Part I

A Brief Overview

Electromagnetic Foundations of Electrical Engineering   J. A. Brandão Faria
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Introduction

Ever since Maxwell wrote his *Treatise on Electricity and Magnetism* the subject area of electromagnetism has been evolving at a strong pace. Despite its age, this area of knowledge is both alive and active: not only have its applications exploded in interest, but also new powerful computational methods have been developed for electric and magnetic field evaluation, and, what is more, its theoretical tools have also evolved.

As far as the theoretical aspects are concerned, modern trends point towards the substitution of traditional vector analysis by new, powerful and concise mathematical tools based on geometric algebra formulations[1] (exterior calculus, graded algebra of multivectors, graded algebra of differential forms, etc.). The most salient feature of these new modern tools is that, in addition to an elegant and concise view of electromagnetic phenomena, they provide a perfect match between the laws of electromagnetism and Einstein's theory of relativity.

Geometric algebra formulations of electromagnetism are adequate for advanced courses in the second and third cycle of studies of the Bologna framework. However, for the first cycle of studies, the most familiar vector calculus (Appendix A) formulation is recommended:

$$\text{Maxwell's equations}\begin{cases}\text{curl } \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \text{div } \mathbf{B} = 0 \\ \text{curl } \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \\ \text{div } \mathbf{D} = \rho \end{cases} \tag{I.1}$$

The electromagnetic field equations in (I.1) will be utilized throughout this book for the macroscopic description of a variety of electromagnetic phenomena which concern current electrical engineering problems.

The equation set in (I.1) is an axiomatic one; it cannot be mathematically deduced from any source. In fact, Maxwell's equations are a result of experimental research accumulated until the end of the nineteenth century.

[1] In terms of geometric algebra, all the properties of the electromagnetic field can be condensed into a single equation: $\nabla F = J$.

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What is truly astonishing is that not a single flaw has been detected in these equations since then. On top of that, Maxwell's equations have passed Einstein's relativity challenge intact and unscathed. Indeed, if you consider two distinct reference frames (x, y, z, t) and (x', y', z', t') where the second is moving with a relative speed $\mathbf{v}$, the new equations for the second referential read

$$\begin{cases} \text{curl}' \ \mathbf{E}' = -\dfrac{\partial \mathbf{B}'}{\partial t'} \\ \text{div}' \ \mathbf{B}' = 0 \\ \text{curl}' \ \mathbf{H}' = \mathbf{J}' + \dfrac{\partial \mathbf{D}'}{\partial t'} \\ \text{div}' \ \mathbf{D}' = \rho' \end{cases} \tag{I.2}$$

showing that the electromagnetic field's governing laws conserved its original shape, despite the change of referential. Note that the primed and unprimed quantities in (I.1) and (I.2) are related through the Lorentz transformation (Appendix B).

Quantum theory, another modern physics development, is mainly devoted to the analysis of microscopic phenomena; nonetheless, Maxwell's equations are not divorced from it. The wave–matter duality principle still allows a link to be established between the two formalisms.

As we will see later (in Part IV), the amount of energy carried by a sinusoidal electromagnetic wave of frequency f, crossing a surface S, during one oscillation period $T = 1/f$, is given by

$$W_T = \int_0^T \left(\int_S (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} \ dS \right) dt \tag{I.3}$$

where $\mathbf{n}$ is the unit normal to S directed along the wave propagation direction.

From a quantum theory viewpoint, such energy is evaluated as

$$W_T = N_p h f \tag{I.4}$$

where h is the universal Plank constant, and N_p is the number of photons crossing S during one time period T.

By equating (I.3) and (I.4) you will find the link between the macroscopic field and quantum views:

$$N_p = \frac{1}{hf} \int_0^T \left(\int_S (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} \ dS \right) dt$$

These remarks are intended to show you that, almost 150 years from their establishment, Maxwell's equations are still current, and do not conflict with modern physics developments.