

EIGHTH INTERNATIONAL EDITION

Instructor's solution manual for

# Microelectronic Circuits

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**Chapter 1**

**Solutions to Exercises within the Chapter**

**Ex: 1.1** When output terminals are open-circuited, as in Fig. 1.1a:

For circuit a.  $v_{oc} = v_s(t)$

For circuit b.  $v_{oc} = i_s(t) \times R_s$

When output terminals are short-circuited, as in Fig. 1.1b:

For circuit a.  $i_{sc} = \frac{v_s(t)}{R_s}$

For circuit b.  $i_{sc} = i_s(t)$

For equivalency

$$R_s i_s(t) = v_s(t)$$

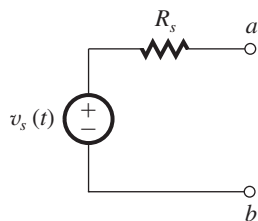


Figure 1.1a

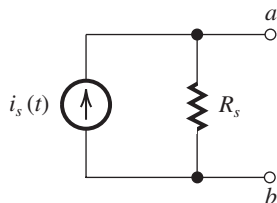
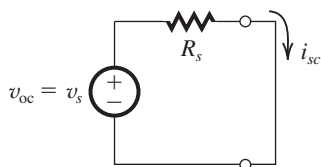


Figure 1.1b

**Ex: 1.2**



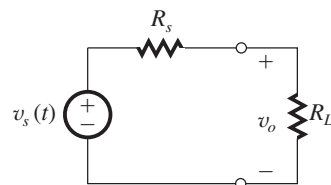
$$v_{oc} = 10 \text{ mV}$$

$$i_{sc} = 10 \text{ } \mu\text{A}$$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10 \text{ mV}}{10 \text{ } \mu\text{A}} = 1 \text{ k}\Omega$$

**Ex: 1.3** Using voltage divider:

$$v_o(t) = v_s(t) \times \frac{R_L}{R_s + R_L}$$



Given  $v_s(t) = 10 \text{ mV}$  and  $R_s = 1 \text{ k}\Omega$ .

If  $R_L = 100 \text{ k}\Omega$

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1} = 9.9 \text{ mV}$$

If  $R_L = 10 \text{ k}\Omega$

$$v_o = 10 \text{ mV} \times \frac{10}{10 + 1} \simeq 9.1 \text{ mV}$$

If  $R_L = 1 \text{ k}\Omega$

$$v_o = 10 \text{ mV} \times \frac{1}{1 + 1} = 5 \text{ mV}$$

If  $R_L = 100 \text{ } \Omega$

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1 \text{ K}} \simeq 0.91 \text{ mV}$$

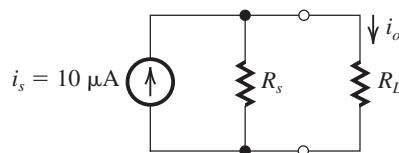
For  $v_o = 0.8 v_s$ ,

$$\frac{R_L}{R_L + R_s} = 0.8$$

Since  $R_s = 1 \text{ k}\Omega$ ,

$$R_L = 4 \text{ k}\Omega$$

**Ex: 1.4** Using current divider:



$$i_o = i_s \times \frac{R_s}{R_s + R_L}$$

Given  $i_s = 10 \text{ } \mu\text{A}$ ,  $R_s = 100 \text{ k}\Omega$ .

For

$$R_L = 1 \text{ k}\Omega, i_o = 10 \text{ } \mu\text{A} \times \frac{100}{100 + 1} = 9.9 \text{ } \mu\text{A}$$

For

$$R_L = 10 \text{ k}\Omega, i_o = 10 \text{ } \mu\text{A} \times \frac{100}{100 + 10} \simeq 9.1 \text{ } \mu\text{A}$$

For

$$R_L = 100 \text{ k}\Omega, i_o = 10 \text{ } \mu\text{A} \times \frac{100}{100 + 100} = 5 \text{ } \mu\text{A}$$

$$\text{For } R_L = 1 \text{ M}\Omega, i_o = 10 \text{ } \mu\text{A} \times \frac{100 \text{ K}}{100 \text{ K} + 1 \text{ M}}$$

$$\simeq 0.9 \text{ } \mu\text{A}$$

$$\text{For } i_o = 0.8 i_s, \frac{100}{100 + R_L} = 0.8$$

$$\Rightarrow R_L = 25 \text{ k}\Omega$$



$$\text{Ex: 1.5 } f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 10^3 \text{ rad/s}$$

$$\text{Ex: 1.6 (a) } T = \frac{1}{f} = \frac{1}{60} \text{ s} = 16.7 \text{ ms}$$

$$\text{(b) } T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000 \text{ s}$$

$$\text{(c) } T = \frac{1}{f} = \frac{1}{10^6} \text{ s} = 1 \mu\text{s}$$

**Ex: 1.7** If 6 MHz is allocated for each channel, then 470 MHz to 608 MHz will accommodate

$$\frac{806 - 470}{6} = 23 \text{ channels}$$

Since the broadcast band starts with channel 14, it will go from channel 14 to channel 36.

$$\text{Ex: 1.8 } P = \frac{1}{T} \int_0^T \frac{v^2}{R} dt$$

$$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$$

Alternatively,

$$P = P_1 + P_3 + P_5 + \dots$$

$$= \left(\frac{4V}{\sqrt{2\pi}}\right)^2 \frac{1}{R} + \left(\frac{4V}{3\sqrt{2\pi}}\right)^2 \frac{1}{R}$$

$$+ \left(\frac{4V}{5\sqrt{2\pi}}\right)^2 \frac{1}{R} + \dots$$

$$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots\right)$$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches  $\pi^2/8$ ; thus  $P$  becomes  $V^2/R$  as found from direct calculation.

Fraction of energy in fundamental

$$= 8/\pi^2 = 0.81$$

Fraction of energy in first five harmonics

$$= \frac{8}{\pi^2} \left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.93$$

Fraction of energy in first seven harmonics

$$= \frac{8}{\pi^2} \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right) = 0.95$$

Fraction of energy in first nine harmonics

$$= \frac{8}{\pi^2} \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}\right) = 0.96$$

Note that 90% of the energy of the square wave is in the first three harmonics, that is, in the fundamental and the third harmonic.

**Ex: 1.9** (a)  $D$  can represent 15 equally-spaced values between 0 and 3.75 V. Thus, the values are spaced 0.25 V apart.

$$v_A = 0 \text{ V} \Rightarrow D = 0000$$

$$v_A = 0.25 \text{ V} \Rightarrow D = 0000$$

$$v_A = 1 \text{ V} \Rightarrow D = 0000$$

$$v_A = 3.75 \text{ V} \Rightarrow D = 0000$$

$$\text{(b) (i) 1 level spacing: } 2^0 \times +0.25 = +0.25 \text{ V}$$

$$\text{(ii) 2 level spacings: } 2^1 \times +0.25 = +0.5 \text{ V}$$

$$\text{(iii) 4 level spacings: } 2^2 \times +0.25 = +1.0 \text{ V}$$

$$\text{(iv) 8 level spacings: } 2^3 \times +0.25 = +2.0 \text{ V}$$

(c) The closest discrete value represented by  $D$  is +1.25 V; thus  $D = 0101$ . The error is -0.05 V, or  $-0.05/1.3 \times 100 = -4\%$ .

$$\text{Ex: 1.10 Voltage gain} = 20 \log 100 = 40 \text{ dB}$$

$$\text{Current gain} = 20 \log 1000 = 60 \text{ dB}$$

$$\text{Power gain} = 10 \log A_p = 10 \log (A_v A_i)$$

$$= 10 \log 10^5 = 50 \text{ dB}$$

$$\text{Ex: 1.11 } P_{dc} = 15 \times 8 = 120 \text{ mW}$$

$$P_L = \frac{(6/\sqrt{2})^2}{1} = 18 \text{ mW}$$

$$P_{\text{dissipated}} = 120 - 18 = 102 \text{ mW}$$

$$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$$

$$\text{Ex: 1.12 } v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5} \text{ V} = 10 \mu\text{V}$$

$$P_L = v_o^2/R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11} \text{ W}$$

With the buffer amplifier:

$$v_o = 1 \times \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

$$= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10+10} = 0.25 \text{ V}$$

$$P_L = \frac{v_o^2}{R_L} = \frac{0.25^2}{10} = 6.25 \text{ mW}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = \frac{0.25 \text{ V}}{1 \text{ V}} = 0.25 \text{ V/V}$$

$$= -12 \text{ dB}$$

$$\text{Power gain } (A_p) \equiv \frac{P_L}{P_i}$$

where  $P_L = 6.25 \text{ mW}$  and  $P_i = v_i i_i$ ,

$$v_i = 0.5 \text{ V and}$$

$$i_i = \frac{1 \text{ V}}{1 \text{ M}\Omega + 1 \text{ M}\Omega} = 0.5 \mu\text{A}$$

Exercise 1-3

Thus,

$$P_i = 0.5 \times 0.5 = 0.25 \mu\text{W}$$

and

$$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$$

$$10 \log A_p = 44 \text{ dB}$$

**Ex: 1.13** Open-circuit (no load) output voltage =  $A_{vo}v_i$

Output voltage with load connected

$$= A_{vo}v_i \frac{R_L}{R_L + R_o}$$

$$0.8 = \frac{1}{R_o + 1} \Rightarrow R_o = 0.25 \text{ k}\Omega = 250 \Omega$$

**Ex: 1.14**  $A_{vo} = 40 \text{ dB} = 100 \text{ V/V}$

$$P_L = \frac{v_o^2}{R_L} = \left( A_{vo}v_i \frac{R_L}{R_L + R_o} \right)^2 / R_L$$

$$= v_i^2 \times \left( 100 \times \frac{1}{1 + 1} \right)^2 / 1000 = 2.5 v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10,000}$$

$$A_p \equiv \frac{P_L}{P_i} = \frac{2.5 v_i^2}{10^{-4} v_i^2} = 2.5 \times 10^4 \text{ W/W}$$

$$10 \log A_p = 44 \text{ dB}$$

**Ex: 1.15** Without stage 3 (see figure)

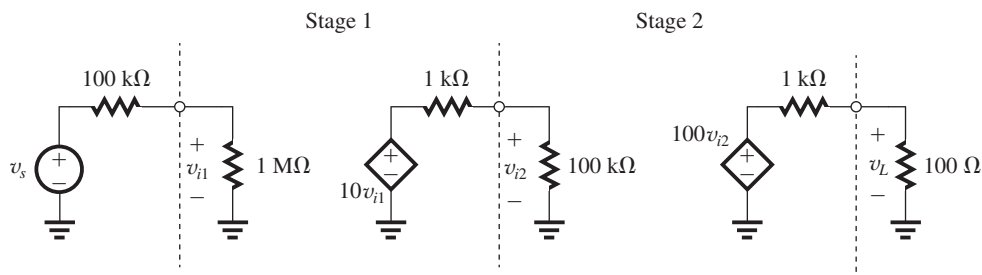
$$\frac{v_L}{v_s} = \left( \frac{1 \text{ M}\Omega}{100 \text{ k}\Omega + 1 \text{ M}\Omega} \right) (10) \left( \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 1 \text{ k}\Omega} \right)$$

$$\times (100) \left( \frac{100}{100 + 1 \text{ k}\Omega} \right)$$

$$\frac{v_L}{v_s} = (0.909)(10)(0.9901)(100)(0.0909)$$

$$= 81.8 \text{ V/V}$$

This figure belongs to Exercise 1.15.



**Ex: 1.16** Refer the solution to Example 1.3 in the text.

$$\frac{v_{i1}}{v_s} = 0.909 \text{ V/V}$$

$$v_{i1} = 0.909 v_s = 0.909 \times 1 = 0.909 \text{ mV}$$

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 9.9 \times 0.909 = 9 \text{ V/V}$$

$$v_{i2} = 9 \times v_s = 9 \times 1 = 9 \text{ mV}$$

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 90.9 \times 9.9 \times 0.909$$

$$= 818 \text{ V/V}$$

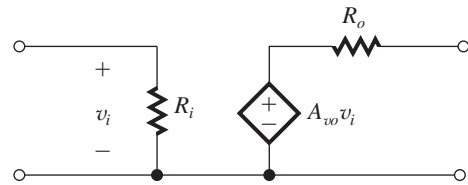
$$v_{i3} = 818 v_s = 818 \times 1 = 818 \text{ mV}$$

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

$$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744 \text{ V/V}$$

$$v_L = 744 \times 1 \text{ mV} = 744 \text{ mV}$$

**Ex: 1.17** Using voltage amplifier model, the three-stage amplifier can be represented as



$$R_i = 1 \text{ M}\Omega$$

$$R_o = 10 \Omega$$

$$A_{vo} = A_{v1} \times A_{v2} \times A_{v3} = 9.9 \times 90.9 \times 1 = 900 \text{ V/V}$$

The overall voltage gain

$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

For  $R_L = 10 \Omega$ :

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{10}{10 + 10} = 409 \text{ V/V}$$

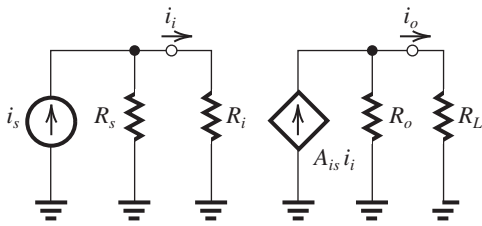
For  $R_L = 1000 \Omega$ :

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810 \text{ V/V}$$

$\therefore$  Range of voltage gain is from 409 V/V to 810 V/V.

**Ex: 1.18**



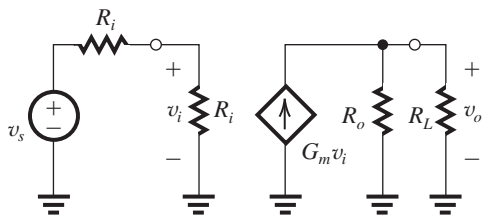
$$i_i = i_s \frac{R_s}{R_s + R_i}$$

$$i_o = A_{is} i_i \frac{R_o}{R_o + R_L} = A_{is} i_s \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Thus,

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

**Ex: 1.19**



$$v_i = v_s \frac{R_i}{R_i + R_s}$$

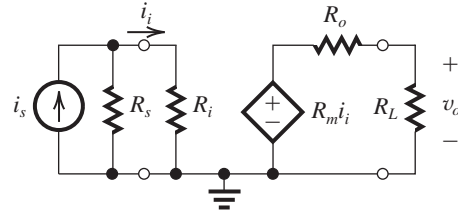
$$v_o = G_m v_i (R_o \parallel R_L)$$

$$= G_m v_s \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

**Ex: 1.20** Using the transresistance circuit model, the circuit will be



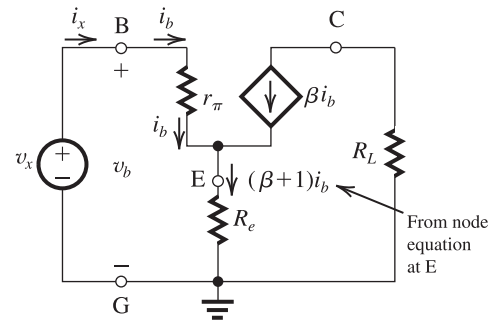
$$\frac{i_i}{i_s} = \frac{R_s}{R_i + R_s}$$

$$v_o = R_m i_i \times \frac{R_L}{R_L + R_o}$$

$$\frac{v_o}{i_i} = R_m \frac{R_L}{R_L + R_o}$$

$$\begin{aligned} \text{Now } \frac{v_o}{i_s} &= \frac{v_o}{i_i} \times \frac{i_i}{i_s} = R_m \frac{R_L}{R_L + R_o} \times \frac{R_s}{R_i + R_s} \\ &= R_m \frac{R_s}{R_s + R_i} \times \frac{R_L}{R_L + R_o} \end{aligned}$$

**Ex: 1.21**



$$v_b = i_b r_\pi + (\beta + 1) i_b R_e$$

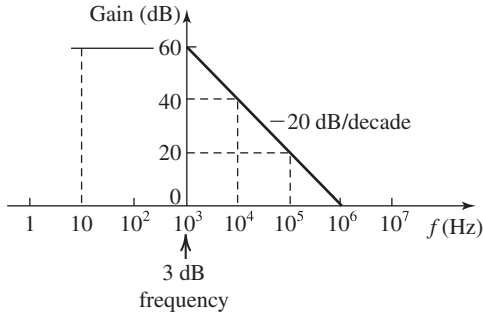
$$= i_b [r_\pi + (\beta + 1) R_e]$$

But  $v_b = v_x$  and  $i_b = i_x$ , thus

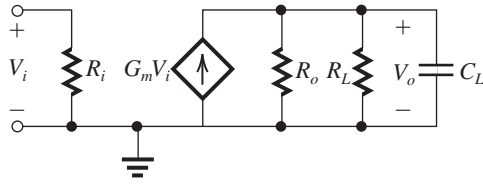
$$R_{in} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R_e$$

**Ex: 1.22**

$f$	Gain
10 Hz	60 dB
10 kHz	40 dB
100 kHz	20 dB
1 MHz	0 dB



**Ex: 1.23**



$$V_o = G_m V_i [R_o \parallel R_L \parallel C_L]$$

$$= \frac{G_m V_i}{\frac{1}{R_o} + \frac{1}{R_L} + sC_L}$$

Thus, 
$$\frac{V_o}{V_i} = \frac{G_m}{\frac{1}{R_o} + \frac{1}{R_L}} \times \frac{1}{1 + \frac{sC_L}{\frac{1}{R_o} + \frac{1}{R_L}}}$$

$$\frac{V_o}{V_i} = \frac{G_m (R_L \parallel R_o)}{1 + sC_L (R_L \parallel R_o)}$$

which is of the STC LP type.

$$\omega_0 = \frac{1}{C_L (R_L \parallel R_o)}$$

$$= \frac{1}{4.5 \times 10^{-9} (10^3 \parallel R_o)}$$

For  $\omega_0$  to be at least  $\omega\pi \times 40 \times 10^3$ , the highest value allowed for  $R_o$  is

$$R_o = \frac{10^3}{2\pi \times 40 \times 10^3 \times 10^3 \times 4.5 \times 10^{-9} - 1}$$

$$= \frac{10^3}{1.131 - 1} = 7.64 \text{ k}\Omega$$

The dc gain is

$$G_m (R_L \parallel R_o)$$

To ensure a dc gain of at least 40 dB (i.e., 100), the minimum value of  $G_m$  is

$$\Rightarrow R_L \geq 100 / (10^3 \parallel 7.64 \times 10^3) = 113.1 \text{ mA/V}$$

**Ex: 1.24** Refer to Fig. E1.24

$$\frac{V_2}{V_s} = \frac{R_i}{R_s + \frac{1}{sC} + R_i} = \frac{R_i}{R_s + R_i} \frac{s}{s + \frac{1}{C(R_s + R_i)}}$$

which is an HP STC function.

$$f_{3dB} = \frac{1}{2\pi C(R_s + R_i)} \leq 100 \text{ Hz}$$

$$C \geq \frac{1}{2\pi(1+9)10^3 \times 100} = 0.16 \mu\text{F}$$

**Ex: 1.25**  $T = 50 \text{ K}$

$$n_i = BT^{3/2} e^{-E_g/(2kT)}$$

$$= 7.3 \times 10^{15} (50)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 50)}$$

$$\simeq 9.6 \times 10^{-39} / \text{cm}^3$$

$T = 350 \text{ K}$

$$n_i = BT^{3/2} e^{-E_g/(2kT)}$$

$$= 7.3 \times 10^{15} (350)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 350)}$$

$$= 4.15 \times 10^{11} / \text{cm}^3$$

**Ex: 1.26**  $N_D = 10^{17} / \text{cm}^3$

From Exercise 1.26,  $n_i$  at

$$T = 350 \text{ K} = 4.15 \times 10^{11} / \text{cm}^3$$

$$n_n = N_D = 10^{17} / \text{cm}^3$$

$$p_n \simeq \frac{n_i^2}{N_D}$$

$$= \frac{(4.15 \times 10^{11})^2}{10^{17}}$$

$$= 1.72 \times 10^6 / \text{cm}^3$$

**Ex: 1.27** At 300 K,  $n_i = 1.5 \times 10^{10} / \text{cm}^3$

$$p_p = N_A$$

Want electron concentration

$$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4 / \text{cm}^3$$

$$\therefore N_A = p_p = \frac{n_i^2}{n_p}$$

$$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$$

$$= 1.5 \times 10^{16} / \text{cm}^3$$

**Ex: 1.28** (a)  $v_{n\text{-drift}} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to  $E$ .

We use

$$v_{n\text{-drift}} = 1350 \times \frac{1}{2 \times 10^{-4}} \because 1 \mu\text{m} = 10^{-4} \text{ cm}$$

$$= 6.75 \times 10^6 \text{ cm/s} = 6.75 \times 10^4 \text{ m/s}$$

(b) Time taken to cross 2- $\mu\text{m}$

$$\text{length} = \frac{2 \times 10^{-6}}{6.75 \times 10^4} \simeq 30 \text{ ps}$$

(c) In  $n$ -type silicon, drift current density  $J_n$  is

$$J_n = qn\mu_n E$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1 \text{ V}}{2 \times 10^{-4}}$$

$$= 1.08 \times 10^4 \text{ A/cm}^2$$

(d) Drift current  $I_n = AJ_n$

$$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$$

$$= 27 \mu\text{A}$$

The resistance of the bar is

$$R = \rho \times \frac{L}{A}$$

$$= qn\mu_n \times \frac{L}{A}$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{2 \times 10^{-4}}{0.25 \times 10^{-8}}$$

$$= 37.0 \text{ k}\Omega$$

Alternatively, we may simply use the preceding result for current and write

$$R = V/I_n = 1 \text{ V}/27 \mu\text{A} = 37.0 \text{ k}\Omega$$

Note that  $0.25 \mu\text{m}^2 = 0.25 \times 10^{-8} \text{ cm}^2$ .

**Ex: 1.29**  $J_n = qD_n \frac{dn(x)}{dx}$

From Fig. E1.29,

$$n_0 = 10^{17}/\text{cm}^3 = 10^5/(\mu\text{m})^3$$

$$D_n = 35 \text{ cm}^2/\text{s} = 35 \times (10^4)^2 (\mu\text{m})^2/\text{s}$$

$$= 35 \times 10^8 (\mu\text{m})^2/\text{s}$$

$$\frac{dn}{dx} = \frac{10^5 - 0}{0.5} = 2 \times 10^5 \mu\text{m}^{-4}$$

$$J_n = qD_n \frac{dn(x)}{dx}$$

$$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 2 \times 10^5$$

$$= 112 \times 10^{-6} \text{ A}/\mu\text{m}^2$$

$$= 112 \mu\text{A}/\mu\text{m}^2$$

For  $I_n = 1 \text{ mA} = J_n \times A$

$$\Rightarrow A = \frac{1 \text{ mA}}{J_n} = \frac{10^3 \mu\text{A}}{112 \mu\text{A}/(\mu\text{m})^2} \simeq 9 \mu\text{m}^2$$

**Ex: 1.30** Using Eq. (1.44),

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

$$D_n = \mu_n V_T = 1350 \times 25.9 \times 10^{-3}$$

$$\simeq 35 \text{ cm}^2/\text{s}$$

$$D_p = \mu_p V_T = 480 \times 25.9 \times 10^{-3}$$

$$\simeq 12.4 \text{ cm}^2/\text{s}$$

**Ex: 1.31** Equation (1.49)

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$$

$$= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) V_0}$$

$$W^2 = \frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) V_0$$

$$V_0 = \frac{1}{2} \left( \frac{q}{\epsilon_s} \right) \left( \frac{N_A N_D}{N_A + N_D} \right) W^2$$

**Ex: 1.32** In a  $p^+n$  diode  $N_A \gg N_D$

Equation (1.49)  $W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$

We can neglect the term  $\frac{1}{N_A}$  as compared to  $\frac{1}{N_D}$ , thus

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D} \cdot V_0}$$

Equation (1.50)  $x_n = W \frac{N_A}{N_A + N_D}$

$$\simeq W \frac{N_A}{N_A}$$

$$= W$$

Equation (1.51),  $x_p = W \frac{N_D}{N_A + N_D}$

since  $N_A \gg N_D$

$$\simeq W \frac{N_D}{N_A} = W \left/ \left( \frac{N_A}{N_D} \right) \right.$$

Equation (1.52),  $Q_J = Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W$

$$\simeq Aq \frac{N_A N_D}{N_A} W$$

$$= AqN_D W$$

Equation (1.53),  $Q_J = A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) V_0}$

$$\begin{aligned} &\simeq A\sqrt{2\epsilon_s q \left(\frac{N_A N_D}{N_A}\right)} V_0 \text{ since } N_A \gg N_D \\ &= A\sqrt{2\epsilon_s q N_D} V_0 \end{aligned}$$

**Ex: 1.33** In Example 1.10,  $N_A = 10^{18}/\text{cm}^3$  and  $N_D = 10^{16}/\text{cm}^3$

In the  $n$ -region of this  $pn$  junction

$$\begin{aligned} n_n &= N_D = 10^{16}/\text{cm}^3 \\ p_n &= \frac{n_i^2}{n_n} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3 \end{aligned}$$

As one can see from above equation, to increase minority-carrier concentration ( $p_n$ ) by a factor of 2, one must lower  $N_D$  ( $= n_n$ ) by a factor of 2.

**Ex: 1.34**

Equation (1.64)  $I_S = Aqn_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$

since  $\frac{D_p}{L_p}$  and  $\frac{D_n}{L_n}$  have approximately

similar values, if  $N_A \gg N_D$ , then the term  $\frac{D_n}{L_n N_A}$  can be neglected as compared to  $\frac{D_p}{L_p N_D}$

$$\therefore I_S \cong Aqn_i^2 \frac{D_p}{L_p N_D}$$

**Ex: 1.35**  $I_S = Aqn_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$

$$= 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\times \left( \frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right)$$

$$= 1.46 \times 10^{-14} \text{ A}$$

$$I = I_S(e^{V/V_T} - 1)$$

$$\simeq I_S e^{V/V_T} = 1.45 \times 10^{-14} e^{0.605/(25.9 \times 10^{-3})}$$

$$= 0.2 \text{ mA}$$

**Ex: 1.36**  $W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} (V_0 - V_F)$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{18}} + \frac{1}{10^{16}} \right)} (0.814 - 0.605)$$

$$= 1.66 \times 10^{-5} \text{ cm} = 0.166 \text{ } \mu\text{m}$$

**Ex: 1.37**  $W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} (V_0 + V_R)$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{18}} + \frac{1}{10^{16}} \right)} (0.814 + 2)$$

$$= 6.08 \times 10^{-5} \text{ cm} = 0.608 \text{ } \mu\text{m}$$

Using Eq. (1.52),

$$\begin{aligned} Q_J &= Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W \\ &= 10^{-4} \times 1.6 \times 10^{-19} \left( \frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \times 6.08 \times \\ &\quad 10^{-5} \text{ cm} \\ &= 9.63 \text{ pC} \end{aligned}$$

Reverse current  $I = I_S = Aqn_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$

$$= 10^{-14} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\times \left( \frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right)$$

$$= 7.3 \times 10^{-15} \text{ A}$$

**Ex: 1.38** Equation (1.69),

$$\begin{aligned} C_{j0} &= A\sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \left(\frac{1}{V_0}\right)} \\ &= 10^{-4} \sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)} \\ &\quad \sqrt{\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right) \left(\frac{1}{0.814}\right)} \\ &= 3.2 \text{ pF} \end{aligned}$$

Equation (1.70),

$$\begin{aligned} C_j &= \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \\ &= \frac{3.2 \times 10^{-12}}{\sqrt{1 + \frac{2}{0.814}}} \\ &= 1.72 \text{ pF} \end{aligned}$$

**Ex: 1.39**  $C_d = \frac{dQ}{dV} = \frac{d}{dV}(\tau_T I)$

$$= \frac{d}{dV}[\tau_T \times I_S(e^{V/V_T} - 1)]$$

$$= \tau_T I_S \frac{d}{dV} (e^{V/V_T} - 1)$$

$$= \tau_T I_S \frac{1}{V_T} e^{V/V_T}$$

$$= \frac{\tau_T}{V_T} \times I_S e^{V/V_T}$$

$$\cong \left(\frac{\tau_T}{V_T}\right) I$$

Exercise 1–8

**Ex: 1.40** Equation (1.73),

$$\begin{aligned}\tau_p &= \frac{L_p^2}{D_p} \\ &= \frac{(5 \times 10^{-4})^2}{10} \\ &= 25 \text{ ns}\end{aligned}$$

Equation (3.57),

$$C_d = \left( \frac{\tau_T}{V_T} \right) I$$

In Example 1.6,  $N_A = 10^{18}/\text{cm}^3$ ,

$$N_D = 10^{16}/\text{cm}^3$$

Assuming  $N_A \gg N_D$ ,

$$\tau_T \simeq \tau_p = 25 \text{ ns}$$

$$\therefore C_d = \left( \frac{25 \times 10^{-9}}{25.9 \times 10^{-3}} \right) 0.1 \times 10^{-3}$$

$$= 96.5 \text{ pF}$$

**Solutions to End-of-Chapter Problems**

1.1 (a)  $V = IR = 5 \text{ mA} \times 1 \text{ k}\Omega = 5 \text{ V}$

$$P = I^2R = (5 \text{ mA})^2 \times 1 \text{ k}\Omega = 25 \text{ mW}$$

(b)  $R = V/I = 5 \text{ V}/1 \text{ mA} = 5 \text{ k}\Omega$

$$P = VI = 5 \text{ V} \times 1 \text{ mA} = 5 \text{ mW}$$

(c)  $I = P/V = 100 \text{ mW}/10 \text{ V} = 10 \text{ mA}$

$$R = V/I = 10 \text{ V}/10 \text{ mA} = 1 \text{ k}\Omega$$

(d)  $V = P/I = 1 \text{ mW}/0.1 \text{ mA}$

$$= 10 \text{ V}$$

$$R = V/I = 10 \text{ V}/0.1 \text{ mA} = 100 \text{ k}\Omega$$

(e)  $P = I^2R \Rightarrow I = \sqrt{P/R}$

$$I = \sqrt{1000 \text{ mW}/1 \text{ k}\Omega} = 31.6 \text{ mA}$$

$$V = IR = 31.6 \text{ mA} \times 1 \text{ k}\Omega = 31.6 \text{ V}$$

Note: V, mA, k $\Omega$ , and mW constitute a consistent set of units.

1.2 (a)  $I = \frac{V}{R} = \frac{5 \text{ V}}{1 \text{ k}\Omega} = 5 \text{ mA}$

(b)  $R = \frac{V}{I} = \frac{5 \text{ V}}{1 \text{ mA}} = 5 \text{ k}\Omega$

(c)  $V = IR = 0.1 \text{ mA} \times 10 \text{ k}\Omega = 1 \text{ V}$

(d)  $I = \frac{V}{R} = \frac{1 \text{ V}}{100 \Omega} = 0.01 \text{ A} = 10 \text{ mA}$

Note: Volts, milliamps, and kilohms constitute a consistent set of units.

1.3 (a)  $P = I^2R = (20 \times 10^{-3})^2 \times 1 \times 10^3$   
 $= 0.4 \text{ W}$

Thus,  $R$  should have a  $\frac{1}{2}$ -W rating.

(b)  $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3$   
 $= 1.6 \text{ W}$

Thus, the resistor should have a 2-W rating.

(c)  $P = I^2R = (1 \times 10^{-3})^2 \times 100 \times 10^3$   
 $= 0.1 \text{ W}$

Thus, the resistor should have a  $\frac{1}{8}$ -W rating.

(d)  $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3$   
 $= 0.16 \text{ W}$

Thus, the resistor should have a  $\frac{1}{4}$ -W rating.

(e)  $P = V^2/R = 20^2/(1 \times 10^3) = 0.4 \text{ W}$

Thus, the resistor should have a  $\frac{1}{2}$ -W rating.

(f)  $P = V^2/R = 11^2/(1 \times 10^3) = 0.121 \text{ W}$

Thus, a rating of  $\frac{1}{8}$  W should theoretically

suffice, though  $\frac{1}{4}$  W would be prudent to allow for inevitable tolerances and measurement errors.

1.4 See figure on next page, which shows how to realize the required resistance values.

1.5 Shunting the 10 k $\Omega$  by a resistor of value of  $R$  results in the combination having a resistance  $R_{\text{eq}}$ ,

$$R_{\text{eq}} = \frac{10R}{R + 10}$$

Thus, for a 1% reduction,

$$\frac{R}{R + 10} = 0.99 \Rightarrow R = 990 \text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R + 10} = 0.95 \Rightarrow R = 190 \text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R + 10} = 0.90 \Rightarrow R = 90 \text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R + 10} = 0.50 \Rightarrow R = 10 \text{ k}\Omega$$

Shunting the 10 k $\Omega$  by

(a) 1 M $\Omega$  results in

$$R_{\text{eq}} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9 \text{ k}\Omega$$

a 1% reduction;

(b) 100 k $\Omega$  results in

$$R_{\text{eq}} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09 \text{ k}\Omega$$

a 9.1% reduction;

(c) 10 k $\Omega$  results in

$$R_{\text{eq}} = \frac{10}{10 + 10} = 5 \text{ k}\Omega$$

a 50% reduction.

1.6 Use voltage divider to find  $V_O$

$$V_O = 5 \frac{2}{2 + 3} = 2 \text{ V}$$

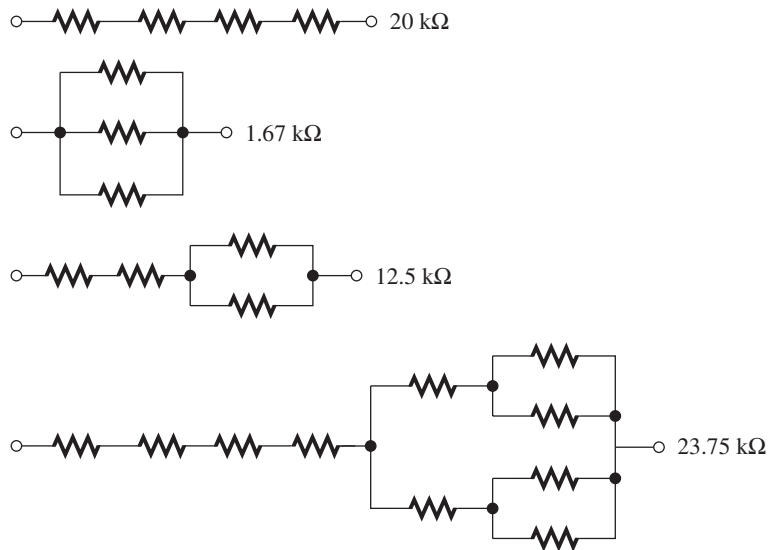
Equivalent output resistance  $R_O$  is

$$R_O = (2 \text{ k}\Omega \parallel 3 \text{ k}\Omega) = 1.2 \text{ k}\Omega$$



This figure belongs to Problem 1.4.

All resistors are 5 kΩ



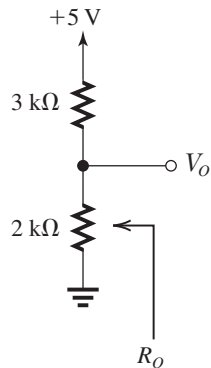
The extreme values of  $V_O$  for  $\pm 5\%$  tolerance resistor are

$$V_{Omin} = 5 \frac{2(1 - 0.05)}{2(1 - 0.05) + 3(1 + 0.05)}$$

$$= 1.88 \text{ V}$$

$$V_{Omax} = 5 \frac{2(1 + 0.05)}{2(1 + 0.05) + 3(1 - 0.05)}$$

$$= 2.12 \text{ V}$$



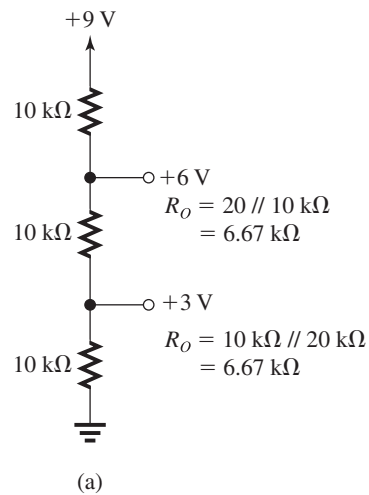
The extreme values of  $R_O$  for  $\pm 5\%$  tolerance resistors are  $1.2 \times 1.05 = 1.26 \text{ k}\Omega$  and  $1.2 \times 0.95 = 1.14 \text{ k}\Omega$ .

1.7  $V_O = V_{DD} \frac{R_2}{R_1 + R_2}$

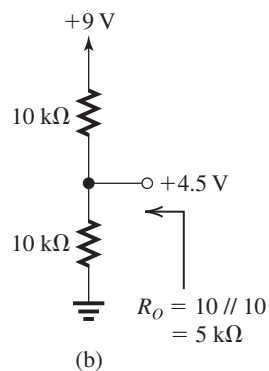
To find  $R_O$ , we short-circuit  $V_{DD}$  and look back into node X,

$$R_O = R_2 \parallel R_1 = \frac{R_1 R_2}{R_1 + R_2}$$

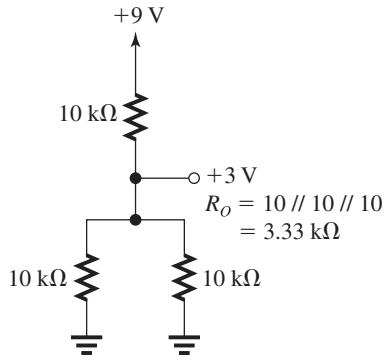
1.8



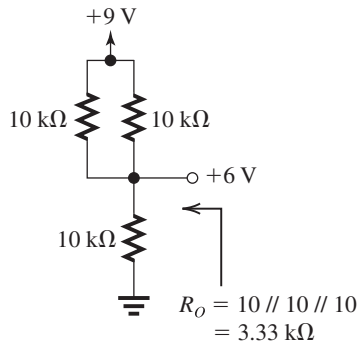
(a)



(b)



(c)



(d)

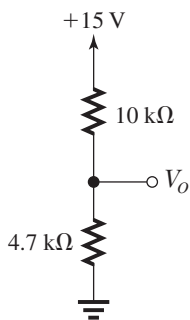
Voltage generated:

+3V [two ways: (a) and (c) with (c) having lower output resistance]

+4.5 V (b)

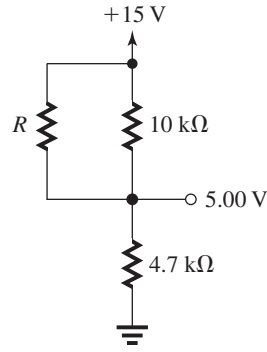
+6V [two ways: (a) and (d) with (d) having a lower output resistance]

1.9



$$V_o = 15 \frac{4.7}{10 + 4.7} = 4.80 \text{ V}$$

To increase  $V_o$  to 10.00 V, we shunt the 10-k $\Omega$  resistor by a resistor  $R$  whose value is such that  $10 \parallel R = 2 \times 4.7$ .



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

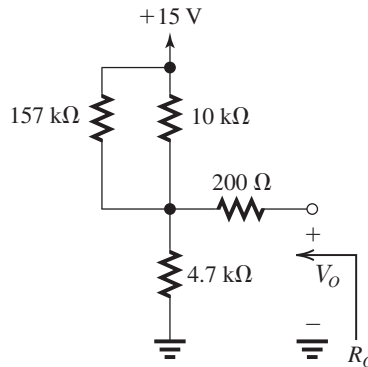
$$\Rightarrow R = 156.7 \approx 157 \text{ k}\Omega$$

Now,

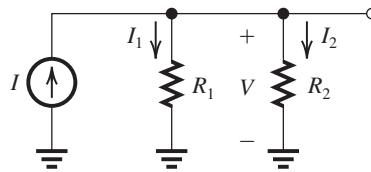
$$R_o = 10 \text{ k}\Omega \parallel R \parallel 4.7 \text{ k}\Omega$$

$$= 9.4 \parallel 4.7 = \frac{9.4}{3} = 3.133 \text{ k}\Omega$$

To make  $R_o = 3.33$ , we add a series resistance of approximately 200  $\Omega$ , as shown below,



1.10



$$V = I(R_1 \parallel R_2)$$

$$= I \frac{R_1 R_2}{R_1 + R_2}$$

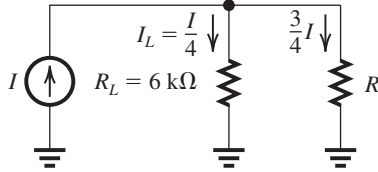
$$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = I \frac{R_1}{R_1 + R_2}$$

**1.11** Connect a resistor  $R$  in parallel with  $R_L$ . To make  $I_L = I/4$  (and thus the current through  $R$ ,  $3I/4$ ),  $R$  should be such that

$$6I/4 = 3IR/4$$

$$\Rightarrow R = 2 \text{ k}\Omega$$



**1.12** The parallel combination of the resistors is  $R_{\parallel}$  where

$$\frac{1}{R_{\parallel}} = \sum_{i=1}^N 1/R_i$$

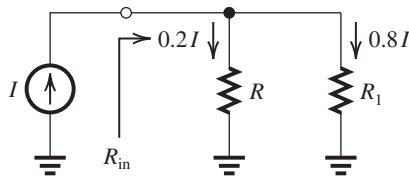
The voltage across them is

$$V = I \times R_{\parallel} = \frac{I}{\sum_{i=1}^N 1/R_i}$$

Thus, the current in resistor  $R_k$  is

$$I_k = V/R_k = \frac{I/R_k}{\sum_{i=1}^N 1/R_i}$$

**1.13**



To make the current through  $R$  equal to  $0.2I$ , we shunt  $R$  by a resistance  $R_1$  having a value such that the current through it will be  $0.8I$ ; thus

$$0.2IR = 0.8IR_1 \Rightarrow R_1 = \frac{R}{4}$$

The input resistance of the divider,  $R_{in}$ , is

$$R_{in} = R \parallel R_1 = R \parallel \frac{R}{4} = \frac{1}{5}R$$

Now if  $R_1$  is 10% too high, that is, if

$$R_1 = 1.1 \frac{R}{4}$$

the problem can be solved in two ways:

(a) Connect a resistor  $R_2$  across  $R_1$  of value such that  $R_2 \parallel R_1 = R/4$ , thus

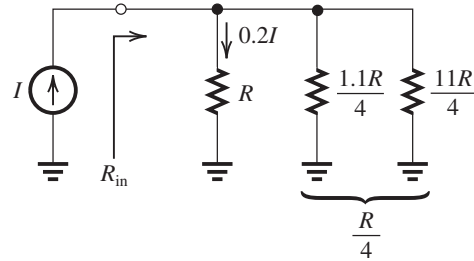
$$\frac{R_2(1.1R/4)}{R_2 + (1.1R/4)} = \frac{R}{4}$$

$$1.1R_2 = R_2 + \frac{1.1R}{4}$$

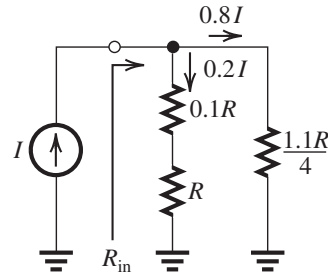
$$\Rightarrow R_2 = \frac{11R}{4} = 2.75 R$$

$$R_{in} = R \parallel \frac{1.1R}{4} \parallel \frac{11R}{4}$$

$$= R \parallel \frac{R}{4} = \frac{R}{5}$$



(b) Connect a resistor in series with the load resistor  $R$  so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of  $R$  (i.e.,  $0.1R$ ).



$$R_{in} = 1.1R \parallel \frac{1.1R}{4}$$

$$= \frac{1.1R}{5}$$

that is, 10% higher than in case (a).

**1.14** For  $R_L = 10 \text{ k}\Omega$ , when signal source generates 0–0.5 mA, a voltage of 0–2 V may appear across the source

To limit  $v_s \leq 1 \text{ V}$ , the net resistance has to be  $\leq 2 \text{ k}\Omega$ . To achieve this we have to shunt  $R_L$  with a resistor  $R$  so that  $(R \parallel R_L) \leq 2 \text{ k}\Omega$ .

$$R \parallel R_L \leq 2 \text{ k}\Omega.$$

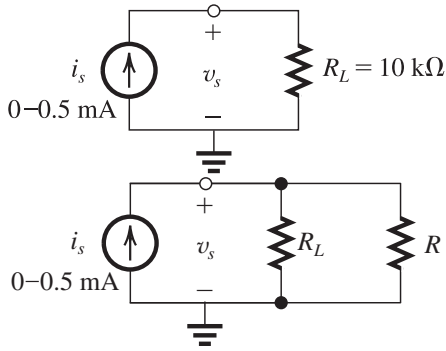
$$\frac{RR_L}{R + R_L} \leq 2 \text{ k}\Omega$$

$$\text{For } R_L = 10 \text{ k}\Omega$$

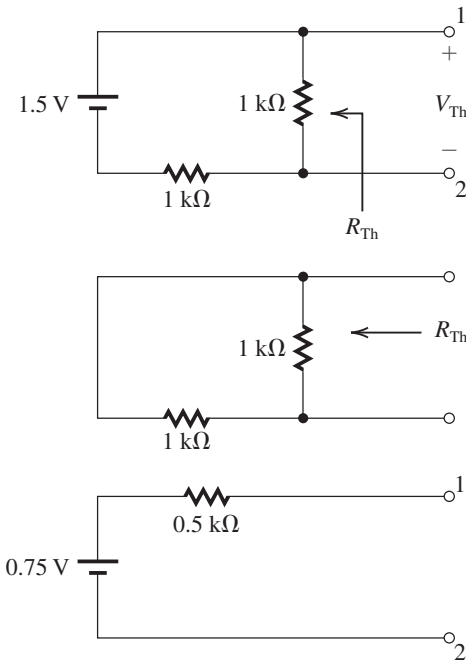
$$R \leq 2.5 \text{ k}\Omega$$

The resulting circuit needs only one additional resistance of  $2 \text{ k}\Omega$  in parallel with  $R_L$  so that

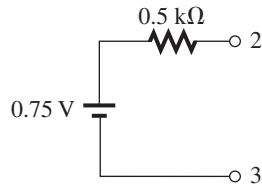
$v_s \leq 1$  V. The circuit is a current divider, and the current through  $R_L$  is now 0-0.1 mA.



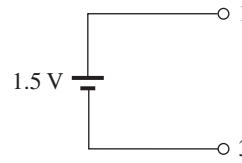
1.15 (a) Between terminals 1 and 2:



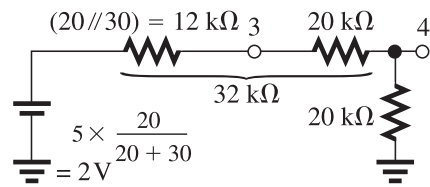
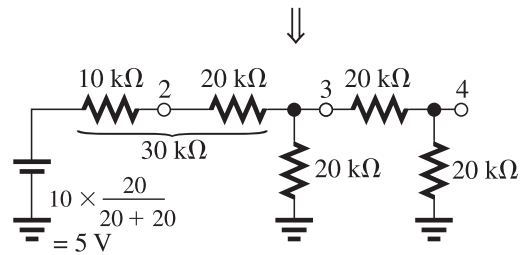
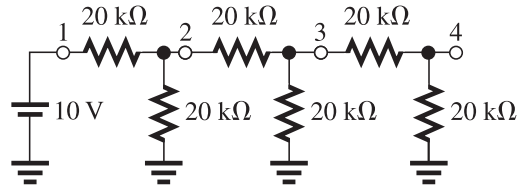
(b) Same procedure is used for (b) to obtain



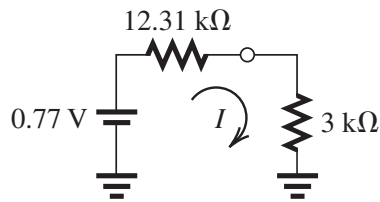
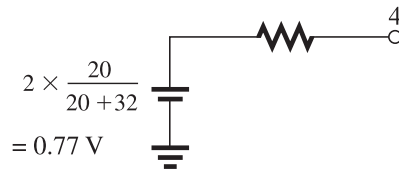
(c) Between terminals 1 and 3, the open-circuit voltage is 1.5 V. When we short circuit the voltage source, we see that the Thévenin resistance will be zero. The equivalent circuit is then



1.16



Thévenin equivalent:  $(20 // 32) = 12.31$  kΩ

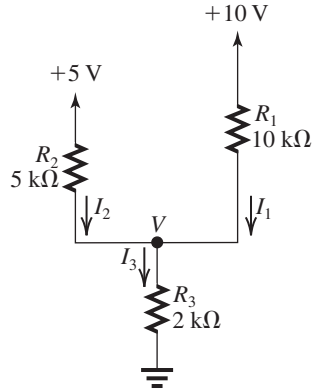


Now, when a resistance of 3 kΩ is connected between node 4 and ground,

$$I = \frac{0.77}{12.31 + 3}$$

$$= 0.05 \text{ mA}$$

1.17



(a) Node equation at the common mode yields

$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across  $R_1$  and  $R_3$  equals 10 V, we write

$$10 = I_1 R_1 + I_3 R_3$$

$$= 10I_1 + (I_1 + I_2) \times 2$$

$$= 12I_1 + 2I_2$$

That is,

$$12I_1 + 2I_2 = 10 \quad (1)$$

Similarly, the voltage drops across  $R_2$  and  $R_3$  add up to 5 V, thus

$$5 = I_2 R_2 + I_3 R_3$$

$$= 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 5 \quad (2)$$

Equations (1) and (2) can be solved together by multiplying Eq. (2) by 6:

$$12I_1 + 42I_2 = 30 \quad (3)$$

Now, subtracting Eq. (1) from Eq. (3) yields

$$40I_2 = 20$$

$$\Rightarrow I_2 = 0.5 \text{ mA}$$

Substituting in Eq. (2) gives

$$2I_1 = 5 - 7 \times 0.5 \text{ mA}$$

$$\Rightarrow I_1 = 0.75 \text{ mA}$$

$$I_3 = I_1 + I_2$$

$$= 0.75 + 0.5$$

$$= 1.25 \text{ mA}$$

$$V = I_3 R_3$$

$$= 1.25 \times 2 = 2.5 \text{ V}$$

To summarize:

$$I_1 = 0.75 \text{ mA} \quad I_2 = 0.5 \text{ mA}$$

$$I_3 = 1.25 \text{ mA} \quad V = 2.5 \text{ V}$$

(b) A node equation at the common node can be written in terms of  $V$  as

$$\frac{10 - V}{R_1} + \frac{5 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{10 - V}{10} + \frac{5 - V}{5} = \frac{V}{2}$$

$$\Rightarrow 0.8V = 2$$

$$\Rightarrow V = 2.5 \text{ V}$$

Now,  $I_1$ ,  $I_2$ , and  $I_3$  can be easily found as

$$I_1 = \frac{10 - V}{10} = \frac{10 - 2.5}{10}$$

$$= 0.75 \text{ mA}$$

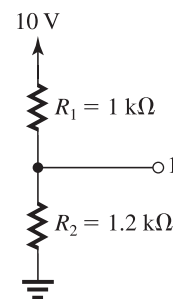
$$I_2 = \frac{5 - V}{5} = \frac{5 - 2.5}{5}$$

$$= 0.5 \text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{2.5}{2} = 1.25 \text{ mA}$$

Method (b) is much preferred, being faster, more insightful, and less prone to errors. In general, one attempts to identify the lowest possible number of variables and write the corresponding minimum number of equations.

1.18 Find the Thévenin equivalent of the circuit to the left of node 1.

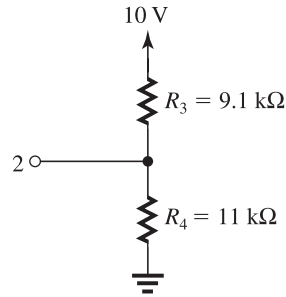


Between node 1 and ground,

$$R_{\text{Th}} = (1 \text{ k}\Omega \parallel 1.2 \text{ k}\Omega) = 0.545 \text{ k}\Omega$$

$$V_{Th} = 10 \times \frac{1.2}{1 + 1.2} = 5.45 \text{ V}$$

Find the Thévenin equivalent of the circuit to the right of node 2.

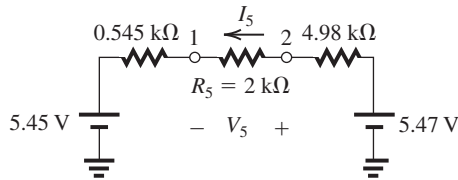


Between node 2 and ground,

$$R_{Th} = 9.1 \text{ k}\Omega \parallel 11 \text{ k}\Omega = 4.98 \text{ k}\Omega$$

$$V_{Th} = 10 \times \frac{11}{11 + 9.1} = 5.47 \text{ V}$$

The resulting simplified circuit is



$$I_5 = \frac{5.47 - 5.45}{4.98 + 2 + 0.545}$$

$$= 2.66 \mu\text{A}$$

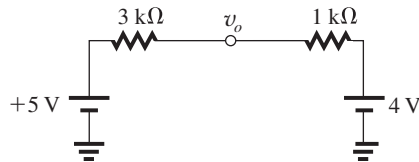
$$V_5 = 2.66 \mu\text{A} \times 2 \text{ k}\Omega$$

$$= 5.32 \text{ mV}$$

**1.19** We first find the Thévenin equivalent of the source to the right of  $v_o$ .

$$V = 4 \times 1 = 4 \text{ V}$$

Then, we may redraw the circuit in Fig. P1.19 as shown below



Then, the voltage at  $v_o$  is found from a simple voltage division.

$$v_o = 4 + (5 - 4) \times \frac{1}{3 + 1} = 4.25 \text{ V}$$

**1.20** Refer to Fig. P1.20. Using the voltage divider rule at the input side, we obtain

$$\frac{v_\pi}{v_s} = \frac{r_\pi}{r_\pi + R_s} \quad (1)$$

At the output side, we find  $v_o$  by multiplying the current  $g_m v_\pi$  by the parallel equivalent of  $r_o$  and  $R_L$ ,

$$v_o = -g_m v_\pi (r_o \parallel R_L) \quad (2)$$

Finally,  $v_o/v_s$  can be obtained by combining Eqs. (1) and (2) as

$$\frac{v_o}{v_s} = -\frac{r_\pi}{r_\pi + R_s} g_m (r_o \parallel R_L)$$

**1.21** (a)  $T = 10^{-4} \text{ ms} = 10^{-7} \text{ s}$

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^7 \text{ rad/s}$$

(b)  $f = 1 \text{ GHz} = 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

(c)  $\omega = 6.28 \times 10^2 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

(d)  $T = 10 \text{ s}$

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

(e)  $f = 60 \text{ Hz}$

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$

$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

(f)  $\omega = 1 \text{ krad/s} = 10^3 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

(g)  $f = 1900 \text{ MHz} = 1.9 \times 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 5.26 \times 10^{-10} \text{ s}$$

$$\omega = 2\pi f = 1.194 \times 10^{10} \text{ rad/s}$$

**1.22** (a)  $Z = R + \frac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$$= (1 - j1.59) \text{ k}\Omega$$

$$(b) Y = \frac{1}{R} + j\omega C$$

$$= \frac{1}{10^4} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$$

$$= 10^{-4}(1 + j6.28) \Omega$$

$$Z = \frac{1}{Y} = \frac{10^4}{1 + j6.28}$$

$$= \frac{10^4(1 - j6.28)}{1 + 6.28^2}$$

$$= (247.3 - j1553) \Omega$$

$$(c) Y = \frac{1}{R} + j\omega C$$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$$

$$= 10^{-5}(1 + j0.628)$$

$$Z = \frac{10^5}{1 + j0.628}$$

$$= (71.72 - j45.04) \text{ k}\Omega$$

$$(d) Z = R + j\omega L$$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$

$$= 100 + j6.28 \times 100$$

$$= (100 + j628), \Omega$$

**1.23** (a)  $Z = 1 \text{ k}\Omega$  at all frequencies

$$(b) Z = 1 / j\omega C = -j \frac{1}{2\pi f \times 10 \times 10^{-9}}$$

$$\text{At } f = 60 \text{ Hz, } Z = -j265 \text{ k}\Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = -j159 \Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = -j0.016 \Omega$$

$$(c) Z = 1 / j\omega C = -j \frac{1}{2\pi f \times 10 \times 10^{-12}}$$

$$\text{At } f = 60 \text{ Hz, } Z = -j0.265 \text{ G}\Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = -j0.16 \text{ M}\Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = -j15.9 \Omega$$

$$(d) Z = j\omega L = j2\pi fL = j2\pi f \times 10 \times 10^{-3}$$

$$\text{At } f = 60 \text{ Hz, } Z = j3.77 \Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = j6.28 \text{ k}\Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = j62.8 \text{ M}\Omega$$

$$(e) Z = j\omega L = j2\pi fL = j2\pi f(1 \times 10^{-6})$$

$$f = 60 \text{ Hz, } Z = j0.377 \text{ m}\Omega$$

$$f = 100 \text{ kHz, } Z = j0.628 \Omega$$

$$f = 1 \text{ GHz, } Z = j6.28 \text{ k}\Omega$$

$$\mathbf{1.24} Y = \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1 - \omega^2 LC}{j\omega L}$$

$$\Rightarrow Z = \frac{1}{Y} = \frac{j\omega L}{1 - \omega^2 LC}$$

The frequency at which  $|Z| = \infty$  is found letting the denominator equal zero:

$$1 - \omega^2 LC = 0$$

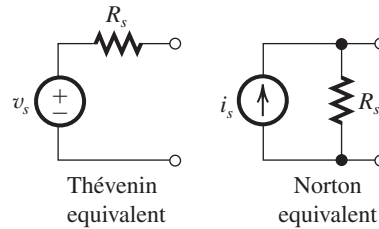
$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

At frequencies just below this,  $\angle Z = +90^\circ$ .

At frequencies just above this,  $\angle Z = -90^\circ$ .

Since the impedance is infinite at this frequency, the current drawn from an ideal voltage source is zero.

**1.25**



$$v_{oc} = v_s$$

$$i_{sc} = i_s$$

$$v_s = i_s R_s$$

Thus,

$$R_s = \frac{v_{oc}}{i_{sc}}$$

$$(a) v_s = v_{oc} = 1 \text{ V}$$

$$i_s = i_{sc} = 0.1 \text{ mA}$$

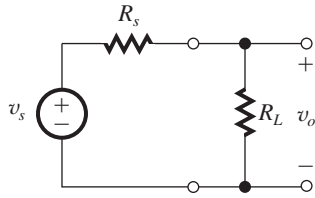
$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{1 \text{ V}}{0.1 \text{ mA}} = 10 \text{ k}\Omega$$

$$(b) v_s = v_{oc} = 0.1 \text{ V}$$

$$i_s = i_{sc} = 1 \mu\text{A}$$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{0.1 \text{ V}}{1 \mu\text{A}} = 0.1 \text{ M}\Omega = 100 \text{ k}\Omega$$

## 1.26



$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

$$v_o = v_s \left/ \left( 1 + \frac{R_s}{R_L} \right) \right.$$

Thus,

$$\frac{v_s}{1 + \frac{R_s}{100}} = 40 \quad (1)$$

and

$$\frac{v_s}{1 + \frac{R_s}{10}} = 10 \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$\frac{1 + (R_s/10)}{1 + (R_s/100)} = 4$$

$$\Rightarrow R_s = 50 \text{ k}\Omega$$

Substituting in Eq. (2) gives

$$v_s = 60 \text{ mV}$$

The Norton current  $i_s$  can be found as

$$i_s = \frac{v_s}{R_s} = \frac{60 \text{ mV}}{50 \text{ k}\Omega} = 1.2 \mu\text{A}$$

**1.27** The nominal values of  $V_L$  and  $I_L$  are given by

$$V_L = \frac{R_L}{R_S + R_L} V_S$$

$$I_L = \frac{V_S}{R_S + R_L}$$

After a 10% increase in  $R_L$ , the new values will be

$$V_L = \frac{1.1R_L}{R_S + 1.1R_L} V_S$$

$$I_L = \frac{V_S}{R_S + 1.1R_L}$$

(a) The nominal values are

$$V_L = \frac{200}{5 + 200} \times 1 = 0.976 \text{ V}$$

$$I_L = \frac{1}{5 + 200} = 4.88 \mu\text{A}$$

After a 10% increase in  $R_L$ , the new values will be

$$V_L = \frac{1.1 \times 200}{5 + 1.1 \times 200} = 0.978 \text{ V}$$

$$I_L = \frac{1}{5 + 1.1 \times 200} = 4.44 \mu\text{A}$$

These values represent a 0.2% and 9% change, respectively. Since the load voltage remains relatively more constant than the load current, a Thévenin source is more appropriate here.

(b) The nominal values are

$$V_L = \frac{50}{5 + 50} \times 1 = 0.909 \text{ V}$$

$$I_L = \frac{1}{5 + 50} = 18.18 \text{ mA}$$

After a 10% increase in  $R_L$ , the new values will be

$$V_L = \frac{1.1 \times 50}{5 + 1.1 \times 50} = 0.917 \text{ V}$$

$$I_L = \frac{1}{5 + 1.1 \times 50} = 16.67 \text{ mA}$$

These values represent a 1% and 8% change, respectively. Since the load voltage remains relatively more constant than the load current, a Thévenin source is more appropriate here.

(c) The nominal values are

$$V_L = \frac{0.1}{2 + 0.1} \times 1 = 47.6 \text{ mV}$$

$$I_L = \frac{1}{2 + 0.1} = 0.476 \text{ mA}$$

After a 10% increase in  $R_L$ , the new values will be

$$V_L = \frac{1.1 \times 0.1}{2 + 1.1 \times 0.1} = 52.1 \text{ mV}$$

$$I_L = \frac{1}{2 + 1.1 \times 0.1} = 0.474 \text{ mA}$$

These values represent a 9% and 0.4% change, respectively. Since the load current remains relatively more constant than the load voltage, a Norton source is more appropriate here. The Norton equivalent current source is

$$I_S = \frac{V_S}{R_S} = \frac{1}{2} = 0.5 \text{ mA}$$

(d) The nominal values are

$$V_L = \frac{16}{150 + 16} \times 1 = 96.4 \text{ mV}$$



$$I_L = \frac{1}{150 + 16} = 6.02 \text{ mA}$$

After a 10% increase in  $R_L$ , the new values will be

$$V_L = \frac{1.1 \times 16}{150 + 1.1 \times 16} = 105 \text{ mV}$$

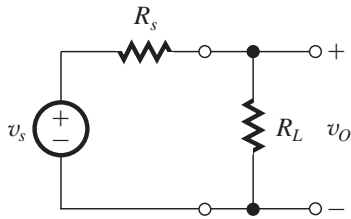
$$I_L = \frac{1}{150 + 1.1 \times 16} = 5.97 \text{ mA}$$

These values represent a 9% and 1% change, respectively. Since the load current remains relatively more constant than the load voltage, a Norton source is more appropriate here. The Norton equivalent current source is

$$I_S = \frac{V_S}{R_S} = \frac{1}{150} = 6.67 \text{ mA}$$

1.28

$$\begin{aligned} P_L &= v_o^2 \times \frac{1}{R_L} \\ &= v_s^2 \frac{R_L^2}{(R_L + R_S)^2} \times \frac{1}{R_L} \\ &= v_s^2 \frac{R_L}{(R_L + R_S)^2} \end{aligned}$$

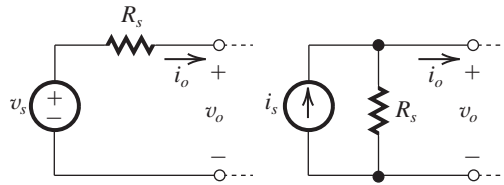


Since we are told that the power delivered to a 16Ω speaker load is 75% of the power delivered to a 32Ω speaker load,

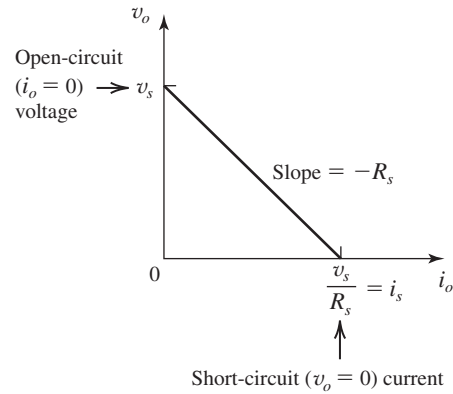
$$\begin{aligned} P_L(R_L = 16\Omega) &= 0.75 \times P_L(R_L = 32\Omega) \\ \frac{16}{(R_S + 32)^2} &= 0.75 \times \frac{32}{(R_S + 32)^2} \\ \frac{\sqrt{16}}{R_S + 32} &= \frac{\sqrt{24}}{R_S + 32} \\ \Rightarrow (\sqrt{24} - \sqrt{16})R_S &= \sqrt{16} \times 32 - \sqrt{24} \times 16 \\ 0.9R_S &= 49.6 \\ R_S &= 55.2\Omega \end{aligned}$$

1.29 The observed output voltage is 1 mV/°C, which is one half the voltage specified by the sensor, presumably under open-circuit conditions: that is, without a load connected. It follows that that sensor internal resistance must be equal to  $R_L$ , that is, 5 kΩ.

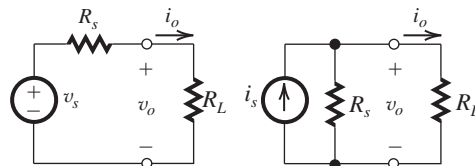
1.30



$$v_o = v_s - i_o R_S$$



1.31



$R_L$  represents the input resistance of the processor

For  $v_o = 0.95v_s$ ,

$$0.95 = \frac{R_L}{R_L + R_S} \Rightarrow R_L = 19R_S$$

For  $i_o = 0.95i_s$ ,

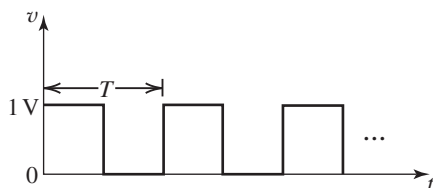
$$0.95 = \frac{R_S}{R_S + R_L} \Rightarrow R_L = R_S/19$$

1.32

Case	$\omega$ (rad/s)	$f$ (Hz)	$T$ (s)
a	$3.14 \times 10^{10}$	$5 \times 10^9$	$0.2 \times 10^{-9}$
b	$2 \times 10^9$	$3.18 \times 10^8$	$3.14 \times 10^{-9}$
c	$6.28 \times 10^{10}$	$1 \times 10^{10}$	$1 \times 10^{-10}$
d	$3.77 \times 10^2$	60	$1.67 \times 10^{-2}$
e	$6.28 \times 10^4$	$1 \times 10^4$	$1 \times 10^{-4}$
f	$6.28 \times 10^5$	$1 \times 10^5$	$1 \times 10^{-5}$

- 1.33 (a)  $v = 10 \sin(2\pi \times 10^3 t)$ , V  
 (b)  $v = 120\sqrt{2} \sin(2\pi \times 60)$ , V  
 (c)  $v = 0.1 \sin(2000t)$ , V  
 (d)  $v = 0.1 \sin(2\pi \times 10^3 t)$ , V

1.34 Comparing the given waveform to that described by Eq. (1.2), we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform looks as follows:



- Average value = 0.5 V  
 Peak-to-peak value = 1 V  
 Lowest value = 0 V  
 Highest value = 1 V  
 Period  $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3}$  s  
 Frequency  $f = \frac{1}{T} = 1$  kHz

- 1.35 (a)  $V_{\text{peak}} = 117 \times \sqrt{2} = 165$  V  
 (b)  $V_{\text{rms}} = 33.9/\sqrt{2} = 24$  V  
 (c)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  V  
 (d)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  kV

1.36 The two harmonics have the ratio  $126/98 = 9/7$ . Thus, these are the 7th and 9th harmonics. From Eq. (1.2), we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of  $98/7$ , or 14 kHz, and peak amplitude of  $63 \times 7 = 441$  mV. The rms value of the fundamental will be  $441/\sqrt{2} = 312$  mV. To find the peak-to-peak amplitude of the square wave, we note that  $4V/\pi = 441$  mV. Thus,

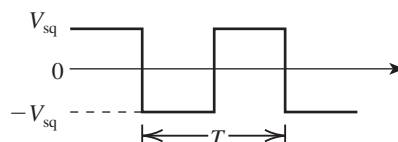
Peak-to-peak amplitude  
 $= 2V = 441 \times \frac{\pi}{2} = 693$  mV  
 Period  $T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4$   $\mu$ s

1.37 The rms value of a symmetrical square wave with peak amplitude  $\hat{V}$  is simply  $\hat{V}$ . Taking the root-mean-square of the first 5 sinusoidal terms in Eq. (1.2) gives an rms value of,

$$\frac{4\hat{V}}{\pi\sqrt{2}} \sqrt{1^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{9}\right)^2} = 0.980\hat{V}$$

which is 2% lower than the rms value of the square wave.

1.38 If the amplitude of the square wave is  $V_{\text{sq}}$ , then the power delivered by the square wave to a resistance  $R$  will be  $V_{\text{sq}}^2/R$ . If this power is to be equal to that delivered by a sine wave of peak amplitude  $\hat{V}$ , then



$$\frac{V_{\text{sq}}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus,  $V_{\text{sq}} = \hat{V}/\sqrt{2}$ . This result is independent of frequency.

### 1.39

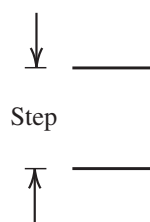
Decimal	Binary
0	0
6	110
11	1011
28	11100
59	111011

1.40 (a) For  $N$  bits there will be  $2^N$  possible levels, from 0 to  $V_{FS}$ . Thus there will be  $(2^N - 1)$  discrete steps from 0 to  $V_{FS}$  with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is



$$\frac{1}{2} \times \text{step size} = \frac{1}{2} \frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

$$(c) \frac{5 \text{ V}}{2^N - 1} \leq 2 \text{ mV}$$

$$2^N - 1 \geq 2500$$

$$2^N \geq 2501 \Rightarrow N = 12,$$

For  $N = 12$ ,

$$\text{Resolution} = \frac{5}{2^{12} - 1} = 1.2 \text{ mV}$$

$$\text{Quantization error} = \frac{1.2}{2} = 0.6 \text{ mV}$$

#### 1.41

$b_3$	$b_2$	$b_1$	$b_0$	Value Represented
0	0	0	0	+0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	-0
1	0	0	1	-1
1	0	1	0	-2
1	0	1	1	-3
1	1	0	0	-4
1	1	0	1	-5
1	1	1	0	-6
1	1	1	1	-7

Note that there are two possible representations of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range  $\pm 3.5 \text{ V}$  can be represented.

Input	Steps	Code
+2.5 V	+5	0101
-3.0 V	-6	1110
+2.7	+5	0101
-2.8	-6	1110

**1.42** (a) When  $b_i = 1$ , the  $i$ th switch is in position 1 and a current ( $V_{ref}/2^i R$ ) flows to the output. Thus  $i_O$  will be the sum of all the currents corresponding to “1” bits, that is,

$$i_O = \frac{V_{ref}}{R} \left( \frac{b_1}{2^1} + \frac{b_2}{2^2} + \dots + \frac{b_N}{2^N} \right)$$

(b)  $b_N$  is the LSB

$b_1$  is the MSB

$$(c) i_{Omax} = \frac{10 \text{ V}}{10 \text{ k}\Omega} \left( \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} \right)$$

$$= 0.99609375 \text{ mA}$$

Corresponding to the LSB changing from 0 to 1 the output changes by  $(10/10) \times 1/2^8 = 3.91 \mu\text{A}$ .

**1.43** There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be  $44,100 \times 16 = 7.056 \times 10^5$  bits per second.

**1.44** Each pixel requires  $8 + 8 + 8 = 24$  bits to represent it. We will approximate a megapixel as  $10^6$  pixels, and a Gbit as  $10^9$  bits. Thus, each image requires  $24 \times 10 \times 10^6 = 2.4 \times 10^8$  bits. The number of such images that fit in 16 Gbits of memory is

$$\lfloor \frac{2.4 \times 10^8}{16 \times 10^9} \rfloor = \lfloor 66.7 \rfloor = 66$$

$$\mathbf{1.45} \text{ (a) } A_v = \frac{v_O}{v_I} = \frac{10 \text{ V}}{100 \text{ mV}} = 100 \text{ V/V}$$

or  $20 \log 100 = 40 \text{ dB}$

$$A_i = \frac{i_O}{i_I} = \frac{v_O/R_L}{i_I} = \frac{10 \text{ V}/100 \Omega}{100 \mu\text{A}} = \frac{0.1 \text{ A}}{100 \mu\text{A}}$$

$$= 1000 \text{ A/A}$$

$$\text{or } 20 \log 1000 = 60 \text{ dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 100 \times 1000$$

$$= 10^5 \text{ W/W}$$

$$\text{or } 10 \log 10^5 = 50 \text{ dB}$$

$$(b) A_v = \frac{v_o}{v_i} = \frac{1 \text{ V}}{10 \mu\text{V}} = 1 \times 10^5 \text{ V/V}$$

$$\text{or } 20 \log 1 \times 10^5 = 100 \text{ dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{1 \text{ V}/10 \text{ k}\Omega}{100 \text{ nA}}$$

$$= \frac{0.1 \text{ mA}}{100 \text{ nA}} = \frac{0.1 \times 10^{-3}}{100 \times 10^{-9}} = 1000 \text{ A/A}$$

$$\text{or } 20 \log A_i = 60 \text{ dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

$$= 1 \times 10^5 \times 1000$$

$$= 1 \times 10^8 \text{ W/W}$$

$$\text{or } 10 \log A_p = 80 \text{ dB}$$

$$(c) A_v = \frac{v_o}{v_i} = \frac{5 \text{ V}}{1 \text{ V}} = 5 \text{ V/V}$$

$$\text{or } 20 \log 5 = 14 \text{ dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{5 \text{ V}/10 \Omega}{1 \text{ mA}}$$

$$= \frac{0.5 \text{ A}}{1 \text{ mA}} = 500 \text{ A/A}$$

$$\text{or } 20 \log 500 = 54 \text{ dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

$$= 5 \times 500 = 2500 \text{ W/W}$$

$$\text{or } 10 \log A_p = 34 \text{ dB}$$

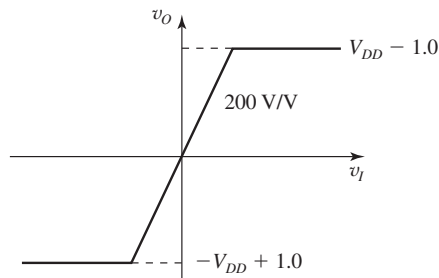
**1.46** For  $\pm 5 \text{ V}$  supplies:

The largest undistorted sine-wave output is of 4-V peak amplitude or  $4/\sqrt{2} = 2.8 \text{ V}_{\text{rms}}$ . Input needed is  $14 \text{ mV}_{\text{rms}}$ .

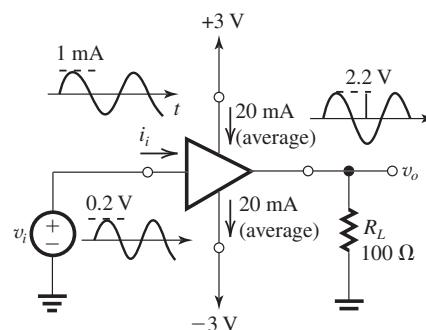
For  $\pm 10\text{-V}$  supplies, the largest undistorted sine-wave output is of 9-V peak amplitude or  $6.4 \text{ V}_{\text{rms}}$ . Input needed is  $32 \text{ mV}_{\text{rms}}$ .

For  $\pm 15\text{-V}$  supplies, the largest undistorted sine-wave output is of 14-V peak amplitude or

$9.9 \text{ V}_{\text{rms}}$ . The input needed is  $9.9 \text{ V}/200 = 49.5 \text{ mV}_{\text{rms}}$ .



**1.47**



$$A_v = \frac{v_o}{v_i} = \frac{2.2}{0.2}$$

$$= 11 \text{ V/V}$$

$$\text{or } 20 \log 11 = 20.8 \text{ dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V}/100 \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

$$\text{or } 20 \log A_i = 26.8 \text{ dB}$$

$$A_p = \frac{p_o}{p_i} = \frac{(2.2/\sqrt{2})^2/100}{\frac{0.2}{\sqrt{2}} \times \frac{10^{-3}}{\sqrt{2}}}$$

$$= 242 \text{ W/W}$$

$$\text{or } 10 \log A_p = 23.8 \text{ dB}$$

$$\text{Supply power} = 2 \times 3 \text{ V} \times 20 \text{ mA} = 120 \text{ mW}$$

$$\text{Output power} = \frac{v_{\text{rms}}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100 \Omega} = 24.2 \text{ mW}$$

$$\text{Input power} = \frac{24.2}{242} = 0.1 \text{ mW (negligible)}$$

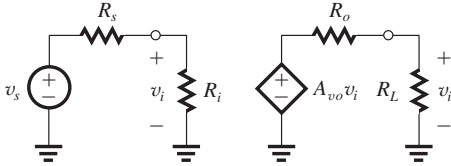
Amplifier dissipation  $\simeq$  Supply power – Output power

$$= 120 - 24.2 = 95.8 \text{ mW}$$

$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

$$\begin{aligned} 1.48 \quad v_o &= A_{vo} v_i \frac{R_L}{R_L + R_o} \\ &= A_{vo} \left( v_s \frac{R_i}{R_i + R_s} \right) \frac{R_L}{R_L + R_o} \end{aligned}$$



Thus,

$$\frac{v_o}{v_s} = A_{vo} \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o}$$

(a)  $A_{vo} = 100, R_i = 10R_s, R_L = 10R_o:$

$$\begin{aligned} \frac{v_o}{v_s} &= 100 \times \frac{10R_s}{10R_s + R_s} \times \frac{10R_o}{10R_o + R_o} \\ &= 82.6 \text{ V/V or } 20 \log 82.6 = 38.3 \text{ dB} \end{aligned}$$

(b)  $A_{vo} = 100, R_i = R_s, R_L = R_o:$

$$\frac{v_o}{v_s} = 100 \times \frac{1}{2} \times \frac{1}{2} = 25 \text{ V/V or } 20 \log 25 = 28 \text{ dB}$$

(c)  $A_{vo} = 100 \text{ V/V}, R_i = R_s/10, R_L = R_o/10:$

$$\begin{aligned} \frac{v_o}{v_s} &= 100 \frac{R_s/10}{(R_s/10) + R_s} \frac{R_o/10}{(R_o/10) + R_o} \\ &= 0.826 \text{ V/V or } 20 \log 0.826 = -1.7 \text{ dB} \end{aligned}$$

1.49 (a)

$$\frac{v_o}{v_s} = \frac{v_i}{v_s} \times \frac{v_o}{v_i}$$

This figure belongs to Problem 1.49.

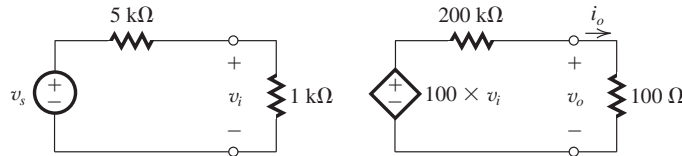


Figure 1

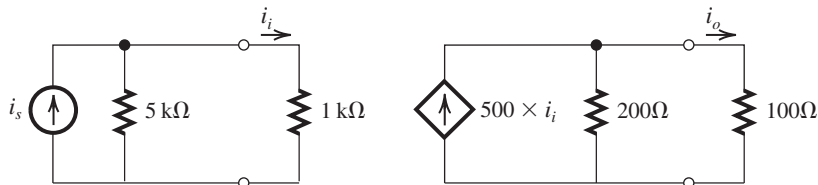


Figure 2

$$\begin{aligned} &= \frac{1}{5 + 1} \times 100 \times \frac{100}{200 + 100} \\ &= 5.56 \text{ V/V} \end{aligned}$$

Much of the amplifier's 100 V/V gain is lost in the source resistance and amplifier's output resistance. If the source were connected directly to the load, the gain would be

$$\frac{v_o}{v_s} = \frac{0.1}{5 + 0.1} = 0.0196 \text{ V/V}$$

This is a factor of 284× smaller than the gain with the amplifier in place!

(b)

The equivalent current amplifier has a dependent current source with a value of

$$\begin{aligned} \frac{100 \text{ V/V}}{200\Omega} \times i_i &= \frac{100 \text{ V/V}}{200\Omega} \times 1000\Omega \times v_i \\ &= 500 \times i_i \end{aligned}$$

$$\text{Thus, } \frac{i_o}{i_s} = \frac{i_i}{i_s} \times \frac{i_o}{i_i}$$

$$\begin{aligned} &= \frac{5}{5 + 1} \times 500 \times \frac{200}{200 + 100} \\ &= 277.8 \text{ A/A} \end{aligned}$$

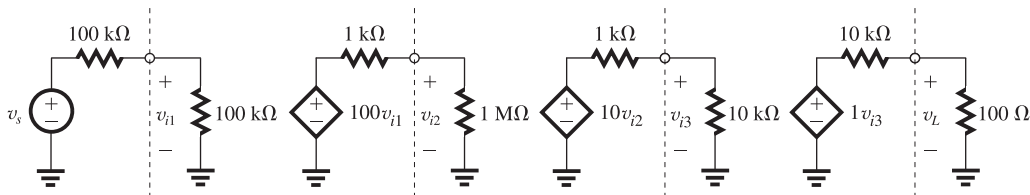
Using the voltage amplifier model, the current gain can be found as follows,

$$\begin{aligned} \frac{i_o}{i_s} &= \frac{i_i}{i_s} \times \frac{v_i}{i_i} \times \frac{i_o}{v_i} \\ &= \frac{5}{5 + 1} \times 1000 \times \frac{100 \text{ V/V}}{200 + 100} \\ &= 277.8 \text{ A/A} \end{aligned}$$

1.50 In Example 1.3, when the first and the second stages are interchanged, the circuit looks like the figure above, and

$$\frac{v_{i1}}{v_s} = \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 100 \text{ k}\Omega} = 0.5 \text{ V/V}$$

This figure belongs to Problem 1.50.



$$A_{v1} = \frac{v_{i2}}{v_{i1}} = 100 \times \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 1 \text{ k}\Omega}$$

$$= 99.9 \text{ V/V}$$

$$A_{v2} = \frac{v_{i3}}{v_{i2}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 1 \text{ k}\Omega}$$

$$= 9.09 \text{ V/V}$$

$$A_{v3} = \frac{v_L}{v_{i3}} = 1 \times \frac{100 \Omega}{100 \Omega + 10 \Omega} = 0.909 \text{ V/V}$$

$$\text{Total gain} = A_v = \frac{v_L}{v_{i1}} = A_{v1} \times A_{v2} \times A_{v3}$$

$$= 99.9 \times 9.09 \times 0.909 = 825.5 \text{ V/V}$$

The voltage gain from source to load is

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i1}} \times \frac{v_{i1}}{v_s} = A_v \cdot \frac{v_{i1}}{v_s}$$

$$= 825.5 \times 0.5$$

$$= 412.7 \text{ V/V}$$

The overall voltage has reduced appreciably. This is because the input resistance of the first stage,  $R_m$ , is comparable to the source resistance  $R_s$ . In Example 1.3 the input resistance of the first stage is much larger than the source resistance.

**1.51** The equivalent circuit at the output side of a current amplifier loaded with a resistance  $R_L$  is shown. Since

$$i_o = (A_{is}i_i) \frac{R_o}{R_o + R_L}$$

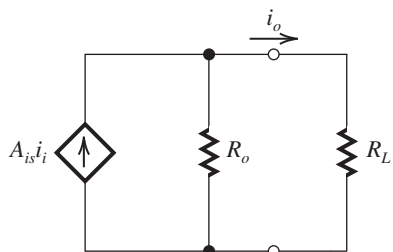
we can write

$$1 = (A_{is}i_i) \frac{R_o}{R_o + 1} \quad (1)$$

and

$$0.5 = (A_{is}i_i) \frac{R_o}{R_o + 12} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have



$$2 = \frac{R_o + 12}{R_o + 1} \Rightarrow R_o = 10 \text{ k}\Omega$$

$$A_{is}i_i = 1 \times \frac{10 + 1}{10} = 1.1 \text{ mA}$$

**1.52**

The current gain is

$$\frac{i_o}{i_i} = \frac{R_m}{R_o + R_L}$$

$$= \frac{5000}{10 + 1000}$$

$$= 4.95 \text{ A/A} = 13.9 \text{ dB}$$

The voltage gain is

$$\frac{v_o}{v_s} = \frac{i_i}{v_s} \times \frac{i_o}{i_i} \times \frac{v_o}{i_o}$$

$$= \frac{1}{R_s + R_i} \times \frac{i_o}{i_i} \times R_L$$

$$= \frac{1}{1000 + 100} \times 4.95 \times 1000$$

$$= 4.90 \text{ V/V} = 13.8 \text{ dB}$$

The power gain is

$$\frac{v_o i_o}{v_s i_i} = 4.95 \times 4.90$$

$$= 24.3 \text{ W/W} = 27.7 \text{ dB}$$

**1.53**

$$G_m = 60 \text{ mA/V}$$

$$R_o = 20 \text{ k}\Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$v_i = v_s \frac{R_i}{R_s + R_i}$$

$$= v_s \frac{2}{1 + 2} = \frac{2}{3} v_s$$

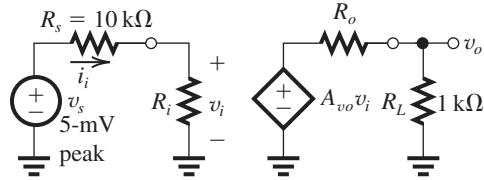
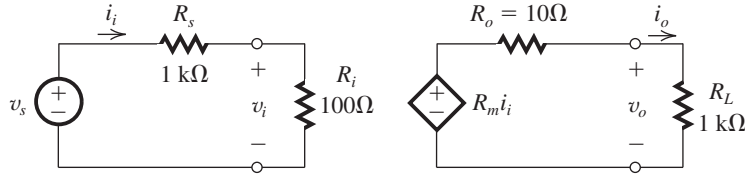
$$v_o = G_m v_i (R_L \parallel R_o)$$

$$= 60 \frac{20 \times 1}{20 + 1} v_i$$

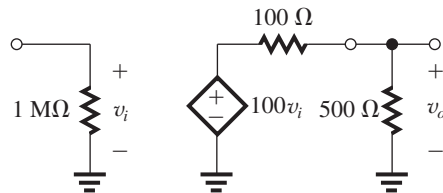
$$= 60 \frac{20}{21} \times \frac{2}{3} v_s$$

$$\text{Overall voltage gain} \equiv \frac{v_o}{v_s} = 38.1 \text{ V/V}$$

This figure belongs to Problem 1.52.



1.54



$$20 \log A_{vo} = 40 \text{ dB} \Rightarrow A_{vo} = 100 \text{ V/V}$$

$$A_v = \frac{v_o}{v_i} = 100 \times \frac{500}{500 + 100} = 83.3 \text{ V/V}$$

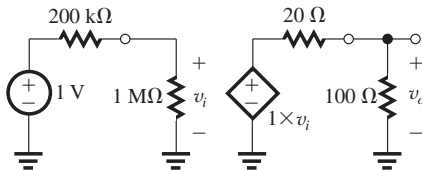
$$\text{or } 20 \log 83.3 = 38.4 \text{ dB}$$

$$A_p = \frac{v_o^2/500 \Omega}{v_i^2/1 \text{ M}\Omega} = A_v^2 \times 10^4 = 1.39 \times 10^7 \text{ W/W}$$

$$\text{or } 10 \log (1.39 \times 10^7) = 71.4 \text{ dB.}$$

For a peak output sine-wave current of 20 mA, the peak output voltage will be  $20 \text{ mA} \times 500 \Omega = 10 \text{ V}$ . Correspondingly  $v_i$  will be a sine wave with a peak value of  $10 \text{ V}/A_v = 10/83.3$ , or an rms value of  $10/(83.3 \times \sqrt{2}) = 0.085 \text{ V}$ . Corresponding output power =  $(10/\sqrt{2})^2/500 \Omega = 0.1 \text{ W}$

1.55



$$v_o = 1 \text{ V} \times \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 200 \text{ k}\Omega} \times 1 \times \frac{100 \Omega}{100 \Omega + 20 \Omega}$$

$$= \frac{1}{1.2} \times \frac{100}{120} = 0.69 \text{ V}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = 0.69 \text{ V/V or } -3.2 \text{ dB}$$

$$\text{Current gain} = \frac{v_o/100 \Omega}{v_s/1.2 \text{ M}\Omega} = 0.69 \times 1.2 \times 10^4 = 8280 \text{ A/A or } 78.4 \text{ dB}$$

$$\text{Power gain} = \frac{v_o^2/100 \Omega}{v_s^2/1.2 \text{ M}\Omega} = 5713 \text{ W/W}$$

$$\text{or } 10 \log 5713 = 37.6 \text{ dB}$$

(This takes into account the power dissipated in the internal resistance of the source.)

1.56 (a) Case S-A-B-L (see figure on next page):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ib}} \times \frac{v_{ib}}{v_{ia}} \times \frac{v_{ia}}{v_s} =$$

$$\left(10 \times \frac{100}{100 + 1000}\right) \times \left(100 \times \frac{10}{10 + 10}\right) \times \left(\frac{100}{100 + 100}\right)$$

$$\frac{v_o}{v_s} = 22.7 \text{ V/V and gain in dB } 20 \log 22.7 =$$

$$27.1 \text{ dB}$$

(b) Case S-B-A-L (see figure on next page):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ia}} \cdot \frac{v_{ia}}{v_{ib}} \cdot \frac{v_{ib}}{v_s} =$$

$$= \left(100 \times \frac{100}{100 + 10 \text{ K}}\right) \times \left(10 \times \frac{100 \text{ K}}{100 \text{ K} + 1 \text{ K}}\right) \times$$

$$\left(\frac{10 \text{ K}}{10 \text{ K} + 100 \text{ K}}\right)$$

$$\frac{v_o}{v_s} = 0.89 \text{ V/V and gain in dB is } 20 \log 0.89 =$$

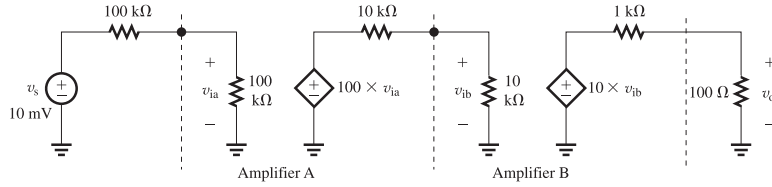
-1 dB. Obviously, case a is preferred because it provides higher voltage gain.

1.57 Each of stages #1, 2, ..., (n - 1) can be represented by the equivalent circuit:

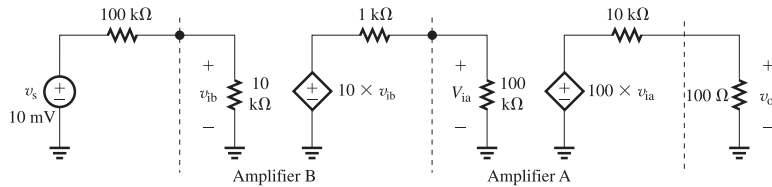
$$\frac{v_o}{v_s} = \frac{v_{i1}}{v_s} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i3}}{v_{i2}} \times \dots \times \frac{v_{in}}{v_{i(n-1)}} \times \frac{v_o}{v_{in}}$$

where

This figure belongs to 1.56, part (a).



This figure belongs to 1.56, part (b).



$$\frac{v_{i1}}{v_s} = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 0.5 \text{ V/V}$$

$$\frac{v_o}{v_{in}} = 10 \times \frac{200 \Omega}{1 \text{ k}\Omega + 200 \Omega} = 1.67 \text{ V/V}$$

$$\frac{v_{i2}}{v_{i1}} = \frac{v_{i3}}{v_{i2}} = \dots = \frac{v_{in}}{v_{i(n-1)}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 9.09 \text{ V/V}$$

Thus,

$$\frac{v_o}{v_s} = 0.5 \times (9.09)^{n-1} \times 1.67 = 0.833 \times (9.09)^{n-1}$$

For  $v_s = 5 \text{ mV}$  and  $v_o = 3 \text{ V}$ , the gain  $\frac{v_o}{v_s}$  must be  $\geq 600$ , thus

$$0.833 \times (9.09)^{n-1} \geq 600$$

$$\Rightarrow n = 4$$

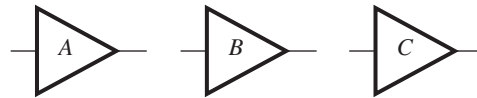
Thus four amplifier stages are needed, resulting in

$$\frac{v_o}{v_s} = 0.833 \times (9.09)^3 = 625.7 \text{ V/V}$$

and correspondingly

$$v_o = 625.7 \times 5 \text{ mV} = 3.13 \text{ V}$$

**1.58** Deliver 0.5 W to a 100-Ω load. Source is 30 mV rms with 0.5-MΩ source resistance. Choose from these three amplifier types:



$$R_i = 1 \text{ M}\Omega \quad R_i = 10 \text{ k}\Omega \quad R_i = 10 \text{ k}\Omega$$

$$A_v = 10 \text{ V/V} \quad A_v = 100 \text{ V/V} \quad A_v = 1 \text{ V/V}$$

$$R_o = 10 \text{ k}\Omega \quad R_o = 1 \text{ k}\Omega \quad R_o = 20 \Omega$$

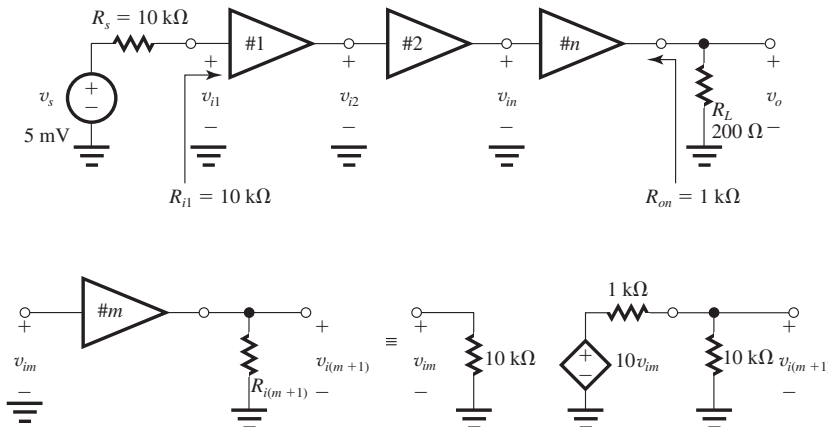
Choose order to eliminate loading on input and output:

A, first, to minimize loading on 0.5-MΩ source  
 B, second, to boost gain  
 C, third, to minimize loading at 100-Ω output.

We first attempt a cascade of the three stages in the order A, B, C (see figure above), and obtain

$$\frac{v_{i1}}{v_s} = \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 0.5 \text{ M}\Omega} = \frac{1}{1.5}$$

This figure belongs to 1.57.





$$\Rightarrow v_{i1} = 30 \times \frac{1}{1.5} = 20 \text{ mV}$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 5$$

$$\Rightarrow v_{i2} = 20 \times 5 = 100 \text{ mV}$$

$$\frac{v_{i3}}{v_{i2}} = 100 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 1 \text{ k}\Omega} = 90.9$$

$$\Rightarrow v_{i3} = 100 \text{ mV} \times 90.9 = 9.09 \text{ V}$$

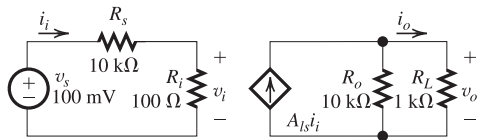
$$\frac{v_o}{v_{i3}} = 1 \times \frac{100 \text{ }\Omega}{100 \text{ }\Omega + 20 \text{ }\Omega} = 0.833$$

$$\Rightarrow v_o = 9.09 \times 0.833 = 7.6 \text{ V}$$

$$P_o = \frac{v_{o\text{rms}}^2}{R_L} = \frac{7.6^2}{100} = 0.57 \text{ W}$$

which exceeds the required 0.5 W. Also, the signal throughout the amplifier chain never drops below 20 mV (which is greater than the required minimum of 10 mV).

**1.59**



(a) Current gain =  $\frac{i_o}{i_i}$

$$= A_{is} \frac{R_o}{R_o + R_L}$$

$$= 100 \frac{10}{11}$$

$$= 90.9 \text{ A/A} = 39.2 \text{ dB}$$

(b) Voltage gain =  $\frac{v_o}{v_s} = \frac{i_o R_L}{i_i (R_s + R_i)}$

$$= \frac{i_o}{i_i} \frac{R_L}{R_s + R_i}$$

$$= 90.9 \times \frac{1}{10 + 0.1}$$

$$= 9 \text{ V/V} = 19.1 \text{ dB}$$

(c) Power gain =  $A_p = \frac{v_o i_o}{v_s i_i}$

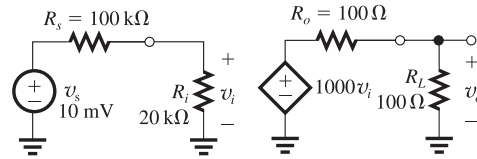
$$= 9 \times 90.9$$

$$= 818 \text{ W/W} = 29.1 \text{ dB}$$

**1.60**

(a)

$$v_o = 10 \text{ mV} \times \frac{20}{20 + 100} \times 1000 \times \frac{100}{100 + 100}$$

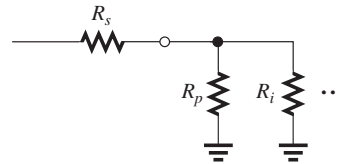


$$= 833 \text{ mV}$$

(b)  $\frac{v_o}{v_s} = \frac{833 \text{ mV}}{10 \text{ mV}} = 83.3 \text{ V/V}$

(c)  $\frac{v_o}{v_i} = 1000 \times \frac{100}{100 + 100} = 500 \text{ V/V}$

(d)



Connect a resistance  $R_p$  in parallel with the input and select its value from

$$\frac{(R_p \parallel R_i)}{(R_p \parallel R_i) + R_s} = \frac{1}{2} \frac{R_i}{R_i + R_s}$$

$$\Rightarrow 1 + \frac{R_s}{R_p \parallel R_i} = 12 \Rightarrow R_p \parallel R_i = \frac{R_s}{11} = \frac{100}{11}$$

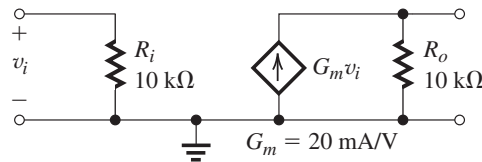
$$\Rightarrow \frac{1}{R_p} + \frac{1}{R_i} = \frac{11}{100}$$

$$R_p = \frac{1}{0.11 - 0.05} = 16.7 \text{ k}\Omega$$

**1.61** To obtain the weighted sum of  $v_1$  and  $v_2$

$$v_o = 10v_1 + 20v_2$$

we use two transconductance amplifiers and sum their output currents. Each transconductance amplifier has the following equivalent circuit:

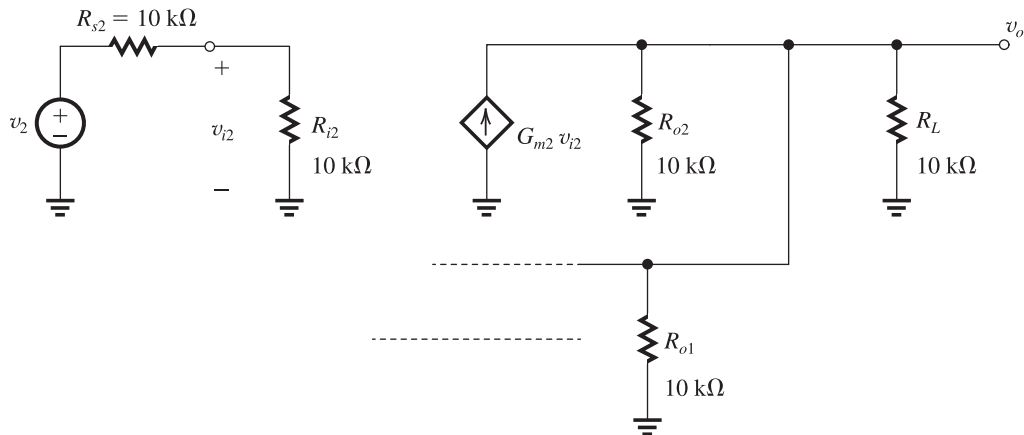


Consider first the path for the signal requiring higher gain, namely  $v_2$ . See figure at top of next page.

The parallel connection of the two amplifiers at the output and the connection of  $R_L$  means that the total resistance at the output is

$$10 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = \frac{10}{3} \text{ k}\Omega.$$

This figure belongs to Problem 1.61.



Thus the component of  $v_o$  due to  $v_2$  will be

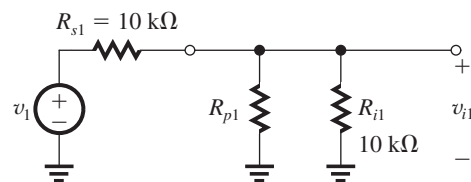
$$v_{o2} = v_2 \frac{10}{10 + 10} \times G_{m2} \times \frac{10}{3}$$

$$= v_2 \times 0.5 \times 20 \times \frac{10}{3} = 33.3 v_2$$

To reduce the gain seen by  $v_2$  from 33.3 to 20, we connect a resistance  $R_p$  in parallel with  $R_L$ ,

$$\left( \frac{10}{3} \parallel R_p \right) = 2 \text{ k}\Omega \Rightarrow R_p = 5 \text{ k}\Omega$$

We next consider the path for  $v_1$ . Since  $v_1$  must see a gain factor of only 10, which is half that seen by  $v_2$ , we have to reduce the fraction of  $v_1$  that appears at the input of its transconductance amplifier to half that that appears at the input of the  $v_2$  transconductance amplifier. We just saw that  $0.5 v_2$  appears at the input of the  $v_2$  transconductance amplifier. Thus, for the  $v_1$  transconductance amplifier, we want  $0.25 v_1$  to appear at the input. This can be achieved by shunting the input of the  $v_1$  transconductance amplifier by a resistance  $R_{p1}$  as in the figure in the next column.



The value of  $R_{p1}$  can be found from

$$\frac{(R_{p1} \parallel R_{i1})}{(R_{p1} \parallel R_{i1}) + R_{s1}} = 0.25$$

Thus,

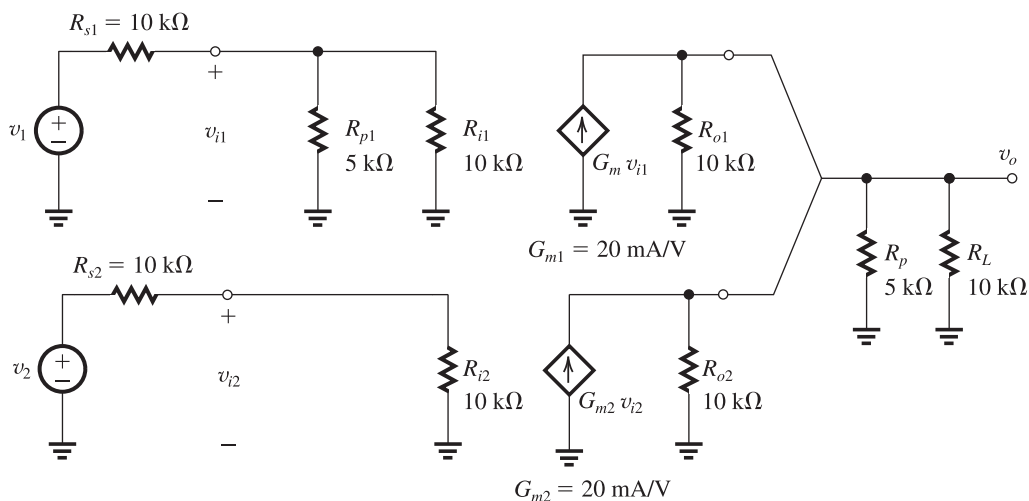
$$1 + \frac{R_{s1}}{(R_{p1} \parallel R_{i1})} = 4$$

$$\Rightarrow R_{p1} \parallel R_{i1} = \frac{R_{s1}}{3} = \frac{10}{3}$$

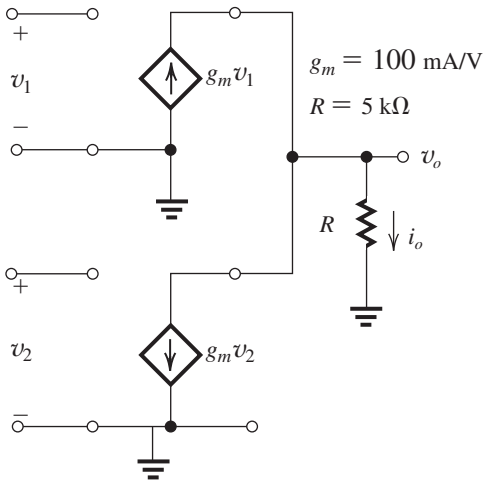
$$R_{p1} \parallel 10 = \frac{10}{3}$$

$$\Rightarrow R_{p1} = 5 \text{ k}\Omega$$

The final circuit will be as follows:



1.62



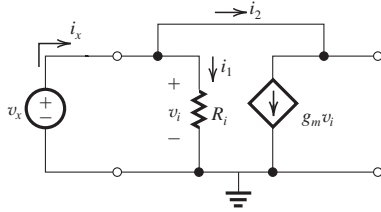
$$i_o = g_m v_1 - g_m v_2$$

$$v_o = i_o R_L = g_m R (v_1 - v_2)$$

$$v_1 = v_2 = 1 \text{ V} \quad \therefore v_o = 0 \text{ V}$$

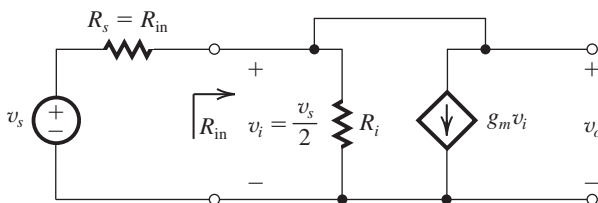
$$\left. \begin{matrix} v_1 = 1.01 \text{ V} \\ v_2 = 0.99 \text{ V} \end{matrix} \right\} \therefore v_o = 100 \times 5 \times 0.02 = 10 \text{ V}$$

1.63 (a)



$$\left. \begin{matrix} i_x = i_1 + i_2 \\ i_1 = v_i / R_i \\ i_2 = g_m v_i \\ v_i = v_x \end{matrix} \right\} \begin{matrix} i_x = v_x / R_i + g_m v_x \\ i_x = v_x \left( \frac{1}{R_i} + g_m \right) \\ \frac{v_x}{i_x} = \frac{1}{1/R_i + g_m} \\ = \frac{R_i}{1 + g_m R_i} = R_{in} \end{matrix}$$

(b)



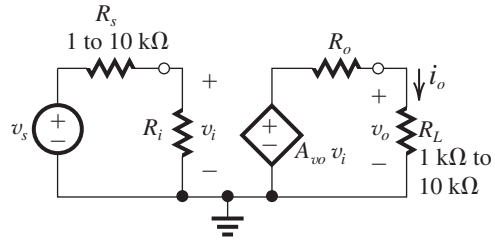
When driven by a source with source resistance  $R_{in}$  as shown in the figure above,

$$v_i = \frac{R_{in}}{R_s + R_{in}} \times v_s = \frac{R_{in}}{R_{in} + R_{in}} \times v_s = 0.5 \times v_s$$

Thus,

$$\frac{v_o}{v_s} = 0.5 \frac{v_o}{v_i}$$

1.64 Voltage amplifier:



For  $R_s$ , varying in the range 1 k $\Omega$  to 10 k $\Omega$  and  $\Delta v_o$  limited to 10%, select  $R_i$  to be sufficiently large:

$$R_i \geq 10 R_{smax}$$

$$R_i = 10 \times 10 \text{ k}\Omega = 100 \text{ k}\Omega = 1 \times 10^5 \Omega$$

For  $R_L$  varying in the range 1 k $\Omega$  to 10 k $\Omega$ , the load voltage variation limited to 10%, select  $R_o$  sufficiently low:

$$R_o \leq \frac{R_{Lmin}}{10}$$

$$R_o = \frac{1 \text{ k}\Omega}{10} = 100 \Omega = 1 \times 10^2 \Omega$$

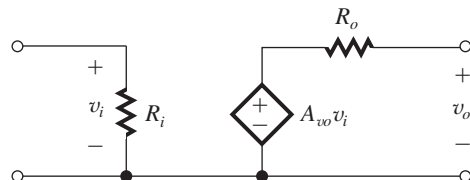
Now find  $A_{vo}$ :

$$v_{omin} = 10 \text{ mV} \times \frac{R_i}{R_i + R_{smax}} \times A_{vo} \frac{R_{Lmin}}{R_o + R_{Lmin}}$$

$$1 = 10 \times 10^{-3} \times \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 10 \text{ k}\Omega}$$

$$\times A_{vo} \times \frac{1 \text{ k}\Omega}{100 \Omega + 1 \text{ k}\Omega}$$

$$\Rightarrow A_{vo} = 121 \text{ V/V}$$



Values for the voltage amplifier equivalent circuit are

$$R_i = 1 \times 10^5 \Omega, A_{vo} = 121 \text{ V/V, and}$$

$$R_o = 1 \times 10^2 \Omega$$

**1.65** Transresistance amplifier:

To limit  $\Delta v_o$  to 10% corresponding to  $R_s$  varying in the range 1 k $\Omega$  to 10 k $\Omega$ , we select  $R_i$  sufficiently low;

$$R_i \leq \frac{R_{s\min}}{10}$$

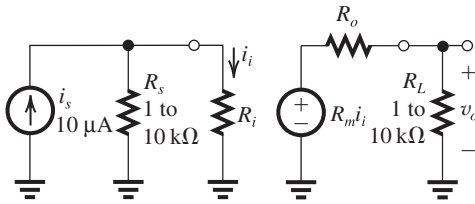
Thus,  $R_i = 100 \Omega = 1 \times 10^2 \Omega$

To limit  $\Delta v_o$  to 10% while  $R_L$  varies over the range 1 k $\Omega$  to 10 k $\Omega$ , we select  $R_o$  sufficiently low;

$$R_o \leq \frac{R_{L\min}}{10}$$

Thus,  $R_o = 100 \Omega = 1 \times 10^2 \Omega$

Now, for  $i_s = 10 \mu\text{A}$ ,

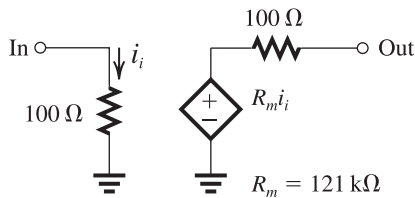


$$v_{o\min} = 10^{-5} \frac{R_{s\min}}{R_{s\min} + R_i} R_m \frac{R_{L\min}}{R_{L\min} + R_o}$$

$$1 = 10^{-5} \frac{1000}{1000 + 100} R_m \frac{1000}{1000 + 100}$$

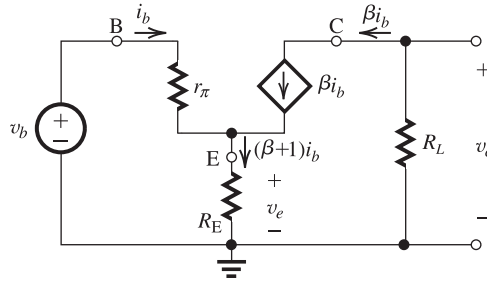
$$\Rightarrow R_m = 1.21 \times 10^5 \Omega$$

$$= 121 \text{ k}\Omega$$



**1.66**

The node equation at  $E$  yields the current through  $R_E$  as  $(\beta i_b + i_b) = (\beta + 1)i_b$ . The voltage  $v_c$  can be found in terms of  $i_b$  as



$$v_c = -\beta i_b R_L \tag{1}$$

The voltage  $v_b$  can be related to  $i_b$  by writing for the input loop:

$$v_b = i_b r_\pi + (\beta + 1)i_b R_E$$

Thus,

$$v_b = [r_\pi + (\beta + 1)R_E]i_b \tag{2}$$

Dividing Eq. (1) by Eq. (2) yields

$$\frac{v_c}{v_b} = -\frac{\beta R_L}{r_\pi + (\beta + 1)R_E} \quad \text{Q.E.D}$$

The voltage  $v_e$  is related to  $i_b$  by

$$v_e = (\beta + 1)i_b R_E$$

That is,

$$v_e = [(\beta + 1)R_E]i_b \tag{3}$$

Dividing Eq. (3) by Eq. (2) yields

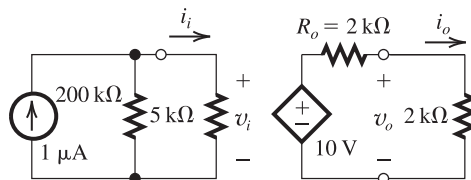
$$\frac{v_e}{v_b} = \frac{(\beta + 1)R_E}{(\beta + 1)R_E + r_\pi}$$

Dividing the numerator and denominator by  $(\beta + 1)$  gives

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi/(\beta + 1)]} \quad \text{Q.E.D}$$

**1.67**  $R_o = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10 \text{ V}}{5 \text{ mA}} = 2 \text{ k}\Omega$

$$v_o = 10 \times \frac{2}{2 + 2} = 5 \text{ V}$$



$$A_v = \frac{v_o}{v_i} = \frac{10(2/4)}{1 \times 10^{-6} \times (200 \parallel 5) \times 10^3}$$

1025 V/V or 60.2 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{v_i/R_i}$$

$$= \frac{v_o}{v_i} \frac{R_i}{R_L} = 1025 \times \frac{5 \text{ k}\Omega}{2 \text{ k}\Omega}$$

= 2562.5 A/A or 62.8 dB

The overall current gain can be found as

$$\frac{i_o}{i_s} = \frac{v_o/R_L}{1 \mu\text{A}} = \frac{5 \text{ V}/2 \text{ k}\Omega}{1 \mu\text{A}}$$

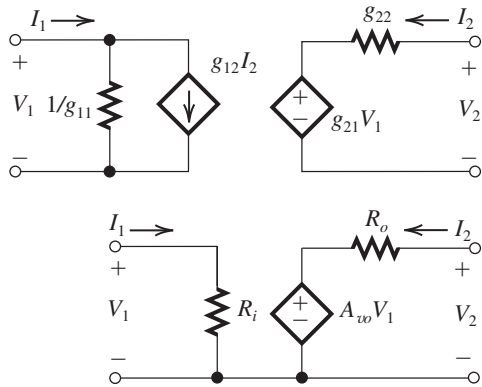
$$= \frac{2.5 \text{ mA}}{1 \mu\text{A}} = 2500 \text{ A/A}$$

or 68 dB.

$$A_p = \frac{v_o^2/R_L}{i_i^2 R_i} = \frac{5^2/(2 \times 10^3)}{\left(10^{-6} \times \frac{200}{200+5}\right)^2 5 \times 10^3}$$

= 2.63 × 10<sup>6</sup> W/W or 64.2 dB

**1.68**



The correspondences between the current and voltage variables are indicated by comparing the two equivalent-circuit models above. At the outset we observe that at the input side of the *g*-parameter model, we have the controlled current source  $g_{12}I_2$ . This has no correspondence in the equivalent-circuit model of Fig. 1.16(a). It represents internal feedback, internal to the amplifier circuit. In developing the model of Fig. 1.16(a), we assumed that the amplifier is unilateral (i.e., has no internal feedback, or that

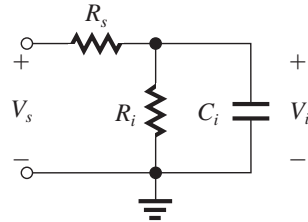
the input side does not know what happens at the output side). If we neglect this internal feedback, that is, assume  $g_{12} = 0$ , we can compare the two models and thus obtain:

$$R_i = 1/g_{11}$$

$$A_{vo} = g_{21}$$

$$R_o = g_{22}$$

**1.69**



$$\frac{V_i}{V_s} = \frac{\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}}{R_s + \left(\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}\right)} = \frac{\frac{R_i}{1 + sC_i R_i}}{R_s + \left(\frac{R_i}{1 + sC_i R_i}\right)}$$

$$= \frac{R_i}{R_s + sC_i R_i R_s + R_i}$$

$$\frac{V_i}{V_s} = \frac{R_i}{(R_s + R_i) + sC_i R_i R_s} = \frac{\frac{R_i}{(R_s + R_i)}}{1 + s\left(\frac{C_i R_i R_s}{R_s + R_i}\right)}$$

which is a low-pass STC function with

$$K = \frac{R_i}{R_s + R_i} \text{ and } \omega_0 = 1/[C_i(R_i \parallel R_s)].$$

For  $R_s = 10 \text{ k}\Omega$ ,  $R_i = 40 \text{ k}\Omega$ , and  $C_i = 5 \text{ pF}$ ,

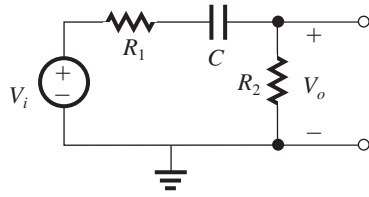
$$\omega_0 = \frac{1}{5 \times 10^{-12} \times (40 \parallel 10) \times 10^3} = 25 \text{ Mrad/s}$$

$$f_0 = \frac{25}{2\pi} = 4 \text{ MHz}$$

The dc gain is

$$K = \frac{40}{10 + 40} = 0.8 \text{ V/V}$$

1.70 Using the voltage-divider rule.



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$

$$T(s) = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{s}{s + \frac{1}{C(R_1 + R_2)}} \right)$$

which from Table 1.2 is of the high-pass type with

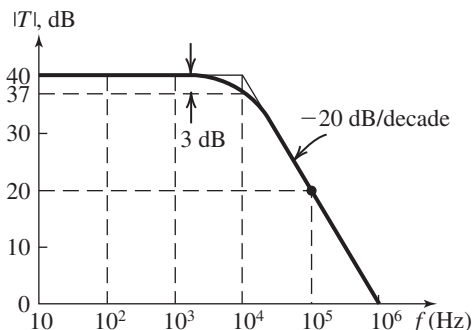
$$K = \frac{R_2}{R_1 + R_2} \quad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

As a further verification that this is a high-pass network and  $T(s)$  is a high-pass transfer function, see that as  $s \rightarrow 0$ ,  $T(s) \rightarrow 0$ ; and as  $s \rightarrow \infty$ ,  $T(s) = R_2/(R_1 + R_2)$ . Also, from the circuit, observe as  $s \rightarrow \infty$ ,  $(1/sC) \rightarrow 0$  and  $V_o/V_i = R_2/(R_1 + R_2)$ . Now, for  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 40 \text{ k}\Omega$  and  $C = 1 \text{ }\mu\text{F}$ ,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 1 \times 10^{-6} (10 + 40) \times 10^3} = 3.18 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10 + 40} \frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

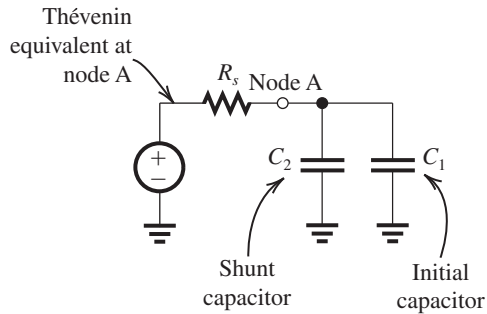
1.71 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of  $10^4 \text{ Hz}$ . From our knowledge of the Bode plots for low-pass STC networks [Fig. 1.23(a)], we can complete the table entries and sketch the amplifier frequency response.



$f(\text{Hz})$	$ T (\text{dB})$	$\angle T(^{\circ})$
0	40	0
100	40	0
1000	40	0
$10^4$	37	$-45^{\circ}$
$10^5$	20	$-90^{\circ}$
$10^6$	0	$-90^{\circ}$

1.72  $R_s = 100 \text{ k}\Omega$ , since the 3-dB frequency is reduced by a very high factor (from 5 MHz to 100 kHz)  $C_2$  must be much larger than  $C_1$ . Thus, neglecting  $C_1$  we find  $C_2$  from

$$100 \text{ kHz} \simeq \frac{1}{2\pi C_2 R_s}$$



$$= \frac{1}{2\pi C_2 \times 10^5} \Rightarrow C_2 = 15.9 \text{ pF}$$

If the original 3-dB frequency (5 MHz) is attributable to  $C_1$ , then

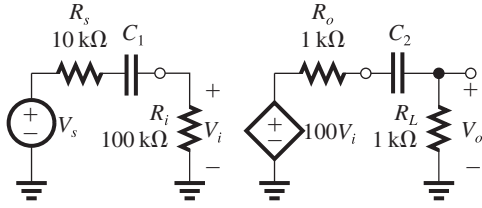
$$5 \text{ MHz} = \frac{1}{2\pi C_1 R_s} \Rightarrow C_1 = \frac{1}{2\pi \times 5 \times 10^6 \times 10^5} = 0.32 \text{ pF}$$

1.73 For the input circuit, the corner frequency  $f_{01}$  is found from

$$f_{01} = \frac{1}{2\pi C_1 (R_s + R_i)}$$

For  $f_{01} \leq 100 \text{ Hz}$ ,

$$\frac{1}{2\pi C_1 (10 + 100) \times 10^3} \leq 100$$



$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 1.4 \times 10^{-8} \text{ F}$$

Thus we select  $C_1 = 1 \times 10^{-7} \text{ F} = 0.1 \mu\text{F}$ . The actual corner frequency resulting from  $C_1$  will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2(R_o + R_L)}$$

For  $f_{02} \leq 100 \text{ Hz}$ ,

$$\frac{1}{2\pi C_2(1 + 1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

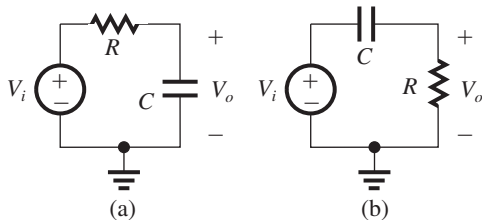
Select  $C_2 = 1 \times 10^{-6} = 1 \mu\text{F}$ .

This will place the corner frequency at

$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right) \left(1 + \frac{s}{2\pi f_{02}}\right)}$$

**1.74** Circuits of Fig. 1.22:



For (a)  $V_o = V_i \left( \frac{1/sC}{1/sC + R} \right)$

$$\frac{V_o}{V_i} = \frac{1}{1 + sCR}$$

which is of the form shown for the low-pass function in Table 1.2 with  $K = 1$  and  $\omega_0 = 1/RC$ .

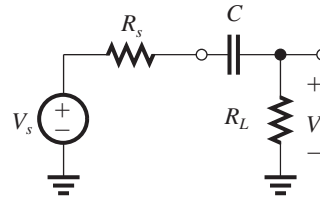
For (b)  $V_o = V_i \left( \frac{R}{R + \frac{1}{sC}} \right)$

$$\frac{V_o}{V_i} = \frac{sRC}{1 + sCR}$$

$$\frac{V_o}{V_i} = \frac{s}{s + \frac{1}{RC}}$$

which is of the form shown in Table 1.2 for the high-pass function, with  $K = 1$  and  $\omega_0 = 1/RC$ .

**1.75** Using the voltage divider rule,



$$\frac{V_i}{V_s} = \frac{R_L}{R_L + R_s + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_s} \frac{s}{s + \frac{1}{C(R_L + R_s)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_s} \quad \omega_0 = \frac{1}{C(R_L + R_s)}$$

For  $f_0 \leq 100 \text{ Hz}$

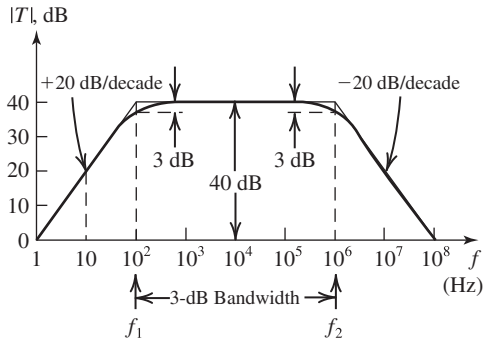
$$\frac{1}{2\pi C(R_L + R_s)} \leq 100$$

$$\Rightarrow C \geq \frac{1}{2\pi \times 100(20 + 5) \times 10^3}$$

Thus, the smallest value of  $C$  that will do the job is  $C = 0.064 \mu\text{F}$  or  $64 \text{ nF}$ .

**1.76** From our knowledge of the Bode plots of STC low-pass and high-pass networks, we see that this amplifier has a midband gain of 40 dB, a low-frequency response of the high-pass STC type with  $f_{3dB} = 10^2 \text{ Hz}$ , and a high-frequency

response of the low-pass STC type with  $f_{3dB} = 10^6$  Hz. We thus can sketch the amplifier frequency response and complete the table entries as follows.



$f$ (Hz)	1	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$	$10^7$	$10^8$
$ T $ (dB)	0	20	37	40	40	40	37	20	0

**1.77** Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects, since the buffer amplifiers have infinite input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{ dB}}}{\omega_0}\right)^2}} = -1$$

$$\Rightarrow 1 + \left(\frac{\omega_{1\text{ dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{ dB}} = 0.51\omega_0$$

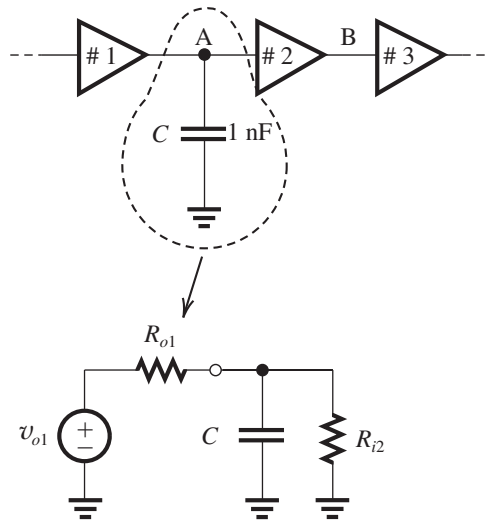
$$\omega_{1\text{ dB}} = 0.51/CR$$

**1.78** Since when  $C$  is connected to node A the 3-dB frequency is reduced by a large factor, the value of  $C$  must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that  $C$  is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$200 \text{ kHz} = \frac{1}{2\pi C(R_{o1} \parallel R_{i2})}$$

$$\Rightarrow (R_{o1} \parallel R_{i2}) = \frac{1}{2\pi \times 200 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 0.8 \text{ k}\Omega$$



Now  $R_{i2} = 100 \text{ k}\Omega$ .

Thus  $R_{o1} \simeq 0.8 \text{ k}\Omega$

Similarly, for node B,

$$20 \text{ kHz} = \frac{1}{2\pi C(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow R_{o2} \parallel R_{i3} = \frac{1}{2\pi \times 20 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 7.96 \text{ k}\Omega$$

$$R_{o2} = 8.65 \text{ k}\Omega$$

The designer should connect a capacitor of value  $C_p$  to node B where  $C_p$  can be found from



$$10 \text{ kHz} = \frac{1}{2\pi C_p (R_{o2} \parallel R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 7.96 \times 10^3}$$

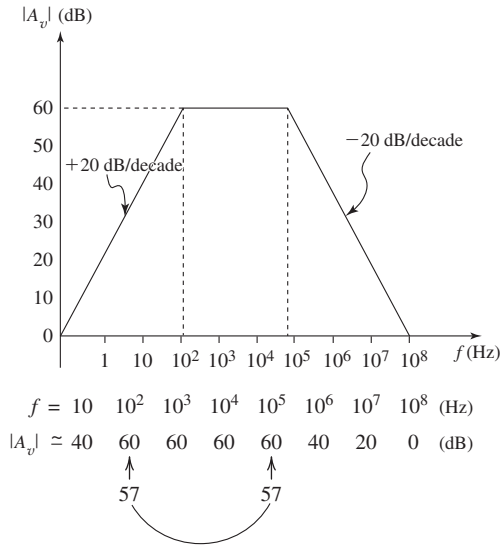
$$= 2 \text{ nF}$$

Note that if she chooses to use node A, she would need to connect a capacitor 10 times larger!

**1.79** The LP factor  $1/(1 + jf/10^5)$  results in a Bode plot like that in Fig. 1.23(a) with the 3-dB frequency  $f_0 = 10^5$  Hz. The high-pass factor  $1/(1 + 10^2/jf)$  results in a Bode plot like that in Fig. 1.24(a) with the 3-dB frequency

$$f_0 = 10^2 \text{ Hz.}$$

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then shifting the resulting plot vertically by 60 dB (corresponding to the factor 1000 in the numerator). The result is as follows:



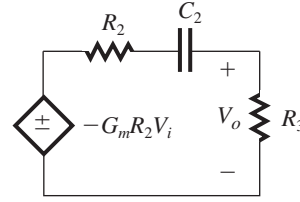
$$\text{Bandwidth} = 10^5 - 10^2 = 99,900 \text{ Hz}$$

**1.80**  $T_i(s) = \frac{V_i(s)}{V_s(s)} = \frac{1/sC_1}{1/sC_1 + R_1} = \frac{1}{sC_1R_1 + 1}$

LP with a 3-dB frequency

$$f_{0i} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi \cdot 10^{-11} \cdot 10^5} = 159 \text{ kHz}$$

For  $T_o(s)$ , the following equivalent circuit can be used:



$$T_o(s) = \frac{V_o}{V_i} = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

$$= -G_m (R_2 \parallel R_3) \frac{s}{s + \frac{1}{C_2(R_2 + R_3)}}$$

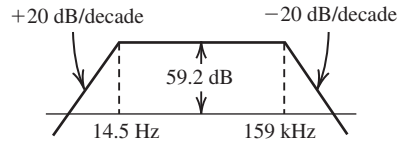
which is an HP, with

$$\text{3-dB frequency} = \frac{1}{2\pi C_2 (R_2 + R_3)}$$

$$= \frac{1}{2\pi \cdot 100 \times 10^{-9} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

$$\therefore T(s) = T_i(s)T_o(s)$$

$$= \frac{1}{1 + \frac{s}{2\pi \times 159 \times 10^3}} \times -909.1 \times \frac{s}{s + (2\pi \times 14.5)}$$



$$\text{Bandwidth} = 159 \text{ kHz} - 14.5 \text{ Hz} \approx 159 \text{ kHz}$$

**1.81**  $V_i = V_s \frac{R_i}{R_s + R_i}$  (1)

(a) To satisfy constraint (1), namely,

$$V_i \geq \left(1 - \frac{x}{100}\right) V_s$$

we substitute in Eq. (1) to obtain

$$\frac{R_i}{R_s + R_i} \geq 1 - \frac{x}{100}$$
 (2)

Thus

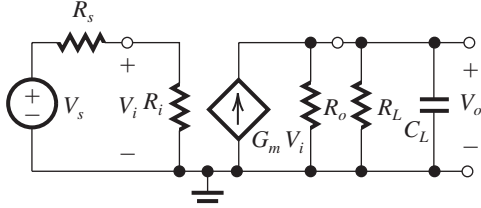
$$\frac{R_s + R_i}{R_i} \leq \frac{1}{1 - \frac{x}{100}}$$

$$\frac{R_s}{R_i} \leq \frac{1}{1 - \frac{x}{100}} - 1 = \frac{\frac{x}{100}}{1 - \frac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_s} \geq \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in



$$R_i \geq R_s \left( \frac{100}{x} - 1 \right) \quad (3)$$

(b) The 3-dB frequency is determined by the parallel RC circuit at the output

$$f_0 = \frac{1}{2\pi} \omega_0 = \frac{1}{2\pi} \frac{1}{C_L (R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right)$$

To obtain a value for  $f_0$  greater than a specified value  $f_{3dB}$  we select  $R_o$  so that

$$\frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right) \geq f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \frac{1}{R_L}} \quad (4)$$

(c) To satisfy constraint (c), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_s + R_i} G_m (R_o \parallel R_L)$$

For the dc gain to be greater than a specified value  $A_0$ ,

$$\frac{R_i}{R_s + R_i} G_m (R_o \parallel R_L) \geq A_0$$

The first factor on the left-hand side is (from constraint (2)) greater or equal to  $(1 - x/100)$ .

Thus

$$G_m \geq \frac{A_0}{\left(1 - \frac{x}{100}\right) (R_o \parallel R_L)} \quad (5)$$

Substituting  $R_s = 10 \text{ k}\Omega$  and  $x = 10\%$  in (3) results in

$$R_i \geq 10 \left( \frac{100}{100} - 1 \right) = 90 \text{ k}\Omega$$

Substituting  $f_{3dB} = 2 \text{ MHz}$ ,  $C_L = 20 \text{ pF}$ , and

$R_L = 10 \text{ k}\Omega$  in Eq. (4) results in

$$R_o \leq \frac{1}{2\pi \times 2 \times 10^6 \times 20 \times 10^{-12} - \frac{1}{10^4}}$$

$$= 6.61 \text{ k}\Omega$$

Substituting  $A_0 = 100$ ,  $x = 10\%$ ,  $R_L = 10 \text{ k}\Omega$ , and  $R_o = 6.61 \text{ k}\Omega$ , Eq. (5) results in

$$G_m \geq \frac{100}{\left(1 - \frac{10}{100}\right) (10 \parallel 6.61) \times 10^3} = 27.9 \text{ mA/V}$$

**1.82** Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \quad \text{and} \quad Z_2 = R_2 \parallel \frac{1}{sC_2}$$

It is obviously more convenient to work in terms of admittances. Therefore we express  $V_o/V_i$  in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute  $Y_1 = (1/R_1) + sC_1$  and  $Y_2 = (1/R_2) + sC_2$  to obtain

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)}}$$

This transfer function will be independent of frequency ( $s$ ) if the second factor reduces to unity.

This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

which can be simplified as follows:

$$\frac{C_1 + C_2}{C_1} = R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1), can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ( $C_1 R_1 = C_2 R_2$ ), its transmission can be determined either by its two resistors  $R_1$ ,  $R_2$  or by its two capacitors.  $C_1$ ,  $C_2$ , and the transmission is *not* a function of frequency.

**1.83** The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of  $5.7^\circ$  at  $f = 100$  Hz. Using the equation for  $\angle T(j\omega)$  from Table 1.2, we obtain

$$\tan^{-1} \frac{f_0}{100} = 5.7^\circ \Rightarrow f_0 = 10 \text{ Hz}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of  $-5.7^\circ$  at  $f = 1$  kHz. Using the relationship for  $\angle T(j\omega)$  given in Table 1.2, we obtain for the LP STC circuit.

$$-\tan^{-1} \frac{10^3}{f_0} = -5.7^\circ \Rightarrow f_0 \simeq 10 \text{ kHz}$$

At  $f = 100$  Hz, the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{10}{100}\right)^2}} = -0.04 \text{ dB}$$

Similarly, at the drop in gain  $f = 1$  kHz is caused by the LP STC network. The drop in gain is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{1000}{10,000}\right)^2}} = -0.04 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at  $f = 10$  Hz and  $f = 10$  kHz.

**1.84** Use the expression in Eq. (1.26), with

$$B = 7.3 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2};$$

$$k = 8.62 \times 10^{-5} \text{ eV/K}; \text{ and } E_g = 1.12 \text{ V}$$

we have

$$T = -55^\circ \text{C} = 218 \text{ K:}$$

$$n_i = 2.68 \times 10^6 \text{ cm}^{-3}; \frac{N}{n_i} = 1.9 \times 10^{16}$$

That is, one out of every  $1.9 \times 10^{16}$  silicon atoms is ionized at this temperature.

$$T = 0^\circ \text{C} = 273 \text{ K:}$$

$$n_i = 1.52 \times 10^9 \text{ cm}^{-3}; \frac{N}{n_i} = 3.3 \times 10^{13}$$

$$T = 20^\circ \text{C} = 293 \text{ K:}$$

$$n_i = 8.60 \times 10^9 \text{ cm}^{-3}; \frac{N}{n_i} = 5.8 \times 10^{12}$$

$$T = 75^\circ \text{C} = 348 \text{ K:}$$

$$n_i = 3.70 \times 10^{11} \text{ cm}^{-3}; \frac{N}{n_i} = 1.4 \times 10^{11}$$

$$T = 125^\circ \text{C} = 398 \text{ K:}$$

$$n_i = 4.72 \times 10^{12} \text{ cm}^{-3}; \frac{N}{n_i} = 1.1 \times 10^{10}$$

**1.85** Use Eq. (1.26) to find  $n_i$ ,

$$n_i = BT^{3/2} e^{-E_g/2kT}$$

Substituting the values given in the problem,

$$\begin{aligned} n_i &= 3.56 \times 10^{14} (300)^{3/2} e^{-1.42/(2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 2.2 \times 10^6 \text{ carriers/cm}^3 \end{aligned}$$

**1.86** The concentration of free carriers (both electrons and holes) in intrinsic silicon is found in Example 3.1 to be  $1.5 \times 10^{10}$  carriers/cm<sup>3</sup> at room temperature. Multiplying this by the volume of the wafer gives

$$1.5 \times 10^{10} \times \frac{\pi \times 15^2 \times 0.3}{4} =$$

$$7.95 \times 10^{10} \text{ free electrons}$$

**1.87** Since  $N_A \gg n_i$ , we can write

$$p_p \approx N_A = 5 \times 10^{18} \text{ cm}^{-3}$$

Using Eq. (1.27), we have

$$n_p = \frac{n_i^2}{p_p} = 45 \text{ cm}^{-3}$$

**1.88** Hole concentration in intrinsic Si =  $n_i$

$$\begin{aligned} n_i &= BT^{3/2} e^{-E_g/2kT} \\ &= 7.3 \times 10^{15} (300)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 1.5 \times 10^{10} \text{ holes/cm}^3 \end{aligned}$$

In phosphorus-doped Si, hole concentration drops below the intrinsic level by a factor of  $10^8$ .

$\therefore$  Hole concentration in P-doped Si is

$$p_n = \frac{1.5 \times 10^{10}}{10^8} = 1.5 \times 10^2 \text{ cm}^{-3}$$

Now,  $n_n \approx N_D$  and  $p_n n_n = n_i^2$

$$n_n = n_i^2 / p_n = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^2}$$

$$= 1.5 \times 10^{18} \text{ cm}^{-3}$$

$$N_D = n_n = 1.5 \times 10^{18} \text{ atoms/cm}^3$$

**1.89**  $T = 27^\circ\text{C} = 273 + 27 = 300 \text{ K}$

At 300 K,  $n_i = 1.5 \times 10^{10} / \text{cm}^3$

Phosphorus-doped Si:

$$n_n \approx N_D = 10^{17} / \text{cm}^3$$

$$p_n = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 / \text{cm}^3$$

Hole concentration =  $p_n = 2.25 \times 10^3 / \text{cm}^3$

$T = 125^\circ\text{C} = 273 + 125 = 398 \text{ K}$

At 398 K,  $n_i = BT^{3/2} e^{-E_g/2kT}$

$$= 7.3 \times 10^{15} \times (398)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 398)}$$

$$= 4.72 \times 10^{12} / \text{cm}^3$$

$$p_n = \frac{n_i^2}{N_D} = 2.23 \times 10^8 / \text{cm}^3$$

At 398 K, hole concentration is

$$p_n = 2.23 \times 10^8 / \text{cm}^3$$

**1.90** (a) The resistivity of silicon is given by Eq. (1.41).

For intrinsic silicon,

$$p = n = n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Using  $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$\mu_p = 480 \text{ cm}^2/\text{V} \cdot \text{s}$ , and  $q = 1.6 \times 10^{-19} \text{ C}$  we have

$$\rho = 2.28 \times 10^5 \Omega\text{-cm.}$$

Using  $R = \rho \cdot \frac{L}{A}$  with  $L = 0.001 \text{ cm}$  and

$A = 3 \times 10^{-8} \text{ cm}^2$ , we have

$$R = 7.6 \times 10^9 \Omega.$$

(b)  $n_n \approx N_D = 5 \times 10^{16} \text{ cm}^{-3}$ ;

$$p_n = \frac{n_i^2}{n_n} = 4.5 \times 10^3 \text{ cm}^{-3}$$

Using  $\mu_n = 1200 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$\mu_p = 400 \text{ cm}^2/\text{V} \cdot \text{s}$ , we have

$$\rho = 0.10 \Omega\text{-cm}; R = 3.33 \text{ k}\Omega.$$

(c)  $n_n \approx N_D = 5 \times 10^{18} \text{ cm}^{-3}$ ;

$$p_n = \frac{n_i^2}{n_n} = 45 \text{ cm}^{-3}$$

Using  $\mu_n = 1200 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$\mu_p = 400 \text{ cm}^2/\text{V} \cdot \text{s}$ , we have

$$\rho = 1.0 \times 10^{-3} \Omega\text{-cm}; R = 33.3 \Omega.$$

As expected, since  $N_D$  is increased by 100, the resistivity decreases by the same factor.

(d)  $p_p \approx N_A = 5 \times 10^{16} \text{ cm}^{-3}$ ;  $n_p = \frac{n_i^2}{n_n}$

$$= 4.5 \times 10^3 \text{ cm}^{-3}$$

$$\rho = 0.31 \Omega\text{-cm}; R = 10.42 \text{ k}\Omega$$

(e) Since  $\rho$  is given to be  $2.8 \times 10^{-6} \Omega\text{-cm}$ , we directly calculate  $R = 9.33 \times 10^{-2} \Omega$ .

**1.91** Cross-sectional area of Si bar

$$= 5 \times 4 = 20 \mu\text{m}^2$$

Since  $1 \mu\text{m} = 10^{-4} \text{cm}$ , we get

$$= 20 \times 10^{-8} \text{cm}^2$$

$$\text{Current } I = Aq(p\mu_p + n\mu_n)E$$

$$= 20 \times 10^{-8} \times 1.6 \times 10^{-19}$$

$$(10^{16} \times 500 + 10^4 \times 1200) \times \frac{1 \text{ V}}{10 \times 10^{-4}}$$

$$= 160 \mu\text{A}$$

**1.92** Use Eq. (1.45):  $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T$  and  $D_p = \mu_p V_T$  where  $V_T = 25.9 \text{ mV}$ .

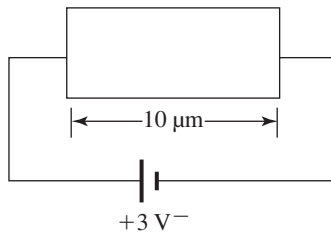
Doping Concentration (carriers/cm <sup>3</sup> )	$\mu_n$ cm <sup>2</sup> /V · s	$\mu_p$ cm <sup>2</sup> /V · s	$D_n$ cm <sup>2</sup> /s	$D_p$ cm <sup>2</sup> /s
Intrinsic	1350	480	35	12.4
$10^{16}$	1200	400	31	10.4
$10^{17}$	750	260	19.4	6.7
$10^{18}$	380	160	9.8	4.1

**1.93** Electric field:

$$E = \frac{3 \text{ V}}{10 \mu\text{m}} = \frac{3 \text{ V}}{10 \times 10^{-6} \text{ m}}$$

$$= \frac{3 \text{ V}}{10 \times 10^{-4} \text{ cm}}$$

$$= 3000 \text{ V/cm}$$



$$v_{p\text{-drift}} = \mu_p E = 480 \times 3000$$

$$= 1.44 \times 10^6 \text{ cm/s}$$

$$v_{n\text{-drift}} = \mu_n E = 1350 \times 3000$$

$$= 4.05 \times 10^6 \text{ cm/s}$$

$$\frac{v_n}{v_p} = \frac{4.05 \times 10^6}{1.44 \times 10^6} = 2.8125 \text{ or}$$

$$v_n = 2.8125 v_p$$

Or, alternatively, it can be shown as

$$\frac{v_n}{v_p} = \frac{\mu_n E}{\mu_p E} = \frac{\mu_n}{\mu_p} = \frac{1350}{480}$$

$$= 2.8125$$

**1.94**  $J_{\text{drift}} = q(n\mu_n + p\mu_p)E$

Here  $n = N_D$ , and since it is  $n$ -type silicon, one can assume  $p \ll n$  and ignore the term  $p\mu_p$ . Also,

$$E = \frac{1 \text{ V}}{10 \mu\text{m}} = \frac{1 \text{ V}}{10 \times 10^{-4} \text{ cm}} = 10^3 \text{ V/cm}$$

Need  $J_{\text{drift}} = 2 \text{ mA}/\mu\text{m}^2 = qN_D\mu_n E$

$$\frac{2 \times 10^{-3} \text{ A}}{10^{-8} \text{ cm}^2} = 1.6 \times 10^{-19} N_D \times 1350 \times 10^3$$

$$\Rightarrow N_D = 9.26 \times 10^{17} / \text{cm}^3$$

**1.95**

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 / \text{cm}^3$$

From Fig. P1.95,

$$\frac{dp}{dx} = -\frac{10^8 p_{n0} - p_{n0}}{W} \simeq -\frac{10^8 p_{n0}}{50 \times 10^{-7}}$$

since  $1 \text{ nm} = 10^{-7} \text{ cm}$

$$\frac{dp}{dx} = -\frac{10^8 \times 2.25 \times 10^4}{50 \times 10^{-7}}$$

$$= -4.5 \times 10^{17}$$

Hence

$$J_p = -qD_p \frac{dp}{dx}$$

$$= -1.6 \times 10^{-19} \times 12 \times (-4.5 \times 10^{17})$$

$$= 0.864 \text{ A/cm}^2$$

**1.96** From Table 1.3,

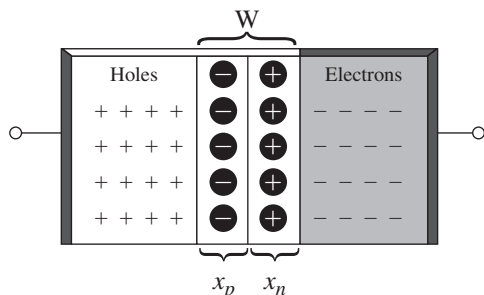
$V_T$  at 300 K = 25.9 mV

Using Eq. (1.46), built-in voltage  $V_0$  is obtained:

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 25.9 \times 10^{-3} \times$$

$$\ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.754 \text{ V}$$



Depletion width

$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_0} \leftarrow \text{Eq. (1.50)}$$

$W =$

$$\sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{1}{10^{17}} + \frac{1}{10^{16}}\right) \times 0.754}$$

$$= 0.328 \times 10^{-4} \text{ cm} = 0.328 \text{ } \mu\text{m}$$

Use Eqs. (1.51) and (1.52) to find  $x_n$  and  $x_p$ :

$$x_n = W \frac{N_A}{N_A + N_D} = 0.328 \times \frac{10^{17}}{10^{17} + 10^{16}}$$

$$= 0.298 \text{ } \mu\text{m}$$

$$x_p = W \frac{N_D}{N_A + N_D} = 0.328 \times \frac{10^{16}}{10^{17} + 10^{16}}$$

$$= 0.03 \text{ } \mu\text{m}$$

Use Eq. (1.53) to calculate charge stored on either side:

$$Q_J = Aq \left(\frac{N_A N_D}{N_A + N_D}\right) W, \text{ where junction area}$$

$$= 100 \text{ } \mu\text{m}^2 = 100 \times 10^{-8} \text{ cm}^2$$

$$Q_J = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \left(\frac{10^{17} \cdot 10^{16}}{10^{17} + 10^{16}}\right)$$

$$\times 0.328 \times 10^{-4}$$

$$\text{Hence, } Q_J = 4.8 \times 10^{-14} \text{ C}$$

**1.97** Equation (1.49):

$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_0}$$

Since  $N_A \gg N_D$ , we have

$$W \simeq \sqrt{\frac{2\epsilon_s}{q} \frac{1}{N_D} V_0}$$

$$V_0 = \frac{qN_D}{2\epsilon_s} \cdot W^2$$

Here  $W = 0.2 \text{ } \mu\text{m} = 0.2 \times 10^{-4} \text{ cm}$

$$\text{So } V_0 = \frac{1.6 \times 10^{-19} \times 10^{16} \times (0.2 \times 10^{-4})^2}{2 \times 1.04 \times 10^{-12}}$$

$$= 0.31 \text{ V}$$

$$Q_J = Aq \left(\frac{N_A N_D}{N_A + N_D}\right) W \cong AqN_D W$$

since  $N_A \gg N_D$ , we have  $Q_J = 3.2 \text{ fC}$ .

**1.98** Using Eq. (1.46) and  $N_A = N_D$

$$= 5 \times 10^{16} \text{ cm}^{-3} \text{ and } n_i = 1.5 \times 10^{10} \text{ cm}^{-3},$$

we have  $V_0 = 778 \text{ mV}$ .

Using Eq. (1.50) and

$\epsilon_s = 11.7 \times 8.854 \times 10^{-14} \text{ F/cm}$ , we have  $W = 2 \times 10^{-5} \text{ cm} = 0.2 \text{ } \mu\text{m}$ . The extension of the depletion width into the  $n$  and  $p$  regions is given in Eqs. (1.51) and (1.52), respectively:

$$x_n = W \cdot \frac{N_A}{N_A + N_D} = 0.1 \text{ } \mu\text{m}$$

$$x_p = W \cdot \frac{N_D}{N_A + N_D} = 0.1 \text{ } \mu\text{m}$$

Since both regions are doped equally, the depletion region is symmetric.

Using Eq. (1.53) and

$A = 20 \text{ } \mu\text{m}^2 = 20 \times 10^{-8} \text{ cm}^2$ , the charge magnitude on each side of the junction is

$$Q_J = 1.6 \times 10^{-14} \text{ C.}$$

**1.99** Using Eq. (1.47) or (1.48), we have charge stored:  $Q_J = qAx_n N_D$ .

Here  $x_n = 0.1 \text{ } \mu\text{m} = 0.1 \times 10^{-4} \text{ cm}$

$$A = 10 \mu\text{m} \times 10 \mu\text{m} = 10 \times 10^{-4} \text{ cm} \\ \times 10 \times 10^{-4} \text{ cm} \\ = 100 \times 10^{-8} \text{ cm}^2$$

So,

$$Q_J = 1.6 \times 10^{-19} \times 100 \times 10^{-8} \times 0.1 \times 10^{-4} \times 10^{18} \\ = 1.6 \text{ pC}$$

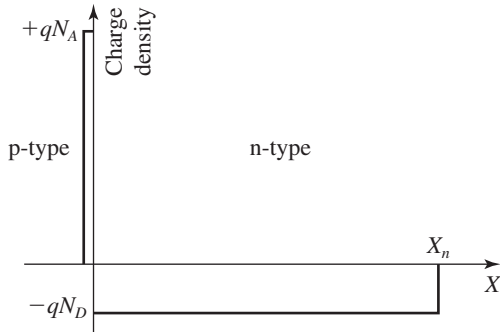
**1.100**  $V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

If  $N_A$  or  $N_D$  is increased by a factor of 10, then new value of  $V_0$  will be

$$V_0 = V_T \ln\left(\frac{10 N_A N_D}{n_i^2}\right)$$

The change in the value of  $V_0$  is  $V_T \ln 10 = 59.6 \text{ mV}$ .

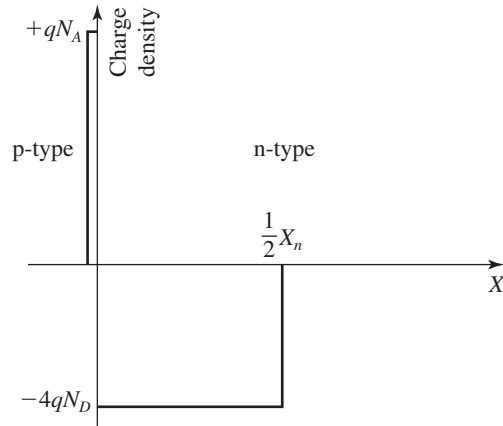
**1.101** This is a one-sided junction, with the depletion layer extending almost entirely into the more lightly doped ( $n$ -type) material. A thin space-charge region in the  $p$ -type region stores the same total charge with higher much higher charge density.



Increasing  $N_D$  by  $4\times$  will increase the charge density on the  $n$ -type side of the junction by  $4\times$ . We see from Eq. (1.45) that since  $N_A \gg N_D \gg n_i$  (i.e. at least a couple of orders of magnitude) a  $4\times$  increase in  $N_D$  will have comparatively little effect on  $V_0$ . Thus, we can assume  $V_0$  is unchanged in Eq. (1.49), and the junction width (residing almost entirely in the  $n$ -type material) is

$$W \approx \sqrt{\frac{2\epsilon_s}{q} \frac{1}{N_A} V_0} \propto \frac{1}{\sqrt{N_D}}$$

A  $4\times$  increase in  $N_D$  therefore results in a  $2\times$  decrease in the width of the depletion region on the  $n$ -type side of the junction, as illustrated below.



**1.102** The area under the triangle is equal to the built-in voltage.

$$V_0 = \frac{1}{2} E_{\text{max}} W$$

Using Eq. (1.49):

$$V_0 = \frac{E_{\text{max}}}{2} \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right)} \times \sqrt{V_0} \\ \Rightarrow E_{\text{max}} = \sqrt{\frac{2qN_A N_D}{\epsilon_s(N_A + N_D)}} V_0$$

Substituting Eq. (1.45) for  $V_0$ ,

$$E_{\text{max}} = \sqrt{\frac{2kTN_A N_D}{\epsilon_s(N_A + N_D)} \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

Finally, we can substitute for  $n_i$  using Eq. (1.26)

$$E_{\text{max}} = \sqrt{\frac{2kTN_A N_D}{\epsilon_s(N_A + N_D)} \left[ \ln(N_A N_D) - 2 \ln B - 3 \ln T + \frac{E_g}{kT} \right]}$$

**1.103** Using Eq. (1.46) with  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $N_D = 10^{16} \text{ cm}^{-3}$ , and  $n_i = 1.5 \times 10^{10}$ , we have  $V_0 = 754 \text{ mV}$

Using Eq. (1.55) with  $V_R = 5 \text{ V}$ , we have  $W = 0.907 \mu\text{m}$ .

Using Eq. (1.56) with  $A = 1 \times 10^{-6} \text{ cm}^2$ , we have  $Q_J = 13.2 \times 10^{-14} \text{ C}$ .

**1.104** Equation (1.65):

$$I_S = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

$$A = 100 \mu\text{m}^2 = 100 \times 10^{-8} \text{ cm}^2$$

$$I_S = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\left( \frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{17}} \right)$$

$$= 7.85 \times 10^{-17} \text{ A}$$

$$I \cong I_S e^{V/V_T}$$

$$= 7.85 \times 10^{-17} \times e^{750/25.9}$$

$$\cong 0.3 \text{ mA}$$

**1.105** Equation (1.54):

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

$$= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0 \left( 1 + \frac{V_R}{V_0} \right)}$$

$$= W_0 \sqrt{1 + \frac{V_R}{V_0}}$$

Equation (1.55):

$$Q_J = A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) \cdot (V_0 + V_R)}$$

$$= A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) V_0 \cdot \left( 1 + \frac{V_R}{V_0} \right)}$$

$$= Q_{J0} \sqrt{1 + \frac{V_R}{V_0}}$$

**1.106** Equation (1.62):

$$I = A q n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

$$\text{Here } I_p = A q n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

$$I_n = A q n_i^2 \frac{D_n}{L_n N_A} (e^{V/V_T} - 1)$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n} \cdot \frac{L_n}{L_p} \cdot \frac{N_A}{N_D}$$

$$= \frac{10}{20} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}}$$

$$\frac{I_p}{I_n} = 100$$

$$\text{Now } I = I_p + I_n = 100 I_n + I_n \cong 1 \text{ mA}$$

$$I_n = \frac{1}{101} \text{ mA} = 0.0099 \text{ mA}$$

$$I_p = 1 - I_n = 0.9901 \text{ mA}$$

$$\mathbf{1.107} \quad n_i = B T^{3/2} e^{-E_g/2kT}$$

At 300 K,

$$n_i = 7.3 \times 10^{15} \times (300)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)}$$

$$= 1.4939 \times 10^{10} / \text{cm}^2$$

$$n_i^2 \text{ (at 300 K)} = 2.232 \times 10^{20}$$

At 305 K,

$$n_i = 7.3 \times 10^{15} \times (305)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 305)}$$

$$= 2.152 \times 10^{10}$$

$$n_i^2 \text{ (at 305 K)} = 4.631 \times 10^{20}$$

$$\text{so } \frac{n_i^2 \text{ (at 305 K)}}{n_i^2 \text{ (at 300 K)}} = 2.152$$

Thus  $I_S$  approximately doubles for every 5°C rise in temperature.

**1.108** Equation (1.63)

$$I = A q n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

$$\text{So } I_p = A q n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

$$I_n = A q n_i^2 \frac{D_n}{L_n N_A} (e^{V/V_T} - 1)$$

For  $p^+ - n$  junction  $N_A \gg N_D$ , thus  $I_p \gg I_n$  and

$$I \simeq I_p = A q n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

For this case using Eq. (1.65):

$$I_S \simeq A q n_i^2 \frac{D_p}{L_p N_D} = 10^4 \times 10^{-8} \times 1.6 \times 10^{-19} \\ \times (1.5 \times 10^{10})^2 \frac{10}{10 \times 10^{-4} \times 10^{17}}$$

$$= 3.6 \times 10^{-16} \text{ A}$$

$$I = I_S (e^{V/V_T} - 1) = 1.0 \times 10^{-3}$$



$$3.6 \times 10^{-16} \left( e^{V/(25.9 \times 10^{-3})} - 1 \right) = 1.0 \times 10^{-3}$$

$$\Rightarrow V = 0.742 \text{ V}$$

$$\mathbf{1.109} \quad V_Z = 12 \text{ V}$$

Rated power dissipation of diode = 0.25 W.

If continuous current “ $I$ ” raises the power dissipation to half the rated value, then

$$12 \text{ V} \times I = \frac{1}{2} \times 0.25 \text{ W}$$

$$I = 10.42 \text{ mA}$$

Since breakdown occurs for only half the time, the breakdown current  $I$  can be determined from

$$I \times 12 \times \frac{1}{2} = 0.25 \text{ W}$$

$$\Rightarrow I = 41.7 \text{ mA}$$

$$\mathbf{1.110} \quad \text{Equation (1.73), } C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

$$\text{For } V_R = 1 \text{ V, } C_j = \frac{0.4 \text{ pF}}{\left(1 + \frac{1}{0.75}\right)^{1/3}}$$

$$= 0.3 \text{ pF}$$

$$\text{For } V_R = 10 \text{ V, } C_j = \frac{0.4 \text{ pF}}{\left(1 + \frac{10}{0.75}\right)^{1/3}}$$

$$= 0.16 \text{ pF}$$

$$\mathbf{1.111} \quad \text{Equation (1.81):}$$

$$C_d = \left(\frac{\tau_T}{V_T}\right)I$$

$$5 \text{ pF} = \left(\frac{\tau_T}{25.9 \times 10^{-3}}\right) \times 1 \times 10^{-3}$$

$$\tau_T = 5 \times 10^{-12} \times 25.9$$

$$= 129.5 \text{ ps}$$

For  $I = 0.1 \text{ mA}$ :

$$C_d = \left(\frac{\tau_T}{V_T}\right) \times I$$

$$= \left(\frac{129.5 \times 10^{-12}}{25.9 \times 10^{-3}}\right) \times 0.1 \times 10^{-3} = 0.5 \text{ pF}$$

$$\mathbf{1.112} \quad \text{Equation (1.72):}$$

$$C_{j0} = A \sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \left(\frac{1}{V_0}\right)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 25.9 \times 10^{-3} \times \ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.754 \text{ V}$$

$$C_{j0} = 100 \times 10^{-8}$$

$$\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right) \left(\frac{10^{17} \times 10^{16}}{10^{17} + 10^{16}}\right) \frac{1}{0.754}}$$

$$= 31.6 \text{ fF}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} = \frac{31.6 \text{ fF}}{\sqrt{1 + \frac{3}{0.754}}}$$

$$= 14.16 \text{ fF}$$

$$\mathbf{1.113} \quad \text{Equation (1.66):}$$

$$\alpha = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}$$

Equation (1.68):

$$C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}}$$

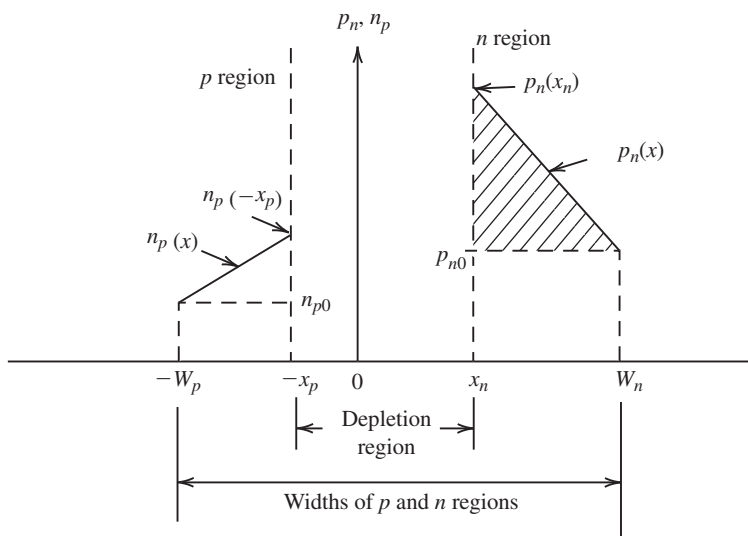
Substitute for  $\alpha$  from Eq. (1.66):

$$C_j = \frac{A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}}{2\sqrt{V_0 + V_R}} \times \frac{\sqrt{\epsilon_s}}{\sqrt{\epsilon_s}}$$

$$= A\epsilon_s \times \frac{1}{\sqrt{\frac{2\epsilon_s}{q} \left(\frac{N_A + N_D}{N_A N_D}\right) (V_0 + V_R)}}$$

$$= \epsilon_s A \frac{1}{\sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) (V_0 + V_R)}}$$

$$= \frac{\epsilon_s A}{W}$$



1.114 (a) See figure at top of page.

(b) The current  $I = I_p + I_n$ .

Find current component  $I_p$ :

$$p_n(x_n) = p_{n0}e^{V/V_T} \text{ and } p_{n0} = \frac{n_i^2}{N_D}$$

$$I_p = AJ_p = AqD_p \frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{p_n(x_n) - p_{n0}}{W_n - x_n} = \frac{p_{n0}e^{V/V_T} - p_{n0}}{W_n - x_n}$$

$$= p_{n0} \frac{(e^{V/V_T} - 1)}{W_n - x_n}$$

$$= \frac{n_i^2}{N_D} \frac{(e^{V/V_T} - 1)}{(W_n - x_n)}$$

$$\therefore I_p = AqD_p \frac{dp}{dx}$$

$$= Aqn_i^2 \frac{D_p}{(W_n - x_n)N_D} \times (e^{V/V_T} - 1)$$

Similarly,

$$I_n = Aqn_i^2 \frac{D_n}{(W_p - x_p)N_A} \times (e^{V/V_T} - 1)$$

$$I = I_p + I_n$$

$$= Aqn_i^2 \left[ \frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right]$$

$$\times (e^{V/V_T} - 1)$$

The excess charge,  $Q_p$ , can be obtained by multiplying the area of the shaded triangle of the  $p_n(x)$  distribution graph by  $Aq$ .

$$Q_p = Aq \times \frac{1}{2} [p_n(x_n) - p_{n0}] (W_n - x_n)$$

$$= \frac{1}{2} Aq [p_{n0}e^{V/V_T} - p_{n0}] (W_n - x_n)$$

$$= \frac{1}{2} Aqp_{n0} (e^{V/V_T} - 1) (W_n - x_n)$$

$$= \frac{1}{2} Aq \frac{n_i^2}{N_D} (W_n - x_n) (e^{V/V_T} - 1)$$

$$= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} \cdot I_p$$

$$\simeq \frac{1}{2} \frac{W_n^2}{D_p} \cdot I_p \text{ for } W_n \gg x_n$$

(c) For  $Q \simeq Q_p$ ,  $I \simeq I_p$ ,

$$Q \simeq \frac{1}{2} \frac{W_n^2}{D_p} I$$

$$\text{Thus, } \tau_T = \frac{1}{2} \frac{W_n^2}{D_p}, \text{ and}$$

$$C_d = \frac{dQ}{dV} = \tau_T \frac{dI}{dV}$$

$$\text{But } I = I_S (e^{V/V_T} - 1)$$

$$\frac{dI}{dV} = \frac{I_S e^{V/V_T}}{V_T} \simeq \frac{I}{V_T}$$

$$\text{so } C_d \simeq \tau_T \cdot \frac{I}{V_T}.$$

$$(d) C_d = \frac{1}{2} \frac{W_n^2}{10} \frac{1 \times 10^{-3}}{25.9 \times 10^{-3}} = 8 \times 10^{-12} \text{ F}$$

Solve for  $W_n$ :

$$W_n = 0.64 \mu\text{m}$$

**Chapter 2**

**Solutions to Exercises within the Chapter**

**Ex: 2.1** The minimum number of terminals required by a single op amp is 5: two input terminals, one output terminal, one terminal for positive power supply, and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amps can all share one terminal for positive power supply and one terminal for negative power supply.

**Ex: 2.2** Relevant equations are:  
 $v_3 = A(v_2 - v_1)$ ;  $v_{Id} = v_2 - v_1$ ,  
 $v_{Icm} = \frac{1}{2}(v_1 + v_2)$

(a)  
 $v_1 = v_2 - \frac{v_3}{A} = 0 - \frac{4}{10^3} = -0.004 \text{ V} = -4 \text{ mV}$   
 $v_{Id} = v_2 - v_1 = 0 - (-0.004) = +0.004 \text{ V}$   
 $= 4 \text{ mV}$

$v_{Icm} = \frac{1}{2}(-4 \text{ mV} + 0) = -2 \text{ mV}$

(b)  $-10 = 10^3(2 - v_1) \Rightarrow v_1 = 2.01 \text{ V}$   
 $v_{Id} = v_2 - v_1 = 2 - 2.01 = -0.01 \text{ V} = -10 \text{ mV}$   
 $v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.01 + 2) = 2.005 \text{ V}$   
 $\simeq 2 \text{ V}$

(c)  
 $v_3 = A(v_2 - v_1) = 10^3(1.998 - 2.002) = -4 \text{ V}$   
 $v_{Id} = v_2 - v_1 = 1.998 - 2.002 = -4 \text{ mV}$   
 $v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.002 + 1.998) = 2 \text{ V}$

(d)  
 $-1.2 = 10^3[v_2 - (-1.2)] = 10^3(v_2 + 1.2)$   
 $\Rightarrow v_2 = -1.2012 \text{ V}$   
 $v_{Id} = v_2 - v_1 = -1.2012 - (-1.2)$   
 $= -0.0012 \text{ V} = -1.2 \text{ mV}$   
 $v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-1.2 + (-1.2012)]$   
 $\simeq -1.2 \text{ V}$

**Ex: 2.3** From Fig. E2.3 we have:  $v_3 = \mu v_d$  and

$v_d = (G_m v_2 - G_m v_1)R = G_m R(v_2 - v_1)$

Therefore:

$v_3 = \mu G_m R(v_2 - v_1)$

That is, the open-loop gain of the op amp is  $A = \mu G_m R$ . For  $G_m = 20 \text{ mA/V}$  and

$\mu = 50$ , we have:

$A = 50 \times 20 \times 5 = 5000 \text{ V/V}$ , or equivalently,  $74 \text{ dB}$ .

**Ex: 2.4** The gain and input resistance of the inverting amplifier circuit shown in Fig. 2.5 are  $-\frac{R_2}{R_1}$  and  $R_1$ , respectively. Therefore, we have:

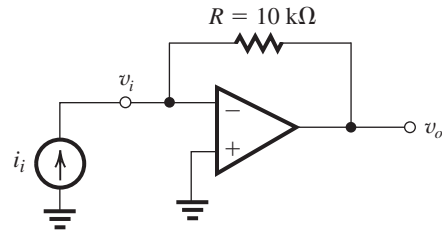
$R_1 = 100 \text{ k}\Omega$  and

$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10 R_1$

Thus:

$R_2 = 10 \times 100 \text{ k}\Omega = 1 \text{ M}\Omega$

**Ex: 2.5**



From Table 1.1 we have:

$R_m = \left. \frac{v_o}{i_i} \right|_{i_o=0}$ ; that is, output is open circuit

The negative input terminal of the op amp (i.e.,  $v_i$ ) is a virtual ground, thus  $v_i = 0$ :

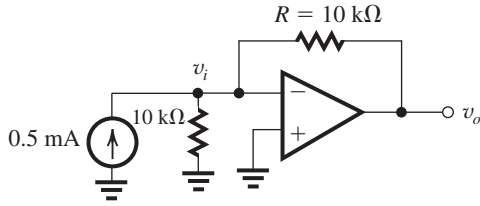
$v_o = v_i - R i_i = 0 - R i_i = -R i_i$

$R_m = \left. \frac{v_o}{i_i} \right|_{i_o=0} = -\frac{R i_i}{i_i} = -R \Rightarrow R_m = -R$   
 $= -10 \text{ k}\Omega$

$R_i = \frac{v_i}{i_i}$  and  $v_i$  is a virtual ground ( $v_i = 0$ ),

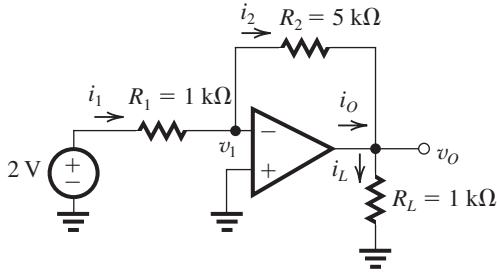
thus  $R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0 \Omega$

Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is  $R_o = 0 \Omega$ .



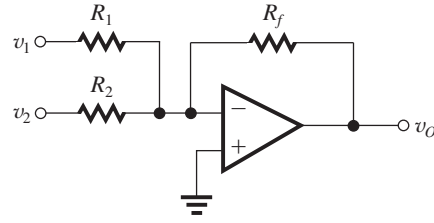
Connecting the signal source shown in Fig. E2.5 to the input of this amplifier, we have:  $v_i$  is a virtual ground that is  $v_i = 0$ , thus the current flowing through the 10-k $\Omega$  resistor connected between  $v_i$  and ground is zero. Therefore,  $v_o = v_i - R \times 0.5 \text{ mA} = 0 - 10 \text{ k}\Omega \times 0.5 \text{ mA} = -5 \text{ V}$ .

**Ex: 2.6**



$v_1$  is a virtual ground, thus  $v_1 = 0 \text{ V}$   
 $i_1 = \frac{2 \text{ V} - v_1}{R_1} = \frac{2 - 0}{1 \text{ k}\Omega} = 2 \text{ mA}$   
 Assuming an ideal op amp, the current flowing into the negative input terminal of the op amp is zero. Therefore,  $i_2 = i_1 \Rightarrow i_2 = 2 \text{ mA}$   
 $v_o = v_1 - i_2 R_2 = 0 - 2 \text{ mA} \times 5 \text{ k}\Omega = -10 \text{ V}$   
 $i_L = \frac{v_o}{R_L} = \frac{-10 \text{ V}}{1 \text{ k}\Omega} = -10 \text{ mA}$   
 $i_o = i_L - i_2 = -10 \text{ mA} - 2 \text{ mA} = -12 \text{ mA}$   
 Voltage gain =  $\frac{v_o}{2 \text{ V}} = \frac{-10 \text{ V}}{1 \text{ V}} = -5 \text{ V/V}$  or 14 dB  
 Current gain =  $\frac{i_L}{i_1} = \frac{-10 \text{ mA}}{2 \text{ mA}} = -5 \text{ A/A}$  or 14 dB  
 Power gain  
 $= \frac{P_L}{P_i} = \frac{-10(-10 \text{ mA})}{2 \text{ V} \times 2 \text{ mA}} = 25 \text{ W/W}$  or 14 dB  
 Note that power gain in dB is  $10 \log_{10} \left| \frac{P_L}{P_i} \right|$ .

**Ex: 2.7**



For the circuit shown above we have:

$$v_o = - \left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 \right)$$

Since it is required that  $v_o = -(v_1 + 4v_2)$ ,

we want to have:

$$\frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 4$$

It is also desired that for a maximum output voltage of 4 V, the current in the feedback resistor not exceed 1 mA.

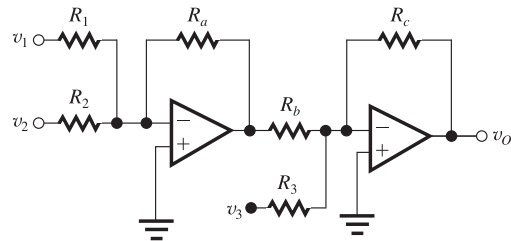
Therefore

$$\frac{4 \text{ V}}{R_f} \leq 1 \text{ mA} \Rightarrow R_f \geq \frac{4 \text{ V}}{1 \text{ mA}} \Rightarrow R_f \geq 4 \text{ k}\Omega$$

Let us choose  $R_f$  to be 4 k $\Omega$ , then

$$R_1 = R_f = 4 \text{ k}\Omega \quad \text{and} \quad R_2 = \frac{R_f}{4} = 1 \text{ k}\Omega$$

**Ex: 2.8**



$$v_o = \left( \frac{R_a}{R_1} \right) \left( \frac{R_c}{R_b} \right) v_1 + \left( \frac{R_a}{R_2} \right) \left( \frac{R_c}{R_b} \right) v_2 - \left( \frac{R_c}{R_3} \right) v_3$$

We want to design the circuit such that

$$v_o = 2v_1 + v_2 - 4v_3$$

Thus we need to have

$$\left( \frac{R_a}{R_1} \right) \left( \frac{R_c}{R_b} \right) = 2, \quad \left( \frac{R_a}{R_2} \right) \left( \frac{R_c}{R_b} \right) = 1, \quad \text{and} \quad \frac{R_c}{R_3} = 4$$

From the above three equations, we have to find six unknown resistors; therefore, we can arbitrarily choose three of these resistors. Let us choose  $R_a = R_b = R_c = 10 \text{ k}\Omega$ .

Then we have

$$R_3 = \frac{R_c}{4} = \frac{10}{4} = 2.5 \text{ k}\Omega$$

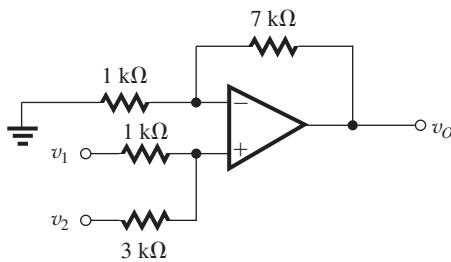
$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \Rightarrow \frac{10}{R_1} \times \frac{10}{10} = 2$$

$$\Rightarrow R_1 = 5 \text{ k}\Omega$$

$$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1$$

$$\Rightarrow R_2 = 10 \text{ k}\Omega$$

**Ex: 2.9** Using the superposition principle to find the contribution of  $v_1$  to the output voltage  $v_o$ , we set  $v_2 = 0$



$v_+$  (the voltage at the positive input of the op amp

$$\text{is: } v_+ = \frac{3}{1+3} v_1 = 0.75 v_1$$

$$\text{Thus } v_o = \left(1 + \frac{7 \text{ k}\Omega}{1 \text{ k}\Omega}\right) v_+ = 8 \times 0.75 v_1 = 6 v_1$$

To find the contribution of  $v_2$  to the output voltage  $v_o$  we set  $v_1 = 0$ .

$$\text{Then } v_+ = \frac{1}{1+3} v_2 = 0.25 v_2$$

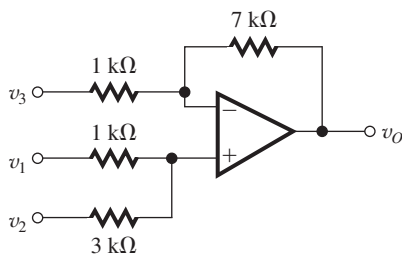
Hence

$$v_o = \left(1 + \frac{7 \text{ k}\Omega}{1 \text{ k}\Omega}\right) v_+ = 8 \times 0.25 v_2 = 2 v_2$$

Combining the contributions of  $v_1$  and  $v_2$

to  $v_o$ , we have  $v_o = 6 v_1 + 2 v_2$

**Ex: 2.10**



Using the superposition principle to find the contribution of  $v_1$  to  $v_o$ , we set  $v_2 = v_3 = 0$ . Then we have (refer to the solution of Exercise 2.9):  $v_o = 6 v_1$

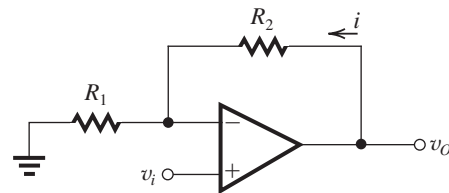
To find the contribution of  $v_2$  to  $v_o$ , we set  $v_1 = v_3 = 0$ , then:  $v_o = 2 v_2$

To find the contribution of  $v_3$  to  $v_o$  we set  $v_1 = v_2 = 0$ , then

$$v_o = -\frac{7 \text{ k}\Omega}{1 \text{ k}\Omega} v_3 = -7 v_3$$

Combining the contributions of  $v_1, v_2$ , and  $v_3$  to  $v_o$  we have:  $v_o = 6 v_1 + 4 v_2 - 7 v_3$ .

**Ex: 2.11**



$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$$

If  $v_o = 10 \text{ V}$ , then it is desired that  $i = 10 \mu\text{A}$ .

Thus,

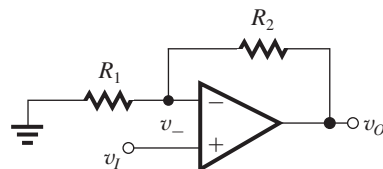
$$i = \frac{10 \text{ V}}{R_1 + R_2} = 10 \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10 \text{ V}}{10 \mu\text{A}}$$

$R_1 + R_2 = 1 \text{ M}\Omega$  and

$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5 \text{ M}\Omega$

**Ex: 2.12**

(a)



$$v_i - v_- = v_o/A \Rightarrow v_- = v_i - v_o/A \quad (1)$$

But from the voltage divider across  $v_o$ ,

$$v_- = v_o \frac{R_1}{R_1 + R_2} \quad (2)$$

Equating Eq. (1) and Eq. (2) gives

$$v_o \frac{R_1}{R_1 + R_2} = v_i - \frac{v_o}{A}$$

which can be manipulated to the form

$$\frac{v_O}{v_I} = \frac{1 + (R_2/R_1)}{1 + \frac{1 + (R_2/R_1)}{A}}$$

(b) For  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 9 \text{ k}\Omega$  the ideal value for the closed-loop gain is  $1 + \frac{9}{1}$ , that is, 10. The actual closed-loop gain is  $G = \frac{10}{1 + 10/A}$ .

If  $A = 10^3$ , then  $G = 9.901$  and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For  $v_I = 1 \text{ V}$ ,  $v_O = G \times v_I = 9.901 \text{ V}$  and

$$v_O = A(v_+ - v_-) \Rightarrow v_+ - v_- = \frac{v_O}{A} = \frac{9.901}{1000}$$

$\simeq 9.9 \text{ mV}$

If  $A = 10^4$ , then  $G = 9.99$  and  $\epsilon = -0.1\%$ .

For  $v_I = 1 \text{ V}$ ,  $v_O = G \times v_I = 9.99 \text{ V}$ ,

therefore,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.99}{10^4} = 0.999 \text{ mV} \simeq 1 \text{ mV}$$

If  $A = 10^5$ , then  $G = 9.999$  and  $\epsilon = -0.01\%$

For  $v_I = 1 \text{ V}$ ,  $v_O = G \times v_I = 9.999$  thus,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.999}{10^5} = 0.09999 \text{ mV}$$

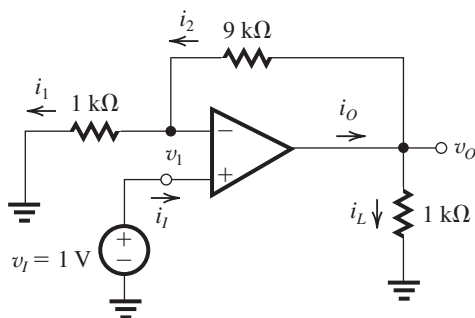
$\simeq 0.1 \text{ mV}$

**Ex: 2.13**

$i_I = 0 \text{ A}$ ,  $v_1 = v_I = 1 \text{ V}$

$$i_1 = \frac{v_1}{1 \text{ k}\Omega} = \frac{1 \text{ V}}{1 \text{ k}\Omega} = 1 \text{ mA}$$

$i_2 = i_1 = 1 \text{ mA}$



$v_O = v_1 + i_2 \times 9 \text{ k}\Omega = 1 + 1 \times 9 = 10 \text{ V}$

$$i_L = \frac{v_O}{1 \text{ k}\Omega} = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

$i_O = i_L + i_2 = 11 \text{ mA}$

$$\frac{v_O}{v_I} = \frac{10 \text{ V}}{1 \text{ V}} = 10 \text{ V/V or } 20 \text{ dB}$$

$$\frac{i_L}{i_I} = \frac{10 \text{ mA}}{0} = \infty$$

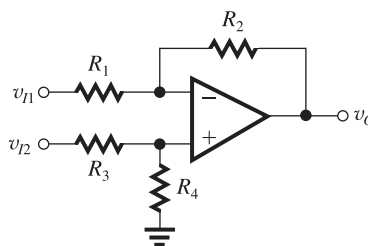
$$\frac{P_L}{P_I} = \frac{v_O \times i_L}{v_I \times i_I} = \frac{10 \times 10}{1 \times 0} = \infty$$

**Ex: 2.14**

(a) Load voltage =  $\frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ M}\Omega} \times 1 \text{ V} \simeq 1 \text{ mV}$

(b) Load voltage = 1 V

**Ex: 2.15**



(a)  $R_1 = R_3 = 2 \text{ k}\Omega$ ,  $R_2 = R_4 = 200 \text{ k}\Omega$

Since  $R_4/R_3 = R_2/R_1$  we have:

$$A_d = \frac{v_O}{v_{I2} - v_{I1}} = \frac{R_2}{R_1} = \frac{200}{2} = 100 \text{ V/V}$$

(b)  $R_{id} = 2R_1 = 2 \times 2 \text{ k}\Omega = 4 \text{ k}\Omega$

Since we are assuming the op amp is ideal,

$R_o = 0 \Omega$

(c)

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The worst-case common-mode gain (i.e., the largest  $A_{cm}$ ) occurs when the resistor tolerances are such that the quantity in parentheses is maximum. This in turn occurs when  $R_2$  and  $R_3$  are at their highest possible values (each one percent above nominal) and  $R_1$  and  $R_4$  are at their lowest possible values (each one percent below nominal), resulting in

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{1.01 \times 1.01}{0.99 \times 0.99} \right)$$

$$|A_{cm}| \simeq \frac{R_4}{R_3 + R_4} \times 0.04 \simeq \frac{200}{202} \times 0.04 \simeq 0.04 \text{ V/V}$$

The corresponding CMRR is

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.04} = 2500$$

or 68 dB.

**Ex: 2.16** We choose  $R_3 = R_1$  and  $R_4 = R_2$ . Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of 20 k $\Omega$ , we require

$$A_d = \frac{R_2}{R_1} = 10 \text{ and}$$

$$R_{id} = 2R_1 = 20 \text{ k}\Omega \Rightarrow R_1 = 10 \text{ k}\Omega \text{ and}$$

$$R_2 = A_d R_1 = 10 \times 10 \text{ k}\Omega = 100 \text{ k}\Omega$$

Therefore,  $R_1 = R_3 = 10 \text{ k}\Omega$  and

$$R_2 = R_4 = 100 \text{ k}\Omega.$$

**Ex: 2.17** Given  $v_{lcm} = +5 \text{ V}$

$$v_{ld} = 10 \sin \omega t \text{ mV}$$

$$2R_1 = 1 \text{ k}\Omega, R_2 = 0.5 \text{ M}\Omega$$

$$R_3 = R_4 = 10 \text{ k}\Omega$$

$$v_{11} = v_{lcm} - \frac{1}{2} v_{ld} = 5 - \frac{1}{2} \times 0.01 \sin \omega t$$

$$= 5 - 0.005 \sin \omega t \text{ V}$$

$$v_{12} = v_{lcm} + \frac{1}{2} v_{ld}$$

$$= 5 + 0.005 \sin \omega t \text{ V}$$

$$v_{-}(\text{op amp } A_1) = v_{11} = 5 - 0.005 \sin \omega t \text{ V}$$

$$v_{-}(\text{op amp } A_2) = v_{12} = 5 + 0.005 \sin \omega t \text{ V}$$

$$v_{ld} = v_{12} - v_{11} = 0.01 \sin \omega t$$

$$v_{O1} = v_{11} - R_2 \times \frac{v_{ld}}{2R_1}$$

$$= 5 - 0.005 \sin \omega t - 500 \text{ k}\Omega \times \frac{0.01 \sin \omega t}{1 \text{ k}\Omega}$$

$$= (5 - 5.005 \sin \omega t) \text{ V}$$

$$v_{O2} = v_{12} + R_2 \times \frac{v_{ld}}{2R_1}$$

$$= (5 + 5.005 \sin \omega t) \text{ V}$$

$$v_{+}(\text{op amp } A_3) = v_{O2} \times \frac{R_4}{R_3 + R_4} = v_{O2} \frac{10}{10 + 10}$$

$$= \frac{1}{2} v_{O2} = \frac{1}{2} (5 + 5.005 \sin \omega t)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

$$v_{-}(\text{op amp } A_3) = v_{+}(\text{op amp } A_3)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

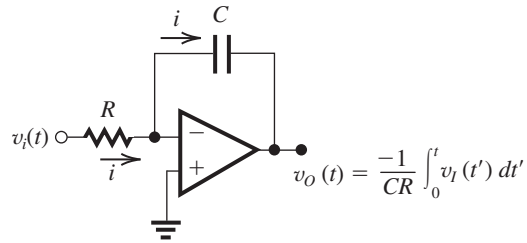
$$v_O = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) v_{ld}$$

$$\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega} \left( 1 + \frac{0.5 \text{ M}\Omega}{0.5 \text{ k}\Omega} \right) \times 0.01 \sin \omega t$$

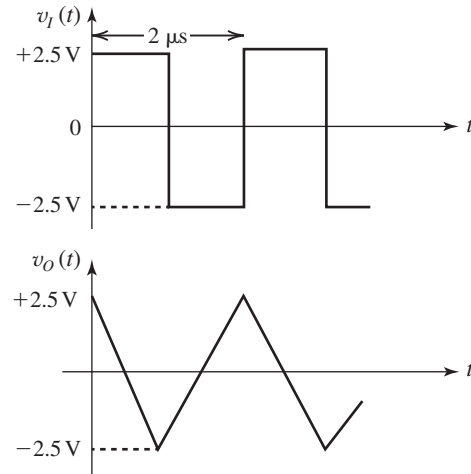
$$= 1(1 + 1000) \times 0.01 \sin \omega t$$

$$= 10.01 \sin \omega t \text{ V}$$

**Ex: 2.18**



The signal waveforms will be as shown.

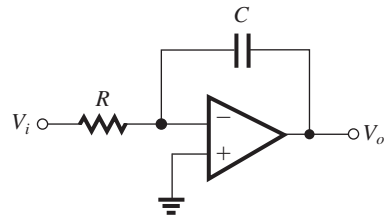


When  $v_i = +2.5 \text{ V}$ , the current through the capacitor will be in the direction indicated,  $i = 2.5 \text{ V}/R$ , and the output voltage will decrease linearly from  $+2.5 \text{ V}$  to  $-2.5 \text{ V}$ . Thus in  $(T/2)$  seconds, the capacitor voltage changes by  $5 \text{ V}$ . The charge equilibrium equation can be expressed as

$$i(T/2) = C \times 5 \text{ V}$$

$$\frac{2.5}{R} \frac{T}{2} = 5C \Rightarrow CR = \frac{2.5T}{10} = \frac{1}{4} \times 2 \times 10^{-6} = 0.5 \mu\text{s}$$

**Ex: 2.19**



The input resistance of this inverting integrator is  $R$ ; therefore,  $R = 10 \text{ k}\Omega$ .

Since the desired integration time constant is  $10^{-3}$  s, we have:  $CR = 10^{-3}$  s  $\Rightarrow$

$$C = \frac{10^{-3} \text{ s}}{10 \text{ k}\Omega} = 0.1 \text{ }\mu\text{F}$$

From Eq. (2.27) the transfer function of this integrator is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

For  $\omega = 10$  rad/s, the integrator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = \frac{1}{1 \times 10^{-3}} = 100 \text{ V/V}$$

and phase  $\phi = 90^\circ$ .

For  $\omega = 1$  rad/s, the integrator transfer function has magnitude

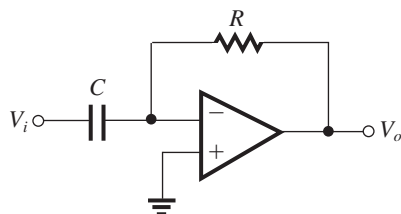
$$\left| \frac{V_o}{V_i} \right| = \frac{1}{1 \times 10^{-3}} = 1000 \text{ V/V}$$

and phase  $\phi = 90^\circ$ .

The frequency at which the integrator gain magnitude is unity is

$$\omega_{\text{int}} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000 \text{ rad/s}$$

**Ex: 2.20**



$C = 0.01 \text{ }\mu\text{F}$  is the input capacitance of this differentiator. We want  $CR = 10^{-2}$  s (the time constant of the differentiator); thus,

$$R = \frac{10^{-2}}{0.01 \text{ }\mu\text{F}} = 1 \text{ M}\Omega$$

From Eq. (2.33), the transfer function of the differentiator is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Thus, for  $\omega = 10$  rad/s the differentiator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = 10 \times 10^{-2} = 0.1 \text{ V/V}$$

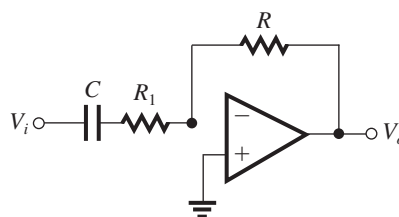
and phase  $\phi = -90^\circ$ .

For  $\omega = 10^3$  rad/s, the differentiator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = 10^3 \times 10^{-2} = 10 \text{ V/V}$$

and phase  $\phi = -90^\circ$ .

If we add a resistor in series with the capacitor to limit the high-frequency gain of the differentiator to 100, the circuit would be:



At high frequencies the capacitor  $C$  acts like a short circuit. Therefore, the high-frequency gain of this circuit is:  $\frac{R}{R_1}$ . To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_1} = 100 \Rightarrow R_1 = \frac{R}{100} = \frac{1 \text{ M}\Omega}{100} = 10 \text{ k}\Omega$$

**Ex: 2.21**

Refer to the model in Fig. 2.29 and observe that

$$v_+ - v_- = V_{OS} + v_2 - v_1 = V_{OS} + v_{Id}$$

and since  $v_O = v_3 = A(v_+ - v_-)$ , then

$$v_O = A(v_{Id} + V_{OS}) \tag{1}$$

where  $A = 10^4$  V/V and  $V_{OS} = 5$  mV. From Eq. (1) we see that  $v_{Id} = 0$  results in  $v_O = 50$  V, which is impossible; thus the op amp saturates and  $v_O = +10$  V. This situation pertains for  $v_{Id} \geq -4$  mV. If  $v_{Id}$  decreases below  $-4$  mV, the op-amp output decreases correspondingly. For instance,  $v_{Id} = -4.5$  mV results in  $v_O = +5$  V;  $v_{Id} = -5$  mV results in  $v_O = 0$  V;  $v_{Id} = -5.5$  mV results in  $v_O = -5$  V; and  $v_{Id} = -6$  mV results in  $v_O = -10$  V, at which point the op amp saturates at the negative level of  $-10$  V. Further decreases in  $v_{Id}$  have no effect on the output voltage. The result is the transfer characteristic sketched in Fig. E2.21. Observe that the linear range of the characteristic is now centered around  $v_{Id} = -5$  mV rather than the ideal situation of  $v_{Id} = 0$ ; this shift is obviously a result of the input offset voltage  $V_{OS}$ .



**Ex: 2.22** (a) The inverting amplifier of  $-1000$  V/V gain will exhibit an output dc offset voltage of  $\pm V_{OS}(1 + R_2/R_1) = \pm 3 \text{ mV} \times (1 + 1000) = \pm 3.03 \text{ V}$ . Now, since the op-amp saturation levels are  $\pm 10 \text{ V}$ , the room left for output signal swing is approximately  $\pm 7 \text{ V}$ . Thus to avoid op-amp saturation and the attendant clipping of the peak of the output sinusoid, we must limit the peak amplitude of the input sine wave to approximately  $7 \text{ V}/1000 = 7 \text{ mV}$ .

(b) If at room temperature ( $25^\circ\text{C}$ ),  $V_{OS}$  is trimmed to zero and (i) the circuit is operated at a constant temperature, the peak of the input sine wave can be increased to  $10 \text{ mV}$ . (ii) However, if the circuit is to operate in the temperature range of  $0^\circ\text{C}$  to  $75^\circ\text{C}$  (i.e., at a temperature that deviates from room temperature by a maximum of  $50^\circ\text{C}$ ), the input offset voltage will drift from by a maximum of  $10 \mu\text{V}/^\circ\text{C} \times 50^\circ\text{C} = 500 \mu\text{V}$  or  $0.5 \text{ mV}$ . This will reduce the allowed peak amplitude of the input sinusoid to  $9.5 \text{ mV}$ .

**Ex: 2.23**

(a) If the amplifier is capacitively coupled in the manner of Fig. 2.32(a), then the input offset voltage  $V_{OS}$  will see a unity-gain amplifier [Fig. 2.32(b)] and the dc offset voltage at the output will be equal to  $V_{OS}$ , that is,  $3 \text{ mV}$ . Thus, almost the entire output range of  $\pm 10 \text{ V}$  will be available for signal swing, allowing a sine-wave input of approximately  $10\text{-mV}$  peak without the risk of output clipping. Obviously, in this case there is no need for output trimming.

(b) We need to select a value of the coupling capacitor  $C$  that will place the 3-dB frequency of the resulting high-pass STC circuit at  $1000 \text{ Hz}$ , thus

$$1000 = \frac{1}{2\pi CR_1}$$

$$\Rightarrow C = \frac{1}{2\pi \times 1000 \times 1 \times 10^3} = 0.16 \mu\text{F}$$

**Ex: 2.24** From Eq. (2.35) we have:

$$V_O = I_{B1}R_2 \simeq I_B R_2$$

$$= 100 \text{ nA} \times 1 \text{ M}\Omega = 0.1 \text{ V}$$

From Eq. (2.37) the value of resistor  $R_3$  (placed in series with positive input to minimize the output offset voltage) is

$$R_3 = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \text{ k}\Omega \times 1 \text{ M}\Omega}{10 \text{ k}\Omega + 1 \text{ M}\Omega}$$

$$= 9.9 \text{ k}\Omega$$

$$R_3 = 9.9 \text{ k}\Omega \simeq 10 \text{ k}\Omega$$

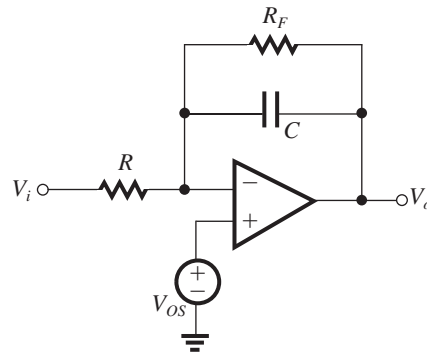
With this value of  $R_3$ , the new value of the output dc voltage [using Eq. (2.38)] is:

$$V_O = I_{OS}R_2 = 10 \text{ nA} \times 10 \text{ k}\Omega = 0.01 \text{ V}$$

**Ex: 2.25** Using Eq. (2.39) we have:

$$v_O = V_{OS} + \frac{V_{OS}}{CR}t \Rightarrow 12 = 2 \text{ mV} + \frac{2 \text{ mV}}{1 \text{ ms}}t$$

$$\Rightarrow t = \frac{12 \text{ V} - 2 \text{ mV}}{2 \text{ mV}} \times 1 \text{ ms} \simeq 6 \text{ s}$$



With the feedback resistor  $R_F$ , to have at least  $\pm 10 \text{ V}$  of output signal swing available, we have to make sure that the output voltage due to  $V_{OS}$  has a magnitude of at most  $2 \text{ V}$ . From Eq. (2.34), we know that the output dc voltage due to  $V_{OS}$  is

$$V_O = V_{OS} \left( 1 + \frac{R_F}{R} \right) \Rightarrow 2 \text{ V} = 2 \text{ mV} \left( 1 + \frac{R_F}{10 \text{ k}\Omega} \right)$$

$$1 + \frac{R_F}{10 \text{ k}\Omega} = 1000 \Rightarrow R_F \simeq 10 \text{ M}\Omega$$

The corner frequency of the resulting STC network is  $\omega_0 = \frac{1}{CR_F}$

We know  $RC = 1 \text{ ms}$  and

$$R = 10 \text{ k}\Omega \Rightarrow C = 0.1 \mu\text{F}$$

$$\text{Thus } \omega_0 = \frac{1}{0.1 \mu\text{F} \times 10 \text{ M}\Omega} = 1 \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} = 0.16 \text{ Hz}$$

**Ex: 2.26**

$$20 \log A_0 = 106 \text{ dB} \Rightarrow A_0 = 200,000 \text{ V/V}$$

$$f_i = 3 \text{ MHz}$$

$$f_b = f_i/A_0 = \frac{3 \text{ MHz}}{200,000} = 15 \text{ Hz}$$

At  $f_b$ , the open-loop gain drops by 3 dB below its value at dc; thus it becomes 103 dB.

For  $f \gg f_b$ ,  $|A| \simeq f_i/f$ ; thus

$$\text{At } f = 300 \text{ Hz, } |A| = \frac{3 \text{ MHz}}{300 \text{ Hz}} = 10^4 \text{ V/V}$$

or 80 dB

$$\text{At } f = 3 \text{ kHz, } |A| = \frac{3 \text{ MHz}}{3 \text{ kHz}} = 10^3 \text{ V/V}$$

or 60 dB

At  $f = 12 \text{ kHz}$ , which is two octaves higher than 3 kHz, the gain will be  $2 \times 6 = 12 \text{ dB}$  below its value at 3 kHz; that is,  $60 - 12 = 48 \text{ dB}$ .

$$\text{At } f = 60 \text{ kHz, } |A| = \frac{3 \text{ MHz}}{60 \text{ kHz}} = 50 \text{ V/V}$$

or 34 dB

**Ex: 2.27**

$$A_0 = 10^6 \text{ V/V or } 120 \text{ dB}$$

The gain falls off at the rate of 20 dB/decade. Thus, it reaches 40 dB at a frequency four decades higher than  $f_b$ ,

$$10^4 f_b = 100 \text{ kHz} \Rightarrow f_b = 10 \text{ Hz}$$

The unity-gain frequency  $f_i$  will be two decades higher than 100 kHz, that is,

$$f_i = 100 \times 100 \text{ kHz} = 10 \text{ MHz}$$

Alternatively, we could have found  $f_i$  from the gain-bandwidth product

$$f_i = A_0 f_b = 10^6 \times 10 \text{ Hz} = 10 \text{ MHz}$$

At a frequency  $f \gg f_b$ ,

$$|A| = f_i/f$$

$$\text{For } f = 10 \text{ kHz, } |A| = \frac{10 \text{ MHz}}{10 \text{ kHz}} = 10^3 \text{ V/V or } 60 \text{ dB}$$

**Ex: 2.28**

$$20 \log A_0 = 106 \text{ dB} \Rightarrow A_0 = 200,000 \text{ V/V}$$

$$f_i = 20 \text{ MHz}$$

For a noninverting amplifier with a nominal dc gain of 100,

$$1 + \frac{R_2}{R_1} = 100$$

Since the nominal dc gain is much lower than  $A_0$ ,

$$f_{3\text{dB}} \simeq f_i / \left(1 + \frac{R_2}{R_1}\right) = \frac{20 \text{ MHz}}{100} = 200 \text{ kHz}$$

**Ex: 2.29** For the input voltage step of magnitude  $V$  the output waveform will still be given by the exponential waveform of Eq. (2.56) if

$$\omega_i V \leq SR$$

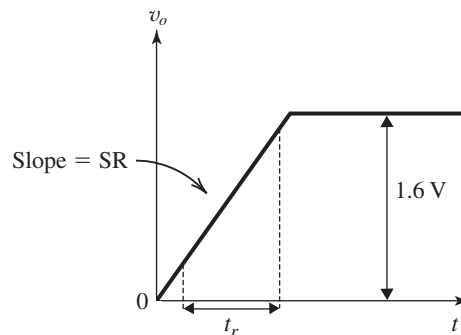
that is,  $V \leq \frac{SR}{\omega_i} \Rightarrow V \leq \frac{SR}{2\pi f_i}$  resulting in

$$V \leq 0.16 \text{ V}$$

From Appendix F we know that the 10% to 90% rise time of the output waveform of the form of Eq. (2.56) is  $t_r \simeq 2.2 \times \text{time constant} = \frac{2.2}{\omega_i}$ .

Thus,  $t_r \simeq 0.35 \mu\text{s}$

If an input step of amplitude 1.6 V (10 times as large compared to the previous case) is applied, the output will be slew-rate limited and thus linearly rising with a slope equal to the slew rate, as shown in the following figure.



$$t_r = \frac{0.9 \times 1.6 - 0.1 \times 1.6}{1 \text{ V}/\mu\text{s}}$$

$$\Rightarrow t_r = 1.28 \mu\text{s}$$

**Ex: 2.30** From Eq. (2.57) we have:

$$f_M = \frac{SR}{2\pi V_{O \text{ max}}} = 318 \text{ kHz}$$

Using Eq. (2.58), for an input sinusoid with frequency  $f = 5 f_M$ , the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O \text{ max}} \left(\frac{f_M}{5f_M}\right) = 5 \times \frac{1}{5} = 1 \text{ V (peak)}$$

**Solutions to End-of-Chapter Problems**

**2.1** Refer to Fig. P2.1.

$$v_+ = v_I \times \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ M}\Omega} = v_I \frac{1}{1001}$$

$$v_O = Av_+ = Av_I \frac{1}{1001}$$

$$A = 1001 \frac{v_O}{v_I}$$

$$= 1001 \times \frac{4}{1}$$

$$A = 4004 \text{ V/V}$$

**2.2**

#	$v_1$	$v_2$	$v_d = v_2 - v_1$	$v_O$	$v_O/v_d$
1	0.00	0.00	0.00	0.00	-
2	1.00	1.00	0.00	0.00	-
3	(a)	1.00	(b)	1.00	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	(c)	(d)	-5.10	

Experiments 4, 5, and 6 show that the gain is approximately 100 V/V.

The missing entry for experiment #3 can be predicted as follows:

(a)  $v_d = \frac{v_O}{A} = \frac{1.00}{100} = 0.01 \text{ V}$ .

(b)  $v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99 \text{ V}$

The missing entries for experiment #7:

(c)  $v_d = \frac{-5.10}{100} = -0.051 \text{ V}$

(d)  $v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049 \text{ V}$

All the results seem to be reasonable.

**2.3**

$v_I = v_{Icm} - \frac{v_{Id}}{2}$	$v_2 = v_{Icm} + \frac{v_{Id}}{2}$	$v_{Id} = v_2 - v_I$	$v_{Icm} = \frac{I}{2}(v_I + v_2)$
0.90	0.90	<b>0</b>	<b>0.90</b>
<b>0.75</b>	<b>1.25</b>	0.50	1.00
-1.20	<b>-1.00</b>	<b>0.20</b>	-1.10
-0.15	0.05	<b>0.20</b>	<b>-0.05</b>
<b>0.10</b>	<b>-0.10</b>	-0.20	0.00

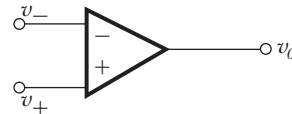
**2.4** The minimum number of pins required by a dual op amp is 8. Each op amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power supplies.

Similarly, the minimum number of pins required by a quad op amp is 14:

$$4 \times 2 + 4 \times 1 + 2 = 14$$

**2.5**  $v_O = -2.000 \text{ V}$

$v_- = -1.000 \text{ V}$



For ideal op amp,

$$v_+ = v_- = -1.000 \text{ V}$$

Measured voltage at positive input = -1.005 V

$$\text{Amplifier gain } A = \frac{v_O}{v_+ - v_-}$$

$$= \frac{-2.000}{-1.005 - (-1.000)}$$

$$= 400 \text{ V/V}$$

**2.6**  $i = G_m(v_2 - v_1)$

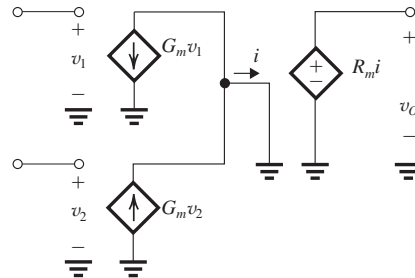
$$v_O = R_m i$$

$$= R_m G_m (v_2 - v_1)$$

$$\text{Gain } A = \frac{v_O}{v_2 - v_1} = R_m G_m$$

$$= 1 \times 10^6 \times 40 \times 10^{-3}$$

$$= 40,000 \text{ V/V}$$



2.7  $v_{cm} = 2 \sin(2\pi 60)t = \frac{1}{2}(v_1 + v_2)$

$v_d = 0.005 \sin(2\pi 1000)t = v_2 - v_1$

$v_1 = v_{cm} - v_d/2$   
 $= \sin(120\pi)t - 0.0025 \sin(2000\pi t)$

$v_2 = v_{cm} + v_d/2$   
 $= \sin(120\pi t) + 0.0025 \sin(2000\pi t)$

2.8 Refer to Fig. E2.3.

$v_d = R(G_{m2}v_2 - G_{m1}v_1)$

$v_o = v_3 = \mu v_d = \mu R(G_{m2}v_2 - G_{m1}v_1)$

$v_o =$   
 $\mu R(G_{m2}v_2 + \frac{1}{2}\Delta G_m v_2 - G_{m1}v_1 + \frac{1}{2}\Delta G_m v_1)$

$v_o = \mu R G_m \underbrace{(v_2 - v_1)}_{v_d} + \frac{1}{2} \mu R \Delta G_m \underbrace{(v_1 + v_2)}_{2v_{1cm}}$

we have  $v_o = A_d v_{1d} + A_{cm} v_{1cm}$

$\Rightarrow A_d = \mu R G_m, A_{cm} = \mu R \Delta G_m$

$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 20 \log \frac{G_m}{\Delta G_m}$

For a  $CMRR \geq 60$  dB,

$\frac{G_m}{\Delta G_m} \geq 1000 \Rightarrow \frac{\Delta G_m}{G_m} \leq 0.1\%$

2.9

Circuit	$v_o/v_i$ (V/V)	$R_{in}$ (k $\Omega$ )
a	$\frac{-100}{20} = -5$	20
b	-5	20
c	-5	20
d	-5	20

Note that in circuit (b) the 20-k $\Omega$  load resistance has no effect on the closed-loop gain because of the zero output resistance of the ideal op amp. In circuit (c), no current flows in the 20-k $\Omega$  resistor connected between the negative input terminal and ground (because of the virtual ground at the inverting input terminal). In circuit (d), zero current flows in the 20-k $\Omega$  resistor connected in series with the positive input terminal.

2.10

- (a)  $G = -2$  V/V    (b)  $G = -10$  V/V  
 (c)  $G = -0.5$  V/V    (d)  $G = -50$  V/V  
 (e)  $G = -5$  V/V

2.11 (a)  $G = -1$  V/V

$= \frac{-R_2}{R_1} \Rightarrow R_1 = R_2 = 10$  k $\Omega$

(b)  $G = -2$  V/V

$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10$  k $\Omega, R_2 = 20$  k $\Omega$

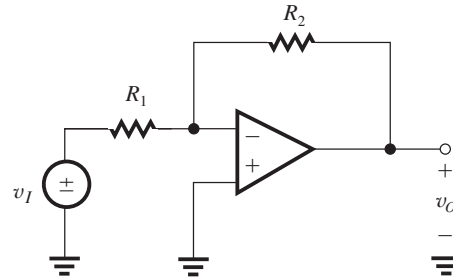
(c)  $G = -5$  V/V

$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10$  k $\Omega, R_2 = 50$  k $\Omega$

(d)  $G = -100$  V/V

$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10$  k $\Omega, R_2 = 1$  M $\Omega$

2.12 Gain is 46 dB



46 dB = 20 log |G|

|G| = 200

$\therefore \frac{v_o}{v_i} = -200$  V/V =  $-\frac{R_2}{R_1}$

$\Rightarrow R_2 = 200R_1 \leq 1$  M $\Omega$

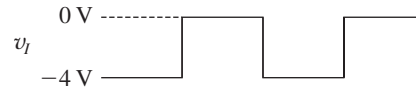
For the largest possible input resistance, choose

$R_2 = 1$  M $\Omega$

$R_1 = \frac{1 \text{ M}\Omega}{200} = 5$  k $\Omega$

$R_{in} = R_1 = 5$  k $\Omega$

2.13

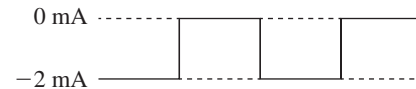


Average = -2 V

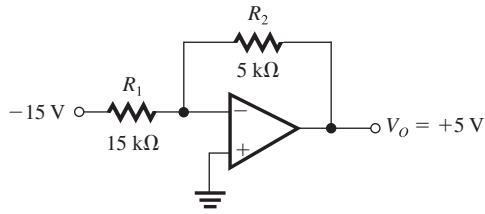
Highest = 0 V

Lowest = -4 V

The current through  $R_1$  is  $V_i/R_1$ .



2.14 The circuit will be as follows:



If  $R_1$  and  $R_2$  have  $\pm 1\%$  tolerance, then  $V_O$  will exhibit  $\pm 2\%$  variability and thus will be in the range of  $5 \times 1.02 = 5.1$  V to  $5 \times 0.98 = 4.9$  V.

Variation of the  $-15$ -V supply by  $\pm 1\%$  results in a  $\pm 1\%$  variation in the output voltage. Thus the total variation in the output voltage can be  $\pm 3\%$ , resulting in  $V_O$  in the range 4.85 V to 5.15 V.

2.15 (a) Choose  $R_1 = 1$  kΩ and  $R_2 = 200$  kΩ.

$$(b) G = -\frac{R_2/R_1}{1 + \frac{R_2/R_1}{A}}$$

$$= -\frac{200}{1 + \frac{200}{5000}} = -192.3 \text{ V/V}$$

To restore the gain to  $-200$  V/V, we need to change the effective value of  $R_1$  to  $R'_1$ ,

$$-200 = -\frac{200/R'_1}{1 + \frac{200/R'_1}{5000}}$$

Thus,

$$\frac{200}{R'_1} = 200 + \frac{200}{5000} \left(1 + \frac{200}{R'_1}\right)$$

$$0.96 \times \frac{200}{R'_1} = 200.04$$

$$\Rightarrow R'_1 = 0.960 \text{ k}\Omega$$

This effective value can be realized by shunting  $R_1$  (1 kΩ) with  $R_a$ ,

$$\frac{1}{0.960} = \frac{1}{1} + \frac{1}{R_a} \Rightarrow R_a = 24 \text{ k}\Omega$$

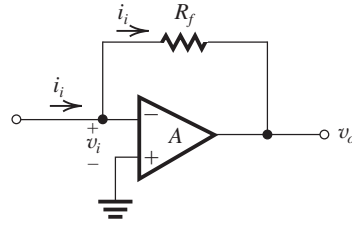
(c) From Appendix J, we find the closest available 1% resistor as either 23.7 kΩ or 24.3 kΩ.

2.16

(a) For  $A = \infty$ :  $v_i = 0$

$$v_o = -i_i R_f$$

$$R_m = \frac{v_o}{i_i} = -R_f$$



$$R_{in} = \frac{v_i}{i_i} = 0$$

(b) For  $A$  finite:

$$v_i = -\frac{v_o}{A}, \quad v_o = v_i - i_i R_f$$

$$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_f \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_f}{1 + \frac{1}{A}}$$

$$R_i = \frac{v_i}{i_i} = \frac{R_f}{1 + A}$$

2.17 From Example 2.2,

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

Here  $R_1 = R_2 = R_4 = 1$  MΩ

$$\therefore \frac{v_o}{v_i} = -\left(1 + 1 + \frac{1}{R_3}\right) = -\left(2 + \frac{1}{R_3}\right)$$

$$\frac{v_o}{v_i} + 2 = -\frac{1 \text{ M}\Omega}{R_3}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{\left(\frac{v_o}{v_i} + 2\right)}$$

$$(a) \frac{v_o}{v_i} = -100 \text{ V/V}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{(-100 + 2)} = 10.2 \text{ k}\Omega$$

$$(b) \frac{v_o}{v_i} = -10 \text{ V/V}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{(-10 + 2)} = 125 \text{ k}\Omega$$

$$(c) \frac{v_o}{v_i} = -2 \text{ V/V}, R_3 = -\frac{1 \text{ M}\Omega}{(-2 + 2)} = \infty$$

$$2.18 (a) I_1 = \frac{1 \text{ V}}{10 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_2 = I_1 = 0.1 \text{ mA}$$

$$-I_2 \times 10 \text{ k}\Omega = -I_3 \times 100 \Omega \equiv V_x$$

$$\therefore I_3 = I_2 \times \frac{10 \text{ k}\Omega}{100 \Omega}$$

$$= 0.1 \times 100$$

$$= 10 \text{ mA}$$

Now  $I_L = I_2 + I_3 = 10.1 \text{ mA}$

and  $V_x = -I_2 \times 10 \text{ k}\Omega = -0.1 \times 10 \text{ k}\Omega = -1 \text{ V}$

(b)  $V_x = R_L I_L + V_O = 10.1 R_L + V_O$

$$R_L = \frac{V_x - V_O}{10.1 \text{ mA}} = \frac{-1 - (-13)}{10.1} = 1.19 \text{ k}\Omega$$

(c)  $100 \Omega \leq R_L \leq 1 \text{ k}\Omega$

$I_L$  stays fixed at 10.1 mA.

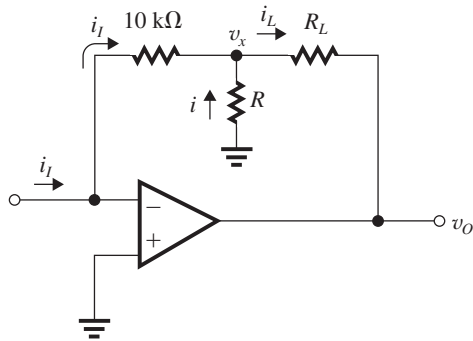
$V_O = V_x - R_L I_L = -1 - R_L \times 10.1$

$R_L = 100 \Omega$ ,  $V_O = -2.01 \text{ V}$ ;  $R_L = 1 \text{ k}\Omega$ ,

$V_O = -11.1 \text{ V}$

$\therefore -11.1 \text{ V} \leq V_O \leq -2.01 \text{ V}$

**2.19**



(a)  $v_x = -i_L \times 10$

$i = -v_x/R = i_L(10/R)$

$$i_L = i_L + i = i_L \left( 1 + \frac{10}{R} \right)$$

Thus,

$$\frac{i_L}{i_L} = 1 + \frac{10}{R}$$

For  $\frac{i_L}{i_L} = 11 \Rightarrow R = 1 \text{ k}\Omega$ .

(b)  $R_{in} = 0 \Omega$  (because of the virtual ground at the input).  $R_o = \infty \Omega$  (because  $i_L$  is independent of the value of  $R_L$ ).

(c) If  $i_L$  is in the direction shown in the figure above, the maximum allowable value of  $i_L$  will be determined by  $v_O$  reaching  $-12 \text{ V}$ , at which point

$$v_x = -i_{Lmax} \times 10$$

and

$$v_O = -12 = v_x - i_{Lmax} \times 1 = -10 i_{Lmax} - 11 i_{Lmax}$$

$$\Rightarrow i_{Lmax} = \frac{12}{21} = 0.57 \text{ mA}$$

If  $i_L$  is in a direction opposite to that shown in the figure, then

$$v_x = 10i_L$$

$$v_O = v_x + i_L R_L = 10i_L + 11i_L = 21i_L$$

The maximum value of  $i_L$  will result in  $v_O = +12 \text{ V}$ . Thus

$$12 = 21i_{Lmax} \Rightarrow i_{Lmax} = \frac{12}{21} = 0.57 \text{ mA}$$

Thus, the allowable range of  $i_L$  is

$$-0.57 \text{ mA} \leq i_L \leq +0.57 \text{ mA}$$

(d) Since  $R_{in} = 0$ , the value of the source resistance will have no effect on the resulting  $i_L$ ,

$$i_L = 0.2 \times 11 = 2.2 \text{ mA}$$

**2.20**  $v_O = -(2v_1 + 4v_2 + 8v_3)$

$R_1, R_2, R_3 \geq 1 \text{ k}\Omega$

$$\frac{R_f}{R_1} = 2, \frac{R_f}{R_2} = 4, \frac{R_f}{R_3} = 8$$

$$R_3 = 1 \text{ k}\Omega \Rightarrow R_f = 8 \text{ k}\Omega$$

$$R_2 = 2 \text{ k}\Omega$$

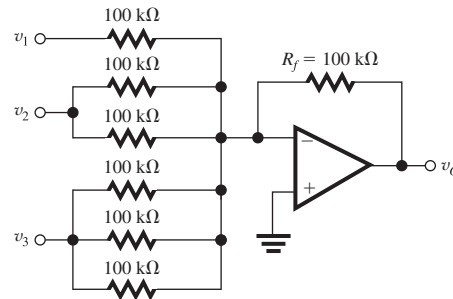
$$R_1 = 4 \text{ k}\Omega$$

**2.21** (a)  $v_O = -(v_1 + 2v_2 + 3v_3)$

$$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100 \text{ k}\Omega,$$

$$\frac{R_f}{R_2} = 2 \Rightarrow R_2 = 50 \text{ k}\Omega$$

$$\frac{R_f}{R_3} = 1 \Rightarrow R_3 = \frac{100}{3} \text{ k}\Omega$$



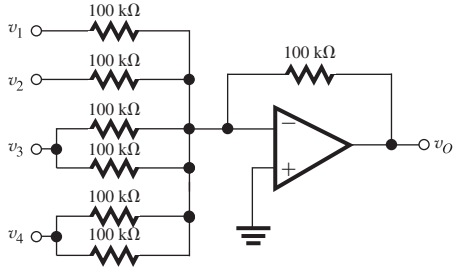
$$R_{i1} = 100 \text{ k}\Omega$$

$$R_{i2} = 50 \text{ k}\Omega$$

$$R_{i3} = 33.3 \text{ k}\Omega$$

(b)  $v_O = -(v_1 + v_2 + 2v_3 + 2v_4)$

$$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100 \text{ k}\Omega$$



$$\frac{R_f}{R_2} = 1 \Rightarrow R_2 = 100 \text{ k}\Omega$$

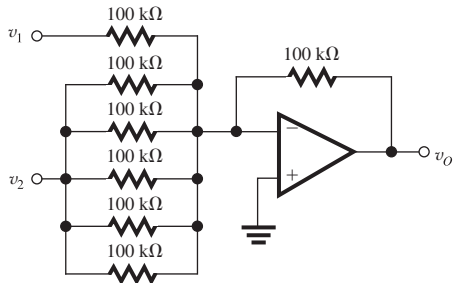
$$\frac{R_f}{R_3} = 2 \Rightarrow R_3 = \frac{100}{2} \text{ k}\Omega$$

$$\frac{R_f}{R_4} = 2 \Rightarrow R_4 = \frac{100}{2} \text{ k}\Omega$$

$$R_{i1} = 100 \text{ k}\Omega, R_{i2} = 100 \text{ k}\Omega,$$

$$R_{i3} = 50 \text{ k}\Omega, R_{i4} = 50 \text{ k}\Omega$$

(c)  $v_o = -(v_1 + 5v_2)$



$$R_1 = 100 \text{ k}\Omega$$

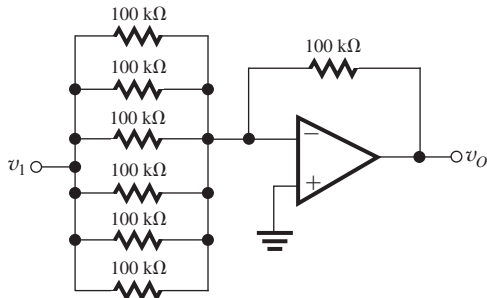
$$R_2 = \frac{100 \text{ k}\Omega}{5}$$

$$R_{i1} = 100 \text{ k}\Omega$$

$$R_{i2} = 20 \text{ k}\Omega$$

(d)  $v_o = -6v_1$

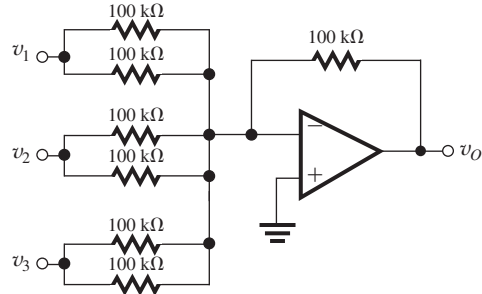
$$R_1 = \frac{100 \text{ k}\Omega}{6}$$



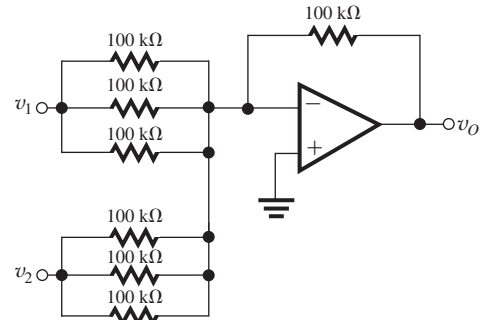
$$R_{i1} = \frac{100}{6} = 1.67 \text{ k}\Omega$$

Suggested configurations:

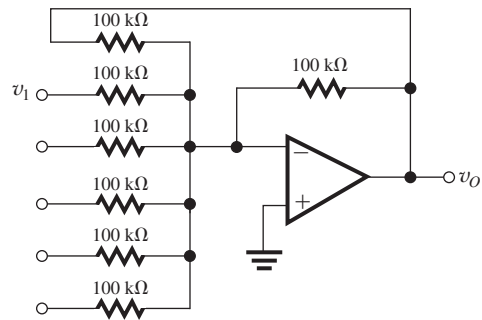
$$v_o = -(2v_1 + 2v_2 + 2v_3)$$



$$v_o = -(3v_1 + 3v_2)$$



In order to have coefficient = 0.5, connect one of the input resistors to  $v_o$ .  $\frac{v_o}{v_1} = 0.5$ .

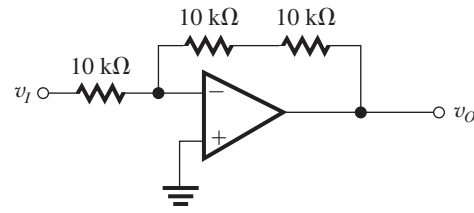
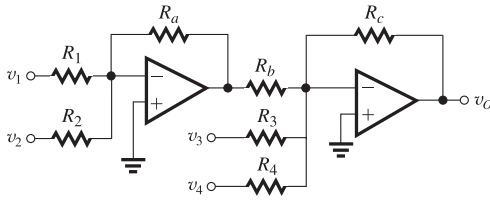


2.22 Using the circuit of Fig. 2.11:

To obtain  $v_o = v_1 + 2v_2 - 3v_3 - 5v_4$ , we can arbitrarily select  $R_c = R_b$ , then

$$\frac{R_a}{R_1} = 1 \quad \text{and} \quad \frac{R_a}{R_2} = 2$$

If we select  $R_2 = 10 \text{ k}\Omega$ , then  $R_a = 20 \text{ k}\Omega$  and  $R_1 = 20 \text{ k}\Omega$ .



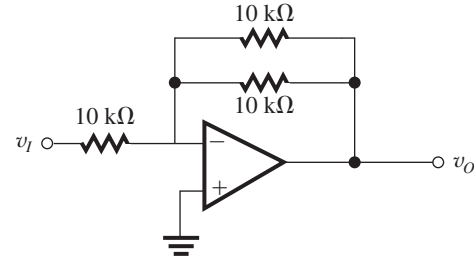
For the second summer, to obtain

$$\frac{v_O}{v_3} = -3 \quad \text{then} \quad \frac{R_c}{R_3} = 3$$

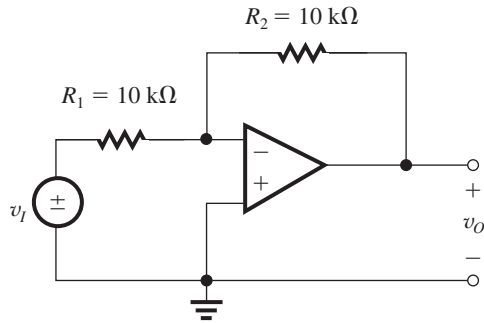
and

$$\frac{v_O}{v_4} = -5 \quad \text{requires} \quad \frac{R_c}{R_4} = 5$$

We can select  $R_c = R_b = 60 \text{ k}\Omega$ , resulting in  $R_3 = 20 \text{ k}\Omega$  and  $R_4 = 12 \text{ k}\Omega$ .



**2.23**



Closed-loop gain is

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega} = -1 \text{ V/V}$$

For  $v_I = +1.00 \text{ V}$ ,

$$v_O = -1 \times 1.00$$

$$= -1.00 \text{ V}$$

The two resistors are 1% resistors

$$\left| \frac{v_O}{v_I} \right|_{\min} = \frac{10(1 - 0.01)}{10(1 + 0.01)} = 0.98 \text{ V/V}$$

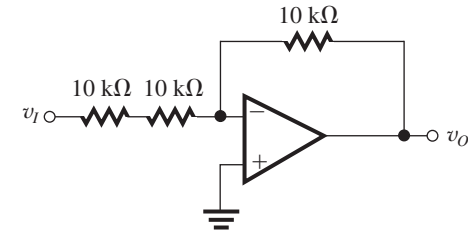
$$\left| \frac{v_O}{v_I} \right|_{\max} = \frac{10(1 + 0.01)}{10(1 - 0.01)} = 1.02 \text{ V/V}$$

Thus the measured output voltage will range from  $-0.98 \text{ V}$  to  $-1.02 \text{ V}$ .

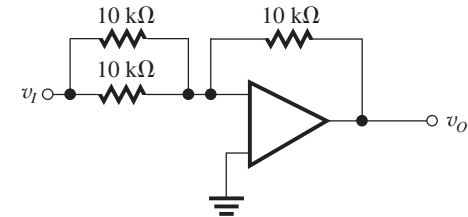
**2.24** There are four possibilities:

$$\frac{v_O}{v_I} = -2 \text{ V/V} \quad R_{\text{in}} = 10 \text{ k}\Omega \text{ (largest gain magnitude)}$$

$$\frac{v_O}{v_I} = -0.5 \text{ V/V} \quad R_{\text{in}} = 10 \text{ k}\Omega \text{ (smallest gain magnitude)}$$



$$\frac{v_O}{v_I} = -0.5 \text{ V/V} \quad R_{\text{in}} = 20 \text{ k}\Omega \text{ (smallest gain magnitude)}$$



$$\frac{v_O}{v_I} = -2 \text{ V/V} \quad R_{\text{in}} = 5 \text{ k}\Omega \text{ (largest gain magnitude)}$$

**2.25**

$$\frac{v_O}{v_I} = -10 \text{ V/V} = -\frac{R_2}{R_1}$$

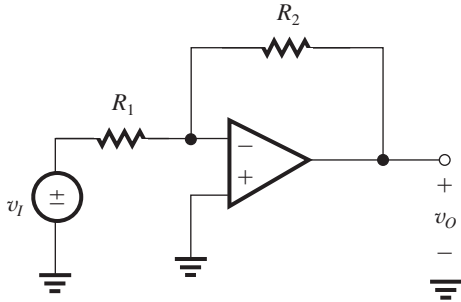
$$\Rightarrow R_2 = 10R_1$$

Total resistance used is  $110 \text{ k}\Omega$

$$\therefore R_1 + R_2 = 110 \text{ k}\Omega$$

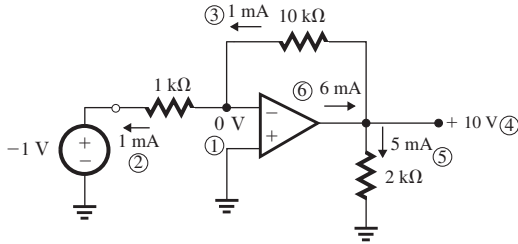
$$R_1 + 10R_1 = 110 \text{ k}\Omega$$





$R_1 = 10 \text{ k}\Omega$  and  $R_2 = 10R_1 = 100 \text{ k}\Omega$

**2.26** The circled numbers indicate the order of the analysis steps. The additional current supplied by the op amp comes from the power supplies (not shown).



**2.27** The inverting amplifier has a gain of  $-10 \text{ V/V}$  resulting in an output voltage of  $+10 \text{ V}$ . Thus, there is  $2 \text{ mA}$  current in  $R_L$ . With  $R_2 = 10 \text{ k}\Omega$ , as shown in Fig. P2.17, there is  $1 \text{ mA}$  current in  $R_2$  for a total of  $3 \text{ mA}$  coming from the ideal op-amp output. If the output current must be limited to  $2.5 \text{ mA}$  with the same gain, the output voltage must remain  $+10 \text{ V}$  and the current in  $R_2$  must be limited to  $2.5 - 2 = 0.5 \text{ mA}$ . This requires,

$$R_2 = \frac{10}{0.5} = 20 \text{ k}\Omega$$

To keep the same gain,  $-R_2/R_1 = -10$ . Hence,

$$R_1 = \frac{R_2}{10} = 2 \text{ k}\Omega$$

**2.28**  $R_1 = R_{1\text{nominal}}(1 \pm x\%)$

$R_2 = R_{2\text{nominal}}(1 \pm x\%)$

$$|G| = \frac{R_2}{R_1} = \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \frac{1 \pm x\%}{1 \pm x\%}$$

Thus,

$$|G|_{\text{max}} \simeq \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 + 2x\%)$$

$$|G|_{\text{min}} \simeq \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 - 2x\%)$$

Thus, the tolerance of the closed-loop gain is  $\pm 2x\%$ .

For  $G_{\text{nominal}} = -100$  and  $x = 1$ , the closed-loop gain will be in the range  $-98 \text{ V/V}$  to  $-102 \text{ V/V}$ .

$$2.29 \quad G = -\frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

For  $A = 500$ ,  $G = -50$ , and  $R_2 = 100 \text{ k}\Omega$ ,

$$-50 = -\frac{100/R_1}{1 + \frac{1 + 100/R_1}{500}}$$

$$\text{Thus, } \frac{100}{R_1} = 50 + 0.1 \left( 1 + \frac{100}{R_1} \right)$$

$$0.9 \times \frac{100}{R_1} = 50.1$$

$$\Rightarrow R_1 = 1.796 \text{ k}\Omega \simeq 1.8 \text{ k}\Omega$$

The input  $2\text{-k}\Omega$  resistor can be shunted by a resistance  $R_a$  to obtain an equivalent  $R_1$  of  $1.796 \text{ k}\Omega$ ,

$$\frac{1}{1.796} = \frac{1}{R_a} + \frac{1}{2}$$

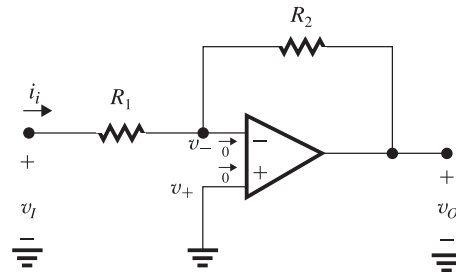
$$\Rightarrow R_a = 17.61 \text{ k}\Omega \simeq 18 \text{ k}\Omega$$

**2.30** Output voltage ranges from  $-10 \text{ V}$  to  $+10 \text{ V}$  and open-loop gain is  $5000 \text{ V/V}$

$\therefore$  Voltage at the inverting input terminal will vary from  $\frac{-10}{5000}$  to  $\frac{10}{5000}$  (i.e.,  $-2 \text{ mV}$  to  $+2 \text{ mV}$ ).

Thus the virtual ground will depart by a maximum of  $\pm 2 \text{ mV}$ .

**2.31**



$$v_o = -Av_- = v_- - i_i R_2$$

$$i_i R_2 = v_- (1 + A)$$

$$v_- = \frac{i_i R_2}{1 + A}$$

$$\text{Again } v_i = i_i R_1 + v_-$$

$$= i_i R_1 + i_i \frac{R_2}{1 + A}$$

$$\text{So } R_{\text{in}} = \frac{v_I}{i_i} = R_1 + \frac{R_2}{A+1} \quad \text{Q.E.D.}$$

**2.32** Refer to Fig. P2.32.

$$\frac{V_o}{V_i} = \frac{-R_2/R_1'}{1 + \frac{R_2/R_1'}{A}} \quad (1)$$

where

$$R_1' = R_1 \parallel R_c$$

Thus,

$$\frac{1}{R_1'} = \frac{1}{R_1} + \frac{1}{R_c}$$

Substituting in Eq. (1),

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \frac{1 + \frac{R_1}{R_c}}{1 + \frac{\frac{R_2}{R_1} + \frac{R_2}{R_c}}{A}}$$

To make  $\frac{V_o}{V_i} = -\frac{R_2}{R_1}$ , we have to make

$$\frac{R_1}{R_c} = \frac{1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}}{A}$$

That is,

$$A \frac{R_1}{R_c} = 1 + G + G \frac{R_1}{R_c}$$

which yields

$$\frac{R_c}{R_1} = \frac{A - G}{1 + G} \quad \text{Q.E.D.}$$

**2.33** (a) Equation (2.5):

$$G = \frac{v_o}{v_I} = \frac{-R_2/R_1}{1 + \left(1 + \frac{R_2}{R_1}\right) / A}$$

$$\text{and } G_{\text{nominal}} = -\frac{R_2}{R_1}$$

Gain error

$$\begin{aligned} \epsilon &= \left| \frac{G - G_{\text{nominal}}}{G_{\text{nominal}}} \right| = \left| \frac{G}{G_{\text{nominal}}} - 1 \right| \\ &= \left| \frac{1}{1 + \left(\frac{1 + R_2/R_1}{A}\right)} - 1 \right| = \left| \frac{-\frac{1 + R_2/R_1}{A}}{1 + \frac{1 + R_2/R_1}{A}} \right| \\ \epsilon &= \frac{1}{1 + \frac{A}{R_2}} \end{aligned}$$

$$\frac{1}{\epsilon} = 1 + \frac{A}{1 + \frac{R_2}{R_1}}$$

Solving for A, we get

$$\begin{aligned} A &= \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{\epsilon} - 1\right) \\ &= (1 - G_{\text{nominal}}) \left(\frac{1}{\epsilon} - 1\right) \end{aligned}$$

$$(b) \quad G_{\text{nominal}} = -100 = -\frac{R_2}{R_1}$$

$$R_{\text{in}} = 1 \text{ k}\Omega = R_1$$

$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 100R_1 = 100 \text{ k}\Omega$$

Again  $G_{\text{nominal}} = -100$  and  $\epsilon_{\text{max}} = 10\%$

$$\begin{aligned} \therefore A &= (1 - G_{\text{nominal}}) \left(\frac{1}{\epsilon} - 1\right) \\ &= [1 - (-100)] \left(\frac{1}{0.1} - 1\right) \end{aligned}$$

$$= 101 \times 9$$

$$A = 909 \text{ V/V}$$

$$\mathbf{2.34} \text{ (a) } |G| = \frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \quad (1)$$

If A is reduced by  $\Delta A$ ,  $|G|$  will be correspondingly reduced by  $\Delta |G|$ ,

$$|G| - \Delta |G| = \frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A - \Delta A}} \quad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$1 - \frac{\Delta |G|}{|G|} = \frac{1 + \frac{1 + R_2/R_1}{A}}{1 + \frac{1 + R_2/R_1}{A - \Delta A}}$$

Thus

$$\begin{aligned} \frac{\Delta |G|}{|G|} &= \frac{\left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{A - \Delta A} - \frac{1}{A}\right)}{1 + \frac{1 + R_2/R_1}{A - \Delta A}} \\ &= \frac{\left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{A}\right) \left(\frac{1}{1 - \frac{\Delta A}{A}} - 1\right)}{1 + \frac{R_2/R_1}{A - \Delta A}} \end{aligned}$$

$$\text{for } \frac{\Delta A}{A} \ll 1 \text{ and } 1 + \frac{R_2}{R_1} \ll A,$$

$$\frac{\Delta |G|}{|G|} \approx \frac{1 + \frac{R_2}{R_1}}{A} \frac{\Delta A}{A}$$

Thus,

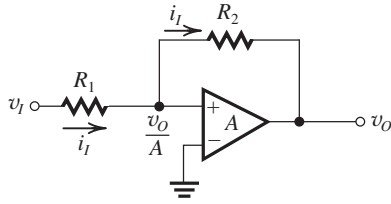
$$\frac{\Delta |G| / |G|}{\Delta A / |A|} \approx \frac{1 + R_2/R_1}{A} \quad \text{Q.E.D}$$

$$(b) \frac{R_2}{R_1} = 100, \frac{\Delta A}{A} = 0.1, \frac{\Delta |G|}{|G|} \leq 0.001$$

$$A \geq (1 + 100) \times 0.1 / 0.001$$

That is,  $A_{\min} \approx 10^4 \text{ V/V}$ .

2.35



Looking at the figure above, the input current may be written as

$$i_I = \frac{v_I - v_O/A}{R_1}$$

The same current flows through  $R_2$  resulting in

$$\begin{aligned} \frac{v_O}{A} - v_O &= i_I \times R_2 \\ \frac{v_O}{A} - v_O &= \frac{v_I - v_O/A}{R_1} \times R_2 \\ \frac{v_O}{A} \left( 1 + \frac{R_2}{R_1} \right) - v_O &= v_I \times \frac{R_2}{R_1} \\ \Rightarrow \frac{v_O}{v_I} &= \frac{R_2/R_1}{\frac{1}{A} \left( 1 + \frac{R_2}{R_1} \right) - 1} \end{aligned}$$

The denominator of the closed-loop gain expression becomes zero, and the closed-loop gain infinite, if  $A = 1 + (R_2/R_1)$ .

2.36

$$R_2/R_1 = 500, R_2 = 100 \text{ k}\Omega \Rightarrow R_1 = 200 \Omega$$

$$(a) R_{\text{in}} = R_1 = 200 \Omega$$

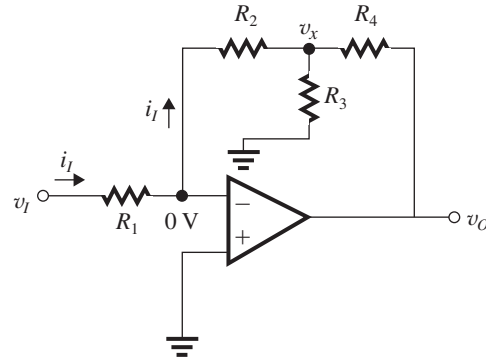
$$(b) \frac{v_O}{v_I} = \frac{-R_2}{R_1} \left( 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3} \right) = -500$$

$$\text{If } R_2 = R_1 = R_4 = 100 \text{ k}\Omega \Rightarrow R_3 = \frac{100 \text{ k}\Omega}{500 - 2} \approx 200 \Omega$$

$$R_{\text{in}} = R_1 = 100 \text{ k}\Omega$$

$$2.37 \quad i_I = \frac{v_I}{R_1}, v_x = -i_I R_2 = -\frac{v_I}{R_1} R_2$$

$$\text{So } \frac{v_x}{v_I} = -\frac{R_2}{R_1}$$



Now, because of the virtual ground at the negative input terminal of the op amp,  $v_O$  appears across the series combination of  $R_4$  and  $(R_2 \parallel R_3)$ ; thus

$$\begin{aligned} v_x &= v_O \frac{(R_2 \parallel R_3)}{(R_2 \parallel R_3) + R_4} \\ &= v_O \frac{R_2 R_3}{R_2 R_3 + R_2 R_4 + R_3 R_4} \\ \frac{v_O}{v_x} &= \frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 R_3} \\ &= 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2} \\ \frac{v_O}{v_I} &= \frac{v_O}{v_x} \frac{v_x}{v_I} \\ &= -\frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2} \right) \quad \text{Q.E.D.} \end{aligned}$$

2.38 (a)  $R_1 = R$

$$R_2 = (R \parallel R) + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_3 = (R_2 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

$$R_4 = (R_3 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

See Fig. 1 on next page.

(b)

See Fig. 2 on next page. We utilize the results of (a) above as follows: At node 1 we have a resistance  $R$  to ground and, looking left, a resistance  $R$ . These two resistances must carry equal currents  $\Rightarrow I_1 = I$ . A node equation at node 1 results in the current through  $(R/2)$  as  $2I$ . At node 2, we have a resistance  $R$  to ground and an equal resistance looking to the left. These two resistances must carry equal currents  $\Rightarrow I_2 = 2I$ .

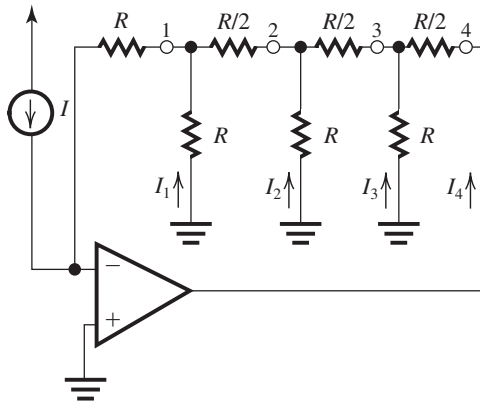


Figure 1

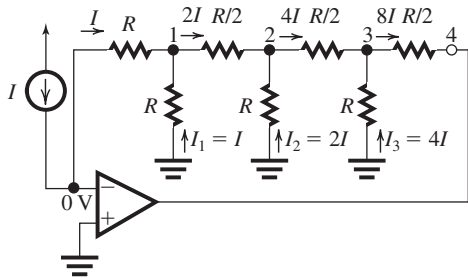


Figure 2

A node equation at 2 results in the current leaving node 2 as  $4I$ . We continue in the same fashion to find  $I_3 = 4I$  and the current from 3 to 4 as  $8I$ .

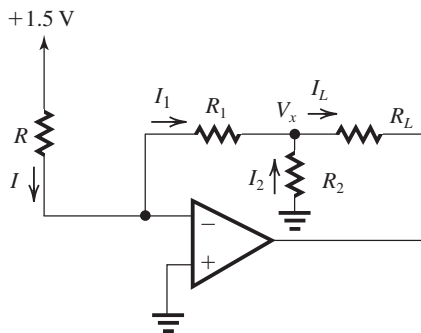
(c)  $V_1 = -I_1R = -IR$

$V_2 = -I_2R = -2IR$

$V_3 = -I_3R = -4IR$

$V_4 = V_3 - \frac{R}{2} \times 8I = -8IR$

**2.39**



The circuit shown above provides a constant current through  $R_L$  that is independent of the value of  $R_L$ . The current through  $R_L$  is determined by the input current and the ratio of the two resistors  $R_1$  and  $R_2$ . Specifically,  $I_1 = I$ ,  $V_x = -I_1R_1$ ,  $I_2 = -V_x/R_2 = I_1(R_1/R_2) = I(R_1/R_2)$ , thus

$$I_L = I + I \frac{R_1}{R_2} = \left(1 + \frac{R_1}{R_2}\right) I$$

which is independent of the value of  $R_L$ . Now, for our specific design:

$I_L = 3.1 \text{ mA}$

$I = 0.1 \text{ mA}$

$R = \frac{1.5\text{V}}{0.1 \text{ mA}} = 15 \text{ k}\Omega$

$3.1 = 0.1 \left(1 + \frac{R_1}{R_2}\right)$

$\Rightarrow \frac{R_1}{R_2} = 30$

Choosing  $R_2 = 500 \Omega \Rightarrow R_1 = 15 \text{ k}\Omega$ . The circuit will work properly as long as the op amp does not saturate (i.e., as long as  $V_O \leq -10 \text{ V}$ ).

But

$V_O = V_x - I_L R_L$

$= -IR_1 - I_L R_L$

$= -0.1 \times 15 - 3.1 R_L$

The maximum allowed value for  $R_L$  can now be found by substituting  $V_O = -10 \text{ V}$ , thus

$R_L = 2.74 \text{ k}\Omega$

**2.40**

To obtain an input resistance of  $100 \text{ k}\Omega$ , we select  $R_1 = 100 \text{ k}\Omega$ . From Example 2.2 we have

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left[1 + \frac{(1-x)R_4}{R_2} + \frac{(1-x)R_4}{R_3 + xR_4}\right]$$

The minimum gain magnitude is obtained when  $x = 1$ ,

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} = -1$$

Thus,  $R_2 = 100 \text{ k}\Omega$ .

The maximum gain magnitude is obtained when  $x = 0$ ,

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left[1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right] = -100$$

$$\Rightarrow 1 + \frac{100}{100} + \frac{100}{R_3} = 100$$

$$\Rightarrow R_3 = \frac{100}{98} = 1.02 \text{ k}\Omega$$

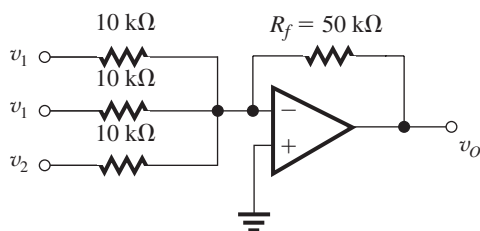
When the potentiometer is set exactly in the middle,  $x = 0.5$  and

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left[1 + \frac{0.5R_4}{R_2} + \frac{0.5R_4}{R_3 + 0.5R_4}\right]$$

$$= -\frac{100}{100} \left[1 + \frac{0.5 \times 100}{100} + \frac{0.5 \times 100}{1.02 + 0.5 \times 100}\right]$$

$$= -2.48 \text{ V/V}$$

## 2.41



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_1 + \frac{R_f}{R_3}v_2\right)$$

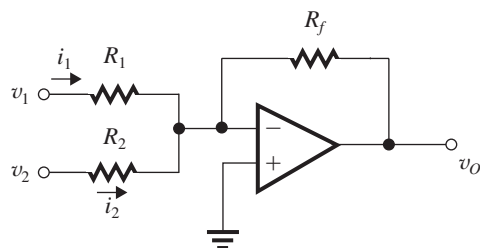
$$= -\left(\frac{50}{10}v_1 + \frac{50}{10}v_1 + \frac{50}{10}v_2\right)$$

$$= -(10v_1 + 5v_2)$$

Now  $v_1 = 1$  V and  $v_2 = -1$  V

$$v_o = -(10 \times 1 - 5) = -5$$
 V

## 2.42



The output of the weighted summer circuit is

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)$$

$$v_o = -\left(2v_1 + \frac{v_2}{2}\right)$$

$$i_1 = \frac{v_1}{R_1} \text{ and } i_2 = \frac{v_2}{R_2}$$

Since  $i_1, i_2 \leq 50 \mu\text{A}$  for 1-V input signals

$$\therefore R_1, R_2 \geq 20 \text{ k}\Omega$$

Here  $\frac{R_f}{R_1} = 2$ , if  $R_1 = 20 \text{ k}\Omega$ ,  $R_f = 40 \text{ k}\Omega$

$$\frac{R_f}{R_2} = \frac{1}{2} \Rightarrow R_2 = 2R_f = 80 \text{ k}\Omega$$

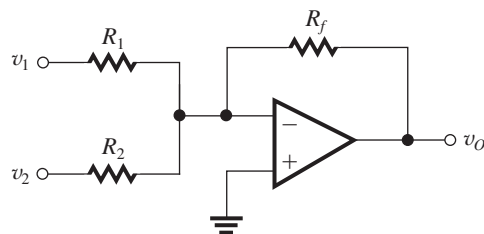
## 2.43

For  $v_1$  and  $v_2$ , we assume

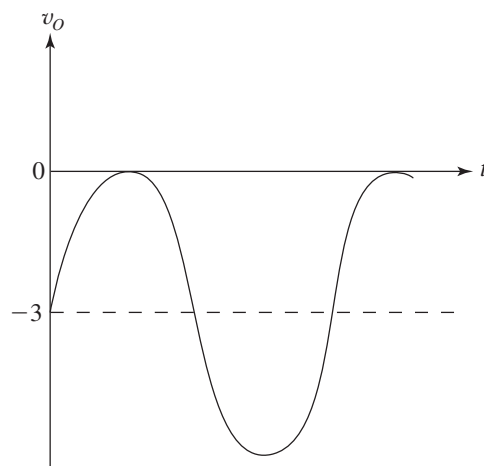
$$v_1 = 3 \sin \omega t$$

$$v_2 = 1.5$$
 V

The output signal required is



$$v_o = -3 \sin \omega t - 3$$



For the summer circuit, we should have

$$\frac{R_f}{R_1} = 1 \text{ and } \frac{R_f}{R_2} = 2$$

Select  $R_f = 2R_2 = 20 \text{ k}\Omega$ .

Thus  $R_2 = 10 \text{ k}\Omega$ ,

and  $R_1 = 20 \text{ k}\Omega$ .

**2.44** This is a weighted summer circuit:

$$v_o = -\left(\frac{R_f}{R_0}v_o + \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

We may write:

$$v_o = 5V \times a_0, \quad v_2 = 5V \times a_2,$$

$$v_1 = 5V \times a_1, \quad v_3 = 5V \times a_3,$$

$$v_o = -R_f \left(\frac{5}{80}a_0 + \frac{5}{40}a_1 + \frac{5}{20}a_2 + \frac{5}{10}a_3\right)$$

$$v_o = -R_f \left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2}\right)$$

$$v_o = -\frac{R_f}{16} (2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$$

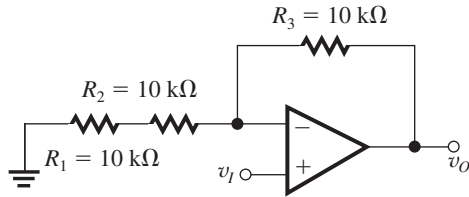
For  $-12 \text{ V} \leq v_o \leq 0$ ,

$$\frac{R_f}{16} (2^0 \times 1 + 2 \times 1 + 2^2 \times 1 + 2^3 \times 1)$$

$$= \frac{15R_f}{16} = 12$$

$$\Rightarrow R_f = 12.8 \text{ k}\Omega$$

2.45

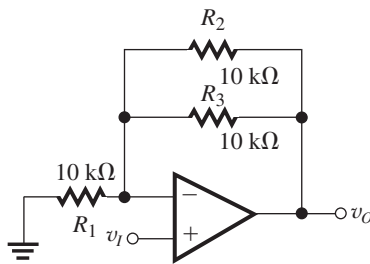


Short-circuit  $R_2$ :

$$\frac{v_o}{v_i} = 2$$

Short-circuit  $R_3$ :

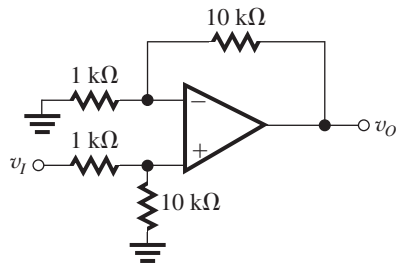
$$\frac{v_o}{v_i} = 1$$



2.46  $v_+ = v_i \frac{R_4}{R_3 + R_4}$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.47

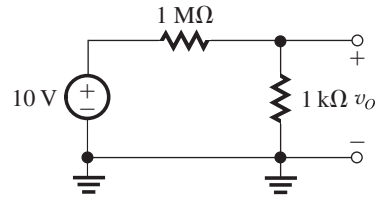


$$v_o = v_i \frac{10}{1 + 10} \left(1 + \frac{10}{1}\right)$$

$$v_o = 10v_i$$

$$R_{in} = 11 \text{ k}\Omega$$

2.48



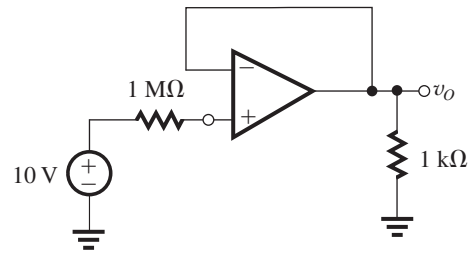
(a) Source is connected directly.

$$v_o = 10 \times \frac{1}{1001} \approx 10 \text{ mV}$$

$$i_L = \frac{v_o}{1 \text{ k}\Omega} = \frac{10 \text{ mV}}{1 \text{ k}\Omega} = 10 \mu\text{A}$$

Current supplied by the source is  $10 \mu\text{A}$ .

(b) Inserting a buffer.



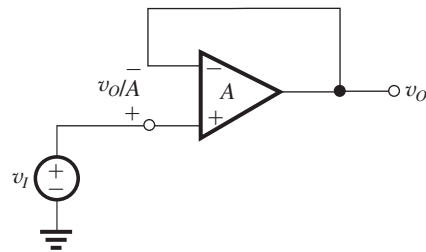
$$v_o = 10 \text{ V}$$

$$i_L = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

Current supplied by the source is 0.

The load current  $i_L$  comes from the power supply of the op amp.

2.49



$$v_o = v_i - \frac{v_o}{A}$$

$$\frac{v_o}{v_i} = \frac{1}{1 + \frac{1}{A}}$$

Error of gain magnitude

$$\frac{\left| \frac{v_O}{v_I} - 1 \right|}{1} = -\frac{1}{A + 1}$$

$A$ (V/V)	1000	100	10
$\frac{v_O}{v_I}$ (V/V)	0.999	0.990	0.909
Gain error	-0.1%	-1%	-9.1%

**2.50** For an inverting amplifier

$$R_{in} = R_1, \quad G = -\frac{R_2}{R_1}$$

for a noninverting amplifier:

$$R_{in} = \infty \quad G = 1 + \frac{R_2}{R_1}$$

Case	Gain (V/V)	$R_{in}$	$R_1$	$R_2$
a	-10	10 k $\Omega$	10 k $\Omega$	100 k $\Omega$
b	-1	100 k $\Omega$	100 k $\Omega$	100 k $\Omega$
c	-2	100 k $\Omega$	100 k $\Omega$	200 k $\Omega$
d	+1	$\infty$	$\infty$	0
e	+2	$\infty$	100 k $\Omega$	100 k $\Omega$
f	+11	$\infty$	10 k $\Omega$	100 k $\Omega$
g	-0.5	20 k $\Omega$	20 k $\Omega$	10 k $\Omega$

**2.51** For a gain of  $1 + (R_2/R_1) = 20$  V/V and a gain error of 1%, Eq. (2.12) provides the minimum required op-amp gain  $A$ .

$$1 = \frac{20}{A + 20} \times 100$$

$$A + 20 = 2000$$

$$\Rightarrow A = 1980$$

**2.52** For a non inverting amplifier [Eq. (2.11)]:

$$G = \frac{G_0}{1 + \frac{G_0}{A}} \quad \epsilon = \frac{G_0 - G}{G_0} \times 100$$

for an inverting amplifier (Eq. 2.5):

$$G = \frac{G_0}{1 + \frac{1 - G_0}{A}} \quad \epsilon = \frac{|G_0| - |G|}{|G_0|} \times 100$$

Case	$G_0$ (V/V)	$A$ (V/V)	$G$ (V/V)	$\epsilon$ (%)
a	-1	10	-0.83	17
b	1	10	0.91	9
c	-1	100	-0.98	2
d	10	10	5	50
e	-10	100	-9	10
f	-10	1000	-9.89	1.1
g	+1	2	0.67	33

**2.53** Refer to Fig. P2.53. When potentiometer is set to the bottom:

$$v_O = v_+ = -15 + \frac{30 \times 25}{25 + 100 + 25} = -10 \text{ V}$$

and to the top:

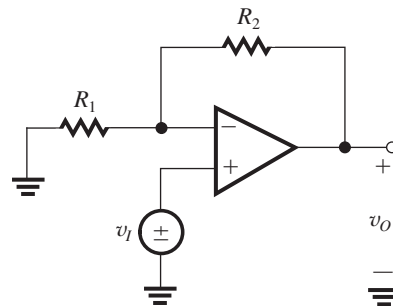
$$v_O = -15 + \frac{30 \times 25}{25 + 100 + 25} = +10 \text{ V}$$

$$\Rightarrow -10 \text{ V} \leq v_O \leq +10 \text{ V}$$

Pot has 20 turns, and for each turn:

$$\Delta v_O = \frac{2 \times 10}{20} = 1 \text{ V}$$

**2.54**



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1}$$

$$(a) \frac{v_O}{v_I} = 1 = 1 + \frac{R_2}{R_1}$$

Set  $R_2 = 0 \Omega$  and eliminate  $R_1$

$$(b) \frac{v_O}{v_I} = 2 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = 1; \text{ set } R_1 = R_2 = 10 \text{ k}\Omega$$

$$(c) \frac{v_O}{v_I} = 21 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = 20; \text{ set } R_1 = 10 \text{ k}\Omega, R_2 = 200 \text{ k}\Omega$$

$$(d) \frac{v_O}{v_I} = 100 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = 99; \text{ set } R_1 = 10 \text{ k}\Omega, R_2 = 990 \text{ k}\Omega$$

**2.55**  $V_- = V_+ = V$ ; thus the current in the moving-coil meter will be  $I = \frac{V}{R}$ , independent of the resistance of the meter. To obtain  $I = 100 \mu\text{A}$  when  $V = 10 \text{ V}$ , we select

$$R = \frac{10}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

The meter resistance does not affect the voltmeter calibration.

**2.56** Refer to the circuit in Fig. P2.56:

(a) Using superposition, we first set  $v_{p1} = v_{p2}, \dots = 0$ . The output voltage that results in response to  $v_{N1}, v_{N2}, \dots, v_{Nn}$  is

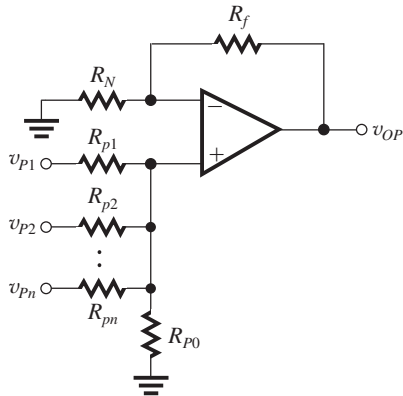
$$v_{ON} = - \left[ \frac{R_f}{R_{N1}} v_{N1} + \frac{R_f}{R_{N2}} v_{N2} + \dots + \frac{R_f}{R_{Nn}} v_{Nn} \right]$$

Then we set  $v_{N1} = v_{N2} = \dots = 0$ , then:

$$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \dots \parallel R_{Nn}$$

The circuit simplifies to that shown below.

$$v_{OP} = \left( 1 + \frac{R_f}{R_N} \right) \times \left( v_{p1} \frac{1/R_{P1}}{\frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \dots + \frac{1}{R_{Pn}}} + v_{p2} \frac{1/R_{P2}}{\frac{1}{R_{P0}} + \dots + \frac{1}{R_{Pn}}} + \dots + v_{pn} \frac{1/R_{Pn}}{\frac{1}{R_{P0}} + \dots + \frac{1}{R_{Pn}}} \right)$$



$$v_{OP} = \left( 1 + \frac{R_f}{R_N} \right) \left( v_{p1} \frac{R_P}{R_{P1}} + v_{p2} \frac{R_P}{R_{P2}} + \dots + \frac{R_P}{R_{Pn}} v_{pn} \right)$$

where

$$R_P = R_{P0} \parallel R_{P1} \parallel \dots \parallel R_{Pn}$$

when all inputs are present:

$$v_O = v_{ON} + v_{OP}$$

$$= - \left( \frac{R_f}{R_{N1}} v_{N1} + \frac{R_f}{R_{N2}} v_{N2} + \dots \right) + \left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_P}{R_{P1}} v_{N1} + \frac{R_P}{R_{P2}} v_{N2} + \dots \right)$$

(b)  $v_O = -4v_{N1} + v_{P1} + 3v_{P2}$

$$\frac{R_f}{R_{N1}} = 4, R_{N1} = 10 \text{ k}\Omega \Rightarrow R_f = 40 \text{ k}\Omega$$

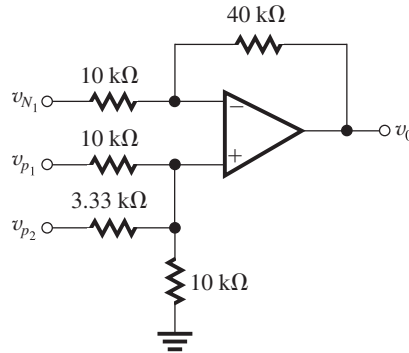
$$\left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_P}{R_{P1}} \right) = 1 \Rightarrow 5 \frac{R_P}{R_{P1}} = 1 \quad (1)$$

$$\left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_P}{R_{P2}} \right) = 3 \Rightarrow 5 \frac{R_P}{R_{P2}} = 3 \quad (2)$$

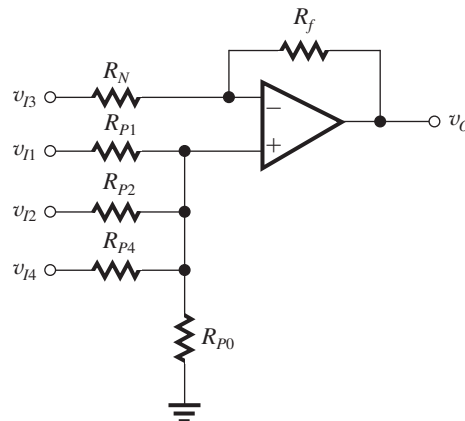
Substituting for  $\frac{1}{R_P}, \frac{1}{R_P} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}}$  in

Eqs. (1) and (2) and selecting (arbitrarily)

$R_{P0} = 10 \text{ k}\Omega$  results in  $R_{P1} = 10 \text{ k}\Omega$  and  $R_{P2} = 3.33 \text{ k}\Omega$ . The result is the following circuit:



**2.57**





Adapting the expression given in Problem 2.56 to the circuit above yields

$$v_O = -\frac{R_f}{R_N} v_{I3} + \left(1 + \frac{R_f}{R_N}\right) \left(\frac{R_P}{R_{P1}} v_{I1} + \frac{R_P}{R_{P2}} v_{I2} + \frac{R_P}{R_{P4}} v_{I4}\right)$$

where  $R_P = R_{P0} \parallel R_{P1} \parallel R_{P2} \parallel R_{P3}$ .

We require

$$v_O = -9v_{I3} + v_{I1} + 2v_{I2} + 4v_{I4}$$

Equating the coefficients of  $v_{I3}$ , we have

$$\frac{R_f}{R_N} = 9$$

Selecting  $R_N = 10 \text{ k}\Omega \Rightarrow R_f = 90 \text{ k}\Omega$ .

Equating the coefficients of  $v_{I1}$  provides

$$\left(1 + \frac{R_f}{R_N}\right) \frac{R_P}{R_{P1}} = 1$$

Thus,

$$10 \frac{R_P}{R_{P1}} = 1 \Rightarrow \frac{R_P}{R_{P1}} = 0.1 \quad (1)$$

Similarly, equating the coefficients of  $v_{I2}$  gives

$$10 \frac{R_P}{R_{P2}} = 2 \Rightarrow \frac{R_P}{R_{P2}} = 0.2 \quad (2)$$

and equating the coefficients of  $v_{I4}$  gives

$$10 \frac{R_P}{R_{P4}} = 4 \Rightarrow \frac{R_P}{R_{P4}} = 0.4 \quad (3)$$

Now, summing Eqs. (1), (2), and (3) provides

$$R_P \left(\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}}\right) = 0.7 \quad (4)$$

But,

$$\frac{1}{R_P} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}}$$

Thus,

$$\frac{1}{R_P} - \frac{1}{R_{P0}} = \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}} \quad (5)$$

Equations (4) and (5) can be combined to obtain

$$1 - \frac{R_P}{R_{P0}} = 0.7 \Rightarrow R_{P0} = \frac{R_P}{0.3} \quad (6)$$

Selecting

$$R_{P0} = 10 \text{ k}\Omega$$

$$\text{Equation (6)} \Rightarrow R_P = 3 \text{ k}\Omega$$

$$\text{Equation (1)} \Rightarrow R_{P1} = 30 \text{ k}\Omega$$

$$\text{Equation (2)} \Rightarrow R_{P2} = 15 \text{ k}\Omega$$

$$\text{Equation (3)} \Rightarrow R_{P4} = 7.5 \text{ k}\Omega$$

**2.58** Refer to Fig. P2.58. Setting  $v_2 = 0$ , we obtain the output component due to  $v_1$  as

$$v_{O1} = -10v_1$$

Setting  $v_1 = 0$ , we obtain the output component due to  $v_2$  as

$$v_{O2} = v_2 \left(1 + \frac{10R}{R}\right) \left(\frac{10R}{10R + R}\right) = 10v_2$$

The total output voltage is

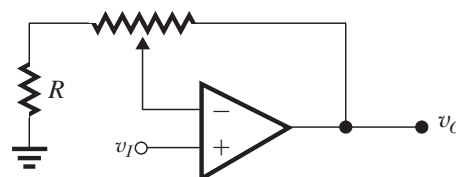
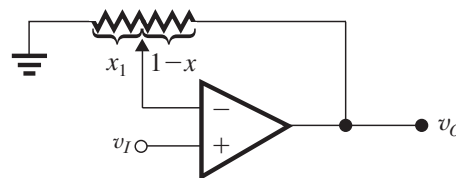
$$v_O = v_{O1} + v_{O2} = 10(v_2 - v_1)$$

For  $v_1 = 10 \sin(2\pi \times 60t) - 0.1 \sin(2\pi \times 1000t)$

$$v_2 = 10 \sin(2\pi \times 60t) + 0.1 \sin(2\pi \times 1000t)$$

$$v_O = 2 \sin(2\pi \times 1000t)$$

### 2.59



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1} = 1 + \frac{1-x}{x} = 1 + \frac{1}{x} - 1$$

$$\therefore \frac{v_O}{v_I} = \frac{1}{x}$$

$$0 \leq x \leq 1 \Rightarrow \infty \geq \frac{v_O}{v_I} \geq 1$$

Add a resistor as shown:

$$\frac{v_O}{v_I} = 1 + \frac{(1-x) \times 10 \text{ k}\Omega}{x \times 10 \text{ k}\Omega + R}$$

$$\text{For } \left(\frac{v_O}{v_I}\right)_{\max} = 11$$

$$x = 0, \frac{v_O}{v_I} = 11 = 1 + \frac{10 \text{ k}\Omega}{R}$$

$$10 = \frac{10 \text{ k}\Omega}{R}$$

$$R = 1 \text{ k}\Omega$$

**2.60** An ideal op amp has infinite gain and, thus zero differential input. The op-amp inputs are tied directly to the sinusoidal signal. Thus, the common-mode input is  $V_{ICM} = \sin(\omega t)$ .

In an inverting configuration, the ideal op amp again ensures zero differential input. Since the positive op-amp input terminal is grounded, the common-mode input is zero.

Voltage followers must tolerate a common-mode input whereas the inverting configuration does not.

**2.61**  $A = 100 \text{ V/V}$   $1 + \frac{R_2}{R_1} = 10 \text{ V/V}$

If  $R_1 = 10 \text{ k}\Omega \Rightarrow R_2 = 90 \text{ k}\Omega$

According to Eq. (2.11):

$$G = \frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + R_2/R_1}{A}}$$

$$G = \frac{1 + 90/10}{1 + \frac{1 + 90/10}{100}} = \frac{10}{1.1} = 9.09 \text{ V/V}$$

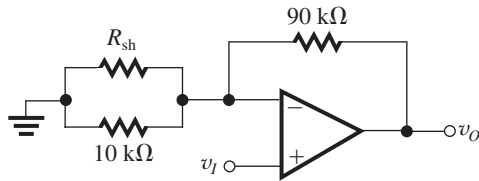
In order to compensate the gain drop, we can shunt  $R_1$  with a resistor  $R_{sh}$ . The value of  $R_{sh}$  can be obtained from

$$10 = \frac{1 + \left(\frac{90}{10} + \frac{90}{R_{sh}}\right)}{1 + \frac{90}{10} + \frac{90}{R_{sh}}}$$

$$10 + 0.1\left(10 + \frac{90}{R_{sh}}\right) = 10 + \frac{90}{R_{sh}}$$

$$1 + \frac{9}{R_{sh}} = \frac{90}{R_{sh}}$$

$$\Rightarrow R_{sh} = 81 \text{ k}\Omega$$



If  $A = 200$ , then:

$$G_{\text{uncompensated}} = \frac{1 + \frac{90}{10}}{1 + \frac{1 + 90/10}{200}} = \frac{10}{1.05} = 9.52 \text{ V/V}$$

$$G_{\text{compensated}} = \frac{1 + \frac{90}{10} + \frac{90}{81}}{1 + \frac{90}{10} + \frac{90}{200}}$$

$$= \frac{11.11}{1.056} = 10.52 \text{ V/V}$$

**2.62**

$$G = \frac{G_0}{1 + \frac{G_0}{A}}, \frac{G_0 - G}{G_0} \times 100 = \frac{G_0/A \times 100}{1 + \frac{G_0}{A}} \leq x$$

$$\text{or } \frac{1 + \frac{G_0}{A}}{\frac{G_0}{A}} \geq \frac{100}{x} \Rightarrow \frac{A}{G_0} \geq \underbrace{\left(\frac{100}{x} - 1\right)}_F$$

$$\Rightarrow A \geq G_0 F, \text{ where } F = \frac{100}{x} - 1 \simeq \frac{100}{x}$$

$x$	0.01	0.1	1	10
$F$	$10^4$	$10^3$	$10^2$	10

Thus for:

$x = 0.01:$	$G_0 \text{ (V/V)}$	1	10	$10^2$	$10^3$	$10^4$
	$A \text{ (V/V)}$	$10^4$	$10^5$	$10^6$	$10^7$	$10^8$
					too high to be practical	

$x = 0.1:$	$G_0 \text{ (V/V)}$	1	10	$10^2$	$10^3$	$10^4$
	$A \text{ (V/V)}$	$10^3$	$10^4$	$10^5$	$10^6$	$10^7$

$x = 1:$	$G_0 \text{ (V/V)}$	1	10	$10^2$	$10^3$	$10^4$
	$A \text{ (V/V)}$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$

$x = 10:$	$G_0 \text{ (V/V)}$	1	10	$10^2$	$10^3$	$10^4$
	$A \text{ (V/V)}$	10	$10^2$	$10^3$	$10^4$	$10^5$

**2.63** If we assume  $R_3 = R_1, R_4 = R_2$ , then

Eq. (2.20):  $R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10 \text{ k}\Omega$

(Refer to Fig. 2.16.)

(a)  $A_d = \frac{R_2}{R_1} = 1 \text{ V/V} \Rightarrow R_2 = 10 \text{ k}\Omega$

$R_1 = R_2 = R_3 = R_4 = 10 \text{ k}\Omega$

(b)  $A_d = \frac{R_2}{R_1} = 5 \text{ V/V} \Rightarrow R_2 = 50 \text{ k}\Omega = R_4$

$R_1 = R_3 = 10 \text{ k}\Omega$

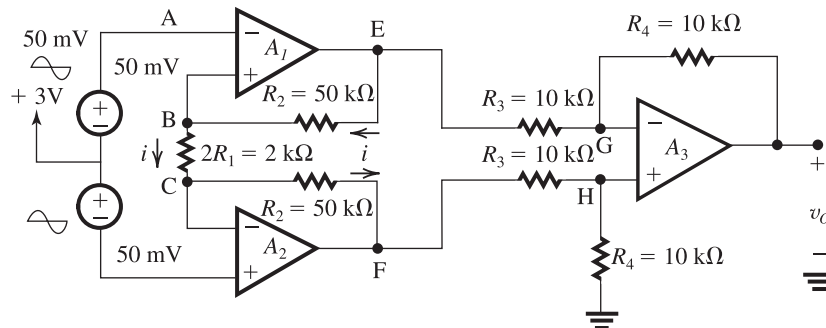
(c)  $A_d = \frac{R_2}{R_1} = 100 \text{ V/V} \Rightarrow R_2 = 1 \text{ M}\Omega = R_4$

$R_1 = R_3 = 10 \text{ k}\Omega$

(d)  $A_d = \frac{R_2}{R_1} = 0.5 \text{ V/V} \Rightarrow R_2 = 5 \text{ k}\Omega = R_4$

$R_1 = R_3 = 10 \text{ k}\Omega$

This figure belongs to Problem 2.64.



2.64 Refer to the figure above:

$$v_B = v_A = 3 + 0.05 \sin \omega t, \text{ V}$$

$$v_C = v_D = 3 - 0.05 \sin \omega t, \text{ V}$$

Current through  $R_2$ ,  $2R_1$ , and  $R_2$  is

$$i = (v_B - v_C)/2R_1$$

$$= 0.05 \sin \omega t, \text{ mA}$$

$$v_E = v_B + iR_2 = 3 + 2.55 \sin \omega t, \text{ V}$$

$$v_F = v_C - iR_2 = 3 - 2.55 \sin \omega t, \text{ V}$$

$$v_G = v_H = \frac{1}{2}v_F = 1.5 - 1.275 \sin \omega t, \text{ V}$$

$$v_O = (v_F - v_E) \times 1 = -5.1 \sin \omega t, \text{ V}$$

2.65 (a) Refer to the circuit in Fig. 2.20(a).

$$v_{I1} = v_{Icm} - \frac{1}{2}v_{Id}$$

$$v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$$

$$v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{I1}$$

$$= \left(1 + \frac{R_2}{R_1}\right)v_{Icm} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{O2} = \left(1 + \frac{R_2}{R_1}\right)v_{I2}$$

$$= \left(1 + \frac{R_2}{R_1}\right)v_{Icm} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = \left(1 + \frac{R_2}{R_1}\right)v_{Icm}$$

$$A_{d1} = 1 + \frac{R_2}{R_1}$$

$$A_{cm1} = 1 + \frac{R_2}{R_1}$$

$$\text{CMRR} = 20 \log \left| \frac{A_{d1}}{A_{cm1}} \right| = 0 \text{ dB}$$

(b) Refer to the circuit in Fig. 2.20(b):

$$v_{I1} = v_{Icm} - \frac{1}{2}v_{Id}$$

$$v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$$

$$v_{-(A1)} = v_{I1} = v_{Icm} - \frac{1}{2}v_{Id}$$

$$v_{-(A2)} = v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$$

Current through  $R_1$  in the upward direction is

$$i = \frac{v_{-(A2)} - v_{-(A1)}}{2R_1} = \frac{v_{Id}}{2R_1}$$

$$v_{O1} = v_{-(A1)} - iR_2 = v_{Icm} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{O2} = v_{-(A2)} + iR_2 = v_{Icm} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{Id}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = v_{Icm}$$

$$A_{d1} = 1 + \frac{R_2}{R_1}$$

$$A_{cm1} = 1$$

$$\text{CMRR} = 20 \log \left| \frac{A_{d1}}{A_{cm1}} \right| = 20 \log \left(1 + \frac{R_2}{R_1}\right)$$

Comment: In circuit (a), the first stage amplifies the differential signal and the common-mode signal equally. On the other hand, in circuit (b),

the first stage amplifies the differential signal by

$$\left(1 + \frac{R_2}{R_1}\right)$$

and the common-mode signal by

$$\text{unity, thus providing a substantial CMRR. Circuit (a) is useless as a differential amplifier!}$$

2.66 Adding a 100-k $\Omega$  potentiometer (whose resistance can be set to any desired value  $R_{iv}$  in the range of 0 to 100 k $\Omega$ ) in series with a fixed resistance  $R_{if}$  makes

$$2R_1 = R_{1f} + R_{1v}$$

The resulting differential gain will be

$$A_d = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_{1f} + R_{1v}} \right)$$

The minimum gain of 2 is obtained with  $R_{1v}$ , at its maximum value of 100 kΩ. Thus

$$2 = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_{1f} + 100} \right) \quad (1)$$

The maximum gain is obtained with  $R_{1v} = 0$ . Thus

$$1000 = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_{1f}} \right) \quad (2)$$

Selecting  $R_3 = R_4$  at a convenient value of, say, 10 kΩ, then Eqs. (1) and (2) can be solved to obtain  $R_{1f} = 100.2 \Omega$  and  $R_2 = 100.1 \text{ k}\Omega$ . Practical values of 100 Ω and 100 kΩ can be used, and the gain range required will be covered.

**2.67** (a)  $\frac{v_B}{v_A} = 1 + \frac{10}{10} = 2 \text{ V/V}$

$$\frac{v_C}{v_A} = -\frac{20}{10} = -2 \text{ V/V}$$

(b)  $v_O = v_B - v_C = 4v_A \Rightarrow \frac{v_O}{v_A} = 4 \text{ V/V}$

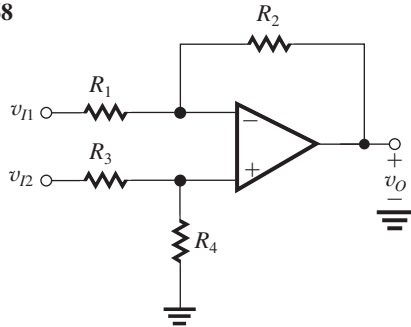
(c)  $v_B$  and  $v_C$  can be  $\pm 4 \text{ V}$ , or 8 V<sub>P-P</sub>.

$$-8 \text{ V} \leq v_O \leq 8 \text{ V}, \text{ or } 16 \text{ V}_{P-P}$$

$$v_{O_{rms}} = \frac{8}{\sqrt{2}} = 5.66 \text{ V}$$

This corresponds to 4 V<sub>P-P</sub> or 1.414 V rms at  $V_I$

**2.68**



$$R_1 = R_3 = 5 \text{ k}\Omega, R_2 = R_4 = 100 \text{ k}\Omega$$

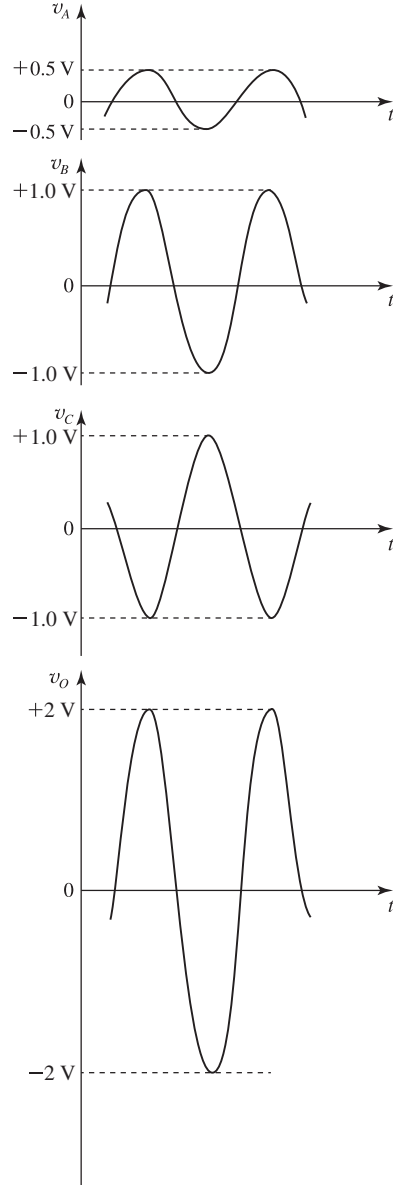
Equation (2.15):  $\frac{R_4}{R_3} = \frac{R_2}{R_1} = 20$

From Eq. (2.16),

$$v_O = \frac{R_2}{R_1} v_{id}$$

$$A_d = \frac{v_O}{v_{id}} = \frac{R_2}{R_1} = 20 \text{ V/V}$$

This figure belongs to Problem 2.67



From Eq. (2.20)

$$R_{id} = 2R_1 = 2 \times 5 \text{ k}\Omega = 10 \text{ k}\Omega$$

The two resistance ratios  $\frac{R_2}{R_1}$  and  $\frac{R_4}{R_3}$  differ by 1%.

$$\therefore \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

Now for this case,  $A_{cm}$  can be found from Eq. (2.19)

$$A_{cm} = \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \times \frac{R_3}{R_4} \right) \approx \frac{100}{100 + 5} \times \left( 1 - 0.99 \frac{R_4}{R_3} \times \frac{R_3}{R_4} \right)$$

$$A_{cm} = 0.0095$$

Neglecting the effect of resistance variation on  $A_d$ ,

$$A_d = \frac{R_2}{R_1} = \frac{100}{5} = 20 \text{ V/V}$$

$$\text{CMRR} = 20 \log \left| \frac{A_d}{A_{cm}} \right|$$

$$= 20 \log \left| \frac{20}{0.0095} \right|$$

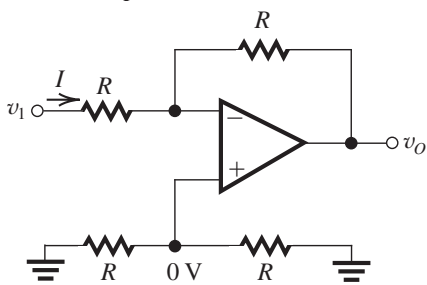
$$= 66.4 \text{ dB}$$

**2.69** Refer to Fig. P2.69:

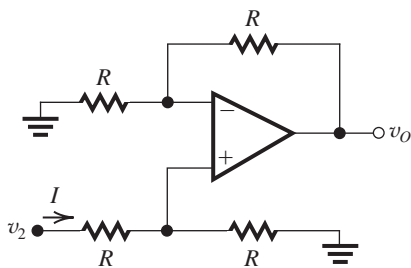
Considering that  $v_- = v_+$ :

$$v_1 + \frac{v_O - v_1}{2} = \frac{v_2}{2} \Rightarrow v_O = v_2 - v_1$$

$$v_1 \text{ only: } R_i = \frac{v_1}{I} = R$$



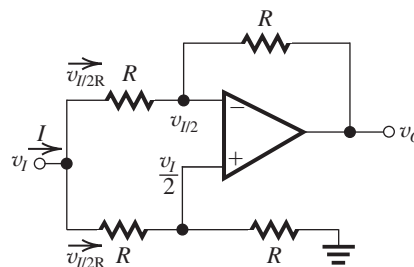
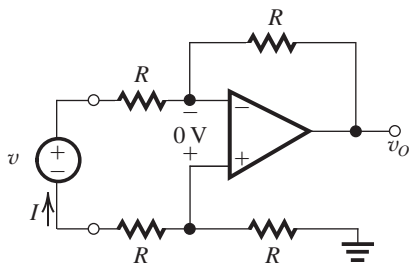
$$v_2 \text{ only: } R_i = \frac{v_2}{I} = 2R$$



$v$  between 2 terminals:

$$R_i = \frac{v}{I} = 2R$$

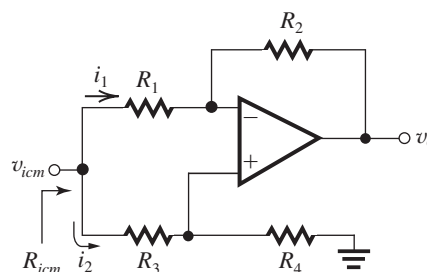
$v_i$  connected to both input terminals



$$I = \frac{v_I}{2R} + \frac{v_I}{2R} = \frac{v_I}{R}$$

$$R_i = R$$

**2.70**



When  $R_2/R_1 = R_4/R_3$ , the output voltage  $v_O$  will be zero. Thus,

$$i_1 = \frac{v_{icm}}{R_1 + R_2}$$

and

$$i_2 = \frac{v_{icm}}{R_3 + R_4}$$

Thus,

$$i_i = v_{icm} \left[ \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \right]$$

and

$$R_{icm} = (R_1 + R_2) \parallel (R_3 + R_4) \quad \text{Q.E.D.}$$

**2.71** From Eq. (2.19),

$$A_{cm} = \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The second factor in this expression in effect determines  $A_{cm}$ . The largest,  $|A_{cm}|$ , will occur when  $R_2$  and  $R_3$  are at their lowest (or highest) values and  $R_1$  and  $R_4$  are at their highest (or lowest) values, as this will provide the maximum deviation of  $\left( \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$  from unity. Thus,

$$R_2 = R_{2\text{nominal}}(1 - \epsilon)$$

$$R_3 = R_{3\text{nominal}}(1 - \epsilon)$$

$$R_1 = R_{1\text{nominal}}(1 + \epsilon)$$

$$R_4 = R_{4\text{nominal}}(1 + \epsilon)$$

where

$$\frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} = \frac{R_{4\text{nominal}}}{R_{3\text{nominal}}} = K$$

Substituting in the expression for  $A_{cm}$ ,

$$A_{cm} = \frac{R_{4\text{nominal}}(1 + \epsilon)}{R_{4\text{nominal}}(1 + \epsilon) + R_{3\text{nominal}}(1 - \epsilon)} \times \left[ 1 - \frac{(1 - \epsilon)^2}{(1 + \epsilon)^2} \right]$$

For  $\epsilon \ll 1$ ,

$$A_{cm} \simeq \frac{K}{K + 1} \times 4\epsilon$$

Since

$$A_d = K$$

$$\text{CMRR} = \left| \frac{A_d}{A_{cm}} \right| = \left( \frac{K + 1}{4\epsilon} \right)$$

which in dB becomes

$$\text{CMRR} = 20 \log \left[ \frac{K + 1}{4\epsilon} \right] \quad \text{Q.E.D.}$$

For  $A_d = 100 \text{ V/V}$  and  $\epsilon = 0.01$ ,

$$\text{CMRR} = 20 \log \left( \frac{101}{0.04} \right) = 68 \text{ dB}$$

To obtain  $\text{CMRR} = 80 \text{ dB}$ ,

$$\frac{101}{4\epsilon} = 10^4$$

$$\epsilon = \frac{101}{4 \times 10^4} \simeq 0.25 \times 10^{-2}$$

That is, the resistor tolerance should be a maximum of 0.25%.

**2.72** (a) Refer to Fig. P2.72 and Eq. (2.19):

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{R_2 R_3}{R_1 R_4} \right) = \frac{100}{100 + 100} \left( 1 - \frac{100 \cdot 100}{100 \cdot 100} \right)$$

$$A_{cm} = 0$$

Refer to Eq. (2.17):  $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

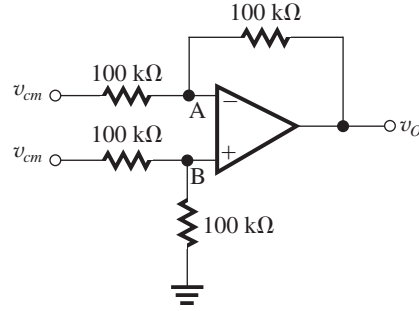
$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

(b)  $v_A = v_B$

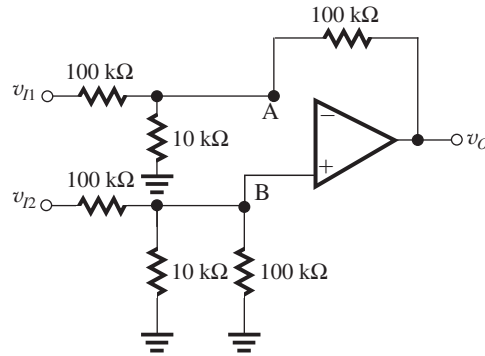
$$v_A = v_{Icm} \frac{100}{100 + 100}$$

$$v_A = \frac{v_{Icm}}{2} \text{ and } v_B = \frac{v_{Icm}}{2}$$

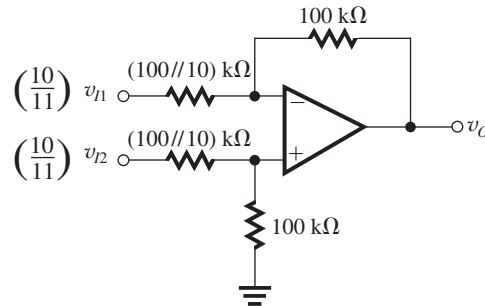
$$\Rightarrow -5 \text{ V} \leq v_{Icm} \leq 5 \text{ V}$$



(c) The circuit becomes as shown below:



Applying Thévenin's theorem to  $v_{I1}$  together with the associated (100-kΩ, 10-kΩ) voltage divider, and similarly to  $v_{I2}$  and the associated (100-kΩ, 10-kΩ) voltage divider, we obtain the following circuit:



$$v_O = \frac{100 \text{ k}\Omega}{(100 \parallel 10) \text{ k}\Omega} \left( \frac{10}{11} v_{I2} - \frac{10}{11} v_{I1} \right)$$

$$= 10(v_{I2} - v_{I1})$$

For  $v_{I1} = v_{I2} = v_{Icm}$ ,  $v_O = 0$ , thus

$$A_{cm} = 0$$

For  $v_{I2} - v_{I1} = v_{Id}$ ,

$$v_O = 10 v_{Id}$$

Thus,  $A_d = 10$ .

To obtain the input common-mode range, we note that for  $v_{I1} = v_{I2} = v_{Icm}$ ,

$$v_+ = v_- = \frac{10}{11} v_{lcm} \times \frac{100}{100 + (100 \parallel 10)}$$

$$= 0.833 v_{lcm}$$

For  $v_+$  and  $v_-$  in the range

$$-2.5 \text{ V} \leq v_+, v_- \leq +2.5 \text{ V}$$

the range of  $v_{lcm}$  will be

$$-3 \text{ V} \leq v_{lcm} \leq +3 \text{ V}$$

**2.73** Refer to Fig. P2.69. Using superposition:

$$v_o = v_{o1} + v_{o2}$$

Calculate  $v_{o1}$ :  $v_+ = \frac{\beta v_{o1}}{2} = v_-$

$$\frac{v_1 - \frac{\beta v_{o1}}{2}}{R} = \frac{\frac{\beta v_{o1}}{2} - v_o}{R} \Rightarrow v_{o1} = \frac{v_1}{\beta - 1}$$

Calculate  $v_{o2}$ :

$$v_- = \frac{v_{o2}}{2} = v_+ \Rightarrow v_2 - \frac{v_{o2}}{2} = \frac{v_{o2}}{2} - \beta v_{o2}$$

$$\Rightarrow v_{o2} = \frac{v_2}{1 - \beta}$$

$$v_o = v_{o1} + v_{o2} = \frac{v_1}{\beta - 1} + \frac{v_2}{1 - \beta}$$

$$= \frac{1}{1 - \beta} (v_2 - v_1)$$

$$A_d = \frac{v_o}{v_2 - v_1} = \frac{1}{1 - \beta} \quad \text{Q.E.D.}$$

$$A_d = 10 \text{ V/V} \Rightarrow \beta = 0.9 = \frac{R_6}{R_5 + R_6}$$

$$R_{ld} = 2R = 2 \text{ M}\Omega \Rightarrow R = 1 \text{ M}\Omega$$

$$R_5 + R_6 \leq \frac{R}{100} \Rightarrow R_5 + R_6 \leq 10 \text{ k}\Omega$$

Selecting  $R_6 = 6.8 \text{ k}\Omega$

$$\frac{6.8}{6.8 + R_5} = 0.9 \Rightarrow R_5 = 756 \Omega$$

**2.74** See partial analysis on circuit diagram below.

From input loop:

$$v_{ld} = 2i_1 R_1 \tag{1}$$

From the loop containing  $R_2$ ,  $+$ ,  $-$ ,  $R_2$ :

$$v_G = i_1 R_2 + 0 + i_1 R_2 = 2i_1 R_2$$

Thus, we can find the current through  $R_G$  as  $v_G/R_G = 2i_1(R_2/R_G)$ . Finally, from the loop containing  $v_o$ ,  $R_2$ ,  $R_G$ , and  $R_2$ :

$$v_o = -2i_1 \left( 1 + 2 \frac{R_2}{R_G} \right) R_2 - 2i_1 R_2$$

$$= -4i_1 R_2 \left( 1 + \frac{R_2}{R_G} \right)$$

Substituting for  $2i_1$  from Eq. (1), we obtain the gain as

$$\frac{v_o}{v_{ld}} = -2 \left( \frac{R_2}{R_1} \right) \left( 1 + \frac{R_2}{R_G} \right) \quad \text{Q.E.D.}$$

**2.75**

We solve this problem by replacing the network in the dashed box above with its Thévenin equivalent. The resistance seen looking into the network is dominated by the  $100\Omega$  resistor to ground, which is much smaller than the other resistances in parallel with it. Thus,

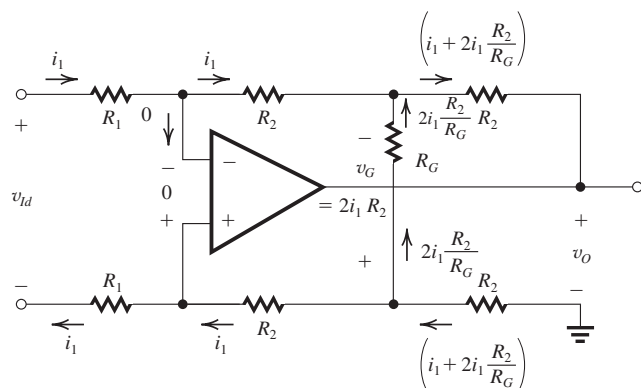
$$R_{Th} = 100 \Omega$$

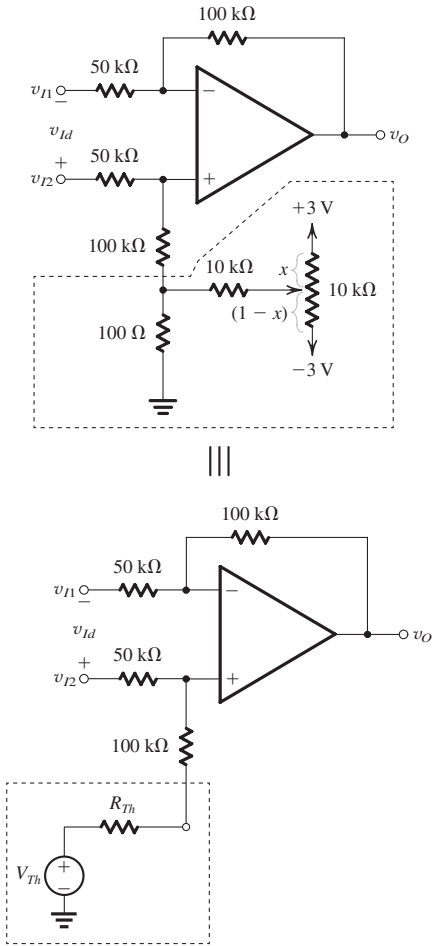
The open-circuit voltage at the output of the network can be shown to be

$$V_{Th} = 0.01 \frac{(3 - 6x)}{(1 + x - x^2)}$$

The resulting circuit is the superposition of a difference amplifier with respect to  $v_{ld}$  having a

This figure belongs to Problem 2.74.





gain of 2 (neglecting  $R_{Th} \ll 100 \text{ k}\Omega$ ), and a non-inverting configuration with respect to  $V_{Th}$  having a gain,

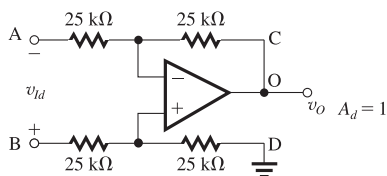
$$\frac{50}{50 + 100 + 0.1} \left(1 + \frac{100}{50}\right) \approx 1$$

Thus, the output is

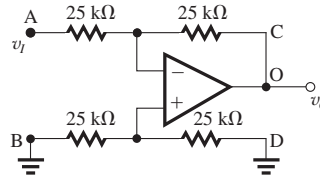
$$v_O = 2v_{Id} + V_{Th} = 2v_{Id} + 0.01 \frac{(3 - 6x)}{(1 + x - x^2)}$$

The change in  $v_O$  as  $X$  goes from 0 to 1 is  $-60 \text{ mV}$ . This range can be increased by increasing the value of the  $100 \text{ }\Omega$  resistor.

**2.76** (a) Refer to Eq. (2.17):  $A_d = \frac{R_2}{R_1} = 1$ . Connect C and O together, and D to ground.

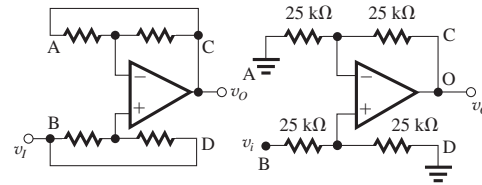


(b) (i)  $\frac{v_O}{v_i} = -1 \text{ V/V}$



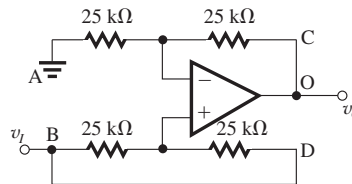
(ii)  $\frac{v_O}{v_i} = +1 \text{ V/V}$

Two possibilities:

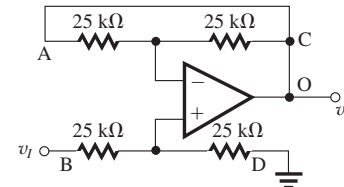


The circuit on the left ideally has infinite input resistance

(iii)  $\frac{v_O}{v_i} = +2 \text{ V/V}$



(iv)  $\frac{v_O}{v_i} = +\frac{1}{2} \text{ V/V}$



**2.77** Referring to Fig. 2.20(a), the feedback resistor around amplifier  $A_2$  is increased to  $R_2 + \Delta R_2$ . With a common-mode input  $v_{Icm}$ , the output of  $A_1$  is

$$v_{Icm} \left(1 + \frac{R_2}{R_1}\right)$$

whereas the output of  $A_2$  is

$$v_{Icm} \left(1 + \frac{R_2 + \Delta R_2}{R_1}\right)$$



Thus, the first stage produces a differential output  $v_{lcm} \Delta R_2 / R_1$ . This is amplified by the final difference-amplifier stage having a gain  $R_4 / R_3$  resulting in an output

$$v_o = v_{lcm} \frac{\Delta R_2 R_4}{R_1 R_3}$$

$$\Rightarrow \frac{v_o}{v_{lcm}} = \frac{\Delta R_2 R_4}{R_1 R_3}$$

The overall common-mode gain can be made zero by modifying the corresponding resistor  $R_1$  so that the two noninverting stages at the input have the same gain. That is,

$$1 + \frac{R_2}{R_1} = 1 + \frac{R_2 + \Delta R_2}{R_1 + \Delta R_1}$$

$$\Rightarrow \Delta R_1 = \frac{R_1}{R_2} \Delta R_2$$

**2.78** (a) Refer to Fig. 2.20(a).

The gain of the first stage is  $\left(1 + \frac{R_2}{R_1}\right) = 101$ . If the op amps of the first stage saturate at  $\pm 12$  V then  $-12 \text{ V} \leq 101 v_{lcm} \leq +12 \text{ V}$

$$\Rightarrow -0.12 \text{ V} \leq v_{lcm} \leq 0.12 \text{ V}$$

As explained in the text, the disadvantage of circuit in Fig. 2.20(a) is that  $v_{lcm}$  is amplified by a gain equal to  $\left(1 + \frac{R_2}{R_1}\right)$  in the first stage and therefore a very small  $v_{lcm}$  range is acceptable to avoid saturation.

(b) In Fig. 2.20(b), when  $v_{lcm}$  is applied,  $v_-$  for both  $A_1$  and  $A_2$  is the same and therefore no current flows through  $2R_1$ . This means the voltage at the output of  $A_1$  and  $A_2$  is the same as  $v_{lcm}$ .

$$\Rightarrow -12 \text{ V} \leq v_{lcm} \leq 12 \text{ V}$$

This circuit allows for a much larger range of  $v_{lcm}$ .

**2.79** Ideally,

$$A_d = \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1}\right) \quad (1)$$

$$A_{cm} = 0$$

$$\text{CMRR} = \infty$$

For  $R_2 = R_3 = R_4 = 100 \text{ k}\Omega$ , and  $2R_1 = 10 \text{ k}\Omega$ .

$$A_d = 1 \left(1 + \frac{100}{5}\right) = 21 \text{ V/V}$$

$$A_{cm} = 0$$

$$\text{CMRR} = \infty$$

If all resistors have  $\pm 1\%$  tolerance, the differential gain will be slightly affected; Eq. (1) indicates that in the worst case  $A_d$  can deviate by approximately  $\pm 4\%$  of the nominal value. The common-mode gain, however, undergoes dramatic change because of the significant effect of resistor tolerances on the operation of the difference amplifier in the second stage. Equation (2.19) can be employed to evaluate the worst-case common-mode gain. For our case,

$$A_{cm2} = 0.5[1 - (1 \pm 0.04)] = \pm 0.02$$

The common-mode gain of the first stage will remain approximately unity. Thus the  $\pm 1\%$  resistor tolerances will mainly affect the common-mode gain of the instrumentation amplifier, increasing it in the worst case to

$$|A_{cm}| = 0.02$$

Correspondingly, the CMRR will be reduced to

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|}$$

$$\simeq 20 \log \left(\frac{21}{0.02}\right) = 60.4 \text{ dB}$$

as opposed to the ideal infinite value!

If  $2R_1$  is reduced to  $1 \text{ k}\Omega$ ,  $A_d$  increases to  $201 \text{ V/V}$  while  $A_{cm}$  remains unchanged at  $0.02 \text{ V/V}$ . Thus, CMRR increases to about  $80 \text{ dB}$ . We conclude that increasing the gain of the first stage increases CMRR.

**2.80** See analysis on the circuit diagrams below.

Note that circuit (a) has the advantage of infinite input resistance. However, it has the limitation that the load impedance must be floating. This constraint is removed in circuit (b), but the input resistance is finite ( $2R_1$ ).

$$\mathbf{2.81} \quad \frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR}$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega CR} \quad \angle\phi = +90^\circ$$

For  $C = 1 \text{ nF}$  and  $R = 10 \text{ k}\Omega$ ,

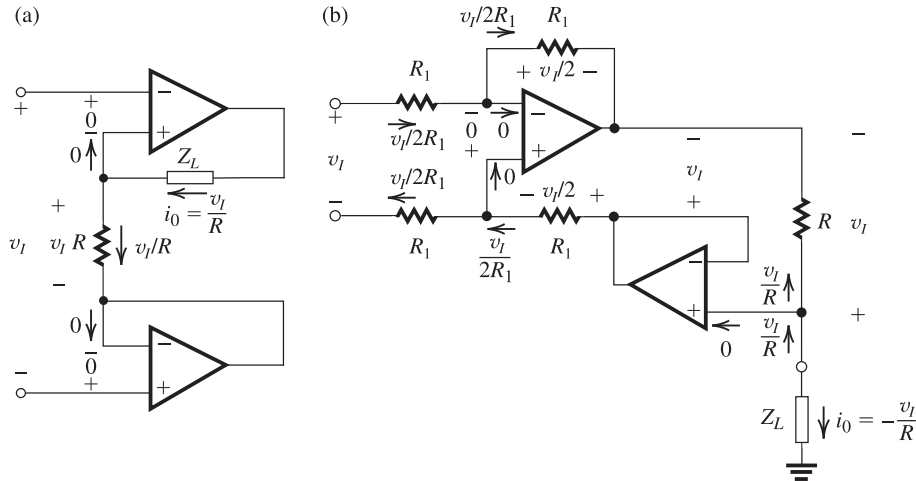
$$CR = 1 \times 10^{-9} \times 10 \times 10^4 = 10^{-4} \text{ s}$$

$$(a) \quad |V_o/V_i| = 1 \text{ at } \omega = \frac{1}{CR} = 10^4 \text{ rad/s.}$$

$$\text{Correspondingly, } f = \frac{10^4}{2\pi} = 1.59 \text{ kHz.}$$

(b) At  $f = 1.59 \text{ kHz}$ , the output sine wave leads the input sine wave by  $90^\circ$ .

This figure belongs to Problem 2.80.



(c) If the frequency is lowered by a factor of 10, the gain increases by a factor of 10 and, correspondingly, the output voltage increases by a factor of 10.

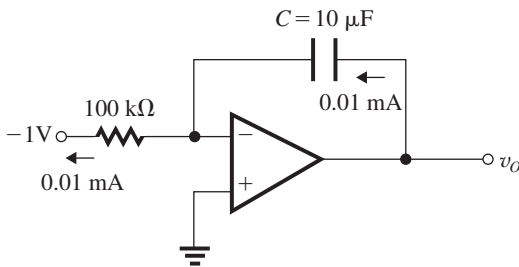
(d) The phase relation between input and output remains unchanged.

2.82  $CR = 1$  s and  $R_{in} = 100$  k $\Omega$ .

$$\Rightarrow R = 100 \text{ k}\Omega$$

$$C = \frac{1}{100 \times 10^3} = 10 \text{ }\mu\text{F}$$

When a dc voltage of  $-1$  V is applied, a dc current of  $1 \text{ V}/100 \text{ k}\Omega = 0.01 \text{ mA}$  will flow as shown in the figure below.



The capacitor voltage  $v_o$  will rise linearly from its initial value of  $-10$  V. Thus,

$$v_o = \frac{It}{C} - 10$$

$$= \frac{10^{-5}t}{10 \times 10^{-6}} - 10 = t - 10, \text{ V}$$

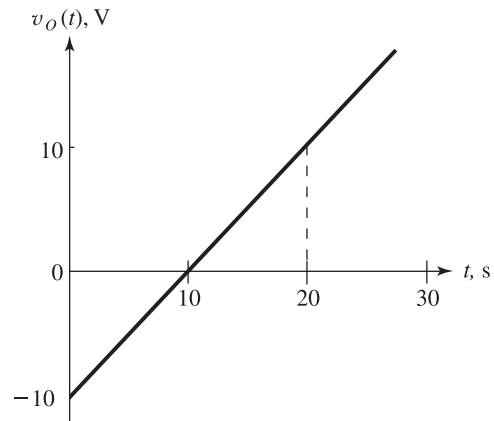
Thus  $v_o$  will reach  $0$  V at

$$t = 10 \text{ s}$$

and will reach  $10$  V at

$$t = 20 \text{ s}$$

See figure below.



2.83 The current drawn from the input of an inverting integrator with ideal op amp is

$$i_t = \frac{v_i}{R}$$

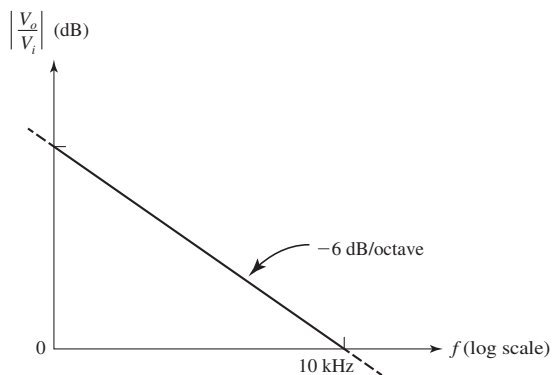
In this case

$$0.01 = \frac{0.1}{R} \Rightarrow R = \frac{0.1}{0.01} = 10 \text{ k}\Omega$$

The change in output voltage over  $1 \text{ ms}$  is

$$-1 \text{ V} = -\frac{1}{CR} \int_0^{10^{-3}} 0.1 dt$$

$$C = \frac{1}{10^4} \int_0^{10^{-3}} 0.1 dt = \frac{1}{10^4} \times 10^{-4} = 10 \text{ nF}$$



The integrator frequency is

$$\omega_{\text{int}} = \frac{1}{CR} = 10 \text{ kHz}$$

$$2.84 \quad 2\pi \times 100 \times 10^3 = \frac{1}{CR} \Rightarrow CR = 1.59 \mu\text{s}$$

For  $R_{\text{in}} = 10 \text{ k}\Omega$ ,  $R = 10 \text{ k}\Omega$ , and

$$C = \frac{1.59 \times 10^{-6}}{10^4} = 159 \text{ pF}$$

To limit the dc gain to 40 dB (i.e., 100 V/V), we connect a resistance  $R_F$  across  $C$  (as in Fig. 2.25) with  $R_F = 100 R = 1 \text{ M}\Omega$ .

The resulting low-pass filter will have a 3-dB frequency of

$$f_{3\text{dB}} = \frac{1}{2\pi CR_F} = \frac{1}{2\pi \times 159 \times 10^{-12} \times 10^6} = 1 \text{ kHz}$$

When a 10- $\mu\text{s}$ , 1-V pulse (see Fig. 1) is applied at the input, a current of  $1 \text{ V}/10 \text{ k}\Omega = 0.1 \text{ mA}$  flows into the integrator. Now we consider two cases: with and without  $R_F$ .

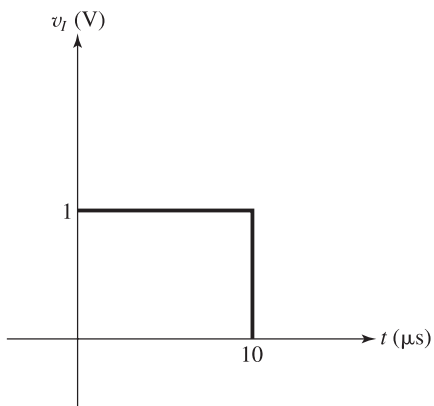


Figure 1

(a) For an integrator without  $R_F$ , the 0.1-mA current flows through  $C$  and the output voltage decreases linearly from 0 V as

$$\begin{aligned} v_O(t) &= -\frac{It}{C} \\ &= -\frac{0.1 \times 10^{-3}}{159 \times 10^{-12}} t \\ &= -0.63 \times 10^6 t, \text{ V} \end{aligned}$$

At the end of the pulse,  $t = 10 \mu\text{s}$ , resulting in

$$\begin{aligned} v_O(10 \mu\text{s}) &= -0.63 \times 10^6 \times 10 \times 10^{-6} \\ &= -6.3 \text{ V} \end{aligned}$$

See Fig. 2.

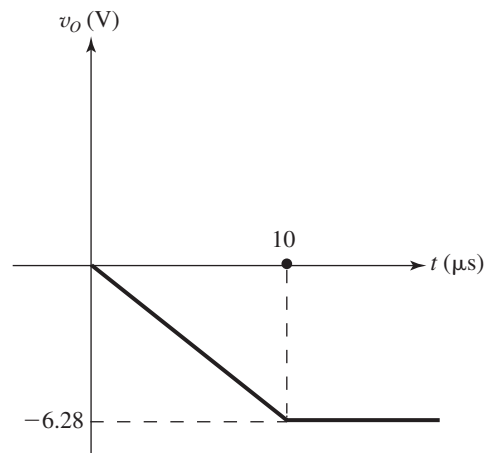


Figure 2

(b) For an integrator with  $R_F$ , the 0.1-mA current flows through the parallel combination of  $C$  and  $R_F$ . The result is

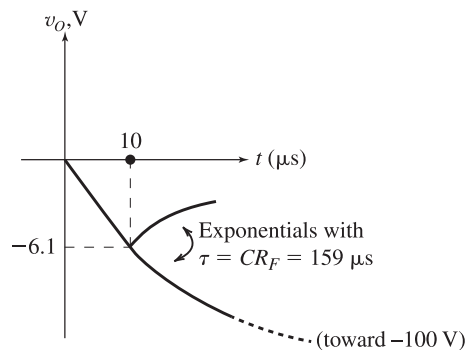


Figure 3

$$v_O(t) = v_{O\text{final}} - (v_{O\text{final}} - v_{O\text{initial}})e^{-t/\tau}$$

where

$$v_{O\text{final}} = -IR_F = -0.1 \times 10^{-3} \times 10^6 = -100 \text{ V}$$

$$v_{O\text{initial}} = 0$$

$$\tau = CR_F = 159 \times 10^{-12} \times 10^6 = 159 \mu\text{s}$$

Thus,

$$v_o(t) = -100(1 - e^{-t/159}), \text{ V}$$

where  $t$  is in  $\mu\text{s}$ . At the end of the pulse,  
 $t = 10 \mu\text{s}$ ,

$$v_o(10 \mu\text{s}) = -100(1 - e^{-10/159}) = -6.1 \text{ V}$$

Beyond  $t = 10 \mu\text{s}$ , the capacitor discharges through  $R_F$ . Thus, including  $R_F$  results in the nonideal integrator response shown in Fig. 3.

**2.85**

For a time constant of  $CR = 10^{-5}$  s with  
 $C = 10^{-9}$  F,

$$R = \frac{10^{-5}}{10^{-9}} = 10 \text{ k}\Omega$$

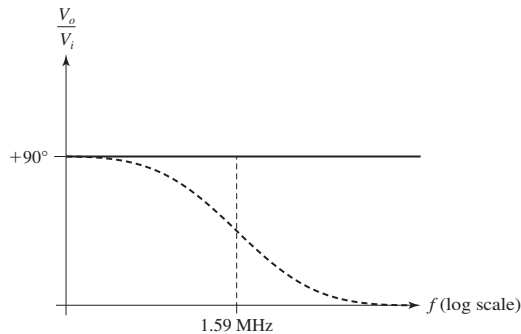
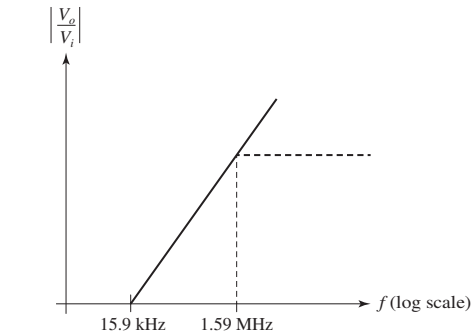
The magnitude response of the ideal differentiator is shown in Fig. 2.28, and the phase response is constant at  $+90^\circ$ . The 0-dB frequency is

$$\frac{1}{2\pi \times CR} = \frac{1}{2\pi \times 10^{-5}} = 15.9 \text{ kHz}$$

The magnitude and phase responses are plotted with the solid line.

With a series resistor  $R_i$  connected to  $C$ , the frequency response becomes

$$-\frac{R}{R_i + 1/sC} = -\frac{-sCR}{1 + sCR_i}$$



At high frequencies, the capacitor acts as a short-circuit, and the differentiator becomes an inverting configuration with a gain  $-\frac{R}{R_i}$ . Thus, to

provide a high-frequency gain magnitude of 100 V/V,

$$\left| \frac{R}{R_i} \right| = 100 \Rightarrow R_i = \frac{R}{100} = \frac{10^4}{100} = 100 \Omega$$

The magnitude and phase response of this modified circuit is shown in the dashed line in the plot. A corner frequency appears in the magnitude response at  $1/(2\pi CR_i) = 1.59$  MHz. The phase response decreases to  $0^\circ$  around 1.59 MHz, with a value of  $45^\circ$  at 1.59 MHz.

**2.86**  $|T| = \frac{1}{\omega RC}$ . If  $|T| = 100$  V/V at

$f = 10$  kHz, then for  $|T| = 1$  V/V,  $f$  has to be  $10 \text{ kHz} \times 100 = 1 \text{ MHz}$ .

Also,

$$RC = \frac{1}{\omega_{\text{int}}} = \frac{1}{2\pi \times 1 \text{ MHz}} = 0.159 \mu\text{s}$$

**2.87** For  $0 \leq t \leq 0.5$  ms:

$$v_o(t) = 0 - \frac{1}{RC} \int_0^t v_i dt$$

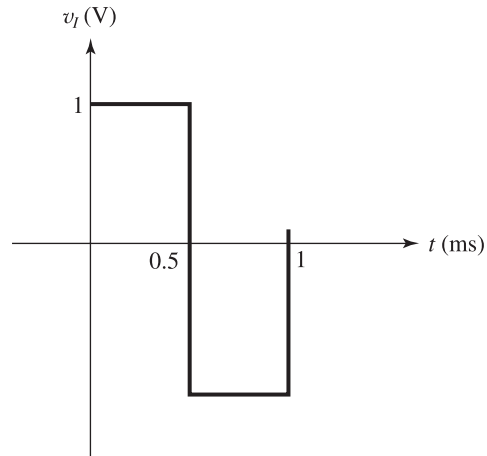
$$v_o(t) = 0 - \frac{t}{RC} = -\frac{t}{1 \text{ ms}}$$

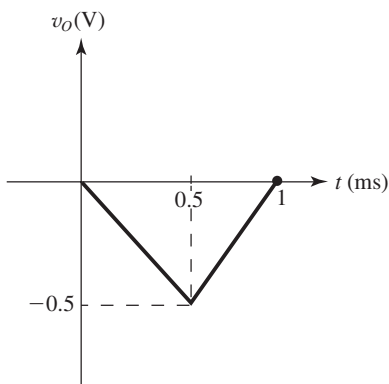
$$v_o(0.5 \text{ ms}) = -0.5 \text{ V}$$

for  $0.5 \leq t \leq 1$  ms:

$$v_o(t) = v_o(0.5 \text{ ms}) - \frac{1}{RC} \int_{0.5}^t -1 dt$$

$$v_o(t) = -0.5 + \frac{1}{RC} (t - 0.5)$$





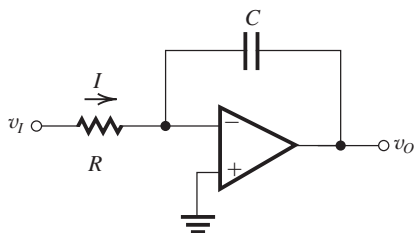
$$v_o(1 \text{ ms}) = -0.5 + \frac{0.5}{1} = 0 \text{ V}$$

Another way of thinking about this circuit is as follows:

for  $0 \leq t \leq 0.5 \text{ ms}$ , a current  $I = \frac{1 \text{ V}}{R}$  flows through  $R$  and  $C$  in the direction indicated on the diagram. At time  $t$  we write:

$$I \cdot t = -Cv_o(t) \Rightarrow v_o(t) = \frac{-I}{C}t = \frac{-1}{RC}t$$

which indicates that the output voltage is linearly decreasing, reaching  $-0.5 \text{ V}$  at  $t = 0.5 \text{ ms}$ .

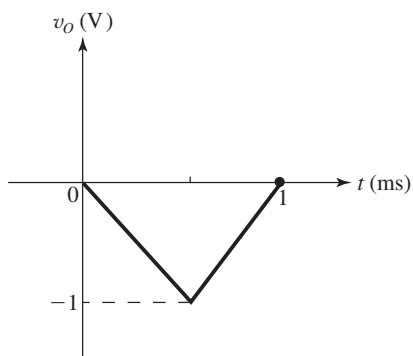


Then for  $0.5 \leq t \leq 1 \text{ ms}$ , the current flows in the opposite direction,  $v_o$  rises linearly, reaching  $0$  at  $t = 1 \text{ ms}$ .

For  $v_i = \pm 2 \text{ V}$ :

We obtain the following waveform (assuming time constant is the same).

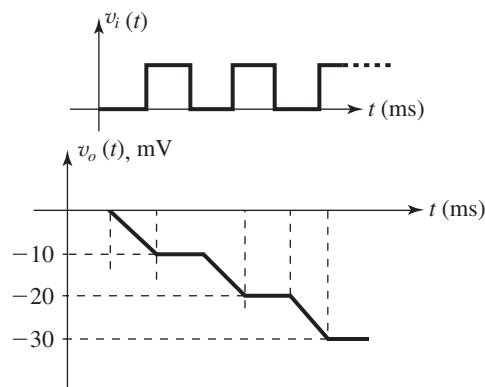
If  $CR$  is also doubled, then the waveform becomes the same as the first case.



**2.88** Each pulse provides a constant current of  $\frac{1 \text{ V}}{R}$  through the capacitor and thus deposits a charge of  $\frac{1 \text{ V}}{R} \times 10 \mu\text{s}$  on the capacitor, resulting in a change of the output voltage of

$$-\frac{1 \times 10 \times 10^{-6}}{RC} = -\frac{10^{-5}}{10^{-3}} = -0.01 \text{ V}$$

Therefore a total of 100 pulses are required to cause a change of  $-1 \text{ V}$  in  $v_o(t)$ .



**2.89** The input resistance of Fig. 2.23 is  $R_1 = 10 \text{ k}\Omega$ . Using the results from Example 2.4 for dc gain  $K$  and 3-dB frequency  $\omega_0$ ,

$$K = -\frac{R_2}{R_1}$$

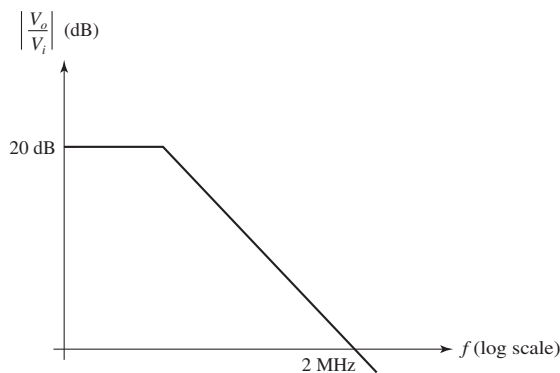
$$-10 = -\frac{R_2}{10}$$

$$\Rightarrow R_2 = 100 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{C_2 R_2}$$

$$2\pi \times 2 \times 10^5 = \frac{1}{C_2 \times 10^5}$$

$$\Rightarrow C_2 = \frac{1}{10^5 \times 2\pi \times 10^5} = 15.9 \text{ pF}$$



**2.90** Equation (2.5) can be generalized as follows:

$$\frac{V_o}{V_i} = -\frac{Z_2/Z_1}{1 + \frac{1 + Z_2/Z_1}{A}}$$

For  $Z_1 = R$ ,  $Z_2 = 1/sC$ , and  $A = A_0$ ,

$$\begin{aligned} \frac{V_o}{V_i} &= -\frac{1/sCR}{1 + \frac{1}{A_0} + \frac{1}{sA_0CR}} \\ &= -\frac{1}{CR(1 + \frac{1}{A_0})} \frac{1}{s + \frac{1}{(A_0 + 1)CR}} \\ &= -\frac{A_0/[(A_0 + 1)CR]}{s + \frac{1}{(A_0 + 1)CR}} \end{aligned}$$

which is low-pass STC function. The pole (or 3-dB) frequency is

$$\omega_p = \frac{1}{(A_0 + 1)CR}$$

The ideal integrator has  $\omega_p = 0$ . Observe that as  $A_0 \rightarrow \infty$ ,  $\omega_p \rightarrow 0$ . The dc gain is  $-A_0$ , which is the dc gain of the op amp.

If an ideal Miller integrator is fed with a  $-1$ -V pulse signal of width  $T = CR$ , the output voltage can be found as follows: The  $-1$ -V pulse will cause a current  $I = 1 \text{ V}/R$  to be drawn through  $R$  and  $C$ . The capacitor voltage, which is  $v_o$ , will rise linearly according to

$$v_o = \frac{1}{C}It = \frac{1}{CR}t$$

Thus, at  $t = T$  (the end of the pulse) the output voltage reaches  $1 \text{ V}$  and then stays constant at this value.

If the integrator is made with an op amp having a finite  $A_0 = 1000$ , the response to the  $-1$ -V step will be that of an STC low-pass circuit. Thus,

$$v_o = v_{o\text{final}} - (v_{o\text{final}} - v_{o\text{initial}})e^{-t/\tau}$$

where

$$v_{o\text{final}} = -1 \text{ V} \times \text{dc gain}$$

$$= -1 \text{ V} \times -1000$$

$$= 1000 \text{ V}$$

$$v_{o\text{initial}} = 0 \text{ V}$$

$$\tau = \frac{1}{\omega_p} = (A_0 + 1)CR = 1001CR$$

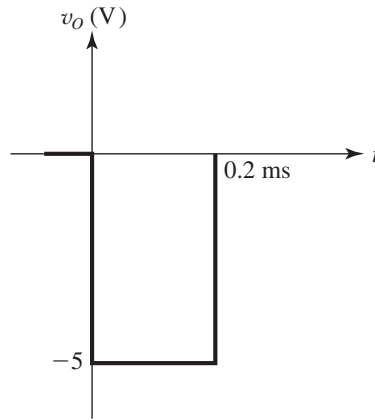
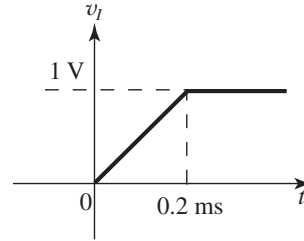
Thus,

$$v_o = 1000(1 - e^{-t/1000CR})$$

At  $t = T$ , which is equal to  $CR$ ,

$$\begin{aligned} v_o(T) &= 1000(1 - e^{-0.001}) \\ &= -0.9995 \simeq -1 \text{ V} \end{aligned}$$

**2.91**



$$v_o = -CR \frac{dv_i}{dt}$$

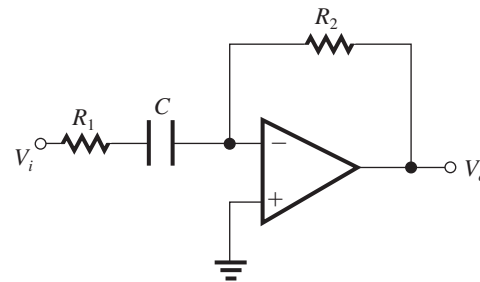
Therefore:

For  $0 \leq t \leq 0.2 \text{ ms}$ :

$$v_o = -1 \text{ ms} \times \frac{1 \text{ V}}{0.2 \text{ ms}} = -5 \text{ V}$$

and  $v_o = 0$  otherwise.

**2.92**



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1 + \frac{1}{sC}}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{(R_2/R_1)s}{s + \frac{1}{CR_1}}$$

which is that of an STC high-pass type.

$$\text{High-frequency gain } (s \rightarrow \infty) = -\frac{R_2}{R_1}$$

$$\text{3-dB frequency } (\omega_{3dB}) = \frac{1}{CR_1}$$

For a high-frequency input resistance of 1 kΩ, we select  $R_1 = 1 \text{ k}\Omega$ . For a high-frequency gain of 40 dB,

$$\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100 \text{ k}\Omega$$

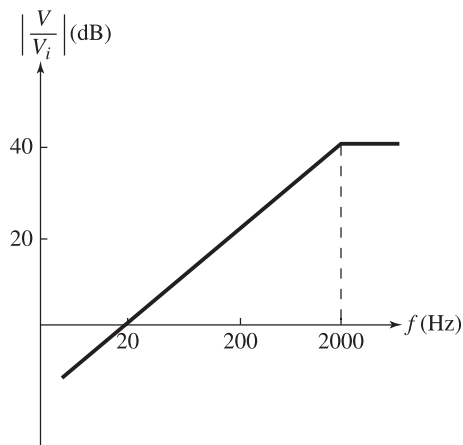
For  $f_{3dB} = 2 \text{ kHz}$ ,

$$\frac{1}{2\pi CR_1} = 2 \times 10^3$$

$$\Rightarrow C = 79 \text{ nF}$$

The magnitude of the transfer function reduces from 40 dB to unity (0 dB) in two decades. Thus

$$f \text{ (unity gain)} = \frac{f_{3dB}}{100} = \frac{2000}{100} = 20 \text{ Hz}$$



**2.93** Refer to the circuit in Fig. P2.93.

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2}$$

$$= -\frac{1}{\left(R_1 + \frac{1}{sC_1}\right) \left(\frac{1}{R_2} + sC_2\right)}$$

$$= -\frac{R_2/R_1}{\left(1 + \frac{1}{sC_1 R_1}\right) (1 + sC_2 R_2)}$$

$$\frac{V_o}{V_i}(j\omega) = -\frac{R_2/R_1}{[1 + (\omega_1/j\omega)][1 + j(\omega/\omega_2)]}$$

where

$$\omega_1 = \frac{1}{C_1 R_1} \quad \text{and} \quad \omega_2 = \frac{1}{C_2 R_2} \quad \text{Q.E.D.}$$

Assuming  $\omega_2 \gg \omega_1$ , then

(a) For  $\omega \ll \omega_1$ ,

$$\frac{V_o}{V_i} \approx -\frac{R_2/R_1}{1 + (\omega_1/j\omega)}$$

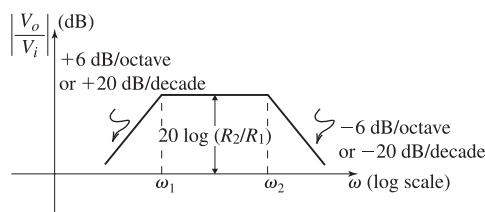
(b) For  $\omega_1 \ll \omega \ll \omega_2$

$$\frac{V_o}{V_i} \approx -(R_2/R_1)$$

(c) For  $\omega \gg \omega_2$

$$\frac{V_o}{V_i} \approx -\frac{R_2/R_1}{1 + j(\omega/\omega_2)}$$

The resulting Bode plot will be as shown:



Design: Gain of 40 dB  $\Rightarrow \frac{R_2}{R_1} = 100$

$$f_1 = 200 \text{ Hz} \Rightarrow \frac{1}{2\pi C_1 R_1} = 200$$

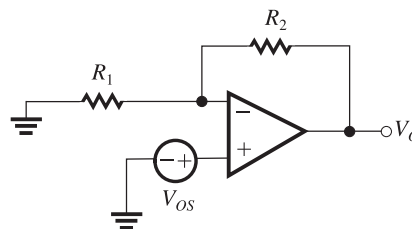
$$f_2 = 200 \text{ kHz} \Rightarrow \frac{1}{2\pi C_2 R_2} = 200 \times 10^3$$

Input resistance (at  $\omega \gg \omega_1$ ) = 2 kΩ

$$\Rightarrow R_1 = 2 \text{ k}\Omega$$

Thus,  $R_1 = 2 \text{ k}\Omega$ ,  $R_2 = 200 \text{ k}\Omega$ ,  $C_1 \approx 0.4 \text{ }\mu\text{F}$ , and  $C_2 \approx 4 \text{ pF}$ .

**2.94** Inverting configuration:



$$V_o = V_{os} \left(1 + \frac{R_2}{R_1}\right)$$

$$-0.2 = V_{os} \left(1 + \frac{R_2}{R_1}\right)$$

$$= V_{OS} \left( 1 + \frac{100}{2} \right)$$

$$\Rightarrow V_{OS} \simeq 4 \text{ mV}$$

**2.95** Input offset voltage = 3 mV

$$\begin{aligned} \text{Output dc offset voltage} &= 3 \text{ mV} \times \text{closed loop gain} \\ &= 3 \text{ mV} \times 1000 \\ &= 3 \text{ V} \end{aligned}$$

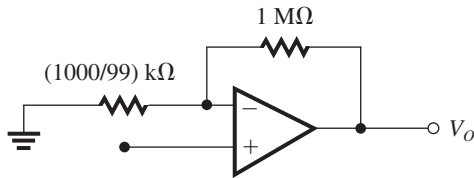
The maximum amplitude of an input sinusoid that results in an output peak amplitude of 12 - 3 = 9 V is given by

$$v_i = \frac{9}{1000} = 9 \text{ mV}$$

If amplifier is capacitively coupled then

$$v_{i \text{ max}} = \frac{12}{1000} = 12 \text{ mV}$$

**2.96**



$$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10.1 \text{ k}\Omega$$

(a)  $V_O = 200 \times 10^{-9} \times 1 \times 10^6 = 0.2 \text{ V}$

(b) Largest output offset is

$$V_O = 1 \text{ mV} \times 100 + 0.2 \text{ V} = 0.3 \text{ V}$$

(c) For bias current compensation, we connect a resistor  $R_3$  in series with the positive input terminal of the op amp, with  $R_3 = R_1 \parallel R_2$ ,

$$R_3 = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega \simeq 10 \text{ k}\Omega$$

$$I_{OS} = \frac{200}{10} = 20 \text{ nA}$$

The offset current alone will result in an output offset voltage of

$$I_{OS} \times R_2 = 20 \times 10^{-9} \times 1 \times 10^6 = 20 \text{ mV}$$

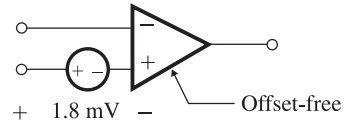
(d)  $V_O = 100 \text{ mV} + 20 \text{ mV} = 120 \text{ mV} = 0.12 \text{ V}$

**2.97**  $V_{OS} = \pm 2 \text{ mV}$

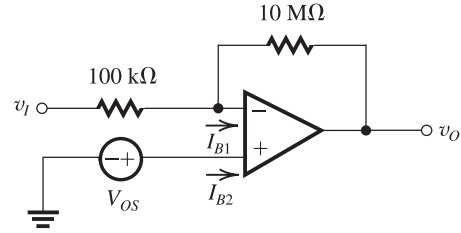
$$\begin{aligned} V_O &= 0.01 \sin \omega t \times 100 + V_{OS} \times 100 \\ &= 1 \sin \omega t \pm 0.2 \text{ V} \end{aligned}$$

**2.98**  $V_{OS} = \frac{-1.8 \text{ V}}{1001} \simeq -1.8 \text{ mV}$

Thus,



**2.99**



(a)  $I_B = (I_{B1} + I_{B2})/2$

Open input:

$$v_O = v_+ + R_2 I_{B1} = V_{OS} + R_2 I_{B1}$$

$$5.3 = V_{OS} + 10,000 I_{B1} \tag{1}$$

Input connected to ground:

$$\begin{aligned} v_O &= v_+ + R_2 \left( I_{B1} + \frac{V_{OS}}{R_1} \right) \\ &= V_{OS} \left( 1 + \frac{R_2}{R_1} \right) + R_2 I_{B1} \end{aligned}$$

$$5 = V_{OS} \times 101 + 10,000 I_{B1} \tag{2}$$

Equations (1), (2)

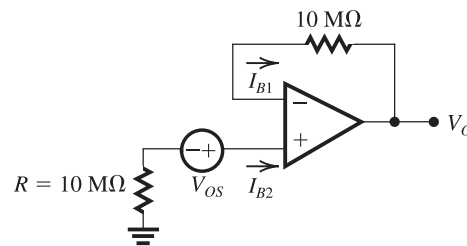
$$\Rightarrow 100 V_{OS} = -0.3 \Rightarrow V_{OS} = -3 \text{ mV}$$

$$\Rightarrow I_{B1} = 530 \text{ nA}$$

$$I_B \simeq I_{B1} = 530 \text{ nA}$$

and both flow into the op-amp input terminals.

(b)  $V_{OS} = -3 \text{ mV}$



(c) In this case, Since  $R$  is very large, we may ignore  $V_{OS}$  compared to the voltage drop across  $R$ .



$V_{OS} \ll R I_B$ , Also Eq. (2.46) holds:

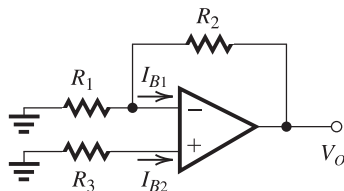
$$R_3 = R_1 \parallel R_2$$

therefore from Eq. (2.40):

$$V_O = I_{OS} \times R_2 \Rightarrow I_{OS} = \frac{-0.6}{10 \text{ M}\Omega}$$

$$I_{OS} = -60 \text{ nA}$$

### 2.100



$$R_2 = 100 \text{ k}\Omega$$

$$R_1 = \frac{100 \text{ k}\Omega}{9}$$

$$R_3 = 5 \text{ k}\Omega$$

$$I_{B1} = 2 \pm 0.1, \mu\text{A}, V_{OS} = 0$$

$$I_{B2} = 2 \pm 0.1, \mu\text{A}$$

From Eq. (2.36):

$$V_O = -I_{B2}R_3 + R_2 \left( I_{B1} - I_{B2} \frac{R_3}{R_1} \right)$$

Thus,

$$V_O = I_{B1}R_2 - I_{B2}R_3 \left( 1 + \frac{R_2}{R_1} \right) \quad (1)$$

The maximum value of  $V_O$  is obtained when  $I_{B1} = 2.1 \mu\text{A}$  and  $I_{B2} = 1.9 \mu\text{A}$ ,

$$\begin{aligned} V_{O\max} &= 2.1 \times 100 - 1.9 \times 5 \left( 1 + \frac{100}{100/9} \right) \\ &= 210 - 95 = 115 \text{ mV} \end{aligned}$$

The minimum value of  $V_O$  is obtained when  $I_{B1} = 1.9 \mu\text{A}$  and  $I_{B2} = 2.1 \mu\text{A}$ ,

$$\begin{aligned} V_{O\min} &= 1.9 \times 100 - 2.1 \times 5 \times 10 \\ &= 190 - 105 = 85 \text{ mV} \end{aligned}$$

Thus the dc offset at the output will be in the range of 85 mV to 115 mV. The bulk of the dc offset at the output, that due to  $I_B$ , can be reduced to zero by making the dc resistances seen by the two input terminals equal. Currently, the positive input terminal sees a resistance  $R_3 = 5 \text{ k}\Omega$  and the negative input terminal sees a resistance equal to  $R_1 \parallel R_2 = \frac{100}{9} \parallel 100 = 10 \text{ k}\Omega$ . Thus the two resistances can be made equal by connecting a

$5\text{-k}\Omega$  resistance in series with  $R_3$ . The resulting dc offset voltage at the output will be

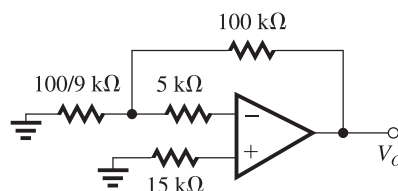
$$V_O = I_{OS}R_2 = 0.2 \times 100 = 20 \text{ mV}$$

Since  $I_{OS}$  can be of either polarity,

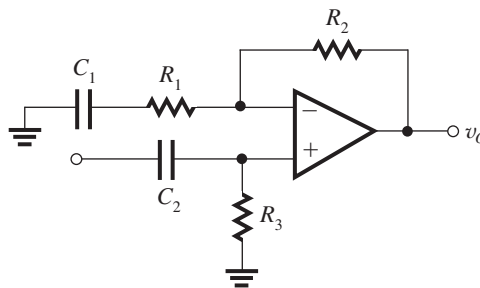
$$V_O = \pm 20 \text{ mV}$$

The same result could have been found by replacing  $R_3$  in Eq. (1) by  $(R_3 + R_4)$  where  $R_4 = 5 \text{ k}\Omega$ .

If the signal source resistance is  $15 \text{ k}\Omega$ , then the resistances can be equalized by adding a  $5\text{-k}\Omega$  resistor in series with the negative input lead of the op amp.



