equivalent cost at the present value of the Joule losses.

2.4.1 Evaluation of CJ

This is the cost due to Joule losses and it is composed of two parts: the energy charge and the charge of the additional supply capacity to provide for the losses.

The cost due to energy charge (CE) is obtained considering first the energy losses (EL) [Wh] for the first year, expressed as:

$$EL = (I_{\max}^2 R l N_p N_c) T, \quad (2.15)$$

where I_{\max} is the maximum load on the cable during the first year, R is the apparent AC resistance of the conductor, considering skin, proximity effects and losses in metal screens and armour, l is the length of the cable. N_p is the number of phase conductor per circuit and N_c is the number of circuits carrying the same value and type of load. T represents the number of

hours per year that the maximum current I_{\max} would need to flow in order to produce the same total yearly energy losses at the actual, variable, load current. The operating time at maximum Joule loss can be found by applying:

$$T = \int_0^{8760} \frac{I(t)^2}{I_{\max}^2} dt, \quad (2.16)$$

t being the time in hours and $I(t)$ the load current as a function of time. Finally, the cost of the first year's losses is represented by:

$$CE = (I_{\max}^2 R l N_p N_c) TP, \quad (2.17)$$

where P is the cost of one watt-hour of energy at the relevant voltage level.

The cost of additional supply capacity (CA) [u/year] is:

$$CA = (I_{\max}^2 R l N_p N_c) D, \quad (2.18)$$

where D is the demand charge per year and u is the arbitrary currency unit.

Considering CE and CA , the overall cost (OC) [cu] of the first year's losses is the sum of these two and, if the costs are paid at the end of year, the overall cost should be considered at its present value, therefore:

$$OC = \frac{(I_{\max}^2 R l N_p N_c) (TP + D)}{(1 + i/100)}. \quad (2.19)$$

The present value of energy costs [cu] during N years of operation, discounted to the date of purchase is:

$$CJ = (I_{\max}^2 R l N_p N_c) (TP + D) \cdot \frac{Q}{(1 + i/100)}, \quad (2.20)$$

where Q is a coefficient taking into account the increase in load, the increase in cost of energy over N years and the discount rate, i .

$$Q = \sum_{n=1}^N (r^{n-1})$$

$$r = \frac{(1 + a/100)^2 \cdot (1 + b/100)}{(1 + i/100)}, \quad (2.21)$$

and a and b are the increase in load per year and the increase in the cost of energy per year.

The total cost is [cu] given by the sum of CI and CJ .

$$CT = CI + I_{\max}^2 R l F [\text{cu}], \quad (2.22)$$

where F [cu/W] is expressed by

$$F = N_p N_c (TP + D) \frac{Q}{(1 + i/100)}. \quad (2.23)$$

2.5 Determination of Economic Conductor Sizes

To evaluate the economic size of a conductor, two approaches can be employed: the first one analyses the economic current range for each conductor in a series of sizes and the second one considers the economic conductor size for a given load.

2.5.1 Economic Current Range for Each Conductor in a Series of Sizes

For a given installation condition, all conductor sizes have an economic current range [A] that presents an upper and a lower limit. For a given conductor size, the upper and lower limits are:

$$I_{\text{low}}^{\text{max}} = \sqrt{\frac{CI - CI_1}{Fl(R_1 - R)}}$$

$$I_{\text{up}}^{\text{max}} = \sqrt{\frac{CI_2 - CI}{Fl(R - R_2)}}, \quad (2.24)$$

where CI is the installed cost of the length of cable whose conductor size is being considered, expressed in cu, an arbitrary currency unit. R [Ω/m] is the AC resistance per unit length of the conductor size being considered, CI_1 [cu] is the installed cost of the next smallest standard conductor, R_1 [Ω/m] is the AC resistance per unit length of the next smallest standard conductor, CI_2 [cu] is the installed cost of the next largest standard conductor and R_2 [Ω/m] is the AC resistance per unit length of the next largest standard conductor.

2.5.2 Economic Conductor Size for a Given Load

The economic conductor size is the cross-section that minimizes the total cost function [cu]:

$$CT(S) = CI(S) + I_{\text{max}}R(S)lF^2. \quad (2.25)$$

The equation for the relationship between $CI(S)$ and conductor size can be derived from known costs of standard cable sizes. The apparent conductor resistance [Ω/m] can be expressed as a function of the cross-section (see IEC 60287-1-1):

$$R(S) = \frac{\rho_{20}(1 + y_p + y_s)(1 + \lambda_1 + \lambda_2)[1 + \alpha_{20}(\theta_m - 20)]}{S} 10^6, \quad (2.26)$$

where ρ_{20} [Ωm] is the DC resistivity of the conductor, y_p is the proximity effect, see IEC 60287-1-1 [3], y_s is the skin effect, see IEC 60287-1-1, λ_1 represents the sheath loss factor, see IEC 60287-1-1, λ_2 is the armour loss factor, see IEC 60287-1-1 [3], α_{20} [K^{-1}] is the temperature coefficient of resistivity for the particular conductor material at 20°C, θ_m [$^{\circ}\text{C}$] is the conductor temperature and S [mm^2] is the cross-sectional area of the cable conductor.

If a linear model can be fitted to the values of initial cost [cu] for the type of cable and installation under consideration, then:

$$CI(S) = l(AS + C), \quad (2.27)$$

where A [cu/m, cu/mm²] is the variable component of cost, related to the conductor size, C [cu/m] is the constant component of cost, unaffected by size of cable and l [m] represents the length of cable.

The optimum size [mm] can be obtained deriving the equation of $CT(S)$:

$$S_{ec} = 1000 \left[\frac{I_{\max}^2 F \rho_{20} (1 + y_p + y_s) (1 + \lambda_1 + \lambda_2) [1 + \alpha_{20} (\theta_m - 20)]}{A} \right]^{0.5}. \quad (2.28)$$

This section will not be exactly equal to a standard size: larger and smaller standard sizes must be calculated to choose the most economical one.

2.6 Summary

Ampacity calculations are required to establish the maximum current-capacity that a cable can tolerate without risking deterioration or damage. Technical criteria are necessary to carry out the ampacity calculations and the procedures generally used for the selection of a cable size lead to the minimum cross-sectional area. The initial cost is therefore minimized but the cost of the losses that will occur during the life of the cable is not minimized. In the last decade the cost of energy in Western Countries has been increasing fast and newer insulating materials allow operation at higher temperatures than before. Instead of just minimizing the initial cost, the sum of the initial cost of losses over the economic life of the cable should be also minimized.

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