Chapter 5

# Decomposition of a Determinist Flux Observer for the Induction Machine: Cartesian and Reduced Order Structures

## 5.1. Introduction

In order to precisely control the torque of an induction actuator, its flux must also be controlled. Unfortunately, its direct measure is tricky and relatively expensive. It is rebuilt through current and/or voltage measures.

There are two methods for flux reconstruction. The first one, estimators, uses an analytical model of the machine connecting the flux and measurable variables [VER 88]. This reconstruction is sensitive to disruptions and model errors. To offset this lack of precision, we must add complex control techniques [WAN 97]. Another solution is to use closed loop models, called observers [VER 88]. The loop gain then minimizes the estimation error. We can note that there are deterministic and stochastic strengths of observers [PIE 00] (Chapter 4). The second considers measurement and calculation of noises (e.g. Kalman filter). But this advantage is offset by a very delicate determination of gains and a very calculation time-intensive implementation [DU 95].

Two specific structures of flux observers are presented in this chapter to facilitate the real-time implementation of the estimation algorithm. The first

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structure, the Cartesian observer [BOU 95a], proposes a breakdown into two sub-observers based on the axes of the two-phase reference frame. These coupled subsystems facilitate the synthesis of observation gains and lead to a simplification of discrete algorithms. The second structure, the reduced order observer [NIL 89], proposes a breakdown of the state vector into a measurable part and a non-measurable part (to estimate). The second part alone is used.

This chapter is divided into three sections. First (section 5.2), models of the induction machine are presented briefly to establish the common foundations of the two structures studied. Section 5.3 is dedicated to Cartesian observers. Following an assessment of the different possible models, a specific structure is detailed. Section 5.4 focuses on reduced order observers. After a review of the different possibilities, the most traditional structure is studied.

## 5.2. Estimation models for the induction machine

The goal of this preliminary part is to define the models of the induction machine that can be used to define Cartesian or reduced order observers for estimating its fluxes. A more detailed description of the different induction machine models is proposed in Chapter 4. We will only present here the data necessary to clarify this chapter. The hypotheses and validity ranges of the different structures studied will be deduced from the models presented.

### 5.2.1. *Park model of the induction machine*

#### 5.2.1.1. *A dynamic model for control*

Control and estimation of the induction machine require a dynamic model precise and simple enough to consider real-time implementation. The Park transformation enables us to use an equivalent orthonormal two-phase model satisfying this objective. The criterion for speed of resolution, linked to the model's simplicity, is all the more important as the use of Cartesian or reduced order observers is meant to make real-time implementation easier while ensuring good estimation robustness.

The Park transformation represents a change in reference between the true three-phase variables and those of the equivalent two-phase model. The use of an orthonormal reference for all the (stator and rotor) variables leads to certain simplifications of the model. Other simplifications are obtained by a judicious choice of the reference (see Chapter 4).

5.2.1.2. *Reference changes*

Different three-phase references must first be taken into consideration (Figure 5.1): reference ($1_S$, $2_S$, $3_S$), supporting stator windings and the reference ($1_R$, $2_R$, $3_R$), supporting those of the rotor (true or fictitious windings like those of the squirrel-cage machine). Reference ($1_R$, $2_R$, $3_R$) runs at rotation speed $\omega_{r/s}$[1] in relation to the stator. This rotation speed (often called electrical angular frequency $\omega_e$) is directly linked to that of shaft Ω through the number of pole pairs $n_p$. The angular position of the rotor reference in relation to that of stator $\theta_{r/s}$ (electrical position) is linked to the angular position of rotor $\theta$:

$$\omega_{r/s} = \omega_e = n_p\Omega = \frac{d}{dt}\theta_{r/s} = n_p \frac{d}{dt}\theta \qquad [5.1]$$

We define a rotating orthonormal reference (d, q) common to all variables used (Figure 5.1). It has an angular position (angular frequency) $\theta_{d/s}$[2] ($\omega_{d/s}$) in relation to reference ($1_S$, $2_S$, $3_S$), and an angular position (angular frequency) $\theta_{d/r}$[3] ($\omega_{d/r}$) in relation to ($1_R$, $2_R$, $3_R$). The different positions (angular frequencies) are thus connected:

$$\theta_{d/s} = \theta_{d/r} + \theta_{r/s} \text{ or } \omega_{d/s} = \omega_{d/r} + n_p\Omega \qquad [5.2]$$

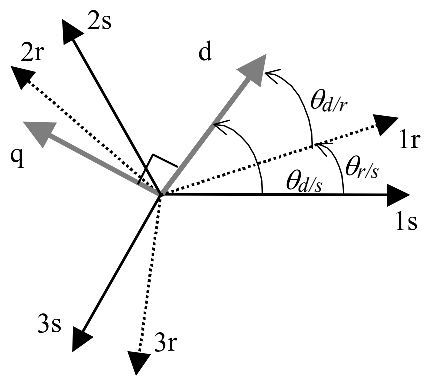


**Figure 5.1.** *Different study references*

1. index *r/s* for angular frequency (phase shift) of rotor (r) in relation to stator (s).
2. index *d/s* for angular frequency (phase shift) of axis *d* in relation to stator (s).
3. index *d/r* for angular frequency (phase shift) of axis *d* in relation to rotor (r).

Angular frequency $\omega_{d/s}$ corresponds to $\omega_s$ (angular frequency of stator voltage) in steady state sinusoidal mode only, which explains the use of different indices.

In the case of reduced and Cartesian observers, the reference linked to the stator ($\alpha_S$, $\beta_S$) is the preferred study reference. It is deducted from the general reference by canceling the phase-difference angle between axis *d* and axis $1_S$, hence:

$$\begin{cases} \omega_{d/s} = 0 \\ \omega_{d/r} = -n_p \Omega \end{cases} \qquad [5.3]$$

The Park transformation makes a reference change between an original three-phase reference and a two-phase orthonormal reference shifted by angular position $\rho$ [CHA 83].

We will only present the simplified version that does not take into consideration the homopolar component (hypothesis of balanced three-phase variables):

$$\left[P(\rho)\right] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\rho & \cos\left(\rho - \frac{2\pi}{3}\right) & \cos\left(\rho + \frac{2\pi}{3}\right) \\ -\sin\rho & -\sin\left(\rho - \frac{2\pi}{3}\right) & -\sin\left(\rho + \frac{2\pi}{3}\right) \end{bmatrix} \qquad [5.4]$$

Two Park transformations are then necessary to define an equivalent model in a single reference (d, q), one in relation to the stator three-phase reference and the other in relation to the rotor three-phase reference (two different variable angles):

$$\underline{x}_{dq} = \left[P\left(\theta_{d/s}\right)\right] \underline{x}_{1s2s3s} \qquad \underline{x}_{dq} = \left[P\left(\theta_{d/r}\right)\right] \underline{x}_{1r2r3r} \qquad [5.5]$$

in reference ($\alpha_S$, $\beta_S$) $\theta_{d/s} = 0$.

#### 5.2.1.3. *The model in reference $(\alpha_S, \beta_S)$*

The dynamic model in any reference (d, q) is obtained by the two appropriate transformations [5.5] and described in Chapter 4.

The model in reference ($\alpha_S$, $\beta_S$) is obtained by using the angles of transformation from [5.3].

The first equation system involves stator voltage, $V_s$ (rotor $V_r$), stator current, $I_s$ (rotor $I_r$), and stator flux, $\Phi_s$ (rotor $\Phi_r$), through the stator resistance, $R_s$ (rotor $R_r$):

$$\begin{cases} v_{s\alpha} = R_s i_{s\alpha} + \dfrac{d}{dt}\Phi_{s\alpha} \\ v_{s\beta} = R_s i_{s\beta} + \dfrac{d}{dt}\Phi_{s\beta} \\ v_{r\alpha} = R_r i_{r\alpha} + \dfrac{d}{dt}\Phi_{r\alpha} + n_p\Omega\Phi_{r\beta} \\ v_{r\beta} = R_r i_{r\beta} + \dfrac{d}{dt}\Phi_{r\beta} - n_p\Omega\Phi_{r\alpha} \end{cases} \qquad [5.6]$$

The second equation system expresses the creation of fluxes according to the different currents through stator $L_S$, rotor $L_r$, cyclic inductances and mutual cyclic inductance between the stator and rotor $M_{sr}$:

$$\begin{cases} \Phi_{s\alpha} = L_s i_{s\alpha} + M_{sr} i_{r\alpha} \\ \Phi_{s\beta} = L_s i_{s\beta} + M_{sr} i_{r\beta} \\ \Phi_{r\alpha} = M_{sr} i_{s\alpha} + L_r i_{r\alpha} \\ \Phi_{r\beta} = M_{sr} i_{s\beta} + L_r i_{r\beta} \end{cases} \qquad [5.7]$$

### 5.2.2. *State models for Cartesian and reduced observers*

#### 5.2.2.1. *State representation of an induction machine*

The different state representations of the induction machine are presented in Chapter 4. In the hypothesis of mechanical and electromagnetic mode separation, the induction machine is a four-order state model.