### 2.101



$$R_2 = R_3 = 100 \text{ k}\Omega$$

$$1 + \frac{R_2}{R_1} = 100$$

$$R_1 = \frac{100 \text{ k}\Omega}{99} = 1.01 \text{ k}\Omega$$

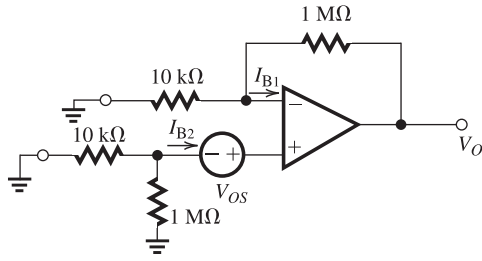
$$\begin{aligned} \frac{1}{R_1 C_1} &= 2\pi \times 100 \Rightarrow C_1 = \frac{1}{1.01 \times 2\pi \times 10^5} \\ &= 1.58 \mu\text{F} \end{aligned}$$

$$\begin{aligned} \frac{1}{R_3 C_2} &= 2\pi \times 10 \Rightarrow C_2 = \frac{1}{2\pi \times 10^6} \\ &= 0.16 \mu\text{F} \end{aligned}$$

### 2.102

The component of  $V_O$  due to  $V_{OS}$  is

$$V_{O1} = V_{OS} \left( 1 + \frac{1 \text{ M}\Omega}{10 \text{ k}\Omega} \right)$$



$$= 5(1 + 100) = 505 \text{ mV}$$

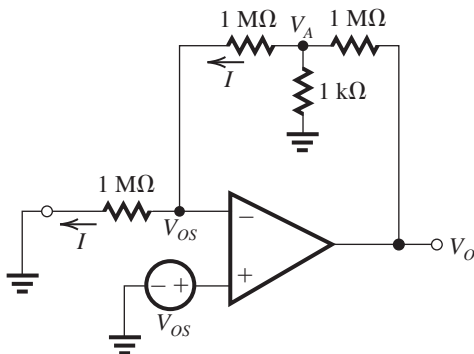
Since the two op-amp input terminals see equal resistances of  $(10 \text{ k}\Omega \parallel 1 \text{ M}\Omega)$ , the input bias current  $I_B$  will not result in a dc voltage at the output; however, the input offset current  $I_{OS}$  will give rise to a dc output offset of

$$\begin{aligned} V_{O2} &= I_{OS} \times 1 \text{ M}\Omega \\ &= 0.2 \mu\text{A} \times 1 \text{ M}\Omega = 0.2 \text{ V} \end{aligned}$$

Thus,

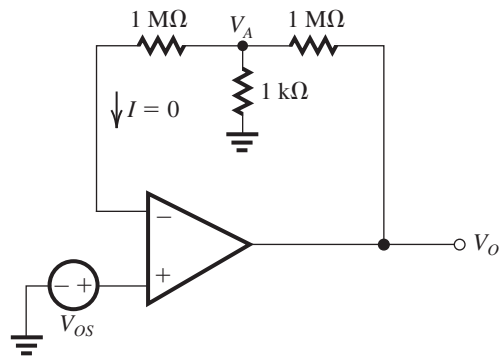
$$\begin{aligned} V_O &= V_{O1} + V_{O2} = 0.505 + 0.2 \\ &= 0.705 \text{ V} \end{aligned}$$

**2.103**



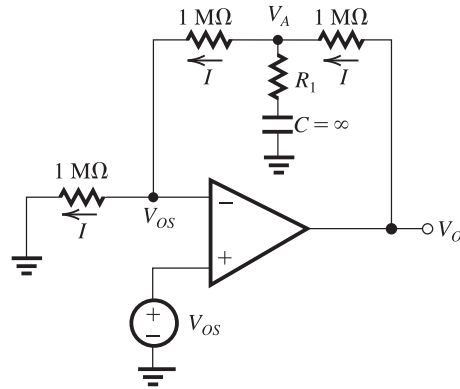
$$\begin{aligned} v_- &= v_+ = V_{OS} \\ V_A &= 2V_{OS} = 6 \text{ mV} \\ I &= \frac{V_{OS}}{1 \text{ M}\Omega} = \frac{6 \text{ mV}}{1 \text{ M}\Omega} = 6 \text{ nA} \\ V_O &= V_A + 1 \text{ M}\Omega \times \left( I + \frac{V_A}{1 \text{ k}\Omega} \right) \\ &= 2V_{OS} + 1 \text{ M}\Omega \times \left( \frac{V_{OS}}{1 \text{ M}\Omega} + \frac{2V_{OS}}{1 \text{ k}\Omega} \right) \\ &= 2003 V_{OS} \\ &= 2003 \times 3 \text{ mV} \\ &\cong 6 \text{ V} \end{aligned}$$

For capacitively coupled input,



$$\begin{aligned} v_+ &= v_- = V_{OS} \\ I &= 0 \quad V_A = V_{OS} \\ V_O &= V_A + 1 \text{ M}\Omega \times \frac{V_{OS}}{1 \text{ k}\Omega} = V_{OS} + 1000V_{OS} \\ &= 1001V_{OS} \\ &= 1001 \times 3 \text{ mV} \simeq 3 \text{ V} \end{aligned}$$

A large capacitor placed in series with the 1-kΩ resistor results in



$$\begin{aligned} v_+ &= v_- = V_{OS} \\ \text{No dc current flows through } R_1, C \text{ branch} \\ \therefore V_O &= V_A + V_{OS} \\ &= 2V_{OS} + V_{OS} \\ &= 3V_{OS} \\ &= 3 \times 3 \text{ mV} \\ &= 9 \text{ mV} \end{aligned}$$

**2.104** At  $0^\circ\text{C}$ , we expect  
 $\pm 20 \times 25 \times 1000 \mu\text{V} = \pm 500 \text{ mV} = \pm 0.5 \text{ V}$   
 At  $75^\circ\text{C}$ , we expect  
 $\pm 20 \times 50 \times 1000 \mu = \pm 1 \text{ V}$

We expect these quantities to have opposite polarities.

**2.105**  $R_3 = R_1 \parallel R_2 = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega = 9.09 \text{ k}\Omega$   
 Now, with the input grounded and assuming  $V_{OS} = 0$ , the measured  $+0.3\text{-V}$  at the output is entirely due to  $I_{OS}$ , that is,

$$0.3 = I_{OS}R_2 = I_{OS} \times 1 \text{ M}\Omega$$

Thus,

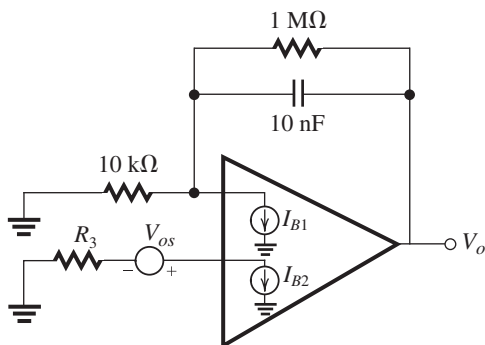
$$I_{OS} = 0.3 \text{ }\mu\text{A}$$

If  $V_{OS} = \pm 1 \text{ mV}$ , then it alone will result in an output voltage of

$$V_{OS} \left( 1 + \frac{R_2}{R_1} \right) = V_{OS} \times 101 = \pm 101 \text{ mV or } \simeq \pm 0.1 \text{ V}$$

If  $V_{OS}$  is positive,  $0.1 \text{ V}$  of the output  $0.3\text{-V}$  offset will be due to  $V_{OS}$ , leaving  $0.2 \text{ V}$  as the result of  $I_{OS}$ ; thus in this case,  $I_{OS} = \frac{0.2 \text{ V}}{1 \text{ M}\Omega} = 0.2 \text{ }\mu\text{A}$ . On the other hand, if  $V_{OS}$  is negative, then  $-0.1 \text{ V}$  of the output  $0.3 \text{ V}$  is due to  $V_{OS}$ , with the result that  $I_{OS}$  must be causing  $0.4 \text{ V}$  of output offset. In this case,  $I_{OS} = \frac{0.4 \text{ V}}{1 \text{ M}\Omega} = 0.4 \text{ }\mu\text{A}$ . Thus, the possible range of  $I_{OS}$  is  $0.2 \text{ }\mu\text{A}$  to  $0.4 \text{ }\mu\text{A}$ .

### 2.106



(a)  $R_3 = R \parallel R_F = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega$

$$\Rightarrow R_3 = 9.9 \text{ k}\Omega$$

(b) As discussed in Section 2.8.2, the dc output voltage of the integrator when the input is

$$\text{grounded is } V_O = V_{OS} \left( 1 + \frac{R_F}{R} \right) + I_{OS}R_F$$

$$V_O = 2 \text{ mV} \left( 1 + \frac{1 \text{ M}\Omega}{10 \text{ k}\Omega} \right) + 20 \text{ nA} \times 1 \text{ M}\Omega$$

$$= 0.202 \text{ V} + 0.02 \text{ V}$$

$$V_O = 0.222 \text{ V}$$

**2.107** At very low frequencies, the gain is  $A_0$ , thus

$$20 \log A_0 = 98 \text{ dB} \Rightarrow A_0 \simeq 80,000 \text{ V/V}$$

At  $f = 100 \text{ kHz}$ , the gain is  $40 \text{ dB}$  or  $100 \text{ V/V}$ . Thus

$$f_i = 100 \text{ kHz} \times 100 = 10 \text{ MHz}$$

$$\text{Since } f_i = A_0 f_b \Rightarrow f_b = \frac{10 \text{ MHz}}{80,000} = 125 \text{ Hz.}$$

**2.108**  $f = 10 \text{ kHz}$   $|A| = 20 \times 10^3$

$$f = 100 \text{ kHz} \quad |A| = 4 \times 10^3$$

Thus, a change of a decade in  $f$  does *not* result in a factor of 10 reduction in gain; in fact, the gain reduces by only a factor of 5. It follows that the first frequency ( $10 \text{ kHz}$ ) is less than  $f_b$ . Therefore, we must use the exact expression for  $|A|$ , that is,

$$|A| = \frac{A_0}{\sqrt{1 + (f/f_b)^2}}$$

Substituting the given data, we obtain

$$\frac{A_0}{\sqrt{1 + (10/f_b)^2}} = 20 \times 10^3 \quad (1)$$

$$\frac{A_0}{\sqrt{1 + (100/f_b)^2}} = 4 \times 10^3 \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\sqrt{\frac{1 + (100/f_b)^2}{1 + (10/f_b)^2}} = 5$$

$$\Rightarrow 1 + \frac{100^2}{f_b^2} = 25 \left( 1 + \frac{10^2}{f_b^2} \right) = 25 + \frac{2500}{f_b^2}$$

$$\frac{10,000}{f_b^2} - \frac{2500}{f_b^2} = 24$$

$$f_b = \sqrt{\frac{7500}{24}} = 17.68 \text{ kHz}$$

Now, substituting in Eq. (1) yields

$$A_0 = 20 \times 10^3 \sqrt{1 + \left( \frac{10}{17.68} \right)^2} = 22,976 \text{ V/V}$$

and the unity-gain frequency is

$$f_i = A_0 f_b = 22,976 \times 10^3 \times 17.68 \times 10^3 = 406.2 \text{ MHz}$$

**2.109** Using Eq. (2.51),

$$f_{3\text{dB}} = \frac{f_i}{1 + R_2/R_1} = \frac{2000}{5} = 400 \text{ kHz}$$

From Eq. (2.53), the closed-loop gain is

$$|G| = \frac{5}{\sqrt{1 + \left( \frac{f}{f_{3\text{dB}}} \right)^2}}$$

Solving for  $|G| = 5 \times 0.99$  yields  $f = 56.8 \text{ kHz}$ .

The amplifier behaves like a low-pass STC

circuit with a time constant

$$\tau = \frac{1}{2\pi \times 400 \times 10^3} = 398 \text{ ns.}$$

$$t_r = 2.2\tau = 875 \text{ ns}$$

**2.110**  $f_i = A_0 f_b$

$A_0$	$f_b$ (Hz)	$f_i$ (Hz)
$10^5$	$10^2$	$10^7$
$10^6$	1	$10^6$
$10^5$	$10^3$	$10^8$
$10^7$	$10^{-1}$	$10^6$
$2 \times 10^5$	10	$2 \times 10^6$

**2.111** The gain drops by 20 dB at  $f \simeq 10f_b$ . Thus

(a)  $A_0 = 2 \times 10^5 \text{ V/V}$

$$f_b = \frac{5 \times 10^2}{10} = 50 \text{ Hz}$$

$$f_i = A_0 f_b = 2 \times 10^5 \times 50 = 10^7 \text{ Hz} = 10 \text{ MHz}$$

(b)  $A_0 = 20 \times 10^5 \text{ V/V}$

$$f_b = \frac{10}{10} = 1 \text{ Hz}$$

$$f_i = A_0 f_b = 20 \times 10^5 \times 1 = 2 \text{ MHz}$$

(c)  $A_0 = 1800 \text{ V/V}$

$$f_b = \frac{0.1 \text{ MHz}}{10} = 10 \text{ kHz}$$

$$f_i = A_0 f_b = 1800 \times 10 = 18 \text{ MHz}$$

(d)  $A_0 = 100 \text{ V/V}$

$$f_b = \frac{0.1 \text{ GHz}}{10} = 10 \text{ MHz}$$

$$f_i = A_0 f_b = 100 \times 10 = 1 \text{ GHz}$$

(e)  $A_0 = 25 \text{ V/mV} = 25 \times 10^3 \text{ V/V}$

$$f_b = \frac{250}{10} = 25 \text{ kHz}$$

$$f_i = A_0 \times f_b = 25 \times 10^3 \times 25 \times 10^3 = 625 \text{ MHz}$$

**2.112**  $G_{\text{nominal}} = -50 \Rightarrow \frac{R_2}{R_1} = 50$

$$A_0 = 10^4$$

$$f_i = 10^8 \text{ Hz}$$

$$f_{3\text{dB}} \text{ of closed-loop amplifier} = \frac{f_i}{1 + \frac{R_2}{R_1}}$$

$$= \frac{10^8}{51} = 1.96 \text{ MHz}$$

$$G = -\frac{50}{1 + j\frac{f}{f_{3\text{dB}}}}$$

$$|G| = \frac{50}{\sqrt{1 + (f/f_{3\text{dB}})^2}}$$

For  $f = 0.1f_{3\text{dB}}$ ,  $|G| = \frac{50}{\sqrt{1.01}} = 49.75 \text{ V/V}$

For  $f = 10f_{3\text{dB}}$ ,  $|G| = \frac{50}{\sqrt{1 + 100}} = 4.975 \text{ V/V}$

which is a 20-dB reduction.

**2.113**  $f_i = 20 \text{ MHz}$  and closed-loop gain

$$1 + \frac{R_2}{R_1} = 100 \text{ V/V}$$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{20 \text{ MHz}}{100} = 200 \text{ kHz}$$

$$G(j\omega) = \frac{100}{1 + j\frac{f}{f_{3\text{dB}}}}$$

$$\Rightarrow \phi = -\tan^{-1} \frac{f}{f_{3\text{dB}}}$$

For  $\phi = -6^\circ$

$$f = f_{3\text{dB}} \times \tan 6^\circ = 21 \text{ kHz}$$

$$\phi = 84^\circ, \quad f = f_{3\text{dB}} \times \tan 84^\circ = 1.9 \text{ MHz}$$

**2.114** (a)  $G = -50 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 50$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{51}$$

For  $f_{3\text{dB}} = 100 \text{ kHz} \Rightarrow f_i = 100 \times 51 = 5.1 \text{ MHz}$

(b)  $G = +50 \text{ V/V} \Rightarrow 1 + \frac{R_2}{R_1} = 50$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{50}$$

For  $f_{3\text{dB}} = 100 \text{ kHz}$ ,  $f_i = 5 \text{ MHz}$

(c)  $G = +2 \text{ V/V} \Rightarrow 1 + \frac{R_2}{R_1} = 2$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{2}$$

For  $f_{3\text{dB}} = 5 \text{ MHz}$ ,  $f_i = 10 \text{ MHz}$

(d)  $G = -2 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 2$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{3}$$

For  $f_{3\text{dB}} = 5$  MHz,  $f_i = 15$  MHz

$$(e) \quad G = -1000 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 1000$$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{1001}$$

For  $f_{3\text{dB}} = 10$  kHz,  $f_i = 10 \times 1001 = 10.1$  MHz

$$(f) \quad G = +1 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 0$$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = f_i$$

For  $f_{3\text{dB}} = 1$  MHz,  $f_i = 1$  MHz

$$(g) \quad G = -1 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 1$$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{2}$$

For  $f_{3\text{dB}} = 1$  MHz,  $f_i = 2$  MHz

$$\mathbf{2.115} \quad \text{Gain} = 1 + \frac{R_2}{R_1} = 96 \text{ V/V}$$

$$f_{3\text{dB}} = 8 \text{ kHz}$$

$$f_i = 96 \times 8 = 768 \text{ kHz}$$

$$\text{for } f_{3\text{dB}} = 32 \text{ kHz}$$

$$\text{Gain} = \frac{768}{32} = 24 \text{ V/V}$$

$$\mathbf{2.116} \quad 1 + \frac{R_2}{R_1} = 10 \Rightarrow R_1 = 1 \text{ k}\Omega \text{ and } R_2 = 9 \text{ k}\Omega$$

When a 100-mV (i.e., 0.1-V) step is applied at the input, the output will be

$$v_o = 0.1 \times 10(1 - e^{-t/\tau}), \text{ V}$$

where

$$\tau = \frac{1}{\omega_{3\text{dB}}}$$

$v_o$  reaches 1% of the 1-V final value at time  $t$ ,

$$1 - e^{-t/\tau} = 0.99$$

$$e^{-t/\tau} = 0.01$$

$$t = 4.6\tau$$

For  $t$  to be 200 ns,

$$\tau = \frac{200}{4.6} = 43.49 \text{ ns}$$

Thus we require a closed-loop 3-dB frequency

$$\omega_{3\text{dB}} = \frac{1}{\tau} \text{ or}$$

$$f_{3\text{dB}} = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 43.49 \times 10^{-9}} = 3.66 \text{ MHz}$$

Correspondingly, the op amp must have an  $f_i$  of

$$f_i = f_{3\text{dB}} \left(1 + \frac{R_2}{R_1}\right) = 36.6 \text{ MHz}$$

**2.117** (a) Assume two identical stages, each with a gain function:

$$G = \frac{G_0}{1 + j\frac{\omega}{\omega_1}} = \frac{G_0}{1 + j\frac{f}{f_1}}$$

$$G = \frac{G_0}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$$

overall gain of the cascade is

$$\frac{G_0^2}{1 + \left(\frac{f}{f_1}\right)^2}$$

The gain will drop by 3 dB when

$$1 + \left(\frac{f_{3\text{dB}}}{f_1}\right)^2 = \sqrt{2}$$

$$f_{3\text{dB}} = f_1 \sqrt{\sqrt{2} - 1} \quad \text{Q.E.D.}$$

$$(b) \quad 40 \text{ dB} = 20 \log G_0 \Rightarrow G_0 = 100 = 1 + \frac{R_2}{R_1}$$

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

(c) Each stage should have 20-dB gain or

$$1 + \frac{R_2}{R_1} = 10 \text{ and therefore a 3-dB frequency of}$$

$$f_1 = \frac{2 \times 10^6}{10} = 2 \times 10^5 \text{ Hz}$$

$$\text{The overall } f_{3\text{dB}} = 2 \times 10^5 \sqrt{\sqrt{2} - 1}$$

$$= 128.7 \text{ kHz,}$$

which is 6.4 times greater than the bandwidth achieved using a single op amp, as in case (b) above.

**2.118**  $f_i = 100 \times 5 = 500$  MHz if a single op amp is used.

With an op amp that has only  $f_i = 50$  MHz, multiple stages in series are required. Note that the magnitude response of  $K$  non-inverting stages in series, each having a dc gain of  $G_0$ , will be

$$|G(f)|^K = \left[ \frac{G_0}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}} \right]^K$$

where for each stage,  
 $f_{3dB} = f_i/G_0 = 50/G_0$  MHz. Thus

$$|G(f)|^K = \left[ \frac{G_0}{\sqrt{1 + \left(\frac{G_0 f}{50}\right)^2}} \right]^K$$

To ensure the 3dB-frequency of the cascade is over 5 MHz, we require

$$|G(5)|^K = \left[ \frac{G_0}{\sqrt{1 + \left(\frac{G_0}{10}\right)^2}} \right]^K > \frac{1}{\sqrt{2}}$$

while maintaining an overall dc gain of  $G_0^K = 100$ . These requirements can be met with  $K = 3$  (but not with  $K = 2$ ). Specifically, with  $K = 3$  each stage has a dc gain of  $G_0 = 100^{1/3} = 4.64$  V/V and 3-dB frequency

$$f_{3dB} = \frac{50}{4.64} = 10.8 \text{ MHz}$$

The overall 3-dB frequency is the frequency at which the gain of each stage is 1 dB below its dc value. That is,

$$\left[ \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}} \right] = 10^{-1/20} = 0.891$$

$$\left(\frac{f}{f_{3dB}}\right)^2 = 0.2589$$

$$\Rightarrow f = \sqrt{0.2589} f_{3dB} = 5.5 \text{ MHz}$$

**2.119** (a)  $\frac{R_2}{R_1} = K, \quad f_{3dB} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{f_i}{1 + K}$

GBP = Gain  $\times$   $f_{3dB}$

$$\text{GBP} = K \frac{f_i}{1 + K} = \frac{K}{K + 1} f_i$$

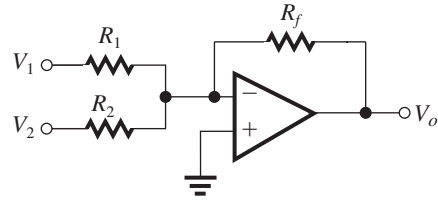
(b)  $1 + \frac{R_2}{R_1} = K, \quad f_{3dB} = \frac{f_i}{K}$

$$\text{GBP} = K \frac{f_i}{K} = f_i$$

For the same closed-loop gain, the noninverting configuration realizes a higher GBP, and it is independent of the closed-loop gain and equal to  $f_i$  of the op amp.

**2.120**

With a finite op-amp gain  $A$ , the voltage at the negative op-amp input terminal is  $-V_O/A$ . A nodal equation at the negative op-amp input terminal with  $V_2 = 0$  is



$$\frac{V_1 + V_O/A}{R_1} + \frac{V_O/A}{R_2} + \frac{V_O + V_O/A}{R_f} = 0$$

$$\Rightarrow V_O = -\frac{R_f}{R_1} \left[ \frac{\frac{A}{1 + R_f/R_1 \parallel R_2}}{1 + \frac{A}{1 + R_f/R_1 \parallel R_2}} \right] V_1$$

With  $V_1 = 0$ , a similar analysis yields

$$V_O = -\frac{R_f}{R_2} \left[ \frac{\frac{A}{1 + R_f/R_1 \parallel R_2}}{1 + \frac{A}{1 + R_f/R_1 \parallel R_2}} \right] V_2$$

Combining these with superposition yields,

$$V_O = -\left(\frac{R_f}{R_1} + \frac{R_f}{R_2}\right) \left[ \frac{\frac{A}{1 + R_f/R_1 \parallel R_2}}{1 + \frac{A}{1 + R_f/R_1 \parallel R_2}} \right] \quad (1)$$

In this case, since the ideal op-amp circuit will result in  $V_O = -(V_1 + 2V_2)$ , we know  $R_f/R_1 = 1$  and  $R_f/R_2 = 2$ . Thus,

$$1 + \frac{R_f}{R_1 \parallel R_2} = 1 + \frac{R_f}{R_1} + \frac{R_f}{R_2} = 4$$

Substitution into (1) yields the final result,

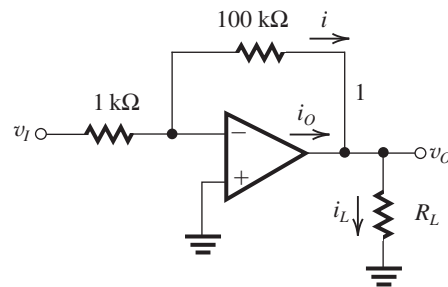
$$V_O = -(V_1 + 2V_2) \frac{A/4}{1 + A/4}$$

**2.121** The peak value of the largest possible sine wave that can be applied at the input without

output clipping is  $\frac{\pm 14 \text{ V}}{100} = 0.14 \text{ V} = 140 \text{ mV}$ .

Thus the rms value =  $\frac{140}{\sqrt{2}} \simeq 100 \text{ mV}$

**2.122**



(a)  $R_L = 1 \text{ k}\Omega$

$$\text{for } v_{O\max} = 10 \text{ V: } V_p = \frac{10}{100}$$

$$V_p = 0.1 \text{ V}$$

When the output is at its peak,

$$i_L = \frac{10}{1 \text{ k}\Omega} = 10 \text{ mA}$$

$$i = \frac{-10}{100 \text{ k}\Omega} = -0.1 \text{ mA; therefore}$$

$$i_O = 10 + 0.1 = 10.1 \text{ mA is just over } i_{O\max} = 10 \text{ mA.}$$

Therefore, the input amplitude must be reduced slightly to ensure an undistorted output. With  $V_p = 99 \text{ mV}$ , the output peak is  $v_o = 9.9 \text{ V} < v_{O\max}$  and the output current is

$$i_O = 9.9 + \frac{9.9}{100} = 10 \text{ mA} = i_{O\max}$$

At the negative peak of the output voltage,  $v_o = 9.9 \text{ V}$ ,  $i_L = -9.9 \text{ mA}$ ,  $i = 0.099 \text{ mA}$ , and  $i_O = -10 \text{ mA} = i_{O\max}$ , the maximum allowed.

$$(b) R_L = 200 \Omega$$

If output is at its peak:

$$i_L = \frac{10 \text{ V}}{0.2} = 50 \text{ mA}$$

which exceeds  $i_{O\max} = 20 \text{ mA}$ . Therefore  $v_o$  cannot go as high as  $10 \text{ V}$ . Instead:

$$10 \text{ mA} = \frac{v_o}{200 \Omega} + \frac{v_o}{100 \text{ k}\Omega} \Rightarrow v_o = \frac{10}{5.01} \simeq 2 \text{ V}$$

$$V_p = \frac{2}{100} = 0.02 \text{ V} = 20 \text{ mV}$$

$$(c) R_L = ?, i_{O\max} = 10 \text{ mA} = \frac{10 \text{ V}}{R_{L\min}} + \frac{10 \text{ V}}{100 \text{ k}\Omega}$$

$$10 - 0.1 = \frac{10}{R_{L\min}} \Rightarrow R_{L\min} = 1.01 \text{ k}\Omega$$

**2.123** Op-amp slew rate =  $1.2 \text{ V}/\mu\text{s}$ .

For the input pulse to rise  $2 \text{ V}$ , it will take

$$\frac{2}{1.2} = 1.67 \mu\text{s}.$$

$\therefore$  The minimum pulse width =  $W = 1.67 \mu\text{s}$

The output will be a triangular with  $2\text{-V}$  peak and  $1.2 \text{ V}/\mu\text{s}$  slopes.

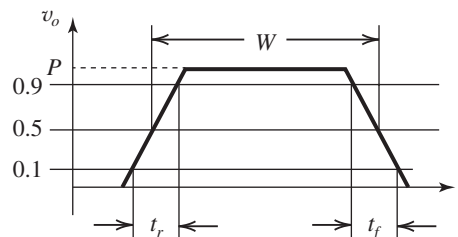
**2.124**

$$W = 2 \mu\text{s}$$

$$t_r + t_f = 0.2 W = 0.4 \mu\text{s}$$

$$t_r = t_f = 0.2 \mu\text{s}$$

$$\text{SR} = \frac{(0.9 - 0.1)P}{t_r} = \frac{0.8 \times 10}{0.2} = 40 \text{ V}/\mu\text{s}$$



**2.125** (a) This inverting amplifier has a 3-dB frequency given by Eq. (2.51)

$$f_{3\text{dB}} = \frac{f_i}{1 + 5} = 0.5 \text{ MHz}$$

The frequency at which the amplifier gain drops by 1 dB is found from

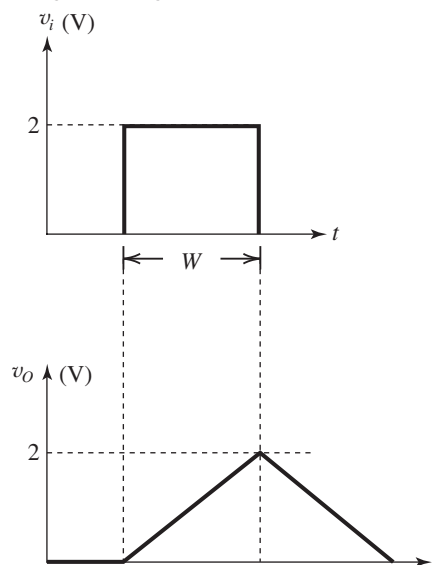
$$\left[ \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\text{dB}}}\right)^2}} \right] = 10^{-1/20} = 0.891$$

$$\left(\frac{f}{f_{3\text{dB}}}\right)^2 = 0.2589$$

$$\Rightarrow f = \sqrt{0.2589} f_{3\text{dB}} = 254 \text{ kHz}$$

(b) An output sinusoid with peak amplitude  $V_o$  at an angular frequency  $\omega$  has a maximum rate-of-change of  $\omega V_o$ . Equating this with the op-amp slew rate at  $\omega = 2\pi \times 254 \text{ kHz}$  gives the maximum amplitude of an undistorted sinewave at that frequency,

This figure belongs to Problem 2.123.



$$2\pi \times 2.54 \times 10^5 V_{o\max} = 10^6$$

$$\Rightarrow V_{o\max} = \frac{10^6}{2\pi \times 2.54 \times 10^5} = 0.627 \text{ V}$$

(c) At the peak of the output sinusoid, the feedback resistor (having a value of  $5 \times 20 = 100 \text{ k}\Omega$ ) will draw only  $0.627/100 = 0.006 \text{ mA}$ . This is negligible compared to the op-amp output current limit of  $\pm 1 \text{ mA}$ . Thus, we focus on the current required to charge and discharge the load capacitance at the output zero-crossings, where the op amp is on the verge of slew rate limiting at  $1 \text{ V/s}$ . The required current is,

$$C \times \frac{dv_O}{dt} = C \times 10^6$$

Equating this with the op-amp output current limit gives the largest capacitive load that can be driven in this way,

$$C_{\max} \times 10^6 = 10^{-3} \Rightarrow C_{\max} = 10^{-9} = 1 \text{ nF}$$

$$\mathbf{2.126} \quad v_O = 10 \sin \omega t \Rightarrow \frac{dv_O}{dt}$$

$$= 10\omega \cos \omega t \Rightarrow \left. \frac{dv_O}{dt} \right|_{\max}$$

$$= 10\omega$$

The highest frequency at which this output is possible is that for which

$$\left. \frac{dv_O}{dt} \right|_{\max} = \text{SR} \Rightarrow 10\omega_{\max} = 40 \times 10^6 \Rightarrow \omega_{\max}$$

$$= 4 \times 10^6 \text{ rad/s}$$

$$\Rightarrow f_{\max} = 637 \text{ kHz}$$

$$\mathbf{2.127} \quad (\text{a}) \quad V_i = 0.5, \quad V_o = 10 \times 0.5 = 5 \text{ V}$$

Output distortion will be due to slew-rate limitation and will occur at the frequency for

$$\text{which } \left. \frac{dv_O}{dt} \right|_{\max} = \text{SR}$$

$$\omega_{\max} \times 5 = 10 \times 10^6$$

$$\omega_{\max} = 2 \times 10^6 \text{ rad/s and } f_{\max} = 318.3 \text{ kHz}$$

(b) The output will distort at the value of  $V_i$  that results in  $\left. \frac{dv_O}{dt} \right|_{\max} = \text{SR}$ .

$$\omega \times 10V_{i\max} = \text{SR}$$

$$V_{i\max} = \frac{10 \times 10^6}{2\pi \times 200 \times 10^3 \times 10} = 0.795 \text{ V}$$

$$(\text{c}) \quad V_i = 50 \text{ mV} \quad V_o = 500 \text{ mV} = 0.5 \text{ V}$$

Slew rate begins at the frequency for which  $\omega \times 0.5 = \text{SR}$

$$\Rightarrow f = \frac{10 \times 10^6}{2\pi \times 0.5} = 3.18 \text{ MHz}$$

However, the small-signal 3-dB frequency is

$$f_{3\text{dB}} = \frac{f_i}{1 + \frac{R_2}{R_1}} = \frac{20 \times 10^6}{10} = 2 \text{ MHz}$$

Thus the useful frequency range is limited to 2 MHz.

(d) For  $f = 50 \text{ kHz}$ , the slew-rate limitation occurs at the value of  $V_i$  given by

$$\omega_i \times 10V_i = \text{SR} \Rightarrow V_i = \frac{10 \times 10^6}{2\pi \times 50 \times 10^3 \times 10} = 3.18 \text{ V}$$

Such an input voltage, however, would ideally result in an output of 31.8 V, which exceeds  $V_{O\max}$ .

$$\text{Thus } V_{i\max} = \frac{V_{O\max}}{10} = 1 \text{ V peak.}$$



**Chapter 3**

**Solutions to Exercises within the Chapter**

**Ex: 3.1** Refer to Fig. 3.3(a). For  $v_I \geq 0$ , the diode conducts and presents a zero voltage drop. Thus  $v_O = v_I$ . For  $v_I < 0$ , the diode is cut off, zero current flows through  $R$ , and  $v_O = 0$ . The result is the transfer characteristic in Fig. E3.1.

**Ex: 3.2** See Fig. 3.3(a) and 3.3(b) During the positive half of the sinusoid, the diode is forward biased, so it conducts resulting in  $v_D = 0$ . During the negative half cycle of the input signal  $v_I$ , the diode is reverse biased. The diode does not conduct, resulting in no current flowing in the circuit. So  $v_O = 0$  and  $v_D = v_I - v_O = v_I$ . This results in the waveform shown in Fig. E3.2.

**Ex: 3.3**  $\hat{i}_D = \frac{\hat{v}_I}{R} = \frac{10 \text{ V}}{5 \text{ k}\Omega} = 2 \text{ mA}$

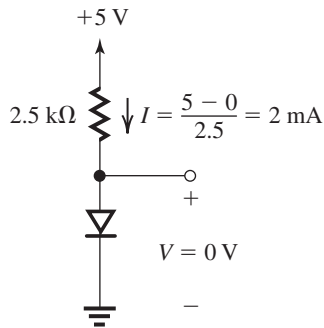
dc component of  $v_O = \frac{1}{\pi} \hat{v}_O$

$= \frac{1}{\pi} \hat{v}_I = \frac{10}{\pi}$

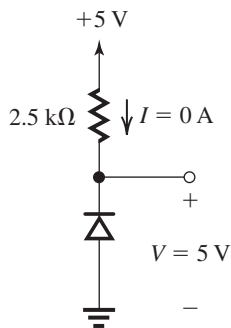
$= 3.18 \text{ V}$

**Ex: 3.4**

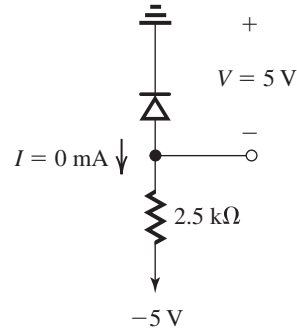
(a)



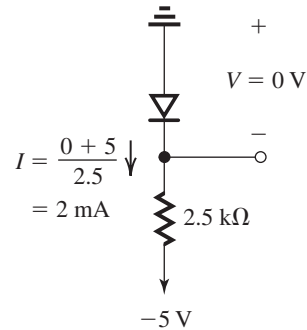
(b)



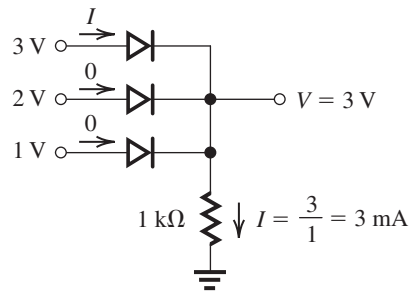
(c)



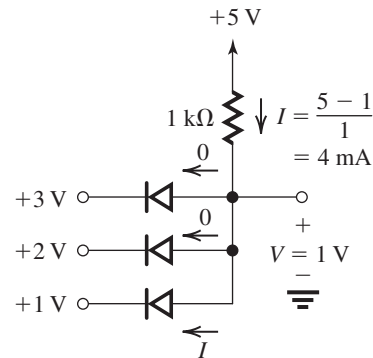
(d)



(e)



(f)



**Ex: 3.5**  $V_{\text{avg}} = \frac{10}{\pi}$

$50 + R = \frac{10}{\frac{\pi}{1 \text{ mA}}} = \frac{10}{\pi} \text{ k}\Omega$

$\therefore R = 3.133 \text{ k}\Omega$

Exercise 3-2

**Ex: 3.6** The maximum current arises when  $|v_f| = 20$  V. In this case,

$$i_D = \frac{20 - 5}{R}$$

To ensure this is 50 mA,

$$R = \frac{20 - 5}{50} = 0.3 \text{ k}\Omega = 300\Omega$$

**Ex: 3.7 Equation (3.5)**

$$V_2 - V_1 = 2.3 V_T \log\left(\frac{I_2}{I_1}\right)$$

At room temperature  $V_T = 25$  mV

$$\begin{aligned} V_2 - V_1 &= 2.3 \times 25 \times 10^{-3} \times \log\left(\frac{10}{0.1}\right) \\ &= 120 \text{ mV} \end{aligned}$$

**Ex: 3.8**  $i = I_S e^{v/V_T}$

$$1 \text{ (mA)} = I_S e^{0.7/V_T} \quad (1)$$

Dividing (1) by (2), we obtain

$$i \text{ (mA)} = e^{(v-0.7)/V_T}$$

$$\Rightarrow v = 0.7 + 0.025 \ln(i)$$

where  $i$  is in mA. Thus,

for  $i = 0.1$  mA,

$$v = 0.7 + 0.025 \ln(0.1) = 0.64 \text{ V}$$

and for  $i = 10$  mA,

$$v = 0.7 + 0.025 \ln(10) = 0.76 \text{ V}$$

**Ex: 3.9**  $i_D = I_S e^{v/V_T}$

$$\Rightarrow I_S = i_D e^{-v/V_T} = 0.25 \times e^{-300/25}$$

$$= 1.23 \times 10^6 \text{ mA} = 1.23 \times 10^{-9} \text{ A}$$

**Ex: 3.10**  $\Delta T = 125 - 25 = 100^\circ\text{C}$

$$I_S = 10^{-14} \times 1.15^{\Delta T}$$

$$= 1.17 \times 10^{-8} \text{ A}$$

**Ex: 3.11** At  $20^\circ\text{C}$   $I = \frac{1 \text{ V}}{1 \text{ M}\Omega} = 1 \mu\text{A}$

Since the reverse leakage current doubles for every  $10^\circ\text{C}$  increase, at  $40^\circ\text{C}$

$$I = 4 \times 1 \mu\text{A} = 4 \mu\text{A}$$

$$\Rightarrow V = 4 \mu\text{A} \times 1 \text{ M}\Omega = 4.0 \text{ V}$$

$$\text{@ } 0^\circ\text{C} \quad I = \frac{1}{4} \mu\text{A}$$

$$\Rightarrow V = \frac{1}{4} \times 1 = 0.25 \text{ V}$$

**Ex: 3.12** a. Use iteration:

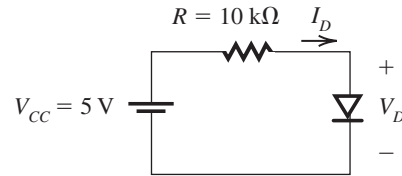
Diode has 0.7 V drop at 1 mA current.

Assume  $V_D = 0.7$  V

$$I_D = \frac{5 - 0.7}{10 \text{ k}\Omega} = 0.43 \text{ mA}$$

Use Eq. (3.5) and note that

$$V_1 = 0.7 \text{ V}, \quad I_1 = 1 \text{ mA}$$



$$V_2 - V_1 = 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

$$V_2 = V_1 + 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

First iteration

$$\begin{aligned} V_2 &= 0.7 + 2.3 \times 25 \times 10^{-3} \log\left(\frac{0.43}{1}\right) \\ &= 0.679 \text{ V} \end{aligned}$$

Second iteration

$$I_2 = \frac{5 - 0.679}{10 \text{ k}\Omega} = 0.432 \text{ mA}$$

$$V_2 = 0.7 + 2.3 \times 25.3 \times 10^{-3} \log\left(\frac{0.432}{1}\right)$$

$$= 0.679 \text{ V} \simeq 0.68 \text{ V}$$

we get almost the same voltage.

∴ The iteration yields

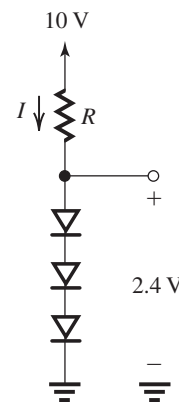
$$I_D = 0.43 \text{ mA}, \quad V_D = 0.68 \text{ V}$$

b. Use constant voltage drop model:

$$V_D = 0.7 \text{ V} \quad \text{constant voltage drop}$$

$$I_D = \frac{5 - 0.7}{10 \text{ k}\Omega} = 0.43 \text{ mA}$$

**Ex: 3.13**



Diodes have 0.7 V drop at 1 mA

$$\therefore 1 \text{ mA} = I_S e^{0.7/V_T} \quad (1)$$

At a current  $I$  (mA),

$$I = I_S e^{V_D/V_T} \quad (2)$$

Using (1) and (2), we obtain

$$I = e^{(V_D - 0.7)/V_T}$$

For an output voltage of 2.4 V, the voltage drop across each diode =  $\frac{2.4}{3} = 0.8 \text{ V}$

Now  $I$ , the current through each diode, is

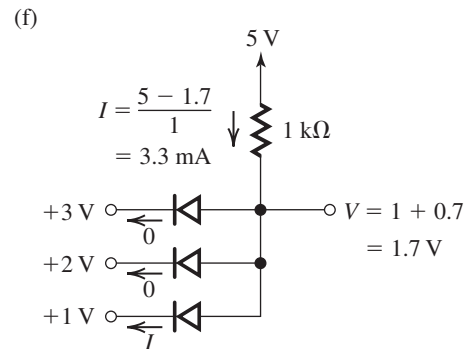
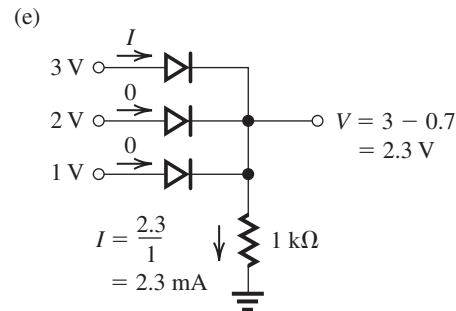
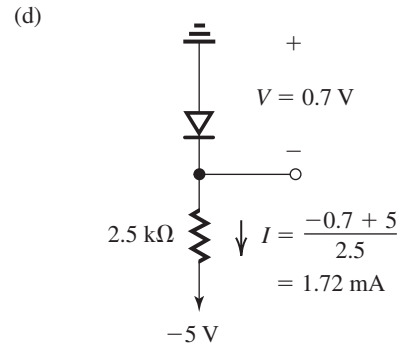
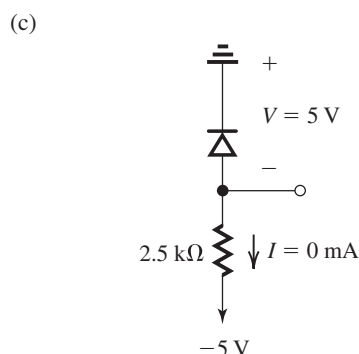
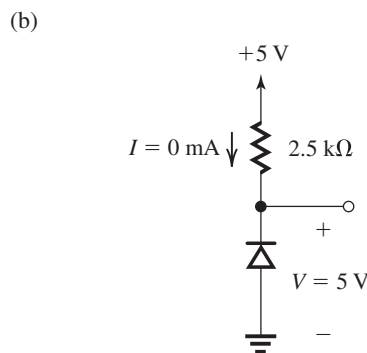
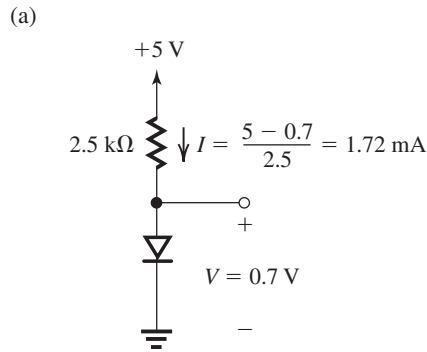
$$I = e^{(0.8 - 0.7)/0.025}$$

$$= 54.6 \text{ mA}$$

$$R = \frac{10 - 2.4}{54.6 \times 10^{-3}}$$

$$= 139 \Omega$$

**Ex: 3.14**



**Ex: 3.15** With a reverse voltage, the diode in Fig. 3.10 will not conduct. Thus, the voltage drop on  $R$  will be zero, and the reverse voltage on the diode is  $-V_D = -V_{DD}$ . To ensure we respect the peak inverse voltage, we require  $V_{DD} = V_D > -30 \text{ V}$ . Hence, the minimum voltage on  $V_{DD}$  is  $-30 \text{ V}$ .

**Ex: 3.16** When conducting a reverse current of 20 mA, the reverse voltage is

$$\begin{aligned} V_Z &= V_{ZT} + \Delta I_Z r_z = \\ &= 3.5 \text{ V} + (20 \text{ mA} - 10 \text{ mA}) 10 \Omega \\ &= 3.6 \text{ V} \end{aligned}$$

The maximum current,  $I_{\max}$ , is the current at 200 mW power dissipation.

$$\begin{aligned} 200 \text{ mW} &= I_{\max} (3.5 \text{ V} + 10 \Omega (I_{\max} - 10 \text{ mA})) \\ 0.2 &= 3.5 I_{\max} + 10 I_{\max}^2 - 0.1 I_{\max} \end{aligned}$$

$$\Rightarrow I_{\max}^2 + 0.34I_{\max} - 0.02 = 0$$

$$\Rightarrow I_{\max} = \frac{1}{2} \left( -0.34 + \sqrt{0.34^2 + 4 \times 0.02} \right)$$

$$= 51 \text{ mA}$$

**Ex: 3.17**  $r_d = \frac{V_T}{I_D}$

$$I_D = 0.1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{0.1 \times 10^{-3}} = 250 \Omega$$

$$I_D = 1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{1 \times 10^{-3}} = 25 \Omega$$

$$I_D = 10 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{10 \times 10^{-3}} = 2.5 \Omega$$

**Ex: 3.18** For small signal model,

$$\Delta i_D = \Delta v_D / r_d \tag{1}$$

where  $r_d = \frac{V_T}{I_D}$

For exponential model,

$$i_D = I_S e^{V/V_T}$$

$$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1) / V_T} = e^{\Delta v_D / V_T}$$

$$\Delta i_D = i_{D2} - i_{D1} = i_{D1} e^{\Delta v_D / V_T} - i_{D1}$$

$$= i_{D1} (e^{\Delta v_D / V_T} - 1) \tag{2}$$

In this problem,  $i_{D1} = I_D = 1 \text{ mA}$ .

Using Eqs. (1) and (2) with  $V_T = 25 \text{ mV}$ , we obtain

	$\Delta v_D$ (mV)	$\Delta i_D$ (mA) small signal	$\Delta i_D$ (mA) exponential model
a	-10	-0.4	-0.33
b	-5	-0.2	-0.18
c	+5	+0.2	+0.22
d	+10	+0.4	+0.49

**Ex: 3.19**

(a) In this problem,  $\frac{\Delta V_O}{\Delta i_L} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20 \Omega$ .

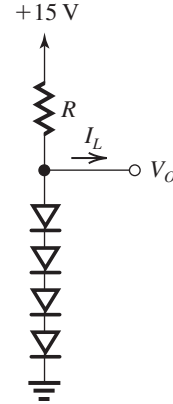
$\therefore$  Total small-signal resistance of the four diodes =  $20 \Omega$

$\therefore$  For each diode,  $r_d = \frac{20}{4} = 5 \Omega$ .

But  $r_d = \frac{V_T}{I_D} \Rightarrow 5 = \frac{25 \text{ mV}}{I_D}$ .

$\therefore I_D = 5 \text{ mA}$

and  $R = \frac{15 - 3}{5 \text{ mA}} = 2.4 \text{ k}\Omega$ .



(b) For  $V_O = 3 \text{ V}$ , voltage drop across each diode =  $\frac{3}{4} = 0.75 \text{ V}$

$$i_D = I_S e^{V/V_T}$$

$$I_S = \frac{i_D}{e^{V/V_T}} = \frac{5 \times 10^{-3}}{e^{0.75/0.025}} = 4.7 \times 10^{-16} \text{ A}$$

(c) If  $i_D = 5 - i_L = 5 - 1 = 4 \text{ mA}$ .

Across each diode the voltage drop is

$$V_D = V_T \ln \left( \frac{I_D}{I_S} \right)$$

$$= 25 \times 10^{-3} \times \ln \left( \frac{4 \times 10^{-3}}{4.7 \times 10^{-16}} \right)$$

$$= 0.7443 \text{ V}$$

Voltage drop across 4 diodes

$$= 4 \times 0.7443 = 2.977 \text{ V}$$

so change in  $V_O = 3 - 2.977 = 23 \text{ mV}$ .

**Ex: 3.20** When the diode current is halved, the voltage changes by

$$\Delta V_Z = r_z \Delta I_Z = 80 \Omega \times \frac{-5 \text{ mA}}{2} = -200 \text{ mV}$$

$$\Rightarrow V_Z = 6 - 0.2 = 5.8 \text{ V}$$

When the diode current is doubled,

$$\Delta V_Z = r_z \Delta I_Z = 80 \Omega \times 5 \text{ mA} = 400 \text{ mV}$$

$$\Rightarrow V_Z = 6 + 0.4 = 6.4 \text{ V}$$

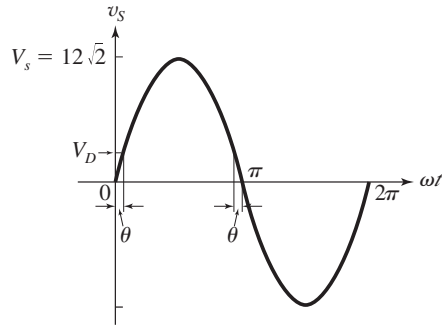
Finally, the value of  $V_{Z0}$  is that obtained by using the model at zero current.

$$V_Z = V_{Z0} + r_z I_Z$$

$$\Rightarrow V_{Z0} = V_Z - r_z I_Z = 6 \text{ V} - 80 \Omega \times 5 \text{ mA}$$

$$= 5.6 \text{ V}$$

**Ex: 3.21**



(a) The diode starts conduction at

$$v_s = V_D = 0.7 \text{ V}$$

$$v_s = V_s \sin \omega t, \text{ here } V_s = 12\sqrt{2}$$

At  $\omega t = \theta$ ,

$$v_s = V_s \sin \theta = V_D = 0.7 \text{ V}$$

$$12\sqrt{2} \sin \theta = 0.7$$

$$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \simeq 2.4^\circ$$

Conduction starts at  $\theta$  and stops at  $180 - \theta$ .

$$\therefore \text{Total conduction angle} = 180 - 2\theta = 175.2^\circ$$

$$(b) v_{O,\text{avg}} = \frac{1}{2\pi} \int_{\theta}^{(\pi-\theta)} (V_s \sin \phi - V_D) d\phi$$

$$= \frac{1}{2\pi} [-V_s \cos \phi - V_D \phi]_{\phi=\theta}^{\phi=\pi-\theta}$$

$$= \frac{1}{2\pi} [V_s \cos \theta - V_s \cos(\pi - \theta) - V_D(\pi - 2\theta)]$$

But  $\cos \theta \simeq 1$ ,  $\cos(\pi - \theta) \simeq -1$ , and

$$\pi - 2\theta \simeq \pi$$

$$v_{O,\text{avg}} = \frac{2V_s}{2\pi} - \frac{V_D}{2}$$

$$= \frac{V_s}{\pi} - \frac{V_D}{2}$$

For  $V_s = 12\sqrt{2}$  and  $V_D = 0.7 \text{ V}$

$$v_{O,\text{avg}} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05 \text{ V}$$

(c) The peak diode current occurs at the peak diode voltage.

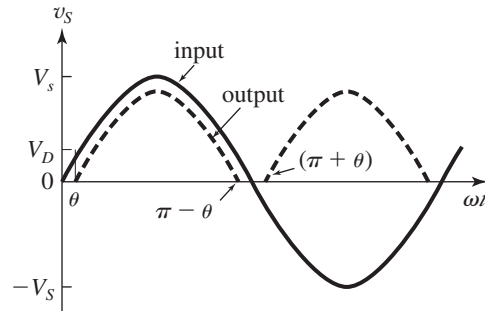
$$\therefore \hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

$$= 163 \text{ mA}$$

$$\text{PIV} = +V_s = 12\sqrt{2}$$

$$\simeq 17 \text{ V}$$

**Ex: 3.22**



(a) As shown in the diagram, the output is zero between  $(\pi - \theta)$  to  $(\pi + \theta)$

$$= 2\theta$$

Here  $\theta$  is the angle at which the input signal reaches  $V_D$ .

$$\therefore V_s \sin \theta = V_D$$

$$\theta = \sin^{-1}\left(\frac{V_D}{V_s}\right)$$

$$2\theta = 2 \sin^{-1}\left(\frac{V_D}{V_s}\right)$$

(b) Average value of the output signal is given by

$$V_O = \frac{1}{2\pi} \left[ 2 \times \int_{\theta}^{(\pi-\theta)} (V_s \sin \phi - V_D) d\phi \right]$$

$$= \frac{1}{\pi} [-V_s \cos \phi - V_D \phi]_{\phi=\theta}^{\phi=\pi-\theta}$$

$$\simeq 2 \frac{V_s}{\pi} - V_D, \quad \text{for } \theta \text{ small.}$$

(c) Peak current occurs when  $\phi = \frac{\pi}{2}$ .

Peak current

$$= \frac{V_s \sin(\pi/2) - V_D}{R} = \frac{V_s - V_D}{R}$$

If  $v_s$  is 12 V(rms),

$$\text{then } V_s = \sqrt{2} \times 12 = 12\sqrt{2}$$

$$\text{Peak current} = \frac{12\sqrt{2} - 0.7}{100} \simeq 163 \text{ mA}$$

Nonzero output occurs for angle  $= 2(\pi - 2\theta)$

The fraction of the cycle for which  $v_o > 0$  is

$$= \frac{2(\pi - 2\theta)}{2\pi} \times 100$$

$$= \frac{2 \left[ \pi - 2 \sin^{-1} \left( \frac{0.7}{12\sqrt{2}} \right) \right]}{2\pi} \times 100$$

$$\approx 97.4 \%$$

Average output voltage  $V_O$  is

$$V_O = 2 \frac{V_s}{\pi} - V_D = \frac{2 \times 12\sqrt{2}}{\pi} - 0.7 = 10.1 \text{ V}$$

Peak diode current  $\hat{i}_D$  is

$$\hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

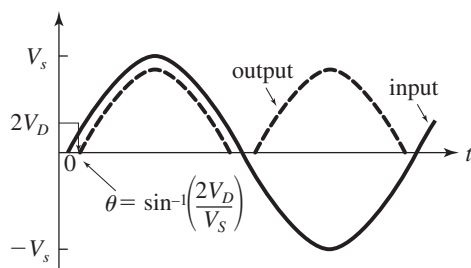
$$= 163 \text{ mA}$$

$$\text{PIV} = V_s - V_D + V_s$$

$$= 12\sqrt{2} - 0.7 + 12\sqrt{2}$$

$$= 33.2 \text{ V}$$

**Ex: 3.23**



$$\begin{aligned} \text{(a) } V_{O,\text{avg}} &= \frac{1}{2\pi} \int (V_s \sin \phi - 2V_D) d\phi \\ &= \frac{2}{2\pi} [-V_s \cos \phi - 2V_D \phi]_{\phi=\theta}^{\pi-\theta} \\ &= \frac{1}{\pi} [V_s \cos \theta - V_s \cos(\pi - \theta) - 2V_D(\pi - 2\theta)] \end{aligned}$$

But  $\cos \theta \approx 1$ ,

$$\cos(\pi - \theta) \approx -1$$

$$\pi - 2\theta \approx \pi. \text{ Thus}$$

$$\Rightarrow V_{O,\text{avg}} \approx \frac{2V_s}{\pi} - 2V_D$$

$$= \frac{2 \times 12\sqrt{2}}{\pi} - 1.4 = 9.4 \text{ V}$$

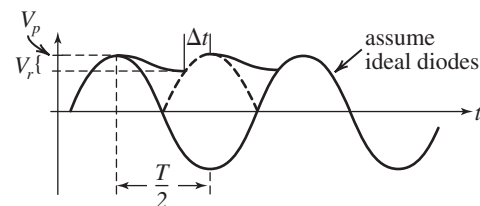
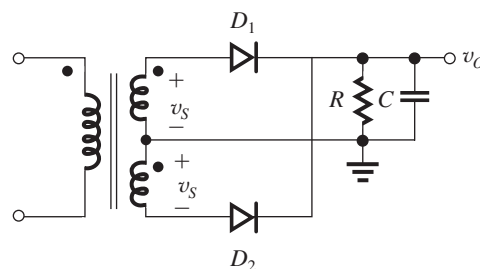
$$\text{(b) Peak diode current} = \frac{\text{Peak voltage}}{R}$$

$$= \frac{V_s - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$$

$$= 156 \text{ mA}$$

$$\text{PIV} = V_s - V_D = 12\sqrt{2} - 0.7 = 16.3 \text{ V}$$

**Ex: 3.24** Full-wave peak rectifier:



The ripple voltage is the amount of voltage reduction during capacitor discharge that occurs when the diodes are not conducting. The output voltage is given by

$$v_o = V_p e^{-t/RC}$$

$V_p - V_r = V_p e^{-\frac{T/2}{RC}}$  ← discharge is only half the period. We also assumed  $\Delta t \ll \frac{T}{2}$ .

$$V_r = V_p \left( 1 - e^{-\frac{T/2}{RC}} \right)$$

$$e^{-\frac{T/2}{RC}} \approx 1 - \frac{T/2}{RC}, \quad \text{for } CR \gg T/2$$

$$\text{Thus } V_r \approx V_p \left( 1 - 1 + \frac{T/2}{RC} \right)$$

$$V_r = \frac{V_p}{2fRC} \quad \text{(a) Q.E.D.}$$

To find the average diode current, note that the charge supplied to  $C$  during conduction is equal to the charge lost during discharge.

$$Q_{\text{SUPPLIED}} = Q_{\text{LOST}}$$

$$i_{C,\text{av}} \Delta t = CV_r \quad \text{SUB (a)}$$

$$(i_{D,\text{av}} - I_L) \Delta t = C \frac{V_p}{2fRC} = \frac{V_p}{2fR}$$

$$= \frac{V_p \pi}{\omega R}$$

$$i_{D,\text{av}} = \frac{V_p \pi}{\omega \Delta t R} + I_L$$

where  $\omega \Delta t$  is the conduction angle.

Note that the conduction angle has the same expression as for the half-wave rectifier and is given by Eq. (3.28),

$$\omega\Delta t \cong \sqrt{\frac{2V_r}{V_p}} \quad (b)$$

Substituting for  $\omega\Delta t$ , we get

$$\Rightarrow i_{D,av} = \frac{\pi V_p}{\sqrt{\frac{2V_r}{V_p}} \cdot R} + I_L$$

Since the output is approximately held at  $V_p$ ,

$$\frac{V_p}{R} \approx I_L \cdot \text{Thus}$$

$$\Rightarrow i_{D,av} \cong \pi I_L \sqrt{\frac{V_p}{2V_r}} + I_L$$

$$= I_L \left[ 1 + \pi \sqrt{\frac{V_p}{2V_r}} \right] \quad \text{Q.E.D.}$$

If  $t = 0$  is at the peak, the maximum diode current occurs at the onset of conduction or at  $t = -\omega\Delta t$ .

During conduction, the diode current is given by

$$i_D = i_C + i_L$$

$$i_{D,max} = C \left. \frac{dv_S}{dt} \right|_{t=-\omega\Delta t} + i_L$$

$$\text{assuming } i_L \text{ is const. } i_L \cong \frac{V_p}{R} = I_L$$

$$= C \frac{d}{dt} (V_p \cos \omega t) + I_L$$

$$= -C \sin \omega t \times \omega V_p + I_L$$

$$= -C \sin(-\omega\Delta t) \times \omega V_p + I_L$$

For a small conduction angle

$$\sin(-\omega\Delta t) \approx -\omega\Delta t. \text{ Thus}$$

$$\Rightarrow i_{D,max} = C\omega\Delta t \times \omega V_p + I_L$$

Sub (b) to get

$$i_{D,max} = C \sqrt{\frac{2V_r}{V_p}} \omega V_p + I_L$$

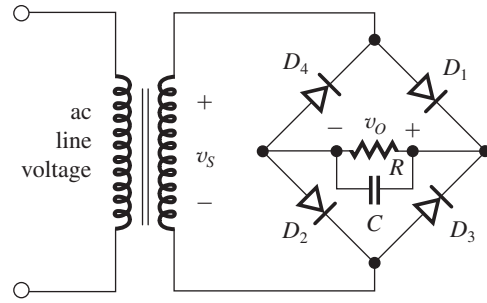
Substituting  $\omega = 2\pi f$  and using (a) together with  $V_p/R \cong I_L$  results in

$$i_{D,max} = I_L \left[ 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right] \quad \text{Q.E.D.}$$

**Ex: 3.25**

The output voltage,  $v_O$ , can be expressed as

$$v_O = (V_p - 2V_D) e^{-t/RC}$$



At the end of the discharge interval

$$v_O = V_p - 2V_D - V_r$$

The discharge occurs almost over half of the time period  $\cong T/2$ .

For time constant  $RC \gg \frac{T}{2}$

$$e^{-t/RC} \cong 1 - \frac{T}{2} \times \frac{1}{RC}$$

$$\therefore V_p - 2V_D - V_r = (V_p - 2V_D) \left( 1 - \frac{T}{2} \times \frac{1}{RC} \right)$$

$$\Rightarrow V_r = (V_p - 2V_D) \times \frac{T}{2RC}$$

Here  $V_p = 12\sqrt{2}$  and  $V_r = 1$  V

$$V_D = 0.8 \text{ V}$$

$$T = \frac{1}{f} = \frac{1}{60} \text{ s}$$

$$1 = (12\sqrt{2} - 2 \times 0.8) \times \frac{1}{2 \times 60 \times 100 \times C}$$

$$C = \frac{(12\sqrt{2} - 1.6)}{2 \times 60 \times 100} = 1281 \mu\text{F}$$

Without considering the ripple voltage, the dc output voltage

$$= 12\sqrt{2} - 2 \times 0.8 = 15.4 \text{ V}$$

If ripple voltage is included, the output voltage is

$$= 12\sqrt{2} - 2 \times 0.8 - \frac{V_r}{2} = 14.9 \text{ V}$$

$$I_L = \frac{14.9}{100 \Omega} \cong 0.15 \text{ A}$$

The conduction angle  $\omega\Delta t$  can be obtained using Eq. (3.28) but substituting  $V_p = 12\sqrt{2} - 2 \times 0.8$ :

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 1}{12\sqrt{2} - 2 \times 0.8}}$$

$$= 0.36 \text{ rad} = 20.7^\circ$$

The average and peak diode currents can be calculated using Eqs. (3.31) and (3.32):

$$i_{D\text{av}} = I_L \left( 1 + \pi \sqrt{\frac{V_p}{2V_r}} \right), \text{ where } I_L = \frac{14.9 \text{ V}}{100 \Omega},$$

$$V_p = 12\sqrt{2} - 2 \times 0.8, \text{ and } V_r = 1 \text{ V}; \text{ thus}$$

$$i_{D\text{av}} = 1.45 \text{ A}$$

$$i_{D\text{max}} = I \left( 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right)$$

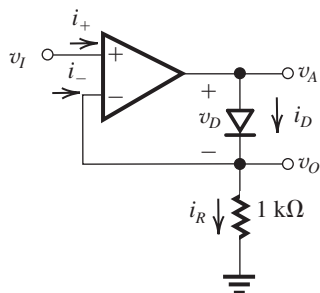
$$= 2.76 \text{ A}$$

PIV of the diodes

$$= V_S - V_{D0} = 12\sqrt{2} - 0.8 = 16.2 \text{ V}$$

To provide a safety margin, select a diode capable of a peak current of 3.5 to 4A and having a PIV rating of 20 V.

**Ex: 3.26**



The diode has 0.7 V drop at 1 mA current.

$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{1 \text{ mA}} = e^{(v_D - 0.7)/V_T}$$

$$\Rightarrow v_D = V_T \ln\left(\frac{i_D}{1 \text{ mA}}\right) + 0.7 \text{ V}$$

$$\text{For } v_I = 10 \text{ mV}, \quad v_O = v_I = 10 \text{ mV}$$

It is an ideal op amp, so  $i_+ = i_- = 0$ .

$$\therefore i_D = i_R = \frac{10 \text{ mV}}{1 \text{ k}\Omega} = 10 \mu\text{A}$$

$$v_D = 25 \times 10^{-3} \ln\left(\frac{10 \mu\text{A}}{1 \text{ mA}}\right) + 0.7 = 0.58 \text{ V}$$

$$v_A = v_D + 10 \text{ mV}$$

$$= 0.58 + 0.01$$

$$= 0.59 \text{ V}$$

$$\text{For } v_I = 1 \text{ V}$$

$$v_O = v_I = 1 \text{ V}$$

$$i_D = \frac{v_O}{1 \text{ k}\Omega} = \frac{1}{1 \text{ k}\Omega} = 1 \text{ mA}$$

$$v_D = 0.7 \text{ V}$$

$$V_A = 0.7 \text{ V} + 1 \text{ k}\Omega \times 1 \text{ mA}$$

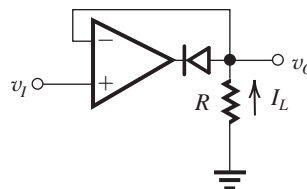
$$= 1.7 \text{ V}$$

For  $v_I = -1 \text{ V}$ , the diode is cut off.

$$\therefore v_O = 0 \text{ V}$$

$$v_A = -12 \text{ V}$$

**Ex: 3.27**



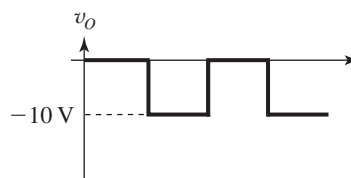
$v_I > 0 \sim$  diode is cut off, loop is open, and the opamp is saturated:

$$v_O = 0 \text{ V}$$

$v_I < 0 \sim$  diode conducts and closes the negative feedback loop:

$$v_O = v_I$$

**Ex: 3.28** Reversing the diode results in the peak output voltage being clamped at 0 V:



Here the dc component of  $v_O = V_O = -5 \text{ V}$

**Ex: 3.29** The capacitor voltage accounts for the shift of the voltage waveforms from  $v_I$  to  $v_O$ . Thus, from Fig. 3.30, we see the capacitor voltage is  $V_p + V_{CC} = 8 \text{ V}$ . The diode's peak inverse voltage arises at the peaks of  $v_O$ ,

$$\text{PIV} = v_{O\text{max}} - V_{CC}$$

$$= V_{CC} + 2V_p - V_{CC}$$

$$= 2V_p = 6 \text{ V}$$

**Ex: 3.30**  $C_{j0} = 100 \text{ fF}$ ,  $V_0 = 3 \text{ V}$ ,  $m = 3$ . Using Equation (3.47),

$$\text{at } V_R = 1 \text{ V: } C_j = \frac{100 \text{ fF}}{\left(1 + \frac{1}{3}\right)^3} = 42.2 \text{ fF, and}$$

$$\text{at } V_R = 3 \text{ V: } C_j = \frac{100 \text{ fF}}{\left(1 + \frac{3}{3}\right)^3} = 12.5 \text{ fF.}$$

**Ex: 3.31** The reverse current is



Exercise 3-9

$$-i_D = I_D + i_P = I_D + R \times P$$

At an incident light power of  $P = 1 \text{ mW}$ ,

$$-i_D = 10^{-4} + 0.5 \times 1 = 0.5 \text{ mA}$$

At an incident light power of  $P = 1 \mu\text{W}$ ,

$$-i_D = 10^{-4} + 0.5 \times 10^{-3} = 6 \times 10^{-4} \text{ mA} = 0.6 \mu\text{A}$$

**Ex: 3.32** Neglecting dark current,

$$i_P = R \times P$$

$t$

$$10^{-6} = 0.3 \times 0.01 \times A$$

$$\Rightarrow A = \frac{10^{-6}}{0.3 \times 10^{-2}} = 3.33 \times 10^{-4} \text{ m}^2 = 3.33 \text{ cm}^2$$

The capacitance is  $10 \text{ pF per mm}^2$  or, equivalently,  $1 \text{ nF per cm}^2$ . Thus,

$$C_j = 1 \times 3.33 = 3.33 \text{ nF}$$

**Ex: 3.33**

$$R = \frac{9 - 3 \times 1.8}{20} = 0.18 \text{ k}\Omega = 180\Omega$$

**Ex: 3.34**

$$R = \frac{9 - 3 \times 2.2}{20} = 0.12 \text{ k}\Omega = 120\Omega$$

**Solutions to End-of-Chapter Problems**

**3.1** Refer to Fig. P3.1.

(a) Diode is conducting, thus

$$V = -3 \text{ V}$$

$$I = \frac{+3 - (-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

(b) Diode is reverse biased, thus

$$I = 0$$

$$V = +3 \text{ V}$$

(c) Diode is conducting, thus

$$V = +3 \text{ V}$$

$$I = \frac{+3 - (-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

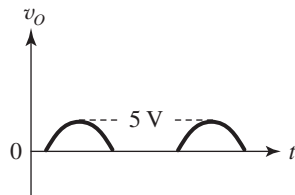
(d) Diode is reverse biased, thus

$$I = 0$$

$$V = -3 \text{ V}$$

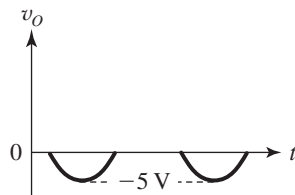
**3.2**

(a)



$$V_{p+} = 5 \text{ V} \quad V_{p-} = 0 \text{ V} \quad f = 1 \text{ kHz}$$

(b)



$$V_{p+} = 0 \text{ V} \quad V_{p-} = -5 \text{ V} \quad f = 1 \text{ kHz}$$

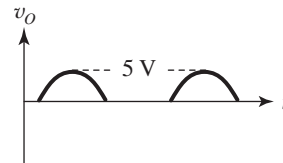
(c)



$$v_o = 0 \text{ V}$$

Neither  $D_1$  nor  $D_2$  conducts, so there is no output.

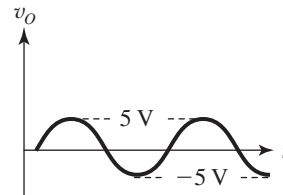
(d)



$$V_{p+} = 5 \text{ V}, \quad V_{p-} = 0 \text{ V}, \quad f = 1 \text{ kHz}$$

Both  $D_1$  and  $D_2$  conduct when  $v_i > 0$

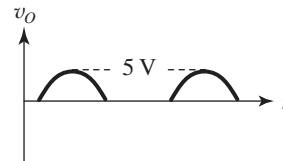
(e)



$$V_{p+} = 5 \text{ V}, \quad V_{p-} = -5 \text{ V}, \quad f = 1 \text{ kHz}$$

$D_1$  conducts when  $v_i > 0$  and  $D_2$  conducts when  $v_i < 0$ . Thus the output follows the input.

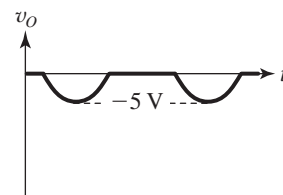
(f)



$$V_{p+} = 5 \text{ V}, \quad V_{p-} = 0 \text{ V}, \quad f = 1 \text{ kHz}$$

$D_1$  is cut off when  $v_i < 0$

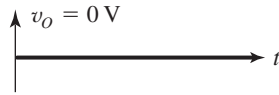
(g)



$$V_{p+} = 0 \text{ V}, \quad V_{p-} = -5 \text{ V}, \quad f = 1 \text{ kHz}$$

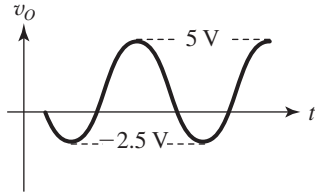
$D_1$  shorts to ground when  $v_i > 0$  and is cut off when  $v_i < 0$  whereby the output follows  $v_i$ .

(h)



$v_o = 0 \text{ V}$  ~ The output is always shorted to ground as  $D_1$  conducts when  $v_I > 0$  and  $D_2$  conducts when  $v_I < 0$ .

(i)



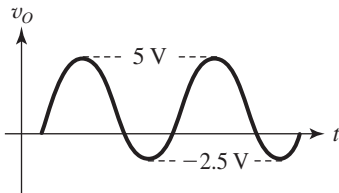
$V_{p+} = 5 \text{ V}$ ,  $V_{p-} = -2.5 \text{ V}$ ,  $f = 1 \text{ kHz}$

When  $v_I > 0$ ,  $D_1$  is cut off and  $v_o$  follows  $v_I$ .

When  $v_I < 0$ ,  $D_1$  is conducting and the circuit becomes a voltage divider where the negative peak is

$$\frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ k}\Omega} \times -5 \text{ V} = -2.5 \text{ V}$$

(j)

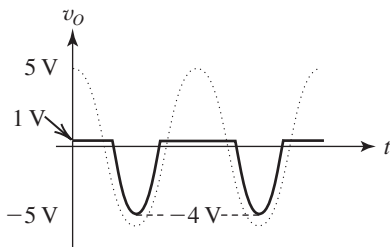


$V_{p+} = 5 \text{ V}$ ,  $V_{p-} = -2.5 \text{ V}$ ,  $f = 1 \text{ kHz}$

When  $v_I > 0$ , the output follows the input as  $D_1$  is conducting.

When  $v_I < 0$ ,  $D_1$  is cut off and the circuit becomes a voltage divider.

(k)



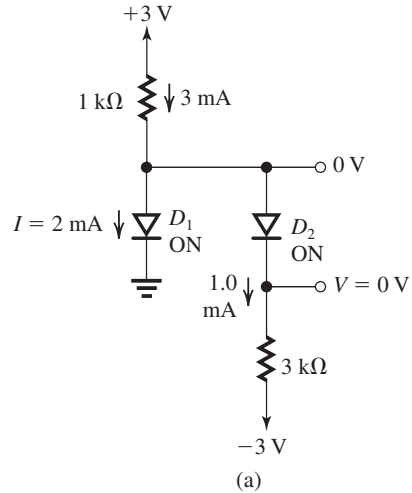
$V_{p+} = 1 \text{ V}$ ,  $V_{p-} = -4 \text{ V}$ ,  $f = 1 \text{ kHz}$

When  $v_I > 0$ ,  $D_1$  is cut off and  $D_2$  is conducting. The output becomes 1 V.

When  $v_I < 0$ ,  $D_1$  is conducting and  $D_2$  is cut off. The output becomes:

$$v_o = v_I + 1 \text{ V}$$

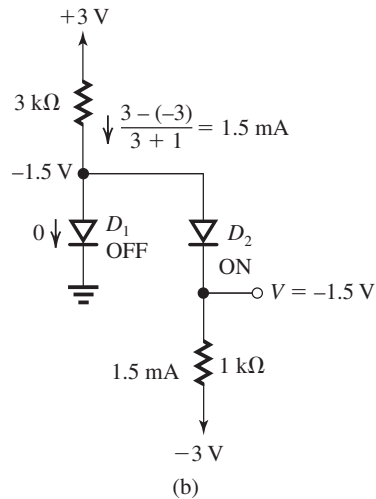
3.3



(a) If we assume that both  $D_1$  and  $D_2$  are conducting, then  $V = 0 \text{ V}$  and the current in  $D_2$  will be  $[0 - (-3)]/3 = 1 \text{ mA}$ . The current in the  $1 \text{ k}\Omega$  will be  $(3 - 0)/1 = 3 \text{ mA}$ . A node equation at the common anodes node yields a 2 mA current in  $D_1$ . Thus, we obtain the results shown in Fig. (a):

$$I = 2 \text{ mA}$$

$$V = 0 \text{ V}$$



(b) In (b), the two resistors are interchanged. With some reasoning, we can see that with  $D_2$

conducting, the intermediate node  $V$  will be below 0 V due to the voltage division between the two resistors. Thus,  $D_1$  is off giving the results shown in Fig. (b):

$$I = 0 \text{ mA}$$

$$V = -1.5 \text{ V}$$

$$3.4 \quad R \geq \frac{120\sqrt{2}}{40} \geq 4.2 \text{ k}\Omega$$

The largest reverse voltage appearing across the diode is equal to the peak input voltage:

$$120\sqrt{2} = 169.7 \text{ V}$$

3.5 The maximum reverse voltage arises when

$$v_I = V_+ - 10 = 3 - 10 = 7 \text{ V}$$

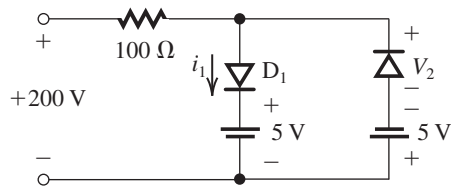
The maximum forward current arises when

$$v_I = I_{\max} \times R + V_+ = 50 \times 0.1 + 3 = 8 \text{ V}$$

Thus, the safe operating range is

$$-7 \text{ V} \leq v_I \leq 8 \text{ V}.$$

3.6



Assuming ideal diodes with  $v_I = +200 \text{ V}$ , the diode  $D_1$  is conducting with zero voltage drop across it. The peak current is thus

$$i_1 = \frac{200 - 5}{100} = 1.95 \text{ A}$$

and the peak voltage is

$$v_2 = 5 - (-5) = 10 \text{ V}$$

3.7

(a) The diode is reverse biased, thus

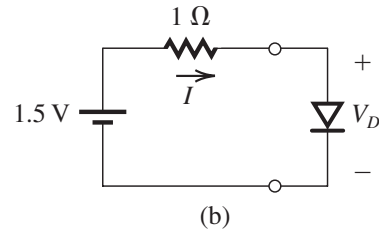
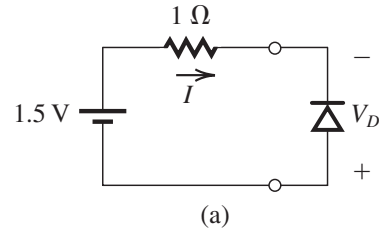
$$I = 0 \text{ A}$$

$$V_D = -1.5 \text{ V}$$

(b) The diode is forward biased, thus

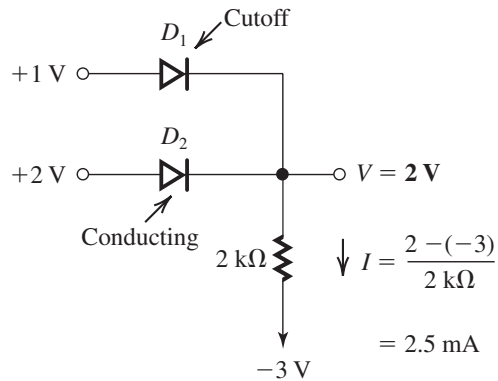
$$V_D = 0 \text{ V}$$

$$I = \frac{1.5 \text{ V}}{1 \Omega} = 1.5 \text{ A}$$

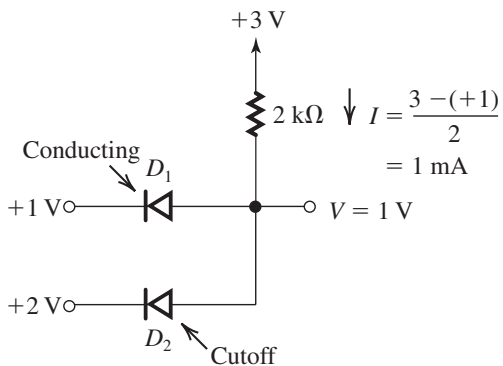


3.8

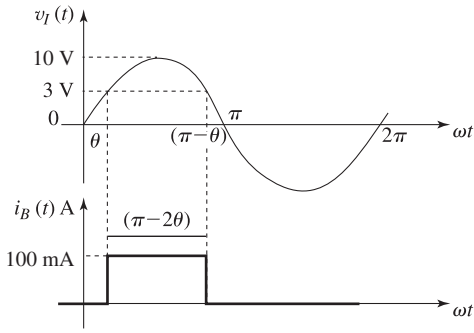
(a)



(b)



3.9



From Fig. P4.5 we see that when  $v_I < V_B$ ; that is,  $v_I < 3\text{ V}$ ,  $D_1$  will be conducting the current  $I$  and  $i_B$  will be zero. When  $v_I$  exceeds the battery voltage ( $3\text{ V}$ ),  $D_1$  cuts off and  $D_2$  conducts, thus steering  $I$  into the battery. Thus,  $i_B$  will have the waveform shown in the figure. Its peak value will be  $60\text{ mA}$ . To obtain the average value, we first determine the conduction angle of  $D_2$ ,  $(\pi - 2\theta)$ , where

$$\theta = \sin^{-1}\left(\frac{3}{6}\right) = 30^\circ$$

Thus

$$\pi - 2\theta = 180^\circ - 60^\circ = 120^\circ$$

The average value of  $i_B$  will be

$$i_B|_{av} = \frac{60 \times 120^\circ}{360^\circ} = 20\text{ mA}$$

If the peak value of  $v_I$  is reduced by 10%, i.e. from  $6\text{ V}$  to  $5.4\text{ V}$ , the peak value of  $i_B$  does not change. The conduction angle of  $D_2$ , however, changes since  $\theta$  now becomes

$$\theta = \sin^{-1}\left(\frac{3}{5.4}\right) = 33.75^\circ$$

and thus

$$\pi - 2\theta = 112.5^\circ$$

Thus the average value of  $i_B$  becomes

$$i_B|_{av} = \frac{60 \times 112.5^\circ}{360^\circ} = 18.75\text{ mA}$$

3.10

A	B	X	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

$$X = AB, \quad Y = A + B$$

$X$  and  $Y$  are the same for

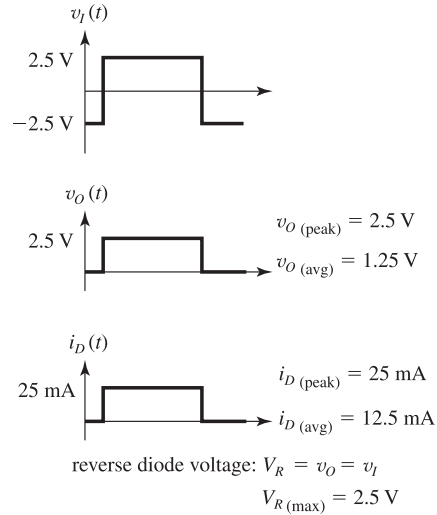
$$A = B$$

$X$  and  $Y$  are opposite if  $A \neq B$

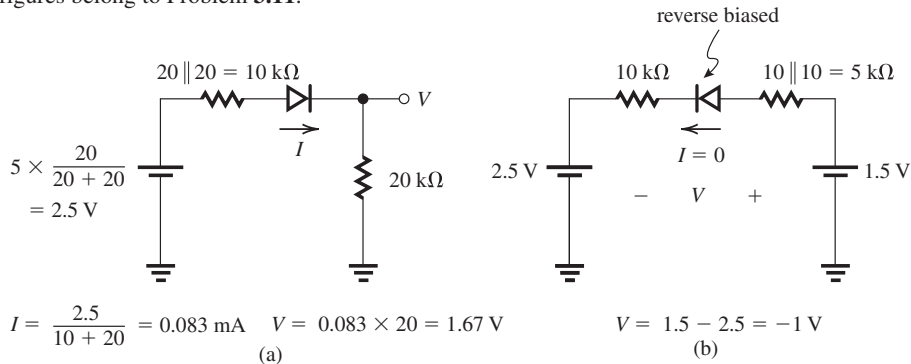
3.11 The analysis is shown on the circuit diagrams below.

3.12 For  $v_I > 0\text{ V}$ :  $D$  is on,  $v_O = v_I$ ,  $i_D = v_I/R$

For  $v_I < 0\text{ V}$ :  $D$  is off,  $v_O = 0$ ,  $i_D = 0$



These figures belong to Problem 3.11.

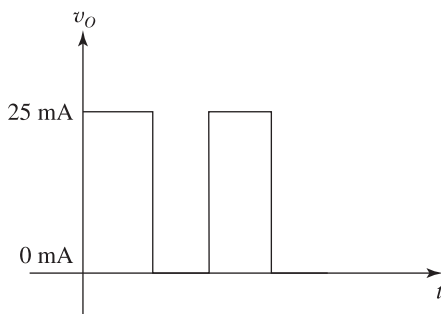
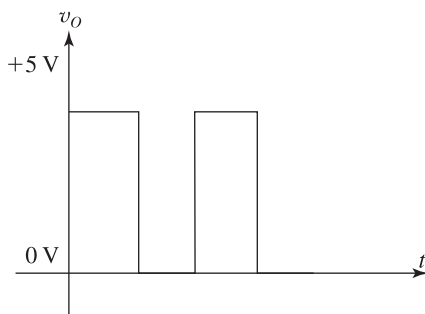
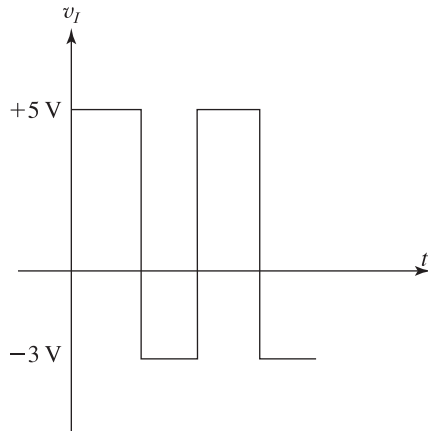


3.13

Using an ideal diode model,

$$i_{D(\text{peak})} = \frac{v_{O(\text{peak})}}{R} = \frac{5}{0.2} = 25 \text{ mA}$$

$$i_{D(\text{avg})} = \frac{i_{D(\text{peak})}}{2} = 12.5 \text{ mA}$$



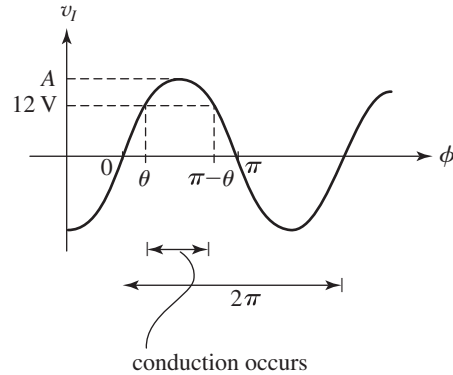
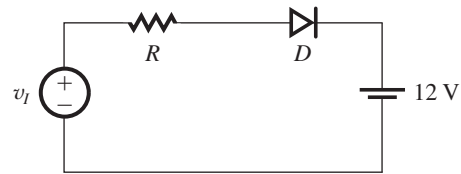
3.14 The peak current is given by

$$\frac{24 - 12}{R} = 50$$

$$\Rightarrow R = \frac{24 - 12}{50} = 0.24 \text{ k}\Omega = 240\Omega$$

3.15

$v_I = A \sin \theta = 12 \sim$  conduction through  $D$  occurs



For a conduction angle  $(\pi - 2\theta)$  that is 25% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{4}$$

$$\theta = \frac{\pi}{4}$$

$$A = 12 / \sin \theta = 17 \text{ V}$$

$\therefore$  Peak-to-peak sine wave voltage

$$= 2A = 34 \text{ V}$$

Given the average diode current to be

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{A \sin \phi - 12}{R} d\phi = 100 \text{ mA}$$

$$\frac{1}{2\pi} \left( \frac{-17 \cos \phi - 12\phi}{R} \right)_{\phi=0.25\pi}^{\phi=0.75\pi} = 0.1$$

$$R = 8.3 \Omega$$

$$\text{Peak diode current} = \frac{A - 12}{R} = 0.6 \text{ A}$$

$$\text{Peak reverse voltage} = A + 12 = 29 \text{ V}$$

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt, choose  $A = 17$  so the peak-to-peak sine wave voltage = 34 V and  $R = 8 \Omega$ .

$$\text{Conduction starts at } v_I = A \sin \theta = 12$$

$$17 \sin \theta = 12$$

$$\theta = \left( \frac{\pi}{4} \right) \text{ rad}$$

Conduction stops at  $\pi - \theta$ .

$\therefore$  Fraction of cycle that current flows is

$$\frac{\pi - 2\theta}{2\pi} \times 100 = 25\%$$

Average diode current =

$$\frac{1}{2\pi} \left( \frac{-17 \cos \phi - 12\phi}{8} \right)_{\phi=\pi/4}^{\phi=3\pi/4} = 103 \text{ mA}$$

Peak diode current

$$= \frac{17 - 12}{8} = 0.625 \text{ A}$$

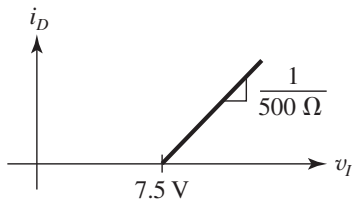
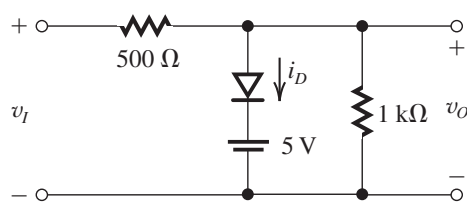
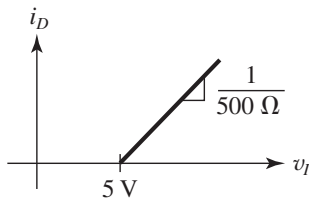
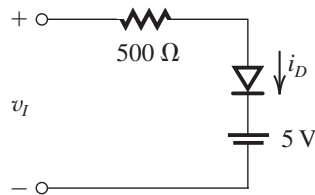
Peak reverse voltage =

$$A + 12 = 29 \text{ V}$$

**3.16**

V	RED	GREEN	
3 V	ON	OFF	- $D_1$ conducts
0	OFF	OFF	- No current flows
-3 V	OFF	ON	- $D_2$ conducts

**3.17**



**3.18**  $I = I_S e^{V_D/V_T}$

$$10^{-3} = I_S e^{0.7/V_T} \tag{1}$$

For  $V_D = 0.71 \text{ V}$ ,

$$I = I_S e^{0.71/V_T} \tag{2}$$

Combining (1) and (2) gives

$$I = 10^{-3} e^{(0.71 - 0.7)/0.025} = 1.49 \text{ mA}$$

For  $V_D = 0.8 \text{ V}$ ,

$$I = I_S e^{0.8/V_T} \tag{3}$$

Combining (1) and (3) gives

$$I = 10^{-3} \times e^{(0.8 - 0.7)/0.025} = 54.6 \text{ mA}$$

Similarly, for  $V_D = 0.69 \text{ V}$  we obtain

$$I = 10^{-3} \times e^{(0.69 - 0.7)/0.025} = 0.67 \text{ mA}$$

and for  $V_D = 0.6 \text{ V}$  we have

$$I = 10^{-3} e^{(0.6 - 0.7)/0.025} = 18.3 \text{ }\mu\text{A}$$

To increase the current by a factor of 10,  $V_D$  must be increased by  $\Delta V_D$ ,

$$10 = e^{\Delta V_D/0.025} \Rightarrow \Delta V_D = 0.025 \ln 10 = 57.6 \text{ mV}$$

**3.19**  $I_S$  can be found by using  $I_S = I_D \cdot e^{-V_D/V_T}$ .

Let a decrease by a factor of 10 in  $I_D$  result in a decrease of  $V_D$  by  $\Delta V$ :

$$I_D = I_S e^{V_D/V_T}$$

$$\frac{I_D}{10} = I_S e^{(V_D - \Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$10 = e^{\Delta V/V_T} \Rightarrow \Delta V \simeq 60 \text{ mV}$$

Thus the result in each case is a decrease in the diode voltage by 60 mV.

(a)  $V_D = 0.700 \text{ V}$ ,  $I_D = 1 \text{ A}$   
 $\Rightarrow I_S = 6.91 \times 10^{-13} \text{ A}$ ;

10% of  $I_D$  gives  $V_D = 0.64 \text{ V}$

(b)  $V_D = 0.650 \text{ V}$ ,  $I_D = 1 \text{ mA}$   
 $\Rightarrow I_S = 5.11 \times 10^{-15} \text{ A}$ ;

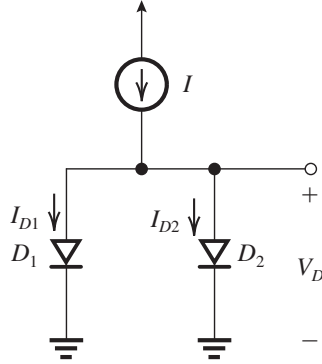
10% of  $I_D$  gives  $V_D = 0.59 \text{ V}$

(c)  $V_D = 0.650 \text{ V}$ ,  $I_D = 10 \text{ }\mu\text{A}$   
 $\Rightarrow I_S = 5.11 \times 10^{-17} \text{ A}$ ;

10% of  $I_D$  gives  $V_D = 0.59 \text{ V}$

(d)  $V_D = 0.700 \text{ V}$ ,  $I_D = 100 \text{ mA}$   
 $\Rightarrow I_S = 6.91 \times 10^{-14} \text{ A}$ ;  
 10% of  $I_D$  gives  $V_D = 0.64 \text{ V}$

**3.20**



$$I_{D1} = I_{S1} e^{V_D/V_T} \quad (1)$$

$$I_{D2} = I_{S2} e^{V_D/V_T} \quad (2)$$

Summing (1) and (2) gives

$$I_{D1} + I_{D2} = (I_{S1} + I_{S2}) e^{V_D/V_T}$$

But

$$I_{D1} + I_{D2} = I$$

Thus

$$I = (I_{S1} + I_{S2}) e^{V_D/V_T} \quad (3)$$

From Eq. (3) we obtain

$$V_D = V_T \ln\left(\frac{I}{I_{S1} + I_{S2}}\right)$$

Also, Eq. (3) can be written as

$$I = I_{S1} e^{V_D/V_T} \left(1 + \frac{I_{S2}}{I_{S1}}\right) \quad (4)$$

Now using (1) and (4) gives

$$I_{D1} = \frac{I}{1 + (I_{S2}/I_{S1})} = I \frac{I_{S1}}{I_{S1} + I_{S2}}$$

We similarly obtain

$$I_{D2} = \frac{I}{1 + (I_{S1}/I_{S2})} = I \frac{I_{S2}}{I_{S1} + I_{S2}}$$

**3.21** We can write a node equation at the anodes:

$$I_{D2} = I_1 - I_2 = 5 \text{ mA}$$

$$I_{D1} = I_2 = 3 \text{ mA}$$

We can write the following equation for the diode voltages:

$$V = V_{D2} - V_{D1}$$

If  $D_2$  has saturation current  $I_S$ , then  $D_1$ , which is 20 times larger, has saturation current  $20I_S$ . Thus we can write

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

$$I_{D1} = 20I_S e^{V_{D1}/V_T}$$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{5}{3} = \frac{1}{20} e^{(V_{D2}-V_{D1})/V_T} = \frac{1}{20} e^{V/V_T}$$

$$\Rightarrow V = 0.025 \ln\left(\frac{100}{3}\right) = 87.7 \text{ mV}$$

To instead achieve  $V = 60 \text{ mV}$ , we need

$$\frac{I_{D2}}{I_{D1}} = \frac{I_1 - I_2}{I_2} = \frac{1}{20} e^{0.06/0.025} = 0.55$$

Solving the above equation with  $I_1$  still at  $8 \text{ mA}$ , we find  $I_2 = 5.16 \text{ mA}$ .

**3.22** Given two different voltage/current measurements for a diode, we can write

$$I_{D1} = I_S e^{V_{D1}/V_T}$$

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

Taking the ratio of the above two equations, we have

$$\begin{aligned} \frac{I_{D1}}{I_{D2}} &= I_S e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2} \\ &= V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right) \end{aligned}$$

For  $I_D = 1 \text{ mA}$ , we have

$$\Delta V = V_T \ln\left(\frac{1 \times 10^{-3} \text{ A}}{10 \text{ A}}\right) = -230 \text{ mV}$$

$$\Rightarrow V_D = 570 \text{ mV}$$

For  $I_D = 3 \text{ mA}$ , we have

$$\Delta V = V_T \ln\left(\frac{3 \times 10^{-3} \text{ A}}{10 \text{ A}}\right) = -202 \text{ mV}$$

$$\Rightarrow V_D = 598 \text{ mV}$$

Assuming  $V_D$  changes by  $-2 \text{ mV}$  per  $1^\circ\text{C}$  increase in temperature, we have, for  $\pm 20^\circ\text{C}$  changes:

$$\text{For } I_D = 1 \text{ mA}, 530 \text{ mV} \leq V_D \leq 610 \text{ mV}$$

$$\text{For } I_D = 3 \text{ mA}, 558 \text{ mV} \leq V_D \leq 638 \text{ mV}$$

Thus the overall range of  $V_D$  is between  $530 \text{ mV}$  and  $638 \text{ mV}$ .

**3.23**  $V_T = \frac{kT}{q}$

where  $k = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$

$$T = 273 + x^\circ\text{C}$$



$$q = 1.60 \times 10^{-19} \text{ C}$$

Thus

$$V_T = 8.62 \times 10^{-5} \times (273 + x^\circ\text{C}), \text{ V}$$

$x [^\circ\text{C}]$	$V_T [\text{mV}]$
-55	18.8
0	23.5
+55	28.3
+125	34.3

for  $V_T = 25 \text{ mV}$  at  $17^\circ\text{C}$

$$\mathbf{3.24} \quad i = I_S e^{v/0.025}$$

$$\therefore 10,000 I_S = I_S e^{v/0.025}$$

$$v = 0.230 \text{ V}$$

At  $v = 0.7 \text{ V}$ ,

$$i = I_S e^{0.7/0.025} = 1.45 \times 10^{12} I_S$$

$$\mathbf{3.25} \quad I_1 = I_S e^{0.7/V_T} = 10^{-3}$$

$$i_2 = I_S e^{0.5/V_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5-0.7}{0.025}}$$

$$i_2 = 0.335 \mu\text{A}$$

$$\mathbf{3.26} \quad I_S \text{ can be found by using } I_S = I_D \cdot e^{-V_D/V_T}.$$

Let an increase by a factor of 10 in  $I_D$  result in an increase of  $V_D$  by  $\Delta V$ :

$$I_D = I_S e^{V_D/V_T}$$

$$10I_D = I_S e^{(V_D+\Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$10 = e^{\Delta V/V_T} \Rightarrow \Delta V \simeq 60 \text{ mV}$$

Thus the result is an increase in the diode voltage by 60 mV.

Similarly, at  $I_D/10$ ,  $V_D$  is reduced by 60 mV.

$$\text{(a) } V_D = 0.70 \text{ V, } I_D = 10 \text{ mA} \\ \Rightarrow I_S = 6.91 \times 10^{-15} \text{ A;}$$

$$I_D \times 10 \text{ gives } V_D = 0.76 \text{ V}$$

$$I_D/10 \text{ gives } V_D = 0.64 \text{ V}$$

$$\text{(b) } V_D = 0.70 \text{ V, } I_D = 1 \text{ mA} \\ \Rightarrow I_S = 6.91 \times 10^{-16} \text{ A;}$$

$$I_D \times 10 \text{ gives } V_D = 0.76 \text{ V}$$

$$I_D/10 \text{ gives } V_D = 0.64 \text{ V}$$

$$\text{(c) } V_D = 0.80 \text{ V, } I_D = 10 \text{ A} \\ \Rightarrow I_S = 1.27 \times 10^{-13} \text{ A;}$$

$$I_D \times 10 \text{ gives } V_D = 0.86 \text{ V}$$

$$I_D/10 \text{ gives } V_D = 0.74 \text{ V}$$

$$\text{(d) } V_D = 0.70 \text{ V, } I_D = 1 \text{ mA} \\ \Rightarrow I_S = 6.91 \times 10^{-16} \text{ A;}$$

$$I_D \times 10 \text{ gives } V_D = 0.76 \text{ V}$$

$$I_D/10 \text{ gives } V_D = 0.64 \text{ V}$$

$$\text{(e) } V_D = 0.6 \text{ V, } I_D = 10 \mu\text{A} \\ \Rightarrow I_S = 3.78 \times 10^{-16} \text{ A}$$

$$I_D \times 10 \text{ gives } V_D = 0.66 \text{ V}$$

$$I_D/10 \text{ gives } V_D = 0.54 \text{ V}$$

**3.27** The voltage across three diodes in series is 2.0 V; thus the voltage across each diode must be 0.667 V. Using  $I_D = I_S e^{V_D/V_T}$ , the required current  $I$  is found to be 3.9 mA.

If 1 mA is drawn away from the circuit,  $I_D$  will be 2.9 mA, which would give  $V_D = 0.794 \text{ V}$ , giving an output voltage of 1.98 V. The change in output voltage is  $-22 \text{ mV}$ .

**3.28** Connecting an identical diode in parallel would reduce the current in each diode by a factor of 2. Writing expressions for the currents, we have

$$I_D = I_S e^{V_D/V_T}$$

$$\frac{I_D}{2} = I_S e^{(V_D-\Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$2 = e^{\Delta V/V_T} \Rightarrow \Delta V = 17.3 \text{ mV}$$

Thus the result is a decrease in the diode voltage by 17.3 mV.

**3.29** We can write the following node equation at the diode anodes:

$$I_{D2} = 10 \text{ mA} - V/R$$

$$I_{D1} = V/R$$

We can write the following equation for the diode voltages:

$$V = V_{D2} - V_{D1}$$

We can write the following diode equations:

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

$$I_{D1} = I_S e^{V_{D1}/V_T}$$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - V/R}{V/R} = e^{(V_{D2}-V_{D1})/V_T} = e^{V/V_T}$$

To achieve  $V = 50 \text{ mV}$ , we need

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - 0.05/R}{0.05/R} = e^{0.05/0.025} = 7.39$$

Solving the above equation, we have

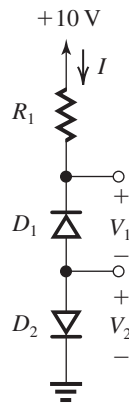
$$R = 42 \Omega$$

**3.30** For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

$T = -20^\circ\text{C}$  corresponds to a temperature decrease of 40°C, which results in an increase of the diode voltage by 80 mV. Thus  $V_D = 770 \text{ mV}$ .

$T = +85^\circ\text{C}$  corresponds to a temperature increase of 65°C, which results in a decrease of the diode voltage by 130 mV. Thus  $V_D = 560 \text{ mV}$ .

**3.31**



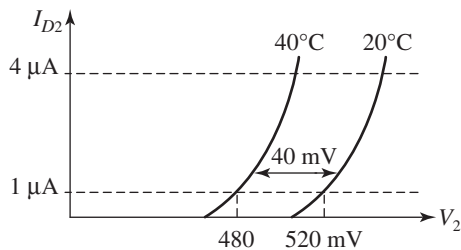
At 20°C:

$$V_{R1} = V_2 = 520 \text{ mV}$$

$$R_1 = 520 \text{ k}\Omega$$

$$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1 \mu\text{A}$$

Since the reverse current doubles for every 10°C rise in temperature, at 40°C,  $I = 4 \mu\text{A}$



$$V_2 = 480 + 2.3 \times 1 \times 25 \log 4$$

$$= 514.6 \text{ mV}$$

$$V_{R1} = 4 \mu\text{A} \times 520 \text{ k}\Omega = 2.08 \text{ V}$$

$$\text{At } 0^\circ\text{C}, I = \frac{1}{4} \mu\text{A}$$

$$V_2 = 560 - 2.3 \times 1 \times 25 \log 4$$

$$= 525.4 \text{ mV}$$

$$V_{R1} = \frac{1}{4} \times 520 = 0.13 \text{ V}$$

**3.32** For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

A decrease in  $V_D$  by 100 mV corresponds to a junction temperature increase of 50°C.

The power dissipation is given by

$$PD = (10 \text{ A})(0.6 \text{ V}) = 6 \text{ W}$$

The thermal resistance is given by

$$\frac{\Delta T}{P_D} = \frac{50^\circ\text{C}}{6 \text{ W}} = 8.33^\circ\text{C/W}$$

**3.33** Given two different voltage/current measurements for a diode, we have

$$\frac{I_{D1}}{I_{D2}} = I_S e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2} = V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right)$$

For the first diode, with  $I_D = 0.1 \text{ mA}$  and

$V_D = 700 \text{ mV}$ , we have

$$I_D = 1 \text{ mA:}$$

$$\Delta V = V_T \ln\left(\frac{1.0}{0.1}\right) = 57.6 \text{ mV} \Rightarrow V_D = 757.6 \text{ mV}$$

$$I_D = 3 \text{ mA:}$$

$$\Delta V = V_T \ln\left(\frac{3}{0.1}\right) = 85 \text{ mV} \Rightarrow V_D = 785 \text{ mV}$$

For the second diode, with

$I_D = 1 \text{ A}$  and  $V_D = 700 \text{ mV}$ , we have

$$I_D = 1.0 \text{ mA:}$$

$$\Delta V = V_T \ln\left(\frac{0.001}{1}\right) = -173 \text{ mV} \Rightarrow V_D = 527 \text{ mV}$$

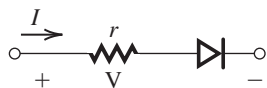
$$I_D = 3 \text{ mA:}$$

$$\Delta V = V_T \ln\left(\frac{0.003}{1}\right) = -145 \text{ mV} \Rightarrow V_D = 555 \text{ mV}$$

For both  $I_D = 1.0 \text{ mA}$  and  $I_D = 3 \text{ mA}$ , the difference between the two diode voltages is approximately 230 mV. Since, for a fixed diode

current, the diode voltage changes with temperature at a constant rate (-2 mV per °C temp. increase), this voltage difference will be independent of temperature!

3.34



$$V = Ir + 2.3V_T \log\left(\frac{I}{I_S}\right)$$

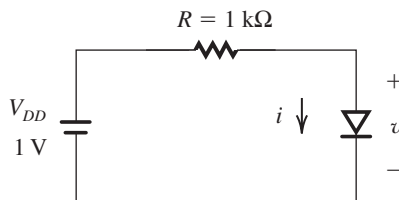
Thus, a change in current, from  $I_1$  to  $I_2$ , results in a change in voltage,

$$V_2 - V_1 = r(I_2 - I_1) + 2.3V_T \log\frac{I_2}{I_1}$$

In this case,  $r = 10\Omega$ ,  $I_1 = 1$  mA and  $I_2 = 2$  mA. Thus,

$$V_2 - V_1 = 10 \times (2 - 1) + 2.3V_T \log\left(\frac{2}{1}\right) = 27.3 \text{ mV}$$

3.35



$$I_S = 10^{-15} \text{ A} = 10^{-12} \text{ mA}$$

Calculate some points

$$v = 0.6 \text{ V}, \quad i = I_S e^{v/V_T} = 10^{-12} e^{0.6/0.025} \approx 0.03 \text{ mA}$$

$$v = 0.65 \text{ V}, \quad i \approx 0.2 \text{ mA}$$

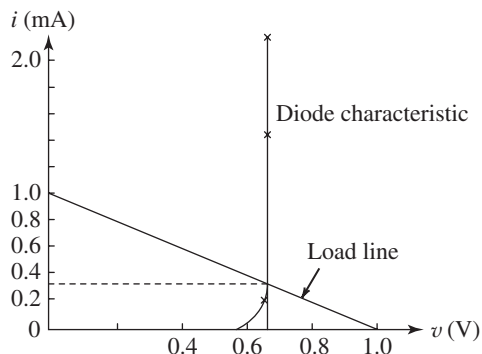
$$v = 0.7 \text{ V}, \quad i \approx 1.45 \text{ mA}$$

Make a sketch showing these points and load line and determine the operating point. The points for the load line are obtained using

$$I_D = \frac{V_{DD} - V_D}{R}$$

From this sketch one can see that the operating point must lie between  $v = 0.65$  V to  $v = 0.7$  V

$$\text{For } i = 0.3 \text{ mA}, \quad v = V_T \ln\left(\frac{i}{I_S}\right)$$

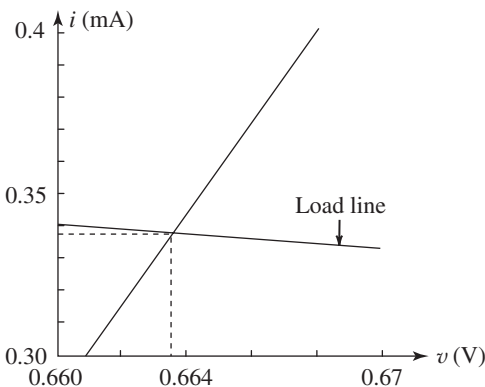


$$= 0.025 \times \ln\left(\frac{3}{10^{-12}}\right)$$

$$= 0.661 \text{ V}$$

$$\text{For } i = 0.4 \text{ mA}, \quad v = 0.668 \text{ V}$$

Now we can refine the diagram to obtain a better estimate



From this graph we get the operating point

$$i = 0.338 \text{ mA}, \quad v = 0.6635 \text{ V}$$

Now we compare graphical results with the exponential model.

At  $i = 0.338$  mA

$$v = V_T \ln\left(\frac{i}{I_S}\right) = 0.025 \times \ln\left(\frac{0.338}{10^{-12}}\right)$$

$$= 0.6637 \text{ V}$$

The difference between the exponential model and graphical results is  $= 0.6637 - 0.6635$

$$= 0.0002 \text{ V}$$

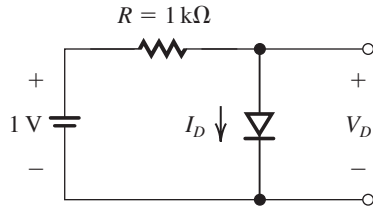
$$= 0.2 \text{ mV}$$

3.36

$$I_S = 10^{-15} \text{ A} = 10^{-12} \text{ mA}$$

Use the iterative analysis procedure:

$$1. \quad V_D = 0.7 \text{ V}, \quad I_D = \frac{1 - 0.7}{1 \text{ K}} = 0.3 \text{ mA}$$



$$2. V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.025 \ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607 \text{ V}$$

$$I_D = \frac{1 - 0.6607}{1 \text{ K}} = 0.3393 \text{ mA}$$

$$3. V_D = 0.025 \ln\left(\frac{0.3393}{10^{-12}}\right) = 0.6638 \text{ V}$$

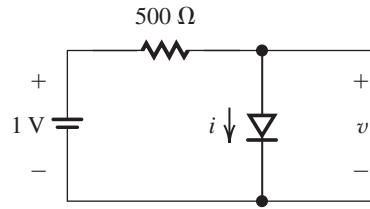
$$I_D = \frac{1 - 0.6638}{1 \text{ K}} = 0.3362 \text{ mA}$$

$$4. V_D = 0.025 \ln\left(\frac{0.3362}{10^{-12}}\right) = 0.6635 \text{ V}$$

$$I_D = \frac{1 - 0.6635}{1 \text{ k}\Omega} = 0.3365 \text{ mA}$$

Stop here as we are getting almost same value of  $I_D$  and  $V_D$

**3.37**



$$a) I_D = \frac{1 - 0.7}{0.5 \text{ k}\Omega} = 0.6 \text{ mA}$$

b) Diode has 0.7 V drop at 1 mA current. Use Eq. (3.5):

$$v_2 = v_1 + 2.3V_T \log\left(\frac{i_2}{i_1}\right)$$

$$1. v = 0.7 \text{ V}$$

$$i = \frac{1 - 0.7}{0.5 \text{ k}\Omega} = 0.6 \text{ mA}$$

$$2. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6}{1}\right) = 0.6872 \text{ V}$$

$$i = \frac{1 - 0.6872}{0.5} = 0.6255 \text{ mA}$$

$$3. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6255}{1}\right) = 0.6882 \text{ V}$$

$$i = \frac{1 - 0.6882}{0.5 \text{ k}\Omega} = 0.6235 \text{ mA}$$

$$4. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6235}{1}\right)$$

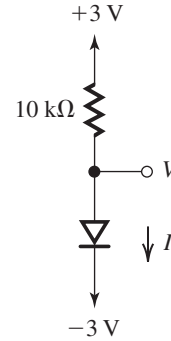
$$= 0.6882 \text{ V}$$

$$i = \frac{1 - 0.6882}{0.5 \text{ k}\Omega} = 0.6235 \text{ mA}$$

Stop as we are getting the same result.

**3.38**

(a)

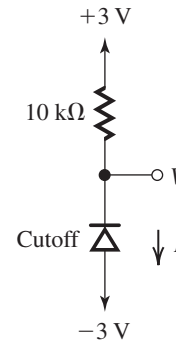


$$V = -3 + 0.7 = -2.3 \text{ V}$$

$$I = \frac{3 + 2.3}{10}$$

$$= 0.53 \text{ mA}$$

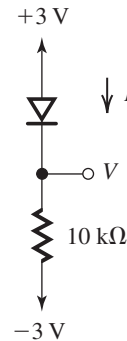
(b)



$$I = 0 \text{ A}$$

$$V = 3 - I(10) = 3 \text{ V}$$

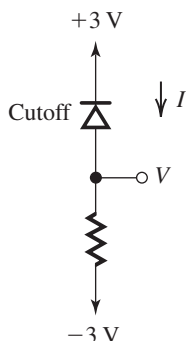
(c)



$$V = 3 - 0.7 = 2.3 \text{ V}$$

$$I = \frac{2.3 + 3}{10} = 0.53 \text{ mA}$$

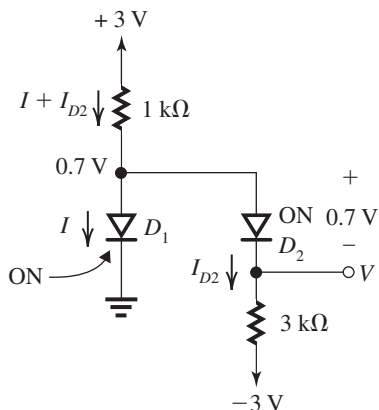
(d)



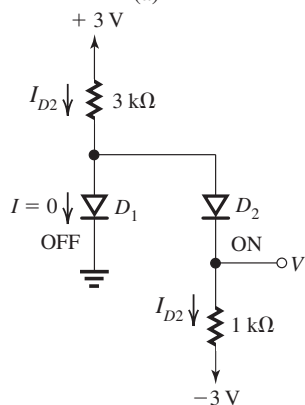
$$I = 0 \text{ A}$$

$$V = -3 \text{ V}$$

3.39



(a)



(b)

(a) Assuming a current flow from the positive to the negative supply voltages,  $D_2$  will be conducting. A voltage division between the two resistors would place forward bias on  $D_1$ , so let's

assume  $D_1$  is also conducting, placing +0.7 V at its anode. Thus, as shown in Fig. (a),

$$V = 0 \text{ V}$$

$$I_{D2} = \frac{0 - (-3)}{3} = 1 \text{ mA}$$

$$I + I_{D2} = \frac{3 - 0.7}{1} = 2.3 \text{ mA}$$

$$\Rightarrow I = 2.3 - 1 = 1.3 \text{ mA}$$

These results are consistent with our assumptions.

(b) Again assuming  $D_2$  is on, a voltage division between the two resistors will negatively bias  $D_1$ . Hence, we assume  $D_1$  is off.

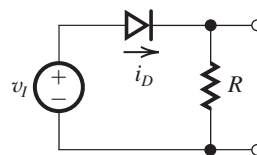
$$I = 0$$

$$I_{D2} = \frac{3 - 0.7 - (-3)}{3 + 1} = 1.325 \text{ mA}$$

$$V = -3 + 1.325 \times 1 = -1.675 \text{ V}$$

Thus, the anode of  $D_1$  is at  $-1.675 + 0.7 = -0.975 \text{ V}$  which confirms that it is reverse-biased.

3.40



$$i_{D,\text{peak}} = \frac{v_{i,\text{peak}} - 0.7}{R} \leq 40 \text{ mA}$$

$$R \geq \frac{120\sqrt{2} - 0.7}{40} = 4.23 \text{ k}\Omega$$

$$\text{Reverse voltage} = 120\sqrt{2} = 169.7 \text{ V.}$$

The design is essentially the same since the supply voltage  $\gg 0.7 \text{ V}$

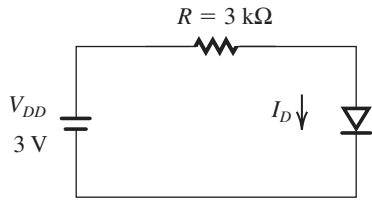
3.41 Eq. (3.8),  $V_Z = V_{Z0} + r_z I_Z$ , also applies to the test voltage and current. Thus,

$$V_{ZT} = V_{Z0} + r_z I_{ZT}$$

We can use this equation to complete the table as follows.

	$V_{ZT}$	$I_{ZT}$	$r_z$	$V_{Z0}$	$V_Z @ I_Z = 10 \text{ mA}$
(a)	3 V	2 mA	50 $\Omega$	2.9 V	3.4 V
(b)	5 V	5 mA	100 $\Omega$	4.5 V	5.5 V
(c)	6 V	2 mA	150 $\Omega$	5.7 V	7.2 V
(d)	9.1 V	1 mA	100 $\Omega$	9 V	10 V

3.42



a)  $I_D = \frac{3 - 0.7}{3} = 0.767 \text{ mA}$

b)  $I_D = I_S e^{V_D/V_T} \Rightarrow I_S = I_D e^{-V_D/V_T}$   
 $I_S = 0.767 \times e^{-700/25} = 5.3 \times 10^{-16} \text{ A}$

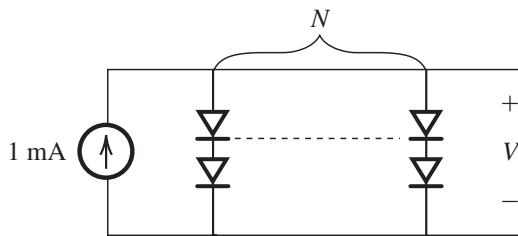
c)  $I_S$  increases by a factor of 100  
 $\Rightarrow \Delta V_D = V_T \left[ \ln \left( \frac{I_D}{100 I_S} \right) - \ln \left( \frac{I_D}{I_S} \right) \right]$   
 $= -V_T \ln 100 = -115 \text{ mV}$   
 $\Rightarrow I_D = \frac{3 - (0.7 - 0.115)}{3} = \frac{3 - 0.585}{3}$   
 $= 0.805 \text{ mA}$

We can iterate the analysis to improve the accuracy. With  $I_D = 0.805 \text{ mA}$ ,

$$V_D = V_T \ln \left( \frac{I_D}{100 I_S} \right) = 25 \times \ln \left( \frac{0.805 \times 10^{-3}}{5.3 \times 10^{-14}} \right) = 586 \text{ mV}$$

This is almost identical to the value computed above. Thus, the current increases by  $0.805 - 0.767 = 0.038 \text{ mA} = 38 \mu\text{A}$ .

3.43 Available diodes have 0.7 V drop at 2 mA current since  $2V_D = 1.4 \text{ V}$  is close to 1.3 V, use  $N$  parallel pairs of diodes to split the 1 mA current evenly, as shown in the figure next.



The voltage drop across each pair of diodes is 1.3 V.  $\therefore$  Voltage drop across each diode  $= \frac{1.3}{2} = 0.65 \text{ V}$ . Using

$$I_2 = I_1 e^{(V_2 - V_1)/V_T}$$

$$= 2e^{(0.65 - 0.7)/0.025}$$

$$= 0.2707 \text{ mA}$$

Thus current through each branch is 0.2707 mA.

The 1 mA will split in  $= \frac{1}{0.2707} = 3.69$  branches.

Choose  $N = 4$ .

There are 4 pairs of diodes in parallel.

$\therefore$  We need 8 diodes.

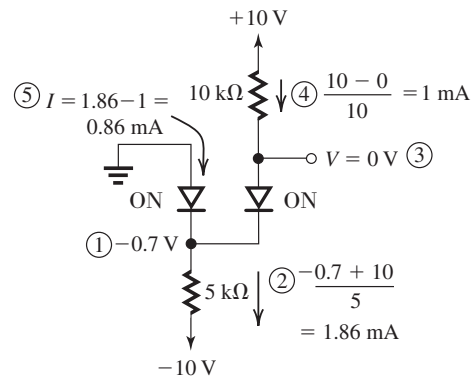
Current through each pair of diodes  $= \frac{1 \text{ mA}}{4} = 0.25 \text{ mA}$

$\therefore$  Voltage across each pair  $= 2 \left[ 0.7 + 0.025 \ln \left( \frac{0.25}{2} \right) \right]$   
 $= 1.296 \text{ V}$

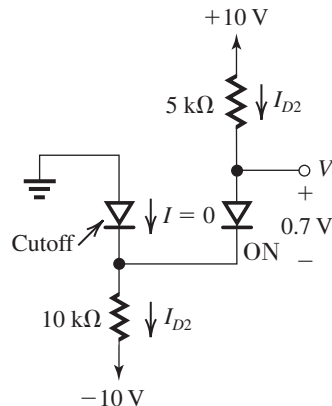
SPECIAL NOTE: There is another possible design utilizing only 6 diodes and realizing a voltage of 1.313 V. It consists of the series connection of 4 parallel diodes and 2 parallel diodes.

3.44 Refer to Example 3.2.

(a)



(b)

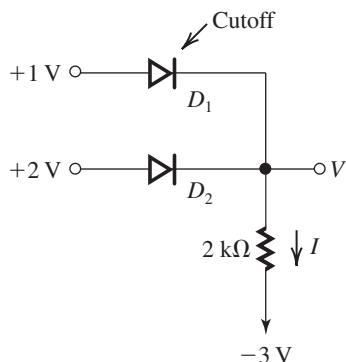


$$I_{D2} = \frac{10 - (-10) - 0.7}{15} = 1.29 \text{ mA}$$

$$V_D = -10 + 1.29(10) + 0.7 = 3.6 \text{ V}$$

3.45

(a)



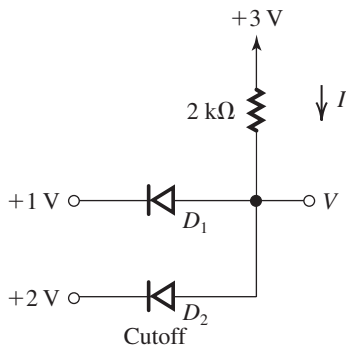
$$V = 2 - 0.7$$

$$= 1.3 \text{ V}$$

$$I = \frac{1.3 - (-3)}{2}$$

$$= 2.15 \text{ mA}$$

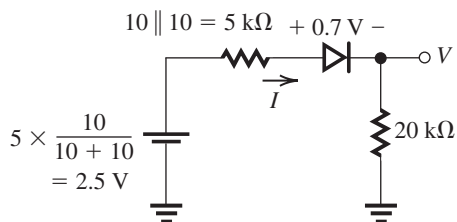
(b)



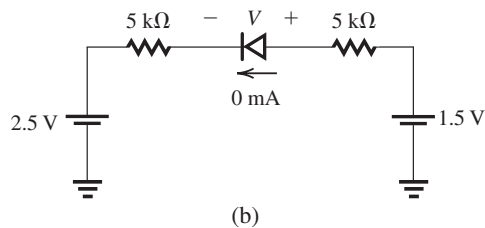
$$V = 1 + 0.7 = 1.7 \text{ V}$$

$$I = \frac{3 - 1.7}{2} = 0.65 \text{ mA}$$

3.46



(a)



$$(a) I = \frac{2.5 - 0.7}{5 + 20} = 0.072 \text{ mA}$$

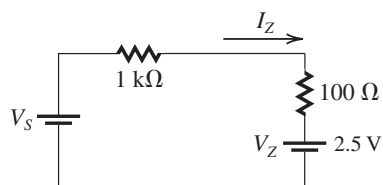
$$V = 0.072 \times 20 = 1.44 \text{ V}$$

(b) The diode will be cut off, thus

$$I = 0$$

$$V = 1.5 - 2.5 = -1 \text{ V}$$

$$3.47 V_{Z0} = 3 \text{ V} - 5 \text{ mA} \times 100\Omega = 2.5 \text{ V.}$$



The power dissipated in the zener is

$$P_Z = V_Z I_Z = (2.5 \text{ V} + 100\Omega \times I_Z) \times I_Z$$

We use this to find the current at  $P_{Z\text{max}} = 40 \text{ mW}$ ,

$$40 \text{ mW} = 2.5 \text{ V} \times I_{Z\text{max}} + 100\Omega \times I_{Z\text{max}}^2$$

$$\Rightarrow 100I_{Z\text{max}}^2 + 2.5I_{Z\text{max}} - 0.04 = 0$$

$$\Rightarrow I_{Z\text{max}} = \frac{-2.5 + \sqrt{2.5^2 + 4 \times 100 \times 0.04}}{2 \times 100}$$

$$= 0.0111 \text{ A} = 11.1 \text{ mA}$$

This current corresponds to

$$V_S = 2.5 + 1.1 \times 11.1 = 14.71 \text{ V and}$$

$$V_Z = 2.5 + 0.1 \times 11.1 = 3.61 \text{ V}$$

3.48 The voltage change per degree Celsius of the series-connected diodes is the sum of the two diode voltage changes. Thus, the temperature coefficient is  $+1.5 - 2 = -0.5 \text{ mV}/^\circ\text{C}$ .

3.49 Use the exponential diode model to find the percentage change in the current.

$$i_D = I_S e^{v/V_T}$$

$$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta v/V_T}$$

For +5 mV change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{5/25} = 1.221$$

% change

$$= \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{1.221 - 1}{1} \times 100 = 22.1\%$$

For -5 mV change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{-5/25} = 0.818$$

$$\% \text{ change} = \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{0.818 - 1}{1} \times 100 = -18.1\%$$

Maximum allowable voltage signal change when the current change is limited to  $\pm 10\%$  is found as follows:

The current varies from 0.9 to 1.1

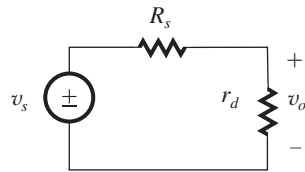
$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/V_T}$$

$$\text{For } 0.9, \Delta V = 25 \ln(0.9) = -2.63 \text{ mV}$$

$$\text{For } 1.1, \Delta V = 25 \ln(1.1) = +2.38 \text{ mV}$$

For  $\pm 10\%$  current change the voltage signal change is from -2.63 mV to +2.38 mV

**3.50** The dc current  $I$  flows through the diode giving rise to the diode resistance  $r_d = \frac{V_T}{I}$  and the small-signal equivalent circuit is represented by



$$v_o = v_s \times \frac{r_d}{r_d + R_s} = v_s \frac{V_T/I}{\frac{V_T}{I} + R_s} = v_s \frac{V_T}{V_T + IR_s}$$

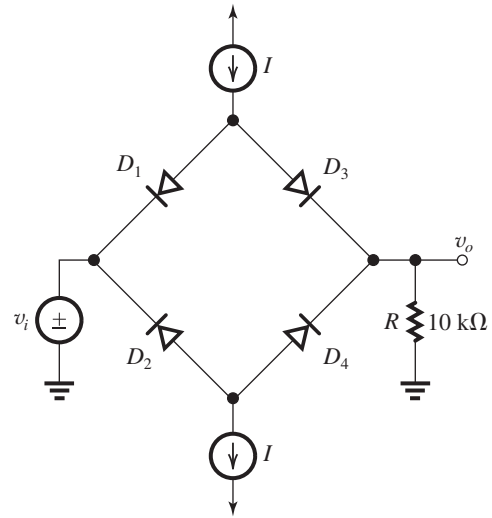
$$\text{Now, } v_o = 10 \text{ mV} \times \frac{25 \text{ mV}}{25 \text{ mV} + 10^3 I}$$

$I$	$v_o$
1 mA	0.24 mV
0.1 mA	2.0 mV
1 $\mu$ A	9.6 mV

$$\text{For } v_o = \frac{1}{2} v_s = v_s \times \frac{0.025}{0.025 + 10^3 I}$$

$$\Rightarrow I = 25 \mu\text{A}$$

**3.51**



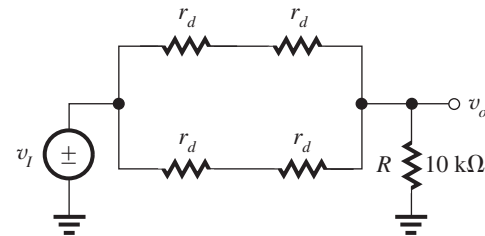
(a) The current through each diode is  $\frac{I}{2}$ :

$$r_d = \frac{V_T}{I} = \frac{2V_T}{I} = \frac{0.05}{I}$$

From the equivalent circuit

$$\frac{v_o}{v_i} = \frac{R}{R + (2r_d \parallel 2r_d)} = \frac{R}{R + r_d}$$

$I$	$r_d$	$\frac{v_o}{v_i}$
0 $\mu$ A	$\infty$	0
1 $\mu$ A	50 k $\Omega$	0.167
10 $\mu$ A	5 k $\Omega$	0.667
100 $\mu$ A	500 $\Omega$	0.952
1 mA	50 $\Omega$	0.995
10 mA	5 $\Omega$	0.9995



Equivalent Circuit

(b) For signal current to be limited to  $\pm 10\%$  of  $I$  ( $I$  is the biasing current), the change in diode voltage can be obtained from the equation

$$\frac{i_D}{I} = e^{\Delta V_D/V_T} = 0.9 \text{ to } 1.1$$

$$\Delta v_D = -2.63 \text{ mV to } +2.32 \text{ mV}$$

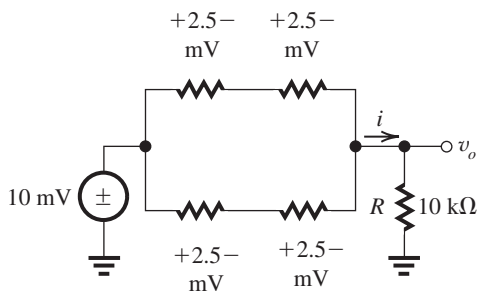


$\simeq \pm 2.5 \text{ mV}$

so the signal voltage across each diode is limited to 2.5 mV when the diode current remains within 10% of the dc bias current.

$\therefore v_o = 10 - 2.5 - 2.5 = 5 \text{ mV}$

and  $i = \frac{5 \text{ mV}}{10 \text{ k}\Omega} = 0.5 \mu\text{A}$



The current through each diode

$= \frac{0.5}{2} \mu\text{A} = 0.25 \mu\text{A}$

The signal current  $i$  is  $0.5 \mu\text{A}$ , and this is 10% of the dc biasing current.

$\therefore$  DC biasing current  $I = 0.5 \times 10 = 5 \mu\text{A}$

(c) Now  $I = 1 \text{ mA}$ .

$\therefore I_D = 0.5 \text{ mA}$

Maximum current derivation 10%.

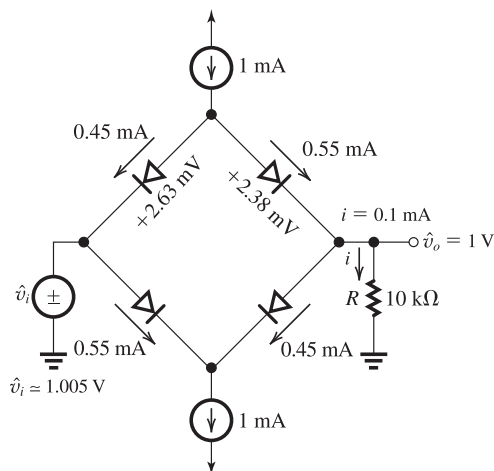
$\therefore i_d = \frac{0.5}{10} = 0.05 \text{ mA}$

and  $i = 2i_d = 0.1 \text{ mA}$ .

$\therefore$  Maximum  $v_o = i \times 10 \text{ k}\Omega$

$= 0.1 \times 10$

$= 1 \text{ V}$



From the results of (a) above, for  $I = 1 \text{ mA}$ ,  $v_o/v_i = 0.995$ ; thus the maximum input signal will be

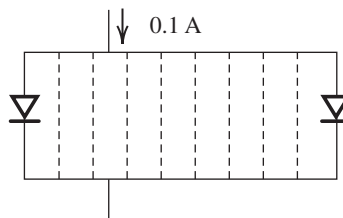
$\hat{v}_i = \hat{v}_o/0.995 = 1/0.995$

$= 1.005 \text{ V}$

The same result can be obtained from the figure above where the signal across the two series diodes is 5 mV, thus

$\hat{v}_i = \hat{v}_o + 5 \text{ mV} = 1 \text{ V} + 5 \text{ mV} = 1.005 \text{ V}$ .

3.52



Ten diode connected in parallel and fed with a total current of 0.1 A. So the current through each diode

$= \frac{0.1}{10} = 0.01 \text{ A}$

Small signal resistance of each diode

$= \frac{V_T}{i_D} = \frac{25 \text{ mV}}{0.01 \text{ A}} = 2.5 \Omega$

Equivalent resistance,  $R_{eq}$ , of 10 diodes connected in parallel is given by

$R_{eq} = \frac{2.5}{10} = 0.25 \Omega$

If there is one diode conducting 0.1 A current, then the small signal resistance of this diode

$= \frac{25 \text{ mV}}{0.1 \text{ A}} = 0.25 \Omega$

This value is the same as of 10 diodes connected in parallel.

If  $0.2 \Omega$  is the resistance for making connection, the resistance in each branch  $= r_d + 0.2 = 2.5 + 0.2 = 2.7 \Omega$

For a parallel combination of 10 diodes, equivalent resistance,  $R_{eq}$ , is

$R_{eq} = \frac{2.7}{10} = 0.27 \Omega$

If there is a single diode conducting all the 0.1 A current, the connection resistance needed for the single diode will be  $= 0.27 - 0.25 = 0.02 \Omega$ .

3.53 As shown in Problem 3.50,

$$\frac{v_o}{v_i} = \frac{V_T}{V_T + R_S I} = \frac{0.025}{0.025 + 10^4 I} \quad (1)$$

Here  $R_S = 10 \text{ k}\Omega$

The current changes are limited  $\pm 10\%$ . Using exponential model, we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta v/V_T} = 0.9 \text{ to } 1.1$$

$$\Delta v = 25 \times 10^{-3} \ln\left(\frac{i_{D2}}{i_{D1}}\right) \text{ and here}$$

For 0.9,  $\Delta v = -2.63 \text{ mV}$

For 1.1,  $\Delta v = 2.38 \text{ mV}$

The variation is  $-2.63 \text{ mV}$  to  $2.38 \text{ mV}$  for  $\pm 10\%$  current variation. Thus the largest symmetrical output signal allowed is  $2.38 \text{ mV}$  in amplitude.

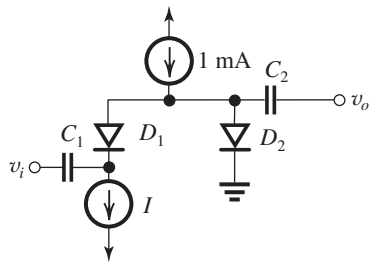
To obtain the corresponding input signal, we divide this by  $(v_o/v_i)$ :

$$\hat{v}_s = \frac{2.38 \text{ mV}}{v_o/v_i} \quad (2)$$

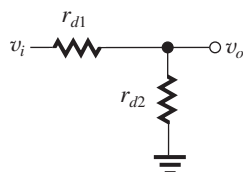
Now for the given values of  $(v_o/v_i)$  calculate  $I$  and  $\hat{v}_s$  using Equations (1) and (2)

$\frac{v_o}{v_i}$	$I$ in mA	$\hat{v}_s$ in mV
0.5	0.0025	4.76
0.1	0.0225	23.8
0.01	0.2475	238
0.001	2.4975	2380

3.54



When both  $D_1$  and  $D_2$  are conducting, the small-signal model is



where we have replaced the large capacitors  $C_1$  and  $C_2$  by short circuits:

$$\frac{v_o}{v_i} = \frac{r_{d2}}{r_{d1} + r_{d2}} = \frac{\frac{V_T}{1 \text{ m} - I}}{\frac{V_T}{I} + \frac{V_T}{1 \text{ m} - I}} = \frac{I}{1 \text{ m}}$$

Thus

$$\frac{v_o}{v_i} = I, \quad \text{where } I \text{ is in mA}$$

$$\text{Now } I = 0 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 0$$

$$I = 1 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 1 \times 10^{-3} = 0.001 \text{ V/V}$$

$$I = 10 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 10 \times 10^{-3} = 0.01 \text{ V/V}$$

$$I = 100 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 100 \times 10^{-3} = 0.1 \text{ V/V}$$

$$I = 500 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 500 \times 10^{-3} = 0.5 \text{ V/V}$$

$$I = 600 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 600 \times 10^{-3} = 0.6 \text{ V/V}$$

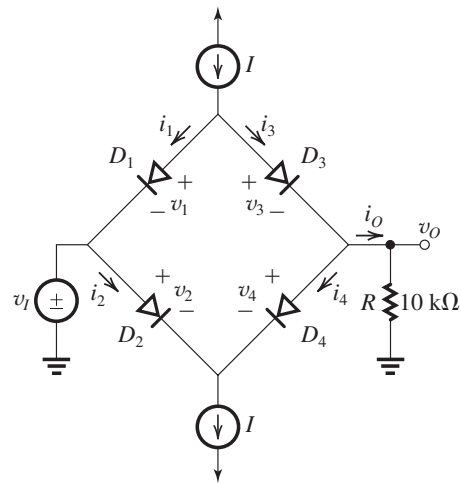
$$I = 900 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 900 \times 10^{-3} = 0.9 \text{ V/V}$$

$$I = 990 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 990 \times 10^{-3} = 0.99 \text{ V/V}$$

$$I = 1 \text{ mA}$$

$$= 1000 \text{ }\mu\text{A}, \frac{v_o}{v_i} = 1000 \times 10^{-3} = 1 \text{ V/V}$$

3.55



$$I = 1 \text{ mA}$$

Each diode exhibits  $0.7 \text{ V}$  drop at  $1 \text{ mA}$  current. Using diode exponential model we have

$$v_2 - v_1 = V_T \ln\left(\frac{i_2}{i_1}\right)$$

and  $v_1 = 0.7 \text{ V}$ ,  $i_1 = 1 \text{ mA}$

$$\Rightarrow v = 0.7 + V_T \ln\left(\frac{i}{1}\right)$$

$$= 700 + 25 \ln(i)$$

Calculation for different values of  $v_O$ :

$v_O = 0$ ,  $i_O = 0$ , and the current  $I = 1 \text{ mA}$  divide equally between the  $D_3, D_4$  side and the  $D_1, D_2$  side.

$$i_1 = i_2 = i_3 = i_4 = \frac{I}{2} = 0.5 \text{ mA}$$

$$v = 700 + 25 \ln(0.5) \simeq 683 \text{ mV}$$

$$v_1 = v_2 = v_3 = v_4 = 683 \text{ mV}$$

From the circuit, we have

$$v_I = -v_1 + v_3 + v_O = -683 + 683 + 0 = 0 \text{ V}$$

For  $v_O = 1 \text{ V}$ ,  $i_O = \frac{1}{10 \text{ k}\Omega} = 0.1 \text{ mA}$

Because of symmetry of the circuit, we obtain

$$i_3 = i_2 = \frac{I}{2} + \frac{i_O}{2} = 0.5 + 0.05 = 0.55 \text{ mA and}$$

$$i_4 = i_1 = 0.45 \text{ mA}$$

$$v_3 = v_2 = 700 + 25 \ln\left(\frac{i_3}{1}\right) = 685 \text{ mV}$$

$$v_4 = v_1 = 700 + 25 \ln\left(\frac{i_4}{1}\right) = 680 \text{ mV}$$

$v_O$ (V)	$i_O$ (mA)	$i_3 = i_2$ (mA)	$i_4 = i_1$ (mA)	$v_3 = v_2$ (mV)	$v_4 = v_1$ (mV)	$v_I = -v_1 + v_3 + v_O$ (V)
0	0	0.5	0.5	683	683	0
+1	0.1	0.55	0.45	685	680	1.005
+2	0.2	0.6	0.4	~687	677	2.010
+5	0.5	0.75	0.25	~693	665	5.028
+9	0.9	0.95	0.05	~699	~625	9.074
+9.9	0.99	0.995	0.005	~700	568	10.09
9.99	0.999	0.9995	0.0005	~700	510	10.18
10	1	1	0	700	0	10.7

$$v_I = -v_1 + v_2 + v_O = -0.680$$

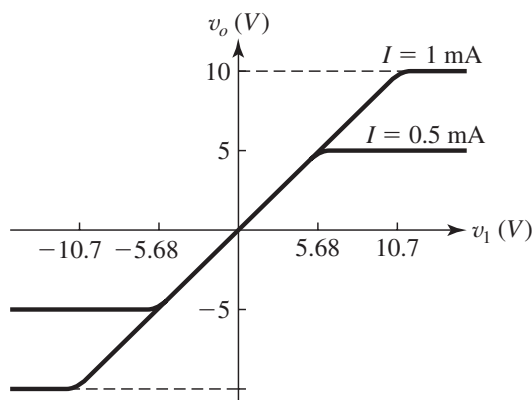
$$+0.685 + 1 = 1.005 \text{ V}$$

Similarly, other values are calculated as shown in the table. The largest values of  $v_O$  on positive and negative side are  $+10 \text{ V}$  and  $-10 \text{ V}$ , respectively. This restriction is imposed by the current  $I = 1 \text{ mA}$

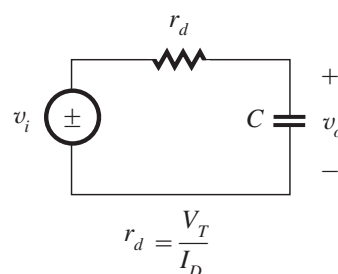
A similar table can be generated for the negative values. It is symmetrical.

For  $v_I > +10.7$ ,  $v_O$  will be saturated at  $+10 \text{ V}$  and it is because  $I = 1 \text{ mA}$ . Similarly, for  $v_I < -10.7 \text{ V}$ ,  $v_O$  will be saturated at  $-10 \text{ V}$ .

For  $I = 0.5 \text{ mA}$ , the output will saturate at  $0.5 \text{ mA} \times 10 \text{ k}\Omega = 5 \text{ V}$ .



**3.56** Representing the diode by the small-signal resistances, the circuit is



$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{r_d + \frac{1}{sC}} = \frac{1}{1 + sCr_d}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega Cr_d}$$

$$\text{Phase shift} = -\tan^{-1}\left(\frac{\omega Cr_d}{1}\right)$$

$$= -\tan^{-1}\left(\omega C \frac{V_T}{I}\right)$$

For phase shift of  $-45^\circ$ , we obtain

$$-45 = -\tan^{-1}\left(2\pi \times 100 \times 10^3 \times 10 \times 10^{-9} \times \frac{0.025}{I}\right)$$

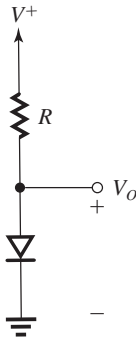
$$\Rightarrow I = 157 \mu\text{A}$$

Now  $I$  varies from  $\frac{157}{10} \mu\text{A}$  to  $157 \times 10 \mu\text{A}$

Range is  $15.7 \mu\text{A}$  to  $1570 \mu\text{A}$

Range of phase shift is  $-84.3^\circ$  to  $-5.71^\circ$

3.57

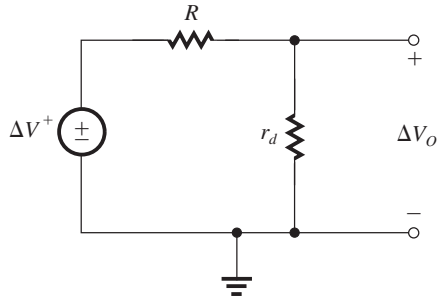


$$(a) \frac{\Delta V_O}{\Delta V^+} = \frac{r_d}{R + r_d} = \frac{V_T/I}{R + \frac{V_T}{I}}$$

$$= \frac{V_T}{IR + V_T}$$

For no load,  $I = \frac{V^+ - V_O}{R} = \frac{V^+ - 0.7}{R}$ .

$$\therefore \frac{\Delta V_O}{\Delta V^+} = \frac{V_T}{V_T + (V^+ - 0.7)}$$



Small-signal model

(b) If  $m$  diodes are in series, we obtain

$$\frac{\Delta V_O}{\Delta V^+} = \frac{mr_d}{mr_d + R} = \frac{mV_T}{mV_T + IR}$$

$$= \frac{mV_T}{mV_T + (V^+ - 0.7m)}$$

(c) For  $m = 1$

$$\frac{\Delta V_O}{\Delta V^+} = \frac{V_T}{V_T + V^+ - 0.7} = 1.75 \text{ mV/V}$$

For  $m = 4$

$$\frac{\Delta V_O}{\Delta V^+} = \frac{mV_T}{mV_T + 15 - m \times 0.7} = 8.13 \text{ mV/V}$$

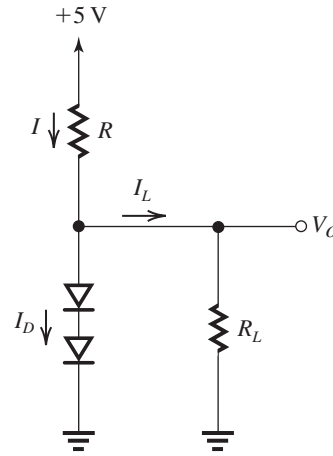
3.58

Diode has 0.7 V drop at 1 mA current

$$V_O = 1.5 \text{ V when } R_L = 1.5 \text{ k}\Omega$$

$$I_D = I_S e^{V/V_T}$$

$$1 \times 10^{-3} = I_S e^{0.7/0.025}$$



$$\Rightarrow I_S = 6.91 \times 10^{-16} \text{ A}$$

Voltage drop across each diode =  $\frac{1.5}{2} = 0.75 \text{ V}$ .

$$\therefore I_D = I_S e^{V/V_T} = 6.91 \times 10^{-16} \times e^{0.75/0.025}$$

$$= 7.38 \text{ mA}$$

$$I_L = 1.5/1.5 = 1 \text{ mA}$$

$$I = I_D + I_L = 7.39 \text{ mA} + 1 \text{ mA}$$

$$= 8.39 \text{ mA}$$

$$\therefore R = \frac{5 - 1.5}{8.39 \text{ mA}} = 417 \Omega$$

Use a small-signal model to find voltage  $\Delta V_O$  when the value of the load resistor,  $R_L$ , changes:

$$r_d = \frac{V_T}{I_D} = \frac{0.025}{7.39} = 3.4 \Omega$$

When load is disconnected, all the current  $I$  flows through the diode. Thus

$$\Delta I_D = 1 \text{ mA}$$

$$\Delta V_O = \Delta I_D \times 2r_d$$

$$= 1 \times 2 \times 3.4$$

$$= 6.8 \text{ mV}$$

With  $R_L = 1 \text{ k}\Omega$ ,

$$I_L \simeq \frac{1.5 \text{ V}}{1} = 1.5 \text{ mA}$$

$$\Delta I_L = 0.5 \text{ mA}$$

$$\Delta I_D = -0.5 \text{ mA}$$

$$\Delta V_O = -0.5 \times 2 \times 3.4$$

$$= -3.4 \text{ mV}$$

With  $R_L = 750 \Omega$ ,

$$I_L \simeq \frac{1.5}{0.75} = 2 \text{ mA}$$

$$\Delta I_L = 1 \text{ mA}$$

$$\Delta I_D = -1 \text{ mA}$$

$$\Delta V_O = -1 \times 2 \times 3.4$$

$$= -6.8 \text{ mV}$$

With  $R_L = 500 \Omega$ ,

$$I_L \simeq \frac{1.5}{0.5} = 3 \text{ mA}$$

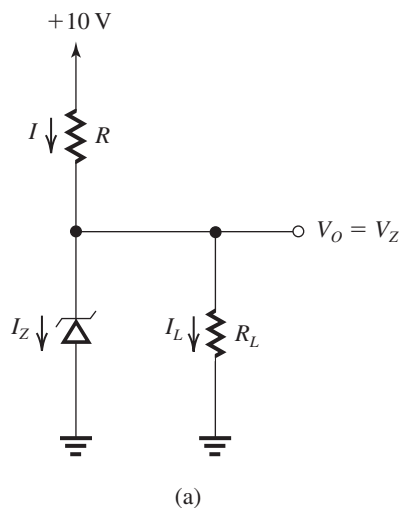
$$\Delta I_L = 2.0 \text{ mA}$$

$$\Delta I_D = -2.0 \text{ mA}$$

$$\Delta V_O = -2 \times 2 \times 3.4$$

$$= -13.6 \text{ mV}$$

3.59



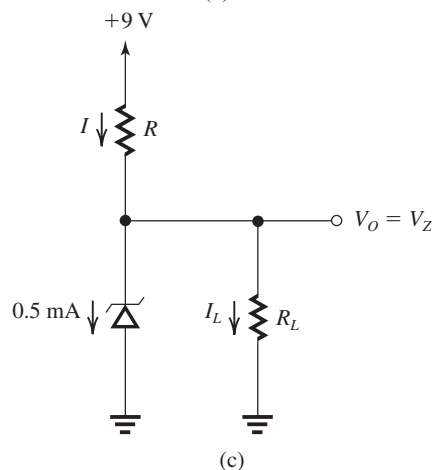
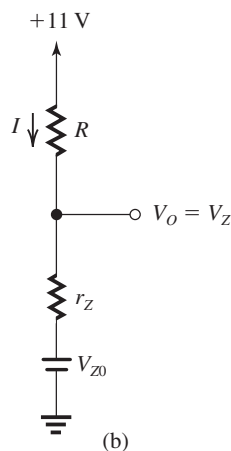
To obtain  $V_O = 7.5 \text{ V}$ , we must arrange for  $I_Z = 10 \text{ mA}$  (the current at which the zener is specified).

Now,

$$I_L = \frac{V_O}{R_L} = \frac{7.5}{1.5} = 5 \text{ mA}$$

Thus

$$I = I_Z + I_L = 10 + 5 = 15 \text{ mA}$$



and

$$R = \frac{10 - V_O}{I} = \frac{10 - 7.5}{15} = 167 \Omega$$

When the supply undergoes a change  $\Delta V_S$ , the change in the output voltage,  $\Delta V_O$ , can be determined from

$$\begin{aligned} \frac{\Delta V_O}{\Delta V_S} &= \frac{(R_L \parallel r_z)}{(R_L \parallel r_z) + R} \\ &= \frac{1.5 \parallel 0.03}{(1.5 \parallel 0.03) + 0.167} = 0.15 \end{aligned}$$

For  $\Delta V_S = +1 \text{ V}$  (10% high),  $\Delta V_O = +0.15 \text{ V}$  and  $V_O = 7.65 \text{ V}$ .

For  $\Delta V_S = -1 \text{ V}$  (10% low),  $\Delta V_O = -0.15 \text{ V}$  and  $V_O = 7.35 \text{ V}$ .

When the load is removed and  $V_S = 11 \text{ V}$ , we can use the zener model to determine  $V_O$ . Refer to Fig. (b). To determine  $V_{Z0}$ , we use

$$\begin{aligned} V_Z &= V_{Z0} + I_{ZT} r_z \\ 7.5 &= V_{Z0} + 0.01 \times 30 \end{aligned}$$

$$\Rightarrow V_{Z0} = 7.2 \text{ V}$$

From Fig. (b) we have

$$I = \frac{11 - 7.2}{0.167 + 0.03} = 19.3 \text{ mA}$$

Thus

$$V_O = V_{Z0} + I r_z$$

$$= 7.2 + 0.0193 \times 30 = 7.78 \text{ V}$$

To determine the smallest allowable value of  $R_L$  while  $V_S = 9 \text{ V}$ , refer to Fig. (c). Note that  $I_Z = 0.5 \text{ mA}$ , thus

$$V_Z = V_{ZK} \simeq V_{Z0} = 7.2 \text{ V}$$

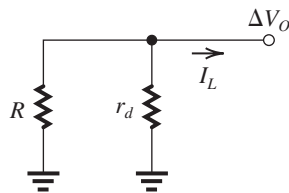
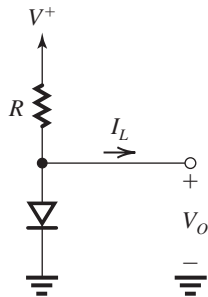
$$I = \frac{9 - 7.2}{0.167} = 10.69 \text{ mA}$$

$$I_L = I - I_Z = 10.69 - 0.5 = 10.19 \text{ mA}$$

$$R_L = \frac{V_O}{I_L} = \frac{7.2}{10.19} = 707 \ \Omega$$

$$V_O = 7.2 \text{ V}$$

**3.60**



Small-signal model

(a) From the small-signal model

$$\Delta V_O = -I_L (r_d \parallel R)$$

$$\frac{\Delta V_O}{I_L} = -(r_d \parallel R)$$

(b) At no load,  $I_D = \frac{V^+ - 0.7}{R}$

$$r_d = \frac{V_T}{I_D}$$

$$\frac{\Delta V_O}{I_L} = -(r_d \parallel R) = -\frac{1}{\frac{1}{r_d} + \frac{1}{R}}$$

$$= -\frac{1}{\frac{I_D}{V^+ - 0.7} + \frac{I_D}{mV_T}}$$

$$= -\frac{V_T}{I_D} \times \frac{1}{\frac{V^+ - 0.7}{mV_T} + 1}$$

$$= -\frac{V_T}{I_D} \times \frac{V^+ - 0.7}{V^+ - 0.7 + mV_T}$$

For  $\frac{\Delta V_O}{I_L} \leq 5 \frac{\text{mV}}{\text{mA}}$

i.e.,  $\frac{V_T}{I_D} \times \frac{V^+ - 0.7}{V^+ - 0.7 + mV_T} \leq 5 \frac{\text{mV}}{\text{mA}}$

$$\left[ \frac{25}{I_D} \times \frac{15 - 0.7}{0.025 + 15 - 0.7} \right] \leq 5 \frac{\text{mV}}{\text{mA}}$$

$$I_D \geq 4.99 \text{ mA}$$

$$I_D \simeq 5 \text{ mA}$$

$$R = \frac{V^+ - 0.7}{I_D} = \frac{15 - 0.7}{5 \text{ mA}}$$

$$R = 2.86 \text{ k}\Omega$$

Diode should be a 5-mA unit; that is, it conducts 5 mA at  $V_D = 0.7 \text{ V}$ , thus  $5 \times 10^{-3} = I_S e^{0.7/0.025}$ .

$$\Rightarrow I_S = 3.46 \times 10^{-15} \text{ A}$$

(c) For  $m$  diodes connected in series we have

$$I_D = \frac{V^+ - 0.7m}{R}$$

and  $r_d = \frac{V_T}{I_D}$

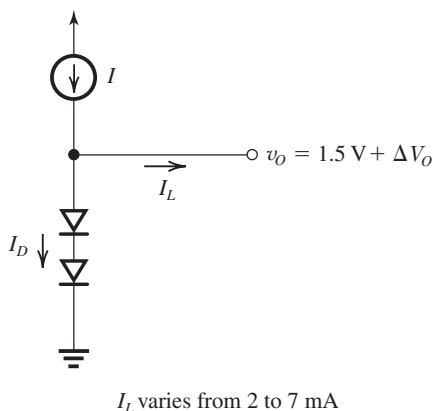
So now  $\frac{\Delta V_O}{I_L} = -(R \parallel mr_d) = -\frac{1}{\frac{1}{R} + \frac{1}{mr_d}}$

$$= -\frac{1}{\frac{I_D}{V^+ - 0.7m} + \frac{I_D}{mV_T}}$$

$$= -\frac{mV_T}{I_D} \frac{1}{\frac{V^+ - 0.7m}{mV_T} + 1}$$

$$= -\frac{mV_T}{I_D} \frac{V^+ - 0.7m}{V^+ - 0.7m + mV_T}$$

## 3.61



To supply a load current of 2 to 7 mA, the current  $I$  must be greater than 7 mA. So  $I$  can be only 10 mA or 15 mA.

Now let us evaluate  $\Delta V_o$  for both 10-mA and 15-mA sources. For the 10-mA source:

Since  $I_L$  varies from 2 to 7 mA, the current  $I_D$  will vary from 8 to 3 mA.

Correspondingly, the voltage across each diode changes by  $\Delta V_D$  where

$$\frac{3}{8} = e^{\Delta V_D / V_T}$$

$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{3}{8}\right) = -24.5 \text{ mV}$$

and the output voltage changes by

$$\Delta V_o = 2 \times \Delta V_D = -49 \text{ mV}$$

With  $I = 15$  mA, the diodes current changes from 13 to 8 mA. Correspondingly, the voltage across each diode changes by  $\Delta V_D$  where

$$\frac{8}{13} = e^{\Delta V_D / V_T}$$

$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{8}{13}\right) = -12.1 \text{ mV}$$

and the output voltage changes by

$$\Delta V_o = 2 \times \Delta V_D = -24.2 \text{ mV}$$

which is less than half that obtained with the 10-mA supply. Thus, from the point of view of reducing the change in  $V_o$  as  $I_L$  changes, we choose the 15-mA supply. Note, however, that the price paid is increased power dissipation.

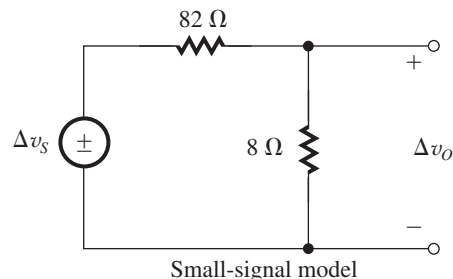
3.62 (a) Three 6.8-V zeners provide  $3 \times 6.8 = 20.4$  V with  $3 \times 10 = 30$ - $\Omega$  resistance. Neglecting  $R$ , we have

$$\text{Load regulation} = -30 \text{ mV/mA.}$$

(b) For 5.1-V zeners we use 4 diodes to provide 20.4 V with  $4 \times 30 = 120$ - $\Omega$  resistance.

$$\text{Load regulation} = -120 \text{ mV/mA}$$

## 3.63



From the small-signal model we obtain

$$\frac{\Delta v_o}{\Delta v_s} = \frac{8}{8 + 82} = \frac{8}{90}$$

Now  $\Delta v_s = 1.0$  V.

$$\therefore \Delta v_o = \frac{8}{90} \Delta v_s = \frac{8}{90} \times 1.0$$

$$= 88.9 \text{ mV}$$

3.64 (a)  $V_{ZT} = V_{Z0} + r_z I_{ZT}$

$$10 = V_{Z0} + 7(0.025)$$

$$\Rightarrow V_{Z0} = 9.825 \text{ V}$$

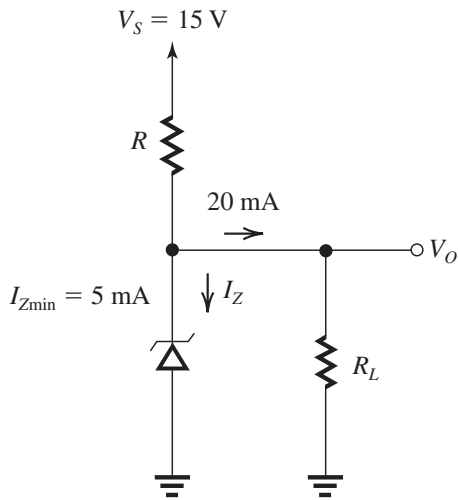
(b) The minimum zener current of 5 mA occurs when  $I_L = 20$  mA and  $V_S$  is at its minimum of  $20(1 - 0.25) = 15$  V. See the circuit below.

$$R \leq \frac{15 - V_{Z0}}{20 + 5}$$

where we have used the minimum value of  $V_S$ , the maximum value of load current, and the minimum required value of zener diode current, and we assumed that at this current  $V_Z \simeq V_{Z0}$ . Thus,

$$R \leq \frac{15 - 9.825 + 7}{25}$$

$$\leq 207 \Omega.$$



$$20 + \frac{1}{4}(20) = 25 \text{ V.}$$

$$V_Z = V_{Z0} + r_z I_Z$$

$$= 9.825 + 7 \times \frac{25 - V_Z}{207}$$

$$207V_Z = 207(9.825) + 7(25) - 7V_Z$$

$$\Rightarrow V_Z = 10.32 \text{ V}$$

$$I_{Z\max} = \frac{25 - 10.32}{0.207} = 70.9 \text{ mA}$$

$$P_Z = 10.32 \times 70.9$$

$$= 732 \text{ mW}$$

∴ use  $R = 207 \Omega$

(c) Line regulation  $= \frac{7}{207 + 7} = 33 \frac{\text{mV}}{\text{V}}$

±25% change in  $v_S \equiv \pm 5 \text{ V}$

$V_O$  changes by  $\pm 5 \times 33 = \pm 0.165 \text{ mV}$

corresponding to  $\frac{\pm 0.165}{10} \times 100 = \pm 1.65\%$

(d) Load regulation  $= -(r_z \parallel R)$

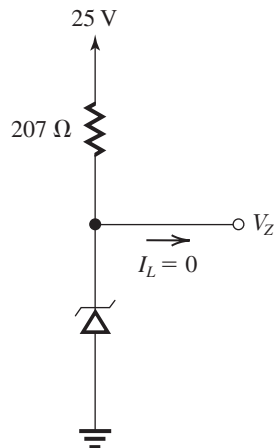
$$= -(7 \parallel 207) = -6.77 \Omega$$

or  $-6.77 \text{ V/A}$

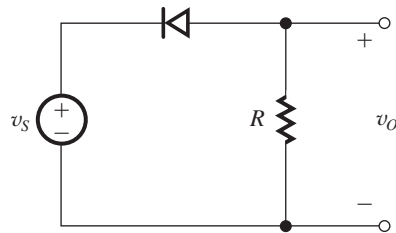
$$\Delta V_O = -6.77 \times 20 \text{ mA} = -135.4 \text{ mV}$$

corresponding to  $-\frac{0.1354}{10} \times 100 = -1.35\%$

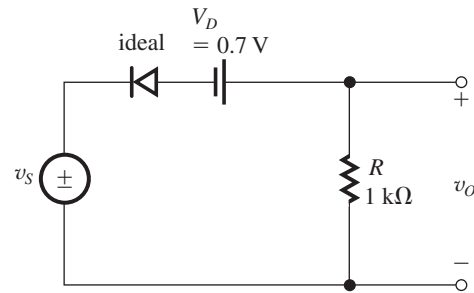
(e) The maximum zener current occurs at no load  $I_L = 0$  and the supply at its largest value of



3.65

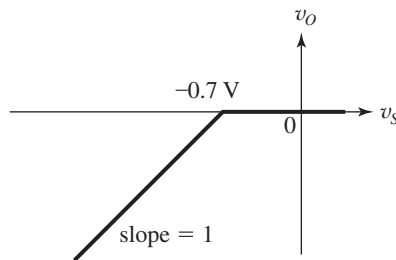


Using the constant voltage drop model:



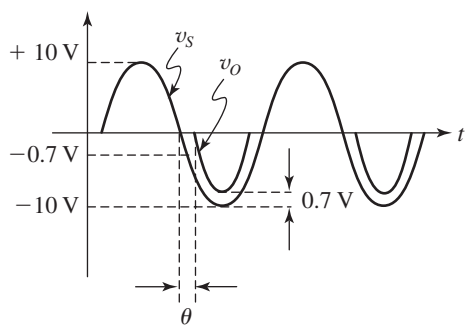
(a)  $v_O = v_S + 0.7 \text{ V}$ , For  $v_S \leq -0.7 \text{ V}$

$v_O = 0$ , for  $v_S \geq -0.7 \text{ V}$





(b)



(c) The diode conducts at an angle

$$\theta = \sin^{-1}\left(\frac{0.7}{10}\right) = 4^\circ \text{ and stops}$$

at  $\pi - \theta = 176^\circ$

Thus the conduction angle is  $\pi - 2\theta$

$= 172^\circ$  or 3 rad.

$$v_{O,\text{avg}} = \frac{-1}{2\pi} \int_{\theta}^{\pi-\theta} (10 \sin \phi - 0.7) d\phi$$

$$= \frac{-1}{2\pi} [-10 \cos \phi - 0.7\phi]_{\theta}^{\pi-\theta}$$

$$= -2.85 \text{ V}$$

(d) Peak current in diode is

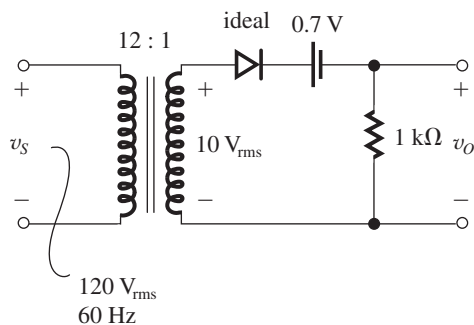
$$\frac{10 - 0.7}{1} = 9.3 \text{ mA}$$

(e) PIV occurs when  $v_s$  is at its the peak and

$v_o = 0$ .

PIV = 10 V

3.66



$$\hat{v}_O = 10\sqrt{2} - 0.7 = 13.44 \text{ V}$$

Conduction begins at

$$10\sqrt{2} \sin \theta = 0.7$$

$$\theta = \sin^{-1}\left(\frac{0.7}{10\sqrt{2}}\right) = 2.84^\circ$$

$$= 0.0495 \text{ rad}$$

Conduction ends at  $\pi - \theta$ .

$$\therefore \text{Conduction angle} = \pi - 2\theta = 3.04 \text{ rad}$$

The diode conducts for

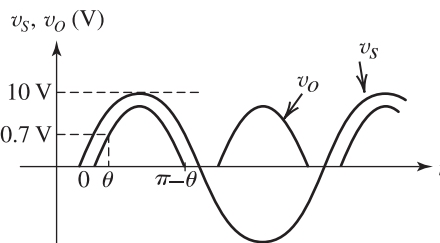
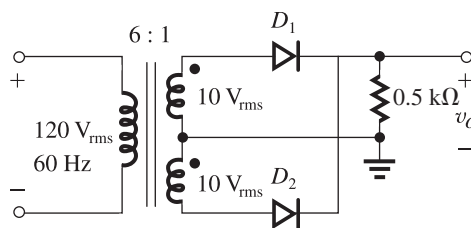
$$\frac{3.04}{2\pi} \times 100 = 48.4\% \text{ of the cycle}$$

$$v_{O,\text{avg}} = \frac{1}{2\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2} \sin \phi - 0.7) d\phi$$

$$= 4.15 \text{ V}$$

$$i_{D,\text{avg}} = \frac{v_{O,\text{avg}}}{R} = 4.15 \text{ mA}$$

3.67



$$\hat{v}_O = 10\sqrt{2} - V_D = 13.44 \text{ V}$$

$$\text{Conduction starts at } \theta = \sin^{-1} \frac{0.7}{10\sqrt{2}} =$$

$$2.84^\circ = 0.05 \text{ rad}$$

and ends at  $\pi - \theta$ . Conduction angle =  $\pi - 2\theta = 3.04 \text{ rad}$  in each half cycle. Thus the

fraction of a cycle for which one of the two diodes conduct =  $\frac{2(3.04)}{2\pi} \times 100 = 96.8\%$

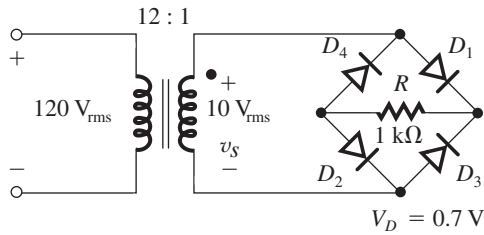
Note that during 96.8% of the cycle there will be conduction. However, each of the two diodes conducts for only half the time, i.e., for 48.4% of the cycle.

$$v_{O,avg} = \frac{1}{\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2}\sin\phi - 0.7)d\phi$$

$$= 8.3 \text{ V}$$

$$i_{L,avg} = \frac{8.3}{0.5 \text{ k}\Omega} = 16.6 \text{ mA}$$

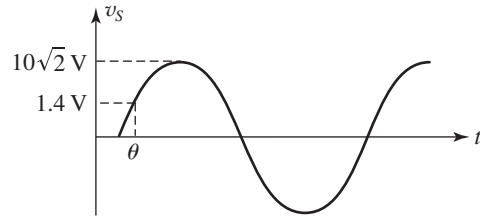
**3.68**



Peak voltage across  $R = 10\sqrt{2} - 2V_D$

$$= 10\sqrt{2} - 1.4$$

$$= 12.74 \text{ V}$$



$$\theta = \sin^{-1} \frac{1.4}{10\sqrt{2}} = 5.68^\circ = 0.1 \text{ rad}$$

Fraction of cycle that  $D_1$  &  $D_2$  conduct is

$$\frac{\pi - 2\theta}{2\pi} \times 100 = 46.8\%$$

Note that  $D_3$  &  $D_4$  conduct in the other half cycle so that there is  $2(46.8) = 93.6\%$  conduction interval.

$$v_{O,avg} = \frac{2}{2\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2}\sin\phi - 2V_D) d\phi$$

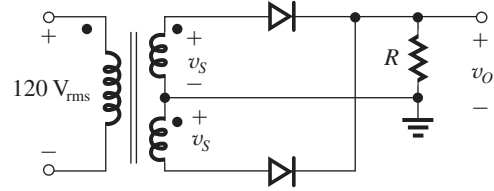
$$= \frac{1}{\pi} \left[ -12\sqrt{2} \cos\phi - 1.4\phi \right]_{\theta}^{\pi-\theta}$$

$$= \frac{2(12\sqrt{2} \cos\theta) - 1.4(\pi - 2\theta)}{\pi}$$

$$= 7.65 \text{ V}$$

$$i_{R,avg} = \frac{v_{O,avg}}{R} = \frac{7.65}{1} = 7.65 \text{ mA}$$

**3.69**



Refer to Fig. 3.22.

For  $V_D \ll V_s$ , conduction angle  $\simeq \pi$ , and

$$v_{O,avg} = \frac{2}{\pi} V_s - V_D = \frac{2}{\pi} V_s - 0.7$$

(a) For  $v_{O,avg} = 10 \text{ V}$

$$V_s = \frac{\pi}{2} \times 10.7 = 16.8 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{16.8} = 10.1 \text{ to } 1$$

(b) For  $v_{O,avg} = 100 \text{ V}$

$$V_s = \frac{\pi}{2} \times 100.7 = 158.2 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{158.2} = 1.072 \text{ to } 1$$

**3.70** Refer to Fig. 3.23.

For  $2V_D \ll V_s$

$$V_{O,avg} = \frac{2}{\pi} V_s - 2V_D = \frac{2}{\pi} V_s - 1.4$$

(a) For  $V_{O,avg} = 10 \text{ V}$

$$10 \text{ V} = \frac{2}{\pi} \cdot V_s - 1.4$$

$$\therefore \hat{V}_s = 11.4 \left( \frac{\pi}{2} \right) = 17.9 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{17.9} = 9.5 \text{ to } 1$$

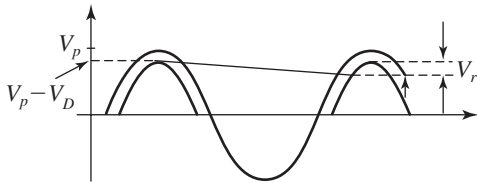
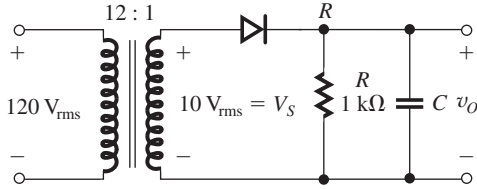
(b) For  $V_{O,avg} = 100 \text{ V}$

$$100 \text{ V} = \frac{2}{\pi} \cdot V_s - 1.4$$

$$\Rightarrow V_s = 101.4 \left( \frac{\pi}{2} \right) = 159 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{159} = 1.07 \text{ to } 1$$

3.71



(i)  $V_r \cong (V_p - V_D) \frac{T}{CR}$  [Eq. (3.28)]

$$0.1 (V_p - V_D) = (V_p - V_D) \frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = 166.7 \mu\text{F}$$

(ii) For

$$V_r = 0.01 (V_p - V_D) = \frac{(V_p - V_D) T}{CR}$$

$$C = 1667 \mu\text{F}$$

(a) (i)  $v_{O,avg} = V_p - V_D - \frac{1}{2} V_r$

$$= 10\sqrt{2} - 0.7 - \frac{1}{2} (10\sqrt{2} - 0.7) 0.1$$

$$= (10\sqrt{2} - 0.7) \left( 1 - \frac{0.1}{2} \right)$$

$$= 12.77 \text{ V}$$

(ii)  $v_{O,avg} = (10\sqrt{2} - 0.7) \left( 1 - \frac{0.01}{2} \right)$

$$= 13.37 \text{ V}$$

(b) (i) Using Eq (3.30), we have the conduction angle =

$$\omega \Delta t \cong \sqrt{2V_r / (V_p - V_D)}$$

$$= \sqrt{\frac{2 \times 0.1 (V_p - 0.7)}{(V_p - 0.7)}}$$

$$= \sqrt{0.2}$$

$$= 0.447 \text{ rad}$$

$\therefore$  Fraction of cycle for

$$\text{conduction} = \frac{0.447}{2\pi} \times 100$$

$$= 7.1\%$$

(ii)  $\omega \Delta t \cong \sqrt{2 \times 0.01 \frac{(V_p - 0.7)}{V_p - 0.7}} = 0.141 \text{ rad}$

$$\text{Fraction of cycle} = \frac{0.141}{2\pi} \times 100 = 2.24\%$$

(c) (i) Use Eq (3.31):

$$i_{D,avg} = I_L \left( 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right)$$

$$= \frac{v_{O,avg}}{R} \left( 1 + \pi \sqrt{\frac{2(V_p - V_D)}{0.1(V_p - V_D)}} \right)$$

$$= \frac{12.77}{10^3} \left( 1 + \pi \sqrt{\frac{2}{0.1}} \right)$$

$$= 192 \text{ mA}$$

(ii)  $i_{D,avg} = \frac{13.37}{10^3} (1 + \pi \sqrt{200})$

$$= 607 \text{ mA}$$

(d) Adapting Eq. (3.32), we obtain

(i)  $i_{D,peak} = I_L \left( 1 + 2\pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right)$

$$= \frac{12.77}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.1}} \right)$$

$$= 371 \text{ mA}$$

(ii)  $i_{D,peak} = \frac{13.37}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.01}} \right)$

$$= 1201 \text{ mA} \cong 1.2 \text{ A}$$

$$3.72 \text{ (i) } V_r = 0.1 (V_p - V_D) = \frac{(V_p - V_D)}{2fCR}$$

The factor of 2 accounts for discharge occurring only during half of the period,  $T/2 = \frac{1}{2f}$ .

$$C = \frac{1}{(2fR)0.1} = \frac{1}{2(60)10^3 \times 0.1} = 83.3 \mu\text{F}$$

$$\text{(ii) } C = \frac{1}{2(60) \times 10^3 \times 0.01} = 833 \mu\text{F}$$

$$\text{(a) (i) } V_O = V_p - V_D - \frac{1}{2}V_r$$

$$= (V_p - V_D) \left(1 - \frac{0.1}{2}\right)$$

$$= (13.44) \left(1 - \frac{0.1}{2}\right)$$

$$= 12.77 \text{ V}$$

$$\text{(ii) } V_O = (13.44) \left(1 - \frac{0.01}{2}\right) = 13.37 \text{ V}$$

$$\text{(b) (i) Fraction of cycle} = \frac{2\omega\Delta t}{2\pi} \times 100$$

$$= \frac{\sqrt{2V_r/(V_p - V_D)}}{\pi} \times 100$$

$$= \frac{1}{\pi} \sqrt{2(0.1)} \times 100 = 14.2\%$$

$$\text{(ii) Fraction of cycle} = \frac{2\sqrt{2(0.01)}}{2\pi} \times 100$$

$$= 4.5\%$$

(c) Use Eq. (3.34):

$$\text{(i) } i_{D, \text{avg}} = I_L \left(1 + \pi \sqrt{\frac{V_p - V_D}{2V_r}}\right)$$

$$= \frac{12.77}{1} \left(1 + \pi \sqrt{\frac{1}{2(0.1)}}\right) = 102.5 \text{ mA}$$

$$\text{(ii) } i_{D, \text{avg}} = \frac{13.37}{1} \left(1 + \pi \frac{1}{\sqrt{2(0.01)}}\right) = 310 \text{ mA}$$

(d) Use Eq. (3.35):

$$\text{(i) } \hat{i}_D = I_L \left(1 + 2\pi \frac{1}{\sqrt{2(0.1)}}\right) = 192 \text{ mA}$$

$$\text{(ii) } \hat{i}_D = I_L \left(1 + 2\pi \frac{1}{\sqrt{0.02}}\right) = 607 \text{ mA}$$

$$3.73 \text{ (i) } V_r = 0.1 (V_p - V_D \times 2) = \frac{V_p - 2V_D}{2fCR}$$

$$C = \frac{(V_p - 2V_D)}{(V_p - 2V_D)} \frac{1}{2(0.1)fR} = 83.3 \mu\text{F}$$

$$\text{(ii) } C = \frac{1}{2(0.01)fR} = 833 \mu\text{F}$$

$$\text{(a) } V_O = V_p - 2V_D - \frac{1}{2}V_r$$

$$\text{(i) } V_O = V_p - 2V_D - \frac{1}{2}(V_p - 2V_D) \times 0.1$$

$$= (V_p - 2V_D) \times 0.95$$

$$= (10\sqrt{2} - 2 \times 0.7) \times 0.95 = 12.1 \text{ V}$$

$$\text{(ii) } V_O = (10\sqrt{2} - 2 \times 0.7) \times 0.995 = 12.68 \text{ V}$$

$$\text{(b) (i) Fraction of cycle} = \frac{2\omega\Delta t}{2\pi} \times 100$$

$$= \frac{\sqrt{2(0.1)}}{\pi} \times 100 = 14.2\%$$

$$\text{(ii) Fraction of cycle} = \frac{\sqrt{2(0.01)}}{\pi} \times 100 = 4.5\%$$

$$\text{(c) (i) } i_{D, \text{avg}} = \frac{12.1}{1} \left(1 + \pi \sqrt{\frac{1}{0.2}}\right) = 97 \text{ mA}$$

$$\text{(ii) } i_{D, \text{avg}} = \frac{12.68}{1} \left(1 + \pi/\sqrt{0.02}\right) = 249 \text{ mA}$$

$$\text{(d) (i) } \hat{i}_D = \frac{12.1}{1} \left(1 + 2\pi \sqrt{\frac{1}{0.2}}\right) = 182 \text{ mA}$$

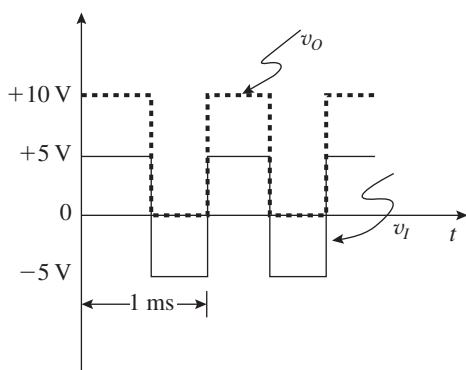
$$\text{(ii) } \hat{i}_D = \frac{12.68}{1} \left(1 + 2\pi \sqrt{\frac{1}{0.02}}\right) = 576 \text{ mA}$$

**3.74** Refer to Fig. P3.74. When  $v_I$  is positive,  $v_A$  goes positive, turning on the diode and closing the negative feedback loop around the op amp. The result is that  $v_- = v_I$ ,  $v_O = 2v_- = 2v_I$ , and  $v_A = v_O + 0.7$ . Thus

$$\text{(a) } v_I = +1 \text{ V, } v_- = +1 \text{ V, } v_O = +2 \text{ V, and } v_A = +2.7 \text{ V.}$$

$$\text{(b) } v_I = +3 \text{ V, } v_- = +3 \text{ V, } v_O = +6 \text{ V, and } v_A = +6.7 \text{ V.}$$

When  $v_I$  goes negative,  $v_A$  follows, the diode turns off, and the feedback loop is opened. The op amp saturates with  $v_A = -13 \text{ V}$ ,  $v_- = 0 \text{ V}$  and  $v_O = 0 \text{ V}$ . Thus



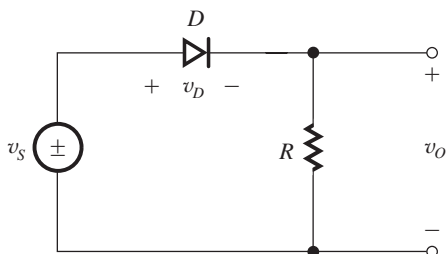
(c)  $v_I = -1\text{ V}$ ,  $v_- = 0\text{ V}$ ,  $v_O = 0\text{ V}$ , and  $v_A = -13\text{ V}$ .

(d)  $v_I = -3\text{ V}$ ,  $v_- = 0\text{ V}$ ,  $v_O = 0\text{ V}$ , and  $v_A = -13\text{ V}$ .

Finally, if  $v_I$  is a symmetrical square wave of 1-kHz frequency, 5-V amplitude, and zero average, the output will be zero during the negative half cycles of the input and will equal twice the input during the positive half cycles. See figure above.

Thus,  $v_O$  is a square wave with 0-V and +10-V levels, i.e. 5-V average and, of course, the same frequency (1 kHz) as the input.

3.75



$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{i_D(1\text{ mA})} = e^{[v_D - v_D(\text{at } 1\text{ mA})]/V_T}$$

$$v_D - v_D(\text{at } 1\text{ mA}) = V_T \ln \left[ \frac{i_D}{1\text{ mA}} \right]$$

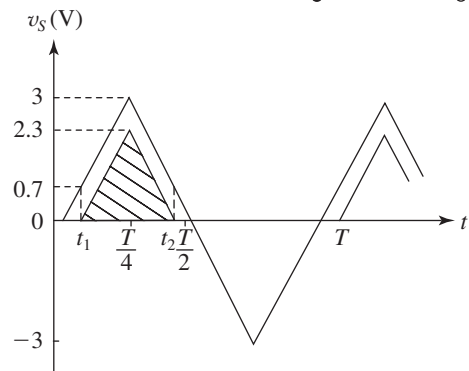
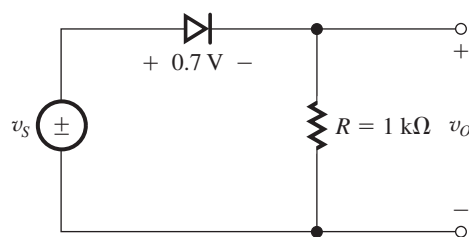
$$v_D = v_D(\text{at } 1\text{ mA}) + V_T \ln \left[ \frac{v_O/R}{1} \right]$$

$$v_O = v_S - v_D$$

$$= v_S - v_D(\text{at } 1\text{ mA}) - V_T \ln \left( \frac{v_O}{R} \right)$$

where  $R$  is in  $\text{k}\Omega$ .

3.76



First find  $t_1$  and  $t_2$

$$\frac{3}{T} = \frac{0.7}{t_1}$$

$$\Rightarrow t_1 = 0.0583 T$$

$$t_2 = \frac{T}{2} - t_1$$

$$= \frac{T}{2} - 0.0583 T$$

$$t_2 = 0.442 T$$

$$v_O(\text{ave.}) = \frac{1}{T} \times \text{area of shaded triangle}$$

$$= \frac{1}{T} \times (3 - 0.7) \times \left( \frac{T}{4} - t_1 \right)$$

$$= \frac{1}{T} \times 2.3 \times T \left( \frac{1}{4} - 0.0583 \right)$$

$$= 0.441\text{ V}$$

3.77  $120\sqrt{2} \pm 10\%$ ;  $20\sqrt{2} \pm 10\%$

$$\Rightarrow \text{Turns ratio} = 6:1$$

$$v_S = \frac{20\sqrt{2}}{2} \pm 10\%$$

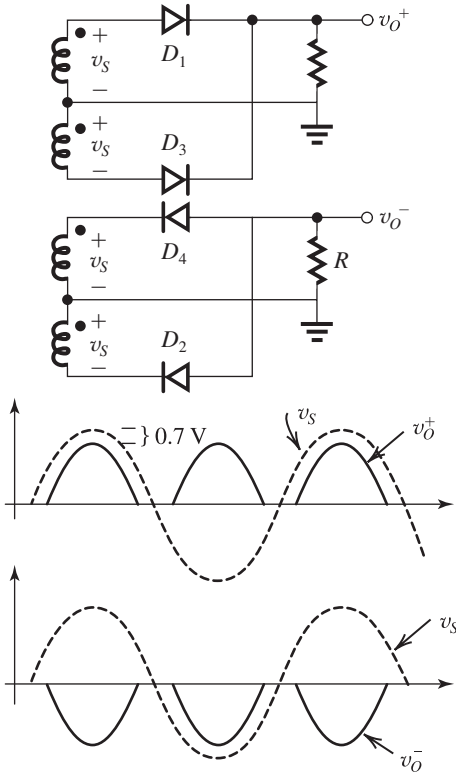
$$\text{PIV} = 2V_S - V_D$$

$$= 2 \times \frac{20\sqrt{2}}{2} \times 1.1 - 0.7$$

$$= 30.4 \text{ V}$$

Using a factor of 1.5 for safety, we select a diode having a PIV rating of approximately 45 V.

**3.78** The circuit is a full-wave rectifier with center tapped secondary winding. The circuit can be analyzed by looking at  $v_o^+$  and  $v_o^-$  separately.



$$v_{o, \text{avg}} = \frac{1}{2\pi} \int (V_s \sin\phi - 0.7) d\phi = 12$$

$$= \frac{2V_s}{\pi} - 0.7 = 12$$

where we have assumed  $V_s \gg 0.7 \text{ V}$  and thus the conduction angle (in each half cycle) is almost  $\pi$ .

$$V_s = \frac{12 + 0.7}{2} \pi = 19.95 \text{ V}$$

Thus voltage across secondary winding

$$= 2V_s \simeq 40 \text{ V}$$

Looking at  $D_4$ ,

$$\text{PIV} = V_s - V_o^-$$

$$= V_s + (V_s - 0.7)$$

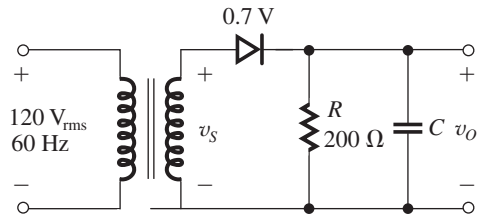
$$= 2V_s - 0.7$$

$$= 39.2 \text{ V}$$

If choosing a diode, allow a safety margin by moving a factor of 1.5, thus

$$\text{PIV} \simeq 60 \text{ V}$$

**3.79**



$$V_o = 12 \text{ V} \pm 1 \text{ V (ripple)}$$

$$R_L = 200 \Omega$$

$$(a) V_o = V_p - V_D - 1$$

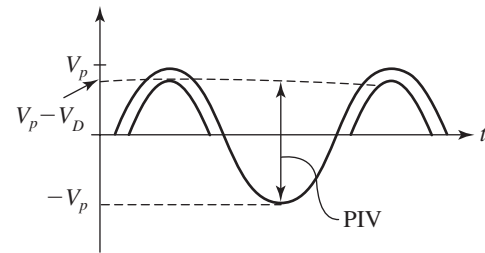
$$\Rightarrow V_p = 13 + 0.7 = 13.7 \text{ V}$$

$$V_{rms} = \frac{13.7}{\sqrt{2}} = 9.7 \text{ V}$$

$$(b) V_r = \frac{V_p - V_D}{fCR}$$

$$2 = \frac{13.7 - 0.7}{60 \times C \times 200}$$

$$\Rightarrow C = \frac{13}{2 \times 60 \times 200} = 542 \mu\text{F}$$



(c) When the diode is cut off, the maximum reverse voltage across it will occur when  $v_s = -V_p$ . At this time,  $v_o = V_o$  and the maximum reverse voltage will be

$$\text{Maximum reverse voltage} = V_o + V_p$$

$$= 12 + 13.7 = 25.7 \text{ V}$$

Using a factor of safety of 1.5 we obtain

$$\text{PIV} = 1.5 \times 25.7$$

$$= 38.5 \text{ V}$$

$$(d) i_{D_{\text{av}}} = I_L \left[ 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{V_O}{R_L} \left[ 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{2(13.7 - 0.7)}{2}} \right]$$

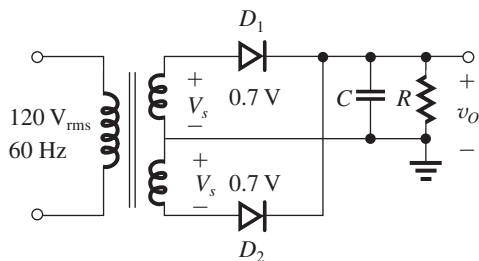
$$= 739 \text{ mA}$$

$$(e) i_{D_{\text{max}}} = I_L \left[ 1 + 2\pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{2(13.7 - 0.7)}{2}} \right]$$

$$= 1.42 \text{ A}$$

**3.80**

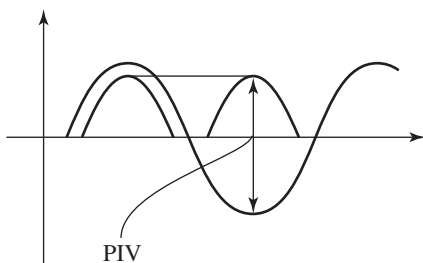


$$(a) V_O = V_p - V_D - 1$$

$$\Rightarrow V_p = V_O + V_D + 1 = 13 + 0.7 = 13.7 \text{ V}$$

$$V_{\text{rms}} = \frac{13.7}{\sqrt{2}} = 9.7 \text{ V}$$

This voltage appears across each half of the transformer secondary. Across the entire secondary we have  $2 \times 9.7 = 19.4 \text{ V}$  (rms).



$$(b) V_r = \frac{V_p - V_D}{2fCR}$$

$$2 = \frac{13.7 - 0.7}{2 \times 60 \times 200 \times C}$$

$$\Rightarrow C = \frac{12}{2 \times 2 \times 60 \times 200} = 271 \mu\text{F}$$

(c) Maximum reverse voltage across  $D_1$  occurs when  $v_s = -V_p$ . At this point  $v_o = V_O$ . Thus maximum reverse voltage =  $V_O + V_p = 12 + 13.7 = 25.7$ . The same applies to  $D_2$ .

In specifying the PIV for the diodes, one usually uses a factor of safety of about 1.5,

$$\text{PIV} = 1.5 \times 25.7 = 38.5 \text{ V}$$

$$(d) i_{D_{\text{av}}} = I_L \left[ 1 + \pi \sqrt{\frac{V_p - V_D}{2 V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{13.7 - 0.7}{2 \times 2}} \right]$$

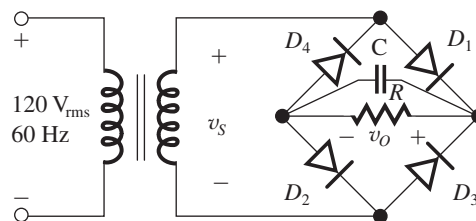
$$= 399 \text{ mA}$$

$$(e) i_{D_{\text{max}}} = I_L \left[ 1 + 2\pi \sqrt{\frac{V_p - V_D}{2 V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{13.7 - 0.7}{2 \times 2}} \right]$$

$$= 739 \text{ mA}$$

**3.81**



$$(a) V_O = V_p - 2V_D - 1$$

$$\Rightarrow V_p = V_O + 2V_D + 1 = 12 + 2 \times 0.7 + 1 = 14.4 \text{ V}$$

$$V_{\text{rms}} = \frac{14.4}{\sqrt{2}} = 10.2 \text{ V}$$

$$(b) V_r = \frac{V_p - 2 V_D}{2fCR}$$

$$\Rightarrow C = \frac{14.4 - 1.4}{2 \times 2 \times 60 \times 200} = 271 \mu\text{F}$$

(c) The maximum reverse voltage across  $D_1$  occurs when  $V_s = -V_p = -14.4$  V. At this time  $D_3$  is conducting, thus

$$\begin{aligned} \text{Maximum reverse voltage} &= -V_p + V_{D3} \\ &= -14.4 + 0.7 = -13.7 \text{ V} \end{aligned}$$

The same applies to the other three diodes. In specifying the PIV rating for the diode we use a factor of safety of 1.5 to obtain

$$\text{PIV} = 1.5 \times 13.7 = 20.5 \text{ V}$$

$$\begin{aligned} \text{(d) } i_{D_{\text{av}}} &= I_L \left[ 1 + \pi \sqrt{\frac{V_p - 2V_D}{2V_r}} \right] \\ &= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{14.4 - 1.4}{2 \times 2}} \right] \end{aligned}$$

$$= 400 \text{ mA}$$

$$\begin{aligned} \text{(e) } i_{D_{\text{max}}} &= I_L \left[ 1 + 2\pi \sqrt{\frac{V_p - 2V_D}{2V_r}} \right] \\ &= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{14.4 - 0.7}{2 \times 2}} \right] \end{aligned}$$

$$= 740 \text{ mA}$$

**3.82**

During the diode's off interval (which is almost equal to  $T$ ) the capacitor discharges and its voltage is given by

$$v_O(t) = 11.3 e^{-t/CR}$$

where  $C = 100 \mu\text{F}$  and  $R = 100 \Omega$ , thus

$$CR = 100 \times 10^{-6} \times 100 = 0.01 \text{ s}$$

At the end of the discharge interval,  $t \simeq T$  and

$$v_O = 11.3 e^{-T/CR}$$

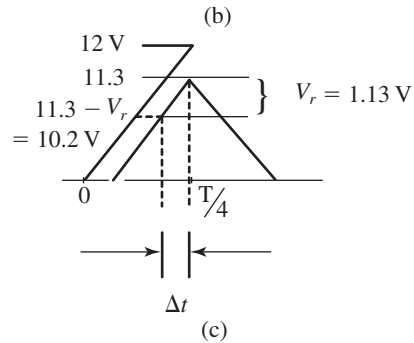
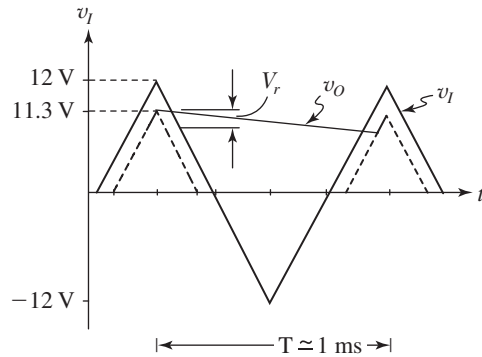
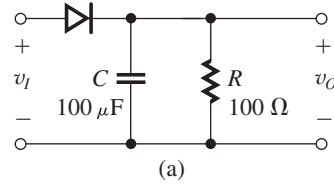
Since  $T = 0.001$  s is much smaller than  $CR$ ,

$$v_O \simeq 11.3 \left( 1 - \frac{T}{CR} \right)$$

The ripple voltage  $V_r$  can be found as

$$\begin{aligned} V_r &= 11.3 - 11.3 \left( 1 - \frac{T}{CR} \right) \\ &= \frac{11.3T}{CR} = \frac{11.3 \times 0.001}{0.01} = 1.13 \text{ V} \end{aligned}$$

The average dc output voltage is



$$v_O = 11.3 - \frac{V_r}{2} = 11.3 - \frac{1.13}{2} = 10.74 \text{ V}$$

To obtain the interval during which the diode conducts,  $\Delta t$ , refer to Fig. (c).

$$\begin{aligned} \frac{12}{T/4} &= \frac{V_r}{\Delta t} \\ \Rightarrow \Delta t &= \frac{V_r \times (T/4)}{12} = \frac{1.13 \times 1}{12 \times 4} \\ &= 23.5 \mu\text{s} \end{aligned}$$

Now, using the fact that the charge gained by the capacitor when the diode is conducting is equal to the charge lost by the capacitor during its discharge interval, we can write

$$\begin{aligned} i_{C_{\text{av}}} \times \Delta t &= C V_r \\ \Rightarrow i_{C_{\text{av}}} &= \frac{C V_r}{\Delta t} = \frac{100 \times 10^{-6} \times 1.13}{23.5 \times 10^{-6}} = 4.8 \text{ A} \end{aligned}$$

$$i_{D_{\text{av}}} = i_{C_{\text{av}}} + i_{L_{\text{av}}}$$

where  $i_{L_{\text{av}}}$  is the average current through  $R$  during the short interval  $\Delta t$ . This is approximately

$$\frac{11.3}{R} = \frac{11.3}{100} = 0.113 \text{ A. Thus}$$



$$i_{D_{av}} = 4.8 + 0.113 = 4.913 \text{ A}$$

Finally, to obtain the peak diode current, we use

$$\begin{aligned} i_{D_{max}} &= i_{C_{max}} + i_{L_{max}} \\ &= C \frac{dv_I}{dt} + \frac{11.3}{R} \\ &= C \times \frac{12}{T/4} + \frac{11.3}{R} \\ &= 100 \times 10^{-6} \times \frac{12 \times 4}{1 \times 10^{-3}} + \frac{11.3}{100} \\ &= 4.8 + 0.113 = 4.913 \text{ A} \end{aligned}$$

which is equal to the average value. This is a result of the linear  $v_I$  which gives rise to a constant capacitor current during the diode conduction interval. Thus  $i_{C_{max}} = i_{C_{av}} = 4.8 \text{ A}$ . Also, the maximum value of  $i_L$  is approximately equal to its average value during the short interval  $\Delta t$ .

**3.83** Refer to Fig. P3.78 and let a capacitor  $C$  be connected across each of the load resistors  $R$ . The two supplies  $v_O^+$  and  $v_O^-$  are identical. Each is a full-wave rectifier similar to that of the tapped-transformer circuit. For each supply,

$$V_O = 12 \text{ V}$$

$$V_r = 1 \text{ V (peak to peak)}$$

Thus

$$v_O = 12 \pm 0.5 \text{ V}$$

It follows that the peak value of  $v_S$  must be  $12.5 + 0.7 = 13.2 \text{ V}$  and the total rms voltage across the secondary will be

$$= \frac{2 \times 13.2}{\sqrt{2}} = 18.7 \text{ V (rms)}$$

$$\text{Transformer turns ratio} = \frac{120}{18.7} = 6.43:1$$

To deliver 100-mA dc current to each load,

$$R = \frac{12}{0.1} = 120 \ \Omega$$

Now, the value of  $C$  can be found from

$$V_r = \frac{V_p - 0.7}{2fCR}$$

$$1 = \frac{12.5}{2 \times 60 \times C \times 120}$$

$$\Rightarrow C = 868 \ \mu\text{F}$$

To specify the diodes, we determine  $i_{D_{av}}$  and  $i_{D_{max}}$ ,

$$\begin{aligned} i_{D_{av}} &= I_L(1 + \pi\sqrt{(V_p - 0.7)/2 V_r}) \\ &= 0.1(1 + \pi\sqrt{12.5/2}) \\ &= 785 \text{ mA} \end{aligned}$$

$$\begin{aligned} i_{D_{max}} &= I_L(1 + 2\pi\sqrt{(V_p - 0.7)/2 V_r}) \\ &= 0.1(1 + 2\pi\sqrt{12.5/2}) \\ &= 1.671 \text{ A} \end{aligned}$$

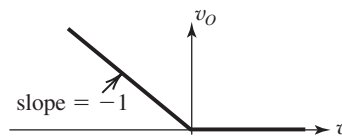
To determine the required PIV rating of each diode, we determine the maximum reverse voltage that appears across one of the diodes, say  $D_1$ . This occurs when  $v_S$  is at its maximum negative value  $-V_p$ . Since the cathode of  $D_1$  will be at  $+12.5 \text{ V}$ , the maximum reverse voltage across  $D_1$  will be  $12.5 + 13.2 = 25.7 \text{ V}$ . Using a factor of safety of 1.5, then each of the four diodes must have

$$\text{PIV} = 1.5 \times 25.7 = 38.6 \text{ V}$$

**3.84**  $v_I > 0$ :  $D_1$  conducts and  $D_2$  cutoff

$v_I < 0$ :  $D_1$  cutoff,

$$D_2 \text{ conducts} \sim \frac{v_O}{v_I} = -1$$



(a)  $v_I = +1 \text{ V}$

$$v_O = 0 \text{ V}$$

$$v_A = -0.7 \text{ V}$$

Keeps  $D_2$  off so no current flows through  $R$

$$\Rightarrow v_- = 0 \text{ V}$$

Virtual ground as feedback loop is closed through  $D_1$

(b)  $v_I = +3 \text{ V}$

$$v_O = 0 \text{ V}$$

$$v_A = -0.7 \text{ V}$$

$$v_- = 0 \text{ V}$$

(c)  $v_I = -1 \text{ V}$

$v_O = +1 \text{ V}$

$v_A = 1.7 \text{ V}$

$v_- = 0 \text{ V}$

~ Virtual ground as negative feedback loop is closed through  $D_2$  and  $R$ .

(d)  $v_I = -3 \text{ V} \Rightarrow v_O = +3 \text{ V}$

$v_A = +3.7 \text{ V}$

$v_- = 0 \text{ V}$

**3.85** The input is an AM signal with carrier frequency  $\omega_0 = 2\pi \times 10^6 \text{ rad/s}$  and time-varying amplitude  $V_p(t) = 1.5 + \sin(2\pi \times 10^3 t)$  Volts. With  $CR = 0.1 \text{ ms}$ , the output time constant is much longer than the period of the carrier,  $1/10^6 = 1 \text{ s}$ . Thus, the output drops very little between successive peaks of the carrier. However, the time constant is much smaller than the period of  $V_p(t)$  so that  $v_O$  will follow changes in  $V_p(t)$ . A sketch appears in Fig. 1.

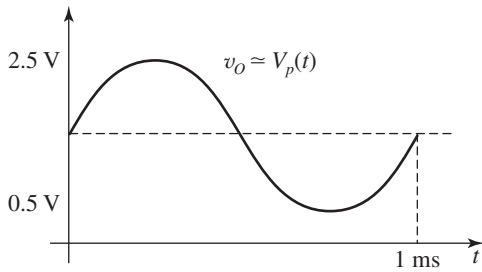


Figure 1

With  $CR = 0.1 \text{ s}$ , the output time constant is much smaller than the period of the carrier ( $1 \text{ s}$ ). Thus, the circuit behaves as a precision rectifier. A sketch appears in Fig. 2.

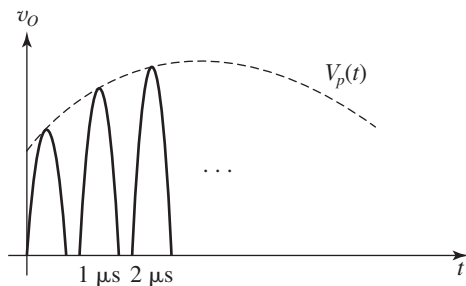
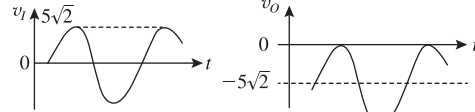
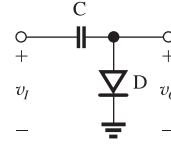


Figure 2

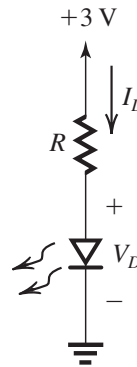
**3.86**



From the figure we see that

$v_{Oav} = -5\sqrt{2} = -7.07 \text{ V}$

**3.87**



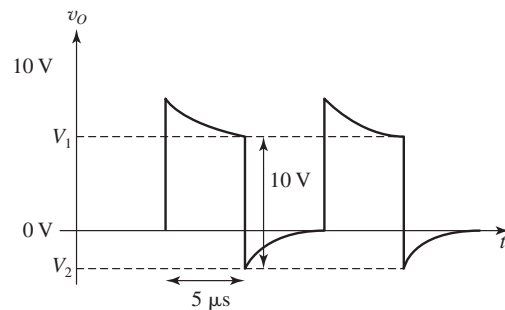
(a)  $V_D = 2.2 \text{ V}, I_D = 10 \text{ mA}$ .

$\Rightarrow R = \frac{3 - 2.2}{10} = 80\Omega$

(b)  $V_D = 1.8 \text{ V}, I_D = 10 \text{ mA}$ .

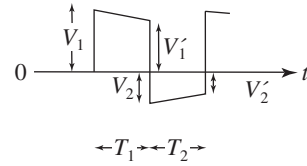
$\Rightarrow R = \frac{3 - 1.8}{10} = 120\Omega$

**3.88**



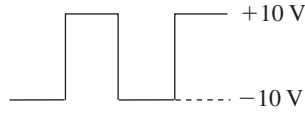
$$V_1 = 5 + 5e^{-5/10} = 8.03 \text{ V}$$

$$V_2 = V_1 - 10 = -1.97 \text{ V}$$

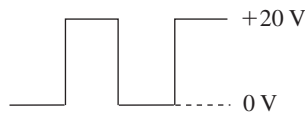


**3.89**

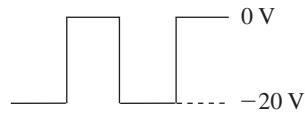
(a)



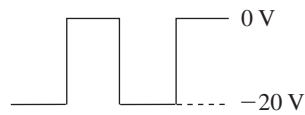
(b)



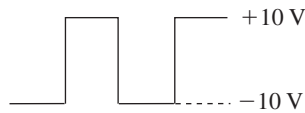
(c)



(d)



(e)



(f) Here there are two different time constants involved. To calculate the output levels, we shall consider the discharge and charge wave forms.

During  $T_1$ ,  $v_o = V_1 e^{-t/RC}$

At  $t = T_1 = T$ ,  $v_o = V_1'$   
 $= V_1 e^{-T/RC}$

where for  $T \ll CR$

$$V_1' \simeq V_1(1 - T/CR) = V_1(1 - \alpha)$$

where  $\alpha \ll 1$

During the interval  $T_2$ , we have

$$|v_o| = |V_2| e^{-t/(CR/2)}$$

At the end of  $T_2$ ,  $t = T$ , and  $v_o = |V_2'|$

where

$$|V_2'| = |V_2| e^{-T/(CR/2)}$$

$$\simeq |V_2| \left(1 - \frac{T}{RC/2}\right) = |V_2| (1 - 2\alpha)$$

Now

$$V_1' + |V_2| = 20 \Rightarrow V_1 + |V_2| - \alpha V_1 = 20 \quad (1)$$

and

$$|V_2'| + V_1 = 20 \Rightarrow V_1 + |V_2| - 2\alpha |V_2| = 20 \quad (2)$$

From (1) and (2) we find that

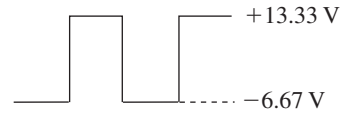
$$V_1 = 2|V_2|$$

Then using (1) and neglecting  $\alpha V_1$  yields

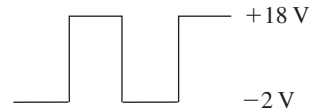
$$3|V_2| = 20 \Rightarrow |V_2| = 6.67 \text{ V}$$

$$V_1 = 13.33 \text{ V}$$

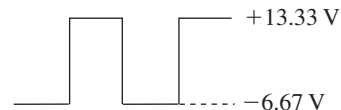
The result is



(g)



(h) Using a method similar to that employed for case (f) above, we obtain



**3.90**  $C_{j0} = 150$  fF,  $V_0 = 3$  V,  $m = 3$ . Using Equation (1.71),

$$C_j = \frac{150 \text{ fF}}{\left(1 + \frac{V_R}{3}\right)^3}$$

$$\Rightarrow V_R = 3 \left(\frac{150}{C_j}\right)^{1/3} - 3$$

at  $C_j = 80$  fF:  $V_R = 3 \left(\frac{150}{80}\right)^{1/3} - 3 = 0.70$  V,  
and

at  $C_j = 20$  fF:  $V_R = 3 \left(\frac{150}{20}\right)^{1/3} - 3 = 2.87$  V.

Thus, the range of required bias voltages is  
 $0.70 \text{ V} < V_R < 2.87 \text{ V}$ .

**3.91** (a) Assuming an ideal op amp, there is a virtual short-circuit between input terminals so that the photodiode reverse voltage is,

$$V_R = v_- = v_+ = 3 \text{ V}$$

Thus,

$$v_O = 3 - i_D \times 250 = 3 + (I_D + i_P) \times 250 = 3 + (I_D + R \times P) \times 250$$

(b) With no illumination,  $P = 0$  and

$$v_O = 3 + I_D \times 250 = 3 + 10^{-7} \times 250 \cong 3 \text{ V}$$

(c) With  $P = 10$  W,

$$v_O = 3 + (10^{-7} + 10^{-2} \times 0.5) \times 250 = 4.25 \text{ V}$$

**Chapter 4**

**Solutions to Exercises within the Chapter**

**Ex: 4.1**  $i_C = I_S e^{v_{BE}/V_T}$

$$v_{BE2} - v_{BE1} = V_T \ln \left[ \frac{i_{C2}}{i_{C1}} \right]$$

$$v_{BE2} = 700 + 25 \ln \left[ \frac{0.1}{1} \right]$$

$$= 642 \text{ mV} = 0.64 \text{ V}$$

$$v_{BE3} = 700 + 25 \ln \left[ \frac{10}{1} \right]$$

$$= 758 \text{ mV} = 0.76 \text{ V}$$

**Ex: 4.2**  $\therefore \alpha = \frac{\beta}{\beta + 1}$

$$\frac{50}{50 + 1} < \alpha < \frac{150}{150 + 1}$$

$$0.980 < \alpha < 0.993$$

**Ex: 4.3**  $I_C = I_E - I_B$

$$= 1.460 \text{ mA} - 0.01446 \text{ mA}$$

$$= 1.446 \text{ mA}$$

$$\alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99$$

$$\beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100$$

$$I_C = I_S e^{v_{BE}/V_T}$$

$$I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{700/25}}$$

$$= \frac{1.446}{e^{28}} \text{ mA} = 10^{-15} \text{ A}$$

**Ex: 4.4**  $\beta = \frac{\alpha}{1 - \alpha}$  and  $I_C = 10 \text{ mA}$

$$\text{For } \alpha = 0.99, \beta = \frac{0.99}{1 - 0.99} = 99$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1 \text{ mA}$$

$$\text{For } \alpha = 0.98, \beta = \frac{0.98}{1 - 0.98} = 49$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2 \text{ mA}$$

**Ex: 4.5** Given:

$$I_S = 10^{-16} \text{ A}, \beta = 100, I_C = 1 \text{ mA}$$

We write

$$I_{SE} = I_S / \alpha = I_S \times \left( 1 + \frac{1}{\beta} \right)$$

$$= 10^{-16} \times 1.01 = 1.01 \times 10^{-16} \text{ A}$$

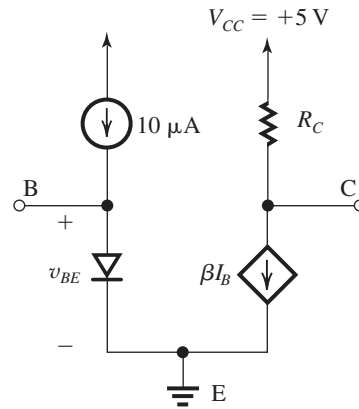
$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-16}}{100} = 10^{-18} \text{ A}$$

$$V_{BE} = V_T \ln \left[ \frac{I_C}{I_S} \right] = 25 \ln \left[ \frac{1 \text{ mA}}{10^{-16} \text{ A}} \right]$$

$$= 25 \times 29.9336$$

$$= 748 \text{ mV}$$

**Ex: 4.6**



$$v_{BE} = 690 \text{ mV}$$

$$I_C = 1 \text{ mA}$$

For active range  $V_C \geq V_B$ ,

$$R_{C\text{max}} = \frac{V_{CC} - 0.690}{I_C}$$

$$= \frac{5 - 0.69}{1}$$

$$= 4.31 \text{ k}\Omega$$

**Ex: 4.7**  $I_S = 10^{-15} \text{ A}$

$$\text{Area}_C = 100 \times \text{Area}_E$$

$$I_{SC} = 100 \times I_S = 10^{-13} \text{ A}$$

**Ex: 4.8**  $i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$

for  $i_C = 0$

$$I_S e^{v_{BE}/V_T} = I_{SC} e^{v_{BC}/V_T}$$

$$\frac{I_{SC}}{I_S} = \frac{e^{v_{BE}/V_T}}{e^{v_{BC}/V_T}}$$

$$= e^{(v_{BE} - v_{BC})/V_T}$$

$$\therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln \left[ \frac{I_{SC}}{I_S} \right]$$

For collector Area = 100 × Emitter area

$$V_{CE} = 25 \ln \left[ \frac{100}{1} \right] = 115 \text{ mV}$$

**Ex: 4.9**  $I_C = I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}$

$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T} + I_{SC} e^{V_{BC}/V_T}$$

$$\beta_{\text{forced}} = \left. \frac{I_C}{I_B} \right|_{\text{sat}} < \beta$$

$$= \beta \frac{I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}}{I_S e^{V_{BE}/V_T} + \beta I_{SC} e^{V_{BC}/V_T}}$$

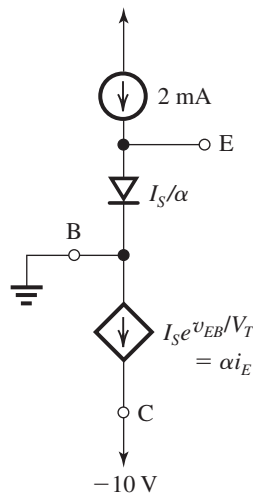
$$= \beta \frac{I_S e^{(V_{BE}-V_{BC})/V_T} - I_{SC}}{I_S e^{(V_{BE}-V_{BC})/V_T} + \beta I_{SC}}$$

$$= \beta \frac{e^{V_{CEsat}/V_T} - I_{SC}/I_S}{e^{V_{CEsat}/V_T} + \beta I_{SC}/I_S} \quad \text{Q.E.D.}$$

$$\beta_{\text{forced}} = 100 \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100}$$

$$= 100 \times 0.2219 \approx 22.2$$

**Ex: 4.10**



$$I_E = \frac{I_S}{\alpha} e^{V_{BE}/V_T}$$

$$2 \text{ mA} = \frac{51}{50} 10^{-14} e^{V_{BE}/V_T}$$

$$V_{BE} = 25 \ln \left[ \frac{2}{10^3} \times \frac{50}{51} \times 10^{14} \right]$$

$$= 650 \text{ mV}$$

$$I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 2$$

$$= 1.96 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2 \text{ } \mu\text{A}$$

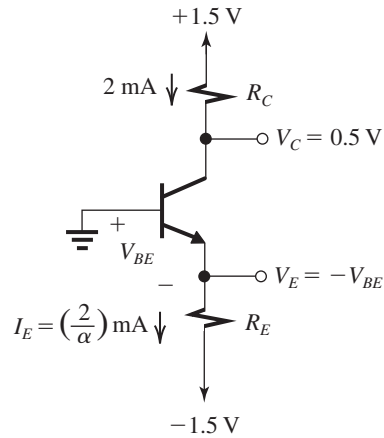
**Ex: 4.11**  $I_C = I_S e^{V_{BE}/V_T} = 1.5 \text{ A}$

$$\therefore V_{BE} = V_T \ln [1.5/10^{-11}]$$

$$= 25 \times 25.734$$

$$= 643 \text{ mV}$$

**Ex: 4.12**



$$R_C = \frac{1.5 - V_C}{I_C} = \frac{1.5 - 0.5}{2}$$

$$= 0.5 \text{ k}\Omega = 500 \text{ } \Omega$$

Since at  $I_C = 1 \text{ mA}$ ,  $V_{BE} = 0.8 \text{ V}$ , then at  $I_C = 2 \text{ mA}$ ,

$$V_{BE} = 0.8 + 0.025 \ln \left( \frac{2}{1} \right)$$

$$= 0.8 + 0.017$$

$$= 0.817 \text{ V}$$

$$V_E = -V_{BE} = -0.817 \text{ V}$$

$$I_E = \frac{2 \text{ mA}}{\alpha} = \frac{2}{0.99} = 2.02 \text{ mA}$$

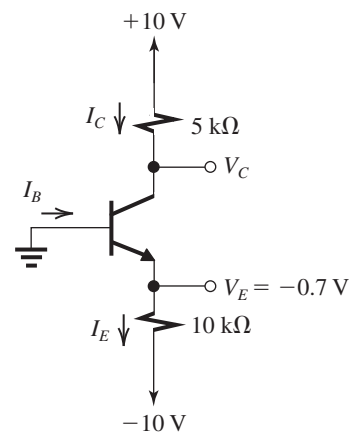
$$I_E = \frac{V_E - (-1.5)}{R_E}$$

Thus,

$$R_E = \frac{-0.817 + 1.5}{2.02} = 0.338 \text{ k}\Omega$$

$$= 338 \text{ } \Omega$$

**Ex: 4.13**



Exercise 4-3

$$I_E = \frac{V_E - (-10)}{10} = \frac{-0.7 + 10}{10}$$

$$= 0.93 \text{ mA}$$

Assuming active-mode operation,

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.93}{50 + 1} = 0.0182 \text{ mA}$$

$$= 18.2 \mu\text{A}$$

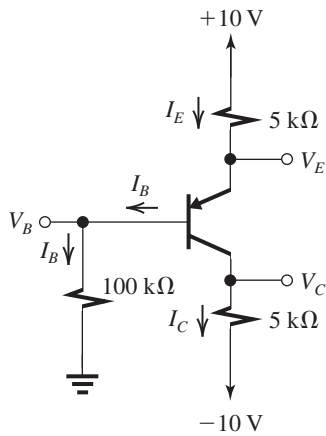
$$I_C = I_E - I_B = 0.93 - 0.0182 = 0.91 \text{ mA}$$

$$V_C = 10 - I_C \times 5$$

$$= 10 - 0.91 \times 5 = 5.45 \text{ V}$$

Since  $V_C > V_B$ , the transistor is operating in the active mode, as assumed.

**Ex: 4.14**



$$V_B = 1.0 \text{ V}$$

Thus,

$$I_B = \frac{V_B}{100 \text{ k}\Omega} = 0.01 \text{ mA}$$

$$V_E = +1.7 \text{ V}$$

Thus,

$$I_E = \frac{10 - V_E}{5 \text{ k}\Omega} = \frac{10 - 1.7}{5} = 1.66 \text{ mA}$$

and

$$\beta + 1 = \frac{I_E}{I_B} = \frac{1.66}{0.01} = 166$$

$$\Rightarrow \beta = 165$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{165}{165 + 1} = 0.994$$

Assuming active-mode operation,

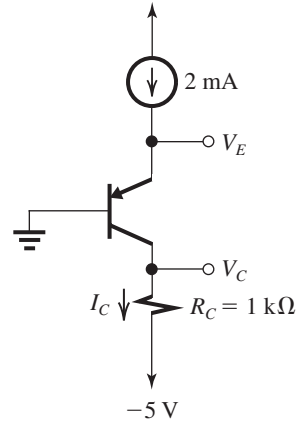
$$I_C = \alpha I_E = 0.994 \times 1.66 = 1.65 \text{ mA}$$

and

$$V_C = -10 + 1.65 \times 5 = -1.75 \text{ V}$$

Since  $V_C < V_B$ , the transistor is indeed operating in the active mode.

**Ex: 4.15**



The transistor is operating at a constant emitter current. Thus, a change in temperature of  $+30^\circ\text{C}$  results in a change in  $V_{EB}$  by

$$\Delta V_{EB} = -2 \text{ mV} \times 30 = -60 \text{ mV}$$

Thus,

$$\Delta V_E = -60 \text{ mV}$$

Since the collector current remains unchanged at  $\alpha I_E$ , the collector voltage does not change:

$$\Delta V_C = 0 \text{ V}$$

**Ex: 4.16** Refer to Fig. 4.19(a):

$$i_C = I_S e^{v_{BE}/V_T} + \frac{v_{CE}}{r_o} \quad (1)$$

Now using Eqs. (4.21) and (4.22), we can express  $r_o$  as

$$r_o = \frac{V_A}{I_S e^{v_{BE}/V_T}}$$

Substituting in Eq. (1), we have

$$i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

which is Eq. (4.18). Q.E.D.

**Ex: 4.17**  $r_o = \frac{V_A}{I_C} = \frac{100}{I_C}$

At  $I_C = 0.1 \text{ mA}$ ,  $r_o = 1 \text{ M}\Omega$

At  $I_C = 1 \text{ mA}$ ,  $r_o = 100 \text{ k}\Omega$

At  $I_C = 10 \text{ mA}$ ,  $r_o = 10 \text{ k}\Omega$

**Ex: 4.18**  $\Delta I_C = \frac{\Delta V_{CE}}{r_o}$

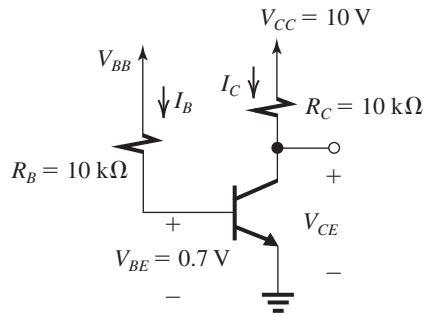
where

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$\Delta I_C = \frac{11 - 1}{100} = 0.1 \text{ mA}$$

Thus,  $I_C$  becomes 1.1 mA.

**Ex: 4.19**



(a) For operation in the active mode with  $V_{CE} = 5 \text{ V}$ ,

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 5}{10} = 0.5 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = 0.01 \text{ mA}$$

$$V_{BB} = V_{BE} + I_B R_B$$

$$= 0.7 + 0.01 \times 10 = 0.8 \text{ V}$$

(b) For operation at the edge of saturation,

$$V_{CE} = 0.3 \text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{10} = 0.97 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.97}{50} = 0.0194 \text{ mA}$$

$$V_{BB} = V_B + I_B R_B$$

$$= 0.7 + 0.0194 \times 10 = 0.894 \text{ V}$$

(c) For operation deep in saturation with  $\beta_{\text{forced}} = 10$ , we have

$$V_{CE} \simeq 0.2 \text{ V}$$

$$I_C = \frac{10 - 0.2}{10} = 0.98 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{0.98}{10} = 0.098 \text{ mA}$$

$$V_{BB} = V_B + I_B R_B$$

$$= 0.7 + 0.098 \times 10 = 1.68 \text{ V}$$

**Ex: 4.20** For  $V_{BB} = 0 \text{ V}$ ,  $I_B = 0$  and the transistor is cut off. Thus,

$$I_C = 0$$

and

$$V_C = V_{CC} = +10 \text{ V}$$

**Ex: 4.21** Refer to the circuit in Fig. 4.22 and let  $V_{BB} = 1.7 \text{ V}$ . The current  $I_B$  can be found from

$$I_B = \frac{V_{BB} - V_B}{R_B} = \frac{1.7 - 0.7}{10} = 0.1 \text{ mA}$$

Assuming operation in the active mode,

$$I_C = \beta I_B = 50 \times 0.1 = 5 \text{ mA}$$

Thus,

$$V_C = V_{CC} - R_C I_C$$

$$= 10 - 1 \times 5 = 5 \text{ V}$$

which is greater than  $V_B$ , verifying that the transistor is operating in the active mode, as assumed.

(a) To obtain operation at the edge of saturation,  $R_C$  must be increased to the value that results in  $V_{CE} = 0.3 \text{ V}$ :

$$R_C = \frac{V_{CC} - 0.3}{I_C} = \frac{10 - 0.3}{5} = 1.94 \text{ k}\Omega$$

(b) Further increasing  $R_C$  results in the transistor operating in saturation. To obtain saturation-mode operation with  $V_{CE} = 0.2 \text{ V}$  and  $\beta_{\text{forced}} = 10$ , we use

$$I_C = \beta_{\text{forced}} \times I_B$$

$$= 10 \times 0.1 = 1 \text{ mA}$$

The value of  $R_C$  required can be found from

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{10 - 0.2}{1} = 9.8 \text{ k}\Omega$$

**Ex: 4.22** Refer to the circuit in Fig. 4.23(a) with the base voltage raised from 4 V to  $V_B$ . If at this value of  $V_B$ , the transistor is at the edge of saturation then,

$$V_C = V_B - 0.4 \text{ V}$$

Since  $I_C \simeq I_E$ , we can write

$$\frac{10 - V_C}{R_C} = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E}$$

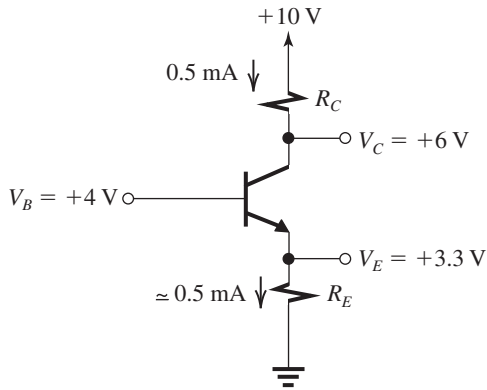


Thus,

$$\frac{10 - (V_B - 0.4)}{4.7} = \frac{V_B - 0.7}{3.3}$$

$$\Rightarrow V_B = +4.7 \text{ V}$$

**Ex: 4.23**



To establish a reverse-bias voltage of 2 V across the CBJ,

$$V_C = +6 \text{ V}$$

From the figure we see that

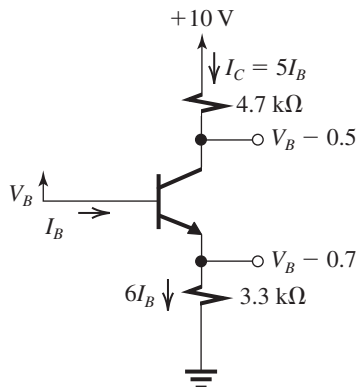
$$R_C = \frac{10 - 6}{0.5} = 8 \text{ k}\Omega$$

and

$$R_E = \frac{3.3}{0.5} = 6.6 \text{ k}\Omega$$

where we have assumed  $\alpha \approx 1$ .

**Ex: 4.24**



The figure shows the circuit with the base voltage at  $V_B$  and the BJT operating in saturation with  $V_{CE} = 0.2 \text{ V}$  and  $\beta_{\text{forced}} = 5$ .

$$I_C = 5I_B = \frac{10 - (V_B - 0.5)}{4.7} \quad (1)$$

$$I_E = 6I_B = \frac{V_B - 0.7}{3.3} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\frac{5}{6} = \frac{10.5 - V_B}{V_B - 0.7} \times \frac{3.3}{4.7}$$

$$\Rightarrow V_B = +5.18 \text{ V}$$

**Ex: 4.25** Refer to the circuit in Fig. 4.26(a). The largest value for  $R_C$  while the BJT remains in the active mode corresponds to

$$V_C = +0.4 \text{ V}$$

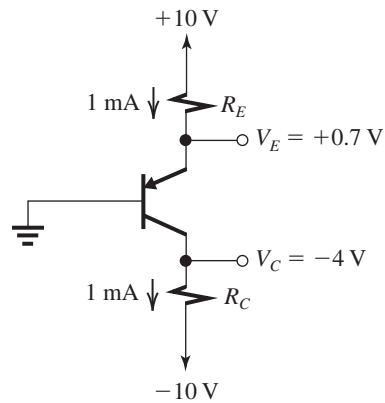
Since the emitter and collector currents remain unchanged, then from Fig. 4.26(b) we obtain

$$I_C = 4.65 \text{ mA}$$

Thus,

$$R_C = \frac{V_C - (-10)}{I_C} = \frac{+0.4 + 10}{4.65} = 2.24 \text{ k}\Omega$$

**Ex: 4.26**



For a 4-V reverse-biased voltage across the CBJ,

$$V_C = -4 \text{ V}$$

Refer to the figure.

$$I_C = 1 \text{ mA} = \frac{V_C - (-10)}{R_C}$$

$$\Rightarrow R_C = \frac{-4 + 10}{1} = 6 \text{ k}\Omega$$

$$R_E = \frac{10 - V_E}{I_E}$$

Assuming  $\alpha = 1$ ,

$$R_E = \frac{10 - 0.7}{1} = 9.3 \text{ k}\Omega$$

**Ex: 4.27** Refer to the circuit in Fig. 4.27:

$$I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

To ensure that the transistor remains in the active mode for  $\beta$  in the range 50 to 150, we need to select  $R_C$  so that for the highest collector current possible, the BJT reaches the edge of saturation, that is,  $V_{CE} = 0.3 \text{ V}$ . Thus,

$$V_{CE} = 0.3 = 10 - R_C I_{C_{\max}}$$

where

$$\begin{aligned} I_{C_{\max}} &= \beta_{\max} I_B \\ &= 150 \times 0.043 = 6.45 \text{ mA} \end{aligned}$$

Thus,

$$R_C = \frac{10 - 0.3}{6.45} = 1.5 \text{ k}\Omega$$

For the lowest  $\beta$ ,

$$\begin{aligned} I_C &= \beta_{\min} I_B \\ &= 50 \times 0.043 = 2.15 \text{ mA} \end{aligned}$$

and the corresponding  $V_{CE}$  is

$$\begin{aligned} V_{CE} &= 10 - R_C I_C = 10 - (1.5 \times 2.15) \\ &= 6.775 \text{ V} \end{aligned}$$

Thus,  $V_{CE}$  will range from 0.3 V to 6.8 V.

**Ex: 4.28** Refer to the solution of Example 4.10.

$$\begin{aligned} I_E &= \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta + 1)]} \\ &= \frac{5 - 0.7}{3 + (33.3/51)} = 1.177 \text{ mA} \end{aligned}$$

$$I_C = \alpha I_E = 0.98 \times 1.177 = 1.15 \text{ mA}$$

Thus the current is reduced by

$$\Delta I_C = 1.28 - 1.15 = 0.13 \text{ mA}$$

which is a  $-10\%$  change.

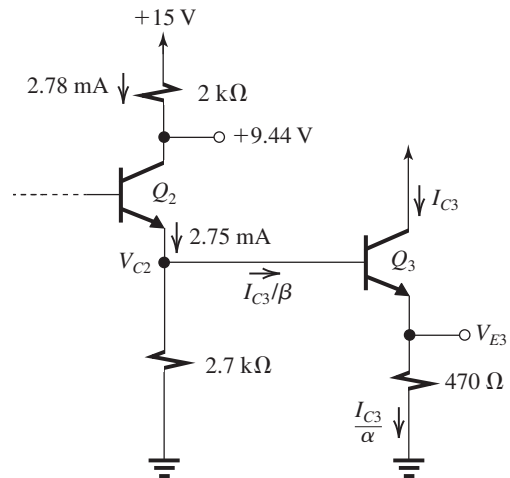
**Ex: 4.29** Refer to the circuit in Fig. 4.30(b). The total current drawn from the power supply is

$$I = 0.103 + 1.252 + 2.78 = 4.135 \text{ mA}$$

Thus, the power dissipated in the circuit is

$$P = 15 \text{ V} \times 4.135 \text{ mA} = 62 \text{ mW}$$

**Ex: 4.30**



From the figure we see that

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47$$

$$V_{C2} = V_{E3} + 0.7 = \frac{I_{C3}}{\alpha} \times 0.47 + 0.7 \quad (1)$$

A node equation at the collector of  $Q_2$  yields

$$2.75 = \frac{V_{C2}}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting for  $V_{C2}$  from Eq. (1), we obtain

$$2.75 = \frac{(0.47 I_{C3}/\alpha) + 0.7}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting  $\alpha = 0.99$  and  $\beta = 100$  and solving for  $I_{C3}$  results in

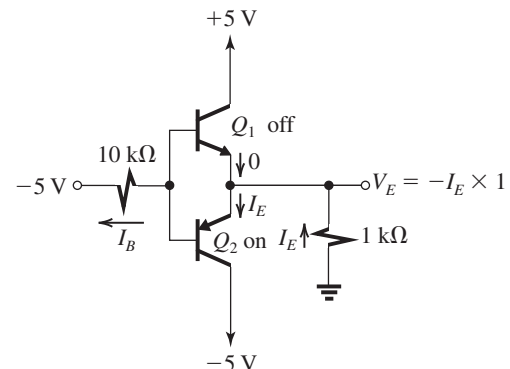
$$I_{C3} = 13.4 \text{ mA}$$

Now,  $V_{E3}$  and  $V_{C2}$  can be determined:

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47 = \frac{13.4}{0.99} \times 0.47 = +6.36 \text{ V}$$

$$V_{C2} = V_{E3} + 0.7 = +7.06 \text{ V}$$

**Ex: 4.31**



Exercise 4-7

From the figure we see that  $Q_1$  will be off and  $Q_2$  will be on. Since the base of  $Q_2$  will be at a voltage higher than  $-5$  V, transistor  $Q_2$  will be operating in the active mode. We can write a loop equation for the loop containing the  $10\text{-k}\Omega$  resistor, the EBJ of  $Q_2$  and the  $1\text{-k}\Omega$  resistor:

$$-I_E \times 1 - 0.7 - I_B \times 10 = -5$$

Substituting  $I_B = I_E/(\beta + 1) = I_E/101$  and rearranging gives

$$I_E = \frac{5 - 0.7}{\frac{10}{101} + 1} = 3.9 \text{ mA}$$

Thus,

$$V_E = -3.9 \text{ V}$$

$$V_{B2} = -4.6 \text{ V}$$

$$I_B = 0.039 \text{ mA}$$

**Ex: 4.32** With the input at  $+10$  V, there is a strong possibility that the conducting transistor

$Q_1$  will be saturated. Assuming this to be the case, the analysis steps will be as follows:

$$V_{CEsat}|_{Q_1} = 0.2 \text{ V}$$

$$V_E = 5 \text{ V} - V_{CEsat} = +4.8 \text{ V}$$

$$I_{E1} = \frac{4.8 \text{ V}}{1 \text{ k}\Omega} = 4.8 \text{ mA}$$

$$V_{B1} = V_E + V_{BE1} = 4.8 + 0.7 = +5.5 \text{ V}$$

$$I_{B1} = \frac{10 - 5.5}{10} = 0.45 \text{ mA}$$

$$I_{C1} = I_{E1} - I_{B1} = 4.8 - 0.45 = 4.35 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7$$

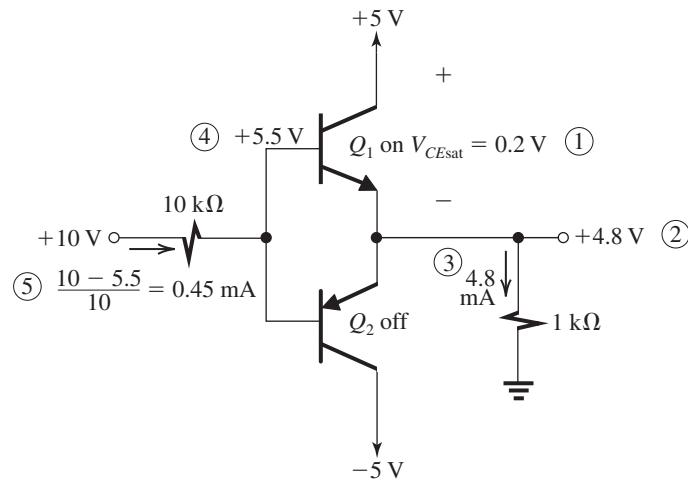
which is lower than  $\beta_{\text{min}}$ , verifying that  $Q_1$  is indeed saturated.

Finally, since  $Q_2$  is off,

$$I_{C2} = 0$$

**Ex: 4.33**  $V_O = +10 - BV_{BCO} = 10 - 70 = -60 \text{ V}$

This figure belongs to Exercise 4.32.



**Solutions to End-of-Chapter Problems**

- 4.1**
- Active
  - Saturation
  - Active
  - Saturation
  - Active
  - Cutoff

**4.2** The EB junctions have a 4:1 area ratio.

$$I_C = I_S e^{V_{BE}/V_T}$$

$$0.5 \times 10^{-3} = I_{S1} \times e^{0.75/0.025}$$

$$\Rightarrow I_{S1} = 4.7 \times 10^{-17} \text{ A}$$

$$I_{S2} = 4I_{S1} = 1.87 \times 10^{-16} \text{ A}$$

**4.3** Old technology:

$$10^{-3} = 2 \times 10^{-15} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-15}}\right) = 0.673 \text{ V}$$

New technology:

$$10^{-3} = 2 \times 10^{-18} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-18}}\right) = 0.846 \text{ V}$$

**4.4**  $I_B = 10 \mu\text{A}$

$$I_C = 800 \mu\text{A}$$

$$\beta = \frac{I_C}{I_B} = 80$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{80}{81} = 0.988$$

**4.5** Transistor is operating in active region:

$$\beta = 50 \rightarrow 300$$

$$I_B = 10 \mu\text{A}$$

$$I_C = \beta I_B = 0.5 \text{ mA} \rightarrow 3 \text{ mA}$$

$$I_E = (\beta + 1)I_B = 0.51 \text{ mA} \rightarrow 3.01 \text{ mA}$$

Maximum power dissipated in transistor is

$$\begin{aligned} & I_B \times 0.7 \text{ V} + I_C \times V_C \\ &= 0.01 \times 0.7 + 3 \times 10 \simeq 30 \text{ mW} \end{aligned}$$

**4.6** For  $i_B = 10 \mu\text{A}$ ,

$$i_C = i_E - i_B = 1000 - 10 = 990 \mu\text{A}$$

$$\beta = \frac{i_C}{i_B} = \frac{990}{10} = 99$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{99}{100} = 0.99$$

For  $i_B = 20 \mu\text{A}$ ,

$$i_C = i_E - i_B = 1000 - 20 = 980 \mu\text{A}$$

$$\beta = \frac{i_C}{i_B} = \frac{980}{20} = 49$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{49}{50} = 0.98$$

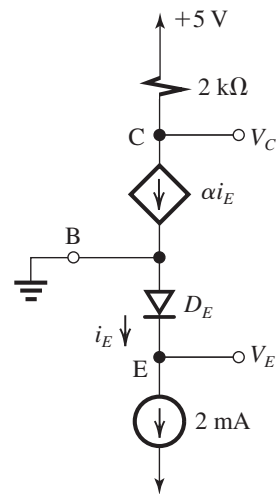
For  $i_B = 50 \mu\text{A}$ ,

$$i_C = i_E - i_B = 1000 - 50 = 950 \mu\text{A}$$

$$\beta = \frac{i_C}{i_B} = \frac{950}{50} = 19$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{19}{20} = 0.95$$

**4.7**



The figure shows the circuit, where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$I_{SE} = \frac{I_S}{\alpha} = \frac{5 \times 10^{-15}}{0.99} = 5.05 \times 10^{-15} \text{ A}$$

The voltage at the emitter  $V_E$  is

$$\begin{aligned} V_E &= -V_{DE} \\ &= -V_T \ln(I_E/I_{SE}) \\ &= -0.025 \ln\left(\frac{2 \times 10^{-3}}{5.05 \times 10^{-15}}\right) \\ &= -0.668 \text{ V} \end{aligned}$$

The voltage at the collector  $V_C$  is found from

$$\begin{aligned} V_C &= 5 - I_C \times 2 \\ &= 5 - \alpha I_E \times 2 \\ &= 5 - 0.99 \times 2 \times 2 = 1.04 \text{ V} \end{aligned}$$

4.8.

$v_{CE}$ (V)	$v_{BC}$ (V)	$i_{BC}$ ( $\mu\text{A}$ )	$i_{BE}$ ( $\mu\text{A}$ )	$i_B$ ( $\mu\text{A}$ )	$i_C$ (mA)	$i_C/i_B$
0.4	0.3	0.016	14.46	14.48	1.446	100
0.3	0.4	0.89	14.46	15.35	1.445	94
0.2	0.5	48.5	14.46	62.96	1.398	22.2

The equations utilized are

$$v_{BC} = v_{BE} - v_{CE} = 0.7 - v_{CE} \quad (1)$$

$$i_{BC} = I_{SC} e^{v_{BC}/V_T} = 10^{-13} e^{v_{BC}/0.025} \quad (2)$$

$$i_{BE} = I_{SB} e^{v_{BE}/V_T} = 10^{-17} e^{0.7/0.025} \quad (3)$$

$$i_B = i_{BC} + i_{BE} \quad (4)$$

$$i_C = I_S e^{v_{BE}/V_T} - i_{BC} = 10^{-15} e^{0.7/0.025} - i_{BC} \quad (5)$$

Performing these calculations for  $v_{CE} = 0.4$  V, 0.3 V, and 0.2 V, we obtain the results shown in the table above.

To obtain  $v_{CE}$  that results in  $i_C = 0$ , we first use Eq. (5),

$$0 = 10^{-15} e^{0.7/0.025} - i_{BC} \Rightarrow i_{BC} = 1.446 \text{ mA}$$

Substituting in Eq. (2), we obtain

$$1.446 \times 10^{-3} = 10^{-13} e^{v_{BC}/0.025} \Rightarrow v_{BC} = 0.585 \text{ V}$$

Finally, substituting in Eq. (1) gives

$$0.0585 = 0.7 - v_{CE} \Rightarrow v_{CE} = 0.115 \text{ V}$$

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-14}}{50} = 2 \times 10^{-16} \text{ A}$$

$$V_{EB} = 0.025 \ln\left(\frac{10 \times 10^{-6}}{2 \times 10^{-16}}\right)$$

$$= 0.616 \text{ V}$$

Thus,

$$V_B = -V_{EB} = -0.616 \text{ V}$$

The collector current can be found as

$$I_C = \beta I_B = 50 \times 10 = 500 \mu\text{A} = 0.5 \text{ mA}$$

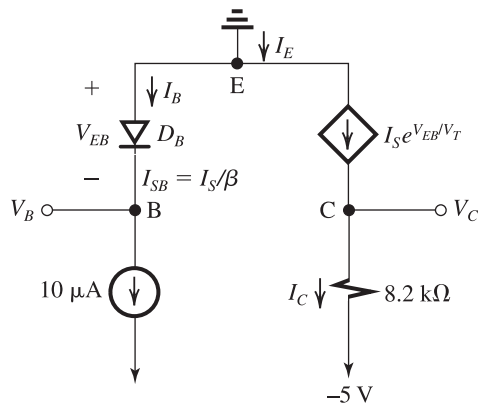
The collector voltage can now be obtained from

$$V_C = -5 + I_C \times 8.2 = -5 + 0.5 \times 8.2 = -0.9 \text{ V}$$

The emitter current can be found as

$$I_E = I_B + I_C = 10 + 500 = 510 \mu\text{A} = 0.51 \text{ mA}$$

4.9



The emitter–base voltage  $V_{EB}$  is found as the voltage drop across the diode  $D_B$ , whose scale current is  $I_{SB} = I_S/\beta$ , it is conducting a 10- $\mu\text{A}$  current. Thus,

$$V_{EB} = V_T \ln\left(\frac{10 \mu\text{A}}{I_{SB}}\right)$$

where

4.10

$$I_B = \frac{I_E}{\beta + 1} = \frac{5}{20 + 1} = 0.238 \text{ A} = 238 \text{ mA}$$

$$I_C = I_S e^{V_{EB}/V_T}$$

$$\alpha I_E = I_S e^{V_{EB}/V_T}$$

where

$$\alpha = \frac{20}{21} = 0.95$$

$$I_S = \alpha I_E e^{-V_{EB}/V_T}$$

$$= 0.95 \times 5 e^{-(0.8/0.025)}$$

$$= 6 \times 10^{-14} \text{ A}$$

A transistor that conducts  $I_C = 1$  mA with

$V_{EB} = 0.70$  V has a scale current

$$I_S = 1 \times 10^{-3} e^{-0.70/0.025} = 6.9 \times 10^{-16} \text{ A}$$

The emitter–base junction areas of these two transistors will have the same ratio as that of their scale currents, thus

$$\frac{\text{EBJ area of first transistor}}{\text{EBJ area of second transistor}} = \frac{6 \times 10^{-14}}{6.9 \times 10^{-16}} = 87$$

$$4.11 \quad I_C = I_S e^{V_{BE}/V_T}$$

$$200 \times 10^{-6} = I_S e^{30}$$

$$\Rightarrow I_S = 1.87 \times 10^{-17} \text{ A}$$

For the transistor that is 32 times larger,

$$I_S = 32 \times 1.87 \times 10^{-17}$$

$$= 6 \times 10^{-16} \text{ A}$$

At  $V_{BE} = 30 V_T$ , the larger transistor conducts a current of

$$I_C = 32 \times 200 \mu\text{A} = 6.4 \text{ mA}$$

At  $I_C = 1 \text{ mA}$ , the base–emitter voltage of the larger transistor can be found as

$$1 \times 10^{-3} = 6 \times 10^{-16} e^{V_{BE}/V_T}$$

$$V_{BE} = V_T \ln\left(\frac{1 \times 10^{-3}}{6 \times 10^{-16}}\right) = 0.704 \text{ V}$$

$$4.12 \quad \frac{I_{S1}}{I_{S2}} = \frac{A_{E1}}{A_{E2}} = \frac{200 \times 200}{0.4 \times 0.4} = 250,000$$

$$I_{C1} = I_{S1} e^{V_{BE1}/V_T}$$

$$I_{C2} = I_{S2} e^{V_{BE2}/V_T}$$

For  $I_{C1} = I_{C2}$  we have

$$e^{(V_{BE2}-V_{BE1})/V_T} = \frac{I_{S1}}{I_{S2}} = 250,000$$

$$V_{BE2} - V_{BE1} = 0.025 \ln(250,000)$$

$$= 0.31 \text{ V}$$

$$4.13 \quad I_{C1} = 10^{-13} e^{700/25} = 0.145 \text{ A} = 145 \text{ mA}$$

$$I_{C2} = 10^{-18} e^{700/25} = 1.45 \mu\text{A}$$

For the first transistor 1 to conduct a current of 1.45  $\mu\text{A}$ , its  $V_{BE}$  must be

$$V_{BE1} = 0.025 \ln\left(\frac{1.45 \times 10^{-6}}{10^{-13}}\right)$$

$$= 0.412 \text{ V}$$

$$4.14 \quad 5 \times 10^{-3} = I_S e^{0.76/0.025} \quad (1)$$

$$I_C = I_S e^{0.70/0.025} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$I_C = 5 \times 10^{-3} e^{-0.06/0.025}$$

$$= 0.45 \text{ mA}$$

For  $I_C = 5 \mu\text{A}$ ,

$$5 \times 10^{-6} = I_S e^{V_{BE}/0.025} \quad (3)$$

Dividing Eq. (3) by Eq. (1) yields

$$10^{-3} = e^{(V_{BE}-0.76)/0.025}$$

$$V_{BE} = 0.76 + 0.025 \ln(10^{-3})$$

$$= 0.587 \text{ V}$$

4.15

$\alpha$	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.999
$\beta = \frac{\alpha}{1-\alpha}$	1	4	9	19	49	99	199	999

4.16

$\beta$	1	2	10	20	50	100	200	500	1000
$\alpha = \frac{\beta}{\beta+1}$	0.5	0.67	0.91	0.95	0.98	0.99	0.995	0.998	0.999

$$4.17 \quad \beta = \frac{\alpha}{1-\alpha} \quad (1)$$

$$\alpha \rightarrow \alpha + \Delta\alpha$$

$$\beta \rightarrow \beta + \Delta\beta$$

$$\beta + \Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} \quad (2)$$

Subtracting Eq. (1) from Eq. (2) gives

$$\Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} - \frac{\alpha}{1 - \alpha}$$

$$\Delta\beta = \frac{\Delta\alpha}{(1 - \alpha - \Delta\alpha)(1 - \alpha)} \quad (3)$$

Dividing Eq. (3) by Eq. (1) gives

$$\frac{\Delta\beta}{\beta} = \left(\frac{\Delta\alpha}{\alpha}\right) \left(\frac{1}{1 - \alpha - \Delta\alpha}\right)$$

For  $\Delta\alpha \ll 1$ , the second factor on the right-hand side is approximately equal to  $\beta$ . Thus

$$\frac{\Delta\beta}{\beta} \simeq \beta \left(\frac{\Delta\alpha}{\alpha}\right) \quad \text{Q.E.D.}$$

$$\text{For } \frac{\Delta\beta}{\beta} = -10\% \text{ and } \beta = 100,$$

$$\frac{\Delta\alpha}{\alpha} \simeq \frac{-10\%}{100} = -0.1\%$$

$$4.18 \quad i_C = I_S e^{v_{BE}/V_T}$$

$$= 5 \times 10^{-15} e^{0.7/0.025} = 7.2 \text{ mA}$$

$i_B$  will be in the range  $\frac{7.2}{50} \text{ mA}$  to  $\frac{7.2}{200} \text{ mA}$ , that is, 144  $\mu\text{A}$  to 36  $\mu\text{A}$ .

$i_E$  will be in the range  $(7.2 + 0.144) \text{ mA}$  to  $(7.2 + 0.036) \text{ mA}$ , that is, 7.344 mA to 7.236 mA.

4.19

Transistor	a	b	c	d	e
$V_{BE}$ (mV)	700	690	580	780	820
$I_C$ (mA)	1.000	1.000	0.230	10.10	73.95
$I_B$ ( $\mu$ A)	10	20	5	120	1050
$I_E$ (mA)	1.010	1.020	0.235	10.22	75
$\alpha$	0.99	0.98	0.979	0.988	0.986
$\beta$	100	50	46	84	70
$I_S$ (A)	$6.9 \times 10^{-16}$	$1.0 \times 10^{-15}$	$1.9 \times 10^{-14}$	$2.8 \times 10^{-16}$	$4.2 \times 10^{-16}$

4.20 First we determine  $I_S$ ,  $\beta$ , and  $\alpha$ :

$$1 \times 10^{-3} = I_S e^{700/25}$$

$$\Rightarrow I_S = 6.91 \times 10^{-16} \text{ A}$$

$$\beta = \frac{I_C}{I_B} = \frac{1 \text{ mA}}{10 \mu\text{A}} = 100$$

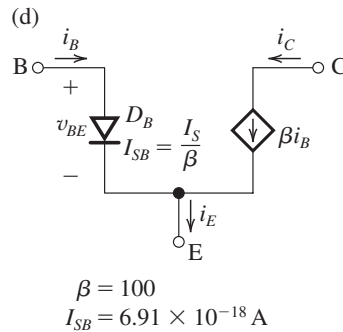
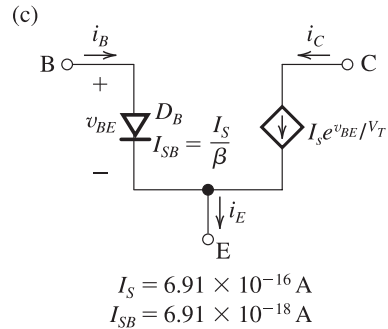
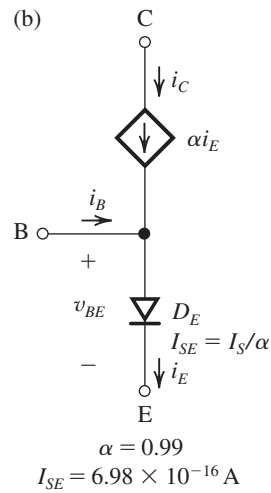
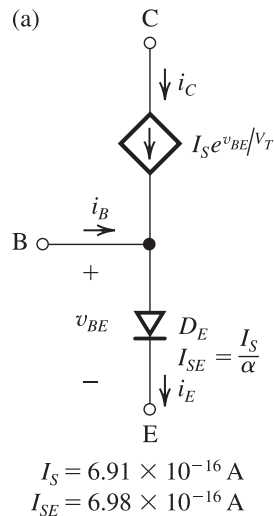
$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

Then we can determine  $I_{SE}$  and  $I_{SB}$ :

$$I_{SE} = \frac{I_S}{\alpha} = 6.98 \times 10^{-16} \text{ A}$$

$$I_{SB} = \frac{I_S}{\beta} = 6.91 \times 10^{-18} \text{ A}$$

The figure below shows the four large-signal models, corresponding to Fig. 4.5(a) to (d), together with their parameter values.



**4.21** Refer to the circuit in Fig. 4.6(b).

$$I_{SB} = \frac{I_S}{\beta} = \frac{5 \times 10^{-15}}{50} = 10^{-16} \text{ A}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5 \times 10^{-3}}{50} = 10^{-5} \text{ A}$$

$$V_B = V_{BE} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

$$= 0.025 \ln\left(\frac{10^{-5}}{10^{-16}}\right)$$

$$= 0.633 \text{ V}$$

We can determine  $R_B$  from

$$R_B = \frac{V_{CC} - V_B}{I_B}$$

$$= \frac{5 - 0.633}{10^{-5}} = 437 \text{ k}\Omega$$

To obtain  $V_{CE} = 1 \text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$= \frac{5 - 1}{0.5} = 8 \text{ k}\Omega$$

**4.22**  $I_S = 10^{-15} \text{ A}$

Thus, a forward-biased EBJ conducting a current of 1 mA will have a forward voltage drop  $V_{BE}$ :

$$V_{BE} = V_T \ln\left(\frac{I}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.691 \text{ V}$$

$$I_{SC} = 100I_S = 10^{-13} \text{ A}$$

Thus, a forward-biased CBJ conducting a 1-mA current will have a forward voltage drop  $V_{BC}$ :

$$V_{BC} = V_T \ln\left(\frac{1 \times 10^{-3}}{1 \times 10^{-13}}\right) = 0.576 \text{ V}$$

When forward-biased with 0.5 V, the emitter-base junction conducts

$$I = I_S e^{0.5/0.025}$$

$$= 10^{-15} e^{0.5/0.025} = 0.49 \text{ }\mu\text{A}$$

and the CBJ conducts

$$I = I_{SC} e^{0.5/0.025}$$

$$= 10^{-13} e^{0.5/0.025} = 48.5 \text{ }\mu\text{A}$$

**4.23** Dividing Eq. (4.14) by Eq. (4.15) and substituting  $i_C/i_B = \beta_{\text{forced}}$  gives

$$\beta_{\text{forced}} = \frac{I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}}{(I_S/\beta) e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T}}$$

Dividing the numerator and denominator of the right-hand side by  $I_{SC} e^{v_{BC}/V_T}$  and replacing  $v_{BE} - v_{BC}$  by  $V_{CE\text{sat}}$  gives

$$\beta_{\text{forced}} = \frac{\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} - 1}{\frac{1}{\beta} \left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} + 1}$$

This equation can be used to obtain  $e^{V_{CE\text{sat}}/V_T}$  and hence  $V_{CE\text{sat}}$  as

$$\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} = \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}$$

$$\Rightarrow V_{CE\text{sat}} = V_T \ln\left[\frac{I_{SC}}{I_S} \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}\right] \quad \text{Q.E.D.}$$

For  $\beta = 100$  and  $I_{SC}/I_S = 100$ , we can use this equation to obtain  $V_{CE\text{sat}}$  corresponding to the given values of  $\beta_{\text{forced}}$ . The results are as follows:

$\beta_{\text{forced}}$	50	10	5	1	0
$V_{CE\text{sat}}$ (V)	0.231	0.178	0.161	0.133	0.115

**4.24** At  $i_C = 1 \text{ mA}$ ,  $v_{EB} = 0.7 \text{ V}$

At  $i_C = 10 \text{ mA}$ ,

$$v_{EB} = 0.7 + V_T \ln\left(\frac{10}{1}\right)$$

$$= 0.7 + 0.025 \ln(10) = 0.758 \text{ V}$$

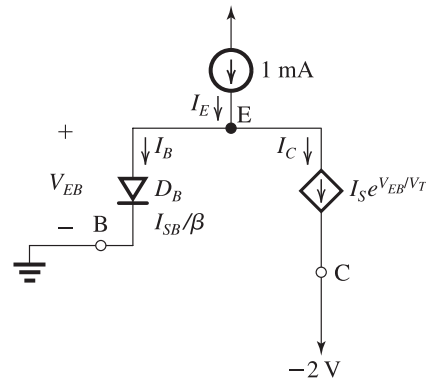
At  $i_C = 100 \text{ mA}$ ,

$$v_{EB} = 0.7 + 0.025 \ln\left(\frac{100}{1}\right)$$

$$= 0.815 \text{ V}$$

Note that  $v_{EB}$  increases by about 60 mV for every decade increase in  $i_C$ .

**4.25**



Referring to the figure, we see that

$$I_E = I_B + I_C = \frac{I_C}{\beta} + I_C$$



Thus,

$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{10}} = 0.909 \text{ mA}$$

$$I_B = 0.091 \text{ mA}$$

For direction of flow, refer to the figure.

$$V_{EB} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

where

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-15}}{10} = 10^{-16} \text{ A}$$

$$V_{EB} = 0.025 \ln\left(\frac{0.091 \times 10^{-3}}{10^{-16}}\right) = 0.688 \text{ V}$$

Thus,

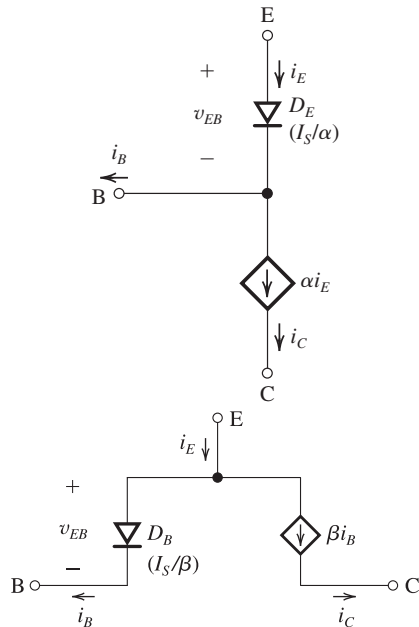
$$V_E = V_B + V_{EB} = 0 + 0.688 = 0.688 \text{ V}$$

If a transistor with  $\beta = 1000$  is substituted,

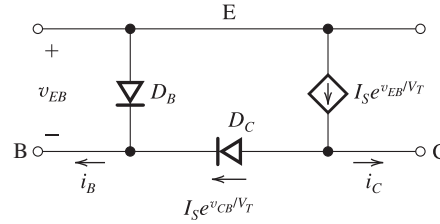
$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{1000}} = 0.999 \text{ mA}$$

Thus,  $I_C$  changes by  $0.999 - 0.909 = 0.09 \text{ mA}$ , a 9.9% increase.

**4.26** The two missing large-signal equivalent circuits for the *pnp* transistor are those corresponding to the *npn* equivalent circuits in Fig. 4.5(b) and 4.5(d). They are shown in the figure.



**4.27**



**4.28** (a) Refer to Fig. P4.28(a).

$$I_1 = \frac{10.7 - 0.7}{5 \text{ k}\Omega} = 2 \text{ mA}$$

Assuming operation in the active mode,

$$I_C = \alpha I_1 \simeq I_1 = 2 \text{ mA}$$

$$V_2 = -10.7 + I_C \times 5 = -10.7 + 2 \times 5 = -0.7 \text{ V}$$

Since  $V_2$  is lower than  $V_B$ , which is 0 V, the transistor is operating in the active mode, as assumed.

(b) Refer to Fig. P4.28(b).

Since  $V_C = -4 \text{ V}$  is lower than  $V_B = -2.7 \text{ V}$ , the transistor is operating in the active mode.

$$I_C = \frac{-4 - (-10)}{2.4 \text{ k}\Omega} = 2.5 \text{ mA}$$

$$I_E = \frac{I_C}{\alpha} \simeq I_C = 2.5 \text{ mA}$$

$$V_3 = +12 - I_E \times 5.6 = 12 - 2.5 \times 5.6 = -2 \text{ V}$$

(c) Refer to Fig. P4.28(c) and use

$$I_C = \frac{0 - (-10)}{20} = 0.5 \text{ mA}$$

Assuming active-mode operation, and utilizing the fact that  $\beta$  is large,  $I_B \simeq 0$  and

$$V_4 \simeq 2 \text{ V}$$

Since  $V_C < V_B$ , the transistor is indeed operating in the active region.

$$I_5 = I_E = \frac{I_C}{\alpha} \simeq I_C = 0.5 \text{ mA}$$

(d) Refer to Fig. P4.28(d). Since the collector is connected to the base with a 10-k $\Omega$  resistor and  $\beta$  is assumed to be very high, the voltage drop across the 10-k $\Omega$  resistor will be close to zero and the base voltage will be equal to that of the collector:

$$V_B = V_7$$

This also implies active-mode operation. Now,

$$V_E = V_B - 0.7$$

Thus,

$$V_E = V_7 - 0.7$$

$$I_6 = \frac{V_E - (-10)}{3}$$

$$= \frac{V_7 - 0.7 + 10}{3} = \frac{V_7 + 9.3}{3} \quad (1)$$

Since  $I_B = 0$ , the collector current will be equal to the current through the 9.1-k $\Omega$  resistor,

$$I_C = \frac{+10 - V_7}{9.1} \quad (2)$$

Since  $\alpha \approx 1$ ,  $I_C = I_E = I_6$  resulting in

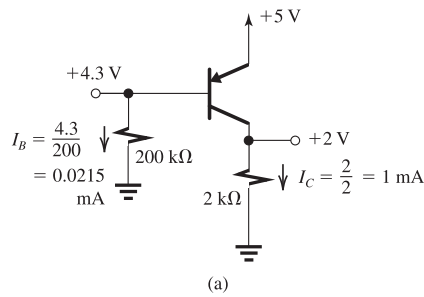
$$\frac{10 - V_7}{9.1} = \frac{V_7 + 9.3}{3}$$

$$\Rightarrow V_7 = -4.5 \text{ V}$$

and

$$I_6 = \frac{V_7 + 9.3}{3} = \frac{-4.5 + 9.3}{3} = 1.6 \text{ mA}$$

4.29 (a)



Since  $V_C$  is lower than  $V_B$ , the transistor is operating in the active region. From the figure above, we see that

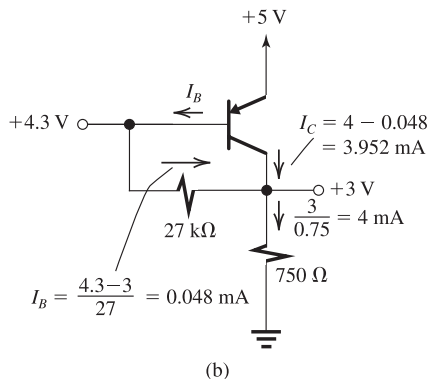
$$I_C = 1 \text{ mA}$$

$$I_B = 0.0215 \text{ mA}$$

Thus,

$$\beta \equiv \frac{I_C}{I_B} = \frac{1}{0.0215} = 46.5$$

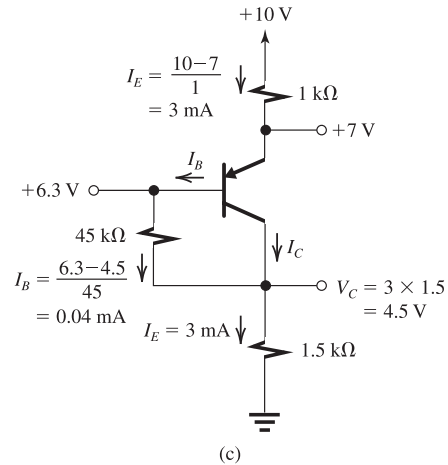
(b)



Observe that with  $V_C$  at 3 V and  $V_B$  at 4.3 V, the transistor is operating in the active region. Refer to the analysis shown in the figure, which leads to

$$\beta \equiv \frac{I_C}{I_B} = \frac{3.952}{0.048} = 82.3$$

(c)



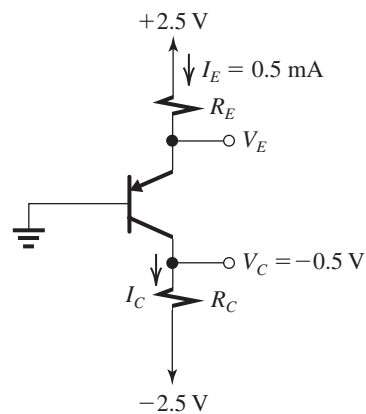
Observe that the transistor is operating in the active region and note the analysis performed on the circuit diagram. Thus,

$$I_C = I_E - I_B = 3 - 0.04 = 2.96 \text{ mA}$$

and

$$\beta \equiv \frac{I_C}{I_B} = \frac{2.96}{0.04} = 74$$

4.30



From the figure we see that  $V_C = -0.5 \text{ V}$  is lower than the base voltage ( $V_B = 0 \text{ V}$ ); thus the transistor will be operating in the active mode.

$$I_C = \alpha I_E = \left( \frac{\beta}{\beta + 1} \right) I_E = \frac{100}{100 + 1} \times 0.5$$

$$= 0.495 \text{ mA}$$

$$R_C = \frac{V_C - (-2.5)}{I_C}$$

$$= \frac{-0.5 + 2.5}{0.495} = 4.04 \text{ k}\Omega \simeq 4 \text{ k}\Omega$$

The transistor  $V_{EB}$  can be found from

$$V_{EB} = 0.64 + V_T \ln\left(\frac{0.5 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.68 \text{ V}$$

Thus,

$$V_E = +0.68 \text{ V}$$

and

$$R_E = \frac{2.5 - 0.68}{0.5} = 3.64 \text{ k}\Omega$$

The maximum allowable value for  $R_C$  while the transistor remains in the active mode corresponds to  $V_C = +0.4 \text{ V}$ . Thus,

$$R_{C\text{max}} = \frac{0.4 - (-2.5)}{0.495} = 5.86 \text{ k}\Omega$$

**4.31** Refer to the circuit in Fig. P4.31. Since  $V_C = 0 \text{ V}$  is equal to  $V_B$ , the transistor will be operating in the active mode. The transistor  $V_{BE}$  can be found from

$$V_{BE} = 0.7 + 0.025 \ln\left(\frac{0.5 \text{ mA}}{1 \text{ mA}}\right)$$

$$= 0.68 \text{ V}$$

Thus,

$$V_E = -0.68 \text{ V}$$

$$I_E = \frac{I_C}{\alpha} = I_C \left(\frac{\beta + 1}{\beta}\right) = 0.5 \times \frac{101}{100}$$

$$= 0.505 \text{ mA}$$

The required value of  $R_E$  can be found from

$$R_E = \frac{V_E - (-1.5)}{I_E}$$

$$R_E = \frac{-0.68 + 1.5}{0.505} = 1.62 \text{ k}\Omega$$

To establish  $V_C = 0 \text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{1.5 - 0}{0.5} = 3 \text{ k}\Omega$$

**4.32** Since the BJT is operating at a constant emitter current, its  $|V_{BE}|$  decreases by 2 mV for every  $^\circ\text{C}$  rise in temperature. Thus,

$$|V_{BE}| \text{ at } 0^\circ\text{C} = 0.7 + 0.002 \times 25 = 0.75 \text{ V}$$

$$|V_{BE}| \text{ at } 100^\circ\text{C} = 0.7 - 0.002 \times 75 = 0.55 \text{ V}$$

**4.33**  $r_o = 1/\text{slope}$

$$= 1/(0.8 \times 10^{-5})$$

$$= 125 \text{ k}\Omega$$

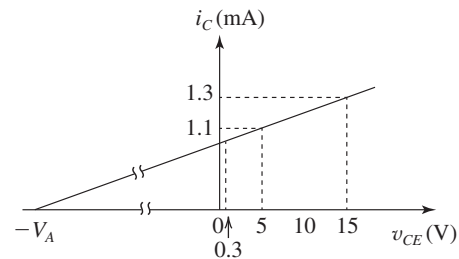
$$r_o = \frac{V_A}{I_C}$$

$$125 \text{ k}\Omega = \frac{V_A}{1 \text{ mA}} \Rightarrow V_A = 125 \text{ V}$$

At  $I_C = 10 \text{ mA}$ ,

$$r_o = \frac{V_A}{I_C} = \frac{125 \text{ V}}{10 \text{ mA}} = 12.5 \text{ k}\Omega$$

**4.34**



Slope of  $i_C$ - $v_{CE}$  line corresponding to  $v_{BE} = 710 \text{ mV}$  is

$$\text{Slope} = \frac{1.3 - 1.1}{15 - 5} = \frac{0.2 \text{ mA}}{10 \text{ V}} = 0.02 \text{ mA/V}$$

Near saturation,  $V_{CE} = 0.3 \text{ V}$ , thus

$$i_C = 1.1 - 0.02 \times (5 - 0.3)$$

$$= 1.006 \simeq 1 \text{ mA}$$

$i_C$  will be 1.2 mA at,

$$v_{CE} = 5 + \frac{1.2 - 1.1}{0.02} = 10 \text{ V}$$

The intercept of the  $i_C$ - $v_{CE}$  straight line on the  $i_C$  axis will be at

$$i_C = 1.1 - 5 \times 0.02 = 1 \text{ mA}$$

Thus, the Early voltage is obtained as

$$\text{Slope} = \frac{i_C(\text{at } v_{CE} = 0)}{V_A}$$

$$\Rightarrow V_A = \frac{1}{0.02} = 50 \text{ V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega$$

which is the inverse of the slope of the  $i_C$ - $v_{CE}$  line.

**4.35** Refer to the circuit in Fig. P4.35.

(a) For active-mode operation with  $V_C = 2 \text{ V}$ :

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 2}{1} = 8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{8}{50} = 0.16 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.16 \times 10 + 0.7 = 2.3 \text{ V}$$

(b) For operation at the edge of saturation:

$$V_{CE} = 0.3 \text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{1} = 9.7 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{9.7}{50} = 0.194 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.194 \times 10 + 0.7 = 2.64 \text{ V}$$

(c) For operation deep in saturation with  $\beta_{\text{forced}} = 10$ :

$$V_{CE} = 0.2 \text{ V}$$

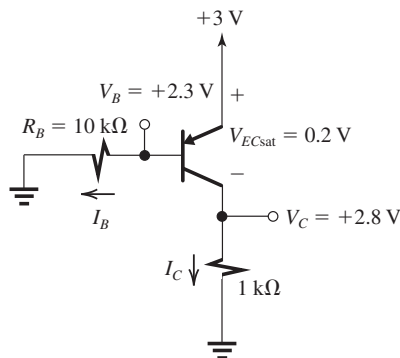
$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.2}{1} = 9.8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{9.8}{10} = 0.98 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.98 \times 10 + 0.7 = 10.5 \text{ V}$$

4.36



Assume saturation-mode operation. From the figure we see that

$$I_C = \frac{V_C}{1 \text{ k}\Omega} = \frac{2.8}{1} = 2.8 \text{ mA}$$

$$I_B = \frac{V_B}{R_B} = \frac{2.3}{10} = 0.23 \text{ mA}$$

Thus,

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B} = \frac{2.8}{0.23} = 12.2$$

Since 12.2 is lower than the transistor  $\beta$  of 50, we have verified that the transistor is operating in saturation, as assumed.

$$V_C = V_{CC} - V_{ECsat} = 3 - 0.2 = 2.8 \text{ V}$$

To operate at the edge of saturation,

$$V_{EC} = 0.3 \text{ V} \quad \text{and} \quad I_C/I_B = \beta = 50$$

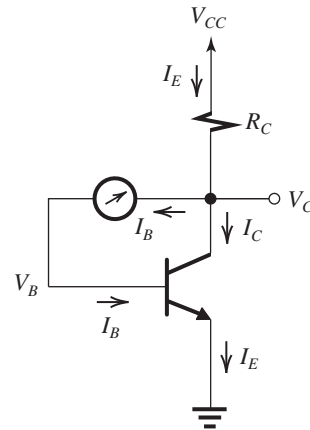
Thus,

$$I_C = \frac{3 - 0.3}{1} = 2.7 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{2.7}{50} = 0.054 \text{ mA}$$

$$R_B = \frac{2.3}{I_B} = \frac{2.3}{0.054} = 42.6 \text{ k}\Omega$$

4.37



Since the meter resistance is small,  $V_C \simeq V_B$  and the transistor is operating in the active region. To obtain  $I_E = 1 \text{ mA}$ , we arrange that  $V_{BE} = 0.7 \text{ V}$ . Since  $V_C \simeq V_B$ ,  $V_C$  must be set to 0.7 by selecting  $R_C$  according to

$$V_C = 0.7 = V_{CC} - I_E R_C$$

Thus,

$$0.7 = 9 - 1 \times R_C$$

$$\Rightarrow R_C = 8.3 \text{ k}\Omega$$

Since the meter reads full scale when the current flowing through it (in this case,  $I_B$  is  $50 \mu\text{A}$ ), a full-scale reading corresponds to

$$\beta \equiv \frac{I_C}{I_B} \simeq \frac{1 \text{ mA}}{50 \mu\text{A}} = 20$$

If the meter reads 1/5 of full scale, then

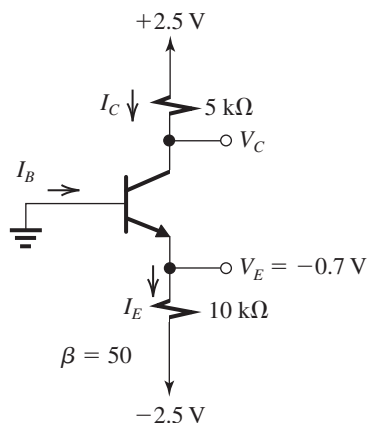
$$I_B = 10 \mu\text{A} \text{ and}$$

$$\beta = \frac{1 \text{ mA}}{10 \mu\text{A}} = 100$$

A meter reading of 1/10 full scale indicates that

$$\beta = \frac{1 \text{ mA}}{5 \mu\text{A}} = 200$$

4.38



$$I_E = \frac{V_E - (-2.5)}{10}$$

$$= \frac{-0.7 + 2.5}{10} = 0.18 \text{ mA}$$

Assuming the transistor is operating in the active mode, we obtain

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.18}{50 + 1} = 3.5 \mu\text{A}$$

$$I_C = \left(\frac{\beta}{\beta + 1}\right)I_E = \frac{50}{51} \times 0.18 = 0.176 \text{ mA}$$

$$V_C = +2.5 - I_C R_C$$

$$= 2.5 - 0.176 \times 5 = 1.62 \text{ V}$$

Since  $V_C > V_B$ , active-mode operation is verified.

4.39 Refer to Fig. 4.15(a) with  $R_C = 5.1 \text{ k}\Omega$  and  $R_E = 6.8 \text{ k}\Omega$ . Assuming  $V_{BE} \simeq 0.7 \text{ V}$ , then  $V_E = -0.7 \text{ V}$ , and

$$I_E = \frac{-0.7 - (-15)}{6.8} = 2.1 \text{ mA}$$

$$I_C = \alpha I_E \simeq 2.1 \text{ mA}$$

$$V_C = 15 - 2.1 \times 5.1 \simeq 4.3 \text{ V}$$

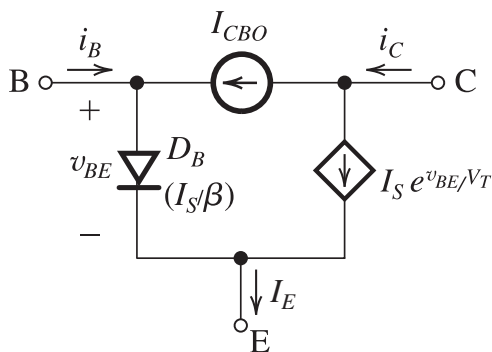
4.40 In all circuits shown in Fig. P4.40, we assume active-mode operation and verify that this is the case at the end of the solution. The solutions are indicated on the corresponding

circuit diagrams on the next page; the order of the steps is shown by the circled numbers.

4.41  $I_{CBO}$  approximately doubles for every  $10^\circ\text{C}$  rise in temperature. A change in temperature from  $25^\circ\text{C}$  to  $125^\circ\text{C}$ —that is, an increase of  $100^\circ\text{C}$ —results in 10 doublings or, equivalently, an increase by a factor of  $2^{10} = 1024$ . Thus  $I_{CBO}$  becomes

$$I_{CBO} = 10 \text{ nA} \times 1024 = 10.24 \mu\text{A}$$

4.42



From the figure we can write

$$i_B = \left(\frac{I_S}{\beta}\right)e^{v_{BE}/V_T} - I_{CBO} \quad (1)$$

$$i_C = I_S e^{v_{BE}/V_T} + I_{CBO} \quad (2)$$

$$i_E = I_S \left(1 + \frac{1}{\beta}\right)e^{v_{BE}/V_T} \quad (3)$$

When the base is left open-circuited,  $i_B = 0$  and Eq. (1) yields

$$I_{CBO} = \left(\frac{I_S}{\beta}\right)e^{v_{BE}/V_T}$$

or equivalently,

$$I_S e^{v_{BE}/V_T} = \beta I_{CBO} \quad (4)$$

Substituting for  $I_S e^{v_{BE}/V_T}$  in Eqs. (2) and (3) gives

$$i_C = i_E = (\beta + 1)I_{CBO}$$

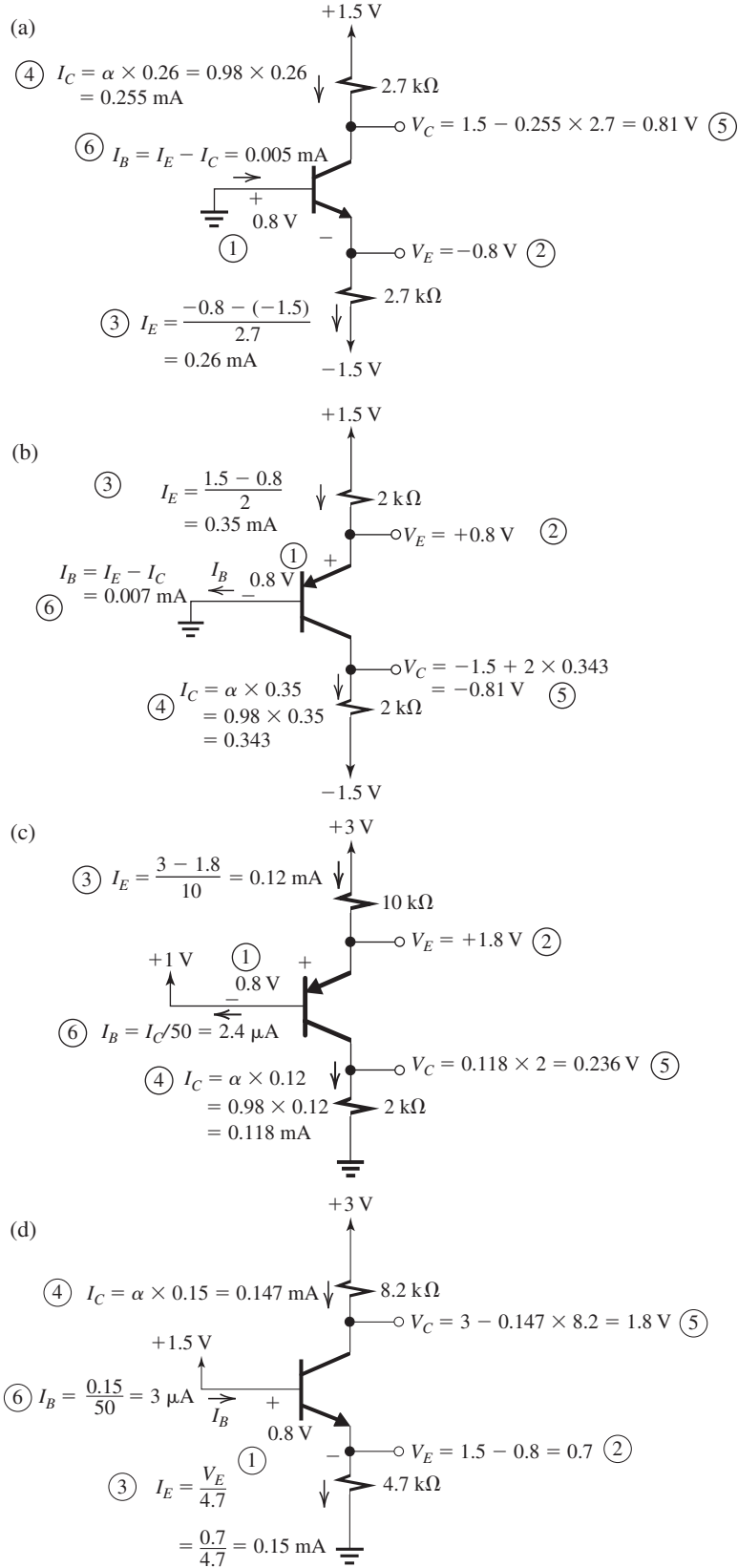
4.43 (a) If the junction temperature rises to  $50^\circ\text{C}$ , which is an increase of  $30^\circ\text{C}$ , the EB voltage decreases to

$$v_{EB} = 692 - 2 \times 30 = 632 \text{ mV}$$

(b) First, we evaluate  $V_T$  at  $20^\circ\text{C}$  and at  $50^\circ\text{C}$ :

$$V_T = \frac{kT}{q}$$

This figure belongs to Problem 4.40.



where  $k = 8.62 \times 10^{-5}$  eV/K.

Thus,

At 20°C,  $T = 293$  K and  $V_T = 8.62 \times 10^{-5} \times 293 = 25.3$  mV

At 50°C,  $T = 323$  K and  $V_T = 8.62 \times 10^{-5} \times 323 = 27.8$  mV

If the transistor is operated at  $v_{BE} = 700$  mV, then (i) At 20°C,  $i_E$  becomes

$$i_E = 0.5e^{(700-692)/25.3} = 0.69 \text{ mA}$$

(ii) At 50°C,  $i_E$  becomes

$$i_E = 0.5e^{(700-632)/27.8} = 5.77 \text{ mA}$$

**4.44**  $v_{BE} = 0.7$  V at  $i_C = 10$  mA

For  $v_{BE} = 0.5$  V,

$$i_C = 10e^{(0.5-0.7)/0.025} = 3.35 \text{ } \mu\text{A}$$

At a current  $I_C$  and a BE voltage  $V_{BE}$ , the slope of the  $i_C$ - $v_{BE}$  curve is  $I_C/V_T$ . Thus,

$$\text{Slope at } V_{BE} \text{ of } 700 \text{ mV} = \frac{10 \text{ mA}}{25 \text{ mV}} = 400 \text{ mA/V}$$

$$\text{Slope at } V_{BE} \text{ of } 500 \text{ mV} = \frac{3.35 \text{ } \mu\text{A}}{25 \text{ mV}} = 0.134 \text{ mA/V}$$

$$\text{Ratio of slopes} = \frac{400}{0.134} \approx 3000$$

**4.45** Use Eq. (4.18):

$$i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

with  $I_S = 10^{-15}$  A and  $V_A = 100$  V, to get

$$i_C = 10^{-15} e^{v_{BE}/0.025} \left( 1 + \frac{v_{CE}}{100} \right)$$

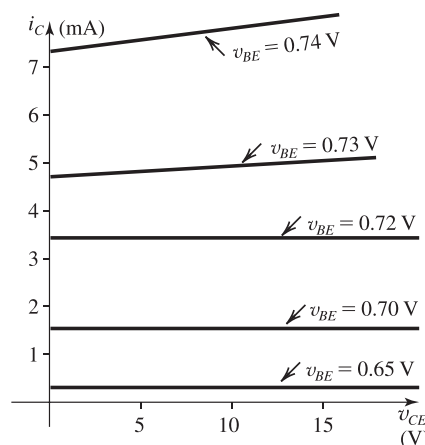
$v_{BE}$	0.65 V	0.70 V	0.72 V	0.73 V	0.74 V
$v_{CE}$ (V)	$i_C$ (mA)	$i_C$ (mA)	$i_C$ (mA)	$i_C$ (mA)	$i_C$ (mA)
0	0.196	1.45	3.21	4.81	7.16
15	0.225	1.67	3.70	5.52	8.24

To find the intercept of the straight-line characteristics on the  $i_C$  axis, we substitute  $v_{CE} = 0$  and evaluate

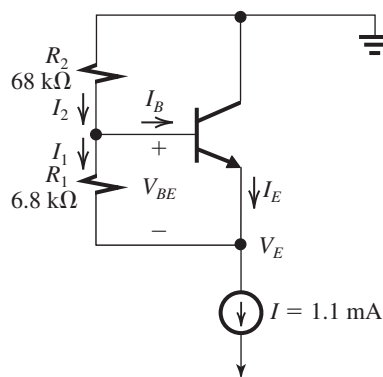
$$i_C = 10^{-15} e^{v_{BE}/V_T} \text{ A}$$

for the given value of  $v_{BE}$ . The slope of each straight line is equal to this value divided by 100 V ( $V_A$ ). Thus we obtain

$v_{BE}$ (V)	0.65	0.70	0.72	0.73	0.74
Intercept (mA)	0.2	1.45	3.22	4.80	7.16
Slope (mA/V)	0.002	0.015	0.032	0.048	0.072



**4.46**



At 25°C, assume  $I_E = 1$  mA. Thus,

$$V_{BE} = 0.68 \text{ V}$$

$$I_1 = \frac{V_{BE}}{R_1} = \frac{0.68 \text{ V}}{6.8 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_E = I - I_1 = 1.1 - 0.1 = 1 \text{ mA}$$

which is the value assumed.

$$I_2 = I_1 + I_B = I_1 + \frac{I_E}{\beta + 1}$$

$$= 0.1 + \frac{1}{101} = 0.11 \text{ mA}$$

Note that the currents in  $R_1$  and  $R_2$  differ only by the small base current, 0.01 mA. Had  $I_1$  and  $I_2$  been equal, then we would have had

$$I_1 R_1 = V_{BE}$$

$$I_2 R_2 \approx I_1 R_2 = V_{BE} \frac{R_2}{R_1}$$

$$\begin{aligned}
 V_E &= -(I_1 R_1 + I_2 R_2) \\
 &= -V_{BE} \left( 1 + \frac{R_2}{R_1} \right) \quad (1) \\
 &= -V_{BE} \left( 1 + \frac{6.8}{0.68} \right) = -11 V_{BE} = -7.48 \text{ V}
 \end{aligned}$$

which gives this circuit the name “ $V_{BE}$  multiplier.” A more accurate value of  $V_E$  can be obtained by taking  $I_B$  into account:

$$\begin{aligned}
 V_E &= -(I_1 R_1 + I_2 R_2) \\
 &= - \left( V_{BE} + \frac{R_2}{R_1} V_{BE} + I_B R_2 \right) \\
 &= - \left( 1 + \frac{R_2}{R_1} \right) V_{BE} - I_B R_2 \quad (2) \\
 &= -7.48 - 0.01 \times 68 = -8.16 \text{ V}
 \end{aligned}$$

As temperature increases, an approximate estimate for the temperature coefficient of  $V_E$  can be obtained by assuming that  $I_E$  remains constant and ignoring the temperature variation of  $\beta$ . Thus, we would be neglecting the temperature change of the  $(I_B R_2)$  terms in Eq. (2). From Eq. (2) we can obtain the temperature coefficient of  $V_E$  by utilizing the fact the  $V_{BE}$  changes by  $-2 \text{ mV}/^\circ\text{C}$ . Thus,

$$\begin{aligned}
 &\text{Temperature coefficient of } V_E \\
 &= - \left( 1 + \frac{R_2}{R_1} \right) \times -2 \\
 &= -11 \times -2 = +22 \text{ mV}/^\circ\text{C}
 \end{aligned}$$

At  $75^\circ\text{C}$ , which is a temperature increase of  $50^\circ\text{C}$ ,

$$V_E = -8.16 + 0.022 \times 50 = -7.06 \text{ V}$$

As a check on our assumption of constant  $I_E$ , let us find the value of  $I_E$  at  $75^\circ\text{C}$ :

$$\begin{aligned}
 I_1(75^\circ\text{C}) &= \frac{V_{BE}(75^\circ\text{C})}{R_1} \\
 &= \frac{0.68 - 2 \times 10^{-3} \times 50}{6.8} \\
 &= 0.085 \text{ mA}
 \end{aligned}$$

$$\begin{aligned}
 I_E(75^\circ\text{C}) &= I - I_1(75^\circ\text{C}) \\
 &= 1.1 - 0.085 = 1.015 \text{ mA}
 \end{aligned}$$

which is reasonably close to the assumed value of 1 mA.

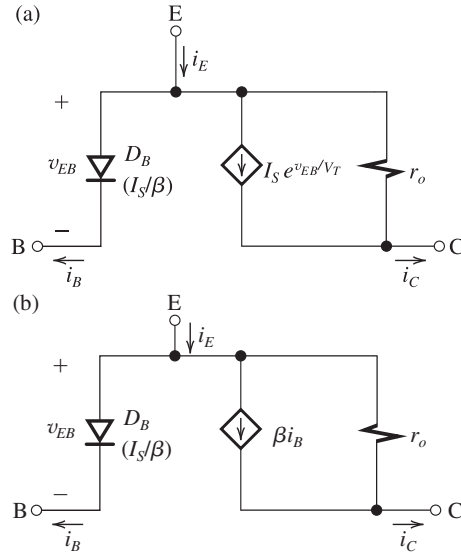
$$4.47 \quad r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{I_C}$$

Thus,

$$\text{At } I_C = 1 \text{ mA}, \quad r_o = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega$$

$$\text{At } I_C = 100 \text{ }\mu\text{A}, \quad r_o = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

4.48 The equivalent circuits shown in the figure correspond to the circuits in Fig. 4.19.



$$4.49 \quad \beta = \frac{i_C}{i_B} = \frac{1 \text{ mA}}{10 \text{ }\mu\text{A}} = 100$$

$$\beta_{ac} = \left. \frac{\Delta i_C}{\Delta i_B} \right|_{v_{CE} \text{ constant}} = \frac{0.08 \text{ mA}}{1.0 \text{ }\mu\text{A}} = 80$$

$$\Delta i_C = \Delta i_B \times \beta_{ac} + \frac{\Delta v_{CE}}{r_o}$$

where

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k}\Omega$$

Thus,

$$\begin{aligned}
 \Delta i_C &= 2 \times 80 + \frac{2}{100} \times 10^3 = 180 \text{ }\mu\text{A} \\
 &= 0.18 \text{ mA} \quad \text{and} \quad i_C = 1.18 \text{ mA}
 \end{aligned}$$

4.50 Refer to the circuit in Fig. P4.35 (with  $V_{BB} = V_{CC}$ ) and to the BJT equivalent circuit of Fig. 4.21.

$$I_C = \frac{V_{CC} - 0.2}{R_C}$$

$$I_B = \frac{V_{CC} - 0.7}{R_B}$$

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B}$$

Thus,

$$\beta_{\text{forced}} = \left( \frac{V_{CC} - 0.2}{V_{CC} - 0.7} \right) \left( \frac{R_B}{R_C} \right) \quad (1)$$

$$P_{\text{dissipated}} = V_{CC}(I_C + I_B)$$



$$= V_{CC}(\beta_{\text{forced}}I_B + I_B)$$

$$= (\beta_{\text{forced}} + 1)V_{CC}I_B \quad (2)$$

For  $V_{CC} = 5 \text{ V}$  and  $\beta_{\text{forced}} = 10$  and  $P_{\text{dissipated}} \leq 20 \text{ mW}$ , we can proceed as follows:

Using Eq. (1) we can determine  $(R_B/R_C)$ :

$$10 = \left(\frac{5 - 0.2}{5 - 0.7}\right) \left(\frac{R_B}{R_C}\right)$$

$$\Rightarrow \frac{R_B}{R_C} = 8.96 \quad (3)$$

Using Eq. (2), we can find  $I_B$ :

$$(10 + 1) \times 5 \times I_B \leq 20 \text{ mW}$$

$$\Rightarrow I_B \leq 0.36 \text{ mA}$$

Thus,

$$\frac{V_{CC} - 0.7}{R_B} \leq 0.36 \text{ mA}$$

$$\Rightarrow R_B \geq 11.9 \text{ k}\Omega$$

From the table of 1% resistors in Appendix J we select

$$R_B = 12.1 \text{ k}\Omega$$

Substituting in Eq. (3), we have

$$R_C = 1.35 \text{ k}\Omega$$

From the table of 1% resistors in Appendix J we select

$$R_C = 1.37 \text{ k}\Omega$$

For these values:

$$I_C = \frac{5 - 0.2}{1.37} = 3.5 \text{ mA}$$

$$I_B = \frac{5 - 0.7}{12.1} = 0.36 \text{ mA}$$

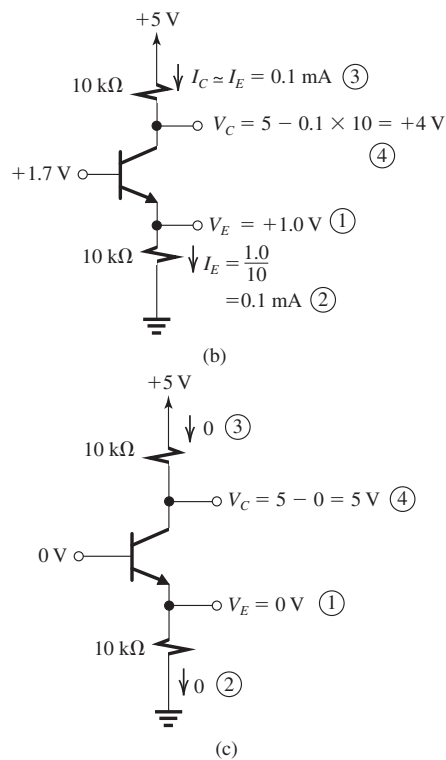
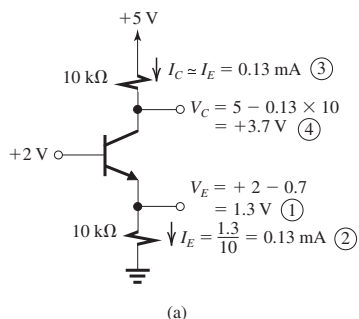
Thus,

$$\beta_{\text{forced}} = \frac{3.5}{0.36} = 9.7$$

$$P_{\text{dissipated}} = V_{CC}(I_C + I_B)$$

$$= 5 \times 3.86 = 19.3 \text{ mW}$$

**4.51**



The analysis and the results are given on the circuit diagrams of Figs. a through c. The circled numbers indicate the order of the analysis steps.

**4.52** Refer to the circuit in Fig. P4.52.

$$V_E = 1 \text{ V}$$

$$I_E = \frac{2.5 - 1}{10} = 0.15 \text{ mA}$$

$$V_B = V_E - 0.7 = 0.3 \text{ V}$$

$$I_B = \frac{V_B}{100 \text{ k}\Omega} = \frac{0.3}{100} = 0.003 \text{ mA}$$

$$I_C = I_E - I_B = 0.15 - 0.003 = 0.147 \text{ mA}$$

$$V_C = -2.5 + 10 \times 0.147 = -1.03 \text{ V}$$

Observe that  $V_C < V_B$ , confirming our implicit assumption that the transistor is operating in the active region.

$$\beta = \frac{I_C}{I_B} = \frac{0.147}{0.003} = 49$$

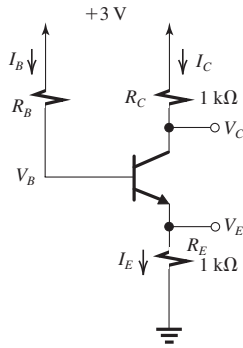
$$\alpha = \frac{\beta}{\beta + 1} = \frac{49}{49 + 1} = 0.98$$

**4.53**

A loop equation for the EB loop yields

$$3 = I_B R_B + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{3 - 0.7}{R_E + \frac{R_B}{\beta + 1}}$$



$$I_E = \frac{2.3}{1 + \frac{R_B}{101}}$$

(a) For  $R_B = 100 \text{ k}\Omega$ ,

$$I_E = \frac{2.3}{1 + \frac{100}{101}} = 1.16 \text{ mA}$$

$$V_E = I_E R_E = 1.16 \times 1 = 1.16 \text{ V}$$

$$V_B = V_E + 0.7 = 1.86 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 1.16 = 1.15 \text{ mA}$$

$$V_C = 3 - 1.15 \times 1 = +1.85 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the transistor is operating in the active region, as assumed.

(b) For  $R_B = 10 \text{ k}\Omega$ ,

$$I_E = \frac{2.3}{1 + \frac{10}{101}} = 2.09 \text{ mA}$$

$$V_E = 2.09 \times 1 = 2.09 \text{ V}$$

$$V_B = 2.09 + 0.7 = 2.79 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 2.09 = 2.07 \text{ mA}$$

$$V_C = 3 - 2.07 = +0.93 \text{ V}$$

Since  $V_C < V_B - 0.4$ , the transistor is operating in saturation, contrary to our original assumption. Therefore, we need to redo the analysis assuming saturation-mode operation, as follows:

$$V_B = V_E + 0.7$$

$$V_C = V_E + V_{CEsat} = V_E + 0.2$$

$$I_B = \frac{3 - V_B}{R_B} = \frac{3 - V_E - 0.7}{10} = \frac{2.3 - V_E}{10} \quad (1)$$

$$I_C = \frac{3 - V_C}{R_C} = \frac{3 - V_E - 0.2}{1} = 2.8 - V_E \quad (2)$$

$$I_E = \frac{V_E}{R_E} = \frac{V_E}{1} = V_E \quad (3)$$

Substituting from Eqs. (1), (2), and (3) into

$$I_E = I_B + I_C$$

gives

$$V_E = 0.23 - 0.1 V_E + 2.8 - V_E$$

$$\Rightarrow V_E = 1.44 \text{ V}$$

$$V_C = 1.64 \text{ V}$$

$$V_B = 2.14 \text{ V}$$

$$I_B = \frac{3 - 2.14}{10} = 0.086 \text{ mA}$$

$$I_C = \frac{3 - 1.64}{1} = 1.36 \text{ mA}$$

Thus,

$$\frac{I_C}{I_B} = \frac{1.36}{0.086} = 15.8$$

which is lower than the value of  $\beta$ , verifying saturation-mode operation.

(c) For  $R_B = 1 \text{ k}\Omega$ , we assume saturation-mode operation:

$$V_B = V_E + 0.7$$

$$V_C = V_E + 0.2$$

$$I_B = \frac{3 - (V_E + 0.7)}{1} = 2.3 - V_E$$

$$I_C = \frac{3 - (V_E + 0.2)}{1} = 2.8 - V_E$$

$$I_E = \frac{V_E}{1} = V_E$$

These values can be substituted into

$$I_E = I_B + I_C$$

to obtain

$$V_E = 2.3 - V_E + 2.8 - V_E$$

$$\Rightarrow V_E = 1.7 \text{ V}$$

$$V_B = 2.4 \text{ V}$$

$$V_C = 1.9 \text{ V}$$

Now checking the currents,

$$I_B = \frac{3 - 2.4}{1} = 0.6 \text{ mA}$$

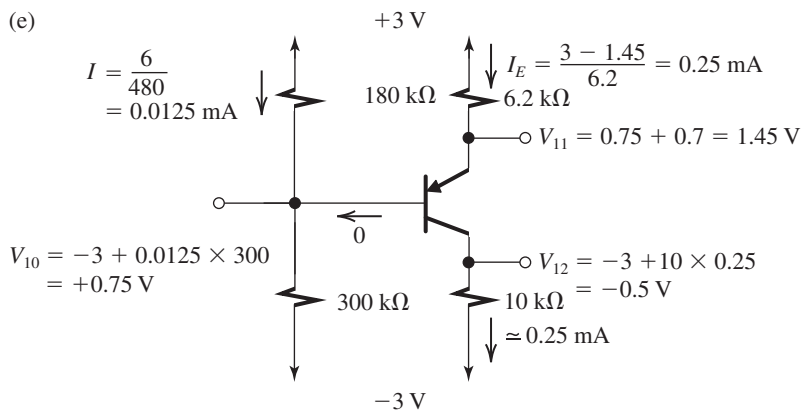
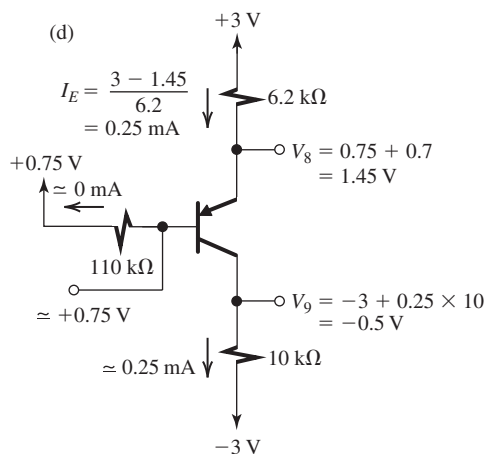
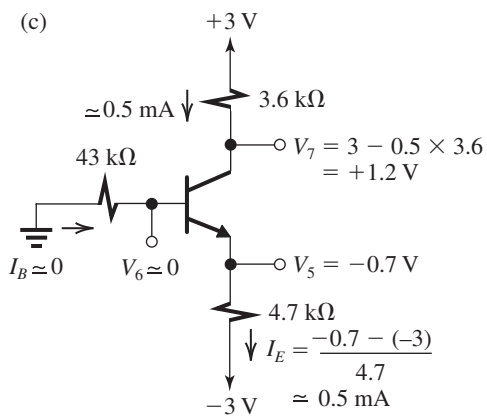
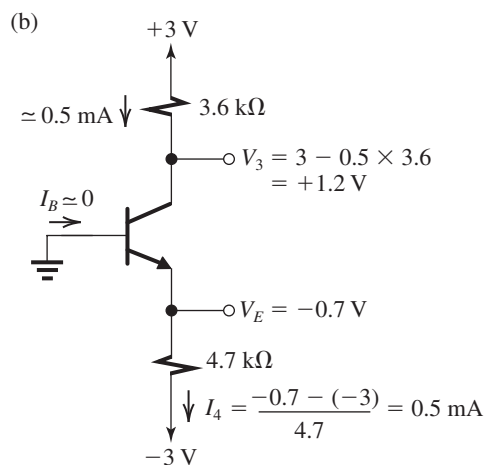
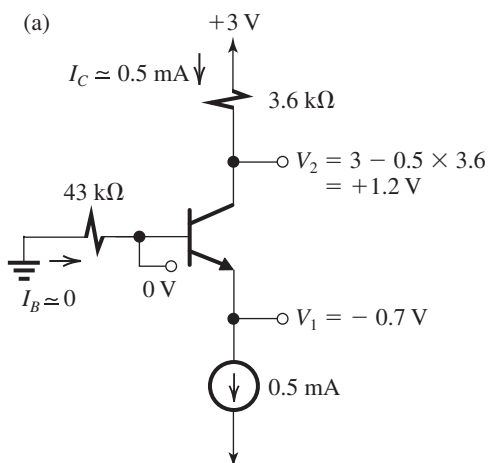
$$I_C = \frac{3 - 1.9}{1} = 1.1 \text{ mA}$$

Thus, the transistor is operating at a forced  $\beta$  of

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{1.1}{0.6} = 1.83$$

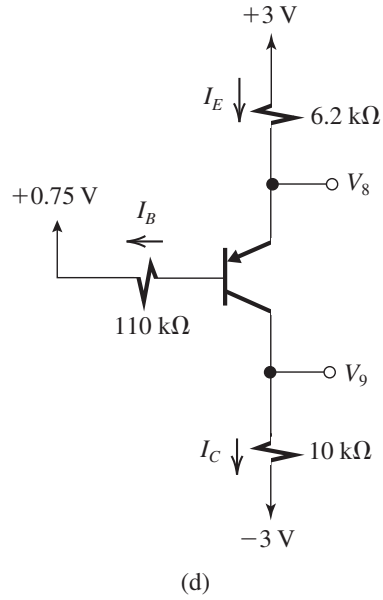
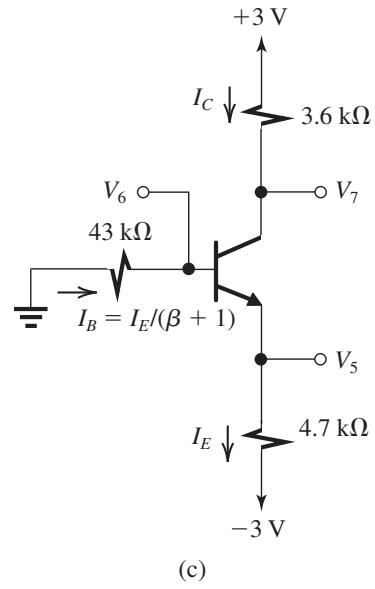
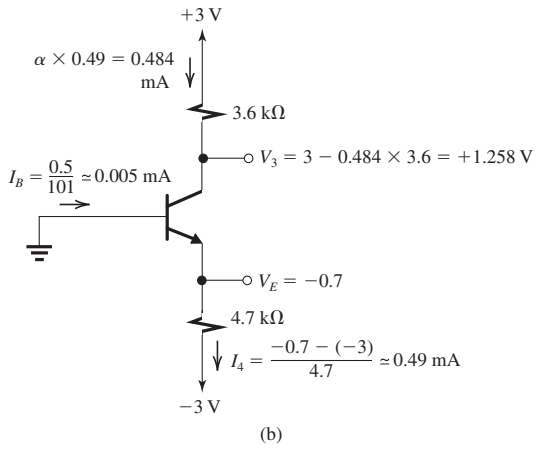
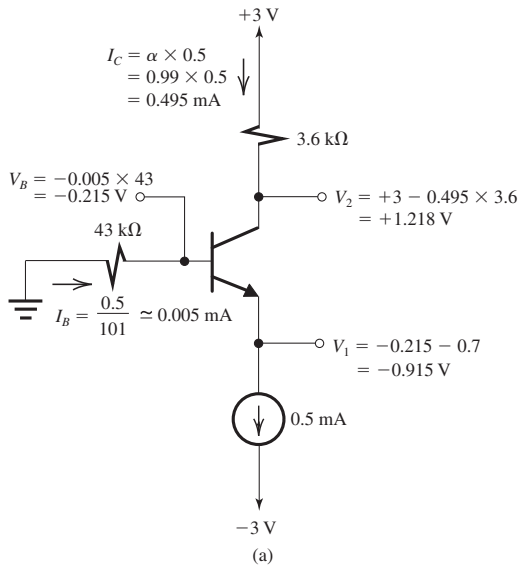
which is much lower than the value of  $\beta$ , confirming operation in saturation.

4.54

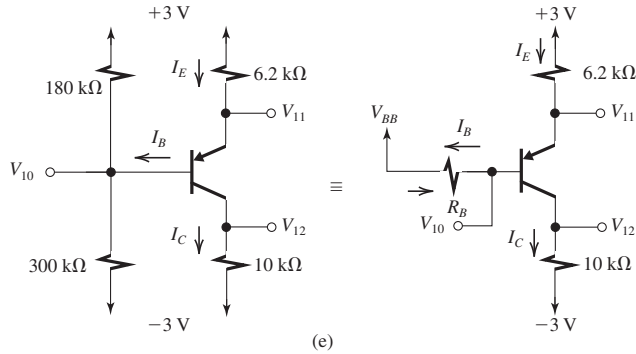


4.55 (a)

See solution and answer on the figure, which corresponds to Fig. P4.54(a).



This figure belongs to Problem 4.55, part (e).



(b)

See solution and answer on the figure, which corresponds to Fig. P4.61(b).

(c)

Writing an equation for the loop containing the BEJ of the transistor leads to

$$I_E = \frac{3 - 0.7}{4.7 + \frac{43}{101}} = 0.449 \text{ mA}$$

$$V_5 = -3 + 0.449 \times 4.7 = -0.9 \text{ V}$$

$$V_6 = -0.9 + 0.7 = -0.2 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.449 = 0.444 \text{ mA}$$

$$V_7 = 3 - 0.444 \times 3.6 = +1.4 \text{ V}$$

(d)

An equation for the loop containing the EBJ of the transistor yields

$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{110}{101}} = 0.213 \text{ mA}$$

$$V_8 = +3 - 0.213 \times 6.2 = +1.7 \text{ V}$$

$$V_B = 1 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.213 = 0.21 \text{ mA}$$

$$V_9 = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

(e) See figure on the previous page. First, we use Thévenin's theorem to replace the voltage divider feeding the base with  $V_{BB}$  and  $R_B$ :

$$V_{BB} = -3 + \frac{6}{480} \times 300 = +0.75 \text{ V}$$

$$R_B = 180 \parallel 300 = 112.5 \text{ k}\Omega$$

Next we write an equation for the loop containing the EBJ to obtain

$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{112.5}{101}} = 0.212 \text{ mA}$$

$$V_{11} = +3 - 0.212 \times 6.2 = +1.7 \text{ V}$$

$$V_{10} = 1.7 - 0.7 = +1 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.212 = 0.21 \text{ mA}$$

$$V_{12} = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

Note that these results are nearly identical to those for case (d) above.

**4.56** Refer to Fig. P4.56. To obtain  $I_E = 0.2 \text{ mA}$  and  $V_E = 2 \text{ V}$  we select

$$R_E = \frac{2 \text{ V}}{0.2 \text{ mA}} = 10 \text{ k}\Omega$$

Now, the base voltage has to be

$$V_B = V_E + 0.7 = 2.7 \text{ V}$$

For  $I_{B2} = 0.1 \text{ mA}$ , the value of  $R_{B2}$  must be

$$R_{B2} = \frac{V_B}{I_{B2}} = \frac{2.7 \text{ V}}{0.1 \text{ mA}} = 27 \text{ k}\Omega$$

The base current  $I_B$  is

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.2}{100 + 1} = 0.002 \text{ mA}$$

and the current  $I_{B1}$  through  $R_{B1}$  is

$$I_{B1} = I_{B2} + I_B = 0.1 + 0.002 = 0.102 \text{ mA}$$

The required value of  $R_{B1}$  can now be found as

$$R_{B1} = \frac{9 - 2.7}{0.102} = 61.8 \text{ k}\Omega$$

The collector current  $I_C$  is

$$I_C = \alpha I_E = \left(\frac{\beta}{\beta + 1}\right) I_E = 0.99 \times 0.2 = 0.198 \text{ mA}$$

To obtain  $V_C = +5 \text{ V}$ , we select  $R_C$  as

$$R_C = \frac{9 - 5}{0.198} = 20.2 \text{ k}\Omega$$

Consulting the table of 5% resistors, in Appendix J, we find the nearest resistance values to be:

$$R_E = 10 \text{ k}\Omega, \quad R_{B1} = 62 \text{ k}\Omega,$$

$$R_{B2} = 27 \text{ k}\Omega, \quad \text{and} \quad R_C = 20 \text{ k}\Omega$$

We can use these values to determine

$$I_E = \frac{9 \times \left(\frac{27}{27 + 62}\right) - 0.7}{10 + \frac{27 \parallel 62}{101}} \simeq 0.2 \text{ mA}$$

$$V_E = 0.2 \times 10 = 2 \text{ V}$$

$$V_B = 2.7 \text{ V}$$

$$I_{B2} = \frac{2.7 \text{ V}}{27 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_C = 0.198 \text{ mA}$$

$$V_C = 9 - 0.198 \times 20 = 5.04 \text{ V}$$

Which are equal or very close to the desired values!

**4.57** Refer to the circuit in Fig. P4.57.

(a) For  $v_I = 0$ , both transistors are cut off and all currents are zero. Thus

$$V_B = 0 \text{ V} \quad \text{and} \quad V_E = 0 \text{ V}$$

(b) For  $v_I = -2$  V,  $Q_1$  will be off and  $Q_2$  will be on, and the circuit reduces to that in Fig. 1. Since  $V_B > -2$  V,  $V_C$  will be lower than  $V_B$ , and  $Q_2$  will be operating in the active region. Thus

$$I_E = \frac{2 - 0.7}{1 + \frac{10}{51}} = 1.1 \text{ mA}$$

$$V_E = -I_E \times 1 = -1.1 \text{ V}$$

$$V_B = V_E - 0.7 = -1.8 \text{ V}$$

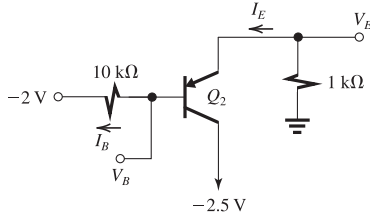


Figure 1

(c) For  $v_I = +2.5$  V,  $Q_1$  will be conducting and  $Q_2$  will be cut off, and the circuit reduces to that in Fig. 2.

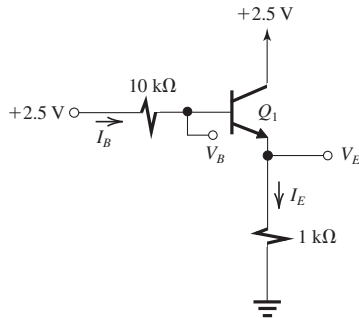


Figure 2

Since  $V_B$  will be lower than +2.5 V,  $V_C$  will be higher than  $V_B$ , and the transistor will be operating in the active mode. Thus,

$$I_E = \frac{2.5 - 0.7}{1 + \frac{10}{51}} = 1.5 \text{ mA}$$

$$V_E = I_E \times 1 = +1.5 \text{ V}$$

$$V_B = V_E + 0.7 = +2.2 \text{ V}$$

(d) For  $v_I = +5$ ,  $Q_1$  will be on and  $Q_2$  will be cut off, and the circuit reduces to that in Fig. 3.

Here we do not know whether  $Q_1$  is operating in the active mode or in saturation. Assuming active-mode operation, we obtain

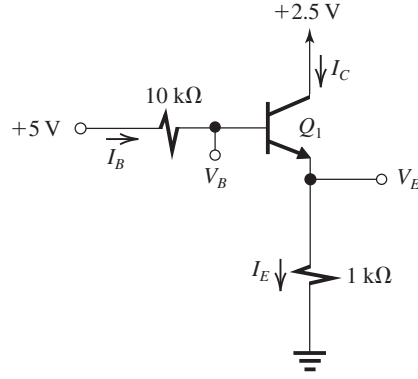


Figure 3

$$I_E = \frac{5 - 0.7}{1 + \frac{10}{51}} = 3.6 \text{ mA}$$

$$V_E = 3.6 \times 1 = +3.6 \text{ V}$$

$$V_B = +4.3$$

which is greater than  $V_C$ . This is an impossible situation, indicating that  $Q_1$  must be saturated. Assuming saturation-mode operation, we obtain

$$V_E = V_C - V_{CEsat} = 2.5 - 0.2 = +2.3 \text{ V}$$

$$I_E = \frac{V_E}{1 \text{ k}\Omega} = 2.3 \text{ mA}$$

$$V_B = V_E + 0.7 = 3.0 \text{ V}$$

$$I_B = \frac{5 - 3}{10} = 0.2 \text{ mA}$$

$$I_C = I_E - I_B = 2.3 - 0.2 = 2.1 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.1}{0.2} = 10.5$$

which is lower than  $\beta$ , verifying that  $Q_1$  is operating in saturation.

4.58

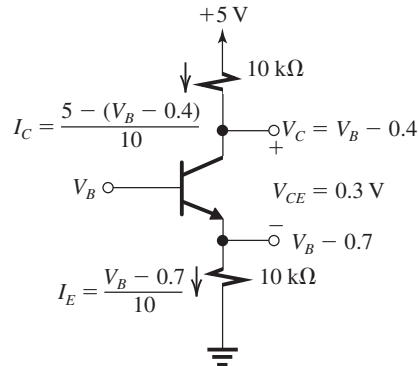


Figure 1

Figure 1 shows the circuit with the value of  $V_B$  that results in operation at the edge of saturation. Since  $\beta$  is very high,

$$I_C \simeq I_E$$

$$\frac{5 - (V_B - 0.4)}{10} = \frac{V_B - 0.7}{10}$$

$$\Rightarrow V_B = 3.05 \text{ V}$$

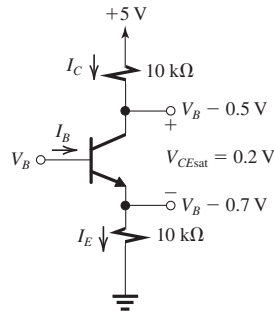


Figure 2

Figure 2 shows the circuit with the value of  $V_B$  that results in the transistor operating in saturation, with

$$I_E = \frac{V_B - 0.7}{10} = 0.1V_B - 0.07$$

$$I_C = \frac{5 - (V_B - 0.5)}{10} = 0.55 - 0.1V_B$$

$$I_B = I_E - I_C = 0.2V_B - 0.62$$

For  $\beta_{\text{forced}} = 3$ ,

$$\frac{I_C}{I_B} = 3$$

$$\frac{0.55 - 0.1V_B}{0.2V_B - 0.62} = 3$$

$$\Rightarrow V_B = 3.44 \text{ V}$$

**4.59** Refer to the circuit in Fig. P4.59.

(a) For  $V_B = -1 \text{ V}$ ,

$$V_E = V_B - V_{BE} = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{1} = \frac{-1.7 + 3}{1} = 1.3 \text{ mA}$$

Assuming active-mode operation, we have

$$I_C = \alpha I_E \simeq I_E = 1.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 1.3 = +1.7 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the transistor is operating in the active mode as assumed.

(b) For  $V_B = 0 \text{ V}$ ,

$$V_E = 0 - V_{BE} = -0.7 \text{ V}$$

$$I_E = \frac{-0.7 - (-3)}{1} = 2.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \simeq I_E = 2.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 2.3 = +0.7 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the BJT is operating in the active mode, as assumed.

(c) For  $V_B = +1 \text{ V}$ ,

$$V_E = 1 - 0.7 = +0.3 \text{ V}$$

$$I_E = \frac{0.3 - (-3)}{1} = 3.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \simeq I_E = 3.3 \text{ mA}$$

$$V_C = 3 - 3.3 \times 1 = -0.3 \text{ V}$$

Now  $V_C < V_B - 0.4$ , indicating that the transistor is operating in saturation, and our original assumption is incorrect. It follows that

$$V_C = V_E + V_{CE(sat)}$$

$$= 0.3 + 0.2 = 0.5 \text{ V}$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - 0.5}{1} = 2.5 \text{ mA}$$

$$I_B = I_E - I_C = 3.3 - 2.5 = 0.8 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.5}{0.8} = 3.1$$

(d) When  $V_B = 0 \text{ V}$ ,  $I_E = 2.3 \text{ mA}$ . The emitter current becomes  $0.23 \text{ mA}$  at

$$V_B = -3 + 0.23 \times 1 + 0.7 = -2.07 \text{ V}$$

(e) The transistor will be at the edge of conduction when  $I_E \simeq 0$  and  $V_{BE} = 0.5 \text{ V}$ , that is,

$$V_B = -3 + 0.5 = -2.5 \text{ V}$$

In this case,

$$V_E = -3 \text{ V}$$

$$V_C = +3 \text{ V}$$

(f) The transistor reaches the edge of saturation when  $V_{CE} = 0.3 \text{ V}$  but  $I_C = \alpha I_E \simeq I_E$ :

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + 0.3 = V_B - 0.4$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - V_B + 0.4}{1} = 3.4 - V_B$$

Since

$$I_C \simeq I_E$$

$$3.4 - V_B = V_B + 2.3$$

$$V_B = 0.55 \text{ V}$$

For this value,

$$V_E = 0.55 - 0.7 = -0.15 \text{ V}$$

$$V_C = -0.15 + 0.3 = +0.15 \text{ V}$$

(g) For the transistor to operate in saturation with  $\beta_{\text{forced}} = 2$ ,

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + V_{CE\text{sat}} = V_B - 0.7 + 0.2 = V_B - 0.5$$

$$I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B$$

$$I_B = I_E - I_C = 2 V_B - 1.2$$

$$\frac{I_C}{I_B} = \frac{3.5 - V_B}{2 V_B - 1.2} = 2$$

$$\Rightarrow V_B = +1.18 \text{ V}$$

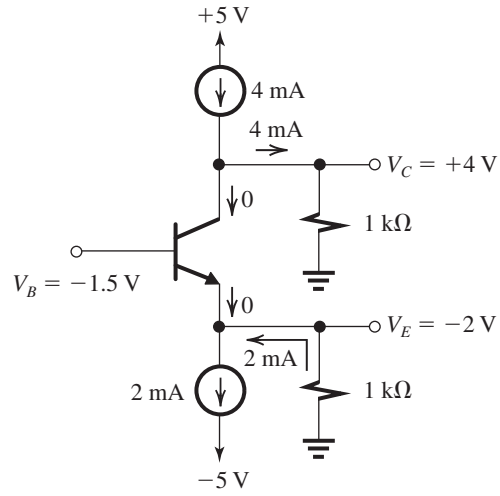


Figure 2

(c)

4.60 (a)  $V_B = 0 \text{ V}$

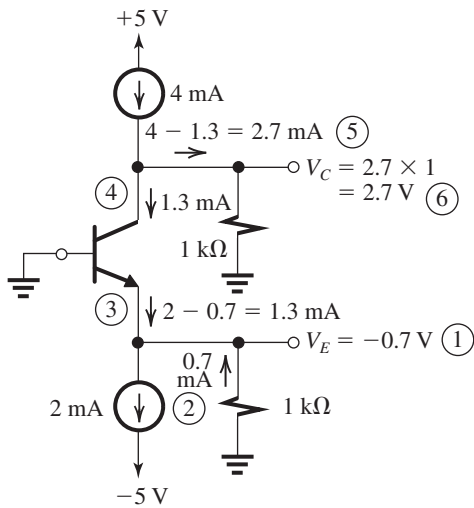


Figure 1

The analysis is shown on the circuit diagram in Fig. 1. The circled numbers indicate the order of the analysis steps.

(b) The transistor cuts off at the value of  $V_B$  that causes the 2-mA current of the current source feeding the emitter to flow through the 1-k $\Omega$  resistor connected between the emitter and ground. The circuit under these conditions is shown in Fig. 2.

Observe that  $V_E = -2 \text{ mA} \times 1 \text{ k}\Omega = -2 \text{ V}$ ,  $I_E = 0$ , and  $V_B = V_E + 0.5 = -1.5 \text{ V}$ . Since  $I_C = 0$ , all the 4 mA supplied by the current source feeding the collector flows through the collector 1-k $\Omega$  resistor, resulting in  $V_C = +4 \text{ V}$ .

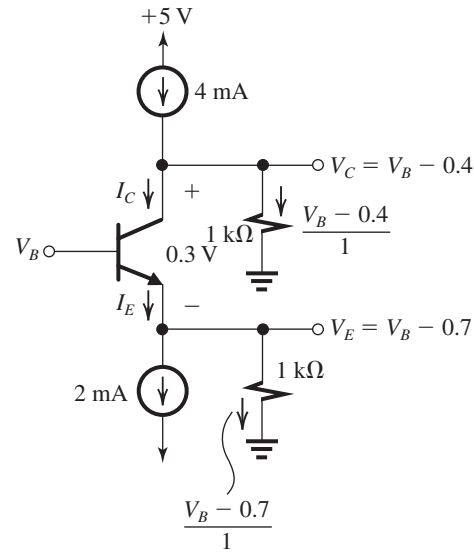


Figure 3

Figure 3 shows the transistor at the edge of saturation. Here  $V_{CE} = 0.3 \text{ V}$  and  $I_C = \alpha I_E \simeq I_E$ .

A node equation at the emitter gives

$$I_E = 2 + V_B - 0.7 = V_B + 1.3 \text{ mA}$$

A node equation at the collector gives

$$I_C = 4 - (V_B - 0.4) = 4.4 - V_B \text{ mA}$$

Imposing the condition  $I_C \simeq I_E$  gives

$$4.4 - V_B = V_B + 1.3$$

$$\Rightarrow V_B = +1.55 \text{ V}$$

Correspondingly,

$$V_E = +0.85 \text{ V}$$

$$V_C = +1.15 \text{ V}$$



4.61

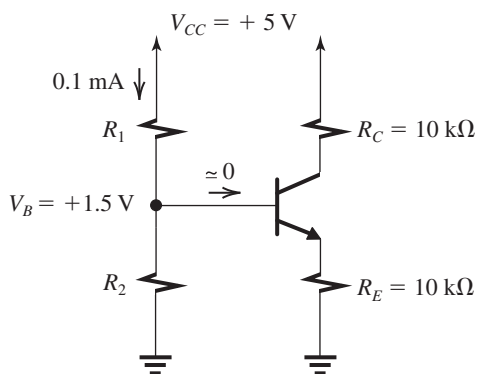


Figure 1

From Fig. 1 we see that

$$R_1 + R_2 = \frac{V_{CC}}{0.1 \text{ mA}} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$V_{CC} \frac{R_2}{R_1 + R_2} = 1.5$$

$$5 \times \frac{R_2}{50} = 1.5$$

$$\Rightarrow R_2 = 15 \text{ k}\Omega$$

$$R_1 = 50 - 15 = 35 \text{ k}\Omega$$

For  $\beta = 100$ , to obtain the collector current, we replace the voltage divider with its Thévenin equivalent, consisting of

$$V_{BB} = 5 \times \frac{R_2}{R_1 + R_2} = 5 \times \frac{15}{35 + 15} = 1.5 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 15 \parallel 35 = 10.5 \text{ k}\Omega$$

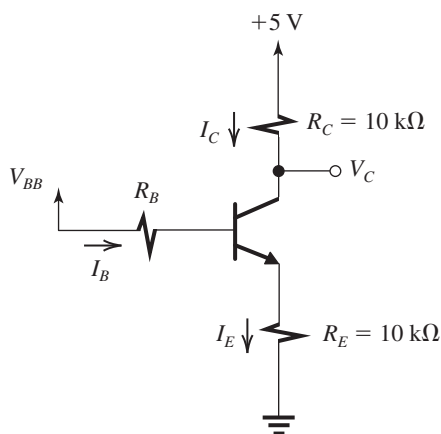


Figure 2

Refer to Fig. 2. Assuming active-mode operation, we can write a loop equation for the base-emitter loop:  $V_{BB} = I_B R_B + V_{BE} + I_E R_E$

$$1.5 = \frac{I_E}{\beta + 1} \times 10.5 + 0.7 + I_E \times 10$$

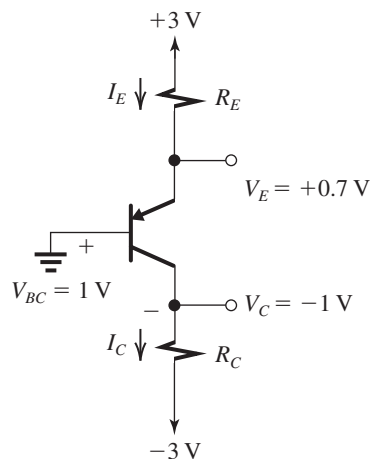
$$\Rightarrow I_E = \frac{1.5 - 0.7}{10 + \frac{10.5}{101}} = 0.079 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.079 = 0.078 \text{ mA}$$

$$V_C = +5 - 0.078 \times 10 = +4.22 \text{ V}$$

Since  $V_B = I_E R_E + V_{BE} = 0.79 + 0.7 = 1.49 \text{ V}$ , we see that  $V_C > V_B - 0.4$ , and thus the transistor is operating in the active region, as assumed.

4.62



Refer to the figure. To obtain  $I_E = 0.5 \text{ mA}$  we select  $R_E$  according to

$$R_E = \frac{3 - 0.7}{0.5} = 4.6 \text{ k}\Omega$$

To obtain  $V_C = -1 \text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{-1 - (-3)}{0.5} = 4 \text{ k}\Omega$$

where we have utilized the fact that  $\alpha \simeq 1$  and thus  $I_C \simeq I_E = 0.5 \text{ mA}$ . From the table of 5% resistors in Appendix J we select

$$R_E = 4.7 \text{ k}\Omega \quad \text{and} \quad R_C = 3.9 \text{ k}\Omega$$

For these values,

$$I_E = \frac{3 - 0.7}{4.7} = 0.49 \text{ mA}$$

$$I_C \simeq I_E = 0.49 \text{ mA}$$

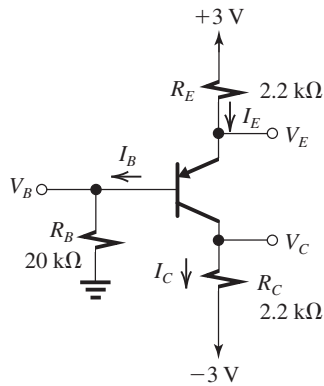
$$V_{BC} = 0 - V_C = -(-3 + 0.49 \times 3.9) = -1.1 \text{ V}$$

4.63

Writing a loop equation for the EBJ loop, we have

$$3 = I_E R_E + V_{EB} + I_B R_B \quad (1)$$

$$= I_E \times 2.2 + 0.7 + \frac{I_E}{\beta + 1} \times 20$$



$$\Rightarrow I_E = \frac{3 - 0.7}{2.2 + \frac{20}{41}} = 0.86 \text{ mA}$$

$$V_E = 3 - 0.86 \times 2.2 = +1.11 \text{ V}$$

$$V_B = V_E - 0.7 = +0.41 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.86 = 0.84 \text{ mA}$$

$$V_C = -3 + 0.84 \times 2.2 = -1.15 \text{ V}$$

Since  $V_C < V_B + 0.4$ , the transistor is operating in the active mode, as assumed. Now, if  $R_B$  is increased to 100 kΩ, the loop equation [Eq. (1)] yields

$$I_E = \frac{3 - 0.7}{2.2 + \frac{100}{41}} = 0.5 \text{ mA}$$

$$V_E = 3 - 0.5 \times 2.2 = +1.9 \text{ V}$$

$$V_B = V_E - V_{EB} = 1.9 - 0.7 = +1.2 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.5 = 0.48 \text{ mA}$$

$$V_C = -3 + 0.48 \times 2.2 = -1.9 \text{ V}$$

Since  $V_C < V_B + 0.4$ , the transistor is operating in the active mode, as assumed. If with  $R_B = 100 \text{ k}\Omega$ , we need the voltages to remain at the values obtained with  $R_B = 20 \text{ k}\Omega$ , the transistor must have a  $\beta$  value determined as follows. For  $I_E$  to remain unchanged,

$$\frac{3 - 0.7}{2.2 + \frac{20}{41}} = \frac{3 - 0.7}{2.2 + \frac{100}{\beta + 1}}$$

$$\Rightarrow \frac{20}{41} = \frac{100}{\beta + 1}$$

$$\beta + 1 = \frac{410}{2} = 205$$

$$\beta = 204$$

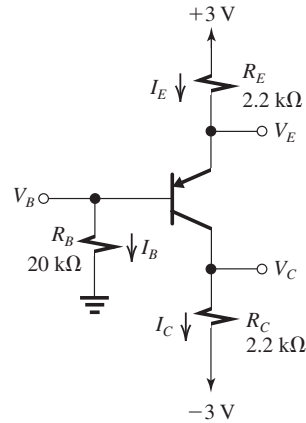
## 4.64

Assume active-mode operation:

$$I_E = \frac{3 - V_{EB}}{R_E + \frac{R_B}{\beta + 1}}$$

$$I_E = \frac{3 - 0.7}{2.2 + \frac{20}{51}} = 0.887 \text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.887}{51} = 0.017 \text{ mA}$$



$$I_C = I_E - I_B = 0.887 - 0.017 = 0.870 \text{ mA}$$

$$V_B = I_B R_B = 0.017 \times 20 = 0.34 \text{ V}$$

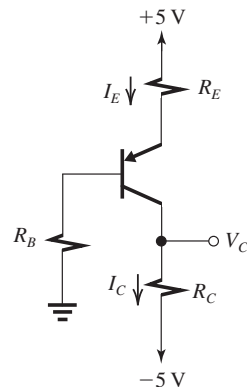
$$V_E = V_B + V_{EB} = 0.34 + 0.7 = 1.04 \text{ V}$$

$$V_C = -3 + I_C R_C = -3 + 0.87 \times 2.2 = -1.09 \text{ V}$$

Thus,  $V_C < V_B + 0.4$ , which means active-mode operation, as assumed. The maximum value of  $R_C$  that still guarantees active-mode operation is that which causes  $V_C$  to be 0.4 V above  $V_B$ : that is,  $V_C = 0.34 + 0.4 = 0.74 \text{ V}$ . Correspondingly,

$$R_{C\text{max}} = \frac{0.74 - (-3)}{0.87} = 4.3 \text{ k}\Omega$$

## 4.65



We require  $I_E$  to be nominally 1 mA (i.e., at  $\beta = 100$ ) and to remain within  $\pm 10\%$  as  $\beta$  varies from 50 to 150. Writing an equation for the loop containing the EBJ results in

$$I_E = \frac{5 - 0.7}{R_E + \frac{R_B}{\beta + 1}}$$

Thus,

$$\frac{4.3}{R_E + \frac{R_B}{101}} = 1 \quad (1)$$

$$\frac{4.3}{R_E + \frac{R_B}{51}} = I_{E\min} \quad (2)$$

$$\frac{4.3}{R_E + \frac{R_B}{151}} = I_{E\max} \quad (3)$$

If we set  $I_{E\min} = 0.9$  mA and solve Eqs. (1) and (2) simultaneously, we obtain

$$R_E = 3.81 \text{ k}\Omega$$

$$R_B = 49.2 \text{ k}\Omega$$

Substituting these values in Eqs. (2) and (3) gives

$$I_{E\min} = 0.9 \text{ mA}$$

$$I_{E\max} = 1.04 \text{ mA}$$

Obviously, this is an acceptable design.

Alternatively, if we set  $I_{E\max}$  in Eq. (3) to 1.1 mA and solve Eqs. (1) and (3) simultaneously, we obtain

$$R_E = 3.1 \text{ k}\Omega$$

$$R_B = 119.2 \text{ k}\Omega$$

Substituting these values in Eqs. (2) and (3) gives

$$I_{E\min} = 0.8 \text{ mA}$$

$$I_{E\max} = 1.1 \text{ mA}$$

Obviously this is not an acceptable design ( $I_{E\min}$  is 20% lower than nominal). Therefore, we shall use the first design. Specifying the resistor values to the nearest kilohm results in

$$R_E = 4 \text{ k}\Omega$$

$$R_B = 50 \text{ k}\Omega$$

To obtain the value of  $R_C$ , we note that at the nominal emitter current value of 1 mA,

$$V_C = -1 \text{ V},$$

$$I_C = \alpha I_E = 0.99 \text{ mA}$$

$$R_C = \frac{-1 - (-5)}{0.99} = 4.04 \text{ k}\Omega$$

Specified to the nearest kilohm,

$$R_C = 4 \text{ k}\Omega$$

Finally, for our design we need to determine the range obtained for collector current and collector voltage for  $\beta$  ranging from 50 to 150 with a nominal value of 100. We compute the nominal value of  $I_E$  from

$$I_{Enominal} = \frac{4.3}{4 + \frac{50}{101}} = 0.96 \text{ mA}$$

We utilize Eqs. (2) and (3) to compute  $I_{E\min}$  and  $I_{E\max}$ ,

$$I_{E\min} = \frac{4.3}{4 + \frac{50}{51}} = 0.86 \text{ mA}$$

$$I_{E\max} = \frac{4.3}{4 + \frac{50}{151}} = 0.99 \text{ mA}$$

Thus,

$$\frac{I_{E\max}}{I_{Enominal}} = \frac{0.99}{0.96} = 1.03$$

$$\frac{I_{E\min}}{I_{Enominal}} = \frac{0.86}{0.96} = 0.9$$

which meet our specifications. The collector currents are

$$I_{Cnominal} = 0.99 \times 0.96 = 0.95 \text{ mA}$$

$$I_{C\min} = 0.99 \times 0.86 = 0.85 \text{ mA}$$

$$I_{C\max} = 0.99 \times 0.99 = 0.98 \text{ mA}$$

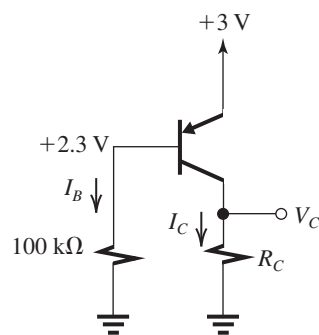
and the collector voltages are

$$V_{Cnominal} = -5 + 0.95 \times 4 = -1.2 \text{ V}$$

$$V_{C\min} = -5 + 0.85 \times 4 = -1.6 \text{ V}$$

$$V_{C\max} = -5 + 0.98 \times 4 = -1.1 \text{ V}$$

#### 4.66



$$I_B = \frac{2.3 \text{ V}}{100 \text{ k}\Omega} = 0.023 \text{ mA}$$

Since  $V_C = 2\text{ V}$  is lower than  $V_B$ , which is  $+2.3\text{ V}$ , the transistor will be operating in the active mode. Thus,

$$I_C = \beta I_B = 50 \times 0.023 = 1.15\text{ mA}$$

To obtain  $V_C = 2\text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{V_C}{I_C} = \frac{2\text{ V}}{1.15\text{ mA}} = 1.74\text{ k}\Omega$$

Now, if the transistor is replaced with another having  $\beta = 100$ , then

$$I_C = 100 \times 0.023 = 2.3\text{ mA}$$

which would imply

$$V_C = 2.3 \times 1.74 = 4\text{ V}$$

which is impossible because the base is at  $2.3\text{ V}$ . Thus the transistor must be in the saturation mode and

$$\begin{aligned} V_C &= V_E - V_{ECsat} \\ &= 3 - 0.2 = 2.8\text{ V} \end{aligned}$$

**4.67** (a)  $\beta = \infty$

The analysis and results are given in the circuit diagram in Fig. 1 below. The circled numbers indicate the order of the analysis steps.

(b)  $\beta = 100$

By reference to Fig. 2, we can write an equation for the loop containing the EBJ of  $Q_1$  as follows:

$$3 = I_{E1} \times 9.1 + 0.7 + I_{B1} \times 100$$

Substituting  $I_{B1} = I_{E1}/(\beta + 1) = I_{E1}/101$  and rearranging, we obtain

$$I_{E1} = \frac{3 - 0.7}{9.1 + \frac{100}{101}} = 0.228\text{ mA}$$

This figure belongs to Problem 4.67.

