



**Electricals For You**

اللجنة الأكاديمية لقسم الهندسة الكهربائية - HU

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لجنة كلية الهندسة - الجامعة الهاشمية

## Chapter 1

### 1.1

$$n_i = BT^{3/2} e^{-E_g/2kT}$$

(a) Silicon

$$\begin{aligned} \text{(i)} \quad n_i &= (5.23 \times 10^{15})(250)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(250)}\right] \\ &= 2.067 \times 10^{19} \exp[-25.58] \\ \underline{n_i} &= \underline{1.61 \times 10^8 \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad n_i &= (5.23 \times 10^{15})(350)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(350)}\right] \\ &= 3.425 \times 10^{19} \exp[-18.27] \\ \underline{n_i} &= \underline{3.97 \times 10^{11} \text{ cm}^{-3}} \end{aligned}$$

(b) GaAs

$$\begin{aligned} \text{(i)} \quad n_i &= (2.10 \times 10^{14})(250)^{3/2} \exp\left[\frac{-1.4}{2(86 \times 10^{-6})(250)}\right] \\ &= (8.301 \times 10^{17}) \exp[-32.56] \\ \underline{n_i} &= \underline{6.02 \times 10^3 \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad n_i &= (2.10 \times 10^{14})(350)^{3/2} \exp\left[\frac{-1.4}{2(86 \times 10^{-6})(350)}\right] \\ &= (1.375 \times 10^{18}) \exp[-23.26] \\ \underline{n_i} &= \underline{1.09 \times 10^8 \text{ cm}^{-3}} \end{aligned}$$

### 1.2

$$\begin{aligned} \text{a.} \quad n_i &= BT^{3/2} \exp\left(\frac{-E_g}{2kT}\right) \\ 10^{12} &= 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right) \\ 1.91 \times 10^{-4} &= T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right) \end{aligned}$$

By trial and error,  $T \approx 368 \text{ K}$

$$\text{b.} \quad n_i = 10^9 \text{ cm}^{-3}$$

$$\begin{aligned} 10^9 &= 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right) \\ 1.91 \times 10^{-7} &= T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right) \end{aligned}$$

By trial and error,  $T \approx 268^\circ \text{ K}$

**1.3**

Silicon

$$\begin{aligned} \text{(a)} \quad n_i &= (5.23 \times 10^{15})(100)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(100)}\right] \\ &= (5.23 \times 10^{18}) \exp[-63.95] \\ \underline{n_i} &= \underline{8.79 \times 10^{-10} \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad n_i &= (5.23 \times 10^{15})(300)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(300)}\right] \\ &= (2.718 \times 10^{19}) \exp[-21.32] \\ \underline{n_i} &= \underline{1.5 \times 10^{10} \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad n_i &= (5.23 \times 10^{15})(500)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(500)}\right] \\ &= (5.847 \times 10^{19}) \exp[-12.79] \\ \underline{n_i} &= \underline{1.63 \times 10^{14} \text{ cm}^{-3}} \end{aligned}$$

Germanium.

$$\begin{aligned} \text{(a)} \quad n_i &= (1.66 \times 10^{15})(100)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(100)}\right] = (1.66 \times 10^{18}) \exp[-38.37] \\ \underline{n_i} &= \underline{35.9 \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad n_i &= (1.66 \times 10^{15})(300)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(300)}\right] = (8.626 \times 10^{18}) \exp[-12.79] \\ \underline{n_i} &= \underline{2.40 \times 10^{13} \text{ cm}^{-3}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad n_i &= (1.66 \times 10^{15})(500)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(500)}\right] = (1.856 \times 10^{19}) \exp[-7.674] \\ \underline{n_i} &= \underline{8.62 \times 10^{15} \text{ cm}^{-3}} \end{aligned}$$

**1.4**

$$\text{(a) n-type; } n_o = 10^{15} \text{ cm}^{-3}; \quad p_o = \frac{n_i^2}{n_o} = \frac{(2.4 \times 10^{13})^2}{10^{15}} = 5.76 \times 10^{11} \text{ cm}^{-3}$$

$$\text{(b) n-type; } n_o = 10^{15} \text{ cm}^{-3}; \quad p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

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**1.5**

(a) p-type;  $p_o = 10^{16} \text{ cm}^{-3}$ ;  $n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \text{ cm}^{-3}$

(b) p-type;  $p_o = 10^{16} \text{ cm}^{-3}$ ;  $n_o = \frac{n_i^2}{p_o} = \frac{(2.4 \times 10^{13})^2}{10^{16}} = 5.76 \times 10^{10} \text{ cm}^{-3}$

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**1.6**

(a) n-type

(b)  $n_o = N_d = 5 \times 10^{16} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

(c)  $n_o = N_d = 5 \times 10^{16} \text{ cm}^{-3}$

From Problem 1.1(a)(ii)  $n_i = 3.97 \times 10^{11} \text{ cm}^{-3}$

$$p_o = \frac{(3.97 \times 10^{11})^2}{5 \times 10^{16}} = 3.15 \times 10^6 \text{ cm}^{-3}$$

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**1.7**

(a) p-type;  $p_o = 5 \times 10^{16} \text{ cm}^{-3}$ ;  $n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$

(b) p-type;  $p_o = 5 \times 10^{16} \text{ cm}^{-3}$ ;  $n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{5 \times 10^{16}} = 6.48 \times 10^{-5} \text{ cm}^{-3}$

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**1.8**

(a) Add boron atoms

(b)  $N_a = p_o = 2 \times 10^{17} \text{ cm}^{-3}$

(c)  $n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{17}} = 1.125 \times 10^3 \text{ cm}^{-3}$

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**1.9**

(a)  $n_o = 5 \times 10^{15} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} \Rightarrow p_o = 4.5 \times 10^4 \text{ cm}^{-3}$$

(b)  $n_o > p_o \Rightarrow$  n-type

(c)  $n_o \cong N_d = 5 \times 10^{15} \text{ cm}^{-3}$

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**1.10**

a. Add Donors

$$N_d = 7 \times 10^{15} \text{ cm}^{-3}$$

b. Want  $p_o = 10^6 \text{ cm}^{-3} = n_i^2 / N_d$

$$\text{So } n_i^2 = (10^6)(7 \times 10^{15}) = 7 \times 10^{21}$$

$$= B^2 T^3 \exp\left(\frac{-E_g}{kT}\right)$$

$$7 \times 10^{21} = (5.23 \times 10^{15})^2 T^3 \exp\left(\frac{-1.1}{(86 \times 10^{-6})(T)}\right)$$

By trial and error,  $T \approx 324^\circ \text{ K}$

**1.11**

$$(a) I = A \sigma E = (10^{-5})(1.5)(10) \Rightarrow I = 0.15 \text{ mA}$$

$$(b) I = \frac{AE}{\rho} \Rightarrow E = \frac{I\rho}{A} = \frac{(1.2 \times 10^{-3})(0.4)}{(2 \times 10^{-4})} = 2.4 \text{ V/cm}$$

**1.12**

$$J = \sigma E \Rightarrow \sigma = \frac{J}{E} = \frac{120}{18} = 6.67 (\Omega \text{-cm})^{-1}$$

$$\sigma \cong e\mu_n N_d \Rightarrow N_d = \frac{\sigma}{e\mu_n} = \frac{(6.67)}{(1.6 \times 10^{-19})(1250)} = 3.33 \times 10^{16} \text{ cm}^{-3}$$

**1.13**

$$(a) \rho \cong \frac{1}{e\mu_n N_d} \Rightarrow N_d = \frac{1}{e\mu_n \rho} = \frac{1}{(1.6 \times 10^{-19})(1250)(0.65)} = 7.69 \times 10^{15} \text{ cm}^{-3}$$

$$(b) J = \frac{E}{\rho} \Rightarrow E = \rho J = (0.65)(160) = 104 \text{ V/cm}$$

**1.14**

$$(a) \sigma \cong e\mu_n N_d \Rightarrow N_d = \frac{\sigma}{e\mu_n} = \frac{1.5}{(1.6 \times 10^{-19})(1000)} = 9.375 \times 10^{15} \text{ cm}^{-3}$$

$$(b) N_a = \frac{\sigma}{e\mu_p} = \frac{0.8}{(1.6 \times 10^{-19})(400)} = 1.25 \times 10^{16} \text{ cm}^{-3}$$

**1.15**

$$(a) \text{ For n-type, } \sigma \cong e\mu_n N_d = (1.6 \times 10^{-19})(8500) N_d$$

$$\text{For } 10^{15} \leq N_d \leq 10^{19} \text{ cm}^{-3} \Rightarrow 1.36 \leq \sigma \leq 1.36 \times 10^4 (\Omega \text{-cm})^{-1}$$

$$(b) J = \sigma E = \sigma(0.1) \Rightarrow 0.136 \leq J \leq 1.36 \times 10^3 \text{ A/cm}^2$$

**1.16**

$$D_n = (0.026)(1250) = 32.5 \text{ cm}^2/\text{s}; \quad D_p = (0.026)(450) = 11.7 \text{ cm}^2/\text{s}$$

$$J_n = eD_n \frac{dn}{dx} = (1.6 \times 10^{-19})(32.5) \left( \frac{10^{16} - 10^{12}}{0 - 0.001} \right) = -52 \text{ A/cm}^2$$

$$J_p = -eD_p \frac{dp}{dx} = -(1.6 \times 10^{-19})(11.7) \left( \frac{10^{12} - 10^{16}}{0 - 0.001} \right) = -18.72 \text{ A/cm}^2$$

Total diffusion current density

$$J = -52 - 18.72 = -70.7 \text{ A/cm}^2$$

**1.17**

$$\begin{aligned} J_p &= -eD_p \frac{dp}{dx} \\ &= -eD_p (10^{15}) \left( \frac{-1}{L_p} \right) \exp\left(\frac{-x}{L_p}\right) \\ J_p &= \frac{(1.6 \times 10^{-19})(15)(10^{15})}{10 \times 10^{-4}} \exp\left(\frac{-x}{L_p}\right) \end{aligned}$$

$$J_p = 2.4 e^{-x/L_p}$$

(a)  $x = 0 \quad J_p = 2.4 \text{ A/cm}^2$

(b)  $x = 10 \text{ } \mu\text{m} \quad J_p = 2.4 e^{-1} = 0.883 \text{ A/cm}^2$

(c)  $x = 30 \text{ } \mu\text{m} \quad J_p = 2.4 e^{-3} = 0.119 \text{ A/cm}^2$

**1.18**

a.  $N_a = 10^{17} \text{ cm}^{-3} \Rightarrow p_o = 10^{17} \text{ cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{17}} \Rightarrow n_o = 3.24 \times 10^{-5} \text{ cm}^{-3}$$

b.  $n = n_o + \delta n = 3.24 \times 10^{-5} + 10^{15} \Rightarrow n = 10^{15} \text{ cm}^{-3}$

$$p = p_o + \delta p = 10^{17} + 10^{15} \Rightarrow p = 1.01 \times 10^{17} \text{ cm}^{-3}$$

**1.19**  $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$

(a) (i)  $V_{bi} = (0.026) \ln\left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.661 \text{ V}$

(ii)  $V_{bi} = (0.026) \ln\left[\frac{(5 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.739 \text{ V}$

(iii)  $V_{bi} = (0.026) \ln\left[\frac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2}\right] = 0.937 \text{ V}$

$$(b) \quad (i) \quad V_{bi} = (0.026) \ln \left[ \frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.8 \times 10^6)^2} \right] = 1.13 \text{ V}$$

$$(ii) \quad V_{bi} = (0.026) \ln \left[ \frac{(5 \times 10^{17})(10^{15})}{(1.8 \times 10^6)^2} \right] = 1.21 \text{ V}$$

$$(iii) \quad V_{bi} = (0.026) \ln \left[ \frac{(10^{18})(10^{18})}{(1.8 \times 10^6)^2} \right] = 1.41 \text{ V}$$

**1.20**

$$V_{bi} = V_T \ln \left( \frac{N_a N_d}{n_i^2} \right)$$

or

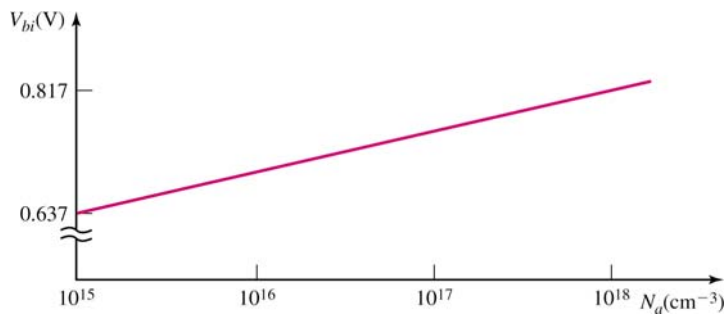
$$N_a = \frac{(n_i^2)}{N_d} \exp \left( \frac{V_{bi}}{V_T} \right) = \frac{(1.5 \times 10)^2}{10^{16}} \exp \left( \frac{0.712}{0.026} \right) = 1.76 \times 10^{16} \text{ cm}^{-3}$$

**1.21**

$$V_{bi} = V_T \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.026) \ln \left[ \frac{N_a (10^{16})}{(1.5 \times 10^{10})^2} \right]$$

$$\text{For } N_a = 10^{15} \text{ cm}^{-3}, \quad V_{bi} = 0.637 \text{ V}$$

$$\text{For } N_a = 10^{18} \text{ cm}^{-3}, \quad V_{bi} = 0.817 \text{ V}$$



**1.22**

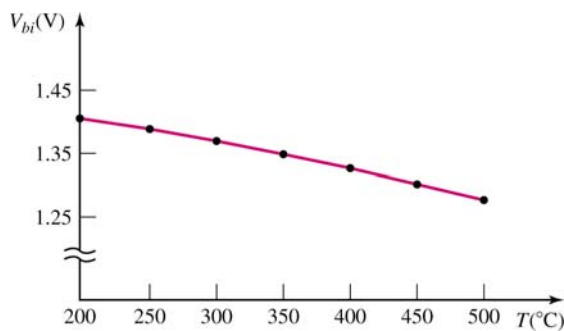
$$kT = (0.026) \left( \frac{T}{300} \right)$$

	$kT$	$(T)^{3/2}$
200	0.01733	2828.4
250	0.02167	3952.8
300	0.026	5196.2
350	0.03033	6547.9
400	0.03467	8000.0
450	0.0390	9545.9
500	0.04333	11,180.3

$$n_i = (2.1 \times 10^{14})(T^{3/2}) \exp\left(\frac{-1.4}{2(86 \times 10^{-6})(T)}\right)$$

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$T$	$n_i$	$V_{bi}$
200	1.256	1.405
250	$6.02 \times 10^3$	1.389
300	$1.80 \times 10^6$	1.370
350	$1.09 \times 10^8$	1.349
400	$2.44 \times 10^9$	1.327
450	$2.80 \times 10^{10}$	1.302
500	$2.00 \times 10^{11}$	1.277



**1.23**

$$C_j = C_{j0} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = (0.026) \ln \left[ \frac{(1.5 \times 10^{16})(4 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.684 \text{ V}$$

(a)  $C_j = (0.4) \left(1 + \frac{1}{0.684}\right)^{-1/2} = 0.255 \text{ pF}$

(b)  $C_j = (0.4) \left(1 + \frac{3}{0.684}\right)^{-1/2} = 0.172 \text{ pF}$

(c)  $C_j = (0.4) \left(1 + \frac{5}{0.684}\right)^{-1/2} = 0.139 \text{ pF}$

**1.24**

(a)  $C_j = C_{j0} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$

For  $V_R = 5 \text{ V}$ ,  $C_j = (0.02) \left(1 + \frac{5}{0.8}\right)^{-1/2} = 0.00743 \text{ pF}$

For  $V_R = 1.5 \text{ V}$ ,  $C_j = (0.02) \left(1 + \frac{1.5}{0.8}\right)^{-1/2} = 0.0118 \text{ pF}$



$$C_j(\text{avg}) = \frac{0.00743 + 0.0118}{2} = 0.00962 \text{ pF}$$

$$v_c(t) = v_c(\text{final}) + (v_c(\text{initial}) - v_c(\text{final}))e^{-t/\tau}$$

where

$$\tau = RC = RC_j(\text{avg}) = (47 \times 10^3)(0.00962 \times 10^{-12})$$

or

$$\tau = 4.52 \times 10^{-10} \text{ s}$$

$$\text{Then } v_c(t) = 1.5 = 0 + (5 - 0)e^{-t_1/\tau}$$

$$\frac{5}{1.5} = e^{+t_1/\tau} \Rightarrow t_1 = \tau \ln\left(\frac{5}{1.5}\right)$$

$$t_1 = 5.44 \times 10^{-10} \text{ s}$$

$$(b) \text{ For } V_R = 0 \text{ V, } C_j = C_{j0} = 0.02 \text{ pF}$$

$$\text{For } V_R = 3.5 \text{ V, } C_j = (0.02) \left(1 + \frac{3.5}{0.8}\right)^{-1/2} = 0.00863 \text{ pF}$$

$$C_j(\text{avg}) = \frac{0.02 + 0.00863}{2} = 0.0143 \text{ pF}$$

$$\tau = RC_j(\text{avg}) = 6.72 \times 10^{-10} \text{ s}$$

$$v_c(t) = v_c(\text{final}) + (v_c(\text{initial}) - v_c(\text{final}))e^{-t/\tau}$$

$$3.5 = 5 + (0 - 5)e^{-t_2/\tau} = 5(1 - e^{-t_2/\tau})$$

$$\text{so that } t_2 = 8.09 \times 10^{-10} \text{ s}$$

### 1.25

$$C_j = C_{j0} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}; \quad V_{bi} = (0.026) \ln \left[ \frac{(5 \times 10^{15})(10^{17})}{(1.5 \times 10^{10})^2} \right] = 0.739 \text{ V}$$

For  $V_R = 1 \text{ V}$ ,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{1}{0.739}}} = 0.391 \text{ pF}$$

For  $V_R = 3 \text{ V}$ ,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{3}{0.739}}} = 0.267 \text{ pF}$$

For  $V_R = 5 \text{ V}$ ,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{5}{0.739}}} = 0.215 \text{ pF}$$

$$(a) f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.391 \times 10^{-12})}} \Rightarrow f_o = 6.57 \text{ MHz}$$

$$(b) f_o = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.267 \times 10^{-12})}} \Rightarrow f_o = 7.95 \text{ MHz}$$

$$(c) \quad f_o = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.215 \times 10^{-12})}} \Rightarrow f_o = 8.86 \text{ MHz}$$

**1.26**

$$a. \quad I = I_S \left[ \exp\left(\frac{V_D}{V_T}\right) - 1 \right] - 0.90 = \exp\left(\frac{V_D}{V_T}\right) - 1$$

$$\exp\left(\frac{V_D}{V_T}\right) = 1 - 0.90 = 0.10$$

$$V_D = V_T \ln(0.10) \Rightarrow \underline{V_D = -0.0599 \text{ V}}$$

b.

$$\left| \frac{I_F}{I_R} \right| = \frac{I_S \left[ \exp\left(\frac{V_F}{V_T}\right) - 1 \right]}{I_S \left[ \exp\left(\frac{V_R}{V_T}\right) - 1 \right]} = \left| \frac{\exp\left(\frac{0.2}{0.026}\right) - 1}{\exp\left(\frac{-0.2}{0.026}\right) - 1} \right|$$

$$= \left| \frac{2190}{-1} \right|$$

$$\underline{\frac{I_F}{I_R} = 2190}$$

$$1.27 \quad I_D = I_S \left[ \exp\left(\frac{V_D}{V_T}\right) - 1 \right]$$

$$(a) \quad (i) \quad I_D = (10^{-11}) \exp\left(\frac{0.3}{0.026}\right) \Rightarrow 1.03 \mu\text{A}$$

$$(ii) \quad I_D = (10^{-11}) \exp\left(\frac{0.5}{0.026}\right) \Rightarrow 2.25 \text{ mA}$$

$$(iii) \quad I_D = (10^{-11}) \exp\left(\frac{0.7}{0.026}\right) \Rightarrow 4.93 \text{ A}$$

$$(iv) \quad I_D = (10^{-11}) \left[ \exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-12} \text{ A}$$

$$(v) \quad I_D = (10^{-11}) \left[ \exp\left(\frac{-0.20}{0.026}\right) - 1 \right] \cong -10^{-11} \text{ A}$$

$$(vi) \quad I_D = -(10^{-11}) \text{ A}$$

$$(b) \quad (i) \quad I_D = (10^{-13}) \exp\left(\frac{0.3}{0.026}\right) \Rightarrow 0.0103 \mu\text{A}$$

$$(ii) \quad I_D = (10^{-13}) \exp\left(\frac{0.5}{0.026}\right) \Rightarrow 22.5 \mu\text{A}$$

$$(iii) \quad I_D = (10^{-13}) \exp\left(\frac{0.7}{0.026}\right) \Rightarrow 49.3 \text{ mA}$$

$$(iv) \quad I_D = (10^{-13}) \left[ \exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-14} \text{ A}$$

$$(v) \quad I_D \cong -10^{-13} \text{ A}$$

$$(vi) \quad I_D \cong -10^{-13} \text{ A}$$

**1.28**  $V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

$$(a) (i) \quad V_D = (0.026) \ln\left(\frac{10 \times 10^{-6}}{10^{-11}}\right) = 0.359 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{100 \times 10^{-6}}{10^{-11}}\right) = 0.419 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-11}}\right) = 0.479 \text{ V}$$

$$(ii) \quad -5 \times 10^{-12} = 10^{-11} \left[ \exp\left(\frac{V_D}{0.026}\right) - 1 \right] \Rightarrow V_D = -0.018 \text{ V}$$

$$(b) (i) \quad V_D = (0.026) \ln\left(\frac{10 \times 10^{-6}}{10^{-13}}\right) = 0.479 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{100 \times 10^{-6}}{10^{-13}}\right) = 0.539 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-13}}\right) = 0.599 \text{ V}$$

$$(ii) \quad -10^{-14} = 10^{-13} \left[ \exp\left(\frac{V_D}{0.026}\right) - 1 \right] \Rightarrow V_D = -0.00274 \text{ V}$$

**1.29**

$$(a) \quad 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$$

$$I_S = 2.03 \times 10^{-15} \text{ A}$$

(b)

$V_D$	$I_D (A)(n=1)$	$I_D (A)(n=2)$
0.1	$9.50 \times 10^{-14}$	$1.39 \times 10^{-14}$
0.2	$4.45 \times 10^{-12}$	$9.50 \times 10^{-14}$
0.3	$2.08 \times 10^{-10}$	$6.50 \times 10^{-13}$
0.4	$9.75 \times 10^{-9}$	$4.45 \times 10^{-12}$
0.5	$4.56 \times 10^{-7}$	$3.04 \times 10^{-11}$
0.6	$2.14 \times 10^{-5}$	$2.08 \times 10^{-10}$
0.7	$10^{-3}$	$1.42 \times 10^{-9}$

**1.30**

(a)

$$I_S = 10^{-12} \text{ A}$$

$V_D(\text{V})$	$I_D(\text{A})$	$\log_{10} I_D$
0.10	$4.68 \times 10^{-11}$	-10.3
0.20	$2.19 \times 10^{-9}$	-8.66
0.30	$1.03 \times 10^{-7}$	-6.99
0.40	$4.80 \times 10^{-6}$	-5.32
0.50	$2.25 \times 10^{-4}$	-3.65
0.60	$1.05 \times 10^{-2}$	-1.98
0.70	$4.93 \times 10^{-1}$	-0.307

(b)

$$I_S = 10^{-14} \text{ A}$$

$V_D(\text{V})$	$I_D(\text{A})$	$\log_{10} I_D$
0.10	$4.68 \times 10^{-13}$	-12.3
0.20	$2.19 \times 10^{-11}$	-10.66
0.30	$1.03 \times 10^{-9}$	-8.99
0.40	$4.80 \times 10^{-8}$	-7.32
0.50	$2.25 \times 10^{-6}$	-5.65
0.60	$1.05 \times 10^{-4}$	-3.98
0.70	$4.93 \times 10^{-3}$	-2.31

**1.31**

a.

$$\frac{I_{D2}}{I_{D1}} = 10 = \exp\left(\frac{V_{D2} - V_{D1}}{V_T}\right)$$

$$\Delta V_D = V_T \ln(10) \Rightarrow \Delta V_D = 59.9 \text{ mV} \approx 60 \text{ mV}$$

b.  $\Delta V_D = V_T \ln(100) \Rightarrow \Delta V_D = 119.7 \text{ mV} \approx 120 \text{ mV}$

**1.32**

(a) (i)  $V_D = (0.026) \ln\left(\frac{2}{2 \times 10^{-9}}\right) = 0.539 \text{ V}$

(ii)  $V_D = (0.026) \ln\left(\frac{20}{2 \times 10^{-9}}\right) = 0.599 \text{ V}$

(b) (i)  $I_D = (2 \times 10^{-9}) \exp\left(\frac{0.4}{0.026}\right) \Rightarrow 9.60 \text{ mA}$

(ii)  $I_D = (2 \times 10^{-9}) \exp\left(\frac{0.65}{0.026}\right) \Rightarrow 144 \text{ A}$

**1.33**

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = (0.026) \ln\left(\frac{2 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.6347 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{2 \times 10^{-3}}{5 \times 10^{-12}}\right) = 0.5150 \text{ V}$$

$$\underline{0.5150 \leq V_D \leq 0.6347 \text{ V}}$$

**1.34**

(a)  $1.5 \times 10^{-3} = I_S \exp\left(\frac{0.30}{0.026}\right) \Rightarrow I_S = 1.46 \times 10^{-8} \text{ A}$

(b) (i)  $I_D = (1.462 \times 10^{-8}) \exp\left(\frac{0.35}{0.026}\right) \Rightarrow I_D = 10.3 \text{ mA}$

(ii)  $I_D = (1.462 \times 10^{-8}) \exp\left(\frac{0.25}{0.026}\right) \Rightarrow I_D = 0.219 \text{ mA}$

**1.35**

(a)  $I_D = (10^{-22}) \exp\left(\frac{0.8}{0.026}\right) \Rightarrow 2.31 \text{ nA}$

$$I_D = (10^{-22}) \exp\left(\frac{1.0}{0.026}\right) \Rightarrow 5.05 \mu\text{A}$$

$$I_D = (10^{-22}) \exp\left(\frac{1.2}{0.026}\right) \Rightarrow 11.1 \text{ mA}$$

$$I_D = (10^{-22}) \left[ \exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-23} \text{ A}$$

For  $V_D = -0.20 \text{ V}$ ,  $I_D = -10^{-22} \text{ A}$

For  $V_D = -2 \text{ V}$ ,  $I_D = -10^{-22} \text{ A}$

(b)

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{0.8}{0.026}\right) \Rightarrow 115 \text{ pA}$$

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{1.0}{0.026}\right) \Rightarrow 0.253 \mu\text{A}$$

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{1.2}{0.026}\right) \Rightarrow 0.554 \text{ mA}$$

$$I_D = (5 \times 10^{-24}) \left[ \exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -2.68 \times 10^{-24} \text{ A}$$

For  $V_D = -0.20 \text{ V}$ ,  $I_D = -5 \times 10^{-24} \text{ A}$

For  $V_D = -2 \text{ V}$ ,  $I_D = -5 \times 10^{-24} \text{ A}$

**1.36**

$I_S$  doubles for every 5C increase in temperature.

$$I_S = 10^{-12} \text{ A at } T = 300\text{K}$$

$$\text{For } I_S = 0.5 \times 10^{-12} \text{ A} \Rightarrow T = 295 \text{ K}$$

$$\text{For } I_S = 50 \times 10^{-12} \text{ A}, (2)^n = 50 \Rightarrow n = 5.64$$

Where n equals number of 5C increases.

$$\text{Then } \Delta T = (5.64)(5) = 28.2 \text{ K}$$

$$\text{So } 295 \leq T \leq 328.2 \text{ K}$$

**1.37**

$$\frac{I_S(T)}{I_S(-55)} = 2^{\Delta T/5}, \quad \Delta T = 155^\circ \text{ C}$$

$$\frac{I_S(100)}{I_S(-55)} = 2^{155/5} = 2.147 \times 10^9$$

$$V_T @ 100^\circ \text{C} \Rightarrow 373^\circ \text{K} \Rightarrow V_T = 0.03220$$

$$V_T @ -55^\circ \text{C} \Rightarrow 216^\circ \text{K} \Rightarrow V_T = 0.01865$$

$$\begin{aligned} \frac{I_D(100)}{I_D(-55)} &= (2.147 \times 10^9) \times \frac{\exp\left(\frac{0.6}{0.0322}\right)}{\exp\left(\frac{0.6}{0.01865}\right)} \\ &= \frac{(2.147 \times 10^9)(1.237 \times 10^8)}{(9.374 \times 10^{13})} \end{aligned}$$

$$\frac{I_D(100)}{I_D(-55)} = 2.83 \times 10^3$$

**1.38**

(a)  $V_{PS} = I_D R + V_D$

$$2.8 = I_D(10^6) + V_D; \quad I_D = (5 \times 10^{-11}) \exp\left(\frac{V_D}{0.026}\right)$$

By trial and error,

$$V_D = 0.282 \text{ V}, \quad I_D = 2.52 \mu \text{ A}$$

(b)

$$I_D \cong -5 \times 10^{-11} \text{ A}, \quad V_D = -2.8 \text{ V}$$

**1.39**

$$10 = I_D (2 \times 10^4) + V_D \text{ and } V_D = (0.026) \ln \left( \frac{I_D}{10^{-12}} \right)$$

Trial and error.

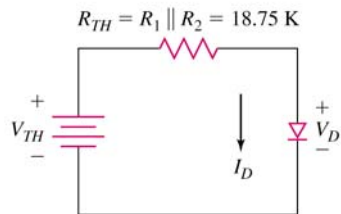
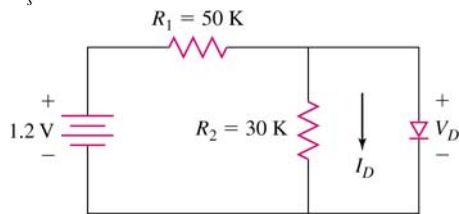
$V_D$ (v)	$I_D$ (A)	$V_D$ (v)
0.50	$4.75 \times 10^{-4}$	0.5194
0.517	$4.7415 \times 10^{-4}$	0.5194
0.5194	$4.740 \times 10^{-4}$	0.5194

$$\underline{V_D = 0.5194 \text{ V}}$$

$$\underline{I_D = 0.4740 \text{ mA}}$$

**1.40**

$$I_s = 5 \times 10^{-13} \text{ A}$$



$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (1.2) = \left( \frac{30}{80} \right) (1.2) = 0.45 \text{ V}$$

$$0.45 = I_D R_{TH} + V_D, \quad V_D = V_T \ln \left( \frac{I_D}{I_s} \right)$$

By trial and error:

$$\underline{I_D = 2.56 \mu\text{A}}, \quad \underline{V_D = 0.402 \text{ V}}$$

**1.41**

(a)  $I_{D1} = I_{D2} = 1 \text{ mA}$

(i)  $V_{D1} = V_{D2} = (0.026) \ln \left( \frac{10^{-3}}{10^{-13}} \right) = 0.599 \text{ V}$

(ii)  $V_{D1} = (0.026) \ln \left( \frac{10^{-3}}{5 \times 10^{-14}} \right) = 0.617 \text{ V}$

$V_{D2} = (0.026) \ln \left( \frac{10^{-3}}{5 \times 10^{-13}} \right) = 0.557 \text{ V}$

(b)  $V_{D1} = V_{D2}$

(i)  $I_{D1} = I_{D2} = \frac{I_i}{2} = 0.5 \text{ mA}$

$V_{D1} = V_{D2} = (0.026) \ln \left( \frac{0.5 \times 10^{-3}}{10^{-13}} \right) = 0.581 \text{ V}$

(ii)  $\frac{I_{D1}}{I_{D2}} = \frac{I_{S1}}{I_{S2}} = \frac{5 \times 10^{-14}}{5 \times 10^{-13}} = 0.10$

So  $I_{D1} = 0.10 I_{D2}$

$I_{D1} + I_{D2} = 1.1 I_{D2} = 1 \text{ mA}$

So  $I_{D2} = 0.909 \text{ mA}$ ,  $I_{D1} = 0.0909 \text{ mA}$

Now

$V_{D1} = (0.026) \ln \left( \frac{0.0909 \times 10^{-3}}{5 \times 10^{-14}} \right) = 0.554 \text{ V}$

$V_{D2} = (0.026) \ln \left( \frac{0.909 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.554 \text{ V}$

**1.42**

(a)  $I_{D3} = (6 \times 10^{-14}) \exp \left( \frac{0.635}{0.026} \right) \Rightarrow 2.426 \text{ mA}$

$I_R = \frac{0.635}{1} = 0.635 \text{ mA}$

$I_{D1} = I_{D2} = 2.426 + 0.635 = 3.061 \text{ mA}$

$V_{D1} = V_{D2} = (0.026) \ln \left( \frac{3.061 \times 10^{-3}}{6 \times 10^{-14}} \right) = 0.641 \text{ V}$

$V_I = 2(0.641) + 0.635 = 1.917 \text{ V}$

(b)  $I_{D3} = 2.426 \text{ mA}$

$I_R = \frac{0.635}{0.5} = 1.27 \text{ mA}$

$I_{D1} = I_{D2} = 2.426 + 1.27 = 3.696 \text{ mA}$

$V_{D1} = V_{D2} = (0.026) \ln \left( \frac{3.696 \times 10^{-3}}{6 \times 10^{-14}} \right) = 0.6459 \text{ V}$

$V_I = 2(0.6459) + 0.635 = 1.927 \text{ V}$



**1.43**

(a) Assume diode is conducting.

Then,  $V_D = V_\gamma = 0.7 \text{ V}$

So that  $I_{R2} = \frac{0.7}{30} \Rightarrow 23.3 \mu\text{A}$

$$I_{R1} = \frac{1.2 - 0.7}{10} \Rightarrow 50 \mu\text{A}$$

Then  $I_D = I_{R1} - I_{R2} = 50 - 23.3$

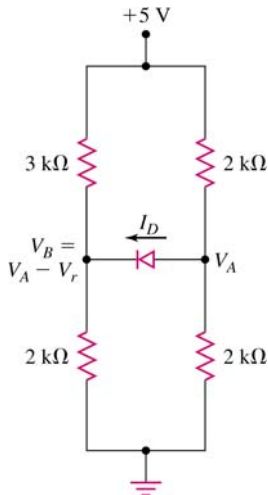
Or  $I_D = 26.7 \mu\text{A}$

(b) Let  $R_1 = 50 \text{ k}\Omega$  Diode is cutoff.

$$V_D = \frac{30}{30 + 50} \cdot (1.2) = 0.45 \text{ V}$$

Since  $V_D < V_\gamma$ ,  $I_D = 0$

**1.44**



At node  $V_A$ :

$$(1) \quad \frac{5 - V_A}{2} = I_D + \frac{V_A}{2}$$

At node  $V_B = V_A - V_\gamma$

$$(2) \quad \frac{5 - (V_A - V_r)}{2} + I_D = \frac{(V_A - V_r)}{2}$$

$$\text{So } \frac{5 - (V_A - V_r)}{3} + \left[ \frac{5 - V_A}{2} - \frac{V_A}{2} \right] = \frac{V_A - V_r}{2}$$

Multiply by 6:

$$10 - 2(V_A - V_r) + 15 - 6V_A = 3(V_A - V_r)$$

$$25 + 2V_r + 3V_r = 11V_A$$

(a)  $V_r = 0.6 \text{ V}$

$$11V_A = 25 + 5(0.6) = 28 \Rightarrow V_A = 2.545 \text{ V}$$

$$\text{From (1) } I_D = \frac{5 - V_A}{2} - \frac{V_A}{2} = 2.5 - V_A \Rightarrow I_D \text{ Neg.} \Rightarrow \underline{I_D = 0}$$

$$\text{Both (a), (b) } \underline{I_D = 0}$$

$$V_A = 2.5, V_B = \frac{2}{5} \cdot 5 = 2 \text{ V} \Rightarrow \underline{V_D = 0.50 \text{ V}}$$

**1.45**

$$\text{(a) } V_O = I_i(1); I_D = 0; \text{ for } 0 \leq I_i \leq 0.7 \text{ mA}$$

$$V_O = 0.7 \text{ V}; I_D = (I_i - 0.7) \text{ mA}; \text{ for } I_i \geq 0.7 \text{ mA}$$

$$\text{(b) } V_O = I_i(1); I_D = 0; \text{ for } 0 \leq I_i \leq 1.7 \text{ mA}$$

$$V_O = 1.7 \text{ V}; I_D = (I_i - 1.7) \text{ mA}; \text{ for } I_i \geq 1.7 \text{ mA}$$

$$\text{(c) } V_O = 0.7 \text{ V}; I_{D1} = I_i; I_{D2} = 0; \text{ for } 0 \leq I_i \leq 2 \text{ mA}$$

**1.46**

Minimum diode current for  $V_{PS}$  (min)

$$I_D(\text{min}) = 2 \text{ mA}, V_D = 0.7 \text{ V}$$

$$I_2 = \frac{0.7}{R_2}, I_1 = \frac{5 - 0.7}{R_1} = \frac{4.3}{R_1}$$

$$\text{We have } I_1 = I_2 + I_D$$

$$\text{so (1) } \frac{4.3}{R_1} = \frac{0.7}{R_2} + 2$$

Maximum diode current for  $V_{PS}$  (max)

$$P = I_D V_D \quad 10 = I_D (0.7) \Rightarrow I_D = 14.3 \text{ mA}$$

$$I_1 = I_2 + I_D$$

or

$$\text{(2) } \frac{9.3}{R_1} = \frac{0.7}{R_2} + 14.3$$

$$\text{Using Eq. (1), } \frac{9.3}{R_1} = \frac{4.3}{R_1} - 2 + 14.3 \Rightarrow \underline{R_1 = 0.41 \text{ k}\Omega}$$

$$\text{Then } \underline{R_2 = 82.5\Omega} \quad 82.5\Omega$$

**1.47**

$$\text{(a) (i) } I = \frac{5 - 0.7}{20} = 0.215 \text{ mA}, V_O = 0.7 \text{ V}$$

$$\text{(ii) } I = \frac{5 - 0.6}{20} = 0.220 \text{ mA}, V_O = 0.6 \text{ V}$$

$$\text{(b) (i) } I = \frac{5 - 0.7 - (-5)}{40} = 0.2325 \text{ mA}, V_O = (0.2325)(20) - 5 = -0.35 \text{ V}$$

$$\text{(iii) } I = \frac{5 - 0.6 - (-5)}{40} = 0.235 \text{ mA}, V_O = (0.235)(20) - 5 = -0.30 \text{ V}$$

- (c) (i)  $I = \frac{2 - 0.7 - (-8)}{25} = 0.372 \text{ mA}$ ,  $V_o = 2 - (0.372)(5) = 0.14 \text{ V}$   
 (ii)  $I = \frac{2 - 0.6 - (-8)}{25} = 0.376 \text{ mA}$ ,  $V_o = 2 - (0.376)(5) = 0.12 \text{ V}$   
 (d) (i)  $I = 0$ ,  $V_o = -5 \text{ V}$   
 (ii)  $I = 0$ ,  $V_o = -5 \text{ V}$

**1.48**

- (a)  $I = \frac{5 - V_o}{20}$ ,  $I = (5 \times 10^{-14}) \exp\left(\frac{V_D}{0.026}\right)$   
 By trial and error,  $V_D = V_o = 0.5775 \text{ V}$ ,  $I = 0.221 \text{ mA}$   
 (b)  $I = \frac{10 - V_D}{40}$ ,  $V_o = 5 - I(20) - V_D$   
 $I = 0.2355 \text{ mA}$ ,  $V_D = 0.579 \text{ V}$ ,  $V_o = -0.289$   
 (c)  $I = \frac{10 - V_D}{25}$ ,  $V_o = 2 - I(5)$   
 $I = 0.3763 \text{ mA}$ ,  $V_D = 0.5913 \text{ V}$ ,  $V_o = 0.1185$   
 (d)  $I = -5 \times 10^{-14} \text{ A}$ ,  $V_o \cong -5 \text{ V}$

**1.49**

- (a) Diode forward biased  $V_D = 0.7 \text{ V}$   
 $5 = (0.4)(4.7) + 0.7 + V \Rightarrow V = 2.42 \text{ V}$   
 (b)  $P = I \cdot V_D = (0.4)(0.7) \Rightarrow P = 0.28 \text{ mW}$

**1.50**

- (a)  $I_{R2} = I_{D1} = \frac{0.65}{1} = 0.65 \text{ mA} = I_{D1}$   
 $I_{D2} = 2(0.65) = 1.30 \text{ mA}$   
 $I_{D2} = \frac{V_i - 2V_r - V_o}{R_1} = \frac{5 - 3(0.65)}{R_1} = 1.30 \Rightarrow R_1 = 2.35 \text{ K}$   
 (b)  $I_{R2} = \frac{0.65}{1} = 0.65 \text{ mA}$   
 $I_{D2} = \frac{8 - 3(0.65)}{2} \Rightarrow I_{D2} = 3.025 \text{ mA}$   
 $I_{D1} = I_{D2} - I_{R2} = 3.025 - 0.65$   
 $I_{D1} = 2.375 \text{ mA}$

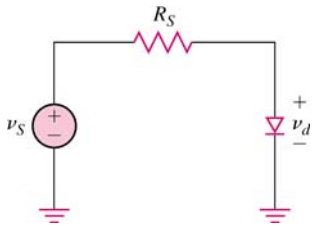
**1.51**

- a.  $\tau_d = \frac{V_T}{I_{DQ}} = \frac{(0.026)}{1} = 0.026 \text{ k}\Omega = 26\Omega$   
 $i_d = 0.05I_{DQ} = 50 \mu\text{A}$  peak-to-peak  
 $v_d = i_d \tau_d = (26)(50) \mu\text{A} \Rightarrow v_d = 1.30 \text{ mV}$  peak-to-peak
- b. For  $I_{DQ} = 0.1 \text{ mA} \Rightarrow \tau_d = \frac{(0.026)}{0.1} = 260\Omega$   
 $i_d = 0.05I_{DQ} = 5 \mu\text{A}$  peak-to-peak  
 $v_d = i_d \tau_d = (260)(5) \mu\text{V} \Rightarrow v_d = 1.30 \text{ mV}$  peak-to-peak

**1.52**

- (a)  $r_d = \frac{V_T}{I_{DQ}} = \frac{0.026}{0.026} = 1 \text{ k}\Omega$
- (b)  $r_d = \frac{0.026}{0.26} \Rightarrow 100\Omega$
- (c)  $r_d = \frac{0.026}{2.6} \Rightarrow 10\Omega$

**1.53**



- a. diode resistance  $r_d = V_T/I$

$$v_d = \left( \frac{r_d}{r_d + R_s} \right) v_s = \left( \frac{V_T/I}{\frac{V_T}{I} + R_s} \right) v_s$$

$$v_d = \left( \frac{V_T}{V_T + IR_s} \right) v_s = v_o$$

- b.  $R_s = 260\Omega$

$$I = 1 \text{ mA}, \frac{v_o}{v_s} = \left( \frac{V_T}{V_T + IR_s} \right) = \frac{0.026}{0.026 + (1)(0.26)} \Rightarrow \frac{v_o}{v_s} = 0.0909$$

$$I = 0.1 \text{ mA}, \frac{v_o}{v_s} = \frac{0.026}{0.026 + (0.1)(0.26)} \Rightarrow \frac{v_o}{v_s} = 0.50$$

$$I = 0.01 \text{ mA}, \frac{v_o}{v_s} = \frac{0.026}{0.026 + (0.01)(0.26)} \Rightarrow \frac{v_o}{v_s} = 0.909$$

**1.54**

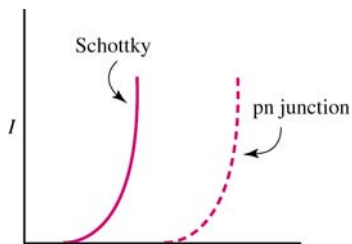
pn junction diode

$$V_D = (0.026) \ln \left( \frac{0.72 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.548 \text{ V}$$

Schottky diode

$$V_D = (0.026) \ln \left( \frac{0.72 \times 10^{-3}}{5 \times 10^{-8}} \right) = 0.249 \text{ V}$$

**1.55**



Schottky:  $I \cong I_S \exp \left( \frac{V_a}{V_T} \right)$

$$V_a = V_T \ln \left( \frac{I}{I_S} \right) = (0.026) \ln \left( \frac{0.5 \times 10^{-3}}{5 \times 10^{-7}} \right) = 0.1796 \text{ V}$$

Then

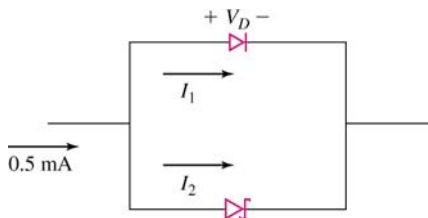
$$V_a \text{ of pn junction} = 0.1796 + 0.30 = 0.4796$$

$$I_S = \frac{I}{\exp \left( \frac{V_a}{V_T} \right)} = \frac{0.5 \times 10^{-3}}{\exp \left( \frac{0.4796}{0.026} \right)}$$

$$I_S = 4.87 \times 10^{-12} \text{ A}$$

**1.56**

(a)



$$I_1 + I_2 = 0.5 \times 10^{-3}$$

$$5 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) + 10^{-12} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

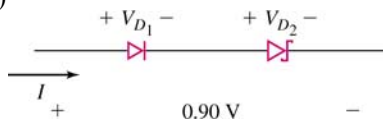
$$5.0001 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

$$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{5.0001 \times 10^{-8}}\right) \Rightarrow \underline{V_D = 0.2395}$$

Schottky diode,  $I_2 = 0.49999$  mA

pn junction,  $I_1 = 0.00001$  mA

(b)



$$I = 10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{V_{D2}}{V_T}\right)$$

$$V_{D1} + V_{D2} = 0.9$$

$$10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{0.9 - V_{D1}}{V_T}\right)$$

$$= 5 \times 10^{-8} \exp\left(\frac{0.9}{V_T}\right) \exp\left(\frac{-V_{D1}}{V_T}\right)$$

$$\exp\left(\frac{2V_{D1}}{V_T}\right) = \left(\frac{5 \times 10^{-8}}{10^{-12}}\right) \exp\left(\frac{0.9}{0.026}\right)$$

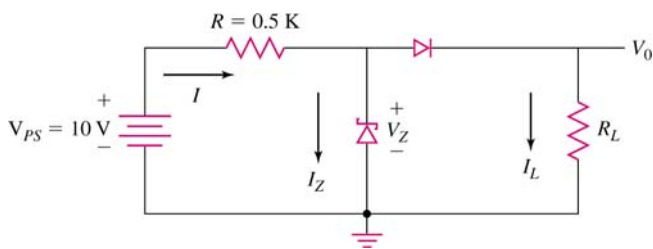
$$2V_{D1} = V_T \ln\left(\frac{5 \times 10^{-8}}{10^{-12}}\right) + 0.9 = 1.1813$$

$$\underline{V_{D1} = 0.5907} \text{ pn junction}$$

$$\underline{V_{D2} = 0.3093} \text{ Schottky diode}$$

$$I = 10^{-12} \exp\left(\frac{0.5907}{0.026}\right) \Rightarrow \underline{I = 7.35 \text{ mA}}$$

1.57



$$V_Z = V_{Z0} = 5.6 \text{ V at } I_Z = 0.1 \text{ mA}$$

$$r_z = 10 \Omega$$

$$I_Z r_Z = (0.1)(10) = 1 \text{ mV}$$

$$V_{Z0} = 5.599$$

a.  $R_L \rightarrow \infty \Rightarrow$

$$I_Z = \frac{10 - 5.599}{R + r_Z} = \frac{4.401}{0.50 + 0.01} = 8.63 \text{ mA}$$

$$V_Z = V_{Z0} + I_Z r_Z = 5.599 + (0.00863)(10)$$

$$\underline{V_Z = V_0 = 5.685 \text{ V}}$$

b.  $V_{PS} = 11 \text{ V} \Rightarrow I_Z = \frac{11 - 5.599}{0.51} = 10.59 \text{ mA}$

$$V_Z = V_0 = 5.599 + (0.01059)(10) = 5.7049 \text{ V}$$

$$V_{PS} = 9 \text{ V} \Rightarrow I_Z = \frac{9 - 5.599}{0.51} = 6.669 \text{ mA}$$

$$V_Z = V_0 = 5.599 + (0.006669)(10) = 5.66569 \text{ V}$$

$$\Delta V_0 = 5.7049 - 5.66569 \Rightarrow \underline{\Delta V_0 = 0.0392 \text{ V}}$$

c.  $I = I_Z + I_L$

$$I_L = \frac{V_0}{R_L}, \quad I = \frac{V_{PS} - V_0}{R}, \quad I_Z = \frac{V_0 - V_{Z0}}{r_Z}$$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 5.599}{0.010} + \frac{V_0}{2}$$

$$\frac{10}{0.50} + \frac{5.599}{0.010} = V_0 \left[ \frac{1}{0.50} + \frac{1}{0.010} + \frac{1}{2} \right]$$

$$20.0 + 559.9 = V_0 (102.5)$$

$$\underline{V_0 = 5.658 \text{ V}}$$

### 1.58

(a)  $I_Z = \frac{10 - 6.8}{0.5} = 6.4 \text{ mA}$

$$P = I_Z V_Z = (6.4)(6.8) = 43.5 \text{ mW}$$

(b)  $I_Z = (0.1)(6.4) = 0.64 \text{ mA}$

$$I_L = 6.4 - 0.64 = 5.76 \text{ mA}$$

$$I_L = \frac{V_Z}{R_L} \Rightarrow R_L = \frac{V_Z}{I_Z} = \frac{6.8}{5.76} = 1.18 \text{ k}\Omega$$

### 1.59

$$I_Z r_Z = (0.1)(20) = 2 \text{ mV}$$

$$V_{Z0} = 6.8 - 0.002 = 6.798 \text{ V}$$

a.  $R_L = \infty$

$$I_Z = \frac{10 - 6.798}{0.5 + 0.02} \Rightarrow I_Z = 6.158 \text{ mA}$$

---

$$V_0 = V_Z = V_{Z0} + I_Z r_Z = 6.798 + (0.006158)(20)$$
$$\underline{V_0 = 6.921 \text{ V}}$$

b.  $I = I_Z + I_L$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 6.798}{0.020} + \frac{V_0}{1}$$
$$\frac{10}{0.30} + \frac{6.798}{0.020} = V_0 \left[ \frac{1}{0.50} + \frac{1}{0.020} + \frac{1}{1} \right]$$
$$359.9 = V_0 (53)$$
$$V_0 = 6.791 \text{ V}$$
$$\Delta V_0 = 6.791 - 6.921$$
$$\underline{\Delta V_0 = -0.13 \text{ V}}$$

---

**1.60**

For  $V_D = 0$ ,  $\underline{I_{SC} = 0.1 \text{ A}}$

For  $I_D = 0$   $V_D = V_T \ln \left( \frac{0.2}{5 \times 10^{-14}} + 1 \right)$

$$\underline{V_D = V_{DC} = 0.754 \text{ V}}$$

---

**1.61**  $V_D = 0$ ,  $I_D = 0.2 \text{ A}$

$$V_D = 0.60 \text{ V}, I_D = 0.1995 \text{ A}$$

$$V_D = 0.65 \text{ V}, I_D = 0.1964 \text{ A}$$

$$V_D = 0.70 \text{ V}, I_D = 0.1754 \text{ A}$$

$$V_D = 0.72 \text{ V}, I_D = 0.1468 \text{ A}$$

$$V_D = 0.74 \text{ V}, I_D = 0.0853 \text{ A}$$

$$V_D = 0.7545 \text{ V}, I_D = 0$$

---

**1.62**

(a)  $0.16 = 0.20 - (5 \times 10^{-14}) \left[ \exp \left( \frac{V_D}{0.026} \right) - 1 \right] \Rightarrow V_D = 0.7126 \text{ V}$

(b)  $P = (0.16)(0.7126) = 0.114 \text{ W}$

---



## Chapter 2

### 2.1

(a) For  $v_I > 0.6$  V,  $v_o = \left(\frac{1000}{1020}\right)(v_I - 0.6)$

For  $v_I < 0.6$  V,  $v_o = 0$

(b) (ii)  $v_o = 0 = \left(\frac{1000}{1020}\right)[10 \sin(\omega t)_1 - 0.6]$

Then  $\sin(\omega t)_1 = \frac{0.6}{10} = 0.06 \Rightarrow (\omega t)_1 = 3.44^\circ \Rightarrow 0.01911\pi$  rad

Also  $(\omega t)_2 = 180 - 3.44 = 176.56^\circ \Rightarrow 0.9809\pi$  rad

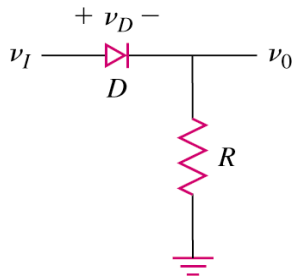
Now

$$\begin{aligned} v_o(\text{avg}) &= \frac{1}{T} \int_0^T v_o(t) dt = \frac{1}{2\pi} \int_0^{2\pi} [10 \sin x - 0.6] dx \\ &= \frac{1}{2\pi} \left[ -10 \cos x \Big|_{0.01911\pi}^{0.9809\pi} - 0.6x \Big|_{0.01911\pi}^{0.9809\pi} \right] \\ &= \frac{1}{2\pi} [(-10)(-0.9982 - 0.9982) - 0.6(0.9809\pi - 0.01911\pi)] \\ v_o(\text{avg}) &= 2.89 \text{ V} \end{aligned}$$

(iii)  $v_o(\text{peak}) = \left(\frac{1000}{1020}\right) \left[ 10 \sin\left(\frac{\pi}{2}\right) - 0.6 \right] = 9.2157 \text{ V}; i_d(\text{max}) = 9.2157 \text{ mA}$

(iv)  $PIV = 10 \text{ V}$

### 2.2



$$v_o = v_I - v_D$$

$$v_D = V_T \ln\left(\frac{i_D}{I_S}\right) \text{ and } i_D = \frac{v_o}{R}$$

$$v_o = v_I - V_T \ln\left(\frac{v_o}{I_S R}\right)$$

**2.3**

(a)  $v_s = 120\sqrt{2}\left(\frac{1}{10}\right) = 16.97 \text{ V (peak)}$

$v_o(\text{peak}) = 16.27 \text{ V}$

(b)  $i_D(\text{peak}) = \frac{16.27}{2} = 8.14 \text{ mA}$

(c)  $v_o = 16.97 \sin \omega t - 0.7$

$$\sin(\omega t)_1 = \frac{0.7}{16.97} = 0.04125 \Rightarrow (\omega t)_1 = 2.364^\circ$$

$$(\omega t)_2 = 180 - 2.364 = 177.64^\circ$$

$$\% = \left(\frac{177.64 - 2.364}{360}\right) \times 100\% = 48.7\%$$

(d)

$$v_o(\text{avg}) = \frac{1}{2\pi} \int_{0.01313\pi}^{0.9869\pi} [16.97 \sin x - 0.7] dx$$

$$= \frac{1}{2\pi} \left[ (-16.97) \cos x \Big|_{0.01313\pi}^{0.9869\pi} - 0.7x \Big|_{0.01313\pi}^{0.9869\pi} \right]$$

$$= \frac{1}{2\pi} [(-16.97)(-0.99915 - 0.99915) - 0.7(0.9738\pi)]$$

$$v_o(\text{avg}) = 5.06 \text{ V}$$

(e)  $i_D(\text{avg}) = \frac{v_o(\text{avg})}{2} = \frac{5.06}{2} = 2.53 \text{ mA}$

**2.4**

(a)  $v_R(t) = 15 \sin \omega t - 0.7 - 9 = 15 \sin \omega t - 9.7$

$$(\omega t)_1 = \sin^{-1}\left(\frac{9.7}{15}\right) = 40.29^\circ \Rightarrow 0.2238\pi \text{ rad}$$

$$(\omega t)_2 = 180 - 40.29 = 139.71^\circ \Rightarrow 0.7762\pi \text{ rad}$$

$$v_R(\text{avg}) = \frac{1}{2\pi} \int_{0.2238\pi}^{0.7762\pi} [15 \sin x - 9.7] dx$$

$$= \frac{1}{2\pi} \left[ (-15) \cos x \Big|_{0.2238\pi}^{0.7762\pi} - 9.7x \Big|_{0.2238\pi}^{0.7762\pi} \right] = \frac{1}{2\pi} [(-15)(-0.7628 - 0.7628) - 9.7(0.5523\pi)]$$

$$v_R(\text{avg}) = 0.9628 \text{ V}$$

$$i_D(\text{avg}) = 0.8 = \frac{0.9628}{R} \Rightarrow R = 1.20 \Omega$$

(b)

$$\% = \left(\frac{139.71 - 40.29}{360}\right) \times 100\% = 27.6\%$$

**2.5**

(a)  $i(\text{peak}) = \frac{v_R(\text{peak})}{R} \Rightarrow R = \frac{15 - 9.7}{1.2} = 4.417 \Omega$

(b)  $v_R(t) = 15 \sin \omega t - 9.7$   
 $(\omega t)_1 = 0.2238\pi$  ;  $(\omega t)_2 = 0.7762\pi$

$$v_R(\text{avg}) = \frac{1}{\pi} \int_{0.2238\pi}^{0.7762\pi} [15 \sin x - 9.7] dx$$

Or from Problem 2.4,  $v_R(\text{avg}) = 2(0.9628) = 1.9256 \text{ V}$

$$i_D(\text{avg}) = \frac{v_R(\text{avg})}{R} = \frac{1.9256}{4.417} = 0.436 \text{ A}$$

(c)  $\% = \left( \frac{139.71 - 40.29}{360} \right) \times 100\% = 27.6\%$

**2.6**

(a)  $v_s(\text{peak}) = 12 + 0.7 = 12.7 \text{ V}$

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{12.7} = 13.4$$

(b)  $R = \frac{12}{0.2} = 60 \Omega$

$$C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(60)(0.25)} \Rightarrow 6667 \mu\text{F}$$

(c)  $PIV = 2v_s(\text{max}) - V_\gamma = 2(12.7) - 0.7 = 24.7 \text{ V}$

**2.7**

$$v_o = v_s - 2V_\gamma \Rightarrow v_s(\text{max}) = v_o(\text{max}) + 2V_\gamma$$

a. For  $v_o(\text{max}) = 25 \text{ V} \Rightarrow v_s(\text{max}) = 25 + 2(0.7) = 26.4 \text{ V}$

$$\frac{N_1}{N_2} = \frac{160}{26.4} \Rightarrow \frac{N_1}{N_2} = 6.06$$

b. For  $v_o(\text{max}) = 100 \text{ V} \Rightarrow v_s(\text{max}) = 101.4 \text{ V}$

$$\frac{N_1}{N_2} = \frac{160}{101.4} \Rightarrow \frac{N_1}{N_2} = 1.58$$

From part (a)  $PIV = 2v_s(\text{max}) - V_\gamma = 2(26.4) - 0.7$

or  $PIV = 52.1 \text{ V}$  or, from part (b)  $PIV = 2(101.4) - 0.7$  or  $PIV = 202.1 \text{ V}$

**2.8**

(a)  $v_s(\text{max}) = 12 + 2(0.7) = 13.4 \text{ V}$

$$v_s(\text{rms}) = \frac{13.4}{\sqrt{2}} \Rightarrow v_s(\text{rms}) = 9.48 \text{ V}$$

(b)

$$V_r = \frac{V_M}{2f R_C} \Rightarrow C = \frac{V_M}{2f V_r R}$$

$$C = \frac{12}{2(60)(0.3)(150)} \Rightarrow C = 2222 \mu\text{F}$$

(c)

$$i_{d, \text{ peak}} = \frac{V_M}{R} \left[ 1 + \pi \sqrt{\frac{2V_M}{V_r}} \right]$$

$$= \frac{12}{150} \left[ 1 + \pi \sqrt{\frac{2(12)}{0.3}} \right]$$

$$\underline{i_{d, \text{ peak}} = 2.33 \text{ A}}$$

**2.9**

(a)

$$v_s(\text{max}) = 12 + 0.7 = 12.7 \text{ V}$$

$$v_s(\text{rms}) = \frac{v_s(\text{max})}{\sqrt{2}} \Rightarrow v_s(\text{rms}) = 8.98 \text{ V}$$

(b)

$$V_r = \frac{V_M}{fRC} \Rightarrow C = \frac{V_M}{fRV_r} = \frac{12}{(60)(150)(0.3)} \quad \text{or} \quad C = 4444 \mu\text{F}$$

(c)

$$\text{For the half-wave rectifier} \quad i_{D, \text{ max}} = \frac{V_M}{R} \left( 1 + 4\pi \sqrt{\frac{V_M}{2V_r}} \right) = \frac{12}{150} \left( 1 + 4\pi \sqrt{\frac{12}{2(0.3)}} \right) \quad \text{or} \quad \underline{i_{D, \text{ max}} = 4.58 \text{ A}}$$

**2.10**

(a)  $v_o(\text{peak}) = 10 - 0.7 = 9.3 \text{ V}$

(b)  $C = \frac{V_M}{fRV_r} = \frac{9.3}{(60)(500)(0.5)} \Rightarrow 620 \mu\text{F}$

(c)  $PIV = 10 + 9.3 = 19.3 \text{ V}$

**2.11**

(a)  $10.3 \leq v_o \leq 12.3 \text{ V}$

(b)  $V_r = \frac{V_M}{fRC} = \frac{12.3}{(60)(1000)(350 \times 10^{-6})} = 0.586 \text{ V}$

$$V_r = \frac{10.3}{(60)(1000)(350 \times 10^{-6})} = 0.490 \text{ V}$$

So  $0.490 \leq V_r \leq 0.586 \text{ V}$

(c)  $C = \frac{V_M}{fRV_r} = \frac{12.3}{(60)(1000)(0.4)} \Rightarrow 513 \mu\text{F}$

