

## Wave Propagation in Fluids

# Wave Propagation in Fluids

*Models and Numerical Techniques*

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Vincent Guinot

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## Introduction

### **What is wave propagation?**

In a kitchen or in a bathroom, the number of times we turn a tap every day is countless. So is the number of times we watch the liquid stream impacting the sink. The circular flow pattern where the fast and shallow water film diverging from the impact point changes into a deeper, bubbling flow is too familiar to deserve attention. Very few people looking at the circular, bubbling pattern – referred to as a hydraulic jump by hydraulics specialists – are aware that they are contemplating a shock wave.

Closing the tap too quickly may result in a thud sound. This is the audible manifestation of the well-known water hammer phenomenon, a train of pressure waves propagating in the metal pipes as fast as hundreds or thousands of meters per second. The water hammer phenomenon is known to cause considerable damage to hydropower duct systems or water supply networks under the sudden operation of valves, pumps or turbines. The sound is heard because the vibrations of the duct system communicate with the ambient atmosphere, and from there with the operator's ears.

Everyone has once thrown stones into a pond, watching the concentric ripples propagate on the surface. Less visible and much slower than the ripples is the moving groundwater that displaces a pollutant front in a journey that may last for years.

As ubiquitous and familiar as wave propagation may be, the phenomenon is often poorly understood. The reason why intuition so often fails to grasp the mechanisms of wave propagation may lie in the commonly shared, instinctive perception that waves are made of matter. This, however, is not true. In the example of the hydraulic jump in the sink, the water molecules move across an immobile

wave. In the example of the ripples propagating on the free surface of a pond, the waves travel while the water remains immobile.

Waves appear when an object or a system (e.g. the molecules in a fluid, a rigid metallic structure) reacts to a perturbation and transmits it to its neighbors. In many cases, as in the example of the water ripples, the initially perturbed system returns to its initial equilibrium state, while the waves keep propagating. In this respect, waves may be seen as information. The ripples propagating in a pond are a sign that the water molecules “inform” their neighbors that the equilibrium state has been perturbed. A sound is nothing other than information about a perturbation occurring in the atmosphere.

Numerical techniques for wave propagation simulation have been the subject of intensive research over the last 50 years. The advent of fast computers has led to the development of efficient numerical techniques. Engineers and consultants now use simulation software packages for wave propagation on a daily basis. Whether for the purpose of acoustics, aerodynamics, flood wave propagation or contaminant transport studies, computer-based simulation tools have become indispensable to almost all domains of engineering. Such tools, however, remain instruments operated by human beings to execute tedious, repetitive operations previously carried out by hand. They cannot, and hopefully never will, replace the expert’s judgment and experience. Human presence remains necessary for the sound assessment of the relevance and accuracy of modeling results. Such an assessment, however, is possible only provided that the very specific type of reasoning required for the correct understanding of wave propagation phenomena has been acquired.

The main purpose of this book is to contribute to a better understanding of wave propagation phenomena and the most commonly used numerical techniques for its simulation. The first three chapters deal with the physics and mathematics of wave propagation. Chapters 4, 5 and 10 provide insight into more theoretical notions, used in specific numerical techniques. Chapters 6 to 9 are devoted to finite difference, finite volume and finite element techniques. Chapter 11 is devoted to practical advice for the modeler. Basic notions of linear algebra and numerical methods are presented in Appendices A to C. The various formulae used in the present book are summarized in Appendix D.

### **What is the intended readership of this book?**

This book is intended for students of professional and research master’s programs and those engaged in doctoral studies, the curriculum of which contains hydraulics and/or fluid mechanics-related subjects. Engineers and developers in the

field of fluid mechanics and hydraulics are also a potential target group. This book was written with the following objectives:

(i) To introduce the physics of wave propagation, the governing assumptions and the derivation of the governing equations (in other words, the modeling process) in various domains of fluid mechanics. The application fields are as diverse as contaminant transport, open channel and free surface hydraulics, or aerodynamics.

(ii) To explain how the behavior of the physical systems can be analyzed using very simple mathematical techniques, thus allowing practical problems to be solved.

(iii) To introduce the main families of numerical techniques used in most simulation software packages. As today's practicing engineers cannot afford not to master modeling packages, a basic knowledge of the existing numerical techniques appears as an indispensable engineering skill.

### **How should this book be read?**

Most of the chapters are made up of three parts:

- the first part of the chapter is devoted to the theoretical notions applied in the remainder of the chapter;
- the second part deals with the application of these theoretical notions to various hydraulics and fluid mechanics equations;
- the third part provides a summary of the key points developed in the chapter, as well as suggestions of application exercises.

The main purpose of the application exercises is to test the reader's ability to reuse the notions developed in the chapter and apply them to practical problems. The solutions to the exercises may be accessed at the following URL: <http://vincentguinot.free.fr/waves/exercises.htm>.

Try to resist the temptation to read the solution immediately. Solving the exercise by yourself should be the primary objective. The solution text is provided only as an aid, in case you cannot find a way to start and for you to check the validity of your reasoning after completing the exercise.