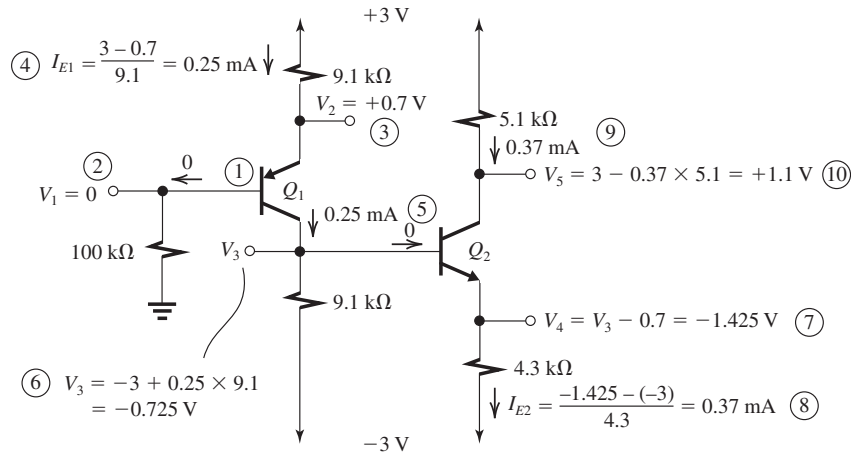


Figure 1 ( $\beta = \infty$ )

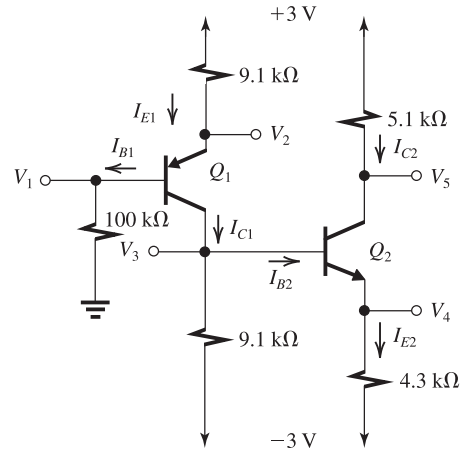


Figure 2 ( $\beta = 100$ )

Thus,

$$I_{B1} = \frac{I_{E1}}{101} = 0.0023\text{ mA}$$

$$V_1 = 0.0023 \times 100$$

$$= 0.23\text{ V}$$

$$V_2 = V_1 + 0.7 = 0.93\text{ V}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.228 = 0.226\text{ mA}$$

Then we write a node equation at  $C_1$ :

$$I_{C1} = I_{B2} + \frac{V_3 - (-3)}{9.1}$$

Substituting for  $I_{C1} = 0.226\text{ mA}$ ,  $I_{B2} = I_{E2}/101$ , and  $V_3 = V_4 + 0.7 = -3 + I_{E2} \times 4.3 + 0.7$  gives

$$0.226 = \frac{I_{E2}}{101} + \frac{-3 + 4.3I_{E2} + 0.7 + 3}{9.1}$$

$$= \frac{I_{E2}}{101} + \frac{4.3I_{E2}}{9.1} + \frac{0.7}{9.1}$$

$$\Rightarrow I_{E2} = 0.31\text{ mA}$$

$$I_{B2} = 0.0031$$

$$I_{C1} - I_{B2} = 0.226 - 0.0031 = 0.223 \text{ mA}$$

$$V_3 = -3 + 0.223 \times 9.1 = -0.97 \text{ V} \approx -1 \text{ V}$$

$$V_4 = V_3 - 0.7 = -1.7 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.31 = 0.3 \text{ mA}$$

$$V_5 = +3 - 0.3 \times 5.1 = +1.47 \text{ V}$$

$$R_2 = \frac{5 - V_3}{0.5} = \frac{5 - 0}{0.5} = 10 \text{ k}\Omega$$

$$V_4 = 0 + 0.7 = 0.7 \text{ V}$$

$$R_3 = \frac{5 - V_4}{0.5} = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

$$R_4 = \frac{V_5 - (-5)}{0.5} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

$$V_6 = V_5 - 0.7 = -2 - 0.7 = -2.7 \text{ V}$$

$$R_6 = \frac{V_6 - (-5)}{1} = \frac{-2.7 + 5}{1} = 2.3 \text{ k}\Omega$$

$$R_5 = \frac{5 - V_7}{1} = \frac{5 - 1}{1} = 4 \text{ k}\Omega$$

Consulting the table of 5% resistors in Appendix J, we select the following resistor values:

$$R_1 = 8.2 \text{ k}\Omega \quad R_2 = 10 \text{ k}\Omega \quad R_3 = 8.2 \text{ k}\Omega$$

**4.68** Figure 1 below shows the circuit with  $\beta = \infty$ ; the required voltage values are indicated. The resistor values are obtained as follows:

$$V_2 = -0.7 \text{ V}$$

$$R_1 = \frac{V_2 - (-5)}{0.5 \text{ mA}}$$

$$\Rightarrow R_1 = 8.6 \text{ k}\Omega$$

This figure belongs to Problem 4.68.

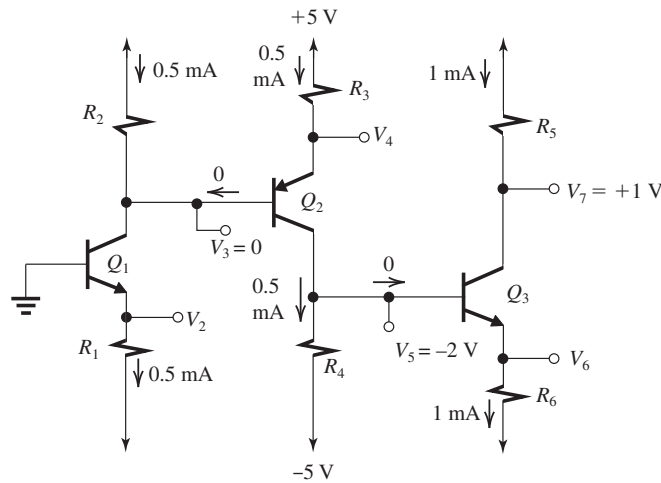


Figure 1

This figure belongs to Problem 4.68.

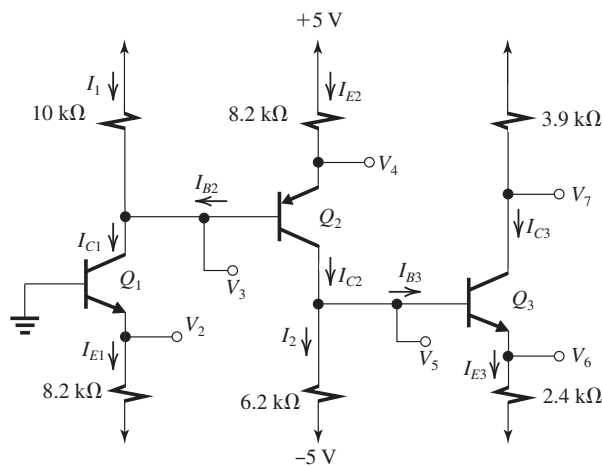


Figure 2

$$R_4 = 6.2 \text{ k}\Omega \quad R_5 = 3.9 \text{ k}\Omega \quad R_6 = 2.4 \text{ k}\Omega$$

The circuit with the selected resistor values is shown in Fig. 2. Analysis of the circuit proceeds as follows:

$$V_2 = -0.7 \text{ V}$$

$$I_{E1} = \frac{V_2 - (-5)}{8.2} = \frac{-0.7 + 5}{8.2} = 0.524 \text{ mA}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.524 = 0.52 \text{ mA}$$

The current  $I_1$  through the 10-k $\Omega$  resistor is given by

$$I_1 = I_{C1} - I_{B2} = I_{C1} - \frac{I_{E2}}{101}$$

Noting that the voltage drop across the 10 k $\Omega$  resistor is equal to  $(I_{E2} \times 8.2 + 0.7)$ , we can write

$$I_1 \times 10 = 8.2I_{E2} + 0.7$$

Thus,

$$10 \left( 0.52 - \frac{I_{E2}}{101} \right) = 8.2I_{E2} + 0.7$$

$$\Rightarrow I_{E2} = 0.542 \text{ mA}$$

$$V_4 = 5 - 0.542 \times 8.2 = 0.56 \text{ V}$$

$$V_3 = 0.56 - 0.7 = -0.14 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.542 = 0.537 \text{ mA}$$

$$I_2 = I_{C2} - I_{B3} = 0.537 - \frac{I_{E3}}{101}$$

Since the voltage drop across the 6.2-k $\Omega$  resistor is equal to  $(0.7 + I_{E3} \times 2.4)$ ,

$$I_2 \times 6.2 = 0.7 + 2.4I_{E3}$$

$$6.2 \left( 0.537 - \frac{I_{E3}}{101} \right) = 0.7 + 2.4I_{E3}$$

$$\Rightarrow I_{E3} = 1.07 \text{ mA}$$

$$V_6 = -5 + 1.07 \times 2.4 = -2.43 \text{ V}$$

$$V_5 = V_6 + 0.7 = -1.73 \text{ V}$$

$$I_{C3} = \alpha \times I_{E3} = 0.99 \times 1.07 = 1.06 \text{ mA}$$

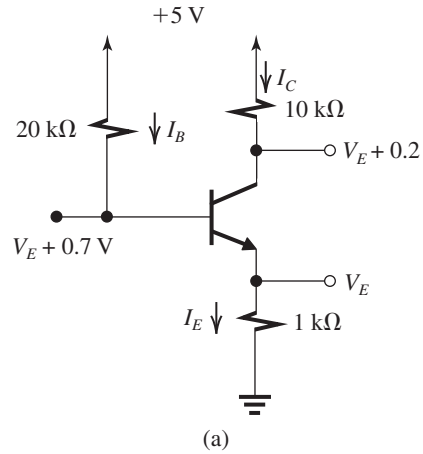
$$V_7 = -3.9 \times 1.06 = 0.87 \text{ V}$$

**4.69** (a) Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P4.69(a). Thus we can write

$$I_E = \frac{V_E}{1} = V_E$$

$$I_C = \frac{5 - (V_E + 0.2)}{10} = 0.5 - 0.1(V_E + 0.2)$$

$$I_B = \frac{5 - (V_E + 0.7)}{20} = 0.25 - 0.05(V_E + 0.7)$$



Now, imposing the constraint

$$I_E = I_C + I_B$$

results in

$$V_E = 0.5 - 0.1(V_E + 0.2) + 0.25 - 0.05(V_E + 0.7)$$

$$\Rightarrow V_E = 0.6 \text{ V}$$

$$V_C = 0.8 \text{ V}$$

$$V_B = 1.3 \text{ V}$$

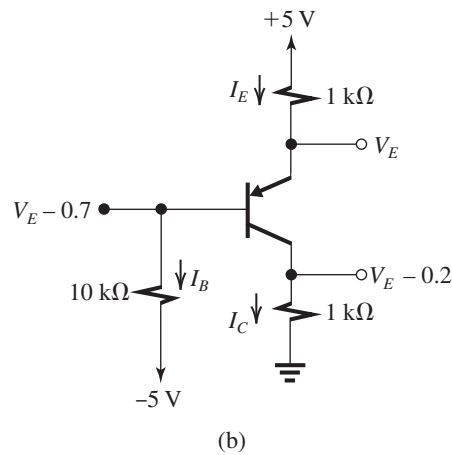
$$I_C = \frac{5 - 0.8}{10} = 0.42 \text{ mA}$$

$$I_B = \frac{5 - 1.3}{20} = 0.185 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{0.42}{0.185} = 2.3$$

which is less than the value of  $\beta_1$  verifying saturation-mode operation.

(b)



Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P4.69(b). We can obtain the currents as follows:

$$I_E = \frac{5 - V_E}{1} = 5 - V_E$$

$$I_C = \frac{V_E - 0.2}{1} = V_E - 0.2$$

$$I_B = \frac{V_E - 0.7 - (-5)}{10} = 0.1 V_E + 0.43$$

Imposing the constraint

$$I_E = I_B + I_C$$

results in  $5 - V_E = V_E - 0.2 + 0.1 V_E + 0.43$

$$\Rightarrow V_E = +2.27 \text{ V}$$

$$V_C = +2.07 \text{ V}$$

$$V_B = 1.57 \text{ V}$$

$$I_C = \frac{2.07}{1} = 2.07 \text{ mA}$$

$$I_B = \frac{1.57 - (-5)}{10} = 0.657 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.07}{0.657} = 3.2$$

which is lower than the value of  $\beta$ , verifying saturation-mode operation.

(c) We will assume that both  $Q_3$  and  $Q_4$  are operating in saturation. To begin the analysis shown in the figure, which corresponds to Fig. P4.69(c), we denote the voltage at the collector of  $Q_3$  as  $V$  and then obtain the voltages at all other nodes in terms of  $V$ , utilizing the fact that a saturated transistor has  $|V_{CE}| = 0.2 \text{ V}$  and of course  $|V_{BE}| = 0.7 \text{ V}$ . Note that the choice of the collector node to begin the analysis is arbitrary; we could have selected any other node

and denoted its voltage as  $V$ . We next draw a circle around the two transistors to define a “supernode.” A node equation for the supernode will be

$$I_{E3} + I_{C4} = I_{B3} + I + I_{E4} \quad (1)$$

where

$$I_{E3} = \frac{5 - (V + 0.2)}{10} = 0.48 - 0.1V \quad (2)$$

$$I_{C4} = \frac{5 - (V - 0.5)}{30} = 0.183 - 0.033V \quad (3)$$

$$I_{B3} = \frac{(V - 0.5)}{10} = 0.1V - 0.05 \quad (4)$$

$$I = \frac{V}{20} = 0.05V \quad (5)$$

$$I_{E4} = \frac{V - 0.7}{10} = 0.1V - 0.07 \quad (6)$$

Substituting from Eqs. (2)–(6) into Eq. (1) gives

$$0.48 - 0.1V + 0.183 - 0.033V$$

$$= 0.1V - 0.05 + 0.05V + 0.1V - 0.07$$

$$\Rightarrow V = 2.044 \text{ V}$$

Thus

$$V_{C3} = V = 2.044 \text{ V}$$

$$V_{C4} = V - 0.5 = 1.54 \text{ V}$$

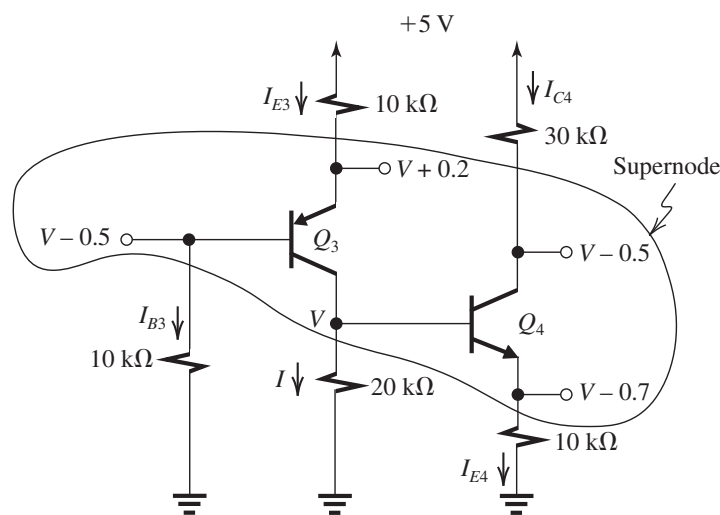
Next we determine all currents utilizing Eqs. (2)–(6):

$$I_{E3} = 0.276 \text{ mA} \quad I_{C4} = 0.116 \text{ mA}$$

$$I_{B3} = 0.154 \text{ mA} \quad I = 0.102 \text{ mA}$$

$$I_{E4} = 0.134$$

This figure belongs to Problem 4.69, part (c).



(c)

The base current of  $Q_4$  can be obtained from

$$I_{B4} = I_{E4} - I_{C4} = 0.134 - 0.116 = 0.018 \text{ mA}$$

Finally, the collector current of  $Q_3$  can be found as

$$I_{C3} = I + I_{B4} = 0.102 + 0.018 = 0.120$$

The forced  $\beta$  values can now be found as

$$\beta_{\text{forced3}} = \frac{I_{C3}}{I_{B3}} = \frac{0.120}{0.154} = 0.8$$

$$\beta_{\text{forced4}} = \frac{I_{C4}}{I_{B4}} = \frac{0.116}{0.018} = 6.4$$

Both  $\beta_{\text{forced}}$  values are well below the  $\beta$  value of 50, verifying that  $Q_3$  and  $Q_4$  are in deep saturation.



**Chapter 5**

**Solutions to Exercises within the Chapter**

**Ex: 5.1**

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.625 \text{ fF}/\mu\text{m}^2$$

$$\mu_n = 450 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$k'_n = \mu_n C_{ox} = 388 \text{ }\mu\text{A}/\text{V}^2$$

$$v_{OV} = v_{GS} - V_t = 0.5 \text{ V}$$

$$g_{DS} = \frac{1}{1 \text{ k}\Omega} = k'_n \frac{W}{L} v_{OV} \Rightarrow \frac{W}{L} = 5.15$$

$$L = 0.18 \text{ }\mu\text{m}, \text{ so } W = 0.93 \text{ }\mu\text{m}$$

**Ex: 5.2**

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{1.4 \text{ nm}} = 24.6 \text{ fF}/\mu\text{m}^2$$

$$\mu_n = 216 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$k'_n = \mu_n C_{ox} = 531 \text{ }\mu\text{A}/\text{V}^2$$

$$i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2$$

$$50 = \frac{1}{2} \times 531 \times 10 \times v_{OV}^2$$

$$\therefore v_{OV} = 0.14 \text{ V}$$

$$v_{GS} = V_t + v_{OV} = 0.49 \text{ V}$$

$$v_{DS, \text{min}} = v_{OV} = 0.14 \text{ V}$$

**Ex: 5.3**  $i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2$  in saturation

Change in  $i_D$  is:

(a) double  $L$ , 0.5

(b) double  $W$ , 2

(c) double  $v_{OV}$ ,  $2^2 = 4$

(d) double  $v_{DS}$ , no change (ignoring length modulation)

(e) changes (a)-(d), 4

Case (c) would cause leaving saturation if

$$v_{DS} < 2v_{OV}$$

**Ex: 5.4** For saturation  $v_{DS} \geq v_{OV}$ , so  $v_{DS}$  must be changed to  $2v_{OV}$

$$i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2, \text{ so } i_D \text{ increases by a factor of 4.}$$

**Ex: 5.5**  $v_{OV} = 0.5 \text{ V}$

$$g_{DS} = k'_n \frac{W}{L} v_{OV} = \frac{1}{1 \text{ k}\Omega}$$

$$\therefore k_n = k'_n \frac{W}{L} = \frac{1}{1 \times 0.5} = 2 \text{ mA}/\text{V}^2$$

At  $v_{DS} = 0.2 \text{ V}$ ,  $v_{DS} < v_{OV}$ , thus the transistor is operating in the triode region,

$$i_D = k_n (v_{OV} v_{DS} - \frac{1}{2} v_{DS}^2) \\ = 2(0.5 \times 0.2 - \frac{1}{2} \times 0.2^2) = 0.16 \text{ mA}$$

At  $v_{DS} = 0.5 \text{ V}$ ,  $v_{DS} = v_{OV}$ , thus the transistor is operating in saturation,

$$i_D = \frac{1}{2} k_n v_{OV}^2 = \frac{1}{2} \times 2 \times 0.5^2 = 0.25 \text{ mA}$$

At  $v_{DS} = 1 \text{ V}$ ,  $v_{DS} > v_{OV}$  and the transistor is operating in saturation with  $i_D = 0.25 \text{ mA}$ .

**Ex: 5.6**  $V_A = V'_A L = 5 \times 0.8 = 4 \text{ V}$

$$\lambda = \frac{1}{V_A} = 0.25 \text{ V}^{-1}$$

$$v_{DS} = 0.8 \text{ V} > v_{OV} = 0.2 \text{ V}$$

$$\Rightarrow \text{Saturation: } i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2 (1 + \lambda v_{DS})$$

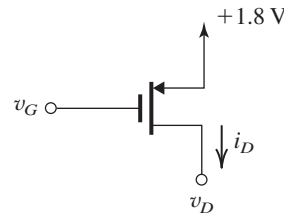
$$i_D = \frac{1}{2} \times 400 \times \frac{16}{0.8} \times 0.2^2 (1 + 0.25 \times 0.8) \\ = 0.192 \text{ mA}$$

$$r_o = \frac{V_A}{i_D} = \frac{4}{0.16} = 25 \text{ k}\Omega$$

where  $i_D$  is the value of  $i_D$  without channel-length modulation taken into account.

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{1 \text{ V}}{25 \text{ k}\Omega} = 0.04 \text{ mA} = 40 \text{ }\mu\text{A}$$

**Ex: 5.7**



$$V_{tp} = -0.5 \text{ V}$$

$$k'_p = 100 \text{ }\mu\text{A}/\text{V}^2$$

$$\frac{W}{L} = 10 \Rightarrow k_p = 1 \text{ mA}/\text{V}^2$$

(a) Conduction occurs for  $V_{SG} \geq |V_{tp}| = 0.5 \text{ V}$

$$\Rightarrow v_G \leq 1.8 - 0.5 = +1.3 \text{ V}$$

(b) Triode region occurs for  $v_{DG} \geq |V_{tp}| = 0.5 \text{ V}$

$$\Rightarrow v_D \geq v_G + 0.5$$

(c) Conversely, for saturation

$$v_{DG} \leq |V_{tp}| = 0.5 \text{ V}$$

$$\Rightarrow v_D \leq v_G + 0.5$$

(d) Given  $\lambda \cong 0$ ,

$$i_D = \frac{1}{2} k'_p \frac{W}{L} |v_{OV}|^2 = 50 \text{ } \mu\text{A}$$

$$\therefore |v_{OV}| = 0.32 \text{ V} = v_{SG} - |V_{tp}|$$

$$\Rightarrow v_{SG} = |v_{OV}| + |V_{tp}| = 0.82 \text{ V}$$

$$v_G = 1.8 - v_{SG} = 0.98 \text{ V}$$

$$v_D \leq v_G + 0.5 = 1.48 \text{ V}$$

(e) For  $\lambda = -0.2 \text{ V}^{-1}$  and  $|v_{OV}| = 0.32 \text{ V}$ ,

$$i_D = 50 \text{ } \mu\text{A} \text{ and } r_o = \frac{1}{|\lambda| i_D} = 100 \text{ k}\Omega$$

(f) At  $v_D = +1 \text{ V}$ ,  $v_{SD} = 0.8 \text{ V}$ ,

$$i_D = \frac{1}{2} k'_n \frac{W}{L} |v_{OV}|^2 (1 + |\lambda| |v_{SD}|)$$

$$= 50 \text{ } \mu\text{A} (1.16) = 58 \text{ } \mu\text{A}$$

At  $v_D = 0 \text{ V}$ ,  $v_{SD} = 1.8 \text{ V}$ ,

$$i_D = 50 \text{ } \mu\text{A} (1.36) = 68 \text{ } \mu\text{A}$$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} = \frac{1 \text{ V}}{10 \text{ } \mu\text{A}} = 100 \text{ k}\Omega$$

which is the same value found in (c).

**Ex: 5.8**

$$R_D = \frac{V_{DD} - v_D}{I_D} = \frac{1 - 0.2}{0.1} = 8 \text{ k}\Omega$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2 \Rightarrow$$

$$100 = \frac{1}{2} \times 400 \times \frac{5}{0.4} V_{OV}^2 \Rightarrow$$

$$V_{OV} = 0.2 \text{ V} \Rightarrow V_{GS} = V_{OV} + V_t = 0.2 + 0.4 = 0.6 \text{ V}$$

$$V_S = -0.6 \text{ V} \Rightarrow R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-0.6 - (-1)}{0.1}$$

$$R_S = 4 \text{ k}\Omega$$

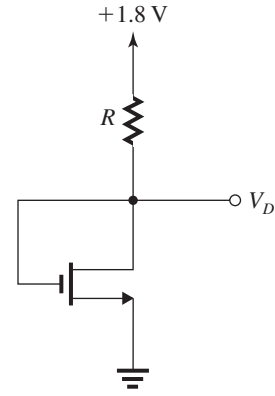
**Ex: 5.9**

$$V_m = 0.5 \text{ V}$$

$$\mu_n C_{ox} = 0.4 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{0.72 \text{ } \mu\text{m}}{0.18 \text{ } \mu\text{m}} = 4.0$$

$$\lambda = 0$$



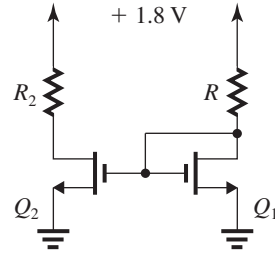
Saturation mode ( $v_{GD} = 0 < V_m$ ):

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_D - V_m)^2 = 0.032 \text{ mA}$$

$$V_D = 0.7 \text{ V} = 1.8 - I_D R$$

$$\therefore R = \frac{1.8 - 0.7}{0.032 \text{ mA}} = 34.4 \text{ k}\Omega$$

**Ex: 5.10**



Since  $Q_2$  is identical to  $Q_1$  and their  $V_{GS}$  values are the same,

$$I_{D2} = I_{D1} = 0.032 \text{ mA}$$

For  $Q_2$  to operate at the triode-saturation boundary, we must have

$$V_{D2} = V_{OV} = 0.7 - 0.5 = 0.2 \text{ V}$$

$$\therefore R_2 = \frac{1.8 \text{ V} - 0.2 \text{ V}}{0.032 \text{ mA}} = 50 \text{ k}\Omega$$

**Ex: 5.11**  $R_D = 6.55 \times 2 = 13.1 \text{ k}\Omega$

$V_{GS} = 2 \text{ V}$ , assume triode region:

$$\left. \begin{aligned} I_D &= k'_n \frac{W}{L} \left[ (V_{GS} - V_m) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \\ I_D &= \frac{V_{DD} - V_{DS}}{R} \end{aligned} \right\} \Rightarrow$$

$$\frac{2 - V_{DS}}{13.1} = 2 \times \left[ (2 - 0.5) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$\Rightarrow V_{DS}^2 - 3.076 V_{DS} + 0.15 = 0$$

$\Rightarrow V_{DS} = 0.05 \text{ V} < V_{OV} \Rightarrow$  triode region

$$I_D = \frac{2 - 0.05}{13.1} = 0.15 \text{ mA}$$

**Ex: 5.12** As indicated in Example 5.6,

$V_D \geq V_G - V_t$  for the transistor to be in the saturation region.

$$V_{D\min} = V_G - V_t = 5 - 1 = 4 \text{ V}$$

$$I_D = 0.5 \text{ mA} \Rightarrow R_{D\max} = \frac{V_{DD} - V_{D\min}}{I_D}$$

$$= \frac{10 - 4}{0.5} = 12 \text{ k}\Omega$$

**Ex: 5.13**

$$I_D = 0.32 \text{ mA} = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.8 \text{ V}$$

$$V_{GS} = 0.8 + 1 = 1.8 \text{ V}$$

$$V_G = V_S + V_{GS} = 1.6 + 1.8 = 3.4 \text{ V}$$

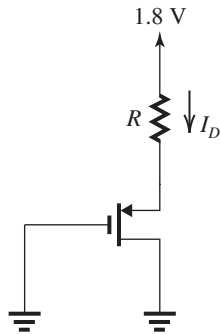
$$R_{G2} = \frac{V_G}{I} = \frac{3.4}{1 \mu\text{A}} = 3.4 \text{ M}\Omega$$

$$R_{G1} = \frac{5 - 3.4}{1 \mu\text{A}} = 1.6 \text{ M}\Omega$$

$$R_S = \frac{V_S}{0.32} = 5 \text{ k}\Omega$$

$$V_D = 3.4 \text{ V, then } R_D = \frac{5 - 3.4}{0.32} = 5 \text{ k}\Omega$$

**Ex: 5.14**



$$V_{tp} = -0.4 \text{ V}$$

$$k'_p = 0.1 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{10 \mu\text{m}}{0.18 \mu\text{m}} \Rightarrow k_p = 5.56 \text{ mA/V}^2$$

$$V_{SG} = |V_{tp}| + |V_{OV}|$$

$$= 0.4 + 0.6 = 1 \text{ V}$$

$$V_S = +1 \text{ V}$$

Since  $V_{DG} = 0$ , the transistor is operating in saturation, and

$$I_D = \frac{1}{2} k_p V_{OV}^2 = 1 \text{ mA}$$

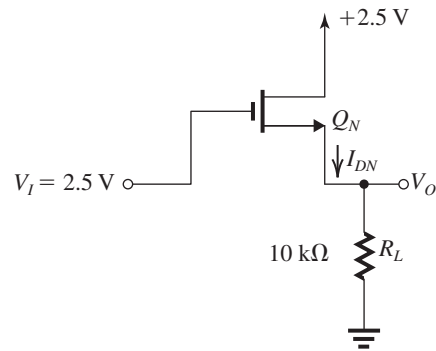
$$\therefore R = \frac{1.8 - V_S}{I_D} = \frac{1.8 - 1}{1} = 0.8 \text{ k}\Omega$$

$$= 800 \Omega$$

**Ex: 5.15**  $V_I = 0$ : since the circuit is perfectly symmetrical,  $V_O = 0$  and therefore  $V_{GS} = 0$ , which implies that the transistors are turned off and  $I_{DN} = I_{DP} = 0$ .

$V_I = 2.5 \text{ V}$ : if we assume that the NMOS is turned on, then  $V_O$  would be less than  $2.5 \text{ V}$ , and this implies that PMOS is off ( $V_{SGP} < 0$ ).

$$I_{DN} = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$



$$I_{DN} = \frac{1}{2} \times 1 \times (2.5 - V_O - 1)^2$$

$$I_{DN} = 0.5(1.5 - V_O)^2$$

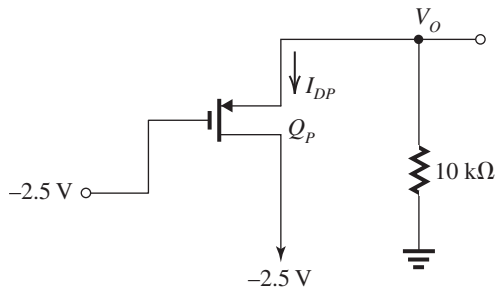
$$\text{Also: } V_O = R_L I_{DN} = 10 I_{DN}$$

$$I_{DN} = 0.5(1.5 - 10 I_{DN})^2$$

$$\Rightarrow 100 I_{DN}^2 - 32 I_{DN} + 2.25 = 0 \Rightarrow I_{DN}$$

$$= 0.104 \text{ mA} \quad I_{DP} = 0$$

$$V_O = 10 \times 0.104 = 1.04 \text{ V}$$



Exercise 5-4

$V_I = -2.5$  V: Again if we assume that  $Q_P$  is turned on, then  $V_O > -2.5$  V and  $V_{GSN} < 0$ , which implies that the NMOS  $Q_N$  is turned off.

$$I_{DN} = 0$$

Because of the symmetry,

$$I_{DP} = 0.104 \text{ mA},$$

$$V_O = -I_{DP} \times 10 \text{ k}\Omega$$

$$= -1.04 \text{ V}$$

$$\begin{aligned} \text{Ex: 5.16 } V_I &= 0.8 + 0.4 \left[ \sqrt{0.7 + 3} - \sqrt{0.7} \right] \\ &= 1.23 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Ex: 5.17 } v_{DS\min} &= v_{GS} + |V_I| \\ &= 1 + 2 = 3 \text{ V} \end{aligned}$$

$$\begin{aligned} I_D &= \frac{1}{2} \times 2 [1 - (-2)]^2 \\ &= 9 \text{ mA} \end{aligned}$$

**Solutions to End-of-Chapter Problems**

$$5.1 \quad k'_n = \mu_n C_{ox}$$

$$= \frac{\text{m}^2}{\text{V} \cdot \text{s}} \frac{\text{F}}{\text{m}^2} = \frac{\text{F}}{\text{V} \cdot \text{s}} = \frac{\text{C/V}}{\text{V} \cdot \text{s}} = \frac{\text{C}}{\text{s}} \frac{1}{\text{V}^2}$$

$$= \frac{\text{A}}{\text{V}^2}$$

Since  $k_n = k'_n W/L$  and  $W/L$  is dimensionless,  $k_n$  has the same dimensions as  $k'_n$ ; that is,  $\text{A/V}^2$ .

5.2 The transistor size will be minimized if  $W/L$  is minimized. To start with, we minimize  $L$  by using the smallest feature size,

$$L = 0.18 \mu\text{m}$$

$$r_{DS} = \frac{1}{k'_n (W/L) (v_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k'_n (W/L) v_{OV}}$$

Two conditions need to met for  $v_{OV}$  and  $r_{DS}$

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6} (W/L) v_{OV,1}}$$

$$= 250 \Rightarrow (W/L) v_{OV,1} = 10$$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6} (W/L) v_{OV,2}}$$

$$= 1000 \Rightarrow (W/L) v_{OV,2} = 2.5$$

If condition 1 is met, condition 2 will be met since the over-drive voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease  $W/L$  as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints. This requires our using the largest possible  $v_{GS,1}$  voltage.

$v_{GS,1} = 1.8 \text{ V}$  so  $v_{OV,1} = 1.8 - 0.5 = 1.3 \text{ V}$ , and

$$W/L = \frac{10}{v_{OV,1}} = \frac{10}{1.3} = 7.69$$

Condition 2 now can be used to find  $v_{GS,2}$

$$v_{OV,2} = \frac{2.5}{W/L} = \frac{2.5}{7.69} = 0.325$$

$$\Rightarrow v_{GS,2} = 0.825 \text{ V} \Rightarrow 0.825 \text{ V} \leq v_{GS} \leq 1.8 \text{ V}$$

$$5.3 \quad L_{\min} = 0.18 \mu\text{m}$$

$$t_{ox} = 4 \text{ nm}$$

$$\mu_n = 450 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} = 450 \times 10^{-4} \frac{\text{m}^2}{\text{V} \cdot \text{s}}$$

$$(a) \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}}$$

$$= 8.625 \times 10^{-3} \frac{\text{F}}{\text{m}^2} \text{ or } \left( \frac{\text{pF}}{\mu\text{m}^2} \right)$$

$$k'_n = \mu_n C_{ox} = 388 \mu\text{A/V}^2$$

$$(b) \quad \text{For } \frac{W}{L} = \frac{2.4}{0.18}, \quad k_n = 5.17 \text{ mA/V}^2$$

$$\therefore 0.1 \text{ mA} = i_D = \frac{1}{2} k_n v_{OV}^2$$

$$v_{OV} = 0.2 \text{ V}$$

$$v_{GS} = 0.7 \text{ V}$$

$$v_{DS} \geq 0.2 \text{ V}$$

$$(c) \quad g_{DS} = \frac{1}{100 \Omega} = k_n v_{OV}$$

$$\therefore v_{OV} = 0.39 \text{ V.}$$

$$v_{GS} = 0.89 \text{ V.}$$

$$5.4 \quad V_{tp} = -0.5 \text{ V}$$

$$(a) \quad |v_{SG}| = |V_{tp}| + |v_{OV}|$$

$$= 0.5 + 0.4 = 0.9 \text{ V}$$

$$\Rightarrow v_G = -0.9 \text{ V}$$

(b) For the  $p$ -channel transistor to operate in saturation, the drain voltage must not exceed the gate voltage by more than  $|V_{tp}|$ . Thus

$$v_{D\max} = -0.9 + 0.5 = -0.4 \text{ V}$$

Put differently,  $v_{SD}$  must be at least equal to  $|v_{OV}|$ , which in this case is  $0.4 \text{ V}$ . Thus  $v_{D\max} = -0.4 \text{ V}$ .

(c) In (b), the transistor is operating in saturation, thus

$$i_D = \frac{1}{2} k_p |v_{OV}|^2$$

$$0.2 = \frac{1}{2} \times k_p \times 0.4^2$$

$$\Rightarrow k_p = 2.5 \text{ mA/V}^2$$

For  $v_D = -20 \text{ mV}$ , the transistor will be operating in the triode region. Thus

$$i_D = k_p \left[ v_{SD} |v_{OV}| - \frac{1}{2} v_{SD}^2 \right]$$

$$= 2.5 \left[ 0.02 \times 0.4 - \frac{1}{2} (0.02)^2 \right]$$

$$= 0.02 \text{ mA}$$

For  $v_D = -2 \text{ V}$ , the transistor will be operating in saturation, thus

$$i_D = \frac{1}{2} k_p |v_{OV}|^2 = \frac{1}{2} \times 2.5 \times 0.4^2 = 0.2 \text{ mA}$$

**5.5**  $t_{ox} = 6 \text{ nm}$ ,  $\mu_n = 460 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  
 $V_t = 0.5 \text{ V}$ , and  $W/L = 10$ .

$$k_n = \mu_n C_{ox} \frac{W}{L} = 460 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{6 \times 10^{-9}} \times 10$$

$$= 2.645 \text{ mA/V}^2$$

(a)  $v_{GS} = 2.5 \text{ V}$  and  $v_{DS} = 1 \text{ V}$

$$v_{OV} = v_{GS} - V_t = 2 \text{ V}$$

Thus  $v_{DS} < v_{OV} \Rightarrow$  triode region,

$$I_D = k_n \left[ v_{DS} v_{OV} - \frac{1}{2} v_{DS}^2 \right]$$

$$= 2.645 \left[ 1 \times 2 - \frac{1}{2} \times 1 \right] = 4 \text{ mA}$$

(b)  $v_{GS} = 2 \text{ V}$  and  $v_{DS} = 1.5 \text{ V}$

$$v_{OV} = v_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

Thus,  $v_{DS} = v_{OV} \Rightarrow$  saturation region,

$$i_D = \frac{1}{2} k_n v_{OV}^2 = \frac{1}{2} \times 2.645 \times 1.5^2$$

$$= 3 \text{ mA}$$

(c)  $v_{GS} = 2.5 \text{ V}$  and  $v_{DS} = 0.2 \text{ V}$

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus,  $v_{DS} < v_{OV} \Rightarrow$  triode region,

$$i_D = k_n \left[ v_{DS} v_{OV} - \frac{1}{2} v_{DS}^2 \right]$$

$$= 2.645 \left[ 0.2 \times 2 - \frac{1}{2} \times 0.2^2 \right] = 1 \text{ mA}$$

(d)  $v_{GS} = v_{DS} = 2.5 \text{ V}$

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus,  $v_{DS} > v_{OV} \Rightarrow$  saturation region,

$$i_D = \frac{1}{2} k_n v_{OV}^2$$

$$= \frac{1}{2} \times 2.645 \times 2^2 = 5.3 \text{ mA}$$

**5.6**  $t_{ox} = 2 \sim 10 \text{ nm}$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$\epsilon_{ox} = 34.5 \text{ pF/m}$$

$$C_{ox}^{-1} = 58 \sim 290 \text{ m}^2/\text{F} \left( \frac{\mu\text{m}^2}{\text{pF}} \right)$$

For 10 pF:

$$\text{Area} = 580 \sim 2900 \text{ } (\mu\text{m}^2)$$

so

$$d = 24 \sim 54 \text{ } \mu\text{m}$$

**5.7**  $C_{ox} = 25 \text{ fF}/\mu\text{m}^2$ ,  $v_{OV} = 0.15 \text{ V}$

$$L = 0.065 \text{ } \mu\text{m}, v_{DS} = 0 \text{ V}$$

$$W = 0.65 \text{ } \mu\text{m}$$

$$Q = C_{ox} \cdot W \cdot L \cdot v_{OV} = 0.16 \text{ fC}$$

**5.8** With  $v_{DS}$  small compared to  $V_{OV}$ , Eq. (5.13a) applies:

$$r_{DS} = \frac{1}{(\mu_n C_{ox}) \left( \frac{W}{L} \right) (v_{OV})}$$

(a)  $v_{OV}$  is doubled  $\rightarrow r_{DS}$  is halved. Factor = 0.5

(b)  $W$  is doubled  $\rightarrow r_{DS}$  is halved. Factor = 0.5

(c)  $W$  and  $L$  are doubled  $\rightarrow r_{DS}$  is unchanged. Factor = 1.0

(d) If oxide thickness  $t_{ox}$  is halved. Since

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

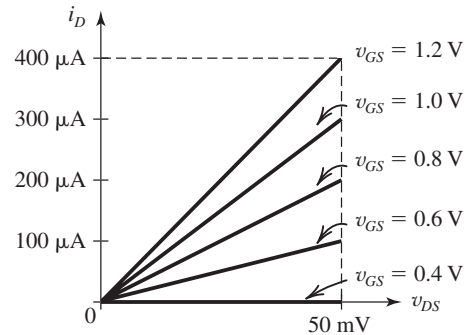
then  $C_{ox}$  is doubled. If  $W$  and  $L$  are also halved,  $r_{DS}$  is halved. Factor = 0.5.

**5.9**  $k_n = 10 \text{ mA/V}^2$ ,  $V_m = 0.4 \text{ V}$ ,

small  $v_{DS}$

$$i_D = k_n (v_{GS} - V_t) v_{DS} = k_n v_{OV} v_{DS}$$

$$g_{DS} = \frac{1}{r_{DS}} = k_n v_{OV}$$



This table belongs to Exercise 5.9.

$V_{GS}$ (V)	$V_{OV}$ (V)	$g_{DS}$ (mA/V)	$r_{DS}$ ( $\Omega$ )
0.4	0	0	$\infty$
0.6	0.2	2.0	500
0.8	0.4	4.0	250
1.0	0.6	6	167
1.2	0.8	8	125

$$\mathbf{5.10} \quad t_{ox} = 1.4 \text{ nm}, V_t = 0.35 \text{ V}$$

$$L_{\min} = 65 \text{ nm, small } v_{DS},$$

$$k'_n = 540 \mu\text{A/V}^2, 0 < v_{GS} < 1.0 \text{ V.}$$

$$r_{DS} = \frac{1}{k'_n \left(\frac{W}{L}\right) (v_{GS} - V_t)}$$

The lowest  $r_{DS}$  is obtained at the largest  $v_{GS}$ , i.e.,  
 $v_{GS} = 1 \text{ V}$ , thus

$$100 = \frac{1}{540 \times 10^{-6} \left(\frac{W}{L}\right) (1 - 0.35)}$$

$$\Rightarrow \frac{W}{L} = 28.5$$

$$\Rightarrow W = 28.5 \times 0.065$$

$$= 1.85 \mu\text{m}$$

$$\begin{aligned} \mathbf{5.11} \quad r_{ds} &= 1 / \left. \frac{\partial i_D}{\partial v_{DS}} \right|_{v_{DS} = v_{DS}} \\ &= \left[ \frac{\partial}{\partial v_{DS}} \left( k_n \left( V_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \right) \right) \right]^{-1} \\ &= \left[ k_n \frac{\partial}{\partial v_{DS}} (V_{OV} v_{DS}) - 1/2 k_n \frac{\partial}{\partial v_{DS}} (v_{DS}^2) \right]^{-1} \\ &= \left[ k_n \left( V_{OV} - \frac{1}{2} \cdot 2v_{DS} \right) \right]^{-1} \\ &= \frac{1}{k_n (V_{OV} - v_{DS})} \end{aligned}$$

$$\text{If } v_{DS} = 0 \Rightarrow r_{ds} = \frac{1}{k_n V_{OV}}$$

$$\text{If } v_{DS} = 0.2V_{OV} \Rightarrow r_{ds} = \frac{1.25}{k_n V_{OV}}$$

$$\begin{aligned} \text{If } v_{DS} = 0.5V_{OV} \Rightarrow r_{ds} &= \frac{1}{k_n (V_{OV} - 0.5V_{OV})} \\ &= 1/k_n (0.5V_{OV}) = \frac{2}{k_n V_{OV}} \end{aligned}$$

$$\begin{aligned} \text{If } v_{DS} = 0.8V_{OV} \Rightarrow r_{ds} &= \frac{1}{k_n (V_{OV} - 0.8V_{OV})} \\ &= 1/k_n (0.2V_{OV}) = \frac{5}{k_n V_{OV}} \end{aligned}$$

$$\text{If } v_{DS} = V_{OV},$$

$$r_{ds} = \frac{1}{0} \Rightarrow \infty$$

$$\mathbf{5.12} \quad V_{DS \text{ sat}} = V_{OV}$$

$$V_{OV} = V_{GS} - V_t = 1 - 0.5 = 0.5 \text{ V}$$

$$\Rightarrow V_{DS \text{ sat}} = 0.5 \text{ V}$$

In saturation:

$$i_D = \frac{1}{2} k'_n \left(\frac{W}{L}\right) V_{OV}^2 = \frac{1}{2} k_n V_{OV}^2$$

$$i_D = \frac{1}{2} \times \frac{4 \text{ mA}}{\text{V}^2} \times (0.5 \text{ V})^2$$

$$i_D = 0.5 \text{ mA}$$

$$\mathbf{5.13} \quad i_D = \frac{1}{2} k'_n \frac{W}{L} |V_{OV}|^2 \quad k'_n = \mu_n C_{ox}$$

For equal drain currents:

$$\mu_n C_{ox} \frac{W_n}{L} = \mu_p C_{ox} \frac{W_p}{L}$$

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = 2.5$$

$$\mathbf{5.14} \quad \text{For small } v_{DS}, i_D \simeq k'_n \frac{W}{L} (v_{GS} - V_t) v_{DS},$$

$$r_{DS} = \frac{v_{DS}}{i_D} = \frac{1}{k'_n \frac{W}{L} (v_{GS} - V_t)}$$

$$= \frac{1}{400 \times 10^{-6} \times 20 \times (1.8 - 0.5)}$$

$$r_{DS} = 96.2 \Omega \quad v_{DS} = r_{DS} \times i_D = 19.2 \text{ mV}$$

For the same performance of a  $p$ -channel device:

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 4 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L} \times 4$$

$$= 20 \times 4 \Rightarrow \frac{W_p}{L} = 80$$

$\mathbf{5.15}$  For triode-region operation with  $v_{DS}$  small,

$$i_D \simeq k_n (v_{GS} - V_t) v_{DS}$$

Thus

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n (v_{GS} - V_t)}$$

$$1 = \frac{1}{k_n (1.2 - 0.8)} = \frac{1}{0.4 k_n}$$

$$\Rightarrow k_n = 2.5 \text{ mA/V}$$

$$r_{DS} = \frac{1}{2.5 (V_{GS} - 0.8)} \quad (\text{k}\Omega)$$

$$0.2 = \frac{1}{2.5 (V_{GS} - 0.8)}$$

$$\Rightarrow V_{GS} = 2.8 \text{ V}$$

For a device with twice the value of  $W$ ,  $k_n$  will be twice as large and the resistance values will be half as large:  $500 \Omega$  and  $100 \Omega$ , respectively.

**5.16**  $V_m = 0.5 \text{ V}$ ,  $k_n = 1.6 \text{ mA/V}^2$

$$I_D = 0.05 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V and } V_{DS} \geq 0.25 \text{ V}$$

$$V_{GS} = 0.5 + 0.25 = 0.75 \text{ V}$$

$$I_D = 0.2 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V and } V_{DS} \geq 0.5 \text{ V}$$

$$V_{GS} = 0.5 + 0.5 = 1 \text{ V}$$

**5.17** For  $V_{GS} = V_{DS} = 1 \text{ V}$ , the MOSFET is operating in saturation,

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$0.4 = \frac{1}{2} k_n (1 - V_t)^2 \tag{1}$$

$$0.1 = \frac{1}{2} k_n (0.8 - V_t)^2 \tag{2}$$

Dividing Eq. (1) by Eq. (2) and taking square roots gives

$$2 = \frac{1 - V_t}{0.8 - V_t}$$

$$\Rightarrow V_t = 0.6 \text{ V}$$

Substituting in Eq. (1), we have

$$0.4 = \frac{1}{2} k_n \times 0.4^2$$

$$\Rightarrow k_n = 5 \text{ mA/V}^2$$

**5.18**  $i_D = k_n (v_{GS} - V_t) v_{DS}$

$$25 = k_n (1 - V_t) \times 0.05 \tag{1}$$

$$50 = k_n (1.5 - V_t) \times 0.05 \tag{2}$$

Dividing Eq. (2) by Eq. (1), we have

$$2 = \frac{1.5 - V_t}{1 - V_t}$$

$$\Rightarrow V_t = 0.5 \text{ V}$$

Substituting in Eq. (1) yields

$$25 = k_n \times 0.5 \times 0.05$$

$$\Rightarrow k_n = 1000 \mu\text{A/V}^2$$

For  $k'_n = 50 \mu\text{A/V}^2$

$$\frac{W}{L} = 20$$

For  $v_{GS} = 2 \text{ V}$  and  $v_{DS} = 0.1 \text{ V}$ ,

$$i_D = k_n \left[ (v_{GS} - V_t) v_{DS} - \frac{1}{2} v_{DS}^2 \right]$$

$$= 1 \left[ (2 - 0.5) \times 0.1 - \frac{1}{2} \times 0.1^2 \right]$$

$$= 0.145 \text{ mA} = 145 \mu\text{A}$$

For  $v_{GS} = 2 \text{ V}$ , pinch-off will occur for

$$v_{DS} = v_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

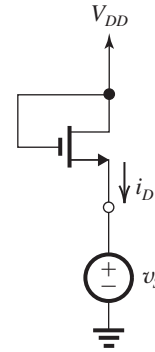
and the resulting drain current will be

$$i_D = \frac{1}{2} k_n (v_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1 \times (2 - 0.5)^2$$

$$= 1.125 \text{ mA}$$

**5.19**



$v_{GD} = 0 \Rightarrow$  saturation

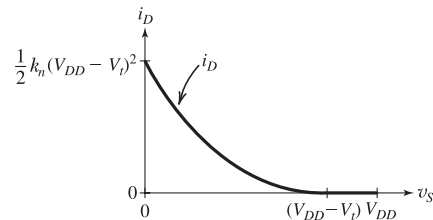
$$i_D = \frac{1}{2} k_n (v_{GS} - V_t)^2$$

$$v_{GS} = V_{DD} - v_S$$

$$\therefore i_D = \frac{1}{2} k_n [(V_{DD} - V_t) - v_S]^2$$

$$0 \leq v_S \leq (V_{DD} - V_t)$$

$$i_D = 0, v_S \geq (V_{DD} - V_t)$$





5.20  $V_{DS} = V_D - V_S$      $V_{GS} = V_G - V_S$

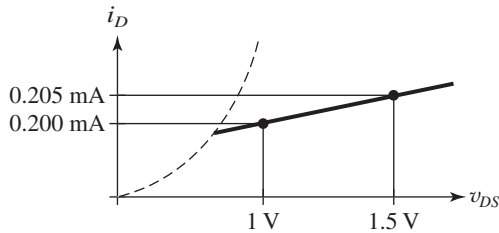
$V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$

According to Table 5.1, three regions are possible.

Case	$V_S$	$V_G$	$V_D$	$V_{GS}$	$V_{OV}$	$V_{DS}$	Region of operation
a	+1.0	+1.0	+2.0	0	-1.0	+1.0	Cutoff
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	Sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	Sat.
d	+1.0	+1.5	0	+0.5	-0.5	-1.0	Sat.*
e	0	+2.5	1.0	+2.5	+1.5	+1.0	Triode
f	+1.0	+1.0	+1.0	0	-1.0	0	Cutoff
g	-1.0	0	0	+1.0	0	+1.0	Sat.
h	-1.5	0	0	+1.5	+0.5	+1.5	Sat.
i	-1.0	0	+1.0	+1.0	0	+2.0	Sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	Triode

\* With the source and drain interchanged:  $V_S = 0$ ,  $V_G = +1.5$ ,  $V_D = +1$ ,  $V_{GS} = +1.5$ ,  $V_{OV} = 0.5$ ,  $V_{DS} = +1$

5.21



$$r_o = \left. \frac{\Delta v_{DS}}{\Delta i_D} \right|_{v_{GS} \text{ const.}} = \frac{0.5}{0.005} = 100 \text{ k}\Omega$$

$$V_A \cong I_D r_o = 0.2 \times 100 = 20 \text{ V}$$

$$\lambda = \frac{1}{V_A} = 0.05 \text{ V}^{-1}$$

5.22  $V_A = V'_A L$ , where  $V'_A$  is completely process dependent. Also,  $r_o = \frac{V_A}{i_D}$ . Therefore, to achieve desired  $r_o$  (which is 5 times larger), we should increase  $L$  ( $L = 5 \times 0.18 = 0.9 \mu\text{m}$ ).

To keep  $i_D$  unchanged, the  $\frac{W}{L}$  ratio must stay unchanged. Therefore:

$$W = 5 \times 10 \times 0.18 = 9 \mu\text{m} \text{ (so } \frac{W}{L} \text{ is kept at 10)}$$

$$V_A = r_o i_D = 20 \text{ k}\Omega \times 0.1 \text{ mA} = 2 \text{ V (for the standard device)}$$

$$V_A = 5 \times 2 = 10 \text{ V (for the new device)}$$

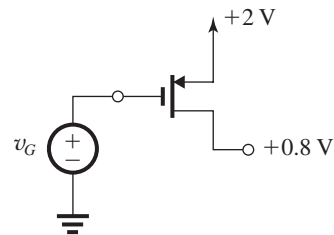
5.23

NMOS	1	2	3	4
$\lambda$	$0.05 \text{ V}^{-1}$	$0.1 \text{ V}^{-1}$	$0.05 \text{ V}^{-1}$	$0.2 \text{ V}^{-1}$
$V_A$	20 V	10 V	20 V	5 V
$I_D$	0.5 mA	0.4 mA	0.2 mA	0.01 mA
$r_o$	40 k $\Omega$	25 k $\Omega$	100 k $\Omega$	500 k $\Omega$

5.24 PMOS with  $V_{tp} = -1 \text{ V}$

Case	$V_S$	$V_G$	$V_D$	$V_{SG}$	$ V_{OV} $	$V_{SD}$	Region of operation
a	+2	+2	0	0	0	2	Cutoff
b	+2	+1	0	+1	0	2	Cutoff-Sat.
c	+2	0	0	+2	1	2	Sat.
d	+2	0	+1	+2	1	1	Sat-Triode
e	+2	0	+1.5	+2	1	0.5	Triode
f	+2	0	+2	+2	1	0	Triode

5.25



$$V_{tp} = -0.4 \text{ V}$$

$$v_G = +2 \text{ V} \rightarrow 0 \text{ V}$$

As  $v_G$  reaches +1.6 V, the transistor begins to conduct and enters the saturation region, since  $v_{DG}$  will be negative. The transistor continues to operate in the saturation region until  $v_G$  reaches +0.4 V, at which point  $v_{DG}$  will be 0.4 V, which is equal to  $|V_{tp}|$ , and the transistor enters the triode region. As  $v_G$  goes below +0.4 V, the transistor continues to operate in the triode region.

This figure belongs to 5.26, part (a).

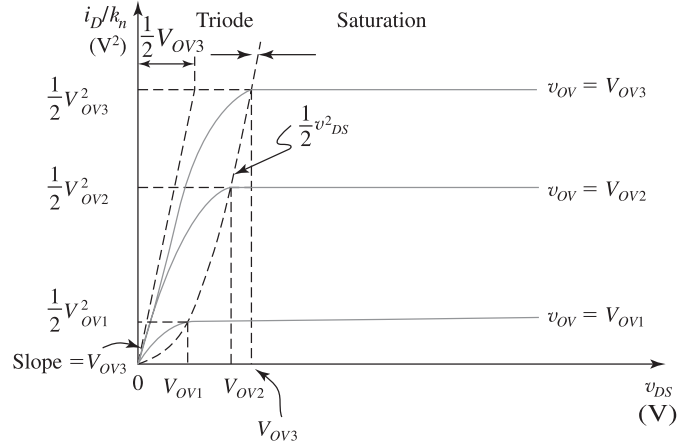


Fig. 1

5.26  $i_D = k_n \left[ v_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \right]$

$$\frac{i_D}{k_n} = v_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \quad (1)$$

Figure 1 shows graphs for  $i_D/k_n$  versus  $v_{DS}$  for various values of  $v_{OV}$ . Since the right-hand side of Eq. (1) does not have any MOSFET parameters, these graphs apply for any  $n$ -channel MOSFET with the assumption that  $\lambda = 0$ . They also apply to  $p$ -channel devices with  $v_{DS}$  replaced by  $v_{SD}$ ,  $k_n$  by  $k_p$ , and  $v_{OV}$  with  $|v_{OV}|$ . The slope of each graph at  $v_{DS} = 0$  is found by differentiating Eq. (1) relative to  $v_{DS}$  with  $v_{OV} = V_{OV}$  and then substituting  $v_{DS} = 0$ . The result is

$$\left. \frac{d(i_D/k_n)}{dv_{DS}} \right|_{v_{DS}=0, v_{OV}=V_{OV}} = V_{OV}$$

Figure 1 shows the tangent at  $v_{DS} = 0$  for the graph corresponding to  $v_{OV} = V_{OV3}$ . Observe that it intersects the horizontal line  $i_D/k_n = \frac{1}{2} V_{OV3}^2$  at  $v_{DS} = \frac{1}{2} V_{OV3}$ . Finally, observe that the curve representing the boundary between the triode region and the saturation region has the equation

$$i_D/k_n = \frac{1}{2} v_{DS}^2$$

Figure 2 shows the graph for the relationship

$$i_D/k_n = \frac{1}{2} v_{OV}^2$$

which describes the MOSFET's operation in the saturation region, that is,

$$v_{DS} \geq v_{OV}$$

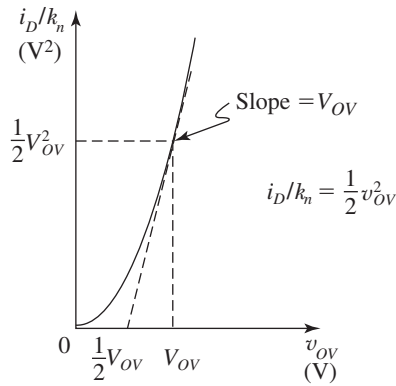


Fig. 2

Here also observe that this relationship (and graph) is universal and represents any MOSFET. The slope at  $v_{OV} = V_{OV}$  is

$$\left. \frac{d(i_D/k_n)}{dv_{OV}} \right|_{v_{OV}=V_{OV}} = V_{OV}$$

Replacing  $k_n$  by  $k_p$  and  $v_{OV}$  by  $|v_{OV}|$  adapts this graph to PMOS transistors.

5.27  $k'_n = 0.4 \text{ mA/V}^2$  and  $V_t = 0.5 \text{ V}$

For  $v_{GS} = v_{DS} = 1.8 \text{ V}$ , the MOSFET is operating in saturation. Thus, to obtain  $i_D = 2 \text{ mA}$ , we write

$$2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times (1.8 - 0.5)^2$$

$$\Rightarrow \frac{W}{L} = 5.92$$

For  $L = 0.18 \text{ }\mu\text{m}$

$$W = 1.07 \text{ }\mu\text{m}$$

**5.28** For the channel to remain continuous,

$$v_{DS} \leq v_{GS} - V_t$$

Thus for  $v_{GS} = 1.0 \text{ V}$  to  $1.8 \text{ V}$  and  $V_t = 0.5$ ,

$$v_{DS} \leq 1 - 0.5$$

That is,  $v_{DS\text{max}} = 0.5 \text{ V}$ .

**5.29**  $\frac{W}{L} = \frac{20}{1} = 20 \quad k'_n = 100 \mu\text{A/V}^2$

$$k_n = k'_n \left( \frac{W}{L} \right) = 100 \times 20 = 2000 \mu\text{A/V}^2 = 2 \text{ mA/V}^2$$

For operation as a linear resistance,

$$i_D = k_n(v_{GS} - V_t)v_{DS}$$

and

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n(v_{GS} - V_t)} = \frac{1}{2(v_{GS} - 0.8)}$$

At  $v_{GS} = 1.0 \text{ V}$ ,

$$r_{DS} = \frac{1}{2(1 - 0.8)} = 2.5 \text{ k}\Omega$$

At  $v_{GS} = 4.8 \text{ V}$ ,

$$r_{DS} = \frac{1}{2(4.8 - 0.8)} = 0.125 \text{ k}\Omega$$

Thus,  $r_{DS}$  will vary in the range of  $2.5 \text{ k}\Omega$  to  $125 \Omega$ .

- (a) If  $W$  is halved,  $k_n$  will be halved and  $r_{DS}$  will vary in the range of  $5 \text{ k}\Omega$  to  $250 \Omega$ .
- (b) If  $L$  is halved,  $k_n$  will be doubled and  $r_{DS}$  will vary in the range of  $1.25 \text{ k}\Omega$  to  $62.5 \Omega$ .
- (c) If both  $W$  and  $L$  are halved,  $k_n$  will remain unchanged and  $r_{DS}$  will vary in the original range of  $2.5 \text{ k}\Omega$  to  $125 \Omega$ .

**5.30 (a)** Refer to Fig. P5.30. For saturation-mode operation of an NMOS transistor,  $v_{DG} \geq -V_m$ ; thus  $v_{DG} = 0$  results in saturation-mode operation. Similarly, for a  $p$ -channel MOSFET, saturation-mode operation is obtained for  $v_{GD} \geq -|V_{tp}|$ , which includes  $v_{GD} = 0$ . Thus, the diode-connected MOSFETs of Fig. P5.30 have the  $i-v$  relationship

$$i = \frac{1}{2} k' \left( \frac{W}{L} \right) (v - |V_t|)^2 \quad (1)$$

where  $k'$  represents  $k'_n$  in the NMOS case and  $k'_p$  in the PMOS case.

(b) If either of the MOSFETs in Fig. P5.30 is biased to operate at  $v = |V_t| + |V_{OV}|$ , then its

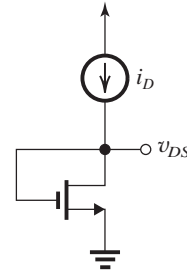
incremental resistance  $r$  at the bias point can be obtained by differentiating Eq. (1) relative to  $v$  and then substituting  $v = |V_t| + |V_{OV}|$  as follows:

$$\frac{\partial i}{\partial v} = k' \left( \frac{W}{L} \right) (v - |V_t|)$$

$$\left. \frac{\partial i}{\partial v} \right|_{v=|V_t|+V_{OV}} = k' \left( \frac{W}{L} \right) V_{OV}$$

$$r = 1 / \left[ \frac{\partial i}{\partial v} \right] = 1 / \left( k' \frac{W}{L} V_{OV} \right) \quad \text{Q.E.D.}$$

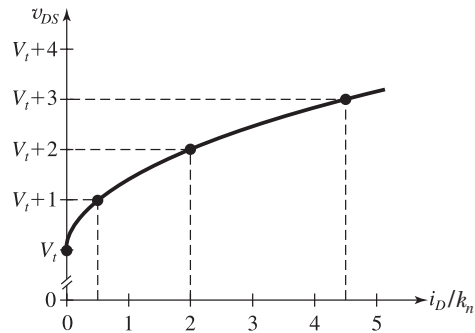
**5.31**



$$v_{DS} = v_{GS}$$

$$i_D = \frac{1}{2} k_n (v_{DS} - V_t)^2$$

$$\therefore v_{DS} = \sqrt{\frac{2i_D}{k_n}} + V_t$$



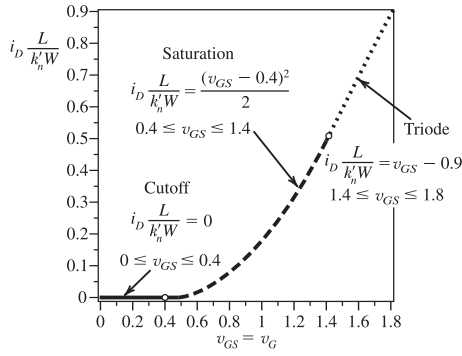
**5.32** The cutoff-saturation boundary is determined by  $v_{GS} = V_t$ , thus  $v_{GS} = 0.4 \text{ V}$  at the boundary (see figure on next page).

The saturation-triode boundary is determined by  $v_{GD} = V_t$ , and  $v_{DS} = V_{DD} = 1 \text{ V}$ , and since  $v_{GS} = v_{GD} + v_{DS}$ , one has  $v_{GS} = 0.4 + 1.0 = 1.4 \text{ V}$  at the boundary.

**5.33 (a)** Let  $Q_1$  have a ratio  $(W/L)$  and  $Q_2$  have a ratio  $1.03 (W/L)$ . Thus

$$I_{D1} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - V_t)^2$$

This figure belongs to Problem 5.32.



$$I_{D2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) \times 1.03 \times (1 - V_t)^2$$

Thus,

$$\frac{I_{D2}}{I_{D1}} = 1.03$$

That is, a 3% mismatch in the  $W/L$  ratios results in a 3% mismatch in the drain currents.

(b) Let  $Q_1$  have a threshold voltage  $V_t = 0.6$  V and  $Q_2$  have a threshold voltage  $V_t + \Delta V_t = 0.6 + 0.01 = 0.61$  V.

Thus

$$I_{D1} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - 0.6)^2$$

$$I_{D2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - 0.61)^2$$

and

$$\frac{I_{D2}}{I_{D1}} = \frac{(1 - 0.61)^2}{(1 - 0.6)^2} = 0.95$$

That is, a 10-mV mismatch in the threshold voltage results in a 5% mismatch in drain currents.

$$\mathbf{5.34} \quad r_o = \frac{V_A}{i_D} = \frac{20}{i_D}, \quad 0.1 \text{ mA} \leq i_D \leq 1 \text{ mA}$$

$$\Rightarrow 20 \text{ k}\Omega \leq r_o \leq 200 \text{ k}\Omega$$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta v_{DS}}{r_o} = \frac{1}{r_o}$$

$$\text{At } i_D = 0.1 \text{ mA}, \quad \Delta i_D = 5 \mu\text{A}, \quad \frac{\Delta i_D}{i_D} = 5\%$$

$$\text{At } i_D = 1 \text{ mA}, \quad \Delta i_D = 50 \mu\text{A}, \quad \frac{\Delta i_D}{i_D} = 5\%$$

**5.35**  $L = 1.0 \mu\text{m} = 5 \times$  minimum. Thus

$$\lambda = \frac{0.5 \text{ V}^{-1}}{5} = 0.1 \text{ V}^{-1}$$

If  $v_{DS}$  is increased from 1 V to 2 V, the drain current will change from

$$I_D = 100 \mu\text{A} = I'_D (1 + \lambda \times 1) = 1.1 I'_D$$

to

$$I_D + \Delta I_D = I'_D (1 + \lambda \times 2) = 1.2 I'_D$$

where  $I'_D$  is the drain current without channel-length modulation taken into account.

Thus

$$I'_D = \frac{100}{1.1}$$

and

$$100 + \Delta I_D = 1.2 I'_D = \frac{1.2 \times 100}{1.1} = 109 \mu\text{A}$$

$$\Rightarrow \Delta I_D = 9 \mu\text{A} \text{ or } 9\%$$

To reduce  $\Delta I_D$  by a factor of 2, we need to reduce  $\lambda$  by a factor of 2, which can be obtained by doubling the channel length to  $2 \mu\text{m}$ .

$$\mathbf{5.36} \quad V_A = V'_A L = 5 \times 0.54 = 2.7 \text{ V}$$

$$\lambda = \frac{1}{V_A} = \frac{1}{2.7} = 0.37 \text{ V}^{-1}$$

$$i_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) v_{OV}^2 (1 + \lambda v_{DS})$$

$$= \frac{1}{2} \times 0.4 \times \left( \frac{5.4}{0.54} \right) \times 0.25^2 (1 + 0.37 \times 1)$$

$$= 0.17 \text{ mA}$$

$$r_o = \frac{V_A}{\frac{1}{2} k'_n \left( \frac{W}{L} \right) v_{OV}^2}$$

$$= \frac{2.7}{\frac{1}{2} \times 0.4 \times \left( \frac{5.4}{0.54} \right) \times 0.25^2}$$

$$= 21.6 \text{ k}\Omega$$

$$\Delta i_D = \frac{\Delta v_{DS}}{r_o} = \frac{0.5 \text{ V}}{21.6 \text{ k}\Omega} = 0.023 \text{ mA}$$

**5.37** Quadrupling  $W$  and  $L$  keeps the current  $i'_D$  unchanged. However, the quadrupling of  $L$  increases  $V_A$  by a factor of 4 and hence increases  $r_o$  by a factor of 4.

Halving  $V_{OV}$  results in decreasing  $i'_D$  by a factor of 4. Thus, this alone increases  $r_o$  by a factor of 4. The overall increase in  $r_o$  is by a factor of  $4 \times 4 = 16$ .

**5.38** Refer to the circuit in Fig. P5.33 and let  $V_{D1} = 2$  V and  $V_{D2} = 2.5$  V. If the two devices are matched,

$$I_{D1} = \frac{1}{2} k_n (1 - V_t)^2 \left( 1 + \frac{2}{V_A} \right)$$

$$I_{D2} = \frac{1}{2} k_n (1 - V_t)^2 \left( 1 + \frac{2.5}{V_A} \right)$$

$$\Delta I_D = I_{D2} - I_{D1} = \frac{1}{2} k_n (1 - V_t)^2 \left( \frac{0.5}{V_A} \right)$$

$$\frac{\Delta I_D}{\frac{1}{2}k_n(1 - V_t)^2} \simeq 0.01 = \frac{0.5}{V_A}$$

$\Rightarrow V_A = 50 \text{ V}$  (or larger to limit the mismatch in  $I_D$  to 1%).

If  $V'_A = 100 \text{ V}/\mu\text{m}$ , the minimum required channel length is  $0.5 \mu\text{m}$ .

### 5.39

$$k_p = k'_p \left( \frac{W}{L} \right) = 100 \mu\text{A}/\text{V}^2$$

$$V_{tp} = -0.5 \text{ V} \quad \lambda = -0.1 \text{ V}^{-1}$$

$$v_G = 0, \quad v_S = +1 \text{ V} \Rightarrow v_{SG} = 1 \text{ V}$$

$$|v_{OV}| = v_{SG} - |V_{tp}| = 1 - 0.5 = 0.5$$

• For  $v_D = +0.8 \text{ V}$ ,  $v_{SD} = 0.2 \text{ V} < |v_{OV}| \Rightarrow$  triode-region operation,

$$\begin{aligned} i_D &= k_p \left[ v_{SD}|v_{OV}| - \frac{1}{2}v_{SD}^2 \right] \\ &= 100 \left( 0.2 \times 0.5 - \frac{1}{2} \times 0.2^2 \right) = 8 \mu\text{A} \end{aligned}$$

• For  $v_D = +0.6 \text{ V}$ ,  $v_{SD} = 0.4 \text{ V} < |v_{OV}| \Rightarrow$  triode-region operation,

$$\begin{aligned} i_D &= k_p \left[ v_{SD}|v_{OV}| - \frac{1}{2}v_{SD}^2 \right] \\ &= 100 \left( 0.4 \times 0.5 - \frac{1}{2} \times 0.4^2 \right) = 12 \mu\text{A} \end{aligned}$$

• For  $v_D = +0.5 \text{ V}$ ,  $v_{SD} = 0.5 \text{ V} = |v_{OV}| \Rightarrow$  saturation-mode operation,

$$\begin{aligned} i_D &= \frac{1}{2}k_p|v_{OV}|^2(1 + |\lambda|v_{SD}) \\ &= \frac{1}{2} \times 100 \times 0.25(1 + 0.1 \times 0.5) = 13.13 \mu\text{A} \end{aligned}$$

• For  $v_D = 0 \text{ V}$ ,  $v_{SD} = 1 \text{ V} > |v_{OV}| \Rightarrow$  saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 0.25(1 + 0.1 \times 1) = 13.75 \mu\text{A}$$

• For  $v_D = -1 \text{ V}$ ,  $v_{SD} = 2 \text{ V} > |v_{OV}| \Rightarrow$  saturation-mode operation,

$$i_D = \frac{1}{2} \times 100 \times 0.25(1 + 0.1 \times 2) = 15 \mu\text{A}$$

**5.40**  $|V_{tp}| = 0.4 \text{ V}$ ,  $|V_A| = 4 \text{ V}$

$$|v_{GS}| = 1 \text{ V}, \quad |v_{DS}| = 1.6 \text{ V}$$

$$i_D = 2 \text{ mA}$$

$$|v_{OV}| = |v_{GS}| - |V_{tp}| = 0.6 \text{ V}$$

$$|v_{DS}| > |v_{OV}| \Rightarrow \text{saturation mode}$$

$$v_{GS} = -1 \text{ V}$$

$$v_{SG} = +1 \text{ V}$$

$$v_{DS} = -1.6 \text{ V}$$

$$v_{SD} = +1.6 \text{ V}$$

$$V_{tp} = -0.4 \text{ V}$$

$$V_A = -4 \text{ V}$$

$$\lambda = -0.25 \text{ V}^{-1}$$

$$i_D = \frac{1}{2}k_p(v_{GS} - V_{tp})^2(1 + \lambda v_{DS})$$

$$2 = \frac{1}{2}k_p[-1 - (-0.4)]^2(1 - 0.25 \times -1.6)$$

$$\Rightarrow k_p = 7.94 \text{ mA}/\text{V}^2$$

**5.41** Case (a): Since  $V_{GS} > 0$  and  $V_{DS} > 0$ , transistor 1 must be NMOS. Also, since in both situations  $V_{DG}$  is positive, the NMOS transistor is operating in saturation and we can obtain  $V_t$  from

$$\frac{(1 - V_t)^2}{(1.5 - V_t)^2} = \frac{100}{400} \Rightarrow V_t = 0.5 \text{ V}$$

To find  $k_n$  we write

$$100 = \frac{1}{2}k_n(1 - 0.5)^2 \Rightarrow k_n = 800 \mu\text{A}/\text{V}^2$$

Case (b): Here  $V_{SG}$  and  $V_{SD}$  are both greater than zero, thus transistor 2 must be PMOS. Also, since in both situations  $V_{GD}$  is positive, the PMOS transistor must be operating in saturation and we can find  $|V_t|$  from,

$$\frac{(2 - |V_t|)^2}{(3 - |V_t|)^2} = \frac{50}{450} \Rightarrow |V_t| = 1.5 \text{ V}$$

To find  $k_p$  we write

$$50 = \frac{1}{2}k_p(2 - 1.5)^2 \Rightarrow k_p = 400 \mu\text{A}/\text{V}^2$$

Case (c): Since  $V_{SG}$  and  $V_{SD}$  are both positive, transistor 3 must be PMOS. At the second operating point,  $V_{GD}$  is positive and thus the transistor is operating in saturation. However, at the first operating point,  $V_{GD}$  is  $-1 \text{ V}$  and we cannot tell whether the transistor is in saturation or in the triode region. Assuming saturation-mode operation in both cases, we can find  $|V_t|$  from

$$\frac{(2 - |V_t|)^2}{(3 - |V_t|)^2} = \frac{200}{800} \Rightarrow |V_t| = 1 \text{ V}$$

We thus see that at the first operating point  $V_{DG} = |V_t|$  confirming operation in saturation, as already assumed (actually, at the boundary

This table belongs to Problem 5.41.

Case	Transistor	$V_S$ (V)	$V_G$ (V)	$V_D$ (V)	$I_D$ ( $\mu$ A)	Type	Mode	$\mu C_{ox} \frac{W}{L}$ ( $\mu$ A/V <sup>2</sup> )	$V_t$ (V)
a	1	0	1	2.5	100	NMOS	Sat.	800	0.5
		0	1.5	2.5	400		Sat.		
b	2	5	3	-4.5	50	PMOS	Sat.	400	-1.5
		5	2	-0.5	450		Sat.		
c	3	5	3	4	200	PMOS	Sat.	400	-1
		5	2	0	800		Sat.		
d	4	-2	0	0	72	NMOS	Sat.	100	+0.8
		-4	0	-3	270		Triode		

between saturation and triode). We can find  $k_p$  by writing

$$200 = \frac{1}{2} k_p (2-1)^2 \Rightarrow k_p = 400 \mu\text{A/V}^2$$

Case (d): Since  $V_{GS} > 0$  and  $V_{DS} > 0$ , transistor 4 must be NMOS. At the first operating point,  $V_{DG} = 0$  and the transistor must be in saturation. At the second operating point,  $V_{DG}$  is negative by 3 V and it is not clear whether the transistor is in saturation or in triode region. Assuming saturation in both cases,

$$\frac{(2 - V_t)^2}{(4 - V_t)^2} = \frac{72}{270} \Rightarrow V_t = -0.136$$

Since  $V_t$  must be positive, we conclude that our assumption of saturation-mode operation in the second case is not correct, and the transistor must be operating in triode. Thus

$$\frac{\frac{1}{2}(2 - V_t)^2}{(4 - V_t) \times 1 - \frac{1}{2} \times 1^2} = \frac{72}{270} \Rightarrow V_t = 0.8 \text{ V or } 2.7 \text{ V}$$

The second solution does not make physical sense as it results in  $I_D = 0$  at the first operating point. Thus

$$V_t = 0.8 \text{ V}$$

and  $k_n$  can be found from

$$72 = \frac{1}{2} k_n (2-0.8)^2 \Rightarrow k_n = 100 \mu\text{A/V}^2$$

The results are provided in the filled-out table above.

**5.42** Refer to the circuits in Fig. P5.42.

(a)  $V_1 = V_{DS} = V_{GS} = 1 \text{ V}$

(b)  $V_2 = +1 - V_{DS} = 1 - 1 = 0 \text{ V}$

(c)  $V_3 = V_{SD} = V_{SG} = 1 \text{ V}$

(d)  $V_4 = +1.25 - V_{SG} = 1.25 - 1 = 0.25 \text{ V}$

Now place a resistor  $R$  in series with the drain. For the circuits in (a) and (b) to remain in saturation,  $V_D$  must not fall below  $V_G$  by more than  $V_t$ .

Thus,

$$IR \leq V_t$$

$$R_{\max} = \frac{V_t}{I} = \frac{0.5}{0.1} = 5 \text{ k}\Omega$$

For the circuits in (c) and (d) to remain in saturation,  $V_D$  must not exceed  $V_G$  by more than  $|V_t|$ . Thus

$$IR \leq |V_t|$$

which yields  $R_{\max} = 5 \text{ k}\Omega$ .

Now place a resistor  $R_S$  in series with the MOSFET source. The voltage across the current source becomes

(a)  $V_{CS} = 2.5 - V_{DS} - IR_S$  (1)

To keep  $V_{CS}$  at least at 0.5 V, the maximum  $R_S$  can be found from

$$0.5 = 2.5 - 1 - 0.1 \times R_{S\max}$$

$$\Rightarrow R_{S\max} = 10 \text{ k}\Omega$$

$$V_1 = 2.5 - 0.5 = 2 \text{ V}$$

(b)  $V_{CS} = 1 - V_{DS} - IR_S - (-1.5)$

$$= 2.5 - V_{DS} - IR_S$$

which is identical to Eq. (1). Thus

$$R_{S\max} = 10 \text{ k}\Omega$$

$$V_2 = -1.5 + 0.5 = -1 \text{ V}$$

(c)  $V_{CS} = 2.5 - I_{RS} - V_{SD}$

which yields

$R_{Smax} = 10 \text{ k}\Omega$

$V_3 = 2.5 - 0.5 = 2 \text{ V}$

(d)  $V_{CS} = 1.25 - I_{RS} - V_{SD} - (-1.25)$

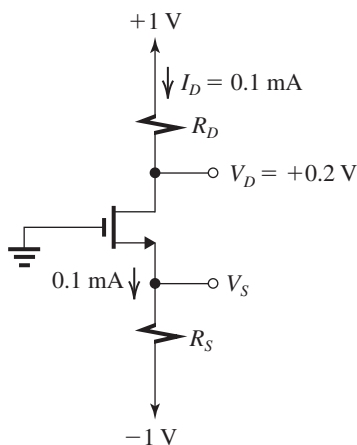
$= 2.5 - V_{SD} - I_{RS}$

which yields

$R_{Smax} = 10 \text{ k}\Omega$

$V_4 = -1.25 + 0.5 = -0.75 \text{ V}$

5.43



Since  $V_{DG} > 0$ , the MOSFET is in saturation.

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$

$0.1 = \frac{1}{2} \times 0.4 \times \frac{4}{0.5} \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.25 \text{ V}$

$V_{GS} = V_t + V_{OV} = 0.5 + 0.25 = 0.75$

$V_S = 0 - V_{GS} = -0.75 \text{ V}$

$R_S = \frac{V_S - (-1)}{I_D} = \frac{-0.75 + 1}{0.1} = 2.5 \text{ k}\Omega$

$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.2}{0.1} = \frac{0.8}{0.1} = 8 \text{ k}\Omega$

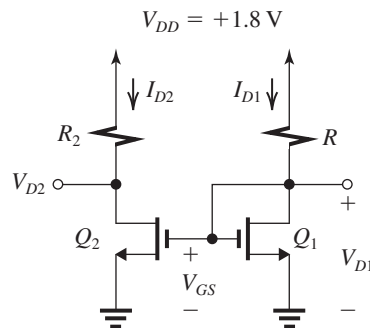
5.44

(a)  $I_{D1} = 50 \mu\text{A}$

$0.05 = \frac{1}{2} \times 0.4 \times \frac{1.44}{0.36} V_{OV}^2$

$\Rightarrow V_{OV} = 0.25 \text{ V}$

$V_{GS1} = V_t + V_{OV}$



$= 0.5 + 0.25 = 0.75 \text{ V}$

$V_{D1} = V_{GS1} = 0.75 \text{ V}$

$R = \frac{V_{DD} - V_{D1}}{I_{D1}} = \frac{1.8 - 0.75}{0.05} = 21 \text{ k}\Omega$

(b) Note that both transistors operate at the same  $V_{GS}$  and  $V_{OV}$ , and

$I_{D2} = 0.5 \text{ mA}$

But

$I_{D2} = \frac{1}{2} k_n \left( \frac{W_2}{L_2} \right) V_{OV}^2$

$0.5 = \frac{1}{2} \times 0.4 \times \frac{W_2}{0.36} \times 0.25^2$

$\Rightarrow W_2 = 14.4 \mu\text{m}$

which is 10 times  $W_1$ , as needed to provide  $I_{D2} = 10I_{D1}$ . Since  $Q_2$  is to operate at the edge of saturation,

$V_{DS2} = V_{OV}$

Thus,

$V_{D2} = 0.25 \text{ V}$

and

$R_2 = \frac{V_{DD} - V_{D2}}{I_{D2}} = \frac{1.8 - 0.25}{0.5} = 3.1 \text{ k}\Omega$

5.45 Refer to Fig. P5.45. Both  $Q_1$  and  $Q_2$  are operating in saturation at  $I_D = 0.1 \text{ mA}$ . For  $Q_1$ ,

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} V_{OV1}^2$

$0.1 = \frac{1}{2} \times 0.4 \times \frac{W_1}{L_1} (0.6 - 0.4)^2$

$\Rightarrow \frac{W_1}{L_1} = 12.5$

$W_1 = 12.5 \times 0.18 = 2.25 \mu\text{m}$

For  $Q_2$ , we have

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W_2}{L_2} \right) V_{OV2}^2$$

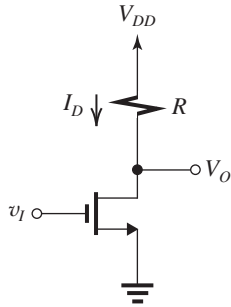
$$0.1 = \frac{1}{2} \times 0.4 \times \frac{W_2}{L_2} (1.4 - 0.6 - 0.4)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 3.125$$

$$W_2 = 3.125 \times 0.18 = 0.56 \mu\text{m}$$

$$R = \frac{1.8 - 1.4}{0.1} = 4 \text{ k}\Omega$$

5.46



Assuming linear operation in the triode region, we can write

$$I_D = \frac{V_O}{r_{DS}} = \frac{50 \text{ mV}}{50 \Omega} = 1 \text{ mA}$$

$$I_D = k'_n \left( \frac{W}{L} \right) (V_{GS} - V_t) V_{DS}$$

$$1 = 0.5 \times \frac{W}{L} \times (1.3 - 0.4) \times 0.05$$

$$\Rightarrow \frac{W}{L} = 44.4$$

$$R = \frac{V_{DD} - V_O}{I_D} = \frac{1.3 - 0.05}{1}$$

$$= 1.25 \text{ k}\Omega$$

5.47 (a) Refer to Fig. P5.47(a): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 5 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = |V_t| + V_{OV} = 0.5 + 0.2 = 0.7 \text{ V}$$

$$V_1 = 0 - V_{GS} = -0.7 \text{ V}$$

$$V_2 = 1 - 0.1 \times 10 = 0 \text{ V}$$

Since  $V_{DG} = 0 \text{ V}$ , the MOSFET is indeed in saturation.

Refer to Fig. P5.47(b): The transistor is operating in saturation, thus

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 5 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = 0.7 \text{ V}$$

$$\Rightarrow V_3 = 0.7 \text{ V}$$

Refer to Fig. P5.47(c): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2} k_p |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 5 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.2 \text{ V}$$

$$V_{SG} = |V_t| + |V_{OV}| = 0.5 + 0.2 = 0.7 \text{ V}$$

$$V_4 = V_{SG} = 0.7 \text{ V}$$

$$V_5 = -1 + I_D \times 10$$

$$= -1 + 0.1 \times 10 = 0 \text{ V}$$

Since  $V_{DG} = 0$ , the MOSFET is indeed in saturation.

Refer to Fig. P5.47(d): Both transistors are operating in saturation at equal  $|V_{OV}|$ . Thus

$$0.1 = \frac{1}{2} \times 5 \times |V_{OV}|^2 \Rightarrow |V_{OV}| = 0.2 \text{ V}$$

$$V_{SG} = |V_t| + |V_{OV}| = 0.7 \text{ V}$$

$$V_6 = 1 - V_{SG} = 1 - 0.7 = 0.3 \text{ V}$$

$$V_7 = +1 - 2 V_{SG} = 1 - 2 \times 0.7 = -0.4 \text{ V}$$

(b) Circuit (a): The 0.1-mA current source can be replaced with a resistance  $R$  connected between the MOSFET source and the  $-1\text{-V}$  supply with

$$R = \frac{V_1 - (-1)}{0.1 \text{ mA}} = \frac{-0.7 + 1}{0.1} = 3 \text{ k}\Omega$$

We use the nearest 1% resistor which is 3.01 k $\Omega$ .

Circuit (b): The 0.1-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{1 - V_3}{0.1 \text{ mA}} = \frac{1 - 0.7}{0.1} = 3 \text{ k}\Omega$$

We use the nearest 1% resistor which is 3.01 k $\Omega$ .

Circuit (c): The 0.1-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{1 - V_4}{0.1 \text{ mA}} = \frac{1 - 0.7}{0.1} = 3 \text{ k}\Omega$$

We use the nearest 1% resistor which is 3.01 k $\Omega$ .



Circuit (d): The 0.1-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{V_7 - (-1)}{0.1 \text{ mA}} = \frac{0.6}{0.1} = 6 \text{ k}\Omega$$

We use the nearest 1% resistor, which is 6.04 k $\Omega$ .

**5.48** (a) Refer to the circuit in Fig. P5.48(a). Since the two NMOS transistors are identical and have the same  $I_D$ , their  $V_{GS}$  values will be equal. Thus

$$V_{GS} = \frac{2}{2} = 1 \text{ V}$$

$$V_2 = 1 \text{ V}$$

$$V_{OV} = V_{GS} - V_t = 0.6 \text{ V}$$

$$\begin{aligned} I_1 = I_D &= \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2 \\ &= \frac{1}{2} \times 400 \times \frac{1}{0.2} \times 0.6^2 \\ &= 360 \text{ }\mu\text{A} \end{aligned}$$

(b) Refer to the circuit in Fig. P5.48(b). Here  $Q_N$  and  $Q_P$  have the same  $I_D = I_3$ . Thus

$$I_3 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OVN}^2 \quad (1)$$

$$I_3 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) V_{OVp}^2 \quad (2)$$

Equating Eqs. (1) and (2) and using  $\mu_n C_{ox} = 4\mu_p C_{ox}$  gives  $4V_{OVN}^2 = V_{OVp}^2$ :

$$|V_{OVp}| = 2 V_{OVN}$$

Now,

$$V_{GSN} = V_{OVN} + V_t = V_{OVN} + 0.4$$

$$V_{SGP} = |V_{OVp}| + |V_t| = 2 V_{OVN} + 0.4$$

But

$$V_{SGP} + V_{GSN} = 2$$

$$3V_{OVN} + 0.8 = 2$$

$$\Rightarrow V_{OVN} = 0.4 \text{ V}$$

$$|V_{OVp}| = 0.8 \text{ V}$$

$$V_{GSN} = 0.8 \text{ V}$$

$$V_{SGP} = 1.2 \text{ V}$$

$$V_4 = V_{GSN} = 0.8 \text{ V}$$

$$I_3 = \frac{1}{2} \times 400 \times \frac{1}{0.2} \times 0.4^2 = 160 \text{ }\mu\text{A}$$

(c) Refer to Fig. P5.48(c). Here the width of the PMOS transistor is made 4 times larger than that

of the NMOS transistor. This compensates for the factor 4 in the process transconductance parameter, resulting in  $k_p = k_n$ , and the two transistors are matched. The solution will be identical to that for (a) above with

$$V_5 = \frac{2}{2} = 1 \text{ V}$$

$$I_6 = 360 \text{ }\mu\text{A}$$

**5.49** Refer to the circuit in Fig. P5.49.

(a)  $Q_1$  and  $Q_2$  are matched. Thus, from symmetry, we see that the 200- $\mu\text{A}$  current will split equally between  $Q_1$  and  $Q_2$ :

$$I_{D1} = I_{D2} = 100 \text{ }\mu\text{A}$$

$$V_1 = V_2 = 2.5 - 0.1 \times 20 = 0.5 \text{ V}$$

To find  $V_3$ , we determine  $V_{GS}$  of either  $Q_1$  and  $Q_2$  (which, of course, are equal),

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_t)^2$$

$$100 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$$\Rightarrow V_{GS} = 0.983 \text{ V}$$

Thus,

$$V_3 = -0.983 \text{ V}$$

(b) With  $V_{GS1} = V_{GS2}$ , but  $(W/L)_1 = 1.5(W/L)_2$ , transistor  $Q_1$  will carry a current 1.5 times that in  $Q_2$ , that is,

$$I_{D1} = 1.5I_{D2}$$

But,

$$I_{D1} + I_{D2} = 200 \text{ }\mu\text{A}$$

Thus

$$I_{D1} = 120 \text{ }\mu\text{A}$$

$$I_{D2} = 80 \text{ }\mu\text{A}$$

$$V_1 = 2.5 - 0.12 \times 20 = 0.1 \text{ V}$$

$$V_2 = 2.5 - 0.08 \times 20 = 0.9 \text{ V}$$

To find  $V_3$ , we find  $V_{GS}$  from the  $I_D$  equation for either  $Q_1$  or  $Q_2$ ,

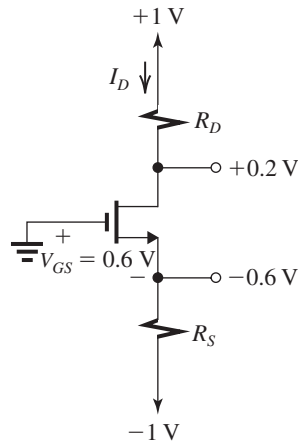
$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_t)^2$$

$$120 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$$\Rightarrow V_{GS} = 1.01 \text{ V}$$

$$V_3 = -1.01 \text{ V}$$

5.50



$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 0.4 \times \frac{W}{L} (1.3 - 0.4)^2$$

$$= 0.162 \left( \frac{W}{L} \right)$$

$$V_D = 1.3 - I_D R_D = 1.3 - 0.162 \left( \frac{W}{L} \right) R_D$$

For the MOSFET to be at the edge of saturation, we must have

$$V_D = V_{OV} = 1.3 - 0.4 = 0.9$$

Thus

$$0.9 = 1.3 - 0.162 \left( \frac{W}{L} \right) R_D$$

$$\Rightarrow \left( \frac{W}{L} \right) R_D \simeq 2.5 \text{ k}\Omega \quad \text{Q.E.D}$$

Since  $V_{DG} > 0$ , the MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2} k'_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 4 \times (0.6 - 0.4)^2$$

$$= 0.08 \text{ mA}$$

$$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.2}{0.08} = \frac{0.8}{0.08} = 10 \text{ k}\Omega$$

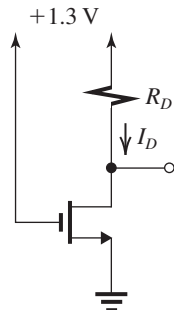
$$R_S = \frac{-0.6 - (-1)}{I_D} = \frac{-0.6 + 1}{0.08} = 5 \text{ k}\Omega$$

For  $I_D$  to remain unchanged from 0.08 mA, the MOSFET must remain in saturation. This in turn can be achieved by ensuring that  $V_D$  does not fall below  $V_G$  (which is zero) by more than  $V_t$  (0.4 V). Thus

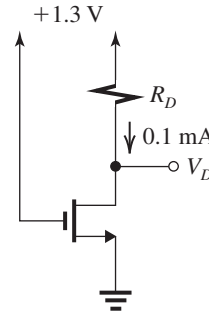
$$1 - I_D R_{D\max} = -0.4$$

$$R_{D\max} = \frac{1.4}{0.08} = 17.5 \text{ k}\Omega$$

5.51



5.52



$$V_{OV} = V_{GS} - V_t$$

$$= 1.3 - 0.4 = 0.9$$

To operate at the edge of saturation, we must have

$$V_D = V_{OV} = 0.9 \text{ V}$$

Thus,

$$R_D = \frac{1.3 - 0.9}{0.1} = 4 \text{ k}\Omega$$

5.53

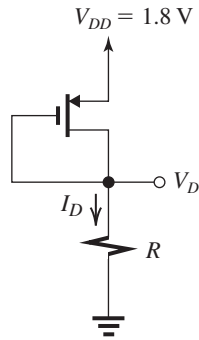
$$I_D = 180 \mu\text{A} \quad \text{and} \quad V_D = 1 \text{ V}$$

$$R = \frac{V_D}{I_D} = \frac{1}{0.18} = 5.6 \text{ k}\Omega$$

Transistor is operating in saturation with

$$|V_{OV}| = 1.8 - V_D - |V_t| = 1.8 - 1 - 0.5 = 0.3 \text{ V:}$$

$$I_D = \frac{1}{2} k'_p \frac{W}{L} |V_{OV}|^2$$



$$180 = \frac{1}{2} \times 100 \times \frac{W}{L} \times 0.3^2$$

$$\Rightarrow \frac{W}{L} = 40$$

$$W = 40 \times 0.18 = 7.2 \mu\text{m}$$

**5.54** Refer to the circuit in Fig. P5.54. All three transistors are operating in saturation with  $I_D = 90 \mu\text{A}$ . For  $Q_1$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} (V_{GS1} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_1}{L_1} (0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 22.2$$

$$W_1 = 22.2 \times 0.5 = 11.1 \mu\text{m}$$

For  $Q_2$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_2}{L_2} (V_{GS2} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_2}{L_2} (1.5 - 0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 50$$

$$W_2 = 50 \times 0.5 = 25 \mu\text{m}$$

For  $Q_3$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_3}{L_3} (V_{GS3} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_3}{L_3} (2.5 - 1.5 - 0.5)^2$$

$$\Rightarrow \frac{W_3}{L_3} = 8$$

$$W_3 = 8 \times 0.5 = 4 \mu\text{m}$$

**5.55** Refer to the circuits in Fig. 5.24:

$$V_{GS} = 5 - 6I_D$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1.5 \times (5 - 6I_D - 1.5)^2$$

which results in the following quadratic equation in  $I_D$ :

$$36I_D^2 - 43.33I_D + 12.25 = 0$$

The physically meaningful root is

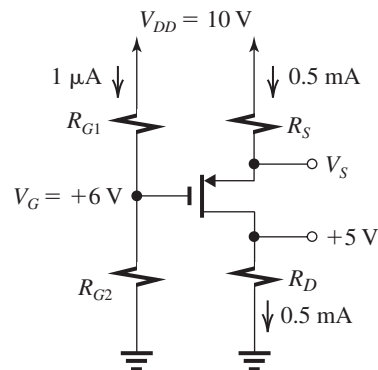
$$I_D = 0.454 \text{ mA}$$

This should be compared to the value of 0.5 mA found in Example 5.6. The difference of about 10% is relatively small, given the large variations in  $k_n$  and  $V_t$  (50% increase in each). The new value of  $V_D$  is

$$V_D = V_{DD} - R_D I_D = 10 - 6 \times 0.454 = +7.28 \text{ V}$$

as compared to +7 V found in Example 5.6. We conclude that this circuit is quite tolerant to variations in device parameters.

### 5.56



Refer to the circuit in the figure above.

$$R_{G1} = \frac{V_{DD} - V_G}{1 \mu\text{A}}$$

$$= \frac{10 - 6}{1} = 4 \text{ M}\Omega$$

$$R_{G2} = \frac{6}{1 \mu\text{A}} = 6 \text{ M}\Omega$$

$$R_D = \frac{5 \text{ V}}{0.5 \text{ mA}} = 10 \text{ k}\Omega$$

To determine  $V_S$ , we use

$$I_D = \frac{1}{2} k'_p \left( \frac{W}{L} \right) (V_{SG} - |V_t|)^2$$

$$0.5 = \frac{1}{2} \times 4 \times (V_{SG} - 1.5)^2$$

$$\Rightarrow V_{SG} = 2 \text{ V}$$

Thus,

$$V_S = V_G + V_{SG} = 6 + 2 = 8 \text{ V}$$

$$R_S = \frac{10 - 8}{0.5} = 4 \text{ k}\Omega$$

**5.57** (a) Refer to Fig. P5.57(a): The MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$10 = \frac{1}{2} \times 2000 \times V_{OV}^2 \Rightarrow V_{OV} = 0.1 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.1 = 0.6 \text{ V}$$

$$V_1 = 0 - V_{GS} = -0.6 \text{ V}$$

(b) Refer to Fig. P5.57(b): The MOSFET is operating in saturation. Thus

$$100 = \frac{1}{2} \times 2000 \times V_{OV}^2 \Rightarrow V_{OV} = 0.316 \text{ V}$$

$$V_{GS} = 0.5 + 0.316 = 0.816 \text{ V}$$

$$V_2 = -0.816 \text{ V}$$

(c) Refer to Fig. P5.57(c): The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 2 \times V_{OV}^2 \Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = 0.5 + 1 = 1.5 \text{ V}$$

$$V_3 = -1.5 \text{ V}$$

(d) Refer to Fig. P5.57(d): The MOSFET is operating in saturation. Thus

$$10 = \frac{1}{2} \times 2000 \times V_{OV}^2 \Rightarrow V_{OV} = 0.1 \text{ V}$$

$$V_{GS} = 0.5 + 0.1 = 0.6 \text{ V}$$

$$V_4 = 0.6 \text{ V}$$

(e) Refer to Fig. P5.57(e): The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 2 \times V_{OV}^2 \Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = 0.5 + 1 = 1.5 \text{ V}$$

$$V_5 = V_{GS} = 1.5 \text{ V}$$

(f) Refer to Fig. P5.57(f). To simplify our solution, we observe that this circuit is that in Fig. P5.57(d) with the 10- $\mu\text{A}$  current source replaced with a 140-k $\Omega$  resistor. Thus  $V_6 = V_4 = +0.6 \text{ V}$

and, as a check,  $I_D = \frac{2 - 0.6}{140} = 0.01 \text{ mA} = 10 \mu\text{A}$ .

(g) Refer to Fig. P5.57(g). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.57(e) with the 1-mA current source replaced with a 0.5-k $\Omega$  resistor. Thus  $V_7 = V_5 = 1.5 \text{ V}$  and, as a check,  $I_D = \frac{2 - 1.5}{0.5} = 1 \text{ mA}$ .

(h) Refer to Fig. P5.57(h). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.57(a) with the 10- $\mu\text{A}$  current source replaced with a 140-k $\Omega$  resistor. Thus  $V_8 = V_1 = -0.6 \text{ V}$  and, as a check,  $I_D = \frac{-0.6 + 2}{140} = 0.01 \text{ mA} = 10 \mu\text{A}$ .

**5.58** (a) Refer to the circuit in Fig. P5.58(a). Transistor  $Q_1$  is operating in saturation. Assume that  $Q_2$  also is operating in saturation,

$$V_{GS2} = 0 - V_2 = -V_2$$

and

$$V_2 = -1 + I_D \times 4$$

$$\Rightarrow I_D = 0.25(V_2 + 1)$$

Now,

$$I_D = \frac{1}{2} k_n (V_{GS2} - V_t)^2$$

Substituting  $I_D = 0.25(V_2 + 1)$  and  $V_{GS2} = -V_2$ ,

$$0.25(V_2 + 1) = \frac{1}{2} \times 5(-V_2 - 0.4)^2$$

$$0.1(V_2 + 1) = V_2^2 + 0.8 V_2 + 0.16$$

$$V_2^2 + 0.7 V_2 + 0.06 = 0$$

$$\Rightarrow V_2 = -0.6 \text{ V}$$

Thus,

$$I_D = 0.25(V_2 + 1) = 0.1 \text{ mA}$$

and

$$V_{GS2} = 0.6 \text{ V}$$

Since  $Q_1$  is identical to  $Q_2$  and is conducting the same  $I_D$ , then

$$V_{GS1} = 0.6 \text{ V}$$

$$\Rightarrow V_1 = 0.5 - 0.6 = -0.1 \text{ V}$$

which confirms that  $Q_2$  is operating in saturation, as assumed.

(b) Refer to the circuit in Fig. P5.58(b). From symmetry, we see that

$$V_4 = +1 \text{ V}$$

Now, consider the part of the circuit consisting of  $Q_2$  and the 4-k $\Omega$  resistor. We observe the similarity of this part with the circuit between the gate of  $Q_2$  and the -1 V supply in Fig. P5.58(a). It follows that for the circuit in Fig. P5.58(b), we can use the solution of part (a) above to write

$$I_{D2} = 0.1 \text{ mA} \quad \text{and} \quad V_{GS2} = 0.6 \text{ V}$$

Thus,

$$V_3 = V_4 - V_{GS2} = 1 - 0.6 = 0.4 \text{ V}$$

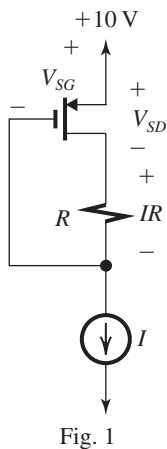
Since  $Q_1$  is conducting an equal  $I_D$  and has the same  $V_{GS}$ ,

$$I_{D1} = 0.1 \text{ mA} \quad \text{and} \quad V_{GS1} = 0.6 \text{ V}$$

$$\Rightarrow V_3 = V_4 + V_{GS1} = 1 + 0.6 = 1.6 \text{ V}$$

We could, of course, have used the circuit's symmetry, observed earlier, to write this final result.

**5.59**



(a) From Fig. 1 we see that

$$V_{DG} = IR$$

Since for the PMOS transistor to operate in saturation,

$$V_{DG} \leq |V_{tp}|$$

It follows the

$$IR \leq |V_{tp}| \quad \text{Q.E.D}$$

(b) (i)  $R = 0$ , the condition above is satisfied and

$$I_D = I = \frac{1}{2} k_p |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 1 \text{ V}$$

$$V_{SG} = |V_{tp}| + |V_{OV}| = 1 + 1 = 2 \text{ V}$$

$$V_G = 10 - 2 = 8 \text{ V}$$

$$V_D = V_G = 8 \text{ V}$$

$$V_{SD} = 2 \text{ V}$$

$$(ii) R = 10 \text{ k}\Omega$$

$$IR = 0.1 \times 10 = 1 \text{ V}$$

which just satisfies the condition for saturation-mode operation in (a) above.

Obviously  $I_D$  and  $|V_{OV}|$  will be the same as in (i) above.

$$V_{SG} = 2 \text{ V}$$

$$V_G = 8 \text{ V}$$

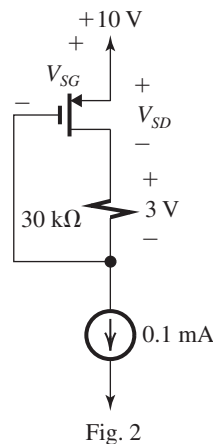
$$V_D = V_G + IR = 8 + 1 = 9 \text{ V}$$

$$V_{SD} = 1 \text{ V}$$

$$(iii) R = 30 \text{ k}\Omega$$

$$IR = 0.1 \times 30 = 3 \text{ V}$$

which is greater than  $|V_{tp}|$ . Thus the condition in (a) above is not satisfied and the MOSFET is operating in the triode region.



From Fig. 2, we see that

$$V_{SD} = V_{SG} - 3$$

Now, for triode-mode operation,

$$I_D = k_p \left[ (V_{SG} - |V_{tp}|) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$$

$$0.1 = 0.2 \left[ (V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2} (V_{SG} - 3)^2 \right]$$

$$\Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$$

$$\Rightarrow V_{SG} = 3.24 \text{ V}$$

$$V_{SD} = V_{SG} - 3 = 0.24 \text{ V}$$

(iv)  $R = 100 \text{ k}\Omega$

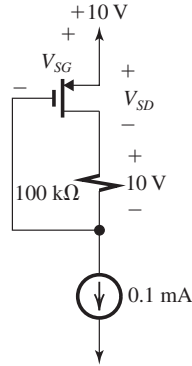


Fig. 3

Here also (see Fig. 3) the MOSFET will be operating in the triode region, and

$$V_{SD} = V_{SG} - 10 \text{ V}$$

Since we expect  $V_{SD}$  to be very small, we can neglect the  $V_{SD}^2$  term in the expression for  $I_D$  and write

$$I_D \simeq k_p(V_{SG} - |V_t|)V_{SD}$$

$$0.1 = 0.2(V_{SG} - 1)(V_{SG} - 10)$$

$$\Rightarrow V_{SG}^2 - 11V_{SG} + 9.5 = 0$$

$$\Rightarrow V_{SG} = 10.055 \text{ V}$$

$$V_{SD} = V_{SG} - 10 = 0.055 \text{ V}$$

**5.60** Refer to the circuit in Fig. P5.60. First consider  $Q_1$  and  $Q_2$ . Both are operating in saturation and since they are identical, they have equal  $V_{GS}$ :

$$V_{GS1} = V_{GS2} = \frac{2}{2} = 1 \text{ V}$$

Thus,

$$I_{D2} = I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_t)^2$$

$$= \frac{1}{2} \times 400 \times \frac{0.5}{0.5} (1 - 0.5)^2$$

$$= 50 \mu\text{A}$$

Now,  $Q_3$  has the same  $V_{GS}$  as  $Q_1$  and is matched to  $Q_1$ . Thus if we assume that  $Q_3$  is operating in saturation, we have

$$I_{D3} = I_{D1} = 50 \mu\text{A}$$

Thus,

$$I_2 = 50 \mu\text{A}$$

This is the same current that flows through  $Q_4$ , which is operating in saturation and is matched to  $Q_3$ . Thus

$$V_{GS4} = V_{GS3} = V_{GS1} = 1 \text{ V}$$

Thus,

$$V_2 = 2 - 1 = 1 \text{ V}$$

This is equal to the voltage at the gate of  $Q_3$ ; thus  $Q_3$  is indeed operating in saturation, as assumed.

If  $Q_3$  and  $Q_4$  have  $W = 5 \mu\text{m}$ , nothing changes for  $Q_1$  and  $Q_2$ . However,  $Q_3$ , which has the same  $V_{GS}$  as  $Q_1$  but has 10 times the width, will have a drain current 10 times larger than  $Q_1$ .

Thus

$$I_2 = I_{D3} = 10 I_{D1} = 10 \times 50 \mu\text{A}$$

$$= 0.5 \text{ mA}$$

Transistor  $Q_4$  will carry  $I_2$  but will retain the same  $V_{GS}$  as before, thus  $V_2$  remains unchanged at 1 V.

**5.61** See Table on next page.

**5.62** Using Eq. (5.30), we can write

$$V_t = V_{t0} + \gamma \left[ \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

where

$$V_{t0} = 1.0 \text{ V}$$

$$\gamma = 0.5 \text{ V}^{1/2}$$

$$2\phi_f = 0.6 \text{ V}$$

and

$$V_{SB} = 0 \text{ to } 4 \text{ V}$$

At

$$V_{SB} = 0, \quad V_t = V_{t0} = 1.0 \text{ V}$$

At

$$V_{SB} = 4 \text{ V},$$

$$V_t = 1 + 0.5 \left[ \sqrt{0.6 + 4} - \sqrt{0.6} \right]$$

$$= 1.69 \text{ V}$$

If the gate oxide thickness is increased by a factor of 4,  $C_{ox}$  will decrease by a factor of 4 and Eq. (5.31) indicates that  $\gamma$  will increase by a factor of 4, becoming 2. Thus at  $V_{SB} = 4 \text{ V}$ ,

$$V_t = 1 + 2 \left[ \sqrt{0.6 + 4} - \sqrt{0.6} \right]$$

$$= 3.74 \text{ V}$$

This table belongs to Problem 5.61.

$L$ ( $\mu\text{m}$ )	0.5	0.25	0.18	0.13
$t_{ox}$ (nm)	10	5	3.6	2.6
$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5(\text{pF}/\mu\text{m})}{t_{ox}(\text{nm})} = \frac{34.5}{t_{ox}(\text{nm})}(\text{fF}/\mu\text{m}^2)$ $\epsilon_{ox} = 34.5 \text{ pF/m}$	3.45	6.90	9.58	13.3
$k'_n \left( \frac{\mu\text{A}}{\text{V}^2} \right) = \mu_n(\text{cm}^2/\text{V}\cdot\text{s}) \times C_{ox}(\text{fF}/\mu\text{m}^2) \times 10^{-1}$ ( $\mu_n = 500 \text{ cm}^2/\text{V}\cdot\text{s}$ )	173	345	479	665
$k_n \left( \frac{\text{mA}}{\text{V}^2} \right) = k'_n \left( \frac{\text{W}}{\text{L}} \right)$ for $L = L_{\min}$ and $W/L = 10$	1.73	3.45	4.79	6.65
$A(\mu\text{m}^2) = W(\mu\text{m}) \times L(\mu\text{m})$ for $\frac{W}{L} = 10$	2.50	0.625	0.324	0.169
$V_{DD}$ (V)	5	2.5	1.8	1.3
$V_t$ (V)	0.7	0.5	0.4	0.4
$I_D$ (mA) for $V_{GS} = V_{DS} = V_{DD}$ , $I_D = \frac{1}{2}k_n(V_{DD} - V_t)^2$	16	6.90	4.69	2.69
$P$ (mW) $P = V_{DD}I_D$	80	17.3	8.44	3.50
$\frac{P}{A} \left( \frac{\text{mW}}{\mu\text{m}^2} \right)$	32	27.7	26.1	20.7
Devices/Chip	$n$	$4n$	$7.72n$	$14.8n$

## 5.63

(a) At  $20^\circ\text{C}$ ,  $V_t = 0.5 \text{ V}$  and  $k_n = 1 \text{ mA/V}^2$ , thus

$$i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$0.5 = \frac{1}{2} \times 1 (V_{GS} - 0.5)^2$$

$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

(b) At  $50^\circ\text{C}$ ,  $\Delta T = 30^\circ\text{C}$ , thus

$$\Delta V_t = -2 \times 30 = -60 \text{ mV} \Rightarrow V_t = 0.5 - 0.06$$

$$= 0.44 \text{ V}$$

$$\Delta k'_n = -0.3\% \times 30 = -9\% \Rightarrow \Delta k_n = -9\%$$

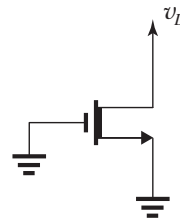
$$\Rightarrow k_n = 1 - 0.09 = 0.91 \text{ mA/V}^2$$

Thus,

$$0.5 = \frac{1}{2} \times 0.91 (V_{GS} - 0.44)^2$$

$$\Rightarrow V_{GS} = 0.44 + \sqrt{\frac{2 \times 0.5}{0.91}} = 1.488 \text{ V}$$

## 5.64



The NMOS depletion-type MOSFET has the same  $i-v$  characteristics as the enhancement-type NMOS except that  $V_m$  is negative, for the depletion device:

$$i_D = k_n \left[ (v_{GS} - V_m)v_{DS} - \frac{1}{2}v_{DS}^2 \right],$$

$$\text{for } v_{DS} \leq v_{GS} - V_m$$

$$i_D = \frac{1}{2} k_n (v_{GS} - V_m)^2,$$

$$\text{for } v_{DS} \geq v_{GS} - V_m$$

For our case,  $v_{GS} = 0$ ,  $V_m = -3$  V, and  $k_n = 2$  mA/V<sup>2</sup>. Thus

$$i_D = 2 \left( 3v_D - \frac{1}{2}v_D^2 \right), \quad \text{for } v_D \leq 3 \text{ V}$$

$$i_D = \frac{1}{2} \times 2 \times 9 = 9 \text{ mA}, \quad \text{for } v_D \geq 3 \text{ V}$$

For

$$v_D = 0.1 \text{ V}, \quad i_D = 2 \left( 3 \times 0.1 - \frac{1}{2} \times 0.1^2 \right) = 0.59 \text{ mA (triode)}$$

For

$$v_D = 1 \text{ V}, \quad i_D = 2 \left( 3 \times 1 - \frac{1}{2} \times 1 \right) = 5 \text{ mA (triode)}$$

For

$$v_D = 3 \text{ V}, \quad i_D = 9 \text{ mA (saturation)}$$

For

$$v_D = 5 \text{ V}, \quad i_D = 9 \text{ mA (saturation)}$$

**5.65** Here the scaling factor is

$$S = \frac{5 \text{ } \mu\text{m}}{32 \text{ nm}} = 156.25$$

Therefore, Moore's law predicts that the number of transistors on a chip of equal area, fabricated in the 32-nm process, would be

$$20,000 \times S^2 = 20,000 \times 156.25^2$$

$$4.88 \times 10^8 \text{ or } 488 \text{ million transistors.}$$

$$\mathbf{5.66} \quad |V_t| = |V_{t0}| + \gamma \left[ \sqrt{2\phi_f + |V_{SB}|} - \sqrt{2\phi_f} \right]$$

$$= 0.7 + 0.5 \left[ \sqrt{0.75 + 3} - \sqrt{0.75} \right]$$

$$= 1.24 \text{ V}$$

Thus,

$$V_t = -1.24 \text{ V}$$

$$\mathbf{5.67} \quad (\text{a}) \quad i_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2} \frac{\partial k_n'}{\partial T} \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$-k_n' \left( \frac{W}{L} \right) (V_{GS} - V_t) \frac{\partial V_t}{\partial T}$$

$$\frac{\partial i_D / i_D}{\partial T} = \frac{\partial k_n' / k_n'}{\partial T} - \frac{2}{V_{GS} - V_t} \frac{\partial V_t}{\partial T}$$

(b) For

$$\frac{\partial V_t}{\partial T} = -2 \text{ mV}/^\circ\text{C} = -0.002 \text{ V}/^\circ\text{C}$$

and

$$\frac{\partial i_D / i_D}{\partial T} = -0.002/^\circ\text{C}, \quad V_{GS} = 5 \text{ V}$$

and  $V_t = 1$  V

$$-0.002 = \frac{\partial k_n' / k_n'}{\partial T} - \frac{2 \times -0.002}{5 - 1}$$

$$\Rightarrow \frac{\partial k_n' / k_n'}{\partial T} = -0.003/^\circ\text{C}$$

or  $-0.3\%/^\circ\text{C}$

$$\mathbf{5.68} \quad i_D = k_n \left[ (v_{GS} - V_m)v_{DS} - \frac{1}{2}v_{DS}^2 \right],$$

for  $v_{DS} \leq v_{GS} - V_m$

$$i_D = \frac{1}{2} k_n (v_{GS} - V_m)^2 (1 + \lambda v_{DS}),$$

for  $v_{DS} \geq v_{GS} - V_m$

For our case,

$$V_m = -2 \text{ V}, \quad k_n = 0.2 \text{ mA/V}^2, \quad \lambda = 0.02 \text{ V}^{-1}$$

and  $v_{GS} = 0$ . Thus

$$i_D = 0.2 \left( 2v_{DS} - \frac{1}{2}v_{DS}^2 \right), \quad \text{for } v_{DS} \leq 2 \text{ V}$$

$$i_D = 0.4(1 + 0.02v_{DS}), \quad \text{for } v_{DS} \geq 2 \text{ V}$$

For  $v_{DS} = 1$  V,

$$i_D = 0.2 \left( 2 - \frac{1}{2} \right) = 0.3 \text{ mA}$$

For  $v_{DS} = 2$  V,

$$i_D = 0.4(1 + 0.02 \times 2) = 0.416 \text{ mA}$$

For  $v_{DS} = 3$  V,

$$i_D = 0.4(1 + 0.02 \times 3) = 0.424 \text{ mA}$$

For  $v_{DS} = 10$  V,

$$i_D = 0.4(1 + 0.02 \times 10) = 0.48 \text{ mA}$$

If the device width  $W$  is doubled,  $k_n$  is doubled, and each of the currents above will be doubled. If both  $W$  and  $L$  are doubled,  $k_n$  remains unchanged. However,  $\lambda$  is divided in half; thus for  $v_{DS} = 2$  V,  $i_D$  becomes 0.408 mA; for  $v_{DS} = 3$  V,  $i_D$  becomes 0.412 mA; and for  $v_{DS} = 10$  V,  $i_D$  becomes 0.44 mA.

### 5.69

The depletion-type MOSFET operates in the triode region when  $v_{DS} \leq v_{GS} - V_t$ ; that is,  $v_{DG} \leq -V_t$ , where  $V_t$  is negative. In the case shown in Fig. 1,  $v_{DG} = 0$ . Thus the condition for triode-mode operation is satisfied, and



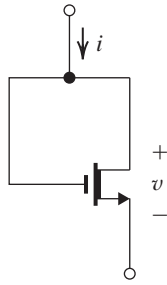


Fig. 1

$$i_D = k_n \left[ (v_{GS} - V_t)v_{DS} - \frac{1}{2}v_{DS}^2 \right]$$

which applies when the channel is not depleted, that is, when  $v_{GS} \geq V_t$ . For our case,

$$i = k_n \left[ (v - V_t)v - \frac{1}{2}v^2 \right], \quad \text{for } v \geq V_t$$

Thus,

$$i = \frac{1}{2}k_n(v^2 - 2V_tv), \quad \text{for } v \geq V_t$$

For  $v \leq V_t$ , the source and the drain exchange roles, as indicated in Fig. 2.

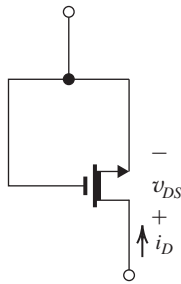


Fig. 2

Here  $v_{GS} = 0$  and  $v_{DS} = -v$ ; thus  $v_{DS} \geq -V_t$ . Thus the device is operating in saturation, and

$$i_D = \frac{1}{2}k_n(0 - V_t)^2$$

$$i_D = \frac{1}{2}k_nV_t^2$$

But  $i = -i_D$ ; thus

$$i = -\frac{1}{2}k_nV_t^2, \quad \text{for } v \leq V_t$$

Figure 3 is a sketch of the  $i-v$  relationship for the case  $V_t = -2$  V and  $k_n = 2$  mA/V<sup>2</sup>.

Here

$$i = v(v + 4), \quad \text{for } v \geq -2$$

and

$$i = -4 \text{ mA}, \quad \text{for } v \leq -2$$

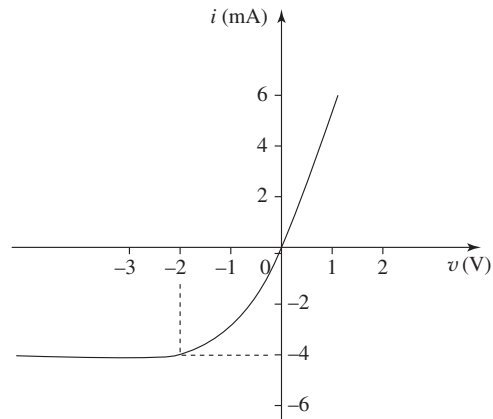


Fig. 3

**Chapter 6**

**Solutions to Exercises within the Chapter**

**Ex: 6.1** Refer to Fig. 6.2(a) and 6.2(b).

Coordinates of point A:  $V_i$  and  $V_{DD}$ ; thus 0.4 V and 1.8 V. To determine the coordinates of point B, we use Eqs. (6.7) and (6.8) as follows:

$$V_{OV}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D}$$

$$= \frac{\sqrt{2 \times 4 \times 17.5 \times 1.8 + 1} - 1}{4 \times 17.5}$$

$$= 0.213 \text{ V}$$

Thus,

$$V_{GS}|_B = V_i + V_{OV}|_B = 0.4 + 0.213 = 0.613 \text{ V}$$

and

$$V_{DS}|_B = V_{OV}|_B = 0.213 \text{ V}$$

Thus, coordinates of B are 0.613 V and 0.213 V. At point C, the MOSFET is operating in the triode region, thus

$$i_D = k_n \left[ (v_{GS}|_C - V_t) v_{DS}|_C - \frac{1}{2} v_{DS}|_C^2 \right]$$

If  $v_{DS}|_C$  is very small,

$$i_D \simeq k_n (v_{GS}|_C - V_t) v_{DS}|_C$$

$$= 4(1.8 - 0.4) v_{DS}|_C$$

$$= 5.6 v_{DS}|_C, \text{ mA}$$

But

$$i_D = \frac{V_{DD} - v_{DS}|_C}{R_D} \simeq \frac{V_{DD}}{R_D} = \frac{1.8}{17.5} = 0.1 \text{ mA}$$

Thus,  $v_{DS}|_C = \frac{0.1}{5.6} = 0.018 \text{ V} = 18 \text{ mV}$ , which is indeed very small, as assumed.

**Ex: 6.2** Refer to Example 6.1 and Fig. 6.4(a).

Design 1:

$$V_{OV} = 0.2 \text{ V}, V_{GS} = 0.6 \text{ V}$$

$$I_D = 0.08 \text{ mA}$$

Now,

$$A_v = -k_n V_{OV} R_D$$

Thus,

$$-10 = -0.4 \times 10 \times 0.2 \times R_D$$

$$\Rightarrow R_D = 12.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 12.5 \times 0.08 = 0.8 \text{ V}$$

Design 2:

$$R_D = 17.5 \text{ k}\Omega$$

$$A_v = -k_n V_{OV} R_D$$

$$-10 = -0.4 \times 10 \times V_{OV} \times 17.5$$

Thus,

$$V_{OV} = 0.14 \text{ V}$$

$$V_{GS} = V_i + V_{OV} = 0.4 + 0.14 = 0.54 \text{ V}$$

$$I_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.14^2 = 0.04 \text{ mA}$$

$$R_D = 17.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 17.5 \times 0.04 = 1.1 \text{ V}$$

**Ex: 6.3**

$$A_v = -\frac{I_C R_C}{V_T}$$

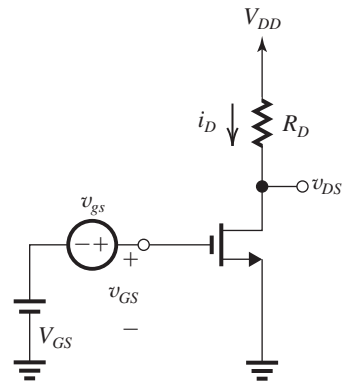
$$-320 = -\frac{1 \times R_C}{0.025} \Rightarrow R_C = 8 \text{ k}\Omega$$

$$V_C = V_{CC} - I_C R_C$$

$$= 10 - 1 \times 8 = 2 \text{ V}$$

Since the collector voltage is allowed to decrease to +0.3 V, the largest negative swing allowed at the output is  $2 - 0.3 = 1.7 \text{ V}$ . The corresponding input signal amplitude can be found by dividing 1.7 V by the gain magnitude (320 V/V), resulting in 5.3 mV.

**Ex: 6.4**



$$V_{DD} = 5 \text{ V}$$

$$V_{GS} = 2 \text{ V}$$

$$V_t = 1 \text{ V}$$

$$\lambda = 0$$

$$k_n' = 20 \mu\text{A}/\text{V}^2$$

$$R_D = 10 \text{ k}\Omega$$

$$\frac{W}{L} = 20$$

(a)  $V_{GS} = 2 \text{ V} \Rightarrow V_{OV} = 1 \text{ V}$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = 200 \mu\text{A} = 0.2 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D R_D = +3 \text{ V}$$

(b)  $g_m = k'_n \frac{W}{L} V_{OV} = 400 \mu\text{A/V} = 0.4 \text{ mA/V}$

(c)  $A_v = \frac{v_{ds}}{v_{gs}} = -g_m R_D = -4 \text{ V/V}$

(d)  $v_{gs} = 0.2 \sin \omega t \text{ V}$

$$v_{ds} = -0.8 \sin \omega t \text{ V}$$

$$v_{DS} = V_{DS} + v_{ds} \Rightarrow 2.2 \text{ V} \leq v_{DS} \leq 3.8 \text{ V}$$

(e) Using Eq. (6.28), we obtain

$$i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n (V_{GS} - V_t) v_{gs} + \frac{1}{2} k_n v_{gs}^2$$

$$i_D = 200 + 80 \sin \omega t$$

$$+ 8 \sin^2 \omega t, \mu\text{A}$$

$$= [200 + 80 \sin \omega t + (4 - 4 \cos 2\omega t)]$$

$$= 204 + 80 \sin \omega t - 4 \cos 2\omega t, \mu\text{A}$$

Thus,  $I_D$  shifts by  $4 \mu\text{A}$  and

$$2\text{HD} = \frac{\hat{i}_{2\omega}}{\hat{i}_{\omega}} = \frac{4 \mu\text{A}}{80 \mu\text{A}} = 0.05 \text{ (5\%)}$$

**Ex: 6.5**

(a)  $V_{GS} = 1.5 \text{ V} \Rightarrow V_{OV} = 1.5 - 1 = 0.5 \text{ V}$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 60 \times 40 \times 0.5^2$$

$$I_D = 300 \mu\text{A} = 0.3 \text{ mA}$$

$$g_m = \frac{2 \times 0.3}{0.5} = 1.2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{15}{0.3} = 50 \text{ k}\Omega$$

(b)  $I_D = 0.5 \text{ mA} \Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$

$$= \sqrt{2 \times 60 \times 40 \times 0.5 \times 10^3}$$

$$g_m = 1.55 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{15}{0.5} = 30 \text{ k}\Omega$$

**Ex: 6.6**

$$I_D = 0.1 \text{ mA}, g_m = 1 \text{ mA/V}, k'_n = 500 \mu\text{A/V}^2$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.1}{1} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k'_n V_{OV}^2}$$

$$= \frac{2 \times 0.1}{\frac{500}{1000} \times 0.2^2} = 10$$

**Ex: 6.7**

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$$

Same bias conditions, so same  $V_{OV}$  and also same  $L$  and  $g_m$  for both PMOS and NMOS.

$$\mu_n C_{ox} W_n = \mu_p C_{ox} W_p \Rightarrow \frac{\mu_p}{\mu_n} = 0.4 = \frac{W_n}{W_p}$$

$$\Rightarrow \frac{W_p}{W_n} = 2.5$$

**Ex: 6.8**

$$I_D = \frac{1}{2} k'_p \frac{W}{L} (V_{SG} - |V_t|)^2$$

$$= \frac{1}{2} \times 60 \times \frac{16}{0.8} \times (1.6 - 1)^2$$

$$I_D = 216 \mu\text{A}$$

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 216}{1.6 - 1} = 720 \mu\text{A/V}$$

$$= 0.72 \text{ mA/V}$$

$$|\lambda| = 0.04 \Rightarrow |V'_A| = \frac{1}{|\lambda|} = \frac{1}{0.04} = 25 \text{ V}/\mu\text{m}$$

$$r_o = \frac{|V'_A| \times L}{I_D} = \frac{25 \times 0.8}{0.216} = 92.6 \text{ k}\Omega$$

**Ex: 6.9**

$$g_m r_o = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}}$$

$$V_A = V'_A \times L = 6 \times 3 \times 0.18 = 3.24 \text{ V}$$

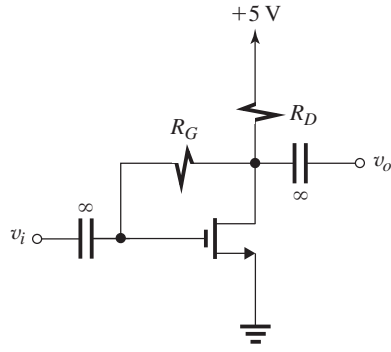
$$g_m r_o = \frac{2 \times 3.24}{0.2} = 32.4 \text{ V/V}$$

**Ex: 6.10**

Refer to the solution of Example 6.3. From Eq. (6.47),  $A_v \equiv \frac{v_o}{v_i} = -g_m R_D$  (note that  $R_L$  is absent).

Thus,

$$g_m R_D = 25$$



Substituting for  $g_m = k_n V_{OV}$ , we have

$$k_n V_{OV} R_D = 25$$

where  $k_n = 1 \text{ mA/V}^2$ , thus

$$V_{OV} R_D = 25 \quad (1)$$

Next, consider the bias equation

$$V_{GS} = V_{DS} = V_{DD} - R_D I_D$$

Thus,

$$V_i + V_{OV} = V_{DD} - R_D I_D$$

Substituting  $V_i = 0.7 \text{ V}$ ,  $V_{DD} = 5 \text{ V}$ , and

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2 = \frac{1}{2} V_{OV}^2$$

we obtain

$$0.7 + V_{OV} = 5 - \frac{1}{2} V_{OV}^2 R_D \quad (2)$$

Equations (1) and (2) can be solved to obtain

$$V_{OV} = 0.319 \text{ V}$$

and

$$R_D = 78.5 \text{ k}\Omega$$

The dc current  $I_D$  can be now found as

$$I_D = \frac{1}{2} k_n V_{OV}^2 = 50.9 \text{ }\mu\text{A}$$

To determine the required value of  $R_G$  we use Eq. (6.48), again noting that  $R_L$  is absent:

$$R_{in} = \frac{R_G}{1 + g_m R_D}$$

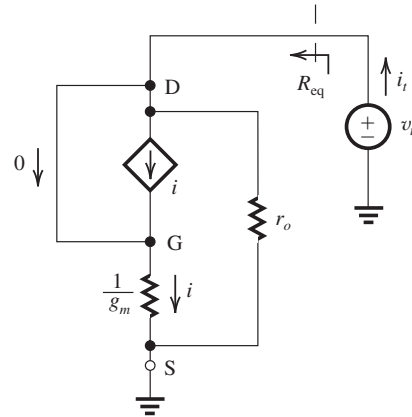
$$0.5 \text{ M}\Omega = \frac{R_G}{1 + 25}$$

$$\Rightarrow R_G = 13 \text{ M}\Omega$$

Finally, the maximum allowable input signal  $\hat{v}_i$  can be found as follows:

$$\hat{v}_i = \frac{V_i}{|A_v| + 1} = \frac{0.7 \text{ V}}{25 + 1} = 27 \text{ mV}$$

**Ex: 6.11**



$$i_t = \frac{v_i}{r_o} + i = \frac{v_i}{r_o} + g_m v_i$$

$$\therefore R_{eq} = \frac{v_i}{i_t} = r_o \parallel \frac{1}{g_m}$$

**Ex: 6.12**

$$\text{Given: } g_m = \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_C = I_C}$$

where  $I_C = I_S e^{v_{BE}/V_T}$

$$\left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_C = I_C} = \frac{I_S e^{v_{BE}/V_T}}{V_T} = \frac{I_C}{V_T}$$

Thus,

$$g_m = \frac{I_C}{V_T}$$

**Ex: 6.13**

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mV}} = 20 \text{ mA/V}$$

**Ex: 6.14**

$$I_C = 0.5 \text{ mA (constant)}$$

For  $\beta = 50$ :

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mV}} = 20 \text{ mA/V}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = 10 \text{ }\mu\text{A}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{20} = 2.5 \text{ k}\Omega$$

For  $\beta = 200$ ,

$$g_m = \frac{I_C}{V_T} = 20 \text{ mA/V}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5 \text{ mA}}{200} = 2.5 \text{ }\mu\text{A}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{20} = 10 \text{ k}\Omega$$

**Ex: 6.15**

$$\beta = 100 \quad I_C = 1 \text{ mA}$$

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} \simeq \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

**Ex: 6.16**

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$= -40 \times 10$$

$$= -400 \text{ V/V}$$

$$V_C = V_{CC} - I_C R_C$$

$$= 15 - 1 \times 10 = 5 \text{ V}$$

$$v_C(t) = V_C + v_c(t)$$

$$= V_C + A_v v_{be}(t)$$

$$= 5 - 400 \times 0.005 \sin \omega t$$

$$= 5 - 2 \sin \omega t$$

$$i_B(t) = I_B + i_b(t)$$

where

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 10 \mu\text{A}$$

$$\text{and } i_b(t) = \frac{g_m v_{be}(t)}{\beta}$$

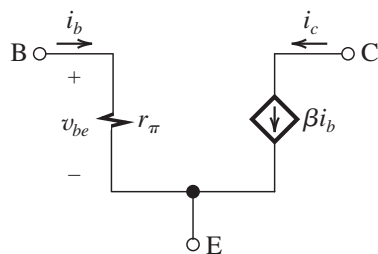
$$= \frac{40 \times 0.005 \sin \omega t}{100}$$

$$= 2 \sin \omega t, \mu\text{A}$$

Thus,

$$i_B(t) = 10 + 2 \sin \omega t, \mu\text{A}$$

**Ex: 6.17**



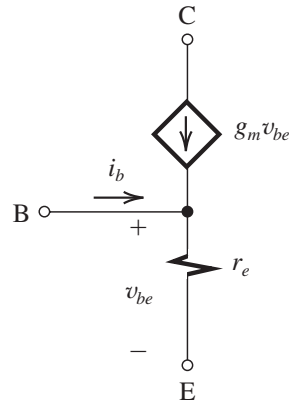
$$i_c = \beta i_b = \beta \frac{v_{be}}{r_\pi}$$

$$= \left( \frac{\beta}{r_\pi} \right) v_{be} = g_m v_{be}$$

$$i_e = i_b + \beta i_b = (\beta + 1) i_b = (\beta + 1) \frac{v_{be}}{r_\pi}$$

$$= \frac{v_{be}}{r_\pi / (\beta + 1)} = \frac{v_{be}}{r_e}$$

**Ex: 6.18**



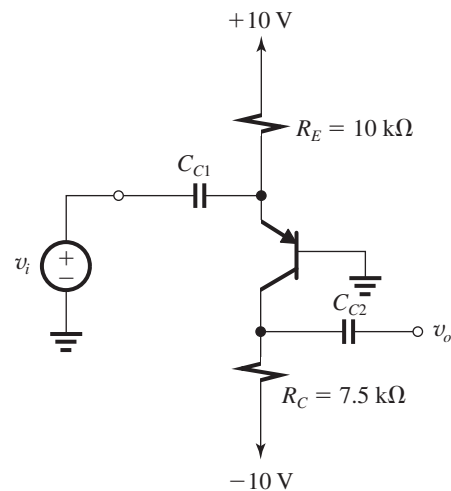
$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be} \left( \frac{1}{r_e} - g_m \right)$$

$$= v_{be} \left( \frac{1}{r_\pi / \beta + 1} - \frac{\beta}{r_\pi} \right)$$

$$= v_{be} \left( \frac{\beta + 1}{r_\pi} - \frac{\beta}{r_\pi} \right) = \frac{v_{be}}{r_\pi}$$

**Ex: 6.19**



Exercise 6-5

$$I_E = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.93 = 0.92 \text{ mA}$$

$$V_C = -10 + I_C R_C = -10 + 0.92 \times 7.5 = -3.1 \text{ V}$$

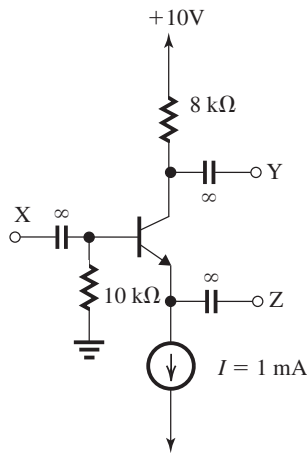
$$A_v = \frac{v_o}{v_i} = \frac{\alpha R_C}{r_e}$$

$$\text{where } r_e = \frac{25 \text{ mV}}{0.93 \text{ mA}} = 26.9 \Omega$$

$$A_v = \frac{0.99 \times 7.5 \times 10^3}{26.9} = 276.2 \text{ V/V}$$

$$\text{For } \hat{v}_i = 10 \text{ mV}, \hat{v}_o = 276.2 \times 10 = 2.76 \text{ V}$$

Ex: 6.20



$$I_E = 1 \text{ mA}$$

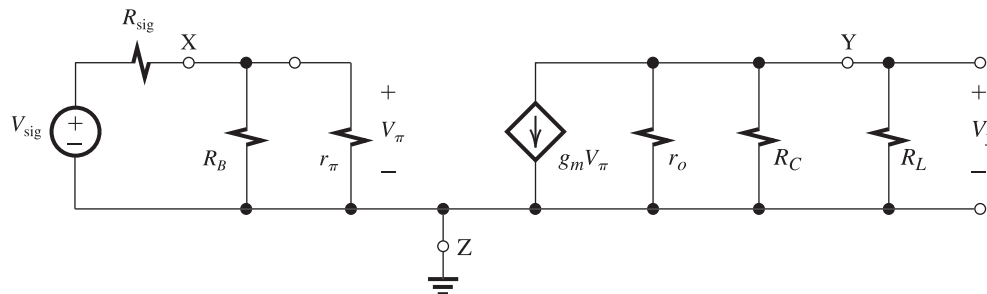
$$I_C = \frac{100}{101} \times 1 = 0.99 \text{ mA}$$

$$I_B = \frac{1}{101} \times 1 = 0.0099 \text{ mA}$$

$$(a) V_C = 10 - 8 \times 0.99 = 2.08 \approx 2.1 \text{ V}$$

$$V_B = -10 \times 0.0099 = -0.099 \approx -0.1 \text{ V}$$

This figure belongs to Exercise 6.20c.



$$V_E = -0.1 - 0.7 = -0.8 \text{ V}$$

$$(b) g_m = \frac{I_C}{V_T} = \frac{0.99}{0.025} \approx 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} \approx 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99} = 101 \approx 100 \text{ k}\Omega$$

(c) See figure below.

$$R_{sig} = 2 \text{ k}\Omega \quad R_B = 10 \text{ k}\Omega \quad r_\pi = 2.5 \text{ k}\Omega$$

$$g_m = 40 \text{ mA/V}$$

$$R_C = 8 \text{ k}\Omega \quad R_L = 8 \text{ k}\Omega \quad r_o = 100 \text{ k}\Omega$$

$$\frac{V_y}{V_{sig}} = \frac{V_\pi}{V_{sig}} \times \frac{V_y}{V_\pi}$$

$$= \frac{R_B \parallel r_\pi}{(R_B \parallel r_\pi) + R_{sig}} \times -g_m (R_C \parallel R_L \parallel r_o)$$

$$= \frac{10 \parallel 2.5}{(10 \parallel 2.5) + 2} \times -40(8 \parallel 8 \parallel 100)$$

$$= -0.5 \times 40 \times 3.846 = -77 \text{ V/V}$$

If  $r_o$  is neglected,  $\frac{V_y}{V_{sig}} = -80$ , for an error of 3.9%.

Ex: 6.21

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{0.25} = 2 \text{ mA/V}$$

$$R_{in} = \infty$$

$$A_{vo} = -g_m R_D = -2 \times 20 = -40 \text{ V/V}$$

$$R_o = R_D = 20 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -40 \times \frac{20}{20 + 20}$$

$$= -20 \text{ V/V}$$

$$G_v = A_v = -20 \text{ V/V}$$

$$\hat{v}_i = 0.1 \times 2V_{OV} = 0.1 \times 2 \times 0.25 = 0.05 \text{ V}$$

$$\hat{v}_o = 0.05 \times 20 = 1 \text{ V}$$

**Ex: 6.22**

$$I_C = 0.5 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \text{ k}\Omega$$

$$A_{vo} = -g_m R_C = -20 \times 10 = -200 \text{ V/V}$$

$$R_o = R_C = 10 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -200 \times \frac{5}{5 + 10}$$

$$= -66.7 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v = \frac{5}{5 + 5} \times -66.7$$

$$= -33.3 \text{ V/V}$$

$$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{sig} = 2 \times 5 = 10 \text{ mV}$$

$$\hat{v}_o = 10 \times 33.3 = 0.33 \text{ V}$$

Although a larger fraction of the input signal reaches the amplifier input, linearity considerations cause the output signal to be in fact smaller than in the original design!

**Ex: 6.23** Refer to the solution to Exercise 6.21.

If  $\hat{v}_{sig} = 0.2 \text{ V}$  and we wish to keep

$\hat{v}_{gs} = 50 \text{ mV}$ , then we need to connect a

resistance  $R_s = \frac{3}{g_m}$  in the source lead. Thus,

$$R_s = \frac{3}{2 \text{ mA/V}} = 1.5 \text{ k}\Omega$$

$$G_v = A_v = -\frac{R_D \parallel R_L}{\frac{1}{g_m} + R_s}$$

$$= -\frac{20 \parallel 20}{0.5 + 1.5} = -5 \text{ V/V}$$

$$\hat{v}_o = G_v \hat{v}_{sig} = 5 \times 0.2 = 1 \text{ V (unchanged)}$$

**Ex: 6.24**

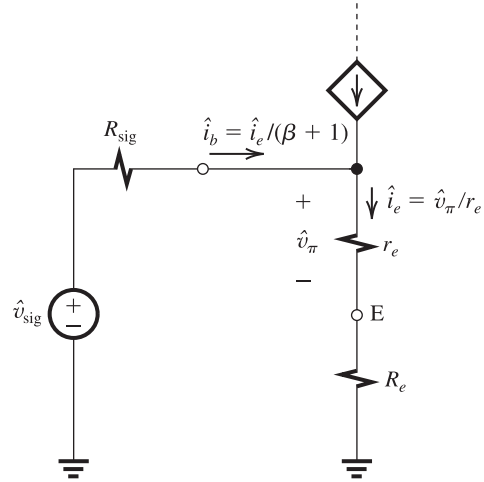
From the following figure we see that

$$\hat{v}_{sig} = \hat{i}_b R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{i}_e}{\beta + 1} R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{v}_\pi}{(\beta + 1)r_e} R_{sig} + \hat{v}_\pi + \frac{\hat{v}_\pi}{r_e} R_e$$

$$\hat{v}_{sig} = \hat{v}_\pi \left( 1 + \frac{R_e}{r_e} + \frac{R_{sig}}{r_\pi} \right) \quad \text{Q.E.D.}$$



For  $I_C = 0.5 \text{ mA}$  and  $\beta = 100$ ,

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25}{0.5} \simeq 50 \Omega$$

$$r_\pi = (\beta + 1)r_e \simeq 5 \text{ k}\Omega$$

For  $\hat{v}_{sig} = 100 \text{ mV}$ ,  $R_{sig} = 10 \text{ k}\Omega$  and with  $\hat{v}_\pi$  limited to  $10 \text{ mV}$ , the value of  $R_e$  required can be found from

$$100 = 10 \left( 1 + \frac{R_e}{50} + \frac{10}{5} \right)$$

$$\Rightarrow R_e = 350 \Omega$$

$$R_{in} = (\beta + 1)(r_e + R_e) = 101 \times (50 + 350)$$

$$= 40.4 \text{ k}\Omega$$

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)(r_e + R_e)}$$

$$= -100 \frac{10}{10 + 101 \times 0.4} = -19.8 \text{ V/V}$$

**Ex: 6.25**

$$\frac{1}{g_m} = R_{sig} = 100 \Omega$$

$$\Rightarrow g_m = \frac{1}{0.1 \text{ k}\Omega} = 10 \text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus,

$$10 = \frac{2I_D}{0.2}$$

$$\Rightarrow I_D = 1 \text{ mA}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_D$$

$$= 0.5 \times 10 \times 2 = 10 \text{ V/V}$$

**Ex: 6.26**

$$I_C = 1 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{in} = r_e = 25 \Omega$$

$$A_{vo} = g_m R_C = 40 \times 5 = 200 \text{ V/V}$$

$$R_o = R_C = 5 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = 200 \times \frac{5}{5 + 5} = 100 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$$

$$= \frac{25}{25 + 5000} \times 100 = 0.5 \text{ V/V}$$

**Ex: 6.27**

$$R_{in} = r_e = 50 \Omega$$

$$\Rightarrow I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{50 \Omega} = 0.5 \text{ mA}$$

$$I_C \simeq I_E = 0.5 \text{ mA}$$

$$G_v = \frac{R_C \parallel R_L}{r_e + R_{sig}}$$

$$40 = \frac{R_C \parallel R_L}{(50 + 50)\Omega}$$

$$R_C \parallel R_L = 4 \text{ k}\Omega$$

**Ex: 6.28** Refer to Fig. 6.42(c).

$$R_o = 100 \Omega$$

Thus,

$$\frac{1}{g_m} = 100 \Omega \Rightarrow g_m = 10 \text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus,

$$I_D = \frac{10 \times 0.25}{2} = 1.25 \text{ mA}$$

$$\hat{v}_o = \hat{v}_i \times \frac{R_L}{R_L + R_o} = 1 \times \frac{1}{1 + 0.1} = 0.91 \text{ V}$$

$$\hat{v}_{gs} = \hat{v}_i \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_L} = 1 \times \frac{0.1}{0.1 + 1} = 91 \text{ mV}$$

**Ex: 6.29**

$$R_o = 200 \Omega$$

$$\frac{1}{g_m} = 200 \Omega$$

$$\Rightarrow g_m = 5 \text{ mA/V}$$

But

$$g_m = k'_n \left( \frac{W}{L} \right) V_{OV}$$

Thus,

$$5 = 0.4 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 50$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 50 \times 0.25^2$$

$$= 0.625 \text{ mA}$$

$$R_L = 1 \text{ k}\Omega \text{ to } 10 \text{ k}\Omega$$

Correspondingly,

$$G_v = \frac{R_L}{R_L + R_o} = \frac{R_L}{R_L + 0.2}$$

will range from

$$G_v = \frac{1}{1 + 0.2} = 0.83 \text{ V/V}$$

to

$$G_v = \frac{10}{10 + 0.2} = 0.98 \text{ V/V}$$

**Ex: 6.30**

$$I_C = 5 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$R_{sig} = 10 \text{ k}\Omega \quad R_L = 1 \text{ k}\Omega$$

$$R_{in} = (\beta + 1)(r_e + R_L)$$

$$= 101 \times (0.005 + 1)$$

$$= 101.5 \text{ k}\Omega$$

$$G_{vo} = 1 \text{ V/V}$$

$$R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$$

$$= 5 + \frac{10,000}{101} = 104 \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta + 1}} = \frac{R_L}{R_L + R_{out}}$$

$$= \frac{1}{1 + 0.104} = 0.91 \text{ V/V}$$

$$v_\pi = v_{sig} \frac{r_e}{r_e + R_L + \frac{R_{sig}}{\beta + 1}}$$



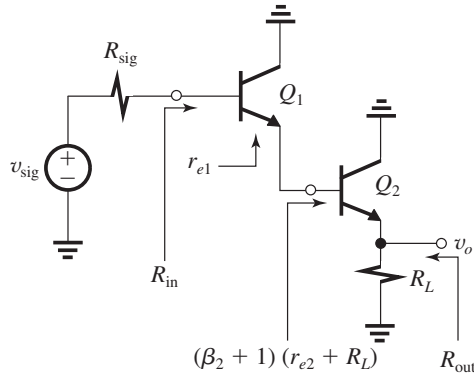
$$\hat{v}_{\text{sig}} = \hat{v}_{\pi} \left[ 1 + \frac{R_L}{r_e} + \frac{R_{\text{sig}}}{(\beta + 1)r_e} \right]$$

$$\hat{v}_{\text{sig}} = 5 \left[ 1 + \frac{1000}{5} + \frac{10,000}{101 \times 5} \right] = 1.1 \text{ V/V}$$

Correspondingly,

$$\hat{v}_o = G_v \times 1.1 = 0.91 \times 1.1 = 1 \text{ V}$$

**Ex: 6.31**



From the figure we can write

$$R_{\text{in}} = (\beta_1 + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)]$$

$$R_{\text{out}} = R_L \parallel \left[ r_{e2} + \frac{r_{e1} + R_{\text{sig}}/(\beta_1 + 1)}{\beta_2 + 1} \right]$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_L + r_{e2} + \frac{r_{e1} + R_{\text{sig}}/(\beta_1 + 1)}{\beta_2 + 1}}$$

For  $I_{E2} = 5 \text{ mA}$ ,  $\beta_1 = \beta_2 = 100$ ,  $R_L = 1 \text{ k}\Omega$ , and  $R_{\text{sig}} = 100 \text{ k}\Omega$ , we obtain

$$r_{e2} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$I_{E1} = \frac{5}{\beta_2 + 1} = \frac{5}{101} \simeq 0.05 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{0.05 \text{ mA}} = 500 \Omega$$

$$R_{\text{in}} = 101 \times (0.5 + 101 \times 1.005) = 10.3 \text{ M}\Omega$$

$$R_{\text{out}} = 1 \parallel \left[ 0.005 + \frac{0.5 + (100/101)}{101} \right] \simeq 20 \Omega$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{1}{1 + 0.005 + \frac{0.5 + (100/101)}{101}}$$

$$= 0.98 \text{ V/V}$$

**Ex: 6.32**

$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$0.5 = \frac{1}{2} \times 1 (V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 2 \text{ V}$$

If  $V_t = 1.5 \text{ V}$ , then

$$I_D = \frac{1}{2} \times 1 \times (2 - 1.5)^2 = 0.125 \text{ mA}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = \frac{0.125 - 0.5}{0.5} = -0.75 = -75\%$$

**Ex: 6.33**

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_D = 6.2 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow 0.5 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 1 + 1 = 2 \text{ V}$$

$$\Rightarrow V_S = -2 \text{ V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-2 - (-5)}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_S = 6.2 \text{ k}\Omega$$

If we choose  $R_D = R_S = 6.2 \text{ k}\Omega$ , then  $I_D$  will change slightly:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2.$$

Also,

$$V_{GS} = -V_S = 5 - R_S I_D$$

Thus,

$$2I_D = (4 - 6.2I_D)^2$$

$$\Rightarrow 38.44I_D^2 - 51.6I_D + 16 = 0$$

$$\Rightarrow I_D = 0.49 \text{ mA}, 0.86 \text{ mA}$$

$I_D = 0.86$  results in  $V_S > 0$  or  $V_S > V_G$ , which is not acceptable. Therefore  $I_D = 0.49 \text{ mA}$  and

$$V_S = -5 + 6.2 \times 0.49 = -1.96 \text{ V}$$

$$V_D = 5 - 6.2 \times 0.49 = +1.96 \text{ V}$$

$R_G$  should be selected in the range of  $1 \text{ M}\Omega$  to  $10 \text{ M}\Omega$ .

**Ex: 6.34**

$$I_D = 0.5 \text{ mA} = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2$$

$$\Rightarrow V_{OV}^2 = \frac{0.5 \times 2}{1} = 1$$

$$\Rightarrow V_{OV} = 1 \text{ V} \Rightarrow V_{GS} = 1 + 1 = 2 \text{ V}$$

$$= V_D \Rightarrow R_D = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$\Rightarrow R_D = 6.2 \text{ k}\Omega$  (standard value). For this  $R_D$  we have to recalculate  $I_D$ :

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2$$

$$= \frac{1}{2} (V_{DD} - R_D I_D - 1)^2$$

$$(V_{GS} = V_D = V_{DD} - R_D I_D)$$

$$I_D = \frac{1}{2} (4 - 6.2 I_D)^2 \Rightarrow I_D \cong 0.49 \text{ mA}$$

$$V_D = 5 - 6.2 \times 0.49 = 1.96 \text{ V}$$

**Ex: 6.35** Refer to Example 6.12.

(a) For design 1,  $R_E = 3 \text{ k}\Omega$ ,  $R_1 = 80 \text{ k}\Omega$ , and  $R_2 = 40 \text{ k}\Omega$ . Thus,  $V_{BB} = 4 \text{ V}$ .

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_1 \parallel R_2}{\beta + 1}}$$

For the nominal case,  $\beta = 100$  and

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{101}} = 1.01 \simeq 1 \text{ mA}$$

For  $\beta = 50$ ,

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{51}} = 0.94 \text{ mA}$$

For  $\beta = 150$ ,

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{151}} = 1.04 \text{ mA}$$

Thus,  $I_E$  varies over a range approximately 10% of the nominal value of 1 mA.

(b) For design 2,  $R_E = 3.3 \text{ k}\Omega$ ,  $R_1 = 8 \text{ k}\Omega$ , and  $R_2 = 4 \text{ k}\Omega$ . Thus,  $V_{BB} = 4 \text{ V}$ . For the nominal case,  $\beta = 100$  and

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4 \parallel 8}{101}} = 0.99 \simeq 1 \text{ mA}$$

For  $\beta = 50$ ,

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4 \parallel 8}{51}} = 0.984 \text{ mA}$$

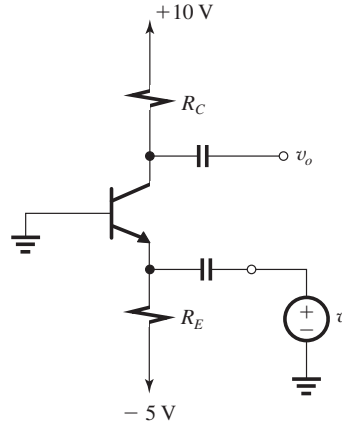
For  $\beta = 150$ ,

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4 \parallel 8}{151}} = 0.995 \text{ mA}$$

Thus,  $I_E$  varies over a range of 1.1% of the nominal value of 1 mA. Note that lowering the resistances of the voltage divider considerably decreases the dependence on the value of  $\beta$ , a

highly desirable result obtained at the expense of increased current and hence power dissipation.

**Ex: 6.36** Refer to Fig. 6.55. Since the circuit is to be used as a common-base amplifier, we can dispense with  $R_B$  altogether and ground the base; thus  $R_B = 0$ . The circuit takes the form shown in the figure below.



To establish  $I_E = 1 \text{ mA}$ ,

$$I_E = \frac{5 - V_{BE}}{R_E}$$

$$1 \text{ mA} = \frac{5 - 0.7}{R_E}$$

$$\Rightarrow R_E = 4.3 \text{ k}\Omega$$

The voltage gain  $\frac{v_o}{v_i} = g_m R_C$ , where

$$g_m = \frac{I_C}{V_T} = 40 \text{ mA/V.}$$

To maximize the voltage gain, we select  $R_C$  as large as possible, consistent with obtaining a  $\pm 2\text{-V}$  signal swing at the collector. To maintain active-mode operation at all times, the collector voltage should not be allowed to fall below the value that causes the CBJ to become forward biased, namely,  $-0.4 \text{ V}$ . Thus, the lowest possible dc voltage at the collector is  $-0.4 \text{ V} + 2 \text{ V} = +1.6 \text{ V}$ .

Correspondingly,

$$R_C = \frac{10 - 1.6}{I_C} \simeq \frac{10 - 1.6}{1 \text{ mA}} = 8.4 \text{ k}\Omega$$

**Ex: 6.37** Refer to Fig. 6.56. For  $I_E = 1 \text{ mA}$  and  $V_C = 2.3 \text{ V}$ ,

$$I_E = \frac{V_{CC} - V_C}{R_C}$$

$$1 = \frac{10 - 2.3}{R_C}$$

$$\Rightarrow R_C = 7.7 \text{ k}\Omega$$

Now, using Eq. (6.147), we obtain

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$1 = \frac{10 - 0.7}{7.7 + \frac{R_B}{101}}$$

$$\Rightarrow R_B = 162 \text{ k}\Omega$$

Selecting standard 5% resistors (Appendix J), we use

$$R_B = 160 \text{ k}\Omega \quad \text{and} \quad R_C = 7.5 \text{ k}\Omega$$

The resulting value of  $I_E$  is found as

$$I_E = \frac{10 - 0.7}{7.5 + \frac{160}{101}} = 1.02 \text{ mA}$$

and the collector voltage will be

$$V_C = V_{CC} - I_E R_C = 2.3 \text{ V}$$

**Ex: 6.38** Refer to Fig. 6.57(b).

$V_S = 3.5 \text{ V}$  and  $I_D = 0.5 \text{ mA}$ ; thus

$$R_S = \frac{V_S}{I_D} = \frac{3.5}{0.5} = 7 \text{ k}\Omega$$

$V_{DD} = 15 \text{ V}$  and  $V_D = 6 \text{ V}$ ; thus

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{15 - 6}{0.5 \text{ mA}} = 18 \text{ k}\Omega$$

To obtain  $V_{OV}$ , we use

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 4V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Thus,

$$V_{GS} = V_t + V_{OV} = 1 + 0.5 = 1.5 \text{ V}$$

We now can obtain the dc voltage required at the gate,

$$V_G = V_S + V_{GS} = 3.5 + 1.5 = 5 \text{ V}$$

Using a current of  $2 \mu\text{A}$  in the voltage divider, we have

$$R_{G2} = \frac{5 \text{ V}}{2 \mu\text{A}} = 2.5 \text{ M}\Omega$$

The voltage drop across  $R_{G1}$  is  $10 \text{ V}$ , thus

$$R_{G1} = \frac{10 \text{ V}}{2 \mu\text{A}} = 5 \text{ M}\Omega$$

This completes the bias design. To obtain  $g_m$  and  $r_o$ , we use

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

**Ex: 6.39** Refer to Fig. 6.57(a) and (c) and to the values found in the solution to Exercise 6.38 above.

$$R_{in} = R_{G1} \parallel R_{G2} = 5 \parallel 2.5 = 1.67 \text{ M}\Omega$$

$$R_o = R_D \parallel r_o = 18 \parallel 200 = 16.5 \text{ k}\Omega$$

$$\begin{aligned} G_v &= -\frac{R_{in}}{R_{in} + R_{sig}} g_m (r_o \parallel R_D \parallel R_L) \\ &= -\frac{1.67}{1.67 + 0.1} \times 2 \times (200 \parallel 18 \parallel 20) \\ &= -17.1 \text{ V/V} \end{aligned}$$

**Ex: 6.40** To reduce  $v_{gs}$  to half its value, the unbypassed  $R_s$  is given by

$$R_s = \frac{1}{g_m}$$

From the solution to Exercise 6.38 above,  $g_m = 2 \text{ mA/V}$ . Thus

$$R_s = \frac{1}{2} = 0.5 \text{ k}\Omega$$

Neglecting  $r_o$ ,  $G_v$  is given by

$$\begin{aligned} G_v &= -\frac{R_{in}}{R_{in} + R_{sig}} \times -\frac{R_D \parallel R_L}{\frac{1}{g_m} + R_s} \\ &= -\frac{1.67}{1.67 + 0.1} \times \frac{18 \parallel 20}{0.5 + 0.5} \\ &= -8.9 \text{ V/V} \end{aligned}$$

**Ex: 6.41** Refer to Fig. 6.58(a). For  $V_B = 5 \text{ V}$  and  $50\text{-}\mu\text{A}$  current through  $R_{B2}$ , we have

$$R_{B2} = \frac{5 \text{ V}}{0.05 \text{ mA}} = 100 \text{ k}\Omega$$

The base current is

$$I_B = \frac{I_E}{\beta + 1} \simeq \frac{0.5 \text{ mA}}{100} = 5 \mu\text{A}$$

The current through  $R_{B1}$  is

$$I_{R_{B1}} = I_B + I_{R_{B2}} = 5 + 50 = 55 \mu\text{A}$$

Since the voltage drop across  $R_{B1}$  is  $V_{CC} - V_B = 10 \text{ V}$ , the value of  $R_{B1}$  can be found from

$$R_{B1} = \frac{10 \text{ V}}{0.055 \mu\text{A}} = 182 \text{ k}\Omega$$

The value of  $R_E$  can be found from

$$\begin{aligned} I_E &= \frac{V_B - V_{BE}}{R_E} \\ \Rightarrow R_E &= \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega \end{aligned}$$

The value of  $R_C$  can be found from

$$V_C = V_{CC} - I_C R_C$$

$$6 = 15 - 0.99 \times 0.5 \times R_C$$

$$R_C \simeq 18 \text{ k}\Omega$$

This completes the bias design. The values of  $g_m$ ,  $r_\pi$ , and  $r_o$  can be found as follows:

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.5} = 200 \text{ k}\Omega$$

**Ex: 6.42** Refer to Fig. 6.58(b) and to the solution of Exercise 6.41 above.

$$R_{in} = R_{B1} \parallel R_{B2} \parallel r_\pi$$

$$= 182 \parallel 100 \parallel 5 = 4.64 \text{ k}\Omega$$

$$R_o = R_C \parallel r_o = 18 \parallel 200 = 16.51 \text{ k}\Omega$$

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L \parallel r_o)$$

$$G_v = -\frac{4.64}{4.64 + 10} \times 20 \times (18 \parallel 200)$$

$$= -57.3 \text{ V/V}$$

**Ex: 6.43** Refer to the solutions of Exercises 6.41 and 6.42 above. With  $R_e$  included (i.e., left unbypassed), the input resistance becomes [refer to Fig. 6.59(b)]

$$R_{in} = R_{B1} \parallel R_{B2} \parallel [(\beta + 1)(r_e + R_e)]$$

Thus,

$$10 = 182 \parallel 100 \parallel [101(0.05 + R_e)]$$

where we have substituted  $r_e = \frac{V_T}{I_E} =$

$\frac{25}{0.5} = 50 \Omega$ . The value of  $R_e$  is found from the equation above to be

$$R_e = 67.2 \Omega$$

The overall voltage gain can be found from

$$G_v = -\alpha \frac{R_{in}}{R_{in} + R_{sig}} \frac{R_C \parallel R_L}{r_e + R_e}$$

$$G_v = -0.99 \times \frac{10}{10 + 10} \frac{18 \parallel 20}{0.05 + 0.0672}$$

$$= -40 \text{ V/V}$$

**Ex: 6.44** Refer to Fig. 6.60(a).

$$R_{in} = 50 \Omega = r_e \parallel R_E \simeq r_e$$

$$r_e = 50 \Omega = \frac{V_T}{I_E}$$

$$\Rightarrow I_E = 0.5 \text{ mA}$$

$$I_C = \alpha I_E \simeq I_E = 0.5 \text{ mA}$$

$$V_C = V_{CC} - R_C I_C$$

For  $V_C = 1 \text{ V}$  and  $V_{CC} = 5 \text{ V}$ , we have

$$1 = 5 - R_C \times 0.5$$

$$\Rightarrow R_C = 8 \text{ k}\Omega$$

To obtain the required value of  $R_E$ , we note that the voltage drop across it is  $(V_{EE} - V_{BE}) = 4.3 \text{ V}$ . Thus,

$$R_E = \frac{4.3}{0.5} = 8.6 \text{ k}\Omega$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

$$= \frac{50 \Omega}{50 \Omega + 50 \Omega} \times 20(8 \parallel 8)$$

$$= 40 \text{ V/V}$$

$$\hat{v}_o = 40 \hat{v}_{sig} = 40 \times 10 \text{ mV} = 0.4 \text{ V}$$

**Ex: 6.45** Refer to Fig. 6.61. Consider first the bias design of the circuit in Fig. 6.61(a). Since the required  $I_E = 1 \text{ mA}$ , the base current

$$I_B = \frac{I_E}{\beta + 1} = \frac{1}{101} \simeq 0.01 \text{ mA. For a dc voltage}$$

drop across  $R_B$  of  $1 \text{ V}$ , we obtain

$$R_B = \frac{1 \text{ V}}{0.01 \text{ mA}} = 100 \text{ k}\Omega$$

The result is a base voltage of  $-1 \text{ V}$  and an emitter voltage of  $-1.7 \text{ V}$ . The required value of  $R_E$  can now be determined as

$$R_E = \frac{-1.7 - (-5)}{I_E} = \frac{3.3}{1 \text{ mA}} = 3.3 \text{ k}\Omega$$

$$R_{in} = R_B \parallel [(\beta + 1)(r_e + (R_E \parallel r_o \parallel R_L))]$$

$$\text{where } r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{in} = 100 \parallel (100 + 1)[0.025 + (3.3 \parallel 100 \parallel 1)]$$

$$= 44.3 \text{ k}\Omega$$

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{44.3}{44.3 + 50} = 0.469 \text{ V/V}$$

$$\frac{v_o}{v_i} = \frac{R_E \parallel r_o \parallel R_L}{r_e + (R_E \parallel r_o \parallel R_L)} = 0.968 \text{ V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.469 \times 0.968 = 0.454 \text{ V/V}$$

$$R_{out} = r_o \parallel R_E \parallel \left[ r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right]$$

$$= 100 \parallel 3.3 \parallel \left[ 0.025 + \frac{100 \parallel 50}{101} \right]$$

$$= 320 \Omega$$

### Solutions to End-of-Chapter Problems

**6.1** Coordinates of point A:  $v_{GS} = V_t = 0.5$  V and  $v_{DS} = V_{DD} = 5$  V.

To obtain the coordinates of point B, we first use Eq. (6.6) to determine  $V_{GS}|_B$  as

$$\begin{aligned} V_{GS}|_B &= V_t + \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \\ &= 0.5 + \frac{\sqrt{2 \times 10 \times 20 \times 5 + 1} - 1}{10 \times 20} \\ &= 0.5 + 0.22 = 0.72 \text{ V} \end{aligned}$$

The vertical coordinate of point B is  $V_{DS}|_B$ ,

$$V_{DS}|_B = V_{GS}|_B - V_t = V_{OV}|_B = 0.22 \text{ V}$$

**6.2** Bias point Q:  $V_{OV} = 0.2$  V and  $V_{DS} = 1$  V.

$$I_{DQ} = \frac{1}{2} k_n V_{OV}^2$$

$$= \frac{1}{2} \times 10 \times 0.04 = 0.2 \text{ mA}$$

$$R_D = \frac{V_{DD} - V_{DS}}{I_{DQ}} = \frac{5 - 1}{0.2} = 20 \text{ k}\Omega$$

Coordinates of point B:

Equation (6.6):

$$\begin{aligned} V_{GS}|_B &= V_t + \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \\ &= 0.5 + \frac{\sqrt{2 \times 10 \times 20 \times 5 + 1} - 1}{10 \times 20} \\ &= 0.5 + 0.22 = 0.72 \text{ V} \end{aligned}$$

Equations (6.7) and (6.8):

$$V_{DS}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.22 \text{ V}$$

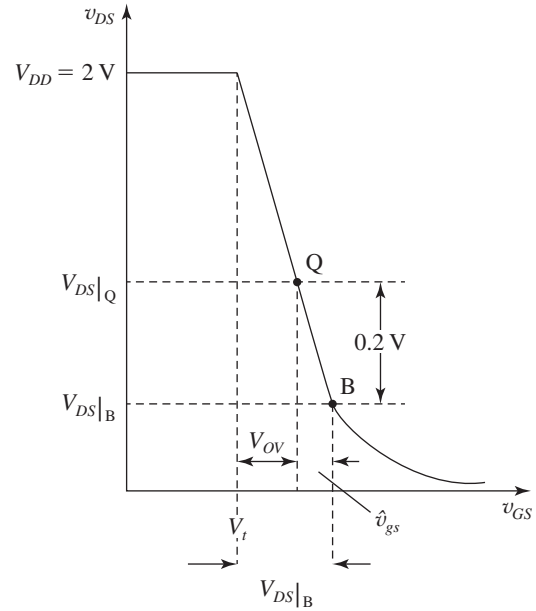
$$A_v = -k_n R_D V_{OV}$$

$$= -10 \times 20 \times 0.2 = -40 \text{ V/V}$$

The lowest instantaneous voltage allowed at the output is  $V_{DS}|_B = 0.22$  V. Thus the maximum allowable negative signal swing at the output is  $V_{DSQ} - 0.22 = 1 - 0.22 = 0.78$  V. The corresponding peak input signal is

$$\hat{v}_{gs} = \frac{0.78 \text{ V}}{|A_v|} = \frac{0.78}{40} = 19.5 \text{ mV}$$

**6.3** (a) To obtain maximum gain while allowing for a  $-0.2$ -V signal swing at the output, we bias the MOSFET at point Q where



$$V_{DS}|_Q = V_{DS}|_B + 0.2 \text{ V} \quad (1)$$

as indicated in the figure above. Now,  $V_{DS}|_B$  is given by Eq. (6.8) [together with Eq. (6.7)],

$$V_{DS}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \quad (2)$$

From the figure we see that

$$V_{DS}|_B = V_{OV} + \hat{v}_{gs}$$

where  $V_{OV} = 0.2$  V (given) and

$$\begin{aligned} \hat{v}_{gs} &= \frac{0.2 \text{ V}}{|A_v|} \\ &= \frac{0.2}{k_n R_D V_{OV}} = \frac{0.2}{k_n R_D \times 0.2} = \frac{1}{k_n R_D} \end{aligned}$$

Thus,

$$V_{DS}|_B = 0.2 + \frac{1}{k_n R_D} \quad (3)$$

Substituting for  $V_{DS}|_B$  from Eq. (2), we obtain

$$\frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.2 + \frac{1}{k_n R_D}$$

Substituting  $V_{DD} = 2$  V, rearranging the equation to obtain a quadratic equation in  $k_n R_D$ , and solving the resulting quadratic equation results in

$$k_n R_D = 79.05$$

which can be substituted into Eq. (3) to obtain

$$V_{DS}|_B = 0.2127 \text{ V}$$

The value of  $V_{DS}$  at the bias point can now be found from Eq. (1) as

$$V_{DS}|_Q = 0.2127 + 0.2 = 0.4127 \text{ V}$$

(b) The gain achieved can be found as

$$A_v = -k_n R_D V_{OV}$$

$$= -79.05 \times 0.2 = -15.81 \text{ V/V}$$

$$\hat{v}_{gs} = \frac{0.2}{|A_v|} = \frac{0.2}{15.81} = 12.7 \text{ mV}$$

(c)  $I_D = 100 \mu\text{A}$

$$R_D = \frac{V_{DD} - V_{DS}|_Q}{I_D}$$

$$= \frac{2 - 0.4127}{0.1} = 15.87 \text{ k}\Omega$$

$$(d) k_n = \frac{79.05}{15.87} = 4.98 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{4.98}{0.4} = 12.45$$

**6.4** Refer to Fig. 6.6.

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T}$$

$$= -\frac{5 - 1}{0.025} = -160 \text{ V/V}$$

The transistor enters saturation when  $v_{CE} \leq 0.3 \text{ V}$ , thus the maximum allowable output voltage swing is  $1 - 0.3 = 0.7 \text{ V}$ . The corresponding maximum input signal permitted  $\hat{v}_{be}$  is

$$\hat{v}_{be} = \frac{0.7 \text{ V}}{|A_v|} = \frac{0.7}{160} = 4.4 \text{ mV}$$

**6.5** To determine  $|A_{v\max}|$ , we use Eq. (6.23),

$$|A_{v\max}| = \frac{V_{CC} - 0.3}{V_T}$$

Then, for  $V_{CE} = \frac{V_{CC}}{2}$  we obtain

$$|A_v| = \frac{V_{CC} - \frac{V_{CC}}{2}}{V_T}$$

$$= \frac{V_{CC}}{2V_T}$$

Finally, if a negative-going output signal swing of 0.4 is required, to obtain maximum gain the transistor must be biased at  $V_{CE} = 0.4 + 0.3 = 0.7 \text{ V}$  and the gain achieved becomes

$$|A_v| = \frac{V_{CC} - 0.7}{V_T}.$$

The results are as follows:

$V_{CC}$	1.0	1.5	2.0	3.0 (V)
$V_{CC} - 0.3$	0.7	1.2	1.7	2.7 (V)
$ A_{v\max} $	28	48	68	108 (V/V)
$V_{CC}/2$	0.5	0.75	1.0	1.5 (V)
$ A_v $	20	30	40	60 (V/V)
$V_{CC} - 0.7$	0.3	0.8	1.3	2.3 (V)
$ A_v $	12	32	52	92 (V/V)

**6.6** See figure at the bottom of this page.

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.2 \times 5}{0.025} = -40 \text{ V/V}$$

**6.7** (a) Using Eq. (6.23) yields

$$|A_{v\max}| = \frac{V_{CC} - 0.3}{V_T} = \frac{3 - 0.3}{0.025} = 108 \text{ V/V}$$

(b) Using Eq. (6.22) with  $A_v = -60$  yields

$$-60 = -\frac{V_{CC} - V_{CE}}{V_T} = -\frac{3 - V_{CE}}{0.025}$$

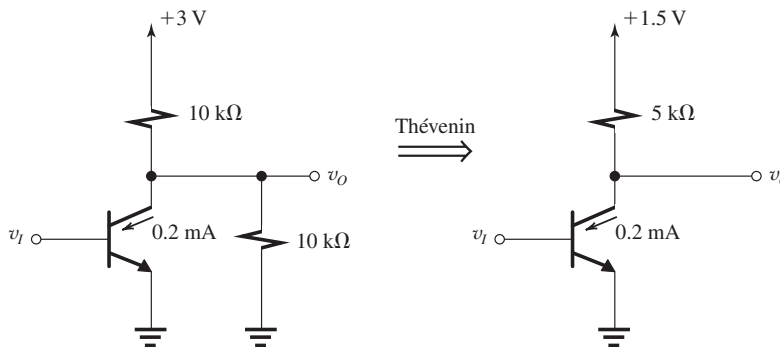
$$\Rightarrow V_{CE} = 1.5 \text{ V}$$

(c)  $I_C = 0.5 \text{ mA}$

$$I_C R_C = V_{CC} - V_{CE} = 3 - 1.5 = 1.5 \text{ V}$$

$$R_C = \frac{1.5}{0.5} = 3 \text{ k}\Omega$$

This figure belongs to Problem 6.6.



(d)  $I_C = I_S e^{V_{BE}/V_T}$   
 $0.5 \times 10^{-3} = 10^{-15} e^{V_{BE}/0.025}$   
 $\Rightarrow V_{BE} = 0.673 \text{ V}$

(e) Assuming linear operation around the bias point, we obtain

$v_{ce} = A_v \times v_{be}$   
 $= -60 \times 5 \sin \omega t = -300 \sin \omega t, \text{ mV}$   
 $= -0.3 \sin \omega t, \text{ V}$

(f)  $i_c = \frac{-v_{ce}}{R_C} = 0.1 \sin \omega t, \text{ mA}$

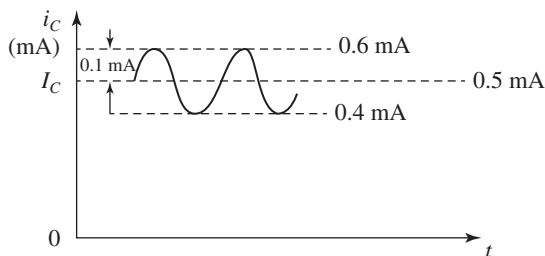
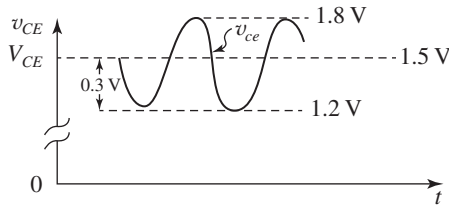
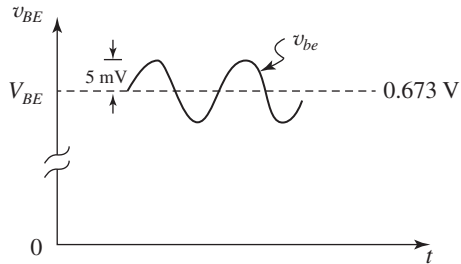
(g)  $I_B = \frac{I_C}{\beta} = \frac{0.5 \text{ mA}}{100} = 0.005 \text{ mA}$

$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} \sin \omega t = 0.001 \sin \omega t, \text{ mA}$

(h) Small-signal input resistance  $\equiv \frac{\hat{v}_{be}}{\hat{i}_b}$

$= \frac{5 \text{ mV}}{0.001 \text{ mA}} = 5 \text{ k}\Omega$

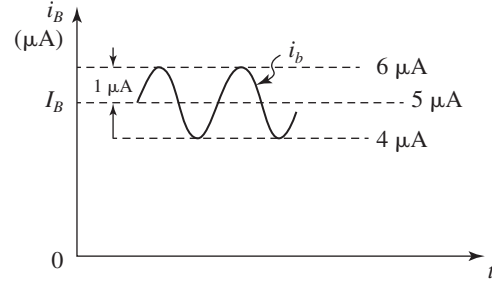
(i)



**6.8**  $V_{DS}|_B = V_{OV}|_B = 0.5 \text{ V}$

Thus,

$I_D|_B = \frac{1}{2} k_n V_{DS}^2|_B = \frac{1}{2} \times 5 \times 0.5^2 = 0.625 \text{ mA}$



The value of  $R_D$  required can now be found as

$R_D = \frac{V_{DD} - V_{DS}|_B}{I_D|_B}$   
 $= \frac{5 - 0.5}{0.625} = 7.2 \text{ k}\Omega$

If the transistor is replaced with another having twice the value of  $k_n$ , then  $I_D|_B$  will be twice as large and the required value of  $R_D$  will be half that used before, that is, 3.6 k $\Omega$ .

**6.9** From Eq. (6.18):

$|A_{v \text{ max}}| = \frac{V_{DD} - V_{OV}|_B}{V_{OV}|_B/2}$

$14 = \frac{2 - V_{OV}|_B}{V_{OV}|_B/2}$

$\Rightarrow V_{OV}|_B = 0.25 \text{ V}$

Now, using Eq. (6.15) at point B, we have

$A_v|_B = -k_n V_{OV}|_B R_D$

Thus,

$-14 = -k_n R_D \times 0.25$

$\Rightarrow k_n R_D = 56$

To obtain a gain of  $-12 \text{ V/V}$  at point Q:

$-12 = -k_n R_D V_{OV}|_Q$

$= -56 V_{OV}|_Q$

Thus,

$V_{OV}|_Q = \frac{12}{56} = 0.214 \text{ V}$

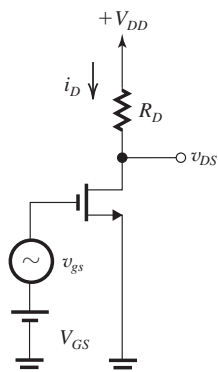
To obtain the required  $V_{DS}|_Q$ , we use Eq. (6.17),

$A_v = -\frac{V_{DD} - V_{DS}|_Q}{V_{OV}|_Q/2}$

$-12 = -\frac{2 - V_{DS}|_Q}{0.214/2}$

$$\Rightarrow V_{DS}|_Q = 0.716 \text{ V}$$

## 6.10



(a) Coordinates of point A:

$$V_{GS}|_A = V_t = 1 \text{ V}, \quad V_{DS}|_A = V_{DD} = 5 \text{ V}$$

Coordinates of point B can be obtained using Eqs. (6.6)–(6.8):

$$V_{OV}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D}$$

$$= \frac{\sqrt{2 \times 1 \times 24 \times 5 + 1} - 1}{1 \times 24} = 0.61 \text{ V}$$

$$V_{GS}|_B = V_t + V_{OV}|_B = 1 + 0.61 = 1.61 \text{ V}$$

$$V_{DS}|_B = V_{OV}|_B = 0.61 \text{ V}$$

(b) At Q,  $V_{OV} = 0.5 \text{ V}$ , thus

$$V_{GS}|_Q = 1 + 0.5 = 1.5 \text{ V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 1 \times 0.5^2 = 0.125 \text{ mA}$$

$$V_{DS}|_Q = V_{DD} - I_D R_D$$

$$= 5 - 0.125 \times 24 = 2 \text{ V}$$

$$A_v = -k_n V_{OV} R_D$$

$$= -1 \times 0.5 \times 24 = -12 \text{ V/V}$$

(c) From part (a) above, the maximum instantaneous input signal while the transistor remains in saturation is 1.61 V and the corresponding output voltage is 0.61 V. Thus, the maximum amplitude of input sine wave is  $(1.61 - 1.5) = 0.11 \text{ V}$ . That is,  $v_{GS}$  ranges from  $1.5 - 0.11 = 1.39 \text{ V}$ , at which

$$i_D = \frac{1}{2} \times 1 \times (1.39 - 1)^2 = 0.076 \text{ mA}$$

and

$$v_{DS} = 5 - 0.076 \times 24 = 3.175 \text{ V}$$

and  $v_{GS} = 1.5 + 0.11 = 1.61 \text{ V}$  at which  $v_{DS} = 0.61 \text{ V}$ .

Thus, the large-signal gain is

$$\frac{0.61 - 3.175}{1.61 - 1.39} = -11.7 \text{ V/V}$$

whose magnitude is slightly less ( $-2.5\%$ ) than the incremental or small-signal gain ( $-12 \text{ V/V}$ ). This is an indication that the transfer characteristic is not a straight line.

6.11  $R_D = 20 \text{ k}\Omega$

$$k'_n = 200 \mu\text{A}/\text{V}^2$$

$$V_{RD} = 1.5 \text{ V}$$

$$V_{GS} = 0.7 \text{ V}$$

$$A_v = -10 \text{ V/V}$$

$$A_v = -k_n V_{OV} R_D$$

$$V_{RD} = I_D R_D = \frac{1}{2} k_n V_{OV}^2 R_D$$

$$\frac{A_v}{V_{RD}} = \frac{-2}{V_{OV}} = \frac{-10}{1.5}$$

$$\therefore V_{OV} = 0.30 \text{ V}$$

$$V_t = V_{GS} - V_{OV} = 0.40 \text{ V}$$

$$k_n = \frac{A_v}{V_{OV} R_D} = \frac{-10}{-0.3 \times 20}$$

$$= 1.67 \text{ mA}/\text{V}^2$$

$$k_n = k'_n \frac{W}{L} = 1.67 \text{ mA}/\text{V}^2$$

$$\therefore \frac{W}{L} = 8.33$$

6.12 At sat/triode boundary

$$v_{GS}|_B = V_{GS} + \hat{v}_{gs}$$

$$v_{DS}|_B = V_{DS} - \hat{v}_o$$

( $\hat{v}_o = \text{max downward amplitude}$ ), we get

$$v_{DS}|_B = v_{GS}|_B - V_t = V_{GS} + \frac{\hat{v}_o}{|A_v|} - V_t$$

$$= V_{DS} - \hat{v}_o$$

$$V_{OV} + \frac{\hat{v}_o}{|A_v|} = V_{DS} - \hat{v}_o$$



$$\hat{v}_o = \frac{V_{DS} - V_{OV}}{1 + \frac{1}{|A_v|}}$$

For  $V_{DD} = 5\text{ V}$ ,  $V_{OV} = 0.5\text{ V}$ , and

$$k'_n \frac{W}{L} = 1\text{ mA/V}^2, \text{ we use}$$

$$A_v = \frac{-2(V_{DD} - V_{DS})}{V_{OV}}$$

and Eq. (1) to obtain

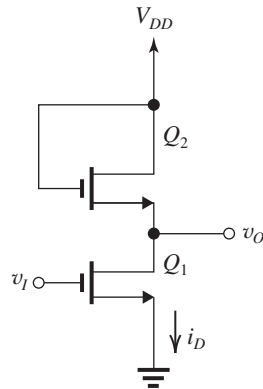
$V_{DS}$	$A_v$	$\hat{v}_o$	$\hat{v}_i$
1 V	-16	471 mV	29.4 mV
1.5 V	-14	933 mV	66.7 mV
2 V	-12	1385 mV	115 mV
2.5 V	-10	1818 mV	182 mV

For  $V_{DS} = 1\text{ V}$ ,  $A_v = -16 = -k_n V_{OV} R_D$

$$\therefore R_D = 32\text{ k}\Omega$$

$$I_D R_D = 4\text{ V}, I_D = 0.125\text{ mA}$$

6.13



given  $V_{i1} = V_{i2} = V_i$

$$\text{For } Q_2, i_D = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_2 [V_{DD} - v_o - V_t]^2$$

For  $V_t \leq v_i \leq v_o + V_t$ ,

$$\text{for } Q_1, i_D = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_1 [v_i - V_t]^2$$

Equating  $i_{D1}$  and  $i_{D2}$

$$\left(\frac{W}{L}\right)_2 [V_{DD} - v_o + V_t]^2$$

$$= \left(\frac{W}{L}\right)_1 [v_i - V_t]^2$$

$$[V_{DD} - v_o - V_t] = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot [v_i - V_t]$$

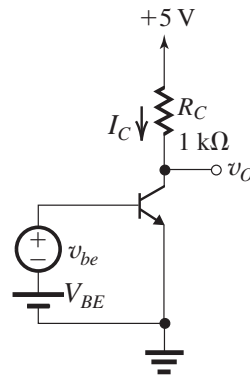
$$(1) \quad v_o = V_{DD} - V_t + V_t \sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$-v_i \sqrt{\frac{(W/L)_1}{(W/L)_2}} \quad \text{Q.E.D.}$$

$$\text{For } \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \sqrt{\frac{\left(\frac{50}{0.5}\right)}{\left(\frac{5}{0.5}\right)}} = \sqrt{10},$$

$$A_v = -\sqrt{10} = -3.16\text{ V/V}$$

6.14



For  $I_C = 0.5\text{ mA}$ , we have

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.5}{0.025} = -20\text{ V/V}$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 5 - 0.5 = 4.5\text{ V}$$

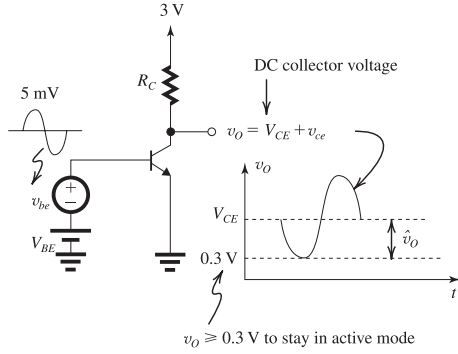
$$\text{max } +\Delta v_o = 5 - 4.5 = 0.5\text{ V}$$

$$\text{max } -\Delta v_o = 4.5 - 0.3 = 4.2\text{ V}$$

Similarly  $\rightarrow$

$I_C$ (mA)	$V_{CE}$ (V)	$A_v$ (V/V)	POS $\Delta v_o$ (V)	Neg $\Delta v_o$ (V)
0.5	4.5	-20	0.5	4.2
1.0	4.0	-40	1.0	3.7
2.5	2.5	-100	2.5	2.2
4.0	1.0	-160	4.0	0.7
4.5	0.5	-180	4.5	0.2

6.15



$$A_v = -\frac{I_C R_C}{V_T} = -\frac{V_{CC} - V_{CE}}{V_T}$$

On the verge of saturation

$$V_{CE} - \hat{v}_{ce} = 0.3 \text{ V}$$

For linear operation,  $v_{ce} = A_v v_{be}$

$$V_{CE} - |A_v| \hat{v}_{be} = 0.3$$

$$(5 - I_C R_C) - |A_v| \times 5 \times 10^{-3} = 0.3$$

But

$$|A_v| = \frac{I_C R_C}{V_T}$$

Thus,

$$I_C R_C = |A_v| V_T$$

and

$$5 - |A_v| V_T - |A_v| \times 5 \times 10^{-3} = 0.3$$

$$|A_v|(0.025 + 0.005) = 5 - 0.3$$

$$|A_v| = 156.67. \text{ Note } A_v \text{ is negative.}$$

$$\therefore A_v = -156.67 \text{ V/V}$$

Now we can find the dc collector voltage.

Referring to the sketch of the output voltage, we see that

$$V_{CE} = 0.3 + |A_v| 0.005 = 1.08 \text{ V}$$

**6.16** To obtain an output signal of peak amplitude  $P$  volts and maximum gain, we bias the transistor at

$$V_{CE} = V_{CEsat} + P$$

The resulting gain will be

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T}$$

which results in  $V_{CC}$  of

$$V_{CC} = V_{CE} + |A_v| V_T$$

Thus the minimum required  $V_{CC}$  will be

$$V_{CCmin} = V_{CEsat} + P + |A_v| V_T$$

but we have to make sure that the amplifier can support a positive peak amplitude of  $P$ , that is,

$$|A_v| V_T \geq P,$$

otherwise the transistor cuts off at or near the positive peaks.

In the results obtained, tabulated below,  $V_{CEsat} = 0.3 \text{ V}$  and  $V_{CC}$  is the nearest 0.5 V to  $V_{CCmin}$ .

Case	$A_v$ (V/V)	$P$ (V)	$ A_v  V_T$	$V_{CCmin}$	$V_{CC}$
a	-20	0.2	0.5	1.0	1.0
b	-50	0.5	1.25	2.05	2.5
c	-100	0.5	2.5	3.3	3.5
d	-100	1.0	2.5	3.8	4.0
e	-200	1.0	5.0	6.3	6.5
f	-500	1.0	12.5	13.8	14.0
g	-500	2.0	12.5	14.8	15.0

**6.17** See figure on next page.

$$\mathbf{6.18} \quad i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

$$I_C = I_S e^{V_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

$$v_{CE} = V_{CC} - R_C i_C$$

$$V_{CE} = V_{CC} - R_C I_C$$

$$\begin{aligned} A_v &= \left. \frac{dv_{CE}}{dv_{BE}} \right|_{v_{BE}=V_{BE}, v_{CE}=V_{CE}} \\ &= -R_C I_S \left( 1 + \frac{V_{CE}}{V_A} \right) e^{V_{BE}/V_T} \left( \frac{1}{V_T} \right) \\ &\quad - R_C I_S e^{V_{BE}/V_T} \left( \frac{dv_{CE}}{dv_{BE}} \right) \left( \frac{1}{V_A} \right) \\ &= -R_C I_C \frac{1}{V_T} - R_C \frac{I_C}{1 + \frac{V_{CE}}{V_A}} \left( \frac{1}{V_A} \right) A_v \end{aligned}$$

Thus,

$$A_v = \frac{-I_C R_C / V_T}{1 + \frac{I_C R_C}{V_A + V_{CE}}} \quad \text{Q.E.D.}$$

Substituting  $I_C R_C = V_{CC} - V_{CE}$ , we obtain

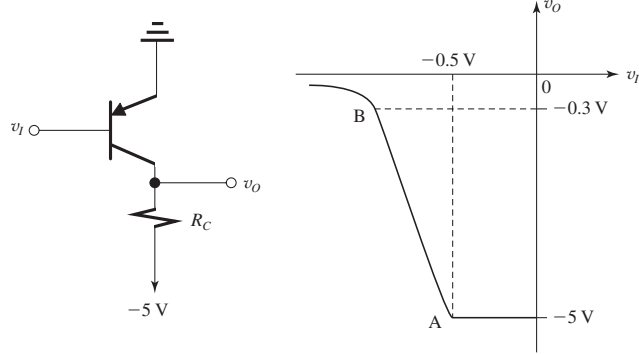
$$A_v = -\frac{(V_{CC} - V_{CE}) / V_T}{\left[ 1 + \frac{V_{CC} - V_{CE}}{V_A + V_{CE}} \right]} \quad \text{Q.E.D.}$$

For  $V_{CC} = 5 \text{ V}$ ,  $V_{CE} = 3 \text{ V}$ , and  $V_A = 100 \text{ V}$ ,

$$A_v \text{ (without the Early effect)} = -\frac{5 - 3}{0.025}$$

$$= -80 \text{ V/V}$$

This figure belongs to Problem 6.17.



$$A_v \text{ (with the Early effect)} = \frac{-80}{1 + \frac{2}{100 + 3}}$$

$$= -78.5 \text{ V/V}$$

**6.19**  $I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{3 - 1}{2} = 1 \text{ mA}$

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T} = -\frac{2}{0.025} = -80 \text{ V/V}$$

Using the small-signal voltage gain with  $\Delta v_{BE} = +5 \text{ mV}$ , we have

$$\Delta v_O = A_v \times \Delta v_{BE} = -80 \times 5 \text{ mV} = -0.4 \text{ V}$$

Using the exponential characteristic yields

$$i_C = I_C e^{v_{BE}/V_T}$$

$$= 1 \times e^{5/25} = 1.22 \text{ mA}$$

Thus,  $\Delta i_C = 0.22 \text{ mA}$  and

$$\Delta v_O = -\Delta i_C R_C$$

$$= -0.22 \times 2 = -0.44 \text{ V}$$

Repeating for  $\Delta v_{BE} = -5 \text{ mV}$  as follows.

Using the small-signal voltage gain:

$$\Delta v_O = -80 \times -5 = +0.4 \text{ V}$$

Using the exponential characteristic:

$$i_C = I_C e^{v_{BE}/V_T}$$

$$= 1 \times e^{-5/25} = 0.82 \text{ mA}$$

Thus,  $\Delta i_C = 0.82 - 1 = -0.18 \text{ mA}$  and  $\Delta v_O = 0.18 \times 2 = 0.36 \text{ V}$

$\Delta v_{BE}$	$\Delta v_O(\text{exp})$	$\Delta v_O(\text{linear})$
+5 mV	-440 mV	-400 mV
-5 mV	+360 mV	+400 mV

Thus, using the small-signal approximation underestimates  $|\Delta v_O|$  for positive  $\Delta v_{BE}$  by about

10% and overestimates  $|\Delta v_O|$  for negative  $\Delta v_{BE}$  by about 10%.

**6.20**  $A_v = -\left(\frac{I_C}{V_T}\right)R_C$

But

$$A_v \equiv \frac{\Delta v_O}{\Delta v_{BE}} = \frac{-\Delta i_C R_C}{\Delta v_{BE}} = -g_m R_C$$

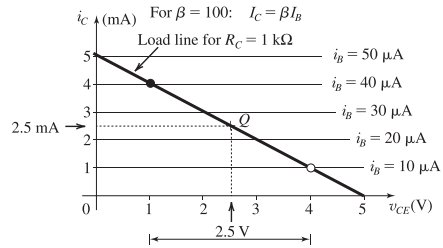
Thus,

$$g_m = I_C/V_T$$

For a transistor biased at  $I_C = 0.5 \text{ mA}$ , we have

$$g_m = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

**6.21**



Peak-to-peak  $v_C$  swing =  $4 - 1 = 3 \text{ V}$

For point Q at  $V_{CC}/2 = 2.5 \text{ V}$ , we obtain

$$V_{CE} = 2.5 \text{ V}, \quad I_C = 2.5 \text{ mA}$$

$$I_B = 25 \mu\text{A}$$

$$I_B = \frac{V_{BB} - 0.7}{R_B} = 25 \mu\text{A}$$

$$\Rightarrow V_{BB} = I_B R_B + 0.7 = 2.5 + 0.7 = 3.2 \text{ V}$$

**6.22** See the graphical construction that follows.

For this circuit:

$$V_{CC} = 10 \text{ V}, \quad \beta = 100,$$

$$R_C = 1 \text{ k}\Omega, \quad V_A = 100 \text{ V},$$

$$I_B = 50 \mu\text{A} \text{ (dc bias),}$$

At  $v_{CE} = 0, i_C = \beta i_B$

$$\begin{aligned} \therefore i_C &= 50 \times 100 \\ &= 5 \text{ mA} \end{aligned}$$

Given the base bias current of  $50 \mu\text{A}$ , the dc or bias point of the collector current  $I_C$ , and voltage  $V_{CE}$  can be found from the intersection of the load line and the transistor line  $L_1$  of  $i_B = 50 \mu\text{A}$ . Specifically:

$$\begin{aligned} \text{Eq. of } L_1 &\Rightarrow i_C = I_C (1 + v_{CE}/V_A) \\ &= 5 (1 + v_{CE}/100) \\ &= 5 + 0.05v_{CE} \end{aligned}$$

$$\text{Load line } \Rightarrow i_C = \frac{V_{CC} - v_{CE}}{R_C} = 10 - v_{CE}$$

$$\therefore 10 - v_{CE} = 5 + 0.05v_{CE}$$

$$V_{CE} = v_{CE} = 4.76 \text{ V}$$

$$I_C = i_C = 10 - v_{CE} = 5.24 \text{ mA}$$

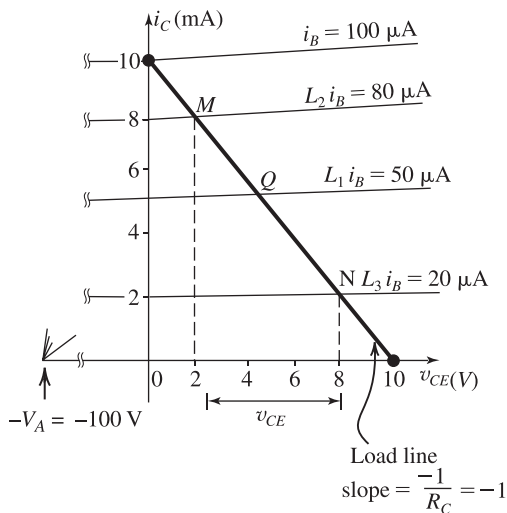
Now for a signal of  $30\text{-}\mu\text{A}$  peak superimposed on  $I_B = 50 \mu\text{A}$ , the operating point moves along the load line between points N and M. To obtain the coordinates of point M, we solve the load line and line  $L_2$  to find the intersection M, and the load line and line  $L_3$  to find N:

For point M:

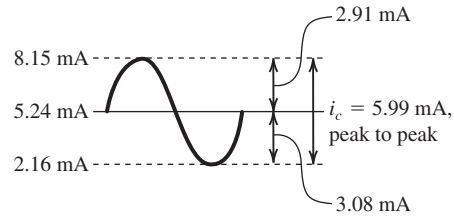
$$\begin{aligned} i_C &= 8 + (8/100)v_{CE} \text{ and } i_C = 10 - v_{CE} \\ \therefore i_C|_M &= 8.15 \text{ mA}, \quad v_{CE}|_M = 1.85 \text{ V} \end{aligned}$$

For point N:

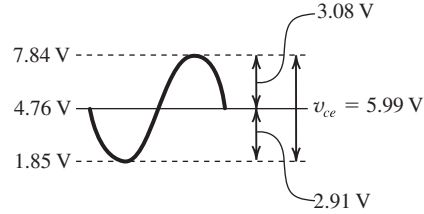
$$\begin{aligned} i_C &= 2 + 0.02v_{CE} \text{ and } i_C = 10 - v_{CE} \\ v_{CE}|_N &= 7.84 \text{ V}, \quad i_C|_N = 2.16 \text{ mA} \end{aligned}$$



Thus the collector current varies as follows:



And the collector voltage varies as follows:



**6.23 (a)**

$$V_{OV} = 0.7 - 0.5 = 0.2 \text{ V}$$

$$\begin{aligned} I_D &= \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 0.4 \times 12.5 \times 0.2^2 \\ &= 0.1 \text{ mA} \end{aligned}$$

$$V_{DS} = 2 - 0.1 \times 15 = 0.5 \text{ V}$$

(b)

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

(c)

$$\begin{aligned} A_{vo} &= -g_m R_D \\ &= -1 \times 15 = -15 \text{ V/V} \end{aligned}$$

(d)

$$v_{ds} = A_{vo} \times v_{gs} = -15 \times 0.015 \sin \omega t$$

$$v_{ds} = -0.225 \sin \omega t, \text{ V}$$

$$\begin{aligned} \hat{v}_{DS} &= V_{DS} + 0.225 = (V_{DD} - R_D I_D) + 0.225 \\ &= (2 - 15 \times 0.1) + 0.225 = 0.5 + 0.225 \\ &= 0.725 \text{ V} \end{aligned}$$

$$\check{v}_{DS} = 0.5 - 0.225 = 0.275 \text{ V}$$

(e) From Eq. (6.28),

$$\begin{aligned} i_D &= 0.1 + k_n V_{OV} v_{gs} + \frac{1}{2} k_n v_{gs}^2, \text{ mA} \\ &= 0.1 + (0.4 \times 12.5) \times 0.2 \times 0.015 \sin \omega t \\ &\quad + \frac{1}{2} \times 0.4 \times 12.5 \times 0.015^2 \sin^2 \omega t \\ &= 0.1 + 0.015 \sin \omega t + 2.5 \\ &\quad \times 0.015^2 \left( \frac{1}{2} - \frac{1}{2} \cos 2\omega t \right) \end{aligned}$$

Thus, the second harmonic distortion is

$$\begin{aligned} &= \frac{2.5 \times \frac{1}{2} \times 0.015^2}{0.015} \times 100 \\ &= 1.9\% \end{aligned}$$

In addition to the component at  $(2\omega)$ , there is a dc shift in the value of  $i_D$  given by

$$2.5 \times 0.015^2 \times 0.5 = 2.8 \times 10^{-4} \text{ mA} = 0.28 \text{ } \mu\text{A}$$

Compared to  $I_D = 100 \text{ } \mu\text{A}$ , this shift is negligible.

**6.24** Refer to Fig. 6.10.

$$\begin{aligned} A_v &= -g_m R_D \\ -10 &= -g_m \times 10 \\ \Rightarrow g_m &= 1 \text{ mA/V} \end{aligned}$$

To obtain maximum gain while allowing for a 0.2-V peak signal at the drain, we bias so that

$$V_{DS} = 0.2 + V_{OV} \tag{1}$$

which indicates that the minimum signal at the drain will be equal to  $V_{OV}$ . Since we can express the gain magnitude of

$$|A_v| = \frac{V_{DD} - V_{DS}}{0.5V_{OV}}$$

Thus,

$$\begin{aligned} 10 &= \frac{1.8 - V_{DS}}{0.5V_{OV}} \\ \Rightarrow V_{DS} &= 1.8 - 5V_{OV} \end{aligned} \tag{2}$$

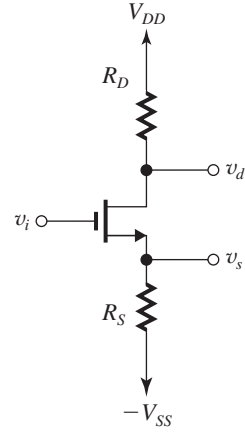
Solving Eqs. (1) and (2) together yields

$$V_{OV} = 0.27 \text{ V}$$

Thus,

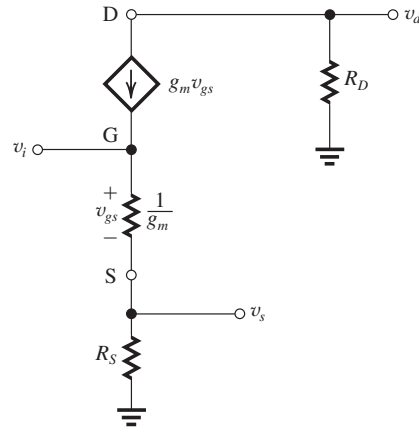
$$\begin{aligned} V_{DS} &= 0.47 \text{ V} \\ I_D &= \frac{1.8 - 0.47}{10} = 0.133 \text{ mA} \\ I_D &= \frac{1}{2} k_n V_{OV}^2 \\ \Rightarrow k_n &= \frac{2 \times 0.133}{0.27^2} = 3.65 \text{ mA/V}^2 \\ k_n &= k'_n \frac{W}{L} \\ 3.65 &= 0.38 \times \frac{W}{L} \\ \Rightarrow \frac{W}{L} &= 9.6 \end{aligned}$$

**6.25**



$$v_i = (g_m v_{gs}) \left( \frac{1}{g_m} + R_S \right)$$

$$v_d = -g_m v_{gs} R_D$$



$$v_s = +g_m v_{gs} R_S$$

$$\therefore \frac{v_s}{v_i} = \frac{R_S}{\frac{1}{g_m} + R_S} = \frac{+g_m R_S}{1 + g_m R_S}$$

$$\frac{v_d}{v_i} = \frac{-R_D}{\frac{1}{g_m} + R_S} = \frac{-g_m R_D}{1 + g_m R_S}$$

**6.26**

$$V_t = 0.8 \text{ V}$$

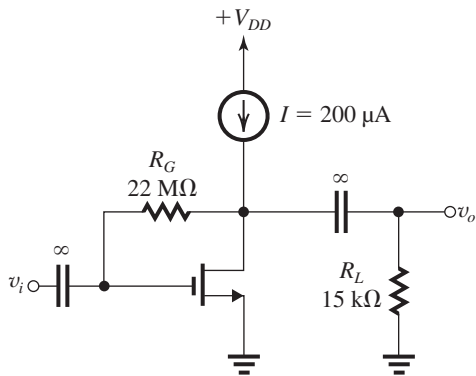
$$V_A = 20 \text{ V}$$

Given  $V_{DS} = V_{GS} = 1 \text{ V}$ . Also,  $I_D = 0.2 \text{ mA}$ .

$$V_{OV} = 0.2 \text{ V}, g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = 100 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = -g_m (R_G \parallel R_L \parallel r_o) = -26.1 \text{ V/V}$$



For  $I_D = 1 \text{ mA}$ :

$$V_{OV} \text{ increases by } \sqrt{\frac{1}{0.2}} = \sqrt{5} \text{ to}$$

$$\sqrt{5} \times 0.2 = 0.45 \text{ V.}$$

$$V_{GS} = V_{DS} = 1.25 \text{ V}$$

$g_m = 4.47 \text{ mA/V}$ ,  $r_o = 20 \text{ k}\Omega$  and

$$\frac{v_o}{v_i} = -38.3 \text{ V/V}$$

**6.27** (a) Open-circuit the capacitors to obtain the bias circuit shown in Fig. 1, which indicates the given values.

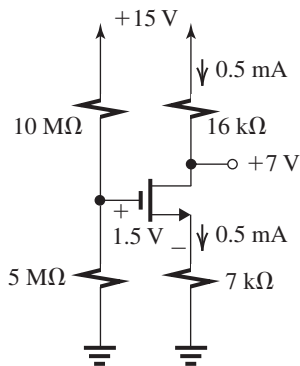


Figure 1

From the voltage divider, we have

$$V_G = 15 \frac{5}{10 + 5} = 5 \text{ V}$$

From the circuit, we obtain

$$\begin{aligned} V_G &= V_{GS} + 0.5 \times 7 \\ &= 1.5 + 3.5 = 5 \text{ V} \end{aligned}$$

which is consistent with the value provided by the voltage divider.

Since the drain voltage (+7 V) is higher than the gate voltage (+5 V), the transistor is operating in saturation.

From the circuit

$$V_D = V_{DD} - I_D R_D = 15 - 0.5 \times 16 = +7 \text{ V, as assumed}$$

Finally,

$$V_{GS} = 1.5 \text{ V, thus } V_{OV} = 1.5 - V_t = 1.5 - 1 = 0.5 \text{ V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 4 \times 0.5^2 = 0.5 \text{ mA}$$

which is equal to the given value. Thus the bias calculations are all consistent.

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

(c) See Fig. 2 on next page

$$(d) R_{in} = 10 \text{ M}\Omega \parallel 5 \text{ M}\Omega = 3.33 \text{ M}\Omega$$

$$\frac{v_{gs}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{3.33}{3.33 + 0.2}$$

$$= 0.94 \text{ V/V}$$

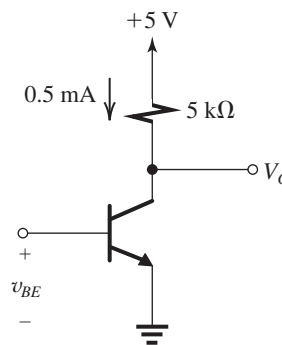
$$\frac{v_o}{v_{gs}} = -g_m(200 \parallel 16 \parallel 16)$$

$$= -2 \times 7.69 = -15.38 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = \frac{v_{gs}}{v_{sig}} \times \frac{v_o}{v_{gs}} = -0.94 \times 15.38$$

$$= -14.5 \text{ V/V}$$

**6.28**



With  $v_{BE} = 0.700 \text{ V}$

$$\begin{aligned} V_C &= V_{CC} - R_C I_C \\ &= 5 - 5 \times 0.5 = 2.5 \text{ V} \end{aligned}$$

For  $v_{BE} = 705 \text{ mV} \Rightarrow v_{be} = 5 \text{ mV}$

$$\begin{aligned} i_C &= I_C e^{v_{be}/V_T} \\ &= 0.5 \times e^{5/25} = 0.611 \text{ mA} \end{aligned}$$

This figure belongs to Problem 6.27, part (c).

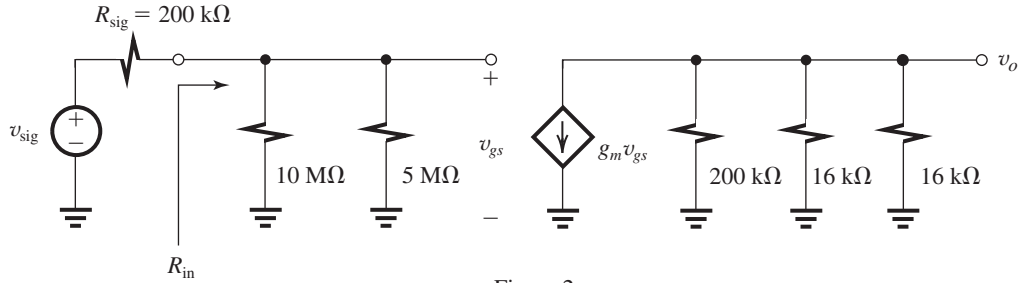


Figure 2

$$v_c = V_{CC} - R_C i_C = 5 - 5 \times 0.611 = 1.95 \text{ V}$$

$$v_{ce} = v_c - V_C = 1.95 - 2.5 = -0.55 \text{ V}$$

$$\text{Voltage gain, } A_v = \frac{v_{ce}}{v_{be}} = -\frac{0.55 \text{ V}}{5 \text{ mV}}$$

$$= -110 \text{ V/V}$$

Using small-signal approximation, we write

$$A_v = -g_m R_C$$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$A_v = -20 \times 5 = -100 \text{ V/V}$$

Thus, the small-signal approximation at this signal level ( $v_{be} = 5 \text{ mV}$ ) introduces an error of  $-9.1\%$  in the gain magnitude.

$$6.29 \quad g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e = \frac{\alpha}{g_m} = \frac{0.99}{40 \text{ mA/V}} \approx 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$A_v = -g_m R_C = -40 \times 5 = -200 \text{ V/V}$$

$$\hat{v}_o = |A_v| \hat{v}_{be} = 200 \times 5 \text{ mV} = 1 \text{ V}$$

6.30

Transistor	a	b	c	d	e	f	g
$\alpha$	1.000	0.990	0.980	1	0.990	0.900	0.940
$\beta$	$\infty$	100	50	$\infty$	100	9	15.9
$I_C$ (mA)	1.00	0.99	1.00	1.00	0.248	4.5	17.5
$I_E$ (mA)	1.00	1.00	1.02	1.00	0.25	5	18.6
$I_B$ (mA)	0	0.010	0.020	0	0.002	0.5	1.10
$g_m$ (mA/V)	40	39.6	40	40	9.92	180	700
$r_e$ ( $\Omega$ )	25	25	24.5	25	100	5	1.34
$r_\pi$ ( $\Omega$ )	$\infty$	2.525 k	1.25 k	$\infty$	10.1 k	50	22.7

$$6.31 \quad I_C = 1 \text{ mA}, \quad \beta = 100, \quad V_A = 100 \text{ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{100 + 1} = 0.99$$

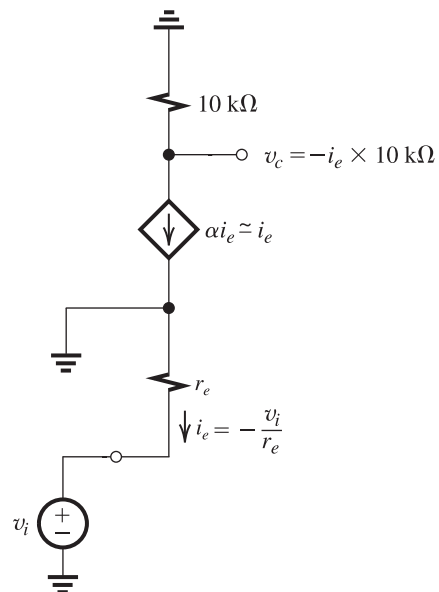
$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25 \text{ mV}}{1 \text{ mA}} = 24.75 \Omega$$

6.32 Refer to Fig. P6.32:

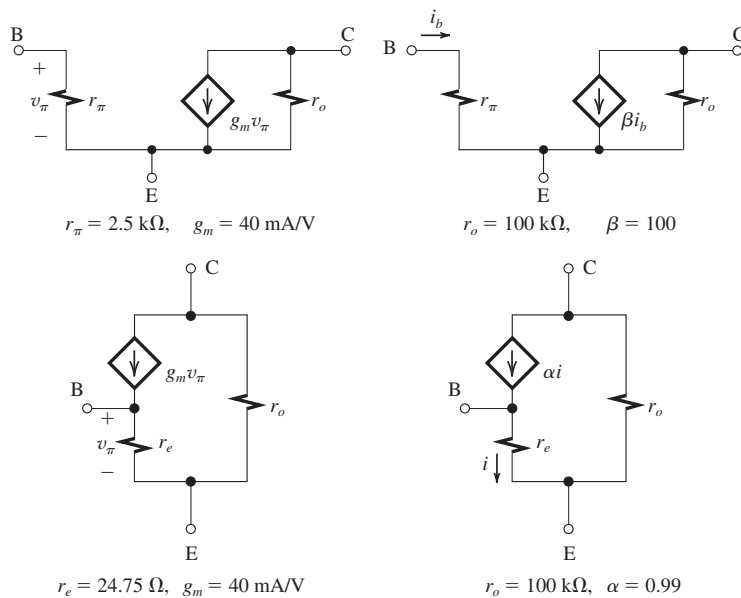
$$V_C = 3 - 0.2 \times 10 = 1 \text{ V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \Omega$$

Replacing the BJT with the  $T$  model of Fig. 6.26(b), we obtain the equivalent circuit shown below.



This figure belongs to Problem 6.31.



$$v_c = -i_e \times 10 \text{ k}\Omega$$

where

$$i_e = -\frac{v_i}{r_e} = -\frac{v_i}{0.125 \text{ k}\Omega}$$

Thus,

$$\begin{aligned} \frac{v_c}{v_i} &= \frac{10 \text{ k}\Omega}{0.125 \text{ k}\Omega} \\ &= 80 \text{ V/V} \end{aligned}$$

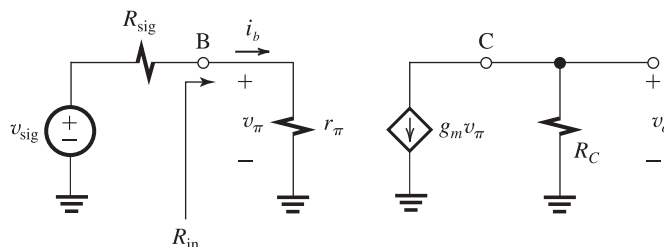
$$\begin{aligned} \frac{v_o}{v_{\text{sig}}} &= \frac{v_o}{v_{\pi}} \frac{v_{\pi}}{v_{\text{sig}}} \\ &= -g_m R_C \frac{r_{\pi}}{r_{\pi} + R_{\text{sig}}} \\ &= -g_m r_{\pi} \frac{R_C}{r_{\pi} + R_{\text{sig}}} \\ &= -\frac{\beta R_C}{r_{\pi} + R_{\text{sig}}} \quad \text{Q.E.D.} \end{aligned}$$

6.33 See figure below.

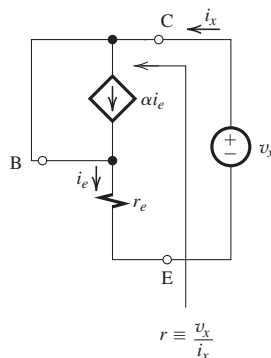
$$\begin{aligned} R_{\text{in}} &\equiv \frac{v_{\pi}}{i_b} = r_{\pi} \\ \frac{v_{\pi}}{v_{\text{sig}}} &= \frac{r_{\pi}}{r_{\pi} + R_{\text{sig}}} \\ v_o &= -g_m v_{\pi} R_C \\ \frac{v_o}{v_{\pi}} &= -g_m R_C \end{aligned}$$

The overall voltage gain can be obtained as follows:

This figure belongs to Problem 6.33.



6.34 Replacing the BJT with the *T* model of Fig. 6.26(b), we obtain the circuit shown below.





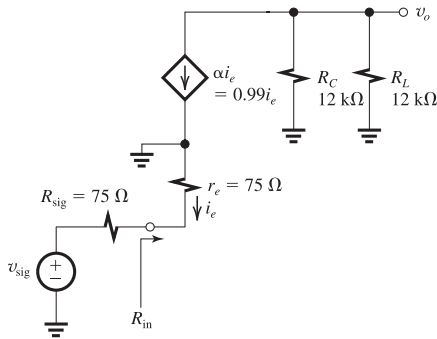
Since  $v_x$  appears across  $r_e$  and  $i_x = i_e = \frac{v_x}{r_e}$ , the small-signal resistance  $r$  is given by

$$r \equiv \frac{v_x}{i_x} = \frac{v_x}{i_e} = r_e$$

**6.35** Refer to Fig. P6.53. The transistor is biased at  $I_E = 0.33$  mA. Thus

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.33 \text{ mA}} = 75 \Omega$$

Replacing the BJT with its  $T$  model results in the following amplifier equivalent circuit.



The input resistance  $R_{in}$  can be found by inspection to be

$$R_{in} = r_e = 75 \Omega$$

To determine the voltage gain ( $v_o/v_i$ ) we first find  $i_e$ :

$$i_e = -\frac{v_i}{R_{sig} + r_e} = -\frac{v_i}{150 \Omega} = -\frac{v_i}{0.15 \text{ k}\Omega}$$

The output voltage  $v_o$  is given by

$$\begin{aligned} v_o &= -\alpha i_e (R_C \parallel R_L) \\ &= -0.99 i_e \times (12 \parallel 12) = -0.99 \times 6 i_e \\ &= -0.99 \times 6 \times \frac{-v_i}{0.15} \end{aligned}$$

Thus,

$$\frac{v_o}{v_i} = 39.6 \text{ V/V}$$

**6.36**

$$r_e = 50 \Omega = \frac{V_T}{I_E}$$

$$\Rightarrow I_E = 0.5 \text{ mA}$$

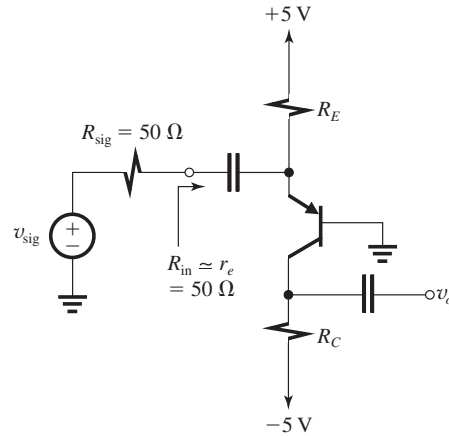
Thus,

$$\frac{5 - V_E}{R_E} = 0.5 \text{ mA}$$

where

$$V_E \simeq 0.7 \text{ V}$$

$$\Rightarrow R_E = 8.6 \text{ k}\Omega$$



To obtain maximum gain and the largest possible signal swing at the output for  $v_{eb}$  of 10 mV, we select a value for  $R_C$  that results in

$$V_C + |A_v| \times 0.01 \text{ V} = +0.4 \text{ V}$$

which is the highest allowable voltage at the collector while the transistor remains in the active region. Since

$$V_C = -5 + I_C R_C \simeq -5 + 0.5 R_C$$

then

$$-5 + 0.5 R_C + g_m R_C \times 0.01 = 0.4$$

Substituting  $g_m = 20$  mA/V results in

$$R_C = 7.7 \text{ k}\Omega$$

The overall voltage gain achieved is

$$\begin{aligned} \frac{v_o}{v_{sig}} &= \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_C \\ &= \frac{50}{50 + 50} \times 20 \times 7.7 \\ &= 77 \text{ V/V} \end{aligned}$$

**6.37** Substituting  $v_{gs} = V_{gs} \sin \omega t$  in Eq. (6.28),

$$\begin{aligned} i_D &= \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n (V_{GS} - V_t) V_{gs} \sin \omega t \\ &\quad + \frac{1}{2} k_n V_{gs}^2 \sin^2 \omega t \\ &= \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n (V_{GS} - V_t) V_{gs} \sin \omega t \\ &\quad + \frac{1}{2} k_n V_{gs}^2 \left( \frac{1}{2} - \frac{1}{2} \cos 2 \omega t \right) \end{aligned}$$

Second-harmonic distortion

$$\begin{aligned} &= \frac{\frac{1}{4} k_n V_{gs}^2}{k_n (V_{GS} - V_t) V_{gs}} \times 100 \\ &= \frac{1}{4} \frac{V_{gs}}{V_{OV}} \times 100 \quad \text{Q.E.D.} \end{aligned}$$

For  $V_{gs} = 10$  mV, to keep the second-harmonic distortion to less than 1%, the minimum overdrive voltage required is

$$V_{OV} = \frac{1}{4} \times \frac{0.01 \times 100}{1} = 0.25 \text{ V}$$

$$\mathbf{6.38} \text{ (a) } I_D = \frac{1}{2} k_n (V_{GS} - V_t^2)$$

$$= \frac{1}{2} \times 5(0.6 - 0.4)^2 = 0.1 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D R_D = 1.8 - 0.1 \times 10 = 0.8 \text{ V}$$

$$\text{(b) } g_m = k_n V_{OV} = 5 \times 0.2 = 1 \text{ mA/V}$$

$$\text{(c) } A_v = -g_m R_D = -1 \times 10 = -10 \text{ V/V}$$

$$\text{(d) } \lambda = 0.1 \text{ V}^{-1}, \quad V_A = \frac{1}{\lambda} = 10 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_v = -g_m (R_D \parallel r_o)$$

$$= -1(10 \parallel 100) = -9.1 \text{ V/V}$$

**6.39**

(a) The bias voltage  $V_{GS} = 0.75 \text{ V}$ . Thus

$$V_{OV} = 0.75 - 0.5 = 0.25 \text{ V}$$

$$g_m = k_n V_{OV} = k'_n \left( \frac{W}{L} \right) V_{OV}$$

$$= 0.4 \times 20 \times 0.25 = 2 \text{ mA/V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 0.4 \times 20 \times 0.25^2$$

$$= 0.25 \text{ mA}$$

$$r_o = \frac{V_A}{I_D} = \frac{5 \text{ V}}{0.25 \text{ mA}} = 20 \text{ k}\Omega$$

(b) The bias current  $I_D = 0.5 \text{ mA}$ . Thus,

$$0.5 = \frac{1}{2} k_n V_{OV}^2 = 0.5 \times 0.4 \times 20 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.35 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.35} = 2.9 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{5 \text{ V}}{0.5 \text{ mA}} = 10 \text{ k}\Omega$$

**6.40** Given  $\mu_n C_{ox} = 250 \mu\text{A/V}^2$ ,

$$V_t = 0.5 \text{ V},$$

$$L = 0.5 \mu\text{m}$$

For  $g_m = 2 \text{ mA/V}^2$  and  $I_D = 0.25 \text{ mA}$ ,

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \Rightarrow \frac{W}{L} = 32$$

$$\therefore W = 16 \mu\text{m}$$

$$V_{OV} = \frac{2I_D}{g_m} = 0.25 \text{ V}$$

$$\therefore V_{GS} = V_{OV} + V_t = 0.75 \text{ V}$$

**6.41** For the NMOS device:

$$I_D = 100 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} \times 400 \times \frac{10}{0.5} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1 \text{ mA}}{0.16} = 1.25 \text{ mA/V}$$

$$V_A = 5L = 5 \times 0.5 = 2.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{2.5}{0.1} = 25 \text{ k}\Omega$$

For the PMOS device:

$$I_D = 100 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} \times 100 \times \frac{10}{0.5} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.316 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.316} = 0.63 \text{ mA/V}$$

$$V_A = 6L = 6 \times 0.5 = 3 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{3}{0.1} = 30 \text{ k}\Omega$$

**6.42** (a) Using the exponential characteristic:

$$i_c = I_C e^{v_{be}/V_T} - I_C$$

$$\text{giving } \frac{i_c}{I_C} = e^{v_{be}/V_T} - 1$$

$v_{be}$ (mV)	$i_c/I_C$ Exponential	$i_c/I_C$ Small signal	Error (%)
+1	+0.041	+0.040	-2.4
-1	-0.039	-0.040	+2.4
+2	+0.083	+0.080	-3.6
-2	-0.077	-0.080	+3.9
+5	+0.221	+0.200	-9.7
-5	-0.181	-0.200	+10.3
+8	+0.377	+0.320	-15.2
-8	-0.274	-0.320	+16.8
+10	+0.492	+0.400	-18.7
-10	-0.330	-0.400	+21.3
+12	+0.616	+0.480	-22.1
-12	-0.381	-0.480	+25.9

(b) Using small-signal approximation:

$$i_c = g_m v_{be} = \frac{I_C}{V_T} \cdot v_{be}$$

$$\text{Thus, } \frac{i_c}{I_C} = \frac{v_{be}}{V_T}$$

See table on the previous page.

For signals at  $\pm 5$  mV, the error introduced by the small-signal approximation is 10%.

The error increases to above 20% for signals at  $\pm 10$  mV.

**6.43** At  $I_C = 0.5$  mA,

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C}$$

where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{100 + 1} = 0.99$$

$$r_e = \frac{0.99 \times 25 \text{ mV}}{0.5 \text{ mA}} \simeq 50 \Omega$$

At  $I_C = 50 \mu\text{A} = 0.05$  mA,

$$g_m = \frac{I_C}{V_T} = \frac{0.05}{0.025} = 2 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{2 \text{ mA/V}} = 50 \text{ k}\Omega$$

$$r_e = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25 \text{ mV}}{0.5 \text{ mA}} \simeq 500 \Omega$$

**6.44** For  $g_m = 30$  mA/V,

$$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 30 \times 0.025 = 0.75 \text{ mA}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta}{30 \text{ mA/V}}$$

For  $r_\pi \geq 3$  k $\Omega$ , we require

$$\beta \geq 90$$

That is,  $\beta_{\min} = 90$ .

$$\mathbf{6.45} \quad r_\pi = \frac{\beta}{g_m}$$

where

$$g_m = \frac{I_C}{V_T}$$

Nominally,  $g_m = 40$  mA/V. However,  $I_C$  varies by  $\pm 20\%$ , so  $g_m$  ranges from 32 mA/V to 48 mA/V. Thus

$$r_\pi = \frac{50 \text{ to } 150}{32 \text{ to } 48 \text{ mA/V}}$$

Thus, the extreme values of  $r_\pi$  are  $\frac{50}{48} = 1.04$  k $\Omega$

and  $\frac{150}{32} = 4.7$  k $\Omega$ .

**6.46**  $V_{CC} = 3$  V,  $V_C = 1$  V,  $R_C = 2$  k $\Omega$

$$I_C = \frac{3 - 1}{2} = 1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$v_{be} = 0.005 \sin \omega t$$

$$i_c = g_m v_{be} = 0.2 \sin \omega t, \text{ mA}$$

$$i_C(t) = I_C + i_c = 1 + 0.2 \sin \omega t, \text{ mA}$$

$$v_C(t) = V_{CC} - R_C i_C$$

$$= 3 - 2(1 + 0.2 \sin \omega t)$$

$$= 1 - 0.4 \sin \omega t, \text{ V}$$

$$i_B(t) = i_C(t)/\beta$$

$$= 0.01 + 0.002 \sin \omega t, \text{ mA}$$

$$A_v = \frac{v_c}{v_{be}} = -\frac{0.4}{0.005} = -80 \text{ V/V}$$

**6.47** Since  $\hat{V}_{be}$  is the maximum value for acceptable linearity, the largest signal at the collector will be obtained by designing for maximum gain magnitude. This in turn is achieved by biasing the transistor at the lowest  $V_{CE}$  consistent with the transistor remaining in the active mode at the negative peak of  $v_o$ .

Thus

$$V_{CE} - |A_v| \hat{V}_{be} = 0.3$$

where we have assumed  $V_{CE\text{sat}} = 0.3$  V. Since

$$V_{CE} = V_{CC} - I_C R_C$$

and

$$|A_v| = g_m R_C = \frac{I_C}{V_T} R_C$$

then

$$V_{CC} - I_C R_C - \frac{\hat{V}_{be}}{V_T} I_C R_C = 0.3 \text{ V}$$

which can be manipulated to yield

$$I_C R_C = \frac{V_{CC} - 0.3}{1 + \frac{\hat{V}_{be}}{V_T}} \quad (1)$$

Since the voltage gain is given by

$$A_v = -\frac{I_C R_C}{V_T}$$

then

$$A_v = -\frac{V_{CC} - 0.3}{V_T + \hat{V}_{be}}$$

For  $V_{CC} = 3\text{ V}$  and  $\hat{V}_{be} = 5\text{ mV}$ ,

$$I_C R_C = \frac{3 - 0.3}{1 + \frac{5}{25}} = 2.25\text{ V}$$

Thus,

$$V_{CE} = V_{CC} - I_C R_C = 3 - 2.25 = 0.75\text{ V}$$

$$\hat{V}_o = V_{CE} - 0.3 = 0.75 - 0.3 = 0.45\text{ V}$$

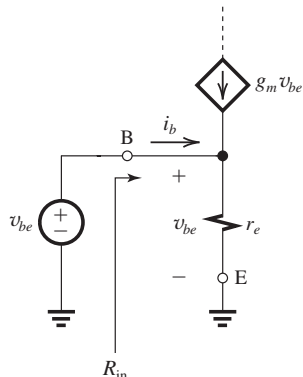
$$A_v = -\frac{3 - 0.3}{0.025 + 0.005} = -90\text{ V/V}$$

Check:

$$A_v = -g_m R_C = -\frac{I_C R_C}{V_T} = -\frac{2.25}{0.025} = -90\text{ V/V}$$

$$\hat{V}_o = |A_v| \times \hat{V}_{be} = 90 \times 5 = 450\text{ mV} = 0.45\text{ V}$$

6.48



$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be} \left( \frac{1}{r_e} - g_m \right)$$

Since

$$r_e = \frac{\alpha}{g_m}$$

$$i_b = v_{be} \left( \frac{g_m}{\alpha} - g_m \right)$$

$$= g_m v_{be} \frac{1 - \alpha}{\alpha}$$

$$= \frac{g_m v_{be}}{\beta}$$

$$R_{in} \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = r_{\pi} \quad \text{Q.E.D.}$$

6.49 Refer to Fig. 6.27.

$$i_c = \alpha i_e = \alpha \frac{v_{be}}{r_e} = \frac{\alpha}{r_e} v_{be}$$

$$= g_m v_{be} \quad \text{Q.E.D.}$$

6.50 The large-signal model of Fig. 4.5(d) is shown in Fig. 1.

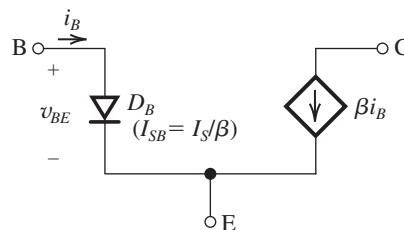


Figure 1

For  $v_{BE}$  undergoing an incremental change  $v_{be}$  from its equilibrium value of  $V_{BE}$ , the current  $i_B$  changes from  $I_B$  by an increment  $i_b$ , which is related to  $v_{be}$  by the incremental resistance of  $D_B$  at the bias current  $I_B$ . This resistance is given by  $V_T/I_B$ , which is  $r_{\pi}$ .

The collector current  $\beta i_B$  changes from  $\beta I_B$  to  $\beta(I_B + i_b)$ . The incremental changes around the equilibrium or bias point are related to each other by the circuit shown in Fig. 2, which is the hybrid- $\pi$  model of Fig. 7.25(b). Q.E.D.

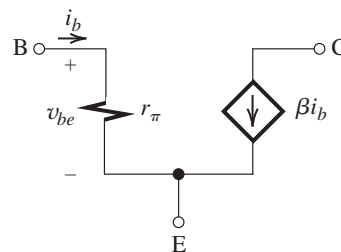


Figure 2

6.51 The large-signal  $T$  model of Fig. 4.5(b) is shown below in Fig. 1.

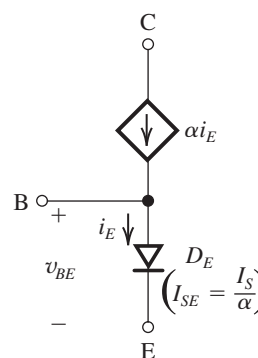


Figure 1

If  $i_E$  undergoes an incremental change  $i_e$  from its equilibrium or bias value  $I_E$ , the voltage  $v_{BE}$  will correspondingly change by an incremental amount  $v_{be}$  (from its equilibrium or bias value  $V_{BE}$ ), which is related to  $i_e$  by the incremental

resistance of diode  $D_E$ . The latter is equal to  $V_T/I_E$ , which is  $r_e$ .

The incremental change  $i_e$  in  $i_E$  gives rise to an incremental change  $\alpha i_e$  in the current of the controlled source.

The incremental quantities can be related by the equivalent circuit model shown in Fig. 2, which is the small-signal  $T$  model of Fig. 6.27(b). Q.E.D.

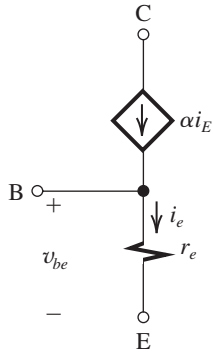


Figure 2

**6.52**  $v_{ce} = |A_v|v_{be}$

$|A_v| = g_m R_C = 50 \times 2 = 100 \text{ V/V}$

For  $v_{ce}$  being 1 V peak to peak,

$v_{be} = \frac{1 \text{ V}}{100} = 0.01 \text{ V peak to peak}$

$i_b = \frac{v_{be}}{r_\pi}$

where

$r_\pi = \frac{\beta}{g_m} = \frac{100}{50} = 2 \text{ k}\Omega$

Thus,

$i_b = \frac{0.01 \text{ V}}{2 \text{ k}\Omega} = 0.005 \text{ mA peak to peak}$

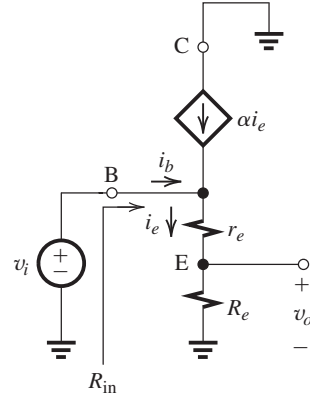
**6.53** Refer to Fig. P6.53. Replacing the BJT with the  $T$  model of Fig. 6.27(b) results in the following amplifier equivalent circuit:

$R_{in} \equiv \frac{v_i}{i_b} = \frac{v_i}{(1 - \alpha)i_e}$

From the circuit we see that

$i_e = \frac{v_i}{r_e + R_e}$

Thus,



$R_{in} = \frac{r_e + R_e}{1 - \alpha}$

But

$1 - \alpha = \frac{1}{\beta + 1}$

Thus,

$R_{in} = (\beta + 1)(r_e + R_e)$  Q.E.D.

From the equivalent circuit, we see that  $v_o$  and  $v_i$  are related by the ratio of the voltage divider formed by  $r_e$  and  $R_e$ :

$\frac{v_o}{v_i} = \frac{R_e}{R_e + r_e}$  Q.E.D.

**6.54** Refer to Fig. P6.54.

$\alpha = \frac{\beta}{\beta + 1} = \frac{200}{201} = 0.995$

$I_C = \alpha \times I_E = 0.995 \times 10 = 9.95 \text{ mA}$

$V_C = I_C R_C = 9.95 \times 0.1 \text{ k}\Omega = 0.995 \text{ V} \simeq 1 \text{ V}$

Replacing the BJT with its hybrid- $\pi$  model results in the circuit shown below.

$g_m = \frac{I_C}{V_T} \simeq \frac{10 \text{ mA}}{0.025 \text{ V}} = 400 \text{ mA/V}$

$r_\pi = \frac{\beta}{g_m} = \frac{200}{400} = 0.5 \text{ k}\Omega$

$R_{ib} = r_\pi = 0.5 \text{ k}\Omega$

$R_{in} = 10 \text{ k}\Omega \parallel 0.5 \text{ k}\Omega = 0.476 \text{ k}\Omega$

$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{0.476}{0.476 + 1} = 0.322 \text{ V/V}$

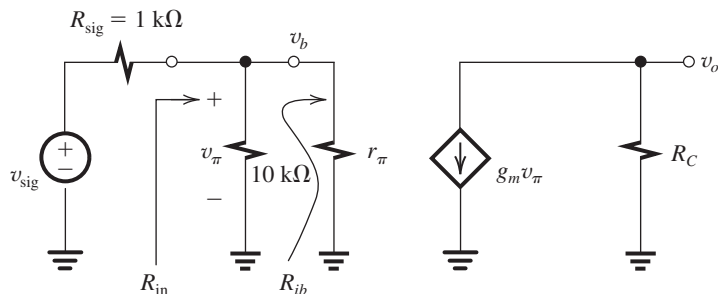
$\frac{v_o}{v_\pi} = -g_m R_C = -400 \times 0.1 = -40 \text{ V/V}$

$\frac{v_o}{v_{sig}} = -40 \times 0.322 = -12.9 \text{ V/V}$

For

$v_o = \pm 0.4 \text{ V/V}$

This figure belongs to Problem 6.54.



$$v_b = v_\pi = \frac{\pm 0.4}{-40} = \mp 0.01 \text{ V} = \mp 10 \text{ mV}$$

$$v_{\text{sig}} = \frac{\pm 0.4}{-12.9} = \mp 31 \text{ mV}$$

**6.55** The largest possible voltage gain is obtained when  $R_L \rightarrow \infty$ , in which case

$$\begin{aligned} \frac{v_o}{v_{\text{sig}}} &= -g_m r_o = -\frac{I_C}{V_T} \frac{V_A}{I_C} \\ &= -\frac{V_A}{V_T} \end{aligned}$$

$$\begin{aligned} \text{For } V_A = 25 \text{ V, } \frac{v_o}{v_{\text{sig}}} &= -\frac{25}{0.025} \\ &= -1000 \text{ V/V} \end{aligned}$$

$$\begin{aligned} \text{For } V_A = 125 \text{ V, } \frac{v_o}{v_{\text{sig}}} &= -\frac{125}{0.025} \\ &= -5000 \text{ V/V} \end{aligned}$$

**6.56** Refer to Fig. 6.31(a):

$$R_{\text{in}} \simeq r_e$$

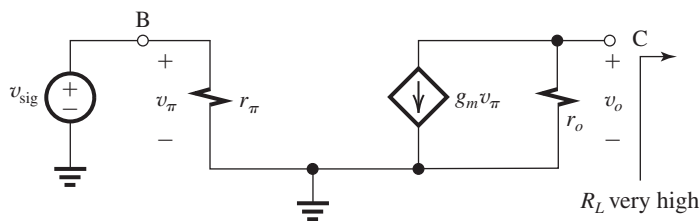
To obtain an input resistance of 75  $\Omega$ ,

$$r_e = 75 \Omega = \frac{V_T}{I_E}$$

Thus,

$$I_E = \frac{25 \text{ mV}}{75 \Omega} = 0.33 \text{ mA}$$

This figure belongs to Problem 6.55.



This current is obtained by raising  $R_E$  to the value found from

$$I_E = \frac{10 - 0.7}{R_E} = 0.33 \text{ mA}$$

$$\Rightarrow R_E = 28.2 \text{ k}\Omega$$

$$\text{Thus, } n = \frac{28.2}{10} = 2.82.$$

The collector resistor becomes

$$2.82 \times 5 = 14.1 \text{ k}\Omega$$

Note that the dc voltage at the collector remains unchanged. The voltage gain now becomes

$$\frac{v_o}{v_i} = \frac{\alpha R_C}{r_e} = \frac{0.99 \times 14.1}{0.075} = 186 \text{ V/V}$$

**6.57** Refer to Fig. P6.57. Since  $\beta$  is very large, the dc base current can be neglected. Thus the dc voltage at the base is determined by the voltage divider,

$$V_B = 5 \frac{100}{100 + 100} = 2.5 \text{ V}$$

and the dc voltage at the emitter will be

$$V_E = V_B - 0.7 = 1.8 \text{ V}$$

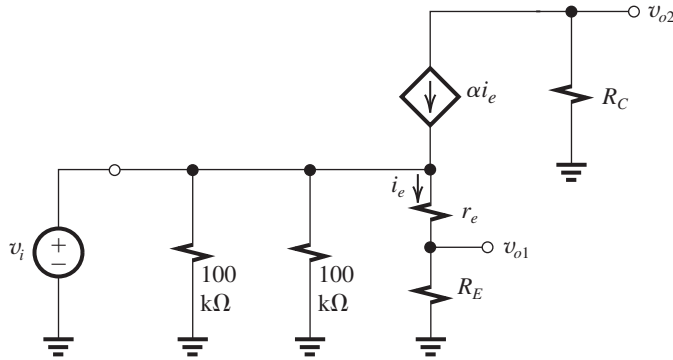
The dc emitter current can now be found as

$$I_E = \frac{V_E}{R_E} = \frac{1.8}{3.6} = 0.5 \text{ mA}$$

and

$$I_C \simeq I_E = 0.5 \text{ mA}$$

This figure belongs to Problem 6.57.



Replacing the BJT with the  $T$  model of Fig. 6.27(b) results in the equivalent circuit model for the amplifier (see figure above).

$$i_e = \frac{v_i}{R_E + r_e}$$

$$v_{o1} = i_e R_E = v_i \frac{R_E}{R_E + r_e}$$

$$\frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e} \quad \text{Q.E.D.}$$

$$v_{o2} = -\alpha i_e R_C = -\alpha \frac{v_i}{R_E + r_e} R_C$$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{R_E + r_e} \quad \text{Q.E.D.}$$

For  $\alpha \simeq 1$ ,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

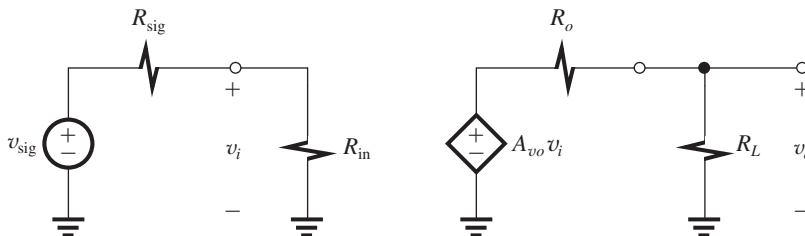
$$\frac{v_{o1}}{v_i} = \frac{3.6}{3.6 + 0.05} = 0.986 \text{ V/V}$$

$$\frac{v_{o2}}{v_i} = -\frac{3.3}{3.6 + 0.05} = -0.904 \text{ V/V}$$

If  $v_{o1}$  is connected to ground,  $R_E$  will in effect be short-circuited at signal frequencies, and  $v_{o2}/v_i$  will become

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{r_e} = -\frac{3.3}{0.05} = -66 \text{ V/V}$$

This figure belongs to Problem 6.58.



$$6.58 \text{ (a)} \quad \frac{R_{in}}{R_{in} + R_{sig}} = 0.95$$

$$\frac{R_{in}}{R_{in} + 100} = 0.95$$

$$\Rightarrow R_{in} = 1.9 \text{ M}\Omega$$

(b) With  $R_L = 2 \text{ k}\Omega$ ,

$$v_o = A_{vo} v_i \frac{2}{2 + R_o}$$

With  $R_L = 1 \text{ k}\Omega$ ,

$$v_o = A_{vo} v_i \frac{1}{1 + R_o}$$

Thus the change in  $v_o$  is

$$\Delta v_o = A_{vo} v_i \left[ \frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right]$$

To limit this change to 5% of the value with  $R_L = 2 \text{ k}\Omega$ , we require

$$\left[ \frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right] / \left( \frac{2}{2 + R_o} \right) = 0.05$$

$$\Rightarrow R_o = \frac{1}{9} \text{ k}\Omega = 111 \Omega$$

$$(c) \quad G_v = 10 = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

$$= \frac{1.9}{1.9 + 0.1} \times A_{vo} \times \frac{2}{2 + 0.111}$$

$$\Rightarrow A_{vo} = 11.1 \text{ V/V}$$

The values found about are limit values; that is, we require

$$R_{in} \geq 1.9 \text{ M}\Omega$$

$$R_o \leq 111 \Omega$$

$$A_{vo} \geq 11.1 \text{ V/V}$$

**6.59**

Refer to Fig. 1.

$$A_{vo} = \left. \frac{v_o}{v_i} \right|_{R_L = \infty} = G_m R_o$$

$$= 10 \text{ mA/V} \times 100 \text{ k}\Omega = 1000 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i}$$

$$= \frac{R_{in}}{R_{in} + R_{sig}} \times G_m (R_L \parallel R_o)$$

$$= \frac{100}{100 + 100} \times 10 \times (100 \parallel 100)$$

$$= 0.5 \times 10 \times 50 = 250 \text{ V/V}$$

**6.60**  $R_{in} = \infty$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$320 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.4 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.32}{0.4} = 1.6 \text{ mA/V}$$

$$A_{vo} = -g_m R_D = -1.6 \times 10 = -16 \text{ V/V}$$

$$R_o = R_D = 10 \text{ k}\Omega$$

$$G_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= -16 \times \frac{10}{10 + 10} = -8 \text{ V/V}$$

$$\text{Peak value of } v_{sig} = \frac{0.2 \text{ V}}{8} = 25 \text{ mV.}$$

**6.61** (a) See figure below.

$$(b) g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.2} = 3 \text{ mA/V}$$

$$R_{D1} = R_{D2} = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

$$G_v = \frac{v_{gs2}}{v_{gs1}} \times \frac{v_o}{v_{gs2}}$$

$$= -g_{m1} R_{D1} \times -g_{m2} (R_{D2} \parallel R_L)$$

$$= 3 \times 10 \times 3 \times (10 \parallel 10)$$

$$= 450 \text{ V/V}$$

This figure belongs to Problem 6.59.

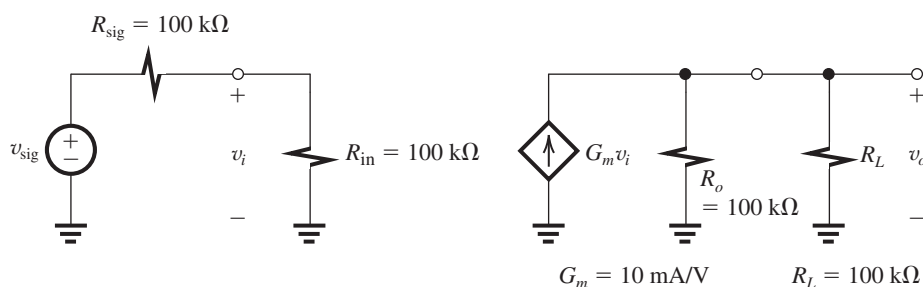
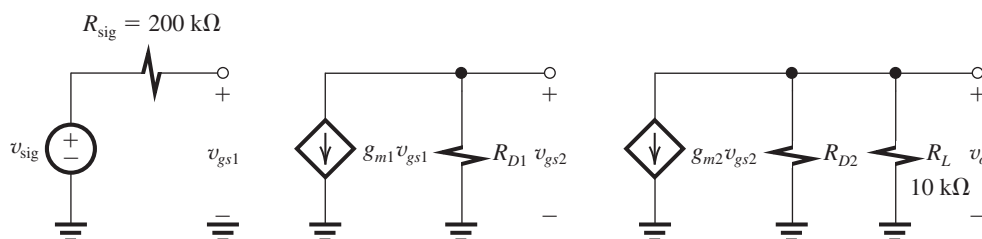


Figure 1

This figure belongs to Problem 6.61.





$$6.62 \quad g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \text{ k}\Omega$$

$$R_o = R_C = 10 \text{ k}\Omega$$

$$A_{vo} = -g_m R_C = -20 \times 10 = -200 \text{ V/V}$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -200 \times \frac{10}{10 + 10}$$

$$= -100 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v$$

$$= \frac{5}{5 + 10} \times -100$$

$$= -33.3 \text{ V/V}$$

For  $\hat{v}_\pi = 5 \text{ mV}$ ,  $\hat{v}_{sig}$  can be found from

$$\hat{v}_\pi = \hat{v}_{sig} \times \frac{R_{in}}{R_{in} + R_{sig}} = \hat{v}_{sig} \times \frac{5}{5 + 10}$$

$$\Rightarrow \hat{v}_{sig} = 15 \text{ mV}$$

Correspondingly,  $\hat{v}_o$  will be

$$\hat{v}_o = G_v \hat{v}_{sig}$$

$$= 15 \times 33.3 = 500 \text{ mV} = 0.5 \text{ V}$$

**6.63** Including  $R_s$  reduced the gain by a factor of 2, thus

$$1 + g_m R_s = 2$$

$$\Rightarrow g_m = \frac{1}{R_s} = \frac{1}{0.2} = 5 \text{ mA/V}$$

The gain without  $R_s$  is  $-20 \text{ V/V}$ . Thus,

$$-g_m R_D = -20 \Rightarrow R_D = \frac{20}{5} = 4 \text{ k}\Omega. \text{ To obtain a gain of } -16 \text{ V/V, we write}$$

$$16 = \frac{20}{1 + g_m R_s} = \frac{20}{1 + 5R_s}$$

$$\Rightarrow R_s = 50 \Omega$$

$$6.64 \quad g_m = \frac{I_C}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 50 \Omega$$

$$R_{in} = (\beta + 1)(r_e + R_e)$$

$$= 101(50 + 200) = 25.25 \text{ k}\Omega$$

$$A_{vo} = -\frac{\alpha R_C}{r_e + R_e} = -\frac{0.99 \times 15}{0.25}$$

$$\simeq -60 \text{ V/V}$$

$$R_o = R_C = 15 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= -60 \times \frac{15}{15 + 15} = -30 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$$

$$= \frac{25.25}{25.25 + 10} \times (-30) = -21.5 \text{ V/V}$$

$$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_b = \hat{v}_\pi \frac{r_e + R_e}{r_e}$$

$$\Rightarrow \hat{v}_b = 5 \times \frac{50 + 200}{50} = 25 \text{ mV}$$

$$\hat{v}_{sig} = \hat{v}_b \frac{R_{sig} + R_{in}}{R_{in}}$$

$$= 25 \times \frac{10 + 25.25}{25.25} = 35 \text{ mV}$$

$$\hat{v}_o = \hat{v}_{sig} \times |G_v|$$

$$= 21.5 \times 35 \simeq 753 \text{ mV}$$

**6.65** Adding a resistance of  $100 \Omega$  in series with the  $100\text{-}\Omega$   $R_{sig}$  changes the input voltage divider ratio from

$$\frac{1/g_m}{(1/g_m) + 100} \text{ to } \frac{1/g_m}{1/g_m + 200}$$

Since this has changed the overall voltage gain from 12 to 10, then

$$\frac{12}{10} = \frac{(1/g_m) + 200}{(1/g_m) + 100}, \text{ where } g_m \text{ is in A/V}$$

$$\Rightarrow g_m = \frac{0.2}{80} \text{ A/V} = 2.5 \text{ mA/V}$$

For  $I_D = 0.25 \text{ mA}$

$$2.5 = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

**6.66** Refer to the circuit in Fig. P6.66. Since  $R_{sig} \gg r_e$ , most of  $i_{sig}$  flows into the emitter of the BJT. Thus

$$i_e \simeq i_{sig}$$

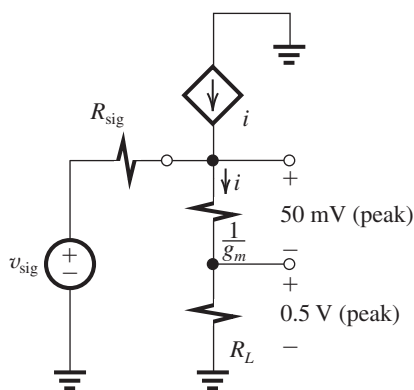
and

$$i_c = \alpha i_e \simeq i_{sig}$$

Thus,

$$v_o = i_c R_C = i_{sig} R_C$$

## 6.67



From the figure above, we have

$$\begin{aligned} \frac{1}{g_m} &= 0.1 \times R_L \\ &= 0.1 \times 2 = 0.2 \text{ k}\Omega \end{aligned}$$

$$g_m = 5 \text{ mA/V}$$

$$g_m = \sqrt{2k_n I_D}$$

$$5 = \sqrt{2 \times 5 \times I_D}$$

$$I_D = 2.5 \text{ mA}$$

At the peak of the sine wave,

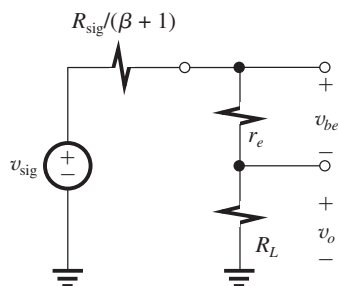
$$i_d = \frac{0.5 \text{ V}}{2 \text{ k}\Omega} = 0.25 \text{ mA, thus}$$

$$i_{D\max} = I_D + 0.25 = 2.75 \text{ mA}$$

$$i_{D\min} = I_D - 0.25 = 2.25 \text{ mA}$$

$$\hat{v}_{\text{sig}} = \hat{v}_{gs} + \hat{v}_o = 0.05 + 0.5 = 0.55 \text{ V}$$

## 6.68



$$\hat{v}_o = 0.5 \text{ V}$$

$$R_L = 2 \text{ k}\Omega$$

$$\hat{v}_{be} = 5 \text{ mV}$$

From the figure above we see that

$$\frac{r_e}{R_L} = \frac{5 \text{ mV}}{500 \text{ mV}}$$

$$\Rightarrow r_e = \frac{R_L}{100} = 20 \Omega$$

$$I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{20 \Omega} = 1.25 \text{ mA}$$

At the peak of the output sine wave, we have

$$\hat{i}_e = \frac{\hat{v}_o}{R_L} = \frac{0.5}{2} = 0.25 \text{ mA}$$

Thus,

$$i_{E\max} = 1.25 + 0.25 = 1.5 \text{ mA}$$

and

$$i_{E\min} = 1.25 - 0.25 = 1.0 \text{ mA}$$

From the figure, we have

$$\begin{aligned} G_v &= \frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_L + r_e + \frac{R_{\text{sig}}}{\beta + 1}} \\ &= \frac{2}{2 + 0.02 + \frac{200}{101}} = 0.5 \text{ V/V} \end{aligned}$$

Thus,

$$\hat{v}_{\text{sig}} = \frac{\hat{v}_o}{G_v} = \frac{0.5 \text{ V}}{0.5 \text{ V/V}} = 1 \text{ V}$$

**6.69** Refer to Fig. 6.48(b). To find  $G_{vo}$ , we remove  $R_L$ . The result is  $v_o = v_{\text{sig}}$  and

$$G_{vo} = 1 \text{ V/V}$$

To obtain  $R_{\text{out}}$ , we write by inspection

$$R_{\text{out}} = r_{e2} + \frac{r_{e1} + R_{\text{sig}}/(\beta_1 + 1)}{\beta_2 + 1}$$

We can find  $G_v$  as

$$G_v = G_{vo} \frac{R_L}{R_L + R_{\text{out}}}$$

To obtain numerical values, we use  $I_{C2} = 10 \text{ mA}$ .

Thus,  $I_{E2} \simeq I_{C2} = 10 \text{ mA}$ , and

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5 \Omega$$

$$I_{E1} = \frac{I_{E2}}{\beta + 1} = \frac{10}{101} \simeq 0.1 \text{ mA}$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25}{0.1} = 250 \Omega$$

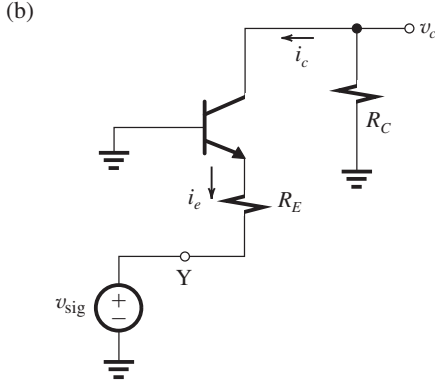
$$\begin{aligned} R_{\text{out}} &= 2.5 + \frac{250 + \frac{10^6}{101}}{101} \\ &= 105 \Omega \end{aligned}$$

$$G_v = 1 \times \frac{1000}{1000 + 105} = 0.9 \text{ V/V}$$

**6.70** (a) Refer to Fig. P6.70.

$$\frac{v_c}{v_{\text{sig}}} = \frac{-i_c R_C}{i_b R_B + i_e (r_e + R_E)}$$

$$\begin{aligned}
 &= -\frac{i_c}{i_b} \frac{R_C}{R_B + \left(\frac{i_e}{i_b}\right)(r_e + R_E)} \\
 &= -\beta \frac{R_C}{R_B + (\beta + 1)(r_e + R_E)} \\
 \frac{v_e}{v_{\text{sig}}} &= \frac{i_e R_E}{i_b R_B + i_e(r_e + R_E)} \\
 &= \frac{R_E}{\frac{R_B}{\beta + 1} + r_e + R_E}
 \end{aligned}$$

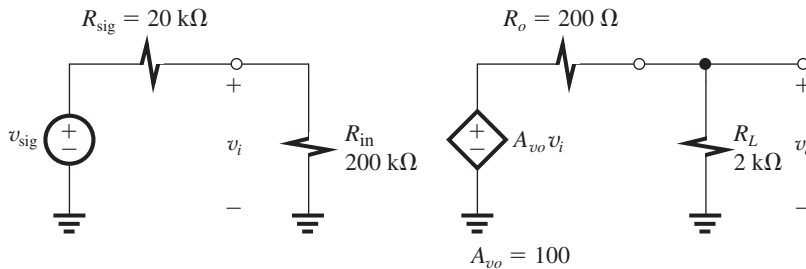


$$\begin{aligned}
 i_e &= -\frac{v_{\text{sig}}}{r_e + R_E} \\
 v_c &= -i_c R_C = -\alpha i_e R_C \\
 \frac{v_c}{v_{\text{sig}}} &= \frac{-i_c R_C}{i_e(r_e + R_E)} = \alpha \frac{R_C}{r_e + R_E}
 \end{aligned}$$

6.71 See figure below.

$$\begin{aligned}
 G_v &= \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_{vo} \frac{R_L}{R_L + R_o} \\
 &= \frac{100}{100 + 20} \times 100 \times \frac{2}{2 + 0.1} \\
 &= 79.4 \text{ V/V} \\
 i_o &= \frac{v_o}{R_L}
 \end{aligned}$$

This figure belongs to Problem 6.71.



$$\begin{aligned}
 i_i &= \frac{v_{\text{sig}}}{R_{\text{sig}} + R_{\text{in}}} \\
 \frac{i_o}{i_i} &= \frac{v_o}{v_{\text{sig}}} \frac{R_{\text{sig}} + R_{\text{in}}}{R_L} \\
 &= G_v \frac{R_{\text{sig}} + R_{\text{in}}}{R_L} \\
 &= 79.4 \times \frac{20 + 100}{2} = 4762 \text{ A/A}
 \end{aligned}$$

6.72 See figure on the next page.

$$G_{vo} = \left. \frac{v_o}{v_{\text{sig}}} \right|_{R_L = \infty}$$

Now, setting  $R_L = \infty$  in the equivalent circuit in Fig. 1(b), we can determine  $G_{vo}$  from

$$G_{vo} = \left. \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \right|_{R_L = \infty} A_{vo}$$

Denoting  $R_{\text{in}}$  with  $R_L = \infty$  as  $R_i$ , we can express  $G_{vo}$  as

$$G_{vo} = \frac{R_i}{R_i + R_{\text{sig}}} A_{vo} \quad \text{Q.E.D.}$$

From the equivalent circuit in Fig. 1(a) on the next page, the overall voltage  $G_v$  can be obtained as

$$G_v = G_{vo} \frac{R_L}{R_L + R_{\text{out}}} \quad \text{Q.E.D.}$$

6.73 Refer to Fig. P6.73. To determine  $R_{\text{in}}$ , we simplify the circuit as shown in Fig. 1 on the next page, where

$$R_{\text{in}} \equiv \frac{v_i}{i_i} = R_1 \parallel R'_{\text{in}}, \quad \text{where } R'_{\text{in}} \equiv \frac{v_i}{i_f}$$

$$v_i = i_f R_f + (i_f - g_m v_i)(R_2 \parallel R_L)$$

$$\text{Thus, } v_i [1 + g_m (R_2 \parallel R_L)] = i_f [R_f + (R_2 \parallel R_L)]$$

$$R'_{\text{in}} \equiv \frac{v_i}{i_f} = \frac{R_f + (R_2 \parallel R_L)}{1 + g_m (R_2 \parallel R_L)}$$

and

$$\begin{aligned}
 R_{\text{in}} &= R_1 \parallel R'_{\text{in}} \\
 &= R_1 \parallel \left[ \frac{R_f + (R_2 \parallel R_L)}{1 + g_m (R_2 \parallel R_L)} \right] \quad \text{Q.E.D.}
 \end{aligned}$$

This figure belongs to Problem 6.72.

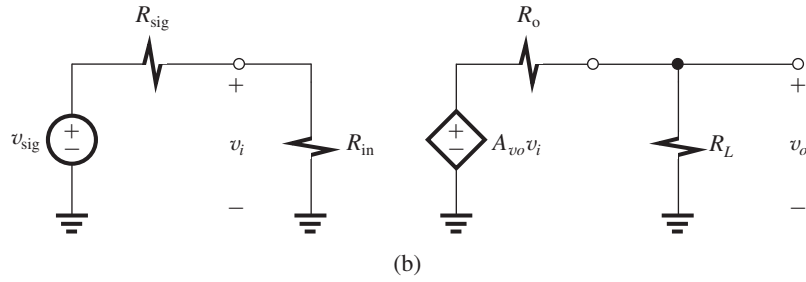
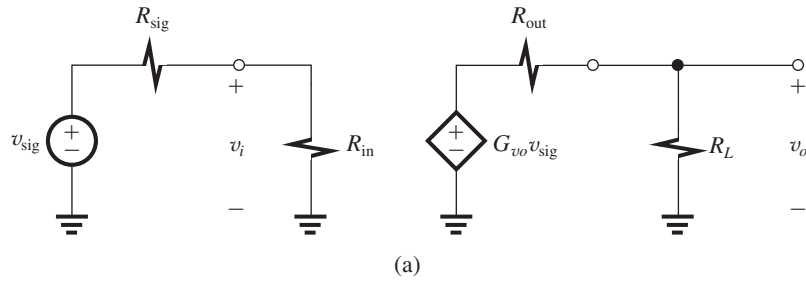


Figure 1

This figure belongs to Problem 6.73.

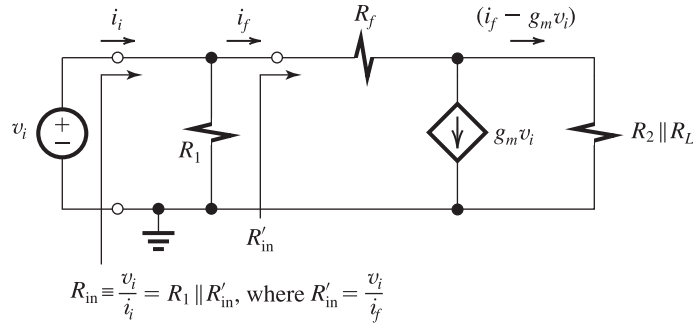


Figure 1

To determine  $A_{vo}$ , we open-circuit  $R_L$  and use the circuit in Fig. 2, where

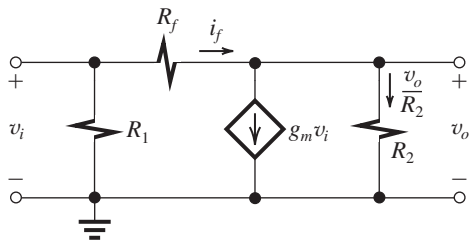


Figure 2

$$i_f = g_m v_i + \frac{v_o}{R_2}$$

$$v_i = i_f R_f + v_o = \left( g_m v_i + \frac{v_o}{R_2} \right) R_f + v_o$$

$$v_i (1 - g_m R_f) = v_o \left( 1 + \frac{R_f}{R_2} \right)$$

Thus,

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{1 - g_m R_f}{1 + \frac{R_f}{R_2}}$$

which can be manipulated to the form

$$A_{vo} = -g_m R_2 \frac{1 - 1/g_m R_f}{1 + (R_2/R_f)} \quad \text{Q.E.D.}$$

Finally, to obtain  $R_o$  we short-circuit  $v_i$  in the circuit of Fig. P6.73. This will disable the controlled source  $g_m v_i$ . Thus, looking between the output terminals (behind  $R_L$ ), we see  $R_2$  in parallel with  $R_f$ ,

$$R_o = R_2 \parallel R_f \quad \text{Q.E.D.}$$

For  $R_1 = 100 \text{ k}\Omega$ ,  $R_f = 1 \text{ M}\Omega$ ,  $g_m = 100 \text{ mA/V}$

$R_2 = 100 \text{ }\Omega$  and  $R_L = 1 \text{ k}\Omega$

$$R_{in} = 100 \parallel \frac{1000 + (0.1 \parallel 1)}{1 + 100(0.1 \parallel 1)} = 100 \parallel 99.1$$

$$= 49.8 \text{ k}\Omega$$

Without  $R_f$  present (i.e.,  $R_f = \infty$ ),  $R_{in} = 100 \text{ k}\Omega$ .

$$A_{vo} = -100 \times 0.1 \frac{1 - (1/100 \times 1000)}{1 + \frac{0.1}{1000}}$$

$$\simeq -10 \text{ V/V}$$

Without  $R_f$ ,  $A_{vo} = -10 \text{ V/V}$  and

$$R_o = 0.1 \parallel 1000 \simeq 0.1 \text{ k}\Omega = 100 \Omega$$

Without  $R_f$ ,  $R_o = 100 \Omega$ .

Thus the only parameter that is significantly affected by the presence of  $R_f$  is  $R_{in}$ , which is reduced by a factor of 2!

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

With  $R_f$ ,

$$G_v = \frac{49.8}{49.8 + 100} \times -10 \times \frac{1}{1 + 0.1}$$

$$= -3 \text{ V/V}$$

Without  $R_f$ ,

$$G_v = \frac{100}{100 + 100} \times -10 \times \frac{1}{1 + 0.1} = -4.5 \text{ V/V}$$

**6.74**  $R_{sig} = 1 \text{ M}\Omega$ ,  $R_L = 10 \text{ k}\Omega$

$$g_m = 2 \text{ mA/V}, R_D = 10 \text{ k}\Omega$$

$$G_v = -g_m(R_D \parallel R_L)$$

$$= -2(10 \parallel 10) = -10 \text{ V/V}$$

**6.75**  $R_D = 2R_L = 30 \text{ k}\Omega$

$$V_{OV} = 0.25 \text{ V}$$

$$G_v = -g_m(R_D \parallel R_L)$$

$$-10 = -g_m(30 \parallel 15)$$

$$\Rightarrow g_m = 1 \text{ mA/V}$$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$1 = \frac{2 \times I_D}{0.25}$$

$$\Rightarrow I_D = 0.125 \text{ mA} = 125 \mu\text{A}$$

If  $R_D$  is reduced to  $15 \text{ k}\Omega$ ,

$$G_v = -g_m(R_D \parallel R_L)$$

$$= -1 \times (15 \parallel 15) = -7.5 \text{ V/V}$$

$$\mathbf{6.76} \quad |G_v| = \frac{R'_L}{(R_{sig}/\beta) + (1/g_m)}$$

$$R'_L = 10 \text{ k}\Omega, R_{sig} = 10 \text{ k}\Omega, g_m = \frac{I_C}{V_T}$$

$$= \frac{1}{0.025} = 40 \text{ mA/V}$$

Nominal  $\beta = 100$

$$\text{(a) Nominal } |G_v| = \frac{10}{(10/100) + 0.025}$$

$$= 80 \text{ V/V}$$

$$\text{(b) } \beta = 50, |G_v| = \frac{10}{(10/50) + 0.025}$$

$$= 44.4 \text{ V/V}$$

$$\beta = 150, |G_v| = \frac{10}{(10/150) + 0.025}$$

$$= 109.1 \text{ V/V}$$

Thus,  $|G_v|$  ranges from  $44.4 \text{ V/V}$  to  $109.1 \text{ V/V}$ .

(c) For  $|G_v|$  to be within  $\pm 20\%$  of nominal (i.e., ranging between  $64 \text{ V/V}$  and  $96 \text{ V/V}$ ), the corresponding allowable range of  $\beta$  can be found as follows:

$$64 = \frac{10}{(10/\beta_{\min}) + 0.025}$$

$$\Rightarrow \beta_{\min} = 76.2$$

$$96 = \frac{10}{(10/\beta_{\max}) + 0.025}$$

$$\Rightarrow \beta_{\max} = 126.3$$

(d) By varying  $I_C$ , we vary the term  $1/g_m$  in the denominator of the  $|G_v|$  expression. If  $\beta$  varies in the range 50 to 150 and we wish to keep  $|G_v|$  within  $\pm 20\%$  of a new nominal value of  $|G_v|$  given by

$$|G_v|_{\text{nominal}} = \frac{10}{(10/100) + (1/g_m)}$$

then

$$0.8 |G_v|_{\text{nominal}} = \frac{10}{(10/50) + (1/g_m)}$$

That is,

$$\frac{8}{0.1 + (1/g_m)} = \frac{10}{0.2 + (1/g_m)}$$

$$\Rightarrow \frac{1}{g_m} = 0.3 \text{ or } g_m = 3.33 \text{ mA/V}$$

$$|G_v|_{\text{nominal}} = \frac{10}{0.1 + 0.3} = 25 \text{ V/V}$$

$$|G_v|_{\min} = \frac{10}{0.2 + 0.3}$$

$$= 20 \text{ V/V } (-20\% \text{ of nominal})$$

We need to check the value obtained for  $\beta = 150$ ,

$$|G_v|_{\max} = \frac{10}{10/150 + 0.3} = 27.3 \text{ V/V}$$

which is less than the allowable value of  $1.2 |G_v|_{\text{nominal}} = 30 \text{ V/V}$ . Thus, the new bias current is

$$I_C = g_m \times V_T = 3.33 \times 0.025 = 0.083 \text{ mA}$$

$$|G_v|_{\text{nominal}} = 25 \text{ V/V}$$

**6.77** (a) See figure below.

$$(b) R_{C1} = R_{C2} = 10 \text{ k}\Omega \quad R_{\text{sig}} = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$\frac{v_{\pi 1}}{v_{\text{sig}}} = \frac{r_{\pi 1}}{r_{\pi 1} + R_{\text{sig}}} = \frac{10}{10 + 10} = 0.5 \text{ V/V}$$

$$\frac{v_{\pi 2}}{v_{\pi 1}} = -g_{m1}(R_{C1} \parallel r_{\pi 2}) = -10(10 \parallel 10)$$

$$= -50 \text{ V/V}$$

$$\frac{v_o}{v_{\pi 2}} = -g_{m2}(R_{C2} \parallel R_L)$$

$$= -10(10 \parallel 10) = -50 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{v_o}{v_{\pi 2}} \times \frac{v_{\pi 2}}{v_{\pi 1}} \times \frac{v_{\pi 1}}{v_{\text{sig}}}$$

$$= -50 \times -50 \times 0.5$$

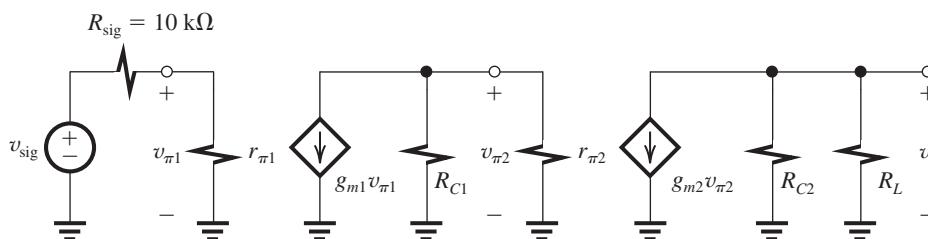
$$= 1250 \text{ V/V}$$

$$\mathbf{6.78} \quad g_m|_{\text{effective}} = \frac{g_m}{1 + g_m R_s}$$

$$2 = \frac{5}{1 + 5R_s}$$

$$\Rightarrow R_s = 0.3 \text{ k}\Omega = 300 \Omega$$

This figure belongs to Problem 6.77.



**6.79** The gain magnitude is reduced by a factor of  $(1 + g_m R_s)$ . Thus, to reduce the gain from  $-10 \text{ V/V}$  to  $-5 \text{ V/V}$ , we write

$$2 = 1 + g_m R_s$$

$$\Rightarrow R_s = \frac{1}{g_m} = \frac{1}{2} = 0.5 \text{ k}\Omega$$

$$\mathbf{6.80} \quad R_{\text{in}} = (\beta + 1)(r_e + R_e)$$

$$15 = 75(r_e + R_e)$$

$$r_e + R_e = \frac{15 \text{ k}\Omega}{75} = 200 \Omega$$

$$\hat{v}_\pi = \hat{v}_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \frac{r_e}{r_e + R_e}$$

$$5 = 150 \times \frac{15}{15 + 30} \left( \frac{r_e}{r_e + R_e} \right)$$

$$\Rightarrow \frac{r_e}{r_e + R_e} = 0.1$$

But  $r_e + R_e = 200 \Omega$ , thus

$$r_e = 20 \Omega$$

which requires a bias current  $I_E$  of

$$I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{20 \Omega} = 1.25 \text{ mA}$$

$$I_C \simeq I_E = 1.25 \text{ mA}$$

$$R_e = 180 \Omega$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$\times \frac{-\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{15}{15 + 30} \times \frac{-0.99 \times 6}{0.2}$$

$$\simeq -10 \text{ V/V}$$

$$\hat{v}_0 = 0.15 \times |G_v| = 1.5 \text{ V}$$

**6.81** Using Eq. (6.113), we have

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{\text{sig}} + (\beta + 1)(r_e + R_e)}$$

$$\simeq -\frac{R_C \parallel R_L}{(R_{\text{sig}}/\beta) + (r_e + R_e)}$$

$$|G_v| = \frac{10}{(10/\beta) + 0.025 + R_e}$$

Without  $R_e$ ,

$$|G_v| = \frac{10}{(10/\beta) + 0.025}$$

For the nominal case,  $\beta = 100$ ,

$$|G_v|_{\text{nominal}} = \frac{10}{0.1 + 0.025} = 80 \text{ V/V}$$

For  $\beta = 50$ ,

$$|G_v|_{\text{low}} = \frac{10}{0.2 + 0.025} = 44.4 \text{ V/V}$$

For  $\beta = 150$ ,

$$|G_v|_{\text{high}} = \frac{10}{(1/15) + 0.025} = 109.1 \text{ V/V}$$

Thus,  $|G_v|$  ranges from 44.4 V/V to 109.1 V/V with a nominal value of 80 V/V. This is a range of  $-44.5\%$  to  $+36.4\%$  of nominal.

To limit the range of  $|G_v|$  to  $\pm 20\%$  of a new nominal value, we connect a resistance  $R_e$  and find its value as follows. With  $R_e$ ,

$$|G_v|_{\text{nominal}} = \frac{10}{(10/100) + 0.025 + R_e}$$

$$= \frac{10}{0.125 + R_e}$$

Now,  $\beta = 50$ ,

$$|G_v|_{\text{low}} = \frac{10}{0.225 + R_e}$$

To limit this value to  $-20\%$  of  $|G_v|_{\text{nominal}}$ , we use

$$\frac{10}{0.225 + R_e} = 0.8 \times \frac{10}{0.125 + R_e}$$

$$\Rightarrow R_e = 0.275 \text{ k}\Omega = 275 \Omega$$

With this value of  $R_e$ ,

$$|G_v|_{\text{nominal}} = \frac{10}{0.125 + 0.275} = 25 \text{ V/V}$$

$$|G_v|_{\text{low}} = \frac{10}{0.225 + 0.275}$$

$$= 20 \text{ V/V } (-20\% \text{ of nominal})$$

$$|G_v|_{\text{high}} = \frac{10}{(1/15) + 0.025 + 0.275}$$

$$= 27.3 \text{ V/V } (+9.1\% \text{ of nominal})$$

$$\mathbf{6.82} \quad R_{\text{in}} = \frac{1}{g_m} = \frac{1}{5 \text{ mA/V}} = 0.2 \text{ k}\Omega$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times g_m (R_D \parallel R_L)$$

$$= \frac{0.2}{0.2 + 0.25} \times 5(5 \parallel 5)$$

$$= 5.6 \text{ V/V}$$

For  $R_{\text{in}} = R_{\text{sig}} = 0.25 \text{ k}\Omega$

$$\frac{1}{g_m} = 0.25 \Rightarrow g_m = 4 \text{ mA/V}$$

Since  $g_m = \sqrt{2k_n I_D}$ , then to change  $g_m$  by a

factor  $\frac{4}{5} = 0.8$ ,  $I_D$  must be changed by a factor of

$$(0.8)^2 = 0.64.$$

**6.83** For  $R_{\text{in}} = R_{\text{sig}} = 50 \Omega$ ,

$r_e = 50 \Omega$

and, with  $\alpha \simeq 1$ ,

$$I_C \simeq \frac{V_T}{r_e} = \frac{25 \text{ mV}}{50 \Omega} = 0.5 \text{ mA}$$

$$g_m = I_C/V_T = 20 \text{ mA/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \parallel R_L)$$

$$G_v = \frac{50}{50 + 50} \times 20 \times (5 \parallel 5) = 25 \text{ V/V}$$

$$\mathbf{6.84} \quad R_{\text{in}} = r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \parallel R_L)$$

$$= \frac{0.125}{0.125 + 0.5} \times 8(10 \parallel 10) = 8 \text{ V/V}$$

$$\hat{v}_\pi = \hat{v}_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$10 = \hat{v}_{\text{sig}} \frac{0.125}{0.125 + 0.5}$$

$$\Rightarrow \hat{v}_{\text{sig}} = 50 \text{ mV}$$

$$\hat{v}_o = G_v \hat{v}_{\text{sig}} = 8 \times 50 = 400 \text{ mV} = 0.4 \text{ V}$$

$$\mathbf{6.85} \quad A_v = \frac{R_L}{R_L + R_o}$$

$$A_v|_{\text{nominal}} = \frac{2}{2 + R_o}$$

$$A_v|_{\text{low}} = \frac{1.5}{1.5 + R_o}$$

$$A_v|_{\text{high}} = \frac{5}{5 + R_o}$$

$$\text{For } A_v|_{\text{high}} = 1.1 A_v|_{\text{nominal}}$$

$$\frac{5}{5 + R_o} = \frac{1.1 \times 2}{2 + R_o}$$

$$\Rightarrow R_o = 0.357 \text{ k}\Omega$$

$$A_v|_{\text{nominal}} = \frac{2}{2.357} = 0.85 \text{ V/V}$$

$$A_v|_{\text{high}} = \frac{5}{5.357} = 0.93$$

(+10% above nominal)

$$A_v|_{\text{low}} = \frac{1.5}{1.5 + 0.357}$$

$$= 0.81 \text{ (-5% from nominal)}$$

$$\frac{1}{g_m} = R_o = 0.357 \text{ k}\Omega$$

$$\Rightarrow g_m = 2.8 \text{ mA/V}$$

To find  $I_D$ , we use

$$g_m = \sqrt{2k_n I_D}$$

$$\Rightarrow I_D = g_m^2 / 2k_n$$

$$= \frac{2.8^2}{2 \times 10} = 0.392 \text{ mA}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.392 = \frac{1}{2} \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.28 \text{ V}$$

**6.86**  $I_C = 5 \text{ mA}$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25}{5} = 5 \Omega$$

$$(a) R_{in} = (\beta + 1)(r_e + R_L)$$

$$= 101 \times (5 + 200) = 20.7 \text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{20.7}{20.7 + 10}$$

$$= 0.67 \text{ V/V}$$

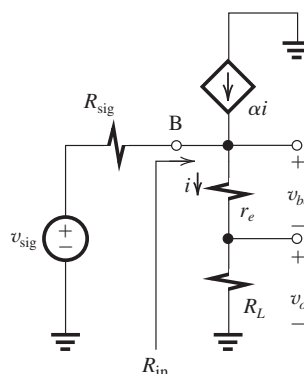
$$\frac{v_o}{v_{sig}} = \frac{v_b}{v_{sig}} \times \frac{v_o}{v_b}$$

$$= 0.67 \times \frac{R_L}{R_L + r_e}$$

$$= 0.67 \times \frac{200}{200 + 5}$$

$$= 0.65 \text{ V/V}$$

(b)



$$\hat{v}_{be} = 10 \text{ mV}$$

$$\hat{v}_o = \frac{R_L}{r_e} \times \hat{v}_{be}$$

$$= \frac{200}{5} \times 10$$

$$= 400 \text{ mV} = 0.4 \text{ V}$$

$$\hat{v}_{sig} = \frac{\hat{v}_o}{G_v} = \frac{0.4}{0.65} = 0.615 \text{ V}$$

(c)  $G_{vo} = 1 \text{ V/V}$

$$R_{out} = r_e + \frac{R_{sig}}{\beta + 1} = 5 + \frac{10,000}{101}$$

$$= 104 \Omega$$

Thus,

$$G_v = G_{vo} \frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{200}{200 + 104} = 0.66 \text{ V/V}$$

which is approximately the same value obtained in (a) above.

For  $R_L = 150 \Omega$ ,

$$G_v = G_{vo} \frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{150}{150 + 104} = 0.59 \text{ V/V}$$

$$\mathbf{6.87} \quad R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$$

$$125 = r_e + \frac{10,000}{\beta + 1} \quad (1)$$

$$225 = r_e + \frac{20,000}{\beta + 1} \quad (2)$$

Subtracting Eq. (1) from Eq. (2), we have

$$100 = \frac{10,000}{\beta + 1}$$



$$\beta + 1 = 100$$

Substituting in Eq. (1) yields

$$125 = r_e + \frac{10,000}{100}$$

$$\Rightarrow r_e = 25 \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \frac{R_{\text{sig}}}{\beta + 1}}$$

$$= \frac{1000}{1000 + 25 + \frac{10,000}{100}} = 0.89 \text{ V/V}$$

**6.88** With the Early effect neglected,

$$G_v = -g_m R_C = -100 \text{ V/V}$$

With the Early effect taken into account, the effective resistance in the collector is reduced from  $R_C$  to  $(R_C \parallel r_o)$ , where

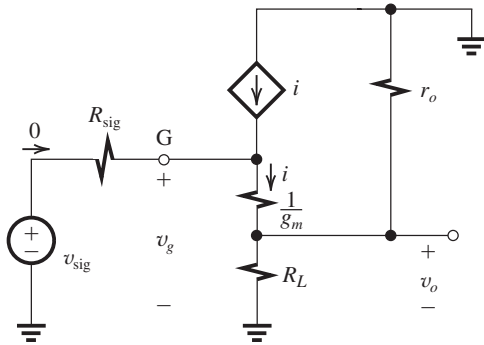
$$r_o = \frac{V_A}{I_C} = \frac{125 \text{ V}}{1 \text{ mA}} = 125 \text{ k}\Omega$$

$$(R_C \parallel r_o) = 10 \parallel 100 = 9.1 \text{ k}\Omega$$

Thus,  $G_v$  becomes

$$G_v = -100 \times \frac{9.1 \text{ k}\Omega}{10 \text{ k}\Omega} = -91 \text{ V/V}$$

**6.89**



$$v_g = v_{\text{sig}}$$

Noting that  $r_o$  appears in effect in parallel with  $R_L$ ,  $v_o$  is obtained as the ratio of the voltage divider formed by  $(1/g_m)$  and  $(R_L \parallel r_o)$ ,

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_o}{v_g} = \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + \frac{1}{g_m}} \quad \text{Q.E.D.}$$

With  $R_L$  removed,

$$G_v = \frac{r_o}{r_o + \frac{1}{g_m}} = 0.98 \quad (1)$$

With  $R_L = 500 \Omega = 0.5 \text{ k}\Omega$ ,

$$G_v = \frac{(0.5 \parallel r_o)}{(0.5 \parallel r_o) + \frac{1}{g_m}} = 0.49 \quad (2)$$

From Eq. (1), we have

$$\frac{1}{g_m} = \frac{r_o}{49}$$

Substituting in Eq. (2) and solving for  $r_o$  gives

$$r_o = 25 \text{ k}\Omega$$

Thus

$$\frac{1}{g_m} = \frac{25}{49} \text{ k}\Omega$$

$$\Rightarrow g_m = 1.96 \text{ mA/V}$$

**6.90** Adapting Eq. (6.114) gives

$$\begin{aligned} G_v &= -\beta \frac{R_C \parallel R_L \parallel r_o}{R_{\text{sig}} + (\beta + 1)r_e} \\ &= -\frac{R_C \parallel R_L \parallel r_o}{\frac{R_{\text{sig}}}{\beta} + \frac{\beta + 1}{\beta} r_e} \\ &= -\frac{R_C \parallel R_L \parallel r_o}{\frac{R_{\text{sig}}}{\beta} + \frac{1}{g_m}} \end{aligned}$$

Thus,

$$|G_v| = \frac{10 \parallel r_o}{0.1 + \frac{1}{g_m}} \quad (1)$$

where  $r_o$  and  $\frac{1}{g_m}$  are in kilohms and are given by

$$r_o = \frac{V_A}{I_C} = \frac{25 \text{ V}}{I_C \text{ mA}} \quad (2)$$

$$\frac{1}{g_m} = \frac{V_T}{I_C} = \frac{0.025 \text{ V}}{I_C \text{ mA}} \quad (3)$$

$I_C$ (mA)	$1/g_m$ (k $\Omega$ )	$r_o$ (k $\Omega$ )	$ G_v $ (V/V)
0.1	0.250	250	27.5
0.2	0.125	125	41.2
0.5	0.050	50	55.6
1.0	0.025	25	57.1
1.25	0.020	20	55.6

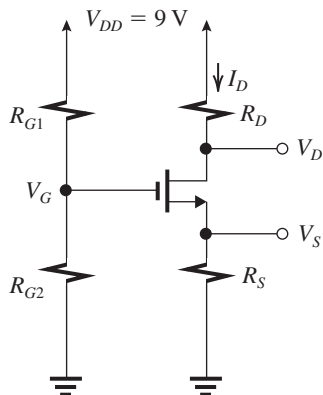
Observe that initially  $|G_v|$  increases as  $I_C$  is increased. However, above about 1 mA this trend reverses because of the effect of  $r_o$ . From the table we see that gain of 50 is obtained for  $I_C$  between 0.2 and 0.5 mA and also for  $I_C$  above 1.25 mA. Practically speaking, one normally uses

the low value to minimize power dissipation. The required value of  $I_C$  is found by substituting for  $r_o$  and  $1/g_m$  from Eqs. (2) and (3), respectively, in Eq. (1) and equating  $G_v$  to 50. The result (after some manipulations) is the quadratic equation.

$$I_C^2 - 2.25I_C + 0.625 = 0$$

The two roots of this equation are  $I_C = 0.325$  mA and 1.925 mA; our preferred choice is  $I_C = 0.325$  mA.

6.91



$$I_D = 0.2 \text{ mA}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.28 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.7 + 0.28 = 0.98 \approx 1 \text{ V}$$

$$\text{Now, selecting } V_S = \frac{V_{DD}}{3} = 3 \text{ V,}$$

$$I_D R_S = 3$$

$$R_S = \frac{3}{0.2} = 15 \text{ k}\Omega$$

Also,

$$I_D R_D = \frac{V_{DD}}{3} = 3 \text{ V}$$

$$\Rightarrow R_D = \frac{3}{0.2} = 15 \text{ k}\Omega$$

$$V_G = V_S + V_{GS}$$

$$= 3 + 1 = 4 \text{ V}$$

Thus the voltage drop across  $R_{G1}$  (5 V) is larger than that across  $R_{G2}$  (4 V). So we select

$$R_{G1} = 22 \text{ M}\Omega$$

and determine  $R_{G2}$  from

$$\frac{R_{G2}}{R_{G1}} = \frac{4 \text{ V}}{5 \text{ V}}$$

$$\Rightarrow R_{G2} = 0.8 R_{G1} = 0.8 \times 22$$

$$= 17.6 \text{ M}\Omega$$

Using only two significant figures, we have

$$R_{G2} = 18 \text{ M}\Omega$$

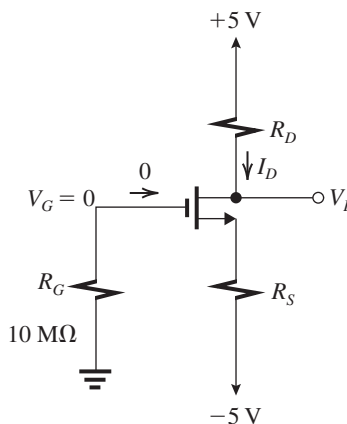
Note that this will cause  $V_G$  to deviate slightly from the required value of 5 V. Specifically,

$$V_G = V_{DD} \frac{R_{G2}}{R_{G2} + R_{G1}}$$

$$= 9 \times \frac{18}{22 + 18} = 4.05 \text{ V}$$

It can be shown that the resulting  $I_D$  will be  $I_D = 0.204$  mA, which is very close to the desired 0.2 mA. Since  $V_D = V_{DD} - I_D R_D \approx +6$  V and  $V_G \approx 4$  V, and the drain voltage can go down to  $V_G - V_t = 3.3$  V, the drain voltage is 2.7 V above the value that causes the MOSFET to leave the saturation region.

6.92



For  $I_D = 0.5$  mA

$$0.5 = \frac{1}{2} k_n V_{OV}^2$$

$$= \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 1 = 2 \text{ V}$$

Since

$$V_G = 0 \text{ V, } V_S = -V_{GS} = -2 \text{ V}$$

which leads to

$$R_S = \frac{V_S - (-5)}{I_C} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

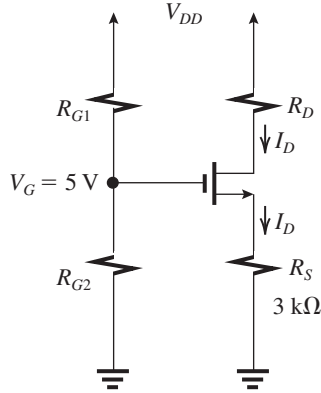
$V_D$  is required to be halfway between cutoff (+5 V) and saturation ( $0 - V_t = -1$  V). Thus

$$V_D = +2 \text{ V}$$

and

$$R_D = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

6.93



$$V_S = I_D R_S = 3I_D$$

$$V_{GS} = 5 - V_S = 5 - 3I_D$$

$$\begin{aligned} I_D &= \frac{1}{2} k_n (V_{GS} - V_t)^2 \\ &= \frac{1}{2} \times 2(5 - 3I_D - 1)^2 \\ &= 16 - 24I_D + 9I_D^2 \\ 9I_D^2 - 25I + 16 &= 0 \end{aligned}$$

$$I_D = 1.78 \text{ mA or } 1 \text{ mA}$$

The first answer is physically meaningless, as it would result in  $V_S = 5.33$  V, which is greater than  $V_G$ , implying that the transistor is cut off. Thus,  $I_D = 1$  mA.

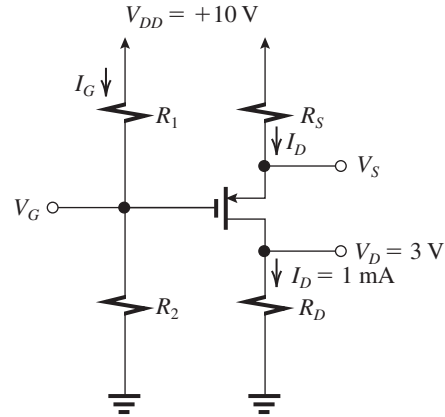
If a transistor for which  $k_n = 3 \text{ mA/V}^2$  is used, then

$$\begin{aligned} I_D &= \frac{1}{2} \times 3(5 - 3I_D - 1)^2 \\ &= 1.5(16 - 24I + 9I_D^2) \\ 9I_D^2 - 24.67I_D + 16 &= 0 \end{aligned}$$

whose physically meaningful solution is

$$I_D = 1.05 \text{ mA}$$

6.94



$$I_D = 1 \text{ mA and } V_D = 3 \text{ V}$$

Thus,

$$R_D = \frac{V_D}{I_D} = \frac{3 \text{ V}}{1 \text{ mA}} = 3 \text{ k}\Omega$$

For the transistor to operate 1 V from the edge of saturation

$$V_D = V_G + |V_t| - 1$$

Thus,

$$3 = V_G + |V_t| - 1$$

$$V_G + |V_t| = 4 \text{ V}$$

(a)  $|V_t| = 1$  V and  $k_p = 0.5 \text{ mA/V}^2$

$$V_G = 3 \text{ V}$$

$$R_2 = \frac{V_G}{I_G} = \frac{3 \text{ V}}{10 \mu\text{A}} = 0.3 \text{ M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{7 \text{ V}}{10 \mu\text{A}} = 0.7 \text{ M}\Omega$$

$$V_D = 3 \text{ V}$$

$$R_D = 3 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k_p (V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.5 (V_{SG} - 1)^2$$

$$\Rightarrow V_{SG} = 3 \text{ V}$$

$$V_S = V_G + 3 = 3 + 3 = 6 \text{ V}$$

$$R_S = \frac{V_{DD} - V_S}{I_D}$$

$$= \frac{10 - 6}{1} = 4 \text{ k}\Omega$$

(b)  $|V_t| = 2$  V and  $k_p = 1.25 \text{ mA/V}^2$

$$V_G = 4 - |V_t| = 2 \text{ V}$$

$$R_2 = \frac{V_G}{I_G} = \frac{2 \text{ V}}{10 \mu\text{A}} = 0.2 \text{ M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{8 \text{ V}}{10 \mu\text{A}} = 0.8 \text{ M}\Omega$$

$$V_D = 3 \text{ V}$$

$$R_D = 3 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k_p (V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 1.25 (V_{SG} - 2)^2$$

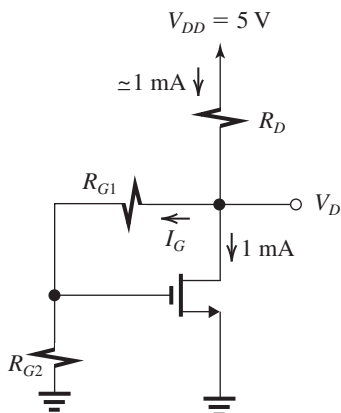
$$V_{SG} = 3.265 \text{ V}$$

$$V_S = V_G + 3.265 = 2 + 3.265$$

$$= 5.265 \text{ V}$$

$$R_S = \frac{10 - 5.265}{1} = 4.7 \text{ k}\Omega$$

6.95



$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$1 = \frac{1}{2} \times 8 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Since the transistor leaves the saturation region of operation when  $v_D < V_{OV}$ , we select

$$V_D = V_{OV} + 2$$

$$V_D = 2.5 \text{ V}$$

Since  $I_G \ll I_D$ , we can write

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2.5}{1} = 2.5 \text{ k}\Omega$$

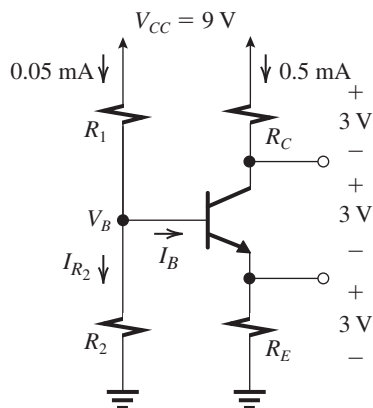
$$V_{GS} = V_t + V_{OV} = 0.8 + 0.5 = 1.3 \text{ V}$$

Thus the voltage drop across  $R_{G2}$  is 1.3 V and that across  $R_{G1}$  is  $(2.5 - 1.3) = 1.2 \text{ V}$ . Thus  $R_{G2}$  is the larger of the two resistances, and we select  $R_{G2} = 22 \text{ M}\Omega$  and find  $R_{G1}$  from

$$\frac{R_{G1}}{R_{G2}} = \frac{1.2}{1.3} \Rightarrow R_{G1} = 20.3 \text{ M}\Omega$$

Specifying all resistors to two significant digits, we have  $R_D = 2.5 \text{ k}\Omega$ ,  $R_{G1} = 22 \text{ M}\Omega$ , and  $R_{G2} = 20 \text{ M}\Omega$ .

6.96



$$I_E = \frac{I_C}{\alpha} \text{ where } \alpha = \frac{\beta}{\beta + 1} = \frac{80}{81} = 0.988$$

$$I_E = \frac{0.5}{0.988} = 0.506 \text{ mA}$$

$$R_E = \frac{V_E}{I_E} = \frac{3 \text{ V}}{0.506 \text{ mA}} = 5.93 \text{ k}\Omega$$

$$R_C = \frac{3}{I_C} = \frac{3 \text{ V}}{0.5 \text{ mA}} = 6 \text{ k}\Omega$$

$$V_B = V_E + 0.7 = 3.7 \text{ V}$$

$$R_1 = \frac{9 - 3.7}{0.05} = 106 \text{ k}\Omega$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{80} = 0.0063 \text{ mA}$$

$$I_{R2} = 0.05 - I_B = 0.05 - 0.0063 = 0.0437 \text{ mA}$$

$$R_2 = \frac{V_B}{I_{R2}} = \frac{3.7}{0.0437} = 84.7 \text{ k}\Omega$$

Consulting the table of 5-% resistors in Appendix J we select:

$$R_E = R_C = 6.2 \text{ k}\Omega$$

$$R_1 = 100 \text{ k}\Omega \quad R_2 = 82 \text{ k}\Omega$$

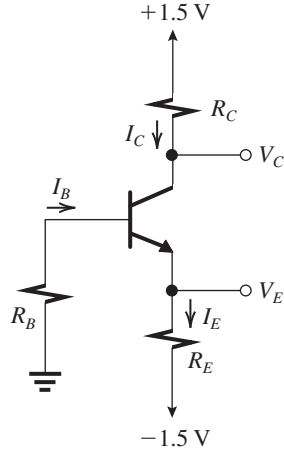
This resistor combination will result in

$$I_E = \frac{9 - \frac{R_2}{R_1 + R_2} - 0.7}{R_E + \left(\frac{R_1 R_2}{R_1 + R_2}\right) / (\beta + 1)}$$

which yields

$$\begin{aligned} I_E &= 0.5 \text{ mA} \\ I_C &= \alpha I_E = 0.49 \text{ mA} \\ V_E &= I_E R_E = 0.5 \times 6.2 = 3.1 \text{ V} \\ V_B &= 3.8 \text{ V} \\ V_C &= V_{CC} - R_C I_C = 9 - 6.2 \times 0.49 \simeq 6 \text{ V} \end{aligned}$$

6.97



$$I_E = \frac{1.5 - 0.7}{R_E + \frac{R_B}{\beta+1}} = \frac{0.8}{R_E + \frac{R_B}{51}} \quad (1)$$

For,

$$\begin{aligned} I_B R_B &= 0.1 I_E R_E \\ R_B &= 0.1 \times \frac{I_E}{I_B} \times R_E = 0.1 \times 51 \times R_E = 5.1 R_E \end{aligned}$$

Substituting in Eq. (1)

$$I_E = \frac{I_C}{\alpha} = \frac{0.5}{50/51} = 0.51 \text{ mA and } R_B = 5.1 R_E$$

gives

$$0.51 = \frac{0.8}{R_E + \frac{5.1}{51} R_E}$$

$$\Rightarrow R_E = 1.43 \text{ k}\Omega$$

$$R_B = 5.1 R_E = 7.3 \text{ k}\Omega$$

$$V_E = -1.5 + I_E R_E = -1.5 + 0.51 \times 1.43 = -0.77 \text{ V}$$

$$V_C = V_E + 1 = +0.23 \text{ V}$$

$$R_C = \frac{1.5 - 0.23}{0.5} = 2.54 \text{ k}\Omega$$

Consulting the table of 5% resistors in Appendix J, we select

$$R_E = 1.5 \text{ k}\Omega \quad R_C = 2.4 \text{ k}\Omega \quad R_B = 7.5 \text{ k}\Omega$$

For  $\beta = \infty$ :

$$I_E = \frac{1.5 - 0.7}{1.5} = 0.53 \text{ mA}$$

$$I_C = \alpha I_E = 0.52 \text{ mA}$$

$$V_E = -1.5 + 0.53 \times 1.5 = -0.7 \text{ V}$$

$$V_B = 0$$

$$V_C = 1.5 - 2.4 \times 0.52 = 0.25 \text{ V}$$

which is 0.95 V above  $V_E$ .

For  $\beta = 50$ :

$$I_E = \frac{1.5 - 0.7}{1.5 + \frac{7.5}{51}} = 0.49 \text{ mA}$$

$$I_C = \alpha I_E = 0.48 \text{ mA}$$

$$V_E = -1.5 + 0.49 \times 1.5 = -0.77 \text{ V}$$

$$V_B = V_E + 0.7 = -0.07 \text{ V}$$

$$I_B R_B = 0.07 \text{ V}$$

$$I_E R_E = 0.49 \times 1.5 = 0.7 \text{ V}$$

Thus,

$$I_B R_B \simeq 0.1 I_E R_E$$

$$V_C = 1.5 - 2.4 \times 0.48 = 0.35 \text{ V}$$

which is 1.1 V above  $V_E$ .

6.98

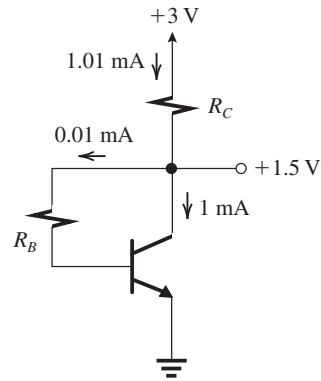


Figure 1

(a) From the circuit diagram of Fig. 1, we can write

$$R_C = \frac{3 - 1.5}{1.01 \text{ mA}} \simeq 1.5 \text{ k}\Omega$$

$$1.5 = 0.01 R_B + V_{BE}$$

$$= 0.01 R_B + 0.7$$

$$\Rightarrow R_B = 80 \text{ k}\Omega$$

(b) Selecting 5% resistors, we have

$$R_C = 1.5 \text{ k}\Omega$$

$$R_B = 82 \text{ k}\Omega$$

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$= \frac{3 - 0.7}{1.5 + \frac{82}{101}} = 0.99 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.99 = 0.98 \text{ mA}$$

$$V_C = 3 - 1.5 \times 0.99 = 1.52 \text{ V}$$

(c)  $\beta = \infty$ :

$$I_C = I_E = \frac{V_{CC} - V_{BE}}{R_C} = \frac{3 - 0.7}{1.5} = 1.53 \text{ mA}$$

$$V_C = 0.7 \text{ V}$$

(d) From the circuit diagram of Fig. 2, we can write

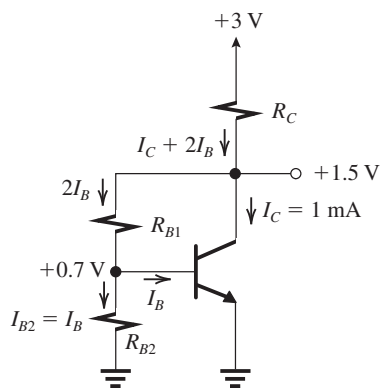


Figure 2

$$I_B = \frac{I_C}{\beta} = \frac{1}{100} = 0.01 \text{ mA}$$

$$R_{B2} = \frac{0.7}{I_{B2}} = \frac{0.7}{0.01}$$

$$= 70 \text{ k}\Omega$$

$$1.5 = 2I_B R_{B1} + 0.7$$

$$0.8 = 2 \times 0.01 \times R_{B1}$$

$$R_{B1} = 40 \text{ k}\Omega$$

$$R_C = \frac{3 - 1.5}{I_C + 2I_B} = \frac{1.5}{1.02} = 1.47 \text{ k}\Omega$$

For  $\beta = \infty$ :

$$I_B = 0, \quad I_{B2} = \frac{0.7}{R_{B2}} = \frac{0.7}{70} = 0.01 \text{ mA}$$

$$I_{B1} = I_{B2} = 0.01 \text{ mA}$$

$$V_C = 0.01 R_{B1} + 0.7 = 0.01 \times 40 + 0.7$$

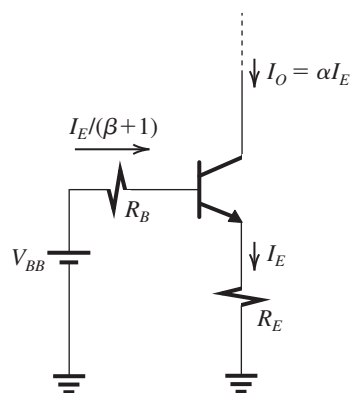
$$= 1.1 \text{ V}$$

$$I_C + 0.01 = \frac{3 - 1.1}{R_C} = \frac{3 - 1.1}{1.47} = 1.29$$

$$I_C = 1.28 \text{ mA}$$

**6.99** Refer to the circuit in Fig. P6.100.

Replacing  $V_{CC}$  together with the voltage divider ( $R_1, R_2$ ) by its Thévenin equivalent results in the circuit shown below.



where

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2}$$

and

$$R_B = (R_1 \parallel R_2)$$

Now,

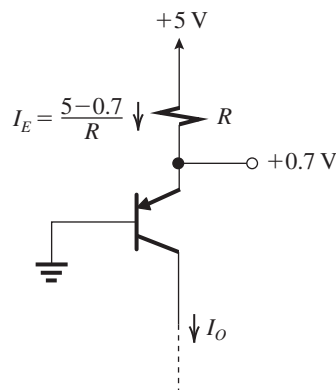
$$V_{BB} = \frac{I_E}{\beta + 1} R_B + V_{BE} + I_E R_E$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + (R_1 \parallel R_2)/(\beta + 1)}$$

$$I_C = \alpha I_E$$

$$= \alpha \frac{V_{CC}[R_2/(R_1 + R_2)] - V_{BE}}{R_E + (R_1 \parallel R_2)/(\beta + 1)} \quad \text{Q.E.D.}$$

**6.100**



$$I_O = \alpha I_E \simeq 0.5 \text{ mA}$$

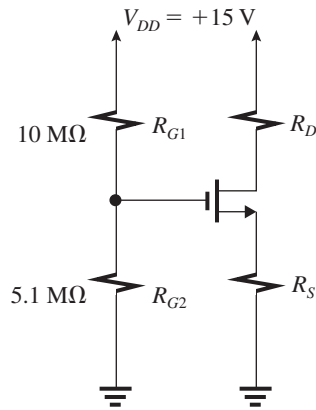
$$I_E = 0.5 \text{ mA}$$

$$\Rightarrow R = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

$$v_{C\max} = 0.7 - V_{EC\text{sat}} = 0.7 - 0.3$$

$$= +0.4 \text{ V}$$

6.101



$$V_G = V_{DD} \frac{R_{G2}}{R_{G1} + R_{G2}}$$

$$= 15 \times \frac{5.1}{10 + 5.1} = 5.07 \text{ V}$$

$$k_n = 0.2 \text{ to } 0.3 \text{ mA/V}^2$$

$$V_t = 1.0 \text{ V to } 1.5 \text{ V}$$

With  $R_S = 0$ ,

$$I_D = \frac{1}{2} k_n (V_G - V_t)^2$$

 $I_{D\max}$  is obtained with  $V_{t\min}$  and  $k_{n\max}$ :

$$I_{D\max} = \frac{1}{2} \times 0.3 (5.07 - 1)^2 = 2.48 \text{ mA}$$

 $I_{D\min}$  is obtained with  $V_{t\max}$  and  $k_{n\min}$ :

$$I_{D\min} = \frac{1}{2} \times 0.2 (5.07 - 1.5)^2 = 1.27 \text{ mA}$$

With  $R_S$  installed and  $V_t = 1 \text{ V}$ ,  $k_n = 0.3 \text{ mA/V}^2$ , we require  $I_D = 1.5 \text{ mA}$ :

$$1.5 = \frac{1}{2} \times 0.3 (V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 4.16 \text{ V}$$

Since  $V_G = 5.07 \text{ V}$ ,

$$V_S = V_G - V_{GS} = 5.07 - 4.16 = 0.91 \text{ V}$$

Thus,

$$R_S = \frac{V_S}{I_D} = \frac{0.91}{1.5} = 607 \Omega$$

From Appendix J, the closest 5% resistor is 620  $\Omega$ . With  $R_S = 620 \Omega$ ,

$$V_S = I_D R_S = 0.62 I_D$$

$$V_{GS} = V_G - V_S = 5.07 - 0.62 I_D$$

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} k_n (5.07 - 0.62 I_D - V_t)^2$$

For  $k_n = 0.3 \text{ mA/V}^2$  and  $V_t = 1$ ,

$$I_D = \frac{1}{2} \times 0.3 (4.07 - 0.62 I_D)^2$$

$$= 0.15 (4.07^2 - 2 \times 4.07 \times 0.62 I_D + 0.62^2 I_D^2)$$

$$0.058 I_D^2 - 1.757 I_D + 2.488 = 0$$

which results in

$$I_D = 28.8 \text{ mA, or } 1.49 \text{ mA}$$

The first value does not make physical sense. Thus,

$$I_D = 1.49 \text{ mA} \simeq 1.5 \text{ mA}$$

which is the maximum value. The minimum value can be obtained by using  $k_n = 0.2 \text{ mA/V}^2$  and  $V_t = 1.5 \text{ V}$  in Eq. (1),

$$I_D = \frac{1}{2} \times 0.2 (3.57 - 0.62 I_D)^2$$

$$= 0.1 (3.57^2 - 2 \times 3.57 \times 0.62 I_D + 0.62^2 I_D^2)$$

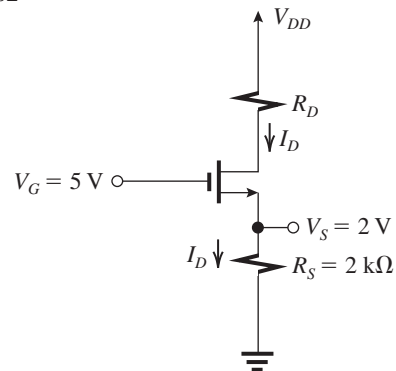
$$0.038 I_D^2 - 1.442 I_D + 1.274 = 0$$

which results in

$$I_D = 37 \text{ mA or } 0.91 \text{ mA}$$

Here again, the physically meaningful answer is  $I_D = 0.91 \text{ mA}$ , which is the minimum value of  $I_D$ . Thus with a 62-k $\Omega$  resistance connected in the source lead, the value of  $I_D$  is limited to the range of 0.91 mA to 1.5 mA.

6.102



$$I_D = \frac{2 \text{ V}}{2 \text{ k}\Omega} = 1 \text{ mA}$$

But

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 2(V_G - V_S - V_t)^2$$

$$1 = (5 - 2 - V_t)^2$$

$$V_t = 2 \text{ V}$$

If  $V_t = 1.5 \text{ V}$ , then we have

$$V_S = I_D R_S = 2I_D$$

$$V_{GS} = V_G - V_S = 5 - 2I_D$$

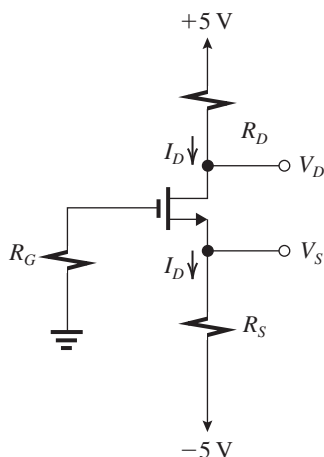
$$I_D = \frac{1}{2} \times 2(5 - 2I_D - 1.5)^2$$

$$4I_D^2 - 15I_D + 12.25 = 0$$

$$I_D = 1.2 \text{ mA}$$

$$V_S = 2.4 \text{ V}$$

**6.103**  $I_D = 0.5 \text{ mA} = \frac{1}{2} \times 4(V_{GS} - 1)^2$



$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

Since  $V_G = 0 \text{ V}$ ,  $V_S = -1.5 \text{ V}$ , and

$$R_S = \frac{-1.5 - (-5)}{0.5} = 7 \text{ k}\Omega$$

Maximum gain is obtained by using the largest possible value of  $R_D$ , that is, the lowest possible value of  $V_D$  that is consistent with allowing negative voltage signal swing at the drain of 1 V. Thus

$$V_D - 1 = v_{D\min} = V_G - V_t = 0 - 1$$

$$\Rightarrow V_D = 0 \text{ V}$$

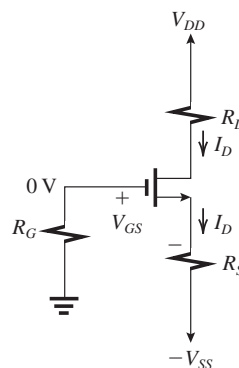
where we have assumed that the signal voltage at the gate is small. Now,

$$V_D = 0 = V_{DD} - I_D R_D$$

$$0 = 5 - 0.5 \times R_D$$

$$\Rightarrow R_D = 10 \text{ k}\Omega$$

**6.104**



(a)  $V_{GS} + I_D R_S = V_{SS}$

But

$$I_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$= K(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{I_D}{K}}$$

Thus,

$$V_t + \sqrt{\frac{I_D}{K}} + I_D R_S = V_{SS}$$

Differentiating relative to  $K$ , we have

$$0 + \frac{1}{2\sqrt{I_D/K}} \left[ \frac{1}{K} \frac{\partial I_D}{\partial K} - \frac{I_D}{K^2} \right] + R_S \frac{\partial I_D}{\partial K} = 0$$

$$\Rightarrow \frac{\partial I_D}{\partial K} \frac{K}{I_D} = \frac{1}{1 + 2\sqrt{KI_D} R_S}$$

$$\Rightarrow S_K^{I_D} = 1/[1 + 2\sqrt{KI_D} R_S] \quad \text{Q.E.D.}$$

(b)  $K = 100 \mu\text{A}/\text{V}^2$ ,  $\frac{\Delta K}{K} = \pm 0.1$ , and  $V_t = 1 \text{ V}$ .

We require  $I_D = 100 \mu\text{A}$  and  $\frac{\Delta I_D}{I_D} = \pm 0.01$ .

Thus,

$$S_K^{I_D} = \frac{\Delta I_D/I_D}{\Delta K/K} = \frac{0.01}{0.10} = 0.1$$

Substituting in the expression derived in (a),

$$0.1 = \frac{1}{1 + 2\sqrt{0.1} \times 0.1 R_S}$$

$$\Rightarrow R_S = 45 \text{ k}\Omega$$

To find  $V_{GS}$ ,

$$I_D = K(V_{GS} - V_t)^2$$

$$100 = 100(V_{GS} - 1)^2$$

$$V_{GS} = 2 \text{ V}$$

$$V_{GS} + I_D R_S = V_{SS}$$

$$2 + 0.1 \times 45 = 6.5 \text{ V}$$



(c) For  $V_{SS} = 5 \text{ V}$  and  $V_{GS} = 2 \text{ V}$ ,

$$I_D R_S = 3 \text{ V}$$

$$R_S = \frac{3}{0.1} = 30 \text{ k}\Omega$$

$$S_K^{I_D} = \frac{1}{1 + 2\sqrt{0.1 \times 0.1 \times 30}} = \frac{1}{7}$$

$$\frac{\Delta I_D}{I_D} = \frac{1}{7} \times \frac{\Delta K}{K} = \frac{1}{7} \times \pm 10\% = \pm 1.4\%$$

**6.105** (a) With a fixed  $V_{GS}$ ,

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$\frac{\partial I_D}{\partial V_t} = -k_n (V_{GS} - V_t)$$

$$S_{V_t}^{I_D} \equiv \frac{\partial I_D}{\partial V_t} \frac{V_t}{I_D} = -\frac{k_n (V_{GS} - V_t) V_t}{I_D}$$

$$= -\frac{k_n (V_{GS} - V_t) V_t}{\frac{1}{2} k_n (V_{GS} - V_t)^2}$$

$$= -\frac{2V_t}{V_{GS} - V_t} = -\frac{2V_t}{V_{OV}} \quad \text{Q.E.D.}$$

For  $V_t = 0.5 \text{ V}$ ,  $\frac{\Delta V_t}{V_t} = \pm 5\%$ , and

$V_{OV} = 0.25 \text{ V}$ , we have

$$\frac{\Delta I_D}{I_D} = S_{V_t}^{I_D} \left( \frac{\Delta V_t}{V_t} \right)$$

$$= -\frac{2 \times 0.5}{0.25} \times \pm 5\%$$

$$= \mp 20\%$$

(b) For fixed bias at the gate  $V_G$  and a resistance  $R_S$  in the source lead, we have

$$V_G = V_{GS} + I_D R_S$$

where  $V_{GS}$  is obtained from

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$V_t + \sqrt{\frac{2I_D}{k_n}} + I_D R_S = V_G$$

Differentiating relative to  $V_t$ , we have

$$1 + \frac{1}{2\sqrt{2I_D/k_n}} \frac{2}{k_n} \frac{\partial I_D}{\partial V_t} + R_S \frac{\partial I_D}{\partial V_t} = 0$$

$$\frac{\partial I_D}{\partial V_t} \left[ \frac{1}{\sqrt{2k_n I_D}} + R_S \right] = -1$$

$$\frac{\partial I_D}{\partial V_t} = -\frac{1}{\frac{1}{\sqrt{2k_n I_D}} + R_S}$$

$$S_{V_t}^{I_D} = \frac{\partial I_D}{\partial V_t} \frac{V_t}{I_D} = -\frac{V_t}{\sqrt{\frac{I_D}{2k_n}} + I_D R_S}$$

But

$$I_D = \frac{1}{2} k_n V_{OV}^2 \Rightarrow V_{OV} = \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$S_{V_t}^{I_D} = -\frac{2V_t}{V_{OV} + 2I_D R_S} \quad \text{Q.E.D.}$$

For  $V_t = 0.5 \text{ V}$ ,  $\frac{\Delta V_t}{V_t} = \pm 5\%$ , and

$V_{OV} = 0.25 \text{ V}$ , to limit  $\frac{\Delta I_D}{I_D}$  to  $\mp 5\%$  we require

$$S_{V_t}^{I_D} = 1$$

Thus

$$-1 = -\frac{2 \times 0.5}{0.25 + 2I_D R_S}$$

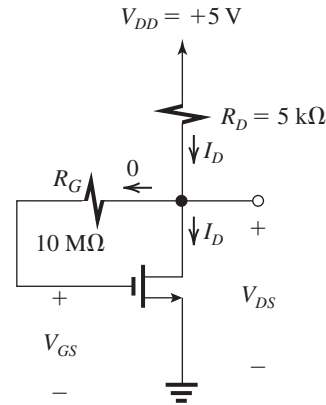
$$\Rightarrow I_D R_S = 0.375 \text{ V}$$

For  $I_D = 0.1 \text{ mA}$ ,

$$R_S = \frac{0.375}{0.1} = 3.75 \text{ k}\Omega$$

$$V_G = V_{GS} + I_D R_S = 0.5 + 0.25 + 0.375 = 1.125 \text{ V}$$

**6.106**



$$V_{GS} = V_{DD} - I_D R_D$$

$$= 5 - 5I_D$$

(a)  $V_t = 0.7 \text{ V}$  and  $k_n = 5 \text{ mA/V}^2$

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$I_D = \frac{1}{2} \times 5(5 - 5I_D - 0.7)^2$$

$$\Rightarrow I_D^2 - 1.736I_D + 0.7396 = 0$$

$$I_D = 0.99 \text{ mA or } 0.75 \text{ mA}$$

The first root results in  $V_{GS} = -0.05 \text{ V}$ , which is physically meaningless. Thus

$$I_D = 0.75 \text{ mA}$$

$$V_G = V_D = 5 - 5 \times 0.75 = 1.25 \text{ V}$$

$$(b) V_t = 1.5 \text{ V and } k_n = 10 \text{ mA/V}^2$$

$$I_D = \frac{1}{2} \times 10(5 - 5I_D - 1.5)^2$$

$$\Rightarrow I_D^2 - 1.408I_D + 0.49 = 0$$

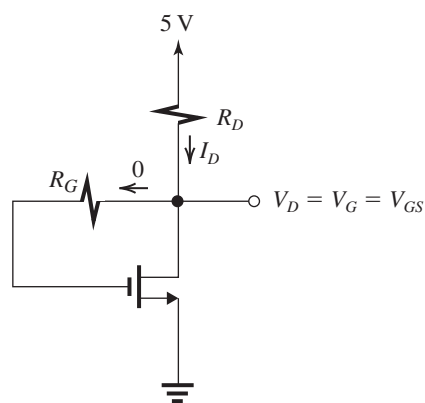
$$I_D = 0.78 \text{ mA or } 0.63 \text{ mA}$$

The first root can be shown to be physically meaningless, thus

$$I_D = 0.63 \text{ mA}$$

$$V_G = V_D = 5 - 5 \times 0.63 = 1.85 \text{ V}$$

## 6.107



$$I_D = 0.2 = \frac{1}{2} \times 10(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = 1.2 \text{ V}$$

$$R_D = \frac{5 - 1.2}{0.2} = 19 \text{ k}\Omega$$

6.108 Refer to Fig. 6.53(a).

$$\frac{R_{B2}}{R_{B1} + R_{B2}} \times 3 = 0.710$$

$$\Rightarrow \frac{R_{B1}}{R_{B2}} = 3.225$$

Given that  $R_{B1}$  and  $R_{B2}$  are 1% resistors, the maximum and minimum values of the ratio  $R_{B1}/R_{B2}$  will be  $3.225 \times 1.02 = 3.2899$  and  $3.225 \times 0.98 = 3.1608$ . The resulting  $V_{BE}$  will be 0.699 V and 0.721 V, respectively.

Correspondingly,  $I_C$  will be

$$I_{C\max} = 1 \times e^{(0.710 - 0.699)/0.025} = 1.55 \text{ mA}$$

and

$$I_{C\min} = 1 \times e^{(0.710 - 0.721)/0.025}$$

$$I_{C\min} = 0.64 \text{ mA}$$

$V_{CE}$  will range from

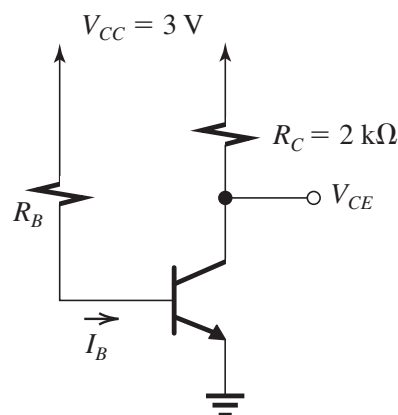
$$V_{CE\min} = 3 - 1.55 \times 2 = -0.1 \text{ V}$$

which is impossible, implying that the transistor will saturate at this value of dc bias!

$$V_{CE\max} = 3 - 0.64 \times 2 = 1.72 \text{ V}$$

It should be clear that this biasing arrangement is useless, since even the small and inevitable tolerances in  $R_{B1}$  and  $R_{B2}$  caused such huge variations in  $I_C$  that in one extreme the transistor left the active mode of operation altogether!

## 6.109



To obtain  $I_C = 1 \text{ mA}$ , we write

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 0.01 \text{ mA}$$

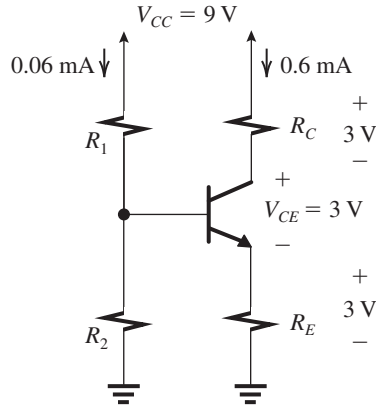
Thus,

$$R_B = \frac{V_{CC} - V_{BE}}{I_B} \simeq \frac{3 - 0.7}{0.01} = 230 \text{ k}\Omega$$

Since  $\beta$  ranges from 50 to 150 and  $I_B$  is fixed at 0.01 mA, the collector current  $I_C$  will range from  $0.01 \times 50 = 0.5 \text{ mA}$  to  $0.01 \times 150 = 1.5 \text{ mA}$ .

Correspondingly,  $V_{CE}$  will range from  $(3 - 0.5 \times 2) = 1 \text{ V}$  to  $(3 - 1.5 \times 2) = 0 \text{ V}$ . The latter value implies that the high- $\beta$  transistor will leave the active region of operation and saturate. Obviously, this bias method is very intolerant of the inevitable variations in  $\beta$ . Thus it is not a good method for biasing the BJT.

## 6.110



Initial design:  $\beta = \infty$

$$R_C = R_E = \frac{3 \text{ V}}{0.6} = 5 \text{ k}\Omega$$

$$R_1 + R_2 = \frac{9}{0.06} = 150 \text{ k}\Omega$$

$$V_B = V_E + V_{BE} = 3 + 0.7 = 3.7 \text{ V}$$

$$R_2 = \frac{3.7}{0.06} = 61.7 \text{ k}\Omega$$

$$R_1 = 150 - 61.7 = 88.3 \text{ k}\Omega$$

Using 5% resistors from Appendix J, and selecting  $R_1$  and  $R_2$  so as to obtain a  $V_{BB}$  that is slightly higher than 3.7 V, we write

$$R_1 = 82 \text{ k}\Omega \text{ and } R_2 = 62 \text{ k}\Omega$$

$$R_E = 5.1 \text{ k}\Omega \text{ and } R_C = 5.1 \text{ k}\Omega$$

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 9 \times \frac{62}{62 + 82} = 3.875$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$R_B = R_1 \parallel R_2 = 62 \parallel 82 = 35.3 \text{ k}\Omega$$

$$I_E = \frac{3.875 - 0.7}{5.1 + \frac{35.3}{91}} = 0.58 \text{ mA}$$

$$V_E = 0.58 \times 5.1 = 3.18$$

$$V_B = 3.88 \text{ V}$$

$$I_C = \alpha I_E = \frac{90}{91} \times 0.58 = 0.57 \text{ mA}$$

$$V_C = 6.1 \text{ V}$$

$$I_{R2} = \frac{V_B}{R_2} = \frac{3.88}{62} = 0.063 \text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.58}{91} = 0.006 \text{ mA and}$$

$$I_{R1} = 0.069 \text{ mA}$$

## 6.111 Refer to Fig. 6.54.

$$(a) I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

$$I_E|_{\text{nominal}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

$$I_E|_{\text{high}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{151}}$$

$$I_E|_{\text{low}} = \frac{V_{BE} - V_{BE}}{R_E + \frac{R_B}{51}}$$

Let's constrain  $I_E|_{\text{low}}$  to be equal to  $I_E|_{\text{nominal}} \times 0.95$  and then check  $I_E|_{\text{high}}$ :

$$0.95 \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}}$$

$$0.95 = \frac{1 + \frac{R_B/R_E}{101}}{1 + \frac{R_B/R_E}{51}}$$

$$\Rightarrow \frac{R_B}{R_E} = 5.73$$

For this value,

$$I_E|_{\text{nominal}} = 0.946 \left( \frac{V_{BB} - V_{BE}}{R_E} \right)$$

$$I_E|_{\text{low}} = 0.90 \left( \frac{V_{BB} - V_{BE}}{R_E} \right) = 0.95 I_E|_{\text{nominal}}$$

$$I_E|_{\text{high}} = 0.963 \left( \frac{V_{BE} - V_{BE}}{R_E} \right) = 1.02 I_E|_{\text{nominal}}$$

Thus, the maximum allowable ratio is

$$\frac{R_B}{R_E} = 5.73$$

$$(b) I_E = \frac{V_{BB} - V_{BE}}{R_E \left( 1 + \frac{R_B/R_E}{\beta + 1} \right)}$$

$$I_E R_E = \frac{V_{BB} - V_{BE}}{1 + \frac{5.73}{\beta + 1}}$$

$$\frac{V_{CC}}{3} = \frac{V_{BB} - V_{BE}}{1 + \frac{5.73}{101}}$$

$$\Rightarrow V_{BB} = V_{BE} + 0.352 V_{CC}$$

$$(c) V_{CC} = 5 \text{ V}$$

$$V_{BB} = 0.7 + 0.352 \times 5 = 2.46 \text{ V}$$

$$R_E = \frac{V_{CC}/3}{I_E} = \frac{5/3}{0.5} = 3.33 \text{ k}\Omega$$

$$R_B = 5.73 \times R_E = 19.08 \text{ k}\Omega$$

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2}$$

$$2.46 = 5 \frac{R_2}{R_1 + R_2}$$

$$2.46R_1 = 5 \frac{R_1 R_2}{R_1 + R_2} = 5R_B$$

$$= 5 \times 19.08$$

$$\Rightarrow R_1 = 38.8 \text{ k}\Omega$$

$$R_2 = 1 \left/ \left( \frac{1}{R_B} - \frac{1}{R_1} \right) \right. = 37.5 \text{ k}\Omega$$

$$(d) V_{CE} = V_{CC} - R_C I_C$$

$$1 = 5 - R_C \times 0.99 \times 0.5$$

$$\Rightarrow R_C = 8.1 \text{ k}\Omega$$

Check design:

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 5 \times \frac{37.5}{37.5 + 38.8}$$

$$= 2.46 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 37.5 \parallel 38.8 = 19.07 \text{ k}\Omega$$

$$I_E|_{\text{nominal}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{101}} = 0.5 \text{ mA}$$

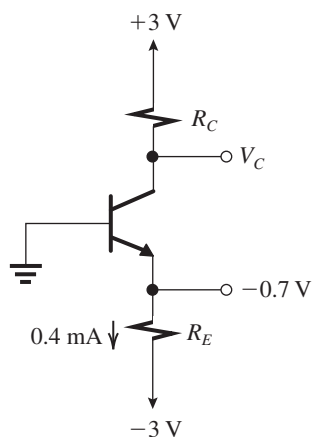
$$I_E|_{\text{low}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{51}} = 0.475 \text{ mA}$$

which is 5% lower than  $I_E|_{\text{nominal}}$ , and

$$I_E|_{\text{high}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{151}} = 0.509 \text{ mA}$$

which is 1.8% higher than  $I_E|_{\text{nominal}}$ .

### 6.112



$$R_E = \frac{-0.7 - (-3)}{0.4}$$

$$= 5.75 \text{ k}\Omega$$

To maximize gain while allowing for  $\pm 1 \text{ V}$  signal swing at the collector, design for the lowest possible  $V_C$  consistent with

$$V_C - 1 = -0.7 + V_{CE\text{sat}}$$

$$= -0.7 + 0.3 = -0.4 \text{ V}$$

$$V_C = 0.6 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{3 - 0.6}{0.39} = 6.2 \text{ k}\Omega$$

As temperature increases from  $25^\circ\text{C}$  to  $125^\circ\text{C}$ , (i.e., by  $100^\circ\text{C}$ ),  $V_{BE}$  decreases by  $2 \text{ mV} \times 100 =$

$$-200 \text{ mV}. \text{ Thus } I_E \text{ increases by } \frac{0.2 \text{ V}}{R_E} =$$

$$\frac{0.2 \text{ V}}{5.75 \text{ k}\Omega} = 0.035 \text{ mA} \text{ to become } 0.435 \text{ mA}. \text{ The}$$

collector current becomes

$$I_C = \frac{\beta}{\beta + 1} \times 0.435$$

where  $\beta$  is the increased value of 150,

$$I_C = \frac{150}{151} \times 0.435 \text{ mA}$$

$$= 0.432 \text{ mA}$$

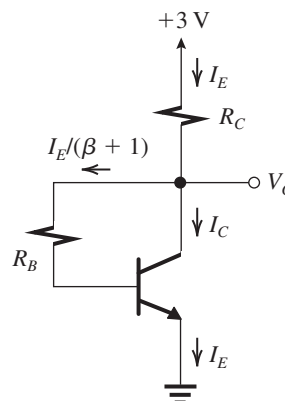
Thus,

$$\Delta I_C = 0.432 - 0.39 = 0.042 \text{ mA}$$

for a percentage increase of

$$\frac{\Delta I_C}{I_C} \times 100 = \frac{0.042}{0.39} \times 100 = 10.8\%$$

### 6.113



$$V_C = V_{CE\text{sat}} + 1 \text{ V}$$

$$= 1.3 \text{ V}$$

$$I_E = \frac{3 - 1.3}{R_C} = 0.5 \text{ mA}$$

$$\Rightarrow R_C = 3.4 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.5}{101} \approx 0.005 \text{ mA}$$

$$V_C = V_{BE} + I_B R_B$$

$$1.3 = 0.7 + 0.005 \times R_B$$

$$\Rightarrow R_B = 120 \text{ k}\Omega$$

Standard 5% resistors:

$$R_C = 3.3 \text{ k}\Omega$$

$$R_B = 120 \text{ k}\Omega$$

If the actual BJT has  $\beta = 50$ , then

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}} = \frac{3 - 0.7}{3.3 + \frac{120}{51}} = 0.41 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.41 \times 3.3 = 1.65 \text{ V}$$

Allowable negative signal swing at the collector is as follows:

$$V_C - V_{CEsat} = 1.65 - 0.3 = 1.35 \text{ V}$$

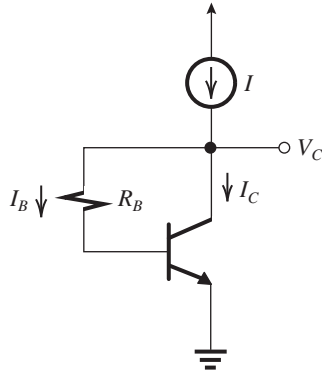
An equal positive swing is *just* possible. For  $\beta = 150$ :

$$I_E = \frac{3 - 0.7}{3.3 + \frac{120}{151}} = 0.56 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.56 \times 3.3 = 1.15 \text{ V}$$

Allowable negative signal swing at the collector =  $1.15 - 0.3 = 0.85 \text{ V}$ . An equal positive swing is possible.

6.114



$$I_C = 1 \text{ mA}$$

$$I = I_C + I_B$$

$$= I_C + \frac{I_C}{\beta} = I_C \left(1 + \frac{1}{\beta}\right)$$

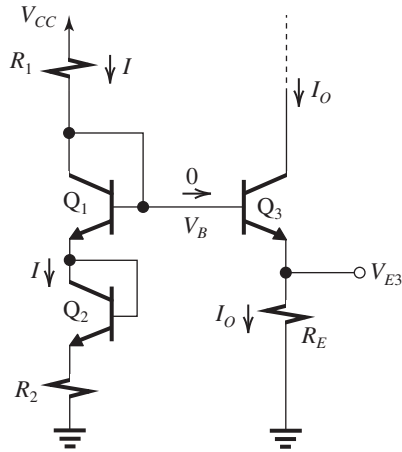
$$I = 1.01 \text{ mA}$$

$$V_C = 1.5 \text{ V} = I_B R_B + V_{BE}$$

$$1.5 = 0.01 \times R_B + 0.7$$

$$R_B = 80 \text{ k}\Omega$$

6.115



$$I = \frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1 + R_2}$$

$$V_B = I R_2 + V_{BE2} + V_{BE1}$$

$$V_{E3} = V_B - V_{BE3}$$

$$V_{E3} = I R_2 + V_{BE2} + V_{BE1} - V_{BE3}$$

$$= (V_{CC} - V_{BE1} - V_{BE2}) \frac{R_2}{R_1 + R_2} + V_{BE1}$$

$$+ V_{BE2} - V_{BE3}$$

$$I_O = \frac{V_{E3}}{R_E} = \frac{1}{R_E} \left[ (V_{CC} - V_{BE1} - V_{BE2}) \frac{R_2}{R_1 + R_2} + V_{BE1} + V_{BE2} - V_{BE3} \right]$$

Now, for  $R_1 = R_2$  and the currents in all junctions equal,

$$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$$

$$I_O = \frac{1}{R_E} \left[ (V_{CC} - 2V_{BE}) \times \frac{1}{2} + V_{BE} \right]$$

$$I_O = \frac{V_{CC}}{2R_E} \quad \text{Q.E.D.}$$

Thus,

$$I_O R_E = \frac{V_{CC}}{2}$$

$$V_B = \frac{V_{CC}}{2} + V_{BE}$$

$$I = (V_B - 2V_{BE})/R_2 = \left( \frac{V_{CC}}{2} - V_{BE} \right) / R_2$$

But since  $I$  must be equal to  $I_O$ , we have

$$\frac{V_{CC}}{2R_E} = \frac{V_{CC}/2 - V_{BE}}{R_2}$$

Thus,

$$R_1 = R_2 = R_E \left( \frac{V_{CC} - 2V_{BE}}{V_{CC}} \right)$$

For  $V_{CC} = 10 \text{ V}$  and  $V_{BE} = 0.7 \text{ V}$ ,

$$R_1 = R_2 = R_E \left( \frac{10 - 1.4}{10} \right) = 0.86R_E$$

To obtain  $I_O = 0.5 \text{ mA}$ ,

$$0.5 = \frac{V_{CC}}{2R_E} = \frac{10}{2R_E}$$

$$\Rightarrow R_E = 10 \text{ k}\Omega$$

$$R_1 = R_2 = 8.6 \text{ k}\Omega$$

**6.116** (a) Refer to Fig. 6.57(a). The dc circuit can be obtained by opening all coupling and bypass capacitors, resulting in the circuit shown in Fig. 1.

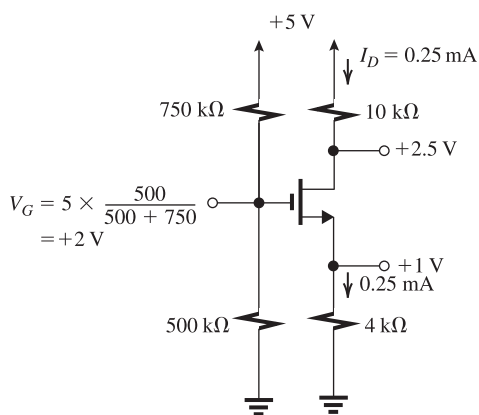


Figure 1

See analysis on figure.

$$V_{GS} = 2 - 1 = 1 \text{ V}$$

$$V_{OV} = V_{GS} - V_t = 1 - 0.75 = 0.25 \text{ V}$$

Since  $V_D$  at 2.5 V is 1.2 V higher than  $V_S + V_{OV} = 1 + 0.25 = 1.25 \text{ V}$ , the transistor is

This figure belongs to Problem 6.116.

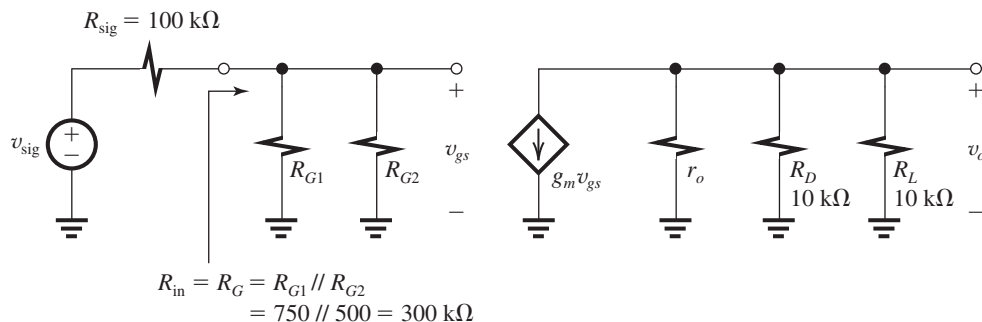


Figure 2

indeed operating in saturation. (Equivalent:  $V_D = 2.5 \text{ V}$  is higher than  $V_G - V_t = 1.25 \text{ V}$ .)

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.25 = \frac{1}{2} k_n \times 0.25^2$$

$$\Rightarrow k_n = 8 \text{ mA/V}^2$$

(b) The amplifier small-signal equivalent-circuit model is shown in Fig. 2.

$$R_{in} = R_{G1} \parallel R_{G2} = 750 \text{ k}\Omega \parallel 500 \text{ k}\Omega = 300 \text{ k}\Omega$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{0.25} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{50}{0.25} = 200 \text{ k}\Omega$$

$$G_v = - \frac{R_{in}}{R_{in} + R_{sig}} g_m (r_o \parallel R_D \parallel R_L)$$

$$= - \frac{300}{300 + 100} \times 2 \times (200 \parallel 10 \parallel 10)$$

$$= -7.3 \text{ V/V}$$

(c)  $V_G = 2 \text{ V}$ ,  $V_D = 2.5 \text{ V}$

$$\hat{v}_G = 2 + \hat{v}_{gs}, \quad \hat{v}_D = 2.5 - |A_v| \hat{v}_{gs}$$

where

$$|A_v| = g_m (r_o \parallel R_D \parallel R_L) = 9.8 \text{ V/V}$$

To remain in saturation,

$$\hat{v}_D \geq \hat{v}_G - V_t$$

$$2.5 - 9.8 \hat{v}_{gs} \geq 2 + \hat{v}_{gs} - 0.75$$

This is satisfied with equality at

$$\hat{v}_{gs} = \frac{2.5 - 1.25}{10.8} = 0.116 \text{ V}$$

The corresponding value of  $\hat{v}_{sig}$  is

$$\hat{v}_{sig} = \hat{v}_{gs} \left( \frac{300 + 100}{300} \right) = 0.154 \text{ V}$$

The corresponding amplitude at the output will be

$$|G_v| \hat{v}_{\text{sig}} = 7.3 \times 0.154 = 1.13 \text{ V}$$

(d) To be able to double  $\hat{v}_{\text{sig}}$  without leaving saturation, we must reduce  $\hat{v}_{gs}$  to half of what would be its new value; that is, we must keep  $\hat{v}_{gs}$  unchanged. This in turn can be achieved by connecting an unbypassed  $R_s$  equal to  $1/g_m$ ,

$$R_s = \frac{1}{2 \text{ mA/V}} = 500 \Omega$$

Since  $\hat{v}_{gs}$  does not change, the output voltage also will not change, thus  $\hat{v}_o = 1.13 \text{ V}$ .