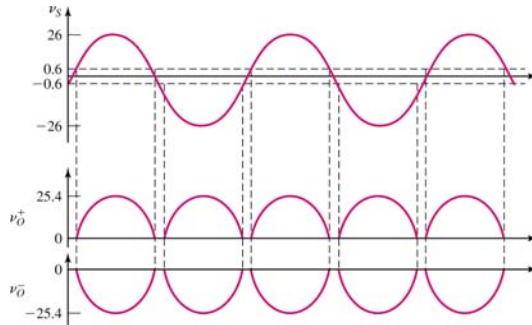
**2.12**

- (a)  $v_s(\text{peak}) = 8.5(\sqrt{2}) = 12.02 \text{ V}$   
 $|V_o|_{\text{max}} = 12.02 - 0.7 = 11.32 \text{ V}$
- (b)  $C = \frac{V_M}{2fRV_r} = \frac{11.32}{2(60)(10)(0.25)} = 0.03773 \text{ F}$
- (c)  $\text{PIV} = 2v_s(\text{peak}) - V_\gamma = 2(12.02) - 0.7 = 23.34 \text{ V}$

**2.13**

- (a)  $v_s(\text{peak}) = 15 + 2(0.7) = 16.4 \text{ V}$   
 $v_s(\text{rms}) = \frac{16.4}{\sqrt{2}} = 11.6 \text{ V}$
- (b)  $C = \frac{V_M}{2fRV_r} = \frac{15}{2(60)(125)(0.35)} = 2857 \mu\text{F}$

**2.14**



**2.15**

- (a)  $v_s = 12.8 \text{ V}$   
 $\frac{N_1}{N_2} = \frac{120\sqrt{2}}{12.8} = 13.3$
- (b)  $R = \frac{12}{0.5} = 24 \Omega$   
 $V_r = 3\% \Rightarrow V_r = (0.03)(12) = 0.36 \text{ V}$   
 $C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(24)(0.36)} = 0.0116 \text{ F}$
- (c)  $i_D(\text{peak}) = \frac{V_M}{R} \left( 1 + \pi \sqrt{\frac{2V_M}{V_r}} \right) = \frac{12}{24} \left( 1 + \pi \sqrt{\frac{2(12)}{0.36}} \right)$   
 $i_D(\text{peak}) = 13.3 \text{ A}$

(d) 
$$i_D(\text{avg}) = \frac{1}{\pi} \sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R} \left( 1 + \frac{\pi}{2} \sqrt{\frac{2V_M}{V_r}} \right) = \frac{1}{\pi} \sqrt{\frac{2(0.36)}{12}} \left( \frac{12}{24} \right) \left( 1 + \frac{\pi}{2} \sqrt{\frac{2(12)}{0.36}} \right)$$

$$i_D(\text{avg}) = 0.539 \text{ A}$$

(e) 
$$PIV = 12.8 + 12 = 24.8 \text{ V}$$

**2.16**

(a) 
$$v_s = 9 + 2(0.8) = 10.6 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{10.6} = 16$$

(b) 
$$R = \frac{9}{0.1} = 90 \Omega$$

$$C = \frac{V_M}{2fRV_r} = \frac{9}{2(60)(90)(0.2)} \Rightarrow 4167 \mu\text{F}$$

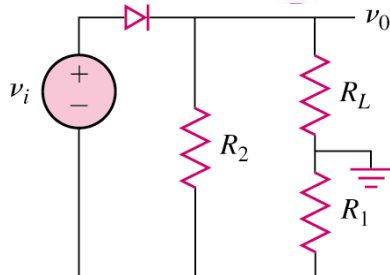
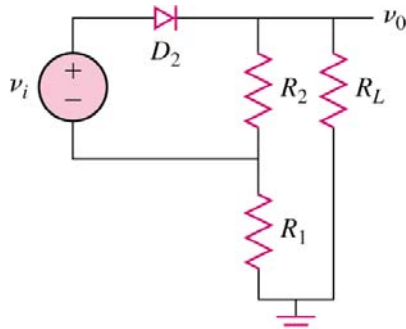
(c) 
$$i_D(\text{peak}) = \frac{V_M}{R} \left( 1 + \pi \sqrt{\frac{2V_M}{V_r}} \right) = \frac{9}{90} \left( 1 + \pi \sqrt{\frac{2(9)}{0.2}} \right) = 3.08 \text{ A}$$

(d) 
$$i_D(\text{avg}) = \frac{1}{\pi} \sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R} \left( 1 + \frac{\pi}{2} \sqrt{\frac{2V_M}{V_r}} \right) = \frac{1}{\pi} \sqrt{\frac{2(0.2)}{9}} \left( \frac{9}{90} \right) \left( 1 + \frac{\pi}{2} \sqrt{\frac{2(9)}{0.2}} \right)$$

$$i_D(\text{avg}) = 0.1067 \text{ A}$$

(e) 
$$PIV = v_s(\text{max}) - V_\gamma = 10.6 - 0.8 = 9.8 \text{ V}$$

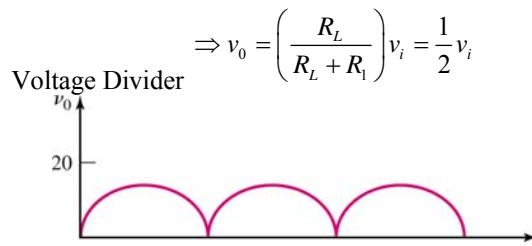
**2.17**



For  $v_i > 0$

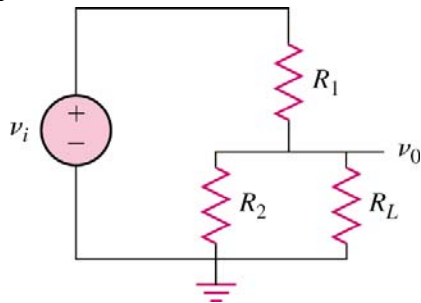
$$V_\gamma = 0$$

Voltage across  $R_L + R_1 = v_i$



**2.18**

For  $v_i > 0$ , ( $V_Z = 0$ )

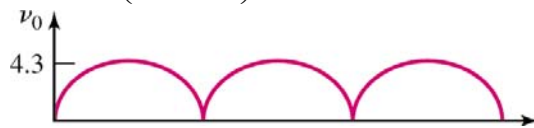


a.

$$v_o = \left( \frac{R_2 \parallel R_L}{R_2 \parallel R_L + R_1} \right) |v_i|$$

$$R_2 \parallel R_L = 2.2 \parallel 6.8 = 1.66 \text{ k}\Omega$$

$$v_o = \left( \frac{1.66}{1.66 + 2.2} \right) v_i = 0.43 |v_i|$$



b.

$$v_o(\text{rms}) = \frac{v_o(\text{max})}{\sqrt{2}} \Rightarrow v_o(\text{rms}) = 3.04 \text{ V}$$

**2.19**

(a)  $I_L = \frac{3.9}{4} = 0.975 \text{ mA}$

$$I_I = \frac{20 - 3.9}{12} = 1.342 \text{ mA}$$

$$I_Z = I_I - I_L = 1.342 - 0.975 = 0.367 \text{ mA}$$

$$P_Z = I_Z V_Z = (0.367)(3.9) = 1.43 \text{ mW}$$

(b)  $I_L = \frac{3.9}{10} = 0.39 \text{ mA}$

$$I_Z = 1.342 - 0.39 = 0.952 \text{ mA}$$

$$P_Z = (0.952)(3.9) = 3.71 \text{ mW}$$

**2.20**

(a)

$$I_Z = \frac{40 - 12}{120} = 0.233 \text{ A}$$

$$P = (0.233)(12) = 2.8 \text{ W}$$

(b)  $I_R = 0.233 \text{ A}$ ,  $I_L = (0.9)(0.233) = 0.21 \text{ A}$

$$0.21 = \frac{12}{R_L} \Rightarrow R_L = 57.1 \Omega$$

So

(c)  $P = (0.1)(0.233)(12) \Rightarrow P = 0.28 \text{ W}$

**2.21**

(a)  $P_Z = I_Z V_Z$

$$4 = I_Z (15.4) \Rightarrow I_Z (\text{max}) = 259.74 \text{ mA}$$

So  $15 \leq I_Z \leq 259.74 \text{ mA}$

(b)  $I_I = \frac{60 - 15.4}{0.15} = 297.33 \text{ mA}$

So  $I_L (\text{max}) = 297.33 - 15 = 282.33 \text{ mA}$

$$I_L (\text{min}) = 297.33 - 259.74 = 37.59 \text{ mA}$$

$$\text{Then } R_L (\text{min}) = \frac{15.4}{0.28233} = 54.55 \Omega$$

$$R_L (\text{max}) = \frac{15.4}{0.03759} = 410 \Omega$$

So  $54.55 \leq R_L \leq 410 \Omega$

**2.22**

a.

$$I_I = \frac{20 - 10}{222} \Rightarrow I_I = 45.0 \text{ mA}$$

$$I_L = \frac{10}{380} \Rightarrow I_L = 26.3 \text{ mA}$$

$$I_Z = I_I - I_L \Rightarrow I_Z = 18.7 \text{ mA}$$

b.

$$P_Z (\text{max}) = 400 \text{ mW} \Rightarrow I_Z (\text{max}) = \frac{400}{10} = 40 \text{ mA}$$

$$\Rightarrow I_L (\text{min}) = I_I - I_Z (\text{max}) = 45 - 40$$

$$\Rightarrow I_L (\text{min}) = 5 \text{ mA} = \frac{10}{R_L}$$

$$\Rightarrow R_L = 2 \text{ k}\Omega$$

(c) For  $R_i = 175 \Omega$   $I_I = 57.1 \text{ mA}$   $I_L = 26.3 \text{ mA}$   $I_Z = 30.8 \text{ mA}$

$$I_Z (\text{max}) = 40 \text{ mA} \Rightarrow I_L (\text{min}) = 57.1 - 40 = 17.1 \text{ mA}$$

$$R_L = \frac{10}{17.1} \Rightarrow R_L = 585 \Omega$$



**2.23**

a. From Eq. (2.30)

$$I_Z(\text{max}) = \frac{500[20-10] - 50[15-10]}{15 - (0.9)(10) - (0.1)(20)}$$

$$= \frac{5000 - 250}{4}$$

$$I_Z(\text{max}) = 1.1875 \text{ A}$$

$$I_Z(\text{min}) = 0.11875 \text{ A}$$

From Eq. (2.28(b))  $R_i = \frac{20-10}{1187.5+50} \Rightarrow R_i = 8.08\Omega$

b.

$$P_Z = (1.1875)(10) \Rightarrow P_Z = 11.9 \text{ W}$$

$$P_L = I_L(\text{max})V_o = (0.5)(10) \Rightarrow P_L = 5 \text{ W}$$

**2.24**

(a)  $I_L = 0$

$$I_Z = \frac{10-5.6}{50+3} \Rightarrow 83.0 \text{ mA}$$

$$V_Z = 5.6 + (0.083)(3) = 5.85 \text{ V} = V_L$$

$$P_Z = I_Z V_Z = (0.083)(5.85) = 0.486 \text{ W}$$

(b)  $\frac{10-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$

$$0.20 + 1.867 = V_L(0.02 + 0.3333 + 0.005)$$

So  $V_L = 5.769 \text{ V}$

Then  $I_L = \frac{5.769}{0.2} = 28.84 \text{ mA}$

$$I_I = \frac{10-5.769}{0.050} = 84.62 \text{ mA}$$

And  $I_Z = I_I - I_L = 55.8 \text{ mA}$

$$P_Z = (0.0558)(5.769) = 0.322 \text{ W}$$

(c)  $I_L = 0$

$$I_Z = \frac{12-5.6}{50+3} \Rightarrow 120.8 \text{ mA}$$

$$V_Z = V_L = 5.6 + (0.1208)(3) = 5.962 \text{ V}$$

$$P_Z = (0.1208)(5.962) = 0.72 \text{ W}$$

(d)  $\frac{12-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$

$$0.24 + 1.867 = V_L(0.02 + 0.333 + 0.005)$$

So  $V_L = 5.88 \text{ V}$

Then  $I_L = \frac{5.88}{0.20} = 29.4 \text{ mA}; I_I = \frac{12-5.88}{0.05} = 122.4 \text{ mA}$

$$I_Z = 122.4 - 29.4 = 93 \text{ mA}$$

$$P_Z = (0.093)(5.88) = 0.547 \text{ W}$$

**2.25**

(a) Set  $I_Z = 10 \text{ mA}$ ;  $I_L = \frac{V_L}{R_L} = \frac{7.5}{1} = 7.5 \text{ mA}$

$$I_I = 10 + 7.5 = 17.5 \text{ mA}$$

$$I_I = 17.5 = \frac{12 - 7.5}{R_i} \Rightarrow R_i = 257 \Omega$$

(b)  $7.5 = V_{Z0} + (0.01)(12) \Rightarrow V_{Z0} = 7.38 \text{ V}$

For  $V_I = (1.1)(12) = 13.2 \text{ V}$

$$\frac{13.2 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.05136 + 0.615 = V_L(0.00389 + 0.0833 + 0.001) \Rightarrow V_L = 7.556 \text{ V}$$

For  $V_I = (0.9)(12) = 10.8 \text{ V}$

$$\frac{10.8 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.04202 + 0.615 = V_L(0.08819) \Rightarrow V_L = 7.450 \text{ V}$$

Then, Source Reg =  $\left( \frac{7.556 - 7.450}{13.2 - 10.8} \right) \times 100\% = 4.42\%$

(c) For  $R_L = 1 \text{ k}\Omega$ ,  $V_L = 7.50 \text{ V}$

For  $R_L = \infty$ ,  $I_Z = \frac{12 - 7.38}{257 + 12} \Rightarrow 17.17 \text{ mA}$

$$V_L = 7.38 + (0.01717)(12) = 7.586 \text{ V}$$

Then, Load Reg =  $\left( \frac{7.586 - 7.50}{7.50} \right) \times 100\% = 1.15\%$

**2.26**

$$\begin{aligned} \% \text{ Reg} &= \frac{V_L(\text{max}) - V_L(\text{min})}{V_L(\text{nom})} \times 100\% \\ &= \frac{V_L(\text{nom}) + I_Z(\text{max})r_z - (V_L(\text{nom}) + I_Z(\text{min})r_z)}{V_L(\text{nom})} \\ &= \frac{[I_Z(\text{max}) - I_Z(\text{min})](3)}{6} = 0.05 \end{aligned}$$

So  $I_Z(\text{max}) - I_Z(\text{min}) = 0.1 \text{ A}$

Now  $I_L(\text{max}) = \frac{6}{500} = 0.012 \text{ A}$ ,  $I_L(\text{min}) = \frac{6}{1000} = 0.006 \text{ A}$

Now  $R_i = \frac{V_{PS}(\text{min}) - V_Z}{I_Z(\text{min}) + I_L(\text{max})}$

or  $280 = \frac{15 - 6}{I_Z(\text{min}) + 0.012} \Rightarrow I_Z(\text{min}) = 0.020 \text{ A}$

$$\text{Then } I_Z(\text{max}) = 0.1 + 0.02 = 0.12 \text{ A} \quad \text{and} \quad R_i = \frac{V_{PS}(\text{max}) - V_Z}{I_Z(\text{max}) + I_L(\text{min})}$$

$$\text{or } 280 = \frac{V_{PS}(\text{max}) - 6}{0.12 + 0.006} \Rightarrow V_{PS}(\text{max}) = 41.3 \text{ V}$$

**2.27**

Using Figure 2.19

a.  $V_{PS} = 20 \pm 25\% \Rightarrow 15 \leq V_{PS} \leq 25 \text{ V}$

For  $V_{PS}(\text{min})$ :

$$I_I = I_Z(\text{min}) + I_L(\text{max}) = 5 + 20 = 25 \text{ mA}$$

$$R_i = \frac{V_{PS}(\text{min}) - V_Z}{I_I} = \frac{15 - 10}{25} \Rightarrow R_i = 200 \Omega$$

b. For  $V_{PS}(\text{max}) \Rightarrow I_I(\text{max}) = \frac{25 - 10}{R_i} \Rightarrow I_I(\text{max}) = 75 \text{ mA}$

For  $I_L(\text{min}) = 0 \Rightarrow I_Z(\text{max}) = 75 \text{ mA}$

$$V_{Z0} = V_Z - I_Z r_Z = 10 - (0.025)(5) = 9.875 \text{ V}$$

$$V_0(\text{max}) = 9.875 + (0.075)(5) = 10.25$$

$$V_0(\text{min}) = 9.875 + (0.005)(5) = 9.90$$

$$\Delta V_0 = 0.35 \text{ V}$$

c.  $\% \text{ Reg} = \frac{\Delta V_0}{V_0(\text{nom})} \times 100\% \Rightarrow \% \text{ Reg} = 3.5\%$

**2.28**

From Equation (2.28(a))

$$R_i = \frac{V_{PS}(\text{min}) - V_Z}{I_Z(\text{min}) + I_L(\text{max})} = \frac{24 - 16}{40 + 400} \quad \text{or} \quad R_i = 18.2 \Omega$$

$$\text{Also } V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$$R \cong R_i + r_c = 18.2 + 2 = 20.2 \Omega$$

Then

$$C = \frac{24}{2(60)(1)(20.2)} \Rightarrow C = 9901 \mu\text{F}$$

**2.29**

$$V_Z = V_{Z0} + I_Z r_Z \quad V_Z(\text{nom}) = 8 \text{ V}$$

$$8 = V_{Z0} + (0.1)(0.5) \Rightarrow V_{Z0} = 7.95 \text{ V}$$

$$I_i = \frac{V_s(\text{max}) - V_Z(\text{nom})}{R_i} = \frac{12 - 8}{3} = 1.333 \text{ A}$$

For  $I_L = 0.2 \text{ A} \Rightarrow I_Z = 1.133 \text{ A}$

For  $I_L = 1 \text{ A} \Rightarrow I_Z = 0.333 \text{ A}$

$$V_L(\max) = V_{Z0} + I_Z(\max)r_z$$

$$= 7.95 + (1.133)(0.5) = 8.5165$$

$$V_L(\min) = V_{Z0} + I_Z(\min)r_z$$

$$= 7.95 + (0.333)(0.5) = 8.1165$$

$$\Delta V_L = 0.4 \text{ V}$$

$$\% \text{ Reg} = \frac{\Delta V_L}{V_0(\text{nom})} = \frac{0.4}{8} \Rightarrow \underline{\% \text{ Reg} = 5.0\%}$$

$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$$R = R_i + r_z = 3 + 0.5 = 3.5 \Omega$$

$$C = \frac{12}{2(60)(3.5)(0.8)} \Rightarrow \underline{C = 0.0357 \text{ F}}$$

Then

### 2.30

For  $-6.3 \leq v_I \leq 3 \text{ V}$ ,  $v_o = v_I$

For  $v_I > 3 \text{ V}$ ,  $I = \frac{v_I - 3}{1.5}$  and  $v_o = v_I - I(0.5)$

$$v_o = v_I - (0.5) \left( \frac{v_I - 3}{1.5} \right) = 0.667v_I + 1.0$$

For  $v_I < -6.3 \text{ V}$ ,  $I = \frac{v_I + 6.3}{2.5}$  and  $v_o = v_I - I(0.5)$

$$v_o = v_I - (0.5) \left( \frac{v_I + 6.3}{2.5} \right) = 0.8v_I - 1.26$$

### 2.31

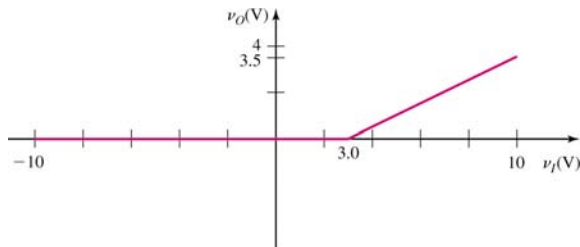
(a) For  $-10 \leq v_I \leq 0$ , both diodes are conducting  $\Rightarrow v_o = 0$

For  $0 \leq v_I \leq 3$ , Zener not in breakdown, so  $i_1 = 0$ ,  $v_o = 0$

For  $v_I > 3$   $i_1 = \frac{v_I - 3}{20} \text{ mA}$

$$v_o = \left( \frac{v_I - 3}{20} \right) (10) = \frac{1}{2}v_I - 1.5$$

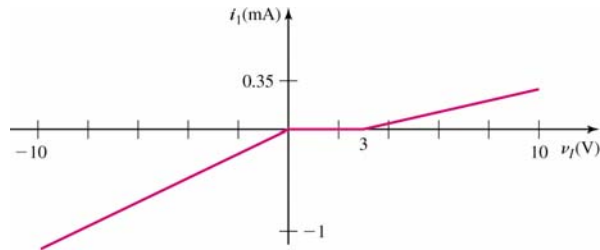
At  $v_I = 10 \text{ V}$ ,  $v_o = 3.5 \text{ V}$ ,  $i_1 = 0.35 \text{ mA}$



(b) For  $v_I < 0$ , both diodes forward biased

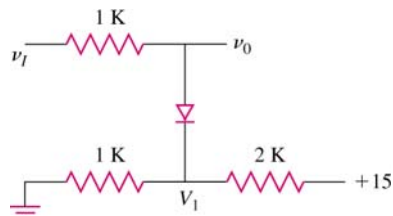
$$-i_1 = \frac{0 - v_I}{10}. \quad \text{At } v_I = -10 \text{ V}, i_1 = -1 \text{ mA}$$

For  $v_I > 3$ ,  $i_1 = \frac{v_I - 3}{20}$ . At  $v_I = 10 \text{ V}$ ,  $i_1 = 0.35 \text{ mA}$



**2.32**

(a)



$$V_1 = \frac{1}{3} \times 15 = 5 \text{ V} \Rightarrow \text{for } v_I \leq 5.7, v_0 = v_I$$

For  $v_I > 5.7 \text{ V}$

$$\frac{v_I - (V_1 + 0.7)}{1} + \frac{15 - V_1}{2} = \frac{V_1}{1}, \quad v_0 = V_1 + 0.7$$

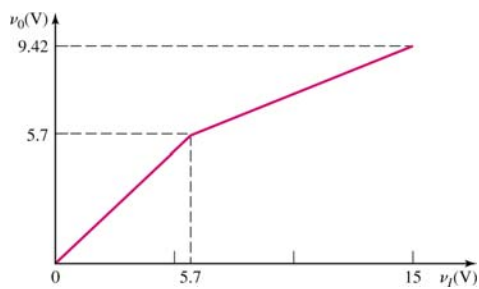
$$\frac{v_I - v_0}{1} + \frac{15 - (v_0 - 0.7)}{2} = \frac{v_0 - 0.7}{1}$$

$$\frac{v_I}{1} + \frac{15.7}{2} + \frac{0.7}{1} = v_0 \left( \frac{1}{1} + \frac{1}{2} + \frac{1}{1} \right) = v_0 (2.5)$$

$$v_I + 8.55 = v_0 (2.5) \Rightarrow v_0 = \frac{1}{2.5} v_I + 3.42$$

$$v_I = 5.7 \Rightarrow v_0 = 5.7$$

$$v_I = 15 \Rightarrow v_0 = 9.42$$

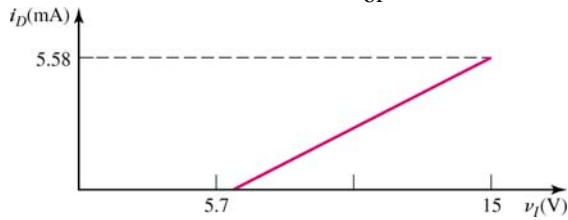


(b)  $i_D = 0$  for  $0 \leq v_I \leq 5.7$

Then for  $v_I > 5.7$  V

$$i_D = \frac{v_I - v_O}{1} = \frac{v_I - \left(\frac{v_I}{2.5} + 3.42\right)}{1} \quad \text{or} \quad i_D = \frac{0.6v_I - 3.42}{1}$$

For  $v_I = 15$ ,  $i_D = 5.58$  mA



**2.33**

(a) (i)  $V_B = 1.8$  V

For  $v_I \geq 1.1$  V,  $v_O = v_I$

For  $v_I \leq 1.1$  V,  $v_O = 1.1$  V

(ii)  $V_B = -1.8$  V

For  $v_I \geq -2.5$  V,  $v_O = v_I$

For  $v_I \leq -2.5$  V,  $v_O = -2.5$  V

(b) (i)  $V_B = 1.8$  V

For  $v_I \geq 2.5$  V,  $v_O = 2.5$  V

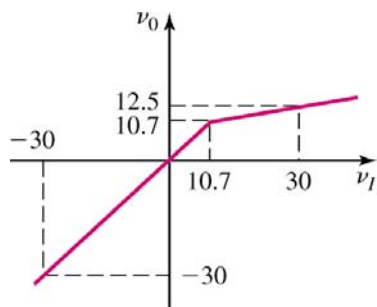
For  $v_I \leq 2.5$  V,  $v_O = v_I$

(ii)  $V_B = -1.8$  V

For  $v_I \geq -1.1$  V,  $v_O = -1.1$  V

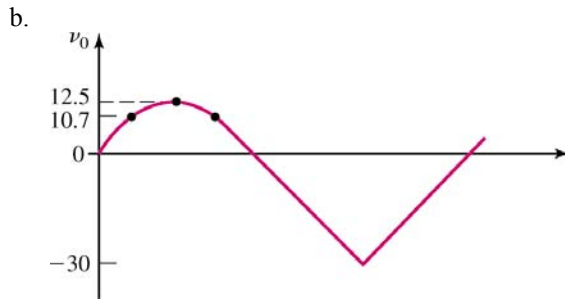
For  $v_I \leq -1.1$  V,  $v_O = v_I$

**2.34**



For  $v_I = 30$  V,  $i = \frac{30 - 10.7}{100 + 10} = 0.175$  A

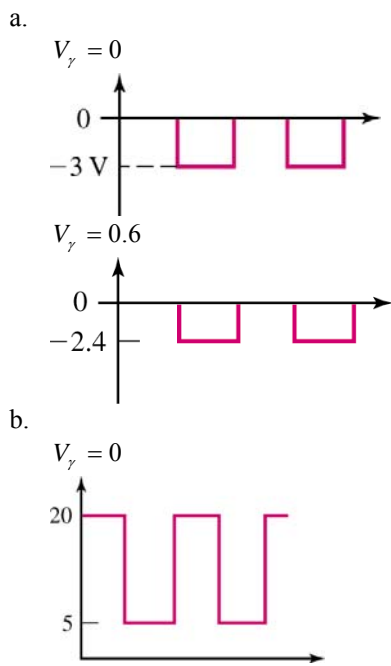
$v_0 = i(10) + 10.7 = 12.5$  V

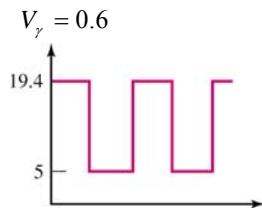


**2.35**

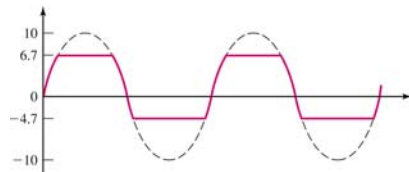
- (a) (i)  $V_B = 5\text{ V}$   
 For  $v_i \geq 5.7\text{ V}$ ,  $v_o = v_i - 5.7$   
 For  $v_i \leq 5.7\text{ V}$ ,  $v_o = 0$
- (ii)  $V_B = -5\text{ V}$   
 For  $v_i \geq -4.3\text{ V}$ ,  $v_o = v_i + 4.3$   
 For  $v_i \leq -4.3\text{ V}$ ,  $v_o = 0$
- (b) (i)  $V_B = 5\text{ V}$   
 For  $v_i \geq 4.3\text{ V}$ ,  $v_o = 0$   
 For  $v_i \leq 4.3\text{ V}$ ,  $v_o = v_i - 4.3$
- (ii)  $V_B = -5\text{ V}$   
 For  $v_i \geq -5.7\text{ V}$ ,  $v_o = 0$   
 For  $v_i \leq -5.7\text{ V}$ ,  $v_o = v_i + 5.7$

**2.36**



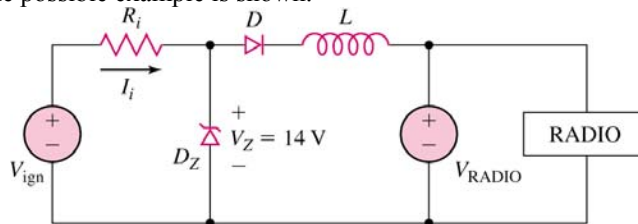


2.37



2.38

One possible example is shown.



$L$  will tend to block the transient signals

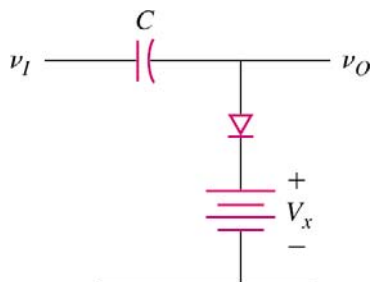
$D_z$  will limit the voltage to  $+14\text{ V}$  and  $-0.7\text{ V}$ .

Power ratings depends on number of pulses per second and duration of pulse.

2.39

- (a) Square wave between  $+40\text{ V}$  and  $0$ .
- (b) Square wave between  $+35\text{ V}$  and  $-5\text{ V}$ .
- (c) Square wave between  $+5\text{ V}$  and  $-35\text{ V}$ .

2.40



- a. For  $V_\gamma = 0 \Rightarrow V_x = 2.7\text{ V}$
- b. For  $V_\gamma = 0.7\text{ V} \Rightarrow V_x = 2.0\text{ V}$



**2.41**

Circuit similar to Figure 2.31(a) with  $V_B = -10$  V.

**2.42**

In steady-state,  $v_o = (10 \sin \omega t + 5)$  V

**2.43**

(i)  $V_B = 5$  V, In steady-state,

$$v_o = (10 \sin \omega t - 5) \text{ V}$$

(ii)  $V_B = -5$  V, In steady-state,

$$v_o = (10 \sin \omega t - 15) \text{ V}$$

**2.44**

a.

$$I_{D1} = \frac{10 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.94 \text{ mA} \quad I_{D2} = 0$$

$$V_o = I_{D1}(9.5) \Rightarrow V_o = 8.93 \text{ V}$$

b.

$$I_{D1} = \frac{5 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.44 \text{ mA} \quad I_{D2} = 0$$

$$V_o = I_{D1}(9.5) \Rightarrow V_o = 4.18 \text{ V}$$

c.

Same as (a)

d.

$$10 = \frac{I}{2}(0.5) + 0.6 + I(9.5) \Rightarrow I = 0.964 \text{ mA}$$

$$V_o = I(9.5) \Rightarrow V_o = 9.16 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.482 \text{ mA}$$

**2.45**

a.

$$I = I_{D1} = I_{D2} = 0 \quad V_o = 10$$

b.

$$10 = I(9.5) + 0.6 + I(0.5) \Rightarrow I = I_{D2} = 0.94 \text{ mA} \quad I_{D1} = 0$$

$$V_o = 10 - I(9.5) \Rightarrow V_o = 1.07 \text{ V}$$

c.

$$10 = I(9.5) + 0.6 + I(0.5) + 5 \Rightarrow I = I_{D2} = 0.44 \text{ mA} \quad I_{D1} = 0$$

$$V_o = 10 - I(9.5) \Rightarrow V_o = 5.82 \text{ V}$$

d.

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) \Rightarrow \underline{I = 0.964 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.482 \text{ mA}}$$

$$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 0.842 \text{ V}}$$

### 2.46

a.

$$V_1 = V_2 = 0 \Rightarrow D_1, D_2, D_3, \text{ on } \underline{V_0 = 4.4 \text{ V}}$$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \underline{I = 0.589 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow \underline{I_{D1} = I_{D2} = 7.6 \text{ mA}}$$

$$I_{D3} = I_{D1} + I_{D2} - I = 2(7.6) - 0.589 \Rightarrow \underline{I_{D3} = 14.6 \text{ mA}}$$

b.

$$V_1 = V_2 = 5 \text{ V } D_1 \text{ and } D_2 \text{ on, } D_3 \text{ off}$$

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) + 5 \Rightarrow \underline{I = 0.451 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.226 \text{ mA}}$$

$$\underline{I_{D3} = 0}$$

$$V_0 = 10 - I(9.5) = 10 - (0.451)(9.5) \Rightarrow \underline{V_0 = 5.72 \text{ V}}$$

c.  $V_1 = 5 \text{ V}, V_2 = 0$   $D_1$  off,  $D_2, D_3$  on  $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \underline{I = 0.589 \text{ mA}}$$

$$I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow \underline{I_{D2} = 7.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 7.6 - 0.589 \Rightarrow \underline{I_{D3} = 7.01 \text{ mA}}$$

d.  $V_1 = 5 \text{ V}, V_2 = 2 \text{ V}$   $D_1$  off,  $D_2, D_3$  on  $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \underline{I = 0.589 \text{ mA}}$$

$$I_{D2} = \frac{4.4 - 0.6 - 2}{0.5} \Rightarrow \underline{I_{D2} = 3.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 3.6 - 0.589 \Rightarrow \underline{I_{D3} = 3.01 \text{ mA}}$$

### 2.47

(a)  $V_1 = 4.4 \text{ V}, I_{D1} = 0.2 = \frac{10 - 0.6 - 4.4}{R_1} \Rightarrow R_1 = 25 \text{ k}\Omega$

$$I_{R2} = 0.2 + 0.3 = 0.5 \text{ mA}$$

$$V_2 = -0.6 \text{ V}, I_{R2} = 0.5 = \frac{4.4 - (-0.6)}{R_2} \Rightarrow R_2 = 10 \text{ k}\Omega$$

$$I_{R3} = 0.5 + 0.5 = 1.0 \text{ mA}$$

$$R_3 = \frac{-0.6 - (-5)}{1.0} = 4.4 \text{ k}\Omega$$

(b) Assume all diodes conducting

$$V_1 = 4.4 \text{ V}, I_{D1} = \frac{10 - 0.6 - 4.4}{10} = 0.5 \text{ mA}$$

$$V_2 = -0.6 \text{ V}, I_{R2} = \frac{4.4 - (-0.6)}{4} = 1.25 \text{ mA}$$

Then  $I_{D2} = 1.25 - 0.5 = 0.75 \text{ mA}$

$$I_{R3} = \frac{-0.6 - (-5)}{2.2} = 2 \text{ mA}$$

Then  $I_{D3} = 2 - 1.25 = 0.75 \text{ mA}$

(c) Diode  $D_2$  cutoff  $\Rightarrow I_{D2} = 0$

$$V_2 = -0.6 \text{ V}, I_{D1} = \frac{10 - 0.6 - (-0.6)}{R_1 + R_2} = \frac{10}{9} = 1.11 \text{ mA}$$

$$V_1 = 10 - 0.6 - (1.11)(3) = 6.07 \text{ V}$$

$$I_{R3} = \frac{-0.6 - (-5)}{2.5} = 1.76 \text{ mA}$$

Then  $I_{D3} = 1.76 - 1.11 = 0.65 \text{ mA}$

(d) Diode  $D_3$  cutoff  $\Rightarrow I_{D3} = 0$

$$V_1 = 4.4 \text{ V}, I_{D1} = \frac{10 - 0.6 - 4.4}{6} = 0.833 \text{ mA}$$

$$I_{R2} = \frac{4.4 - (-5)}{R_2 + R_3} = \frac{9.4}{9} = 1.044 \text{ mA}$$

$$V_2 = (1.044)(6) - 5 = 1.27 \text{ V}$$

Then  $I_{D2} = 1.044 - 0.833 = 0.211 \text{ mA}$

### 2.48

(a)  $I_{D1} = I_{D2} = 2.5 \text{ mA}$

$$I_{D2} = 2.5 = \frac{0.7 - 0.7 - (-2)}{R} \Rightarrow R = 0.8 \text{ k}\Omega$$

(b)  $I_{D1} = 0.2I_{D2}$ ,  $I_{D1} + I_{D2} = 5$

$$0.2I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 4.167 \text{ mA}$$

$$I_{D2} = 4.167 = \frac{2}{R} \Rightarrow R = 0.48 \text{ k}\Omega$$

(c)  $I_{D1} = 5I_{D2}$ ,  $I_{D1} + I_{D2} = 5$

$$5I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 0.833 \text{ mA}$$

$$I_{D2} = 0.833 = \frac{2}{R} \Rightarrow R = 2.4 \text{ k}\Omega$$

**2.49**

(a)  $D_1$  and  $D_2$  on

$$5 + \frac{5 - 0.7 - V_A}{2} = \frac{V_A - 0.7 - (-5)}{1.1}$$

$$5 + 2.15 - 3.909 = V_A \left( \frac{1}{2} + \frac{1}{1.1} \right) \Rightarrow V_A = 2.30 \text{ V}$$

Then  $I_{D1} = \frac{5 - 0.7 - 2.3}{2} = 1.0 \text{ mA}$

$$I_{D2} = \frac{2.3 - 0.7 - (-5)}{1.1} = 6.0 \text{ mA}$$

(b)  $D_1$  cutoff,  $I_{D1} = 0$

$$I_{D2} = 5 \text{ mA}, \quad V_A = 0.7 + (5)(2.5) - 5 = 8.2 \text{ V}$$

(c)  $V_A = 0$ ,  $I_{D1} = \frac{5 - 0.7 - 0}{2} = 2.15 \text{ mA}$

Then  $I_{D2} = 5 + 2.15 = 7.15 \text{ mA}$

$$I_{D2} = 7.15 = \frac{0 - 0.7 - (-5)}{R_2} \Rightarrow R_2 = 0.60 \text{ k}\Omega$$

**2.50**

(a) (i)  $v_I = 5 \text{ V}$ ,  $D_1$  and  $D_2$  on

$$\frac{5 - (v_o + 0.6)}{5} + \frac{5 - v_o}{5} = \frac{v_o}{0.5} + \frac{v_o - 0.6}{0.5}$$

$$0.88 + 1.0 + 1.2 = v_o(0.20 + 0.20 + 2.0 + 2.0) \Rightarrow v_o = 0.7 \text{ V}$$

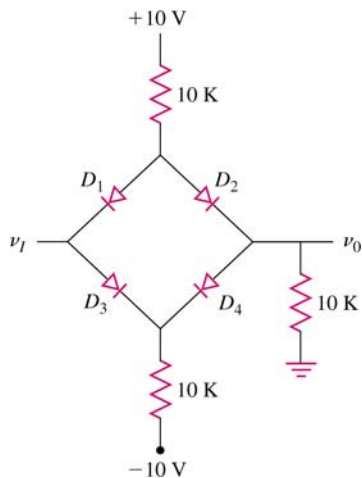
(ii)  $v_I = -5 \text{ V}$

$$v_o = \left( \frac{0.5}{0.5 + 5} \right) v_I = -0.455 \text{ V}$$

(b) (i)  $v_I = 5 \text{ V}$ ,  $v_o = 4.4 \text{ V}$

(ii)  $v_I = -5 \text{ V}$ ,  $v_o = -0.6 \text{ V}$

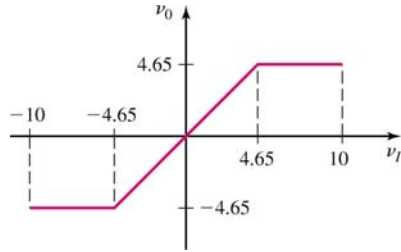
**2.51**



For  $v_I > 0$ , when  $D_1$  and  $D_4$  turn off

$$I = \frac{10 - 0.7}{20} = 0.465 \text{ mA}$$

$$v_0 = I(10 \text{ k}\Omega) = 4.65 \text{ V}$$



$$v_0 = v_I \text{ for } -4.65 \leq v_I \leq 4.65$$

### 2.52

(a) All diodes on

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7}{2} + \frac{V_A - 0.7 - (-5)}{14} + \frac{V_A - 0.7 - (-10)}{24}$$

$$2.439 + 0.35 - 0.307 - 0.3875 = V_A(0.1626 + 0.50 + 0.0714 + 0.0417) \Rightarrow V_A = 2.70 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.70 - 0.7}{2} = 1.0 \text{ mA}$$

$$I_{D2} = \frac{2.70 - 0.7 - (-5)}{14} = 0.50 \text{ mA}$$

$$I_{D3} = \frac{2.70 - 0.7 - (-10)}{24} = 0.50 \text{ mA}$$

(b)  $D_1$  cutoff,  $I_{D1} = 0$

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7 - (-5)}{3.3} + \frac{V_A - 0.7 - (-10)}{5.2}$$

$$2.439 - 1.303 - 1.788 = V_A(0.1626 + 0.303 + 0.1923) \Rightarrow V_A = -0.991 \text{ V}$$

$$\text{Then } I_{D2} = \frac{-0.991 - 0.7 - (-5)}{3.3} = 1.0 \text{ mA}$$

$$I_{D3} = \frac{-0.991 - 0.7 - (-10)}{5.2} = 1.60 \text{ mA}$$

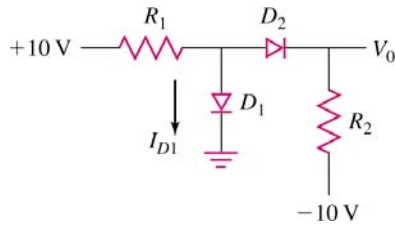
(c)  $D_1$  and  $D_2$  cutoff,  $I_{D1} = I_{D2} = 0$

$$I_{D3} = \frac{15 - 0.7 - (-10)}{R_1 + R_4} = \frac{24.3}{6.15 + 1.32} = 3.25 \text{ mA}$$

$$V_A = 15 - (3.25)(6.15) = -5 \text{ V}$$

**2.53**

a.



$$R_1 = 5 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega$$

$$D_1 \text{ and } D_2 \text{ on} \Rightarrow V_0 = 0$$

$$I_{D1} = \frac{10 - 0.7}{5} - \frac{0 - (-10)}{10} = 1.86 - 1.0$$

$$I_{D1} = 0.86 \text{ mA}$$

b.

$$R_1 = 10 \text{ k}\Omega, R_2 = 5 \text{ k}\Omega, D_1 \text{ off}, D_2 \text{ on } I_{D1} = 0$$

$$I = \frac{10 - 0.7 - (-10)}{15} = 1.287$$

$$V_0 = IR_2 - 10 \Rightarrow V_0 = -3.57 \text{ V}$$

**2.54**

If both diodes on

(a)

$$V_A = -0.7 \text{ V}, V_O = -1.4 \text{ V}$$

$$I_{R1} = \frac{10 - (-0.7)}{10} = 1.07 \text{ mA}$$

$$I_{R2} = \frac{-1.4 - (-15)}{5} = 2.72 \text{ mA}$$

$$I_{R1} + I_{D1} = I_{R2} \Rightarrow I_{D1} = 2.72 - 1.07$$

$$I_{D1} = 1.65 \text{ mA}$$

(b)  $D_1$  off,  $D_2$  on

$$I_{R1} = I_{R2} = \frac{10 - 0.7 - (-15)}{5 + 10} = 1.62 \text{ mA}$$

$$V_O = I_{R2}R_2 - 15 = (1.62)(10) - 15 \Rightarrow V_O = 1.2 \text{ V}$$

$$V_A = 1.2 + 0.7 = 1.9 \text{ V} \Rightarrow D_1 \text{ off},$$

$$I_{D1} = 0$$

**2.55**

(a)  $D_1$  on,  $D_2$  off

$$I_{D1} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$V_O = -15 \text{ V}$$

(b)  $D_1$  on,  $D_2$  off

$$I_{D1} = \frac{10 - 0.7}{5} = 1.86 \text{ mA}$$

$$V_o = -15 \text{ V}$$

---

**2.56**

$$\frac{15 - (V_o + 0.7)}{10} = \frac{V_o + 0.7}{20} + \frac{V_o}{20}$$

$$\frac{15}{10} - \frac{0.7}{10} - \frac{0.7}{20} = V_o \left( \frac{1}{10} + \frac{1}{20} + \frac{1}{20} \right) = V_o \left( \frac{4.0}{20} \right)$$

$$V_o = 6.975 \text{ V}$$

$$I_D = \frac{V_o}{20} \Rightarrow \underline{I_D = 0.349 \text{ mA}}$$

---

**2.57**

(a) Diode is cutoff,  $I_D = 0$ ,  $V_D = 0$

$$V_A = V_B = 3 \text{ V}$$

(b) Diode is conducting,  $V_D = 0.7 \text{ V}$

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.50 + 0.07 + 0.27 = V_B(0.10 + 0.10 + 0.10 + 0.10) \Rightarrow V_B = 2.1 \text{ V and } V_A = 1.4 \text{ V}$$

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + I_D$$

$$\text{So } I_D = \frac{5 - 2.1}{10} - \frac{2.1}{10} = 0.08 \text{ mA}$$

(c) Diode is cutoff,  $I_D = 0$

$$V_A = \frac{1}{2}(5) = 2.5 \text{ V}, \quad V_B = \frac{1}{2}(4) = 2.0 \text{ V}$$

$$V_D = 2 - 2.5 = -0.5 \text{ V}$$

(d) Diode is conducting,  $V_D = 0.7 \text{ V}$

$$\frac{8 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.80 + 0.07 + 0.27 = V_B(0.40) \Rightarrow V_B = 2.85 \text{ V and } V_A = 2.15 \text{ V}$$

$$\text{Then } I_D = \frac{8 - V_B}{10} - \frac{V_B}{10} = \frac{1}{10} [8 - 2(2.85)] = 0.23 \text{ mA}$$

---

**2.58**

$$v_i = 0, D_1 \text{ off}, D_2 \text{ on}$$

$$I = \frac{10 - 2.5}{15} = 0.5 \text{ mA}$$

$$v_o = 10 - (0.5)(5) \Rightarrow v_o = 7.5 \text{ V} \quad \text{for } 0 \leq v_i \leq 7.5 \text{ V}$$

For  $v_i > 7.5 \text{ V}$ , Both  $D_1$  and  $D_2$  on

$$\frac{v_i - v_o}{15} = \frac{v_o - 2.5}{10} + \frac{v_o - 10}{5} \quad \text{or} \quad v_i = v_o(5.5) - 33.75$$

When  $v_o = 10 \text{ V}$ ,  $D_2$  turns off

$$v_i = (10)(5.5) - 33.75 = 21.25 \text{ V}$$

For  $v_i > 21.25 \text{ V}$ ,  $v_o = 10 \text{ V}$

**2.59**

(a) For  $v_i = 0.5 \text{ V}$ ,  $I_{D1} = I_{D2} = I_{D3} = 0$ ,  $v_o = 0.5 \text{ V}$

(b) For  $v_i = 1.5 \text{ V}$ ,  $D_1$  on;  $I_{D2} = I_{D3} = 0$

$$I_{D1} = \frac{1.5 - 0.7}{4 + 8} = 0.0667 \text{ mA}$$

$$v_o = 0.7 + (0.0667)(8) = 1.23 \text{ V}$$

(c) For  $v_i = 3 \text{ V}$ ,  $D_1$  and  $D_2$  conducting,  $I_{D3} = 0$

$$\frac{3 - v_o}{4} = \frac{v_o - 0.7}{8} + \frac{v_o - 1.7}{6}$$

$$0.75 + 0.0875 + 0.2833 = v_o(0.25 + 0.125 + 0.1667) \Rightarrow v_o = 2.069 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.069 - 0.7}{8} = 0.171 \text{ mA}$$

$$I_{D2} = \frac{2.069 - 1.7}{6} = 0.0615 \text{ mA}$$

(d) For  $v_i = 5 \text{ V}$ , all diodes conducting

$$\frac{5 - v_o}{4} = \frac{v_o - 0.7}{8} + \frac{v_o - 1.7}{6} + \frac{v_o - 2.7}{4}$$

$$1.25 + 0.0875 + 0.2833 + 0.675 = v_o(0.25 + 0.125 + 0.1667 + 0.25)$$

$$\text{So } v_o = 2.90 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.90 - 0.7}{8} = 0.275 \text{ mA}$$

$$I_{D2} = \frac{2.90 - 1.7}{6} = 0.20 \text{ mA}$$

$$I_{D3} = \frac{2.90 - 2.7}{4} = 0.05 \text{ mA}$$

**2.60**

(a)  $I_{D2} = 0$  for  $v_i < 4.5 \text{ V}$

$$I_{D2} = 100 \text{ mA} \text{ for } v_i > 4.5 \text{ V}$$

(b)  $I_{D2} = 0$  for  $v_i < 9 \text{ V}$

$$I_{D2} = 100 \text{ mA} \text{ for } v_i > 9 \text{ V}$$



**2.61**

a.  $\underline{V_{01} = V_{02} = 0}$

b.  $\underline{V_{01} = 4.4 \text{ V}, V_{02} = 3.8 \text{ V}}$

c.  $\underline{V_{01} = 4.4 \text{ V}, V_{02} = 3.8 \text{ V}}$

Logic "1" level degrades as it goes through additional logic gates.

---

**2.62**

a.  $\underline{V_{01} = V_{02} = 5 \text{ V}}$

b.  $\underline{V_{01} = 0.6 \text{ V}, V_{02} = 1.2 \text{ V}}$

c.  $\underline{V_{01} = 0.6 \text{ V}, V_{02} = 1.2 \text{ V}}$

Logic "0" signal degrades as it goes through additional logic gates.

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**2.63**

$$(V_1 \text{ AND } V_2) \text{ OR } (V_3 \text{ AND } V_4)$$

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**2.64**

$$I = \frac{10 - 1.5 - 0.2}{R + 10} = 12 \text{ mA} = 0.012$$

$$R + 10 = \frac{8.3}{0.012} = 691.7 \Omega$$

$$\underline{R = 681.7 \Omega}$$

---

**2.65**

$$I = \frac{10 - 1.7 - V_i}{0.75} = 8$$

$$V_i = 10 - 1.7 - 8(0.75) \Rightarrow \underline{V_i = 2.3 \text{ V}}$$

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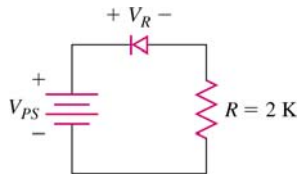
**2.66**

$$R + r_f = \frac{5 - 1.7}{15} = 0.220 \text{ k} \Omega$$

$$r_f = 20 \Omega \Rightarrow R = 200 \Omega$$

---

2.67



$$V_R = 1 \text{ V}, I = 0.8 \text{ mA}$$

$$V_{PS} = 1 + (0.8)(2)$$

$$\underline{V_{PS} = 2.6 \text{ V}}$$

---

2.68

$$I_{ph} = \eta e \Phi A$$

$$0.6 \times 10^{-3} = (1)(1.6 \times 10^{-19})(10^{17}) A$$

$$\underline{A = 3.75 \times 10^{-2} \text{ cm}^2}$$

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## Chapter 3

### 3.1

$$K_n = \frac{k'_n}{2} \cdot \frac{W}{L} = \frac{120}{2} \left( \frac{10}{0.8} \right) \Rightarrow 0.75 \text{ mA/V}^2$$

(a) (i)  $I_D = 0$

(ii)  $I_D = (0.75) \left[ 2(1-0.4)(0.1) - (0.1)^2 \right] \Rightarrow 82.5 \mu\text{A}$

(iii)  $I_D = (0.75) \left[ 2(2-0.4)(0.1) - (0.1)^2 \right] = 0.2325 \text{ mA}$

(iv)  $I_D = (0.75) \left[ 2(3-0.4)(0.1) - (0.1)^2 \right] = 0.3825 \text{ mA}$

(b) (i)  $I_D = 0$

(ii)  $I_D = (0.75)(1-0.4)^2 = 0.27 \text{ mA}$

(iii)  $I_D = (0.75)(2-0.4)^2 = 1.92 \text{ mA}$

(iv)  $I_D = (0.75)(3-0.4)^2 = 5.07 \text{ mA}$

### 3.2

$$I_D = K_n \left[ 2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2 \right]$$

$$0.5 = K_n \left[ 2(0.6)V_{DS} - V_{DS}^2 \right]$$

$$1.0 = K_n \left[ 2(1.0)V_{DS} - V_{DS}^2 \right]$$

Take ratio

$$0.5 = \frac{1.2V_{DS} - V_{DS}^2}{2V_{DS} - V_{DS}^2} \Rightarrow V_{DS} - 0.5V_{DS}^2 = 1.2V_{DS} - V_{DS}^2$$

or  $1 - 0.5V_{DS} = 1.2 - V_{DS}$

which yields  $V_{DS} = 0.4 \text{ V}$

Then  $0.5 = K_n \left[ (1.2)(0.4) - (0.4)^2 \right] \Rightarrow K_n = 1.56 \text{ mA/V}^2$

### 3.3

(a) Enhancement-mode

(b) From Graph  $V_T = 1.5 \text{ V}$

Now

$$0.03 = K_n (2 - 1.5)^2 = 0.25 K_n \Rightarrow K_n = 0.12$$

$$0.15 = K_n (3 - 1.5)^2 = 2.25 K_n \quad K_n = 0.0666$$

$$0.39 = K_n (4 - 1.5)^2 = 6.25 K_n \quad K_n = 0.0624$$

$$0.77 = K_n (5 - 1.5)^2 = 12.25 K_n \quad K_n = 0.0629$$

From last three,  $K_n (\text{Avg}) = 0.0640 \text{ mA/V}^2$

$$i_D (\text{sat}) = 0.0640(3.5 - 1.5)^2 \Rightarrow i_D (\text{sat}) = 0.256 \text{ mA for } V_{GS} = 3.5 \text{ V}$$

(c)  $i_D (\text{sat}) = 0.0640(4.5 - 1.5)^2 \Rightarrow i_D (\text{sat}) = 0.576 \text{ mA for } V_{GS} = 4.5 \text{ V}$

**3.4**

- $V_{GS} = 0$
- a.  $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 0 - (-2.5) = 2.5 \text{ V}$   
 $V_{DS} = 0.5 \text{ V} \Rightarrow$  Biased in nonsaturation  
 i.  $I_D = (1.1) \left[ 2(0 - (-2.5))(0.5) - (0.5)^2 \right] \Rightarrow \underline{I_D = 2.48 \text{ mA}}$   
 $V_{DS} = 2.5 \text{ V} \Rightarrow$  Biased in saturation  
 ii.  $I_D = (1.1)(0 - (-2.5))^2 \Rightarrow \underline{I_D = 6.88 \text{ mA}}$   
 iii.  $V_{DS} = 5 \text{ V}$  Same as (ii)  $\Rightarrow \underline{I_D = 6.88 \text{ mA}}$
- b.  $V_{GS} = 2 \text{ V}$   
 $V_{DS}(\text{sat}) = 2 - (-2.5) = 4.5 \text{ V}$   
 $V_{DS} = 0.5 \text{ V} \Rightarrow$  Nonsaturation  
 i.  $I_D = (1.1) \left[ 2(2 - (-2.5))(0.5) - (0.5)^2 \right] \Rightarrow \underline{I_D = 4.68 \text{ mA}}$   
 $V_{DS} = 2.5 \text{ V} \Rightarrow$  Nonsaturation  
 ii.  $I_D = (1.1) \left[ 2(2 - (-2.5))(2.5) - (2.5)^2 \right] \Rightarrow \underline{I_D = 17.9 \text{ mA}}$   
 $V_{DS} = 5 \text{ V} \Rightarrow$  Saturation  
 iii.  $I_D = (1.1)(2 - (-2.5))^2 \Rightarrow \underline{I_D = 22.3 \text{ mA}}$

**3.5**

- (a)  $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 2.2 - 0.4 = 1.8 \text{ V}$   
 $2.2 = V_{DS} > V_{DS}(\text{sat}) = 1.8 \Rightarrow$  Saturation
- (b)  $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1 - 0.4 = 0.6 \text{ V}$   
 $V_{DS} = -0.6 - (-1) = 0.4 \text{ V} < V_{DS}(\text{sat}) = 0.6 \text{ V} \Rightarrow$  Nonsaturation
- (c)  $V_{GS} = 1 - 1 = 0 \Rightarrow$  Cutoff

**3.6**

- (a)  $V_{SG} = 2.2 - 2.2 = 0 \Rightarrow$  Cutoff
- (b)  $V_{SG} = 2 \text{ V}$ ,  $V_{SD} = 2 - (-1) = 3 \text{ V}$   
 $V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6 \text{ V}$   
 So  $V_{SD} = 3 > V_{SD}(\text{sat}) = 1.6 \Rightarrow$  Saturation
- (c)  $V_{SG} = 2 \text{ V}$ ,  $V_{SD} = 2 - 1 = 1 \text{ V}$   
 $V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6 \text{ V}$   
 So  $V_{SD} = 1 < V_{SD}(\text{sat}) = 1.6 \Rightarrow$  Nonsaturation

**3.7**

$$I_D = \frac{k'_n}{2} \left( \frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$0.5 = \frac{0.12}{2} \left( \frac{W}{L} \right) [0 - (-1.2)]^2 \Rightarrow \left( \frac{W}{L} \right) = 5.79$$

**3.8**

$$k'_n = \mu_n C_{ox} = \frac{\mu_n \epsilon_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85 \times 10^{-14})}{t_{ox}} = \frac{2.071 \times 10^{-10}}{t_{ox}}$$

- (a) 500 A  $k'_n = 41.4 \mu\text{A}/\text{V}^2$   
 (b) 250  $k'_n = 82.8 \mu\text{A}/\text{V}^2$   
 (c) 100  $k'_n = 207 \mu\text{A}/\text{V}^2$   
 (d) 50  $k'_n = 414 \mu\text{A}/\text{V}^2$   
 (e) 25  $k'_n = 828 \mu\text{A}/\text{V}^2$

**3.9**

(a)  $K_n = \frac{W\mu_n \epsilon_{ox}}{2Lt_{ox}} = \frac{(20 \times 10^{-4})(650)(3.9)(8.85 \times 10^{-14})}{2(0.8 \times 10^{-4})(200 \times 10^{-8})} = 1.40 \text{ mA}/\text{V}^2$

(b)  $I_D = K_n (V_{GS} - V_{TN})^2 = (1.40)(2 - 0.4)^2$

Or  $I_D = 3.58 \text{ mA}$

(c)  $V_{DS}(sat) = V_{GS} - V_{TN} = 2 - 0.4 = 1.6 \text{ V}$

**3.10**

(a)  $I_D = \frac{k'_n}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{TN})^2$

$$0.6 = \left(\frac{0.12}{2}\right) \left(\frac{W}{L}\right) (1.4 - 0.8)^2 \Rightarrow \left(\frac{W}{L}\right) = 27.8$$

Or  $W = (27.8)(0.8) = 22.2 \mu\text{m}$

(b)  $I_D = \left(\frac{0.12}{2}\right) (27.8) [2(1.4 - 0.8)(0.4) - (0.4)^2] = 0.534 \text{ mA}$

(c)  $V_{DS}(sat) = V_{GS} - V_{TN} = 1.4 - 0.8 = 0.6 \text{ V}$

**3.11**

$$k'_n = \mu_n C_{ox} = \frac{\mu_n \epsilon_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$$

$$k'_n = 0.1035 \text{ mA}/\text{V}^2$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{TN})^2$$

$$1.2 = \left(\frac{0.1035}{2}\right) \left(\frac{W}{L}\right) (3 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right) = 4.026$$

Then  $W = (4.026)(0.8) = 3.22 \mu\text{m}$

**3.12**

$$I_D = WC_{ox} (V_{GS} - V_{TN}) \mu_{sat}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}} = 1.726 \times 10^{-7} \text{ F/cm}$$

$$I_D = (3.22 \times 10^{-4})(1.726 \times 10^{-7})(3 - 0.6)(2 \times 10^7)$$

$$I_D = 2.67 \text{ mA}$$

**3.13**

$$I_D = \frac{k'_p}{2} \left( \frac{W}{L} \right) (V_{SG} + V_{TP})^2$$

$$0.225 = \left( \frac{0.05}{2} \right) \left( \frac{W}{L} \right) (2 + V_{TP})^2$$

$$0.65 = \left( \frac{0.05}{2} \right) \left( \frac{W}{L} \right) (3 + V_{TP})^2$$

Then  $\sqrt{\frac{0.65}{0.225}} = \frac{3 + V_{TP}}{2 + V_{TP}} = 1.70 \Rightarrow V_{TP} = -0.571 \text{ V}$

And  $0.225 = \left( \frac{0.05}{2} \right) \left( \frac{W}{L} \right) (2 - 0.571)^2 \Rightarrow \left( \frac{W}{L} \right) = 4.41$

**3.14**

$V_S = 5 \text{ V}, V_G = 0 \Rightarrow V_{SG} = 5 \text{ V}$

$V_{TP} = -0.5 \text{ V} \Rightarrow V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 5 - 0.5 = 4.5 \text{ V}$