For a squirrel-cage induction machine, the power is provided through stator voltage corresponding to control vector $\underline{U}$. Output vector $\underline{Y}$ is made up of stator current. The state representation is thus defined with several possibilities in terms of the choice of state vector $\underline{X}$, which will set the dynamic [$A$], control [$B$], and observation [$C$] matrices:

$$\begin{cases} \dfrac{d}{dt}\underline{X} = [A]\underline{X} + [B]\underline{U} \\ \underline{Y} = [C]\underline{X} \end{cases} \qquad [5.8]$$

with:

$$\underline{U} = \underline{V}_{s\alpha\beta} = \begin{bmatrix} V_{s\alpha} & V_{s\beta} \end{bmatrix}^T \quad \underline{Y} = \underline{I}_{s\alpha\beta} = \begin{bmatrix} I_{s\alpha} & I_{s\beta} \end{bmatrix}^T \qquad [5.9]$$

In the context of flux estimation for a squirrel-cage induction machine, the state vectors with rotor current (non-accessible) or those with no flux are not used: only three solutions are appropriate.

### 5.2.2.2. *State vector made up of stator currents and fluxes*

The state vector is made up of axes $\alpha$ and $\beta$ components of the stator current and flux:

$$\underline{X} = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \Phi_{s\alpha} & \Phi_{s\beta} \end{bmatrix}^T \tag{5.10}$$

The matrices of state representation come from this choice:

$$[A] = \begin{bmatrix} -(aR_s + bR_r) & -n_p\Omega & aR_r / L_r & -an_p\Omega \\ n_p\Omega & -(aR_s + bR_r) & an_p\Omega & aR_r / L_r \\ -R_s & 0 & 0 & 0 \\ 0 & -R_s & 0 & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} a & 0 \\ 0 & a \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } [C] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{5.11}$$

$$a = \frac{1}{\sigma L_s} \quad b = \frac{1}{\sigma L_r} \quad c = \frac{1-\sigma}{\sigma M_{sr}} \quad \sigma = 1 - \frac{M_{sr}^2}{L_s L_r} \tag{5.12}$$

This representation is used to estimate the stator flux. We can also note that the output vector corresponds to a part of the state vector: this property is relatively important in terms of the precision and robustness of the different observers studied (see section 5.4).

### 5.2.2.3. *State vector made up of stator currents and rotor fluxes*

The state vector is made up of stator currents and rotor fluxes:

$$\underline{X} = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \Phi_{r\alpha} & \Phi_{r\beta} \end{bmatrix}^T \tag{5.13}$$

New state matrices are obtained:

$$[A]=\begin{bmatrix} -aR_s-(1-\sigma)bR_r & 0 & cR_r/L_r & cn_p\Omega \\ 0 & -aR_s-(1-\sigma)bR_r & -cn_p\Omega & cR_r/L_r \\ R_rM_{sr}/L_r & 0 & -R_r/L_r & -n_p\Omega \\ 0 & R_rM_{sr}/L_r & n_p\Omega & -R_r/L_r \end{bmatrix}$$

$$[B]=\begin{bmatrix} a & 0 \\ 0 & a \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } [C]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad [5.14]$$

This representation also helps in the estimation of the rotor flux. This state model has the same properties as the previous model [5.11] and can therefore lead to a rotor flux reduced observer and good robustness of observers (output vector corresponding to a part of the state vector).

5.2.2.4. *State vector made up of stator fluxes and rotor fluxes*

The state vector is made up of stator and rotor fluxes:

$$\underline{X}=\begin{bmatrix} \Phi_{s\alpha} & \Phi_{s\beta} & \Phi_{r\alpha} & \Phi_{r\beta} \end{bmatrix}^T \qquad [5.15]$$

New state matrices are obtained:

$$[A]=\begin{bmatrix} -aR_s & 0 & cR_s & 0 \\ 0 & -aR_s & 0 & cR_s \\ cR_r & 0 & -bR_r & -n_p\Omega \\ 0 & cR_r & n_p\Omega & bR_r \end{bmatrix}$$

$$[B]=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } [C]=\begin{bmatrix} a & 0 & -c & 0 \\ 0 & a & 0 & -c \end{bmatrix} \qquad [5.16]$$

This representation helps in the estimation of these two fluxes. Even though this model may seem more universal than the previous ones, it cannot result in reduced order observers (no variables can be easily measured in the state vector). In addition, the convergence of the output vector in the case of an observer (difference between the true output and estimated output) is not ensured because the state vector, matrix [C], contains parameters that are sensitive to robustness problems.

### 5.2.3. *Determination of the flux in the reference used by the control*

#### 5.2.3.1. *Different study references*

We show that reference ($\alpha_S$, $\beta_S$) (*d* aligned with a stator phase) is the reference providing the closest dynamic behavior to the real squirrel-cage machine [PIE 88, VUL 98] because it uses a linear transformation for stator variables (zero-angle Park transformation called Concordia transformation).

However, control reference (d, q) is generally different, and it is most often linked to the flux involved. In relation to the control reference, the estimation also gives the Cartesian components of this flux, and we can easily calculate its polar components:

$$\left|\underline{\Phi}\right| = \sqrt{\Phi_d^2 + \Phi_q^2} \qquad \theta_\Phi = \arg\Phi = \mathrm{actg}\left(\Phi_q / \Phi_d\right) \qquad [5.17]$$

#### 5.2.3.2. *Flux estimators*

The notion of flux estimator is associated with a copy of a model (or part of a model) of the induction machine in order to rebuild one (or more) flux with open loop from accessible dimensions (stator voltage and current, and speed or position for the squirrel-cage machine).

In general, a single flux is used in the most common dynamic controls of induction machines. In this case, a single flux must be rebuilt and the use of a part of the state model is sufficient if we choose a state vector containing stator current. We can thus speak of reduced order estimator.

For example, to estimate the rotor flux, reference (d, q) linked to the rotor flux offers the simplest model [LEO 91]. The rotor equations alone are used in [5.5]:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{rd}} = -\dfrac{R_{\mathrm{r}}}{L_{\mathrm{r}}}\Phi_{\mathrm{rd}} + \dfrac{M_{\mathrm{sr}}R_{\mathrm{r}}}{L_{\mathrm{r}}}i_{\mathrm{sd}} \\ \omega_{\mathrm{d/r}} = \dfrac{M_{\mathrm{sr}}R_{\mathrm{r}}}{\Phi_{\mathrm{rd}}L_{\mathrm{r}}}i_{\mathrm{sd}} \end{cases} \qquad [5.18]$$

The angle of transformation of measured stator current is deducted from the integration of angular frequency $\omega_{\mathrm{d/s}}$, which comes from $\omega_{\mathrm{d/r}}$ and rotation speed [5.2].

5.2.3.3. *Flux observers*

Different induction machine models can be used for the estimation of flux. Some require more hypotheses in addition to the Park transformation (mode decoupling, etc.), decreasing their range of validity. The choice of an estimation model depends on the flux considered and accessible variables available on the machine. Most estimators correspond to the use of part of a model chosen. We can thus speak of reduced order estimators. But we must note that these open loop structures raise problems of robustness which are often tricky for induction machine dynamic controls based on a good understanding of one of the fluxes.

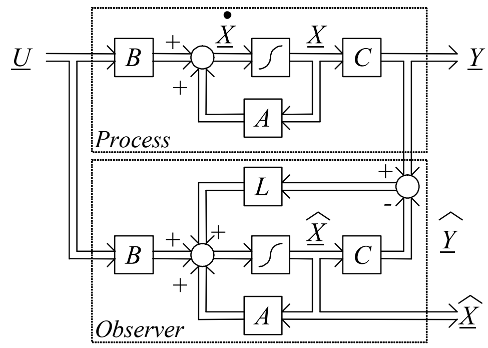


**Figure 5.2.** *Structure of a deterministic state observer*

Flux observers [VER 88] correspond to closed loop estimators (Figure 5.2). They offer better precision and robustness to estimate fluxes. On the other hand, they require a lot of calculating time as compared to estimators. In order to reduce this disadvantage, Cartesian observers and reduced order observers are intermediate solutions.