**6.117** (a) DC bias: Refer to the circuit in Fig. P6.117 with all capacitors eliminated:

$$R_{\text{in}} \text{ at gate} = R_G = 10 \text{ M}\Omega$$

$V_G = 0$ , thus  $V_S = -V_{GS}$ , where  $V_{GS}$  can be obtained from

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.4 \text{ V}$$

$$V_{GS} = V_t + 0.4 = 0.8 + 0.4 = 1.2 \text{ V}$$

$$V_S = -1.2 \text{ V}$$

$$R_S = \frac{-1.2 - (-5)}{0.4} = 9.5 \text{ k}\Omega$$

To remain in saturation, the minimum drain voltage must be limited to  $V_G - V_t = 0 - 0.8 = -0.8 \text{ V}$ . Now, to allow for 0.8-V negative signal swing, we must have

$$V_D = 0 \text{ V}$$

and

$$R_D = \frac{5 - 0}{0.4} = 12.5 \text{ k}\Omega$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.4} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

(c) If terminal Z is connected to ground, the circuit becomes a CS amplifier,

$$G_v = -\frac{v_y}{v_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}}} \times -g_m(r_o \parallel R_D \parallel R_L)$$

$$= -\frac{10}{10 + 1} \times 2 \times (100 \parallel 12.5 \parallel 10)$$

$$= -9.6 \text{ V/V}$$

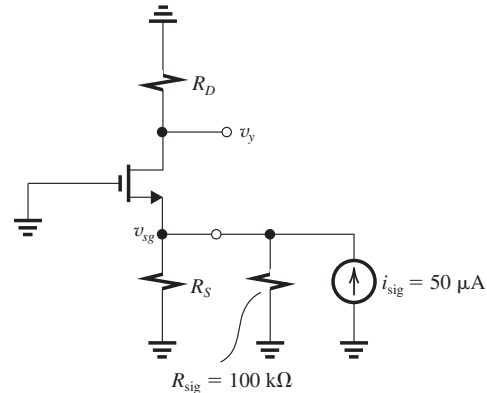
(d) If terminal Y is grounded, the circuit becomes a CD or source-follower amplifier:

$$\begin{aligned} \frac{v_z}{v_x} &= \frac{(R_S \parallel r_o)}{(R_S \parallel r_o) + \frac{1}{g_m}} \\ &= \frac{(9.5 \parallel 100)}{(9.5 \parallel 100) + \frac{1}{2}} = 0.946 \text{ V/V} \end{aligned}$$

Looking into terminal Z, we see  $R_o$ :

$$\begin{aligned} R_o &= R_S \parallel r_o \parallel \frac{1}{g_m} \\ &= 9.5 \parallel 100 \parallel \frac{1}{2} = 473 \Omega \end{aligned}$$

(e) If X is grounded, the circuit becomes a CG amplifier.



The figure shows the circuit prepared for signal calculations.

$$\begin{aligned} v_{sg} &= i_{\text{sig}} \times \left[ R_{\text{sig}} \parallel R_S \parallel \frac{1}{g_m} \right] \\ &= 50 \times 10^{-3} (\text{mA}) \left[ 100 \parallel 9.5 \parallel \frac{1}{2} \right] (\text{k}\Omega) \\ &= 0.024 \text{ V} \end{aligned}$$

$$\begin{aligned} v_y &= (g_m R_D) v_{sg} \\ &= (2 \times 12.5) \times 0.024 = 0.6 \text{ V} \end{aligned}$$

**6.118** (a) Refer to the circuit of Fig. P6.118(a):

$$A_{vo} \equiv \frac{v_{o1}}{v_i} = \frac{10}{10 + \frac{1}{g_m}} = \frac{10}{10 + \frac{1}{10}} = 0.99 \text{ V/V}$$

$$R_o = \frac{1}{g_m} \parallel 10 \text{ k}\Omega = 0.1 \parallel 10 = 99 \Omega$$

(b) Refer to Fig. P6.118(b):

$$R_{\text{in}} = 10 \text{ k}\Omega \parallel \frac{1}{g_m} = 10 \parallel 0.1 = 99 \Omega$$

$$\frac{v_o}{v_{i2}} = \frac{5 \parallel 2}{1/g_m} = 10(5 \parallel 2) = 14.3 \text{ V/V}$$

$$(c) v_{i2} = (A_{vo} v_i) \frac{R_{\text{in}}}{R_{\text{in}} + R_o}$$

$$= 0.99 \times v_i \times \frac{99}{99 + 99}$$

$$\simeq 0.5v_i$$

$$v_o = 14.3 \times v_{i2} = 14.3 \times 0.5v_i$$

$$\frac{v_o}{v_i} = 7.15 \text{ V/V}$$

**6.119** Refer to the circuit of Fig. 6.58.

DC design:

$$V_B = 5 \text{ V}, \quad V_{BE} = 0.7 \text{ V}$$

$$V_E = 4.3 \text{ V}$$

For

$$I_E = 2 \text{ mA}, \quad R_E = \frac{V_E}{I_E} = \frac{4.3}{2} = 2.15 \text{ k}\Omega$$

$$I_{R_2} = 0.2 \text{ mA}, \quad R_{B2} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{2}{101} \simeq 0.02 \text{ mA}$$

$$I_{R_1} = I_{R_2} + I_B = 0.2 + 0.02 = 0.22 \text{ mA}$$

$$R_{B1} = \frac{V_{CC} - V_B}{I_{R_1}} = \frac{15 - 5}{0.22} = 45.5 \text{ k}\Omega$$

Choosing 5% resistors:

$$R_E = 2.2 \text{ k}\Omega, \quad R_{B1} = 47 \text{ k}\Omega, \quad R_{B2} = 24 \text{ k}\Omega$$

For these values,

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}} = 15 \times \frac{24}{24 + 47} = 5.07 \text{ V}$$

$$R_B = R_{B1} \parallel R_{B2} = 47 \parallel 24 = 15.89 \text{ k}\Omega$$

$$I_E = \frac{5.07 - 0.7}{2.2 + \frac{15.89}{101}} = 1.85 \text{ mA}$$

$$V_B = I_E R_E + V_{BE} = 1.85 \times 2.2 + 0.7 = 4.8 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 1.85 = 1.84 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.84}{0.025} = 73.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{73.4} = 1.36 \text{ k}\Omega$$

$$R_{in} = R_{B1} \parallel R_{B2} \parallel r_\pi = 47 \parallel 24 \parallel 1.36 = 1.25 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.25}{1.25 + 2} = 0.385 \text{ V/V}$$

For an overall gain of  $-40 \text{ V/V}$ ,

$$\frac{v_o}{v_\pi} = -\frac{40}{0.385} = -104 \text{ V/V}$$

But

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$-104 = -73.4(R_C \parallel 2)$$

$$(R_C \parallel 2) = 1.416$$

$$R_C = 4.86 \text{ k}\Omega$$

We can select either  $4.7 \text{ k}\Omega$  or  $5.1 \text{ k}\Omega$ . With  $4.7 \text{ k}\Omega$ , the gain will be

$$\frac{v_o}{v_{sig}} = -0.385 \times 73.4 \times (4.7 \parallel 2) = -39.6 \text{ V/V}$$

which is slightly lower than the required  $-40 \text{ V/V}$ , and we will obtain

$$V_C = 15 - 4.7 \times 1.84 = 6.4 \text{ V}$$

allowing for about  $2 \text{ V}$  of negative signal swing at the collector. If we choose  $5.1 \text{ k}\Omega$ , the gain will be

$$\frac{v_o}{v_{sig}} = -0.385 \times 73.4 \times (5.1 \parallel 2) = -40.6 \text{ V/V}$$

which is slightly higher than the required gain, and we will obtain

$$V_C = 15 - 5.1 \times 1.84 = 5.6 \text{ V}$$

which allows for only  $1.2\text{-V}$  negative signal swing.

**6.120** Refer to the circuit of Fig. P6.120.

(a)  $I_E = 0.2 \text{ mA}$ . Writing a loop equation for the base-emitter circuit results in

$$I_B R_{sig} + V_{BE} + I_E R_E = 3$$

$$\frac{I_E}{\beta + 1} R_{sig} + V_{BE} + I_E R_E = 3$$

$$\frac{0.2}{101} \times 5 + 0.7 + 0.2 R_E = 3$$

$$\Rightarrow R_E = 11.5 \text{ k}\Omega$$

(b)  $I_C = \alpha I_E \simeq 0.2 \text{ mA}$

$$V_C = 0.5 = 3 - 0.2 R_C$$

$$\Rightarrow R_C = 12.5 \text{ k}\Omega$$

$$(c) g_m = \frac{I_C}{V_T} = \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

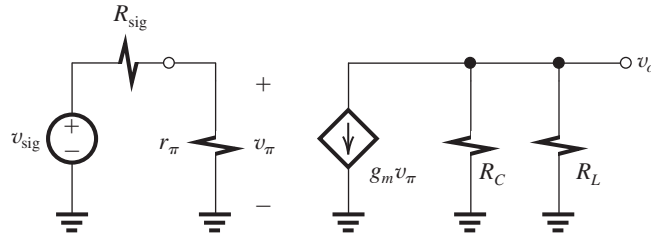
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$G_v = \frac{v_o}{v_{sig}} = -\frac{12.5}{12.5 + 5} \times 8 \times (12.5 \parallel 10)$$

$$= -31.7 \text{ V/V}$$



This figure belongs to Problem 6.120.



**6.121** Refer to the circuit in Fig. P6.121:

$$I_E = 0.2 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

Note that the emitter has a resistance  $R_e = 250 \Omega$ .

$$R_{in} = 100 \text{ k}\Omega \parallel (\beta + 1)(r_e + R_e)$$

$$= 100 \parallel [101 \times (0.125 + 0.25)]$$

$$= 100 \parallel 37.9 = 27.5 \text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{27.5}{27.5 + 10} = 0.733 \text{ V/V}$$

$$\frac{v_o}{v_b} = -\alpha \frac{\text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$\simeq -\frac{10 \parallel 10}{0.125 + 0.25} = -13.33 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = -0.733 \times 13.33 = -9.8 \text{ V/V}$$

For  $v_{be}$  to be limited to 5 mV, the signal between base and ground will be  $\hat{v}_b = \hat{v}_{be} \left( \frac{r_e + R_E}{r_e} \right) =$

$$5 \left( \frac{0.125 + 0.25}{0.125} \right) = 15 \text{ mV. The limit on } v_{sig}$$

can be obtained by dividing the 15 mV by  $v_b/v_{sig}$ ,

$$\hat{v}_{sig} = \frac{15 \text{ mV}}{0.733} = 20.5 \text{ mV}$$

Correspondingly, at the output we have

$$\hat{v}_o = |G_v| \hat{v}_{sig} = 9.8 \times 20.5 = 201 \text{ mV} \simeq 0.2 \text{ V}$$

**6.122** Refer to the circuit in Fig. P6.122.

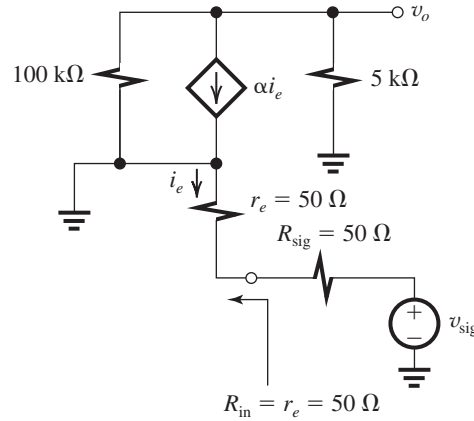
The dc emitter current is equal to 0.5 mA, and

$$I_C = \alpha I_E \simeq 0.5 \text{ mA; also,}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$R_{in} = r_e = 50 \Omega$$

$$i_e = \frac{-v_{sig}}{r_e + R_{sig}} = \frac{-v_{sig}}{50 + 50}$$



$$= \frac{-v_{sig}}{100 \Omega} = \frac{-v_{sig}}{0.1 \text{ k}\Omega}$$

At the output node,

$$v_o = -\alpha i_e (5 \parallel 100)$$

$$= \alpha \frac{v_{sig}}{0.1} (5 \parallel 100)$$

$$\frac{v_o}{v_{sig}} = \alpha \frac{5 \parallel 100}{0.1} \simeq 47.6 \text{ V/V}$$

$$\mathbf{6.123} \text{ (a) } I_E = \frac{3 - 0.7}{1 + \frac{100}{\beta + 1}}$$

$$\beta = 50:$$

$$I_E = \frac{2.3}{1 + \frac{100}{51}} = 0.78 \text{ mA}$$

$$V_E = I_E R_E = 0.78 \text{ V}$$

$$V_B = V_E + 0.7 = 1.48 \text{ V}$$

$$\beta = 200:$$

$$I_E = \frac{2.3}{1 + \frac{100}{201}} = 1.54 \text{ mA}$$

$$V_E = I_E R_E = 1.54 \text{ V}$$

$$V_B = V_E + 0.7 = 2.24 \text{ V}$$

$$\text{(b) } R_{in} = 100 \parallel (\beta + 1)[r_e + (1 \parallel 1)]$$

$$= 100 \parallel (\beta + 1)(r_e + 0.5)$$

$$\beta = 50:$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.78 \text{ mA}} = 32.1 \Omega$$

$$R_{in} = 100 \parallel [51 \times (0.0321 + 0.5)]$$

$$= 21.3 \text{ k}\Omega$$

$$\beta = 200:$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1.54 \text{ mA}} = 16.2 \Omega$$

$$R_{in} = 100 \parallel [201 \times (0.0162 + 0.5)]$$

$$= 50.9 \text{ k}\Omega$$

$$(c) \frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$\frac{v_o}{v_b} = \frac{(1 \parallel 1)}{(1 \parallel 1) + r_e} = \frac{500}{500 + r_e} \quad (r_e \text{ in } \Omega)$$

$$\beta = 50:$$

$$\frac{v_b}{v_{sig}} = \frac{21.3}{21.3 + 10} = 0.68 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 32.1} = 0.94 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.68 \times 0.94 = 0.64 \text{ V/V}$$

$$\beta = 200:$$

$$\frac{v_b}{v_{sig}} = \frac{50.9}{50.9 + 10} = 0.836 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 16.2} = 0.969 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.836 \times 0.969 = 0.81 \text{ V/V}$$

**6.124** Refer to the equivalent circuit in Fig. 6.57(c).

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_D \parallel R_L \parallel r_o)$$

$$= -\frac{R_G}{R_G + R_{sig}} g_m (R_D \parallel R_L \parallel r_o)$$

$$= -\frac{10}{10 + 1} \times 5 \times (10 \parallel 50 \parallel 20)$$

$$= -26.7 \text{ V/V}$$

**6.125** Refer to Fig. P6.118.

(a) DC bias:

$$|V_{OV}| = 0.25 \text{ V} \Rightarrow V_{SG} = |V_{tp}| + |V_{OV}| = 1 \text{ V}$$

Since  $V_G = 0 \text{ V}$ ,  $V_S = V_{SG} = +1 \text{ V}$ , and

$$I_D = \frac{2.5 - 1}{R_S} = 0.5 \text{ mA}$$

$$\Rightarrow R_S = \frac{1.5}{0.5} = 3 \text{ k}\Omega$$

$$(b) G_v = -g_m R_D$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.25} = 4 \text{ mA/V}$$

Thus,

$$-12 = -4R_D \Rightarrow R_D = 3 \text{ k}\Omega$$

$$(c) v_G = 0 \text{ V (dc)} + v_{sig}$$

$$v_{Gmin} = -\hat{v}_{sig}$$

$$\hat{v}_D = V_D + |G_v| \hat{v}_{sig}$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.5 \times 3 = -1 \text{ V}$$

To remain in saturation,

$$\hat{v}_D \leq \check{v}_G + |V_{tp}|$$

$$-1 + 12 \hat{v}_{sig} \leq -\hat{v}_{sig} + 0.75$$

Satisfying this constraint with equality gives

$$\hat{v}_{sig} = 0.135 \text{ V}$$

and the corresponding output voltage

$$\hat{v}_d = |G_v| \hat{v}_{sig} = 1.62 \text{ V}$$

(d) If  $\hat{v}_{sig} = 50 \text{ mV}$ , then

$$V_D + |G_v| \hat{v}_{sig} = -\hat{v}_{sig} + |V_{tp}|$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.5 R_D$$

and

$$|G_v| = g_m R_D = 4R_D$$

Thus

$$-2.5 + 0.5 R_D + 4R_D \hat{v}_{sig} = -\hat{v}_{sig} + |V_{tp}|$$

$$-2.5 + 0.5 R_D + 4R_D \times 0.05 = -0.05 + 0.75$$

$$0.4R_D = 3.15$$

$$\Rightarrow R_D = 4.6 \text{ k}\Omega$$

$$G_v = -g_m R_D = -4 \times 4.6 = -18.4 \text{ V/V}$$

**6.126** Refer to Fig. P7.126.

$$R_{i2} = \frac{1}{g_{m2}} = 50 \Omega$$

$$\Rightarrow g_{m2} = \frac{1}{50} \text{ A/V} = 20 \text{ mA/V}$$

If  $Q_1$  is biased at the same point as  $Q_2$ , then

$$\begin{aligned}
 g_{m1} &= g_{m2} = 20 \text{ mA/V} \\
 i_{d1} &= g_{m1} \times 5 \text{ (mV)} \\
 &= 20 \times 0.005 = 0.1 \text{ mA} \\
 v_{d1} &= i_{d1} \times 50 \ \Omega \\
 &= 0.1 \times 50 = 5 \text{ mV} \\
 v_o &= i_{d1} R_D = 1 \text{ V} \\
 R_D &= \frac{1 \text{ V}}{0.1 \text{ mA}} = 10 \text{ k}\Omega
 \end{aligned}$$

6.127 (a) DC bias:

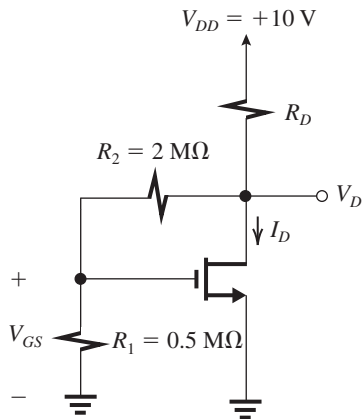


Figure 1

$$\begin{aligned}
 V_{GS} &= V_i + V_{OV} \\
 &= 0.6 + 0.2 = 0.8 \text{ V}
 \end{aligned}$$

From the voltage divider ( $R_1, R_2$ ; see Fig. 1), we can write

$$\begin{aligned}
 V_{GS} &= V_D \frac{R_1}{R_1 + R_2} = V_D \frac{0.5}{0.5 + 2} \\
 V_D &= 5V_{GS} = 5 \times 0.8 = 4 \text{ V} \\
 I_D &= \frac{1}{2} k_n V_{OV}^2 \left( 1 + \frac{V_{DS}}{V_A} \right) \\
 I_D &= \frac{1}{2} \times 5 \times 0.2^2 \left( 1 + \frac{4}{60} \right)
 \end{aligned}$$

This figure belongs to Problem 6.127(c).

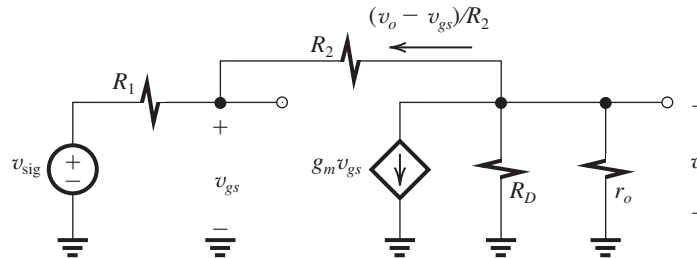


Figure 2

$$= 0.1066 \text{ mA}$$

The current in the voltage divider is

$$I = \frac{V_D}{R_1 + R_2} = \frac{4}{2.5} = 1.6 \ \mu\text{A} = 0.0016 \text{ mA}$$

Thus the current through  $R_D$  will be  $(0.1066 + 0.0016) \simeq 0.108 \text{ mA}$  and

$$R_D = \frac{V_{DD} - V_D}{0.108} = \frac{10 - 4}{0.108} = 55.6 \text{ k}\Omega$$

$$(b) \ g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.107}{0.2} = 1.07 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{60}{0.107} = 561 \text{ k}\Omega$$

(c) Replacing the MOSFET with its hybrid- $\pi$  model, we obtain the small-signal equivalent circuit of the amplifier, shown in Fig. 2.

Node equation at the output:

$$\frac{v_o}{R_D} + \frac{v_o}{r_o} + \frac{v_o - v_{gs}}{R_2} + g_m v_{gs} = 0$$

$$v_o \left( \frac{1}{R_D} + \frac{1}{r_o} + \frac{1}{R_2} \right) = -g_m \left( 1 - \frac{1}{g_m R_2} \right) v_{gs}$$

Thus,

$$v_o = \underbrace{-g_m (R_D \parallel r_o \parallel R_2)}_A \left( 1 - \frac{1}{g_m R_2} \right) v_{gs} \quad (1)$$

Next, we express  $v_{gs}$  in terms of  $v_{sig}$  and  $v_o$  using superposition:

$$v_{gs} = v_{sig} \frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2} \quad (2)$$

Substituting for  $v_{gs}$  from Eq. (2) into Eq. (1) yields

$$v_o = -A v_{sig} \frac{R_2}{R_1 + R_2} - A v_o \frac{R_1}{R_1 + R_2}$$

where

$$A = g_m (R_D \parallel r_o \parallel R_2) \left( 1 - \frac{1}{g_m R_2} \right)$$

Thus,

$$v_o \left( 1 + A \frac{R_1}{R_1 + R_2} \right) = -A \frac{R_2}{R_1 + R_2} v_{\text{sig}}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{-A \frac{R_2}{R_1 + R_2}}{1 + A \frac{R_1}{R_1 + R_2}}$$

$$= \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

Thus,

$$\frac{v_o}{v_{\text{sig}}} = - \frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{g_m(R_D \parallel r_o \parallel R_2)(1 - 1/g_m R_2)}}$$

Q.E.D.

Substituting numerical values yields

$$\frac{v_o}{v_{\text{sig}}} = - \frac{2/0.5}{1 + \frac{1 + (2/0.5)}{1.07(55.6 \parallel 561 \parallel 2000)(1 - 1/1.07 \times 2000)}}$$

$$= - \frac{4}{\left( 1 + \frac{5}{52.8} \right)}$$

$$= -3.65 \text{ V/V}$$

Note that the gain is nearly equal to  $-R_2/R_1 = -4$ , which is the gain of an op amp connected in the inverting configuration.

6.128 (a) DC bias:

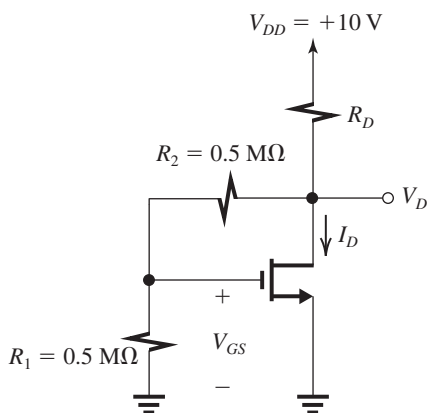


Figure 1

$$V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.6 + 0.2 = 0.8 \text{ V}$$

From the voltage divider ( $R_1, R_2$ ; see Fig. 1), we can write

$$V_{GS} = \frac{R_1}{R_1 + R_2} V_D = \frac{0.5}{0.5 + 0.5} V_D = 0.5 V_D$$

Thus

$$V_D = 2V_{GS} = 1.6 \text{ V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$= \frac{1}{2} \times 5 \times 0.2^2 = 0.1 \text{ mA}$$

$$I_{\text{divider}} = \frac{V_D}{1 \text{ M}\Omega} = \frac{1.6 \text{ V}}{1 \text{ M}\Omega} = 1.6 \mu\text{A}$$

$$I_{RD} = 0.1 + 0.0016 \approx 0.102 \text{ mA}$$

$$R_D = \frac{V_{DD} - V_D}{I_{RD}} = \frac{10 - 1.6}{0.102} = 82.4 \text{ k}\Omega$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

(c) Replacing the MOSFET with its  $T$  model results in the amplifier equivalent circuit shown in Fig. 2. At the output node,

$$v_o = i[R_D \parallel (R_1 + R_2)]$$

$$v_o = iR'_D \tag{1}$$

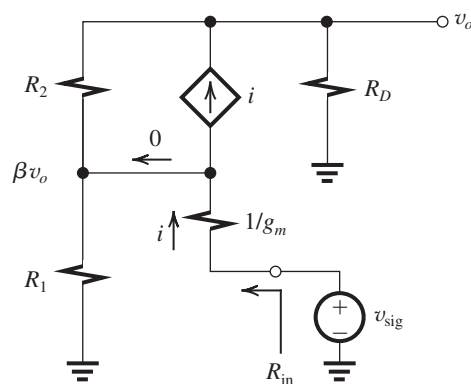


Figure 2

where  $R'_D = R_D \parallel (R_1 + R_2)$ . The voltage at the gate is a fraction  $\beta$  of  $v_o$  with

$$\beta = \frac{R_1}{R_1 + R_2}$$

Now, the current  $i$  can be found from

$$i = \frac{v_{\text{sig}} - \beta v_o}{1/g_m} = g_m v_{\text{sig}} - \beta g_m v_o \tag{2}$$

Substituting for  $i$  from Eq. (2) into Eq. (1) yields

$$v_o = (g_m v_{\text{sig}} - \beta g_m v_o) R'_D$$

Thus

$$\frac{v_o}{v_{\text{sig}}} = \frac{g_m R'_D}{1 + \beta g_m R'_D}$$

$$\begin{aligned}
 &= \frac{1/\beta}{1 + \frac{1/\beta}{g_m R'_D}} \\
 &= \frac{1 + (R_2/R_1)}{1 + \frac{1 + R_2/R_1}{g_m R'_D}} \quad \text{Q.E.D.} \quad (3)
 \end{aligned}$$

The input resistance  $R_{in}$  can be obtained as follows:

$$R_{in} = \frac{v_{sig}}{i}$$

Substituting for  $i$  from Eq. (1) yields

$$R_{in} = \frac{v_{sig}}{v_o} R'_D$$

and replacing  $\frac{v_{sig}}{v_o}$  by the inverse of the gain expression in Eq. (3) gives

$$\begin{aligned}
 R_{in} &= R'_D \left[ \frac{1}{g_m R'_D} + \frac{1}{1 + (R_2/R_1)} \right] \\
 R_{in} &= \frac{1}{g_m} \left[ 1 + g_m R'_D \frac{R_1}{R_1 + R_2} \right] \quad \text{Q.E.D.}
 \end{aligned}$$

(d) Substituting numerical values:

$$\begin{aligned}
 \frac{v_o}{v_{sig}} &= \frac{1 + (0.5/0.5)}{1 + \frac{1 + (0.5/0.5)}{1 \times (82.4 \parallel 1000)}} \\
 &= \frac{2}{1 + \frac{2}{76.13}} = 1.95 \text{ V/V}
 \end{aligned}$$

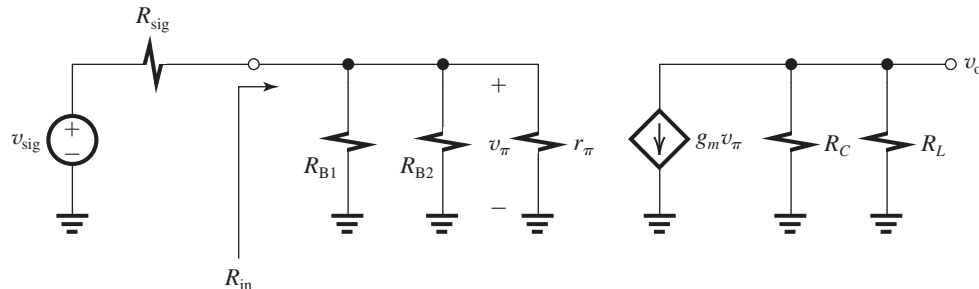
Note that the gain  $\simeq 1 + \frac{R_2}{R_1} = 2$ , similar to that of an op amp connected in the noninverting configuration!

$$\begin{aligned}
 R_{in} &= \frac{1}{1} \left[ 1 + 1 \times (82.4 \parallel 1000) \frac{0.5}{0.5 + 0.5} \right] \\
 &= 39.1 \text{ k}\Omega
 \end{aligned}$$

**6.129** Refer to the circuit of Fig. P6.129.

$$I_C = \frac{\alpha(V_{BB} - V_{BE})}{R_E + \frac{R_B}{\beta + 1}}$$

This figure belongs to Problem 6.129.



where

$$V_{BB} = V_{CC} \frac{R_2}{R_2 + R_1} = 15 \times \frac{15}{15 + 27} = 5.357 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 15 \parallel 27 = 9.643 \text{ k}\Omega$$

$$I_C = \frac{0.99(5.357 - 0.7)}{2.4 + \frac{9.643}{101}} = 1.85 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.85 \text{ mA}}{0.025 \text{ V}} = 74 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{74} = 1.35 \text{ k}\Omega$$

Replacing the BJT with its hybrid- $\pi$  model results in the equivalent circuit shown at the bottom of the page:

$$R_{in} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi = 9.643 \parallel 1.35 = 1.18 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.18}{1.18 + 2} = 0.371 \text{ V/V}$$

$$\begin{aligned}
 \frac{v_o}{v_\pi} &= -g_m(R_C \parallel R_L) \\
 &= -74(3.9 \parallel 2) = -97.83
 \end{aligned}$$

$$\frac{v_o}{v_{sig}} = -0.371 \times 97.83 = -36.3 \text{ V/V}$$

**6.130** Refer to the circuit of Fig. P6.129:

$$I_C = \frac{\alpha(V_{BB} - V_{BE})}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC} \frac{R_2}{R_2 + R_1} = 15 \times \frac{47}{47 + 82} = 5.465 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 47 \parallel 82 = 29.88 \text{ k}\Omega$$

$$I_C = \frac{0.99(5.465 - 0.7)}{7.2 + \frac{29.88}{101}} = 0.63 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.63}{0.025} = 25.2 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{25.2} = 4 \text{ k}\Omega$$

$$R_{\text{in}} = R_1 \parallel R_2 \parallel r_{\pi} = R_B \parallel r_{\pi} \\ = 29.88 \parallel 4 = 3.5 \text{ k}\Omega$$

$$\frac{v_{\pi}}{v_{\text{sig}}} = \frac{3.5}{3.5 + 2} = 0.636 \text{ V/V}$$

$$\frac{v_o}{v_{\pi}} = -g_m(R_C \parallel R_L) \\ = -25.2(12 \parallel 2) = -43.2 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = -0.636 \times 43.2 = -27.5 \text{ V/V}$$

Comparing the results above to those of Problem 6.124, we see that raising the resistance values has indeed resulted in increasing the transmission from source to transistor base, from 0.371 V/V to 0.636 V/V. However, because  $I_C$  has decreased and  $g_m$  has correspondingly decreased, the gain from base to collector has decreased by a larger factor (from 97.83 V/V to 43.2 V/V), with the result that the overall gain has in fact decreased (from 36.3 V/V to 27.5 V/V). Thus, this is not a successful strategy!

**6.131** Refer to the circuit of Fig. P6.131.

DC voltage drop across  $R_B = 0.2 \text{ V}$ ,

$$I_B R_B = 0.2 \text{ V}$$

$$\frac{I}{\beta + 1} R_B = 0.2 \text{ V}$$

$$I R_B = 0.2 \times 101 \quad (1)$$

$$R_{\text{in}} = R_B \parallel r_{\pi} = 10 \text{ k}\Omega$$

$$R_B \parallel \frac{V_T}{I_B} = 10$$

$$R_B \parallel \frac{0.025}{I/(\beta + 1)} = 10$$

$$R_B \parallel \left( \frac{0.025 \times 101}{I} \right) = 10$$

$$\frac{R_B \times \frac{0.025 \times 101}{I}}{R_B + \frac{0.025 \times 101}{I}} = 10$$

$$\frac{0.025 \times 101 R_B}{I R_B + 0.025 \times 101} = 10 \quad (2)$$

Substituting for  $I R_B$  from Eq. (1) yields

$$\frac{0.025 \times 101 R_B}{0.2 \times 101 + 0.025 \times 101} = 10$$

$$\frac{0.025 R_B}{0.225} = 10$$

$$\Rightarrow R_B = 90 \text{ k}\Omega$$

$$I = \frac{0.2 \times 101}{90} = 0.22 \text{ mA}$$

To maximize the open-circuit voltage gain between base and collector while ensuring that the instantaneous collector voltage does not fall below  $(v_B - 0.4)$  when  $v_{be}$  is as high as 5 mV, we impose the constraint

$$V_C - |A_{vo}| \times 0.005 = V_B + 0.005 - 0.4$$

where

$$V_C = V_{CC} - I_C R_C \\ = 5 - 0.99 \times 0.22 R_C$$

$$\simeq 5 - 0.22 R_C$$

$$|A_{vo}| = g_m R_C = \frac{0.99 \times 0.22}{0.025} R_C = 8.7 R_C$$

and

$$V_B = -\frac{0.22}{101} \times 90 = -0.2 \text{ V}$$

Thus,

$$5 - 0.22 R_C - 8.7 R_C \times 0.005 = -0.2 - 0.395$$

$$\Rightarrow R_C = 21.2 \text{ k}\Omega$$

Selecting 5% resistors, we find

$$R_B = 91 \text{ k}\Omega$$

$$R_C = 22 \text{ k}\Omega$$

and specifying  $I$  to one significant digit gives

$$I = 0.2 \text{ mA}$$

$$g_m = \frac{\alpha I_C}{V_T} \simeq \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$A_{vo} = -g_m R_C = -8 \times 22 = -176 \text{ V/V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$R_{\text{in}} = R_B \parallel r_{\pi} = 91 \parallel 12.5 = 11 \text{ k}\Omega$$

$$G_v = -\frac{11}{20 + 11} \times 8(22 \parallel 20) \\ = -29.7 \text{ V/V}$$

**6.132** Refer to the circuit of Fig. P6.132.

(a) DC analysis of each of the two stages:

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 15 \frac{47}{100 + 47} \\ = 4.8 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 100 \parallel 47 = 32 \text{ k}\Omega$$

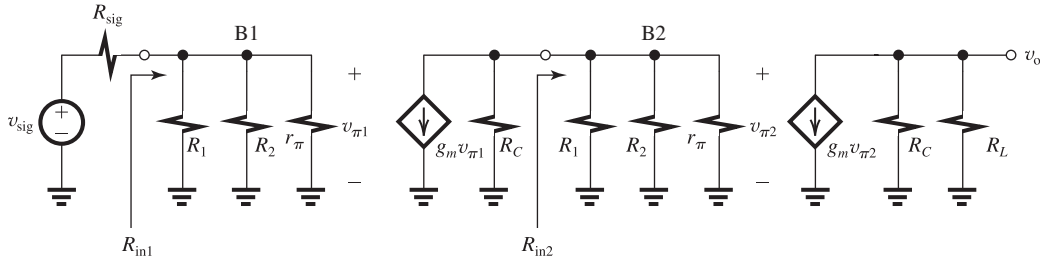
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

$$= \frac{4.8 - 0.7}{3.9 + \frac{32}{101}} = 0.97 \text{ mA} \simeq 1 \text{ mA}$$

$$I_C = \alpha I_E \simeq 1 \text{ mA}$$

$$V_C = V_{CC} - I_C R_C = 15 - 1 \times 6.8 = 8.2 \text{ V}$$

This figure belongs to Problem 6.132.



(b) See figure above.

$$g_m = \frac{I_C}{V_T} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 2.5 \text{ k}\Omega$$

$$R_{sig} = 5 \text{ k}\Omega, R_1 = 100 \text{ k}\Omega, R_2 = 47 \text{ k}\Omega, \\ R_C = 6.8 \text{ k}\Omega, R_L = 2 \text{ k}\Omega$$

$$(c) R_{in1} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi = 32 \parallel 2.5 \\ = 2.32 \text{ k}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2.32}{2.32 + 5} = 0.32 \text{ V/V}$$

$$(d) R_{in2} = R_1 \parallel R_2 \parallel r_\pi = R_{in1} = 2.32 \text{ k}\Omega$$

$$\frac{v_{b2}}{v_{\pi 1}} = \frac{v_{b2}}{v_{\pi 1}} = -g_m(R_C \parallel R_{in2}) \\ = -40(6.8 \parallel 2.32) = -69.2 \text{ V/V}$$

$$(e) \frac{v_o}{v_{b2}} = \frac{v_o}{v_{\pi 2}} = -g_m(R_C \parallel R_L) \\ = -40(6.8 \parallel 2) = -61.8 \text{ V/V}$$

$$(f) \frac{v_o}{v_{sig}} = \frac{v_o}{v_{b2}} \times \frac{v_{b2}}{v_{b1}} \times \frac{v_{b1}}{v_{sig}} = -61.8 \\ \times -69.2 \times 0.32 = 1368.5 \text{ V/V}$$

From Fig. 1 we see that

$$I_C = 0.495 \text{ mA}$$

$$V_C = I_B \times 200 \text{ k}\Omega + I_E \times 0.2 \text{ k}\Omega + V_{BE} \\ = 0.005 \times 200 + 0.5 \times 0.2 + 0.7 \\ = 1.18 \text{ V}$$

(b)

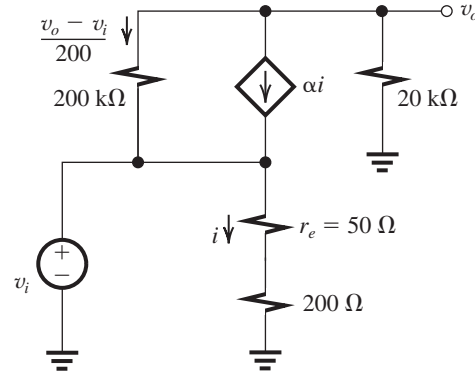


Figure 2

From Fig. 2, we have

$$g_m = \frac{I_C}{V_T} = \frac{0.495}{0.025} \simeq 20 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = 50 \Omega$$

$$i = \frac{v_i}{r_e + R_e} = \frac{v_i}{50 + 200} \\ = \frac{v_i}{250 \Omega} = \frac{v_i}{0.25 \text{ k}\Omega} = 4 v_i, \text{ mA}$$

Node equation at the output:

$$\frac{v_o}{20} + \alpha i + \frac{v_o - v_i}{200} = 0$$

$$\frac{v_o}{20} + 0.99 \times 4 v_i + \frac{v_o}{200} - \frac{v_i}{200} = 0$$

$$v_o \left( \frac{1}{20} + \frac{1}{200} \right) = -v_i \left( 4 \times 0.99 - \frac{1}{200} \right)$$

$$\frac{v_o}{v_i} = -71.9 \text{ V/V}$$

6.133 (a)

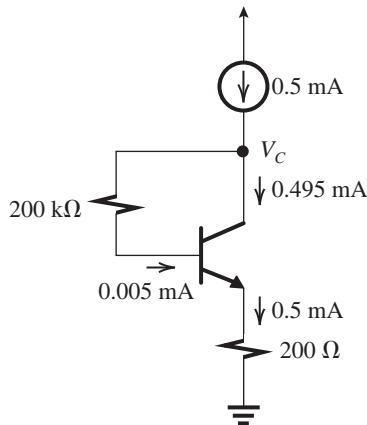


Figure 1

**6.134** Refer to the circuit in Fig. P6.134.

$$I_E = \frac{3 - 0.7}{3.3 + \frac{100}{\beta + 1}}$$

$$= \frac{2.3}{3.3 + \frac{100}{101}} = 0.54 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.54 \text{ mA}} = 46.3 \Omega$$

$$R_{in} = (\beta + 1)[r_e + (3.3 \parallel 2)]$$

$$= 101 \times (0.0463 + 1.245)$$

$$= 130.4 \text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{130.4}{130.4 + 100}$$

$$= 0.566 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{3.3 \parallel 2}{(3.3 \parallel 2) + r_e} = \frac{1.245}{1.245 + 0.0463}$$

$$= 0.964 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.566 \times 0.964 = 0.55 \text{ V/V}$$

$$i_o = \frac{v_o}{2 \text{ k}\Omega}$$

$$i_i = \frac{v_i}{R_{in}} = \frac{v_b}{130.4 \text{ k}\Omega}$$

$$\frac{i_o}{i_i} = \frac{v_o}{v_b} \times \frac{130.4}{2} = 0.964 \times 65.2$$

$$= 62.9 \text{ A/A}$$

$$R_{out} = 3.3 \parallel \left( r_e + \frac{100}{\beta + 1} \right)$$

$$= 3.3 \parallel \left( 0.0463 + \frac{100}{101} \right)$$

$$= 0.789 \text{ k}\Omega = 789 \Omega$$

**6.135** Refer to the circuit in Fig. P6.135.

For dc analysis, open-circuit the two coupling capacitors. Then replace the 9-V source and the two 20-k $\Omega$  resistors by their Thévenin equivalent, namely, a 4.5-V source and a 10-k $\Omega$  series resistance. The latter can be added to the 10-k $\Omega$  resistor that is connected to the base. The result is the circuit shown in Fig. 1, which can be used to calculate  $I_E$ .

$$(a) I_E = \frac{4.5 - 0.7}{2 + \frac{20}{\beta + 1}}$$

$$= \frac{3.8}{2 + \frac{20}{101}} = 1.73 \text{ mA}$$

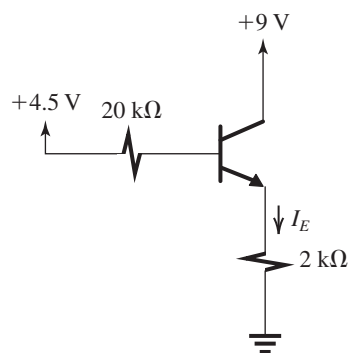


Figure 1

$$I_C = \alpha I_E = 0.99 \times 1.73 \text{ mA}$$

$$= 1.71 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = 68.4 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1.73 \text{ mA}} = 14.5 \Omega$$

$$= 0.0145 \text{ k}\Omega$$

$$r_\pi = (\beta + 1)r_e = 101 \times 0.0145$$

$$= 1.46 \text{ k}\Omega$$

(b) Replacing the BJT with its  $T$  model (without  $r_o$ ) and replacing the capacitors with short circuits results in the equivalent-circuit model shown in Fig. 2.

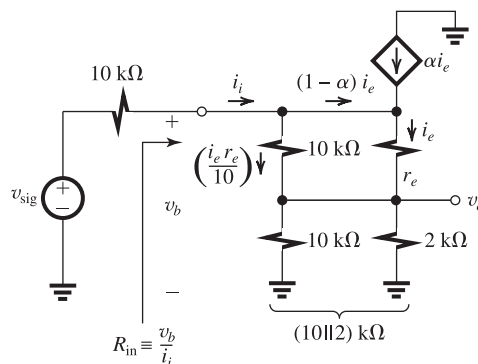


Figure 2

From Fig. 2 we see that

$$v_e = \left( i_e + i_e \frac{r_e}{10} \right) (10 \parallel 2)$$

$$v_b = v_e + i_e r_e = i_e (10 \parallel 2) \left( 1 + \frac{r_e}{10} \right) + i_e r_e$$

$$i_i = (1 - \alpha) i_e + i_e \frac{r_e}{10}$$

$$= \frac{i_e}{\beta + 1} + i_e \frac{r_e}{10}$$

We can now obtain  $R_{in}$  from



$$R_{in} \equiv \frac{v_b}{i_i} = \frac{(10 \parallel 2) \left(1 + \frac{r_e}{10}\right) + r_e}{\frac{1}{\beta + 1} + \frac{r_e}{10}}$$

$$= \frac{(\beta + 1)(10 \parallel 2) \left(1 + \frac{r_e}{10}\right) + (\beta + 1)r_e}{1 + (\beta + 1)\frac{r_e}{10}}$$

$$= \frac{101 \times (10 \parallel 2) \times (1 + 0.00145) + 101 \times 0.0145}{1 + 101 \times 0.00145}$$

$$= \frac{168.577 + 1.4645}{1 + 0.14645} = 148.3 \text{ k}\Omega$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{148.3}{148.3 + 10} = 0.937$$

$$\frac{v_o}{v_b} = \frac{v_e}{v_b} = \frac{i_e \left(1 + \frac{r_e}{10}\right) (10 \parallel 2)}{i_e \left(1 + \frac{r_e}{10}\right) (10 \parallel 2) + i_e r_e}$$

$$= \frac{1.00145 \times (10 \parallel 2)}{1.00145 \times (10 \parallel 2) + 0.0145}$$

$$= 0.991 \text{ V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.937 \times 0.991 = 0.93 \text{ V/V}$$

(c) When  $C_B$  is open-circuited, the equivalent circuit becomes that shown in Fig. 3.

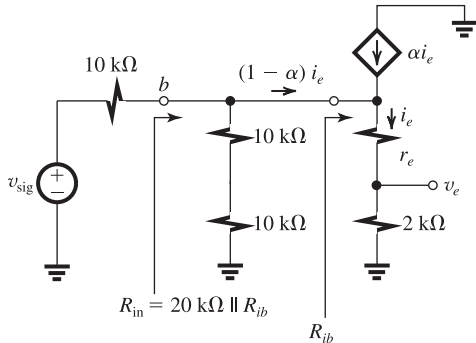


Figure 3

Thus,

$$R_{in} = 20 \text{ k}\Omega \parallel R_{ib}$$

$$= 20 \text{ k}\Omega \parallel (\beta + 1)(r_e + 2)$$

$$= 20 \parallel 101 \times 2.0145$$

$$= 18.21 \text{ k}\Omega$$

which is greatly reduced because of the absence of bootstrapping. The latter causes the lower node of the 10-kΩ base-biasing resistor to rise with the output voltage, thus causing a much reduced signal current in the 10-kΩ resistor and a correspondingly larger effective resistance across the amplifier input.

The reduced  $R_{in}$  will result in a reduction in  $v_b/v_{sig}$ ,

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{18.21}{18.21 + 10}$$

$$= 0.646 \text{ V/V}$$

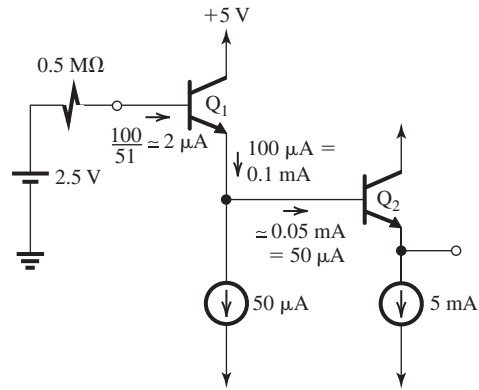
$$\frac{v_o}{v_b} = \frac{2}{2 + 0.0145} = 0.993$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.646 \times 0.993$$

$$= 0.64 \text{ V/V}$$

which is much reduced relative to the value obtained with bootstrapping.

6.136



(a) Applying Thévenin's theorem to the base-biasing circuit of  $Q_1$  results in the dc circuit shown below. From our partial analysis on the figure, we can write

$$I_{E1} = 0.1 \text{ mA}$$

$$I_{E2} = 5 \text{ mA}$$

$$V_{B1} \text{ can be obtained as}$$

$$V_{B1} = 2.5 - 2 \mu\text{A} \times 0.5 \text{ M}\Omega = 1.5 \text{ V}$$

$$\text{and } V_{B2} \text{ can be found as}$$

$$V_{B2} = V_{B1} - 0.7 = 0.8 \text{ V}$$

(b) Refer to the circuit in Fig. P6.136. With a load resistance  $R_L = 1 \text{ k}\Omega$  connected to the output terminal, the voltage gain  $v_o/v_{b2}$  can be found as

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e2}}$$

$$\text{where}$$

$$r_{e2} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$\frac{v_o}{v_{b2}} = \frac{1000}{1000 + 5} = 0.995 \text{ V/V}$$

$$R_{ib2} = (\beta_2 + 1)(r_{e2} + R_L) \\ = 101 \times 1.005 = 101.5 \text{ k}\Omega$$

$$(c) R_{in} = 1 \text{ M}\Omega \parallel 1 \text{ M}\Omega \parallel (\beta + 1)(r_{e1} + R_{ib2})$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega = 0.25 \text{ k}\Omega$$

$$R_{in} = 0.5 \text{ M}\Omega \parallel [51 \times (0.25 + 101.5)] \text{ k}\Omega \\ = 0.5 \text{ M}\Omega \parallel 5.2 \text{ M}\Omega \\ = 456 \text{ k}\Omega$$

$$\frac{v_{e1}}{v_{b1}} = \frac{R_{ib}}{R_{ib} + r_{e1}} = \frac{101.5}{101.5 + 0.25} \\ = 0.9975 \text{ V/V}$$

$$(d) \frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{456}{456 + 100} = 0.82 \text{ V/V}$$

$$(e) \frac{v_o}{v_{sig}} = 0.82 \times 0.9975 \times 0.995 = 0.814 \text{ V/V}$$

**6.137** We need to raise  $f_H$  by a factor of

$$\frac{2 \text{ MHz}}{500 \text{ kHz}} = 4. \text{ Thus}$$

$$1 + g_m R_e = 4$$

$$\Rightarrow R_e = \frac{3}{g_m}$$

Since

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$R_e = \frac{3}{0.04} = 75 \Omega$$

The new value of  $f_L$  will be

$$f_L = \frac{100 \text{ Hz}}{1 + g_m R_e} = \frac{100}{4} = 25 \text{ Hz}$$

and the midband gain will become

$$|A_M| = \frac{100}{1 + g_m R_e} = \frac{100}{4} = 25 \text{ V/V}$$

## Chapter 7

## Solutions to Exercises within the Chapter

**Ex: 7.1** In the current source of Example 7.1 (Fig. 7.1) we have  $I_O = 100 \mu\text{A}$  and we want to reduce the change in output current,  $\Delta I_O$ , corresponding to a 1-V change in output voltage,  $\Delta V_O$ , to 1% of  $I_O$ .

$$\text{That is, } \Delta I_O = \frac{\Delta V_O}{r_{o2}} = 0.01 I_O \Rightarrow \frac{1 \text{ V}}{r_{o2}} = 0.01 \times 100 \mu\text{A} = 1 \mu\text{A}$$

$$r_{o2} = \frac{1 \text{ V}}{1 \mu\text{A}} = 1 \text{ M}\Omega$$

$$r_{o2} = \frac{V_A \times L}{I_O} \Rightarrow 1 \text{ M}\Omega = \frac{20 \times L}{100 \mu\text{A}}$$

$$\Rightarrow L = \frac{100 \text{ V}}{20 \text{ V}/\mu\text{m}} = 5 \mu\text{m}$$

To keep  $V_{OV}$  of the matched transistors the same as that in Example 7.1,  $\frac{W}{L}$  of the transistor should remain the same. Therefore,

$$\frac{W}{5 \mu\text{m}} = \frac{10 \mu\text{m}}{1 \mu\text{m}} \Rightarrow W = 50 \mu\text{m}$$

So the dimensions of the matched transistors  $Q_1$  and  $Q_2$  should be changed to

$$W = 50 \mu\text{m} \text{ and } L = 5 \mu\text{m}$$

**Ex: 7.2** For the circuit of Fig. 7.4 we have

$$I_2 = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1}, I_3 = I_{\text{REF}} \frac{(W/L)_3}{(W/L)_1}$$

$$\text{and } I_5 = I_4 \frac{(W/L)_5}{(W/L)_4}$$

Since all channel lengths are equal, that is,

$$L_1 = L_2 = \dots = L_5 = 1 \mu\text{m}$$

and

$$I_{\text{REF}} = 10 \mu\text{A}, I_2 = 60 \mu\text{A}, I_3 = 20 \mu\text{A}, I_4 = I_3 = 20 \mu\text{A}, \text{ and } I_5 = 80 \mu\text{A},$$

we have

$$I_2 = I_{\text{REF}} \frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{\text{REF}}} = \frac{60}{10} = 6$$

$$I_3 = I_{\text{REF}} \frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{\text{REF}}} = \frac{20}{10} = 2$$

$$I_5 = I_4 \frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{20} = 4$$

To allow the voltage at the drain of  $Q_2$  to go down to within 0.2 V of the negative supply voltage, we need  $V_{OV2} = 0.2 \text{ V}$ :

$$I_2 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 V_{OV2}^2 = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_2 V_{OV2}^2$$

$$60 \mu\text{A} = \frac{1}{2} 200 \frac{\mu\text{A}}{\text{V}^2} \left( \frac{W}{L} \right)_2 (0.2)^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2$$

$$= 15 \times L_2$$

$$W_2 = 15 \mu\text{m}, \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5 \mu\text{m}$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5 \mu\text{m}$$

To allow the voltage at the drain of  $Q_5$  to go up to within 0.2 V of positive supply, we need  $V_{OV5} = 0.2 \text{ V}$ :

$$I_5 = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_5 V_{OV5}^2$$

$$80 \mu\text{A} = \frac{1}{2} 80 \frac{\mu\text{A}}{\text{V}^2} \left( \frac{W}{L} \right)_5 (0.2)^2 \Rightarrow$$

$$\left( \frac{W}{L} \right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50 L_5$$

$$W_5 = 50 \mu\text{m}$$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50 \mu\text{m}}{4} = 12.5 \mu\text{m}$$

Thus:

$$W_1 = 2.5 \mu\text{m}, W_2 = 15 \mu\text{m}, W_3 = 5 \mu\text{m}, W_4 = 12.5 \mu\text{m}, \text{ and } W_5 = 50 \mu\text{m}$$

**Ex: 7.3**  $V_{BE} = V_T \ln(I_{\text{REF}}/I_5)$

$$= 0.025 \ln(10^{-3}/10^{-15})$$

$$= 0.691 \text{ V}$$

From Eq. (7.21) we have

$$I_O = I_{\text{REF}} \left( \frac{m}{1 + \frac{m+1}{\beta}} \right) \left( 1 + \frac{V_O - V_{BE}}{V_{A2}} \right)$$

$$I_O = 1 \text{ mA} \left( \frac{1}{1 + \frac{1+1}{100}} \right) \left( 1 + \frac{5 - 0.691}{100} \right)$$

$$= 1.02 \text{ mA}$$

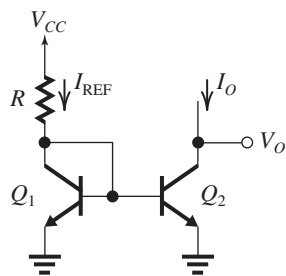
$$I_O = 1.02 \text{ mA}$$

$$R_o = r_{o2} = \frac{V_A}{I_O} = \frac{100 \text{ V}}{1.02 \text{ mA}} = 98 \text{ k}\Omega \simeq 100 \text{ k}\Omega$$

**Ex: 7.4**

From Eq. (7.23), we have

$$I_O = \frac{I_{\text{REF}}}{1 + (2/\beta)} \left( 1 + \frac{V_O - V_{BE}}{V_A} \right)$$



where  $V_{BE} = V_T \ln\left(\frac{I_O}{I_S}\right)$   
 $= 0.025 \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.673 \text{ V}$   
 $0.5 \text{ mA} = \frac{I_{REF}}{1 + (2/100)} \left(1 + \frac{2 - 0.673}{50}\right) \Rightarrow$

$$I_{REF} = 0.5 \text{ mA} \frac{1.02}{1.0265} = 0.497 \text{ mA}$$

$$I_{REF} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{REF}}$$

$$R = \frac{5 - 0.673}{0.497 \text{ mA}} = 8.71 \text{ k}\Omega$$

$$V_{Omin} = V_{CEsat} = 0.3 \text{ V}$$

For  $V_O = 5 \text{ V}$ , From Eq. (7.23) we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)} \left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$

$$I_O = \frac{0.497}{1 + (2/100)} \left(1 + \frac{5 - 0.673 \text{ V}}{50}\right) = 0.53 \text{ mA}$$

**Ex: 7.5**  $I_1 = I_2 = \dots = I_N = I_{C|Q_{REF}}$

At the input node,

$$I_{REF} = I_{C|Q_{REF}} + I_{B|Q_{REF}} + I_{B1} + \dots + I_{BN}$$

$$= I_{C|Q_{REF}} + (N + 1) I_{B|Q_{REF}}$$

$$= I_{C|Q_{REF}} + \frac{(N + 1)}{\beta} I_{C|Q_{REF}}$$

$$\Rightarrow I_{C|Q_{REF}} = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}}$$

Thus,

$$I_1 = I_2 = \dots = I_N = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}} \quad \text{Q.E.D.}$$

For  $\beta = 100$ , to limit the error to 10%,

$$0.1 = \frac{N + 1}{\beta} = \frac{N + 1}{100}$$

$$\Rightarrow N = 9$$

**Ex: 7.6**

$$R_{in} \simeq \frac{1}{g_{m1}}$$

Now,  $R_{in} = 1 \text{ k}\Omega$ , thus

$$g_{m1} = 1 \text{ mA/V}$$

But

$$g_{m1} = \sqrt{2(\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$1 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1} \times 0.1$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$$

To obtain

$$A_{is} \Big|_{\text{ideal}} = 5$$

$$5 = \frac{(W/L)_2}{(W/L)_1}$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 12.5 = 62.5$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{V_{A2}}{5I_{D1}}$$

Thus,

$$40 \text{ k}\Omega = \frac{V_{A2}}{5 \times 0.1}$$

$$\Rightarrow V_{A2} = 20 \text{ V}$$

But

$$V_{A2} = V'_{A2} L_2$$

$$20 = 20 \times L_2$$

$$\Rightarrow L_2 = 1 \text{ }\mu\text{m}$$

Selecting  $L_1 = L_2$ , then

$$L_1 = L_2 = 1 \text{ }\mu\text{m}$$

$$W_1 = 12.5 \text{ }\mu\text{m}$$

$$W_2 = 62.5 \text{ }\mu\text{m}$$

The actual short-circuit current-transfer ratio is given by Eq. (7.31),

$$A_{is} = \frac{5}{1 + \frac{1}{g_{m1} r_{o1}}}$$

Thus the percentage error is

$$\text{Error} = \frac{-1/g_{m1} r_{o1}}{1 + (1/g_{m1} r_{o1})} \times 100\%$$

Substituting,

$$g_{m1} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V'_A L_1}{I_{D1}} = \frac{20 \times 1}{0.1} = 200 \text{ k}\Omega$$

$$\text{Error} = \frac{-1/200}{1 + (1/200)} \times 100 = -0.5\%$$

**Ex: 7.7**

Using Eq. (7.42):

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$$

For  $I_D = 10 \mu\text{A}$ , we have

$$g_m = \sqrt{2(387 \mu\text{A}/\text{V}^2)(10)(10 \mu\text{A})}$$

$$= 0.28 \text{ mA/V}$$

Using Eq. (7.46):

$$A_0 = V_A' \frac{\sqrt{2\mu_n C_{ox} (W/L)}}{\sqrt{I_D}}$$

$$= \frac{5 \text{ V}/\mu\text{m} \sqrt{2(387 \mu\text{A}/\text{V}^2)(10)(0.36)^2}}{\sqrt{10 \mu\text{A}}}$$

$$A_0 = 50 \text{ V/V}$$

Since  $g_m$  varies with  $\sqrt{I_D}$  and  $A_0$  with  $\frac{1}{\sqrt{I_D}}$ ,

for

$$I_D = 100 \mu\text{A} \Rightarrow g_m = 0.28 \text{ mA/V} \left(\frac{100}{10}\right)^{1/2}$$

$$= 0.88 \text{ mA/V}$$

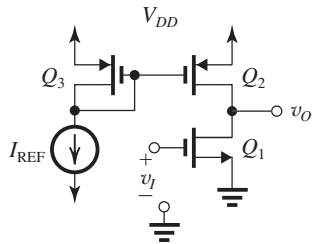
$$A_0 = 50 \left(\frac{10}{100}\right)^{1/2} = 15.8 \text{ V/V}$$

For  $I_D = 1 \text{ mA}$ , we have

$$g_m = 0.28 \text{ mA/V} \left(\frac{1}{0.010}\right)^{1/2} = 2.8 \text{ mA/V}$$

$$A_0 = 50 \left(\frac{0.010}{1}\right)^{1/2} = 5 \text{ V/V}$$

**Ex: 7.8**



Since all transistors have the same

$$\frac{W}{L} = \frac{7.2 \mu\text{m}}{0.36 \mu\text{m}},$$

we have

$$I_{\text{REF}} = I_{D3} = I_{D2} = I_{D1} = 100 \mu\text{A}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1} \sqrt{I_{D1}}$$

$$= \sqrt{2(387 \mu\text{A}/\text{V}^2) \left(\frac{7.2}{0.36}\right) (100 \mu\text{A})}$$

$$= 1.24 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A_n}' L_1}{I_{D1}} = \frac{5 \text{ V}/\mu\text{m} (0.36 \mu\text{m})}{0.1 \text{ mA}} = 18 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A_p}'| L_2}{I_{D2}} = \frac{6 \text{ V}/\mu\text{m} (0.36 \mu\text{m})}{0.1 \text{ mA}} = 21.6 \text{ k}\Omega$$

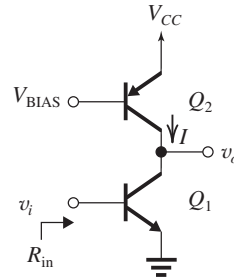
Voltage gain is

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2})$$

$$A_v = -(1.24 \text{ mA/V}) (18 \text{ k}\Omega \parallel 21.6 \text{ k}\Omega)$$

$$= -12.2 \text{ V/V}$$

**Ex: 7.9**



$$I_{C1} = I = 100 \mu\text{A} = 0.1 \text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1 \text{ mA}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$R_{\text{in}} = r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_{o1} = \frac{V_A}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

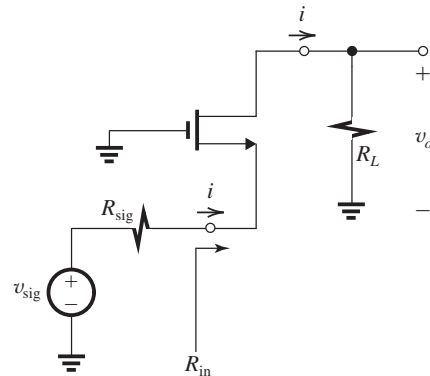
$$r_{o2} = \frac{|V_A|}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$A_0 = g_{m1} r_{o1} = (4 \text{ mA/V}) (500 \text{ k}\Omega) = 2000 \text{ V/V}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2}) = -(4 \text{ mA/V}) \times$$

$$(500 \text{ k}\Omega \parallel 500 \text{ k}\Omega) = -1000 \text{ V/V}$$

**Ex: 7.10** Refer to Fig. 1,



$$v_o = iR_L$$

$$v_{\text{sig}} = i(R_s + R_{\text{in}})$$

Thus,

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_s + R_{\text{in}}} \quad \text{Q.E.D.}$$

**Ex: 7.11** Since  $g_m r_o \gg 1$ , we use Eq. (7.52),

$$R_{\text{in}} \simeq \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

$R_L$	0	$r_o$	$(g_m r_o)r_o$	$\infty$
$R_{\text{in}}$	$\frac{1}{g_m}$	$\frac{2}{g_m}$	$r_o$	$\infty$

**Ex: 7.12** For  $g_m r_o \gg 1$ , we use Eq. (7.57),

$$R_{\text{out}} \simeq r_o + (g_m r_o)R_s$$

to obtain

$R_s$	0	$r_o$	$(g_m r_o)r_o$	$\infty$
$R_{\text{out}}$	$r_o$	$(g_m r_o)r_o$	$(g_m r_o)^2 r_o$	$\infty$

**Ex: 7.13** Substituting  $R_s = r_o$  in Eq. (7.53) gives

$$A_{is} = \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{2}{g_m r_o}}$$

Substituting the given values of  $(g_m r_o)$  we obtain

$g_m r_o$	20	50	100
$A_{is}$ (A/A)	0.95	0.98	0.99

**Ex: 7.14**  $A_{vo}$  remains unchanged at  $g_m r_o$ . With a load resistance  $R_L$  connected,

$$\begin{aligned} A_v &= A_{vo} \frac{R_L}{R_L + R_o} \\ &= (g_m r_o) \frac{R_L}{R_L + (1 + g_m R_s)r_o} \end{aligned}$$

**Ex: 7.15** Since  $g_m r_o \gg 1$  we use Eq. (7.62),

$$R_{\text{in}} \simeq r_{\pi} \parallel \left[ \frac{1}{g_m} + \frac{R_L}{g_m r_o} \right]$$

For  $R_L = 0$ ,

$$R_{\text{in}} \simeq r_{\pi} \parallel \frac{1}{g_m} = r_e$$

For  $R_L = r_o$ ,

$$R_{\text{in}} = r_{\pi} \parallel \left( \frac{1}{g_m} + \frac{1}{g_m} \right) = r_{\pi} \parallel \frac{2}{g_m} \simeq 2r_e$$

For  $R_L = \beta r_o$ ,

$$\begin{aligned} R_{\text{in}} &= r_{\pi} \parallel \left( \frac{1}{g_m} + \frac{\beta r_o}{g_m r_o} \right) = r_{\pi} \parallel \left( \frac{1}{g_m} + r_{\pi} \right) \\ &\simeq r_{\pi} \parallel r_{\pi} = \frac{1}{2} r_{\pi} \end{aligned}$$

For  $R_L = \infty$ ,

$$R_{\text{in}} = r_{\pi}$$

Summary:

$R_L$	0	$r_o$	$\beta r_o$	$\infty$
$R_{\text{in}}$	$r_e$	$2r_e$	$\frac{1}{2} r_{\pi}$	$r_{\pi}$

**Ex: 7.16** Using Eq. (7.69),

$$R_{\text{out}} \simeq r_o + (g_m r_o)(R_e \parallel r_{\pi})$$

we obtain

$R_e$	0	$r_e$	$r_{\pi}$	$r_o$	$\infty$
$R_{\text{out}}$	$r_o$	$2r_o$	$\left( \frac{\beta}{2} + 1 \right) r_o$	$(\beta + 1)r_o$	$(\beta + 1)r_o$

**Ex: 7.17** Using Eq. (7.65) with  $R_e = r_o$ ,

$$\begin{aligned} A_{is} &= \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{1}{g_m r_o} + \frac{1}{g_m (r_o \parallel r_{\pi})}} \\ &= \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{2}{g_m r_o} + \frac{1}{g_m r_{\pi}}} = \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{2}{g_m r_o} + \frac{1}{\beta}} \end{aligned}$$

For  $\beta = 100$  and  $g_m r_o = 100$ ,

$$A_{is} = \frac{1 + \frac{1}{100}}{1 + \frac{2}{100} + \frac{1}{100}} = 0.98 \text{ A/A}$$

For  $\beta = 100$  and  $g_m r_o = 1000$ ,

$$A_{is} = \frac{1 + \frac{1}{1000}}{1 + \frac{2}{1000} + \frac{1}{100}} = 0.99 \text{ A/A}$$

**Ex: 7.18**  $R_o = [1 + g_m (R_e \parallel r_{\pi})]r_o$

where

$$g_m = 4 \text{ mA/V}, \quad r_{\pi} = \frac{\beta}{g_m} = 25 \text{ k}\Omega,$$

$$R_e = 1 \text{ k}\Omega, \quad \text{and } r_o = \frac{V_A}{I_C} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

Thus,

$$R_o = [1 + 4(1 \parallel 25)] \times 200 \simeq 1 \text{ M}\Omega$$

Without emitter degeneration,

$$R_o = r_o = 200 \text{ k}\Omega$$

**Ex: 7.19** If  $L$  is halved ( $L = \frac{0.55 \mu\text{m}}{2}$ ) and  $|V_A| = |V_A'| \cdot L$ , we obtain

$$|V_A| = 5 \text{ V}/\mu\text{m} \left( \frac{0.55 \mu\text{m}}{2} \right) = 1.375 \text{ V}$$

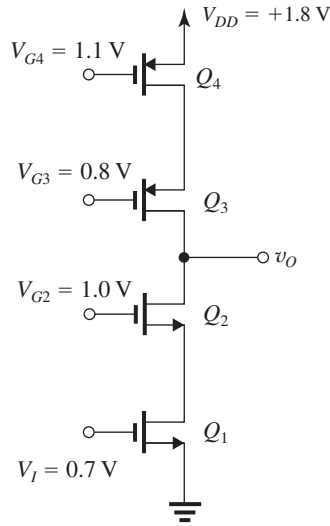
$$R_o = \frac{|V_A|}{|V_{OV}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2 (1.375 \text{ V})^2}{(0.3 \text{ V}) (100 \mu\text{A})} = 126 \text{ k}\Omega$$

Since  $I_D = \frac{1}{2}(\mu_p C_{ox}) \left( \frac{W}{L} \right) |V_{OV}|^2 \left( 1 + \frac{V_{SD}}{|V_A|} \right)$

$$\frac{W}{L} = \frac{2 (100 \mu\text{A})}{90 \mu\text{A}/\text{V}^2 (0.3 \text{ V})^2 \left( 1 + \frac{0.3 \text{ V}}{1.375 \text{ V}} \right)}$$

$$\frac{W}{L} = 20.3$$

**Ex: 7.20**



If all transistors are matched and are obviously operating at the same  $I_D$ , then all  $|V_{OV}|$  will be equal and equal to that of  $Q_1$ , namely,  $|V_{OV}| = 0.7 - 0.5 = 0.2 \text{ V}$

To keep  $Q_2$  in saturation,  $v_{O\min} = V_{G2} - |V_t| = 0.5 \text{ V}$

To keep  $Q_3$  in saturation,  $v_{O\max} = V_{G3} + |V_t| = 1.3 \text{ V}$

Thus, the allowable range of  $v_O$  is from 0.5 V to 1.3 V.

**Ex: 7.21** Refer to Fig. 7.31.

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2 I_D}{|V_{OV}|} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A|}{I_D} = \frac{2}{0.2} = 10 \text{ k}\Omega$$

$$R_{on} = (g_{m2} r_{o2}) r_{o1} = (2 \times 10) \times 10 = 200 \text{ k}\Omega$$

$$R_{op} = (g_{m3} r_{o3}) r_{o4} = (2 \times 10) \times 10 = 200 \text{ k}\Omega$$

$$R_o = R_{on} \parallel R_{op} = 200 \parallel 200 = 100 \text{ k}\Omega$$

$$A_v = -g_{m1} R_o = -2 \times 100 = -200 \text{ V/V}$$

**Ex: 7.22**  $g_{m1} = g_{m2} = g_m$

$$= \frac{I_D}{V_{OV}} = \frac{0.1 \text{ mA}}{(0.2/2) \text{ V}} = 1 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_o = \frac{V_A}{I_D} = \frac{2 \text{ V}}{0.1 \text{ mA}} = 20 \text{ k}\Omega$$

so,  $g_m r_o = 1 \text{ mA/V} (20 \text{ k}\Omega) = 20$

(a) For  $R_L = 20 \text{ k}\Omega$ ,

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2} r_{o2}} = \frac{20 \text{ k}\Omega + 20 \text{ k}\Omega}{1 + 20} = 1.9 \text{ k}\Omega$$

$$\therefore A_{v1} = -g_{m1} (r_{o1} \parallel R_{in2}) = -1 \text{ mA/V} (20 \parallel 1.9) = -1.74 \text{ V/V}$$

or

If we use the approximation of Eq. (7.84),

$$R_{in2} \approx \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{20 \text{ k}\Omega}{20} + \frac{1}{1 \text{ mA/V}} = 2 \text{ k}\Omega$$

then

$$A_{v1} = -1 \text{ mA/V} (20 \text{ k}\Omega \parallel 2 \text{ k}\Omega) = -1.82 \text{ V/V}$$

Continuing, from Eq. (7.81),

$$A_v = -g_{m1} [(g_{m2} r_{o2} r_{o1}) \parallel R_L]$$

$$A_v = -1 \text{ mA/V} \{ [(20) (20 \text{ k}\Omega)] \parallel 20 \text{ k}\Omega \}$$

$$= -19.0 \text{ V/V}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-19.0}{-1.82} = 10.5 \text{ V/V}$$

(b) Now, for  $R_L = 400 \text{ k}\Omega$ ,

$$R_{in2} \simeq \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{400 \text{ k}\Omega}{20} + \frac{1}{1 \text{ mA/V}} = 21 \text{ k}\Omega$$

$$A_{v1} = -1 \text{ mA/V} (20 \text{ k}\Omega \parallel 21 \text{ k}\Omega) = -10.2 \text{ V/V}$$

$$A_v = -1 \text{ mA/V} [(20) (20 \text{ k}\Omega)] \parallel 400 \text{ k}\Omega$$

$$= -200 \text{ V/V}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-200}{-10.2} = 19.6 \text{ V/V}$$

**Ex: 7.23** Referring to Fig. 7.34,

$$R_{op} = (g_{m3}r_{o3}) (r_{o4} \parallel r_{\pi3}) \text{ and}$$

$$R_{on} = (g_{m2}r_{o2}) (r_{o1} \parallel r_{\pi2})$$

The maximum values of these resistances are obtained when  $r_o \gg r_{\pi}$  and are given by

$$R_{on} \Big|_{\max} = (g_{m2}r_{o2}) r_{\pi2}$$

$$R_{op} \Big|_{\max} = (g_{m3}r_{o3}) r_{\pi3}$$

Since  $g_m r_{\pi} = \beta$ ,

$$R_{on} \Big|_{\max} = \beta_2 r_{o2}$$

$$R_{op} \Big|_{\max} = \beta_3 r_{o3}$$

Since  $A_v = -g_{m1}(R_{on} \parallel R_{op})$ ,

$$|A_{v\max}| = g_{m1}(\beta_2 r_{o2} \parallel \beta_3 r_{o3})$$

**Ex: 7.24** For the *nnp* transistors,

$$g_{m1} = g_{m2} = \frac{|I_C|}{|V_T|} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi1} = r_{\pi2} = \frac{\beta}{g_m} = \frac{100}{8 \text{ mA/V}} = 12.5 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{|I_C|} = \frac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$$

From Fig. 7.34,

$$R_{on} = (g_{m2}r_{o2}) (r_{o1} \parallel r_{\pi2})$$

$$= (8 \text{ mA/V}) (25 \text{ k}\Omega) (25 \text{ k}\Omega \parallel 12.5 \text{ k}\Omega)$$

$$R_{on} = 1.67 \text{ M}\Omega$$

For the *npn* transistors,

$$g_{m3} = g_{m4} = \frac{|I_C|}{V_T} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi3} = r_{\pi4} = \frac{\beta}{g_m} = \frac{50}{8 \text{ mA/V}} = 6.25 \text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_A|}{|I_C|} = \frac{4 \text{ V}}{0.2 \text{ mA}} = 20 \text{ k}\Omega$$

$$R_{op} = (g_{m3}r_{o3}) (r_{o4} \parallel r_{\pi3})$$

$$= (8 \text{ mA/V}) (20 \text{ k}\Omega) (20 \text{ k}\Omega \parallel 6.25 \text{ k}\Omega)$$

$$R_{op} = 762 \text{ k}\Omega$$

$$A_v = -g_{m1}(R_{on} \parallel R_{op})$$

$$= -(8 \text{ mA/V}) (1.67 \text{ M}\Omega \parallel 762 \text{ k}\Omega)$$

$$A_v = -4186 \text{ V/V}$$

$A_{v\max}$  occurs when  $r_{o1}$  and  $r_{o4}$  are  $\gg r_{\pi}$ .

Then

$$R_{on} = (g_{m2}r_{o2}) r_{\pi2} = \beta_2 r_{o2}$$

$$R_{on} = 100 (25 \text{ k}\Omega) = 2.5 \text{ M}\Omega$$

$$R_{op} = (g_{m3}r_{o3}) r_{\pi3} = \beta_3 r_{o3}$$

$$R_{op} = 50 (20 \text{ k}\Omega) = 1 \text{ M}\Omega$$

Finally,

$$A_{v\max} = -(8 \text{ mA/V}) (2.5 \text{ M}\Omega \parallel 1.0 \text{ M}\Omega)$$

$$A_{v\max} = -5714 \text{ V/V}$$

**Ex: 7.25**

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2 = 0.4 \text{ mA/V}$$

$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$R_L = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}} = 25 \parallel 25 \parallel 2.5 \text{ k}\Omega$$

$$= 2.083 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + \frac{1}{g_m}} = \frac{2.083}{2.083 + \frac{1}{2}} = 0.81 \text{ V/V}$$

To obtain  $R_{out}$  we use Eq. (7.95),

$$R_{out} = \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \parallel r_{o1} \parallel r_{o3}$$

$$= 0.5 \parallel 2.5 \parallel 25 \parallel 25$$

$$= 0.403 \text{ k}\Omega = 403 \Omega$$

**Ex: 7.26** Refer to the circuit in Fig. 7.36. All transistors are operating at  $I_D = I_{REF} = 100 \mu\text{A}$  and equal  $V_{OV}$ , found from

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{3.6}{0.36} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.227 \text{ V}$$

$$V_{GS} = 0.227 + 0.5 = 0.727 \text{ V}$$

$$V_{Omin} = V_{G3} - V_{I3}$$

$$= V_{GS4} + V_{GS1} - V_{I3}$$

Thus,

$$V_{Omin} = 2V_{GS} - V_t$$

$$= V_t + 2V_{OV}$$

$$= 0.5 + 2 \times 0.227 = 0.95 \text{ V}$$



$$g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 0.1}{0.227} = 0.88 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o2} = (0.88 \times 18) \times 18 = 285 \text{ k}\Omega$$

**Ex: 7.27** For the Wilson mirror from Eq. (7.97), we have

$$\frac{I_O}{I_{REF}} \simeq \frac{1}{1 + \frac{2}{\beta^2}} = 0.9998$$

$$\text{Thus } \frac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 0.02\%$$

whereas for the simple mirror from Eq. (7.18) we have

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}} = 0.98$$

$$\text{Hence } \frac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 2\%$$

For the Wilson current mirror, we have

$$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100 \text{ k}\Omega}{2} = 5 \text{ M}\Omega$$

and for the simple mirror,  $R_o = r_o = 100 \text{ k}\Omega$ .

**Ex: 7.28** For the two current sources designed in Example 7.5, we have

$$g_m = \frac{I_C}{V_T} = \frac{10 \text{ }\mu\text{A}}{25 \text{ mV}} = 0.4 \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{10 \text{ }\mu\text{A}} = 10 \text{ M}\Omega,$$

$$r_n = \frac{\beta}{g_m} = 250 \text{ k}\Omega$$

For the current source in Fig. 7.40(a), we have

$$R_o = r_{o2} = r_o = 10 \text{ M}\Omega$$

For the current source in Fig. 7.40(b), from Eq. (7.105), we have

$$R_{\text{out}} \simeq [1 + g_m (R_E \parallel r_n)] r_o$$

From Example 7.5,  $R_E = R_3 = 11.5 \text{ k}\Omega$ ;

therefore,

$$R_{\text{out}} \simeq \left[ 1 + 0.4 \frac{\text{mA}}{\text{V}} (11.5 \text{ k}\Omega \parallel 250 \text{ k}\Omega) \right] 10 \text{ M}\Omega$$

$$\therefore R_{\text{out}} = 54 \text{ M}\Omega$$

## Solutions to End-of-Chapter Problems

7.3

7.1 Refer to Fig. 7.1.

$$I_O = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2$$

$$80 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.2 = 0.7 \text{ V}$$

$$I_{REF} = 80 \mu\text{A} = \frac{V_{DD} - V_{GS}}{R}$$

$$\Rightarrow R = \frac{1.8 - 0.7}{0.08 \text{ mA}} = 13.75 \text{ k}\Omega$$

For  $Q_2$  to operate in saturation,  $V_O$  must not decrease below  $V_{G2}$  by more than  $V_t$ , thus

$$V_{Omin} = V_G - V_t = V_{GS} - V_t = V_{OV} = 0.2 \text{ V}$$

7.2 Refer to Fig. 7.1.

$$I_{REF} = 10 \mu\text{A} = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_1 V_{OV}^2$$

$$10 = \frac{1}{2} \times 400 \times \left( \frac{1}{0.4} \right) V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.14 = 0.54 \text{ V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.2 - 0.54}{0.01 \text{ mA}} = 66 \text{ k}\Omega$$

Since

$$\frac{I_O}{I_{REF}} = \frac{60}{10} = 6,$$

$$\left( \frac{W}{L} \right)_2 = 6 \left( \frac{W}{L} \right)_1$$

But  $L_1 = L_2$ , thus

$$W_2 = 6 W_1 = 6 \times 1 = 6 \mu\text{m}$$

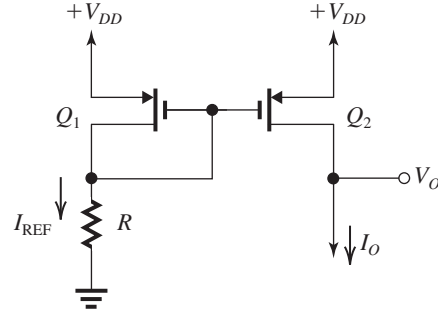
$$V_{Omin} = V_{OV} = 0.2 \text{ V}$$

$$R_o = r_{o2} = \frac{V_A' L_2}{I_O} = \frac{6 \times 0.4}{0.060 \text{ mA}} = 40 \text{ k}\Omega$$

For

$$\Delta V_O = +0.2 \text{ V},$$

$$\Delta I_O = \frac{+0.2 \text{ V}}{R_o} = \frac{+0.2 \text{ V}}{40 \text{ k}\Omega} = 0.005 \text{ mA or } 5 \mu\text{A}$$

Set  $|V_{OV}| = V_{DD} - V_{Omax}$ 

$$= 1.3 - 1.1 = 0.2 \text{ V}$$

$$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 1.3 - 0.4 - 0.2 = 0.7 \text{ V}$$

$$R = \frac{V_G}{I_{D1}} = \frac{0.7 \text{ V}}{80 \mu\text{A}} = 8.75 \text{ k}\Omega$$

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV}|^2$$

thus

$$\frac{W}{L} = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2} = \frac{2 \times 80 \mu\text{A}}{80 \mu\text{A/V}^2 \times 0.2^2} = 50$$

7.4 Refer to the circuit of Fig. P7.4. For  $Q_2$  to operate properly (i.e., in the saturation mode) for drain voltages as high as  $+0.8 \text{ V}$ , and provided its width is the minimum possible, we use

$$|V_{OV}| = 0.2 \text{ V}$$

Note that all three transistors  $Q_1$ ,  $Q_2$ , and  $Q_3$  will be operated at this value of overdrive voltage.

For  $Q_1$ ,

$$I_{D1} = I_{REF} = 20 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_1 |V_{OV}|^2$$

$$20 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_1 \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 10$$

For  $L = 0.5 \mu\text{m}$ ,

$$W_1 = 5 \mu\text{m}$$

Now, for

$$I_2 = 100 \mu\text{A} = 5I_{REF}, \text{ we have}$$

$$\frac{(W/L)_2}{(W/L)_1} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 10 = 50$$

$$W_2 = 50 \times 0.5 = 25 \mu\text{m}$$

For

$$I_3 = 40 \mu\text{A} = 2I_{\text{REF}}, \text{ we obtain}$$

$$\frac{(W/L)_3}{(W/L)_1} = 2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 20$$

$$W_3 = 10 \mu\text{m}$$

We next consider  $Q_4$  and  $Q_5$ . For  $Q_5$  to operate in saturation with the drain voltage as low as  $-0.8 \text{ V}$ , and for it to have the minimum possible  $W/L$ , we operate  $Q_5$  at

$$V_{OV} = 0.2 \text{ V}$$

This is the same overdrive voltage at which  $Q_4$  will be operating. Thus, we can write for  $Q_4$ ,

$$I_4 = I_3 = 40 \mu\text{A}$$

and using

$$I_{D4} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_4 V_{OV}^2$$

$$40 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_4 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = 5$$

$$W_4 = 2.5 \mu\text{m}$$

Finally, since

$$I_5 = 80 \mu\text{A} = 2 I_4,$$

$$\left(\frac{W}{L}\right)_5 = 2 \left(\frac{W}{L}\right)_4$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 10$$

$$W_5 = 5 \mu\text{m}$$

To find the value of  $R$ , we use

$$|V_{SG1}| = |V_{tp}| + |V_{OV1}|$$

$$= 0.5 + 0.2 = 0.7 \text{ V}$$

$$R = \frac{1 - |V_{SG1}|}{I_{\text{REF}}} = \frac{0.3 \text{ V}}{0.02 \text{ mA}}$$

$$= 15 \text{ k}\Omega$$

The output resistance of the current source  $Q_2$  is

$$r_{o2} = \frac{|V_{A2}|}{I_2} = \frac{|V'_{Ap}| \times L}{I_2}$$

$$= \frac{5 \times 0.5}{0.1 \text{ mA}} = 25 \text{ k}\Omega$$

The output resistance of the current sink  $Q_5$  is

$$r_{o5} = \frac{V_{A5}}{I_5} = \frac{V'_{An} \times L}{I_5}$$

$$= \frac{5 \times 0.5}{80} = 31.25 \text{ k}\Omega$$

**7.5** Using Eq. (7.22) with  $m = 1$ ,

$$I_O = I_{\text{REF}} \frac{1}{1 + \frac{2}{\beta}} \left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

where

$$I_{\text{REF}} = 100 \mu\text{A}, \quad \beta = 100, \quad V_{A2} = 35 \text{ V}, \text{ and}$$

$$V_{BE} = V_T \ln \frac{I_{\text{REF}}}{I_S}$$

$$= 0.025 \ln \left(\frac{100 \times 10^{-6}}{6 \times 10^{-18}}\right)$$

$$= 0.761 \text{ V}$$

Thus, at  $V_O = 2 \text{ V}$ ,

$$I_O = 100 \times \frac{1}{1 + \frac{2}{100}} \times \left(1 + \frac{2 - 0.761}{35}\right)$$

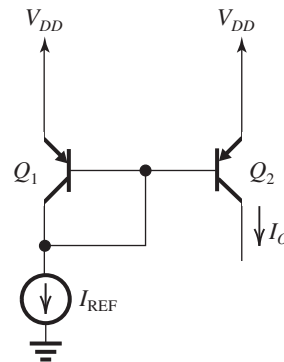
$$= 101.5 \mu\text{A}$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{\text{nominal}}} = \frac{35}{0.1} = 350 \text{ k}\Omega$$

For

$$\Delta V_O = 1 \text{ V}, \quad \Delta I_O = \frac{\Delta V_O}{R_o} = \frac{1 \text{ V}}{350 \text{ k}\Omega} \simeq 2.9 \mu\text{A}$$

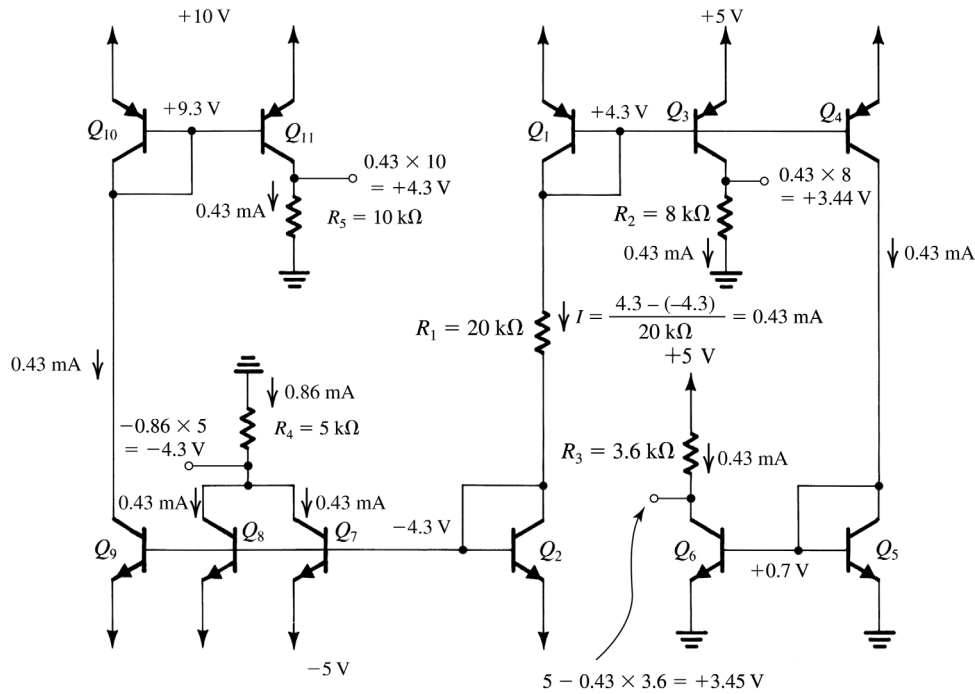
**7.6**



For identical transistors, the transfer ratio is

$$\frac{I_O}{I_{\text{REF}}} = \frac{1}{1 + 2/\beta} = \frac{1}{1 + \frac{2}{50}} = 0.96$$

This figure belongs to Problem 7.7.



7.7 The solution is given in the circuit diagram. Note that the starting point is calculating the current  $I$  in the  $Q_1$ - $R_1$ - $Q_2$  branch. See figure above.

7.8 Refer to Fig. 7.11.

$$I_{C1} \approx I_{REF} = 0.1 \text{ mA}$$

$$V_{BE1} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.642 \text{ V}$$

$$I_{C3} \approx I_{B1} + I_{B2} = 2 I_{B1} = 2 \times \frac{I_{C1}}{\beta}$$

$$= 2 \times \frac{0.1}{100} = 0.002 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.002 \text{ mA}}\right)$$

$$= 0.545 \text{ V}$$

$$V_x = V_{BE3} + V_{BE1} = 1.187 \text{ V}$$

If  $I_{REF}$  is increased to 1 mA,

$$V_{BE1} = 0.7$$

$$I_{C3} \approx 0.02 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1}{0.02}\right) = 0.6 \text{ V}$$

$$V_x = 1.3 \text{ V}$$

Thus,

$$\Delta V_x = 1.3 - 1.187 = 0.113 \text{ V}$$

When  $V_O = V_x$ , the Early effect on  $Q_1$  and  $Q_2$  will be the same, and

$$I_O = I_{REF} / (1 + 2/\beta^2)$$

Thus,  $I_O$  will be

$$I_{REF} = 100 \mu\text{A} \Rightarrow I_O = \frac{100}{1 + (2/100^2)} = 99.98 \mu\text{A}, \text{ for an error of } -0.02 \mu\text{A} \text{ or } -0.02\%.$$

$$I_{REF} = 1 \text{ mA} \Rightarrow I_O = \frac{1}{1 + (2/100^2)} = 0.9998 \text{ mA}, \text{ for an error of } -0.0002 \text{ mA} \text{ or } -0.02\%. \text{ For proper current-source operation, the minimum required voltage at the output is the value needed to keep } Q_3 \text{ in the active region, which is approximately } 0.3 \text{ V.}$$

7.9 Using Eq. (7.28),

$$R_{in} = r_{o1} \parallel \frac{1}{g_{m1}}$$

where

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{V_A' L}{I_{D1}} = \frac{10 \times 0.5}{0.1 \text{ mA}} = 50 \text{ k}\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.5 \times \left(\frac{10}{0.5}\right) \times 0.1} = 1.414 \text{ mA/V}$$

$$\frac{1}{g_{m1}} = 0.71 \text{ k}\Omega$$

Thus,

$$R_{in} = 50 \parallel 0.71 = 0.7 \text{ k}\Omega = 700 \Omega$$

$$A_{is} = \frac{(W/L)_2}{(W/L)_1} = \frac{50/0.5}{10/0.5} = 5 \text{ A/A}$$

$$\begin{aligned} R_o = r_{o2} &= \frac{V_A}{I_{D2}} = \frac{V'_A L}{I_{D2}} \\ &= \frac{10 \times 0.5}{5 \times 0.1} = 10 \text{ k}\Omega \end{aligned}$$

**7.10** Refer to Fig. 7.1.

$$\frac{\Delta I_O}{I_O} = 10\%$$

$$\Delta I_O = 0.1 \times 150 = 15 \mu\text{A}$$

$$\Delta V_O = 1.8 - 0.3 = 1.5 \text{ V}$$

$$r_o = \frac{\Delta V_O}{\Delta I_O} = \frac{1.5 \text{ V}}{15 \mu\text{A}} = 100 \text{ k}\Omega$$

But

$$r_o = \frac{V_A}{I_O} = \frac{V'_A L}{I_O}$$

$$100 = \frac{10 \times L}{0.15} \Rightarrow L = 1.5 \mu\text{m}$$

$$\Rightarrow V_A = 15 \text{ V}$$

$$V_{OV} = V_{DS2\text{min}} = 0.3 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.3 = 0.8 \text{ V}$$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 \left( 1 + \frac{V_{DS}}{V_A} \right)$$

$$150 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.09 \left( 1 + \frac{0.8}{15} \right)$$

$$\Rightarrow \frac{W}{L} = 7.91$$

$$W = 7.91 \times 1.5 = 11.9 \mu\text{m}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.8 - 0.8}{0.15} = 6.7 \text{ k}\Omega$$

**7.11** Refer to Fig. 7.2.

$$I_{REF} = \frac{1}{2} (\mu_n C_{ox}) \left( \frac{W}{L} \right)_1 V_{OV}^2$$

$$25 = \frac{1}{2} \times 387 \times \frac{1.74}{0.54} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$I_{\text{nominal}} = I_{REF} \frac{(W/L)_2}{(W/L)_1} = I_{REF} \left( \frac{W_2}{W_1} \right)$$

$$I_{\text{nominal}} = 25 \times 4 = 100 \mu\text{A}$$

$$V_{CS\text{min}} = V_{OV} = 0.2 \text{ V}$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{\text{nominal}}} = \frac{V'_A L}{I_{\text{nominal}}}$$

$$R_o = \frac{5 \times 0.54}{0.1} = 27 \text{ k}\Omega$$

$$I_O = I_{\text{nominal}} \left[ 1 + \frac{V_O - V_{GS}}{V_{A2}} \right]$$

where

$$V_{GS} = 0.5 + 0.2 = 0.7 \text{ V}$$

$$V_{A2} = 5 \times 0.54 = 2.7 \text{ V}$$

So,

$$I_O = 100 \left( 1 + \frac{V_O - 0.7}{2.7} \right)$$

For

$$V_O = 0.2 \text{ V},$$

$$I_O = 100 \left( 1 + \frac{0.2 - 0.7}{2.7} \right) = 81.5 \mu\text{A}$$

For

$$V_O = 0.7 \text{ V},$$

$$I_O = 100 \left( 1 + \frac{0.7 - 0.7}{2.7} \right) = 100 \mu\text{A}$$

For

$$V_O = 1.2 \text{ V},$$

$$I_O = 100 \left( 1 + \frac{1.2 - 0.7}{2.7} \right) = 118.5 \mu\text{A}$$

For

$$V_O = 1.7 \text{ V},$$

$$I_O = 100 \left( 1 + \frac{1.7 - 0.7}{2.7} \right) = 137 \mu\text{A}$$

**7.12** Referring to Fig. P7.12, we obtain

$$V_{GS1} = V_{GS2}, \text{ so } \frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1}$$

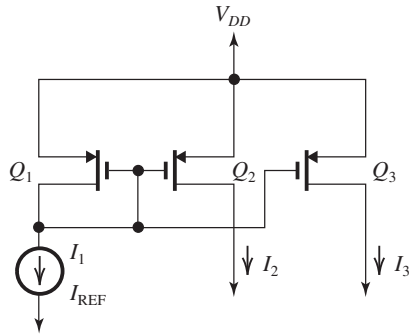
$$\Rightarrow I_{D2} = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

$$I_{D3} = I_{D2}$$

$$V_{GS3} = V_{GS4}, \text{ thus } \frac{I_{D4}}{I_{D3}} = \frac{(W/L)_4}{(W/L)_3}$$

$$\Rightarrow I_O = I_{D4} = I_{REF} \frac{(W/L)_2}{(W/L)_1} \cdot \frac{(W/L)_4}{(W/L)_3}$$

**7.13** Referring to the figure, suppose that  $Q_1$  has  $W = 10 \mu\text{m}$ ,  $Q_2$  has  $W = 20 \mu\text{m}$ , and  $Q_3$  has  $W = 40 \mu\text{m}$ .



(1) With  $Q_1$  diode connected,

$$I_2 = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1} = 100 \mu\text{A} \left( \frac{20}{10} \right) = 200 \mu\text{A}$$

$$I_3 = 100 \mu\text{A} \left( \frac{40}{10} \right) = 400 \mu\text{A}$$

(2) With  $Q_2$  diode connected, and  $W = 20 \mu\text{m}$ ,

$$I_1 = 100 \mu\text{A} \left( \frac{10}{20} \right) = 50 \mu\text{A}$$

$$I_3 = 100 \mu\text{A} \left( \frac{40}{20} \right) = 200 \mu\text{A}$$

(3) If  $Q_3$  with  $W = 40 \mu\text{m}$  is diode connected,

$$I_1 = 100 \mu\text{A} \left( \frac{10}{40} \right) = 25 \mu\text{A}$$

$$I_2 = 100 \mu\text{A} \left( \frac{20}{40} \right) = 50 \mu\text{A}$$

So, with only one transistor diode connected, we can get 25  $\mu\text{A}$ , 50  $\mu\text{A}$ , 200  $\mu\text{A}$ , and 400  $\mu\text{A}$ , or four different currents.

Now, if two transistors are diode connected, the effective width is the sum of the two widths.

(4) If  $Q_1$  and  $Q_2$  are diode connected, then

$W_{\text{eff}} = 20 + 10 = 30 \mu\text{m}$ , so that

$$I_3 = 100 \mu\text{A} \left( \frac{40}{30} \right) = 133 \mu\text{A}$$

(5) If  $Q_2$  and  $Q_3$  are diode connected, then

$W_{\text{eff}} = 20 + 40 = 60 \mu\text{m}$ , so that

$$I_1 = 100 \mu\text{A} \left( \frac{10}{60} \right) = 16.7 \mu\text{A}$$

(6) If  $Q_1$  and  $Q_3$  are diode connected,

$W_{\text{eff}} = 10 + 40 = 50 \mu\text{m}$ , so that

$$I_2 = 100 \mu\text{A} \left( \frac{20}{50} \right) = 40 \mu\text{A}$$

So three different currents are obtained with double-diode connects.

To find  $V_{SG}$ , we use the following for the diode-connected transistor(s):

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) (V_{SG} - |V_{tp}|)^2$$

and substitute  $I_D = I_{\text{REF}} = 100 \mu\text{A}$ . Thus

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{1 \mu\text{m}} \right) (V_{SG} - 0.6)^2$$

$$\Rightarrow V_{SG} = 0.6 + \sqrt{\frac{2}{W(\mu\text{m})}}$$

For the six cases above we obtain

(1)  $W = W_1 = 10 \mu\text{m} \Rightarrow V_{SG} = 1.05 \text{ V}$

(2)  $W = W_2 = 20 \mu\text{m} \Rightarrow V_{SG} = 0.92 \text{ V}$

(3)  $W = W_3 = 40 \mu\text{m} \Rightarrow V_{SG} = 0.82 \text{ V}$

(4)  $W = W_1 + W_2 = 30 \mu\text{m} \Rightarrow V_{SG} = 0.86 \text{ V}$

(5)  $W = W_2 + W_3 = 60 \mu\text{m} \Rightarrow V_{SG} = 0.78 \text{ V}$

(6)  $W = W_1 + W_3 = 50 \mu\text{m} \Rightarrow V_{SG} = 0.80 \text{ V}$

**7.14** Nominally,  $I_O = I_{\text{REF}} = 1 \text{ mA}$

$$r_{o2} = \frac{V_{A2}}{I_O} = \frac{90}{1} = 90 \text{ k}\Omega$$

$$r_{o2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10 - 1}{\Delta I_O} = 90 \Rightarrow \Delta I_O = 0.1 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.1}{1} = 10\% \text{ change}$$

**7.15** (a) If  $I_S = 10^{-17} \text{ A}$  and we ignore base currents, then

$$I_{\text{REF}} = I_S e^{V_{BE}/V_T} \text{ so that}$$

$$V_{BE} = V_T \ln \left( \frac{I_{\text{REF}}}{10^{-17}} \right)$$

For  $I_{\text{REF}} = 10 \mu\text{A}$ ,

$$V_{BE} = 0.025 \ln \left( \frac{10^{-5}}{10^{-17}} \right) = 0.691 \text{ V}$$

For  $I_{\text{REF}} = 10 \text{ mA}$ ,

$$V_{BE} = 0.025 \ln \left( \frac{10^{-2}}{10^{-17}} \right) = 0.863 \text{ V}$$

So for the range of

$$10 \mu\text{A} \leq I_{\text{REF}} \leq 10 \text{ mA},$$

$$10 \mu\text{A} \leq I_O \leq 10 \text{ mA}$$

$$\text{and, } 0.691 \text{ V} \leq V_{BE} \leq 0.863 \text{ V}$$

(b) Accounting for finite  $\beta$ ,

$$I_O = I_{REF} \cdot \frac{1}{1 + 2/\beta}$$

For  $I_{REF} = 10 \mu\text{A}$ ,

$$I_O = \frac{10 \mu\text{A}}{1 + \frac{2}{50}} = 9.62 \mu\text{A}$$

For  $I_{REF} = 0.1 \text{ mA}$ ,

$$I_O = \frac{0.1 \text{ mA}}{1 + \frac{2}{100}} = 0.098 \text{ mA}$$

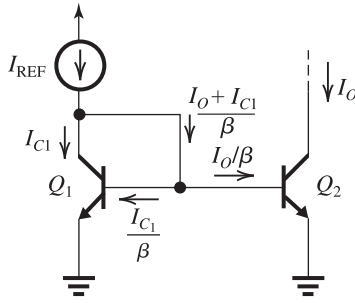
For  $I_{REF} = 1 \text{ mA}$ ,

$$I_O = \frac{1 \text{ mA}}{1 + \frac{2}{100}} = 0.98 \text{ mA}$$

For  $I_{REF} = 10 \text{ mA}$ ,

$$I_O = \frac{10 \text{ mA}}{1 + \frac{2}{50}} = 9.62 \text{ mA}$$

### 7.16



$$I_O = mI_{C1}$$

A node equation at the collector of  $Q_1$  yields

$$I_{REF} = I_{C1} + \frac{I_O + I_{C1}}{\beta}$$

Substituting  $I_{C1} = I_O/m$  results in

$$\frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{m+1}{\beta}} \quad \text{Q.E.D.}$$

For  $\beta = 80$  and the error in the current transfer ratio to be limited to 10%, that is,

$$\frac{m}{1 + \frac{m+1}{\beta}} \geq 0.9m$$

$$\left(1 + \frac{m+1}{\beta}\right) \leq \frac{1}{0.9}$$

$$\frac{m+1}{\beta} \leq \frac{1}{0.9} - 1$$

$$m \leq \beta \left( \frac{1}{0.9} - 1 \right) - 1$$

$$m \leq 80 \left( \frac{1}{0.9} - 1 \right) - 1 = 7.88$$

Thus, the largest current transfer ratio possible is 7.88.

**7.17** Using Eq. (7.21),

$$I_O = I_{REF} \frac{m}{1 + \frac{m+1}{\beta}} \left( 1 + \frac{V_O - V_{BE}}{V_{A2}} \right)$$

This equation can be adapted for the *pn*p mirror of Fig. P7.17 by substituting  $m = 1$ , replacing  $V_O$  with the voltage across  $Q_3$ , namely  $(3 - V_O)$ , replacing  $V_{BE}$  with  $V_{EB}$ , and  $V_{A2}$  with  $|V_A|$ :

$$I_O = I_{REF} \frac{1 + [(3 - V_O - V_{EB})/|V_A|]}{1 + (2/\beta)} \quad (1)$$

Now, substituting  $I_O = 1 \text{ mA}$ ,  $V_O = 1 \text{ V}$ ,  $\beta = 50$ ,  $|V_A| = 50 \text{ V}$ , and

$$V_{EB} = V_T \ln \frac{I_O}{I_S} = 0.025 \ln \left( \frac{10^{-3}}{10^{-15}} \right) = 0.691 \text{ V}$$

results in

$$I_{REF} = \frac{1 \times (1 + 0.04)}{1 + \frac{3 - 1 - 0.691}{50}} = 1.013 \text{ mA}$$

$$R = \frac{V_{CC} - V_{EB}}{I_{REF}} = \frac{3 - 0.691}{1.013} = 2.28 \text{ k}\Omega$$

Maximum allowed voltage  $V_O = 3 - 0.3 = 2.7 \text{ V}$ . For  $V_O = 2.7 \text{ V}$ , Eq. (1) yields

$$I_O = 1.013 \frac{1 + \frac{3 - 2.7 - 0.691}{50}}{1.04} = 0.966 \text{ mA}$$

For  $V_O = -5 \text{ V}$ , Eq. (1) yields

$$I_O = 1.013 \frac{1 + \frac{3 - (-5) - 0.691}{50}}{1.04} = 1.116 \text{ mA}$$

Thus, the change in  $I_O$  is 0.15 mA.

**7.18** Refer to the circuit in Fig. P7.18.

$$V_2 = 2.7 - V_{EB} = 2.7 - 0.7 = +2 \text{ V}$$

$$V_3 = 0 + V_{EB} = +0.7 \text{ V}$$

Thus,  $Q_3$  and  $Q_4$  are operating in the active mode, and each is carrying a collector current of  $I/2$ .

The same current is flowing in  $Q_2$  and  $Q_1$ ; thus

$$V_1 = -2.7 + \frac{I}{2}R$$

But

$$V_1 = -V_{BE1} = -0.7$$

Thus,

$$-0.7 = -2.7 + \frac{1}{2}IR$$

$\Rightarrow IR = 4 \text{ V}$

The current  $I$  splits equally between  $Q_5$  and  $Q_6$ ; thus

$V_4 = -2.7 + \left(\frac{I}{2}\right)R = -2.7 + 2 = -0.7 \text{ V}$

$V_5 = -2.7 + \left(\frac{I}{2}\right)\left(\frac{R}{2}\right) = -2.7 + 1 = -1.7 \text{ V}$

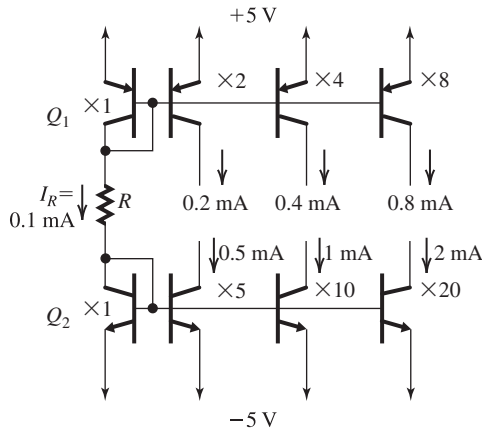
Thus,  $Q_5$  and  $Q_6$  are operating in the active mode as we have implicitly assumed.

Note that the values of  $V_1, V_2, V_3, V_4,$  and  $V_5$  do not depend on the value of  $R$ . Only  $I$  depends on the value of  $R$ :

(a)  $R = 10 \text{ k}\Omega \Rightarrow I = \frac{4}{10} = 0.4 \text{ mA}$

(b)  $R = 100 \text{ k}\Omega \Rightarrow I = \frac{4}{100} = 0.04 \text{ mA}$

**7.19** There are various ways this design could be achieved, but the most straightforward is the one shown:



With this scheme,

$R = \frac{5 - 0.7 - 0.7 - (-5)}{0.1 \text{ mA}} = 86 \text{ k}\Omega$

and each transistor has EBJ areas proportional to the current required. Multiple, parallel transistors are acceptable.

*Note:* This large value of  $R$  is not desirable in integrated form; other designs may be more suitable.

Even without knowing exact circuitry, we can find the total power dissipation as approximately

$P_T = P_{CC} + P_{EE}$

$P_T = 5 \text{ V} (0.1 + 0.2 + 0.4 + 0.8) \text{ mA}$

$+ 5 \text{ V} (0.1 + 0.5 + 1 + 2) \text{ mA}$

$P_T = 7.5 \text{ mW} + 18 \text{ mW} = 25.5 \text{ mW}$

**7.20** (a) Figure 1 shows the current conveyor circuit with  $Y$  connected to a voltage  $V$ ,  $X$  fed with a current source  $I$ , and  $Z$  connected to a voltage  $V_Z$  that keeps  $Q_5$  operating in the active mode. Assuming that all transistors are operating in the active mode and that  $\beta \gg 1$ , so that we can neglect all base currents, we see that the current  $I$  through  $Q_1$  will flow through the two-output mirror  $Q_3, Q_4,$  and  $Q_5$ . The current  $I$  in  $Q_5$  will be drawn from  $Q_2$ , which forms a mirror with  $Q_1$ . Thus  $V_{EB2} = V_{EB1}$  and the voltage that appears at  $X$  will be equal to  $V$ . The current in  $Q_5$  will be equal to  $I$ , thus terminal  $Z$  sinks a constant current  $I$ .

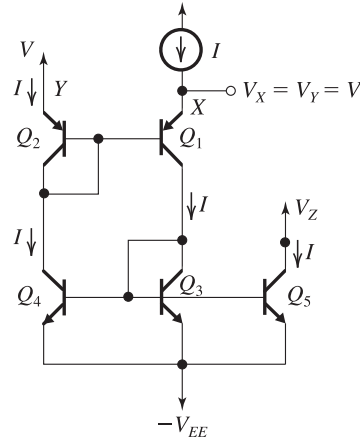


Figure 1

(b)

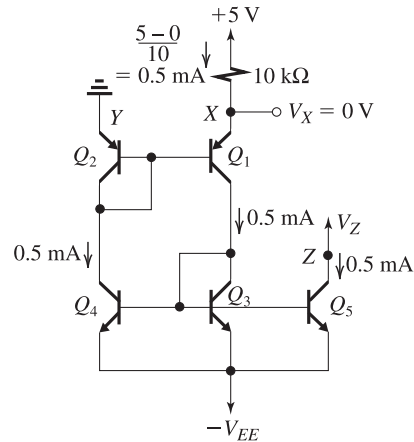


Figure 2

Figure 2 shows the special case of  $V = 0 \text{ V}$ . As before, the voltage at  $X, V_X,$  will be equal to  $V$ . Thus

$V_X = 0$

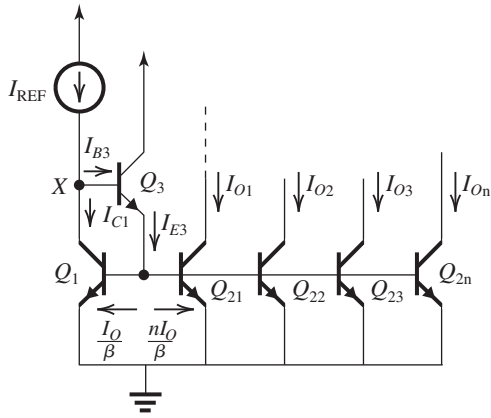


That is, a virtual ground appears at X, and thus the current  $I$  that flows into X can be found from

$$I = \frac{5 - V_X}{10 \text{ k}\Omega} = \frac{5 - 0}{10} = 0.5 \text{ mA}$$

This is the current that will be mirrored to the output, resulting in  $I_Z = 0.5 \text{ mA}$ .

7.21



$$I_{O1} = I_{O2} = I_{O3} \cdots = I_{On} = I_O = I_{C1}$$

The emitter of  $Q_3$  supplies the base currents for all transistors, so

$$I_{E3} = \frac{(n+1)I_O}{\beta}$$

$$I_{REF} = I_{B3} + I_O = \frac{(n+1)I_O}{\beta(\beta+1)} + I_O$$

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{(n+1)}{\beta(\beta+1)}} \approx \frac{1}{1 + \frac{n+1}{\beta^2}}$$

For the deviation from unity to be kept  $\leq 0.2\%$

$$\frac{n+1}{\beta^2} \leq 0.002$$

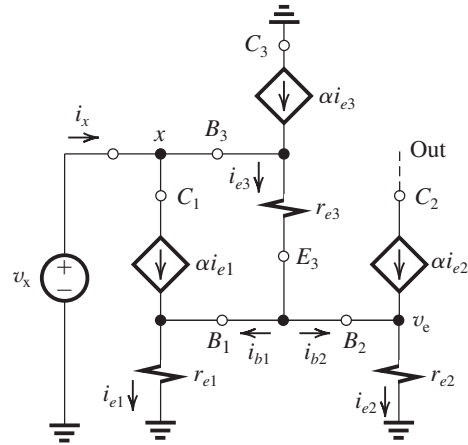
$$\Rightarrow n_{\max} = 0.002 \times 150^2 - 1 = 44$$

7.22 Refer to Fig. 7.11 and observe that  $I_{C1} \approx I_{REF}$  and  $I_{C2} = I_{C1}$ ; thus each of  $Q_1$  and  $Q_2$  is operating at a collector bias current approximately equal to  $I_{REF}$ . Transistor  $Q_3$  is operating at an emitter bias current

$$I_{E3} = I_{B1} + I_{B2} = 2I_B = 2I_C/\beta = \frac{2(1-\alpha)}{\alpha} I_C = \frac{2(1-\alpha)}{\alpha} I_{REF}$$

Replacing each of the three transistors with its  $T$  model and applying an input test voltage  $v_x$  to

determine  $R_{in}$ , we obtain the equivalent circuit shown.



In this equivalent circuit,

$$r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{\alpha V_T}{I_{REF}}$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{\alpha V_T}{2(1-\alpha)I_{REF}}$$

$$i_{e1} = i_{e2} = i_e$$

$$i_{e3} = i_{b1} + i_{b2} = 2(1-\alpha)i_e$$

From the figure we obtain

$$\begin{aligned} i_x &= \alpha i_{e1} + (1-\alpha)i_{e3} \\ &= \alpha i_e + (1-\alpha) \times 2(1-\alpha)i_e \\ &= i_e[\alpha + 2(1-\alpha)^2] \end{aligned}$$

But  $2(1-\alpha)^2 \ll \alpha$ . Thus,

$$i_x \approx \alpha i_e \tag{1}$$

$$\begin{aligned} v_x &= i_{e3} r_{e3} + i_{e1} r_{e1} \\ &= 2(1-\alpha)i_e \frac{\alpha V_T}{2(1-\alpha)I_{REF}} + i_e \frac{\alpha V_T}{I_{REF}} \end{aligned}$$

$$v_x = \alpha i_e \left[ \frac{V_T}{I_{REF}} + \frac{V_T}{I_{REF}} \right]$$

Now, using  $i_x = \alpha i_e$  from Eq. (1), we have

$$v_x = i_x \times \frac{2V_T}{I_{REF}}$$

Thus,

$$R_{in} \equiv \frac{v_x}{i_x} = \frac{2V_T}{I_{REF}} \quad \text{Q.E.D.}$$

For  $I_{REF} = 100 \mu\text{A} = 0.1 \text{ mA}$ ,

$$R_{in} = \frac{2 \times 25 \text{ mV}}{0.1 \text{ mA}} = 500 \Omega$$

$$7.23 \quad A_{is} = 4 = \frac{(W/L)_2}{(W/L)_1}$$

Since  $L_1 = L_2$ , then

$$\frac{W_2}{W_1} = 4$$

$$R_{in} = r_{o1} \parallel \frac{1}{g_{m1}} \simeq \frac{1}{g_{m1}}$$

For

$$R_{in} = 500 \, \Omega \Rightarrow g_{m1} = 2 \text{ mA/V}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

Thus,

$$2 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.2}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$R_o = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V'_A L}{I_{D2}}$$

Thus,

$$20 = \frac{20 L}{4 \times 0.2}$$

$$\Rightarrow L = 0.8 \, \mu\text{m}$$

$$W_1 = 25 \times 0.8 = 20 \, \mu\text{m}$$

$$W_2 = 4 W_1 = 80 \, \mu\text{m}$$

The actual  $A_{is}$  is given by Eq. (7.31),

$$A_{is} = \frac{(W/L)_2}{(W/L)_1} \frac{1}{1 + (1/g_{m1} r_{o1})}$$

where

$$\frac{(W/L)_2}{(W/L)_1} = 4$$

$$g_{m1} = 2 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{V'_A L_1}{I_{D1}} = \frac{20 \times 0.8}{0.2} = 80 \text{ k}\Omega$$

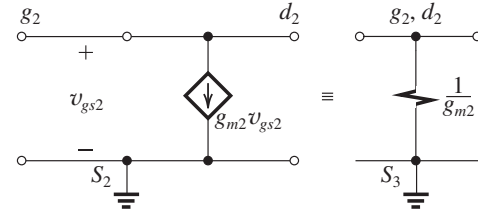
Thus,

$$A_{is} = \frac{4}{1 + \frac{1}{2 \times 80}} = 3.975 \text{ A/A}$$

indicating an error of  $-0.6\%$ .

**7.24** Refer to Fig. P7.24. Consider first the diode-connected transistor  $Q_2$ . From the figure we see that from a small-signal point of view it is equivalent to a resistance  $1/g_{m2}$ . Thus the voltage gain of  $Q_1$  will be

$$\frac{v_{d1}}{v_i} = -g_{m1} \times \frac{1}{g_{m2}} = -\frac{g_{m1}}{g_{m2}}$$



The signal current in the drain of  $Q_1$ ,  $g_{m1} v_i$ , will be mirror in the drain of  $Q_3$ ;

$$i_{d3} = g_{m1} v_i \frac{(W/L)_3}{(W/L)_2} = g_{m1} v_i \frac{W_3}{W_2}$$

which flows through  $R_L$  and produces the output voltage  $v_o$ ,

$$v_o = i_{d3} R_L = g_{m1} v_i \frac{W_3}{W_2} R_L$$

Thus, the small-signal voltage gain will be

$$\frac{v_o}{v_i} = g_{m1} R_L (W_3/W_2)$$

**7.25** Replacing  $Q_1$  and  $Q_2$  with their small-signal hybrid- $\pi$  models results in the equivalent circuit shown in the figure on the next page. Observe that the controlled source  $g_{m1} v_{\pi 1}$  appears across its controlling voltage  $v_{\pi 1}$ ; thus the controlled source can be replaced with a resistance  $(1/g_{m1})$ . The input resistance  $R_{in}$  can now be obtained by inspection as

$$R_{in} = r_{o1} \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2}$$

Since  $r_{o1} \gg r_{\pi 1}$ ,

$$R_{in} \simeq \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2} \quad (1)$$

Now,

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{\beta}{g_{m1}}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{\beta}{g_{m2}}$$

Since  $I_{C2} = m I_{C1}$ ,  $g_{m2} = m g_{m1}$ , thus

$$r_{\pi 2} = \frac{\beta}{m g_{m1}}$$

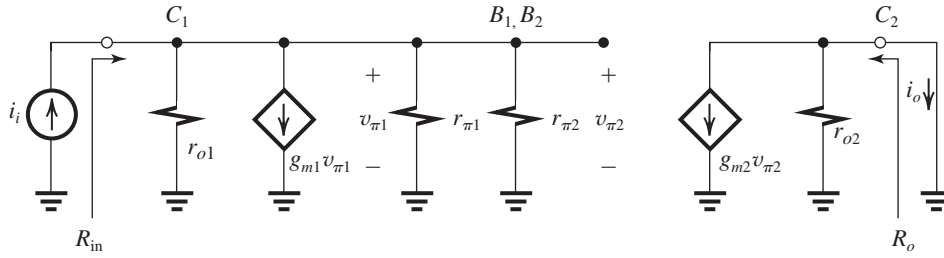
Substituting for  $r_{\pi 1}$  and  $r_{\pi 2}$  into Eq. (1) we obtain

$$R_{in} = 1/g_{m1} \left[ 1 + \frac{m+1}{\beta} \right] \quad (2)$$

The short-circuit output current  $i_o$  is given by

$$i_o = g_{m2} v_{\pi 2}$$

This figure belongs to Problem 7.25.



Since  $v_{\pi 2} = v_{\pi 1} = i_i R_{in}$ , then the short-circuit current gain  $A_{is}$  is given by

$$A_{is} = \frac{i_o}{i_i} = g_{m2} R_{in} = m g_{m1} R_{in} = \frac{m}{1 + \frac{m+1}{\beta}}$$

For situations where  $\beta_1$  and  $\beta_2$  are large, we can neglect  $r_{\pi 1}$  and  $r_{\pi 2}$  in Eqs. (1) and (2) to obtain

$$R_{in} \simeq 1/g_{m1}$$

$$A_{is} \simeq g_{m2}/g_{m1}$$

Finally, by inspection

$$R_o = r_{o2}$$

For  $I = 0.1$  mA,  $m = 5$ ,  $\beta = 100$ , and  $V_A = 30$  V,

$$g_{m1} = 4$$
 mA/V

$$\Rightarrow R_{in} = \frac{1}{4 \left( 1 + \frac{5+1}{100} \right)} = 0.236$$
 k $\Omega$

$$A_{is} = \frac{5}{1 + \frac{5+1}{100}} = 4.72$$
 A/A

$$R_o = \frac{30}{5 \times 0.1} = 60$$
 k $\Omega$

7.26 (a)

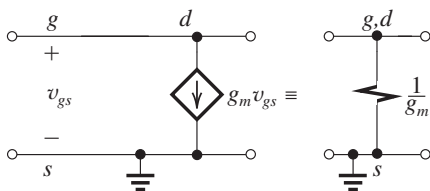


Figure 1

Replacing the MOSFET with its hybrid- $\pi$  model but neglecting  $r_o$  results in the equivalent circuit in Fig. 1. Observing that the controlled-source  $g_m v_{gs}$  appears across its control voltage  $v_{gs}$ , we can replace it by a resistance  $1/g_m$ , as indicated. Thus the small-signal resistance of the

diode-connected MOS transistor is  $1/g_m$ . For the given values,

$$g_m = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right) I_D} = \sqrt{2 \times 0.2 \times 10 \times 0.1} = 0.632$$
 mA/V

$$\frac{1}{g_m} = 1.6$$
 k $\Omega$

(b) Replacing the BJT with its hybrid- $\pi$  model results in the equivalent circuit in Fig. 2 (see next page). Observing that the controlled-source  $g_m v_{\pi}$  appears across its control voltage  $v_{\pi}$ , we can replace it by a resistance  $1/g_m$ , as indicated. Next the two parallel resistances  $1/g_m$  and  $r_{\pi}$  can be combined as

$$\frac{1}{\frac{1}{g_m} + r_{\pi}} \times r_{\pi} = \frac{r_{\pi}}{1 + g_m r_{\pi}} = \frac{r_{\pi}}{\beta + 1} = r_e$$

Thus, the diode-connected BJT has a small-signal resistance  $r_e$ . For the given data,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250$$
  $\Omega$

7.27 Refer to Fig. 7.13(b).

$$g_m = \frac{I_C}{V_T} = \frac{I}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20$$
 mA/V

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{0.5 \text{ mA}} = 200$$
 k $\Omega$

$$R_{in} = r_{\pi} = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5$$
 k $\Omega$

$$A_{vo} = -A_0 = -g_m r_o = -20 \times 200 = -4000$$
 V/V

$$R_o = r_o = 200$$
 k $\Omega$

To raise  $R_{in}$  by a factor of 5 by changing  $I$ , the value of  $I$  must be lowered by the same factor to  $I = 0.1$  mA.

Now,  $g_m$  is reduced by a factor of 5 and  $r_o$  is increased by a factor of 5, keeping  $A_{vo}$  unchanged at  $-4000$  V/V. However,  $R_o$  will be increased to

This figure belongs to Problem 7.26, part (b).

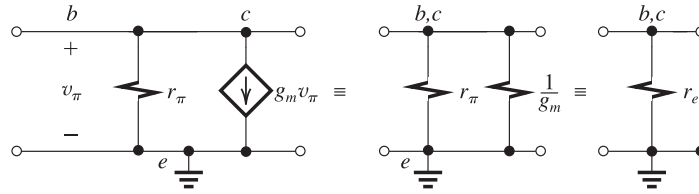
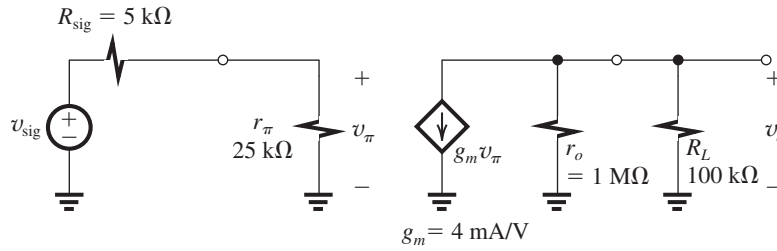


Figure 2

This figure belongs to Problem 7.27.



$$R_o = 5 \times 200 \text{ k}\Omega = 1 \text{ M}\Omega$$

If the amplifier is fed with a signal source having  $R_{\text{sig}} = 5 \text{ k}\Omega$  and a  $100\text{-k}\Omega$  load resistance is connected to the output, the equivalent circuit shown below results.

$$\begin{aligned} \frac{v_o}{v_{\text{sig}}} &= \frac{r_{\pi}}{r_{\pi} + R_{\text{sig}}} \times -g_m(r_o \parallel R_L) \\ &= -\frac{25}{25 + 5} \times 4 (1000 \text{ k}\Omega \parallel 100 \text{ k}\Omega) \\ &= -303 \text{ V/V} \end{aligned}$$

**7.28** From Eq. (7.46) we see that  $A_0$  is inversely proportional to  $\sqrt{I_D}$ . Thus

$$\begin{aligned} I_D = 100 \mu\text{A} \quad A_0 &= 50 \text{ V/V} \\ I_D = 25 \mu\text{A} \quad A_0 &= 100 \text{ V/V} \\ I_D = 400 \mu\text{A} \quad A_0 &= 25 \text{ V/V} \end{aligned}$$

From Eq. (7.42),  $g_m$  is proportional to  $\sqrt{I_D}$ . Thus changing  $I_D$  from  $100 \mu\text{A}$  to  $25 \mu\text{A}$  reduces  $g_m$  by a factor of 2. Changing  $I_D$  from  $100 \mu\text{A}$  to  $400 \mu\text{A}$  increases  $g_m$  by a factor of 2.

$$\mathbf{7.29} \quad A_0 = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}}$$

$$\begin{aligned} 25 &= \frac{2 \times 5 \times L}{0.2} \\ \Rightarrow L &= 0.5 \mu\text{m} \end{aligned}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

$$1 = \frac{2I}{0.2}$$

$$\Rightarrow I = 0.1 \text{ mA}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.04$$

$$\Rightarrow \frac{W}{L} = 12.5$$

**7.30** Refer to Fig. 7.15(a) and Table K.1.

$$\begin{aligned} V_G &= V_{DD} - V_{SG2} \\ &= V_{DD} - (|V_{tp}| + |V_{OV}|) \\ &= 1.8 - (0.5 + 0.2) \\ &= 1.1 \text{ V} \end{aligned}$$

$$I_{D1} = I_{D2} = I = 100 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} (\mu_n C_{ox}) \left( \frac{W}{L} \right)_1 |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 387 \times \left( \frac{W}{L} \right)_1 \times 0.2^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 12.92$$

$$I_{D2} = \frac{1}{2} (\mu_p C_{ox}) \left( \frac{W}{L} \right)_2 |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 86 \times \left( \frac{W}{L} \right)_2 \times 0.2^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = 58.14$$

$$r_{o1} = \frac{|V'_{An}| L}{I_{D1}} = \frac{5L}{0.1} = 50L \text{ k}\Omega$$

$$r_{o2} = \frac{|V'_{Ap}| L}{I_{D2}} = \frac{6L}{0.1} = 60L \text{ k}\Omega$$

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2})$$

where

$$g_{m1} = \frac{2I_{D1}}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$(r_{o1} \| r_{o2}) = (50L \| 60L) = 27.3L \text{ k}\Omega$$

$$A_v = -20 \text{ V/V}$$

Thus,

$$-20 = -1 \times 27.3L$$

$$\Rightarrow L = 0.73 \text{ }\mu\text{m}$$

**7.31** Refer to Fig. P7.31. The gain of the first stage is

$$A_{v1} = -g_{m1}(r_{o1}/2)$$

where  $(r_{o1}/2)$  is the equivalent resistance at the output of  $Q_1$  and includes  $r_{o1}$  in parallel with the output resistance of the current-source load, which is equal to  $r_{o1}$ . Similarly, the gain of the second stage is

$$A_{v2} = -g_{m2}(r_{o2}/2)$$

Now because  $V_{An} = |V_{Ap}| = |V_A|$  and both  $Q_1$  and  $Q_2$  are operating at equal currents  $I$ , we have

$$r_{o1} = r_{o2} = r_o$$

The overall voltage gain  $A_v$  will be

$$A_v = A_{v1}A_{v2}$$

$$A_v = \frac{1}{4} g_{m1}g_{m2}r_o^2$$

If the two transistors are operated at equal overdrive voltages,  $|V_{OV}|$ , both will have equal  $g_m$ ,

$$A_v = \frac{1}{4} (g_m r_o)^2$$

and

$$g_m r_o = \frac{2|V_A|}{|V_{OV}|} = \frac{2 \times 5}{|V_{OV}|} = \frac{10}{|V_{OV}|}$$

$$A_v = 400 = \frac{1}{4} \times \left[ \frac{10}{|V_{OV}|} \right]^2$$

$$\Rightarrow |V_{OV}| = 0.25 \text{ V}$$

**7.32** Refer to Fig. 7.16(a) and to Table K.1

(a)

$$V_I = V_m + V_{OV1}$$

$$= 0.4 + 0.15 = 0.55 \text{ V}$$

For  $Q_1$ ,

$$I_{D1} = \frac{1}{2} (\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 |V_{OV1}|^2$$

$$100 = \frac{1}{2} \times 511 \times \left(\frac{W}{L}\right)_1 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 17.4$$

For  $Q_2$  and  $Q_3$ ,

$$I_{D2,3} = \frac{1}{2} (\mu_p C_{ox}) \left(\frac{W}{L}\right)_{2,3} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 128 \times \left(\frac{W}{L}\right)_{2,3} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = 69.4$$

(b)

$$A_v = -g_{m1}(r_{o1} \| r_{o2})$$

where

$$g_{m1} = \frac{2I_{D1}}{V_{OV}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V_{An} L}{I} = \frac{5 \times 0.4}{0.1} = 20 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{|V_{Ap}| L}{I} = \frac{6 \times 0.4}{0.1} = 24 \text{ k}\Omega$$

Thus,

$$A_v = -1.33(20 \| 24)$$

$$= -14.5 \text{ V/V}$$

(c)

$$V_{OV1} \leq v_o \leq V_{DD} - |V_{OV2}|$$

$$0.15 \text{ V} \leq v_o \leq (1.3 - 0.15) \text{ V}$$

$$0.15 \text{ V} \leq v_o \leq 1.15 \text{ V}$$

(d) To keep the gain unchanged,

$$R_D = r_{o2} = 24 \text{ k}\Omega$$

To keep the signal swing at the output unchanged, we must bias  $Q_1$  so that  $V_{DS1}$  = midpoint of signal swing, thus

$$V_{DS1} = 0.65 \text{ V}$$

We can now find the required value of the power supply,

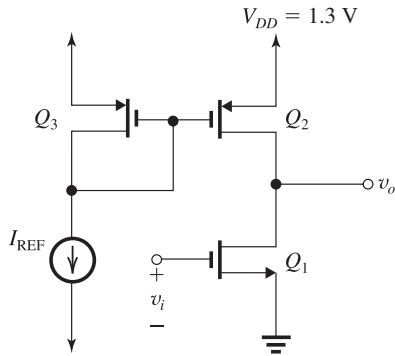
$$V'_{DD} = V_{DS1} + I_D R_D$$

$$= 0.65 + 0.1 \times 24$$

$$= 3.05 \text{ V}$$

which is much larger than the voltages allowed in the 0.13- $\mu\text{m}$  technology.

7.33



For an output of 1.1 V,

$$V_{SD2\min} = |V_{OV2}| = 1.3 - 1.1 = 0.2 \text{ V,}$$

For an output of 0.2 V,

$$V_{DS1\min} = 0.2 \text{ V,}$$

thus

$$V_{OV1} = 0.2 \text{ V}$$

Since  $I_{D2} = I_{D3} = I_{D1} = 50 \mu\text{A}$

and  $I_D = \frac{1}{2} (\mu_p C_{ox}) (W/L) V_{OV}^2$ , we have

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{ox}) |V_{OV}|^2} = \frac{2(50 \mu\text{A})}{(128 \mu\text{A}/\text{V}^2) (0.2 \text{ V})^2} = 19.5$$

For  $Q_1$ ,

$$\left(\frac{W}{L}\right)_1 = \frac{2(50 \mu\text{A})}{(511 \mu\text{A}/\text{V}^2) (0.2 \text{ V})^2} = 4.9$$

$A_v$  must be at least  $-10 \text{ V/V}$

and  $A_v = -g_m (r_{o1} \parallel r_{o2})$

$$g_{m1} = \frac{2I_D}{V_{OV1}} = \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$r_{o1} \parallel r_{o2} = \frac{10}{0.5} = 20 \text{ k}\Omega$$

But

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V'_{An} L}{I_{D1}} = \frac{5L}{0.05} = 100L$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{|V'_{Ap}| L}{I_{D2}} = \frac{6L}{0.05} = 120L$$

Thus,

$$100L \parallel 120L = 20 \text{ k}\Omega$$

$$\Rightarrow L = 0.367 \mu\text{m}$$

If  $L$  is to be an integer multiple of 0.13  $\mu\text{m}$ , then

$$L = 0.39 \mu\text{m}$$

The gain achieved will be  $-10.6 \text{ V/V}$ .

To raise the gain to 20  $\text{V/V}$ ,  $r_{o1} \parallel r_{o2}$  has to be raised to 40  $\text{k}\Omega$ , which requires

$$L = 2 \times 0.367 = 0.734$$

Again, to use a multiple of 0.13  $\mu\text{m}$  we select  $L = 0.78 \mu\text{m}$ .

This doubling of  $L$  will mean a doubling of  $W$  for each of the three transistors. Thus the area ( $WL$ ) of each transistor will increase by a factor of 4 and the total gate area of the circuit will increase by the same factor.

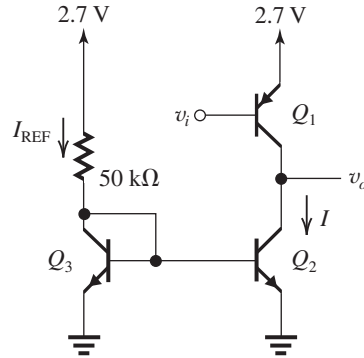
7.34 (a)  $I_{\text{REF}} = I_{C3} = \frac{2.7 - V_{BE3}}{50 \text{ k}\Omega}$

$$I_{\text{REF}} = \frac{2.7 - 0.7}{50}$$

$$= 0.04 \text{ mA}$$

$$\Rightarrow I_{C2} = 5I_{C3}$$

$$I_{C2} = I = 0.2 \text{ mA} \Rightarrow I = 0.2 \text{ mA}$$



(b)  $|V_A| = 20 \text{ V} \Rightarrow r_{o1} = \frac{|V_A|}{I} = \frac{20}{0.2} = 100 \text{ k}\Omega$

$$r_{o2} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

Total resistance at the collector of  $Q_1$  is equal to  $r_{o1} \parallel r_{o2}$ , thus

$$r_{\text{tot}} = 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

(c)  $g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$

$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

(d)  $R_{\text{in}} = r_{\pi 1} = 6.25 \text{ k}\Omega$

$$R_o = r_{o1} \parallel r_{o2} = 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

$$A_v = -g_{m1} R_o = -8 \times 50 = -400 \text{ V/V}$$

7.35 For  $I = 10 \mu\text{A}$ :

$$g_m = \frac{I}{V_T} = \frac{10 \mu\text{A}}{25 \text{ mV}} = 0.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4 \text{ mA/V}} = 250 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10 \text{ V}}{10 \mu\text{A}} = 1 \text{ M}\Omega$$

$$A_0 = g_m r_o = \frac{V_A}{V_I} = \frac{10 \text{ V}}{0.025 \text{ V}} = 400 \text{ V/V}$$

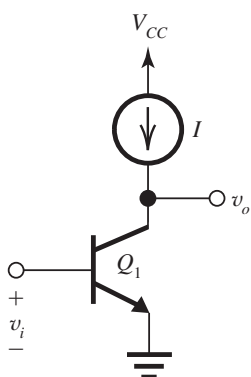
For  $I = 100 \mu\text{A}$  :

$$g_m = \frac{100 \mu\text{A}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{100 \mu\text{A}} = 100 \text{ k}\Omega$$

$$A_0 = 4 \text{ mA/V} \times 100 \text{ k}\Omega = 400 \text{ V/V}$$



For  $I = 1 \text{ mA}$ :

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{1 \text{ mA}} = 10 \text{ k}\Omega$$

$$A_0 = 40 \text{ mA/V} \times 10 \text{ k}\Omega = 400 \text{ V/V}$$

$I$	$g_m$	$r_\pi$	$r_o$	$A_0$
$10 \mu\text{A}$	$0.4 \text{ mA/V}$	$250 \text{ k}\Omega$	$1 \text{ M}\Omega$	$400 \text{ V/V}$
$100 \mu\text{A}$	$4.0 \text{ mA/V}$	$25 \text{ k}\Omega$	$100 \text{ k}\Omega$	$400 \text{ V/V}$
$1 \text{ mA}$	$40 \text{ mA/V}$	$2.5 \text{ k}\Omega$	$10 \text{ k}\Omega$	$400 \text{ V/V}$

7.36

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 10 \times 0.4}{0.2} = 40 \text{ V/V}$$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$1 = \frac{2I_D}{0.2} \Rightarrow I_D = 0.1 \text{ mA}$$

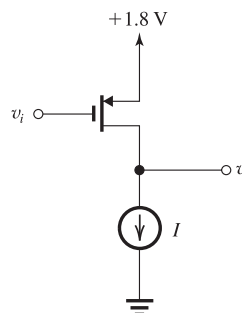
$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 12.5$$

$$W = 5 \mu\text{m}$$

7.37



The highest instantaneous voltage allowed at the drain is that which results in a voltage equal to ( $V_{OV}$ ) across the transistor. Thus

$$v_{Omax} = 1.8 - 0.2 = +1.6 \text{ V}$$

7.38 For the *npn* transistor,

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

For the NMOS transistor,

$$g_m = \frac{2I_D}{V_{OV}}$$

$$4 = \frac{2I_D}{0.25}$$

$$\Rightarrow I_D = 0.5 \text{ mA}$$

7.39

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.05 \text{ mA}}{0.15 \text{ V}} = 0.67 \text{ mA/V}$$

From Table K.1 (in Appendix K),

$$V_A' = 5 \text{ V}/\mu\text{m}$$

Thus,

$$V_A = V_A' L = 5 \times 0.5 = 2.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{2.5}{0.05} = 50 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.67 \times 50 = 33.5 \text{ V/V}$$

From Table K.1,

$$\mu_n C_{ox} = 511 \mu\text{A}/\text{V}^2$$

Since,

$$I_D = \frac{1}{2}(\mu_n C_{ox}) \left(\frac{W}{L}\right) V_{OV}^2$$

then,

$$50 = \frac{1}{2} \times 511 \times \left(\frac{W}{L}\right) \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 8.7$$

$$W = 8.7 \times 0.5 = 4.35 \mu\text{m}$$

**7.40**  $g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$

From Table K.1 (Appendix K), for the 0.18- $\mu\text{m}$  process we have

$$|V_A'| = 5 \text{ V}/\mu\text{m}, \mu_n C_{ox} = 387 \mu\text{A}/\text{V}^2$$

Thus, for our NMOS transistor whose  $L = 0.54 \mu\text{m}$ ,

$$V_A = 5 \times 0.54 = 2.7 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{2.7 \text{ V}}{0.2 \text{ mA}} = 13.5 \text{ k}\Omega$$

$$A_0 = g_m r_o = 2 \times 13.5 = 27 \text{ V/V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$200 = \frac{1}{2} \times 387 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 25.8 \Rightarrow W = 14 \mu\text{m}$$

**7.41** For the BJT cell:

$$g_m = \frac{I_C}{V_T} = \frac{I_C}{0.025 \text{ V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{I_C}$$

$$A_0 = g_m r_o = \frac{V_A}{V_T} = \frac{100 \text{ V}}{0.025 \text{ V}} = 4000 \text{ V/V}$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{100}{g_m}$$

For the MOSFET cell:

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} = \sqrt{2 \times 0.2 \times 40 \times I_D}$$

$$= \sqrt{16I_D} = 4\sqrt{I_D} \text{ mA/V} \quad (I_D \text{ in mA})$$

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{I_D}$$

$$A_0 = g_m r_o = \frac{40}{\sqrt{I_D}} \text{ V/V} \quad (I_D \text{ in mA})$$

$$R_{in} = \infty$$

	BJT Cell		MOSFET Cell	
Bias current	$I_C = 0.1 \text{ mA}$	$I_C = 1 \text{ mA}$	$I_D = 0.1 \text{ mA}$	$I_D = 1 \text{ mA}$
$g_m$ (mA/V)	4	40	1.26	4
$r_o$ (k $\Omega$ )	1000	100	100	10
$A_0$ (V/V)	4000	4000	126	40
$R_{in}$ (k $\Omega$ )	25	2.5	$\infty$	$\infty$

**7.42** Using Eq. (7.46),

$$A_0 = \frac{V_A' \sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

$$20 = \frac{5\sqrt{2} \times 0.4 \times 10 \times 0.54 \times 0.54}{\sqrt{I_D}}$$

$$\Rightarrow I_D = 0.146 \text{ mA}$$

**7.43**  $g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$

$$= \sqrt{2 \times 0.4 \times 10 I_D} = 2.83\sqrt{I_D}$$

$$I_D = 20 \mu\text{A}, \quad g_m = 2.83\sqrt{0.020} = 0.4 \text{ mA/V}$$

$$I_D = 200 \mu\text{A}, \quad g_m = 2.83\sqrt{0.20} = 1.26 \text{ mA/V}$$

$$I_D = 2.0 \text{ mA}, \quad g_m = 2.53\sqrt{2.0} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{I_D} = \frac{1.8}{I_D}$$

$$A_0 = g_m r_o$$

$$I_D = 20 \mu\text{A} \quad r_o = \frac{1.8}{0.020} = 90 \text{ k}\Omega$$

$$A_0 = 0.4 \times 90 = 36 \text{ V/V}$$

$$I_D = 200 \mu\text{A} \quad r_o = \frac{1.8}{0.20} = 9 \text{ k}\Omega$$

$$A_0 = 1.26 \times 9 = 11.4 \text{ V/V}$$

$$I_D = 2.0 \text{ mA} \quad r_o = \frac{1.8}{2.0} = 0.9 \text{ k}\Omega$$

$$A_0 = 4 \times 0.9 = 3.6 \text{ V/V}$$

**7.44**

$$L = 0.36 \mu\text{m}, \quad V_{OV} = 0.25 \text{ V}, \quad I_D = 10 \mu\text{A}$$

(a)  $g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 10}{0.25} = 80 \mu\text{A/V}$



$$r_o = \frac{V_A}{I_D} = \frac{V'_A L}{I_D}$$

From Appendix K, Table K.1,  $V'_A = 5 \text{ V}/\mu\text{m}$ ,

$$r_o = \frac{5 \times 0.36}{10} = 0.18 \text{ M}\Omega$$

$$A_0 = g_m r_o = 80 \times 0.18 = 14.4 \text{ V/V}$$

(b) If  $I_D$  is increased to  $100 \mu\text{A}$  (i.e., by a factor of 10),  $V_{OV}$  increases by a factor of  $\sqrt{10} = 3.16$  to

$$V_{OV} = 0.25 \times 3.16 = 0.79 \text{ V}$$

and  $g_m$  increases by a factor of  $\sqrt{10} = 3.16$  to

$$g_m = 80 \times 3.16 = 253 \mu\text{A/V} = 0.253 \text{ mA/V}$$

and  $r_o$  decreases by a factor of 10 to

$$r_o = \frac{0.18 \text{ M}\Omega}{10} = 18 \text{ k}\Omega$$

Thus,  $A_0$  becomes

$$A_0 = 0.253 \times 18 = 4.55 \text{ V/V}$$

(c) If the device is redesigned with a new value of  $W$  so that it operates at

$$V_{OV} = 0.25 \text{ V for } I_D = 100 \mu\text{A},$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{0.2 \text{ mA}}{0.25 \text{ V}} = 0.8 \text{ mA/V}$$

$$r_o = \frac{V'_A L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.8 \times 18 = 14.4 \text{ V/V}$$

(d) If the redesigned device in (c) is operated at  $10 \mu\text{A}$ ,  $V_{OV}$  decreases by a factor equal to  $\sqrt{10}$  to  $0.08 \text{ V}$ ,  $g_m$  decreases by a factor of  $\sqrt{10}$  to  $0.253 \text{ mA/V}$ ,  $r_o$  increases by a factor of 10 to  $180 \text{ k}\Omega$ , and  $A_0$  becomes

$$0.253 \times 180 = 45.5 \text{ V/V}$$

which is an increase by a factor of  $\sqrt{10}$ .

(e) The lowest value of  $A_0$  is obtained with the first design when operated at  $I_D = 100 \mu\text{A}$ . The resulting  $A_0 = 4.55 \text{ V/V}$ . The highest value of  $A_0$  is obtained with the second design when operated at  $I_D = 10 \mu\text{A}$ . The resulting  $A_0 = 45.5 \text{ V/V}$ . If in any design  $W/L$  is held constant while  $L$  is increased by a factor of 10,  $g_m$  remains unchanged but  $r_o$  increases by a factor of 10, resulting in  $A_0$  increasing by a factor of 10.

#### 7.45

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}} = \frac{2 \times 6 \times 0.5}{0.15} = 40 \text{ V/V}$$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 22.2$$

Thus,

$$W = 22.2 \times 0.5 = 11.1 \mu\text{m}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_o = \frac{V'_A L}{I_D} = \frac{6 \times 0.5}{0.1} = 30 \text{ k}\Omega$$

**7.46** Refer to Fig. 7.15(a) and Table K.1.

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.4 + 0.15 = 0.55 \text{ V}$$

$$V_G = 1.3 - V_{SG2} = 1.3 - 0.55 = 0.75 \text{ V}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 511 \times \left( \frac{W}{L} \right)_1 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 17.4$$

$$I_{D2} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_2 |V_{OV2}|^2$$

$$100 = \frac{1}{2} \times 128 \times \left( \frac{W}{L} \right)_2 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = 69.4$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2})$$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_{o1} = \frac{|V'_A| L}{I_D} = \frac{5 \times 0.4}{0.1} = 20 \text{ k}\Omega$$

$$r_{o2} = \frac{|V'_A| L}{I_D} = \frac{6 \times 0.4}{0.1} = 24 \text{ k}\Omega$$

$$A_v = -1.33 \times (20 \parallel 24) = -14.5 \text{ V/V}$$

#### 7.47

(a) Neglecting the dc current in the feedback network and the Early effect, we see from Fig. 1 that  $I_D = 200 \mu\text{A}$ . Now, using

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

we can determine  $V_{OV}$ :

$$0.2 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.45 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95 \text{ V}$$

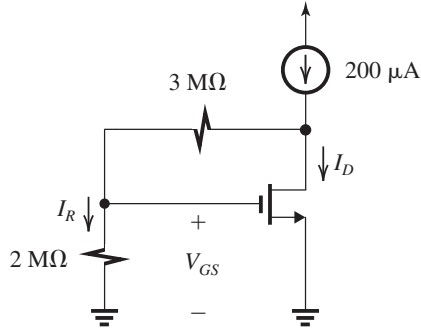


Figure 1

The current in the feedback network can now be found as

$$I_R = \frac{V_{GS}}{2 \text{ M}\Omega} = \frac{0.95}{2} = 0.475 \mu\text{A}$$

which indeed is much smaller than the  $200 \mu\text{A}$  delivered by the current source. Thus, we were justified in neglecting  $I_R$  above.

$$V_{DS} = V_{GS} \frac{3+2}{2} = 2.4 \text{ V}$$

(b) Replacing the MOSFET with its hybrid- $\pi$  model, we obtain the equivalent circuit shown in Fig. 2.

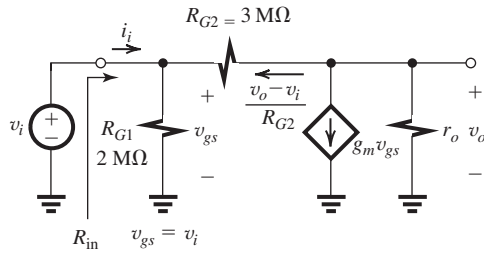


Figure 2

A node equation at the output node yields

$$\frac{v_o}{r_o} + g_m v_{gs} + \frac{v_o - v_i}{R_{G2}} = 0$$

where  $v_{gs} = v_i$ . Thus,

$$v_o \left( \frac{1}{r_o} + \frac{1}{R_{G2}} \right) = -v_i \left( g_m - \frac{1}{R_{G2}} \right)$$

$$\frac{v_o}{v_i} = - \left( g_m - \frac{1}{R_{G2}} \right) (r_o \parallel R_{G2})$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.45} = 0.89 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = - \left( 0.89 - \frac{1}{3000} \right) \times (100 \parallel 3000)$$

$$= -86 \text{ V/V}$$

To obtain the maximum allowable negative signal swing at the output, we note that the MOSFET will remain in saturation as long as  $v_{DG} \geq -V_t$ . Thus at the limit  $v_{DG} = -0.5 \text{ V}$ , or equivalently,

$$v_{G\text{max}} = 0.5 + v_{D\text{min}}$$

$$V_{GS} + |\hat{v}_i| = 0.5 + V_{DS} - |\hat{v}_o|$$

$$0.95 + \frac{|\hat{v}_o|}{|A_v|} = 0.5 + 2.4 - |\hat{v}_o|$$

$$|\hat{v}_o| = \frac{0.5 + 2.4 - 0.95}{1 + \frac{1}{|A_v|}}$$

Substituting  $|A_v| = 86$ , we obtain

$$|\hat{v}_o| = 1.93 \text{ V}$$

An approximate value of  $|\hat{v}_o|$  could have been obtained from

$$v_{O\text{min}} = V_{OV} = 0.45 \text{ V}$$

Thus,

$$V_{DS} - |\hat{v}_o| = V_{OV}$$

$$\Rightarrow |\hat{v}_o| = V_{DS} - V_{OV} = 2.4 - 0.45$$

$$= 1.95 \text{ V}$$

$$|\hat{v}_i| = \frac{|\hat{v}_o|}{86} = 22 \text{ mV}$$

(c) To determine  $R_{in}$ , refer to Fig. 2,

$$i_i = \frac{v_i}{R_{G1}} - \frac{v_o - v_i}{R_{G2}}$$

$$= \frac{v_i}{R_{G1}} - \frac{A_v v_i - v_i}{R_{G2}}$$

$$= v_i \left[ \frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}} \right]$$

$$R_{in} = \frac{v_i}{i_i} = 1 \left/ \left[ \frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}} \right] \right.$$

$$= 1 \left/ \left( \frac{1}{2} + \frac{(1 + 86)}{3} \right) \right. = 33.9 \text{ k}\Omega$$

**7.48** Refer to Fig. 7.16(a).

$$R_o = 50 \text{ k}\Omega = r_{o1} \parallel r_{o2}$$

But

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_{\text{REF}}} = \frac{5}{I_{\text{REF}}}$$

Thus,

$$50 = \frac{1}{2} \times \frac{5 \text{ V}}{I_{\text{REF}} \text{ mA}}$$

$$\Rightarrow I_{\text{REF}} = 50 \mu\text{A}$$

$$A_v = -g_{m1} R_o$$

$$-20 = -g_{m1} \times 50$$

$$\Rightarrow g_{m1} = 0.4 \text{ mA/V}$$

But

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$0.4 = \sqrt{2 \times 0.4 \left(\frac{W}{L}\right)_1 \times 0.05}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 4$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$50 = \frac{1}{2} \times 400 \times 4 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.25 \text{ V}$$

If  $Q_2$  and  $Q_3$  are operated at  $|V_{OV}| = 0.25 \text{ V}$ ,

$$I_{D2} = I_{D3} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{2,3} |V_{OV}|^2$$

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{2,3} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{2,3} = 16$$

7.49 (a)

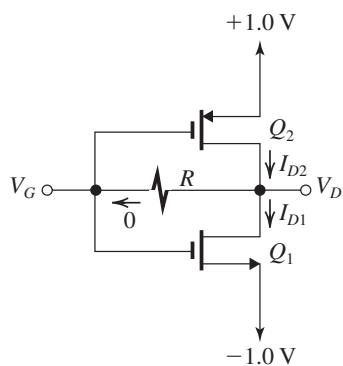


Figure 1

For  $G$  and  $D$  open, from Fig. 1 that since the dc currents into the gates are zero,

$$V_D = V_G$$

Also, since  $Q_1$  and  $Q_2$  are matched and carry equal drain currents,

$$I_{D1} = I_{D2} = I_D$$

$$V_{SG2} = V_{GS1} = \frac{1}{2}[1 - (-1)] = 1 \text{ V}$$

and thus,

$$V_G = 0$$

Thus,

$$I_D = \frac{1}{2} \times 1 \times (1 - 0.5)^2 = 0.125 \text{ mA}$$

(b)

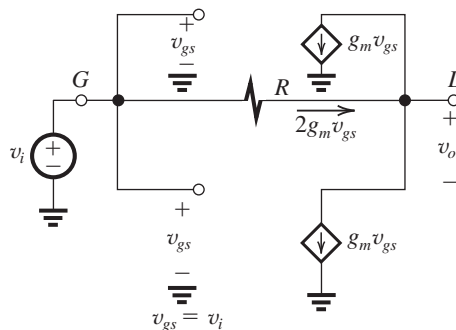


Figure 2

From Fig. 2 we see that

$$v_o = v_i - 2g_m v_{gs} R$$

But

$$v_{gs} = v_i$$

Thus,

$$A_v = \frac{v_o}{v_i} = 1 - 2g_m R$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.125}{1 - 0.5} = 0.5 \text{ mA/V}$$

$$A_v = 1 - 2 \times 0.5 \times 1000 = -999 \text{ V/V}$$

(c)

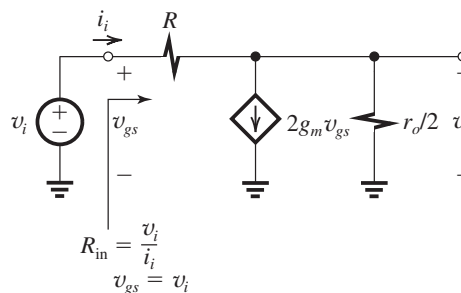


Figure 3

For the circuit in Fig. 3 we can write at the output

$$\frac{v_o}{r_o/2} + 2g_m v_{gs} + \frac{v_o - v_i}{R} = 0$$

Substituting  $v_{gs} = v_i$  and rearranging, we obtain

$$\frac{v_o}{v_i} = -2g_m \frac{1 - \frac{1}{2g_m R}}{\frac{1}{R} + \frac{2}{r_o}}$$

But  $2g_m R \gg 1$ ; thus

$$A_v = \frac{v_o}{v_i} \simeq -2g_m \left( R \parallel \frac{r_o}{2} \right)$$

where

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.125} = 160 \text{ k}\Omega$$

$$A_v = -2 \times 0.5(1000 \parallel 80) = -74.1 \text{ V/V}$$

$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{(v_i - v_o)/R} = R \frac{1}{1 - \frac{v_o}{v_i}}$$

$$= \frac{R}{1 - A_v} = \frac{1000}{1 + 74.1} = 13.3 \text{ k}\Omega$$

$$(d) \frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{13.3}{20 + 13.3} = 0.4 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i} = 0.4 \times -74.1 = -29.6 \text{ V/V}$$

(e) Both  $Q_1$  and  $Q_2$  remain in saturation for output voltages that ensure that the minimum voltage across each transistor is equal to  $|V_{OV}| = 0.5 \text{ V}$ . Thus, the output voltage can range from  $-0.5 \text{ V}$  to  $+0.5 \text{ V}$ .

**7.50** Using Eq. (7.51),

$$R_{in} = \frac{r_o + R_L}{g_m r_o + 1} = \frac{20 + 840}{(2 \times 20) + 1} \simeq 21 \text{ k}\Omega$$

Using Eq. (7.53),

$$A_{is} = \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{1}{g_m r_o} + \frac{1}{g_m R_s}} = \frac{1 + \frac{1}{2 \times 20}}{1 + \frac{1}{2 \times 20} + \frac{1}{2 \times 20}} = 0.976 \text{ A/A}$$

Using Eq. (7.55),

$$R_{out} = r_o + R_s + g_m r_o R_s = 20 + 20 + 2 \times 20 \times 20 = 840 \text{ k}\Omega$$

$$\frac{v_o}{v_{sig}} = \frac{i_o R_L}{i_{sig} R_s} = \left( \frac{i_o}{i_{sig}} \right) \frac{R_L}{R_s} \quad (1)$$

From Fig. 7.18 we see that

$$i_o = A_{is} i_{sig} \frac{R_{out}}{R_{out} + R_L}$$

Thus,

$$\frac{i_o}{i_{sig}} = A_{is} \frac{R_{out}}{R_{out} + R_L}$$

Substituting into Eq. (1) gives

$$\frac{v_o}{v_{sig}} = \left( \frac{R_L}{R_s} \right) A_{is} \frac{R_{out}}{R_{out} + R_L}$$

Thus

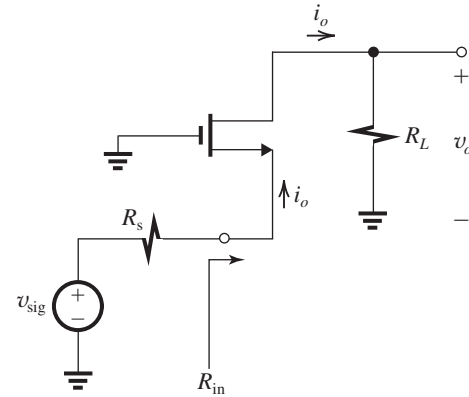


Figure 1

$$\frac{v_o}{v_{sig}} = \frac{840}{20} \times 0.976 \times \frac{840}{840 + 840} = 20.5 \text{ V/V}$$

An alternative and much simpler approach of finding the voltage gain of the circuit in Fig. 1 is to find the current  $i_o$  from

$$i_o = \frac{v_{sig}}{R_s + R_{in}} \quad (2)$$

and then the output voltage  $v_o$  as

$$v_o = i_o R_L \quad (3)$$

Combining Eqs. (2) and (3) gives

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}} = \frac{840}{20 + 21} = 20.5 \text{ V/V}$$

**7.51** Refer to Fig. P7.51.

$$I_D = 0.2 \text{ mA} \quad V_{OV} = 0.2 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$R_{out} = r_o + R_s + g_m r_o R_s$$

$$500 = 100 + R_s(1 + 2 \times 100)$$

$$\Rightarrow R_s = \frac{400}{201} \simeq 2 \text{ k}\Omega$$

$$\begin{aligned}
V_{\text{BIAS}} &= I_D R_S + V_{GS} \\
&= I_D R_S + V_t + V_{OV} \\
&= 0.2 \times 2 + 0.5 + 0.2 \\
&= 1.1 \text{ V}
\end{aligned}$$

**7.52** Refer to Fig. P7.52. To obtain maximum output resistance, we use the largest possible  $R_s$  consistent with  $I_D R_s \leq 0.2 \text{ V}$ . Thus

$$R_s = \frac{0.2 \text{ V}}{0.1 \text{ mA}} = 2 \text{ k}\Omega$$

Now, for  $Q_2$  we have

$$g_m = 2 \text{ mA/V} \quad \text{and} \quad V_A = 5 \text{ V}$$

Thus,

$$r_o = \frac{V_A}{I_D} = \frac{5 \text{ V}}{0.1 \text{ mA}} = 50 \text{ k}\Omega$$

$$\begin{aligned}
R_{\text{out}} &= r_o + R_s + g_m r_o R_s \\
&= 50 + 2 + 2 \times 50 \times 2 \\
&= 252 \text{ k}\Omega
\end{aligned}$$

**7.53** Refer to Fig. P7.53.

$$(a) \quad I_{D1} = I_{D2} = I_{D3} = 100 \mu\text{A}$$

Using  $I_{D1} = \frac{1}{2} k'_n (W/L)_1 V_{OV1}^2$ , we obtain

$$0.1 = \frac{1}{2} \times 4 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.224 \text{ V}$$

$$V_{GS1} = V_t + V_{OV1} = 0.8 + 0.224 = 1.024 \text{ V}$$

$$V_{\text{BIAS}} = V_{GS} + I_{D1} R_s$$

$$= 1.024 + 0.1 \times 0.05 = 1.03 \text{ V}$$

$$(b) \quad g_{m1} = \frac{2 I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.224} = 0.9 \text{ mA/V}$$

All transistors are operating at  $I_D = 0.1 \text{ mA}$  and have  $|V_A| = 20 \text{ V}$ . Thus all have equal values for  $r_o$ :

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$(c) \quad \text{For } Q_2, R_L = r_{o2} = 200 \text{ k}\Omega,$$

$$\begin{aligned}
R_{\text{in}} &= \frac{r_o + R_L}{1 + g_m r_o} \\
&= \frac{200 + 200}{1 + 0.9 \times 200} = 2.2 \text{ k}\Omega
\end{aligned}$$

$$\begin{aligned}
(d) \quad R_{\text{out}} &= r_o + R_s + g_m r_o R_s \\
&= 200 + 0.05 + 0.9 \times 200 \times 0.05 \\
&= 209 \text{ k}\Omega
\end{aligned}$$

$$(e) \quad \frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_s} = \frac{2.2}{2.2 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_{\text{in}}} = \frac{200}{2.2} = 90.9 \text{ V}$$

$$\frac{v_o}{v_{\text{sig}}} = 90.9 \times 0.98 = 89 \text{ V/V}$$

(f) The value of  $v_o$  can range from  $V_{\text{BIAS}} - V_t = 1.03 - 0.8 = 0.23 \text{ V}$  to  $(V_{DD} - V_{OV2})$ . Since  $I_{D2} = I_{D1}$  and  $k_n = k_p$ , then  $V_{OV2} = V_{OV1}$ . Thus the maximum value of  $v_o$  is  $3.3 - 0.224 = 3.076 \text{ V}$ . Thus the peak-to-peak value of  $v_o$  is  $3.076 - 0.23 = 2.85 \text{ V}$ . Correspondingly, the peak-to-peak value of  $v_{\text{sig}}$  will be

$$v_{\text{sig}} (\text{peak to peak}) = \frac{2.85}{89} = 32 \text{ mV}$$

### 7.54

$$I_C = 0.1 \text{ mA} \quad \beta = 100 \quad V_A = 20 \text{ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$g_m r_o = 4 \times 200 = 800 \text{ V/V}$$

Using Eq. (7.61),

$$\begin{aligned}
R_{\text{in}} &= r_\pi \parallel \frac{r_o + R_L}{g_m r_o + 1} \\
&= 25 \parallel \frac{200 + 1000}{800 + 1} = 25 \parallel 1.5 \\
&= 1.4 \text{ k}\Omega
\end{aligned}$$

Using Eq. (7.65),

$$\begin{aligned}
A_{is} &= \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{1}{g_m r_o} + \frac{1}{g_m (R_e \parallel r_\pi)}} \\
&= \frac{1 + \frac{1}{800}}{1 + \frac{1}{800} + \frac{1}{4(25 \parallel 25)}} \\
&= 0.98 \text{ A/A}
\end{aligned}$$

Using Eq. (7.67),

$$\begin{aligned}
R_{\text{out}} &= r_o + (R_e \parallel r_\pi) + (g_m r_o)(R_e \parallel r_\pi) \\
&= 200 + (25 \parallel 25) + 800(25 \parallel 25) \\
&= 10.2 \text{ M}\Omega
\end{aligned}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{i_o R_L}{i_{\text{sig}} R_e} = \left( \frac{R_L}{R_e} \right) \left( \frac{i_o}{i_{\text{sig}}} \right) \quad (1)$$

From Fig. 7.25(b),

$$i_o = A_{is} i_{sig} \frac{R_{out}}{R_{out} + R_L}$$

$$\Rightarrow \frac{i_o}{i_{sig}} = A_{is} \frac{R_{out}}{R_{out} + R_L}$$

Substituting in Eq. (1), we obtain

$$\frac{v_o}{v_{sig}} = \left(\frac{R_L}{R_e}\right) A_{is} \left(\frac{R_{out}}{R_{out} + R_L}\right)$$

$$= \frac{1000}{25} \times 0.98 \times \frac{10.2}{10.2 + 1}$$

$$= 35.7 \text{ V/V}$$

**7.55** Refer to Fig. P7.55.

$$I = I_C = \alpha I_E = 0.99 \times \frac{5 - 0.7}{4.3} \simeq 1 \text{ mA}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{out} = r_o + (R_E \parallel r_\pi)(1 + g_m r_o)$$

where

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$g_m r_o = 40 \times 100 = 4000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R_E = 4.3 \text{ k}\Omega$$

Thus,

$$R_{out} = 100 + (4.3 \parallel 2.5) \times 4001 = 6.4 \text{ M}\Omega$$

For

$$\Delta V_C = 10 \text{ V}$$

$$\Delta I = \frac{10 \text{ V}}{6.4 \text{ M}\Omega} = 1.6 \text{ }\mu\text{A}$$

A very small change indeed!

**7.56** Refer to Fig. 7.27.

$$R_{out} = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

$$\simeq r_o + (R_e \parallel r_\pi)(g_m r_o)$$

$$\frac{R_{out}}{r_o} = 1 + g_m (R_e \parallel r_\pi)$$

$$= 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e}$$

$$= 1 + \frac{\beta R_e}{(\beta/g_m) + R_e}$$

For our case  $\beta = 100$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V, thus}$$

$$\frac{R_{out}}{r_o} = 1 + \frac{100 R_e}{5 + R_e} \quad (1)$$

where  $R_e$  is in kilohms.

(a) For  $R_{out} = 5 r_o$ , Eq. (1) gives

$$R_e = 0.208 \text{ k}\Omega = 208 \text{ }\Omega$$

(b) For  $R_{out} = 10 r_o$ , Eq. (1) gives

$$R_e = 0.495 \text{ k}\Omega \simeq 500 \text{ }\Omega.$$

(c) For  $R_{out} = 50 r_o$ , Eq. (1) gives  $R_e = 4.8 \text{ k}\Omega$ .

From Eq. (1) we see that the maximum value of  $R_{out}/r_o$  is obtained with  $R_e = \infty$  and its value is 101, which is  $(\beta + 1)$ .

### 7.57

(a) Refer to Fig. 7.18(a) and note that the current  $i_o$  that flows through  $R_L$  is equal to the current that enters the source of the transistor, thus

$$i_o = i_{sig} \frac{R_s}{R_s + R_{in}}$$

and the current gain is

$$\frac{i_o}{i_{sig}} = \frac{R_s}{R_s + R_{in}}$$

(b)

$$R_{in} = \frac{r_o + R_L}{g_m r_o + 1} = \frac{20 + 20}{2 \times 20 + 1} = 0.976 \text{ k}\Omega$$

$$\frac{i_o}{i_{sig}} = \frac{20}{20 + 0.976} = 0.95 \text{ A/A}$$

(c) If  $R_L$  is doubled; that is, increases to 40 k $\Omega$ ,

$$R_{in} = \frac{20 + 40}{2 \times 20 + 1} = 1.46 \text{ k}\Omega$$

and

$$\frac{i_o}{i_{sig}} = \frac{20}{20 + 1.46} = 0.93 \text{ A/A}$$

Thus, a change in  $R_L$  of 100% results in a change in the current gain of only 2%.

(d) For  $R_L = 200 \text{ k}\Omega$ ,

$$R_{in} = \frac{20 + 200}{2 \times 20 + 1} = 5.37 \text{ k}\Omega$$

and

$$\frac{i_o}{i_{sig}} = \frac{20}{20 + 5.37} = 0.79 \text{ A/A}$$

Thus, a change of +900% in  $R_L$  results in only a change of  $-0.16$  or  $-17\%$  in the current gain.

(e) The fact that the current gain  $i_o/i_{\text{sig}}$  of the CG amplifier changes very little in the face of large changes in  $R_L$  indicates that it functions as a current buffer.

### 7.58

$$R_s = 25 \text{ k}\Omega \quad R_L = 1 \text{ M}\Omega \quad R_{\text{out}} = 1 \text{ M}\Omega$$

$$R_{\text{out}} \simeq (g_m r_o) R_s$$

Thus,

$$A_0 = g_m r_o \simeq \frac{R_{\text{out}}}{R_s} = \frac{1 \text{ M}\Omega}{25 \text{ k}\Omega} = 40 \text{ V/V}$$

But,

$$A_0 = \frac{2V_A}{V_{OV}}$$

Thus,

$$40 = \frac{2V_A}{0.15} \\ \Rightarrow V_A = 3 \text{ V}$$

Since

$$V_A = V'_A L \\ 3 = 5 \times L \\ \Rightarrow L = 0.6 \text{ }\mu\text{m}$$

### 7.59

(a) Refer to Fig. 7.25(b): At the output we see that

$$i_o = A_{is} i_{\text{sig}} \left( \frac{R_{\text{out}}}{R_{\text{out}} + R_L} \right)$$

Thus,

$$\frac{i_o}{i_{\text{sig}}} = A_{is} \frac{R_{\text{out}}}{R_{\text{out}} + R_L}$$

(b)

$$g_m = 5 \text{ mA/V} \quad r_o = 200 \text{ k}\Omega \quad \beta = 100 \quad R_e = 20 \text{ k}\Omega$$

$$R_L = 200 \text{ k}\Omega$$

$$g_m r_o = 1000 \text{ V/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{5} = 20 \text{ k}\Omega$$

Using Eq. (7.65),

$$A_{is} = \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{1}{g_m r_o} + \frac{1}{g_m (R_e \parallel r_\pi)}} \\ = \frac{1 + \frac{1}{1000}}{1 + \frac{1}{1000} + \frac{1}{5(20 \parallel 20)}} \\ = 0.98 \text{ A/A}$$

Using Eq. (7.67),

$$R_{\text{out}} = r_o + (R_e \parallel r_\pi) + (g_m r_o)(R_e \parallel r_\pi) \\ = 200 + (20 \parallel 20) + 1000(20 \parallel 20) \\ = 10.2 \text{ M}\Omega$$

$$\frac{i_o}{i_{\text{sig}}} = A_{is} \frac{R_{\text{out}}}{R_{\text{out}} + R_L} \\ = 0.98 \times \frac{10.2}{10.2 + 0.2} = 0.96 \text{ A/A}$$

(c) If  $R_L$  is doubled, it becomes 400 k $\Omega$  and the current gain becomes

$$\frac{i_o}{i_{\text{sig}}} = 0.98 \times \frac{10.2}{10.2 + 0.4} = 0.94 \text{ A/A}$$

Thus, in the face of a 100% change in  $R_L$ , the current gain decreases by 0.02 which is approximately 2.1%:

(d) If  $R_L$  is increased by a factor of 10 or by 900%, from 200 k $\Omega$  to 2 M $\Omega$ , the current gain becomes

$$\frac{i_o}{i_{\text{sig}}} = 0.98 \times \frac{10.2}{10.2 + 2} \\ = 0.82 \text{ A/A}$$

which is a decrease of 0.15 or 15%.

(e) A current buffer works to keep the current gain as constant as possible in the face of changes in the load resistance  $R_L$ . From the results of (c) and (d) above we see that the CB amplifier fulfills this requirement admirably!

**7.60** Using Eq. (7.61),

$$R_{\text{in}} = r_\pi \parallel \frac{r_o + R_L}{g_m r_o + 1} \\ = (\beta + 1) r_e \parallel \frac{r_o + R_L}{g_m r_o + 1}$$

To obtain  $R_{\text{in}} = 2r_e$ , we arrange that

$$\frac{r_o + R_L}{g_m r_o + 1} \simeq 2r_e$$

Usually  $g_m r_o \gg 1$ , thus we need

$$\frac{r_o + R_L}{g_m r_o} \simeq 2r_e$$

that is,

$$\frac{1}{g_m} + \frac{R_L}{g_m r_o} \simeq 2r_e \quad (1)$$

Since  $\frac{1}{g_m} \simeq r_e$  then Eq. (1) is satisfied when

$$R_L = r_o$$

**7.61** Refer to the circuit in Fig. P7.55.

$$I = \alpha \frac{3 - V_{BE}}{R_e} \simeq \frac{2.3}{R_e}$$

Thus,

$$R_e = \frac{2.3}{I(\text{mA})} \text{ k}\Omega \quad (1)$$

$$R_o = (g_m r_o) (R_e \parallel r_\pi) \quad (2)$$

But,

$$g_m r_o = \frac{V_A}{V_T}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta}{I/V_T} = \frac{\beta V_T}{I}$$

$$R_e \parallel r_\pi = \frac{1}{\frac{1}{R_e} + \frac{1}{r_\pi}} = \frac{1}{\frac{1}{2.3} + \frac{1}{\beta V_T}}$$

Substituting in Eq. (2),

$$R_o = \frac{1}{I} \frac{V_A/V_T}{\frac{1}{2.3} + \frac{1}{\beta V_T}}$$

That is,

$$R_o = \left(\frac{1}{I}\right) \left(\frac{V_A}{\frac{V_T}{2.3} + \frac{1}{\beta}}\right)$$

Substituting  $V_T = 0.025$  V and the given numerical values  $V_A = 25$  V and  $\beta = 100$ , we get

$$R_o \simeq \frac{1.2}{I(\text{mA})} \text{ M}\Omega \quad (3)$$

We can now use Eqs. (1) and (3) to obtain  $R_e$  and  $R_o$  for each of the given values of  $I$ , obtaining

$I$ (mA)	0.1	0.2	0.5	1.0
$R_e$ (k $\Omega$ )	23	11.5	4.6	2.3
$R_o$ (M $\Omega$ )	12	6	2.4	1.2

**7.62** Using Eq. (7.75),

$$A_{vo} = -A_0^2$$

where

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2 \times 3}{0.15} = 40$$

Thus,

$$A_{vo} = -1600 \text{ V/V}$$

**7.63** Refer to Fig. 7.30

$$R_o = g_{m3} r_{o3} r_{o4}$$

For identical transistors,

$$\begin{aligned} R_o &= (g_m r_o) r_o \\ &= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I} \end{aligned}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{V_{OV}} \quad \text{Q.E.D.}$$

(a)  $I = 0.1$  mA

$$0.1 \times R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 1.6 \text{ M}\Omega$$

To obtain the  $W/L$  values,

$$I = I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 50$$

(b)  $I = 0.5$  mA

$$0.5 R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 320 \text{ k}\Omega$$

$$I = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$500 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 250$$

**7.64** Refer to Fig. 7.28(a).

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$



$$3 = \frac{2I}{0.15}$$

$$\Rightarrow I = 0.225 \text{ mA}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2V_A}{V_{OV}} \frac{V_A}{I} = \frac{2V_A^2}{V_{OV} I}$$

$$150 = \frac{2V_A^2}{0.15 \times 0.225}$$

$$\Rightarrow V_A = 1.6 \text{ V}$$

$$V_A = V'_A L$$

$$L = \frac{V_A}{V'_A} = \frac{1.6}{5} = 0.32 \text{ } \mu\text{m}$$

To obtain  $W/L$ , we use

$$I_D = I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$225 = \frac{1}{2} \times 511 \times \left( \frac{W}{L} \right) \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 39.1$$

To obtain maximum negative signal swing at the output, we select  $V_G$  so that the voltage at the drain of  $Q_1$  is the minimum permitted, which is equal to  $V_{OV}$  (i.e., 0.15 V). Thus

$$V_G = 0.15 + V_{GS2}$$

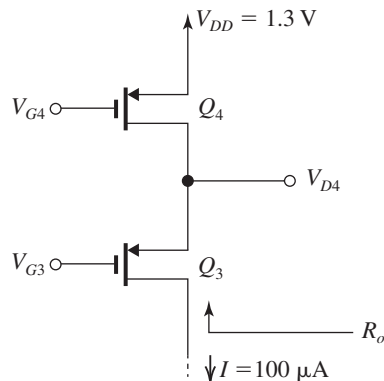
$$= 0.15 + V_{OV2} + V_t$$

$$= 0.15 + 0.15 + 0.4 = 0.7 \text{ V}$$

The minimum permitted output voltage is

$$V_G - V_t = 0.7 - 0.4 = 0.3 \text{ V or } 2V_{OV}.$$

### 7.65



$$V_{SG4} = |V_{tp}| + |V_{OV}|$$

$$= 0.4 + 0.15 = 0.55 \text{ V}$$

Thus,

$$V_{G4} = V_{DD} - V_{SG4} = 1.3 - 0.55 = 0.75 \text{ V}$$

To obtain the largest possible signal swing at the output, we maximize the allowable positive signal swing by setting  $V_{D4}$  at its highest possible value of  $V_{DD} - |V_{OV}| = 1.3 - 0.15 = 1.15 \text{ V}$ .

This will be obtained by selecting  $V_{G3}$  as follows:

$$V_{G3} = V_{D4} - V_{SG3}$$

Since

$$V_{SG3} = V_{SG4} = 0.55 \text{ V}$$

$$V_{G3} = 1.15 - 0.55 = 0.6 \text{ V}$$

The highest allowable voltage at the output will be

$$v_{D3\text{max}} = V_{G3} + |V_{tp}|$$

$$= 0.6 + 0.4 = 1 \text{ V}$$

Since both  $Q_3$  and  $Q_4$  carry the same current  $I = 100 \text{ } \mu\text{A}$  and are operated at the same overdrive voltage,  $|V_{OV}| = 0.15 \text{ V}$ , their  $W/L$  ratios will be the same and can be found from

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 128 \times \left( \frac{W}{L} \right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{3,4} = 69.4$$

To obtain  $R_o$ , we first find  $g_m$  and  $r_o$  of both devices,

$$g_{m3,4} = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_{o3,4} = \frac{|V_A|}{I_D} = \frac{3}{0.1} = 30 \text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o4}$$

$$= 1.33 \times 30 \times 30 = 1.2 \text{ M}\Omega$$

### 7.66 Refer to Fig. 7.31.

$$A_v = -g_{m1} R_o$$

$$-280 = -1 \times R_o \Rightarrow R_o = 280 \text{ k}\Omega$$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}} \Rightarrow I = \frac{1}{2} g_{m1} V_{OV}$$

$$= \frac{1}{2} \times 1 \times 0.25 = 0.125 \text{ mA}$$

All four transistors are operated at the same value of  $I_D$  and the same value of  $|V_{OV}|$ . Also all have the same channel length and  $|V'_A|$ ; thus all  $r_o$  values are equal. Thus

$$R_{on} = R_{op} = 2R_o = 2 \times 280 = 560 \text{ k}\Omega$$

$$560 = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \frac{|V_A|}{I}$$

$$= \frac{2|V_A|^2}{0.25 \times 0.125}$$

$$\Rightarrow V_A = 2.96 \text{ V}$$

$$L = \frac{V_A}{V'_A} = \frac{2.96}{5} = 0.6 \text{ } \mu\text{m}$$

For each of the NMOS devices,

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$125 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 10$$

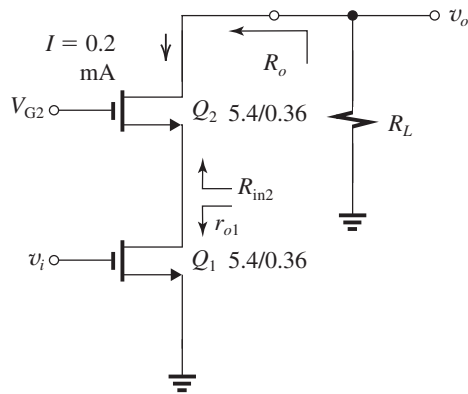
For each of the PMOS transistors,

$$I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$125 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 40$$

**7.67**



$$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.387 \times \frac{5.4}{0.36} \times 0.2} = 1.52 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{V'_A L}{I_D} = \frac{5 \times 0.36}{0.2} = 9 \text{ k}\Omega$$

$$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} = 9 + 9 + 1.52 \times 9 \times 9 = 141 \text{ k}\Omega$$

$$A_v = -g_{m1} (R_o \parallel R_L)$$

$$-100 = -1.52 (R_o \parallel R_L)$$

$$\Rightarrow R_o \parallel R_L = 65.8 \text{ k}\Omega$$

$$\frac{1}{R_o} + \frac{1}{R_L} = \frac{1}{65.8}$$

$$\frac{1}{R_L} = \frac{1}{65.8} - \frac{1}{141} = \frac{1}{123}$$

$$\Rightarrow R_L = 123 \text{ k}\Omega$$

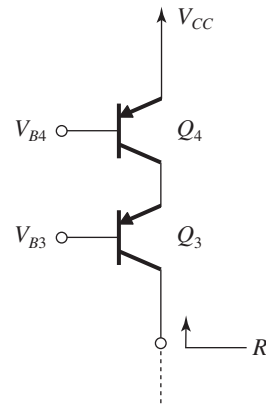
$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{9 + 123}{1 + 1.52 \times 9} = 9 \text{ k}\Omega$$

$$R_{d1} = r_{o1} \parallel R_{in2} = 9 \parallel 9 = 4.5 \text{ k}\Omega$$

$$A_1 = -g_{m1} R_{d1} = -1.52 \times 4.5 = -6.84 \text{ V/V}$$

**7.68**



$$R_o = (g_{m3} r_{o3})(r_{o4} \parallel r_{\pi3})$$

$$I = 0.2 \text{ mA}$$

$$g_{m3} = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_{o3} = r_{o4} = \frac{V_A}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$r_{\pi3} = \frac{\beta}{g_{m3}} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$R_o = (8 \times 25)(25 \parallel 6.25)$$

$$= 1 \text{ M}\Omega$$

**7.69** Refer to Fig. 7.34. When all transistors have equal  $\beta$  and  $r_o$ , and, since they conduct equal currents, they have equal  $g_m$ , then

$$R_{on} = R_{op} = g_m r_o (r_o \parallel r_{\pi})$$

$$R_o = R_{on} \parallel R_{op} = \frac{1}{2} (g_m r_o)(r_o \parallel r_{\pi})$$

$$A_v = -g_m R_o$$

This table belongs to Problem 7.71.

	$L = L_{\min} = 0.18 \mu\text{m}$ $IR_o = 8.1 \text{ V}$				$L = 2L_{\min} = 0.36 \mu\text{m}$ $IR_o = 32.4 \text{ V}$				$L = 3L_{\min} = 0.54 \mu\text{m}$ $IR_o = 72.9 \text{ V}$			
	$g_m$ (mA/V)	$R_o$ (k $\Omega$ )	$A_v$ (V/V)	$2WL$ ( $\mu\text{m}^2$ )	$g_m$ (mA/V)	$R_o$ (k $\Omega$ )	$A_v$ (V/V)	$2WL$ ( $\mu\text{m}^2$ )	$g_m$ (mA/V)	$R_o$ (k $\Omega$ )	$A_{v_o}$ (V/V)	$2WL$ ( $\mu\text{m}^2$ )
$I = 0.01 \text{ mA}$ $W/L = n$	0.1	810	-40.5	$0.065n$	0.1	3,240	-162	$0.26n$	0.1	7,290	-364.5	$0.58n$
$I = 0.1 \text{ mA}$ $W/L = 10n$	1.0	81	-40.5	$0.65n$	1.0	324	-162	$2.6n$	1.0	729	-364.5	$5.8n$
$I = 1.0 \text{ mA}$ $W/L = 100n$	10.0	8.1	-40.5	$6.5n$	10.0	32.4	-162	$26n$	10.0	72.9	-364.5	$58n$

$$= -\frac{1}{2}(g_m r_o)g_m(r_o \parallel r_\pi)$$

$$= -\frac{1}{2}(g_m r_o)\frac{g_m r_o r_\pi}{r_\pi + r_o}$$

$$= -\frac{1}{2}(g_m r_o)\frac{1}{\frac{1}{g_m r_o} + \frac{1}{g_m r_\pi}}$$

Substituting  $g_m r_o = \frac{|V_A|}{V_T}$  and  $g_m r_\pi = \beta$ ,

$$A_v = -\frac{1}{2}\frac{|V_A|/V_T}{(V_T/|V_A|) + (1/\beta)}$$

For  $|V_A| = 5 \text{ V}$  and  $\beta = 50$  we obtain

$$A_v = -\frac{1}{2}\frac{5/0.025}{(0.025/5) + (1/50)} = -4000 \text{ V/V}$$

$$7.70 \quad 50 = g_m r_o$$

$$= A_{02} = \frac{2V_A}{V_{OV}}$$

$$V_A = 50 \times V_{OV}/2$$

$$= 25 \times 0.2 = 5 \text{ V}$$

$$V_A = V'_A L$$

$$5 = 5 \times L \Rightarrow L = 1 \mu\text{m}$$

### 7.71

Refer to Fig. 7.30.

$$R_o = (g_m r_{o3})r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o)r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{|V_{OV}|}$$

Substituting

$$|V_A| = |V'_A| L$$

$$IR_o = \frac{2|V'_A|^2}{|V_{OV}|} L^2 \quad \text{Q.E.D.}$$

Now, for

$$L = 0.18 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.18^2 = 8.1 \text{ V}$$

$$L = 0.36 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.36^2 = 32.4 \text{ V}$$

$$L = 0.54 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.54^2 = 72.9 \text{ V}$$

To fill out the table we use

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2I}{|V_{OV}|} = \frac{2I}{0.2} = 10I$$

$$A_v = g_m(R_o/2)$$

(a) The price paid is the increase in circuit area.

(b) As  $I$  is increased,  $g_m$  increases and hence the current-driving capability of the amplifier, and as we will see later, its bandwidth.

(c) The circuit with the largest area ( $58n$ ) as compared to the circuit with the smallest area ( $0.065n$ ):  $A_v$  is  $364.5/40.5 = 9$  times larger;  $g_m$  is 100 times larger, but  $R_o$  is 11.1 times lower.

### 7.72

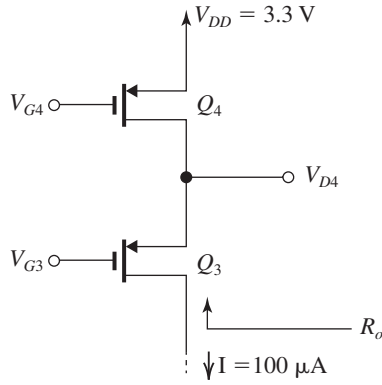
$$V_{SG4} = |V_{tp}| + |V_{OV}|$$

$$= 0.8 + 0.2 = 1 \text{ V}$$

Thus,

$$V_{G4} = V_{DD} - V_{SG4} = 3.3 - 1 = 2.3 \text{ V}$$

To obtain the largest possible signal swing at the output, we maximize the allowable positive



signal swing by setting  $V_{D4}$  at its highest possible value of  $V_{DD} - |V_{OV}| = 3.3 - 0.2 = 3.1$  V. This will be obtained by selecting  $V_{GS}$  as follows:

$$V_{G3} = V_{D4} - V_{SG3}$$

Since

$$V_{SG3} = V_{SG4} = 1 \text{ V}$$

$$V_{G3} = 3.1 - 1 = 2.1 \text{ V}$$

the highest allowable voltage at the output will be

$$\begin{aligned} v_{D3\text{max}} &= V_{G3} + |V_{tp}| \\ &= 2.1 + 0.8 = 2.9 \text{ V} \end{aligned}$$

Since both  $Q_3$  and  $Q_4$  carry the same current  $I = 100 \mu\text{A}$  and are operated at the same overdrive voltage,  $|V_{OV}| = 0.2$  V, their  $W/L$  ratios will be the same and can be found from

$$\begin{aligned} I_D &= \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2 \\ 100 &= \frac{1}{2} \times 60 \times \left( \frac{W}{L} \right)_{3,4} \times 0.2^2 \\ \Rightarrow \left( \frac{W}{L} \right)_{3,4} &= 83.3 \end{aligned}$$

To obtain  $R_o$ , we first find  $g_m$  and  $r_o$  of both devices,

$$g_{m3,4} = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

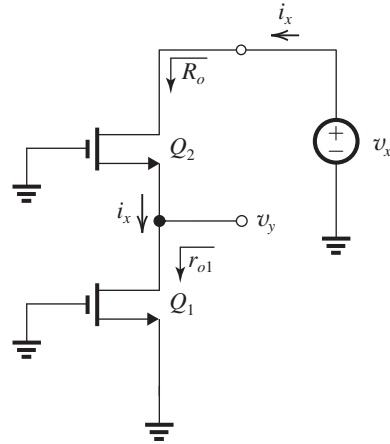
$$r_{o3,4} = \frac{|V_A|}{I_D} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$\begin{aligned} R_o &= (g_{m3} r_{o3}) r_{o4} \\ &= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega \end{aligned}$$

**7.73**

While  $v_x$  appears across  $R_o$ ,  $v_y$  appears across  $r_{o1}$ , Thus,

$$\frac{v_y}{v_x} = \frac{r_{o1}}{R_o}$$

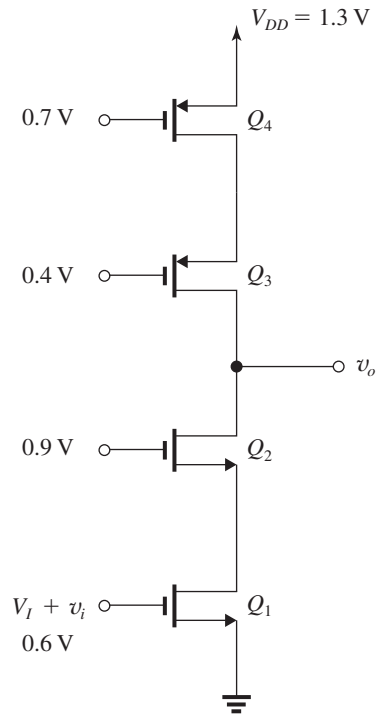


$$= \frac{r_{o1}}{r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1}}$$

For  $g_{m2} r_{o2} \gg 1$  and  $g_{m2} r_{o1} \gg 1$ ,

$$\frac{v_y}{v_x} \simeq \frac{1}{g_{m2} r_{o2}}$$

**7.74**



Since all four transistors have equal transconductance parameters,  $k$ , and all four have the same bias current, their overdrive voltages will be equal. We can obtain  $|V_{OV}|$  by considering either  $Q_1$  or  $Q_4$ . For  $Q_1$ ,

$$V_{GS} = V_t = 0.6 \text{ V} = V_t + V_{OV}$$

Thus,

$$V_{OV} = 0.6 - 0.4 = 0.2 \text{ V}$$

Similarly, for  $Q_4$ ,

$$V_{SG} = V_{DD} - V_{G4} = 1.3 - 0.7 = 0.6 \text{ V}$$

Thus,

$$|V_{OV}| = V_{SG} - |V_t| = 0.6 - 0.4 = 0.2 \text{ V}$$

The maximum allowable voltage at the output is

$$\begin{aligned} v_{O\max} &= V_{G3} + |V_{t3}| \\ &= 0.4 + 0.4 = 0.8 \text{ V} \end{aligned}$$

The minimum allowable voltage at the output is

$$\begin{aligned} v_{O\min} &= V_{G2} - |V_{t2}| \\ &= 0.9 - 0.4 = 0.5 \text{ V} \end{aligned}$$

Thus the output voltage can range from 0.5 V to 0.8 V.

**7.75** Refer to Fig. P7.75.

$$(a) R_1 = r_{o1} = r_o$$

$$R_2 \simeq (g_m r_o) r_o$$

$$R_3 = \frac{R_2 + r_o}{g_m r_o} = \frac{g_m r_o^2 + r_o}{g_m r_o} \simeq r_o$$

$$(b) i_1 = g_m v_i$$

$$i_2 = i_1 \frac{R_3}{R_3 + r_o} = g_m v_i \frac{r_o}{r_o + r_o} = \frac{1}{2} g_m v_i$$

$$i_3 = i_1 - i_2 = \frac{1}{2} g_m v_i$$

$$i_4 = i_3 = \frac{1}{2} g_m v_i$$

$$i_5 = i_4 = \frac{1}{2} g_m v_i$$

$$i_6 = 0 \text{ (because } v_{sg4} = 0)$$

$$i_7 = i_5 = \frac{1}{2} g_m v_i$$

$$(c) v_1 = -i_2 r_o = -\frac{1}{2} (g_m r_o) v_i$$

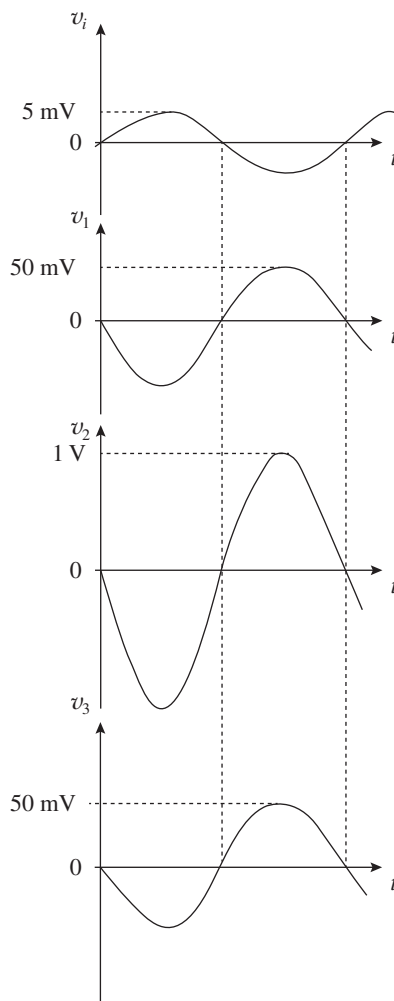
$$v_2 = -i_4 R_2 = -\frac{1}{2} g_m (g_m r_o) r_o v_i$$

$$= -\frac{1}{2} (g_m r_o)^2 v_i$$

$$v_3 = -i_5 R_1 = -\frac{1}{2} g_m v_i r_o = -\frac{1}{2} (g_m r_o) v_i$$

(d)  $v_i$  is a 5-mV peak sine wave.

$$\hat{v}_1 = -\frac{1}{2} \times 20 \times v_i = -10 \times 5 = -50 \text{ mV}$$



Thus,  $v_1$  is a 50-mV peak sine wave that is  $180^\circ$  out of phase with  $v_i$ .

$$\hat{v}_2 = -\frac{1}{2} \times 20^2 \times 5 = -1 \text{ V}$$

Thus,  $v_2$  is a 1-V peak sine wave,  $180^\circ$  out of phase relative to  $v_i$ .

$$\hat{v}_3 = -\frac{1}{2} \times 20 \times 5 = -50 \text{ mV}$$

Thus,  $v_3$  is a 50-mV peak sine wave,  $180^\circ$  out of phase relative to  $v_i$ .

**7.76** Refer to Fig. P7.76.

$$R_o = (g_{m3} r_{o3})(g_{m2} r_{o2}) r_{o1}$$

$$A_{vo} = -g_{m1} R_o$$

$$= -(g_{m1} r_{o1})(g_{m2} r_{o2})(g_{m3} r_{o3})$$

For identical transistors with equal  $g_m$  and  $r_o$ ,

$$R_o = (g_m r_o)^2 r_o$$

$$A_{vo} = -(g_m r_o)^3$$

For

$$g_m = 2 \text{ mA/V and } r_o = 15 \text{ k}\Omega,$$

$$R_o = -(30)^2 \times 15 = 13.5 \text{ M}\Omega$$

$$A_{vo} = -(30)^3 = -2.7 \times 10^4 \text{ V/V}$$

**7.77** When Eq. (7.86) is applied to the case of identical *pn*p transistors, it becomes

$$R_o = (g_m r_o)(r_o \parallel r_\pi)$$

Now,

$$g_m = \frac{I}{V_T} \quad r_o = \frac{|V_A|}{I}$$

$$g_m r_o = |V_A|/V_T$$

$$r_\pi = \frac{\beta}{g_m}$$

Thus,

$$IR_o = \frac{|V_A|}{V_T} \frac{I r_o r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T} \frac{|V_A| r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T} \frac{|V_A|}{1 + \frac{r_o}{r_\pi}}$$

$$= \frac{|V_A|}{V_T} \frac{|V_A|}{1 + \frac{1}{\beta} g_m r_o}$$

$$= \frac{|V_A|}{V_T} \frac{1}{\frac{1}{|V_A|} + \frac{1}{\beta} \frac{1}{V_T}}$$

$$= \frac{|V_A|}{(V_T/|V_A|) + (1/\beta)} \quad \text{Q.E.D.}$$

For  $|V_A| = 5 \text{ V}$  and  $\beta = 50$  we obtain

$$IR_o = \frac{5}{(0.025/5) + (1/50)} = 200 \text{ V}$$

$I$ (mA)	0.1	0.5	1
$R_o$ (k $\Omega$ )	2000	400	200

**7.78** The output resistance of the cascode amplifier (excluding the load) is

$$R_o = g_m r_o (r_o \parallel r_\pi)$$

Thus,

$$A_v = -g_m (R_o \parallel R_L)$$

$$= -g_m (R_o \parallel \beta r_o)$$

For  $|V_A| = 100 \text{ V}$ ,  $\beta = 50$ , and  $I = 0.2 \text{ mA}$  we obtain

$$g_m = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$r_o = \frac{|V_A|}{I} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$R_o = 8 \times 500 \times (500 \parallel 6.25)$$

$$= 24,691 \text{ k}\Omega$$

$$A_v = -8(24.7 \parallel 25) \times 10^3$$

$$= -99.4 \times 10^3 \simeq -10^5 \text{ V/V}$$

**7.79** (a) Refer to the circuit in Fig. P7.79(a).

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.4 \times 25 \times 0.1}$$

$$= 1.41 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_D} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{125}{4} = 31.25 \text{ k}\Omega$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi 2})$$

$$= 4 \times 18 \times (18 \parallel 31.25) = 822.3 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 822.3 = -1159 \text{ V/V}$$

(b) Refer to circuit in Fig. P7.79(b).

$$g_{m1} = g_{m2} = \sqrt{2 \times 0.4 \times 25 \times 0.1}$$

$$= 1.41 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1.41 \times 18 \times 18 = 457 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 457 = -644 \text{ V/V}$$

We observe that the circuit with a BTT cascode transistor provides higher gain.

**7.80** The dc level shift provided by a source follower is equal to its  $V_{GS}$ . Thus, to obtain a dc level shift of 0.9 V, we write

$$V_{GS} = 0.9 \text{ V} = V_t + V_{OV}$$

$$\Rightarrow V_{OV} = 0.9 - 0.6 = 0.3 \text{ V}$$

To obtain the required bias current, we use

$$I = I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{20}{0.5} \times 0.3^2$$

$$I = 0.36 \text{ mA} = 360 \mu\text{A}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.36}{0.3} = 2.4 \text{ mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2.4 = 0.48 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{0.36} = 27.8 \text{ k}\Omega$$

To determine  $A_{vo}$ , we note [refer to Fig. 7.33(b)] that the total effective resistance between the MOSFET source terminal and ground is

$$r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}}. \text{ Denoting this resistance } R, \text{ we have}$$

$$R = r_o \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 27.8 \parallel 27.8 \parallel \frac{1}{0.48}$$

$$= 1.81 \text{ k}\Omega$$

Thus, the open-circuit voltage gain is

$$A_{vo} = \frac{R}{R + \frac{1}{g_m}}$$

$$= \frac{1.81}{1.81 + \frac{1}{2.4}} = 0.81 \text{ V/V}$$

$$R_o = R \parallel \frac{1}{g_m}$$

$$= 1.81 \text{ k}\Omega \parallel \frac{1}{2.4 \text{ mA/V}}$$

$$= 0.339 \text{ k}\Omega$$

When a load resistance of 2 k $\Omega$  is connected to the output, the total resistance between the output node and ground becomes  $R \parallel R_L = 1.81 \parallel 2 = 0.95 \text{ k}\Omega$ . Thus, the voltage gain becomes

$$A_v = \frac{0.95}{0.95 + \frac{1}{2.4}} = 0.7 \text{ V/V}$$

### 7.81

Refer to Fig. 7.36. Each of the four transistors is operating at the same drain current,

$I_D = I_{REF} = 50 \mu\text{A}$  and thus all have the same  $V_{OV}$  which can be determined as follows:

$$I_D = \frac{1}{2} (\mu_n C_{ox}) \left( \frac{W}{L} \right) V_{OV}^2$$

$$50 = \frac{1}{2} \times 511 \times \frac{3.9}{0.39} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

and

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.14 = 0.54 \text{ V}$$

The minimum voltage required at the output is

$$V_{Omin} = 2V_{GS} - V_t = V_t + 2V_{OV}$$

$$= 0.4 + 2 \times 0.14 = 0.68 \text{ V}$$

$$R_o = g_{m3} r_{o3} r_{o2} = (g_m r_o) r_o$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.05}{0.14} = 0.7 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.39}{0.05} = 39 \text{ k}\Omega$$

Thus

$$R_o = (0.7 \times 39) \times 39 = 1.1 \text{ M}\Omega$$

**7.82** Refer to Eq. (7.98),

$$R_o = \beta_3 r_{o3} / 2$$

where

$$r_{o3} = \frac{V_A}{I} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

Thus,

$$R_o = \frac{100 \times 100}{2} = 5 \text{ M}\Omega$$

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{10 \text{ V}}{5 \text{ M}\Omega} = 2 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = \frac{2 \mu\text{A}}{1 \text{ mA}} = 0.002 \quad \text{or } 0.2\%$$

**7.83** For each of the three transistors,

$$I_D = I_{REF} = \frac{1}{2} (\mu_n C_{ox}) \left( \frac{W}{L} \right) V_{OV}^2$$

Thus,

$$100 = \frac{1}{2} \times 511 \times 16 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.16 = 0.56 \text{ V}$$

The voltage at  $G_{1,2} = V_{GS} = 0.56 \text{ V}$

The voltage at  $G_3 = 2V_{GS} = 1.12 \text{ V}$

$$V_{O\min} = V_{G3} - V_t = 1.12 - 0.4 = 0.72 \text{ V}$$

$$R_o = (g_{m3}r_{o3})r_{o2} = (g_m r_o)r_o$$

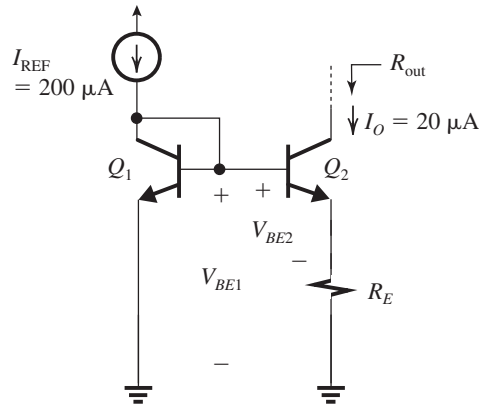
where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.01}{0.16} = 1.25 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V'_A L}{I_D} = \frac{5 \times 4 \times 0.13}{0.1} = 26 \text{ k}\Omega$$

$$R_o = (1.25 \times 26) \times 26 = 845 \text{ k}\Omega$$

**7.84**



(a) Assuming  $\beta$  is high so that we can neglect base currents,

$$I_O R_E = V_T \ln\left(\frac{I_{\text{REF}}}{I_O}\right)$$

Substituting  $I_O = 20 \mu\text{A}$  and  $I_{\text{REF}} = 200 \mu\text{A}$  results in

$$0.02 R_E = 0.025 \ln\left(\frac{200}{20}\right)$$

$$\Rightarrow R_E = 2.88 \text{ k}\Omega$$

(b)  $R_{\text{out}} = (R_E \parallel r_{\pi 2}) + r_{o2} + g_{m2}r_{o2}(R_E \parallel r_{\pi 2})$

where

$$g_{m2} = \frac{0.02}{0.025} = 0.8 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{50}{0.02} = 2500 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_m} = \frac{200}{0.8} = 250 \text{ k}\Omega$$

$$R_{\text{out}} = (2.9 \parallel 250) + 2500 + 0.8 \times 2500$$

$$\times (2.9 \parallel 250)$$

$$= 8.2 \text{ M}\Omega$$

A 5-V change in  $V_O$  gives rise to

$$\Delta I_O = \frac{5}{8.2} = 0.6 \mu\text{A}$$

**7.85** Refer to Fig. 7.36.

$$I_O = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1}$$

$$= 20 \frac{27/0.54}{2.7/0.54} = 200 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$20 = \frac{1}{2} \times 387 \times \frac{2.7}{0.54} \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.14 \text{ V}$$

$$V_{G2} = V_{GS1} = V_t + V_{OV1} = 0.5 + 0.14 = 0.64 \text{ V}$$

$$V_{OV4} = V_{OV1}$$

Thus,

$$V_{GS4} = V_{GS1} = 0.64 \text{ V}$$

$$V_{G3} = 0.64 + 0.64 = 1.28 \text{ V}$$

The lowest voltage at the output while  $Q_3$  remains in saturation is

$$V_{O\min} = V_{G3} - V_{t3}$$

$$= 1.28 - 0.5 = 0.78 \text{ V}$$

Transistors  $Q_2$  and  $Q_3$  are operating at the same  $V_{OV} = 0.14 \text{ V}$ .

Thus,

$$g_{m2} = g_{m3} = \frac{2I_{D2,3}}{V_{OV2,3}} = \frac{2 \times 0.2}{0.14} = 2.9 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{V'_A L}{I_D} = \frac{5 \times 0.54}{0.2} = 13.5 \text{ k}\Omega$$

$$R_o = g_{m3}r_{o3}r_{o2}$$

$$= 2.9 \times 13.5 \times 13.5 = 529 \text{ k}\Omega$$

**7.86**

From the figure (see next page) we see that

$$R_o = (g_{m3}r_{o3})(g_{m2}r_{o2})r_{o1}$$

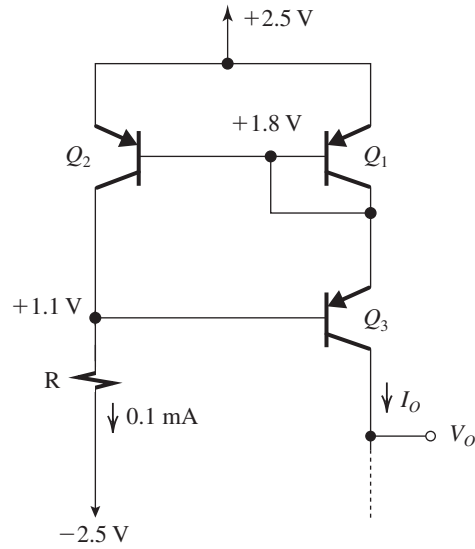
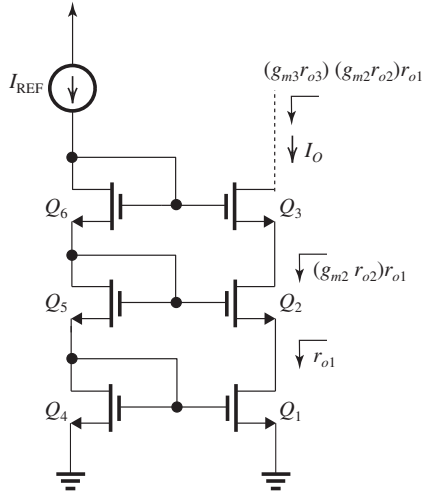
$$\mathbf{7.87} \text{ (a) } I_{O1} = I_{O2} = \frac{1}{2} \frac{I_{\text{REF}}}{1 + \frac{2}{\beta^2}}$$

(b)

The figure shows the required circuit. Observe that the output transistor is split into three



This figure belongs to Problem 7.86.

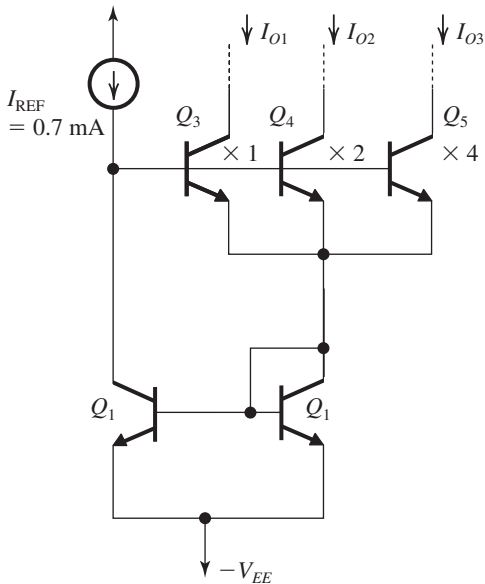


transistors having base–emitter junctions with area ratio 1:2:4. Thus

$$I_{O1} = \frac{0.1}{1 + \frac{2}{\beta^2}} = \frac{0.1}{1 + \frac{2}{50^2}} = 0.0999 \text{ mA}$$

$$I_{O2} = \frac{0.2}{1 + \frac{2}{50^2}} = 0.1998 \text{ mA}$$

$$I_{O4} = \frac{0.4}{1 + \frac{2}{50^2}} = 0.3997 \text{ mA}$$



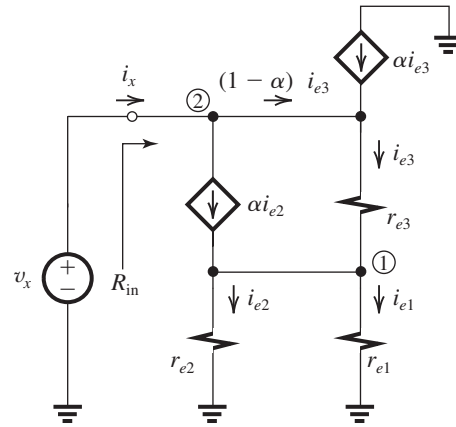
7.88

$$R = \frac{1.1 + 2.5}{0.1} = 36 \text{ k}\Omega$$

$V_{Omax}$  is limited by  $Q_3$  saturating. Thus

$$V_{Omax} = V_{E3} - V_{ECsat} = 1.8 - 0.3 = 1.5 \text{ V}$$

7.89 Replacing each of the transistors in the Wilson mirror of Fig. 7.37(a) with its  $T$  model while neglecting  $r_o$  results in the circuit shown below.



Note that the diode-connected transistor  $Q_1$  reduces to a resistance  $r_{e1}$ . To determine  $R_{in}$ , we have applied a test voltage  $v_x$ . In the following we analyze the circuit to find  $i_x$  and hence  $R_{in}$ , as

$$R_{in} \equiv \frac{v_x}{i_x}$$

Note that all three transistors are operating at equal emitter currents, approximately equal to  $I_{REF}$ . Thus

$$r_{e1} = r_{e2} = r_{e3} = \frac{V_T}{I_{REF}}$$

Analysis of the circuit proceeds as follows. Since  $r_{e1} = r_{e2}$ , we obtain

$$i_{e2} = i_{e1} \quad (1)$$

Node equation at node 1:

$$i_{e3} + \alpha i_{e2} = i_{e1} + i_{e2}$$

Using Eq. (1) yields

$$i_{e3} = (2 - \alpha)i_{e1} \quad (2)$$

Node equation at node 2:

$$i_x = \alpha i_{e2} + (1 - \alpha)i_{e3}$$

Using Eqs. (1) and (2) yields

$$i_x = i_{e1}[\alpha + (1 - \alpha)(2 - \alpha)i_{e3}] \quad (3)$$

Finally,  $v_x$  can be expressed as the sum of the voltages across  $r_{e3}$  and  $r_{e1}$ ,

$$v_x = i_{e3}r_e + i_{e1}r_e$$

Using Eq. (2) yields

$$v_x = i_{e1}r_e(3 - \alpha) \quad (4)$$

Dividing Eq. (4) by Eq. (3) yields

$$R_{in} = \frac{v_x}{i_x} = r_e \frac{3 - \alpha}{2 - 2\alpha + \alpha^2}$$

For  $\alpha \simeq 1$ ,

$$R_{in} = 2r_e = 2 \frac{V_T}{I_{REF}} \quad \text{Q.E.D.}$$

Thus, for  $I_{REF} = 0.2 \text{ mA}$ ,

$$R_{in} = 250 \Omega$$

**7.90** Replacing each of the three transistors in the Wilson current mirror in Fig. 7.38(a) with its  $T$  model results in the circuit in the figure.

Here, we have applied a test voltage  $v_x$  to determine  $R_{in}$ ,

$$R_{in} \equiv \frac{v_x}{i_x}$$

Since all three transistors are identical and are operating at the same  $I_D$ ,

$$g_{m1} = g_{m2} = g_{m3}$$

Now from the figure we see that

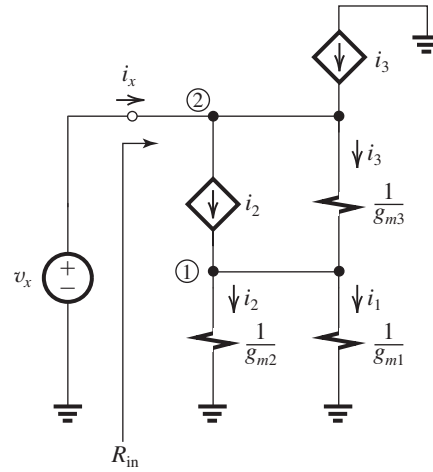
$$i_1 = i_2$$

and

$$i_2 + i_3 = i_2 + i_1$$

Thus

$$i_3 = i_1 = i_2$$



A node equation at node 2 gives

$$i_x + i_3 = i_2 + i_3$$

Thus

$$i_x = i_2$$

The voltage  $v_x$  can be expressed as the sum of the voltages across  $1/g_{m3}$  and  $1/g_{m1}$ :

$$v_x = (i_3/g_{m3}) + (i_1/g_{m1})$$

Substituting  $i_3 = i_2$  and  $i_1 = i_2$ ,  $g_{m1} = g_{m3} = g_m$ , and

$$v_x = 2 i_2/g_m$$

But  $i_2 = i_x$ ; thus

$$v_x = 2 i_x/g_m$$

and thus

$$R_{in} = \frac{2}{g_m} \quad \text{Q.E.D.}$$

**7.91** Refer to circuit in Fig. 7.38(a).

(a) Each of the three transistors is operating at  $I_D = I_{REF}$ . Thus

$$I_{REF} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$180 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.3 \text{ V}$$

$$V_{GS} = V_m + V_{OV} = 0.5 + 0.3 = 0.8 \text{ V}$$

(b)  $Q_1$  is operating at  $V_{DS} = V_{GS} = 0.8 \text{ V}$

$Q_2$  is operating at  $V_{DS} = 2V_{GS} = 1.6 \text{ V}$

Thus,

$$I_{REF} - I_O = \frac{\Delta V_{DS}}{r_o}$$

where

$$r_o = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

$$I_{REF} - I_O = \frac{0.8}{100} = 0.008 \text{ mA} = 8 \mu\text{A}$$

$$I_O = 180 - 8 = 172 \mu\text{A}$$

(c) Refer to Fig. 8.38(c). Since  $Q_1$  and  $Q_2$  are now operating at equal  $V_{DS}$ , we estimate

$$I_O = I_{REF} = 180 \mu\text{A}.$$

(d) The minimum allowable  $V_O$  is the value at which  $Q_3$  leaves the saturation region:

$$V_{Omin} = V_{GS3} - V_t$$

$$= V_{GS3} + V_{GS1} - V_t$$

$$= 0.8 + 0.8 - 0.5 = 1.1 \text{ V}$$

(e) Diode-connected transistor  $Q_4$  has an incremental resistance  $1/g_{m4}$ . Reference to Fig. 7.38(b) indicates that the incremental resistance of  $Q_4$  would appear in series with the gate of  $Q_3$  and thus carries zero current. Thus, including  $Q_4$  has no effect on the value of  $R_o$ , which can be found from Eq. (7.99):

$$R_o = g_{m3} r_{o3} r_{o2}$$

where

$$g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.18}{0.3} = 1.2 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

Thus,

$$R_o = 1.2 \times 100 \times 100 = 12 \text{ M}\Omega$$

(f) For  $\Delta V_O = 1 \text{ V}$ , we obtain

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{1 \text{ V}}{12 \text{ M}\Omega} = 0.08 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = 0.04\%$$

**7.92** Fig. 1 shows the circuit of the modified BJT Wilson mirror.

**7.93** (a) Refer to the circuit in Fig. P7.93.

Neglecting the base currents, we see that all three transistors are operating at  $I_C = 10 \mu\text{A}$ , and thus

$$V_{BE1} = V_{BE2} = V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{10 \mu\text{A}}\right)$$

$$= 0.585 \text{ V}$$

This figure belongs to Problem 7.92.

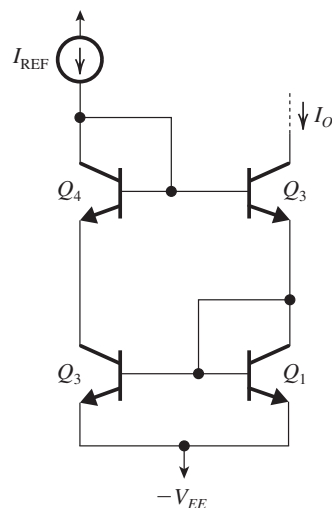


Figure 1

From the circuit we see that the voltage across  $R$  is  $V_{BE} = 0.585 \text{ V}$ , thus

$$I_O R = V_{BE}$$

$$R = \frac{0.585}{0.01} = 58.5 \text{ k}\Omega$$

$$(b) \quad g_{m3} = \frac{0.01}{0.025} = 0.4 \text{ mA/V}$$

$$r_{o3} = \frac{40}{0.01} = 4000 \text{ k}\Omega$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{100}{0.4} = 250 \text{ k}\Omega$$

$$R_{out} = (R \parallel r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (R \parallel r_{\pi 3})$$

$$= (58.5 \parallel 250) + 4000 + 0.4 \times 4000 \times (58.5 \parallel 250)$$

$$= 79.9 \text{ M}\Omega$$

**7.94** Refer to the circuit in Fig. P7.94. Since  $Q_1$  and  $Q_2$  are matched and conducting equal currents  $I$ , their  $V_{GS}$  values will be equal. Thus from the loop  $Q_1$ ,  $Q_6$ ,  $R$ , and  $Q_2$ , we see that

$$IR = V_{EB6}$$

$$= V_T \ln\left(\frac{I}{I_S}\right) \quad \text{Q.E.D.}$$

Now to obtain  $I = 0.2 \text{ mA}$ , we write

$$0.2R = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.2 \text{ mA}}\right)$$

$$\Rightarrow R = 3.3 \text{ k}\Omega$$

**Chapter 8**

**Solutions to Exercises within the Chapter**

**Ex: 8.1** Referring to Fig. 8.3,

If  $R_D$  is doubled to 5 k $\Omega$ ,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2} (5 \text{ k}\Omega) = 0.5 \text{ V}$$

$$V_{CM_{\max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents  $I_{D1}$ , and  $I_{D2}$  are still 0.2 mA each,

$$V_{GS} = 0.82 \text{ V}$$

$$\text{So, } V_{CM_{\min}} = V_{SS} + V_{CS} + V_{GS}$$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

$$-0.28 \text{ V to } +1.0 \text{ V}$$

**Ex: 8.2** (a) The value of  $v_{id}$  that causes  $Q_1$  to conduct the entire current is  $\sqrt{2} V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45 \text{ V}$$

then,  $V_{D1} = V_{DD} - I \times R_D$

$$= 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$V_{D2} = V_{DD} = +1.5 \text{ V}$$

$$v_o = V_{D2} - V_{D1} = +1 \text{ V}$$

(b) For  $Q_2$  to conduct the entire current:

$$v_{id} = -\sqrt{2} V_{OV} = -0.45 \text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5 \text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$v_o = V_{D2} - V_{D1} = -1 \text{ V}$$

(c) Thus the differential output range is

$$+1 \text{ V to } -1 \text{ V.}$$

**Ex: 8.3** Refer to answer table for Exercise 8.3 where values were obtained in the following way:

$$\frac{I}{2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{I}{k'_n V_{OV}^2} = \frac{2}{V_{OV}^2}$$

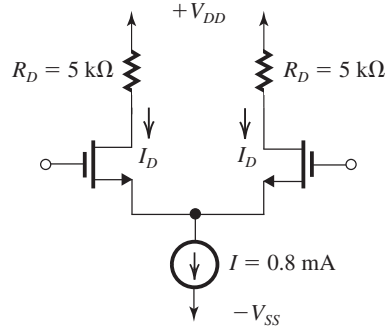
$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{0.4}{V_{OV}} \text{ mA/V}$$

$$\left( \frac{v_{id}/2}{V_{OV}} \right)^2 = 0.1 \rightarrow v_{id} = 2 V_{OV} \sqrt{0.1} =$$

$$0.632 V_{OV}$$

**Ex: 8.4**  $I_D = \frac{I}{2} = \frac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_{OV})^2$$



Thus,

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n \left( \frac{W}{L} \right)}} = \sqrt{\frac{2(0.4 \text{ mA})}{0.2 \text{ (mA/V}^2) (100)}} = 0.2 \text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4 \text{ mA} \times 2}{0.2 \text{ V}} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.4 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$A_d = (4 \text{ mA/V}) (5 \text{ k}\Omega \parallel 50 \text{ k}\Omega) = 18.2 \text{ V/V}$$

**Ex: 8.5** With  $I = 200 \mu\text{A}$ , for all transistors,

$$I_D = \frac{I}{2} = \frac{200 \mu\text{A}}{2} = 100 \mu\text{A}$$

$$L = 2(0.18 \mu\text{m}) = 0.36 \mu\text{m}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A| L}{I_D}$$

$$= \frac{(10 \text{ V}/\mu\text{m})(0.36 \mu\text{m})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

$$\text{Since } I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2,$$

$$\left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2}$$

$$\frac{2(100 \mu\text{A})}{(400 \mu\text{A/V}^2) (0.2 \text{ V})^2} = 12.5$$

$$\left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2}$$

$$\frac{2(100 \mu\text{A})}{(100 \mu\text{A/V}^2) (0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100 \mu\text{A})(2)}{0.2 \text{ V}} = 1 \text{ mA/V,}$$

so

$$A_d = g_m (r_{o1} \parallel r_{o3}) = 1(\text{mA/V}) (36 \text{ k}\Omega \parallel 36 \text{ k}\Omega) = 18 \text{ V/V}$$

**Ex: 8.6**  $L = 2 (0.18 \mu\text{m}) = 0.36 \mu\text{m}$

$$\text{All } r_o = \frac{|V'_A| \cdot L}{I_D}$$

The drain current for all transistors is

$$I_D = \frac{I}{2} = \frac{200 \mu\text{A}}{2} = 100 \mu\text{A}$$

$$r_o = \frac{(10 \text{ V}/\mu\text{m})(0.36 \mu\text{m})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

Referring to Fig. 8.13(a),

Since  $I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$  for all NMOS transistors,

$$\begin{aligned} \left(\frac{W}{L}\right)_1 &= \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 \\ &= \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100 \mu\text{A})}{400 \mu\text{A}/\text{V}^2 (0.2 \text{ V})^2} = 12.5 \end{aligned}$$

$$\begin{aligned} \left(\frac{W}{L}\right)_5 &= \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 \\ &= \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100 \mu\text{A})}{100 \mu\text{A}/\text{V}^2 (0.2 \text{ V})^2} = 50 \end{aligned}$$

For all transistors,

$$g_m = \frac{|I_D|}{|V_{OV}|/2} = \frac{(0.1 \text{ mA})(2)}{(0.2 \text{ V})} = 1 \text{ mA/V}$$

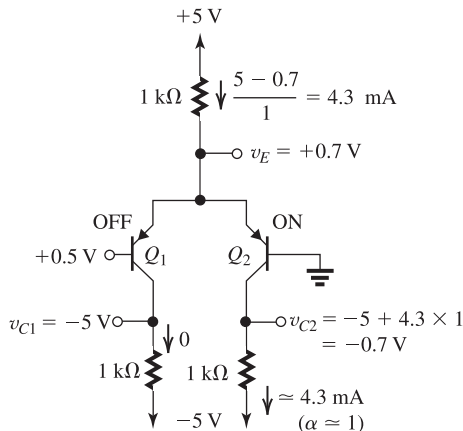
From Fig. 8.13(b),

$$\begin{aligned} R_{on} &= (g_{m3} r_{o3}) r_{o1} = (1 \times 36) \times 36 \\ &= 1.296 \text{ M}\Omega \end{aligned}$$

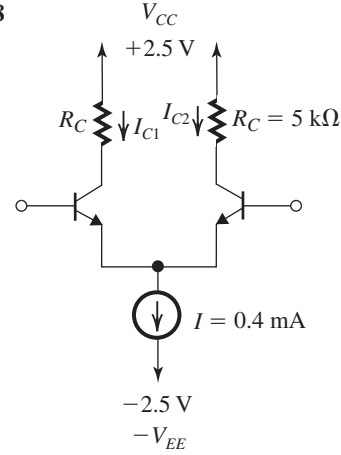
$$\begin{aligned} R_{op} &= (g_{m5} r_{o5}) r_{o7} = (1 \times 36) \times 36 \\ &= 1.296 \text{ M}\Omega \end{aligned}$$

$$\begin{aligned} A_d &= g_{m1} (R_{on} \parallel R_{op}) \\ &= (1 \text{ mA/V}) (1.296 \text{ M}\Omega \parallel 1.296 \text{ M}\Omega) \\ &= 648 \text{ V/V} \end{aligned}$$

**Ex: 8.7**



**Ex: 8.8**



$$I_{C1} = I_{C2} \simeq I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.4 \text{ mA}}{2}$$

$$= 0.2 \text{ mA}$$

$$V_{CM\text{max}} \simeq V_C + 0.4 \text{ V}$$

$$= V_{CC} - I_C R_C + 0.4 \text{ V}$$

$$= 2.5 - 0.2 \text{ mA} (5 \text{ k}\Omega) + 0.4 \text{ V} = +1.9 \text{ V}$$

$$V_{CM\text{min}} = -V_{EE} + V_{CS} + V_{BE}$$

$$V_{CM\text{min}} = -2.5 \text{ V} + 0.3 \text{ V} + 0.7 \text{ V} = -1.5 \text{ V}$$

Input common-mode range is  $-1.5 \text{ V}$  to  $+1.9 \text{ V}$

**Ex: 8.9** Substituting  $i_{E1} + i_{E2} = I$  in Eq. (8.45) yields

$$i_{E1} = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$0.99 I = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$v_{B1} - v_{B2} = -V_T \ln\left(\frac{1}{0.99} - 1\right)$$

$$= -25 \ln(1/99)$$

$$= 25 \ln(99) = 115 \text{ mV}$$

**Ex: 8.10** (a) The DC current in each transistor is  $0.5 \text{ mA}$ . Thus  $V_{BE}$  for each will be

$$V_{BE} = 0.7 + 0.025 \ln\left(\frac{0.5}{1}\right)$$

$$= 0.683 \text{ V}$$

$$\Rightarrow v_E = 5 - 0.683 = +4.317 \text{ V}$$

$$(b) g_m = \frac{I_C}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$\begin{aligned} (c) i_{C1} &= 0.5 + g_{m1} \Delta v_{BE1} \\ &= 0.5 + 20 \times 0.005 \sin(2\pi \times 1000t) \\ &= 0.5 + 0.1 \sin(2\pi \times 1000t), \text{ mA} \end{aligned}$$

$$i_{C2} = 0.5 - 0.1 \sin(2\pi \times 1000t), \text{ mA}$$

$$\begin{aligned} \text{(d) } v_{C1} &= (V_{CC} - I_C R_C) - 0.1 \\ &\times R_C \sin(2\pi \times 1000t) \\ &= (15 - 0.5 \times 10) - 0.1 \times 10 \sin(2\pi \times 1000t) \end{aligned}$$

$$= 10 - 1 \sin(2\pi \times 1000t), \text{ V}$$

$$v_{C2} = 10 + 1 \sin(2\pi \times 1000t), \text{ V}$$

$$\text{(e) } v_{C2} - v_{C1} = 2 \cdot \sin(2\pi \times 1000t), \text{ V}$$

$$\begin{aligned} \text{(f) Voltage gain} &\equiv \frac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}} \\ &= \frac{2 \text{ V peak}}{0.01 \text{ V peak}} = 200 \text{ V/V} \end{aligned}$$

**Ex: 8.11** The transconductance for each transistor is

$$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$$

$$I_D = \frac{I}{2} = \frac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$$

Thus,

$$g_m = \sqrt{2 \times 0.2 \times 100 \times 0.4} = 4 \text{ mA/V}$$

The differential gain for matched

$$R_D \text{ values is } A_d = \frac{v_{O2} - v_{O1}}{v_{id}} = g_m R_D$$

If we ignore the 1% here, then we obtain

$$A_d = g_m R_D = (4 \text{ mA/V}) (5 \text{ k}\Omega) = 20 \text{ V/V}$$

$$|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \left( \frac{5}{2 \times 25} \right) (0.01) = 0.001 \text{ V/V}$$

$$\text{CMRR (dB)} = 20 \log \frac{|A_d|}{|A_{CM}|} = 20 \log \left( \frac{20}{0.001} \right)$$

$$= 86 \text{ dB}$$

**Ex: 8.12** From Exercise 8.11,

$$g_m = 4 \text{ mA/V}$$

Using Eq. (8.85) and the fact that  $R_{SS} = 25 \text{ k}\Omega$ , we obtain

$$\text{CMRR} = \frac{(2 g_m R_{SS})}{\left( \frac{\Delta g_m}{g_m} \right)} = \frac{2(4 \text{ mA/V})(25 \text{ k}\Omega)}{0.01}$$

$$= 20,000$$

$$\text{CMRR (dB)} = 20 \log_{10} (20,000) = 86 \text{ dB}$$

**Ex: 8.13** If the output of a MOS differential amplifier is taken single-endedly, then

$$|A_d| = \frac{1}{2} g_m R_D$$

(that is, half the gain obtained with the output taken differentially), and from Eq. (8.74) we have

$$|A_{cm}| \simeq \frac{R_D}{2R_{SS}}$$

Thus,

$$\text{CMRR} \equiv \frac{|A_d|}{|A_{cm}|} = g_m R_{SS}$$

Substituting  $g_m = 4 \text{ mA/V}$ ,

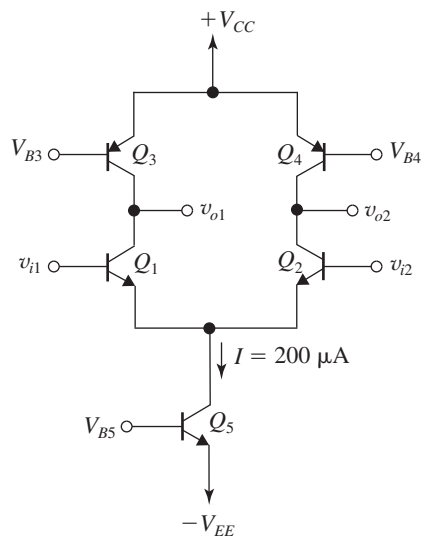
$R_D = 5 \text{ k}\Omega$ , and  $R_{SS} = 25 \text{ k}\Omega$ ,

$$|A_d| = 10 \text{ V/V}$$

$$|A_{cm}| = 0.1 \text{ V/V}$$

$$\text{CMRR} = 100 \text{ or } 40 \text{ dB}$$

**Ex: 8.14**



$$I = 200 \mu\text{A}$$

Since  $\beta \gg 1$ ,

$$I_{C1} \approx I_{C2} \approx \frac{I}{2} = \frac{200 \mu\text{A}}{2} = 100 \mu\text{A}$$

$$g_{m1} = g_{m2} = g_m = \frac{I_C}{V_T} = \frac{100 \mu\text{A}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$R_{C1} = R_{C2} = R_C = r_o = \frac{|V_A|}{I_C}$$

$$= \frac{10 \text{ V}}{100 \mu\text{A}} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_A}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 0.25 \text{ k}\Omega$$

$$\begin{aligned} |A_d| &= \frac{R_C \parallel r_o}{r_e} = \frac{100 \text{ k}\Omega \parallel 100 \text{ k}\Omega}{0.25 \text{ k}\Omega} \\ &= 200 \text{ V/V} \end{aligned}$$

$$R_{id} = 2r_{\pi}, \quad r_{\pi} = \frac{\beta}{g_m} = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$R_{id} = 2(25 \text{ k}\Omega) = 50 \text{ k}\Omega$$

$$R_{EE} = \frac{V_A}{I} = \frac{10 \text{ V}}{200 \mu\text{A}} = 50 \text{ k}\Omega$$

If the total load resistance is assumed to be mismatched by 1%, then we have

$$|A_{cm}| = \frac{R_C}{2R_{EE}} \frac{\Delta R_C}{R_C} \\ = \frac{100}{2 \times 50} \times 0.01 = 0.01 \text{ V/V}$$

$$\text{CMRR (dB)} = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| = 20 \log_{10} \left| \frac{200}{0.01} \right| \\ = 86 \text{ dB}$$

Using Eq. (8.96), we obtain

$$R_{icm} = \beta R_{EE} \cdot \frac{1 + \frac{R_C}{\beta r_o}}{1 + \frac{R_C + 2R_{EE}}{r_o}} \\ = 100 \times 50 \times \frac{1 + \frac{100 \times 100}{100 + 2 \times 50}}{1 + \frac{100}{100}}$$

$$R_{icm} \simeq 1.68 \text{ M}\Omega$$

**Ex: 8.15** From Exercise 8.4:

$$V_{OV} = 0.2 \text{ V}$$

Using Eq. (8.97) we obtain  $V_{OS}$  due to  $\Delta R_D/R_D$  as:

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \cdot \left( \frac{\Delta R_D}{R_D} \right) \\ = \frac{0.2}{2} \times 0.02 = 0.002 \text{ V} \quad \text{i.e. } 2 \text{ mV}$$

To obtain  $V_{OS}$  due to  $\Delta k_n$ ,

use Eq. (8.98),

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta k_n}{k_n} \right) \\ \Rightarrow V_{OS} = \left( \frac{0.2}{2} \right) \times 0.02 = 0.002 \\ \Rightarrow 2 \text{ mV}$$

The offset voltage arising from  $\Delta V_t$  is obtained from Eq. (8.99):

$$V_{OS} = \Delta V_t = 2 \text{ mV}$$

Finally, from Eq. (8.100) the total input offset is

$$V_{OS} = \left[ \left( \frac{V_{OV}}{2} \frac{\Delta R_D}{R_D} \right)^2 + \left( \frac{V_{OV}}{2} \frac{\Delta k_n}{k_n} \right)^2 + (\Delta V_t)^2 \right]^{1/2}$$

$$= \sqrt{(2 \times 10^{-3})^2 + (2 \times 10^{-3})^2 + (2 \times 10^{-3})^2} \\ = \sqrt{3 \times (2 \times 10^{-3})^2} \\ = 3.5 \text{ mV}$$

**Ex: 8.16** From Eq. (8.103), we get

$$V_{OS} = V_T \sqrt{\left( \frac{\Delta R_C}{R_C} \right)^2 + \left( \frac{\Delta I_S}{I_S} \right)^2} \\ = 25 \sqrt{(0.02)^2 + (0.1)^2} \\ = 2.55 \text{ mV} \\ I_B = \frac{100}{2(\beta + 1)} = \frac{100}{2 \times 101} \simeq 0.5 \mu\text{A}$$

$$I_{OS} = I_B \left( \frac{\Delta \beta}{\beta} \right) \\ = 0.5 \times 0.1 \mu\text{A} = 50 \text{ nA}$$

**Ex: 8.17**  $I_D = \frac{1}{2} I = 0.4 \text{ mA}$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$g_{m1,2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$G_{md} = g_{m1,2} = 4 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A_n}}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{A_p}|}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25 \text{ k}\Omega$$

$$A_d = G_m R_o = 4 \times 25 = 100 \text{ V/V}$$

**Ex: 8.18**  $G_{md} = g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{0.4 \text{ mA}}{0.025 \text{ V}}$

$$= 16 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{V_A}{I_C} = \frac{V_A}{I/2} = \frac{100}{0.4} = 250 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 250 \parallel 250 = 125 \text{ k}\Omega$$

$$A_d = G_m R_o = 16 \times 125 = 2000 \text{ V/V}$$

$$R_{id} = 2r_{\pi} = 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{160}{16} = 20 \text{ k}\Omega$$

**Ex: 8.19** From Exercise 8.17, we get

$$I_D = 0.4 \text{ mA}$$

$$V_{OV} = 0.2 \text{ V} \quad g_{m1,2} = 4 \text{ mA/V}$$

$$G_{md} = 4 \text{ mA/V} \quad A_d = 100 \text{ V/V}$$

Now,

$$R_{SS} = 25 \text{ k}\Omega$$

$$g_{m3} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_p I_D}$$

$$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4 \text{ mA/V}$$

$$|A_{cm}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005 \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005}$$

$$= 20,000 \text{ or } 20 \log 20,000 = 86 \text{ dB}$$

**Ex: 8.20** Refer to Fig. (8.37).

(a) Using Eq. (8.142), we obtain

$$I_6 = \frac{(W/L)_6}{(W/L)_4} (I/2)$$

$$\Rightarrow 128 = \frac{(W/L)_6}{11.1} \times 64$$

$$\text{thus, } (W/L)_6 = 22.2$$

Using Eq. (8.143), we get

$$I_7 = \frac{(W/L)_7}{(W/L)_5} I$$

$$\Rightarrow 128 = \frac{(W/L)_7}{88.8} \times 128$$

$$\text{thus, } (W/L)_7 = 88.8$$

(b) For  $Q_1$ ,

$$\frac{I}{2} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$\Rightarrow V_{OV1} = \sqrt{\frac{64}{\frac{1}{2} \times 128 \times 44.4}} = 0.15 \text{ V}$$

Similarly for  $Q_2$ ,  $V_{OV2} = 0.15 \text{ V}$

For  $Q_6$ ,

$$128 = \frac{1}{2} \times 512 \times 22.2 V_{OV6}^2$$

$$\Rightarrow V_{OV6} = 0.15 \text{ V}$$

$$(c) \quad g_m = \frac{2I_D}{V_{OV}}$$

	$I_D$	$V_{OV}$	$g_m$
$Q_1$	64 $\mu\text{A}$	0.15 V	0.853 mA/V
$Q_2$	64 $\mu\text{A}$	0.15 V	0.853 mA/V
$Q_6$	128 $\mu\text{A}$	0.15 V	1.71 mA/V

$$(d) \quad r_{o2} = 3.2/0.064 = 50 \text{ k}\Omega$$

$$r_{o4} = 3.2/0.064 = 50 \text{ k}\Omega$$

$$r_{o6} = 3.2/0.128 = 25 \text{ k}\Omega$$

$$r_{o7} = 3.2/0.128 = 25 \text{ k}\Omega$$

(e) Eq. (8.140):

$$A_1 = -g_{m1} (r_{o2} \parallel r_{o4})$$

$$= -0.853 (50 \parallel 50) = -21.3 \text{ V/V}$$

Eq. (8.141):

$$A_2 = -g_{m6} (r_{o6} \parallel r_{o7})$$

$$= -21.3 \text{ V/V}$$

Overall voltage gain is

$$A_1 \times A_2 = -21.3 \times -21.3 = 454 \text{ V/V}$$

**Ex: 8.21**  $R_{id} = 20.2 \text{ k}\Omega$

$$A_{vo} = 8513 \text{ V/V}$$

$$R_o = 152 \Omega$$

With  $R_S = 10 \text{ k}\Omega$  and  $R_L = 1 \text{ k}\Omega$ ,

$$G_v = \frac{20.2}{20.2 + 10} \times 8513 \times \frac{1}{(1 + 0.152)}$$

$$= 4943 \text{ V/V}$$

**Ex: 8.22**  $\frac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101 \text{ A/A}$

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7 + 303.5} = 0.0492 \text{ A/A}$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100 \text{ A/A}$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3 + 234.8} = 0.0126 \text{ A/A}$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100 \text{ A/A}$$

$$\frac{i_{b5}}{i_{c2}} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40 + 5.05} =$$

$$0.8879 \text{ A/A}$$

$$\frac{i_{c2}}{i_1} = \beta_2 = 100 \text{ A/A}$$

Thus the overall current gain is

$$\frac{i_{e8}}{i_1} = 101 \times 0.0492 \times 100 \times 0.0126 \times 100$$

$$\times 0.8879 \times 100$$

$$= 55,599 \text{ A/A}$$

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = \frac{R_6}{R_{i1}} \cdot \frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2} \times 55599 = 8257 \text{ V/V}$$



**Solutions to End-of-Chapter Problems****8.1** Refer to Fig. 8.2.

$$(a) \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_m + V_{OV} = 0.4 + 0.2 = 0.6 \text{ V}$$

$$(b) V_{CM} = 0$$

$$V_S = 0 - V_{GS} = -0.6 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

$$V_O = 0 \text{ V}$$

$$(c) V_{CM} = +0.4 \text{ V}$$

$$V_S = 0.4 - V_{GS} = 0.4 - 0.6 = -0.2 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

$$V_O = 0 \text{ V}$$

Since  $V_{CM} = 0.4 \text{ V}$  and  $V_D = 0.6 \text{ V}$ ,  $V_{GD} = -0.2 \text{ V}$ , which is less than  $V_m$  ( $0.4 \text{ V}$ ), indicating that our implicit assumption of saturation-mode operation is justified.

$$(d) V_{CM} = -0.1 \text{ V}$$

$$V_S = -0.1 - V_{GS} = -0.1 - 0.6 = -0.7 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

$$V_O = 0 \text{ V}$$

(e) The highest value of  $V_{CM}$  for which  $Q_1$  and  $Q_2$  remain in saturation is

$$V_{CM\max} = V_{D1,2} + V_m$$

$$= 0.6 + 0.4 = 1.0 \text{ V}$$

(f) To maintain the current-source operating properly, we need to keep a minimum voltage of  $0.2 \text{ V}$  across it, thus

$$V_{S\min} = -V_{SS} + V_{CS} = -1 + 0.2 = -0.8 \text{ V}$$

$$V_{CM\min} = V_{S\min} + V_{GS}$$

$$= -0.8 + 0.6$$

$$= -0.2 \text{ V}$$

$$(g) -0.2 \text{ V to } +1.0 \text{ V}$$

**8.2** Refer to Fig. P8.2.(a) For  $V_{G1} = V_{G2} = 0 \text{ V}$ ,

$$I_{D1} = I_{D2} = \frac{1}{2} \times 0.2 = 0.1 \text{ mA}$$

$$I_{D1,2} = \frac{1}{2} k_p \left( \frac{W}{L} \right) |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 5 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.2 \text{ V}$$

$$V_{SG} = |V_{tp}| + |V_{OV}|$$

$$= 0.4 + 0.2 = 0.6 \text{ V}$$

$$V_S = 0 + V_{SG} = +0.6 \text{ V}$$

$$V_{D1} = V_{D2} = -V_{SS} + I_D R_D$$

$$= -0.9 + 0.1 \times 4$$

$$= -0.5 \text{ V}$$

Since for each of  $Q_1$  and  $Q_2$ ,

$$V_{SD} = 0.6 - (-0.5)$$

$$= 1.1 \text{ V}$$

which is greater than  $|V_{OV}|$ ,  $Q_1$  and  $Q_2$  are operating in saturation as implicitly assumed.

(b) The highest value of  $V_{CM}$  is limited by the need to keep a minimum of  $0.2 \text{ V}$  across the current source, thus

$$V_{CM\max} = +0.9 - 0.2 - V_{SG}$$

$$= +0.9 - 0.2 - 0.6 = +0.1 \text{ V}$$

The lowest value of  $V_{CM}$  is limited by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$V_{CM\min} = V_{D1,2} - |V_{tp}|$$

$$= -0.5 - 0.4 = -0.9 \text{ V}$$

Thus,

$$-0.9 \text{ V} \leq V_{ICM} \leq +0.1 \text{ V}$$

**8.3** Refer to the circuit in Fig. P8.3.For  $v_{G1} = v_{G2} = 0 \text{ V}$ ,

$$I_{D1} = I_{D2} = \frac{0.4}{2} = 0.2 \text{ mA}$$

To obtain

$$V_{D1} = V_{D2} = +0.1 \text{ V}$$

$$V_{DD} - I_{D1,2} R_D = 0.1$$

$$0.9 - 0.2 R_D = 0.1$$

$$\Rightarrow R_D = 4 \text{ k}\Omega$$

For  $Q_1$  and  $Q_2$ ,

$$I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \left( \frac{W}{L} \right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{1,2} = 44.4$$

For  $Q_3$ ,

$$0.4 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right)_3 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = 88.8$$

Since  $Q_3$  and  $Q_4$  form a current mirror with

$$I_{D3} = 4I_{D4},$$

$$\left( \frac{W}{L} \right)_4 = \frac{1}{4} \left( \frac{W}{L} \right)_3 = 22.2$$

$$V_{GS4} = V_{GS3} = V_m + V_{OV} = 0.4 + 0.15$$

$$= 0.55 \text{ V}$$

$$R = \frac{0.9 - (-0.9) - 0.55}{0.1}$$

$$= 12.5 \text{ k}\Omega$$

The lower limit on  $V_{CM}$  is determined by the need to keep  $Q_3$  operating in saturation. For this to happen, the minimum value of  $V_{DS3}$  is

$$V_{OV} = 0.15 \text{ V. Thus,}$$

$$V_{ICM\min} = -V_{SS} + V_{OV3} + V_{GS1,2}$$

$$= -0.9 + 0.15 + 0.4 + 0.15$$

$$= -0.2 \text{ V}$$

The upper limit on  $V_{CM}$  is determined by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$V_{ICM\max} = V_{D1,2} + V_m$$

$$= 0.1 + 0.4 = 0.5 \text{ V}$$

Thus,

$$-0.2 \text{ V} \leq V_{ICM} \leq +0.5 \text{ V}$$

**8.4** Refer to Fig. 8.5.

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$1 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$\frac{1}{2} \times 0.25 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right) 0.25^2$$

$$\Rightarrow \frac{W}{L} = 10$$

**8.5** For  $v_{id} = 0.1 \text{ V}$

$$\left( \frac{v_{id}/2}{V_{OV}} \right)^2 = 0.04$$

$$\frac{v_{id}/2}{V_{OV}} = 0.2$$

$$\frac{0.1/2}{V_{OV}} = 0.2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$2 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

$$A_d = \frac{1 \text{ V}}{0.1 \text{ V}} = 10$$

$$g_m R_D = 10$$

$$\Rightarrow R_D = \frac{10}{2} = 5 \text{ k}\Omega$$

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.2 \times \frac{W}{L} \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 40$$

**8.6** Since the quiescent power dissipation is

$$P = (V_{DD} + V_{SS}) \times I$$

then the maximum allowable  $I$  is

$$I = \frac{0.3 \text{ mW}}{1.2 \text{ V}} = 0.25 \text{ mA}$$

We shall utilize this value. The value of  $V_{OV}$  can be found from

$$\sqrt{2} V_{OV} = 0.2 \text{ V}$$

$$\Rightarrow V_{OV} = \frac{0.2}{\sqrt{2}} = 0.14 \text{ V}$$

The realized value of  $g_m$  will be

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$= \frac{0.25}{0.14} = 1.8 \text{ mA/V}$$

To obtain a differential gain  $A_d$  of 8 V/V,

$$A_d = g_m R_D$$

$$8 = 1.8 \times R_D$$

$$\Rightarrow R_D = 4.4 \text{ k}\Omega$$

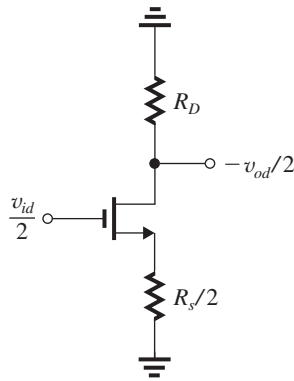
Finally, the required value of  $W/L$  can be determined from

$$I_D = \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$0.125 = \frac{1}{2} \times 0.5 \times \frac{W}{L} \times 0.14^2$$

$$\Rightarrow \frac{W}{L} = 25.5$$

8.7



From symmetry, a virtual ground appears at the midpoint of  $R_s$ . Thus, the differential half circuit will be as shown in the figure, and

$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{R_D}{\frac{1}{g_m} + \frac{R_s}{2}}$$

For  $R_s = 0$ ,

$$A_d = \frac{R_D}{1/g_m} = g_m R_D,$$

as expected.

To reduce the gain to half this value, we use

$$\frac{R_s}{2} = \frac{1}{g_m}$$

$$\Rightarrow R_s = \frac{2}{g_m}$$

8.8 Refer to Fig. P8.8.

The value of  $R$  is found as follows:

$$R = \frac{V_{G6} - V_{G7}}{I_{REF}}$$

$$= \frac{0.8 - (-0.8)}{0.2} = 8 \text{ k}\Omega$$

Since  $I = I_{REF}$ ,  $Q_3$  and  $Q_6$  are matched and are operating at

$$|V_{OV}| = 1.5 - 0.8 - 0.5 = 0.2 \text{ V}$$

Thus,

$$0.2 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{6,3} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 = 100$$

Each of  $Q_4$  and  $Q_5$  is conducting a dc current of  $(I/2)$  while  $Q_7$  is conducting a dc current  $I_{REF} = I$ . Thus  $Q_4$  and  $Q_5$  are matched and their  $W/L$  ratios are equal while  $Q_7$  has twice the  $(W/L)$  ratio of  $Q_4$  and  $Q_5$ . Thus,

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{4,5} V_{OV4,5}^2$$

where

$$V_{OV4,5} = -0.8 - (-1.5) - 0.5 = 0.2 \text{ V}$$

thus,

$$0.1 = \frac{1}{2} \times 0.25 \times \left(\frac{W}{L}\right)_{4,5} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 20$$

and

$$\left(\frac{W}{L}\right)_7 = 40$$

$$r_{o4} = r_{o5} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o4,5})$$

$$50 = g_{m1,2}(100 \parallel 100)$$

$$\Rightarrow g_{m1,2} = 1 \text{ mA/V}$$

But

$$g_{m1,2} = \frac{2(I/2)}{|V_{OV1,2}|}$$

$$1 = \frac{0.2}{|V_{OV1,2}|}$$

$$\Rightarrow |V_{OV1,2}| = 0.2 \text{ V}$$

The  $(W/L)$  ratio for  $Q_1$  and  $Q_2$  can now be determined from

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{1,2} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 50$$

A summary of the results is provided in the table below.

Transistor	W/L	$I_D$ (mA)	$ V_{GS} $ (V)
$Q_1$	50	0.1	0.7
$Q_2$	50	0.1	0.7
$Q_3$	100	0.2	0.7
$Q_4$	20	0.1	0.7
$Q_5$	20	0.1	0.7
$Q_6$	100	0.2	0.7
$Q_7$	40	0.2	0.7

8.9 Refer to Fig. 8.13.

All transistors have the same channel length and are carrying a dc current  $I/2$ . Thus all transistors have the same  $r_o = \frac{|V_A|}{I/2}$ . Also, all transistors are operating at the same  $|V_{OV}|$  and have equal dc currents, thus all have the same

$g_m = \frac{2(I/2)}{|V_{OV}|} = I/|V_{OV}|$ . Thus all transistors have equal intrinsic gain  $g_m r_o = 2|V_A|/|V_{OV}|$ . Now, the gain  $A_d$  is given by

$$\begin{aligned} A_d &= g_m(R_{on} \parallel R_{op}) \\ &= \frac{1}{2} g_m R_{on} \\ &= \frac{1}{2} g_m (g_m r_o) r_o = \frac{1}{2} (g_m r_o)^2 \end{aligned}$$

Thus,

$$\begin{aligned} A_d &= \frac{1}{2} \left[ \frac{2|V_A|}{|V_{OV}|} \right]^2 \\ &= 2(|V_A|/|V_{OV}|)^2 \quad \text{Q.E.D.} \end{aligned}$$

To obtain  $A_d = 500$  V/V while operating all transistors at  $|V_{OV}| = 0.2$  V, we use

$$\begin{aligned} 500 &= 2 \frac{|V_A|^2}{0.04} \\ \Rightarrow |V_A| &= 3.16 \text{ V} \end{aligned}$$

Since  $|V_A'| = 5 \text{ V}/\mu\text{m}$ , the channel length  $L$  (for all transistors) must be

$$3.16 = 5 \times L$$

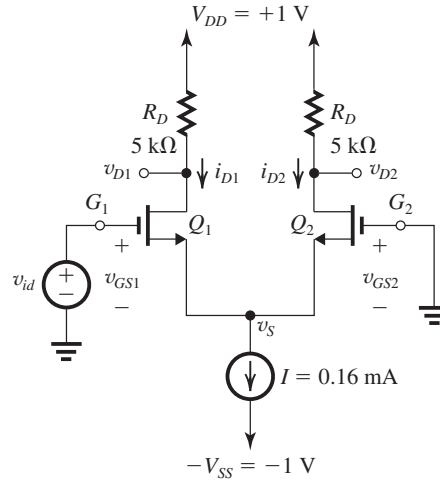
$$L = 0.632 \mu\text{m}$$

To obtain the highest possible  $g_m$ , we operate at the highest possible  $I$  consistent with limiting the power dissipation (in equilibrium) to 0.5 mW.

Thus,

$$I = \frac{0.5 \text{ mW}}{(0.9 + 0.9)\text{V}} = 0.28 \text{ mA}$$

8.10



(a) For  $i_{D1} = i_{D2} = 0.08$  mA,

$$v_{G1} = v_{G2}$$

Thus,

$$v_{id} = 0 \text{ V}$$

$$i_{D1} = i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$v_{GS1} = v_{GS2} = 0.2 + 0.4 = 0.6 \text{ V}$$

$$v_S = -0.6 \text{ V}$$

$$v_{D1} = v_{D2} = V_{DD} - i_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = 0.6 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = 0 \text{ V}$$

(b) For  $i_{D1} = 0.12$  mA and  $i_{D2} = 0.04$  mA,

$$i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS2} - V_m)^2$$

$$0.04 = \frac{1}{2} \times 0.4 \times 10 \times (v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.541 \text{ V}$$

Thus,

$$v_S = -0.541 \text{ V}$$

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS1} - V_m)^2$$

$$0.12 = \frac{1}{2} \times 0.4 \times 10 (v_{id} - v_S - V_m)^2$$

$$= \frac{1}{2} \times 0.4 \times 10 (v_{id} + 0.541 - 0.4)^2$$

Case	$i_{D1}(\text{mA})$	$i_{D2}(\text{mA})$	$v_{id}(\text{V})$	$v_S(\text{V})$	$v_{D1}(\text{V})$	$v_{D2}(\text{V})$	$v_O$
a	0.08	0.08	0	-0.6	+0.6	+0.6	0
b	0.12	0.04	+0.104	-0.541	+0.4	+0.8	+0.4
c	0.16	0	+0.283	-0.4	+0.2	+1.0	+0.8
d	0.04	0.12	-0.104	-0.645	+0.8	+0.4	-0.4
e	0	0.16	-0.283	-0.683	+1.0	+0.2	-0.8

$$\Rightarrow v_{id} = 0.104 \text{ V}$$

$$v_{GS1} = 0.104 - (-0.541) = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = 0.8 - 0.4 = 0.4 \text{ V}$$

(c)  $i_{D1} = 0.16 \text{ mA}$  and  $i_{D2} = 0$  with  $Q_2$  just cutting off, thus

$$v_{GS2} = V_m = 0.4 \text{ V}$$

$$\Rightarrow v_S = -0.4 \text{ V}$$

$$i_{D1} = \frac{1}{2} \times 0.4 \times 10 (v_{GS1} - V_m)^2$$

$$0.16 = \frac{1}{2} \times 0.4 \times 10 (v_{id} + 0.4 - 0.4)^2$$

$$\Rightarrow v_{id} = 0.283 \text{ V}$$

which is  $\sqrt{2}V_{OV}$ , as derived in the text.

$$v_{GS1} = 0.283 - (-0.4) = 0.683 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.16 \times 5 = +0.2 \text{ V}$$

Note that since  $v_{G1} = v_{id} = 0.283 \text{ V}$ ,  $Q_1$  is still operating in saturation, as implicitly assumed.

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0 \times 5 = 1 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = 1 - 0.2 = 0.8 \text{ V}$$

(d)  $i_{D1} = 0.04 \text{ mA}$  and  $i_{D2} = 0.12 \text{ mA}$ . Since this split of the current  $I$  is the complement of that in case (b) above, the value of  $v_{id}$  must be the negative of that found in (b). Thus,

$$v_{id} = -0.104 \text{ V}$$

$$v_{GS1} = 0.541 \text{ V}$$

$$v_S = -0.645 \text{ V}$$

$$v_{GS2} = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_{D2} = 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = -0.4 \text{ V}$$

(e)  $i_{D1} = 0$  ( $Q_1$  just cuts off) and  $i_{D2} = 0.16 \text{ mA}$ . This case is the complement of that in (c) above, thus

$$v_{GS1} = V_m = 0.4 \text{ V}$$

$$v_{GS2} = 0.683 \text{ V}$$

$$v_S = -0.683 \text{ V}$$

$$v_{id} = -0.683 + 0.4 = -0.283 \text{ V}$$

which is  $-\sqrt{2}V_{OV}$ , as derived in the text.

$$v_{D1} = V_{DD} - i_{D1}R_D = 1 - 0 \times 5 = 1 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1 - 0.16 \times 5 = 0.2 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = -0.8 \text{ V}$$

### Summary

A summary of the results is shown in the table above.

### 8.11 Refer to Fig. P8.2.

To determine  $V_{OV}$ , consider the case  $v_{id} = 0$ ,

$$0.1 = \frac{1}{2} \times 5 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.2 \text{ V}$$

With  $v_{G2} = 0$  and  $v_{G1} = v_{id}$ , to steer the current from one side of the differential pair to the other,  $v_{id}$  must be the ends of the range

$$-\sqrt{2}|V_{OV}| \leq v_{id} \leq \sqrt{2}|V_{OV}|$$

that is,

$$-0.283 \text{ V} \leq v_{id} \leq +0.283 \text{ V}$$

At  $v_{id} = -0.283 \text{ V}$ ,  $Q_2$  just cuts off, thus

$$v_S = |V_{tp}| = 0.4 \text{ V}$$

and

$$v_{SG1} = 0.4 - (-0.283) = 0.683 \text{ V}$$

At this value of  $v_{SG1}$ ,

$$i_{D1} = \frac{1}{2} \times 5 \times (0.683 - 0.4)^2$$

$$= 0.2 \text{ mA}$$

which is the entire bias current.

$$v_{D1} = -0.9 + 0.2 \times 4 = -0.1 \text{ V}$$

Thus,  $v_{DG1} = -0.1 - (-0.283) = +0.183$  which is less than  $|V_{tp}|$ ; thus  $Q_1$  is still operating in saturation, as implicitly assumed.

$$v_{D2} = -0.9 \text{ V}$$

Finally,

$$v_O = v_{D2} - v_{D1} = -0.8 \text{ V}$$

At  $v_{id} = +0.283 \text{ V}$ ,  $Q_1$  just cuts off, thus

$$v_{SG1} = |V_{tp}| = 0.4 \text{ V and}$$

$$v_S = +0.283 + 0.4 = +0.683 \text{ V}$$

and thus

$$v_{SG2} = 0.683 \text{ V}$$

which results in

$$i_{D2} = \frac{1}{2} \times 5 (0.683 - 0.4)^2$$

$$= 0.2 \text{ mA}$$

which is the entire bias current. Here,

$$v_{D1} = -0.9 \text{ V}$$

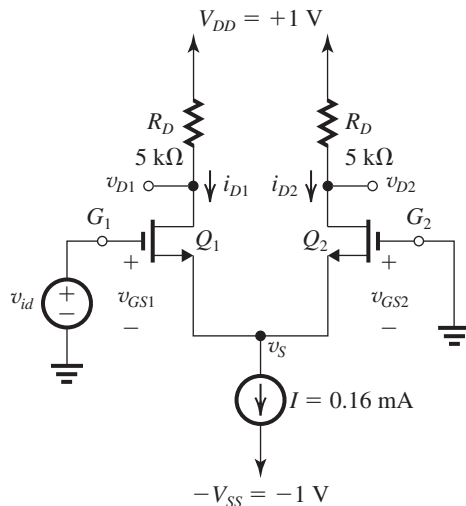
$$v_{D2} = -0.9 + 0.2 \times 4 = -0.1 \text{ V}$$

which verifies that  $Q_2$  is operating in saturation, as implicitly assumed.

Finally,

$$v_O = v_{D2} - v_{D1} = +0.8 \text{ V}$$

### 8.12



For  $i_{D1} = 0.09 \text{ mA}$  and  $i_{D2} = 0.07 \text{ mA}$ ,

$$i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS2} - V_{tn})^2$$

$$0.07 = \frac{1}{2} \times 0.4 \times 10 (v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.587 \text{ V}$$

and

$$v_S = -0.587 \text{ V}$$

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS1} - V_{tn})^2$$

$$0.09 = \frac{1}{2} \times 0.4 \times 10 (v_{GS1} - 0.4)^2$$

$$\Rightarrow v_{GS1} = 0.612 \text{ V}$$

$$v_{id} = v_S + v_{GS1} = -0.587 + 0.612$$

$$= 0.025 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2} R_D$$

$$= 1 - 0.07 \times 5 = 0.65 \text{ V}$$

$$v_{D1} = 1 - 0.09 \times 5 = 0.55 \text{ V}$$

$$v_O = v_{D2} - v_{D1} = 0.65 - 0.55 = 0.10 \text{ V}$$

$$\text{Voltage gain} = \frac{v_O}{v_{id}} = \frac{0.10}{0.025} = 4 \text{ V/V}$$

To obtain the complementary split in current, that is,  $i_{D1} = 0.07 \text{ mA}$  and  $i_{D2} = 0.09 \text{ mA}$ ,

$$v_{id} = -0.025 \text{ V}$$

**8.13** From Exercise 8.3 and the accompanying table, we note that  $|v_{id}|_{\max}$  is proportional to  $V_{OV}$ :

$$\frac{|v_{id}|_{\max}}{V_{OV}} = \frac{0.126}{0.2} = 0.63$$

Thus, to obtain  $|v_{id}|_{\max} = 220 \text{ mV} = 0.22 \text{ V}$  at the same level of linearity, we use

$$V_{OV} = \frac{0.22}{0.63} = 0.35 \text{ V}$$

For this value of  $V_{OV}$ , the required  $(W/L)$  can be found from

$$0.2 = \frac{1}{2} \times 0.2 \times \left( \frac{W}{L} \right) \times 0.35^2$$

$$\Rightarrow \frac{W}{L} = 16.3$$

The value of  $g_m$  is

$$g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 0.2}{0.35} = 1.14 \text{ mA/V}$$

**8.14** Refer to Eq. (8.23). For

$$\left( \frac{v_{id}/2}{V_{OV}} \right)^2 \leq k$$

$$\Rightarrow \left( \frac{v_{id}/2}{V_{OV}} \right) \leq \sqrt{k} \quad (1)$$

$$\Delta I = I \left( \frac{v_{id}/2}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2}$$

$$\Delta I_{\max} = I \sqrt{k} \sqrt{1 - k}$$

Thus,

$$\frac{\Delta I_{\max}}{I/2} = 2\sqrt{k(1-k)} \quad \text{Q.E.D.} \quad (2)$$

and the corresponding value of  $v_{id}$  is found from Eq. (1) as

$$v_{id\max} = 2\sqrt{k}V_{OV} \quad \text{Q.E.D.} \quad (3)$$

Equations (2) and (3) can be used to evaluate  $\frac{\Delta I_{\max}}{I/2}$  and  $\frac{v_{id\max}}{V_{OV}}$  for various values of  $k$ :

$k$	0.01	0.1	0.2
$\frac{v_{id\max}}{V_{OV}}$	0.2	0.632	0.894
$\frac{\Delta I_{\max}}{I/2}$	0.2	0.6	0.8

### 8.15 Switching occurs at

$$v_{id} = \sqrt{2}V_{OV}$$

Thus,

$$0.3 = \sqrt{2}V_{OV}$$

$$\Rightarrow V_{OV} = 0.212 \text{ V}$$

Now, to obtain full current switching at  $v_{id} = 0.5 \text{ V}$ ,  $V_{OV}$  must be increased to

$$V_{OV} = 0.212 \times \frac{0.5}{0.3} = 0.353 \text{ V}$$

Since  $I_D$  is proportional to  $V_{OV}^2$  the current  $I_D$  and hence the bias current  $I$  must be increased by the ratio  $(0.353/0.212)^2$ , then  $I$  must be

$$I = 200 \times \left( \frac{0.353}{0.212} \right)^2 = 554.5 \mu\text{A}$$

### 8.16 Equations (8.23) and (8.24):

$$i_{D1} = \frac{I}{2} + \frac{I}{2} \left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2} \quad (8.23)$$

$$i_{D2} = \frac{I}{2} - \frac{I}{2} \left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2} \quad (8.24)$$

(a) For 10% increase above the equilibrium value of  $\frac{I}{2}$ ,

$$\left( \frac{I}{2} \right) \left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2} = 0.1 \times \frac{I}{2}$$

$$\left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \frac{1}{4} \left( \frac{v_{id}}{V_{OV}} \right)^2} = 0.1$$

$$\Rightarrow \frac{v_{id}}{V_{OV}} \simeq 0.1$$

$$v_{id} \simeq 0.1V_{OV}$$

(b) In Eqs. (8.23) and (8.24) let

$$i_{D1} = \left( \frac{I}{2} \right) + \left( \frac{I}{2} \right) \times \Delta$$

$$i_{D2} = \left( \frac{I}{2} \right) - \left( \frac{I}{2} \right) \times \Delta$$

where

$$\Delta = \left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2}$$

If  $v_{id}$  is such that

$$\frac{i_{D1}}{i_{D2}} = m$$

then

$$m = \frac{1 + \Delta}{1 - \Delta}$$

$$\Rightarrow \Delta = \frac{m - 1}{m + 1}$$

For  $m = 1$ ,  $\Delta = 0$  and  $v_{id} = 0$

For  $m = 2$ ,

$$\Delta = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

$$\left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \frac{1}{4} \left( \frac{v_{id}}{V_{OV}} \right)^2} = \frac{1}{3}$$

Squaring both sides, we obtain a quadratic equation in  $\left( \frac{v_{id}}{V_{OV}} \right)^2$  which can be solved to obtain

$$v_{id} = 0.338V_{OV}$$

For  $m = 1.1$ ,

$$\Delta = \frac{1.1 - 1}{1.1 + 1} = \frac{0.1}{2.1} \simeq 0.05$$

Thus,

$$\left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \frac{1}{4} \left( \frac{v_{id}}{V_{OV}} \right)^2} = 0.05$$

$$\Rightarrow v_{id} \simeq 0.05V_{OV}$$

For  $m = 1.01$

$$\Delta = \frac{1.01 - 1}{1.01 + 1} \simeq 0.005$$

$$\left( \frac{v_{id}}{V_{OV}} \right) \sqrt{1 - \frac{1}{4} \left( \frac{v_{id}}{V_{OV}} \right)^2} = 0.005$$

$$v_{id} \simeq 0.005V_{OV}$$

For  $m = 20$ ,

$$\Delta = \frac{m-1}{m+1} = \frac{19}{21} = 0.905 \text{ V}$$

Thus,

$$\left(\frac{v_{id}}{V_{OV}}\right) \sqrt{1 - \frac{1}{4} \left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.905$$

$$\Rightarrow v_{id} = 1.072V_{OV}$$

$$\mathbf{8.17} \quad 0.1 = \frac{1}{2} \times 0.2 \times 32V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.18 \text{ V}$$

$$g_m = \frac{2 \times (0.2/2)}{0.18} = 1.11 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_m(R_D \parallel r_o)$$

$$= 1.11 \times (10 \parallel 100) = 10.1 \text{ V/V}$$

**8.18** To limit the power dissipation to 1 mW,

$$P = (V_{DD} + V_{SS})I$$

Thus, the maximum value we can use for  $I$  is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

Using this value, we obtain

$$V_D = V_{DD} - \frac{I}{2} R_D$$

$$0.2 = 1 - 0.25 \times R_D$$

$$\Rightarrow R_D = 3.2 \text{ k}\Omega$$

$$A_d = g_m R_D$$

$$10 = g_m \times 3.2$$

$$g_m = \frac{10}{3.2} = 3.125 \text{ mA/V}$$

But

$$g_m = \frac{2 \times (I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$3.125 = \frac{0.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

To obtain  $W/L$ , we use

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.16^2$$

$$\Rightarrow \frac{W}{L} = 48.8 \simeq 50$$

$$\mathbf{8.19} \quad (a) \quad A_d = g_m R_D$$

$$20 = g_m \times 47$$

$$\Rightarrow g_m = \frac{20}{47} = 0.426 \text{ mA/V}$$

$$(b) \quad g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$0.426 = \frac{I}{0.2}$$

$$\Rightarrow I = 0.085 \text{ mA} = 85 \mu\text{A}$$

(c) Across each  $R_D$  the dc voltage is

$$\frac{I}{2} R_D = \frac{0.085}{2} \times 47 = 2 \text{ V}$$

(d) The peak sine-wave signal of each gate source is 5 mV, thus at each drain the peak sine wave is

$$A_d \times 5 = 20 \times 5 = 100 \text{ mV} = 0.1 \text{ V}$$

(e) The minimum voltage at each drain will be

$$v_{D\min} = V_{DD} - R_D I_D - V_{\text{peak}}$$

$$= V_{DD} - 2 - 0.1$$

For the transistor to remain in saturation

$$v_{D\min} \geq v_{G\max} - V_m$$

where

$$v_{G\max} = V_{CM} + V_{\text{peak}}(\text{input})$$

$$= 0.5 + 0.005 = 0.505 \text{ V}$$

Thus,

$$V_{DD} - 2.1 \geq 0.505 - 0.5$$

$$V_{DD} \geq 2.105 \text{ V}$$

Thus, the lowest value of  $V_{DD}$  is 2.11 V.

**8.20** For a CS amplifier biased at a current  $I_D$  and utilizing a drain resistance  $R_D$ , the voltage gain is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I_D} R_D \quad (1)$$

For a differential pair biased with a current  $I$  and utilizing drain resistances  $R_D$ , the differential gain is

$$A_d = g_m R_D$$

where



$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} \left(\frac{I}{2}\right)}$$

Thus

$$A_d = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I/2} R_D \quad (2)$$

Equating the gains from Eqs. (1) and (2), we get

$$I = 2I_D$$

That is, the differential pair must be biased at a current twice that of the CS amplifier. Since both circuits use equal power supplies, the power dissipation of the differential pair will be twice that of the CS amplifier.

**8.21** Since both circuits use the same supply voltages and dissipate equal powers, their currents must be equal, that is,

$$I_D = I$$

where  $I_D$  is the bias current of the CS amplifier and  $I$  is the bias current of the differential pair. The gain of the CS amplifier is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{CS} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{CS} I_D} R_D \quad (1)$$

The gain of the differential amplifier is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right)}$$

Thus,

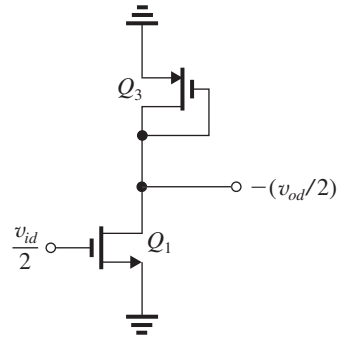
$$A_d = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right)} R_D \quad (2)$$

Equating the gains in Eqs. (1) and (2) and substituting  $I_D = I$  gives

$$\begin{aligned} \sqrt{\left(\frac{W}{L}\right)_{CS}} &= \sqrt{\left(\frac{W}{L}\right)_{\text{diff}}} \times \frac{1}{2} \\ \Rightarrow \left(\frac{W}{L}\right)_{\text{diff}} &= 2 \left(\frac{W}{L}\right)_{CS} \end{aligned}$$

If all transistors have the same channel length, each of the differential pair transistors must be twice as wide as the transistor in the CS amplifier.

## 8.22



(a) The figure shows the differential half-circuit. Recalling that the incremental (small-signal) resistance of a diode-connected transistor is given by  $\left(\frac{1}{g_m} \parallel r_o\right)$ , the equivalent load resistance of  $Q_1$  will be

$$R_D = \frac{1}{g_{m3}} \parallel r_{o3}$$

and the differential gain of the amplifier in Fig. P8.22 will be

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1} \left[ \frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \right]$$

Since both sides of the amplifier are matched, this expression can be written in a more general way as

$$A_d = g_{m1,2} \left[ \frac{1}{g_{m3,4}} \parallel r_{o3,4} \parallel r_{o1,2} \right]$$

(b) Neglecting  $r_{o1,2}$  and  $r_{o3,4}$  (much larger than  $1/g_{m3,4}$ ),

$$\begin{aligned} A_d &\simeq \frac{g_{m1,2}}{g_{m3,4}} \\ &= \frac{\sqrt{2\mu_n C_{ox} (W/L)_{1,2} (I/2)}}{\sqrt{2\mu_p C_{ox} (W/L)_{3,4} (I/2)}} \\ &= \sqrt{\frac{\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}} \end{aligned}$$

(c)  $\mu_n = 4\mu_p$  and all channel lengths are equal,

$$A_d = 2 \sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

For  $A_d = 10$ ,

$$10 = 2 \sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

$$\Rightarrow \frac{W_{1,2}}{W_{3,4}} = 25$$

**8.23** Refer to Fig. P8.23.

(a) With  $v_{G1} = v_{G2} = 0$ ,

$$v_{GS1} = v_{GS2} = V_{OV1,2} + V_m$$

Thus

$$V_{S1} = V_{S2} = -(V_{OV1,2} + V_m)$$

(b) For the situation in (a),  $V_{DS}$  of  $Q_3$  is zero, thus zero current flows in  $Q_3$ . Transistor  $Q_3$  will have an overdrive voltage of

$$\begin{aligned} V_{OV3} &= V_C - V_{S1,2} - V_m \\ &= V_C + (V_{OV1,2} + V_m) - V_m \\ &= V_C + V_{OV1,2} \end{aligned}$$

(c) With  $v_{G1} = v_{id}/2$  and  $v_{G2} = -v_{id}/2$  where  $v_{id}$  is a small signal, a small signal will appear between drain and source of  $Q_3$ . Transistor  $Q_3$  will be operating in the triode region and its drain-source resistance  $r_{DS}$  will be given by

$$r_{DS} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_3 V_{OV3}}$$

Thus,

$$R_s = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_3 V_{OV3}}$$

Now,

$$g_{m1,2} = (\mu_n C_{ox}) \left(\frac{W}{L}\right)_{1,2} V_{OV1,2}$$

$$g_{m3} = (\mu_n C_{ox}) \left(\frac{W}{L}\right)_3 V_{OV3}$$

$$\text{For } \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_{1,2},$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right) = \frac{g_{m1,2}}{V_{OV1,2}}$$

Thus,

$$R_s = \frac{1}{\frac{g_{m1,2}}{V_{OV1,2}} \times V_{OV3}} = \frac{1}{g_{m1,2}} \frac{V_{OV1,2}}{V_{OV3}}$$

$$(d) (i) R_s = \frac{1}{g_{m1,2}}$$

$$V_{OV3} = V_{OV1,2}$$

But

$$V_{OV3} = V_C + V_{OV1,2}$$

$$\Rightarrow V_C = 0$$

$$(ii) R_s = \frac{0.5}{g_{m1,2}}$$

$$\Rightarrow V_{OV3} = 2 V_{OV1,2}$$

But

$$V_{OV3} = V_C + V_{OV1,2}$$

$$\Rightarrow V_C = V_{OV1,2}$$

**8.24** Refer to Fig. P8.24.

(a) With  $v_{G1} = v_{G2} = 0$  V,

$$V_{S1} = V_{S2} = -V_{GS1,2} = -(V_t + V_{OV})$$

The current through  $Q_3$  and  $Q_4$  will be zero because the voltage across them ( $v_{DS3} + v_{DS4}$ ) is zero.

Because the voltages at their gates are zero and at their sources are  $-(V_t + V_{OV})$ , each of  $Q_3$  and  $Q_4$  will be operating at an overdrive voltage equal to  $V_{OV}$ . Thus each of  $Q_3$  and  $Q_4$  will have an  $r_{DS}$  given by

$$r_{DS3,4} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4} V_{OV}} \quad (1)$$

Since

$$g_{m1,2} = \mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} V_{OV} \quad (2)$$

substituting from (2) into (1) gives

$$r_{DS3,4} = \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}$$

and since

$$R_s = r_{DS3} + r_{DS4}$$

then

$$R_s = \frac{2}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}} \quad (3)$$

(b) With  $v_{G1} = v_{id}/2$  and  $v_{G2} = -v_{id}/2$  where  $v_{id}$  is a small signal,

$$\begin{aligned} A_d &\equiv \frac{v_{od}}{v_{id}} \\ &= \frac{2 R_D}{\frac{1}{g_{m1}} + R_s + \frac{1}{g_{m2}}} \end{aligned}$$

Using (3), we obtain

$$\begin{aligned} A_d &= \frac{R_D}{\frac{1}{g_{m1,2}} + \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}} \\ &= \frac{g_{m1,2} R_D}{1 + \frac{(W/L)_{1,2}}{(W/L)_{3,4}}} \end{aligned}$$

**8.25** Refer to Fig. P8.25.

(a) Since the dc voltages  $V_{GS1}$  and  $V_{GS2}$  are equal,  $Q_1$  and  $Q_2$  will be operating at the same value of

$V_{OV}$  and their dc currents  $I_{D1}$  and  $I_{D2}$  will have the same ratio at their  $(W/L)$  ratios, that is,

$$I_{D1} = I/3$$

$$I_{D2} = 2I/3$$

(b)  $Q_1$  and  $Q_2$  will be operating at the same  $V_{OV}$ , obtained as follows:

$$\frac{I}{3} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$\Rightarrow V_{OV} = \sqrt{\frac{2I}{3\mu_n C_{ox} \left( \frac{W}{L} \right)}}$$

$$(c) A_d \equiv \frac{v_{od}}{v_{id}}$$

$$= \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

where

$$g_{m1} = \frac{2 \times (I/3)}{V_{OV}} = \frac{2I}{3V_{OV}}$$

$$g_{m2} = \frac{2 \times (2I/3)}{V_{OV}} = \frac{4I}{3V_{OV}}$$

$$A_d = \frac{2R_D}{\left( \frac{3}{2} + \frac{3}{4} \right) (V_{OV}/I)} = \frac{8}{9} \frac{IR_D}{V_{OV}}$$

**8.26** Refer to Fig. 8.15(a).

The current  $I$  will split equally between  $Q_1$  and  $Q_2$ . Thus,

$$I_{E1} = I_{E2} = 0.2 \text{ mA}$$

$$I_{C1} = I_{C2} = \alpha \times 0.2 = 0.99 \times 0.2 = 0.198 \text{ mA}$$

$$V_{BE1} = V_{BE2} = 0.7 + 0.025 \ln\left(\frac{0.198}{1}\right)$$

$$= 0.660 \text{ V}$$

$$V_{E1} = V_{E2} = -1 - 0.66 = -1.66 \text{ V}$$

$$V_{C1} = V_{C2} = V_{CC} - I_{C1,2} R_C$$

$$= 2.5 - 0.198 \times 5 = +1.51 \text{ V}$$

**8.27** (a) Refer to Fig. 8.15(a).

$$I_{E1} = I_{E2} = \frac{I}{2} = 10 \mu\text{A}$$

$$I_{C1} = I_{C2} = \alpha \times 10 = 0.98 \times 10 = 9.8 \mu\text{A}$$

$$V_{BE1} = V_{BE2} = 0.690 + 0.025 \ln\left(\frac{9.8 \times 10^{-3}}{1}\right)$$

$$= 0.574 \text{ V}$$

Thus,

$$V_E = -0.574 \text{ V}$$

$$V_{C1} = V_{C2} = V_{CC} - I_C R_C$$

$$= 1.2 - 9.8 \times 10^{-3} \times 82$$

$$\simeq 0.4 \text{ V}$$

(b) Refer to Fig. 8.15(a).

The maximum value of  $V_{CM}$  is limited by the need to keep  $Q_1$  and  $Q_2$  in the active mode. This is achieved by keeping  $v_{CE1,2} \geq 0.3 \text{ V}$ .

Since  $V_{C1,2} = 0.4 \text{ V}$ ,

$$V_{E_{\max}} = 0.4 - 0.3 = 0.1 \text{ V}$$

and

$$V_{CM_{\max}} = V_{BE1,2} + V_{E_{\max}}$$

$$V_{CM_{\max}} = 0.574 + 0.1 = 0.674 \text{ V}$$

The minimum value of  $V_{CM}$  is dictated by the need to keep the current source operating properly, i.e., to keep 0.3 V across it, thus

$$V_{E_{\min}} = -1.2 + 0.3 = -0.9 \text{ V}$$

and

$$V_{CM_{\min}} = V_{E_{\min}} + V_{BE1,2}$$

$$= -0.9 + 0.574 = -0.326 \text{ V}$$

Thus, the input common-mode range is

$$-0.326 \text{ V} \leq V_{ICM} \leq +0.674 \text{ V}$$

(c) Refer to Fig. 8.15(d).

$$i_{E1} = 11 \mu\text{A}, \quad i_{E2} = 9 \mu\text{A}$$

$$i_{C1} = 10.78 \mu\text{A}, \quad i_{C2} = 8.82 \mu\text{A}$$

$$v_{BE1} = 0.69 + 0.025 \ln\left(\frac{10.78 \times 10^{-3}}{1}\right)$$

$$= 0.5767 \text{ V}$$

$$v_{BE2} = 0.69 + 0.025 \ln\left(\frac{8.82 \times 10^{-3}}{1}\right)$$

$$= 0.5717 \text{ V}$$

Thus,

$$v_{B1} = v_{BE1} - v_{BE2}$$

$$= 0.5767 - 0.5717 = 0.005 \text{ V}$$

$$= 5 \text{ mV}$$

**8.28** Require  $v_O = 1 \text{ V}$  when  $v_{id} = 10 \text{ mV}$  and  $I = 1 \text{ mA}$ .

Using Eq. (8.48), we obtain

$$i_{E1} = \frac{1 \text{ (mA)}}{1 + e^{-10/25}} = 0.599 \text{ mA}$$

$$i_{E2} = I - i_{E1} = 1 - 0.599 = 0.401 \text{ mA}$$

$$\begin{aligned} v_o &= v_{C2} - v_{C1} \\ &= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C) \\ &= (i_{C1} - i_{C2})R_C \\ &\simeq (i_{E1} - i_{E2})R_C \\ &= 0.198R_C \end{aligned}$$

For  $v_o = 1\text{ V}$ , we have

$$R_C = \frac{1}{0.198} = 5.05\text{ k}\Omega$$

$$\begin{aligned} V_{C1} &= V_{C2} = V_{CC} - \frac{I}{2}R_C \\ &= 5 - 0.5 \times 5.05 \simeq 2.5\text{ V} \end{aligned}$$

With a signal of  $10\text{ mV}$  applied, the voltage at one collector rises to  $3\text{ V}$  and at the other falls to  $2\text{ V}$ . To ensure that the transistors remain in the active region, the maximum common-mode input voltage must be limited to  $(2 + 0.4) = +2.4\text{ V}$ .

$$8.29 \quad g_m = \frac{I_C}{V_T} = \frac{\alpha \times 0.2}{0.025} \simeq 8\text{ mA/V}$$

$$R_{id} = 2r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{160}{8} = 40\text{ k}\Omega$$

$$8.30 \quad R_{id} = 2r_\pi = 20\text{ k}\Omega$$

$$r_\pi = 10\text{ k}\Omega$$

$$\frac{\beta}{g_m} = 10\text{ k}\Omega$$

$$\frac{100}{g_m} = 10$$

$$\Rightarrow g_m = 10\text{ mA/V}$$

$$A_d = 100 = g_m R_C$$

$$R_C = \frac{100}{g_m} = \frac{100}{10} = 10\text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{I/2}{V_T}$$

$$\Rightarrow I = 2V_T g_m$$

$$= 2 \times 0.025 \times 10 = 0.5\text{ mA}$$

$$8.31 \quad v_{id} = 10\text{ mA/V}$$

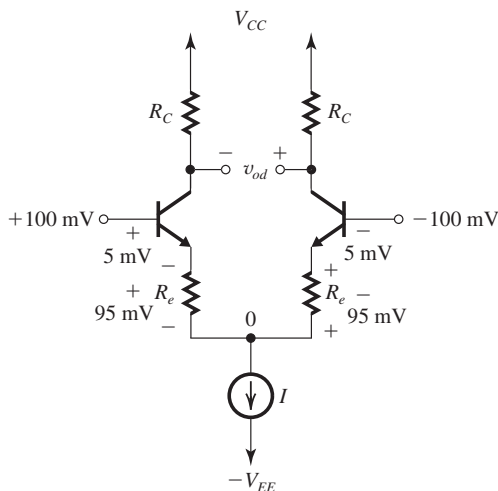
Input signal to half-circuit =  $5\text{ mV}$ . For  $I = 200\text{ }\mu\text{A}$ , the bias current of the half-circuit is  $100\text{ }\mu\text{A}$  and,

$$r_e = \frac{25\text{ mV}}{0.1\text{ mA}} = 250\text{ }\Omega$$

$$\begin{aligned} \text{Gain of half-circuit} &= -\frac{R_C}{r_e} = -\frac{10}{0.25} \\ &= -40\text{ V/V} \end{aligned}$$

At each collector we expect a signal of  $40 \times 5\text{ mV} = 200\text{ mV}$ . Between the two collectors, the signal will be  $400\text{ mV}$ .

### 8.32



$v_{id} = 200\text{ mV}$  appears across  $(2r_e + 2R_e)$ . Thus the signal across  $(r_e + R_e)$  is  $100\text{ mV}$ . Since the signal across  $r_e$  is  $5\text{ mV}$ , it follows that the signal across  $R_e$  must be  $100 - 5 = 95\text{ mV}$  and thus

$$R_e = 19r_e$$

(See signal distribution on the figure above.)

The input resistance  $R_{id}$  is

$$\begin{aligned} R_{id} &= (\beta + 1)(2r_e + 2R_e) \\ &= 2(100 + 1)(r_e + R_e) \\ &= 2 \times 101 \times (r_e + 19r_e) \\ &= 2 \times 101 \times 20r_e \end{aligned}$$

To obtain  $R_{id} = 400\text{ k}\Omega$ ,

$$\begin{aligned} 400 &= 2 \times 101 \times 20 \times r_e \\ \Rightarrow r_e &\simeq 100\text{ }\Omega \end{aligned}$$

Since

$$r_e = \frac{V_T}{I_E},$$

$$100 = \frac{25\text{ mV}}{I_E}$$

$$\Rightarrow I_E = 0.25\text{ mA}$$

$$I = 0.5\text{ mA}$$

$$R_e = 19r_e = 19 \times 0.1 = 1.9\text{ k}\Omega$$

$$\text{Gain} = \frac{\alpha \times 2R_C}{2r_e + 2R_e}$$

$$\approx \frac{R_C}{r_e + R_e}$$

But the gain required is

$$\text{Gain} = \frac{v_{od}}{v_{id}} = \frac{2 \text{ V}}{0.2 \text{ V}} = 10 \text{ V/V}$$

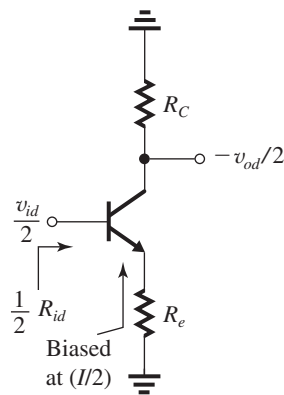
Thus,

$$10 = \frac{R_C}{0.1 + 1.9}$$

$$\Rightarrow R_C = 20 \text{ k}\Omega$$

The determination of a suitable value of  $V_{CC}$  requires information on the required input common-mode range (which is not specified). Suffice it to say that the dc voltage drop across  $R_C$  is 5 V and that each collector swings  $\pm 1$  V. A supply voltage  $V_{CC} = 10$  V will certainly be sufficient.

8.33



Both circuits have the same differential half-circuit shown in the figure. Thus, for both

$$A_d = \frac{\alpha R_C}{r_e + R_e}$$

$$R_{id} = (\beta + 1)(2r_e + 2R_e) = 2(\beta + 1)(r_e + R_e)$$

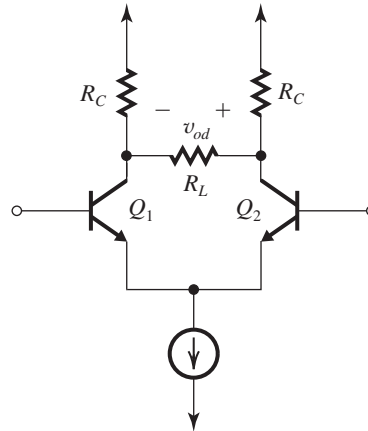
With  $v_{id} = 0$ , the dc voltage appearing at the top end of the bias current source will be

(a)  $V_{CM} - V_{BE} - \left(\frac{I}{2}\right)R_e$

(b)  $V_{CM} - V_{BE}$

Since circuit (b) results in a larger voltage across the current source and given that the minimum value of  $V_{CM}$  is limited by the need to keep a certain specified minimum voltage across the current source, we see that circuit (b) will allow a larger negative  $V_{CM}$ .

8.34



$$A_d = \alpha \frac{\text{Total resistance between collectors}}{\text{Total resistance in the emitter circuit}} = \alpha \frac{(2R_C \parallel R_L)}{2r_e}$$

8.35  $R_{id} = 2r_\pi = 2 \frac{\beta}{g_m}$

$$g_m = \frac{I_C}{V_T} \approx \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$R_{id} = \frac{2 \times 100}{8} = 25 \text{ k}\Omega$$

$$G_v = \frac{R_{id}}{R_{id} + R_{sig}} \frac{\alpha(2R_C \parallel R_L)}{2r_e}$$

$$G_v = \frac{R_{id}}{R_{id} + R_{sig}} \times \frac{1}{2} g_m(2R_C \parallel R_L)$$

$$= \frac{25}{25 + 100} \times \frac{1}{2} \times 8 \times (40 \parallel 40)$$

$$= 16 \text{ V/V}$$

8.36 Refer to Fig. P8.36.

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25 \text{ k}\Omega}{2r_e + 500 \Omega}$$

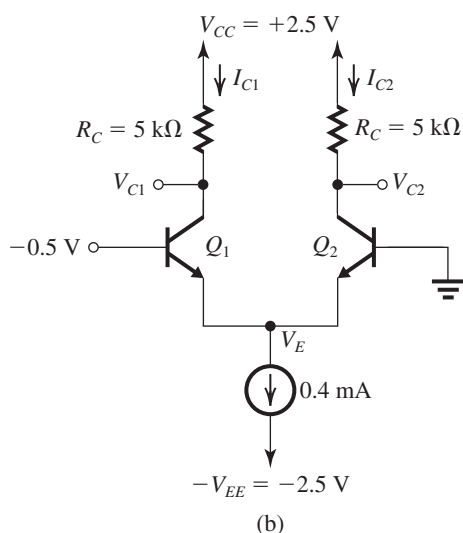
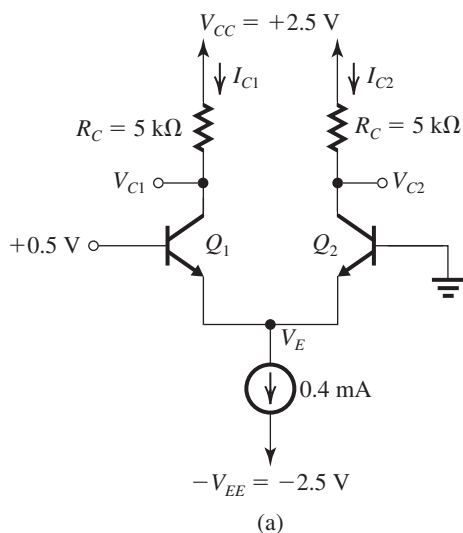
$$= \frac{0.99 \times 25 \text{ k}\Omega}{500 \Omega + 500 \Omega} \approx 25 \text{ V/V}$$

$$R_{in} = (\beta + 1)(2r_e + 500 \Omega)$$

$$= 101 \times (2 \times 250 \Omega + 500 \Omega)$$

$$= 101 \text{ k}\Omega$$

8.37



(a) For  $v_{B1} = +0.5$  V,  $Q_1$  conducts all the current  $I$  (0.4 mA) while  $Q_2$  cuts off. Thus  $Q_1$  will have a  $V_{BE}$  obtained as follows:

$$V_{BE1} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.4}{1}\right)$$

$$= 0.677 \text{ V}$$

Thus,

$$V_E = +0.5 - 0.677 = -0.177 \text{ V}$$

which indicates that  $V_{BE2} = +0.177$  V, too small to turn  $Q_2$  on.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0.99 \times 0.4 \times 5$$

$$= +0.52 \text{ V}$$

$$V_{C2} = V_{CC} - I_{C2} \times R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

Observe that  $Q_1$  is operating in the active mode, as implicitly assumed, and the current source has a voltage of 2.323 V across it, more than sufficient for its proper operation.

(b) With  $v_{B1} = -0.5$  V,  $Q_1$  turns off and  $Q_2$  conducts all the bias current (0.4 mA) and thus exhibits a  $V_{BE}$  of 0.677 V, thus

$$V_E = -0.677 \text{ V}$$

which indicated that  $V_{BE1} = +0.177$  V, which is too small to turn  $Q_1$  on. Also, note that the current source has a voltage of  $-0.677 + 2.5 = 1.823$  V across it, more than sufficient for its proper operation.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

$$V_{C2} = 2.5 - 0.99 \times 0.4 \times 5 = +0.52 \text{ V}$$

**8.38** Refer to Fig. 8.15(a) and assume the current source  $I$  is implemented with a single BJT that requires a minimum of 0.3 V for proper operation. Thus, the minimum voltage allowed at the emitters of  $Q_1$  and  $Q_2$  is  $-2.5 \text{ V} + 0.3 \text{ V} = -2.2 \text{ V}$ . Now, since each of  $Q_1$  and  $Q_2$  is conducting a current of 0.2 mA, their  $V_{BE}$  voltages will be equal:

$$V_{BE1,2} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.2}{1}\right)$$

$$= 0.660 \text{ V}$$

Thus, the minimum allowable  $V_{CM}$  is

$$V_{CM\min} = -2.2 + 0.660 = -1.54 \text{ V}$$

The upper limit on  $V_{CM}$  is dictated by the need to keep  $Q_1$  and  $Q_2$  operating in the active mode, thus

$$V_{CM\max} = 0.4 + V_{C1,2}$$

$$= 0.4 + (2.5 - 0.99 \times 0.2 \times 5)$$

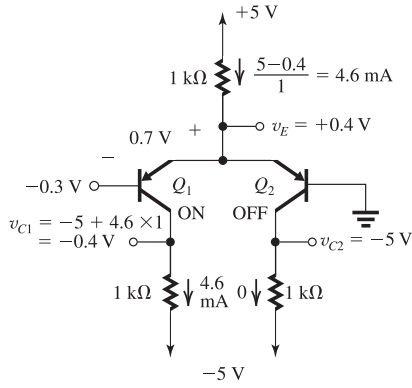
$$= +1.91 \text{ V}$$

Thus, the input common-mode range is

$$-1.54 \text{ V} \leq V_{ICM} \leq 1.91 \text{ V}$$

**8.39**

The solution is given on the circuit diagram on the next page.



**8.40** Refer to Fig. 8.15(a) with  $V_{CC}$  replaced by  $(V_{CC} + v_r)$ .

$$v_{C1} = (V_{CC} + v_r) - \alpha \frac{I}{2} R_C$$

$$= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r$$

$$v_{C2} = (V_{CC} + v_r) - \alpha \frac{I}{2} R_C$$

$$= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r$$

$$v_{od} \equiv v_{C2} - v_{C1} = 0$$

Thus, while  $v_{C1}$  and  $v_{C2}$  will include a ripple component  $v_r$ , the difference output voltage  $v_{od}$  will be ripple free. Thus, the differential amplifier rejects the undesirable supply ripple.

**8.41** Refer to Fig. 8.14.

(a)  $V_{CMmax} = V_{CC} - \frac{I}{2} R_C$

(b) For  $V_{CC} = 2\text{ V}$  and  $V_{CMmax} = 1\text{ V}$ ,

$$1 = 2 - \frac{1}{2} (I R_C)$$

$$\Rightarrow I R_C = 2\text{ V}$$

(c)  $I_B = \frac{I/2}{\beta + 1} \leq 2\text{ }\mu\text{A}$

$$I \leq 2 \times 101 \times 2 = 404\text{ }\mu\text{A}$$

Select

$$I = 0.4\text{ mA}$$

then

$$R_C = \frac{2}{0.4} = 5\text{ k}\Omega$$

This table belongs to Problem 8.42.

$v_{id}(\text{mV})$	2	5	8	10	20	30	40
$\left[ \frac{\Delta i_{E1}}{I} / v_{id} \right] (\text{V}^{-1})$	9.99	9.97	9.92	9.87	9.50	8.95	8.30

**8.42**  $\frac{\Delta i_{E1}}{I} = \frac{i_{E1} - (I/2)}{I}$   
 $= \frac{i_{E1}}{I} - 0.5$

Using Eq. (8.34), we obtain

$$\frac{\Delta i_{E1}}{I} = \frac{1}{1 + e^{-v_{id}/V_T}} - 0.5$$

We use this equation to obtain the results in the table below. Observe that for  $v_{id} < 10\text{ mV}$  the proportional transconductance gain is nearly constant at about 10. The gain decreases as  $v_{id}$  further increases, indicating nonlinear operation. This is especially pronounced for  $v_{id} > 20\text{ mV}$ .

**8.43** Refer to Fig. 8.14.

$$v_o = v_{C2} - v_{C1}$$

$$= (V_{CC} - i_{C2} R_C) - (V_{CC} - i_{C1} R_C)$$

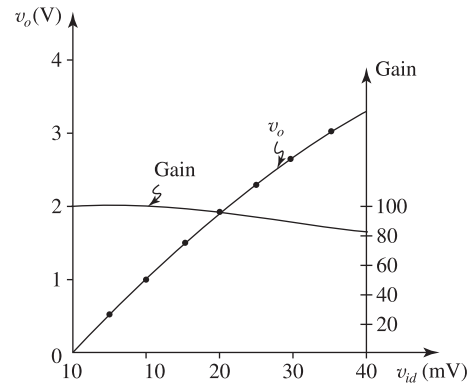
$$= R_C (i_{C1} - i_{C2})$$

Using Eqs. (8.48) and (8.49) and assuming  $\alpha \approx 1$ , so that  $i_{C1} \approx i_{E1}$  and  $i_{C2} \approx i_{E2}$ , we get

$$v_o = I R_C \left[ \frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}} \right]$$

$$= 5 \left[ \frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}} \right]$$

This relationship can be used to obtain the data in the table on the next page



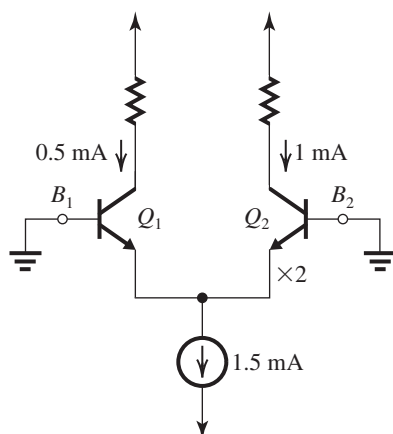
The figure shows  $v_o$  versus  $v_{id}$  and the gain versus  $v_{id}$ . Observe that the transfer characteristic is nearly linear and the gain is nearly constant for  $v_{id} \leq 10\text{ mV}$ . As  $v_{id}$  increases, the transfer

This table belongs to Problem 8.43.

$v_{id}$ (mV)	2	5	10	15	20	25	30	35	40
$v_o$ (V)	0.2	0.498	0.987	1.457	1.90	2.311	2.685	3.022	3.320
Gain = $\frac{v_{od}}{v_{id}}$	100	99.7	98.7	97.1	95.0	92.4	89.5	86.3	83.0

characteristic bends and the gain is reduced. However, for  $v_{id}$  even as large as 20 mV, the gain is only 5% below its ideal value of 100.

#### 8.44



Since  $Q_2$  has twice the EBJ area of  $Q_1$ , the 1.5-mA bias current will split in the same ratio, that is,

$$i_{E2} = 2 i_{E1}$$

Thus,

$$i_{E2} = 1 \text{ mA} \quad \text{and} \quad i_{E1} = 0.5 \text{ mA}$$

To equalize the collector currents, we apply a signal

$$v_{id} = v_{B1} - v_{B2}$$

Now,

$$i_{C1} = \frac{I_S}{\alpha} e^{(v_{B1} - v_E)/V_T} \quad (1)$$

$$i_{C2} = \frac{2I_S}{\alpha} e^{(v_{B2} - v_E)/V_T} \quad (2)$$

where we have denoted the scale current of  $Q_1$  by  $I_S$  and that of  $Q_2$  as  $2I_S$ . Dividing (1) by (2), we get

$$\frac{i_{C1}}{i_{C2}} = \frac{1}{2} e^{(v_{B1} - v_{B2})/V_T}$$

For  $i_{C1} = i_{C2}$ , we obtain

$$v_{B1} - v_{B2} = V_T \ln 2$$

$$= 25 \ln 2 = 17.3 \text{ mV}$$

$$8.45 \text{ (a)} \quad V_{BE} = 0.69 + 0.025 \ln\left(\frac{0.1}{1}\right)$$

$$= 0.632 \text{ V}$$

(b) Using Eq. (8.50), we obtain

$$i_{C1} = \alpha i_{E1} \approx \frac{I}{1 + e^{-v_{id}/V_T}}$$

For  $v_{id} = 20 \text{ mV}$ ,

$$i_{C1} = \frac{200 \mu\text{A}}{1 + e^{-20/25}} = 138 \mu\text{A}$$

$$i_{C2} = 200 - 138 = 62 \mu\text{A}$$

(c) For  $v_{id} = 200 \text{ mV}$  while  $i_{C1} = 138 \mu\text{A}$  and  $i_{C2} = 62 \mu\text{A}$ : Since  $i_{C1}$  and  $i_{C2}$  have not changed,  $v_{BE1}$  and  $v_{BE2}$  also would not change. Thus,

$$v_{B1} - v_{B2} = v_{BE1} + i_{E1}R_e - i_{E2}R_e - v_{BE2}$$

$$= (v_{BE1} - v_{BE2}) + R_e (i_{E1} - i_{E2})$$

$$200 = 20 + R_e (i_{C1} - i_{C2})$$

$$= 20 + R_e (138 - 62)$$

$$\Rightarrow R_e = \frac{180 \text{ mV}}{76 \mu\text{A}} = 2.37 \text{ k}\Omega$$

(d) Without  $R_e$ ,

$$v_{id} = 20 \text{ mV} \rightarrow i_{C1} - i_{C2} = 76 \mu\text{A}$$

$$G_m = \frac{76 \mu\text{A}}{20 \text{ mV}} = 3.8 \text{ mA/V}$$

With  $R_e$ ,

$$v_{id} = 200 \text{ mV} \rightarrow i_{C1} - i_{C2} = 76 \mu\text{A}$$

$$G_m = \frac{76 \mu\text{A}}{200 \text{ mV}} = 0.38 \text{ mA/V}$$

Thus, the effective  $G_m$  has been reduced by a factor of 10, which is the same factor by which the allowable input signal has been increased while maintaining the same linearity.

$$8.46 \text{ (a)} \quad r_e = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \Omega$$

The 0.2-V differential input signal appears across  $(2r_e + 2R_e)$ , thus

$$i_e = \frac{200 \text{ mV}}{200 + 2 \times 400} = 0.2 \text{ mA}$$

$$v_{be} = 0.2 \times 100 = 20 \text{ mV}$$



(b) The total emitter current in one transistor is

$$\frac{I}{2} + i_e = 0.45 \text{ mA and in the other transistor}$$

$$\frac{I}{2} - i_e = 0.05 \text{ mA.}$$

(c) At one collector the signal voltage is  $-\alpha i_e R_C \simeq -i_e R_C = -0.2 \times 10 = -2 \text{ V}$  and at the other collector the signal voltage is  $+2 \text{ V}$ .

(d) Voltage gain  $= \frac{4 \text{ V}}{0.2 \text{ V}} = 20 \text{ V/V}$

**8.47** (a) The maximum allowable value of the bias current  $I$  is found as

$$I = \frac{P}{(V_{CC} + V_{EE})} = \frac{1 \text{ mW}}{5 \text{ V}} = 0.2 \text{ mA}$$

We choose to operate at this value of  $I$ . Thus

$$g_m = \frac{I_C}{V_T} = \frac{\alpha(0.2/2)}{0.025} \simeq 4 \text{ mA/V}$$

$$A_d = g_m R_C$$

$$60 = 4 \times R_C$$

$$\Rightarrow R_C = 15 \text{ k}\Omega$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 2.5 - \frac{0.2}{2} \times 15$$

$$= +1 \text{ V}$$

(b)  $R_{id} = 2r_{\pi} = 2 \frac{\beta}{g_m}$

$$= 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

(c)  $v_{od} = A_d \times v_{id}$

$$= 60 \times 10 = 600 \text{ mV} = 0.6 \text{ V}$$

Thus, there will be  $\pm 0.3 \text{ V}$  signal swing at each collector. That is, the voltage at each collector will range between  $0.7 \text{ V}$  and  $+1.3 \text{ V}$ .

(d) To maintain the BJT in the active mode at all times, the maximum allowable  $V_{CM}$  is limited to

$$V_{CM\max} = 0.4 + v_{C\min}$$

$$= 0.4 + 0.7 = 1.1 \text{ V}$$

**8.48** (a) Consider transistor  $Q_1$ ,

$$v_{C1\min} = (V_{CC} - \frac{I}{2} R_C) - A_d \left( \frac{\hat{v}_{id}}{2} \right) \quad (1)$$

where

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C$$

$$= \frac{I R_C}{2 V_T}$$

Thus,

$$\frac{I R_C}{2} = A_d V_T \quad (2)$$

Substituting from (2) into (1), we obtain

$$v_{C1\min} = V_{CC} - A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right) \quad (3)$$

Since

$$v_{B1} = V_{CM\max} + \frac{\hat{v}_{id}}{2}$$

to keep  $Q_1$  in the active mode,

$$v_{B1} \leq 0.4 + v_{C1\min}$$

Thus,

$$V_{CM\max} + \frac{\hat{v}_{id}}{2} = 0.4 + V_{CC} - A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right)$$

$$\Rightarrow V_{CM\max} = V_{CC} + 0.4 - \frac{\hat{v}_{id}}{2} -$$

$$A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right) \quad \text{Q.E.D.} \quad (4)$$

(b)  $V_{CC} = 2.5 \text{ V}, \quad \hat{v}_{id} = 10 \text{ mV},$

$$A_d = 50 \text{ V/V},$$

$$V_{CM\max} = 2.5 + 0.4 - 0.005 - 50(2.5 + 5) \times 10^{-3} \\ \simeq 1.4 \text{ V}$$

$$\hat{v}_{od} = A_d \times \hat{v}_{id} = 50 \times 10 = 500 \text{ mV}$$

$$= 0.5 \text{ V}$$

Using Eq. (2), we obtain

$$I R_C = 2 A_d V_T = 2 \times 50 \times 0.025$$

$$= 2.5 \text{ V}$$

To limit the power dissipation in the quiescent state to  $1 \text{ mW}$ , the bias current must be limited to

$$I = \frac{P_{\max}}{V_{CC} + V_{EE}} = \frac{1}{5} = 0.2 \text{ mA}$$

Using this value for  $I$ , we get

$$R_C = \frac{2.5}{0.2} = 12.5 \text{ k}\Omega$$

(c) To obtain  $V_{CM\max} = 1 \text{ V}$ , we use Eq. (4) to determine the allowable value of  $A_d$ ,

$$1 = 2.5 + 0.4 - 0.005 - A_d(2.5 + 5) \times 10^{-3}$$

$$\Rightarrow A_d = 63.2 \text{ V/V}$$

Thus, by reducing  $V_{CM\max}$  from  $1.4 \text{ V}$  to  $1 \text{ V}$ , we are able to increase the differential gain from  $50 \text{ V/V}$  to  $63.2 \text{ V/V}$ .

**8.49**  $A_d = g_m R_C$

$$= \frac{I_C}{V_T} R_C$$

$$\simeq \frac{(I/2)}{V_T} R_C$$

$$= \frac{IR_C}{2V_T}$$

$$= \frac{4}{2 \times 0.025} = 80 \text{ V/V}$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2}R_C$$

$$= 5 - 2 = 3 \text{ V}$$

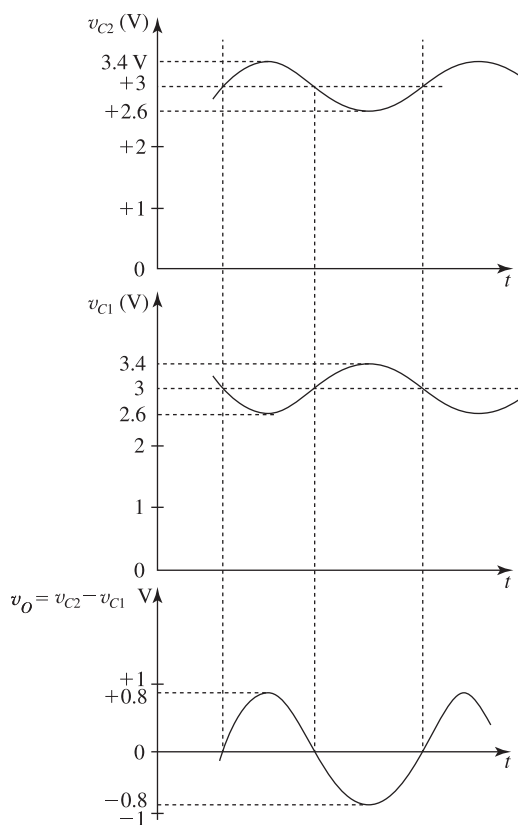
$$v_{C1} = 3 - 80 \times 0.005 \sin(\omega t)$$

$$= 3 - 0.4 \sin(\omega t)$$

$$v_{C2} = 3 + 0.4 \sin(\omega t)$$

$$v_O = v_{C2} - v_{C1} = 0.8 \sin(\omega t)$$

The waveforms are sketched in the figure below.



**8.50** See figure on next page. The circuit together with its equivalent half-circuit is shown in the figure.

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

For

$$r_{o1,2} = r_{o3,4} = \frac{V_A}{\alpha(I/2)} \approx \frac{2V_A}{I}$$

$$g_{m1,2} = \frac{I_{C1,2}}{V_T} \approx \frac{I}{2V_T}$$

$$A_d = \frac{I}{2V_T} \left( \frac{2V_A}{I} \parallel \frac{2V_A}{I} \right)$$

$$= \frac{I}{2V_T} \times \frac{V_A}{I} = \frac{V_A}{2V_T}$$

$$= \frac{20}{2 \times 0.025} = 400 \text{ V/V}$$

**8.51** Refer to Fig. P8.51(a).

$$\frac{I}{2}R_e = 3V_T$$

$$\Rightarrow R_e = \frac{6V_T}{I} \quad (1)$$

$$\alpha \left( \frac{I}{2} \right) R_C = 80V_T$$

$$R_C = \frac{160V_T}{\alpha I} \quad (2)$$

$$A_d = \alpha \frac{\text{Total resistance in collector circuit}}{\text{Total resistance in emitter circuit}}$$

$$A_d = \alpha \frac{2R_C}{2r_e + 2R_e} = \alpha \frac{R_C}{r_e + R_e}$$

Substituting for  $R_C$  from (2), for  $R_e$  from (1), and for  $r_e = V_T/(I/2)$ , we obtain

$$A_d = \frac{\alpha(160V_T/\alpha I)}{(2V_T/I) + (6V_T/I)}$$

$$= \frac{160}{2 + 6} = 20 \text{ V/V}$$

**8.52** See figure on next page.

$$\frac{v_{id}}{v_{sig}} = \frac{R_{id}}{R_{id} + R_{sig}} \quad (1)$$

where

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

thus,

$$\frac{v_{id}}{v_{sig}} = \frac{2(\beta + 1)(r_e + R_e)}{2(\beta + 1)(r_e + R_e) + R_{sig}} \quad (2)$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha \times \text{Total resistance between collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{2\alpha R_C}{2r_e + 2R_e}$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha R_C}{r_e + R_e} \quad (3)$$

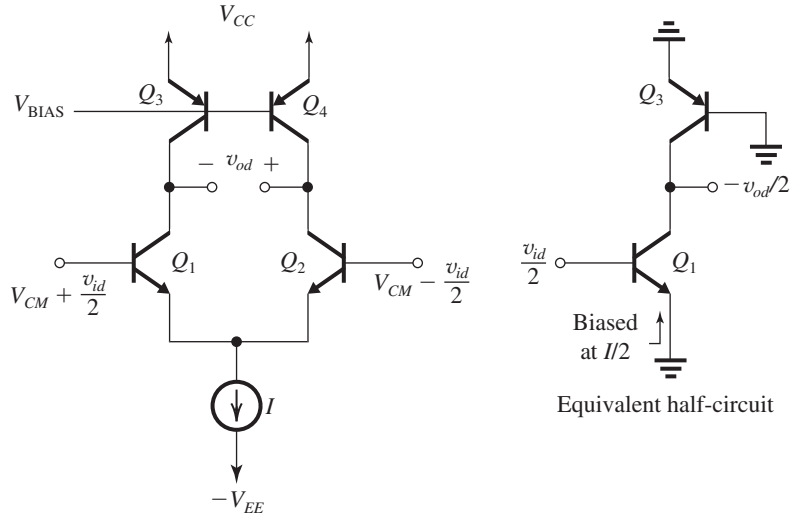
Using (2) and (3), we get

$$G_v \equiv \frac{v_{od}}{v_{sig}} = \frac{2\alpha(\beta + 1)R_C}{2(\beta + 1)(r_e + R_e) + R_{sig}}$$

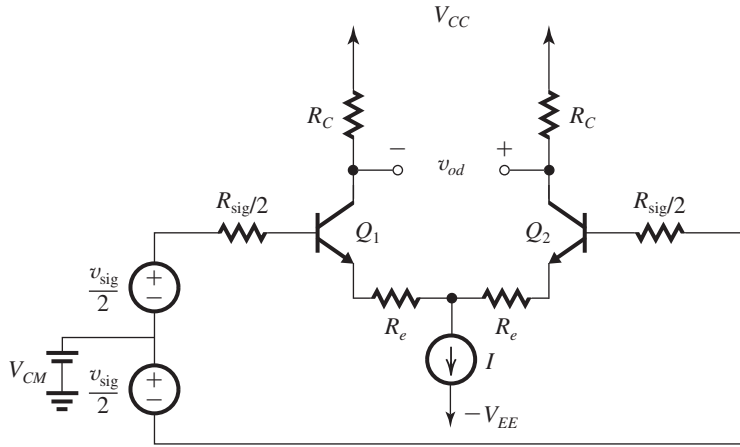
Since  $\alpha = \frac{\beta}{\beta + 1}$ ,  $\alpha(\beta + 1) = \beta$ , we have

$$G_v = \frac{2\beta R_C}{2(\beta + 1)(r_e + R_e) + R_{sig}} \quad (4)$$

This figure belongs to Problem 8.50.



This figure belongs to Problem 8.52.



If  $v_{id} = 0.5 v_{sig}$ , then from (1) we obtain

$$R_{id} = R_{sig}$$

Substituting for  $R_{sig} = R_{id} = 2(\beta + 1)(r_e + R_e)$  into Eq. (4) gives

$$G_v = \frac{2\beta R_C}{4(\beta + 1)(r_e + R_e)} = \frac{1}{2} \frac{\alpha R_C}{r_e + R_e} \quad (5)$$

If  $\beta$  is doubled to  $2\beta$  while  $R_{sig}$  remains at its old value, we get

$$R_{sig} = 2(\beta + 1)(r_e + R_e) \quad (6)$$

then the new value of  $G_v$  is obtained by replacing  $\beta$  by  $2\beta$  in Eq. (4) and substituting for  $R_{sig}$  from (6):

$$G_v = \frac{4\beta R_C}{2(2\beta + 1)(r_e + R_e) + 2(\beta + 1)(r_e + R_e)} \\ \simeq \frac{4R_C}{6(r_e + R_e)} = \frac{2}{3} \frac{R_C}{r_e + R_e}$$

Thus the gain increases from approximately

$$\frac{1}{2} \frac{R_C}{(r_e + R_e)} \text{ to } \frac{2}{3} \frac{R_C}{(r_e + R_e)}.$$

**8.53** Refer to Fig. P8.53.

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 20}{2r_e + 2 \times 0.25}$$

where

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

Thus,

$$\frac{v_o}{v_i} = \frac{0.99 \times 20}{2 \times 0.25 + 2 \times 0.25} \simeq 20 \text{ V/V}$$

$$R_{in} = (\beta + 1)(2r_e + 2R_e)$$

$$= 2 \times 101 \times (0.25 + 0.25)$$

$$= 101 \text{ k}\Omega$$

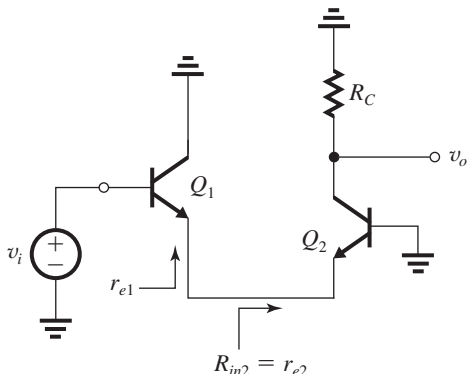
**8.54** (a) Refer to the circuit in Fig. P8.54. As a differential amplifier, the voltage gain is found from

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times R_C}{2r_e}$$

$$= \frac{\alpha R_C}{2r_e}$$

(b) The circuit in Fig. P8.54 can be considered as the cascade connection of an emitter follower  $Q_1$  (biased at an emitter current  $I/2$ ) and a common-gate amplifier  $Q_2$  (also biased at an emitter current of  $I/2$ ). Referring to the figure below:



$$\frac{v_{e1,2}}{v_i} = \frac{r_{e2}}{r_{e1} + r_{e2}} = \frac{1}{2}$$

$$\frac{v_o}{v_{e1,2}} = \frac{\alpha R_C}{r_{e2}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{\alpha R_C}{r_{e2}} = \frac{\alpha R_C}{2r_e}$$

which is identical to the expression found in (a) above.

**8.55**  $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$

$$= \sqrt{2 \times 5 \times 0.1} = 1 \text{ mA/V}$$

$$|A_d| = g_m R_D = 1 \times 12 = 12 \text{ V/V}$$

$$|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{12}{2 \times 100} \times 0.01 = 6 \times 10^{-4} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 2 \times 10^4 \text{ or } 86 \text{ dB}$$

**8.56**  $|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \frac{\Delta(W/L)}{W/L}$

$$|A_d| = g_m R_D$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 2g_m R_{SS} \left/ \frac{\Delta(W/L)}{W/L} \right.$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.1/2)}{0.2} = 0.5 \text{ mA/V}$$

For CMRR of 80 dB, the CMRR is  $10^4$ ; thus

$$10^4 = 2 \times 0.5 \times R_{SS}/0.02$$

$$R_{SS} = 200 \text{ k}\Omega$$

For the current source transistor to have

$$r_o = 200 \text{ k}\Omega,$$

$$200 = \frac{V'_A \times L}{0.1 \text{ mA}}$$

$$L = \frac{200 \times 0.1}{5} = 4 \mu\text{m}$$

**8.57** Refer to Fig. P8.57.

(a)  $\frac{v_o}{v_{id}} = \frac{\alpha \times \text{Total resistance across which } v_o \text{ appears}}{\text{Total resistance in the emitter}}$

$$= \alpha \times \frac{2 \text{ k}\Omega}{r_{e1} + r_{e2}}$$

To determine  $r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E}$ , where  $I_E$  is the dc emitter current of each of  $Q_1$  and  $Q_2$ , we use

$$V_E = V_B - V_{BE} = 0 - 0.7$$

$$= -0.7 \text{ V}$$

$$2I_E = \frac{-0.7 - (-5)}{4.3} = 1 \text{ mA}$$

$$I_E = 0.5 \text{ mA}$$

$$r_{e1} = r_{e2} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

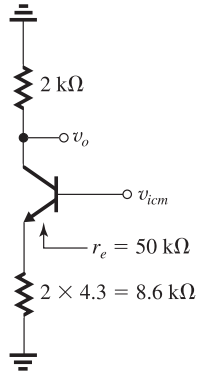
$$\frac{v_o}{v_{id}} = \alpha \times \frac{2 \text{ k}\Omega}{0.1 \text{ k}\Omega} \simeq 20 \text{ V/V}$$

(b)

The common-mode half-circuit is shown in the figure,

$$\frac{v_o}{v_{icm}} = - \frac{\alpha \times 2 \text{ k}\Omega}{(0.05 + 8.6) \text{ k}\Omega}$$

$$\simeq -0.23 \text{ V/V}$$



$$\left| \frac{v_o}{v_{icm}} \right| = 0.23 \text{ V/V}$$

$$(c) \text{ CMRR} = \frac{|v_o/v_{id}|}{|v_o/v_{icm}|} = \frac{20}{0.23} = 86.5$$

or 38.7 dB

$$(d) v_o = -0.023 \sin 2\pi \times 60t + 0.2 \sin 2\pi \times 1000 t \text{ volts}$$

$$8.58 (a) g_m = \frac{I_C}{V_T} \simeq \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$A_d = g_m R_C = 4 \times 25 = 100 \text{ V/V}$$

$$(b) R_{id} = 2r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

$$(c) |A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \frac{25}{2 \times 500} \times 0.01$$

$$= 2.5 \times 10^{-4} \text{ V/V}$$

$$(d) \text{ CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{2.5 \times 10^{-4}} = 4 \times 10^5$$

or 112 dB

$$(e) r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 500 \frac{1 + (25/(100 \times 1000))}{1 + \frac{25 + 1000}{1000}}$$

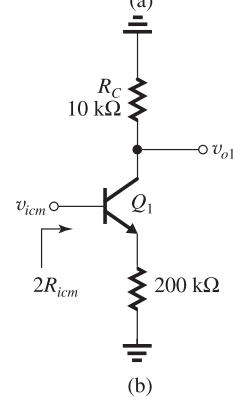
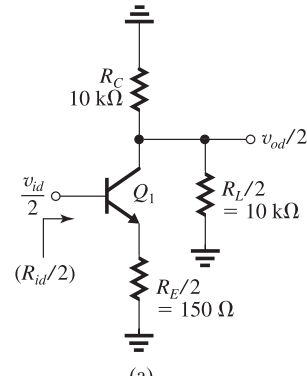
$$\simeq 25 \text{ M}\Omega$$

**8.59**

Figure (a) shows the differential half-circuit.

$$I_E = 0.5 \text{ mA}, \quad I_C = \alpha I_E \simeq 0.5 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$



$$r_e = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$A_d = \frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$\simeq \frac{10 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{(50 + 150) \Omega}$$

$$= \frac{5}{0.2} = 25 \text{ V/V}$$

We have neglected  $r_o$  because its equivalent value at the output will be  $r_o[1 + (R_e/r_e)] = 200[1 + (150/50)] = 800 \text{ k}\Omega$  which is much greater than the effective load resistance of  $5 \text{ k}\Omega$ .

$$R_{id} = 2 \times (\beta + 1)(50 \Omega + 150 \Omega)$$

$$= 2 \times 101 \times 0.2 \text{ (k}\Omega) = 40.4 \text{ k}\Omega$$

$$|A_{cm}| \simeq \left( \frac{R_C}{2R_{SS}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$|A_{cm}| = \frac{10}{200} \times 0.02 = 0.001 \text{ V/V}$$

To obtain  $R_{icm}$ , we use Eq. (8.98):

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

where  $2R_{EE} = 200 \text{ k}\Omega$ , thus  $R_{EE} = 100 \text{ k}\Omega$  and

$$R_{icm} = 100 \times 100 \frac{1 + (10/(100 \times 200))}{1 + \frac{10 + 200}{200}}$$

$$= 4.88 \text{ M}\Omega$$

**8.60** Refer to Fig. P8.2.

$$I_D = 0.25 \text{ mA} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV}|^2$$

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.353 \text{ V}$$

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.25}{0.353} = 1.416 \text{ mA/V}$$

$$|A_d| = g_m R_D = 1.416 \times 4 = 5.67 \text{ V/V}$$

$$|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{4}{2 \times 30} \times 0.02$$

$$= 1.33 \times 10^{-3} \text{ V/V}$$

$$\text{CMRR} = 4252.5 \text{ or } 72.6 \text{ dB}$$

**8.61** Refer to Fig. P8.61.

(a) Assume  $v_{id} = 0$  and the two sides of the differential amplifier are matched. Thus,

$$I_{D1} = I_{D2} = 0.01 \text{ mA}$$

$$I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.01 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

$$V_{CM} = V_{GS} + 0.02 \text{ mA} \times R_{SS}$$

$$= V_t + V_{OV} + 0.02 \times R_{SS}$$

$$= 0.4 + 0.14 + 0.4$$

$$= 0.94 \text{ V}$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.01}{0.14} = 0.14 \text{ mA/V}$$

$$A_d = g_m R_D$$

$$15 = 0.14 \times R_D$$

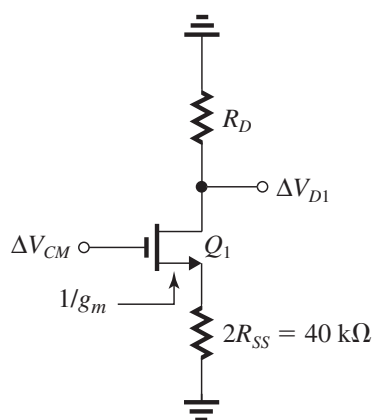
$$\Rightarrow R_D = 107 \text{ k}\Omega$$

$$(c) V_{D1} = V_{D2} = V_{DD} - I_D R_D$$

$$= 2 - 0.01 \times 107 = 0.93 \text{ V}$$

(d)

The figure shows the common-mode half-circuit,



$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = - \frac{R_D}{\frac{1}{g_m} + 2R_{SS}}$$

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = - \frac{107}{\frac{1}{0.14} + 40} = -2.26 \text{ V/V}$$

(e) For  $Q_1$  and  $Q_2$  to enter the triode region

$$V_{CM} + \Delta V_{CM} = V_t + V_{D1} + \Delta V_{D1}$$

Substituting  $V_{CM} = 0.94$ ,  $V_t = 0.4$  V,  $V_{D1} = 0.93$  V, and  $\Delta V_{D1} = -2.26\Delta V_{CM}$  results in

$$0.94 + \Delta V_{CM} = 0.4 + 0.93 - 2.26\Delta V_{CM}$$

$$\Rightarrow \Delta V_{CM} = 0.12 \text{ V}$$

With this change,  $V_{CM} = 1.06$  V and  $V_{D1,2} = 0.66$  V; thus  $V_{CM} = V_t + V_{D1,2}$ .

**8.62** The new deliberate mismatch  $\Delta R_D/R_D$  cancels the two existing mismatch terms in the expression for  $A_{cm}$  given in the problem statement so as to reduce  $A_{cm}$  to zero. Thus,

$$\frac{R_D}{2R_{SS}} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\frac{5}{2 \times 25} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = -0.02 \text{ or } -2\%$$

(Note the sign of the change is usually determined experimentally.)

**8.63** It is required to raise the CMRR by 40 dB, that is, by a factor of 100. Thus, the cascoding of the bias current source must raise its output resistance  $R_{SS}$  by a factor of 100. Thus the cascode transistor must have  $A_0 = 100$ . Since

$$A_0 = g_m r_o = \frac{2I}{V_{OV}} \frac{V_A}{I} = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2V_A}{0.2}$$

$$\Rightarrow V_A = 10 \text{ V}$$

$$V_A = V'_A \times L$$

$$10 = 5 \times L$$

$$\Rightarrow L = 2 \text{ } \mu\text{m}$$

$$\mathbf{8.64} \quad R_{EE} = \frac{V_A}{I} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

For the transistors in the differential pair, we have

$$r_o = \frac{V_A}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

For  $R_C \ll r_o$ ,

$$R_{icm} \simeq \beta R_{EE} \left/ \left( 1 + \frac{2R_{EE}}{r_o} \right) \right.$$

$$= \frac{50 \times 100}{1 + \frac{2 \times 100}{200}} = 2.5 \text{ M}\Omega$$

**8.65** For the differential-pair transistors, we have

$$I_C \simeq 0.1 \text{ mA}$$

$$g_m = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$(a) \quad A_d = g_m R_C = 4 \times 10 = 40 \text{ V/V}$$

where we have neglected the effect of  $r_o$  since  $r_o \gg R_C$ .

(b) If the bias current is realized using a simple current source,

$$R_{EE} = r_o|_{\text{current source}} = \frac{V_A}{I} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$|A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \left( \frac{10}{2 \times 100} \right) \times 0.1$$

$$= 5 \times 10^{-3} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{40}{5 \times 10^{-3}} = 8 \times 10^3$$

or 78 dB

(c) If the bias current  $I$  is generated using a Wilson mirror,

$$R_{EE} = R_o|_{\text{Wilson mirror}}$$

$$= \frac{1}{2} \beta r_o$$

where  $r_o$  is that of the transistors in the Wilson mirror, then

$$r_o = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$R_{EE} = \frac{1}{2} \times 100 \times 100 = 5 \text{ M}\Omega$$

$$|A_{cm}| = \left( \frac{10}{2 \times 5,000} \right) \times 0.1$$

$$= 1 \times 10^{-4} \text{ V/V}$$

$$\text{CMRR} = \frac{40}{1 \times 10^{-4}} = 4 \times 10^5$$

or 112 dB

**8.66** See figure on next page.

$$v_{be1} = 2.5 \sin(\omega t), \text{ mV and}$$

$$v_{be2} = -2.5 \sin(\omega t), \text{ mV}$$

$$v_{c1} \simeq V_{CC} - \left( \frac{I}{2} \right) R_C - g_m R_C \times 2.5 \times 10^{-3} \sin(\omega t)$$

where

$$g_m = \frac{I/2}{V_T} = \frac{I \text{ mA}}{0.05 \text{ V}}$$

Thus,

$$v_{c1} = 5 - \frac{I}{2} \times 10 - \frac{I}{0.05} \times 10 \times 2.5 \times 10^{-3} \sin(\omega t)$$

$$= 5 - 5I - 0.5I \sin(\omega t) \quad (\text{V})$$

Similarly,

$$v_{c2} = 5 - 5I + 0.5I \sin(\omega t) \quad (\text{V})$$

To ensure operation in the active mode at all times with  $v_{CB} = 0 \text{ V}$ , we use

$$v_{c1\text{min}} = 0.005$$

$$5 - 5.5I = 0.005$$

$$\Rightarrow I \simeq 0.9 \text{ mA}$$

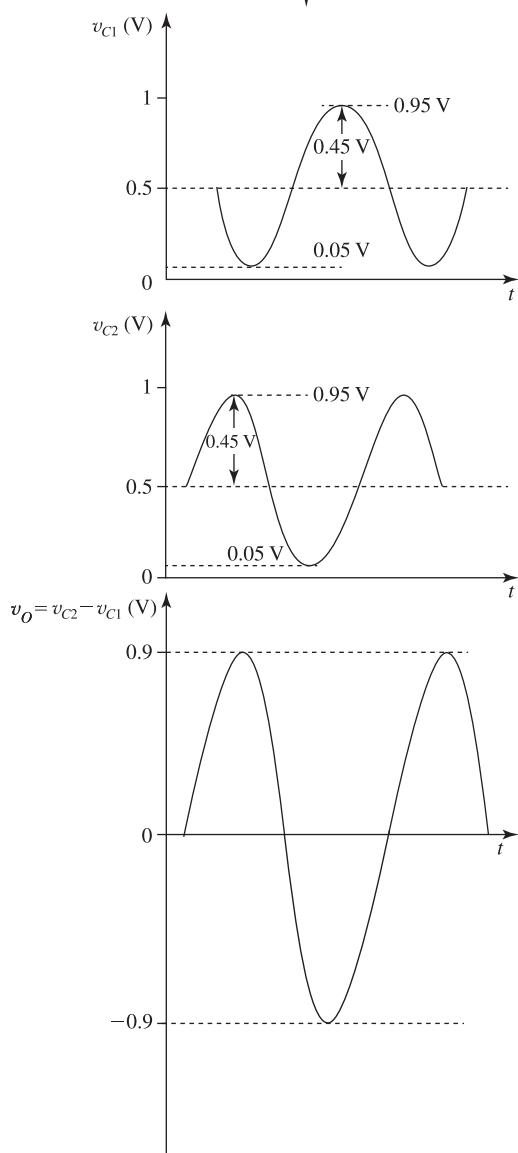
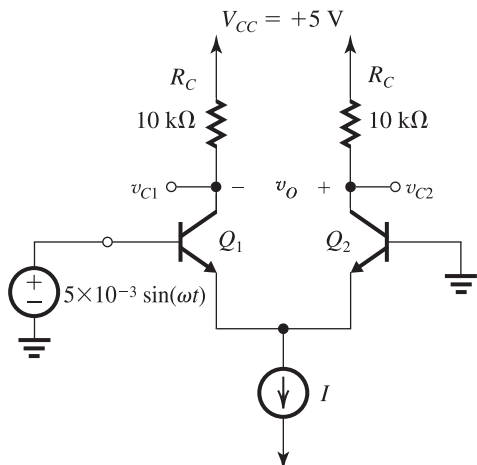
With this value of bias current, we obtain

$$g_m = \frac{0.9}{0.05} = 18 \text{ mA/V}$$

$$A_d = g_m R_C = 18 \times 10 = 180 \text{ V/V}$$

At each collector there will be a sine wave of  $180 \times 2.5 = 450 \text{ mV} = 0.45 \text{ V}$  amplitude superimposed on the dc bias voltage of  $5 - 0.45 \times 10 = 0.5 \text{ V}$ . Between the two collectors there will be a sine wave with 0.9 V peak amplitude. The figure illustrates the waveforms obtained.

This figure belongs to Problem 8.66.



8.67  $\frac{v_{o1}}{v_{id}} = -100 \text{ V/V}$   $\frac{v_{o2}}{v_{id}} = +100 \text{ V/V}$

$\frac{v_{o1,2}}{v_{icm}} = -0.1 \text{ V/V}$

$R_{id} = 10 \text{ k}\Omega$

$I = 2 \text{ mA}$

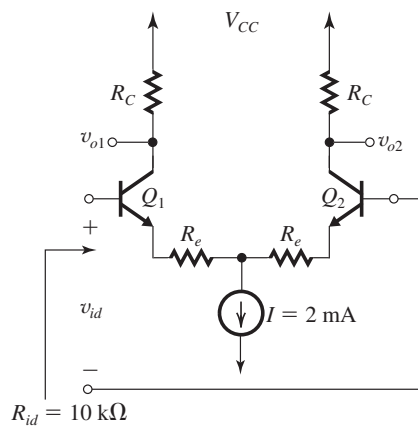
$I_{E1} = I_{E2} = 1 \text{ mA}$

$r_{e1} = r_{e2} = 25 \Omega$

$g_{m1} = g_{m2} = 40 \text{ mA/V}$

$r_{\pi1} = r_{\pi2} = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$

Since  $R_{id} > r_{\pi}$ , we need emitter resistances, as shown in the figure.



$10 = (\beta + 1)(2r_e + 2R_e)$

$r_e + R_e = \frac{10}{2 \times 101} \simeq 50 \Omega$

$R_e = 25 \Omega$

$\frac{v_{o1}}{v_{id}} = -\frac{\alpha R_C}{2(r_e + R_e)}$

$-100 = \frac{-\alpha R_C}{2(0.025 + 0.025)}$

$\Rightarrow R_C \simeq 10 \text{ k}\Omega$

To allow for  $\pm 2 \text{ V}$  swing at each collector,

$V_{CC} - \frac{I}{2}R_C - 2 \geq 0$

assuming that  $V_{CM} = 0 \text{ V}$ . Thus,

$V_{CC} = \frac{2}{2} \times 10 + 2 = 12 \text{ V}$

We can use  $V_{CC} = 15 \text{ V}$  to allow for  $V_{ICM}$  as high as  $+3 \text{ V}$ .

$|A_{cm}| \text{ (to each collector)} \simeq \frac{R_C}{2R_{EE}}$

For  $|A_{cm}| = 0.1$ ,

$0.1 = \frac{10}{2R_{EE}}$

$\Rightarrow R_{EE} = 50 \text{ k}\Omega$



This is the minimum value of  $R_o$  of the bias current source. If the current source is realized by a simple current mirror, we obtain

$$R_{EE} = r_o = \frac{V_A}{I}$$

Thus,

$$50 = \frac{V_A}{2}$$

$$\Rightarrow V_A = 100 \text{ V}$$

The common-mode input resistance is

$$R_{icm} \simeq \beta R_{EE} \frac{1 + R_C/\beta r_o}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

where  $r_o$  is the output resistance of each of  $Q_1$  and  $Q_2$ ,

$$r_o = \frac{V_A}{I/2} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$R_{icm} = 100 \times 50 \frac{1 + (10/(100 \times 100))}{1 + \frac{10 + 100}{100}}$$

$$= 2.4 \text{ M}\Omega$$

**8.68** If the output is taken single-endedly, then

$$|A_{cm}| = \frac{R_C}{2R_{EE}}$$

$$|A_d| = \frac{1}{2} g_m R_C$$

$$\text{CMRR}_s = \frac{|A_{cm}|}{|A_d|} = g_m R_{EE}$$

If the output is taken differentially, then

$$|A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$|A_d| = g_m R_C$$

$$\text{CMRR}_d = 2g_m R_{EE} / \left( \frac{\Delta R_C}{R_C} \right)$$

Thus,

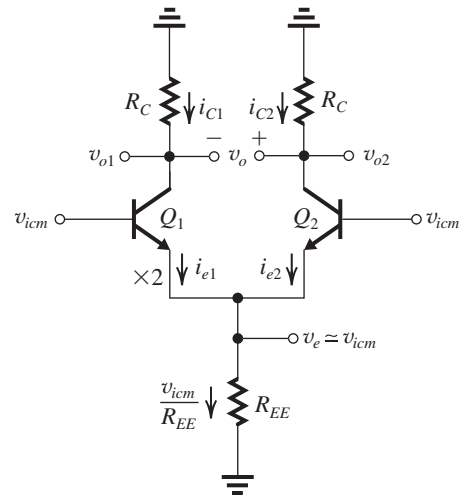
$$\frac{\text{CMRR}_d}{\text{CMRR}_s} = \frac{2}{\Delta R_C/R_C}$$

$$20 \log \frac{2}{\Delta R_C/R_C} = 46 \text{ dB}$$

$$\Rightarrow \frac{\Delta R_C}{R_C} = 0.01 = 1\%$$

**8.69** If  $Q_1$  has twice the base-emitter junction area of  $Q_2$ , the bias current  $I$  will split  $\frac{2}{3} I$  in  $Q_1$  and  $\frac{1}{3} I$  in  $Q_2$ . This is because with  $B_1$  and  $B_2$  grounded the two transistors will have equal

$V_{BE}$ 's. Thus their currents must be related by the ratio of their scale currents  $I_S$ , which are proportional to the junction areas.