$V_D = 0 \Rightarrow V_{SD} = 5 \text{ V} \Rightarrow$  Biased in saturation

a.  $I_D = 2(5 - 0.5)^2 \Rightarrow \underline{I_D = 40.5 \text{ mA}}$

$V_D = 2 \text{ V} \Rightarrow V_{SD} = 3 \text{ V} \Rightarrow$  Nonsaturation

b.  $I_D = 2[2(5 - 0.5)(3) - (3)^2] \Rightarrow \underline{I_D = 36 \text{ mA}}$

$V_D = 4 \text{ V} \Rightarrow V_{SD} = 1 \text{ V} \Rightarrow$  Nonsaturation

c.  $I_D = 2[2(5 - 0.5)(1) - (1)^2] \Rightarrow \underline{I_D = 16 \text{ mA}}$

d.  $V_D = 5 \text{ V} \Rightarrow V_{SD} = 0 \Rightarrow \underline{I_D = 0}$

**3.15**

(a) Enhancement-mode

(b) From Graph  $V_{TP} = +0.5 \text{ V}$

$0.45 = k_p (2 - 0.5)^2 = 2.25 K_p \Rightarrow$	$K_p = 0.20$
$1.25 = k_p (3 - 0.5)^2 = 6.25 K_p$	0.20
$2.45 = k_p (4 - 0.5)^2 = 12.25 K_p$	0.20
$4.10 = k_p (5 - 0.5)^2 = 20.25 K_p$	0.202
<b>Avg <math>K_p = 0.20 \text{ mA/V}^2</math></b>	

$$i_D(\text{sat}) = 0.20 (3.5 - 0.5)^2 = 1.8 \text{ mA}$$

$$(c) \quad i_D(\text{sat}) = 0.20 (4.5 - 0.5)^2 = 3.2 \text{ mA}$$

**3.16**

$$V_{SD}(\text{sat}) = V_{SG} + V_{TP}$$

$$(a) \quad V_{SD}(\text{sat}) = -1 + 2 \Rightarrow V_{SD}(\text{sat}) = 1 \text{ V}$$

$$(b) \quad V_{SD}(\text{sat}) = 0 + 2 \Rightarrow V_{SD}(\text{sat}) = 2 \text{ V}$$

$$(c) \quad V_{SD}(\text{sat}) = 1 + 2 \Rightarrow V_{SD}(\text{sat}) = 3 \text{ V}$$

$$I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2 = \frac{k'_p}{2} \cdot \frac{W}{L} \cdot [V_{SD}(\text{sat})]^2$$

$$(a) \quad I_D = \left(\frac{0.040}{2}\right)(6)(1)^2 \Rightarrow \underline{I_D = 0.12 \text{ mA}}$$

$$(b) \quad I_D = \left(\frac{0.040}{2}\right)(6)(2)^2 \Rightarrow \underline{I_D = 0.48 \text{ mA}}$$

$$(c) \quad I_D = \left(\frac{0.040}{2}\right)(6)(3)^2 \Rightarrow \underline{I_D = 1.08 \text{ mA}}$$

**3.17**

$$K_p = \frac{k'_p}{2} \left(\frac{W}{L}\right) = \frac{50}{2} \left(\frac{12}{0.8}\right) \Rightarrow K_p = 0.375 \text{ mA/V}^2$$

$$(a) \text{ Nonsaturation}$$

$$I_D = (0.375)[2(2 - 0.5)(0.2) - (0.2)^2] = 0.21 \text{ mA}$$

$$(b) \text{ Nonsaturation}$$

$$I_D = (0.375)[2(2 - 0.5)(0.8) - (0.8)^2] = 0.66 \text{ mA}$$

$$(c) \text{ Nonsaturation}$$

$$I_D = (0.375)[2(2 - 0.5)(1.2) - (1.2)^2] = 0.81 \text{ mA}$$

$$(d) \text{ Saturation}$$

$$I_D = (0.375)(2 - 0.5)^2 = 0.844 \text{ mA}$$

$$(e) \text{ Saturation}$$

$$I_D = (0.375)(2 - 0.5)^2 = 0.844 \text{ mA}$$

**3.18**

$$k'_p = \mu_p C_{ox} = \frac{\mu_p \epsilon_{ox}}{t_{ox}} = \frac{(250)(3.9)(8.85 \times 10^{-14})}{t_{ox}} = \frac{8.629 \times 10^{-11}}{t_{ox}}$$

$$(a) \quad t_{ox} = 500 \text{ \AA} \Rightarrow k'_p = 17.3 \text{ } \mu\text{A/V}^2$$

$$(b) \quad 250 \text{ \AA} \Rightarrow k'_p = 34.5 \text{ } \mu\text{A/V}^2$$

$$(c) \quad 100 \text{ \AA} \Rightarrow k'_p = 86.3 \text{ } \mu\text{A/V}^2$$

$$(d) \quad 50 \text{ \AA} \Rightarrow k'_p = 173 \text{ } \mu\text{A/V}^2$$

$$(e) \quad 25 \text{ \AA} \Rightarrow k'_p = 345 \text{ } \mu\text{A/V}^2$$

**3.19**

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}} = 6.90 \times 10^{-8} \text{ F/cm}^2$$

$$k'_n = (\mu_n C_{ox}) = (675)(6.90 \times 10^{-8}) \Rightarrow 46.6 \mu\text{A/V}^2$$

$$k'_p = (\mu_p C_{ox}) = (375)(6.90 \times 10^{-8}) \Rightarrow 25.9 \mu\text{A/V}^2$$

PMOS:

$$I_D = \frac{k'_p}{2} \left( \frac{W}{L} \right)_p (V_{SG} + V_{TP})^2$$

$$0.8 = \left( \frac{0.0259}{2} \right) \left( \frac{W}{L} \right)_p (5 - 0.6)^2 \Rightarrow \left( \frac{W}{L} \right)_p = 3.19$$

$$L = 4 \mu\text{m} \Rightarrow \underline{W_p = 12.8 \mu\text{m}}$$

$$K_p = \left( \frac{0.0259}{2} \right) (3.19) \Rightarrow \underline{K_p = 41.3 \mu\text{A/V}^2 = K_n}$$

Want  $K_n = K_p$

$$\frac{k'_n}{2} \left( \frac{W}{L} \right)_n = \frac{k'_p}{2} \left( \frac{W}{L} \right)_p = 41.3$$

$$\left( \frac{46.6}{2} \right) \left( \frac{W}{L} \right)_n = 41.3 \Rightarrow \left( \frac{W}{L} \right)_n = 1.77$$

$$L = 4 \mu\text{m} \Rightarrow \underline{W_n = 7.09 \mu\text{m}}$$

**3.20**

$$V_{GS} = 2 \text{ V}, \quad I_D = (0.2)(2 - 1.2)^2 = 0.128 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.128)} \Rightarrow \underline{r_o = 781 \text{ k}\Omega}$$

$$V_{GS} = 4 \text{ V}, \quad I_D = (0.2)(4 - 1.2)^2 = 1.57 \text{ mA}$$

$$r_o = \frac{1}{(0.01)(1.57)} \Rightarrow \underline{r_o = 63.7 \text{ k}\Omega}$$

$$V_A = \frac{1}{\lambda} = \frac{1}{(0.01)} \Rightarrow \underline{V_A = 100 \text{ V}}$$

**3.21**

$$I_D = \frac{k'_n}{2} \left( \frac{W}{L} \right) (V_{GS} - V_{TN})^2 = \left( \frac{0.12}{2} \right) (4)(2 - 0.5)^2 = 0.54 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_{DQ}} \Rightarrow \lambda = \frac{1}{r_o I_{DQ}} = \frac{1}{(200 \times 10^3)(0.54 \times 10^{-3})} = 0.00926 \text{ V}^{-1}$$

$$\text{Then } V_A = \frac{1}{\lambda} = 108 \text{ V}$$



**3.22**

$$V_{TN} = V_{TNO} + \gamma \left[ \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$\Delta V_{TN} = 2 = (0.8) \left[ \sqrt{2\phi_f + V_{SB}} - \sqrt{2(0.35)} \right]$$

$$2.5 + 0.837 = \sqrt{2(0.35) + V_{SB}} \Rightarrow \underline{V_{SB} = 10.4 \text{ V}}$$

**3.23**

$$V_{TN} = V_{TNo} + r \left[ \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$= 0.75 + 0.6 \left[ \sqrt{2(0.37) + 3} - \sqrt{2(0.37)} \right]$$

$$= 0.75 + 0.6 [1.934 - 0.860]$$

$$V_{TN} = 1.39 \text{ V}$$

$$V_{DS}(\text{sat}) = 2.5 - 1.39 = 1.11 \text{ V}$$

Sat Region  $I_D = (15) \left( \frac{0.08}{2} \right) (2.5 - 1.39)^2$

(a)  $\underline{I_D = 0.739 \text{ mA}}$

Non-Sat  $I_D = (15) \left( \frac{0.08}{2} \right) \left[ 2(2.5 - 1.39)(0.25) - (0.25)^2 \right]$

(b)  $\underline{I_D = 0.296 \text{ mA}}$

**3.24**

$$E_{ox} = 6 \times 10^6 \text{ V/cm}$$

(a) (i)  $V_G = E_{ox} t_{ox} = (6 \times 10^6) (120 \times 10^{-8}) = 7.2 \text{ V}$

(ii)  $V_G = \frac{7.2}{3} = 2.4 \text{ V}$

(b) (i)  $V_G = (6 \times 10^6) (200 \times 10^{-8}) = 12 \text{ V}$

(ii)  $V_G = \frac{12}{3} = 4 \text{ V}$

**3.25**

Want (3)(24) =  $E_{ox} t_{ox} = (6 \times 10^6) t_{ox}$

$$\underline{t_{ox} = 1.2 \times 10^{-5} \text{ cm} = 1200 \text{ Angstroms}}$$

**3.26**

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{18}{18 + 32} \right) (10) = 3.6 \text{ V}$$

Assume transistor biased in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$3.6 - V_{GS} = (0.5)(2)(V_{GS} - 0.8)^2$$

$$= V_{GS}^2 - 1.6V_{GS} + 0.64$$

$$V_{GS}^2 - 0.6V_{GS} - 2.96 = 0$$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(2.96)}}{2} \Rightarrow \underline{V_{GS} = 2.046 \text{ V}}$$

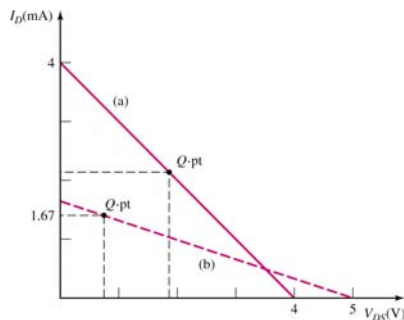
$$I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.046}{2} \Rightarrow \underline{I_D = 0.777 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D (R_D + R_S)$$

$$= 10 - (0.777)(4 + 2) \Rightarrow \underline{V_{DS} = 5.34 \text{ V}}$$

$$\underline{V_{DS} > V_{DS}(\text{sat})}$$

**3.27**



(a)  $V_{GS} = 4 \text{ V}$     $V_{DS}(\text{sat}) = 4 - 0.8 = 3.2 \text{ V}$

If Sat  $I_D = 0.25(4 - 0.8)^2 = 2.56$

$V_{DS} = 1.44$     $\times$

Non-Sat

$$4 = I_D R_D + V_{DS} = K_n R_D [2(V_{GS} - V_T)V_{DS} - V_{DS}^2] + V_{DS}$$

$$4 = (0.25)(1) [2(4 - 0.8)V_{DS} - V_{DS}^2] + V_{DS}$$

$$4 = 2.6V_{DS} - 0.25V_{DS}^2$$

$$0.25V_{DS}^2 - 2.6V_{DS} + 4 = 0$$

$$V_{DS} = \frac{2.6 \pm \sqrt{6.76 - 4}}{2(0.25)} = 1.88 \text{ V}$$

$$I_D = \frac{4 - 1.88}{1} = 2.12 \text{ mA}$$

(b) Non-Sat region

$$5 = I_D R_D + V_{DS} = K_n R_D [2(V_{GS} - V_T)V_{DS} - V_{DS}^2] + V_{DS}$$

$$5 = (0.25)(3)[2(5 - 0.8)V_{DS} - V_{DS}^2] + V_{DS}$$

$$5 = 7.3V_{DS} - 0.75V_{DS}^2$$

$$0.75 V_{DS}^2 - 7.3V_{DS} + 5 = 0$$

$$V_{DS} = \frac{7.3 \pm \sqrt{53.29 - 15}}{2(0.75)}$$

$$V_{DS} = 0.741 \text{ V}$$

$$I_D = \frac{5 - 0.741}{3} = 1.42 \text{ mA}$$

3.28

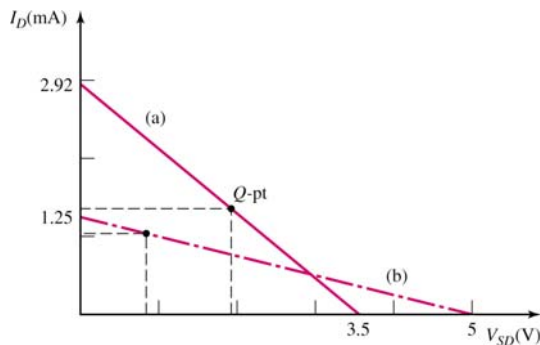
$$0.8 = \left(\frac{0.12}{2}\right)(80)(V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.808 \text{ V}$$

$$V_{GS} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$0.80825 = \frac{1}{R_1}(200)(1.8) \Rightarrow R_1 = 445 \text{ k}\Omega$$

$$R_1 \parallel R_2 = R_{in} = 200 \text{ k}\Omega \Rightarrow R_2 = 363 \text{ k}\Omega$$

3.29



(a)  $V_{SG} = V_{DD} = 3.5$      $V_{SD}(\text{sat}) = 3.5 - 0.8 = 2.7 \text{ V}$

If biased in Sat region,  $I_D = (0.2)(3.5 - 0.8)^2$   
 $= 1.46 \text{ mA}$

$V_{SD} = 3.5 - (1.46)(1.2) = 1.75 \text{ V} \quad \times$

Biased in Non-Sat Region.

$$3.5 = V_{SD} + I_D R_D = V_{SD} + K_p R_D [2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2]$$

$$3.5 = V_{SD} + (0.2)(1.2)[2(3.5 - 0.8)V_{SD} - V_{SD}^2]$$

$$3.5 = V_{SD} + 1.296 V_{SD} - 0.24 V_{SD}^2$$

$$0.24 V_{SD}^2 - 2.296 V_{SD} + 3.5 = 0$$

$$V_{SD} = \frac{+2.296 \pm \sqrt{5.272 - 3.36}}{2(0.24)} \text{ use - sign } \underline{V_{SD} = 1.90 \text{ V}}$$

$$I_D = (0.2)[2(3.5 - 0.8)(1.9) - (1.9)^2] = 0.2[10.26 - 3.61]$$

$$I_D = \frac{3.5 - 1.90}{1.2} = 1.33 \text{ mA}$$

$$\underline{I_D = 1.33 \text{ mA}}$$

(b)  $V_{SG} = V_{DD} = 5 \text{ V}$   $V_{SD}(\text{sat}) = 5 - 0.8 = 4.2 \text{ V}$

If Sat Region  $I_D = (0.2)(5 - 0.8)^2 = 3.53 \text{ mA}$ ,  $V_{SD} < 0$

Non-Sat Region.

$$5 = V_{SD} + K_p R_D [2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2]$$

$$5 = V_{SD} + (0.2)(4)[2(5 - 0.8)V_{SD} - V_{SD}^2]$$

$$5 = V_{SD} + 6.72 V_{SD} - 0.8 V_{SD}^2$$

$$0.8 V_{SD}^2 - 7.72 V_{SD} + 5 = 0$$

$$V_{SD} = \frac{7.72 \pm \sqrt{59.598 - 16}}{2(0.8)} \text{ use - sign } \underline{V_{SD} = 0.698 \text{ V}}$$

$$I_D = \frac{5 - 0.698}{4} \Rightarrow \underline{I_D = 1.08 \text{ mA}}$$

### 3.30

$$V_G = \left(\frac{22}{22+8}\right)(6) - 3 = 1.40 \text{ V}$$

$$3 = K_p R_S (V_{SG} + V_{TP})^2 + V_{SG} + V_G$$

$$3 = (0.5)(0.5)(V_{SG}^2 - 1.6V_{SG} + 0.64) + V_{SG} + 1.40$$

$$0.25V_{SG}^2 + 0.6V_{SG} - 1.44 = 0 \Rightarrow V_{SG} = 1.483 \text{ V}$$

$$I_D = (0.5)(1.483 - 0.8)^2 = 0.2332 \text{ mA}$$

$$V_{SD} = 6 - (0.2332)(0.5 + 5) = 4.72 \text{ V}$$

**3.31**

$$V_G = 0, \quad V_{SG} = V_S$$

Assume saturation region

$$I_D = 0.4 = K_p (V_{SG} + V_{TP})^2$$

$$0.4 = (0.2)(V_S - 0.8)^2$$

$$V_S = \sqrt{\frac{0.4}{0.2}} + 0.8 \Rightarrow \underline{V_S = 2.21 \text{ V}}$$

$$V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3 \text{ V}$$

$$V_{SD} = V_S - V_D = 2.21 - (-3) \Rightarrow \underline{V_{SD} = 5.21 \text{ V}}$$

$$\underline{V_{SD} > V_{SD}(\text{sat})}$$

**3.32**

$$I_D = \frac{V_{DD} - V_{DSQ} - V_{RS}}{R_D} = \frac{3.3 - 1.6 - 0.8}{5} = 0.18 \text{ mA}$$

$$0.18 = \left(\frac{0.12}{2}\right) \left(\frac{W}{L}\right) (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right) = 18.75$$

$$R_S = \frac{0.8}{0.18} = 4.44 \text{ k}\Omega$$

$$V_G = 0.8 + 0.8 = 1.6 = \left(\frac{R_2}{R_1 + R_2}\right) (3.3) \quad (3.3)$$

$$\text{Now } I_R = (0.05)(0.18) \Rightarrow 9 \mu\text{A}$$

$$\text{So } R_1 + R_2 = \frac{3.3}{0.009} = 367 \text{ k}\Omega$$

$$\text{Then } 1.6 = \left(\frac{R_2}{367}\right) (3.3) \Rightarrow R_2 = 178 \text{ k}\Omega \quad \text{and } R_1 = 189 \text{ k}\Omega$$

**3.33**

$$I_D = \frac{0.2}{1} = 0.2 \text{ mA}$$

$$V_{DS} = 1.8 - (0.2)(4 + 1) = 0.8 \text{ V}$$

$$\text{Now } V_{DS}(\text{sat}) = 0.8 - 0.4 = 0.4 \text{ V}$$

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN} \Rightarrow 0.4 = V_{GS} - 0.4 \Rightarrow V_{GS} = 0.8 \text{ V}$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{TN})^2$$

$$0.2 = \left(\frac{0.12}{2}\right) \left(\frac{W}{L}\right) (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right) = 20.8$$

$$\text{Now } V_G = V_{GS} + I_D R_S = 0.8 + (0.2)(1) = 1.0 \text{ V}$$

$$V_G = 1 = \frac{1}{R_1} \cdot R_m \cdot V_{DD} = \frac{1}{R_1} (200)(1.8) \Rightarrow R_1 = 360 \text{ k}\Omega$$

$$R_1 \parallel R_2 = R_m = 200 \text{ k}\Omega \Rightarrow R_2 = 450 \text{ k}\Omega$$

**3.34**

$$(a) \quad 0.35 = \left(\frac{0.12}{2}\right)(50)(V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.742 \text{ V}$$

$$(b) \quad V_{DS} = 1.8 - (0.35)(2) = 1.1 \text{ V}$$

$$V_{DS}(sat) = V_{GS} - V_{TN} = 0.742 - 0.4 = 0.342 \text{ V}$$

$$V_{DS} > V_{DS}(sat) \Rightarrow \text{Saturation}$$

**3.35**

$$V_G = \left(\frac{6}{6+14}\right)(10) - 5 = -2 \text{ V}$$

$$V_G = V_{GS} + I_D R_S - 5 = V_{GS} + \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_S (V_{GS} - V_{TN})^2 - 5$$

$$5 - 2 = V_{GS} + \left(\frac{0.12}{2}\right)(25)(0.5)(V_{GS}^2 - 0.8V_{GS} + 0.16)$$

$$\text{Or } 0.75V_{GS}^2 + 0.4V_{GS} - 2.88 = 0 \Rightarrow V_{GS} = 1.71 \text{ V}$$

$$I_D = \left(\frac{0.12}{2}\right)(25)(1.71 - 0.4)^2 = 2.58 \text{ mA}$$

$$V_{DS} = 10 - (2.58)(1.2 + 0.5) = 5.62 \text{ V}$$

**3.36**

$$\text{Let } \left(\frac{W}{L}\right) = 20 \text{ for example, then}$$

$$0.20 = \left(\frac{0.05}{2}\right)(20)(V_{SG} - 0.6)^2 \Rightarrow V_{SG} = 1.232 \text{ V}$$

$$\text{Then } V_{RS} = 1.232 = (0.2)R_S \Rightarrow R_S = 6.16 \text{ k}\Omega$$

$$R_D = \frac{6 - 1.232 - 3}{0.2} = 8.84 \text{ k}\Omega$$

$$V_G = 3 - 1.232 - 1.232 = 0.536 \text{ V}$$

$$I_R = \frac{6}{R_1 + R_2} = (0.1)(0.2) = 0.02 \text{ mA}, \Rightarrow R_1 + R_2 = 300 \text{ k}\Omega$$

$$V_G = 0.536 = \left(\frac{R_2}{R_1 + R_2}\right)(6) - 3 = \left(\frac{R_2}{300}\right)(6) - 3$$

$$\text{Or } R_2 = 176.8 \text{ k}\Omega \text{ and } R_1 = 123.2 \text{ k}\Omega$$

**3.37**

$$I_Q = 50 = 500(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 1.516 \text{ V}}$$

$$(a) \quad (i) \quad V_{DS} = 5 - (-1.516) \Rightarrow \underline{V_{DS} = 6.516 \text{ V}}$$

$$I_Q = 1 = (0.5)(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 2.61 \text{ V}}$$

$$(ii) \quad V_{DS} = 5 - (-2.61) \Rightarrow \underline{V_{DS} = 7.61 \text{ V}}$$

- (b) (i) Same as (a)  $V_{GS} = V_{DS} = 1.516 \text{ V}$   
 (ii)  $V_{GS} = V_{DS} = 2.61 \text{ V}$

**3.38**

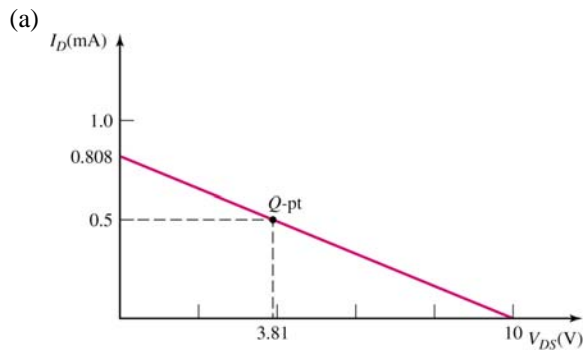
$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$0.25 = (0.2)(V_{GS} - 0.6)^2$$

$$V_{GS} = \sqrt{\frac{0.25}{0.2}} + 0.6 \Rightarrow V_{GS} = 1.72 \text{ V} \Rightarrow \underline{V_S = -1.72 \text{ V}}$$

$$V_D = 9 - (0.25)(24) \Rightarrow \underline{V_D = 3 \text{ V}}$$

**3.39**



$$R_D = \frac{5 - 1}{0.5} \Rightarrow \underline{R_D = 8 \text{ K}}$$

$$I_{DQ} = 0.5 = 0.25(V_{GS} - 1.4)^2 \Rightarrow V_{GS} = 2.81 \text{ V}$$

$$R_S = \frac{-2.81 - (-5)}{0.5} \Rightarrow \underline{R_S = 4.38 \text{ K}}$$

- (b) Let  $R_D = 8.2 \text{ K}$ ,  $R_S = 4.3 \text{ K}$   
 Now  $\frac{-V_{GS} - (-5)}{4.3} = I_D = 0.25(V_{GS} - 1.4)^2$   
 $5 - V_{GS} = 1.075(V_{GS}^2 - 2.8 V_{GS} + 1.96)$   
 $1.075 V_{GS}^2 - 2.01 V_{GS} - 2.89 = 0$   
 $V_{GS} = \frac{2.01 \pm \sqrt{4.04 + 12.427}}{2(1.075)} \Rightarrow V_{GS} = 2.82 \text{ V}$   
 $I_D = 0.25(2.82 - 1.4)^2 \Rightarrow I_D = 0.504 \text{ mA}$   
 $V_{DS} = 10 - (0.504)(8.2 + 4.3) \Rightarrow \underline{V_{DS} = 3.70 \text{ V}}$

(c) If  $R_s = 4.3 + 10\% = 4.73 \text{ K}$   
 $5 - V_{GS} = 1.18(V_{GS}^2 - 2.8V_{GS} + 1.96)$   
 $1.18 V_{GS}^2 - 2.31 V_{GS} - 2.68 = 0$   
 $V_{GS} = \frac{2.31 \pm \sqrt{5.336 + 12.65}}{2(1.18)} = 2.78 \text{ V}$   
 $I_D = (0.25)(2.78 - 1.4)^2 \Rightarrow I_D = 0.476 \text{ mA}$   
 If  $R_s = 4.3 - 10\% = 3.87 \text{ K}$   
 $5 - V_{GS} = (0.9675)(V_{GS}^2 - 2.8V_{GS} + 1.96)$   
 $0.9675V_{GS}^2 - 1.71V_{GS} - 3.10 = 0$   
 $V_{GS} = \frac{1.71 \pm \sqrt{2.924 + 12.0}}{2(0.9675)} = 2.88 \text{ V}$   
 $I_D = (0.25)(2.88 - 1.4)^2 = 0.548 \text{ mA}$

**3.40**

$$I_D = 0.10 = \frac{9 - 2.5}{R} \Rightarrow R = 65 \text{ k}\Omega$$

$$I_D = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

Now  $V_{SG} = V_{SD} = 2.5 \text{ V}$

$$0.10 = \left(\frac{0.05}{2}\right)\left(\frac{W}{L}\right)(2.5 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right) = 1.235$$

And  $W = (1.235)(0.8) = 0.988 \mu\text{m}$

**3.41**

$$5 = I_{DQ}R_s + V_{SDQ} = I_{DQ}(2) + 2.5$$

$$I_{DQ} = 1.25 \text{ mA}$$

$$I_R = \frac{10}{R_1 + R_2} = (1.25)(0.1) \Rightarrow R_1 + R_2 = 80 \text{ k}\Omega$$

$$I_{DQ} = K_p (V_{SG} + V_{TP})^2$$

$$1.25 = 0.5(V_{SG} + 1.5)^2 \Rightarrow \sqrt{\frac{1.25}{0.5}} - 1.5 = V_{SG}$$

$$V_{SG} = 0.0811 \text{ V}$$

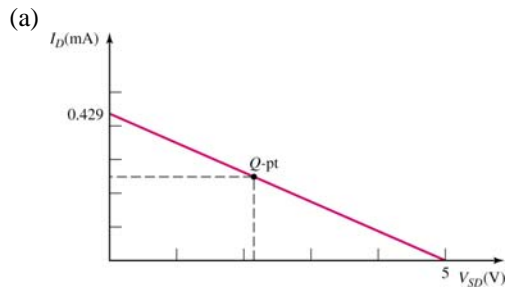
$$V_G = V_s - V_{SG} = 2.5 - 0.0811 = 2.42 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$

$$2.42 = \left(\frac{R_2}{80}\right)(10) - 5 \Rightarrow R_2 = 59.4 \text{ k}\Omega, \quad R_1 = 20.6 \text{ k}\Omega$$



3.42



$$R_D = \frac{V_D - (-5)}{I_{DQ}} = \frac{5 - 2}{0.25} \Rightarrow R_D = 12 \text{ K}$$

$$I_D = \left(\frac{W}{L}\right) \left(\frac{k'_p}{2}\right) (V_{SG} + V_{TP})^2$$

$$0.25 = (15) \left(\frac{0.035}{2}\right) (V_{SG} - 1.2)^2 \Rightarrow V_{SG} = 2.18 \text{ V}$$

$$R_S = \frac{5 - 2.18}{0.25} \Rightarrow R_S = 11.3 \text{ K}$$

$$V_{SD} = 2.18 - (-2) = 4.18 \text{ V}$$

(b)

$$k'_p = 35 + 5\% = 36.75 \mu\text{A}/\text{V}^2$$

$$I_D = (15) \left(\frac{0.03675}{2}\right) (V_{SG} - 1.2)^2 = \frac{5 - V_{SG}}{11.3}$$

$$3.11(V_{SG}^2 - 2.4V_{SG} + 1.44) = 5 - V_{SG}$$

$$3.11V_{SG}^2 - 6.46V_{SG} - 0.522 = 0$$

$$V_{SG} = \frac{6.46 \pm \sqrt{41.73 + 6.49}}{2(3.11)} = 2.155 \text{ V}$$

$$I_D = \frac{5 - 2.155}{11.3} = 0.252 \text{ mA}$$

$$V_{SD} = 10 - (0.252)(12 + 11.3) = 4.13 \text{ V}$$

$$k'_p = 35 - 5\% = 33.25 \mu\text{A}/\text{V}^2$$

$$I_D = (15) \left(\frac{0.03325}{2}\right) (V_{SG} - 1.2)^2 = \frac{5 - V_{SG}}{11.3}$$

$$2.82(V_{SG}^2 - 2.4V_{SG} + 1.44) = 5 - V_{SG}$$

$$2.82V_{SG}^2 - 5.77V_{SG} - 0.939 = 0$$

$$V_{SG} = \frac{5.77 \pm \sqrt{33.29 + 10.59}}{2(2.82)} = 2.198 \text{ V}$$

$$I_D = \frac{5 - 2.198}{11.3} = 0.248 \text{ mA}$$

$$V_{SD} = 10 - (0.248)(12 + 11.3) = 4.22 \text{ V}$$

**3.43**

$$I_D = \frac{-V_{SD} - (-10)}{R_D} \Rightarrow 5 = \frac{-6 + 10}{R_D} \Rightarrow \underline{R_D = 0.8 \text{ k}\Omega}$$

$$I_D = K_p (V_{SG} + V_{TP})^2 \Rightarrow 5 = 3(V_{SG} - 1.75)^2$$

$$V_{SG} = \sqrt{\frac{5}{3}} + 1.75 = 3.04 \text{ V} \Rightarrow V_G = -3.04$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = -3.04$$

$$R_{in} = R_1 \parallel R_2 = 80 \text{ k}\Omega$$

$$\frac{1}{R_1} \cdot (80)(10) = 5 - 3.04 \Rightarrow \underline{R_1 = 408 \text{ k}\Omega}$$

$$\frac{408R_2}{408 + R_2} = 80 \Rightarrow \underline{R_2 = 99.5 \text{ k}\Omega}$$

**3.44**

(a) Both  $M_1$  and  $M_2$  in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2} (4)(v_I - 0.4)^2 = \frac{k'_n}{2} (1)[0 - (-0.6)]^2 \Rightarrow v_I = 0.7 \text{ V}$$

(b) (i)  $v_I = 0.6 \text{ V}$ ;  $M_1$  in saturation,  $M_2$  in nonsaturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2} (4)(0.6 - 0.4)^2 = \frac{k'_n}{2} (1)[2(0 - (-0.6))(5 - v_o) - (5 - v_o)^2]$$

$$\text{We find } (5 - v_o)^2 - 1.2(5 - v_o) + 0.16 = 0 \Rightarrow (5 - v_o) = 0.153$$

$$\text{So } v_o = 4.85 \text{ V}$$

(ii)  $v_I = 1.5 \text{ V}$ ;  $M_1$  in nonsaturation,  $M_2$  in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2} (4)[2(1.5 - 0.4)v_o - v_o^2] = \frac{k'_n}{2} (1)(0 - (-0.6))^2$$

$$\text{We find } v_o^2 - 2.2v_o + 0.09 = 0 \Rightarrow v_o = 0.0417 \text{ V}$$

**3.45**

$M_1$  in nonsaturation,  $M_2$  in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2} \left( \frac{W}{L} \right)_1 [2(3 - 0.4)(0.025) - (0.025)^2] = \frac{k'_n}{2} (1)[0 - (-0.6)]^2$$

$$\left( \frac{W}{L} \right)_1 (0.1294) = 0.36 \Rightarrow \left( \frac{W}{L} \right)_1 = 2.78$$

**3.46**

(a) Transistors matched

$$V_{GS1} = V_{GS2} = 2.5 \text{ V}, \quad V_O = 2.5 \text{ V}$$

$$I_D = \left(\frac{0.12}{2}\right)(30)(2.5 - 0.4)^2 = 7.938 \text{ mA}$$

(b)  $I_{D1} = I_{D2}$

$$\frac{k'_n}{2} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = \frac{k'_n}{2} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$V_{GS2} = 5 - V_{GS1}$$

$$\text{Then } \sqrt{\frac{30}{15}}(V_{GS1} - 0.4) = (5 - V_{GS1} - 0.4)$$

Which yields  $V_{GS1} = 2.14 \text{ V}$ ,  $V_{GS2} = V_O = 2.86 \text{ V}$

$$I_D = \left(\frac{0.12}{2}\right)(30)(2.14 - 0.4)^2 = 5.45 \text{ mA}$$

(c)  $I_{D1} = I_{D2}$

$$(15)(V_{GS1} - 0.4)^2 = (30)(V_{GS2} - 0.4)^2$$

$$0.7071(V_{GS1} - 0.4) = 5 - V_{GS1} - 0.4$$

Which yields  $V_{GS1} = 2.86 \text{ V}$ ,  $V_{GS2} = V_O = 2.14 \text{ V}$

$$I_D = \left(\frac{0.12}{2}\right)(15)(2.86 - 0.4)^2 = 5.45 \text{ mA}$$

**3.47**

(a)  $V_1 = V_{GS3} = 2.5 \text{ V}$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_3 (2.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 3.69$$

$$V_2 = 6 = V_{GS2} + V_1 = V_{GS2} + 2.5 \Rightarrow V_{GS2} = 3.5 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_2 (3.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 1.59$$

$$V_{GS1} = 9 - V_2 = 9 - 6 = 3 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_1 (3 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 2.31$$

(b) (i)  $k'_n = (120)(1.05) = 126 \mu \text{ A/V}^2$

No change;  $V_1 = 2.5 \text{ V}$ ,  $V_2 = 6 \text{ V}$

(ii)  $k'_n = (120)(0.95) = 114 \mu \text{ A/V}^2$

No change;  $V_1 = 2.5 \text{ V}$ ,  $V_2 = 6 \text{ V}$

(c)  $k'_{n1} = 114 \mu \text{ A/V}^2$ ,  $k'_{n2} = k'_{n3} = 126 \mu \text{ A/V}^2$

$$I_{D2} = I_{D3}$$

$$\left(\frac{0.126}{2}\right)(1.59)(V_{GS2} - 0.6)^2 = \left(\frac{0.126}{2}\right)(3.69)(V_{GS3} - 0.6)^2$$

Now  $V_{GS3} = V_1$ ,  $V_{GS2} = V_2 - V_1$

$$\text{So } (V_2 - V_1 - 0.6) = 1.523(V_1 - 0.6)$$

$$\text{And } V_2 = 2.523V_1 - 0.3138$$

$$\text{Also } I_{D1} = I_{D3}$$

$$\left(\frac{0.114}{2}\right)(2.31)(V_{GS1} - 0.6)^2 = \left(\frac{0.126}{2}\right)(3.69)(V_{GS3} - 0.6)^2$$

$$\text{We have } V_{GS1} = 9 - V_2 = 9 - (2.523V_1 - 0.3138) = 9.3138 - 2.523V_1$$

$$\text{Also } V_{GS3} = V_1$$

$$\text{Then } (0.13167)(9.3138 - 2.523V_1 - 0.6)^2 = (0.23247)(V_1 - 0.6)^2$$

$$8.7138 - 2.523V_1 = 1.3287(V_1 - 0.6)$$

$$\text{Which yields } V_1 = 2.469 \text{ V and } V_2 = 5.916 \text{ V}$$

### 3.48

$M_L$  in saturation,  $M_D$  in nonsaturation

$$I_{DD} = I_{DL}$$

$$\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D [2(5 - 0.6)(0.15) - (0.15)^2] = \left(\frac{k'_n}{2}\right)(2)(5 - 0.15 - 0.6)^2$$

$$\left(\frac{W}{L}\right)_D (1.2975) = 36.125 \Rightarrow \left(\frac{W}{L}\right)_D = 27.8$$

### 3.49

$M_L$  in saturation,  $M_D$  in nonsaturation

$$I_{DD} = I_{DL}$$

$$\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D [2(5 - 0.6)(0.10) - (0.10)^2] = \left(\frac{k'_n}{2}\right)(2)[0 - (-1.2)]^2$$

$$\left(\frac{W}{L}\right)_D (0.87) = 2.88 \Rightarrow \left(\frac{W}{L}\right)_D = 3.31$$

### 3.50

(a) Transistor in nonsaturation

$$I_D = \frac{3 - 0.08}{30} = 0.09733 \text{ mA}$$

$$0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right) [2(2.6 - 0.4)(0.08) - (0.08)^2]$$

$$0.09733 = \left(\frac{W}{L}\right)(0.020736) \Rightarrow \left(\frac{W}{L}\right) = 4.69$$

(b)  $0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right) [2(3 - 0.4)(0.08) - (0.08)^2]$

$$0.09733 = \left(\frac{W}{L}\right)(0.024576) \Rightarrow \left(\frac{W}{L}\right) = 3.96$$

**3.51**

$$5 = I_D R_D + V_\gamma + V_{DS}$$

$$5 = (12)R_D + 1.6 + 0.15 \Rightarrow R_D = 271 \Omega$$

$$I_D = 12 = \left(\frac{0.08}{2}\right) \frac{W}{L} \left[2(5 - 0.6)(0.15) - (0.15)^2\right] \Rightarrow \left(\frac{W}{L}\right) = 231$$

**3.52**

$$5 = V_{SD} + I_D R_D + V_\gamma$$

$$5 = 0.20 + (15)R_D + 1.6 \Rightarrow R_D = 213 \Omega$$

$$I_D = 15 = \left(\frac{0.04}{2}\right) \left(\frac{W}{L}\right) \left[2(5 - 0.6)(0.20) - (0.20)^2\right]$$

$$15 = \left(\frac{W}{L}\right)(0.0344) \Rightarrow \left(\frac{W}{L}\right) = 436$$

**3.53**

(a)  $I_R = \frac{5 - 0.15}{50} = 0.097 \text{ mA}$

$$I_D = \frac{0.097}{2} = \left(\frac{0.12}{2}\right) \left(\frac{W}{L}\right) \left[2(5 - 0.6)(0.15) - (0.15)^2\right]$$

$$0.0485 = \left(\frac{W}{L}\right)(0.07785) \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 0.623$$

(b)  $M_1$  nonsaturation,  $M_2$  cutoff

$$I_R = I_D = \frac{5 - V_o}{50} = \left(\frac{0.12}{2}\right) (0.623) \left[2(5 - 0.6)V_o - V_o^2\right]$$

We find  $1.869V_o^2 - 17.45V_o + 5 = 0 \Rightarrow V_o = 0.297 \text{ V}$

**3.54**

(a)  $V_{DS2}(\text{sat}) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.9 \text{ V}$

$$I_{Q1} = 125 = \left(\frac{120}{2}\right) \left(\frac{W}{L}\right)_2 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 8.33$$

$$V_{GS3} = V_{GS2} = 0.9 \text{ V}$$

$$I_{REF1} = 225 = \left(\frac{120}{2}\right) \left(\frac{W}{L}\right)_3 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 15$$

$$M_1 \text{ and } M_2 \text{ matched, so } \left(\frac{W}{L}\right)_1 = 8.33$$

(b)  $V_{GS1} = 0.9 \text{ V} \Rightarrow V_{D1} = V_{DS1} - V_{GS1} = 2 - 0.9 = 1.1 \text{ V}$

$$R_D = \frac{2.5 - 1.1}{0.125} = 11.2 \text{ k}\Omega$$

**3.55**

(a)  $V_{SDB}(\text{sat}) = 0.8 = V_{SGB} + V_{TP} = V_{SGB} - 0.4 \Rightarrow V_{SGB} = 1.2 \text{ V}$

$$I_{Q2} = 200 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_B (1.2 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_B = 12.5$$

$$V_{SGC} = V_{SGB} = 1.2 \text{ V}$$

$$I_{REF2} = 125 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_C (1.2 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_C = 7.81$$

$M_A$  and  $M_B$  matched, so  $\left(\frac{W}{L}\right)_A = 12.5$

(b)  $V_{SGA} = 1.2 \text{ V}$

$$V_{DA} = V_{SGA} - V_{SDA} = 1.2 - 4 = -2.8 \text{ V}$$

$$R_D = \frac{-2.8 - (-5)}{0.20} = 11 \text{ k}\Omega$$

**3.56**

$$V_{DS2}(\text{sat}) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.6 \Rightarrow V_{GS2} = 1.1 \text{ V}$$

$$I_Q = 0.35 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_2 (1.1 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 23.3$$

$$V_{GS3} = V_{GS2} = 1.1 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_3 (1.1 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 10$$

$$V_{GS4} = 5 - 1.1 = 3.9 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_4 (3.9 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 0.23$$

$M_1$  and  $M_2$  matched, so  $\left(\frac{W}{L}\right)_1 = 23.3$

$$V_{GS1} = 1.1 \text{ V}; V_{D1} = V_{DS1} - V_{GS1} = 3.5 - 1.1 = 2.4 \text{ V}$$

$$R_D = \frac{5 - 2.4}{0.35} = 7.43 \text{ k}\Omega$$

**3.57**

$$V_{DS}(\text{sat}) = V_{GS} - V_P$$

So  $\underline{V_{DS} > V_{DS}(\text{sat}) = -V_P}$ ,  $\underline{I_D = I_{DSS}}$

**3.58**

$$V_{DS}(\text{sat}) = V_{GS} - V_P = \underline{V_{GS} + 3 = V_{DS}(\text{sat})}$$

a.  $V_{GS} = 0 \Rightarrow \underline{I_D = I_{DSS} = 6 \text{ mA}}$

b.  $I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6 \left(1 - \frac{-1}{-3}\right)^2 \Rightarrow \underline{I_D = 2.67 \text{ mA}}$

c.  $I_D = 6 \left(1 - \frac{-2}{-3}\right)^2 \Rightarrow \underline{I_D = 0.667 \text{ mA}}$

d.  $\underline{I_D = 0}$

**3.59**

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{V_P}\right)^2$$

$$0.30 = I_{DSS} \left(1 - \frac{3}{V_P}\right)^2$$

$$\frac{2.8}{0.30} = \frac{\left(1 - \frac{1}{V_P}\right)^2}{\left(1 - \frac{3}{V_P}\right)^2} = 9.33$$

$$\frac{\left(1 - \frac{1}{V_P}\right)}{\left(1 - \frac{3}{V_P}\right)} = 3.055$$

$$1 - \frac{1}{V_P} = 3.055 - \frac{9.165}{V_P}$$

$$\frac{8.165}{V_P} = 2.055 \Rightarrow \underline{V_P = 3.97 \text{ V}}$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{3.97}\right)^2 = I_{DSS} (0.560) \Rightarrow \underline{I_{DSS} = 5.0 \text{ mA}}$$

**3.60**

$$V_S = -V_{GS}, V_{SD} = V_S - V_{DD}$$

Want  $V_{SD} \geq V_{SD}(\text{sat}) = V_P - V_{GS}$

$$V_S - V_{DD} \geq V_P - V_{GS} \Rightarrow V_{GS} - V_{DD} \geq V_P - V_{GS} \Rightarrow \underline{V_{DD} \leq -V_P}$$

So  $\underline{V_{DD} \leq -2.5 \text{ V}}$

$$I_D = 2 = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 6 \left(1 - \frac{V_{GS}}{2.5}\right)^2 \Rightarrow V_{GS} = 1.06 \text{ V} \Rightarrow \underline{V_S = -1.06 \text{ V}}$$

**3.61**

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$18.5 = K_n (0.35 - V_{TN})^2$$

$$86.2 = K_n (0.5 - V_{TN})^2$$

Then

$$\frac{18.5}{86.2} = 0.2146 = \frac{(0.35 - V_{TN})^2}{(0.50 - V_{TN})^2} \Rightarrow \underline{V_{TN} = 0.221 \text{ V}}$$

$$18.5 = K_n (0.35 - 0.221)^2 \Rightarrow \underline{K_n = 1.11 \text{ mA/V}^2}$$

**3.62**

$$I_D = K (V_{GS} - V_{TN})^2$$

$$250 = K (0.75 - 0.24)^2 \Rightarrow \underline{K = 0.961 \text{ mA/V}^2}$$

**3.63**

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = -\frac{V_{GS}}{R_S}$$

$$10 \left(1 - \frac{V_{GS}}{-5}\right)^2 = -\frac{V_{GS}}{0.2}$$

$$2 \left(1 + \frac{2V_{GS}}{5} + \frac{V_{GS}^2}{25}\right) = -V_{GS}$$

$$\frac{2}{25}V_{GS}^2 + \frac{9}{5}V_{GS} + 2 = 0$$

$$2V_{GS}^2 + 45V_{GS} + 50 = 0$$

$$V_{GS} = \frac{-45 \pm \sqrt{(45)^2 - 4(2)(50)}}{2(2)} \Rightarrow \underline{V_{GS} = -1.17 \text{ V}}$$

$$I_D = -\frac{V_{GS}}{R_S} = \frac{1.17}{0.2} \Rightarrow \underline{I_D = 5.85 \text{ mA}}$$

$$V_D = 20 - (5.85)(2) = 8.3 \text{ V}$$

$$V_{DS} = V_D - V_S = 8.3 - 1.17 \Rightarrow \underline{V_{DS} = 7.13 \text{ V}}$$



**3.64**

$$\begin{aligned}V_{DS} &= V_{DD} - V_S \\8 &= 10 - V_S \Rightarrow V_S = 2 \text{ V} = I_D R_S = (5) R_S \Rightarrow \underline{R_S = 0.4 \text{ k}\Omega} \\I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \\5 &= I_{DSS} \left(1 - \frac{-1}{V_P}\right)^2 \quad \text{Let } \underline{I_{DSS} = 10 \text{ mA}} \\5 &= 10 \left(1 - \frac{-1}{V_P}\right)^2 \Rightarrow \underline{V_P = -3.41 \text{ V}} \\V_G &= V_{GS} + V_S = -1 + 2 = 1 \text{ V} \\V_G &= \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \\1 &= \frac{1}{R_1} (500)(10) \Rightarrow \underline{R_1 = 5 \text{ M}\Omega} \\ \frac{5R_2}{5 + R_2} &= 0.5 \Rightarrow \underline{R_2 = 0.556 \text{ M}\Omega}\end{aligned}$$

**3.65**

$$\begin{aligned}I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \\5 &= 8 \left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow \underline{V_{GS} = 0.838 \text{ V}} \\V_{SD} &= V_{DD} - I_D (R_S + R_D) \\ &= 20 - (5)(0.5 + 2) \Rightarrow \underline{V_{SD} = 7.5 \text{ V}} \\V_S &= 20 - (5)(0.5) = 17.5 \text{ V} \\V_G &= V_S + V_{GS} = 17.5 + 0.838 = 18.3 \text{ V} \\V_G &= \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \\18.3 &= \frac{1}{R_1} (100)(20) \Rightarrow \underline{R_1 = 109 \text{ k}\Omega} \\ \frac{109R_2}{109 + R_2} &= 100 \Rightarrow \underline{R_2 = 1.21 \text{ M}\Omega}\end{aligned}$$

**3.66**

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 = 7 \left( 1 - \frac{V_{GS}}{3} \right)^2 \Rightarrow \underline{V_{GS} = 0.465 \text{ V}}$$

$$V_{SD} = V_{DD} - I_D (R_S + R_D)$$

$$6 = 12 - (5)(0.3 + R_D) \Rightarrow \underline{R_D = 0.9 \text{ k}\Omega}$$

$$V_S = 12 - (5)(0.3) = 10.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 10.5 + 0.465 = 10.965 \text{ V}$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$10.965 = \left( \frac{R_2}{100} \right) (12) \Rightarrow \underline{R_2 = 91.4 \text{ k}\Omega} \Rightarrow \underline{R_1 = 8.6 \text{ k}\Omega}$$

**3.67**

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{60}{140 + 60} \right) (20) \Rightarrow \underline{V_G = 6 \text{ V}}$$

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S}$$

$$(8)(2) \left( 1 - \frac{V_{GS}}{(-4)} \right)^2 = 6 - V_{GS}$$

$$16 \left( 1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16} \right) = 6 - V_{GS}$$

$$V_{GS}^2 + 9V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-9 \pm \sqrt{(9)^2 - 4(10)}}{2} \Rightarrow \underline{V_{GS} = -1.30}$$

$$I_D = 8 \left( 1 - \frac{(-1.30)}{(-4)} \right)^2 \Rightarrow \underline{I_D = 3.65 \text{ mA}}$$

$$\begin{aligned} V_{DS} &= V_{DD} - I_D (R_S + R_D) \\ &= 20 - (3.65)(2 + 2.7) \end{aligned}$$

$$V_{DS} = 2.85 \text{ V}$$

$$\begin{aligned} V_{DS} &> V_{DS}(\text{sat}) = V_{GS} - V_P \\ &= -1.30 - (-4) \\ &= 2.7 \text{ V (Yes)} \end{aligned}$$

**3.68**

$$\begin{aligned}V_{DS} &= V_{DD} - I_D (R_S + R_D) \\5 &= 12 - I_D (0.5 + 1) \Rightarrow \underline{I_D = 4.67 \text{ mA}} \\V_S &= I_D R_S = (4.67)(0.5) \Rightarrow V_S = 2.33 \text{ V} \\V_G &= \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{20}{450 + 20} \right) (12) \Rightarrow V_G = 0.511 \text{ V} \\V_{GS} &= V_G - V_S = 0.511 - 2.33 \Rightarrow \underline{V_{GS} = -1.82 \text{ V}} \\I_D &= I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 \\4.67 &= 10 \left( 1 - \frac{(-1.82)}{V_P} \right)^2 \Rightarrow \underline{V_P = -5.75 \text{ V}}\end{aligned}$$

**3.69**

$$\begin{aligned}I_D &= I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2, \quad V_{GS} = 0 \\I_D &= I_{DSS} = 4 \text{ mA} \\R_D &= \frac{V_{DD} - V_{DS}}{I_D} = \frac{10 - 3}{4} \Rightarrow \underline{R_D = 1.75 \text{ k}\Omega}\end{aligned}$$

**3.70**

$$\begin{aligned}V_{SD} &= V_{DD} - I_D R_S \\10 &= 20 - (1) R_S \Rightarrow \underline{R_S = 10 \text{ k}\Omega} \\R_1 + R_2 &= \frac{V_{DD}}{I} = \frac{20}{0.1} = 200 \text{ k}\Omega \\I_D &= I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 \\1 &= 2 \left( 1 - \frac{V_{GS}}{2} \right)^2 \Rightarrow \underline{V_{GS} = 0.586 \text{ V}} \\V_G &= V_S + V_{GS} = 10 + 0.586 = 10.586 \\V_G &= \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} \\10.586 &= \left( \frac{R_2}{200} \right) (20) \Rightarrow \underline{R_2 = 106 \text{ k}\Omega} \\R_1 &= 94 \text{ k}\Omega\end{aligned}$$

**3.71**

$$\begin{aligned}V_{DS} &= V_{DD} - I_D (R_S + R_D) \\2 &= 3 - (0.040)(10 + R_D) \Rightarrow \underline{R_D = 15 \text{ k}\Omega} \\I_D &= K (V_{GS} - V_{TN})^2 \\40 &= 250(V_{GS} - 0.20)^2 \Rightarrow \underline{V_{GS} = 0.60 \text{ V}} \\V_G &= V_{GS} + V_S = 0.60 + (0.040)(10) = 1.0 \text{ V} \\V_G &= \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} \\1 &= \left( \frac{R_2}{150} \right) (3) \Rightarrow \underline{R_2 = 50 \text{ k}\Omega} \\R_1 &= \underline{100 \text{ k}\Omega}\end{aligned}$$

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**3.72**

$$\begin{aligned}V_O &= 0.70 \text{ V} \Rightarrow V_{DS} = 0.70 > V_{DS}(\text{sat}) = V_{GS} - V_{TN} \\ \text{For } 0.75 - 0.15 &= 0.6 \\ \text{Biased in the saturation region} \\ I_D &= \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.7}{50} \Rightarrow \underline{I_D = 46 \mu\text{A}} \\ I_D &= K (V_{GS} - V_{TN})^2 \Rightarrow 46 = K (0.75 - 0.15)^2 \Rightarrow \underline{K = 128 \mu\text{A}/\text{V}^2}\end{aligned}$$

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## Chapter 4

### 4.1

$$(a) \quad (i) \quad g_m = 2\sqrt{\frac{k'_n}{2}\left(\frac{W}{L}\right)}I_{DQ}$$

$$0.5 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)}(0.5) \Rightarrow \left(\frac{W}{L}\right) = 2.5$$

$$(ii) \quad I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$$

$$0.5 = \left(\frac{0.1}{2}\right)(2.5)(V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 2.4 \text{ V}$$

$$(b) \quad (i) \quad 0.5 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)}(0.15) \Rightarrow \left(\frac{W}{L}\right) = 8.33$$

$$(ii) \quad 0.15 = \left(\frac{0.1}{2}\right)(8.33)(V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 1.0 \text{ V}$$

### 4.2

$$(a) \quad (i) \quad g_m = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)}I_{DQ}$$

$$1.2 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)}(0.15) \Rightarrow \left(\frac{W}{L}\right) = 120$$

$$(ii) \quad I_{DQ} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)(V_{SGQ} + V_{TP})^2$$

$$0.15 = \left(\frac{0.04}{2}\right)(120)(V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 0.85 \text{ V}$$

$$(b) \quad (i) \quad 1.2 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)}(0.50) \Rightarrow \left(\frac{W}{L}\right) = 36$$

$$(ii) \quad 0.50 = \left(\frac{0.04}{2}\right)(36)(V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 1.43 \text{ V}$$

### 4.3

$$I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

$$\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$$

$$3.4[1 + 5\lambda] = 3.0[1 + 10\lambda]$$

$$3.4 - 3.0 = \lambda(3 \cdot 10 - (3.4) \cdot 5) \Rightarrow \lambda = \underline{0.0308}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{5}{0.4} = 12.5 \text{ k}\Omega$$

4.4

$$r_o = \frac{1}{\lambda I_D}$$
$$I_D = \frac{1}{\lambda r_o} = \frac{1}{(0.012)(100)} \Rightarrow I_D = \underline{0.833 \text{ mA}}$$

4.5

(a)  $I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$   
 $I_D = I_{D0} (1 + \lambda V_{DS})$   
Then  $\frac{0.258}{0.250} = \frac{(1 + 3.3\lambda)}{(1 + 1.5\lambda)}$   
Or  $1.032(1 + 1.5\lambda) = 1 + 3.3\lambda \Rightarrow \lambda = 0.01826 \text{ V}^{-1}$   
 $0.250 = I_{D0} [1 + (0.01826)(1.5)] \Rightarrow I_{D0} = 0.2433 \text{ mA}$   
 $r_o = \frac{1}{\lambda I_{D0}} = \frac{1}{(0.01826)(0.2433)} = 225 \text{ k}\Omega$

(b)  $I_D = (0.2433)[1 + (0.01826)(5)] = 0.2655 \text{ mA}$

4.6

(a)

(i)  $r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.05)} = 1000 \text{ K}$

(ii)  $r_o = \frac{1}{(0.02)(0.5)} = 100 \text{ K}$

(b)

(i)  $\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{1000} = 0.001 \text{ mA} = 1.0 \mu\text{A}$   
 $\frac{\Delta I_D}{I_D} = \frac{1.0}{50} \Rightarrow 2\%$   
 $\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{100} = 0.01 \Rightarrow 10 \mu\text{A}$   
 $\frac{\Delta I_D}{I_D} = \frac{10}{500} \Rightarrow 2\%$

(ii)  $\frac{\Delta I_D}{I_D} = \frac{10}{500} \Rightarrow 2\%$

4.7

$$I_D = 1.0 \text{ mA}$$
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(1)} = 100 \text{ K}$$

**4.8**

(a)  $V_{DD} = I_{DQ}R_D + V_{DSQ}$   
 $3.3 = I_{DQ}(5) + 1.5 \Rightarrow I_{DQ} = 0.36 \text{ mA}$   
 $I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$   
 $0.36 = \left(\frac{0.1}{2}\right)(40)(V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 0.824 \text{ V}$

(b)  $K_n = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right) = \left(\frac{0.1}{2}\right)(40) = 2 \text{ mA/V}^2$   
 $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.36)} = 1.697 \text{ mA/V}$   
 $r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.025)(0.36)} = 111.1 \text{ k}\Omega$   
 $A_v = -g_m(r_o \parallel R_D) = -(1.697)(111.1 \parallel 5) = -8.12$

**4.9**

(a)  $A_v = -g_m R_D$   
 $-3.8 = -g_m(10) \Rightarrow g_m = 0.38 \text{ mA/V}$   
 $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$   
 $0.38 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.12)} \Rightarrow \left(\frac{W}{L}\right) = 6.02$

(b)  $-5 = -g_m(10) \Rightarrow g_m = 0.50 \text{ mA/V}$   
 $0.50 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.12)} \Rightarrow \left(\frac{W}{L}\right) = 10.4$

**4.10**

(a)  $V_{DD} = I_{DQ}R_D + V_{DSQ}$   
 $5 = (0.5)R_D + 3 \Rightarrow R_D = 4 \text{ k}\Omega$   
 $I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$   
 $0.5 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)(1.2 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right) = 34.7$

(b)  $g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.08}{2}\right)(34.7)(0.5)} = 1.666 \text{ mA/V}$   
 $r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.5)} = 133.3 \text{ k}\Omega$

(c)  $A_v = -g_m(r_o \parallel R_D) = -(1.666)(133.3 \parallel 4) = -6.47$

**4.11**

$$K_n v_{gs}^2 = K_n [V_{gs} \sin \omega t]^2 = K_n V_{gs}^2 \sin^2 \omega t$$

$$\sin^2 \omega t = \frac{1}{2} [1 - \cos 2\omega t]$$

$$\text{So } K_n v_{gs}^2 = \frac{K_n V_{gs}^2}{2} [1 - \cos 2\omega t]$$

$$\text{Ratio of signal at } 2\omega \text{ to that at } \omega: \frac{\frac{K_n V_{gs}^2}{2} \cdot \cos 2\omega t}{2K_n (V_{GSQ} - V_{TN}) V_{gs} \cdot \sin \omega t}$$

$$\text{The coefficient of this expression is then: } \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$$

**4.12**

$$0.01 = \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$$

$$\text{So } V_{gs} = (0.01)(4)(3-1) \Rightarrow \underline{V_{gs} = 0.08 \text{ V}}$$

**4.13**

$$(a) \quad V_{GS} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \left( \frac{60}{60 + 240} \right) (3.3) = 0.66 \text{ V}$$

$$I_{DQ} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (V_{GS} - V_{TN})^2 = \left( \frac{0.1}{2} \right) (80) (0.66 - 0.4)^2 = 0.270 \text{ mA}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D = 3.3 - (0.270)(8) = 1.14 \text{ V}$$

$$(b) \quad g_m = 2 \sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left( \frac{0.1}{2} \right) (80) (0.270)} = 2.078 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.27)} = 185 \text{ k}\Omega$$

$$(c) \quad A_v = -g_m (r_o \parallel R_D) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{Si}} \right)$$

$$\text{We find } r_o \parallel R_D = 185 \parallel 8 = 7.668 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 60 \parallel 240 = 48 \text{ k}\Omega$$

$$\text{So } A_v = -(2.078)(7.668) \left( \frac{48}{48 + 2} \right) = -15.3$$

**4.14**

$$A_v = -g_m (r_o \parallel R_D)$$

$$-10 = -g_m (100 \parallel 5) \Rightarrow \underline{g_m = 2.1 \text{ mA/V}}$$



**4.15**

(a)  $V_G = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \left( \frac{175}{175 + 225} \right) (5) = 2.1875 \text{ V}$

$2.1875 = V_{GS} + I_D R_S = V_{GS} + K_n R_S (V_{GS} - V_{TN})^2$

$2.1875 = V_{GS} + (1)(1)(V_{GS}^2 - 1.6V_{GS} + 0.64)$

or  $V_{GS}^2 - 0.6V_{GS} - 1.5475 = 0 \Rightarrow V_{GS} = 1.58 \text{ V}$

$I_{DQ} = K_n (V_{GS} - V_{TN})^2 = (1)(1.58 - 0.8)^2 = 0.608 \text{ mA}$

$V_{DSQ} = V_{DD} - I_{DQ} (R_S + R_D) = 5 - (0.608)(1 + 4) = 1.96 \text{ V}$

(b)  $A_v = \frac{-g_m R_D}{1 + g_m R_S}$

$g_m = 2\sqrt{(1)(0.608)} = 1.56 \text{ mA/V}$

$A_v = \frac{-(1.56)(4)}{1 + (1.56)(1)} = -2.44$

(c)  $A_v = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_S} = \frac{-(1.56)(R_D \parallel R_L)}{1 + (1.56)(1)} = -0.6094(R_D \parallel R_L)$

$-(0.75)(2.44) = -(0.6094)(R_D \parallel R_L) \Rightarrow R_D \parallel R_L = 3.0 \text{ k}\Omega$

$4 \parallel R_L = 3 \Rightarrow R_L = 12 \text{ k}\Omega$

**4.16**

(a)  $V_{DSQ} = V_{DD} - I_{DQ} (R_S + R_D)$

$5 = 12 - (2)(R_S + R_D) \Rightarrow R_S + R_D = 3.5 \text{ k}\Omega$

$R_S = 0.5 \text{ k}\Omega$ , then  $R_D = 3 \text{ k}\Omega$

$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$

$2 = 1.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.355 \text{ V}$

$V_G = V_{GSQ} + I_{DQ} R_S = 2.355 + (2)(0.5) = 3.355 \text{ V}$

$V_G = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$3.355 = \frac{1}{R_1} (250)(12) \Rightarrow R_1 = 894 \text{ k}\Omega$

$R_1 \parallel R_2 = 894 \parallel R_2 = 250 \Rightarrow R_2 = 347 \text{ k}\Omega$

(b)  $g_m = 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V}$

$A_v = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_S} = \frac{-(3.464)(3 \parallel 10)}{1 + (3.464)(0.5)} = -2.93$

**4.17**

- (a) From Problem 4.16;
- $R_S = 0.5 \text{ k}\Omega$ ,  $R_D = 3 \text{ k}\Omega$ ,
- $R_1 = 894 \text{ k}\Omega$ ,  $R_2 = 347 \text{ k}\Omega$

(b)  $g_m = 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V}$   
 $A_v = -g_m(R_D \parallel R_L) = -(3.464)(3 \parallel 10) = -7.99$

**4.18**

(a)  
 $A_v = -g_m R_D$   
 $-15 = -2R_D \Rightarrow R_D = 7.5 \text{ K}$

(b)  
 $A_v = \frac{-g_m R_D}{1 + g_m R_S}$   
 $-5 = \frac{-(2)(7.5)}{1 + (2)R_S} \Rightarrow R_S = 1 \text{ K}$

**4.19**

(a)  $A_v = \frac{-g_m R_D}{1 + g_m R_S}$   
 $-8 = \frac{-g_m R_D}{1 + g_m (1)}$   
 (1)  $-16 = -g_m R_D$   
 (2)  $8 = \frac{16}{1 + g_m (1)} \Rightarrow g_m = 1 \text{ mA/V}$

Then  $R_D = 16 \text{ K}$

$A_v = -10 = \frac{-(1)(16)}{1 + (1)R_S}$

(b)  $R_S = 0.6 \text{ K}$

**4.20**

(a)  $I_{DQ} = I_Q = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$   
 $0.5 = \left(\frac{0.1}{2}\right)(50)(V_{GSQ} - 0.8)^2 \Rightarrow V_{GSQ} = 1.247 \text{ V}$   
 $V_{DSQ} = V^+ - I_{DQ}R_D - (-V_{GSQ}) = 5 - (0.5)(6) + 1.247 = 3.25 \text{ V}$

(b)  $g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(50)(0.5)} = 2.236 \text{ mA/V}$

$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$

$A_v = -g_m(r_o \parallel R_D) = -(2.236)(100 \parallel 6) = -12.7$

$$(c) A_v = -g_m (r_o \parallel R_D \parallel R_L) = -(2.236)(100 \parallel 6 \parallel 20) = -9.86$$

$$(d) A_v = -g_m (r_o \parallel R_D \parallel R_L) = -(2.236)(100 \parallel 6 \parallel 6) = -6.51$$

#### 4.21

$$(a) V_{DSQ} = 5 - (-5) - I_{DQ}(R_S + R_D)$$

$$5.5 = 10 - (0.1)(R_S + R_D) \Rightarrow R_S + R_D = 45 \text{ k}\Omega$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$(0.1) = (0.85)(V_{GSQ} - 0.8)^2 \Rightarrow V_{GSQ} = 1.143 \text{ V}$$

$$V_{GS} + I_D R_S = 5$$

$$1.143 + (0.1)R_S = 5 \Rightarrow R_S = 38.6 \text{ k}\Omega$$

$$\Rightarrow R_D = 6.43 \text{ k}\Omega$$

$$(b) g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.85)(0.1)} = 0.583 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$(c) A_v = -g_m (R_D \parallel r_o \parallel R_L) = -(0.583)(6.43 \parallel 500 \parallel 40) = -3.19$$

#### 4.22

$$(a) V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$2 = 3.3 - (0.5)(R_S + R_D) \Rightarrow R_S + R_D = 2.6 \text{ k}\Omega$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$0.5 = 2(V_{GSQ} - (-0.8))^2 \Rightarrow V_{GSQ} = -0.3 \text{ V}$$

$$V_{GSQ} + I_{DQ} R_S = 0$$

$$-0.3 + (0.5)R_S = 0 \Rightarrow R_S = 0.6 \text{ k}\Omega$$

$$\Rightarrow R_D = 2 \text{ k}\Omega$$

$$(b) g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.5)} = 2 \text{ mA/V}$$

$$A_v = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_S} = \frac{-(2)(2 \parallel 10)}{1 + (2)(0.6)} = -1.52$$

#### 4.23

$$(a) V_{DD} = I_{DQ} R_D + V_{DSQ} + I_{DQ} R_S \text{ and } V_{GS} + I_{DQ} R_S = 0$$

$$\text{Then } V_{DD} = K_n R_D (V_{GS} - V_{TN})^2 + V_{DSQ} - V_{GS}$$

$$5 = (2)(2)(V_{GS} + 0.8)^2 + 2.5 - V_{GS}$$

Which yields

$$4V_{GS}^2 + 5.4V_{GS} + 0.06 = 0, \Rightarrow V_{GS} = -0.0112 \text{ V}$$

$$\text{and } I_{DQ} = 2(-0.0112 + 0.8)^2 = 1.244 \text{ mA}$$

$$5 = (1.244)(2) + 2.5 + (1.244)R_S \Rightarrow R_S = 8.99 \Omega \cong 9 \Omega$$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(1.244)} = 3.155 \text{ mA/V}$$

$$A_v = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_S} = \frac{-(3.155)(2 \parallel 2)}{1 + (3.155)(0.009)} = -3.07$$

**4.24**

a.

$$5 = I_{DQ} R_S + V_{SDQ} + I_{DQ} R_D - 5$$

$$5 = I_{DQ} R_S + 6 + I_{DQ} (10) - 5$$

$$I_{DQ} = \frac{4}{R_S + 10}$$

1.  $V_S = V_{SDQ} + I_{DQ} R_D - 5 = V_{SGQ}$

2.  $1 + I_{DQ} (10) = V_{SGQ}$

3.  $I_{DQ} = K_p (V_{SGQ} - 2)^2$

Choose  $R_S = 10 \text{ k}\Omega \Rightarrow I_{DQ} = \frac{4}{20} = 0.20 \text{ mA}$

$$V_{SGQ} = 1 + (0.2)(10) = 3 \text{ V}$$

$$0.20 = K_p (3 - 2)^2 \Rightarrow \underline{K_p = 0.20 \text{ mA/V}^2}$$

b.

$$I_{DQ} = (0.20)(3 - 2)^2 = 0.20 \text{ mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$A_v = -g_m (R_D \parallel R_L) = -(0.4)(10 \parallel 10) \Rightarrow \underline{A_v = -2.0}$$

c.