## 5.3. Cartesian observers

This section is dedicated to Cartesian observers which are simply a specific mode of representation of complete order observers for the induction machine. First, the structure of these flux rebuilders is presented, as well as the different models that can be used. The study of a specific Cartesian observer is then detailed to illustrate more concretely these rarely used structures.

### 5.3.1. *Principle and structure of Cartesian observers*

#### 5.3.1.1. *Breakdown of a complete observer*

A complete observer is based on an four-order state machine model (see section 4.2 of Chapter 4). We can break it down into two order-two sub-observers according to each axis of the study reference (Cartesian components). These sub-observers are then coupled by coupling matrices $[K_\alpha]$ and $[K_\beta]$ (Figure 5.3).

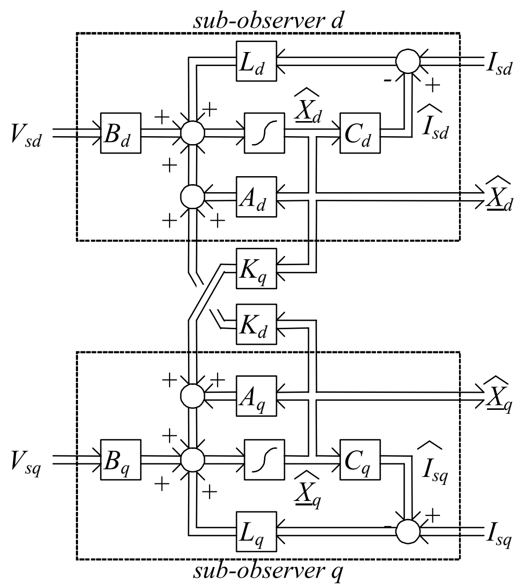


**Figure 5.3.** *Structure of a Cartesian observer*

5.3.1.2. *Coupled sub-observers*

The state vector is broken down into two state vectors and each one only contains the variables of the same axis. Each sub-observer then has one of the Cartesian components of the desired flux. Control input of axis $\alpha$ sub-observer (and of $\beta$) is voltage $V_{s\alpha}$ ($V_{r\beta}$) and its output is $i_{s\alpha}$ ($i_{r\beta}$), respectively. On the other hand, the different state vectors lead to several possibilities for the two state sub-vectors with two dimensions.

The equations of each sub-observer reveal additional input in relation to the traditional observer:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_\alpha = \left[A_\alpha\right]\underline{\hat{X}}_\alpha + \underline{B}_\alpha v_{s\alpha} + \underline{L}_\alpha(i_{s\alpha} - \hat{i}_{s\alpha}) + \left[K_\alpha\right]\underline{\hat{X}}_\beta \\ \hat{I}_{s\alpha} = \underline{C}_\alpha^T\underline{\hat{X}}_\alpha \end{cases}$$
$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_\beta = \left[A_\beta\right]\underline{\hat{X}}_\beta + \underline{B}_\beta v_{s\beta} + \underline{L}_\beta(i_{s\beta} - \hat{i}_{s\beta}) + \left[K_\beta\right]\underline{\hat{X}}_\alpha \\ \hat{I}_{s\beta} = \underline{C}_\beta^T\underline{\hat{X}}_\beta \end{cases} \qquad [5.19]$$

These additional terms, $[K_\alpha]\ \underline{X}_\alpha$ and $[K_\beta]\ \underline{X}_\beta$, correspond to couplings between the two axes (thus between the two sub-observers). The two gain matrices $\underline{L}_\alpha$ and $\underline{L}_\beta$ have to be determined to ensure good dynamics and precision of sub-observers.

We can make the connection between this Cartesian representation and the representation in a complex form sometimes used [VER 88]. The latter is based on a two-dimension state vector with components that are complex variables associated with reference ($\alpha_S$, $\beta_S$):

$$\underline{X} = \left[\left(x_{1\alpha} + jx_{1\beta}\right) \quad \left(x_{2\alpha} + jx_{2\beta}\right)\right]^T \qquad [5.20]$$

We then obtain an order two equivalent complex model representing the real order four global models. However, variables are not grouped by axis, and cannot induce coupling matrices at the basis of the properties of the Cartesian observer. The synthesis and discretization methods turn out to be very different [BOC 91].

5.3.1.3. *Characteristics of Cartesian observers*

In the continuous field, the two sub-observers can be considered as independent. Because of this, their synthesis becomes simplified (e.g. pole placement for a second order system). We can observe that the discretization of this type of algorithm will require additional hypotheses linked to coupling input (see example in section 5.3.4).

The real-time implementation is thus made easier because it must then solve two two-order state systems instead of a single four-order system (see the example discussed).

### 5.3.2. *Different Cartesian observers*

The three state vectors considered for the flux estimation can lead to Cartesian observers for a squirrel-cage induction machine. Three study references can be used first: reference ($\alpha_S$, $\beta_S$) linked to the stator (direct access to stator current), the reference linked to the rotor flux or the reference linked to the stator flux (direct estimation of their polar coordinates). Reference ($\alpha_S$, $\beta_S$) is the most widely used because it is based on the prior knowledge of the angle. We will only limit ourselves to this case.

#### 5.3.2.1. *Cartesian observer associated with the stator current and stator flux*

This observer is based on model [5.11] where the state vector has the components of axis $\alpha_S$ and $\beta_S$ of the stator currents and stator fluxes [5.10] in reference to reference frame ($\alpha_S$, $\beta_S$):

$$\underline{X}_\alpha = \begin{bmatrix} i_{s\alpha} & \Phi_{s\alpha} \end{bmatrix}^T \qquad \underline{X}_\beta = \begin{bmatrix} i_{s\beta} & \Phi_{s\beta} \end{bmatrix}^T \tag{5.21}$$

The new matrices of this specific representation are deducted from global representation [5.11]:

$$\begin{bmatrix} A_\alpha \end{bmatrix} = \begin{bmatrix} A_\beta \end{bmatrix} = \begin{bmatrix} -(aR_s + bR_r) & aR_r / L_r \\ -R_s & 0 \end{bmatrix} \quad \begin{bmatrix} K_\alpha \end{bmatrix} = -\begin{bmatrix} K_\beta \end{bmatrix} = \begin{bmatrix} -n_p\Omega & -an_p\Omega \\ 0 & 0 \end{bmatrix}$$

$$\underline{B}_\alpha = \underline{B}_\beta = \begin{bmatrix} a \\ 1 \end{bmatrix} \text{and } \underline{C}_\alpha = \underline{C}_\beta = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5.22}$$

We can notice the symmetry of both axes $\alpha$ and $\beta$ sub-models. This property can reveal equality between the two observation $\underline{L}_\alpha = \underline{L}_\beta$ gain matrices. On the other hand, the new dynamic matrices [$A_\alpha$] and [$A_\beta$] are stationary (no term linked to speed). The non-stationarities were reported in coupling matrices [$K_\alpha$] and [$K_\beta$].

This Cartesian observer can help in a robust estimation of stator flux for controls based on flux control. This structure has actually been used for DTC control of an induction machine requiring very good knowledge of the stator flux [BEL 00].

5.3.2.2. *Cartesian observer associated with the stator current and rotor flux*

This observer is based on state model [5.14] where the state vector is made up of the stator currents and rotor fluxes [5.13] in reference ($\alpha_S$, $\beta_S$):

$$\underline{X}_\alpha = \begin{bmatrix} i_{s\alpha} & \Phi_{r\alpha} \end{bmatrix}^T \quad \underline{X}_\beta = \begin{bmatrix} i_{s\beta} & \Phi_{r\beta} \end{bmatrix}^T \tag{5.23}$$

The new matrices of this representation are deducted from global representation [5.14] considering the simplifications caused by reference ($\alpha_S$, $\beta_S$):

$$\left[A_\alpha\right] = \left[A_\beta\right] = \begin{bmatrix} -aR_s - (1-\sigma)bR_r & cR_r / L_r \\ R_r M_{sr} / L_r & -R_r / L_r \end{bmatrix} \quad \left[K_\alpha\right] = -\left[K_\beta\right] = \begin{bmatrix} 0 & cn_p\Omega \\ 0 & -n_p\Omega \end{bmatrix}$$

$$\underline{B}_\alpha = \underline{B}_\beta = \begin{bmatrix} a \\ 0 \end{bmatrix} \text{and } \underline{C}_\alpha = \underline{C}_\beta = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5.24}$$

We observe the same properties of symmetry and reporting of non-stationarities as the previous model [5.22]. This observer can help in a robust estimation of rotor flux for controls based on flux control.