With a common-mode input signal  $v_{icm}$  applied, as shown in the figure, the current  $(v_{icm}/R_{EE})$  will split between  $Q_1$  and  $Q_2$  in the same ratio as that of their base-emitter junction areas, thus

$$i_{e1} = \frac{2}{3} \frac{v_{icm}}{R_{EE}}$$

and

$$i_{e2} = \frac{1}{3} \frac{v_{icm}}{R_{EE}}$$

Thus,

$$v_{o1} = -i_{c1} R_C \simeq -i_{e1} R_C = -\frac{2}{3} \frac{R_C}{R_{EE}} v_{icm}$$

and

$$v_{o2} = -\frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

With the output taken differentially, we have

$$v_o = v_{o2} - v_{o1} = \frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

$$A_{cm} = \frac{1}{3} \frac{R_C}{R_{EE}} = \frac{1}{3} \times \frac{15}{400} = 0.0125 \text{ V/V}$$

**8.70** (a)  $g_m = \sqrt{2k_n I_D} = \sqrt{k_n I}$

$$A_d = g_m R_D = \sqrt{k_n I} R_D \tag{1}$$

$$V_{OV} = \sqrt{\frac{I/2}{\frac{1}{2} k_n}} = \sqrt{\frac{I}{k_n}}$$

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

Thus,

$$V_{OS} = \frac{1}{2} \sqrt{I/k_n} \left( \frac{\Delta R_D}{R_D} \right) \quad (2)$$

Equations (1) and (2) can be combined to obtain the following relationship between  $A_d$  and  $V_{OS}$ :

$$A_d = \left( \frac{2k_n R_D}{\Delta R_D / R_D} \right) V_{OS} \quad (3)$$

For the numerical values given, Eq. (3) becomes

$$A_d = 4 \times 10^3 V_{OS} \quad (4)$$

(b) For each value of  $V_{OS}$  we use Eq. (2) to determine  $I$  and then Eq. (4) to determine  $A_d$ . The results are as follows:

$V_{OS}$ (mV)	1	2	3	4	5
$I$ (mA)	0.04	0.16	0.36	0.64	1.00
$A_d$ (V/V)	4	8	12	16	20

We observe that by accepting a larger offset we are able to obtain a higher gain. Observe that the gain realized is proportional to the offset voltage one is willing to accept. Also, observe that the bias current required increases by a factor equal to the square of the increase in gain.

**8.71** The offset voltage due to  $\Delta V_T$  is

$$V_{OS} = \pm 5 \text{ mV}$$

The offset voltage due to  $\Delta R_D$  is

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) = \frac{0.3}{2} \times 0.02 = 3 \text{ mV}$$

The offset voltage due to  $\Delta(W/L)$  is

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \frac{\Delta(W/L)}{(W/L)} = \frac{0.3}{2} \times 0.02 = 3 \text{ mV}$$

The worst-case offset voltage will be when all three components add up,

$$V_{OS} = 5 + 3 + 3 = 11 \text{ mV}$$

The major contribution to the total is the variability of  $V_T$ .

To compensate for a total offset of 11 mV by appropriately varying  $R_D$ , we need to change  $R_D$  by  $\Delta R_D$  obtained from

$$11 \text{ mV} = \left( \frac{V_{OV}}{2} \right) \times \frac{\Delta R_D}{R_D}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{11 \times 2}{300} = 0.0733$$

or 7.33%

$$\mathbf{8.72} \quad V_{OS} = V_T \left( \frac{\Delta R_C}{R_C} \right)$$

$$= 25 \times 0.1 = 2.5 \text{ mV}$$

$$\mathbf{8.73} \quad V_{OS} = V_T \left( \frac{\Delta I_S}{I_S} \right)$$

$$= 25 \times 0.1 = 2.5 \text{ mV}$$

**8.74** With a resistance  $R_E$  connected in the emitter of each transistor, we follow a development similar to that used for the MOSFET and the BJT differential amplifiers except that we replace  $g_m$  by  $G_m = \frac{\alpha}{r_e + R_E}$ . Thus,

$$V_{OS} = \left( \frac{\alpha I / 2}{G_m} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \frac{I}{2} (r_e + R_E) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \frac{I}{2} \left( \frac{V_T}{I/2} + R_E \right) \left( \frac{\Delta R_C}{R_C} \right)$$

which yields,

$$V_{OS} = \left( V_T + \frac{1}{2} I R_E \right) \left( \frac{\Delta R_C}{R_C} \right)$$

**8.75** Refer to Fig. P8.75.

$$(a) \quad R_{C1} = 1.04 \times 5 = 5.20 \text{ k}\Omega$$

$$R_{C2} = 0.96 \times 5 = 4.80 \text{ k}\Omega$$

To equalize the total resistance in each collector, we adjust the potentiometer so that

$$R_{C1} + x \times 1 \text{ k}\Omega = R_{C2} + (1 - x) \times 1 \text{ k}\Omega$$

$$5.2 + x = 4.8 + 1 - x$$

$$\Rightarrow x = 0.3 \text{ k}\Omega$$

(b) If the area of  $Q_1$  and hence  $I_{S1}$  is 5% larger than nominal, then we have

$$I_{S1} = 1.05 I_S$$

and the area of  $Q_2$  and hence  $I_{S2}$  is 5% smaller than nominal,

$$I_{S2} = 0.95 I_S$$

Thus,

$$I_{E1} = 0.5 \times 1.05 = 0.525 \text{ mA}$$

$$I_{E2} = 0.5 \times 0.95 = 0.475 \text{ mA}$$

Assuming  $\alpha \simeq 1$ , we obtain

$$I_{C1} = 0.525 \text{ mA} \quad I_{C2} = 0.475 \text{ mA}$$

To reduce the resulting offset to zero, we adjust the potentiometer so that

$$V_{C1} = V_{C2}$$

$$\begin{aligned} \Rightarrow V_{CC} - (R_{C1} + x)I_{C1} &= V_{CC} - (R_{C2} + 1 - x)I_{C2} \\ I_{C1}(R_{C1} + x) &= I_{C2}(R_{C2} + 1 - x) \\ 0.525(5 + x) &= 0.475(5 + 1 - x) \\ \Rightarrow x &= 0.225 \end{aligned}$$

$$\begin{aligned} \mathbf{8.76} \quad I_{B\max} &= \frac{400}{2 \times 81} \simeq 2.5 \mu\text{A} \\ I_{B\min} &= \frac{400}{2 \times 201} = 1 \mu\text{A} \\ I_{OS\max} &= \frac{200}{81} - \frac{200}{201} \simeq 1.5 \mu\text{A} \end{aligned}$$

$$\begin{aligned} \mathbf{8.77} \quad g_m &= \sqrt{2 k'_n(W/L)I_D} \\ &= \sqrt{k'_n(W/L)I} \\ A_d &= g_m R_D \\ V_{OV} &= \frac{2 I_D}{g_m} = \frac{I}{g_m} \\ V_{OS} &= \left(\frac{V_{OV}}{2}\right) \left(\frac{\Delta R_D}{R_D}\right) \end{aligned}$$

For  $I = 160 \mu\text{A}$ , we have

$$\begin{aligned} g_m &= \sqrt{4 \times 0.16} = 0.8 \text{ mA/V} \\ A_d &= 0.8 \times 10 = 8 \text{ V/V} \\ V_{OV} &= \frac{0.16}{0.8} = 0.2 \text{ V} \\ V_{OS} &= \frac{0.2}{2} \times 0.02 = 2 \text{ mV} \end{aligned}$$

For  $I = 360 \mu\text{A}$ , we have

$$\begin{aligned} g_m &= \sqrt{4 \times 0.36} = 1.2 \text{ mA/V} \\ A_d &= 1.2 \times 10 = 12 \text{ V/V} \\ V_{OV} &= \frac{0.36}{1.2} = 0.3 \text{ V} \\ V_{OS} &= \frac{0.3}{2} \times 0.02 = 3 \text{ mV} \end{aligned}$$

Thus by increasing the bias current, both the gain and the offset voltage increase, and by the same factor (1.5).

$$\begin{aligned} \mathbf{8.78} \quad V_{OV} &= \sqrt{\frac{I/2}{\frac{1}{2}k'_n(W/L)}} = \sqrt{\frac{I}{k'_n(W/L)}} \\ &= \sqrt{\frac{0.1}{0.2 \times 10}} = 0.224 \text{ V} \\ \frac{\Delta R_D}{R_D} = 0.04 &\Rightarrow V_{OS} = \left(\frac{V_{OV}}{2}\right) \left(\frac{\Delta R_D}{R_D}\right) \\ &= \frac{0.224}{2} \times 0.04 = 4.5 \text{ mV} \end{aligned}$$

$$\begin{aligned} \frac{\Delta(W/L)}{(W/L)} = 0.04 &\Rightarrow V_{OS} = \left(\frac{V_{OV}}{2}\right) \left(\frac{\Delta(W/L)}{(W/L)}\right) \\ &= \frac{0.224}{2} \times 0.04 = 4.5 \text{ mV} \end{aligned}$$

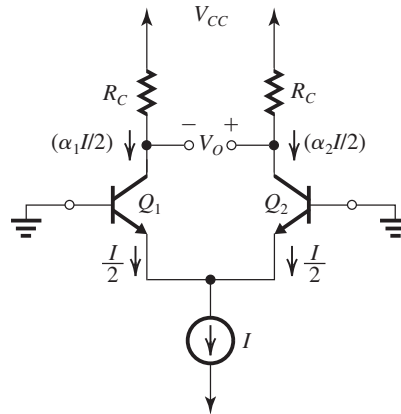
$$\Delta V_t = 5 \text{ mV} \Rightarrow V_{OS} = \Delta V_t = 5 \text{ mV}$$

$$\text{Worst-case } V_{OS} = 4.5 + 4.5 + 5 = 14 \text{ mV}$$

If the three components are independent,

$$V_{OS} = \sqrt{4.5^2 + 4.5^2 + 5^2} = 8.1 \text{ mV}$$

**8.79**



The current  $I$  splits equally between the two emitters. However, the unequal  $\beta$ 's will mean unequal  $\alpha$ 's. Thus, the two collector currents will be unequal,

$$I_{C1} = \alpha_1 I/2$$

$$I_{C2} = \alpha_2 I/2$$

and the collector voltages will be unequal,

$$V_{C1} = V_{CC} - \alpha_1 (I/2) R_C$$

$$V_{C2} = V_{CC} - \alpha_2 (I/2) R_C$$

Thus a differential output voltage  $V_O$  develops:

$$V_O = V_{C2} - V_{C1}$$

$$= \frac{1}{2} I R_C (\alpha_1 - \alpha_2)$$

The input offset voltage  $V_{OS}$  can be obtained by dividing  $V_O$  by the differential gain  $A_d$ :

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C = \frac{I R_C}{2 V_T}$$

Thus,

$$V_{OS} = V_T (\alpha_1 - \alpha_2)$$

Substituting, we obtain

$$\alpha_1 = \frac{\beta_1}{\beta_1 + 1}$$

and

$$\alpha_2 = \frac{\beta_2}{\beta_2 + 1}$$

$$V_{OS} = V_T \left( \frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1} \right)$$

$$= V_T \frac{\beta_1 \beta_2 + \beta_1 - \beta_1 \beta_2 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$= V_T \frac{\beta_1 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$\simeq V_T \frac{\beta_1 - \beta_2}{\beta_1 \beta_2}$$

$$= V_T \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right) \quad \text{Q.E.D.}$$

For  $\beta_1 = 50$  and  $\beta_2 = 100$ , we have

$$V_{OS} = 25 \left( \frac{1}{100} - \frac{1}{50} \right) = -0.25 \text{ mV}$$

**8.80** For the MOS amplifier:

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{200}{2} \times 0.04$$

$$= 4 \text{ mV}$$

For the BJT amplifier:

$$V_{OS} = V_T \left( \frac{\Delta R_C}{R_C} \right)$$

$$= 25 \times 0.04 = 1 \text{ mV}$$

If in the MOS amplifier the width of each device is increased by a factor of 4 while the bias current is kept constant,  $V_{OV}$  will be reduced by a factor of 2. Thus  $V_{OS}$  becomes

$$V_{OS} = 2 \text{ mV}$$

**8.81** Since the only difference between the two sides of the differential pair is the mismatch in  $V_A$ , we can write

$$I_{C1} = I_C \left( 1 + \frac{V_{CE1}}{V_{A1}} \right)$$

$$I_{C2} = I_C \left( 1 + \frac{V_{CE2}}{V_{A2}} \right)$$

$$I_{C1} + I_{C2} = \alpha I$$

$$I_C \left( 2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}} \right) = \alpha I$$

$$\Rightarrow I_C = \alpha I / \left( 2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}} \right)$$

$$I_{C1} = \frac{\alpha I}{2} \frac{1 + \frac{V_{CE1}}{V_{A1}}}{1 + \frac{V_{CE1}}{2V_{A1}} + \frac{V_{CE2}}{2V_{A2}}}$$

For  $\frac{V_{CE1}}{V_{A1}} \ll 1$  and  $\frac{V_{CE2}}{V_{A2}} \ll 1$  we have

$$I_{C1} \simeq \frac{\alpha I}{2} \left( 1 + \frac{1}{2} \frac{V_{CE1}}{V_{A1}} - \frac{1}{2} \frac{V_{CE2}}{V_{A2}} \right)$$

$$I_{C2} \simeq \frac{\alpha I}{2} \left( 1 + \frac{1}{2} \frac{V_{CE2}}{V_{A2}} - \frac{1}{2} \frac{V_{CE1}}{V_{A1}} \right)$$

We can express  $I_{C1}$  and  $I_{C2}$  as

$$I_{C1} = \frac{\alpha I}{2} (1 + \Delta)$$

$$I_{C2} = \frac{\alpha I}{2} (1 - \Delta)$$

where

$$\Delta = \frac{1}{2} \left( \frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}} \right)$$

(1)

With  $V_{OS}$  applied,

$$\left( \frac{\alpha I}{2} \right) \Delta = g_m (V_{OS}/2)$$

Substituting  $g_m = \frac{\alpha I/2}{V_T}$  gives

$$V_{OS} = 2V_T \Delta$$

Substituting for  $\Delta$  from Eq. (1) gives

$$V_{OS} = V_T \left( \frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}} \right)$$

As a first-order approximation, we can assume

$$V_{CE1} \simeq V_{CE2} = 10 \text{ V}$$

and substitute  $V_{A1} = 100 \text{ V}$  and  $V_{A2} = 200 \text{ V}$  to determine  $V_{OS}$  as

$$V_{OS} = 25 \left( \frac{10}{100} - \frac{10}{200} \right)$$

$$= 25 \times 0.05 = 1.25 \text{ mV}$$

### 8.82

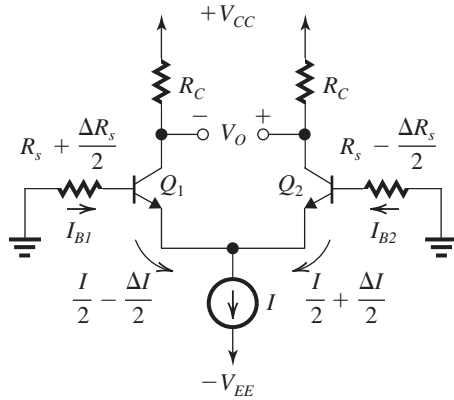
Refer to the figure and note that,

$$I_{B1} = I_B - \Delta I_B$$

$$I_{B2} = I_B + \Delta I_B$$

where

$$I_B = \frac{I}{2(\beta + 1)} \quad (1)$$



$$\Delta I_B = \frac{\Delta I}{2(\beta + 1)} \quad (2)$$

Equating the incremental changes in the voltage between each grounded base and emitter, we write

$$\begin{aligned} I_B \left( \frac{\Delta R_s}{2} \right) - \Delta I_B R_s - \Delta I_B \left( \frac{\Delta R_s}{2} \right) - \left( \frac{\Delta I}{2} \right) r_e \\ = I_B \left( -\frac{\Delta R_s}{2} \right) + \Delta I_B R_s + \Delta I_B \left( \frac{\Delta R_s}{2} \right) + \left( \frac{\Delta I}{2} \right) r_e \end{aligned}$$

Substituting for  $I_B$  from Eq. (1) and for  $\Delta I_B$  from Eq. (2) and collecting terms gives

$$\Delta I = \frac{I \Delta R_s}{2(\beta + 1)} \frac{1}{r_e + \frac{R_s}{\beta + 1} + \frac{\Delta R_s}{2(\beta + 1)}}$$

Neglecting the third term in the denominator of the second factor,

$$\Delta I \approx \frac{I \Delta R_s}{2(\beta + 1)} \frac{1}{r_e + \frac{R_s}{\beta + 1}} \quad (3)$$

The offset voltage at the output is

$$V_O = \Delta I R_C$$

The input offset voltage  $V_{OS}$  is obtained by dividing  $V_O$  by  $A_d = g_m R_C$ , thus

$$V_{OS} = \frac{\Delta I}{g_m}$$

Using Eq. (3) gives

$$V_{OS} = \frac{I \Delta R_s}{2(\beta + 1)g_m} \frac{1}{r_e + \frac{R_s}{\beta + 1}}$$

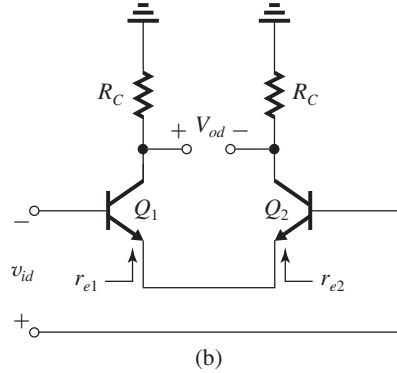
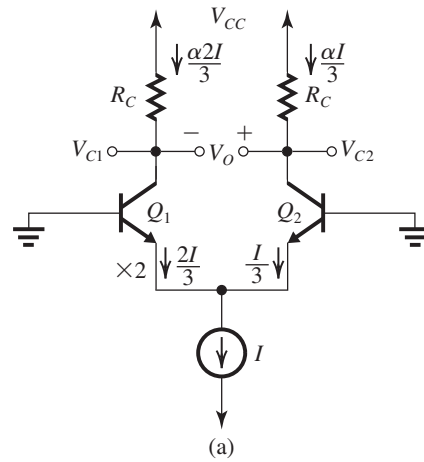
$$\text{Substituting } g_m = \frac{\alpha}{r_e} = \frac{\beta}{(\beta + 1)r_e},$$

$$V_{OS} = \frac{I \Delta R_s}{2\beta} \frac{1}{1 + \frac{R_s}{(\beta + 1)r_e}}$$

$$\text{But, } (\beta + 1)r_e = r_\pi = \frac{\beta}{g_m}, \text{ thus}$$

$$V_{OS} = \left( \frac{I}{2\beta} \right) \Delta R_s / \left[ 1 + \frac{g_m R_s}{\beta} \right] \quad \text{Q.E.D.}$$

8.83



From Fig. (a) we see that the transistor with twice the area ( $Q_1$ ) will carry twice the current in the other transistor ( $Q_2$ ). Thus

$$I_{E1} = \frac{2I}{3}, \quad I_{E2} = \frac{I}{3}$$

$$I_{C1} = \frac{\alpha 2I}{3}, \quad I_{C2} = \frac{\alpha I}{2}$$

Thus,

$$V_{C1} = V_{CC} - \frac{\alpha 2I}{3} R_C$$

$$V_{C2} = V_{CC} - \frac{\alpha I}{3} R_C$$

and the dc offset voltage at the output will be

$$V_O = V_{C2} - V_{C1}$$

$$V_O = \frac{1}{3} \alpha I R_C$$

To reduce this output voltage to zero, we apply a dc input voltage  $v_{id}$  in the direction shown in Fig. (b). The voltage  $v_{id}$  is required to produce  $v_{od}$  in the direction shown which is opposite in direction to  $V_O$  and of course  $|v_{od}| = |V_O|$ , thus

$$A_d v_{id} = \frac{1}{3} \alpha I R_C \quad (1)$$

The gain  $A_d$  is found as follows:

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times 2R_C}{r_{e1} + r_{e2}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{2I/3} = \frac{3V_T}{2I} = \frac{1.5V_T}{I}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{V_T}{I/3} = \frac{3V_T}{I}$$

thus,

$$A_d = \frac{2\alpha R_C}{4.5 V_T/I} = \frac{2\alpha I R_C}{4.5 V_T} \quad (2)$$

Substituting in Eq. (1) gives

$$v_{id} = 0.75 V_T = 18.75 \text{ mV}$$

Now, using large signal analysis:

$$v_{id} = V_{B2} - V_{B1} = (V_{B2} - V_E) - (V_{B1} - V_E)$$

$$I_{C1} = I_{S1} e^{(V_{B1} - V_E)/V_T} \quad (3)$$

$$I_{C2} = I_{S2} e^{(V_{B2} - V_E)/V_T} \quad (4)$$

where  $I_{S1} = 2 I_{S2}$ .

To make  $I_{C1} = I_{C2}$ ,

$$I_{S1} e^{(V_{B1} - V_E)/V_T} = I_{S2} e^{(V_{B2} - V_E)/V_T}$$

$$e^{(V_{B2} - V_{B1})/V_T} = 2$$

$$V_{B2} - V_{B1} = V_T \ln 2$$

Thus,

$$v_{id} = 17.3 \text{ mV}$$

which is reasonably close to the approximate value obtained using small-signal analysis.

**8.84** A 2-mV input offset voltage corresponds to a difference  $\Delta R_C$  between the two collector resistances,

$$2 = V_T \frac{\Delta R_C}{R_C}$$

$$= 25 \times \frac{\Delta R_C}{20}$$

$$\Rightarrow \Delta R_C = 1.6 \text{ k}\Omega$$

Thus a 2-mV offset can be nulled out by adjusting one of the collector resistances by 1.6 k $\Omega$ . If the adjustment mechanism raises one  $R_C$  and lowers the other, then each need to be adjusted by only (1.6 k $\Omega$ /2) = 0.8 k $\Omega$ .

If a potentiometer is used (as in Fig. P8.84), the total resistance of the potentiometer must be at least 1.6 k $\Omega$ . If specified to a single digit, we use 2 k $\Omega$ .

$$\mathbf{8.85} \quad G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{I}{0.25}$$

$$R_o = r_{o2} \parallel r_{o4}$$

For

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V'_A|L}{I/2}$$

$$= \frac{2 \times 5 \times 0.5}{I} = \frac{5}{I}$$

$$R_o = \frac{1}{2} \times \frac{5}{I} = \frac{2.5}{I}$$

Thus,

$$A_d = G_m R_o = \frac{I}{0.25} \times \frac{2.5}{I} = 10 \text{ V/V}$$

$$\mathbf{8.86} \quad \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times 20 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

$$g_{m1,2} = \frac{2 \times (I/2)}{V_{OV}} = \frac{2 \times 0.1}{0.16} = 1.25 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V'_A| \times L}{I/2} = \frac{6 \times 0.5}{0.1}$$

$$= 30 \text{ k}\Omega$$

$$A_d = g_{m1,2} (r_{o2} \parallel r_{o4})$$

$$= 1.25 \times (30 \parallel 30)$$

$$= 18.8 \text{ V/V}$$

$$\mathbf{8.87} \quad A_d = g_{m1,2} (r_{o2} \parallel r_{o4})$$

$$g_{m1,2} = \sqrt{2k'_n \left( \frac{W}{L} \right) I_D}$$

$$= \sqrt{4I} = 2\sqrt{I}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I} = \frac{2 \times 5}{I} = \frac{10}{I}$$

$$A_d = 2\sqrt{I} \times \frac{1}{2} \times \frac{10}{I} = \frac{10}{\sqrt{I}}$$

$$20 = \frac{10}{\sqrt{I}}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

$$\mathbf{8.88} \quad G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV1,2}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{V_{An}}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Ap}|}{I/2} = \frac{12}{0.1} = 120 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 200 \parallel 120 = 75 \text{ k}\Omega$$

$$A_d = G_m R_o = 1 \times 75 = 75 \text{ V/V}$$

The gain is reduced by a factor of 2 with  $R_L = R_o = 75 \text{ k}\Omega$ .

**8.89**  $I_{E1} = I_{E2} = 0.25 \text{ mA}$

$$I_{C1} = I_{C2} \simeq 0.25 \text{ mA}$$

$$g_{m1,2} = \frac{I_{C1,2}}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_C} = \frac{10 \text{ V}}{0.25 \text{ mA}} = 40 \text{ k}\Omega$$

$$R_{id} = 2 r_{\pi} = 2 \frac{\beta}{g_m} = 2 \times \frac{100}{10} = 20 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 40 \parallel 40 = 20 \text{ k}\Omega$$

$$G_m = g_{m1,2} = 10 \text{ mA/V}$$

$$A_d = G_m R_o = 10 \times 20 = 200 \text{ V/V}$$

If  $R_L = R_{id} = 20 \text{ k}\Omega$ , then

$$\begin{aligned} G_v &= 200 \times \frac{R_L}{R_L + R_o} \\ &= 200 \times \frac{20}{20 + 20} = 100 \text{ V/V} \end{aligned}$$

**8.90** Refer to Fig. P8.90.

$$A_d = G_{md} R_o$$

where

$$G_{md} = g_{m1,2} \simeq \frac{I/2}{V_T}$$

and

$$R_o = R_{o4} \parallel R_{o7}$$

Here  $R_{o4}$  is the output resistance of the cascode amplifier (looking into the collector of  $Q_4$ ), thus

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \parallel r_{\pi4})$$

Usually  $r_{\pi4} \ll r_{o2}$ ,

$$R_{o4} \simeq g_{m4} r_{\pi4} r_{o4} = \beta_4 r_{o4}$$

The resistance  $R_{o7}$  is the output resistance of the Wilson mirror and is given by

$$R_{o7} = \frac{1}{2} \beta_7 r_{o7}$$

Thus

$$R_o = (\beta_4 r_{o4}) \parallel \left( \frac{1}{2} \beta_7 r_{o7} \right)$$

Since all  $\beta$  and  $r_o$  are equal, we obtain

$$\begin{aligned} R_o &= (\beta r_o) \parallel \left( \frac{1}{2} \beta r_o \right) \\ &= \frac{1}{3} \beta r_o \end{aligned}$$

and

$$A_d = \frac{1}{3} \beta g_m r_o \quad \text{Q.E.D.}$$

For  $\beta = 100$  and  $V_A = 20 \text{ V}$ , we have

$$g_m r_o = \frac{I_C}{V_T} \frac{V_A}{I_C} = \frac{V_A}{V_T} = \frac{20}{0.025} = 800$$

$$A_d = \frac{1}{3} \times 100 \times 800 = 2.67 \times 10^4 \text{ V/V}$$

**8.91** (a) For  $Q_1$  and  $Q_2$ ,

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.5 \times \left( \frac{W}{L} \right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 17.8$$

For  $Q_3$  and  $Q_4$ ,

$$\frac{I}{2} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 0.125 \times \left( \frac{W}{L} \right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 71.1$$

$$\begin{aligned} \text{(b) } G_{md} = g_{m1,2} &= \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{0.2}{0.15} \\ &= 1.33 \text{ mA/V} \end{aligned}$$

$$A_d = G_{md} R_o$$

$$60 = 1.33 \times R_o$$

$$\Rightarrow R_o = 45 \text{ k}\Omega$$

But

$$R_o = r_{o2} \parallel r_{o4}$$

and  $r_{o2} = r_{o4}$  ( $Q_2$  and  $Q_4$  have the same  $I_D = \frac{I}{2}$  and the same  $V_A$ ). Thus

$$r_{o2} = r_{o4} = 90 \text{ k}\Omega = \frac{|V_A|}{I/2}$$

$$|V_A| = \frac{I}{2} \times 90 \text{ k}\Omega = 9 \text{ V}$$

$$9 = |V_A'| L = 15 L$$

$$\Rightarrow L = 0.6 \text{ }\mu\text{m}$$

$$\text{(c) } v_{Omin} = V_{CM} - V_m$$

$$= 0 - 0.4 = -0.4 \text{ V}$$

$$v_{O\max} = V_{DD} - |V_{OV}| = 0.8 - 0.15 = 0.65 \text{ V}$$

Thus,

$$-0.4 \text{ V} \leq v_o \leq 0.65 \text{ V}$$

$$(d) R_{SS} = \frac{|V_A|}{I} = \frac{9}{0.2} = 45 \text{ k}\Omega$$

The CMRR can be obtained using Eq. (8.138):

$$\text{CMRR} = (2g_{m1,2}R_{SS})[g_{m3}(r_o || r_{o3})]$$

$$= (g_m R_{SS})(g_m r_o)$$

$$= (1.33 \times 45)(1.33 \times 90)$$

$$= 7200 \text{ or } 77 \text{ dB}$$

### 8.92

$$G_{md} = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$= \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{6}{0.1} = 60 \text{ k}\Omega$$

$$R_o = r_{o2} || r_{o4} = 60 || 60 = 30 \text{ k}\Omega$$

$$A_d = G_{md}R_o = 1 \times 30 = 30 \text{ V/V}$$

$$R_{SS} = \frac{|V_A|}{I} = \frac{6}{0.2} = 30 \text{ k}\Omega$$

Using Eq. (8.134),

$$R_{im} = \frac{1}{g_{m3}} || r_{o3} = \frac{1}{g_{m1,2}} || r_o$$

$$= \frac{1}{1 \text{ mA/V}} || 60 \text{ k}\Omega$$

$$= 0.984 \text{ k}\Omega$$

Using Eq. (8.135),

$$A_m = \frac{1}{1 + \frac{1}{g_{m3}r_{o3}}} = \frac{1}{1 + \frac{1}{1 \times 60}}$$

$$= 0.9836 \text{ A/A}$$

Using Eq. (8.136),

$$G_{mcm} \simeq -\frac{1}{2R_{SS}} \frac{1}{g_{m3}(r_o || r_{o3})}$$

$$= -\frac{1}{2 \times 30} \times \frac{1}{1 \times 0.5 \times 60}$$

$$= 5.56 \times 10^{-4} \text{ mA/V}$$

Using Eq. (8.137),

$$A_{cm} = -\frac{1}{2g_{m3}R_{SS}}$$

$$= -\frac{1}{2 \times 1 \times 30} = 0.0167 \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{30}{0.0167} = 1800$$

or 65.1 dB

### 8.93 $G_m = 2 \text{ mA/V}$

With  $R_L = \infty$ ,

$$A_d = G_m R_o$$

and

$$v_o = G_m R_o v_{id}$$

With  $R_L = 20 \text{ k}\Omega$ ,

$$v_o = G_m R_o v_{id} \frac{R_L}{R_L + R_o}$$

$$= G_m R_o \frac{20}{20 + R_o} v_{id} = \frac{1}{2} G_m R_o v_{id}$$

Thus,

$$R_o = 20 \text{ k}\Omega$$

$$A_d \text{ (with } R_L = \infty) = G_m R_o = 2 \times 20 = 40 \text{ V/V}$$

### 8.94

For  $Q_1$ ,  $Q_2$ ,  $Q_3$  and  $Q_4$ :

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_t + |V_{OV}|$$

$$= 0.5 + 0.2 = 0.7 \text{ V}$$

For  $Q_5$ ,  $Q_6$ ,  $Q_7$ , and  $Q_8$ :

$$I_D = 0.2 \text{ mA}$$

$$0.2 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.28 \text{ V}$$

$$V_{GS} = 0.5 + 0.28 = 0.78 \text{ V}$$

From the figure we see that for each transistor to operate at  $V_{DS}$  at least equal to  $V_{GS}$ , the total power supply is given by

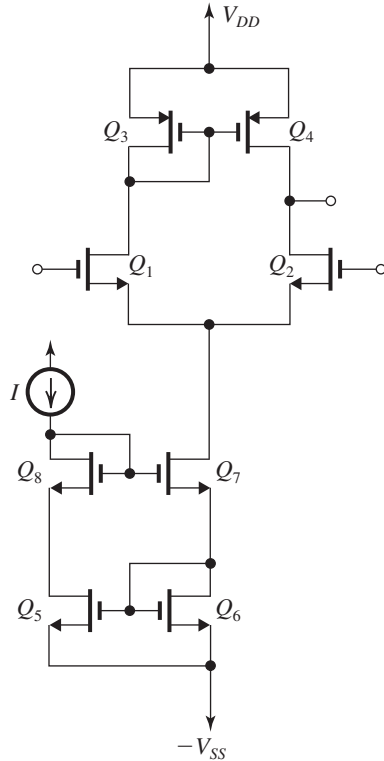
$$V_{DD} + V_{SS} = V_{DS4} + V_{DS2} + V_{DS7} + V_{DS6}$$

$$= V_{GS4} + V_{GS2} + V_{GS7} + V_{GS6}$$

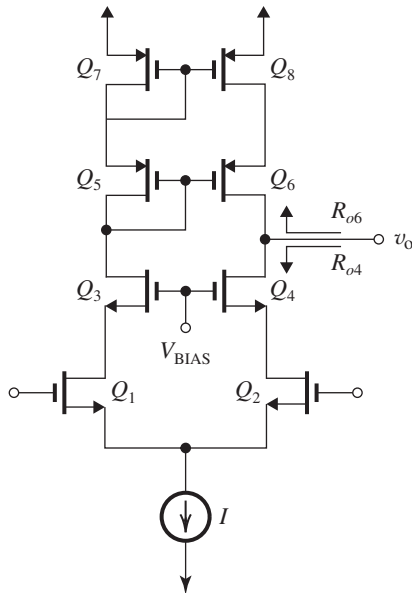


$$= 0.7 + 0.7 + 0.78 + 0.78$$

$$= 2.96 \approx 3.0 \text{ V}$$



8.95



(a) See figure.

$$(b) A_d = g_{m1,2}(R_{o4} \parallel R_{o6})$$

$$g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$R_{o6} = g_{m6}r_{o6}r_{o8}$$

Since all transistors are operated at a bias current ( $I/2$ ) and have the same overdrive voltage  $|V_{OV}|$  and the same Early voltage,  $|V_A|$ , all have the same  $g_m = I/|V_{OV}|$  and the same

$$r_o = \frac{|V_A|}{I/2} = 2|V_A|/I. \text{ Thus,}$$

$$R_{o6} = g_m r_o^2$$

$$R_{o4} = g_{m4}r_{o4}r_{o2} = g_m r_o^2$$

$$A_d = g_m(g_m r_o^2 \parallel g_m r_o^2)$$

$$= \frac{1}{2}(g_m r_o)^2$$

$$g_m r_o = \frac{I}{|V_{OV}|} \times \frac{2|V_A|}{I} = \frac{2|V_A|}{|V_{OV}|}$$

$$A_d = 2(|V_A|/|V_{OV}|)^2 \quad \text{Q.E.D.}$$

For  $|V_{OV}| = 0.2 \text{ V}$  and  $|V_A| = 10 \text{ V}$ , we have

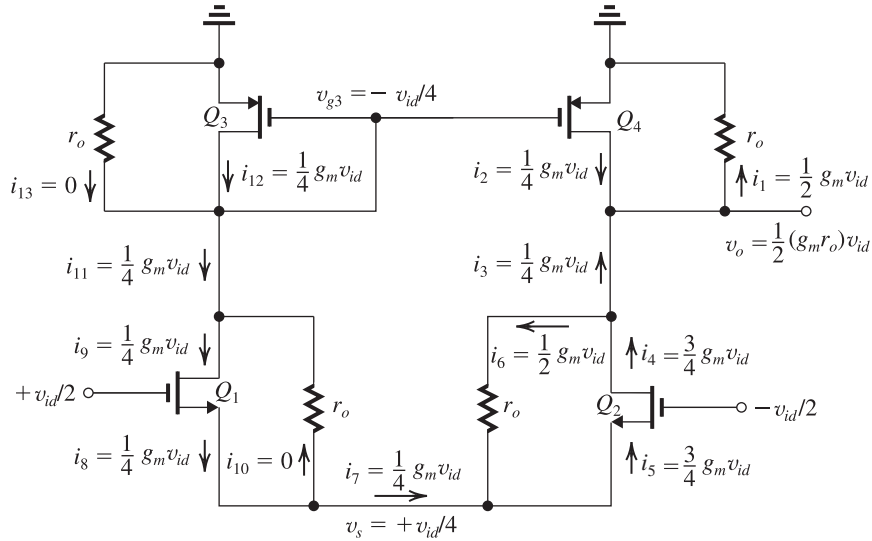
$$A_d = 2\left(\frac{10}{0.2}\right)^2 = 5000 \text{ V/V}$$

**8.96** The currents  $i_1$  to  $i_{13}$  are shown on the circuit diagram. Observe that  $i_{11} = i_7 = i_3$  (the current that enters a transistor exits at the other end!). Also observe that the mirror  $Q_3$  and  $Q_4$  is indeed functioning properly as the drain currents of  $Q_3$  and  $Q_4$  are equal ( $i_{12} = i_2 = \frac{1}{4}g_m v_{id}$ ). However, the currents in their  $r_o$ 's are far from being equal!

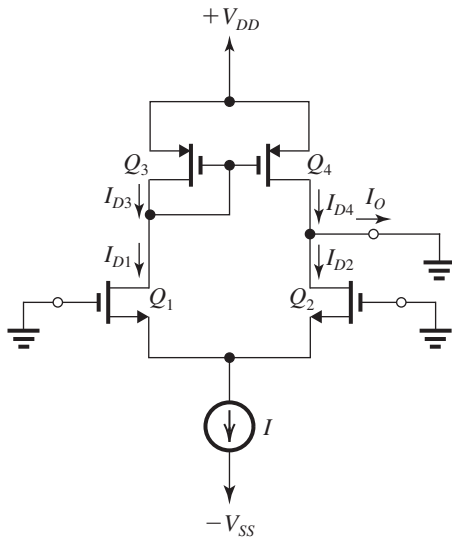
There are some inconsistencies that result from the approximations made to obtain the results shown in Fig. P8.96, namely,  $g_m r_o \gg 1$ . Note for instance that although we find the current in  $r_o$  of  $Q_2$  to be  $\frac{1}{2}g_m v_{id}$ , the voltages at the two ends of  $r_o$  are  $\frac{1}{2}(g_m r_o)v_{id}$  and  $v_{id}/4$ ; thus the current must be  $v_{id}\left(\frac{1}{2}g_m r_o - \frac{1}{4}\right)/r_o$ , which is approximately  $\frac{1}{2}g_m v_{id}$ .

The purpose of this problem is to show the huge imbalance that exists in this circuit. In fact,  $Q_1$  has  $|v_{gs}| = \frac{1}{4}v_{id}$  while  $Q_2$  has  $|v_{gs}| = \frac{3}{4}v_{id}$ . This imbalance results from the fact that the current mirror is *not* a balanced load. Nevertheless, we know that this circuit provides a reasonably high common-mode rejection.

This figure belongs to Problem 8.96.



8.97



(a) Let

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_A + \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_A - \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A$$

$Q_1$  and  $Q_2$  have equal values of  $V_{GS}$  and thus of  $V_{OV}$ , thus

$$I_{D1} = \frac{1}{2} k'_n \left[ \left(\frac{W}{L}\right)_A + \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A \right] V_{OV}^2$$

$$= \frac{1}{2} k'_n \left(\frac{W}{L}\right)_A \left[ 1 + \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right] V_{OV}^2$$

Since, in the ideal case

$$I_{D1} = \frac{I}{2} = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_A V_{OV}^2$$

$$I_{D1} = \frac{I}{2} \left[ 1 + \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right]$$

Similarly, we can show that

$$I_{D2} = \frac{I}{2} \left[ 1 - \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right]$$

The current mirror causes

$$I_{D4} = I_{D3} = I_{D1}$$

Thus,

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D1} - I_{D2}$$

$$= \frac{I}{2} \frac{\Delta(W/L)_A}{(W/L)_A}$$

The input offset voltage is

$$V_{OS} = \frac{I_O}{G_{md}}$$

where

$$G_{md} = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

Thus,

$$V_{OS} = (V_{OV}/2) \frac{\Delta(W/L)_A}{(W/L)_A} \quad \text{Q.E.D.}$$

$$(b) \quad I_{D1} = I_{D2} = \frac{I}{2}$$

$$I_{D3} = I_{D1}$$

If the  $(W/L)$  ratios of the mirror transistors have a mismatch  $\Delta(W/L)_M$ , the current transfer ratio of

the mirror will have an error of  $[\Delta(W/L)_M/(W/L)_M]$ . Thus

$$I_{D4} = I_{D3} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right]$$

At the output node, we have

$$\begin{aligned} I_O &= I_{D4} - I_{D2} \\ &= I_{D3} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right] - I_{D2} \\ &= I_{D1} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right] - I_{D2} \\ &= \frac{I}{2} \frac{\Delta(W/L)_M}{(W/L)_M} \end{aligned}$$

and the corresponding  $V_{OS}$  will be

$$\begin{aligned} V_{OS} &= \frac{I_O}{G_{md}} = \frac{I_O}{I/V_{OV}} \\ &= \left( \frac{V_{OV}}{2} \right) \frac{\Delta(W/L)_M}{(W/L)_M} \quad \text{Q.E.D.} \end{aligned}$$

$$(c) \quad V_{OS}|_{Q_1, Q_2 \text{ mismatch}} = \left( \frac{0.2}{2} \right) \times 0.02 = 2 \text{ mV}$$

$$V_{OS}|_{Q_3, Q_4 \text{ mismatch}} = \left( \frac{0.2}{2} \right) \times 0.02 = 2 \text{ mV}$$

Worst-case  $V_{OS} = 2 + 2 = 4 \text{ mV}$

**8.98** Refer to Fig. P8.90.

$$\begin{aligned} (a) \quad V_{B7} &= +5 - V_{EB6} - V_{EB7} = 5 - 0.7 - 0.7 \\ &= +3.6 \text{ V} \end{aligned}$$

$$v_{O\max} = V_{B7} + 0.4 = +4 \text{ V}$$

(b) The dc bias voltage should be

$$\begin{aligned} V_O &= v_{O\max} - 1.5 \\ &= 4 - 1.5 = +2.5 \text{ V} \end{aligned}$$

(c) For  $v_O$  to swing negatively (i.e., below the dc bias value of 2.5 V) by 1.5 V, that is, to +1 V with  $Q_4$  remaining in saturation,  $V_{BIAS}$  should be

$$\begin{aligned} V_{BIAS} &= v_{O\min} + 0.4 \\ &= 1.4 \text{ V} \end{aligned}$$

(d) With  $V_{BIAS} = 1.4 \text{ V}$ , the bias voltage at the collectors of  $Q_1$  and  $Q_2$  is

$$\begin{aligned} V_{C1,2} &= V_{BIAS} - V_{BE3,4} \\ &= 1.4 - 0.7 = +0.7 \text{ V} \end{aligned}$$

The upper limit on  $V_{CM}$  is 0.4 V above  $V_{C1,2}$ :

$$V_{CM\max} = 0.7 + 0.4 = +1.1 \text{ V}$$

**8.99** Refer to Fig. P8.99,

$$\begin{aligned} \Delta i &= \frac{\alpha I}{2} - \frac{\alpha I/2}{1 + \frac{2}{\beta_P}} \\ &= \frac{\alpha I}{2} \frac{2/\beta_P}{1 + \frac{2}{\beta_P}} \\ &\simeq \frac{\alpha I}{\beta_P} \end{aligned}$$

To reduce this output current to zero, an input voltage  $V_{OS}$  has to be applied with a value of

$$V_{OS} = -\frac{\Delta i}{G_{md}} = -\frac{\alpha I/\beta_P}{(\alpha I/2)/V_T}$$

Thus,

$$V_{OS} = -\frac{2V_T}{\beta_P} \quad \text{Q.E.D.}$$

For  $\beta_P = 50$ ,

$$V_{OS} = -\frac{2 \times 25}{50} = -1 \text{ mV}$$

If the simple current mirror is replaced by the Wilson current mirror,

$$\Delta i = \frac{\alpha I}{2} - \frac{\alpha I/2}{1 + \frac{2}{\beta_P^2}}$$

which leads to

$$\Delta i \simeq \frac{\alpha I}{\beta_P^2}$$

and the offset voltage becomes

$$V_{OS} = -\frac{2V_T}{\beta_P^2}$$

which for  $\beta_P = 50$  gives

$$V_{OS} = -0.02 \text{ mV}$$

a negligible quantity!

**8.100** The overdrive voltage,  $|V_{OV}|$ , at which  $Q_1$  and  $Q_2$  are operating is found from

$$\frac{I}{2} = \frac{1}{2} k'_p (W/L) |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 6.4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.18 \text{ V}$$

$$G_{md} = g_{m1,2} = \frac{2(I/2)}{|V_{OV}|}$$

$$= \frac{0.2}{0.18} = 1.13 \text{ mA/V}$$

$$r_{o2} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Anpn}|}{I/2} = \frac{30}{0.1} = 300 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 100 \text{ k}\Omega \parallel 300 \text{ k}\Omega = 75 \text{ k}\Omega$$

$$A_d = G_{md}R_o = 1.13 \times 75 = 85 \text{ V/V}$$

**8.101** The CMRR is given by Eq. (8.138):

$$\text{CMRR} = (2g_{m1}R_{SS})[g_{m3}(r_o \parallel r_{o3})]$$

(a) Current source is implemented with a simple current mirror:

$$R_{SS} = r_o \parallel Q_S = \frac{|V_A|}{I}$$

$$g_{m1,2} = g_{m3} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$r_{o2} = r_{o3} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

Thus,

$$\begin{aligned} \text{CMRR} &= \frac{2I}{V_{OV}} \times \frac{|V_A|}{I} \times \frac{1}{2} \times \frac{I}{V_{OV}} \times \frac{2|V_A|}{I} \\ &= 2 \left( \frac{V_A}{V_{OV}} \right)^2 \quad \text{Q.E.D.} \end{aligned}$$

(b) Current source is implemented with the modified Wilson mirror in Fig. P8.101:

$$R_{SS} = g_{m7}r_{o7}r_{o9}$$

Transistor  $Q_7$  has the same  $k'(W/L)$  as  $Q_1$  and  $Q_2$ , but  $Q_7$  carries a current  $I$  twice that of  $Q_1$  and  $Q_2$ . Thus

$$V_{OV7} = \sqrt{2}V_{OV1,2} = \sqrt{2}V_{OV}$$

and

$$g_{m7} = \frac{2I}{V_{OV7}} = \frac{2I}{\sqrt{2}V_{OV}} = \frac{\sqrt{2}I}{V_{OV}}$$

$$r_{o7} = r_{o9} = \frac{V_A}{I}$$

Thus,

$$R_{SS} = \frac{\sqrt{2}I}{V_{OV}} \left( \frac{V_A}{I} \right)^2 = \frac{\sqrt{2}V_A^2}{V_{OV}I}$$

and

$$\begin{aligned} \text{CMRR} &= \frac{2I}{V_{OV}} \times \frac{\sqrt{2}V_A^2}{V_{OV}I} \times \frac{1}{2} \times \frac{I}{V_{OV}} \times \frac{2|V_A|}{I} \\ &= 2\sqrt{2} \left( \left| \frac{V_A}{V_{OV}} \right| \right)^3 \quad \text{Q.E.D.} \end{aligned}$$

For  $k'(W/L) = 4 \text{ mA/V}^2$  and  $I = 160 \mu\text{A}$ ,

$$0.080 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.2 \text{ V}$$

For  $|V_A| = 5 \text{ V}$ :

For case (a),

$$\text{CMRR} = 2 \times \left( \frac{5}{0.2} \right)^2 = 1250 \text{ or } 62 \text{ dB}$$

For case (b),

$$\text{CMRR} = 2\sqrt{2} \left( \frac{5}{0.2} \right)^3 = 4.42 \times 10^4$$

or 93 dB

**8.102** To determine  $G_{mcm}$  we use Eq. (8.133) and assume  $\epsilon$  to be negligibly small, thus

$$G_{mcm} = \frac{1}{2R_{SS}} \left[ (A_m - 1) - \frac{R_{im}}{2r_o} (A_m + 1) \right]$$

where  $R_{SS} = 100 \text{ k}\Omega$ ,  $A_m = 0.98 \text{ A/A}$ ,  $R_{im} = 1.63 \text{ k}\Omega$ , and  $r_o = R_{od} = 50 \text{ k}\Omega$ . Thus,

$$\begin{aligned} G_{mcm} &= \frac{1}{2 \times 100} \left[ (0.98 - 1) - \frac{1.63}{2 \times 50} (0.98 + 1) \right] \\ &= 2.61 \times 10^{-4} \text{ mA/V} \end{aligned}$$

The common-mode gain can now be found as

$$\begin{aligned} |A_{cm}| &= |G_{mcm}| (R_{od} \parallel R_{om}) \\ &= 2.61 \times 10^{-4} \times (50 \parallel 75) \\ &= 7.83 \times 10^{-3} \text{ V/V} \end{aligned}$$

The CMRR can be found as

$$\begin{aligned} \text{CMRR} &= \frac{|A_d|}{|A_{cm}|} = \frac{30}{7.83 \times 10^{-3}} \\ &= 3.83 \times 10^3 \text{ or } 71.7 \text{ dB} \end{aligned}$$

**8.103** To obtain  $\text{CMRR} = 80 \text{ dB}$  or  $\text{CMRR} = 10^4$  we must design for a common-mode gain  $|A_{cm}|$  of

$$|A_{cm}| = \frac{|A_d|}{\text{CMRR}} = \frac{40}{10^4} = 4 \times 10^{-3} \text{ V/V}$$

But,

$$|A_{cm}| = G_{mcm}(R_{od} \parallel R_{om})$$

Thus,

$$\begin{aligned} 4 \times 10^{-3} &= G_{mcm} \times (40 \parallel 40) \\ \Rightarrow G_{mcm} &= 2 \times 10^{-4} \text{ mA/V} \end{aligned}$$

Now, using Eq. (8.133) with  $\epsilon$  assumed negligibly small and  $r_o = R_{od}$ , we obtain

$$\begin{aligned} 2 \times 10^{-4} &= \frac{1}{2R_{SS}} \times [(0.98 - 1) - \\ &\frac{0.5}{2 \times 40} (0.98 + 1)] \Rightarrow R_{SS} = 81 \text{ k}\Omega \end{aligned}$$

**8.104** Refer to Fig. P8.104.

$$(a) I_{D1} = I_{D2} = 100 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{1,2} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 12.5$$

$$I_{D3} = I_{D4} = 100 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 50$$

$$I_{D5} = I_{D7} = I_{D8} = 200 \mu\text{A}$$

Thus

$$200 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{5,7,8} V_{OV}^2$$

$$= \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{5,7,8} \times 0.04$$

$$\left( \frac{W}{L} \right)_5 = \left( \frac{W}{L} \right)_7 = \left( \frac{W}{L} \right)_8 = 25$$

$$I_{D6} = 200 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_6 |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_6 \times 0.04$$

$$\left( \frac{W}{L} \right)_6 = 100$$

The results are summarized in the following table:

Transistor	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$W/L$	12.5	12.5	50	50	25	100	25	25

Ideally, the dc voltage at the output is zero.

(b) The upper limit of  $V_{ICM}$  is determined by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$\begin{aligned} V_{ICM\max} &= V_{D1} + V_t \\ &= V_{DD} - |V_{SG4}| + V_t \\ &= 0.9 - |V_t| - |V_{OV4}| + V_t \\ &= 0.9 - 0.2 = +0.7 \text{ V} \end{aligned}$$

The lower limit of  $V_{ICM}$  is determined by the need to keep  $Q_5$  in saturation,

$$\begin{aligned} V_{ICM\min} &= -0.9 + |V_{OV5}| + |V_{GS1}| \\ &= -0.9 + 0.2 + 0.2 + 0.4 = -0.1 \text{ V} \end{aligned}$$

Thus

$$-0.1 \text{ V} \leq V_{ICM} \leq +0.7 \text{ V}$$

$$(c) v_{O\max} = V_{DD} - |V_{OV6}| \\ = 0.9 - 0.2 = +0.7 \text{ V}$$

$$v_{O\min} = -V_{SS} + |V_{OV7}| \\ = -0.9 + 0.2 = -0.7 \text{ V}$$

Thus

$$-0.7 \text{ V} \leq v_o \leq +0.7 \text{ V}$$

$$(d) A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4})$$

where

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.1 \text{ mA}} = \frac{6}{0.1} = 60 \text{ k}\Omega$$

$$A_1 = -1 \times (60 \parallel 60) = -30 \text{ V/V}$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

where

$$g_{m6} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.2} = \frac{6}{0.2} = 30 \text{ k}\Omega$$

$$A_2 = -2 \times (30 \parallel 30) = -30 \text{ V/V}$$

$$A_0 = A_1 A_2 = 30 \times 30 = 900 \text{ V/V}$$

**8.105** If  $(W/L)_7$  becomes 48/0.8,  $I_{D7}$  will become

$$\begin{aligned} I_{D7} &= I_{D8} \frac{(W/L)_7}{(W/L)_8} \\ &= I_{\text{REF}} \frac{(24/0.4)}{(20/0.4)} \\ &= 100 \times 1.2 = 120 \mu\text{A} \end{aligned}$$

Thus  $I_{D7}$  will exceed  $I_{D6}$  by 20  $\mu\text{A}$ , which will result in a systematic offset voltage,

$$V_O = 20 \mu\text{A}(r_{o6} \parallel r_{o7})$$

where

$$r_{o6} = 50 \text{ k}\Omega$$

and  $r_{o7}$  now becomes

$$r_{o7} = \frac{5}{0.12} = 41.7 \text{ k}\Omega$$

Thus

$$\begin{aligned} V_O &= 20 \times 10^{-3} \times (50 \parallel 41.7) \\ &= 455 \text{ mV} \end{aligned}$$

The corresponding input offset voltage will be

$$\begin{aligned} V_{OS} &= \frac{V_O}{A_0} \\ &= \frac{455}{625} = 0.73 \text{ mV} \end{aligned}$$

**8.106** Refer to Fig. P8.106.

(a) With the inputs grounded and the output at 0 V dc, we have

$$I_{E1} = I_{E2} = \frac{1}{2} \times 0.4 = 0.2 \text{ mA}$$

$$I_{E3} = I_{E4} \simeq 0.2 \text{ mA}$$

$$I_{E5} \simeq 0.5 \text{ mA}$$

$$I_{E6} = 1 \text{ mA}$$

(b) The short-circuit transconductance of the first stage is

$$G_m = g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

The voltage gain of the first stage can be obtained by multiplying  $G_m$  by the total resistance at the output node of the stage, i.e., the common collectors of  $Q_2$  and  $Q_4$  and the base of  $Q_5$ . Since  $r_{o2} = r_{o4} = \infty$ , the resistance at this node is equal to the input resistance of  $Q_5$  which is  $R_{\pi 5}$ ,

$$r_{\pi 5} = \frac{\beta}{g_{m5}}$$

where

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

thus

$$r_{\pi 5} = \frac{100}{20} = 5 \text{ k}\Omega$$

Thus the voltage gain of the first stage is given by

$$A_1 \equiv \frac{v_{b5}}{v_{id}} = -G_m r_{\pi 5} = -8 \times 5 = -40 \text{ V/V}$$

The voltage gain of the second stage is

$$A_2 \equiv \frac{v_{c5}}{v_{b5}} = -g_{m5} R_C$$

where  $R_{C5}$  is the total resistance in the collector of  $Q_5$ . Since  $r_{o5} = \infty$ ,  $R_{C5}$  is simply the input resistance of the emitter follower  $Q_6$ , we have

$$R_{C5} = R_{i6} = (\beta + 1)(r_{e6} + R_L)$$

where

$$r_{e6} = \frac{V_T}{I_{E6}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{i6} = (100 + 1)(0.025 + 1)$$

$$= 103.5 \text{ k}\Omega$$

Thus

$$A_2 = -20 \times 103.5 = -2070 \text{ V/V}$$

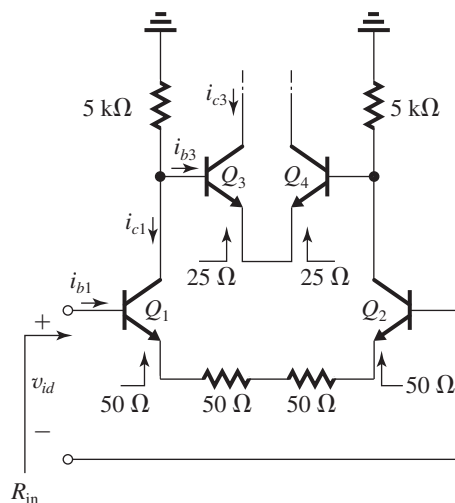
The gain of the third stage is given by

$$A_3 \equiv \frac{v_o}{v_5} = \frac{R_L}{R_L + r_{e6}} = \frac{1}{1 + 0.025} = 0.976 \text{ V/V}$$

The overall voltage gain can now be obtained as

$$A_0 \equiv \frac{v_o}{v_{id}} = A_1 A_2 A_3 = -40 \times -2070 \times 0.976 = 8.07 \times 10^4 \text{ V/V}$$

**8.107**



$$R_{in2} = 2(\beta + 1)(25 + 25)$$

$$= 2 \times 101 \times 50 \simeq 10 \text{ k}\Omega$$

$$\text{Effective load of first stage} = R_{in2} \parallel (5 + 5)$$

$$= 10 \parallel 10 = 5 \text{ k}\Omega$$

$$A_1 =$$

$$\propto \frac{\text{Total resistance between collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{5 \text{ k}\Omega}{4 \times 50 \Omega} = 25 \text{ V/V}$$

$$R_{in} = (\beta + 1)(4 \times 50 \Omega)$$

$$= 101 \times 200 \simeq 20 \text{ k}\Omega$$

$$\frac{i_{c1}}{i_{b1}} = \beta_1 = 100$$

$$\frac{i_{b3}}{i_{c1}} = \frac{(5 + 5)}{(5 + 5) + R_{in2}} = \frac{10}{10 + 10} = 0.5$$

$$\frac{i_{c3}}{i_{b3}} = \beta_3 = 100$$

Thus

$$\frac{i_{c3}}{i_{b1}} = \frac{i_{c3}}{i_{b3}} \times \frac{i_{b3}}{i_{c1}} \times \frac{i_{c1}}{i_{b1}} = 100 \times 0.5 \times 100$$

$$= 5000 \text{ A/A}$$

**8.108** Refer to Fig. 8.38. With  $R_5$  replaced with a 1-mA constant-current source with a high output

resistance, the total resistance in the collector of  $Q_7$  now becomes the input resistance of  $Q_8$ , which is

$$R_{i4} = (\beta + 1)(r_{e8} + R_6) = 101 \times (0.005 + 3) = 303.5 \text{ k}\Omega$$

Thus the gain of the third stage now becomes

$$A_3 = -\alpha \frac{303.5}{2.3 + 0.025} \simeq -130.5 \text{ V/V}$$

and the overall voltage gain increases to

$$\frac{v_o}{v_{id}} = 8513 \times \frac{130.5}{6.42} = 1.73 \times 10^5 \text{ V/V}$$

(b) The output resistance now becomes

$$R_o = 3 \text{ k}\Omega \parallel \left( r_{e8} + \frac{\text{very large resistance}}{\beta + 1} \right) \simeq 3 \text{ k}\Omega$$

When the amplifier is loaded with  $R_L = 100 \Omega$ ,

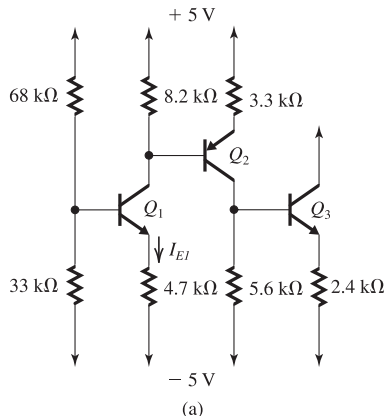
$$G_v = 1.73 \times 10^5 \frac{R_L}{R_L + R_o} = 1.73 \times 10^5 \times \frac{100}{3000 + 100} = 5581 \text{ V/V}$$

If the original amplifier is loaded in  $R_L = 100 \Omega$ ,

$$G_v = 8513 \times \frac{100}{152 + 100} = 3378 \text{ V/V}$$

Thus, although the output resistance of the original amplifier is much lower than that of the modified one, the overall voltage gain realized when the original amplifier is loaded in 100- $\Omega$  resistance is much lower than that obtained with the modified design. Thus, replacing the 15.7-k $\Omega$  resistance with a constant-current source is an excellent modification to make!

**8.109 (a)**



Refer to Fig. (a) for the dc analysis. Replacing the 68 k $\Omega$ -33 k $\Omega$  divider network by its Thévenin equivalent, we obtain

$$V_{BB} = -5 \text{ V} + \frac{33}{33 + 68} \times 10 \text{ V} = -1.73 \text{ V}$$

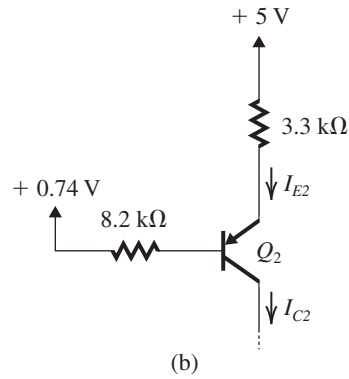
$$R_{BB} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega = 22.2 \text{ k}\Omega$$

Now, we can determine  $I_{E1}$  from

$$I_{E1} = \frac{V_{BB} - (-5) - 0.7}{4.7 + \frac{R_{BB}}{\beta + 1}} = \frac{-1.73 + 5 - 0.7}{4.7 + \frac{22.2}{101}} = 0.52 \text{ mA}$$

$$I_{C1} = \alpha_1 \times 0.52 = 0.99 \times 0.52 \simeq 0.52 \text{ mA}$$

The collector current  $I_{C1}$  and the 8.2-k $\Omega$  resistor it feeds can be replaced by a Thévenin equivalent as shown in Fig. (b). Thus



$$I_{E2} = \frac{5 - 0.74 - 0.7}{3.3 + \frac{8.2}{101}} = 1.05 \text{ mA}$$

$$I_{C2} \simeq 1.04 \text{ mA}$$

$$I_{C2} \simeq 1.04 \text{ mA}$$

The collector current  $I_{C2}$  and the 5.6-k $\Omega$  resistance it feeds can be replaced by a Thévenin equivalent as shown in Fig. (c). Thus

$$I_{E3} = \frac{0.824 - 0.7 - (-5)}{2.4 + \frac{5.6}{101}} = 2.1 \text{ mA}$$

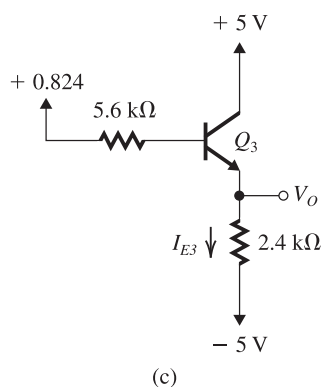
$$= 2.1 \text{ mA}$$

$$V_O = -5 + 2.1 \times 2.4 = 0 \text{ V}$$

$$(b) R_{in} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega \parallel r_{\pi 1}$$

where

$$r_{\pi 1} = \frac{\beta}{g_{m1}}$$



$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.52}{0.025} = 20.8 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{20.8} = 4.81 \text{ k}\Omega$$

$$R_{in} = 68 \text{ k}\Omega \parallel 33 \parallel 4.81 \approx 4 \text{ k}\Omega$$

$$R_{out} = 2.4 \text{ k}\Omega \parallel \left( r_{e3} + \frac{5.6 \text{ k}\Omega}{\beta + 1} \right)$$

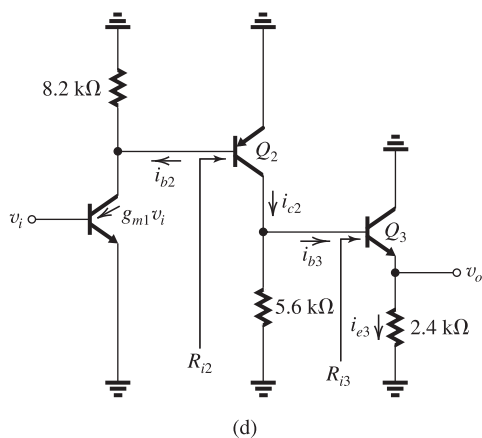
where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{2.1 \text{ mA}} = 11.9 \Omega$$

$$R_{out} = 2.4 \parallel \left( 0.0119 + \frac{5.6}{101} \right)$$

$$= 65.5 \Omega$$

(c) Refer to Fig. (d)



$$i_{c1} = g_{m1} v_i = 20.8 v_i$$

$$R_{i2} = r_{\pi 2} = \frac{\beta}{g_{m2}}$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.04 \text{ mA}}{0.025 \text{ V}} = 41.6 \text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{41.6} = 2.4 \text{ k}\Omega$$

$$i_{b2} = g_{m1} v_i \frac{8.2}{8.2 + 2.4} = 16.1 v_i$$

$$i_{c2} = \beta_2 i_{b2} = 100 \times 16.1 v_i = 1610 v_i$$

$$R_{i3} = (\beta + 1)(r_{e3} + 2.4 \text{ k}\Omega)$$

$$= 101(0.0119 + 2.4) = 243.6 \text{ k}\Omega$$

$$i_{b3} = i_{c2} \times \frac{5.6}{5.6 + 243.6} = 0.0225 i_{c2}$$

$$= 0.0225 \times 1610 v_i = 36.18 v_i$$

$$i_{e3} = (\beta + 1) i_{b3}$$

$$= 101 \times 36.18 = 3654 v_i$$

$$v_o = i_{e3} \times 2.4 \text{ k}\Omega$$

$$= 3654 \times 2.4 v_i = 8770 v_i$$

Thus

$$\frac{v_o}{v_i} = 8770 \text{ V/V}$$

**8.110** Refer to Fig. 8.37.

$W_6$  can be determined using Eq. (8.144):

$$\frac{(W/L)_6}{(W/L)_4} = 2 \frac{(W/L)_7}{(W/L)_5}$$

$$\frac{(W/0.5)_6}{(6.25/0.5)} = 2 \frac{(50/0.5)}{(50/0.5)}$$

$$\Rightarrow W_6 = 12.5 \mu\text{m}$$

For all devices we can evaluate  $I_D$  as follows:

$$I_{D8} = I_{REF} = 50 \mu\text{A}$$

$$I_{D5} = I_{REF} \frac{(W/L)_5}{(W/L)_8} = 4 I_{REF} = 200 \mu\text{A}$$

$$I = I_{D5} = 200 \mu\text{A}$$

$$I_{D1} = I_{D2} = \frac{1}{2} I_{D5} = 100 \mu\text{A}$$

$$I_{D3} = I_{D4} = I_{D1} = 100 \mu\text{A}$$

$$I_{D6} = I_{D7} = 4 I_{REF} = 200 \mu\text{A}$$

With  $I_D$  in each device known, we can use

$$I_{Di} = \frac{1}{2} \mu C_{ox} \left( \frac{W}{L} \right)_i |V_{OVi}|^2$$

to determine  $|V_{OVi}|$  and then

$$|V_{GSi}| = |V_{OVi}| + |V_t|$$

The values of  $g_{mi}$  and  $r_{oi}$  can then be determined from

$$g_{mi} = \frac{2I_{Di}}{|V_{OVi}|}$$

$$r_{oi} = \frac{|V_A|}{I_{Di}}$$

$$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$$



This table belongs to Problem 8.110.

	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$I_D$ ( $\mu\text{A}$ )	100	100	100	100	200	200	200	50
$ V_{OV} $ (V)	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
$ V_{GS} $ (V)	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
$g_m$ (mA/V)	0.95	0.95	0.95	0.95	1.9	1.9	1.9	0.48
$r_o$ (k $\Omega$ )	60	60	60	60	30	30	30	120

$$= -0.95 \times (60 \parallel 60) = -28.5 \text{ V/V}$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -1.9 \times (30 \parallel 30) = -28.5 \text{ V/V}$$

$$A_0 = A_1 A_2 = -28.5 \times -28.5 = 812 \text{ V/V}$$

The upper limit of  $V_{ICM}$  is determined by the need to keep  $Q_5$  in saturation, thus

$$\begin{aligned} V_{ICM\max} &= V_{DD} - |V_{OV5}| - |V_{SG1}| \\ &= 0.9 - 0.21 - 0.61 = +0.08 \text{ V} \end{aligned}$$

The lower limit of  $V_{ICM}$  is determined by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$\begin{aligned} V_{ICM\min} &= V_{G3} - |V_t| \\ &= -V_{SS} + |V_{GS3}| - |V_t| \\ &= -0.9 + 0.61 - 0.4 = -0.69 \text{ V} \end{aligned}$$

Thus

$$-0.69 \text{ V} \leq V_{ICM} \leq +0.08 \text{ V}$$

The output voltage range is

$$-V_{SS} + V_{OV6} \leq v_O \leq V_{DD} - |V_{OV7}|$$

that is,

$$-0.69 \text{ V} \leq v_O \leq +0.69 \text{ V}$$

The results are summarized in the table at the top of the page.

**8.111** (a) Increasing  $(W/L)_1$  and  $(W/L)_2$  by a factor of 4 reduces  $|V_{OV1,2}|$  by a factor of 2. Thus  $g_{m1,2} = 2I_D/|V_{OV1,2}|$  increase by a factor of 2.

(b)  $A_1$  is proportional to  $g_{m1,2}$ , thus  $A_1$  increases by a factor of 2 and the overall voltage gain increases by a factor of 2.

(c) Since the input offset voltage is proportional to  $|V_{OV1,2}|$ , it will decrease by a factor of 2. This, however, does not apply to  $V_{OS}$  due to  $\Delta V_t$ .

**8.112** Refer to Fig. 8.37 and let the two input terminals be grounded. Then,

$$I_{D1} = I_{D2} = \frac{I}{2}$$

If  $Q_3$  has a threshold voltage  $V_t$  and  $Q_4$  has a threshold voltage  $V_t + \Delta V_t$  then

$$I_{D3} = \frac{I}{2} = \frac{1}{2} k_{n3}(V_{GS3} - V_t)^2$$

$$\Rightarrow V_{GS3} = V_t + \sqrt{I/k_{n3}}$$

$$I_{D4} = \frac{1}{2} k_{n4}(V_{GS4} - V_t - \Delta V_t)^2$$

Since  $k_{n4} = k_{n3}$  and  $V_{GS4} = V_{GS3}$ , we have

$$I_{D4} = \frac{1}{2} k_{n3}(V_{GS3} - V_t - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3}(\sqrt{I/k_{n3}} - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3} \frac{I}{k_{n3}} \left(1 - \frac{\Delta V_t}{\sqrt{I/k_{n3}}}\right)^2$$

$$= \frac{I}{2} \left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

The output current of the first stage will be

$$I_O = I_{D2} - I_{D4}$$

$$= \frac{I}{2} - \frac{I}{2} \left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

For  $\frac{\Delta V_t}{V_{OV3}} \ll 1$  we obtain

$$I_O \simeq \frac{I}{2} - \frac{I}{2} \left(1 - \frac{2\Delta V_t}{V_{OV3}}\right)$$

$$= \frac{2(I/2)}{V_{OV3}} \Delta V_t$$

$$= g_{m3} \Delta V_t \quad \text{Q.E.D.}$$

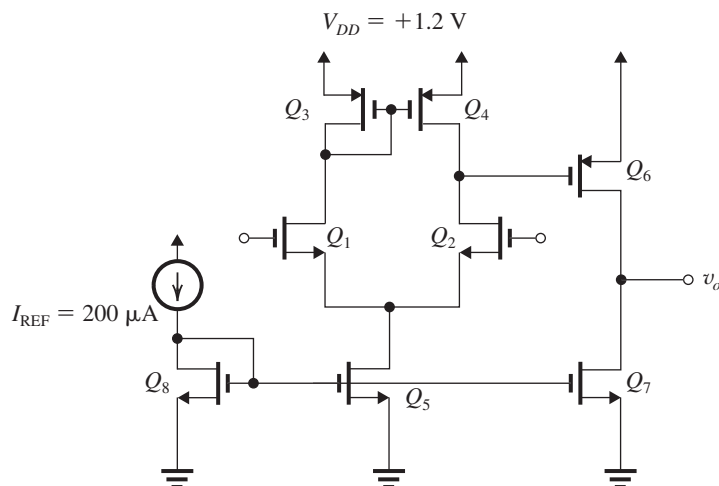
The corresponding input offset voltage will be

$$V_{OS} = \frac{I_O}{g_{m1,2}}$$

$$= \frac{g_{m3} \Delta V_t}{g_{m1,2}}$$

$$V_{OS} = \frac{g_{m3}}{g_{m1,2}} \Delta V_t$$

This figure belongs to Problem 8.113.



### 8.113

(a) For  $Q_1 - Q_4$  to conduct a current of  $200 \mu\text{A}$ , we have

$$I_{D1,2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$200 = \frac{1}{2} \times 540 \times \left( \frac{W}{L} \right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{1,2} = 32.9$$

$$I_{D3,4} = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{3,4} = 178$$

Transistor  $Q_5$  must carry a current of  $400 \mu\text{A}$ , thus

$$400 = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_5 V_{OV}^2$$

$$= \frac{1}{2} \times 540 \times \left( \frac{W}{L} \right)_5 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_5 = 65.8$$

Similarly,  $Q_7$  is required to conduct a current of  $400 \mu\text{A}$ , thus

$$\left( \frac{W}{L} \right)_7 = \left( \frac{W}{L} \right)_5 = 65.8$$

Transistor  $Q_8$  conducts a current of  $200 \mu\text{A}$ , thus

$$\left( \frac{W}{L} \right)_8 = \frac{1}{2} \left( \frac{W}{L} \right)_5 = 32.9$$

Finally,  $Q_6$  must conduct a current equal to that of  $Q_7$ , that is,  $400 \mu\text{A}$ , thus

$$400 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_6 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_6 = 356$$

The results are summarized in the following table:

Transistor	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$I_D$ ( $\mu\text{A}$ )	200	200	200	200	400	400	400	200
$W/L$	32.9	32.9	178	178	65.8	356	65.8	32.9

(b) The upper limit on  $V_{ICM}$  is determined by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$\begin{aligned} V_{ICM\max} &= V_{D1,2} + |V_t| \\ &= V_{DD} - |V_t| - |V_{OV}| + |V_t| \\ &= 1.2 - 0.15 = 1.05 \text{ V} \end{aligned}$$

The lower limit on  $V_{ICM}$  is determined by the need to keep  $Q_5$  in saturation, thus

$$\begin{aligned} V_{ICM} &= |V_{OV5}| + V_{GS1,2} \\ &= 0.15 + 0.15 + 0.35 = 0.65 \text{ V} \end{aligned}$$

Thus

$$0.65 \text{ V} \leq V_{ICM} \leq 1.05 \text{ V}$$

(c)  $0.15 \text{ V} \leq v_o \leq (1.2 - 0.15)$

that is,

$$0.15 \text{ V} \leq v_o \leq 1.05 \text{ V}$$

$$(d) g_{m1,2} = \frac{2 \times 0.2}{0.15} = 2.67 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.2 \text{ mA}} = \frac{2.4}{0.2} = 12 \text{ k}\Omega$$

$$\begin{aligned}
 A_1 &= -g_{m1,2}(r_{o2} \parallel r_{o4}) = 2.67(12 \parallel 12) \\
 &= -16 \text{ V/V} \\
 g_{m6} &= \frac{2 \times 0.4}{0.15} = 5.33 \text{ mA/V} \\
 r_{o6} = r_{o7} &= \frac{|V_A|}{0.4 \text{ mA}} = \frac{2.4}{0.4} = 6 \text{ k}\Omega \\
 A_2 &= -g_{m6}(r_{o6} \parallel r_{o7}) \\
 &= -5.33(6 \parallel 6) = 16 \text{ V/V} \\
 A_0 &= A_1 A_2 = -16 \times -16 = 256 \text{ V/V}
 \end{aligned}$$

**8.114** Refer to Fig. 8.38. From Example 8.7, we obtain

$$\begin{aligned}
 I_{C1} &= I_{C2} = 0.25 \text{ mA} \\
 I_{C4} &= I_{C5} = 1 \text{ mA} \\
 I_{C7} &= 1 \text{ mA} \\
 I_{C8} &= 5 \text{ mA}
 \end{aligned}$$

Thus

$$\begin{aligned}
 r_{e1} = r_{e2} &\simeq \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \Omega \\
 r_{e4} = r_{e5} &= \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega
 \end{aligned}$$

With 100- $\Omega$  resistance in the emitter of each of  $Q_1$  and  $Q_2$ , we have

$$\begin{aligned}
 R_{id} &= (\beta + 1)(2r_{e1,2} + 2R_{e1,2}) \\
 &= 101 \times (2 \times 0.1 + 2 \times 0.1) \\
 &= 40.4 \text{ k}\Omega
 \end{aligned}$$

Thus,  $R_{id}$  increases by a factor of 2. With 25- $\Omega$  resistance in the emitter of each of  $Q_4$  and  $Q_5$ , the input resistance of the second stage becomes

$$\begin{aligned}
 R_{i2} &= (\beta + 1)(2r_{e4,5} + 2R_{e4,5}) \\
 &= 101 (2 \times 0.025 + 2 \times 0.025) \\
 &= 10.1 \text{ k}\Omega
 \end{aligned}$$

Thus,  $R_{i2}$  is increased by a factor of 2. The gain of the first stage will be

$$\begin{aligned}
 \frac{v_{o1}}{v_{id}} &= \\
 &\frac{\alpha \times \text{Total resistance between the collectors} \\
 &\text{of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2} \\
 &\simeq \frac{40 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{2 \times 0.1 + 2 \times 0.1} = 20 \text{ V/V}
 \end{aligned}$$

Thus the gain of the first stage decreases but only slightly. Of course, the two 100- $\Omega$  resistances in the emitters reduce the gain but some of the

reduction is mitigated by the increase in  $R_{i2}$ , which increases the effective load resistance of the first stage.

The gain of the second stage will now be

$$A_2 = \frac{v_{o2}}{v_{o1}} = -\alpha \frac{3 \text{ k}\Omega \parallel R_{i3}}{2 \times 0.025 + 2 \times 0.025}$$

From Example 8.14,  $R_{i3} = 234.8 \text{ k}\Omega$ , thus

$$A_2 \simeq -\frac{3 \parallel 234.8}{0.1} = -29.6 \text{ V/V}$$

which is half the value without the two 25- $\Omega$  emitter resistances. The gain of the third stage remains unchanged at  $-6.42 \text{ V/V}$ , and the gain of the fourth stage remains unchanged at  $1 \text{ V/V}$ . Thus the overall voltage gain becomes

$$\begin{aligned}
 \frac{v_o}{v_{id}} &= A_1 A_2 A_3 A_4 \\
 &= 20 \times -29.6 \times -6.42 \times 1 \\
 &= 3800.6 \text{ V/V}
 \end{aligned}$$

which is slightly less than half the gain obtained without the emitter resistances. This is the price paid for doubling  $R_{id}$ .

**8.115** The output resistance is mostly determined by  $R_5$ . To reduce  $R_o$  by a factor of 2, we use

$$\begin{aligned}
 R_o &= \frac{152}{2} = R_6 \parallel \left[ r_{e6} + \frac{R_5}{\beta + 1} \right] \\
 76 &= 3000 \parallel \left[ 5 + \frac{R_5}{101} \right] \\
 \Rightarrow R_5 &= 7.37 \text{ k}\Omega
 \end{aligned}$$

This change in  $R_5$  will affect the gain of the third stage, which will now become

$$\begin{aligned}
 A_3 &= -\frac{R_5 \parallel (\beta + 1)(r_{e8} + R_6)}{R_4 + r_{e7}} \\
 &= -\frac{7.37 \parallel (101)(0.005 + 3)}{2.3 + 0.025} \\
 &= -3.1 \text{ V/V}
 \end{aligned}$$

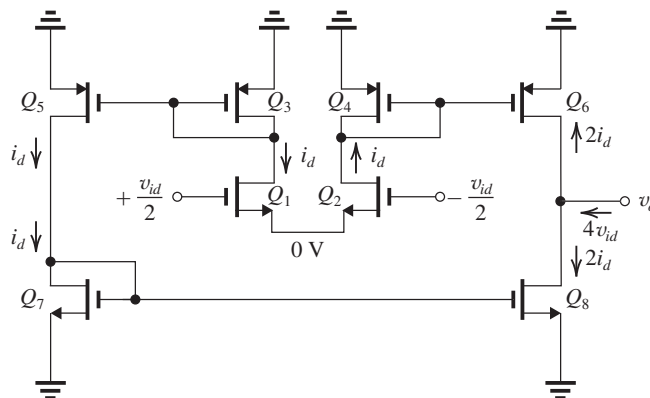
which is about half the original value (not surprising since  $R_5$  is about half its original value). Thus, the amplifier gain will be reduced by a factor of 2. To restore the gain of the third stage to its original value, we can reduce  $R_4$ . This will, however, change  $R_{i3}$  and will reduce the gain of the second stage, though only slightly. For instance, to restore the gain of the third stage to  $-6.42 \text{ V/V}$ , we use

$$\begin{aligned}
 \frac{2.3 + 0.025}{R_4 + 0.025} &= \frac{6.42}{3.1} \\
 \Rightarrow R_4 &= 1.085 \text{ k}\Omega
 \end{aligned}$$

This table belongs to Problem 8.116.

Transistor	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$I_D$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$I$	$\frac{I}{2}$	$I$
$g_m$	$\frac{I}{ V_{OV} }$	$\frac{I}{ V_{OV} }$	$\frac{I}{ V_{OV} }$	$\frac{I}{ V_{OV} }$	$\frac{I}{ V_{OV} }$	$\frac{2I}{ V_{OV} }$	$\frac{I}{ V_{OV} }$	$\frac{2I}{ V_{OV} }$
$r_o$	$\frac{2 V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{ V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{ V_A }{I}$

This figure belongs to Problem 8.116, part (b).



Now  $R_{i3} = 101 \times (1.085 + 0.025) = 112 \text{ k}\Omega$  and the gain of the second stage becomes

$$A_2 = -\frac{3 \text{ k}\Omega \parallel 112 \text{ k}\Omega}{30 \Omega} = -58.4 \text{ V/V}$$

which is a slight decrease in magnitude from the original value of  $-59.2 \text{ V/V}$ .

**8.116 (a)** Refer to Fig. P8.116. The current  $I_D$  in each of the eight transistors can be found by inspection. Then,  $g_m$  of each transistor can be determined as  $2I_D/|V_{OV}|$  and  $r_o$  as  $|V_A|/I_D$ . The results are given in the table at the top of the page.

(b) See figure above. Observe that at the output node the total signal current is  $4i_d$  where

$$i_d = g_{m1,2} \frac{v_{id}}{2} \\ = \frac{I}{2|V_{OV}|} v_{id}$$

and since the output resistance is

$$R_o = r_{o6} \parallel r_{o8} = \frac{1}{2} \frac{|V_A|}{I}$$

then

$$v_o = 4 i_d R_o = 4 \times \frac{I}{2|V_{OV}|} \times \frac{1}{2} \frac{|V_A|}{I} \times v_{id}$$

Thus

$$A_d \equiv \frac{v_o}{v_{id}} = \frac{|V_A|}{|V_{OV}|} \quad \text{Q.E.D.}$$

(c) See figure on next page. With  $v_{icm}$  applied to both input terminals, we can replace each of  $Q_1$  and  $Q_2$  with an equivalent circuit composed of a controlled current,  $v_{icm}/2R_{SS}$  in parallel with a very large output resistance ( $R_{o1}$  and  $R_{o2}$  which are equal). The resistances  $R_{o1}$  and  $R_{o2}$  will be much larger than the input resistance of each of the mirrors  $Q_3 - Q_5$  and  $Q_4 - Q_6$  and thus we can neglect  $R_{o1}$  and  $R_{o2}$  altogether. The short-circuit output current of the  $Q_4 - Q_6$  mirror will be

$$i_{o6} = \frac{g_{m6}}{g_{m4}} \left( 1 - \frac{1}{g_{m4} r_{o4}} \right) \frac{v_{icm}}{2R_{SS}} \\ = \left( 1 - \frac{|V_{OV}|}{2|V_A|} \right) \left( \frac{v_{icm}}{R_{SS}} \right)$$

and the output resistance will be  $r_{o6}$ . The short-circuit output current of the  $Q_3 - Q_5$  mirror will be

$$i_{o5} = \frac{g_{m5}}{g_{m3}} \left( 1 - \frac{1}{g_{m3} r_{o3}} \right) \frac{v_{icm}}{2R_{SS}} \\ = \left( 1 - \frac{|V_{OV}|}{2|V_A|} \right) \left( \frac{v_{icm}}{2R_{SS}} \right)$$

and the output resistance will be  $r_{o5}$ . Since  $r_{o5}$  is much larger than the input resistance of the  $Q_7 - Q_8$  mirror ( $\approx 1/g_{m7}$ ), most of  $i_{o5}$  will flow into  $Q_7$ , resulting in an output short-circuit current  $i_{o8}$ :

$$\begin{aligned}
 i_{o8} &= \frac{g_{m8}}{g_{m7}} \left(1 - \frac{1}{g_{m7}r_{o7}}\right) i_{o5} \\
 &= 2 \left(1 - \frac{1}{g_{m7}r_{o7}}\right) i_{o5} \\
 &= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right) \left(1 - \frac{1}{g_{m7}r_{o7}}\right) \frac{v_{icm}}{R_{SS}}
 \end{aligned}$$

and the output resistance is  $r_{o8}$ . Thus, at the output node we have a net current

$$\begin{aligned}
 i_{o6} - i_{o8} &= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right) \left(\frac{1}{g_{m7}r_{o7}}\right) \left(\frac{v_{icm}}{R_{SS}}\right) \\
 &\simeq \left(\frac{1}{g_{m7}r_{o7}}\right) \left(\frac{v_{icm}}{R_{SS}}\right)
 \end{aligned}$$

This current flows into the output resistance ( $r_{o6} \parallel r_{o8}$ ) and thus produces an output voltage

$$v_o = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} v_{icm}$$

and the common-mode gain becomes

$$|A_{cm}| = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} \quad \text{Q.E.D.}$$

(d)  $R_{SS} = \frac{|V_A|}{I}$

$$A_d = \left| \frac{V_A}{V_{OV}} \right|$$

$$|A_{cm}| = \frac{\frac{1}{2}|V_A|/I}{|V_A|/I} \frac{1}{[I/|V_{OV}|][2|V_A|/I]}$$

$$|A_{cm}| = \frac{1}{2} \times \frac{1}{2} \frac{|V_{OV}|}{|V_A|} = \frac{1}{4} \left| \frac{V_{OV}}{V_A} \right|$$

$$\text{CMRR} = 4 \left| \frac{V_A}{V_{OV}} \right|^2 \quad \text{Q.E.D.}$$

(e) The upper limit on  $V_{ICM}$  is determined by  $Q_1$  and  $Q_2$  remaining in saturation, thus

$$\begin{aligned}
 V_{ICM_{\max}} &= V_{DD} - |V_{SG}| + |V_t| \\
 &= V_{DD} - |V_{OV}|
 \end{aligned}$$

The lower limit on  $V_{ICM}$  is determined by the need to keep the bias current source in saturation, i.e. maintaining a minimum voltage across it of  $|V_{OV}|$ , thus

$$\begin{aligned}
 V_{ICM_{\min}} &= -V_{DD} + |V_{OV}| + |V_{GS}| \\
 &= -V_{DD} + 2|V_{OV}| + |V_t|
 \end{aligned}$$

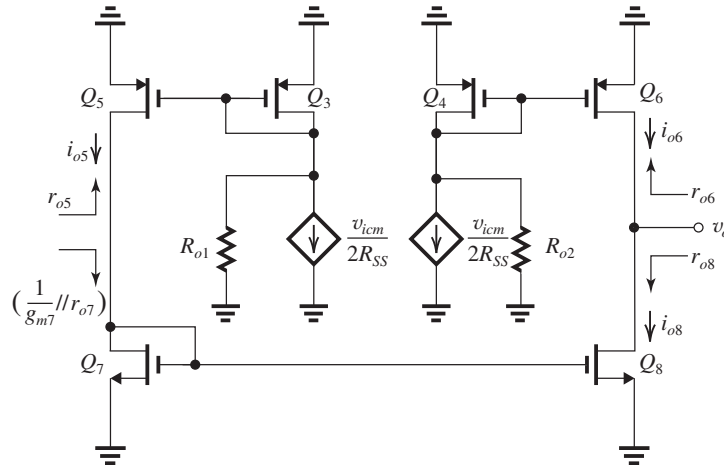
Thus

$$-V_{DD} + |V_t| + 2|V_{OV}| \leq V_{ICM} \leq V_{DD} - |V_{OV}|$$

The output linear range is

$$-V_{DD} + |V_{OV}| \leq v_o \leq +V_{DD} - |V_{OV}|$$

This figure belongs to Problem 8.116, part (c).



## Chapter 9

## Solutions to Exercises within the Chapter

$$\text{Ex: 9.1 } C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11} \text{ F/m}}{3 \times 10^{-9} \text{ m}}$$

$$= 11.5 \times 10^{-3} \text{ F/m}^2$$

$$= 11.5 \text{ fF}/\mu\text{m}^2$$

$$C_{ov} = WL_{ov}C_{ox}$$

$$= 1.5 \times 0.03 \times 11.5 = 0.52 \text{ fF}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov}$$

$$= \frac{2}{3} \times 1.5 \times 0.15 \times 11.5 + 0.52$$

$$= 2.25 \text{ fF}$$

$$C_{gd} = C_{ov} = 0.52 \text{ fF}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} = \frac{1}{\sqrt{1 + \frac{0.2}{0.9}}} = 0.9 \text{ fF}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} = \frac{1}{\sqrt{1 + \frac{0.8 + 0.2}{0.9}}} = 0.7 \text{ fF}$$

$$\text{Ex: 9.2 } g_m = \sqrt{2k'_n(W/L)I_D}$$

$$= \sqrt{2 \times 0.5 \times (1.5/0.15) \times 0.1} = 1 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1 \times 10^{-3}}{2\pi(2.25 + 0.52) \times 10^{-15}}$$

$$f_T = 57.5 \text{ GHz}$$

$$\text{Ex: 9.3 } C_{de} = \tau_F g_m$$

where

$$\tau_F = 20 \text{ ps}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

Thus,

$$C_{de} = 20 \times 10^{-12} \times 40 \times 10^{-3} = 0.8 \text{ pF}$$

$$C_{je} \simeq 2C_{je0}$$

$$= 2 \times 20 = 40 \text{ fF}$$

$$C_\pi = C_{de} + C_{je}$$

$$= 0.8 + 0.04 = 0.84 \text{ pF}$$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$= \frac{20}{\left(1 + \frac{2}{0.5}\right)^{0.33}} = 12 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47 \text{ GHz}$$

$$\text{Ex: 9.4 } |\beta| = 10 \text{ at } f = 50 \text{ MHz}$$

Thus,

$$f_T = 10 \times 50 = 500 \text{ MHz}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$$= 12.7 \text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$$

$$\text{Ex: 9.5 } C_\pi = C_{de} + C_{je}$$

$$10.7 = C_{de} + 2$$

$$\Rightarrow C_{de} = 8.7 \text{ pF}$$

Since  $C_{de}$  is proportional to  $g_m$  and hence  $I_C$ , at  $I_C = 0.1 \text{ mA}$ ,

$$C_{de} = 0.87 \text{ pF}$$

and

$$C_\pi = 0.87 + 2 = 2.87 \text{ pF}$$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}} = 130.7 \text{ MHz}$$

$$\text{Ex: 9.6 } A_M = -g_m R'_L$$

where

$$g_m = 2 \text{ mA/V}$$

$$R'_L = R_L \parallel r_o = 10 \parallel 20 = 6.67 \text{ k}\Omega$$

Thus,

$$A_M = -2 \times 6.67 = -13.3 \text{ V/V}$$

$$C_{eq} = (1 + g_m R'_L)C_{gd}$$

$$= (1 + 13.3) \times 5 = 71.5 \text{ fF}$$

$$C_{in} = C_{gs} + C_{eq} = 20 + 71.5 = 91.5$$

$$f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

$$= \frac{1}{2\pi \times 91.5 \times 10^{-15} \times 20 \times 10^3}$$

$$= 86.9 \text{ MHz}$$

$$\text{Ex: 9.7 } f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

For  $f_H \geq 100 \text{ MHz}$ ,

$$C_{in} \leq \frac{1}{2\pi \times 100 \times 10^6 \times 20 \times 10^3} = 79.6 \text{ fF}$$

But,

$$C_{in} = C_{gs} + C_{eq} = 20 + C_{eq}$$

Thus,

$$C_{eq} \leq 59.6 \text{ fF}$$

$$C_{gd}(1 + g_m R'_L) \leq 59.6$$

$$C_{gd} \leq \frac{59.6}{1 + 20} = 2.8 \text{ fF}$$

**Ex: 9.8** Refer to Example 9.2.

To move  $f_i$  from 10.6 GHz to 3 GHz,  $f_H$  must be moved from 530.5 MHz to

$$f_H = 530.5 \times \frac{3}{10.6} = 150.1 \text{ MHz}$$

Since,

$$f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

then

$$C_L + C_{gd} = \frac{1}{2\pi \times 150.1 \times 10^6 \times 10 \times 10^3} = 106 \text{ fF}$$

$$C_L = 106 - 5 = 101 \text{ fF}$$

**Ex: 9.9** To reduce the midband gain to half the value found, we reduce  $R'_L$  by the same factor, thus

$$R'_L = \frac{4.76}{2} = 2.38 \text{ k}\Omega$$

But,

$$R'_L = R_L \parallel r_o$$

$$2.38 = R_L \parallel 100$$

$$\Rightarrow R_L = 2.44 \text{ k}\Omega$$

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 7 + 1(1 + 40 \times 2.38)$$

$$= 103.2 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 103.2 \times 10^{-12} \times 1.67 \times 10^3}$$

$$= 923 \text{ MHz}$$

Thus, by accepting a reduction in gain by a factor of 2, the bandwidth is increased by a factor of  $923/480 = 1.9$ , approximately the same factor as the reduction in gain.

$$\text{Ex: 9.10 } T(s) = \frac{1000}{1 + \frac{s}{2\pi \times 10^5}}$$

$$\text{GB} = 1000 \times 100 \times 10^3 = 10^8 \text{ Hz}$$

$$\text{Ex: 9.11 } |A_M| = g_m R'_L = 2 \times 10 = 20 \text{ V/V}$$

$$\text{GB} = |A_M| f_H$$

$$= 20 \times 56.8 = 1.14 \text{ GHz}$$

$$\text{Ex: 9.12 } |A_M| = \frac{1}{2} \times 20 = 10 \text{ V/V}$$

$$R_{gs} = 20 \text{ k}\Omega$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 10) + 5 = 225 \text{ k}\Omega$$

$$R_{C_L} = R'_L = 5 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs} = 20 \times 10^{-15} \times 20 \times 10^3 = 400 \text{ ps}$$

$$\tau_{gd} = C_{gd} R_{gd} = 5 \times 10^{-15} \times 225 \times 10^3 = 1125 \text{ ps}$$

$$\tau_{C_L} = C_L R_{C_L} = 25 \times 10^{-15} \times 5 \times 10^3 = 125 \text{ ps}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= 400 + 1125 + 125$$

$$= 1650 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 1650 \times 10^{-12}} = 96.5 \text{ MHz}$$

$$\text{GB} = 10 \times 96.5 = 0.965 \text{ GHz}$$

$$\text{Ex: 9.13 } g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Since  $I_D$  is increased by a factor of 4,  $g_m$  doubles:

$$g_m = 2 \times 2 = 4 \text{ mA/V}$$

Since  $R'_L$  is  $r_o/2$ , increasing  $I_D$  by a factor of four results in  $r_o$  and hence  $R'_L$  decreasing by a factor of 4, thus

$$R'_L = \frac{1}{4} \times 10 = 2.5 \text{ k}\Omega$$

$$|A_M| = g_m R'_L = 4 \times 2.5 = 10 \text{ V/V}$$

$$R_{gs} = R_{sig} = 20 \text{ k}\Omega$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 10) + 2.5$$

$$= 222.5 \text{ k}\Omega$$

$$R_{C_L} = R'_L = 2.5 \text{ k}\Omega$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= C_{gs} R_{sig} + C_{gd} R_{gd} + C_L R_{C_L}$$

$$= 20 \times 10^{-15} \times 20 \times 10^3 + 5 \times 10^{-15} \times 222.5 \times 10^3 + 25 \times 10^{-15} \times 2.5 \times 10^3$$

$$= 400 + 1112.5 + 62.5$$

$$= 1575 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 1575 \times 10^{-12}} = 101 \text{ MHz}$$

$$\text{GB} = |A_M| f_H$$

$$= 10 \times 101 = 1.01 \text{ GHz}$$

**Ex: 9.14** (a)  $g_m = 40 \text{ mA/V}$

$$r_\pi = \frac{200}{40} = 5 \text{ k}\Omega$$

$$r_{on} = \frac{V_{An}}{I} = \frac{130}{1} = 130 \text{ k}\Omega$$

$$r_{op} = \frac{|V_{Ap}|}{I} = \frac{50}{1} = 50 \text{ k}\Omega$$

$$R'_L = r_{on} \parallel r_{op} = 130 \parallel 50 = 36.1 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R'_L$$

$$= -\frac{5}{5 + 36} \times 40 \times 36.1$$

$$= -176 \text{ V/V}$$

(b)  $C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$$= 16 + 0.3(1 + 40 \times 36.1)$$

$$= 450 \text{ pF}$$

$$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}}$$

$$= 5 \parallel 36 = 4.39 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 450 \times 10^{-12} \times 4.39 \times 10^3}$$

$$= 80.6 \text{ kHz}$$

(c)  $R_\pi = R'_{\text{sig}} = 4.39 \text{ k}\Omega$

$$R_\mu = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$$

$$= 4.39(1 + 40 \times 36.1) + 36.1$$

$$= 6.38 \text{ M}\Omega$$

$$R_{CL} = R'_L = 36.1 \text{ k}\Omega$$

$$\tau_H = C_\pi + C_\mu R_\mu + C_L R_{CL}$$

$$= 16 \times 4.39 + 0.3 \times 6.38 \times 10^3 + 5 \times 36.1$$

$$= 70.2 + 1914 + 180.5$$

$$= 2164.7 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 2164.7 \times 10^{-9}}$$

$$= 73.5 \text{ kHz}$$

$$(d) f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 0.3 \times 10^{-12}} = 21.2 \text{ GHz}$$

(e)  $\text{GB} = 175 \times 73.5 = 12.9 \text{ MHz}$

$$\text{Ex: 9.15 } R_{\text{in}} = \frac{R_L + r_o}{1 + g_m r_o}$$

$$= \frac{800 + 20}{1 + 40} \simeq 20 \text{ k}\Omega$$

$$G_v = \frac{R_L}{R_{\text{sig}} + R_{\text{in}}} = \frac{800}{20 + 20} = 20 \text{ V/V}$$

$$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}} = 20 \parallel 20 = 10 \text{ k}\Omega$$

$$R_{gd} = R_L \parallel R_o$$

$$= 800 \parallel 840 = 410 \text{ k}\Omega$$

$$\tau_H = C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd}$$

$$= 20 \times 10^{-15} \times 10 \times 10^3 + (5 + 25) \times 10^{-15} \times 410 \times 10^3$$

$$= 200 + 12,300$$

$$= 12,500 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 12,500 \times 10^{-12}} = 12.7 \text{ MHz}$$

Thus, while the midband gain has been increased substantially (by a factor of 21), the bandwidth has been substantially lowered (by a factor of 20.7). Thus, the high-frequency advantage of the CG amplifier is completely lost!

**Ex: 9.16** (a)  $A_{CS} = -g_m(R_L \parallel r_o)$

$$= -g_m(r_o \parallel r_o) = -\frac{1}{2} g_m r_o$$

$$= -\frac{1}{2} \times 40 = -20 \text{ V/V}$$

$$A_{\text{cascode}} = -g_m(R_L \parallel R_o)$$

$$= -g_m(r_o \parallel g_m r_o r_o)$$

$$\simeq -g_m r_o = -40 \text{ V/V}$$

Thus,

$$\frac{A_{\text{cascode}}}{A_{CS}} = 2$$

(b) For the CS amplifier,

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd}$$

where

$$R_{gs} = R_{\text{sig}}$$

$$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$$

$$\simeq R_{\text{sig}}(1 + g_m R'_L)$$

$$= R_{\text{sig}} \left( 1 + \frac{1}{2} g_m r_o \right)$$

$$= R_{\text{sig}} \left( 1 + \frac{1}{2} \times 40 \right) = 21 R_{\text{sig}}$$



$$\begin{aligned}\tau_H &= C_{gs}R_{sig} + C_{gd} \times 21R_{sig} \\ &= C_{gs}R_{sig} + 0.25C_{gs} \times 21R_{sig} \\ &= 6.25C_{gs}R_{sig}\end{aligned}$$

$$f_H = \frac{1}{2\pi \times 6.25C_{gs}R_{sig}}$$

For the cascode amplifier,

$$\tau_H \simeq R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})]$$

where

$$R_{d1} = r_{o1} \parallel R_{in2} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{\frac{g_m}{2} + r_o} = \frac{2}{\frac{g_m}{2} + r_o} r_o$$

$$= \frac{2r_o}{2 + g_m r_o} = \frac{2r_o}{2 + 40} = \frac{r_o}{21}$$

$$\tau_H = C_{gs}R_{sig} \left[ 1 + 0.25 \left( 1 + \frac{g_m r_o}{21} \right) \right]$$

$$= C_{gs}R_{sig} \left[ 1 + 0.25 \left( 1 + \frac{40}{21} \right) \right]$$

$$= 1.73C_{gs}R_{sig}$$

$$f_H = \frac{1}{2\pi \times 1.73 C_{gs}R_{sig}}$$

Thus,

$$\frac{f_H(\text{cascode})}{f_H(\text{CS})} = \frac{6.25}{1.73} = 3.6$$

$$(c) \frac{f_i(\text{cascode})}{f_i(\text{CS})} = 2 \times 3.6 = 7.2$$

**Ex: 9.17**  $g_m = 40 \text{ mA/V}$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \text{ k}\Omega$$

$$A_0 = g_m r_o$$

$$= 40 \times 130 = 5200 \text{ V/V}$$

$$R_{o1} = r_{o1} = 130 \text{ k}\Omega$$

$$R_{in2} = r_{\pi 2} \parallel \frac{r_{o2} + R_L}{g_{m2} r_{o2}}$$

$$= 5 \parallel \frac{130 + 50}{5200}$$

$$= 35 \text{ }\Omega$$

$$R_o \simeq \beta_2 r_{o2} = 200 \times 130 = 26 \text{ M}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + R_{sig}} g_m (R_o \parallel R_L)$$

$$= -\frac{5}{5 + 36} 40(26,000 \parallel 50)$$

$$A_M = -244 \text{ V/V}$$

$$R'_{sig} = r_{\pi 1} \parallel R_{sig}$$

$$= 5 \parallel 36 = 4.39 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 4.39 \text{ k}\Omega$$

$$R_{c1} = r_{o1} \parallel R_{in2}$$

$$= 130 \text{ k}\Omega \parallel 35 \text{ }\Omega \simeq 35 \text{ }\Omega$$

$$R_{\mu 1} = R'_{sig}(1 + g_{m1}R_{c1}) + R_{c1}$$

$$= 4.39(1 + 40 \times 0.035) + 0.035$$

$$= 10.6 \text{ k}\Omega$$

$$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + C_{\pi 2}R_{c1}$$

$$+ (C_L + C_{\mu 2})(R_L \parallel R_o)$$

$$= 16 \times 4.39 + 0.3 \times 10.6 + 16 \times 0.035$$

$$+ (5 + 0.3)(50 \parallel 26,000)$$

$$= 70.24 + 3.18 + 0.56 + 264.5$$

$$= 338.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 338.5 \times 10^{-9}} = 470 \text{ kHz}$$

$$f_i = |A_M|f_H = 244 \times 470 = 113.8 \text{ MHz}$$

Thus, in comparison to the CE amplifier of Exercise 9.19, we see that  $|A_M|$  has increased from 175 V/V to 242 V/V,  $f_H$  has increased from 73.5 kHz to 470 kHz, and  $f_i$  has increased from 12.9 MHz to 113.8 MHz.

To have  $f_H$  equal to 1 MHz,

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 1 \times 10^6} = 159.2 \text{ ns}$$

Thus,

$$159.2 = 70.24 + 3.18 + 0.56$$

$$+ (C_L + C_\mu)(50 \parallel 26,000)$$

$$\Rightarrow C_L + C_\mu = 1.71 \text{ pF}$$

Thus,  $C_L$  must be reduced to 1.41 pF.

**Ex: 9.18** From Eq. (9.103), we obtain

$$R_{gs} = \frac{R_{sig}}{g_m R'_L + 1} + \frac{R'_L}{g_m R'_L + 1} = \frac{R_{sig} + R'_L}{g_m R'_L + 1}$$

$$R_{gd} = R_{sig}$$

$$R_{cL} = \frac{R'_L}{g_m R'_L + 1}$$

**Ex: 9.19** From Example 9.8, we get

$$\tau_H = b_1 = 104 \text{ ps}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 104 \times 10^{-12}} = 1.53 \text{ GHz}$$

This is lower than the exact value found in Example 9.8 (i.e., 1.86 GHz) by about 18%, still not a bad estimate!

**Ex: 9.20**  $g_m = 40 \text{ mA/V}$

$$r_e = 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R_{\text{sig}} = 1 \text{ k}\Omega$$

$$R'_L = R_L \parallel r_o = 1 \parallel 100 = 0.99 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \frac{R'_{\text{sig}}}{\beta + 1}}$$

$$= \frac{0.99}{0.99 + 0.025 + (1/101)} = 0.97 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 15.9 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 13.9 \text{ pF}$$

$$f_Z = \frac{1}{2\pi C_\pi r_e} = \frac{1}{2\pi \times 13.9 \times 10^{-12} \times 25}$$

$$= 458 \text{ MHz}$$

$$b_1 = \frac{\left[ C_\pi + C_\mu \left( 1 + \frac{R'_L}{r_e} \right) \right] R_{\text{sig}} + \left[ C_\pi + C_L \left( 1 + \frac{R_{\text{sig}}}{r_\pi} \right) \right] R'_L}{1 + \frac{R'_L}{r_e} + \frac{R_{\text{sig}}}{r_\pi}}$$

$$= \frac{\left[ 13.9 + 2 \left( 1 + \frac{0.99}{0.025} \right) \right] \times 1 + (13.9 + 0)0.99}{1 + \frac{0.99}{0.025} + \frac{1}{2.5}}$$

$$= 2.66 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{C_\pi C_\mu R_L R_{\text{sig}}}{1 + \frac{R'_L}{r_e} + \frac{R_{\text{sig}}}{r_\pi}} = \frac{13.9 \times 2 \times 0.99 \times 1}{1 + \frac{0.99}{0.025} + \frac{1}{2.5}}$$

$$= 0.671 \times 10^{-18}$$

$\omega_{p1}$  and  $\omega_{p2}$  are the roots of the equation

$$1 + b_1 s + b_2 s^2 = 0$$

Solving we obtain,

$$f_{p1} = 67.2 \text{ MHz}$$

$$f_{p2} = 562 \text{ MHz}$$

Since  $f_{p1} \ll f_{p2}$ ,

$$f_H \simeq f_{p1} = 67.2 \text{ MHz}$$

**Ex: 9.21** (a)  $I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

(b)  $A_d = g_m(R_D \parallel r_o)$

where

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$A_d = 4(5 \parallel 50) = 4 \times 4.545$$

$$= 18.2 \text{ V/V}$$

(c)  $f_H = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 4.545 \times 10^3}$$

$$= 292 \text{ MHz}$$

(d)  $\tau_{gs} = C_{gs} R_{\text{sig}} = 50 \times 10 = 500 \text{ ps}$

$$\tau_{gd} = C_{gd} R_{gd} = C_{gd} [R_{\text{sig}} (1 + g_m R'_L) + R'_L]$$

$$= 10 [10(1 + 18.2) + 4.545]$$

$$= 1965.5 \text{ ps}$$

$$\tau_{C_L} = (C_L + C_{db}) R'_L = 110 \times 4.545 = 500 \text{ ps}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= 500 + 1965.5 + 500 = 2965.5 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 2965.5 \times 10^{-12}}$$

$$= 53.7 \text{ MHz}$$

**Ex: 9.22**  $f_Z = \frac{1}{2\pi R_{SS} C_{SS}}$

$$= \frac{1}{2\pi \times 75 \times 10^3 \times 0.4 \times 10^{-12}}$$

$$= 5.3 \text{ MHz}$$

Thus, the 3-dB frequency of the CMRR is 5.3 MHz.

**Ex: 9.23**  $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$$g_{m1,2} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$A_d = 20(200 \parallel 200) = 2000 \text{ V/V}$$

The dominant high-frequency pole is that introduced at the output node,

$$\begin{aligned} f_H &= \frac{1}{2\pi C_L(r_{o2} \parallel r_{o4})} \\ &= \frac{1}{2\pi \times 2 \times 10^{-12} \times 100 \times 10^3} \\ &= 0.8 \text{ MHz} \end{aligned}$$

**Ex: 9.24** (a)  $A_M = -g_m R'_L$

where

$$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10 \text{ k}\Omega$$

$$A_M = -2 \times 10 = -20 \text{ V/V}$$

$$\begin{aligned} \tau_H &= C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R'_L \\ &= C_{gs}R_{sig} + C_{gd}[R_{sig}(1 + g_m R'_L) + R'_L] + C_L R'_L \\ &= 20 \times 20 + 5[20(1 + 20) + 10] + 5 \times 10 \\ &= 400 + 2150 + 50 \\ &= 2600 \text{ ps} \end{aligned}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 2600 \times 10^{-12}} \\ &= 61.2 \text{ MHz} \end{aligned}$$

$$GB = |A_M|f_H$$

$$= 20 \times 61.2$$

$$= 1.22 \text{ GHz}$$

$$(b) G_m = \frac{g_m}{1 + g_m R_s} = \frac{2}{1 + 2} = 0.67 \text{ mA/V}$$

$$R_o \simeq r_o(1 + g_m R_s)$$

$$= 20 \times 3 = 60 \text{ k}\Omega$$

$$R'_L = R_L \parallel R_o = 20 \parallel 60 = 15 \text{ k}\Omega$$

$$A_M = -G_m R'_L$$

$$= -0.67 \times 15 = -10 \text{ V/V}$$

$$R_{gd} = R_{sig}(1 + G_m R'_L) + R'_L$$

$$= 20(1 + 10) + 15$$

$$= 235 \text{ k}\Omega$$

$$R_{CL} = R'_L = 15 \text{ k}\Omega$$

$$R_{gs} = \frac{R_{sig} + R_s + R_{sig}R_s/(r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)}$$

where

$$R_s = \frac{2}{g_m} = 1 \text{ k}\Omega$$

$$R_{gs} = \frac{20 + 1 + \frac{20 \times 1}{20 + 20}}{1 + 2 \times \frac{20}{20 + 20}}$$

$$= 10.75 \text{ k}\Omega$$

$$\begin{aligned} \tau_H &= C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{CL} \\ &= 20 \times 10.75 + 5 \times 235 + 5 \times 15 \\ &= 215 + 1175 + 75 = 1465 \text{ ps} \end{aligned}$$

$$f_H = \frac{1}{2\pi \times 1465 \times 10^{-12}} = 109 \text{ MHz}$$

$$GB = 10 \times 109 = 1.1 \text{ GHz}$$

**Ex: 9.25** Refer to Fig. 9.35(b).

$$A_M = \frac{2r_\pi}{2r_\pi + R_{sig}} \times \frac{1}{2} \times g_m R_L$$

where

$$g_m = 20 \text{ mA/V}$$

$$r_\pi = \frac{100}{20} = 5 \text{ k}\Omega$$

$$A_M = \frac{10}{10 + 10} \times \frac{1}{2} \times 20 \times 10 = 50 \text{ V/V}$$

$$f_{p1} = \frac{1}{2\pi \left( \frac{C_\pi}{2} + C_\mu \right) (2r_\pi \parallel R_{sig})}$$

$$= \frac{1}{2\pi \left( \frac{6}{2} + 2 \right) \times 10^{-12} (10 \parallel 10) \times 10^3}$$

$$= 6.4 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi C_\mu R_L}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$$= 8 \text{ MHz}$$

$$T(s) = \frac{50}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$|T(j\omega)| = \frac{50}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{p1}}\right)^2\right] \left[1 + \left(\frac{\omega}{\omega_{p2}}\right)^2\right]}}$$

At  $\omega = \omega_H$ ,  $|T| = 50/\sqrt{2}$ , thus

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right] \left[1 + \left(\frac{\omega_H}{\omega_{p2}}\right)^2\right]$$

$$\begin{aligned}
 &= 1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2 \left(\frac{\omega_H}{\omega_{P2}}\right)^2 \\
 &\frac{\omega_H^4}{\omega_{P1}^2 \omega_{P2}^2} + \omega_H^2 \left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}\right) - 1 = 0 \\
 &\frac{f_H^4}{f_{P1}^2 f_{P2}^2} + f_H^2 \left(\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}\right) - 1 = 0 \\
 &\frac{f_H^4}{6.4^2 \times 8^2} + f_H^2 \left(\frac{1}{6.4^2} + \frac{1}{8^2}\right) - 1 = 0 \\
 &\Rightarrow f_H = 4.6 \text{ MHz (Exact value)}
 \end{aligned}$$

Using Eq. (9.150), an approximate value for  $f_H$  can be obtained:

$$\begin{aligned}
 f_H &\simeq 1/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}} \\
 &= 1/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}
 \end{aligned}$$

**Ex: 9.26**  $A_M = -\frac{R_G}{R_G + R_{\text{sig}}} g_m (R_D \parallel R_L)$

$$\begin{aligned}
 &= -\frac{10}{10 + 0.1} \times 2(10 \parallel 10) \\
 &= -9.9 \text{ V/V} \\
 f_{P1} &= \frac{1}{2\pi C_{C1}(R_{\text{sig}} + R_G)} \\
 &= \frac{1}{2\pi \times 1 \times 10^{-6}(0.1 + 10) \times 10^6} \\
 &= 0.016 \text{ Hz} \\
 f_{P2} &= \frac{g_m + 1/R_S}{2\pi C_S} \\
 &= \frac{(2 + 0.1) \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 334.2 \text{ Hz} \\
 f_{P3} &= \frac{1}{2\pi C_{C2}(R_D + R_L)} \\
 &= \frac{1}{2\pi \times 1 \times 10^{-6}(10 + 10) \times 10^3}
 \end{aligned}$$

$$\begin{aligned}
 &= 8 \text{ Hz} \\
 f_Z &= \frac{1}{2\pi C_S R_S} \\
 &= \frac{1}{2\pi \times 1 \times 10^{-6} \times 10 \times 10^3} \\
 &= 15.9 \text{ Hz}
 \end{aligned}$$

Since the highest-frequency pole is  $f_{P2} = 334.2$  and the next highest-frequency singularity is  $f_Z$  at 15.9 Hz, the lower 3-dB frequency  $f_L$  will be

$$f_L \simeq f_{P2} = 334.2 \text{ Hz}$$

**Ex: 9.27** Refer to Fig. 9.42.

$$\begin{aligned}
 \tau_{C1} &= C_{C1} [R_{\text{sig}} + (R_B \parallel r_\pi)] \\
 &= 1 \times 10^{-6} [5 + (100 \parallel 2.5)] \times 10^3 \\
 &= 7.44 \text{ ms} \\
 \tau_{CE} &= C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right) \right] \\
 \beta &= g_m r_\pi = 40 \times 2.5 = 100 \\
 r_e &\simeq 1/g_m = 25 \Omega \\
 \tau_{CE} &= 1 \times 10^{-6} \left[ 5 \parallel \left( 0.025 + \frac{100 \parallel 5}{101} \right) \right] \times 10^3 \\
 \tau_{CE} &= 0.071 \text{ ms} \\
 \tau_{C2} &= C_{C2} (R_C + R_L) \\
 &= 1 \times 10^{-6} (8 + 5) \times 10^3 \\
 &= 13 \text{ ms} \\
 f_L &= \frac{1}{2\pi} \left[ \frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}} \right] \\
 &= \frac{1}{2\pi} \left[ \frac{1}{7.44} + \frac{1}{0.071} + \frac{1}{13} \right] \times 10^3 \\
 &= 2.28 \text{ kHz} \\
 f_Z &= \frac{1}{2\pi C_E R_E} \\
 &= \frac{1}{2\pi \times 1 \times 10^{-6} \times 5 \times 10^3} = 31.8 \text{ Hz}
 \end{aligned}$$

Since  $f_Z$  is much lower than  $f_L$  it will have a negligible effect on  $f_L$ .

**Solutions to End-of-Chapter Problems**

$$\begin{aligned}
\mathbf{9.1} \quad C_{ox} &= \frac{\epsilon_{ox}}{t_{ox}} \\
&= \frac{3.45 \times 10^{-11} \text{ F/m}}{4 \times 10^{-9} \text{ m}} = 0.86 \times 10^{-2} \text{ F/m}^2 \\
&= 0.86 \times 10^{-2} \times 10^{-12} \text{ F}/\mu\text{m}^2 \\
&= 8.6 \text{ fF}/\mu\text{m}^2 \\
k'_n &= \mu_n C_{ox} \\
&= 450 \times 10^8 (\mu\text{m}^2/\text{V}\cdot\text{s}) \\
&\quad \times 8.6 \times 10^{-15} \text{ F}/\mu\text{m}^2 \\
&= 387 \mu\text{A}/\text{V}^2 \\
I_D &= \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 (1 + \lambda V_{DS}) \\
200 &= \frac{1}{2} \times 387 \times 40 \times V_{OV}^2 (1 + 0.1 \times 0.7) \\
\Rightarrow V_{OV} &= 0.155 \text{ V} \\
g_m &= \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.155} = 2.6 \text{ mA/V} \\
\chi &= \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} \\
&= \frac{0.5}{2\sqrt{0.85 + 0.35}} = 0.23 \\
g_{mb} &= \chi g_m = 0.6 \text{ mA/V} \\
r_o &= \frac{|V_A|}{I_D} = \frac{1}{|\lambda| I_D} = \frac{1}{0.1 \times 0.2} = 50 \text{ k}\Omega \\
C_{gs} &= \frac{2}{3} W L C_{ox} + W L_{ov} C_{ox} \\
&= \frac{2}{3} \times 12 \times 0.3 \times 8.6 + 12 \times 0.03 \times 8.6 \\
&= 20.64 + 3.1 = 23.7 \text{ fF} \\
C_{gd} &= W L_{ov} C_{ox} = 12 \times 0.03 \times 8.6 \\
&= 3.1 \text{ fF} \\
C_{sb} &= \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}} \\
&= \frac{5}{\sqrt{1 + \frac{0.35}{0.9}}} = 4.2 \text{ fF} \\
C_{db} &= \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}} \\
&= \frac{5}{\sqrt{1 + \frac{1.05}{0.9}}} = 3.4 \text{ fF}
\end{aligned}$$

$$\begin{aligned}
f_T &= \frac{g_m}{2\pi(C_{gs} + C_{gd})} \\
&= \frac{2.6 \times 10^{-3}}{2\pi(23.7 + 3.1) \times 10^{-15}} = 15.4 \text{ GHz}
\end{aligned}$$

$$\begin{aligned}
\mathbf{9.2} \quad g_m &= \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.3} = 1.33 \text{ mA/V} \\
f_T &= \frac{g_m}{2\pi(C_{gs} + C_{gd})} \\
&= \frac{1.33 \times 10^{-3}}{2\pi \times (25 + 5) \times 10^{-15}} = 7.1 \text{ GHz}
\end{aligned}$$

$$\mathbf{9.3} \quad f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$C_\pi = 8 \text{ pF}$$

$$C_\mu = 1 \text{ pF}$$

Thus,

$$f_T = \frac{20 \times 10^{-3}}{2\pi \times (8 + 1) \times 10^{-12}} = 353.7 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta} = \frac{353.7}{100} = 3.54 \text{ MHz}$$

**9.4** For  $f \gg f_\beta$ ,

$$|\beta| \simeq \frac{f_T}{f}$$

At  $f = 50 \text{ MHz}$  and  $I_C = 0.2 \text{ mA}$ ,

$$|\beta| = 10 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 500 \text{ MHz}$$

At  $f = 50 \text{ MHz}$  and  $I_C = 1.0 \text{ mA}$ ,

$$|\beta| = 12 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 600 \text{ MHz}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$C_\pi = C_{de} + C_{je}$$

$$= \tau_F g_m + C_{je}$$

$$C_\mu = 0.1 \text{ pF}$$

At  $I_C = 0.2 \text{ mA}$ ,  $g_m = \frac{0.2}{0.025} = 8 \text{ mA/V}$ , thus

$$500 \times 10^6 = \frac{8 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_{\pi} = 2.45 \text{ pF}$$

$$\tau_F \times 8 \times 10^{-3} + C_{je} = 2.45 \times 10^{-12} \quad (1)$$

At  $I_C = 1 \text{ mA}$ ,  $g_m = \frac{1}{0.025} = 40 \text{ mA/V}$ , thus

$$600 \times 10^6 = \frac{40 \times 10^{-3}}{2\pi(C_{\pi} + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_{\pi} = 10.51 \text{ pF}$$

$$\tau_F \times 40 \times 10^{-3} + C_{je} = 10.51 \times 10^{-12} \quad (2)$$

Solving (1) together with (2) yields

$$\tau_F = 252 \text{ ps}$$

$$C_{je} = 0.43 \text{ pF}$$

$$\mathbf{9.5} \quad f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$10 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_{\pi} + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_{\pi} = 0.54 \text{ pF}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$f_{\beta} = \frac{f_T}{\beta_0} = \frac{10 \times 10^9}{120} = 83.3 \text{ MHz}$$

$$\mathbf{9.6} \quad f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

For  $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component ( $WL_{ov}C_{ox}$ ) is small, we get

$$C_{gs} \simeq \frac{2}{3}WLC_{ox} \quad (2)$$

The transconductance  $g_m$  is given by

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} \quad (3)$$

Substituting from (2) and (3) into (1), we get

$$f_T = \frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}}{2\pi \times \frac{2}{3}WLC_{ox}}$$

$$= \frac{1.5}{\pi L} \sqrt{\frac{\mu_n I_D}{2C_{ox}WL}} \quad \text{Q.E.D.}$$

We observe that for a given device,  $f_T$  is proportional to  $\sqrt{I_D}$ ; thus to obtain faster operation the MOSFET is operated at a higher  $I_D$ .

Also, we observe that  $f_T$  is inversely proportional to  $L\sqrt{WL}$ ; thus faster operation is obtained from smaller devices.

$$\mathbf{9.7} \quad f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

For  $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component is small, we get

$$C_{gs} \simeq \frac{2}{3}WLC_{ox} \quad (2)$$

The transconductance  $g_m$  can be expressed as

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV} \quad (3)$$

Substituting from (2) and (3) into (1), we obtain

$$f_T = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}}{2\pi \times \frac{2}{3}WLC_{ox}}$$

$$= \frac{3\mu_n V_{OV}}{4\pi L^2}$$

We note that for a given channel length,  $f_T$  can be increased by operating the MOSFET at a higher  $V_{OV}$ .

For  $L = 0.5 \mu\text{m}$  and  $\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{s}$ ,

we have

$$V_{OV} = 0.2 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.2}{4\pi \times 0.5^2}$$

$$= 5.73 \text{ GHz}$$

$$V_{OV} = 0.4 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.4}{4\pi \times 0.5^2}$$

$$= 11.46 \text{ GHz}$$

$$\mathbf{9.8} \quad A_0 = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}}$$

$$A_0 = \frac{2 \times 5 \times L}{0.2} = 50L, \text{ V/V} \quad (L \text{ in } \mu\text{m})$$

$$f_T \simeq \frac{3\mu_n V_{OV}}{4\pi L^2} = \frac{3 \times 400 \times 10^8 \times 0.2}{4\pi L^2}$$

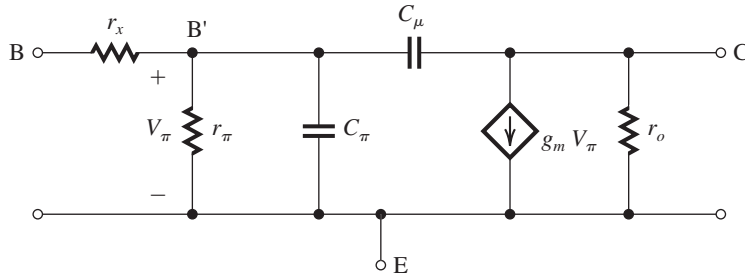
$$f_T = \frac{1.91}{L^2}, \text{ GHz} \quad (L \text{ in } \mu\text{m})$$

The expressions for  $A_0$  and  $f_T$  can be used to obtain their values for different values of  $L$ .

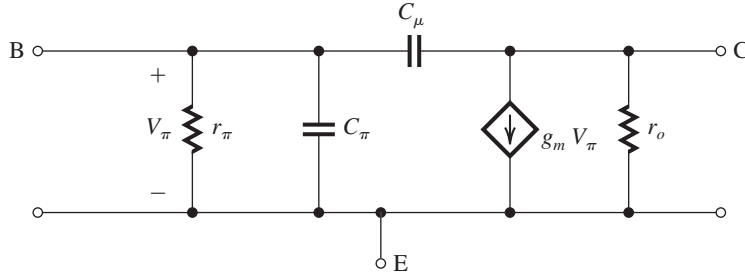
The results are given in the following table.

$L$	$L_{\min}$ <b>0.13 <math>\mu\text{m}</math></b>	$2L_{\min}$ <b>0.26 <math>\mu\text{m}</math></b>	$3L_{\min}$ <b>0.39 <math>\mu\text{m}</math></b>	$4L_{\min}$ <b>0.52 <math>\mu\text{m}</math></b>	$5L_{\min}$ <b>0.65 <math>\mu\text{m}</math></b>
$A_0$ (V/V)	6.5	13	19.5	26	32.5
$f_T$ (GHz)	113	28.3	12.6	7.1	4.5

This figure belongs to Problem 9.9.



This figure belongs to Problem 9.10.



9.9 See figure above.  $C_\pi = C_{de} + C_{je}$

where  $C_{de}$  is proportional to  $I_C$ .

At  $I_C = 0.5$  mA,

$$8 = C_{de} + 2 \Rightarrow C_{de} = 6 \text{ pF}$$

At  $I_C = 0.25$  mA,  $C_{de} = \frac{1}{2} \times 6 = 3$  pF, and  $C_\pi = 3 + 2 = 5$  pF.

Also, at  $I_C = 0.25$  mA,  $g_m = 10$  mA/V. Thus  $f_T$  at  $I_C = 0.25$  mA is

$$f_T = \frac{10 \times 10^{-3}}{2\pi(5 + 1) \times 10^{-12}} = 265.3 \text{ MHz}$$

9.10 See figure above.

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50 \text{ k}\Omega$$

$$C_{de} = \tau_F g_m = 30 \times 10^{-12} \times 40 \times 10^{-3} = 1.2 \text{ pF}$$

$$C_{je0} = 20 \text{ fF}$$

$$C_\pi = C_{de} + 2C_{je0} = 1.2 + 2 \times 0.02 = 1.24 \text{ pF}$$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$C_\mu = \frac{30}{\left(1 + \frac{2}{0.75}\right)^{0.5}} = 15.7 \text{ fF}$$

$$\begin{aligned} f_T &= \frac{g_m}{2\pi(C_\pi + C_\mu)} \\ &= \frac{40 \times 10^{-3}}{2\pi(1.24 + 0.016) \times 10^{-12}} \\ &= 5.1 \text{ GHz} \end{aligned}$$

9.11 For  $f \gg f_\beta$ ,

$$|\beta| \simeq \frac{f_T}{f}$$

$$50 = \frac{5000 \text{ MHz}}{f}$$

$$\Rightarrow f = 100 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{5000 \text{ MHz}}{200} = 25 \text{ MHz}$$

9.12 To complete the table we use the following relationships:

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{I_E \text{ (mA)}}$$

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} \simeq \frac{I_E}{V_T} = \frac{I_E \text{ (mA)}}{0.025 \text{ V}}$$

$$r_\pi = \frac{\beta_0}{g_m \text{ (mA/V)}}, \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$f_\beta = \frac{1}{2\pi(C_\pi + C_\mu)r_\pi}$$

$$f_\beta = \frac{f_T}{\beta_0}$$

The results are summarized in the table on the next page.

This table belongs to Problem 9.12.

Transistor	$I_E$ (mA)	$r_e$ ( $\Omega$ )	$g_m$ (mA/V)	$r_\pi$ (k $\Omega$ )	$\beta_0$	$f_T$ (MHz)	$C_\mu$ (pF)	$C_\pi$ (pF)	$f_\beta$ (MHz)
(a)	2	12.5	80	12.5	100	500	2	23.5	5
(b)	1	25	40	3.13	125	500	2	10.7	4
(c)	1	25	40	2.5	100	500	2	10.7	5
(d)	10	2.5	400	0.25	100	500	2	125.3	5
(e)	0.1	250	4	25	100	150	2	2.2	1.5
(f)	1	25	40	0.25	10	500	2	10.7	50
(g)	1.25	20	50	0.2	10	800	1	9	80

9.13 The frequency of the pole is

$$f_P = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-12}}$$

$$= 1.59 \text{ MHz}$$

9.14  $A_M = -g_m(r_o \parallel R_L)$

$$= -4(20 \parallel 20)$$

$$= -40 \text{ V/V}$$

$$f_H = \frac{1}{2\pi R_{sig} C_{in}} \text{ where } C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$= 25 + 5(1 + 40)$$

$$= 230 \text{ fF}$$

$$f_H = \frac{1}{2\pi \times 20 \times 10^3 \times 230 \times 10^{-15}}$$

$$= 34.6 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}}$$

$$= \frac{4 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}}$$

$$= 127.3 \text{ GHz}$$

9.15 (a)  $V_o = -AV_i$

If the current flowing through  $R_{sig}$  is denoted  $I_i$ , we obtain

$$Y_{in} = \frac{I_i}{V_i} = \frac{sC(V_i - V_o)}{V_i}$$

$$= sC \left( 1 - \frac{V_o}{V_i} \right)$$

$$= sC(1 + A)$$

Thus,

$$C_{in} = C(1 + A)$$

(b) 
$$\frac{V_i(s)}{V_{sig}(s)} = \frac{1/sC_{in}}{R_{sig} + \frac{1}{sC_{in}}}$$

$$= \frac{1}{1 + sC_{in}R_{sig}}$$

$$\frac{V_o(s)}{V_{sig}(s)} = -\frac{A}{1 + sC_{in}R_{sig}}$$

(c) DC gain = 40 dB = 100 V/V,

$$\Rightarrow A = 100 \text{ V/V}$$

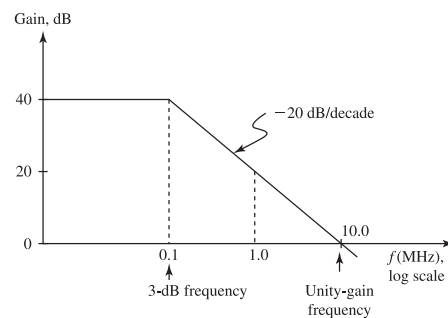
$$f_{3dB} = \frac{1}{2\pi C_{in}R_{sig}}$$

$$100 \times 10^3 = \frac{1}{2\pi C_{in} \times 1 \times 10^3}$$

$$\Rightarrow C_{in} = 1591.5 \text{ pF}$$

$$C = \frac{C_{in}}{A + 1} = \frac{1591.5}{101} = 15.8 \text{ pF}$$

(d) The Bode plot is shown in the figure below. From the figure we see that the gain reduces to unity two decades higher than  $f_{3dB}$ , that is at 10 MHz.



9.16 Refer to Example 9.1. If the transistor is replaced with another whose  $W$  is half that of the original transistor, we obtain

$$W_2 = \frac{1}{2}W_1$$

Since

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$



then

$$C_{gs2} = \frac{1}{2} C_{gs1} = 10 \text{ fF}$$

Also,

$$C_{gd} = W L_{ov} C_{ox}$$

thus,

$$C_{gd2} = \frac{1}{2} C_{gd1} = 2.5 \text{ fF}$$

Since

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

then

$$g_{m2} = \frac{1}{\sqrt{2}} g_{m1} = 1.41 \text{ mA/V}$$

Since

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

then

$$V_{OV2} = \sqrt{2} V_{OV1}$$

Finally,

$$r_{o2} = r_{o1} = 20 \text{ k}\Omega$$

Thus,

$$R'_{L2} = R'_{L1} = 10 \text{ k}\Omega$$

Thus,

$$\begin{aligned} C_{in2} &= C_{gs2} + (g_{m2} R'_{L2} + 1) C_{gd2} \\ &= 10 + (1.41 \times 10 + 1) \times 2.5 \\ &= 47.75 \text{ fF} \end{aligned}$$

This should be compared to  $C_{in1} = 125 \text{ fF}$ . Thus,

This figure belongs to Problem 9.17.

$$\begin{aligned} f_{H2} &= \frac{1}{2\pi C_{in2} R_{sig}} \\ &= \frac{1}{2\pi \times 47.75 \times 10^{-15} \times 20 \times 10^3} \\ &= 166.7 \text{ MHz} \end{aligned}$$

in comparison to  $f_{H1} = 63.7 \text{ MHz}$

$$|A_{M2}| = g_{m2} R'_{L2}$$

$$= 1.41 \times 10$$

$$= 14.1 \text{ V/V}$$

in comparison to  $|A_{M1}| = 20 \text{ V/V}$ .

$$GB_2 = 14.1 \times 166.7 = 2.35 \text{ GHz}$$

in comparison to  $GB_1 = 20 \times 63.7$

$$= 1.27 \text{ GHz}.$$

$$\mathbf{9.17} \quad A_M = -g_m R'_L$$

$$= -2 \times 20 = -40 \text{ V/V}$$

$$f_H = \frac{1}{2\pi (C_L + C_{gd}) R'_L}$$

$$\begin{aligned} &= \frac{1}{2\pi (100 + 10) \times 10^{-15} \times 20 \times 10^3} \\ &= 72.3 \text{ MHz} \end{aligned}$$

$$f_i = |A_M| f_H$$

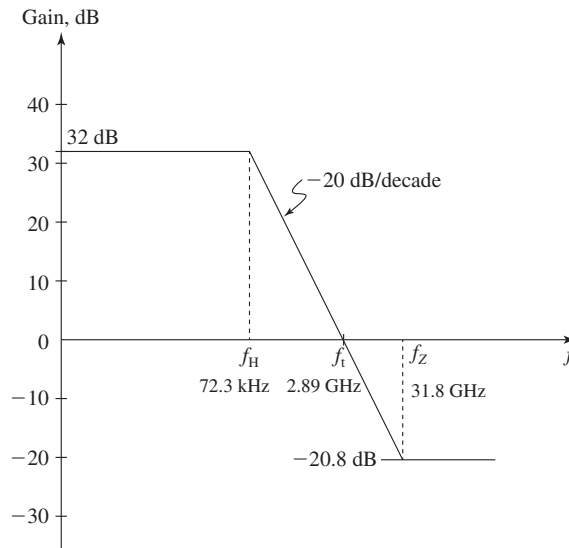
$$= 40 \times 72.3 = 2.89 \text{ GHz}$$

$$f_z = \frac{g_m}{2\pi C_{gd}} = \frac{2 \times 10^{-3}}{2\pi \times 10 \times 10^{-15}}$$

$$= 31.8 \text{ GHz}$$

A Bode plot for the gain magnitude is shown in the figure. Since from Eq. (9.43),

$$A(s) = -g_m R'_L \frac{1 - s(C_{gd}/g_m)}{1 + s(C_L + C_{gd})R'_L}$$



we find that as  $s \rightarrow \infty$ , the gain magnitude becomes

$$|A| = \frac{C_{gd}}{C_L + C_{gd}} = \frac{10}{100 + 10} = \frac{1}{11}$$

or  $-20.8$  dB as indicated in the figure.

$$\mathbf{9.18} \quad g_m R'_L = 50$$

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 10 + 1(1 + 50)$$

$$= 61 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 61 \times 10^{-12} \times 5 \times 10^3}$$

$$= 522 \text{ kHz}$$

**9.19** Figure (a) on the next page shows the high-frequency equivalent circuit. A simplified version is shown in Fig. (b) on the next page, from which we get

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_{in}}{R_{in} + R_{sig}} (g_m R'_L)$$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.8 \text{ mA}}{0.025 \text{ V}} = 32 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{160}{32} = 5 \text{ k}\Omega$$

$$R_{in} = R_B \| r_\pi = R_{B1} \| R_{B2} \| r_\pi$$

$$= 68 \| 27 \| 5 = 4 \text{ k}\Omega$$

$$R'_{sig} = R_{sig} \| R_{in} = 10 \| 4 = 2.86 \text{ k}\Omega$$

$$R'_L = R_L \| R_C = 10 \| 4.7 = 3.2 \text{ k}\Omega$$

$$g_m R'_L = 32 \times 3.2 = 102.4$$

$$A_M = -\frac{4}{4 + 10} \times 102.4$$

$$= -29.3 \text{ V/V}$$

Next we determine  $C_\pi$ ,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$10^9 = \frac{32 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 5.1 \text{ pF}$$

Since,

$$C_\mu = 0.5 \text{ pF}$$

$$C_\pi = 5.1 - 0.5 = 4.6 \text{ pF}$$

Finally,

$$f_H = \frac{1}{2\pi R'_{sig} C_{in}}$$

where

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 4.6 + 0.5(1 + 102.4)$$

$$= 56.3 \text{ pF}$$

Thus,

$$f_H = \frac{1}{2\pi \times 2.86 \times 10^3 \times 56.3 \times 10^{-12}}$$

$$= 988 \text{ kHz}$$

$$\mathbf{9.20} \quad R_{in} = \frac{R}{1 - K}$$

$$= \frac{100 \text{ k}\Omega}{1 - 0.9} = 1000 \text{ k}\Omega = 1 \text{ M}\Omega$$

**9.21** Using Miller's theorem, we obtain

$$Z_{in} = \frac{Z}{1 - A}, \quad Z_{out} = \frac{Z}{1 - \frac{1}{A}}$$

For

$$Z = \frac{1}{j\omega C}$$

$$Z_{in} = \frac{1}{j\omega C(1 - A)} \Rightarrow C_{in} = C(1 - A)$$

$$Z_{out} = \frac{1}{j\omega C\left(1 - \frac{1}{A}\right)} \Rightarrow C_{out} = C\left(1 - \frac{1}{A}\right)$$

$$\text{(a) } A = -1000 \text{ V/V}, \quad C = 1 \text{ pF}$$

$$C_{in} = 1(1 + 1000) = 1001 \text{ pF}$$

$$C_{out} = 1\left(1 + \frac{1}{1000}\right) = 1.001 \text{ pF}$$

$$\text{(b) } A = -10 \text{ V/V}, \quad C = 10 \text{ pF}$$

$$C_{in} = 10(1 + 10) = 110 \text{ pF}$$

$$C_{out} = 10\left(1 + \frac{1}{10}\right) = 11 \text{ pF}$$

$$\text{(c) } A = -1 \text{ V/V}, \quad C = 10 \text{ pF}$$

$$C_{in} = 10(1 + 1) = 20 \text{ pF}$$

$$C_{out} = 10(1 + 1) = 20 \text{ pF}$$

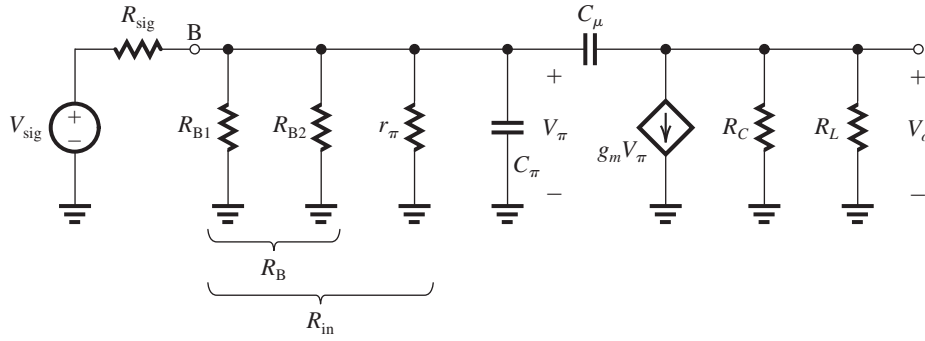
$$\text{(d) } A = +1 \text{ V/V}, \quad C = 10 \text{ pF}$$

$$C_{in} = C(1 - 1) = 0$$

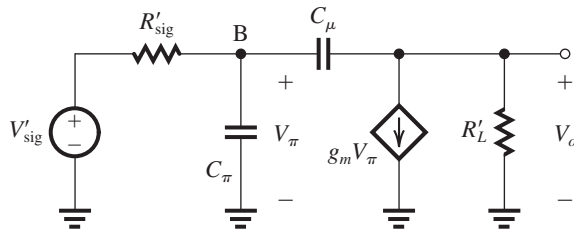
$$C_{out} = C(1 - 1) = 0$$

$$\text{(e) } A = +10 \text{ V/V}, \quad C = 10 \text{ pF}$$

This figure belongs to Problem 9.19.



(a)



$$V'_{sig} = V_{sig} \frac{R_{in}}{R_{in} + R_{sig}} \quad R'_{sig} = R_{sig} \parallel R_{in}$$

$$R_{in} = R_B \parallel r_{\pi} = R_{B1} \parallel R_{B2} \parallel r_{\pi} \quad R'_L = R_C \parallel R_L$$

(b)

$$C_{in} = 10(1 - 10) = -90 \text{ pF}$$

$$C_{out} = 10\left(1 - \frac{1}{10}\right) = 9 \text{ pF}$$

The  $-90 \text{ pF}$  input capacitance can be used to cancel an equal ( $+90 \text{ pF}$ ) capacitance between the input node and ground.

$$\begin{aligned} 9.22 \quad f_P &= \frac{1}{2\pi C(R_{sig} \parallel R_{in})} \\ &= \frac{1}{2\pi \times 0.1 \times 10^{-12}(1 \parallel 1) \times 10^6} = 3.18 \text{ MHz} \end{aligned}$$

$$\begin{aligned} 9.23 \quad C_{in} &= C_{gs} + C_{gd}(1 + g_m R'_L) \\ &= 1 + 0.1(1 + 39) \\ &= 5 \text{ pF} \end{aligned}$$

$$\begin{aligned} f_{sdB} &= \frac{1}{2\pi C_{in} R_{sig}} \\ &= \frac{1}{2\pi \times 5 \times 10^{-12} R_{sig}} \end{aligned}$$

For  $f_{sdB} > 1 \text{ MHz}$ ,

$$R_{sig} < \frac{1}{2\pi \times 5 \times 10^{-12} \times 1 \times 10^6} = 31.8 \text{ k}\Omega$$

$$\begin{aligned} 9.24 \quad C_{in} &= 0.2(1 + 1000) \\ &= 200.2 \text{ pF} \end{aligned}$$

$$\frac{V_o(s)}{V_{sig}(s)} = -\frac{1000}{1 + sC_{in}R_{sig}}$$

$$\begin{aligned} f_{3dB} &= \frac{1}{2\pi C_{in} R_{sig}} \\ &= \frac{1}{2\pi \times 200.2 \times 10^{-12} \times 1 \times 10^3} \\ &= 795 \text{ kHz} \end{aligned}$$

The gain falls off at the rate of 20 dB/decade. For the gain to reach 0 dB (unity), the gain has to fall by 60 dB. This requires three decades or a factor of 1000, thus

$$f_{\text{unity gain}} = 795 \times 1000 = 795 \text{ MHz}$$

$$9.25 \quad f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

For  $f_H \geq 6 \text{ MHz}$

$$C_{in} \leq \frac{1}{2\pi f_H R_{sig}} = \frac{1}{2\pi \times 6 \times 10^6 \times 1 \times 10^3}$$

$$C_{in} \leq 26.5 \text{ pF}$$

But,

$$C_{in} = C_{gs} + (1 + g_m R'_L) C_{gd}$$

$$= 5 + (1 + g_m R'_L) \times 1, \text{ pF}$$

$$= 6 + g_m R'_L, \text{ pF}$$

For  $C_{in} \leq 26.5 \text{ pF}$  we have

$$g_m R'_L \leq 20.5$$

$$R'_L \leq \frac{20.5}{5} = 4.1 \text{ k}\Omega$$

Corresponding to  $R'_L = 4.1 \text{ k}\Omega$ , we have

$$|A_M| = g_m R'_L = 20.5 \text{ V/V}$$

$$GB = |A_M| f_H$$

$$= 20.5 \times 6 = 123 \text{ MHz}$$

If  $f_H = 2 \text{ MHz}$ , we obtain

$$C_{in} = 26.5 \times 3 = 79.5 \text{ pF}$$

$$g_m R'_L = 79.5 - 6 = 73.5$$

Thus,

$$|A_M| = 73.5 \text{ V/V}$$

$$GB = 73.5 \times 2 = 147 \text{ MHz}$$

$$\mathbf{9.26} \quad A_M = -\frac{R_{in}}{R_{in} + R_{sig}} g_m R'_L$$

where

$$R'_L = R_D \parallel R_L \parallel r_o$$

$$= 8 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 4.1 \text{ k}\Omega$$

$$A_M = -\frac{100}{100 + 100} \times 3 \times 4.1$$

$$= -6.1 \text{ V/V}$$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L) \quad (1)$$

$$= 1 + 0.2(1 + 3 \times 4.1)$$

$$= 3.66 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad (2)$$

where

$$R'_{sig} = R_{sig} \parallel R_{in}$$

$$= 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3}$$

$$= 870 \text{ kHz}$$

To double  $f_H$  by changing  $R_{in}$ , Eq. (2) indicates that  $R'_{sig}$  must be halved:

$$R'_{sig} = 25 \text{ k}\Omega$$

which requires  $R_{in}$  to be changed to  $R_{in2}$ ,

$$25 \text{ k}\Omega = 100 \parallel R_{in2}$$

$$\Rightarrow R_{in2} = 33.3 \text{ k}\Omega$$

This change will cause  $|A_M|$  to become

$$|A_{M2}| = \frac{33.3}{33.3 + 100} \times 3 \times 4.1$$

$$= 3.1 \text{ V/V}$$

which is about half the original value.

To double  $f_H$  by changing  $R_L$ , Eq. (2) indicates that  $C_{in}$  must be halved:

$$C_{in2} = \frac{1}{2} \times 3.66 = 1.83 \text{ pF}$$

Using Eq. (1), we obtain

$$1.83 = 1 + 0.2(1 + g_m R'_{L2})$$

$$\Rightarrow g_m R'_{L2} = 3.15$$

Thus,

$$R'_{L2} = 1.05 \text{ k}\Omega$$

and  $R_{L2}$  can be found from

$$1.05 = R_L \parallel 8 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$\Rightarrow R_L = 1.24 \text{ k}\Omega$$

and the midband gain becomes

$$|A_{M2}| = \frac{100}{100 + 100} \times 3.15 = 1.6 \text{ V/V}$$

which is about a quarter of the original gain.

Clearly, changing  $R_{in}$  is the preferred course of action!

$$\mathbf{9.27} \quad g_m = \sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.4 \times 12.5 \times 0.1}$$

$$= 1 \text{ mA/V}$$

$$r_{o1} = \frac{|V_{A1}|}{I_{D1}} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

The total resistance at the output node,  $R'_L$ , is given by

$$R'_L = r_{o1} \parallel r_{o2} = 50 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 25 \text{ k}\Omega$$

$$A_M = -g_{m1} R'_L$$

$$= -1 \times 25 = -25 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

where

$$\begin{aligned} C_{in} &= C_{gs} + C_{gd}(1 + g_{m1}R'_L) \\ &= 30 + 5(1 + 25) \\ &= 160 \text{ fF} \end{aligned}$$

Thus,

$$\begin{aligned} f_H &= \frac{1}{2\pi \times 160 \times 10^{-15} \times 20 \times 10^3} \\ &= 49.7 \text{ MHz} \\ f_Z &= \frac{g_m}{2\pi C_{gd}} = \frac{1 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} \\ &= 31.8 \text{ GHz} \end{aligned}$$

**9.28** Refer to Fig. 9.12(c)

$$\begin{aligned} f_i &= \frac{g_m}{2\pi(C_L + C_{gd})} \\ 10^9 &= \frac{2 \times 10^{-3}}{2\pi(C_L + C_{gd})} \\ \Rightarrow C_L + C_{gd} &= 318.3 \text{ fF} \end{aligned}$$

Since the gain levels off at high frequencies at:  $-20$  dB, then

$$\frac{f_Z}{f_i} = 10$$

But

$$\frac{f_Z}{f_i} = \frac{C_L + C_{gd}}{C_{gd}}$$

Thus

$$\begin{aligned} \frac{C_{gd}}{C_L + C_{gd}} &= 0.1 \\ \Rightarrow C_{gd} &= 31.83 \text{ fF} \end{aligned}$$

and

$$C_L = 318.3 - 31.83 = 286.5 \text{ fF}$$

Since,

$$f_i = |A_M| f_H$$

then,

$$f_H = \frac{10^9}{50} = 20 \text{ MHz}$$

$$\mathbf{9.29} \quad f_i = \frac{g_m}{2\pi(C_L + C_{gd})}$$

$$\begin{aligned} C_L + C_{gd} &= \frac{g_m}{2\pi f_i} \\ &= \frac{2 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 0.159 \text{ pF} \end{aligned}$$

To reduce  $f_i$  to 1 GHz, an additional capacitance of 0.159 pF must be connected to the output node. (Doubling the effective capacitance at the output node reduces  $f_i$  by a factor of 2.)

**9.30** Refer to Fig. P9.27. To determine  $g_{m1}$  we use

$$\begin{aligned} g_{m1} &= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}} \\ &= \sqrt{2 \times 0.090 \times \frac{100}{1.6} \times 0.1} \\ &= 1.06 \text{ mA/V} \end{aligned}$$

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

$$R'_L = r_{o1} \parallel r_{o2} = 128 \parallel 192 = 76.8 \text{ k}\Omega$$

$$\begin{aligned} A_M &= -g_{m1}R'_L \\ &= -1.06 \times 76.8 = -81.4 \text{ V/V} \end{aligned}$$

$$\begin{aligned} C_L &= C_{db1} + C_{db2} + C_{gd2} \\ &= 20 + 36 + 15 = 71 \text{ fF} \end{aligned}$$

$$f_H = \frac{1}{2\pi(C_L + C_{gd1})R'_L}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi(71 + 15) \times 10^{-15} \times 76.8 \times 10^3} \\ &= 24.1 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_Z &= \frac{g_{m1}}{2\pi C_{gd1}} = \frac{1.06 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}} \\ &= 11.2 \text{ GHz} \end{aligned}$$

$$\mathbf{9.31} \quad A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + R_{sig}} g_m R'_L$$

where

$$\begin{aligned} R'_L &= r_o \parallel R_L \\ &= 100 \text{ k}\Omega \parallel 10 \text{ k}\Omega \\ &= 9.1 \text{ k}\Omega \end{aligned}$$

and

$$r_\pi = \beta/g_m = 100/40 = 2.5 \text{ k}\Omega$$

$$\begin{aligned} A_M &= -\frac{2.5}{2.5 + 10} \times 40 \times 9.1 \\ &= -72.8 \text{ V/V} \end{aligned}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$\begin{aligned} C_{in} &= C_{\pi} + C_{\mu}(1 + g_m R'_L) \\ &= 10 + 0.5 \times (1 + 40 \times 9.1) \\ &= 192.5 \text{ pF} \end{aligned}$$

and

$$\begin{aligned} R'_{sig} &= r_{\pi} \parallel R_{sig} \\ &= 2.5 \parallel 10 \\ &= 2 \text{ k}\Omega \\ f_H &= \frac{1}{2\pi \times 192.5 \times 10^{-12} \times 2 \times 10^3} \\ &= 413 \text{ kHz} \end{aligned}$$

**9.32 (a)**  $A_M =$

$$-\frac{r_{\pi}}{r_{\pi} + R_{sig}} g_m R'_L$$

For  $R_{sig} \gg r_{\pi}$ ,

$$A_M \simeq -\frac{r_{\pi}}{R_{sig}} g_m R'_L = -\beta R'_L / R_{sig} \quad \text{Q.E.D.}$$

(b)  $C_{in} = C_{\pi} + (g_m R'_L + 1)C_{\mu}$

For  $g_m R'_L \gg 1$  and  $g_m R'_L C_{\mu} \gg C_{\pi}$ ,

$$\begin{aligned} C_{in} &\simeq g_m R'_L C_{\mu} \\ f_H &= \frac{1}{2\pi C_{in} R'_{sig}} \end{aligned}$$

where

$$\begin{aligned} R'_{sig} &= r_{\pi} \parallel R_{sig} \\ &\simeq r_{\pi} \end{aligned}$$

Thus,

$$f_H \simeq \frac{1}{2\pi g_m R'_L C_{\mu} r_{\pi}}$$

$$f_H = \frac{1}{2\pi C_{\mu} \beta R'_L} \quad \text{Q.E.D.}$$

(c)  $GB = |A_M| f_H$

$$= \beta \frac{R'_L}{R_{sig}} \frac{1}{2\pi C_{\mu} \beta R'_L} = \frac{1}{2\pi C_{\mu} R_{sig}} \quad \text{Q.E.D.}$$

For  $R_{sig} = 25 \text{ k}\Omega$  and  $C_{\mu} = 1 \text{ pF}$ ,

$$GB = \frac{1}{2\pi \times 1 \times 10^{-12} \times 25 \times 10^3} = 6.37 \text{ MHz}$$

For  $\beta = 100$ ,

(i)  $R'_L = 25 \text{ k}\Omega$ :

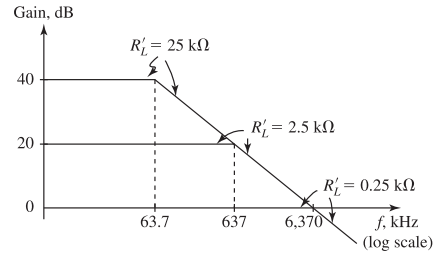
$$A_M = -100 \times \frac{25}{25} = -100 \text{ V/V}$$

$$f_H = \frac{GB}{|A_M|} = \frac{6.37 \text{ MHz}}{100 \text{ V/V}} = 63.7 \text{ kHz}$$

(ii)  $R'_L = 2.5 \text{ k}\Omega$ :

$$A_M = -100 \times \frac{2.5}{25} = -10 \text{ V/V}$$

$$f_H = \frac{GB}{|A_M|} = \frac{6.37 \text{ MHz}}{10 \text{ V/V}} = 637 \text{ kHz}$$



The Bode plots are shown in the figure.

If the midband gain is unity,

$$f_H = GB = 6.37 \text{ MHz}$$

This is obtained when  $R'_L$  is

$$\begin{aligned} 1 &= 100 \times \frac{R'_L}{25} \\ \Rightarrow R'_L &= 0.25 \text{ k}\Omega = 250 \Omega \end{aligned}$$

**9.33**

From the figure we see that the controlled current-source  $g_m V_{\pi}$  appears across its control voltage  $V_{\pi}$ , thus we can replace the current source with a resistance  $1/g_m$ . Now, the parallel equivalent of  $r_{\pi}$  and  $1/g_m$  is

$$\frac{r_{\pi}(1/g_m)}{r_{\pi} + \frac{1}{g_m}} = \frac{r_{\pi}}{g_m r_{\pi} + 1} = \frac{r_{\pi}}{\beta + 1} = r_e$$

Thus, the equivalent circuit simplifies to that of  $r_e$  in parallel with  $C_{\pi}$ ,

$$Z_i(s) = \frac{1}{\frac{1}{r_e} + sC_{\pi}} = \frac{r_e}{1 + sC_{\pi}r_e}$$

$$Z_i(j\omega) = \frac{r_e}{1 + j\omega C_{\pi}r_e}$$

$Z_i(j\omega)$  will have a  $45^\circ$  phase at

$$\omega_{45} C_{\pi} r_e = 1$$

$$\Rightarrow \omega_{45} = \frac{1}{C_{\pi} r_e}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

At high bias currents,

$$C_{\pi} \gg C_{\mu}$$

and

$$f_T \simeq \frac{g_m}{2\pi C_\pi}$$

Since  $g_m \simeq 1/r_e$ , we have

$$f_T \simeq \frac{1}{2\pi C_\pi r_e}$$

Thus,

$$f_{45^\circ} \simeq f_T = 400 \text{ MHz}$$

If the bias current is reduced to the value that results in  $C_\pi \simeq C_\mu$ ,

$$f_T = \frac{g_m}{2\pi \times 2C_\pi} = \frac{g_m}{4\pi C_\pi}$$

Again,  $g_m \simeq \frac{1}{r_e}$ , thus

$$f_T \simeq \frac{1}{4\pi C_\pi r_e}$$

It follows that in this case,

$$f_{45^\circ} = \frac{1}{2} f_T = 200 \text{ MHz}$$

**9.34** Figure 1 shows the amplifier high-frequency equivalent circuit. A node equation at the output provides

$$\left(\frac{1}{r_o} + sC_L\right)V_o + g_m V_\pi + sC_\mu(V_o - V_\pi) = 0$$

Replacing  $V_\pi$  by  $V_i$  and collecting terms results in

$$V_o \left[ \frac{1}{r_o} + s(C_L + C_\mu) \right] = -V_i (g_m - sC_\mu)$$

$$\Rightarrow \frac{V_o}{V_i} = -g_m r_o \frac{1 - s(C_\mu/g_m)}{1 + s(C_L + C_\mu)r_o} \quad \text{Q.E.D.}$$

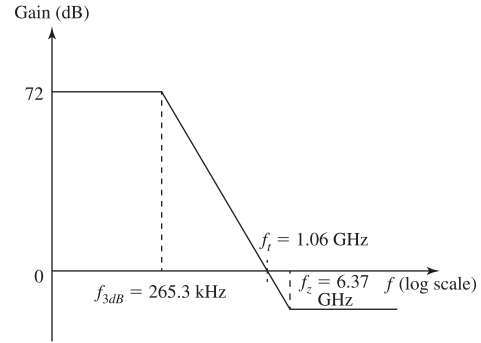


Figure 2

For  $I_C = 200 \mu\text{A} = 0.2 \text{ mA}$  and  $V_A = 100 \text{ V}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

DC gain =  $-g_m r_o = 8 \times 500 = -4000 \text{ V/V}$

$$f_{3dB} = \frac{1}{2\pi(C_L + C_\mu)r_o} = \frac{1}{2\pi(1 + 0.2) \times 10^{-12} \times 500 \times 10^3} = 265.3 \text{ kHz}$$

$$f_z = \frac{g_m}{2\pi C_\mu} = \frac{8 \times 10^{-3}}{2\pi \times 0.2 \times 10^{-12}} = 6.37 \text{ GHz}$$

This figure belongs to Problem 9.34, part (a).

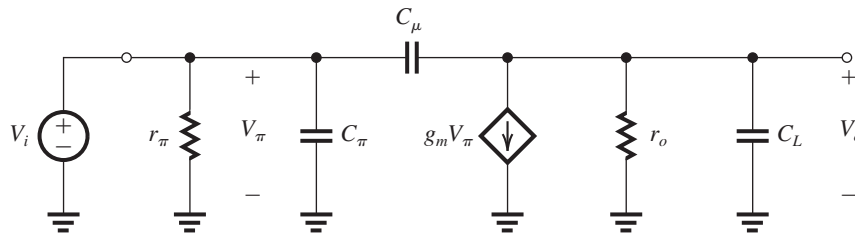
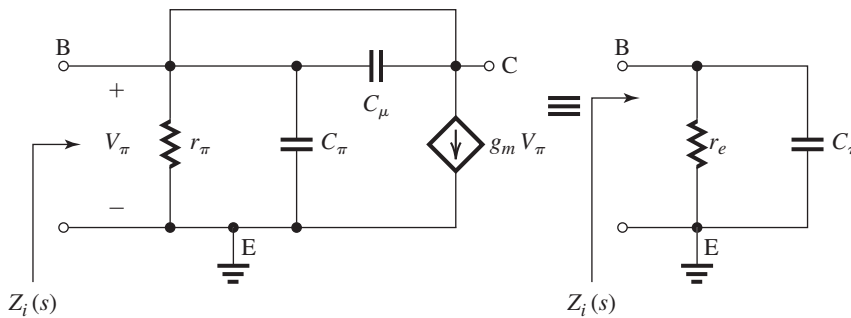


Figure 1



$$f_t = |A_{dc}|f_{3dB}$$

$$= 4000 \times 265.3 = 1.06 \text{ GHz}$$

The Bode plot is shown in Figure 2.

**9.35**

(a) Refer to Fig. 1.

$$I_i = \frac{V_i - V_o}{R} = \frac{V_i - 2V_i}{R} = -\frac{V_i}{R}$$

Thus,

$$R_{in} \equiv \frac{V_i}{I_i} = -R$$

(b) Replacing the signal source with its equivalent Norton's form results in the circuit in Fig. 2. Observe that  $R_{eq} = \infty$  when  $R_{sig} = R$ . In this case,

$$I_L = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$$

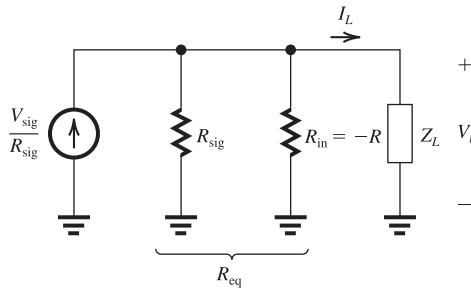


Figure 2

(c) If  $Z_L = \frac{1}{sC}$ ,

$$V_i = I_L Z_L$$

$$= \frac{V_{sig}}{R} \times \frac{1}{sC}$$

$$= \frac{1}{sCR} V_{sig}$$

and

This figure belongs to Problem 9.35.

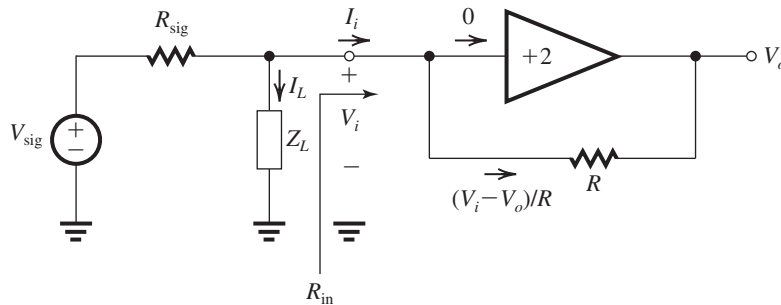


Figure 1

$$V_o = 2V_i = \frac{2}{sCR} V_{sig}$$

Thus,

$$\frac{V_o}{V_{sig}} = \frac{2}{sCR}$$

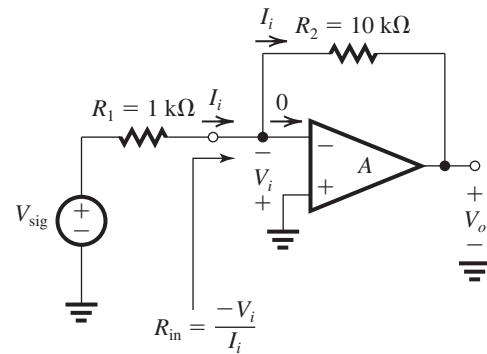
which is the transfer function of an ideal noninverting integrator.

**9.36**

From the figure we see that

$$V_o = AV_i \tag{1}$$

From Miller's theorem, we have



$$R_{in} = \frac{R_2}{1 - \left(\frac{V_o}{-V_i}\right)} = \frac{R_2}{1 + \frac{V_o}{V_i}} = \frac{R_2}{1 + A} \tag{2}$$

Using the voltage divider rule at the input, we get

$$-V_i = V_{sig} \frac{R_{in}}{R_{in} + R_1}$$

$$\Rightarrow V_i = -V_{sig} \frac{R_{in}}{R_{in} + R_1} \tag{3}$$

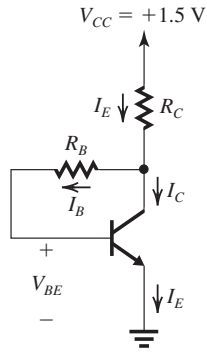
For each value of  $A$  we use Eq. (2) to determine  $R_{in}$ , Eq. (3) to determine  $V_i$  (for  $V_{sig} = 1 \text{ V}$ ), Eq. (1) to determine  $V_o$ , and finally we calculate the



value of  $V_o/V_{sig}$ . The results are given in the table below.

A (V/V)	$R_{in}$ (k $\Omega$ )	$V_i$ (V)	$V_o$ (V)	$V_o/V_{sig}$ (V/V)
10	$9.091 \times 10^{-1}$	-0.476	-4.76	-4.76
100	$9.901 \times 10^{-2}$	-0.0900	-9.00	-9.00
1000	$9.990 \times 10^{-3}$	$-9.89 \times 10^{-3}$	-9.89	-9.89
10,000	$9.999 \times 10^{-4}$	$-9.989 \times 10^{-4}$	-9.99	-9.99

9.37



(a) For the dc analysis, refer to the figure.

$$V_{CC} = I_E R_C + I_B R_B + V_{BE}$$

$$1.5 = I_E \times 1 + \frac{I_E}{\beta + 1} \times 47 + 0.7$$

$$\Rightarrow I_E = \frac{1.5 - 0.7}{1 + \frac{100}{101}} = 0.546 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.546 = 0.54 \text{ mA}$$

(b)  $g_m = \frac{I_C}{V_T} = \frac{0.54 \text{ mA}}{0.025 \text{ V}} = 21.6 \text{ mA/V}$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{21.6} = 4.63 \text{ k}\Omega$$

(c)  $\frac{V_o}{V_b} = -g_m(R_C \parallel R_L)$

$$= -21.6(1 \parallel 1) = -10.8 \text{ V/V}$$

(d) Using Miller's theorem, the component of  $R_{in}$  due to  $R_B$  can be found as

$$R_{in1} = \frac{R_B}{1 - (V_o/V_b)}$$

$$= \frac{47 \text{ k}\Omega}{1 - (-10.8)} = 4 \text{ k}\Omega$$

$$R_{in} = R_{in1} \parallel r_\pi$$

$$= 4 \parallel 4.63 = 2.14 \text{ k}\Omega$$

(e)  $G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{V_o}{V_b}$

$$= \frac{2.14}{2.14 + 1} \times -10.8 = -7.4 \text{ V/V}$$

(f)  $C_{in} = C_\pi + \left(1 + \left|\frac{V_o}{V_b}\right|\right) C_\mu$

where

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \times 600 \times 10^6}$$

$$C_\pi + C_\mu = 5.73 \text{ pF}$$

$$C_\pi = 5.73 - 0.8 = 4.93 \text{ pF}$$

$$C_{in} = 4.93 + (1 + 10.8) \times 0.8$$

$$= 14.37 \text{ pF}$$

(g)  $R'_{sig} = R_{in} \parallel R_{sig}$

$$= 2.14 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.68 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.68 \times 10^3}$$

$$= 16.3 \text{ MHz}$$

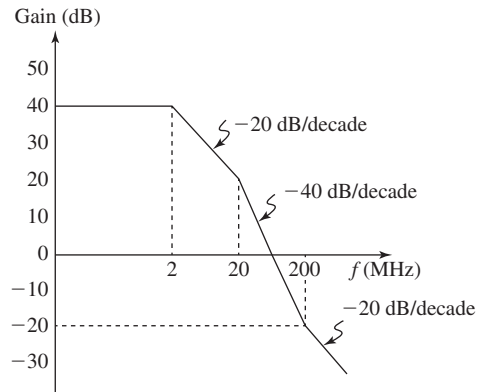
9.38  $A_M = 40 \text{ dB} \Rightarrow 100 \text{ V/V}$

$$A(s) = \frac{100}{\left(1 + \frac{s}{2\pi \times 2 \times 10^6}\right) \left(1 + \frac{s}{2\pi \times 20 \times 10^6}\right)}$$

Since  $f_{p1} \ll f_{p2} \ll f_Z$ , we have

$$f_{3dB} \approx f_{p1} = 2 \text{ MHz}$$

The Bode plot is shown in the figure.



9.39  $\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{CL}$

where

$$C_{gs} = 30 \text{ fF}$$

$$R_{gs} = R'_{sig} = 10 \text{ k}\Omega$$

$$C_{gd} = 5 \text{ fF}$$

$$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$$

$$= 10(1 + 2 \times 20) + 20$$

$$= 430 \text{ k}\Omega$$

$$C_L = 30 \text{ fF}$$

$$R_{CL} = R'_L = 20 \text{ k}\Omega$$

Thus,

$$\tau_H = 30 \times 10 + 5 \times 430 + 30 \times 20$$

$$= 3050 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 3050 \times 10^{-12}}$$

$$= 52.2 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{2 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} = 63.7 \text{ GHz}$$

$$\mathbf{9.40} \quad A_M = -g_m R'_L$$

$$= -4 \times 20 = -80 \text{ V/V}$$

$$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$$

$$= 2 + 0.1(80 + 1)$$

$$= 10.1 \text{ pF}$$

Using the Miller approximation, we obtain

$$f_H \simeq \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 10.1 \times 10^{-12} \times 20 \times 10^3}$$

$$= 788 \text{ kHz}$$

Using the open-circuit time constants, we get

$$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}}$$

$$= 2 \times 20 = 40 \text{ ns}$$

$$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 80) + 20 = 1640 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1640 = 164 \text{ ns}$$

$$\tau_{CL} = C_L R'_L$$

$$= 2 \times 20 = 40 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 40 + 164 + 40 = 244 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 244 \times 10^{-9}} = 652 \text{ kHz}$$

The estimate obtained using the open-circuit time constants is more appropriate as it takes into account the effect of  $C_L$ . We note from Eq. (1) that  $\tau_{CL}$  is 16.4% of  $\tau_H$ , thus  $C_L$  has a significant effect on the determination of  $f_H$ .

$$\mathbf{9.41} \quad \tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}}$$

$$= 5 \times 10 = 50 \text{ ns}$$

$$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$$

$$= 10(1 + 5 \times 10) + 10$$

$$= 520 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 1 \times 520 = 520 \text{ ns}$$

$$R_{CL} = C_L R'_L$$

$$= 5 \times 10 = 50 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 50 + 520 + 50 = 620 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} \quad (1)$$

$$= \frac{1}{2\pi \times 620 \times 10^{-9}} = 257 \text{ kHz}$$

The interaction of  $R_{\text{sig}}$  with the input capacitance contributes all of  $\tau_{gs}$  (50 ns) and a significant part of  $\tau_{gd}$ , namely

$$C_{gd}[R_{\text{sig}}(1 + g_m R'_L)] = 1 \times 10(1 + 50) = 510 \text{ ns}$$

Thus, the total contribution of  $R_{\text{sig}}$  is

$$50 + 510 = 560 \text{ ns}$$

which is  $\frac{560}{620} = 90.3\%$  of  $\tau_H$ . To double  $f_H$ , we must reduce  $\tau_H$  to half of its value:

$$\tau_H = \frac{1}{2} \times 620 = 310 \text{ ns}$$

Now,

$$\tau_H = R_{\text{sig}}[C_{gs} + C_{gd}(1 + g_m R'_L)] + C_{gd} R'_L + C_L R'_L$$

$$310 = R_{\text{sig}}[5 + 1(1 + 50)] + 1 \times 10 + 5 \times 10$$

$$\Rightarrow R_{\text{sig}} = 4.46 \text{ k}\Omega$$

$$\mathbf{9.42} \quad A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R'_L$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

Thus,

$$A_M = -\frac{2.5}{2.5 + 1} \times 40 \times 5$$

$$= -143 \text{ V/V}$$

Using the Miller effect, we obtain

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 10 + 0.3(1 + 40 \times 5)$$

$$= 70.3 \text{ pF}$$

$$R'_{sig} = r_\pi \parallel R_{sig}$$

$$= 2.5 \parallel 1 = 0.71 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 70.3 \times 10^{-12} \times 0.71 \times 10^3}$$

$$= 3.2 \text{ MHz}$$

Using the open-circuit time constants, we get

$$\tau_\pi = C_\pi R_\pi = C_\pi R'_{sig}$$

$$= 10 \times 0.71 = 7.1 \text{ ns}$$

$$\tau_\mu = C_\mu [R'_{sig}(g_m R'_L + 1) + R'_L]$$

$$= 0.3[0.71(40 \times 5 + 1) + 5]$$

$$= 44.3 \text{ ns}$$

$$\tau_{C_L} = C_L R'_L = 3 \times 5 = 15 \text{ ns}$$

$$\tau_H = \tau_\pi + \tau_\mu + \tau_{C_L}$$

$$= 7.1 + 44.3 + 15 = 66.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 66.4 \times 10^{-9}} = 2.47 \text{ MHz}$$

This is a more realistic estimate of  $f_H$  as it takes into account the effect of  $C_L$ .

**9.43** (a)  $A(s) = 1000 \frac{1}{1 + s/(2\pi \times 10^5)}$

(b) Figure 1 shows the Bode plot for the gain magnitude and phase.

(c)  $GB = 1000 \times 100 \text{ kHz} = 100 \text{ MHz}$

(d) The unity-gain frequency  $f_t$  is

$$f_t = 100 \text{ MHz}$$

(e) Figure 2 shows the magnitude response when a second pole at 1 MHz appears in the transfer function. The unity-gain frequency  $f_t$  now is

$$f_t = 10 \text{ MHz}$$

which is different from the gain-bandwidth product,

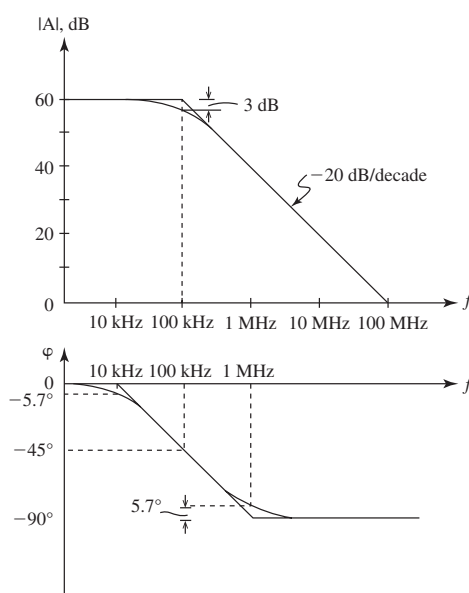


Figure 1

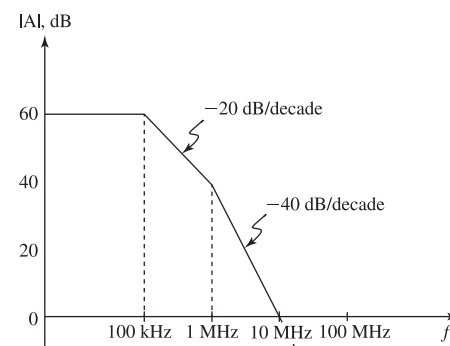


Figure 2

$$GB = 100 \text{ MHz}$$

**9.44**  $A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$

$$= -\frac{0.5}{0.5 + 0.1} \times 5 \times 10$$

$$= -41.7 \text{ V/V}$$

$$\tau_{gs} = C_{gs} R_{gs}$$

$$= C_{gs} R'_{sig}$$

$$= C_{gs} (R_{sig} \parallel R_G)$$

$$= 1 \times 10^{-12} \times (100 \parallel 500) \times 10^3$$

$$= 83.3 \text{ ns}$$

$$\tau_{gd} = C_{gd} R_{gd}$$

$$= C_{gd} [R'_{sig} (1 + g_m R'_L) + R'_L]$$

$$= 0.2 \times 10^{-12} [(100 \parallel 500)(1 + 5 \times 10) + 10] \times 10^3$$

$$= 851.7 \text{ ns}$$

$$\begin{aligned} \tau_{CL} &= C_L R'_L \\ &= 20 \times 10^{-12} \times 10 \times 10^3 \\ &= 200 \text{ ns} \\ \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\ &= 83.3 + 851.7 + 200 \\ &= 1135 \text{ ns} \\ f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 1135 \times 10^{-9}} \\ &= 140 \text{ kHz} \end{aligned}$$

**9.45** The figure below shows the equivalent circuit of the two-stage amplifier where we have modeled each stage as a transconductance amplifier. At node 1:

$$\begin{aligned} R_{eq1} &= R_{sig} \parallel R_{in1} \\ &= 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 5 \text{ k}\Omega \\ C_{eq1} &= C_{in1} + C_{sig} = 10 + 10 = 20 \text{ pF} \end{aligned}$$

Thus,

$$\begin{aligned} f_{p1} &= \frac{1}{2\pi C_{eq1} R_{eq1}} \\ f_{p1} &= \frac{1}{2\pi \times 20 \times 10^{-12} \times 5 \times 10^3} \\ &= 1.59 \text{ MHz} \end{aligned}$$

At node 2:

$$\begin{aligned} R_{eq2} &= R_{o1} \parallel R_{in2} = 2 \text{ k}\Omega \parallel 10 \text{ k}\Omega \\ &= 1.67 \text{ k}\Omega \\ C_{eq2} &= C_{o1} + C_{in2} \\ &= 2 + 10 = 12 \text{ pF} \end{aligned}$$

$$\begin{aligned} f_{p2} &= \frac{1}{2\pi C_{eq2} R_{eq2}} \\ &= \frac{1}{2\pi \times 12 \times 10^{-12} \times 1.67 \times 10^3} \\ &= 7.94 \text{ MHz} \end{aligned}$$

At node 3:

$$R_{eq3} = R_{o2} \parallel R_L = 2 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.67 \text{ k}\Omega$$

$$C_{eq3} = C_{o2} + C_L = 2 + 7 = 9 \text{ pF}$$

$$f_{p3} = \frac{1}{2\pi C_{eq3} R_{eq3}}$$

$$\begin{aligned} f_{p3} &= \frac{1}{2\pi \times 9 \times 10^{-12} \times 0.67 \times 10^3} \\ &= 26.4 \text{ MHz} \end{aligned}$$

Thus, the three poles have frequencies 1.59 MHz, 7.94 MHz, and 26.4 MHz. Since the frequency of the second pole is more than two octaves higher than that of the first pole, the 3-dB frequency will be mostly determined by  $f_{p1}$ ,

$$f_{3dB} \simeq f_{p1} = 1.59 \text{ MHz}$$

**9.46** To lower  $f_H$  from 56.8 MHz (see Example 9.5) to 40 MHz,  $\tau_H$  must be increased to

$$\begin{aligned} \tau_H &= \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 40 \times 10^6} \\ &= 3979 \text{ ps} \end{aligned}$$

Now,

$$\begin{aligned} \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\ 3979 &= 400 + 2150 + \tau_{CL} \\ \Rightarrow \tau_{CL} &= 1429 \text{ ps} \end{aligned}$$

But,

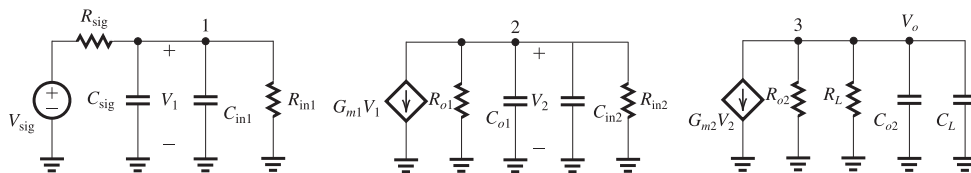
$$\begin{aligned} \tau_{CL} &= C_L R'_L \\ 1429 &= C_L \times 10 \\ \Rightarrow C_L &= 142.9 \text{ fF} \end{aligned}$$

Thus, the original  $C_L$  of 25 fF must be increased by

$$142.9 - 25 = 118 \text{ fF}$$

$$\begin{aligned} \mathbf{9.47} \quad R'_L &= r_o \parallel R_L = 20 \text{ k}\Omega \parallel 12 \text{ k}\Omega \\ &= 7.5 \text{ k}\Omega \\ \tau_{gs} &= C_{gs} R_{gs} = C_{gs} R_{sig} \\ &= 0.2 \times 100 = 20 \text{ ns} \end{aligned}$$

This figure belongs to Problem 9.45.



$$\begin{aligned}\tau_{gd} &= C_{gd}[R_{\text{sig}}(g_m R'_L + 1) + R'_L] \\ &= 0.2[100(1.5 \times 7.5 + 1) + 7.5] \\ &= 246.5 \text{ ns}\end{aligned}$$

$$(a) C_L = 0$$

$$\tau_{CL} = 0$$

$$\tau_H = \tau_{gs} + \tau_{gd} = 20 + 246.5 = 266.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 266.5 \times 10^{-9}} = 597 \text{ kHz}$$

$$(b) C_L = 10 \text{ pF}$$

$$\tau_{CL} = C_L R'_L = 10 \times 7.5 = 75 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 20 + 246.5 + 75 = 341.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 341.5 \times 10^{-9}} = 466 \text{ kHz}$$

$$(c) C_L = 50 \text{ pF}$$

$$\tau_{CL} = C_L R'_L = 50 \times 7.5 = 375 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 20 + 246.5 + 375$$

$$= 641.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 641.5 \times 10^{-9}} = 248 \text{ kHz}$$

Using the Miller approximation, since  $C_L$  is not taken into account, then for all three cases we obtain

$$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$$

$$= 0.2 + 0.2(1.5 \times 7.5 + 1)$$

$$= 2.65 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 2.65 \times 10^{-12} \times 100 \times 10^3}$$

$$= 600 \text{ kHz}$$

which is very close to the estimate obtained using the method of open-circuit time constants for the case  $C_L = 0$ . However, as  $C_L$  is increased, the estimate obtained using the Miller approximation becomes less and less realistic, which is due to the fact that it does not take  $C_L$  into account.

#### 9.48 CS amplifier with:

$$R'_{\text{sig}} = r_o/2$$

$$R'_L = r_o/2$$

$$C_{gs} = C_{gd} = 0.1 \text{ pF}$$

Using the Miller effect, we get

$$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$$

$$= 0.1 + 0.1 \left( \frac{1}{2} g_m r_o + 1 \right)$$

$$= 0.1 \left( \frac{1}{2} g_m r_o + 2 \right)$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 0.1 \left( \frac{1}{2} g_m r_o + 2 \right) (r_o/2) \times 10^{-9}}$$

$$= \frac{1}{0.1\pi \left( \frac{1}{2} g_m r_o + 2 \right) r_o \times 10^{-9}}$$

where  $r_o$  is in  $\text{k}\Omega$ .

For initial design,

$$g_m = 2 \text{ mA/V}, \quad r_o = 20 \text{ k}\Omega$$

$$f_H = \frac{1}{0.1\pi \left( \frac{1}{2} \times 40 + 2 \right) \times 20 \times 10^{-9}}$$

$$= 7.23 \text{ MHz}$$

(i) For the case,  $I$  is reduced by a factor of 4:

Since

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

reducing  $I_D$  by a factor of 4 reduces  $g_m$  by a factor of 2,

$$g_m = 1 \text{ mA/V}$$

Since

$$r_o = \frac{V_A}{I_D},$$

reducing  $I_D$  by a factor of 4, increases  $r_o$  by a factor of 4,

$$r_o = 80 \text{ k}\Omega$$

Thus,

$$f_H = \frac{1}{0.1\pi \left( \frac{1}{2} \times 80 + 2 \right) \times 80 \times 10^{-9}}$$

$$= 0.95 \text{ MHz}$$

(ii) For the case,  $I$  is increased by a factor of 4,

$g_m$  increases by a factor of 2,

$$g_m = 4 \text{ mA/V}$$

and  $r_o$  decreases by a factor of 4,

$$r_o = 5 \text{ k}\Omega$$

Thus,

$$f_H = \frac{1}{0.1\pi \left( \frac{1}{2} \times 20 + 2 \right) \times 5 \times 10^{-9}}$$

$$= 53.1 \text{ MHz}$$

**9.49**  $R_{\text{sig}} = 5 \text{ k}\Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}} = 5 \text{ k}\Omega \parallel 5 \text{ k}\Omega$$

$$= 2.5 \text{ k}\Omega$$

$$\tau_H = C_\pi R_\pi = C_\pi R'_{\text{sig}}$$

$$= 10 \times 2.5 = 25 \text{ ns}$$

$$\tau_\mu = C_\mu R_\mu$$

$$= C_\mu [R'_{\text{sig}} (1 + g_m R'_L) + R'_L]$$

$$= 1 \times [2.5(1 + 20 \times 5) + 5]$$

$$= 257.5 \text{ ns}$$

$$\tau_{CL} = C_L R'_L$$

$$= 10 \times 5 = 50 \text{ ns}$$

$$\tau_H = \tau_\pi + \tau_\mu + \tau_{CL}$$

$$= 25 + 257.5 + 50 = 332.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 332.5 \times 10^{-9}} = 479 \text{ kHz}$$

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R'_L$$

$$= -\frac{5}{5 + 5} \times 20 \times 5$$

$$= -50 \text{ V/V}$$

**9.50** Refer to Fig. 9.18(c).

$$\frac{V_o}{V_{\text{sig}}} = \frac{1/g_m}{\frac{1}{g_m} + R_{\text{sig}}} g_m R_L$$

$$= \frac{1/5}{\frac{1}{5} + 1} \times 5 \times 10$$

$$= 8.3 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_{gs} \left( R_{\text{sig}} \parallel \frac{1}{g_m} \right)}$$

$$f_{P1} = \frac{1}{2\pi \times 4 \times 10^{-12} \left( 1 \parallel \frac{1}{5} \right) \times 10^3}$$

$$= 239 \text{ MHz}$$

$$f_{P2} = \frac{1}{2\pi (C_L + C_{gd}) R_L}$$

$$= \frac{1}{2\pi (2 + 0.2) \times 10^{-12} \times 10 \times 10^3}$$

$$= 7.23 \text{ MHz}$$

Since  $f_{P1} \gg f_{P2}$ ,  $f_{P2}$  will be dominant and

$$f_H \simeq f_{P2} = 7.23 \text{ MHz}$$

**9.51**  $r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$

$$R_{\text{sig}} = r_o/2 = 50 \text{ k}\Omega$$

$$R_L = r_o = 100 \text{ k}\Omega$$

$$g_m r_o = 1.5 \times 100 = 150$$

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{100 + 100}{1 + 150} = 1.32 \text{ k}\Omega$$

$$R_o = R_{\text{sig}} + r_o + g_m r_o R_{\text{sig}}$$

$$= 50 + 100 + 150 \times 50 = 7650 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} (R_{\text{in}} \parallel R_{\text{sig}})$$

$$\tau_{gs} = 0.2(1.32 \parallel 50)$$

$$= 0.26 \text{ ns}$$

$$R_{gd} = R_L \parallel R_o$$

$$= 100 \parallel 7650 = 98.7 \text{ k}\Omega$$

$$\tau_{gd} = (C_{gd} + C_L + C_{db}) R_{gd}$$

$$= (0.015 + 0.03 + 0.02) \times 98.7$$

$$= 6.42 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} = 0.26 + 6.42$$

$$= 6.68 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 6.68 \times 10^{-9}} = 23.8 \text{ MHz}$$

**9.52**  $R_o = r_{o2} + r_{o1} + (g_{m2} r_{o2}) r_{o1}$

$$= 2r_o + g_m r_o^2$$

$$= 2 \times 20 + 2 \times 20 \times 20 = 840 \text{ k}\Omega$$

$$A_v = -g_{m1} (R_o \parallel R_L)$$

$$= -2(840 \parallel 1000)$$

$$= -913 \text{ V/V}$$

Using Eq. (9.92), we obtain

$$\tau_H = R_{\text{sig}} [C_{gs1} + C_{gd1} (1 + g_{m1} R_{d1})]$$

$$+ R_{d1} (C_{gd1} + C_{db1} + C_{gs2})$$

$$+ (R_L \parallel R_o)(C_L + C_{db2} + C_{gd2})$$

where we have assumed that  $C_L$  does not include  $C_{db2}$ . Now,

$$R_{d1} = r_{o1} \parallel R_{in2}$$

$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_m r_{o2}}$$

$$= \frac{20 + 1000}{1 + 2 \times 20} = 24.9 \text{ k}\Omega$$

$$R_{d1} = 20 \parallel 24.9 = 11.1 \text{ k}\Omega$$

Thus,

$$\begin{aligned} \tau_H &= 100[20 + 5(1 + 2 \times 11.1)] \\ &\quad + 11.1(5 + 5 + 20) \\ &\quad + (1000 \parallel 840)(20 + 5 + 5) \end{aligned}$$

$$= 13600 + 333 + 13696$$

$$= 27,629 \text{ ps} = 27.63 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 27.63 \times 10^{-9}} = 5.76 \text{ MHz}$$

$$\mathbf{9.53} \quad 20 \log |A_M| = 74 \text{ dB}$$

$$\Rightarrow |A_M| = 5000$$

$$R_o \simeq (g_m r_o) r_o$$

$$R_L = R_o$$

$$|A_M| = g_m(R_L \parallel R_o)$$

$$5000 = \frac{1}{2} g_m R_o$$

$$= \frac{1}{2} (g_m r_o)^2$$

$$\Rightarrow g_m r_o = 100$$

$$\frac{2V_A}{V_{OV}} = 100$$

$$\Rightarrow V_{OV} = \frac{2 \times 10}{100} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times 50 \times 0.2^2$$

$$= 0.2 \text{ mA}$$

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$f_t = \frac{2 \times 10^{-3}}{2\pi(1 + 0.1) \times 10^{-12}}$$

$$= 289.4 \text{ MHz}$$

$$f_{3dB} = \frac{f_t}{|A_M|} = \frac{289.4}{5000} = 57.9 \text{ kHz}$$

If the cascode transistor is removed,

$$A_M = -g_m(r_o \parallel R_L)$$

where

$$R'_L = r_o \parallel R_L$$

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{0.2 \text{ mA}} = 50 \text{ k}\Omega$$

$$R_L = (g_m r_o) r_o = (2 \times 50) \times 50 = 5000 \text{ k}\Omega$$

$$R'_L = 50 \parallel 5000 = 49.5 \text{ k}\Omega$$

$$A_M = -2 \times 49.5 = -99 \text{ V/V}$$

$$f_{3dB} = \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

$$= \frac{1}{2\pi(1 + 0.1) \times 10^{-12} \times 49.5 \times 10^3}$$

$$= 2.9 \text{ MHz}$$

$$f_t = |A_M| f_{3dB}$$

$$= 99 \times 2.9$$

$$= 287.1 \text{ MHz}$$

which is almost equal to  $f_t$  of the cascode. Thus, removing the cascode transistor lowers  $|A_M|$  by a factor of almost 50 and increases  $f_{3dB}$  by the same factor, keeping  $f_t$  constant.

**9.54** Refer to Fig. 9.22.

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m (\beta r_o \parallel R_L)$$

$$= -\frac{2.5}{2.5 + 0.05 + 5} 40(100 \times 100 \parallel 2)$$

$$\simeq -26.5 \text{ (V/V)}$$

$$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$$

$$= 2.5 \parallel (0.05 + 5) = 1.67 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 1.67 \text{ k}\Omega$$

$$R_{c1} = r_{o1} \parallel \left[ r_{e2} \frac{r_{o2} + R_L}{r_{o2} + R_L / (\beta_2 + 1)} \right]$$

$$= 100 \parallel \left[ 0.025 \frac{100 + 2}{100 + \frac{2}{101}} \right]$$

$$= 25.5 \text{ }\Omega$$

$$R_{\mu 1} = R'_{sig} (1 + g_{m1} R_{c1}) + R_{c1}$$

$$\begin{aligned}
 &= 1.67(1 + 40 \times 0.0255) + 0.0255 \\
 &= 3.4 \text{ k}\Omega \\
 R_o &= \beta_2 r_{o2} = 100 \times 100 = 10,000 \text{ k}\Omega \\
 \tau_H &= C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{cs1} + C_{\pi 2}) R_{c1} \\
 &\quad + (C_L + C_{cs2} + C_{\mu 2})(R_L \parallel R_o) \\
 &= 10 \times 1.67 + 2 \times 3.4 + (0 + 10) \times 0.0255 \\
 &\quad + (0 + 0 + 2)(2 \parallel 10,000) \\
 &= 16.7 + 6.8 + 0.255 + 4 = 27.8 \text{ ns} \\
 f_H &= \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 27.8 \times 10^{-9}} \\
 &= 5.7 \text{ MHz}
 \end{aligned}$$

**9.55** Replacing the BJT with its high-frequency  $T$  model while neglecting  $r_o$  results in the equivalent circuit shown in the figure below.

(a) There are two separate poles, one at the input given by

$$f_{p1} = \frac{1}{2\pi C_{\pi} (R_{\text{sig}} \parallel r_e)}$$

and the other at the output, given by

$$f_{p2} = \frac{1}{2\pi (C_L + C_{\mu}) R_L} \quad \text{Q.E.D.}$$

(b)  $I_C = 1 \text{ mA}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 25 \Omega$$

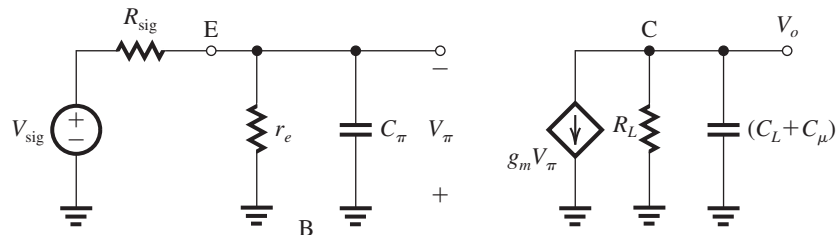
$$f_{p1} = \frac{1}{2\pi \times 10 \times 10^{-12} (1 \parallel 0.025) \times 10^3} = 652.5 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi (1 + 1) \times 10^{-12} \times 10 \times 10^3} = 7.96 \text{ MHz}$$

Since  $f_{p2} \ll f_{p1}$ ,  $f_{p2}$  will be dominant and

$$f_H \simeq f_{p2} = 7.96 \text{ MHz}$$

This figure belongs to Problem **9.55**.



$$\begin{aligned}
 f_T &= \frac{g_m}{2\pi (C_{\pi} + C_{\mu})} \\
 &= \frac{40 \times 10^{-3}}{2\pi (10 + 1) \times 10^{-12}} \\
 &= 579 \text{ MHz}
 \end{aligned}$$

**9.56** Refer to Fig. 9.18 with

$$R_L = r_o$$

$$R_{\text{sig}} = r_o/2$$

$$C_L = C_{gs}$$

Now,

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{r_o + r_o}{1 + g_m r_o}$$

$$\simeq \frac{2r_o}{g_m r_o} = \frac{2}{g_m}$$

$$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}}$$

$$\begin{aligned}
 &= \frac{r_o}{2} \parallel \frac{2}{g_m} \\
 &= \frac{\frac{r_o}{2} \times \frac{2}{g_m}}{\frac{r_o}{2} + \frac{2}{g_m}} = \frac{r_o}{\frac{1}{2} g_m r_o + 2}
 \end{aligned}$$

$$\simeq \frac{r_o}{\frac{1}{2} g_m r_o} = \frac{2}{g_m}$$

$$\tau_{gs} = C_{gs} R_{gs} \simeq \frac{2C_{gs}}{g_m}$$

$$R_o = r_o + R_{\text{sig}} + g_m r_o R_{\text{sig}}$$

$$R_o = r_o + \frac{1}{2} r_o + \frac{1}{2} g_m r_o r_o$$

$$\simeq \frac{1}{2} g_m r_o^2$$

$$R_{gd} = R_L \parallel R_o$$

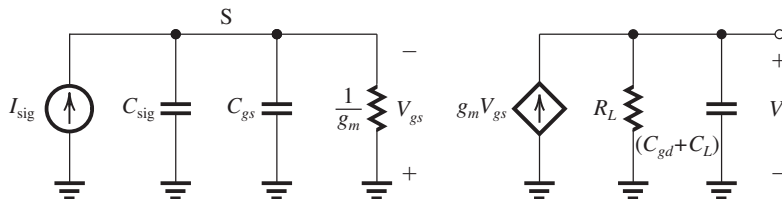
$$= r_o \parallel \frac{1}{2} g_m r_o^2$$

$$\simeq r_o$$

$$\tau_{gd} = (C_L + C_{gd}) R_{gd}$$



This figure belongs to Problem 9.58.



$$= (C_{gs} + C_{gd})r_o$$

$$\tau_H = \tau_{gs} + \tau_{gd}$$

$$= \frac{2C_{gs}}{g_m} + (C_{gs} + C_{gd})r_o$$

Since  $g_m r_o \gg 1$ , we obtain

$$\tau_H \simeq (C_{gs} + C_{gd})r_o$$

and

$$f_H = \frac{1}{2\pi\tau_H}$$

$$f_H = \frac{1}{2\pi(C_{gs} + C_{gd})r_o}$$

Since

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

then

$$f_H = \frac{f_T}{g_m r_o} \quad \text{Q.E.D.}$$

**9.57** Refer to Example 9.9. To reduce  $f_H$  to 200 MHz,  $\tau_H$  must become

$$\begin{aligned} \tau_H &= \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 200 \times 10^6} \\ &= 795.8 \text{ ps} \end{aligned}$$

Since  $\tau_{gs}$  remains constant at 26.6 ps,  $\tau_{CL}$  must be increased to

$$\tau_{CL} = 795.8 - 26.6 = 769.2 \text{ ps}$$

But,

$$\tau_{CL} = (C_{gd} + C_L)R_{gd}$$

thus,

$$769.2 = (5 + C_L) \times 18.7$$

$$\Rightarrow C_L + 5 = 41.1 \text{ fF}$$

$$C_L = 36.1 \text{ fF}$$

Thus, the amount of additional capacitance to be connected at the output is

$$36.1 - 25 = 11.1 \text{ fF}$$

**9.58** The figure above shows the high-frequency equivalent circuit of the amplifier where the transistor has been replaced with its  $T$  model with  $r_o$  neglected. The transfer function  $V_o/I_{\text{sig}}$  can be found as follows:

$$V_{gs} = -I_{\text{sig}}/Y_{\text{in}}$$

where

$$Y_{\text{in}} = g_m + s(C_{\text{sig}} + C_{gs})$$

thus

$$V_{gs} = -\frac{I_{\text{sig}}}{g_m + s(C_{\text{sig}} + C_{gs})} \quad (1)$$

$$V_o = -g_m V_{gs} \frac{1}{\frac{1}{R_L} + s(C_L + C_{gd})} \quad (2)$$

Combining Eqs. (1) and (2), we get

$$\frac{V_o}{I_{\text{sig}}} = \frac{R_L}{\left[1 + s \frac{C_{\text{sig}} + C_{gs}}{g_m}\right] [1 + s(C_L + C_{gd})R_L]}$$

Thus, the two poles have the frequencies

$$\omega_{P1} = \frac{g_m}{C_{\text{sig}} + C_{gs}}$$

and

$$\omega_{P2} = \frac{1}{(C_L + C_{gd})R_L}$$

**9.59** (a)  $A_M = -g_m R'_L$

where

$$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10 \text{ k}\Omega$$

Thus,

$$A_M = -4 \times 10 = -40 \text{ V/V}$$

$$\begin{aligned} \tau_{gs} &= C_{gs} R_{gs} \\ &= C_{gs} R_{\text{sig}} = 2 \times 20 = 40 \text{ ns} \end{aligned}$$

$$\begin{aligned} R_{gd} &= R_{\text{sig}}(1 + g_m R'_L) + R'_L \\ &= 20(1 + 4 \times 10) + 10 \\ &= 830 \text{ k}\Omega \end{aligned}$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.3 \times 830 = 249 \text{ ns}$$

$$\tau_{CL} = C_L R'_L = 1 \times 10 = 10 \text{ ns}$$

$$\begin{aligned}\tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\ &= 40 + 249 + 10 = 299 \text{ ns} \\ f_H &= \frac{1}{2\pi\tau_H} \\ &= \frac{1}{2\pi \times 299 \times 10^{-9}} = 532 \text{ kHz} \\ \text{GB} &\equiv |A_M|f_H = 40 \times 532 = 21.3 \text{ MHz}\end{aligned}$$

$$(b) A_M = -g_{m1}(R_o \parallel R_L)$$

where

$$\begin{aligned}R_o &= r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1} \\ &= 2r_o + g_m r_o^2 \\ &= 2 \times 20 + 4 \times 20 \times 20 = 1640 \text{ k}\Omega \\ A_M &= -4(1640 \parallel 20) = -79 \text{ V/V} \\ R_{in2} &= \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}} \\ &= \frac{20 + 20}{1 + 4 \times 20} \simeq 0.49 \text{ k}\Omega \\ R_{d1} &= r_{o1} \parallel R_{in2} = 20 \parallel 0.49 = 0.48 \text{ k}\Omega\end{aligned}$$

Using Eq. (9.92), we obtain

$$\begin{aligned}\tau_H &= R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1}) \\ &\quad + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) \\ &\quad + (R_L \parallel R_o)(C_L + C_{gd2})] \\ &= 20[2 + 0.3(1 + 4 \times 0.48)] \\ &\quad + 0.48(0.3 + 0.2 + 2) \\ &\quad + (20 \parallel 1640)(1 + 0.3)\end{aligned}$$

$$\tau_H = 57.5 + 1.2 + 25.7 = 84.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 84.4 \times 10^{-9}} = 1.89 \text{ MHz}$$

$$\text{GB} \equiv |A_M|f_H = 79 \times 1.89 = 149 \text{ MHz}$$

Note the increase in gain, bandwidth, and GB.

**9.60** (a) For the CS amplifier,

$$|A_M| = g_m(r_o \parallel r_o) = \frac{1}{2}g_m r_o$$

$$\begin{aligned}f_H &= \frac{1}{2\pi C_{in}R_{sig}} \\ &= \frac{1}{2\pi \left[ C_{gs} + C_{gd} \left( \frac{1}{2}g_m r_o + 1 \right) \right] R_{sig}}\end{aligned}\quad (1)$$

For the cascode amplifier,

$$\begin{aligned}|A_M| &= g_m(R_o \parallel r_o) \\ &= g_m[(g_m r_o)r_o \parallel r_o]\end{aligned}$$

$$\simeq g_m r_o$$

Thus, the gain increases by a factor of 2.

$$f_H = \frac{1}{2\pi C_{in}R_{sig}}$$

where

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R_{d1})$$

$$R_{d1} = r_o \parallel R_{in2}$$

$$= r_o \parallel \frac{R_L + r_o}{g_m r_o}$$

$$R_{d1} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{g_m} \simeq \frac{2}{g_m}$$

$$C_{in} = C_{gs} + C_{gd} \left( 1 + g_m \times \frac{2}{g_m} \right)$$

$$= C_{gs} + 3C_{gd}$$

$$f_H = \frac{1}{2\pi(C_{gs} + 3C_{gd})R_{sig}} \quad (2)$$

From (1) and (2), the ratio  $N$  of  $f_H$  of the cascode amplifier to  $f_H$  of the CS amplifier is

$$N = \frac{C_{gs} + C_{gd} \left( \frac{1}{2}g_m r_o + 1 \right)}{C_{gs} + 3C_{gd}}$$

Thus,

$$N \simeq \frac{C_{gs} + \frac{1}{2}(g_m r_o)C_{gd}}{C_{gs} + 3C_{gd}} \quad \text{Q.E.D.}$$

$$(b) 50 = \frac{1}{2}g_m r_o$$

$$\Rightarrow g_m r_o = 100$$

$$N = \frac{C_{gs} + \frac{1}{2} \times 100 \times 0.1C_{gs}}{C_{gs} + 3 \times 0.1C_{gs}}$$

$$= \frac{1 + 5}{1 + 0.3} = 4.6$$

$$(c) g_m r_o = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2 \times 10}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.2^2$$

$$= 0.08 \text{ mA} = 80 \mu\text{A}$$

**9.61** (a) From Fig. 9.29, we have

$$f_i = \frac{g_m}{2\pi(C_L + C_{gd})} = \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{2\pi(C_L + C_{gd})} \sqrt{I_D} \quad \text{Q.E.D.} \quad (1)$$

$$(b) \quad g_m = \sqrt{2\mu_n C_{ox}(W/L)} \sqrt{I_D} \quad (2)$$

$$V_{OV} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}(W/L)}} \quad (3)$$

$$r_o = \frac{V_A}{I_D} \quad (4)$$

$$R_o = (g_m r_o) r_o \quad (5)$$

$$\begin{aligned} A_M &= -g_m(R_o \parallel R_L) \\ &= -g_m(R_o \parallel R_o) \\ &= -\frac{1}{2}g_m R_o \end{aligned} \quad (6)$$

Substituting

$$\mu_n C_{ox} = 0.4 \text{ mA/V}^2, \quad W/L = 20, \quad C_L = 20 \text{ fF},$$

$C_{gd} = 5 \text{ fF}$  and  $V_A = 10 \text{ V}$  in Eqs. (1)–(6), we obtain the results in the table on the next page.

**9.62** (a) Gain from base to collector of

$$Q_1 = -1. \text{ Thus,}$$

$$\begin{aligned} C_{in} &= C_{\pi 1} + C_{\mu 1}(1 + 1) \\ &= C_{\pi 1} + 2C_{\mu 1} \end{aligned}$$

$$\begin{aligned} f_{P1} &= \frac{1}{2\pi R'_{sig} C_{in}} \\ &= \frac{1}{2\pi R'_{sig} (C_{\pi 1} + 2C_{\mu 1})} \quad \text{Q.E.D.} \end{aligned}$$

At the output node, the total capacitance is  $(C_L + C_{c2} + C_{\mu 2})$  and since  $r_o$  is large,  $R_o$  will be very large, thus the total resistance will be  $R_L$ . Thus the pole introduced at the output node will have a frequency  $f_{P2}$ ,

$$f_{P2} = \frac{1}{2\pi(C_L + C_{c2} + C_{\mu 2})R_L} \quad \text{Q.E.D.}$$

(b)  $I = 1 \text{ mA}$

$$g_m = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

(i)  $R_{sig} = 1 \text{ k}\Omega$

$$R'_{sig} = r_{\pi} \parallel R_{sig} = 2.5 \parallel 1 = 0.71 \text{ k}\Omega$$

$$f_{P1} = \frac{1}{2\pi \times 0.71 \times 10^3 (10 + 2 \times 2) \times 10^{-12}}$$

$$= 16 \text{ MHz}$$

$$\begin{aligned} f_{P2} &= \frac{1}{2\pi(0 + 0 + 2) \times 10^{-12} \times 2 \times 10^3} \\ &\simeq 40 \text{ MHz} \end{aligned}$$

$$f_H = 1 / \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1 / \sqrt{\frac{1}{16^2} + \frac{1}{40^2}} = 14.9 \text{ MHz}$$

(ii)  $R_{sig} = 10 \text{ k}\Omega$

$$R'_{sig} = r_{\pi} \parallel R_{sig} = 2.5 \parallel 10 = 2 \text{ k}\Omega$$

$$\begin{aligned} f_{P1} &= \frac{1}{2\pi \times 2 \times 10^3 (10 + 4) \times 10^{-12}} \\ &= 5.7 \text{ MHz} \end{aligned}$$

$$f_{P2} = 40 \text{ MHz}$$

$$f_H = 1 / \sqrt{\frac{1}{5.7^2} + \frac{1}{40^2}} = 5.6 \text{ MHz}$$

**9.63** Refer to Fig. 9.22.

$$I_C = 0.1 \text{ mA}$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_D} = \frac{100 \text{ V}}{0.1 \text{ mA}} = 1000 \text{ k}\Omega$$

$$r_e \simeq \frac{1}{g_m} = 0.25 \text{ k}\Omega$$

$$A_M = -\frac{r_{\pi}}{r_{\pi} + R_{sig}} g_m (\beta r_o \parallel R_L)$$

$$= -\frac{r_{\pi}}{r_{\pi} + r_{\pi}} g_m (\beta r_o \parallel \beta r_o)$$

$$= -\frac{1}{4} \beta g_m r_o$$

$$= -\frac{1}{4} \times 100 \times 4 \times 1000$$

$$= -100,000 \text{ V/V}$$

$$R'_{sig} = r_{\pi} \parallel R_{sig} = r_{\pi} \parallel r_{\pi} = \frac{1}{2} r_{\pi} = 12.5 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 12.5 \text{ k}\Omega$$

$$R_{c1} = r_o \parallel r_{\pi} \parallel \frac{r_o + R_L}{g_m r_o}$$

$$= 1000 \parallel 25 \parallel \frac{1000 + 100 \times 1000}{4 \times 1000}$$

$$= 1000 \parallel 25 \parallel 25.25$$

$$= 1000 \parallel 12.56 = 12.4 \text{ k}\Omega$$

This table belongs to Problem 9.61, part (b).

$I_D$ (mA)	$f_i$ (GHz)	$V_{OV}$ (V)	$g_m$ (mA/V)	$r_o$ (k $\Omega$ )	$R_o$ (M $\Omega$ )	$A_M$ (V/V)	$f_H$ (MHz)
0.1	8	0.16	1.26	100	12.6	−7938	1
0.2	11.5	0.22	1.80	50	4.5	−4050	2.8
0.5	18	0.35	2.83	20	1.13	−1600	11.3

$$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1}R_{c1}) + R_{c1}$$

$$= 12.5(1 + 4 \times 12.4) + 12.4 = 645 \text{ k}\Omega$$

$$R_o = \beta r_o = 100 \times 1000 = 100 \text{ M}\Omega$$

$$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + (C_{cs1} + C_{\pi 2})R_{c1}$$

$$+ (C_L + C_{c2} + C_{\mu 2})(R_L \parallel R_o)$$

To determine  $C_{\pi}$ , we use

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

$$1 \times 10^9 = \frac{4 \times 10^{-3}}{2\pi(C_{\pi} + C_{\mu})}$$

$$\Rightarrow C_{\pi} + C_{\mu} = 0.64 \text{ pF}$$

$$C_{\pi} = 0.64 - 0.1 = 0.54 \text{ pF}$$

$$\tau_H = 0.54 \times 12.5 + 0.1 \times 645 + 0.54 \times 12.4$$

$$+ 0.1 \times \frac{1}{2} \times 100 \times 1000$$

$$= 6.8 + 64.5 + 6.7 + 5000 \text{ ns}$$

Obviously the last term, which is due to the pole at the output node, is dominant. The frequency of the output pole is

$$f_P = \frac{1}{2\pi \times 5000 \times 10^{-9}} = 31.8 \text{ kHz}$$

$$f_H \simeq f_P = 31.8 \text{ MHz}$$

Because the other poles are at much higher frequencies, an estimate of the unity-gain frequency can be found as

$$f_i = |A_M|f_P = 10^5 \times 31.8 \times 10^3 = 3.18 \text{ GHz}$$

This estimate of  $f_i$  is not very good (too high!). The other three poles have frequencies much lower than 3.18 GHz and will cause the gain to decrease faster, reaching the 0 dB value at a frequency lower than 3.18 GHz. Also note that  $f_T$  of the BJTs is 1 GHz and the models we use for the BJT do not hold at frequencies approaching  $f_T$ .

$$\mathbf{9.64} \quad A_M = \frac{R'_L}{R'_L + \frac{1}{g_m}}$$

where

$$R'_L = R_L \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 2 \parallel 20 = 1.82 \text{ k}\Omega$$

$$A_M = \frac{1.82}{1.82 + \frac{1}{5}} = 0.9 \text{ V/V}$$

$$R_o = r_o \parallel \frac{1}{g_m} = 20 \parallel \frac{1}{5} \simeq 0.2 \text{ k}\Omega = 200 \Omega$$

$$f_Z = \frac{g_m}{2\pi C_{gs}}$$

$$= \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

Next, we evaluate  $b_1$  and  $b_2$ :

$$b_1 = \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{\text{sig}} + \left( \frac{C_{gs} + C_L}{g_m R'_L + 1} \right) R'_L$$

$$= \left( 0.1 + \frac{2}{5 \times 1.82 + 1} \right) 20 + \left( \frac{2 + 1}{5 \times 1.82 + 1} \right) \times 1.82$$

$$= 5.96 + 0.54 = 6.50 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}}{g_m R'_L + 1} R_{\text{sig}} R'_L$$

$$= \frac{(2 + 0.1) \times 1 + 2 \times 0.1}{5 \times 1.82 + 1} \times 20 \times 1.82$$

$$= 8.3 \times 10^{-18}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.3}}{6.5} = 0.44$$

Thus, the poles are real and their frequencies can be obtained by finding the roots of the polynomial  $(1 + b_1 s + b_2 s^2)$

$$= 1 + 6.5 \times 10^{-9} s + 8.3 \times 10^{-18} s^2$$

which are

$$\omega_{P1} = 0.21 \times 10^9 \text{ rad/s}$$

and

$$\omega_{P2} = 0.57 \times 10^9 \text{ rad/s}$$

Thus,

$$f_{P1} = \frac{\omega_{P1}}{2\pi} = 33.4 \text{ MHz}$$

$$f_{P2} = \frac{\omega_{P2}}{2\pi} = 90.7 \text{ MHz}$$

Since the two poles are relatively close to each other, an estimate of  $f_H$  can be obtained using

$$f_H = 1 / \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 31.6 \text{ MHz}$$

**9.65** Refer to Fig. 9.23(c). Replacing  $C_{gs}$  with an input capacitance between  $G$  and ground, we get

$$C_{eq} = C_{gs}(1 - K)$$

where

$$K = \frac{g_m R'_L}{1 + g_m R'_L}$$

then

$$C_{eq} = C_{gs} / (1 + g_m R'_L)$$

and the total input capacitance becomes

$$C_{in} = C_{gd} + C_{eq}$$

$$= C_{gd} + \frac{C_{gs}}{1 + g_m R'_L}$$

The frequency of the input pole is

$$f_{P1} = \frac{1}{2\pi R_{sig} \left( C_{gd} + \frac{C_{gs}}{1 + g_m R'_L} \right)}$$

$$f_H \simeq f_{P1}$$

**9.66** Refer to Fig. 9.25.

$$I_C = 1 \text{ mA}$$

$$g_m = 40 \text{ mA/V}, \quad r_e = 25 \Omega$$

$$r_\pi = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20 \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$2 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 3.2 \text{ pF}$$

$$C_\pi = 3.2 - 0.1 = 3.1 \text{ pF}$$

$$R'_L = R_L \parallel r_o = 1 \parallel 20 = 0.95 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \frac{R_{sig}}{\beta + 1}}$$

$$A_M = \frac{0.95}{0.95 + 0.025 + \frac{1}{101}}$$

$$= 0.96 \text{ V/V}$$

$$f_z = \frac{1}{2\pi C_\pi r_e}$$

$$= \frac{1}{2\pi \times 3.1 \times 10^{-12} \times 25}$$

$$\simeq 2 \text{ GHz}$$

$$b_1 = \frac{\left[ C_\pi + C_\mu \left( 1 + \frac{R'_L}{r_e} \right) \right] R_{sig} + \left[ C_\pi + C_L \left( 1 + \frac{R'_{sig}}{r_\pi} \right) \right] R'_L}{1 + \frac{R'_L}{r_e} + \frac{R_{sig}}{r_\pi}}$$

$$= \frac{\left[ 3.1 + 0.1 \left( 1 + \frac{0.95}{0.025} \right) \right] \times 1 + (3.1 + 0) \times 0.95}{1 + \frac{0.95}{0.025} + \frac{1}{2.5}} \times 10^{-9}$$

$$= 0.25 \times 10^{-9}$$

$$b_2 = \frac{[(C_\pi + C_\mu)C_L + C_\pi C_\mu]R'_L R_{sig}}{1 + \frac{R'_L}{r_e} + \frac{R_{sig}}{r_\pi}}$$

$$= \frac{(0 + 3.1 \times 0.1) \times 0.95 \times 1}{1 + \frac{0.95}{0.025} + \frac{1}{2.5}} \times 10^{-18}$$

$$= 7.5 \times 10^{-21}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{7.5 \times 10^{-21}}}{0.25 \times 10^{-9}} = 0.346$$

Thus, the poles are real and their frequencies can be found as the roots of the polynomial

$$(1 + b_1 s + b_2 s^2)$$

$$= 1 + 0.25 \times 10^{-9} s + 7.5 \times 10^{-21} s^2$$

$$= \left( 1 + \frac{s}{\omega_{P1}} \right) \left( 1 + \frac{s}{\omega_{P2}} \right)$$

$$\Rightarrow \omega_{P1} = 4.65 \times 10^9 \text{ rad/s}$$

$$\omega_{P2} = 28.7 \times 10^9 \text{ rad/s}$$

Thus,

$$f_{P1} = 740 \text{ MHz}$$

$$f_{P2} = 4.6 \text{ GHz}$$

Thus,

$$f_{3dB} \simeq f_{P1} = 740 \text{ MHz}$$

$$\mathbf{9.67} \quad f_H \simeq f_{P1} \simeq \frac{1}{2\pi b_1}$$

where

$$b_1 = \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{sig} + \left( \frac{C_{gs} + C_L}{g_m R'_L + 1} \right) R'_L$$

For  $C_L = 0$ ,

$$b_1 = C_{gd} R_{sig} + \frac{C_{gs}}{g_m R'_L + 1} (R_{sig} + R'_L)$$

For  $R_{sig} \gg R'_L$ ,

$$b_1 \simeq C_{gd}R_{\text{sig}} + \frac{C_{gs}}{g_m R'_L + 1} R_{\text{sig}}$$

$$= \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{\text{sig}}$$

and

$$f_H = \frac{1}{2\pi R_{\text{sig}} \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right)} \quad \text{Q.E.D.}$$

For the given numerical values,

$$f_H = \frac{1}{2\pi \times 100 \times 10^3 \left[ 0.1 + \frac{12}{5 \times (2 \parallel 20) + 1} \right]} \times 10^{-12}$$

$$= 1.23 \text{ MHz}$$

**9.68** For a maximally flat response we have

$$Q = \frac{1}{\sqrt{2}}$$

$$\omega_0 = \omega_{3dB} = 2\pi \times 10^6 \text{ rad/s}$$

Thus, the transfer function will be

$$\frac{V_o(s)}{V_i(s)} = \frac{\text{dc gain} \times \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{0.8(2\pi \times 10^6)^2}{s^2 + s\sqrt{2} \times 2\pi \times 10^6 + (2\pi \times 10^6)^2}$$

$$= \frac{31.58 \times 10^{12}}{s^2 + s \ 8.886 \times 10^6 s + 39.48 \times 10^{12}}$$

**9.69** With  $g_{mb} = 0$  and  $r_o$  large, we obtain

$$R'_L \simeq R_L$$

and

$$A_M = \frac{g_m R_L}{g_m R_L + 1}$$

For  $A_M = 0.9$ ,

$$0.9 = \frac{g_m R_L}{g_m R_L + 1}$$

$$\Rightarrow g_m R_L = 9$$

Now, for a maximally-flat response,  $Q = 1/\sqrt{2}$ .

Using the expression for  $Q$  in Eq. (9.111), we get

$$Q = \frac{\sqrt{g_m R_L + 1} \sqrt{[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]R_{\text{sig}}R_L}}{[C_{gs} + C_{gd}(g_m R_L + 1)]R_{\text{sig}} + (C_{gs} + C_L)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{9 + 1} \sqrt{[(10 + 1)10 + 10 \times 1]} \times 100 \times R_L}{[10 + 1(9 + 1)] \times 100 + (10 + 10)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{R_L}}{10 \left( 1 + \frac{R_L}{100} \right)}$$

$$\Rightarrow \left( \frac{R_L}{100} \right)^2 - 4 \left( \frac{R_L}{100} \right) + 1 = 0$$

This equation results in two solutions,

$$R_L = 27 \text{ k}\Omega \text{ and } R_L = 373 \text{ k}\Omega$$

The second answer is not very practical as it implies the transistor is operating at  $g_m = 9/373 = 0.024 \text{ mA/V}$ , a very small transconductance!. We will pursue only the first answer. Thus,

$$R_L = 27 \text{ k}\Omega$$

$$g_m = 0.33 \text{ mA/V}$$

and the 3-dB frequency is found using Eq. (9.110):

$$f_{3dB} = f_0 = \frac{1}{2\pi \sqrt{b_2}}$$

$$\omega_{3dB} = \sqrt{\frac{g_m R_L + 1}{R_{\text{sig}} R_L [(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]}}$$

$$= \sqrt{\frac{9 + 1}{100 \times 27[(10 + 1) \times 10 + 10 \times 1]} \times 10^6 \times 10^{-24}}$$

$$= 5.55 \text{ Mrad/s}$$

$$f_{3dB} = 884 \text{ kHz}$$

**9.70**  $I = 0.4 \text{ mA}$

$$(a) I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times 16 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.25}$$

$$= 1.6 \text{ mA/V}$$

$$(b) r_o = \frac{V_A}{I_D} = \frac{20}{0.2}$$

$$= 100 \text{ k}\Omega$$

$$R_D \parallel r_o = 10 \parallel 100$$

$$= 9.1 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$= 1.6 \times 9.1 = 14.5 \text{ V/V}$$

(c)  $R_{\text{sig}}$  small and the frequency response is determined by the output pole:

$$f_{p2} = \frac{1}{2\pi (C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$$

$$= \frac{1}{2\pi (100 + 5 + 5) \times 10^{-15} \times 9.1 \times 10^3}$$

$$= 159 \text{ MHz}$$

$$f_H \simeq 159 \text{ MHz}$$

$$(d) R_{\text{sig}} = 40 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs}$$

$$= C_{gs} R_{\text{sig}}$$

$$= 40 \times 10^{-15} \times 40 \times 10^3$$

$$= 1.6 \text{ ns}$$

$$R_{gd} = R_{\text{sig}}(g_m R'_L + 1) + R'_L$$

$$= 40(1.6 \times 9.1 + 1) + 9.1$$

$$= 631.5 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 5 \times 631.5 = 3.16 \text{ ns}$$

$$\tau_{CL} = (C_L + C_{db}) R'_L$$

$$= (100 + 5) \times 9.1$$

$$= 955.5 \text{ ps} = 0.96 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 1.6 + 3.16 + 0.96 = 5.72 \text{ ns}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 5.72 \times 10^{-9}}$$

$$= 27.8 \text{ MHz}$$

**9.71** The common-mode gain will have a zero at

$$f_Z = \frac{1}{2\pi R_{SS} C_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \times 1 \times 10^{-12}}$$

$$= 1.59 \text{ MHz}$$

Thus, the CMRR will have two poles, one at  $f_Z$ , i.e. at 1.59 MHz, and the other at the dominant pole of  $A_d$ , 20 MHz. Thus, the 3-dB frequency of CMRR will be approximately equal to  $f_Z$ ,

$$f_{3dB} = 1.59 \text{ MHz}$$

$$\mathbf{9.72} \quad g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$$

$$= \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 1(100 \parallel 100) = 50 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

$$= \frac{1}{2\pi C_L (r_{o2} \parallel r_{o4})}$$

$$= \frac{1}{2\pi \times 0.2 \times 10^{-12} (100 \parallel 100) \times 10^3}$$

$$= \frac{1}{2\pi \times 0.2 \times 50 \times 10^{-9}} = 15.9 \text{ MHz}$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m}$$

where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|} = 1 \text{ mA/V}$$

$$f_{P2} = \frac{1 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 1.59 \text{ GHz}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = 2 \times 1.59 = 3.18 \text{ GHz}$$

**9.73** At low frequencies,

$$A_d = 100 \text{ V/V}$$

$$A_{cm} = 0.1 \text{ V/V}$$

$$\text{CMRR} = \frac{A_d}{A_{cm}} = 1000 \text{ or } 60 \text{ dB}$$

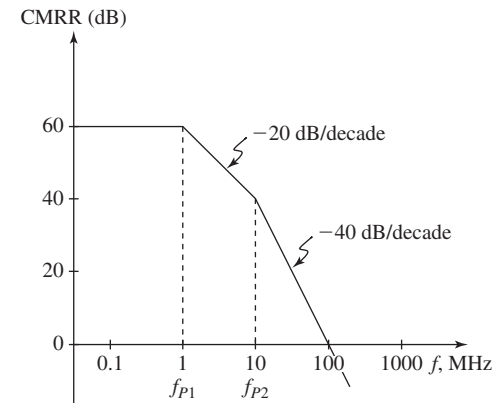
The first pole of CMRR is coincident with the zero of the common-mode gain,

$$f_{P1} = 1 \text{ MHz}$$

The second pole is coincident with the dominant pole of the differential gain,

$$f_{P2} = 10 \text{ MHz}$$

A sketch for the Bode plot for the gain magnitude is shown in the figure.



**9.74** The common-mode gain will have a zero at

$$f_Z = \frac{1}{2\pi \times 1 \times 10^6 \times 1 \times 10^{-12}}$$

= 159 kHz

Thus, the CMRR will have two poles: The first will be coincident with the zero of  $A_{cm}$ ,

$f_{p1} = 159 \text{ kHz}$

and the second will be coincident with the pole of  $A_d$ ,

$f_{p2} = 2 \text{ MHz}$

**9.75**  $R_{SS} = \frac{V_A}{I} = \frac{40}{0.1}$

= 400 k $\Omega$

$C_{SS} = 100 \text{ fF}$

$f_z = \frac{1}{2\pi C_{SS} R_{SS}}$

=  $\frac{1}{2\pi \times 100 \times 10^{-15} \times 400 \times 10^3} = 4 \text{ MHz}$

If  $V_{OV}$  of the current source is reduced by a factor of 2 while  $I$  remains unchanged,  $(W/L)$  must be increased by a factor of 4. Assume  $L$  remains unchanged,  $W$  must be increased by a factor of 4. Since  $C_{SS}$  is proportional to  $W$ , its value will be quadrupled:

$C_{SS} = 400 \text{ fF}$

The output resistance  $R_{SS}$  will remain unchanged. Thus,  $f_z$  will decrease by a factor of 4 to become

$f_z = 1 \text{ MHz}$

**9.76**  $g_m = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$

=  $\frac{10 \times 10^{-3}}{2\pi \times 500 \times 10^6}$

= 3.2 pF

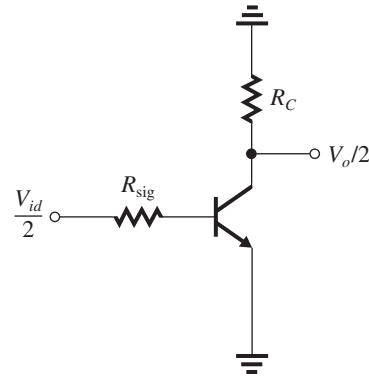
$C_\pi = 3.2 - 0.5 = 2.7 \text{ pF}$

(a)

The figure shows the differential half-circuit and its high-frequency equivalent circuit.

(b)  $A_d \equiv \frac{V_o}{V_{id}} = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R_C$

=  $-\frac{10}{10 + 0.1 + 10} \times 10 \times 10$



= -49.8 V/V

(c)  $C_{in} = C_\pi + C_\mu(1 + g_m R_C)$

= 2.7 + 0.5(1 + 10 × 10)

= 53.2 pF

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$R'_{sig} = r_\pi \parallel (R_{sig} + r_x)$

= 10  $\parallel$  10.1  $\simeq$  5 k $\Omega$

$f_H = \frac{1}{2\pi \times 53.2 \times 10^{-12} \times 5 \times 10^3} = 598 \text{ kHz}$

GB =  $|A_d| f_H = 49.8 \times 598 = 29.8 \text{ MHz}$

**9.77**  $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$g_{m1,2} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$

$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$

$A_d = \frac{I}{|V_{OV}|} \left( \frac{2|V_A|}{I} \parallel \frac{2|V_A|}{I} \right)$

$A_d = \frac{|V_A|}{|V_{OV}|}$

$f_{p1} = \frac{1}{2\pi C_L R_o}$

where

$R_o = r_{o2} \parallel r_{o4} = \frac{|V_A|}{I} f_{p1} = \frac{I}{2\pi C_L |V_A|}$  (1)

$f_{p2} = \frac{g_{m3}}{2\pi C_m}$

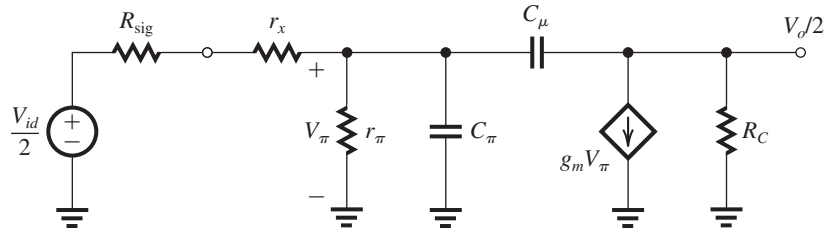
where

$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$

$C_m = \frac{C_L}{4}$



This figure belongs to Problem 9.76.



$$f_{p2} = \frac{4I}{2\pi C_L |V_{OV}|} \quad (2)$$

$$f_Z = \frac{2g_m^3}{2\pi C_m} = 2f_{p2} = \frac{8I}{2\pi C_L |V_{OV}|} \quad (3)$$

Dividing (2) by (1), we obtain

$$\frac{f_{p2}}{f_{p1}} = 4 \frac{|V_A|}{|V_{OV}|} = 4 A_d \quad \text{Q.E.D.}$$

Since  $f_{p2} = 4 A_d f_{p1}$ , the unity-gain frequency  $f_i$  is equal to GB, thus

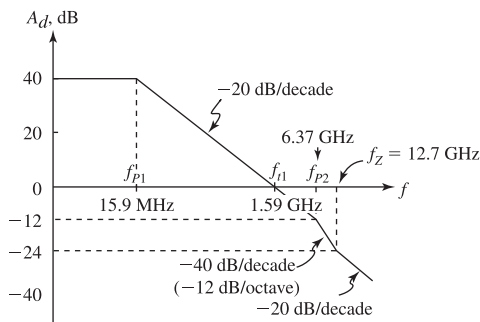
$$\begin{aligned} f_i &= A_d f_{p1} \\ &= \frac{|V_A|}{|V_{OV}|} \frac{I}{2\pi C_L |V_A|} \\ f_i &= \frac{I/|V_{OV}|}{2\pi C_L} \\ &= \frac{g_m}{2\pi C_L} \quad \text{Q.E.D.} \end{aligned}$$

For the numerical values given, we have

$$A_d = \frac{20}{0.2} = 100 \text{ V/V}$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$\begin{aligned} f_{p1} &= \frac{I}{2\pi C_L |V_A|} \\ &= \frac{0.2 \times 10^{-3}}{2\pi \times 100 \times 10^{-15} \times 20} \\ &= 15.9 \text{ MHz} \end{aligned}$$



$$f_i = 15.9 \times 100 = 1.59 \text{ GHz}$$

$$\begin{aligned} f_{p2} &= 4A_d f_{p1} = 4 \times 100 \times 15.9 \\ &= 6.37 \text{ GHz} \end{aligned}$$

$$f_Z = 2f_{p2} = 12.7 \text{ GHz}$$

A sketch of the Bode plot for  $|A_d|$  is shown in the figure.

**9.78** See figure below. The mirror high-frequency equivalent circuit is shown in the figure. Note that we have neglected  $r_o$ . The model of the diode-connected transistor  $Q_1$  reduces to  $r_{e1}$  in parallel with  $C_{\pi 1}$ .

To obtain the current-transfer function  $I_o(s)/I_i(s)$ , we first determine  $V_\pi$  in terms of  $I_i$ . Observe that the short-circuit at the output causes  $C_{\mu 2}$  to appear in parallel with  $C_{\pi 1}$  and  $C_{\pi 2}$ . Thus,

$$V_\pi = \frac{1}{I_i(s) \left( \frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}} \right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})} \quad (1)$$

At the output node we have

$$I_o(s) = g_{m2} V_\pi - sC_{\mu 2} V_\pi \quad (2)$$

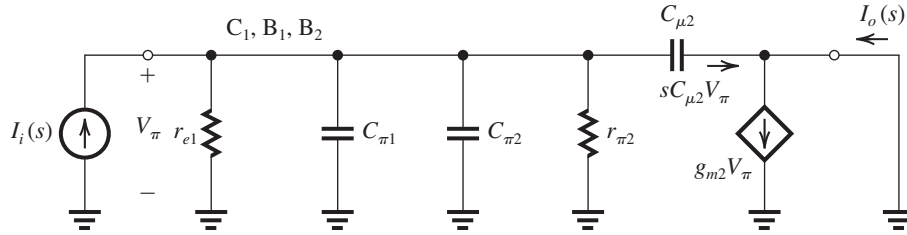
Combining Eqs. (1) and (2) gives

$$\frac{I_o(s)}{I_i(s)} = \frac{g_{m2} - sC_{\mu 2}}{\left( \frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}} \right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

Since the two transistors are operating at approximately equal dc bias currents, their small-signal parameters will be equal, thus

$$\begin{aligned} \frac{I_o(s)}{I_i(s)} &= \frac{g_m - sC_\mu}{\frac{1}{r_e} \left( 1 + \frac{1}{\beta + 1} \right) + s(2C_\pi + C_\mu)} \\ &= \frac{g_m r_e}{1 + \frac{1}{\beta + 1}} \frac{1 - s(C_\mu/g_m)}{1 + s \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta + 1} \right) \right]} \\ &= \alpha \frac{1 - s(C_\mu/g_m)}{1 + \frac{1}{\beta + 1} \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta + 1} \right) \right]} \end{aligned}$$

This figure belongs to Problem 9.78.



$$= \frac{1}{1 + 2/\beta} \frac{1 - s(C_\mu/g_m)}{1 + s \left[ (2C_\pi + C_\mu)r_e \left( 1 + \frac{1}{\beta + 1} \right) \right]}$$

Thus we see that the low-frequency transmission is

$$\frac{I_o(0)}{I_i} = \frac{1}{1 + \frac{2}{\beta}}$$

as expected. The pole is at  $f_p$ ,

$$f_p \simeq \frac{1}{2\pi(2C_\pi + C_\mu)r_e}$$

and the zero is at

$$f_z = \frac{g_m}{2\pi C_\mu}$$

For the numerical values given,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq 25 \Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$$= 12.7 \text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$$

$$f_p = \frac{1}{2\pi(2 \times 10.7 + 2) \times 10^{-12} \times 25}$$

$$= \frac{1 \times 10^{12}}{2\pi \times 23.4 \times 25}$$

$$= 272 \text{ MHz}$$

$$f_z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 3.18 \text{ GHz}$$

**9.79** (a)  $A_M = -g_m R'_L$

where

$$R'_L = R_L \parallel r_o$$

$$= 40 \parallel 40 = 20 \text{ k}\Omega$$

$$A_M = -5 \times 20 = -100 \text{ V/V}$$

$$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{sig}$$

$$= 2 \times 20 = 40 \text{ ns}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 5 \times 20) + 20$$

$$= 2040 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 2040$$

$$= 204 \text{ ns}$$

$$\tau_{CL} = C_L R'_L$$

$$= 1 \times 20 = 20 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 40 + 204 + 20 = 264 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 264 \times 10^{-9}} = 603 \text{ kHz}$$

$$\text{GB} = 100 \times 603 = 60.3 \text{ MHz}$$

(b) With  $R_s = 400 \Omega$ ,

$$G_m = \frac{g_m}{1 + g_m R_s}$$

$$= \frac{5}{1 + 5 \times 0.4} = 1.67 \text{ mA/V}$$

$$R_o = r_o(1 + g_m R_s)$$

$$= 40(1 + 5 \times 0.4) = 120 \text{ k}\Omega$$

$$R'_L = R_L \parallel R_o = 40 \parallel 120 = 30 \text{ k}\Omega$$

$$A_M = -G_m R'_L$$

$$= -1.67 \times 30 = -50 \text{ V/V}$$

$$R_{gd} = R_{sig}(1 + G_m R'_L) + R'_L$$

$$= 20(1 + 1.67 \times 30) + 30$$

$$= 1050 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1050 = 105 \text{ ns}$$

$$\tau_{CL} = C_L R_{CL}$$

$$= C_L R'_L$$

$$\begin{aligned}
 &= 1 \times 30 = 30 \text{ ns} \\
 R_{gs} &= \frac{R_{sig} + R_s + R_{sig}R_s/(r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)} \\
 &= \frac{20 + 0.4 + 20 \times 0.4/(40 + 40)}{1 + 5 \times 0.4 \left( \frac{40}{40 + 40} \right)} \\
 &= 10.25 \text{ k}\Omega \\
 \tau_{gs} &= C_{gs}R_{gs} = 2 \times 10.25 = 20.5 \text{ ns} \\
 \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\
 &= 20.5 + 105 + 30 = 155.5 \text{ ns} \\
 f_H &= \frac{1}{2\pi \tau_H} \\
 &= \frac{1}{2\pi \times 155.5 \times 10^{-9}} = 1.02 \text{ MHz} \\
 \text{GB} &= 51.2 \text{ MHz}
 \end{aligned}$$

**9.80** (a)  $\text{GB} = |A_M|f_H$

$$\begin{aligned}
 &= \frac{1}{2\pi C_{gd}R_{sig}} \\
 &= \frac{1}{2\pi \times 0.2 \times 10^{-12} \times 100 \times 10^3} \\
 &= 7.96 \text{ MHz}
 \end{aligned}$$

(b)  $|A_M| = 20 \text{ V/V}$

$$f_H = \frac{7.96}{20} = 398 \text{ kHz}$$

(c)  $A_0 = g_m r_o$

$$100 = 5 \times r_o$$

$$\Rightarrow r_o = 20 \text{ k}\Omega$$

$$G_m = \frac{g_m}{1 + g_m R_s} = \frac{5}{1 + g_m R_s}$$

$$R_o = r_o(1 + g_m R_s) = 20(1 + g_m R_s)$$

$$R'_L = R_L \parallel R_o = 20 \parallel 20(1 + g_m R_s)$$

$$A_M = -G_m R'_L$$

$$20 = \frac{5}{1 + g_m R_s} [20 \parallel 20(1 + g_m R_s)]$$

$$4(1 + g_m R_s) = \frac{20 \times 20(1 + g_m R_s)}{20 + 20(1 + g_m R_s)}$$

$$\Rightarrow 1 + g_m R_s = 4$$

$$\Rightarrow R_s = \frac{3}{g_m} = 0.6 \text{ k}\Omega = 600 \Omega$$

**9.81** (a) Refer to Fig. P9.81(a). Since the total resistance at the drain is  $r_o$ , we have

$$A_M = -g_m r_o \quad \text{Q.E.D.}$$

$$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R_{sig}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= R_{sig}(1 + g_m r_o) + r_o$$

$$\tau_{gd} = C_{gd}R_{gd} = C_{gd}[R_{sig}(1 + g_m r_o) + r_o]$$

$$\tau_{CL} = C_L R'_L = C_L r_o$$

Thus,

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= C_{gs}R_{sig} + C_{gd}[R_{sig}(1 + g_m r_o) + r_o]$$

$$+ C_L r_o \quad \text{Q.E.D.}$$

For the given numerical values,

$$A_M = -1 \times 20 = -20 \text{ V/V}$$

$$\tau_H = 20 \times 20 + 5[20(1 + 1 \times 20) + 20] + 10 \times 20$$

$$= 400 + 2200 + 200 = 2800 \text{ ps} = 2.8 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 2.8 \times 10^{-9}}$$

$$= 56.8 \text{ MHz}$$

$$\text{GB} = 20 \times 56.8 = 1.14 \text{ GHz}$$

(b) From Fig. 1 we see that

This figure belongs to Problem **9.81**, part (b).

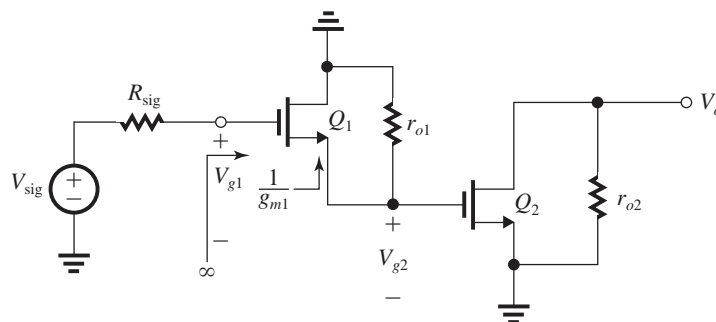


Figure 1

$$\frac{V_{g1}}{V_{sig}} = 1$$

$$\frac{V_{g2}}{V_{g1}} = \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}$$

$$\frac{V_o}{V_{g2}} = -g_{m2}r_{o2}$$

Thus,

$$A_M = 1 \times \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}} \times -g_{m2}r_{o2}$$

$$= -\frac{r_{o1}}{1/g_{m1} + r_{o1}} (g_{m2}r_{o2}) \quad \text{Q.E.D.}$$

Next we evaluate the open-circuit time constants. Refer to Fig. 2.

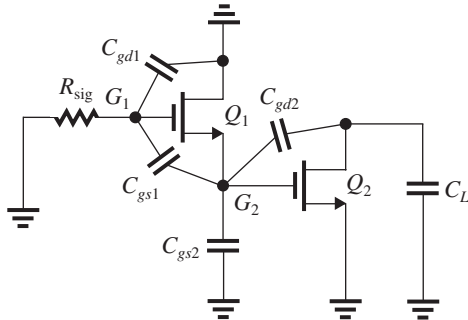


Figure 2

$C_{gd1}$ : Capacitor  $C_{gd1}$  is between  $G_1$  and ground and thus sees the resistance  $R_{sig}$ ,

$$R_{gd1} = R_{sig}$$

$$\tau_{gd1} = C_{gd1}R_{sig}$$

$C_{gs1}$ : To find the resistance  $R_{gs1}$  seen by capacitor  $C_{gs1}$ , we replace  $Q_1$  with its hybrid- $\pi$  equivalent circuit with  $V_{sig}$  set to zero,  $C_{gd1} = 0$ , and  $C_{gs1}$  replaced by a test voltage  $V_x$ . The resulting equivalent circuit is shown in Fig. 3.

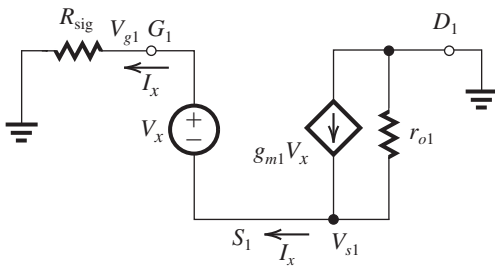


Figure 3

Analysis of the circuit in Fig. 3 proceeds as follows:

$$V_{g1} = I_x R_{sig}$$

$$V_{s1} = V_{g1} - V_x = I_x R_{sig} - V_x$$

Node equation at  $S_1$ ,

$$I_x = g_{m1} V_x - \frac{V_{s1}}{r_{o1}}$$

$$= g_{m1} V_x - \frac{I_x R_{sig} - V_x}{r_{o1}}$$

$$I_x \left( 1 + \frac{R_{sig}}{r_{o1}} \right) = V_x \left( g_{m1} + \frac{1}{r_{o1}} \right)$$

Thus,

$$R_{gs1} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + r_{o1}}{1 + g_{m1} r_{o1}}$$

$$\tau_{gs1} = C_{gs1} R_{gs1}$$

$$= C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_{m1} r_{o1}}$$

$C_{gs2}$ : Capacitor  $C_{gs2}$  sees the resistance between  $G_2$  and ground, which is the output resistance of source follower  $Q_1$ ,

$$R_{gs2} = \frac{1}{g_{m1}} \parallel r_{o1}$$

Thus,

$$\tau_{gs2} = C_{gs2} \left( \frac{1}{g_{m1}} \parallel r_{o1} \right)$$

$C_{gd2}$ : Transistor  $Q_2$  operates as a CS amplifier with an equivalent signal-source resistance equal to the output resistance of the source follower  $Q_2$ , that is,  $\left( \frac{1}{g_{m1}} \parallel r_{o1} \right)$  and with a gain from gate to drain of  $g_{m2}r_{o2}$ . Thus, the formula for  $R_{gd}$  in a CS amplifier can be adapted as follows:

$$R_{gd2} = \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_{m2}r_{o2}) + r_{o2}$$

and thus,

$$\tau_{gd2} = C_{gd2} \left[ \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_{m2}r_{o2}) + r_{o2} \right]$$

$C_L$ : Capacitor  $C_L$  sees the resistance between  $D_2$ , and ground which is  $r_{o2}$ ,

$$\tau_{CL} = C_L r_{o2}$$

Summing  $\tau_{gd1}$ ,  $\tau_{gs1}$ ,  $\tau_{gs2}$ ,  $\tau_{gd2}$  and  $\tau_{CL}$  gives  $\tau_H$  in the problem statement. Q.E.D.

For the given numerical values:

$$A_M = -\frac{20}{1 + 20} (1 \times 20)$$

$$= -19 \text{ V/V}$$

$$\tau_{gd1} = C_{gd1} R_{sig} = 5 \times 20 = 100 \text{ ps}$$

$$\begin{aligned}\tau_{gs1} &= C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_m r_{o1}} \\ &= 20 \frac{20 + 20}{1 + 1 \times 20} = 38 \text{ ps}\end{aligned}$$

$$\begin{aligned}\tau_{gs2} &= C_{gs2} \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) \\ &= 20 \times (1 \parallel 20) = 19 \text{ ps}\end{aligned}$$

$$\begin{aligned}\tau_{gd2} &= C_{gd2} \left[ \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_m r_{o2}) + r_{o2} \right] \\ &= 5[(1 \parallel 20)(1 + 20) + 20] \\ &= 200 \text{ ps}\end{aligned}$$

$$\tau_{CL} = C_L r_{o2} = 10 \times 20 = 200 \text{ ps}$$

$$\tau_H = 100 + 38 + 19 + 200 + 200 = 557 \text{ ps}$$

$$\begin{aligned}f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 557 \times 10^{-12}} \\ &= 286 \text{ MHz}\end{aligned}$$

$$GB = 19 \times 286 = 5.43 \text{ GHz}$$

Thus, while the dc gain remained approximately the same both  $f_H$  and GB increased by a factor of about 5!

$$\begin{aligned}\mathbf{9.82} \quad g_m &= \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} \\ &= \frac{I}{V_{OV}} = \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}\end{aligned}$$

$$\begin{aligned}\frac{V_o}{V_{sig}} &= \frac{R_D}{2/g_m} \\ &= \frac{1}{2} g_m R_D = \frac{1}{2} \times 1 \times 50 = 25 \text{ V/V}\end{aligned}$$

The high-frequency analysis can be performed in an analogous manner to that used in the text for the bipolar circuit. Refer to Fig. 9.35(b) and adapt the circuit for the MOS case. Thus,

$$\begin{aligned}f_{P1} &= \frac{1}{2\pi R_{sig} \left( \frac{C_{gs}}{2} + C_{gd} \right)} \\ &= \frac{1}{2\pi \times 100 \times 10^3 \left( \frac{4}{2} + 0.5 \right) \times 10^{-12}} \\ &= 637 \text{ kHz}\end{aligned}$$

and

$$\begin{aligned}f_{P2} &= \frac{1}{2\pi R_D C_\mu} \\ &= \frac{1}{2\pi \times 50 \times 10^3 \times 0.5 \times 10^{-12}}\end{aligned}$$

$$= 6.37 \text{ MHz}$$

Since  $f_{P2} \simeq 10f_{P1}$ , the pole at  $f_{P1}$  will dominate and

$$f_H \simeq f_{P1} = 637 \text{ kHz}$$

**9.83** Refer to Eqs. (9.133)–(9.136). For our case,

$$G_m = \frac{g_m}{1 + g_m R_s} \quad (1)$$

$R_o = \text{very large}$

$$R'_L = R_L \parallel R_o = R_L$$

$$A_M = -G_m R_L = \frac{-g_m R_L}{1 + g_m R_s} \quad (2)$$

Refer to Eqs. (9.135)–(9.138). For our case,

$$R_{gd} = R_{sig}(1 + G_m R_L) + R_L \quad (3)$$

$$R_{gs} = \frac{R_{sig} + R_s}{1 + g_m R_s} \quad (4)$$

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

For the numerical values given:

(a)  $R_s = 0$

$$G_m = g_m = 5 \text{ mA/V}$$

$$A_M = -g_m R_L = -5 \times 5 = -25 \text{ V/V}$$

$$R_{gd} = 100(1 + 5 \times 5) + 5 = 2605 \text{ k}\Omega$$

$$\tau_H = 10 \times 100 + 2 \times 2605 = 6210 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 6210 \times 10^{-9}} = 25.6 \text{ kHz}$$

$$GB = 25.6 \times 25 = 641 \text{ kHz}$$

(b)  $R_s = 100 \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.1} = 3.33 \text{ mA/V}$$

$$A_M = -3.33 \times 5 = -16.7 \text{ V/V}$$

$$R_{gd} = 100(1 + 3.33 \times 5) + 5$$

$$= 1771.7 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 0.1}{1 + 5 \times 0.1} = 66.7 \text{ k}\Omega$$

$$\tau_H = 10 \times 66.7 + 2 \times 1771.7 = 4211 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 4211 \times 10^{-9}} = 37.8 \text{ kHz}$$

$$GB = 631 \text{ kHz}$$

(c)  $R_s = 200 \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.2} = 2.5 \text{ mA/V}$$

$$A_M = -2.5 \times 5 = -12.5 \text{ V/V}$$

$$R_{gd} = 100(1 + 2.5 \times 5) + 5 = 1355 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 0.2}{1 + 5 \times 0.2} = 50.1 \text{ k}\Omega$$

$$\tau_H = 10 \times 50.1 + 2 \times 1355 = 3211 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 3211 \times 10^{-9}} = 49.6 \text{ kHz}$$

$$\text{GB} = 49.6 \times 12.5 = 620 \text{ kHz}$$

A summary of the results is provided in the following table:

	$R_s = 0$	$R_s = 100 \text{ }\Omega$	$R_s = 200 \text{ }\Omega$
$ A_M  \text{ (V/V)}$	25	16.7	12.5
$f_H \text{ (kHz)}$	25.6	37.8	49.6
GB (kHz)	641	631	620

Observe that increasing  $R_s$  trades off gain for bandwidth while GB remains approximately constant.

$$\mathbf{9.84} \quad G_m = \frac{g_m}{1 + g_m R_s} = \frac{g_m}{1 + k}$$

$$R'_L = R_L \parallel R_o$$

$$= r_o \parallel r_o(1 + g_m R_s)$$

$$= r_o \parallel r_o(1 + k)$$

$$= \frac{r_o \times r_o(1 + k)}{r_o + r_o(1 + k)}$$

$$= r_o \frac{1 + k}{2 + k}$$

$$A_M = -G_m R'_L = -\frac{g_m r_o}{2 + k}$$

Thus,

$$A_M = \frac{-A_0}{2 + k} \quad \text{Q.E.D.}$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s / (r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)}$$

$$= \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s / 2r_o}{1 + \frac{1}{2} g_m R_s}$$

For  $R_{\text{sig}} \gg R_s$ ,

$$R_{gs} \simeq \frac{R_{\text{sig}}(1 + R_s/2r_o)}{1 + (k/2)}$$

For  $r_o \gg R_s$ ,

$$R_{gs} \simeq \frac{R_{\text{sig}}}{1 + (k/2)}$$

$$\tau_{gs} = C_{gs} R_{gs} = \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)}$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L$$

Utilizing the expressions for  $R'_L$  and  $G_m R'_L$  derived earlier, we obtain

$$R_{gd} = R_{\text{sig}} \left[ 1 + \frac{A_0}{2 + k} \right] + r_o \left( \frac{1 + k}{2 + k} \right)'$$

$$\tau_{gd} = C_{gs} R_{gd} = C_{gd} R_{\text{sig}} \left( 1 + \frac{A_0}{2 + k} \right) + C_{gd} r_o \left( \frac{1 + k}{2 + k} \right)$$

$$\tau_{cL} = C_L R'_L$$

$$= C_L r_o \frac{1 + k}{2 + k}$$

Thus,

$$\begin{aligned} \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{cL} \\ &= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}} \left( 1 + \frac{A_0}{2 + k} \right) \\ &\quad + C_{gd} r_o \left( \frac{1 + k}{2 + k} \right) + C_L r_o \left( \frac{1 + k}{2 + k} \right) \\ &= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}} \left( 1 + \frac{A_0}{2 + k} \right) \\ &\quad + (C_L + C_{gd}) r_o \left( \frac{1 + k}{2 + k} \right) \quad \text{Q.E.D.} \end{aligned}$$

**9.85** Substituting the given numerical values in the expressions for  $A_M$  and  $\tau_H$  given in the statement for Problem 9.84 and noting that  $A_0 = g_m r_o = 5 \times 40 = 200$ , we obtain

$$A_M = -\frac{200}{2 + k} \text{ V/V}$$

$$\tau_H = \frac{40}{1 + (k/2)} + 2 + \frac{400}{2 + k} + 44 \frac{1 + k}{2 + k}, \text{ ns}$$

$$f_H = 1/2\pi \tau_H$$

$$\text{GB} = |A_M| f_H$$

To obtain  $f_H = 2 \text{ MHz}$ , we see from the table (next page) that

$$k = 11$$

Thus,

$$1 + g_m R_s = 11$$

$$\Rightarrow R_s = \frac{10}{5} = 2 \text{ k}\Omega$$

The gain achieved is

$$|A_M| = 15.4 \text{ V/V}$$

$k$	$ A_M $ , V/V	$\tau_H$ ns	$f_H$ (MHz)	GB (MHz)
0	100	264	0.603	60.3
1	66.7	191.3	0.832	55.6
2	50	155	1.03	51.5
3	40	133.2	1.19	47.6
4	33.3	118.7	1.34	44.6
5	28.6	108.3	1.47	42.0
6	25	100.5	1.58	39.5
7	22.2	94.4	1.69	37.5
8	20	89.6	1.78	35.6
9	18.2	85.7	1.86	33.9
10	16.7	82.3	1.93	32.2
11	15.4	79.6	2.00	30.8
12	14.3	77.2	2.06	29.5
13	13.3	75.1	2.12	28.2
14	12.5	73.3	2.17	27.1
15	11.8	71.6	2.22	26.2

**9.86** At an emitter bias current of 0.1 mA,  $Q_1$  and  $Q_2$  have

$$g_m = 4 \text{ mA/V}$$

$$r_e = 250 \text{ } \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{4 \times 10^{-3}}{2\pi \times 200 \times 10^6} = 3.2 \text{ pF}$$

$$C_\mu = 0.2 \text{ pF}$$

$$C_\pi = 3 \text{ pF}$$

To determine  $R_{in}$  and the voltage gain  $A_M$ , refer to the circuit in Fig. 9.33(a). Here, however,  $R_L$  is  $r_{o2}$ .

$$R_{in2} = r_{\pi2} = 25 \text{ k}\Omega$$

$$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{o1} \parallel R_{in2})]$$

$$= 101[0.25 + (1000 \parallel 25)]$$

$$\simeq 2.5 \text{ M}\Omega$$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2.5 \text{ M}\Omega}{2.5 \text{ M}\Omega + 10 \text{ k}\Omega} \simeq 1 \text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{(R_{in2} \parallel r_{o1})}{(R_{in2} \parallel r_{o1}) + r_{e1}}$$

$$= \frac{25 \parallel 1000}{(25 \parallel 1000) + 0.25} = 0.99 \simeq 1 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2}r_{o2} = -4 \times 1000 = -4000 \text{ V/V}$$

Thus,

$$A_M = \frac{V_o}{V_{sig}} = -4000 \text{ V/V}$$

To determine  $f_H$  we use the method of open-circuit time constants. Figure 9.33(b) shows the circuit with  $V_{sig} = 0$  and the four capacitances indicated. Again, recall that here  $R_L = r_{o2}$ . Also, in our present circuit there is a capacitance  $C_L$  at the output.

Capacitance  $C_{\mu1}$  sees a resistance  $R_{\mu1}$ ,

$$R_{\mu1} = R_{sig} \parallel R_{in}$$

$$= 10 \text{ k}\Omega \parallel 2.5 \text{ M}\Omega \simeq 10 \text{ k}\Omega$$

To find the resistance  $R_{\pi1}$  we refer to the circuit in Fig. 9.33(c) where  $R_{in2}$  is considered to include  $r_{o2}$ ,

$$R_{in2} = 25 \text{ k}\Omega \parallel 1000 \text{ k}\Omega = 24.4 \text{ k}\Omega$$

We use the formula for  $R_{\pi1}$  given in Example 9.10:

$$R_{\pi1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{\pi1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 347 \text{ } \Omega$$

Capacitance  $C_{\pi2}$  sees a resistance  $R_{\pi2}$ :

$$R_{\pi2} = R_{in2} \parallel R_{out1}$$

$$= r_{\pi2} \parallel r_{o1} \parallel \left[ r_{e1} + \frac{R_{sig}}{\beta_1 + 1} \right]$$

$$= 25 \parallel 1000 \parallel \left[ 0.25 + \frac{10}{101} \right]$$

$$= 344 \text{ } \Omega$$

Capacitance  $C_{\mu2}$  sees a resistance  $R_{\mu2}$ :

$$R_{\mu2} = (1 + g_{m2}r_{o2})(R_{in2} \parallel R_{out1}) + r_{o2}$$

$$= (1 + 4 \times 1000) \times 0.344 + 1000$$

$$= 2376 \text{ k}\Omega$$

We can determine  $\tau_H$  from

$$\tau_H = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_{\mu2}R_{\mu2}$$

$$+ C_{\pi2}R_{\pi2} + C_L r_o$$

$$= 0.2 \times 10 + 3 \times 0.347 + 0.2 \times 2376$$

$$\begin{aligned}
 &+ 3 \times 0.344 + 1 \times 1000 \\
 \tau_H &= 2 + 1 + 475.2 + 1 + 1000 \\
 &= 1479.2 \text{ ns}
 \end{aligned}$$

Observe that there are two dominant capacitances: the most significant is  $C_L$  and the second most significant is  $C_{\mu 2}$ .

$$\begin{aligned}
 f_H &= \frac{1}{2\pi\tau_H} \\
 &= \frac{1}{2\pi \times 1479.2 \times 10^{-9}} = 107.6 \text{ kHz}
 \end{aligned}$$

$$9.87 \quad g_m = \frac{I_C}{V_T} \simeq \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$R_{in} = 2r_\pi = 6 \text{ k}\Omega$$

$$\begin{aligned}
 \frac{V_o}{V_{sig}} &= \frac{R_{in}}{R_{in} + R_{sig}} \frac{\alpha R_L}{2r_e} \\
 &\simeq \frac{6}{6 + 12} \times \frac{10}{2 \times 0.025} = 66.7 \text{ V/V}
 \end{aligned}$$

$$\begin{aligned}
 C_\pi + C_\mu &= \frac{g_m}{2\pi f_T} \\
 &= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6} = 12.7 \text{ pF}
 \end{aligned}$$

$$C_\mu = 0.5 \text{ pF}$$

$$C_\pi = 12.2 \text{ pF}$$

$$R'_{sig} = R_{sig} \parallel 2r_\pi = 12 \parallel 6 = 4 \text{ k}\Omega$$

$$\begin{aligned}
 f_{p1} &= \frac{1}{2\pi R'_{sig} \left( \frac{C_\pi}{2} + C_\mu \right)} \\
 &= \frac{1}{2\pi \times 4 \times 10^3 \left( \frac{12.2}{2} + 0.5 \right) \times 10^{-12}} \\
 &= 6 \text{ MHz}
 \end{aligned}$$

$$\begin{aligned}
 f_{p2} &= \frac{1}{2\pi R_L C_\mu} \\
 &= \frac{1}{2\pi \times 10 \times 10^3 \times 0.5 \times 10^{-12}} \\
 &= 31.8 \text{ MHz}
 \end{aligned}$$

Thus,  $f_{p1}$  is the dominant pole and

$$f_H \simeq f_{p1} = 6 \text{ MHz}$$

**9.88** Using an approach analogous to that utilized for the BJT circuit (Fig. 9.35), we see that there is a pole at the input with frequency  $f_{p1}$ :

$$f_{p1} = \frac{1}{2\pi R_{sig} \left( \frac{C_{gs}}{2} + C_{gd} \right)}$$

$$f_{p1} = \frac{1}{2\pi \times 20 \times 10^3 \left( \frac{2}{2} + 0.1 \right) \times 10^{-12}}$$

$$= 7.2 \text{ MHz},$$

and a pole at the output with frequency  $f_{p2}$ ,

$$\begin{aligned}
 f_{p2} &= \frac{1}{2\pi (C_{gd} + C_L) R_L} \\
 &= \frac{1}{2\pi \times (0.1 + 1) \times 10^{-12} \times 20 \times 10^3} \\
 &= 7.2 \text{ MHz}
 \end{aligned}$$

Thus,

$$f_{p1} = f_{p2} = 7.2 \text{ MHz}$$

The midband gain  $A_M$  is obtained as

$$\begin{aligned}
 A_M &= \frac{R_L}{2/g_m} = \frac{1}{2} g_m R_L \\
 &= \frac{1}{2} \times 5 \times 20 = 50 \text{ V/V}
 \end{aligned}$$

Thus, the amplifier transfer function is

$$\frac{V_o(s)}{V_{sig}(s)} = \frac{50}{\left( 1 + \frac{s}{2\pi \times 7.2 \times 10^6} \right)^2}$$

$$\left| \frac{V_o}{V_{sig}} \right| = \frac{50}{1 + \left( \frac{\omega}{2\pi \times 7.2 \times 10^6} \right)^2}$$

$$\text{At } \omega = \omega_{3dB}, \left| \frac{V_o}{V_i} \right| = \frac{50}{\sqrt{2}}, \text{ thus}$$

$$\sqrt{2} = 1 + \left( \frac{\omega_{3dB}}{2\pi \times 7.2 \times 10^6} \right)^2$$

$$\begin{aligned}
 f_{3dB} &= \sqrt{\sqrt{2} - 1} \times 7.2 \text{ MHz} \\
 &= 4.6 \text{ MHz}
 \end{aligned}$$

**9.89** (a) For each of  $Q_1$  and  $Q_2$ ,

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_m r_o = 100$$

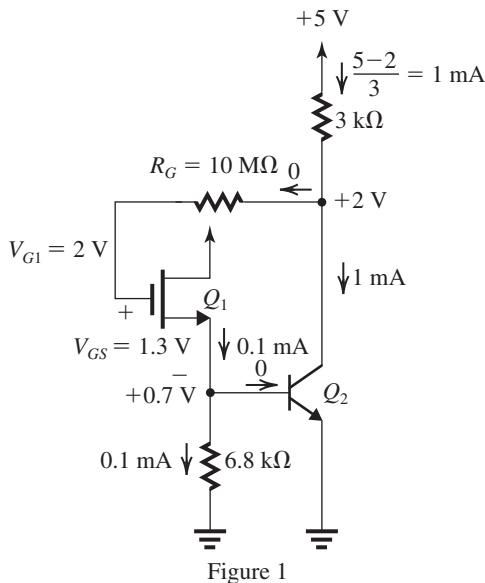
Since the output resistance of each of the current sources is equal to  $r_o$ , the gain of each stage is  $(g_m r_o / 2)$  and the overall voltage gain is

$$\frac{V_o}{V_{sig}} = (g_m r_o)^2 = 2500 \text{ V/V}$$



(b)  $\tau_{gs1} = C_{gs}R_{sig}$   
 $= 20 \times 10 = 200 \text{ ps}$   
 $R_{gd1} = R_{sig}(1 + \frac{1}{2}g_{m1}r_{o1}) + (r_{o1}/2)$   
 $= 10(1 + 50) + 50$   
 $= 560 \text{ k}\Omega$   
 $\tau_{gd1} = C_{gd1}R_{gd1} = 5 \times 560 = 2800 \text{ ps}$   
 At the drain of  $Q_1$  we have  $(C_{db1} + C_{gs2})$  and the resistance seen is  $r_o/2$ :  
 $\tau_{d1} = (C_{db1} + C_{gs2})(r_o/2)$   
 $= (5 + 20) \times 50 = 1250 \text{ ps}$   
 $R_{gd2} = (\frac{r_{o1}}{2}) \left(1 + \frac{1}{2}g_{m2}r_{o2}\right) + (r_{o2}/2)$   
 $= 50(1 + 50) + 50 = 2600 \text{ k}\Omega$   
 $\tau_{gd2} = C_{gd2}R_{gd2} = 5 \times 2600 = 13,000 \text{ ps}$   
 $\tau_{d2} = C_{db2}(r_{o2}/2)$   
 $= 5 \times 50 = 250 \text{ ps}$   
 $\tau_H = \tau_{gs1} + \tau_{gd1} + \tau_{d1} + \tau_{gd2} + \tau_{d2}$   
 $= 200 + 2800 + 1250 + 13,000 + 250$   
 $= 17,500 \text{ ps} = 17.5 \text{ ns}$   
 $f_H = \frac{1}{2\pi\tau_H}$   
 $= \frac{1}{2\pi \times 17.5 \times 10^{-9}} = 9.1 \text{ MHz}$

9.90 (a)



The dc analysis is shown in Fig. 1. It is based on  $V_{S1} = V_{BE2} = 0.7 \text{ V}$ . Neglecting  $I_{B2}$ , we obtain

$$I_{D1} = \frac{0.7 \text{ V}}{6.8 \text{ k}\Omega} \simeq 0.1 \text{ mA} \quad \text{Q.E.D.}$$

$$I_{D1} = \frac{1}{2}k'_n(W/L)V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} \simeq 0.3 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 0.3 = 1.3 \text{ V}$$

$$V_{G1} = 0.7 + 1.3 = 2 \text{ V}$$

$$V_{C1} = V_{G1} = 2 \text{ V}$$

$$I_{C2} = \frac{5 - 2}{3} = 1 \text{ mA} \quad \text{Q.E.D.}$$

(b)  $g_{m1} = \frac{2I_{D1}}{V_{OV}} = \frac{2 \times 0.1}{0.3} = 0.67 \text{ mA/V}$

$$C_{gs} = C_{gd} = 1 \text{ pF}$$

$$g_{m2} = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$C_{\pi 2} + C_{\mu 2} = \frac{g_{m2}}{2\pi f_{T2}}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 10.6 \text{ pF}$$

$$C_{\mu 2} = 0.8 \text{ pF}$$

$$C_{\pi 2} = 9.8 \text{ pF}$$

(c)  $Q_1$  acts as a source follower, thus

$$\frac{V_{b2}}{V_i} = \frac{6.8 \text{ k}\Omega \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + (6.8 \text{ k}\Omega \parallel r_{\pi 2})}$$

$$= \frac{(6.8 \parallel 5)}{1.5 + (6.8 \parallel 5)} = 0.66 \text{ V/V}$$

Neglecting  $R_G$ , we obtain

$$\frac{V_o}{V_{b2}} = -g_{m2}(3 \text{ k}\Omega \parallel 1 \text{ k}\Omega)$$

$$= -40(3 \parallel 1) = -30 \text{ V/V}$$

Thus,

$$\frac{V_o}{V_i} = 0.66 \times -30$$

$$\simeq -20 \text{ V/V}$$

Using Miller's theorem, the input resistance  $R_{in}$  is found as

$$R_{in} = \frac{R_G}{1 - \frac{V_o}{V_i}} = \frac{10 \text{ M}\Omega}{1 - (-20)}$$

This figure belongs to Problem 9.90, part (d).

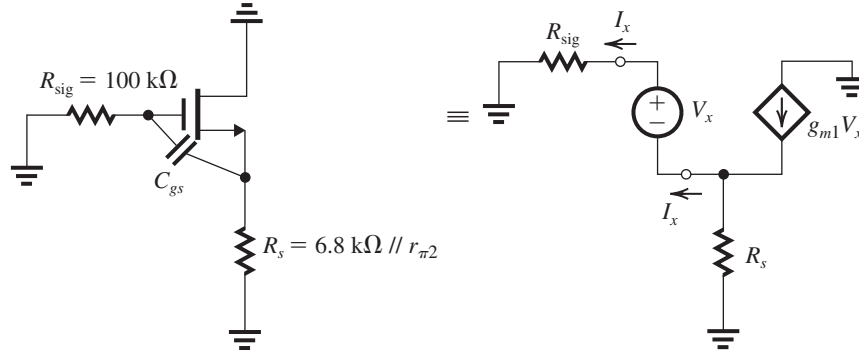


Figure 2

$$= 476 \text{ k}\Omega$$

$$\frac{V_i}{V_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$= \frac{476}{476 + 100} = 0.83 \text{ V/V}$$

$$\frac{V_o}{V_{\text{sig}}} = 0.83 \times -20 = -16.5 \text{ V/V}$$

$$(c) \tau_{gd1} = C_{gd1}(R_{\text{in}} \parallel R_{\text{sig}})$$

$$= 1 \times 10^{-12}(476 \parallel 100) \times 10^3$$

$$= 82.6 \text{ ns}$$

To determine the resistance  $R_{gs}$  seen by  $C_{gs}$ , refer to Fig. 2.

We can show that

$$R_{gs} \equiv \frac{V_x}{I_x} = \frac{R_{\text{sig}} + R_s}{1 + g_{m1}R_s}$$

where

$$R_s = 6.8 \text{ k}\Omega \parallel r_{\pi 2}$$

$$= 6.8 \parallel 5 = 2.88 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 2.88}{1 + 0.67 \times 2.88} = 35.1 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs}R_{gs} = 1 \times 10^{-12} \times 35.1 \times 10^3 = 35.1 \text{ ns}$$

$$\tau_{\pi 2} = C_{\pi 2} \left( r_{\pi 2} \parallel 6.8 \text{ k}\Omega \parallel \frac{1}{g_{m1}} \right)$$

$$= 9.8 \times 10^{-12} \times 0.99 \times 10^3$$

$$= 9.7 \text{ ns}$$

$$R_{\mu 2} = \left( \frac{1}{g_{m1}} \parallel 6.8 \text{ k}\Omega \parallel r_{\pi 2} \right) [1 + g_{m2}(3 \parallel 1)] + (3 \parallel 1)$$

$$= (1.5 \parallel 6.8 \parallel 5) \left( 1 + 40 \times \frac{3}{4} \right) + 0.75$$

$$= 31.4 \text{ k}\Omega$$

$$\tau_{\mu 2} = C_{\mu 2}R_{\mu 2} = 0.8 \times 31.4 = 25.1 \text{ ns}$$

$$\tau_H = \tau_{gd} + \tau_{gs} + \tau_{\pi 2} + \tau_{\mu 2}$$

$$= 82.6 + 35.1 + 9.7 + 25.1$$

$$= 152.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 152.5 \times 10^{-9}} = 1 \text{ MHz}$$

**9.91** All transistors are operating at  $I_E = 0.5 \text{ mA}$ . Thus,

$$g_m \simeq 20 \text{ mA/V}$$

$$r_e \simeq 50 \Omega$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$r_o = \text{very high (neglect)}$$

$$C_{\pi} + C_{\mu} = \frac{g_m}{2\pi f_T} = \frac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 8 \text{ pF}$$

$$C_{\mu} = 2 \text{ pF}$$

$$C_{\pi} = 6 \text{ pF}$$

(a) CE amplifier

$$A_M = -\frac{r_{\pi}}{r_{\pi} + R_{\text{sig}}} g_m R_L$$

$$= -\frac{5}{5 + 10} \times 20 \times 10 = -66.7 \text{ V/V}$$

Since  $C_L = 0$ , we can obtain a good estimate of  $f_H$  using the Miller approximation:

$$C_{\text{in}} = C_{\pi} + C_{\mu}(g_m R_L + 1)$$

$$= 6 + 2(20 \times 10 + 1)$$

$$= 408 \text{ pF}$$

$$f_H = \frac{1}{2\pi R_{\text{sig}} C_{\text{in}}}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 408 \times 10^{-12}}$$

$$= 39 \text{ kHz}$$

(b) This is a cascode amplifier. Refer to Fig. 9.22 for the analysis equations.a

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m (\beta r_o \parallel R_L)$$

$$\simeq -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R_L$$

$$= -66.7 \text{ V/V (same as the CE in (a))}$$

$$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}} = 5 \parallel 10 = 3.33 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{\text{sig}}$$

$$\tau_{\pi 1} = C_{\pi 1} R_{\pi 1} = 6 \times 3.33 = 20 \text{ ns}$$

$$R_{c1} = r_{\pi 2} \parallel \frac{1}{g_{m2}} = r_{e2} = 50 \Omega$$

$$R_{\mu 1} = R'_{\text{sig}} (1 + g_{m1} R_{c1}) + R_{c1}$$

$$= 3.33(1 + 40 \times 0.05) + 0.05$$

$$= 3.33(1 + 2) + 0.05 = 10.05 \text{ k}\Omega$$

$$\tau_{\mu 1} = C_{\mu 1} R_{\mu 1} = 2 \times 10.05 = 20.1 \text{ ns}$$

$$\tau_{c1} = C_{\pi 2} R_{c1} = 6 \times 0.05 = 0.3 \text{ ns}$$

$$\tau_{\mu 2} = C_{\mu 2} R_L = 2 \times 10 = 20 \text{ ns}$$

$$\tau_H = 20 + 20.1 + 0.3 + 20 = 60.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 60.4 \times 10^{-9}}$$

$$= 2.6 \text{ MHz}$$

(c) This is a CC-CB cascade similar to the circuit analyzed in Fig. 9.35. There are two poles: one at the input,

$$f_{P1} = \frac{1}{2\pi (R_{\text{sig}} \parallel 2r_\pi) \left( \frac{C_\pi}{2} + C_\mu \right)}$$

$$f_{P1} = \frac{1}{2\pi (10 \parallel 10) \times 10^3 (3 + 2) \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 5 \times 5 \times 10^{-9}}$$

$$= 6.4 \text{ MHz}$$

and one at the output,

$$f_{P2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 2 \times 10^{-12}}$$

$$= 8 \text{ MHz}$$

Since the two poles are relatively close to each other, we use the root-sum-of-the-squares formula to obtain an estimate for  $f_H$ :

$$f_H = 1/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}$$

$$A_M = \frac{R_L}{2r_e + \frac{R_{\text{sig}}}{\beta + 1}}$$

$$= \frac{10}{2 \times 0.05 + \frac{10}{101}} \simeq 50 \text{ V/V}$$

(d) This is a CC-CE cascade similar to the circuit analyzed in Example 9.10.

$$R_{\text{in}} = (\beta_1 + 1)(r_{e1} + r_{\pi 2})$$

$$= 101(0.05 + 5) = 510 \text{ k}\Omega$$

$$\frac{V_{b1}}{V_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{510}{510 + 10}$$

$$= 0.98 \text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5}{5 + 0.05} = 0.99 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2} R_L = -20 \times 10 = -200 \text{ V/V}$$

$$A_M = \frac{V_o}{V_{\text{sig}}} = -0.98 \times 0.99 \times 200$$

$$= -194 \text{ V/V}$$

$$R_{\mu 1} = R_{\text{sig}} \parallel R_{\text{in}}$$

$$= 10 \parallel 510 = 9.81 \text{ k}\Omega$$

$$\tau_{\mu 1} = C_{\mu 1} R_{\mu 1} = 2 \times 9.81 = 19.6 \text{ ns}$$

$$R_{\pi 1} = \frac{R_{\text{sig}} + R_{\text{in}2}}{1 + \frac{R_{\text{sig}}}{r_{\pi 1}} + \frac{R_{\text{in}2}}{r_{e1}}}$$

$$= \frac{10 + 5}{1 + \frac{10}{5} + \frac{5}{0.05}} = 0.15 \text{ k}\Omega$$

$$\tau_{\pi 1} = C_{\pi 1} R_{\pi 1} = 6 \times 0.15 = 0.9 \text{ ns}$$

$$R_{\pi 2} = R_{\text{in}2} \parallel R_{\text{out}1}$$

$$= r_{\pi 2} \parallel \left( \frac{R_{\text{sig}}}{\beta_1 + 1} + r_{e1} \right)$$

$$= 5 \parallel \left( \frac{10}{101} + 0.05 \right) = 0.15 \text{ k}\Omega$$

$$\tau_{\pi 2} = C_{\pi 2} R_{\pi 2} = 6 \times 0.15 = 0.9 \text{ ns}$$

$$R_{\mu 2} = (1 + g_{m2} R_L)(R_{\text{in}2} \parallel R_{\text{out}1}) + R_L$$

$$= (1 + 20 \times 10) \left[ 5 \parallel \left( \frac{10}{101} + 0.05 \right) \right] + 10$$

$$= 39.1 \text{ k}\Omega$$

$$\tau_{\mu 2} = C_{\mu 2} R_{\mu 2} = 2 \times 39.1 = 78.2 \text{ ns}$$

$$\tau_H = 19.6 + 0.9 + 0.9 + 78.2 = 99.6 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 99.6 \times 10^{-9}} = 1.6 \text{ MHz}$$

(e) This is a folded cascode amplifier (i.e., one in which the cascode transistor is *pn*p). The analysis is identical to that for (b) above.

$$A_M = -66.7 \text{ V/V}$$

$$f_H = 2.6 \text{ MHz}$$

(f) This is a CC-CB cascade. The analysis is identical to that for case (c) above.

$$A_M = 50 \text{ V/V}$$

$$f_H = 5 \text{ MHz}$$

### Summary of Results

Case	Configuration	$A_M$ (V/V)	$f_H$ (MHz)	GB (MHz)
a	CE	-66.7	0.039	2.6
b	Cascode	-66.7	2.6	173
c	CC-CB	50	5	250
d	CC-CE	-194	1.6	310
e	Folded Cascode	-66.7	2.6	173
f	CC-CB	50	5	250

**9.92** Refer to Fig. 9.37(b).

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}} + \frac{1}{sC_{C1}}}$$

where

$$R_G = R_{G1} \parallel R_{G2} = 2 \text{ M}\Omega \parallel 1 \text{ M}\Omega = 667 \text{ k}\Omega$$

and

$$R_{\text{sig}} = 200 \text{ k}\Omega$$

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}}} \frac{s}{s + \frac{1}{C_{C1}(R_G + R_{\text{sig}})}}$$

Thus,

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

We required

$$f_{P1} \leq 10 \text{ Hz}$$

thus we select  $C_{C1}$  so that

$$\frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})} \leq 10$$

$$C_{C1} \geq \frac{1}{2\pi \times 10 \times (667 + 200) \times 10^3} = 18.4 \text{ nF}$$

$$\Rightarrow C_{C1} = 20 \text{ nF}$$

**9.93** Refer to Fig. 9.37.

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} \times g_m(R_D \parallel R_L)$$

where

$$R_G = R_{G1} \parallel R_{G2} = 47 \text{ M}\Omega \parallel 10 \text{ M}\Omega$$

$$= 8.246 \text{ M}\Omega$$

$$R_{\text{sig}} = 100 \text{ k}\Omega, \quad g_m = 5 \text{ mA/V}, \quad R_D = 4.7 \text{ k}\Omega \text{ and } R_L = 10 \text{ k}\Omega.$$

Thus,

$$A_M = -\frac{8.246}{8.246 + 0.1} \times 5(4.7 \parallel 10)$$

$$= -15.8 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

$$= \frac{1}{2\pi \times 0.01 \times 10^{-6}(8.246 + 0.1) \times 10^6}$$

$$= 1.9 \text{ Hz}$$

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$= \frac{5 \times 10^{-3} + 0.5 \times 10^{-3}}{2\pi \times 10 \times 10^{-6}} = 87.5 \text{ Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

$$= \frac{1}{2\pi \times 10 \times 10^{-6} \times 2 \times 10^3} = 8 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6}(4.7 + 10) \times 10^3} = 10.8 \text{ Hz}$$

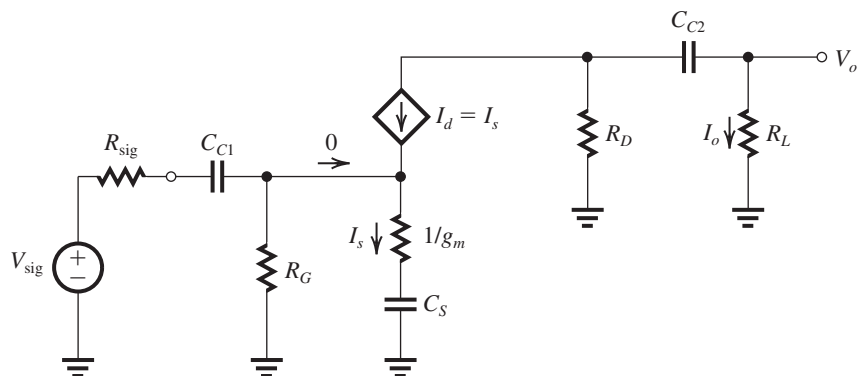
Since

$$f_{P2} \gg f_{P1}, f_{P3}, f_Z,$$

$$f_L \simeq f_{P2} = 87.5 \text{ Hz}$$

**9.94** See figure on next page. Replacing the MOSFET with its  $T$  model results in the circuit shown in the figure.

This figure belongs to Problem 9.94.



$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} \times g_m(R_D \parallel R_L)$$

$$= -\frac{2}{2 + 0.5} \times 3(20 \parallel 10)$$

$$= -16 \text{ V/V}$$

To minimize the total capacitance we select  $C_S$  so as to place  $f_{p2}$  (usually the highest-frequency low-frequency pole) at 100 Hz. Thus,

$$100 = \frac{g_m}{2\pi C_S}$$

$$= \frac{3 \times 10^{-3}}{2\pi C_S}$$

$$\Rightarrow C_S = 4.8 \mu\text{F}$$

Select  $C_S = 5 \mu\text{F}$ .

Of the two remaining poles, the one caused by  $C_{C2}$  has associated relatively low-valued resistances ( $R_D$  and  $R_L$  are much lower than  $R_G$ ), thus to minimize the total capacitance we place  $f_{p3}$  at 10 Hz and  $f_{p1}$  at 1 Hz. Thus,

$$10 = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$\equiv \frac{1}{2\pi C_{C2}(20 + 10) \times 10^3}$$

$$\Rightarrow C_{C2} = 0.53 \mu\text{F}$$

Select  $C_{C2} = 1 \mu\text{F}$ .

$$1 = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

$$1 = \frac{1}{2\pi C_{C1}(2 + 0.5) \times 10^6}$$

$$\Rightarrow C_{C1} = 63.7 \text{ nF}$$

Select  $C_{C1} = 100 \text{ nF} = 0.1 \mu\text{F}$ .

With the selected capacitor values, we obtain

$$f_{p1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 2.5 \times 10^6} = 0.64 \text{ Hz}$$

$$f_{p2} = \frac{3 \times 10^{-3}}{2\pi \times 5 \times 10^{-6}} = 95.5 \text{ Hz}$$

$$f_Z = 0 \text{ (dc)}$$

$$f_{p3} = \frac{1}{2\pi \times 1 \times 10^{-6}(20 + 10) \times 10^3} = 5.3 \text{ Hz}$$

Since  $f_{p2} \gg f_{p1}$  and  $f_{p3}$ , we have

$$f_L \simeq f_{p2} = 95.5 \text{ Hz}$$

**9.95** The amplifier in Fig. P9.95 will have the equivalent circuit in Fig. 9.41(b) except with  $R_E = \infty$  (i.e., omitted). Also, the equivalent circuits in Fig. 9.42 can be used to determine the three short-circuit time constants, again with  $R_E = \infty$ . Since the amplifier is operating at  $I_C \simeq I_E = 100 \mu\text{A} = 0.1 \text{ mA}$  and  $\beta = 100$ ,

$$r_e = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

Using the equivalent circuit in Fig. 9.42(b), we get

$$\tau_{CE} = C_E \left( r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right)$$

To make  $C_E$  responsible for 80% of  $f_L$ , we use

$$\frac{1}{\tau_{CE}} = 0.8 \omega_L = 0.8 \times 2\pi f_L$$

$$\Rightarrow \tau_{CE} = \frac{1}{0.8 \times 2\pi \times 100} \simeq 2 \text{ ms}$$

Thus,

$$C_E \left[ 250 + \frac{(200 \parallel 20) \times 10^3}{101} \right] = 2 \times 10^{-3}$$

$$\Rightarrow C_E = 4.65 \mu\text{F}$$

Select,  $C_E = 5 \mu\text{F}$ .

Using the information in Fig. 9.42(a), we determine  $\tau_{C1}$  as

$$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$$

To make the contribution of  $C_{C1}$  to the determination of  $f_L$  equal to 10%, we use

$$\frac{1}{\tau_{C1}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C1} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C1}[(200 \parallel 25) \times 10^3 + 20 \times 10^3] = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 0.38 \text{ } \mu\text{F}$$

Select  $C_{C1} = 0.5 \text{ } \mu\text{F}$ .

For  $C_{C2}$  we use the information in Fig. 9.42(c) to determine  $\tau_{C2}$ :

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

To make the contribution of  $C_{C2}$  to the determination of  $f_L$  equal to 10%, we use

$$\frac{1}{\tau_{C2}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C2} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C2}(20 + 10) \times 10^3 = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C2} = 0.53 \text{ } \mu\text{F}$$

Although, to be conservative we should select  $C_{C2} = 1 \text{ } \mu\text{F}$ ; in this case we can select

$$C_{C2} = 0.5 \text{ } \mu\text{F}$$

because the required value is very close to  $0.5 \text{ } \mu\text{F}$  and because we have selected  $C_{C1}$  and  $C_E$  larger than the required values. The resulting  $f_L$  will be

$$f_L = \frac{1}{2\pi} \left[ \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C2}} \right]$$

$$\tau_{CE} = 5 \times 10^{-6} \times \left[ 250 + \frac{(200 \parallel 20) \times 10^3}{101} \right]$$

$$= 2.15 \text{ ms}$$

$$\tau_{C1} = 0.5 \times 10^{-6} [(200 \parallel 25) \times 10^3 + 20 \times 10^3]$$

$$= 21.1 \text{ ms}$$

$$\tau_{C2} = 0.5 \times 10^{-6} (20 + 10) \times 10^3 = 15 \text{ ms}$$

$$f_L = \frac{1}{2\pi} \left[ \frac{10^3}{2.15} + \frac{10^3}{21.1} + \frac{10^3}{15} \right]$$

$$= 92.2 \text{ Hz}$$

which is lower (hence more conservative) than the required value of 100 Hz.

$$C_{\text{total}} = 5 + 0.5 + 0.5 = 6.0 \text{ } \mu\text{F}$$

### 9.96

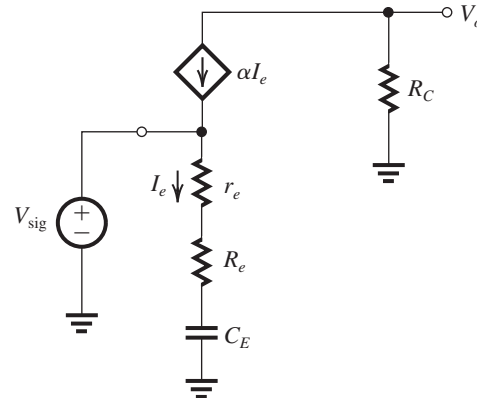


Figure 1

Replacing the BJT with its  $T$  model results in the circuit shown in Fig. 1.

$$(a) I_e = \frac{V_{\text{sig}}}{r_e + R_e + \frac{1}{sC_E}}$$

$$V_o = -\alpha I_e R_C$$

Thus,

$$\frac{V_o}{V_{\text{sig}}} = -\frac{\alpha R_C}{r_e + R_e} \frac{s}{s + \frac{1}{C_E(r_e + R_e)}} \quad (1)$$

From this expression we obtain

$$A_M = -\frac{\alpha R_C}{r_e + R_e} \quad (2)$$

and

$$f_L = f_P = \frac{1}{2\pi C_E(r_e + R_e)} \quad (3)$$

(b) From Eq. (2) we see that

$$|A_M| = \frac{\alpha R_C}{r_e} \frac{1}{1 + \frac{R_e}{r_e}}$$

Thus, including  $R_e$  reduces the gain magnitude by the factor  $\left(1 + \frac{R_e}{r_e}\right)$ .

(c) From Eq. (3), we obtain

$$f_L = \frac{1}{2\pi C_E r_e} \frac{1}{1 + \frac{R_e}{r_e}}$$

Thus, including  $R_e$  reduces  $f_L$  by the factor  $\left(1 + \frac{R_e}{r_e}\right)$ . This is the same factor by which the magnitude of the gain is reduced. Thus,  $R_e$  can be used to tradeoff gain for decreasing  $f_L$  (that is, increasing the amplifier bandwidth).

(d)  $I = 0.25 \text{ mA}$ ,  $R_C = 10 \text{ k}\Omega$ ,  $C_E = 10 \text{ }\mu\text{F}$

$$r_e = \frac{V_T}{I} = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \text{ }\Omega$$

For  $R_e = 0$ :

$$|A_M| = \frac{\alpha R_C}{r_e} \simeq \frac{10 \text{ k}\Omega}{100 \text{ }\Omega} = 100 \text{ V/V}$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6} \times 100} = 159.2 \text{ Hz}$$

To lower  $f_L$  by a factor of 10, we use

$$1 + \frac{R_e}{r_e} = 10$$

$$\Rightarrow R_e = 900 \text{ }\Omega$$

The gain now becomes

$$|A_M| = \frac{100}{1 + \frac{R_e}{r_e}} = \frac{100}{10} = 10 \text{ V/V}$$

See Fig. 2 for the Bode plot.

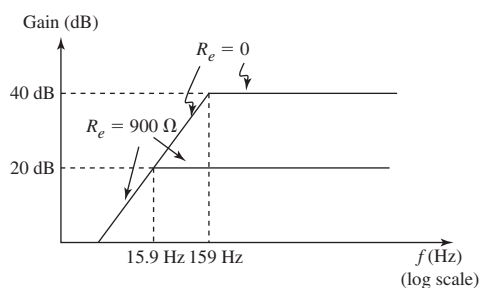
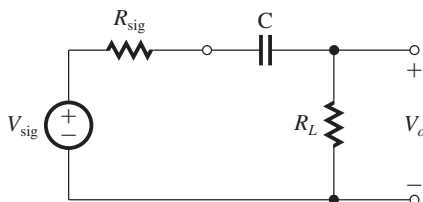


Figure 2

9.97 The circuit is shown in the figure below.



(a) Using the voltage divider rule, we find the transfer function

$$\frac{V_o}{V_{\text{sig}}} = \frac{R_L}{R_L + R_{\text{sig}} + \frac{1}{sC}}$$

which can be manipulated to the form

$$\frac{V_o}{V_{\text{sig}}} = \frac{R_L}{R_L + R_{\text{sig}}} \frac{s}{s + \frac{1}{C(R_L + R_{\text{sig}})}} \quad (1)$$

(b) At very high frequency, i.e., as  $s \rightarrow \infty$ ,

$$\frac{V_o}{V_{\text{sig}}} = \frac{R_L}{R_L + R_{\text{sig}}} = \frac{4}{4 + 1} = 0.8 \text{ V/V}$$

which is the voltage-divider ratio obtained when  $C$  acts as a short circuit.

(c) From Eq. (1), the 3-dB frequency can be found as the frequency of the real pole, that is,

$$\begin{aligned} \omega_{3\text{dB}} &= \frac{1}{C(R_L + R_{\text{sig}})} \\ &= \frac{1}{1 \times 10^{-6}(4 + 1) \times 10^3} \\ &= 200 \text{ rad/s} \end{aligned}$$

$$f_{3\text{dB}} = \frac{200}{2\pi} = 31.8 \text{ Hz}$$

(d) From Eq. (1), at  $s = 0$ ,

$$\frac{V_o}{V_{\text{sig}}} = 0$$

9.98 Refer to Fig. 9.37(b).

$$V_o = -I_d \frac{R_D}{R_D + \frac{1}{sC_2} + R_L} \times R_L$$

$$\frac{V_o}{I_d} = -\frac{R_D R_L}{R_D + R_L} \frac{s}{s + \frac{1}{C_2(R_D + R_L)}}$$

$$f_{p3} = \frac{1}{2\pi C_2(R_D + R_L)}$$

where

$$R_D = 10 \text{ k}\Omega \text{ and } R_L = 10 \text{ k}\Omega$$

To make  $f_{p3} \leq 10 \text{ Hz}$ ,

$$\frac{1}{2\pi C_2(R_D + R_L)} \leq 10$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 10 \times (10 + 10) \times 10^3} = 0.8 \text{ }\mu\text{F}$$

Select,  $C_2 = 0.8 \text{ }\mu\text{F}$ .

9.99 Refer to Fig. 9.37(b).

$$I_s = \frac{V_g}{\frac{1}{g_m} + Z_S}$$

$$I_s = \frac{g_m V_g Y_S}{Y_S + g_m}$$

$$\frac{I_s}{V_g} = \frac{g_m \left( \frac{1}{R_S} + sC_S \right)}{g_m + \frac{1}{R_S} + sC_S}$$

$$= g_m \frac{s + 1/C_S R_S}{s + \frac{g_m + 1/R_S}{C_S}}$$

Thus,

$$f_{p2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$f_z = \frac{1}{2\pi C_S R_S}$$

where

$$g_m = 5 \text{ mA/V and } R_S = 1.8 \text{ k}\Omega$$

To make  $f_{p2} \leq 100 \text{ Hz}$ ,

$$\frac{g_m + 1/R_S}{2\pi C_S} \leq 100$$

$$\Rightarrow C_S \geq \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 100} = 8.8 \text{ }\mu\text{F}$$

Select  $C_S = 10 \text{ }\mu\text{F}$ .

Thus,

$$f_{p2} = \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 10 \times 10^{-6}} = 88.4 \text{ Hz}$$

and

$$f_z = \frac{1}{2\pi \times 10 \times 10^{-6} \times 1.8 \times 10^3} = 8.84 \text{ Hz}$$

**9.100**

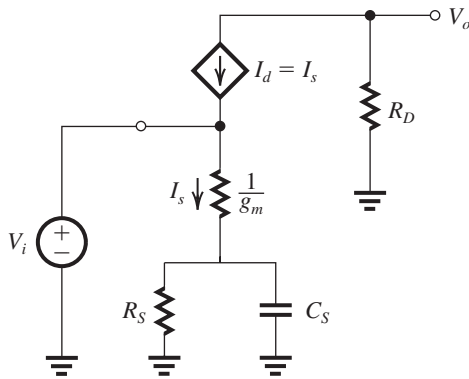


Figure 1

Replacing the MOSFET with its  $T$  model results in the circuit shown in Fig. 1.

(a)  $A_M \equiv \frac{V_o}{V_i} = -g_m R_D$

$$-20 = -2 \times R_D$$

$$\Rightarrow R_D = 10 \text{ k}\Omega$$

(b)  $f_p = \frac{g_m + 1/R_S}{2\pi C_S}$

$$100 = \frac{2 \times 10^{-3} + (1/4.5 \times 10^3)}{2\pi C_S}$$

$$\Rightarrow C_S = 3.54 \text{ }\mu\text{F}$$

(c)  $f_z = \frac{1}{2\pi C_S R_S} =$

$$\frac{1}{2\pi \times 3.54 \times 10^{-6} \times 4.5 \times 10^3} = 10 \text{ Hz}$$

(d) Since  $f_p \gg f_z$ ,

$$f_L \simeq f_p = 100 \text{ Hz}$$

(e) The Bode plot for the gain is shown in Fig. 2.

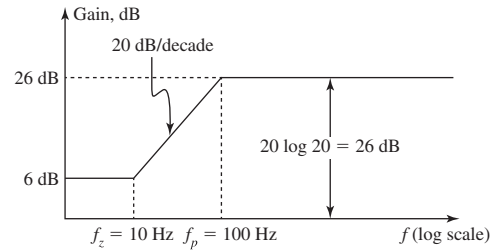


Figure 2

Observe that the dc gain is 6 dB, i.e. 2 V/V. This makes perfect sense since from Fig. 1 we see that at dc, capacitor  $C_S$  behaves as open circuit and the gain becomes

$$\text{DC gain} = -\frac{R_D}{\frac{1}{g_m} + R_S} = -\frac{10 \text{ k}\Omega}{\left(\frac{1}{2} + 4.5\right)}$$

$$= -2 \text{ V/V}$$

**9.101** Refer to Fig. 9.41.

In the midband,

$$R_{in} = R_{B1} \parallel R_{B2} \parallel r_\pi$$

where

$$R_{B1} = 33 \text{ k}\Omega, R_{B2} = 22 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.3 \text{ mA}}{0.025 \text{ V}} = 12 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.3 \text{ mA}} = 83.3 \text{ }\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10 \text{ k}\Omega$$

Thus,

$$R_{in} = 33 \parallel 22 \parallel 10 = 5.7 \text{ k}\Omega$$

$$A_M = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

where



$$R_{\text{sig}} = 5 \text{ k}\Omega, R_C = 4.7 \text{ k}\Omega, \text{ and } R_L = 5.6 \text{ k}\Omega$$

Thus,

$$A_M = -\frac{5.7}{5.7+5} \times 12(4.7 \parallel 5.6) \\ = -16.3 \text{ V/V}$$

Using the method of short-circuit time constants and the information in Fig. 9.42, we obtain

$$\begin{aligned} \tau_{C1} &= C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}] \\ &= C_{C1}(R_{\text{in}} + R_{\text{sig}}) \\ &= 1 \times 10^{-6}(5.7 + 5) \times 10^3 = 10.7 \text{ ms} \\ \tau_{CE} &= C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right) \right] \\ &= 20 \times \\ &10^{-6} \left[ 3.9 \times 10^3 \parallel \left( 83.3 + \frac{(33 \parallel 22 \parallel 5) \times 10^3}{121} \right) \right] \\ &= 2.2 \text{ ms} \\ \tau_{C2} &= C_{C2}(R_C + R_L) \\ &= 1 \times 10^{-6}(4.7 + 5.6) \times 10^3 = 10.3 \text{ ms} \\ f_L &\simeq \frac{1}{2\pi} \left( \frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}} \right) \\ &= \frac{1}{2\pi} \left( \frac{1}{10.7 \times 10^{-3}} + \frac{1}{2.2 \times 10^{-3}} + \frac{1}{10.3 \times 10^{-3}} \right) \\ &= 102.7 \text{ Hz} \end{aligned}$$

**9.102** Refer to the data given in the statement for Problem 9.8.

$$R_B = R_{B1} \parallel R_{B2} = 33 \text{ k}\Omega \parallel 22 \text{ k}\Omega = 13.2 \text{ k}\Omega$$

$$I_C \simeq I_E \simeq 0.3 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.3 \text{ mA}}{0.025 \text{ V}} = 12 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.3 \text{ mA}} = 83.3 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10 \text{ k}\Omega$$

From Fig. 9.40, we have

$$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$$

For  $C_{C1}$  to contribute 10% of  $f_L$ , we use

$$\begin{aligned} \frac{1}{\tau_{C1}} &= 0.1\omega_L = 0.1 \times 2\pi f_L \\ &= 0.1 \times 2\pi \times 50 \\ \Rightarrow \tau_{C1} &= 31.8 \text{ ms} \end{aligned}$$

Thus,

$$C_{C1}[(13.2 \parallel 10) + 5] \times 10^3 = 31.8 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 3 \mu\text{F}$$

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

For  $C_{C2}$  to contribute 10% of  $f_L$ , we use

$$\begin{aligned} \frac{1}{\tau_{C2}} &= 0.1\omega_L = 0.1 \times 2\pi f_L \\ &= 0.1 \times 2\pi \times 50 \\ \Rightarrow \tau_{C2} &= 31.8 \text{ ms} \end{aligned}$$

Thus,

$$\begin{aligned} C_{C2}(4.7 + 5.6) \times 10^3 &= 31.8 \times 10^{-3} \\ \Rightarrow C_{C2} &= 3.09 \mu\text{F} \simeq 3 \mu\text{F} \end{aligned}$$

Finally,

$$\tau_{CE} = C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right) \right]$$

For  $C_E$  to contribute 80% of  $f_L$ , we use

$$\begin{aligned} \frac{1}{\tau_{CE}} &= 0.8\omega_L = 0.8 \times 2\pi f_L \\ &= 0.8 \times 2\pi \times 50 \\ \Rightarrow \tau_{CE} &= 3.98 \text{ ms} \end{aligned}$$

Thus,

$$\begin{aligned} C_E \left[ 3900 \parallel \left( 83.3 + \frac{(13.2 \parallel 5) \times 1000}{121} \right) \right] \\ &= 3.98 \times 10^{-3} \\ \Rightarrow C_E &= 36.2 \mu\text{F} \end{aligned}$$

**9.103** Using the information in Fig. 9.42, we get

$$\begin{aligned} \tau_{C1} &= C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}] \\ &= C_{C1}[(10 \parallel 1) + 5] \times 10^3 \\ &= C_{C1} \times 5.91 \times 10^3 \\ \tau_{CE} &= C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right) \right] \end{aligned}$$

where

$$r_e = \frac{r_\pi}{\beta + 1} = \frac{1000}{101} \simeq 10 \Omega$$

$$\begin{aligned} \tau_{CE} &= \\ C_E \left[ 1.5 \times 10^3 \parallel \left( 10 + \frac{(10 \parallel 5) \times 1000}{101} \right) \right] \\ &= C_E \times 41.8 \end{aligned}$$

For  $C_{C1}$  and  $C_E$  to contribute equally to  $f_L$ ,

$$\tau_{C1} = \tau_{CE}$$

Thus,

$$\begin{aligned} C_{C1} \times 5.91 \times 10^3 &= C_E \times 41.8 \\ \Rightarrow \frac{C_E}{C_{C1}} &= 141.4 \end{aligned}$$

This figure belongs to Problem 9.104, part (a).

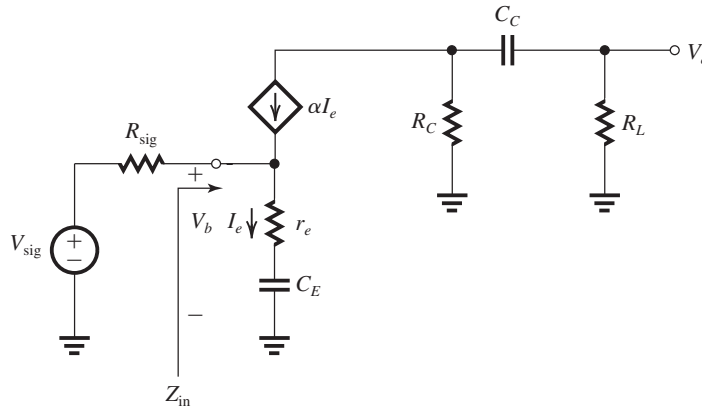


Figure 1

**9.104** Replacing the BJT with its  $T$  model results in the equivalent circuit shown in Fig. 1 above.

(a) At midband,  $C_E$  and  $C_C$  act as short circuits. Thus

$$R_{in} = (\beta + 1)r_e$$

$$\frac{V_o}{V_{sig}} = -\frac{(\beta + 1)r_e}{(\beta + 1)r_e + R_{sig}} g_m(R_C \parallel R_L)$$

$$= -\frac{\beta(R_C \parallel R_L)}{(\beta + 1)r_e + R_{sig}}$$

(b) Because the controlled current source  $\alpha I_e$  is ideal, it effectively separates the input circuit from the output circuit. The result is that the poles caused by  $C_E$  and  $C_C$  do not interact. The pole due to  $C_E$  will have frequency  $\omega_{PE}$ :

$$\omega_{PE} = \frac{1}{C_E \left[ r_e + \frac{R_{sig}}{\beta + 1} \right]}$$

and the pole due to  $C_C$  will have a frequency  $\omega_{PC}$

$$\omega_{PC} = \frac{1}{C_C(R_C + R_L)}$$

(c) The overall voltage transfer function can be expressed as

$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s + \omega_{PE}} \frac{s}{s + \omega_{PC}}$$

(d)  $r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$

$$A_M = -\frac{100(10 \text{ k}\Omega \parallel 10 \text{ k}\Omega)}{101 \times 25 \times 10^{-3} \text{ k}\Omega + 10 \text{ k}\Omega}$$

$$= -40 \text{ V/V}$$

(e) To minimize the total capacitance we choose to make the pole caused by  $C_E$  the dominant one and make its frequency equal to  $f_L = 100 \text{ Hz}$ ,

$$2\pi \times 100 = \frac{1}{C_E \left[ 25 + \frac{10,000}{101} \right]}$$

$$\Rightarrow C_E = 12.83 \mu\text{F}$$

Placing the pole due to  $C_C$  at 10 Hz, we obtain

$$2\pi \times 10 = \frac{1}{C_C(10 + 10) \times 10^3}$$

$$\Rightarrow C_C = 0.8 \mu\text{F}$$

(f) A Bode plot for the gain magnitude is shown in Fig. 2.

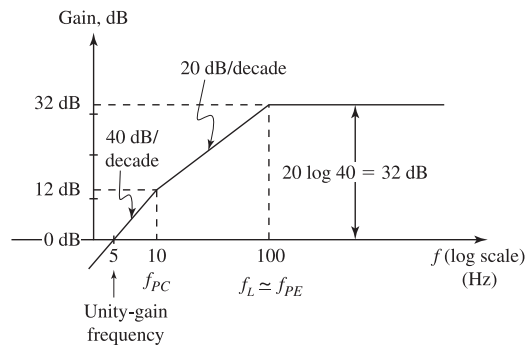


Figure 2

The gain at  $f_{p2}$  (10 Hz) is 12 dB. Since the gain decreases by 40 dB/decade or equivalently 12 dB/octave, it reaches 0 dB (unity magnitude) at  $f = f_{PC}/2 = 5 \text{ Hz}$ .

## Chapter 10

## Solutions to Exercises within the Chapter

**Ex. 10.1** (b)  $\beta = 1/A_f|_{\text{ideal}} = \frac{1}{10} = 0.1 \text{ V/V}$

$$0.1 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{R_2}{R_1} = 9$$

(c)  $A = 100 \quad \beta = 0.1$

$$A\beta = 10$$

$$1 + A\beta = 11 \text{ or } 20.8 \text{ dB}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{100}{11} = 9.091 \text{ V/V}$$

$A_f$  differs from the ideal value of 10 V/V by -9.1%.

(d)  $10 = \frac{A}{1 + A\beta} = \frac{100}{1 + 100\beta}$

$$\Rightarrow \beta = 0.09 \text{ V/V}$$

$$0.09 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{R_2}{R_1} = 10.11$$

(e)

$$V_o = A_f V_s = 10 \times 1 = 10 \text{ V}$$

$$V_f = \beta V_o = 0.09 \times 10 = 0.9 \text{ V}$$

$$V_i = V_s - V_f = 1 - 0.9 = 0.1 \text{ V}$$

(f)

$$A = 0.8 \times 100 = 80$$

$$A_f = \frac{80}{1 + 80 \times 0.09}$$

$$= 9.76 \text{ V/V}$$

which is a 2.44% decrease.

**Ex. 10.2** (b)

$$\beta = 1/A_f|_{\text{ideal}} = 1/10^3$$

$$\beta = 0.001 \text{ V/V}$$

$$0.001 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{R_2}{R_1} = 999$$

(c)  $A\beta = 10^4 \times 10^{-3} = 10$

$$1 + A\beta = 11 \text{ or } 20.8 \text{ dB}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{10^4}{11} = 909.1 \text{ V/V}$$

(d)

$$1000 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 0.0009$$

$$0.0009 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{R_2}{R_1} = 1110.1$$

(e)

$$V_o = A_f V_s = 1000 \times 0.01 = 10 \text{ V}$$

$$V_f = \beta V_o = 0.0009 \times 10 = 0.009 \text{ V}$$

$$V_i = V_s - V_f = 0.01 - 0.009 = 0.001 \text{ V}$$

(f)

$$A = 0.8 \times 10^4$$

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 0.0009}$$

$$= 975.6 \text{ V/V}$$

which is a 2.44% decrease.

**Ex. 10.3** To constrain the corresponding change in  $A_f$  to 0.1%, we need an amount-of-feedback of at least

$$1 + A\beta = \frac{10\%}{0.1\%} = 100$$

Thus the largest obtainable closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{100} = 10 \text{ V/V}$$

Each amplifier in the cascade will have a nominal gain of 10 V/V and a maximum variability of 0.1%; thus the overall voltage gain will be  $(10)^3 = 1000 \text{ V/V}$  and the maximum variability will be 0.3%.

**Ex. 10.4**

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1$$

$$A\beta = 10^4 \times 0.1 = 1000$$

$$1 + A\beta = 1001$$

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = \frac{10^4}{1 + 10^4 \times 0.1} = 9.99 \text{ V/V}$$

$$f_{Hf} = f_H(1 + A\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

**Ex. 10.5** (a) Refer to Fig. 10.9(c).

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b)

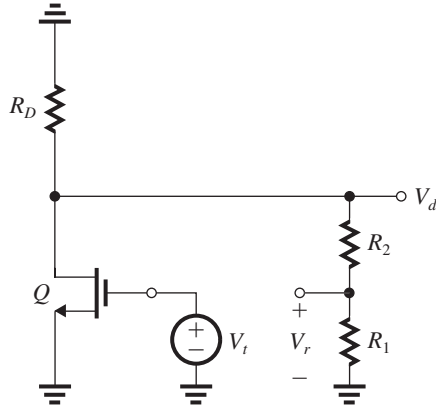


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain  $A\beta$ . Observe that we have eliminated the input signal  $V_s$ , and opened the loop at the gate of  $Q$  where the input impedance is infinite obviating the need for a termination resistance at the right-hand side of the break. Now we need to analyze the circuit to determine

$$A\beta \equiv -\frac{V_r}{V_i}$$

First, we write for the gain of the CS amplifier  $Q$ ,

$$\frac{V_d}{V_i} = -g_m[R_D \parallel (R_1 + R_2)] \quad (1)$$

then we use the voltage-divider rule to find  $V_r$ ,

$$\frac{V_r}{V_d} = \frac{R_1}{R_1 + R_2} \quad (2)$$

Combining Eqs. (1) and (2) gives

$$A\beta \equiv -\frac{V_r}{V_i} = g_m[R_D \parallel (R_1 + R_2)] \frac{R_1}{R_1 + R_2}$$

which can be simplified to

$$A\beta = g_m \frac{R_D R_1}{R_D + R_1 + R_2}$$

$$(c) A = \frac{A\beta}{\beta}$$

$$= g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

$$(d) \beta = \frac{R_1}{R_1 + R_2} = \frac{20}{20 + 80} = 0.2 \text{ V/V}$$

$$A\beta = 4 \frac{10 \times 20}{10 + 20 + 80} = 7.27$$

$$A = \frac{7.27}{0.2} = 36.36 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{36.36}{1 + 36.36 \times 0.2} = 4.4 \text{ V/V}$$

If  $A\beta$  were  $\gg 1$ , then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{0.2} = 5 \text{ V/V}$$

**Ex. 10.6** From the solution of Example 10.5,

$$1 + A\beta = 60.1$$

Thus,

$$f_{Hf} = (1 + A\beta)f_H$$

$$= 60.1 \times 1$$

$$= 60.1 \text{ kHz}$$

**Ex. 10.7** Refer to Fig. E10.7. The 1-mA bias current will split equally between the emitters of  $Q_1$  and  $Q_2$ , thus

$$I_{E1} = I_{E2} = 0.5 \text{ mA}$$

Transistor  $Q_3$  will be operating at an emitter current

$$I_{E3} = 5 \text{ mA}$$

determined by the 5-mA current source. Since the dc component of  $V_s = 0$ , the negative feedback will force the dc voltage at the output to be approximately zero.

The  $\beta$  circuit is shown in Fig. 1 (on next page) together with the determination of  $\beta$  and of the loading effects of the  $\beta$  circuit on the  $A$  circuit,

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1 \text{ V/V}$$

$$R_{11} = R_1 \parallel R_2 = 1 \parallel 9 = 0.9 \text{ k}\Omega$$

$$R_{22} = R_1 + R_2 = 1 + 9 = 10 \text{ k}\Omega$$

The  $A$  circuit is shown in Fig. 2 (on next page).

$$r_{e1} = r_{e2} = \frac{V_T}{I_{E1,2}} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$i_e = \frac{V_i}{r_{e1} + r_{e2} + \frac{R_s + R_{11}}{\beta + 1}}$$

Exercise 10-3

These figures belong to Exercise 10.7.

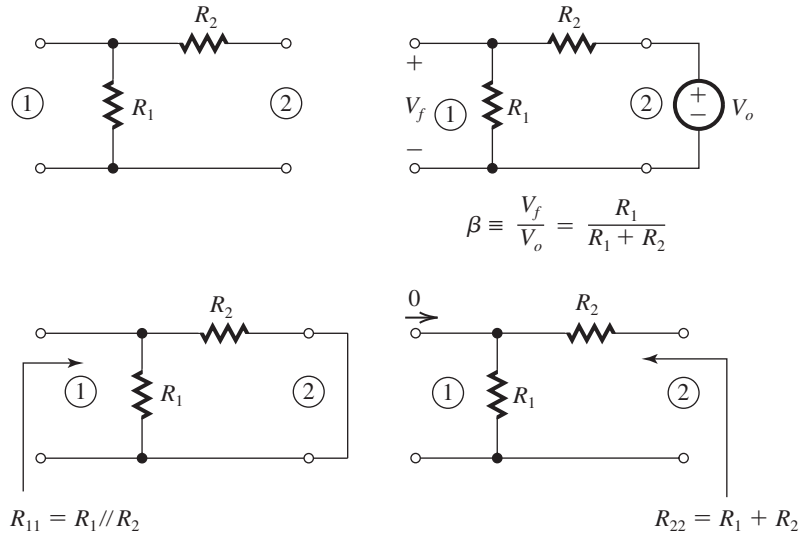


Figure 1

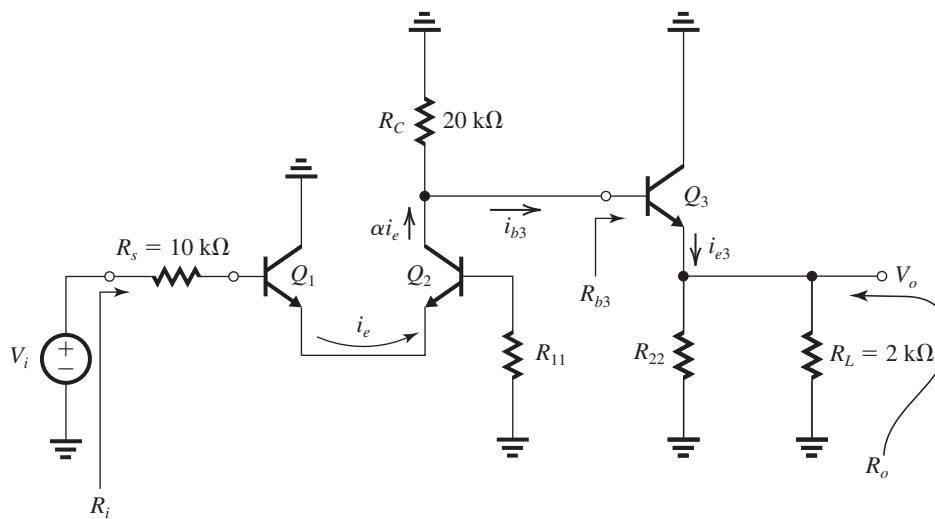


Figure 2

$$i_e = \frac{V_i}{0.05 + 0.05 + \frac{10 + 0.9}{101}} \Rightarrow i_e = 4.81V_i \quad (1)$$

$$R_{b3} = (\beta + 1)[r_{e3} + (R_{22} \parallel R_L)] = 101[0.005 + (10 \parallel 2)] = 168.84 \text{ k}\Omega$$

$$i_{b3} = \alpha i_e \frac{R_C}{R_C + R_{b3}} = 0.99 i_e \frac{20}{20 + 168.84}$$

$$\Rightarrow i_{b3} = 0.105 i_e \quad (2)$$

$$V_o = i_{e3}(R_{22} \parallel R_L) = (\beta + 1)i_{b3}(R_{22} \parallel R_L) = i_{b3} \times 101(10 \parallel 2) \Rightarrow V_o = 168.33i_{b3} \quad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{V_o}{V_i} = 85 \text{ V/V}$$

$$\beta = 0.1 \text{ V/V}$$

Exercise 10-4

$$A\beta = 8.5$$

$$1 + A\beta = 9.5$$

$$A_f = \frac{85}{9.5} = 8.95 \text{ V/V}$$

From the A circuit, we have

$$R_i = R_s + R_{11} + (\beta + 1)(r_{e1} + r_{e2})$$

$$= 10 + 0.9 + 101 \times 0.1$$

$$= 21 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 21 \times 9.5 = 199.5 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s = 199.5 - 10 = 189.5 \text{ k}\Omega$$

From the A circuit, we have

$$R_o = R_L \parallel R_{22} \parallel \left( r_{e5} + \frac{R_C}{\beta + 1} \right)$$

$$= 2 \parallel 10 \parallel \left( 0.005 + \frac{20}{101} \right)$$

$$= 181 \text{ }\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{181}{9.5} = 19.1 \text{ }\Omega$$

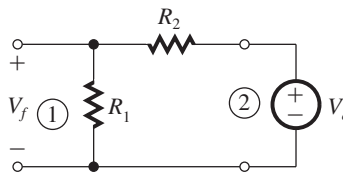
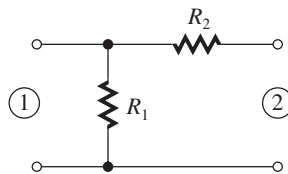
$$R_{of} = R_L \parallel R_{out}$$

$$19.1 = 2 \text{ k}\Omega \parallel R_{out}$$

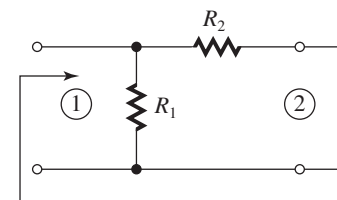
$$\Rightarrow R_{out} = 19.2 \text{ }\Omega$$

**Ex. 10.8** Figure 1 below shows the  $\beta$  circuit together with the determination of  $\beta$ ,  $R_{11}$  and  $R_{22}$ .

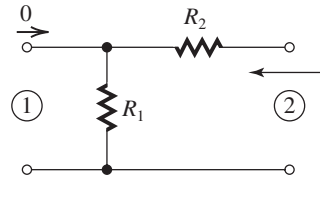
$$\beta = \frac{R_1}{R_1 + R_2}$$



$$\beta \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2}$$



$$R_{11} = R_1 \parallel R_2$$



$$R_{22} = R_1 + R_2$$

$$R_{11} = R_1 \parallel R_2$$

Figure 1

$$R_{22} = R_1 + R_2$$

Figure 2 shows the A circuit. We can write

$$V_o = g_m(R_D \parallel R_{22})V_i$$

Thus,

$$A \equiv \frac{V_o}{V_i} = g_m[R_D \parallel (R_1 + R_2)]$$

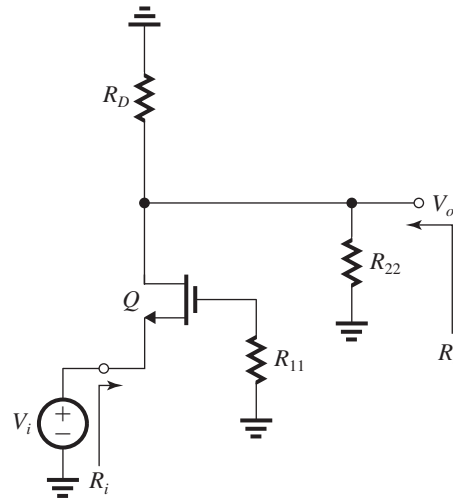


Figure 2

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

From A circuit, we have

$$R_i = \frac{1}{g_m}$$

$$R_o = R_D \parallel R_{22}$$

$$R_{in} = R_{if} = R_i(1 + A\beta)$$

$$R_{in} = \frac{1}{g_m}(1 + A\beta)$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{out} = \frac{R_D \parallel (R_1 + R_2)}{1 + A\beta}$$

Comparison with the results of Exercise 10.5 shows that the expressions for  $A$  and  $\beta$  are identical. However,  $R_{in}$  and  $R_{out}$  cannot be determined using the method of Exercise 10.5.

**Ex. 10.9**  $A_f|_{ideal} = 10 \text{ A/A}$

$$\beta = \frac{1}{A_f|_{ideal}} = 0.1 \text{ A/A}$$

$$A = 1000 \text{ A/A}$$

$$1 + A\beta = 1 + 1000 \times 0.1 = 101$$

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{101} = 9.9 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1000 \Omega}{101} = 9.9 \Omega$$

$$R_{of} = R_o(1 + A\beta) = 100 \text{ k}\Omega \times 101 = 10.1 \text{ M}\Omega$$

**Ex. 10.10** Refer to the solution of Example 10.7. If  $\mu$  is reduced from 1000 V/V to 100 V/V,  $A$  will be reduced by the same factor (10) to become

$$A = 64.9 \text{ mA/V}$$

and the loop gain will be correspondingly reduced,

$$A\beta = 64.9$$

Thus, the amount of feedback becomes

$$1 + A\beta = 65.9$$

and the closed-loop gain  $A_f$  becomes

$$A_f = \frac{A}{1 + A\beta} = \frac{64.9}{65.9} = 0.985 \text{ mA/V}$$

The input resistance becomes

$$R_{in} = R_{if} = (1 + A\beta)R_i$$

$$= 65.9 \times 101 = 6.7 \text{ M}\Omega$$

The output resistance becomes

$$R_{out} = R_{of} = (1 + A\beta)R_o$$

$$= 65.9 \times 61 = 4 \text{ M}\Omega$$

**Ex. 10.11** Refer to the solution of Example 10.7. To double  $A_f|_{ideal}$  we need to reduce  $\beta$  to half its

value. Since  $\beta = R_F$ , the feedback resistance  $R_F$  becomes

$$R_F = 500 \Omega$$

Now, since  $A$  is given by

$$A = \frac{R_{id}}{R_{id} + R_F} \frac{\mu}{\frac{1}{g_m} + (R_F \parallel r_{o2})} \frac{r_{o2}}{R_F + r_{o2}}$$

the value of  $A$  becomes

$$A = \frac{100}{100 + 0.5} \frac{1000}{0.5 + (0.5 \parallel 20)} \frac{20}{0.5 + 20}$$

$$= 982.7 \text{ mA/V}$$

Thus,  $A_f$  becomes

$$A_f = \frac{A}{1 + A\beta} = \frac{982.7}{1 + 982.7 \times 0.5}$$

$$= 1.996 \text{ mA/V}$$

**Ex. 10.12** To obtain  $A_f|_{ideal} = 100 \text{ mA/V}$ , the feedback factor  $\beta$  must be

$$\beta = \frac{1}{100} = 0.01 \text{ V/mA} = 10 \Omega$$

But,

$$\beta = \frac{R_{E1}R_{E2}}{R_{E1} + R_{E2} + R_F}$$

For  $R_{E1} = R_{E2} = 100 \Omega$ ,

$$10 = \frac{100 \times 100}{100 + 100 + R_F}$$

$$\Rightarrow R_F = 800 \Omega$$

Since the ideal value of  $(I_o/V_s)$  is 100 mA/V, the ideal value of the voltage gain is,

$$\frac{V_o}{V_s} = -\frac{I_o R_C}{V_s} = -100 \times 0.6$$

$$= -60 \text{ V/V}$$

**Ex. 10.13** See figure on next page. Figure 1 shows the circuit prepared for the determination of the loop gain,

$$A\beta \equiv -\frac{V_r}{V_i}$$

We will trace the signal around the loop as follows:

$$I_{e2} = g_{m2}V_i \tag{1}$$

$$I_{b3} = I_{e2} \frac{R_{C2}}{R_{C2} + R_{i3}} \tag{2}$$

where

$$R_{i3} = (\beta + 1)\{r_{e3} + [R_{E2} \parallel (R_F + (R_{E1} \parallel r_{e1}))]\} \tag{3}$$

$$I_{e3} = (\beta + 1)I_{b3} \tag{4}$$

This figure belongs to Exercise 10.13.

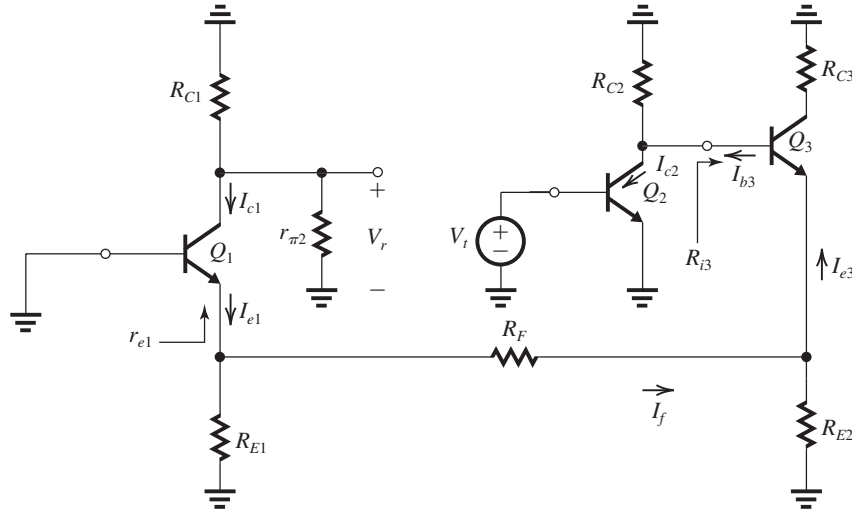


Figure 1

$$I_f = I_{e3} \frac{R_{E2}}{R_{E2} + R_F + (R_{E1} \parallel r_{e1})} \quad (5)$$

$$I_{e1} = I_f \frac{R_{E1}}{R_{E1} + r_{e1}} \quad (6)$$

$$I_{c1} = \alpha I_{e1} \quad (7)$$

$$V_r = -I_{c1}(R_{C1} \parallel r_{\pi2}) \quad (8)$$

Combining (1)–(8) gives  $V_r$  in terms of  $V_t$  and hence  $A\beta \equiv -V_r/V_t$ . We shall do this numerically using the values in Example 10.8:

$$g_{m2} = 40 \text{ mA/V}, R_{C2} = 5 \text{ k}\Omega, \beta = 100,$$

$$r_{e3} = 6.25 \Omega, R_{E1} = R_{E2} = 100 \Omega, R_F = 640 \Omega,$$

$$r_{e1} = 41.7 \Omega, \alpha_1 = 0.99, R_{C1} = 9 \text{ k}\Omega, \text{ and } r_{\pi2} = 2.5 \text{ k}\Omega$$

$$R_{i3} = 101 \{ 0.00625 + [0.1 \parallel (0.64 + (0.1 \parallel 0.0417))] \}$$

$$= 9.42 \text{ k}\Omega$$

$$I_{c2} = 40V_t \quad (9)$$

$$I_{b3} = 0.347I_{c2} \quad (10)$$

$$I_{e3} = 101I_{b3} \quad (11)$$

$$I_f = 0.13I_{e3} \quad (12)$$

$$I_{e1} = 0.706I_f \quad (13)$$

$$I_{c1} = 0.99I_{e1} \quad (14)$$

$$V_r = -1.957I_{c1} \quad (15)$$

Combining (9)–(15), we obtain

$$A\beta = -\frac{V_r}{V_t} = 249.2$$

**Ex. 10.14**

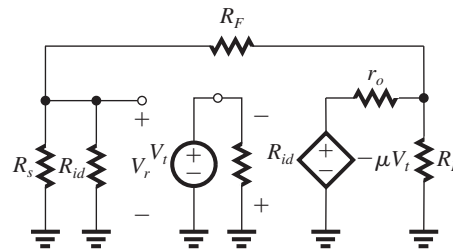


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

Using the voltage-divider rule, we can write by inspection

$$V_r = -\mu V_t \frac{R_L \parallel [R_F + (R_s \parallel R_{id})]}{r_o + \{R_L \parallel [R_F + (R_s \parallel R_{id})]\}} \frac{(R_s \parallel R_{id})}{R_F + (R_s \parallel R_{id})}$$

$$V_r = -\mu V_t \frac{R_L(R_s \parallel R_{id})}{r_o[R_L + R_F + (R_s \parallel R_{id})] + R_L[R_F + (R_s \parallel R_{id})]}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} = \frac{\mu R_L(R_{id} \parallel R_s)}{r_o[R_L + R_F + (R_{id} \parallel R_s)] + R_L[R_F + (R_{id} \parallel R_s)]}$$

Q.E.D.

Using the numerical values in Example 10.9, we get

$$A\beta = 1035.2$$



This figure belongs to Exercise 10.15.

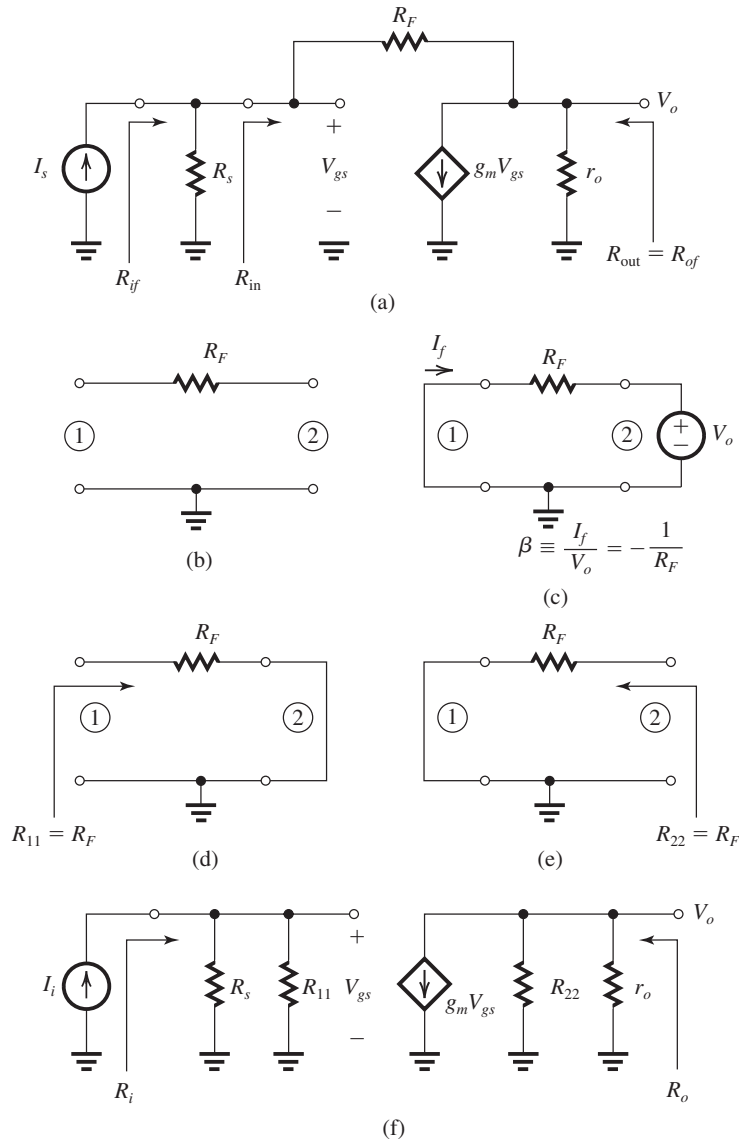


Figure 1

**Ex. 10.15** Figure 1(a) above shows the feedback amplifier circuit. The  $\beta$  circuit is shown in Fig. 1(b), and the determination of  $\beta$  is shown in Fig. 1(c),

$$\beta = -\frac{1}{R_F}$$

(a)

$$A_f|_{ideal} = \frac{1}{\beta} = -R_F$$

(b) The determination of  $R_{11}$  and  $R_{22}$  is illustrated in Figs. 1(d) and (e), respectively:

$$R_{11} = R_{22} = R_F$$

Finally, the  $A$  circuit is shown in Fig. 1(f). We can write by inspection

$$R_i = R_s \parallel R_{11} = R_s \parallel R_F$$

$$R_o = r_o \parallel R_{22} = r_o \parallel R_F$$

$$V_{gs} = I_i R_i$$

$$V_o = -g_m V_{gs} (r_o \parallel R_{22})$$

Thus,

$$A \equiv \frac{V_o}{I_i} = -g_m (R_s \parallel R_F) (r_o \parallel R_F)$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$A_f = \frac{g_m(R_s \parallel R_F)(r_o \parallel R_F)}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F} \quad \text{Q.E.D.}$$

$$(c) R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m(r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{in}}$$

thus,

$$\frac{1}{R_{in}} = \frac{1}{R_F} [1 + g_m(r_o \parallel R_F)]$$

$$\Rightarrow R_{in} = \frac{R_F}{1 + g_m(r_o \parallel R_F)} \quad \text{Q.E.D.}$$

$$(d) R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{r_o \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{out}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m(R_s \parallel R_F)}{R_F}$$

$$\Rightarrow R_{out} = r_o \parallel \frac{R_F}{1 + g_m(R_s \parallel R_F)} \quad \text{Q.E.D.}$$

$$(e) A = -5(1 \parallel 10)(20 \parallel 10)$$

$$A = -30.3 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1 \text{ mA/V}$$

$$A\beta = 3.03$$

$$1 + A\beta = 4.03$$

$$A_f = \frac{A}{1 + A\beta} = -\frac{30.3}{4.03} = -7.52 \text{ k}\Omega$$

(Compare to the ideal value of  $-10 \text{ k}\Omega$ ).

$$R_i = R_s \parallel R_F = 1 \parallel 10 = 909 \Omega$$

$$R_o = r_o \parallel R_F = 20 \parallel 10 = 6.67 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 226 \Omega$$

$$R_{in} = 1 \parallel \left[ \frac{1}{R_{if}} - \frac{1}{R_s} \right] = 291 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{6.67}{4.03} = 1.66 \text{ k}\Omega$$

$$R_{out} = R_{of} = 1.66 \text{ k}\Omega$$

**Ex. 10.16** Refer to the solution of Example 10.10.

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$= -100 \frac{100}{\frac{1}{5} + (10 \parallel 90 \parallel 20)} \frac{20}{20 + (10 \parallel 90)}$$

$$= -1076.4 \text{ A/A}$$

$$\beta = -0.1$$

$$A_f = \frac{A}{1 + A\beta} = -\frac{1076.4}{1 + 1076.4 \times 0.1}$$

$$= -\frac{1076.4}{1 + 107.64} = -\frac{1076.4}{108.64}$$

$$= -9.91 \text{ A/A}$$

$$R_i = 100 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{100}{108.64}$$

$$= 921 \Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 929 \times 108.64$$

$$= 101 \text{ M}\Omega$$

**Ex. 10.17** Refer to the solution of Example 10.10. With  $R_2 = 0$ ,  $\beta = -\frac{1}{1+0} = -1$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = -1 \text{ A/A}$$

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$$

$$\Rightarrow R_i = R_1$$

$$R_o = r_{o2} + (R_1 \parallel R_2) + (g_m r_{o2})(R_1 \parallel R_2)$$

$$\Rightarrow R_o = r_{o2}$$

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$\Rightarrow A = -\mu g_m R_1$$

$$A\beta = \mu g_m R_1$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_m R_1}{1 + \mu g_m R_1}$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{in} = \frac{R_1}{1 + \mu g_m R_1} \simeq \frac{1}{\mu g_m}$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= r_{o2}(1 + \mu g_m R_1)$$

$$\Rightarrow R_{out} \simeq \mu(g_m r_{o2})R_1$$

**Ex. 10.18** The feedback shifts the pole by a factor equal to the amount of feedback:

$$1 + A_0\beta = 1 + 10^5 \times 0.01 = 1001$$

The pole will be shifted to a frequency

$$f_{Pf} = f_p(1 + A_0\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

If  $\beta$  is changed to a value that results in a nominal closed-loop gain of 1, then we obtain

$$\beta = 1$$

and

$$1 + A_0\beta = 1 + 10^5 \times 1 \simeq 10^5$$

then the pole will be shifted to a frequency

$$f_{Pf} = 10^5 \times 100 = 10 \text{ MHz}$$

**Ex. 10.19** From Eq. (10.40), we see that the poles coincide when

$$(\omega_{p1} + \omega_{p2})^2 = 4(1 + A_0\beta)\omega_{p1}\omega_{p2}$$

$$(10^4 + 10^6)^2 = 4(1 + 100\beta) \times 10^4 \times 10^6$$

$$\Rightarrow 1 + 100\beta = 25.5$$

$$\Rightarrow \beta = 0.245$$

The corresponding value of  $Q = 0.5$ . This can also be verified by substituting in Eq. (10.42).

A maximally flat response is obtained when  $Q = 1/\sqrt{2}$ . Substituting in Eq. (10.42), we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^4 \times 10^6}}{10^4 + 10^6}$$

$$\Rightarrow \beta = 0.5$$

In this case, the low-frequency closed-loop gain is

$$\begin{aligned} A_f(0) &= \frac{A_0}{1 + A_0\beta} \\ &= \frac{100}{1 + 100 \times 0.5} = 1.96 \text{ V/V} \end{aligned}$$

**Ex. 10.20** The closed-loop poles are the roots of the characteristic equation

$$1 + A(s)\beta = 0$$

$$1 + \left( \frac{10}{1 + \frac{s}{10^4}} \right) \beta = 0$$

To simplify matters, we normalize  $s$  by the factor  $10^4$ , thus obtaining the normalized

complex-frequency variable  $S = s/10^4$ , and the characteristic equation becomes

$$(S + 1)^3 + 10^3\beta = 0 \tag{1}$$

This equation has three roots, a real one and a pair that can be complex conjugate. The real pole can be found from

$$\begin{aligned} (S + 1)^3 &= -10^3\beta \\ \Rightarrow S &= -1 - 10\beta^{1/3} = -(1 + 10\beta^{1/3}) \end{aligned} \tag{2}$$

Dividing the characteristic polynomial in (1) by  $(S + 1 + 10\beta^{1/3})$  gives a quadratic whose two roots are the remaining poles of the feedback amplifier. After some straightforward but somewhat tedious algebra, we obtain

$$\begin{aligned} S^2 + (10\beta^{1/3} - 2)S + (1 + 100\beta^{2/3} - 10\beta^{1/3}) \\ = 0 \end{aligned} \tag{3}$$

The pair of poles can now be obtained as

$$S = (-1 + 5\beta^{1/3}) \pm j5\sqrt{3}\beta^{1/3} \tag{4}$$

Equations (1) and (3) describe the three poles shown in Fig. E10.20.