Choose  $R_S = 20 \text{ k}\Omega \Rightarrow I_{DQ} = \frac{4}{30} = 0.133 \text{ mA}$

$$V_{SGQ} = 1 + (0.133)(10) = 2.33 \text{ V}$$

$$0.133 = K_p (2.33 - 2)^2 \Rightarrow \underline{K_p = 1.22 \text{ mA/V}^2}$$

$$g_m = 2\sqrt{(1.22)(0.133)} = 0.806 \text{ mA/V}$$

$$A_v = -(0.806)(10 \parallel 10) \Rightarrow \underline{A_v = -4.03}$$

A larger gain can be achieved.

**4.25**

(a)

$$I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.25 = 0.8(V_{SGQ} - 0.5)^2$$

$$V_{SGQ} = 1.059 \text{ V}$$

$$R_s = \frac{3 - 1.059}{0.25} \Rightarrow R_s = 7.76 \text{ K}$$

$$V_D = V_s - V_{SDQ} = 1.059 - 1.5 = -0.441 \text{ V}$$

$$R_D = \frac{-0.441 - (-3)}{0.25} \Rightarrow R_D = 10.2 \text{ K}$$

(b)

$$A_v = -g_m (R_D \parallel R_L)$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.8)(0.25)} = 0.8944 \text{ mA/V}$$

$$A_v = -(0.8944)(10.2 \parallel 2)$$

$$A_v = -1.50$$

(c)

$$\Delta V_o = \Delta I (R_D \parallel R_L) = 0.25(10.2 \parallel 2) = 0.418$$

$$\text{So } \Delta V_o = 0.836 \text{ peak-to-peak}$$

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**4.26**

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.2 = 2\sqrt{K_n (6)} \Rightarrow K_n = 0.202 \text{ mA/V}^2$$

$$6 = 0.202(2.8 - V_{TN})^2 \Rightarrow V_{TN} = -2.65 \text{ V}$$

$$V_{DSQ} = 18 - I_{DQ}(R_S + R_D)$$

$$R_S + R_D = \frac{18 - 10}{6} = 1.33 \text{ k}\Omega \Rightarrow R_S = 1.33 - R_D$$

$$A_v = -\frac{g_m (R_D \parallel R_L)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2 \left( \frac{R_D \cdot 1}{R_D + 1} \right)}{1 + (2.2)(1.33 - R_D)}$$

$$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$$

$$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$$

$$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$$

$$2.2R_D^2 + 0.47R_D - 3.93 = 0$$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)} \Rightarrow R_D = 1.23 \text{ k}\Omega, \quad R_S = 0.10 \text{ k}\Omega$$

$$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4 \text{ V}$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1} (100)(18) = 3.4 \Rightarrow R_1 = 529 \text{ k}\Omega$$

$$\frac{529R_2}{529 + R_2} = 100 \Rightarrow R_2 = 123 \text{ k}\Omega$$

**4.27**

(a)  $V_S = V_{SGQ} = V_{SDQ} + I_{DQ}R_D - 9$

$$V_{SGQ} = 5 + I_{DQ}(4) - 9 = K_p(4)(V_{SGQ} + V_{TP})^2 - 4$$

$$V_{SGQ} = 8(V_{SGQ}^2 - 2.4V_{SGQ} + 1.44) - 4$$

Or  $8V_{SGQ}^2 - 20.2V_{SGQ} + 7.52 = 0 \Rightarrow V_{SGQ} = 2.071 \text{ V}$

$$I_{DQ} = I_Q = K_p(V_{SGQ} + V_{TP})^2 = 2(2.071 - 1.2)^2 = 1.518 \text{ mA}$$

(b)  $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.518)} = 3.485 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.03)(1.518)} = 22 \text{ k}\Omega$$

$$A_v = -g_m(R_D \parallel r_o) = -(3.485)(4 \parallel 22) = -11.8$$

(c)  $A_v = -g_m(R_D \parallel r_o \parallel R_L) = -(3.485)(4 \parallel 22 \parallel 8) = -8.29$

**4.28**

(a)

$$I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.5 = 0.25 (V_{SGQ} + 0.8)^2$$

$$V_{SGQ} = 0.614 \text{ V} = V_S$$

$$R_S = \frac{10 - 0.614}{0.5} \Rightarrow R_S = 18.8 \text{ K}$$

$$V_D = V_S - V_{SDQ} = 0.614 - 3 = -2.386 \text{ V}$$

$$R_D = \frac{-2.386 - (-10)}{0.5} \Rightarrow R_D = 15.2 \text{ K}$$

(b)

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.5)} = 0.7071 \text{ mA/V}$$

$$A_v = -(0.7071)(15.2)$$

$$A_v = -10.7$$

**4.29**

$$A_v = -g_m (R_D \parallel R_L)$$

$$V_{DSQ} = V_{DD} - I_{DQ} (R_S + R_D)$$

$$10 = 20 - (1)(R_S + R_D) \Rightarrow R_S + R_D = 10 \text{ k}\Omega$$

Let  $R_D = 8 \text{ k}\Omega$ ,  $R_S = 2 \text{ k}\Omega$

$$A_v = -10 = -g_m (8 \parallel 20)$$

$$g_m = 1.75 \text{ mA/V} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n (1)} \Rightarrow K_n = 0.766 \text{ mA/V}^2$$

$$V_S = I_{DQ} R_S = (1)(2) = 2 \text{ V}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow 1 = 0.766 (V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.14 \text{ V}$$

$$V_G = V_{GS} + V_S = 3.14 + 2 = 5.14$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \Rightarrow \frac{1}{R_1} (200)(20) = 5.14 \Rightarrow R_1 = 778 \text{ k}\Omega$$

$$\frac{778 R_2}{778 + R_2} = 200 \Rightarrow R_2 = 269 \text{ k}\Omega$$

**4.30**

(a) 
$$A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(5)(100)}{1 + (5)(100)} = 0.998$$

$$R_o = \frac{1}{g_m} \parallel r_o = \frac{1}{5} \parallel 100 = 0.2 \parallel 100 \Rightarrow R_o \cong 200 \Omega$$

$$(b) A_v = \frac{g_m (r_o \parallel R_S)}{1 + g_m (r_o \parallel R_S)}$$

We have  $r_o \parallel R_S = 100 \parallel 5 = 4.762 \text{ k}\Omega$

$$A_v = \frac{(5)(4.762)}{1 + (5)(4.762)} = 0.960$$

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**4.31**

$$A_v = \frac{g_m (R_L \parallel r_o)}{1 + g_m (R_L \parallel r_o)}$$

$$0.98 = \frac{g_m r_o}{1 + g_m r_o} \Rightarrow g_m r_o = 49$$

$$\text{Also } 0.49 = \frac{g_m (R_L \parallel r_o)}{1 + g_m (R_L \parallel r_o)} = \frac{g_m \left( \frac{R_L r_o}{R_L + r_o} \right)}{1 + g_m \left( \frac{R_L r_o}{R_L + r_o} \right)}$$

$$0.49 = \frac{g_m (R_L r_o)}{R_L + r_o + g_m (R_L r_o)}$$

$$0.49 = \frac{(49)(1)}{1 + r_o + (49)(1)} = \frac{49}{50 + r_o}$$

$$\underline{r_o = 50 \text{ K}}$$

$$\underline{g_m = 0.98 \text{ mA/V}}$$

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**4.32**

(a)

$$A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(2)(25)}{1 + (2)(25)}$$

$$A_v = 0.98$$

$$R_o = \frac{1}{g_m} \parallel r_o = \frac{1}{2} \parallel 25 = 0.5 \parallel 25$$

$$\underline{R_o = 0.49 \text{ K}}$$

(b)

$$A_v = \frac{g_m (r_o \parallel R_L)}{1 + g_m (r_o \parallel R_L)} = \frac{2(25 \parallel 2)}{1 + 2(25 \parallel 2)} = \frac{2(1.852)}{1 + 2(1.852)}$$

$$\underline{A_v = 0.787}$$

---



**4.33**

$$(a) \quad I_{DQ} = \frac{0 - (-1.5)}{10} = 0.15 \text{ mA}$$

$$I_{DQ} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$0.15 = \left( \frac{0.1}{2} \right) (80) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 0.594 \text{ V}$$

$$(b) \quad g_m = 2 \sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left( \frac{0.1}{2} \right) (80) (0.15)} = 1.549 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

We find  $r_o \parallel R_S = 333 \parallel 10 = 9.708 \text{ k}\Omega$

$$\text{Then } A_v = \frac{g_m (r_o \parallel R_S)}{1 + g_m (r_o \parallel R_S)} = \frac{(1.549)(9.708)}{1 + (1.549)(9.708)} = 0.938$$

$$(c) \quad R_o = \frac{1}{g_m} \parallel r_o \parallel R_S = \frac{1}{1.549} \parallel 333 \parallel 10 = 0.6456 \parallel 9.708$$

or  $R_o = 605 \Omega$

**4.34**

$$(a) \quad I_{DQ} = \frac{V_{DD} - V_{DSQ}}{R_S} = \frac{2.5 - 1.25}{0.5} = 2.5 \text{ mA}$$

$$(b) \quad A_v = \frac{g_m R_S}{1 + g_m R_S}$$

$$0.85 = \frac{g_m (0.5)}{1 + g_m (0.5)} \Rightarrow g_m = 11.33 \text{ mA/V}$$

$$g_m = 2 \sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}}$$

$$11.33 = 2 \sqrt{\left( \frac{0.1}{2} \right) \left( \frac{W}{L} \right) (2.5)} \Rightarrow \left( \frac{W}{L} \right) = 257$$

$$(c) \quad I_{DQ} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$2.5 = \left( \frac{0.1}{2} \right) (257) (V_{GSQ} - 0.6)^2 \Rightarrow V_{GSQ} = 1.041 \text{ V}$$

$$V_{IQ} = V_{GSQ} + V_o = 1.041 + 1.25 = 2.291 \text{ V}$$

**4.35**

(a)  $P = I_Q V_{DD} \Rightarrow 2.5 = I_Q (2.5) \Rightarrow I_Q = 1 \text{ mA}$

(b)  $R_o = \frac{1}{g_m} \parallel r_o$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(1)} = 50 \text{ k}\Omega$$

So  $0.5 = \frac{1}{g_m} \parallel 50 \Rightarrow g_m = 1.98 \text{ mA/V}$

$$g_m = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) I_{DQ}}$$

$$1.98 = 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right) (1)} \Rightarrow \left(\frac{W}{L}\right) = 19.6$$

(c)  $A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(1.98)(50)}{1 + (1.98)(50)} = 0.990$

(d)  $g_m = 2 \sqrt{\left(\frac{0.1}{2}\right) (100)(1)} = 4.472 \text{ mA/V}$

$$R_o = \frac{1}{g_m} \parallel r_o = \frac{1}{4.472} \parallel 50 = 0.2236 \parallel 50 \Rightarrow R_o = 223 \Omega$$

**4.36**

(a)  $V_G = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{DD} = \left(\frac{350}{350 + 850}\right) (10) = 2.917 \text{ V}$

$$10 = I_{DQ} R_S + V_{SGQ} + V_G$$

Now  $K_p = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right) = \left(\frac{0.04}{2}\right) (80) = 1.6 \text{ mA/V}^2$

So  $10 = (1.6)(4)(V_{SGQ} - 1.2)^2 + V_{SGQ} + 2.917$

We find  $6.4V_{SGQ}^2 - 14.36V_{SGQ} + 2.133 = 0 \Rightarrow V_{SGQ} = 2.084 \text{ V}$

$$I_{DQ} = 1.6(2.084 - 1.2)^2 = 1.25 \text{ mA}$$

$$V_{SDQ} = 10 - (1.25)(4) = 5 \text{ V}$$

(b)  $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1.6)(1.25)} = 2.828 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.05)(1.25)} = 16 \text{ k}\Omega$$

$$r_o \parallel R_S \parallel R_L = 16 \parallel 4 \parallel 4 = 1.778 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)} = \frac{(2.828)(1.778)}{1 + (2.828)(1.778)} = 0.834$$

(c)  $A_g = \frac{i_o}{v_i} = \frac{i_o}{v_o} \cdot \frac{v_o}{v_i} = \frac{1}{R_L} \cdot \frac{v_o}{v_i} = \frac{1}{4} (0.834) = 0.2085 \text{ mA/V}$

$$(d) R_o = \frac{1}{g_m} \| R_s \| r_o = \frac{1}{2.828} \| 4 \| 16 = 0.3536 \| 3.2$$

$$R_o = 318 \Omega$$

**4.37**

(a) (i)  $K_n = \frac{k'_n}{2} \cdot \frac{W}{L} = \left(\frac{0.1}{2}\right)(20) = 1 \text{ mA/V}^2$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(5)} = 4.472 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(5)} = 10 \text{ k}\Omega$$

$$r_o \| R_L = 10 \| 4 = 2.857 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \| R_L)}{1 + g_m (r_o \| R_L)} = \frac{(4.472)(2.857)}{1 + (4.472)(2.857)} = 0.927$$

(ii)  $R_o = \frac{1}{g_m} \| r_o = \frac{1}{4.472} \| 10$

$$R_o = 219 \Omega$$

(b) (i)  $g_m = 2\sqrt{(1)(2)} = 2.828 \text{ mA/V}$

$$r_o = \frac{1}{(0.02)(2)} = 25 \text{ k}\Omega$$

$$r_o \| R_L = 25 \| 4 = 3.448 \text{ k}\Omega$$

$$A_v = \frac{(2.828)(3.448)}{1 + (2.828)(3.448)} = 0.907$$

(ii)  $R_o = \frac{1}{g_m} \| r_o = \frac{1}{2.828} \| 25$

$$R_o = 349 \Omega$$

**4.38**

a.

$$A_v = \frac{g_m R_L}{1 + g_m R_L} \Rightarrow 0.95 = \frac{g_m (4)}{1 + g_m (4)}$$

$$0.95 = 4(1 - 0.95)g_m \Rightarrow g_m = 4.75 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$

$$4.75 = 2\sqrt{(0.030)\left(\frac{W}{L}\right)(4)} \Rightarrow \frac{W}{L} = 47.0$$

$$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$

b.  $4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow \underline{I_Q = 3.13 \text{ mA}}$

**4.39**

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$5 = 5(V_{GS} + 2)^2 \Rightarrow V_{GS} = -1 \text{ V} \Rightarrow V_S = -V_{GS} = 1 \text{ V}$$

$$I_{DQ} = \frac{V_S - (-5)}{R_S} \Rightarrow R_S = \frac{1+5}{5} \Rightarrow \underline{R_S = 1.2 \text{ k}\Omega}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)}$$

$$= \frac{(10)(20 \parallel 1.2 \parallel 2)}{1 + (10)(20 \parallel 1.2 \parallel 2)} \Rightarrow \underline{A_v = 0.878}$$

$$R_o = \frac{1}{g_m} \parallel r_o \parallel R_S = \frac{1}{10} \parallel 20 \parallel 1.2 \Rightarrow \underline{R_o = 91.9 \Omega}$$

**4.40**

(a)  $V_S = I_{DQ} R_S - 5 = (5)(1) - 5 = 0 \Rightarrow V_{GS} = 0$

$$I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) (V_{GSQ} - V_{TN})^2$$

$$5 = \left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right) [0 - (-2)]^2 \Rightarrow \left(\frac{W}{L}\right) = 25$$

(b)  $g_m = 2\sqrt{\left(\frac{0.1}{2}\right) (25)(5)} = 5 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$r_o \parallel R_S = 20 \parallel 1 = 0.9524 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \parallel R_S)}{1 + g_m (r_o \parallel R_S)} = \frac{(5)(0.9524)}{1 + (5)(0.9524)} = 0.826$$

(c)  $R_o = \frac{1}{g_m} \parallel r_o \parallel R_S = \frac{1}{5} \parallel 20 \parallel 1 \Rightarrow R_o = 165 \Omega$

(d)  $r_o \parallel R_S \parallel R_L = 20 \parallel 1 \parallel 2 = 0.6452 \text{ k}\Omega$

$$A_v = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)} = \frac{(5)(0.6452)}{1 + (5)(0.6452)} = 0.763$$

**4.41**

$$R_o = \frac{1}{g_m} \parallel R_S$$

Output resistance determined primarily by  $g_m$

$$\text{Set } \frac{1}{g_m} = 0.2 \text{ k}\Omega \Rightarrow g_m = 5 \text{ mA/V}$$

$$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow 5 = 2\sqrt{(4)I_{DQ}} \Rightarrow I_{DQ} = 1.56 \text{ mA}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$1.56 = 4(V_{GS} + 2)^2$$

$$V_{GS} = -1.38 \text{ V}, \quad V_S = -V_{GS} = 1.38 \text{ V}$$

$$R_S = \frac{1.38 - (-5)}{1.56} \Rightarrow R_S = 4.09 \text{ k}\Omega$$

$$A_v = \frac{g_m (R_S \parallel R_L)}{1 + g_m (R_S \parallel R_L)} = \frac{5(4.09 \parallel 2)}{1 + 5(4.09 \parallel 2)} \Rightarrow A_v = 0.870$$

**4.42**

(a)  $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(10)} = 14.14 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(10)} = 10 \text{ k}\Omega$$

$$A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(14.14)(10)}{1 + (14.14)(10)} = 0.993$$

(b)  $R_o = \frac{1}{g_m} \parallel r_o = \frac{1}{14.14} \parallel 10 = 0.07072 \parallel 10 \Rightarrow R_o = 70.2 \Omega$

(c)  $A_v = \frac{g_m (r_o \parallel R_L)}{1 + g_m (r_o \parallel R_L)}$

$$0.90 = \frac{(14.14)(r_o \parallel R_L)}{1 + (14.14)(r_o \parallel R_L)} \Rightarrow (r_o \parallel R_L) = 0.6365 \text{ k}\Omega$$

$$10 \parallel R_L = 0.6365 \Rightarrow R_L = 680 \Omega$$

**4.43**

$$|\Delta i_D| = I_{DQ} = \frac{-1}{R_S \parallel R_L} \cdot \Delta v_0$$

$$\Delta v_0 = -I_{DQ} \cdot R_S \parallel R_L = -\frac{I_{DQ} \cdot R_S R_L}{R_S + R_L}$$

$$v_0 (\text{min}) = -\frac{I_{DQ} R_S}{1 + \frac{R_S}{R_L}}$$


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$$A_v = \frac{g_m (R_S \parallel R_L)}{1 + g_m (R_S \parallel R_L)} = \frac{v_0}{v_i}$$

$$v_i = \frac{-I_{DQ} (R_S \parallel R_L) [1 + g_m (R_S \parallel R_L)]}{g_m (R_S \parallel R_L)}$$

$$v_i (\text{min}) = -\frac{I_{DQ}}{g_m} [1 + g_m (R_S \parallel R_L)]$$

**4.44**

(a)

$$V_{DD} = V_{DSQ} + I_{DQ}R_S$$

$$3 = 1.5 + (0.25)R_S \Rightarrow \underline{R_S = 6 \text{ K}}$$

$$V_S = 1.5 \text{ V}$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$0.25 = 0.5 (V_{GSQ} - 0.4)^2$$

$$V_{GSQ} = 1.107 \text{ V}$$

$$V_G = V_{GSQ} + V_S + 1.107 + 1.5 = 2.607 \text{ V}$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} - R_L - V_{DD}$$

$$2.607 = \frac{1}{R_1} (300)(3) \Rightarrow \underline{R_1 = 345.2 \text{ K}} \Rightarrow \underline{R_2 = 2291 \text{ K}}$$

(b)

$$A_v = \frac{g_m R_S}{1 + g_m R_S} \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.25)} = 0.7071 \text{ mA/V}$$

$$A_v = \frac{(0.7071)(6)}{1 + (0.7071)(6)} \Rightarrow \underline{A_v = 0.809}$$

$$R_o = \frac{1}{g_m} \parallel R_S = \frac{1}{(0.7071)} \parallel 6 = 1.414 \parallel 6$$

$$\underline{R_o = 1.14 \text{ K}}$$

**4.45**

$$R_i = \frac{1}{g_m} = 0.5 \Rightarrow g_m = \frac{1}{0.5} = 2 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$2 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow \left(\frac{W}{L}\right) = 80$$

$$A_v = g_m R_D$$

$$20 = (2)R_D \Rightarrow \underline{R_D = 10 \text{ k}\Omega}$$

**4.46**

(a)  $R_o = R_D = 500 \text{ }\Omega$

(b)  $V_{GSQ} = 1.2 \text{ V}$

$$I_{DQ} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{2.2 - (V_{DS}(sat) + 0.3)}{0.5} = \frac{2.2 - (1.2 - 0.4 + 0.3)}{0.5}$$

$$I_{DQ} = 2.2 \text{ mA}$$

$$\begin{aligned}
 \text{(c)} \quad I_{DQ} &= K_n (V_{GSQ} - V_{TN})^2 \\
 2.2 &= K_n (1.2 - 0.4)^2 \Rightarrow K_n = 3.438 \text{ mA/V}^2 \\
 g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3.438)(2.2)} = 5.5 \text{ mA/V} \\
 R_i &= \frac{1}{g_m} = \frac{1}{5.5} \Rightarrow R_i = 182 \Omega \\
 \text{(d)} \quad A_v &= g_m R_D = (5.5)(0.5) = 2.75
 \end{aligned}$$

**4.47**

$$\begin{aligned}
 K_n &= \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right) = \left(\frac{0.1}{2}\right)(80) = 4 \text{ mA/V}^2 \\
 g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(0.5)} = 2.828 \text{ mA/V} \\
 A_v &= g_m R_D = (2.828)(4) = 11.3 \\
 R_i &= 10 \parallel \frac{1}{g_m} = 10 \parallel \frac{1}{2.828} = 10 \parallel 0.3536 \Rightarrow R_i = 342 \Omega
 \end{aligned}$$

**4.48**

a.

$$\begin{aligned}
 V_{GS} + I_{DQ} R_S &= 5 \\
 I_{DQ} &= \frac{5 - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2 \\
 5 - V_{GS} &= (10)(3)(V_{GS}^2 - 2V_{GS} + 1) \\
 30V_{GS}^2 - 59V_{GS} + 25 &= 0 \\
 V_{GS} &= \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow \underline{V_{GS} = 1.35 \text{ V}} \\
 I_{DQ} &= (3)(1.35 - 1)^2 \Rightarrow \underline{I_{DQ} = 0.365 \text{ mA}} \\
 V_{DSQ} &= 10 - (0.365)(5 + 10) \Rightarrow \underline{V_{DSQ} = 4.53 \text{ V}}
 \end{aligned}$$

b.

$$\begin{aligned}
 g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow \underline{g_m = 2.093 \text{ mA/V}} \\
 r_0 &= \frac{1}{\lambda I_{DQ}} = \frac{1}{(0)(0.365)} \Rightarrow \underline{r_0 = \infty}
 \end{aligned}$$

c.

$$A_v = g_m (R_D \parallel R_L) = (2.093)(5 \parallel 4) \Rightarrow \underline{A_v = 4.65}$$

**4.49**

a.

$$I_{DQ} = K_p (V_{SG} + V_{TP})^2$$

$$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.225 \text{ V}$$

$$5 = I_{DQ}R_s + V_{SG} \Rightarrow R_s = \frac{5 - 2.225}{0.75} \Rightarrow \underline{R_s = 3.70 \text{ k}\Omega}$$

$$V_{SDQ} = 10 - I_{DQ}(R_s + R_D)$$

$$6 = 10 - (0.75)(3.70 + R_D) \Rightarrow \underline{R_D = 1.63 \text{ k}\Omega}$$

b.

$$R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.225 \text{ mA/V}$$

$$R_i = \frac{1}{1.225} \Rightarrow \underline{R_i = 0.816 \text{ k}\Omega}$$

$$R_o = R_D \Rightarrow \underline{R_o = 1.63 \text{ k}\Omega}$$

c.

$$i_o = \left( \frac{R_D}{R_D + R_L} \right) \left( \frac{R_s}{R_s + [1/g_m]} \right) \cdot i_i$$

$$i_o = \left( \frac{1.63}{1.63 + 2} \right) \left( \frac{3.70}{3.70 + 0.816} \right) i_i$$

$$i_o = 0.368 i_i = \underline{i_o = 1.84 \sin \omega t (\mu\text{A})}$$

$$v_o = i_o R_L = (1.84)(2) \sin \omega t \Rightarrow \underline{v_o = 3.68 \sin \omega t (\text{mV})}$$

**4.50**

(a)  $V_O = (V_{DS}(\text{sat}) + 0.25) - V_{GS}$

$$V_O = V_{GS} - V_{TN} + 0.25 - V_{GS} = -0.4 + 0.25 = -0.15 \text{ V}$$

$$R_D = \frac{1.8 - (-0.15)}{2} = 0.975 \text{ k}\Omega$$

(b)  $A_v = g_m R_D$

$$6 = g_m (0.975) \Rightarrow g_m = 6.154 \text{ mA/V}$$

$$g_m = 2\sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}}$$

$$6.154 = 2\sqrt{\left( \frac{0.1}{2} \right) \left( \frac{W}{L} \right) (2)} \Rightarrow \left( \frac{W}{L} \right) = 94.7$$

(c)  $I_{DQ} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$

$$2 = \left( \frac{0.1}{2} \right) (94.7) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 1.05 \text{ V}$$



**4.51**

- (a)  $I_{DQ} = I_Q = K_n (V_{GSQ} - V_{TN})^2$   
 $2 = 4(V_{GSQ} - 0.6)^2 \Rightarrow V_{GSQ} = 1.307 \text{ V}$   
 $V_D = V_{DSQ} - V_{GSQ} = 3.5 - 1.307 = 2.193 \text{ V}$   
 $R_D = \frac{3.3 - 2.193}{2} = 0.554 \text{ k}\Omega$
- (b)  $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(2)} = 5.657 \text{ mA/V}$   
 $R_i = \frac{1}{g_m} = \frac{1}{5.657} \Rightarrow R_i = 177 \Omega$
- (c)  $A_v = g_m (R_D \parallel R_L) = (5.657)(0.554 \parallel 4) = 2.75$

**4.52**

- (a)  $I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$   
 $1.2 = 2.5(V_{SGQ} - 0.8)^2 \Rightarrow V_{SGQ} = 1.493 \text{ V}$   
 $R_S = \frac{V^+ - V_{SGQ}}{I_{DQ}} = \frac{3.3 - 1.493}{1.2} = 1.51 \text{ k}\Omega$   
 $V_{SDQ} = 6.6 - I_{DQ} (R_S + R_D) \Rightarrow 3 = 6.6 - 1.2(1.51 + R_D) \Rightarrow R_D = 1.49 \text{ k}\Omega$
- (b)  $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2.5)(1.2)} = 3.464 \text{ mA/V}$   
 $A_v = g_m (R_D \parallel R_L) = (3.464)(1.49 \parallel 4) = 3.76$

**4.53**

- (a)  $|A_v| = \sqrt{\frac{K_D}{K_L}} = \sqrt{\frac{(W/L)_D}{(W/L)_L}} = 5$   
 So  $\left(\frac{W}{L}\right)_D = 25$   
 From Example 4.11,  
 $V_{GSDr} = \frac{(3.3 - 0.6) + (0.6)(1 + 5)}{1 + 5} = 1.05 \text{ V}$   
 $V_{GSDQ} = \frac{1.05 - 0.6}{2} + 0.6 = 0.825 \text{ V}$
- (b)  $I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D (V_{GSDQ} - V_{TN})^2 = \left(\frac{0.1}{2}\right) (25)(0.825 - 0.6)^2$   
 $I_{DQ} = 0.0633 \text{ mA}$   
 Now  $I_{DD} = I_{DL}$   
 $\left(\frac{W}{L}\right)_D (V_{GSDQ} - V_{TN})^2 = \left(\frac{W}{L}\right)_L (V_{GSLQ} - V_{TN})^2$   
 $\sqrt{\frac{25}{1}} (V_{GSDQ} - V_{TN}) = V_{DD} - V_O - V_{TN}$   
 $5(0.825 - 0.6) = 3.3 - V_O - 0.6$   
 Or  $V_{DSQ} = V_O = 1.575 \text{ V}$

**4.54**

- (a) Neglect  $\lambda$  in dc analysis.

Transition points:

For point B,

$$V_{oB} = V_{DD} - |V_{TNL}| = 5 - 0.8 = 4.2 \text{ V}$$

For point A,

$$I_{DD} = I_{DL}$$

$$K_{nD}(V_{GSDQ} - V_{TND})^2 = K_{nL}(V_{GSLQ} - V_{TNL})^2$$

$$(1.2)(V_{GSDQ} - 0.6)^2 = (0.2)[0 - (-0.8)]^2$$

$$\text{So } V_{GSDQ} = \sqrt{\frac{0.2}{1.2}}(0.8) + 0.6 = 0.9266 \text{ V}$$

$$\text{Then } V_{oA} = V_{GSDQ} - V_{TND} = 0.9266 - 0.6 = 0.3266 \text{ V}$$

For point A:  $V_{oA} = 0.3266 \text{ V}$ ,  $V_{GSDQ} = 0.9266 \text{ V}$

For point B:  $V_{oB} = 4.2 \text{ V}$ ,  $V_{GSDQ} = 0.9266 \text{ V}$

- (b)  $V_{GSDQ} = 0.9266 \text{ V}$ ,

$$V_{DSDQ} = \frac{4.2 - 0.3266}{2} + 0.3266 = 2.2633 \text{ V}$$

- (c)  $I_{DQ} = K_{nD}(V_{GSDQ} - V_{TND})^2 = (1.2)(0.9266 - 0.6)^2 = 0.128 \text{ mA}$

- (d)  $A_v = -g_{mD}(r_{oD} \parallel r_{oL})$

$$r_{oD} = r_{oL} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.128)} = 390.6 \text{ k}\Omega$$

$$g_{mD} = 2\sqrt{K_{nD}I_{DQ}} = 2\sqrt{(1.2)(0.128)} = 0.7838 \text{ mA/V}$$

$$A_v = -(0.7838)(390.6 \parallel 390.6) = -153$$

**4.55**

- (a)  $V_{TN} = 0.6 \text{ V}$

$$I_D = K_n(V_{DS} - V_{TN})^2$$

$$0.5 = K_n(1.5 - 0.6)^2 \Rightarrow K_n = 0.6173 \text{ mA/V}^2$$

$$\frac{1}{r} = \frac{dI_D}{dV_{DS}} = 2K_n(V_{DS} - V_{TN})$$

$$\text{Then } r = \frac{1}{2K_n(V_{DS} - V_{TN})} = \frac{1}{2(0.6173)(1.5 - 0.6)} \Rightarrow r = 900 \Omega$$

- (b)  $I_D = (0.6173)(3 - 0.6)^2 = 3.56 \text{ mA}$

$$r = \frac{1}{2(0.6173)(3 - 0.6)} \Rightarrow r = 337 \Omega$$

**4.56**

a.

$$I_{DQ} = K_{nD} (V_{GS} - V_{TND})^2 = (0.5)(0 - (-1))^2$$

$$I_{DQ} = 0.5 \text{ mA}$$

$$I_{DQ} = K_{nL} (V_{GSL} - V_{TNL})^2 = K_{nL} (V_{DD} - V_o - V_{TNL})^2$$

$$0.5 = 0.030(10 - V_o - 1)^2$$

$$\sqrt{\frac{0.5}{0.030}} = 9 - V_o \Rightarrow \underline{V_o = 4.92 \text{ V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_{nD} (V_i - V_{TND})^2 = K_{nL} (V_{DD} - V_o - V_{TNL})^2$$

$$\sqrt{\frac{K_{nD}}{K_{nL}}} (V_i - V_{TND}) = V_{DD} - V_o - V_{TNL}$$

$$V_o = V_{DD} - V_{TNL} - \sqrt{\frac{K_{nD}}{K_{nL}}} (V_i - V_{TND})$$

$$A_v = \frac{dV_o}{dV_i} = -\sqrt{\frac{K_{nD}}{K_{nL}}} = -\sqrt{\frac{(W/L)_D}{(W/L)_L}}$$

$$A_v = -\sqrt{\frac{500}{30}} \Rightarrow \underline{A_v = -4.08}$$

**4.57**

(a)

$$I_{DQ} = K_L (V_{GSL} - V_{TNL})^2 = K_L (V_{DSL} - V_{TNL})^2$$

$$I_D = (0.1)(4 - 1)^2 = 0.9 \text{ mA}$$

$$I_{DQ} = K_D (V_{GSD} - V_{TND})^2$$

$$0.9 = (1)(V_{GSD} - 1)^2 \Rightarrow V_{GSD} = 1.95 \text{ V}$$

$$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4 \Rightarrow \underline{V_{GG} = 5.95 \text{ V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_D (V_{GSD} - V_{TND})^2 = K_L (V_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{K_D}{K_L}} (V_{GG} + V_i - V_o - V_{TND}) = V_o - V_{TNL}$$

$$V_o \left( 1 + \sqrt{\frac{K_D}{K_L}} \right) = \sqrt{\frac{K_D}{K_L}} (V_{GG} + V_i - V_{TND}) + V_{TNL}$$

$$A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D / K_L}}{1 + \sqrt{K_D / K_L}} \Rightarrow \underline{A_v = \frac{1}{1 + \sqrt{K_L / K_D}}}$$

(c) From Problem 4.55.

$$R_{LD} = \frac{1}{2K_L(V_{DSL} - V_{TNL})}$$

$$= \frac{1}{2(0.1)(4-1)} = 1.67 \text{ k}\Omega \quad \dots$$

$$g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90 \text{ mA/V}$$

$$A_v = \frac{g_m(R_{LD} \parallel R_L)}{1 + g_m(R_{LD} \parallel R_L)} = \frac{(1.90)(1.67 \parallel 4)}{1 + (1.90)(1.67 \parallel 4)} \Rightarrow A_v = 0.691$$

#### 4.58

a. From Problem 4.57.

$$A_v = \frac{g_m(R_{LD} \parallel R_L)}{1 + g_m(R_{LD} \parallel R_L)} = \frac{(1.90)(1.67 \parallel 10)}{1 + (1.90)(1.67 \parallel 10)}$$

$$A_v = 0.731$$

b.

$$R_0 = \frac{1}{g_m} \parallel R_{LD} = \frac{1}{1.90} \parallel 1.67 = 0.526 \parallel 1.67$$

$$R_0 = 0.40 \text{ k}\Omega$$

#### 4.59

(a)  $A_v = -g_{mD}(r_{oD} \parallel r_{oL})$

$$r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.04)(0.5)} = 50 \text{ k}\Omega$$

$$r_{oD} \parallel r_{oL} = 100 \parallel 50 = 33.33 \text{ k}\Omega$$

$$\text{Then } -40 = -g_{mD}(33.33) \Rightarrow g_{mD} = 1.20 \text{ mA/V}$$

$$g_{mD} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D (I_{DQ})}$$

$$1.20 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_D (0.5)} \Rightarrow \left(\frac{W}{L}\right)_D = 14.4$$

(b)  $I_{DQ} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_L (V_{SGQ} + V_{TP})^2$

$$0.5 = \left(\frac{0.04}{2}\right)(50)(V_{SGQ} - 0.4)^2 \Rightarrow V_{SGQ} = 1.107 \text{ V}$$

$$V_{SGQ} = V^+ - V_B$$

$$1.107 = 2.5 - V_B \Rightarrow V_B = 1.393 \text{ V}$$

$$(c) \quad I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D (V_{GSDQ} - V_{TN})^2$$

$$0.5 = \left(\frac{0.1}{2}\right)(14.4)(V_{GSDQ} - 0.4)^2 \Rightarrow V_{GSDQ} = 1.233 \text{ V}$$

**4.60**

(a) (i)  $r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.25)} = 100 \text{ k}\Omega$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$$

$$r_{oD} \parallel r_{oL} = 100 \parallel 200 = 66.67 \text{ k}\Omega$$

$$A_v = -g_{mD}(r_{oD} \parallel r_{oL})$$

$$-25 = -g_{mD}(66.67) \Rightarrow g_{mD} = 0.375 \text{ mA/V}$$

$$g_{mD} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_D I_{DQ}}$$

$$0.375 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_D (0.25)} \Rightarrow \left(\frac{W}{L}\right)_D = 7.03$$

(ii)  $I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L (V_B - V_{TN})^2$

$$0.25 = \left(\frac{0.1}{2}\right)(10)(V_B - 0.4)^2 \Rightarrow V_B = 1.107 \text{ V}$$

(iii)  $I_{DQ} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_D (V_{SGDQ} + V_{TP})^2$

$$0.25 = \left(\frac{0.04}{2}\right)(7.03)(V_{SGDQ} - 0.6)^2 \Rightarrow V_{SGDQ} = 1.933 \text{ V}$$

(b) (i)  $r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.1)} = 250 \text{ k}\Omega$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{oD} \parallel r_{oL} = 250 \parallel 500 = 166.7 \text{ k}\Omega$$

$$A_v = -g_{mD}(r_{oD} \parallel r_{oL})$$

$$-25 = -g_{mD}(166.7) \Rightarrow g_{mD} = 0.15 \text{ mA/V}$$

$$g_{mD} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_D I_{DQ}}$$

$$0.15 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_D (0.1)} \Rightarrow \left(\frac{W}{L}\right)_D = 2.81$$

$$(ii) I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L (V_B - V_{TN})^2$$

$$0.1 = \left(\frac{0.1}{2}\right)(10)(V_B - 0.4)^2 \Rightarrow V_B = 0.874 \text{ V}$$

$$(iii) I_{DQ} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_D (V_{SGDQ} + V_{TP})^2$$

$$0.1 = \left(\frac{0.04}{2}\right)(2.81)(V_{SGDQ} - 0.6)^2 \Rightarrow V_{SGDQ} = 1.934 \text{ V}$$

**4.61**

$$K_{n1} = \left(\frac{85}{2}\right)(50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1}I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.9220$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -(0.922)(200 \parallel 133.3)$$

$$\underline{A_v = -73.7}$$

**4.62**

$$K_{p1} = \frac{k'_p}{2}\left(\frac{w}{L}\right) = \left(\frac{40}{2}\right)(50) \Rightarrow 1.0 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -(0.6325)(133.3 \parallel 200)$$

$$\underline{A_v = -50.6}$$

**4.63**

(a)  $I_{DD} = I_{DL}$

$$K_{nD}(V_{GSD} - V_{TND})^2 = K_{nL}(V_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{2}{0.5}}(V_I - V_O - 0.4) = V_O - 0.4$$

$$2V_I - 0.8 = 3V_O - 0.4$$

$$V_O = \left(\frac{2}{3}\right)V_I - \left(\frac{1}{3}\right)(0.4)$$

For  $V_I = 0.8 \text{ V}$ ,  $V_O = 0.4 \text{ V}$

For  $V_I = 2.5 \text{ V}$ ,  $V_O = 1.533 \text{ V}$

$$(b) \quad I_D = K_{nL} (V_{GSL} - V_{TNL})^2 = (0.5)(V_O - 0.4)^2$$

$$I_D = (0.5) \left[ \left( \frac{2}{3} \right) V_I - \left( \frac{1}{3} \right) (0.4) - 0.4 \right]^2 = (0.5) \left[ \left( \frac{2}{3} \right) V_I - 0.533 \right]^2$$

For  $V_I = 0.8 \text{ V}$ ,  $I_D = 0$ ; For  $V_I = 2.5 \text{ V}$ ,  $I_D = 0.642 \text{ mA}$

$$(c) \quad \text{From (a), voltage gain} = \text{constant} = 2/3 = 0.667$$

#### 4.64

$$(a) \quad V_{SD}(\text{sat}) = V_{SG} + V_{TP} = (2.5 - 1) - 0.4 = 1.1 \text{ V}$$

$$V_{SD} = V^+ - V_O \Rightarrow V_O(\text{max}) = V^+ - V_{SD}(\text{sat}) = 2.5 - 1.1 = 1.4 \text{ V}$$

$$(b) \quad I_D = \left( \frac{k'_p}{2} \right) \left( \frac{W}{L} \right)_L (V_{SGL} + V_{TP})^2 = \left( \frac{0.04}{2} \right) (5) (2.5 - 1 - 0.4)^2 = 0.121 \text{ mA}$$

$$(c) \quad I_D = \left( \frac{k'_p}{2} \right) \left( \frac{W}{L} \right)_D (V_{SGD} + V_{TP})^2$$

$$0.121 = \left( \frac{0.04}{2} \right) (50) (V_{SGD} - 0.4)^2 \Rightarrow V_{SGD} = 0.748 \text{ V}$$

$$(d) \quad g_{mD} = 2 \sqrt{\left( \frac{k'_p}{2} \right) \left( \frac{W}{L} \right)_D I_D} = 2 \sqrt{\left( \frac{0.04}{2} \right) (50) (0.121)} = 0.6957 \text{ mA/V}$$

$$r_{oD} = r_{oL} = \frac{1}{\lambda I_D} = \frac{1}{(0.025)(0.121)} = 330.6 \text{ k}\Omega$$

$$A_v = \frac{g_{mD} (r_{oD} \parallel r_{oL})}{1 + g_{mD} (r_{oD} \parallel r_{oL})} = \frac{(0.6957)(165.3)}{1 + (0.6957)(165.3)} = 0.9914$$

#### 4.65

$$(a) \quad I_{DQ} = K_n (V_{GSDQ} - V_{TN})^2$$

$$1 = 2(V_{GSDQ} - 0.6)^2 \Rightarrow V_{GSDQ} = 1.307 \text{ V}$$

$$I_{DQ} = K_p (V_{SGLQ} + V_{TP})^2$$

$$1 = 0.5(V_{SGLQ} - 0.6)^2 \Rightarrow V_{SGLQ} = 2.014 \text{ V}$$

$$V_O = 3.3 - 2.014 = 1.286 \text{ V}$$

$$V_{DSDQ} = V_O - V_S = 1.286 - (-1.307) = 2.593 \text{ V}$$

$$(b) \quad I_d = I_l$$

$$g_{mD} V_i = g_{mL} V_o$$

$$A_v = \frac{V_o}{V_i} = \frac{g_{mD}}{g_{mL}} = \sqrt{\frac{K_n}{K_p}}$$

$$(c) \quad A_v = \sqrt{\frac{2}{0.5}} = 2$$

**4.66**

(a)

$$K_{n1} = \left(\frac{85}{2}\right)(50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1}I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.922 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

(b)

$$R_{i1} = \frac{1}{g_{m1}} = \frac{1}{0.922} = 1.085 \text{ K}$$

$$V_{gs1} = -\left(\frac{R_{i1}}{R_{i1} + 0.050}\right)V_i = -\left(\frac{1.085}{1.085 + 0.050}\right)V_i = -0.956V_i$$

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) \cdot \frac{V_{gs1}}{V_i} = +(0.956)(0.922)(200)(133.3)$$

$$\underline{A_v = 70.5}$$

(c)

$$R_i = 0.05 + \frac{1}{g_{m1}} = 0.05 + \frac{1}{0.922} \Rightarrow \underline{R_i = 1.135 \text{ K}}$$

(d)

$$R_o \approx r_{o1} \parallel r_{o2} = 200 \parallel 133.7 \Rightarrow \underline{R_o \approx 80 \text{ K}}$$

**4.67**

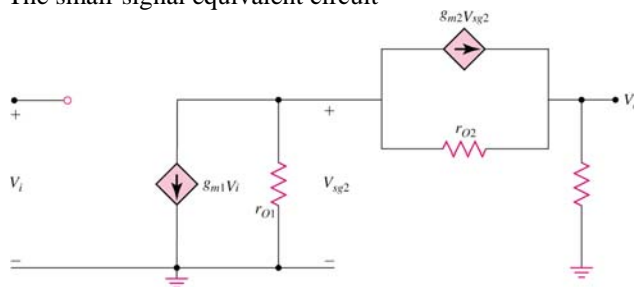
(a)

$$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.1)} = 100 \text{ K}$$

(b) The small-signal equivalent circuit



$$(1) \quad g_{m1}V_i + \frac{V_{sg2}}{r_{o1}} + g_{m2}V_{sg2} + \frac{V_{sg2} - V_o}{r_{o2}} = 0$$



(2)

$$\frac{V_o}{r_o} + \frac{V_o - V_{sg2}}{r_{o2}} = g_{m2} V_{sg2}$$

$$V_o \left( \frac{1}{r_o} + \frac{1}{r_{o2}} \right) = V_{sg2} \left( \frac{1}{r_{o2}} + g_{m2} \right)$$

$$V_o \left( \frac{1}{50} + \frac{1}{100} \right) = V_{sg2} \left( \frac{1}{100} + 0.8944 \right) \Rightarrow V_{sg2} = V_o (0.03317)$$

(1)

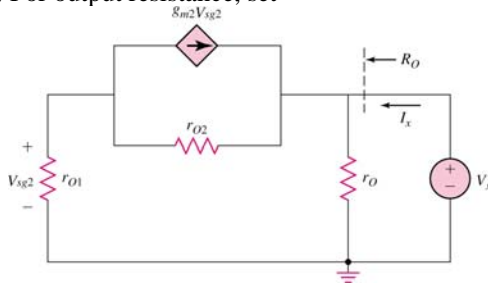
$$g_{m1} V_i + V_{sg2} \left( \frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) = \frac{V_o}{r_{o2}}$$

$$0.8944 V_i + V_o (0.03317) \left( \frac{1}{100} + 0.8944 + \frac{1}{100} \right) = \frac{V_o}{100}$$

$$0.8944 V_i = V_o (0.01 - 0.03033)$$

$$\frac{V_o}{V_i} = -44$$

(c) For output resistance, set  $V_i = 0$ .



(1)

$$g_{m2} V_{sg2} + I_x = \frac{V_x}{r_o} + \frac{V_x - V_{sg2}}{r_{o2}}$$

(2)

$$\frac{V_{sg2}}{r_{o1}} + g_{m2} V_{sg2} + \frac{V_{sg2} - V_x}{r_{o2}} = 0$$

(2)

$$V_{sg2} \left( \frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) = \frac{V_x}{r_{o2}}$$

$$V_{sg2} \left( \frac{1}{100} + 0.8944 + \frac{1}{100} \right) = \frac{V_x}{100}$$

$$V_{sg2} = V_x (0.010936)$$

(1)

$$I_x = V_x \left( \frac{1}{r_o} + \frac{1}{r_{o2}} \right) - V_{sg2} \left( \frac{1}{r_{o2}} + g_{m2} \right)$$

$$I_x = V_x \left( \frac{1}{50} + \frac{1}{100} \right) - V_x (0.010936) \left( \frac{1}{100} + 0.8944 \right)$$

$$I_x = V_x (0.03 - 0.0098905)$$

$$R_o = \frac{V_x}{I_x} = 49.7 \text{ K}$$

**4.68**

$$\begin{aligned}
 \text{(a)} \quad I_{DQ1} &= K_{n1}(V_{GSQ1} - V_{TN1})^2 \\
 0.2 &= 0.2(V_{GSQ1} - 0.6)^2 \Rightarrow V_{GSQ1} = 1.6 \text{ V} \\
 I_{DQ2} &= K_{p2}(V_{SGQ2} + V_{TP2})^2 \\
 0.5 &= 1.0(V_{SGQ2} - 0.6)^2 \Rightarrow V_{SGQ2} = 1.307 \text{ V} \\
 R_{S1} &= \frac{0.6}{0.2} = 3 \text{ k}\Omega \\
 V_{D1} &= 0.6 + 2 = 2.6 \text{ V} \\
 R_{D1} &= \frac{5 - 2.6}{0.2} = 12 \text{ k}\Omega \\
 V_{G1} &= V_{GSQ1} + 0.6 = 1.6 + 0.6 = 2.2 \text{ V} \\
 V_{G1} = 2.2 &= \left( \frac{R_2}{R_1 + R_2} \right) (5) = \frac{1}{R_1} \cdot R_{in} \cdot (5) = \frac{1}{R_1} (400)(5)
 \end{aligned}$$

$$\text{Or } R_1 = 909 \text{ k}\Omega \text{ and } R_1 \parallel R_2 = R_{in} = 400 \text{ k}\Omega \Rightarrow R_2 = 714 \text{ k}\Omega$$

$$V_{S2} = V_{D1} + V_{SGQ2} = 2.6 + 1.307 = 3.907 \text{ V}$$

$$R_{S2} = \frac{5 - 3.907}{0.5} = 2.19 \text{ k}\Omega$$

$$V_{D2} = V_{S2} - 3 = 3.907 - 3 = 0.907 \text{ V}$$

$$R_{D2} = \frac{0.907}{0.5} = 1.81 \text{ k}\Omega$$

$$\text{(b)} \quad g_{m1} = 2\sqrt{K_{n1}I_{DQ1}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2}I_{DQ2}} = 2\sqrt{(1)(0.5)} = 1.414 \text{ mA/V}$$

$$A_v = (-g_{m1}R_{D1})(-g_{m2}R_{D2}) = g_{m1}g_{m2}R_{D1}R_{D2}$$

$$A_v = (0.4)(1.414)(12)(1.81) = 12.3$$

**4.69**

$$\begin{aligned}
 \text{(a)} \quad I_{DQ1} &= K_{n1}(V_{GS1} - V_{TN1})^2 \\
 0.1 &= 0.2(V_{GS1} - 0.6)^2 \Rightarrow V_{GSQ1} = 1.307 \text{ V} \\
 I_{DQ2} &= K_{p2}(V_{SGQ2} + V_{TP})^2 \\
 0.25 &= 1.0(V_{SGQ2} - 0.6)^2 \Rightarrow V_{SGQ2} = 1.10 \text{ V} \\
 V_{G1} &= V_{GSQ1} + I_{DQ1}R_{S1} = 1.307 + (0.1)(1) = 1.407 \text{ V} \\
 V_{G1} &= \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \\
 1.407 &= \frac{1}{R_1} (250)(3.3) \Rightarrow R_1 = 586 \text{ k}\Omega \\
 R_1 \parallel R_2 &= R_{in} = 250 \text{ k}\Omega \Rightarrow R_2 = 436 \text{ k}\Omega \\
 V_{D1} &= I_{DQ1}R_{S1} + V_{DSQ1} = (0.1)(1) + 1.2 = 1.3 \text{ V} \\
 R_{D1} &= \frac{3.3 - 1.3}{0.1} = 20 \text{ k}\Omega
 \end{aligned}$$

$$V_{S2} = V_{D1} + V_{SGQ2} = 1.3 + 1.1 = 2.4 \text{ V}$$

$$R_{S2} = \frac{3.3 - 2.4}{0.25} = 3.6 \text{ k}\Omega$$

$$V_{D2} = V_{S2} - V_{SDQ2} = 2.4 - 1.8 = 0.6 \text{ V}$$

$$R_{D2} = \frac{0.6}{0.25} = 2.4 \text{ k}\Omega$$

$$(b) \quad g_{m1} = 2\sqrt{K_{n1}I_{DQ1}} = 2\sqrt{(0.2)(0.1)} = 0.2828 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2}I_{DQ2}} = 2\sqrt{(1)(0.25)} = 1.0 \text{ mA/V}$$

$$A_v = g_{m1}g_{m2}R_{D1}R_{D2} = (0.2828)(1.0)(20)(2.4) = 13.6$$

#### 4.70

$$K_{p1} = \left(\frac{0.04}{2}\right)(20) = 0.4 \text{ mA/V}^2$$

$$K_{n2} = \left(\frac{0.1}{2}\right)(80) = 4.0 \text{ mA/V}^2$$

$$(a) \quad R_{S1} = \frac{0.6}{0.1} = 6 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1 = 0.2 \text{ V}$$

$$R_{D1} = \frac{0.2 - (-1.8)}{0.1} = 20 \text{ k}\Omega$$

$$I_{DQ1} = K_{p1}(V_{SGQ1} + V_{TP})^2$$

$$0.1 = 0.4(V_{SGQ1} - 0.4)^2 \Rightarrow V_{SGQ1} = 0.90 \text{ V}$$

$$I_{DQ2} = K_{n2}(V_{GSQ2} - V_{TN})^2$$

$$0.3 = 4(V_{GSQ2} - 0.4)^2 \Rightarrow V_{GSQ2} = 0.6739 \text{ V}$$

$$V_{G1} = 1.8 - 0.6 - V_{SGQ1} = 1.8 - 0.6 - 0.9 = 0.3 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)(3.6) - 1.8$$

$$0.3 = \frac{1}{R_1}(200)(3.6) - 1.8 \Rightarrow R_1 = 343 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 200 \text{ k}\Omega \Rightarrow R_2 = 480 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1.0 = 0.2 \text{ V}$$

$$V_{S2} = V_{D1} - V_{GSQ2} = 0.2 - 0.6739 = -0.4739 \text{ V}$$

$$R_{S2} = \frac{-0.4739 - (-1.8)}{0.3} = 4.42 \text{ k}\Omega$$

$$(b) \quad A_v = \left(\frac{-g_{m1}R_{D1}}{1 + g_{m1}R_{S1}}\right)\left(\frac{g_{m2}R_{S2}}{1 + g_{m2}R_{S2}}\right)$$

$$g_{m1} = 2\sqrt{K_{p1}I_{DQ1}} = 2\sqrt{(0.4)(0.1)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{DQ2}} = 2\sqrt{(4)(0.3)} = 2.191 \text{ mA/V}$$

$$A_v = \frac{-(0.4)(20)}{1 + (0.4)(6)} \cdot \frac{(2.191)(4.42)}{1 + (2.191)(4.42)} = -2.13$$

$$(c) R_o = \frac{1}{g_{m2}} \parallel R_{S2} = \frac{1}{2.191} \parallel 4.42 = 0.4564 \parallel 4.42 \Rightarrow R_o = 414 \Omega$$

**4.71**

(a)

$$I_{DQ1} = \frac{10 - V_{GS1}}{R_{S2}} = K_{n1} (V_{GS1} - V_{TN1})^2$$

$$10 - V_{GS1} = (4)(10)(V_{GS1}^2 - 4V_{GS1} + 4)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)} \Rightarrow V_{GS1} = 2.435 \text{ V}$$

$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow I_{DQ1} = 0.757 \text{ mA}$$

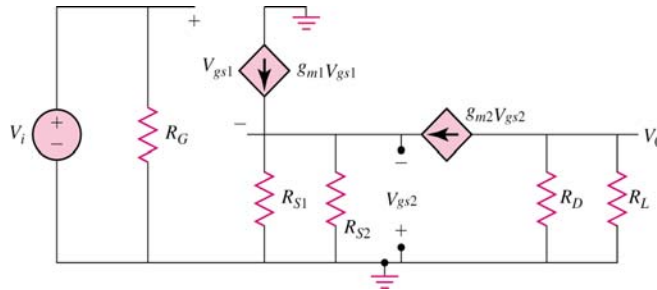
$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow V_{DSQ1} = 12.4 \text{ V}$$

Also  $I_{DQ2} = 0.757 \text{ mA}$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow V_{DSQ2} = 8.65 \text{ V}$$

(b)  $g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow g_{m1} = g_{m2} = 3.48 \text{ mA/V}$

c.



$$V_o = -(g_{m2}V_{gs2})(R_D \parallel R_L)$$

$$V_{gs2} = (-g_{m1}V_{gs1} - g_{m2}V_{gs2})(R_{S1} \parallel R_{S2})$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

$$V_{gs2} + g_{m2}V_{gs2}(R_{S1} \parallel R_{S2}) = -g_{m1}(V_i + V_{gs2})(R_{S1} \parallel R_{S2})$$

$$V_{gs2} + g_{m2}V_{gs2}(R_{S1} \parallel R_{S2}) + g_{m1}V_{gs2}(R_{S1} \parallel R_{S2}) = -g_{m1}V_i(R_{S1} \parallel R_{S2})$$

$$V_{gs2} = \frac{-g_{m1}V_i(R_{S1} \parallel R_{S2})}{1 + g_{m2}(R_{S1} \parallel R_{S2}) + g_{m1}(R_{S1} \parallel R_{S2})}$$

$$A_v = \frac{V_o}{V_i} = \frac{g_{m1}g_{m2}(R_{S1} \parallel R_{S2})(R_D \parallel R_L)}{1 + (g_{m1} + g_{m2})(R_{S1} \parallel R_{S2})}$$

$$A_v = \frac{(3.48)^2(10 \parallel 10)(5 \parallel 2)}{1 + (3.48 + 3.48)(10 \parallel 10)} \Rightarrow A_v = 2.42$$

**4.72**

a.

$$I_{DQ} = 3 \text{ mA}$$

$$V_{S1} = I_{DQ}R_S - 5 = (3)(1.2) - 5 = -1.4 \text{ V}$$

$$I_{DQ} = K_1(V_{GS} - V_{TN})^2$$

$$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.225 \text{ V}$$

$$V_{G1} = V_{GS} + V_{S1} = 2.225 - 1.4 = 0.825 \text{ V}$$

$$V_{G1} = \left( \frac{R_3}{R_1 + R_2 + R_3} \right) (5) \Rightarrow 0.825 = \left( \frac{R_3}{500} \right) (5) \Rightarrow \underline{R_3 = 82.5 \text{ k}\Omega}$$

$$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.225 = 3.325 \text{ V}$$

$$V_{G2} = \left( \frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) (5) \Rightarrow 3.325 = \left( \frac{R_2 + R_3}{500} \right) (5)$$

$$R_2 + R_3 = 332.5 \Rightarrow \underline{R_2 = 250 \text{ k}\Omega}$$

$$R_1 = 500 - 250 - 82.5 \Rightarrow \underline{R_1 = 167.5 \text{ k}\Omega}$$

$$V_{D2} = V_{D1} + V_{DSQ2} = 1.1 + 2.5 = 3.6 \text{ V}$$

$$R_D = \frac{5 - 3.6}{3} \Rightarrow \underline{R_D = 0.467 \text{ k}\Omega}$$

b.

$$A_v = -g_{m1}R_D$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90 \text{ mA/V}$$

$$A_v = -(4.90)(0.467) \Rightarrow \underline{A_v = -2.29}$$

**4.73**

a.

$$V_{S1} = I_{DQ}R_S - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$$

$$I_{DQ} = K_1(V_{GS1} - V_{TN})^2$$

$$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618 \text{ V}$$

$$V_{G1} = V_{GS1} + V_{S1} = 2.618 \text{ V} = IR_S = (0.1)R_3 \Rightarrow \underline{R_3 = 26.2 \text{ k}\Omega}$$

$$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12 \text{ V}$$

$$= (0.1)(R_2 + R_3)$$

$$R_2 + R_3 = 61.2 \text{ k}\Omega \Rightarrow \underline{R_2 = 35 \text{ k}\Omega}$$

$$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0 \text{ V}$$

$$R_D = \frac{10 - 7}{5} \Rightarrow \underline{R_D = 0.6 \text{ k}\Omega}$$

$$R_1 = \frac{10 - 6.12}{0.1} \Rightarrow \underline{R_1 = 38.8 \text{ k}\Omega}$$

b.

$$A_v = -g_{m1} R_D$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.944 \text{ mA/V}$$

$$A_v = -(8.944)(0.6) \Rightarrow \underline{A_v = -5.37}$$

**4.74**

a.