5.3.2.3. *Cartesian observer associated with the stator flux and rotor flux*

This observer is based on state model [5.16] where the state vector is made up of the stator currents and rotor fluxes [5.15] in reference ($\alpha_S$, $\beta_S$):

$$\underline{X}_\alpha = \underline{\Phi}_\alpha = \begin{bmatrix} \Phi_{s\alpha} & \Phi_{r\alpha} \end{bmatrix}^T \qquad \underline{X}_\beta = \underline{\Phi}_\beta = \begin{bmatrix} \Phi_{s\beta} & \Phi_{r\beta} \end{bmatrix}^T \tag{5.25}$$

The new matrices of this representation are deducted from the global representation [5.16] considering the simplifications caused by reference ($\alpha_S$, $\beta_S$):

$$\left[A_\alpha\right] = \left[A_\beta\right] = \begin{bmatrix} -aR_s & cR_s \\ \mathrm{c}R_r & -bR_r \end{bmatrix} [K_\alpha - \left[K_\beta\right] = \begin{bmatrix} 0 & 0 \\ 0 & -n_p\Omega \end{bmatrix}$$

$$\underline{B}_\alpha = \underline{B}_\beta = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{and } \underline{C}_\alpha = \underline{C}_\beta = \begin{bmatrix} a & -c \end{bmatrix} \tag{5.26}$$

We observe the same properties of symmetry and reporting of non-stationarities as the previous models [5.22] and [5.24].

This Cartesian observer can help in a robust estimation of stator and rotor fluxes for controls based on the control of one of those fluxes. This structure was actually implemented for a Cartesian vector control of stator flux in reference ($\alpha_S$, $\beta_S$) [BOU 95a].

Even though this observer seems to be more universal than the first two (reconstruction of both fluxes), we must realize that observation matrices $\underline{C}_\alpha$ and $\underline{C}_\beta$ contain parameters sensitive to robustness problems. Because of this, the convergence of currents estimated with true currents does not condition the convergence of fluxes. It is, however, clearly improved in relation to that of an estimator type structure [BOU 99].

### 5.3.3. *Synthesis of the Cartesian observer linked to stator and rotor fluxes*

In order to present a complete study of Cartesian observer, we chose the structure with a state vector that contains the stator and rotor fluxes in reference ($\alpha_S$, $\beta_S$).

#### 5.3.3.1. *Observability*

The observability of this structure by the Kalman criterion leads to writing the four line determinant of the matrix of observability because the system is of order four [BUH 88]. We choose to study the determinant of the first four lines:

$$\Delta = c^4 (p\Omega)^2 R_s R_r [a^2 (n_p \Omega)^2 + (ab - c^2)^2 R_r^2] \tag{5.27}$$

It demonstrates the inobservabilty of this zero speed system (zero determinant) and rigorous observability for any other operation point (constant $ab - c^2$, image of magnetic leaks, strictly positive for an induction machine).

This result is consistent because, at zero speed, coupling disappears (see [5.26]): the two sub-observers are then completely independent (and we demonstrate their rigorous observability for this operation point [BOU 95a]). It thus seems that the system can be observed locally: $\underline{\Phi}_\alpha$ is observable through $I_{s\alpha}$ and $\underline{\Phi}_\beta$ through $I_{s\beta}$.

#### 5.3.3.2. *Calculation of observer gains*

In order to conserve the symmetry properties of model [5.18] for the Cartesian observer, gain matrices are chosen identical:

$$\underline{L}_\alpha = \underline{L}_\beta = \begin{bmatrix} l_1 & l_2 \end{bmatrix}^T \tag{5.28}$$

The estimation errors of the two sub-observers are deducted from the system's state model [5.16] and from the Cartesian observer's state model [5.28]:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\varepsilon}_{\varphi\alpha} = \left[A_\alpha - L_\alpha C_\alpha\right]\underline{\varepsilon}_{\varphi\alpha} + \left[K_\alpha\right]\underline{\varepsilon}_{\varphi\beta} \\ \dfrac{d}{dt}\underline{\varepsilon}_{\varphi\beta} = \left[A_\beta - L_\beta C_\beta\right]\underline{\varepsilon}_{\varphi\beta} + \left[K_\beta\right]\underline{\varepsilon}_{\varphi\alpha} \end{cases} \qquad [5.29]$$

By imposing $\underline{L}_\alpha = \underline{L}_\beta$, we impose the dynamics of each estimation error (action on the equivalent dynamic matrix $[A_\mathrm{i} - L_\mathrm{i}C_\mathrm{i}]$). The system associated with the error of observation is made up of two coupled sub-systems (Figure 5.4). We can express the transfer function of the system by:

$$\varepsilon_{\varphi\alpha}(p) = \left[sI - A_\alpha + L_\alpha C\alpha\right]^{-1}\left[K_\alpha\right]\left[sI - A_\beta + L_\beta C_\beta\right]^{-1}\left[K_\beta\right]\varepsilon_{\varphi\alpha}(p) \qquad [5.30]$$

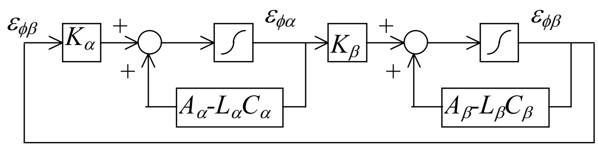


**Figure 5.4.** *Block diagram of the error of observation*

We then deduce the characteristic equation of the estimation error, which will set the convergence dynamics of estimated values with the true values:

$$\left[s^2 + s\left(x + jn_\mathrm{p}\Omega\right) + \left(k + jn_\mathrm{p}\Omega\right)z\right]\left[s^2 + s\left(x - jn_\mathrm{p}\Omega\right) + \left(k - jn_\mathrm{p}\Omega\right)z\right] = 0$$

with:

$$x = a(R_\mathrm{s} + l_1) + bR_\mathrm{r} - cl_2, \quad z = a(R_\mathrm{s} + l_1), \quad k = \frac{ab - c^2}{a}R_\mathrm{s} \qquad [5.31]$$

We obtain two combined complex pole pairs. A complex study helps us choose a judicious placement of the four poles (Figure 5.5) [BOU 95a].

We align the poles with the same real part to avoid operating in the unstable zone (combined complex double poles), and we set the converge dynamics chosen by the real associated part. This strategy leads to the following gains:

$$\begin{cases} l_1 = \dfrac{k + \sqrt{k^2 + (n_\mathrm{p}\Omega)^2}}{2a} - R_\mathrm{s} \\ l_2 = \dfrac{bR_\mathrm{r} - a(R_\mathrm{s} + l_1)}{c} \end{cases} \tag{5.32}$$

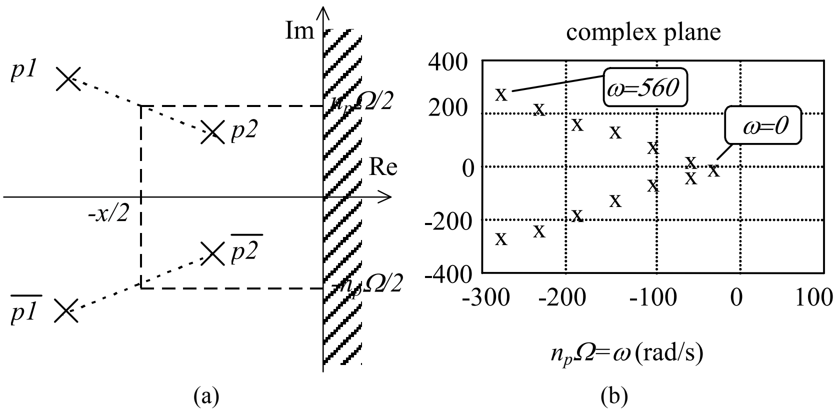


**Figure 5.5.** *Cartesian observer pole: (a) general case; (b) choice of gains used*

This choice leads to a stable solution and a sufficiently quick convergence of the estimation error in the range of speed used (Figure 5.5).

We notice that a different gain calculation is necessary for zero speed because of the completely decoupled structures in this case. In practice, following sensor problems, we operate in estimator mode for low speeds (with empirical determination of commutation velocity) [BOU 95a].