From Eq. (2) we see that the pair of complex poles lie on the  $j\omega$  axis for the value of  $\beta$  that makes the coefficient of  $S$  equal to zero, thus

$$\beta_{cr} = \left( \frac{2}{10} \right)^3 = 0.008$$

**Ex. 10.21**

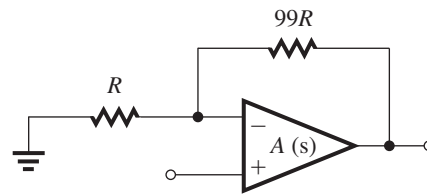


Figure 1

From Fig. 1, we can easily obtain the loop gain as

$$\begin{aligned} A\beta &= A(s) \times 0.01 \\ &= \frac{10^5}{1 + \frac{s}{2\pi \times 10}} \times 0.01 \\ &= \frac{1000}{1 + \frac{s}{2\pi \times 10}} \end{aligned}$$

From this single-pole response (low-pass STC response) we can find the unity-gain frequency by inspection as

$$f_1 = f_P \times 1000 \\ = 10^4 \text{ Hz}$$

The phase angle at  $f_1$  will be  $-90^\circ$  and thus the phase margin is  $90^\circ$ .

**Ex. 10.22** From Eq. (10.48), we obtain

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + e^{-j\theta}| \\ = 1/|1 + \cos \theta - j \sin \theta|$$

(a) For  $\text{PM} = 30^\circ$ ,  $\theta = 180 - 30 = 150^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 150^\circ - j \sin 150^\circ| \\ = 1.93$$

(b) For  $\text{PM} = 60^\circ$ ,  $\theta = 180 - 60 = 120^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 120^\circ - j \sin 120^\circ| \\ = 1$$

(c) For  $\text{PM} = 90^\circ$ ,  $\theta = 180 - 90 = 90^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 90^\circ - j \sin 90^\circ| \\ = 1/\sqrt{2} = 0.707$$

**Ex. 10.23** For a phase margin of  $45^\circ$ ,  $\phi = -135^\circ$ . Locating the point  $\phi = -135^\circ$  on the phase plot in Fig. 10.38, we find the corresponding point on the gain plot at 80 dB, that is,

$$20 \log(1/\beta) = 80 \text{ dB}$$

$$\Rightarrow \beta = 10^{-4}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}} \\ = 9.09 \times 10^3 \\ = 79.2 \text{ dB} \simeq 80 \text{ dB}$$

**Ex. 10.24** To obtain stable performance for closed-loop gains as low as 20 dB (which is 80 dB below  $A_0$ , or equivalently  $10^4$  below  $A_0$ ), we must place the new dominant pole at  $1 \text{ MHz}/10^4 = 100 \text{ Hz}$ .

**Ex. 10.25** The frequency of the first pole must be lowered from 1 MHz to a new frequency

$$f'_D = \frac{10 \text{ MHz}}{10^4} = 1000 \text{ Hz}$$

that is, by a factor of 1000. Thus, the capacitance at the controlling node must be increased by a factor of 1000.

**Solutions to End-of-Chapter Problems**

**10.1**  $A_f = \frac{A}{1 + A\beta}$

$$200 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 4.9 \times 10^{-3}$$

If  $A$  changes to  $10^3$ , then we get

$$A_f = \frac{1000}{1 + 10^3 \times 4.9 \times 10^{-3}}$$

$$= \frac{1000}{5.9} = 169.5$$

$$\text{Percentage change in } A_f = \frac{169.5 - 200}{200} \times 100$$

$$= -15.3\%$$

**10.2**  $A_f|_{\text{ideal}} = 100$

$$\beta = \frac{1}{A_f|_{\text{ideal}}} = 0.01$$

$$A_f = \frac{A}{1 + A\beta} = \frac{1}{\beta} \frac{A\beta}{1 + A\beta}$$

$$A_f = A_f|_{\text{ideal}} \frac{A\beta}{1 + A\beta} = \frac{A_f|_{\text{ideal}}}{1 + (1/A\beta)}$$

For  $A\beta \gg 1$ ,

$$A_f \simeq A_f|_{\text{ideal}} \left(1 - \frac{1}{A\beta}\right)$$

Thus, the deviation is

$$-\frac{100}{A\beta} \%$$

To limit the deviation to 1%,  $A\beta|_{\text{minimum}} = 100$  and  $A_{\text{minimum}} = 10^4$

**10.3** The direct connection of the output terminal to the inverting input terminal results in  $V_f = V_o$  and thus

$$\beta = 1$$

If  $A = 1000$ , then the closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 1000 \times 1} = 0.999 \text{ V/V}$$

Amount of feedback =  $1 + A\beta$

$$= 1 + 1000 \times 1 = 1001$$

or 60 dB

For  $V_s = 1 \text{ V}$ , we obtain

$$V_o = A_f V_s = 0.999 \times 1 = 0.999 \text{ V}$$

$$V_i = V_s - V_o = 1 - 0.999$$

$$= 0.001 \text{ V}$$

If  $A$  becomes  $1000(1 - 0.1) = 900 \text{ V/V}$ , then we get

$$A_f = \frac{900}{1 + 900 \times 1} = 0.99889$$

Thus,  $A_f$  changes by

$$= \frac{0.99889 - 0.999}{0.999} \times 100 = -0.011\%$$

**10.4** (a) Figure 1 shows the situation with  $A$  set to  $\infty$ . We see that,

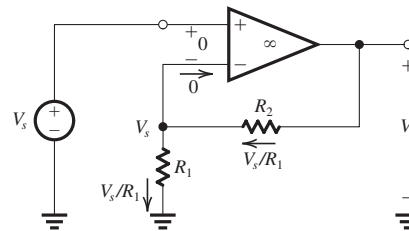


Figure 1

$$V_o = \frac{V_s}{R_1} (R_1 + R_2)$$

Thus,

$$A_f|_{\text{ideal}} = \frac{V_o}{V_s} = 1 + \frac{R_2}{R_1}$$

and,

$$\beta = 1/A_f|_{\text{ideal}} = \frac{R_1}{R_1 + R_2}$$

(b)

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{A}{1 + A\beta}$$

$$\Rightarrow \beta = 0.1 - \frac{1}{A} \tag{1}$$

But,

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + R_2}$$

$$\Rightarrow R_2 = 10\left(\frac{1}{\beta} - 1\right) \tag{2}$$

Eqns. (1) and (2) can be used to determine the values of  $\beta$  and  $R_2$  required in each of the following cases:

(i)

$$A = 1000 \text{ V/V},$$

$$\beta = 0.1 - 0.001 = 0.099 \text{ V/V}$$

$$R_2 = 10\left(\frac{1}{0.099} - 1\right) = 91 \text{ k}\Omega$$

(ii)

$$A = 100 \text{ V/V}$$

$$\beta = 0.1 - 0.01 = 0.09 \text{ V/V}$$

$$R_2 = 101.1 \text{ k}\Omega$$

(iii)

$$A = 20 \text{ V/V}$$

$$\beta = 0.1 - 0.05 = 0.05$$

$$R_2 = 190 \text{ k}\Omega$$

(c) (i)

$$A = 0.9 \times 1000 = 900 \text{ V/V}$$

$$A_f = \frac{900}{1 + 900 \times 0.099} = 9.99 \text{ V/V}$$

Thus  $A_f$  changes by  $-0.11\%$ .

(ii)

$$A = 0.9 \times 100 = 90 \text{ V/V}$$

$$A_f = \frac{90}{1 + 90 \times 0.09} = 9.89 \text{ V/V}$$

Thus,  $A_f$  changes by  $-1.1\%$ .

(iii)

$$A = 0.9 \times 20 = 18 \text{ V/V}$$

$$A_f = \frac{18}{1 + 18 \times 0.05} = 9.47 \text{ V/V}$$

Thus  $A_f$  changes by  $-5.3\%$ . We conclude that as  $A$  becomes smaller and hence the amount of feedback ( $1 + A\beta$ ) is lower, the desensitivity of the feedback amplifier to changes in  $A$  decreases. In other words, the negative feedback becomes less effective as  $(1 + A\beta)$  decreases.

$$\mathbf{10.5} \quad A_f|_{\text{ideal}} = 10$$

$$\beta = \frac{1}{A_f|_{\text{ideal}}} = 0.1$$

$$20 \log(1 + A\beta) = 40 \text{ dB}$$

$$\Rightarrow 1 + A\beta = 100$$

$$\Rightarrow A = 990$$

$$A_f = \frac{A}{1 + A\beta} = \frac{990}{1 + 990 \times 0.1} = 9.9$$

$$\mathbf{10.6} \quad A = \frac{V_o}{V_i} = \frac{5 \text{ V}}{10 \text{ mV}} = 500 \text{ V/V}$$

$$V_f = V_s - V_i = 1 - 0.01 = 0.99 \text{ V}$$

$$\beta = \frac{V_f}{V_o} = \frac{0.99}{5} = 0.198 \text{ V/V}$$

**10.7** For each value of  $A$  given, we have three different values of  $\beta$ : 0.00, 0.50, and 1.00. To obtain  $A_f$ , we use

$$A_f = \frac{A}{1 + A\beta}$$

The results obtained are as follows.

Case	$A$ (V/V)	$A_f$ (V/V) for $\beta = 0.00$	$A_f$ (V/V) for $\beta = 0.50$	$A_f$ (V/V) for $\beta = 1.00$
(a)	1	1	0.667	0.500
(b)	10	10	1.667	0.909
(c)	100	100	1.961	0.990
(d)	1000	1000	1.996	0.999
(e)	10,000	10,000	1.9996	0.9999

$$\mathbf{10.8} \quad A = \frac{5 \text{ V}}{2 \text{ mV}} = 2500 \text{ V/V}$$

$$A_f = \frac{5 \text{ V}}{100 \text{ mV}} = 50 \text{ V/V}$$

Amount of feedback  $\equiv 1 + A\beta$ 

$$= \frac{A}{A_f} = \frac{2500}{50} = 50$$

or 34 dB

$$A\beta = 49$$

$$\beta = \frac{49}{2500} = 0.0196 \text{ V/V}$$

$$\mathbf{10.9} \quad A_{\text{nominal}} = 1000$$

$$A_{\text{low}} = 500$$

$$A_{\text{high}} = 1500$$

If we apply negative feedback with a feedback factor  $\beta$ , then

$$A_{f, \text{nominal}} = \frac{1000}{1 + 1000\beta}$$

$$A_{f, \text{low}} = \frac{500}{1 + 500\beta}$$

$$A_{f, \text{high}} = \frac{1500}{1 + 1500\beta}$$

It is required that

$$A_{f, \text{low}} \geq 0.99A_{f, \text{nominal}} \quad (1)$$

and

$$A_{f, \text{high}} \leq 1.01A_{f, \text{nominal}} \quad (2)$$

If we satisfy condition (1) with equality, we can determine the required value of  $\beta$ . We must then check that condition (2) is satisfied. Thus,

$$\frac{500}{1 + 500\beta} = 0.99 \times \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.098$$

For this value of  $\beta$ , we obtain

$$A_{f, \text{nominal}} = \frac{1000}{1 + 1000 \times 0.098}$$

$$= 10.101$$

$$A_{f, \text{low}} = \frac{500}{1 + 500 \times 0.098} = 10$$

$$A_{f, \text{high}} = \frac{1500}{1 + 1500 \times 0.098} = 10.135$$

Thus, the low value of the closed-loop gain is 0.101 below nominal or  $-1\%$ , and the high value is 0.034 above nominal or  $0.34\%$ . Thus, our amplifier meets specification and the nominal value of closed-loop gain is 10.1. This is the highest possible closed-loop gain that can be obtained while meeting specification.

Now, if three closed-loop amplifiers are placed in cascade, the overall gain obtained will be

$$\text{Nominal Gain} = (10.1)^3 = 1030$$

$$\text{Lowest Gain} = 10^3 = 1000$$

$$\text{Highest Gain} = (10.135)^3 = 1041$$

Thus, the lowest gain will be approximately  $3\%$  below nominal, and the highest gain will be  $1\%$  above nominal.

**10.10** From Eq. (11.10), we have

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1 + A\beta}$$

Since  $-40$  dB is 0.01, we have

$$0.01 = \frac{1}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

For

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{5}$$

we have

$$1 + A\beta = 5$$

$$\Rightarrow A\beta = 4$$

**10.11** For  $A = 1000$  V/V, we have

$$A_f = 10 = \frac{1000}{1 + A\beta}$$

$$\Rightarrow \text{Densensitivity factor} \equiv 1 + A\beta = 100$$

$$A\beta = 99$$

$$\beta = \frac{99}{1000} = 0.099 \text{ V/V}$$

For  $A = 500$  V/V, we have

$$A_f = 10 = \frac{500}{1 + A\beta}$$

$$\Rightarrow \text{Densensitivity factor} \equiv 1 + A\beta = 50$$

$$\beta = \frac{49}{500} = 0.098 \text{ V/V}$$

If the  $A = 1000$  amplifiers have a gain uncertainty of  $\pm 10\%$ , the gain uncertainty of the closed-loop amplifiers will be

$$= \frac{\pm 10\%}{100} = \pm 0.1\%$$

If we require a gain uncertainty of  $\pm 0.1\%$  using the  $A = 500$  amplifiers, then

$$\pm 0.1\% = \frac{\text{Gain uncertainty of } A = 500 \text{ amplifiers}}{50}$$

$$\Rightarrow \text{Gain uncertainty} = \pm 5\%$$

**10.12**  $A_f = 10$  V/V

$$1 + A\beta = \frac{\pm 10\%}{\pm 0.1\%} = 100$$

$$10 = \frac{A}{100}$$

$$\Rightarrow A = 1000 \text{ V/V}$$

$$\beta = \frac{100 - 1}{1000} = 0.099 \text{ V/V}$$

**10.13** The open-loop gain varies from  $A$  to  $10A$  with temperature and time. Correspondingly,  $A_f$  varies from  $(25 - 1\%)$  i.e.  $24.75$  V/V to  $(25 + 1\%)$  or  $25.25$  V/V. Substituting these quantities into the formula for the closed-loop gain

$$A_f = \frac{A}{1 + A\beta}$$

we obtain

$$24.75 = \frac{A}{1 + A\beta} \quad (1)$$

$$25.25 = \frac{10A}{1 + 10A\beta} \quad (2)$$

Dividing Eq. (2) by Eq. (1), we obtain

$$1.02 = 10 \frac{1 + A\beta}{1 + 10A\beta}$$

$$1.02 + 10.2A\beta = 10 + 10A\beta$$

$$\Rightarrow A\beta = 44.9$$

Substituting in (1) yields

$$A = 24.75 \times 45.9 = 1136$$

and

$$\beta = \frac{44.9}{1136} = 0.0395$$

### 10.14

$$A_0 = 10^5 \text{ V/V}$$

$$f_{3\text{dB}} = 100 \text{ Hz}$$

(a)

$$A_{f0} = 10^3 = \frac{10^5}{1 + A_0\beta}$$

$$\Rightarrow 1 + A_0\beta = 100 \text{ or } 40 \text{ dB}$$

$$3\beta = 9.9 \times 10^{-4} \simeq 0.001 \text{ V/V}$$

$$f_H = (1 + A_0\beta)f_{3\text{dB}} = 100 \times 100 \\ = 10 \text{ kHz}$$

(b)

$$A_{f0} = 10^2 = \frac{10^5}{1 + A_0\beta}$$

$$1 + A_0\beta = 1000$$

$$\beta = 9.99 \times 10^{-3} \simeq 0.01 \text{ V/V}$$

$$f_H = 1000 \times 100 \\ = 100 \text{ kHz}$$

(c)

$$A_{f0} = 10 = \frac{10^5}{1 + A_0\beta}$$

$$1 + A\beta = 10^4$$

$$\beta = 9.999 \times 10^{-2} \simeq 0.1 \text{ V/V}$$

$$f_H = 10^4 \times 100 \\ = 1 \text{ MHz}$$

(d)

$$A_{f0} = 1 = \frac{10^5}{1 + A_0\beta}$$

$$1 + A\beta = 10^5$$

$$\beta = 9.9999 \times 10^{-1} \simeq 1 \text{ V/V}$$

$$f_H = 10^5 \times 100 \\ = 10 \text{ MHz}$$

### 10.15

$$A_{f0} = 10 \text{ V/V}$$

$$f_H \geq 10 \text{ MHz}$$

$$A_0 = 1000 \text{ V/V}$$

$$A_{f0} = \frac{A_0}{1 + A_0\beta}$$

$$\Rightarrow 1 + A_0\beta = \frac{A_0}{A_{f0}} = \frac{1000}{10} = 100$$

$$\beta = \frac{99}{1000} = 0.099 \text{ V/V}$$

$$f_{3\text{dB}} = \frac{10 \text{ MHz}}{100} = 0.1 \text{ MHz} = 100 \text{ kHz}$$

Thus, the open-loop amplifier 3-dB frequency must be at least 100 kHz.

**10.16** The closed-loop gain for the first (high-gain) segment is

$$A_{f1} = \frac{1000}{1 + 1000\beta} \quad (1)$$

and that for the second segment is

$$A_{f2} = \frac{100}{1 + 100\beta} \quad (2)$$

We require

$$\frac{A_{f1}}{A_{f2}} = 1.1$$

Thus, dividing Eq. (1) by Eq. (2) yields

$$1.1 = 10 \frac{1 + 100\beta}{1 + 1000\beta}$$

$$1.1 + 1100\beta = 10 + 1000\beta$$

$$\Rightarrow \beta = 0.089$$

$$A_{f1} = \frac{1000}{1 + 1000 \times 0.089} = 11.1 \text{ V/V}$$

$$A_{f2} = \frac{100}{1 + 100 \times 0.089} = 10.1 \text{ V/V}$$

The first segment ends at  $|v_O| = 10 \text{ mV} \times 1000 = 10 \text{ V}$ . This corresponds to

$$v_S = \frac{10 \text{ V}}{A_{f1}} = \frac{10}{11.1} = 0.9 \text{ V}$$

The second segment ends at  $|v_O| = 10 + 0.05 \times 100 = 15 \text{ V}$ . This corresponds to

$$v_S = 0.9 + \frac{15 - 10}{A_{f2}}$$

$$= 0.9 + \frac{5}{10.1} = 1.4 \text{ V}$$

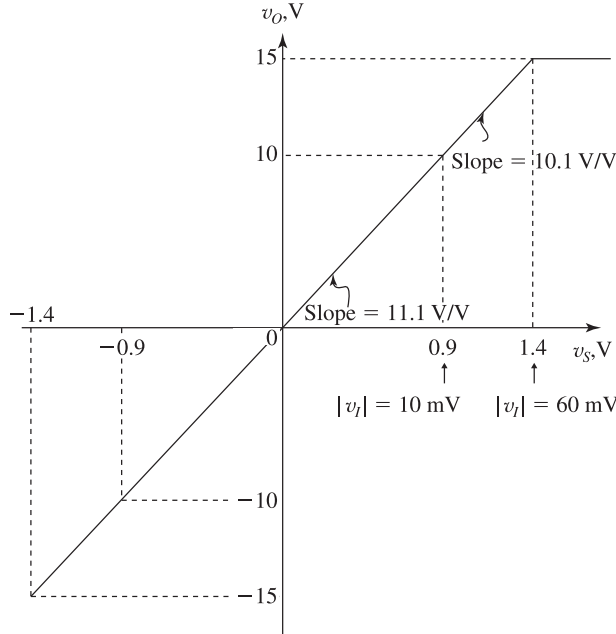
Thus, the transfer characteristic of the feedback amplifier can be described as follows:

$$\text{For } |v_S| \leq 0.9 \text{ V, } v_O/v_S = 11.1 \text{ V/V}$$

$$\text{For } 0.9 \text{ V} \leq |v_S| \leq 1.4 \text{ V, } v_O/v_S = 10.1 \text{ V/V}$$



This figure belongs to Problem 10.16.



For  $|v_s| \geq 1.4 \text{ V}$ ,  $v_o = \pm 15 \text{ V}$

The transfer characteristic is shown in the figure above.

**10.17** Let the gain of the ideal (nonvarying) driver amplifier be denoted  $\mu$ . Then, the open-loop gain  $A$  will vary from  $2\mu$  to  $12\mu$ . Correspondingly, the closed-loop gain will vary from  $95 \text{ V/V}$  to  $105 \text{ V/V}$ . Substituting these quantities into the closed-loop gain expression, we obtain

$$95 = \frac{2\mu}{1 + 2\mu\beta} \quad (1)$$

$$105 = \frac{12\mu}{1 + 12\mu\beta} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$1.105 + 1.105 \times 12\mu\beta = 6 + 12\mu\beta$$

$$\Rightarrow \mu\beta = 3.885$$

Substituting in Eq. (1) yields

$$\mu = \frac{95(1 + 2 \times 3.885)}{2} = 416.6 \text{ V/V}$$

$$\beta = \frac{3.885}{416.6} = 9.33 \times 10^{-3} \text{ V/V}$$

If  $A_f$  is to be held to within  $\pm 0.5\%$ , Eqs. (1) and (2) are modified to

$$99.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (3)$$

$$100.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (4)$$

Dividing (4) by (3) yields

$$1.01 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 41.58$$

Substituting into (3) provides

$$\mu = \frac{99.5(1 + 2 \times 41.58)}{2}$$

$$= 4187 \text{ V/V}$$

which is about a factor of 10 higher than the gain required in the less constrained case. The value of  $\beta$  required is

$$\beta = \frac{41.58}{4187} = 9.93 \times 10^{-3} \text{ V/V}$$

Repeating for  $A_f = 10 \text{ V/V}$  (a factor of 10 lower than the original case):

(a) For  $\pm 5\%$  maximum variability, Eqs. (1) and (2) become

$$9.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (5)$$

$$10.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (6)$$

Dividing (6) by (5) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 3.885$$

which is identical to the first case considered, and

$$\mu = \frac{9.5(1 + 2 \times 3.885)}{2} = 41.66 \text{ V/V}$$

which is a factor of 10 lower than the value required when the gain required was 100. The feedback factor  $\beta$  is

$$\beta = \frac{3.885}{41.66} = 9.33 \times 10^{-2} \text{ V/V}$$

which is a factor of 10 higher than the case with  $A_f = 100$ .

(b) Finally, for the case  $A_f = 10 \pm 0.5\%$  we can write by analogy

$$\mu\beta = 41.58$$

$$\mu = 418.7 \text{ V/V}$$

$$\beta = 9.93 \times 10^{-2} \text{ V/V}$$

**10.18** If we use one stage, the amount of feedback required is

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{100} = 10$$

Thus the closed-loop amplifier will have a variability of

$$\text{Variability of } A_f = \frac{\pm 30\%}{10} = \pm 3\%$$

which does not meet specifications. Next, we try using two stages. For a nominal gain of 100, each stage will be required to have a nominal gain of 10. Thus, for each stage the amount of feedback required will be

$$1 + A\beta = \frac{1000}{10} = 100$$

Thus, the closed-loop gain of each stage will have a variability of

$$= \frac{\pm 30\%}{100} = \pm 0.3\%$$

and the cascade of two stages will thus show a variability of  $\pm 0.6\%$ , well within the required  $\pm 1\%$ . Thus two stages will suffice.

We next investigate the design in more detail. Each stage will have a nominal gain of 10 and thus

$$1 + A\beta = \frac{1000}{10} = 100$$

$$\Rightarrow A\beta = 99$$

$$\Rightarrow \beta = 0.099$$

Since  $A$  ranges from 700 V/V to 1300 V/V, the gain of each stage will range from

$$A_{f, \text{low}} = \frac{700}{1 + 700 \times 0.099} = 9.957 \text{ V/V}$$

and a high value of

$$A_{f, \text{high}} = \frac{1300}{1 + 1300 \times 0.099} = 10.023 \text{ V/V}$$

Thus, the cascade of two stages will have a range of

$$\text{Lowest gain} = 9.957^2 = 99.14 \text{ V/V}$$

$$\text{Highest gain} = 10.023^2 = 100.46 \text{ V/V}$$

which is  $-0.86\%$  to  $+0.46\%$  of the nominal 100 V/V gain, well within the required  $\pm 1\%$ .

**10.19** If the nominal open-loop gain is  $A$ , then we require that as  $A$  drops to  $(A/2)$  the closed-loop gain drops from 10 to a minimum of 9.8. Substituting these values in the expression for the closed-loop gain, we obtain

$$10 = \frac{A}{1 + A\beta} \quad (1)$$

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$1.02 = \frac{2 \left( 1 + \frac{1}{2}A\beta \right)}{1 + A\beta}$$

$$1.02 = \frac{2 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{1}{1 + A\beta}$$

$$\Rightarrow 1 + A\beta = \frac{1}{0.02} = 50$$

Substituting in Eq. (1) gives

$$A = 10 \times 50 = 500 \text{ V/V}$$

and

$$\beta = \frac{50 - 1}{500} = 0.098 \text{ V/V}$$

If  $\beta$  is accurate to within  $\pm 1\%$ , to ensure that the minimum closed-loop gain realized is 9.8 V/V, we have

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A \times 0.098 \times 1.01}$$

$$\Rightarrow A = 653.4 \text{ V/V}$$

$$\mathbf{10.20} \quad A_f = \frac{A}{1 + A\beta}$$

$$100 = \frac{A}{1 + A\beta} \quad (1)$$

$$99 = \frac{0.1A}{1 + 0.1A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$\begin{aligned} 1.01 &= \frac{10(1 + 0.1A\beta)}{1 + A\beta} \\ &= \frac{10 + A\beta}{1 + A\beta} \\ &= 1 + \frac{9}{1 + A\beta} \\ \Rightarrow \frac{9}{1 + A\beta} &= 0.01 \\ 1 + A\beta &= 900 \\ A\beta &= 899 \end{aligned}$$

Substituting  $(1 + A\beta) = 900$  into Eq. (1) yields

$$A = 100 \times 900 = 90,000 \text{ V/V}$$

The value of  $\beta$  is

$$\beta = \frac{899}{90,000} = 9.989 \times 10^{-3} \text{ V/V}$$

If  $A$  were increased tenfold, i.e.,  $A = 900,000$ , we obtain

$$A_f = \frac{900,000}{1 + 8990} = 100.1 \text{ V/V}$$

If  $A$  becomes infinite, we get

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} \\ &= \frac{1}{\frac{1}{A} + \beta} = \frac{1}{\beta} \\ &= \frac{1}{9.989 \times 10^{-3}} = 100.11 \text{ V/V} \end{aligned}$$

$$\mathbf{10.21} \quad A = A_M \frac{s}{s + \omega_L}$$

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} \\ &= \frac{A_M s / (s + \omega_L)}{1 + A_M \beta s / (s + \omega_L)} \\ &= \frac{A_M s}{s + \omega_L + s A_M \beta} \\ &= \frac{A_M s}{s(1 + A_M \beta) + \omega_L} \\ &= \frac{A_M}{1 + A_M \beta} \frac{s}{s + \omega_L / (1 + A_M \beta)} \end{aligned}$$

Thus,

$$\begin{aligned} A_{Mf} &= \frac{A_M}{1 + A_M \beta} \\ \omega_{Lf} &= \frac{\omega_L}{1 + A_M \beta} \end{aligned}$$

Thus, both the midband gain and the 3-dB frequency are lowered by the amount of feedback,  $(1 + A_M \beta)$ .

**10.22** Let's first try  $N = 2$ . The closed-loop gain of each stage must be

$$A_f = \sqrt{1000} = 31.6 \text{ V/V}$$

Thus, the amount-of-feedback in each stage must be

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{31.6} = 31.6$$

The 3-dB frequency of each stage is

$$\begin{aligned} f_{3dB}|_{\text{stage}} &= (1 + A\beta)f_H \\ &= 31.6 \times 20 = 632 \text{ kHz} \end{aligned}$$

Thus, the 3-dB frequency of the cascade amplifier is

$$f_{3dB}|_{\text{cascade}} = 632\sqrt{2^{1/2} - 1} = 406.8 \text{ kHz}$$

which is less than the required 1 MHz.

Next, we try  $N = 3$ . The closed-loop gain of each stage is

$$A_f = (1000)^{1/3} = 10 \text{ V/V}$$

and thus each stage will have an amount-of-feedback

$$1 + A\beta = \frac{1000}{10} = 100$$

which results in a stage 3-dB frequency of

$$\begin{aligned} f_{3dB}|_{\text{stage}} &= (1 + A\beta)f_H \\ &= 100 \times 20 = 2000 \text{ kHz} \\ &= 2 \text{ MHz} \end{aligned}$$

The 3-dB frequency of the cascade amplifier will be

$$\begin{aligned} f_{3dB}|_{\text{cascade}} &= 2\sqrt{2^{1/3} - 1} \\ &= 1.02 \text{ MHz} \end{aligned}$$

which exceeds the required value of 1 MHz.

Thus, we need three identical stages, each with a closed-loop gain of 10 V/V, an amount-of-feedback of 100, and a loop gain

$$A\beta = 99$$

Thus,

$$\beta = 0.099 \text{ V/V}$$

### 10.23

Refer to Fig. 1 (next page). For  $v_I = +0.7 \text{ V}$ , we have  $v_O = 0$  and

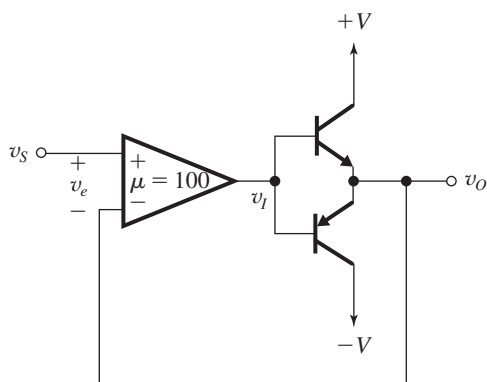


Figure 1

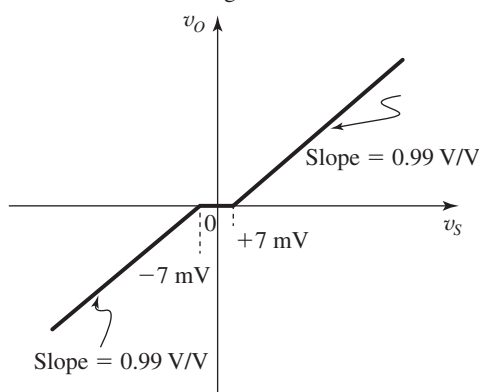


Figure 2

$$v_e = \frac{v_I}{\mu} = \frac{+0.7}{100} = +7 \text{ mV}$$

Similarly, for  $v_I = -0.7 \text{ V}$ , we obtain  $v_o = 0$  and

$$v_e = \frac{v_I}{\mu} = \frac{-0.7}{100} = -7 \text{ mV}$$

Thus, the limits of the deadband are now  $\pm 7 \text{ mV}$ .

Outside the deadband, the gain of the feedback amplifier, that is,  $v_o/v_s$ , can be determined by noting that the open-loop gain  $A \equiv v_o/v_e = 100 \text{ V/V}$  and the feedback factor  $\beta = 1$ , thus

$$\begin{aligned} A_f &\equiv \frac{v_o}{v_s} = \frac{A}{1 + A\beta} \\ &= \frac{100}{1 + 100 \times 1} = 0.99 \text{ V/V} \end{aligned}$$

The transfer characteristic is depicted in Fig. 2.

**10.24** Refer to Fig. 10.13.

(a) The ideal closed-loop gain is given by

$$A_f = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

(b) From Example 10.4, we obtain

$$\begin{aligned} A\beta &= \mu \frac{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]}{\{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]\} + r_o} \\ &\times \frac{R_1 \parallel (R_{id} + R_s)}{[R_1 \parallel (R_{id} + R_s)] + R_2} \times \frac{R_{id}}{R_{id} + R_s} \\ A\beta &= 1000 \frac{10 \parallel [90 + 10 \parallel (100 + 100)]}{\{10 \parallel [90 + 10 \parallel (100 + 100)]\} + 1} \\ &\times \frac{10 \parallel (100 + 100)}{[10 \parallel (100 + 100)] + 90} \times \frac{100}{100 + 100} \\ &= 1000 \times 0.9009 \times 0.0957 \times 0.5 \end{aligned}$$

$$= 43.11$$

$$A = \frac{A\beta}{\beta} = \frac{43.11}{0.1} = 431.1 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{431.1}{1 + 43.11} = 9.77 \text{ V/V}$$

(c) To obtain  $A_f = 9.9 \text{ V/V}$ , we use

$$\begin{aligned} 9.9 &= \frac{A}{1 + A\beta} \\ &= \frac{A}{1 + A \times 0.1} \\ \Rightarrow A &= 1010 \text{ V/V} \end{aligned}$$

Thus  $\mu$  must be increased by the factor

$$\begin{aligned} \frac{1010}{431.1} &= 2.343 \text{ to become} \\ \mu &= 2343 \text{ V/V} \end{aligned}$$

**10.25** (a) Setting the open-loop gain to  $\infty$ , the voltage  $V_{gs1}$  reduces to zero and the current in the source of  $Q_1(I_{d1})$  becomes zero. This isolates the feedback network which is the voltage divider  $(R_F, R_{S1})$ . Thus,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

and the ideal value of the closed-loop gain is

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_F}{R_{S1}}$$

$$10 = 1 + \frac{R_F}{0.1}$$

$$\Rightarrow R_F = 0.9 \text{ k}\Omega$$

(b)

Figure 1 on the next page shows the circuit for determining the loop gain. Observe that we have broken the loop at the gate of  $Q_2$  where the input resistance is infinite, obviating the need for adding a termination resistance. Also, observe that as usual we have set  $V_s = 0$ . To determine the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

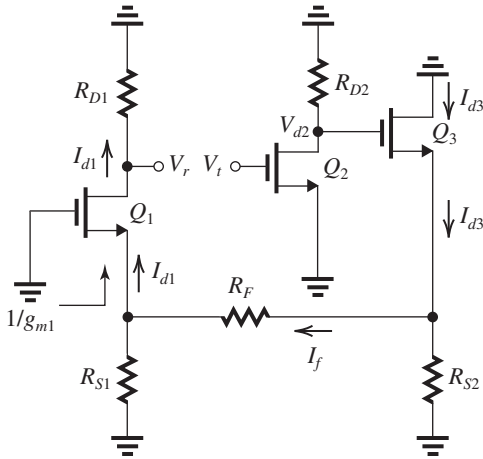


Figure 1

we write the following equations:

$$V_{d2} = -g_{m2}R_{D2}V_t \quad (1)$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{g_{m3}} + \left\{ R_{S2} \parallel \left[ R_F + \left( R_{S1} \parallel \frac{1}{g_{m1}} \right) \right] \right\}} \quad (2)$$

$$I_f = I_{d3} \frac{R_{S2}}{\left[ R_F + \left( R_{S1} \parallel \frac{1}{g_{m1}} \right) \right] + R_{S2}} \quad (3)$$

$$I_{d1} = I_f \frac{R_{S1}}{R_{S1} + \frac{1}{g_{m1}}} \quad (4)$$

$$V_r = I_{d1}R_{D1} \quad (5)$$

Substituting the numerical values in (1)–(5), we obtain

$$V_{d2} = -4 \times 10V_t = -40V_t \quad (6)$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{4} + \left\{ 0.1 \parallel \left[ 0.9 + \left( 0.1 \parallel \frac{1}{4} \right) \right] \right\}} \quad (7)$$

$$I_{d3} = 2.935V_{d2} \quad (7)$$

$$I_f = I_{d3} \frac{0.1}{\left[ 0.9 + \left( 0.1 \parallel \frac{1}{4} \right) \right] + 0.1} \quad (8)$$

$$I_f = 0.0933I_{d3} \quad (8)$$

$$I_{d1} = I_f \frac{0.1}{0.1 + \frac{1}{4}} = 0.286I_f \quad (9)$$

$$V_r = 10I_{d1} \quad (10)$$

Combining (6)–(10) gives

$$V_r = -31.33V_t$$

$$\Rightarrow A\beta = 31.33$$

$$A = \frac{A\beta}{\beta} = \frac{31.33}{0.1} = 313.3 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{313.3}{1 + 31.33} = 9.7 \text{ V/V}$$

Thus,  $A_f$  is 0.3 V/V lower than the ideal value of 10 V/V, a difference of -3%. The circuit could be adjusted to make  $A_f$  exactly 10 by changing  $\beta$  through varying  $R_F$ . Specifically,

$$10 = \frac{313.3}{1 + 313.3\beta}$$

$$\Rightarrow \beta = 0.0968$$

But,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$0.0968 = \frac{0.1}{0.1 + R_F}$$

$$\Rightarrow R_F = 933 \Omega$$

(an increase of 33  $\Omega$ ).

**10.26** All MOSFETs are operating at  $I_D = 100 \mu\text{A} = 0.1 \text{ mA}$  and  $|V_{OV}| = 0.2 \text{ V}$ , thus

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

All devices have

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

(a)

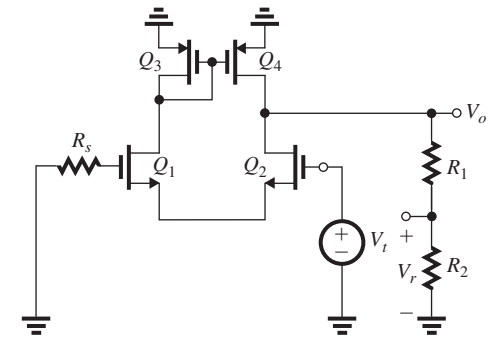


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain  $A\beta$ .

$$V_o = -g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]V_t \quad (1)$$

$$V_r = \frac{R_2}{R_1 + R_2}V_o = \beta V_o$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]\beta$$

$$= 1(100 \parallel 100 \parallel 1000)\beta$$

$$= 47.62\beta$$

Thus,

$$A = 47.62 \text{ V/V}$$

$$(b) A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{47.62}{1 + 47.62\beta}$$

$$\Rightarrow \beta = 0.179 \text{ V/V}$$

$$\frac{R_2}{R_1 + R_2} = 0.179$$

$$\Rightarrow R_2 = 179 \text{ k}\Omega$$

$$R_1 = 821 \text{ k}\Omega$$

**10.27** Because the op amp has an infinite input resistance and a zero output resistance, this circuit is a direct implementation of the ideal feedback structure and thus

$$A = 1000 \text{ V/V}$$

and

$$\beta = \frac{R_1}{R_1 + R_2}$$

The ideal closed-loop gain is

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

$$\beta = \frac{10}{10 + 90} = 0.1 \text{ V/V}$$

$$A\beta = 1000 \times 0.1 = 100$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 100} = 9.9 \text{ V/V}$$

To obtain  $A_f$  that is exactly 10, we use

$$10 = \frac{1000}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

$$\beta = 0.099$$

$$0.099 = \frac{R_1}{R_1 + R_2}$$

$$0.099 = \frac{10}{10 + R_2}$$

$$\Rightarrow R_2 = 91 \text{ k}\Omega$$

**10.28** Refer to Fig. 10.9.

$$(a) \beta = \frac{R_1}{R_1 + R_2}$$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$5 = 1 + \frac{R_2}{1}$$

$$\Rightarrow R_2 = 4 \text{ k}\Omega$$

(b) From Example 10.3, we have

$$A\beta = (g_{m1}R_{D1})(g_{m2}R_{D2}) \frac{1}{1 + g_{m1}R_1} \times$$

$$\frac{R_1}{R_{D2} + R_2 + \left(R_1 \parallel \frac{1}{g_{m1}}\right)}$$

=

$$(4 \times 10)(4 \times 10) \frac{1}{1 + 4 \times 1} \times \frac{1}{10 + 4 + (1 \parallel 0.25)}$$

$$= 22.54$$

$$A = \frac{A\beta}{\beta} = \frac{22.54}{0.2} = 112.7 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{112.7}{1 + 22.54} = 4.79 \text{ V/V}$$

**10.29** Refer to Fig. 10.9(c) and to the expressions for  $\beta$ ,  $A\beta$ , and  $A$  given in the answer section of Exercise 10.5.

$$A = g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

where

$$g_m = 4 \text{ mA/V}$$

$$R_D = 10 \text{ k}\Omega$$

$$R_1 + R_2 = 1 \text{ M}\Omega \text{ (the potentiometer resistance)}$$

Thus,

$$A = 4 \times \frac{10 \times 1000}{10 + 1000} = 39.6 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{39.6}{1 + 39.6\beta}$$

$$\Rightarrow \beta = 0.175 \text{ V/V}$$

$$0.175 = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_1 = 0.175 \times 1000 = 175 \text{ k}\Omega$$

**10.30** (a) Setting the open-loop gain to  $\infty$ , results in  $V_{be1} = 0$ ,  $I_{e1} = 0$  and  $I_{b1} = 0$ , which isolates the feedback network as the voltage divider ( $R_1, R_2$ ), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b)

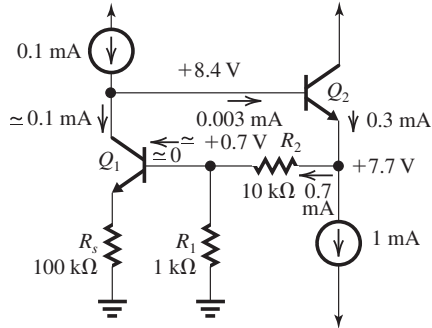


Figure 1

The dc analysis is shown in Fig. 1, from which we see that

$$I_{E1} \approx 0.1 \text{ mA}$$

$$I_{E2} \approx 0.3 \text{ mA}$$

$$V_{E2} = +7.7 \text{ V}$$

(c) Setting  $V_s = 0$  and eliminating dc sources, the feedback amplifier circuit simplifies to that shown in Fig. 2.

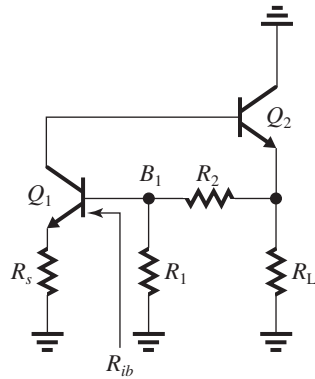


Figure 2

Now, breaking the feedback loop at the base of  $Q_1$  while terminating the right-hand side of the circuit (behind the break) in the resistance  $R_{ib}$ ,

$$R_{ib} = (\beta_1 + 1)(r_{e1} + R_s)$$

results in the circuit in Fig. 3 which we can use to determine the loop gain  $A\beta$  as follows:

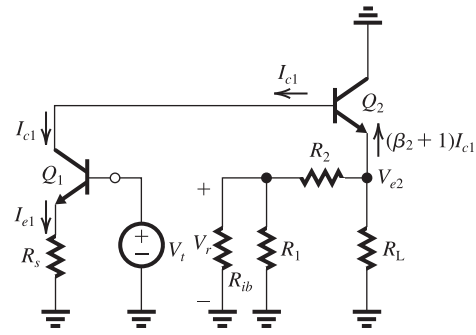


Figure 3

$$I_{e1} = \frac{V_t}{r_{e1} + R_s} \quad (1)$$

$$I_{c1} = \alpha_1 I_{e1} \quad (2)$$

$$V_{e2} = -(\beta_2 + 1)I_{c1} \{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\} \quad (3)$$

$$V_r = V_{e2} \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2} \quad (4)$$

Combining (1) to (4), we can determine  $A\beta$  as

$$A\beta \equiv -\frac{V_r}{V_t} = \alpha_1 \frac{(\beta_2 + 1) \{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\}}{r_{e1} + R_s} \times \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2}$$

Substituting

$$\alpha_1(\beta_2 + 1) = \alpha(\beta + 1) = \beta = 100$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$R_s = 100 \Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_{ib} = 101(0.25 + 0.1) = 35.35 \text{ k}\Omega$$

we obtain

$$A\beta = \frac{100 \{1 \parallel [10 + (1 \parallel 35.35)]\}}{0.25 + 0.1}$$

$$\times \frac{1 \parallel 35.35}{(1 \parallel 35.35) + 10}$$

$$= 23.2$$

$$(d) A = \frac{A\beta}{\beta} = \frac{23.2}{(1/11)} = 255.2 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{255.2}{1 + 23.2} = 10.55 \text{ V/V}$$

**10.31** (a) The feedback circuit consists of the voltage divider ( $R_F$ ,  $R_E$ ). Thus,

$$\beta = \frac{R_E}{R_E + R_F}$$

and,

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_F}{R_E}$$

Thus,

$$25 = 1 + \frac{R_F}{0.05}$$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(b) Figure 1 on next page shows the feedback amplifier circuit prepared for determining the loop gain  $A\beta$ . Observe that we have eliminated all dc sources, set  $V_s = 0$ , and broken the loop at the base of  $Q_2$ . We have terminated the broken loop in a resistance  $r_{\pi 2}$ . To determine the loop gain

$$A\beta \equiv \frac{V_r}{V_i}$$

we write the following equations:

$$I_{c2} = g_{m2}V_i \quad (1)$$

$$I_{b3} = I_{c2} \frac{R_{C2}}{R_{C2} + (\beta_3 + 1)[R_F + (R_E \parallel r_{e1})]} \quad (2)$$

$$I_{e3} = (\beta_3 + 1)I_{b3} \quad (3)$$

$$I_{e1} = I_{e3} \frac{R_E}{R_E + r_{e1}} \quad (4)$$

$$V_r = -\alpha_1 I_{e1}(R_{C1} \parallel r_{\pi 2}) \quad (5)$$

Substituting

$$\alpha_1 = 0.99$$

$$R_{C1} = 2 \text{ k}\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{100}{80} = 1.25 \text{ k}\Omega$$

$$R_E = 0.05 \text{ k}\Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega = 0.025 \text{ k}\Omega$$

$$\beta_3 = 100$$

$$R_{C2} = 1 \text{ k}\Omega$$

$$R_F = 1.2 \text{ k}\Omega$$

we obtain

$$I_{c2} = 80V_i \quad (6)$$

$$I_{b3} = I_{c2} \frac{1}{1 + 101[1.2 + (0.05 \parallel 0.025)]}$$

$$= 8.072 \times 10^{-3} I_{c2} \quad (7)$$

$$I_{e3} = 101 I_{b3} \quad (8)$$

$$I_{e1} = I_{e3} \frac{50}{50 + 25} = 0.667 I_{e3} \quad (9)$$

$$V_r = -0.99(2 \parallel 1.25) I_{e1}$$

$$V_r = -0.7615 I_{e1} \quad (10)$$

Combining (6)–(10) results in

$$A\beta = 33.13$$

$$A = \frac{A\beta}{\beta} = \frac{33.13}{1/25} = 828.2 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{828.2}{1 + 33.13} = 24.3 \text{ V/V}$$

This figure belongs to Problem 10.31, part (b).

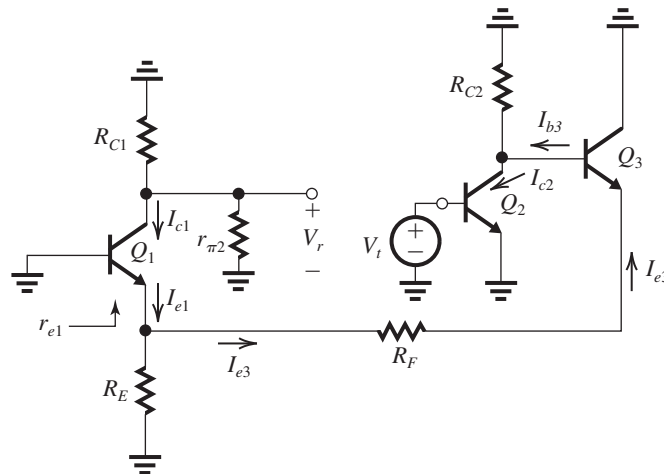


Figure 1



This figure belongs to Problem 10.33, part (b).

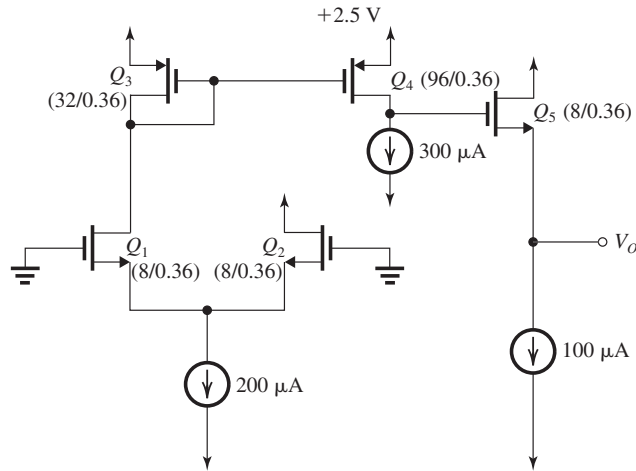


Figure 1

This figure belongs to Problem 10.33, part (d).

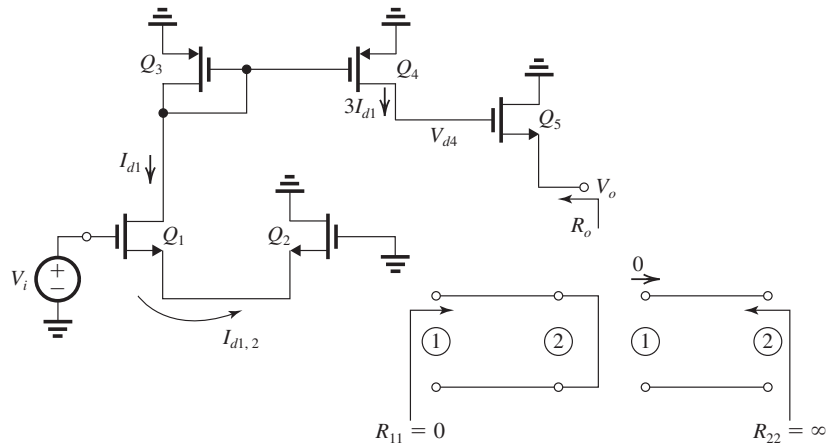


Figure 2

**10.32**  $R_i = 2 \text{ k}\Omega$

$R_o = 2 \text{ k}\Omega$

$A = 1000 \text{ V/V}$

$\beta = 0.1 \text{ V/V}$

Loop Gain  $\equiv A\beta = 1000 \times 0.1 = 100$

$1 + A\beta = 101$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{101} = 9.9 \text{ V/V}$$

$R_{if} = R_i(1 + A\beta)$

$= 2 \times 101 = 202 \text{ k}\Omega$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{2}{101} = 19.8 \Omega$$

**10.33** (a) Refer to Fig. P10.33. Assume that for some reason  $v_S$  increases. This will increase the differential input signal ( $v_S - v_O$ ) applied to the differential amplifier. The drain current of  $Q_1$  will increase, and this increase will be mirrored in the drain current of  $Q_4$ . The increase in  $i_{D4}$  will cause the voltage at the gate of  $Q_5$  to rise. Since  $Q_5$  is operating as a source follower, the voltage at its source,  $v_O$ , will follow and increase. This will cause the differential input signal ( $v_S - v_O$ ) to decrease, thus counteracting the originally assumed change. Thus, the feedback is negative.

(b) Figure 1 above shows the circuit prepared for dc analysis. We see that

$I_{D1} = I_{D2} = 100 \mu\text{A}$

$I_{D3} = 100 \mu\text{A}$

$$I_{D4} = 300 \mu\text{A}$$

$$I_{D5} = 100 \mu\text{A}$$

For  $Q_1$  and  $Q_2$ , use

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \frac{8}{0.36} V_{OV1,2}^2$$

$$\Rightarrow V_{OV1,2} = 0.15 \text{ V}$$

For  $Q_3$ , use

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV3}|^2$$

$$100 = \frac{1}{2} \times 100 \times \frac{32}{0.36} |V_{OV3}|^2$$

$$\Rightarrow |V_{OV3}| = 0.15 \text{ V}$$

Since  $V_{SG4} = V_{SG3}$ , we have

$$|V_{OV4}| = |V_{OV3}| = 0.15 \text{ V}$$

Finally for  $Q_5$ , use

$$100 = \frac{1}{2} \times 400 \times \frac{8}{0.36} \times V_{OV5}^2$$

$$\Rightarrow V_{OV5} = 0.15 \text{ V}$$

If perfect matching pertains, then

$$V_{D4} = V_{D3} = V_{DD} - V_{SG3}$$

$$= 1 - |V_t| - |V_{OV3}|$$

$$= 1 - 0.35 - 0.15 = 0.5 \text{ V}$$

$$V_O = V_{D4} - V_{GS5}$$

$$= V_{D4} - V_t - V_{OV5}$$

$$= 0.5 - 0.35 - 0.15 = 0 \text{ V}$$

which is as stated in the Problem statement.

$$(c) \quad g_{m1} = g_{m2} = g_{m3} = \frac{2I_D}{|V_{OV}|}$$

$$= \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$g_{m4} = \frac{2 \times 0.3}{0.15} = 4 \text{ mA/V}$$

$$g_{m5} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = \frac{|V_A|}{I_D} = \frac{|V'_A| \times L}{I_D}$$

$$= \frac{10 \times 0.36}{0.1} = 36 \text{ k}\Omega$$

$$r_{o4} = \frac{3.6}{0.3} = 12 \text{ k}\Omega$$

$$r_{o5} = \frac{3.6}{0.1} = 36 \text{ k}\Omega$$

(d) Figure 2 on the previous page shows the  $A$  circuit. Observe that since the  $\beta$  network is simply a wire connecting the output node to the gate of  $Q_2$ , we have  $R_{11} = 0$  and  $R_{22} = \infty$ . To determine  $A$ , we write

$$I_{d1,2} = \frac{V_i}{2/g_{m1,2}} = \frac{1}{2} g_{m1,2} V_i$$

Since  $\left(\frac{W}{L}\right)_4 = 3\left(\frac{W}{L}\right)_3$ , the drain current of  $Q_4$  will be

$$I_{d4} = 3I_{d1} = \frac{3}{2} g_{m1,2} V_i$$

The voltage at the drain of  $Q_4$  will be

$$V_{d4} = I_{d4} r_{o4}$$

$$= \frac{3}{2} g_{m1,2} r_{o4} V_i$$

Finally,  $V_o$  is related to  $V_{d4}$  as

$$\frac{V_o}{V_{d4}} = \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = \frac{3}{2} g_{m1,2} r_{o4} \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Substituting numerical values, we obtain

$$A = \frac{3}{2} \times 1.33 \times 12 \times \frac{36}{36 + 0.25}$$

$$= 23 \text{ V/V}$$

The output resistance  $R_o$  is

$$R_o = r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= 36 \parallel 0.25 = 0.24 \text{ k}\Omega$$

$$= 240 \Omega$$

$$(e) \quad A_f = \frac{A}{1 + A\beta}$$

$$= \frac{23}{1 + 23} = 0.958 \text{ V/V}$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{240}{1 + 23} = 10 \Omega$$

$$R_{out} = R_{of} = 10 \Omega$$

(f) To obtain a closed-loop gain of 5 V/V, we connect a voltage divider in the feedback loop, as shown in Fig. 3 (next page).

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

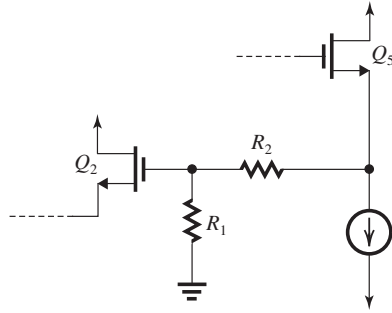


Figure 3

$$5 = \frac{23}{1 + 23\beta}$$

$$\Rightarrow \beta = 0.1565$$

Selecting  $R_1 = 1 \text{ M}\Omega$ , we obtain

$$0.1565 = \frac{1}{1 + R_2}$$

$$\Rightarrow R_2 = 5.39 \text{ M}\Omega$$

Note that by selecting large values for  $R_1$  and  $R_2$ , we have ensured that their loading effect on the  $A$  circuit would be negligible.

**10.34** Refer to the solution to Problem 10.30.

$$(a) \beta = \frac{R_1}{R_1 + R_2}$$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b) From the solution to Problem 10.30, we have

$$I_{E1} \simeq 0.1 \text{ mA}$$

$$I_{E2} \simeq 0.3 \text{ mA}$$

$$V_{E2} = +7.7 \text{ V}$$

(c) The  $A$  circuit is shown in Fig. 1.

$$I_{e1} = \frac{V_i}{R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}}$$

$$V_o = I_{e2} [R_L \parallel (R_1 + R_2)]$$

$$= (\beta_2 + 1)\alpha_1 V_i \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Since  $\beta_1 = \beta_2 = \beta$  and  $\alpha = \frac{\beta}{\beta + 1}$ , we have

$$A \equiv \frac{V_o}{V_i} = \beta \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Substituting  $\beta = 100$ ,  $R_L = 1 \text{ k}\Omega$ ,  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $R_s = 0.1 \text{ k}\Omega$ , and  $r_{e1} = 0.25 \text{ k}\Omega$  gives

$$A = 100 \frac{1 \parallel 11}{0.1 + 0.25 + \frac{1 \parallel 10}{101}}$$

$$= 255.3 \text{ V/V}$$

$$R_i = R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}$$

$$= 0.1 + 0.25 + \frac{1 \parallel 10}{101} = 0.359 \text{ k}\Omega$$

$$R_o = R_L \parallel (R_1 + R_2)$$

$$= 1 \parallel 11 = 0.917 \text{ k}\Omega$$

$$(d) \beta = \frac{R_1}{R_1 + R_2}$$

$$= \frac{1}{1 + 10} = \frac{1}{11}$$

$$(e) \frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$1 + A\beta = 1 + \frac{255.3}{11} = 24.21$$

$$A_f = \frac{255.3}{24.21} = 10.55 \text{ V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 0.359 \times 24.21 = 8.69 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s$$

$$= 8.69 - 0.1 = 8.59 \text{ k}\Omega$$

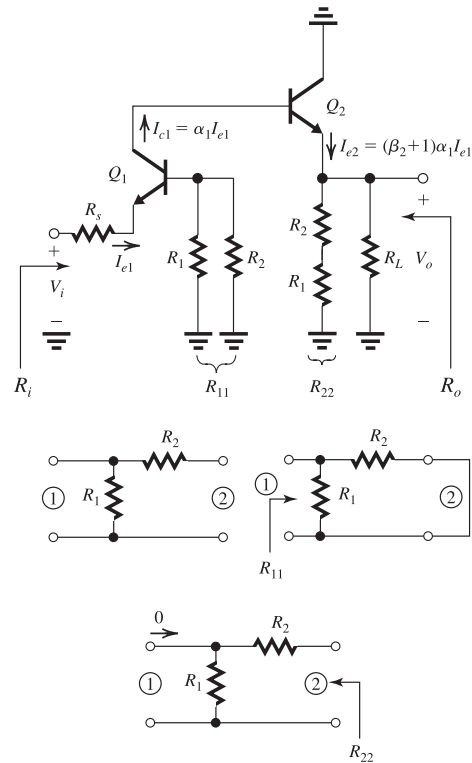


Figure 1

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.917 \text{ k}\Omega}{24.21} = 37.9 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$37.9 = R_{out} \parallel 1000$$

$$\Rightarrow R_{out} = 39.4 \text{ }\Omega$$

The value of  $A_f$  (10.5 V/V) is 0.45 less than the ideal value of 11, which is about 4%.

**10.35** (a) Refer to Fig. P10.35. Let  $V_s$  increase by a small increment. Since  $Q_1$  is operating in effect as a CS amplifier, a negative incremental voltage will appear at its drain. Transistor  $Q_2$  is also operating as a CS amplifier; thus a positive incremental voltage will appear at its drain. Transistor  $Q_3$  is operating as a source follower; thus the signal at its source (which is the output voltage) will follow that at its gate and thus will be positive. The end result is that we are feeding back through the voltage divider ( $R_2, R_1$ ) a positive incremental signal that will appear across  $R_1$  and thus at the source of the  $Q_1$ . This signal, being of the same polarity as the originally assumed change in the signal at the gate of  $Q_1$  ( $V_s$ ), will *subtract* from the original change, causing a *smaller* signal to appear across the gate-source terminals of  $Q_1$ . Hence, the feedback is negative.

(b) Setting the open-loop gain to  $\infty$ , the voltage  $V_{gs1}$  reduces to zero, which in turn causes the current in the source of  $Q_1$  to be zero. Thus the feedback voltage across  $R_1$  is simply determined by the voltage divider ratio of ( $R_1, R_2$ ), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$\beta = \frac{2}{2 + 18} = 0.1 \text{ V/V}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$A_f|_{\text{ideal}} = 1 + \frac{18}{2} = 10 \text{ V/V}$$

(c)  $V_{G1} = 0.9 \text{ V}$

$$V_{S1} = V_{G1} - V_{GS1}$$

$$= V_{G1} - V_{r1} - V_{OV1}$$

$$= 0.9 - 0.5 - 0.2 = 0.2 \text{ V}$$

$$V_{G2} = V_{DD} - V_{SG2}$$

$$= V_{DD} - |V_{r2}| - |V_{OV2}|$$

$$= 1.80 - 0.5 - 0.2 = 1.1 \text{ V}$$

Thus, current source  $I_1$  will have 0.7-V drop across it, more than sufficient for its proper operation. Since  $V_{S1} = 0.2 \text{ V}$  the dc current through  $R_1$  will be

$$I_{R1} = \frac{V_{S1}}{R_1} = \frac{0.2 \text{ V}}{2 \text{ k}\Omega} = 0.1 \text{ mA}$$

Now, a node equation at  $S_1$  reveals that because  $I_{D1} = 0.1 \text{ mA}$  and  $I_{R1} = 0.1 \text{ mA}$ , the dc current in  $R_2$  will be zero. Thus, it will have a zero voltage drop across it and

$$V_{S3} = V_{S1} = 0.2 \text{ V}$$

Thus, current source  $I_3$  will have across it the minimum voltage required to keep it operating properly. Finally,

$$\begin{aligned} V_{G3} &= V_{S3} + V_{GS3} \\ &= V_{S3} + V_{r3} + V_{OV3} \\ &= 0.2 + 0.5 + 0.2 = 0.9 \text{ V} \end{aligned}$$

Thus, current source  $I_2$  will have across it a voltage more than sufficient to keep it operating properly.

(d)

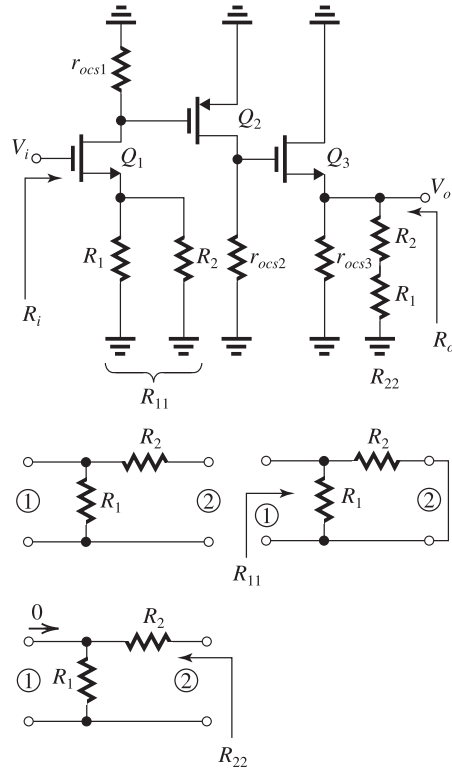


Figure 1

Figure 1 shows the  $A$  circuit as well as the  $\beta$  circuit and how the loading-effect resistances  $R_{11}$  and  $R_{22}$  are determined.

To determine  $A$ , let's first determine the small-signal parameters of all transistors as well as  $r_o$  of each of the three current sources.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I_{D1}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs1} = \frac{|V_A|}{I_1} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs2} = \frac{|V_A|}{I_2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m3} = \frac{2I_{D3}}{V_{OV3}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3} = \frac{|V_A|}{I_{D3}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs3} = \frac{|V_A|}{I_3} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Refer to the  $A$  circuit.

Transistor  $Q_1$  is a CS amplifier with a resistance  $R_{11}$  in its source:

$$R_s = R_{11} = R_1 \parallel R_2 = 2 \parallel 18 = 1.8 \text{ k}\Omega$$

Transistor  $Q_1$  will have an effective transconductance:

$$G_{m1} = \frac{g_{m1}}{1 + g_{m1}R_s} = \frac{1}{1 + 1 \times 1.8} = 0.36 \text{ mA/V}$$

The output resistance of  $Q_1$  will be

$$\begin{aligned} R_{o1} &= (1 + g_{m1}R_s)r_{o1} \\ &= (1 + 1 \times 1.8) \times 100 = 280 \text{ k}\Omega \end{aligned}$$

The total resistance at the drain of  $Q_1$  is

$$\begin{aligned} R_{d1} &= r_{ocs1} \parallel R_{o1} \\ &= 100 \parallel 280 = 73.7 \text{ k}\Omega \end{aligned}$$

Thus, the voltage gain of the first stage is

$$\begin{aligned} A_1 &= -G_{m1}R_{d1} \\ &= -0.36 \times 73.7 = -26.5 \text{ V/V} \end{aligned}$$

The gain of the second stage is

$$\begin{aligned} A_2 &= -g_{m2}(r_{ocs2} \parallel r_{o2}) \\ &= -1(100 \parallel 100) = -50 \text{ V/V} \end{aligned}$$

To determine the gain of the third stage, we first determine the total resistance between the source of  $Q_3$  and ground:

$$R_{s3} = r_{ocs3} \parallel r_{o3} \parallel (R_1 + R_2)$$

$$\begin{aligned} R_{s3} &= 100 \parallel 100 \parallel 20 \\ &= 14.3 \text{ k}\Omega \end{aligned}$$

Thus,

$$\begin{aligned} A_3 &= \frac{R_{s3}}{R_{s3} + \frac{1}{g_{m3}}} \\ &= \frac{14.3}{14.3 + \frac{1}{1}} = 0.935 \text{ V/V} \end{aligned}$$

The overall voltage gain  $A$  can now be found as

$$\begin{aligned} A &= A_1 A_2 A_3 \\ &= -26.5 \times -50 \times 0.935 = 1240 \text{ V/V} \end{aligned}$$

(e) We already found  $\beta$  in (b) as

$$\beta = 0.1 \text{ V/V}$$

(f)  $1 + A\beta = 1 + 1240 \times 0.1 = 125$

$$A_f = \frac{A}{1 + A\beta} = \frac{1240}{125} = 9.92 \text{ V/V}$$

which is lower by 0.08 or 0.8% than the ideal value obtained in (b).

$$(g) R_{of} = \frac{R_o}{1 + A\beta}$$

To obtain  $R_o$  refer to the output part of the  $A$  circuit.

$$\begin{aligned} R_o &= (R_1 + R_2) \parallel r_{ocs3} \parallel r_{o3} \parallel \frac{1}{g_{m3}} \\ &= 20 \parallel 100 \parallel 100 \parallel 1 \\ &= 935 \Omega \\ R_{of} &= \frac{935}{125} = 7.5 \Omega \end{aligned}$$

Note: This problem, though long, is extremely valuable as it exercises the student's knowledge in many aspects of amplifier design.

**10.36** (a) Refer to Fig. P10.26. Let  $V_s$  increase by a positive increment. This will cause the drain current of  $Q_1$  to increase. The increase in  $I_{d1}$  will be fed to the  $Q_3 - Q_4$  mirror, which will provide a corresponding increase in the drain current of  $Q_4$ . The latter current will cause the voltage at the output node to rise. A fraction of the increase in  $V_o$  is applied through the divider ( $R_1, R_2$ ) to the gate of  $Q_2$ . The increase in the voltage of the gate of  $Q_2$  will *subtract* from the initially assumed increase of the voltage of the gate of  $Q_1$ , resulting in a smaller increase in the differential voltage applied to the ( $Q_1, Q_2$ ) pair. Thus, the feedback counteracts the originally assumed change, verifying that it is negative.

This figure belongs to Problem 10.36, part (c).

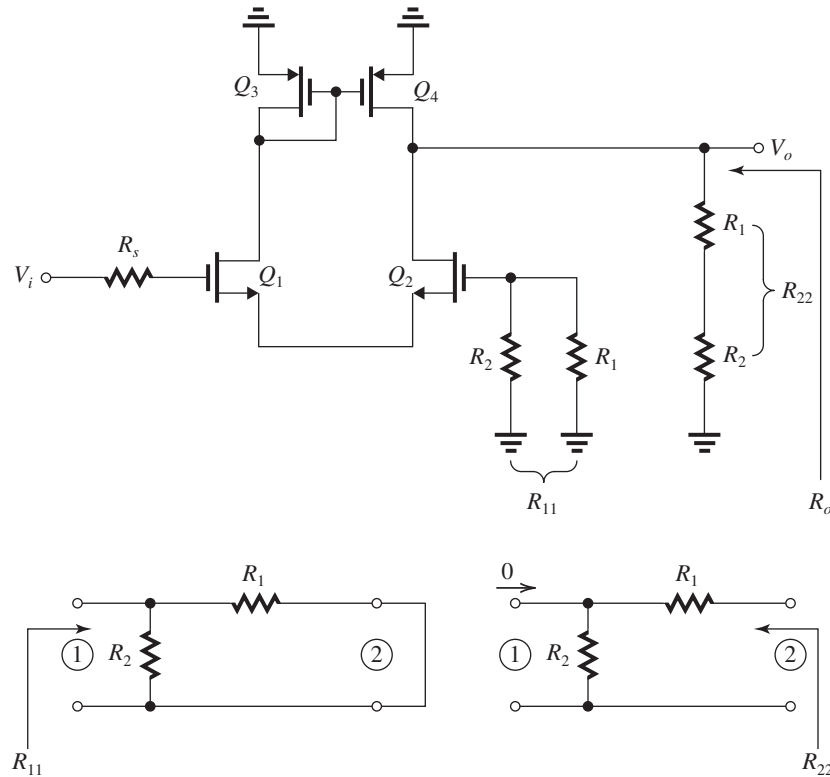


Figure 1

(b) The negative feedback will cause the dc voltage at the gate of  $Q_2$  to be approximately equal to the dc voltage at the gate of  $Q_1$ , that is, zero. Now, with  $V_{G2} \approx 0$ , the dc current in  $R_2$  will be zero and similarly the dc current in  $R_1$  will be zero, resulting in  $V_o = 0$  V dc.

(c) Figure 1 shows the  $A$  circuit. It also shows how the loading effect of the  $\beta$  network on the  $A$  circuit, namely  $R_{11}$  and  $R_{22}$ , are found. The gain of the  $A$  circuit can be written by inspection as

$$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22})$$

where

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D3,4}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$R_{22} = R_1 + R_2 = 1 \text{ M}\Omega$$

$$A = 1(100 \parallel 100 \parallel 1000) = 47.62 \text{ V/V}$$

This is identical to the value found in the solution to Problem 10.26.

$$(d) \frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{47.62}{1 + 47.62\beta}$$

$$\Rightarrow \beta = 0.179$$

Thus,

$$\frac{R_2}{R_1 + R_2} = 0.179$$

$$R_2 = 0.179 \text{ M}\Omega = 179 \text{ k}\Omega$$

$$R_1 = 1000 - 179 = 821 \text{ k}\Omega$$

Again, these values are identical to those found in Problem 10.26.

(e) Refer to Fig. 1 above.

$$R_o = R_{22} \parallel r_{o2} \parallel r_{o4}$$

$$= 1000 \parallel 100 \parallel 100 = 47.62 \text{ k}\Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{47.62}{1 + 47.62 \times 0.179}$$

$$= 5 \text{ k}\Omega$$

This value cannot be found using the loop-gain analysis method of Problem 10.26.

(f) With  $R_L = 10\text{ k}\Omega$ ,

$$\frac{V_o}{V_s} = 5 \times \frac{R_L}{R_L + R_{\text{out}}}$$

$$5 \times \frac{10}{10 + 5} = 3.33\text{ V/V}$$

(g) As an alternative to (f), we shall redo the analysis of the A circuit in (c) above with  $R_L = 10\text{ k}\Omega$  included:

$$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22} \parallel R_L)$$

$$= 1(100 \parallel 100 \parallel 1000 \parallel 10)$$

$$= 8.26\text{ V/V}$$

Using  $\beta = 0.179$ , we obtain

$$A_f = \frac{8.26}{1 + 8.26 \times 0.179} = 3.33\text{ V/V}$$

which is identical to the value found in (f) above.

**10.37** Since the output voltage is sampled, the resistance-with-feedback is lower. The reduction is by the factor  $(1 + A\beta)$ , thus

$$1 + A\beta = 200$$

$$A\beta = 199$$

$$R_{of} = \frac{R_o}{200}$$

$$\Rightarrow R_o = 200 \times 100 = 20,000\ \Omega$$

$$= 20\text{ k}\Omega$$

**10.38** 
$$A = \frac{A_0}{1 + \frac{s}{\omega_H}}$$

$$1 + A\beta = 1 + \frac{A_0\beta}{1 + \frac{s}{\omega_H}}$$

$$Z_{if} = R_i(1 + A\beta)$$

$$= R_i + \frac{A_0\beta R_i}{1 + \frac{s}{\omega_H}}$$

Thus,  $Z_{if}$  consists of a resistance  $R_i$  in series with an admittance  $Y$ ,

$$Y = \frac{1}{A_0\beta R_i} + \frac{s}{A_0\beta R_i\omega_H}$$

which is a resistance  $(A_0\beta R_i)$  in parallel with a capacitance  $1/A_0\beta R_i\omega_H$ . The equivalent circuit is shown in Fig. 1.

$$Z_{of} = \frac{R_o}{1 + A\beta}$$

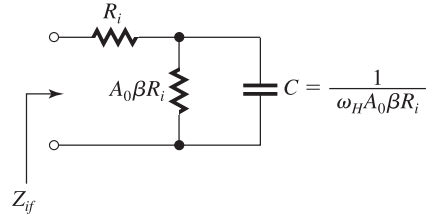


Figure 1

$$= \frac{R_o}{1 + \frac{A_0\beta}{1 + \frac{s}{\omega_H}}}$$

Thus, the output admittance  $Y_{of}$  is

$$Y_{of} \equiv \frac{1}{Z_{of}} = \frac{1}{R_o} + \frac{A_0\beta}{R_o \left(1 + \frac{s}{\omega_H}\right)}$$

which consists of a resistance  $R_o$  in parallel with an impedance  $Z$  given by

$$Z = \frac{R_o}{A_0\beta} + s \frac{R_o}{A_0\beta\omega_H}$$

which consists of a resistance  $(R_o/A_0\beta)$  in series with an inductance  $L = R_o/A_0\beta\omega_H$ . The equivalent circuit of  $Z_{of}$  is shown in Fig. 2.

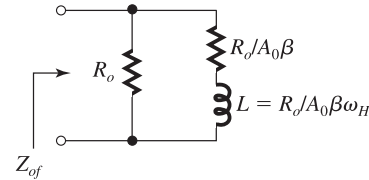


Figure 2

**10.39**  $A = 1000\text{ V/V}$

$$R_i = 1\text{ k}\Omega$$

$$R_{if} = 10\text{ k}\Omega$$

Thus, the connection at the input is a series one, and

$$1 + A\beta = \frac{10}{1} = 10$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{10} = 100\text{ V/V}$$

To implement a unity-gain voltage follower, we use  $\beta = 1$ . Thus the amount of feedback is

$$1 + A\beta = 1 + 1000 = 1001$$

and the input resistance becomes

$$R_{if} = (1 + A\beta)R_i$$

$$= 1001 \times 1 = 1001\text{ k}\Omega = 1.001\text{ M}\Omega$$

**10.40** (a)  $\beta = 1$

$$A_f|_{\text{ideal}} = 1 \text{ V/V}$$

(b) Substituting  $R_1 = \infty$  and  $R_2 = 0$  in the expression for  $A$  in Example 11.5, we obtain

$$A = \mu \frac{R_L}{R_L + r_o} \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = A \times 1 = A$$

$$(c) A = 10^4 \times \frac{2}{2+1} \times \frac{100}{100+10}$$

$$= 6060.6 \text{ V/V}$$

$$A\beta = 6060.6$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{6060.6}{1 + 6060.6} = 0.9998 \text{ V/V}$$

From Example 11.5 with  $R_1 = \infty$  and  $R_2 = 0$ , we have

$$R_i = R_s + R_{id} = 10 + 100 = 110 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 110 \times 6061.6 = 667 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s \simeq 667 \text{ M}\Omega$$

$$R_o = r_o \parallel R_L = 1 \parallel 2 = 0.67 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.67 \text{ k}\Omega}{6061.6} = 0.11 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$R_{out} \simeq 0.11 \text{ }\Omega$$

**10.41** (a) Figure 1 on the next page shows the  $A$  circuit and the circuit for determining  $\beta$  as well as the determination of the loading effects of the  $\beta$  circuit.

(b) If  $A\beta$  is large, then

$$A_f \equiv \frac{V_o}{V_s} \simeq \frac{1}{\beta}$$

Since

$$\beta = \frac{R_E}{R_F + R_E}$$

we have

$$A_f = \frac{R_F + R_E}{R_E} \quad \text{Q.E.D.}$$

$$(c) 25 = 1 + \frac{R_F}{50 \text{ }\Omega}$$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(d) Refer to the  $A$  circuit in Fig. 1. The voltage gain of  $Q_1$  is given by

$$\frac{V_{c1}}{V_i} = -\alpha_1 \frac{R_{C1} \parallel r_{\pi 2}}{r_{e1} + R_{11}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \text{ }\Omega$$

$$R_{11} = R_E \parallel R_F = 50 \text{ }\Omega \parallel 1200 \text{ }\Omega = 48 \text{ }\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} \simeq \frac{I_{E2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ mA}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{80} = \frac{100}{80} = 1.5 \text{ k}\Omega$$

$$\alpha_1 = 0.99 \simeq 1$$

$$\frac{V_{c1}}{V_i} = -10 = -\frac{R_{C1} \parallel 1.5}{0.025 + 0.048}$$

$$\Rightarrow R_{C1} = 1.42 \text{ k}\Omega$$

Next consider the second stage composed of the CE transistor  $Q_2$ . The load resistance of the second stage is composed of  $R_{C2}$  in parallel with the input resistance of emitter-follower  $Q_3$ . The latter resistance is given by

$$R_{i3} = (\beta_3 + 1)(r_{e3} + R_{22})$$

where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \text{ }\Omega$$

$$R_{22} = R_F + R_E = 1.2 + 0.05 = 1.25 \text{ k}\Omega$$

Thus,

$$R_{i3} = 101 \times 1.25 = 126.3 \text{ k}\Omega$$

$$A_2 \equiv \frac{V_{c2}}{V_{b2}} = -g_{m2}(R_{C2} \parallel R_{i3})$$

$$-50 = -80(R_{C2} \parallel 126.3)$$

$$\Rightarrow R_{C2} = 628 \text{ }\Omega$$

(e)  $A = A_1 A_2 A_3$

where

$$A_3 = \frac{R_{22}}{R_{22} + r_{e3}} = \frac{1.25}{1.25 + 0.005} = 0.996 \text{ V/V}$$

$$A \equiv -10 \times -50 \times 0.996$$

$$= 498 \text{ V/V}$$

$$A_f \equiv \frac{V_o}{V_s} = \frac{498}{1 + 498 \times \frac{50}{1250}}$$

$$= 23.8 \text{ V/V}$$

(f) Refer to the  $A$  circuit in Fig. 1.

$$R_i = (\beta_1 + 1)(r_{e1} + R_{11})$$

$$R_i = 101(0.025 + 0.048)$$

$$= 7.37 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$



This figure belongs to Problem 10.41, part (a).

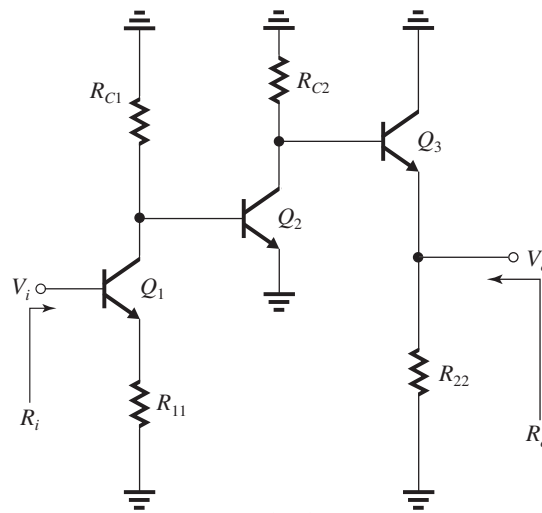
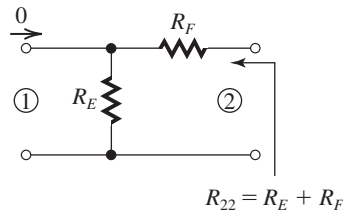
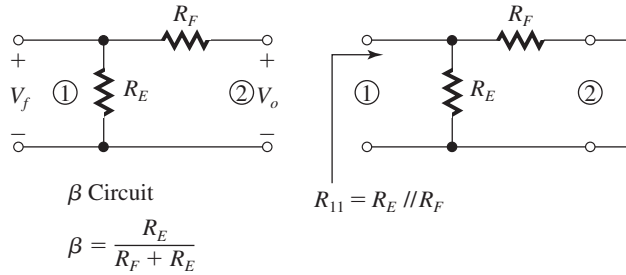


Figure 1

where

$$1 + A\beta = 1 + \frac{498}{25} = 20.92$$

$$R_{if} = 7.37 \times 20.92 = 154 \text{ k}\Omega$$

$$R_o = R_{22} \parallel \left[ r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right]$$

$$= 1.25 \parallel \left[ 0.005 + \frac{0.628}{101} \right]$$

$$= 11.1 \text{ }\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{11.1}{20.92} = 0.53 \text{ }\Omega$$

**10.42** (a) Refer to Fig. P10.42. If  $V_s$  increases, the output of  $A_1$  will decrease and this will cause the output of  $A_2$  to increase. This, in turn, causes the output of  $A_3$ , which is  $V_o$ , to increase. A portion of the positive increment in  $V_o$  is fed back to the positive input terminal of  $A_1$  through the voltage divider ( $R_2, R_1$ ). The increased voltage at the positive input terminal of  $A_1$  counteracts the originally assumed increase at the negative input terminal, verifying that the feedback is negative.

(b) Setting the open-loop gain to  $\infty$  results in  $V_1 = 0$  and thus the input current of the amplifier is zero. This isolates the feedback circuit which is the voltage divider ( $R_1, R_2$ ). Thus,

$$\beta = \frac{R_1}{R_1 + R_2}$$

This figure belongs to Problem 10.42, part (c).

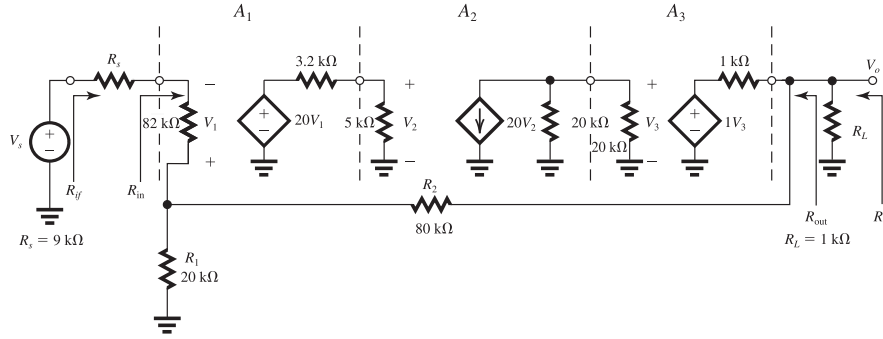


Figure 1

This figure belongs to Problem 10.42, part (d).

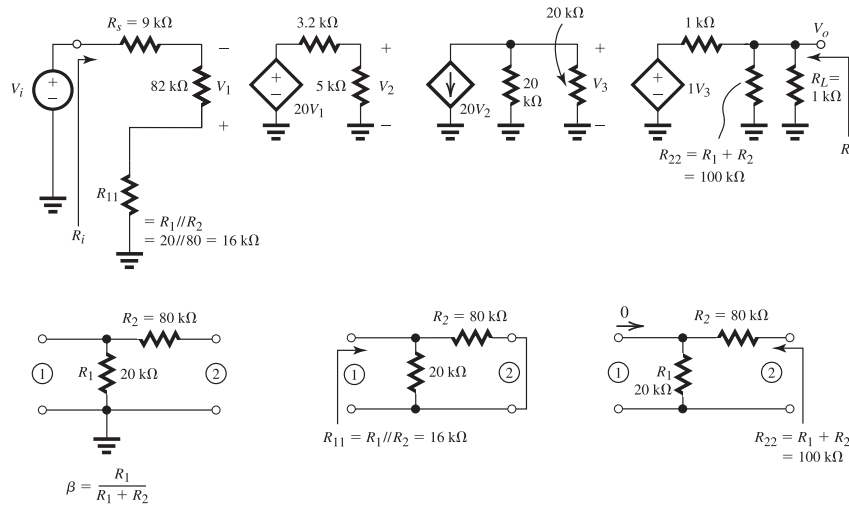


Figure 2

and

$$A_f|_{\text{ideal}} = \frac{1}{\beta}$$

Thus, to obtain an ideal closed-loop gain of 5 V/V we need  $\beta = 0.2$ :

$$0.2 = \frac{20}{20 + R_2}$$

$$\Rightarrow R_2 = 80 \text{ k}\Omega$$

(c) Figure 1 shows the small-signal equivalent circuit of the feedback amplifier.

(d) Figure 2 above shows the  $A$  circuit and the  $\beta$  circuit together with the determination of its loading effects,  $R_{i1}$ , and  $R_{22}$ . We can write

$$\frac{V_1}{V_i} = -\frac{82}{82 + 9 + 16} = -0.766 \text{ V/V}$$

$$V_2 = 20V_1 \times \frac{5}{3.2 + 5} = 12.195V_1$$

$$V_3 = -20V_2(20 \parallel 20) = -200V_2$$

$$V_o = V_3 \frac{1 \parallel 100}{(1 \parallel 100) + 1} = 0.497V_3$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.497 \times -200 \times 12.195 \times -0.766$$

$$= 928.5 \text{ V/V}$$

$$(e) \beta = \frac{20}{20 + 80} = 0.2 \text{ V/V}$$

$$1 + A\beta = 1 + 928.5 \times 0.2 = 186.7$$

$$(f) A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{928.5}{186.7} = 4.97 \text{ V/V}$$

which is nearly equal to the ideal value of 5 V/V.

(g) From the  $A$  circuit,

$$R_i = 9 + 82 + 16 = 107 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 107 \times 186.7 = 19.98 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s \simeq 19.98 \text{ M}\Omega$$

(h) From the  $A$  circuit,

$$R_o = R_L \parallel R_{22} \parallel 1 \text{ k}\Omega$$

$$= 1 \parallel 100 \parallel 1 = 497.5 \text{ }\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{497.5}{186.7} = 2.66 \text{ }\Omega$$

$$R_{out} \parallel R_L = R_{of}$$

$$R_{out} \parallel 1000 = 2.66 \text{ }\Omega$$

$$R_{out} \simeq 2.66 \text{ }\Omega$$

$$(i) f_{Hf} = f_H(1 + A\beta)$$

$$= 100 \times 186.7$$

$$= 18.67 \text{ kHz}$$

(j) If  $A_1$  drops to half its nominal value,  $A$  will drop to half its nominal value:

$$A = \frac{1}{2} \times 928.5 = 464.25$$

and  $A_f$  becomes

$$A_f = \frac{464.25}{1 + 464.25 \times 0.2} = 4.947 \text{ V/V}$$

Thus, the percentage change in  $A_f$  is

$$= \frac{4.947 - 4.97}{4.97} = -0.47\%$$

**10.43** Recalling that:

- shunt connection lowers resistance, and
- series connection raises resistance, then,

(a) Shunt-Series

(b) Series-Series

(c) Shunt-Shunt

$$\mathbf{10.44} \quad A_f|_{\text{ideal}} = 10 \text{ mA/V}$$

$$\beta = \frac{1}{A_f|_{\text{ideal}}} = 0.1 \text{ V/mA}$$

$$A = 1 \text{ A/V} = 1000 \text{ mA/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 1000 \times 0.1} = \frac{1000}{1 + 100}$$

$$= 9.9 \text{ mA/V}$$

$$R_{if} = (1 + A\beta)R_i$$

$$= 101 \times 10 \text{ k}\Omega = 1.01 \text{ M}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{100 \text{ }\Omega}{101} = 0.99 \text{ }\Omega$$

**10.45** Refer to Fig. P10.45(a).

(a) Setting  $A = \infty$  results in the voltage between the two input terminals of the amplifier  $\mu$  being zero. Thus the current into these two terminals also become zero. As a result, the voltage across  $R_F$  equals  $V_s$  and the current through  $R_F$ ; namely  $V_s/R_F$  flows into the source terminal of  $Q$ . Thus

$$I_o = \frac{V_s}{R_F}$$

$$\Rightarrow A_f|_{\text{ideal}} \equiv \frac{I_o}{V_s} = \frac{1}{R_F}$$

(b) For

$$A_f|_{\text{ideal}} = 10 \text{ mA/V},$$

$$R_F = \frac{1}{A_f|_{\text{ideal}}} = \frac{1}{10} = 0.1 \text{ V/mA}$$

That is,

$$R_F = 0.1 \text{ k}\Omega = 100 \text{ }\Omega$$

Note that

$$\beta = R_F = 100 \text{ }\Omega$$

(c) Refer to Fig. P10.45(b). The voltage at the gate of  $Q$  will be

$$V_g = -\mu V_t$$

The current in the source of  $Q$  will be

$$I_d$$

$$= \frac{V_g}{\frac{1}{g_m} + R_F}$$

$$= \frac{-\mu V_t}{\frac{1}{g_m} + R_F}$$

The returned voltage  $V_r$  can now be found as

$$V_r = I_d R_F = \frac{-\mu R_F}{\frac{1}{g_m} + R_F} V_t$$

and the loop gain is

$$A\beta \equiv -\frac{V_r}{V_t} = \frac{\mu R_F}{\frac{1}{g_m} + R_F}$$

(d)

$$A\beta = \frac{1000 \times 0.1}{0.5 + 0.1} = 166.7$$

$$A = \frac{A\beta}{\beta} = \frac{166.7}{0.1 \text{ k}\Omega} = 1667 \text{ mA/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1667}{1 + 166.7}$$

$$= 9.94 \text{ mA/V}$$

which is very close (−0.6%) to the ideal value of 10 mA/V.

**10.46** (a) Refer to Fig. P10.46. With  $A$  set to  $\infty$ ,  $I_i = 0$  and  $V_i = 0$ . Thus,  $I_s$  flows through  $R_F$  and establishes a voltage of  $-I_s R_F$  at node  $X$ . Writing a node equation at  $X$  provides

$$I_o = -I_s + \frac{V_x}{R_M} = -I_s - \frac{I_s R_F}{R_M}$$

Thus, the ideal  $A_f$  is

$$A_f|_{\text{ideal}} \equiv \frac{I_o}{I_s} = -\left(1 + \frac{R_F}{R_M}\right) \quad \text{Q.E.D.}$$

For  $R_F = 1 \text{ k}\Omega$ ,

$$-5 = -\left(1 + \frac{1}{R_M}\right)$$

$$\Rightarrow R_M = 0.25 \text{ k}\Omega$$

(b)

$$\beta = \frac{1}{A_f|_{\text{ideal}}} = -\frac{1}{1 + \frac{R_F}{R_M}}$$

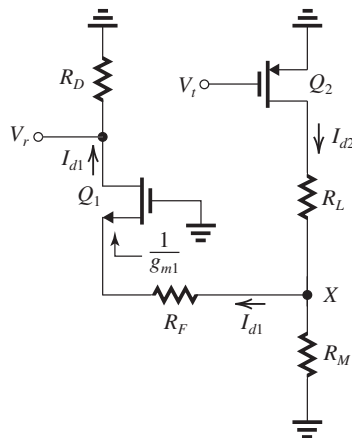


Figure 1

$$= -\frac{R_M}{R_M + R_F}$$

(c) Figure 1 shows the circuit for determining the loop gain. We see that the returned voltage  $V_r$  is given by

$$V_r = I_{d1} R_D \quad (1)$$

The current  $I_{d1}$  can be found by applying the current-divider rule at node  $X$ ,

$$I_{d1} = \frac{R_M}{R_M + R_F + \frac{1}{g_{m1}}} I_{d2} \quad (2)$$

Finally,  $I_{d2}$  can be determined as

$$I_{d2} = -g_{m2} V_t \quad (3)$$

Combining Eqns. (1)-(3) enables us to find the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t} = \frac{g_{m2} R_D R_M}{R_M + R_F + \frac{1}{g_{m1}}} \quad (4)$$

Dividing  $A\beta$  by  $\beta = -R_M/(R_M + R_F)$  gives

$$A = -\frac{g_{m2} R_D (R_M + R_F)}{R_M + R_F + \frac{1}{g_{m1}}} \quad (5)$$

which can be written in the alternate form

$$A = -\frac{g_{m2} R_D}{1 + 1/[g_{m2}(R_M + R_F)]} \quad \text{Q.E.D.} \quad (6)$$

(d)

$$A_f = \frac{A}{1 + A\beta}$$

Substituting for  $A$  from Eq. (6) and for  $\beta = -R_M/(R_M + R_F)$  we obtain

$$A_f = -\frac{g_{m2} R_D}{1 + \frac{g_{m2} R_D R_M + (1/g_{m1})}{R_M + R_F}} \quad (7)$$

(e) Substituting the given numerical values into Eq. (7) gives

$$-5 = -\frac{4 \times 10}{1 + \frac{4 \times 10 R_M + (1/4)}{R_M + 1}}$$

which gives the required value of  $R_M$  as

$$R_M = 0.205 \text{ k}\Omega = 205 \Omega$$

This value is about 25% lower than the value that results in an ideal  $A_f$  of  $-5 \text{ V/V}$ . This is because of the limited amount of loop gain available here.

$$\mathbf{10.47} \quad A \equiv \frac{V_o}{I_s} = 100 \text{ V/mA}$$

$$R_i = R_o = 1 \text{ k}\Omega$$

To reduce both the input and output resistances, we need shunt-shunt feedback. For  $R_{if} = R_{of} = 10 \Omega$ , we need

$$1 + A\beta = \frac{R_i}{R_{if}} = \frac{R_o}{R_{of}} = \frac{1000}{10} = 100$$

Thus,  $A\beta = 99$  and,

$$\beta = \frac{99}{100} = 0.99 \text{ mA/V}$$

The resulting closed-loop gain is

$$A_f = \frac{A}{1 + A\beta} = \frac{100}{100} = 1 \text{ V/mA}$$

**10.48**

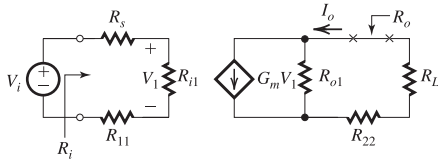


Figure 1

Figure 1 shows the *A* circuit where

$$G_m = 0.6 \text{ A/V}$$

$$R_{i1} = 10 \text{ k}\Omega$$

$$R_{o1} = 100 \text{ k}\Omega$$

$$\beta = 200 \Omega$$

$$R_{22} = 200 \Omega$$

$$R_{11} = 10 \text{ k}\Omega$$

$$R_s = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

To determine *A*,

$$A \equiv \frac{I_o}{V_i}$$

we write

$$V_1 = V_i \frac{R_{i1}}{R_{i1} + R_s + R_{11}} = V_i \frac{10}{10 + 10 + 10} = \frac{1}{3} V_i \quad (1)$$

$$I_o = G_m V_1 \frac{R_{o1}}{R_{o1} + R_L + R_{22}} = 0.6 \times \frac{100}{100 + 10 + 0.2} V_i = 0.544 V_i \quad (2)$$

Combining (1) and (2), we obtain

$$I_o = 0.544 \times \frac{1}{3} V_i = 0.1815 V_i$$

$$A = 0.1815 \text{ A/V}$$

$$A_f = \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{0.1815}{1 + 0.1815 \times 200} = \frac{0.1815}{1 + 36.3} = 4.87 \text{ mA/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$R_i$  is obtained from the *A* circuit as

$$R_i = R_s + R_{i1} + R_{11} = 10 + 10 + 10 = 30 \text{ k}\Omega$$

Thus,

$$R_{if} = 30 \times 37.3 = 1.119 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s = 1.119 - 0.010 = 1.109 \text{ M}\Omega$$

$$\simeq 1.11 \text{ M}\Omega$$

$$R_{of} = R_o(1 + A\beta)$$

where  $R_o$  is obtained from the *A* circuit as

$$R_o = R_L + R_{o1} + R_{22} = 10 + 100 + 0.2 = 110.2 \text{ k}\Omega$$

$$R_{of} = 110.2 \times 37.3 = 4.11 \text{ M}\Omega$$

$$R_{out} = R_{of} - R_L = 4.11 - 0.01 = 4.1 \text{ M}\Omega$$

**10.49** Refer to Fig. P10.45(a).

$$A_f|_{\text{ideal}} \equiv \frac{I_o}{V_s} = 10 \text{ mA/V}$$

$$\Rightarrow \beta = 1/A_f|_{\text{ideal}} = 0.1 \text{ V/mA}$$

The feedback network consists of the resistance  $R_F$ , as shown in Fig. 1. Thus,

$$\beta = R_F$$

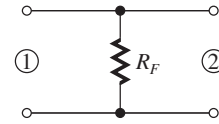


Figure 1

$$\Rightarrow R_F = 0.1 \text{ V/mA} = 0.1 \text{ k}\Omega = 100 \Omega$$

Figure 2 shows the *A* circuit where

$$R_{11} = R_{22} = R_F = 0.1 \text{ k}\Omega$$

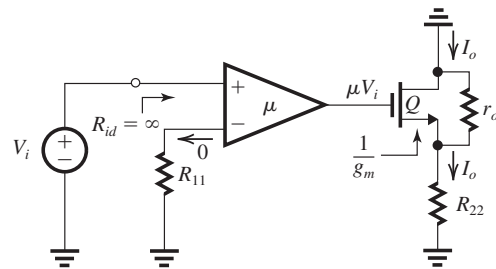


Figure 2

From Fig. 2 we find

$$I_o = \frac{\mu V_i}{\frac{1}{g_m} + (r_o \parallel R_{22})} \frac{r_o}{r_o + R_{22}}$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{\mu}{\frac{1}{g_m} + (r_o \parallel R_{22})} \frac{r_o}{r_o + R_{22}}$$

For  $g_m = 5 \text{ mA/V}$  and  $r_o = 20 \text{ k}\Omega$ ,

$$A = \frac{\mu}{0.2 + (20 \parallel 0.1)} \frac{20}{20 + 0.1}$$

$$= 3.32 \mu \text{ mA/V}$$

$$A\beta = 3.32 \mu \times 0.1 = 0.332 \mu$$

$$1 + A\beta = 1 + 0.332 \mu \quad (1)$$

From Fig. 2,

$$R_o = r_o + R_{22} + g_m r_o R_{22}$$

$$= 20 + 0.1 + 5 \times 20 \times 0.1$$

$$= 30.1 \text{ k}\Omega$$

For  $R_{of} = 5 \text{ M}\Omega$ , we required

$$1 + A\beta = \frac{R_{of}}{R_o} = \frac{5000}{30.1} = 166.1$$

Using Eq. (1) we find

$$1 + 0.332 \mu = 166.1$$

$$\Rightarrow \mu = 497 \text{ V/V}$$

$$A = 3.32 \times \mu = 1650 \text{ mA/V}$$

The actual value of  $A_f$  is

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1650}{1 + 1650 \times 0.1} = \frac{1650}{1 + 165}$$

$$= 9.94 \text{ mA/V}$$

which is very close to the ideal value of  $10 \text{ mA/V}$ .

**10.50** (a) Refer to Fig. P10.50. With  $A$  set to  $\infty$ , the voltage between the two input terminals of amplifier  $A_1$  becomes zero and the current into the terminals also becomes zero. Thus the voltage across  $R_F$  becomes equal to  $V_s$  and the current through  $R_F$ , i.e.,  $V_s/R_F$  flows through  $R_L$ , thus

$$I_o = \frac{V_s}{R_F}$$

which means that

$$A_f|_{\text{ideal}} \equiv \frac{I_o}{V_s} = \frac{1}{R_F}$$

(b)

$$\beta = 1/A_f|_{\text{ideal}} = R_F$$

The  $\beta$  circuit is shown in Fig. 1.

(c) From Fig. 1,

$$R_{11} = R_{22} = R_F$$

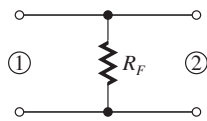


Figure 1

(d) The  $A$  circuit is shown in Fig. 2.

(e) From Fig. 2, we write

$$V_{id} = -V_i \frac{R_{id}}{R_s + R_{id} + R_{11}} \quad (1)$$

$$V_{gs} = A_1 V_{id} \quad (2)$$

$$I_o = -g_m V_{gs} \frac{r_{o2}}{r_{o2} + R_L + R_{22}} \quad (3)$$

Combining Eqns. (1)–(3), we find

$$A \equiv \frac{I_o}{V_i} =$$

$$A_1 g_m \frac{R_{id}}{R_s + R_{id} + R_{11}} \frac{r_{o2}}{r_{o2} + R_L + R_{22}} \quad (4)$$

For  $R_{id} \gg R_s + R_{11}$  and  $r_{o2} \gg R_L + R_{22}$ ,

$$A \simeq A_1 g_m$$

(f)

$$A_f = \frac{A}{1 + A\beta} = \frac{A_1 g_m}{1 + A_1 g_m R_F}$$

$$R_{if} = (1 + A\beta)R_i$$

where from Fig. 2,

$$R_i = R_s + R_{id} + R_{22}$$

$$= R_s + R_{id} + R_F$$

Thus,

$$R_{if} = (1 + A_1 g_m R_F)(R_s + R_{id} + R_F)$$

$$R_{in} = R_{if} - R_s$$

$$R_{in} = A_1 g_m R_F R_s + (1 + A_1 g_m R_F)(R_{id} + R_F)$$

For  $R_{id} \gg (R_F + R_s)$

$$R_{in} \simeq (1 + A_1 g_m R_F)R_{id}$$

From Fig. 2,

$$R_o = R_L + R_{22} + r_{o2} = R_L + R_F + r_{o2}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= (1 + A_1 g_m R_F)(R_L + R_F + r_{o2})$$

$$R_{out} = R_{of} - R_L$$

For  $r_{o2} \gg (R_L + R_F)$ ,

$$R_{out} \simeq (1 + A_1 g_m R_F)r_{o2}$$

(g)

$$A_f = \frac{200 \times 2}{1 + 200 \times 2 \times 0.2}$$

$$= \frac{400}{1 + 80} = 4.94 \text{ mA/V}$$

$$R_{in} = (1 + 200 \times 2 \times 0.2) \times 100 = 8100 \text{ k}\Omega$$

$$= 8.1 \text{ M}\Omega$$

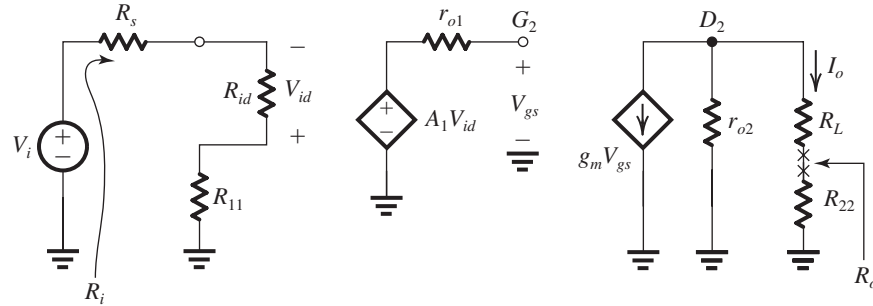


Figure 2

$$R_{out} = 81 \times 20 = 1620 \text{ k}\Omega$$

$$= 1.62 \text{ M}\Omega$$

**10.51** (a) Since  $V_s$  has a zero dc component, the gate of  $Q_1$  is at zero dc voltage. The negative feedback will force the gate of  $Q_2$  to be approximately at the same dc voltage as that at the gate of  $Q_1$ , thus

$$V_{G2} = 0$$

$$V_{D1} = 1.2 - V_{SG3}$$

$$= 1.2 - |V_i| - |V_{OV3}|$$

$$= 1.2 - 0.4 - 0.2 = +0.6 \text{ V}$$

$$V_{D2} = V_{G2} + V_{GS5}$$

$$= 0 + V_i + V_{OV5}$$

$$= 0.6 \text{ V}$$

(b)

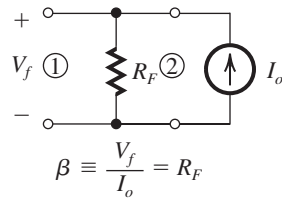


Figure 1

The feedback network is shown in Fig. 1, from which we find

$$\beta = R_F = 10 \text{ k}\Omega$$

For  $A\beta \gg 1$ ,

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_F}$$

$$A_f|_{ideal} = \frac{1}{10 \text{ k}\Omega} = 0.1 \text{ mA/V}$$

(c) From the  $\beta$  circuit in Fig. 1 and noting that the feedback topology is series-series, the loading effects of the feedback network are

$$R_{11} = R_{22} = R_F = 10 \text{ k}\Omega$$

Figure 2 shows the A circuit. We can write

$$\frac{V_{gs5}}{V_i} = g_{m1,2}(r_{o2} \parallel r_{o4}) \quad (1)$$

$$I_o = \frac{V_{gs5}}{(1/g_{m5}) + (R_{22} \parallel r_{o5})} \frac{r_{o5}}{r_{o5} + R_{22}}$$

Thus,

$$A \equiv \frac{I_o}{V_s} = \frac{g_{m1,2}(r_{o2} \parallel r_{o4})}{(1/g_{m5}) + (R_F \parallel r_{o5})} \frac{r_{o5}}{r_{o5} + R_F}$$

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D2,4}}$$

$$= \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$g_{m5} = \frac{2I_{D5}}{V_{OV5}} = \frac{2 \times 0.8}{0.2} = 8 \text{ mA/V}$$

$$r_{o5} = \frac{|V_A|}{I_{D5}} = \frac{20}{0.8} = 25 \text{ k}\Omega$$

$$A = \frac{1 \times (200 \parallel 200)}{0.125 + (10 \parallel 25)} \frac{25}{25 + 10}$$

$$= 9.83 \text{ mA/V}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{9.83}{1 + 9.83 \times 10} = 0.099 \text{ mA/V}$$

(d) From the A circuit, we have

$$R_o = r_{o5} + R_{22} + g_{m5}r_{o5}R_{22}$$

$$R_o = 25 + 10 + 8 \times 25 \times 10 = 2035 \text{ k}\Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 2.035 \times (1 + 9.83 \times 10)$$

$$= 202 \text{ M}\Omega$$

$$(e) V_o = I_o R_F$$

This figure belongs to Problem 10.51, part (b).

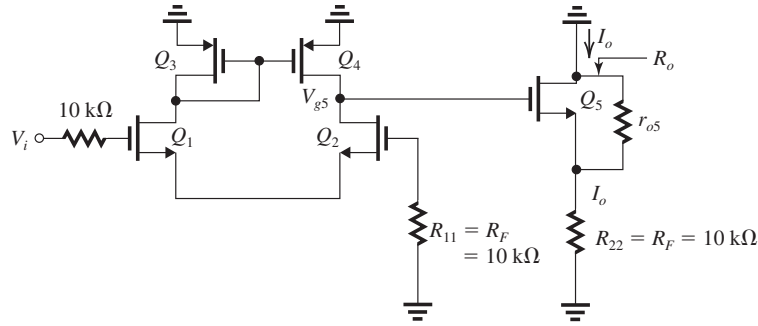


Figure 2

$$= A_f V_s R_F$$

$$\frac{V_o}{V_s} = A_f R_F = 0.099 \times 10 = 0.99 \text{ V/V}$$

$$R_{\text{out}} = \frac{\text{Output resistance at source of } Q_5}{1 + A\beta}$$

$$\simeq \frac{1/g_{m5}}{1 + A\beta}$$

$$= \frac{125 \Omega}{1 + 9.83 \times 10} = 1.26 \Omega$$

**10.52** Refer to Fig. 10.24(a).

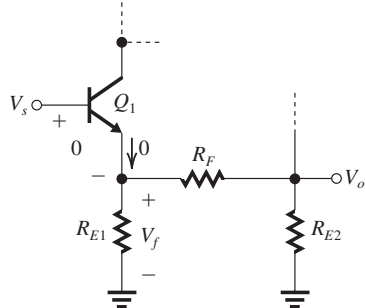


Figure 1

(a) To obtain  $A_f|_{\text{ideal}}$ , we set  $A = \infty$ . This results in  $V_{be1} = 0$  and thus  $I_{e1} = 0$ . This situation is illustrated in Fig. 1 from which we find that

$$V_f = V_s$$

and,

$$V_f = V_o \frac{R_{E1}}{R_{E1} + R_F}$$

Thus,

$$A_f|_{\text{ideal}} \equiv \frac{V_o}{V_s} = \frac{R_{E1} + R_F}{R_{E1}} = 1 + \frac{R_F}{R_{E1}}$$

and

$$\beta = 1/A_f|_{\text{ideal}} = \frac{R_{E1}}{R_{E1} + R_F}$$

For  $R_{E1} = 100 \Omega$  and  $R_F = 640 \Omega$ ,

$$A_f|_{\text{ideal}} = 1 + \frac{640}{100} = 7.4 \text{ V/V}$$

$$\beta = \frac{100}{100 + 640} = 0.135 \text{ V/V}$$

(b)

$$A\beta = 246.3$$

$$A = \frac{246.3}{0.135} = 1824.4 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{1824.4}{1 + 246.3} = 7.38 \text{ V/V}$$

which is very close to the ideal value of 7.40 V/V.

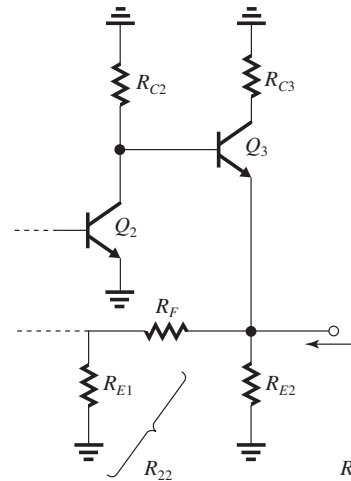


Figure 2

Figure 2 shows part of the  $A$  circuit. The output resistance  $R_o$  can be found as

$$R_o = R_{E2} \parallel (R_F + R_{E1}) \parallel \left( r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right)$$

From Example 10.8 we know that  $Q_3$  is operating at a bias current of 4 mA, thus  $r_{e3} = 6.25 \Omega$ .



Substituting this and the other given component values we obtain,

$$R_o = 100 \parallel 740 \parallel \left( 6.25 + \frac{5000}{101} \right) = 34.1 \, \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{34.1}{247.3} = 0.14 \, \Omega$$

**10.53** (a) Figure 1 shows a portion of the circuit when  $A = \infty$ . Note that  $V_{gs1} = 0$  and  $I_{d1} = 0$ . The analysis shown on the circuit provides

$$I_o = \frac{V_s}{R_{S1}} + \frac{V_o}{R_{S2}} = \frac{V_s}{R_{S1}} + \frac{V_s \left( 1 + \frac{R_F}{R_{S1}} \right)}{R_{S2}}$$

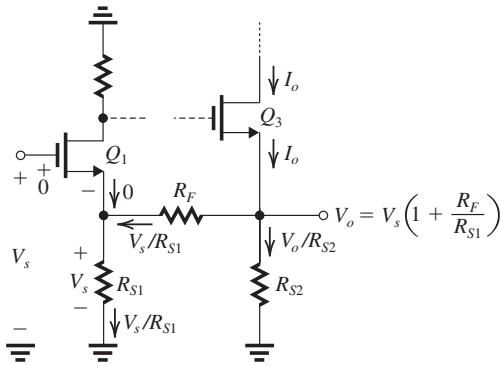


Figure 1

Thus,

$$A_f|_{ideal} \equiv \frac{I_o}{V_s} = \frac{1}{R_{S1}} + \frac{1}{R_{S2}} + \frac{R_F}{R_{S1}R_{S2}}$$

To obtain  $A_f|_{ideal} = 100 \text{ mA/V}$ ,

$$100 = \frac{1}{0.1} + \frac{1}{0.1} + \frac{R_F}{0.1 \times 0.1}$$

$$\Rightarrow R_F = 0.8 \text{ k}\Omega = 800 \, \Omega$$

(b) The  $\beta$  circuit is shown in Fig. 2.

$$\beta = \frac{1}{A_f|_{ideal}} = \frac{1}{100 \text{ mA/V}} = 0.01 \text{ V/mA}$$

$$R_{11} = R_{S1} \parallel (R_F + R_{S2}) = 100 \parallel 900 = 90 \, \Omega$$

$$R_{22} = R_{S2} \parallel (R_F + R_{S1}) = 90 \, \Omega$$

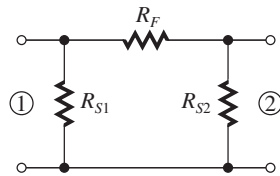


Figure 2

(c) The  $A$  circuit is shown in Fig. 3. Neglecting  $r_{o1}$  and  $r_{o2}$ , we find

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{\frac{1}{g_{m1}} + R_{11}} = -\frac{10}{0.2 + 0.09} = -34.5 \text{ V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -5 \times 10 = -50 \text{ V/V}$$

Finally, neglecting  $r_{o3}$ , we find

$$I_o = \frac{V_{d2}}{\frac{1}{g_{m3}} + R_{22}} = \frac{V_{d2}}{0.2 + 0.09} = 3.45 \text{ mA/V}$$

Combining the three results, we find

$$A \equiv \frac{I_o}{V_i} = -34.5 \times -50 \times 3.45 = 5951 \text{ mA/V}$$

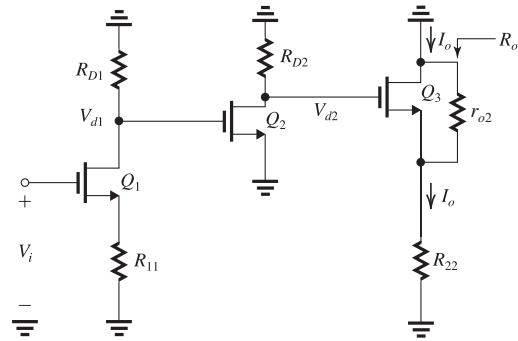


Figure 3

$$(d) A\beta = 5951 \times 0.01 = 59.51$$

$$1 + A\beta = 60.51$$

$$A_f = \frac{A}{1 + A\beta} = \frac{5951}{60.51} = 98.3 \text{ mA/V}$$

which is 1.7% lower than the ideal value of 100 mA/V. To restore  $A_f$  to exactly 100 mA/V we can increase  $R_F$ . This lowers  $\beta$  and thus increases  $A_f$ .

$$(e) R_o = r_{o3} + R_{22} + g_{m3}r_{o3}R_{22} = 20 + 0.09 + 5 \times 20 \times 0.09 = 29.1 \text{ k}\Omega$$

$$R_{of} = R_o(1 + A\beta) = 29.1 \times 60.51 = 1.76 \text{ M}\Omega$$

$$R_{out1} = R_{of} = 1.76 \text{ M}\Omega$$

**10.54** (a) Figure 1 below shows the circuit with  $A$  set to  $\infty$ . Straightforward analysis proceeds as follows:

$$V_x = V_s + \frac{V_s}{R_3} \times R_2 = \left( 1 + \frac{R_2}{R_3} \right) V_s$$

$$I_o = \frac{V_s}{R_3} + \frac{V_x}{R_1} = \frac{V_s}{R_3} + \frac{1}{R_1} \left( 1 + \frac{R_2}{R_3} \right) V_s$$

$$\Rightarrow A_f|_{\text{ideal}} = \frac{I_o}{V_s} = \frac{1}{R_1} + \frac{1}{R_3} + \frac{R_2}{R_1 R_3}$$

$$100 = \frac{1}{0.1} + \frac{1}{0.1} + \frac{R_2}{0.01}$$

$$\Rightarrow R_2 = 800 \Omega$$

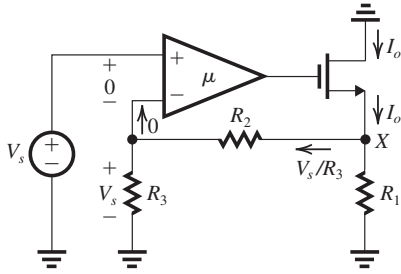


Figure 1

(b)  $\beta = 1/A_f|_{\text{ideal}} = 0.01 \text{ V/mA}$

The  $\beta$  circuit is shown in Fig. 2.

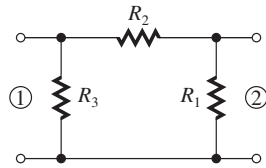


Figure 2

(c)  $R_{11} = R_3 \parallel (R_2 + R_1) = 100 \parallel (800 + 100) = 90 \Omega$

$$R_{22} = R_1 \parallel (R_2 + R_3) = 90 \Omega$$

(d) Figure 3 shows the A circuit.

$$V_{id} = V_i \frac{R_{id}}{R_{id} + R_{11}}$$

$$V_g = \mu V_{id}$$

$$I_o = \frac{V_g}{\frac{1}{g_m} + (R_{22} \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + R_{22}}$$

Combining Eqns. (1)–(3), we find

$$\begin{aligned} A &\equiv \frac{I_o}{V_i} = \mu \frac{R_{id}}{R_{id} + R_{11}} \frac{1}{\frac{1}{g_m} + (R_{22} \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + R_{22}} \\ &= \mu \frac{100}{100 + 0.09} \frac{1}{0.5 + (0.09 \parallel 20)} \frac{20}{20 + 0.09} \\ &= 1.687 \mu \text{ mA/V} \end{aligned}$$

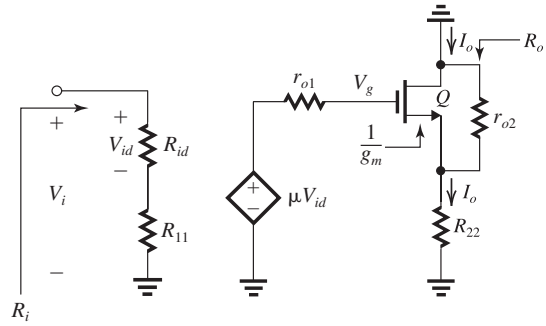


Figure 3

(e)  $1 + A\beta = 100$

$$A\beta = 99$$

$$\Rightarrow A = 9900 \text{ mA/V}$$

$$1.687 \mu = 9900$$

$$\Rightarrow \mu = 5868 \text{ V/V}$$

(f)

$$A_f = \frac{A}{1 + A\beta} = \frac{9900}{100} = 99 \text{ mA/V}$$

which is 1% below the ideal value.

(g) From Fig. 3,

$$R_i = R_{id} + R_{11} = 100 + 0.09 = 100.09 \text{ k}\Omega$$

$$R_o = r_{o2} + R_{22} + g_m r_{o2} R_{22}$$

$$= 20 + 0.09 + 2 \times 20 \times 0.09 = 23.69 \text{ k}\Omega$$

$$R_{in} = R_{if} = (1 + A\beta)R_i = 100 \times 100.09 \simeq 10 \text{ M}\Omega$$

$$R_{out} = R_{of} = (1 + A\beta)R_o = 100 \times 23.69 =$$

$$2.37 \text{ M}\Omega$$

10.55 (a)

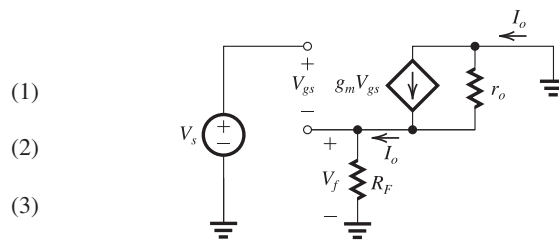


Figure 1

Figure 1 shows the small-signal equivalent circuit of the feedback amplifier. Observe that the resistance  $R_F$  senses the output current  $I_o$  and provides a voltage  $I_o R_F$  that is subtracted from  $V_s$ . Thus the feedback network is composed of the resistance  $R_F$ , as shown in Fig. 2. Because the feedback is of the series-series type, the loading

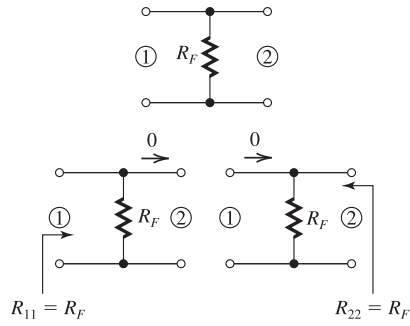


Figure 2

resistances  $R_{11}$  and  $R_{22}$  are determined as indicated in Fig. 2,

$$R_{11} = R_F$$

$$R_{22} = R_F$$

(b) The  $\beta$  circuit is shown in Fig. 2 and

$$\beta = R_F$$

Figure 3 shows the  $A$  circuit.

(c)

To determine  $A = I_o/V_i$ , we write

$$V_{gs} = V_i$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F}$$

$$A \equiv \frac{I_o}{V_i} = g_m \frac{r_o}{r_o + R_F}$$

$$1 + A\beta = 1 + \frac{g_m r_o R_F}{r_o + R_F}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{g_m r_o / (r_o + R_F)}{1 + g_m r_o R_F / (r_o + R_F)}$$

$$= \frac{g_m}{1 + g_m R_F + \frac{R_F}{r_o}}$$

From the  $A$  circuit in Fig. 3, we have

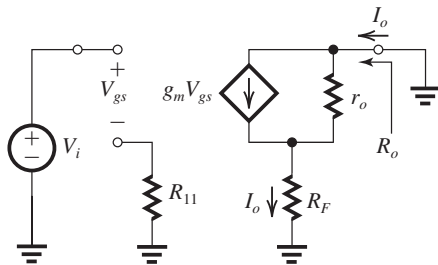


Figure 3

$$R_o = r_o + R_F$$

$$R_{of} = (1 + A\beta)R_o$$

$$= \left(1 + \frac{g_m r_o R_F}{r_o + R_F}\right)(r_o + R_F)$$

$$= r_o + R_F + g_m r_o R_F$$

which is a familiar relationship!

**10.56** Figure 1 on the next page shows the equivalent circuit with  $V_s = 0$  and a voltage  $V_x$  applied to the collector for the purpose of determining the output resistance  $R_o$ ,

$$R_o \equiv \frac{V_x}{I_x}$$

Some of the analysis is displayed on the circuit diagram. Since the current entering the emitter node is equal to  $I_x$ , we can write for the emitter voltage

$$V_e = I_x [R_e \parallel (r_\pi + R_b)] \quad (1)$$

The base current can be obtained using the current-divider rule applied to  $R_e$  and  $(r_\pi + R_b)$  as

$$I_b = -I_x \frac{R_e}{R_e + r_\pi + R_b} \quad (2)$$

The voltage from collector to ground is equal to  $V_x$  and can be expressed as the sum of the voltage drop across  $r_o$  and  $V_e$ ,

$$V_x = (I_x - \beta I_b)r_o + V_e$$

Substituting for  $V_e$  from (1) and for  $I_b$  from (2), we obtain

$$\begin{aligned} R_o &= \frac{V_x}{I_x} = r_o + [R_e \parallel (r_\pi + R_b)] \\ &+ \frac{R_e \beta r_o}{R_e + r_\pi + R_b} \\ &= r_o + [R_e \parallel (r_\pi + R_b)] \left[1 + r_o \frac{\beta}{r_\pi + R_b}\right] \end{aligned}$$

Since  $\beta = g_m r_\pi$ , we obtain

$$R_o = r_o + [R_e \parallel (r_\pi + R_b)] \left[1 + g_m r_o \frac{r_\pi}{r_\pi + R_b}\right] \quad \text{Q.E.D.}$$

For  $R_b = 0$ ,

$$R_o = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

The maximum value of  $R_o$  will be obtained when  $R_e \gg r_\pi$ . If  $R_e$  approaches infinity (zero signal current in the emitter),  $R_o$  approaches the theoretical maximum:

$$\begin{aligned} R_{o\max} &= r_o + r_\pi(1 + g_m r_o) \\ &= r_o + r_\pi + \beta r_o \\ &\approx \beta r_o \end{aligned} \quad (3)$$

This figure belongs to Problem 10.56.

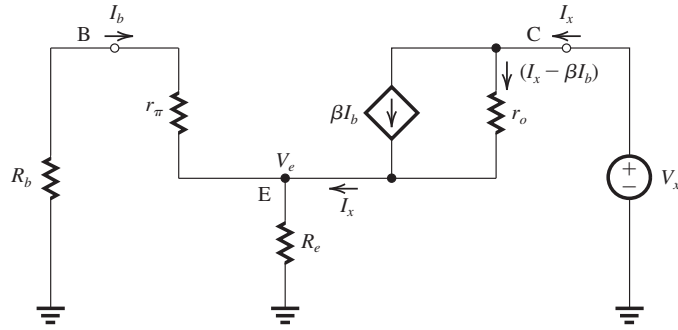


Figure 1

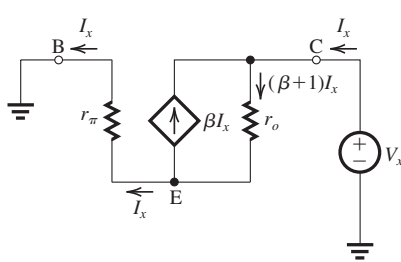


Figure 2

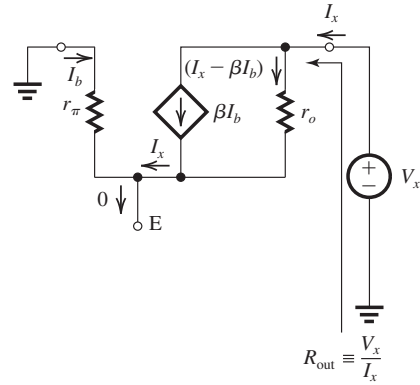


Figure 1

The situation that pertains in the circuit when  $R_e = \infty$  is illustrated in Fig. 2. Observe that since the signal current in the emitter is zero, the base current will be equal to the collector current ( $I_x$ ) and in the direction indicated. The controlled-source current will be  $\beta I_x$ , and this current adds to  $I_x$  to provide a current  $(\beta + 1)I_x$  in the output resistance  $r_o$ . A loop equation takes the form

$$V_x = (\beta + 1)I_x r_o + I_x r_\pi$$

and thus

$$R_o \equiv \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

which is identical to the result in Eq. (3).

**10.57**

Figure 1 shows the situation that pertains in the transistor when  $\mu$  is so large that  $V_b \approx 0$  and  $I_e \approx 0$ . Observe that

$$I_b = -I_x$$

Writing a loop equation for the C-E-B, we obtain

$$V_x = (I_x - \beta I_b)r_o - I_b r_\pi$$

Substituting  $I_b = -I_x$ , we obtain

$$R_{out} = \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

or if  $\beta$  is denoted  $h_{fe}$ ,

$$R_{out} = r_\pi + (h_{fe} + 1)r_o \quad \text{Q.E.D.}$$

Thus, for large amounts of feedback,  $R_{out}$  is limited to this value, which is approximately  $h_{fe}r_o$  independent of the amount of feedback. This phenomenon does not occur in MOSFET circuits where  $h_{fe} = \infty$ .

**10.58** Figure 1 on the next page shows the  $\beta$  circuit together with the determination of  $\beta$ ,  $R_{11}$  and  $R_{22}$ ,

$$\beta = R_F$$

$$R_{11} = R_{22} = R_F$$

Figure 2 shows the  $A$  circuit. To determine  $A \equiv I_o/V_i$ , we write for  $Q_1$

$$\frac{V_{d1}}{V_i} = -\frac{R_D}{(1/g_{m1}) + R_F} \quad (1)$$

and for  $Q_2$

$$I_o = -g_{m2}V_{d1} \quad (2)$$

Combining (1) and (2) results in

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F}$$

$$A\beta = \frac{g_{m2}R_D R_F}{(1/g_{m1}) + R_F}$$

This figure belongs to Problem 10.58.

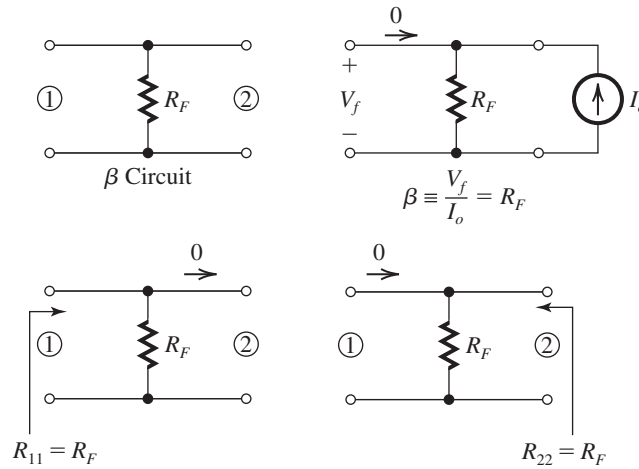


Figure 1

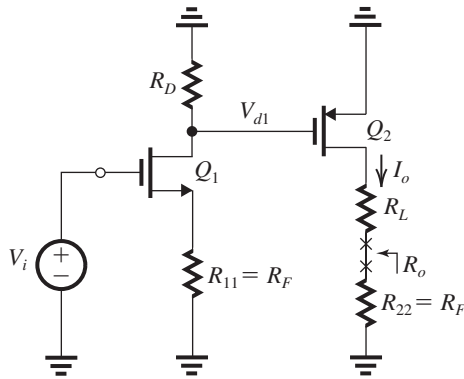


Figure 2

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$\Rightarrow A_f = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F + g_{m2}R_D R_F}$$

From the  $A$  circuit, breaking the loop at  $XX$  gives

$$R_o = R_F + R_L + r_{o2}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= \left[ 1 + \frac{g_{m2} + R_D R_F}{(1/g_{m1}) + R_F} \right] [R_F + R_L + r_{o2}]$$

For

$$g_{m1} = g_{m2} = 4 \text{ mA/V}, \quad R_D = 20 \text{ k}\Omega,$$

$$r_{o2} = 20 \text{ k}\Omega, \quad R_F = 100 \text{ }\Omega, \text{ and } R_L = 1 \text{ k}\Omega,$$

we obtain

$$A = \frac{4 \times 20}{0.25 + 0.1} = 228.6 \text{ mA/V}$$

$$\beta = 0.1 \text{ k}\Omega$$

$$A\beta = 22.86$$

$$1 + A\beta = 23.86$$

$$A_f = \frac{228.6}{23.86} = 9.56 \text{ mA/V}$$

$$R_o = 0.1 + 1 + 20 = 21.1 \text{ k}\Omega$$

$$R_{of} = 23.86 \times 21.1 = 503.4 \text{ k}\Omega$$

10.59

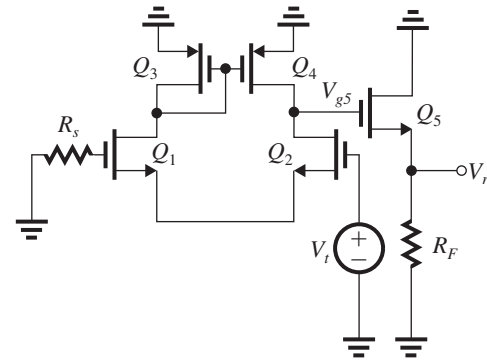


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain  $A\beta$ :

$$A\beta \equiv -\frac{V_r}{V_i}$$

First we write for the gain of differential amplifier

$$\frac{V_{gs5}}{V_i} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \quad (1)$$

Next we write for the source follower,

$$\frac{V_r}{V_{gs5}} = \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})} \quad (2)$$

This figure belongs to Problem 10.60

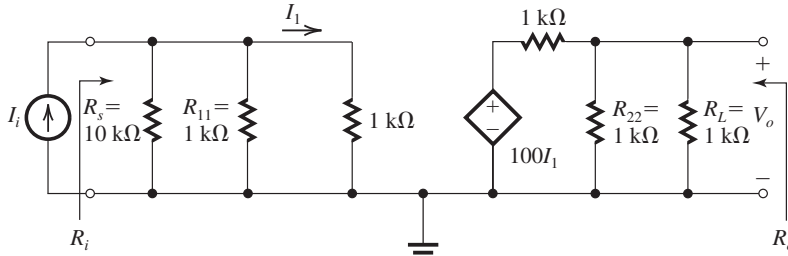


Figure 1

Combining (1) and (2) yields

$$A\beta = -\frac{V_o}{I_i} = g_{m1,2}(r_{o2} \parallel r_{o4}) \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})}$$

Q.E.D.

**10.60** The A circuit is shown in Fig. 1.

$$R_i = R_s \parallel R_{11} \parallel 1 \text{ k}\Omega$$

$$= 10 \parallel 1 \parallel 1 = 0.476 \text{ k}\Omega$$

$$I_1 = I_i \frac{(1 \parallel 10)}{(1 \parallel 10) + 1} = 0.476 I_i$$

$$V_o = 100 I_1 \frac{R_L \parallel R_{22}}{(R_L \parallel R_{22}) + 1}$$

$$= 100 I_1 \frac{1 \parallel 1}{(1 \parallel 1) + 1} = 33.3 I_1$$

Thus,

$$V_o = 33.3 \times 0.476 I_i$$

$$\Rightarrow A \equiv \frac{V_o}{I_i} = 15.87 \text{ V/mA}$$

$$R_o = R_L \parallel R_{22} \parallel 1 = 1 \parallel 1 \parallel 1 = 0.333 \text{ k}\Omega$$

$$A_f = \frac{A}{1 + A\beta} = \frac{15.87}{1 + 15.87 \times 1}$$

$$= \frac{15.87}{16.87} = 0.94 \text{ V/mA}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{476 \Omega}{16.87} = 28.2 \Omega$$

$$R_{in} = 1 / \left[ \frac{1}{R_{if}} - \frac{1}{R_s} \right] = 1 / \left[ \frac{1}{28.2} - \frac{1}{10,000} \right]$$

$$= 28.3 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{333 \Omega}{16.87} = 19.7 \Omega$$

$$R_{out} = 1 / \left[ \frac{1}{R_{of}} - \frac{1}{R_L} \right] = 1 / \left[ \frac{1}{19.7} - \frac{1}{1000} \right]$$

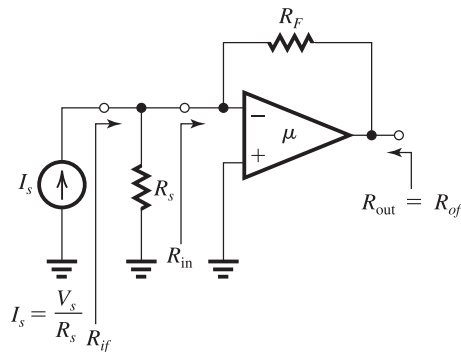
$$= 20.1 \Omega$$

**10.61** (a) Refer to the feedback network shown in Fig. 10.27(c). Thus,

$$\beta = -\frac{1}{R_F}$$

If  $A\beta \gg 1$ , then we have

$$A_f = \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_F$$



and the voltage gain realized will be

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \simeq -\frac{R_F}{R_s}$$

If  $R_s = 2 \text{ k}\Omega$ , to obtain  $V_o/V_s \simeq -10 \text{ V/V}$ , we required

$$R_F = 10 \times R_s = 20 \text{ k}\Omega$$

(b) Refer to the solution to Example 10.9.

$$\beta = -\frac{1}{R_F} = -\frac{1}{20 \text{ k}\Omega} = -0.05 \text{ mA/V}$$

and

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$= 100 \parallel 20 \parallel 2 = 1.786 \text{ k}\Omega$$

With  $R_L = \infty$ , we get

$$A \equiv \frac{V_o}{I_i} = -10^3 \times 1.786 \times \frac{20}{20 + 2}$$

$$= -1623.6 \text{ k}\Omega$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$1 + A\beta = 1 + 1623.6 \times 0.05$$

$$= 82.18$$

$$A_f = \frac{V_o}{I_s} = -\frac{1623.6}{82.18}$$

$$= -19.76 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{19.76}{2}$$

$$= -9.88 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.786}{82.18} = 21.7 \text{ }\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$21.7 \text{ }\Omega = 2000 \text{ }\Omega \parallel R_{in}$$

$$R_{in} \simeq 21.7 \text{ }\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

where with  $R_L = \infty$  we get

$$R_o = r_o \parallel R_f = 2 \parallel 20 = 1.818 \text{ k}\Omega$$

$$R_{out} = R_{of} = \frac{1.818}{82.18} = 22.1 \text{ }\Omega$$

(c)  $f_{Hf} = f_H(1 + A\beta)$

$$= 1 \times 82.18$$

$$= 82.18 \text{ kHz}$$

10.62 (a)

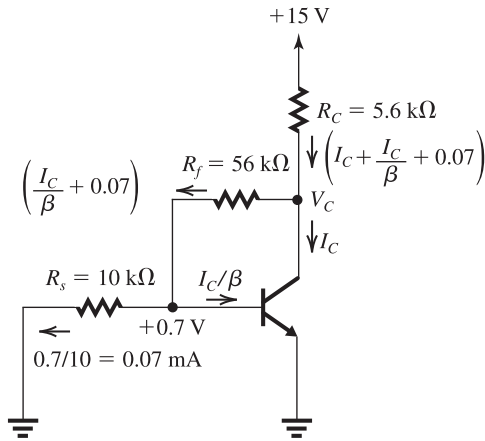


Figure 1

Figure 1 illustrates the dc analysis. We can express the dc collector voltage  $V_C$  in two alternative ways:

$$V_C = +15 - R_C \left( I_C + \frac{I_C}{\beta} + 0.07 \right)$$

and

$$V_C = 0.7 + R_f \left( \frac{I_C}{\beta} + 0.07 \right)$$

Equating these two expressions yields

$$15 - 5.6(I_C + 0.01I_C + 0.07)$$

$$= 0.7 + 56(0.01I_C + 0.07)$$

$$\Rightarrow I_C = 1.6 \text{ mA}$$

$$V_C \simeq 5.5 \text{ V}$$

(b) Figure 2 on the next page shows the small-signal equivalent circuit of the amplifier where

$$g_m = \frac{1.6 \text{ mA}}{0.025 \text{ V}} = 64 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{64} = 1.56 \text{ k}\Omega$$

(c) Figure 3 on the next page shows the  $A$  circuit. It includes the loading effects of the feedback network:

$$R_{11} = R_{22} = R_f$$

Also, observe that

$$\beta = -\frac{1}{R_f}$$

From the  $A$  circuit in Fig. 3, we have

$$R_i = R_s \parallel R_f \parallel r_\pi \tag{1}$$

$$V_\pi = I_i R_i \tag{2}$$

$$V_o = -g_m V_\pi (R_C \parallel R_f) \tag{3}$$

Combining (1), (2) and (3) gives

$$A \equiv \frac{V_o}{I_i} = -g_m (R_s \parallel R_f \parallel r_\pi) (R_C \parallel R_f)$$

$$R_i = 10 \text{ k}\Omega \parallel 56 \text{ k}\Omega \parallel 1.56 \text{ k}\Omega$$

$$= 1.32 \text{ k}\Omega$$

$$A = -64 \times 1.32 \times (5.6 \parallel 56)$$

$$= -430 \text{ k}\Omega$$

From the  $A$  circuit, we have

$$R_o = R_C \parallel R_f$$

$$= 5.6 \text{ k}\Omega \parallel 56 \text{ k}\Omega$$

$$= 5.1 \text{ k}\Omega$$

(d)  $\beta = -\frac{1}{R_f} = -\frac{1}{56 \text{ k}\Omega}$

$$A\beta = \frac{430}{56} = 7.68$$

$$1 + A\beta = 8.68$$

(e)  $A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$

These figures belong to Problem 10.62.

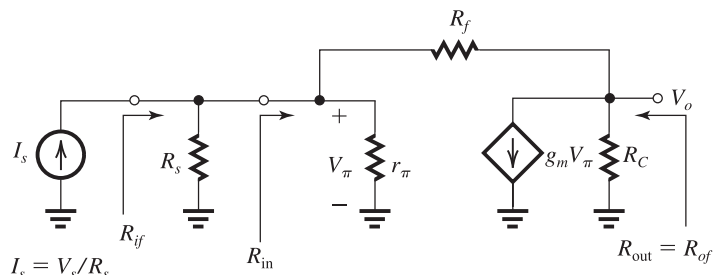


Figure 2

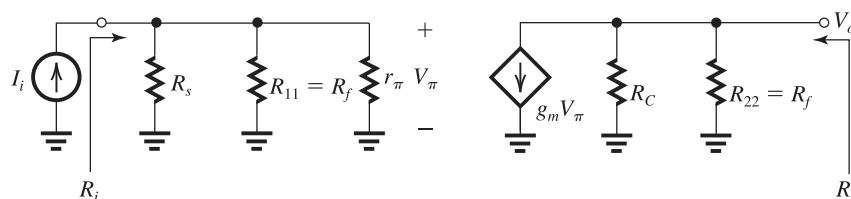


Figure 3

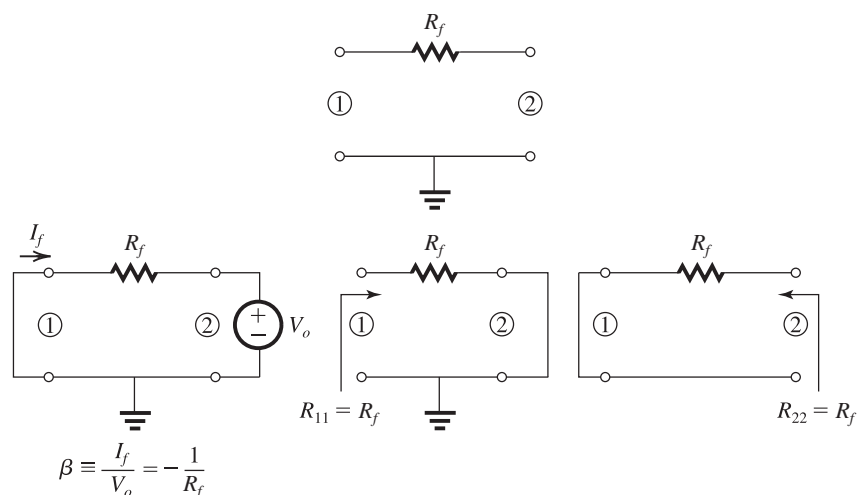


Figure 4

$$= -\frac{430}{8.68} = -49.5 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.32 \text{ k}\Omega}{8.68} = 152 \text{ }\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$152 \text{ }\Omega = 10 \text{ k}\Omega \parallel R_{in}$$

$$R_{in} = 155 \text{ }\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5.1 \text{ k}\Omega}{8.68} = 588 \text{ }\Omega$$

$$(f) \frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{49.5}{10} = -4.95 \text{ V/V}$$

Refer to Fig. P10.62 and assume the gain of the BJT to be infinite so that the signal voltage at its base is zero (virtual ground). In this case, we have

$$\frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{56}{10} = -5.6 \text{ V/V}$$

Thus, the actual gain magnitude ( $\approx 5 \text{ V/V}$ ) is only about 12% below the ideal value; not bad for a single transistor op amp!



## 10.63

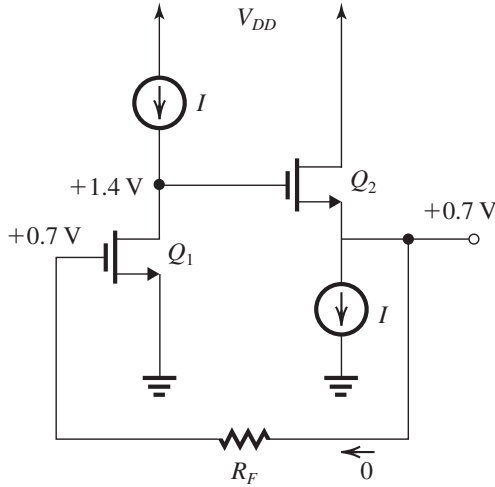


Figure 1

(a) See Figure 1.

$$V_{G1} = V_{GS1} = V_i + V_{OV}$$

$$= 0.5 + 0.2 = +0.7 \text{ V}$$

(because the dc voltage across  $R_F$  is zero)

$$V_O = V_{G1}$$

$$V_O = +0.7 \text{ V}$$

$$V_{D1} = V_O + V_{GS2}$$

$$= 0.7 + 0.5 + 0.2$$

$$= +1.4 \text{ V}$$

$$(b) \quad g_{m1,2} = \frac{2I}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$r_{o1,2} = \frac{V_A}{I} = \frac{16 \text{ V}}{0.4 \text{ mA}} = 40 \text{ k}\Omega$$

(c) Figure 2 on the next page shows the  $\beta$  circuit and the determination of its loading effects,

$$R_{11} = R_{22} = R_F$$

Figure 2 shows also the  $A$  circuit. We can write

$$V_{g1} = I_i R_i \quad (1)$$

where

$$R_i = R_{11} = R_F \quad (2)$$

$$V_{d1} = -g_{m1} r_{o1} \quad (3)$$

$$\frac{V_o}{V_{d1}} = \frac{R_{22} \parallel r_{o2}}{(R_{22} \parallel r_{o2}) + \frac{1}{g_{m2}}} \quad (4)$$

Combining Eqs. (1)–(4) results in

$$A \equiv \frac{V_o}{I_i} = -g_{m1} r_{o1} R_F \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(d) From Fig. 3 on the next page we see that

$$\beta = -\frac{1}{R_F}$$

$$A\beta = g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$1 + A\beta = 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$(e) \quad A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{g_{m1} r_{o1} R_F (R_F \parallel r_{o2})}{(R_F \parallel r_{o2}) + 1/g_{m2} + (g_{m1} r_{o1})(R_F \parallel r_{o2})}$$

(f)  $R_i = R_F$

$$R_{in} = R_{if} = R_i / (1 + A\beta)$$

$$= R_F / \left[ 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}} \right]$$

$$R_{out} = R_{of} = R_o / (1 + A\beta)$$

where from the  $A$  circuit we have

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{out} = \left( R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right) / \left[ 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}} \right]$$

$$(g) \quad A = -4 \times 40 \times 10 \frac{10 \parallel 40}{(10 \parallel 40) + 0.25}$$

$$= -1551.5 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = 155.15$$

$$1 + A\beta = 156.15$$

$$A_f = -\frac{1551.5}{156.15} = -9.94 \text{ k}\Omega$$

$$R_i = R_F = 10 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_F}{1 + A\beta} = \frac{10,000 \Omega}{156.15} = 64 \Omega$$

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_o = 10 \parallel 40 \parallel 0.25 = 242 \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{242}{156.15} = 1.55 \Omega$$

This figure belongs to Problem 10.63, part (c).

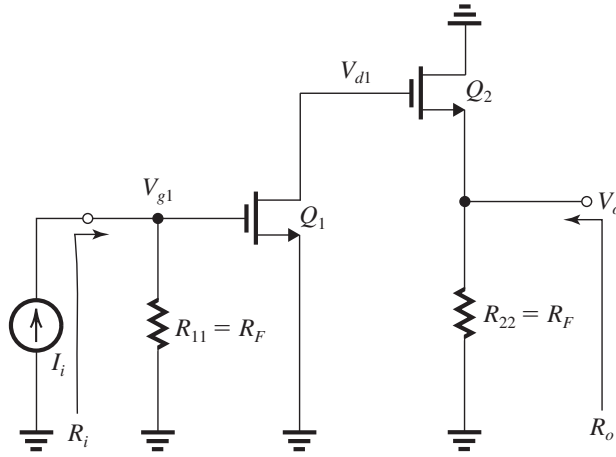
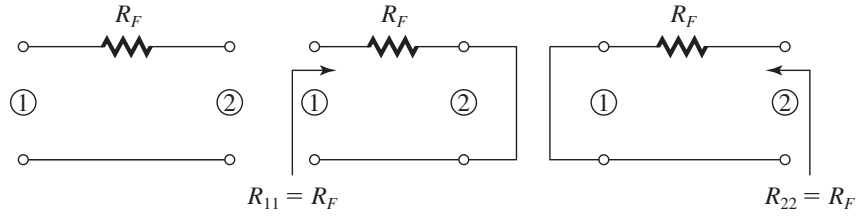


Figure 2

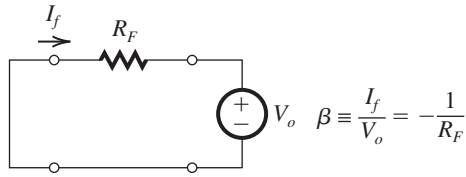


Figure 3

**10.64** Figure 1 on the next page shows the feedback network with a voltage  $V_o$  applied to port 2 to determine  $\beta$ :

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

For  $A\beta \gg 1$ , we have

$$\frac{V_o}{I_s} \equiv A_f \simeq \frac{1}{\beta} = -R_F$$

Thus, for  $\frac{V_o}{I_s} \simeq -10 \text{ k}\Omega$ , we select

$$R_F = 10 \text{ k}\Omega$$

The loading of the feedback network on the  $A$  circuit can be determined as shown in Fig. 1:

$$R_{11} = R_{22} = R_F$$

Figure 2 shows the  $A$  circuit. For the CG amplifier  $Q_1$ , we can write

$$R_i = R_F \parallel \frac{1}{g_{m1}} \quad (1)$$

$$V_{sg} = I_i R_i \quad (2)$$

$$V_{d1} = g_{m1} V_{sg} R_{D1} \quad (3)$$

Combining (1)–(3) yields

$$V_{d1} = (g_{m1} R_{D1}) \left( R_F \parallel \frac{1}{g_{m1}} \right) I_i \quad (4)$$

For the CS stage  $Q_2$ , we can write

$$\frac{V_o}{V_{d1}} = -g_{m2} (R_{D2} \parallel R_F) \quad (5)$$

Combining (4) and (5), we obtain the open-loop gain  $A$ :

$$A \equiv \frac{V_o}{I_s} = -(g_{m1} R_{D1}) \left( R_F \parallel \frac{1}{g_{m1}} \right) g_{m2} (R_{D2} \parallel R_F)$$

Substituting  $g_{m1} = g_{m2} = 4 \text{ mA/V}$ ,  $R_{D1} = R_{D2} = 10 \text{ k}\Omega$ , and  $R_F = 10 \text{ k}\Omega$  gives

$$A = -(4 \times 10) \times (10 \parallel 0.25) \times 4 \times (10 \parallel 10)$$

$$A = -195 \text{ k}\Omega$$

$$A\beta = \frac{195}{10} = 19.5$$

$$1 + A\beta = 20.5$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{195}{20.5} = -9.52 \text{ k}\Omega$$

This figure belongs to Problem 10.64, part (a).

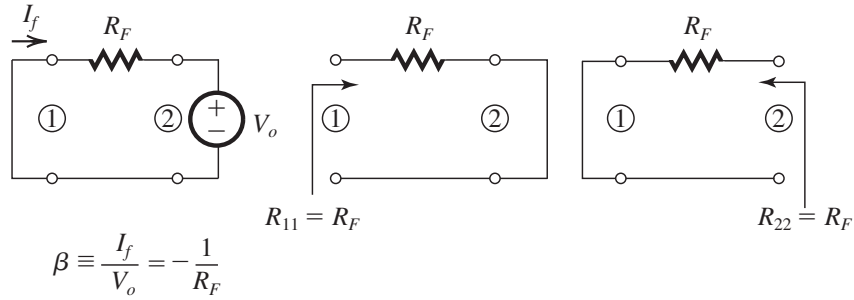


Figure 1

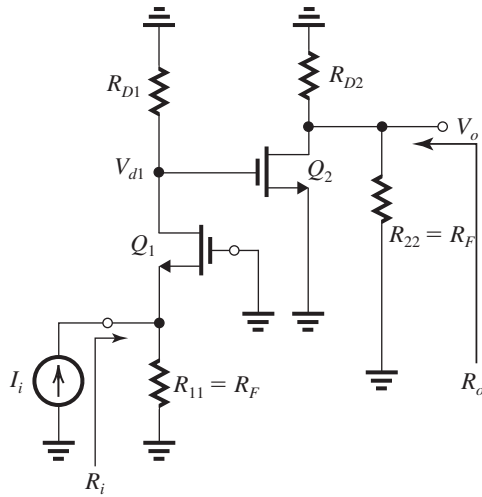


Figure 2

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

From Eq. (1), we obtain

$$R_i = 10 \parallel 0.25 = 244 \Omega$$

$$R_{in} = \frac{244}{20.5} = 11.9 \Omega$$

From the A circuit,

$$R_o = R_{D2} \parallel R_F = 10 \parallel 10 = 5 \text{ k}\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{5000}{20.5} = 244 \Omega$$

**10.65**

Figure 1 shows the circuit prepared for the determination of the loop gain  $A\beta$ :

$$A\beta = -\frac{V_r}{V_t}$$

An expression for  $V_r$  can be written by inspection as

$$V_r = -g_m V_t [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F}$$

Thus,

$$A\beta = g_m [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F} \quad (1)$$

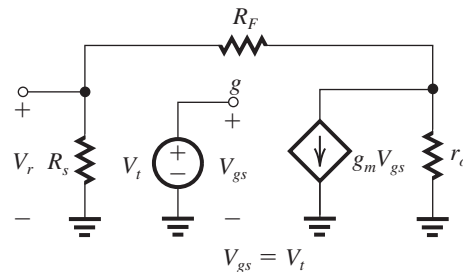


Figure 1

The feedback network ( $\beta$  circuit) is shown in Fig. 2 fed with  $V_o$  at port 2 and with port 1 short-circuited to determine  $\beta$ :

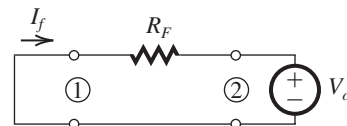


Figure 2

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F} \quad (2)$$

Equations (1) and (2) can now be used to determine A:

$$A = -g_m [r_o \parallel (R_s + R_F)] (R_s \parallel R_F)$$

Using the numerical values given in Exercise 10.15(e), we obtain

$$A = -5[20 \parallel (1 + 10)](1 \parallel 10) = -32.3 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -0.1 \text{ mA/V}$$

$$A\beta = 3.23$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{32.3}{1 + 3.23} = -7.63 \text{ k}\Omega$$

Compare these results to those found in Exercise 11.19:  $A = -32.3 \text{ k}\Omega$  ( $-30.3 \text{ k}\Omega$ ),  $\beta = -0.1 \text{ mA/V}$  ( $-0.1 \text{ mA/V}$ ),  $A\beta = -3.23$  ( $-3.03$ ), and  $A_f = -7.63 \text{ k}\Omega$  ( $-7.52 \text{ k}\Omega$ ). The slight differences are due to the approximation used in the systematic analysis method.

**10.66** Figure 1 shows the small-signal-equivalent circuit of the feedback amplifier of Fig. E10.15. Analysis to determine  $V_o/I_s$  proceeds as follows:

Writing a node equation at the output node provides

$$g_m V_{gs} + \frac{V_o}{r_o} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$\Rightarrow V_{gs} = -V_o \frac{\frac{1}{r_o} + \frac{1}{R_F}}{g_m - \frac{1}{R_F}}$$

$$\Rightarrow V_{gs} = -V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)} \quad (1)$$

Writing a node equation at node G provides

$$I_s - \frac{V_{gs}}{R_s} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$I_s - \frac{V_{gs}}{(R_s \parallel R_F)} + \frac{V_o}{R_F} = 0 \quad (2)$$

Substituting for  $V_{gs}$  from (1) into (2), we obtain

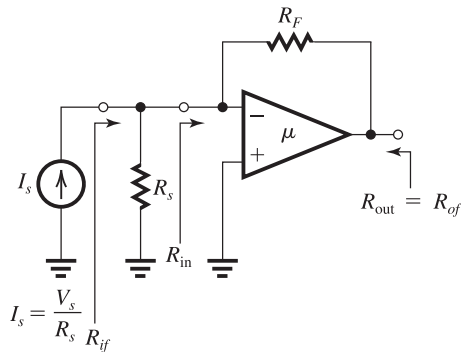


Figure 1

$$I_s + V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)} + \frac{V_o}{R_F} = 0 \Rightarrow \frac{V_o}{I_s} = -\frac{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)}{1 + \left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)/R_F} \quad \text{Q.E.D.}$$

The feedback analysis method assumes

$$g_m \gg \frac{1}{R_F}$$

**10.67**

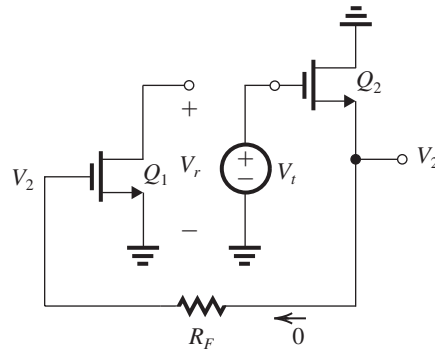


Figure 1

Figure 1 shows the circuit of Fig. P10.63 prepared for determining the loop gain  $A\beta$ :

$$A\beta \equiv -\frac{V_r}{V_t}$$

The gain of the source-follower  $Q_2$  can be found as

$$\frac{V_2}{V_t} = \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \quad (1)$$

The gain of the CS transistor  $Q_1$  can be found as

$$V_r = -g_{m1} r_{o1} V_2 \quad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \quad (3)$$

This expression differs from that obtained in Problem 10.63 utilizing the general feedback analysis method. To determine the numerical difference, we evaluate  $A\beta$  in Eq. (3) using

$$g_{m1} = g_{m2} = 4 \text{ mA/V}, \quad r_{o1} = r_{o2} = 40 \text{ k}\Omega$$

$$A\beta = 4 \times 40 \times \frac{40}{40 + 0.25}$$

$$= 159$$

which is larger than the value found in Problem 10.63 (155.15) by about 2.5%. This small difference is a result of the approximations involved in the general method. The more accurate result for  $A\beta$  is the one obtained here. However, the loop-gain method does not make it possible to determine the input and output resistances of the feedback amplifier.

10.68

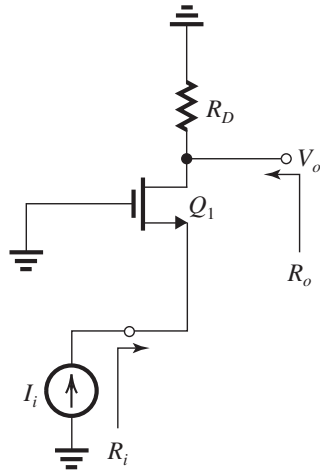


Figure 1

The  $A$  circuit is shown in Fig. 1.

$$R_i = \frac{1}{g_{m1}} = \frac{1}{5} = 0.2 \text{ k}\Omega$$

$$A \equiv \frac{V_o}{I_i} = R_D = 10 \text{ k}\Omega$$

$$R_o = R_D = 10 \text{ k}\Omega$$

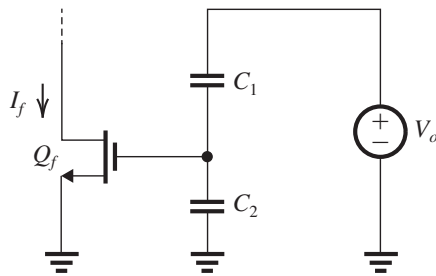


Figure 2

The  $\beta$  circuit is shown in Fig. 2.

$$\beta \equiv \frac{I_f}{V_o} = \frac{C_1}{C_1 + C_2} g_{mf} = \frac{0.9}{0.9 + 0.1} \times 2$$

$$= 1.8 \text{ mA/V}$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= \frac{10}{1 + 10 \times 1.8} = 0.53 \text{ k}\Omega \text{ (V/mA)}$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta} = \frac{200 \Omega}{19} = 10.5 \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{10 \text{ k}\Omega}{19} = 526 \Omega$$

10.69 (a)

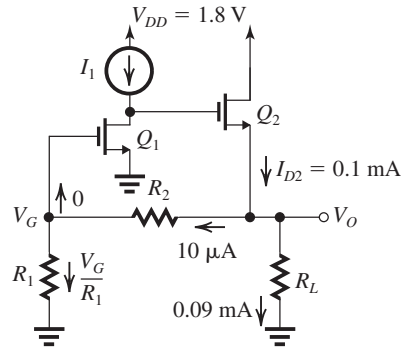


Figure 1

Figure 1 shows the circuit for the purpose of performing a dc design.

$$I_{D1} = 100 \mu\text{A} \Rightarrow I_1 = 100 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_1 \times 0.2^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 12.5$$

$$V_{G1} = V_m + V_{OV1}$$

$$= 0.4 + 0.2 = 0.6 \text{ V}$$

Since  $I_{R2,R1} = 10 \mu\text{A}$ , we have

$$R_1 = \frac{0.6 \text{ V}}{0.01 \text{ mA}} = 60 \text{ k}\Omega$$

$$I_{RL} = I_{D2} - I_{R2,R1}$$

$$= 0.1 - 0.01 = 0.09 \text{ mA}$$

$$V_o = 0.09 \times 10 = 0.9 \text{ V}$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 V_{OV2}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right)_2 \times 0.2^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = 12.5$$

$$R_2 = \frac{V_o - V_G}{0.01 \text{ mA}}$$

$$= \frac{0.9 - 0.6}{0.01} = 30 \text{ k}\Omega$$

$$V_{GS2} = V_m + V_{OV2} = 0.6 \text{ V}$$

$$V_{D1} = V_{G2} = 0.9 + 0.6 = 1.5 \text{ V}$$

(b) Converting the signal source to its Norton's form results in the circuit shown in Fig. 2. Now, setting the open-loop gain to  $\infty$ , the input voltage  $V_{gs1}$  becomes zero and thus the currents in  $R_1$  and  $R_s$  become zero. As a result, the current  $I_s = V_s/R_s$  flows through  $R_2$  and provides  $V_o = -I_s R_2 = -\frac{V_s}{R_s} R_2$

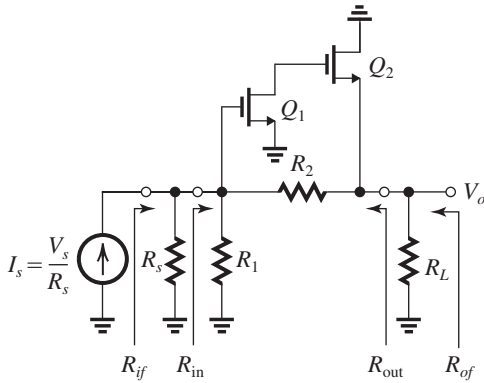


Figure 2

Thus, the ideal value of the voltage gain is

$$\frac{V_o}{V_s} = -\frac{R_2}{R_s}$$

The ideal value of the transresistance gain,

$$A_f|_{\text{ideal}} \equiv \frac{V_o}{I_s} = -R_2$$

Thus,

$$\beta = 1/A_f|_{\text{ideal}} = -\frac{1}{R_2}$$

The  $\beta$  circuit is shown in Fig. 3.

(c)

$$-5 = \frac{R_2}{R_s} = \frac{30}{R_s}$$

$$\Rightarrow R_s = 6 \text{ k}\Omega$$

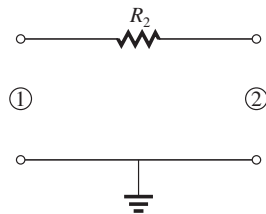


Figure 3

(d)

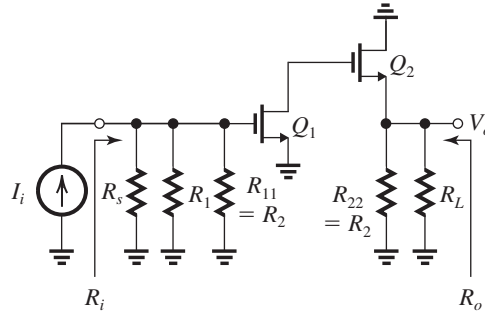


Figure 4

The A circuit is shown in Fig. 4.

$$R_i = R_s \parallel R_1 \parallel R_2$$

$$= 6 \parallel 60 \parallel 30 = 4.62 \text{ k}\Omega$$

$$V_{gs1} = I_i R_i = 4.62 I_i \tag{1}$$

$$V_{d1} = -g_{m1} V_{gs1} r_{o1}$$

where

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Thus,

$$V_{d1} = -100 V_{gs1} \tag{2}$$

$$\frac{V_o}{V_{d1}} = \frac{R_L \parallel R_2 \parallel r_{o2}}{(R_L \parallel R_2 \parallel r_{o2}) + 1/g_{m2}}$$

where

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_{D2}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Thus,

$$\frac{V_o}{V_{d1}} = \frac{(10 \parallel 30 \parallel 100)}{(10 \parallel 30 \parallel 100) + 1} = 0.875 \text{ V/V} \tag{3}$$

Combining (1)–(3), we obtain

$$A = \frac{V_o}{I_i} = -4.62 \times 100 \times 0.875$$

$$= -404 \text{ k}\Omega$$

$$R_o = R_L \parallel R_2 \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$= 10 \parallel 30 \parallel 100 \parallel 1$$

$$= 875 \text{ }\Omega$$

$$(e) A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{404}{1 + (404/30)}$$

$$= -\frac{404}{14.47} = -27.9 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{27.9}{6} = -4.65 \text{ V/V}$$

$$(f) R_{if} = \frac{R_i}{1 + A\beta} = \frac{4.62}{14.46} = 0.32 \text{ k}\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$0.32 = 6 \parallel R_{in}$$

$$\Rightarrow R_{in} = 337 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{875}{14.46} = 60.5 \Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$60.5 = R_{out} \parallel 10,000 \Rightarrow R_{out} = 61 \Omega$$

**10.70** (a) Converting the signal source to its Norton's form, we obtain the circuit shown in Fig. 1(a) on the next page.

This is a shunt–shunt feedback amplifier with the feedback network consisting of the resistor  $R_f$ . To determine  $\beta$ , we use the arrangement shown in Fig. 1(b) on the next page,

$$\beta = -\frac{1}{R_f}$$

Now, for  $A\beta \gg 1$ , the closed-loop gain becomes

$$A_f \equiv \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_f$$

The voltage gain  $\frac{V_o}{V_s}$  is obtained as

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$\frac{V_o}{V_s} \simeq -\frac{R_f}{R_s} \quad \text{Q.E.D.}$$

(b) To obtain a closed-loop voltage gain of approximately  $-100 \text{ V/V}$ , we use

$$-100 = -\frac{R_f}{R_s}$$

For  $R_s = 10 \text{ k}\Omega$ , we obtain

$$R_f = 1 \text{ M}\Omega$$

Now consider the amplifier stage shown in Fig. P10.70(b). First, we determine the dc bias point as follows:

$$I_E = \frac{15 \times \frac{10}{10 + 15} - 0.7}{4.7 + \frac{10 \parallel 15}{101}} = 1.11 \text{ mA}$$

$$I_C = 1.11 \times 0.99 = 1.1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.1}{0.025} = 44 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{44} = 2.27 \text{ k}\Omega$$

$$R_{in} = 10 \parallel 15 \parallel 2.27 = 1.65 \text{ k}\Omega$$

$$R_{out} = 7.5 \text{ k}\Omega$$

$$A_{vo} = -g_m \times 7.5 \text{ k}\Omega$$

$$= -44 \times 7.5 = -330 \text{ V/V}$$

Figure 2(a) on the next page shows the equivalent circuit of the amplifier stage. Figure 2(b) shows the  $A$  circuit of the feedback amplifier made up of the cascade of three stages. Observe that we have included  $R_s$  and  $R_L$  as well as  $R_{11}$  and  $R_{22}$ . The overall gain  $A \equiv V_o/I_s$  can be obtained as follows:

$$R_i = R_s \parallel R_f \parallel 1.65 \text{ k}\Omega$$

$$= 10 \parallel 1000 \parallel 1.65 = 1.414 \text{ k}\Omega$$

$$V_1 = I_i R_i = 1.414 I_i \quad (1)$$

$$V_2 = -330 V_1 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_1 \quad (2)$$

$$V_3 = -330 V_2 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_2 \quad (3)$$

$$V_o = -330 V_3 \times \frac{1 \parallel 1000}{(1 \parallel 1000) + 7.5} = -38.8 V_3 \quad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{V_o}{I_i} = -1.94 \times 10^5 \text{ k}\Omega$$

Since

$$\beta = -\frac{1}{R_f} = -\frac{1}{1 \text{ M}\Omega} = -\frac{1}{1000 \text{ k}\Omega}$$

we have

$$A\beta = 194$$

and

$$1 + A\beta = 195$$

Thus,

$$A_f \equiv \frac{V_o}{I_s} = -\frac{1.94 \times 10^5}{195}$$

$$= -995 \text{ k}\Omega$$

and the voltage gain realized is

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = \frac{-995}{10} = -99.5 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

This figure belongs to Problem 10.70, part (a).

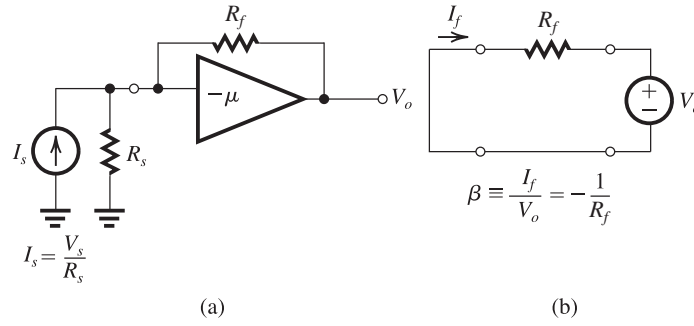


Figure 1

This figure belongs to Problem 10.70, part (b).

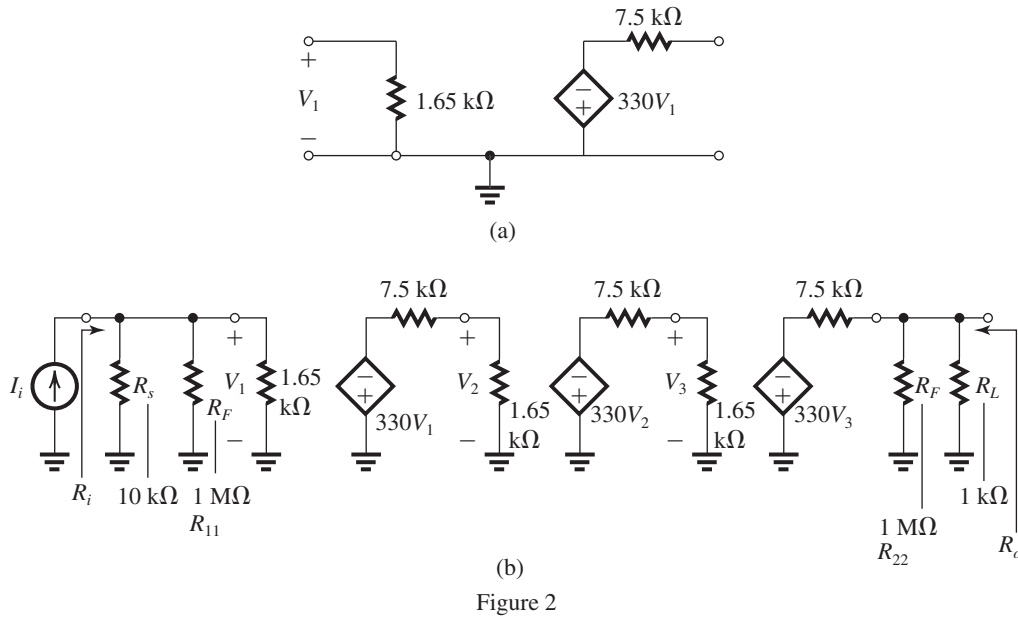


Figure 2

$$= \frac{1414 \Omega}{195} = 7.25 \Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$R_{in} \simeq 7.3 \Omega$$

From the A circuit, we have

$$R_o = R_L \parallel R_F \parallel 7.5 \text{ k}\Omega$$

$$R_o = 1 \parallel 1000 \parallel 7.5 = 881.6 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{881.6}{195} = 4.5 \Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$\Rightarrow R_{out} \simeq 4.5 \Omega$$

**10.71** Refer to Fig. 10.30(c), which shows the determination of  $\beta$ ,

$$\beta = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \tag{1}$$

Refer to Fig. 10.30(e), which shows the A circuit. The input resistance  $R_i$  is given by

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$$

For our case here,  $R_s = R_{id} = \infty$ , thus

$$R_i = R_1 + R_2$$

For  $R_{in} = R_{if} = 1 \text{ k}\Omega$ , we have

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$\Rightarrow R_i = R_{if}(1 + A\beta)$$

Thus

$$R_1 + R_2 = 1 \text{ k}\Omega \times (1 + A\beta)$$

Since  $1 + A\beta$  is 40 dB, that is,

$$1 + A\beta = 100$$



we have

$$R_1 + R_2 = 1 \times 100 = 100 \text{ k}\Omega \quad (2)$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$-100 = \frac{A}{100}$$

$$\Rightarrow A = -10^4 \text{ A/A}$$

$$\beta = \frac{A\beta}{A} = \frac{99}{-10^4} = -0.0099$$

Using Eqs. (1) and (2), we obtain

$$-0.0099 = -\frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_1 = 0.0099 \times 100 = 0.99 \text{ k}\Omega$$

$$R_2 = 100 - 0.99 = 99.01 \text{ k}\Omega$$

From Example 10.10, we obtain

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$-10^4 =$$

$$-\mu \frac{100}{0.2 + (0.99 \parallel 99.01 \parallel 20)} \frac{20}{20 + (0.99 \parallel 99.01)}$$

$$\mu = 119 \text{ V/V}$$

From Example 10.10, we have

$$R_o = r_{o2} + (R_1 \parallel R_2) + g_{m2}r_{o2}(R_1 \parallel R_2)$$

$$R_o = 20 + (0.99 \parallel 99.01)(1 + 5 \times 20)$$

$$= 119 \text{ k}\Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta) = 119 \times 100 = 11.9 \text{ M}\Omega$$

**10.72 (a)**

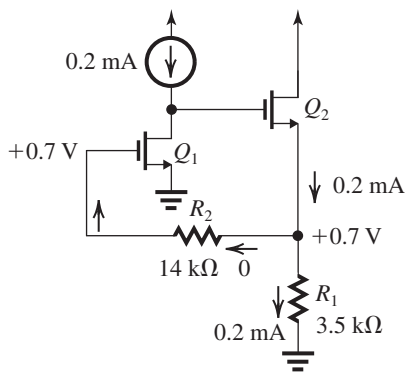


Figure 1

Figure 1 shows the dc analysis. It starts by noting that  $I_{D1} = 0.2 \text{ mA}$ . Thus  $V_{OV1} = 0.2 \text{ V}$  and

$$V_{G1} = V_{GS1} = V_t + V_{OV1} = 0.5 + 0.2$$

$$= 0.7 \text{ V}$$

Since the dc current through  $R_2$  is zero, the dc voltage drop across it will be zero, thus

$$V_{S2} = +0.7 \text{ V}$$

and

$$I_{R1} = \frac{0.7 \text{ V}}{3.5 \text{ k}\Omega} = 0.2 \text{ mA}$$

Thus,  $Q_2$  is operating at

$$I_D = 0.2 \text{ mA}$$

Q.E.D.

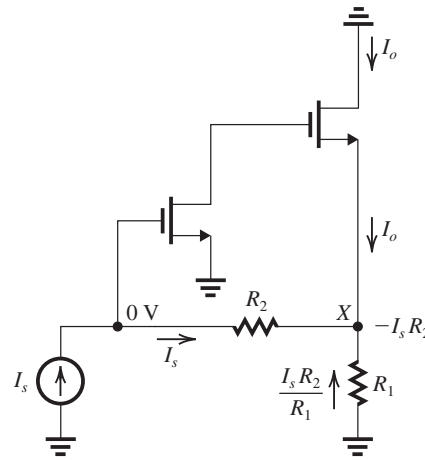


Figure 2

(b) To determine the ideal closed-loop gain, we set  $A = \infty$ . This results in  $V_i = 0$  as shown in Fig. 2. A node equation at X provides,

$$I_o = -I_s \left(1 + \frac{R_2}{R_1}\right)$$

Thus,

$$A_f|_{\text{ideal}} \equiv \frac{I_o}{I_s} = -\left(1 + \frac{R_2}{R_1}\right)$$

Substituting  $R_1 = 3.5 \text{ k}\Omega$  and  $R_2 = 14 \text{ k}\Omega$ ,

$$A_f|_{\text{ideal}} = -5 \text{ A/A}$$

and

$$\beta = \frac{1}{A_f|_{\text{ideal}}} = -0.2 \text{ A/A}$$

$$(c) \quad g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}}$$

$$= \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

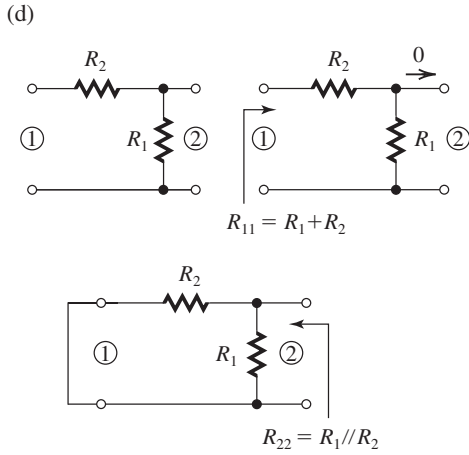


Figure 3

Figure 3 shows the  $\beta$  circuit and the determination of its loading effects,  $R_{11}$  and  $R_{22}$ ,

$$R_{11} = R_1 + R_2 = 3.5 + 14 = 17.5 \text{ k}\Omega$$

$$R_{22} = R_1 \parallel R_2 = 3.5 \parallel 14 = 2.8 \text{ k}\Omega$$

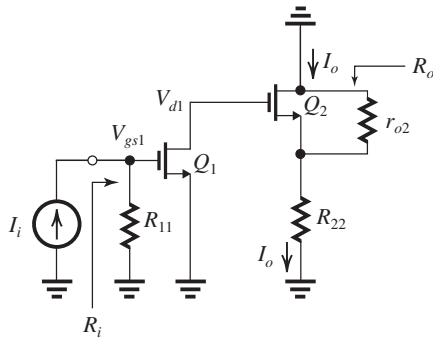


Figure 4

Figure 4 shows the  $A$  circuit. To obtain  $A = I_o/I_i$ , we write

$$V_{gs1} = I_i R_{11} \quad (1)$$

$$V_{d1} = -g_{m1} r_{o1} V_{gs1} \quad (2)$$

$$I_o = \frac{V_{d1}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} \frac{r_{o2}}{r_{o2} + R_{22}} \quad (3)$$

Combining (1)–(3) yields

$$A = \frac{I_o}{I_i} = - \frac{R_{11}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + R_{22}}$$

$$A = - \frac{17.5}{0.5 + (50 \parallel 2.8)} \times 2 \times 50 \times \frac{50}{50 + 2.8}$$

$$A = -525.8 \text{ A/A}$$

$$R_i = R_{11} = 17.5 \text{ k}\Omega$$

$$R_o = r_{o2} + R_{22} + g_{m2} r_{o2} R_{22}$$

$$= 50 + 2.8 + 2 \times 50 \times 2.8$$

$$= 332.8 \text{ k}\Omega$$

$$(e) A\beta = -525.8 \times -0.2$$

$$= 105.16$$

$$1 + A\beta = 106.16$$

$$A_f = - \frac{525.8}{106.16} = -4.95 \text{ A/A}$$

$$(f) R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{17.5 \text{ k}\Omega}{106.16} = 164.8 \text{ }\Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 332.8 \times 106.16 = 35.3 \text{ M}\Omega$$

**10.73** (a) Figure 1 on the next page shows the  $\beta$  network as well as the determination of its loading effects on the  $A$  circuit:

$$R_{11} = R_F + R_M$$

$$R_{22} = R_M \parallel R_F$$

Figure 2 on the next page shows the  $A$  circuit. Some of the analysis is shown on the diagram.

$$R_i = R_{11} \parallel \frac{1}{g_{m1}} \quad (1)$$

$$V_{sg1} = I_i R_i \quad (2)$$

$$V_{gs2} = g_{m1} V_{sg1} R_D \quad (3)$$

$$I_o = -g_{m2} V_{gs2} \quad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{I_o}{I_i} = - \left( R_{11} \parallel \frac{1}{g_{m1}} \right) (g_{m1} R_D) g_{m2} \quad (5)$$

(b) The  $\beta$  circuit prepared for the determination of  $\beta$  is shown in Fig. 3.

$$\beta \equiv \frac{I_f}{I_o} = - \frac{R_M}{R_M + R_F} \quad (6)$$

(c) From (5)–(6), we obtain

$$A\beta = \frac{R_M}{R_M + R_F} \left( R_{11} \parallel \frac{1}{g_{m1}} \right) (g_{m1} R_D) g_{m2}$$

(d)  $g_{m1} = g_{m2} = 5 \text{ mA/V}$ ,  $R_D = 20 \text{ k}\Omega$

$R_M = 10 \text{ k}\Omega$ , and  $R_F = 90 \text{ k}\Omega$ , thus

$$R_{11} = R_F + R_M = 90 + 10 = 100 \text{ k}\Omega$$

$$R_{22} = R_M \parallel R_F = 10 \parallel 90 = 9 \text{ k}\Omega$$

$$A = -(100 \parallel 0.2) \times (5 \times 20) \times 5 = -99.8 \text{ A/A}$$

$$\beta = - \frac{10}{10 + 90} = -0.1 \text{ A/A}$$

These figures belong to Problem 10.73, part (a).

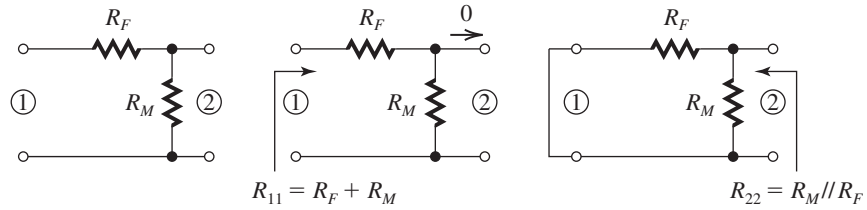


Figure 1

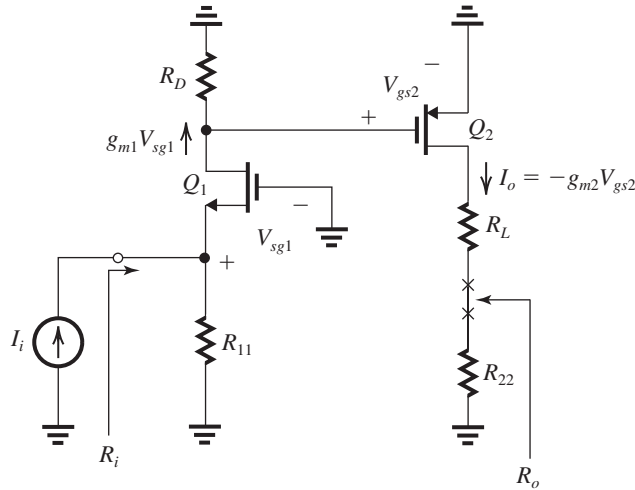


Figure 2

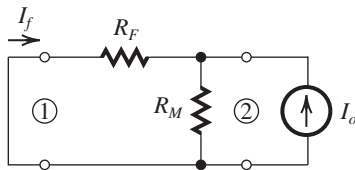


Figure 3

$$\begin{aligned}
 &= (R_M \parallel R_F) + R_L + r_{o2} \\
 &= (10 \parallel 90) + 1 + 20 \\
 &= 30 \text{ k}\Omega \\
 R_{of} &= R_o(1 + A\beta) = 30 \times 10.98 \\
 &= 329.4 \text{ k}\Omega \\
 R_{out} &= R_{of} - R_L = 328.4 \text{ k}\Omega
 \end{aligned}$$

$$A\beta = 9.98$$

$$1 + A\beta = 10.98$$

$$A_f \equiv \frac{I_o}{I_s} = \frac{A}{1 + A\beta} = -\frac{99.8}{10.98} = -9.1 \text{ A/A}$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

where

$$R_i = 100 \text{ k}\Omega \parallel 0.2 \text{ k}\Omega \approx 0.2 \text{ k}\Omega$$

$$R_{in} = \frac{200 \Omega}{10.98} = 18.2 \Omega$$

(e) Breaking the output loop of the A circuit between XX, we find

$$R_o = R_{22} + R_L + r_{o2}$$

### 10.74

The dc analysis is shown in Fig. 1, from which we see that

$$I_{C1} = 0.1 \text{ mA}$$

$$I_{C2} = 10 \text{ mA}$$

Thus,

$$g_{m1} = 4 \text{ mA/V}$$

$$r_{\pi 1} = 25 \text{ k}\Omega$$

$$g_{m2} = 400 \text{ mA/V}$$

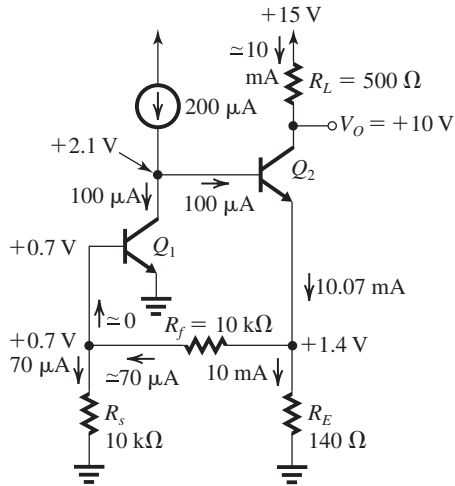


Figure 1

This is a shunt-series feedback amplifier with an ideal closed-loop gain  $A_f|_{\text{ideal}}$  of

$$A_f|_{\text{ideal}} = - \left( 1 + \frac{R_f}{R_E} \right)$$

$$= - \left( 1 + \frac{10}{0.14} \right) = -72.4 \text{ A/A}$$

The  $\beta$  circuit is shown in Fig. 2 on the next page together with the determination of its loading effects and of  $\beta$ .

$$R_{11} = R_f + R_E = 10.14 \text{ k}\Omega$$

$$R_{22} = R_f \parallel R_E = 10 \parallel 0.14 = 0.138 \text{ k}\Omega$$

$$\beta = - \frac{R_E}{R_f + R_E} = - \frac{0.14}{10 + 0.14} = 0.0138 \text{ A/A}$$

The A circuit is shown in Fig. 3 on the next page.

$$R_i = R_s \parallel R_{11} \parallel r_{\pi 1}$$

$$= 10 \parallel 10.14 \parallel 25$$

$$= 4.19 \text{ k}\Omega$$

$$V_{\pi 1} = I_i R_i$$

$$I_o = -\beta_2 g_{m1} V_{\pi 1}$$

$$\Rightarrow A = \frac{I_o}{I_i} = -\beta_2 g_{m1} R_i$$

$$A = -100 \times 4 \times 4.19 = -1676 \text{ A/A}$$

$$A\beta = -1676 \times 0.0138 = 23.13$$

$$1 + A\beta = 24.13$$

$$A_f = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$I_s = \frac{V_s}{R_s}$$

$$A_f = - \frac{1676}{24.13} = -69.5 \text{ A/A}$$

$$\frac{V_o}{V_s} = \frac{-I_o R_L}{I_s R_s} = 69.5 \times \frac{0.5}{10} = 3.48 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4190 \Omega}{24.13} = 173.6 \Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$173.6 \Omega = 10,000 \Omega \parallel R_{in}$$

$$\Rightarrow R_{in} = 176.7 \Omega$$

### 10.75

Figure 1 shows the shunt-series feedback amplifier circuit of Fig. 10.30(a) prepared for determining the loop gain,

$$A\beta \equiv - \frac{V_r}{V_i}$$

Observe that here  $R_s = R_{id} = \infty$ . Thus  $I_o$  can be obtained as

$$I_o = \frac{V_i}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1} \quad (1)$$

The voltage  $V_r$  can be obtained as

$$V_r = I_o R_1 \times -\mu = -\mu R_1 I_o \quad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv - \frac{V_r}{V_i} = \mu \frac{R_1}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1}$$

For

$$\mu = 1000 \text{ V/V}, \quad R_1 = 10 \text{ k}\Omega,$$

$$g_m = 5 \text{ mA/V}, \quad \text{and } r_{o2} = 20 \text{ k}\Omega$$

we obtain

$$A\beta = 1000 \times \frac{10}{0.2 + (20 \parallel 10)} \frac{20}{20 + 10}$$

$$= 970.9$$

which is slightly lower than the value found in Example 10.10 (1076.4), the difference being about  $-10\%$ . This is a result of the assumptions and approximations made in the general feedback analysis method.

From Example 10.10 (or directly from the  $\beta$  circuit) we have

$$\beta = -0.1 \text{ A/A}$$

Thus,

$$A = -9709$$

These figures belong to Problem 10.74.

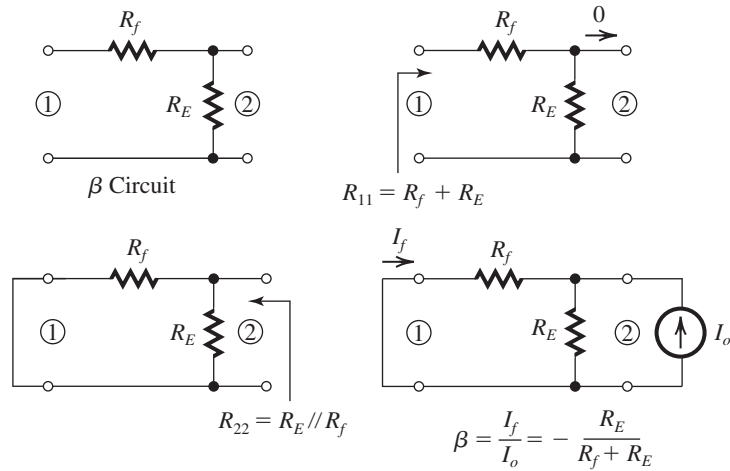


Figure 2

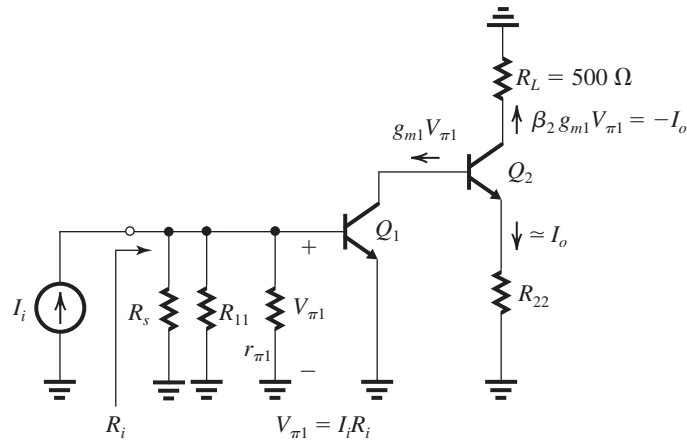


Figure 3

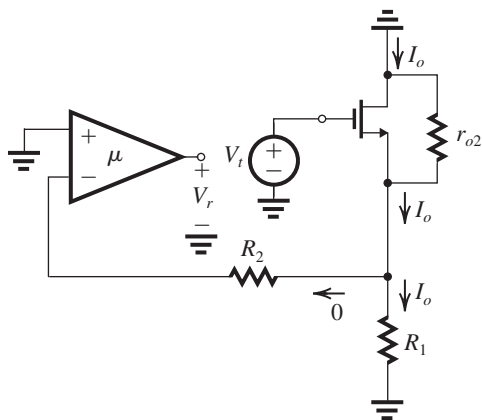


Figure 1

and

$$A_f = -\frac{9709}{971.9} = -9.99 \text{ A/A}$$

which is identical to the value obtained in Example 11.10. Thus while  $A\beta$  and  $A$  differ slightly for the earlier results,  $A_f$  is identical; an illustration of the power of negative feedback!

**10.76 (a)** If  $\mu$  is a very large, a virtual ground will appear at the input terminal. Thus the input resistance  $R_{in} = V_- / I_i = 0$ . Since no current flows in  $R_s$ , or into the amplifier input terminal, all the current  $I_s$  will flow in the transistor source terminal and hence into the drain, thus

$$I_o = I_s$$

These figures belong to Problem 10.76, part (b).

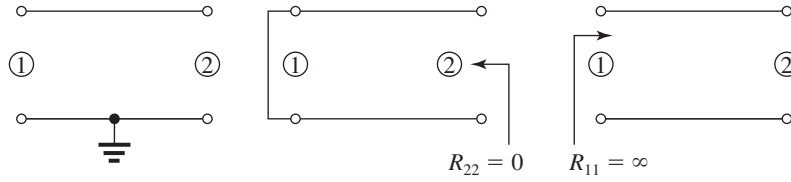


Figure 1

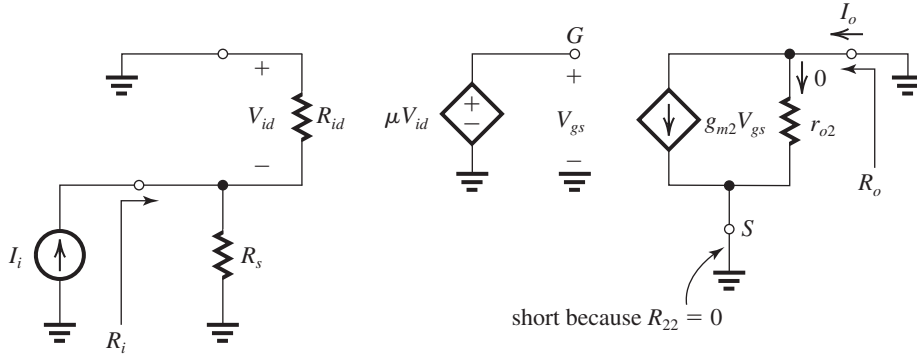


Figure 2

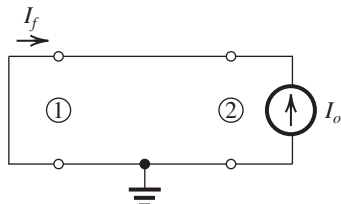


Figure 3

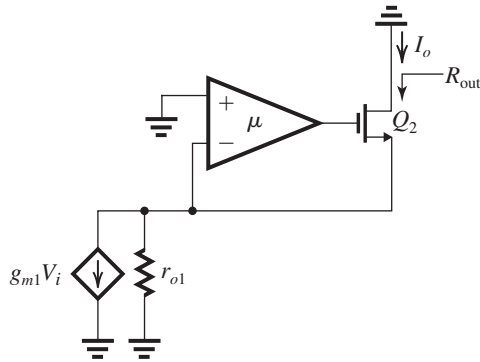


Figure 4

and

$$\frac{I_o}{I_s} = 1$$

(b) This is a shunt-series feedback amplifier in which the feedback circuit consists of a wire, as shown in Fig. 1. As indicated,

$$R_{11} = \infty$$

$$R_{22} = 0$$

The A circuit is shown in Fig. 2, for which we can write

$$V_{id} = -I_i(R_s \parallel R_{id}) \approx -I_i R_s \quad (1)$$

(since  $R_{id}$  is very large)

$$V_{gs} = \mu V_{id} \quad (2)$$

$$I_o = g_{m2} V_{gs} \quad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{I_o}{I_i} = -\mu g_{m2} R_s$$

$$R_i = R_s$$

$$R_o = r_{o2}$$

(c)

From Fig. 3 we find

$$\beta \equiv \frac{I_f}{I_o} = -1$$

$$(d) A\beta = \mu g_{m2} R_s$$

$$A_f = \frac{A}{1 + A\beta} = -\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

Note that the negative sign is due to our assumption that  $I_s$  flows into the input node (see Fig. 2 for the way  $I_i$  is applied). If instead  $I_s$  is flowing out of the input node, as indicated in Fig. P10.76, then

$$A_f \equiv \frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

If  $\mu$  is large so that  $\mu g_{m2} R_s \gg 1$ ,

$$A_f \simeq 1$$

$$(e) R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{R_s}{\mu g_{m2} R_s}$$

$$R_{if} = R_{in} \parallel R_s$$

$$\frac{1}{R_{if}} = \frac{1}{R_{in}} + \frac{1}{R_s}$$

$$\frac{1}{R_s} + \mu g_{m2} = \frac{1}{R_{in}} + \frac{1}{R_s}$$

$$\Rightarrow R_{in} = \frac{1}{\mu g_{m2}}$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= r_{o2}(1 + \mu g_{m2} R_s)$$

(f)

Figure 4 shows the circuit of the cascode amplifier in Fig. P10.76 with  $V_G$  replaced by signal ground, and  $Q_1$  replaced by its equivalent circuit at the drain. This circuit looks identical to that of Fig. P10.76(a). Thus we can write

$$I_o \simeq g_{m1} v_i$$

$$R_{out} = r_{o2}(1 + \mu g_{m2} r_{o1}) = r_{o2} + \mu(g_{m2} r_{o2}) r_{o1}$$

$$\simeq \mu(g_{m2} r_{o2}) r_{o1}$$

Compared to the case of a regular cascode, we see that while  $I_o \simeq g_{m1} V_i$  as in the regular cascode, utilizing the "super" CG transistor results in increasing the output resistance by the additional factor  $\mu$ !

**10.77** (a) Refer to Fig. P10.77. Let  $I_s$  increase. This will cause the voltage at the input node, which is the voltage at the positive input of the amplifier  $\mu$ , to increase. The amplifier output voltage will correspondingly increase. Thus,  $V_{gs}$  of  $Q_1$  increases and  $I_{o1}$  increases. This counteracts the originally assumed change of increased current into the input node. Thus, the feedback is negative.

(b) The negative feedback will cause the dc voltage at the positive input terminal of the amplifier to be equal to  $V_{BIAS}$ . Thus, the voltage at the drain of  $Q_1$  will be equal to  $V_{BIAS}$ . For  $Q_1$  to operate in the active mode, the minimum voltage at the drain must be equal to  $V_{OV}$  which is 0.2 V. Thus the minimum value of  $V_{BIAS}$  must be 0.2 V.

(c)

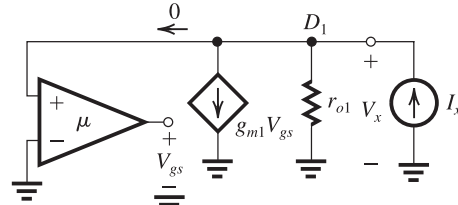


Figure 1

Figure 1 shows the circuit for determining  $R_{in}$ .

$$R_{in} \equiv \frac{V_x}{I_x}$$

A node equation at  $D_1$  gives

$$V_x = (I_x - g_{m1} V_{gs}) r_{o1}$$

$$\Rightarrow V_x = I_x r_{o1} - g_{m1} V_{gs} r_{o1}$$

But,

$$V_{gs} = \mu V_x,$$

thus

$$V_x = I_x r_{o1} - \mu g_{m1} r_{o1} V_x$$

$$\Rightarrow R_{in} \equiv \frac{V_x}{I_x} = \frac{r_{o1}}{1 + \mu g_{m1} r_{o1}}$$

Since  $\mu g_{m1} r_{o1} \gg 1$ , we have

$$R_{in} \simeq \frac{1}{\mu g_{m1}}$$

(d) Since the drain of  $Q_2$  is outside the feedback loop, we have

$$R_{out} = r_{o2}$$

**10.78** See figures on the next two pages.

(a) Refer to Fig. (a).

(b) Refer to Fig. (b).

(c)  $R_{outz} = r_{o2} \parallel r_{o4}$

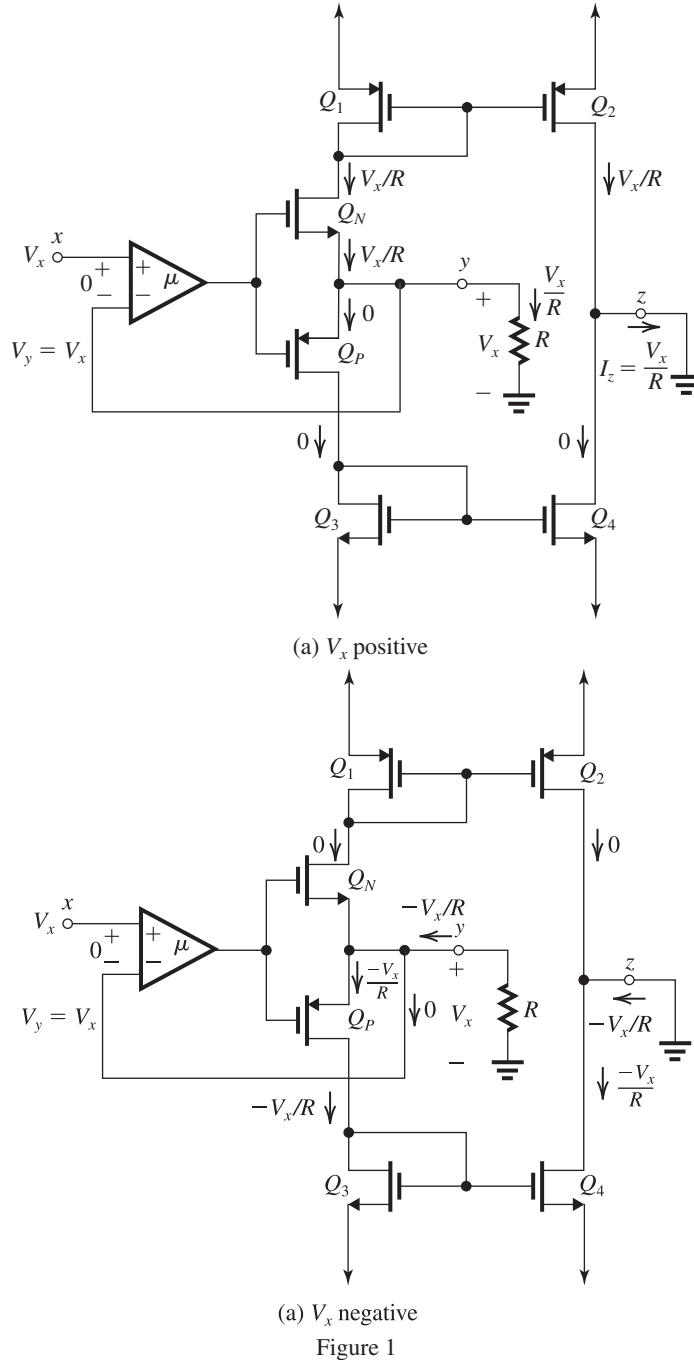
**10.79** (a) Refer to the circuit in Fig. P10.79. Observe that the feedback signal is capacitively coupled and so are the signal source and  $R_L$ ; thus, these do not enter into the dc bias calculations and the feedback does not affect the bias. The dc emitter current in  $Q_1$  can be determined from

$$I_{E1} = \frac{12 \times \frac{15}{100 + 15} - 0.7}{0.870 + \frac{100 \parallel 15}{101}} = 0.865 \text{ mA}$$

$$I_{C1} = 0.99 \times 0.865 = 0.86 \text{ mA}$$

Next consider  $Q_2$  and let its emitter current be  $I_{E2}$ . The base current of  $Q_2$  will be

These figures belong to Problem 10.78.



$I_{E2}/(\beta + 1) \simeq 0.01 I_{E2}$ . The current through  $R_{C1}$  will be  $(I_{C1} + I_{B2}) = (0.86 + 0.01 I_{E2})$ . We can thus write the following equation:

$$12 = (0.86 + 0.01 I_{E2}) \times 10 + 0.7 + 3.4 \times I_{E2}$$

$$\Rightarrow I_{E2} = \frac{12 - 8.6 - 0.7}{3.4 + 0.1} = \frac{2.7}{3.5} = 0.77 \text{ mA}$$

$$I_{C2} = 0.76 \text{ mA}$$

The small-signal parameters of  $Q_1$  and  $Q_2$  can now be obtained as

$$g_{m1} = \frac{0.86}{0.025} = 34.4 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{34.4} = 2.91 \text{ k}\Omega$$

$$g_{m2} = \frac{0.76}{0.025} = 30.4 \text{ mA/V}$$



These figures belong to Problem 10.78.

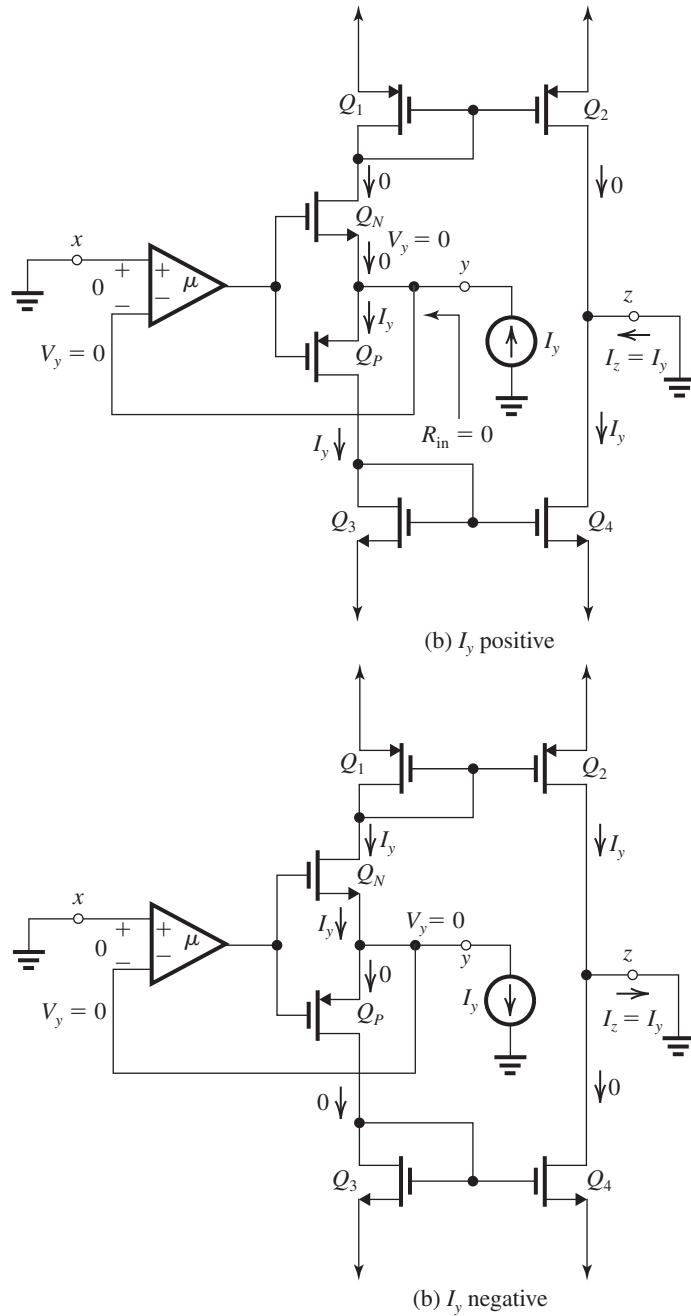


Figure 2

$$r_{\pi 2} = \frac{100}{30.4} = 3.3 \text{ k}\Omega$$

(b) The equivalent circuit of the feedback amplifier is shown in Fig. 1 on the next page, where

$$R_s = 10 \text{ k}\Omega$$

$$I_s = \frac{V_s}{R_s}$$

$$R_B = R_{B1} \parallel R_{B2} = 13 \text{ k}\Omega$$

(c) To obtain  $A_f|_{\text{ideal}}$ , we set  $A = \infty$ . This results in  $V_{\pi 1}$  (Fig. 1) being zero. Thus  $I_s$  flows into  $R_f$  and establishes a voltage  $-I_s R_f$  at node X. Writing a node equation at X provides

$$I_o = -I_s \left( 1 + \frac{R_f}{R_{E2}} \right)$$

Thus,

This figure belongs to Problem 10.79, part (b).

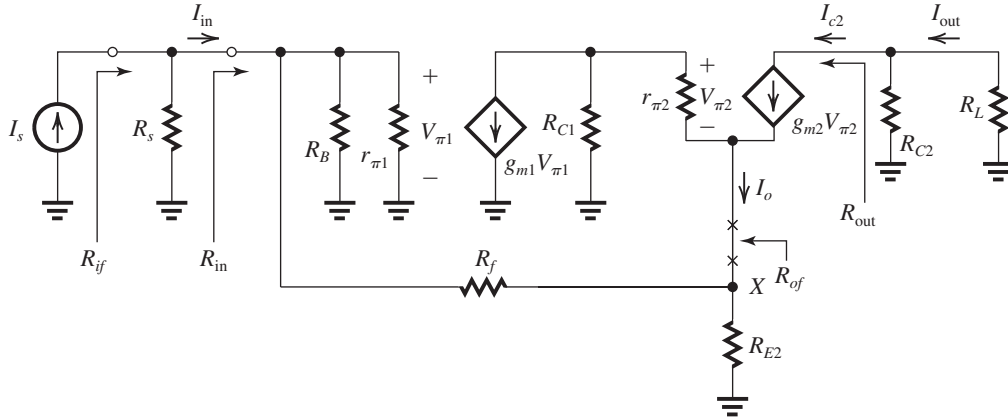


Figure 1

These figures belong to Problem 10.79, part (c).

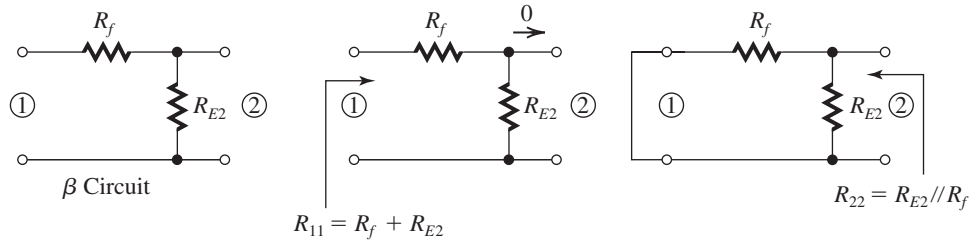


Figure 2

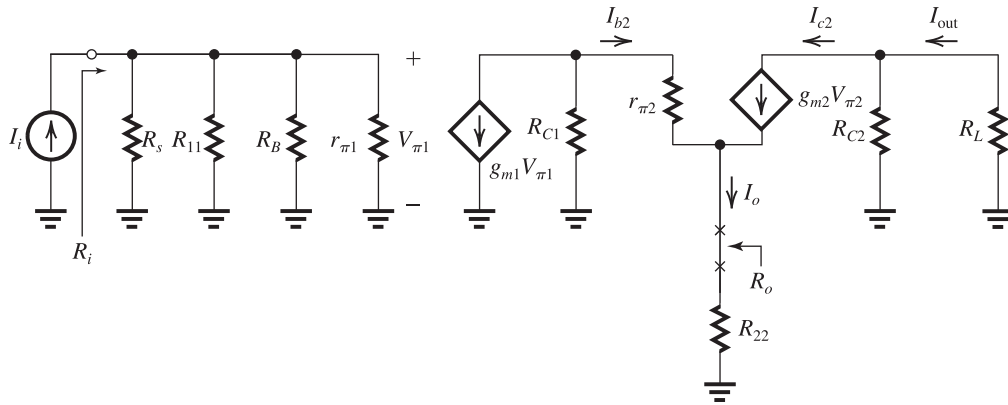


Figure 3

$$A_f|_{\text{ideal}} \equiv \frac{I_o}{I_s} = -(1 + \frac{R_f}{R_{E2}})$$

For  $R_f = 10 \text{ k}\Omega$  and  $R_{E2} = 3.4 \text{ k}\Omega$ .

$$A_f|_{\text{ideal}} = -(1 + \frac{10}{3.4}) = 3.94 \text{ A/A}$$

Now,

$$I_{\text{out}} = I_{c2} \frac{R_{C2}}{R_{C2} + R_L} = \alpha I_{e2} \frac{R_{C2}}{R_{C2} + R_L}$$

$$= \alpha I_o \frac{R_{C2}}{R_{C2} + R_L} = 0.99 \times \frac{8}{8 + 1} I_o$$

$$I_{\text{out}} = 0.88 I_o$$

In the ideal situation,

$$I_{\text{in}} = I_s$$

Thus, the ideal value of  $I_{\text{out}}/I_{\text{in}}$  is

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{0.88 I_o}{I_s} = 0.88 \times 3.94 = 3.47 \text{ A/A}$$

(d)

$$\beta = 1/A_f|_{\text{ideal}} = \frac{1}{-3.94} = -0.254 \text{ A/A}$$

The  $\beta$  circuit is shown in Fig. 2 on the previous page.

(e) The determination of the loading effects of the  $\beta$  circuit on the  $A$  circuit is shown in Fig. 2:

$$R_{11} = R_f + R_{E2} = 10 + 3.4 = 13.4 \text{ k}\Omega$$

$$R_{22} = R_{E2} \parallel R_f = 3.4 \parallel 10 = 2.54 \text{ k}\Omega$$

The  $A$  circuit is shown in Fig. 3 on the previous page.

Analysis of the  $A$  circuit to determine  $A \equiv I_o/I_s$  proceeds as follows:

$$\begin{aligned} R_i &= R_s \parallel R_{11} \parallel R_B \parallel r_{\pi 1} \\ &= 10 \parallel 13.4 \parallel 13 \parallel 2.91 = 1.68 \text{ k}\Omega \\ V_{\pi 1} &= I_i R_i \end{aligned} \quad (1)$$

$$I_{b2} = -g_{m1} V_{\pi 1} \frac{R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1)R_{22}} \quad (2)$$

$$I_o = I_{e2} = (\beta + 1)I_{b2} \quad (3)$$

Combining Eqs. (1)–(3) results in

$$\begin{aligned} A \equiv \frac{I_o}{I_i} &= -\frac{(\beta + 1)R_i g_{m1} R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1)R_{22}} \\ &= \frac{101 \times 1.68 \times 34.4 \times 10}{10 + 3.3 + 101 \times 2.54} \\ &= -216.3 \text{ A/A} \end{aligned}$$

Breaking the emitter loop of  $Q_2$  at  $XX$  gives

$$\begin{aligned} R_o &= R_{22} + \frac{r_{\pi 2} + R_{C1}}{\beta + 1} \\ &= 2.54 + \frac{3.3 + 10}{101} = 2.67 \text{ k}\Omega \end{aligned}$$

$$(f) A\beta = -216.3 \times -0.254 = 54.9$$

$$1 + A\beta = 55.9$$

$$A_f = \frac{I_o}{I_s} = -\frac{216.3}{55.9} = -3.87 \text{ A/A}$$

which is very close to the ideal value of 3.94 found in (c) above.

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.68 \text{ k}\Omega}{55.9} = 30.1 \text{ }\Omega$$

$$\begin{aligned} R_{of} &= R_o(1 + A\beta) = 2.67 \times 55.9 \\ &= 149.2 \text{ k}\Omega \end{aligned}$$

$$(g) R_{if} = R_s \parallel R_{in}$$

$$30.1 \text{ }\Omega = 10 \text{ k}\Omega \parallel R_{in}$$

$$R_{in} = 30.2 \text{ }\Omega$$

$$I_{in} = I_s \frac{R_s}{R_s + R_{in}} \simeq I_s$$

$$I_{out} = I_{c2} \frac{R_{C2}}{R_{C2} + R_L} = \alpha I_o \frac{R_{C2}}{R_{C2} + R_L}$$

$$\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_s} = \frac{I_o}{I_s} \times \alpha \frac{R_{C2}}{R_{C2} + R_L}$$

$$\begin{aligned} \Rightarrow \frac{I_{out}}{I_{in}} &= -3.87 \times 0.99 \times \frac{8}{8 + 1} \\ &= -3.41 \text{ A/A} \end{aligned}$$

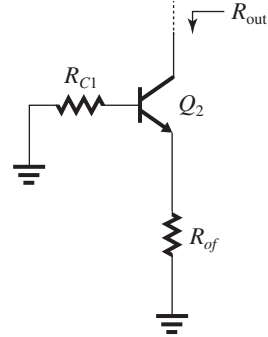


Figure 4

To determine  $R_{out}$ , consider the circuit in Fig. 4. Using the formula given at the end of Example 11.8, adapted to our case here, we get

$$R_{out} = r_{o2} + [R_{of} \parallel (r_{\pi 2} + R_{C1})] \left[ 1 + g_{m2} r_{o2} \frac{r_{\pi 2}}{r_{\pi 2} + R_{C1}} \right]$$

where

$$r_{o2} = \frac{75 \text{ V}}{0.76 \text{ mA}} = 98.7 \text{ k}\Omega$$

$$\begin{aligned} R_{out} &= 98.7 + [149.2 \parallel (3.3 + 10)] \left[ 1 + 30.3 \times 98.7 \times \frac{3.3}{3.3 + 10} \right] \\ &= 98.7 + 12.21 \times 743 = 9.17 \text{ M}\Omega \end{aligned}$$

Check:

Maximum possible  $R_{out} \simeq \beta r_o \simeq 10 \text{ M}\Omega$

So, our result is reasonable.

$$\mathbf{10.80} \quad A(s) = \frac{10^4}{(1 + \frac{s}{100})(1 + \frac{s}{10^4})^2}$$

$$\phi = -(\tan^{-1} \frac{\omega}{100} + 2 \tan^{-1} \frac{\omega}{10^4})$$

The phase angle will be  $-180^\circ$  at  $\omega \simeq 10^4 \text{ rad/s}$ .

Check:

$$\begin{aligned} \phi(\omega = 10^4 \text{ rad/s}) &= -(\tan^{-1} 100 + 2 \tan^{-1} 1) \\ &= -(89.4^\circ + 90^\circ) \simeq -180^\circ \end{aligned}$$

Thus,

$$\omega_{180} \simeq 10^4 \text{ rad/s}$$

$$\text{At } \omega = \omega_{180},$$

$$|A| = \frac{10^4}{\sqrt{1+100^2(1+1)}} \simeq 50$$

$$|A\beta| = 50\beta$$

Thus,

$$\beta_{\max} = \frac{1}{50} = 0.02$$

and

$$A_f|_{\min} = \frac{10^4}{1+10^4 \times 0.02} \simeq 50$$

$$\mathbf{10.81} \quad A(s)\beta(s) = \frac{10^4 k}{\left(1 + \frac{s}{10^3}\right)^3}$$

$$A(j\omega)\beta(j\omega) = \frac{10^4 k}{\left(1 + j\frac{\omega}{10^3}\right)^3}$$

$$|A(j\omega)\beta(j\omega)| = \frac{10^4 k}{\left(1 + \frac{\omega^2}{10^6}\right)^{3/2}}$$

$$\phi(\omega) = -3 \tan^{-1}\left(\frac{\omega}{10^3}\right)$$

$$-180^\circ = -3 \tan^{-1}\frac{\omega_{180}}{10^3}$$

$$\Rightarrow \omega_{180} = 10^3 \tan 60 = 1732 \text{ rad/s}$$

$$|A(j\omega_{180})\beta(j\omega_{180})| = \frac{10^4 k}{\left(1 + \frac{1732^2}{10^6}\right)^{3/2}}$$

$$= 0.125 \times 10^4 k$$

For stable operation,

$$|A(j\omega_{180})\beta(j\omega_{180})| < 1$$

$$\Rightarrow 0.125 \times 10^4 k < 1$$

$$k < 8 \times 10^{-4}$$

$$\mathbf{10.82} \quad A(s) = \frac{10^4}{\left(1 + \frac{s}{10^4}\right)\left(1 + \frac{s}{10^5}\right)^2}$$

$$A(j\omega) = \frac{10^4}{\left(1 + j\frac{\omega}{10^4}\right)\left(1 + j\frac{\omega}{10^5}\right)^2}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2 \tan^{-1}\left(\frac{\omega}{10^5}\right)$$

$$-180 = -\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) - 2 \tan^{-1}\left(\frac{\omega_{180}}{10^5}\right)$$

By trial and error we find

$$\omega_{180} = 1.095 \times 10^5 \text{ rad/s}$$

At this frequency,

$$|A| = \frac{10^4}{\sqrt{1+10.95^2(1+1.095^2)}} = 413.6$$

$$= 413.6$$

For stable operation,

$$|A|\beta_{cr} < 1$$

$$\beta_{cr} < 2.42 \times 10^{-3}$$

Thus, oscillation will commence for

$$\beta \geq 2.42 \times 10^{-3}$$

$$\mathbf{10.83} \quad A_f(0) = \frac{A_0}{1+A_0\beta}$$

where

$$A_0 = \frac{1 \text{ MHz}}{10 \text{ Hz}} = 10^5 \text{ V/V}$$

Thus,

$$A_f(0) = \frac{10^5}{1+10^5 \times 0.1} \simeq 10 \text{ V/V}$$

$$f_{3dB} = 10(1+A_0\beta)$$

$$= 10(1+10^5 \times 0.1) \simeq 10^5 \text{ Hz}$$

Unity-gain frequency of closed-loop amplifier

$$= A_f(0) \times f_{3dB}$$

$$= 10 \times 10^5 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

Thus, the pole shifts by a factor equal to the amount-of-feedback,  $(1+A_0\beta)$ .

**10.84** (a) The closed-loop poles become coincident when  $Q = 0.5$ . Using Eq. (10.42), we obtain

$$Q = \frac{\sqrt{(1+A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$0.5 = \frac{\sqrt{(1+A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$\Rightarrow 1 + A_0\beta = 0.5^2 \frac{(\omega_{P1} + \omega_{P2})^2}{\omega_{P1}\omega_{P2}}$$

$$= 0.5^2 \times \frac{(2\pi)^2(10^4 + 10^5)^2}{(2\pi)^2 \times 10^4 \times 10^5}$$

$$= 0.5^2 \times \frac{11^2}{10} = 3.025$$

$$\beta = 2.025 \times 10^{-4}$$

$$\omega_c = \frac{1}{2}(\omega_{P1} + \omega_{P2})$$

$$= \frac{1}{2} \times 2\pi(f_{P1} + f_{P2})$$

$$f_c = \frac{1}{2} \times (10^4 + 10^5) = 5.5 \times 10^4 \text{ Hz}$$

$$(b) \quad A_f(0) = \frac{A_0}{1+A_0\beta}$$

$$= \frac{10^4}{1 + 2.205 \times 10^{-4} \times 10^4} = 3306 \text{ V/V}$$

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta}$$

where

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$A_f(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) + A_0\beta}$$

$$= \frac{A_0}{(1 + A_0\beta) + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}}$$

$$A_f(j\omega_c) = \frac{A_0}{(1 + A_0\beta) + j\left(\frac{\omega}{\omega_{p1}} + \frac{\omega}{\omega_{p2}}\right) - \left(\frac{\omega}{\omega_{p1}}\right)\left(\frac{\omega}{\omega_{p2}}\right)}$$

$$A_f(j\omega_c) = \frac{10^4}{3.025 + j(5.5 + 0.55) - 5.5 \times 0.55}$$

$$A_f(j\omega_c) = \frac{10^4}{j 6.05}$$

$$|A_f(j\omega_c)| = \frac{10^4}{6.05} = 1653 \text{ V/V}$$

(c)  $Q = 0.5$ .

(d) If  $\beta = 2.025 \times 10^{-3} \text{ V/V}$ . Using Eq. (10.40), we obtain

$$s = -\frac{1}{2} \times 2\pi(10^4 + 10^5)$$

$$\pm \frac{1}{2} \times 2\pi \sqrt{(10^4 + 10^5)^2 - 4(1 + 10^4 \times 2.025 \times 10^{-3}) \times 10^4 \times 10^5}$$

$$\frac{s}{2\pi} = -5.5 \times 10^4$$

$$\pm 0.5 \sqrt{121 \times 10^8 - 4 \times 21.25 \times 10^9}$$

$$= -5.5 \times 10^4 \pm 0.5 \times 10^4 \sqrt{121 - 40 \times 21.25}$$

$$= -5.5 \times 10^4 \pm j0.5 \times 10^4 \times 27$$

$$= (-5.5 \pm j13.25) \times 10^4 \text{ Hz}$$

Using Eq. (10.42), we obtain

$$Q = \frac{\sqrt{(1 + 10^4 \times 2.025 \times 10^{-3})10^4 \times 10^5}}{10^4 + 10^5}$$

$$= 1.325$$

$$\mathbf{10.85} \quad A_0 = 10^4 \text{ V/V}$$

$$f_P = 100 \text{ Hz}$$

$$f_{Hf} = f_P(1 + A_0\beta) = 10 \text{ kHz}$$

$$\Rightarrow 1 + A_0\beta = \frac{10 \times 10^3}{100} = 100$$

$$\Rightarrow \beta = \frac{99}{10^4} = 0.0099 \text{ V/V}$$

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{10^4}{100} = 100 \text{ V/V}$$

$$A_f(s) = \frac{A_f(0)}{1 + \frac{s}{\omega_{Hf}}}$$

$$A_f(s) = \frac{100}{1 + s/2\pi \times 10^4}$$

$$\mathbf{10.86} \quad A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099$$

To obtain a maximally flat response,

$$Q = 0.707$$

Using Eq. (10.42), we obtain

$$0.707 = \frac{\sqrt{100 \times 1 \times f_{P2}}}{1 + f_{P2}}$$

$$\frac{1}{2} = \frac{100 f_{P2}}{(1 + f_{P2})^2}$$

$$f_{P2}^2 + 2 f_{P2} + 1 = 200 f_{P2}$$

$$f_{P2}^2 - 198 f_{P2} + 1 = 0$$

$$f_{P2} \simeq 198 \text{ kHz}$$

(the other solution is a very low frequency which obviously does not make physical sense).

The 3-dB frequency of the closed-loop amplifier is  $f_0$ , which can be obtained from Eq. (10.40) and the graphical construction of Fig. 10.34:

$$\frac{f_0}{2Q} = \frac{1}{2}(f_{P1} + f_{P2})$$

$$f_0 = \frac{1}{\sqrt{2}}(1 + 198) = 140.7 \text{ kHz}$$

$$\mathbf{10.87} \quad A_{f0} = 10$$

Maximally flat response with  $f_{sdB} = f_0 = 1 \text{ kHz}$ .

Let each stage in the cascade have a dc gain  $K$  and a 3-dB frequency  $f_P$ , thus

$$A(s) = \frac{K^2}{\left(1 + \frac{s}{\omega_P}\right)^2}$$

Using the expression for  $Q$  in Eq. (10.42),

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{p1}\omega_{p2}}}{\omega_{p1} + \omega_{p2}}$$

and substituting  $Q = 1/\sqrt{2}$ ,  $\omega_{p1} = \omega_{p2} = \omega_p$ , and  $A_0 = K^2$ , we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{1 + K^2\beta}}{2}$$

$$1 + K^2\beta = 2$$

and

$$K^2\beta = 1$$

Now,

$$A_{f0} = \frac{A_0}{1 + A_0\beta}$$

$$10 = \frac{K^2}{1 + K^2\beta}$$

$$= \frac{K^2}{2}$$

$$\Rightarrow K^2 = 20 \Rightarrow K = \sqrt{20} = 4.47 \text{ V/V}$$

$$\beta = \frac{1}{20} = 0.05 \text{ V/V}$$

$$f_0 = \sqrt{(1 + A_0\beta)f_p^2}$$

$$= \sqrt{2f_p^2}$$

$$= \sqrt{2}f_p$$

$$\Rightarrow f_p = \frac{f_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$= 0.707 \text{ kHz}$$

$$A_f(s) = \frac{10\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$A_f(s) = \frac{10(2\pi \times 10^3)^2}{s^2 + s\frac{2\pi \times 10^3}{1/\sqrt{2}} + (2\pi \times 10^3)^2}$$

**10.88** Let each stage have the transfer function

$$T(s) = \frac{K}{1 + \frac{s}{\omega_p}}$$

where  $\omega_p = 2\pi \times 100 \times 10^3 \text{ rad/s}$

$$A(s) = \left( \frac{K}{1 + \frac{s}{\omega_p}} \right)^3$$

$$\beta = 1$$

Thus the characteristic equation is given by

$$1 + A(s)\beta = 0$$

$$1 + \frac{K^3}{\left(1 + \frac{s}{\omega_p}\right)^3} = 0$$

To simplify matters, let  $\frac{s}{\omega_p} = S$ , where  $S$  is a normalized frequency variable, thus

$$(1 + S)^3 + K^3 = 0 \quad (1)$$

This equation has three roots, which are the poles of the feedback amplifier. One of the roots will be real and the other two can be complex conjugate depending on the value of  $K$ . The real pole can be directly obtained from Eq. (1) as

$$(1 + S_1)^3 = -K^3$$

$$(1 + S_1) = -K$$

$$S_1 = -(1 + K) \quad (2)$$

Now we need to obtain the two other poles. The characteristic equation in (1) can be written as

$$S^3 + 3S^2 + 3S + (1 + K^3) = 0 \quad (3)$$

Equivalently it can be written as

$$(S + 1 + K)(S^2 + aS + b) = 0 \quad (4)$$

Equating the coefficients of corresponding terms in (3) and (4), we can find  $a$  and  $b$  and thus obtain the quadratic factor

$$S^2 + (2 - K)S + (1 - K + K^2) = 0 \quad (5)$$

This equation can now be easily solved to obtain the pair of complex conjugate poles as

$$S_{2,3} = -1 + \frac{K}{2} \pm j\frac{\sqrt{3}}{2}K$$

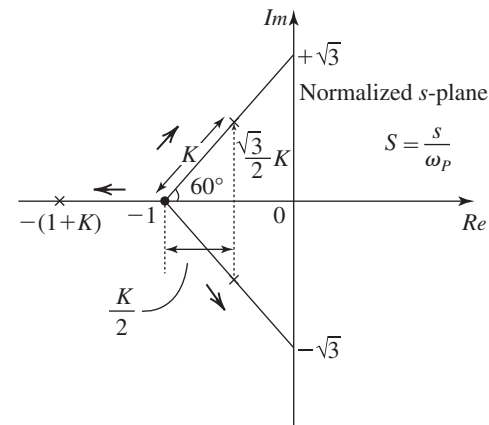


Figure 1

Figure 1 shows the root locus of the three poles in the normalized  $s$  plane (normalized relative to  $\omega_p = 2\pi \times 10^5$ ). Observe that as  $K$  increases the real pole  $S_1$  moves outwardly on the negative real axis. The pair of conjugate poles move on straight

lines with  $60^\circ$  angles to the horizontal. These two poles reach the  $j\omega$  axis (which is the boundary for stable operation) at  $K = 2$  at which point

$$S_{2,3} = \pm j\sqrt{3}$$

Thus the minimum value of  $K$  from which oscillations occur is  $K = 2$ . Oscillations will be at

$$\omega = \sqrt{3} \times 2\pi f_p$$

or

$$f = \sqrt{3} \times 100 \text{ kHz}$$

$$= 173.2 \text{ kHz}$$

**10.89**

$$A(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi \times 10}\right) \left(1 + \frac{s}{2\pi \times 10^3}\right)}$$

$$\beta = 0.01$$

$$A\beta(j\omega) = \frac{1000}{\left(1 + j\frac{\omega}{2\pi \times 10}\right) \left(1 + j\frac{\omega}{2\pi \times 10^3}\right)} \quad (1)$$

$$|A\beta| = \frac{1000}{\sqrt{\left[1 + \left(\frac{\omega}{2\pi \times 10}\right)^2\right] \left[1 + \left(\frac{\omega}{2\pi \times 10^3}\right)^2\right]}} \quad (2)$$

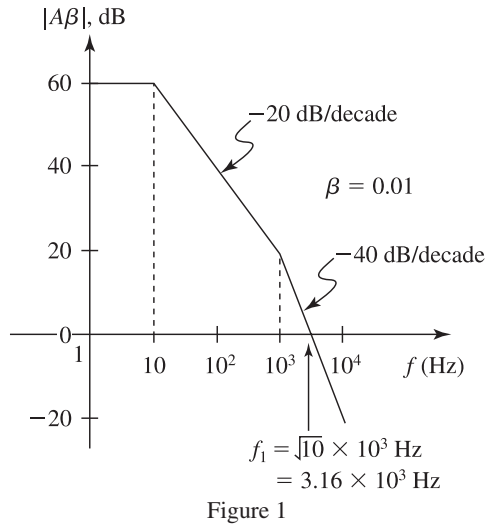


Figure 1 shows a sketch of the Bode plot for  $|A\beta|$ . Observe that the unity-gain frequency will occur on the  $-40$  dB/decade line. The value of  $f_1$  can be obtained from the Bode plot as follows: The  $-40$  dB/decade line represents gain drop proportional to  $1/f^2$ . For a drop by only 20 dB (a factor of 10) the change in frequency is by a factor  $\sqrt{10}$ , thus

$$f_1 = \sqrt{10} \times 10^3 \text{ Hz} = 3.16 \times 10^3 \text{ Hz}$$

However, the Bode plot results are usually approximate. If we require a more exact value for  $f_1$  we need to iterate a couple of times using the exact equation in (2). The result is

$$f_1 = 3.085 \times 10^3 \text{ Hz}$$

The phase angle can now be obtained using (1) as follows:

$$\begin{aligned} \phi &= -\tan^{-1} \frac{\omega_1}{2\pi \times 10} - \tan^{-1} \frac{\omega_1}{2\pi \times 10^3} \\ &= -\tan^{-1} \frac{f_1}{10} - \tan^{-1} \frac{f_1}{10^3} \\ &= -\tan^{-1}(3.085 \times 10^2) - \tan^{-1}(3.085) \\ &= -89.81^\circ - 72.03^\circ = -161.84 \end{aligned}$$

Thus,

$$\text{Phase margin} = 180^\circ - 161.84 = 18.15^\circ$$

To obtain a phase margin of  $45^\circ$ :

The phase shift due to the first pole will be  $\simeq 90^\circ$ . Thus, the phase shift due to the second pole must be  $\simeq -45^\circ$ , thus

$$-45^\circ = -\tan^{-1} \frac{f_1}{10^3}$$

$$f_1 \simeq 10^3 \text{ rad/s.}$$

Since  $f_1$  is two decades above  $f_p$  we need  $A_0\beta$  to be 100. Thus,  $\beta$  will now be

$$\beta = 100/10^5 = 10^{-3}$$

Figure 2 shows a sketch of the Bode plot for  $|A\beta|$  in this case. The closed-loop gain is 60 dB.

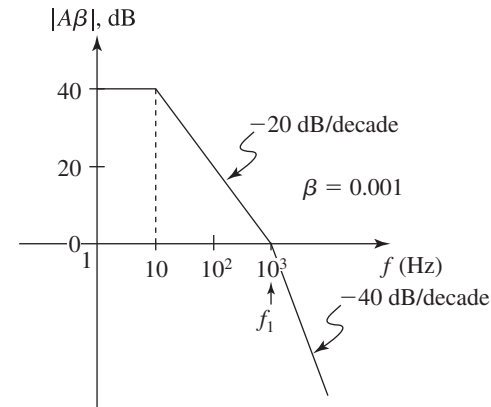


Figure 2

**10.90** Eq. (10.48) gives

$$\frac{|A_f(j\omega_1)|}{A_f(0)} = \frac{1}{|1 + e^{-j\theta}|}$$

Thus

$$P = \frac{1}{|1 + e^{-j\theta}|}$$

$$= \frac{1}{|1 + \cos \theta - j \sin \theta|}$$

$$\Rightarrow P^2 = \frac{1}{(1 + \cos \theta)^2 + \sin^2 \theta}$$

$$= \frac{1}{1 + (\cos^2 \theta + \sin^2 \theta) + 2 \cos \theta}$$

$$P^2 = \frac{1}{2 + 2 \cos \theta} = \frac{1}{2(1 + \cos \theta)}$$

Now, the phase margin  $\phi$  is

$$\phi = 180^\circ - \theta$$

Thus,

$$\theta = 180^\circ - \phi$$

and,

$$P^2 = \frac{1}{2[1 + \cos(180^\circ - \phi)]}$$

$$= \frac{1}{2(1 - \cos \phi)}$$

which can be written as

$$\cos \phi = 1 - \frac{1}{2P^2}$$

or equivalently

$$\phi = \cos^{-1}\left(1 - \frac{1}{2P^2}\right)$$

Q.E.D.

(b) We use this equation to obtain the following results:

$P$	$\phi$
1.05	56.9°
1.10	54.1°
0.1 dB $\equiv$ 1.0115	59.2°
1.0 dB $\equiv$ 1.122	52.9°
3 dB $\equiv$ 1.414	41.4°

This figure belongs to Problem 10.91.

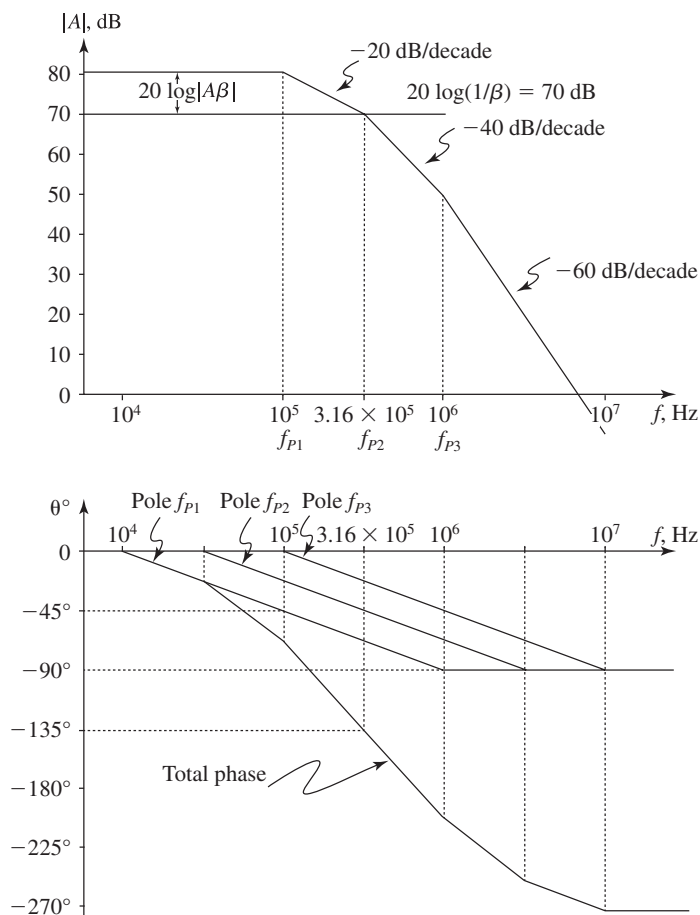


Figure 1



**10.91** Figure 1 on page 10–60 shows magnitude and phase Bode plots for the amplifier specified in this problem. From the phase plot we find that  $\theta = -135^\circ$  (which corresponds to a phase margin of  $45^\circ$ ) occurs at

$$f = 3.16 \times 10^5 \text{ Hz}$$

At this frequency,  $|A|$  is 70 dB. The  $\beta$  horizontal straight line drawn at 70-dB level gives

$$20 \log \left( \frac{1}{\beta} \right) = 70 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-4}$$

Correspondingly,

$$A_f = \frac{10^4}{1 + 10^4 \times 3.16 \times 10^{-4}} = 2.4 \times 10^3 \text{ V/V}$$

or 67.6 dB

**10.92**  $A_0 = 10^5$  with a single pole at

$$f_p = 10 \text{ Hz}$$

For a unity-gain buffer,  $\beta = 1$ , thus

$$A_0\beta = 10^5 \text{ and } f_p = 10 \text{ Hz}$$

Since the loop gain rolls off at a uniform slope of  $-20 \text{ dB/decade}$ , it will reach the 0 dB line ( $|A\beta| = 1$ ) five decades beyond 10 Hz. Thus the unity-gain frequency will be

$$f_1 = 10^5 \times 10 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

The phase shift will be that resulting from a single pole,  $-90^\circ$ , resulting in a phase margin:

$$\text{Phase margin} = 180^\circ - 90^\circ = 90^\circ$$

**10.93** Figure 1 is a replica of Fig. 10.38 except here we locate on the phase plot the points at which the phase margin is  $90^\circ$  and  $135^\circ$ , respectively. Drawing a vertical line from each of those points and locating the intersection with the  $|A|$  line enables us to determine the maximum  $\beta$  that can be used in each case. Thus, for  $\text{PM} = 90^\circ$ , we have

This figure belongs to Problem 10.93.

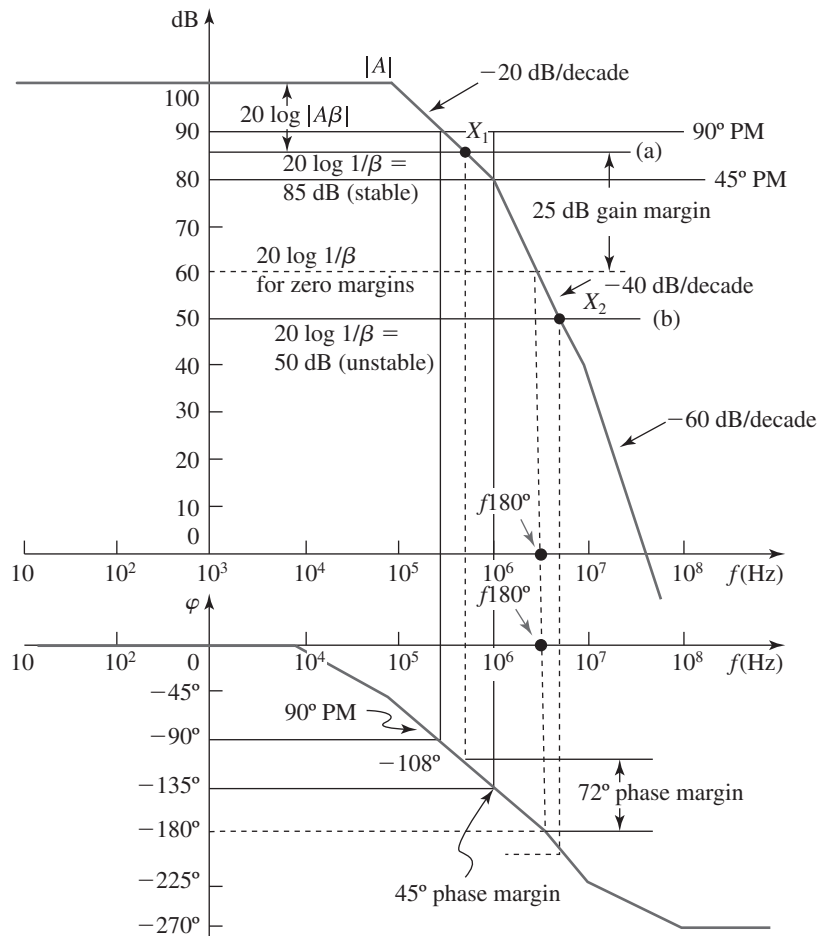


Figure 1

$$20 \log \frac{1}{\beta} = 90 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-5}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 3.16 \times 10^{-5}}$$

$$= 2.4 \times 10^4 \text{ V/V or } 87.6 \text{ dB}$$

and for PM = 45°, we have

$$20 \log \frac{1}{\beta} = 80 \text{ dB}$$

$$\Rightarrow \beta = 10^{-4} \text{ V/V}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}}$$

$$= 9.09 \times 10^3 \text{ V/V}$$

or 79.2 dB

**10.94** Figure 1 on the next page shows the Bode plot for the amplifier gain and for a differentiator. Observe that following the rate-of-closure rule the intersection of the two graphs is arranged so that the maximum difference in slopes is 20 dB/decade.

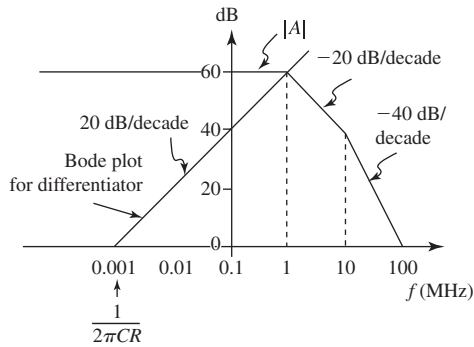


Figure 1

Thus, to guarantee stability,

$$\frac{1}{2\pi CR} \leq 0.001 \text{ MHz or } 1 \text{ kHz}$$

$$\Rightarrow CR \geq \frac{1}{2\pi \times 10^3} = 0.159 \text{ ms}$$

**10.95** Refer to Fig. 11.39.

(a) For  $\beta = 0.001$ ,

$$20 \log \frac{1}{\beta} = 60 \text{ dB}$$

A horizontal line at the 60-dB level will intersect the vertical line at  $f_{p2} = 10^6$  Hz at a point  $Z_1$ .

Drawing a line with a slope of  $-20$  dB/decade from  $Z_1$  will intersect the 100-dB horizontal line at a frequency two decade lower than  $f_{p2}$ , thus the frequency to which the 1st pole must be moved is

$$f'_D = \frac{f_{p2}}{100} = \frac{10^6}{100} = 10 \text{ kHz}$$

(b) For  $\beta = 0.1$ ,

$$20 \log \frac{1}{\beta} = 20 \text{ dB}$$

Following a process similar to that for (a) above, the first pole must be lowered to

$$f'_D = \frac{10^6}{10^4} = 100 \text{ Hz}$$

**10.96** (a) Fig. 1, graph 1 is the Bode plot for the gain of the uncompensated amplifier.

(b) We assume that the Miller compensation will shift the first pole to a new, much lower frequency  $\omega'_{p1}$ , and will shift the second pole to a frequency  $\omega'_{p2}$  higher than  $\omega_{p3}$ . Thus the gain of the compensated amplifier will fall off, starting at  $\omega'_{p1}$ , with a uniform slope of  $-20$  dB/decade until it reaches 0 dB at the second pole, which is now that with frequency  $\omega_{p3}$ . The frequency of the new dominant pole can be found from

$$\omega'_{p1} = \frac{\omega_{p3}}{10^6} = \frac{10^8}{10^6} = 100 \text{ rad/s}$$

and the unity-gain frequency is

$$\omega_t = \omega_{p3} = 10^8 \text{ rad/s}$$

(c) Refer to Fig. 10.41.

$$\omega_{p1} = \frac{1}{C_1 R_1}$$

$$\Rightarrow R_1 = \frac{1}{\omega_{p1} C_1} = \frac{1}{10^5 \times 1 \times 10^{-12}} = 10^7 \Omega$$

$$\omega_{p2} = \frac{1}{C_2 R_2}$$

$$\Rightarrow R_2 = \frac{1}{\omega_{p2} C_2} = \frac{1}{10^7 \times 0.05 \times 10^{-12}} = 2 \times 10^6 \Omega$$

The value of  $C_f$  can be found using Eq. (10.58),

$$C_f = \frac{1}{\omega'_{p1} g_m R_1 R_2} = \frac{1}{10^2 \times 10 \times 10^{-3} \times 10^7 \times 2 \times 10^6} = 0.05 \text{ pF}$$

The new frequency of the second pole can be found using Eq. (10.59),

This figure belongs to Problem 10.96.

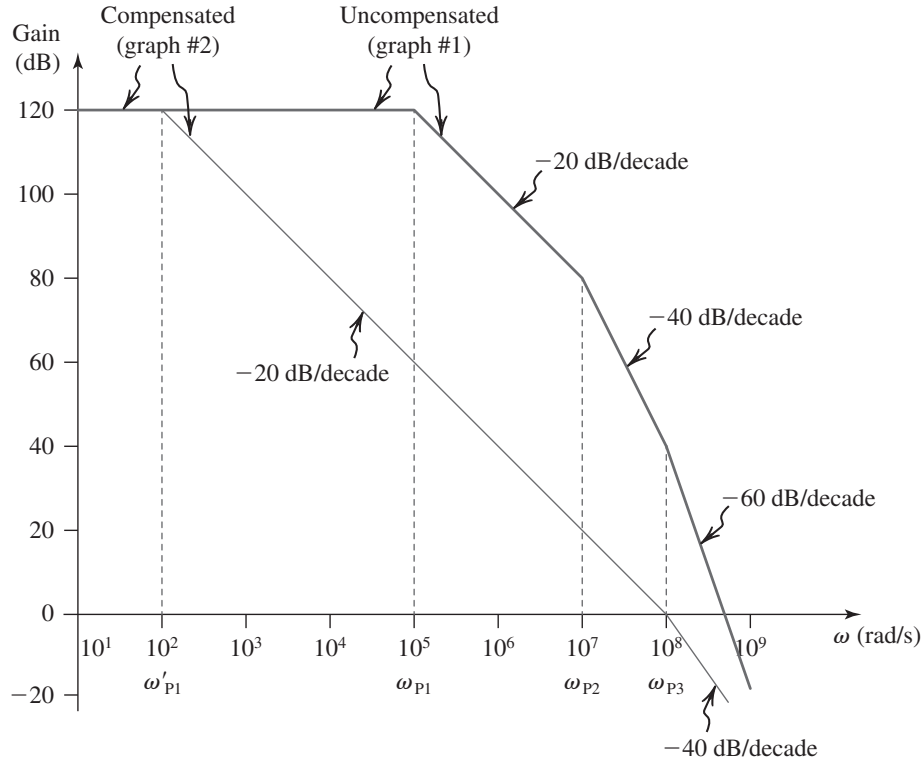


Figure 1

$$\omega'_{p2} = \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)} = \frac{10 \times 10^{-3} \times 0.05 \times 10^{-12}}{[1 \times 0.05 + 0.05(1 + 0.05)] \times 10^{-24}} \simeq 5 \times 10^9 \text{ rad/s}$$

which indeed is higher than  $\omega_{P3}$ .

**10.97** The new pole must be placed at

$$f_D = \frac{1 \text{ MHz}}{10^4} = 100 \text{ Hz}$$

In this way the modified open-loop gain will decrease at the uniform rate of  $-20$  dB/decade, thus reaching 0 dB in four decades, that is at 1 MHz where the original pole exists. At 1 MHz, the slope changes to  $-40$  dB/decade, but our amplifier will be guaranteed to be stable with a closed-loop gain at low as unity.

**10.98** We must move the 1 MHz pole to a new location,

$$f_D = \frac{20 \text{ MHz}}{10^4} = 2 \text{ kHz}$$

This reduction in frequency by a factor of  $\frac{1 \text{ MHz}}{2 \text{ kHz}} = 500$  will require that the total

capacitance at the controlling node must become 500 times what it originally was.

**10.99**  $f_{P1} = \frac{1}{2\pi C_1 R_1}$

$$10^5 = \frac{1}{2\pi \times 150 \times 10^{-12} \times R_1}$$

$$\Rightarrow R_1 = 10.61 \text{ k}\Omega$$

$$f_{P2} = \frac{1}{2\pi C_2 R_2}$$

$$10^6 = \frac{1}{2\pi \times 5 \times 10^{-12} \times R_2}$$

$$\Rightarrow R_2 = 31.83 \text{ k}\Omega$$

First, we determine an approximate value of  $f'_{P2}$  from Eq. (10.59):

$$f'_{P2} = \frac{g_m C_f}{2\pi [C_1 C_2 + C_f (C_1 + C_2)]}$$

Assume that  $C_f \gg C_2$ , then

$$f'_{P2} \simeq \frac{g_m}{2\pi (C_1 + C_2)}$$

$$= \frac{40 \times 10^{-3}}{2\pi (150 + 5) \times 10^{-12}}$$

$$= 41.1 \text{ MHz}$$

which is much greater than  $f_{p3}$ . Thus, we use  $f_{p3}$  to determine the new location of  $f_{p1}$ ,

$$f'_{p1} = \frac{2 \times 10^6}{10^4} = 200 \text{ Hz}$$

Using Eq. (10.58), we obtain

$$f'_{p1} = \frac{1}{2\pi g_m R_2 C_f R_1}$$

$$200 = \frac{1}{2\pi \times 40 \times 10^{-3} \times 31.83 \times 10^3 \times C_f \times 10.61 \times 10^3}$$

$$\Rightarrow C_f = 58.9 \text{ pF}$$

Since  $C_f$  is indeed much greater than  $C_2$ , the pole at the output will have the frequency already calculated:

$$f'_{p2} \simeq 41.1 \text{ MHz}$$

$$\mathbf{10.100} \quad R_1 = R_2 = R$$

$$C_1 = 10C$$

$$C_2 = C$$

$$C_f \gg C$$

$$g_{m1}R = 100$$

$$\omega_{p1} = \frac{1}{10CR} = \frac{0.1}{CR} \quad (1)$$

$$\omega_{p2} = \frac{1}{CR} \quad (2)$$

$$\omega'_{p1} = \frac{1}{g_m R C_f R} = \frac{1}{100 C_f R}$$

Thus,

$$\omega'_{p1} = \frac{0.01}{C_f R} \quad (3)$$

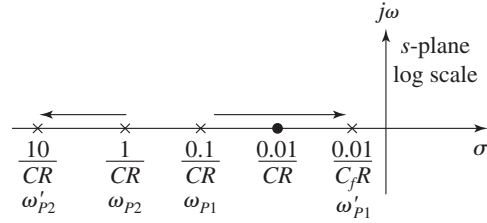


Figure 1

$$\omega'_{p2} = \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)}$$

$$= \frac{g_m C_f}{10C^2 + 11CC_f} = \frac{g_m C_f}{C(10C + 11C_f)}$$

Since  $C_f \gg C$ , we have

$$\omega'_{p2} \simeq \frac{g_m C_f}{11CC_f} = \frac{g_m}{11C}$$

Substituting  $g_m = 100/R$ , we obtain

$$\omega'_{p2} = \frac{100}{11CR} \simeq \frac{10}{CR} \quad (4)$$

Equations (1)–(4) provide a summary of pole splitting: The two initial poles with frequencies  $\frac{0.1}{CR}$  and  $\frac{1}{CR}$ , a decade apart in frequency, are split further apart. The lower frequency pole is moved to a frequency  $\frac{0.01}{C_f R}$  which is more than a decade lower (because  $C_f \gg C$ ) and the higher frequency pole is moved to a frequency  $\frac{10}{CR}$  which is a decade higher. This is further illustrated in Fig. 1.

## Chapter 11

## Solutions to Exercises within the Chapter

**Ex: 11.1** To allow for  $v_O$  to reach  $-V_{CC} + V_{CEsat} = -15 + 0.2 = -14.8$  V, with  $Q_1$  just cutting off (i.e.  $i_{E1} = 0$ ),

$$I = \frac{14.8 \text{ V}}{R_L} = \frac{14.8}{1 \text{ k}\Omega} = 14.8 \text{ mA}$$

The value of  $R$  can now be found from

$$I = \frac{V_R}{R} = \frac{V_{CC} - V_D}{R}$$

$$14.8 = \frac{15 - 0.7}{R}$$

$$\Rightarrow R = \frac{14.3}{14.8} = 0.97 \text{ k}\Omega$$

The resulting output signal swing will be  $-14.8$  V to  $+14.8$  V. The minimum current in  $Q_1 = 0$ . The maximum current in  $Q_1 = 14.8 + 14.8 = 29.6$  mA

**Ex: 11.2** At  $v_O = -10$  V, we have

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 - 10 = 4.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{4.8}{1}\right)$$

$$= 0.64 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= -10 + 0.64 = -9.36 \text{ V}$$

At  $v_O = 0$  V, we have

$$i_L = 0 \text{ mA}$$

$$i_{E1} = I = 14.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{14.8}{1}\right)$$

$$= 0.67 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 0 + 0.67 = 0.67 \text{ V}$$

At  $v_O = 10$  V

$$i_L = \frac{10}{1} = 10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 + 10 = 24.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{24.8}{1}\right)$$

$$= 0.68 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 10 + 0.68 = 10.68 \text{ V}$$

At  $v_O = -10$  V, we have

$$i_{E1} = 4.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{4.8 \text{ mA}} = 5.2 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0052} = 0.995 \text{ V/V}$$

At  $v_O = 0$  V,

$$i_{E1} = 14.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{14.8 \text{ mA}} = 1.7 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0017} = 0.998 \text{ V/V}$$

At  $v_O = +10$  V,

$$i_{E1} = 24.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{24.8 \text{ mA}} = 1.0 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.001} = 0.999 \text{ V/V}$$

**Ex: 11.3**

$$\text{a. } P_L = \frac{(\hat{V}_o/\sqrt{2})^2}{R_L} = \frac{(8/\sqrt{2})^2}{100} = 0.32 \text{ W}$$

$$P_S = 2V_{CC} \times I = 2 \times 10 \times 100 \times 10^{-3}$$

$$= 2 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_L}{P_S} \times 100$$

$$= \frac{0.32}{2} \times 100$$

$$= 16\%$$

$$\text{Ex: 11.4 (a) } P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

$$= \frac{1}{2} \frac{(4.5)^2}{4} = 2.53 \text{ W}$$

$$\text{(b) } P_{S+} = P_{S-} = V_{CC} \times \frac{1}{\pi} \frac{\hat{V}_o}{R_L}$$

$$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15 \text{ W}$$

$$\text{(c) } \eta = \frac{P_L}{P_S} \times 100 = \frac{2.53}{2 \times 2.15} \times 100$$

$$= 59\%$$

$$\text{(d) Peak input currents} = \frac{1}{\beta + 1} \frac{\hat{V}_o}{R_L}$$

$$= \frac{1}{51} \times \frac{4.5}{4}$$

$$= 22.1 \text{ mA}$$

(e) Using Eq. (11.22), we obtain

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91 \text{ W}$$

**Ex: 11.5** (a) The quiescent power dissipated in each transistor  $= I_Q \times V_{CC}$

Total power dissipated in the two transistors

$$= 2I_Q \times V_{CC}$$

$$= 2 \times 2 \times 10^{-3} \times 15$$

$$= 60 \text{ mW}$$

(b)  $I_Q$  is increased to 10 mA

At  $v_O = 0$ , we have  $i_N = i_P = 10 \text{ mA}$

From Eq. (11.31), we obtain

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} = \frac{25}{10 + 10} = 1.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 1.25}$$

$$\frac{v_o}{v_i} = 0.988 \text{ at } v_O = 0 \text{ V}$$

At  $v_O = 10 \text{ V}$ , we have

$$i_L = \frac{10 \text{ V}}{100 \Omega} = 0.1 \text{ A} = 100 \text{ mA}$$

Use Eq. (11.27) to calculate  $i_N$ :

$$i_N^2 - i_N i_L - I_Q^2 = 0$$

$$i_N^2 - 100 i_N - 10^2 = 0$$

$$\Rightarrow i_N = 101.0 \text{ mA}$$

Using Eq. (11.26), we obtain

$$i_P = \frac{I_Q^2}{i_N} \simeq 1 \text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{101.0 + 1} \simeq 0.2451 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.2451} \simeq 1$$

$$\% \text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In Example 11.3,  $I_Q = 2 \text{ mA}$ , and for  $v_O = 0$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{2 + 2} = 6.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 6.25} = 0.94$$

$$v_O = 10 \text{ V}$$

$$i_L = \frac{10 \text{ V}}{100 \Omega} = 100 \text{ mA}$$

Again calculate  $i_N$  (for  $I_Q = 2 \text{ mA}$ ) using Eq. (11.27) ( $i_N = 100.04 \text{ mA}$ ):

$$i_P = \frac{I_Q^2}{i_N} = \frac{2^2}{100.04} = 0.04 \text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{100.04 + 0.04} = 0.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} \simeq 1$$

$$\% \text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For  $I_Q = 10 \text{ mA}$ , change is 1.2%

For  $I_Q = 2 \text{ mA}$ , change is 6%

(c) The quiescent power dissipated in each transistor  $= I_Q \times V_{CC}$

$$\text{Total power dissipated} = 2 \times 10 \times 10^{-3} \times 15$$

$$= 300 \text{ mW}$$

**Ex: 11.6** From Example 11.5, we have  $V_{CC} = 15 \text{ V}$ ,  $R_L = 100 \Omega$

$Q_N$  and  $Q_P$  matched and  $I_S = 10^{-13} \text{ A}$  and  $\beta = 50$ ,  $I_{\text{Bias}} = 3 \text{ mA}$

$$\text{For } v_O = 10 \text{ V, we have } i_L = \frac{10}{100} = 0.1 \text{ A}$$

As a first approximation,  $i_N \simeq 0.1 \text{ A}$ ,

$$i_P = 0, i_{BN} \simeq \frac{0.1 \text{ A}}{50 + 1} \simeq 2 \text{ mA}$$

$$i_D = I_{\text{Bias}} - i_{BN} = 3 - 2 = 1 \text{ mA}$$

$$V_{BB} = 2V_T \ln \left( \frac{10^{-3}}{\frac{1}{3} \times 10^{-13}} \right) \quad (1)$$

This  $\frac{1}{3}$  is because biasing diodes have  $\frac{1}{3}$  area of the output devices.

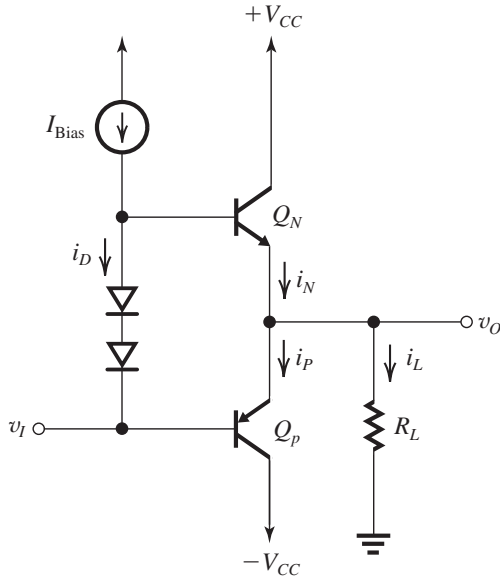
But  $V_{BB} = V_{BEN} + V_{BEP}$

$$= V_T \ln \left( \frac{i_N}{I_S} \right) + V_T \ln \left( \frac{i_N - i_L}{I_S} \right)$$

$$= V_T \ln \left[ \frac{i_N (i_N - i_L)}{I_S^2} \right] \quad (2)$$

Equating Eqs. 1 and 2, we obtain

$$2V_T \ln \left( \frac{10^{-3}}{\frac{1}{3} \times 10^{-13}} \right) = V_T \ln \left[ \frac{i_N (i_N - i_L)}{I_S^2} \right]$$



$$\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_N(i_N - 0.1)}{(10^{-13})^2}$$

$$i_N(i_N - 0.1) = 9 \times 10^{-6}$$

If \$i\_N\$ is in mA, then

$$i_N(i_N - 100) = 9$$

$$i_N^2 - 100 i_N - 9 = 0$$

$$\Rightarrow i_N = 100.1 \text{ mA}$$

$$i_P = i_N - i_L = 0.1 \text{ mA}$$

$$\text{For } v_O = -10 \text{ V and } i_L = \frac{-10}{100} = -0.1 \text{ A}$$

$$= -100 \text{ mA:}$$

As a first approximation assume \$i\_P \cong 100 \text{ mA}\$,

$$i_N \cong 0. \text{ Since } i_N = 0, \text{ current through diodes} \\ = 3 \text{ mA}$$

$$\therefore V_{BB} = 2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \quad (3)$$

$$\text{But } V_{BB} = V_T \ln\left(\frac{i_N}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \\ = V_T \ln\left(\frac{i_P + i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \quad (4)$$

Here \$i\_L = -0.1 \text{ A}\$

Equating Eqs. (3) and (4), we obtain

$$2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) =$$

$$V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_P(i_P - 0.1)}{(10^{-13})^2}$$

$$i_P(i_P - 0.1) = 81 \times 10^{-6}$$

Expressing currents in mA, we have

$$i_P(i_P - 100) = 81$$

$$i_P^2 - 100 i_P - 81 = 0$$

$$\Rightarrow i_P = 100.8 \text{ mA}$$

$$i_N = i_P + i_L = 0.8 \text{ mA}$$

**Ex: 11.7** \$\Delta I\_C = g\_m \times 2 \text{ mV}/^\circ\text{C} \times 5^\circ\text{C}\$, mA

where \$g\_m\$ is in mA/mV

$$g_m = \frac{10 \text{ mA}}{25 \text{ mV}} = 0.4 \text{ mA/mV}$$

$$\text{Thus, } \Delta I_C = 0.4 \times 2 \times 5 = 4 \text{ mA}$$

**Ex: 11.8** Refer to Fig. 11.14.

(a) To obtain a terminal voltage of 1.2 V, and since \$\beta\_1\$ is very large, it follows that

$$V_{R1} = V_{R2} = 0.6 \text{ V.}$$

Thus \$I\_{C1} = 1 \text{ mA}\$

$$I_R = \frac{1.2 \text{ V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5 \text{ mA}$$

Thus, \$I = I\_{C1} + I\_R = 1.5 \text{ mA}\$

(b) For \$\Delta V\_{BB} = +50 \text{ mV}\$:

$$V_{BB} = 1.25 \text{ V } I_R = \frac{1.25}{2.4} = 0.52 \text{ mA}$$

$$V_{BE} = \frac{1.25}{2} = 0.625 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$$

$$= 2.72 \text{ mA}$$

$$I = 2.72 + 0.52 = 3.24 \text{ mA}$$

For \$\Delta V\_{BB} = +100 \text{ mV}\$, we have

$$V_{BB} = 1.3 \text{ V, } I_R = \frac{1.3}{2.4} = 0.54 \text{ mA}$$

$$V_{BE} = \frac{1.3}{2} = 0.65 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$$

$$= 7.39 \text{ mA}$$

$$I = 7.39 + 0.54 = 7.93 \text{ mA}$$

For \$\Delta V\_{BB} = +200 \text{ mV}\$:

$$V_{BB} = 1.4 \text{ V}, \quad I_R = \frac{1.4}{2.4} = 0.58 \text{ mA}$$

$$V_{BE} = 0.7 \text{ V}$$

$$I_{C1} = 1 \times e^{0.1/0.025} = 54.60 \text{ mA}$$

$$I = 54.60 + 0.58 = 55.18 \text{ mA}$$

For  $\Delta V_{BB} = -50 \text{ mV}$ :

$$V_{BB} = 1.15 \text{ V}, \quad I_R = \frac{1.15}{2.4} = 0.48 \text{ mA}$$

$$V_{BE} = \frac{1.15}{2}$$

$$= 0.575$$

$$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37 \text{ mA}$$

$$I = 0.48 + 0.37 = 0.85 \text{ mA}$$

For  $\Delta V_{BB} = -100 \text{ mV}$ :

$$V_{BB} = 1.1 \text{ V} \quad I_R = \frac{1.1}{2.4} = 0.46 \text{ mA}$$

$$V_{BE} = 0.55 \text{ V}$$

$$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13 \text{ mA}$$

$$I = 0.46 + 0.13 = 0.59 \text{ mA}$$

For  $\Delta V_{BB} = -200 \text{ mV}$ :

$$V_{BB} = 1.0 \text{ V} \quad I_R = \frac{1}{2.4} = 0.417 \text{ mA}$$

$$V_{BE} = 0.5 \text{ V}$$

$$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018 \text{ mA}$$

$$I = 0.43 \text{ mA}$$

**Ex: 11.9** (a) From symmetry we see that all transistors will conduct equal currents and have equal  $V_{BE}$ 's. Thus,

$$v_O = 0 \text{ V}$$

If  $V_{BE} \simeq 0.7 \text{ V}$ , then

$$V_{E1} = 0.7 \text{ V} \text{ and } I_1 = \frac{15 - 0.7}{5} = 2.86 \text{ mA}$$

If we neglect  $I_{B3}$ , then

$$I_{C1} \simeq 2.86 \text{ mA}$$

At this current,  $|V_{BE}|$  is given by

$$|V_{BE}| = 0.025 \ln\left(\frac{2.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right) \simeq 0.63 \text{ V}$$

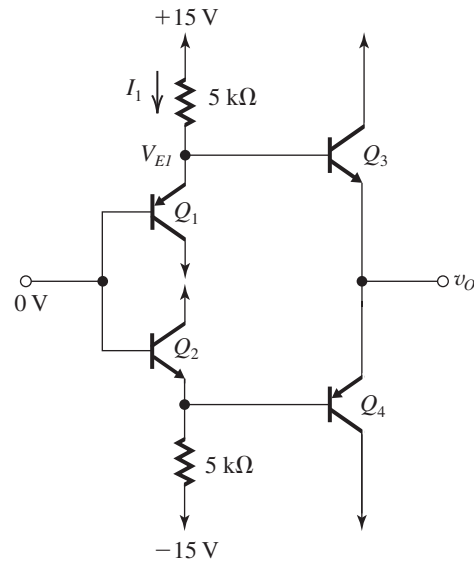
Thus  $V_{E1} = 0.63 \text{ V}$  and  $I_1 = 2.87 \text{ mA}$

No more iterations are required and

$$i_{C1} = i_{C2} = i_{C3} = i_{C4} \simeq 2.87 \text{ mA}$$

(b) For  $v_I = +10 \text{ V}$ :

To start the iterations, let  $V_{BE1} \simeq 0.7 \text{ V}$

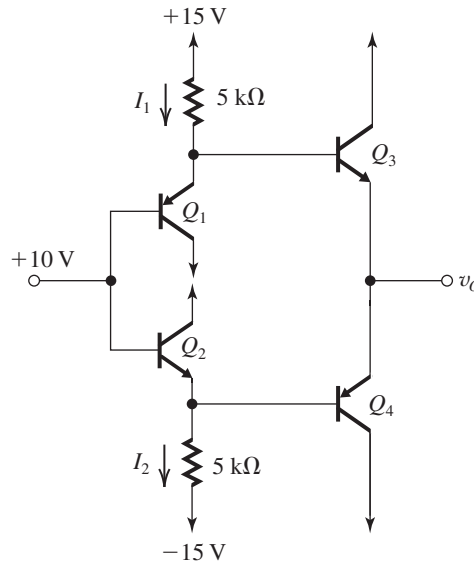


Thus,

$$V_{E1} = 10.7 \text{ V}$$

and

$$I_1 = \frac{15 - 10.7}{5} = 0.86 \text{ mA}$$



Neglecting  $I_{B3}$ , we obtain

$$I_{C1} \simeq I_{E1} \simeq I_1 = 0.86 \text{ mA}$$

But at this current

$$|V_{BE1}| = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{0.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.6 \text{ V}$$



Exercise 11-5

Thus,  $V_{E1} = +10.6$  V and  $I_1 = 0.88$  mA. No further iterations are required and  $I_{C1} \simeq 0.88$  mA.

To find  $I_{C2}$ , we use an identical procedure:

$$V_{BE2} \simeq 0.7$$
 V

$$V_{E2} = 10 - 0.7 = +9.3$$
 V

$$I_2 = \frac{9.3 - (-15)}{5} = 4.86$$
 mA

$$V_{BE2} = 0.025 \ln\left(\frac{4.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.643$$
 V

$$V_{E2} = 10 - 0.643 = +9.357$$
 V

$$I_2 = 4.87$$
 mA

$$I_{C2} \simeq 4.87$$
 mA

Finally,

$$I_{C3} = I_{C4} = 3.3 \times 10^{-14} e^{V_{BE}/V_T}$$

where

$$V_{BE} = \frac{V_{E1} - V_{E2}}{2} = 0.62$$
 V

$$\text{Thus, } I_{C3} = I_{C4} = 1.95$$
 mA

The symmetry of the circuit enables us to find the values for  $v_I = -10$  V as follows:

$$I_{C1} = 4.87$$
 mA  $I_{C2} = 0.88$  mA

$$I_{C3} = I_{C4} = 1.95$$
 mA

$$\text{For } v_I = +10 \text{ V, we have } v_O = V_{E1} - V_{BE3} \\ = 10.6 - 0.62 = +9.98$$
 V

$$\text{For } v_I = -10 \text{ V, we have } v_O = V_{E1} - V_{BE3} \\ = -9.357 - 0.62 = -9.98$$
 V

(c) For  $v_I = +10$  V, we have

$$v_O \simeq 10$$
 V

$$I_L \simeq 100$$
 mA

$$I_{C3} \simeq 100$$
 mA

$$I_{B3} = \frac{100}{201} \simeq 0.5$$
 mA

Assuming that  $|V_{BE1}|$  has not changed much from 0.6 V, then

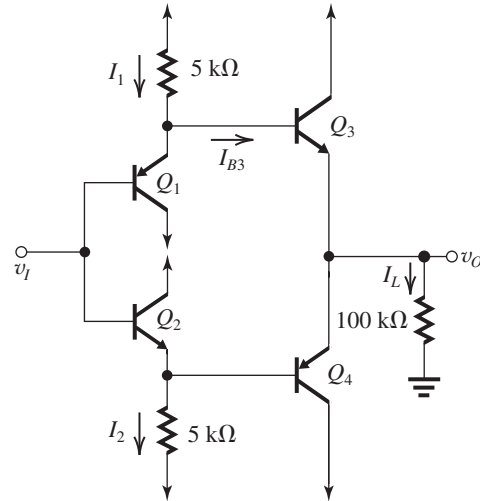
$$V_{E1} \simeq 10.6$$
 V

$$I_1 = \frac{15 - 10.6}{5} = 0.88$$
 mA

$$I_{E1} = I_1 - I_{B3} = 0.88 - 0.5 = 0.38$$
 mA

$$I_{C1} \simeq 0.38$$
 mA

$$|V_{BE1}| = 0.025 \ln\left(\frac{0.38 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$



$$= 0.58$$
 V

$$V_{E1} = 10.58$$
 V

$$I_1 = \frac{15 - 10.58}{5} = 0.88$$
 mA

$$\text{Thus, } I_{C1} \simeq 0.38$$
 mA

Now for  $Q_2$  we have

$$V_{BE2} = 0.643$$
 V

$$V_{E2} = 10 - 0.643 = 9.357$$
 V

$$I_2 = 4.87$$
 mA

$$I_{B4} \simeq 0$$

$$I_{C2} \simeq 4.87$$
 mA (as in (b))

Assuming that  $I_{C3} \simeq 100$  mA, we have

$$V_{BE3} = 0.025 \ln\left(\frac{100 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.72$$
 V

$$\text{Thus, } v_O = V_{E1} - V_{BE3}$$

$$= 10.58 - 0.72 = +9.86$$
 V

$$|V_{BE4}| = v_O - V_{E2}$$

$$9.86 - 9.36 = 0.5$$
 V

$$\text{Thus, } I_{C4} = 3.3 \times 10^{-14} e^{0.5/0.025}$$

$$\simeq 0.02$$
 mA

From symmetry we find the values for the case

$v_I = -10$  V as:

$$I_{C1} = 4.87$$
 mA,  $I_{C2} = 0.38$  mA

$$I_{C3} = 0.02$$
 mA,  $I_{C4} = 100$  mA

$$v_O = -9.86$$
 V

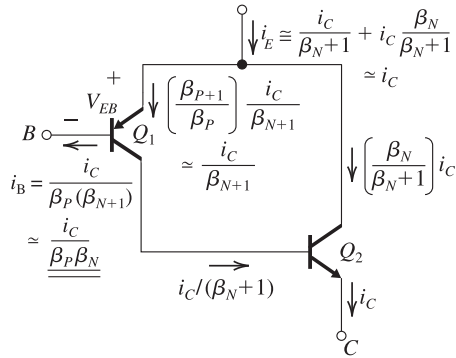
**Ex: 11.10** For  $Q_1$ :

$$i_{C1} = I_{SP} e^{v_{EB}/V_T}$$

$$\frac{i_C}{\beta_N + 1} = I_{SP} e^{v_{EB}/V_T}$$

$$i_C \approx \beta_N I_{SP} e^{v_{EB}/V_T}$$

Thus, effective scale current =  $\beta_N I_{SP}$



(b) Effective current gain  $\equiv \frac{i_C}{i_B} = \beta_P \beta_N$

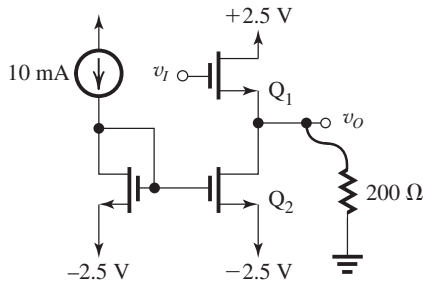
$$= 20 \times 50 = 1000$$

$$100 \times 10^{-3} = 50 \times 10^{-14} e^{v_{EB}/0.025}$$

$$v_{EB} = 0.025 \ln(2 \times 10^{11})$$

$$= 0.651 \text{ V}$$

**Ex: 11.11**



$$v_O = 0 \text{ V} \Rightarrow I_{D1} = 10 \text{ mA}$$

$$\Rightarrow V_{GS1} = V_m + V_{OV1}$$

$$= v_m + \sqrt{\frac{2I_{D1}}{k_n}}$$

$$= 0.5 \text{ V} + \sqrt{\frac{20 \text{ mA}}{50 \text{ mA/V}^2}}$$

$$= 1.13 \text{ V}$$

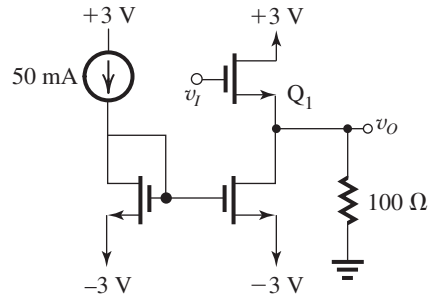
$$\Rightarrow v_I = 1.13 \text{ V}$$

From Eq. (11.38),

$$R_L \geq \frac{V_{SS} - V_{OV2}}{I} = \frac{2.5 \text{ V} - 0.63 \text{ V}}{10 \text{ mA}}$$

$$= 187 \Omega$$

**Ex: 11.12**



When  $v_O = 0 \text{ V}$ ,

$$I_{D1} = 50 \text{ mA}$$

$$g_{m1} = \sqrt{2k_n I_{D1}} = \sqrt{2 \times 80 \times 50} = 89 \text{ mA/V}$$

$$A_v = \frac{R_L}{R_L + 1/g_{m1}}$$

$$= \frac{100}{100 + 1/0.089}$$

$$= 0.899 \text{ V/V}$$

$$V_{OV2} = \sqrt{\frac{2I}{k_n}}$$

$$= \sqrt{\frac{2 \times 50}{80}}$$

$$= 1.12 \text{ V}$$

$$\Rightarrow v_{Omin} = -2.5 + 1.12$$

$$= -1.38 \text{ V}$$

We must also ensure Equation (11.38) is satisfied.

$$I \geq \frac{V_{SS} - V_{OV2}}{R_L}$$

$$50 \text{ mA} \geq \frac{1.38 \text{ V}}{100 \Omega} = 13.8 \text{ mA}$$

Thus, when  $v_O = v_{Omin}$ :

$$I_{D1} = 50 \text{ mA} - \frac{1.38 \text{ V}}{100 \Omega}$$

$$= 36.2 \text{ mA}$$

$$\Rightarrow g_{m1} = \sqrt{2k_n I_{D1}}$$

$$= \sqrt{2 \times 80 \times 36.2}$$

$$= 76.1 \text{ mA/V}$$

$$\Rightarrow A_v = \frac{R_L}{R_L + 1/g_{m1}}$$

$$= \frac{100}{100 + 1/0.0761}$$

$$= 0.884 \text{ V/V}$$

The percentage change is therefore,

$$\left| \frac{0.884 - 0.899}{0.899} \right| \times 100\% = 1.7\%$$

Exercise 11-7

**Ex: 11.13** According to Equation (11.44),

$$R_{out} \cong \frac{1}{\mu g_{mp}}$$

$$\Rightarrow \mu \cong \frac{1}{g_{mp} R_{out}}$$

$$= \frac{1}{0.05 \times 1}$$

$$= 20 \text{ V/V}$$

$$g_{mp} = \frac{2I}{V_{OV1}}$$

$$\Rightarrow I = \frac{g_{mp} V_{OV1}}{2}$$

$$= \frac{50 \times 0.3}{2}$$

$$= 7.5 \text{ mA}$$

**Ex: 11.14**  $I_Q = 0.1 \times \frac{2 \text{ V}}{100\Omega} = 2 \text{ mA}$

$$g_{mn} = g_{mp} = \frac{2I_Q}{V_{OV1,2}} = \frac{2 \times 2}{0.3} = 13.33 \text{ mA/V}$$

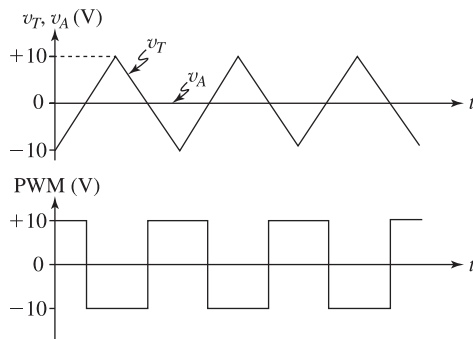
According to Equation (11.45),

$$R_{out} = \frac{1}{\mu(g_{mp} + g_{mn})}$$

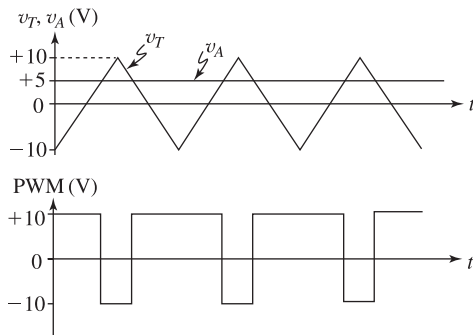
$$= \frac{1}{10 \times (13.3 + 13.3)}$$

$$= 3.75\Omega$$

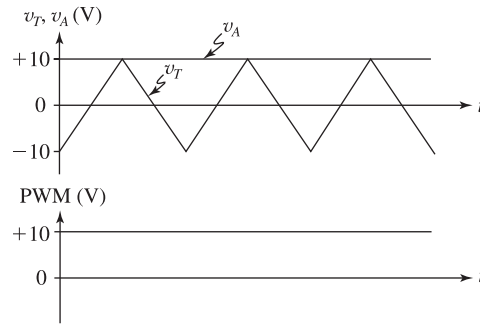
**Ex: 11.15**



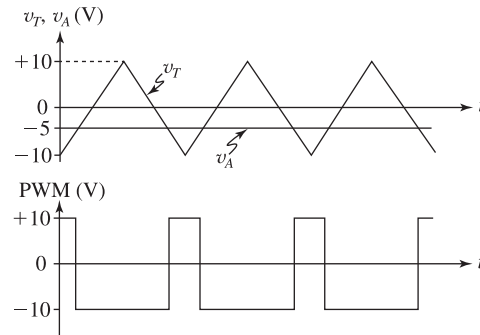
(a)  $v_A = 0$ :  $D = 50\%$ ; Average = 0



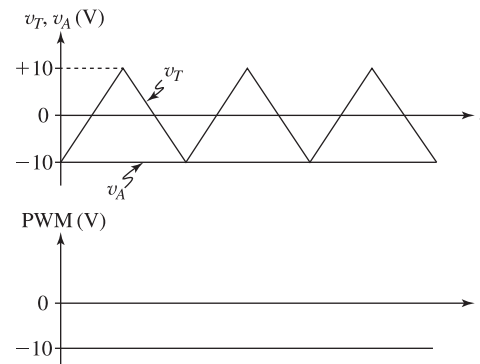
(b)  $v_A = +5 \text{ V}$ :  $D = 75\%$ ; Average = +5 V



(c)  $v_A = +10 \text{ V}$ :  $D = 100\%$ ; Average = +10 V



(d)  $v_A = -5 \text{ V}$ :  $D = 25\%$ ; Average = -5 V



(e)  $v_A = -10 \text{ V}$ :  $D = 0\%$ ; Average = -10 V

**Ex: 11.16**

$$f_s = 10 \times \text{highest frequency in audio signal}$$

$$= 10 \times 20 = 200 \text{ kHz}$$

Since  $f_s$  is a decade higher than  $f_p$ , the gain will have fallen by 40 dB. Thus the PWM component at  $f_s$  will be attenuated by 40 dB.

**Ex: 11.17** Maximum peak amplitude =  $V_{DD}$

$$\text{Maximum power delivered to } R_L = \frac{(V_{DD}/\sqrt{2})^2}{R_L}$$

$$= \frac{V_{DD}^2}{2R_L}$$

Exercise 11-8

For  $V_{DD} = 35 \text{ V}$  and  $R_L = 8 \ \Omega$ :

Peak amplitude = 35 V

Maximum power =  $\frac{35^2}{2 \times 8} = 76.6 \text{ W}$

Power delivered by power supplies

$$= \frac{P_L}{\eta} = \frac{76.6}{0.9} = 85.1 \text{ W}$$

**Solutions to End-of-Chapter Problems**

$$\mathbf{11.1} \quad I = \frac{0 - (-V_{CC}) - V_D}{R}$$

$$= \frac{10 - 0.7}{1} = 9.3 \text{ mA}$$

$$\text{Upper limit on } v_O = V_{CC} - V_{CEsat}$$

$$= 10 - 0.3 = 9.7 \text{ V}$$

$$\text{Corresponding input} = 9.7 + 0.7 = 10.4 \text{ V}$$

$$\text{Lower limit on } v_O = -IR_L = -9.3 \times 1$$

$$= -9.3 \text{ V}$$

$$\text{Corresponding input} = -9.3 + 0.7 = -8.6 \text{ V}$$

If the EBJ area of  $Q_3$  is twice as large as that of  $Q_2$ , then

$$I = \frac{1}{2} \times 9.3 = 4.65 \text{ mA}$$

There will be no change in  $v_{Omax}$  and in the corresponding value of  $v_I$ . However,  $v_{Omin}$  will now become

$$v_{Omin} = -IR_L$$

$$= -4.65 \times 1 = -4.65 \text{ V}$$

and the corresponding value of  $v_I$  will be

$$v_I = -4.65 + 0.7 = -3.95 \text{ V}$$

If the EBJ area of  $Q_3$  is made half as big as that of  $Q_2$ , then

$$I = 2 \times 9.3 = 18.6 \text{ mA}$$

There will be no change in  $v_{Omax}$  and in the corresponding value of  $v_I$ . However,  $v_{Omin}$  will now become

$$v_{Omin} = -V_{CC} + V_{CEsat}$$

$$= -10 + 0.3 = -9.7 \text{ V}$$

and the corresponding value of  $v_I$  will be

$$v_I = -9.7 + 0.7 = -9 \text{ V}$$

**11.2** Refer to Fig. 11.3. For a load resistance of  $100 \Omega$  and  $v_O$  ranging between  $-5 \text{ V}$  and  $+5 \text{ V}$ , the maximum current through  $Q_1$  is

$$I + \frac{5}{0.1} = I + 50, \text{ mA and the minimum current}$$

$$\text{is } I - \frac{5}{0.1} = I - 50, \text{ mA.}$$

For a current ratio of 15, we have

$$\frac{I + 50}{I - 50} = 15$$

$$\Rightarrow I = 57.1 \text{ mA}$$

$$R = \frac{9.3 \text{ V}}{57.1 \text{ mA}} = 163 \Omega$$

The incremental voltage gain is  $A_v = \frac{R_L}{R_L + r_{e1}}$

For  $R_L = 100 \Omega$ ;

At  $v_O = +5 \text{ V}$ ,  $i_{E1} = 57.1 + 50 = 107.1 \text{ mA}$

$$r_{e1} = \frac{25}{107.1} = 0.233 \Omega$$

$$A_v = \frac{100}{100 + 0.233} = 0.998 \text{ V/V}$$

At  $v_O = 0 \text{ V}$ ,  $i_{E1} = 57.1 \text{ mA}$

$$r_{e1} = \frac{25}{57.1} = 0.438 \Omega$$

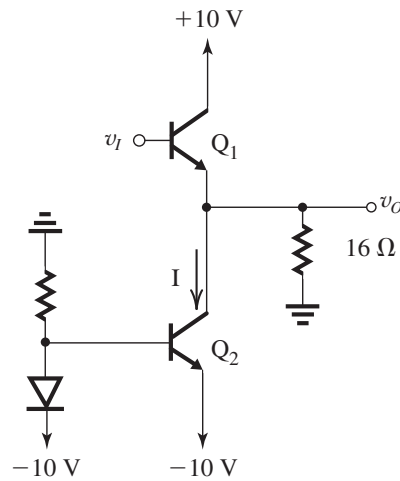
$$A_v = \frac{100}{100.438} = 0.996 \text{ V/V}$$

At  $v_O = -5 \text{ V}$ ,  $i_{E1} = 57.1 - 50 = 7.1 \text{ mA}$

$$r_{e1} = \frac{25}{7.1} = 3.52 \Omega$$

$$A_v = \frac{100}{103.52} = 0.966 \text{ V/V}$$

Thus the incremental gain changes by  $0.998 - 0.966 = 0.032$  or about 3% over the range of  $v_O$ .

**11.3**

The minimum gain will be at  $v_O = -5 \text{ V}$ .

$$A_v = \frac{R_L}{R_L + r_{e1}}$$

$$= \frac{R_L}{R_L + V_T/i_{E1}}$$

$$\Rightarrow 0.98 = \frac{16}{16 + 0.025/i_{E1}}$$

$$0.98 \left( 16 + \frac{0.025}{i_{E1}} \right) = 16$$

$$\frac{0.025}{i_{E1}} = \frac{16}{0.98} - 16$$

$$\Rightarrow i_{E1} = \frac{0.025}{\frac{16}{0.98} - 16} = 76.5 \text{ mA}$$

$$\Rightarrow I = i_{E1} + \frac{\hat{V}_O}{R_L}$$

$$= 0.0765 + \frac{5}{16}$$

$$= 0.389 \text{ A}$$

The power dissipation, assuming  $Q_3$  and  $Q_2$  are matched, is

$$P_{in} = 10 \times 0.389 + (-10) \times (-0.389)$$

$$+ (-10) \times (-0.389) = 11.67 \text{ W}$$

The power delivered to the load is

$$P_{out} = \frac{1}{2} \frac{\hat{V}_O^2}{R_L} = \frac{15^2}{216} = 0.78 \text{ W}$$

Thus, the efficiency is

$$\eta = \frac{0.78}{11.67} = 0.067 \text{ or } 6.7\%$$

#### 11.4 The minimum required value of $V_{CC}$ is

$$V_{CC} = \hat{V}$$

and the minimum required value of  $I$  is

$$I = \frac{\hat{V}}{R_L}$$

From Eq. (12.10),

$$\eta = \frac{1}{4} \left( \frac{\hat{V}_o}{IR_L} \right) \left( \frac{\hat{V}_o}{V_{CC}} \right)$$

$$= \frac{1}{4} \left( \frac{\hat{V}}{\hat{V}} \right) \left( \frac{\hat{V}}{\hat{V}} \right) = 0.25$$

or 25%

**11.5** First we determine the bias current  $I$  as follows:

$$I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

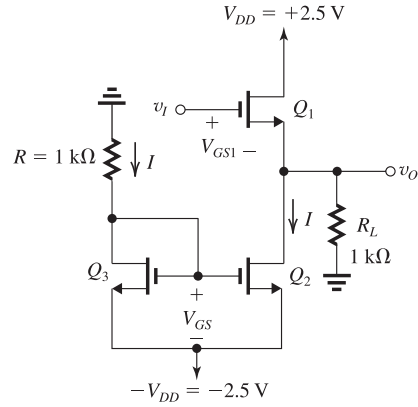
But

$$V_{GS} = 2.5 - IR = 2.5 - I$$

Thus

$$I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (2.5 - I - V_t)^2$$

$$= \frac{1}{2} \times 20(2.5 - I - 0.5)^2$$



$$I = 10(2 - I)^2$$

$$\Rightarrow I^2 - 4.1I + 4 = 0$$

$$I = 1.6 \text{ mA and } V_{GS} = 0.9 \text{ V}$$

The upper limit on  $v_O$  is determined by  $Q_1$  leaving the saturation region (and entering the triode region). This occurs when  $v_I$  exceeds  $V_{DD}$  by  $V_t$  volts:

$$v_{I\max} = 2.5 + 0.5 = 3 \text{ V}$$

To obtain the corresponding value of  $v_O$ , we must find the corresponding value of  $V_{GS1}$ , as follows:

$$v_O = v_I - V_{GS1}$$

$$i_L = \frac{v_O}{R_L} = \frac{v_I - V_{GS1}}{R_L} = \frac{v_I - V_{GS1}}{1}$$

$$i_L = 3 - V_{GS1}$$

$$i_{D1} = i_L + I = 3 - V_{GS1} + 1.6$$

$$= 4.6 - V_{GS1}$$

But,

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS1} - V_t)^2$$

$$4.6 - V_{GS1} = \frac{1}{2} \times 20(V_{GS1} - 0.5)^2$$

$$\Rightarrow V_{GS1}^2 - 0.9V_{GS1} - 0.21 = 0$$

$$V_{GS1} = 1.09 \text{ V}$$

$$v_{O\max} = v_{I\max} - V_{GS1}$$

$$= 3 - 1.09 = +1.91 \text{ V}$$

The lower limit of  $v_O$  is determined either by  $Q_1$  cutting off,

$$v_O = -IR_L = -1.6 \times 1 = -1.6 \text{ V}$$

or by  $Q_2$  leaving saturation,

$$v_O = -V_{DD} + V_{OV2}$$

where

$$V_{OV2} = V_{GS2} - V_t = 0.9 - 0.5 = 0.4 \text{ V}$$

Thus,

$$v_o = -2.5 + 0.4 = -2.1 \text{ V}$$

We observe that  $Q_1$  will cut off before  $Q_2$  leaves saturation, thus

$$v_{O\min} = -1.6 \text{ V}$$

and the corresponding value of  $v_i$  will be

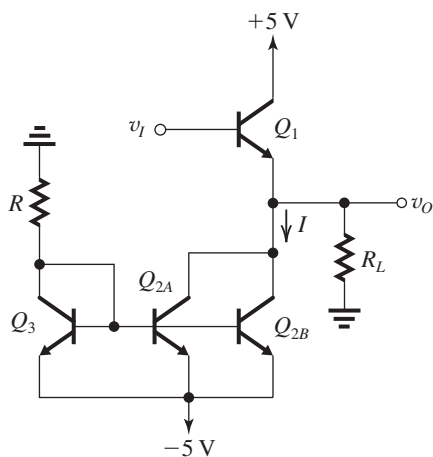
$$\begin{aligned} v_{i\min} &= v_{O\min} + V_t \\ &= -1.6 + 0.5 = -1.1 \text{ V} \end{aligned}$$

**11.6** Refer to Fig. 11.3. With  $V_{CC} = +5 \text{ V}$ , the upper limit on  $v_o$  is  $4.7 \text{ V}$ , which is greater than the required value of  $+3 \text{ V}$ . To obtain a lower limit of  $-3 \text{ V}$ , we select  $I$  so that

$$IR_L = 3$$

$$\Rightarrow I = 3 \text{ mA}$$

Since we are provided with four devices, we can minimize the total supply current by paralleling two devices to form  $Q_2$  as shown below.



The resulting supply current will be  $3 \times \frac{I}{2}$  rather than  $2I$  which is the value obtained in the circuit of Fig. 12.2. Then the supply current is  $4.5 \text{ mA}$ . The value of  $R$  is found from

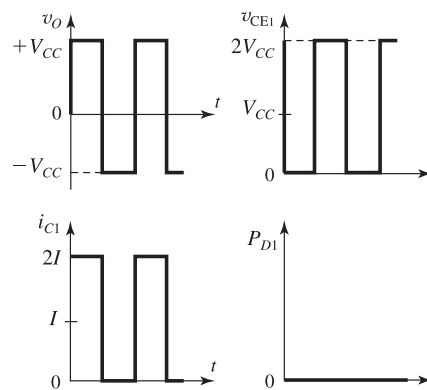
$$R = \frac{4.3 \text{ V}}{1.5 \text{ mA}} = 2.87 \text{ k}\Omega$$

In a practical design we would select a standard value for  $R$  that results in  $I$  somewhat larger than  $3 \text{ mA}$ . Say,  $R = 2.7 \text{ k}\Omega$ . In this case  $I = 3.2 \text{ mA}$ .

$$\begin{aligned} \text{Power from negative supply} &= 3 \times 1.6 \times 5 \\ &= 24 \text{ mW.} \end{aligned}$$

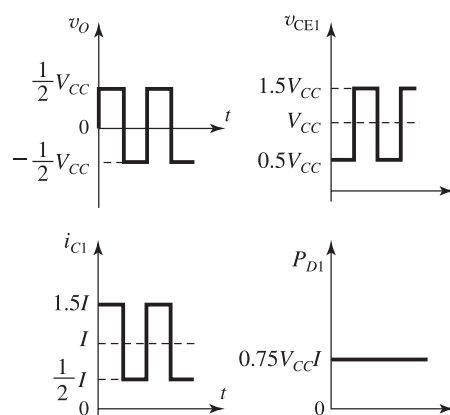
**11.7** Refer to Figs. 11.3 and 11.5.

For  $v_o$  being a square wave of  $\pm V_{CC}$  levels:



$P_{D1}|_{\text{average}} = 0$ . For the corresponding sine wave curve [Fig. 11.5], we have  $P_{D1}|_{\text{avg}} = \frac{1}{2} V_{CC} I$ .

For  $v_o$  being a square wave of  $\pm V_{CC}/2$  levels:



$$P_{D1}|_{\text{average}} = 0.75 V_{CC} I$$

For a sine-wave output of  $V_{CC}/2$  peak amplitude:

$$v_o = \frac{1}{2} V_{CC} \sin \theta$$

$$i_{C1} = I + \frac{\frac{1}{2} V_{CC}}{R_L} \sin \theta = I + \frac{1}{2} I \sin \theta$$

$$v_{CE1} = V_{CC} - \frac{1}{2} V_{CC} \sin \theta$$

$$P_{D1} = \left( V_{CC} - \frac{1}{2} V_{CC} \sin \theta \right) \left( I + \frac{1}{2} I \sin \theta \right)$$

$$= V_{CC} I - \frac{1}{4} V_{CC} I \sin^2 \theta$$

$$= V_{CC} I - \frac{1}{4} V_{CC} I \times \frac{1}{2} (1 - \cos 2\theta)$$

$$= \frac{7}{8} V_{CC} I + \frac{1}{8} V_{CC} I \cos 2\theta$$

$$P_{D1}|_{\text{average}} = \frac{7}{8} V_{CC} I$$

**11.8** In all cases, the average voltage across  $Q_2$  is equal to  $V_{CC}$ . Thus, since  $Q_2$  conducts a constant current  $I$ , its average power dissipation is  $V_{CC}I$ .

**11.9**  $V_{CC} = 16, 12, 10,$  and  $8$  V

$$I = 100 \text{ mA}, R_L = 100 \Omega$$

$$\hat{V}_o = 8 \text{ V}$$

$$\begin{aligned} \eta &= \frac{1}{4} \left( \frac{\hat{V}_o}{R_L} \right) \left( \frac{\hat{V}_o}{V_{CC}} \right) \\ &= \frac{1}{4} \left( \frac{8}{10} \right) \left( \frac{8}{V_{CC}} \right) = \frac{1.6}{V_{CC}} \end{aligned}$$

$V_{CC}$	16	12	10	8
$\eta$	10%	13.3%	16%	20%

**11.10** Refer to Figs. 11.6 and 11.7. A 10% loss in peak amplitude is obtained when the amplitude of the input signal is 5 V.

**11.11** For  $V_{CC} = 5$  V and  $R_L = 8 \Omega$ , the maximum sine-wave output power occurs when

$$\begin{aligned} \hat{V}_o &= V_{CC} \text{ and is } P_{L\max} = \frac{1}{2} \frac{V_{CC}^2}{R_L} \\ &= \frac{1}{2} \times \frac{25}{8} = 1.56 \text{ W} \end{aligned}$$

Correspondingly,

$$\begin{aligned} P_{S+} &= P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC} \\ &= \frac{1}{\pi} \times \frac{5}{8} \times 5 = 0.995 \text{ W} \end{aligned}$$

for a total supply power of

$$P_S = 2 \times 0.995 = 1.99 \text{ W}$$

The power conversion efficiency  $\eta$  is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{1.56}{1.99} \times 100 = 78.5\%$$

For  $\hat{V}_o = 2.5$  V,

$$P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{6.25}{8} = 0.39 \text{ W}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC}$$

$$= \frac{1}{\pi} \times \frac{2.5}{8} \times 5 = 0.497 \text{ W}$$

$$P_S = 0.995 \text{ W}$$

$$\eta = \frac{0.39}{0.995} \times 100 = 39\%$$

Thus, the efficiency reduces to half its maximum value.

**11.12**  $V_{CC} = 10$  V

For maximum  $\eta$ ,

$$\hat{V}_o = V_{CC} = 10 \text{ V}$$

The output voltage that results in maximum device dissipation is given by Eq. (11.20),

$$\begin{aligned} \hat{V}_o &= \frac{2}{\pi} V_{CC} \\ &= \frac{2}{\pi} \times 10 = 6.37 \text{ V} \end{aligned}$$

If operation is always at full output voltage,  $\eta = 78.5\%$  and thus

$$\begin{aligned} P_{\text{dissipation}} &= (1 - \eta) P_S \\ &= (1 - \eta) \frac{P_L}{\eta} = \frac{1 - 0.785}{0.785} P_L = 0.274 P_L \end{aligned}$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \times 0.274 P_L = 0.137 P_L$$

For a rated device dissipation of 2 W, and using a factor of 2 safety margin,

$$P_{\text{dissipation/device}} = 1 \text{ W}$$

$$= 0.137 P_L$$

$$\Rightarrow P_L = 7.3 \text{ W}$$

$$7.3 = \frac{1}{2} \times \frac{100}{R_L}$$

$$\Rightarrow R_L = 6.85 \Omega \text{ (i.e. } R_L \geq 6.85 \Omega \text{)}$$

The corresponding output power (i.e., greatest possible output power) is 7.3 W.

If operation is allowed at  $\hat{V}_o = \frac{1}{2} V_{CC} = 5$  V,

$$\eta = \frac{\pi}{4} \frac{\hat{V}_o}{V_{CC}} \text{ (Eq. 11.15)}$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \frac{1 - \eta}{\eta} P_L = 0.772 P_L$$

$$1 = 0.772 P_L$$

$$\Rightarrow P_L = 1.3 \text{ W}$$

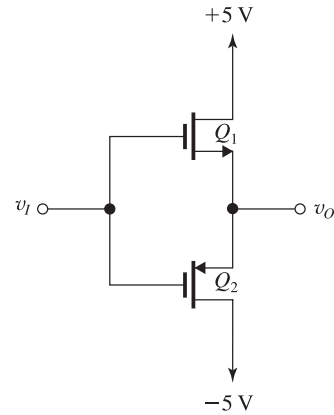
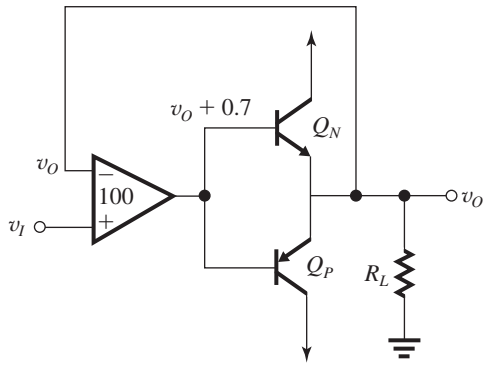
$$= \frac{1}{2} \frac{5^2}{R_L}$$

$$\Rightarrow R_L = 9.62 \Omega \text{ (i.e., } \geq 9.62 \Omega \text{)}$$

**11.13**

With  $v_I$  sufficiently positive so that  $Q_N$  is conducting, the situation shown obtains. Then,





$$(v_I - v_O) \times 100 = v_O + 0.7$$

$$\Rightarrow v_O = \frac{1}{1.01}(v_I - 0.007)$$

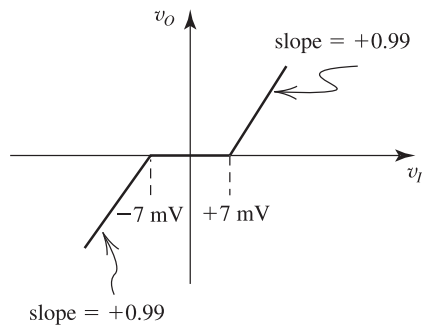
This relationship applies for  $v_I \geq 0.007$ . Similarly, for  $v_I$  sufficiently negative so that  $Q_P$  conducts, the voltage at the output of the amplifier becomes  $v_O - 0.7$ , thus

$$(v_I - v_O) \times 100 = v_O - 0.7$$

$$\Rightarrow v_O = \frac{1}{1.01}(v_I + 0.007)$$

This relationship applies for  $v_I \leq -0.007$ .

The result is the transfer characteristic



Without the feedback arrangement, the deadband becomes  $\pm 700$  mV and the slope change a little (to nearly +1 V/V).

### 11.14

Devices have  $|V_t| = 0.5$  V

$$\mu C_{ox} \frac{W}{L} = 2 \text{ mA/V}^2$$

For  $R_L = \infty$ , the current is normally zero, so

$$V_{GS} = V_t$$

$$\therefore v_O = v_I - V_{GS1} = 5 - 0.5 = 4.5 \text{ V}$$

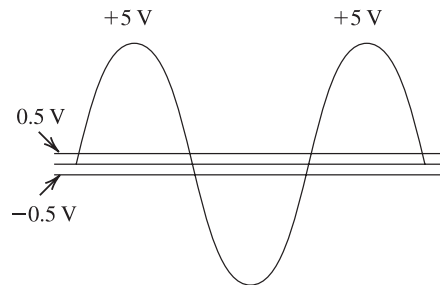
The peak output voltage will be 4.5 V

$$\sin \theta = \frac{0.5}{5} \Rightarrow \theta = 5.74^\circ$$

Crossover interval =  $4\theta = 22.968$

$$= \frac{22.96}{360} \times 100$$

$$= 6.4\%$$

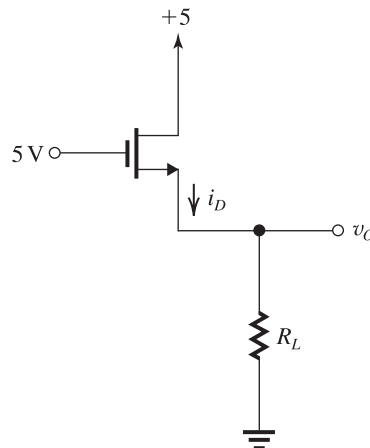


For  $v_I = 5$  V,  $v_O = 2.5$  V:

$$\therefore V_{GS} = 5 - 2.5 = 2.5 \text{ V}$$

$$i_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 2 \times (2.5 - 0.5)^2$$



$$i_D = 4 \text{ mA and } R_L = \frac{2.5 \text{ V}}{4 \text{ mA}} = 625 \Omega$$

$$\mathbf{11.15} \quad P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

$$50 = \frac{1}{2} \frac{\hat{V}_o^2}{8}$$

$$\Rightarrow \hat{V}_o = 28.3 \text{ V}$$

$$V_{CC} = 28.3 + 4 = 32.3 \rightarrow 33 \text{ V}$$

$$\text{Peak current from each supply} = \frac{\hat{V}_o}{R_L} = \frac{28.3}{8}$$

$$= 3.54 \text{ A}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi} \times 3.54 \times 33 = 37.2 \text{ W}$$

Thus,

$$P_S = 2 \times 37.2 = 74.4 \text{ W}$$

$$\eta = \frac{P_L}{P_S} = \frac{50}{74.4} = 67.2\%$$

Using Eq. (12.22), we obtain

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{33^2}{\pi^2 \times 8} = 13.8 \text{ W}$$

$$\mathbf{11.16} \quad P_L = \frac{\hat{V}_o^2}{R_L}$$

$$P_{S+} = P_{S-} = \frac{1}{2} \left( \frac{\hat{V}_o}{R_L} \right) V_{SS}$$

$$P_S = \frac{\hat{V}_o}{R_L} V_{SS}$$

$$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2 / R_L}{\hat{V}_o V_{SS} / R_L} = \frac{\hat{V}_o}{V_{SS}}$$

$$\eta_{\max} = 1 (100\%), \text{ obtained for } \hat{V}_o = V_{SS}$$

$$P_{L\max} = \frac{V_{SS}^2}{R_L}$$

$$P_{\text{dissipation}} = P_S - P_L$$

$$= \frac{\hat{V}_o}{R_L} V_{SS} - \frac{\hat{V}_o^2}{R_L}$$

$$\frac{\partial P_{\text{dissipation}}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$$

$$= 0 \text{ for } \hat{V}_o = \frac{V_{SS}}{2}$$

$$\text{Correspondingly, } \eta = \frac{V_{SS}/2}{V_{SS}} = \frac{1}{2} \text{ or } 50\%$$

**11.17** The gain in the deadband is zero. As  $v_I$  exceeds 0.5 V and increases, the current in  $Q_N$  increases and its  $r_e$  decreases, thus

$$A_v = \frac{R_L}{R_L + r_{eN}}$$

increases, and it approaches unity for large  $v_I$ . A similar situation occurs for negative  $v_I$ , resulting in the symmetrical graph shown in Fig. 1.

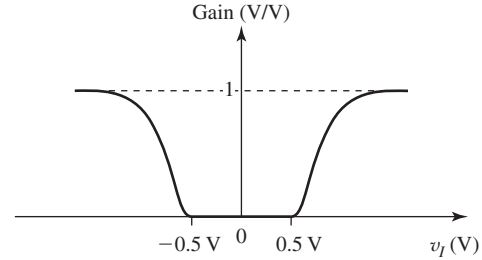


Figure 1

$$\mathbf{11.18} \quad V_{BB} = 2V_T \ln(I_Q/I_S)$$

$$= 2 \times 0.025 \ln(10^{-3}/10^{-14})$$

$$= 1.266 \text{ V}$$

At  $v_I = 0$ ,  $i_N = i_P = I_Q = 1 \text{ mA}$ , we have

$$r_{eN} = r_{eP} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{\text{out}} = r_{eN} \parallel r_{eP} = 12.5 \Omega$$

$$A_v = \frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 12.5}$$

$$= 0.889 \text{ V/V}$$

At  $v_O = 10 \text{ V}$ , we have

$$i_L = \frac{10}{100} = 0.1 \text{ A} = 100 \text{ mA}$$

To obtain  $i_N$ , we use Eq. (12.27):

$$i_N^2 - i_L i_N - I_Q^2 = 0$$

$$i_N^2 - 100 i_N - 1 = 0$$

$$\Rightarrow i_N = 100.01 \text{ mA}$$

$$i_P = i_N - i_L = 0.01 \text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} \simeq \frac{25 \text{ mV}}{100 \text{ mA}} = 0.25 \Omega$$

$$A_v = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.25} = 0.998 \text{ V/V}$$

$$\mathbf{11.19} \quad A_v = \frac{R_L}{R_L + R_{\text{out}}} \text{ and } R_{\text{out}} = \frac{r_e}{2} = \frac{V_I}{2I_Q}$$

Now  $A_v \geq 0.97$  for  $R_L \geq 100 \Omega$

$$\therefore 0.97 = \frac{100}{100 + R_{\text{out}}}$$

$$\Rightarrow R_{\text{out}} \simeq 3 \Omega$$

This table belongs to Problem 11.20.

$v_O$ (V)	$i_L$ (mA)	$i_N$ (mA)	$i_P$ (mA)	$v_{BEN}$ (V)	$v_{EBP}$ (V)	$v_i$ (V)	$v_O/v_I$ (V/V)	$R_{out}$ ( $\Omega$ )	$v_o/v_i$ (V/V)	$i_I$ (mA)	$R_{in}$ ( $\Omega$ )
+10.0	100	100.04	0.04	0.691	0.495	10.1	0.99	0.25	1.00	2	5050
+5.0	50	50.08	0.08	0.673	0.513	5.08	0.98	0.50	1.00	1	5080
+1.0	10	10.39	0.39	0.634	0.552	1.041	0.96	2.32	0.98	0.2	5205
+0.5	5	5.70	0.70	0.619	0.567	0.526	0.95	4.03	0.96	0.1	5260
+0.2	2	3.24	1.24	0.605	0.581	0.212	0.94	5.58	0.95	0.04	5300
+0.1	1	2.56	1.56	0.599	0.587	0.106	0.94	6.07	0.94	0.02	5300
0	0	2	2	0.593	0.593	0	-	6.25	0.94	0	
-0.1	-1	1.56	2.56	0.587	0.599	-0.106	0.94	6.07	0.94	-0.02	5300
-0.2	-2	1.24	3.24	0.581	0.605	-0.212	0.94	5.58	0.95	-0.04	5300
-0.5	-5	0.70	5.70	0.567	0.619	-0.526	0.95	4.03	0.96	-0.1	5260
-1.0	-10	0.39	10.39	0.552	0.634	-1.041	0.96	2.32	0.98	-0.2	5205
-5.0	-50	0.08	50.08	0.513	0.673	-5.08	0.98	0.50	1.00	-1	5080
-10.0	-100	0.04	100.04	0.495	0.691	-10.1	0.99	0.25	1.00	-2	5050

$$R_{out} = 3 = \frac{V_T}{2I_Q}$$

$$I_Q = \frac{V_T}{6} = \frac{25 \times 10^{-3}}{6} = 4.17 \text{ mA}$$

$$V_{BB} = 2V_{BE} = 2 \left[ 0.7 + V_T \ln \left( \frac{4.17}{100} \right) \right] = 1.24 \text{ V}$$

11.20 The current  $i_I$  can be obtained as

$$i_I = \frac{i_N}{\beta_N + 1} - \frac{i_P}{\beta_P + 1} = \frac{i_L}{\beta + 1}$$

where  $\beta_N = \beta_P = \beta = 49$

Using values of  $v_I$  from the table above, one can evaluate  $R_{in}$  as

$$R_{in} = \frac{v_I}{i_I}$$

Using the resistance reflection rule

$$R_{in} \simeq (\beta + 1)R_L = 50 \times 100 = 5000 \Omega$$

For large input signal, the two values of  $R_{in}$  are somewhat the same. For the small values of  $v_I$ , the calculated value in the table is larger.

11.21 At  $i_L = 0$ , we have  $i_N = i_P = I_Q$  and

$$R_{out} = \frac{1}{2} \frac{V_T}{I_Q}$$

Thus,

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} = \frac{100}{100 + \frac{12.5}{I_Q}} \quad (1)$$

where  $I_Q$  is in mA.

For  $i_L = 50$  mA, we have

$$i_N \simeq 50 \text{ mA and } i_P \simeq 0$$

Thus,

$$R_{out} \simeq r_{eN} = \frac{V_T}{i_N} = \frac{25 \text{ mV}}{50 \text{ mA}} = 0.5 \Omega$$

$$\frac{v_o}{v_i} = \frac{100}{100 + 0.5} = 0.995 \text{ V/V}$$

To limit the variation to 5%, we use

$$\frac{v_o}{v_i} \Big|_{i_L=0} = 0.995 - 0.05 = 0.945 \text{ V/V}$$

Substituting this value in Eq. (1) yields

$$I_Q = 2.15 \text{ mA}$$

11.22  $\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}}$  and

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{V_T}{I_Q + I_Q} \text{ at } v_O = 0$$

$$(a) \epsilon = 1 - \frac{v_o}{v_i} \Big|_{v_O=0}$$

$$= 1 - \frac{R_L}{R_L + R_{out}} = 1 - \frac{R_L}{R_L + \frac{V_T}{2I_Q}} =$$

$$\frac{V_T/2I_Q}{R_L + (V_T/2I_Q)}$$

$$\epsilon = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)} = \frac{V_T}{2R_L I_Q + V_T}$$

If  $2I_Q R_L \gg V_T$ , then we have

$$\epsilon \simeq \frac{V_T}{2I_Q R_L} \quad \text{Q.E.D.}$$

(b) Quiescent power dissipation =  $2V_{CC}I_Q = P_D$

(c)  $\epsilon \times$  Quiescent power dissipation =

$$\frac{V_T}{2I_Q R_L} \times 2V_{CC}I_Q = V_T \times \left(\frac{V_{CC}}{R_L}\right)$$

$$\therefore \epsilon P_D = V_T \left(\frac{V_{CC}}{R_L}\right)$$

(d)  $\epsilon P_D = V_T \frac{V_{CC}}{R_L} = 25 \times 10^{-3} \times \frac{10}{100}$

= 2.5 mW

$$P_D = \frac{2.5 \times 10^{-3}}{\epsilon}$$

$\epsilon$	$P_D$ (mW)	$I_Q$ (mA)
0.05	50	2.5
0.02	125	6.25
0.01	250	12.5

**11.23**  $I_Q = 1$  mA

For output of -1 V, we have

$$i_L = -\frac{1}{100} = -10 \text{ mA}$$

Using Eq. (12.27), we obtain

$$i_N^2 - i_L i_N - I_Q^2 = 0$$

$$i_N^2 + 10i_N - 1 = 0$$

$$i_N = 0.1 \text{ mA}$$

$$i_p = 10.1 \text{ mA}$$

Thus  $v_{EBP}$  increases by  $V_T \ln \frac{10.1}{1} = 0.06$  V

and the input step must be -1.06 V.

Largest possible positive output from 6 to 10, i.e., 4 V

Largest negative output from 6 to 0, i.e., 6 V

**11.24**  $R_{out} = r_e/2 = 8 \Omega$

$$\Rightarrow r_e = 16 \Omega$$

$$I_Q = \frac{V_T}{r_e} = \frac{25}{16} = 1.56 \text{ mA}$$

$$\text{Thus, } n = \frac{1.56}{0.2} = 7.8$$

**11.25** Since the peak positive output current is 250 mA, the base current of  $Q_N$  can be as high as

$$\frac{250}{\beta_N + 1} = \frac{250}{51} \simeq 5 \text{ mA. We select}$$

$I_{BIAS} = 6$  mA, thus providing the multiplier with a minimum current of 1 mA.

Under quiescent conditions ( $v_O = 0$  and  $i_L = 0$ ) the base current of  $Q_N$  can be neglected.

Selecting  $I_R = 0.5$  mA leaves  $I_{C1} = 5.5$  mA. To obtain a quiescent current of 2 mA in the output transistors,  $V_{BB}$  should be

$$V_{BB} = 2V_T \ln \frac{2 \times 10^{-3}}{10^{-15}} = 1.19 \text{ V}$$

Thus,

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38 \text{ k}\Omega$$

At a collector current of 5.5 mA,  $Q_1$  has

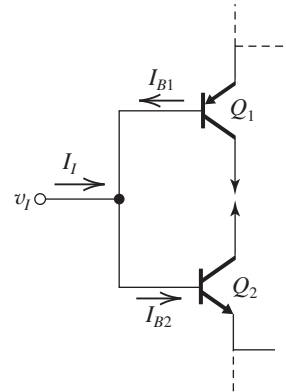
$$V_{BEL} = V_T \ln \frac{5.5 \times 10^{-3}}{10^{-14}} = 0.676 \text{ V}$$

The value of  $R_1$  can now be determined as

$$R_1 = \frac{0.676}{0.5} = 1.35 \text{ k}\Omega \text{ and}$$

$$R_2 = 2.58 - 1.35 = 1.03 \text{ k}\Omega$$

**11.26** (a) For  $R_L = \infty$ :



At  $v_I = 0$  V, we have

$$I_{B1} = I_{B2} = \frac{2.87}{200}$$

$$I_I = I_{B2} - I_{B1} = 0$$

At  $v_I = +10$  V, we have

$$I_{B1} = \frac{0.88}{200} \text{ mA} = 4.4 \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} \text{ mA} = 24.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 20 \mu\text{A}$$

At  $v_I = -10$  V, we have

$$I_{B1} = \frac{4.87}{200} \text{ mA} = 24.4 \mu\text{A}$$

$$I_{B2} = \frac{0.88}{200} \text{ mA} = 4.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = -20 \mu\text{A}$$

(b) For  $R_L = 100 \Omega$ :

At  $v_I = 0 \text{ V}$ , we have  $I_I = 0$

At  $v_I = +10 \text{ V}$ , we have

$$I_{B1} = \frac{0.38}{200} = 1.9 \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} = 24.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 22.5 \mu\text{A}$$

At  $v_I = -10 \text{ V}$ , we have  $I_I = -22.5 \mu\text{A}$

**11.27** Circuit operating near  $v_I = 0$  and is fed with a signal source having zero resistance.

The resistance looking as shown by the arrow  $X$  is

$$= R_1 \parallel r_{e1}$$

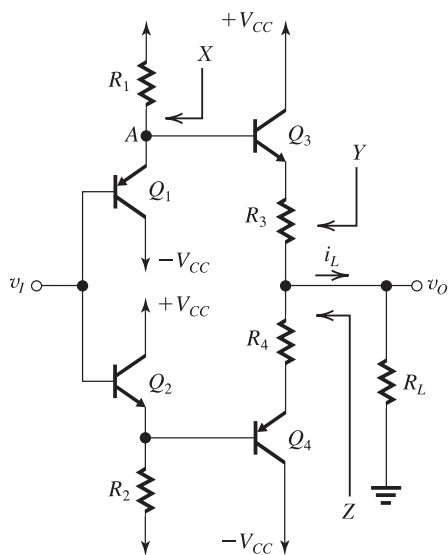
This resistance is reflected from base to the emitter of  $Q_3$  as  $(R_1 \parallel r_{e1}) / (\beta_3 + 1)$ .

The resistance seen by arrow  $Y$ , from the upper half of the circuit

$$= R_3 + r_{e3} + (R_1 \parallel r_{e1}) / (\beta_3 + 1)$$

A similar resistance is seen by the arrow  $Z$  and both of these resistances (seen by arrows  $Y$  and arrow  $Z$ ) are in parallel, therefore

$$R_{\text{out}} = \frac{1}{2} [R_3 + r_{e3} + (R_1 \parallel r_{e1}) / (\beta_3 + 1)]$$



**11.28** (a)  $v_I = 0$  and transistors have  $\beta = 100$ .

$$v_O = 0 \text{ V}$$

$$I_Q = I_{E3} = I_{E4} = I_{E1} = I_{E2} \simeq 1 \text{ mA}$$

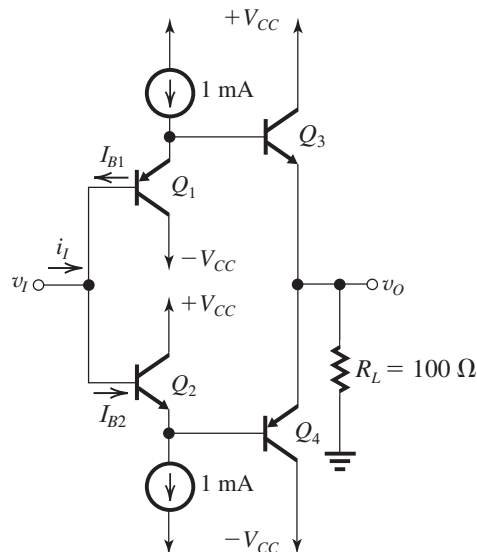
$$\text{More precisely, } \frac{I_{E3}}{\beta + 1} + I_{E1} = 1 \text{ mA}$$

thus,

$$I_Q \left( \frac{1}{\beta + 1} + 1 \right) = 1$$

$$\Rightarrow I_Q \simeq 0.99 \text{ mA}$$

Input bias current is zero because  $I_{B1} = I_{B2}$ .



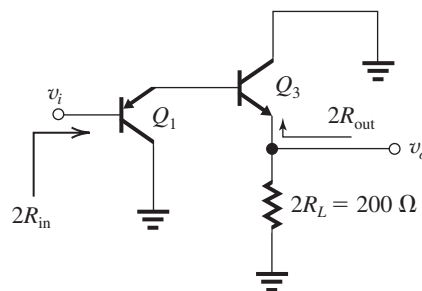
(b) From the equivalent half circuit, we have

$$2R_{\text{in}} = (\beta_1 + 1) [r_{e1} + (\beta_3 + 1) (r_{e3} + 2R_L)]$$

$$r_{e1} = r_{e3} = \frac{V_T}{I_E} = \frac{25}{1} = 25 \Omega$$

$$2R_{\text{in}} = (100 + 1)[25 + (100 + 1)(25 + 2 \times 100)]$$

$$\Rightarrow R_{\text{in}} = 1.15 \text{ M}\Omega$$



$$A_v = \frac{v_o}{v_i} = \frac{2R_L}{2R_L + r_{e3} + \frac{r_{e1}}{\beta_3 + 1}}$$

$$= \frac{200}{200 + 25 + \frac{25}{101}}$$

$$\simeq 0.89 \text{ V/V}$$

$$2R_{\text{out}} = r_{e3} + \frac{r_{e1}}{\beta + 1}$$

$$= 25 + \frac{25}{101}$$

$$R_{\text{out}} = 12.6 \Omega$$

**11.29**  $I_Q \simeq I_{\text{BIAS}} = 1 \text{ mA}$ , neglecting the base current of  $Q_N$ . More precisely,

$$I_Q = I_{\text{BIAS}} - \frac{I_Q}{\beta + 1}$$

$$\Rightarrow I_Q = \frac{I_{\text{BIAS}}}{1 + \frac{1}{\beta + 1}} \simeq 0.98 \times 1 = 0.98 \text{ mA}$$

The largest positive output is obtained when all of  $I_{\text{BIAS}}$  flows into the base of  $Q_N$ , resulting in

$$v_o = (\beta_N + 1)I_{\text{BIAS}}R_L$$

$$= 51 \times 1 \times 100 \Omega = 5.1 \text{ V}$$

The largest possible negative output voltage is limited by the saturation of  $Q_P$  to

$$-10 + V_{\text{ECsat}} = -10 \text{ V}$$

To achieve a maximum positive output of 10 V without changing  $I_{\text{BIAS}}$ ,  $\beta_N$  must be

$$10 = (\beta_N + 1) \times 1 \times 10^{-3} \times 100 \Omega$$

$$\Rightarrow \beta_N = 99$$

Alternatively, if  $\beta_N$  is held at 50,  $I_{\text{BIAS}}$  must be increased so that

$$10 = 51 \times I_{\text{BIAS}} \times 10^{-3} \times 100 \Omega$$

$$\Rightarrow I_{\text{BIAS}} = 1.96 \text{ mA}$$

for which

$$I_Q = \frac{I_{\text{BIAS}}}{1 + \frac{1}{\beta + 1}} = 1.92 \text{ mA}$$

**11.30** Figure 1(a) shows the small-signal equivalent circuit of the class AB circuit in Fig. 11.13. Here, each of  $Q_N$  and  $Q_P$  has been replaced with its hybrid- $\pi$  model, and the small resistances of the diodes have been neglected. As well, we have not included  $r_o$  of each of  $Q_N$  and  $Q_P$ .

This figure belongs to Problem 11.30.

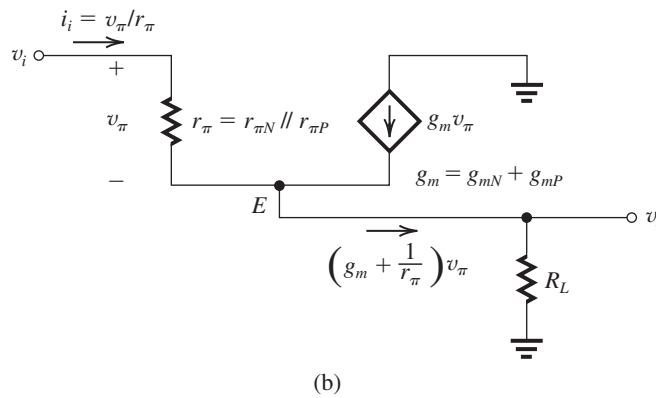
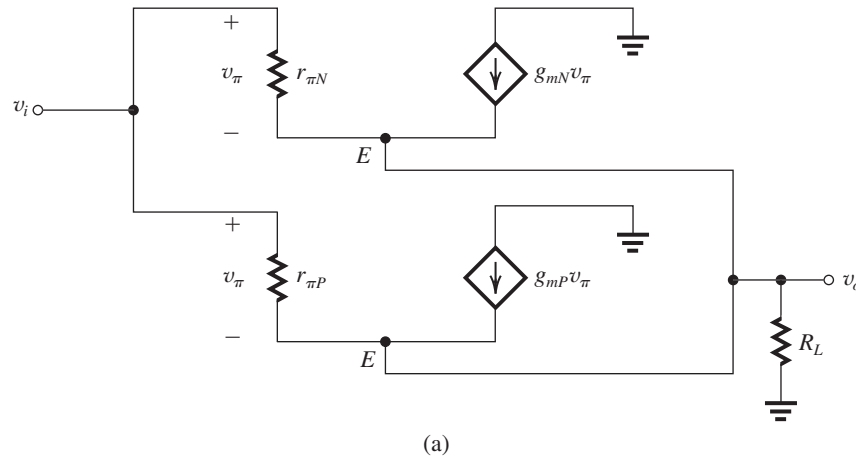


Figure 1

The circuit in Fig. 1(a) can be simplified to that in Fig. 1(b) where

$$r_\pi = r_{\pi N} \parallel r_{\pi P} \quad (1)$$

$$g_m = g_{mN} + g_{mP} \quad (2)$$

Since  $g_m \simeq \frac{1}{r_e}$ , then from (2) we obtain

$$\frac{1}{r_e} = \frac{1}{r_{eN}} + \frac{1}{r_{eP}}$$

or, equivalently,

$$r_e = r_{eN} \parallel r_{eP} \quad (3)$$

We observe that the circuit in Fig. 1(b) is the equivalent circuit of an emitter follower with the small-signal parameters  $r_\pi$ ,  $g_m$ , and  $r_e$  given in Eqs. (1), (2), and (3). Furthermore, its  $\beta$  is given by

$$\beta = g_m r_\pi = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P}) \quad (4)$$

For the circuit in Fig. 1(b), we can write

$$v_i = v_\pi + v_o \quad (5)$$

$$v_o = \left( g_m + \frac{1}{r_\pi} \right) v_\pi R_L \quad (6)$$

Equations (5) and (6) can be used to obtain the incremental (or small-signal) gain,

$$\frac{v_o}{v_i} = \frac{\left( g_m + \frac{1}{r_\pi} \right) R_L}{\left( g_m + \frac{1}{r_\pi} \right) R_L + 1}$$

But,

$$g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

Thus,

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{R_L/r_e}{R_L/r_e + 1} \\ \Rightarrow \frac{v_o}{v_i} &= \frac{R_L}{R_L + r_e} = \frac{R_L}{R_L + (r_{eN} \parallel r_{eP})} \quad \text{Q.E.D.} \quad (7) \end{aligned}$$

The input resistance is found as follows:

$$R_{\text{in}} = \frac{v_i}{i_i} = \frac{v_i}{v_\pi/r_\pi}$$

Substituting for  $v_i$  from (5) together with utilizing (7) gives

$$\begin{aligned} R_{\text{in}} &= \frac{v_\pi \left[ 1 + \left( g_m + \frac{1}{r_\pi} \right) R_L \right]}{v_\pi/r_\pi} \\ &= r_\pi + (g_m r_\pi + 1) R_L \\ &= r_\pi + (\beta + 1) R_L \end{aligned}$$

$$\begin{aligned} &= (\beta + 1)(R_L + r_e) \\ &\simeq \beta[R_L + (r_{eN} \parallel r_{eP})] \quad \text{Q.E.D.} \quad (8) \end{aligned}$$

**11.31** Refer to Fig. P11.31. Neglecting the small resistances of  $D_1$  and  $D_2$ , we can write for the voltage gain of the CE amplifier transistor  $Q_3$ ,

$$\frac{v_{c3}}{v_i} = -g_{m3} R_{\text{in}} \quad (1)$$

where  $R_{\text{in}}$  is the input resistance of the class AB circuit, given in the statement of Problem 11.30 as

$$R_{\text{in}} \simeq \beta[R_L + (r_{eN} \parallel r_{eP})] \quad (2)$$

where

$$\beta = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P}) \quad (3)$$

The voltage gain of the class AB circuit is given in the statement of Problem 11.30 as

$$\frac{v_o}{v_{c3}} = \frac{R_L}{R_L + (r_{eN} \parallel r_{eP})} \quad (4)$$

Now, we can combine (1), (2), and (4) to obtain the voltage gain of the circuit in Fig. P11.31 as

$$\begin{aligned} \frac{v_o}{v_i} &= -g_{m3} \beta [R_L + (r_{eN} \parallel r_{eP})] \frac{R_L}{R_L + (r_{eN} \parallel r_{eP})} \\ \Rightarrow \frac{v_o}{v_i} &= -g_{m3} \beta R_L \end{aligned}$$

where  $\beta$  is given by Eq. (3).

**11.32** The total resistance in the base circuit (while neglecting the small resistances of  $D_1$  and  $D_2$ ) is  $R_{\text{BIAS}} \parallel r_{o3}$ . Thus, utilizing the equivalent emitter follower for the class AB stage, we obtain for the output resistance

$$R_{\text{out}} = r_e + \frac{(R_{\text{BIAS}} \parallel r_{o3})}{\beta + 1}$$

where

$$r_e = r_{eN} \parallel r_{eP}$$

and

$$\beta = (g_{mN} + g_{mP})(r_{\pi N} \parallel r_{\pi P})$$

**11.33** At  $20^\circ\text{C}$ ,  $I_Q = 1\text{mA} = I_S e^{(0.6/0.025)}$

$$\Rightarrow I_S \text{ (at } 20^\circ\text{C)} = 3.78 \times 10^{-11} \text{ mA}$$

$$\text{At } 70^\circ\text{C}, I_S = 3.78 \times 10^{-11} (1.14)^{50}$$

$$= 2.64 \times 10^{-8} \text{ mA}$$

$$\text{At } 70^\circ\text{C}, V_T = 25 \frac{273 + 70}{273 + 20} = 29.3 \text{ mV}$$

$$\text{Thus, } I_Q \text{ (at } 70^\circ\text{C)} = 2.64 \times 10^{-8} e^{0.6/0.0293}$$

$$= 20.7 \text{ mA}$$

Additional current =  $20.7 - 1 = 19.7$  mA

Additional power =  $2 \times 20 \times 19.7 = 788$  mW

Additional temperature rise =  $10 \times 0.788$   
=  $7.9^\circ\text{C}$ ,

At  $77.9^\circ\text{C}$ :

$$V_T = \frac{25}{293} (273 + 77.9) = 29.9 \text{ mV}$$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.9} e^{(0.6/0.0299)}$$

$$= 38.7 \text{ mA}$$

etc., etc.

**11.34** (a)  $V_{BE} = 0.7$  V at 1 mA

At 0.5 mA,

$$V_{BE} = 0.7 + 0.025 \ln \frac{0.5}{1} = 0.683 \text{ V}$$

$$\text{Thus, } R_1 = \frac{0.683}{0.5} = 1.365 \text{ k}\Omega$$

and  $R_2 = 1.365 \text{ k}\Omega$

$$V_{BB} = 2V_{BE} = 1.365 \text{ V}$$

(b) For  $I_{\text{bias}} = 2$  mA,  $I_C$  increases to nearly 1.5 mA for which

$$V_{BE} = 0.7 + 0.025 \ln \frac{1.5}{1} = 0.710 \text{ V}$$

Note that  $I_R = \frac{0.710}{1.365} = 0.52$  mA is very nearly equal to the assumed value of 0.50 mA, thus no further iterations are required.

$$V_{BB} = 2V_{BE} = 1.420 \text{ V}$$

(c) For  $I_{\text{bias}} = 10$  mA, assume that  $I_R$  remains constant at 0.5 mA, thus  $I_{C1} = 9.5$  mA

$$\text{and } V_{BE} = 0.7 + 0.025 \ln \frac{9.5}{1} = 0.756 \text{ V}$$

at which

$$I_R = \frac{0.755}{1.365} = 0.554 \text{ mA}$$

Thus,

$$I_{C1} = 10 - 0.554 = 9.45 \text{ mA}$$

$$\text{and } V_{BE} = 0.7 + 0.025 \ln \frac{9.45}{1} = 0.756 \text{ V}$$

$$\text{Thus, } V_{BB} = 2 \times 0.756 = 1.512 \text{ V}$$

(d) Now for  $\beta = 100$ ,

$$I_{R1} = \frac{0.756}{1.365} = 0.554 \text{ mA}$$

$$I_{R2} = 0.554 + \frac{9.45}{101} = 0.648 \text{ mA}$$

$$I_C = 10 - 0.648 = 9.352 \text{ mA}$$

$$\text{Thus, } V_{BE} = 0.7 + 0.025 \ln \frac{9.352}{1} = 0.756 \text{ V}$$

$$V_{BB} = 0.756 + I_{R2}R_2$$

$$= 0.756 + 0.648 \times 1.365$$

$$= 1.641 \text{ V}$$

**11.35** Figure 1 on the next page shows the  $V_{BE}$  multiplier together with its small-signal equivalent circuit prepared for determining the incremental terminal resistance  $r$ ,

$$r \equiv \frac{v_x}{i_x}$$

Now,

$$i_x = g_m v_\pi + \frac{v_\pi}{R_1 \parallel r_\pi} \quad (1)$$

$$v_x = v_\pi + \frac{v_\pi}{R_1 \parallel r_\pi} R_2 \quad (2)$$

Dividing (2) by (1) gives

$$r = \frac{1 + R_2/(R_1 \parallel r_\pi)}{g_m + \frac{1}{R_1 \parallel r_\pi}}$$

$$= \frac{R_2 + (R_1 \parallel r_\pi)}{1 + g_m(R_1 \parallel r_\pi)}$$

For  $R_1 = R_2 = 1.2 \text{ k}\Omega$ ,  $I_C = 1$  mA, and  $\beta = 100$ , we have

$$g_m = 40 \text{ mA/V}$$

$$r_\pi = \frac{100}{40} = 2.5 \text{ k}\Omega$$

Thus,

$$r = \frac{1.2 + (1.2 \parallel 2.5)}{1 + 40(1.2 \parallel 2.5)} = 60.2 \text{ }\Omega$$

### 11.36

At  $v_I = 5$  V, we have

$$V_{E1} = +5.7 \text{ V}$$

$$I_{R1} = \frac{V_{CC} - V_{E1}}{R_1} = \frac{10 - 5.7}{R_1} = \frac{4.3}{R_1}$$

To allow for  $I_{B3} = 10$  mA if needed while reducing  $I_{E1}$  by no more than half, then  $I_{R1}$  must be  $2 \times 10 = 20$  mA. Thus,

$$R_1 = \frac{V_{R1}}{I_{R1}} = \frac{4.3}{20} = 0.215 \text{ k}\Omega = 215 \text{ }\Omega$$

Similarly,

$$R_2 = 0.215 \text{ k}\Omega = 215 \text{ }\Omega$$

Next, we determine the values of  $R_3$  and  $R_4$ : At  $v_I = 0$ , assume  $V_{EB1} = 0.7$ . Then

$$V_{E1} = 0.7 \text{ V}$$



This figure belongs to Problem 11.35.