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$4 = 6 \left(1 - \frac{V_{GS}}{(-3)}\right)^2$$

$$V_{GS} = (-3) \left[1 - \sqrt{\frac{4}{6}}\right] \Rightarrow \underline{V_{GS} = -0.551 \text{ V}}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D$$

$$6 = 10 - (4) R_D \Rightarrow \underline{R_D = 1 \text{ k}\Omega}$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_P)} \left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(6)}{3} \left(1 - \frac{-0.551}{-3}\right) \Rightarrow \underline{g_m = 3.265 \text{ mA/V}}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(4)} \Rightarrow \underline{r_o = 25 \text{ k}\Omega}$$

c.  $A_v = -g_m (r_o \parallel R_D) = -(3.265)(25 \parallel 1) \Rightarrow \underline{A_v = -3.14}$

**4.75**

$$V_{GS} + I_{DQ}(R_{S1} + R_{S2}) = 0$$

$$I_{DQ} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$V_{GS} + I_{DSS}(R_{S1} + R_{S2}) \left( 1 - \frac{V_{GS}}{V_P} \right)^2 = 0$$

$$V_{GS} + (2)(0.1 + 0.25) \left( 1 - \frac{V_{GS}}{V_P} \right)^2 = 0$$

$$V_{GS} + 0.7 \left( 1 - \frac{2V_{GS}}{(-2)} + \frac{V_{GS}^2}{(-2)^2} \right) = 0$$

$$0.175V_{GS}^2 + 1.7V_{GS} + 0.7 = 0$$

$$V_{GS} = \frac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)} \Rightarrow V_{GS} = -0.4314 \text{ V}$$

$$g_m = \frac{2I_{DSS}}{-V_P} \left( 1 - \frac{V_{GS}}{V_P} \right) = \frac{2(2)}{-2} \left( 1 - \frac{-0.431}{-2} \right) \Rightarrow g_m = 1.569 \text{ mA/V}$$

$$A_v = \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_{S1}} = \frac{-(1.569)(8 \parallel 4)}{1 + (1.569)(0.1)} \Rightarrow A_v = -3.62$$

$$A_i = \frac{i_o}{i_i} = \frac{(v_o / R_L)}{(v_i / R_G)} = \frac{v_o}{v_i} \cdot \frac{R_G}{R_L} = (-3.62) \left( \frac{50}{4} \right) \Rightarrow A_i = -45.2$$

**4.76**

$$I_{DQ} = \frac{I_{DSS}}{2} = 4 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = 10 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$10 = 20 - (4)(R_S + R_D) \Rightarrow R_S + R_D = 2.5 \text{ k}\Omega$$

$$V_S = 2 \text{ V} = I_{DQ}R_S = 4R_S \Rightarrow R_S = 0.5 \text{ k}\Omega, \quad R_D = 2.0 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4 = 8 \left( 1 - \frac{V_{GS}}{(-4.2)} \right)^2 \Rightarrow V_{GS} = (-4.2) \left( 1 - \sqrt{\frac{4}{8}} \right) \Rightarrow V_{GS} = -1.23 \text{ V}$$

$$V_G = V_S + V_{GS} = 2 - 1.23$$

$$V_G = 0.77 \text{ V} = \left( \frac{R_2}{R_1 + R_2} \right) (20) = \left( \frac{R_2}{100} \right) (20) \Rightarrow R_2 = 3.85 \text{ k}\Omega, \quad R_1 = 96.2 \text{ k}\Omega$$

**4.77**

a.

$$I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$$

$$R_S = \frac{12-6}{5} \Rightarrow \underline{R_S = 1.2 \text{ k}\Omega}$$

$$I_{DQ} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 = 10 \left( 1 - \frac{V_{GS}}{-5} \right)^2 \Rightarrow V_{GS} = (-5) \left( 1 - \sqrt{\frac{5}{10}} \right) \Rightarrow V_{GS} = -1.464 \text{ V}$$

$$V_G = V_S + V_{GS} = 6 - 1.464 = 4.536 \text{ V}$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$4.536 = \frac{1}{R_1} (100)(12) \Rightarrow \underline{R_1 = 265 \text{ k}\Omega}$$

$$\frac{265R_2}{265 + R_2} = 100 \Rightarrow \underline{R_2 = 161 \text{ k}\Omega}$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_P)} \left( 1 - \frac{V_{GS}}{V_P} \right) = \frac{2(10)}{5} \left( 1 - \frac{-1.46}{-5} \right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)}$$

$$A_v = \frac{(2.83)(20 \parallel 1.2 \parallel 0.5)}{1 + (2.83)(20 \parallel 1.2 \parallel 0.5)} \Rightarrow \underline{A_v = 0.495}$$

$$R_0 = \frac{1}{g_m} \parallel R_S = \frac{1}{2.83} \parallel 1.2 = 0.353 \parallel 1.2 \Rightarrow \underline{R_0 = 0.273 \text{ k}\Omega}$$



4.78

a.

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{110}{110 + 90} \right) (10) = 5.5 \text{ V}$$

$$I_{DQ} = \frac{10 - (V_G - V_{GS})}{R_S} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$10 - 5.5 + V_{GS} = (2)(5) \left( 1 - \frac{V_{GS}}{1.75} \right)^2$$

$$4.5 + V_{GS} = 10 \left( 1 - 1.143V_{GS} + 0.3265V_{GS}^2 \right)$$

$$3.265V_{GS}^2 - 12.43V_{GS} + 5.5 = 0$$

$$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)} \Rightarrow V_{GS} = 0.511 \text{ V}$$

$$I_{DQ} = (2) \left( 1 - \frac{0.511}{1.75} \right)^2 \Rightarrow I_{DQ} = 1.00 \text{ mA}$$

$$V_{SDQ} = 10 - (1.00)(5) \Rightarrow V_{SDQ} = 5.0 \text{ V}$$

b.

$$g_m = \frac{2I_{DSS}}{V_P} \left( 1 - \frac{V_{GS}}{V_P} \right) = \frac{2(2)}{1.75} \left( 1 - \frac{0.511}{1.75} \right) \Rightarrow g_m = 1.618 \text{ mA/V}$$

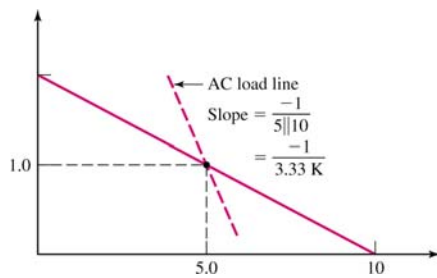
$$A_v = \frac{g_m (R_S \parallel R_L)}{1 + g_m (R_S \parallel R_L)} = \frac{(1.618)(5 \parallel 10)}{1 + (1.618)(5 \parallel 10)} \Rightarrow A_v = 0.844$$

$$A_i = \frac{i_o}{i_i} = \frac{(v_o / R_L)}{(v_i / R_i)} = A_v \cdot \left( \frac{R_i}{R_L} \right)$$

$$R_i = R_1 \parallel R_2 = 90 \parallel 110 = 49.5 \text{ k}\Omega$$

$$A_i = (0.844) \left( \frac{49.5}{10} \right) \Rightarrow A_i = 4.18$$

c.



$$\Delta i_d = 1.0 \text{ mA}$$

$$v_{sd} = (3.33)(1.0) = 3.33 \text{ V}$$

Maximum swing in output voltage = 6.66 V peak-to-peak

4.79

$$I_{DQ} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4 = 8 \left( 1 - \frac{V_{GS}}{4} \right)^2 \Rightarrow V_{GS} = 4 \left( 1 - \sqrt{\frac{4}{8}} \right) \Rightarrow V_{GS} = 1.17 \text{ V}$$

$$V_{SDQ} = V_{DD} - I_{DQ} (R_S + R_D)$$

$$7.5 = 20 - 4(R_S + R_D) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$$

$$g_m = \frac{2I_{DSS}}{V_P} \left( 1 - \frac{V_{GS}}{V_P} \right) = \frac{2(8)}{4} \left( 1 - \frac{1.17}{4} \right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$R_S = 3.125 - R_D$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-3(1 + g_m R_S) = -g_m R_D$$

$$3[1 + (2.83)(3.125 - R_D)] = (2.83) R_D$$

$$9.844 - 2.83R_D = 0.9433R_D \Rightarrow \underline{R_D = 2.61 \text{ k}\Omega} \quad \underline{R_S = 0.516 \text{ k}\Omega}$$

$$V_S = 20 - (4)(0.516) \Rightarrow V_S = 17.94 \text{ V}$$

$$V_G = V_S - V_{GS} = 17.94 - 1.17 = 16.77 \text{ V}$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{R_2}{400} \right) (20) \Rightarrow \underline{R_2 = 335 \text{ k}\Omega} \quad \underline{R_1 = 65 \text{ k}\Omega}$$

---

## Chapter 5

### 5.1

(a)  $i_E = (1 + \beta)i_B \Rightarrow 1 + \beta = \frac{325}{2.8} = 116 \Rightarrow \beta = 115$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{115}{116} = 0.9914$$

$$i_C = i_E - i_B = 325 - 2.8 = 322 \mu\text{A}$$

(b)  $1 + \beta = \frac{1.80}{0.020} = 90 \Rightarrow \beta = 89$

$$\alpha = \frac{89}{90} = 0.9889$$

$$i_C = 1.80 - 0.02 = 1.78 \text{ mA}$$

---

### 5.2

(a)  $\alpha = \frac{i_C}{i_E} = \frac{726}{732} = 0.9918$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9918}{1 - 0.9918} = 121$$

$$i_B = i_E - i_C = 732 - 726 = 6 \mu\text{A}$$

(b)  $\alpha = \frac{2.902}{2.961} = 0.9801$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.980074}{1 - 0.980074} = 49.19$$

$$i_B = 2.961 - 2.902 \Rightarrow i_B = 59 \mu\text{A}$$

---

### 5.3

(a) For  $\beta = 110$ :  $\alpha = \frac{\beta}{1 + \beta} = \frac{110}{111} = 0.99099$

For  $\beta = 180$ :  $\alpha = \frac{180}{181} = 0.99448$

$$0.99099 \leq \alpha \leq 0.99448$$

(b)  $I_C = \beta I_B = 110(50 \mu\text{A}) \Rightarrow I_C = 5.50 \text{ mA}$

or  $I_C = 180(50 \mu\text{A}) \Rightarrow I_C = 9.00 \text{ mA}$

so  $5.50 \leq I_C \leq 9.0 \text{ mA}$

---

**5.4**

(a)  $i_B = \frac{i_E}{1+\beta} = \frac{1.25}{151} \Rightarrow i_B = 8.28 \mu\text{A}$

$$i_C = \left(\frac{\beta}{1+\beta}\right) \cdot i_E = \left(\frac{150}{151}\right)(1.25) = 1.242 \text{ mA}$$

$$\alpha = \frac{150}{151} = 0.9934$$

(b)  $i_B = \frac{4.52}{81} \Rightarrow i_B = 55.8 \mu\text{A}$

$$i_C = \left(\frac{80}{81}\right)(4.52) = 4.46 \text{ mA}$$

$$\alpha = \frac{80}{81} = 0.9877$$

**5.5**

(a)

$\alpha$	$\beta = \frac{\alpha}{1-\alpha}$
0.9	9
0.95	19
0.98	49
0.99	99
0.995	199
0.999	999

(b)

$\beta$	$\alpha = \frac{\beta}{1+\beta}$
20	0.9524
50	0.9804
100	0.9901
150	0.9934
220	0.9955
400	0.9975

**5.6**

(a)  $I_B = \frac{I_E}{1+\beta} = \frac{1.2}{81} \Rightarrow I_B = 14.8 \mu\text{A}$

$$I_C = \left(\frac{\beta}{1+\beta}\right) \cdot I_E = \left(\frac{80}{81}\right)(1.2) = 1.185 \text{ mA}$$

$$\alpha = \frac{80}{81} = 0.9877$$

$$V_C = 5 - (1.185)(2) = 2.63 \text{ V}$$

$$(b) I_B = \frac{0.8}{81} \Rightarrow I_B = 9.88 \mu\text{A}$$

$$I_C = \left(\frac{80}{81}\right)(0.8) = 0.790 \text{ mA}$$

$$\alpha = 0.9877$$

$$V_C = 5 - (0.790)(2) = 3.42 \text{ V}$$

$$(c) (i) I_B = \frac{1.2}{121} \Rightarrow I_B = 9.92 \mu\text{A}$$

$$I_C = \left(\frac{120}{121}\right)(1.2) = 1.19 \text{ mA}$$

$$\alpha = \frac{120}{121} = 0.9917$$

$$V_C = 5 - (1.19)(2) = 2.62 \text{ V}$$

$$(ii) I_B = \frac{0.8}{121} \Rightarrow I_B = 6.61 \mu\text{A}$$

$$I_C = \left(\frac{120}{121}\right)(0.8) = 0.7934 \text{ mA}$$

$$\alpha = 0.9917$$

$$V_C = 5 - (0.7934)(2) = 3.41 \text{ V}$$

---

### 5.7

$$I_E = I_{Eo} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$0.80 \times 10^{-3} = (5 \times 10^{-14}) \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\text{Then } V_{BE} = (0.026) \ln\left(\frac{0.80 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.6109 \text{ V}$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9910}{1-0.9910} = 110$$

$$I_C = \alpha I_E = (0.9910)(0.80) = 0.7928 \text{ mA}$$

$$I_B = \frac{I_E}{1+\beta} = \frac{0.80}{111} \Rightarrow I_B = 7.21 \mu\text{A}$$

$$V_C = 5 - I_C R_C = 5 - (0.7928)(2) = 3.41 \text{ V}$$

---

5.8

$$I_B = \frac{0.75}{61} \Rightarrow 12.3 \mu\text{A}$$

$$I_C = (0.75) \left( \frac{60}{61} \right) = 0.738 \text{ mA}$$

$$\alpha = \frac{60}{61} = 0.9836$$

$$V_C = I_C R_C - 10 = (0.738)(5) - 10$$

(a)  $V_C = -6.31 \text{ V}$

$$I_B = \frac{1.5}{61} \Rightarrow 24.6 \mu\text{A}$$

$$I_C = (1.5) \left( \frac{60}{61} \right) = 1.475 \text{ mA}$$

$$\alpha = \left( \frac{60}{61} \right) = 0.9836$$

(b)  $V_C = (1.475)(5) - 10 \Rightarrow V_C = -2.625 \text{ V}$

(c) Yes,  $V_C < 0$  in both cases so that B-C junction is reverse biased.

5.9

(a)  $V_C = I_C(5) - 10$

$$-1.2 = I_C(5) - 10 \Rightarrow I_C = 1.76 \text{ mA}$$

$$I_E = \frac{I_C}{\alpha} = \frac{1.76}{0.986} = 1.785 \text{ mA}$$

$$I_B = I_E - I_C = 1.785 - 1.76 \Rightarrow I_B = 25 \mu\text{A}$$

(b)  $I_E = I_{E0} \exp\left(\frac{V_{EB}}{V_T}\right)$

$$1.785 \times 10^{-3} = 2 \times 10^{-15} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$V_{EB} = (0.026) \ln\left(\frac{1.785 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.7154 \text{ V}$$

5.10

$$i_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = (5 \times 10^{-15}) \exp\left(\frac{0.615}{0.026}\right)$$

$$i_C = 93.7 \mu\text{A}$$

$$i_B = \frac{93.7}{125} = 0.7495 \mu\text{A}$$

$$i_E = (126)(0.7495) = 94.44 \mu\text{A}$$

**5.11**

Device 1:  $i_E = I_{Eo1} e^{v_{EB}/V_T} \Rightarrow 0.5 \times 10^{-3} = I_{Eo1} e^{0.650/0.026}$

So that

$$\underline{I_{Eo1} = 6.94 \times 10^{-15} \text{ A}}$$

Device 2:  $12.2 \times 10^{-3} = I_{Eo2} e^{0.650/0.026}$

Or

$$\underline{I_{Eo2} = 1.69 \times 10^{-13} \text{ A}}$$

$$\text{Ratio of areas} = \frac{I_{Eo2}}{I_{Eo1}} = \frac{1.69 \times 10^{-13}}{6.94 \times 10^{-15}} \Rightarrow \underline{\text{Ratio} = 24.4}$$

**5.12**

For transistor A:

$$v_{BE}(A) = V_T \ln\left(\frac{I_C}{I_{SA}}\right) = (0.026) \ln\left(\frac{275 \times 10^{-6}}{8 \times 10^{-16}}\right) = 0.6906 \text{ V}$$

For transistor B:

$$I_{SB} = 4I_{SA} = 4(8 \times 10^{-16}) = 3.2 \times 10^{-15} \text{ A}$$

$$v_{BE}(B) = (0.026) \ln\left(\frac{275 \times 10^{-6}}{3.2 \times 10^{-15}}\right) = 0.6546 \text{ V}$$

**5.13**

(a)  $i_C = I_{Co} \left(1 + \frac{v_{CE}}{V_A}\right)$

$$0.6 = I_{Co} \left(1 + \frac{2}{80}\right) \Rightarrow I_{Co} = 0.58537 \text{ mA}$$

At  $v_{CE} = 5 \text{ V}$

$$i_C = (0.58537) \left(1 + \frac{5}{80}\right) = 0.622 \text{ mA}$$

(b)  $r_o = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{5 - 2}{0.621956 - 0.60} \Rightarrow r_o = 137 \text{ k}\Omega$

**5.14**

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}} = \frac{60}{\sqrt[3]{100}}$$

$$\underline{BV_{CE0} = 12.9 \text{ V}}$$

**5.15**

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$56 = \frac{220}{\sqrt[3]{\beta}} \Rightarrow \sqrt[3]{\beta} = \frac{220}{56} = 3.93$$

$$\underline{\beta = 60.6}$$

**5.16**

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$BV_{CB0} = (BV_{CE0})\sqrt[3]{\beta} = (50)\sqrt[3]{50}$$

$$\underline{BV_{CB0} = 184 \text{ V}}$$

**5.17**

$$I_E = \frac{-0.7 - (-10)}{5} = 1.86 \text{ mA}$$

$$I_C = (1.86)\left(\frac{75}{76}\right) = 1.836 \text{ mA}$$

$$V_C = -0.7 + 4 = 3.3 \text{ V}$$

(a)  $R_C = \frac{10 - 3.3}{1.836} \Rightarrow R_C = 3.65 \text{ K}$

$$I_B = \frac{0.5}{76} = 0.00658 \text{ mA}$$

$$V_B = I_B R_B = (0.00658)(25) \Rightarrow V_B = 0.164 \text{ V}$$

$$I_C = (0.5)\left(\frac{75}{76}\right) = \underline{0.493 \text{ mA}}$$

(b)  $R_C = \frac{-1 - (-5)}{0.493} \Rightarrow R_C = 8.11 \text{ K}$

$$0 = \frac{I_E}{76}(10) + 0.7 + I_E(4) - 8$$

$$7.3 = I_E(4 + 0.132) \Rightarrow I_E = 1.767 \text{ mA}$$

$$I_C = (1.767)\left(\frac{75}{76}\right) = \underline{1.744 \text{ mA}}$$

$$V_{CE} = 8 - (1.744)(4) - [(1.767)(4) - 8]$$

(c)  $= 16 - 6.972 - 7.068 \Rightarrow V_{CE} = 1.96 \text{ V}$

$$5 = I_E(10) + \left(\frac{I_E}{76}\right)(20) + 0.7 + I_E(2) = I_E(10 + 0.263 + 2) + 0.7$$

$$I_E = 0.3506 \text{ mA} \Rightarrow \underline{I_B = 4.61 \mu\text{A}} \quad V_C = 5 - (0.3506)(10)$$

(d)  $\underline{V_C = 1.49 \text{ V}}$



**5.18**

For Fig. P5.17(a),  $R_E = 5 + 5\% = 5.25 \text{ k}\Omega$

$$I_E = \frac{-0.7 - (-10)}{5.25} = 1.77 \text{ mA}$$

$$I_C = 1.75 \text{ mA}$$

$$R_C = \frac{10 - 3.3}{1.75} = 3.83 \text{ K}$$

$$R_E = 5 - 5\% = 4.75 \text{ K}$$

$$I_E = \frac{-0.7 - (-10)}{4.75} = 1.96 \text{ mA}$$

$$I_C = 1.93 \text{ mA}$$

$$R_C = \frac{10 - 3.3}{1.93} = 3.47 \text{ K}$$

So  $1.75 \leq I_C \leq 1.93 \text{ mA}$      $3.47 \leq R_C \leq 3.83 \text{ K}$

For Fig. P5.17(c),  $R_E = 4 + 5\% = 4.2 \text{ k}\Omega$

$$I_B = \frac{8 - 0.7}{10 + (76)(4.2)} = 0.0222 \text{ mA} \quad \underline{I_C = 1.66 \text{ mA}}$$

$$I_E = 1.69 \text{ mA}$$

$$V_{CE} = 16 - (1.66)(4) - (1.69)(4.2) \\ = 16 - 6.64 - 7.098 \Rightarrow \underline{V_{CE} = 2.26 \text{ V}}$$

$$R_E = 4 - 5\% = 3.8 \text{ K}$$

$$I_B = \frac{8 - 0.7}{10 + (76)(3.8)} = 0.0244 \quad I_C = 1.83 \text{ mA}$$

$$I_E = 1.86 \text{ mA}$$

$$V_{CE} = 16 - (1.83)(4) - (1.86)(3.8) \\ = 16 - 7.32 - 7.068$$

$$V_{CE} = 1.61 \text{ V}$$

So  $1.66 \leq I_C \leq 1.83 \text{ mA}$      $1.61 \leq V_{CE} \leq 2.26 \text{ V}$

**5.19**

(a)  $V_{CC} = I_C R_C + V_{CE}$

$$I_C = \frac{2.5 - 1.1}{4} = 0.35 \text{ mA}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$V_{BE} = V_{BB} = (0.026) \ln\left(\frac{0.35 \times 10^{-3}}{5 \times 10^{-16}}\right) = 0.7091 \text{ V}$$

---

$$(b) \quad I_E = \frac{V_{CC} - V_{CE}}{R_E} = \frac{2.5 - 1.1}{2} = 0.7 \text{ mA}$$
$$I_C = \left( \frac{\beta}{1 + \beta} \right) \cdot I_E = \left( \frac{90}{91} \right) (0.70) = 0.6923 \text{ mA}$$
$$V_{BE} = (0.026) \ln \left( \frac{0.6923 \times 10^{-3}}{5 \times 10^{-16}} \right) = 0.7269 \text{ V}$$
$$V_{BB} = V_{BE} + I_E R_E = 0.7269 + (0.7)(2) = 2.127 \text{ V}$$

---

**5.20**

(a)  $I_C = 0$ ,  $V_{CE} = 2 \text{ V}$

(b)  $I_C = \beta I_B = (120)(2) \Rightarrow I_C = 0.24 \text{ mA}$   
 $V_{CE} = 2 - (0.24)(4) = 1.04 \text{ V}$

(c)  $I_E = \frac{1.4 - 0.7}{2} = 0.35 \text{ mA}$   
 $I_C = \left( \frac{120}{121} \right) (0.35) = 0.3471 \text{ mA}$   
 $V_{CE} = 2 - (0.3471)(4) - (0.35)(2) = -0.088 \text{ V}$  - Not possible  
Transistor in Saturation  
 $V_{CE} = 0.2 \text{ V}$   
 $V_E = 0.7 \text{ V} \Rightarrow V_C = 0.9 \text{ V}$   
 $I_C = \frac{2 - 0.9}{4} = 0.275 \text{ mA}$

---

**5.21**

(a)  $I_E = \frac{2 - (0.7 + 0.2)}{1.5} = 0.7333 \text{ mA}$   
 $I_C = \left( \frac{\beta}{1 + \beta} \right) \cdot I_E = \left( \frac{120}{121} \right) (0.7333) = 0.7273 \text{ mA}$   
 $V_{EC} = V_E = 0.9 \text{ V}$

(b)  $I_C = \beta I_B = (120)(15) \Rightarrow I_C = 1.8 \text{ mA}$  - Not possible  
Transistor in saturation  
 $V_{EC} = 0.2 \text{ V}$   
 $I_E = \frac{2 - 0.2}{1.5} = 1.2 \text{ mA}$   
 $I_C = I_E - I_B = 1.2 - 0.015 = 1.185 \text{ mA}$

(c) Transistor cutoff  
 $I_C = 0$ ,  $V_{EC} = 2 \text{ V}$

---

**5.22**

$$(a) \quad I_{BQ} = \frac{V_{BB} - V_{BE}(on)}{R_B} \Rightarrow R_B = \frac{1.3 - 0.7}{0.005} = 120 \text{ k}\Omega$$

$$I_{CQ} = \beta I_{BQ} = (100)(0.005) = 0.5 \text{ mA}$$

$$R_C = \frac{3 - 1.5}{0.5} = 3 \text{ k}\Omega$$

$$(b) \quad \text{For } \beta = 75, \quad I_{CQ} = (75)(0.005) = 0.375 \text{ mA}$$

$$V_{CE} = 3 - (0.375)(3) = 1.875 \text{ V}$$

$$\text{For } \beta = 125, \quad I_{CQ} = (125)(0.005) = 0.625 \text{ mA}$$

$$V_{CE} = 3 - (0.625)(3) = 1.125 \text{ V}$$

$$\text{So } 1.125 \leq V_{CE} \leq 1.875 \text{ V}$$

**5.23**

(a)

$$V_B = -I_B R_B \Rightarrow I_B = \frac{-V_B}{R_B} = \frac{-(-1)}{500}$$

$$I_B = 2.0 \mu\text{A}$$

$$V_E = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{R_E} = \frac{-1.7 + 3}{4.8} = 0.2708 \text{ mA}$$

$$\frac{I_E}{I_B} = (1 + \beta) = \frac{0.2708}{0.002} = 135.4 \Rightarrow \underline{\beta = 134.4}$$

$$\alpha = \frac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9926}$$

$$I_C = \beta I_B \Rightarrow \underline{I_C = 0.269 \text{ mA}}$$

$$V_{CE} = 3 - V_E = 3 - (-1.7) \Rightarrow \underline{V_{CE} = 4.7 \text{ V}}$$

(b)

$$I_E = \frac{5 - 4}{2} \Rightarrow I_E = 0.5 \text{ mA}$$

$$4 = 0.7 + I_B R_B + (I_B + I_C) R_C - 5$$

$$I_B + I_C = I_E$$

$$I_B + I_C = I_E$$

$$4 = 0.7 + I_B (100) + (0.5)(8) - 5$$

$$I_B = 0.043 \Rightarrow \frac{I_E}{I_B} = (1 + \beta) = \frac{0.5}{0.043} = 11.63$$

$$\underline{\beta = 10.63}, \quad \alpha = \frac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9140}$$

**5.24**

$$(a) \quad I_C = \frac{5 - V_B}{10}, \quad I_E = \frac{(V_B - 0.7) + 5}{3}, \quad I_C = \left( \frac{\beta}{1 + \beta} \right) \cdot I_E = \left( \frac{90}{91} \right) \cdot I_E$$

$$\text{Then } \frac{5 - V_B}{10} = \left( \frac{90}{91} \right) \left( \frac{V_B + 4.3}{3} \right) \Rightarrow V_B = -2.136 \text{ V}$$

$$I_E = \frac{-2.136 - 0.7 + 5}{3} = 0.721 \text{ mA}$$

$$(b) \quad V_{CE} = 10 - I_C(10) - I_E(3)$$

$$2 = 10 - I_E \left[ 3 + \left( \frac{90}{91} \right) (10) \right] = 10 - I_E(12.89)$$

$$\text{Then } I_E = 0.6206 \text{ mA}$$

$$\text{And } V_B = 0.7 + (0.6206)(3) - 5 = -2.438 \text{ V}$$

**5.25**

$$(a) \quad I_E = \frac{3.3 - 0.85}{10} = 0.245 \text{ mA}$$

$$I_B = \frac{0.85 - 0.7}{50} \Rightarrow I_B = 3 \mu\text{A}$$

$$I_C = I_E - I_B = 0.245 - 0.003 = 0.242 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.242}{0.003} = 80.67$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{80.67}{81.67} = 0.9878$$

$$V_C = (0.242)(10) - 3.3 = -0.88 \text{ V}$$

$$V_{EC} = 0.85 - (-0.88) = 1.73 \text{ V}$$

$$(b) \quad \beta = (80.67)(1.10) = 88.73$$

$$I_B = \frac{V_E - 0.7}{50}, \quad I_E = \frac{3.3 - V_E}{10} = (89.73) \left( \frac{V_E - 0.7}{50} \right), \Rightarrow V_E = 0.8371 \text{ V}$$

$$I_E = \frac{3.3 - 0.8371}{10} = 0.2463 \text{ mA}$$

$$I_C = \left( \frac{88.73}{89.73} \right) (0.2463) = 0.2435 \text{ mA}$$

$$V_C = (0.2435)(10) - 3.3 = -0.8645 \text{ V}$$

$$V_{EC} = 0.8371 - (-0.8645) = 1.70 \text{ V}$$

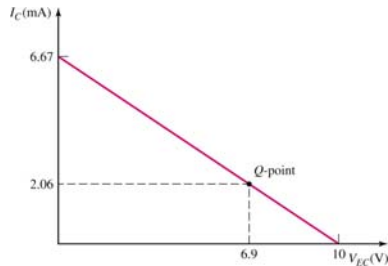
**5.26**

$$I_B = \frac{5 - 0.7}{250} \Rightarrow 17.2 \mu\text{A}$$

$$I_C = (120)(0.0172) = 2.064 \text{ mA}$$

$$V_C = (2.064)(1.5) - 5 = -1.90 \text{ V}$$

$$V_{EC} = 5 - (-1.90) \Rightarrow V_{EC} = 6.90 \text{ V}$$



**5.27**

$$I_C = \left(\frac{50}{51}\right)(1) = 0.98 \text{ mA}$$

$$V_C = I_C R_C - 9 = (0.98)(4.7) - 9 \text{ or } \underline{V_C = -4.39 \text{ V}}$$

$$I_B = \frac{1}{51} = 0.0196 \text{ mA}$$

$$V_E = I_B R_B + V_{EB}(\text{on}) = (0.0196)(50) + 0.7 \text{ or } \underline{V_E = 1.68 \text{ V}}$$

**5.28**

$$I_C = \left(\frac{50}{51}\right)(0.5) = 0.49 \text{ mA}, I_B = \frac{0.5}{51} = 0.0098 \text{ mA}$$

$$V_E = I_B R_B + V_{EB}(\text{on}) = (0.0098)(50) + 0.7 \text{ or } V_E = 1.19 \text{ V}$$

$$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.70 \text{ V}$$

$$\text{Then } V_{EC} = V_E - V_C = 1.19 - (-6.7) = 7.89 \text{ V}$$

$$P_Q = I_C V_{EC} + I_B V_{EB} = (0.49)(7.89) + (0.0098)(0.7) \text{ or } \underline{P_Q = 3.87 \text{ mW}}$$

$$\text{Power Dissipated} = P_S = I_Q(9 - V_E) = (0.5)(9 - 1.19)$$

$$\text{Or } \underline{P_S = 3.91 \text{ mW}}$$

**5.29**

$$I_{E1} = I_{E2} = \frac{I}{2} \Rightarrow \underline{I_{E1} = I_{E2} = 0.5 \text{ mA}}$$

$$I_{C1} = I_{C2} \approx 0.5 \text{ mA}$$

$$V_{C1} = V_{C2} = 5 - (0.5)(4) \Rightarrow \underline{V_{C1} = V_{C2} = 3 \text{ V}}$$

**5.30**

$$R_E = 0 \quad I_B = \frac{2 - 0.7}{R_B} = \frac{1.3}{R_B}$$

$$I_C = (80) \left(\frac{1.3}{R_B}\right) = \frac{5 - 2}{R_C} = 0.8 \Rightarrow R_C = 3.75 \text{ K}$$

(a)  $R_B = 130 \text{ K}$

$$R_E = 1 \text{ K} \quad I_B = \frac{0.8}{80} = 0.010 \text{ mA} \quad I_E = 0.8 \left( \frac{81}{80} \right) = 0.81 \text{ mA}$$

$$2 = (0.010)(R_B) + 0.7 + (0.81)(1) \Rightarrow \underline{R_B = 49 \text{ K}}$$

$$(b) \quad 5 = (0.8)R_C + 2 + (0.81)(1) \Rightarrow \underline{R_C = 2.74 \text{ K}}$$

$$(c) \quad \text{For part (a)} \quad I_B = \frac{2 - 0.7}{130} = 0.01 \text{ mA}$$

$$I_C = (120)(0.01) \Rightarrow \underline{I_C = 1.20 \text{ mA}}$$

$$V_{CE} = 5 - (1.2)(3.75) \Rightarrow \underline{V_{CE} = 0.5 \text{ V}}$$

For part (b)  $2 = I_B(49) + 0.7 + (121)I_B(1)$

$$I_B = 0.00765 \text{ mA}, I_E = 0.925 \text{ mA}, I_C = 0.918 \text{ mA}$$

$$V_{CE} = 5 - (0.918)(2.74) - (0.925)(1) \Rightarrow \underline{V_{CE} = 1.56 \text{ V}}$$

Including  $R_E$  result in smaller changes in Q-point values.

### 5.31

$$(a) \quad R_C = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{9 - 4.5}{0.25} = 18 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.25}{80} \Rightarrow I_{BQ} = 3.125 \mu\text{A}$$

$$R_B = \frac{9 - 0.7}{0.003125} \Rightarrow R_B = 2.656 \text{ M}\Omega$$

$$(b) \quad I_{CQ} = (120)(0.003125) = 0.375 \text{ mA}$$

$$V_{CEQ} = 9 - (0.375)(18) = 2.25 \text{ V}$$

### 5.32

$$(a) \quad I_C = I_E = 0, \quad V_C = 6 \text{ V}$$

$$(b) \quad I_E = \frac{0.9 - 0.7}{1} = 0.2 \text{ mA}, \quad I_C = \left( \frac{150}{151} \right) (0.2) = 0.1987 \text{ mA}$$

$$V_C = 6 - (0.1987)(10) = 4.013 \text{ V}$$

$$(c) \quad I_E = \frac{1.5 - 0.7}{1} = 0.8 \text{ mA}$$

Transistor in saturation

$$V_C = 1.5 - 0.7 + 0.2 = 1 \text{ V}$$

$$I_C = \frac{6 - 1}{10} = 0.5 \text{ mA}$$

$$(d) \quad I_E = \frac{2.2 - 0.7}{1} = 1.5 \text{ mA}$$

$$V_C = 2.2 - 0.7 + 0.2 = 1.7 \text{ V}$$

$$I_C = \frac{6 - 1.7}{10} = 0.43 \text{ mA}$$

**5.33**

$$V_{BB} = 0.$$

$$\text{Cutoff } V_0 = \left( \frac{R_L}{R_C + R_L} \right) V_{CC} = \left( \frac{10}{10 + 5} \right) (5)$$

a.  $V_0 = 3.33 \text{ V}$

$$V_{BB} = 1 \text{ V}$$

$$I_B = \frac{1 - 0.7}{50} \Rightarrow 6 \text{ } \mu\text{A}$$

$$I_C = \beta I_B = (75)(6) \Rightarrow I_C = 0.45 \text{ mA}$$

$$\frac{5 - V_0}{5} = I_C + \frac{V_0}{10}$$

$$1 - 0.45 = V_0 \left( \frac{1}{5} + \frac{1}{10} \right) \Rightarrow V_0 = 1.83 \text{ V}$$

b.

c. Transistor in saturation  $V_0 = V_{CE}(\text{sat}) = 0.2 \text{ V}$

**5.34**

(a)  $\beta = 100$

$$I_Q = 0.1 \text{ mA} \quad I_C = \left( \frac{100}{101} \right) (0.1) = 0.0990 \text{ mA}$$

(i)  $V_o = 5 - (0.099)(5) \Rightarrow V_o = 4.505 \text{ V}$

$$I_Q = 0.5 \text{ mA} \quad I_C = \left( \frac{100}{101} \right) (0.5) = 0.495 \text{ mA}$$

(ii)  $V_o = 5 - (0.495)(5) \Rightarrow V_o = 2.525 \text{ V}$

$$I_Q = 2 \text{ mA} \text{ Transistor is in saturation}$$

(iii)  $V_o = -V_{BE}(\text{sat}) + V_{CE}(\text{sat}) = -0.7 + 0.2 \Rightarrow V_o = -0.5 \text{ V}$

(b)  $\beta = 150$

$$I_Q = 0.1 \text{ mA} \quad I_C = \left( \frac{150}{151} \right) (0.1) = 0.09934 \text{ mA}$$

$$V_o = 5 - (0.09934)(5) \Rightarrow V_o = 4.503 \text{ V}$$

(i)  $\% \text{ change} = \frac{4.503 - 4.505}{4.503} \times 100\% = -0.044\%$

$$I_Q = 0.5 \text{ mA} \quad I_C = \left( \frac{150}{151} \right) (0.5) = 0.4967 \text{ mA}$$

$$V_o = 5 - (0.4967)(5) \Rightarrow V_o = 2.517 \text{ V}$$

(ii)  $\% \text{ change} = \frac{2.517 - 2.525}{2.525} \times 100\% = -0.32\%$

$$I_Q = 2 \text{ mA} \text{ Transistor in saturation}$$

(iii)  $V_o = -8.5 \text{ V}$     No change

**5.35**

(a)  $I_C = \frac{5-4}{5} = 0.20 \text{ mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(0.20) = 0.2017 \text{ mA}$$

(b)  $I_C = \frac{5-2}{5} = 0.60 \text{ mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(0.60) = 0.605 \text{ mA}$$

(c)  $I_C = \frac{5-0}{5} = 1.0 \text{ mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(1.0) = 1.008 \text{ mA}$$

**5.36**

For  $I_Q = 0$ , then  $\underline{P_Q = 0}$

For  $I_Q = 0.5 \text{ mA}$ ,  $I_C = \left(\frac{50}{51}\right)(0.5) = 0.49 \text{ mA}$

$$I_B = \frac{0.5}{51} = 0.0098 \text{ mA}, V_B = 0.490 \text{ V}, V_E = 1.19 \text{ V}$$

$$V_C = (0.49)(4.7) - 9 = -6.70 \text{ V} \Rightarrow V_{EC} = 7.89 \text{ V}$$

$$P \cong I_C V_{EC} = (0.49)(7.89) \Rightarrow \underline{P = 3.87 \text{ mW}}$$

For  $I_Q = 1.0 \text{ mA}$ , Using the same calculations as above, we find  $P = 5.95 \text{ mW}$

For  $I_Q = 1.5 \text{ mA}$ ,  $P = 6.26 \text{ mW}$

For  $I_Q = 2 \text{ mA}$ ,  $P = 4.80 \text{ mW}$

For  $I_Q = 2.5 \text{ mA}$ ,  $P = 1.57 \text{ mW}$

For  $I_Q = 3 \text{ mA}$ , Transistor is in saturation.

$$0.7 + I_B(50) = 0.2 + I_C(4.7) - 9$$

$$I_E = I_Q = I_B + I_C \Rightarrow I_B = 3 - I_C$$

$$\text{Then, } 0.7 + (3 - I_C)(50) = 0.2 + I_C(4.7) - 9$$

Which yields  $I_C = 2.916 \text{ mA}$  and  $I_B = 0.084 \text{ mA}$

$$P = I_B V_{EB} + I_C V_{EC} = (0.084)(0.7) + (2.916)(0.2) \quad \text{or} \quad P = 0.642 \text{ mW}$$

**5.37**

$$I_E = \frac{V_{EE} - V_{EB}(\text{on})}{R_E} = \frac{9 - 0.7}{4} \Rightarrow \underline{I_E = 2.075 \text{ mA}}$$

$$I_C = \alpha I_E = (0.9920)(2.075) \Rightarrow \underline{I_C = 2.06 \text{ mA}}$$

$$V_{BC} + I_C R_C = V_{CC}$$

$$V_{BC} = 9 - (2.06)(2.2) \Rightarrow \underline{V_{BC} = 4.47 \text{ V}}$$



**5.38**

(a)  $I_C = \frac{12-6}{2.2} = 2.727 \text{ mA}$ ,  $I_B = \frac{2.727}{80} = 0.03409 \text{ mA}$

$$I_{R2} = \frac{0.7 - (-12)}{100} = 0.127 \text{ mA}$$

$$I_{R1} = I_B + I_{R2} = 0.1611 \text{ mA}$$

$$V_I = I_{R1}R_1 + 0.7 = (0.1611)(15) + 0.7 = 3.12 \text{ V}$$

(b) For  $V_{CEQ} = 9 \text{ V}$ ,  $I_C = \frac{12-9}{2.2} = 1.364 \text{ mA}$ ,  $I_B = 0.01705 \text{ mA}$

$$I_{R1} = 0.01705 + 0.127 = 0.14405 \text{ mA}$$

$$V_I = (0.14405)(15) + 0.7 = 2.86 \text{ V}$$

For  $V_{CEQ} = 3 \text{ V}$ ,  $I_C = \frac{12-3}{2.2} = 4.0909 \text{ mA}$ ,  $I_B = 0.05114 \text{ mA}$

$$I_{R1} = 0.05114 + 0.127 = 0.1781 \text{ mA}$$

$$V_I = (0.1781)(15) + 0.7 = 3.37 \text{ V}$$

So  $2.86 \leq V_I \leq 3.37 \text{ V}$

**5.39**

For  $V_{CE} = 4.5$

$$I_{CQ} = \frac{5-4.5}{1} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{25} = 0.02 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-5)}{100} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.02 = 0.077 \text{ mA}$$

$$V_I = I_{R1}R_1 + V_{BE}(\text{on}) = (0.077)(15) + 0.7 = 1.86 \text{ V}$$

For  $V_{CE} = 1.0$

$$I_{CQ} = \frac{5-1}{1} = 4 \text{ mA}$$

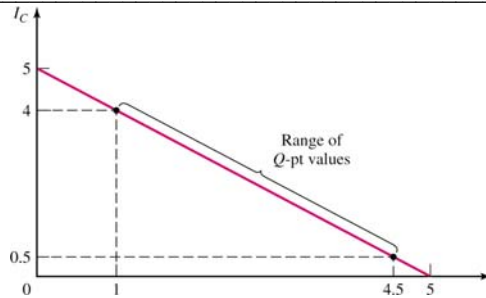
$$I_{BQ} = \frac{4}{25} = 0.16 \text{ mA}$$

$$I_{R2} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.16 = 0.217 \text{ mA}$$

$$V_I = (0.217)(15) + 0.7 \Rightarrow 3.96 \text{ V}$$

So  $1.86 \leq V_I \leq 3.96 \text{ V}$



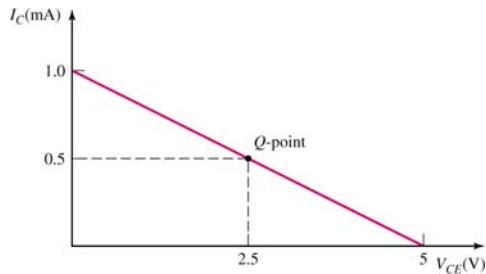
**5.40**

$$R_C = \frac{5 - 2.5}{0.5} = 5 \text{ K}$$

$$I_B = \frac{0.5}{120} = 0.00417 \text{ mA}$$

$$R_B = \frac{5 - 0.7}{0.00417} = 1032 \text{ K}$$

(a)



Choose  $R_C = 5.1 \text{ K}$

$$R_B = 1 \text{ M}\Omega$$

(b)

For  $R_B = 1 \text{ M}\Omega + 10\% = 1.1 \text{ M}$ ,  $R_C = 5.1 \text{ k} + 10\% = 5.61 \text{ K}$

$$I_{BQ} = \frac{5 - 0.7}{1.1} = 3.91 \mu\text{A} \Rightarrow I_{CQ} = 0.469 \text{ mA}$$

$$V_{CEQ} = 2.37 \text{ V}$$

$$R_B = 1 \text{ M}\Omega + 10\% = 1.1 \text{ M}, R_C = 5.1 \text{ K} - 10\% = 4.59 \text{ K}$$

$$I_{BQ} = 3.91 \mu\text{A} \Rightarrow I_{CQ} = 0.469 \text{ mA}$$

$$V_{CEQ} = 2.85 \text{ V}$$

$$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega \quad R_C = 5.1 \text{ k} + 10\% = 5.61 \text{ K}$$

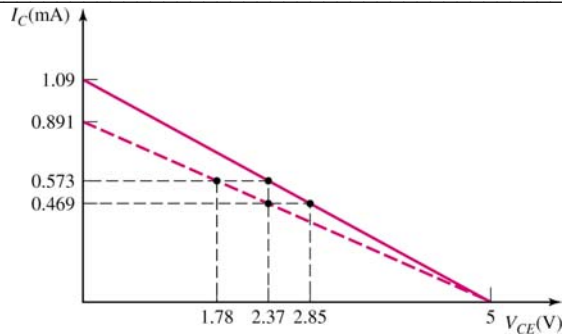
$$I_{BQ} = \frac{5 - 0.7}{0.90} = 4.78 \mu\text{A} \Rightarrow I_C = 0.573 \text{ mA}$$

$$V_{CEQ} = 1.78 \text{ V}$$

$$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega \quad R_C = 5.1 \text{ k} - 10\% = 4.59 \text{ K}$$

$$I_{BQ} = 4.78 \mu\text{A} \Rightarrow I_C = 0.573 \text{ mA}$$

$$V_{CEQ} = 2.37 \text{ V}$$



**5.41**

$$V_{E2} = 5 - V_{BE2} \quad V_{E1} = 5 - V_{BE1}$$

$$V_O = V_{E2} - V_{E1} = (5 - V_{BE2}) - (5 - V_{BE1})$$

$$V_O = V_{BE1} - V_{BE2}$$

We have  $V_{BE1} = V_T \ln \left( \frac{I_{E1}}{I_{EO}} \right)$

$$V_{BE2} = V_T \ln \left( \frac{I_{E2}}{I_{EO}} \right)$$

$$V_O = V_T \left[ \ln \left( \frac{I_{E1}}{I_{EO}} \right) - \ln \left( \frac{I_{E2}}{I_{EO}} \right) \right]$$

$$V_O = V_T \ln \left( \frac{I_{E1}}{I_{E2}} \right) = V_T \ln \left( \frac{10_I}{I} \right)$$

$$\boxed{V_O = \frac{kT}{e} \ln(10)}$$

**5.42**

(a) (i)  $I_C = \frac{5-4}{4} = 0.25 \text{ mA}$ ,  $I_B = \frac{0.25}{120} = 0.002083 \text{ mA}$

$$V_I = (0.002083)(200) + 0.7 = 1.117 \text{ V}$$

(ii)  $I_C = 0.25 \text{ mA}$ ,  $I_E = \left( \frac{121}{120} \right) (0.25) = 0.252 \text{ mA}$

$$V_I = (0.002083)(200) + 0.7 + (0.252)(1) = 1.369 \text{ V}$$

(b) (i)  $I_C = \frac{5-2.5}{4} = 0.625 \text{ mA}$ ,  $I_B = \frac{0.625}{120} = 0.005208 \text{ mA}$

$$V_I = (0.005208)(200) + 0.7 = 1.742 \text{ V}$$

(ii)  $I_E = \left( \frac{121}{120} \right) (0.625) = 0.6302 \text{ mA}$

$$V_I = 1.742 + (0.6302)(1) = 2.372 \text{ V}$$

(c) Transistor biased in saturation

$$3.5 = I_B(200) + 0.7 + I_E(1)$$

$$I_C = \frac{5-V_O}{4}, \quad I_E = \frac{V_O - 0.2}{1}$$

$$I_B = I_E - I_C = \frac{V_o - 0.2}{1} - \left( \frac{5 - V_o}{4} \right) = V_o(1.25) - 1.45$$

$$\text{Then } 3.5 = [V_o(1.25) - 1.45](200) + 0.7 + (V_o - 0.2)$$

$$V_o = 1.167 \text{ V}$$

**5.43**

For  $4.3 \leq V_I \leq 5$  Q is cutoff  $I_C = 0$

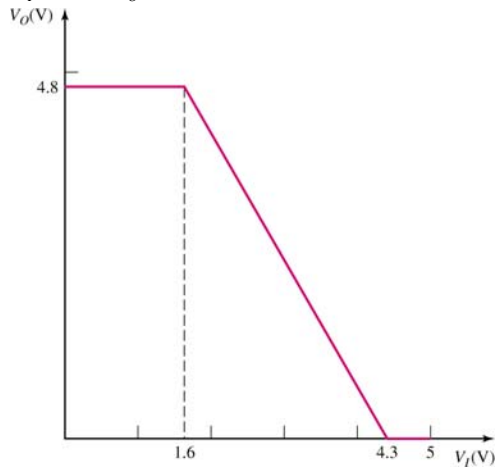
$$V_o = 0$$

If Q reaches saturation,  $V_o = 4.8$

$$I_C = \frac{4.8}{4} = 1.2 \text{ mA}$$

$$I_B = \frac{1.2}{80} = 0.015 = \frac{5 - 0.7 - V_I}{180} \Rightarrow V_I = 1.6$$

So  $V_I \leq 1.6$ ,  $V_o = 4.8$



**5.44**

(a) For  $V_I \geq 4.3$ , Q is off and  $V_O = 0$

$$5 = \left(\frac{101}{100}\right)I_C(1) + 0.2 + I_C(4) \Rightarrow I_C = 0.958 \text{ mA}$$

When transistor enters saturation,

$$V_O = 3.832 \text{ V}$$

$$I_B = 0.00958 \text{ mA}$$

$$5 = \left(\frac{101}{100}\right)(0.958)(1) + 0.7 + (0.00958)(180) + V_I$$

$$V_I = 5 - 0.7 - 0.9676 - 1.7244 \Rightarrow V_I = 1.61 \text{ V}$$

For  $V_I = 0$ , transistor in saturation

$$5 = I_E(1) + 0.2 + I_C(4) \Rightarrow 5 = I_C(1) + I_B(1) + 0.2 + I_C(4)$$

$$5 = I_E(1) + 0.7 + I_B(180) \quad 5 = I_C(1) + I_B(1) + 0.7 + I_B(180)$$

$$I_E = I_C + I_B$$

$$4.8 = 5I_C + I_B(1)$$

$$4.3 = I_C + 181I_B$$

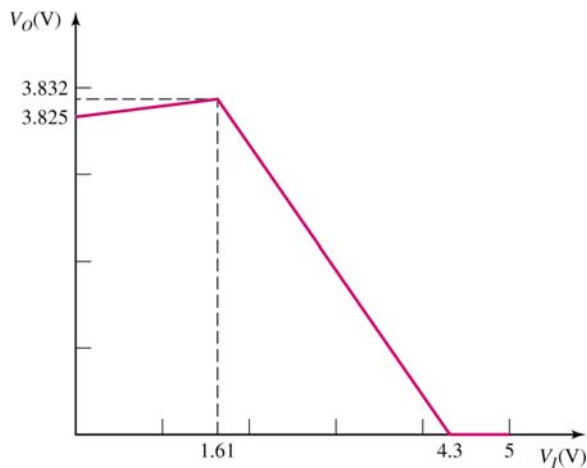
$$I_B = 4.8 - 5I_C$$

$$4.3 = I_C + (181)(4.8 - 5I_C)$$

$$904I_C = 864.5$$

$$I_C = 0.956 \text{ mA}$$

$$V_O = 3.825 \text{ V}$$



**5.45**

$$I_C = \frac{V_{CC} - V_{CE}(sat)}{R_C} = \frac{5 - 0.2}{0.2} = 24 \text{ mA}$$

$$\frac{I_C}{I_B} = 20 \Rightarrow I_B = \frac{24}{20} = 1.2 \text{ mA}$$

$$I_B = \frac{V_I - V_{BE}(on)}{R_B} \Rightarrow R_B = \frac{5 - 0.7}{1.2} = 3.58 \text{ k}\Omega$$

**5.46**

(a)  $V_I = 0, \Rightarrow I_B = I_C = I_E = 0, V_O = 0$

(b)  $V_I = 2.5 \text{ V},$

$$V_I = I_B R_B + V_{BE}(on) + I_E R_E, \text{ and } I_E = (1 + \beta)I_B$$

$$\text{Then } I_B = \frac{V_I - V_{BE}(on)}{R_B + (1 + \beta)R_E} = \frac{2.5 - 0.7}{10 + (51)(0.5)} \Rightarrow I_B = 50.7 \mu\text{A}$$

$$I_C = (50)(0.0507) = 2.535 \text{ mA}, \quad I_E = (51)(0.0507) = 2.586 \text{ mA}$$

$$V_O = I_E R_E = (2.586)(0.5) = 1.293 \text{ V}$$

(c)  $V_I = 5 \text{ V},$  Transistor in saturation

$$V_O = 2.8 \text{ V}, \quad I_E = \frac{2.8}{0.5} = 5.6 \text{ mA}$$

$$I_B = \frac{V_I - V_{BE}(on) - V_O}{R_B} = \frac{5 - 0.7 - 2.8}{10} = 0.15 \text{ mA}$$

$$I_C = I_E - I_B = 5.6 - 0.15 = 5.45 \text{ mA}$$

**5.47**

$$I_C = \frac{V_O}{R_C} = \frac{8.8}{0.5} = 17.6 \text{ mA}$$

$$\frac{I_C}{I_B} = 25 \Rightarrow I_B = \frac{17.6}{25} = 0.704 \text{ mA}$$

$$I_B = \frac{9 - V_{EB}(on) - V_I}{R_B} \Rightarrow R_B = \frac{9 - 0.7 - 5}{0.704} = 4.69 \text{ k}\Omega$$

**5.48**

(a)  $I_{CQ} = \frac{3 - 1.6}{2} = 0.7 \text{ mA}, \quad I_{BQ} = \frac{0.7}{120} \Rightarrow I_{BQ} = 5.833 \mu\text{A}$

$$R_B = \frac{V_{BB} - V_{BE}(on)}{I_{BQ}} = \frac{1 - 0.7}{0.005833} = 51.4 \text{ k}\Omega$$

(b)  $\Delta V_O = 3.3 - 0.2 = 2.8 \text{ V},$  peak-to-peak

(c)  $\frac{\Delta V_O}{\Delta V_I} = \frac{3 - 1.6}{0.7 - 1.0} = A_v = -4.67$

(d)  $\Delta V_I(\text{max}) = \frac{\Delta V_O(\text{max})}{|A_v|} = \frac{2.8}{4.667} = 0.6 \text{ V}$

So  $v_i = 0.6 \text{ V},$  peak-to-peak

**5.49**

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.15}{120} \Rightarrow I_{BQ} = 1.25 \mu\text{A}$$

$$I_{EQ} = I_{CQ} \left( \frac{1 + \beta}{\beta} \right) = (0.15) \left( \frac{121}{120} \right) = 0.15125 \text{ mA}$$

We have  $R_{TH} = 200 \text{ k}\Omega$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + I_{EQ} R_E = (0.00125)(200) + 0.7 + (0.15125)(2) = 1.2525 \text{ V}$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$\text{So } 1.2525 = \frac{1}{R_1} (200)(2.5)$$

$$\Rightarrow R_1 = 399 \text{ k}\Omega \text{ and } R_2 = 401 \text{ k}\Omega$$

$$V_{CEQ} = 2.5 - I_{CQ} R_C - I_{EQ} R_E = 2.5 - (0.15)(6) - (0.15125)(2) = 1.30 \text{ V}$$

**5.50**

$$I_{EQ} = \left( \frac{1 + \beta}{\beta} \right) \cdot I_{CQ} = \left( \frac{151}{150} \right) (0.20) = 0.2013 \text{ mA}, \quad I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.20}{150} \Rightarrow I_{BQ} = 1.33 \mu\text{A}$$

$$V_{CC} = I_{CQ} R_C + V_{CEQ} + I_{EQ} R_E$$

$$2.5 = (0.20)R_C + 1.5 + (0.2013)(1) \Rightarrow R_C = 4 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + I_{EQ} R_E = (0.00133)(120) + 0.7 + (0.2013)(1) = 1.061 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} \Rightarrow 1.061 = \frac{1}{R_1} (120)(2.5)$$

$$\text{So } R_1 = 283 \text{ k}\Omega \text{ and } R_2 = 208 \text{ k}\Omega$$

**5.51**

$$R_{TH} = R_1 \parallel R_2 = 20 \parallel 15 = 8.57 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{15}{15 + 20} \right) (10) = 4.29 \text{ V}$$

$$V_{CC} = I_{EQ} R_E + V_{EB}(on) + \frac{I_{EQ}}{1 + \beta} \cdot R_{TH} + V_{TH}$$

$$10 = I_{EQ} (1) + 0.7 + I_{EQ} \left( \frac{8.57}{101} \right) + 4.29$$

$$\text{Then } I_{EQ} = \frac{10 - 0.7 - 4.29}{1 + \frac{8.57}{101}} = \frac{5.01}{1.085} \Rightarrow I_{EQ} = 4.62 \text{ mA}$$

$$V_B = \frac{I_{EQ}}{1 + \beta} \cdot R_{TH} + V_{TH} = \left( \frac{4.62}{101} \right) (8.57) + 4.29 \Rightarrow V_B = 4.68 \text{ V}$$

**5.52**

(a)

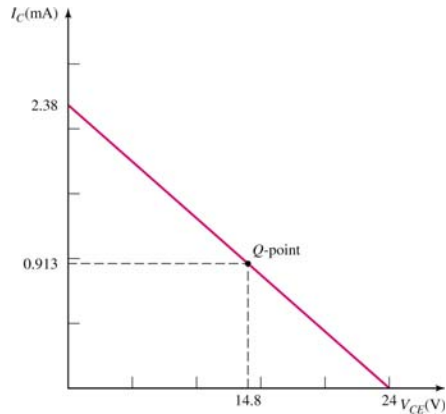
$$R_{TH} = 42 \parallel 58 = 24.36 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{42}{100} \right) (24) = 10.08 \text{ V}$$

$$I_{BQ} = \frac{10.08 - 0.7}{24.36 + (126)(10)} = \frac{9.38}{1284.36} \Rightarrow I_{BQ} = 7.30 \mu\text{A}$$

$$I_{CQ} = 0.913 \text{ mA}, \quad I_{EQ} = 0.9202 \text{ mA}$$

$$V_{CEQ} = 14.8 \text{ V}$$



(b)

$$R_1 + 5\% = 60.9, R_2 + 5\% = 44.1 \quad R_{TH} = 25.58 \text{ K} \quad V_{TH} = 10.08$$

$$I_{BQ} = \frac{10.08 - 0.7}{25.58 + 126(10)} = \frac{9.38}{1285.58} \Rightarrow 7.30 \mu\text{A}$$

$$I_{CQ} = 0.912 \text{ mA} \quad I_{EQ} = 0.919$$

$$V_{CEQ} = 14.81$$

$$R_1 + 5\% = 60.9, R_2 - 5\% = 39.90 \quad R_{TH} = 24.11 \text{ K} \quad V_{TH} = 9.50$$

$$I_{BQ} = \frac{9.50 - 0.7}{24.11 + (126)(10)} = \frac{8.8}{1284.11} = 6.85 \mu\text{A}$$

$$I_{CQ} = 0.857 \text{ mA} \quad I_{EQ} = 0.8635 \text{ mA}$$

$$V_{CEQ} = 15.37 \text{ V}$$

$$R_1 - 5\% = 55.1 \text{ K} \quad R_2 + 5\% = 44.1 \text{ K} \quad R_{TH} = 24.50 \text{ K} \quad V_{TH} = 10.67 \text{ V}$$

$$I_{BQ} = \frac{10.67 - 0.7}{24.50 + 1260} = \frac{9.97}{1284.5} = 7.76 \mu\text{A}$$

$$I_{CQ} = 0.970 \text{ mA} \quad I_{EQ} = 0.978 \text{ mA}$$

$$V_{CEQ} = 14.22 \text{ V}$$

$$R_1 - 5\% = 55.1 \text{ K} \quad R_2 - 5\% = 39.90 \quad R_{TH} = 23.14 \text{ K} \quad V_{TH} = 10.08$$

$$I_{BQ} = \frac{10.08 - 0.7}{23.14 + 1260} = \frac{9.38}{1283.14} = 7.31 \mu\text{A}$$

$$I_{CQ} = 0.914 \text{ mA} \quad I_{EQ} = 0.9211 \text{ mA}$$

$$V_{CEQ} = 14.79 \text{ V}$$

So we have  $0.857 \leq I_{CQ} \leq 0.970 \text{ mA}$

$14.22 \leq V_{CEQ} \leq 15.37 \text{ V}$

### 5.53

(a)  $R_{TH} = R_1 \parallel R_2 = 96 \parallel 24 = 19.2 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{24}{24 + 96} \right) (9) = 1.80 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta)R_E} = \frac{1.80 - 0.7}{19.2 + (81)(1)} \Rightarrow I_{BQ} = 10.98 \mu\text{A}$$



$$I_{CQ} = \beta I_{BQ} = (80)(0.01098) = 0.8782 \text{ mA}, \quad I_{EQ} = (81)(0.01098) = 0.8892 \text{ mA}$$

$$V_{CEQ} = 9 - (0.8782)(5.25) - (0.8892)(1) = 3.50 \text{ V}$$

(b)  $I_{BQ} = \frac{1.80 - 0.7}{19.2 + (121)(1)} \Rightarrow I_{BQ} = 7.846 \mu\text{A}$

$$I_{CQ} = (120)(0.007846) = 0.9415 \text{ mA}, \quad I_{EQ} = (121)(0.007846) = 0.9494 \text{ mA}$$

$$V_{CEQ} = 9 - (0.9415)(5.25) - (0.9494)(1) = 3.108 \text{ V}$$

For  $I_{CQ}$ :  $\left(\frac{0.9415 - 0.8782}{0.8782}\right) \times 100\% = 7.21\%$