5.3.3.3. *Equivalence with a complete order observer*

The equations of the Cartesian observer [5.19] can be rewritten with a four-order state vector:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}} = [A']\underline{\hat{X}} + [B]\underline{V}_\mathrm{s} + [L](\underline{I}_\mathrm{s} - \underline{\hat{I}}_\mathrm{s}) + [K]\underline{\hat{X}} \\ \underline{\hat{I}}_\mathrm{s} = [C]\underline{\hat{X}} \end{cases} \tag{5.33}$$

$$[A'] = \begin{bmatrix} -aR_s & cR_s & 0 & 0 \\ cR_r & -bR_r & 0 & 0 \\ 0 & 0 & -aR_s & cR_s \\ 0 & 0 & cR_r & -bR_r \end{bmatrix} \quad [K] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -n_p\mathrm{W} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -n_p\Omega \end{bmatrix}$$

$$[L] = \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \\ 0 & l_1 \\ 0 & l_2 \end{bmatrix} \qquad [5.34]$$

This new format demonstrates that the Cartesian observer is actually a complete order observer with a specific gain matrix. The use of the Cartesian representation can be seen as a simple way to achieve the synthesis of a complete order observer.

### 5.3.4. *Discretization of the Cartesian observer linked to stator and rotor fluxes*

The properties of the Cartesian observer will be taken advantage of during its discretization for real-time implementation.

#### 5.3.4.1. *Discretization of a complete order observer*

A discrete state model must be used for real-time implementation. It is deducted from the continuous state equation [5.8] using the hypothesis that input vector $\underline{U}$ is constant between two sampling periods [BUH 86]. We then obtain the equation at the differences between moments $kT_{\text{ech}}$ and $(k + 1)\, T_{\text{ech}}$:

$$\begin{cases} \underline{X}_{k+1} = [F]\underline{\hat{X}}_k + [H]\underline{U}_k \\ \underline{Y}_k = [C]\underline{X}_k \end{cases} \qquad [5.35]$$

$$[F] = \exp\left([A]T_{\text{ech}}\right) \quad [\mathrm{H}] = [A]^{-1}[F - I][B] \qquad [5.36]$$

This discretization is applied to the complete order observer considering that its dynamic matrix is $[A–LC]$ and that there are two input vectors $\underline{V}_s$ and $\underline{I}_s$:

$$\begin{cases} \underline{\hat{X}}_{k+1} = [F]\underline{\hat{X}}_k + [H_v]\underline{V}_{s_k} + [H_i]\underline{I}_{s_k} \\ \hat{I}_{s_k} = [C]\underline{\hat{X}}_k \end{cases} \qquad [5.37]$$

Beyond the state transition matrix [$F$], two discrete controls must be calculated: [$H_{\text{v}}$] in relation to voltage and [$H_{\text{i}}$] in relation to the current, which is a return variable. Because of the non-stationarity of matrix [$A$] and the velocity dependence of observation gains [$L$] [5.32], the three discrete matrices will need to be calculated in real-time (with traditional approximations), or discretized by range (determination of velocity ranges for which the matrices are almost stationary) [BOU 96]. Real-time implementation is then relatively intensive in calculation time.

5.3.4.2. *Cartesian observer discretization*

In order to decrease calculation time, we use the properties of the Cartesian observer by discretizing its characteristic equation in its initial form [5.19] (and by using the error of observation as input[4]) [BOU 96]:

$$\begin{cases} \underline{\hat{X}}_{\alpha k+1} = [F_\alpha]\underline{\hat{X}}_{\alpha k} + [H_{\alpha\text{v}}]\underline{V}_{\text{s}\alpha k} + [H_{\alpha\text{e}}]\varepsilon_{\text{s}\alpha k} + [H_{\alpha x}]\underline{\hat{X}}_{\beta_k} \\ \hat{I}_{\text{s}\alpha k} = \underline{C}_\alpha \underline{\hat{X}}_{\alpha k} \end{cases}$$
$$\begin{cases} \underline{\hat{X}}_{\beta_{k+1}} = [F_\beta]\underline{\hat{X}}_{\beta_k} + [H_{\beta\text{v}}]\underline{V}_{\text{s}\beta_k} + [H_{\beta\text{e}}]\varepsilon_{\text{s}\beta_k} + [H_{\beta x}]\underline{\hat{X}}_{\beta_k} \\ \hat{I}_{\text{s}\beta_k} = \underline{C}_\beta \underline{\hat{X}}_{\beta_k} \end{cases} \qquad [5.38]$$

$$\begin{aligned} &[F_i] = \exp([A_i]T_{\text{ech}}) \quad [\text{Z}_\text{i}] = [A_i]^{-1}[F_i - I] \\ &[\text{H}_{iv}] = [Z_i]\,[B_i] \quad [\text{H}_{ie}] = [Z_i]\,[L_i] \quad [\text{H}_{ix}] = [Z_i]\,[K_i] \end{aligned} \qquad [5.39]$$

We must then note that the quasi-stationarity hypotheses between two sampling periods of input sampling must also be applied to the current error and to the two state vectors. This structure can therefore only be used for sampling periods adapted to time constants of the system.

The Cartesian form first enables the discretization of two-order systems. In addition, as dynamic matrices [$A_\alpha$] and [$A_\beta$] are stationary, state transition matrices can be discretized offline, decreasing calculation time (in real time) and increasing precision (use of more complex and precise methods). Matrices [$H_{\text{ie}}$] and [$H_{\text{ix}}$] still have to be calculated in real time, however. But by breaking down the calculation, so that matrix [$Z_{\text{i}}$] calculated offline appears, these calculations come down to simple multiplications and additions.

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4. Since main input $V_{\text{S}}$ is most often found in modulated form (obtained by PWM), we can use its reference value obtained from the control (if the modulation period is low compared to the system's dynamic). This avoids a measurement which includes a filter.

We show that for the same observer (same gain values) the discretization of the Cartesian form requires half as many basic operations as that of the complete form, while increasing its precision [BOU 96].

### 5.3.5. *Validation of the Cartesian observer for stator and rotor fluxes*

#### 5.3.5.1. *Specifications*

The Cartesian observer was implemented in a stator flux Cartesian vector control for a 500 W induction machine (see parameters in Appendix). We can then have an estimation of the stator flux Cartesian components directly controlled by their reference values. A 68,000 Motorola microprocessor was used for the control of two machines, the sampling period was set at $T_{ech}$ = 500 μs. In addition, the modulation period of Pulse Width Modulation (PWM) was set at $T_{mod}$ = 500 μs. This control made it possible to experimentally validate a double drive for the motorization of a mobile robot [BOU 95b].

In order to harmonize the test with the different observers proposed, a specific trajectory is used to test the behavior in the four quadrants of the velocity torque plan (Figure 5.6). The flux is first imposed at its nominal value. At $t$ = 1 s, a speed trapezoid is imposed, and we can note a decrease of the flux at high speed, making it possible to also test the flux reduction mode. Between $t$ = 6 s and $t$ = 13 s, a nominal step torque is imposed: this leads to speed disruption. In the preliminary test presented, the control uses the Cartesian observer without parametric variations in relation to simulation models.

#### 5.3.5.2. *Robustness test*

Prior to real-time implementation, robustness tests must enable us to validate observer performances.

Second, parametric variations are only done in the observer (and not in the vector control) in order to illustrate the robustness of this estimation structure:

$$\Delta R\text{s} = 50\%$$
$$\Delta R\text{r} = 50\%$$
$$\Delta L\text{s} = 20\%$$

These parametric variations were defined according to identification errors and evolutions during the operation: effect of temperature (resistances) and magnetic level (inductances). The amplitude of the rotor flux in the machine is used as a reference and the gap with an estimated flux and an observed flux is drawn

(Figure 5.7). The amplitude of the estimated flux is obtained by the traditional rotor flux estimator. The observed flux comes from the Cartesian observer. We notice greater estimator sensitivity to parametric variations.

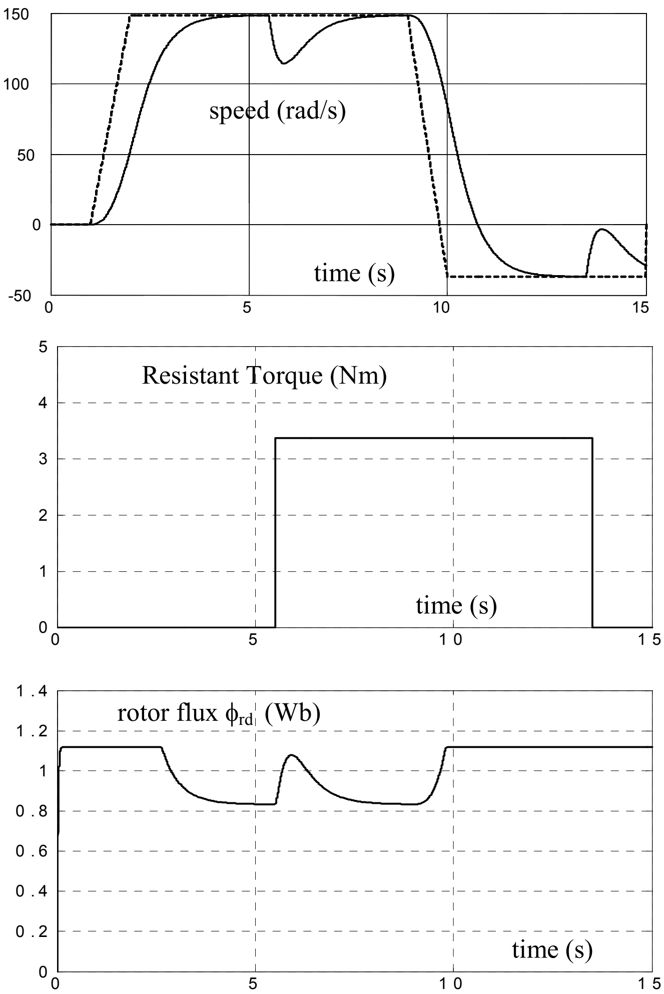


**Figure 5.6.** *Test profile without parametric variation*

Improvement can be achieved by an online identification of sensitive parameters with the use of extended observers [DU 95] and adaptive structures, etc.