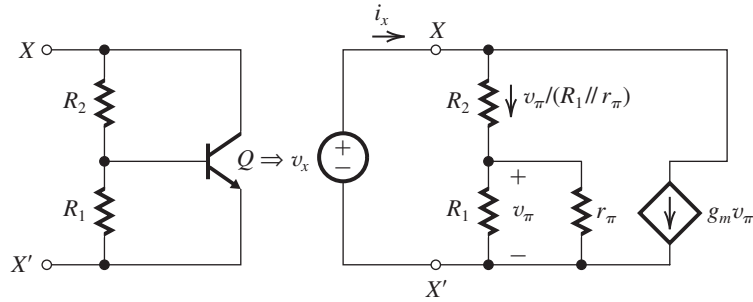
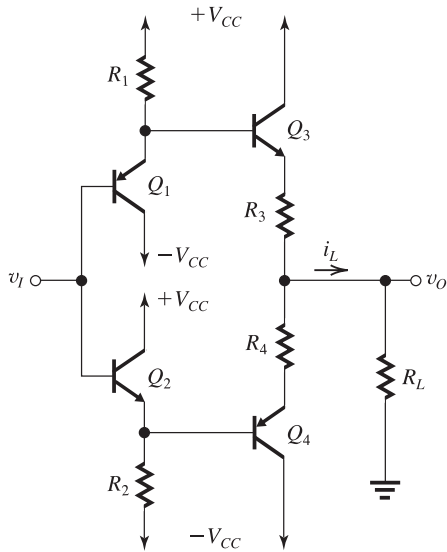


Figure 1



$$I_{R1} = \frac{10 - 0.7}{0.215} = 43.3 \text{ mA}$$

$$V_{EB1} = 0.7 + 0.025 \times \ln\left(\frac{43.3}{10}\right)$$

$$= 0.737 \text{ V}$$

$$V_{E1} = 0.737 \text{ V}$$

Meanwhile  $Q_3$  will be conducting  $I_Q = 40 \text{ mA}$ . Since  $I_{S3} = 3I_{S1}$  then  $Q_3$  has  $V_{BE} = 0.7 \text{ V}$  at  $I_C = 30 \text{ mA}$ . At  $40 \text{ mA}$ ,

$$V_{BE3} = 0.7 + 0.025 \times \ln\left(\frac{40}{30}\right)$$

$$= 0.707 \text{ V}$$

For  $v_o = 0$ ,

$$V_{E1} - V_{BE3} - I_{E3}R_3 = 0$$

$$0.737 - 0.707 - 40R_3 = 0$$

$$\Rightarrow R_3 = 0.75 \Omega$$

Similarly,

$$R_4 = 0.75 \Omega$$

$$R_{\text{out}} = \frac{1}{2} \left[ R_3 + r_{e3} + \frac{R_1 \parallel r_{e1}}{\beta_3 + 1} \right]$$

where

$$r_{e3} = \frac{25 \text{ mV}}{40 \text{ mA}} = 0.625 \Omega$$

$$r_{e1} = \frac{25 \text{ mV}}{20 \text{ mA}} = 1.25 \Omega$$

$$R_{\text{out}} = \frac{1}{2} \left[ 0.75 + 0.625 + \frac{215 \parallel 1.25}{51} \right]$$

$$R_{\text{out}} = 0.7 \Omega$$

Next, consider the situation when

$$v_i = +1 \text{ V and } R_L = 2 \Omega$$

Let  $v_o \approx 1 \text{ V}$ , then

$$i_L = \frac{1 \text{ V}}{2 \Omega} = 0.5 \text{ A} = 500 \text{ mA}$$

Now if we assume that  $i_{E4} \approx 0$ , then

$$i_{E3} = i_L = 500 \text{ mA}$$

$$V_{BE3} = 0.7 + 0.025 \ln \frac{500}{30}$$

$$= 0.770 \text{ V}$$

$$i_{B3} = \frac{500}{51} \approx 10 \text{ mA}$$

Assuming that  $V_{EB1} \approx 0.7 \text{ V}$ , then

$$v_{E1} = 1 + 0.7 = 1.7 \text{ V}$$

$$i_{R1} = \frac{10 - 1.7}{0.215} = 38.6 \text{ mA}$$

$$i_{E1} = i_{R1} - i_{B2} = 38.6 - 10 = 28.6 \text{ mA}$$

$$V_{EB1} = 0.7 + 0.025 \ln \frac{28.6}{10}$$

$$= 0.726 \text{ V}$$

$$V_{E1} = 1.726 \text{ V}$$

$$i_L = \frac{V_{E1} - V_{BE3}}{R_3 + R_L}$$

$$= \frac{1.726 - 0.770}{0.75 + 2}$$

$$= 0.348 \text{ A}$$

$$v_O = i_L R_L$$

$$= 0.348 \times 2 = 0.695 \text{ V}$$

Let's check that  $i_{E4}$  is zero. The voltage at the base of  $Q_4$  is

$$V_{B4} = 1 - V_{BE2}$$

$$\simeq 1 - 0.74 = 0.26 \text{ V}$$

The voltage across  $R_4$  and  $V_{EB4}$  is

$$= v_O - 0.26 = 0.695 - 0.26 = 0.435 \text{ V}$$

which is sufficiently small to keep  $Q_4$  cut off, verifying our assumption that  $i_{E4} \simeq 0$ .

Let's now do more iterations to refine our estimate of  $v_O$ :

$$i_L = 0.35 \text{ A}$$

$$i_{B3} = \frac{0.35}{51} \simeq 7 \text{ mA}$$

$$i_{E1} = \frac{10 - 1 - 0.726}{0.215} - 7 = 31.5 \text{ mA}$$

$$V_{EB1} = 0.7 + 0.025 \ln\left(\frac{31.5}{10}\right)$$

$$= 0.729 \text{ V}$$

$$V_{E1} = 1 + 0.729 = 1.729 \text{ V}$$

$$i_{E3} = i_L = 350 \text{ mA}$$

$$V_{BE3} = 0.7 + 0.025 \ln\left(\frac{350}{30}\right)$$

$$= 0.761 \text{ V}$$

$$i_L = \frac{V_{E1} - V_{BE3}}{R_3 + R_2}$$

$$= \frac{1.729 - 0.761}{0.75 + 2} = 0.352 \text{ A}$$

$$v_O = i_L R_L$$

$$= 0.352 \times 2 = 0.704 \text{ V}$$

### 11.37

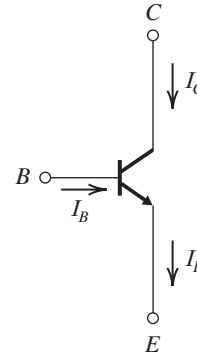
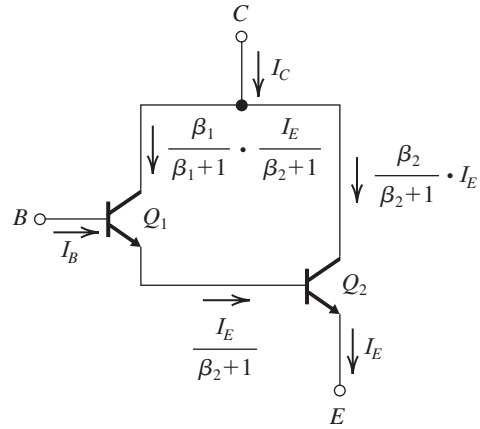
(a) For the composite transistor, we have

$$\beta = \frac{I_C}{I_B}$$

Refer to the diagram.

$$I_B = \frac{1}{\beta_1 + 1} \frac{I_E}{\beta_2 + 1}$$

$$I_C = \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} I_E + \frac{\beta_2}{(\beta_2 + 1)} I_E$$



$$= \frac{\beta_1 + \beta_2 (\beta_1 + 1)}{(\beta_1 + 1)(\beta_2 + 1)} \cdot I_E$$

For the composite transistor,  $\beta$  is given by

$$\beta = \frac{I_C}{I_B} = \frac{\frac{\beta_1 + \beta_2 (\beta_1 + 1)}{(\beta_1 + 1)(\beta_2 + 1)} \times I_E}{\frac{1}{(\beta_1 + 1)(\beta_2 + 1)} \cdot I_E}$$

$$= \beta_1 + \beta_2 (\beta_1 + 1)$$

$$\simeq \beta_1 \beta_2 \text{ since } \beta_1 \gg 1 \text{ and } \beta_2 \gg 1$$

(b) Refer to the diagram.

Operating current of  $Q_2$

$$= I_{C2} = \frac{\beta_2}{\beta_2 + 1} I_E = \frac{\beta_2}{\beta_2 + 1} \times \frac{\beta + 1}{\beta} I_C$$

$$\text{where } \beta = \beta_1 \beta_2$$

$$\simeq I_C \text{ since } \beta_2 \gg 1 \text{ and } \beta \gg 1$$

Operating current of  $Q_1$

$$= I_{C1} = \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} I_E$$

$$= \frac{\beta_1}{(\beta_1 + 1)(\beta_2 + 1)} \cdot \frac{\beta + 1}{\beta} I_C$$

$$\simeq \frac{I_C}{\beta_2} \text{ since } \beta \gg 1, \beta_2 \gg 1 \text{ and } \beta_1 \gg 1.$$

(c) Again refer to the diagram and part (b).

$$V_{BE} = V_{BE2} + V_{BE1} = V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

From part (b),  $I_{C2} \simeq I_C$  and  $I_{C1} \simeq \frac{I_C}{\beta_2}$

$$\begin{aligned} \therefore V_{BE} &= V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{1}{\beta_2} \frac{I_C}{I_S}\right) \\ &= V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{I_C}{I_S}\right) + V_T \ln\left(\frac{1}{\beta_2}\right) \end{aligned}$$

$$V_{BE} = 2V_T \ln\left(\frac{I_C}{I_S}\right) - V_T \ln(\beta_2)$$

$$(d) \quad r_{\pi \text{eq}} = (\beta_1 + 1)[r_{e1} + (\beta_2 + 1)r_{e2}]$$

$$\text{Here, } r_{e2} = \frac{V_T}{I_{E2}} \simeq \frac{V_T}{I_{C2}} \simeq \frac{V_T}{I_C}$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{V_T}{I_{C1}} \simeq \frac{V_T}{I_C/\beta_2} = \beta_2 r_{e2}$$

$$r_{\pi \text{eq}} \simeq (\beta_1 + 1)[\beta_2 r_{e2} + \beta_2 r_{e2}]$$

$$= 2(\beta_1 + 1)\beta_2 r_{e2}$$

$$\cong 2\beta_1\beta_2 r_{e2}$$

$$= 2\beta_1\beta_2 \frac{V_T}{I_C}$$

(e) To find  $g_{\text{meq}}$ , apply a signal  $v_{be}$  and find the corresponding current  $i_c$ :

$$i_c = i_{c1} + i_{c2} = g_{m1}v_{be1} + g_{m2}v_{be2}$$

$$= g_{m1}v_{be} \frac{r_{e1}}{r_{e1} + (\beta_2 + 1)r_{e2}}$$

$$+ g_{m2} \frac{(\beta_2 + 1)r_{e2}}{r_{e1} + (\beta_2 + 1)r_{e2}} \cdot v_{be}$$

$$\simeq v_{be} \frac{1}{\beta_2 r_{e2} + (\beta_2 + 1)r_{e2}}$$

$$+ \frac{\beta_2}{\beta_2 r_{e2} + (\beta_2 + 1)r_{e2}} \cdot v_{be}$$

$$\therefore g_m r_e \simeq 1,$$

$$i_c \simeq v_{bc} \frac{1}{2\beta_2 r_{e2}} + v_{be} \frac{\beta_2}{2\beta_2 r_{e2}}$$

$$\simeq v_{bc} \frac{(\beta_2 + 1)}{2\beta_2 r_{e2}}$$

$$\simeq v_{bc} \frac{1}{2r_{e2}}$$

$$g_{\text{meq}} = \frac{i_c}{v_{be}} = \frac{1}{2r_{e2}}$$

$$= \frac{1}{2} \frac{I_C}{V_T}$$

**11.38 (a)**

From Figure 1 we can write

$$5 = I_{C2}R_C + \frac{I_{C2}}{\beta^2}R_B + 1.4$$

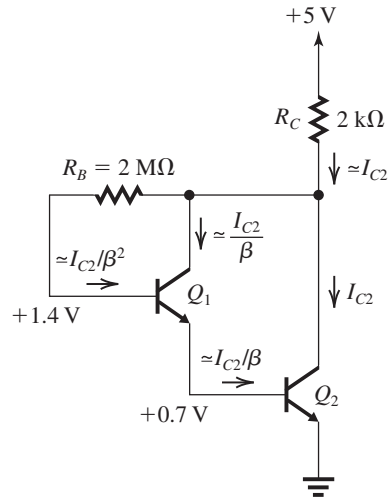


Figure 1

$$\Rightarrow I_{C2} = \frac{5 - 1.4}{R_C + \frac{R_B}{\beta^2}}$$

$$= \frac{3.6}{2 + \frac{2000}{10,000}} = 1.64 \text{ mA}$$

$$I_{C1} \simeq \frac{I_{C2}}{\beta} = \frac{1.64}{100} = 0.0164 \text{ mA}$$

(b)

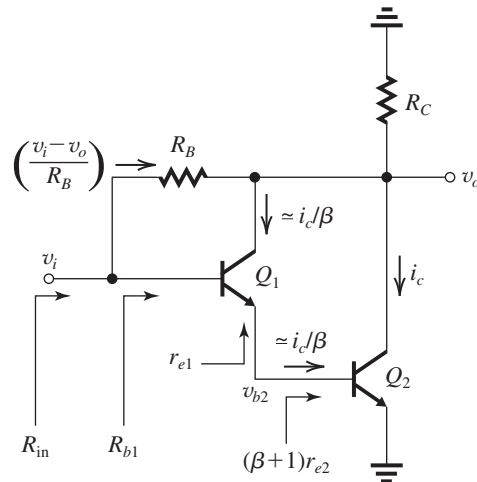


Figure 2

$$v_{b2} = v_i \frac{(\beta + 1)r_{e2}}{(\beta + 1)r_{e2} + r_{e1}}$$

where

$$r_{e2} = \frac{V_T}{I_{E2}} \simeq \frac{V_T}{I_{C2}} = \frac{25 \text{ mV}}{1.64 \text{ mA}} = 15.2 \Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{V_T}{I_{C1}} = \frac{25 \text{ mV}}{0.0164 \text{ mA}} = 1.52 \text{ k}\Omega$$

$$v_{b2} = v_i \frac{101 \times 15.2}{101 \times 15.2 + 1520} = 0.5v_i$$

$$i_c = g_{m2}v_{b2} \\ = g_{m2} \times 0.5v_i$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.64}{0.025} = 65.6 \text{ mA/V}$$

$$i_c = 65.6 \times 0.5v_i = 32.8v_i$$

Writing a node equation at the output provides

$$\frac{v_o}{R_C} + i_c + \frac{v_o - v_i}{R_B} = 0$$

Substituting  $i_c = 32.8v_i$ , we obtain

$$v_o \left( \frac{1}{R_C} + \frac{1}{R_B} \right) = -v_i \left[ \left( 1 + \frac{1}{\beta} \right) 32.8 - \frac{1}{R_B} \right]$$

$$A_v \equiv \frac{v_o}{v_i} = - \frac{\left( 1 + \frac{1}{\beta} \right) 32.8 - \frac{1}{R_B}}{\frac{1}{R_C} + \frac{1}{R_B}} \\ = - \frac{1.01 \times 32.8 - (1/2000)}{\frac{1}{2} + \frac{1}{2000}} \\ = -66.2 \text{ V/V}$$

$$(c) R_{b1} = (\beta + 1)[r_{e1} + (\beta + 1)r_{e2}] \\ = 101[1.52 + 101 \times 0.0152] \\ = 318.7 \text{ k}\Omega$$

The component of  $R_{in}$  arising from  $R_B$  can be found as

$$R_{i2} = \frac{v_i}{(v_i - v_o)/R_B} \\ = \frac{R_B}{1 - (v_o/v_i)} = \frac{2000}{1 - (-66.2)} = 29.8 \text{ k}\Omega$$

Thus

$$R_{in} = R_{ib} \parallel R_{i2} \\ = 318.7 \parallel 29.8 = 27.2 \text{ k}\Omega$$

**11.39** (a) DC Analysis:

$$1 \text{ mA} = 0.0214 + \frac{I_{E2}}{1010} + I_{E2}$$

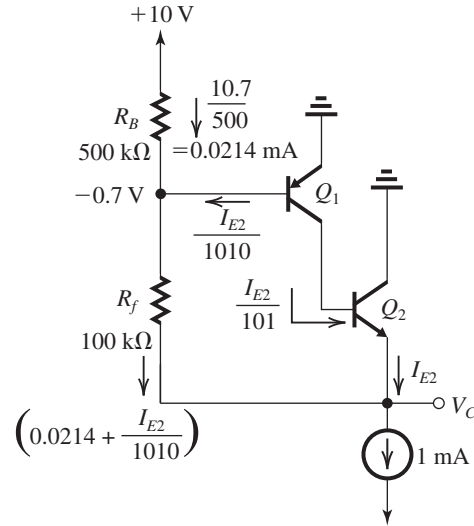
$$\Rightarrow I_{E2} = 0.978 \text{ mA}$$

$$I_{C2} = 0.99 \times 0.978 = 0.97 \text{ mA}$$

$$I_{C1} = \frac{0.978}{101} = 9.7 \mu\text{A}$$

$$V_C = -0.7 - 100 \left( 0.0214 + \frac{0.978}{1010} \right)$$

$$= -2.94 \text{ V}$$



(b) Small-signal parameters:

$$g_{m1} = \frac{9.7 \times 10^{-6}}{25 \times 10^{-3}} = 0.388 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 25.77 \text{ k}\Omega$$

$$r_{o1} = \frac{|V_A|}{I_{C1}} = \frac{100}{9.7 \mu\text{A}} = 10.31 \text{ M}\Omega$$

$$g_{m2} = \frac{0.97 \times 10^{-3}}{25 \times 10^{-3}} = 38.8 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = 2.58 \text{ k}\Omega$$

$$r_{o2} = |V_A|/I_{C2} = 103.1 \text{ k}\Omega$$

Node equation at  $b_2$ :

$$g_{m1}v_{\pi 1} + \frac{v_{b2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

But  $v_{b2} = v_o + v_{\pi 2}$ , then

$$g_{m1}v_{\pi 1} + \frac{v_o + v_{\pi 2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

$$\Rightarrow v_{\pi 2} \left( \frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}} \right) = - \left( \frac{v_o}{r_{o1}} + g_{m1}v_{\pi 1} \right)$$

$$\text{or, } v_{\pi 2} = - \frac{\frac{v_o}{r_{o1}} + g_{m1}v_{\pi 1}}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}}$$

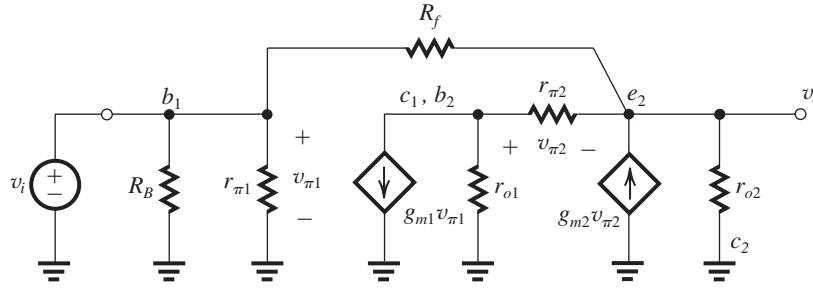
Node equation at output:

$$\frac{v_o}{r_{o2}} + \frac{v_o - v_{\pi 1}}{R_f} = g_{m2}v_{\pi 2} + \frac{1}{r_{\pi 2}}v_{\pi 2}$$

$$= \left( g_{m2} + \frac{1}{r_{\pi 2}} \right) v_{\pi 2}$$

$$= - \frac{\left( g_{m2} + \frac{1}{r_{\pi 2}} \right) \left[ \frac{v_o}{r_{o1}} + g_{m1}v_{\pi 1} \right]}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}}$$

This figure belongs to Problem 11.39, part (b).



Substituting  $v_{\pi 1} = v_i$  and collecting terms, we obtain

$$\begin{aligned} v_o & \left[ \frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{\left( g_{m2} + \frac{1}{r_{\pi 2}} \right)}{r_{o1} \left( \frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}} \right)} \right] \\ & = -v_i \left[ \frac{g_{m1} \left( g_{m2} + \frac{1}{r_{\pi 2}} \right)}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}} - \frac{1}{R_f} \right] \\ & \frac{v_o}{v_i} = - \frac{\frac{g_{m1} \left( g_{m2} + \frac{1}{r_{\pi 2}} \right)}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}} - \frac{1}{R_f}}{\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{\left( g_{m2} + \frac{1}{r_{\pi 2}} \right)}{r_{o1} \left( \frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}} \right)}} \end{aligned}$$

Since  $r_{\pi 2} \ll r_{o1}$ , we have

$$\begin{aligned} \frac{v_o}{v_i} & \simeq - \frac{g_{m1} (g_{m2} r_{\pi 2} + 1) - \frac{1}{R_f}}{\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{1}{r_{o1}} (g_{m2} r_{\pi 2} + 1)} \\ & = - \frac{g_{m1} (\beta_2 + 1) - \frac{1}{R_f}}{\left( \frac{1}{r_{o2}} + \frac{1}{R_f} \right) + \frac{1}{r_{o1}} (\beta_2 + 1)} \end{aligned}$$

Since  $\frac{1}{R_f} \ll g_{m1} (\beta_2 + 1)$ , we have

$$\frac{v_o}{v_i} \simeq - \frac{g_{m1} (\beta_2 + 1)}{\left( \frac{1}{r_{o2}} + \frac{1}{R_f} \right) + \frac{1}{r_{o1}} (\beta_2 + 1)}$$

Substituting  $\beta_2 = \beta_N$  and noting that  $\beta_N \gg 1$ , we obtain

$$\begin{aligned} \frac{v_o}{v_i} & \simeq -g_{m1} \frac{1}{\frac{1}{\beta_N} \left( \frac{1}{r_{o2}} + \frac{1}{R_f} \right) + \frac{1}{r_{o1}}} \\ & = -g_{m1} [r_{o1} \parallel \beta_N (r_{o2} \parallel R_f)] \quad \text{Q.E.D.} \end{aligned}$$

(c)

$$\begin{aligned} \frac{v_o}{v_i} & = -0.388 [10.31 \times 10^3 \parallel 100 (103.1 \parallel 100)] \\ & = -1320 \text{ V/V} \\ R_{\text{in}} & = R_B \parallel r_{\pi 1} \parallel \left[ v_i / \left( \frac{v_i - v_o}{R_f} \right) \right] \\ & = 500 \parallel 25.77 \parallel \left[ \frac{R_f}{1 - \frac{v_o}{v_i}} \right] \\ & = 500 \parallel 25.77 \parallel \frac{100}{1 + 1320} \\ & = 500 \parallel 25.77 \parallel 0.0757 \\ & = 75.5 \Omega \end{aligned}$$

**11.40** First consider the situation in the quiescent state and find  $V_{BB}$ .

$$I_{Q2} = I_{Q4} = 2 \text{ mA}$$

$$\begin{aligned} V_{BE2} = V_{BE4} & = 0.7 + 0.025 \ln \left( \frac{2}{10} \right) \\ & = 0.660 \text{ V} \end{aligned}$$

For  $Q_1$  and  $Q_3$ , we have

$$I_C = \frac{2}{\beta} = \frac{2}{100} = 0.02 \text{ mA}$$

$$\begin{aligned} V_{BE1} = |V_{BE3}| & = 0.7 + 0.025 \ln \left( \frac{0.02}{1} \right) \\ & = 0.602 \text{ V} \end{aligned}$$

$$I_{B1} = \frac{0.02 \text{ mA}}{\beta} = \frac{0.02}{100} = 0.2 \mu\text{A}$$

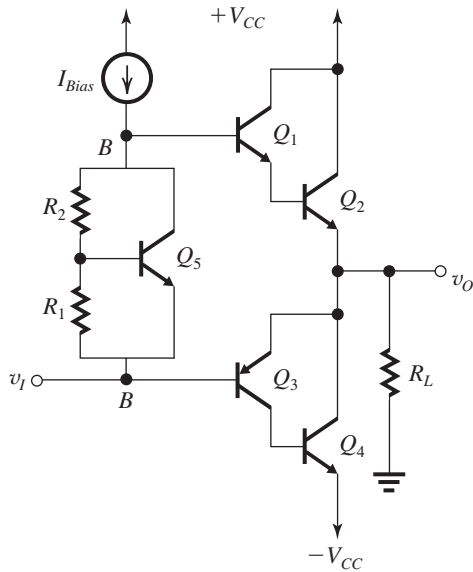
$$\begin{aligned} I_{\text{Bias}} & = 100 \times \text{Base current in } B_1 \\ & = 100 \times 0.2 = 20 \mu\text{A} \end{aligned}$$

$$I_{R1,R2} = \frac{1}{10} \times 20 \mu\text{A} = 2 \mu\text{A}$$

$$\text{and } I_{C5} = 20 - 2 = 18 \mu\text{A}$$

$$V_{BE5} = 0.7 + 0.025 \ln \left( \frac{18 \mu}{1 \text{ m}} \right) \simeq 0.600 \text{ V}$$

$$V_{BB} = V_{BE1} + V_{BE2} + |V_{BE3}|$$



$$= 0.602 + 0.660 + 0.602$$

$$= 1.864 \text{ V}$$

$$R_1 + R_2 = \frac{1.864}{2 \mu\text{A}} = 932 \text{ k}\Omega$$

$$R_1 = \frac{0.600}{2 \mu\text{A}} = 300 \text{ k}\Omega$$

$$R_2 = 932 - 300 = 632 \text{ k}\Omega$$

Now find  $v_I$  for  $v_O = 10 \text{ V}$  and  $R_L = 1 \text{ k}\Omega$ .

$Q_2$  is conducting most of the current and  $Q_4$  conducting a negligible current.

$$\therefore I_{C2} \simeq I_L = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

$\therefore$  The current through each of  $Q_1$  and  $Q_2$  increases by a factor of  $\frac{10}{2} = 5$

$$\text{Thus } V_{BE2} = 0.66 + 0.025 \ln 5 = 0.700 \text{ V}$$

$$V_{BE1} = 0.602 + 0.025 \ln 5 = 0.642 \text{ V}$$

$$\text{and } I_{B1} = 5 \times 0.2 \mu\text{A} = 1 \mu\text{A}$$

$\therefore$  The current through the multiplier is  $I_{B\text{ias}} - 1 = 20 - 1 = 19 \mu\text{A}$ . Assuming most of the decrease occurs in  $I_{C5}$ , we obtain

$$\therefore I_{C5} = 18 - 1 = 17 \mu\text{A}$$

$$V_{BE5} = 0.7 + 0.025 \ln \left( \frac{17 \mu\text{A}}{1 \mu\text{A}} \right) = 0.598 \text{ V}$$

$\therefore V_{BB1}$ , the voltage across the multiplier is

$$V_{BB} = 0.598 \times \frac{932}{300} = 1.858 \text{ V}$$

It follows that  $V_{EB3}$  becomes

$$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516 \text{ V}$$

i.e.  $V_{EB3}$  has decreased by  $0.600 - 0.516 = 0.084 \text{ V}$

Correspondingly,  $I_{C3}$  will decrease by a factor of

$$e^{\frac{-0.084}{0.025}} = 0.035.$$

$$\therefore I_{C4} \text{ becomes } 0.035 \times 2 = 0.07 \text{ mA}$$

This value is close to zero, no iteration required.

$$\therefore v_I = 10 + 0.7 + 0.642 - 1.858$$

$$v_I = 9.484 \text{ V}$$

Now find  $v_I$  for  $v_O = -10 \text{ V}$  and  $R_L = 1 \text{ k}\Omega$ .

$$i_L = \frac{-10}{1 \text{ k}\Omega} = -10 \text{ mA}$$

Assume that current through  $Q_2$  is almost zero.

$$\therefore I_{C4} \simeq 10 \text{ mA}$$

The current through  $Q_4$  increases by a factor of 5 (relative to the quiescent value).

$\therefore$  The current through  $Q_3$  must also

increase by the same factor. Thus

$$|V_{BE3}| = 0.602 + 0.025 \ln 5 = 0.642 \text{ V}$$

$|V_{BE3}|$  has increased by  $0.642 - 0.602 = 0.04 \text{ V}$ . Since  $Q_1$  and  $Q_2$  are almost cut off, all of the  $I_{B\text{ias}}$  now flows through the  $V_{BE}$  multiplier. That is an increase of  $0.2 \mu\text{A}$ . Assuming that most of the increase occurs in  $I_{C5}$ ,  $V_{BE5}$  becomes

$$V_{BE5} = 0.7 + 0.025 \ln \left( \frac{18.2 \mu\text{A}}{1 \mu\text{A}} \right) \simeq 0.600 \text{ V}$$

The voltage  $V_{BE5}$  remains almost constant, and the voltage across the multiplier will remain almost constant. Thus the increase in  $|V_{EB3}|$  will result in an equal decrease in  $|V_{BE1}| + |V_{BE2}|$ , i.e.

$$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$$

The current through each of  $Q_1$  and  $Q_2$  decreases by the same factor, let it be  $m$ ; then

$$0.025 \ln m + 0.025 \ln m = -0.04 \text{ V}$$

$$\Rightarrow m = 0.45$$

$$\text{Thus } I_{C2} = 0.45 \times 2 = 0.9 \text{ mA}$$

Now do iteration

$$I_{C4} = 10.9$$

$$I_{C4} \text{ has increased by a factor of } \frac{10.9}{2} = 5.45$$

$$\therefore |V_{BE3}| = 0.602 + 0.025 \ln 5.45$$

$$= 0.644$$

$$v_I = v_O + |V_{EB3}|$$

$$v_I \simeq -10.644 \text{ V}$$

**11.41** (a) Refer to the circuit in Fig. P11.41. While  $D_1$  is conducting, the voltage at the emitter

of  $Q_3$  is  $(V_{CC1} - V_D)$ . For  $Q_3$  to turn on, the voltage at its base must be at least equal to  $V_{CC1} = 35$  V. This will occur when  $v_I$  reaches the value

$$\begin{aligned} v_I &= V_{CC1} - V_{Z1} - V_{BB} \\ &= 35 - 3.3 - 1.2 = 30.5 \text{ V} \end{aligned}$$

This is the positive threshold at which  $Q_3$  is turned on.

(b) The power dissipated in the circuit is given by Eq. (11.19):

$$P_D = \frac{2}{\pi} \frac{\hat{V}_o}{R_L} V_{CC} - \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

For 95% of the time,  $\hat{V}_o = 30$  V,  $V_{CC} = 35$  V,

$$\begin{aligned} P_D &= \frac{1}{R_L} \left[ \frac{2}{\pi} \times 30 \times 35 - \frac{1}{2} \times 30^2 \right] \\ &= \frac{218.5}{R_L} \end{aligned}$$

For 5% of the time,  $\hat{V}_o = 65$  V,  $V_{CC} = 70$  V,

$$\begin{aligned} P_D &= \frac{1}{R_L} \left[ \frac{2}{\pi} \times 65 \times 70 - \frac{1}{2} \times 65^2 \right] \\ &= \frac{784.1}{R_L} \end{aligned}$$

Thus, the total power dissipation is

$$\begin{aligned} P_D &= \frac{218.5}{R_L} \times 0.95 + \frac{784.1}{R_L} \times 0.05 \\ &= \frac{246.8}{R_L} \end{aligned} \quad (1)$$

This should be compared to the power dissipation of a class AB output stage operated from  $\pm 70$  V. Here,

$$\begin{aligned} P_D \text{ (for 95% of the time)} \\ &= \frac{1}{R_L} \left[ \frac{2}{\pi} \times 30 \times 70 - \frac{1}{2} \times 30^2 \right] \\ &= \frac{886.9}{R_L} \end{aligned}$$

$$\begin{aligned} P_D \text{ for 5% of the time)} \\ &= \frac{1}{R_L} \left[ \frac{2}{\pi} \times 65 \times 70 - \frac{1}{2} \times 65^2 \right] \\ &= \frac{784.1}{R_L} \end{aligned}$$

$$\begin{aligned} \text{Total dissipation} &= \frac{886.9}{R_L} \times 0.95 + \frac{784.1}{R_L} \times 0.05 \\ &= \frac{881.8}{R_L} \end{aligned} \quad (2)$$

The results in (1) and (2) indicate that using the Class G circuit in Fig. P11.41 results in a reduction in  $P_D$  by a factor of 3.6!

**11.42** When  $V_{BE5} = 100 \times 10^{-3} \times R_{E1}$ , then  $I_{C5} = 2$  mA

$$\begin{aligned} V_{BE5} &= V_T \ln \left( \frac{I_{C5}}{I_S} \right) \\ &= 25 \times 10^{-3} \ln \left( \frac{2 \times 10^{-3}}{10^{-14}} \right) \end{aligned}$$

$$= 0.651 \text{ V}$$

$$100 \times 10^{-3} R_{E1} = 0.651$$

$$R_{E1} = 6.51 \text{ } \Omega$$

If peak output current = 60 mA,

$$V_{BE5} = R_{E1} \times 60 \text{ mA}$$

$$= 6.51 \times 60 \times 10^{-3}$$

$$= 0.391 \text{ V}$$

$$i_{C5} = I_S e^{V_{BE5}/V_T}$$

$$= 10^{-14} e^{0.434/25 \times 10^{-3}}$$

$$\cong 0.35 \mu\text{A}$$

**11.43** Refer to Fig. P11.43.

$$2 \times 10^{-3} = 10^{-14} e^{V_{EB5}/V_T}$$

$$V_{EB5} = 0.025 \ln(2 \times 10^{11})$$

$$= 0.650 \text{ V}$$

$$R = \frac{0.650 \text{ V}}{100 \text{ mA}} = 6.5 \text{ } \Omega$$

For a normal peak output current of 75 mA, we have

$$V_{EB5} = 6.5 \times 75 = 487.5 \text{ mV}$$

$$I_{C5} = 10^{-14} \times e^{487.5/25}$$

$$= 2.9 \mu\text{A}$$

**11.44**  $I_{D2} = I = 15$  mA

At  $v_O = 0$  V,  $i_L = 0$

$$i_{D1} = I + i_L = 15 \text{ mA}$$

Substitute  $i_{D1} = 15$  mA into

$$i_{D1} = \frac{1}{2} \times 50 (v_I = 0.5)^2$$

$$\Rightarrow v_I = 1.27 \text{ V}$$

$$g_m = \sqrt{2k_n i_{D1}}$$

$$= \sqrt{2 \times 50 \times 15}$$

$$= 38.7 \text{ mA/V}$$

$$A_v = \frac{R_L}{R_L + 1/g_m}$$

$$= \frac{150}{150 + 1/0.0387}$$

$$= 0.853 \text{ V/V}$$

$$\text{At } v_O = +1 \text{ V, } i_L = \frac{v_O}{R_L}$$

$$= \frac{1}{150}$$

$$= 6.67 \text{ mA}$$

$$i_{D1} = I + i_L$$

$$= 15 + 6.67 = 21.67 \text{ mA}$$

Substitute  $i_{D1} = 21.67 \text{ mA}$  into

$$i_{D1} = \frac{1}{2} \times 50(v_I - 1 - 0.5)^2$$

$$\Rightarrow v_I = 2.43 \text{ V}$$

$$g_m = \sqrt{2k_n i_{D1}}$$

$$= \sqrt{2 \times 50 \times 21.67}$$

$$= 46.5 \text{ mA/V}$$

$$A_v = \frac{R_L}{R_L + 1/g_m}$$

$$= \frac{150}{150 + 1/0.0465}$$

$$= 0.875 \text{ V/V}$$

$$\text{At } v_O = -1 \text{ V, } i_L = \frac{v_O}{R_L}$$

$$= \frac{-1}{150}$$

$$= -6.67 \text{ mA}$$

$$i_{D1} = I + i_L$$

$$= 15 - 6.67 = 8.33 \text{ mA}$$

Substitute  $i_{D1} = 8.33 \text{ mA}$  into

$$i_{D1} = \frac{1}{2} \times 50(v_I - (-1) - 0.5)^2$$

$$\Rightarrow v_I = -0.08 \text{ V}$$

$$g_m = \sqrt{2k_n i_{D1}}$$

$$= \sqrt{2 \times 50 \times 8.33}$$

$$= 28.9 \text{ mA/V}$$

$$A_v = \frac{R_L}{R_L + 1/g_m}$$

$$= \frac{150}{150 + 1/0.0289}$$

$$= 0.813 \text{ V/V}$$

**11.45** For the MOS source follower, the output resistance is approximately

$$R_{\text{out}} \cong \frac{1}{g_{m1}} = \frac{V_{OV1}}{2I}$$

$$\Rightarrow I = \frac{V_{OV1}}{2R_{\text{out}}}$$

$$= \frac{350 \text{ mV}}{2 \times 5\Omega}$$

$$= 35 \text{ mA}$$

For the BJT emitter follower, the output resistance is

$$R_{\text{out}} \cong \frac{V_T}{I}$$

$$\Rightarrow I = \frac{V_T}{R_{\text{out}}}$$

$$= \frac{25 \text{ mV}}{5\Omega}$$

$$= 5 \text{ mA}$$

So, the MOS source follower requires 7 times more bias current to achieve the same output resistance.

**11.46** Refer to Fig. 11.23. Consider the situation when  $Q_N$  is conducting the maximum current of 20 mA,

$$20 = \frac{1}{2} k_n v_{OVN}^2$$

$$= \frac{1}{2} \times 200 v_{OVN}^2$$

$$\Rightarrow v_{OVN} = 0.45 \text{ V}$$

Thus,

$$v_{O\text{min}} = -V_{SS} + v_{OVN}$$

$$= -2.5 + 0.45 = -2.05 \text{ V}$$

Because  $Q_N$  and  $Q_P$  are matched, a similar situation pertains when  $Q_P$  is supplying maximum current, and

$$v_{O\text{max}} = +2.05 \text{ V}$$

Thus, the output voltage swing realized is  $\pm 2.05 \text{ V}$ .

**11.47** From Eq. (11.45), we obtain

$$R_{\text{out}} = 1/\mu(g_{mp} + g_{mn})$$

where

$$g_{mp} = g_{mn} = \frac{2I_Q}{|V_{OV}|} = \frac{2 \times 2}{0.2} = 20 \text{ mA/V}$$

$$R_{\text{out}} = \frac{1}{5(20 + 20)} = \frac{1}{200} \text{ k}\Omega = 5 \Omega$$

**11.48** (a) First, we obtain

$$|\text{Gain error}| = \frac{1}{2\mu g_m R_L} \quad (1)$$

From Eq. (11.45), we get

$$R_{\text{out}} = \frac{1}{\mu(g_{mn} + g_{mp})}$$



For  $g_{mn} = g_{mp} = g_m$ , we have

$$R_{\text{out}} = \frac{1}{2\mu g_m} \quad (2)$$

Combining (1) and (2) yields

$$|\text{Gain error}| = \frac{R_{\text{out}}}{R_L} \quad \text{Q.E.D.}$$

(b) For  $R_L = 100 \Omega$  and  $|\text{Gain error}| = 3\%$ ,

$$R_{\text{out}} = 0.03 \times 100 = 3 \Omega$$

But,

$$R_{\text{out}} = \frac{1}{2\mu g_m}$$

$$3 = \frac{1}{2 \times 5 \times g_m}$$

$$\Rightarrow g_m = \frac{1}{30} = 33.3 \text{ mA/V}$$

Using

$$g_m = \frac{2I_Q}{V_{OV}}$$

we obtain

$$33.3 = \frac{2 \times 2.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = \frac{5}{33.3} = 0.15 \text{ V}$$

**11.49**  $i_{DP}$  and  $i_{DN}$  are given by Eqs. (11.46) and (11.47) as

$$i_{DP} = I_Q \left( 1 - \mu \frac{v_O - v_I}{V_{OV}} \right)^2 \quad (1)$$

$$i_{DN} = I_Q \left( 1 + \mu \frac{v_O - v_I}{V_{OV}} \right)^2 \quad (2)$$

Equation (1) shows that  $Q_P$  turns off and  $i_{DP} = 0$  when

$$\mu \frac{v_O - v_I}{V_{OV}} = 1$$

Substituting this into Eq. (2) gives

$$i_{DN} = I_Q(1 + 1)^2 = 4I_Q$$

Since  $i_L = -i_{DN}$ , we have

$$v_O = i_L R_L = -4I_Q R_L \quad \text{Q.E.D.}$$

Similarly, Eq. (2) shows that  $Q_N$  turns off and

$$i_{DN} = 0 \text{ when}$$

$$\mu \frac{v_O - v_I}{V_{OV}} = -1$$

substituting this into Eq. (1) gives

$$i_{DP} = I_Q(1 + 1)^2 = 4I_Q$$

Since in this case  $i_L = i_{DP}$ , then

$$v_O = i_L R_L = 4I_Q R_L \quad \text{Q.E.D.}$$

Thus, one of the two transistors turns off when  $|i_L|$  reaches  $4I_Q$ .

$$\mathbf{11.50} \text{ (a) } I_Q = \frac{1}{2} k' \frac{W}{L} V_{OV}^2$$

$$1.5 = \frac{1}{2} \times 0.1 \left( \frac{W}{L} \right)_P (0.15)^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_P = 1333.3$$

$$\left( \frac{W}{L} \right)_N = \frac{(W/L)_P}{k'_n/k'_p}$$

$$\left( \frac{W}{L} \right)_N = \frac{1333.3}{2.5} = 533.3$$

$$\text{(b) } g_m = \frac{2I_Q}{V_{OV}} = \frac{2 \times 1.5}{0.15} = 20 \text{ mA/V}$$

$$R_{\text{out}} = \frac{1}{2\mu g_m} \text{ (where } g_{mn} = g_{mp} = g_m)$$

$$2.5 = \frac{1}{2\mu \times 20 \times 10^{-3}}$$

$$\Rightarrow \mu = 10 \text{ V/V}$$

$$\text{(c) Gain} = \frac{R_L}{R_L + R_{\text{out}}}$$

$$= \frac{50}{50 + 2.5} = 0.952$$

$$\text{Gain error} = 0.952 - 1 = -0.048$$

or  $-5\%$

(d) In the quiescent state the dc voltage at the output of each amplifier must be of the value that causes the current in  $Q_N$  and  $Q_P$  to be  $I_Q$ . Thus, for the  $Q_P$  amplifier the output voltage is

$$V_{DD} - V_{SG} = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 2.5 - 0.5 - 0.15 = 1.85 \text{ V}$$

Similarly, the voltage at the output of the  $Q_N$  amplifier must be

$$-V_{SS} + V_{GS} = -2.5 + 0.5 + 0.15$$

$$= -1.85 \text{ V}$$

(e)  $Q_P$  will be supplying all the load current when  $Q_N$  cuts off. From Eq. (11.47) we see that  $Q_N$  cuts off when

$$\mu \frac{v_O - v_I}{V_{OV}} = -1$$

Substituting this in Eq. (12.46), we find the current  $i_{DP}$  to be

$$i_{DP} = I_Q(1 + 1)^2 = 4I_Q$$

Since in this situation

$$i_L = i_{DP}$$

This figure belongs to Problem 11.50, part (f).

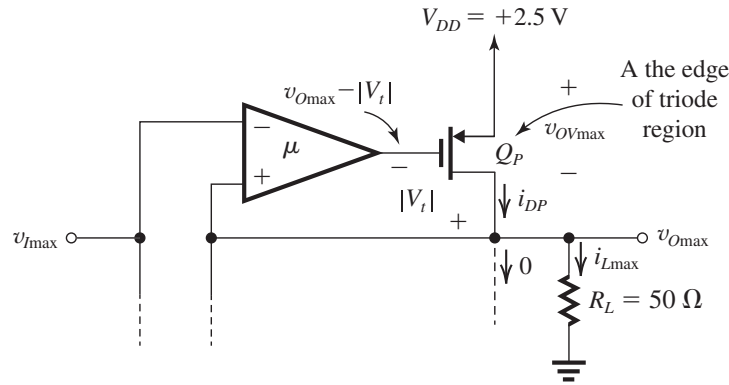
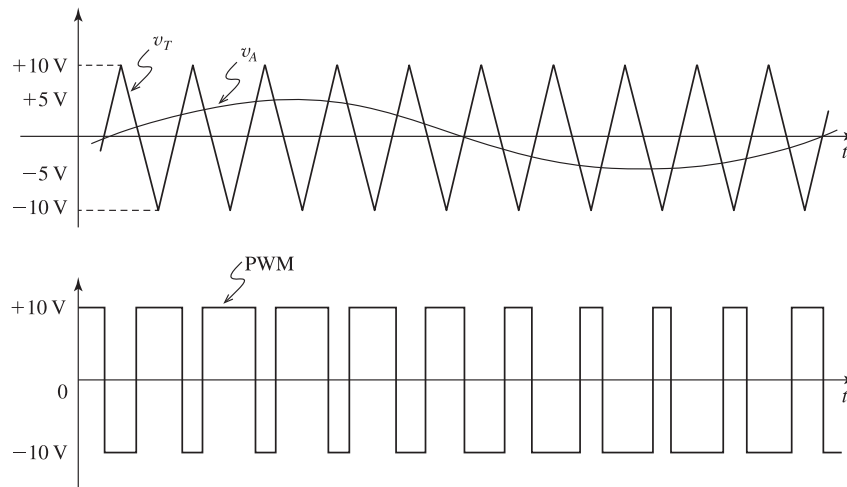


Figure 1

This figure belongs to Problem 11.51.



then

$$i_L = 4I_Q$$

and

$$v_O = 4I_Q R_L$$

$$= 4 \times 1.5 \times 10^{-3} \times 50 = 0.3 \text{ V}$$

Similarly, when  $v_O = -0.3 \text{ V}$ ,  $Q_P$  will cut off and all the current ( $4I_Q = 6 \text{ mA}$ ) will be supplied by  $Q_N$ .

(f) The situation at  $v_O = v_{O,max}$  is illustrated in Fig. 1. Analysis of this circuit provides

$$i_{DP} = \frac{1}{2} \times k'_n \left( \frac{W}{L} \right)_n [2.5 - (v_{O,max} - 0.5) - 0.5]^2$$

$$\frac{v_{O,max}}{R_L} = \frac{1}{2} \times 0.25 \times 533.3 (2.5 - v_{O,max})^2$$

$$\Rightarrow v_{O,max} = 1.77 \text{ V}$$

Similarly,

$$v_{O,min} = -1.77 \text{ V}$$

11.51 See figure.

11.52

$$\text{Average} = +10 \times 0.65 - 10 \times 0.35 = +3 \text{ V}$$

If duty cycle changed to 0.35, the average becomes

$$= +10 \times 0.35 - 10 \times 0.65 = -3 \text{ V}$$

11.53 (a) Maximum peak voltage across

$$R = V_{DD}$$

Maximum power supplied to load

$$= \frac{(V_{DD}/\sqrt{2})^2}{R} = \frac{V_{DD}^2}{2R}$$

(b) Power loss =  $4f_s C V_{DD}^2$

$$\eta = \frac{P_L}{P_L + P_{loss}}$$

$$= \frac{V_{DD}^2/2R}{(V_{DD}^2/2R) + 4f_s C V_{DD}^2}$$

$$= \frac{1}{1 + 8f_s CR}$$

For  $f_s = 250$  kHz,  $C = 1000$  pf and  $R = 16 \Omega$

$$\eta = \frac{1}{1 + 8 \times 250 \times 10^3 \times 1000 \times 10^{-12} \times 16} = 97\%$$

**11.54** The peak power dissipation in  $Q_1$  arises at  $v_O = 0$  V.

$$P_{1\max} = 200 \text{ mA} \times 10 \text{ V} = 2 \text{ W}$$

The maximum collector current in  $Q_1$  arises at  $v_O = +5$  V.

$$i_{C1\max} = 200 \text{ mA} + \frac{5 \text{ V}}{50 \Omega} = 300 \text{ mA}$$

The maximum collector-emitter voltage arises on  $Q_1$  when  $v_O = -5$  V.

$$v_{CE1\max} = +10 \text{ V} - (-5 \text{ V}) = 15 \text{ V}$$

The maximum power dissipation on  $Q_2$  arises when  $v_O = +5$  V.

$$P_{2\max} = 200 \text{ mA} \times (5 \text{ V} - (-10 \text{ V})) = 3 \text{ W}$$

The collector current in  $Q_2$  is approximately constant at 200 mA. The maximum collector-emitter voltage arises at  $v_O = +5$  V.

$$v_{CE2\max} = +5 \text{ V} - (-10 \text{ V}) = 15 \text{ V}$$

Thus, we must specify the transistors to handle the maximum current of  $Q_1$ , with a factor-of-2 safety margin,

$$I_{C\max} = 2 \times 300 = 600 \text{ mA}$$

and we must specify the breakdown voltage to be more than the maximum collector-emitter voltage of either  $Q_1$  or  $Q_2$ , with a factor-of-2 safety margin.

$$BV_{CEO} = 2 \times 15 = 30 \text{ V}$$

### 11.55

(a) Figure 1 shows the SOA boundaries.

(b) For the CS configuration in Fig. P12.55,

$$V_{DS} = V_{DD} - RI_D \quad (1)$$

We see that maximum  $V_{DS}$  occurs when  $I_D = 0$  and the resulting maximum  $V_{DS}$  is

$$V_{DS\max} = V_{DD}$$

Writing (1) in the alternative form

$$I_D = \frac{V_{DD} - V_{DS}}{R} \quad (2)$$

shows that maximum  $I_D$  is obtained when  $V_{DS} = 0$  and the resulting maximum  $I_D$  is

$$I_{D\max} = \frac{V_{DD}}{R}$$

The power dissipation in the transistor is given by

$$P_D = V_{DS}I_D = (V_{DD} - RI_D)I_D$$

$P_D$  will be maximum when

$$\frac{\partial P_D}{\partial I_D} = 0$$

that is,

$$V_{DD} - 2RI_D = 0$$

$$\Rightarrow RI_D = \frac{V_{DD}}{2}$$

or

$$V_{DS} = \frac{V_{DD}}{2}$$

The corresponding  $P_{D\max}$  is

$$P_{D\max} = V_{DS}I_D = \frac{V_{DD}}{2} \frac{V_{DD}}{2R} = \frac{V_{DD}^2}{4R}$$

(c) For  $V_{DD} = 40$  V,  $v_{DS\max} = 40$  V. Now, since  $V_{DS}$  and  $I_D$  are related by the linear relationship in (1) or (2), the straight line representing this relationship on the  $i_D - v_{DS}$  plane must pass by the point  $v_{DS} = 40$  V and  $i_D = 0$ . Now we are searching for the straight line with maximum slope that clears the hyperbola and intersects the vertical axis at 5 A or less. For this case, this straight line is the one joining the points (40, 0) and (0, 5). It is a tangent to the hyperbola at  $v_{DS} = \frac{V_{DD}}{2} = 20$  V, which is the point of maximum power dissipation. For this straight line

$$R = \frac{40 \text{ V}}{5 \text{ A}} = 8 \Omega$$

$$I_{D\max} = 5 \text{ A}$$

$$P_{D\max} = \frac{V_{DD}^2}{4R} = \frac{40^2}{4 \times 8} = 50 \text{ W}$$

(d) For  $V_{DD} = 30$  V: Following a process similar to that in (c), we find

$$R = \frac{30 \text{ V}}{5 \text{ A}} = 6 \Omega$$

$$I_{D\max} = 5 \text{ A}$$

$$P_{D\max} = \frac{30^2}{4 \times 6} = 37.5 \text{ W}$$

The locus of the operating point is shown in Fig. 1.

(e) For  $V_{DD} = 15$  V, we have

$$R = \frac{15 \text{ V}}{5 \text{ A}} = 3 \Omega$$

$$I_{D\max} = 5 \text{ A}$$

$$P_{D\max} = \frac{15^2}{4 \times 3} = 18.75 \text{ W}$$

The locus of the operating point is shown in Fig. 1.

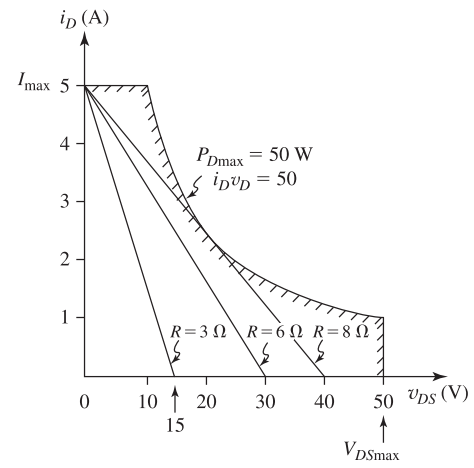


Figure 1

## Chapter 12

## Solutions to Exercises within the Chapter

**Ex: 12.1** Using Eq. (12.2), we obtain

$$\begin{aligned} V_{ICMmin} &= -V_{SS} + V_m + V_{OV3} - |V_{tp}| \\ &= -1.65 + 0.5 + 0.3 - 0.5 \\ &= -1.35 \text{ V} \end{aligned}$$

Using Eq. (12.3), we get

$$\begin{aligned} V_{ICMmax} &= V_{DD} - |V_{OV3}| - |V_{tp}| - |V_{OV1}| \\ &= 1.65 - 0.3 - 0.5 - 0.3 \\ &= +0.55 \text{ V} \end{aligned}$$

Thus,

$$-1.35 \text{ V} \leq V_{ICM} \leq +0.55 \text{ V}$$

Using Eq. (12.5), we obtain

$$-V_{SS} + V_{OV6} \leq v_O \leq V_{DD} - |V_{OV7}|$$

Thus

$$\begin{aligned} -1.65 + 0.5 \leq v_O \leq 1.65 - 0.5 \\ \Rightarrow -1.15 \text{ V} \leq v_O \leq +1.15 \text{ V} \end{aligned}$$

In a unity-gain configuration, the output is connected to the gate of  $Q_1$  and must, therefore, respect both limits. The overlap is

$$-1.15 \text{ V} \leq v_O \leq +0.55 \text{ V}$$

**Ex: 12.2** For all devices, we have

$$|V_{An}| = 5 \text{ V}/\mu\text{m} \times 0.5 \mu\text{m} = 2.5 \text{ V}$$

$$|V_{Ap}| = 6 \text{ V}/\mu\text{m} \times 0.5 \mu\text{m} = 3 \text{ V}$$

Using Eq. (12.11), we get

$$\begin{aligned} A_1 &= -\frac{2}{|V_{OV1}|} \bigg/ \left[ \frac{1}{|V_{A2}|} + \frac{1}{V_{A4}} \right] \\ &= -\frac{2}{0.2 \text{ V}} \bigg/ \left[ \frac{1}{3 \text{ V}} + \frac{1}{2.5 \text{ V}} \right] = -13.6 \text{ V/V} \end{aligned}$$

Using Eq. (12.16), we obtain

$$\begin{aligned} A_2 &= -\frac{2}{|V_{OV6}|} \bigg/ \left[ \frac{1}{V_{A6}} + \frac{1}{|V_{A4}|} \right] \\ &= -\frac{2}{0.3 \text{ V}} \bigg/ \left[ \frac{1}{2.5 \text{ V}} + \frac{1}{3 \text{ V}} \right] = -9.1 \text{ V/V} \end{aligned}$$

The total gain is

$$\begin{aligned} A_v &= A_1 A_2 \\ &= -13.6 \times -9.1 \\ &= 123.8 \text{ V/V} \end{aligned}$$

We find the op-amp output resistance as follows,

$$r_{o6} = \frac{V_{An}}{I_{D6}} = \frac{2.5 \text{ V}}{0.1 \text{ mA}} = 25 \text{ k}\Omega$$

$$r_{o7} = \frac{|V_{Ap}|}{I_{D6}} = \frac{3 \text{ V}}{0.1 \text{ mA}} = 30 \text{ k}\Omega$$

$$R_o = r_{o6} \parallel r_{o7} = 25 \parallel 30 = 13.6 \text{ k}\Omega$$

**Ex: 12.3** The feedback is of the voltage sampling type (i.e., the connection at the output is a shunt one), thus

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

where

$$R_o = r_{o6} \parallel r_{o7}$$

$$A = g_{m1}(r_{o2} \parallel r_{o4})g_{m6}(r_{o6} \parallel r_{o7})$$

$$\beta = 1$$

Thus,

$$R_{out} = \frac{r_{o6} \parallel r_{o7}}{1 + g_{m1}(r_{o2} \parallel r_{o4})g_{m6}(r_{o6} \parallel r_{o7})}$$

Usually,

$$A \gg 1$$

Thus,

$$R_{out} \simeq \frac{1}{g_{m6}[g_{m1}(r_{o2} \parallel r_{o4})]}$$

**Ex: 12.4** Using Eq. (12.31), we get

$$\begin{aligned} f_i &= \frac{G_{m1}}{2\pi C_C} \\ \Rightarrow C_C &= \frac{G_{m1}}{2\pi f_i} \\ &= \frac{0.35 \times 10^{-3}}{2\pi \times 20 \times 10^6} \\ &= 2.8 \text{ pF} \end{aligned}$$

From Eq. (12.26), we have

$$\begin{aligned} f_z &= \frac{G_{m2}}{2\pi C_C} \\ &= \frac{1.0 \times 10^{-3}}{2\pi \times 2.8 \times 10^{-12}} \\ &\simeq 57 \text{ MHz} \end{aligned}$$

From Eq. (12.30), we have

$$\begin{aligned} f_{P2} &= \frac{G_{m2}}{2\pi C_2} \\ &= \frac{1.0 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 80 \text{ MHz} \end{aligned}$$

Thus,  $f_i$  is lower than  $f_z$  and  $f_{P2}$ .

**Ex: 12.5** (a) Using Eq. (12.31), we have

$$\begin{aligned} f_i &= \frac{G_{m1}}{2\pi C_C} \\ \Rightarrow C_C &= \frac{G_{m1}}{2\pi f_i} \\ &= \frac{1 \times 10^{-3}}{2\pi \times 100 \times 10^6} \\ &= 1.6 \text{ pF} \end{aligned}$$

$$A_0 = G_{m1}(r_{o2} \parallel r_{o4})G_{m2}(r_{o6} \parallel r_{o7})$$

$$= 1(100 \parallel 100) \times 2(40 \parallel 40)$$

$$= 50 \times 2 \times 20 = 2000 \text{ V/V}$$

$$f_{3dB} = \frac{f_i}{A_0} = \frac{100 \times 10^6}{2000} = 50 \text{ kHz}$$

(b) From Eq. (12.29), we have

$$R = \frac{1}{G_{m2}} = \frac{1}{2 \times 10^{-3}} = 500 \text{ } \Omega$$

(c) From Eq. (12.30), we have

$$\begin{aligned} f_{P2} &= \frac{G_{m2}}{2\pi C_2} \\ &= \frac{2 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 318 \text{ MHz} \end{aligned}$$

$$\phi_{P2} = -\tan^{-1} \frac{f_i}{f_{P2}}$$

$$= -\tan^{-1} \frac{100}{318}$$

$$= -17.4^\circ$$

$$PM = 90 - 17.4 = 72.6^\circ$$

**Ex: 12.6** Using Eq. (12.42), we obtain

$$SR = V_{OV1}\omega_t$$

$$= 0.2 \times 2\pi \times 100 \times 10^6$$

$$= 126 \text{ V}/\mu\text{s}$$

Using Eq. (12.40),

$$SR = \frac{I}{C_C}$$

$$\Rightarrow I = 126 \times 10^6 \times 1.6 \times 10^{-12}$$

$$= 200 \text{ } \mu\text{A}$$

**Ex: 12.7**

$$G_{m2} = g_{m6} = \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.5}{0.25} = 4 \text{ mA/V}$$

$$f_{P2} = \frac{G_{m2}}{2\pi \times C_2} = \frac{4}{2\pi \times 15 \times 10^{-12}} = 42 \text{ MHz}$$

$$\tan^{-1} \frac{f_i}{f_{P2}} = 20^\circ$$

$$\Rightarrow f_i = f_{P2} \times \tan 20^\circ = 15.3 \text{ MHz}$$

$$G_{m1} = g_{m1} = \frac{2(I/2)}{V_{OV}} = \frac{2 \times 0.3}{0.25} = 1.2 \text{ mA/V}$$

$$f_i = \frac{G_{m1}}{2\pi \times C_C} \Rightarrow C_C = \frac{G_{m1}}{2\pi \times f_i}$$

$$= \frac{1.2 \times 10^{-3}}{2\pi \times 15.3 \times 10^6} = 12.5 \text{ pF}$$

**Ex: 12.8**

$$I_{REF} = \frac{0.2 \text{ mA}}{10} = 20 \text{ } \mu\text{A} = I_{D8}$$

$$I_{D8} = \frac{1}{2}k'_p \left(\frac{W}{L}\right)_8 V_{OV}^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_8 = \frac{2I_{D8}}{k'_p \times V_{OV}^2}$$

$$= \frac{2 \times 20 \times 10^{-6}}{86 \times 10^{-6} \times 0.5^2} = 1.86$$

$$W_8 = 1.86 \times L_8 = 1.86 \times 0.6 = 1.1 \text{ } \mu\text{m}$$

$$\frac{W_5}{W_8} = \frac{I_{D5}}{I_{D8}} \Rightarrow W_5 = \frac{200}{20} \times 1.1 = 11 \text{ } \mu\text{m}$$

$$\frac{W_6}{W_8} = \frac{I_{D6}}{I_{D8}} \Rightarrow W_6 = \frac{400}{20} \times 1.1 = 22 \text{ } \mu\text{m}$$

$$|V_{GS8}| = |V_{tp}| + |V_{OV}| = 0.5 + 0.5 = 1.0 \text{ V}$$

The required resistor value is

$$R = \frac{1 \text{ V} - 1 \text{ V} - (-1 \text{ V})}{I_{REF}} = \frac{1 \text{ V}}{20 \text{ } \mu\text{A}} = 50 \text{ k}\Omega$$

**Ex: 12.9** Total bias current =  $300 \text{ } \mu\text{A} = 2I_B$

$$\Rightarrow I_B = 150 \text{ } \mu\text{A}$$

$$I_B = I_{D1} + I_{D3}$$

$$150 = I_{D1} + 0.25I_{D1}$$

$$\Rightarrow I_{D1} = 120 \text{ } \mu\text{A}$$

$$I = I_{D1} + I_{D2}$$

$$= 120 + 120 = 240 \text{ } \mu\text{A}$$

$$I_{D3,4} = 0.25I_{D1,2} = 0.25 \times 120$$

$$= 30 \text{ } \mu\text{A}$$

**Ex: 12.10** Using Eq. (12.50), we get

$$V_{ICM\max} = V_{DD} - |V_{OV9}| + V_m$$

$$= 1.65 - 0.3 + 0.5 = +1.85 \text{ V}$$

Using Eq. (12.51), we obtain

$$V_{ICM\min} = -V_{SS} + V_{OV11} + V_{OV1} + V_m$$

$$V_{ICM\min} = -1.65 + 0.3 + 0.3 + 0.5$$

$$= -0.55 \text{ V}$$

Thus,

$$-0.55 \text{ V} \leq V_{ICM} \leq +1.85 \text{ V}$$

Using Eq. (12.54), we get

$$\begin{aligned} V_{Omax} &= V_{DD} - |V_{OV10}| - |V_{OV4}| \\ &= 1.65 - 0.3 - 0.3 = +1.05 \text{ V} \end{aligned}$$

Using Eq. (12.55), we obtain

$$\begin{aligned} V_{Omin} &= -V_{SS} + V_{OV7} + V_{OV5} + V_m \\ &= -1.65 + 0.3 + 0.3 + 0.5 \\ &= -0.55 \text{ V} \end{aligned}$$

Thus,

$$-0.55 \text{ V} \leq v_o \leq +1.05 \text{ V}$$

**Ex: 12.11**  $G_m = g_{m1} = g_{m2}$

$$\begin{aligned} G_m &= \frac{2(I/2)}{V_{OV1}} = \frac{I}{V_{OV1}} \\ &= \frac{0.24}{0.2} = 1.2 \text{ mA/V} \\ r_{o2} &= \frac{|V_A|}{I/2} = \frac{20}{0.12} = 166.7 \text{ k}\Omega \\ r_{o4} &= \frac{|V_A|}{I_{D4}} = \frac{|V_A|}{I_B - \frac{I}{2}} = \frac{20}{0.150 - 0.120} \\ &= \frac{20}{0.03} = 666.7 \text{ k}\Omega \\ r_{o10} &= \frac{|V_A|}{I_B} \\ &= \frac{20}{0.15} = 133.3 \text{ k}\Omega \\ g_{m4} &= \frac{2I_{D4}}{|V_{OV}|} = \frac{2 \times 0.03}{0.2} = 0.3 \text{ mA/V} \\ R_{o4} &= (g_{m4}r_{o4})(r_{o2} \parallel r_{o10}) \\ &= (0.3 \times 666.7)(166.7 \parallel 133.3) \\ &= 14.8 \text{ M}\Omega \\ g_{m6} &= \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.03}{0.2} = 0.3 \text{ mA/V} \\ r_{o6} = r_{o8} &= \frac{|V_A|}{I_{D6,8}} = \frac{20}{0.03} = 666.7 \text{ k}\Omega \\ R_{o6} &= g_{m6}r_{o6}r_{o8} \\ &= 0.3 \times 666.7 \times 666.7 = 133.3 \text{ M}\Omega \\ R_o &= R_{o4} \parallel R_{o6} \\ &= 14.8 \parallel 133.3 = 13.3 \text{ M}\Omega \\ A_v &= G_m R_o = 1.2 \times 13.3 \times 10^3 = 16,000 \text{ V/V} \end{aligned}$$

**Ex: 12.12** (a) The NMOS input stage operates over the following input common-mode range:

$$-V_{SS} + 2V_{OV} + V_m \leq V_{ICM} \leq V_{DD} - |V_{OV}| + V_m$$

that is,

$$\begin{aligned} (-2.5 + 0.6 + 0.7) &\leq V_{ICM} \leq (2.5 - 0.3 + 0.7) \\ -1.2 \text{ V} &\leq V_{ICM} \leq +2.9 \text{ V} \end{aligned}$$

(b) The PMOS input stage operates over the following input common-mode range:

$$-V_{SS} + V_{OV} - |V_{tp}| \leq V_{ICM} \leq V_{DD} - 2|V_{OV}| - |V_{tp}|$$

that is,

$$\begin{aligned} (-2.5 + 0.3 - 0.7) &\leq V_{ICM} \leq (2.5 - 0.6 - 0.7) \\ -2.9 \text{ V} &\leq V_{ICM} \leq +1.2 \text{ V} \end{aligned}$$

(c) The overlap range is

$$-1.2 \text{ V} \leq V_{ICM} \leq +1.2 \text{ V}$$

(d) The input common-mode range is

$$-2.9 \text{ V} \leq V_{ICM} \leq +2.9 \text{ V}$$

**Ex: 12.13** Denote the  $(W/L)$  of the transistors in the wide-swing mirror by  $(W/L)_M$ . Transistor  $Q_4$  has

$$\begin{aligned} (W/L)_5 &= \frac{1}{4}(W/L)_M \\ I_{REF} &= \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_5 V_{OV5}^2 \\ &= \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_M \times \frac{1}{4}V_{OV5}^2 \\ &= \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_M (V_{OV5}/2)^2 \end{aligned}$$

Thus,

$$\frac{V_{OV5}}{2} = V_{OV}$$

where  $V_{OV}$  is the overdrive voltage for each of the mirror transistors. Thus,

$$V_5 = V_m + 2V_{OV}$$

which is the value of  $V_{BIAS}$  needed in the circuit of Fig. 12.13(b).

**Ex: 12.14**  $I_{S2} = 2I_{S1}$

Using Eq. (12.72), we obtain

$$\begin{aligned} I &= \frac{V_T}{R_2} \ln \frac{I_{S2}}{I_{S1}} \\ 0.01 &= \frac{0.025}{R_2} \ln 2 \\ \Rightarrow R_2 &= 1.73 \text{ k}\Omega \end{aligned}$$

$$R_3 = R_4 = \frac{0.2 \text{ V}}{0.01 \text{ mA}} = 20 \text{ k}\Omega$$

**Ex: 12.15** To obtain  $I_5 = 10 \mu\text{A}$ , transistor  $Q_5$  must have the same emitter area as  $Q_2$  and

$$R_5 = R_2 = 1.73 \text{ k}\Omega$$

To obtain  $I_6 = 5 \mu\text{A}$ , transistor  $Q_6$  must have one-half the emitter area of  $Q_2$  and

$$R_6 = 2R_2 = 3.46 \text{ k}\Omega$$

To obtain  $I_7 = 20 \mu\text{A}$ , transistor  $Q_7$  must have double the emitter area of  $Q_3$  and

$$R_7 = 0.5R_3 = 10 \text{ k}\Omega$$

To obtain  $I_8 = 10 \mu\text{A}$ , transistor  $Q_8$  must have the same emitter area as  $Q_3$  and

$$R_8 = R_3 = 20 \text{ k}\Omega$$

**Ex: 12.16** Refer to Fig. 12.23. At node X,

$$I_{REF} = \frac{2I}{1 + \frac{2}{\beta_P}} + \frac{2I}{\beta_P}$$

$$I_{REF} = 2I \left[ \frac{\beta_P + 1 + \frac{2}{\beta_P}}{\beta_P \left( 1 + \frac{2}{\beta_P} \right)} \right]$$

$$\simeq 2I \frac{\beta_P + 1}{\beta_P + 2}$$

$$\simeq 2I$$

Thus,

$$I = \frac{I_{REF}}{2} = 9.5 \mu\text{A}$$

resulting in

$$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 9.5 \mu\text{A}$$

**Ex: 12.17** Figure 1 on the next page shows the determination of the loop gain of the feedback circuit that stabilizes the bias currents of the first stage of the 741 op amp. Note that since  $I_{REF}$  is assumed to be constant, we have shown its incremental value at node X to be zero. Observe that this circuit shows only incremental quantities. The analysis shown provides the returned current signal as

$$I_r = -I_t \frac{\beta_P}{1 + \frac{2}{\beta_P}}$$

For  $\beta_P \gg 1$ , we have

$$I_r \simeq -\beta_P I_t$$

and the loop gain  $A\beta$  is

$$A\beta \equiv -\frac{I_r}{I_t} = \beta_P$$

$$\text{Ex: 12.18 } I_B = \frac{1}{2}(I_{B1} + I_{B2})$$

$$= \frac{1}{2} \left( \frac{I}{\beta_N + 1} + \frac{I}{\beta_N + 1} \right)$$

$$= \frac{I}{\beta_N + 1} \simeq \frac{I}{\beta_N}$$

$$= \frac{9.5}{200} = 47.5 \text{ nA}$$

$$I_{OS} = 0.1 \times I_B = 4.75 \text{ nA}$$

**Ex: 12.19**

$$V_{C1} = V_{CC} - V_{EB8} = 15 - 0.6 = 14.4 \text{ V}$$

$Q_1$  and  $Q_2$  saturate when  $V_{ICM}$  exceeds  $V_{C1}$  by 0.3 V. Thus,

$$V_{ICM\text{max}} = +14.7 \text{ V}$$

$$\text{Ex: 12.20 } r_e = \frac{V_T}{I_E} \simeq \frac{25 \text{ mV}}{9.5 \mu\text{A}} = 2.63 \text{ k}\Omega$$

$$g_{m1} \simeq \frac{1}{r_e} = 0.38 \text{ mA/V}$$

$$G_{m1} = \frac{1}{2} g_{m1} = 0.19 \text{ mA/V}$$

$$R_{id} = (\beta_N + 1) \times 4r_e$$

$$= 201 \times 4 \times 2.63$$

$$= 2.1 \text{ M}\Omega$$

$$\text{Ex: 12.21 } r_{o4} = \frac{|V_{Ap}|}{I} = \frac{50 \text{ V}}{9.5 \mu\text{A}} = 5.26 \text{ M}\Omega$$

$$g_{m4} = 0.38 \text{ mA/V}$$

$$r_{e2} = 2.63 \text{ k}\Omega$$

$$r_{\pi4} = \frac{\beta_P}{g_{m4}} = \frac{50}{0.38} = 131.6 \text{ k}\Omega$$

$$R_{o4} = r_{o4} [1 + g_{m4} (r_{e2} \parallel r_{\pi4})]$$

$$= 5.26 [1 + 0.38 (2.63 \parallel 131.6)]$$

$$= 10.4 \text{ M}\Omega$$

10.5 M $\Omega$  is obtained by neglecting  $r_{\pi4}$ .

$$r_{o6} = \frac{V_{An}}{I} = \frac{125 \text{ V}}{9.5 \mu\text{A}} = 13.16 \text{ M}\Omega$$

$$g_{m6} = 0.38 \text{ mA/V}$$

$$R_6 = 1 \text{ k}\Omega$$

$$r_{\pi6} = \frac{200}{0.38} = 526.3 \text{ k}\Omega$$

$$R_{o6} = r_{o6} [1 + g_{m6} (R_2 \parallel r_{\pi6})]$$

$$= 13.16 [1 + 0.38 (1 \parallel 526.3)]$$



This figure belongs to Exercise 12.17.

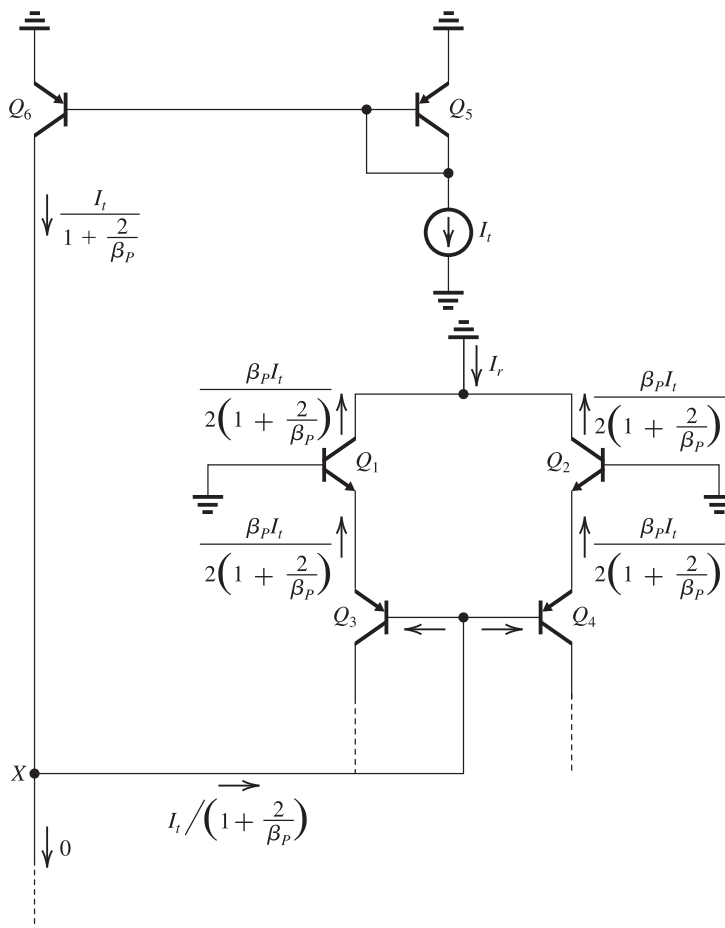


Figure 1

= 18.2 MΩ

$R_{o1} = 0.5 \times R_{o4}$

= 5.3 MΩ

**Ex: 12.22**  $|A_{vo}| = G_{m1} R_{o1}$

Using  $G_{m1}$  given in the answer to Exercise 12.20,

$G_{m1} = 0.19 \text{ mA/V}$

and  $R_{o1}$  given in the answer to Exercise 12.21,

$R_{o1} = 5.3 \text{ M}\Omega$

we obtain

$|A_{vo}| = G_{m1} R_{o1}$

=  $0.19 \times 5.3 = 1007 \text{ V/V}$

**Ex: 12.23**

$R_{o6} = r_{o6} = \frac{|V_{Ap}|}{I_{C6}} = \frac{50 \text{ V}}{19 \mu\text{A}} = 2.63 \text{ M}\Omega$

$R_{o7} = r_{o7} [1 + g_{m7} (R_4 \parallel r_{\pi 7})]$

where

$r_{o7} = \frac{V_{An}}{I_{C7}} = \frac{125 \text{ V}}{19 \mu\text{A}} = 6.58 \text{ M}\Omega$

$g_{m7} = \frac{I_{C7}}{V_T} = \frac{19 \mu\text{A}}{0.025 \text{ V}} = 0.76 \text{ mA/V}$

$R_4 = 5 \text{ k}\Omega$

$r_{\pi 7} = \frac{\beta_N}{g_{m7}} = \frac{200}{0.76} = 263.2 \text{ k}\Omega$

$R_{o7} = 6.58 [1 + 0.76 (5 \parallel 263.2)]$

= 31.1 MΩ

$R_o = R_{o6} \parallel R_{o7}$

=  $2.63 \parallel 31.1 = 2.43 \text{ M}\Omega$

**Ex: 12.24** Using Eq. (12.84), we obtain

$G_{mcm} = \frac{\epsilon_m}{2R_o}$

=  $\frac{5.5 \times 10^{-3}}{2 \times 2.43 \times 10^6} = 1.13 \times 10^{-6} \text{ mA/V}$

$$\text{CMRR} = \frac{G_{m1}}{G_{mcm}} = \frac{0.19}{1.13 \times 10^{-6}} = 1.68 \times 10^5$$

or 104.5 dB

Without common-mode feedback, the CMRR is reduced by a factor equal to  $\beta_P$ . Equivalently,

$$\begin{aligned} \text{CMRR} &= 104.5 - 20 \log \beta_P \\ &= 104.5 - 20 \log 50 \\ &= 70.5 \text{ dB} \end{aligned}$$

**Ex: 12.25** Refer to the circuit in Fig. 12.29.

(a) Current gain from  $v_{IP}$  to output

$$\begin{aligned} &= (\beta_1 + 1)(\beta_2 + 1)\beta_P \\ &\simeq \beta_1\beta_2\beta_P = \beta_N\beta_P^2 \end{aligned}$$

Current gain from  $v_{IN}$  to output =  $(\beta_3 + 1)\beta_N$

$$\simeq \beta_3\beta_N = \beta_N^2$$

(b) For  $i_L = +10$  mA,

current needed at  $v_{IP}$  input

$$= \frac{10}{\beta_N\beta_P^2} = \frac{10}{40 \times 10^2} = 2.5 \mu\text{A}$$

For  $i_L = -10$  mA,

$$\text{current needed at } v_{IN} \text{ input} = \frac{10}{\beta_N^2} = \frac{10}{40^2}$$

$$= 6.25 \mu\text{A}$$

**Ex: 12.26**  $I_Q = 0.4$  mA,  $I = 10$   $\mu\text{A}$ ,

$$\frac{I_{S7}}{I_{S10}} = 10, \quad \frac{I_{S7}}{I_{S11}} = 2$$

Using Eq. (12.93), we obtain

$$0.4 \times 10^3 = 2 \left( \frac{I_{\text{REF}}^2}{10} \right) \times 10 \times 2$$

where  $I_{\text{REF}}$  is in  $\mu\text{A}$ . Thus,

$$I_{\text{REF}} = 10 \mu\text{A}$$

For  $i_L = -10$  mA, then

$$i_P = 10 + i_N$$

Using Eq. (12.94), we get

$$\frac{i_N(10 + i_N)}{i_N + 10 + i_N} = 0.2$$

$$\Rightarrow i_N^2 - 9.6i_N + 2 = 0$$

$$\Rightarrow i_N = 0.2 \text{ mA}$$

$$i_P = 10.2 \text{ mA}$$

**Solutions to End-of-Chapter Problems**

**12.1** Using Eq. (12.2), we get

$$\begin{aligned} V_{ICMmin} &= -V_{SS} + V_m + V_{OV3} - |V_{tp}| \\ &= -1 + 0.4 + 0.2 - 0.4 = -0.8 \text{ V} \end{aligned}$$

Using Eq. (12.3), we obtain

$$\begin{aligned} V_{ICMmax} &= V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}| \\ &= 1 - 0.2 - 0.4 - 0.2 = +0.2 \text{ V} \end{aligned}$$

Thus,

$$-0.8 \text{ V} \leq V_{ICM} \leq +0.2 \text{ V}$$

Using Eq. (12.5), we get

$$-V_{SS} + V_{OV6} \leq v_o \leq V_{DD} - |V_{OV7}|$$

Thus,

$$-0.8 \text{ V} \leq v_o \leq +0.8 \text{ V}$$

**12.2** For NMOS devices, we have

$$V_A = 25 \times 0.3 = 7.5 \text{ V}$$

For PMOS devices,

$$|V_A| = 20 \times 0.3 = 6 \text{ V}$$

Using Eq. (12.11),

$$\begin{aligned} A_1 &= -\frac{2}{|V_{OV1}|} \bigg/ \left[ \frac{1}{|V_{A2}|} + \frac{1}{V_{A4}} \right] \\ &= -\frac{2}{0.15} \bigg/ \left( \frac{1}{6} + \frac{1}{7.5} \right) \\ &= -44.4 \text{ V/V} \end{aligned}$$

Using Eq. (12.16), we obtain

$$\begin{aligned} A_2 &= -\frac{2}{V_{OV6}} \bigg/ \left[ \frac{1}{V_{A6}} + \frac{1}{|V_{A7}|} \right] \\ &= -\frac{2}{0.2} \bigg/ \left[ \frac{1}{7.5} + \frac{1}{6} \right] \\ &= -33.3 \text{ V/V} \end{aligned}$$

$$A = A_1 A_2 = 1478.5 \text{ V/V}$$

$$r_{o6} = \frac{7.5}{0.3} = 25 \text{ k}\Omega$$

$$r_{o7} = \frac{6}{0.3} = 20 \text{ k}\Omega$$

$$R_o = r_{o6} \parallel r_{o7} = 11.1 \text{ k}\Omega$$

For a unity-gain voltage amplifier using this op amp, we have

$$\begin{aligned} R_{out} = R_{of} &= \frac{R_o}{1 + A\beta} \\ &= \frac{11.1 \text{ k}\Omega}{1 + 1481.5 \times 1} \\ &= 7.5 \text{ }\Omega \end{aligned}$$

**12.3** For all transistors, we have

$$|V_A| = 20 \times 0.3 = 6 \text{ V}$$

Using Eq. (12.11), we get

$$A_1 = -\frac{2}{|V_{OV}|} \bigg/ \frac{2}{|V_A|} = -\frac{6}{|V_{OV}|}$$

Using Eq. (12.16), we obtain

$$A_2 = -\frac{2}{|V_{OV}|} \bigg/ \frac{2}{|V_A|} = -\frac{6}{|V_{OV}|}$$

$$A = A_1 A_2 = \frac{36}{|V_{OV}|^2}$$

$$1600 = \frac{36}{|V_{OV}|^2}$$

$$\Rightarrow |V_{OV}| = 0.15 \text{ V}$$

**12.4** For the op amp to not have a systematic offset voltage, the condition in Eq. (12.1) must be satisfied, that is,

$$\frac{(W/L)_6}{(W/L)_4} = 2 \frac{(W/L)_7}{(W/L)_5}$$

$$\frac{W/0.3}{6/0.3} = 2 \frac{45/0.3}{30/0.3}$$

$$\Rightarrow W = 18 \text{ }\mu\text{m}$$

Refer to Fig. 12.1:

$$I_{D8} = I_{REF} = 40 \text{ }\mu\text{A}$$

$$I = I_{D5} = I_{REF} \frac{W_5}{W_8} = 40 \times \frac{30}{6} = 200 \text{ }\mu\text{A}$$

$$I_{D7} = I_{REF} \frac{W_7}{W_8} = 40 \times \frac{45}{6} = 300 \text{ }\mu\text{A}$$

$$I_{D6} = 300 \text{ }\mu\text{A}$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = 100 \text{ }\mu\text{A}$$

The overdrive voltage at which each transistor is operating is determined from

$$I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} V_{OV}^2$$

Then  $V_{GS}$  is found from

$$|V_{GS}| = |V_t| + |V_{OV}|$$

The transconductance at which each transistor is operating is obtained from

$$g_m = \frac{2I_D}{V_{OV}}$$

The output resistance of each transistor is found from

$$r_o = \frac{|V_A|}{I_D}$$

The results are summarized in the following table:

	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$I_D$ ( $\mu\text{A}$ )	100	100	100	100	200	300	300	40
$ V_{OV} $ (V)	0.15	0.15	0.19	0.19	0.24	0.19	0.24	0.24
$ V_{GS} $ (V)	0.6	0.6	0.64	0.64	0.69	0.64	0.69	0.69
$g_m$ (mA/V)	1.33	1.33	1.05	1.05	1.67	3.16	2.5	0.33
$r_o$ (k $\Omega$ )	150	150	150	150	75	50	50	375

$$A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= -1.33(150 \parallel 150) = -100 \text{ V/V}$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -3.16(50 \parallel 50) = -79 \text{ V/V}$$

$$A = A_1 A_2 = 7900 \text{ V/V}$$

Using Eq. (12.2), we obtain

$$V_{ICM\min} = -V_{SS} + V_m + V_{OV3} - |V_{tp}|$$

$$V_{ICM\min} = -1 + 0.45 + 0.19 - 0.45$$

$$= -0.81 \text{ V}$$

Using Eq. (12.3), we get

$$V_{ICM\max} = V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}|$$

$$= 1 - 0.24 - 0.45 - 0.15$$

$$= +0.16 \text{ V}$$

Thus,

$$-0.8 \text{ V} \leq V_{ICM} \leq +0.16 \text{ V}$$

Using Eq. (12.5), we obtain

$$-V_{SS} + V_{OV6} \leq v_o \leq V_{DD} - |V_{OV7}|$$

Thus,

$$-1 + 0.19 \leq v_o \leq 1 - 0.24$$

$$-0.81 \text{ V} \leq v_o \leq 0.76 \text{ V}$$

**12.5** From Eq. (12.31), we obtain

$$f_i = \frac{G_{m1}}{2\pi C_C}$$

Thus,

$$C_C = \frac{G_{m1}}{2\pi f_i} = \frac{0.8 \times 10^{-3}}{2\pi \times 120 \times 10^6} = 1.06 \text{ pF}$$

From Eq. (12.30), we get

$$f_{p2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{2.4 \times 10^{-3}}{2\pi \times 1.2 \times 10^{-12}} = 318.3 \text{ MHz}$$

From Eq. (12.26), we get

$$f_z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{2.4 \times 10^{-3}}{2\pi \times 1.06 \times 10^{-12}} = 360 \text{ MHz}$$

**12.6** (a)  $A = G_{m1} R_1 G_{m2} R_2$

$$= 1 \times 100 \times 2 \times 50 = 10,000 \text{ V/V}$$

(b) Without  $C_C$  connected:

$$\omega_{p1} = \frac{1}{C_1 R_1} = \frac{1}{0.1 \times 10^{-12} \times 100 \times 10^3}$$

$$= 10^8 \text{ rad/s}$$

$$\omega_{p2} = \frac{1}{C_2 R_2} = \frac{1}{2 \times 10^{-12} \times 50 \times 10^3}$$

$$= 10^7 \text{ rad/s}$$

Figure 1 on the next page shows a Bode plot for the gain magnitude.

(c) With  $C_C$  connected:

Using Eq. (12.30), we obtain

$$\omega_{p2} = \frac{G_{m2}}{C_2}$$

$$= \frac{2 \times 10^{-3}}{2 \times 10^{-12}} = 10^9 \text{ rad/s}$$

This figure belongs to Problem 12.6, part (b).

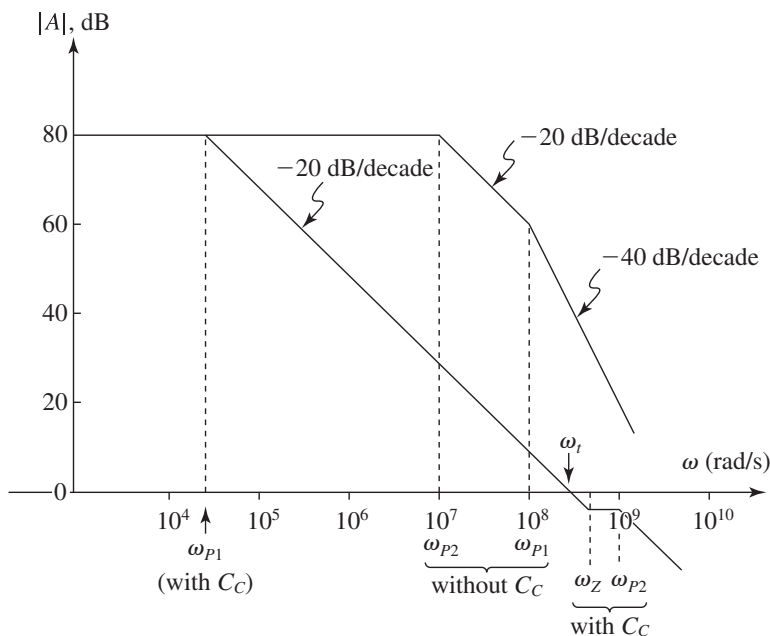


Figure 1

For  $\omega_t$  two octaves below  $\omega_{p2}$ , we have

$$\omega_t = \frac{10^9}{4} \text{ rad/s}$$

Using Eq. (12.31), we get

$$\omega_t = \frac{G_{m2}}{C_C}$$

Thus,

$$\frac{10^9}{4} = \frac{1 \times 10^{-3}}{C_C}$$

$$\Rightarrow C_C = 4 \text{ pF}$$

Now using Eq. (12.29), we obtain

$$\begin{aligned} \omega_{p1} &= \frac{1}{R_1 C_C G_{m2} R_2} \\ &= \frac{1}{100 \times 10^3 \times 4 \times 10^{-12} \times 2 \times 10^{-3} \times 50 \times 10^3} \\ &= \frac{10^5}{4} \text{ rad/s} = 25,000 \text{ rad/s} \end{aligned}$$

Using Eq. (12.26), we get

$$\begin{aligned} \omega_z &= \frac{G_{m2}}{C_C} \\ &= \frac{2 \times 10^{-3}}{4 \times 10^{-12}} = \frac{10^9}{2} \text{ rad/s} = 5 \times 10^8 \text{ rad/s} \end{aligned}$$

The Bode plot for the gain magnitude with  $C_C$  connected is shown in Fig. 1.

**12.7 (a)** Using Eq. (12.31), we get

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\begin{aligned} C_C &= \frac{G_{m1}}{2\pi f_t} \\ &= \frac{1 \times 10^{-3}}{2\pi \times 100 \times 10^6} = 1.59 \text{ pF} \end{aligned}$$

$$(b) f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 318 \text{ MHz}$$

$$\begin{aligned} f_z &= \frac{G_{m2}}{2\pi C_C} = \frac{2 \times 10^{-3}}{2\pi \times 1 \times 1.59 \times 10^{-12}} \\ &= 200 \text{ MHz} \end{aligned}$$

To obtain  $f_{P1}$ , we need to know the dc gain of the op amp,  $A_0$ , then

$$f_{P1} = \frac{f_t}{A_0}$$

The value of  $A_0$  is not specified in the problem statement!

$$\begin{aligned} \text{(c) } \phi_{p2} &= -\tan^{-1}\left(\frac{f_i}{f_{p2}}\right) \\ &= -\tan^{-1}\left(\frac{100}{318}\right) = -17.5^\circ \\ \phi_Z &= -\tan^{-1}\left(\frac{f_i}{f_Z}\right) \\ \phi_Z &= -\tan^{-1}\left(\frac{100}{200}\right) = -26.6^\circ \end{aligned}$$

$$\phi_{\text{total}} = 90^\circ + 17.5 + 26.6 = 134^\circ$$

$$\text{Phase margin} = 180 - 134 = 46^\circ$$

(d) From Eq. (12.39), for

$$f_Z = \infty$$

we select

$$R = \frac{1}{G_{m2}} = \frac{1}{2} = 0.5 \text{ k}\Omega = 500 \Omega$$

$$\text{Phase margin} = 180^\circ - (90^\circ + 17.5^\circ) = 72.5^\circ$$

(e) To obtain a phase margin of  $85^\circ$ , we need the left-half plane zero to provide at  $f_i$  a phase angle of  $85^\circ - 72.5^\circ = 12.5^\circ$ . Thus,

$$\begin{aligned} 12.5^\circ &= \tan^{-1}\left(\frac{f_i}{f_Z}\right) \\ f_Z &= \frac{f_i}{\tan 12.5^\circ} = \frac{100}{\tan 12.5^\circ} = 451 \text{ MHz} \end{aligned}$$

From Eq. (12.39), we have

$$-f_Z = \frac{1}{2\pi C_C \left(\frac{1}{G_{m2}} - R\right)}$$

$$\Rightarrow R = 722 \Omega$$

**12.8** If the phase margin is  $80^\circ$ , the phase due to the second pole  $f_{p2}$  at the unity gain frequency  $f_i$  must be  $10^\circ$ . Thus,

$$\tan^{-1} \frac{f_i}{f_{p2}} = 10^\circ$$

Since  $f_i = 1 \text{ MHz}$ ,

$$f_{p2} = \frac{1 \text{ MHz}}{\tan 10^\circ} = 5.67 \text{ MHz}$$

**12.9** For a phase margin of  $85^\circ$  with a closed loop gain of 100, the phase at  $f_i$  due to the pole at 5 MHz must be at most  $5^\circ$ ; thus,

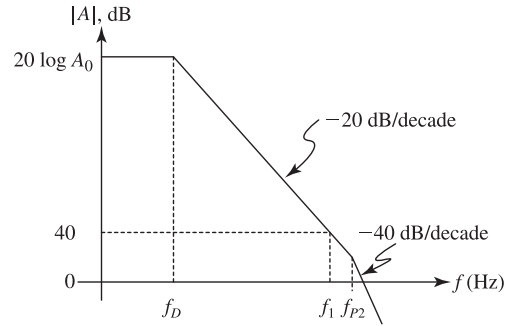


Figure 1

$$\tan^{-1} \frac{f_i}{5 \text{ MHz}} = 5^\circ$$

$$\Rightarrow f_i = 5 \times \tan 5^\circ = 437 \text{ kHz}$$

Thus, the new dominant pole must be at  $f_D$ ,

$$f_D \times \frac{A_0}{100} = 437$$

$$f_D \times \frac{243,147}{100} = 437$$

$$\Rightarrow f_D = 180 \text{ Hz}$$

To find the required value of  $C_C$ , we use the following equation to determine  $C_{in}$ :

$$\begin{aligned} C_{in} &= C_C(1 + |A_2|) \\ &= C_C \times 516 \end{aligned}$$

Then,

$$f_D = \frac{1}{2\pi C_{in} R_i}$$

where

$$R_i = 2.5 \text{ M}\Omega$$

$$180 = \frac{1}{2\pi \times 516 C_C \times 2.5 \times 10^6}$$

$$\Rightarrow C_C = 0.7 \text{ pF}$$

**12.10**  $C_2 = 0.7 \text{ pF}$ .

For a phase margin of  $72^\circ$ , the phase due to  $f_{p2}$  at  $f_i$  must be  $18^\circ$ ; thus,

$$\frac{f_i}{f_{p2}} = \tan 18^\circ$$

$$\Rightarrow f_{p2} = \frac{100}{\tan 18^\circ} = 307.8 \text{ MHz}$$

But

$$\begin{aligned} f_{P2} &= \frac{G_{m2}}{2\pi C_2} \\ \Rightarrow G_{m2} &= 2\pi f_{P2} C_2 \\ &= 2\pi \times 307.8 \times 10^6 \times 0.7 \times 10^{-12} \\ &= 1.35 \text{ mA/V} \end{aligned}$$

Thus,

$$g_{m6} = 1.35 \text{ mA/V}$$

For the transmission zero to be at  $\infty$ ,

$$\begin{aligned} R &= \frac{1}{G_{m2}} = \frac{1}{1.35 \times 10^{-3}} = 739 \Omega \\ \text{SR} &= 2\pi f_i |V_{OV1,2}| \\ &= 2\pi \times 100 \times 10^6 \times 0.15 \\ &= 94.2 \text{ V}/\mu\text{s} \\ \text{SR} &= \frac{I}{C_C} \\ \Rightarrow C_C &= \frac{I}{\text{SR}} = \frac{100 \times 10^{-6}}{94.2 \times 10^6} = 1.06 \text{ pF} \end{aligned}$$

**12.11**  $G_{m1} = 1 \text{ mA}$ ,  $G_{m2} = 5 \text{ mA/V}$

(a) Using Eq. (12.31), we obtain

$$\begin{aligned} f_i &= \frac{G_{m1}}{2\pi C_C} \\ \Rightarrow C_C &= \frac{G_{m1}}{2\pi f_i} = \frac{1 \times 10^{-3}}{2\pi \times 80 \times 10^6} \\ &= 2 \text{ pF} \end{aligned}$$

(b) Phase margin =

$$90^\circ - \tan^{-1}\left(\frac{f_i}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_i}{f_Z}\right)$$

where

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

and

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

For a PM of  $70^\circ$ , we have

$$\tan^{-1}\left(\frac{f_i}{f_{P2}}\right) + \tan^{-1}\left(\frac{f_i}{f_Z}\right) = 20^\circ$$

But,

$$f_Z = \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

and

$$\tan^{-1}\left(\frac{f_i}{f_Z}\right) = \tan^{-1}\left(\frac{80}{398}\right) = 11.4^\circ$$

Thus,

$$\tan^{-1}\left(\frac{f_i}{f_{P2}}\right) = 20 - 11.4^\circ = 8.6^\circ$$

$$\frac{f_i}{f_{P2}} = \tan 8.6^\circ$$

$$\Rightarrow f_{P2} = \frac{80}{\tan 8.6^\circ} = 529 \text{ MHz}$$

$$\frac{G_{m2}}{2\pi C_2} = 529 \times 10^6$$

$$C_2 = \frac{5 \times 10^{-3}}{2\pi \times 529 \times 10^6} = 1.51 \text{ pF}$$

This is the maximum value that  $C_2$  can have; if  $C_2$  is larger, then  $f_{P2}$  will be lower; and the phase it introduces at  $f_i$  will increase, causing the phase margin to drop below  $70^\circ$ .

**12.12**  $G_{m1} = 0.8 \text{ mA/V}$ ,  $G_{m2} = 2 \text{ mA/V}$

(a) Using Eq. (12.31), we obtain

$$\begin{aligned} f_i &= \frac{G_{m1}}{2\pi C_C} \\ \Rightarrow C_C &= \frac{G_{m1}}{2\pi f_i} = \frac{0.8 \times 10^{-3}}{2\pi \times 100 \times 10^6} = 1.27 \text{ pF} \end{aligned}$$

(b) Phase margin =

$$90^\circ - \tan^{-1}\left(\frac{f_i}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_i}{f_Z}\right)$$

$$60^\circ = 90 - \tan^{-1}\left(\frac{f_i}{f_{P2}}\right) - \tan^{-1}\left(\frac{f_i}{f_Z}\right)$$

Thus,

$$\tan^{-1}\left(\frac{f_i}{f_{P2}}\right) + \tan^{-1}\left(\frac{f_i}{f_Z}\right) = 30^\circ$$

where

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$f_Z = \frac{1}{2\pi C_C \left(\frac{1}{G_{m2}} - R\right)}$$

$$= \frac{1}{2\pi \times 1.27 \times 10^{-12}(0.5 - 0.5) \times 10^3} = \infty$$

Thus,

$$\tan^{-1}\left(\frac{f_i}{f_{p2}}\right) = 30^\circ$$

$$f_{p2} = \frac{f_i}{\tan 30} = 173.2 \text{ MHz}$$

We now can obtain  $C_2$  from

$$173.2 \times 10^6 = \frac{2 \times 10^{-3}}{2\pi C_2}$$

$$\Rightarrow C_2 = \frac{2 \times 10^{-3}}{2\pi \times 173.2 \times 10^6} = 1.84 \text{ pF}$$

**12.13** From Eq. (12.19), we have

$$\text{CMRR} = [g_{m1}(r_{o2} \parallel r_{o4})] [2g_{m3}R_{SS}]$$

where

$$g_{m1} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = |V_A|/(I/2) = \frac{2|V_A|}{I}$$

$$g_{m3} = \frac{I}{|V_{OV}|}$$

$$R_{SS} = r_{o5} = \frac{|V_A|}{I}$$

Thus,

$$\text{CMRR} = \frac{I}{|V_{OV}|} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{|V_{OV}|} \times \frac{|V_A|}{I}$$

$$= 2 \frac{|V_A|^2}{|V_{OV}|^2}$$

For CMRR = 72 dB = 4000, we have

$$4000 = 2 \times \frac{|V_A|^2}{0.15^2}$$

$$\Rightarrow |V_A| = 6.7 \text{ V}$$

Since  $|V_A| = |V_A'|L$ , we have

$$6.7 = 15L$$

$$\Rightarrow L = 0.45 \text{ } \mu\text{m}$$

$$A_{\hat{v}} = \left| \frac{V_A}{V_{OV}} \right|^2 = \left( \frac{6.7}{0.15} \right)^2 = 2000 \text{ V/V}$$

**12.14**  $G_{m1} = 1 \text{ mA/V}$

$G_{m2} = 3 \text{ mA/V}$

$C_1 = 0.2 \text{ pF}$

$C_2 = 3 \text{ pF}$

Using Eq. (12.31), we obtain

$$f_i = \frac{G_{m1}}{2\pi C_C}$$

Thus,

$$C_C = \frac{G_{m1}}{2\pi f_i} = \frac{1 \times 10^{-3}}{2\pi \times 50 \times 10^6} = 3.18 \text{ pF}$$

Using Eq. (12.30), we get

$$f_{p2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{3 \times 10^{-3}}{2\pi \times 3 \times 10^{-12}}$$

$$= 159.2 \text{ MHz}$$

Using Eq. (12.26), we obtain

$$f_z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{3 \times 10^{-3}}{2\pi \times 3.18 \times 10^{-12}}$$

$$= 150 \text{ MHz}$$

Thus  $f_i$  is lower than  $f_{p2}$  and  $f_z$ .

**12.15** Since  $C_2$  is increased by 5 $\times$ ,  $\omega_{p2}$  is decreased by 1/5. To maintain the same phase margin,

$$f_i = \frac{150 \times 10^6}{5} = 30 \text{ MHz}$$

The compensation capacitance must increase,

$$C_C = 0.8 \times 10^{-12} \times 5 = 4 \text{ pF}$$

requiring an additional external compensation capacitor of value 4 – 0.8 = 3.2 pF.

**12.16**  $G_{m1} = 0.3 \text{ mA/V}$

$G_{m2} = 0.6 \text{ mA/V}$

$r_{o2} = r_{o4} = 222 \text{ k}\Omega$



$$r_{o6} = r_{o7} = 111 \text{ k}\Omega$$

$$C_2 = 1 \text{ pF}$$

$$(a) A = G_{m1}(r_{o2} \parallel r_{o4})G_{m2}(r_{o6} \parallel r_{o7})$$

$$= 0.3(222 \parallel 222) \times 0.6(111 \parallel 111)$$

$$= 33.3 \times 33.3 = 1109 \text{ V/V}$$

$$(b) f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{0.6 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 95.5 \text{ MHz}$$

$$(c) R = \frac{1}{G_{m2}} = \frac{1}{0.6 \times 10^{-3}} = 1.67 \text{ k}\Omega$$

$$(d) \text{Phase margin} = 180 - 90 - \tan^{-1}\left(\frac{f_i}{f_{P2}}\right)$$

$$80^\circ = 90 - \tan^{-1}\left(\frac{f_i}{f_{P2}}\right)$$

$$f_i = f_{P2} \tan 10^\circ$$

$$= 95.5 \times 0.176 = 16.8 \text{ MHz}$$

Using Eq. (12.26), we obtain

$$C_C = \frac{G_{m1}}{2\pi f_i} = \frac{0.3 \times 10^{-3}}{2\pi \times 16.8 \times 10^6} = 2.84 \text{ pF}$$

The dominant pole will be at a frequency

$$f_{P1} = \frac{f_i}{\text{DC Gain}} = \frac{16.8 \times 10^6}{1109}$$

$$= 15.1 \text{ kHz}$$

(e) Since

$$f_i = \frac{G_{m1}}{2\pi C_C}$$

to double  $f_i$ ,  $C_C$  must be reduced by a factor of 2,

$$C_C = \frac{2.84}{2} = 1.42 \text{ pF}$$

At the new  $f_i = 2 \times 16.8 = 33.6 \text{ MHz}$ , we have

$$\phi_{P2} = -\tan^{-1}\frac{f_i}{f_{P2}}$$

$$= -\tan^{-1}\left(\frac{33.6}{95.5}\right) = -19.4^\circ$$

To reduce this phase lag to  $-10^\circ$ , we need to change  $R$  so that the zero moves to the negative

real axis and introduces a phase lead of  $9.4^\circ$ .

Thus,

$$\tan^{-1}\frac{f_i}{f_Z} = 9.4^\circ$$

$$f_Z = \frac{f_i}{\tan 9.4^\circ} = \frac{33.6}{0.166} = 203 \text{ MHz}$$

$$f_Z = \frac{1}{2\pi C_C \left(R - \frac{1}{G_{m2}}\right)}$$

$$\Rightarrow R - \frac{1}{G_{m2}} = \frac{1}{2\pi \times 203 \times 10^6 \times 1.42 \times 10^{-12}}$$

$$= 552 \Omega$$

$$R = 1670 + 552 = 2222 \Omega$$

$$= 2.22 \text{ k}\Omega$$

**12.17** Using Eq. (12.41), we obtain

$$\text{SR} = 2\pi f_i V_{OV1,2}$$

$$= 2\pi \times 100 \times 10^6 \times 0.2$$

$$= 125.6 \text{ V}/\mu\text{s}$$

Using Eq. (12.40),

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow C_C = \frac{I}{\text{SR}} = \frac{100 \times 10^{-6}}{125.6 \times 10^6}$$

$$= 0.8 \text{ pF}$$

**12.18** The phase introduced at  $f_i = 1 \text{ MHz}$  by each of the coincident second and third poles must be  $5^\circ$ . Thus,  $f_{P2} = f_{P3}$  can be obtained from

$$\tan^{-1}\frac{f_i}{f_{P2}} = 5^\circ$$

$$\Rightarrow f_{P2} = f_{P3} = \frac{1 \text{ MHz}}{\tan 5^\circ} = 11.4 \text{ MHz}$$

$$\mathbf{12.19} \quad f_P = \frac{f_i}{A_0} = \frac{5 \text{ MHz}}{10^6} = 5 \text{ Hz}$$

But,

$$f_P = \frac{1}{2\pi CR}$$

where

$$C = (1 + |A|)C_C$$

$$= (1 + 1000) \times 50$$

$$= 50.05 \text{ nF}$$

$$5 = \frac{1}{2\pi \times 50.05 \times 10^{-9} \times R}$$

$$\Rightarrow R = 636 \text{ k}\Omega$$

**12.20** DC gain  $A_0 = G_{m1}R$

$$= 2 \times 10^{-3} \times 2 \times 10^7$$

$$= 4 \times 10^4 \text{ V/V}$$

$$20 \log A_0 = 92 \text{ dB}$$

$$f_P = \frac{1}{2\pi C_C R}$$

$$= \frac{1}{2\pi \times 100 \times 10^{-12} \times 2 \times 10^7}$$

$$= 79.6 \text{ Hz} \simeq 80 \text{ Hz}$$

$$f_t = A_0 f_P = 4 \times 10^4 \times 80$$

$$= 3.2 \text{ MHz}$$

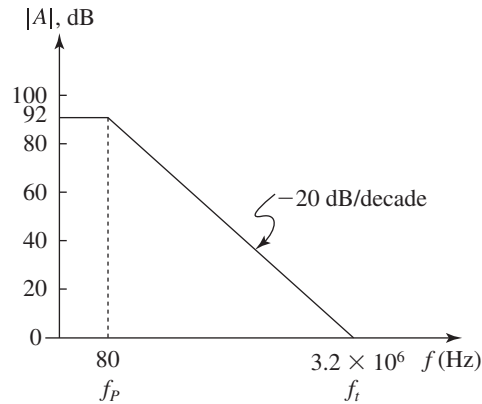


Figure 1

Figure 1 shows a sketch of the Bode plot for the magnitude of the open-loop gain of the op amp.

$$SR = \frac{I}{C_C}$$

But,

$$G_{m1} = \frac{I}{2V_T}$$

Thus,

$$I = 2V_T G_{m1}$$

and

$$SR = 2V_T \frac{G_{m1}}{C_C}$$

$$= 2 \times 25 \times 10^{-3} \times \frac{2 \times 10^{-3}}{100 \times 10^{-12}}$$

$$= 1 \text{ V}/\mu\text{s}$$

**12.21** For a sine-wave output, we have

$$v_O = \hat{V}_o \sin \omega t$$

$$\frac{dv_O}{dt} = \omega \hat{V}_o \cos \omega t$$

$$\left. \frac{dv_O}{dt} \right|_{\max} = \omega \hat{V}_o$$

$$10 \times 10^6 = \omega_M \times 10$$

$$\omega_M = 10^6 \text{ rad/s}$$

$$f_M = \frac{10^6}{2\pi} = 159.2 \text{ kHz}$$

If the topology is similar to that of the 741, then we can use the equation

$$SR = 4V_T \omega_t$$

$$\Rightarrow \omega_t = \frac{SR}{4V_T} = \frac{10 \times 10^6}{4 \times 25 \times 10^{-3}}$$

$$= 10^8 \text{ rad/s}$$

$$f_t = \frac{10^8}{2\pi} = 15.9 \text{ MHz}$$

**12.22** Assuming  $A_0\beta \gg 1$ , the closed-loop bandwidth is approximately

$$\omega_{P1}(1 + A_0\beta) \approx \omega_{P1}A_0\beta$$

With a uniform  $-20\text{dB/decade}$  frequency response,  $\omega_t = A_0\omega_{P1}$ . Substituting into the equation above, the closed loop bandwidth is seen to be  $\omega_t\beta$ .

**12.23**  $SR = 60 \text{ V}/\mu\text{s}$ ,  $f_t = 60 \text{ MHz}$

(a) Using Eq. (12.41), we obtain

$$SR = 2\pi f_t |V_{O1}|$$

$$\Rightarrow |V_{OV1}| = \frac{60 \times 10^6}{2\pi \times 60 \times 10^6} = 0.16 \text{ V}$$

(b) Using Eq. (12.40), we get

$$SR = \frac{I}{C_C}$$

$$\Rightarrow C_C = \frac{I}{SR} = \frac{120 \times 10^{-6}}{60 \times 10^6} = 2 \text{ pF}$$

(c) For  $Q_1$  and  $Q_2$ , we have

$$I_{D1,2} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{1,2} |V_{OV1,2}|^2$$

$$60 = \frac{1}{2} \times 60 \times \left(\frac{W}{L}\right)_{1,2} \times 0.16^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 78.1$$

**12.24** Figure 1 below shows the circuit.

**12.25** (a) From Eq. (12.49), we have

$$PSRR^- = g_{m1}(r_{o2} \parallel r_{o4})g_{m6}r_{o6}$$

where

$$g_{m1} = \frac{2 \times \frac{I}{2}}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

$$g_{m6} = \frac{2I_{D6}}{|V_{OV}|}$$

$$r_{o6} = \frac{|V_A|}{I_{D6}}$$

Thus,

$$\begin{aligned} PSRR^- &= \frac{I}{|V_{OV}|} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times \frac{2I_{D6}}{|V_{OV}|} \times \frac{|V_A|}{I_{D6}} \\ &= 2 \left| \frac{V_A}{V_{OV}} \right|^2 \quad \text{Q.E.D.} \end{aligned}$$

(b) A  $PSRR^-$  of 72 dB means

$$PSRR^- = 4000$$

Thus,

$$4000 = 2 \frac{|V_A|^2}{0.15^2}$$

$$\Rightarrow |V_A| = 6.71 \text{ V}$$

Now,

$$|V_A| = |V'_A|L$$

$$6.71 = 15L$$

$$\Rightarrow L = 0.45 \text{ } \mu\text{m}$$

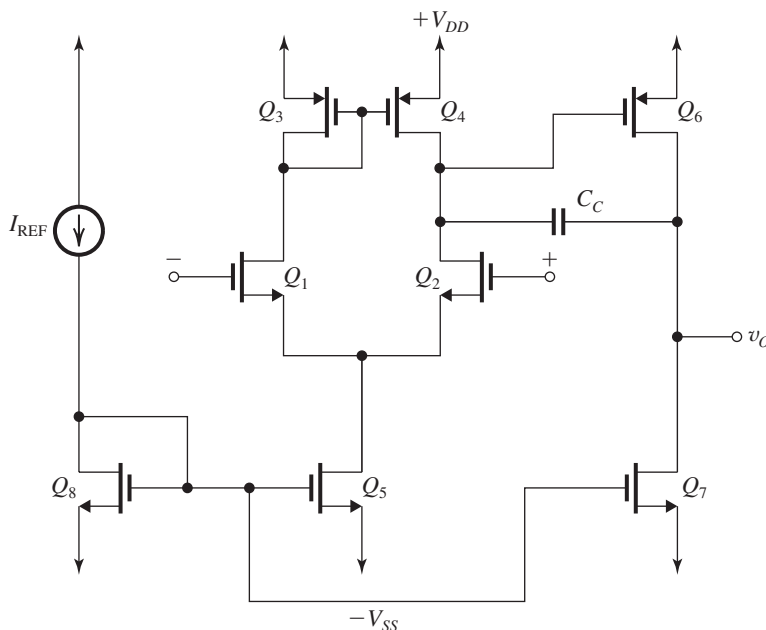


Figure 1

This table belongs to Problem 12.28.

Transistor	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$	$Q_9$	$Q_{10}$	$Q_{11}$
$W/L$	25	25	25	25	6.25	6.25	6.25	6.25	125	125	50

$$12.26 \quad 2I_B \times 2 = 1 \text{ mW}$$

$$I_B = \frac{10^{-3}}{4} = 0.25 \text{ mA} = 250 \text{ } \mu\text{A}$$

$$I_{D1} = 4I_{D3}$$

$$I_{D1} + I_{D3} = I_B$$

$$5I_{D3} = 250 \text{ } \mu\text{A}$$

$$I_{D3} = 50 \text{ } \mu\text{A}, \quad I_{D4} = 50 \text{ } \mu\text{A}$$

$$I_{D1} = 200 \text{ } \mu\text{A}, \quad I_{D2} = 200 \text{ } \mu\text{A}$$

$$I = 400 \text{ } \mu\text{A}$$

$$12.27 \quad V_{\text{BIAS1}} = V_{DD} - |V_{OV9}| - |V_{OV3}| - |V_{tp}|$$

$$= 1 - 0.15 - 0.15 - 0.4 = +0.3 \text{ V}$$

$$V_{\text{BIAS2}} = V_{DD} - |V_{OV9}| - |V_{tp}|$$

$$= 1 - 0.15 - 0.4 = +0.45 \text{ V}$$

$$V_{\text{BIAS3}} = -V_{SS} + V_{OV11} + V_m$$

$$= -1 + 0.15 + 0.4 = -0.45 \text{ V}$$

$$V_{\text{ICMmax}} = V_{DD} - |V_{OV9}| + V_m$$

$$= 1 - 0.15 + 0.4 = +1.25 \text{ V}$$

$$V_{\text{ICMmin}} = -V_{SS} + |V_{OV11}| + V_{OV1} + V_m$$

$$= -1 + 0.15 + 0.15 + 0.4 = -0.3 \text{ V}$$

Thus,

$$-0.3 \text{ V} \leq V_{\text{ICM}} \leq +1.25 \text{ V}$$

$$v_{O\text{max}} = V_{\text{BIAS1}} + |V_{tp}|$$

$$= 0.3 + 0.4 = +0.7 \text{ V}$$

$$v_{O\text{min}} = -V_{SS} + V_{OV7} + V_m + V_{OV5}$$

$$= -1 + 0.15 + 0.4 + 0.15 = -0.3 \text{ V}$$

Thus,

$$-0.3 \text{ V} \leq v_O \leq +0.7 \text{ V}$$

$$12.28 \quad I_{D1} = I_{D2} = \frac{I}{2} = 0.2 \text{ mA}$$

$$0.2 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 25$$

$$I_{D3} = I_{D4} = I_B - \frac{I}{2} = 250 - 200 = 50 \text{ } \mu\text{A}$$

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 25$$

$$I_{D5} = I_{D6} = I_{D7} = I_{D8} = 50 \text{ } \mu\text{A}$$

$$50 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{5-8} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= 6.25$$

$$I_{D9} = I_{D10} = I_B = 250 \text{ } \mu\text{A}$$

$$250 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{9,10} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_{10} = 125$$

$$I_{D11} = I = 400 \text{ } \mu\text{A}$$

$$400 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{11} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_{11} = 50$$

Summary: See table above.

12.29 (a) Refer to Fig. 12.12. For the NMOS input stage

$$V_{\text{ICMmax}} = V_{DD} - |V_{OV}| + V_m$$

$$= 1 - 0.15 + 0.45 = +1.3 \text{ V}$$

$$\begin{aligned} V_{ICMmin} &= -V_{SS} + V_{OV} + V_{OV} + V_m \\ &= -1 + 0.15 + 0.15 + 0.45 \\ &= -0.25 \text{ V} \end{aligned}$$

Thus,

$$-0.25 \text{ V} \leq V_{ICM} \leq +1.3 \text{ V}$$

(b) For the PMOS input stage

$$\begin{aligned} V_{ICMmax} &= V_{DD} - |V_{OV}| - |V_{tp}| \\ &= 1 - 0.15 - 0.15 - 0.45 \text{ V} \\ &= +0.25 \text{ V} \end{aligned}$$

$$\begin{aligned} V_{ICMmin} &= -V_{SS} + V_{OV} - |V_{tp}| \\ &= -1 + 0.15 - 0.45 \\ &= -1.3 \text{ V} \end{aligned}$$

Thus,

$$-1.3 \text{ V} \leq V_{ICM} \leq +0.25 \text{ V}$$

(c) The overlap range is

$$-0.25 \text{ V} \leq V_{ICM} \leq +0.25 \text{ V}$$

(d)  $-1.3 \text{ V} \leq V_{ICM} \leq +1.3 \text{ V}$

$$\mathbf{12.30} \quad G_m = g_{m1} = g_{m2} = \frac{2(I/2)}{V_{OV}}$$

$$= \frac{I}{V_{OV}} = \frac{0.4}{0.2} = 2 \text{ mA/V}$$

$$I_{D4} = I_B - \frac{I}{2} = 0.25 - 0.2 = 0.05 \text{ mA}$$

$$g_{m4} = \frac{2I_{D4}}{|V_{OV}|} = \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$r_{o4} = \frac{|V_A|}{I_{D4}} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{|V_A|}{I/2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

$$r_{o10} = \frac{|V_A|}{I_{D10}} = \frac{|V_A|}{I_B} = \frac{10}{0.25} = 40 \text{ k}\Omega$$

$$\begin{aligned} R_{o4} &= (g_{m4}r_{o4}) (r_{o2} \parallel r_{o10}) \\ &= 0.5 \times 200 (50 \parallel 40) \\ &= 2.22 \text{ M}\Omega \end{aligned}$$

$$I_{D6} = 50 \mu\text{A} = 0.05 \text{ mA}$$

$$g_{m6} = \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$r_{o6} = \frac{|V_A|}{I_{D6}} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$r_{o8} = \frac{|V_A|}{I_{D8}} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$\begin{aligned} R_{o6} &= g_{m6}r_{o6}r_{o8} \\ &= 0.5 \times 200 \times 200 = 20 \text{ M}\Omega \end{aligned}$$

$$\begin{aligned} R_o &= R_{o4} \parallel R_{o6} \\ &= 2.22 \parallel 20 = 2 \text{ M}\Omega \end{aligned}$$

$$\begin{aligned} A_v &= G_m R_o \\ &= 2 \times 2000 = 4000 \text{ V/V} \end{aligned}$$

For the closed-loop amplifier:

$$A = A_v = 4000$$

$$\beta = \frac{C}{C + 9C} = 0.1$$

$$\frac{V_o}{V_i} = A_f = \frac{A}{1 + A\beta} = \frac{4000}{1 + 4000 \times 0.1}$$

$$= \frac{4000}{401} = 9.975 \text{ V/V}$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{2 \text{ M}\Omega}{401} \simeq 5 \text{ k}\Omega$$

$$\mathbf{12.31} \quad \text{SR} = \frac{I_B}{C_L}$$

$$10 \times 10^6 = \frac{I_B}{10 \times 10^{-12}}$$

$$\Rightarrow I_B = 10^{-4} \text{ A} = 0.1 \text{ mA} = 100 \mu\text{A}$$

$$\frac{I}{2} = 3 \left( I_B - \frac{I}{2} \right)$$

$$\frac{I}{2} (1 + 3) = 3I_B = 300$$

$$I = 150 \mu\text{A}$$

Now,

$$f_i = \frac{G_m}{2\pi C_L}$$

where

$$G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$$

$$= \frac{0.15 \text{ mA}}{0.15 \text{ V}} = 1 \text{ mA/V}$$

Thus,

$$f_t = \frac{1 \times 10^{-3}}{2\pi \times 10^{-12}}$$

$$= 15.92 \text{ MHz}$$

Phase due to the two nondominant poles at  $f_t$

$$= -2 \tan^{-1} \left( \frac{15.92}{50} \right) = -35.3^\circ$$

Thus,

$$\text{Phase margin} = 90 - 35.3 = 54.7^\circ$$

To increase the phase margin to  $75^\circ$ , the phase due to the two nondominant poles must be reduced to  $90 - 75 = 15^\circ$ , i.e. each should contribute  $7.5^\circ$ , thus we must reduce  $f_t$  to the value obtained as follows:

$$\tan^{-1} \left( \frac{f_t}{50 \text{ MHz}} \right) = 7.5^\circ$$

$$f_t = 50 \times \tan 7.5^\circ = 6.58 \text{ MHz}$$

This is achieved by increasing  $C_L$ ,

$$6.58 \times 10^6 = \frac{1 \times 10^{-3}}{2\pi C_L}$$

$$\Rightarrow C_L = \frac{10^{-3}}{2\pi \times 7.92 \times 10^6} = 24.2 \text{ pF}$$

The new value of slew-rate will be

$$\text{SR} = \frac{I_B}{C_L} = \frac{0.1 \times 10^{-3}}{24.2 \times 10^{-12}} = 4.13 \text{ V}/\mu\text{s}$$

**12.32** Refer to Fig. 12.9. When  $V_{id}$  is sufficiently large to cause  $Q_1$  to cut off and  $Q_2$  to conduct all of  $I$ ,  $Q_3$  will carry a current  $I_B$ . However,  $Q_4$  will carry  $(I_B - I)$ . The current  $I_B$  in  $Q_3$  will be mirrored in the drain of  $Q_6$ . Thus, at the output node the current available to charge  $C_L$  will be

$$I_O = I_B - (I_B - I) = I$$

and the slew rate becomes

$$\text{SR} = \frac{I}{C_L}$$

**12.33**

$$\begin{aligned} \frac{V_f}{V_{id}} &= G_m \times R_o \parallel (R_S + 1/sC_L) \\ &= \frac{G_m R_o (R_S + 1/sC_L)}{R_o + R_S + 1/sC_L} \end{aligned}$$

$$= \frac{G_m R_o (1 + sR_S C_L)}{1 + s(R_o + R_S)C_L}$$

$$\frac{V_o}{V_{id}} = \frac{V_f}{V_{id}} \times \frac{V_o}{V_f}$$

$$= \frac{G_m R_o (1 + sR_S C_L)}{1 + s(R_o + R_S)C_L} \times \frac{1/sC_L}{R_S + 1/sC_L}$$

$$= \frac{G_m R_o}{1 + s(R_o + R_S)C_L}$$

The introduction of  $R_S$  slightly reduces the dominant pole frequency to  $1/(R_o + R_S)C_L$ , and thus also reduces the unity-gain frequency,  $\omega_t$ , and slightly increases phase margin.

Much more important, connecting the feedback network to  $V_f$  instead of  $V_o$  introduces a left-half-plane zero at  $-1/R_S C_L$ , which has a phase lead and thus adds to the phase margin  $\tan^{-1}(\omega_t R_S C_L)$ .

$$\mathbf{12.34} \quad A = 80 \text{ dB} \equiv 10^4 \text{ V/V}$$

$$f_t = 20 \text{ MHz}, \quad C_L = 10 \text{ pF}$$

$$I_B = I$$

$$|V_A| = 12 \text{ V}$$

Refer to Figs. 12.9 and 12.10. For  $I = I_B$ , the dc operating currents of the 11 transistors are as follows:

$$Q_1 - Q_8: \frac{I}{2}$$

$$Q_9, Q_{10}, \text{ and } Q_{11}: I$$

Thus, for  $Q_1 - Q_8$ , we have

$$g_m = \frac{I}{|V_{OV}|}$$

and

$$r_o = \frac{2|V_A|}{I}$$

while, for  $Q_9 - Q_{11}$ ,

$$r_o = \frac{|V_A|}{I}$$

Now,

$$G_m = g_{m1,2} = \frac{I}{V_{OV}}$$

$$R_{o4} = (g_{m4} r_{o4}) (r_{o2} \parallel r_{o10})$$

$$= \frac{I}{|V_{OV}|} \times \frac{2|V_A|}{I} \left[ \frac{2|V_A|}{I} \parallel \frac{|V_A|}{I} \right]$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{2}{3} \frac{|V_A|}{I}$$

$$= \frac{4|V_A|^2}{3|V_{OV}|I}$$

$$R_{o6} = g_{m6}r_{o6}r_{o8}$$

$$= \frac{I}{|V_{OV}|} \frac{2|V_A|}{I} \frac{2|V_A|}{I}$$

$$= \frac{4|V_A|^2}{|V_{OV}|I}$$

$$R_o = R_{o4} \parallel R_{o6}$$

$$= \left[ \frac{4}{3} \frac{|V_A|^2}{|V_{OV}|I} \right] \parallel \left[ \frac{|V_A|^2}{|V_{OV}|I} \right]$$

$$= \frac{|V_A|^2}{|V_{OV}|I}$$

The voltage gain can now be found as

$$A = G_m R_o = g_{m1,2} R_o$$

$$= \frac{I}{|V_{OV}|} \frac{|V_A|^2}{|V_{OV}|I}$$

$$= \frac{|V_A|^2}{|V_{OV}|^2}$$

$$10,000 = \left| \frac{V_A}{V_{OV}} \right|^2$$

$$\Rightarrow \frac{|V_A|}{|V_{OV}|} = 100$$

$$\Rightarrow |V_{OV}| = \frac{12}{100} = 0.12 \text{ V}$$

To obtain  $f_i = 20 \text{ MHz}$ , we use

$$20 \times 10^6 = \frac{g_{m1,2}}{2\pi \times 10 \times 10^{-12}}$$

$$g_{m1,2} = 2\pi \times 10 \times 10^{-12} \times 20 \times 10^6$$

$$= 1.257 \times 10^{-3} \text{ A/V}$$

Thus,

$$\frac{I}{|V_{OV}|} = 1.257 \times 10^{-3}$$

$$\Rightarrow I = 1.257 \times 0.12 \times 10^{-3}$$

$$= 0.15 \text{ mA} = 150 \mu\text{A}$$

$$I_B = I = 150 \mu\text{A}$$

$$\text{SR} = \frac{I_B}{C_L} = \frac{150 \times 10^{-6}}{10 \times 10^{-12}}$$

$$= 15 \text{ V}/\mu\text{s}$$

For  $Q_1$  and  $Q_2$ , we have

$$I_D = \frac{I}{2} = 75 \mu\text{A} = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$75 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{1,2} \times 0.12^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 26$$

For  $Q_3$  and  $Q_4$ , we have

$$I_D = I_B - \frac{I}{2} = 150 - 75 = 75 \mu\text{A}$$

Thus,

$$75 = \frac{1}{2} \times \frac{400}{2.5} \times \left( \frac{W}{L} \right)_{3,4} \times 0.12^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 65.1$$

For  $Q_5$ ,  $Q_6$ ,  $Q_7$ , and  $Q_8$ , we have

$$I_D = I_B = 75 \mu\text{A}$$

$$75 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{5-8} \times 0.12^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_5 = \left( \frac{W}{L} \right)_6 = \left( \frac{W}{L} \right)_7 = \left( \frac{W}{L} \right)_8 = 26$$

For  $Q_9$  and  $Q_{10}$ , we have

$$I_D = I_B = 150 \mu\text{A}$$

$$150 = \frac{1}{2} \times \frac{400}{2.5} \times \left( \frac{W}{L} \right)_{9,10} \times 0.12^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_9 = \left( \frac{W}{L} \right)_{10} = 130.2$$

For  $Q_{11}$ , we have

$$I_D = I = 150 \mu\text{A}$$

$$150 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{11} \times 0.12^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{11} = 52$$

See table on the next page for a summary.

Summary (Approximate Values):

Transistor	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$	$Q_9$	$Q_{10}$	$Q_{11}$
$W/L$	26	26	65	65	26	26	26	26	130	130	52

This figure belongs to Problem 12.35.

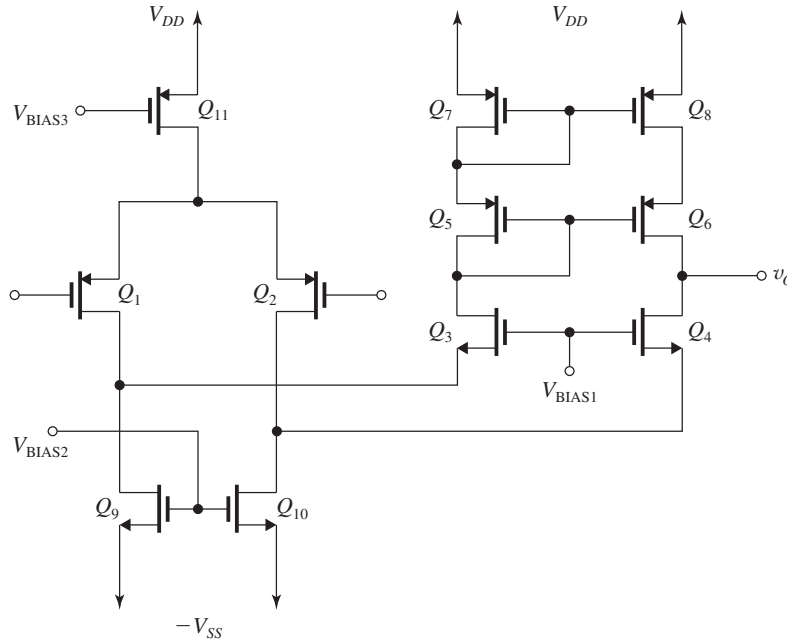


Figure 1

**12.35** Figure 1 above shows a folded-cascode single-stage op amp circuit that is complementary to that in Fig. 12.10.

**12.36** (a) Figure 1 on the next page shows the dc bias currents of all transistors as well as the various dc node voltages. Since  $Q_{12}$  and  $Q_{13}$  have  $V_G = +0.4$  V and  $V_S = +0.85$  V, both will be cut off.

(b) Figure 2 on the next page shows the currents and voltages during slewing. We assume that  $V_{id}$  is sufficiently large to steer  $I = 320$   $\mu$ A into  $Q_1$  and turn  $Q_2$  off. Now, at the drain of  $Q_1$  we can write

$$I_{D12} + I_{D9} = 320 \mu\text{A} \quad (1)$$

The current through  $Q_{14}$  becomes

$$I_{D14} = 20 \mu\text{A} + I_{D12} \quad (2)$$

Also, since  $Q_9$  has  $(W/L)$  equal to ten times that of  $Q_{14}$ , we have

$$I_{D9} = 10 I_{D14} \quad (3)$$

Solving (1), (2), and (3) together yields

$$I_{D12} = 10.9 \mu\text{A} \approx 11 \mu\text{A}$$

$$I_{D14} \approx 31 \mu\text{A}$$

$$I_{D9} \approx 310 \mu\text{A}$$

Thus,

$$I_{D10} = I_{D9} \approx 310 \mu\text{A}$$

$$I_{D4} = I_{D10} = 310 \mu\text{A}$$

$$I_O = 310 \mu\text{A}$$

Thus, the current available to charge  $C_L$  has increased from  $I_B = 200$   $\mu$ A to 310  $\mu$ A, a factor of 1.65. The slew rate will increase by the same factor. However, it is possible that the large current that  $Q_4$  now conducts can raise the voltage at its source and thus at the drain of  $Q_{10}$  to a value that drives  $Q_{10}$  into the triode region. In this case, the current that  $Q_{10}$  provides will be reduced and the full increase in SR will not be realized.

To determine the voltage at the drain of  $Q_1$ , we first find  $V_{G14}$ ,



This figure belongs to Problem 12.36, part (a).

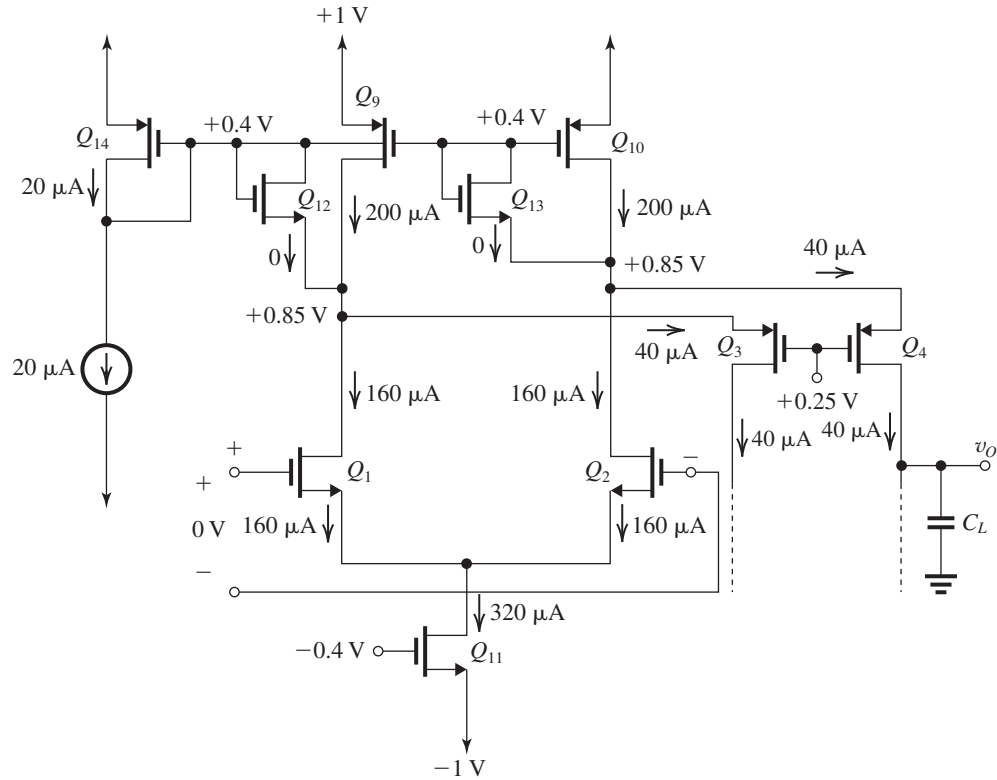


Figure 1

This figure belongs to Problem 12.36, part (b).

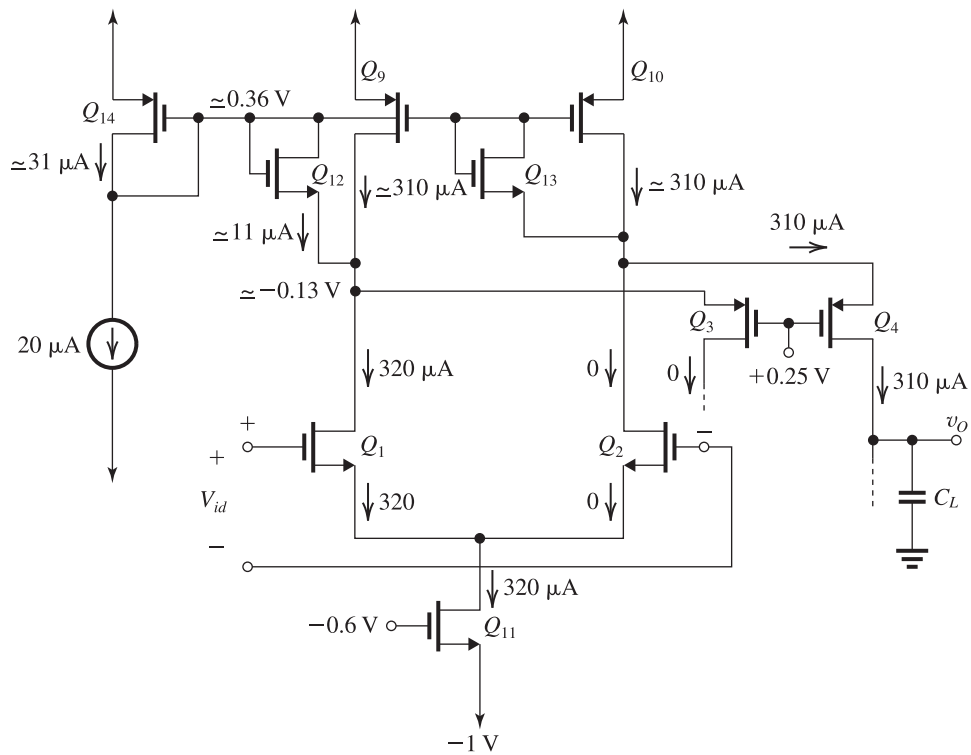


Figure 2

$$V_{G14} = V_{DD} - V_{SG14}$$

When  $I_{D4}$  was 20  $\mu\text{A}$ ,  $|V_{OV14}|$  was 0.15 V. Now  $I_{D4} = 31 \mu\text{A}$  and thus  $|V_{OV14}|$  becomes

$$|V_{OV14}| = 0.15 \times \sqrt{\frac{31}{20}} = 0.19 \text{ V}$$

Thus,

$$V_{SG14} = 0.45 + 0.19 = 0.64 \text{ V}$$

and

$$V_G = 1 - 0.64 = +0.36 \text{ V}$$

Next, we estimate  $V_{GS12}$ . Since  $I_{D12}$  is only 11  $\mu\text{A}$  and assuming  $Q_{11}$  is, say, similar to  $Q_1$ , then

$$V_{OV12} = 0.15 \sqrt{\frac{11}{160}}$$

$$= 0.04 \text{ V}$$

Thus,

$$V_{GS12} = 0.45 + 0.04 = 0.49 \text{ V}$$

It follows that

$$V_{D1} = 0.36 - 0.49 = -0.13 \text{ V}$$

which is sufficient to keep  $Q_1$  and  $Q_{11}$  in the saturation region.

**12.37** First we determine  $V_{OV}$ :

$$90 = \frac{1}{2} \times 400 \times 20 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.15 \text{ V}$$

$$V_{BIAS} = V_t + 2V_{OV} = 0.45 + 2 \times 0.15$$

$$= 0.75 \text{ V}$$

Figure 1 shows the voltages at the various nodes in the mirror circuit. The minimum voltage allowable at the output terminal is

$$v_{Omin} = V_{BIAS} - V_m$$

$$= 0.75 - 0.45 = 0.3 \text{ V}$$

which is  $2V_{OV}$ .

The output resistance is

$$R_o \simeq g_{m3}r_{o3}r_{o1}$$

where

$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{10}{0.09} = 111.1 \text{ k}\Omega$$

$$g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.09}{0.15} = 1.2 \text{ mA/V}$$

$$R_o = 1.2 \times 111.1 \times 111.1 = 14.8 \text{ M}\Omega$$

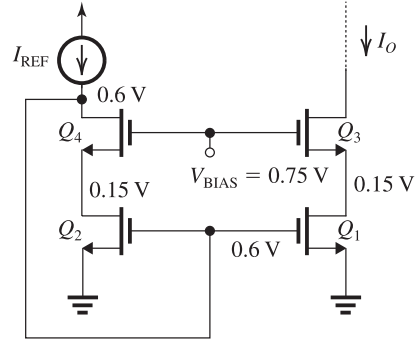


Figure 1

**12.38** Since  $Q_3$  is operating in the common-gate configuration and since the resistance in its drain is low, the input resistance at its source is  $1/g_{m3}$ . This resistance appears in parallel with  $r_{o1}$  which is much larger. Thus, the total resistance at this node is  $\simeq 1/g_{m3}$  and since the total capacitance is  $C_P$ , the pole introduced will have a frequency

$$f_P \simeq \frac{1}{2\pi C_P/g_{m3}} = \frac{g_{m3}}{2\pi C_P} \quad \text{Q.E.D.}$$

Now,

$$f_i = \frac{g_{m1}}{2\pi C_L}$$

where  $g_{m1} = g_{m3}$  because all transistors are operating at the same values of  $I_D$  and  $|V_{OV}|$ .

For a phase margin of  $80^\circ$  the phase at  $f_i$  introduced by the pole at  $f_P$  must be only  $10^\circ$ ,

$$\tan^{-1}\left(\frac{f_i}{f_P}\right) = 10^\circ$$

$$f_P = \frac{f_i}{\tan 10^\circ} = \frac{f_i}{0.176}$$

$$\frac{g_{m3}}{2\pi C_P} = \frac{g_{m1}}{2\pi C_L \times 0.176}$$

$$\Rightarrow C_P = 0.176C_L$$

This is the largest value that  $C_P$  can have.

**12.39**

Refer to Fig. 1 next page.

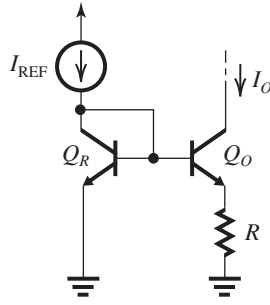


Figure 1

$$I_{\text{REF}} = 0.3 \text{ mA}, I_O = 10 \text{ } \mu\text{A}$$

$$\begin{aligned} V_{\text{BER}} &= V_T \ln \frac{I_{\text{REF}}}{I_S} \\ &= 25 \ln \left( \frac{0.3 \times 10^{-3}}{10^{-14}} \right) = 603 \text{ mV} \end{aligned}$$

$$\begin{aligned} V_{\text{BEO}} &= V_T \ln \frac{I_O}{I_S} \\ &= 25 \ln \left( \frac{10 \times 10^{-6}}{10^{-14}} \right) \end{aligned}$$

$$= 518 \text{ mV}$$

$$V_{\text{BER}} - V_{\text{BEO}} = 603 - 518 = 85 \text{ mV}$$

$$R = \frac{85 \text{ mV}}{10 \text{ } \mu\text{A}} = 8.5 \text{ k}\Omega$$

$$\mathbf{12.40} \quad I = 5 \text{ } \mu\text{A}$$

$$\frac{I_{S2}}{I_{S1}} = 4$$

Using the following equation, we obtain

$$I = \frac{V_T}{R_2} \ln \left( \frac{I_{S2}}{I_{S1}} \right)$$

$$5 \times 10^{-3} = \frac{0.025}{R_2} \ln 4$$

$$\Rightarrow R_2 = 6.93 \text{ k}\Omega$$

$$R_3 = R_4 = \frac{0.15 \text{ V}}{0.005 \text{ mA}} = 30 \text{ k}\Omega$$

**12.41** Using Eq. (12.72),

$$I = \frac{V_T}{R_1} \ln \left( \frac{I_{S2}}{I_{S1}} \right) \Rightarrow R_1 = \frac{V_T}{I} \ln \left( \frac{I_{S2}}{I_{S1}} \right)$$

$$= \frac{25 \times 10^{-3}}{15 \times 10^{-6}} \times \ln 4$$

$$= 2.31 \text{ k}\Omega$$

The voltage across  $R_3$  is

$$V_{R3} = IR_3 = 0.1 \text{ V}$$

The values of  $R_3$  and  $R_4$  are matched, so

$$R_4 = R_3 = \frac{0.1}{15 \times 10^{-6}} = 6.67 \text{ k}\Omega$$

The area of  $Q_7$  must be larger than that of  $Q_3$  by a factor

$$\frac{I_7}{I} = \frac{45}{15} = 3$$

and its emitter resistance must be scaled by the same factor

$$R_7 = \frac{R_3}{3} = \frac{6.67 \text{ k}\Omega}{3} = 2.22 \text{ k}\Omega$$

The area of  $Q_8$  must be larger than that of  $Q_3$  by a factor

$$\frac{I_8}{I} = \frac{150}{15} = 10$$

and its emitter resistance must be scaled by the same factor

$$R_8 = \frac{R_3}{10} = \frac{6.67 \text{ k}\Omega}{10} = 667 \Omega$$

**12.42** The maximum output voltage is limited by the voltage drop across current source  $I_2$  and  $V_{\text{BE}6}$ .

$$v_{O \text{ max}} = V_{\text{CC}} - V_{I2} - V_{\text{BE}6} = 5 - 0.3 - 0.6 = 4.1 \text{ V}$$

The minimum output voltage is limited by the voltage drop on  $R_2$ , saturation of  $Q_5$ , and  $V_{\text{BE}7}$ .

$$\begin{aligned} v_{O \text{ min}} &= I_2 R_2 + V_{\text{CE sat}5} + V_{\text{BE}7} = \\ &0.2 \times 0.3 + 0.1 + 0.6 = 0.76 \text{ V} \end{aligned}$$

**12.43** Refer to Fig. 12.16.

$$V_{\text{ICMmax}} = V_{\text{CC}} - 0.1 - 0.7 = 3 - 0.8$$

$$= +2.2 \text{ V}$$

$$V_{\text{ICMmin}} = \frac{1}{2} I R_C - 0.6$$

$$= \frac{1}{2} \times 0.02 \times 35 - 0.6$$

$$= -0.25 \text{ V}$$

Thus,

$$-0.25 \text{ V} \leq V_{\text{ICM}} \leq +2.2 \text{ V}$$

$$A_v = g_m R_C$$

where

$$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-6}}{25 \times 10^{-3}} = 0.4 \text{ mA/V}$$

$$A_v = 0.4 \times 35 = 14 \text{ V/V}$$

**12.44**  $g_m = \frac{I/2}{V_T} = \frac{20 \mu\text{A}}{25 \text{ mV}} = 0.8 \text{ mA/V}$

For  $A_d = 10 \text{ V/V}$ , we have

$$10 = g_m R_C$$

$$\Rightarrow R_C = 12.5 \text{ k}\Omega$$

$$\frac{I}{2} R_C = 20 \times 10^{-3} \times 12.5 = 0.25 \text{ V}$$

$$V_{ICM\min} = 0.8 \text{ V}$$

$$V_{ICM\max} = V_{CC} - \frac{I}{2} R_C + 0.6$$

$$= 3 - 0.25 + 0.6 = 3.35 \text{ V}$$

Thus,

$$0.8 \text{ V} \leq V_{ICM} \leq 3.35 \text{ V}$$

$$R_{id} = 2r_\pi = 2 \frac{\beta_N}{g_m}$$

$$= 2 \frac{40}{0.8} = 100 \text{ k}\Omega$$

To increase  $R_{id}$  by a factor of 4,  $g_m$  and hence  $I$  must be reduced by a factor of 4, thus  $I_{C6}$  becomes

$$I_{C6} = 10 \mu\text{A}$$

To keep the gain and the permissible range of  $V_{ICM}$  unchanged,  $R_C$  must be increased by a factor of 4, thus  $R_C$  becomes

$$R_C = 50 \text{ k}\Omega$$

**12.45** Differential input breakdown voltage

$$= 0.7 + 0.7 + 50 + 7$$

$$= 58.4 \text{ V}$$

where we have assumed that a forward conducting transistor exhibits  $|V_{BE}| = 0.7 \text{ V}$ .

**12.46** Refer to Fig. 12.23. For  $\beta_P$  large, a node equation at X yields

$$I_{C6} \simeq I_{REF}$$

If the ratio of the area of  $Q_6$  to that of  $Q_5$  is  $n$ , then

$$I_{C6} = n \times 2I$$

Thus,

$$2nI = I_{REF}$$

For  $I = 10 \mu\text{A}$  and  $I_{REF} = 40 \mu\text{A}$ , we have

$$n = 2$$

**12.47**  $V_{BE1} = V_T \ln \frac{I_1}{I_{S1}}$

$$V_{BE2} = V_T \ln \frac{I_1}{I_{S2}}$$

$$V_{BE3} = V_T \ln \frac{I_3}{I_{S3}}$$

$$V_{BE4} = V_T \ln \frac{I_3}{I_{S4}}$$

$$V_{BE3} + V_{BE4} = V_{BE1} + V_{BE2}$$

$$V_T \ln \frac{I_3}{I_{S3}} + V_T \ln \frac{I_3}{I_{S4}} = V_T \ln \frac{I_1}{I_{S1}} + V_T \ln \frac{I_1}{I_{S2}}$$

$$V_T \ln \frac{I_3^2}{I_{S3} I_{S4}} = V_T \ln \frac{I_1^2}{I_{S1} I_{S2}}$$

$$\Rightarrow \frac{I_3^2}{I_{S3} I_{S4}} = \frac{I_1^2}{I_{S1} I_{S2}}$$

$$\Rightarrow I_3 = I_1 \sqrt{\frac{I_{S3} I_{S4}}{I_{S1} I_{S2}}} \quad \text{Q.E.D.}$$

$$150 = I_1 \sqrt{3 \times 3} = 3I_1$$

$$\Rightarrow I_1 = 50 \mu\text{A}$$

**12.48**

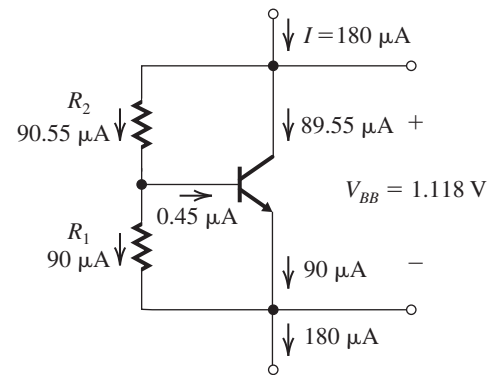


Figure 1

Refer to Fig. 1.

$$I_E = I_{R1} = \frac{180}{2} = 90 \mu\text{A}$$

$$I_B = \frac{90}{201} = 0.45 \mu\text{A}$$

$$I_C = \frac{\beta_N}{\beta_N + 1} I_E$$

$$= \frac{200}{201} \times 90 = 89.55 \mu\text{A}$$

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$= 25 \ln \frac{89.55 \times 10^{-6}}{10^{-14}}$$

$$= 573 \text{ mV}$$

$$R_1 = \frac{573 \text{ mV}}{90 \mu\text{A}} = 6.37 \text{ k}\Omega$$

$$I_{R2} = I_{R1} + I_B = 90 + 0.45 = 90.45 \mu\text{A}$$

$$V_{R2} = V_{BB} - V_{R1} = 1.118 - 0.573$$

$$= 0.545 \text{ V}$$

$$R_2 = \frac{545 \text{ mV}}{90.45 \mu\text{A}} = 6.03 \text{ k}\Omega$$

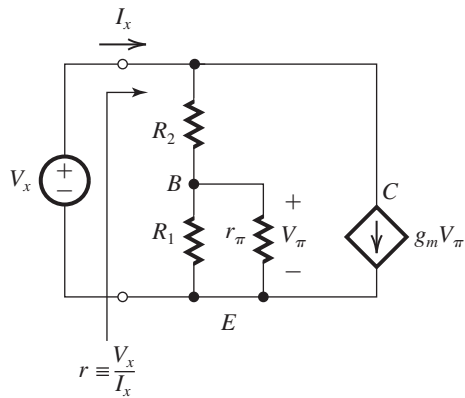


Figure 2

To determine the incremental resistance between the two terminals of the  $V_{BE}$  multiplier, we replace the transistor with its hybrid- $\pi$  model, as shown in Fig. 2. Here

$$g_m = \frac{I_C}{V_T} = \frac{89.55 \mu\text{A}}{25 \text{ mV}} = 3.6 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{3.6} = 55.6 \text{ k}\Omega$$

$$R_1 \parallel r_\pi = 6.37 \parallel 55.6 = 5.7 \text{ k}\Omega$$

$$V_\pi = V_x \frac{R_1 \parallel r_\pi}{(R_1 \parallel r_\pi) + R_2}$$

$$= V_x \frac{5.7}{5.7 + 6.03} = 0.49 V_x$$

$$I_x = \frac{V_x}{5.7 + 6.03} + g_m \times 0.49 V_x$$

$$= V_x(0.085 + 1.764)$$

$$r \equiv \frac{V_x}{I_x} = \frac{1}{0.085 + 1.764} = 0.541 \text{ k}\Omega$$

$$= 541 \Omega$$

**12.49** (a) Refer to Fig. P12.49.

$$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 0.05 \text{ mA}$$

$$I_{C5} = 1 \text{ mA}$$

$$I_{C7} = I_{C6} = I_{C5} = 1 \text{ mA}$$

(b) For  $Q_1$  and  $Q_2$ , we have

$$g_m = \frac{0.05 \text{ mA}}{0.025 \text{ V}} = 2 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{2} = 50 \text{ k}\Omega$$

$$R_{id} = 2r_\pi = 100 \text{ k}\Omega$$

(c) Figure 1 shows the small-signal analysis where

$$i_e = \frac{v_i}{2r_{e1,2}}$$

$$v_o = (\beta + 2)\beta\alpha i_e R_L$$

$$A_v = \frac{v_o}{v_i} = \frac{(\beta + 2)\beta\alpha R_L}{2r_{e1,2}}$$

$$A_v \simeq \frac{1}{2}\beta^2 \frac{R_L}{r_{e1,2}}$$

where

$$r_{e1,2} = \frac{25 \text{ mV}}{0.05 \text{ mA}} = 0.5 \text{ k}\Omega$$

$$A_v = \frac{1}{2} 100^2 \times \frac{5}{0.5} = 5 \times 10^4 \text{ V/V}$$

or 94 dB

(d)

Replacing the second stage with an amplifier having a large negative gain, we obtain the

This figure belongs to Problem 12.49, part (c).

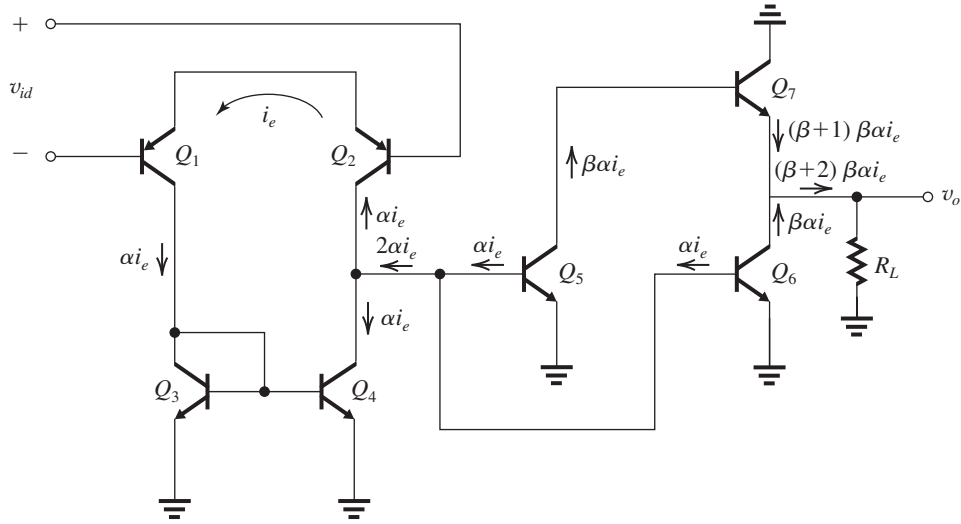


Figure 1

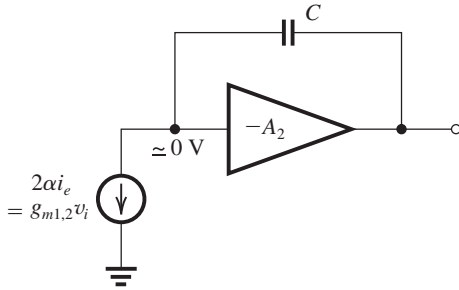


Figure 2

equivalent circuit shown in Fig. 2. From this equivalent circuit we see that the gain is approximately given by

$$A(s) = \frac{g_{m1,2}}{sC}$$

Thus, the unity gain frequency  $\omega_t$  is given by

$$\omega_t = \frac{g_{m1,2}}{C}$$

and the 3-dB frequency  $\omega_p$  is

$$\omega_p = \frac{\omega_t}{A_0} = \frac{g_{m1,2}}{A_0 C}$$

$$f_p = \frac{g_{m1,2}}{2\pi A_0 C}$$

For  $f_p = 100$  Hz and substituting  $g_{m1,2} = 2$  mA/V and  $A_0 = 5 \times 10^4$ , we find

$$C = \frac{2 \times 10^{-3}}{2\pi \times 100 \times 5 \times 10^4}$$

$$= 63.7 \text{ pF}$$

**12.50** For  $I_5 = 10 \mu\text{A} = I$ , then

$$\frac{Q_5 \text{ emitter area}}{Q_1 \text{ emitter area}} = 1$$

For  $I_6 = 40 \mu\text{A} = 4I$ , then

$$\frac{Q_6 \text{ emitter area}}{Q_1 \text{ emitter area}} = 4$$

If we connect a resistance  $R_6$  in the emitter of  $Q_6$ , then  $I_6$  changes to a new value determined as follows:

$$V_{BE6} + I_6 R_6 = V_{BE1}$$

$$I_6 R_6 = V_{BE1} - V_{BE6}$$

$$= V_T \ln \frac{I}{I_{S1}} - V_T \ln \frac{I_6}{I_{S6}}$$

But  $I_6$  is to be equal to  $I$ , thus

$$I R_6 = V_T \ln \frac{I_{S6}}{I_{S1}}$$

$$R_6 = \frac{V_T}{I} \ln 4$$

$$\Rightarrow R_6 = \frac{0.025}{0.01} \ln 4 = 3.47 \text{ k}\Omega$$

If the  $V_{\text{BIAS1}}$  line has a low incremental resistance to ground, then

$$R_{o5} = r_{o5} = \frac{V_{An}}{I_5} = \frac{30 \text{ V}}{10 \mu\text{A}} = 3 \text{ M}\Omega$$

$$R_{o6} = r_{o6} + (R_6 \parallel r_{\pi6})(1 + g_{m6} r_{o6})$$

where

$$r_{o6} = \frac{30 \text{ V}}{10 \mu\text{A}} = 3 \text{ M}\Omega$$

$$g_{m6} = \frac{10 \mu\text{A}}{0.025 \text{ V}} = 0.4 \text{ mA/V}$$

$$r_{\pi6} = \frac{\beta_N}{g_{m6}} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

$$R_{o6} = 3 + (3.47 \parallel 100) \times 10^{-3} (1 + 1200)$$

$$R_{o6} = 3 + 4 = 7 \text{ M}\Omega$$

Thus, increasing the BEJ area by a factor of 4 and adding a resistance  $R_6$  to restore the current to the desired value of  $10 \mu\text{A}$  increases the output resistance by a factor of about 2.5!

**12.51** (a) The bias current  $I$  of the differential pair is given by the equation

$$I = \frac{V_T}{R_5} \ln\left(\frac{I_{S5}}{I_{S1}}\right) \quad (1)$$

The voltage gain of the differential pair is given by

$$A_d = g_m R_C$$

where  $g_m$  is the transconductance of each of the two transistors in the differential pair,

$$g_m = \frac{I/2}{V_T} = \frac{I}{2V_T}$$

Thus,

$$A_d = \frac{IR_C}{2V_T} \quad (3)$$

Substituting for  $I$  from Eq. (1) into Eq. (3), we obtain

$$A_d = \frac{1}{2} \frac{R_C}{R_5} \ln\left(\frac{I_{S5}}{I_{S2}}\right) \quad (4)$$

which indicates that  $A_d$  will be independent of temperature!

$$(b) \quad I = 20 \mu\text{A}, \quad A_d = 10 \text{ V/V}, \quad \frac{I_{S5}}{I_{S1}} = 4$$

Using Eq. (1), we obtain

$$20 \times 10^{-3} = \frac{0.025}{R_5} \ln 4$$

$$\Rightarrow R_5 = 1.73 \text{ k}\Omega$$

Using Eq. (4), we get

$$10 = \frac{1}{2} \frac{R_C}{1.73} \ln 4$$

$$\Rightarrow R_C = 25 \text{ k}\Omega$$

**12.52** Due to the large value of  $\beta$ , we may assume  $r_{\pi4} \gg R_1$  and, thus,

$$R_o = r_{o4} + R_1(1 + g_{m4}r_{o4})$$

For the values given,

$$g_{m4} = \frac{I_1/2}{V_T} = \frac{50}{2 \times 25} = 1 \text{ mA/V}$$

$$r_{o4} = \frac{125 \text{ V}}{25 \mu\text{A}} = 5 \text{ M}\Omega$$

$$\begin{aligned} \Rightarrow R_o &= 5 \times 10^6 + 400(1 + 1 \times 10^{-3} \times 5 \times 10^6) \\ &= 7 \text{ M}\Omega \end{aligned}$$

To increase  $R_o$  to  $14 \text{ M}\Omega$ , we must increase  $R_1$  so that

$$5 \text{ M}\Omega + R_1(1 + 5000) = 14 \text{ M}\Omega$$

$$\Rightarrow R_1 = 1.8 \text{ k}\Omega$$

**12.53** (a) If  $Q_1$  has emitter resistor  $R_1$  and conducts a current  $I_E$ , the resistor in the emitter of  $Q_2$  is  $R_1 + \Delta R_1$  and it will conduct a current  $I_E - \Delta I_E$  so that

$$V_{BE1} + I_E R_1 = V_{BE2} + (I_E - \Delta I_E)(R_1 + \Delta R_1)$$

$$V_{BE1} - V_{BE2} = I_E \Delta R_1 - \Delta I_E (R_1 + \Delta R_1)$$

Using a small-signal approximation, we recognize the change in  $V_{BE}$  on the left-hand side of the expression is  $\Delta I_E r_e$ .

$$\Delta I_E r_e = I_E \Delta R_1 - \Delta I_E (R_1 + \Delta R_1)$$

$$\Delta I_E r_e + \Delta I_E (R_1 + \Delta R_1) = I_E \Delta R_1$$

$$\frac{\Delta I_E}{I_E} = \frac{\Delta R_1}{R_1 + \Delta R_1 + r_e}$$

(b) To reduce the output current of the first stage of the op amp in Fig. 12.15 to zero, we have to apply an input voltage given by

$$\begin{aligned} V_{OS} &= \frac{\Delta I_E}{g_{m1,2}} \\ &= \Delta I_E \times \frac{V_T}{I_E} \\ &= \frac{\Delta R_1 V_T}{R_1 + \Delta R_1 + r_e} \end{aligned}$$

(c) The emitter current of the mirror determines the collector current of the differential pair. Thus,

an input offset voltage  $V_{OS}$  results in an output current

$$\begin{aligned}\Delta I_E &= g_{m1} V_{OS} \\ &= I_{C1} V_{OS} / V_T \\ &= I_E V_{OS} / V_T\end{aligned}$$

The resistor mismatch required to produce an equal magnitude but opposite polarity output current is found combining this and the result of part (a).

$$\begin{aligned}\Delta I_E &= I_E \frac{\Delta R_1}{R_1 + \Delta R_1 + r_e} = I_E \frac{V_{OS}}{V_T} \\ \Rightarrow \frac{\Delta R_1}{R_1} &= \frac{V_{OS}}{V_T} \frac{1 + r_e/R_1}{1 - V_{OS}/V_T}\end{aligned}$$

**12.54** (a) Refer to Fig. 12.15.

$$V_{ICM\min} = V_{C1} - 0.6$$

$$= 0.7 - 0.6 = 0.1 \text{ V}$$

$$V_{ICM\max} = V_{CC} - 0.1 - 0.7$$

$$= 3 - 0.8 = 2.2 \text{ V}$$

Thus,

$$0.1 \text{ V} \leq V_{ICM} \leq 2.2 \text{ V}$$

(b)

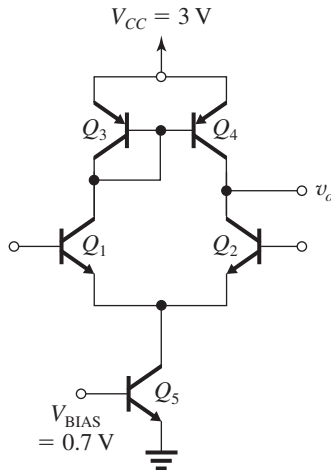


Figure 1

Figure 1 shows the complementary circuit to that in Fig. 12.33(a).

Here,

$$0.8 \text{ V} \leq V_{ICM} \leq 2.9 \text{ V}$$

**12.55** Refer to Fig. 12.19(a), which shows the differential half-circuit of the differential amplifier of Fig. 12.18.

$$R_{id} = 2r_{\pi 1} = 2 \frac{\beta_p}{g_{m1}}$$

where

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{4 \times 10^{-6}}{25 \times 10^{-3}} = 0.16 \text{ mA/V}$$

Thus,

$$R_{id} = \frac{2 \times 10}{0.16} = 125 \text{ k}\Omega$$

The short-circuit transconductance  $G_{m1}$  can be found from Fig. 12.19(b):

$$G_{m1} = \frac{i_o}{v_{id}/2}$$

At node X we have four resistances to ground:

$$r_{o1} = \frac{|V_{Ap}|}{I_{C1}} = \frac{20 \text{ V}}{4 \mu\text{A}} = 5 \text{ M}\Omega$$

$$R_7 = 22 \text{ k}\Omega$$

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30 \text{ V}}{8 \mu\text{A}} = 3.75 \text{ M}\Omega$$

$$r_{e7} \simeq \frac{1}{g_{m7}} = \frac{V_T}{I_{C7}} = \frac{25 \text{ mV}}{8 \mu\text{A}} = 3.125 \text{ k}\Omega$$

Obviously,  $r_{o1}$  and  $r_{o7}$  are much larger than  $r_{e7}$  and  $R_7$ . Then, the portion of  $g_{m1}(v_{id}/2)$  that flows into the emitter proper of  $Q_7$  can be found from

$$\begin{aligned}i_{e7} &\simeq g_{m1} \left( \frac{V_{id}}{2} \right) \frac{R_7}{R_7 + r_{e7}} \\ &= g_{m1} \left( \frac{V_{id}}{2} \right) \frac{22}{22 + 3.125} \\ &= 0.876 g_{m1} \left( \frac{V_{id}}{2} \right)\end{aligned}$$

Thus,

$$\begin{aligned}G_{m1} &\equiv \frac{i_o}{V_{id}/2} = \frac{\alpha i_{e7}}{V_{id}/2} \\ &= 0.876 g_{m1} = 0.137 \text{ mA/V}\end{aligned}$$

The total resistance between the output node and ground for the circuit in Fig. 12.38(a) is

$$R = R_{o9} \parallel R_{o7} \parallel (R_L/2)$$

The resistance  $R_{o9}$  is the output resistance of  $Q_9$ , which has an emitter-degeneration resistance  $R_9$ . Thus,

$$R_{o9} = r_{o9} + (R_9 \parallel r_{\pi 9})(1 + g_{m9} r_{o9})$$



where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{20 \text{ V}}{8 \mu\text{A}} = 2.5 \text{ M}\Omega$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{8 \mu\text{A}}{25 \text{ mV}} = 0.32 \text{ mA/V}$$

$$r_{\pi9} = \frac{\beta_P}{g_{m9}} = \frac{10}{0.32} = 31.25 \text{ k}\Omega$$

Thus,

$$\begin{aligned} R_{o9} &= 12.5 + (33 \parallel 31.25) \\ &\quad \times 10^{-3}(1 + 0.32 \times 2.5 \times 10^3) \\ &= 15.3 \text{ M}\Omega \end{aligned}$$

The resistance  $R_{o7}$  is the output resistance of  $Q_7$ , which has an emitter-degeneration resistance ( $R_7 \parallel r_{o1}$ )  $\simeq R_7$ . Thus,

$$R_{o7} = r_{o7} + (R_7 \parallel r_{\pi7})(1 + g_{m7}r_{o7})$$

where

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30 \text{ V}}{8 \mu\text{A}} = 3.75 \text{ M}\Omega$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{8 \mu\text{A}}{25 \text{ mV}} = 0.32 \text{ mA/V}$$

$$r_{\pi7} = \frac{\beta_N}{g_{m7}} = \frac{40}{0.32} = 125 \text{ k}\Omega$$

Thus,

$$\begin{aligned} R_{o7} &= 3.75 + (22 \parallel \\ &125) \times 10^{-3}(1 + 0.32 \times 3.75 \times 10^3) \\ &= 26.2 \text{ M}\Omega \\ \frac{R_L}{2} &= \frac{1.5}{2} = 0.75 \text{ M}\Omega \end{aligned}$$

The load resistance  $R$  can now be found as

$$R = 15.3 \parallel 26.2 \parallel 0.75 = 0.696 \text{ M}\Omega$$

Finally, we can find the voltage gain as

$$\begin{aligned} A_v &= \frac{v_{od}/2}{v_{id}/2} = G_{m1}R \\ &= 0.137 \times 0.696 \times 10^3 = 95.4 \text{ V/V} \end{aligned}$$

### 12.56 $I_{C1} = I$

$$I_{C7} = I_{C9} = 2I$$

From Fig. 12.18 we see that the current through  $R_7$  is approximately  $(I_{C1} + I_{C7})$ , that is,  $3I$ . Thus,

$$R_7 = \frac{0.2}{3I}$$

Since  $Q_3$  and  $Q_4$  are cut off, the current through  $R_9$  is equal to  $I_{E9}$  or approximately  $I_{C9}$ , thus

$$R_9 = \frac{0.3}{2I}$$

To determine the short-circuit transconductance  $G_{m1}$ , refer to Fig. 12.19(b).

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{I}{V_T}$$

$$G_{m1} = \frac{i_o}{v_{id}/2}$$

At node  $X$  we have four resistances in parallel, namely,  $r_{o1}$ ,  $R_7$ ,  $r_{o7}$ , and  $r_{e7}$ :

$$r_{o1} = \frac{|V_{Ap}|}{I_{C1}} = \frac{20}{I}$$

$$R_7 = \frac{0.2}{3I} = \frac{0.067}{I}$$

$$r_{o7} = \frac{V_{An}}{I_{C7}} = \frac{30}{2I} = \frac{15}{I}$$

$$r_{e7} \simeq \frac{V_T}{I_{C7}} = \frac{0.025}{2I} = \frac{0.0125}{I}$$

Thus,  $r_{o1}$  and  $r_{o7}$  are much greater than  $r_{e7}$  and  $R_7$ , and the portion of  $g_{m1}\left(\frac{v_{id}}{2}\right)$  that flows into the emitter proper of  $Q_7$  is given by

$$\begin{aligned} i_{e7} &\simeq g_{m1}\left(\frac{v_{id}}{2}\right) \frac{R_7}{R_7 + r_{e7}} \\ &= \left(\frac{I}{V_T}\right) \left(\frac{v_{id}}{2}\right) \frac{0.067}{0.067 + 0.0125} \\ &= 0.84\left(\frac{I}{V_T}\right) \left(\frac{v_{id}}{2}\right) \end{aligned}$$

The output short-circuit current  $i_o$  will be

$$i_o \simeq i_{e7} = 0.84\left(\frac{I}{V_T}\right) \left(\frac{v_{id}}{2}\right)$$

Thus,

$$G_{m1} = 0.84 \frac{I}{V_T} \simeq 33.6I$$

To obtain the output resistance  $R$ ,

$$R = R_{o9} \parallel R_{o7}$$

we determine  $R_{o9}$  as follows:

$$R_{o9} = r_{o9} + (R_9 \parallel r_{\pi9})(1 + g_{m9}r_{o9})$$

where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{20}{2I} = \frac{10}{I}$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{2I}{0.025} = 80I$$

$$g_{m9}r_{o9} = 800$$

$$r_{\pi9} = \frac{\beta_P}{g_{m9}} = \frac{10}{80I} = \frac{0.125}{I}$$

Thus,

$$\begin{aligned} R_{o9} &= \frac{10}{I} + \left( \frac{0.15}{I} \parallel \frac{0.125}{I} \right) \times 801 \\ &= \frac{64.6}{I} \end{aligned}$$

We next determine  $R_{o7}$  as follows:

$$R_{o7} = r_{o7} + (R_7 \parallel r_{\pi7})(1 + g_{m7}r_{o7})$$

where

$$r_{o7} = \frac{15}{I}$$

$$R_7 = \frac{0.067}{I}$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{2I}{V_T}$$

$$g_{m7}r_{o7} = 1200$$

$$r_{\pi7} = \frac{\beta_N}{g_{m7}} = \frac{40}{2I/V_T} = \frac{0.5}{I}$$

Thus,

$$\begin{aligned} R_{o7} &= \frac{15}{I} + \left( \frac{0.067}{I} \parallel \frac{0.5}{I} \right) \times 1201 \\ &= \frac{86}{I} \end{aligned}$$

We now can determine the output resistance  $R$  as

$$R = R_{o9} \parallel R_{o7} = \frac{64.6}{I} \parallel \frac{86}{I} = \frac{36.9}{I}$$

The open-circuit voltage gain can be obtained as

$$\begin{aligned} A_{vo} &= G_{m1}R \\ &= 0.84 \left( \frac{I}{V_T} \right) \left( \frac{36.9}{I} \right) \\ &= 1240 \text{ V/V} \end{aligned}$$

With a load resistance  $R_L$ , we have

$$\begin{aligned} A_v &= A_{vo} \frac{R_L}{R_L + R} \\ &= 1240 \frac{R_L}{R_L + \frac{36.9}{I}} \\ &= 1240 \frac{IR_L}{IR_L + 36.9} \end{aligned}$$

For  $R_L = 1 \text{ M}\Omega$  and  $I$  in  $\mu\text{A}$ , we have

$$A_v = 1240 \frac{I}{I + 36.9}$$

From this equation we can obtain

$$I = \frac{36.9}{\frac{1240}{A_v} - 1}$$

Thus, for  $A_v = 150 \text{ V/V}$ , the required value of  $I$  is

$$I = \frac{36.9}{\frac{1240}{150} - 1} = 5.1 \mu\text{A}$$

and for  $A_v = 300 \text{ V/V}$ , we require

$$I = \frac{36.9}{\frac{1240}{300} - 1} = 11.8 \mu\text{A}$$

**12.57** (a) Refer to Fig. 12.20. Break the loop at the input of the CMF circuit and apply a common-mode input signal  $\Delta V_{CM}$ . The CMF circuit will respond by causing a change  $\Delta V_B$  in its output voltage that can be found from its transfer characteristic as

$$\Delta V_B = \Delta V_{CM}$$

Now, a change  $\Delta V_B$  in the base voltage of  $Q_7$  and  $Q_8$  results in

$$\Delta I_{E8} = \Delta I_{E7} = \frac{\Delta V_B}{r_{e7} + R_7}$$

The corresponding change in the collector voltages of  $Q_7$  and  $Q_8$  will be

$$\Delta v_{O2} = \Delta v_{O1} = -\Delta I_{C7}R_o$$

Now,

$$\Delta I_{C7} \simeq \Delta I_{E7}$$

and

$$R_o = R_{o7} \parallel R_{o9}$$

thus

$$\Delta v_{O1} = -\frac{\Delta V_B}{r_{e7} + R_7} (R_{o7} \parallel R_{o9})$$

This is the returned voltage, thus

$$A\beta \equiv -\frac{\Delta v_{O1}}{\Delta V_{CM}} = \frac{R_{o7} \parallel R_{o9}}{r_{e7} + R_7} \quad \text{Q.E.D.} \quad (1)$$

(b) From Example 12.4, we have

$$R_{o7} = 23 \text{ M}\Omega, R_{o9} = 12.9 \text{ M}\Omega,$$

$$r_{e7} \simeq \frac{V_T}{I_{C7}} = \frac{25 \text{ mV}}{10 \mu\text{A}} = 2.5 \text{ k}\Omega,$$

$$R_7 = 20 \text{ k}\Omega$$

thus

$$A\beta = \frac{(23 \parallel 12.9) \times 10^3}{2.5 + 20} = 367.3$$

(c) For a change  $\Delta I = 0.3 \mu\text{A}$ , the corresponding change in  $V_{CM}$  without feedback is

$$\Delta V_{CM} = \Delta I (R_{o7} \parallel R_{o9})$$

The negative feedback reduces this change by the amount of negative feedback  $1 + A\beta \simeq A\beta$ , thus the actual  $\Delta V_{CM}$  becomes

$$\Delta V_{CM} \simeq \frac{\Delta I (R_{o7} \parallel R_{o9})}{A\beta}$$

Substituting for  $A\beta$  from Eq. (1), we obtain

$$\begin{aligned} \Delta V_{CM} &= \Delta I (r_{e7} + R_7) \\ &= 0.3 \times 10^{-6} (2.5 + 20) \\ &= 6.75 \text{ mV} \end{aligned}$$

which is identical to the value found in Example 12.4.

**12.58** Refer to Fig. 12.23.

(a) A node equation at X yields

$$\begin{aligned} \frac{2I}{1 + 2/\beta_P} + \frac{2I}{\beta_P} &= I_{REF} \\ 2I \frac{\beta_P + 1 + \frac{2}{\beta_P}}{\beta_P \left(1 + \frac{2}{\beta_P}\right)} &= I_{REF} \end{aligned}$$

$$I = \frac{I_{REF}}{2} \left[ \frac{\beta_P(\beta_P + 2)}{\beta_P^2 + \beta_P + 2} \right]$$

For  $\beta_P = 50$ , we have

$$I = \frac{I_{REF}}{2} \times 1.019$$

For  $\beta_P = 20$ , we have

$$I = \frac{I_{REF}}{2} \times 1.043$$

Thus,  $I$  increases by  $\frac{I_{REF}}{2} \times 0.024$ , which is 2.4%.

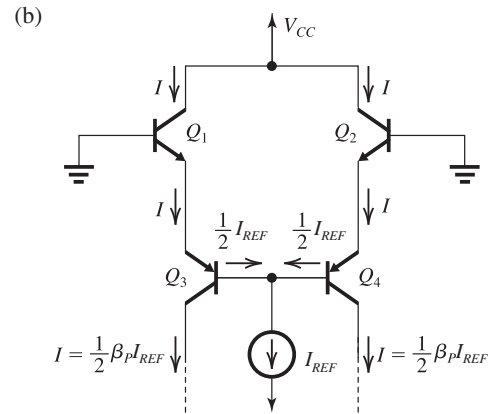


Figure 1

Figure 1 shows the suggested alternative design. As shown, here

$$I \simeq \frac{1}{2} \beta_P I_{REF}$$

For  $\beta_P = 50$ , we have

$$I = 25 I_{REF}$$

For  $\beta_P = 20$ , we have

$$I = 10 I_{REF}$$

Thus,  $I$  changes by  $-15 I_{REF}$ , which is  $-60\%$  change! This is a result of the absence of the desensitivity effect of negative feedback.

**12.59** Refer to Fig. 12.23.

For  $I_{S6} = 2I_{S5}$ , the collector current of  $Q_6$  will be

$$I_{REF} = \frac{4I}{1 + \frac{2}{\beta_P}}$$

If  $\beta_P$  is large, a node equation at X yields

$$4I \simeq I_{REF}$$

$$\Rightarrow I = \frac{1}{4}I_{REF} = \frac{19}{4} = 4.75 \mu\text{A}$$

To establish  $I_{C1} = I_{C2} = 9.5 \mu\text{A}$ , we need to design the Widlar source to provide

$I_{REF} = 38 \mu\text{A}$ . We may do so using transistors  $Q_{1-5}$  in Fig. 12.14 taking  $I_{REF}$  at the collector of  $Q_5$ . Since all resistor values and all transistor sizes are the same (except  $Q_1$ ), assuming relatively large  $\beta$ , all collector currents in the Widlar circuit are  $I_{REF}$ . From Eq. (12.72),

$$\begin{aligned} R_2 &= \frac{V_T}{I_{REF}} \ln \left( \frac{I_{S2}}{I_{S1}} \right) \\ &= \frac{0.025}{38 \times 10^{-6}} \ln 10 \\ &= 1.5 \text{ k}\Omega \end{aligned}$$

**12.60**  $2I = 40 \mu\text{A}$

Assuming

$$I_{C1} = I_{C2} = I = 20 \mu\text{A}$$

then

$$I_{B1} = \frac{20}{150} = 133 \text{ nA}$$

$$I_{B2} = \frac{20}{220} = 90.9 \text{ nA}$$

$$I_B = \frac{1}{2}(I_{B1} + I_{B2}) = 112 \text{ nA}$$

$$I_{OS} = |I_{B1} - I_{B2}| = 42.4 \text{ nA}$$

**12.61** For  $I_B = 60 \text{ nA}$ , we have

$$\beta_N \simeq \frac{9.5 \mu\text{A}}{60 \text{ nA}} = 158.3$$

$$I_{OS} = |I_{B1} - I_{B2}|$$

For  $I_{OS} = 5 \text{ nA}$ ,  $I_{B1}$  and  $I_{B2}$  must not differ by more than  $5 \text{ nA}$ , thus

$$\frac{9.5 \mu\text{A}}{\beta_{N\min}} = 62.5 \text{ nA}$$

and

$$\frac{9.5 \mu\text{A}}{\beta_{N\max}} = 57.5 \text{ nA}$$

Thus,

$$152 \leq \beta_N \leq 165$$

**12.62**

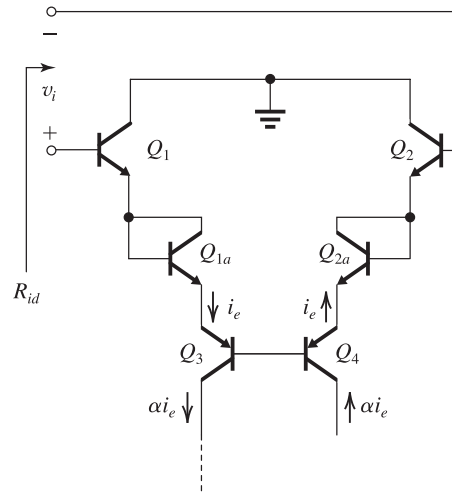


Figure 1

Figure 1 shows the input stage with the two extra diode-connected transistors  $Q_{1a}$  and  $Q_{2a}$ . Since these devices are simply in series with  $Q_1 - Q_4$ , they will have the same dc bias current, namely  $9.5 \mu\text{A}$ . Thus, each of  $Q_{1a}$  and  $Q_{2a}$  will have an incremental resistance equal to  $r_e$  of each of  $Q_1$  to  $Q_4$ ,

$$r_e = \frac{25 \text{ mV}}{9.5 \mu\text{A}} = 2.63 \text{ k}\Omega$$

The input differential resistance  $R_{id}$  now becomes

$$\begin{aligned} R_{id} &= (\beta_N + 1) \times 6r_e \\ &= 201 \times 6 \times 2.63 \\ &= 3.2 \text{ M}\Omega \end{aligned}$$

The effective transconductance of the input stage,  $G_{m1}$ , now becomes

$$\begin{aligned} G_{m1} &\equiv \frac{2\alpha i_e}{v_{id}} \\ &= \frac{2\alpha i_e}{6i_e r_e} = \frac{1}{3}g_{m1} \\ &= \frac{1}{3} \frac{9.5 \mu\text{A}}{25 \text{ mV}} = 0.13 \text{ mA/V} \end{aligned}$$

To find  $R_{o4}$ , refer to the circuit in Fig. 2.

$$R_{o4} = r_{o4}[1 + g_{m4}(2r_e \parallel r_{\pi 4})]$$

where

$$r_{o4} = \frac{50 \text{ V}}{9.5 \mu\text{A}} = 5.26 \text{ M}\Omega$$

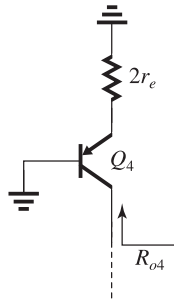


Figure 2

$$g_{m4} = 0.38 \text{ mA/V}$$

$$r_{\pi4} = \frac{50}{0.38} = 131.6 \text{ k}\Omega$$

$$R_{o4} = 5.26[1 + 0.38(5.26 \parallel 131.6)]$$

	Original Design	Modified Design
$R_{id}$ (k $\Omega$ )	2.1	3.2
$G_{m1}$ (mA/V)	0.19	0.13
$R_{o4}$ (M $\Omega$ )	10.5	15.4
$R_{o1}$ (M $\Omega$ )	6.7	8.3
$ A_{vo} $ (V/V)	1273	1083

$$= 15.4 \text{ M}\Omega$$

$$R_{o1} = R_{o4} \parallel R_{o6}$$

$$= 15.4 \parallel 18.2 = 8.33 \text{ M}\Omega$$

$$\text{Open-circuit voltage gain} = G_{m1}R_{o1}$$

$$= 0.13 \times 8.33 \times 10^3 = 1083 \text{ V/V Comparison}$$

Thus the input resistance increases but the gain decreases: The additional diodes introduce negative feedback in the input stage; same effect as adding a resistance in the emitter of a common-emitter amplifier.

**12.63** From Eq. (12.86) we have

$$\text{CMRR} = g_{m1}(R_{o6} \parallel R_{o7})/\epsilon_m$$

where

$$g_{m1} = 0.38 \text{ mA/V}$$

$$R_{o6} = 2.63 \text{ M}\Omega$$

$$R_{o7} = 31.1 \text{ M}\Omega$$

$$\epsilon_m = 1 - 0.995 = 0.005$$

Thus,

$$\text{CMRR} = 0.38(2.63 \parallel 31.3) \times 10^3/0.005$$

$$= 1.84 \times 10^5$$

or 105.3 dB

**12.64** Please note that an error occurred in the first printing of the text:  $Q_9$  is biased at  $19 \mu\text{A}$ . With a resistance  $R$  in the emitter of  $Q_9$ ,  $R_{o9}$  becomes

$$R_{o9} = r_{o9}[1 + g_{m9}(R \parallel r_{\pi9})]$$

where

$$r_{o9} = \frac{|V_{Ap}|}{I_{C9}} = \frac{50 \text{ V}}{19 \mu\text{A}} = 2.63 \text{ M}\Omega$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{19 \mu\text{A}}{0.025 \text{ V}} = 0.76 \text{ mA/V}$$

$$r_{\pi9} = \frac{\beta_P}{g_{m9}} = \frac{50}{0.76} = 65.8 \text{ k}\Omega$$

Thus, to obtain  $R_{o9} = R_{o10} = 31.1 \text{ M}\Omega$ , we use

$$31.1 = 2.63[1 + 0.76(R \parallel 65.8)]$$

$$\Rightarrow R = 18.2 \text{ k}\Omega$$

Thus,  $R_o$  to the left of node Y becomes

$$R_o = 31.1 \text{ M}\Omega \parallel 31.1 \text{ M}\Omega = 15.55 \text{ M}\Omega$$

## 12.65

Figure 1 shows the input stage with the approach suggested for determining  $G_{mcm}$ . Here

$$R_f = R_o(1 + A\beta) = R_o(1 + \beta_P) \simeq \beta_P R_o$$

A node equation at the common bases of  $Q_3$  and  $Q_4$  yields

$$\frac{2i}{\beta_P} = \frac{v_{icm}}{R_f}$$

$$\Rightarrow i = \frac{\beta_P}{2R_f} v_{icm}$$

$$= \frac{\beta_P}{2\beta_P R_o} v_{icm}$$

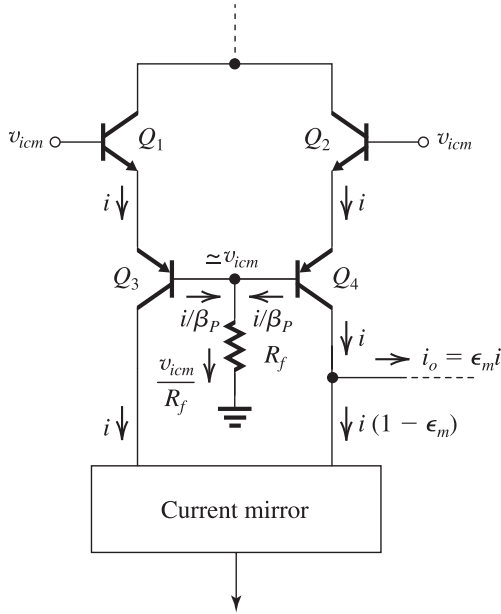


Figure 1

$$= \frac{v_{icm}}{2R_o}$$

Thus,

$$i_o = \epsilon_m i = \frac{\epsilon_m}{2R_o} v_{icm}$$

and

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{\epsilon_m}{2R_o}$$

which is the same result obtained by the alternative approach of Example 12.5.

**12.66** Refer to Fig. 12.23.

(a) A node equation at X yields

$$\frac{2I}{1 + 2/\beta_P} + \frac{2I}{\beta_P} = I_{REF}$$

$$2I \frac{\beta_P + 1 + \frac{2}{\beta_P}}{\beta_P \left(1 + \frac{2}{\beta_P}\right)} = I_{REF}$$

$$I = \frac{I_{REF}}{2} \left[ \frac{\beta_P(\beta_P + 2)}{\beta_P^2 + \beta_P + 2} \right]$$

For  $\beta_P = 50$ , we have

$$I = \frac{I_{REF}}{2} \times 1.019$$

For  $\beta_P = 20$ , we have

$$I = \frac{I_{REF}}{2} \times 1.043$$

Thus,  $I$  increases by  $\frac{I_{REF}}{2} \times 0.024$ , which is 2.4%.

(b)

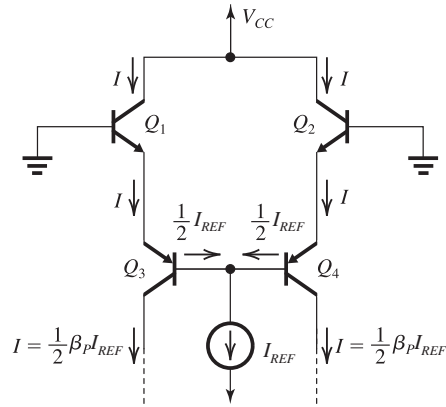


Figure 1

Figure 1 shows the suggested alternative design. As shown, here

$$I \simeq \frac{1}{2} \beta_P I_{REF}$$

For  $\beta_P = 50$ , we have

$$I = 25I_{REF}$$

For  $\beta_P = 20$ , we have

$$I = 10I_{REF}$$

Thus,  $I$  changes by  $-15I_{REF}$ , which is  $-60\%$  change! This is a result of the absence of the desensitivity effect of negative feedback.

**12.67** (a)  $v_o$  can range to within 0.1 V (the saturation voltage) of ground and  $V_{CC}$ , thus

$$0.1 \text{ V} \leq v_o \leq 2.9 \text{ V}$$

(b) For  $i_L = 0$ , the output resistance is

$$R_o = r_{oN} \parallel r_{oP}$$

where

$$r_{oN} = \frac{V_{An}}{I_Q} = \frac{30 \text{ V}}{0.6 \text{ mA}} = 50 \text{ k}\Omega$$

$$r_{oP} = \frac{|V_{Ap}|}{I_Q} = \frac{20 \text{ V}}{0.6 \text{ mA}} = 33.3 \text{ k}\Omega$$

Thus,

$$R_o = 50 \parallel 33.3 = 20 \text{ k}\Omega$$

$$(c) R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{20 \text{ k}\Omega}{1 + 10^5} \simeq 0.2 \text{ }\Omega$$

(d) For  $i_L = 12 \text{ mA}$ , we have

$$i_N = \frac{I_Q}{2} = 0.3 \text{ mA}$$

$$i_P = 12 + 0.3 = 12.3 \text{ mA}$$

$$r_{oN} = \frac{30 \text{ V}}{0.3 \text{ mA}} = 100 \text{ k}\Omega$$

$$r_{oP} = \frac{20 \text{ V}}{12.3 \text{ mA}} = 1.63 \text{ k}\Omega$$

$$R_o = 100 \parallel 1.63 = 1.6 \text{ k}\Omega$$

(e) For  $i_L = -12 \text{ mA}$ , we have

$$i_P = 0.3 \text{ mA}$$

$$i_N = 12.3 \text{ mA}$$

$$r_{oN} = \frac{30 \text{ V}}{12.3 \text{ mA}} = 2.44 \text{ k}\Omega$$

$$r_{oP} = \frac{20 \text{ V}}{0.3 \text{ mA}} = 66.7 \text{ k}\Omega$$

$$R_o = 2.44 \parallel 66.7 = 2.4 \text{ k}\Omega$$

$$\mathbf{12.68} \quad I_Q = 0.6 \text{ mA} = 600 \text{ }\mu\text{A}$$

$$I = 12 \text{ }\mu\text{A}$$

$$\frac{I_{SN}}{I_{S10}} = 8$$

$$\frac{I_{S7}}{I_{S11}} = 4$$

Using Eq. (12.93), we have

$$600 = 2 \left( \frac{I_{REF}^2}{12} \right) \times 8 \times 4$$

$$\Rightarrow I_{REF} = 10.6 \text{ }\mu\text{A}$$

The minimum current in each transistor is about 0.3 mA.

**12.69** Refer to Fig. 12.30.

$$v_{B7} = v_{BEN} = V_T \ln \left( \frac{i_N}{I_{SN}} \right) \quad (1)$$

$$i_4 = \frac{v_{EBP} - v_{EB4}}{R_4} \quad (2)$$

$$v_{B6} = v_{BES} + i_5 R_5$$

But,

$$i_5 = i_4 \text{ and } R_5 = R_4$$

thus

$$v_{B6} = v_{BES} + i_4 R_4$$

Using Eq. (2), we obtain

$$\begin{aligned} v_{B6} &= v_{BES} + v_{EBP} - v_{EB4} \\ &= (v_{BES} - v_{EB4}) + v_{EBP} \\ &= V_T \ln \left( \frac{I_{S4}}{I_{S5}} \right) + V_T \ln \left( \frac{i_P}{I_{SP}} \right) \\ &= V_T \ln \left( \frac{I_{S4} i_P}{I_{S5} I_{SP}} \right) \end{aligned} \quad (3)$$

Now, using the given relationship

$$\frac{I_{SP}}{I_{S4}} = \frac{I_{SN}}{I_{S5}}$$

in Eq. (3), we get

$$v_{B6} = V_T \ln \left( \frac{i_P}{I_{SN}} \right) \quad (4)$$

Using Eqs. (1) and (4), we obtain

$$v_{B6} - v_{B7} = V_T \ln \left( \frac{i_P}{i_N} \right)$$

This is the differential voltage input for the differential amplifier  $Q_6 - Q_7$ . Thus,

$$\begin{aligned} i_{C6} &= \frac{I}{1 + e^{(v_{B6} - v_{B7})/V_T}} \\ &= \frac{I}{1 + \frac{i_P}{i_N}} \\ &= \frac{i_N}{i_P + i_N} I \quad \text{Q.E.D.} \end{aligned}$$

Similarly,

$$\begin{aligned} i_{C7} &= \frac{I}{1 + e^{(v_{B7} - v_{B6})/V_T}} \\ &= \frac{I}{1 + \frac{i_N}{i_P}} \\ &= \frac{i_P}{i_P + i_N} I \quad \text{Q.E.D.} \end{aligned}$$

$$\mathbf{12.70} \quad v_E = v_{EB7} + v_{BE7}$$

Since  $Q_7$  conducts a current  $i_{C7}$  given by Eq. (12.88),

$$i_{C7} = I \frac{i_P}{i_P + i_N}$$

and  $Q_N$  conducts a current  $i_N$ , then

$$\begin{aligned} v_E &= V_T \ln \left( \frac{I i_P}{i_P + i_N} \frac{1}{I_{S7}} \right) + V_T \ln \left( \frac{i_N}{I_{SN}} \right) \\ &= V_T \ln \left[ \frac{i_P i_N}{i_P + i_N} \frac{I}{I_{SN} I_{S7}} \right] \quad \text{Q.E.D.} \end{aligned}$$



**Chapter 13**

*Solutions to Exercises within the Chapter*

**Ex: 13.1**  $A = -20 \log|T|$  [dB]

$$\begin{array}{cccccccc} |T| = & 1 & 0.99 & 0.9 & 0.8 & 0.7 & 0.5 & 0.1 & 0 \\ A \simeq & 0 & 0.1 & 1 & 2 & 3 & 6 & 20 & \infty \end{array}$$

**Ex: 13.2**  $A_{\max} = -20 \log 0.9 \simeq 0.9$  dB

$$A_{\min} = 20 \log \left[ \frac{1}{0.01} \right] = 40 \text{ dB}$$

**Ex: 13.3** (a)

$$T(s) = \frac{K}{s^3 + b_2s^2 + b_1s + b_0}$$

This is a low-pass filter with 3 transmission zeros at  $s = \infty$ .

(b)

$$T(s) = \frac{Ks}{s^3 + b_2s^2 + b_1s + b_0}$$

There is one transmission zero at  $s = 0$ , and two transmission zeros at  $s = \infty$ .  $\Rightarrow$  Bandpass filter.

(c)

$$T(s) = \frac{Ks^2}{s^3 + b_2s^2 + b_1s + b_0}$$

There are two transmission zeros at  $s = 0$ , and one transmission zero at  $s = \infty$ .  $\Rightarrow$  Bandpass.

(d)

$$T(s) = \frac{Ks^3}{s^3 + b_2s^2 + b_1s + b_0}$$

Three transmission zeros at  $s = 0$ .  $\Rightarrow$  Highpass.

**Ex: 13.4**

$$T(s) = k \frac{(s + j2)(s - j2)}{\left(s + \frac{1}{2} + j\sqrt{\frac{3}{2}}\right)\left(s + \frac{1}{2} - j\sqrt{\frac{3}{2}}\right)}$$

$$= k \frac{(s^2 + 4)}{s^2 + s + \frac{1}{4} + \frac{3}{4}}$$

$$= k \frac{(s^2 + 4)}{s^2 + s + 1}$$

$$T(0) = k \frac{4}{1} = 1$$

$$k = \frac{1}{4}$$

$$\therefore T(s) = \frac{1}{4} \frac{(s^2 + 4)}{s^2 + s + 1}$$

**Ex: 13.5**

$$\begin{aligned} T(s) &= a_3 \frac{s(s^2 + 4)}{(s + 0.1 + j0.8)(s + 0.1 - j0.8)} \times \\ &\quad \frac{1}{(s + 0.1 + j1.2)(s + 0.1 - j1.2)} \\ &= \frac{a_3s(s^2 + 4)}{(s^2 + 0.2s + 0.65)(s^2 + 0.2s + 1.45)} \end{aligned}$$

**Ex: 13.6**

$$T(s) = -\frac{R_2}{R_1} \frac{1}{1 + \frac{s}{\omega_0}}$$

where  $R_2/R_1 = 10$  and  $\omega_0 = 10^5$  rad/s. The  $|T|$  will be reduced by 3 dB from the value at dc, at  $\omega = \omega_0 = 10^5$  rad/s. Thus,

$$\omega_p = 10^5 \text{ rad/s}$$

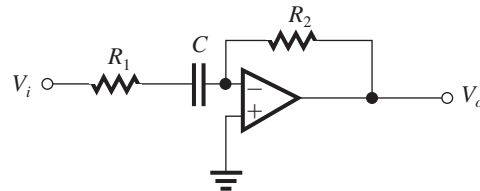
The  $|T|$  at  $\omega = \omega_s = 3\omega_0$  is

$$|T(j\omega_s)| = \frac{10}{\sqrt{1 + \left(\frac{3\omega_0}{\omega_0}\right)^2}}$$

Thus, relative to  $|T(0)|$ , the transmission will be lower by  $20 \log \sqrt{1 + 3^2}$ , which is 10 dB. Thus,

$$A(\omega_s) = 10 \text{ dB}$$

**Ex: 13.7**



$$10^4 = \frac{1}{CR_1}, R_1 = 10 \text{ k}\Omega$$

$$C = 0.01 \text{ }\mu\text{F} = 10 \text{ nF}$$

$$\text{H.F. gain} = \frac{-R_2}{R_1} = -10$$

$$R_2 = 100 \text{ k}\Omega$$

$$\text{Ex: 13.8 } T(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

For maximally flat response,  $Q = 1/\sqrt{2}$ , thus

$$T(s) = \frac{\omega_0^2}{s^2 + s\sqrt{2}\omega_0 + \omega_0^2}$$

$$T(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\sqrt{2}\omega\omega_0}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 2\omega^2\omega_0^2}}$$

$$= \frac{\omega_0^2}{\sqrt{\omega_0^4 + \omega^4}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^4}}$$

At  $\omega = \omega_0$ ,

$$|T(j\omega_0)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

which is 3 dB below the value at  $\omega = 0$  (0 dB). Q.E.D.

**Ex: 13.9**  $T(s) = \frac{sK\left(\frac{\omega_0}{Q}\right)}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

where  $K$  is the center-frequency gain. For  $\omega_0 = 10^5$  rad/s and 3-dB bandwidth =  $10^3$  rad/s, we have

$$3\text{-dB BW} = \frac{\omega_0}{Q}$$

$$10^3 = \frac{10^5}{Q}$$

$$\Rightarrow Q = 100$$

Also, for a center-frequency gain of 10, we have

$$K = 10$$

Thus,

$$T(s) = \frac{10^4 s}{s^2 + 10^3 s + 10^{10}}$$

**Ex: 13.10**

$$T(s) = k \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

For  $\omega_n = 1.2$  rad/s,  $\omega_0 = 1$  rad/s,  $Q = 5$ , and dc

$$\text{gain} = k \frac{\omega_n^2}{\omega_0^2} = 1, \text{ i.e., } k = 0.694,$$

$$T(s) = 0.694 \frac{s^2 + 1.44}{s^2 + 0.2s + 1}$$

$$T(\infty) = 0.694$$

**Ex: 13.11**  $|T(j\omega)| = 1/\sqrt{1 + \epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}$

$\omega_p = \omega_{3\text{dB}}$ ,  $\epsilon = 1$ , and  $N = 5$ ,

$$|T(j\omega)| = 1/\sqrt{1 + \left(\frac{\omega}{\omega_{3\text{dB}}}\right)^{10}}$$

$$= 1/\sqrt{1 + \left(\frac{f}{f_{3\text{dB}}}\right)^{10}}$$

$$|T(j\omega_s)| = 1/\sqrt{1 + \left(\frac{30}{10}\right)^{10}}$$

$$A(\omega_s) = -20 \log |T(j\omega_s)|$$

$$= 47.7 \text{ dB}$$

**Ex: 13.12**

$$\epsilon = \sqrt{10^{\frac{A_{\text{max}}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

$$A(\omega_s) = -20 \log |T(j\omega_s)|$$

$$= 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right]$$

Thus,

$$10 \log [1 + 0.5088^2 \times 1.5^{2N}] \geq 30$$

$$N = 10: \text{ LHS} = 29.35 \text{ dB}$$

$$N = 11: \text{ LHS} = 32.87 \text{ dB}$$

$\therefore$  Use  $N = 11$  and obtain

$$A_{\text{min}} = 32.87 \text{ dB}$$

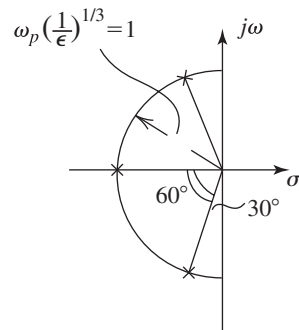
**Ex: 13.13** The real pole is at  $s = -1$

The complex conjugate poles are at

$$s = -\cos 60^\circ \pm j \sin 60^\circ$$

$$= -0.5 \pm j\sqrt{\frac{3}{2}}$$

Note that  $\omega_0 = 1$  rad/s and  $Q = 1$ .



$$T(s) = \frac{1}{(s+1)\left(s+0.5+j\sqrt{\frac{3}{2}}\right)\left(s+0.5-j\sqrt{\frac{3}{2}}\right)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$

DC gain = 1

**Ex: 13.14**

$$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{0.5}{10}} - 1} = 0.3493$$

$$A(\omega_s) = 10 \log \left[ 1 + \epsilon^2 \cosh^2 \left( N \cosh^{-1} \frac{\omega_s}{\omega_p} \right) \right]$$

$$= 10 \log [1 + 0.3493^2 \cosh^2(7 \cosh^{-1} 2)]$$

$$= 64.9 \text{ dB}$$

For  $A_{\max} = 1 \text{ dB}$ ,  $\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$

$$A(\omega_s) = 10 \log [1 + 0.5088^2 \cosh^2(7 \cosh^{-1} 2)]$$

$$= 68.2 \text{ dB}$$

This is an increase of 3.3 dB

**Ex: 13.15**  $\epsilon = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

(a) For the Chebyshev filter:

$$A(\omega_s)$$

$$= 10 \log [1 + 0.5088^2 \cosh^2(N \cosh^{-1} 1.5)]$$

$$\geq 50 \text{ dB}$$

$$N = 7.4 \therefore \text{choose } N = 8$$

Excess attenuation =

$$10 \log [1 + 0.5088^2 \cosh^2(8 \cosh^{-1} 1.5)] - 50$$

$$= 55 - 50 = 5 \text{ dB}$$

(b) For a Butterworth filter

$$\epsilon = 0.5088$$

$$A(\omega_s) = 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right]$$

$$= 10 \log (1 + 0.5088^2 (1.5)^{2N}) \geq 50$$

$$N = 15.9 \therefore \text{choose } N = 16$$

Excess attenuation =

$$10 \log \left( [1 + 0.5088^2 (1.5)^{32}] - 50 \right) = 0.5 \text{ dB}$$

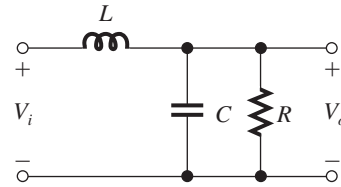
**Ex: 13.16** Maximally flat  $\Rightarrow Q = \frac{1}{\sqrt{2}}$

$$\omega_0 = 2\pi \times 100 \times 10^3$$

Arbitrarily selecting  $R = 1 \text{ k}\Omega$ , we get

$$Q = \omega_0 CR \Rightarrow C = \frac{1}{\sqrt{2} \times 2\pi 10^5 \times 10^3}$$

$$= 1125 \text{ pF}$$



Also  $Q = \frac{R}{\omega_0 L}$

$$\therefore L = \frac{R}{\omega_0 Q} = \frac{10^3}{2\pi 10^5 \times \frac{1}{\sqrt{2}}} = 2.25 \text{ mH}$$

**Ex: 13.17** Refer to Fig. 13.19(c).

$$\omega_0 = 1/\sqrt{LC} = 10^5 \text{ rad/s}$$

$$3\text{-dB BW} = \frac{\omega_0}{Q} = \frac{1}{CR} = 10^3 \text{ rad/s}$$

Selecting  $R = 100 \text{ k}\Omega$ ,

$$C = \frac{1}{10^3 R} = \frac{1}{10^3 \times 10^5} = 10^{-8} \text{ F} = 10 \text{ nF}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{10^{10} \times 10^{-8}} = 0.01 \text{ H} = 10 \text{ mH}$$

**Ex: 13.18** Refer to Fig. 13.20(a). Selecting,

$$R_1 = R_2 = R_3 = R_5 = R$$

and

$$C_4 = C$$

gives

$$Z_{in} = sCR^2$$

Thus,

$$L = CR^2$$

To obtain  $L = 100 \text{ mH}$ , we select  $C = 1 \text{ nF}$ .

Thus,

$$100 \times 10^{-3} = 1 \times 10^{-9} R^2$$

$$\Rightarrow R = 10^4 = 10 \text{ k}\Omega$$

The values of  $C$  and  $R$  are practically convenient.

**Ex: 13.19**

$$f_0 = 10 \text{ kHz}, \Delta f_{3\text{dB}} = 500 \text{ Hz}$$

$$Q = \frac{f}{\Delta f_{3\text{dB}}} = \frac{10^4}{500} = 20$$

Using Eqs. (13.53) and (13.54) and selecting,

$$C = 1.2 \text{ nF},$$

we get,

$$R = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 13.26 \text{ k}\Omega$$

$$R_6 = QR = 20 \times 13.26 = 265 \text{ k}\Omega$$

Now,

$$K = \text{center-frequency gain} = 10$$

Thus,

$$1 + r_2/r_1 = 10$$

Selecting  $r_1 = 10 \text{ k}\Omega$ , we obtain  $r_2 = 90 \text{ k}\Omega$ .

**Ex: 13.20** Refer to the KHN circuit in Fig. 13.24.

Choosing  $C = 1 \text{ nF}$ , we obtain

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 10^4 \times 10^{-9}} = 15.9 \text{ k}\Omega$$

Using Eq. (13.65) and selecting  $R_1 = 10 \text{ k}\Omega$ , we get

$$R_f = R_1 = 10 \text{ k}\Omega$$

Using Eq. (13.66) and setting  $R_2 = 10 \text{ k}\Omega$ , we obtain

$$R_3 = R_2(2Q - 1) = 10(2 \times 2 - 1) = 30 \text{ k}\Omega$$

$$\text{High-frequency gain} = K = 2 - \frac{1}{Q} = 1.5 \text{ V/V}$$

The transfer function to the output of the first integrator is

$$\frac{V_{bp}}{V_i} = -\frac{1}{sCR} \times \frac{V_{hp}}{V_i} = \frac{sK/(CR)}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Thus the center-frequency gain is given by

$$\frac{K}{CR \omega_0} = KQ = 1.5 \times 2 = 3 \text{ V/V}$$

**Ex: 13.21** Refer to Fig. 13.25 (b)

$$CR = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{2\pi 10^4 \times 10^4} = 1.59 \text{ nF}$$

$$R_d = QR = 20 \times 10 = 200 \text{ k}\Omega$$

$$\text{Center frequency gain} = KQ = 1$$

$$\therefore K = \frac{1}{Q} = \frac{1}{20}$$

$$R_g = R/K = 20R = 200 \text{ k}\Omega$$

**Ex: 13.22** Refer to Fig. 13.27(a) and Eqns. (13.80)–(13.85). Selecting

$$C_1 = C_2 = C = 1 \text{ nF},$$

and,

$$R_3 = R$$

then,

$$R_4 = \frac{R}{4Q^2} = \frac{R}{4 \times 25} = \frac{R}{100}$$

and

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 5}{10^5} = 10^{-4} \text{ s}$$

$$R = \frac{10^{-4}}{10^{-9}} = 10^5 \Omega = 100 \text{ k}\Omega$$

$$R_3 = R = 100 \text{ k}\Omega$$

$$R_4 = \frac{R}{100} = 1 \text{ k}\Omega$$

The center-frequency gain is given by Eq. (13.85),

$$K = -2Q^2 = -2 \times 25 = -50 \text{ V/V}$$

**Ex: 13.23** Refer to Fig. 13.29 and Eqns.

(13.94)–(13.101). Selecting

$$R_1 = R_2 = R$$

$$C_4 = C$$

then

$$C_3 = \frac{C}{4Q^2} = \frac{C}{4 \times \frac{1}{2}} = \frac{C}{2}$$

Thus, if we select  $C = 1 \text{ nF}$  then

$$C_4 = 1 \text{ nF}$$

$$C_3 = 0.5 \text{ nF}$$

Now,

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1/\sqrt{2}}{10^5} = 1.414 \times 10^{-5} \text{ s}$$

Thus,

$$R = \frac{1.414 \times 10^{-5}}{1 \times 10^{-9}} = 1.414 \times 10^4 = 14.14 \text{ k}\Omega$$

$$\Rightarrow R_1 = R_2 = 14.14 \text{ k}\Omega$$

Finally, from the transfer function in Eq. (13.94)

$$\text{DC gain} = 1 \text{ V/V}$$

**Ex: 13.24** From Eq. (13.108),

$$C_3 = C_4 = \omega_0 T_c C$$

$$= 2\pi 10^4 \times \frac{1}{200 \times 10^3} \times 20$$

$$= 6.283 \text{ pF}$$

From Eq. (13.110),

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = 0.314 \text{ pF}$$

From Eq. (13.111),

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = 1$$

$$C_6 = C_5 = 0.314 \text{ pF}$$

**Ex: 13.25** Pole frequency  $f_0 = 1 \text{ kHz}$

$$\begin{aligned} \text{Center-frequency gain} &= \frac{1}{\text{Amplifier gain}} \\ &= \frac{1}{2} \text{ V/V} \end{aligned}$$

**Ex: 13.26** Refer to Fig. 13.34(a).

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 10^{-3} \times 10 \times 10^{-9}}} = 10^6 \text{ rad/s}$$

The loop gain at  $\omega_0 = (1 + \frac{r_2}{r_1}) \times 1 = 1.01$ .

Thus, the condition of oscillation is satisfied.

**Ex: 13.27**  $\omega_0 = \frac{1}{\sqrt{LC}}$

(a)

$$L \rightarrow 1.01L$$

$$\omega_0 + \Delta\omega_0 = \frac{1}{\sqrt{1.01LC}} = \frac{0.995}{\sqrt{LC}}$$

Thus,

$$\frac{\Delta\omega_0}{\omega_0} \times 100 = -0.5\%$$

(b)

$$C \rightarrow 1.01C$$

$$\omega_0 + \Delta\omega_0 = \frac{1}{\sqrt{L \times 1.01C}} = \frac{0.995}{\sqrt{LC}}$$

$$\frac{\Delta\omega_0}{\omega_0} \times 100 = -0.5\%$$

(c)

$$R \rightarrow 1.01R$$

Since  $\omega_0$  is *not* a function of  $R$ , changing  $R$  has no effect on  $\omega_0$ , that is,  $\Delta\omega_0 = 0$ .

**Ex: 13.28** Refer to Fig. 13.37. Noting that the sinusoid at the positive input terminal of the op amp has a peak amplitude of 0.9 V, to obtain a 5-V peak amplitude at the output, the closed-loop gain of the op amp must be

$$1 + \frac{r_2}{r_1} = \frac{5}{0.9} = 5.6$$

$$\Rightarrow \frac{r_2}{r_1} = 4.6$$

**Ex: 13.29** (a)  $L(s) = \left(1 + \frac{R_2}{R_1}\right) \frac{Z_p}{Z_p + Z_s}$

$$= \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + Z_s Y_p}$$

$$= \left(1 + \frac{20.3}{10}\right) \frac{1}{1 + \left(R + \frac{1}{sC}\right) \left(\frac{1}{R} + sC\right)}$$

$$= \frac{3.03}{3 + sCR + \frac{1}{sCR}}$$

where  $R = 10 \text{ k}\Omega$  and  $C = 16 \text{ nF}$

Thus,

$$L(s) = \frac{3.03}{3 + s16 \times 10^{-5} + \frac{1}{s \times 16 \times 10^{-5}}}$$

The closed-loop poles are found by setting  $L(s) = 1$ ; that is, they are the values of  $s$ , satisfying

$$3 + s \times 16 \times 10^{-5} + \frac{1}{s \times 16 \times 10^{-5}} = 3.03$$

$$\Rightarrow s = \frac{10^5}{16} (0.015 \pm j)$$

(b) The frequency of oscillation is  $(10^5/16) \text{ rad/s}$  or approximately 1 kHz.

(c) Using Eq. (13.123), the positive peak amplitude can be found as

$$\begin{aligned} \hat{v}_{o+} &= \left[ \frac{1}{3} \times 15 + \left(1 + \frac{1}{3}\right) \times 0.7 \right] / \left( \frac{2}{3} - \frac{1}{3} \times \frac{1}{3} \right) \\ &= 10.68 \text{ V} \end{aligned}$$

Since the limiter circuit is symmetric, the negative peak amplitude also will be 10.68 V. Thus, the peak-to-peak amplitude of the output sinusoid will be

$$\hat{v}_{op-p} = 2 \times 10.68 = 21.36 \text{ V}$$

**Ex: 13.30** (a) For oscillations to start, we need from Eq. (13.121),  $R_2/R_1 = 2$ ; thus the potentiometer should be set so that its resistance to ground is 20 k $\Omega$ .

(b) From Eq. (13.120),

$$\begin{aligned} f_0 &= \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10 \times 10^3 \times 16 \times 10^{-9}} \\ &= 1 \text{ kHz} \end{aligned}$$

**Ex: 13.31** Figure 1 shows the circuit together with the analysis details. The current  $I$  can be found as

This figure belongs to Exercise 13.31.

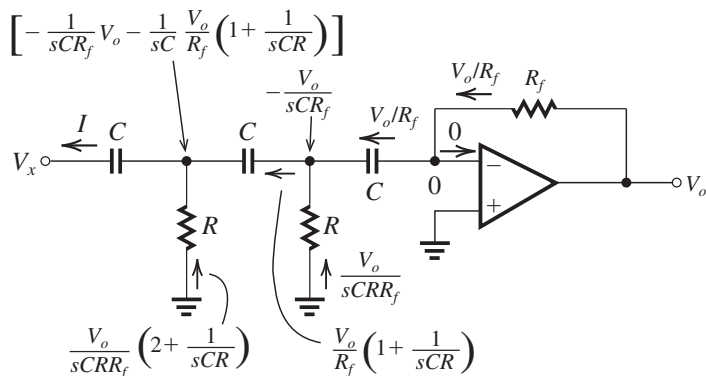


Figure 1

$$I = \frac{V_o}{R_f} \left(1 + \frac{1}{sCR}\right) + \frac{V_o}{sCRR_f} \left(2 + \frac{1}{sCR}\right)$$

Finally,  $V_x$  can be found from

$$\begin{aligned} V_x &= -\frac{1}{sCR_f} V_o \left(2 + \frac{1}{sCR}\right) - \frac{I}{sC} \\ &= -\frac{1}{sCR_f} V_o \left(2 + \frac{1}{sCR}\right) - \frac{V_o}{sCR_f} \left(1 + \frac{1}{sCR}\right) \\ &\quad - \frac{V_o}{s^2 C^2 RR_f} \left(2 + \frac{1}{sCR}\right) \\ -\frac{V_x}{V_o} &= \frac{3}{sCR_f} + \frac{4}{s^2 C^2 RR_f} + \frac{1}{s^3 C^3 R^2 R_f} \\ \frac{V_o}{V_x} &= -\frac{s^2 C^2 RR_f}{4 + s3CR + \frac{1}{sCR}} \end{aligned}$$

For  $s = j\omega$ ,

$$\frac{V_o}{V_x} = \frac{\omega^2 C^2 RR_f}{4 + j\left(3\omega CR - \frac{1}{\omega CR}\right)}$$

**Ex: 13.32** From the loop-gain expression

$$\frac{V_o}{V_x} = \frac{\omega^2 C^2 RR_f}{4 + j\left(3\omega CR - \frac{1}{\omega CR}\right)}$$

we see that the phase will be zero at

$$3\omega_0 CR = \frac{1}{\omega_0 CR}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{3}CR}$$

At this frequency, we have

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{\omega_0^2 C^2 RR_f}{4} \\ &= \frac{1}{12} \frac{R_f}{R} \end{aligned}$$

Thus the minimum value that  $R_f$  must have for oscillations to start is

$$R_f = 12R$$

Using the values in Fig. 13.42, we obtain

$$\omega_0 = \frac{1}{\sqrt{3}} \frac{1}{16 \times 10^{-9} \times 10 \times 10^3} = 3.608 \text{ krad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{3.608 \times 10^3}{2\pi} = 574.3 \text{ Hz}$$

$$R_f \geq 12 \times 10 = 120 \text{ k}\Omega$$

$$\text{Ex: 13.33 } \omega_0 = \frac{1}{CR} \Rightarrow CR = \frac{1}{2\pi 10^3}$$

For  $C = 16 \text{ nF}$ , we have  $R = 10 \text{ k}\Omega$ .

The peak-to-peak amplitude of the square wave is 1.4 V, which has a fundamental-frequency component of  $\frac{4 \times 1.4}{\pi} = 1.8 \text{ V}$  peak-to-peak. Since the output is twice as large as the voltage across the resonator, its peak-to-peak amplitude is 3.6 V.

**Ex: 13.34** Refer to Fig. 13.46(a). The admittance seen by the transistor between drain and source is

$$Y = sC_1 + \frac{1}{sL + \frac{1}{sC_2}}$$

$$= \frac{s^2 LC_1 + \frac{C_1}{C_2} + 1}{sL + \frac{1}{sC_2}}$$

$$= \frac{s^2 + 1/\left[L \frac{C_1 C_2}{C_1 + C_2}\right]}{s \frac{1}{C_1} + \frac{1}{sLC_1 C_2}}$$

$$Y(j\omega) = \frac{-\omega^2 + 1/\left[L \frac{C_1 C_2}{C_1 + C_2}\right]}{j \frac{\omega}{C_1} \left(1 - \frac{1}{\omega^2 LC_2}\right)}$$

from which we see that at  
 $\omega = \omega_0 = 1/\sqrt{L} \frac{C_1 C_2}{C_1 + C_2}$  the numerator is zero  
 and  $Y(j\omega_0) = 0$ . Q.E.D.

**Ex: 13.35** Refer to the circuit in Fig. 13.47(a).

$$\begin{aligned} R &= R_{\text{coil}} \parallel r_o \parallel R_L \\ &= \frac{Q}{\omega_0 C_1} \parallel 100 \times 10^3 \parallel 100 \times 10^3 \\ &= \frac{100}{10^6 \times 0.01 \times 10^{-6}} \parallel 10^5 \parallel 10^5 \\ &= 10^4 \parallel 10^5 \parallel 10^5 \\ &= 8.33 \text{ k}\Omega \end{aligned}$$

For sustained oscillations, we have

$$\begin{aligned} g_m R &= \frac{C_2}{C_1} \\ C_2 &= 0.01 \times 5 \times 8.33 \\ &= 0.42 \text{ }\mu\text{F} \\ \omega_0^2 L \frac{C_1 C_2}{C_1 + C_2} &= 1 \\ 10^{12} \times L \times \frac{0.01 \times 0.42}{0.01 + 0.42} \times 10^{-6} &= 1 \\ \Rightarrow L &= 102 \text{ }\mu\text{H} \end{aligned}$$

To allow oscillations to grow in amplitude, we need

$$g_m R > \frac{C_2}{C_1}$$

which can be achieved by using a somewhat smaller  $C_2$  than the value found above.

**Ex: 13.36**  $\omega_0 = 10 \text{ Grad/s} = 10 \times 10^9 = \frac{1}{\sqrt{LC}}$

$$\begin{aligned} 10^{10} &= \frac{1}{\sqrt{10 \times 10^{-9} \times C}} \\ \Rightarrow C &= 1 \text{ pF} \\ R_p &= \omega_0 L Q \\ &= 10^{10} \times 10 \times 10^{-9} \times 10 = 1000 \text{ }\Omega \\ &= 1 \text{ k}\Omega \\ g_m|_{\text{min}} &= \frac{1}{(R_p \parallel r_o)} = \frac{1}{(1 \parallel 10) \times 10^3} \\ &= 1.1 \text{ mA/V} \end{aligned}$$

**Ex: 13.37** From Eq. (13.144), we have

$$\begin{aligned} f_s &= \frac{1}{2\pi \sqrt{LC_s}} = \frac{1}{2\pi \sqrt{0.52 \times 0.012 \times 10^{-12}}} \\ &= 2.015 \text{ MHz} \end{aligned}$$

From Eq. (13.145), we have

$$\begin{aligned} f_p &= \frac{1}{2\pi \sqrt{L \frac{C_s C_p}{C_s + C_p}}} \\ &= \frac{1}{2\pi \sqrt{0.52 \times \frac{0.012 \times 4 \times 10^{-12}}{0.012 + 4}}} \\ &= 2.018 \text{ MHz} \\ Q &= \frac{\omega_0 L}{r} \cong \frac{\omega_s L}{r} \\ &= \frac{2\pi \times 2.015 \times 10^6 \times 0.52}{120} \\ &\cong 55,000 \end{aligned}$$

**Ex: 13.38**  $V_{TH} = |V_{TL}| = \beta |L|$

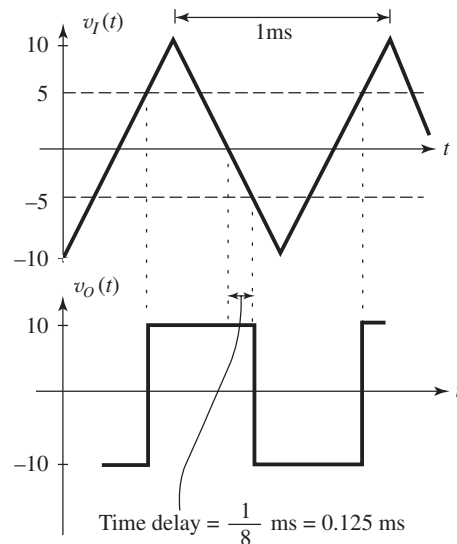
$$\begin{aligned} 5 &= \frac{R_1}{R_1 + R_2} \times 13 \\ \frac{R_2}{R_1} &= 1.6 \\ R_2 &= 16 \text{ k}\Omega \end{aligned}$$

**Ex: 13.39**  $V_{TH} = V_{TL} = \frac{R_1}{R_2} |L|$

$$\begin{aligned} 5 &= \frac{R_1}{R_2} \times 10 \\ R_2 &= 2R_1 \end{aligned}$$

Possible choice:  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 20 \text{ k}\Omega$

**Ex: 13.40**



**Ex: 13.41**

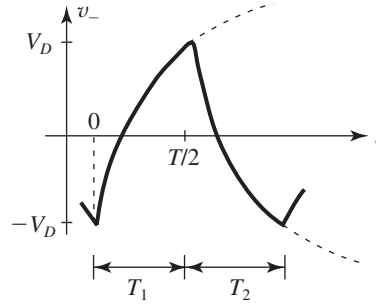
$$\beta = \frac{R_1}{R_1 + R_2} = \frac{100}{100 + 1000} = 0.091 \text{ V/V}$$

$$T = 2\tau \ln \frac{1 + \beta}{1 - \beta}$$

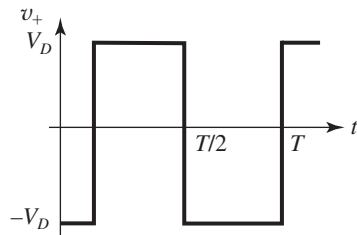
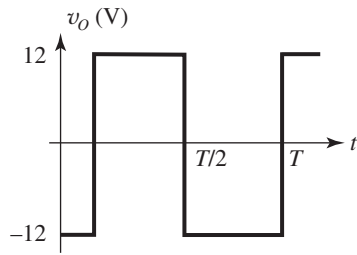
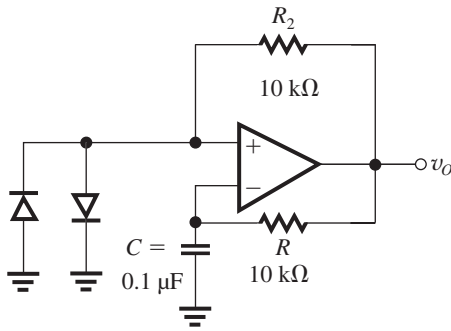
$$= 2 \times 0.01 \times 10^{-6} \times 10^6 \times \ln \left( \frac{1.091}{1 - 0.091} \right)$$

$$= 0.00365 \text{ s}$$

$$f_0 = \frac{1}{T} = 274 \text{ Hz}$$



**Ex: 13.42**



$$T_1 = T_2 = T/2$$

During  $T_1$ , we have

$$v_-(t) = 12 - (12 + V_D) e^{-t/\tau}$$

$$v_- = V_D \text{ at } t = T/2$$

$$V_D = 12 - (12 + V_D) e^{-T/2\tau}$$

$$T = 2\tau \ln \left( \frac{12 + V_D}{12 - V_D} \right)$$

$$= 2 \times 0.1 \times 10^{-6} \times 10 \times 10^3 \times \ln \left( \frac{12 + V_D}{12 - V_D} \right)$$

$$f = \frac{1}{T} = \frac{500}{\ln \left( \frac{12 + V_D}{12 - V_D} \right)} \text{ Hz}$$

$$f|_{25^\circ\text{C}} = \frac{500}{\ln \left( \frac{12.7}{11.3} \right)}$$

$$= 4281 \text{ Hz}$$

$$\text{At } 0^\circ\text{C}, V_D = 0.7 + 0.05 = 0.75 \text{ V}$$

$$f|_{0^\circ\text{C}} = \frac{500}{\ln \left( \frac{12.75}{11.25} \right)} = 3995 \text{ Hz}$$

$$\text{At } 50^\circ\text{C}, V_D = 0.7 - 0.05 = 0.65 \text{ V}$$

$$f|_{50^\circ\text{C}} = \frac{500}{\ln \left( \frac{12.65}{11.35} \right)} = 4611 \text{ Hz}$$

$$\text{At } 100^\circ\text{C}, V_D = 0.7 - 0.15 = 0.55 \text{ V}$$

$$f|_{100^\circ\text{C}} = \frac{500}{\ln \left( \frac{12.55}{11.45} \right)} = 5451 \text{ Hz}$$

**Ex: 13.43** To obtain a triangular waveform with 10-V peak-to-peak amplitude, we should have

$$V_{TH} = -V_{TL} = 5 \text{ V}$$

$$\text{But } V_{TL} = -L_+ \times \frac{R_1}{R_2}$$

$$\text{Thus, } -5 = -10 \times \frac{10}{R_2}$$

$$\Rightarrow R_2 = 20 \text{ k}\Omega$$

For 1-kHz frequency,  $T = 1 \text{ ms}$ .

Thus,

$$T/2 = 0.5 \times 10^{-3} = CR \frac{V_{TH} - V_{TL}}{L_+}$$

$$= 0.01 \times 10^{-6} \times R \times 10/10$$

$$\Rightarrow R = 50 \text{ k}\Omega$$



## Solutions to End-of-Chapter Problems

$$13.1 \quad T(s) = \frac{\omega_0}{s + \omega_0}, \quad T(j\omega) = \frac{\omega_0}{j\omega + \omega_0}$$

$$|T(j\omega)| = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\phi(\omega) \equiv \tan^{-1} \left[ \frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))} \right]$$

$$= -\tan^{-1}(\omega/\omega_0)$$

$$G = 20 \log_{10} |T(j\omega)|$$

$$A = -20 \log_{10} |T(j\omega)|$$

$\omega$	$ T(j\omega) $ [V/V]	$G$ [dB]	$A$ [dB]	$\phi$ [degrees]
0	1	0	0	0
$0.5\omega_0$	0.8944	-0.97	0.97	-26.57
$\omega_0$	0.7071	-3.01	3.01	-45.0
$2\omega_0$	0.4472	-6.99	6.99	-63.43
$5\omega_0$	0.1961	-14.1	14.1	-78.69
$10\omega_0$	0.0995	-20.0	20.0	-84.29
$100\omega_0$	0.010	-40.0	40.0	-89.43

13.2 At  $\omega = 0$ , we have

$$20 \log |T| = 0 \text{ dB}$$

$$\Rightarrow |T| = 1 \text{ V/V}$$

At  $\omega = \omega_p$ , we have

$$20 \log |T| = -A_{\max} = -0.2 \text{ dB}$$

$$\Rightarrow |T| = 0.977 \text{ V/V}$$

At  $\omega = \omega_s$ , we have

$$20 \log |T| = -A_{\min} = -60 \text{ dB}$$

$$\Rightarrow |T| = 0.001 \text{ V/V}$$

13.3 Refer to Fig. 13.3.

$$A_{\max} = 20 \log 1.05 = 0.42 \text{ dB}$$

$$A_{\min} = 20 \log \left( \frac{1}{0.0005} \right)$$

$$= 66 \text{ dB}$$

$$\text{Selectivity factor} \equiv \frac{f_s}{f_p}$$

$$= \frac{5}{4} = 1.25$$

## 13.4

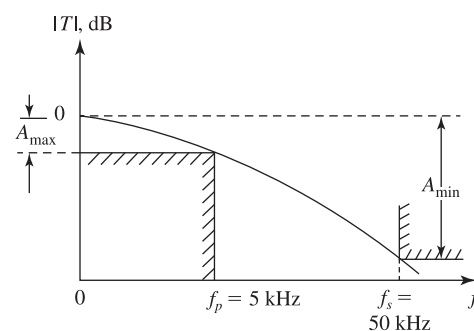


Figure 1

Refer to Fig. 1.

$$T(s) = \frac{2\pi \times 10^4}{s + 2\pi \times 10^4}$$

$$T(j\omega) = \frac{1}{1 + j\frac{\omega}{2\pi \times 10^4}}$$

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{10^4}\right)^2}}$$

At  $f = f_p = 5 \text{ kHz}$ , we have

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{5 \times 10^3}{10^4}\right)^2}} = 0.894$$

Thus,

$$A_{\max} = -20 \log 0.894 = 0.97 \text{ dB}$$

At  $f = f_s = 10 f_p = 50 \text{ kHz}$ , we have

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{50 \times 10^3}{10^4}\right)^2}} = 0.196$$

$$A_{\min} = 20 \log \left( \frac{1}{0.196} \right) = 14.15 \text{ dB}$$

13.5 See Fig. 1.

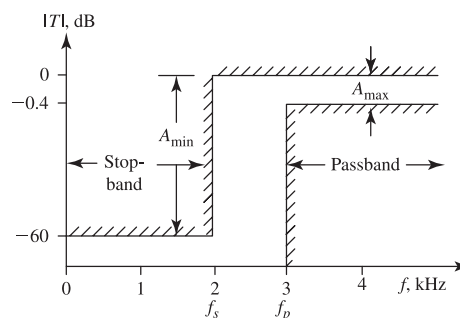


Figure 1

$$13.6 \quad T(s) = \frac{2\pi \times 10^4}{s + 2\pi \times 10^4}$$

$$T(j\omega) = \frac{2\pi \times 10^4}{2\pi \times 10^4 + j\omega}$$

$$= \frac{1}{1 + j[\omega/2\pi \times 10^4]}$$

$$|T(j\omega)| = 1 / \sqrt{1 + \left(\frac{\omega}{2\pi \times 10^4}\right)^2}$$

$$\phi(\omega) = -\tan^{-1}(\omega/2\pi \times 10^4)$$

(a)  $f = 1$  kHz

$$\omega = 2\pi \times 10^3 \text{ rad/s}$$

$$|T| = 1/\sqrt{1 + 0.01} \simeq 0.995 \text{ V/V}$$

$$\phi = -\tan^{-1}(0.1) = -5.7^\circ$$

Peak amplitude of output sinusoid = 0.995 V

Phase of output relative to that of input =  $-5.7^\circ$ .

(b)  $f = 10$  kHz

$$\omega = 2\pi \times 10^4 \text{ rad/s}$$

$$|T| = 1/\sqrt{2} = 0.707 \text{ V/V}$$

$$\phi = -\tan^{-1}(1) = -45^\circ$$

Peak amplitude of output sinusoid = 0.707 V

Phase of output relative to that of input =  $-45^\circ$ .

(c)  $f = 100$  kHz

$$\omega = 2\pi \times 10^5 \text{ rad/s}$$

$$|T| = 1/\sqrt{1 + 100} \simeq 0.1 \text{ V/V}$$

$$\phi = -\tan^{-1}(10) = -84.3^\circ$$

Peak amplitude of output sinusoid = 0.1 V

Phase of output relative to that of input =  $-84.3^\circ$ .

(d)  $f = 1$  MHz

$$\omega = 2\pi \times 10^6 \text{ rad/s}$$

$$|T| = 1/\sqrt{1 + 10^4} = 0.01 \text{ V/V}$$

$$\phi = -\tan^{-1}(100) = -89.4^\circ$$

Peak amplitude of output sinusoid = 0.01 V

Phase of output relative to that of input =  $-89.4^\circ$ .

$$13.7 \quad T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$T(j\omega) = [j(2\omega - \omega^3) + (1 - 2\omega^2)]^{-1}$$

$$|T(j\omega)| = [(2\omega - \omega^3)^2 + (1 - 2\omega^2)^2]^{-\frac{1}{2}}$$

$$= [4\omega^2 - 4\omega^4 + \omega^6 + 1 - 4\omega^2 + 4\omega^4]^{-\frac{1}{2}}$$

$$= [1 + \omega^6]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{1 + \omega^6}} \quad \text{Q.E.D.}$$

For phase angle:

$$\phi(\omega) = \tan^{-1} \left[ \frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))} \right]$$

$$= -\tan^{-1} \left[ \frac{2\omega - \omega^3}{1 - 2\omega^2} \right]$$

For  $\omega = 0.1$  rad/s:

$$|T(j\omega)| = (1 + 0.1^6)^{-1/2} \simeq 1$$

For  $\omega = 0.2$  rad/s,

$$|T(j\omega)| = (1 + 0.2^6)^{-1/2} \simeq 1$$

For  $\omega = 0.5$  rad/s,

$$|T(j\omega)| = (1 + 0.5^6)^{-1/2} = 0.9923$$

For  $\omega = 1$  rad/s,

$$|T(j\omega)| = (1 + 1^6)^{-1/2} = 0.7071$$

For  $\omega = 2$  rad/s,

$$|T(j\omega)| = (1 + 2^6)^{-1/2} = 0.1240$$

For  $\omega = 5$  rad/s,

$$|T(j\omega)| = (1 + 5^6)^{-1/2} = 0.008$$

For  $\omega = 10$  rad/s,

$$|T(j\omega)| = (1 + 10^6)^{-1/2} = 0.001$$

$$13.8 \quad \tau = 1 \text{ s} \Rightarrow \omega_0 = \frac{1}{\tau} = 1 \text{ rad/s}$$

Thus,

$$T(s) = \frac{1}{s+1}$$

where we have also used the given information on the dc transmission being unity.

$$|T| = \frac{1}{\sqrt{1 + \omega^2}}$$

Since  $A_{\max} = 2$  dB, we have

$$20 \log|T(j\omega_p)| = -2 \text{ dB}$$

$$\Rightarrow |T(j\omega_p)| = 0.794$$

Thus,

$$\frac{1}{\sqrt{1 + \omega_p^2}} = 0.794$$

$$\Rightarrow \omega_p = 0.765 \text{ rad/s}$$

Since  $A_{\min} = 12 \text{ dB}$ , we have

$$20 \log|T(j\omega_s)| = -12 \text{ dB}$$

$$\Rightarrow |T(j\omega_s)| = 0.25$$

Thus,

$$\frac{1}{\sqrt{1 + \omega_s^2}} = 0.25$$

$$\Rightarrow \omega_s = 3.85 \text{ rad/s}$$

$$\text{Selectivity factor} = \frac{\omega_s}{\omega_p} = \frac{3.85}{0.765} = 5$$

13.9 See Fig. 1.

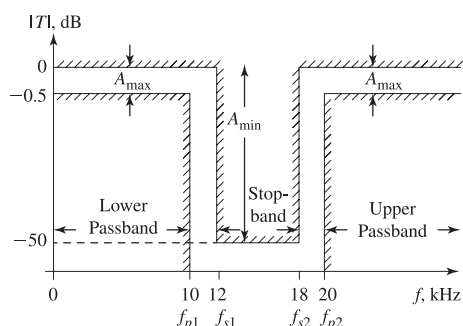


Figure 1

13.10 This is a third-order low-pass filter. Its transfer function is given by

$$T(s) = \frac{k(s^2 + 4)}{(s + 1)(s + 0.5 - j1) \times (s + 0.5 + j1)}$$

$$= \frac{k(s^2 + 4)}{(s + 1)(s^2 + s + 1.25)}$$

$$T(0) = \frac{4k}{1.25} = 1$$

$$\Rightarrow k = \frac{1.25}{4} = 0.3125$$

Thus,

$$T(s) = \frac{0.3125(s^2 + 4)}{(s + 1)(s^2 + s + 1.25)}$$

13.11

Figure 1 shows the location of the five poles in the  $s$  plane. To find  $\omega_0$  and  $Q$  of the two pairs of complex poles, we refer to Fig. 13.9 which shows the definition of  $\omega_0$  and  $Q$ . Now, for the  $(p_1, p_1^*)$  pair,

$$\omega_{01} = 10^5 \text{ rad/s}$$

$$\frac{\omega_{01}}{2Q_1} = 10^5 \cos 72^\circ$$

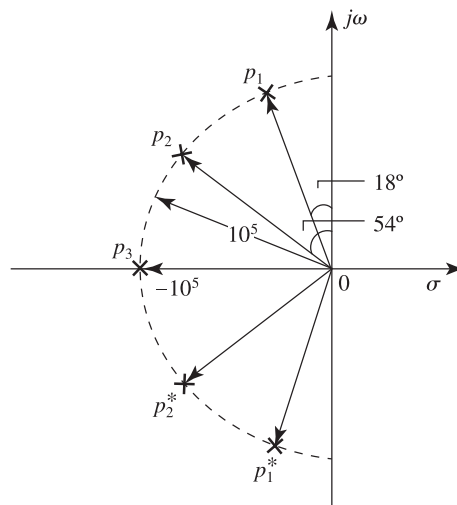


Figure 1

$$\Rightarrow \frac{10^5}{2Q_1} = 10^5 \times 0.309$$

$$\Rightarrow Q_1 = 1.618$$

For the pair  $(p_2, p_2^*)$ ,

$$\omega_{02} = 10^5 \text{ rad/s}$$

$$\frac{\omega_{02}}{2Q_2} = 10^5 \cos 36^\circ$$

$$\Rightarrow Q_2 = 0.618$$

The real-axis pole gives rise to the denominator factor  $(s + \omega_0)$ ; that is,  $(s + 10^5)$ . Thus, the denominator of  $T(s)$  can be written as

$$D(s) = (s + \omega_0) \left(s^2 + s \frac{\omega_0}{Q_1} + \omega_0^2\right) \left(s^2 + s \frac{\omega_0}{Q_2} + \omega_0^2\right)$$

where  $\omega_0 = 10^5 \text{ rad/s}$ ,  $Q_1 = 1.618$ , and  $Q_2 = 0.618$ .

(a) If all the transmission zeros are at  $s = \infty$  and the dc gain is unity,

$$T(s) = \frac{\omega_0^5}{(s + \omega_0) \left(s^2 + s \frac{\omega_0}{Q_1} + \omega_0^2\right) \left(s^2 + s \frac{\omega_0}{Q_2} + \omega_0^2\right)}$$

(b) If all the transmission zeros are at  $s = 0$  and  $T(\infty) = 1$ ,

$$T(s) = \frac{s^5}{(s + \omega_0) \left(s^2 + s \frac{\omega_0}{Q_1} + \omega_0^2\right) \left(s^2 + s \frac{\omega_0}{Q_2} + \omega_0^2\right)}$$

13.12 Since there are four poles (and four zeros) this is a fourth-order ( $N = 4$ ) filter. The complex-conjugate pole pair with  $\omega_0 = 10^3 \text{ rad/s}$  and  $Q = 1$  gives rise to the denominator factor  $(s^2 + s 10^3 + 10^6)$ .

The complex-conjugate pole pair with  $\omega_0 = 1.2 \times 10^3$  and  $Q = 12$  gives rise to the denominator factor

$$\left( s^2 + s \frac{1.2 \times 10^3}{12} + 1.44 \times 10^6 \right)$$

Now, since there are two zeros at  $s = 0$  and two zeros at  $s = \infty$ , the transfer function is

$$T(s) = \frac{ks^2}{(s^2 + s 10^3 + 10^6)(s^2 + s 10^2 + 1.44 \times 10^6)}$$

The value of  $k$  can be determined from

$$|T(j10^3)| = 1$$

Now,

$$T(j\omega) = \frac{-k\omega^2}{[(10^6 - \omega^2) + j10^3\omega][(1.44 \times 10^6 - \omega^2) + j10^2\omega]}$$

$$T(j10^3) = \frac{-k \times 10^6}{(j10^6)[(0.44 \times 10^6) + j10^5]}$$

$$|T(j10^3)| = \frac{k}{\sqrt{(0.44 \times 10^6)^2 + 10^{10}}}$$

Forcing this to equal one, gives

$$k = 4.512 \times 10^5$$

**13.13** Refer to Fig. 13.8.

Input resistance =  $R_1$

Thus, to obtain an input resistance of 10 k $\Omega$ , we select

$$R_1 = 10 \text{ k}\Omega$$

$$|\text{DC gain}| = 10 = \frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 10 R_1 = 100 \text{ k}\Omega$$

$$CR_2 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 10 \times 10^3}$$

$$\Rightarrow C = \frac{1}{2\pi \times 10 \times 10^3 \times 100 \times 10^3} = 159.2 \text{ pF}$$

**13.14** Refer to Fig. E13.7.

H.F. Input resistance =  $R_1$

Thus,

$$R_1 = 100 \text{ k}\Omega$$

$$CR_1 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 100}$$

$$\Rightarrow C = \frac{1}{2\pi \times 100 \times 100 \times 10^3}$$

$$= 15.92 \text{ nF}$$

$$\text{High-frequency gain} = -\frac{R_2}{R_1} = -1$$

$$\Rightarrow R_2 = R_1 = 120 \text{ k}\Omega$$

$$\mathbf{13.15} \quad T(s) = \frac{k\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

For a maximally flat response,  $Q = 1/\sqrt{2}$ , for a dc gain of 10,  $k = 10$ , and for  $\omega_0 = 10^4$ , we obtain

$$T(s) = \frac{10^9}{s^2 + s 1.414 \times 10^4 + 10^8}$$

$$\mathbf{13.16} \quad T(s) = \frac{s^2}{s^2 + s \frac{2\pi \times 10^4}{2.5} + (2\pi \times 10^4)^2}$$

$$\mathbf{13.17} \quad T(s) = \frac{k s \left( \frac{\omega_0}{Q} \right)}{s^2 + s \left( \frac{\omega_0}{Q} \right) + \omega_0^2}$$

where  $\omega_0 = 2\pi \times 10^5$ ,

$$Q = \frac{\omega_0}{BW} = \frac{2\pi \times 10^5}{2\pi \times 2000} = 50$$

$k$  = center-frequency gain = 5

Thus,

$$T(s) = \frac{s \times 5 \times 2\pi \times 2000}{s^2 + s \times 2\pi \times 2000 + (2\pi \times 10^5)^2}$$

$$= \frac{6.283 \times 10^4 s}{s^2 + 1.257 \times 10^4 s + 3.948 \times 10^{11}}$$

$$\mathbf{13.18} \quad T(s) = \frac{k[s^2 + (1.25 \times 10^4)^2]}{s^2 + s \frac{10^4}{2} + 10^8}$$

where  $k$  is found from,

$$T(0) = 1 = \frac{k \times 1.25^2 \times 10^8}{10^8}$$

$$\Rightarrow k = 0.64$$

$$T(s) = \frac{0.64(s^2 + 1.5625 \times 10^8)}{s^2 + 5 \times 10^3 s + 10^8}$$

$$T(\infty) = 0.64$$

**13.19**

Refer to Fig. 1 (next page).

$$V_+ = V_i \frac{1/sC}{R + (1/sC)}$$

$$= V_i \frac{1}{sCR + 1}$$

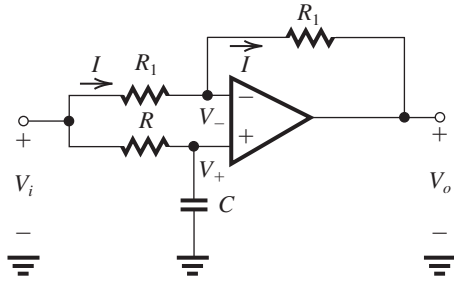


Figure 1

$$V_- = V_+ = V_i \frac{1}{sCR + 1}$$

$$I = \frac{V_i - V_-}{R_1} = \frac{V_i}{R_1} \left( 1 - \frac{1}{sCR + 1} \right)$$

$$= \frac{V_i}{R_1} \frac{sCR}{sCR + 1}$$

$$V_o = V_- - IR_1$$

$$= \frac{V_i}{sCR + 1} - \frac{sRCV_i}{sCR + 1}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{sCR - 1}{sCR + 1}$$

or

$$T(s) = \frac{V_o}{V_i} = -\frac{s - 1/CR}{s + 1/CR} = -\frac{s - \omega_0}{s + \omega_0}$$

where

$$\omega_0 = \frac{1}{CR}$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$= \frac{\omega_0 - j\omega}{\omega_0 + j\omega}$$

$$= \frac{1 - j(\omega/\omega_0)}{1 + j(\omega/\omega_0)}$$

$$|T(j\omega)| = \sqrt{\frac{1 + (\omega/\omega_0)^2}{1 + (\omega/\omega_0)^2}} = 1$$

$$\phi(\omega) = -2 \tan^{-1} \left( \frac{\omega}{\omega_0} \right)$$

$$\Rightarrow \frac{\omega}{\omega_0} = \tan \left[ -\frac{1}{2} \phi(\omega) \right] \quad (1)$$

Thus, for a given phase shift  $\phi$ , we can use Eq. (1) to determine  $(\omega/\omega_0)$ . For  $\omega = 5 \times 10^3$  rad/s, we can then determine the required value of  $\omega_0$ . Finally, for  $C = 10$  nF, the required value for  $R$  can be found from

$$R = \frac{1}{\omega_0 \times 10 \times 10^{-9}} = \frac{10^8}{\omega_0}$$

The results obtained are as follows:

$\phi$	$-30^\circ$	$-60^\circ$	$-90^\circ$	$-120^\circ$	$-150^\circ$
$\omega/\omega_0$	0.268	0.577	1	1.732	3.732
$\omega_0$ (krad/s)	18.66	8.66	5	2.89	1.34
$R$ (k $\Omega$ )	5.36	11.55	20	34.60	74.63

$$13.20 \quad T(s) = \frac{k(s^2 + 4)}{(s + 0.25 + j)(s + 0.25 - j)}$$

$$= \frac{k(s^2 + 4)}{s^2 + 0.5s + 1.0625}$$

$$T(0) = \frac{4k}{1.0625} = 1$$

$$\Rightarrow k = \frac{1.0625}{4} = 0.2656$$

$$T(s) = \frac{0.2656(s^2 + 4)}{s^2 + 0.5s + 1.0625}$$

$$T(\infty) = 0.2656$$

### 13.21

$$T(s) = \frac{s(s^2 + 10^6)(s^2 + 9 \times 10^6)}{s^6 + b_5s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0}$$

Note that we started with the numerator factors (which represent the given transmission zeros).

We used the fact that there is one transmission zero at  $s = \infty$  to write the denominator sixth-order polynomial. Thus,

$$N = 6$$

A sketch of the magnitude response,  $|T|$ , is given in Fig. 1.

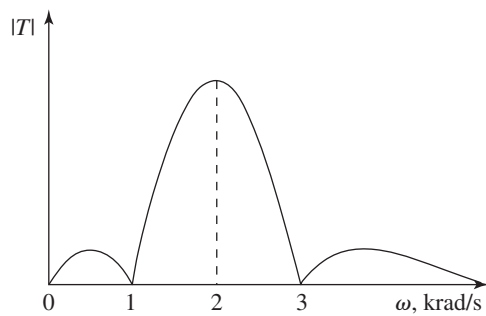


Figure 1

### 13.22

Refer to Fig. 1 (next page).

$$I_1 = \frac{V_o}{1} = V_o$$

$$I_2 = sCV_o = s \times 1 \times V_o = sV_o$$

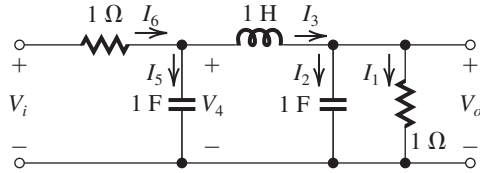


Figure 1

$$\begin{aligned}
 I_3 &= I_1 + I_2 = (s + 1)V_o \\
 V_4 &= V_o + sLI_3 \\
 &= V_o + s \times 1(s + 1)V_o \\
 &= (s^2 + s + 1)V_o \\
 I_5 &= s \times 1 \times V_4 \\
 &= s(s^2 + s + 1)V_o \\
 I_6 &= I_3 + I_5 = (s + 1)V_o + s(s^2 + s + 1)V_o \\
 &= (s^3 + s^2 + 2s + 1)V_o \\
 V_i &= V_4 + I_6 \times 1 \\
 &= (s^2 + s + 1)V_o + (s^3 + s^2 + 2s + 1)V_o \\
 &= (s^3 + 2s^2 + 3s + 2)V_o \\
 \frac{V_o(s)}{V_i(s)} &= \frac{1}{s^3 + 2s^2 + 3s + 2}
 \end{aligned}$$

All the transmission zeros are at  $s = \infty$ . To find the poles, we have to factor the third-order denominator polynomial. Toward this end, we find by inspection that one of the zeros of the denominator polynomial is at  $s = -1$ . Thus, the polynomial will have a factor  $(s + 1)$  and can be written as

$$s^3 + 2s^2 + 3s + 2 = (s + 1)(s^2 + as + b)$$

where by equating corresponding terms on both sides we find that

$$b = 2$$

$$a + 1 = 2 \Rightarrow a = 1$$

Thus,

$$s^3 + 2s^2 + 3s + 2 = (s + 1)(s^2 + s + 2)$$

and the poles are at

$$s = -1$$

and at the roots of

$$s^2 + s + 2 = 0$$

which are

$$s = \frac{-1 \pm \sqrt{1 - 8}}{2}$$

$$= -0.5 \pm j\left(\frac{\sqrt{7}}{2}\right)$$

$$= -0.5 \pm j1.323$$

**13.23** Figure 1 (on the next page) shows the bandpass filter realized as the cascade of a first-order low-pass filter and a first-order high-pass filter. The component values are determined as follows:

$$R_{in} = R_1$$

To make  $R_{in}$  as large as possible while satisfying the constraint that no resistance is larger than 100 k $\Omega$ , we select

$$R_1 = 100 \text{ k}\Omega$$

The low-frequency gain of the low-pass circuit is  $(-R_2/R_1)$ . With  $R_1 = 100 \text{ k}\Omega$ , the maximum gain obtained is unity and is achieved by selecting

$$R_2 = 100 \text{ k}\Omega$$

This implies that the required gain of 12 dB or 4 V/V must be all realized in the high-pass circuit.

The upper 3-dB frequency of the bandpass filter is the 3-dB frequency of the low-pass circuit, that is,

$$50 \times 10^3 = \frac{1}{2\pi C_1 R_2}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 50 \times 10^3 \times 100 \times 10^3}$$

$$= 31.8 \text{ pF}$$

Next, we consider the high-pass circuit. The high-frequency ( $f \gg 50 \text{ Hz}$ ) gain of this circuit is  $(-R_4/R_3)$ . To obtain a gain of  $-4 \text{ V/V}$ , we select

$$R_4 = 100 \text{ k}\Omega$$

$$R_3 = \frac{100}{4} = 25 \text{ k}\Omega$$

The lower 3-dB frequency of the bandpass filter (50 Hz) is the 3-dB frequency of the high-pass circuit, thus

$$50 = \frac{1}{2\pi C_2 R_3}$$

$$\Rightarrow C_2 = \frac{1}{2\pi \times 50 \times 25 \times 10^3}$$

$$= 0.127 \text{ }\mu\text{F}$$

**13.24** (a) Refer to the op-amp-RC circuit in Fig. P13.24. Assuming an ideal op amp, we have

$$T(s) = \frac{V_o}{V_i} = -\frac{Z_2(s)}{Z_1(s)}$$

Since both  $Z_1$  and  $Z_2$  have a parallel structure, it is far more convenient to work in terms of  $Y_1(s)$  and  $Y_2(s)$ , thus

$$T(s) = -\frac{Y_1(s)}{Y_2(s)}$$

This figure belongs to Problem 13.23.

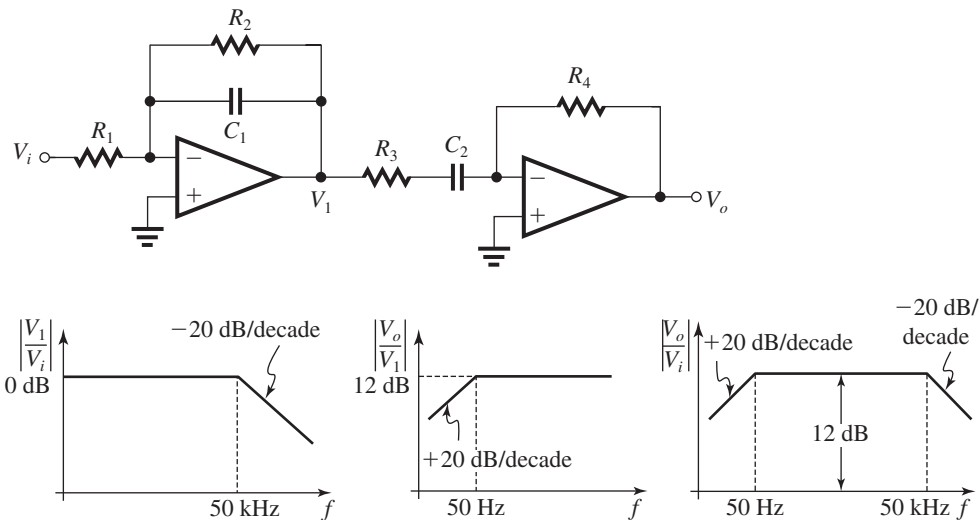


Figure 1

$$= -\frac{\frac{1}{R_1} + s C_1}{\frac{1}{R_2} + s C_2}$$

$$= -\left(\frac{C_1}{C_2}\right) \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{C_2 R_2}}$$

Thus,

$$\omega_z = \frac{1}{C_1 R_1}$$

$$\omega_p = \frac{1}{C_2 R_2}$$

$$\text{DC gain} = T(0) = -\frac{R_2}{R_1}$$

$$\text{HF gain} = T(\infty) = -\frac{C_1}{C_2}$$

(b) Low-frequency input resistance =  $R_1$

Thus,

$$R_1 = 10 \text{ k}\Omega$$

$$\text{DC gain} = -\frac{R_2}{R_1} = -1$$

$$\Rightarrow R_2 = R_1 = 10 \text{ k}\Omega$$

$$f_z = \frac{\omega_z}{2\pi} = \frac{1}{2\pi C_1 R_1}$$

$$100 = \frac{1}{2\pi C_1 \times 10 \times 10^3}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 10^6} = 0.16 \text{ }\mu\text{F}$$

$$f_p = \frac{1}{2\pi C_2 R_2}$$

$$10 \times 10^3 = \frac{1}{2\pi C_2 \times 10 \times 10^3}$$

$$C_2 = \frac{1}{2\pi \times 10^8} = 1.6 \text{ nF}$$

Figure 1 shows a sketch of the magnitude of the transfer function. Note that the magnitude of the high-frequency gain is  $C_1/C_2 = 100$  or 40 dB.

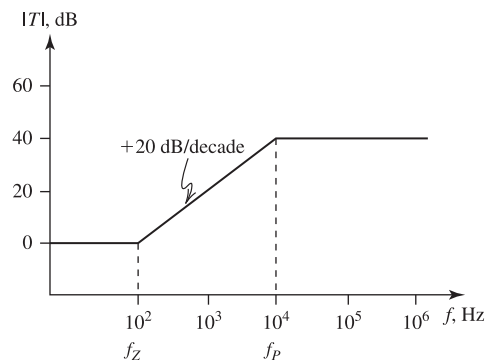


Figure 1

13.25 Refer to Fig. 13.10.

$$T(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{10^8}{s^2 + 5000 s + 10^8}$$

$$\omega_{\max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

$$= 10^4 \sqrt{1 - \frac{1}{2 \times 4}} = 9354 \text{ rad/s}$$

Since the dc gain is unity, we have

$$|a_0/\omega_0^2| = 1$$

$$|T_{\max}| = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} = \frac{2}{\sqrt{1 - \frac{1}{16}}} = 2.066$$

**13.26** There are many possible solutions. However, we will investigate only the two whose  $|T|$  are shown in Fig. 1.

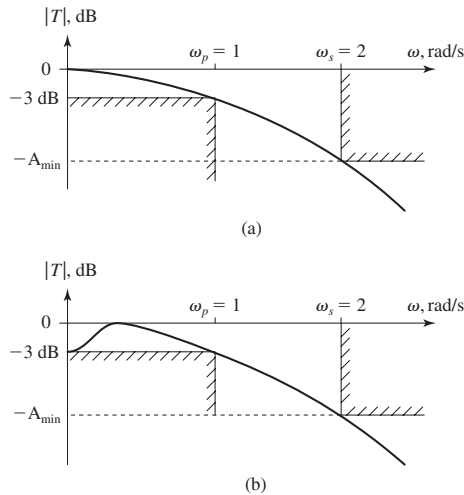


Figure 1

(a) Here the response is "maximally flat" and is obtained when  $Q = 1/\sqrt{2}$ ,

$$T(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

In this case  $\omega_0 = \omega_{3dB} = \omega_p = 1 \text{ rad/s}$ . Thus,

$$T(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$|T(j\omega)| = \frac{1}{\sqrt{(1 - \omega^2)^2 + 2\omega^2}}$$

$$= \frac{1}{\sqrt{1 + \omega^4}}$$

Thus, at the stopband edge,  $\omega = \omega_s = 2 \text{ rad/s}$ ,

$$|T(j\omega_s)| = \frac{1}{\sqrt{1 + 2^4}} = 0.2425$$

Thus,  $A_{\min}$  is

$$A_{\min} = -20 \log |T(j\omega_s)| = -20 \log 0.2425$$

$$= 12.3 \text{ dB}$$

(b) In this case the passband response is equiripple.

$$T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{dc gain} = \frac{a_0}{\omega_0^2}$$

But from Fig. 1(b),

$$\text{dc gain} = \frac{1}{\sqrt{2}}, \text{ thus}$$

$$\frac{a_0}{\omega_0^2} = \frac{1}{\sqrt{2}} \Rightarrow a_0 = 0.707 \omega_0^2$$

We can determine  $Q$  by noting from Fig. 13.10 that at the peak,

$$|T| = \frac{|a_0| Q}{\omega_0^2 \sqrt{1 - \frac{1}{4Q^2}}} = \frac{0.707Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

From Fig. 1(b) we see that at the peak,  $|T| = 1$ , thus

$$\frac{0.707Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 1$$

Solving this equation results in

$$Q = 1.3$$

Finally, we can determine  $\omega_0$  by forcing the condition that

$$|T(j1)| = 0.707$$

which can be seen from Fig. 1(b). Thus,

$$T(s) = \frac{0.707 \omega_0^2}{s^2 + s \frac{\omega_0}{1.3} + \omega_0^2}$$

$$|T(j1)| = \frac{0.707 \omega_0^2}{\sqrt{(\omega_0^2 - 1) + \frac{\omega_0^2}{1.69}}} = 0.707$$

Solving this equation results in

$$\omega_0 = 0.8427$$

The value of  $A_{\min}$  can be found as follows:

$$|T(j\omega_s)| = |T(j2)| = \frac{0.707 \omega_0^2}{\sqrt{(\omega_0^2 - 4)^2 + \frac{4\omega_0^2}{1.69}}}$$

substituting  $\omega_0 = 0.8427$  gives

$$|T(j2)| = 0.14$$

$$A_{\min} = -20 \log |T(j2)| = 17 \text{ dB}$$

**13.27** For a maximally flat response, we have

$$Q = 1/\sqrt{2}$$

and

$$\omega_{3dB} = \omega_0$$

Thus,



$$\omega_0 = 1 \text{ rad/s}$$

If the high-frequency gain is unity, then we have

$$a_2 = 1$$

and

$$T(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

The two zeros are at  $s = 0$ . The poles are complex conjugate and given by

$$\begin{aligned} s &= -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{\left(1 - \frac{1}{4Q^2}\right)} \\ &= -\frac{1}{2 \times \frac{1}{\sqrt{2}}} \pm j \sqrt{\left(1 - \frac{1}{4 \times \frac{1}{2}}\right)} \\ &= -\frac{1}{\sqrt{2}} \pm j \frac{1}{\sqrt{2}} \\ &= -0.707 \pm j0.707 \end{aligned}$$

**13.28** Poles are at  $-0.5 \pm j\sqrt{3}/2$ . Thus,

$$\omega_0 = \sqrt{0.5^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1 \text{ rad/s}$$

$$\frac{\omega_0}{2Q} = 0.5$$

$$\Rightarrow Q = \frac{\omega_0}{2 \times 0.5} = \frac{1}{1} = 1$$

High-frequency gain =  $a_2 = 1$ .

$$T(s) = \frac{s^2}{s^2 + s + 1}$$

**13.29** (a) A second-order bandpass filter with a center-frequency gain of unity (arbitrary) has the transfer function

$$T(s) = \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Thus,

$$\begin{aligned} |T(j\omega)| &= \frac{\omega\omega_0/Q}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\omega_0/Q)^2}} \\ &= 1/\sqrt{1 + Q^2 \frac{(\omega_0^2 - \omega^2)^2}{\omega^2\omega_0^2}} \end{aligned} \quad (1)$$

From Fig. 1 we see that at each  $|T|$  (below the peak value) there are two frequencies  $\omega_1 < \omega_0$  and  $\omega_2 > \omega_0$  with the same  $|T|$ . The relationship between  $\omega_1$  and  $\omega_2$  can be determined by considering the second term in the denominator of Eq. (1), as follows:

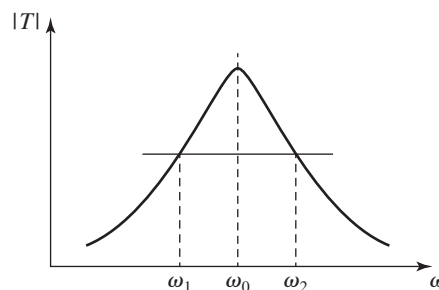


Figure 1

$$\frac{\omega_0^2 - \omega_1^2}{\omega_1\omega_0} = \frac{\omega_2^2 - \omega_0^2}{\omega_2\omega_0}$$

Cross multiplying and collecting terms results in

$$\omega_1\omega_2 = \omega_0^2 \quad \text{Q.E.D.}$$

(b) Refer to Fig. 2. Since the two edges of the passband must be geometrically symmetric around  $\omega_0$ , we have

$$\begin{aligned} \omega_0 &= \sqrt{\omega_{P1}\omega_{P2}} \\ &= \sqrt{8100 \times 10,000} \\ &= 9000 \text{ rad/s} \end{aligned}$$

The  $Q$  factor can now be found from

$$\begin{aligned} 3\text{-dB BW} &= \frac{\omega_0}{Q} \\ 10,000 - 8100 &= \frac{9000}{Q} \\ \Rightarrow Q &= \frac{9000}{1900} = 4.74 \end{aligned}$$

The geometric symmetry of  $|T|$  enables us to find  $\omega_{s2}$  from

$$\begin{aligned} \omega_{s1}\omega_{s2} &= \omega_0^2 \\ \Rightarrow \omega_{s2} &= \frac{(9000)^2}{3000} = 27,000 \text{ rad/s} \end{aligned}$$

Using Eq. (1), we obtain

$$\begin{aligned} A_{\min} &= A(\omega_{s1}) = 10 \log \left[ 1 + Q^2 \frac{(\omega_0^2 - \omega_{s1}^2)^2}{\omega_{s1}^2 \omega_0^2} \right] \\ &= 10 \log \left[ 1 + 4.74^2 \frac{(9000^2 - 3000^2)^2}{3000^2 \times 9000^2} \right] \\ &= 22.1 \text{ dB} \end{aligned}$$

Figure 2 (next page) shows a sketch of  $|T|$ .

**13.30** (a)

$$\begin{aligned} T(s) &= a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2} \\ |T(j\omega)| &= |a_2| \frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}} \end{aligned}$$

This figure belongs to Problem 13.29, part (b).

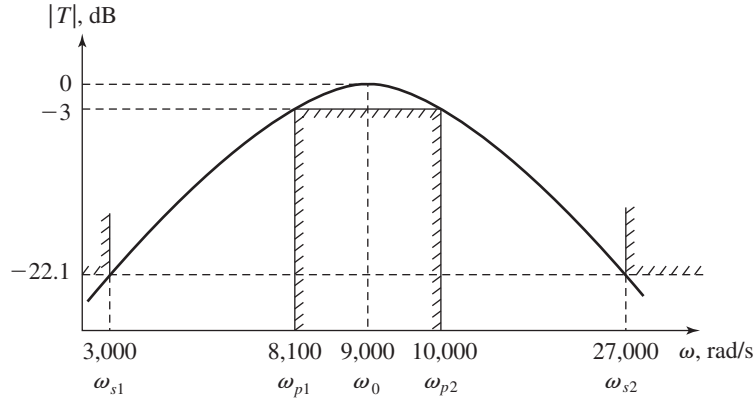


Figure 2

$$= |a_2| / \sqrt{1 + \frac{\omega^2 \omega_0^2}{Q^2(\omega_0^2 - \omega^2)^2}} \quad (1)$$

Refer to Fig. 1 and note that at any value of  $|T|$  there are two frequencies,  $\omega_1$  and  $\omega_2$ , with this gain value. From Eq. (1)

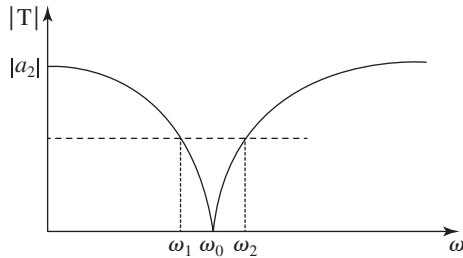


Figure 1

$$\frac{\omega_1^2 \omega_0^2}{Q^2(\omega_0^2 - \omega_1^2)^2} = \frac{\omega_2^2 \omega_0^2}{Q^2(\omega_2^2 - \omega_0^2)^2}$$

$$\Rightarrow \frac{\omega_1}{\omega_0^2 - \omega_1^2} = \frac{\omega_2}{\omega_2^2 - \omega_0^2}$$

$$\omega_1 \omega_2^2 - \omega_1 \omega_0^2 = \omega_2 \omega_0^2 - \omega_1^2 \omega_2$$

$$\omega_1 \omega_2 (\omega_1 + \omega_2) = (\omega_1 + \omega_2) \omega_0^2$$

$$\Rightarrow \omega_1 \omega_2 = \omega_0^2 \quad \text{Q.E.D.}$$

Now if  $\omega_1$  and  $\omega_2$  differ by  $BW_a$ ,

$$\omega_2 - \omega_1 = BW_a$$

and if the attenuation over this band of frequencies is to be greater than  $A$  dB then using Eq. (1), we have

$$10 \log_{10} \left[ 1 + \frac{\omega_0^2 \omega_1^2}{(\omega_0^2 - \omega_1^2)^2 Q^2} \right] \geq A$$

$$\frac{\omega_1 \omega_0}{(\omega_0^2 - \omega_1^2) Q} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q} \frac{\omega_1 \omega_0}{(\omega_1 \omega_2 - \omega_1^2)} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q} \frac{\omega_0}{\omega_2 - \omega_1} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q} \frac{\omega_0}{BW_a} \geq \sqrt{10^{A/10} - 1}$$

$$Q \leq \frac{\omega_0}{BW_a \sqrt{10^{A/10} - 1}} \quad \text{Q.E.D.}$$

(b)

$$\omega_0 = 2\pi \times 60 \text{ rad/s}$$

$$BW_a = 2\pi \times 6 \text{ rad/s}$$

$$A = 20 \text{ dB}$$

Thus,

$$Q = \frac{60}{6\sqrt{10^2 - 1}} \approx 1$$

Thus,

$$T(s) = \frac{s^2 + (2\pi \times 60)^2}{s^2 + s \frac{(2\pi \times 60)}{1} + (2\pi \times 60)^2}$$

$$13.31 \quad T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_n = 1 \text{ rad/s} \quad \omega_0 = 1.3 \text{ rad/s} \quad Q = 3$$

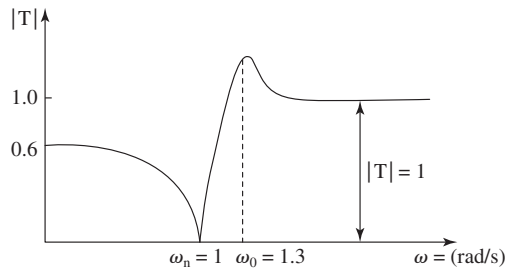


Figure 1

$a_2 = \text{high-frequency gain} = 1$

Thus,

$$T(s) = \frac{s^2 + 1}{s^2 + 0.433s + 1.69}$$

$$T(0) = \frac{1}{1.69} = 0.591$$

A sketch of  $|T|$  is shown in Fig. 1 above.

**13.32** For an  $N$ th-order Butterworth filter, we have from Eq. (13.19)

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

At the 3-dB frequency  $\omega_{3\text{dB}}$  we have:

$$\epsilon^2 \left(\frac{\omega_{3\text{dB}}}{\omega_p}\right)^{2N} = 1 \Rightarrow \omega_{3\text{dB}} = \left(\frac{1}{\epsilon^2}\right)^{1/2N} \omega_p. \text{ Thus,}$$

from Eq. (13.23) the attenuation at  $\omega = 1.8\omega_{3\text{dB}}$  is:

$$\begin{aligned} A &= 10 \log \left[ 1 + \epsilon^2 \left(\frac{1.8\omega_{3\text{dB}}}{\omega_p}\right)^{2N} \right] \\ &= 10 \log \left[ 1 + \epsilon^2 \left(1.8 \frac{1}{\epsilon^2}\right)^{1/2N} \right]^{2N} \\ &= 10 \log(1 + 1.8^{2N}) \end{aligned}$$

For the case  $N = 7$ ,

$$A = 10 \log(1 + 1.8^{14}) = 35.7 \text{ dB}$$

**13.33**  $A_{\text{max}} = 0.5 \text{ dB}$ ,  $N = 5$ ,  $\omega_p = 10^3 \text{ rad/s}$

Using Eq. (13.22), we obtain

$$\begin{aligned} \epsilon &= \sqrt{10^{A_{\text{max}}/10} - 1} = \sqrt{10^{0.05} - 1} \\ &= 0.3493 \end{aligned}$$

The natural modes can be determined by reference to Fig. 13.15(a):

$$\begin{aligned} \omega_0 &= \omega_p \left(\frac{1}{\epsilon}\right)^{1/N} \\ \omega_0 &= 10^3 \times \left(\frac{1}{0.3493}\right)^{1/5} \\ &= 1.234 \times 10^3 \text{ rad/s} \\ p_1, p_1^* &= \omega_0 \left[ \sin\left(\frac{\pi}{10}\right) \pm j \cos\left(\frac{\pi}{10}\right) \right] \\ &= \omega_0 (-0.309 \pm j0.951) \\ &= 1.234 \times 10^3 (0.309 \pm j0.951) \\ p_2, p_2^* &= \omega_0 \left[ -\sin\left(\frac{3\pi}{10}\right) \pm j \cos\left(\frac{3\pi}{10}\right) \right] \\ &= 1.234 \times 10^3 (-0.809 \pm j0.588) \\ p_3 &= -\omega_0 = -1.234 \times 10^3 \end{aligned}$$

**13.34**  $f_p = 10 \text{ kHz}$ ,  $A_{\text{max}} = 3 \text{ dB}$ ,  
 $f_s = 20 \text{ kHz}$ ,  $A_{\text{min}} = 20 \text{ dB}$

Using Eq. (13.22), we obtain

$$\begin{aligned} \epsilon &= \sqrt{10^{A_{\text{max}}/10} - 1} \\ &= \sqrt{10^{3/10} - 1} = 1 \end{aligned}$$

Using Eq. (13.23), we have

$$A(\omega_s) = 10 \log[1 + \epsilon^2 (\omega_s/\omega_p)^{2N}]$$

Thus,

$$\begin{aligned} A_{\text{min}} &\geq 10 \log[1 + \epsilon^2 (\omega_s/\omega_p)^{2N}] \\ 20 &\geq 10 \log(1 + 2^{2N}) \end{aligned}$$

For  $N = 3$ ,

$$10 \log(1 + 2^6) = 18.1 \text{ dB}$$

For  $N = 4$ ,

$$10 \log(1 + 2^8) = 24.1 \text{ dB}$$

Thus,

$$N = 4$$

The poles can be determined by reference to Fig. 13.15(a):

$$\omega_0 = \omega_p \left(\frac{1}{\epsilon}\right)^{1/4} = \omega_p = 2\pi \times 10^4 \text{ rad/s}$$

$$\begin{aligned} p_1, p_1^* &= \omega_0 \left[ -\sin\left(\frac{\pi}{8}\right) \pm j \cos\left(\frac{\pi}{8}\right) \right] \\ &= 2\pi \times 10^4 (-0.383 \pm j0.924) \end{aligned}$$

$$\begin{aligned} p_2, p_2^* &= \omega_0 \left[ -\sin\left(\frac{3\pi}{8}\right) \pm j \cos\left(\frac{3\pi}{8}\right) \right] \\ &= 2\pi \times 10^4 (-0.924 \pm j0.383) \end{aligned}$$

Thus,

$$T(s) = \frac{\omega_0^4}{(s^2 + 0.765\omega_0s + \omega_0^2)(s^2 + 1.848\omega_0s + \omega_0^2)}$$

where  $\omega_0 = 2\pi \times 10^4$  and where we have assumed the dc gain to be unity.

Using Eq. (13.19), we obtain

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_p}\right)^8}}$$

At  $f = 30 \text{ kHz} = 3f_p$ , the attenuation is

$$\begin{aligned} A &= -20 \log|T| = 10 \log(1 + 3^8) \\ &= 38.2 \text{ dB} \end{aligned}$$

**13.35**

(a) See Fig. 1(a).

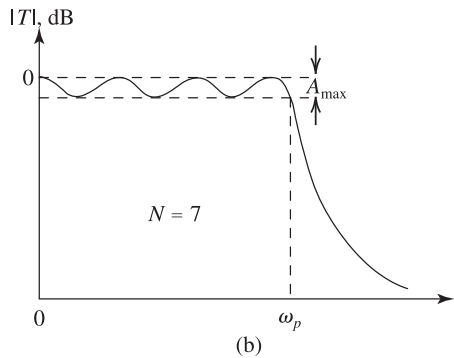
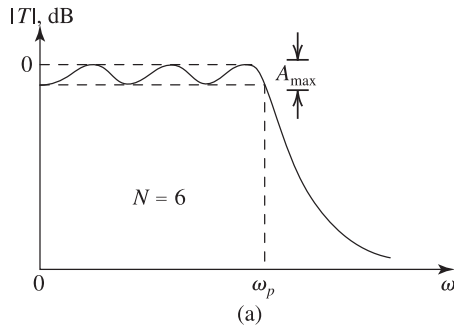


Figure 1

(b) See Fig. 1(b).

**13.36**

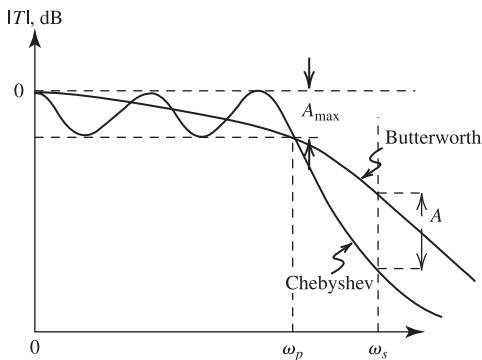


Figure 1

From Fig. 1 we see that at the stopband edge,  $\omega_s$ , the Chebyshev filter provides  $A$ (dB) greater attenuation than the Butterworth filter of the same order and having the same  $A_{max}$ .

**13.37**  $N = 5, A_{max} = 1$  dB

$$\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$$

Using Eq. (13.32),

$$A(\omega_s) = 10 \log [1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\omega_s/\omega_p))]$$

$$= 10 \log [1 + 0.5088^2 \cosh^2(5 \cosh^{-1}2)]$$

$$= 45.3 \text{ dB}$$

**13.38**

$$A_{max} = 1 \text{ dB} \Rightarrow \epsilon = \sqrt{10^{1/10} - 1} = 0.5088$$

$$f_p = 3.4 \text{ kHz}, f_s = 4 \text{ kHz} \Rightarrow \frac{f_s}{f_p} = 1.176$$

$$A_{min} = 35 \text{ dB}$$

(a) To obtain the required order  $N$ , we use Eq. (13.32),

$$A(\omega_s) = 10 \log [1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\omega_s/\omega_p))]$$

Thus,

$$10 \log [1 + 0.5088^2 \cosh^2(N \cosh^{-1}1.176)] \geq 35$$

We attempt various values for  $N$  as follows:

$N$	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	39.0 dB

Use  $N = 10$ .

$$\text{Excess attenuation} = 39 - 35 = 4 \text{ dB}$$

(b) The poles can be determined using Eq. (13.33), namely

$$p_k/\omega_p = -\sin\left(\frac{2k-1}{N} \frac{\pi}{2}\right) \sinh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right)$$

$$+j \cos\left(\frac{2k-1}{N} \frac{\pi}{2}\right) \cosh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right)$$

$$k = 1, 2, \dots, N$$

First we determine

$$\sinh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right)$$

$$= \sinh\left(\frac{1}{10} \sinh^{-1} \frac{1}{0.5088}\right) = 0.1433$$

and

$$\cosh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right) = 1.0102$$

Thus,

$$p_1/\omega_p = -0.1433 \sin\left(\frac{\pi}{20}\right) + j1.0102 \cos\left(\frac{\pi}{20}\right)$$

$$= -0.0224 + j0.9978$$

$$p_2/\omega_p = -0.1433 \sin\left(\frac{3\pi}{20}\right) + j1.0102 \cos\left(\frac{3\pi}{20}\right)$$

$$= -0.0651 + j0.9001$$

$$p_3/\omega_p = -0.1433 \sin\left(\frac{5\pi}{20}\right) + j1.0102 \cos\left(\frac{5\pi}{20}\right)$$

$$= -0.1013 + j0.7143$$

$$p_4/\omega_p = -0.1433 \sin\left(\frac{7\pi}{20}\right) + j1.0102 \cos\left(\frac{7\pi}{20}\right)$$

$$= -0.1277 + j0.4586$$

$$p_5/\omega_p = -0.1433 \sin\left(\frac{9\pi}{20}\right) + j1.0102 \cos\left(\frac{9\pi}{20}\right)$$

$$= -0.1415 + j0.1580$$

$$p_6 = p_5^*, p_7 = p_4^*, p_8 = p_3^*, p_9 = p_2^*, p_{10} = p_1^*$$

Each pair of complex conjugate poles,

$$p_k, p_k^* = \omega_p(-\Sigma_k \pm j\Omega_k)$$

gives rise to a quadratic factor in the denominator of  $T(s)$  given by

$$s^2 + s\omega_p(2\Sigma_k) + \omega_p^2(\Sigma_k^2 + \Omega_k^2)$$

where

$$\omega_p = 2\pi \times 3.4 \times 10^3 \text{ rad/s}$$

Thus, we obtain for the five pole pairs:

$$p_1, p_1^*: (s^2 + s 0.0448\omega_p + 0.9961\omega_p^2)$$

$$p_2, p_2^*: (s^2 + s 0.1302\omega_p + 0.8144\omega_p^2)$$

$$p_3, p_3^*: (s^2 + s 0.2026\omega_p + 0.5205\omega_p^2)$$

$$p_4, p_4^*: (s^2 + s 0.2554\omega_p + 0.2266\omega_p^2)$$

$$p_5, p_5^*: (s^2 + s 0.2830\omega_p + 0.0450\omega_p^2)$$

The transfer function  $T(s)$  can now be written as

$$T(s) = \frac{B}{\text{Product of five quadratic terms}}$$

The value of  $B$  determines the required dc gain, specifically

$$\begin{aligned} \text{DC gain} &= \frac{B}{\omega_p^{10} \times 0.9961 \times 0.8144 \times 0.5205 \times 0.2266 \times 0.0450} \\ &= \frac{B}{4.31 \times 10^{-3} \omega_p^{10}} \end{aligned}$$

$$\text{For DC gain} = \frac{1}{\sqrt{1 + \epsilon^2}} = \frac{1}{\sqrt{1 + 0.5088^2}}$$

$$= 0.891$$

we select

$$\begin{aligned} B &= 4.31 \times 10^{-3} \times \omega_p^{10} \times 0.891 \\ &= 3.84 \times 10^{-3} \times (2\pi \times 3.4 \times 10^3)^{10} \\ &= 7.60 \times 10^{40} \end{aligned}$$

$$\mathbf{13.39} \quad A_{\max} = 0.5 \text{ dB}, \quad A_{\min} \geq 20 \text{ dB}, \quad \frac{\omega_s}{\omega_p} = 1.7$$

Using Eq. (13.31), we obtain

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1}$$

$$= \sqrt{10^{0.05} - 1} = 0.3493$$

Using Eq. (13.32), we have

$$A(\omega_s) = 10 \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$

$$= 10 \log[1 + 0.3493^2 \times 1.7^{2N}]$$

$$\text{For } N = 5, \quad A(\omega_s) = 14.08 \text{ dB}$$

$$\text{For } N = 6, \quad A(\omega_s) = 18.58 \text{ dB}$$

$$\text{For } N = 7, \quad A(\omega_s) = 23.15 \text{ dB}$$

Thus, to meet the  $A_s \geq 20 \text{ dB}$  specification, we use

$$N = 7$$

in which case the actual minimum stopband attenuation realized is

$$A_{\min} = 23.15 \text{ dB}$$

If  $A_{\min}$  is to be exactly 20 dB, we can use Eq. (13.32) to obtain the new value of  $\epsilon$  as follows:

$$20 = 10 \log[1 + \epsilon^2 \times 1.7^{14}]$$

$$\epsilon = \sqrt{\frac{100 - 1}{1.7^{14}}} = 0.2425$$

Now, using Eq. (13.30) we can determine the value to which  $A_{\max}$  can be reduced as

$$A_{\max} = 20 \log\sqrt{1 + \epsilon^2}$$

$$A_{\max} = 20 \log\sqrt{1 + 0.2425^2}$$

$$= 0.25 \text{ dB}$$

**13.40** Using Eq. (13.32), we have

$$A(\omega_s) = 10 \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$

For large  $A(\omega_s)$ , we can neglect the unity term in this expression to obtain

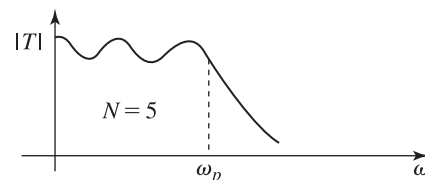
$$A(\omega_s) \simeq 10 \log[\epsilon^2(\omega_s/\omega_p)^{2N}]$$

Substituting  $A(\omega_s) \geq A_{\min}$  we obtain

$$20 \log \epsilon + 20N \log(\omega_s/\omega_p) \geq A_{\min}$$

$$\Rightarrow N \geq \frac{A_{\min} - 20 \log \epsilon}{20 \log(\omega_s/\omega_p)} \quad \text{Q.E.D.}$$

**13.41**



$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2 \left[ N \cos^{-1} \left( \frac{\omega}{\omega_p} \right) \right]}}$$

for  $\omega < \omega_p$ .

Peaks are obtained when

$$\cos^2 \left[ N \cos^{-1} \left( \frac{\hat{\omega}}{\omega_p} \right) \right] = 0$$

$$\cos^2 \left[ 5 \cos^{-1} \left( \frac{\hat{\omega}}{\omega_p} \right) \right] = 0$$

$$5 \cos^{-1} \left( \frac{\hat{\omega}}{\omega_p} \right) = (2k + 1) \frac{\pi}{2}, \quad k = 0, 1, 2$$

$$\therefore \hat{\omega} = \omega_p \cos \left[ \frac{(2k + 1) \pi}{10} \right], \quad k = 0, 1, 2$$

$$\hat{\omega}_1 = \omega_p \cos \left( \frac{\pi}{10} \right) = 0.95 \omega_p$$

$$\hat{\omega}_2 = \omega_p \cos \left( \frac{3}{10} \pi \right) = 0.59 \omega_p$$

$$\hat{\omega}_3 = \omega_p \cos \left( \frac{5}{10} \pi \right) = 0$$

Valleys are obtained when

$$\cos^2 \left[ N \cos^{-1} \left( \frac{\check{\omega}}{\omega_p} \right) \right] = 1$$

$$5 \cos^{-1} \left( \frac{\check{\omega}}{\omega_p} \right) = k\pi, \quad k = 0, 1, 2$$

$$\therefore \check{\omega} = \omega_p \cos \left( \frac{k\pi}{5} \right), \quad k = 0, 1, 2$$

$$\check{\omega}_1 = \omega_p \cos 0 = \omega_p$$

$$\check{\omega}_2 = \omega_p \cos \frac{\pi}{5} = 0.81 \omega_p$$

$$\check{\omega}_3 = \omega_p \cos \frac{2\pi}{5} = 0.31 \omega_p$$

**13.42** (a) Consider first the Butterworth filter.

For the given  $A_{\max} = 1$  dB we find the corresponding value of  $\epsilon$  using Eq. (13.31) as

$$\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$$

Next we use Eq. (13.32) to determine the attenuation at  $\omega_s = 2\omega_p$ , as

$$\begin{aligned} A(\omega_s) &= 10 \log_{10} (1 + 0.5088^2 \times 2^{12}) \\ &= 30.3 \text{ dB} \end{aligned}$$

(b) We next consider the Chebyshev filter. Its  $\epsilon$  is the same as that for the Butterworth filter, that is,

$$\epsilon = 0.5088$$

The attenuation at  $\omega_s = 2\omega_p$  can be determined using Eq. (13.32) as

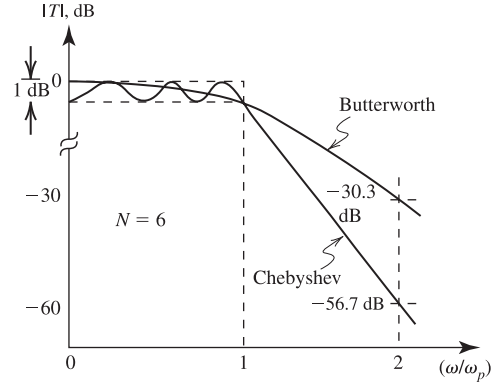


Figure 1

$$\begin{aligned} A(\omega_s) &= 10 \log [1 + 0.5088^2 \cosh^2 (6 \cosh^{-1} 2)] \\ &= 56.7 \text{ dB} \end{aligned}$$

which is nearly twice as much as the attenuation provided by the Butterworth filter. A sketch of the transmission of both filter types is shown in Fig. 1.

Note that we have expanded the vertical scale to show the details of the passband.

**13.43** Refer to Fig. 13.18(c). Using the voltage divider rule, we obtain

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{Z_{RC}}{Z_{RC} + sL} \\ &= \frac{1}{1 + sLY_{RC}} \\ &= \frac{1}{1 + sL \left( sC + \frac{1}{R} \right)} \end{aligned}$$

$$\begin{aligned} T(s) &= \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 LC + s \frac{L}{R} + 1} \\ &= \frac{1/LC}{s^2 + s \frac{1}{CR} + \frac{1}{LC}} \end{aligned}$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad [\text{which is Eq. (13.39)}]$$

$$\frac{\omega_0}{Q} = \frac{1}{CR} \Rightarrow Q = \omega_0 CR$$

[which is Eq. (13.40)]      Q.E.D.

**13.44**  $Q = \omega_0 CR$

$$5 = 10^5 \times C \times 10 \times 10^3$$

$$\Rightarrow C = 5 \text{ nF}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow L = 1/\omega_0^2 C$$

$$L = \frac{1}{10^{10} \times 5 \times 10^{-9}} = 20 \text{ mH}$$

**13.45** Refer to Fig. 1. Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{LR}}{Z_{LR} + \frac{1}{sC}}$$

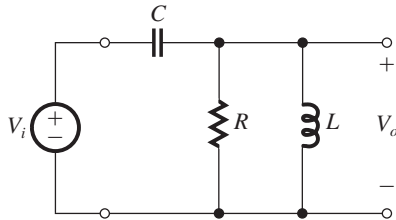


Fig. 1

$$= \frac{1}{1 + \frac{1}{sC} Y_{LR}}$$

$$= \frac{1}{1 + \frac{1}{sC} \left( \frac{1}{R} + \frac{1}{sL} \right)}$$

$$= \frac{1}{1 + \frac{1}{sCR} + \frac{1}{s^2 LC}}$$

$$= \frac{s^2}{s^2 + s \frac{1}{CR} + \frac{1}{LC}}$$

which is a high-pass function with

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

and

$$\frac{\omega_0}{Q} = \frac{1}{CR}$$

$$\Rightarrow Q = \frac{\omega_0}{CR}$$

The expressions for  $\omega_0$  and  $Q$  are identical to those in Eqns. (13.39) and (13.40), respectively.

For  $\omega_0 = 10^6$  rad/s and  $Q = 1$ , selecting  $R = 1 \text{ k}\Omega$  gives

$$1 = 10^6 \times C \times 10^3$$

$$\Rightarrow C = 1 \text{ nF}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$= \frac{1}{10^{12}} \times 10^{-9} = 1 \text{ mH}$$

**13.46** Refer to the circuit in Fig. 13.19(e) and let  $L_2 = \infty$ . Using the voltage divider rule gives

$$\frac{V_o}{V_i} = \frac{Z_{R,C_2}}{Z_{L_1,C_1} + Z_{R,C_2}}$$

$$= \frac{Y_{L_1,C_1}}{Y_{L_1,C_1} + Y_{R,C_2}}$$

$$= \frac{sC_1 + \frac{1}{sL_1}}{sC_1 + \frac{1}{sL_1} + sC_2 + \frac{1}{R}}$$

$$= \frac{s^2 + \frac{1}{L_1 C_1}}{s^2 \left(1 + \frac{C_2}{C_1}\right) + s \frac{1}{C_1 R} + \frac{1}{L_1 C_1}}$$

$$= \frac{C_1}{C_1 + C_2} \frac{s^2 + \frac{1}{L_1 C_1}}{s^2 + s \frac{1}{(C_1 + C_2)R} + \frac{1}{L_1(C_1 + C_2)}}$$

which is the transfer function of a second-order notch filter with

$$\omega_n = \frac{1}{\sqrt{L_1 C_1}} \quad (1)$$

$$\omega_0 = \frac{1}{\sqrt{L_1(C_1 + C_2)}} \quad (2)$$

$$Q = \omega_0(C_1 + C_2)R \quad (3)$$

$$\text{High-frequency gain} = \frac{C_1}{C_1 + C_2} \quad (4)$$

$$\text{DC gain} = 1 \quad (5)$$

From Eqns. (1) and (2) we see that  $\omega_n > \omega_0$

verifying that this is a low-pass notch (LPN) circuit. For the given required values and selecting  $R = 1 \text{ k}\Omega$  we obtain from Eq. (3),

$$2 = 10^5 \times (C_1 + C_2) \times 10^3$$

$$\Rightarrow C_1 + C_2 = 20 \text{ nF}$$

Dividing Eq. (1) by Eq. (2) and squaring the result gives

$$\frac{C_1 + C_2}{C_1} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

Thus,

$$\frac{C_1 + C_2}{C_1} = 2$$

$$\Rightarrow C_1 = \frac{1}{2} (C_1 + C_2) = 10 \text{ nF}$$

and

$$C_2 = 10 \text{ nF}$$

Finally, we can use Eq. (1) to determine the required value of  $L_1$  as

$$L_1 = \frac{1}{\omega_n^2 C_1} = \frac{1}{2 \times 10^{10} \times 10 \times 10^{-9}}$$

$$\Rightarrow L_1 = 5 \text{ mH}$$

**13.47** Refer to the circuit in Fig. 13.19(c). Since, at the center frequency, the  $LC$  circuit acts as an open circuit, the circuit reduces to  $R$ . We can change the gain to 0.5, by splitting  $R$  into two equal parts, each equal to  $2R$ . The voltage divider ( $2R$ ,  $2R$ ) will provide at  $\omega_0$ , a gain of 0.5. The  $Q$  factor will be determined by the parallel equivalent of  $2R$  and  $2R$ . Since the latter is equivalent to  $R$ ,  $Q$  will remain unchanged, as required. The resulting circuit is shown in Fig. 1.

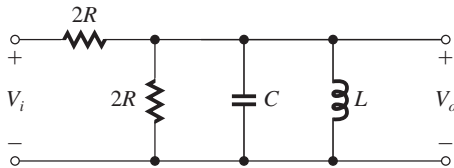


Figure 1

**13.48** (a) If  $L$  became  $1.01L$ , we obtain

$$\omega_0 = \frac{1}{\sqrt{1.01LC}} \simeq \frac{1}{1.005\sqrt{LC}} \simeq \frac{0.995}{\sqrt{LC}}$$

Thus,  $\omega_0$  decreases by 0.5%.

(b) If  $C$  increases by 1%, similar analysis as in

(a) results in  $\omega_0$  decreasing by 0.5%.

(c)  $\omega_0$  is independent of the value of  $R \Rightarrow$  no change.

**13.49** (a) As  $\omega$  reaches 0 (dc), we get

$$\frac{V_o}{V_i} = \frac{1/sC_2}{\frac{1}{sC_1} + \frac{1}{sC_2}} = \frac{C_1}{C_1 + C_2}$$

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

As  $\omega$  reaches  $\infty$ ,

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

No transmission zeros are introduced.

$$(b) \frac{V_o}{V_i} = \frac{Z_{R,C_2}}{Z_{R,C_2} + \frac{1}{sC_1}}$$

$$= \frac{1}{1 + \frac{1}{sC_1} Y_{R,C_2}}$$

$$= \frac{1}{1 + \frac{1}{sC_1} \left( \frac{1}{R} + sC_2 \right)}$$

$$= \frac{1}{\left( 1 + \frac{C_2}{C_1} \right) + \frac{1}{sC_1 R}}$$

$$= \frac{s}{s \left( 1 + \frac{C_2}{C_1} \right) + \frac{1}{C_1 R}}$$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{1}{1 + \frac{C_2}{C_1}} = \frac{C_1}{C_1 + C_2}$$

There is a transmission zero at  $s = 0$ , due to  $C_1$ .

$$(c) \frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2)} = \frac{L_2}{L_1 + L_2}$$

$$\frac{V_o}{V_i}(0) = \frac{L_2}{L_1 + L_2}$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

No transmission zeros.

$$(d) \frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2) + R}$$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

There is a transmission zero at 0 (dc) due to  $L_2$ .

**13.50** Refer to the circuit in Fig. 13.19(e) and let  $C_2 = 0$ . Using the voltage-divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{L_2,R}}{Z_{L_2,R} + Z_{L_1,C_1}}$$

$$= \frac{Y_{L_1,C_1}}{Y_{L_1,C_1} + Y_{L_2,R}}$$

$$= \frac{sC_1 + \frac{1}{sL_1}}{sC_1 + \frac{1}{sL_1} + \frac{1}{sL_2} + \frac{1}{R}}$$

$$= \frac{s^2 + \frac{1}{L_1 C_1}}{s^2 + s \frac{1}{C_1 R} + \left( \frac{1}{L_1} + \frac{1}{L_2} \right) \frac{1}{C_1}}$$

which is a second-order notch function with



This figure belongs to Problem 13.51.

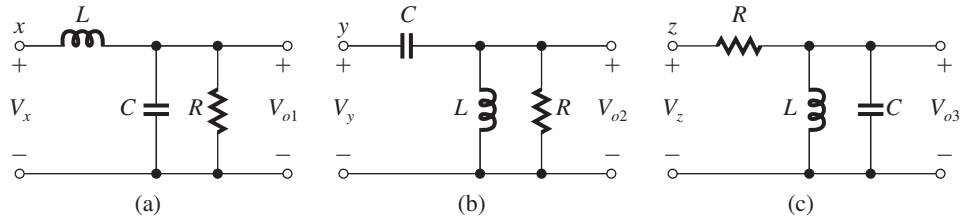


Figure 1

$$\omega_n = \frac{1}{\sqrt{L_1 C_1}} \quad (1)$$

$$\omega_0 = \frac{1}{\sqrt{\left(\frac{L_1 L_2}{L_1 + L_2}\right) C_1}} \quad (2)$$

$$Q = \omega_0 C_1 R \quad (3)$$

$$\text{High-frequency gain} = 1 \quad (4)$$

$$\text{Low-frequency gain} = \frac{L_2}{L_1 + L_2} \quad (5)$$

From Eqns. (1) and (2), we see that

$$\omega_n < \omega_0$$

verifying that this is a high-pass notch. For the given required values and selecting  $R = 1 \text{ k}\Omega$  we obtain using Eq. (3),

$$2 = 10^5 \times C_1 \times 10^3$$

$$\Rightarrow C_1 = 20 \text{ nF}$$

Using Eq. (1) we find  $L_1$  as

$$L_1 = \frac{1}{\omega_n^2 C_1} = \frac{1}{\frac{1}{2} \times 10^{10} \times 20 \times 10^{-9}}$$

$$\Rightarrow L_1 = 10 \text{ mH}$$

Dividing Eqn. (1) by Eqn. (2) and squaring the result gives

$$\left(\frac{\omega_n}{\omega_0}\right)^2 = \frac{L_2}{L_1 + L_2}$$

$$\Rightarrow L_2 = L_1 = 10 \text{ mH}$$

**13.51** Using superposition, we find the resulting  $T(s)$  in each of the three cases and then sum the results. Thus (see Figure 1 above):

(a) When  $x$  is lifted off ground and connected to  $V_x$ , the circuit becomes as shown in Fig. 1(a), and the transfer function becomes that of a low-pass filter with  $\omega_0 = 1/\sqrt{LC}$ ,  $Q = \omega_0 CR$ , and dc gain of unity, thus

$$\frac{V_{o1}}{V_x} = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \quad (1)$$

(b) With  $y$  lifted off ground and connected to  $V_y$ , the resulting circuit [shown in Fig. 1(b)], is that of a high-pass filter with  $\omega_0 = 1/\sqrt{LC}$ ,  $Q = \omega_0 CR$ , and a high-frequency gain of unity, thus

$$\frac{V_{o2}}{V_y} = \frac{s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \quad (2)$$

(c) With  $z$  lifted off ground and connected to  $V_z$ , the resulting circuit shown in Fig. 1(c) is that of a bandpass filter with  $\omega_0 = 1/\sqrt{LC}$ ,  $Q = \omega_0 CR$ , and a center-frequency gain of unity, thus

$$\frac{V_{o3}}{V_z} = \frac{s\frac{\omega_0}{Q}}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2} \quad (3)$$

Now summing (1), (2), and (3) gives

$$V_o = \frac{s^2 V_y + s\left(\frac{\omega_0}{Q}\right) V_z + \omega_0^2 V_x}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$\mathbf{13.52} \quad L = C_4 R_1 R_3 R_5 / R_2$$

Selecting  $R_1 = R_2 = R_3 = R_5 = R = 10 \text{ k}\Omega$ , we have

$$L = C_4 R^2 = C_4 \times 10^8$$

Thus,

$$C_4 = L \times 10^{-8}$$

(a) For  $L = 15 \text{ H}$ , we have

$$C_4 = 15 \times 10^{-8} \text{ F} = 0.15 \text{ }\mu\text{F}$$

(b) For  $L = 1.5 \text{ H}$ , we have

$$C_4 = 0.015 \text{ }\mu\text{F} = 15 \text{ nF}$$

(c) For  $L = 0.15 \text{ H}$ , we have

$$C_4 = 1.5 \text{ nF}$$

**13.53** For a fifth-order Butterworth filter with  $A_{\max} = 3 \text{ dB}$ , and thus  $\epsilon = 1$ , and  $\omega_p = 10^3 \text{ rad/s}$ ,

This figure belongs to Problem 13.53.

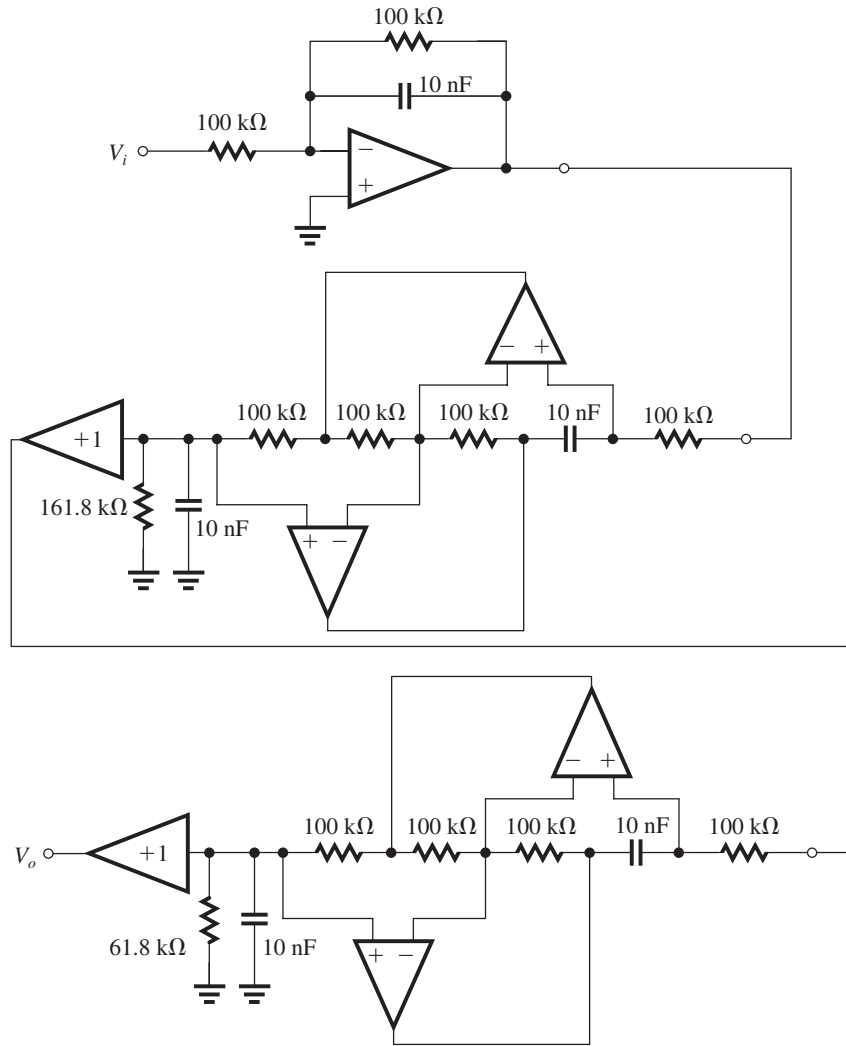


Figure 1

the poles can be determined using the graphical construct of Fig. 13.19(a), thus

The pole pair  $p_1, p_1^*$  has  $\omega_0 = 10^3$  rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{\pi}{10}\right) = 10^3 \times 0.309$$

$$\Rightarrow Q = \frac{1}{2 \times 0.309} = 1.618$$

The pole pair  $p_2, p_2^*$  has  $\omega_0 = 10^3$  rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{3\pi}{10}\right) = 10^3 \times 0.809$$

$$\Rightarrow Q = \frac{1}{2 \times 0.809} = 0.618$$

The real-axis pole  $p_3$  is at a

$$s = -10^3 \text{ rad/s}$$

Thus, the transfer function  $T(s)$  is

$$T(s) = \frac{10^3}{s + 10^3} \times \frac{10^6}{s^2 + s \frac{10^3}{1.618} + 10^6} \times \frac{10^6}{s^2 + s \frac{10^3}{0.618} + 10^6}$$

The first-order factor,

$$T_1(s) = \frac{10^3}{s + 10^3}$$

can be realized by the circuit in Fig. 13.22(a) with  $R_1 = R_2 = 100 \text{ k}\Omega$  (arbitrary but convenient value).

$$CR_1 = \frac{1}{\omega_0}$$

$$\Rightarrow C = \frac{1}{10^3 \times 100 \times 10^3} = 0.01 \mu\text{F} = 10 \text{ nF}$$

The second-order transfer function

$$T_2(s) = \frac{10^6}{s^2 + s \frac{10^3}{1.618} + 10^6}$$

can be realized by the circuit in Fig. 13.22(a) with

$$C_4 = C_6 = C = 10 \text{ nF (practical value)}$$

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{10^3 \times 10 \times 10^{-9}} = 100 \text{ k}\Omega$$

$$R_6 = Q/\omega_0 C = Q \times 100 = 161.8 \text{ k}\Omega$$

$$K = 1$$

The second-order transfer function

$$T_3(s) = \frac{10^6}{s^2 + s \frac{10^3}{0.618} + 10^6}$$

can be realized using the circuit in Fig. 13.22(a) with

$$C_4 = C_6 = C = 10 \text{ nF (practical value)}$$

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C} = 100 \text{ k}\Omega$$

$$R_6 = Q \left( \frac{1}{\omega_0 C} \right) = 0.618 \times 100 = 61.8 \text{ k}\Omega$$

$$K = 1$$

The complete filter circuit is obtained as the cascade connection of the three filter sections, as shown in Fig. 1 on the previous page.

**13.54** Figure 1 shows the circuit realization of the high-pass filter. The component values can be found as follows:

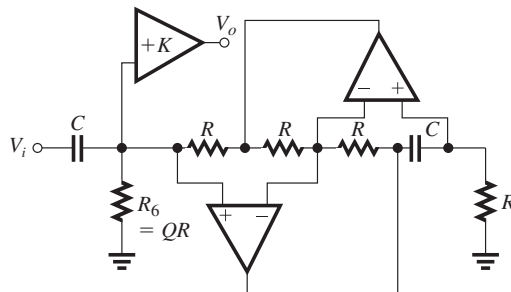


Figure 1

Select  $R = 100 \text{ k}\Omega$

$$C = \frac{1}{\omega_0 R} = \frac{1}{10^3 \times 100 \times 10^3} = 10 \text{ nF}$$

$$R_6 = QR = 5 \times 100 = 500 \text{ k}\Omega$$

$$K = 1$$

**13.55** Refer to Fig. 13.22(c) and let  $C_{62} = 0$  and  $C_{61} = C$ . We will use superposition to find the component of  $V_o$  due to  $V_i$  injected through  $C$ ,  $V_{o1}$ , and that due to  $V_i$  injected through  $R_{51}$ ,  $V_{o2}$ . To find  $V_{o1}$ , connect  $x_1$  to ground. This results in  $R_{51}$  and  $R_{52}$  appearing in parallel with the combination  $R_5 = R_{51} \parallel R_{52} = R$ . The output  $V_{o1}$  will be the high-pass junction,

$$\frac{V_{o1}}{V_i} = \frac{K s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \quad (1)$$

where  $\omega_0 = 1/CR$ . Next, we determine  $V_{o2}$  by connecting  $y_1$  to ground. Replacing  $V_i$  together with the voltage divider ( $R_{51}$ ,  $R_{52}$ ) with its Norton's equivalent, i.e., a signal  $[V_i \frac{R_{52}}{R_{51} + R_{52}}]$ , fed through a resistance  $R_{51} \parallel R_{52} = R_5 = R$ , we obtain at the output the low-pass function

$$\frac{V_{o2}}{V_i} = \frac{R_{52}}{R_{52} + R_{51}} \frac{K \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \quad (2)$$

Combining Eqns. (1) and (2) gives

$$V_o = V_{o1} + V_{o2} \text{ as}$$

$$\frac{V_o}{V_i} = K \frac{s^2 + \left( \frac{R_{52}}{R_{51} + R_{52}} \right) \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

which is a notch function with

$$\omega_n = \omega_0 \sqrt{\frac{R_{52}}{R_{51} + R_{52}}}$$

Thus,  $\omega_n < \omega_0$ , verifying that the function realized is a low-pass notch (LPN). Q.E.D.

To obtain the component values we select  $R = 10 \text{ k}\Omega$ , thus

$$C = \frac{1}{\omega_0 R} = \frac{1}{10^5 \times 10 \times 10^3} = 1 \text{ nF}$$

$$R_6 = QR = 5 \times 10 = 50 \text{ k}\Omega$$

$$\frac{R_{52}}{R_{51} + R_{52}} = \left( \frac{\omega_n}{\omega_0} \right)^2 = 0.64 \quad (3)$$

Since,

$$\frac{R_{51} R_{52}}{R_{51} + R_{52}} = R_5 = R = 10 \text{ k}\Omega \quad (4)$$

Eqns. (3) and (4) together yield

$$R_{51} = \frac{10}{0.64} = 15.625 \text{ k}\Omega$$

and

$$R_{52} = 27.78 \text{ k}\Omega$$

Finally, to obtain a high-frequency gain of 5 V/V, select  $K = 5$ .

This figure belongs to Problem 13.57.

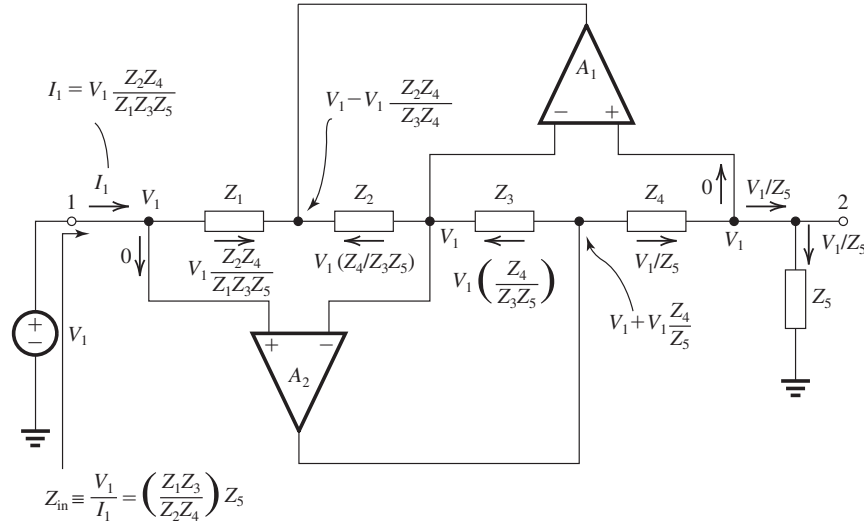


Figure 1

**13.56** Refer to Fig. 13.22(b). Selecting  $R = 2 \text{ k}\Omega$ .

$$C = \frac{1}{\omega_0 R} = \frac{1}{2\pi \times 100 \times 10^3 \times 2 \times 10^3} = 796 \text{ pF}$$

$$Q = \frac{f_0}{BW} = \frac{100}{1} = 100$$

$$R_6 = QR = 100 \times 2 = 200 \text{ k}\Omega$$

$$K = \text{center-frequency gain} = 10$$

**13.57** (a) Figure 1 above shows the analysis. It is based on assuming ideal op amps that exhibit virtual short circuits between their input terminals, and draw zero currents into their input terminals. Observe that:

(1) The circuit is fed at port-1 with a voltage  $V_1$ . The virtual short circuit between the input terminals of each of  $A_2$  and  $A_1$  cause the voltage at port-2 to be

$$V_2 = V_1 \tag{1}$$

(2) With port-2 terminated in an impedance  $Z_5$ , the current that flows out of port-2 is

$$I_2 = \frac{V_2}{Z_5}$$

(3) Following the analysis indicated, we find the input current into port-1 as

$$I_1 = V_1 \frac{Z_2 Z_4}{Z_1 Z_3 Z_5}$$

This current could have been written as

$$I_1 = V_1 \frac{Z_2 Z_4}{Z_1 Z_3} I_2 \tag{2}$$

(4) Equations (1) and (2) describe the operation of the circuit: It propagates the voltage applied to port-1,  $V_1$ , and to port-2,  $V_2 = V_1$ ; and whatever current is drawn out of port-2 is multiplied by the function  $(Z_2 Z_4 / Z_1 Z_3)$  and appears at port-1.

(5) The result of the two actions in (4) is that the input impedance looking into port-1 becomes

$$Z_{11} \equiv \frac{V_1}{I_1} = \frac{V_2}{I_2 (Z_2 Z_4 / Z_1 Z_3)} = \left( \frac{Z_1 Z_3}{Z_2 Z_4} \right) \left( \frac{V_2}{I_2} \right)$$

or

$$Z_{11} = \left( \frac{Z_1 Z_3}{Z_2 Z_4} \right) Z_5$$

(b) If port-1 is terminated in an impedance  $Z_6$ , the input impedance looking into port-2 can be obtained by invoking the symmetry of the circuit, thus

$$Z_{22} = \left( \frac{Z_2 Z_4}{Z_1 Z_3} \right) Z_6$$

Thus, the circuit behaves as an impedance transformer with the transformation ratio from

port-2 to port-1 being  $\left( \frac{Z_1 Z_3}{Z_2 Z_4} \right)$  and from port-1 to port-2 being  $\left( \frac{Z_2 Z_4}{Z_1 Z_3} \right)$ .

Since  $Z_1, Z_2, Z_3,$  and  $Z_4$  can be arbitrary functions of  $s$ , the transformation ratio can be an arbitrary function of  $s$ . Thus the circuit can be used as an impedance converter. For instance, the particular selection of impedances in Fig. 13.20(a) results in the transformation ratio from port-2 to port-1

This figure belongs to Problem 13.58.

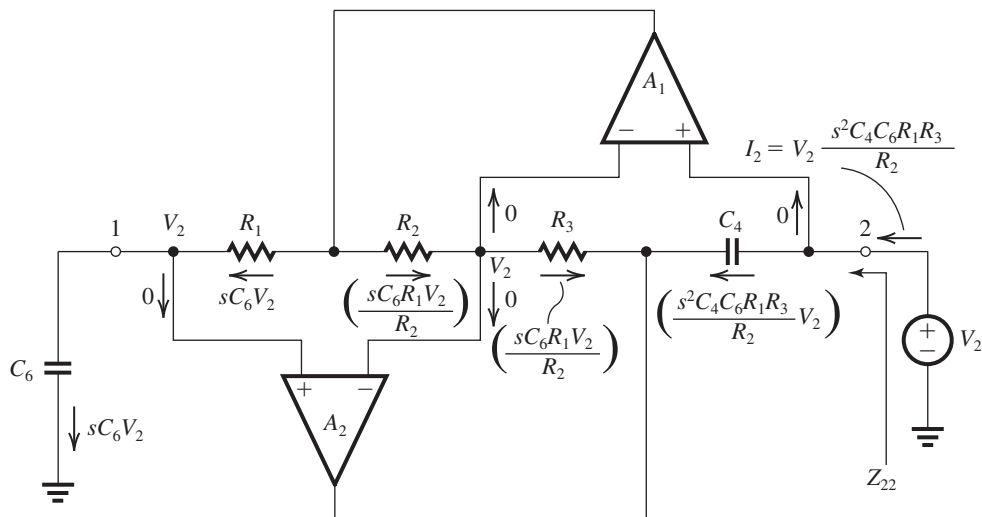


Figure 1

This figure belongs to Problem 13.59.

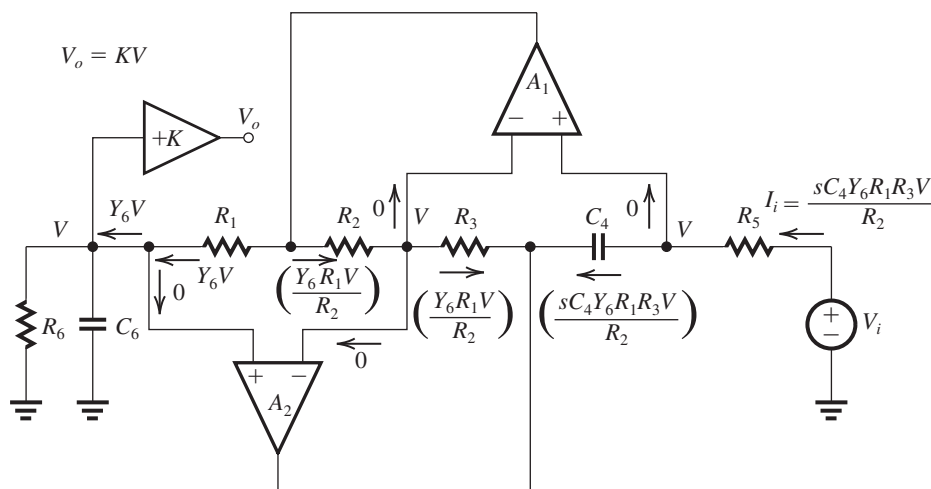


Figure 1

being  $(sC_4R_1R_3/R_2)$ . As a result, the circuit in Fig. 13.20(a) converts a resistance  $R_5$  into an inductance  $L = C_4R_1R_3R_5/R_2$ . Other conversion functions are possible and the circuit is known as a Generalized Impedance Converter or GIC.

**13.58** Figure 1 above shows the suggested circuit together with the analysis. The input impedance looking into port-2 is

$$Z_{22}(s) \equiv \frac{V_2}{I_2} = \frac{R_2}{s^2 C_4 C_6 R_1 R_3}$$

For  $s = j\omega$  we have

$$Z_{22}(j\omega) = -\frac{R_2}{\omega^2 C_4 C_6 R_1 R_2}$$

which is a negative resistance whose magnitude depends on frequency  $\omega$ ; thus it is called a Frequency Dependent Negative Resistance or FDNR.

**13.59** Figure 1 (above) shows the circuit together with the analysis details. At the input we can write

$$\begin{aligned} V_i &= V + I_i R_5 \\ &= V + \frac{s C_4 Y_6 R_1 R_3 R_5}{R_2} V \end{aligned}$$

where

$$V = \frac{V_o}{K}$$

and

$$Y_6 = \frac{1}{R_6} + sC_6$$

Thus,

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{KV}{V_i} \\ &= \frac{K}{1 + sC_4 \left( sC_6 + \frac{1}{R_6} \right) \frac{R_1 R_3 R_5}{R_2}} \\ &= \frac{K R_2 / C_4 C_6 R_1 R_3 R_5}{s^2 + s \frac{1}{C_6 R_6} + \frac{1}{C_4 C_6 R_1 R_3 R_5}} \end{aligned}$$

which is a low-pass function with

$$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$$

$$Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

DC gain =  $K$

**13.60**

$$(a) T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

Replacing  $s$  by  $s/10^5$ , we obtain

$$\begin{aligned} T(s) &= \frac{0.4508 \left( \frac{s^2}{10^{10}} + 1.6996 \right)}{\left( \frac{s}{10^5} + 0.7294 \right) \left( \frac{s^2}{10^{10}} + \frac{s}{10^5} 0.2786 + 1.0504 \right)} \\ &= \frac{0.4508 \times 10^5 (s^2 + 1.6996 \times 10^{10})}{(s + 0.7294 \times 10^5) \times (s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10})} \end{aligned}$$

(b) First-order section:

This figure belongs to Problem 13.60, part (b).

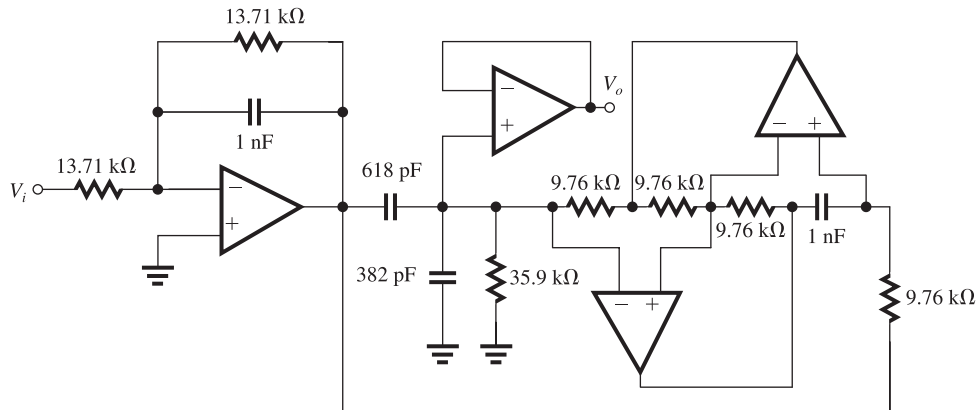


Figure 1

$$T_1(s) = \frac{0.7294 \times 10^5}{s + 0.7294 \times 10^5}$$

where the dc gain is made unity. This function can be realized using the circuit of Fig. 13.8 with

$C = 1$  nF (arbitrary but convenient value)

$$\begin{aligned} R_2 &= \frac{1}{\omega_0 C} = \frac{1}{0.7294 \times 10^5 \times 1 \times 10^{-9}} \\ &= 13.71 \text{ k}\Omega \end{aligned}$$

For dc gain of unity, we have

$$R_1 = R_2 = 13.71 \text{ k}\Omega$$

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2 + 1.6996 \times 10^{10})}{s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10}}$$

We use the circuit in Fig. 13.22(c) with  $R_{52} = \infty$ .

Thus,  $R_{51} = R_5$ . Selecting

$$C_4 = C_{61} + C_{62} = C = 1 \text{ nF}$$

and

$$R_1 = R_2 = R_3 = R_5 = R$$

then

$$\begin{aligned} R &= \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504 \times 10^5 \times 1 \times 10^{-9}}} \\ &= 9.76 \text{ k}\Omega \end{aligned}$$

From Eq. (13.57),

$$\omega_n = \frac{1}{\sqrt{C_{61} C R^2}} = \frac{1}{C R} \sqrt{\frac{C}{C_{61}}} = \omega_0 \sqrt{\frac{C}{C_{61}}}$$

$$C_{61} = C \left( \frac{\omega_0}{\omega_n} \right)^2$$

$$= 1 \times 10^{-9} \frac{1.0504 \times 10^{10}}{1.6996 \times 10^{10}} = 0.618 \text{ nF}$$

$$= 618 \text{ pF}$$

$$C_{62} = 1 - 0.618 = 0.382 \text{ nF} = 382 \text{ pF}$$

$$Q = \frac{\sqrt{1.0504}}{0.2786} = 3.679$$

$$R_6 = QR = 3.679 \times 9.76 = 35.9 \text{ k}\Omega$$

Finally,

$$K = \text{DC gain} = 1$$

The complete circuit is shown in Fig. 1 on the previous page.

**13.61** Bandpass with  $f_0 = 2 \text{ kHz}$ , 3-dB bandwidth of 50 Hz, thus

$$Q = \frac{f_0}{BW} = \frac{2 \text{ kHz}}{50 \text{ Hz}} = 40$$

Refer to the circuit in Fig. 13.24(a). Using

$$C = 10 \text{ nF}$$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}} \\ = 7.96 \text{ k}\Omega$$

Select

$$R_1 = 10 \text{ k}\Omega$$

then

$$R_f = R_1 = 10 \text{ k}\Omega$$

Select

$$R_2 = 1 \text{ k}\Omega$$

then

$$R_3 = (2Q - 1)R_2 = (80 - 1) \times 1 = 79 \text{ k}\Omega$$

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{40} = 1.975$$

$$\text{Center-frequency gain} = KQ = 1.975 \times 40 = 79$$

**13.62** Refer to Fig. 13.25(b) and Eq. (13.69). For a dc gain of 10,

$$K = 10$$

For an input resistance of 1 k $\Omega$ ,

$$R_g = \frac{R}{K} = 1 \text{ k}\Omega$$

Thus,

$$R = 10 \text{ k}\Omega$$

Now,

$$C = \frac{1}{\omega_0 R} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^3} \\ = 1.59 \text{ nF}$$

$$R_d = QR = 20 \times 10 = 200 \text{ k}\Omega$$

**13.63** Refer to Fig. 13.26 and Eq. (13.70). To obtain a notch filter we select

$$R_1 = R_3 = \infty$$

which results in

$$T(s) = - \left( \frac{C_1}{C} \right) \frac{s^2 + \frac{1}{CC_1RR_2}}{s^2 + s\frac{1}{QR} + \frac{1}{C^2R^2}}$$

Thus,

$$\omega_0 = \frac{1}{CR} \quad (1)$$

$$\text{High-frequency Gain} \equiv G = \frac{C_1}{C} \quad (2)$$

$$\omega_n = \frac{1}{\sqrt{CC_1RR_2}} = \frac{1}{CR} \sqrt{\frac{C}{C_1}} \sqrt{\frac{R}{R_2}}$$

Thus

$$\omega_n = \omega_0 \sqrt{\frac{1}{G}} \sqrt{\frac{R}{R_2}} \quad (3)$$

For required value of  $\omega_0$ ,  $\omega_n$  and  $G$ , we select a convenient value for  $C$  and use Eq. (1) to find  $R$ ,

$$R = \frac{1}{\omega_0 C}$$

and Eq. (2) to find  $C_1$ ,

$$C_1 = G \times C$$

and Eq. (3) to find  $R_2$ ,

$$R_2 = \left( \frac{R}{G} \right) \left( \frac{\omega_0}{\omega_n} \right)^2$$

We note that these design equations apply for any notch filter.

**13.64** Refer to Fig. 1 (next page). We can write expressions for the output of each of three op amps as follows:

$$V_o = -V_i \frac{sC_1 + \frac{1}{R_1}}{sC + \frac{1}{QR}} - V_2 \frac{\frac{1}{R}}{sC + \frac{1}{QR}} \quad (1)$$

$$V_1 = -\frac{1}{sCR} V_o - \frac{1}{sCR_2} V_i \quad (2)$$

$$V_2 = -V_1 - \left( \frac{r}{R_3} \right) V_i \quad (3)$$

Substituting for  $V_1$  from (2) into (3) gives

$$V_2 = \frac{1}{sCR} V_o + \left( \frac{1}{sCR_2} - \frac{r}{R_3} \right) V_i \quad (4)$$

Substituting for  $V_2$  from (4) into (1) and rearranging gives

This figure belongs to Problem 13.64.

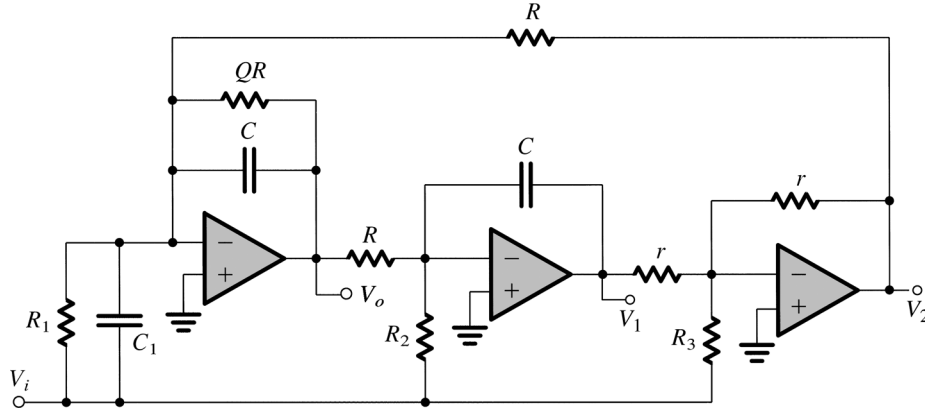


Figure 1

$$\frac{V_o}{V_i} = \frac{s^2 \left(\frac{C_1}{C}\right) + s \frac{1}{C} \left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2 RR_2}}{s^2 + s \frac{1}{QCR} + \frac{1}{C^2 R^2}}$$

which is identical to Eq. (13.70). Q.E.D.

**13.65** Refer to Fig. 13.26 and Eq. (13.70). To obtain a notch filter, we select

$$R_1 = R_3 = \infty$$

which makes the transfer function

$$T(s) = - \left(\frac{C_1}{C}\right) \frac{s^2 + \frac{1}{CC_1 RR_2}}{s^2 + s \frac{1}{QCR} + \frac{1}{C^2 R^2}}$$

Thus,

$$\omega_0 = \frac{1}{CR} \tag{1}$$

$$\text{DC Gain} = \frac{R}{R_2} \tag{2}$$

$$\omega_n = \frac{1}{\sqrt{(CR)(C_1 R_2)}} = \omega_0 \sqrt{\frac{C}{C_1}} \sqrt{\frac{R}{R_2}} \tag{3}$$

For  $\omega_0 = 10^5$  rad/s,  $Q = 10$ , dc gain = 1, and  $\omega_n = 1.3 \times 10^5$  rad/s, and using  $C = 10$  nF and  $r = 20$  k $\Omega$ , we find

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^5 \times 10 \times 10^{-9}} = 1 \text{ k}\Omega$$

$$R_d = QR = 10 \times 1 = 10 \text{ k}\Omega$$

From Eq. (2),

$$1 = \frac{R}{R_2}$$

$$\Rightarrow R_2 = R = 1 \text{ k}\Omega$$

From Eq. (3),

$$\omega_n = \omega_0 \sqrt{\frac{C}{C_1}} \sqrt{\frac{R}{R_2}}$$

$$\Rightarrow C_1 = C \left(\frac{\omega_0}{\omega_n}\right)^2 = 10 \times \frac{1}{1.3^2} = 5.92 \text{ nF}$$

**13.66** (a) Refer to Fig. 13.24. The output voltage of the summer is given by

$$V_o = - \left[ \left(\frac{R_F}{R_H}\right) V_{hp} + \left(\frac{R_F}{R_B}\right) V_{bp} + \frac{R_F}{R_L} V_{lp} \right] \tag{1}$$

where from Eq. (13.59),

$$V_{hp} = \frac{Ks^2}{s^2 + s \left(\frac{\omega_0}{Q}\right) + \omega_0^2} V_i \tag{2}$$

and from Eq. (13.62),

$$V_{bp} = - \left(\frac{\omega_0}{s}\right) = - \frac{K\omega_0 s}{s^2 + s \left(\frac{\omega_0}{Q}\right) + \omega_0^2} V_i \tag{3}$$

and from Eq. (13.63),

$$V_{lp} = \left(\frac{\omega_0^2}{s^2}\right) V_{hp} = \frac{K\omega_0^2}{s^2 + s \left(\frac{\omega_0}{Q}\right) + \omega_0^2} V_i \tag{4}$$

Substituting from Eqns. (2)-(4) into Eq. (1) and dividing by  $V_i$ , we obtain

$$\frac{V_o}{V_i} = -K \frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s \left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

(b) To obtain a notch function we select  $R_B = \infty$ , thus

$$\frac{V_o}{V_i} = -K \left(\frac{R_F}{R_H}\right) \frac{s^2 + (R_H/R_L)\omega_0^2}{s^2 + s \left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$\omega_n^2 = \left(\frac{R_H}{R_L}\right) \omega_0^2 \tag{5}$$

$$\text{DC gain} = G = -K \left(\frac{R_F}{R_L}\right)$$

But,



This figure belongs to Problem 13.67.

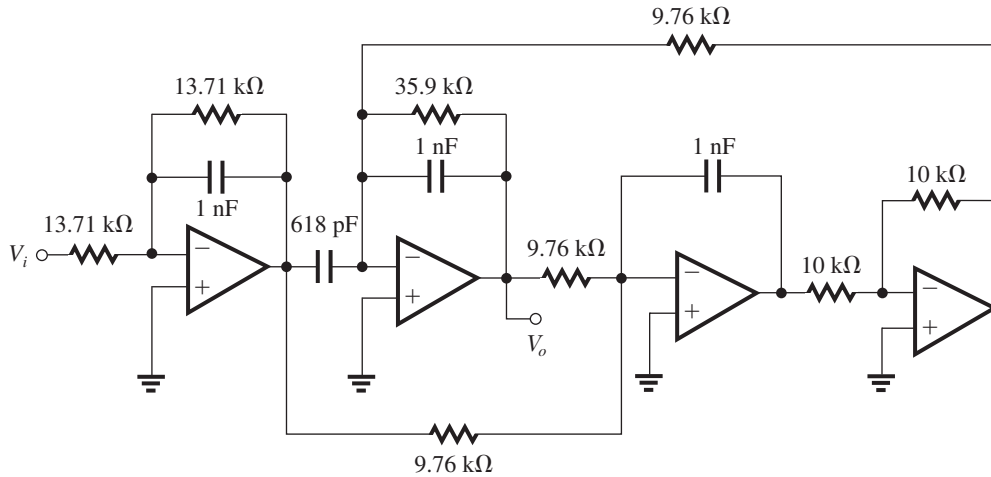


Figure 1

$$K = 2 - \frac{1}{Q}$$

Thus,

$$G = -\left(2 - \frac{1}{Q}\right) \left(\frac{R_F}{R_L}\right) \quad (6)$$

From Eq. (5),

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2 \quad (7)$$

and from Eq. (6),

$$\frac{R_F}{R_L} = -\frac{G}{2 - \frac{1}{Q}} \quad (8)$$

Selecting  $R_L$  as an arbitrary but convenient value we can use (7) and (8) to determine the required values of  $R_H$  and  $R_F$ , respectively.

**13.67**

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

(a) Replacing  $s$  by  $s/10^5$ , we obtain

$$T(s) = \frac{0.4508 \times 10^5 (s^2 + 1.6996 \times 10^{10})}{(s + 0.7294 \times 10^5)(s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10})}$$

(b) First-order section:

$$T_1(s) = \frac{0.7294 \times 10^5}{s + 0.7294 \times 10^5}$$

which is made to have a dc gain of unity, as required. This function can be realized by the circuit in Fig. 13.26. Selecting

$$C = 1 \text{ nF (arbitrary but convenient)}$$

we have

$$\begin{aligned} R_2 &= \frac{1}{\omega_0 C} \\ &= \frac{1}{0.7294 \times 10^5 \times 1 \times 10^{-9}} \\ &= 13.71 \text{ k}\Omega \end{aligned}$$

For a dc gain of unity, we have

$$R_1 = R_2 = 13.71 \text{ k}\Omega$$

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2 + 1.6996 \times 10^{10})}{s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10}}$$

where the dc gain is unity. Refer to Fig. 13.26 and Eq. (13.70).

Selecting

$$C = 1 \text{ nF}$$

then

$$\begin{aligned} R &= \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504} \times 10^5 \times 1 \times 10^{-9}} \\ &= 9.76 \text{ k}\Omega \end{aligned}$$

$$R_d = QR = \frac{\sqrt{1.0504}}{0.2786} \times 9.76 = 35.9 \text{ k}\Omega$$

Select

$$r = 10 \text{ k}\Omega$$

Now,

$$\begin{aligned} C_1 &= C \times \text{high-frequency gain} \\ &= 1 \times 0.618 = 0.618 \text{ nF} = 618 \text{ pF} \end{aligned}$$

$$R_1 = \infty$$

$$R_3 = \infty$$

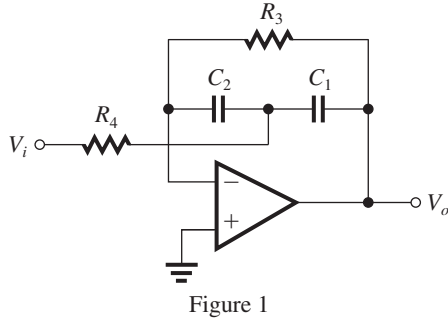
$$R_2 = R \frac{(\omega_0/\omega_n)^2}{\text{HF Gain}}$$

$$= 9.76 \times \frac{1.0504}{1.6996} \times \frac{1}{0.618}$$

$$= 9.76 \text{ k}\Omega$$

The complete circuit is shown in Fig. 1 (previous page).

**13.68**



The circuit is shown in Fig. 1. The transfer function can be obtained using the equation on page 1340 with  $\alpha = 1$ , thus

$$\frac{V_o}{V_i} = \frac{-s/C_1 R_4}{s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}$$

Using the design equations (13.75) and (13.76), we obtain

$$R_3 = R$$

$$R_4 = R/4Q^2$$

$$CR = \frac{2Q}{\omega_0}$$

where

$$\omega_0 = 2\pi \times 10 \times 10^3$$

$$Q = \frac{f_0}{BW} = \frac{10}{2} = 5$$

Thus,

$$CR = \frac{2 \times 5}{2\pi \times 10^4}$$

Selecting

$$C = 10 \text{ nF}$$

we obtain

$$R = \frac{10}{2\pi \times 10^4 \times 10 \times 10^{-9}} = 15.92 \text{ k}\Omega$$

Thus,

$$C_1 = C_2 = 10 \text{ nF}$$

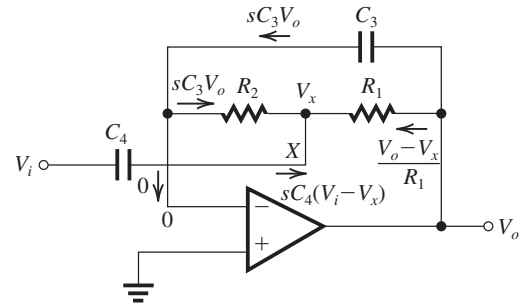
$$R_3 = 15.92 \text{ k}\Omega$$

$$R_4 = \frac{R}{4Q^2} = \frac{15.92}{100} = 159.2 \Omega$$

$$\text{Center-frequency gain} = -\frac{1/C_1 R_4}{\left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3}}$$

$$= -\frac{1}{2} \frac{R_3}{R_4} = -\frac{1}{2} \times 100 = -50 \text{ V/V}$$

**13.69**



$$V_x = -sC_3 V_o R_2 = -sC_3 R_2 V_o \quad (1)$$

A node equation at X gives

$$sC_3 V_o + \frac{V_o - V_x}{R_1} + sC_4 (V_i - V_x) = 0$$

which can be rearranged as

$$V_o (sC_3 + \frac{1}{R_1}) + sC_4 V_i = V_x (\frac{1}{R_1} + sC_4)$$

Substituting for  $V_x$  from Eq. (1) gives

$$V_o (sC_3 + \frac{1}{R_1}) + sC_4 V_i = -sC_3 R_2 (\frac{1}{R_1} + sC_4) V_o$$

which can be rearranged to obtain the transfer function as

$$\frac{V_o}{V_i} = -\frac{sC_4}{sC_3 + \frac{1}{R_1} + sC_3 \frac{R_2}{R_1} + s^2 C_3 C_4 R_2}$$

which can be written in the alternate form

$$\frac{V_o}{V_i} = -\frac{s \frac{1}{C_3 R_2}}{s^2 + s \frac{1}{C_4} (\frac{1}{R_1} + \frac{1}{R_2}) + \frac{1}{C_3 C_4 R_1 R_2}}$$

which is the transfer function of a second-order bandpass filter with

$$\omega_0 = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}} \quad (2)$$

$$Q = \sqrt{\frac{C_4}{C_3}} / (\sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}}) \quad (3)$$

$$\text{Center-frequency gain} = (\frac{C_4}{C_3}) / (1 + \frac{R_2}{R_1}) \quad (4)$$

This figure belongs to Problem 13.70.