For  $V_{CEQ}$ :  $\left(\frac{3.108 - 3.500}{3.500}\right) \times 100\% = -11.2\%$

### 5.54

(a)

$$I_{CQ} \cong I_{EQ} = 0.4 \text{ mA}$$

$$R_C = \frac{3}{0.4} \Rightarrow R_C = 7.5 \text{ k}\Omega; \quad R_E = \frac{3}{0.4} \Rightarrow R_E = 7.5 \text{ k}\Omega$$

$$R_1 + R_2 \cong \frac{9}{(0.2)(0.4)} = 112.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(112.5 - R_2)R_2}{112.5}, \quad I_{BQ} = \frac{0.4}{100} = 0.004 \text{ mA}$$

$$R_2 \left(\frac{9}{112.5}\right) = (0.004) \left[ \frac{(112.5 - R_2)R_2}{112.5} \right] + 0.7 + (101)(0.004)(7.5)$$

We obtain  $R_2(0.08) = 0.004R_2 - 3.56 \times 10^{-5}R_2^2 + 3.73$

From this quadratic, we find  $R_2 = 48 \text{ k}\Omega \Rightarrow R_1 = 64.5 \text{ k}\Omega$

(b) Standard resistor values:

Set  $R_E = R_C = 7.5 \text{ k}\Omega$  and  $R_1 = 62 \text{ k}\Omega$ ,  $R_2 = 47 \text{ k}\Omega$

Now  $R_{TH} = R_1 \parallel R_2 = 62 \parallel 47 = 26.7 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{47}{47 + 62}\right)(9) = 3.88 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

So  $I_{BQ} = \frac{3.88 - 0.7}{26.7 + (101)(7.5)} = 0.00406 \text{ mA}$

Then  $I_{CQ} = 0.406 \text{ mA}$

$$V_{RC} = V_{RE} = (0.406)(7.5) = 3.05 \text{ V}$$

**5.55**

(a)

$$R_{TH} = R_1 \parallel R_2 = 12 \parallel 2 = 1.714 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left( \frac{2}{14} \right) (10) - 5 \Rightarrow V_{TH} = -3.571 \text{ V}$$

(b)

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1 + \beta) I_{BQ} R_E - 5$$

$$-3.57 = I_{BQ} (1.714) + 0.7 + (101) I_{BQ} (0.5) - 5$$

$$I_{BQ} = \frac{5 - 0.7 - 3.571}{1.714 + (101)(0.5)} = \frac{0.729}{52.21} \Rightarrow 13.96 \mu\text{A}$$

$$I_{CQ} = 1.396 \text{ mA}, I_{EQ} = 1.410 \text{ mA}$$

$$V_{CEQ} = 10 - (1.396)(5) - (1.41)(0.5) \Rightarrow V_{CEQ} = 2.32 \text{ V}$$

(d)

$$R_E = 0.5 + 5\% = 0.525 \text{ K} \quad R_C = 5 + 5\% = 5.25 \text{ K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.525)} \Rightarrow 13.32 \mu\text{A}$$

$$I_{CQ} = 1.332 \text{ mA} \quad I_{EQ} = 1.345 \text{ mA}$$

$$V_{CEQ} = 10 - (1.332)(5.25) - (1.345)(0.525) \\ = 10 - 6.993 - 0.7061 \Rightarrow V_{CEQ} = 2.30 \text{ V}$$

$$R_E = 0.5 + 5\% = 0.525 \text{ K} \quad R_C = 5 - 5\% = 4.75 \text{ K}$$

$$I_{CQ} = 1.332 \text{ mA} \quad I_{EQ} = 1.345 \text{ mA}$$

$$V_{CEQ} = 10 - (1.332)(4.75) - (1.345)(0.525) \\ = 10 - 6.327 - 0.7061 \Rightarrow V_{CEQ} = 2.97 \text{ V}$$

$$R_E = 0.5 - 5\% = 0.475 \text{ K} \quad R_C = 5 + 5\% = 5.25 \text{ K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.475)} \Rightarrow 14.67 \mu\text{A}$$

$$I_{CQ} = 1.467 \text{ mA} \quad I_{EQ} = 1.482 \text{ mA}$$

$$V_{CEQ} = 10 - (1.467)(5.25) - (1.482)(0.475) \\ = 10 - 7.70175 - 0.70395 \Rightarrow V_{CEQ} = 1.59 \text{ V}$$

$$R_E = 0.5 - 5\% = 0.475 \text{ K} \quad R_C = 5 - 5\% = 4.75 \text{ K}$$

$$I_{CQ} = 1.467 \text{ mA} \quad I_{EQ} = 1.482 \text{ mA}$$

$$V_{CEQ} = 10 - (1.467)(4.75) - (1.482)(0.475) \\ = 10 - 6.96825 - 0.70395 \Rightarrow V_{CEQ} = 2.33 \text{ V}$$

**5.56**

(a)  $R_{TH} = R_1 \parallel R_2 = 40 \parallel 40 = 20 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V^+ = \left( \frac{40}{40 + 40} \right) (2.5) = 1.25 \text{ V}$$

$$V^+ = I_{EQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$\text{So } I_{BQ} = \frac{V^+ - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1 + \beta)R_E} = \frac{2.5 - 0.7 - 1.25}{20 + (91)(0.7)} \Rightarrow I_{BQ} = 6.57 \mu\text{ A}$$

$$I_{CQ} = 0.5914 \text{ mA}, \quad I_{EQ} = 0.5980 \text{ mA}$$

$$V_{ECQ} = 2.5 - (0.5914)(1.6) - (0.5980)(0.7) = 1.135 \text{ V}$$

$$(b) \quad I_{BQ} = \frac{2.5 - 0.7 - 1.25}{20 + (151)(0.7)} \Rightarrow I_{BQ} = 4.375 \mu\text{ A}$$

$$I_{CQ} = 0.6563 \text{ mA}, \quad I_{EQ} = 0.6607 \text{ mA}$$

$$V_{ECQ} = 2.5 - (0.6563)(1.6) - (0.6607)(0.7) = 0.9874 \text{ V}$$

$$\text{For } I_{CQ}: \left( \frac{0.6563 - 0.5914}{0.5914} \right) \times 100\% = 10.97\%$$

$$\text{For } V_{ECQ}: \left( \frac{0.9874 - 1.135}{1.135} \right) \times 100\% = -13.0\%$$

### 5.57

(a)

$$R_{TH} = 36 \parallel 68 = 23.5 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{36}{36 + 68} \right) (10) = 3.46 \text{ V}$$

$$I_{BQ} = \frac{3.46 - 0.7}{23.5 + (51)(30)} = 0.00178 \text{ mA}$$

$$I_{CQ} = 0.0888 \text{ mA}, \quad I_{EQ} = 0.0906 \text{ mA}$$

$$V_{CEQ} = 10 - (0.0888)(42) - (0.0906)(30) \Rightarrow V_{CEQ} = 3.55 \text{ V}$$

(b)

$$R_1 = 22.7 \text{ k}, \quad R_2 = 12 \text{ k}, \quad R_C = 14 \text{ k}, \quad R_E = 10 \text{ k}$$

$$R_{TH} = 7.85 \text{ k} \quad V_{TH} = 3.46$$

$$I_{BQ} = \frac{3.46 - 0.7}{7.85 + (51)(10)} = 0.00533 \text{ mA}$$

$$I_{CQ} = 0.266 \text{ mA} \quad I_{EQ} = 0.272 \text{ mA}$$

$$V_{CE} = 10 - (0.266)(14) - (0.272)(10)$$

$$\underline{V_{CE} = 3.56 \text{ V}}$$

**5.58**

(a)

$$R_{TH} = 36 \parallel 68 = 23.5 \text{ k}\Omega; V_{TH} = \left( \frac{68}{36+68} \right) (10) - 5 = 1.54 \text{ V}$$

$$5 = (51)I_{BQ}(30) + 0.7 + I_{BQ}(23.5) + 1.54$$

$$I_{BQ} = \frac{2.76}{1553.5} = 1.78 \text{ }\mu\text{A} \Rightarrow I_{CQ} = 0.0888 \text{ mA}$$

$$I_{EQ} = 0.0906 \text{ mA}$$

$$V_{ECQ} = 10 - (0.0906)(30) - (0.0888)(42) \\ = 10 - 2.718 - 3.7296 \Rightarrow \underline{V_{ECQ} = 3.55 \text{ V}}$$

(b)

$$R_{TH} = 12 \parallel 22.7 = 7.85 \text{ k}\Omega$$

$$V_{TH} = 1.54 \text{ V}, R_E = 10 \text{ k}\Omega, R_C = 14 \text{ k}\Omega$$

$$5 = (51)I_{BQ}(10) + 0.7 + I_{BQ}(7.85) + 1.54$$

$$I_{BQ} = \frac{2.76}{517.85} \Rightarrow 5.33 \text{ }\mu\text{A}, I_{CQ} = 0.266 \text{ mA}, I_{EQ} = 0.272 \text{ mA}$$

$$V_{ECQ} = 10 - (0.272)(10) - (0.266)(14) = 3.56 \text{ V}$$

**5.59**

(a)

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.5) = 5.05 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.8}{100} = 0.008 \text{ mA}$$

$$\text{Then } \frac{1}{R_1}(5.05)(10) = (0.008)(5.05) + 0.7 + (101)(0.008)(0.5)$$

$$\text{or } \underline{R_1 = 44.1 \text{ k}\Omega}, \quad \frac{44.1R_2}{44.1 + R_2} = 5.05 \Rightarrow \underline{R_2 = 5.70 \text{ k}\Omega}$$

$$\text{Now } I_{EQ} = \left( \frac{101}{100} \right) (0.8) = 0.808 \text{ mA}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$10 = (0.8)R_C + 5 + (0.808)(0.5)$$

$$\underline{R_C = 5.75 \text{ k}\Omega}$$

(b) For  $75 \leq \beta \leq 150$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left( \frac{5.7}{5.7 + 44.1} \right) (10) = 1.145 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E$$

$$\beta = 75, I_{BQ} = \frac{1.145 - 0.7}{5.05 + (76)(0.5)} = 0.0103 \text{ mA}$$

For

$$\text{Then } I_{CQ} = (75)(0.0103) = 0.775 \text{ mA}$$

$$\beta = 150, I_{BQ} = \frac{1.145 - 0.7}{5.05 + (151)(0.5)} = 0.00552 \text{ mA}$$

For

$$\text{Then } I_{CQ} = 0.829 \text{ mA}$$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.829 - 0.775}{0.80} \times 100\% \Rightarrow \underline{\% \text{ Change} = 6.75\%}$$

(c) For  $R_E = 1 \text{ k}\Omega$

$$R_{TH} = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(10) = (0.008)(10.1) + 0.7 + (101)(0.008)(1)$$

which yields  $R_1 = 63.6 \text{ k}\Omega$

$$\frac{63.6R_2}{63.6 + R_2} = 10.1 \Rightarrow R_2 = 12.0 \text{ k}\Omega$$

And

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left( \frac{12}{12 + 63.6} \right) (10) = 1.587 \text{ V}$$

Now

$$\beta = 75, I_{BQ} = \frac{1.587 - 0.7}{10.1 + (76)(1)} = 0.0103 \text{ mA}$$

For

$$\text{So } I_{CQ} = 0.773 \text{ mA}$$

$$\beta = 150, I_{BQ} = \frac{1.587 - 0.7}{10.1 + (151)(1)} = 0.00551 \text{ mA}$$

For

$$\text{Then } I_{CQ} = 0.826 \text{ mA}$$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.826 - 0.773}{0.8} \times 100\% \Rightarrow \underline{\% \text{ Change} = 6.63\%}$$

### 5.60

$$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$10 = (0.8)(R_C + R_E) + 5 \Rightarrow R_C + R_E = 6.25 \text{ k}\Omega$$

Let  $R_E = 0.875 \text{ k}\Omega$

Then, for bias stable  $R_{TH} = (0.1)(121)(0.875) = 10.6 \text{ k}\Omega$

$$I_{BQ} = \frac{0.8}{120} = 0.00667 \text{ mA}$$

$$\frac{1}{R_1}(10.6)(10) = (0.00667)(10.6) + 0.7 + (121)(0.00667)(0.875)$$

So  $R_1 = 71.8 \text{ k}\Omega$  and  $\frac{71.8R_2}{71.8 + R_2} = 10.6 \Rightarrow R_2 = 12.4 \text{ k}\Omega$

Then  $I_R \cong \frac{10}{71.8+12.4} = 0.119 \text{ mA}$   
This is close to the design specification.

**5.61**

$$I_{CQ} \approx I_{EQ} \Rightarrow V_{CEQ} = V_{CC} - I_{CQ}(R_C + R_E)$$

$$6 = 12 - I_{CQ}(2 + 0.2)$$

$$\underline{I_{CQ} = 2.73 \text{ mA}, \quad I_{BQ} = 0.0218 \text{ mA}}$$

$$V_{CEQ} = 6 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E - 6$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (12) - 6, \quad R_{TH} = R_1 \parallel R_2$$

$$\text{Bias stable} \Rightarrow R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(126)(0.2) = 2.52 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{1}{R_1} \right) (R_{TH})(12) - 6$$

$$\frac{1}{R_1} (2.52)(12) - 6 = (0.0218)(2.52) + 0.7 + (126)(0.0218)(0.2) - 6$$

$$\frac{1}{R_1} (30.24) = 0.7549 + 0.5494$$

$$R_1 = 23.2 \text{ k}\Omega, \quad \frac{23.2R_2}{23.2 + R_2} = 2.52$$

$$\underline{R_2 = 2.83 \text{ k}\Omega}$$

**5.62**

(a)  $R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(0.2) = 2.42 \text{ k}\Omega$

$$V_{CEQ} = 6 - I_{CQ} \left[ R_C + \left( \frac{1 + \beta}{\beta} \right) R_E \right] = 6 - I_{CQ}(2.202)$$

$$2.8 = 6 - I_{CQ}(2.202) \Rightarrow I_{CQ} = 1.453 \text{ mA}$$

Then  $I_{EQ} = 1.465 \text{ mA}, \quad I_{BQ} = 12.11 \mu\text{A}$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E - 3 = (0.01211)(2.42) + 0.7 + (1.465)(0.2) - 3$$

$$\text{Then } V_{TH} = -1.978 \text{ V} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 = \frac{1}{R_1} (2.42)(6) - 3$$

Which yields  $R_1 = 14.2 \text{ k}\Omega$  and  $R_2 = 2.92 \text{ k}\Omega$

(b) For  $R_1 = (1.05)(14.2) = 14.91 \text{ k}\Omega$

$$R_2 = (0.95)(2.92) = 2.774 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 2.34 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{2.774}{2.774 + 14.91} \right) (6) - 3 = -2.059 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E - 3$$

$$\text{So } I_{BQ} = \frac{3 - 2.059 - 0.7}{2.34 + (121)(0.2)} \Rightarrow I_{BQ} = 9.08 \mu\text{A}$$

$$I_{CQ} = 1.090 \text{ mA}, \quad I_{EQ} = 1.099 \text{ mA}$$

$$V_{CEQ} = 6 - (1.09)(2) - (1.099)(0.2) = 3.60 \text{ V}$$

$$\text{For } R_1 = (0.95)(14.2) = 13.49 \text{ k}\Omega$$

$$R_2 = (1.05)(2.92) = 3.066 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 2.50 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{3.066}{3.066 + 13.49} \right) (6) - 3 = -1.889 \text{ V}$$

$$I_{BQ} = \frac{3 - 1.889 - 0.7}{2.50 + (121)(0.2)} \Rightarrow I_{BQ} = 15.39 \mu\text{A}$$

$$I_{CQ} = 1.847 \text{ mA}, \quad I_{EQ} = 1.863 \text{ mA}$$

$$V_{CEQ} = 6 - (1.847)(2) - (1.863)(0.2) = 1.933 \text{ V}$$

$$\text{So } 1.09 \leq I_{CQ} \leq 1.847 \text{ mA}$$

$$1.933 \leq V_{CEQ} \leq 3.60 \text{ V}$$

### 5.63

$$V_{CEQ} \cong V_{CC} - I_{CQ}(R_C + R_E)$$

$$5 = 12 - 3(R_C + R_E) \Rightarrow R_C + R_E = 2.33 \text{ k}\Omega$$

$$\text{Let } \frac{R_E}{R_C} = 0.333 \text{ k}\Omega \text{ and } R_C = 2 \text{ k}\Omega$$

Nominal value of  $\beta = 100$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.333) = 3.36 \text{ k}\Omega$$

$$I_{BQ} = \frac{3}{100} = 0.03 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (12) - 6 = \frac{1}{R_1}(3.36)(12) - 6$$

$$\text{Then } V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$$

$$\frac{1}{R_1}(3.36)(12) - 6 = (0.03)(3.36) + 0.7 + (101)(0.03)(0.333) - 6$$

which yields  $R_1 = 22.3 \text{ k}\Omega$  and  $R_2 = 3.96 \text{ k}\Omega$

$$\text{Now } V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (12) - 6 = \left( \frac{3.96}{3.96 + 22.3} \right) (12) - 6 \text{ or } V_{TH} = -4.19 \text{ V}$$

$$\text{For } \beta = 75, \quad V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$$

$$I_{BQ} = \frac{V_{TH} + 6 - 0.7}{R_{TH} + (1 + \beta)R_E} = \frac{-4.19 + 6 - 0.7}{3.36 + (76)(0.333)} = 0.0387 \text{ mA} \Rightarrow \underline{I_C = 2.90 \text{ mA}}$$

$$\beta = 150, \quad I_{BQ} = \frac{-4.19 + 6 - 0.7}{3.36 + (151)(0.333)} = 0.0207 \text{ mA}$$

For

$$\text{Then } \underline{I_C = 3.10 \text{ mA}}$$

Specifications are met.

**5.64**

$$(a) \quad V^+ = V_{ECQ} + I_{CQ} \left[ R_C + \left( \frac{1+\beta}{\beta} \right) R_E \right]$$

$$3.3 = 1.5 + I_{CQ}(3.022) \Rightarrow I_{CQ} = 0.5956 \text{ mA}$$

$$I_{BQ} = 6.618 \mu\text{A}, \quad I_{EQ} = 0.6022 \text{ mA}$$

$$V^+ = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$3.3 = (0.6022)(2) + 0.7 + (0.006618)(2.4) + V_{TH}$$

$$\text{So } V_{TH} = 1.380 \text{ V} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = \frac{1}{R_1} (2.4)(3.3)$$

$$\text{Which yields } R_1 = 5.74 \text{ k}\Omega \text{ and } R_2 = 4.12 \text{ k}\Omega$$

$$(b) \quad I_{BQ} = \frac{V^+ - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{3.3 - 0.7 - 1.38}{2.4 + (131)(2)} \Rightarrow I_{BQ} = 4.61 \mu\text{A}$$

$$I_{CQ} = 0.60 \text{ mA}, \quad I_{EQ} = 0.6045 \text{ mA}$$

$$V_{ECQ} = 3.3 - (0.60)(1) - (0.6045)(2) = 1.49 \text{ V}$$

**5.65**

$$I_{CQ} = 4.8 \text{ mA} \rightarrow I_{EQ} = 4.84 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$6 = 18 - (4.8)(2) - (4.84)R_E \Rightarrow R_E = 0.496 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(0.496) = 6.0 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$$

$$I_{BQ} = 0.040 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (6.0)(18)$$

$$\frac{1}{R_1} (6.0)(18) = (0.04)(6.0) + 0.70 + (121)(0.04)(0.496)$$

$$\frac{1}{R_1} (108) = 3.34$$

$$R_1 = 32.3 \text{ k}\Omega, \quad \frac{32.3 R_2}{32.3 + R_2} = 6.0$$

$$R_2 = 7.37 \text{ k}\Omega$$

**5.66**

$$\text{For } I_{EQ} \cong I_{CQ}, \quad R_C + R_E = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{2.5 - 1.6}{0.2} = 4.5 \text{ k}\Omega$$

$$\text{So } R_E = 0.5 \text{ k}\Omega$$

$$\text{For } \beta = 100$$

$$R_{TH} = (0.1)(101)(0.5) = 5.05 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.2}{100} \Rightarrow I_{BQ} = 2 \mu\text{A}, \quad I_{EQ} = 0.202 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E = (0.002)(5.05) + 0.7 + (0.202)(0.5) = 0.8111 \text{ V}$$



$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} \Rightarrow 0.8111 = \frac{1}{R_1} (5.05)(2.5)$$

So  $R_1 = 15.6 \text{ k}\Omega$  and  $R_2 = 7.47 \text{ k}\Omega$

For  $\beta = 80$ ,

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{0.8111 - 0.7}{5.05 + (81)(0.5)} \Rightarrow I_{BQ} = 2.439 \mu\text{A}, I_{CQ} = 0.1951 \text{ mA}$$

For  $\beta = 120$ ,

$$I_{BQ} = \frac{0.8111 - 0.7}{5.05 + (121)(0.5)} \Rightarrow I_{BQ} = 1.695 \mu\text{A}, I_{CQ} = 0.2034 \text{ mA}$$

Design is valid

### 5.67

$$I_{CQ} = 1 \text{ mA} \rightarrow I_{EQ} = 1.017 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$5 = 15 - (1)(5) - (1.017)R_E \Rightarrow R_E = 4.92 \text{ k}\Omega$$

$$\text{Bias stable: } R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(61)(4.92) = 30.0 \text{ k}\Omega$$

$$I_{BQ} = \frac{1}{60} = 0.0167 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$\frac{1}{R_1}(30.0)(15) = (0.0167)(30.0) + 0.70 + (1.017)(4.92)$$

$$\frac{1}{R_1}(448.5) = 6.197$$

$$R_1 = 72.5 \text{ k}\Omega, \quad \frac{72.5R_2}{72.5 + R_2} = 30.0$$

$$R_2 = 51.2 \text{ k}\Omega$$

Check: For  $\beta = 45$

$$V_{TH} = \left( \frac{51.2}{51.2 + 72.5} \right) (15) = 6.21 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{6.21 - 0.7}{30 + (46)(4.92)} = 0.0215 \text{ mA}$$

$$I_{CQ} = 0.967 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 3.27\%$$

Check: For  $\beta = 75$

$$I_{BQ} = \frac{6.21 - 0.7}{30.0 + (76)(4.92)} = 0.0136 \text{ mA}$$

$$I_{CQ} = 1.023 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 2.31\%$$

Design criterion is satisfied.

**5.68**

(a)

$$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$3 = (0.1)(5R_E + R_E) + 1.4 \Rightarrow R_E = 2.67 \text{ k}\Omega$$

$$R_C = 13.3 \text{ k}\Omega, \quad I_{BQ} = \frac{100}{120} = 0.833 \text{ }\mu\text{A}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(2.67) = 32.3 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(32.3)(3)$$

$$= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

$$= (0.000833)(32.3) + 0.7 + (121)(0.000833)(2.67)$$

which gives  $R_1 = 97.3 \text{ k}\Omega$ , and  $R_2 = 48.4 \text{ k}\Omega$

(b)

$$I_R \cong \frac{3}{R_1 + R_2} = \frac{3}{97.3 + 48.4} \Rightarrow 20.6 \text{ }\mu\text{A}$$

$$I_{CQ} = 100 \text{ }\mu\text{A}$$

$$P = (I_{CQ} + I_R)V_{CC} = (100 + 20.6)(3)$$

$$\text{or } P = 362 \text{ }\mu\text{W}$$

**5.69**

$$I_E = \frac{5 - V_E}{R_E} = \frac{5}{3} = 1.67 \text{ mA}$$

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E = (0.1)(101)(3) = 30.3 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (4) - 2 = \frac{1}{R_1} \cdot R_{TH} \cdot (4) - 2$$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta} = 0.0165 \text{ mA}$$

$$5 = I_{EQ}R_E + V_{EB}(\text{on}) + I_B R_{TH} + V_{TH}$$

$$5 = (1.67)(3) + 0.7 + (0.0165)(30.3) + \frac{1}{R_1}(30.3)(4) - 2$$

$$0.80 = \frac{1}{R_1}(30.3)(4) \Rightarrow R_1 = 152 \text{ k}\Omega$$

$$\frac{152R_2}{152 + R_2} = 30.3 \Rightarrow R_2 = 37.8 \text{ k}\Omega$$

**5.70**

a.  $R_{TH} = R_1 \parallel R_2 = 10 \parallel 20 = 6.67 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left( \frac{20}{20 + 10} \right) (10) - 5 = 1.67 \text{ V}$$

b.

$$10 = (1 + \beta) I_{BQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{10 - 0.7 - 1.67}{6.67 + (61)(2)} = \frac{7.63}{128.7} \Rightarrow I_{BQ} = 0.0593 \text{ mA}$$

$$I_{CQ} = 3.56 \text{ mA}, \quad I_{EQ} = 3.62 \text{ mA}$$

$$V_E = 10 - I_{EQ} R_E = 10 - (3.62)(2)$$

$$V_E = 2.76 \text{ V}$$

$$V_C = I_{CQ} R_C - 10 = (3.56)(2.2) - 10$$

$$V_C = -2.17 \text{ V}$$

**5.71**

$$V^+ - V^- \cong I_{CQ} (R_C + R_E) + V_{ECQ}$$

$$20 = (0.5)(R_C + R_E) + 8 \Rightarrow (R_C + R_E) = 24 \text{ k}\Omega$$

Let  $\frac{R_E = 10 \text{ k}\Omega}{R_C = 14 \text{ k}\Omega}$  then

Let  $\beta = 60$  from previous problem.

$$R_{TH} = (0.1)(1 + \beta) R_E = (0.1)(61)(10)$$

Or  $R_{TH} = 61 \text{ k}\Omega$

$$I_{BQ} = \frac{0.5}{60} = 0.00833 \text{ mA}$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot 10 - 5$$

Now  $10 = (1 + \beta) I_{BQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$

$$10 = (61)(0.00833)(10) + 0.7 + (0.00833)(61) + \frac{1}{R_1} (61)(10) - 5$$

Then  $R_1 = 70.0 \text{ k}\Omega$  and  $R_2 = 474 \text{ k}\Omega$

$$I_R \cong \frac{10}{R_1 + R_2} = \frac{10}{70 + 474} \Rightarrow 18.4 \mu\text{A}$$

So the  $40 \mu\text{A}$  current limit is met.

**5.72**

(a)  $V_{ECQ} = (V^+ - V^-) - I_{CQ} \left[ R_C + \left( \frac{1 + \beta}{\beta} \right) \cdot R_E \right]$

$$2.7 = 5 - (0.15) \left[ R_C + \left( \frac{81}{80} \right) (2) \right] \Rightarrow R_C = 13.3 \text{ k}\Omega$$

$$I_{EQ} = 0.1519 \text{ mA}, \quad I_{BQ} = 1.875 \mu\text{A}$$

$$R_{TH} = (0.1)(81)(2) = 16.2 \text{ k}\Omega$$

$$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$$

$$2.5 = (0.1519)(2) + 0.7 + (0.001875)(16.2) + V_{TH} \Rightarrow V_{TH} = 1.466 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH}(5) - 2.5 \Rightarrow 1.466 = \frac{1}{R_1} (16.2)(5) - 2.5$$

So  $R_1 = 20.4 \text{ k}\Omega$  and  $R_2 = 78.7 \text{ k}\Omega$

(b) For  $\beta = 60$

$$I_{BQ} = \frac{V^+ - V_{TH} - V_{EB}(on)}{R_{TH} + (1 + \beta)R_E} = \frac{2.5 - 1.466 - 0.7}{16.2 + (61)(2)} \Rightarrow I_{BQ} = 2.417 \mu\text{A}$$

$$I_{CQ} = 0.145 \text{ mA}, \quad I_{EQ} = 0.1474 \text{ mA}$$

$$V_{ECQ} = 5 - (0.145)(13.3) - (0.1474)(2) = 2.777 \text{ V}$$

For  $\beta = 100$

$$I_{BQ} = \frac{2.5 - 1.466 - 0.7}{16.2 + (101)(2)} \Rightarrow I_{BQ} = 1.531 \mu\text{A}$$

$$I_{CQ} = 0.1531 \text{ mA}, \quad I_{EQ} = 0.1546 \text{ mA}$$

$$V_{ECQ} = 5 - (0.1531)(13.3) - (0.1546)(2) = 2.655 \text{ V}$$

$$\text{For } I_{CQ}: \left( \frac{0.1531 - 0.145}{0.15} \right) \times 100\% = 5.4\%$$

$$\text{For } V_{ECQ}: \left( \frac{2.655 - 2.777}{2.70} \right) \times 100\% = -4.52\%$$

### 5.73

a.

$$R_{TH} = 500 \parallel 500 \parallel 70 = 250 \parallel 70 = 54.7 \text{ k}\Omega$$

$$\frac{5 - V_{TH}}{500} + \frac{3 - V_{TH}}{500} = \frac{V_{TH} - (-5)}{70}$$

$$\frac{5}{500} + \frac{3}{500} - \frac{5}{70} = V_{TH} \left( \frac{1}{500} + \frac{1}{500} + \frac{1}{70} \right) \Rightarrow -0.0554 = V_{TH} (0.0183)$$

$$V_{TH} = -3.03 \text{ V}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on) - (-5)}{R_{TH} + (1 + \beta)R_E}$$

$$= \frac{-3.03 - 0.7 + 5}{54.7 + (101)(5)}$$

$$I_{BQ} = 0.00227 \text{ mA}$$

$$I_{CQ} = 0.227 \text{ mA}, \quad I_{EQ} = 0.229$$

$$V_{CEQ} = 20 - (0.227)(50) - (0.229)(5)$$

$$V_{CEQ} = 7.51 \text{ V}$$

**5.74**

$$R_E \cong \frac{V_E}{I_{CQ}} = \frac{1.5}{0.8} = 1.87 \text{ k}\Omega$$

$$10 = (0.8)R_C + 4 + 1.5 \Rightarrow R_C = 5.63 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(1.87) = 22.6 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.8}{120} \Rightarrow I_{BQ} = 6.67 \mu\text{A}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$(0.00667)(22.6) + 0.7 + (0.807)(1.87) = \frac{1}{R_1}(22.6)(10)$$

which yields  $R_1 = 95.8 \text{ k}\Omega$  and  $R_2 = 29.6 \text{ k}\Omega$

**5.75**

$$I_{CQ} = 50 \mu\text{A}, I_{BQ} = 0.625 \mu\text{A}, I_{EQ} = 50.6 \mu\text{A}$$

(a)

$$R_E = \frac{1}{0.0506} = 19.8 \text{ K}$$

$$5 = (0.050)R_C + 5 + (0.0506)(19.8) - 5$$

$$R_C = 80 \text{ K}$$

$$R_{TH} = R_1 \parallel R_2, \text{ Design bias stable circuit}$$

$$R_{TH} = (0.1)(51)(19.8) = 101 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$\text{So } \frac{1}{R_1}(101)(10) - 5 = I_{BQ}(101) + 0.7 + (0.0506)(19.8) - 5$$

$$\frac{1}{R_1}(1010) = 0.0631 + 0.7 + 1$$

$$R_1 = 573 \text{ K} \quad \frac{573 R_2}{573 + R_2} = 101$$

$$R_2 = 123 \text{ K}$$

(b)

$$R_{TH} = 101 \text{ K}, V_{TH} = -3.23 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + 0.7 + (121)(19.8)I_{BQ} - 5$$

$$1.07 = I_{BQ}(101 + 2395.8) \Rightarrow I_{BQ} = 0.429 \mu\text{A}$$

$$I_{CQ} = 0.0514 \text{ mA}, I_{EQ} = 0.0519 \text{ mA}$$

$$V_{CEQ} = 10 - (0.0514)(80) - (0.0519)(19.8) \\ = 10 - 4.11 - 1.03 \Rightarrow V_{CEQ} = 4.86 \text{ V}$$

**5.76**

(a)  $R_E \cong \frac{0.7}{0.5} = 1.4 \text{ k}\Omega$ ,  $I_{BQ} = \frac{0.5}{120} \Rightarrow I_{BQ} = 4.167 \mu\text{A}$   
 $V_{CEQ} = 6 - I_{CQ}R_C - 0.7$   
 $2.5 = 6 - (0.5)R_C - 0.7 \Rightarrow R_C = 5.6 \text{ k}\Omega$   
 $R_{TH} = (0.1)(121)(1.4) = 16.9 \text{ k}\Omega$   
 $V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E - 3$   
 $V_{TH} = (0.004167)(16.9) + 0.7 + (121)(0.004167)(1.4) - 3 = -1.5237 \text{ V}$   
 $V_{TH} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow -1.5237 = \frac{1}{R_1}(16.9)(6) - 3$   
 which yields  $R_1 = 68.7 \text{ k}\Omega$  and  $R_2 = 22.4 \text{ k}\Omega$

(b) For standard resistor values:

Let  $R_E = 1.5 \text{ k}\Omega$ ,  $R_C = 5.6 \text{ k}\Omega$ ,  $R_1 = 68 \text{ k}\Omega$ ,  $R_2 = 22 \text{ k}\Omega$   
 $R_{TH} = R_1 \parallel R_2 = 68 \parallel 22 = 16.62 \text{ k}\Omega$   
 $V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right)(6) - 3 = \left( \frac{22}{22 + 68} \right)(6) - 3 = -1.533 \text{ V}$   
 $I_{BQ} = \frac{3 + V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta)R_E} = \frac{3 - 1.533 - 0.7}{16.62 + (121)(1.5)} \Rightarrow I_{BQ} = 3.87 \mu\text{A}$   
 $I_{CQ} = 0.4646 \text{ mA}$ ,  $I_{EQ} = 0.4684 \text{ mA}$   
 $V_{CEQ} = 6 - (0.4646)(5.6) - (0.4684)(1.5) = 2.70 \text{ V}$

**5.77**

(a)  $R_E \cong \frac{0.7}{0.1} = 7 \text{ k}\Omega$ ,  $R_C + R_E = \frac{6 - 3}{0.1} = 30 \text{ k}\Omega$ ,  $\Rightarrow R_C = 23 \text{ k}\Omega$   
 $R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(111)(7) = 77.7 \text{ k}\Omega$   
 $I_{BQ} = \frac{0.1}{110} \Rightarrow I_{BQ} = 0.909 \mu\text{A}$ ,  $I_{EQ} = 0.1009 \text{ mA}$   
 $V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$   
 $3 = (0.1009)(7) + 0.7 + (0.000909)(77.7) + V_{TH}$ ,  $\Rightarrow V_{TH} = 1.523 \text{ V}$   
 $V_{TH} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow 1.523 = \frac{1}{R_1}(77.7)(6) - 3$   
 which yields  $R_1 = 103 \text{ k}\Omega$  and  $R_2 = 316 \text{ k}\Omega$   
 $I_{BQ} = \frac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1 + \beta)R_E} = \frac{3 - 0.7 - 1.523}{77.7 + (151)(7)} \Rightarrow I_{BQ} = 0.685 \mu\text{A}$   
 $I_{CQ} = 0.1027 \text{ mA}$ ,  $I_{EQ} = 0.1034 \text{ mA}$   
 $V_{ECQ} = 6 - (0.1027)(23) - (0.1034)(7) = 2.914 \text{ V}$

**5.78**

$$\begin{aligned} \text{(a) } V_{ECQ} &= 18 - I_{CQ}R_C - V_{RE} \\ 6 &= 18 - (1.2)R_C - 1.5 \Rightarrow R_C = 8.75 \text{ k}\Omega \\ I_{EQ} &= \left(\frac{76}{75}\right)(1.2) = 1.216 \text{ mA}, \quad I_{BQ} = \frac{1.2}{75} \Rightarrow I_{BQ} = 16 \mu\text{A} \\ R_E &= \frac{1.5}{1.216} = 1.234 \text{ k}\Omega \\ R_{TH} &= (0.1)(1 + \beta)R_E = (0.1)(76)(1.234) = 9.375 \text{ k}\Omega \\ V^+ &= I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH} \\ 9 &= (1.216)(1.234) + 0.7 + (0.016)(9.375) + V_{TH} \Rightarrow V_{TH} = 6.65 \text{ V} \\ V_{TH} &= \frac{1}{R_1}(R_{TH})(18) - 9 \Rightarrow 6.65 = \frac{1}{R_1}(9.375)(18) - 9 \end{aligned}$$

which yields  $R_1 = 10.78 \text{ k}\Omega$  and  $R_2 = 71.8 \text{ k}\Omega$

Set  $R_C = 9.1 \text{ k}\Omega$ ,  $R_E = 1.2 \text{ k}\Omega$ ,  $R_1 = 11 \text{ k}\Omega$ ,  $R_2 = 68 \text{ k}\Omega$

$$\begin{aligned} R_{TH} &= R_1 \parallel R_2 = 11 \parallel 68 = 9.47 \text{ k}\Omega \\ V_{TH} &= \left(\frac{R_2}{R_1 + R_2}\right)(18) - 9 = \left(\frac{68}{68 + 11}\right)(18) - 9 = 6.494 \text{ V} \\ I_{BQ} &= \frac{V^+ - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1 + \beta)R_E} = \frac{9 - 0.7 - 6.494}{9.47 + (76)(1.2)} \Rightarrow I_{BQ} = 17.94 \mu\text{A} \\ I_{CQ} &= 1.345 \text{ mA}, \quad I_{EQ} = 1.363 \text{ mA} \\ V_{ECQ} &= 18 - (1.345)(9.1) - (1.363)(1.2) = 4.12 \text{ V} \end{aligned}$$

**5.79**

$$\begin{aligned} R_{TH} &= R_1 \parallel R_2 = 100 \parallel 40 = 28.6 \text{ k}\Omega \\ V_{TH} &= \left(\frac{R_2}{R_1 + R_2}\right)(10) = \left(\frac{40}{40 + 100}\right)(10) = 2.86 \text{ V} \\ I_{B1} &= \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_{E1}} = \frac{2.86 - 0.7}{28.6 + (121)(1)} \\ I_{B1} &= 0.0144 \text{ mA}, \quad I_{C1} = 1.73 \text{ mA}, \quad I_{E1} = 1.75 \text{ mA} \\ \frac{10 - V_{B2}}{3} &= I_{C1} + I_{B2} \\ I_{E2} &= \frac{V_{B2} - V_{BE}(\text{on}) - (-10)}{5} \\ \frac{10 - V_{B2}}{3} &= I_{C1} + \frac{V_{B2} - 0.7 + 10}{(121)(5)} \\ \frac{10}{3} - 1.73 - \frac{9.3}{605} &= V_{B2} \left(\frac{1}{3} + \frac{1}{(121)(5)}\right) \\ 1.588 &= V_{B2} (0.335) \Rightarrow V_{B2} = 4.74 \text{ V} \end{aligned}$$

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$$I_{E2} = \frac{4.74 - 0.7 - (-10)}{5} \Rightarrow \underline{I_{E2} = 2.808 \text{ mA}}$$

$$\underline{I_{B2} = 0.0232 \text{ mA}}$$

$$\underline{I_{C2} = 2.785 \text{ mA}}$$

$$V_{CEQ1} = 4.74 - (1.75)(1) \Rightarrow \underline{V_{CEQ1} = 2.99 \text{ V}}$$

$$V_{CEQ2} = 10 - (4.74 - 0.7) \Rightarrow \underline{V_{CEQ2} = 5.96 \text{ V}}$$

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**5.80**

$$V_{E1} = -0.7$$

$$I_{R1} = \frac{-0.7 - (-5)}{20} = 0.215 \text{ mA}$$

$$V_{E2} = -0.7 - 0.7 = -1.4$$

$$I_{E2} = \frac{-1.4 - (-5)}{1} \Rightarrow \underline{I_{E2} = 3.6 \text{ mA}}$$

$$\underline{I_{B2} = 0.0444 \text{ mA}}$$

$$\underline{I_{C2} = 3.56 \text{ mA}}$$

$$I_{E1} = I_{R1} + I_{B2} = 0.215 + 0.0444$$

$$\underline{I_{E1} = 0.259 \text{ mA}}$$

$$\underline{I_{B1} = 0.00320 \text{ mA}}$$

$$\underline{I_{C1} = 0.256 \text{ mA}}$$

---

**5.81**

$$V_{B1} = V_{RE} + V_{BE}(on) = 0.5 + 0.7 = 1.2 \text{ V}$$

$$R_3 = \frac{1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_E = \frac{0.5}{0.2} = 2.5 \text{ k}\Omega$$

$$V_{B2} = V_{BE}(on) + V_{CE} + V_{RE} = 0.7 + 1.2 + 0.5 = 2.4 \text{ V}$$

$$R_2 = \frac{V_{B2} - V_{B1}}{I_{R2}} = \frac{2.4 - 1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_1 = \frac{V^+ - V_{B2}}{I_{R1}} = \frac{5 - 2.4}{0.020} = 130 \text{ k}\Omega$$

$$V_{C2} = 2V_{CE} + V_{RE} = 2(1.2) + 0.5 = 2.9 \text{ V}$$

$$R_C = \frac{V^+ - V_{C2}}{I_{C2}} = \frac{5 - 2.9}{0.20} = 10.5 \text{ k}\Omega$$

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**5.82**

$$R_{TH} = 40 \parallel 80 = 26.67 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{40}{40+80} \right) (9) = 3 \text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta_n)R_{E1}} = \frac{3 - 0.7}{26.67 + (121)(2)} \Rightarrow I_{B1} = 8.56 \mu\text{A}$$

$$I_{C1} = 1.027 \text{ mA}, \quad I_{E1} = 1.036 \text{ mA}$$

$$\frac{9 - V_{C1}}{2} + I_{B2} = I_{C1}$$

$$9 = I_{E2}(0.1) + V_{EB}(on) + V_{C1} \Rightarrow I_{B2} = \frac{9 - 0.7 - V_{C1}}{(1 + \beta_p)(0.1)} = \frac{8.3 - V_{C1}}{8.1}$$

So  $\frac{9 - V_{C1}}{2} + \frac{8.3 - V_{C1}}{8.1} = 1.027 \Rightarrow V_{C1} = 7.214 \text{ V}$

$$I_{B2} = \frac{8.3 - 7.214}{8.1} = 0.134 \text{ mA}$$

$$I_{C2} = 10.73 \text{ mA}, \quad I_{E2} = 10.86 \text{ mA}$$

$$V_{E1} = I_{E1}R_{E1} = (1.036)(2) = 2.072 \text{ V}$$

$$V_{CE1} = V_{C1} - V_{E1} = 7.214 - 2.072 = 5.14 \text{ V}$$

$$V_{EC2} = 9 - (10.86)(0.1) - (10.73)(0.2) = 5.77 \text{ V}$$

**5.83**

$$R_{TH} = R_1 \parallel R_2 = 50 \parallel 100 = 33.3 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left( \frac{100}{100 + 50} \right) (10) - 5 = 1.67 \text{ V}$$

$$5 = I_{E1}R_{E1} + V_{EB}(on) + I_{B1}R_{TH} + V_{TH}$$

$$I_{E1} = \left( \frac{101}{100} \right) (0.8) = 0.808 \text{ mA}$$

$$I_{B1} = 0.008 \text{ mA}$$

$$5 = (0.808)R_{E1} + 0.7 + (0.008)(33.3) + 1.67$$

$$R_{E1} = 2.93 \text{ k}\Omega$$

$$V_{E1} = 5 - (0.808)(2.93) = 2.63 \text{ V}$$

$$V_{C1} = V_{E1} - V_{ECQ1} = 2.63 - 3.5 = -0.87 \text{ V}$$

$$V_{E2} = -0.87 - 0.70 = -1.57 \text{ V}$$

$$I_{E2} = \frac{-1.57 - (-5)}{R_{E2}} = 0.808 \Rightarrow \underline{R_{E2} = 4.25 \text{ k}\Omega}$$

$$V_{CEQ2} = 4 \Rightarrow V_{C2} = -1.57 + 4 = 2.43 \text{ V}$$

$$R_{C2} = \frac{5 - 2.43}{0.8} \Rightarrow \underline{R_{C2} = 3.21 \text{ k}\Omega}$$

---

$$I_{RC1} = I_{C1} - I_{B2} = 0.8 - 0.008 = 0.792 \text{ mA}$$

$$R_{C1} = \frac{-0.87 - (-5)}{0.792} \Rightarrow \underline{R_{C1} = 5.21 \text{ k}\Omega}$$

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## Chapter 6

### 6.1

$$\begin{aligned} \text{(a) (i) } g_m &= \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} = 19.23 \text{ mA/V} \\ r_\pi &= \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega \\ r_o &= \frac{V_A}{I_{CQ}} = \frac{150}{0.5} = 300 \text{ k}\Omega \\ \text{(ii) } g_m &= \frac{2}{0.026} = 76.92 \text{ mA/V} \\ r_\pi &= \frac{(180)(0.026)}{2} = 2.34 \text{ k}\Omega \\ r_o &= \frac{150}{2} = 75 \text{ k}\Omega \\ \text{(b) (i) } g_m &= \frac{0.25}{0.026} = 9.615 \text{ mA/V} \\ r_\pi &= \frac{(80)(0.026)}{0.25} = 8.32 \text{ k}\Omega \\ r_o &= \frac{100}{0.25} = 400 \text{ k}\Omega \\ \text{(ii) } g_m &= \frac{0.08}{0.026} = 3.077 \text{ mA/V} \\ r_\pi &= \frac{(80)(0.026)}{0.08} = 26 \text{ k}\Omega \\ r_o &= \frac{100}{0.08} = 1250 \text{ k}\Omega \end{aligned}$$

### 6.2

$$\begin{aligned} \text{(a) } g_m &= \frac{I_{CQ}}{V_T} \Rightarrow I_{CQ} = (95)(0.026) = 2.47 \text{ mA} \\ r_\pi &= \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{2.47} = 1.32 \text{ k}\Omega \\ r_o &= \frac{V_A}{I_{CQ}} = \frac{200}{2.47} = 81 \text{ k}\Omega \\ \text{(b) } I_{CQ} &= g_m V_T = (120)(0.026) = 3.12 \text{ mA} \\ \beta &= g_m r_\pi = (120)(1.2) = 144 \end{aligned}$$

### 6.3

$$g_m = \frac{0.8}{0.026} = 30.77 \text{ mA/V}; \quad g_m = \frac{1.2}{0.026} = 46.15 \text{ mA/V}$$

So  $30.77 \leq g_m \leq 46.15 \text{ mA/V}$

$$r_{\pi} = \frac{(180)(0.026)}{0.8} = 5.85 \text{ k}\Omega ; \quad r_{\pi} = \frac{(90)(0.026)}{1.2} = 1.95 \text{ k}\Omega$$

So  $1.95 \leq r_{\pi} \leq 5.85 \text{ k}\Omega$

#### 6.4

(a)  $i_C = I_{CQ} + g_m v_{be} = 0.12 + \left( \frac{0.12}{0.026} \right) (5 \times 10^{-3} \sin \omega t)$   
 $i_C = 0.12 + 0.0231 \sin \omega t \text{ (mA)}$   
 $v_{CE} = V_{CEQ} - i_C R_C = [3.3 - (0.12)(15)] - (15)(0.0231 \sin \omega t)$   
 $v_{CE} = 1.5 - 0.346 \sin \omega t \text{ (V)}$

(b)  $A_v = \frac{v_{ce}}{v_{be}} = \frac{-0.346}{0.005} = -69.2$

#### 6.5

(a)  $I_{CQ} = \frac{\beta(V_{BB} - V_{BE}(\text{on}))}{R_B} = \frac{(120)(1.10 - 0.7)}{110} = 0.436 \text{ mA}$   
 $g_m = \frac{0.436}{0.026} = 16.78 \text{ mA/V}$   
 $r_{\pi} = \frac{(120)(0.026)}{0.436} = 7.15 \text{ k}\Omega$   
 $r_o = \frac{80}{0.436} = 183 \text{ k}\Omega$

(b)  $A_v = -g_m (R_C \parallel r_o) \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right) = -(16.78)(4 \parallel 183) \left( \frac{7.15}{7.15 + 110} \right)$   
 $A_v = -4.0$

(c)  $v_s = \frac{v_o}{A_v} = \frac{0.5 \sin(100t)}{-4} = -0.125 \sin(100t) \text{ (V)}$

#### 6.6

a.

$$r_{\pi} = 5.4 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.578 \text{ mA}$$

$$V_{CEQ} = \frac{1}{2} V_{CC} = \frac{1}{2} (5) = 2.5 \text{ V}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C \Rightarrow 2.5 = 5.0 - (0.578) R_C \Rightarrow R_C = 4.33 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.578}{120} = 0.00482 \text{ mA}$$

$$V_{BB} = I_{BQ} R_B + V_{BE}(\text{on})$$

$$= (0.00482)(25) + 0.70 \Rightarrow \underline{V_{BB} = 0.820 \text{ V}}$$

b.

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.578} = 5.40 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.578}{0.026} = 22.2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.578} = 173 \text{ k}\Omega$$

$$V_o = -g_m (r_o \parallel R_C) V_{\pi}, \quad V_{\pi} = \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right) V_s$$

$$A_v = -g_m \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right) (r_o \parallel R_C) = -\frac{\beta (r_o \parallel R_C)}{r_{\pi} + R_B}$$

$$A_v = -\frac{(120)[173 \parallel 4.33]}{5.40 + 25} = -\frac{(120)(4.22)}{30.4} \Rightarrow \underline{A_v = -16.7}$$

6.7

$$r_{\pi} = \frac{(120)(0.026)}{0.5} = 6.24 \text{ k}\Omega$$

(a)  $R_i = R_B + r_{\pi} = 50 + 6.24 = 56.24 \text{ k}\Omega$

(b)  $R_i = R_B \parallel r_{\pi} = 100 \parallel 6.24 = 5.87 \text{ k}\Omega$

(c)  $R_i = r_{\pi} = 6.24 \text{ k}\Omega$

6.8

$$r_o = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

(a)  $R_o = R_C \parallel r_o = 4 \parallel 400 = 3.96 \text{ k}\Omega$

(b)  $R_o = R_C \parallel r_o = 10 \parallel 400 = 9.76 \text{ k}\Omega$

(c)  $R_o = R_C \parallel R_L \parallel r_o = 10 \parallel 5 \parallel 400 = 3.333 \parallel 400 = 3.31 \text{ k}\Omega$

6.9

$$I_{CQ} = \frac{10 - 4}{4} = 1.5 \text{ mA}$$

$$I_{BQ} = \frac{1.5}{100} = 0.015 \text{ mA}$$

$$r_{\pi} = \frac{(100)(0.026)}{1.5} = 1.73 \text{ k}\Omega$$

$$i_b = \frac{v_{be}}{r_{\pi}} = \frac{5 \sin \omega t \text{ (mV)}}{1.73 \text{ k}\Omega} = 2.89 \sin \omega t \text{ (\mu A)}$$

So

$$i_B(t) = I_{BQ} + i_b(t) = 15 + 2.89 \sin \omega t \text{ (}\mu\text{ A)}$$

$$i_{C1}(t) = \beta i_B(t) = 1.5 + 0.289 \sin \omega t \text{ (mA)}$$

$$v_{C1}(t) = 10 - i_{C1}(t)R_C = 10 - [1.5 + 0.289 \sin \omega t](4) \text{ (V)}$$

$$v_{C1}(t) = 4 - 1.156 \sin \omega t \text{ (V)}$$

$$A_v = \frac{v_c(t)}{v_{be}(t)} = \frac{-1.156}{0.005} \Rightarrow A_v = -231$$

**6.10**

a.

$$V_{ECQ} = \frac{1}{2}V_{CC} = 5 \text{ V}$$

$$V_{ECQ} = 10 - I_{CQ}R_C \Rightarrow 5 = 10 - (0.5)R_C \Rightarrow R_C = 10 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 0.005$$

$$V_{EB}(\text{on}) + I_{BQ}R_B = V_{BB} = (0.70) + (0.005)(50) \Rightarrow V_{BB} = 0.95 \text{ V}$$

b.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow g_m = 19.2 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} \Rightarrow r_\pi = 5.2 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.5} \Rightarrow r_0 = \infty$$

$$A_v = -\frac{\beta R_C}{r_\pi + R_B} = -\frac{(100)(10)}{5.2 + 50} \Rightarrow A_v = -18.1$$

c.