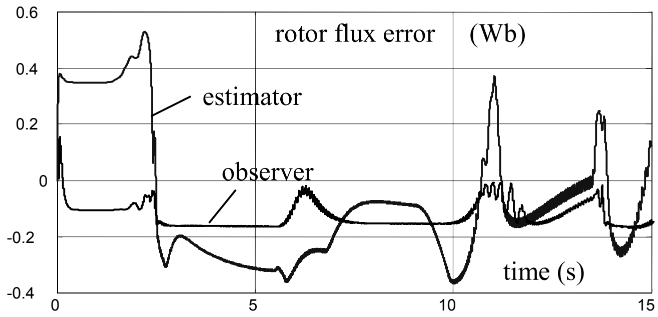


**Figure 5.7.** *Robustness test for the Cartesian observer*

### 5.3.6. *Assessment on Cartesian observers*

A Cartesian observer is simply a specific representation of a complete order observer: two sub-observers in each axis, coupled by coupling matrices. In terms of the flux estimation for an induction machine, three state representations can be used. Reference ($\alpha_S$, $\beta_S$) linked to the stator remains the most judicious, however.

The use of the Cartesian form helps to achieve an easy analytical synthesis of observation gains because the study then involves two identical systems of the second order. In addition, discretization of the Cartesian form provides significant gain in computation time while increasing precision. On the other hand, this structure can only be used for sampling periods short enough to consider the different quasi constant variables between two periods.

## 5.4. Reduced order observers

This part focuses on reduced order observers that only estimate a part of the model state vector. These estimation structures have often been used for two decades because of the real-time implementation possibilities at a time when control processors were limited in performance [LEO 91].

We first present the structure of these flux estimations, as well as the different models that can be used. The study of the most traditional reduced order observer is then detailed to illustrate these flux estimation structures.

### 5.4.1. *Principle and structure of reduced order observers*

#### 5.4.1.1. *Principle of the reduced order observer*

A reduced order observer is an observer limited to only estimating a part of the state vector. The other components of the state vector must be accessible for this to happen. Because of this, the dimension of the observer thus restricted is reduced to the dimension of the state sub-vector made up of the variables to estimate. In fact, the complete order observer is broken down into two parts, one associated with measurable state variables and the other to the variables to estimate: the second part alone is used (after a few modifications).

The state models used for the induction machine are of order four (see section 5.2). We can reduce the estimation of a state vector made up of stator current and flux involved: we focus on the components of the flux, since the current components can be measured. The state equation to solve is of dimension two, meaning real-time implementation (notably in relation to its discretization (see an example in section 5.4.4)).

#### 5.4.1.2. *Breakdown of the complete observer*

State vector $X$ (dimension $n = m + p$) is broken down into two state vectors, one containing variables that cannot be measured (dimensions to estimate) $\underline{X}_1$ (dimension $p$) and one made up of measurable variables $\underline{X}_2$ (dimension $m$). Associated sub-matrices are deducted from the matrices of global representation [5.8]. It is therefore logical to associate output vector $\underline{Y}$ to the state vector made up of accessible dimensions $\underline{X}_2$ [BOR 90]:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{X}_1 = [A_{11}]\underline{X}_1 + [A_{12}]\underline{X}_2 + [B_1]\underline{U} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{X}_2 = [A_{22}]\underline{X}_2 + [A_{21}]\underline{X}_1 + [B_2]\underline{U} \\ \underline{Y} = \underline{X}_2 \end{cases} \qquad [5.40]$$

The system is made up of two state equations, each containing a dynamic matrix and two control matrices. We can make the analogy between the inter-axis coupling matrices $[K_\alpha]$ and $[K_\beta]$ of the Cartesian model [5.19] and those of state inter-variable coupling $[A_{12}]$ and $[A_{21}]$ of this model [5.40].

For the observer equation, vector $\underline{X}_1$ is replaced by its estimate, $\underline{\hat{X}}_1$, whereas $\underline{X}_2$ is not modified. We use the hypothesis that $\underline{X}_2 = \underline{\hat{X}}_2$.

This hypothesis will make it possible to define the reduced order observer and to only use a part of the equations to estimate $\underline{X}_1$ (hence calculation time gain desired).

In order to minimize the calculations, it is logical to limit the action of the return gain $[L]$ on the state equation linked to non-measurable variables (Figure 5.8):

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_1 = [A_{11}]\underline{\hat{X}}_1 + [A_{12}]\underline{X}_2 + [B_1]\underline{U} + [L]\underline{\varepsilon} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_2 = [A_{22}]\underline{X}_2 + [A_{21}]\underline{\hat{X}}_1 + [B_2]\underline{U} \\ \underline{Y} = \underline{X}_2 \end{cases} \quad [5.41]$$

The choice for the estimation error $\underline{\varepsilon}$ (error between an estimated variable and a true variable) remains unresolved. In fact, output vector $\underline{Y}$ (equal to $\underline{X}_2$) can no longer be used because its estimation (i.e. that of $\underline{X}_2$) is equal to its true value by hypothesis. We then use the error of prediction linked to the derivative of $\underline{X}_2$ [BOR 90]:

$$\underline{\varepsilon} = \frac{\mathrm{d}}{\mathrm{d}t}\underline{X}_2 - \frac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_2 \quad [5.42]$$

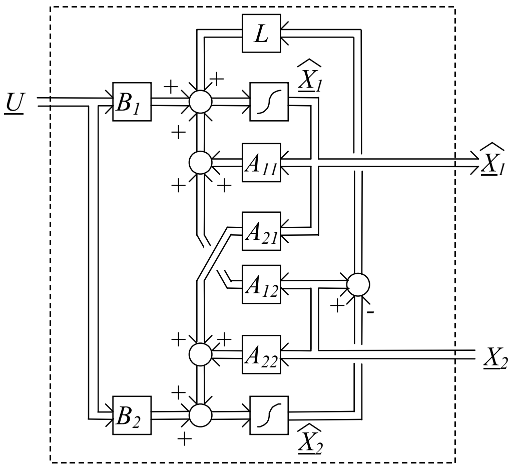


**Figure 5.8.** *Structure of a reduced observer*

This choice provides a new state equation by using the estimation expression of the derivative of $\underline{X}_2$ in [5.41]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{X}}_1 = [A_{11} - LA_{21}]\underline{\hat{X}}_1 + [A_{12} - LA_{22}]\underline{X}_2 + [B_1 - LB_2]\underline{U} + [L]\frac{\mathrm{d}}{\mathrm{d}t}\underline{Y} \qquad [5.43]$$

Since this state equation involves the output derivative (which is clearly not accessible), a judicious variable change is used to make this term disappear:

$$\underline{\hat{W}} = \underline{\hat{X}}_1 - [L]\underline{X}_2 \qquad [5.44]$$

Equation [5.45] is simplified with this variable $\underline{W}$ with dimension $p$:

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\underline{\hat{W}} = [A_{\mathrm{w}}]\underline{\hat{W}} + [B_x]\underline{X}_2 + [B_{\mathrm{u}}]\underline{U} \\ &\qquad [A_{\mathrm{w}}] = [A_{11} - LA_{21}] \qquad [B_{\mathrm{u}}] = [B_1 - LB_2] \\ &\qquad [B_x] = [(A_{11} - LA_{21})L + A_{12} - LA_{22}] \end{aligned} \qquad [5.45]$$

The observer is then synthesized (gain calculation) for a good estimation of $\underline{W}$ through [5.45], then vector $\underline{X}_1$ is rebuilt following the estimation of $\underline{W}$ and the measure of $\underline{X}_2$ [5.44].