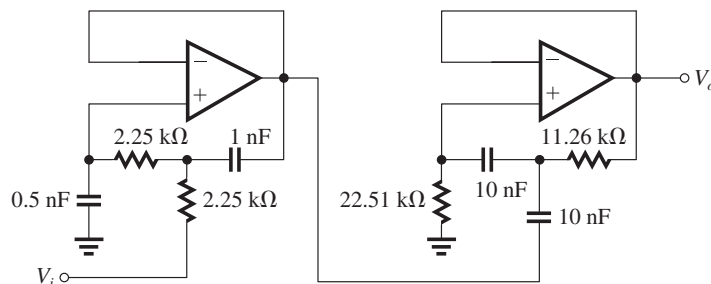


Figure 1

This figure belongs to Problem 13.70.

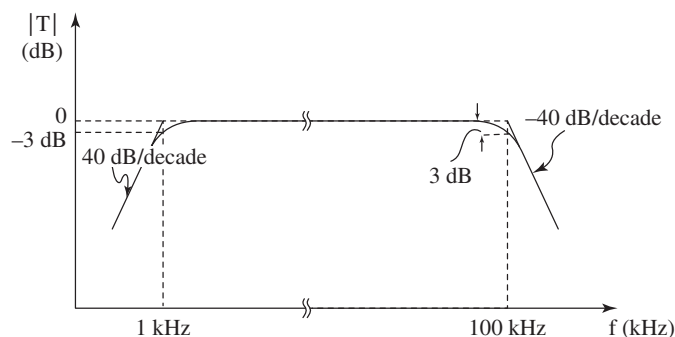


Figure 2

For  $R_1 = R_2 = R$ ,  $C_4 = C$ , and  $C_3 = C/m$ , Eq. (3) gives

$$Q = \frac{\sqrt{m}}{2}$$

$$\Rightarrow m = 4Q^2$$

Eq. (2) gives

$$\omega_0 = 1/\sqrt{\frac{C^2}{m}R^2}$$

$$\Rightarrow CR = \frac{2Q}{\omega_0}$$

Finally,

$$\text{Center-frequency gain} = 2Q^2$$

**13.70** We need to design both circuits to have  $Q = 1/\sqrt{2}$ . Consider first the low-pass circuit in Fig. 13.29 and its design equations (13.98)–(13.101). To obtain  $f_0 = 100$  kHz, that is,  $\omega_0 = 2\pi \times 10^5$  rad/s, and  $Q = 1/\sqrt{2}$ ,

$$CR = \frac{2Q}{\omega_0} = \frac{\sqrt{2}}{2\pi \times 10^5}$$

Selecting  $C = 1$  nF,

$$R = \frac{1}{\sqrt{2}\pi \times 10^5 \times 1 \times 10^{-9}} = 2.25 \text{ k}\Omega$$

Thus,

$$R_1 = R_2 = 2.25 \text{ k}\Omega$$

$$C_4 = 1 \text{ nF}$$

$$C_3 = \frac{C}{4Q^2} = \frac{1 \times 10^{-9}}{4 \times \frac{1}{2}} = 0.5 \text{ nF}$$

Next, consider the high-pass circuit in Fig. 13.28(a) and design Eqns. (13.88)–(13.91). To obtain  $f_0 = 1$  kHz and  $Q = 1/\sqrt{2}$ ,

$$CR = \frac{2 \times \frac{1}{\sqrt{2}}}{2\pi \times 10^3} = \frac{1}{\sqrt{2}\pi \times 10^3}$$

Selecting  $C = 10$  nF,

$$R = \frac{1}{\sqrt{2}\pi \times 10^3 \times 10 \times 10^{-9}} = 22.51 \text{ k}\Omega$$

Thus,

$$C_1 = C_2 = 10 \text{ nF}$$

$$R_3 = 22.51 \text{ k}\Omega$$

$$R_4 = \frac{22.51}{4Q^2} = 11.26 \text{ k}\Omega$$

The overall circuit for the bandpass filter is shown in Fig. 1 above. A sketch of the Bode plot for  $|T|$  is shown in Fig. 2.

### 13.71

The circuit is shown in Fig. 1 (next page). The voltage at node X is given by

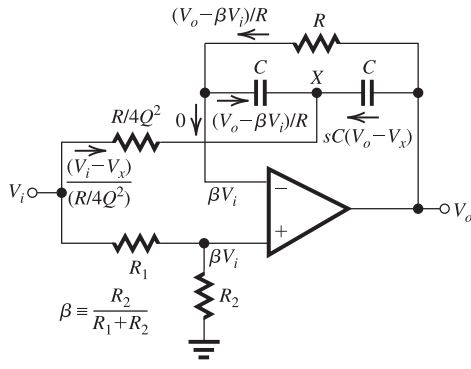


Figure 1

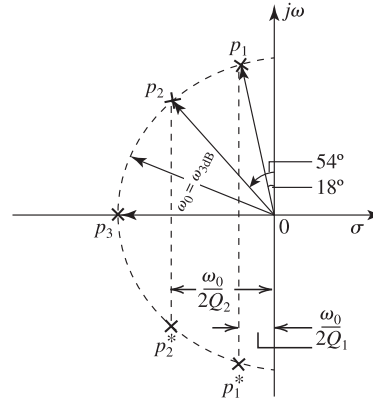


Figure 1

$$V_x = \beta V_i - \frac{1}{sC} \frac{V_o - \beta V_i}{R}$$

$$= \beta V_i \left( 1 + \frac{1}{sCR} \right) - \frac{V_o}{sCR} \quad (1)$$

Writing a node equation at X gives

$$\frac{V_o - \beta V_i}{R} + \frac{V_i - V_x}{R/4Q^2} + sC(V_o - V_x) = 0$$

Substituting for  $V_x$  from Eq. (1) and collecting terms gives

$$\frac{V_o}{V_i} = \beta \frac{s^2 + s \frac{2}{CR} \left[ 1 + 2Q^2 \left( 1 - \frac{1}{\beta} \right) \right] + \frac{4Q^2}{C^2 R^2}}{s^2 + s \frac{2}{CR} + \frac{4Q^2}{C^2 R^2}}$$

We observe that, as expected,

$$\omega_0 = \frac{2Q}{CR}$$

To obtain a notch function, we set

$$\frac{2}{CR} \left[ 1 + 2Q^2 \left( 1 - \frac{1}{\beta} \right) \right] = 0$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{2Q^2}$$

But,  $\beta = \frac{R_2}{R_1 + R_2}$ . Thus,

$$\frac{R_2}{R_1} = 2Q^2$$

### 13.72

Figure 1 shows a graphical construct to determine the poles of the fifth order Butterworth filter. The pair of complex conjugate poles  $p_1$  and  $p_1^*$  have a frequency

$$\omega_{01} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a  $Q$  factor

$$Q_1 = \frac{1}{2 \sin 18^\circ} = 1.618$$

The pair of complex conjugate poles  $p_2$  and  $p_2^*$  have

$$\omega_{02} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a  $Q$  factor,

$$Q_2 = \frac{1}{2 \sin 54^\circ} = 0.618$$

The real-axis pole  $p_3$  is at

$$s = -\omega_0 = -2\pi \times 10^4 \text{ rad/s}$$

The first second-order section can be realized using the circuit in Fig. 13.29. The design equations are (13.98)–(13.101).

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = C/4Q^2$$

Here,  $Q = Q_1 = 1.618$ , thus

$$C_3 = \frac{C}{4 \times 1.618^2} = 0.095C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 1.618}{2\pi \times 10^4 \times 10 \times 10^3} = 5.15 \text{ nF}$$

$$C_3 = 0.492 \text{ nF} = 492 \text{ pF}$$

$$C_4 = 5.15 \text{ nF}$$

The second second-order section also can be realized using the circuit in Fig. 13.29. Here,

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = \frac{C}{4Q^2}$$

where  $Q = Q_2 = 0.618$ . Thus

$$C_3 = \frac{C}{4 \times 0.618^2} = 0.655C$$

This figure belongs to Problem 13.72, part (b).

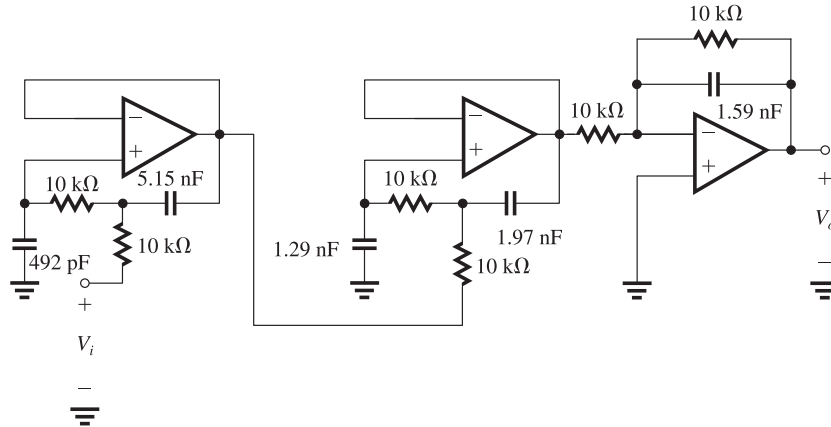


Figure 2

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 0.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 0.618}{2\pi \times 10^4 \times 10^4} = 1.97 \text{ nF}$$

$$C_3 = 1.29 \text{ nF}$$

$$C_4 = 1.97 \text{ nF}$$

The first-order section can be realized using the circuit in Fig. 13.8 with

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0 R}$$

$$= \frac{1}{2\pi \times 10^4 \times 10^4} = 1.59 \text{ nF}$$

The complete circuit is shown in Fig. 2 above.

$$\mathbf{13.73} \quad R_{eq} = \frac{T_c}{C_1} = \frac{1}{f_c C_1} = \frac{1}{100 \times 10^3 C_1}$$

For  $C_1 = 0.1 \text{ pF}$ ,

$$R_{eq} = \frac{1}{100 \times 10^3 \times 0.1 \times 10^{-12}}$$

100 M $\Omega$

For  $C_1 = 0.5 \text{ pF}$ ,

$$R_{eq} = \frac{1}{100 \times 10^3 \times 0.5 \times 10^{-12}}$$

20 M $\Omega$

For  $C_1 = 1 \text{ pF}$ ,

$$R_{eq} = \frac{1}{100 \times 10^3 \times 1 \times 10^{-12}} = 10 \text{ M}\Omega$$

For  $C_1 = 5 \text{ pF}$ ,

$$R_{eq} = \frac{1}{100 \times 10^3 \times 5 \times 10^{-12}} = 2 \text{ M}\Omega$$

For  $C_1 = 10 \text{ pF}$ ,

$$R_{eq} = \frac{1}{100 \times 10^3 \times 10 \times 10^{-12}} = 1 \text{ M}\Omega$$

**13.74** From Eqs. (13.109) and (13.110),

$$C_3 = C_4 = \omega_0 T_c C$$

$$= 2\pi \times 10^4 \times \frac{1}{200 \times 10^3} \times 20 \times 10^{-12}$$

$$= 6.283 \text{ pF}$$

From Eq. (13.112),

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{50} = 0.126 \text{ pF}$$

From Eq. (13.113),

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = 1$$

$$\Rightarrow C_6 = C_5 = 0.126 \text{ pF}$$

**13.75**  $\omega_0 = \omega_{3dB} = 10^3 \text{ rad/s}$

$$Q = 1/\sqrt{2} \text{ and DC gain} = 1$$

$$f_c = 100 \text{ kHz}, C_1 = C_2 = C = 5 \text{ pF}$$

From Eqs. (13.109) and (13.110),

$$C_3 = C_4 = \omega_0 T_c C$$

$$= 10^3 \times \frac{1}{100 \times 10^3} \times 5 \times 10^{-12}$$

$$= 0.05 \text{ pF}$$

From Eq. (13.112),

$$C_5 = \frac{C_4}{Q} = \frac{0.05}{1/\sqrt{2}} = 0.071 \text{ pF}$$

The dc gain of the low-pass circuit is

$$\text{DC gain} = \frac{C_6}{C_4}$$

For DC gain = 1,

$$C_6 = C_4 = 0.05 \text{ pF}$$

**13.76** Change transferred  $\Rightarrow Q = CV$

$$= 10^{-12} (1)$$

$$= 1 \text{ pC}$$

For  $f_0 = 100 \text{ kHz}$ , average current is given by

$$I_{\text{AV}} = \frac{Q}{T} = 1 \text{ pC} \times 100 \times 10^3$$

$$= 0.1 \text{ } \mu\text{A}$$

For each clock cycle, the output will change by the same amount as the change in voltage across  $C_2$ .

$$\therefore \Delta V = Q/C_2 = \frac{1 \text{ pC}}{10 \text{ pF}} = 0.1 \text{ V}$$

For  $\Delta V = 0.1 \text{ V}$  for each Clock cycle, the amplifier will saturate in

$$= \frac{10 \text{ V}}{0.1 \text{ V}} = 100 \text{ cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10 \text{ V}}{(100 \text{ cycles})(1/100 \times 10^3)}$$

$$= 10^4 \text{ V/s}$$

**13.77** From Eq. (13.108),

$$C_3 = C_4 = \omega_0 T_c C$$

$$= 2\pi \times 10^4 \times \frac{1}{500 \times 10^3} \times 20$$

$$= 2.51 \text{ pF}$$

From Eq. (13.110),

$$C_5 = \frac{C_4}{Q} = \frac{2.51}{20} = 0.126 \text{ pF}$$

From Eq. (13.111),

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = 1$$

$$\Rightarrow C_6 = C_5 = 0.126 \text{ pF}$$

**13.78** (a)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$10^7 = \frac{1}{\sqrt{0.1 \times 10^{-3} \times C}}$$

$$C = \frac{1}{10^{-4} \times 10^{14}} = 100 \text{ pF}$$

(b)

$$Q = \omega_0 CR$$

$$50 = 10^7 \times 100 \times 10^{-12} \times R$$

$$\Rightarrow R = 50 \text{ k}\Omega$$

(c)

$$\sigma = \frac{1}{2CR} \left( \frac{r_2}{r_1} \right)$$

$$100 = \frac{1}{2 \times 100 \times 10^{-12} \times 50 \times 10^3} \left( \frac{r_2}{r_1} \right)$$

$$\Rightarrow \frac{r_2}{r_1} = 0.001$$

**13.79** The loop gain  $L(s)$  is given by

$$L(s) = \left( 1 + \frac{r_2}{r_1} \right) T(s)$$

where  $T(s)$  is the transfer function of the RC circuit.  $T(s)$  can be derived as shown in Fig. 1.

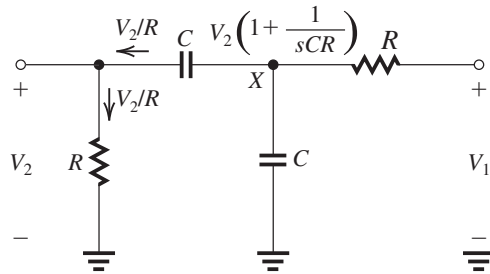


Figure 1

Writing a node equation at X,

$$\frac{V_2}{R} + sCV_2 \left( 1 + \frac{1}{sCR} \right) = \frac{V_1}{R} - \frac{V_2}{R} \left( 1 + \frac{1}{sCR} \right)$$

Rearranging this equation, we obtain the transfer function  $T(s)$  as

$$T(s) \equiv \frac{V_2}{V_1} = \frac{s \frac{1}{CR}}{s^2 + s \frac{3}{CR} + \frac{1}{C^2 R^2}}$$

Thus, the loop gain is

$$L(s) = \frac{s \left( \frac{1}{CR} \right) \left( 1 + \frac{r_2}{r_1} \right)}{s^2 + s \frac{3}{CR} + \frac{1}{C^2 R^2}}$$

$$L(j\omega) = \frac{j\omega \left( \frac{1}{CR} \right) \left( 1 + \frac{r_2}{r_1} \right)}{(-\omega^2 + \frac{1}{C^2 R^2}) + j\omega \frac{3}{CR}}$$

The phase shift will be zero at

$$\omega = \frac{1}{CR}$$

Thus, the frequency of oscillation is

$$\omega_0 = \frac{1}{CR}$$

At this frequency, the loop gain is

$$L(j\omega_0) = \frac{1}{3} \left(1 + \frac{r_2}{r_1}\right)$$

To ensure oscillations start,

$$|L(j\omega_0)| > 1$$

$$\Rightarrow \frac{r_2}{r_1} > 2$$

**13.80** The third harmonic component of  $v_L$  has one-third the amplitude of the fundamental.

Since,

$$|T(j3\omega_0)| = \frac{3\omega_0(\omega_0/Q)}{\sqrt{(\omega_0^2 - 9\omega_0^2)^2 + \frac{9\omega_0^4}{Q^2}}}$$

and  $Q = 100$ ,

$$|T(j3\omega_0)| = 3.75 \times 10^{-3}$$

Thus, at  $v_O$

$$\frac{\text{Amplitude of Third Harmonic}}{\text{Amplitude of Fundamental}} = \frac{1}{3} \times 3.75 \times 10^{-3}$$

$$= 1.25 \times 10^{-3} \text{ or } 0.125\%$$

Repeating for the fifth harmonic; at  $v_L$ , the amplitude of the fifth harmonic is (1/5) that of the fundamental. Since

$$|T(j5\omega_0)| = \frac{5\omega_0(\omega_0/Q)}{\sqrt{(\omega_0^2 - 25\omega_0^2)^2 + \frac{25\omega_0^4}{Q^2}}}$$

and  $Q = 100$ ,

$$|T(j5\omega_0)| = \frac{0.05}{\sqrt{576 + 0.0025}} = 2.08 \times 10^{-3}$$

Thus, at  $v_O$ ,

$$\frac{\text{Amplitude of Fifth Harmonic}}{\text{Amplitude of Fundamental}} = \frac{1}{5} \times 2.08 \times 10^{-3}$$

$$= 4.2 \times 10^{-4} = 0.042\%$$

**13.81** Refer to Fig. 13.37. For  $f_0 = 100$  kHz,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi \times 100 \times 10^3 = \frac{1}{\sqrt{0.1 \times 10^{-3} \times C}}$$

$$\Rightarrow C = 25.3 \text{ nF}$$

To obtain  $Q = 50$ ,

$$Q = \omega_0 CR$$

$$50 = 2\pi \times 100 \times 10^3 \times 25.3 \times 10^{-9} \times R$$

$$\Rightarrow R = 3.145 \text{ k}\Omega$$

Peak amplitude of sine wave across  $LC$  tank

$$= \frac{4 \times 0.7}{\pi} = 0.891 \text{ V}$$

Thus, to obtain a 3-V amplitude at the output,

$$1 + \frac{r_2}{r_1} = \frac{3 \text{ V}}{0.891 \text{ V}} = 3.366$$

$$\Rightarrow r_2 = 2.366 \times 10 = 23.66 \text{ k}\Omega$$

$$r_2 \geq \frac{3 \text{ V} - 0.7 \text{ V}}{1 \text{ mA}} = 2.3 \text{ k}\Omega$$

### 13.82

$$\text{Loop gain} = \frac{sAK(\omega_0/Q)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2} \quad (1)$$

The circuit will oscillate at the frequency for which the phase of the loop gain is zero. From Eq. (1) we see that at  $s = j\omega_0$  the phase will be zero; thus the frequency of oscillation is  $\omega_0$ . Oscillations will start if the magnitude of the loop gain at  $\omega_0$  is greater than one,

$$AK > 1$$

Thus the minimum value of  $K$  is

$$K_{\min} = \frac{1}{A}$$

**13.83** The characteristic equation is obtained as follows:

$$1 - L(s) = 0$$

$$1 - A \frac{Ks\left(\frac{\omega_0}{Q}\right)}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = 0$$

$$\Rightarrow s^2 + s\frac{\omega_0}{Q}(1 - AK) + \omega_0^2 = 0$$

Thus, the poles will be in the left half of the  $s$ -plane at a radial frequency  $\omega_0$  and a horizontal distance from the  $j\omega$  axis of

$$\sigma = -\frac{\omega_0}{2Q}(1 - AK)$$

(a) For the poles to be on the  $j\omega$  axis, we need

$$\sigma = 0$$

$$\Rightarrow AK = 1$$

(b) For the poles in the right half of the  $s$ -plane at a horizontal distance from the  $j\omega$  axis of  $\frac{\omega_0}{2Q}$ , we have

$$\sigma = +\frac{\omega_0}{2Q}$$

which is achieved by making

$$AK = 2$$

**13.84** For oscillations to begin, the total phase shift around the loop at  $\omega_0$  must be 0 or  $360^\circ$ .

Since the phase shift of the frequency-selective network is  $180^\circ$ , the amplifier must have a phase shift of  $180^\circ$ . Also, the loop gain at  $\omega_0$  must be at least unity. Since the frequency-selective network has 12-dB attenuation, the amplifier gain must be at least 12 dB or 4 V/V.

**13.85**

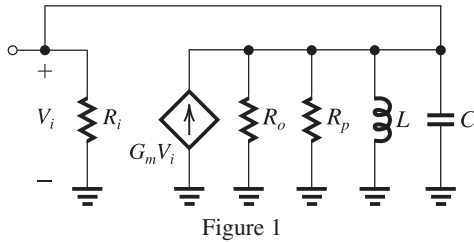


Figure 1 shows the resulting circuit. Here  $R_p$  can be found from

$$R_p = \omega_0 L Q$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6} \times 100 \times 10^{-12}}} = 10^8 \text{ rad/s}$$

Thus,

$$R_p = 10^8 \times 1 \times 10^{-6} \times 50 = 5 \text{ k}\Omega$$

The total parallel resistance can now be found as

$$R = R_i \parallel R_o \parallel R_p = 5 \parallel 5 \parallel 5 = 1.67 \text{ k}\Omega$$

The circuit will oscillate when the loop gain is unity, that is,

$$G_m R = 1 \Rightarrow G_m = \frac{1}{R} = 0.6 \text{ mA/V}$$

and the frequency of oscillation will be

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^8}{2\pi} = 15.92 \text{ MHz}$$

**13.86** Refer to Fig. 1.

$$\alpha \equiv \frac{V_-}{V_o} = \frac{r_1}{r_1 + r_2}$$

$$V_x = \alpha V_o + \frac{1}{sC} \frac{\alpha V_o}{R} = \alpha V_o \left(1 + \frac{1}{sCR}\right) \quad (1)$$

A node equation at X gives

$$\alpha \frac{V_o}{R} + sC V_x + \frac{V_x - V_o}{R} = 0 \quad (2)$$

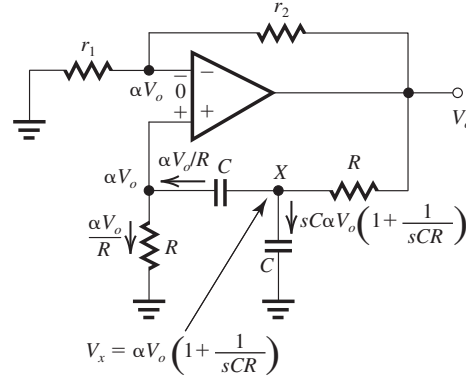


Figure 1

Substituting from Eq. (1) into Eq. (2)

$$\alpha \frac{V_o}{R} + sC \alpha V_o \left(1 + \frac{1}{sCR}\right) + \frac{\alpha V_o}{R} \left(1 + \frac{1}{sCR}\right) - \frac{V_o}{R} = 0$$

Dividing by  $V_o$  and rearranging terms, we obtain

$$s^2 + \frac{s}{\alpha CR} (3\alpha - 1) + \frac{1}{C^2 R^2} = 0$$

For  $s = j\omega$ ,

$$\left(-\omega^2 + \frac{1}{C^2 R^2}\right) + j \frac{\omega}{\alpha CR} (3\alpha - 1) = 0$$

Equating the real part to zero gives the frequency of oscillation

$$\omega_0 = \frac{1}{CR}$$

Equating the imaginary part to zero gives the condition for sustained oscillations as

$$3\alpha - 1 = 0 \Rightarrow \frac{r_2}{r_1} = 2$$

To ensure that oscillations start,  $(r_2/r_1)$  is made slightly greater than 2.

**13.87** We first obtain the loop gain  $L(s)$ . This was done in the solution to Problem 13.79 with the result that

$$L(s) = \frac{s \left(\frac{1}{CR}\right) \left(1 + \frac{r_2}{r_1}\right)}{s^2 + s \frac{3}{CR} + \frac{1}{C^2 R^2}}$$

The characteristic equation is found by equating  $L(s)$  to 1,

$$\frac{s \frac{1}{CR} \left(1 + \frac{r_2}{r_1}\right)}{s^2 + s \frac{3}{CR} + \frac{1}{C^2 R^2}} = 1$$



which can be rearranged into the form

$$s^2 + s \frac{1}{CR} \left( 2 - \frac{r_2}{r_1} \right) + \frac{1}{C^2 R^2} = 0$$

For the poles to be on the  $j\omega$  axis,

$$\frac{r_2}{r_1} = 2$$

which results in oscillations at

$$\omega_0 = \frac{1}{CR}$$

This value of  $(r_2/r_1)$  results in sustained oscillations. To ensure that oscillations start, we make  $r_2/r_1$  slightly greater than 2.

**13.88** For the circuit to oscillate, two conditions must be satisfied:

- (1) The total phase shift around the loop should be 0 or  $360^\circ$ , and
- (2) The loop gain must be at least unity.

Here we have three amplifier stages (Fig. P13.88). Thus the phase angle of each amplifier at the oscillation frequency  $\omega_0$ , must be  $120^\circ$ . Now, for each amplifier stage we have

$$V_o = -g_m V_i \frac{1}{\frac{1}{R} + sC}$$

$$\frac{V_o}{V_i} = -\frac{g_m R}{1 + sCR}$$

At  $s = j\omega_0$ , we have

$$\begin{aligned} \frac{V_o}{V_i}(j\omega_0) &= -\frac{g_m R}{1 + j\omega_0 CR} \\ &= -\frac{g_m R (1 - j\omega_0 CR)}{1 + (\omega_0 CR)^2} \end{aligned}$$

$$= \frac{g_m R}{1 + (\omega_0 CR)^2} (-1 + j\omega_0 CR)$$

For the phase angle to be  $120^\circ$ , we need

$$\tan 60 = \omega_0 CR$$

$$\Rightarrow \omega_0 = \frac{\sqrt{3}}{CR}$$

$$\left| \frac{V_o}{V_i}(j\omega_0) \right| = \frac{g_m R}{\sqrt{1 + (\omega_0 CR)^2}} = \frac{g_m R}{\sqrt{1 + 3}}$$

$$= 0.5g_m R$$

For a loop gain of unity, we have

$$(0.5g_m R)^3 = 1$$

$$g_m R = 2$$

$$g_m |_{\min} = \frac{2}{R}$$

**13.89** The sine wave across the LC circuit will have a peak amplitude of  $4V/\pi$  where  $V$  is the amplitude of the square wave; i.e., 2 V. Thus the peak amplitude of the sine wave will be 2.55 V.

**13.90** Refer to Fig. 13.38.

$$\begin{aligned} \frac{V_a}{V_b} &= \frac{Z_p}{Z_p + Z_s} \\ &= \frac{1}{1 + Z_s Y_p} \\ &= \frac{1}{1 + \left( R + \frac{1}{sC} \right) \left( \frac{1}{R} + sC \right)} \\ &= \frac{1}{1 + 1 + 1 + sCR + \frac{1}{sCR}} \\ &= \frac{s/CR}{s^2 + s(3/CR) + \frac{1}{(CR)^2}} \end{aligned}$$

which is a bandpass function with a center frequency  $\omega_0$  given by

$$\omega_0 = \frac{1}{CR}$$

and a pole- $Q$  of

$$Q = \frac{1}{3}$$

and a center-frequency gain of

$$\text{Gain} = \frac{1}{3}$$

Thus, the oscillation frequency is

$$\omega_0 = \frac{1}{CR}$$

and the condition of oscillation is

$$\frac{1}{3} \left( 1 + \frac{R_2}{R_1} \right) = 1$$

$$\Rightarrow \frac{R_2}{R_1} = 2$$

To ensure that oscillations start, we select  $R_2/R_1$  slightly greater than 2.

**13.91** Refer to Fig. 13.38. Assume that  $R_3 = R_6$  and  $R_4 = R_5$  and consider the magnitude of the positive peak. When  $v_O = \hat{V}_o$ ,  $D_2$  just conducts and clamps node  $b$  to a voltage

$V_b = V_1 + V_D \simeq \frac{\hat{V}_o}{3} + V_D$ . Neglecting the current through  $D_2$ , we can write

$$\frac{\hat{V}_o - V_b}{R_5} = \frac{V_b - (-15)}{R_6}$$

Thus,

$$\hat{V}_o = \left(1 + \frac{R_5}{R_6}\right)V_b + 15\frac{R_5}{R_6}$$

Substituting

$$V_b = \frac{1}{3}\hat{V}_o + V_D$$

we obtain

$$\hat{V}_o = \frac{15\left(\frac{R_5}{R_6}\right) + \left(1 + \frac{R_5}{R_6}\right)V_D}{\frac{2}{3} - \frac{1}{3}\frac{R_5}{R_6}} \quad (1)$$

To obtain an 8-V peak-to-peak output, we use

$$\hat{V}_o = 4 = \frac{15\left(\frac{R_5}{R_6}\right) + \left(1 + \frac{R_5}{R_6}\right) \times 0.7}{\frac{2}{3} - \frac{1}{3}\frac{R_5}{R_6}}$$

$$\Rightarrow \frac{R_5}{R_6} = 0.115$$

Since  $R_5 = 1 \text{ k}\Omega$ , then

$$R_6 = 8.66 \text{ k}\Omega$$

and

$$R_3 = 8.66 \text{ k}\Omega$$

If  $R_3$  and  $R_6$  are open circuited, substituting  $R_6 = \infty$  in Eq. (1) yields

$$\hat{V}_o = 1.5V_D = 1.05 \text{ V}$$

This result can be obtained directly from the circuit: If  $R_3$  and  $R_6$  are open circuited, the positive peak of the output will be the voltage at which  $D_2$  conducts. At this point,  $v_o = \hat{V}_o$  and  $v_1 = \frac{1}{3}\hat{V}_o$ , thus the voltage across  $V_D$  is  $\frac{2}{3}\hat{V}_o$  or, equivalently,

$$\hat{V}_o = 1.5V_D$$

A similar situation occurs at the negative peak at which diode  $D_1$  conducts and the voltage across it will be  $\frac{2}{3}\hat{V}_o$ , etc.

**13.92** Selecting  $R_p = R_s = R = 10 \text{ k}\Omega$ , we can find  $C$  from Eq. (13.88),

$$\omega_0 = \frac{1}{CR}$$

$$2\pi \times 10 \times 10^3 = \frac{1}{C \times 10 \times 10^3}$$

$$\Rightarrow C = 1.59 \text{ nF}$$

Selecting  $R_1 = 10 \text{ k}\Omega$ , we can determine  $R_2$  for sustained oscillations from Eq (13.89),

$$R_2 = 2R_1 = 20 \text{ k}\Omega$$

To ensure oscillations start we select  $R_2$  slightly larger than  $20 \text{ k}\Omega$ .

Next, we design the limiter circuit. Using Eq.

$$(13.11), 3 = \left[\left(\frac{R_5}{R_6}\right) \times 5 + \left(1 + \frac{R_5}{R_6}\right) \times 0.7\right] / \left(\frac{2}{3} - \frac{1}{3}\frac{R_5}{R_6}\right) \\ \Rightarrow \frac{R_5}{R_6} = 0.194$$

Selecting  $R_6 = 10 \text{ k}\Omega \Rightarrow R_5 = 1.94 \text{ k}\Omega$

From symmetry, we select  $R_4 = R_5 = 1.94 \text{ k}\Omega$

$R_3 = R_6 = 10 \text{ k}\Omega$

**13.93** Refer to Fig. 1 (next page).

$$I_1 = \frac{V_1}{R}$$

$$V_o = -R_f I_1 = -\frac{R_f}{R} V_1$$

$$\Rightarrow V_1 = -\left(\frac{R}{R_f}\right)V_o \quad (1)$$

$$I_2 = sCV_1$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + sCV_1$$

$$I_3 = \frac{V_1}{R}(1 + sCR)$$

$$V_2 = V_1 + I_3 R = V_1(2 + sCR)$$

$$I_4 = sCV_2 = sCV_1(2 + sCR)$$

$$I_5 = I_3 + I_4 = \frac{V_1}{R}(1 + sCR) + sCV_1(2 + sCR)$$

$$V_3 = V_2 + I_5 R$$

$$= V_1(2 + sCR) + V_1(1 + sCR) + sCRV_1(2 + sCR)$$

$$= V_1(3 + s4CR + s^2C^2R^2)$$

$$I_6 = sCV_3$$

$$= sCV_1(3 + s4CR + s^2C^2R^2)$$

$$I_7 = I_5 + I_6$$

$$= \frac{V_1}{R}(1 + sCR) + sCV_1(5 + s5CR + s^2C^2R^2)$$

$$V_x = V_3 + I_7 R$$

$$= V_1(3 + s4CR + s^2C^2R^2)$$

$$+ V_1(1 + sCR) + V_1(s5CR + s^25C^2R^2 + s^3C^3R^3)$$

$$= V_1(4 + s10CR + s^26C^2R^2 + s^3C^3R^3)$$

Substituting for  $V_1$  from Eq. (1) and equating  $V_x$  with  $V_o$ , we obtain

$$-\frac{R_f}{R} = 4 + s10CR + s^26C^2R^2 + s^3C^3R^3$$

For  $s = j\omega$

This figure belongs to Problem 13.93.

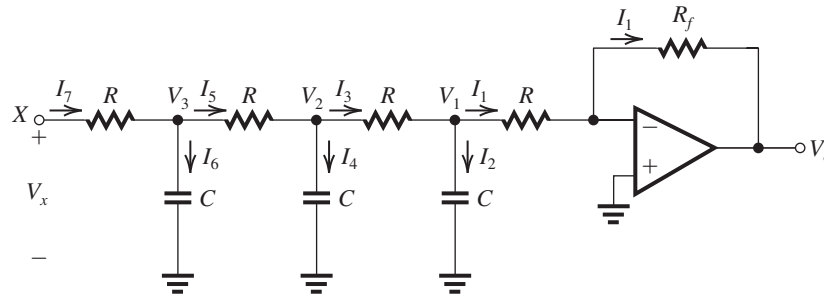


Figure 1

$$-\frac{R_f}{R} = (4 - 6\omega^2 C^2 R^2) + j\omega(10CR - \omega^2 C^3 R^3)$$

Equating the imaginary part on the RHS to zero, we obtain for the frequency of oscillation  $\omega_0$

$$\omega_0 = \frac{\sqrt{10}}{CR}$$

Equating the real parts on both sides gives the condition for sustained oscillations as

$$\frac{R_f}{R} = 6\frac{10}{C^2 R^2} C^2 R^2 - 4 = 56$$

That is,

$$R_f = 56R$$

Numerical values:

$$f_0 = 15 \text{ kHz}$$

$$R = 10 \text{ k}\Omega$$

$$CR = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3}$$

$$C = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3 \times 10 \times 10^3} = 3.36 \text{ nF}$$

$$R_f = 56 \times 10 = 560 \text{ k}\Omega$$

**13.94** Refer to the circuit in Fig. 13.43. Using Eq. (13.53),

$$\omega_0 = \frac{1}{CR}$$

$$\Rightarrow C = \frac{1}{\omega_0 R} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^3}$$

$$= 1.59 \text{ nF}$$

where we used  $R = 10 \text{ k}\Omega$ . Now, for sustained oscillations,

$$R_f = 2R = 20 \text{ k}\Omega$$

However, to ensure oscillation will start, we use a slightly smaller value for  $R_f$ .

To determine the limiter component values, we first derive an expression for the peak amplitude

of  $v_{o1}$ , i.e.  $\hat{v}_{o1}$ . At the time  $v_{o1} = \hat{v}_{o1}$ , diode  $D_2$  is conducting and clamping the voltage at the common node of  $R_3$  and  $R_4$  at  $V_D$  volts. Writing an equation for that node while neglecting the current in the diode relative to the current in  $R_3$  and  $R_4$ , we obtain

$$\frac{\hat{v}_{o1} - V_D}{R_3} = \frac{V_D - (-V)}{R_4}$$

where  $V$  is the magnitude of each of the two supply voltages. This equation can be rearranged to the form

$$\hat{v}_{o1} = V_D + \left(\frac{R_3}{R_4}\right)(V + V_D)$$

Thus, for  $V_D = 0.7 \text{ V}$  and  $V = 15 \text{ V}$ , to obtain  $\hat{v}_{o1} = 5 \text{ V}$ , the required  $(R_3/R_4)$  is found from

$$5 = 0.7 + \left(\frac{R_3}{R_4}\right)(15 + 0.7)$$

$$\Rightarrow \frac{R_3}{R_4} = 0.274$$

For  $R_4 = 10 \text{ k}\Omega$ ,

$$R_3 = 2.74 \text{ k}\Omega$$

$$\mathbf{13.95} \quad \omega_0 = \frac{1}{CR}$$

$$R = \frac{1}{2\pi \times 10 \times 10^3 \times 1.6 \times 10^{-9}} = 9.95 \text{ k}\Omega$$

The square wave  $v_2$  will have a peak-to-peak amplitude

$$V = 1.4 \text{ V}$$

The component at the fundamental frequency  $\omega_0$  will have a peak-to-peak amplitude of  $4V/\pi = 1.78 \text{ V}$ . The filter has a center-frequency gain of 2, thus at  $v_1$  the sine wave will have a peak-to-peak amplitude of approximately 3.6 V.

The output amplitude can be doubled by adding a diode in series with each of the diodes in the limiter.

**13.96** If the closed-loop amplifier in Fig. 13.38 exhibits a phase shift of  $-3^\circ$  for  $\omega$  around  $\omega_0$ , then the loop-gain expression in Eq. (13.119) becomes

$$L(j\omega) = \frac{(1 + R_2/R_1)e^{-j\phi}}{3 + j(\omega CR - 1/\omega CR)}$$

where

$$\phi = \frac{3\pi}{180} = \pi/60$$

Oscillation will occur at the frequency  $\omega_0$  for which the phase angle of  $L(j\omega)$  is  $0^\circ$ :

$$-\phi = \tan^{-1} \frac{1}{3} \left( \omega_0 CR - \frac{1}{\omega_0 CR} \right)$$

$$\omega_0 CR - \frac{1}{\omega_0 CR} = -3 \tan 3^\circ = -0.157$$

$$\Rightarrow \omega_0^2 + \frac{0.157}{CR} \omega_0 - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_0 = \frac{0.925}{CR}$$

**13.97** The characteristic equation can be written using the expression for the loop gain in Eq. (13.118) as follows:

$$1 - L(s) = 0$$

$$1 - \frac{1 + R_2/R_1}{3 + sCR + \frac{1}{sCR}} = 0$$

$$3 + sCR + \frac{1}{sCR} - 1 - \frac{R_2}{R_1} = 0$$

$$s^2 + s \left( 2 - \frac{R_2}{R_1} \right) / CR + \frac{1}{(CR)^2} = 0$$

Thus the poles have

$$\omega_0 = \frac{1}{CR}$$

and

$$Q = \frac{1}{2 - \frac{R_2}{R_1}}$$

The poles will be on the  $j\omega$  axis for  $R_2/R_1 = 2$  and will be in the right half of the  $s$ -plane for  $R_2/R_1 > 2$ . Q.E.D.

The frequency of oscillation is

$$\omega_0 = 1/CR.$$

**13.98** Figure 1 shows the circuit and some of the analysis for the purpose of determining the transfer function of the RC circuit. A node equation at X gives

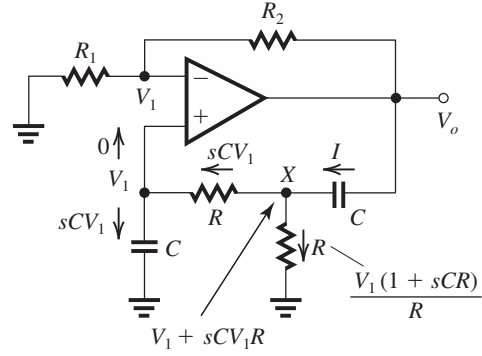


Figure 1

$$I = \frac{V_1}{R} (1 + sCR) + sCV_1$$

For  $V_o$  we now can write

$$V_o = V_1(1 + sCR) + \frac{I}{sC}$$

$$= V_1(1 + sCR) + \frac{V_1}{sCR} (1 + sCR) + V_1$$

$$\Rightarrow \frac{V_1}{V_o} = \frac{s/CR}{s^2 + s \left( \frac{3}{CR} \right) + \frac{1}{(CR)^2}}$$

The loop gain  $L(s)$  can now be found as

$$L(s) = \frac{s \left[ \left( 1 + \frac{R_2}{R_1} \right) / CR \right]}{s^2 + s \frac{3}{CR} + \frac{1}{(CR)^2}}$$

$$L(j\omega) = \frac{j\omega \left[ \left( 1 + \frac{R_2}{R_1} \right) / CR \right]}{\left[ \frac{1}{(CR)^2} - \omega^2 \right] + j \frac{3\omega}{CR}}$$

Zero phase shift will occur at  $\omega = \omega_0$ :

$$\omega_0 = \frac{1}{CR}$$

At  $\omega = \omega_0$ , we have

$$|L(j\omega)| = \frac{1}{3} \left( 1 + \frac{R_2}{R_1} \right)$$

For oscillations to begin, we need

$$\frac{1}{3} \left( 1 + \frac{R_2}{R_1} \right) \geq 1$$

$$\frac{R_2}{R_1} \geq 2$$

**13.99** The circuit in Fig. 1 depicts the situation when  $v_o$  is at its positive peak, i.e.,  $v_o = \hat{V}_o$ . We made use of the fact that for sustained oscillations the closed-loop gain of the amplifier must be 3.

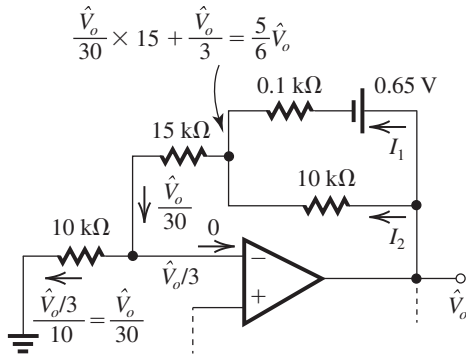


Figure 1

Thus the voltage at the inverting input terminal will be  $(\hat{V}_o/3)$ . Some of the analysis is shown in the figure. We complete the analysis as follows:

$$I_1 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o - 0.65}{0.1} = 10\hat{V}_o - \frac{50}{6}\hat{V}_o - 6.5$$

$$I_2 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o}{10} = 0.1\hat{V}_o - \frac{5}{60}\hat{V}_o$$

$$I_1 + I_2 = \frac{\hat{V}_o}{30}$$

Thus,

$$\hat{V}_o \left( 10 - \frac{50}{6} + 0.1 - \frac{5}{60} \right) - 6.5 = \frac{\hat{V}_o}{30}$$

$$\Rightarrow \hat{V}_o = 3.94 \text{ V}$$

Thus the peak-to-peak of the output sinusoid will be  $2 \times 3.94 = 7.88 \text{ V}$ .

**13.100** First we design the circuit to operate at 10 kHz.

$$\omega_0 = \frac{1}{CR}$$

$$2\pi \times 10 \times 10^3 = \frac{1}{CR}$$

$$\Rightarrow CR = 0.159 \times 10^{-4} \text{ s}$$

For  $R = 10 \text{ k}\Omega$ , we have

$$C = \frac{0.159 \times 10^{-4}}{10 \times 10^3} = 1.59 \text{ nF}$$

Now, refer to Eq. (13.8). If the closed-loop amplifier has an excess phase lag of  $5.7^\circ$ , then the gain will be  $\left(1 + \frac{R_2}{R_1}\right)e^{-j5.7^\circ}$ . Oscillations will occur at the frequency  $\omega_{01}$  at which the phase angle of the denominator is  $-5.7^\circ$ , that is,

$$\tan^{-1} \frac{1}{3} \left( \omega_{01}CR - \frac{1}{\omega_{01}CR} \right) = -5.7^\circ$$

$$\omega_{01}CR - \frac{1}{\omega_{01}CR} = 3 \tan(-5.7^\circ) = -0.3$$

$$\Rightarrow \omega_{01}^2 + \frac{0.3}{CR} \omega_{01} - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_{01} = \frac{0.86}{CR}$$

That is, the frequency of oscillation is reduced by 14% to

$$f_{01} = 0.86f_0 = 8.6 \text{ kHz}$$

To restore operation to  $f_0 = 10 \text{ kHz}$ , we modify the shunt resistor  $R$  to  $R_x$ , as indicated in Fig. 1.

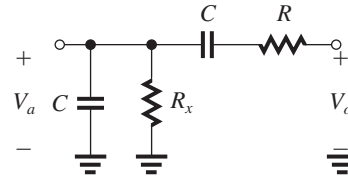


Figure 1

We now require the feedback RC circuit to have a phase shift of  $-(-5.7^\circ) = +5.7^\circ$  at  $f = 10 \text{ kHz}$ . The transfer function of the RC circuit can be found as follows:

$$\begin{aligned} \frac{V_a}{V_o} &= \frac{Z_p}{Z_p + Z_s} \\ &= \frac{1}{1 + Z_s Y_p} \\ &= \frac{1}{1 + \left( R + \frac{1}{sC} \right) \left( \frac{1}{R_x} + sC \right)} \\ &= \frac{1}{\left( 2 + \frac{R}{R_x} \right) + sCR + \frac{1}{sCR_x}} \end{aligned}$$

For  $s = j\omega$ , we have

$$\frac{V_a}{V_o} = \frac{1}{\left( 2 + \frac{R}{R_x} \right) + j \left( \omega CR - \frac{1}{\omega CR_x} \right)}$$

At  $\omega = \omega_0$ , the phase angle of  $\frac{V_a}{V_o}$  must be  $+5.7^\circ$  or equivalently, the phase angle of the denominator must be  $-5.7^\circ$ . Thus,

$$\tan^{-1} \frac{\omega_0 CR - \frac{1}{\omega_0 CR_x}}{2 + \frac{R}{R_x}} = -5.7^\circ$$

$$\omega_0 CR - \frac{1}{\omega_0 CR_x} = \left( 2 + \frac{R}{R_x} \right) \tan(-5.7^\circ)$$

$$= \left( 2 + \frac{R}{R_x} \right) \times -0.0998$$

Now,  $\omega_0 CR = 1$ , thus

$$1 - \frac{R}{R_x} = -0.0998 \left( 2 + \frac{R}{R_x} \right)$$

$$1 + 2 \times 0.0998 = \frac{R}{R_x} (1 - 0.0998) \Rightarrow \frac{R}{R_x} = 1.33$$

$$\Rightarrow R_x = 0.75 R = 7.5 \text{ k}\Omega$$

At  $\omega = \omega_0$  and for  $R_x = 7.5 \text{ k}\Omega$

$$\frac{V_a}{V_o}(\omega_0) = \frac{1}{\left( 2 + \frac{10}{7.5} \right) + j \left( 1 - \frac{10}{7.5} \right)}$$

$$= \frac{1}{3.33 - j0.33}$$

$$\left| \frac{V_a}{V_o}(\omega_0) \right| = \frac{1}{\sqrt{(3.33)^2 + (0.33)^2}} = \frac{1}{3.35}$$

Thus the magnitude of the gain of the amplifier must be 3.35 V/V. Thus,  $R_2/R_1$  must be changed to

$$\frac{R_2}{R_1} = 2.35$$

**13.101** Figure 1 (next page) shows the circuit with the additional resistance  $R$  included. The loop has been broken at the output of the op amp. The analysis will determine  $V_o/V_x$  and equate it to unity, which is the condition for sustained oscillations.

To begin, observe that the voltage  $V_1$  is related to  $V_o$  by

$$\frac{V_o}{V_1} = -\frac{R_f}{R} \quad (1)$$

Also, the current  $I_1$  is given by

$$I_1 = \frac{V_1}{R} \quad (2)$$

We now proceed to determine the various currents and voltages of the RC network as follows:

$$V_2 = V_1 + \frac{1}{sC} I_1$$

$$= V_1 + \frac{1}{sC} \frac{V_1}{R} = V_1 \left( 1 + \frac{1}{sCR} \right)$$

$$I_2 = \frac{V_2}{R} = \frac{V_1}{R} \left( 1 + \frac{1}{sCR} \right)$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + \frac{V_1}{R} \left( 1 + \frac{1}{sCR} \right)$$

$$= \frac{V_1}{R} \left( 2 + \frac{1}{sCR} \right)$$

$$V_3 = V_2 + \frac{I_3}{sC}$$

$$= V_1 \left( 1 + \frac{1}{sCR} \right) + \frac{V_1}{sCR} \left( 2 + \frac{1}{sCR} \right)$$

$$= V_1 \left( 1 + \frac{3}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$I_4 = \frac{V_3}{R} = \frac{V_1}{R} \left( 1 + \frac{3}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$I_5 = I_3 + I_4$$

$$= \frac{V_1}{R} \left( 2 + \frac{1}{sCR} \right) + \frac{V_1}{R} \left( 1 + \frac{3}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$= \frac{V_1}{R} \left( 3 + \frac{4}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$V_x = V_3 + \frac{I_5}{sC}$$

$$= V_1 \left( 1 + \frac{3}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$+ \frac{V_1}{sCR} \left( 3 + \frac{4}{sCR} + \frac{1}{s^2 C^2 R^2} \right)$$

$$= V_1 \left( 1 + \frac{6}{sCR} + \frac{5}{s^2 C^2 R^2} + \frac{1}{s^3 C^3 R^3} \right)$$

Now, by replacing  $V_1$  by the value from Eq. (1), we obtain

$$V_x = -V_o \frac{R}{R_f} \left( 1 + \frac{6}{sCR} + \frac{5}{s^2 C^2 R^2} + \frac{1}{s^3 C^3 R^3} \right)$$

For sustained oscillations  $V_o = V_x$ , thus

$$-\frac{R_f}{R} = 1 + \frac{6}{sCR} + \frac{5}{s^2 C^2 R^2} + \frac{1}{s^3 C^3 R^3}$$

For  $s = j\omega$ , we have

$$-\frac{R_f}{R} = 1 + \frac{6}{j\omega CR} - \frac{5}{\omega^2 C^2 R^2} - \frac{1}{j\omega^3 C^3 R^3}$$

$$= \left( 1 - \frac{5}{\omega^2 C^2 R^2} \right) - j \left( \frac{6}{\omega CR} - \frac{1}{\omega^3 C^3 R^3} \right)$$

Thus, oscillation will occur at the frequency that renders the imaginary part of the RHS zero:

$$\frac{6}{\omega_0 CR} = \frac{1}{\omega_0^3 C^3 R^3}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{6} CR}$$

At this frequency, the real part of the RHS must be equal to  $(-R_f/R)$ :

$$-\frac{R_f}{R} = 1 - \frac{5}{1/6} = -29$$

Thus,

$$R_f = 29R$$

which is the minimum required value for  $R_f$  to obtain sustained oscillations. Numerical values:

$$f_0 = \frac{1}{2\pi \sqrt{6} \times 16 \times 10^{-9} \times 10 \times 10^3}$$

This figure belongs to Problem 13.101.

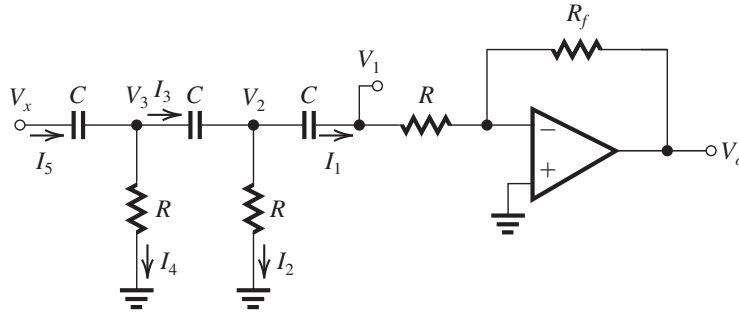


Figure 1

= 406 Hz

$R_f = 290 \text{ k}\Omega$

**13.102** Refer to the circuit in Fig. 13.42. Let  $v_o$  be at its positive peak  $\hat{v}_o$ . At this point, diode  $D_2$  is conducting and clamping the voltage at the common node of  $R_3$  and  $R_4$  to  $V_D$ . Writing an equation for that node while neglecting the diode current, we obtain

$$\frac{\hat{v}_o - V_D}{R_3} = \frac{V_D - (-V)}{R_4}$$

which can be rearranged to the form

$$\hat{v}_o = V_D + \left(\frac{R_3}{R_4}\right)(V + V_D) \quad \text{Q.E.D.}$$

For  $V = 5 \text{ V}$ ,  $V_D = 0.7 \text{ V}$ , and  $\hat{v}_o = 2 \text{ V}$ ,

$$2 = 0.7 + \left(\frac{R_3}{R_4}\right)(5 + 0.7)$$

$$\Rightarrow \frac{R_3}{R_4} = 0.228$$

**13.103** Refer to the circuit in Fig. 13.43 with the limiter eliminated and with the loop broken at X to determine the loop gain

$$L(s) = \frac{V_{o2}}{V_x}$$

To determine  $L(s)$ , we note that it is the product of the transfer functions of the inverting integrator formed around op amp 1:

$$\frac{V_{o1}}{V_x} = -\frac{1}{sCR} \tag{1}$$

and the noninverting integrator formed around op amp 2. To obtain the transfer function of the latter, refer to Fig. 1.

Writing a node equation at the positive input terminal of op amp 2, we obtain

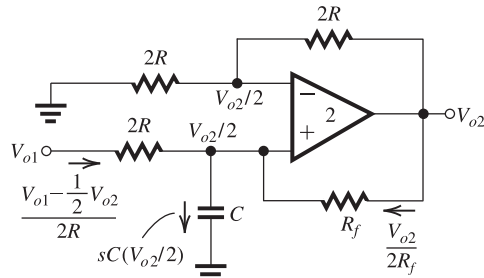


Figure 1

$$\frac{V_{o1} - \frac{1}{2}V_{o2}}{2R} + \frac{V_{o2}}{2R_f} = sC \frac{V_{o2}}{2}$$

$$\Rightarrow \frac{V_{o2}}{V_{o1}} = \frac{1}{sCR + \frac{1}{2} - \frac{R}{R_f}}$$

Substituting for  $R_f = \frac{2R}{1 + \Delta}$ , we obtain

$$\frac{V_{o2}}{V_{o1}} = \frac{1}{sCR - \frac{\Delta}{2}} \tag{2}$$

Using Eqs. (1) and (2), we obtain the loop gain as

$$L(s) = -\frac{1}{sCR} \frac{1}{sCR - \frac{\Delta}{2}}$$

The characteristic equation can now be written as

$$1 - L(s) = 0$$

$$1 + \frac{1}{sCR} \frac{1}{sCR - \frac{\Delta}{2}} = 0$$

$$sCR \left( sCR - \frac{\Delta}{2} \right) + 1 = 0$$

$$s^2 - s \frac{1}{CR} \frac{\Delta}{2} + \frac{1}{C^2R^2} = 0$$

Thus, the poles are

$$s = \frac{1}{2CR} \left( \frac{\Delta}{2} \right) \pm \frac{1}{2} \sqrt{\frac{1}{C^2R^2} \left( \frac{\Delta}{2} \right)^2 - \frac{4}{C^2R^2}}$$

For  $\Delta \ll 4$ , we have

$$s \simeq \frac{1}{CR} \left[ \left( \frac{\Delta}{4} \right) \pm j \right]$$

from which it is obvious that the poles are in the right half of the  $s$ -plane. Q.E.D.

**13.104** Figure 1 shows the circuit of the noninverting integrator. Observe that the voltage at the inverting input terminal of the op amp is  $V_{o2}/2$ . Assuming an ideal op amp, the voltage at the non-inverting input terminal also will be  $V_{o2}/2$ . Writing a node equation at the non-inverting input terminal gives

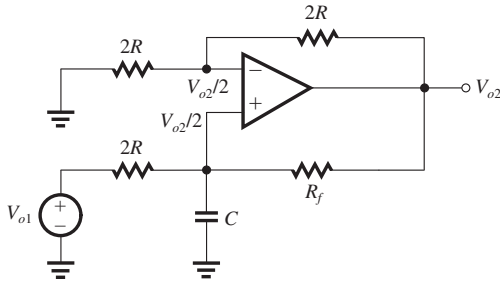


Figure 1

$$\frac{V_{o1} - (V_{o2}/2)}{2R} + \frac{V_{o2} - (V_{o2}/2)}{R_f} = sC \frac{V_{o2}}{2}$$

which can be rearranged and the transfer function ( $V_{o2}/V_{o1}$ ) written as

$$\frac{V_{o2}}{V_{o1}} = \frac{1}{sCR + \left( \frac{1}{2} - \frac{R}{R_f} \right)}$$

For perfect integration, the term between brackets in the denominator must be made to equal zero, that is

$$\frac{R}{R_f} = \frac{1}{2}$$

$$\Rightarrow R_f = 2R \quad \text{Q.E.D.}$$

**13.105** Figure 1(a) shows the filter circuit that is part of the oscillator in Fig. 13.45. We need to derive the transfer function  $V_1/V_2$ . We will do the derivation in two steps:

First, we will show that the input impedance  $Z_{in} \equiv V/I$  is an inductance  $L$  and we will find its value. Thus, the circuit has the equivalent shown in Fig. 1(b). Second, we will derive the transfer function of the circuit in Fig. 1(b) on the next page.

To obtain the input impedance  $Z_{in}$ , we analyze the circuit as shown in Fig. 1(a). The key point

here is that the ideal op amps cause the voltage between the two input terminals of each to be zero. Thus, the voltage  $V$  at the input terminal propagates to the other side of the circuit. The remainder of the analysis is straightforward with the result that the input  $I$  is given by

$$I = V/sCR^2$$

Thus, the input impedance  $Z_{in}$  is

$$Z_{in} \equiv \frac{V}{I} = sCR^2$$

which is that of an inductance  $L$ ,

$$L = CR^2$$

Next, consider the LC circuit in Fig 1(b):

$$\begin{aligned} \frac{V}{V_2} &= \frac{Z_{LC}}{Z_{LC} + QR} \\ &= \frac{1}{1 + QRY_{LC}} \\ &= \frac{1}{1 + QR(sC + \frac{1}{sL})} \\ &= \frac{s \frac{1}{QCR}}{s^2 + s \frac{1}{QCR} + \frac{1}{LC}} \end{aligned}$$

Returning to the circuit in Fig. 1(a) we note that  $V_1 = 2V$ , thus

$$\frac{V_1}{V_2} = \frac{s \frac{2}{QCR}}{s^2 + s \frac{1}{QCR} + \frac{1}{LC}}$$

Substituting for  $L = CR^2$ ,

$$\frac{V_1}{V_2} = \frac{s \frac{2}{QCR}}{s^2 + s \frac{1}{QCR} + \frac{1}{C^2R^2}}$$

which is a bandpass function with  $\omega_0 = 1/CR$  and  $Q$  determined by the resistance  $QR$ . The center-frequency gain is  $2V/V$ . Q.E.D.

**13.106** The different harmonic components will be attenuated relative to the fundamental by the selective response of the bandpass circuit. Let us first determine the magnitude of the transmission of the bandpass filter at a frequency  $\omega$  relative to that at the fundamental frequency  $\omega_0$ ,

$$\begin{aligned} T(s) &= \frac{s \left( \frac{\omega_0}{Q} \right)}{s^2 + s \left( \frac{\omega_0}{Q} \right) + \omega_0^2} \\ |T(j\omega)| &= \frac{\frac{\omega\omega_0}{Q}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left( \frac{\omega\omega_0}{Q} \right)^2}} \end{aligned}$$



This figure belongs to Problem 13.105.

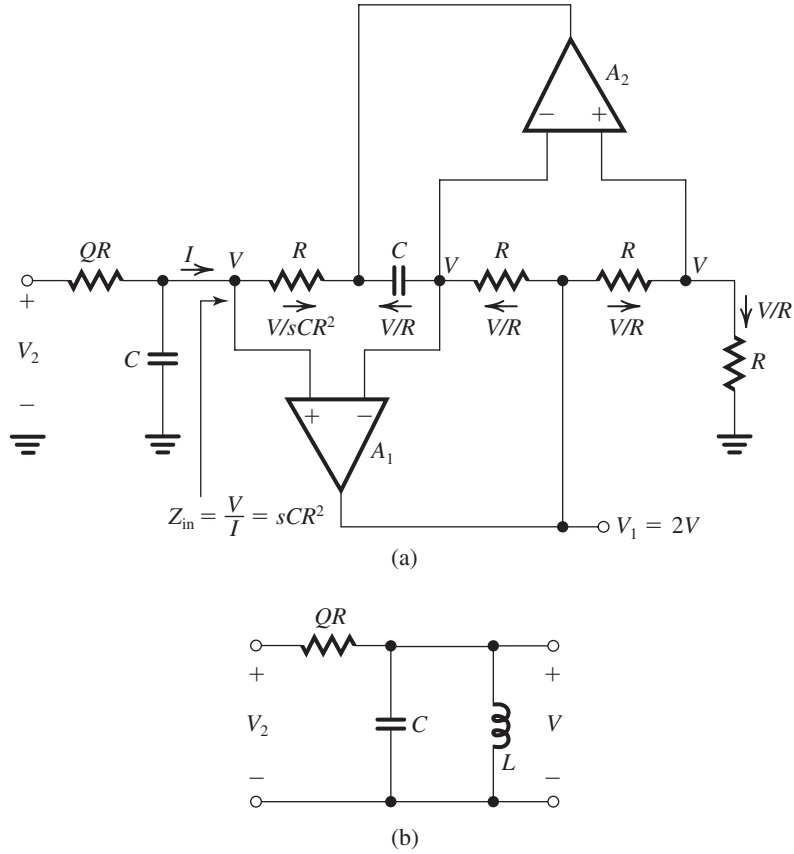


Figure 1

$$= 1/\sqrt{1 + Q^2 \left( \frac{\omega_0^2 - \omega^2}{\omega\omega_0} \right)^2}$$

$$= 1/\sqrt{1 + Q^2 \left( \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2}$$

For the  $n$ th harmonic, we have  $\omega = n\omega_0$ , where  $n = 3, 5, 7, \dots$ . Since  $n$  is large and  $Q = 20$  is large, we obtain

$$|T(j\omega)| \approx 1/Q \left( n - \frac{1}{n} \right)$$

Thus, relative to the fundamental, we have:

(a) The second harmonic = 0.

(b) The third harmonic

$$= \frac{1/3}{20 \left( 3 - \frac{1}{3} \right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$\frac{1/5}{20 \left( 5 - \frac{1}{5} \right)} = 2.08 \times 10^{-3}$$

(d) The rms of harmonics to the tenth:

$$\text{The seventh harmonic} = \frac{1/7}{20 \left( 7 - \frac{1}{7} \right)}$$

$$= 1.04 \times 10^{-3}$$

$$\text{The ninth harmonic} = \frac{1/9}{20 \left( 9 - \frac{1}{9} \right)}$$

$$= 0.625 \times 10^{-3}$$

Thus, the rms of harmonics to the tenth

$$= \sqrt{6.25^2 + 2.08^2 + 1.04^2 + 0.625^2} \times 10^{-3}$$

$$= 6.70 \times 10^{-3}$$

### 13.107

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage at the gate,  $V_g$ , can be expressed as

$$V_g = -s^2 LC_2 V_{gs} \quad (1)$$

The voltage at the source,  $V_s$ , can be expressed as

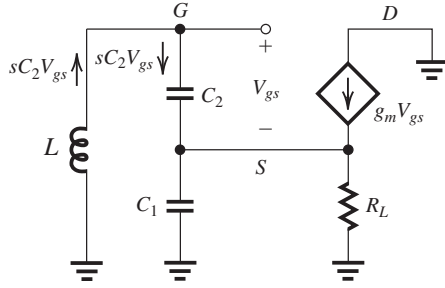


Figure 1

$$V_s = V_g - V_{gs}$$

Thus,

$$V_s = -s^2 LC_2 V_{gs} - V_{gs} \quad (2)$$

A node equation at S provides

$$sC_2 V_{gs} + g_m V_{gs} = \left( \frac{1}{R_L} + sC_1 \right) V_s$$

Substituting for  $V_s$  from Eq. (2), we obtain

$$sC_2 V_{gs} + g_m V_{gs} = - \left( \frac{1}{R_L} + sC_1 \right) (s^2 LC_2 + 1) V_{gs}$$

Dividing by  $V_{gs}$  and collecting terms, we obtain

$$s^3 LC_1 C_2 + s^2 \frac{LC_2}{R_L} + s(C_1 + C_2) + \left( g_m + \frac{1}{R_L} \right) = 0$$

For  $s = j\omega$ , we have

$$j\omega \left[ -\omega^2 LC_1 C_2 + (C_1 + C_2) \right] + \left( g_m + \frac{1}{R_L} - \omega^2 \frac{LC_2}{R_L} \right) = 0 \quad (3)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation  $\omega_0$  is the value of  $\omega$  at which the imaginary part is zero, thus

$$\omega_0^2 = 1 / \left[ L \left( \frac{C_1 C_2}{C_1 + C_2} \right) \right] \quad (4)$$

$$\Rightarrow \omega_0 = 1 / \sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}$$

The condition for sustained oscillations can be found by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{R_L} = \left( \frac{C_1 + C_2}{C_1} \right) \left( \frac{1}{R_L} \right)$$

$$\Rightarrow g_m R_L = \frac{C_2}{C_1}$$

To ensure that oscillations start, we use

$$g_m R_L > \frac{C_2}{C_1}$$

**13.108** Refer to Fig. 13.46(b). The design equations are given by Eqns. (13.141) and (13.136). Using (13.141),

$$\omega_0 = 1 / \sqrt{(L_1 + L_2)C}$$

we can determine  $(L_1 + L_2)$  as

$$\begin{aligned} L_1 + L_2 &= \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi \times 10^7)^2 \times 100 \times 10^{-12}} \\ &= 2.53 \mu\text{H} \end{aligned} \quad (1)$$

Using Eq. (13.136),

$$\frac{L_1}{L_2} = g_m R$$

where

$$R = 20 \parallel 20 = 10 \text{ k}\Omega$$

Thus,

$$\frac{L_1}{L_2} = 2 \times 10 = 20 \quad (2)$$

Eqns. (1) and (2) can be solved together to provide

$$L_1 = 2.41 \mu\text{H}$$

$$L_2 = 0.12 \mu\text{H}$$

**13.109** From Exercise 13.37, we have

$$L = 0.52 \text{ H}$$

$$C_s = 0.012 \text{ pF}$$

$$C_p = 4 \text{ pF}$$

$$C_{\text{eq}} = \frac{C_s \left( C_p + \frac{C_1 C_2}{C_1 + C_2} \right)}{C_s + C_p + \frac{C_1 C_2}{C_1 + C_2}}$$

$$C_2 = 10 \text{ pF} \quad C_1 = 1 \text{ to } 10 \text{ pF}$$

$$C_L = \frac{0.012 \left( 4 + \frac{10 \times 1}{10 + 1} \right)}{\left( 0.012 + 4 + \frac{10}{11} \right)} = 0.01197 \text{ pF}$$

$$C_H = \frac{0.012 \left( 4 + \frac{10 \times 10}{10 + 10} \right)}{\left( 0.012 + 4 + \frac{100}{20} \right)} = 0.01198 \text{ pF}$$

$$\therefore f_{0H} = \frac{1}{2\pi [0.52 \times 0.01197 \times 10^{-12}]^{1/2}}$$

$$= 2.0173 \text{ MHz}$$

$$f_{0L} = \left[ 2\pi (0.52 \times 0.01198 \times 10^{-12})^{1/2} \right]^{-1}$$

$$= 2.0165 \text{ MHz}$$

$$\text{Difference} = 800 \text{ Hz}$$

## 13.110

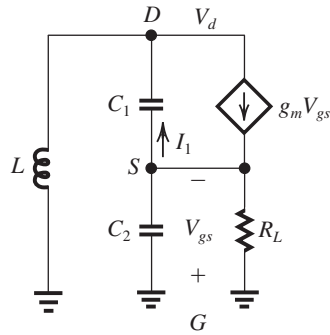


Figure 1

Figure 1 shows the equivalent circuit where we have neglected  $r_o$ . A node equation at  $S$  provides the following expression for  $I_1$ :

$$I_1 = V_{gs} \left( g_m + \frac{1}{R_L} + sC_2 \right)$$

The drain voltage  $V_d$  can now be found as

$$V_d = -V_{gs} - \frac{1}{sC_1} I_1$$

$$V_d = -V_{gs} - \frac{1}{sC_1} \left( g_m + \frac{1}{R_L} + sC_2 \right) V_{gs} \quad (1)$$

A node equation at  $D$  provides

$$\begin{aligned} \frac{V_d}{sL} &= V_{gs} \left( g_m + \frac{1}{R_L} + sC_2 \right) - g_m V_{gs} \\ &= V_{gs} \left( \frac{1}{R_L} + sC_2 \right) \end{aligned}$$

Substituting for  $V_d$  from (1), we obtain

$$\begin{aligned} -V_{gs} - \frac{1}{sC_1} \left( g_m + \frac{1}{R_L} + sC_2 \right) V_{gs} &= \\ V_{gs} sL \left( \frac{1}{R_L} + sC_2 \right) \end{aligned}$$

Dividing by  $V_{gs}$  and collecting terms results in

$$s^3 LC_1 C_2 + s^2 \frac{LC_1}{R_L} + s(C_1 + C_2) + g_m + \frac{1}{R_L} = 0$$

For  $s = j\omega$ , we have

$$j\omega \left[ -\omega^2 LC_1 C_2 + (C_1 + C_2) \right] + \left( -\omega^2 \frac{LC_1}{R_L} + g_m + \frac{1}{R_L} \right) = 0 \quad (2)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation  $\omega_0$  is the value of  $\omega$  at which the imaginary part is zero, thus

$$\omega_0^2 = \frac{1}{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)} \quad (3)$$

or

$$\omega_0 = 1 / \sqrt{L \frac{C_1 C_2}{C_1 + C_2}}$$

The condition for sustained oscillation can be found by equating the real part in Eq. (2) to zero at  $\omega = \omega_0$  and by making use of Eq. (3). Thus,

$$\begin{aligned} -\frac{C_1 + C_2}{C_2} \frac{1}{R_L} + g_m + \frac{1}{R_L} &= 0 \\ \Rightarrow g_m R_L &= \frac{C_1}{C_2} \end{aligned}$$

While to ensure that oscillations start, we make

$$g_m R_L > \frac{C_1}{C_2}$$

**13.111** Figure 1 shows the equivalent circuit where  $r_o$  has been absorbed into  $R_L$ . We have neglected  $R_f$  with the assumption that  $R_f \gg \omega_0 L$ . From Fig. 1, we can write

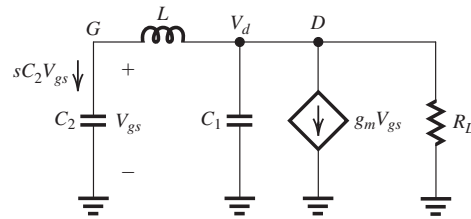


Figure 1

$$V_d = V_{gs} + sL \times sC_2 V_{gs}$$

Thus,

$$V_d = V_{gs} (1 + s^2 LC_2) \quad (1)$$

A node equation at  $D$  provides

$$sC_2 V_{gs} + sC_1 V_d + \frac{V_d}{R_L} + g_m V_{gs} = 0$$

Substituting for  $V_d$  from (1),

$$\begin{aligned} sC_2 V_{gs} + sC_1 (1 + s^2 LC_2) V_{gs} \\ + \frac{1}{R_L} (1 + s^2 LC_2) V_{gs} + g_m V_{gs} &= 0 \end{aligned}$$

Dividing by  $V_{gs}$  and collecting terms gives

$$s^3 LC_1 C_2 + s^2 \frac{LC_2}{R_L} + s(C_1 + C_2) + \left( g_m + \frac{1}{R_L} \right) = 0$$

Replacing  $s$  by  $j\omega$ , we get

$$j\omega \left[ -\omega^2 LC_1 C_2 + (C_1 + C_2) \right] + \left[ -\omega^2 \frac{LC_2}{R_L} + \left( g_m + \frac{1}{R_L} \right) \right] = 0 \quad (2)$$

Equating the imaginary term to zero gives the frequency of oscillation as

$$\omega_0 = 1/\sqrt{L \frac{C_1 C_2}{C_1 + C_2}} \quad (3)$$

The condition of oscillation is found by equating the real term in (2) to zero and making use of (3). The result is

$$g_m R_L = \frac{C_2}{C_1}$$

To ensure that oscillations start, we make

$$g_m R_L > \frac{C_2}{C_1}$$

**13.112 (a)**

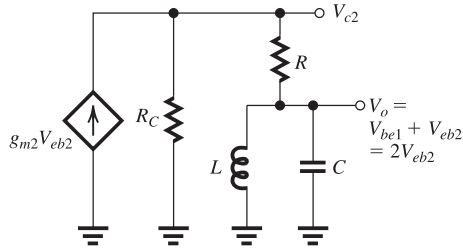


Figure 1

Figure 1 shows the equivalent circuit.

(b) Oscillations will occur at the frequency for which the tuned circuit has infinite impedance. This is because at this frequency the phase shift around the loop will be zero. Thus,

$$\omega_0 = 1/\sqrt{LC}$$

At this frequency, no current flows through  $R$ . Thus the voltage  $V_{c2}$  will be

$$V_{c2} = (g_m2 V_{eb2})R_C$$

and the voltage  $V_o$  will equal  $V_{c2}$ , thus

$$(g_m2 V_{eb2})R_C = V_o = 2V_{eb2}$$

which yields the condition for sustained oscillation as

$$g_m2 R_C = 2$$

For oscillations to start, we impose the condition

$$g_m2 R_C > 2 \quad (1)$$

But

$$g_m2 \approx \frac{I/2}{V_T}$$

Thus, the condition in (1) can be expressed as

$$IR_C > 4V_T$$

that is,

$$IR_C > 0.1 \text{ V}$$

(c) Selecting  $IR_C = 1 \text{ V}$  means that oscillations will start and will grow in amplitude until  $V_o$  is

large enough to cause  $Q_1$  and  $Q_2$  to alternately turn on and off. When this happens the collector current of  $Q_2$  will be 0 in half a cycle and equal to  $I$  in the other half cycle. Thus a square wave voltage of amplitude  $IR_C = 1 \text{ V}$  peak-to-peak will develop at the collector of  $Q_2$ . This square wave voltage is applied through  $R$  to the bandpass filter formed by the RLC circuit. Thus, the sinusoid that develops across the LC circuit—that is,  $V_o$ —will have a frequency  $\omega_0$  and a peak-to-peak amplitude of  $\left(\frac{4}{\pi} \times 1\right) = \frac{4}{\pi} \text{ V}$ . There will be third, fifth, and other odd harmonics, but those will be attenuated because of the selectivity of the bandpass RLC circuit.

**13.113**  $\omega_0 = 20 \text{ Grad/s} = 20 \times 10^9 \text{ rad/s}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$20 \times 10^9 = \frac{1}{\sqrt{5 \times 10^{-9} \times C}}$$

$$\Rightarrow C = 0.5 \text{ pF}$$

$$R_p = \omega_0 L Q$$

$$= 20 \times 10^9 \times 5 \times 10^{-9} \times 10$$

$$= 1000 \Omega = 1 \text{ k}\Omega$$

$$r_o \parallel R_p = 5 \parallel 1 = \frac{5}{6} \text{ k}\Omega$$

$$g_m|_{\min} = \frac{1}{\frac{5}{6} \times 10^3} = 1.2 \text{ mA/V}$$

**13.114**  $V_{TH} = -V_{TL} = 1 \text{ V}$

$$L_+ = -L_- = 5 \text{ V}$$

$$V_{TH} = \beta L_+$$

$$1 = \frac{R_1}{R_1 + R_2} \times 5$$

$$\Rightarrow \frac{R_2}{R_1} = 4$$

For  $R_1 = 10 \text{ k}\Omega$ , we have

$$R_2 = 40 \text{ k}\Omega$$

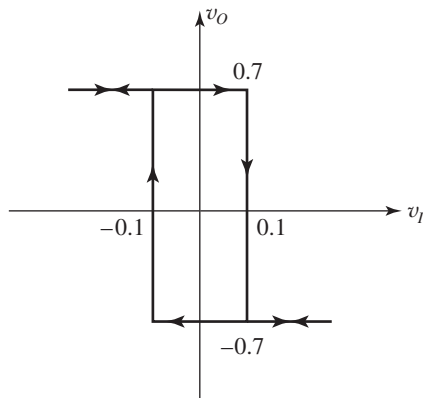
**13.115** See figure next page. Output levels =  $\pm 0.7 \text{ V}$

$$\text{Threshold levels} = \pm \frac{10}{10 + 60} \times 0.7 = 0.1 \text{ V}$$

$$i_{D, \max} = \frac{12 - 0.7}{10} - \frac{0.7}{10 + 60} = 1.12 \text{ mA}$$

**13.116 (a)** A 0.5-V peak sine wave is not large enough to change the state of the circuit. Hence, the output will be either +12 V or -12 V.

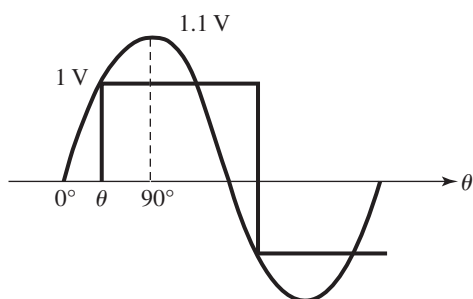
This figure belongs to Problem 13.115.



(b) The 1.1-V peak sine wave will change the state when

$$1.1 \sin \theta = 1$$

$$\theta = 65.40$$



$\therefore$  The output is a symmetric square wave of frequency  $f$ , and it lags the sine wave by an angle of  $65.4^\circ$ . The square wave has a swing of  $\pm 12$  V.

Since  $V_{TH} = -V_{TL} = 1$  V, if the average shifts by an amount so either the +ve or -ve swing is  $< 1$  V, then no change of state will occur. Clearly, if the shift is 0.1 V, the output will be a DC voltage.

$$13.117 \quad T = 2\tau \ln \frac{1 + \beta}{1 - \beta}, \quad \beta = \frac{R_1}{R_1 + R_2} = \frac{10}{26}$$

$$T = 2(5 \times 10^{-9}) (62 \times 10^3) \ln \left( \frac{1 + 10/26}{1 - 10/26} \right)$$

$$T = 0.503 \text{ ms} \Rightarrow f = 1989 \text{ Hz}$$

13.118 Refer to Fig. 13.56. The circuit is shown in Fig. 1 (next page). For the symmetrical triangular wave to have 2-V peak-to-peak amplitude,

$$V_{TH} = -V_{TL} = 1 \text{ V}$$

Also, we have

$$L_+ = -L_- = 5 \text{ V}$$

Using Eq. (13.155),

$$T_2 = \frac{T}{2} = CR \frac{V_{TH} - V_{TL}}{-L_-}$$

where  $T = \frac{1}{f} = \frac{1}{10 \times 10^3} = 10^{-4}$  s and using

$C = 10^{-9}$  F, then

$$0.5 \times 10^{-4} = 10^{-9} \times R \times \frac{1 - (-1)}{-(-5)}$$

$$\Rightarrow R = 125 \text{ k}\Omega$$

For the biastable,

$$V_{TL} = -L_+ \left( \frac{R_1}{R_2} \right)$$

$$-1 = -5 \times \frac{R_1}{R_2}$$

$$\Rightarrow R_2 = 5 R_1$$

Selecting  $R_1 = 20 \text{ k}\Omega$ , we need

$$R_2 = 100 \text{ k}\Omega$$

### 13.119

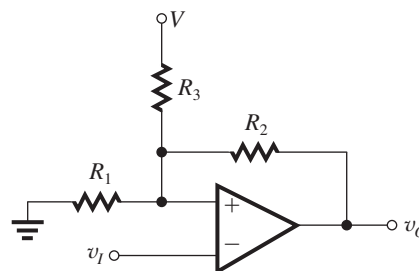


Figure 1

(a) Refer to Fig. 1. With  $v_o = L_+$ , the voltage at the op amp positive input terminal will be  $V_{TH}$ . Now, writing a node equation at the op amp positive input terminal, we have

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_+ - V_{TH}}{R_2}$$

$$V_{TH} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{L_+}{R_2} + \frac{V}{R_3}$$

$$\Rightarrow V_{TH} = \left( \frac{L_+}{R_2} + \frac{V}{R_3} \right) (R_1 \parallel R_2 \parallel R_3)$$

Similarly, we can obtain

$$V_{TL} = \left( \frac{L_-}{R_2} + \frac{V}{R_3} \right) (R_1 \parallel R_2 \parallel R_3)$$

(b)  $L_+ = -L_- = 10$  V,  $V = 15$  V,  $R_1 = 10 \text{ k}\Omega$

$$V_{TH} = 5.1 = \left( \frac{10}{R_2} + \frac{15}{R_3} \right) (R_1 \parallel R_2 \parallel R_3)$$

This figure belongs to Problem 13.118.

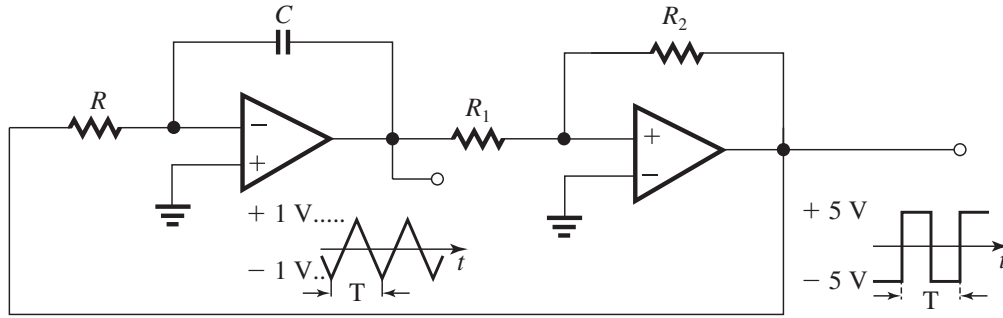


Figure 1

$$\frac{5.1}{R_1} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{10}{R_2} + \frac{15}{R_3}$$

$$0.51 = \frac{4.9}{R_2} + \frac{9.9}{R_3} \quad (1)$$

$$V_{TL} = 4.9 = \left( \frac{-10}{R_2} + \frac{15}{R_3} \right) (R_1 \parallel R_2 \parallel R_3)$$

$$\frac{4.9}{R_1} + \frac{4.9}{R_2} + \frac{4.9}{R_3} = -\frac{10}{R_2} + \frac{15}{R_3}$$

$$0.49 = \frac{-14.9}{R_2} + \frac{10.1}{R_3} \quad (2)$$

Multiplying Eq. (1) by  $\left(\frac{14.9}{4.9}\right)$ , we obtain

$$1.55 = \frac{-14.9}{R_2} + \frac{30.1}{R_3} \quad (3)$$

Adding (2) and (3) gives

$$2.04 = \frac{40.2}{R_3}$$

$$\Rightarrow R_3 = 19.7 \text{ k}\Omega$$

Substituting in Eq. (1), we obtain

$$0.51 = \frac{4.9}{R_2} + \frac{9.9}{19.7}$$

$$\Rightarrow R_2 = \frac{4.9}{0.0076} = 656.7 \text{ k}\Omega$$

**13.120** Using Eq. (13.149),

$$-1 = -5 \times \frac{10}{R_2}$$

$$\Rightarrow R_2 = 50 \text{ k}\Omega$$

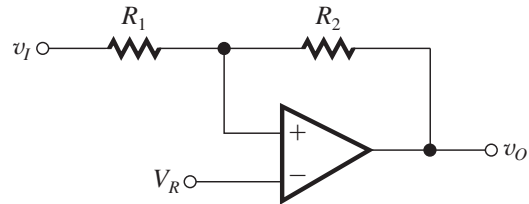
**13.121**

(a) For  $v_I = V_{TL}$  and  $v_O = L_+$  initially

$$\frac{L_+ - V_R}{R_2} = \frac{V_R - V_{TL}}{R_1}$$

$$V_{TL} = V_R + \frac{R_1}{R_2} V_R - \frac{R_1}{R_2} L_+$$

$$\therefore V_{TL} = V_R \left( 1 + \frac{R_1}{R_2} \right) - \frac{R_1}{R_2} L_+$$



Similarly,

$$\frac{L_- - V_R}{R_2} = \frac{V_R - V_{TH}}{R_1}$$

$$V_{TH} = V_R \left( 1 + \frac{R_1}{R_2} \right) - \frac{R_1}{R_2} L_-$$

(b) Given

$$L_+ = -L_- = V$$

$$R_1 = 10 \text{ k}\Omega$$

$$V_{TL} = 0$$

$$V_{TH} = V/10$$

Substituting these values, we get

$$0 = V_R \left( 1 + \frac{10}{R_2} \right) - \left( \frac{10}{R_2} \right) V \quad (1)$$

$$\frac{V}{10} = V_R \left( 1 + \frac{10}{R_2} \right) + \left( \frac{10}{R_2} \right) V \quad (2)$$

Subtracting Eq. (2) from Eq. (1), we obtain

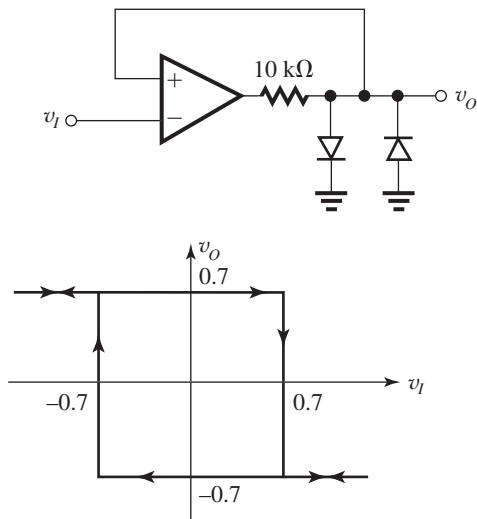
$$-\frac{V}{10} = -\frac{20}{R_2} \times V$$

$$\Rightarrow R_2 = 200 \text{ k}\Omega$$

$$0 = V_R \left( 1 + \frac{10}{200} \right) - \frac{10}{200} V$$

$$V_R = \frac{10/200 \text{ V}}{1 + 10/200} = \frac{V}{21}$$

13.122



13.123 Refer to Fig. 13.55(b) and Eq. (13.153).

$$\beta = \frac{R_1}{R_1 + R_2} = 0.5$$

$$f = 10\text{ kHz} \Rightarrow T = 10^{-4}\text{ s}$$

$$10^{-4} = 2\tau \ln \frac{1 + 0.5}{1 - 0.5}$$

$$\Rightarrow \tau = 0.455 \times 10^{-4}\text{ s}$$

$$= 1 \times 10^{-9} \times R$$

$$\Rightarrow R = 45.5\text{ k}\Omega$$

Choose  $R_1 = R_2 = 10\text{ k}\Omega$

**Chapter 14**

**Solutions to Exercises within the Chapter**

**Ex: 14.1** Observe the two PMOS transistors in series with gates connected to inputs *A* and *B*, and the parallel path with three PMOS transistors in series with gates connected to inputs *C*, *D*, and *E*. Therefore, the PUN realizes the function  $Y = \overline{AB} + \overline{CDE}$ .

**Ex: 14.2**

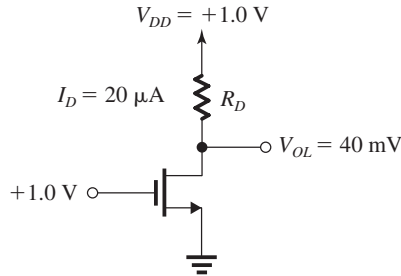


Figure 1

Refer to Fig. 1.

$$r_{DS} = \frac{V_{OL}}{I_D} = \frac{40 \text{ mV}}{20 \mu\text{A}} = 2 \text{ k}\Omega$$

Substituting in the expression given for  $r_{DS}$ , we get

$$2 = \frac{1}{0.540 \times \left(\frac{W}{L}\right) \times (1.0 - 0.35)}$$

$$\Rightarrow \frac{W}{L} = 1.42$$

The value of  $R_D$  can be obtained from

$$R_D = \frac{V_{DD} - V_{OL}}{I_D} = \frac{1.0 - 0.04}{0.02} = 48 \text{ k}\Omega$$

When the switch is open,  $I_D = 0$  and

$$P_{\text{Drawn}} = V_{DD}I_D = 0$$

When the switch is closed,  $I_D = 20 \mu\text{A}$ , and

$$P_{\text{Drawn}} = V_{DD}I_D = 1.0 \times 20 = 20 \mu\text{W}$$

**Ex: 14.3** If  $V_x$  remains unchanged at 0.0225 V, then

$$k_n R_D = \frac{1}{V_x} = \frac{1}{0.0225} = 44.44$$

For  $R_D$  to be 10 k $\Omega$ ,

$$k_n = \frac{44.44}{10} = 4.444 \text{ mA/V}^2$$

Thus,

$$4.444 = 0.54 \times \frac{W}{L}$$

$$\Rightarrow \frac{W}{L} = 8.22$$

The parameters  $V_{OH}$ ,  $V_{OL}$ ,  $V_{IL}$ ,  $V_{IH}$  and thus  $NM_L$  and  $NM_H$  do not depend on the value  $R$  (but on  $V_x$ ) and thus their values will not change.

The current  $I_{DD}$  drawn from the power supply during the low-output interval becomes

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} = \frac{1.0 - 0.033}{10} = 96.7 \mu\text{A}$$

and the power drawn from the supply during the low-output interval is

$$P_D = V_{DD}I_D = 1.0 \times 96.7 = 96.7 \mu\text{W}$$

Since the inverter spends half of the time in this state, we have

$$P_{\text{Daverage}} = \frac{1}{2}P_D = 48 \mu\text{W}$$

**Ex: 14.4**  $k_n R_D = \frac{1}{V_x}$

For  $R_D = 10 \text{ k}\Omega$  and

$$k_n = k'_n \left(\frac{W}{L}\right) = 0.54 \times 1.5 = 0.81 \text{ mA/V}^2$$

we obtain

$$V_x = \frac{1}{0.81 \times 10} = 0.12 \text{ V}$$

Now,

$$V_{OH} = 1.0 \text{ V}$$

From Eq. (14.22) we get

$$V_{OL} = \frac{1.0}{1 + \frac{1.0 - 0.35}{0.12}} = 0.16 \text{ V}$$

From Eq. (14.12) we obtain

$$V_{IL} = V_t + V_x = 0.35 + 0.12 = 0.47 \text{ V}$$

From Eq. (14.20) we have

$$V_{IH} = 0.35 + 1.63\sqrt{1.0 \times 0.12} - 0.12 = 0.80 \text{ V}$$

Thus,

$$NM_L = V_{IL} - V_{OL} = 0.47 - 0.16 = 0.31 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 1.0 - 0.80 = 0.20 \text{ V}$$

During the output-low interval, we have

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} = \frac{1.0 - 0.16}{10} = 84 \mu\text{A}$$

and

$$P_D = 1.0 \times 84 = 84 \mu\text{W}$$

Thus,

$$P_{\text{Daverage}} = \frac{1}{2}P_D = 42 \mu\text{W}$$



**Ex: 14.5** (a) For  $V_M = 0.6 \text{ V} = \frac{1}{2} V_{DD}$ , the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}$$

$$\Rightarrow \frac{W_p}{0.13} = 4$$

$$\Rightarrow W_p = 0.52 \mu\text{m}$$

(b)  $V_{OH} = V_{DD} = 1.2 \text{ V}$

$$V_{OL} = 0 \text{ V}$$

To obtain  $V_{IH}$ , we use Eq. (14.35):

$$\begin{aligned} V_{IH} &= \frac{1}{8}(5V_{DD} - 2V_t) \\ &= \frac{1}{8}(5 \times 1.2 - 2 \times 0.4) \\ &= 0.65 \text{ V} \end{aligned}$$

To obtain  $V_{IL}$ , we use Eq. (14.36):

$$\begin{aligned} V_{IL} &= \frac{1}{8}(3V_{DD} + 2V_t) \\ &= \frac{1}{8}(3 \times 1.2 + 2 \times 0.4) \\ &= 0.55 \text{ V} \end{aligned}$$

The noise margins can now be found as

$$\begin{aligned} NM_H &= V_{OH} - V_{IH} \\ &= 1.2 - 0.65 = 0.55 \text{ V} \\ NM_L &= V_{IL} - V_{OL} \\ &= 0.55 - 0 = 0.55 \text{ V} \end{aligned}$$

(c) With  $v_I = V_{DD}$  and  $v_O$  low, the output resistance is  $r_{DSN}$ :

$$\begin{aligned} r_{DSN} &= 1 / \left[ \mu_n C_{ox} \left( \frac{W}{L} \right)_n (V_{DD} - V_t) \right] \\ &= \frac{1}{0.43 \times 1(1.2 - 0.4)} = 2.9 \text{ k}\Omega \end{aligned}$$

With  $v_I = 0 \text{ V}$  and  $v_O = V_{DD}$ , the output resistance is  $r_{DSP}$ :

$$r_{DSP} = 1 / \left[ \mu_p C_{ox} \left( \frac{W}{L} \right)_p (V_{DD} - |V_t|) \right]$$

$$= \frac{1}{\frac{0.43}{4} \times 4(1.2 - 0.4)} = 2.9 \text{ k}\Omega$$

The two output resistances are equal because the inverter is matched.

(d) Using Eq. (14.39), we obtain

$$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r + 1}$$

where

$$|V_{tp}| = V_{tn} = 0.4 \text{ V}$$

and

$$r = \sqrt{\frac{\mu_p W_p}{\mu_n W_n}} = \sqrt{\frac{1}{4}} \times 1 = \frac{1}{2}$$

Thus,

$$V_M = \frac{0.5(1.2 - 0.4) + 0.4}{0.5 + 1} = 0.53 \text{ V}$$

**Ex: 14.6** To obtain  $V_M = 2.5 \text{ V} = \frac{1}{2} V_{DD}$ , the inverter must be matched, thus

$$\mu_n C_{ox} \left( \frac{W}{L} \right)_n = \mu_p C_{ox} \left( \frac{W}{L} \right)_p$$

$$2\mu_p C_{ox} \left( \frac{W}{L} \right)_n = \mu_p C_{ox} \left( \frac{W}{L} \right)_p$$

$$\left( \frac{W}{L} \right)_p = 2 \left( \frac{W}{L} \right)_n \quad (1)$$

For  $v_I = V_{DD} = 5 \text{ V}$  and  $v_O = 0.2 \text{ V}$ ,  $Q_N$  will be operating in the triode region. Thus,

$$i_D = \mu_n C_{ox} \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$0.2 = 0.05 \times \left( \frac{W}{L} \right)_n \left[ (5 - 1) \times 0.2 - \frac{1}{2} \times 0.2^2 \right]$$

$$\Rightarrow \left( \frac{W}{L} \right)_n = 5.13 \simeq 5$$

$$\left( \frac{W}{L} \right)_p = 2 \times 5.13 = 10.26 \simeq 10$$

**Solutions to End-of-Chapter Problems**

**14.1** (a)  $R_{on} = r_{DSN}$

$$= \frac{1}{(\mu_n C_{ox}) \left(\frac{W}{L}\right)_n (V_{DD} - V_m)} \quad (1)$$

$$= \frac{1}{0.470 \times 1.5(1 - 0.35)} = 2.18 \text{ k}\Omega$$

(b)  $R_{on} = r_{DSP}$

$$= \frac{1}{(\mu_p C_{ox}) \left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp}|)} \quad (2)$$

$$= \frac{1}{0.190 \times 1.5(1 - 0.35)} = 5.40 \text{ k}\Omega$$

(c) From (1) and (2) since  $V_m = -|V_{tp}|$ , then if  $R_{on}$  are to be equal, then

$$(\mu_n C_{ox}) \left(\frac{W}{L}\right)_n = (\mu_p C_{ox}) \left(\frac{W}{L}\right)_p$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} \left(\frac{W}{L}\right)_n$$

$$= \frac{470}{190} \times 1.5 = 3.71$$

**14.2**

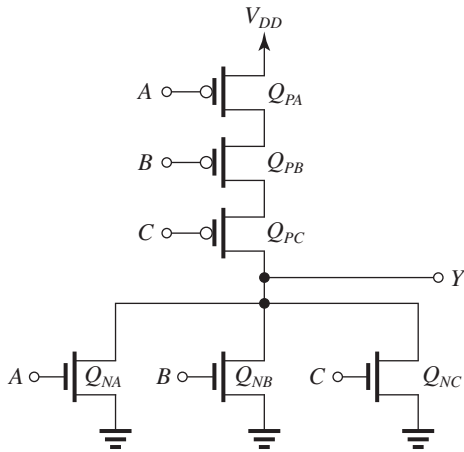


Figure S14.2

$$Y = \overline{A + B + C}$$

$$\Rightarrow \bar{Y} = A + B + C \Rightarrow \text{PDN}$$

$$Y = \bar{A} \bar{B} \bar{C} \Rightarrow \text{PUN}$$

The circuit realization is shown in Fig. S14.2.

**14.3**

Figure S14.3 shows the complete CMOS logic gate where the PUN is obtained as the dual of the

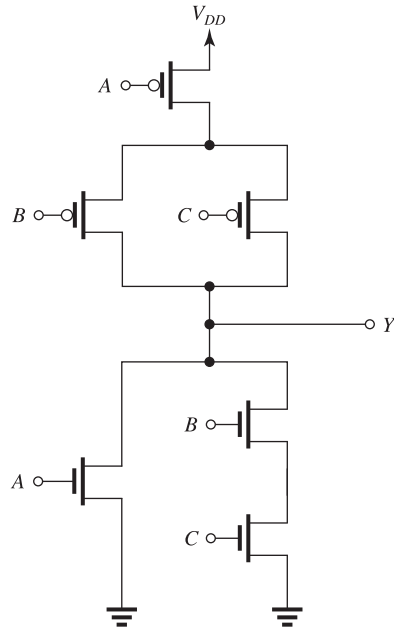


Figure S14.3

given PDN. The function realized can be found from the PDN as

$$\bar{Y} = A + BC$$

or

$$Y = \overline{A + BC}$$

**14.4** The given Boolean expression can be written as

$$\bar{Y} = AB + CDE$$

from which the PDN of the circuit in Fig. S14.4 (next page) can be directly obtained.

The PUN can then be found as the dual of the PDN. The complete circuit is shown in Fig. S14.4.

**14.5** Direct realization of the given expression results in the PUN of the logic circuit shown in Fig. S14.5 (next page). The PDN shown is obtained as the dual of the PUN. Not shown are the two inverters needed to obtain  $\bar{A}$  and  $\bar{B}$ .

**14.6** (a) For  $Q_N$ , we have

$$R_{on} = \frac{1}{(\mu_n C_{ox}) \left(\frac{W}{L}\right)_n (V_{DD} - V_m)}$$

For  $Q_p$ , we have

$$R_{on} = \frac{1}{(\mu_p C_{ox}) \left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp}|)}$$

This figure belongs to Problem 14.4.

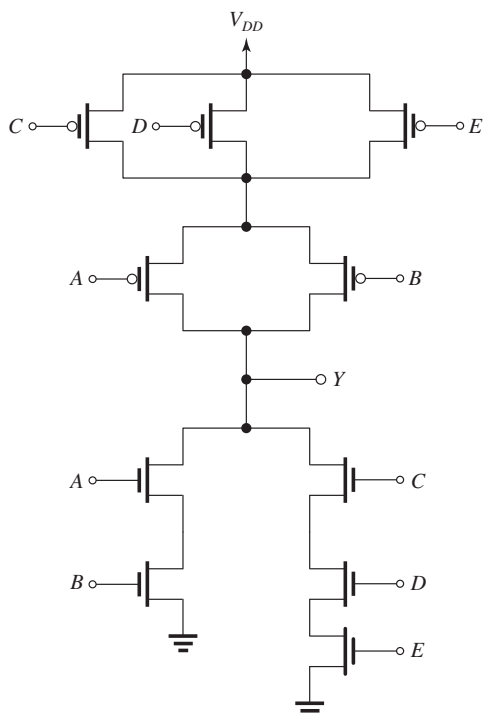


Figure S14.4

This figure belongs to Problem 14.5.

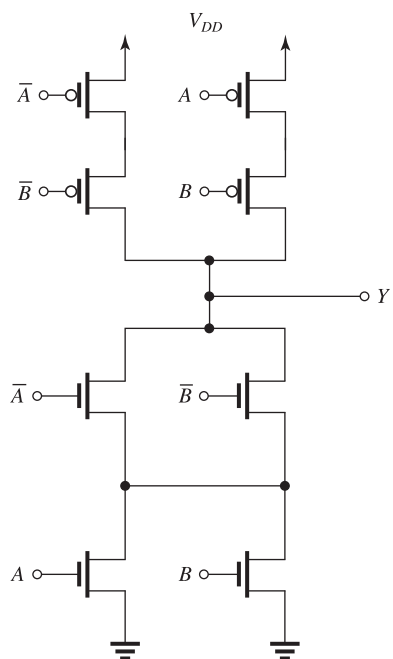


Figure S14.5

Since  $V_m = |V_{tp}|$ , then for  $R_{on}$  of  $Q_p$  to equal  $R_{on}$  of  $Q_n$ ,

$$(\mu_p C_{ox}) \left(\frac{W}{L}\right)_p = (\mu_n C_{ox}) \left(\frac{W}{L}\right)_n$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} \left(\frac{W}{L}\right)_n$$

$$= \frac{500}{125} \times 1.5 = 6.0$$

(b) For both devices, we have

$$R_{on} = \frac{1}{0.5 \times 1.5 \times (1.2 - 0.4)} = 1.67 \text{ k}\Omega$$

14.7

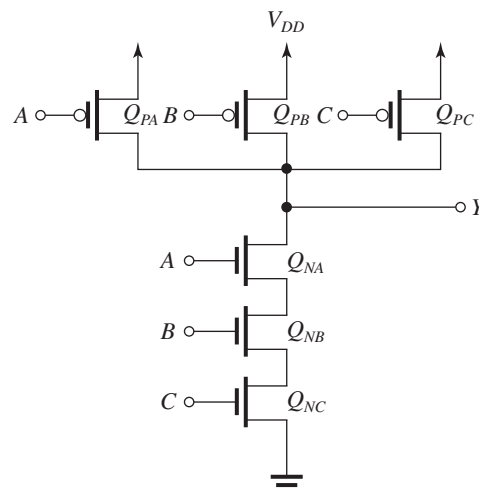


Figure S14.7

$$Y = \overline{ABC}$$

$$\Rightarrow \overline{Y} = ABC \Rightarrow \text{PDN}$$

$$Y = \overline{A} + \overline{B} + \overline{C} \Rightarrow \text{PUN}$$

Figure S14.7 shows the circuit realization.

14.8

Figure S14.8 (next page) shows the complete CMOS circuit where the PUN is obtained as the dual network of the given PDN. The logic function realized can be written from the PDN as

$$\overline{Y} = A(B + C)$$

or equivalently

$$Y = \overline{A(B + C)}$$

14.9

Figure S14.9 (next page) shows the complete CMOS logic circuit where we have obtained the PDN as the dual of the given PUN. The logic function can be written from the PDN as

$$\overline{Y} = A + BCD$$

This figure belongs to Problem 14.8.

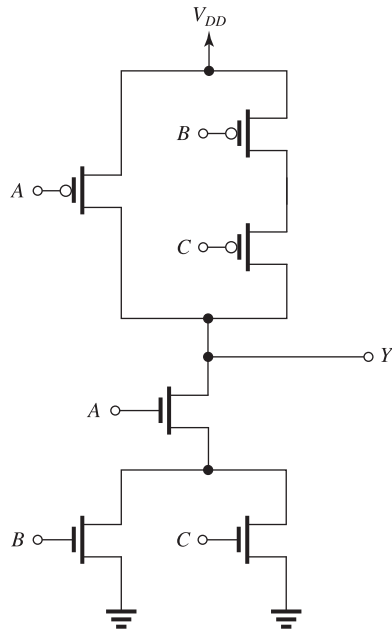


Figure S14.8

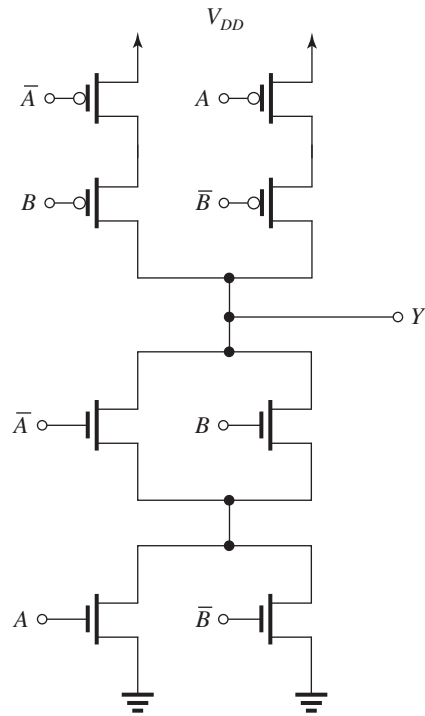


Figure S14.10

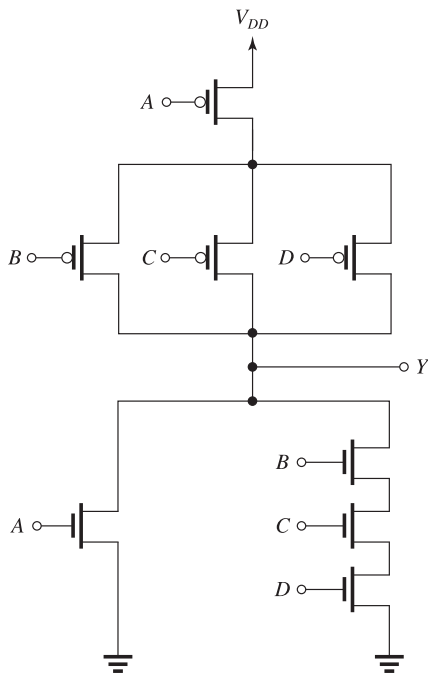


Figure S14.9

or equivalently

$$Y = \overline{A + BCD}$$

**14.10**

Figure S14.10 shows a CMOS realization of the Exclusive-OR function. This circuit is obtained

by utilizing the PUN in Fig. 14.11(a) and then finding the dual PDN. Note that two additional inverters are needed to generate  $\bar{A}$  and  $\bar{B}$ , for a total of 12 transistors.

$$14.11 \quad Y = \bar{A}BC + A\bar{B}C + AB\bar{C} \quad (1)$$

Using this expression to directly synthesize the PUN we obtain the circuit shown in Fig. 1.

This PUN circuit requires 9 transistors plus three inverters for a total of 15 transistors.

This, of course, does not include the transistors required for the PDN which we shall consider shortly. Inspecting the PUN circuit reveals the potential for eliminating two transistors through what is known as "path merging." Specifically the two transistors in the top row that are controlled by  $\bar{A}$  can be merged into a single transistor, and the two transistors in the bottom row that are controlled by  $\bar{C}$  can be merged into a single transistor. The result is the 7-transistor PUN shown in Fig. S14.11(b).

We next consider the realization of the PDN. A straightforward realization can be obtained by finding the dual of the PUN in Fig. S14.11(a). This is shown in Fig. S14.11(c). It requires nine transistors plus three inverters. The latter, of course are the same three inverters needed to obtain the complemented variables in PUN. The circuit in Fig. S14.11(c) does not lend itself to

This figure belongs to Problem 14.11, part (a).

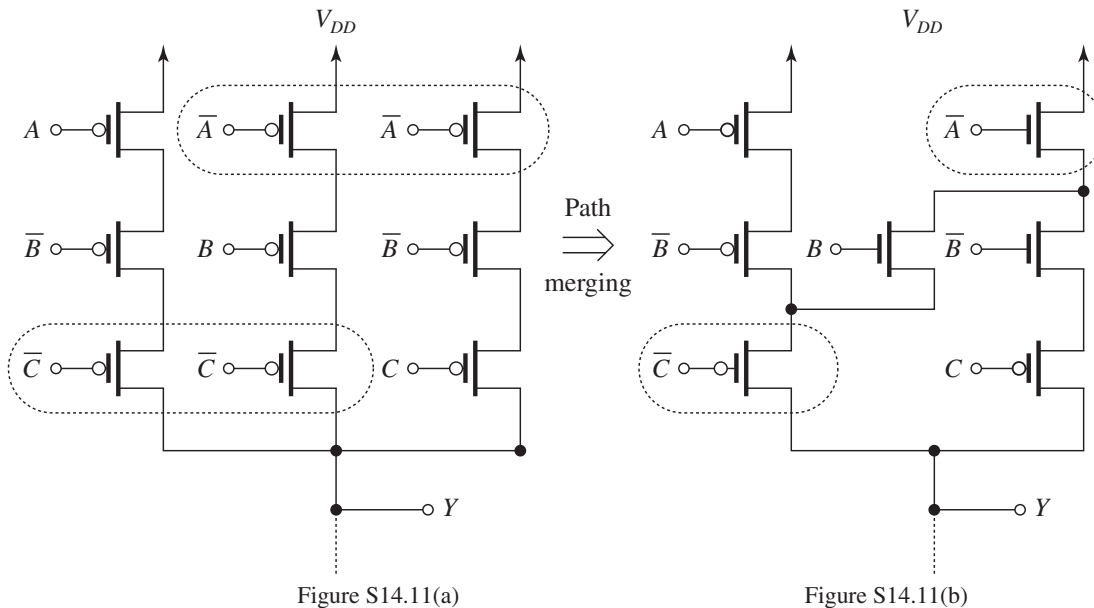


Figure S14.11(a)

Figure S14.11(b)

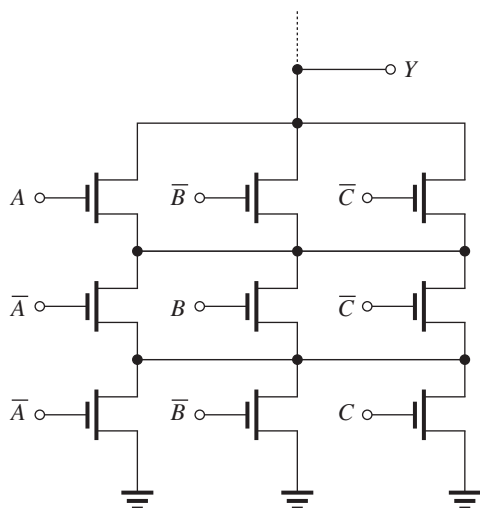


Figure S14.11(c)

path merging, at least not in a straightforward way.

There is, however, an alternative way to synthesize a PDN with a lower number of transistors. We simply obtain  $\bar{Y}$  from the expression in Eq. (1) using DeMorgan's law as follows:

$$\begin{aligned} \bar{Y} &= \overline{ABC} \overline{A\bar{B}\bar{C}} \overline{A\bar{B}C} \\ &= (A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C) \quad (2) \end{aligned}$$

Direct synthesis of Eq. (2) results in the circuit of Fig. S14.11(c). However, further manipulation of the expression in (2) results in a more economical realization, as follows:

$$\begin{aligned} \bar{Y} &= AB(\bar{A} + \bar{B} + C) + A\bar{C}(\bar{A} + \bar{B} + C) \\ &\quad + \bar{B}\bar{A}(\bar{A} + \bar{B} + C) + \bar{B}\bar{C}(\bar{A} + \bar{B} + C) \\ &\quad + \bar{C}\bar{A}(\bar{A} + \bar{B} + C) + \bar{C}B(\bar{A} + \bar{B} + C) \\ &\quad + \bar{C}(\bar{A} + \bar{B} + C) \\ &= ABC + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{B} + \bar{A}\bar{C} \\ &= ABC + \bar{A}(\bar{B} + \bar{C}) + \bar{B}\bar{C} \quad (3) \end{aligned}$$

A direct realization of this expression results in the PDN shown in Fig. S14.11(d). This circuit requires 8 transistors (not counting the inverters).

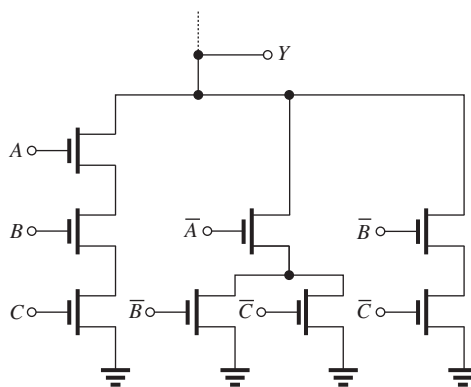


Figure S14.11(d)

**14.12 (a)** Even-parity checker:

$$\bar{Y} = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

See Fig. S14.12(a) on next page.

This figure belongs to Problem 14.12, part (a).

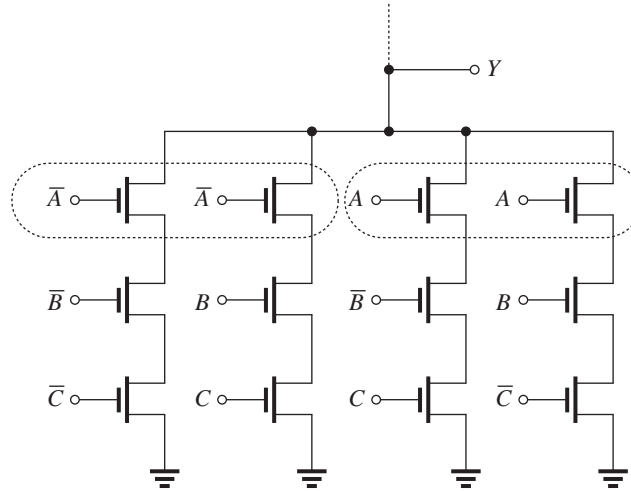


Figure S14.12(a)

This figure belongs to Problem 14.12, part (c).

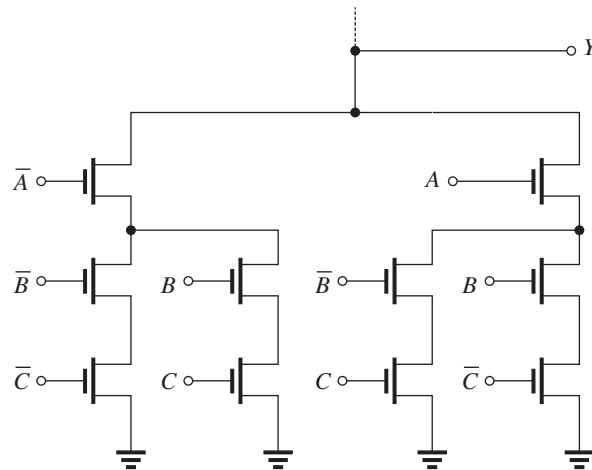


Figure S14.12(b)

(b) This expression can be directly realized with the PDN shown in Fig. S14.12(a). Note that the circuit requires 12 transistors in addition to the three inverters needed to generate  $\bar{A}$ ,  $\bar{B}$ , and  $\bar{C}$ .

(c) From inspection of the PDN in Fig. 1 we see that we can combine the two transistors controlled by  $\bar{A}$  and the two transistors controlled by  $A$ . This results in the PDN realization shown in Fig. 2 which requires 10 transistors, not counting those in the inverters. See Fig. S14.12(b) above.

(d) The PUN in Fig. S14.12(c) (next page) can be obtained as the dual of the PDN in Fig. S14.12(b). Combining the PDN and the PUN gives the complete realization of the even-parity checker.

Note: The number of transistors in the PDN of Fig. S14.12(b) can be reduced by 2 by combining

the two transistors in the bottom row that are controlled by  $C$ , and the two transistors that are controlled by  $\bar{C}$ . The resulting 8-transistor realization is shown in Fig. S14.12(d) on the next page. However, it is not easy to obtain a PUN as a dual of this circuit. See Fig. S14.12(d) on next page.

#### 14.13 Odd-parity checker:

$$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C} \\ = A(\bar{B}\bar{C} + \bar{B}C) + \bar{A}(BC + \bar{B}\bar{C}) \quad (1)$$

The Boolean expression in Eq. (1) can be directly realized by the PUN in Fig. S14.13 on page 14-7. Recall that we use for the switch control variables the complements of the variables in the equation. It requires 10 transistors in addition to the three

This figure belongs to Problem 14.12.

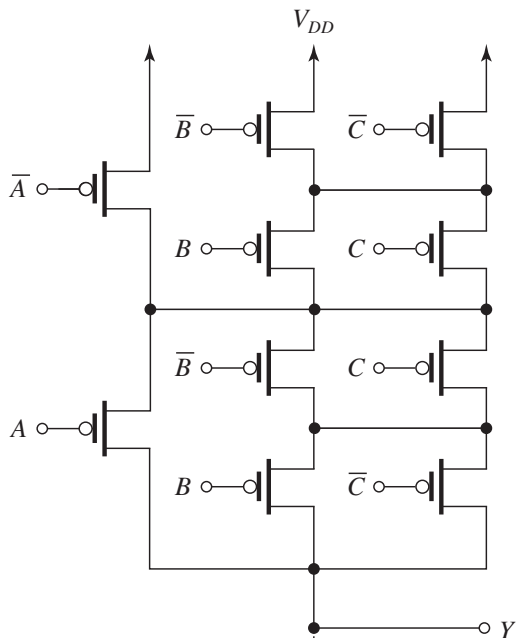


Figure S14.12(c)

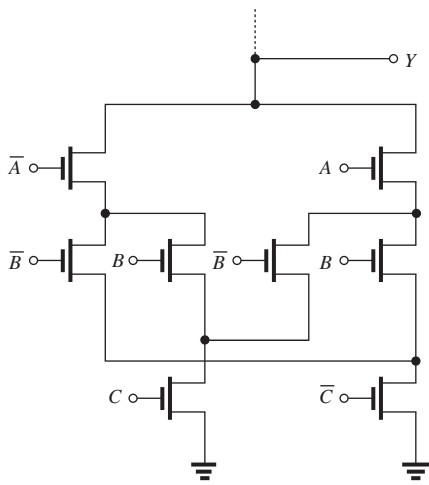


Figure S14.12(d)

inverters needed to provide  $\bar{A}$ ,  $\bar{B}$  and  $\bar{C}$ . The dual of the PUN can be obtained and results in the PDN shown in Fig. S14.13 on the next page.

$$14.14 \quad S = \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC \\ = \bar{A}(B\bar{C} + \bar{B}C) + A(BC + \bar{B}\bar{C})$$

This is the same function as that of the odd-parity checker in Problem 14.12. Thus the realization of the  $S$  function will be identical to that in Fig. S14.12(a) of Problem 14.12.

As for  $C_0$  we write

$$C_0 = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

This expression can be minimized as follows:

$$C_0 = (\bar{A} + A)BC + (\bar{B} + B)AC + (\bar{C} + C)AB \\ = BC + AC + AB = A(B + C) + BC$$

which can be realized directly by the PUN of the circuit in Fig. S14.14 on page 14-8 where the PDN is obtained as the dual network of the PUN. In addition to the 10 transistors, we need three inverters to generate  $\bar{A}$ ,  $\bar{B}$  and  $\bar{C}$ .

14.15 Direct realization of the given expression results in the PUN portion of the circuit shown in Fig. S14.15 on page 14-8. The PDN is obtained as the dual of the PUN. Not shown are the three inverters needed to obtain  $\bar{A}$ ,  $\bar{B}$  and  $\bar{C}$ .

14.16 (a)  $NM_H = V_{OH} - V_{IH}$

$$= 1.5 - 1.2 = 0.3 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 1.1 - 0.3 = 0.8 \text{ V}$$

(b)

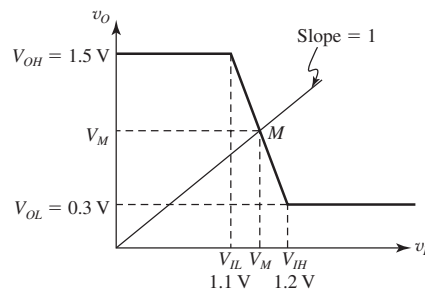


Figure S14.16

Refer to Fig. S14.16. Slope of the VTC in the transition region is:

$$\text{Slope} = \frac{V_{OH} - V_{OL}}{V_{IL} - V_{IH}} \\ = \frac{1.5 - 0.3}{1.1 - 1.2} = -12 \text{ V/V}$$

But the slope can also be expressed as

$$\text{Slope} = \frac{V_M - V_{OH}}{V_M - V_{IH}}$$

Thus,

$$\frac{V_M - 0.3}{V_M - 1.2} = -12$$

$$\Rightarrow V_M = 1.13 \text{ V}$$

(c) The voltage gain in the transition region is equal to the slope found above, thus

$$\text{Gain} = -12 \text{ V/V}$$

This figure belongs to Problem 14.13.

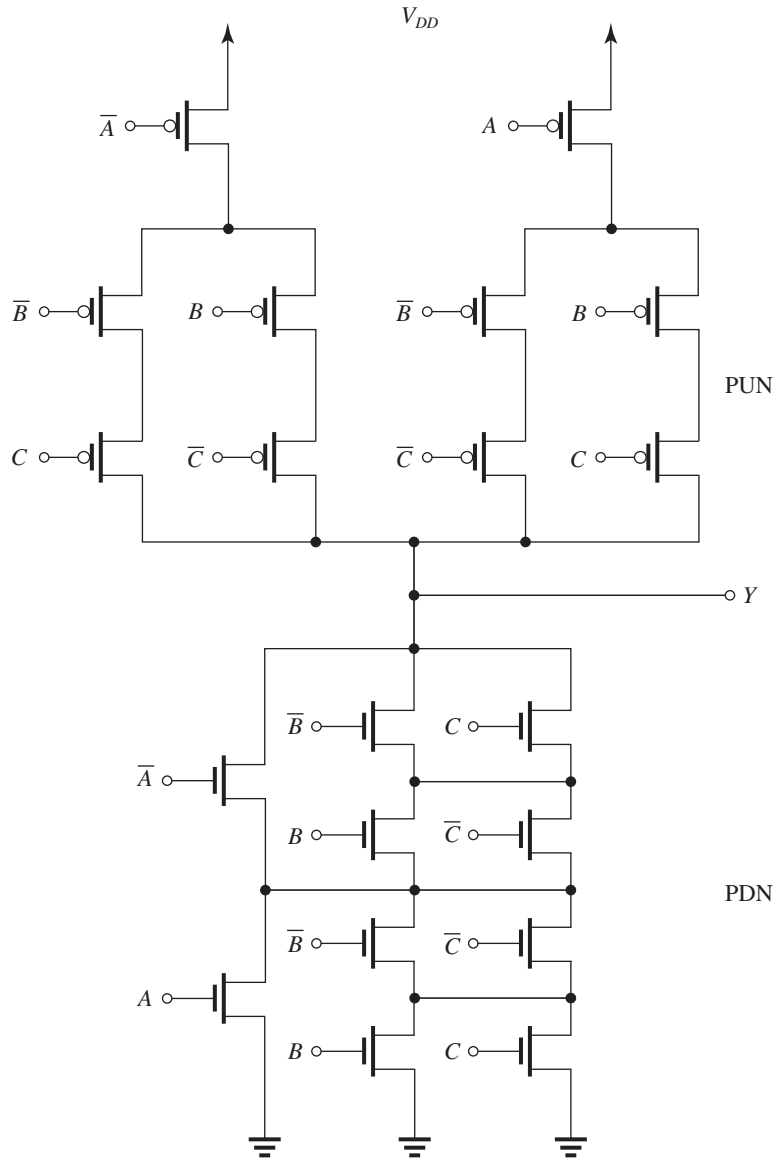


Figure S14.13

14.17 (a) Refer to Fig. 14.16.

$$\begin{aligned}
 V_{OL} &= V_{DD} \frac{R_{on}}{R + R_{on}} \\
 &= 1.8 \times \frac{0.1}{2 + 0.1} = 0.086 \text{ V} \\
 V_{OH} &= V_{DD} = 1.8 \text{ V} \\
 NM_H &= V_{OH} - V_{IH} \\
 &= 1.8 - 0.8 = 1 \text{ V} \\
 NM_L &= V_{IL} - V_{OL} \\
 &= 0.6 - 0.086 = 0.514 \text{ V}
 \end{aligned}$$

(b) Refer to Fig. S14.17.

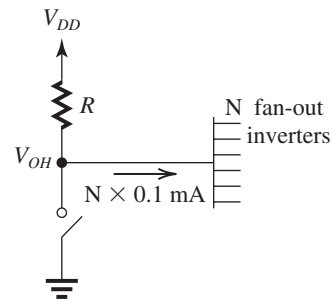


Figure S14.17

$$\begin{aligned}
 V_{OH} &= V_{DD} - N \times 0.1 \times R \\
 &= 1.8 - N \times 0.1 \times 2
 \end{aligned}$$



This figure belongs to Problem 14.14.

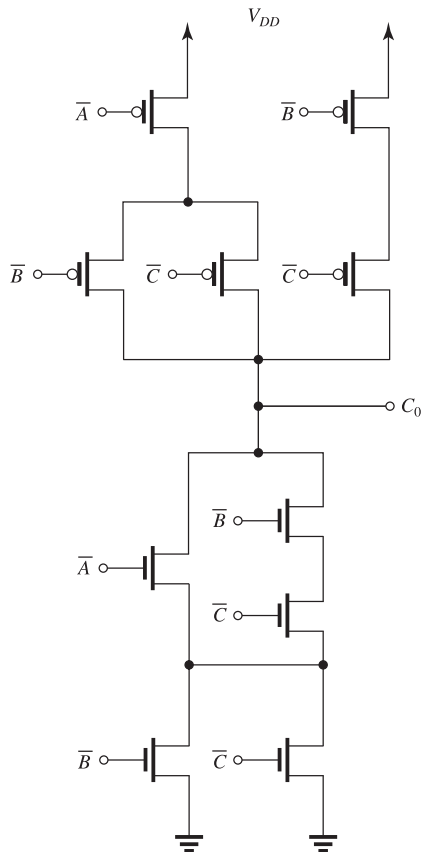


Figure S14.14

$$= 1.8 - 0.2N$$

$$NM_H = 1.8 - 0.2N - 0.8$$

$$= 1 - 0.2N$$

For  $NM_H \geq NM_L$ , we have

$$1 - 0.2N \geq 0.514$$

$$\Rightarrow N \leq 2.43$$

which means

$$N = 2$$

(c) (i) When the inverter output is low,

$$P_D = \frac{V_{DD}^2}{R + R_{on}} = \frac{1.8^2}{2 + 0.1} \approx 1.54 \text{ mW}$$

(ii) When the output is high and the inverter is driving two inverters, the current drawn from the supply is  $2 \times 0.1 = 0.2 \text{ mA}$  and thus the power dissipation is

$$P_D = V_{DD}I_{DD} = 1.8 \times 0.2 = 0.36 \text{ mW}$$

**14.18** For an ideal inverter:

$$V_M = \frac{1}{2}V_{DD} = 0.9 \text{ V}$$

This figure belongs to Problem 14.15.

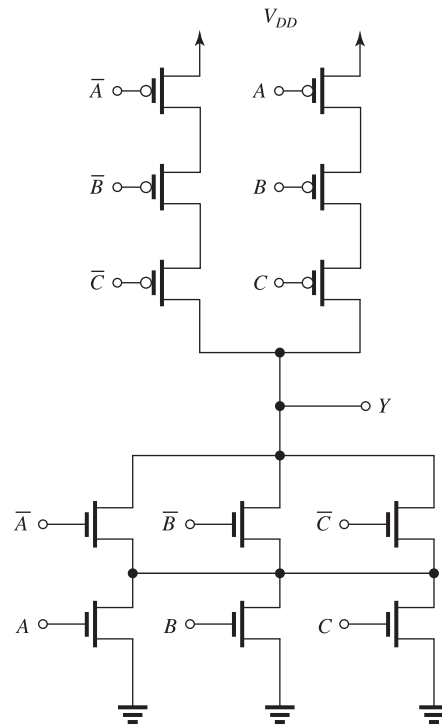


Figure S14.15

$$V_{IL} = V_{IH} = V_M = 0.9 \text{ V}$$

$$V_{OL} = 0 \text{ V}$$

$$V_{OH} = V_{DD} = 1.8 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.9 - 0 = 0.9 \text{ V}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= 1.8 - 0.9 = 0.9 \text{ V}$$

The ideal transfer characteristic is shown in Fig. S14.18, from which we see that

Gain in transition region =  $\infty$

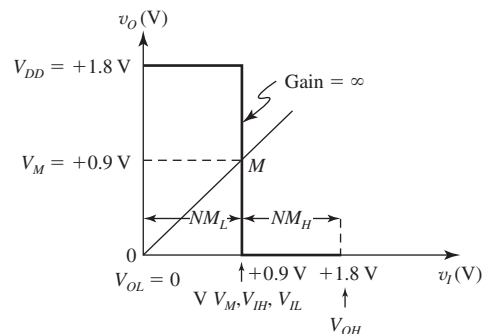


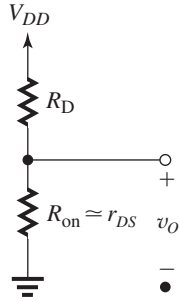
Figure S14.18

**14.19**

*Equivalent circuit for output-low state*

The output-high level for the simple inverter circuit shown in Fig. 14.15 of the text is

$$V_{OH} = V_{DD} \Rightarrow V_{DD} = 1.0 \text{ V.}$$



When the output is low, the current drawn from the supply can be calculated as

$$I = \frac{V_{DD}}{R_D + R_{on}} = 30 \mu\text{A}$$

$$\text{Therefore: } R_D + r_{DS} = \frac{1.0}{30 \times 10^{-6}} = 33.3 \text{ k}\Omega$$

Also:

$$V_{OL} = 0.05 \text{ V} = \frac{r_{DS}}{R_D + r_{DS}} \times V_{DD}$$

$$\Rightarrow r_{DS} = 33.3 \text{ k}\Omega \times \frac{0.05}{1.0} = 1.67 \text{ k}\Omega$$

$$\text{Hence: } R_D = 33.3 \text{ K} - 1.67 \text{ K} = 31.6 \text{ k}\Omega$$

$$r_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)}$$

$$= \frac{1}{540 \times 10^{-6} \times \frac{W}{L} (1.0 - 0.35)}$$

$$= 1.67 \text{ k}\Omega$$

$$\Rightarrow \frac{W}{L} = 1.7$$

When the output is low:

$$P_D = V_{DD} I_{DD} = 1.0 \times 30 \mu\text{A} = 30 \mu\text{W}$$

When the output is high, the transistor is off:  
 $P_D = 0 \text{ W}$

**14.20**

(a)  $v_I = 0$ , so  $Q_N$  is off

$Q_P$  is in the triode region but conducts zero current, so the output voltage is equal to  $V_{DD}$

$$V_{OH} = V_{DD} = 1.0 \text{ V}$$

(b)  $v_I = V_{DD}$ , and the output is likely lower than  $V_t$ , such that  $Q_P$  is in the saturation region and  $Q_N$  is in the triode region

$$i_{DP} = \frac{1}{2} k_p (V_{DD} - V_t)^2$$

$$i_{DN} = k_n \left[ (V_{DD} - V_t) V_{OL} - \frac{V_{OL}^2}{2} \right]$$

Equating  $i_{DP}$  and  $i_{DN}$  yields a quadratic equation that can be solved to obtain

$$V_{OL} = (V_{DD} - V_t) (1 - \sqrt{1 - k_p/k_n})$$

$$\text{(c) } V_{OL} = (1 - 0.35) (1 - \sqrt{1 - 1/5.4}) = 63 \text{ mV}$$

**14.21**

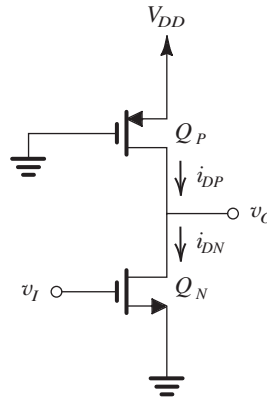


Figure S14.21

Referring to Fig. S14.21, we can assume that  $V_M > V_t$ , such that  $Q_P$  operates in the triode region.  $Q_N$  operates in the saturation region.

$$i_{DP} = k_p \left[ (V_{DD} - V_t)(V_{DD} - V_M) - \frac{(V_{DD} - V_M)^2}{2} \right]$$

$$i_{DN} = \frac{k_n}{2} (V_M - V_t)^2$$

Equating  $i_{DP}$  and  $i_{DN}$

$$\frac{1}{2} k_n (V_M - V_t)^2 =$$

$$k_p \left[ (V_{DD} - V_t)(V_{DD} - V_M) - \frac{(V_{DD} - V_M)^2}{2} \right]$$

$$\Rightarrow \frac{1}{2} r (V_M^2 - 2V_t V_M + V_t^2) =$$

$$\frac{1}{2} V_{DD}^2 - V_{DD} V_t + V_M V_t - \frac{1}{2} V_M^2$$

$$\Rightarrow (r + 1) V_M^2 - 2(r + 1) V_t V_M + (r + 1) V_t^2 = V_{DD}^2 - 2V_{DD} V_t + V_t^2$$

$$\Rightarrow (r + 1)(V_M - V_t)^2 = (V_{DD} - V_t)^2$$

$$\Rightarrow V_M = V_t + \frac{V_{DD} - V_t}{\sqrt{r + 1}}$$

$$14.22 \quad NM_H = V_{OH} - V_{IH}$$

$$= 1.2 - 0.7 = 0.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.5 - 0.1 = 0.4 \text{ V}$$

$$14.23 \quad NM_H = V_{OH} - V_{IH}$$

$$= 0.8V_{DD} - 0.6V_{DD} = 0.2V_{DD}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.4V_{DD} - 0.1V_{DD} = 0.3V_{DD}$$

$$\text{Width of transition region} = V_{IH} - V_{IL}$$

$$= 0.6V_{DD} - 0.4V_{DD} = 0.2V_{DD}$$

For a minimum noise margin of 0.25 V, we have

$$NM_H = 0.25$$

$$\Rightarrow 0.2V_{DD} = 0.25$$

$$\Rightarrow V_{DD} = 1.25 \text{ V}$$

#### 14.24

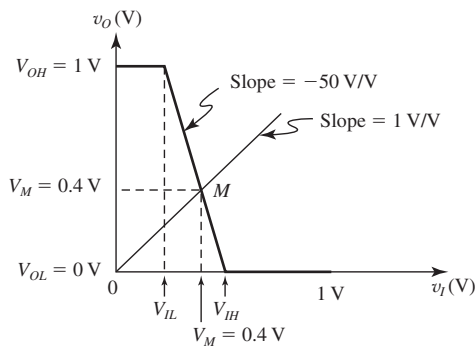


Figure S14.24

Figure S14.24 shows a sketch of the VTC where we have approximated the VTC in the transition region by a straight line with a slope equal to the maximum possible small-signal gain, namely 50 V/V. We can use the geometry of the VTC to determine  $V_{IH}$  and  $V_{IL}$  as follows:

$$|\text{Slope}| = 50$$

$$= \frac{V_M}{V_{IH} - V_M}$$

Thus,

$$50 = \frac{0.4}{V_{IH} - 0.4}$$

$$\Rightarrow V_{IH} = 0.408 \text{ V}$$

Similarly,

$$|\text{Slope}| = \frac{V_{OH}}{V_{IH} - V_{IL}}$$

$$50 = \frac{1}{0.408 - V_{IL}}$$

$$\Rightarrow V_{IL} = 0.388 \text{ V}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= 1 - 0.408 = 0.592 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.388 - 0 = 0.388 \text{ V}$$

Since we approximated the VTC in the transition region by a straight line, the large-signal voltage gain will be equal to the small-signal voltage gain,

$$= -50 \text{ V/V}$$

14.25 Here,  $V_{OH} = 0.9 \text{ V}$ , and  $V_{OL} = 0.0 \text{ V}$

$$\text{Also, } V_{IH} - V_{IL} \leq 0.9/3 = 0.3 \text{ V} \quad (1)$$

Now, the noise margins are “within 30% of one other.” Thus,  $NM_H = (1 \pm 0.3) NM_L$  or  $NM_L = (1 \pm 0.3) NM_H$ . Thus, they remain “within” either  $NM_H = 1.3 NM_L$  or  $NM_L = 1.3 NM_H$ , in which case either  $NM_L = 0.769 NM_H$  or  $NM_H = 0.769 NM_L$

For the former case:

$$0.769(V_{OH} - V_{IH}) = (V_{IL} - V_{OL}) \text{ or}$$

$$0.769(0.9 - V_{IH}) = V_{IL} - 0, \text{ whence}$$

$$V_{IL} = 0.692 - 0.769V_{IH}$$

$$\text{Now, from (1), } V_{IH} = V_{IL} + 0.3$$

Thus,

$$V_{IL} = 0.692 - 0.769(V_{IL} + 0.3)$$

$$= 0.461 - 0.769V_{IL}$$

$$\text{and } V_{IL} = 0.461/1.769 = 0.26 \text{ V}$$

$$\text{whence } V_{IH} = 0.3 + 0.26 = 0.56 \text{ V}$$

Alternatively,  $NM_H = 0.769 NM_L$  and

$$(V_{OH} - V_{IH}) = 0.769(V_{IL} - V_{OL}) \text{ or}$$

$$0.9 - V_{IH} = 0.769V_{IL} - 0 \text{ and}$$

$$V_{IH} = 0.9 - 0.769V_{IL}, \text{ with (1),}$$

$$V_{IL} + 0.3 = 0.9 - 0.769V_{IL}, \text{ and}$$

$$1.769V_{IL} = 0.6, \text{ whence } V_{IL} = 0.34 \text{ V}$$

$$\text{and } V_{IH} = 0.3 + 0.34 = 0.64$$

Thus, overall,  $V_{OH} = 0.9 \text{ V}$ ,  $V_{OL} = 0.0 \text{ V}$ ,

$V_{IH}$  ranges from 0.56 V to 0.64 V, and

$V_{IL}$  ranges from 0.26 V to 0.34 V, in

which case the margins can be as low as

$$NM_L = V_{IL} - V_{OL} = 0.26 \text{ V and}$$

$$NM_H = V_{OH} - V_{IH} = 0.9 - 0.64 = 0.26 \text{ V}$$

and as high as 0.34 V.

**14.26** Refer to Example 14.6 from the text:

$$V_{OH} = V_{DD} = 1.2 \text{ V}$$

The power drawn from the supply during the low-output state is

$$P_{DD} = V_{DD}I_{DD} \Rightarrow 50 \mu\text{W} = 1.2 \times I_{DD}$$

$$\Rightarrow I_{DD} = 41.7 \mu\text{A}$$

In this case:

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} \Rightarrow 41.7 \mu\text{A} = \frac{1.2 - 0.05}{R_D}$$

$$\Rightarrow R_D = 27.6 \text{ k}\Omega$$

In order to determine  $\frac{W}{L}$ , we note that

$$k_n R_D = 1/V_X \text{ or } k'_n \frac{W}{L} R_D = \frac{1}{V_X}$$

Therefore, we need to first calculate  $V_X$  using Eq. (14.22) from the text.

$$V_{OL} = \frac{V_{DD}}{1 + \frac{V_{DD} - V_t}{V_X}} \text{ or equivalently}$$

$$0.05 \text{ V} = \frac{1.2}{1 + \frac{1.2 - 0.4}{V_X}}$$

$$\Rightarrow V_X = \frac{0.8}{23} = 0.035 \text{ V}$$

Hence,  $k'_n \frac{W}{L} R_D = \frac{1}{V_X}$  gives

$$500 \times 10^{-6} \times \frac{W}{L} \times 27.6 \times 10^3 = \frac{1}{0.035} \Rightarrow \frac{W}{L} = 2.1$$

Using Eq. (14.12), we obtain

$$V_{IL} = V_t + V_X = 0.4 + 0.035 = 0.435 \text{ V}$$

From Eq. (14.14) we obtain

$$V_M = V_t + \sqrt{2(V_{DD} - V_t)V_X + V_X^2} - V_X \\ = 0.4 + \sqrt{2(1.2 - 0.4)0.035 + 0.035^2} - 0.035$$

$$V_M = 0.6 \text{ V}$$

From Eq. (14.20) we get

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_X} - V_X$$

$$= 0.4 + 1.63\sqrt{1.2 \times 0.035} - 0.035 = 0.7 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.7 = 0.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 0.435 - 0.05 = 0.385 \text{ V}$$

$$\mathbf{14.27} \quad V_t = 0.3V_{DD}, \quad V_M = V_{DD}/2$$

From Eq. (14.13), we obtain

$$V_x \Big|_{V_M = \frac{V_{DD}}{2}} = \frac{\left(\frac{V_{DD}}{2} - V_t\right)^2}{V_{DD}}$$

$$= \frac{(0.5V_{DD} - 0.3V_{DD})^2}{V_{DD}}$$

$$\Rightarrow V_x = 0.04V_{DD}$$

$$V_{OH} = V_{DD}$$

From Eq. (14.12), we get

$$V_{IL} = V_t + V_x = 0.3V_{DD} + 0.04V_{DD}$$

$$= 0.34V_{DD}$$

From Eq. (14.20), we obtain

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$$

$$= 0.3V_{DD} + 1.63\sqrt{V_{DD} \times 0.04V_{DD}} - 0.04V_{DD}$$

$$= 0.586V_{DD}$$

From Eq. (14.22), we get

$$V_{OL} = \frac{V_{DD}}{1 + [(V_{DD} - V_t)/V_x]}$$

$$= \frac{V_{DD}}{1 + \frac{V_{DD} - 0.3V_{DD}}{0.04V_{DD}}} = 0.054V_{DD}$$

$$NM_H = V_{OH} - V_{IH}$$

$$= V_{DD} - 0.586V_{DD} = 0.414V_{DD}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.34V_{DD} - 0.054V_{DD} = 0.286V_{DD}$$

For  $V_{DD} = 1.0 \text{ V}$ :

$$V_x = 0.04 \text{ V}, \quad V_{OH} = 1.0 \text{ V}, \quad V_{IL} = 0.34 \text{ V},$$

$$V_{IH} = 0.586 \text{ V}, \quad V_{OL} = 0.054 \text{ V},$$

$$NM_H = 0.414 \text{ V}, \quad NM_L = 0.286 \text{ V}$$

$$P_D = V_{DD}I_D$$

$$= V_{DD} \times \frac{V_{DD} - V_{OL}}{R_D}$$

Substituting for  $R_D$  from

$$R_D = \frac{1}{k_n V_x}$$

we obtain

$$P_D = V_{DD}(V_{DD} - 0.054V_{DD}) \times k_n \times 0.04V_{DD}$$

$$P_D = 0.038V_{DD}^3 \times k'_n \left(\frac{W}{L}\right)$$

$$= 0.038 \times 1.0^3 \times 0.54 \times 10^{-3} \left(\frac{W}{L}\right)$$

$$= 0.021 \left(\frac{W}{L}\right), \text{ mW}$$

For  $P_D = 100 \mu\text{W} = 0.1 \text{ mW}$ , we obtain

$$0.1 = 0.021 \left(\frac{W}{L}\right)$$

$$\Rightarrow \frac{W}{L} = 4.9$$

$$R_D = \frac{1}{k_n V_x}$$

$$= \frac{1}{0.54 \times 4.9 \times 0.04}$$

$$= 9.4 \text{ k}\Omega$$

**14.28** (a) To obtain  $V_M = V_{DD}/2$ , the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5$$

$$\Rightarrow W_p = 2.5W_n = 2.5 \times 1.5 \times 65 = 244 \text{ nm}$$

$$\text{Silicon area} = W_n L_n + W_p L_p$$

$$= 1.5 \times 65 \times 65 + 2.5 \times 1.5 \times 65 \times 65$$

$$= 1.5 \times 65 \times 65(1 + 2.5)$$

$$= 22,181 \text{ nm}^2$$

$$(b) V_{OH} = V_{DD} = 1 \text{ V}$$

$$V_{OL} = 0 \text{ V}$$

To obtain  $V_{IH}$ , we use Eq. (14.35):

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 1 - 2 \times 0.35)$$

$$= 0.5375 \text{ V}$$

To obtain  $V_{IL}$ , we use Eq. (14.36):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$= \frac{1}{8}(3 \times 1 + 2 \times 0.35)$$

$$= 0.4625 \text{ V}$$

The noise margins can now be found as

$$NMH = V_{OH} - V_{IH}$$

$$= 1 - 0.5375 = 0.4625 \text{ V}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= 0.4625 - 0 = 0.4625 \text{ V}$$

The noise margins are equal at approximately 0.46 V; a result of the matched design of the inverter.

(c) Since the inverter is matched, the output resistances in the two states will be equal. Thus,

$$r_{DSP} = r_{DSN} = 1 / \left[ (\mu_n C_{ox}) \left(\frac{W}{L}\right)_n (V_{DD} - V_t) \right]$$

$$= \frac{1}{0.47 \times 1.5(1 - 0.35)} = 2.18 \text{ k}\Omega$$

**14.29**  $Q_N$  will be operating in the triode region, thus

$$I_{Dn} = k'_n \left(\frac{W}{L}\right)_n \left[ (V_{DD} - V_m)V_O - \frac{1}{2}V_O^2 \right]$$

For  $V_m = 0.3V_{DD}$  and  $V_O = 0.1V_{DD}$ , we have

$$I_{Dn} = k'_n \left(\frac{W}{L}\right)_n$$

$$\left[ (V_{DD} - 0.3V_{DD}) \times 0.1V_{DD} - \frac{1}{2} \times 0.1^2 V_{DD}^2 \right]$$

$$= k'_n (W/L)_n (0.07V_{DD}^2 - 0.005V_{DD}^2)$$

$$= 0.065k'_n (W/L)_n V_{DD}^2 \quad \text{Q.E.D.}$$

For  $V_{DD} = 1.2 \text{ V}$ ,  $k'_n = 0.5 \text{ mA/V}^2$  and  $I_{Dn} = 0.1 \text{ mA}$ , we have

$$0.1 = 0.065 \times 0.5(W/L)_n \times 1.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 2.14$$

**14.30** The current reaches its peak at

$v_t = V_M = \frac{V_{DD}}{2}$ . At this point, both  $Q_N$  and  $Q_P$  are operating in the saturation region and conducting a current

$$I_{DP} = I_{DN} = \frac{1}{2}k'_n \left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_t\right)^2$$

$$= \frac{1}{2} \times 540 \times 1.5 \left(\frac{1.0}{2} - 0.35\right)^2$$

$$= 9.1 \mu\text{A}$$

**14.31**  $V_{OH} = 2.5 \text{ V}$

$$V_{OL} = 0 \text{ V}$$

(a) For the matched case we have

$$W_p = 3.5W_n$$

$$V_M = \frac{1}{2}V_{DD} = 1.25 \text{ V}$$

$$\begin{aligned} \text{Eq. (14.35): } V_{IH} &= \frac{1}{8}(5V_{DD} - 2V_t) \\ &= \frac{1}{8}(5 \times 2.5 - 2 \times 0.5) \\ &= 1.4375 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Eq. (14.36): } V_{IL} &= \frac{1}{8}(3V_{DD} + 2V_t) \\ &= \frac{1}{8}(3 \times 2.5 + 2 \times 0.5) \\ &= 1.0625 \text{ V} \end{aligned}$$

$$NM_H = NM_L = 1.0625 \text{ V}$$

$$\begin{aligned} \text{Silicon area} &= W_n L_n + W_p L_p \\ &= 1.5 \times 0.25 \times 0.25 + 3.5 \times 1.5 \times 0.25 \times 0.25 \\ &= 4.5 \times 1.5 \times 0.25^2 = 0.42 \mu\text{m}^2 \end{aligned}$$

(b)  $W_p = W_n$  (minimum-size design):

$$\text{Eq. (14.40): } r = \sqrt{\frac{\mu_p}{\mu_n} \frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5}} \times 1 = 0.53$$

$$\begin{aligned} \text{Eq. (14.39): } V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ &= \frac{0.53(2.5 - 0.5) + 0.5}{0.53 + 1} \\ &= 1.02 \text{ V} \end{aligned}$$

Thus,  $V_M$  shifts to the left by 0.23 V. Assuming  $V_{IL}$  shifts by approximately the same amount, then

$$V_{IL} \simeq 1.0625 - 0.23 \simeq 0.83 \text{ V}$$

Since  $NM_L = V_{IL}$ ,  $NM_L$  will be reduced by approximately 22% (relative to the matched case).

$$\begin{aligned} \text{Silicon area} &= W_n L_n + W_p L_p \\ &= 1.5 \times 0.25 \times 0.25 + 1.5 \times 0.25 \times 0.25 \\ &= 3 \times 0.25^2 = 0.19 \mu\text{m}^2 \end{aligned}$$

which is a reduction of 55% relative to the matched case.

(c)  $W_p = 2W_n$  (a compromise design):

$$\begin{aligned} \text{Eq. (14.40): } r &= \sqrt{\frac{\mu_p}{\mu_n} \frac{W_p}{W_n}} = \sqrt{\frac{1}{3.5}} \times \frac{2}{1} \\ &= 0.756 \end{aligned}$$

$$\begin{aligned} \text{Eq. (14.39): } V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ &= \frac{0.756(2.5 - 0.5) + 0.5}{0.756 + 1} \\ &= 1.15 \text{ V} \end{aligned}$$

Thus, relative to the matched case the switching point ( $V_M$ ) is shifted left by  $(1.25 - 1.15) = 0.1$

V. Assuming that  $V_{IL}$  is reduced by approximately the same amount, then

$$V_{IL} = 1.0625 - 0.1 = 0.9625 \text{ V}$$

Thus,  $NM_L$  which equals  $V_{IL}$  is reduced by about 9% (relative to the matched case).

$$\begin{aligned} \text{Silicon area} &= W_n L_n + W_p L_p \\ &= 1.5 \times 0.25 \times 0.25 + 2 \times 1.5 \times 0.25 \times 0.25 \\ &= 3 \times 1.5 \times 0.25^2 \\ &= 0.28 \mu\text{m}^2 \end{aligned}$$

Compared to the matched case, the silicon area is reduced by 33%.

**14.32** For  $v_I = +1.5 \text{ V}$ ,  $Q_N$  will be conducting and operating in the triode region while  $Q_P$  will be off. Thus, the incremental resistance to the left of node A will be  $r_{DSN}$ ,

$$\begin{aligned} r_{DSN} &= \frac{1}{k_n(V_I - V_m)} \\ &= \frac{1}{0.25(1.5 - 0.4)} = 3.64 \text{ k}\Omega \end{aligned}$$

Thus,

$$\begin{aligned} v_a &= 100 \left( \frac{3.64}{3.64 + 100} \right) \\ &= 3.5 \text{ mV} \end{aligned}$$

For  $v_I = -1.5 \text{ V}$ ,  $Q_N$  will be off but  $Q_P$  will be operating in the triode region with a resistance  $r_{DSP}$ ,

$$\begin{aligned} r_{DSP} &= \frac{1}{k_p(V_{SGP} - |V_{tp}|)} \\ &= \frac{1}{0.05(1.5 - 0.4)} = 18.2 \text{ k}\Omega \end{aligned}$$

Thus,

$$v_a = 100 \left( \frac{18.2}{18.2 + 100} \right) = 15.4 \text{ mV}$$

**14.33** From Eq. (14.39) we have

$$\begin{aligned} V_M &= \frac{r(V_{DD} - |V_{tp}|) + V_m}{r + 1} \\ rV_M + V_M &= r(V_{DD} - |V_{tp}|) + V_m \\ r(V_{DD} - |V_{tp}| - V_M) &= V_M - V_m \\ \Rightarrow r &= \frac{V_M - V_m}{V_{DD} - |V_{tp}| - V_M} \quad \text{Q.E.D.} \end{aligned}$$

For  $V_{DD} = 1.0 \text{ V}$ ,  $V_m = |V_{tp}| = 0.35 \text{ V}$ , to obtain  $V_M = 0.6V_{DD}$ , we need

$$r = \frac{0.6 \times 1.0 - 0.35}{1.0 - 0.35 - 0.6 \times 1.0}$$

$$= 5$$

But,

$$r = \sqrt{\frac{\mu_p W_p}{\mu_n W_n}}$$

$$5 = \sqrt{\frac{1}{5.4} \times \frac{W_p}{W_n}}$$

$$\frac{W_p}{W_n} = 5^2 \times 5.4 = 135$$

**14.34** (a)  $W_p = 1.5 \times 2.0 \times 28 \text{ nm} = 84 \text{ nm}$

(b)  $V_{OH} = 0.9 \text{ V}$

$$V_{OL} = 0 \text{ V}$$

$$V_{IH} = \frac{1}{8}(5 \times 0.9 - 2 \times 0.3) = 0.49 \text{ V}$$

$$V_{IL} = \frac{1}{8}(3 \times 0.9 + 2 \times 0.3) = 0.41 \text{ V}$$

$$NM_H = 0.9 - 0.49 = 0.41 \text{ V}$$

$$NM_L = 0.41 - 0 = 0.41 \text{ V}$$

(c)  $r_{DSN} = \frac{1}{0.75 \times 2 \times (0.9 - 0.3)} = 1.11 \text{ k}\Omega$

$$r_{DSP} = \frac{1}{\frac{0.75}{1.5} \times 2 \times 1.5 \times (0.9 - 0.3)}$$

$$= 1.11 \text{ k}\Omega$$

(d)  $r = \sqrt{\frac{1}{1.5}} \times 1 = 0.816$

$$V_M = \frac{0.816 \times (0.9 - 0.3) + 0.3}{1 + 0.816} = 0.43 \text{ V}$$

**14.35** Refer to Example 14.3 except here:

$V_{DD} = 1.3 \text{ V}$ ,  $V_{in} = |V_{tp}| = 0.4 \text{ V}$ ,  $\mu_n = 4 \mu_p$ , and  $\mu_n C_{ox} = 0.5 \text{ mA/V}^2$ . Also,  $Q_N$  and  $Q_P$  have  $L = 0.13 \mu\text{m}$  and  $(W/L)_n = 1.5$ .

(a) For  $V_M = V_{DD}/2 = 0.65 \text{ V}$ , the inverter must be matched, thus

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 4$$

Since  $W_n/L = 1.5$ ,

$$W_n = 1.5 \times 0.13 = 0.195 \mu\text{m}. \text{ Thus,}$$

$$W_p = 4 \times 0.195 = 0.78 \mu\text{m}$$

For this design, the silicon area is

$$A = W_n L + W_p L = L(W_n + W_p) \\ = 0.13(0.195 + 0.78) = 0.127 \mu\text{m}^2$$

(b)  $V_{OH} = V_{DD} = 1.3 \text{ V}$

$$V_{OL} = 0 \text{ V}$$

To obtain  $V_{IH}$ , we use Eq. (14.35):

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}(5 \times 1.3 - 2 \times 0.4)$$

$$= 0.7125 \text{ V}$$

To obtain  $V_{IL}$ , we use Eq. (14.36):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$= \frac{1}{8}(3 \times 1.3 + 2 \times 0.4)$$

$$= 0.5875 \text{ V}$$

We can now compute the noise margins as

$$NM_H = V_{OH} - V_{IH} = 1.3 - 0.7125$$

$$= 0.5875 \text{ V} \simeq 0.59 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 0.5875 - 0$$

$$= 0.5875 \text{ V} \simeq 0.59 \text{ V}$$

For  $v_I = V_{IH} = 0.7125 \text{ V}$ , we can obtain the corresponding value of  $v_O$  by substituting in Eq. (14.34):

$$v_O = V_{IH} - \frac{V_{DD}}{2} = 0.7125 - 0.65 = 0.0625 \text{ V}$$

Thus, the worst-case value of  $V_{OL}$  is

$$V_{O_{\max}} = 0.0625 \simeq 0.06 \text{ V}, \text{ and the noise margin } NM_L \text{ reduces to}$$

$$NM_L = 0.5875 - 0.0625 = 0.5250 \text{ V}$$

or approximately 0.53 V.

From symmetry, we can obtain the value of  $v_O$  corresponding to  $v_I = V_{IL}$  as

$$v_O = V_{DD} - 0.0625$$

$$= 1.3 - 0.0625 = 1.2375 \text{ V} \simeq 1.24 \text{ V}$$

Thus, the worst-case value of  $V_{OH}$  is

$$V_{OH_{\min}} \simeq 1.24 \text{ V}, \text{ and the noise margin } NM_H \text{ is reduced to}$$

$$NM_H = V_{OH_{\min}} - V_{IH}$$

$$= 1.2375 - 0.7125 = 0.5250 \text{ V}$$

or approximately 0.53 V.

Note that the reduction in the noise margin (about 0.06 V) is slight.

(c) The output resistance of the inverter in the low-output state is

$$r_{DSN} = \frac{1}{\mu_n C_{ox} (W/L)_n (V_{DD} - V_{in})}$$

$$= \frac{1}{0.5 \times 1.5(1.3 - 0.4)} = 1.48 \text{ k}\Omega$$

Since  $Q_N$  and  $Q_P$  are matched, the output resistance in the high-output state will be equal, that is,

$$r_{DSP} = r_{DSN} = 1.48 \text{ k}\Omega$$

(d) If the inverter is biased to operate at  $v_I = v_O = V_M = 0.65 \text{ V}$ , then each of  $Q_N$  and  $Q_P$  will be operating at an overdrive voltage  $V_{OV} = V_M - V_t = 0.65 - 0.4 = 0.25 \text{ V}$  and will be conducting equal dc currents  $I_D$  of

$$\begin{aligned} I_D &= \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_n V_{OV}^2 \\ &= \frac{1}{2} \times 500 \times 1.5 \times 0.25^2 \\ &= 23.4 \text{ }\mu\text{A} \end{aligned}$$

Thus,  $Q_N$  and  $Q_P$  will have equal transconductances:

$$g_{mn} = g_{mp} = \frac{2I_D}{V_{OV}} = \frac{2 \times 23.4}{0.25} = 0.19 \text{ mA/V}$$

Transistors  $Q_N$  and  $Q_P$  will have output resistances  $r_{on}$  and  $r_{op}$  given by

$$\begin{aligned} r_{on} &= \frac{V_{An}}{I_D} \\ r_{op} &= \frac{|V_{Ap}|}{I_D} \end{aligned}$$

Since no values are given for  $V_{An}$  and  $V_{Ap}$  we shall use the data in Table K.1, namely

$$V'_{An} = 5 \text{ V}/\mu\text{m} \text{ and } |V'_{Ap}| = 6 \text{ V}/\mu\text{m}$$

Thus,

$$\begin{aligned} V_{An} &= 5 \times 0.13 = 0.65 \text{ V} \\ |V_{Ap}| &= 6 \times 0.13 = 0.78 \text{ V} \\ r_{on} &= \frac{0.65 \text{ V}}{23.4 \text{ }\mu\text{A}} = 27.8 \text{ k}\Omega \\ r_{op} &= \frac{0.78 \text{ V}}{23.4 \text{ }\mu\text{A}} = 33.3 \text{ k}\Omega \end{aligned}$$

We can now compute the voltage gain at  $M$  as

$$\begin{aligned} A_v &= -(g_{mn} + g_{mp})(r_{on} \parallel r_{op}) \\ &= -(0.19 + 0.19)(27.8 \parallel 33.3) \\ &= -5.8 \text{ V/V} \end{aligned}$$

When the straight line at  $M$  of slope  $-5.8 \text{ V/V}$  is extrapolated, it intersects the line  $v_O = 0$  at  $\left[ 0.65 + \frac{0.65}{5.8} \right] = 0.762 \text{ V}$  and the line  $v_O = V_{DD}$

at  $\left[ 0.65 - \frac{0.65}{5.8} \right] = 0.538 \text{ V}$ . Thus the width of the transition region can be considered  $(0.762 - 0.538) = 0.224 \text{ V}$ .

(e) For  $W_p = W_n$ , the parameter  $r$  can be found from Eq. (14.40):

$$r = \sqrt{\frac{\mu_p W_p}{\mu_n W_n}} = \sqrt{\frac{1}{4}} \times 1 = 0.5$$

The corresponding value of  $V_M$  can be determined from Eq. (14.39) as

$$V_M = \frac{0.5(1.3 - 0.4) + 0.4}{0.5 + 1} = 0.57 \text{ V}$$

Thus,  $V_M$  shifts by only  $-0.08 \text{ V}$ . We can estimate the reduction in  $NM_L$  to be approximately equal to the shift in  $V_M$ , that is,  $NM_L$  becomes

$$NM_L = 0.5875 - 0.08 \simeq 0.51 \text{ V}$$

The silicon area for this design can be computed as follows:

$$\begin{aligned} A &= L(W_n + W_p) \\ &= 0.13(1.5 \times 0.13 + 1.5 \times 0.13) \\ &= 0.051 \text{ }\mu\text{m}^2 \end{aligned}$$

This represents a 60% reduction from the matched case!

(f) For  $W_p = 2W_n$ , we have

$$\begin{aligned} r &= \sqrt{\frac{1}{4}} \times 2 = \frac{1}{\sqrt{2}} = 0.707 \\ V_M &= \frac{0.707(1.3 - 0.4) + 0.4}{0.707 + 1} = 0.61 \text{ V} \end{aligned}$$

Thus, relative to the matched case,  $V_M$  is reduced by only  $0.04 \text{ V}$ . Correspondingly,  $NM_L$  will be reduced by approximately an equal amount, thus  $NM_L$  becomes

$$NM_L \simeq 0.59 - 0.04 = 0.55 \text{ V}$$

In this case, the silicon area required is

$$\begin{aligned} A &= L(W_n + W_p) = L \times 3W_n \\ &= 0.13 \times 3 \times 1.5 \times 0.13 \\ &= 0.076 \text{ }\mu\text{m}^2 \end{aligned}$$

which represents a 40% reduction relative to the matched case.



**Chapter 15**

**Solutions to Exercises within the Chapter**

**Ex: 15.1**  $I t_{PLH} = C \frac{V_{DD}}{2}$

$\Rightarrow t_{PLH} = CV_{DD}/2I$

To obtain  $t_{PLH} = 10$  ps with  $C = 10$  fF and  $V_{DD} = 1.2$  V, we need a current  $I$  obtained as follows:

$$10 \times 10^{-12} = \frac{10 \times 10^{-15} \times 1.2}{2I}$$

$$\Rightarrow I = \frac{1.2 \times 10^{-14}}{2 \times 10^{-11}} = 0.6 \text{ mA}$$

**Ex: 15.2**

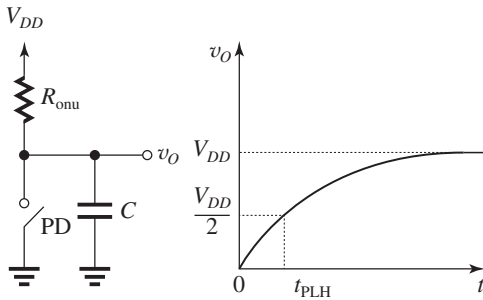


Figure 1(a)

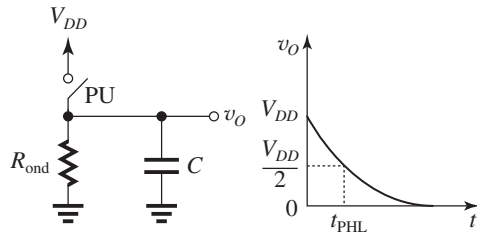


Figure 1(b)

To obtain  $t_{PLH}$ , consider the situation in Fig. 1(a). Here, PD has just opened (at  $t = 0$ ) leaving  $v_O = 0$  V at  $t = 0+$ . Capacitor  $C$  then charges through the on resistance of the pull-up switch,  $R_{onu}$ , toward  $V_{DD}$ , thus

$$\begin{aligned} v_O(t) &= V_{\infty} - (V_{\infty} - V_{0+})e^{-t/\tau} \\ &= V_{DD} - (V_{DD} - 0)e^{-t/\tau} \\ &= V_{DD}(1 - e^{-t/\tau}) \end{aligned}$$

At  $t = t_{PLH}$ ,  $v_O = V_{DD}/2$ , thus

$$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_{PLH}/\tau})$$

$$\Rightarrow e^{-t_{PLH}/\tau} = 0.5$$

$$\Rightarrow t_{PLH} = \tau \ln 2 = 0.69\tau$$

For  $C = 10$  fF and  $R_{onu} = 20$  k $\Omega$ , then

$$\tau = 10 \times 10^{-15} \times 20 \times 10^3 = 200 \text{ ps}$$

and

$$t_{PLH} = 0.69 \times 200 = 138 \text{ ps}$$

Next we determine  $t_{PHL}$  by considering the situation depicted in Fig. 1(b). Here, PU has just opened, leaving  $v_O(0+) = V_{DD}$ . Capacitor  $C$  then discharges through the on resistance of the pull-down switch,  $R_{ond}$ , toward 0 V, thus  $v_O(\infty) = 0$ , thus

$$\begin{aligned} v_O &= 0 - (0 - V_{DD})e^{-t/\tau} \\ &= V_{DD}e^{-t/\tau} \end{aligned}$$

At  $t = t_{PHL}$ ,  $v_O = V_{DD}/2$  and we get

$$\frac{V_{DD}}{2} = V_{DD}e^{-t_{PHL}/\tau}$$

$$\Rightarrow t_{PHL} = \tau \ln 2 = 0.69\tau$$

Here,

$$\tau = CR_{ond}$$

$$= 10 \times 10^{-15} \times 10 \times 10^3 = 100 \text{ ps}$$

Thus,

$$t_{PHL} = 69 \text{ ps}$$

The propagation delay  $t_p$  can now be obtained as

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(138 + 69) = 104 \text{ ps}$$

**Ex: 15.3**

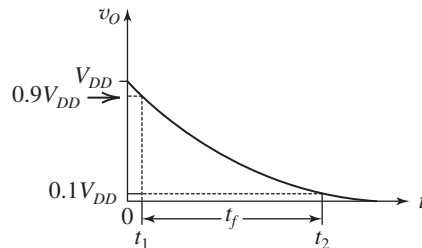


Figure 1

Figure 1 shows the exponential discharge curve and the two points that define the extent of the fall time,  $t_f$ . Here,

$$v_O(t) = V_{DD}e^{-t/\tau}$$

$$v_O(t_1) = 0.9V_{DD} = e^{-t_1/\tau} \quad (1)$$

$$v_O(t_2) = 0.1V_{DD} = e^{-t_2/\tau} \quad (2)$$

Dividing (1) by (2) gives

$$9 = e^{-(t_1-t_2)/\tau}$$

$$9 = e^{t_f/\tau}$$

$$\Rightarrow t_f = \tau \ln 9 = 2.2\tau$$

For  $C = 100$  fF and  $R = 2$  k $\Omega$ ,

$$\tau = 100 \times 10^{-15} \times 2 \times 10^3 = 200 \text{ ps}$$

and

$$t_f = 2.2 \times 200 = 440 \text{ ps} = 0.44 \text{ ns}$$

**Ex: 15.4**  $\alpha_n = 2 / \left[ \frac{7}{4} - \frac{3V_{in}}{V_{DD}} + \left( \frac{V_m}{V_{DD}} \right)^2 \right]$

$$= 2 / \left[ \frac{7}{4} - \frac{3 \times 0.5}{1.8} + \left( \frac{0.5}{1.8} \right)^2 \right] = 2.01$$

$$t_{PHL} = \frac{\alpha_n C}{k'_n (W/L)_n V_{DD}}$$

$$= \frac{2.01 \times 10 \times 10^{-15}}{300 \times 10^{-6} \times 1.5 \times 1.8}$$

$$= 24.8 \text{ ps}$$

$$\alpha_p = 2 / \left[ \frac{7}{4} - \frac{3|V_{ip}|}{V_{DD}} + \left( \frac{V_{ip}}{V_{DD}} \right)^2 \right]$$

$$= 2.01$$

$$t_{PLH} = \frac{\alpha_p C}{k'_p (W/L)_p V_{DD}}$$

$$= \frac{2.01 \times 10 \times 10^{-15}}{75 \times 10^{-6} \times 3 \times 1.8}$$

$$= 49.6 \text{ ps}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$$

$$= \frac{1}{2}(24.8 + 49.6)$$

$$= 37.2 \text{ ps}$$

**Ex: 15.5**  $t_{PHL} = 0.69R_N C$

$$50 \times 10^{-12} = 0.69 \times \frac{12.5 \times 1}{(W/L)_n} \times 10^3 \times 20 \times 10^{-15}$$

$$\Rightarrow (W/L)_n = 3.5$$

$$t_{PLH} = 0.69R_p C$$

$$50 \times 10^{-12} = 0.69 \times \frac{30 \times 1}{(W/L)_p} \times 10^3 \times 20 \times 10^{-15}$$

$$\Rightarrow (W/L)_p = 8.3$$

Note: If the 0.69 factor is replaced by 1 to account for the fact that the pulse edges are not ideal, then

$$(W/L)_n = 5$$

$$(W/L)_p = 12$$

**Ex: 15.6** With an additional 0.1 pF,  $C$  becomes

$$C = 0.866 \text{ fF} + 10 \text{ fF} = 10.866 \text{ fF}$$

Thus,

$$t_{PHL} = 4.18 \times \frac{10.866}{0.866}$$

$$= 52.4 \text{ ps}$$

$$t_{PLH} = 5.58 \times \frac{10.866}{0.866}$$

$$= 70.0 \text{ ps}$$

Thus,

$$t_P = \frac{1}{2}(52.4 + 70.0)$$

$$= 61.2 \text{ ps}$$

**Ex: 15.7**  $C = 2 \times 0.024 + 2 \times 0.024 + 0.03 + 0.03 + 4 \times 0.105 + 4 \times 0.105 + 0.5 = 1.496 \text{ fF}$

$$t_P = (4.88 \text{ ps}) \left( \frac{1.496}{0.866} \right) = 8.43 \text{ ps}$$

**Ex: 15.8** Refer to Example 15.3.

(a)  $C_{\text{int}} = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2}$

$$= 2 \times 0.024 + 2 \times 0.024 + 0.03 + 0.03$$

$$= 0.156 \text{ fF}$$

$$C_{\text{ext}} = C_{g3} + C_{g4} + C_w$$

$$= 0.105 + 0.105 + 0.5$$

$$= 0.71 \text{ fF}$$

(b) To reduce the extrinsic component of  $t_P$  by a factor of 2, we need to scale  $(W/L)_n$  and  $(W/L)_p$  by a factor

$$S = 2$$

(c) The original value of  $t_P = 4.88$  ps is composed of an intrinsic component

$$t_{P,\text{int}} = t_P \times \frac{C_{\text{int}}}{C}$$

$$= 4.88 \times \frac{0.156}{0.866} = 0.88 \text{ ps}$$

This component remains unchanged. The extrinsic component

$$t_{p,\text{ext}} = 4.88 - 0.88 = 4$$

is reduced by a factor of 2. Thus,  $t_p$  becomes

$$t_p = 0.88 + \frac{4}{2} = 2.88 \text{ ps}$$

$$\begin{aligned} \text{(d) Area} &= W_n L + W_p L \\ &= L(W_n + W_p) \end{aligned}$$

By scaling  $W_n$  and  $W_p$  by a factor of 2, the area increases by a factor of 2.

**Ex: 15.9**  $L = 0.18 \text{ } \mu\text{m}$ ,  $n = 1.5$ ,  $p = 3$

(a) Four-input NOR gate: Refer to Fig. 15.8.

$$\text{For NMOS transistors: } W/L = n = 1.5 = \frac{0.27}{0.18}$$

$$\text{For PMOS transistors: } W/L = 4p = 12 = \frac{2.16}{0.18}$$

(b) Four-input NAND gate: Refer to Fig. 15.9.

$$\text{For NMOS transistors: } W/L = 4n = 6 = \frac{1.08}{0.18}$$

$$\text{For PMOS transistors: } W/L = p = 3 = \frac{0.54}{0.18}$$

Area of NOR gate

$$= 4 \times 0.18 \times 0.27 + 4 \times 0.18 \times 2.16 = 1.7496 \text{ } \mu\text{m}^2$$

Area of NAND gate

$$= 4 \times 0.18 \times 1.08 + 4 \times 0.18 \times 0.54 = 1.1664$$

Thus,

$$\frac{\text{NOR area}}{\text{NAND area}} = \frac{1.7496}{1.1664} = 1.5$$

**Ex: 15.10** Refer to Fig. 15.9.

(a) Maximum charging current is the current supplied by the four identical PMOS transistors. Minimum charging current is the current supplied by one of the PMOS transistors. Thus, the ratio of maximum to minimum currents is 4.

(b) There is only one possible configuration for discharging a load capacitance, namely, when all 4 NMOS transistors are conducting. So, as far as capacitor discharge is concerned, the ratio is one.

**Ex: 15.11**  $P_{\text{dyn}} = fCV_{DD}^2$

$$= 1 \times 10^9 \times 0.866 \times 10^{-15} \times 0.9^2$$

$$= 0.70 \text{ } \mu\text{W}$$

**Ex: 15.12**  $P_{\text{dyn}} = fCV_{DD}^2$

$$= 250 \times 10^6 \times 80 \times 10^{-15} \times 1.2^2$$

$$= 28.8 \text{ } \mu\text{W}$$

**Ex: 15.13**  $P_{\text{dyn}} = fCV_{DD}^2$

$C$  decreases by a factor (0.13/0.5) and  $V_{DD}$  decreases from 5 V to 1.2 V; thus for the same  $f$ , the power dissipation will decrease by a factor

$$= \frac{0.5}{0.13} \times \frac{5}{1.2} = 66.8$$

**Ex: 15.14**  $PDP = fCV_{DD}^2 t_p$

When  $f = f_{\text{max}} = 1/2t_p$ ,

$$\begin{aligned} PDP &= \frac{1}{2} CV_{DD}^2 = \frac{1}{2} \times 0.866 \times 10^{-15} \times 0.9^2 \\ &= 0.35 \text{ fJ} \end{aligned}$$

$$\begin{aligned} EDP &= \frac{1}{2} CV_{DD}^2 t_p = 0.35 \times 10^{-15} \times 4.88 \times 10^{-12} \\ &= 1.7 \times 10^{-27} \text{ Js} \end{aligned}$$

**Ex: 15.15** Since dynamic power dissipation is scaled by  $\frac{1}{S^2}$  and propagation delay is scaled by  $\frac{1}{S}$ , hence, PDP is scaled by  $\frac{1}{S^2} \times \frac{1}{S} = \frac{1}{S^3} = \frac{1}{8}$ . Thus, PDP decreases by a factor of 8.

**Ex: 15.16** If  $V_{DD}$  and  $V_i$  are kept constant, the entries in Table 15.1 that change are as follows:

Obviously,  $V_{DD}$  and  $V_i$  do not scale by  $\frac{1}{S}$  anymore. They are kept constant!

$t_p \propto \frac{\alpha C}{k' V_{DD}}$ : since  $\alpha$  is a function of  $\frac{V_i}{V_{DD}}$ , then  $\alpha$  remains unchanged, while  $C$  is scaled by  $\frac{1}{S}$ , and  $k'$  is scaled by  $S$ , therefore  $t_p$  is scaled by

$$\frac{1/S}{S} = \frac{1}{S^2}$$

Energy/Switching cycle, i.e.,  $CV_{DD}^2$ , is scaled by  $\frac{1}{S}$

$P_{\text{dyn}} \propto \frac{CV_{DD}^2}{2t_p}$  and thus is scaled by  $\frac{1/S}{1/S^2} = S$  thus  $P_{\text{dyn}}$  increases.

The power density, i.e.,  $\frac{P_{\text{dyn}}}{\text{device area}}$ , is scaled by

$$\frac{S}{1/S^2} = S^3$$

**Solutions to End-of-Chapter Problems**

**15.1** (a) Capacitor  $C$  is charged to 10 V and the switch closes at  $t = 0$ , thus

$$v_O(0+) = 10 \text{ V}$$

Capacitor  $C$  then discharges through  $R$  exponentially with  $v_O(\infty) = 0$

$$v_O(t) = 0 - (0 - 10) e^{-t/\tau}$$

$$\Rightarrow v_O(t) = 10e^{-t/\tau}$$

(b) For  $C = 100$  pF and  $R = 1$  k $\Omega$ , we have

$$\tau = 100 \times 10^{-12} \times 1 \times 10^3 = 100 \text{ ns}$$

$$t_{PHL} = 0.69\tau = 0.69 \times 100 = 69 \text{ ns}$$

$$t_f = 2.22\tau = 2.2 \times 100 = 220 \text{ ns}$$

**15.2** (a) Switch opens at time  $t = 0$ , thus  $v_O(0+) = 0$  V. The capacitor then charges by a constant current  $I$ , thus

$$It = Cv_O(t)$$

$$\Rightarrow v_O(t) = \frac{I}{C}t$$

(b) For  $I = 1$  mA and  $C = 10$  pF the time  $t$  for  $v_O$  to reach 1 V can be found as

$$1 = \frac{1 \times 10^{-3}}{10 \times 10^{-12}} t$$

$$\Rightarrow t = 10^{-8} \text{ s} = 10 \text{ ns}$$

**15.3**  $V_{OH} = V_{DD}$

At  $t = 0$ ,  $v_I$  goes low and the transistor turns off instantly, thus

$$v_O(0+) = V_{OL}$$

Now capacitor  $C$  charges through  $R$  toward  $v_O(\infty) = V_{DD}$ , thus

$$v_O(t) = V_{DD} - (V_{DD} - V_{OL}) e^{-t/\tau}$$

At  $t = t_{PLH}$ ,

$$v_O = \frac{1}{2}(V_{OL} + V_{OH}) = \frac{1}{2}(V_{OL} + V_{DD}), \text{ thus}$$

$$\frac{1}{2}(V_{OL} + V_{DD}) = V_{DD} - (V_{DD} - V_{OL})e^{-t/\tau}$$

$$\Rightarrow t_{PLH} = 0.69\tau$$

For  $R = 10$  k $\Omega$  and we wish to limit  $\tau_{PLH}$  to 100 ps then the maximum value that  $C$  can have is found from

$$0.69 \times C \times 10 \times 10^3 = 100 \times 10^{-12}$$

$$\Rightarrow C = 1.45 \times 10^{-14} \text{ F}$$

$$= 14.5 \text{ fF}$$

**15.4** (a)  $V_{OL} = 0$  V

$$V_{OH} = V_{DD} = 1.2 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = \frac{V_{DD}}{2} - 0$$

$$= 0.6 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = V_{DD} - \frac{V_{DD}}{2}$$

$$= 0.6 \text{ V}$$

(b) Capacitor  $C$  discharges through  $R_{on}$  of  $P_D$ ,

$$v_O(0+) = V_{DD} = 1.2 \text{ V}$$

$$v_O(\infty) = 0 \text{ V}$$

Thus,

$$v_O(t) = V_{DD}e^{-t/\tau}$$

$$\Rightarrow t_{PHL} = 0.69\tau$$

$$= 0.69 \times 0.1 \times 10^{-12} \times 2 \times 10^3$$

$$= 138 \text{ ps}$$

(c) Here the capacitor charges through  $R_{on}$  of PU toward  $V_{DD}$ . Thus,

$$v_O(0+) = 0, \quad v_O(\infty) = V_{DD},$$

$$v_O(t) = V_{DD}(1 - e^{-t/\tau})$$

At  $t = t_{PLH}$ ,  $v_O(t) = V_{DD}/2$ , thus

$$t_{PLH} = 0.69\tau$$

$$= 0.69 \times 0.1 \times 10^{-12} \times 2 \times 10^3$$

$$= 138 \text{ ps}$$

Finally, we obtain  $\tau_P$  as

$$\tau_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(138 + 138) = 138 \text{ ps}$$

$$\mathbf{15.5} \quad \alpha_n = 2 \sqrt{\left[ \frac{7}{4} - \frac{3V_m}{V_{DD}} + \left( \frac{V_m}{V_{DD}} \right)^2 \right]}$$

$$= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 \times 0.35}{1.0} + \left( \frac{0.35}{1.0} \right)^2 \right]}$$

$$= 2.44$$

$$t_{PHL} = \frac{\alpha_n C}{k'_n(W/L)_n V_{DD}}$$

$$= \frac{2.44 \times 4 \times 10^{-15}}{500 \times 10^{-6} \times 2 \times 1.0}$$

$$= 9.8 \text{ ps}$$

$$\alpha_p = \alpha_n = 2.44$$

$$t_{PLH} = \frac{\alpha_p C}{k'_p(W/L)_p V_{DD}}$$

$$\begin{aligned}
 &= \frac{2.44 \times 4 \times 10^{-15}}{\left(\frac{500}{4}\right) \times 10^{-6} \times 4 \times 1.0} \\
 &= 19.5 \text{ ps} \\
 t_P &= \frac{1}{2}(t_{PHL} + t_{PLH}) \\
 &= \frac{1}{2}(9.8 + 19.5) = 14.7 \text{ ps}
 \end{aligned}$$

$$15.6 \quad R_N = \frac{12.5 \times 1}{1} = 12.5 \text{ k}\Omega$$

$$R_P = \frac{30 \times 1}{1} = 30 \text{ k}\Omega$$

$$\begin{aligned}
 t_{PHL} &= 0.69CR_N \\
 &= 0.69 \times 20 \times 10^{-15} \times 12.5 \times 10^3 \\
 &= 172.5 \text{ ps}
 \end{aligned}$$

$$\begin{aligned}
 t_{PLH} &= 0.69CR_P \\
 &= 0.69 \times 20 \times 10^{-15} \times 30 \times 10^3 \\
 &= 414 \text{ ps}
 \end{aligned}$$

$$\begin{aligned}
 t_P &= \frac{1}{2}(172.5 + 414) \\
 &= 293.3 \text{ ps}
 \end{aligned}$$

15.7

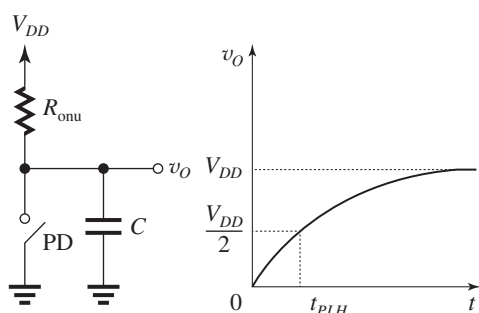


Figure 1(a)

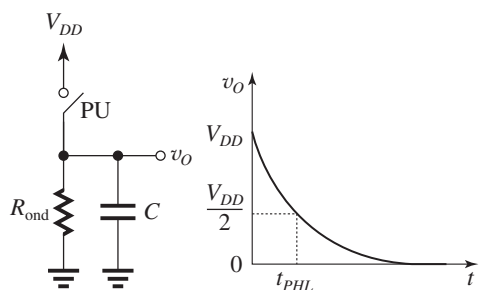


Figure 1(b)

To obtain  $t_{PLH}$ , consider the situation in Fig. 1(a). Here, PD has just opened (at  $t = 0$ ), leaving  $v_O = 0 \text{ V}$  at  $t = 0+$ . Capacitor  $C$  then charges through the "on" resistance of the pull-up switch,  $R_{\text{onu}}$ , toward  $V_{DD}$ , thus

$$\begin{aligned}
 v_O(t) &= V_{\infty} - (V_{\infty} - V_{0+})e^{-t/\tau} \\
 &= V_{DD} - (V_{DD} - 0)e^{-t/\tau} \\
 &= V_{DD}(1 - e^{-t/\tau})
 \end{aligned}$$

At  $t = t_{PLH}$ ,  $v_O = V_{DD}/2$ , thus

$$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_{PLH}/\tau})$$

$$\Rightarrow e^{-t_{PLH}/\tau} = 0.5$$

$$\Rightarrow t_{PLH} = \tau \ln 2 = 0.69\tau$$

For  $C = 20 \text{ fF}$ ,  $R_{\text{onu}} = 2 \text{ k}\Omega$ , then

$$\begin{aligned}
 t_{PLH} &= 0.69 \times 20 \times 10^{-15} \times 2 \times 10^3 \\
 &= 27.6 \text{ ps}
 \end{aligned}$$

Next we determine  $t_{PHL}$  by considering the situation depicted in Fig. 1(b). Here, PU has just opened, leaving  $v_O(0+) = V_{DD}$ . Capacitor  $C$  then discharges through the on resistance of the pull-down switch,  $R_{\text{ond}}$ , toward  $0 \text{ V}$ , thus  $v_O(\infty) = 0$ , thus

$$\begin{aligned}
 v_O &= 0 - (0 - V_{DD})e^{-t/\tau} \\
 &= V_{DD}e^{-t/\tau}
 \end{aligned}$$

At  $t = t_{PHL}$ ,  $v_O = V_{DD}/2$  and we get

$$\frac{V_{DD}}{2} = V_{DD}e^{-t_{PHL}/\tau}$$

$$\Rightarrow t_{PHL} = 0.69\tau$$

Here,

$$\begin{aligned}
 \tau &= CR_{\text{ond}} \\
 &= 20 \times 10^{-15} \times 1 \times 10^3 = 20 \text{ ps}
 \end{aligned}$$

Thus,

$$t_{PHL} = 0.69 \times 20 \simeq 13.8 \text{ ps}$$

The propagation delay  $t_P$  can now be obtained as

$$\begin{aligned}
 t_P &= \frac{1}{2}(t_{PLH} + t_{PHL}) \\
 &= \frac{1}{2}(27.6 + 13.8) = 20.7 \text{ ps}
 \end{aligned}$$

$$15.8 \text{ (a)} \quad t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

Since  $t_P = 45 \text{ ps}$ , then

$$t_{PLH} + t_{PHL} = 90 \text{ ps} \quad (1)$$

Now, since  $I_{\text{charge}}$  is half  $I_{\text{discharge}}$ , then

$$t_{PLH} = 2t_{PHL} \quad (2)$$

Using (1) together with (2) yields

$$t_{PLH} = 60 \text{ ps}$$

$$t_{PHL} = 30 \text{ ps}$$

(b) Since the propagation delay is directly proportional to  $C$ , then the increase in propagation delay by 50%, when the capacitance is increased by 0.1 pF, indicates that the original total capacitance is 0.2 pF.

(c) The reduction of propagation delays by 40% when the load inverter is removed indicates that the load inverter was contributing 40% of the total capacitance found in (b), that is,

$$C_{\text{out}} = 0.12 \text{ pF}$$

$$C_{\text{load}} = 0.08 \text{ pF}$$

**15.9** See figure on next page.

(a) For a rising input, time to the full change of output of second gate is

$$150 + 200 + \frac{250}{2} = 475 \text{ ps}$$

(b) For a falling input, time to the full change of output of the second gate is

$$200 + 150 + \frac{100}{2} = 400 \text{ ps}$$

The propagation delay is

$$\begin{aligned} t_P &= \frac{1}{2}(t_{PLH} + t_{PHL}) \\ &= \frac{1}{2}(200 + 150) = 175 \text{ ps} \end{aligned}$$

$$\begin{aligned} \mathbf{15.10} \quad \alpha_n &= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 V_m}{V_{DD}} + \left( \frac{V_m}{V_{DD}} \right)^2 \right]} \\ &= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 \times 0.35}{1.0} + \left( \frac{0.35}{1.0} \right)^2 \right]} \\ &= 2.44 \end{aligned}$$

For a matched inverter, we have

$$t_{PHL} = t_{PLH} = t_P$$

For  $t_P \leq 25 \text{ ps}$ ,

$$t_{PHL} \leq 25 \text{ ps}$$

But,

$$\begin{aligned} t_{PHL} &= \frac{\alpha_n C}{k'_n (W/L)_n V_{DD}} \\ \frac{2.44 \times 10 \times 10^{-15}}{500 \times 10^{-6} (W/L)_n \times 1.0} &\leq 25 \times 10^{-12} \end{aligned}$$

$$(W/L)_n \geq 1.95$$

$$(W/L)_p \geq 7.8$$

$$\mathbf{15.11} \quad R_N = \frac{12.5 \times 1}{(W/L)_n} = \frac{12.5}{1.5} = 8.33 \text{ k}\Omega$$

$$\begin{aligned} t_{PHL} &= 0.69 C R_N \\ &= 0.69 \times 10 \times 10^{-15} \times 8.33 \times 10^3 \\ &= 57.5 \text{ ps} \end{aligned}$$

$$R_P = \frac{30 \times 1}{(W/L)_p} = \frac{30}{3} = 10 \text{ k}\Omega$$

$$\begin{aligned} t_{PLH} &= 0.69 C R_P \\ &= 0.69 \times 10 \times 10^{-15} \times 10 \times 10^3 \\ &= 69 \text{ ps} \end{aligned}$$

$$t_P = \frac{1}{2}(57.5 + 69) = 63.3 \text{ ps}$$

**15.12** Refer to Example 15.2.

The method of average currents yields

$$t_{PHL} = 15.0 \text{ ps}$$

The method of equivalent resistance yields

$$t_{PHL} = 27.6 \text{ ps}$$

If the discrepancy is entirely due to the reduction in current due to velocity saturation in the NMOS transistor, then the factor by which the current decreases is  $15.0/27.6 = 0.54$ .

The value of  $t_{PLH}$  does not change (in fact there is a slight decrease due to various approximations). We may therefore conclude that the effect of velocity saturation is minimal in the PMOS transistor.

$$\begin{aligned} \mathbf{15.13} \quad \alpha_n &= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 V_m}{V_{DD}} + \left( \frac{V_m}{V_{DD}} \right)^2 \right]} \\ &= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 \times 0.35}{1} + \left( \frac{0.35}{1} \right)^2 \right]} \\ &= 2.43 \end{aligned}$$

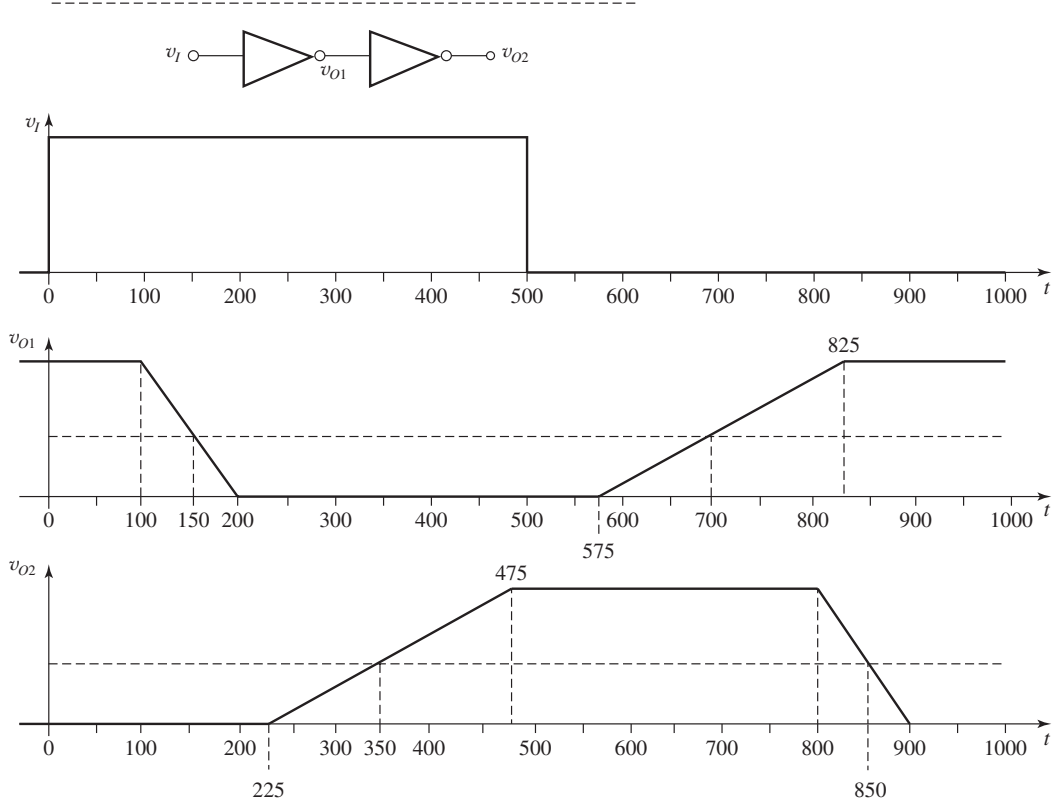
$$\begin{aligned} t_{PHL} &= \frac{\alpha_n C}{k'_n (W/L)_n V_{DD}} \\ &= \frac{2.43 \times 10 \times 10^{-15}}{470 \times 10^{-6} \times 1.5 \times 1} \\ &= 34.4 \text{ ps} \end{aligned}$$

$$t_{PLH} = \frac{\alpha_p C}{k'_p (W/L)_p V_{DD}}$$

Since  $|V_{tp}| = V_m$ , we have

$$\alpha_p = \alpha_n = 2.43$$

This figure belongs to Problem 15.9.



Thus,

$$t_{PLH} = \frac{2.43 \times 10 \times 10^{-15}}{190 \times 10^{-6} \times 3 \times 1} = 42.6 \text{ ps}$$

$$t_p = \frac{1}{2}(34.4 + 42.6) = 38.5 \text{ ps}$$

The theoretical maximum switching frequency is

$$f_{\max} = \frac{1}{2t_p} = \frac{1}{2 \times 38.5 \times 10^{-12}} \simeq 13 \text{ GHz}$$

**15.14** For

$$t_{PHL} = t_{PLH} = t_p \leq 15 \text{ ps}$$

we use

$$t_{PHL} = 0.69CR_N = 0.69C \times \frac{8 \times 1.5}{(W/L)_n} \times 10^3$$

and thus obtain

$$0.69 \times 5 \times 10^{-15} \times \frac{12}{(W/L)_n} \times 10^3 \leq 15 \times 10^{-12}$$

$$\Rightarrow \left(\frac{W}{L}\right)_n \geq 2.76$$

Similarly,

$$t_{PLH} = 0.69 C R_p$$

$$= 0.69 C \times \frac{24 \times 1.5}{(W/L)_p} \times 10^3$$

Thus,

$$0.69 \times 5 \times 10^{-15} \times \frac{36}{(W/L)_p} \times 10^3 \leq 15 \times 10^{-12}$$

$$\Rightarrow (W/L)_p \geq 8.28$$

**15.15**  $W_n = 260 \text{ nm}$

$$W_p = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} \times W_n$$

$$= \frac{500}{125} \times 260 = 1040 \text{ nm}$$

$$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_w$$

where

$$C_{gd1} = 0.3 \times W_n = 0.3 \times 0.26 = 0.078 \text{ fF}$$

$$C_{gd2} = 0.3 \times W_p = 0.3 \times 1.04 = 0.312 \text{ fF}$$

$$C_{db1} = 0.5 \times W_n = 0.5 \times 0.26 = 0.13 \text{ fF}$$

$$C_{db2} = 0.5 \times W_p = 0.5 \times 1.04 = 0.52 \text{ fF}$$

$$C_{g3} = 0.26 \times 0.065 \times 25 + 2 \times 0.3 \times 0.26 \\ = 0.5785 \text{ fF}$$

$$C_{g4} = 1.04 \times 0.065 \times 25 + 2 \times 0.3 \times 1.04 \\ = 2.314 \text{ fF}$$

Thus,

$$C = 2 \times 0.078 + 2 \times 0.312 + 0.13 + 0.52 + \\ 0.5785 + 2.314 + 2$$

$$= 6.32 \text{ fF}$$

$$\alpha_n = 2 \sqrt{\left[ \frac{7}{4} - \frac{3 V_t}{V_{DD}} + \left( \frac{V_t}{V_{DD}} \right)^2 \right]}$$

$$= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 \times 0.35}{1.0} + \left( \frac{0.35}{1.0} \right)^2 \right]}$$

$$= 2.44$$

$$t_{PHL} = \frac{\alpha_n C}{k'_n \left( \frac{W}{L} \right)_n V_{DD}}$$

$$= \frac{2.44 \times 6.32 \times 10^{-15}}{500 \times 10^{-6} \times \left( \frac{260}{65} \right) \times 1.0}$$

$$= 7.7 \text{ ps}$$

Since the inverter is matched,

$$t_{PLH} = t_{PHL} = 7.7 \text{ ps}$$

and

$$t_p = 7.7 \text{ ps}$$

The propagation delay increases by 50% if  $C$  is increased by 50%, that is, by  $6.32/2 = 3.16 \text{ fF}$ .

$$\mathbf{15.16} \quad C = 2.5 \times 0.13 + 2.5 \times 0.13 + 2$$

$$= 2.65 \text{ fF}$$

$$\alpha_n = 2 \sqrt{\left[ \frac{7}{4} - \frac{3 V_m}{V_{DD}} + \left( \frac{V_m}{V_{DD}} \right)^2 \right]}$$

$$= 2 \sqrt{\left[ \frac{7}{4} - \frac{3 \times 0.35}{1.0} + \left( \frac{0.35}{1.0} \right)^2 \right]}$$

$$= 2.44$$

$$t_{PHL} = \frac{\alpha_n C}{k'_n \left( \frac{W}{L} \right)_n V_{DD}}$$

$$= \frac{2.44 \times 2.65 \times 10^{-15}}{500 \times 10^{-6} \times \frac{130}{65} \times 1.0}$$

$$= 6.5 \text{ ps}$$

$$\alpha_p = \alpha_n = 2.44$$

$$t_{PLH} = \frac{\alpha_p C}{k'_p \left( \frac{W}{L} \right)_p V_{DD}}$$

$$= \frac{2.44 \times 2.65 \times 10^{-15}}{\frac{500}{4} \times 10^{-6} \times \frac{130}{65} \times 1.0}$$

$$= 25.9 \text{ ps}$$

$$t_p = \frac{1}{2}(6.5 + 25.9) = 16.2 \text{ ps}$$

If the design is changed to a matched one, then

$$W_p = 4W_n = 4 \times 130 = 520 \text{ nm}$$

$$C = 2.5 \times 0.13 + 2.5 \times 0.52 + 2$$

$$= 3.625 \text{ fF}$$

$$\alpha_n = \alpha_p = 2.44$$

$$t_{PHL} = \frac{2.44 \times 3.625 \times 10^{-15}}{500 \times 10^{-6} \times \frac{130}{65} \times 1.0}$$

$$= 8.8 \text{ ps}$$

$$t_{PLH} = \frac{2.44 \times 3.625 \times 10^{-15}}{\frac{500}{4} \times 10^{-6} \times \frac{520}{65} \times 1.0}$$

$$= 8.8 \text{ ps}$$

$$t_p = \frac{1}{2}(8.8 + 8.8) = 8.8 \text{ ps}$$

**15.17** To reduce  $t_p$  by 40 ps, we need to reduce the extrinsic part by 40 ps. Now the original value of the extrinsic part is

$$t_p = 80 \times \frac{45}{45 + 15} = 60 \text{ ps}$$

A reduction by 40 ps requires the use of a scale factor  $S$ ,

$$S = 3$$

This is the factor by which  $(W/L)_n$  and  $(W/L)_p$  must be scaled. The inverter area will be increased by the same ratio, that is, 3.

**15.18** Figure 1 on the next page shows the CMOS logic gate with the  $(W/L)$  ratios selected so that the worst-case  $t_{PLH}$  and  $t_{PHL}$  are equal to the corresponding values of the basic inverter with  $(W/L)_n = n$  and  $(W/L)_p = p$ . Observe that the worst case for discharging a capacitor occurs through the three series transistors  $Q_{NA}$ ,  $Q_{NC}$ , and  $Q_{ND}$ . To make the equivalent  $W/L$  for these three series transistors equal to  $n$ , we select each of their  $(W/L)$  ratios to be equal to  $3n$ . Finally, for the discharge path ( $Q_{NA}$ ,  $Q_{NB}$ ) to have an equivalent  $W/L$  equal to  $n$ , we selected  $W/L$  of  $Q_{NB}$  equal to  $1.5n$ .

For the PUN, the worst-case charging path is that through  $Q_{PB}$  and one of  $Q_{PC}$  or  $Q_{PD}$ . Thus we select each of these three transistors to have



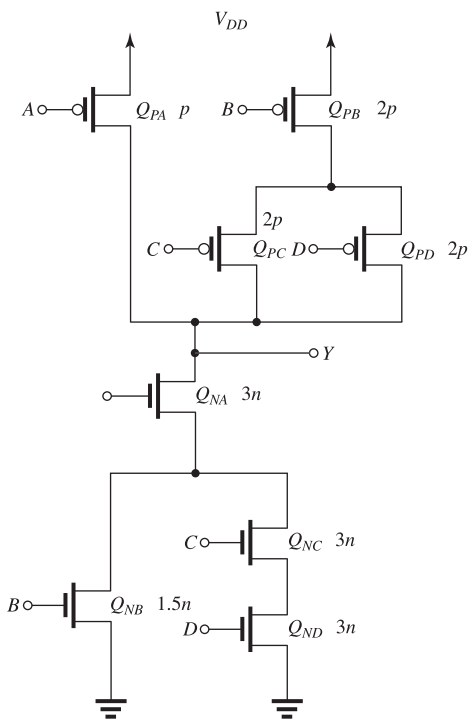


Figure 1

$W/L = 2p$ . Finally, we selected  $W/L$  of  $Q_{PA}$  equal to  $p$ .

15.19

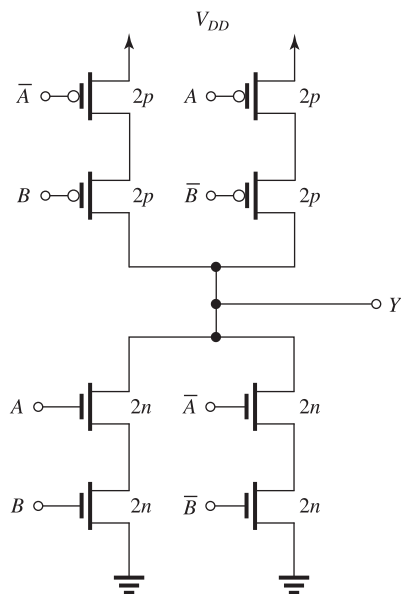


Figure 1

$$n = \frac{56}{28}, p = \frac{84}{28}$$

Figure 1 shows the circuit with the  $W/L$  ratio of each of the eight transistors indicated. Observe

that the worst-case situation for both charging and discharging is two transistors in series. To achieve an equivalent  $W/L$  ratio for each path equal to that of the corresponding transistor in the basic inverter, each transistor is sized at twice that of the inverter. Including the two inverters required to obtain the complemented variable, the area is

$$\begin{aligned} A &= 2W_nL + 2W_pL + 4 \times 2W_nL + 4 \times 2W_pL \\ &= 10L(W_n + W_p) \\ &= 10 \times 28(56 + 84) \\ &= 39,200 \end{aligned}$$

15.20 (a) Examination of Eq. (15.19) reveals that the NMOS transistors  $Q_1$  and  $Q_3$  contribute

$$C_n = 2 C_{gd1} + C_{db1} + C_{g3} \tag{1}$$

and the PMOS transistors  $Q_2$  and  $Q_4$  contribute

$$C_p = 2 C_{gd2} + C_{db2} + C_{g4} \tag{2}$$

The only difference in determining the corresponding capacitances in Eqs. (1) and (2) is the transistor width  $W$ . Thus each of the components in Eq. (2) can be written as the corresponding component in Eq. (1) multiplied by  $(W_p/W_n)$ . Overall, we can write

$$C_p = C_n \frac{W_p}{W_n}$$

and the total capacitance  $C$  can be expressed as

$$\begin{aligned} C &= C_n + C_p + C_w \\ &= C_n + C_n \frac{W_p}{W_n} + C_w \end{aligned}$$

Thus,

$$C = C_n \left( 1 + \frac{W_p}{W_n} \right) + C_w \quad \text{Q.E.D.}$$

$$(b) R_N = \frac{12.5 \times 1}{(W/L)_n} \text{ k}\Omega$$

For  $(W/L)_n = 1$ , we have

$$R_N = 12.5 \text{ k}\Omega$$

Thus,

$$\begin{aligned} t_{PHL} &= 0.69CR_N \\ &= 0.69 \times 12.5 \times 10^3 C \\ &= 8.625 \times 10^3 C \quad \text{Q.E.D.} \end{aligned}$$

$$\begin{aligned} R_P &= \frac{30 \times 1}{(W/L)_p} \text{ k}\Omega \\ &= \frac{30}{(W_p/W_n)(W/L)_n} \text{ k}\Omega \end{aligned}$$

For  $(W/L)_n = 1$ , we have

$$R_p = \frac{30}{W_p/W_n} \text{ k}\Omega$$

and

$$\begin{aligned} t_{PLH} &= 0.69CR_p \\ &= 0.69 \times \frac{30}{W_p/W_n} \times 10^3 C \\ &= \frac{20.7 \times 10^3}{W_p/W_n} C \quad \text{Q.E.D.} \end{aligned}$$

$$\begin{aligned} \text{(c) } t_p &= \frac{1}{2}(t_{PHL} + t_{PLH}) \\ &= \frac{1}{2} \left[ 8.625 \times 10^3 C + \frac{20.7 \times 10^3}{W_p/W_n} C \right] \end{aligned}$$

For  $W_p = W_n$ , we have

$$\begin{aligned} t_p &= 14.66 \times 10^3 C \\ t_p &= 14.66 \times 10^3 \left[ C_n \left( 1 + \frac{W_p}{W_n} \right) + C_w \right] \\ &= 14.66 \times 10^3 (2C_n + C_w) \end{aligned} \quad (3)$$

(d) In the matched case, we have

$$t_{PLH} = t_{PHL}$$

From the results in (b), the required ratio ( $W_p/W_n$ ) can be determined as

$$\begin{aligned} \frac{20.7}{W_p/W_n} &= 8.625 \\ \Rightarrow \frac{W_p}{W_n} &= 2.4 \end{aligned}$$

In this case, we have

$$\begin{aligned} C &= C_n(1 + 2.4) + C_w = 3.4 C_n + C_w \\ \text{and} \\ t_p &= t_{PLH} = 8.625 \times 10^3 (3.4 C_n + C_w) \end{aligned} \quad (4)$$

(e) (i) For  $C_w = 0$ , we have

$$\begin{aligned} W_p = W_n: \quad t_p &= 29.32 \times 10^3 C_n \\ W_p = 2.4 W_n: \quad t_p &= 29.32 \times 10^3 C_n \end{aligned}$$

Thus, in the case where  $C$  is entirely intrinsic, scaling does not affect  $t_p$ . This is what we found in Eq. (15.26).

(ii) For  $C_w \gg C_n$ , we have

$$\begin{aligned} W_p = W_n: \quad t_p &= 14.66 \times 10^3 C_w \\ W_p = 2.4 W_n: \quad t_p &= 8.625 \times 10^3 C_w \end{aligned}$$

Here  $C$  is entirely extrinsic, thus scaling the PMOS transistors has resulted in a decrease in  $t_p$ .

We conclude that using a matched design reduces  $t_p$  only when  $C$  is dominated by external capacitances. The matched design, of course, has the drawback of increased area.

### 15.21

$$\begin{aligned} L &= 0.90 \text{ nm}, \quad W_n = 180 \text{ nm}, \quad W_p = 360 \text{ nm}, \\ n &= 180/90, \quad p = 360/90 \end{aligned}$$

(a) Circuit (a) uses a six-input NOR gate and one inverter.

The six-input NOR requires:

6 NMOS transistors each with  $W/L = n$

and

6 PMOS transistors each with  $W/L = 6p$

The inverter requires

1 NMOS transistor with  $W/L = n$

and

1 PMOS transistor with  $W/L = p$

Thus,

$$\begin{aligned} \text{Area} &= 6W_nL + 6 \times 6W_pL + W_nL + W_pL \\ &= L(7W_n + 37W_p) \\ &= 90(7 \times 180 + 37 \times 360) \\ &= 90 \times 14,510 = 1,305,900 \text{ nm}^2 \end{aligned}$$

(b) Circuit (b) uses two three-input NOR gates and one two-input NAND gate.

Each three-input NOR gate requires

3 NMOS transistors, each with  $W/L = n$

3 PMOS transistors, each with  $W/L = 3p$

The two-input NAND gate requires

2 NMOS transistors, each with  $W/L = 2n$

2 PMOS transistors, each with  $W/L = p$

Thus,

$$\begin{aligned} \text{Area} &= 2 \times 3 \times W_nL + 2 \times 3 \times 3 \times W_pL \\ &\quad + 2 \times 2 \times W_nL + 2 \times W_pL \\ &= L(10W_n + 20W_p) \\ &= 90(10 \times 180 + 20 \times 360) \\ &= 90 \times 9000 \\ &= 810,000 \text{ nm}^2 \end{aligned}$$

Thus circuit (a) required  $1305900/810000 = 1.61$  times the area of circuit (b).

**15.22** When the devices are sized as in Fig. 15.9,  $t_{PLH}$  that results when one PMOS transistor is conducting (worst case) is

$$t_{PLH} = 0.69R_pC$$

$$= 0.69 \times \frac{R_{eff,P} \times (W/L)_p}{p} \times C$$

and  $t_{PHL}$  is obtained by noting that the equivalent  $W/L$  of the discharge path is  $4n/4 = n$  and thus

$$t_{PHL} = 0.69 R_N C$$

$$= 0.69 \times \frac{R_{eff,N} \times (W/L)_n}{n} \times C$$

For the case in which all  $p$ -channel devices have  $W/L = p$  and all  $n$ -channel devices have  $W/L = n$ , we have

$$t_{PLH} = 0.69 \times \frac{R_{eff,P} \times (W/L)_p}{p} \times C$$

which is the same as in the first case. However,

$$t_{PHL} = 0.69 \times \frac{R_{eff,N} \times (W/L)_n}{n/4} \times C$$

$$= 0.69 \times \frac{4 \times R_{eff,N} \times (W/L)_n}{n} \times C$$

which is four times the value obtained in the first case.

15.23

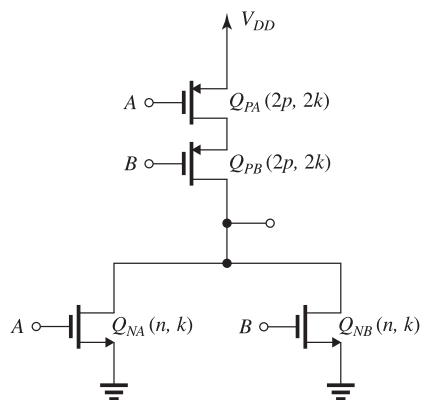


Figure 1

Refer to the circuit in Fig. 1. For  $Q_{NA}$  and  $Q_{NB}$ ,  $(W/L)$  is equal to that of the NMOS transistor in the basic matched inverter. Thus,

$$k_{NA} = k_{NB} = k$$

For  $Q_{PA}$  and  $Q_{PB}$ ,  $(W/L)$  is equal to twice the value of the PMOS transistor of the basic matched inverter. Since for the matched inverter  $k_p = k_n = k$ , here we have

$$k_{PA} = k_{PB} = 2k$$

(a) Figure 2 shows the circuit for the case input A is grounded. Note that  $Q_{NA}$  will be cut-off and has been eliminated. Switching will occur at  $v_I = V$  which will be near  $V_{DD}/2$ . At this point,

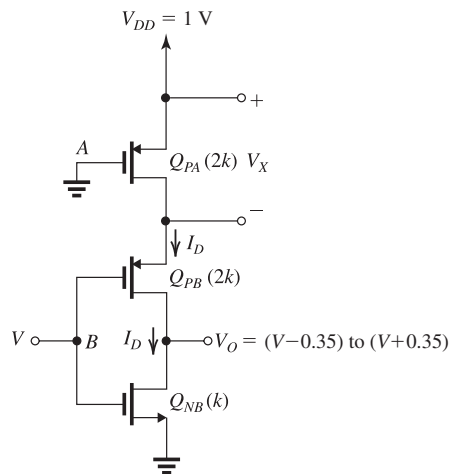


Figure 2

$Q_{NB}$  and  $Q_{PB}$  will be in saturation and  $Q_{PA}$  will be in the triode region with a very small voltage  $V_X$  across it. All transistors will be conducting the same current  $I_D$ . For  $Q_{PA}$  we can write

$$I_D = 2k \left[ (1 - 0.35)V_X - \frac{1}{2}V_X^2 \right]$$

or

$$I_D = k(1.3 V_X - V_X^2) \tag{1}$$

For  $Q_{PB}$  we can write

$$I_D = \frac{1}{2} \times 2k(1 - V_X - V - 0.35)^2$$

or

$$I_D = k(0.65 - V_X - V)^2 \tag{2}$$

Finally, for  $Q_{NB}$  we can write

$$I_D = \frac{1}{2}k(V - 0.35)^2 \tag{3}$$

Next, we solve Eqs. (2) and (3) together to obtain  $V_X$  in terms of  $V$ . Equating Eqs. (2) and (3) gives

$$\begin{aligned} \pm \frac{1}{\sqrt{2}}(V - 0.35) &= 0.65 - V_X - V \\ \pm 0.707(V - 0.35) &= 0.65 - V_X - V \end{aligned} \tag{4}$$

First try the solution corresponding to the + sign on the left-hand side of (4):

$$\begin{aligned} 0.707(V - 0.35) &= 0.65 - V_X - V \\ \Rightarrow V_X &= 0.897 - 1.707 V \end{aligned} \tag{5}$$

Since  $V_X \simeq 0$ , this equation gives

$$V = 0.53 \text{ V}$$

which is reasonable. The other solution gives

$$-0.707(V - 0.35) = 0.65 - V_X - V$$

This figure belongs to Problem 15.23, part (b).

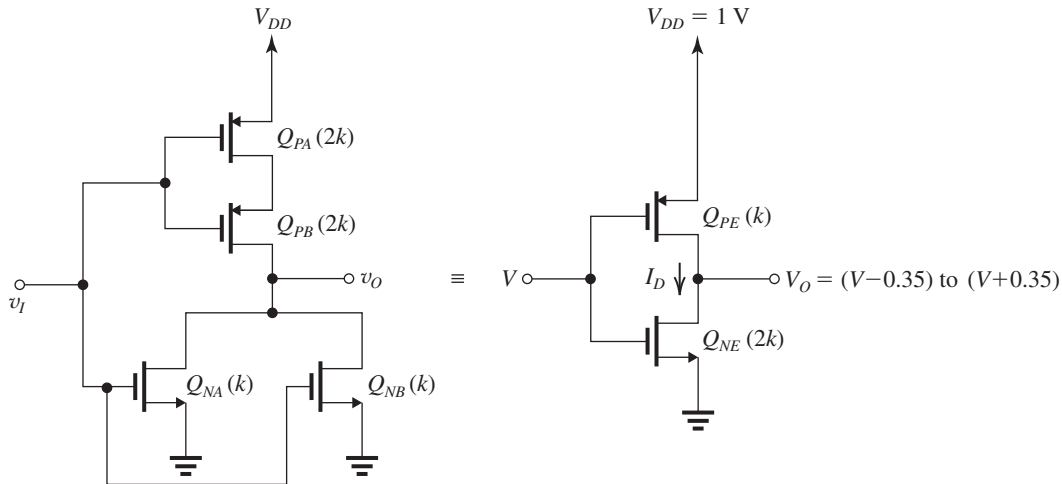


Figure 3

$$\Rightarrow V_X = 0.403 - 0.293 \text{ V}$$

For  $V_X \approx 0$ , this equation gives

$$V = 1.37 \text{ V}$$

which is obviously impossible! Thus Eq. (5) is the solution that is physically meaningful. Next we substitute for  $V_X$ . From Eq. (5) into Eq. (1) to obtain

$$\begin{aligned} I_D &= k \cdot 1.3(0.897 - 1.707 \text{ V}) - k(0.897 - 1.707 \text{ V})^2 \\ &= k(0.361 + 0.843 \text{ V} - 2.914 \text{ V}^2) \end{aligned} \quad (6)$$

Equating this value of  $I_D$  to that in Eq. (3) gives

$$\begin{aligned} (V - 0.35)^2 &= 2(0.361 + 0.843 \text{ V} - 2.914 \text{ V}^2) \\ \Rightarrow 6.83 \text{ V}^2 - 2.39 \text{ V} - 0.6 &= 0 \end{aligned}$$

$$\Rightarrow V = 0.54 \text{ V}$$

This is a reasonable value: It is greater than  $(V_{DD}/2)$ , which is required since  $Q_{NB}$  has a conductance parameter  $k$  while  $Q_{PB}$  has a parameter  $2k$ . Of course,  $V_{SG}$  of  $Q_{PB}$  is smaller than that of  $Q_{NB}$  because of  $V_X$ . The latter, however, is small. It can be found by substituting for  $V = 0.54$  in Eq. (5),

$$V_X = 0.89 - 1.707 \times 0.54 \approx 0 \text{ V}$$

(b) Figure 3 shows the circuit when the corresponding input terminals are connected together. The two parallel NMOS transistors can be replaced by an equivalent NMOS transistor  $Q_{NE}$  having a  $W/L = 2n$  and thus a transconductance parameter  $2k$ . The two series PMOS transistors can be replaced by an

equivalent PMOS transistor  $Q_{PE}$  having a  $W/L = (2p/2) = p$  and thus a transconductance parameter  $k$ . We are now ready to determine the threshold voltage, denoted  $V$  as before. Here, both  $Q_{NE}$  and  $Q_{PE}$  will be operating in saturation and conducting a current  $I_D$ . Thus, for  $Q_{PE}$  we can write

$$I_D = \frac{1}{2}k(V_{DD} - V - 0.35)^2$$

or

$$I_D = \frac{1}{2}k(0.65 - V)^2 \quad (7)$$

and for  $Q_{NE}$ , we can write

$$I_D = \frac{1}{2} \times 2k(V - 0.35)^2$$

or

$$I_D = k(V - 0.35)^2 \quad (8)$$

The value of  $V$  can be obtained by solving (7) and (8) together. Equating (7) and (8) gives

$$\frac{1}{2}(0.65 - V)^2 = (V - 0.35)^2$$

One solution is

$$\Rightarrow \frac{1}{\sqrt{2}}(0.65 - V) = V - 0.35$$

$$\Rightarrow V = 0.47 \text{ V}$$

This is a reasonable, physically meaningful answer and thus there is no need to find the other solution. As expected,  $V$  is lower than  $(V_{DD}/2)$ , which is a result of the fact that  $Q_{NE}$  has a larger (twice as large) transconductance parameter than

$Q_{PE}$ . Thus,  $Q_{NE}$  needs a smaller  $V_{GS}$  than  $V_{SG}$  of  $Q_{PE}$  to provide an equal  $I_D$ .

**15.24** (a) Refer to Fig. 15.11(c). By inspection we see that

$$t_P = \tau_1 + \tau_2 + \dots + \tau_{n-1} + \tau_n$$

But,

$$\tau_1 = \tau_2 = \dots = \tau_{n-1} = xCR$$

and

$$\tau_n = \frac{R}{x^{n-1}} C_L$$

Thus,

$$t_P = (n-1)xRC + \frac{1}{x^{n-1}} RC_L \quad \text{Q.E.D.} \quad (1)$$

(b) Differentiating  $t_P$  in Eq. (1) relative to  $x$  gives

$$\frac{\partial t_P}{\partial x} = (n-1)RC - \frac{(n-1)}{x^n} RC_L$$

Equating  $\frac{\partial t_P}{\partial x}$  to zero gives

$$x^n = \frac{C_L}{C} \quad \text{Q.E.D.} \quad (2)$$

(c) Differentiating  $t_P$  in Eq. (1) relative to  $n$  gives

$$\frac{\partial t_P}{\partial n} = xRC - \frac{1}{x^{n-1}} (\ln x) RC_L$$

Equating  $\frac{\partial t_P}{\partial n}$  to zero gives

$$x^n \left( \frac{C}{C_L} \right) = \ln x \quad \text{Q.E.D.} \quad (3)$$

To obtain the value of  $x$  for optimum performance, we combine the two optimality conditions in (2) and (3). Thus

$$\ln x = 1$$

$$\Rightarrow x = e \quad \text{Q.E.D.}$$

**15.25** (a)  $n = 4$

Minimum delay is obtained when the scaling factor  $x$  is given by

$$x^n = \frac{C_L}{C}$$

Here,

$$x^4 = \frac{1600C}{C} = 1600$$

$$\Rightarrow x = 6.32$$

$$t_P = 4xCR$$

$$= 4 \times 6.32CR = 25.3CR$$

(b) Let the number of the inverters be  $n$ .

Optimum performance is obtained when

$$x^n = \frac{C_L}{C} = 1600$$

and

$$x = e = 2.718$$

$$n = \frac{\ln 1600}{\ln e} = 7.4 \simeq 7$$

Thus, we use 7 inverters. The actual scaling factor required can be found from

$$x^7 = 1600$$

$$\Rightarrow x = (1600)^{1/7} = 2.87$$

The value of  $t_P$  realized will be

$$t_P = 7 \times 2.87CR$$

$$= 20.1CR$$

which represents a reduction in  $t_P$  by about 20.6%. Thus adding three inverters reduces the delay by 20.6%.

$$\mathbf{15.26} \quad E = CV_{DD}^2$$

$$= 10 \times 10^{-15} \times 1.8^2 = 32.4 \text{ fJ}$$

For  $2 \times 10^6$  inverters switched at  $f = 1 \text{ GHz}$ ,

$$P_D = 2 \times 10^6 \times 1 \times 10^9 \times 32.4 \times 10^{-15}$$

$$= 64.8 \text{ W}$$

$$I_{DD} = \frac{P_D}{V_{DD}} = \frac{64.8}{1.8} = 36 \text{ A}$$

**15.27** (a) The currents decrease by a factor of

$$\frac{1.2}{1.8} = 0.67; \text{ that is, the new current values are}$$

0.67 of the old current values. However, the

voltage swing is also reduced by the same factor.

The result is that  $t_P$  remains unchanged. The PDP will be reduced by a factor of 0.44.

Since the maximum operating frequency is

proportional to  $1/t_P$ , it also will remain unchanged.

(b) If current is proportional to  $V_{DD}^2$ , the currents

become  $0.67^2$  of their old values. This together

with the reduction of voltage swing by a factor of

0.67 will result in  $t_P$  increasing by a factor of

$(1/0.67)$  and the maximum operating frequency

being reduced by a factor of 0.67. The PDP

decreases by a factor of 0.67.

**15.28** Each cycle, the inverter draws an average current of

$$I_{av} = \frac{60 + 0}{2} = 30 \mu\text{A}$$

Since  $I_{av} = 150 \mu\text{A}$ , then the average current corresponding to the dynamic power dissipation is  $120 \mu\text{A}$ . Thus,

$$P_{\text{dyn}} = 3.3 \times 120 \times 10^{-6} = 396 \mu\text{W}$$

But,

$$P_{\text{dyn}} = fCV_{DD}^2$$

Thus,

$$396 \times 10^{-6} = 100 \times 10^6 \times 3.3^2 \times C$$

$$\Rightarrow C = 0.36 \text{ pF}$$

$$\mathbf{15.29} \quad P_{\text{dyn}} = fCV_{DD}^2$$

$$= 2.5 \times 10^9 \times 5 \times 10^{-15} \times 1$$

$$= 12.5 \mu\text{W}$$

$$I_{DD} = \frac{12.5 \times 10^{-6}}{1} = 12.5 \mu\text{A}$$

**15.30** Since  $P_{\text{dyn}}$  is proportional to  $V_{DD}^2$ , reducing the power supply from 1.2 V to 1.0 V reduces the power dissipation by a factor of  $\left(\frac{1.0}{1.2}\right)^2 = 0.694$ .

The power dissipation now becomes  $0.694 \times 10 = 6.94 \text{ mW}$ . Since

$P_{\text{dyn}}$  is proportional to  $f$ , reducing  $f$  by the same factor as the supply voltage (0.83) results in reducing the power dissipation *further* by a factor of 0.83, i.e.,

$$\begin{aligned} \text{Additional savings in power} &= (1 - 0.83) \times 6.94 \\ &= 1.16 \text{ mW} \end{aligned}$$

$$\mathbf{15.31} \quad t_{PLH} = 1.3 \text{ ns}, \quad t_{PHL} = 1.2 \text{ ns}$$

$$t_P = \frac{1}{2}(1.3 + 1.2) = 1.25 \text{ ns}$$

$$P_{D_{\text{av}}} = \frac{1}{2}(0.1 + 0.2) = 0.15 \text{ mW}$$

$$PDP = 0.15 \times 10^{-3} \times 1.25 \times 10^{-9} = 0.188 \text{ pJ}$$

**15.32** (a)

Figure 1 shows the circuit as the switch is opened ( $t = 0+$ ). Capacitor  $C$  will charge through  $R$ , and its voltage will increase from the initial value of  $V_{OL}$  to the high value  $V_{OH}$ ,

$$\begin{aligned} v_O &= v_O(\infty) - [v_O(\infty) - v_O(0+)]e^{-t/\tau} \\ &= V_{OH} - (V_{OH} - V_{OL})e^{-t/\tau} \quad \text{Q.E.D.} \end{aligned}$$

where

$$\tau_1 = CR$$

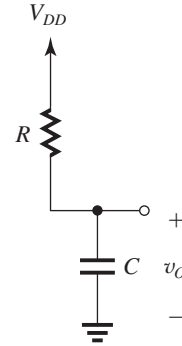


Figure 1

To reach the 50% point,  $\frac{1}{2}(V_{OH} + V_{OL})$  the time required,  $t_{PLH}$ , can be found as follows:

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1}$$

$$(V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1} = \frac{1}{2}(V_{OH} - V_{OL})$$

$$\Rightarrow t_{PLH} = \tau_1 \ln 2$$

$$= 0.69\tau_1 = 0.69CR \quad \text{Q.E.D.}$$

(b) Figure 2(a) on the next page shows the circuit after the switch closes ( $t = 0+$ ). At this instant the capacitor voltage is  $V_{OH}$ . The capacitor then discharges and eventually reaches the low level  $V_{OL}$ . To determine  $\tau_{PHL}$ , we simplify the circuit to that in Fig. 2(b). Using this circuit, we can express  $v_O$  as

$$\begin{aligned} v_O(t) &= v_O(\infty) - [v_O(\infty) - v_O(0)]e^{-t/\tau_2} \\ &= V_{OL} - (V_{OL} - V_{OH})e^{-t/\tau_2} \end{aligned}$$

when

$$\tau_2 = C(R_{\text{on}} \parallel R) \simeq CR_{\text{on}}$$

The value of  $t_{PHL}$  can be found from

$$v_O(t_{PHL}) = \frac{1}{2}(V_{OH} + V_{OL})$$

$$= V_{OL} - (V_{OL} - V_{OH})e^{-t_{PHL}/\tau_2}$$

$$\Rightarrow t_{PHL} = \tau_2 \ln 2$$

$$= 0.69\tau_2$$

$$= 0.69CR_{\text{on}} \quad \text{Q.E.D.}$$

$$(c) \quad t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(0.69CR + 0.69CR_{\text{on}})$$

$$t_P = \frac{1}{2} \times 0.69C(R + R_{\text{on}})$$

Since  $R_{\text{on}} \ll R$ ,

$$t_P \simeq 0.35CR \quad \text{Q.E.D.}$$

This figure belongs to Problem 15.32, part (b).

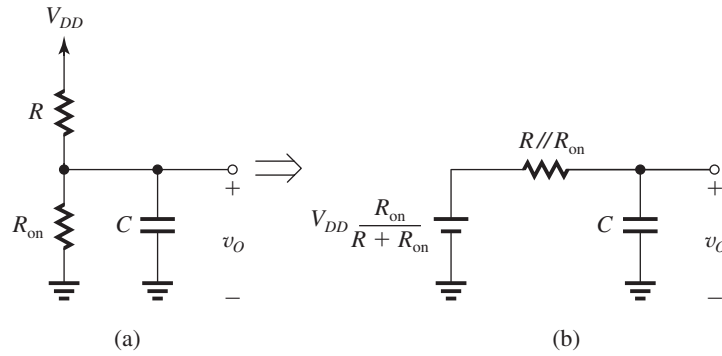


Figure 2

(d) During the low-input state, the switch is open, the current is zero, and the power dissipation is zero.

During the high-input state, the switch is closed and a current

$$I_{DD} = \frac{V_{DD}}{R + R_{on}} \approx \frac{V_{DD}}{R}$$

flows, and the power dissipation is

$$P_D = V_{DD} I_{DD} = \frac{V_{DD}^2}{R}$$

Now, if the inverter spends half the time in each state, the average power dissipation will be

$$P = \frac{1}{2} \frac{V_{DD}^2}{R} \quad \text{Q.E.D.}$$

(e) For  $V_{DD} = 1.8 \text{ V}$  and  $C = 2 \text{ pF}$ , we have

$$t_p = 0.35 \times 2 \times 10^{-12} R$$

If  $t_p$  is to be smaller or equal to  $1.4 \text{ ns}$ , we must have

$$0.35 \times 2 \times 10^{-12} R \leq 1.4 \times 10^{-9}$$

$$\Rightarrow R \leq 2.03 \text{ k}\Omega$$

If  $P$  is to be smaller or equal to  $2 \text{ mW}$ , we must have

$$\frac{1}{2} \times \frac{1.8^2}{R} \leq 2$$

where  $R$  is in  $\text{k}\Omega$ , thus

$$R \geq 1.23 \text{ k}\Omega$$

Thus, to satisfy both constraints,  $R$  must lie in the range

$$1.23 \text{ k}\Omega \leq R \leq 2.03 \text{ k}\Omega$$

Selecting  $R = 1.5 \text{ k}\Omega$  yields

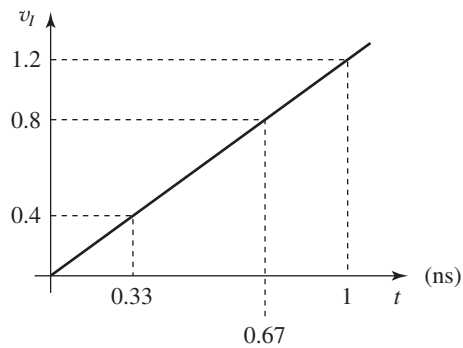
$$\begin{aligned} t_p &= 0.35 \times 2 \times 10^{-12} \times 1.5 \times 10^3 \\ &= 1.04 \text{ ns} \end{aligned}$$

and

$$P = \frac{1}{2} \frac{1.8^2}{1.5} = 1.08 \text{ mW}$$

Both values are within the design specifications.

### 15.33



From Eq. (15.40), we have

$$I_{\text{peak}} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_n \left( \frac{V_{DD}}{2} - V_m \right)^2$$

$$I_{\text{peak}} = \frac{1}{2} \times 500 \frac{\mu\text{A}}{\text{V}^2} \left( \frac{1.2}{2} - 0.4 \right)^2 = 10 \mu\text{A}$$

The time when the input reaches  $V_i$  is

$$\frac{0.4}{1.2} \times 1 = 0.33 \text{ ns}$$

$$\frac{1.2 - 0.4}{1.2} \times 1 = 0.67 \text{ ns}$$

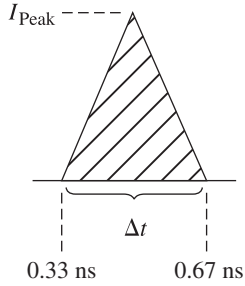
So the base of the triangle is

$$\Delta t = 0.67 - 0.33 = 0.34 \text{ ns}$$

$$\begin{aligned} E &= \frac{1}{2} I_{\text{peak}} \times V_{DD} \times \Delta t \\ &= \frac{1}{2} \times 10 \mu\text{A} \times 1.2 \times 0.34 \text{ ns} \end{aligned}$$

$= 2 \text{ fJ}$

$P = f \times E = 100 \times 10^6 \times 2 \times 10^{-15} = 0.2 \text{ } \mu\text{W}.$



**15.34** (a)  $t_p \propto \frac{\alpha C}{k' V_{DD}}$ , and  $k'$  is scaled by  $S$ , and  $C$  and  $V_{DD}$  are scaled by  $\frac{1}{S}$ ; thus  $t_p$  is scaled by

$$\frac{1}{S} \times \frac{1}{\frac{1}{S}} = \frac{1}{S}$$

$S = 2 \Rightarrow t_p$  is scaled by  $\frac{1}{2}$  ( $t_p$  decreases)

The maximum operating speed is  $\frac{1}{2t_p}$  and therefore is scaled by 2.

$P_{\text{dyn}} = f_{\text{max}} C V_{DD}^2$  and thus is scaled by

$$S \times \frac{1}{S} \times \frac{1}{S^2} = \frac{1}{S^2} = \frac{1}{4} \text{ (} P_{\text{dyn}} \text{ decreases).}$$

Power density =  $\frac{P_{\text{dyn}}}{\text{area}}$  and thus is scaled by

$$\frac{1}{\frac{S^2}{1}} = 1, \text{ i.e., remains unchanged.}$$

PDP is scaled by  $\frac{1}{S^3}$  (power is scaled by  $\frac{1}{S^2}$  and delay by  $\frac{1}{S}$ ) and thus it is scaled by  $\frac{1}{8}$  (PDP decreases).

(b) If  $V_{DD}$  and  $V_m$  remain unchanged while  $S = 2$ , we have

$$t_p = \frac{\alpha C}{k' V_{DD}} \text{ and } \alpha = \frac{2}{\frac{7}{4} - \frac{3V_m}{V_{DD}} + \left(\frac{V_m}{V_{DD}}\right)^2} \text{ so } \alpha$$

remains unchanged and  $t_p$  is scaled by

$$\frac{1}{\frac{1}{S}} = \frac{1}{\frac{1}{2}} = \frac{1}{4}$$

The maximum operating speed is  $\frac{1}{2t_p}$  and therefore is scaled by 4.

$P_{\text{dyn}} = f_{\text{max}} C V_{DD}^2$  and thus is scaled by

$$4 \times \frac{1}{2} \times 1 = 2$$

Power density =  $\frac{P_{\text{dyn}}}{\text{area}}$  is thus scaled by

$$\frac{2}{\frac{1}{S^2}} = \frac{2}{\frac{1}{4}} = 8$$

PDP is scaled by  $2 \times \frac{1}{4} = \frac{1}{2}$

**15.35**  $I_D = \frac{1}{2} k_n (V_{GS} - V_m)^2$

$$0.2 = \frac{1}{2} \times 0.4 (V_{GS} - 0.4)^2$$

$$\Rightarrow V_{GS} = 1.4 \text{ V}$$

(a) For  $V_m = 0.4 + 10\% = 0.44 \text{ V}$ , we have

$$I_D = \frac{1}{2} \times 0.4 (1.4 - 0.44)^2 = 0.184 \text{ mA}$$

For  $V_m = 0.4 - 10\% = 0.36 \text{ V}$ , we have

$$I_D = \frac{1}{2} \times 0.4 (1.4 - 0.36)^2 = 0.216 \text{ mA}$$

Thus,  $I_D$  will range from 0.184 mA to 0.216 mA (i.e.,  $0.2 \pm 8\%$ , mA).

(b) If the time for a 0.1-V change of the capacitor voltage is denoted  $T$ , then

$$I_D T = C \Delta V$$

$$T = \frac{C \Delta V}{I_D}$$

For  $I_D = 0.184 \text{ mA}$ , we have

$$T = \frac{100 \times 10^{-15} \times 0.1}{0.184 \times 10^{-3}} = 54.3 \text{ ps}$$

For  $I_D = 0.216 \text{ mA}$ , we have

$$T = \frac{100 \times 10^{-15} \times 0.1}{0.216 \times 10^{-3}} = 46.3 \text{ ps}$$

Thus, the discharge time ranges from 46.3 ps to 54.3 ps.



## Chapter 16

## Solutions to Exercises within the Chapter

**Ex: 16.1**  $V_i = V_{i0} + \gamma \left( \sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f} \right)$

Since  $V_{OH} = V_{DD} - V_i$ ,

$$V_i = V_{i0} + \gamma \left( \sqrt{V_{DD} - V_i + 2\phi_f} - \sqrt{2\phi_f} \right)$$

Substituting values, we get

$$V_i = 0.5 + 0.3 \left( \sqrt{1.8 - V_i + 0.85} - \sqrt{0.85} \right)$$

$$V_i = 0.5 + 0.3 \left( \sqrt{2.65 - V_i} - 0.3\sqrt{0.85} \right)$$

$$V_i - 0.223 = 0.3\sqrt{2.65 - V_i}$$

Squaring both sides yields

$$V_i^2 - 0.446V_i + 0.05 = 0.09(2.65 - V_i)$$

so that,  $V_i^2 - 0.356V_i - 0.189 = 0$

Solving this quadratic equation yields one practical value for  $V_i$ :

$$V_i = 0.648 \text{ V}$$

$$V_{OH} = V_{DD} - V_i = 1.8 \text{ V} - 0.648 \text{ V}$$

$$= 1.15 \text{ V}$$

**Ex: 16.2** (a) Referring to Fig. 16.7, we obtain

$$V_{OH} = 1 \text{ V}$$

$$V_{OL} = 0 \text{ V}$$

(b) Referring to Fig. 16.7(a), we get

$$\begin{aligned} i_{DN}(0) &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (V_{DD} - V_{i0})^2 \\ &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (1 - 0.35)^2 = 190.1 \mu\text{A} \end{aligned}$$

$$\begin{aligned} i_{DP}(0) &= \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_{i0})^2 \\ &= \frac{1}{2} \times 150 \times \left( \frac{130}{65} \right) (1 - 0.35)^2 = 63.4 \mu\text{A} \end{aligned}$$

Capacitor current is

$$\begin{aligned} i_C(0) &= i_{DN}(0) + i_{DP}(0) = 190.1 + 63.4 \\ &= 253.5 \mu\text{A} \end{aligned}$$

To obtain  $i_{DN}(t_{PLH})$ , we note that this situation is identical to that in Example 16.1 and we can use the result of part (c):

$$i_{DN}(t_{PLH}) = 3.6 \mu\text{A}$$

$$i_{DP}(t_{PLH}) = k'_p \left( \frac{W}{L} \right)_p \times$$

$$\left[ (V_{DD} - V_{i0}) \frac{V_{DD}}{2} - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right]$$

$$= (150) \left( \frac{130}{65} \right) \left[ (1 - 0.35) \left( \frac{1}{2} \right) - \frac{1}{2} \left( \frac{1}{2} \right)^2 \right]$$

$$= 60 \mu\text{A}$$

Thus,  $i_C(t_{PLH}) = 3.6 + 60 = 63.6 \mu\text{A}$

$$i_C|_{av} = \frac{1}{2} (253.5 + 63.6) = 158.6 \mu\text{A}$$

$$t_{PLH} = \frac{C \left( \frac{V_{DD}}{2} \right)}{i_C|_{av}} = \frac{5(10^{-15}) \left( \frac{1}{2} \right)}{158.6(10^{-6})}$$

$$= 15.8 \text{ ps}$$

(c) Referring to Fig. 16.7(b), we obtain

$$\begin{aligned} i_{DN}(0) &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (V_{DD} - V_{i0})^2 \\ &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (1 - 0.35)^2 = 190.1 \mu\text{A} \end{aligned}$$

$$\begin{aligned} i_{DP}(0) &= \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_{i0})^2 \\ &= \frac{1}{2} (150) \left( \frac{130}{65} \right) (1 - 0.35)^2 = 63.4 \mu\text{A} \end{aligned}$$

$$\begin{aligned} i_C(0) &= i_{DN}(0) + i_{DP}(0) = 190.1 + 63.4 \\ &= 253.5 \mu\text{A} \end{aligned}$$

$$i_{DN}(t_{PHL}) =$$

$$\begin{aligned} &k'_n \left( \frac{W}{L} \right)_n \times \left[ (V_{DD} - V_{i0}) \frac{V_{DD}}{2} - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right] \\ &= 450 \left( \frac{130}{65} \right) \left[ (1 - 0.35) \left( \frac{1}{2} \right) - \frac{1}{2} \left( \frac{1}{2} \right)^2 \right] \\ &= 180 \mu\text{A} \end{aligned}$$

To find  $i_{DP}(t_{PHL})$ , we first determine  $V_{tp}$  when

$$v_O = \frac{V_{DD}}{2}, \text{ which corresponds to } V_{SB} = \frac{V_{DD}}{2}$$

$$|V_{tp}| = V_{i0} + \gamma \left[ \sqrt{\frac{V_{DD}}{2} + 2\phi_f} - \sqrt{2\phi_f} \right]$$

$$= 0.35 + 0.25 \left[ \sqrt{\frac{1}{2} + 1} - \sqrt{1} \right] = 0.41 \text{ V}$$

$$\text{Thus, } i_{DP}(t_{PHL}) = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p \left[ \frac{V_{DD}}{2} - |V_{tp}| \right]^2$$

$$= \frac{1}{2} (150) \left( \frac{130}{65} \right) \left( \frac{1}{2} - 0.41 \right)^2 = 1.2 \mu\text{A}$$

$$i_C(t_{PHL}) = i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})$$

$$= 180 + 1.2 = 181.2 \mu\text{A}$$

$$i_C|_{av} = \frac{253.5 \mu\text{A} + 181.2 \mu\text{A}}{2} = 217.4 \mu\text{A}$$

So,

$$t_{PHL} = \frac{C \left( \frac{V_{DD}}{2} \right)}{i_{C|av}} = \frac{5 (10^{-15}) \left( \frac{1}{2} \right)}{217.4 (10^{-6})} = 11.5 \text{ ps}$$

$Q_p$  will turn off when  $v_O = |V_{tp}|$

where

$$|V_{tp}| = V_{t0} + \gamma \left[ \sqrt{V_{DD} - |V_{tp}| + 2\phi_f} - \sqrt{2\phi_f} \right]$$

Solving for  $|V_{tp}|$ , we get

$$|V_{tp}| = 0.35 + 0.25 \left[ \sqrt{1 - |V_{tp}| + 1} - \sqrt{1} \right]$$

$$|V_{tp}| - 0.1 = 0.25 \sqrt{2 - |V_{tp}|}$$

Squaring both sides, and setting one side equal to zero, we have the quadratic equation

$$|V_{tp}|^2 - 0.1375 |V_{tp}| - 0.115 = 0$$

Solving, we get  $|V_{tp}| = 0.41 \text{ V}$

$$(d) t_p = \frac{1}{2} (t_{PLH} + t_{PHL}) = \frac{1}{2} (15.8 \text{ ps} + 11.5 \text{ ps})$$

$\simeq 13.7 \text{ ps}$

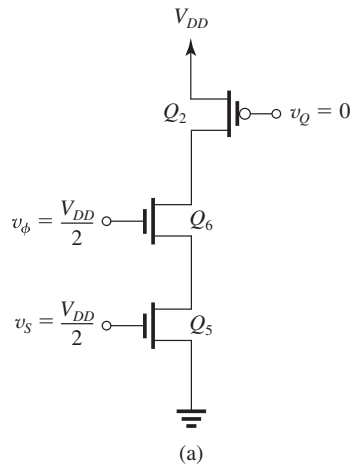
**Ex: 16.3**  $R_{TGAV} = \frac{R_{TG1} + R_{TG2}}{2}$   
 $= \frac{3.9 \text{ k}\Omega + 7.1 \text{ k}\Omega}{2} = 5.5 \text{ k}\Omega$

$$t_{PLH} = 0.69 RC = 0.69 (5.5 \text{ k}\Omega) (5) (10^{-15}) \text{ F}$$

$t_{PLH} = 19.0 \text{ ps}$ , which is close to the value of 15.8 ps obtained in Exercise 16.2.

**Ex: 16.4** When the drain of  $Q_{P1}$  is at 0 V,  $Q_{P1}$  is operating in the saturation region, so

This figure belongs to Exercise 16.5.



$$I_{P1}(0) = \frac{1}{2} k'_p \frac{W}{L} (V_{DD} - |V_{tp}|)^2 = \frac{1}{2} (150)(2)(1.0 - 0.35)^2 = 63.4 \text{ }\mu\text{A}$$

At this point,

$$R_{P1} = V_{DD}/I_{P1}(0) = 1.0/63.4 = 15.8 \text{ k}\Omega$$

When the drain of  $Q_{P1}$  is at  $V_{DD}/2 = 0.5 \text{ V}$ , then  $Q_{P1}$  is operating in the triode region, so

$$I_{P1}(0.5) = k'_p \frac{W}{L} \left( (V_{DD} - |V_{tp}|) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right)$$

$$= (150)(2)((1.0 - 0.35)(0.5) - \frac{1}{2}(0.5)^2)$$

$$= 60 \text{ }\mu\text{A}$$

Here,  $R_{P1} = 0.5/60 = 8.3 \text{ k}\Omega$ .

The average value of  $R_{P1}$  is  $(15.8 + 8.3)/2 = 12.1 \text{ k}\Omega$ .

Assuming a ramp input,

$$t_{PLH} = R_{P1}(C_{out1} + C_{TG1}) + (R_{P1} + R_{TG})(C_{TG2} + C_{m2})$$

$$= 12.1(4 + 2) + (12.1 + 5.5)(2 + 4) = 178.2 \text{ ps}$$

**Ex: 16.5**

Refer to Fig. 1(a). When  $v_\phi = v_S = \frac{V_{DD}}{2}$ ,  $Q_{eq}$  (Fig. 1(b)) will be operating in saturation. From Fig. 1(b) we see that  $Q_2$  will be operating in the triode region. For  $Q_{eq}$  we have

$$I_{Deq} = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_{eq} \left( \frac{V_{DD}}{2} - V_m \right)^2$$

$$= \frac{1}{2} \times 450 \times \frac{1}{2} \left( \frac{W}{L} \right)_{5,6} \left( \frac{1}{2} - 0.35 \right)^2$$

$$= 2.53 \left( \frac{W}{L} \right)_{5,6} \text{ }\mu\text{A}$$

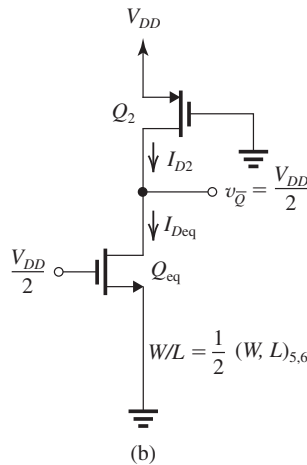


Figure 1

For  $Q_2$ , we have

$$\begin{aligned} I_{D2} &= k'_p \left(\frac{W}{L}\right)_2 \left[ (V_{DD} - |V_{tp}|) \left(\frac{V_{DD}}{2}\right) - \frac{1}{2} \left(\frac{V_{DD}}{2}\right)^2 \right] \\ &= 150 \times \frac{390}{65} \left[ (1 - 0.35) 0.5 - \frac{1}{2} (0.5)^2 \right] \\ &= 180 \mu\text{A} \end{aligned}$$

Equating  $I_{Deq}$  to  $I_{D2}$  gives

$$\left(\frac{W}{L}\right)_{5,6} = \frac{180}{2.53} = 71.1$$

**Ex: 16.6**  $\left(\frac{W}{L}\right)_a \Big|_{\max} = \frac{1}{\left(1 - \frac{V_m}{V_{DD} - V_m}\right)^2} - 1$

$$\left(\frac{W}{L}\right)_a \Big|_{\max} = 2 \times \left[ \frac{1}{\left(1 - \frac{0.35}{1 - 0.35}\right)^2} - 1 \right] = 7.4$$

$$\Rightarrow \left(\frac{W}{L}\right)_a \leq 7.4$$

**Ex: 16.7** (a)  $\left(\frac{W}{L}\right)_a = 7.4$

This case corresponds to  $V_{\bar{Q}} = V_m = 0.35$  V. The value of  $I_5$  can be found using Eq. (16.19), namely

$$I_5 = \frac{1}{2} (\mu_n C_{ox}) \left(\frac{W}{L}\right)_5 (V_{DD} - V_m - V_{\bar{Q}})^2$$

where

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_a = 7.4$$

Thus,

$$\begin{aligned} I_5 &= \frac{1}{2} \times 450 \times 7.4 (1 - 0.35 - 0.35)^2 \\ &= 149.9 \mu\text{A} \end{aligned}$$

We now can find  $\Delta t$  from

$$\Delta t = \frac{C_B \Delta V}{I_5}$$

Thus,

$$\Delta t = \frac{2 \times 10^{-12} \times 0.15}{149.9 \times 10^{-6}} = 2.0 \text{ ns}$$

(b)  $(W/L)_a = 2.0$

We first determine  $V_{\bar{Q}}$  using Eq. (16.21),

$$\frac{V_{\bar{Q}}}{V_{DD} - V_m} = 1 - 1/\sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

where  $(W/L)_5 = 2.0$ , thus

$$\begin{aligned} \frac{V_{\bar{Q}}}{1 - 0.35} &= 1 - 1/\sqrt{1 + \frac{2.0}{2.0}} \\ \Rightarrow V_{\bar{Q}} &= 0.19 \text{ V} \end{aligned}$$

Next, we use Eq. (16.19) to determine  $I_5$  as

$$\begin{aligned} I_5 &= \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_m - V_{\bar{Q}})^2 \\ &= \frac{1}{2} \times 450 \times 2 (1 - 0.35 - 0.19)^2 \\ &= 95.2 \mu\text{A} \end{aligned}$$

Finally, we determine  $\Delta t$  using Eq. (16.25) as

$$\begin{aligned} \Delta t &= \frac{C_B \Delta V}{I_5} \\ &= \frac{2 \times 10^{-12} \times 0.15}{95.2 \times 10^{-6}} = 3.2 \text{ ns} \end{aligned}$$

**Ex: 16.8**

$$\begin{aligned} \left(\frac{W}{L}\right)_p &\leq \left(\frac{W}{L}\right)_a \times \frac{\mu_n}{\mu_p} \left[ 1 - \left(1 - \frac{V_m}{V_{DD} - V_m}\right)^2 \right] \\ \left(\frac{W}{L}\right)_p &\leq \left(\frac{W}{L}\right)_a \times 3 \times \left[ 1 - \left(1 - \frac{0.35}{1 - 0.35}\right)^2 \right] \\ \left(\frac{W}{L}\right)_p &\leq 2.36 \left(\frac{W}{L}\right)_a \text{ or} \\ \left(\frac{W}{L}\right)_p &\leq 2.36 \times 7.4 \Rightarrow \left(\frac{W}{L}\right)_p \leq 17.5 \end{aligned}$$

For minimum area, select

$$W_n = W_p = W_a = 130 \text{ nm}$$

**Ex: 16.9** From Eqs. 16.32 and 16.33, we have

$$\begin{aligned} \Delta V(1) &\simeq \frac{C_S}{C_B} \times \frac{V_{DD}}{2} = \frac{30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2} \\ &= 60 \text{ mV} \\ \Delta V(0) &\simeq -\frac{C_S}{C_B} \times \frac{V_{DD}}{2} \\ &= \frac{-30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2} = -60 \text{ mV} \end{aligned}$$

**Ex: 16.10**  $f = \frac{1}{2 \times 5t_P}$

$$= \frac{1}{2 \times 5 \times 100 \times 10^{-12}} = 1 \text{ GHz}$$

Since  $f \approx V_{DD}$ , reducing  $V_{DD}$  by 10% reduces  $f$  to 900 MHz.

**Ex: 16.11** We first determine  $g_{mn}$  and  $g_{mp}$

$$g_{mn} = \mu_n C_{ox} \left( \frac{W}{L} \right)_n (V_{GS} - V_{tn}) =$$

$$450 \times \frac{130}{65} (0.5 - 0.35) = 135 \mu\text{A/V}.$$

$$g_{mp} = \mu_p C_{ox} \left( \frac{W}{L} \right)_p (V_{SG} - |V_{tp}|) =$$

$$150 \times \frac{390}{65} (0.5 - 0.35) = 135 \mu\text{A/V}.$$

Consequently,

$$G_m = g_{mn} + g_{mp} = 135 + 135 = 270 \mu\text{A/V},$$

and the time constant  $\tau$  is

$$\tau = \frac{C_B}{G_m} = \frac{0.25 \times 10^{-12}}{270 \times 10^{-6}} = 0.93 \text{ ns}.$$

Now  $\Delta t$  can be determined from

$$0.9 \times 1.0 = 0.5 + 0.1e^{\Delta t/0.93},$$

so  $\Delta t = 1.3 \text{ ns}$ .

**Ex: 16.12** Refer to Fig. 16.36.

Our decoder is an extension of that shown:

We have  $M$  bits in the address (as opposed to 3) and correspondingly there will be  $2^M$  word lines.

Now, each of the  $2^M$  word lines is connected to  $M$  NMOS devices and to one PMOS transistor. Thus the total number of devices required is

$$M2^M \text{ (NMOS)} + 2^M \text{ (PMOS)}$$

$$= 2^M (M + 1)$$

**Ex: 16.13** Refer to Fig. 16.38: Our tree decoder will have  $2^N$  bit lines. Thus it will have  $N$  levels: At the first levels there will be 2 transistors, at the second  $2^2$ , ..., at the  $N$ th level there will be  $2^N$  transistors. Thus the total number of transistors can be found as

$$\text{Number} = 2 + 2^2 + 2^3 + \dots + 2^N$$

$$= 2 \underbrace{(1 + 2 + 2^2 + \dots + 2^{N-1})}$$

Geometric series  $r = 2$

$$\text{Sum} = \frac{r^N - 1}{r - 1} = \frac{2^N - 1}{2 - 1}$$

$$= 2^N - 1$$

Thus,

$$\text{Number} = 2(2^N - 1)$$

**Solutions to End-of-Chapter Problems**

**16.1**

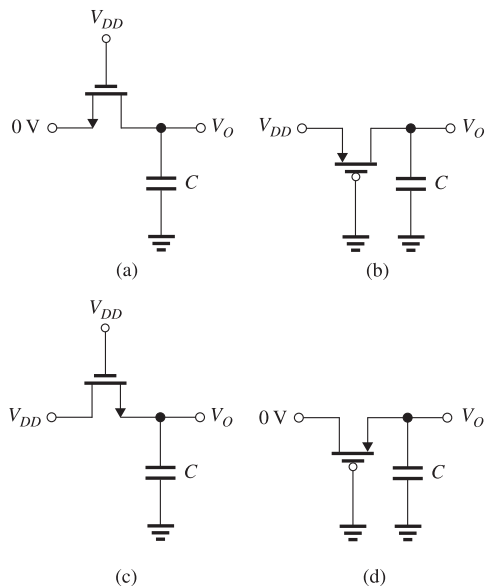


Figure 1

Refer to Fig. 1.

(a)  $V_O = 0 \text{ V}$

(b)  $V_O = V_{DD}$

(c)  $V_O = V_{DD} - V_m$

where

$$V_m = V_{m0} + \gamma \left( \sqrt{V_O + 2\phi_f} - \sqrt{2\phi_f} \right)$$

(d)  $V_O = |V_{tp}|$

$$= |V_{tp0}| + \gamma \left( \sqrt{V_{DD} - V_O + 2\phi_f} - \sqrt{2\phi_f} \right)$$

**16.2**

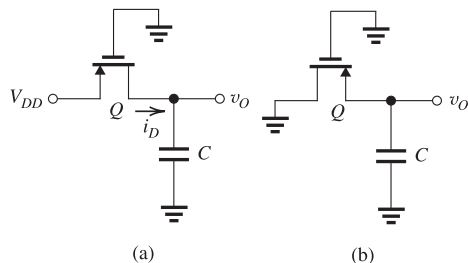


Figure 1

(a) Figure 1(a) shows the situation for  $t \geq 0$ . In this case  $V_i$  remains constant at  $V_{i0}$  and the capacitor charges to  $V_{DD}$ .

Thus,

$$V_{OH} = V_{DD}$$

(b) Figure 1(b) shows the situation for  $t \geq 0$ . Here as  $C$  discharges through  $Q$ , the threshold voltage changes. The discharge current reduces to zero when  $v_O = |V_t|$ . Thus,

$$V_{OL} = |V_t|$$

where

$$|V_t| = |V_{t0}| + \gamma \left[ \sqrt{V_{DD} - V_{OL} + 2\phi_f} - \sqrt{2\phi_f} \right]$$

Substituting  $|V_t| = V_{OL}$ , we obtain

$$V_{OL} = |V_{t0}| + \gamma \left[ \sqrt{V_{DD} - V_{OL} + 2\phi_f} - \sqrt{2\phi_f} \right]$$

(c) Refer to Fig. 1(a).

At  $t = 0$ ,  $v_O = 0$  and  $Q$  will be operating in saturation. Thus,

$$\begin{aligned} i_D(0) &= \frac{1}{2} k_p (V_{DD} - |V_{t0}|)^2 \\ &= \frac{1}{2} \times 125 (1.2 - 0.4)^2 \\ &= 40 \mu\text{A} \end{aligned}$$

At  $t = t_{PLH}$ ,  $v_O = V_{DD}/2 = 0.6 \text{ V}$  and  $Q$  will be operating in the triode region. Thus,

$$\begin{aligned} i_D(t_{PLH}) &= k_p \left[ (V_{DD} - |V_{t0}|) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right] \\ &= 125 \left[ (1.2 - 0.4) \times 0.6 - \frac{1}{2} (0.6)^2 \right] \\ &= 37.5 \mu\text{A} \end{aligned}$$

The average charging current can now be found as

$$i_D|_{av} = \frac{1}{2} (40 + 37.5) = 33.75 \mu\text{A}$$

The propagation delay  $t_{PLH}$  can then be determined from

$$\begin{aligned} t_{PLH} &= \frac{C(V_{DD}/2)}{i_D|_{av}} \\ &= \frac{C \times 0.6}{33.75 \times 10^{-6}} = 17.8 \times 10^3 \text{ C s} \end{aligned}$$

If  $C = 10 \text{ fF}$ , we get

$$t_{PLH} = 17.8 \times 10^3 \times 10 \times 10^{-15} = 178 \text{ ps}$$

**16.3** (a)  $V_{OH} = V_{DD} = 1.2 \text{ V}$

$V_{OL} = 0 \text{ V}$

(b) Refer to Fig. 16.7(a).

$$i_{DN}(0) = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (V_{DD} - V_{t0})^2$$

$$= \frac{1}{2} \times 500 \times 1.5(1.2 - 0.4)^2$$

$$= 240 \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_{i0})^2$$

$$= \frac{1}{2} \times 125 \times 1.5(1.2 - 0.4)^2$$

$$= 60 \mu\text{A}$$

The capacitor charging current  $i_C(0)$  is

$$i_C(0) = i_{DN}(0) + i_{DP}(0)$$

$$= 240 + 60 = 300 \mu\text{A}$$

The current  $i_{DN}(t_{PLH})$  is the value of  $i_D$  at  $v_O = V_{DD}/2 = 0.6$  V. At this point,

$$V_m = V_{i0} + \gamma \left( \sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f} \right)$$

$$= 0.4 + 0.2 \left( \sqrt{0.6 + 0.88} - \sqrt{0.88} \right)$$

$$= 0.456 \text{ V}$$

Now,  $i_{DN}(t_{PLH})$  can be found as

$$i_{DN}(t_{PLH}) = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n \left( \frac{V_{DD}}{2} - V_m \right)^2$$

$$= \frac{1}{2} \times 500 \times 1.5(0.6 - 0.456)^2$$

$$= 7.8 \mu\text{A}$$

To find  $i_{DP}(t_{PLH})$ , we note that  $Q_P$  will be operating in the triode region, thus

$$i_{DP}(t_{PLH}) = k'_p \left( \frac{W}{L} \right)_p \left[ (V_{DD} - V_{i0}) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right]$$

$$= 125 \times 1.5 \left[ (1.2 - 0.4)(0.6) - \frac{1}{2} (0.6)^2 \right]$$

$$= 56.25 \mu\text{A}$$

The capacitor charging current at  $t = t_{PLH}$  will be

$$i_C(t_{PLH}) = i_{DN}(t_{PLH}) + i_{DP}(t_{PLH})$$

$$= 7.8 + 56.25 = 64.1 \mu\text{A}$$

The average charging current is

$$i_{C|av} = \frac{1}{2} [i_C(0) + i_C(t_{PLH})]$$

$$= \frac{1}{2} (300 + 64.1)$$

$$= 182.1 \mu\text{A}$$

Finally,  $t_{PLH}$  can be found as

$$t_{PLH} = \frac{C(V_{DD}/2)}{i_{C|av}}$$

$$= \frac{15 \times 10^{-15} \times 0.6}{182.1 \times 10^{-6}}$$

$$= 49.4 \text{ ps}$$

(c) Refer to Fig. 16.7(b).

$$i_{DN}(0) = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (V_{DD} - V_{i0})^2$$

$$= \frac{1}{2} \times 500 \times 1.5 (1.2 - 0.4)^2$$

$$= 240 \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_{i0})^2$$

$$= \frac{1}{2} \times 125 \times 1.5 (1.2 - 0.4)^2$$

$$= 60 \mu\text{A}$$

The capacitor discharge current  $i_C(0)$  can now be found as

$$i_C(0) = i_{DN}(0) + i_{DP}(0)$$

$$= 240 + 60$$

$$= 300 \mu\text{A}$$

At  $t = t_{PHL}$ ,  $v_O = V_{DD}/2 = 0.6$  V, and  $Q_N$  will be operating in the triode region, thus

$$i_{DN}(t_{PHL}) = k'_n \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_{i0}) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right]$$

$$= 500 \times 1.5 \left[ (1.2 - 0.4) 0.6 - \frac{1}{2} (0.6)^2 \right]$$

$$= 225 \mu\text{A}$$

At  $t = t_{PHL}$ ,  $v_O = V_{DD}/2 = 0.6$  V, and  $Q_P$  will be operating in the saturation region but will have a threshold voltage  $|V_{tp}|$  determined as follows:

$$|V_{tp}| = V_{i0} + \gamma \left[ \sqrt{\frac{1}{2} V_{DD} + 2\phi_f} - \sqrt{2\phi_f} \right]$$

$$= 0.4 + 0.2 \left[ \sqrt{0.6 + 0.88} - \sqrt{0.88} \right]$$

$$= 0.456 \text{ V}$$

$$i_{DP}(t_{PHL}) = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p \left( \frac{V_{DD}}{2} - |V_{tp}| \right)^2$$

$$= \frac{1}{2} \times 125 \times 1.5 (0.6 - 0.456)^2$$

$$= 1.9 \mu\text{A}$$

The capacitor discharge current at  $t = t_{PHL}$  can now be determined as

$$i_C(t_{PHL}) = i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})$$

$$= 225 + 1.9 \simeq 227 \mu\text{A}$$

The average capacitor discharge current can now be found as

$$i_{C|av} = \frac{1}{2}(300 + 227) = 263.5 \mu\text{A}$$

and, finally,  $t_{PHL}$  can be determined from

$$\begin{aligned} t_{PHL} &= \frac{C(V_{DD}/2)}{i_{C|av}} \\ &= \frac{15 \times 10^{-15} \times 0.6}{263.5 \times 10^{-6}} = 34.2 \text{ ps} \end{aligned}$$

Transistor  $Q_P$  will turn off when  $v_O = |V_{tp}|$  where

$$\begin{aligned} |V_{tp}| &= V_{i0} + \gamma \left[ \sqrt{V_{DD} - |V_{tp}| + 2\phi_f} - \sqrt{2\phi_f} \right] \\ &= 0.4 + 0.2 \left[ \sqrt{1.2 - |V_{tp}| + 0.88} - \sqrt{0.88} \right] \\ \Rightarrow |V_{tp}| + 0.2 \sqrt{0.88} - 0.4 &= 0.2 \sqrt{2.08 - |V_{tp}|} \end{aligned}$$

Squaring both sides and collecting terms results in the quadratic equation

$$|V_{tp}|^2 - 0.384|V_{tp}| - 0.0382 = 0$$

whose solution is

$$|V_{tp}| = 0.466 \text{ V}$$

This is the value of  $v_O$  at which  $Q_P$  stops conducting.

$$\begin{aligned} \text{(d) } t_P &= \frac{1}{2}(t_{PLH} + t_{PHL}) \\ &= \frac{1}{2}(49.4 + 34.2) = 41.8 \text{ ps} \end{aligned}$$

**16.4** Refer to Fig. 16.3. Since  $V_{OH}$  is the value of  $v_O$  at which  $Q$  stops conducting,

$$V_{DD} - V_{OH} - V_t = 0$$

then

$$V_{OH} = V_{DD} - V_t$$

where

$$\begin{aligned} V_t &= V_{i0} + \gamma \left( \sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f} \right) \\ &= V_{i0} + \gamma \left( \sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f} \right) \end{aligned}$$

Substituting  $V_{i0} = 0.4 \text{ V}$ ,  $\gamma = 0.2 \text{ V}^{1/2}$ ,  $V_{DD} = 1.2 \text{ V}$ , and  $2\phi_f = 0.88 \text{ V}$ , we obtain

$$\begin{aligned} V_t &= 0.4 + 0.2 \left( \sqrt{1.2 - V_t + 0.88} - \sqrt{0.88} \right) \\ V_t - 0.4 + 0.2\sqrt{0.88} &= 0.2 \sqrt{2.08 - V_t} \\ (V_t - 0.212)^2 &= 0.04(2.08 - V_t) \\ \Rightarrow V_t^2 - 0.384 V_t - 0.038 &= 0 \end{aligned}$$

$$\Rightarrow V_t = 0.466 \text{ V}$$

$$V_{OH} = 1.3 - 0.466$$

$$= 0.834 \text{ V}$$

**16.5** Refer to Fig. 16.3.

We need to find the current  $i_D$  at  $t = 0$  (where  $v_O = 0$ ,  $V_t = V_{i0} = 0.4 \text{ V}$ ) and at  $t = t_{PLH}$  (where

$v_O = \frac{V_{DD}}{2} = 0.6 \text{ V}$ ,  $V_t$  to be determined), as follows:

$$i_D(0) = \frac{1}{2} \times 500 \times 1.5 (1.2 - 0.4)^2 = 240 \mu\text{A}$$

$$V_t \text{ (at } v_O = 0.6 \text{ V)} = 0.4 + 0.2 \left( \sqrt{0.6 + 0.88} - \sqrt{0.88} \right)$$

$$= 0.456 \text{ V}$$

$$i_D(t_{PLH}) = \frac{1}{2} \times 500 \times 1.5 (1.2 - 0.456 - 0.6)^2$$

$$= 7.8 \mu\text{A}$$

We can now compute the average charging current as

$$i_{D|av} = \frac{240 + 7.8}{2} = 123.9 \mu\text{A}$$

and  $t_{PLH}$  can be found as

$$\begin{aligned} t_{PLH} &= \frac{C(V_{DD}/2)}{i_{D|av}} \\ &= \frac{10 \times 10^{-15} \times 0.6}{123.9 \times 10^{-6}} = 48.4 \text{ ps} \end{aligned}$$

**16.6** Refer to the circuit in Fig. 16.4. Observe that, here,  $V_t$  remains constant at  $V_{i0} = 0.4 \text{ V}$ . At  $t = 0$ ,  $Q$  will be operating in saturation, and the drain current will be

$$\begin{aligned} i_D(0) &= \frac{1}{2} \times 500 \times 1.5 (1.2 - 0.4)^2 \\ &= 240 \mu\text{A} \end{aligned}$$

At  $t = t_{PHL}$ ,  $Q$  will be operating in the triode region, and thus

$$\begin{aligned} i_D(t = t_{PHL}) &= 500 \times 1.5 \left[ (1.2 - 0.4) \times 0.6 - \frac{1}{2} \times 0.6^2 \right] \\ &= 225 \mu\text{A} \end{aligned}$$

Thus, the average discharge current is given by

$$i_{D|av} = \frac{1}{2}(240 + 225) = 232.5 \mu\text{A}$$

and  $t_{PHL}$  can be determined as

$$\begin{aligned} t_{PHL} &= \frac{C(V_{DD}/2)}{i_{D|av}} \\ &= \frac{10 \times 10^{-15} \times 0.6}{232.5 \times 10^{-6}} = 25.8 \text{ ps} \end{aligned}$$

**16.7**

From Exercise 16.1 we have the output voltage of the switch as  $1.15 \text{ V}$ . This is the voltage applied to the input of the CMOS inverter, as shown in

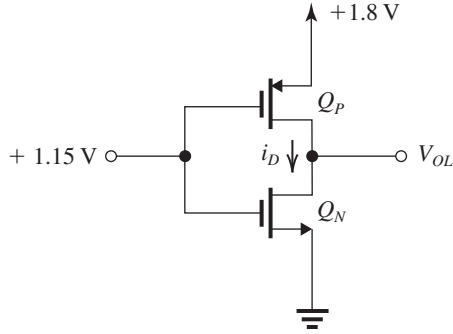


Figure 1

Fig. 1. Because we expect  $V_{OL}$  to be small,  $Q_P$  will be operating in the saturation region and its drain current will be

$$\begin{aligned} i_D &= \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)_p (1.8 - 1.15 - 0.4)^2 \\ &= \frac{1}{2} \times 75 \times \frac{0.54}{0.18} \times 0.0625 \\ &= 7 \mu\text{A} \end{aligned}$$

This is also the current conducted by  $Q_N$  which is operating in the triode region, thus

$$7 = (\mu_n C_{ox})\left(\frac{W}{L}\right)_n \left[ (1.15 - 0.4)V_{OL} - \frac{1}{2}V_{OL}^2 \right]$$

Assuming  $V_{OL}$  to be very small, we obtain

$$\begin{aligned} 7 &\simeq (\mu_n C_{ox})\left(\frac{W}{L}\right)_n \times 0.75V_{OL} \\ 7 &= 300 \times \frac{1}{2} \times \frac{0.54}{0.18} \times 0.75 V_{OL} \\ \Rightarrow V_{OL} &= 0.02 \text{ V} \end{aligned}$$

The static power dissipation is

$$\begin{aligned} P_D &= 7 \mu\text{A} \times 1.8 \text{ V} \\ &= 12.6 \mu\text{W} \end{aligned}$$

**16.8** For the switch gate and input both at  $V_{DD} = 1.0\text{V}$ , the switch output is

$$V_{OH} = V_{DD} - V_t$$

$$\text{where } V_t = V_{t0} + \gamma \left[ \sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f} \right]$$

Substituting for  $V_{OH}$ , we get

$$\begin{aligned} V_t &= V_{t0} + \gamma \left[ \sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f} \right] \\ &= 0.35 + 0.25 \left[ \sqrt{1.0 - V_t + 1.0} - \sqrt{1.0} \right] \end{aligned}$$

so that

$$V_t = 0.1 + 0.25\sqrt{2.0 - V_t}$$

$$V_t - 0.1 = 0.25\sqrt{2.0 - V_t}$$

Squaring both sides, we get

$$V_t^2 - 0.2V_t + 0.01 = 0.125 - 0.0652V_t$$

$$\text{or } V_t^2 - 0.1375V_t - 0.115 = 0$$

Solving this quadratic, we find that

$$V_t = 0.41 \text{ V}$$

$$V_{OH} = V_{DD} - V_t = 1.0 \text{ V} - 0.41 \text{ V} = 0.59 \text{ V}$$

With the input low and the switch gate high,  $V_{OL} \rightarrow 0 \text{ V}$

If  $V_{OH} = 0.59 \text{ V}$ , the PMOS transistor of the inverter is in the saturation region. Since the inverter transistors are matched, we get

$$\left(\frac{W}{L}\right)_p = \frac{k'_n}{k'_p} \left(\frac{W}{L}\right)_n \text{ so that}$$

$$i_{DP} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{t0})^2$$

$$\begin{aligned} i_{DP} &= \frac{1}{2} (150) \left(\frac{130}{65}\right) (3) \times (1.0 - 0.59 - 0.35)^2 \\ &= 1.08 \mu\text{A} \end{aligned}$$

For  $t_{PLH}$ , at  $t = 0$ , we have

$$\begin{aligned} i_D(0) &= \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2 \\ &= \frac{1}{2} (450) \left(\frac{130}{65}\right) (1 - 0.35)^2 = 190.1 \mu\text{A} \end{aligned}$$

At  $v_O = \frac{V_{DD}}{2}$ , we have

$$\begin{aligned} V_t &= V_{t0} + \gamma \left[ \sqrt{\frac{V_{DD}}{2} + 2\phi_f} - \sqrt{2\phi_f} \right] \\ &= 0.35 \text{ V} + 0.25 \left[ \sqrt{\frac{1.0}{2} + 1.0} - \sqrt{1.0} \right] \\ &= 0.41 \text{ V} \end{aligned}$$

$$i_D(t_{PLH}) = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_{DD} - v_O - V_t)^2$$

$$\begin{aligned} &= \frac{1}{2} (450) \left(\frac{130}{65}\right) \left(1 - \frac{1}{2} - 0.41\right)^2 \\ &= 3.6 \mu\text{A} \end{aligned}$$

$$i_D|_{av} = \frac{(190.1 \mu\text{A} + 3.6 \mu\text{A})}{2} = 96.9 \mu\text{A}$$

$$t_{PLH} = \frac{C \left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{10(10^{-15})(0.5)}{96.9 \times 10^{-6}}$$

$$= 51.6 \text{ ps}$$

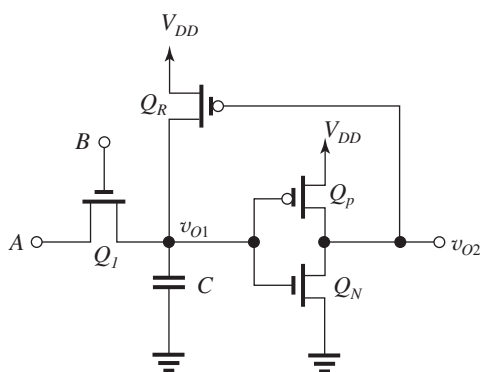
For  $t_{PHL}$ ,  $V_t = V_{t0}$  and

$$i_{D(0)} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_{DD} - V_{t0})^2$$



$$\begin{aligned}
 &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (1 - 0.35)^2 = 190.1 \mu\text{A} \\
 i_D(t_{PHL}) &= \mu_n C_{ox} \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_{t0}) v_O - \frac{1}{2} v_O^2 \right] \\
 &= 450 \left( \frac{130}{65} \right) \times \left[ (1 - 0.35) \left( \frac{1}{2} \right) - \frac{1}{2} \left( \frac{1.0}{2} \right)^2 \right] \\
 &= 180 \mu\text{A} \\
 i_{D|_{av}} &= \frac{1}{2} (190.1 + 180) = 185.1 \mu\text{A} \\
 t_{PHL} &= \frac{C \left( \frac{V_{DD}}{2} \right)}{i_{D|_{av}}} = \frac{10 (10^{-15}) (0.5)}{185.1 \times 10^{-6}} \\
 &= 27.0 \text{ ps}
 \end{aligned}$$

## 16.9



For the inverter, with  
 $v_{O2} = V_{DD} - |V_{t0}| = 1.0 \text{ V} - 0.35 \text{ V} = 0.65 \text{ V}$

$Q_N$  is in the saturation region, so that

$$\begin{aligned}
 i_{DN} &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (v_{O1} - v_{t0})^2 \\
 &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (v_{O1} - 0.35)^2 \\
 &= 450 (v_{O1} - 0.35 \text{ V})^2 \mu\text{A}
 \end{aligned}$$

$Q_P$  is operating in the triode region so

$$\begin{aligned}
 i_{DP} &= k'_p \left( \frac{W}{L} \right)_p \left[ (v_{DD} - v_{O1} - V_{t0}) (v_{SD}) - \frac{1}{2} (v_{SD})^2 \right] \\
 &= (150) \left( \frac{390}{65} \right) \times \left[ (1.0 - v_{O1} - 0.35) (0.35) - \frac{1}{2} (0.35)^2 \right] \\
 &= 900 [0.166 - 0.35 v_{O1}] \mu\text{A}
 \end{aligned}$$

Since  $i_{DP} = i_{DN}$ , we set these equal:

$$\begin{aligned}
 450 (v_{O1} - 0.35)^2 &= 149.6 - 315 v_{O1} \\
 450 (v_{O1}^2 - 0.7 v_{O1} + 0.1225) &= 149.6 - 315 v_{O1}
 \end{aligned}$$

Simplifying, we get

$$\begin{aligned}
 v_{O1}^2 - 0.21 &= 0 \\
 v_{O1} &= \sqrt{0.21} = 0.46 \text{ V} \approx \frac{V_{DD}}{2} \quad \text{Q.E.D.}
 \end{aligned}$$

For  $Q_1$ , we have

$$\begin{aligned}
 V_t &= V_{t0} + \gamma \left[ \sqrt{V_{O1} + 2\phi_f} - \sqrt{2\phi_f} \right] \\
 V_t &= 0.35 + 0.5 \left[ \sqrt{0.46 + 1} - \sqrt{1} \right] \\
 &= 0.45 \text{ V}
 \end{aligned}$$

Capacitor charging current before  $Q_R$  turns on is due to the current supplied by  $Q_1$ .

At  $v_{O1} = 0.46 \text{ V}$ :

$$\begin{aligned}
 i_D &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_1 (V_{DD} - v_{O1} - V_t)^2 \\
 &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (1.0 - 0.46 - 0.45)^2 \\
 &= 3.6 \mu\text{A}
 \end{aligned}$$

At  $v_{O1} = 0 \text{ V}$ , we have

$$\begin{aligned}
 i_D &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_1 (V_{DD} - V_{t0})^2 \\
 &= \frac{1}{2} (450) \left( \frac{130}{65} \right) (1.0 - 0.35)^2 = 190.1 \mu\text{A} \\
 i_{D|_{av}} &= \frac{1}{2} (3.6 \mu\text{A} + 190.1 \mu\text{A}) = 96.9 \mu\text{A} \\
 t_{PLH} &= \frac{C v_{O1}}{i_{D|_{av}}} = \frac{20 (10^{-15}) (0.46)}{96.9 (10^{-6})} = 95 \text{ ps}
 \end{aligned}$$

## 16.10

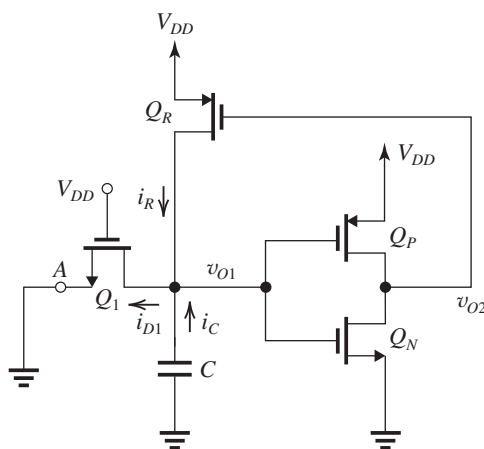


Figure 1

Refer to Fig. 1. At  $t = 0$ ,  $v_{O1} = V_{DD}$ ,  $v_{O2} = 0$  V,  $Q_R$  is conducting but  $i_R = 0$ , and  $Q_1$  is operating in the saturation region with  $i_{D1}$  given by

$$\begin{aligned} i_{D1} &= \frac{1}{2} k'_n \left( \frac{W}{L} \right)_1 (V_{DD} - V_{i0})^2 \\ &= \frac{1}{2} \times 450 \times \frac{130}{65} \times (1.0 - 0.35)^2 \\ &= 190.1 \mu\text{A} \end{aligned}$$

The discharge current  $i_C$  is given by

$$\begin{aligned} i_C &= i_{D1} - i_R \\ &= 190.1 - 0 = 190.1 \mu\text{A} \end{aligned}$$

The inverter begins to switch when  $v_{O1}$  is reduced from  $V_{DD}$  to the value  $V_{IH}$ ,

$$\begin{aligned} V_{IH} &= \frac{1}{8} (5V_{DD} - 2V_t) \\ &= \frac{1}{8} (5 \times 1.0 - 2 \times 0.35) = 0.5375 \text{ V} \end{aligned}$$

At this value of  $v_{O1}$ ,  $Q_1$  will be operating in the triode region and its current will be

$$\begin{aligned} i_{D1} &= k'_n \left( \frac{W}{L} \right)_1 \left[ (V_{DD} - V_{i0})v_{O1} - \frac{1}{2} v_{O1}^2 \right] \\ &= 450 \times \frac{130}{65} \left[ (1.0 - 0.35)0.5375 - \frac{1}{2} \times 0.5375^2 \right] \\ &= 184.4 \mu\text{A} \end{aligned}$$

The maximum current that  $Q_R$  conducts occurs when it is operating in saturation (which does not occur during the discharge process examined here). The maximum drain current of  $Q_R$  is given by

$$i_{DR} = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_{i0})^2$$

Limiting this current to half the value of  $i_{D1}$  found above, we obtain

$$\begin{aligned} \frac{1}{2} \times 184.4 &= \frac{1}{2} \times 150 \times \left( \frac{W}{L} \right)_R \times (1 - 0.35)^2 \\ \Rightarrow \left( \frac{W}{L} \right)_R &= 2.9 \end{aligned}$$

The current conducted by  $Q_R$  at  $v_{O1} = V_{IH} = 0.5375$  V

$$\begin{aligned} i_R &= 150 \times 2.9 \left[ (1 - 0.35)(1 - 0.5375) - \frac{1}{2} (1 - 0.5375)^2 \right] \\ &= 84.2 \mu\text{A} \end{aligned}$$

The discharge current  $i_C$  at this time is given by

$$i_C = 184.4 - 84.2 = 100.2 \mu\text{A}$$

The average value of the discharge current  $i_C$  is given by

$$\begin{aligned} i_C|_{av} &= \frac{1}{2} (190.1 + 100.2) \\ &= 145.2 \mu\text{A} \end{aligned}$$

and  $t_{PHL}$  will be

$$\begin{aligned} t_{PHL} &= \frac{20 \times 10^{-15} (1.0 - 0.5375)}{145.2 \times 10^{-6}} \\ &= 63.7 \text{ ps} \end{aligned}$$

**16.11** Using Eq. (16.10), we have

$$R_{Neq} = \frac{V_{DD} - v_O}{\frac{1}{2} k_n (V_{DD} - V_m - v_O)^2}, \text{ for } v_O \leq$$

$$V_{DD} - V_m$$

For  $v_O = 0$  V,

$$\begin{aligned} R_{Neq}(0) &= \frac{1.2 - 0}{\frac{1}{2} \times 0.5 \times 1.5 (1.2 - 0.4 - 0)^2} \\ &= 5 \text{ k}\Omega \end{aligned}$$

For  $v_O = 0.6$  V,

$$\begin{aligned} R_{Neq}(0.6 \text{ V}) &= \frac{1.2 - 0.6}{\frac{1}{2} \times 0.5 \times 1.5 (1.2 - 0.4 - 0.6)^2} \\ &= 40 \text{ k}\Omega \end{aligned}$$

Using Eq. (16.14),

$$R_{Peq} = \frac{V_{DD} - v_O}{\frac{1}{2} k_p (V_{DD} - |V_{tp}|)^2}, \text{ for } v_O \leq |V_{tp}|$$

we obtain for  $v_O = 0$  V,

$$\begin{aligned} R_{Peq}(0) &= \frac{1.2 - 0}{\frac{1}{2} \times 0.125 \times 1.5 (1.2 - 0.4)^2} \\ &= 20 \text{ k}\Omega \end{aligned}$$

Using Eq. (16.15),

$$R_{Peq} = \frac{1}{k_p \left[ V_{DD} - |V_{tp}| - \frac{1}{2} (V_{DD} - v_O) \right]^2},$$

for  $v_O \geq |V_{tp}|$

we obtain at  $v_O = 0.6$  V,

$$\begin{aligned} R_{Peq}(0.6 \text{ V}) &= \frac{1}{0.125 \times 1.5 \left[ 1.2 - 0.4 - \frac{1}{2} (1.2 - 0.6) \right]^2} \\ &= 10.7 \text{ k}\Omega \end{aligned}$$

Thus, at  $v_O = 0$  V,

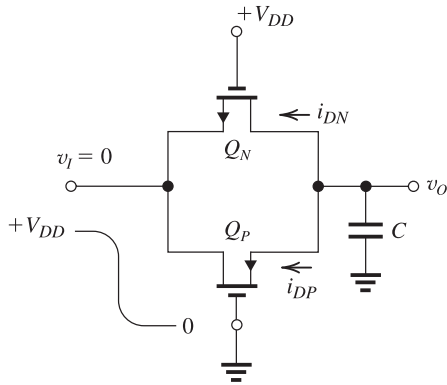
$$R_{TG}(0) = R_{Neq}(0) \parallel R_{Peq}(0)$$

$$\begin{aligned}
 &= 5 \parallel 20 = 4 \text{ k}\Omega \\
 R_{TG}(0.6 \text{ V}) &= R_{Neq}(0.6 \text{ V}) \parallel R_{Peq}(0.6 \text{ V}) \\
 &= 40 \parallel 10.7 = 8.42 \text{ k}\Omega \\
 R_{TG|_{av}} &= \frac{4 + 8.42}{2} = 6.21 \text{ k}\Omega \\
 t_{PLH} &= 0.69CR_{TG|_{av}} \\
 &= 0.69 \times 15 \times 10^{-15} \times 6.21 \times 10^3 = 64.3 \text{ ps}
 \end{aligned}$$

**16.12**  $t_p = 0.69 \sum_{k=1}^n kCR_{TG}$

$$\begin{aligned}
 &= 0.69CR_{TG} \frac{n(n+1)}{2} \text{ (for a step input)} \\
 \text{Substituting } C &= 10 \text{ fF, } R_{TG} = 10 \text{ k}\Omega \text{ and } n = 16, \\
 t_p &= 0.69 \times 10 \times 10^{-15} \times 10 \times 10^3 \times \frac{16 \times 17}{2} \\
 &= 9.4 \text{ ns (for a step input)} \\
 \text{For a ramp input, we have} \\
 t_p &= CR_{TG} \frac{n(n+1)}{2} \\
 &= 10 \times 10^{-15} \times 10 \times 10^3 \times \frac{16 \times 17}{2} \\
 &= 13.6 \text{ ns (ramp input)}
 \end{aligned}$$

**16.13**



Initially,  $C$  is charged so that  $v_o = V_{DD}$ .  
 When  $v_i$  goes low to 0 V,  $Q_N$  is initially in the saturation region with

$$i_{DN} = \frac{1}{2}k_n(V_{DD} - V_{t0})^2$$

until  $v_{DSN} = v_{GS} - V_m = V_{DD} - V_m$

$$R_{Neq} = \frac{v_o - 0}{\frac{1}{2}k_n(V_{DD} - V_{t0})^2} = \frac{2v_o}{k_n(V_{DD} - V_{t0})^2}$$

for  $v_o \geq V_{DD} - V_m$   
 When  $Q_N$  enters the triode region, we get

$$i_{DN} = k_n \left[ (V_{DD} - V_{t0})v_o - \frac{1}{2}v_o^2 \right]$$

for  $v_o \leq V_{DD} - V_{t0}$ .  
 Then,  $R_{Neq} = \frac{v_o}{k_n \left[ (V_{DD} - V_{t0})v_o - \frac{1}{2}v_o^2 \right]}$

$$= \frac{1}{k_n \left[ (V_{DD} - V_{t0}) - \frac{1}{2}v_o \right]}$$

For  $Q_P$ , initially,

$$i_{DP} = \frac{1}{2}k_p(v_o - |V_{tp}|)^2 \text{ so that}$$

$$R_{Peq} = \frac{2v_o}{k_p(v_o - |V_{tp}|)^2}, \text{ for } v_o \geq |V_{tp}|$$

where we will neglect the body effect of  $|V_{tp}|$ .  
 For  $v_o \leq |V_{tp}|$ ,

$$i_{DP} = 0$$

and

$$R_{Peq} = \infty$$

Now, for the process technology specified in Problem 16.3, we have the following:

At  $v_o = V_{DD}$ ,

$$R_{Neq} = \frac{2V_{DD}}{k_n(V_{DD} - V_{t0})^2}$$

$$= \frac{2 \times 1.2}{0.5 \times 1(1.2 - 0.4)^2} = 7.5 \text{ k}\Omega$$

$$R_{Peq} = \frac{2V_{DD}}{k_p(V_{DD} - |V_{tp}|)^2}$$

$$= \frac{2 \times 1.2}{0.125 \times 1(1.2 - 0.4)^2} = 30 \text{ k}\Omega$$

$$R_{TG}(v_o = V_{DD}) = 7.5 \text{ k}\Omega \parallel 30 \text{ k}\Omega$$

$$= 6 \text{ k}\Omega$$

At  $v_o = \frac{V_{DD}}{2}$ ,

$$R_{Neq} = \frac{1}{k_n \left[ V_{DD} - V_{t0} - \frac{1}{2} \left( \frac{V_{DD}}{2} \right) \right]}$$

$$= \frac{1}{0.5 \times 1(1.2 - 0.4 - 0.3)}$$

$$= 4 \text{ k}\Omega$$

$$R_{Peq} = \frac{2(V_{DD}/2)}{k_p \left( \frac{V_{DD}}{2} - |V_{tp}| \right)^2}$$

$$= \frac{2 \times 0.6}{0.125 \times 1(0.6 - 0.4)^2}$$

$$= 240 \text{ k}\Omega$$

Thus,

$$R_{TG} \left( v_O = \frac{V_{DD}}{2} \right) = 4 \text{ k}\Omega \parallel 240 \text{ k}\Omega$$

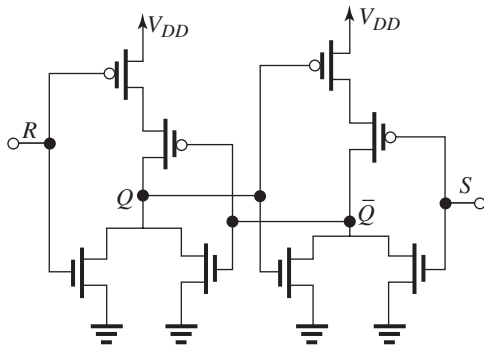
$$= 3.9 \text{ k}\Omega$$

$$R_{TG|av} = \frac{6 + 3.9}{2} = 4.95 \text{ k}\Omega$$

$$t_{PHL} = 0.69CR_{TG|av}$$

$$= 0.69 \times 15 \times 10^{-15} \times 4.95 \times 10^3 = 51.2 \text{ ps}$$

**16.14**



**16.15**

Refer to Fig. 16.11(b) and to Fig. 1 on the next page. Figure 1(a) shows  $v_X$  versus  $v_W$  and Fig. 1(b) shows  $v_Z$  versus  $v_X$ . Both of these VTCs are identical with a gain in the transition region of  $-6 \text{ V/V}$ . The equation of the linear transition region in Fig. 1(a) is

$$v_X = -6(v_W - 0.6) + 0.6 \tag{1}$$

for

$$0.5 \text{ V} \leq v_X \leq 0.7 \text{ V}$$

The two VTCs in Figs. 1(a) and (b) can be used to obtain the VTC of the cascade of the two inverters, that is,  $v_Z$  versus  $v_W$ , which is shown in Figure 1(c). The slope at the midpoint, point C in Fig. 1(c), is  $-6 \times -6 = 36 \text{ V/V}$ . The two switching points, at  $v_W = 0.583 \text{ V}$  and  $0.617 \text{ V}$ , can be determined using Eq. (1) and setting  $v_X = V_{IL}$  or  $V_{IH}$ . The width of the transition region is  $0.617 - 0.583 = 0.033 \text{ V}$ . This value can also be obtained from  $1.2 \text{ V}/36 \text{ V/V} = 0.033 \text{ V}$ .

Finally, we show in Fig. 1(c) the three points A, B, C:

A (0 V, 0 V); B (0.6 V, 0.6 V), C (1.2 V, 1.2 V)

**16.16** Following the procedure used in Example 16.4 and referring to Fig. 16.14(b), we can write

$$I_{Deq} = I_{D2}$$

$$k'_n \times \frac{1}{2} \left( \frac{W}{L} \right)_{5,6} \left[ (V_{DD} - V_m) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right] = k'_p \left( \frac{W}{L} \right)_p \left[ (V_{DD} - |V_{tp}|) \left( \frac{V_{DD}}{2} \right) - \frac{1}{2} \left( \frac{V_{DD}}{2} \right)^2 \right]$$

Since  $V_m = |V_{tp}|$ , this equation reduces to

$$\frac{1}{2} k'_n \left( \frac{W}{L} \right)_{5,6} = k'_p \left( \frac{W}{L} \right)_p \tag{1}$$

Since the inverter is matched, then

$$k'_p \left( \frac{W}{L} \right)_p = k'_n \left( \frac{W}{L} \right)_n \tag{2}$$

Substituting for  $k'_p(W/L)_p$  from Eq. (2) into Eq. (1) gives

$$\left( \frac{W}{L} \right)_{5,6} = 2 \left( \frac{W}{L} \right)_n \quad \text{Q.E.D.}$$

Refer to Fig. 16.14. For  $Q_1$  and  $Q_3$ ,

$$\left( \frac{W}{L} \right)_{1,3} = \frac{65 \text{ nm}}{65 \text{ nm}}$$

$$\left( \frac{W}{L} \right)_{2,4} = \frac{195 \text{ nm}}{65 \text{ nm}}$$

$$\left( \frac{W}{L} \right)_{5-8} = \frac{130 \text{ nm}}{65 \text{ nm}}$$

**16.17** Refer to Fig. P16.17.

(a) When  $\phi$  is high, the transmission gate is conducting (i.e.,  $Q_5$  and  $Q_6$  are conducting) and the line  $D$  is connected to the input terminal of  $G_2$ . Thus  $\bar{Q} = D$  and  $Q = \bar{D}$ . The two feedback loops around  $G_2$  are open because although  $Q_1$  or  $Q_4$  can conduct (if  $D = 1$  and thus  $\bar{Q} = 0$ ,  $Q_1$  can conduct while if  $D = 0$  and thus  $\bar{Q} = 1$ ,  $Q_4$  can conduct) their conduction paths are blocked by  $Q_2$  and  $Q_3$  which remain cut off when  $\phi$  is high.

(b) If  $D$  is high, then  $\bar{Q}$  is low and  $Q$  is high. Now, if  $\phi$  goes low, the transmission gate turns off, thus the input of  $G_2$  is isolated from  $D$ . The high value at the gate of  $G_2$  is maintained by the feedback loop around  $G_2$  consisting of  $Q_1$  and  $Q_2$ , both of which conduct, thus connecting the input node of  $G_2$  to  $V_{DD}$ . Meanwhile,  $Q_3$  and  $Q_4$  are at cut off.

(c) If  $D$  is low, then the voltage at the input of  $G_2$  will be low (0 V) and thus  $\bar{Q}$  will be high and  $Q$

This figure belongs to Exercise 16.15.

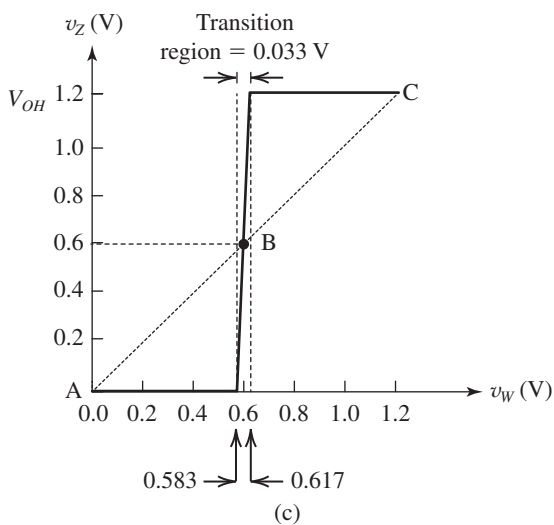
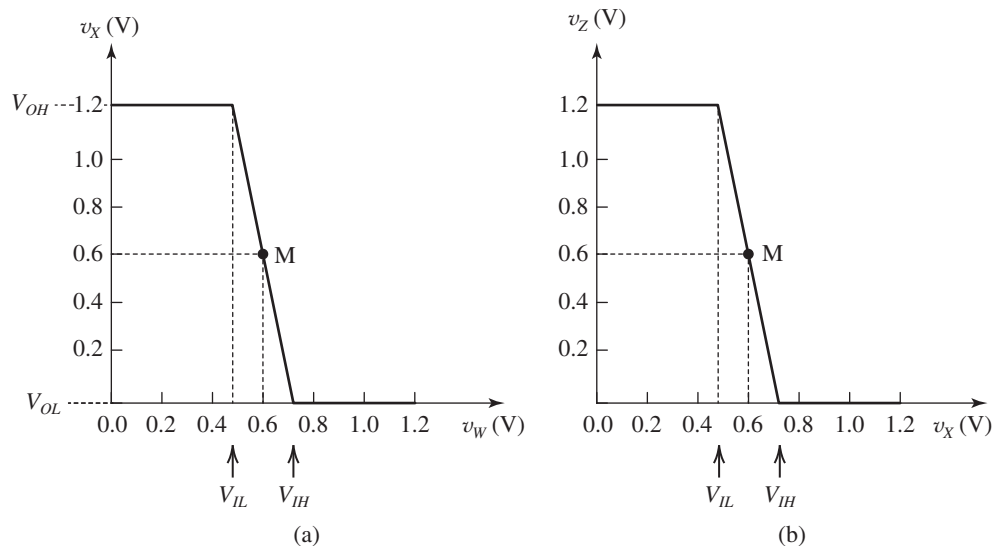


Figure 1

will be low. When  $\phi$  goes low, the transmission gate turns off and the input node of  $G_2$  is isolated from  $D$ . The low voltage at the input node of  $G_2$  is maintained by the feedback loop around  $G_2$  consisting of  $Q_3$  and  $Q_4$ , both of which conduct. Meanwhile,  $Q_1$  and  $Q_2$  are cut off.

(d) No. The circuit connects either  $V_{DD}$  or ground directly to the input of  $G_2$  which establishes the value of  $Q$  and  $\bar{Q}$ . When  $\phi$  goes low, a feedback loop closes around  $G_2$ , thus locking in the value of  $\bar{Q}$  (equal to  $\bar{D}$ ) and thus  $Q$  (equal to  $D$ ).

**16.18** Figure 1 on the next page shows the equivalence of a NAND gate to an OR gate with inverters at the inputs. We can use this equivalence to derive an SR flip-flop utilizing

two cross-coupled NAND gates starting from the one utilizing NOR gates, as shown in Fig. 2 (next page).

The resulting circuit, shown in Fig. 2(d), can be described by the following truth table:

$\bar{R}$	$\bar{S}$	$Q_{n+1}$
0	0	not used
0	1	0
1	0	1
1	1	$Q_n$

Here, the rest state is when both trigger inputs  $\bar{R}$  and  $\bar{S}$  are high. To set the flip-flop we lower  $\bar{S}$  to

These figures belong to Problem 16.18.

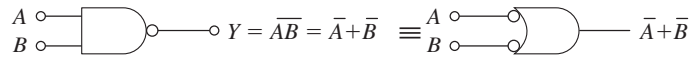


Figure 1

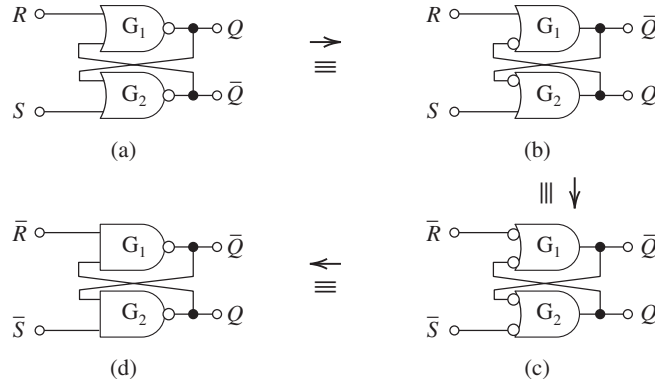


Figure 2

zero. This results in  $Q_{n+1} = 1$ . Conversely, to reset the flip-flop,  $\bar{R}$  is lowered to zero, resulting in  $Q_{n+1} = 0$ . The situation with both  $\bar{R}$  and  $\bar{S}$  lowered to zero results in an undefined output and is thus avoided.

**16.19** From Table 14.2 we obtain for the inverter threshold voltage

$$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_m}{1 + r}$$

where

$$r = \sqrt{\frac{k'_p(W/L)_p}{k'_n(W/L)_n}}$$

For our case, we have

$$k'_n = 3k'_p = 450 \mu\text{A/V}^2$$

$$\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n = \frac{130 \text{ nm}}{65 \text{ nm}}$$

$$V_m = |V_{tp}| = 0.35 \text{ V}$$

$$V_{DD} = 1.0 \text{ V}$$

Thus,

$$r = \sqrt{\frac{1}{3}} \times 1 = 0.577$$

$$V_M = \frac{0.577(1.0 - 0.35) + 0.35}{1.577}$$

$$= 0.46 \text{ V}$$

Now, refer to Fig. 16.14(b) with  $v_{\bar{Q}} = 0.46 \text{ V}$ . Both  $Q_{\text{eq}}$  and  $Q_2$  will be operating in the triode region. Equating their drain currents results in

$$450 \times \frac{1}{2} \left(\frac{W}{L}\right)_5 \left[ (1.0 - 0.35) \times 0.46 - \frac{1}{2} \times 0.46^2 \right]$$

$$= 150 \times \frac{130}{65} \left[ (1.0 - 0.35)(1.0 - 0.46) - \frac{1}{2} \times (1.0 - 0.46)^2 \right]$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 1.42$$

Therefore,

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 1.42$$

**16.20** (a) Refer to Fig. 16.14(b). For  $V_{DD} = 1.2$  and  $V_m = |V_{tp}| = 0.4 \text{ V}$ , both  $Q_{\text{eq}}$  and  $Q_2$  will be operating in the triode region. Equating their drain currents results in

$$500 \times \frac{1}{2} \left(\frac{W}{L}\right)_5 \left[ (1.2 - 0.4) \left(\frac{1.2}{2}\right) - \frac{1}{2} \left(\frac{1.2}{2}\right)^2 \right]$$

$$= 125 \times \frac{0.8}{0.13} \left[ (1.2 - 0.4) \left(\frac{1.2}{2}\right) - \frac{1}{2} \left(\frac{1.2}{2}\right)^2 \right]$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \frac{0.4}{0.13}$$

Thus, the minimum required  $W/L$  ratios are

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= \frac{0.4 \mu\text{m}}{0.13 \mu\text{m}}$$

(b) Selecting

$$\left(\frac{W}{L}\right)_{5-8} = \frac{0.8 \mu\text{m}}{0.13 \mu\text{m}}$$

we can determine the minimum required width of the set and reset pulses as follows: The minimum required pulse width is composed of two components. The first is the time for  $v_{\bar{Q}}$  in the

circuit in Fig. 16.14(a) to fall from  $V_{DD}$  to  $V_{DD}/2$ , where  $V_{DD}/2$  is the threshold voltage of the inverter formed by  $Q_3$  and  $Q_4$  in Fig. 16.15.

The second component is the time for the output of the  $Q_3 - Q_4$  inverter to rise from 0 to  $V_{DD}/2$ . We will denote the first component  $t_{PHL}$  and the second  $t_{PLH}$ .

To determine  $t_{PHL}$  refer to the circuit in Fig. 16.19. The capacitor discharge current  $i_C$  is

$$i_C = i_{D_{eq}} - i_{D2}$$

To determine the average value of  $i_C$ , we calculate  $i_{D_{eq}}$  and  $i_{D2}$  at  $t = 0$  and  $t = t_{PHL}$ . At  $t = 0$ ,  $v_{\bar{Q}} = V_{DD}$ , thus  $Q_2$  is off,

$$i_{D2}(0) = 0$$

and  $Q_{eq}$  is in saturation,

$$\begin{aligned} i_{D_{eq}} &= \frac{1}{2} \times 500 \times \frac{1}{2} \times \frac{0.8}{0.13} \times (1.2 - 0.4)^2 \\ &= 492.3 \mu\text{A} \end{aligned}$$

Thus,

$$i_C(0) = 492.3 - 0 = 429.3 \mu\text{A}$$

At  $t = t_{PHL}$ ,  $v_{\bar{Q}} = V_{DD}/2$ , thus both  $Q_2$  and  $Q_{eq}$  will be in the triode region,

$$\begin{aligned} i_{D2}(t_{PHL}) &= \\ 125 \times \frac{0.8}{0.13} \times \left[ (1.2 - 0.4) \left( \frac{1.2}{2} \right) - 0.5 \left( \frac{1.2}{2} \right)^2 \right] \\ &= 230.8 \mu\text{A} \end{aligned}$$

and

$$\begin{aligned} i_{D_{eq}}(t_{PHL}) &= \\ 500 \times \frac{1}{2} \times \frac{0.8}{0.13} \left[ (1.2 - 0.4) \left( \frac{1.2}{2} \right) - 0.5 \left( \frac{1.2}{2} \right)^2 \right] \\ &= 461.5 \mu\text{A} \end{aligned}$$

Thus,

$$i_C(t_{PHL}) = 461.5 - 230.8 = 230.7 \mu\text{A}$$

and the average value of  $i_C$  over the interval  $t = 0$  to  $t = t_{PHL}$  is

$$\begin{aligned} i_{C|av} &= \frac{i_C(0) + i_C(t_{PHL})}{2} \\ &= \frac{429.3 + 230.7}{2} = 330 \mu\text{A} \end{aligned}$$

We now can calculate  $t_{PHL}$  as

$$\begin{aligned} t_{PHL} &= \frac{C(V_{DD}/2)}{i_{C|av}} = \frac{15 \times 10^{-15} \times 0.6}{330 \times 10^{-6}} \\ &= 27.3 \text{ ps} \end{aligned}$$

Next we determine the time  $t_{PLH}$  for the output of the  $Q_3 - Q_4$  inverter,  $v_Q$ , to rise from 0 to  $V_{DD}/2$ .

For this purpose we use an equation from Example 16.2,

$$t_{PLH} = \frac{\alpha_p C}{k'_p (W/L)_p V_{DD}}$$

where

$$\begin{aligned} \alpha_p &= 2 \left/ \left[ \frac{7}{4} - \frac{3|V_{tp}|}{V_{DD}} + \left( \frac{|V_{tp}|}{V_{DD}} \right)^2 \right] \right. \\ &= \frac{2}{\frac{7}{4} - \frac{3 \times 0.4}{1.2} + \left( \frac{0.4}{1.2} \right)^2} \\ &= 2.32 \end{aligned}$$

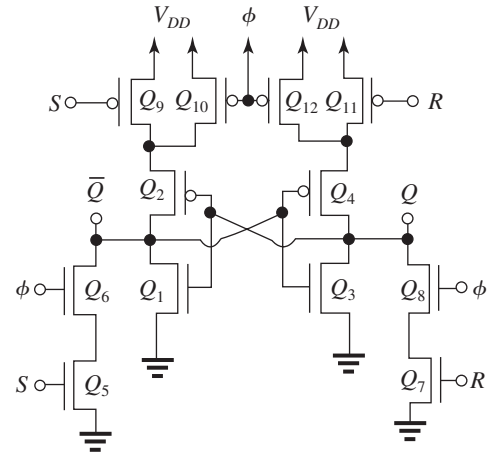
Thus,

$$\begin{aligned} t_{PLH} &= \frac{2.32 \times 15 \times 10^{-15}}{125 \times 10^{-6} \times \left( \frac{0.8}{0.13} \right) \times 1.2} \\ &= 37.7 \text{ ps} \end{aligned}$$

Finally, the minimum required width of the set pulse can be calculated as

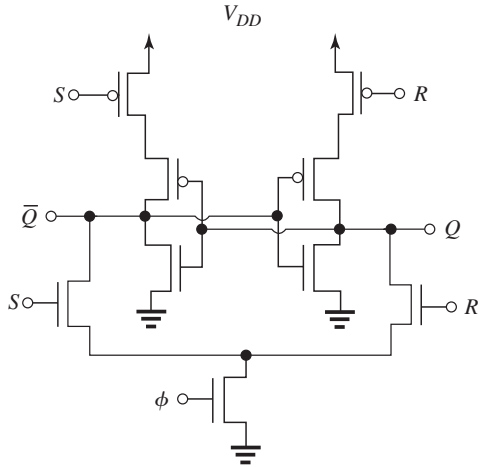
$$\begin{aligned} T_{\min} &= t_{PHL} + t_{PLH} \\ &= 27.3 + 37.7 = 65 \text{ ps} \end{aligned}$$

### 16.21



As noted, this fully complementary circuit uses 12 transistors. However, a 10-T version exists in which  $Q_{10}$  and  $Q_{12}$  are omitted.

Note further that an effective 9-T version exists in which  $Q_6$  and  $Q_8$  are moved below  $Q_5$  and  $Q_7$ , then merged into a single grounded-source device. Note that all of the designs can employ the latter idea to reduce the transistor count by 1. See the sketch below:



This circuit suffers only from the fact that unclocked changes in  $S$  and  $R$  have a secondary input on  $Q/\bar{Q}$  since raising  $S$  or  $R$  disconnects  $Q/\bar{Q}$  from  $V_{DD}$ . In some applications this may lead to system noise sensitivity in which case one or the other or both of  $Q_{10}$ ,  $Q_{12}$  (in the previous sketch) may be added.

16.22

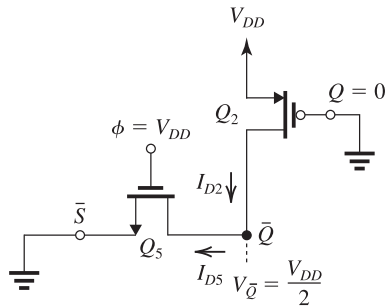


Figure 1

Figure 1 shows the relevant part of the circuit at the point of switching ( $V_{\bar{Q}} = V_{DD}/2$ ). For this situation to be achieved, the current supplied by  $Q_5$ ,  $I_{D5}$ , must at least be equal to that supplied by  $Q_2$ . Since both transistors are operating in the triode region, we can write

$$I_{D5} = I_{D2}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_5 \left[ (V_{DD} - V_m) \left(\frac{V_{DD}}{2}\right) - \frac{1}{2} \left(\frac{V_{DD}}{2}\right)^2 \right]$$

$$= \mu_p C_{ox} \left(\frac{W}{L}\right)_p \left[ (V_{DD} - |V_{tp}|) \left(\frac{V_{DD}}{2}\right) - \frac{1}{2} \left(\frac{V_{DD}}{2}\right)^2 \right]$$

Since  $V_m = |V_{tp}|$ , this equation yields

$$\left(\frac{W}{L}\right)_5 = \left(\frac{\mu_p}{\mu_n}\right) \left(\frac{W}{L}\right)_p$$

This is the minimum required value of  $(W/L)_5$ .

16.23 (a)  $0.18 \mu\text{m}$ :  $V_{DD} = 1.8 \text{ V}$  and  $V_t = 0.5 \text{ V}$

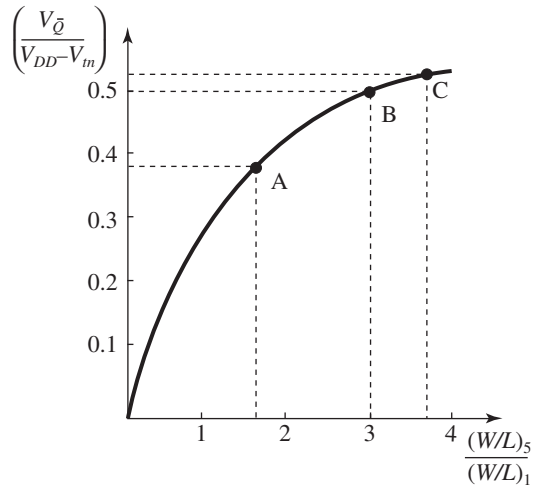
$$\text{A: } \left(\frac{0.5}{1.8 - 0.5}\right) = 0.385 \Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 1.7$$

$$\text{From Example 16.2 } \Rightarrow \frac{(W/L)_5}{(W/L)_1} = 1.64$$

(b)  $0.13 \mu\text{m}$ :  $V_{DD} = 1.2 \text{ V}$  and  $V_t = 0.4 \text{ V}$

$$\text{B: } \left(\frac{0.4}{1.2 - 0.4}\right) = 0.5$$

$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 3$$



$$\left(\frac{W}{L}\right)_5 / \left(\frac{W}{L}\right)_1 = 3$$

(c)  $65 \text{ nm}$ :  $V_{DD} = 1.0 \text{ V}$  and  $V_t = 0.35 \text{ V}$

$$\text{C: } \left(\frac{0.35}{1.0 - 0.35}\right) = 0.538 \Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 3.7$$

$$\text{From Example 16.2 } \Rightarrow \frac{(W/L)_5}{(W/L)_1} = 3.69$$

16.24 From Example 16.2, we obtain

$$\left(\frac{W}{L}\right)_{a,\text{max}} = 1.5 \left[ \frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1 \right]$$



$$= 4.5$$

(a) For  $\left(\frac{W}{L}\right)_a = \frac{1}{3} \times 4.5 = 1.5$ , Eq. (16.22)

yields [note that  $\left(\frac{W}{L}\right)_s = \left(\frac{W}{L}\right)_a$ ]

$$\frac{V_{\bar{Q}}}{1.2 - 0.4} = 1 - 1/\sqrt{1 + \frac{1.5}{1.5}}$$

$$\Rightarrow V_{\bar{Q}} = 0.293 \times 0.8 = 0.23 \text{ V}$$

$$I_5 = \frac{1}{2} \times 500 \times 1.5 (1.2 - 0.4 - 0.23)^2$$

$$= 121.8 \mu\text{A}$$

(b) For  $\left(\frac{W}{L}\right)_a = \frac{2}{3} \times 4.5 = 3$ , Eq. (16.22) yields

$$\frac{V_{\bar{Q}}}{1.2 - 0.4} = 1 - 1/\sqrt{1 + \frac{3}{1.5}}$$

$$\Rightarrow V_{\bar{Q}} = 0.34 \text{ V}$$

$$I_5 = \frac{1}{2} \times 500 \times 3 (1.2 - 0.4 - 0.34)^2$$

$$= 158.7 \mu\text{A}$$

(c) For  $\left(\frac{W}{L}\right)_a = 4.5$ , Eq. (16.22) yields

$$\frac{V_{\bar{Q}}}{1.2 - 0.4} = 1 - 1/\sqrt{1 + \frac{4.5}{1.5}}$$

$$\Rightarrow V_{\bar{Q}} = 0.4 \text{ V (which is equal to } V_m,$$

as should be expected)

$$I_5 = \frac{1}{2} \times 500 \times 4.5 (1.2 - 0.4 - 0.4)^2$$

$$= 180 \mu\text{A}$$

We see that the read delay,  $\Delta t$ , is inversely proportional to  $I_5$ . Thus the third design produces the shortest read delay.

**16.25** Equation (16.23) provides the constraint

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1$$

that is,

$$(W/L)_a \leq 3(W/L)_n \quad (1)$$

Equation (16.29) provides the constraint

$$\frac{(W/L)_p}{(W/L)_a} \leq 4 \left[1 - \left(1 - \frac{0.4}{1.2 - 0.4}\right)^2\right]$$

that is,

$$(W/L)_p \leq 3(W/L)_a \quad (2)$$

Selecting  $(W/L)_n = 1$ , then Eq. (1) gives

$$(W/L)_a \leq 3$$

which permits selecting

$$(W/L)_a = 1$$

Finally, the constraint in Eq. (2) becomes

$$(W/L)_p \leq 3$$

which permits selecting

$$(W/L)_p = 1$$

Thus our minimum-area design has  $L = 0.13 \mu\text{m}$  and

$$\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_a = 1$$

**16.26** From Eq. (16.32) or (16.33), we have

$$\Delta V \simeq \frac{C_S}{C_B} \left(\frac{V_{DD}}{2}\right)$$

$$25 \times 10^{-3} = \frac{35}{C_B} \times \frac{1.2}{2}$$

$$\Rightarrow C_B = 840 \text{ fF}$$

This is the maximum allowable value for  $C_B$ .

Capacitive load due to cells =  $840 - 20 = 820 \text{ fF}$

$$\text{Maximum number of cells} = \frac{820}{0.8}$$

$$= 1025 \text{ or likely } 1024.$$

Number of row address bits requiring

$$= \log_2 1024 = 10$$

If the sense amplifier gain is increased by a factor of 4,  $\Delta V$  is reduced by a factor of 4, and the maximum  $C_B$  increases by a factor of 4. Thus, the number of cells can be increased approximately by a factor of 4 to 4096 bits, requiring two more bits for the word address, for a total of 12 bits.

$$\mathbf{16.27} \quad \frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n} \leq \frac{1}{\left(1 - \frac{V_m}{V_{DD} - V_m}\right)^2} - 1$$

$$= \frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1 = 3$$

$$\left(\frac{W}{L}\right)_a \leq 1.5 \times 3 = 4.5 \text{ or } \left(\frac{W}{L}\right)_a \leq 4.5$$

**16.28** We can use the graph, or more accurately Eq. (16.21), to determine the maximum allowable value of  $(W/L)_5/(W/L)_1$  as follows:

$$\frac{V_{\bar{Q}}}{V_{DD} - V_m} = 1 - 1 / \sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

$$\frac{0.1}{1.0 - 0.35} = 1 - 1 / \sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

$$\frac{(W/L)_5}{(W/L)_1} = 0.397$$

or, more appropriately,

$$\frac{(W/L)_5}{(W/L)_1} \leq 0.397$$

If in our design we use this relationship with the equality sign and recall that  $L_1 = L_5 = 65$  nm and that the minimum width is 65 nm, then we select

$$\left(\frac{W}{L}\right)_5 = 1$$

$$\Rightarrow W_5 = 65 \text{ nm}$$

and thus,

$$(W/L)_1 = \frac{1}{0.397} = 2.52$$

resulting in

$$W_1 = 2.52 \times 65 = 164 \text{ nm}$$

**16.29**

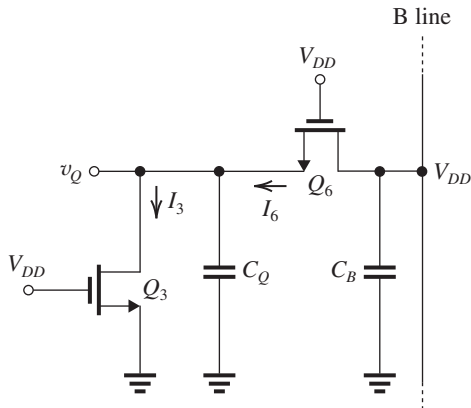


Figure 1

The relevant part of the circuit is shown in Fig. 1. It is assumed that  $Q = 0$ , that is, initially  $v_Q = 0$  V. Also it is assumed that the B line voltage is  $V_{DD}$ . When the word line is selected and the gate of  $Q_6$  is pulled to  $V_{DD}$  volts,  $Q_6$  conducts and operates in the saturation region. Its current  $I_6$  charges  $C_Q$  whose voltage  $v_Q$  rises from 0 V and thus  $Q_3$  conducts and operates in the triode region. Current  $I_6$ , in addition to charging  $C_Q$ ,

also supplies a current equal to  $I_3$ . Equilibrium is reached at the value of  $V_Q$  that makes  $I_3$  equal to  $I_6$  and thus  $C_Q$  stops charging. The design is based on  $V_Q$  being sufficiently small to prevent the latch from changing state (nondestructive readout). Usually, one imposes the condition that  $V_Q \leq V_m$ .

The operation of the circuit is described by equations identical to (16.20)–(16.27) except for  $(W/L)_5$  replaced with  $(W/L)_6$  and  $(W/L)_1$  replaced with  $(W/L)_3$ . The graph in Fig. 16.23 applies here also with the same changes mentioned. Finally, Eq. (16.27) provides the constraint on the  $(W/L)$  ratios that results if  $V_Q$  is to be less or equal to  $V_m$ .

**16.30** Without taking the body effect into account, Eq. (16.23) applies with  $V_m = V_{i0}$ , thus

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1$$

$$\Rightarrow \frac{(W/L)_a}{(W/L)_n} \leq 3$$

The body effect will affect the operation of  $Q_5$  whose  $V_m$  now will be given by

$$V_m = V_{i0} + \gamma \left( \sqrt{V_{SB} + 2\phi_f} - \sqrt{2\phi_f} \right) \quad (1)$$

In equilibrium

$$V_{\bar{Q}} = V_m$$

thus

$$V_{SB} = V_m$$

Equation (1) then becomes

$$V_m = V_{i0} + \gamma \left( \sqrt{V_m + 2\phi_f} - \sqrt{2\phi_f} \right)$$

Thus,

$$V_m = 0.4 + 0.2 \left( \sqrt{V_m + 0.88} - \sqrt{0.88} \right)$$

$$V_m - 0.4 + 0.2\sqrt{0.88} = 0.2\sqrt{V_m + 0.88}$$

$$V_m - 0.212 = 0.2\sqrt{V_m + 0.88}$$

Squaring both sides and collecting terms results in the quadratic equation

$$V_m^2 - 0.464V_m + 0.0097 = 0$$

whose physically meaningful solution is

$$V_m = 0.44 \text{ V}$$

Substituting this value in Eq. (16.20) and

$$V_{\bar{Q}} = V_m = 0.44, \text{ we obtain}$$

$$I_5 = \frac{1}{2} (\mu_n C_{ox}) \left(\frac{W}{L}\right)_5 (1.2 - 0.44 - 0.44)^2$$

$$= 0.0512(\mu_n C_{ox}) \left( \frac{W}{L} \right)_5 \quad (1)$$

Substituting  $V_m = V_{t0} = 0.4$  V and  $V_{\bar{Q}} = 0.44$  V in Eq. (16.21), we obtain

$$\begin{aligned} I_1 &= \mu_n C_{ox} \left( \frac{W}{L} \right)_1 \left[ (1.2 - 0.4)0.44 - \frac{1}{2}(0.44)^2 \right] \\ &= 0.2552(\mu_n C_{ox}) \left( \frac{W}{L} \right)_1 \quad (2) \end{aligned}$$

Equating  $I_5$  from Eq. (1) to  $I_1$  from Eq. (2) gives

$$\frac{(W/L)_5}{(W/L)_1} = 5$$

or

$$\frac{(W/L)_5}{(W/L)_1} \leq 5$$

Thus, the body effect in  $Q_1$  enables us to use a larger  $(W/L)$  for  $Q_5$ . This is a result of the reduced current due to the increased  $V_m$  and also of the increased value allowable for  $V_{\bar{Q}}$ .

**16.31** (a) Using Eq. (16.27), we obtain

$$\frac{(W/L)_a}{(W/L)_n} \leq \frac{1}{\left(1 - \frac{V_m}{V_{DD} - V_m}\right)^2} - 1$$

$$\Rightarrow (W/L)_a \leq 3$$

(b) For  $(W/L)_5 = 1$ ,  $V_{\bar{Q}}$  can be found using Eq. (16.22) as

$$\frac{V_{\bar{Q}}}{V_{DD} - V_m} = 1 - 1/\sqrt{1 + \frac{(W/L)_5}{(W/L)_1}}$$

$$\Rightarrow V_{\bar{Q}} = 0.23$$
 V

The value of  $I_5$  can now be obtained from Eq. (16.20) as

$$I_5 = \frac{1}{2}(\mu_n C_{ox}) \left( \frac{W}{L} \right)_5 (V_{DD} - V_m - V_{\bar{Q}})^2$$

$$= \frac{1}{2} 500 \times 1 (1.2 - 0.4 - 0.23)^2$$

$$= 81.2 \mu\text{A}$$

Finally, the read delay,  $\Delta t$ , can be calculated using Eq. (16.24),

$$\Delta t = \frac{C_{\bar{B}} \Delta V}{I_5}$$

$$= \frac{2 \times 10^{-12} \times 0.2}{81.2 \times 10^{-6}} = 4.93 \text{ ns}$$

(c) If  $(W/L)_5 = 3$  is utilized,  $V_{\bar{Q}}$  will be 0.4 V, and  $I_5$  can be determined using Eq. (16.20) as

$$I_5 = \frac{1}{2} \times 500 \times 3 (1.2 - 0.4 - 0.4)^2$$

$$= 120 \mu\text{A}$$

and the read delay becomes

$$\Delta t = \frac{2 \times 10^{-12} \times 0.2}{120 \times 10^{-6}} = 3.33 \text{ ns}$$

**16.32** By analogy to the description of the process of writing a 0, the relevant part of the circuit for writing a 1 is shown in Fig. 1.

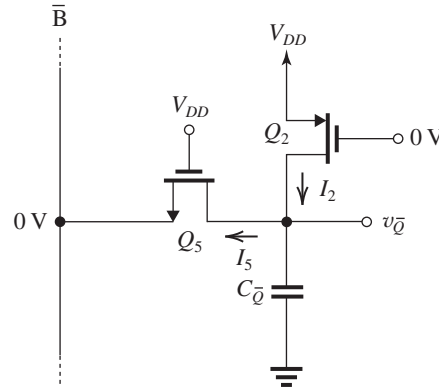


Figure 1

Initially, the cell is storing a zero, thus  $v_{\bar{Q}} = V_{DD}$  and  $v_Q = 0$  V. To write a 1, we lower the  $\bar{B}$  line 0 V and select the cell by raising the word line (the voltage at the gate of the access transistor  $Q_5$ ) to  $V_{DD}$ . We assume that the voltage at the gate of  $Q_2$  (which is  $v_Q$ ) will remain low for a while.

Transistor  $Q_5$  will conduct and its current  $I_5$  will discharge the small capacitor  $C_{\bar{Q}}$  and  $v_{\bar{Q}}$  will decrease. This will allow  $Q_2$  to conduct.

This process continues until  $v_{\bar{Q}}$  is pulled to the threshold voltage of transistor  $Q_3$  which causes the latch to toggle. That is, equilibrium will be reached when

$$V_{\bar{Q}} = V_m$$

and,

$$I_2 = I_5$$

At this point,  $Q_2$  will be operating at the edge of saturation and its current will be given by Eq. (16.26) with  $I_4$  replaced by  $I_2$  and  $(W/L)_4$  replaced by  $(W/L)_2$ . Simultaneously,  $Q_5$  will be operating in the triode region and its current will be given by Eq. (16.29) with  $I_6$  replaced by  $I_5$ ,  $(W/L)_6$  replaced by  $(W/L)_3$ , and  $V_Q$  replaced by  $V_{\bar{Q}}$ .

It follows that the process for writing a 1 is similar to that for writing a 0, and the condition on the  $(W/L)$  ratios will be identical to that in Eq. (16.31).

**16.33** Using Eq. (16.29), we obtain

$$\frac{(W/L)_p}{(W/L)_a} \leq 4 \left[ 1 - \left( 1 - \frac{0.4}{1.2 - 0.4} \right)^2 \right]$$

$$\Rightarrow (W/L)_p \leq 3(W/L)_a$$

**16.34**  $\mu_n \approx 4\mu_p$

$$V_Q \leq V_m$$

(a)  $\left( \frac{0.5}{1.8 - 0.5} \right) = 0.39$ . From Eq. (16.28),

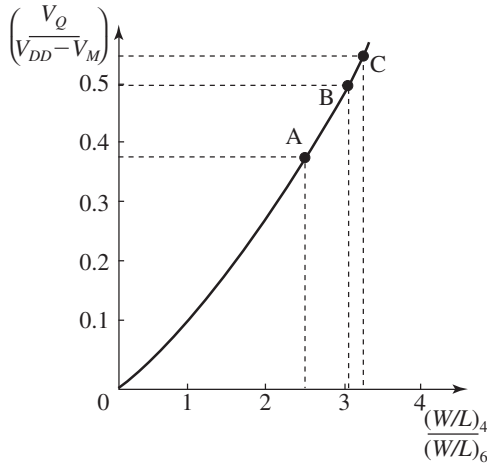
$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 2.5$$

(b)  $\left( \frac{0.4}{1.2 - 0.4} \right) = 0.5$ . From Eq. (16.28),

$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 3.0$$

(c)  $\left( \frac{0.35}{1.0 - 0.35} \right) = 0.54$ . From Eq. (16.28),

$$\rightarrow \frac{(W/L)_4}{(W/L)_6} = 3.15$$



**16.35** The storage capacitor  $C_S$  loses 0.2 V in 12 ms as a result of the leakage current  $I$ , thus

$$I \times 12 \text{ ms} = C_S \times 0.2 \text{ V}$$

$$\Rightarrow I = \frac{25 \times 10^{-15} \times 0.2}{12 \times 10^{-3}}$$

$$= 0.42 \text{ pA}$$

**16.36** A 1-Mbit-square array has 1024 words  $\times$  1024 bits. Thus the capacitance of the bit line,  $C_B$ , is

$$C_B = 1024 \times 0.5 + 12$$

$$= 524 \text{ fF}$$

$$\Delta V(1) \simeq \frac{C_S}{C_B} \left( \frac{V_{DD}}{2} \right)$$

$$= \frac{30}{524} \left( \frac{1.0}{2} \right) = 28.6 \text{ mV}$$

$$\Delta V(0) \simeq -\frac{C_S}{C_B} \left( \frac{V_{DD}}{2} \right) = -28.6 \text{ mV}$$

**16.37**  $f = \frac{1}{2Nt_P}$

$$250 \times 10^6 = \frac{1}{2 \times 9t_P}$$

$$\Rightarrow t_P = 222 \text{ ps}$$

Since  $f \propto V_{DD}$ , a 20% reduction in supply voltage reduces the oscillator frequency to 200 MHz.

**16.38** Refer to Fig. 1 on the next page.

$$T = (3t_{PHL} + 2t_{PLH}) + (2t_{PHL} + 3t_{PLH})$$

$$= (3 \times 200 + 2 \times 300) + (2 \times 200 + 3 \times 300)$$

$$= 1200 + 1300 = 2500 \text{ ps} = 2.5 \text{ ns}$$

$$f = \frac{1}{T} = \frac{1}{2.5 \times 10^{-9}} = 400 \text{ MHz}$$

The output is high for 1.2 of the 2.5-ns period = 48% of the cycle. Note that

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(300 + 200) = 250 \text{ ps}$$

$$T = 2Nt_P = 2 \times 5 \times 250 = 2500 \text{ ps} = 2.5 \text{ ns}$$

as found graphically.

**16.39** From Eq. (16.36), we have

$$v_B = \frac{V_{DD}}{2} + \Delta V(1)e^{(G_m/C_B)t} \quad (1)$$

$$0.9V_{DD} = 0.5V_{DD} + 0.05e^{(G_m/C_B) \times 2 \times 10^{-9}}$$

$$\frac{G_m}{C_B} \times 2 \times 10^{-9} = \ln \left( \frac{0.4 V_{DD}}{0.05} \right)$$

$$G_m = \frac{C_B}{2 \times 10^{-9}} \times \ln \left( \frac{0.4 \times 1.2}{0.05} \right)$$

$$= \frac{1 \times 10^{-12}}{2 \times 10^{-9}} \times 2.26 = 1.13 \text{ mA/V}$$

$$G_m = g_{mn} + g_{mp}$$

Since the inverters are matched, we have

$$g_{mn} = g_{mp} = \frac{G_m}{2} = \frac{1.13}{2} = 0.565 \text{ mA/V}$$

This figure belongs to Problem 16.38.

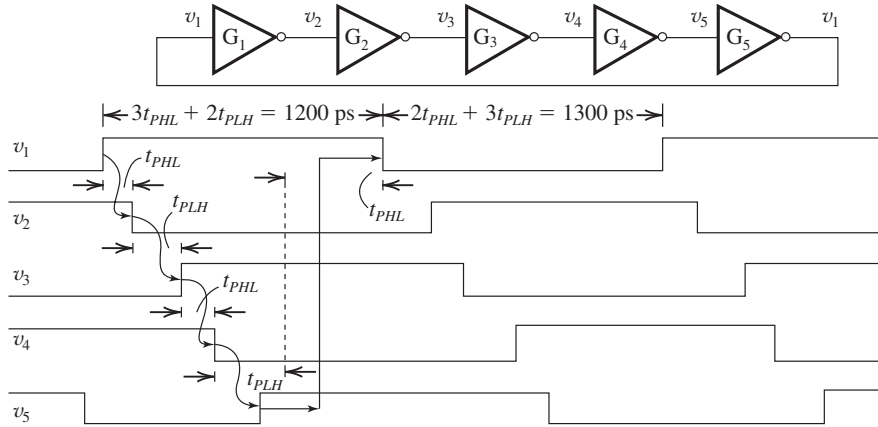


Figure 1

But,

$$g_{mn} = \mu_n C_{ox} \left( \frac{W}{L} \right)_n \left( \frac{V_{DD}}{2} - V_m \right)$$

$$0.565 \times 10^{-3} = 500 \times 10^{-6} \left( \frac{W}{L} \right)_n (0.6 - 0.4)$$

$$\Rightarrow \left( \frac{W}{L} \right)_n = 5.65$$

$$W_n = 5.65 \times 0.13 = 0.73 \mu\text{m}$$

$$W_p = 4W_n = 2.94 \mu\text{m}$$

If the input signal is doubled, we can use Eq. (1) to determine  $t$  as follows:

$$0.9V_{DD} = 0.5V_{DD} + 0.1e^{(G_m/C_B)t}$$

$$\frac{G_m}{C_B} t = \ln \left( \frac{0.4 V_{DD}}{0.1} \right)$$

$$t = \frac{1 \times 10^{-12}}{1.13 \times 10^{-3}} \ln(4 \times 1.2)$$

$$= 1.4 \text{ ns}$$

**16.40** We need four inverters to implement the 0.8-ns delay block.

**16.41** Since  $2^{10} = 1024$ , there are 10 address bits.

The decoder has 1024 output lines. The NOR array requires 20 input lines (for the 10 address bits and their complements).

Each output line has one PMOS transistor; for a total of 1024 PMOS transistors.

Each output line has 10 NMOS transistors for a total of  $1024 \times 10 = 10,240$  NMOS transistors.

The total number of NMOS and PMOS transistors is 11,264.

**16.42** In a 1-Mbit-square array, there are 1024 columns, requiring 10 address bits.

1024 NMOS pass transistors are needed in the multiplexer. The NOR decoder has 1024 output lines; each output line has one PMOS transistor, for a total of 1024 PMOS transistors. Each output line has 10 NMOS transistors, for a total 10,240 NMOS transistors. Total number of transistors =  $1024 + 1024 + 10,240 = 12,288$

**16.43** A square 1-Mbit array has 1024 columns, requiring 10 address bits. Ten levels of pass gates are required.

Using the expression given to the answer of Exercise 16.12, we have

$$\text{Total number of transistors} = 2(2^N - 1)$$

$$= 2(2^{10} - 1) = 2046$$

**16.44** The value of  $G_m$  is given by

$$G_m = g_{mn} + g_{mp}$$

Since the inverters have a matched design, then

$$g_{mn} = g_{mp}$$

and

$$g_{mn} = \mu_n C_{ox} \left( \frac{W}{L} \right)_n \left( \frac{V_{DD}}{2} - V_m \right)$$

$$= 500 \left( \frac{0.26}{0.13} \right) \left( \frac{1.2}{2} - 0.4 \right)$$

$$= 200 \mu\text{A/V} = 0.2 \text{ mA/V}$$

Thus,

$$G_m = 0.4 \text{ mA/V}$$

Assume a read 1 operation,

$$v_B = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

Thus,

$$0.9V_{DD} = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

$$\frac{0.4V_{DD}}{\Delta V(1)} = e^{(G_m/C_B)t_1}$$

$$\Rightarrow \Delta V(1) = 0.4V_{DD}e^{-(G_m/C_B)t_d} \quad (1)$$

Substituting  $V_{DD} = 1.2 \text{ V}$ ,  $G_m = 0.4 \text{ mA/V}$ ,  $C_B = 0.4 \text{ pF}$ , and  $t_d = 1 \text{ ns}$ , we obtain

$$\Delta V(1) = 0.177 \text{ V}$$

Thus the signal between the  $B$  and  $\bar{B}$  lines is

$$2\Delta V(1) = 0.353 \text{ V} = 353 \text{ mV}$$

If the time can be relaxed by 1 ns, then  $t_1 = 2 \text{ ns}$  and

$$\Delta V(1) = 0.065 \text{ V}$$

and the required signal between the  $B$  and  $\bar{B}$  becomes

$$2\Delta V(1) = 130 \text{ mV}$$

If instead we have  $\Delta V(1) = 0.177 \text{ V}$  and  $t_1 = 2 \text{ ns}$ , we can see from Eq. (1) that  $C_B$  can be increased by the same factor  $t_1$  is increased, that is, by a factor of 2. Thus  $C_B$  can become

$$C_B = 2 \times 0.4 = 0.8 \text{ pF}$$

Thus the bit line length can be increased by 100%, i.e., doubled. Doubling the length of the bit line will double the delay time required to charge the bit-line capacitance to 4 ns.

**16.45** (a) Using Eq. (16.36), we write

$$v_B = 0.5V_{DD} + \Delta V(1)e^{(G_m/C_B)t}$$

$$0.9V_{DD} = 0.5V_{DD} + \left(\frac{\Delta V}{2}\right)e^{(G_m/C_B)t_d}$$

$$\Rightarrow t_d = \frac{C_B}{G_m} \ln\left(\frac{0.8V_{DD}}{\Delta V}\right) \quad \text{Q.E.D.} \quad (1)$$

An identical expression can be obtained for the case a 0 is being read.

(b) If  $t_d$  is to be reduced to one-half its original value,  $G_m$  must be increased by a factor of 2. This can be achieved by doubling the width of all transistors (because  $g_{mn}$  and  $g_{mb}$  are proportional to  $(W/L)_n$  and  $(W/L)_p$ , respectively).

(c) Consider Eq. (1) in two situations: (1)  $\Delta V = 0.2 \text{ V}$  and  $G_m = G_{m1}$ , (2)  $\Delta V = 0.1 \text{ V}$  and  $G_m = G_{m2}$ . If in both cases the same value of  $t_d$  results, then

$$\frac{1}{G_{m1}} \ln\left(\frac{0.8 \times 1.2}{0.2}\right) = \frac{1}{G_{m2}} \ln\left(\frac{0.8 \times 1.2}{0.1}\right)$$

$$\Rightarrow \frac{G_{m2}}{G_{m1}} = 1.44$$

To increase  $G_m$  by a factor of 1.44, the widths of all transistors must be increased by a factor of 1.44.