**6.11**

$$v_o = 1.2 \sin \omega t \text{ (V)}$$

$$i_C(t)R_C + v_o = 0 \Rightarrow i_C(t) = \frac{-1.2 \sin \omega t}{2}$$

$$i_C(t) = -0.60 \sin \omega t \text{ (mA)}$$

$$i_b(t) = \frac{i_C(t)}{\beta} = -6 \sin \omega t \text{ (}\mu\text{A)}$$

$$v_{be}(t) = i_b(t) \cdot r_\pi \quad g_m r_\pi = \beta$$

$$r_\pi = \frac{100}{50} = 2 \text{ K}$$

$$v_{be}(t) = -12 \sin \omega t \text{ (mV)}$$

**6.12**

a.

$$I_{CQ} \approx I_{EQ}$$

$$V_{CEQ} = 5 = 10 - I_{CQ}(R_C + R_E)$$

$$= 10 - I_{CQ}(1.2 + 0.2)$$

$$I_{CQ} = 3.57 \text{ mA}$$

$$I_{BQ} = \frac{3.57}{150} = 0.0238 \text{ mA}$$

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(3.02)(10) - 5 = (0.0238)(3.02) + 0.7 + (151)(0.0238)(0.2) - 5$$

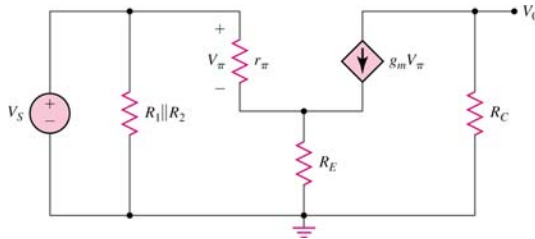
$$\frac{1}{R_1}(30.2) = 1.50 \Rightarrow R_1 = 20.1 \text{ k}\Omega$$

$$\frac{20.1R_2}{20.1 + R_2} = 3.02 \Rightarrow \underline{R_2 = 3.55 \text{ k}\Omega}$$

b.

$$r_\pi = \frac{(150)(0.026)}{3.57} = 1.09 \text{ k}\Omega$$

$$g_m = \frac{3.57}{0.026} = 137 \text{ mA/V}$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = \frac{-(150)(1.2)}{1.09 + (151)(0.2)} \Rightarrow A_v = -5.75$$

**6.13**

(a)  $R_{TH} = R_1 \parallel R_2 = 33 \parallel 50 = 19.88 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{50}{50 + 33} \right) (3.3) = 1.988 \text{ V}$$

$$V_{CC} = I_{BQ}(1 + \beta)R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$\text{Then } I_{BQ} = \frac{3.3 - 0.7 - 1.988}{19.88 + (101)(1)} = 0.005063 \text{ mA}$$

$$I_{CQ} = 0.506 \text{ mA}; \quad I_{EQ} = 0.511 \text{ mA}$$

$$V_{ECQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = 3.3 - (0.506)(2) - (0.511)(1)$$

$$V_{ECQ} = 1.78 \text{ V}$$

$$(b) \quad r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.506} = 5.14 \text{ k}\Omega$$

$$A_v = \frac{-\beta R_C}{r_{\pi} + (1 + \beta)R_E} = \frac{-(100)(2)}{5.14 + (101)(1)} = -1.884$$

$$(c) \quad R_1 = (1.05)(33) = 34.65 \text{ k}\Omega$$

$$R_2 = (0.95)(50) = 47.5 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 34.65 \parallel 47.5 = 20.03 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{47.5}{47.5 + 34.65} \right) (3.3) = 1.908 \text{ V}$$

$$I_{CQ} = (100) \left( \frac{3.3 - 0.7 - 1.908}{20.03 + (101)(1)} \right) = 0.5718 \text{ mA}$$

$$r_{\pi} = \frac{(100)(0.026)}{0.5718} = 4.547 \text{ k}\Omega$$

$$A_v = \frac{-(100)(2)}{4.547 + (101)(1)} = -1.895$$

$$\text{Also } R_1 = (0.95)(33) = 31.35 \text{ k}\Omega$$

$$R_2 = (1.05)(50) = 52.5 \text{ k}\Omega$$

$$R_{TH} = 31.35 \parallel 52.5 = 19.63 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{52.5}{52.5 + 31.35} \right) (3.3) = 2.066 \text{ V}$$

$$I_{CQ} = (100) \left( \frac{3.3 - 0.7 - 2.066}{19.63 + (101)(1)} \right) = 0.4427 \text{ mA}$$

$$r_{\pi} = \frac{(100)(0.026)}{0.4427} = 5.873 \text{ k}\Omega$$

$$A_v = \frac{-(100)(2)}{5.873 + (101)(1)} = -1.871$$

$$\text{So } 1.871 \leq |A_v| \leq 1.895$$



**6.14**

(a)

$$V_{CC} = \left( \frac{1+\beta}{\beta} \right) I_{CQ} R_E + V_{ECQ} + I_{CQ} R_C$$

$$12 = \left( \frac{101}{100} \right) I_{CQ} (1) + 6 + I_{CQ} (2)$$

so that  $I_{CQ} = 1.99 \text{ mA}$

$$I_{BQ} = \frac{1.99}{100} = 0.0199 \text{ mA}$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(12)$$

$$V_{CC} = (1+\beta)I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$12 = (101)(0.0199)(1) + 0.7 + (0.0199)(10.1) + \frac{121.2}{R_1}$$

which yields  $R_1 = 13.3 \text{ k}\Omega$  and  $R_2 = 41.6 \text{ k}\Omega$

$$(b) \quad A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \frac{-(100)(2)}{1.31 + (101)(1)} \Rightarrow A_v = -1.95$$

**6.15**

$$I_{CQ} = 0.25 \text{ mA}, I_{EQ} = 0.2525 \text{ mA}$$

$$I_{BQ} = 0.0025 \text{ mA}$$

$$I_{BQ}R_B + V_{BE}(\text{on}) + I_{EQ}(R_S + R_E) - 5 = 0$$

$$(0.0025)(50) + 0.7 + (0.2525)(0.1 + R_E) = 5$$

$$R_E = 16.4 \text{ k}\Omega$$

$$V_E = -(0.0025)(50) - 0.7 = -0.825 \text{ V}$$

$$V_C = V_{CEQ} + V_E = 3 - 0.825 = 2.175 \text{ V}$$

$$R_C = \frac{5 - 2.175}{0.25} \Rightarrow R_C = 11.3 \text{ k}\Omega$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_S}$$

$$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_v = \frac{-(100)(11.3)}{10.4 + (101)(0.1)} \Rightarrow A_v = -55.1$$

$$R_i = R_B \parallel [r_\pi + (1+\beta)R_S]$$

$$= 50 \parallel [10.4 + (101)(0.1)]$$

$$R_i = 50 \parallel 20.5 \Rightarrow R_i = 14.5 \text{ k}\Omega$$

**6.16**

$$(a) \quad I_{CQ} \cong \frac{V_{CC} - V_{CEQ}}{R_C + R_E} = \frac{9 - 5.20}{2.2 + 2} = 0.905 \text{ mA}$$

$$I_{BQ} \cong 0.00754 \text{ mA}; \quad I_{EQ} \cong 0.9123 \text{ mA}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E \\ = (0.00754)(24.2) + 0.7 + (0.9123)(2) = 2.707 \text{ V}$$

$$\text{Now } V_{TH} = 2.707 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (24.2)(9)$$

$$\text{Or } R_1 = 80.5 \text{ k}\Omega \quad \text{and then } R_2 = 34.6 \text{ k}\Omega$$

$$(b) \quad r_\pi = \frac{(120)(0.026)}{0.905} = 3.448 \text{ k}\Omega$$

$$g_m = \frac{0.905}{0.026} = 34.81 \text{ mA/V}$$

$$r_o = \frac{100}{0.905} = 110 \text{ k}\Omega$$

$$V_\pi = i_s (R_1 \parallel R_2 \parallel r_\pi) = i_s (80.5 \parallel 34.6 \parallel 3.448)$$

$$V_\pi = i_s (3.02)$$

$$v_o = -g_m V_\pi (r_o \parallel R_C \parallel R_L)$$

$$\text{Then } R_m = \frac{v_o}{i_s} = -g_m (3.02) (110 \parallel 2.2 \parallel 1) = -(34.81)(3.02)(110 \parallel 0.6875)$$

$$\text{Or } R_m = -71.8 \text{ V/mA}$$

$$(c) \quad \text{For } \beta = 100,$$

$$I_{BQ} = \frac{2.707 - 0.7}{24.2 + (101)(2)} = 0.008873 \text{ mA}$$

$$I_{CQ} = 0.8873 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.8873} = 2.93 \text{ k}\Omega, \quad g_m = \frac{0.8873}{0.026} = 34.13 \text{ mA/V}, \quad r_o = \frac{100}{0.8873} = 113 \text{ k}\Omega$$

$$V_\pi = i_s (R_{TH} \parallel r_\pi) = i_s (24.2 \parallel 2.93) = i_s (2.614)$$

$$R_m = \frac{v_o}{i_s} = -(34.13)(2.614)(113 \parallel 0.6875) = -61.0 \text{ V/mA}$$

$$\text{For } \beta = 150,$$

$$I_{CQ} = (150) \left( \frac{2.707 - 0.7}{24.2 + (151)(2)} \right) = 0.923 \text{ mA}$$

$$r_\pi = 4.225 \text{ k}\Omega, \quad g_m = 35.5 \text{ mA/V}, \quad r_o = 108 \text{ k}\Omega$$

$$V_\pi = i_s (24.2 \parallel 4.225) = i_s (3.597)$$

$$R_m = \frac{v_o}{i_s} = -(35.5)(3.597)(108 \parallel 0.6875) = -87.2 \text{ V/mA}$$

$$\text{So } 61.0 \leq |R_m| \leq 87.2 \text{ V/mA}$$

**6.17**

(a) (i)  $I_{BQ} = \frac{0.8}{81} = 0.009877 \text{ mA}$ ,  $I_{CQ} = 0.790 \text{ mA}$

$$R_B = \frac{0.2}{0.009877} = 20.25 \text{ k}\Omega$$

$$R_C = \frac{-3 - (-5)}{0.79} = 2.53 \text{ k}\Omega$$

(ii)  $g_m = \frac{0.79}{0.026} = 30.38 \text{ mA/V}$ ,  $r_o = \frac{100}{0.79} = 127 \text{ k}\Omega$

$$r_o \parallel R_C = 127 \parallel 2.53 = 2.48 \text{ k}\Omega$$

$$i_o = -g_m v_s \left( \frac{2.48}{2.48 + 4} \right)$$

$$G_f = \frac{i_o}{v_s} = -(30.38) \left( \frac{2.48}{2.48 + 4} \right) = -11.63 \text{ mA/V}$$

(b) (i)  $I_{BQ} = \frac{0.8}{121} = 0.00661 \text{ mA}$ ,  $I_{CQ} = 0.7934 \text{ mA}$

$$R_B = \frac{0.2}{0.00661} = 30.3 \text{ k}\Omega$$

$$R_C = \frac{-3 - (-5)}{0.7934} = 2.52 \text{ k}\Omega$$

(ii)  $g_m = \frac{0.7934}{0.026} = 30.52 \text{ mA/V}$ ,  $r_o = \frac{80}{0.7934} = 101 \text{ k}\Omega$

$$r_o \parallel R_C = 101 \parallel 2.52 = 2.459 \text{ k}\Omega$$

$$G_f = -(30.52) \left( \frac{2.459}{2.459 + 4} \right) = -11.62 \text{ mA/V}$$

**6.18**

(a) (i)  $I_{BQ} = \frac{0.25}{120} = 0.00208 \text{ mA}$ ,  $I_{EQ} = 0.252 \text{ mA}$

$$5 = (0.00208)(2.5) + 0.7 + (0.252)R_E \Rightarrow R_E = 17.0 \text{ k}\Omega$$

$$V_{CEQ} = 3 = 10 - (0.252)(17) - (0.25)R_C \Rightarrow R_C = 10.9 \text{ k}\Omega$$

(ii)  $g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$ ,  $r_\pi = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$

$$A_v = -g_m (R_C \parallel R_L) \left( \frac{r_\pi}{r_\pi + R_S} \right) = -(9.615)(10.9 \parallel 5) \left( \frac{12.48}{12.48 + 2.5} \right) = -27.5$$

(iii)  $v_o = -(27.49)(5 \times 10^{-3} \sin \omega t) = -0.137 \sin \omega t \text{ (V)}$

(b) (i)  $5 = 0.7 + (0.252)R_E \Rightarrow R_E = 17.1 \text{ k}\Omega$

$$V_{CEQ} = 3 = 10 - (0.252)(17.06) - (0.25)R_C \Rightarrow R_C = 10.8 \text{ k}\Omega$$

(ii)  $g_m = 9.615 \text{ mA/V}$ ,  $r_\pi = 12.48 \text{ k}\Omega$

$$A_v = -g_m (R_C \parallel R_L) = -(9.615)(10.8 \parallel 5) = -32.9$$

(iii)  $v_o = -(32.86)(5 \times 10^{-3} \sin \omega t) = -0.164 \sin \omega t \text{ (V)}$

**6.19**

(a) (i)  $I_{BQ} = \frac{5 - 0.7}{2.5 + (81)(10)} = 0.005292 \text{ mA}, \Rightarrow I_{CQ} = 0.4234 \text{ mA}$

$g_m = \frac{0.4234}{0.026} = 16.28 \text{ mA/V}, r_\pi = \frac{(80)(0.026)}{0.4234} = 4.91 \text{ k}\Omega$

$A_v = -g_m (R_C \parallel R_L) \left( \frac{r_\pi}{r_\pi + R_S} \right) = -(16.28)(5 \parallel 5) \left( \frac{4.91}{4.91 + 2.5} \right) = -26.97$

$i_o = \frac{v_o}{R_L} = \frac{A_v v_s}{R_L}$

or  $G_f = \frac{i_o}{v_s} = \frac{-26.97}{5} = -5.39 \text{ mA/V}$

(ii)  $v_o = -(26.97)(4 \times 10^{-3} \sin \omega t) = -0.108 \sin \omega t \text{ (V)}$

$i_o = (-5.39 \times 10^{-3})(4 \times 10^{-3} \sin \omega t) \Rightarrow -21.6 \sin \omega t \text{ (}\mu\text{A)}$

(b) (i)  $I_{CQ} = (120) \left( \frac{5 - 0.7}{2.5 + (121)(10)} \right) = 0.4256 \text{ mA}$

$g_m = \frac{0.4256}{0.026} = 16.37 \text{ mA/V}, r_\pi = \frac{(120)(0.026)}{0.4256} = 7.33 \text{ k}\Omega$

$A_v = -(16.37)(5 \parallel 5) \left( \frac{7.33}{7.33 + 2.5} \right) = -30.5$

$G_f = \frac{-30.5}{5} = -6.1 \text{ mA/V}$

(ii)  $v_o = -(30.5)(4 \times 10^{-3} \sin \omega t) = -0.122 \sin \omega t \text{ (V)}$

$i_o = -24.4 \sin \omega t \text{ (}\mu\text{A)}$

**6.20**

$R_{TH} = R_1 \parallel R_2 = 27 \parallel 15 = 9.64 \text{ K}$

$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{15}{15 + 27} \right) (9) = 3.214 \text{ V}$

$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{3.214 - 0.7}{9.64 + (101)(1.2)} = \frac{2.514}{130.84}$

$I_{BQ} = 0.0192 \text{ mA} \quad I_{CQ} = 1.9214 \text{ mA}$

$g_m = \frac{1.92}{0.026} = 73.9 \text{ mA/V} \quad r_\pi = \frac{(100)(0.026)}{1.92} = 1.35 \text{ K}$

$r_o = \frac{100}{1.92} = 52.1 \text{ K}$

$$V_o = -g_m V_\pi (r_o \parallel R_C \parallel R_L) \quad V_\pi = \left( \frac{r_\pi \parallel R_{TH}}{r_\pi \parallel R_{TH} + R_S} \right) V_S$$

$$r_\pi \parallel R_{TH} = 1.35 \parallel 9.64 = 1.184 \text{ K}$$

$$V_\pi = \left( \frac{1.184}{1.184 + 10} \right) V_S$$

$$= 0.1059 V_S$$

$$A_v = -(73.9)(0.1059)(52.1 \parallel 2.2 \parallel 2)$$

$$= -(73.9)(0.1059)(52.1 \parallel 1.0476)$$

$$= -(73.9)(0.1059)(1.027)$$

$$A_v = -8.04$$

$$A_I = \frac{I_o}{I_S} = \frac{-g_m V_\pi \left( \frac{r_o \parallel R_C}{r_o \parallel R_C + R_L} \right)}{\frac{V_\pi}{R_{TH} \parallel r_\pi}}$$

$$A_I = -g_m (R_{TH} \parallel r_\pi) \left( \frac{r_o \parallel R_C}{r_o \parallel R_C + R_L} \right)$$

$$r_o \parallel R_C = 52.1 \parallel 2.2 = 2.11 \text{ K}$$

$$R_{TH} \parallel r_\pi = 9.64 \parallel 1.35 = 1.184 \text{ K}$$

$$A_I = -(73.9)(1.184) \left( \frac{2.11}{2.11 + 2} \right)$$

$$A_I = -44.9$$

$$R_i = R_{TH} \parallel r_\pi = 9.64 \parallel 1.35$$

$$R_i = 1.184 \text{ K}$$

### 6.21

a.

$$I_E = 0.35 \text{ mA}, \quad I_B = \frac{0.35}{101} = 0.00347 \text{ mA}$$

$$V_B = -I_B R_B = -(0.00347)(10) \Rightarrow V_B = -0.0347 \text{ V}$$

$$V_E = V_B - V_{BE}(\text{on}) \Rightarrow V_E = -0.735 \text{ V}$$

b.

$$V_C = V_{CEQ} + V_E = 3.5 - 0.735 = 2.77 \text{ V}$$

$$I_C = \left( \frac{\beta}{1 + \beta} \right) \cdot I_E = \left( \frac{100}{101} \right) (0.35) = 0.347 \text{ mA}$$

$$R_C = \frac{V^+ - V_C}{I_C} = \frac{5 - 2.77}{0.347} \Rightarrow R_C = 6.43 \text{ k}\Omega$$

(c)

$$A_v = -g_m \left( \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_S} \right) (R_C \parallel r_o)$$

$$g_m = \frac{0.347}{0.026} = 13.3 \text{ mA/V}, \quad r_o = \frac{100}{0.347} = 288 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.347} = 7.49 \text{ k}\Omega$$

$$R_B \parallel r_\pi = 10 \parallel 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left( \frac{4.28}{4.28 + 0.1} \right) (6.43 \parallel 288) \Rightarrow A_v = -81.7$$

d.

$$A_v = -g_m \left( \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_S} \right) (R_C \parallel r_o)$$

$$R_B \parallel r_\pi = 10 \parallel 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left( \frac{4.28}{4.28 + 0.5} \right) (6.43 \parallel 288) \Rightarrow A_v = -74.9$$

## 6.22

a.

$$R_{TH} = R_1 \parallel R_2 = 6 \parallel 1.5 = 1.2 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V^+ = \left( \frac{1.5}{1.5 + 6} \right) (5) = 1.0 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta) R_E} = \frac{1.0 - 0.7}{1.2 + (181)(0.1)} = 0.0155 \text{ mA}$$

$$I_{CQ} = 2.80 \text{ mA}, \quad I_{EQ} = 2.81$$

$$V_{CEQ} = V^+ - I_{CQ} R_C - I_{EQ} R_E$$

$$= 5 - (2.8)(1) - (2.81)(0.1) \Rightarrow \underline{V_{CEQ} = 1.92 \text{ V}}$$

b.

$$r_\pi = \frac{(180)(0.026)}{2.80} = 1.67 \text{ k}\Omega$$

$$g_m = \frac{2.80}{0.026} = 108 \text{ mA/V}$$

(c)

$$A_v = g_m \left( \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) (R_C \parallel R_L)$$

$$R_1 \parallel R_2 \parallel r_\pi = 6 \parallel 1.5 \parallel 1.67 = 0.698 \text{ k}\Omega$$

$$A_v = (108) \left( \frac{0.698}{0.698 + 0.2} \right) (1 \parallel 1.2) = 45.8$$

**6.23**

a.

$$9 = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_S$$

$$I_{EQ} = 0.75 \text{ mA}, I_{BQ} = \frac{0.75}{81} = 0.00926 \text{ mA}$$

$$I_{CQ} = 0.741 \text{ mA}$$

$$9 = (0.75)R_E + 0.7 + (0.00926)(2) \Rightarrow \underline{R_E = 11.0 \text{ k}\Omega}$$

b.

$$V_E = 9 - (0.75)(11) = 0.75 \text{ V}$$

$$V_C = V_E - V_{ECQ} = 0.75 - 7 = -6.25 \text{ V}$$

$$R_C = \frac{V_C - (-9)}{I_{CQ}} = \frac{9 - 6.25}{0.741} \Rightarrow \underline{R_C = 3.71 \text{ k}\Omega}$$

c.

$$A_v = -g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) (R_C \parallel R_L \parallel r_o)$$

$$r_\pi = \frac{(80)(0.026)}{0.741} = 2.81 \text{ k}\Omega$$

$$r_o = \frac{80}{0.741} = 108 \text{ k}\Omega$$

$$A_v = \frac{-80}{2.81 + 2} (3.71 \parallel 10 \parallel 108)$$

$$A_v = -43.9$$

d.

$$R_i = R_S + r_\pi = 2 + 2.81 \Rightarrow R_i = 4.81 \text{ k}\Omega$$

**6.24**

(a) For  $\beta = 80$ ,  $I_{CQ} = (80) \left[ \frac{4 - 0.7}{5 + (81)(5)} \right] = 0.6439 \text{ mA}$

$$g_m = \frac{0.6439}{0.026} = 24.77 \text{ mA/V}, r_\pi = \frac{(80)(0.026)}{0.6439} = 3.23 \text{ k}\Omega, r_o = \frac{50}{0.6439} = 77.7 \text{ k}\Omega$$

$$V_\pi = \frac{r_\pi \parallel R_B}{r_\pi \parallel R_B + R_S} \cdot v_s = \frac{(3.23 \parallel 5)}{(3.23 \parallel 5) + 1} \cdot v_s = (0.6624)v_s$$

$$A_v = -g_m \cdot \frac{V_\pi}{v_s} \cdot (r_o \parallel R_C \parallel R_L) = -(24.77)(0.6624)(77.7 \parallel 4 \parallel 4) = -32$$

For  $\beta = 120$ ,  $I_{CQ} = (120) \left[ \frac{4 - 0.7}{5 + (121)(5)} \right] = 0.6492 \text{ mA}$

$$g_m = 24.97 \text{ mA/V}, r_\pi = 4.806 \text{ k}\Omega, r_o = 77 \text{ k}\Omega$$

$$V_\pi = \frac{(4.806 \parallel 5)}{(4.806 \parallel 5) + 1} \cdot v_s = (0.710)v_s$$

$$A_v = -(24.97)(0.710)(77 \parallel 4 \parallel 4) = -34.6$$

So  $32 \leq |A_v| \leq 34.6$

(b)  $R_i = R_B \parallel r_\pi$

For  $\beta = 80$ ,  $R_i = 5 \parallel 3.23 = 1.96 \text{ k}\Omega$

For  $\beta = 120$ ,  $R_i = 5 \parallel 4.806 = 2.45 \text{ k}\Omega$

So  $1.96 \leq R_i \leq 2.45 \text{ k}\Omega$

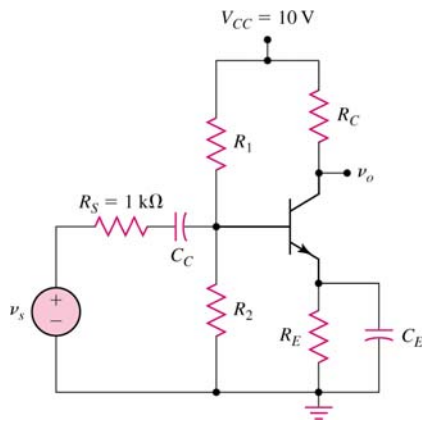
(c)  $R_o = R_C \parallel r_o$

For  $\beta = 80$ ,  $R_o = 4 \parallel 77.7 = 3.804 \text{ k}\Omega$

For  $\beta = 120$ ,  $R_o = 4 \parallel 77 = 3.802 \text{ k}\Omega$

So  $3.802 \leq R_o \leq 3.804 \text{ k}\Omega$

**6.25**



Assume an npn transistor with  $\beta = 100$  and  $V_A = \infty$ . Let  $V_{CC} = 10 \text{ V}$ .

$$|A_v| = \frac{0.5}{0.01} = 50$$

Bias at  $I_{CQ} = 1 \text{ mA}$  and let  $R_E = 1 \text{ k}\Omega$

For a bias stable circuit

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(10) = \frac{101}{R_1}$$

$$I_{BQ} = \frac{1}{100} = 0.01 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E$$

$$\frac{101}{R_1} = (0.01)(10.1) + 0.7 + (101)(0.01)(1)$$

which yields  $R_1 = 55.8 \text{ k}\Omega$ ,  $R_2 = 12.3 \text{ k}\Omega$

Now

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$



$$V_o = -g_m V_\pi R_C$$

$$\text{where } V_\pi = \left( \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) \cdot V_s = \left( \frac{10.1 \parallel 2.6}{10.1 \parallel 2.6 + 1} \right) \cdot V_s = 0.674 V_s$$

$$\text{Then } A_v = \frac{V_o}{V_s} = -(0.674) g_m R_C = -(0.674)(38.46) R_C = -50$$

$$\text{which yields } R_C = 1.93 \text{ k}\Omega$$

With this  $R_C$ , the dc bias is OK.

Finish Design, Set  $R_C = 2 \text{ k}\Omega$ ,  $R_E = 1 \text{ k}\Omega$

$$R_1 = 56 \text{ k}\Omega, R_2 = 12 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 9.88 \text{ K}$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{12}{12 + 56} \right) (10) = 1.765 \text{ V}$$

$$I_{BQ} = \frac{1.765 - 0.7}{9.88 + (101)(1)} = 9.60 \text{ }\mu\text{A}$$

$$I_{CQ} = 0.9605 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.9605} = 2.707 \text{ K} \quad g_m = \frac{0.9605}{0.026} = 36.94$$

$$R_{TH} \parallel r_\pi = 2.125 \text{ K}$$

$$V_\pi = \left( \frac{R_{TH} \parallel r_\pi}{R_{TH} \parallel r_\pi + R_S} \right) \cdot V_i = \left( \frac{2.125}{2.125 + 1} \right) \cdot V_i = (0.680) \cdot V_i$$

$$A_v = -(0.680) g_m R_C = -(0.680)(36.94)(2) = -50.2$$

Design specification met.

## 6.26

a.

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$$I_{CQ} = 1.69 \text{ mA}, I_{EQ} = 1.71 \text{ mA}$$

$$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$$

$$V_{CEQ} = 5.38 \text{ V}$$

b.

$$g_m = \frac{1.69}{0.026} = 65 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.69} = 1.54 \text{ k}\Omega, r_o = \infty$$

(c)

$$A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi + (1 + \beta)R_E} \cdot \frac{R_B \parallel R_{ib}}{R_B \parallel R_{ib} + R_S}$$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 1.54 + (101)(3) = 304.5 \text{ k}\Omega$$

$$R_B \parallel R_{ib} = 10 \parallel 304.5 = 9.68 \text{ k}\Omega$$

Then

$$A_v = \frac{-(100)(6.8 \parallel 6.8)}{1.54 + (101)(3)} \cdot \left( \frac{9.68}{9.68 + 0.5} \right) = -1.06$$

$$i_o = \left( \frac{R_C}{R_C + R_L} \right) (-\beta i_b)$$

$$i_b = \left( \frac{R_B}{R_B + r_\pi + (1 + \beta)R_E} \right) \cdot i_s$$

$$A_i = -(\beta) \left( \frac{R_C}{R_C + R_L} \right) \left( \frac{R_B}{R_B + r_\pi + (1 + \beta)R_E} \right)$$

$$= -(100) \left( \frac{6.8}{6.8 + 6.8} \right) \left( \frac{10}{10 + 1.54 + (101)(3)} \right) \Rightarrow A_i = -1.59$$

(d)  $R_{is} = R_S + R_B \parallel R_{ib} = 0.5 + 10 \parallel 304.5 = 10.2 \text{ k}\Omega$

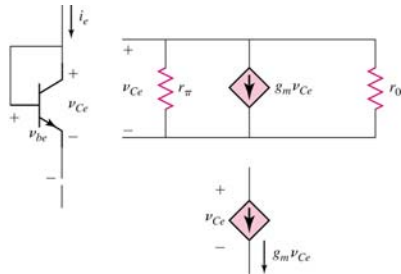
(e)

$$A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi + (1 + \beta)R_E} = \frac{-(100)(6.8 \parallel 6.8)}{1.54 + (101)(3)}$$

$$A_v = -1.12$$

$$A_i = -1.59, \text{ same as part (c)}$$

6.27



$$r = \frac{v_{ce}}{g_m v_{ce}} = \frac{1}{g_m}$$

$$\text{So } r_e = r_\pi \parallel \frac{1}{g_m} \parallel r_o$$

6.28

(a) Set  $\frac{R_C}{R_E} = 12.5 \Rightarrow R_C = 12.5 R_E$

$$\text{Set } V_{CEQ} = 1.5 \cong 3.3 - I_{CQ}(R_C + R_E) = 3.3 - I_{CQ}(13.5 R_E)$$

$$\text{Set } I_{CQ} = 0.1 \text{ mA}, \Rightarrow R_E = 1.33 \text{ k}\Omega \text{ and } R_C = 16.7 \text{ k}\Omega$$

$$\text{We find } I_{BQ} = 0.833 \mu\text{A}$$

$$\text{Set } R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(1.33) = 16.1 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1 + \beta)I_{BQ}R_E = (0.000833)(16.1) + 0.7 + (121)(0.000833)(1.33)$$

$$V_{TH} = 0.8475 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (16.1)(3.3)$$

So  $R_1 = 62.7 \text{ k}\Omega$ ,  $R_2 = 21.7 \text{ k}\Omega$

Actual gain:

$$r_\pi = \frac{(120)(0.026)}{0.1} = 31.2 \text{ k}\Omega$$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 31.2 + (121)(1.33) = 192.1 \text{ k}\Omega$$

$$R_i = R_{TH} \parallel R_{ib} = 16.1 \parallel 192.1 = 14.85 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{-(120)(16.7)}{31.2 + (121)(1.33)} \cdot \left( \frac{14.85}{14.85 + 0.1} \right) = -10.4$$

(b)  $R_{is} = R_s + R_i = 0.1 + 14.85 = 14.95 \text{ k}\Omega$

$$R_o = R_C = 16.7 \text{ k}\Omega$$

### 6.29

$$\frac{100}{5} = 20.$$

Need a voltage gain of 5

Assume a sign inversion from a common-emitter is not important. Use the configuration for Figure 6.31.

Let  $R_s = 0$ . Need an input resistance of

$$R_i = \frac{5 \times 10^{-3}}{0.2 \times 10^{-6}} = 25 \times 10^3 \Rightarrow R_i = 25 \text{ k}\Omega$$

$$R_i = R_{TH} \parallel R_{ib}, \text{ Let } R_{TH} = 50 \text{ k}\Omega, \text{ then } R_{ib} = 50 \text{ k}\Omega$$

$$R_{ib} = r_\pi + (1 + \beta)R_E \cong (1 + \beta)R_E$$

$$\text{For } \beta = 100, R_E \cong \frac{R_{ib}}{1 + \beta} = \frac{50}{101} = 0.495 \text{ k}\Omega$$

Let  $R_E = 0.5 \text{ k}\Omega$ ,  $V_{CC} = 10 \text{ V}$ ,  $I_{CQ} = 0.2 \text{ mA}$

$$\text{Then } I_{BQ} = \frac{0.2}{100} = 0.002 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (50)(10) = (0.002)(50) + 0.7 + (101)(0.002)(0.5)$$

which yields  $R_1 = 555 \text{ k}\Omega$  and  $R_2 = 55 \text{ k}\Omega$

$$\text{Now } A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E}, \quad r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

So

$$-20 = \frac{-(100)R_C}{13 + (101)(0.5)} \Rightarrow R_C = 12.7 \text{ k}\Omega$$

[Note:  $I_{CQ}R_C = (0.2)(12.7) = 2.54 \text{ V}$ . So dc biasing is OK.]

6.30

$$\text{Set } \frac{R_C}{R_E} = 15 \Rightarrow R_C = 15R_E$$

$$\text{Set } R_C = 5 \text{ k}\Omega, \text{ then } R_E = 0.333 \text{ k}\Omega$$

$$\text{Set } I_{CQ} = 0.5 \text{ mA}, \text{ then } V_{ECQ} \cong 5 - (0.5)(5.333) = 2.33 \text{ V}$$

$$r_\pi = 4.68 \text{ k}\Omega, R_{ib} = r_\pi + (1 + \beta)R_E = 4.68 + (91)(0.333) = 35 \text{ k}\Omega$$

$$\text{Set } R_i = 22 \text{ k}\Omega = R_{TH} \parallel R_{ib} = R_{TH} \parallel 35 \Rightarrow R_{TH} \cong 60 \text{ k}\Omega$$

$$\text{Now } I_{BQ} = 0.00556 \text{ mA}, I_{EQ} = 0.506 \text{ mA}$$

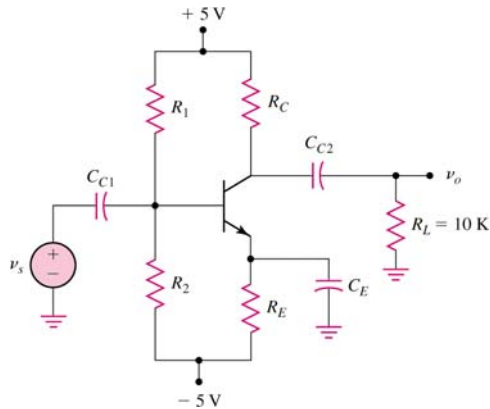
$$V_{CC} = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$5 = (0.506)(0.333) + 0.7 + (0.00556)(60) + V_{TH}$$

$$V_{TH} = 3.798 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (60)(5)$$

$$\text{So that } R_1 = 79 \text{ k}\Omega \text{ and } R_2 = 249 \text{ k}\Omega$$

6.31



$$\beta = 120$$

$$\text{Let } I_{CQ} = 0.35 \text{ mA}, I_{EQ} = 0.353 \text{ mA}$$

$$I_{BQ} = 0.00292 \text{ mA}$$

$$\text{Let } R_E = 2 \text{ k}\Omega. \text{ For } V_{CEQ} = 4 \text{ V} \Rightarrow 10 = 4 + (0.35)R_C + (0.353)(2)$$

$$R_C = 15.1 \text{ k}\Omega, r_\pi = \frac{(120)(0.026)}{0.35} = 8.91 \text{ k}\Omega$$

$$A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi} = -\frac{(120)(15.1 \parallel 10)}{8.91}$$

$$A_v = -81.0$$

For bias stable circuit:

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(24.2)(10) - 5 = (0.00292)(24.2) + 0.7 + (121)(0.00292)(2) - 5$$

$$\frac{1}{R_1}(242) = 1.477, \quad R_1 = 164 \text{ k}\Omega$$

$$\frac{164R_2}{164 + R_2} = 24.2 \Rightarrow R_2 = 28.4 \text{ k}\Omega$$

$$\frac{10}{164 + 28.4} = 0.052, \quad 0.35 + 0.052 = \underline{0.402 \text{ mA}}$$

So bias current specification is met.

**6.32**

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 50 = 19.88 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{50}{50 + 33} \right) (3.3) = 1.988 \text{ V}$$

$$3.3 = (101)I_{BQ}(1) + 0.7 + I_{BQ}(19.88) + 1.988$$

So  $I_{BQ} = 0.005063 \text{ mA}$ ,  $I_{CQ} = 0.5063 \text{ mA}$ ,  $I_{EQ} = 0.5114 \text{ mA}$

$$V_{ECQ} = 3.3 - (0.5114)(1) - (0.5063)(2) = 1.776 \text{ V}$$

Then  $\Delta V_{EC} = 1.776 - 0.5 = 1.276 \text{ V}$ , or  $\Delta V_{EC} = 3 - 1.776 = 1.224 \text{ V}$

So  $\Delta V_{EC} = 2(1.224) = 2.448 \text{ V}$  peak-to-peak

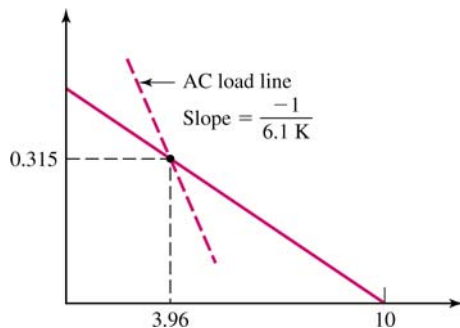
**6.33**

$$I_{BQ} = \frac{5 - 0.7}{50 + (101)(0.1 + 12.9)} = 0.00315 \text{ mA}$$

$$I_{CQ} = 0.315 \text{ mA}, \quad I_{EQ} = 0.319 \text{ mA}$$

$$V_{CEQ} = (5 + 5) - (0.315)(6) - (0.319)(13)$$

$$V_{CEQ} = 3.96 \text{ V}$$



$$\Delta i_c = -\frac{1}{6.1} \Delta v_{eC}$$

$$\text{For } \Delta i_c = 0.315 - 0.05 = 0.265 \Rightarrow |\Delta v_{eC}| = 1.62$$

$$v_{eC}(\text{min}) = 3.96 - 1.62 = 2.34$$

Output signal swing determined by current:

Max. output swing =  $\underline{3.24 \text{ V peak-to-peak}}$

**6.34**

(a)  $5 = (81)I_{BQ}(10) + 0.7 + I_{BQ}(2.5)$

So that  $I_{BQ} = 0.005292 \text{ mA}$ ,  $I_{CQ} = 0.4234 \text{ mA}$ ,  $I_{EQ} = 0.4287 \text{ mA}$

Now  $V_{ECQ} = 10 - (0.4234)(5) - (0.4287)(10) = 3.60 \text{ V}$

$\Delta V_{EC} = \Delta I_C (R_C \parallel R_L) = \Delta I_C (2.5)$

For  $\Delta I_C = 0.4234 \Rightarrow \Delta V_{EC} = 1.06 \text{ V}$

So that  $\Delta V_{EC} = 2(1.06) = 2.12 \text{ V peak-to-peak}$

(b)  $\Delta I_C = 2(0.4234) = 0.847 \text{ mA peak-to-peak}$

**6.35**

$I_{EQ} = 0.8 \text{ mA}$ ,  $I_{CQ} = 0.790 \text{ mA}$ ,  $I_{BQ} = 0.009877 \text{ mA}$

$V_E = 0.7 + (0.009877)(20) = 0.898 \text{ V}$

$V_C = (0.79)(2.5) - 5 = -3.025 \text{ V}$

Then  $V_{ECQ} = V_E - V_C = 3.923 \text{ V}$

$\Delta V_{EC} = \Delta I_C (R_C \parallel R_L) = \Delta I_C (2.5 \parallel 4) = \Delta I_C (1.538)$

For  $\Delta I_C = 0.79 - 0.08 = 0.71 \text{ mA}$ , then  $\Delta V_{EC} = (0.71)(1.538) = 1.09 \text{ V}$

So,  $\Delta I_C = 2(0.71) = 1.42 \text{ mA peak-to-peak}$ ,

$\Delta i_o = \left( \frac{2.5}{2.5 + 4} \right) \Delta I_C = 0.546 \text{ mA peak-to-peak}$

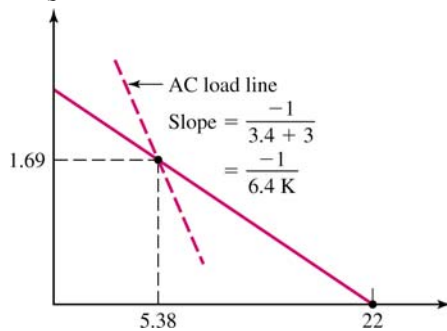
**6.36**

$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$

$I_{CQ} = 1.69 \text{ mA}$ ,  $I_{EQ} = 1.71 \text{ mA}$

$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$

$V_{CEQ} = 5.38 \text{ V}$



$\Delta i_c = -\frac{1}{6.4} \Delta v_{ce}$

For  $v_{ce}(\min) = 1 \text{ V}$ ,  $\Delta v_{ce} = 5.38 - 1 = 4.38 \text{ V} \Rightarrow |\Delta i_c| = \frac{4.38}{6.4} = 0.684 \text{ mA}$

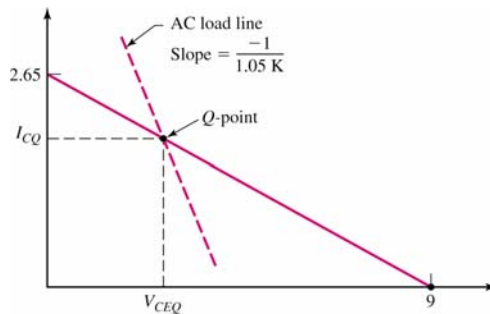
Output swing limited by voltage:

$$\begin{aligned} \Delta v_{ce} &= \text{Max. swing in output voltage} \\ &= \underline{8.76 \text{ V peak-to-peak}} \end{aligned}$$

$$\Delta i_o = \frac{1}{2} \Delta i_c \Rightarrow \Delta i_o = 0.342 \text{ mA}$$

$$\text{or } \Delta i_o = 0.684 \text{ mA (peak-to-peak)}$$

**6.37**



$$r_o = \frac{100}{I_{CQ}}$$

Neglect  $r_o$  as 1st approx.: dc load line  $V_{CE} = 9 - I_C$  (3.4)

$$\Delta I_C = I_{CQ} - 0.1$$

$$\Delta V_{CE} = V_{CEQ} - 1$$

$$\text{Also } \Delta V_{CE} = \Delta I_C (R_C \parallel R_L) = \Delta I_C (1.05)$$

$$\text{Or } V_{CEQ} - 1 = (I_{CQ} - 0.1)(1.05)$$

Substituting the expression for the dc load line.

$$[9 - I_{CQ} (3.4) - 1] = (I_{CQ} - 0.1)(1.05)$$

$$8.105 = I_{CQ} (4.45) \Rightarrow I_{CQ} = 1.821 \text{ mA}$$

$$V_{CEQ} = 2.81 \text{ V}$$

$$I_{BQ} = \frac{1.821}{100} = 0.01821$$

$$R_{TH} = (0.1)(101)(1.2) = 12.12 \text{ K}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (12.12)(9) = (0.01821)(12.12) + 0.7 + (101)(0.01821)(1.2)$$

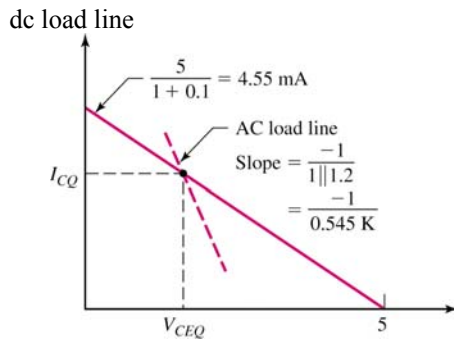
$$= 0.2207 + 0.7 + 2.20705$$

$$R_1 = 34.9 \text{ K}$$

$$R_2 = 18.6 \text{ K}$$

$$\frac{34.9 R_2}{34.9 + R_2} = 12.12$$

**6.38**



For maximum symmetrical swing

$$\Delta i_c = I_{CQ} - 0.25$$

$$\Delta v_{ce} = V_{CEQ} - 0.5 \text{ and } |\Delta i_c| = \frac{1}{0.545 \text{ k}\Omega} \cdot |\Delta v_{ce}|$$

$$I_{CQ} - 0.25 = \frac{V_{CEQ} - 0.5}{0.545}$$

$$V_{CEQ} = 5 - I_{CQ} (1.1)$$

$$0.545(I_{CQ} - 0.25) = [5 - I_{CQ}(1.1)] - 0.5$$

$$(0.545 + 1.1)I_{CQ} = 5 - 0.5 + 0.136$$

$$\underline{I_{CQ} = 2.82 \text{ mA}, \quad I_{BQ} = 0.0157 \text{ mA}}$$

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E$$

$$= (0.1)(181)(0.1) = 1.81 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = I_{BQ} R_{TH} + V_{BE(on)} + (1 + \beta)I_{BQ} R_E$$

$$\frac{1}{R_1}(1.81)(5) = (0.0157)(1.81) + 0.7 + (181)(0.0157)(0.1)$$

$$\frac{1}{R_1}(9.05) = 1.013 \Rightarrow \underline{R_1 = 8.93 \text{ k}\Omega}$$

$$\frac{8.93R_2}{8.93 + R_2} = 1.81 \Rightarrow \underline{R_2 = 2.27 \text{ k}\Omega}$$

**6.39**

$$I_{CQ} = 0.647 \text{ mA}, \quad V_{CEQ} = 10 - (0.647)(9) = 4.18 \text{ V}$$

$$\Delta i_c = I_{CQ} = 0.647 \text{ mA}$$

So  $\Delta v_{ce} = \Delta i_c (4 \parallel 4) = (0.647)(2) = 1.294 \text{ V}$

Voltage swing is well within the voltage specification. Then  $\Delta v_{ce} = 2(1.294) = 2.59 \text{ V}$  peak-to-peak.



**6.40**

$$(a) A_v = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E}$$

$$0.92 = \frac{(121)(0.5)}{r_\pi + (121)(0.5)} \Rightarrow r_\pi = 5.261 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.593 \text{ mA}$$

$$(b) r_o = \frac{V_A}{I_{CQ}} = \frac{20}{0.593} = 33.7 \text{ k}\Omega$$

$$A_v = \frac{(1+\beta)(r_o \parallel R_E)}{r_\pi + (1+\beta)(r_o \parallel R_E)}$$

$$r_o \parallel R_E = 33.7 \parallel 0.5 = 0.4927 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{(121)(0.4927)}{5.261 + (121)(0.4927)} = 0.919$$

$$(c) \text{ For (a), } R_o = R_E \parallel \frac{r_\pi}{1+\beta} = 0.5 \parallel \frac{5.261}{121} = 0.5 \parallel 0.04348 \Rightarrow R_o = 40 \Omega$$

$$\text{For (b), } R_o = R_E \parallel r_o \parallel \frac{r_\pi}{1+\beta} = 0.04 \parallel 33.7 \Rightarrow R_o = 39.95 \Omega$$

**6.41**

$$(a) R_{ib} = r_\pi + (1+\beta)R_E$$

$$A_v = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E} = \frac{(1+\beta)R_E}{R_{ib}}$$

$$R_E = \frac{(A_v)(R_{ib})}{1+\beta} = \frac{(0.95)(50)}{81} = 0.586 \text{ k}\Omega$$

$$50 = r_\pi + (81)(0.586) \Rightarrow r_\pi = 2.5 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(80)(0.026)}{2.5} = 0.832 \text{ mA}$$

$$(b) R_o = R_E \parallel \frac{r_\pi}{1+\beta} = 0.586 \parallel \frac{2.5}{81} = 0.586 \parallel 0.03086$$

$$R_o = 29.3 \Omega$$

**6.42**

$$(a) g_m = 76.92 \text{ mA/V}, \quad r_\pi = 1.04 \text{ k}\Omega, \quad r_o = 25 \text{ k}\Omega$$

$$A_v = \frac{(1+\beta)r_o}{R_S + r_\pi + (1+\beta)r_o} = \frac{(81)(25)}{1 + 1.04 + (81)(25)} = 0.999$$

$$R_i = R_S + r_\pi + (1+\beta)r_o = 1 + 1.04 + (81)(25) \Rightarrow R_i = 2.027 \text{ M}\Omega$$

$$R_o = r_o \parallel \frac{r_\pi + R_S}{1+\beta} = 25 \parallel \left( \frac{1.04 + 1}{81} \right) = 25 \parallel 0.0252 \Rightarrow R_o = 25.2 \Omega$$

$$(b) r_\pi = 10.4 \text{ k}\Omega, \quad r_o = 250 \text{ k}\Omega$$

$$A_v = \frac{(81)(250)}{1 + 10.4 + (81)(250)} = 0.9994$$

$$R_i = 1 + 10.4 + (81)(250) \Rightarrow R_i = 20.26 \text{ M}\Omega$$

$$R_o = 250 \parallel \left( \frac{10.4 + 1}{81} \right) = 250 \parallel 0.1407 \Rightarrow R_o = 141 \Omega$$

**6.43**

(a)  $R_o = \frac{r_\pi + R_S}{1 + \beta}$

$$0.015 = \frac{r_\pi + 0.5}{121} \Rightarrow r_\pi = 1.315 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(120)(0.026)}{1.315} = 2.37 \text{ mA}$$

(b)  $r_o = \frac{50}{2.373} = 21.07 \text{ k}\Omega$

$$A_v = \frac{(1 + \beta)r_o}{R_S + r_\pi + (1 + \beta)r_o} = \frac{(121)(21.07)}{0.5 + 1.315 + (121)(21.07)} = 0.9993$$

**6.44**

a.

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 10 = 5 \text{ k}\Omega$$

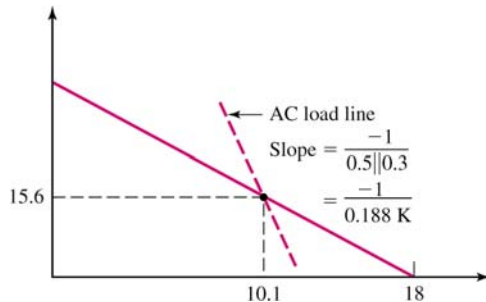
$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (18) - 9 = \left( \frac{10}{10 + 10} \right) (18) - 9 = 0$$

$$I_{BQ} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 0.0869 \text{ mA}$$

$$I_{CQ} = 15.6 \text{ mA}, \quad I_{EQ} = 15.7 \text{ mA}$$

$$V_{CEQ} = 18 - (15.7)(0.5) = 10.1 \text{ V}$$

b.



c.

$$r_\pi = \frac{(180)(0.026)}{15.6} = 0.30 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_\pi + (1 + \beta)(R_E \parallel R_L)} \cdot \left( \frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S} \right)$$

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L) = 0.30 + (181)(0.5 \parallel 0.3) = 34.2 \text{ k}\Omega$$

$$R_1 \parallel R_2 \parallel R_{ib} = 5 \parallel 34.2 = 4.36 \text{ k}\Omega$$

$$A_v = \frac{(181)(0.5 \parallel 0.3)}{0.3 + (181)(0.5 \parallel 0.3)} \cdot \left( \frac{4.36}{4.36 + 1} \right) \Rightarrow A_v = 0.806$$

d.

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L)$$

$$R_{ib} = 0.30 + (181)(0.188) = 34.3 \text{ k}\Omega$$

$$R_o = R_E \parallel \left( \frac{r_{\pi} + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right) = 0.5 \parallel \left( \frac{0.3 + 5 \parallel 1}{181} \right) \Rightarrow R_o = 6.18 \Omega$$

### 6.45

a.

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 10 = 5 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (-10) = -5 \text{ V}$$

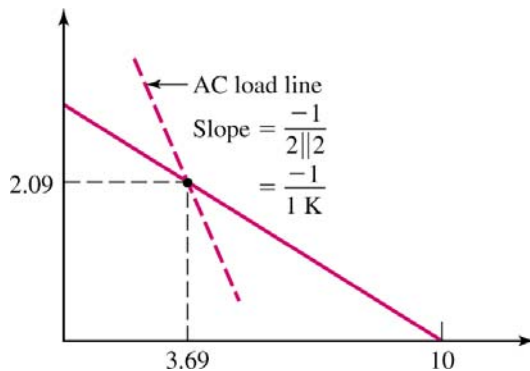
$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1 + \beta) I_{BQ} R_E - 10$$

$$I_{BQ} = \frac{-5 - 0.7 - (-10)}{5 + (121)(2)} = 0.0174 \text{ mA}$$

$$I_{CQ} = 2.09 \text{ mA}, \quad I_{EQ} = 2.11 \text{ mA}$$

$$V_{CEQ} = 10 - (2.09)(1) - (2.11)(2) = 3.69 \text{ V}$$

b.



c.

$$r_{\pi} = \frac{(120)(0.026)}{2.09} = 1.49 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_{\pi} + (1 + \beta)(R_E \parallel R_L)} \cdot \left( \frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S} \right)$$

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L) = 1.49 + (121)(2 \parallel 2)$$

$$R_{ib} = 122.5 \text{ k}\Omega, \quad R_1 \parallel R_2 \parallel R_{ib} = 5 \parallel 122.5 = 4.80 \text{ k}\Omega$$

$$A_v = \frac{(121)(2 \parallel 2)}{1.49 + (121)(2 \parallel 2)} \cdot \left( \frac{4.80}{4.80 + 5} \right) \Rightarrow A_v = 0.484$$

d.

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L)$$

$$R_{ib} = 1.49 + (121)(2 \parallel 2) \Rightarrow R_{ib} = 122 \text{ k}\Omega$$

$$R_o = R_E \parallel \left( \frac{r_{\pi} + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right) = 2 \parallel \left( \frac{1.49 + 5 \parallel 5}{121} \right) \Rightarrow R_o = 32.4 \Omega$$

### 6.46

(a)  $R_{TH} = R_1 \parallel R_2 = 585 \parallel 135 = 109.7 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{135}{135 + 585} \right) (3.3) = 0.61875 \text{ V}$$

$$3.3 = (1 + \beta)I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{3.3 - 0.7 - 0.61875}{109.7 + (91)(12)} = 0.001649 \text{ mA}$$

Then  $I_{CQ} = 0.1484 \text{ mA}$ ,  $I_{EQ} = 0.150 \text{ mA}$

$$V_{ECQ} = 3.3 - (0.15)(12) = 1.5 \text{ V}$$

(c)  $r_{\pi} = \frac{(90)(0.026)}{0.1484} = 15.77 \text{ k}\Omega$ ,  $r_o = \frac{60}{0.1484} = 404 \text{ k}\Omega$

$$A_v = \frac{(1 + \beta)(r_o \parallel R_E \parallel R_L)}{r_{\pi} + (1 + \beta)(r_o \parallel R_E \parallel R_L)}$$

$$r_o \parallel R_E \parallel R_L = 404 \parallel 12 \parallel 4 = 2.978 \text{ k}\Omega$$

$$A_v = \frac{(91)(2.978)}{15.77 + (91)(2.978)} = 0.945$$

$$R_{ib} = r_{\pi} + (1 + \beta)(r_o \parallel R_E \parallel R_L) = 15.77 + (91)(2.978) = 286.8 \text{ k}\Omega$$

$$A_i = (1 + \beta) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{r_o \parallel R_E}{r_o \parallel R_E + R_L} \right) = (91) \left( \frac{109.7}{109.7 + 286.8} \right) \left( \frac{404 \parallel 12}{404 \parallel 12 + 4} \right) = 18.7$$

(d)  $R_{ib} = 286.8 \text{ k}\Omega$

$$R_o = R_E \parallel r_o \parallel \frac{r_{\pi}}{1 + \beta} = 12 \parallel 404 \parallel \frac{15.77}{91} \Rightarrow R_o = 171 \Omega$$

### 6.47

(a)

$$I_{BQ} = \frac{0.5}{81} = 0.00617 \text{ mA}$$

$$V_B = I_{BQ}R_B = (0.00617)(10) \Rightarrow \underline{V_B = 0.0617 \text{ V}}$$

$$V_E = V_B + 0.7 \Rightarrow \underline{V_E = 0.7617 \text{ V}}$$

(b)

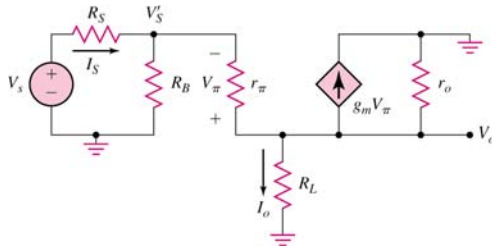
$$I_{CQ} = (0.5) \left( \frac{80}{81} \right) = 0.494 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.494}{0.026} \Rightarrow g_m = 19 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{0.494} \Rightarrow r_\pi = 4.21 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.494} \Rightarrow r_o = 304 \text{ k}\Omega$$

(c)



For  $R_S = 0$

$$V_o = - \left( \frac{V_\pi}{r_\pi} + g_m V_\pi \right) (R_L \parallel r_o)$$

so that 
$$V_\pi = \frac{-V_o}{\left( \frac{1+\beta}{r_\pi} \right) (R_L \parallel r_o)}$$

Now  $V_s + V_\pi = V_o$

$$V_s = V_o - V_\pi = V_o + \frac{V_o}{\left( \frac{1+\beta}{r_\pi} \right) (R_L \parallel r_o)}$$

We find

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(R_L \parallel r_o)}{r_\pi + (1+\beta)(R_L \parallel r_o)} = \frac{(81)(0.5 \parallel 304)}{4.21 + (81)(0.5 \parallel 304)}$$

$$A_v = 0.906$$

$$R_{ib} = r_\pi + (1+\beta)(R_L \parallel r_o) \cong 4.21 + (81)(0.5) = 44.7 \text{ k}\Omega$$

$$I_b = \left( \frac{R_B}{R_B + R_{ib}} \right) \cdot I_s \quad \text{and} \quad I_o = \left( \frac{r_o}{r_o + R_L} \right) (1+\beta) I_b$$

Then

$$A_i = \frac{I_o}{I_s} = (1+\beta) \left( \frac{R_B}{R_B + R_{ib}} \right) \left( \frac{r_o}{r_o + R_L} \right)$$

$$A_i \cong (81) \left( \frac{10}{10 + 44.7} \right) (1) \Rightarrow A_i = 14.8$$

(d)

$$V_s' = \left( \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S} \right) \cdot V_s = \left( \frac{10 \| 44.7}{10 \| 44.7 + 2} \right) \cdot V_s = (0.803)V_s$$

Then  $A_v = (0.803)(0.906) \Rightarrow A_v = 0.728$

$A_i = 14.8$  (unchanged)

### 6.48

(a)

$$I_{CQ} = 1.98 \text{ mA}, \quad r_\pi = \frac{(100)(0.026)}{1.98} = 1.313 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.98} = 50.5 \text{ k}\Omega$$

$$R_o = \frac{r_\pi + R_S}{1 + \beta} \| r_o = \frac{1.31 + 10}{101} \| 50.5 \Rightarrow R_o = 112 \Omega$$

(b) From Equation (6.68)

$$A_v = \frac{(1 + \beta)(r_o \| R_L)}{r_\pi + (1 + \beta)(r_o \| R_L)} \quad r_o = \frac{100}{1.98} = 50.5 \text{ K}$$

(i)

$$R_L = 0.5 \text{ K}$$

$$A_v = \frac{(101)(50.5 \| 0.5)}{1.31 + (101)(50.5 \| 0.5)}$$

$$A_v = \frac{(101)(0.4951)}{1.31 + (101)(0.4951)} \Rightarrow A_v = 0.974$$

(ii)

$$R_L = 5 \text{ K} \quad r_o \| R_L = 50.5 \| 5 = 4.5495$$

$$A_v = \frac{(101)(4.55)}{1.31 + (101)(4.55)} \Rightarrow A_v = 0.997$$

### 6.49

(a)  $\frac{5 - 0.7}{3.3} = I_{EQ} = 1.303 \text{ mA}, \quad I_{CQ} = 1.29 \text{ mA}$

$$V_{ECQ} = 0.7 - (-5) = 5.7 \text{ V}$$

(b)  $r_\pi = \frac{(110)(0.026)}{1.29} = 2.217 \text{ k}\Omega, \quad r_o = \frac{50}{1.29} = 38.76 \text{ k}\Omega$

$$A_v = \frac{(1 + \beta)(r_o \| R_E \| R_L)}{r_\pi + (1 + \beta)(r_o \| R_E \| R_L)}$$

$$r_o \| R_E \| R_L = 38.76 \| 3.3 \| 1 = 0.7525 \text{ k}\Omega$$

$$A_v = \frac{(111)(0.7525)}{2.217 + (111)(0.7525)} = 0.974$$

$$R_{ib} = r_\pi + (1 + \beta)(r_o \| R_E \| R_L) = 2.217 + (111)(0.7525) = 85.7 \text{ k}\Omega$$

$$R_o = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_o = \frac{2.217}{111} \parallel 3.3 \parallel 38.76 \Rightarrow R_o = 19.8 \Omega$$

$$(c) \quad i_s(t) = \frac{v_s(t)}{R_{ib}} = \frac{2.8 \sin \omega t}{85.7} \Rightarrow i_s(t) = 32.7 \sin \omega t \quad (\mu A)$$

$$i_o(t) = (1 + \beta) \left( \frac{R_E \parallel r_o}{R_E \parallel r_o + R_L} \right) \cdot i_s(t) = (111) \left( \frac{3.041}{3.041 + 1} \right) (32.7 \sin \omega t)$$

$$\text{or } i_o(t) = 2.73 \sin \omega t \quad (\text{mA})$$

$$v_o(t) = i_o(t) R_L = 2.73 \sin \omega t \quad (\text{V})$$

$$v_{eb}(t) = -i_s(t) r_\pi = -(32.7 \sin \omega t)(2.217)$$

$$v_{eb}(t) = -72.5 \sin \omega t \quad (\text{mV})$$

**6.50**

a.

$$I_{EQ} = 1 \text{ mA}, \quad V_{CEQ} = V_{CC} - I_{EQ} R_E$$

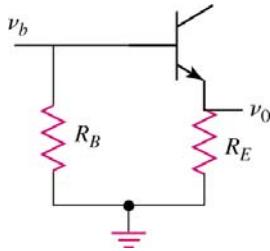
$$5 = 10 - (1)(R_E) \Rightarrow R_E = 5 \text{ k}\Omega$$

$$I_{BQ} = \frac{1}{101} = 0.0099 \text{ mA}$$

$$10 = I_{BQ} R_B + V_{BE}(\text{on}) + I_{EQ} R_E$$

$$10 = (0.0099) R_B + 0.7 + (1)(5) \Rightarrow R_B = 434 \text{ k}\Omega$$

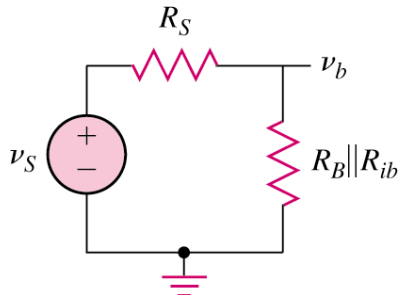
b.



$$r_\pi = \frac{(100)(0.026)}{0.99} = 2.63 \text{ k}\Omega$$

$$\frac{v_o}{v_b} = \frac{(1 + \beta) R_E}{r_\pi + (1 + \beta) R_E} = \frac{(101)(5)}{2.63 + (101)(5)} = 0.995$$

$$\Rightarrow v_b = \frac{v_o}{0.995} = \frac{4}{0.995} \Rightarrow v_b = 4.02 \text{ V peak-to-peak at base}$$



$$R_{ib} = r_{\pi} + (1 + \beta)R_E = 508 \text{ k}\Omega$$

$$R_B \parallel R_{ib} = 434 \parallel 508 = 234 \text{ k}\Omega$$

$$v_b = \frac{R_B \parallel R_{ib}}{R_B \parallel R_{ib} + R_S} \cdot v_s = \frac{234 v_s}{234 + 0.7} = \frac{234}{234.7} v_s$$

$$v_b = 0.997 v_s \Rightarrow v_s = \frac{4.02}{0.997} \Rightarrow v_s = 4.03 \text{ V peak-to-peak}$$

c.