We must notice, however, that this new equation [5.45] is conditional to a stationary matrix [$L$]: in fact, it is based on d ([$L$] $\underline{X}_2$)/dt = [$L$] d $\underline{X}_2$/dt through [5.44]). For a matrix [$L$] function of speed, this equation is no longer valid, except if we use the mode of decoupling hypothesis.

#### 5.4.1.3. *Characteristics of reduced order observers*

The reduced order observer (dimension $p < n$) is based on a section of the state model linked to non-measurable variables. Its synthesis becomes simplified because of a lower order characteristic equation. The same applies to its real-time implementation because of the order reduction of the state equation to discretize [5.45]. A specific complete order (order four) flux observer study and its reduced order (order two) equivalent results in a calculation time gain of five and a memory size gain of ten [SHO 95]. Even though this is a specific case, this study shows the advantage of this type of estimation structure.

However, the robustness of reduced order observers is not as significant as those of complete order structures [GAR 98], while still better than that of estimators [NIL 89]. In fact, the different matrices of the observation equation [5.45] contain many parameters. On the other hand, quasi-stationarity hypotheses similar to those of Cartesian observers will be addressed during the discretization of the reduced order observer (see section 5.4.4).

### 5.4.2. *Different reduced order observers*

Because of the necessity of having measurable variables in the state vector, on the one hand, only the two possibilities containing stator currents can be used. On the other hand, the choice of reference is not restricted. We thus find reduced order flux observers in references ($\alpha_S$, $\beta_S$) [HOR 89], (d, q) linked to rotor flux [NIL 89], (d, q) linked to stator current [ROB 92], ($\alpha_R$, $\beta_R$) [DAS 97], etc.

The most common reference however remains reference ($\alpha_S$, $\beta_S$) [BOU 98, DEL 97, ORL 89, VER 88]. Even though each observer is limited to the estimation of a flux, we can also notice certain specific structures for estimating both by a change of variable linking both fluxes [BEL 88].

In this section, we will concentrate on giving the reduced order observer models in reference ($\alpha_S$, $\beta_S$), which are the most commonly used.

#### 5.4.2.1. *Reduced order stator flux observer*

This observer is therefore based on state model [5.11] where the state vector is made up of axis $\alpha_S$ and $\beta_S$ components of the stator currents and stator fluxes [5.10] because we are limiting ourselves to reference ($\alpha_S$, $\beta_S$):

$$\underline{X}_1 = \underline{\Phi}_{s\alpha\beta} = \begin{bmatrix}\Phi_{s\alpha} & \Phi_{s\beta}\end{bmatrix}^T \quad \underline{X}_2 = \underline{I}_{s\alpha\beta} = \begin{bmatrix}i_{s\alpha} & i_{s\beta}\end{bmatrix}^T \qquad [5.46]$$

The new matrices of this particular representation are deducted from global representation [5.11], considering the simplifications caused by reference ($\alpha_S$, $\beta_S$), or $\omega_{d/s} = 0$ and $\omega_{d/r} = -n_p\Omega$:

$$[A_{11}] = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \qquad [A_{12}] = \begin{bmatrix}0 & -R_s\\ -R_s & 0\end{bmatrix}$$

$$[A_{21}] = \begin{bmatrix}aR_r/L_r & -an_p\Omega\\ -an_p\Omega & aR_r/L_r\end{bmatrix} \qquad [A_{22}] = \begin{bmatrix}-(aR_s+bR_r) & -n_p\Omega\\ -n_p\Omega & -(aR_s+bR_r)\end{bmatrix}$$

$$[B_1] = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad [B_2] = \begin{bmatrix}a & 0\\ 0 & a\end{bmatrix} \qquad [5.47]$$

Even if the dynamic flux sub-matrix $[A_{11}]$ is zero, we must note that the dynamic matrix of the reduced observer is a linear combination of other matrices [5.45] and therefore does not equal zero.

This reduced order observer can help in estimating the stator flux for controls based on flux control.

5.4.2.2. *Reduced order rotor flux observer*

This observer is based on state model [5.14] where the state vector is made up of the stator currents and rotor fluxes [5.13] in reference ($\alpha_S$, $\beta_S$):

$$\underline{X}_1 = \underline{\Phi}_{r\alpha\beta} = \begin{bmatrix} \Phi_{r\alpha} & \Phi_{r\beta} \end{bmatrix}^T \qquad \underline{X}_2 = \underline{I}_{s\alpha\beta} = \begin{bmatrix} i_{s\alpha} & i_{s\beta} \end{bmatrix}^T \qquad [5.48]$$

The new matrices of this representation are deducted from global representation [5.14], considering the simplifications caused by the reference ($\alpha_S$, $\beta_S$):

$$[A_{11}] = \begin{bmatrix} -R_r / L_r & -n_p\Omega \\ n_p\Omega & -R_r / L_r \end{bmatrix} \quad [A_{12}] = \begin{bmatrix} R_r M_{sr} / L_r & 0 \\ 0 & R_r M_{sr} / L_r \end{bmatrix}$$

$$[A_{21}] = \begin{bmatrix} cR_r / L_r & cn_p\Omega \\ -cn_p\Omega & cR_r / L_r \end{bmatrix} \quad [A_{22}] = \begin{bmatrix} -aR_s - (1-\sigma)bR_r & 0 \\ 0 & -aR_s - (1-\sigma)bR_r \end{bmatrix}$$

$$[B_1] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad [B_2] = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \qquad [5.49]$$

A simplification emerges because of the cancellation of the control sub-matrix linked to the current $[B_1]$. This observer can help in a robust estimation of rotor flux for controls based on flux control. This structure was widely used to improve the first vector controls field oriented control, consisting of a rotor flux control in polar coordinates. It enables a robust estimation of this flux for a real-time implementation that is not very significant.

### 5.4.3. *Synthesis of the reduced order rotor flux observer*

5.4.3.1. *Observability*

The observability of global model [5.14] by the Kalman criterion can be expressed through the determinant of the first four lines of the observability matrix:

$$\Delta = c^2 \left[ (n_\mathrm{p}\Omega)^2 + \left( \frac{R_\mathrm{r}}{L_\mathrm{r}} \right)^2 \right] \qquad [5.50]$$

Since this determinant is strictly positive for any value of speed, this state system [5.14] is rigorously observable. Because of this, the reduced state system can be observed for the whole range of speed [BOR 90].

5.4.3.2. *Gain calculation*

After the change in variable [5.44] making the derivative output terms disappear, the reduced order observer of intermediate variable $W$ is managed by equation [5.45]. Its dynamic matrix corresponds to $[A_{11}–LA_{21}]$. Observation gain $[L]$ then makes it possible to set the observer dynamics (i.e., the convergence dynamics of the estimation error).

In order to simplify the calculations, the gain matrix takes the form [BOU 98]:

$$[L] = \frac{1}{c} \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} \qquad [5.51]$$

The observer poles are then deducted from the polynomial characteristic of dynamic observer matrix $[A_{11}-LA_{21}]$:

$$\begin{aligned} & s^2 + s\left[ n_\mathrm{p}\mathrm{W}(l_{21} - l_{12}) + \frac{2 + l_{11} + l_{22}}{L_\mathrm{r} / R_\mathrm{r}} \right] \\ & \quad + \left[ (n_\mathrm{p}\mathrm{W})^2 + \frac{R_\mathrm{r}^2}{L_\mathrm{r}^2} \right] \left[ (1+l_{11})(1+l_{22}) - l_{12}l_{21} \right] = 0 \end{aligned} \qquad [5.52]$$

The two solution poles, $p_1$ and $p_2$, are functions of gains $l_{ij}$, speed and machine parameters. We propose that they be chosen in such a way that we can verify certain criteria linked to the speed of cancellation of the error (Figure 5.9):

- observer stability: $\mathrm{Re}\{p_1\} < 0$ and $\mathrm{Re}\{p_2\} < 0$
- combined complex poles: $p_1 = p_2^*$
- velocity of convergence at zero speed: $\mathrm{Re}\{p_{10}\} = \mathrm{Re}\{p_{20}\} < 1/\tau_\mathrm{des}$
- speed of convergence for any other speed: $\zeta = \sin\psi = 0.707$

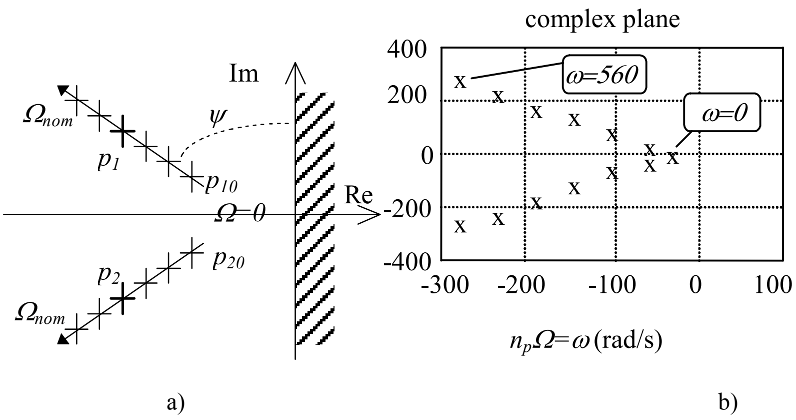


**Figure 5.9.** *Reduced observer pole: (a) general case; (b) choice of gains used*

The resolution of the characteristic equation [5.52] determines the two solution poles. The four criteria defined lead to conditions on the different gains [BOU 98] with one having answer $t_{des}$:

– observer stability: $l_{11} + l_{22} + 2 > 0$

– combined complex poles: $2\,(l_{11}–l_{22}) + 4l_{12}l_{21} > 0$

– velocity of convergence at zero speed: $l_{11} + l_{22} + 2 = 6L_r/(R_r t_{des})$

– velocity of convergence for any other speed: $l_{21} = 1 + l_{22}$

We notice that an anti-symmetrical gain matrix [$L$] verifies all these conditions. This specific form is often used as a starting point in the study because the state model matrices of the process are anti-symmetrical. In order to restrict the field of gain definition, we use this form as a new criterion:

– Anti-symmetrical matrix [$L$]: $l_{11} = l_{22}$ and $l_{12} = – l_{21}$

A particular solution is thus obtained by combining the five relations:

$$\left[L\right] = \frac{1}{c}\begin{bmatrix} l_{11} & -(1+l_{11}) \\ -(1+l_{11}) & l_{11} \end{bmatrix} \quad \text{with} \quad l_{11} = 3\frac{L_r}{R_r t_{des}} - 1 \qquad [5.53]$$

The choice for response time of the observer (error convergence) at zero speed $t_{des}$ completely defines gains.

Speed of convergence is the criterion retained for the observer synthesis. Other criteria are possible, notably a robustness criterion in relation to parameters [DEL 97] (also see Chapter 2).

### 5.4.4. *Discretization of the reduced order rotor flux observer*

The reduced observer [5.40] is discretized by using the observation error as input [BOU 96]:

$$\begin{cases} \underline{\hat{W}}_{k+1} = [F_\mathrm{w}]\underline{\hat{W}}_k + [H_\mathrm{i}]\underline{I}_{sk} + [H_\mathrm{v}]\underline{V}_{sk} \\ \underline{\hat{\Phi}}_{\mathrm{r}k} = \underline{\hat{W}}_k + [L]\underline{I}_{sk} \end{cases} \qquad [5.54]$$

$$\begin{aligned} &[F_\mathrm{w}] = \exp([A_\mathrm{w}]T_\mathrm{ech}) \\ &[\mathrm{H_i}] = [A_\mathrm{w}]^{-1}[F_\mathrm{w} - I]\;[B_\mathrm{i}] \qquad [\mathrm{H_v}] = [A_\mathrm{w}]^{-1}[F_\mathrm{w} - I][B_\mathrm{v}] \end{aligned} \qquad [5.55]$$

The variable-change equation directly involves matrix $[L]$ (equation independent from time). Since matrices $[A_\mathrm{W}]$ and $[B_\mathrm{i}]$ depend on $[A_{11}]$ and $[A_{21}]$, which are non-stationary, discrete matrices $[F_\mathrm{W}]$, $[H_\mathrm{i}]$, and $[H_\mathrm{v}]$ must be calculated in real time (or discretized by range).

We can note however that since the matrices are anti-symmetrical, calculation of the state transition matrix $[F_\mathrm{W}]$ is then simplified (and also the opposite of $[A_\mathrm{W}]$):

$$[F_\mathrm{w}] = \exp(a_\mathrm{w}T_\mathrm{ech})\begin{bmatrix} \cos b_\mathrm{w}T_\mathrm{ech} & -\sin b_\mathrm{w}T_\mathrm{ech} \\ \sin b_\mathrm{w}T_\mathrm{ech} & \cos b_\mathrm{w}T_\mathrm{ech} \end{bmatrix} \qquad [A_\mathrm{w}] = \begin{bmatrix} a_\mathrm{w} & -b_\mathrm{w} \\ b_\mathrm{w} & a_\mathrm{w} \end{bmatrix} \qquad [5.56]$$

We notice that some studies propose the discretization of this type of observer before synthesis, which is done subsequently in the discrete field [DEL 99].

### 5.4.5. *Validation of the reduced order rotor flux observer*

#### 5.4.5.1. *Specifications*

This reduced order observer was implemented in a rotor flux vector control for a 3 kW induction machine with a sampling period of $T_\mathrm{ech}$ = 800 µs [DEL 99].

In order to propose a study consistent with that of the Cartesian observer, we simulate the vector control and the reduced observer for the 500 W reference machine under the same conditions and tests (see section 5.3.5 and parameters in the Appendix).

5.4.5.2. *Robustness test*

The parametric variations defined previously (see section 5.3.5.2) were only applied in the observer (and not in the vector control) in order to bring out its inherent robustness.

The tests proposed (Figure 5.10) show an improvement of flux estimation compared to the reference estimator. An improvement can occur from an online identification of the problem parameters: extended observers [ORL 89], etc.

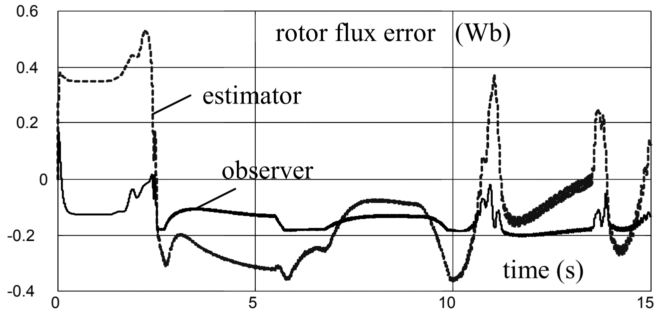


**Figure 5.10.** *Robustness test for the reduced observer*

### 5.4.6. *Assessment on reduced order observers*

A reduced order observer only estimates a part of the state vector by considering the other part accessible. Because of this, on the one hand, only two state representations can be used for the flux estimation of a squirrel-cage induction machine because only stator currents can be measured. On the other hand, different references are used, even if reference ($\alpha_S$, $\beta_S$) linked to the stator remains the most common.

The use of the reduced form helps to achieve an easy analytical synthesis of observation gains because the study then involves a system of the second order. On the one hand, the discretization of this Cartesian form provides an important gain in computation time. On the other hand, it is not as robust as a complete order observer. In addition, this structure can only be used for sampling periods short enough to consider the different quasi-constant currents (additional inputs) between two periods.

### 5.5. Conclusion on Cartesian and reduced order observers

The use of a flux observer for the induction machine provides a robust estimation of the coordinates of this flux. Because of this, they participate in the good modern control performances of induction machines based on precise control of a flux. However, the complete order observer of the machine is a non-stationary four-dimension system. Because of this, its real-time implementation is relatively expensive and intensive in computation time.

In order to reduce this computation time, two techniques of complete observer breakdown are used. In the first one, the Cartesian structure proposes a separation into two coupled sub-observers. This breakdown provides simpler syntheses and especially a less-intensive real-time implementation in calculation time, while conserving the properties of a complete order observer. The second structure is based on the breakdown of the system based on measurable state variables and those that need to be estimated. A reduced order observer is limited to the estimation of the part of the state vector linked to the variables to estimate. Even though its synthesis is simple, and its implementation less expensive, we notice a slight loss in performance. This type of structure has nevertheless often been used in manufacturing processes.

### 5.6. Appendix : parameters of the study induction machine

| | | |
|---|---|---|
| $P$n | = 500 W | |
| $N_n$ | = 1,420 rpm $f_s$ = 50 Hz | |
| $\eta$ | = 72% cos$\phi$ = 0.78 | |
| $n_p$ | = 2 | |
| $I$ | = 4.2 A | $V_\bullet$ = 0 127 |
| $I_Y$ | = 2.4 A | $V_Y$ = 220 V |
| $R_s$ | = 10.75 Ω | $R_r$ = 7 Ω |
| $L_s$ | = $L_r$ = 424 mH | $M_{sr}$ = 397 mH |

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