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L)$$

$$R_{ib} = 2.63 + (101)(5 \parallel 1) = 86.8 \text{ k}\Omega$$

$$R_B \parallel R_{ib} = 434 \parallel 86.8 = 72.3 \text{ k}\Omega$$

$$v_b = \left( \frac{72.3}{72.3 + 0.7} \right) \cdot v_s = 0.99 v_s = (0.99)(4.03)$$

$$v_b = 3.99 \text{ V peak-to-peak}$$

$$v_o = \frac{(1 + \beta)(R_E \parallel R_L)}{r_{\pi} + (1 + \beta)(R_E \parallel R_L)} \cdot v_b$$

$$= \frac{(101)(0.833)}{2.63 + (101)(0.833)} \cdot (3.99)$$

$$v_o = 3.87 \text{ V peak-to-peak}$$

### 6.51

$$R_{TH} = R_1 \parallel R_2 = 40 \parallel 60 = 24 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{60}{60 + 40} \right) (10) = 6 \text{ V}$$

For  $\beta = 75$ ,  $I_{BQ} = \frac{6 - 0.7}{24 + (76)(5)} = 0.0131 \text{ mA}$

$$I_{CQ} = 0.984 \text{ mA}$$

For  $\beta = 150$ ,  $I_{BQ} = \frac{6 - 0.7}{24 + (151)(5)} = 0.00680 \text{ mA}$

$$I_{CQ} = 1.02 \text{ mA}$$

For  $\beta = 75$ ,  $r_{\pi} = \frac{(75)(0.026)}{0.984} = 1.98 \text{ k}\Omega$

For  $\beta = 150$ ,  $r_{\pi} = \frac{(150)(0.026)}{1.02} = 3.82 \text{ k}\Omega$

For  $\beta = 75$ ,  $R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L) = 65.3 \text{ k}\Omega$

For  $\beta = 150$ ,  $R_{ib} = 130 \text{ k}\Omega$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_{\pi} + (1 + \beta)(R_E \parallel R_L)} \cdot \frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S}$$

For  $\beta = 75$ ,  $R_1 \parallel R_2 \parallel R_{ib} = 40 \parallel 60 \parallel 65.3 = 17.5 \text{ k}\Omega$

$$A_v = \frac{(76)(0.833)}{1.98 + (76)(0.833)} \cdot \frac{17.5}{17.5 + 4} = 0.789$$



For  $\beta = 150$ ,  $R_1 \parallel R_2 \parallel R_{ib} = 40 \parallel 60 \parallel 130 = 20.3 \text{ k}\Omega$

$$A_v = \frac{(151)(0.833)}{3.82 + (151)(0.833)} \cdot \frac{20.3}{20.3 + 4} = 0.811$$

So  $0.789 \leq A_v \leq 0.811$

$$A_i = (1 + \beta) \left( \frac{R_E}{R_E + R_L} \right) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$\beta = 75$$

$$A_i = (76) \left( \frac{5}{5+1} \right) \left( \frac{24}{24+65.3} \right) \Rightarrow \underline{A_i = 17.0}$$

$$\beta = 150$$

$$A_i = (151) \left( \frac{5}{6} \right) \left( \frac{24}{24+130} \right) \Rightarrow \underline{A_i = 19.6}$$

$$\underline{17.0 \leq A_i \leq 19.6}$$

### 6.52

(a)

$$9 = \left( \frac{I_E}{1 + \beta} \right) (100) + V_{BE}(\text{on}) + I_E R_E$$

$$I_E = \frac{9 - 0.7}{\left( \frac{100}{1 + \beta} \right) + R_E}$$

$$\beta = 50 \quad I_E = \frac{8.3}{\left( \frac{100}{51} \right) + 1} = 2.803 \text{ mA}$$

$$\beta = 200 \quad I_E = \frac{8.3}{\left( \frac{100}{201} \right) + 1} = 5.543 \text{ mA}$$

$$2.80 \leq I_E \leq 5.54 \text{ mA}$$

$$V_E = I_E R_E, \quad \beta = 50, \quad V_E = 2.80 \text{ V}$$

$$\beta = 200, \quad V_E = 5.54 \text{ V}$$

(b)  $\beta = 50$ ,  $I_{CQ} = 2.748 \text{ mA}$ ,  $r_\pi = 0.473 \text{ K}$

$\beta = 200$ ,  $I_{CQ} = 5.515 \text{ mA}$ ,  $r_\pi = 0.943 \text{ K}$

$$R_i = R_B \parallel \left[ r_\pi + (1 + \beta)(R_E \parallel R_L) \right]$$

$$\beta = 50, \quad R_i = 100 \parallel \left[ 0.473 + (51)(1 \parallel 1) \right] = 100 \parallel 25.97 = 20.6 \text{ k}\Omega$$

$$\beta = 200, \quad R_i = 100 \parallel \left[ 0.943 + (201)(1 \parallel 1) \right] = 100 \parallel 101.4 = 50.3 \text{ k}\Omega$$

From Fig. (6.68)

$$A_v = \frac{(1+\beta)(R_E \parallel R_L)}{r_\pi + (1+\beta)(R_E \parallel R_L)} \cdot \left( \frac{R_i}{R_i + R_S} \right)$$

$$= \frac{(51)(1 \parallel 1)}{0.473 + (51)(1 \parallel 1)} \cdot \left( \frac{20.6}{20.6 + 10} \right)$$

$$\beta = 50 \Rightarrow A_v = 0.661$$

$$\beta = 200 \Rightarrow A_v = \frac{(201)(1 \parallel 1)}{0.943 + (201)(1 \parallel 1)} \cdot \left( \frac{50.3}{50.3 + 10} \right)$$

$$A_v = 0.826$$

**6.53**

$$V_o = (1+\beta)I_b R_L$$

$$I_b = \frac{V_s}{r_\pi + (1+\beta)R_L}$$

so  $A_v = \frac{(1+\beta)R_L}{r_\pi + (1+\beta)R_L}$

For  $\beta = 100$ ,  $R_L = 0.5 \text{ k}\Omega$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

Then  $A_v (\text{min}) = \frac{(101)(0.5)}{5.2 + (101)(0.5)} = 0.9066$

Then  $\beta = 180$ ,  $R_L = 500 \text{ k}\Omega$

$$r_\pi = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega$$

Then  $A_v (\text{max}) = \frac{(181)(500)}{9.36 + (181)(500)} = 0.9999$

**6.54**

(a)  $V_{CEQ} = V_{CC} - I_{EQ}R_E$

Let  $V_{CEQ} = 2.5 \text{ V}$ , then  $2.5 = 5 - I_{EQ}(0.5) \Rightarrow I_{EQ} \cong I_{CQ} = 5 \text{ mA}$ ,  $I_{BQ} = 0.04167 \text{ mA}$

$$r_\pi = \frac{(120)(0.026)}{5} = 0.624 \text{ k}\Omega$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \parallel R_L) = 0.624 + (121)(0.5 \parallel 0.5) = 30.87 \text{ k}\Omega$$

$$A_i = (1+\beta) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{R_E}{R_E + R_L} \right)$$

$$10 = (121) \left( \frac{R_{TH}}{R_{TH} + 30.87} \right) \left( \frac{0.5}{0.5 + 0.5} \right) \Rightarrow R_{TH} = 6.113 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E = (0.04167)(6.113) + 0.7 + (5)(0.5) = 3.455 \text{ V}$$

$$V_{TH} = 3.455 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (6.113)(5)$$

So  $R_1 = 8.85 \text{ k}\Omega$  and  $R_2 = 19.8 \text{ k}\Omega$

$$R_o = R_E \parallel \frac{r_\pi}{1 + \beta} = 0.5 \parallel \frac{0.624}{121} \Rightarrow R_o = 5.10 \Omega$$

$$(b) A_i = (1 + \beta) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{R_E}{R_E + R_L} \right)$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \parallel R_L) = 0.624 + (121)(0.5 \parallel 2) = 49.02 \text{ k}\Omega$$

$$A_i = (121) \left( \frac{6.113}{6.113 + 49.02} \right) \left( \frac{0.5}{0.5 + 2} \right) = 2.68$$

**6.55**

$$R_i = R_{TH} \parallel R_{ib} \text{ where } R_{ib} = r_\pi + (1 + \beta) R_E$$

$$V_{CEQ} = 3.5, \quad I_{CQ} = \frac{5 - 3.5}{2} = 0.75 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.75} = 4.16 \text{ k}\Omega$$

$$R_{ib} = 4.16 + (121)(2) = 246 \text{ k}\Omega$$

$$\text{Then } R_i = 120 = R_{TH} \parallel 246 \Rightarrow R_{TH} = 234 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.75}{120} = 0.00625 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (234)(5) = (0.00625)(234) + 0.7 + (121)(0.00625)(2)$$

$$\text{which yields } R_1 = 318 \text{ k}\Omega \text{ and } R_2 = 886 \text{ k}\Omega$$

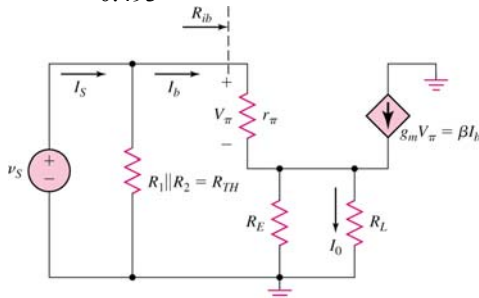
**6.56**

a.

$$\text{Let } R_E = 24 \Omega \text{ and } V_{CEQ} = \frac{1}{2} V_{CC} = 12 \text{ V} \Rightarrow I_{EQ} = \frac{12}{24} = 0.5 \text{ A}$$

$$I_{CQ} = 0.493 \text{ A}, \quad I_{BQ} = 6.58 \text{ mA}$$

$$r_\pi = \frac{(75)(0.026)}{0.493} = 3.96 \Omega$$



$$I_o = (1 + \beta) I_b \left( \frac{R_E}{R_E + R_L} \right)$$

$$I_b = I_s \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L)$$

$$= 3.96 + (76)(24 \parallel 8) \Rightarrow R_{ib} = 460 \ \Omega$$

$$A_i = \frac{I_0}{I_s} = (1 + \beta) \left( \frac{R_E}{R_E + R_L} \right) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$8 = (76) \left( \frac{24}{24 + 8} \right) \left( \frac{R_{TH}}{R_{TH} + 460} \right)$$

$$0.140 = \frac{R_{TH}}{R_{TH} + 460} \Rightarrow \underline{R_{TH} = 74.9 \ \Omega} \text{ (Minimum value)}$$

dc analysis:

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

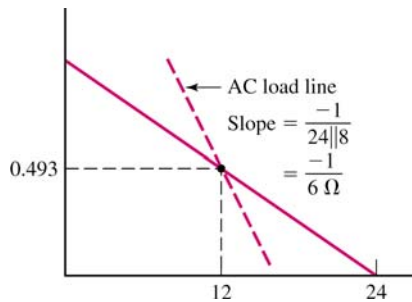
$$= I_{BQ} R_{TH} + V_{BE}(\text{on}) + I_{EQ} R_E$$

$$\frac{1}{R_1} (74.9)(24) = (0.00658)(74.9) + 0.70 + (0.5)(24)$$

$$= 13.19$$

$$\underline{R_1 = 136 \ \Omega}, \quad \frac{136 R_2}{136 + R_2} = 74.9 \Rightarrow \underline{R_2 = 167 \ \Omega}$$

b.



$$\Delta i_c = -\frac{1}{6} \Delta v_{ce}$$

$$\text{For } \Delta i_c = 0.493 \Rightarrow |\Delta v_{ce}| = (0.493)(6) \Rightarrow \text{Max. swing in output voltage for this design}$$

$$= \underline{5.92 \text{ V peak-to-peak}}$$

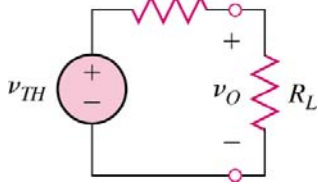
c.

$$R_0 = \frac{r_{\pi}}{1 + \beta} \parallel R_E = \frac{3.96}{76} \parallel 24 = 0.0521 \parallel 24 \Rightarrow \underline{R_0 = 52 \text{ m}\Omega}$$

6.57

The output of the emitter follower is

$$v_o = \left( \frac{R_L}{R_L + R_o} \right) \cdot v_{TH}$$



For  $v_o$  to be within 5% for a range of  $R_L$ , we have

$$\frac{R_L(\min)}{R_L(\min) + R_o} = (0.95) \frac{R_L(\max)}{R_L(\max) + R_o}$$

$$\frac{4}{4 + R_o} = (0.95) \frac{10}{10 + R_o} \text{ which yields } R_o = 0.364 \text{ k}\Omega$$

We have  $R_o = \left( \frac{r_\pi + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right) \parallel R_E \parallel r_o$

The first term dominates

Let  $R_1 \parallel R_2 \parallel R_S \cong R_S$ , then

$$R_o \cong \frac{r_\pi + R_S}{1 + \beta} \Rightarrow 0.364 = \frac{r_\pi + 4}{1 + \beta}$$

$$\text{or } 0.364 = \frac{r_\pi}{1 + \beta} + \frac{4}{1 + \beta} = \frac{\beta V_T}{I_{CQ}(1 + \beta)} + \frac{4}{1 + \beta}$$

$$0.364 \cong \frac{V_T}{I_{CQ}} + \frac{4}{1 + \beta}$$

The factor  $\frac{4}{1 + \beta}$  is in the range of  $\frac{4}{91} = 0.044$  to  $\frac{4}{131} = 0.0305$ . We can set

Or  $I_{CQ} = 0.08125 \text{ mA}$ . To take into account other factors, set  $I_{CQ} = 0.15 \text{ mA}$ ,

$$I_{BQ} = \frac{0.15}{110} = 0.00136 \text{ mA}$$

For  $V_{CEQ} \cong 5 \text{ V}$ , set  $R_E = \frac{5}{0.15} = 33.3 \text{ k}\Omega$

Design a bias stable circuit.

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} (R_{TH}) (10) - 5$$

$$R_{TH} = (0.1)(1 + \beta) R_E = (0.1)(111)(33.3) = 370 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1 + \beta) I_{BQ} R_E - 5$$

$$\text{So } \frac{1}{R_1}(370)(10) - 5 = (0.00136)(370) + 0.7 + (111)(0.00136)(33.3) - 5$$

which yields  $R_1 = 594 \text{ k}\Omega$  and  $R_2 = 981 \text{ k}\Omega$

$$\text{Now } A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_\pi + (1 + \beta)(R_E \parallel R_L)} \cdot \left( \frac{R_{TH} \parallel R_{ib}}{R_{TH} \parallel R_{ib} + R_S} \right)$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \parallel R_L) \text{ and } r_\pi = \frac{\beta V_T}{I_{CQ}}$$

For  $\beta = 90$ ,  $R_L = 4 \text{ k}\Omega$ ,

$$r_\pi = 15.6 \text{ k}\Omega, \quad R_{ib} = 340.6 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3 \parallel 4)}{15.6 + (91)(33.3 \parallel 4)} \cdot \frac{370 \parallel 340.6}{370 \parallel 340.6 + 4} \Rightarrow A_v = 0.9332$$

For  $\beta = 90$ ,  $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 715.4 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3 \parallel 10)}{15.6 + (91)(33.3 \parallel 10)} \cdot \frac{370 \parallel 715.4}{370 \parallel 715.4 + 4} \Rightarrow A_v = 0.9625$$

For  $\beta = 130$ ,  $R_L = 4 \text{ k}\Omega$

$$r_\pi = 22.5 \text{ k}\Omega, \quad R_{ib} = 490 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3 \parallel 4)}{22.5 + (131)(33.3 \parallel 4)} \cdot \frac{370 \parallel 490}{370 \parallel 490 + 4} \Rightarrow A_v = 0.9360$$

For  $\beta = 130$ ,  $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 1030 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3 \parallel 10)}{22.5 + (131)(33.3 \parallel 10)} \cdot \frac{370 \parallel 1030}{370 \parallel 1030 + 4} \Rightarrow A_v = 0.9645$$

$$\text{Now } v_o(\text{min}) = |A_v(\text{min})| \cdot v_s = 3.73 \sin \omega t$$

$$v_o(\text{max}) = |A_v(\text{max})| \cdot v_s = 3.86 \sin \omega t$$

$$\frac{\Delta v_o}{v_o} = 3.5\%$$

### 6.58

$$P_{AVG} = i_L^2(\text{rms}) R_L \Rightarrow 1 = i_L^2(\text{rms})(12)$$

$$\text{so } i_L(\text{rms}) = 0.289 \text{ A} \Rightarrow i_L(\text{peak}) = \sqrt{2}(0.289)$$

$$i_L(\text{peak}) = 0.409 \text{ A}$$

$$v_L(\text{peak}) = i_L(\text{peak}) \cdot R_L = (0.409)(12) = 4.91 \text{ V}$$

$$\text{Need a gain of } \frac{4.91}{5} = 0.982$$

With  $R_S = 10 \text{ k}\Omega$ , we will not be able to meet this voltage gain requirement. Need to insert a buffer or an op-amp voltage follower (see Chapter 9) between  $R_S$  and  $C_{c1}$ .

$$\text{Set } I_{EQ} = 0.5 \text{ A, } V_{CEQ} = \frac{1}{3}(12 - (-12)) = 8 \text{ V}$$

$$24 = I_{EQ}R_E + V_{CEQ} = (0.5)R_E + 8 \Rightarrow \underline{R_E = 32 \Omega}$$

$$\text{Let } \beta = 50, I_{CQ} = \frac{50}{51}(0.5) = 0.49 \text{ A}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.49} = 2.65 \Omega$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \parallel R_L) = 2.65 + (51)(32 \parallel 12)$$

$$R_{ib} = 448 \Omega$$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_\pi + (1 + \beta)(R_E \parallel R_L)} = \frac{(51)(32 \parallel 12)}{2.65 + (51)(32 \parallel 12)} = 0.994$$

So gain requirement has been met.

$$I_{BQ} = \frac{0.49}{50} = 0.0098 \text{ A} = 9.8 \text{ mA}$$

$$\text{Let } I_R \cong \frac{24}{R_1 + R_2} \cong 10I_B = 98 \text{ mA}$$

$$\text{So that } R_1 + R_2 = 245 \Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2}(24) - 12 = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E - 12$$

$$\left(\frac{R_2}{245}\right)(24) = \frac{(0.0098)R_1R_2}{245} + 0.7 + (0.5)(32)$$

$$\text{Now } R_1 = 245 - R_2$$

So we obtain

$$4 \times 10^{-5} R_2^2 + 0.0882 R_2 - 16.7 = 0 \text{ which yields } R_2 = 175 \Omega \text{ and } R_1 = 70 \Omega$$

### 6.59

$$(a) \quad g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$A_v = g_m R_C = (38.46)(2) = 76.9$$

$$(b) \quad A_i = \frac{\beta}{1 + \beta} = \frac{120}{121} = 0.9917$$

$$(c) \quad R_i = \frac{r_\pi}{1 + \beta}, \quad r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$R_i = \frac{3.12}{121} \Rightarrow R_i = 25.8 \Omega$$

$$(d) \quad R_o = R_C = 2 \text{ k}\Omega$$

### 6.60

$$(a) \quad I_{CQ} = \left(\frac{80}{81}\right)(2) = 1.975 \text{ mA}$$

$$g_m = \frac{1.975}{0.026} = 75.97 \text{ mA/V, } r_\pi = \frac{(80)(0.026)}{1.975} = 1.053 \text{ k}\Omega$$

$$A_v = g_m (R_C \parallel R_L) = (75.97)(1.5 \parallel 2.5) = 71.2$$

$$(b) A_i = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{R_C}{R_C + R_L} \right) = \left( \frac{80}{81} \right) \left( \frac{1.5}{1.5 + 2.5} \right) = 0.370$$

$$(c) R_i = \frac{r_\pi}{1 + \beta} = \frac{1.053}{81} \Rightarrow R_i = 13 \Omega$$

$$(d) r_o = \frac{80}{1.975} = 40.5 \text{ k}\Omega$$

$$(i) R_{oc} = r_o = 40.5 \text{ k}\Omega$$

$$(ii) R_o = r_o \parallel R_C \parallel R_L = 40.5 \parallel 1.5 \parallel 2.5 = 0.916 \text{ k}\Omega$$

### 6.61

$$(a) I_{CQ} = \left( \frac{110}{111} \right) (0.5) = 0.4955 \text{ mA}$$

$$g_m = \frac{0.4955}{0.026} = 19.06 \text{ mA/V}, \quad r_\pi = \frac{(110)(0.026)}{0.4955} = 5.77 \text{ k}\Omega$$

$$A_v = g_m \left( \frac{R_C}{R_S} \right) \left[ \frac{r_\pi}{1 + \beta} \parallel R_S \right] = (19.06) \left( \frac{4}{1} \right) \left[ \frac{5.77}{111} \parallel 1 \right] = (76.24) [0.05198 \parallel 1]$$

$$A_v = 3.77$$

$$(b) A_i = \frac{\beta}{1 + \beta} = \frac{110}{111} = 0.991$$

$$(c) R_i = R_S + \frac{r_\pi}{1 + \beta} = 1 + \frac{5.77}{111} = 1.052 \text{ k}\Omega$$

$$(d) R_o = R_C = 4 \text{ k}\Omega$$

### 6.62

$$(a) I_{R2} = \frac{0.7}{2.8} = 0.25 \text{ mA}$$

$$I_{CQ} \cong 1.50 - 0.25 = 1.25 \text{ mA}$$

$$V_C = (0.25)(5 + 2.8) = 1.95 \text{ V} = V_{CEQ}$$

$$(b) g_m = \frac{1.25}{0.026} = 48.08 \text{ mA/V}, \quad r_\pi = \frac{(120)(0.026)}{1.25} = 2.5 \text{ k}\Omega$$

$$A_v = g_m (R_1 \parallel R_L) = (48.08)(5 \parallel 10) = 160.3$$

$$(c) A_i = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{R_1}{R_1 + R_L} \right) = \left( \frac{120}{121} \right) \left( \frac{5}{5 + 10} \right) = 0.3306$$



**6.63**

(a)

$$I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$I_{CQ} = 0.921 \text{ mA}$$

$$V_{ECQ} = 20 - (0.93)(10) - (0.921)(5)$$

$$V_{ECQ} = 6.10 \text{ V}$$

(b)

$$g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V}$$

$$A_v = g_m (R_C \parallel R_L) = (35.42)(5 \parallel 50)$$

$$A_v = 161$$

**6.64**

(a)  $I_{EQ} = 0.93 \text{ mA}$ ,  $I_{CQ} = 0.921 \text{ mA}$

$$V_{ECQ} = 6.10 \text{ V}$$

(b)  $g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V}$   $r_\pi = 2.82 \text{ K}$

From Eq. 6.90

$$A_v = g_m \frac{(R_C \parallel R_L) \left[ \frac{r_\pi}{1 + \beta} \parallel R_E \parallel R_S \right]}{R_S}$$
$$= \frac{(35.42)(50 \parallel 5)}{0.1} \left[ \frac{2.82}{101} \parallel 10 \parallel 0.1 \right]$$

$$A_v = \frac{(35.42)(4.545)}{0.1} [0.0218]$$

$$A_v = 35.1$$

**6.65**

(a)  $I_{EQ} = 1 \text{ mA}$ ,  $I_{CQ} = 0.9917 \text{ mA}$

$$V_C = 5 - (0.9917)(2) = 3.017 \text{ V}$$

$$V_E = -0.7 \text{ V}$$

$$V_{CEQ} = 3.72 \text{ V}$$

(b)

$$A_v = g_m (R_C \parallel R_L)$$

$$g_m = \frac{0.9917}{0.026} = 38.14 \text{ mA/V}$$

$$A_v = (38.14)(2 \parallel 10) \Rightarrow A_v = 63.6$$

6.66

a. Emitter current

$$I_{EQ} = I_{CC} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{101} = 0.00495 \text{ mA}$$

$$V_E = I_{EQ} R_E = (0.5)(1) \Rightarrow V_E = 0.5 \text{ V}$$

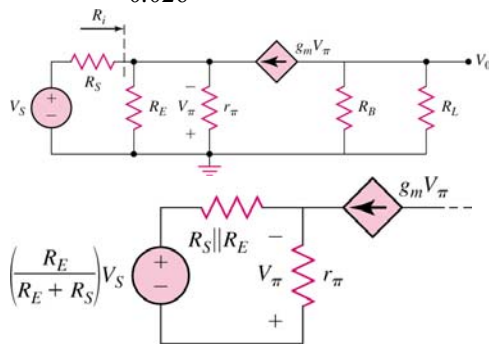
$$V_B = V_E + V_{BE}(\text{on}) = 0.5 + 0.7 \Rightarrow V_B = 1.20 \text{ V}$$

$$V_C = V_B + I_{BQ} R_B = 1.20 + (0.00495)(100) \Rightarrow V_C = 1.7 \text{ V}$$

b.

$$r_\pi = \frac{(100)(0.026)}{(100)(0.00495)} = 5.25 \text{ k}\Omega$$

$$g_m = \frac{(100)(0.00495)}{0.026} = 19.0 \text{ mA/V}$$



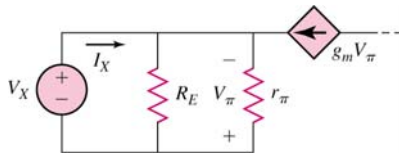
$$V_o = -g_m V_\pi (R_B \parallel R_L)$$

$$V_\pi = \frac{-(R_E \parallel R_{ie})}{R_E \parallel R_{ie} + R_s} V_s = -(0.4971) V_s$$

$$V_o = (19)(0.4971) V_s (100 \parallel 1)$$

$$A_v = 9.37$$

c.



$$I_X = \frac{V_X}{R_E} + \frac{V_X}{r_\pi} - g_m V_\pi, \quad V_\pi = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_i} = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m$$

$$\text{or } R_i = R_E \parallel r_\pi \parallel \frac{1}{g_m} = 1 \parallel 5.253 \parallel \frac{1}{19} = 0.84 \parallel 0.05252$$

$$R_i = 49.4 \Omega$$

**6.67**

(a)  $R_{TH} = R_1 \parallel R_2 = 150 \parallel 50 = 37.5 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{50}{50 + 150} \right) (9) = 2.25 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1 + \beta) I_{BQ} R_E$$

or  $I_{BQ} = \frac{2.25 - 0.7}{37.5 + (126)(3)} = 0.00373 \text{ mA}$ ,  $I_{CQ} = 0.4663 \text{ mA}$ ,  $I_{EQ} = 0.470 \text{ mA}$

$$V_{CEQ} = 9 - (0.4663)(6) - (0.470)(3) = 4.79 \text{ V}$$

(b)  $g_m = \frac{0.4663}{0.026} = 17.93 \text{ mA/V}$ ,  $r_\pi = \frac{(125)(0.026)}{0.4663} = 6.97 \text{ k}\Omega$

$$R_m = \frac{v_o}{i_s} = \left( \frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} \right) \left( \frac{\beta}{1 + \beta} \right) (R_C \parallel R_L)$$

We find  $R_S \parallel R_E = 100 \parallel 3 = 2.913 \text{ k}\Omega$

$$R_{ie} = \frac{r_\pi + R_{TH}}{1 + \beta} = \frac{6.97 + 37.5}{126} = 0.3529 \text{ k}\Omega$$

$$R_m = \left( \frac{2.913}{2.913 + 0.3529} \right) \left( \frac{125}{126} \right) (6 \parallel 4) = 2.12 \text{ V/mA}$$

(c)  $v_s = i_s (R_S \parallel R_E \parallel R_{ie}) = i_s (100 \parallel 3 \parallel 0.3529) = i_s (0.3148)$

So  $i_s = \frac{v_s}{0.3148}$

Then  $A_v = \frac{v_o}{v_s} = \frac{v_o}{i_s} \cdot \frac{i_s}{v_s} = \frac{2.12}{0.3148} = 6.73$

**6.68**

(a)  $V_{CEQ} \cong V_{CC} - I_{CQ} (R_C + R_E)$

Let  $V_{CEQ} = 2.5 \text{ V}$  and  $I_{CQ} = 0.25 \text{ mA}$

Then  $2.5 = 5 - (0.25)(R_C + 0.5) \Rightarrow R_C = 9.5 \text{ k}\Omega$

$$v_o = i_c (R_C \parallel R_L) = \left( \frac{\beta}{1 + \beta} \right) \cdot i_e \cdot (R_C \parallel R_L)$$

$$i_e = \frac{v_s}{R_{ie}} = \frac{v_s}{\left( \frac{r_\pi + R_{TH}}{1 + \beta} \right)}$$

$$A_v = \frac{v_o}{v_s} = \left( \frac{\beta}{r_\pi + R_{TH}} \right) (R_C \parallel R_L)$$

Now  $r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$

$$A_v = 25 = \left( \frac{100}{10.4 + R_{TH}} \right) (9.5 \parallel 12) = \frac{530.2}{10.4 + R_{TH}} \Rightarrow R_{TH} = 10.81 \text{ k}\Omega$$

Also  $I_{BQ} = 0.0025 \text{ mA}$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1 + \beta) I_{BQ} R_E = (0.0025)(10.81) + 0.7 + (101)(0.0025)(0.5)$$

$$V_{TH} = 0.853275 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.81)(5)$$

We obtain  $R_1 = 63.3 \text{ k}\Omega$  and  $R_2 = 13 \text{ k}\Omega$

(b) From part (a),  $I_{CQ} = 0.25 \text{ mA}$ ,  $V_{CEQ} = 2.5 \text{ V}$

(c)  $A_v = g_m (R_C \parallel R_L)$

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$A_v = (9.615)(9.5 \parallel 12) = 51$$

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**6.69**

(a)

$$I_{CQ} = \left(\frac{60}{61}\right)(1) \Rightarrow I_{CQ} = 0.984 \text{ mA}$$

$$V_{CEQ} = I_{BQ} R_B + V_{BE}(\text{on}) = \left(\frac{1}{61}\right)(100) + 0.7$$

$$V_{CEQ} = 2.34 \text{ V}$$

(b)

$$A_v = g_m \frac{(R_B \parallel R_L)}{R_S} \left[ \frac{r_\pi}{1 + \beta} \parallel R_S \right]$$

$$g_m = \frac{0.984}{0.026} = 37.85 \text{ mA/V}$$

$$r_\pi = 1.59 \text{ K}$$

$$A_v = \frac{(37.85)(100 \parallel 2)}{0.05} \left[ \frac{1.59}{61} \parallel 0.05 \right]$$

$$= 1484 [0.0261 \parallel 0.05]$$

$$A_v = 25.4$$

---

**6.70**

$$i_s(\text{peak}) = 2.5 \mu\text{A}, V_o(\text{peak}) = 5 \text{ mV}$$

$$\text{So we need } R_m = \frac{V_o}{i_s} = \frac{5 \times 10^{-3}}{2.5 \times 10^{-6}} = 2 \times 10^3 \Omega \Rightarrow R_m = 2 \text{ k}\Omega$$

We have

$$\frac{V_o}{I_s} = \left( \frac{\beta}{1 + \beta} \right) (R_C \parallel R_L) \left( \frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} \right)$$

$$\text{Let } R_C = 4 \text{ k}\Omega, R_L = 5 \text{ k}\Omega, R_E = 2 \text{ k}\Omega$$

Now  $\beta = 120$ , so we have

$$2 = \left( \frac{120}{121} \right) (4 \parallel 5) \left( \frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} \right) = 2.204 \left( \frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} \right)$$

$$\text{Then } \frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} = 0.9075$$

$$R_S \parallel R_E = 50 \parallel 2 = 1.923 \text{ k}\Omega, \text{ so that } R_{ie} = 0.196 \text{ k}\Omega$$

Assume  $V_{CEQ} = 3 \text{ V}$

$$V_{CC} \cong I_{CQ} (R_C + R_E) + V_{CEQ}$$

$$5 = I_{CQ} (4 + 2) + 3 \Rightarrow I_{CQ} = 0.333 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.333} = 9.37 \text{ k}\Omega$$

$$R_{ie} = \frac{r_\pi + R_{TH}}{1 + \beta} \Rightarrow 0.196 = \frac{9.37 + R_{TH}}{121}$$

which yields  $R_{TH} = 14.35 \text{ k}\Omega$

$$\text{Now } V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + I_{EQ} R_E$$

$$I_{BQ} = \frac{1}{120} = 0.00833 \text{ mA}, I_{EQ} = \left( \frac{121}{120} \right) (1) = 1.008 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (14.35)(5) = (0.00833)(14.35) + 0.7 + (1.008)(2)$$

which yields  $R_1 = 25.3 \text{ k}\Omega$

and  $R_2 = 33.2 \text{ k}\Omega$

**6.71**

a.

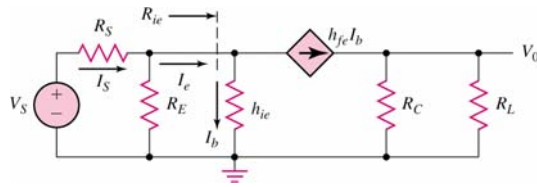
$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

$$I_{CQ} = 1.91 \text{ mA}$$

$$V_{ECQ} = V_{CC} + V_{EB}(\text{on}) - I_C R_C$$

$$= 25 + 0.7 - (1.91)(6.5) \Rightarrow V_{ECQ} = 13.3 \text{ V}$$

b.



Neglect effect  $h_{oe}$

Assume

$$2.45 \leq h_{ie} \leq 3.7 \text{ k}\Omega$$

$$80 \leq h_{fe} \leq 120$$

$$V_o = (h_{fe} I_b) (R_C \parallel R_L)$$

$$R_{ie} = \frac{h_{ie}}{1 + h_{fe}}, \quad I_e = \left( \frac{R_E}{R_E + R_{ie}} \right) \cdot I_s$$

$$I_b = \left( \frac{I_e}{1 + h_{fe}} \right), \quad I_s = \frac{V_s}{R_S + (R_E \parallel R_{ie})}$$

$$A_v = \left( \frac{h_{fe}}{1 + h_{fe}} \right) (R_C \parallel R_L) \left( \frac{R_E}{R_E + R_{ie}} \right) \left( \frac{1}{R_S + R_E \parallel R_{ie}} \right)$$

High gain device:  $h_{ie} = 3.7 \text{ k}\Omega$ ,  $h_{fe} = 120$

$$R_{ie} = \frac{3.7}{121} = 0.0306 \text{ k}\Omega$$

$$R_E \parallel R_{ie} = 10 \parallel 0.0306 = 0.0305 \text{ k}\Omega$$

$$A_v = \left( \frac{120}{121} \right) (6.5 \parallel 5) \left( \frac{10}{10 + 0.0306} \right) \left( \frac{1}{1 + 0.0305} \right) \Rightarrow A_v = 2.711$$

Low gain device:  $h_{ie} = 2.45 \text{ k}\Omega$ ,  $h_{fe} = 80$

$$R_{ie} = \frac{2.45}{81} = 0.03025 \text{ k}\Omega$$

$$R_E \parallel R_{ie} = 10 \parallel 0.03025 = 0.0302 \text{ k}\Omega$$

$$A_v = \left( \frac{80}{81} \right) (6.5 \parallel 5) \left( \frac{10}{10 + 0.03025} \right) \left( \frac{1}{1 + 0.0302} \right) \Rightarrow A_v = 2.70$$

$$\underline{2.70 \leq A_v \leq 2.71} \quad \text{So } A_v \cong \text{constant}$$

c.

$$R_i = R_E \parallel R_{ie}$$

We found  $\underline{0.0302 \leq R_i \leq 0.0305 \text{ k}\Omega}$

Neglecting  $h_{oe}$ ,  $\underline{R_o = R_C = 6.5 \text{ k}\Omega}$

**6.72**

a. Small-signal voltage gain

$$A_v = g_m (R_C \parallel R_L) \Rightarrow 25 = g_m (R_C \parallel 1)$$

For  $V_{ECQ} = 3\text{ V}$ ,  $V_C = -V_{ECQ} + V_{EB}(\text{on}) = -3 + 0.7 = -2.3\text{ V}$

$$V_{CC} - I_{CQ}R_C + V_C = 0 \Rightarrow I_{CQ} = \frac{5 - 2.3}{R_C} = \frac{2.7}{R_C}$$

For  $I_{CQ} = 1\text{ mA}$ ,  $R_C = 2.7\text{ k}\Omega$

$$g_m = \frac{1}{0.026} = 38.5\text{ mA/V}$$

$$A_v = (38.5)(2.7 \parallel 1) = 28.1$$

Design criterion satisfied and  $V_{ECQ}$  satisfied.

$$I_E = \left(\frac{101}{100}\right)(1) = 1.01\text{ mA}$$

$$V_{EE} = I_E R_E + V_{EB}(\text{on}) \Rightarrow R_E = \frac{5 - 0.7}{1.01} \Rightarrow R_E = 4.26\text{ k}\Omega$$

b.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} \Rightarrow r_\pi = 2.6\text{ k}\Omega, g_m = 38.5\text{ mA/V}, r_o = \infty$$

**6.73**

(a)  $g_{m1} = g_{m2} = \frac{1}{0.026} = 38.46\text{ mA/V}$

$$r_{\pi 1} = \frac{(120)(0.026)}{1} = 3.12\text{ k}\Omega, r_{\pi 2} = \frac{(80)(0.026)}{1} = 2.08\text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = \frac{(1 + \beta_1) \left( R_E \parallel \frac{r_{\pi 2}}{1 + \beta_2} \right)}{r_{\pi 1} + (1 + \beta_1) \left( R_E \parallel \frac{r_{\pi 2}}{1 + \beta_2} \right)}$$

We find  $R_E \parallel \frac{r_{\pi 2}}{1 + \beta_2} = 1 \parallel \frac{2.08}{81} = 0.02504\text{ k}\Omega$

Then  $A_{v1} = \frac{(121)(0.02504)}{3.12 + (121)(0.02504)} = 0.4927$

(b)  $A_{v2} = \frac{V_{o2}}{V_{o1}} = g_{m2} R_C = (38.46)(4) = 153.8$

(c)  $A_v = A_{v1} \cdot A_{v2} = (0.4927)(153.8) = 75.8$

**6.74**

(a)  $g_{m1} = \frac{0.5}{0.026} = 19.23\text{ mA/V}$ ,  $g_{m2} = \frac{2}{0.026} = 76.92\text{ mA/V}$

$$r_{\pi 1} = \frac{(100)(0.026)}{0.5} = 5.2\text{ k}\Omega, r_{\pi 2} = \frac{(100)(0.026)}{2} = 1.3\text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = -g_{m1}(R_{C1} \parallel R_{ib2})$$

where  $R_{ib2} = r_{\pi 2} + (1 + \beta_2)R_{E2} = 1.3 + (101)(4) = 405.3 \text{ k}\Omega$

$$A_{v1} = -(19.23)(4 \parallel 405.3) = -76.17$$

(b)  $A_{v2} = \frac{V_{o2}}{V_{o1}} = \frac{(1 + \beta_2)(R_{E2})}{r_{\pi 2} + (1 + \beta_2)R_{E2}} = \frac{(101)(4)}{1.3 + (101)(4)} = 0.9968$

(c)  $A_v = A_{v1} \cdot A_{v2} = (-76.17)(0.9968) = -75.93$

### 6.75

a.

$$V_{TH1} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{20}{20 + 80} \right) (10) = 2.0 \text{ V}$$

$$R_{TH1} = R_1 \parallel R_2 = 20 \parallel 80 = 16 \text{ k}\Omega$$

$$I_{B1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111 \text{ mA}$$

$$I_{C1} = 1.11 \text{ mA}; \quad g_{m1} = \frac{1.11}{0.026} = 42.74 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{1.11} = \infty$$

$$V_{TH2} = \left( \frac{R_4}{R_3 + R_4} \right) \cdot V_{CC} = \left( \frac{15}{15 + 85} \right) (10) = 1.50 \text{ V}$$

$$R_{TH2} = R_3 \parallel R_4 = 15 \parallel 85 = 12.75 \text{ k}\Omega$$

$$I_{B2} = \frac{1.50 - 0.70}{12.75 + (101)(0.5)} = 0.01265 \text{ mA}$$

$$I_{C2} = 1.265 \text{ mA} \Rightarrow g_{m2} = \frac{1.265}{0.026} \Rightarrow g_{m2} = 48.65 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} \Rightarrow r_{\pi 2} = 2.06 \text{ k}\Omega$$

$$r_{o2} = \infty$$

b.

$$A_{v1} = -g_{m1}R_{C1} = -(42.7)(2) \Rightarrow A_{v1} = -85.48$$

$$A_{v2} = -g_{m2}(R_{C2} \parallel R_L) = -(48.5)(4 \parallel 4) \Rightarrow A_{v2} = -97.3$$

c. Input resistance of 2nd stage

$$R_{i2} = R_3 \parallel R_4 \parallel r_{\pi 2} = 15 \parallel 85 \parallel 2.06 = 12.75 \parallel 2.06$$

$$R_{i2} = 1.773 \text{ k}\Omega$$

$$A'_{v1} = -g_{m1}(R_{C1} \parallel R_{i2}) = -(42.7)(2 \parallel 1.773)$$

$$A'_{v1} = -40.17$$



Overall gain:  $A_v = (-40.17)(-97.3) \Rightarrow A_v = 3909$

If we had  $A_{v1} \cdot A_{v2} = (-85.48)(-97.3) = 8317$

Loading effect reduces overall gain

**6.76**

a.

$$V_{TH1} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{12.7}{12.7 + 67.3} \right) (12) = 1.905 \text{ V}$$

$$R_{TH1} = R_1 \parallel R_2 = 12.7 \parallel 67.3 = 10.68 \text{ k}\Omega$$

$$I_{B1} = \frac{1.905 - 0.7}{10.68 + (121)(2)} = 0.00477 \text{ mA}$$

$$I_{C1} = 0.572 \text{ mA}$$

$$g_{m1} = \frac{0.572}{0.026} = 22 \text{ mA/V}$$

$$r_{\pi1} = \frac{(120)(0.026)}{0.572} = 5.45 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{0.572} = \infty$$

$$V_{TH2} = \left( \frac{R_4}{R_3 + R_4} \right) \cdot V_{CC} = \left( \frac{45}{45 + 15} \right) (12) = 9.0 \text{ V}$$

$$R_{TH2} = R_3 \parallel R_4 = 15 \parallel 45 = 11.25 \text{ k}\Omega$$

$$I_{B2} = \frac{9.0 - 0.7}{11.25 + (121)(1.6)} = 0.0405 \text{ mA}$$

$$I_{C2} = 4.86 \text{ mA}$$

$$g_{m2} = \frac{4.86}{0.026} = 187 \text{ mA/V}$$

$$r_{\pi2} = \frac{(120)(0.026)}{4.86} = 0.642 \text{ k}\Omega, \quad r_{o2} = \infty$$

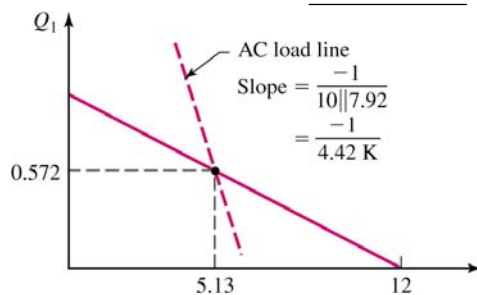
b.

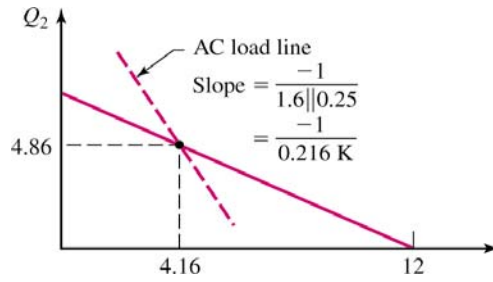
$$I_{E1} = 0.577 \text{ mA}$$

$$V_{CEQ1} = 12 - (0.572)(10) - (0.577)(2) \Rightarrow \underline{V_{CEQ1} = 5.13 \text{ V}}$$

$$I_{E2} = 4.90$$

$$V_{CEQ2} = 12 - (4.90)(1.6) \Rightarrow \underline{V_{CEQ2} = 4.16 \text{ V}}$$





$$R_{i2} = R_3 \parallel R_4 \parallel R_{ib}$$

$$R_{ib} = r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)$$

$$= 0.642 + (121)(1.6 \parallel 0.25) = 26.8 \text{ k}\Omega$$

$$R_{i2} = 15 \parallel 45 \parallel 26.8 = 7.92 \text{ k}\Omega$$

c.

$$A_{v1} = -g_{m1}(R_{C1} \parallel R_{i2}) = -(22)(10 \parallel 7.92) \Rightarrow A_{v1} = -97.2$$

$$A_{v2} = \frac{(1 + \beta)(R_{E2} \parallel R_L)}{r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)} = \frac{(121)(0.216)}{0.642 + (121)(0.216)}$$

$$A_{v2} = 0.976$$

Overall gain,  $A_v = -(97.2)(0.976) = -94.9$

d.

$$R_{iS} = R_1 \parallel R_2 \parallel r_{\pi 1} = 67.3 \parallel 12.7 \parallel 5.45 = 3.61 \text{ k}\Omega$$

$$R_o = \frac{r_{\pi 2} + R_S}{1 + \beta} \parallel R_{E2}$$

where  $R_S = R_3 \parallel R_5 \parallel R_{C1} = 15 \parallel 45 \parallel 10 = 5.29 \text{ k}\Omega$

$$R_o = \frac{0.642 + 5.29}{121} \parallel 1.6 = 0.049 \parallel 1.6 \Rightarrow R_o = 47.6 \Omega$$

e.

$$\Delta i_C = \frac{-1}{0.216 \text{ k}\Omega} \cdot \Delta v_{ce}, \Delta i_C = 4.86$$

$$|\Delta v_{ce}| = (4.86)(0.216) = 1.05 \text{ V}$$

Max. output voltage swing = 2.10 V peak-to-peak

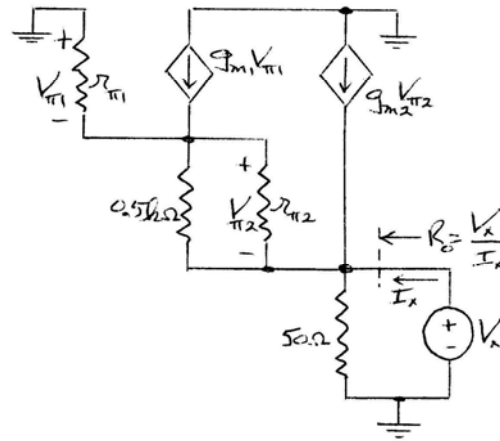
### 6.77

(a)  $I_{R2} = \frac{5 - 2(0.7)}{0.050} = 72 \text{ mA}, I_{R1} = \frac{0.7}{0.5} = 1.4 \text{ mA}$

$$I_{E2} = 72 - 1.4 = 70.6 \text{ mA}, I_{C2} = \left(\frac{80}{81}\right)(70.6) = 69.73 \text{ mA}, I_{B2} = \frac{70.6}{81} = 0.8716 \text{ mA}$$

$$I_{E1} = 1.4 + 0.8716 = 2.2716 \text{ mA}, I_{C1} = \left(\frac{120}{121}\right)(2.2716) = 2.253 \text{ mA}$$

(b)



$$g_{m1} = \frac{2.253}{0.026} = 86.65 \text{ mA/V}, \quad g_{m2} = \frac{69.73}{0.026} = 2681.9 \text{ mA/V}$$

$$r_{\pi1} = \frac{(120)(0.026)}{2.253} = 1.385 \text{ k}\Omega, \quad r_{\pi2} = \frac{(80)(0.026)}{69.73} = 0.02983 \text{ k}\Omega$$

$$V_s = V_{\pi1} + V_{\pi2} + V_o$$

$$V_o = \left[ \left( \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} \right) + g_{m2}V_{\pi2} \right] (0.05)$$

$$V_{\pi2} = \left( \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} \right) (0.5 \parallel r_{\pi2}) = V_{\pi1} \left( \frac{1}{1.385} + 86.65 \right) (0.5 \parallel 0.02983) = V_{\pi1} (2.4595)$$

$$V_o = \left[ V_{\pi1} \left( \frac{1}{1.385} + 86.65 \right) + (2681.9)V_{\pi1} (2.4595) \right] (0.05) = V_{\pi1} (334.175)$$

$$V_s = V_{\pi1} + V_{\pi2} + V_o = V_{\pi1} + V_{\pi1} (2.4595) + V_o$$

$$\text{So } V_{\pi1} = (V_s - V_o)(0.28906)$$

$$\text{And } V_o = (334.175)(V_s - V_o)(0.28906) = 96.596(V_s - V_o)$$

$$A_v = \frac{V_o}{V_s} = 0.990$$

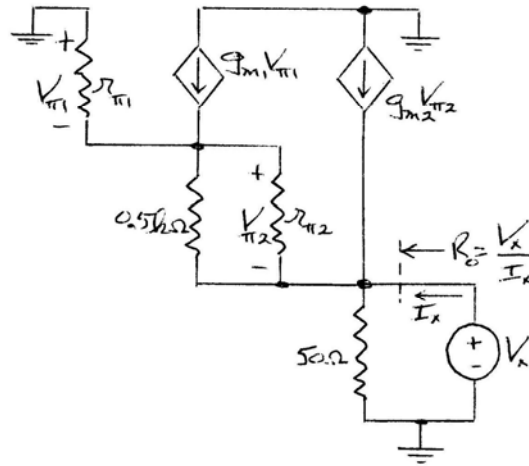
(c) For  $R_{ib}$ :

$$\text{We have } V_o = (0.989754)V_s$$

$$V_{\pi1} = (V_s - V_o)(0.28906) = V_s (1 - 0.989754)(0.28906) = V_s (0.0029618)$$

$$R_{ib} = \frac{V_s}{I_s} = \frac{V_s}{\left( \frac{V_{\pi1}}{r_{\pi1}} \right)} = \frac{r_{\pi1}}{0.0029618} \Rightarrow R_{ib} = 467.6 \text{ k}\Omega$$

For  $R_o$  :



$$(1) \quad V_{\pi 1} + V_{\pi 2} + V_x = 0$$

$$(2) \quad I_x + g_{m2}V_{\pi 2} + \left( \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} \right) = \frac{V_x}{0.05}$$

We had  $V_{\pi 2} = V_{\pi 1}(2.4595)$

$$(1) \quad V_{\pi 1} + V_{\pi 1}(2.4595) + V_x = 0 \Rightarrow V_{\pi 1} = -V_x(0.28906)$$

$$(2) \quad I_x + g_{m2}V_{\pi 1}(2.4595) + V_{\pi 1} \left( \frac{1}{r_{\pi 1}} + g_{m1} \right) = \frac{V_x}{0.05}$$

$$I_x + V_{\pi 1} \left[ (2681.9)(2.4595) + \left( \frac{1}{1.385} + 86.65 \right) \right] = \frac{V_x}{0.05}$$

$$I_x - V_x(0.28906)[6683.5] = \frac{V_x}{0.05}$$

$$R_o = \frac{V_x}{I_x} = 0.512 \, \Omega$$

**6.78**

a.

$$R_{TH} = R_1 \parallel R_2 = 335 \parallel 125 = 91.0 \, \text{k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC}$$

$$= \left( \frac{125}{125 + 335} \right) (10) = 2.717 \, \text{V}$$

$$V_{TH} = I_{B1}R_{TH} + V_{BE1} + V_{BE2} + I_{E2}R_{E2}$$

$$I_{E2} = (1 + \beta)I_{E1} = (1 + \beta)^2 I_{B1}$$

$$I_{B1} = \frac{2.717 - 1.40}{91.0 + (101)^2 (1)} \Rightarrow I_{B1} = 0.128 \mu\text{A}$$

$$I_{C1} = 12.8 \mu\text{A}$$

$$I_{C2} = \beta I_{E1} = \beta(1 + \beta)I_{B1} = (100)(101)(0.128 \mu\text{A})$$

$$I_{C2} = 1.29 \text{ mA}, I_{E2} = 1.31 \text{ mA}$$

$$I_{RC} = I_{C2} + I_{C1} = 1.29 + 0.0128 = 1.30 \text{ mA}$$

$$V_C = 10 - I_{RC}R_C = 10 - (1.30)(2.2) = 7.14 \text{ V}$$

$$V_E = I_{E2}R_{E2} = (1.30)(1) = 1.30 \text{ V}$$

$$V_{CE2} = 7.14 - 1.30 = 5.84 \text{ V}$$

$$V_{CE1} = V_{CE2} - V_{BE2} = 5.84 - 0.7$$

$$V_{CE1} = 5.14 \text{ V}$$

Summary:

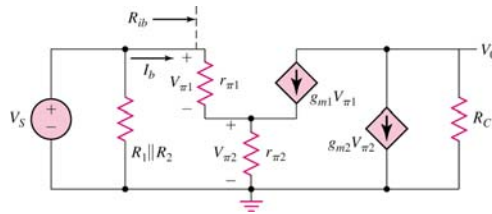
$$I_{C1} = 12.8 \mu\text{A} \quad I_{C2} = 1.29 \text{ mA}$$

$$V_{CE1} = 5.14 \text{ V} \quad V_{CE2} = 5.84 \text{ V}$$

b.

$$g_{m1} = \frac{0.0128}{0.026} = 0.492 \text{ mA/V}$$

$$g_{m2} = \frac{1.292}{0.026} = 49.7 \text{ mA/V}$$



$$V_0 = -(g_{m1}V_{\pi1} + g_{m2}V_{\pi2})R_C$$

$$V_S = V_{\pi1} + V_{\pi2}, \quad V_{\pi1} = V_S - V_{\pi2}$$

$$V_{\pi2} = \left( \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} \right) r_{\pi2}$$

$$V_{\pi2} = V_{\pi1} \left( \frac{1 + \beta}{r_{\pi1}} \right) r_{\pi2}$$

$$V_0 = -[g_{m1}(V_S - V_{\pi2}) + g_{m2}V_{\pi2}]R_C$$

$$V_0 = -[g_{m1}V_S + (g_{m2} - g_{m1})V_{\pi2}]R_C$$

$$V_{\pi2} = (V_S - V_{\pi2})(1 + \beta) \left( \frac{r_{\pi2}}{r_{\pi1}} \right)$$

$$V_{\pi2} \left[ 1 + (1 + \beta) \left( \frac{r_{\pi2}}{r_{\pi1}} \right) \right] = V_S (1 + \beta) \left( \frac{r_{\pi2}}{r_{\pi1}} \right)$$

$$V_0 = - \left\{ g_{m1} V_S + (g_{m2} - g_{m1}) \cdot \frac{V_S (1 + \beta) \left( \frac{r_{\pi 2}}{r_{\pi 1}} \right)}{1 + (1 + \beta) \left( \frac{r_{\pi 2}}{r_{pi 1}} \right)} \right\} R_C$$

$$A_v = \frac{V_0}{V_S}$$

$$= - \left\{ (0.492) + \frac{(49.7 - 0.492)(101) \left( \frac{2.01}{203} \right)}{1 + (101) \left( \frac{2.01}{203} \right)} \right\} 2.2$$

$$A_v = -55.2$$

c.

$$R_{is} = R_1 \parallel R_2 \parallel R_{ib}$$

$$R_{ib} = r_{\pi 1} + (1 + \beta) r_{\pi 2}$$

$$= 203 + (101)(2.01) = 406 \text{ k}\Omega$$

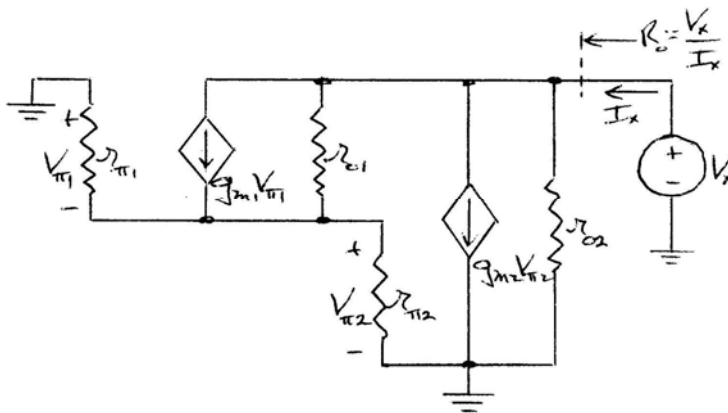
$$R_{is} = 91 \parallel 406 = 74.3 \text{ k}\Omega = R_{is}$$

$$R_0 = R_C = 2.2 \text{ k}\Omega$$

6.79

$$I_{E1} = I_{Bias} + I_{B2} = I_{Bias} + \frac{I_{C2}}{\beta_2}$$

$$I_{C1} = \left( \frac{\beta_1}{1 + \beta_1} \right) \left[ I_{Bias} + \frac{I_{C2}}{\beta_2} \right]$$



(a)  $I_{C2} = I_{Bias} = 1 \text{ mA}$ ,  $I_{C1} = \left( \frac{120}{121} \right) \left[ 1 + \frac{1}{80} \right] = 1.004 \text{ mA}$

$$g_{m1} = 38.62 \text{ mA/V}, \quad r_{\pi 1} = 3.108 \text{ k}\Omega, \quad r_{o1} = 79.68 \text{ k}\Omega$$

$$g_{m2} = 38.46 \text{ mA/V}, \quad r_{\pi 2} = 2.08 \text{ k}\Omega, \quad r_{o2} = 50 \text{ k}\Omega$$

$$(1) I_x = \frac{V_x}{r_{o2}} + g_{m2}V_{\pi2} + \frac{V_x - V_{\pi2}}{r_{o1}} + g_{m1}V_{\pi1}$$

$$(2) V_{\pi2} = \left[ \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_x - V_{\pi2}}{r_{o1}} \right] \cdot r_{\pi2}$$

$$(3) V_{\pi1} + V_{\pi2} = 0 \Rightarrow V_{\pi1} = -V_{\pi2}$$

$$\text{Then (1) } I_x = \frac{V_x}{r_{o2}} + g_{m2}V_{\pi2} + \frac{V_x - V_{\pi2}}{r_{o1}} - g_{m1}V_{\pi2}$$

$$I_x = V_x \left( \frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right) + V_{\pi2} \left( g_{m2} - \frac{1}{r_{o1}} - g_{m1} \right)$$

$$(2) V_{\pi2} = -V_{\pi2} \left( \frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{o1}} \right) \cdot r_{\pi2} + V_x \left( \frac{r_{\pi2}}{r_{o1}} \right)$$

$$V_{\pi2} \left[ 1 + \left( \frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{o1}} \right) \cdot r_{\pi2} \right] = V_x \left( \frac{r_{\pi2}}{r_{o1}} \right)$$

$$\text{Now (2) } V_{\pi2} \left[ 1 + \left( \frac{1}{3.108} + 38.62 + \frac{1}{79.68} \right) (2.08) \right] = V_x \left( \frac{2.08}{79.68} \right)$$

$$V_{\pi2} = V_x (0.00031825)$$

$$\text{Then (1) } I_x = V_x \left( \frac{1}{50} + \frac{1}{79.68} \right) + V_x (0.00031825) \left( 38.46 - \frac{1}{79.68} - 38.62 \right) = V_x (0.32495)$$

$$R_o = \frac{V_x}{I_x} = 30.77 \text{ k}\Omega$$

$$(b) I_{C2} = 1 \text{ mA}, I_{Bias} = 0.2 \text{ mA}, I_{C1} = \left( \frac{120}{121} \right) \left( 0.2 + \frac{1}{80} \right) = 0.2107 \text{ mA}$$

$$g_{m1} = 8.104 \text{ mA/V}, r_{\pi1} = 14.81 \text{ k}\Omega, r_{o1} = 379.7 \text{ k}\Omega$$

$$g_{m2} = 38.46 \text{ mA/V}, r_{\pi2} = 2.08 \text{ k}\Omega, r_{o2} = 50 \text{ k}\Omega$$

$$\text{Now (2) } V_{\pi2} \left[ 1 + \left( \frac{1}{14.81} + 8.104 + \frac{1}{379.7} \right) (2.08) \right] = V_x \left( \frac{2.08}{379.7} \right)$$

$$V_{\pi2} = V_x (0.0003043)$$

$$(1) I_x = V_x \left( \frac{1}{50} + \frac{1}{379.7} \right) + V_x (0.0003043) \left( 38.46 - \frac{1}{379.7} - 8.104 \right) = V_x (0.031867)$$

$$R_o = \frac{V_x}{I_x} = 31.38 \text{ k}\Omega$$

$$(c) I_{C2} = 2 \text{ mA}, I_{Bias} = 0, I_{C1} = \left( \frac{120}{121} \right) \left( \frac{2}{80} \right) = 0.02479 \text{ mA}$$

$$g_{m1} = 0.9536 \text{ mA/V}, r_{\pi1} = 125.9 \text{ k}\Omega, r_{o1} = 3327 \text{ k}\Omega$$

$$g_{m2} = 76.92 \text{ mA/V}, r_{\pi2} = 1.04 \text{ k}\Omega, r_{o2} = 25 \text{ k}\Omega$$

$$\text{Now (2) } V_{\pi2} \left[ 1 + \left( \frac{1}{125.9} + 0.9536 + \frac{1}{3327} \right) (1.04) \right] = V_x \left( \frac{1.04}{3327} \right)$$

$$V_{\pi2} = V_x (0.00015627)$$

$$(1) I_x = V_x \left( \frac{1}{25} + \frac{1}{3327} \right) + V_x (0.00015627) \left( 76.92 - \frac{1}{3327} - 0.9536 \right) = V_x (0.05217)$$

$$R_o = \frac{V_x}{I_x} = 19.17 \text{ k}\Omega$$

**6.80**

(a)  $R_{TH} = R_1 \parallel R_2 = 250 \parallel 75 = 57.69 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{75}{75 + 250} \right) (5) = 1.1538 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta)R_E} = \frac{1.1538 - 0.7}{57.69 + (121)(0.6)} = 0.003483 \text{ mA}$$

$$\Rightarrow I_{CQ} = 0.418 \text{ mA}, \quad I_{EQ} = 0.4214 \text{ mA}$$

$$V_{CEQ} = 5 - (0.418)(5.6) - (0.4214)(0.6) = 2.406 \text{ V}$$

For  $Q$ :  $P_Q = I_{CQ}V_{CEQ} = (0.418)(2.406) = 1.01 \text{ mW}$

For  $R_C$ :  $P_{RC} = I_{CQ}^2 R_C = (0.418)^2 (5.6) = 0.978 \text{ mW}$

For  $R_E$ :  $P_{RE} = I_{EQ}^2 R_E = (0.4214)^2 (0.6) = 0.107 \text{ mW}$

(b)  $R_{ib} = r_\pi + (1 + \beta)R_E$

$$r_\pi = \frac{(120)(0.026)}{0.418} = 7.464 \text{ k}\Omega$$

$$R_{ib} = 7.464 + (121)(0.6) = 80.06 \text{ k}\Omega$$

$$R_i = R_{TH} \parallel R_{ib} = 57.69 \parallel 80.06 = 33.53 \text{ k}\Omega$$

$$R_{is} = R_s + R_i = 0.5 + 33.53 = 34.03 \text{ k}\Omega$$

$$i_s = \frac{v_s}{R_{is}}; \quad i_b = \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \cdot i_s = \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{v_s}{R_{is}} \right)$$

$$i_c = \beta \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{v_s}{R_{is}} \right); \quad i_e = (1 + \beta) \left( \frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left( \frac{v_s}{R_{is}} \right)$$

$$\text{Now } i_c = (120) \left( \frac{57.69}{57.69 + 80.06} \right) \left( \frac{0.1 \sin \omega t}{34.03} \right) = 0.1477 \sin \omega t \text{ (mA)}$$

$$i_e = 0.1489 \sin \omega t \text{ (mA)}$$

For  $R_C$ :  $\bar{P}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C = 0.978 + \frac{1}{2} (0.1477)^2 (5.6) = 1.039 \text{ mW}$

For  $R_E$ :  $\bar{P}_{RE} = I_{EQ}^2 R_E + \frac{1}{2} I_e^2 R_E = 0.107 + \frac{1}{2} (0.1489)^2 (0.6) = 0.1137 \text{ mW}$

For  $Q$ :  $\bar{P}_Q = I_{CQ}V_{CEQ} - \frac{1}{2} I_c^2 R_C - \frac{1}{2} I_e^2 R_E = 1.01 - 0.0611 - 0.00665 = 0.942 \text{ mW}$

**6.81**

(a)  $I_{BQ}R_B + V_{BE}(on) + (1 + \beta)I_{BQ}R_E + V^- = 0$

$$I_{BQ} = \frac{5 - 0.7}{100 + (121)(4)} = 0.007363 \text{ mA}; \quad I_{CQ} = 0.8836 \text{ mA}, \quad I_{EQ} = 0.8909 \text{ mA}$$

$$V_{CEQ} = 10 - (0.8836)(4) - (0.8909)(4) = 2.902 \text{ V}$$

For  $Q$ :  $P_Q = I_{CQ}V_{CEQ} = (0.8836)(2.902) = 2.564 \text{ mW}$



$$\text{For } R_C : P_{RC} = I_{CQ}^2 R_C = (0.8836)^2 (4) = 3.123 \text{ mW}$$

$$\text{For } R_E : P_{RE} = I_{EQ}^2 R_E = (0.8909)^2 (4) = 3.175 \text{ mW}$$

$$(b) \Delta V_{CE} = \Delta I_C (R_C) = (0.8836)(4) = 3.534 \text{ V - Not possible}$$

$$\Delta V_{CE} = 2.902 - 0.5 = 2.402 \text{ V}$$

$$\text{So } \Delta I_C = \frac{2.402}{4} = 0.6005 \text{ mA}$$

$$\bar{P}_{RC} = \frac{1}{2} (\Delta I_C)^2 R_C = \frac{1}{2} (0.6005)^2 (4) = 0.721 \text{ mW}$$

**6.82**

a.

$$I_{BQ} = \frac{10 - 0.7}{50 + (151)(10)} = 0.00596 \text{ mA}$$

$$I_{CQ} = 0.894 \text{ mA}, \quad I_{EQ} = 0.90 \text{ mA}$$

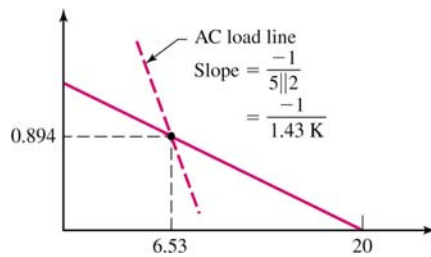
$$V_{ECQ} = 20 - (0.894)(5) - (0.90)(10) \Rightarrow V_{ECQ} = 6.53 \text{ V}$$

$$P_Q \cong I_{CQ} V_{ECQ} = (0.894)(6.53) \Rightarrow P_Q = 5.84 \text{ mW}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.894)^2 (5) \Rightarrow P_{RC} = 4.0 \text{ mW}$$

$$P_{RE} \cong I_{EQ}^2 R_E = (0.90)^2 (10) \Rightarrow P_{RE} = 8.1 \text{ mW}$$

b.



$$\Delta i_C = \frac{-1}{1.43 \text{ k}\Omega} \cdot \Delta v_{ec}$$

$$\Delta i_C = 0.894 \Rightarrow |\Delta v_{ec}| = (0.894)(1.43) = 1.28 \text{ V}$$

$$\Delta i_0 = \left( \frac{5}{5+2} \right) \Delta i_C = 0.639 \text{ mA}$$

$$\bar{P}_{RL} = \frac{1}{2} (0.639)^2 (2) \Rightarrow \bar{P}_{RL} = 0.408 \text{ mW}$$

$$\bar{P}_{RC} = \frac{1}{2} \cdot (0.894 - 0.639)^2 (5) \Rightarrow \bar{P}_{RC} = 0.163 \text{ mW}$$

$$\bar{P}_{RE} = 0$$

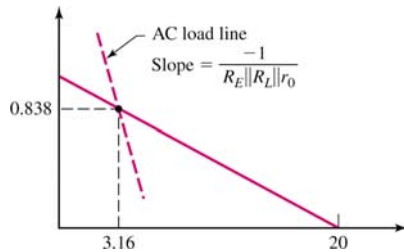
$$\bar{P}_Q = 5.84 - 0.408 - 0.163 \Rightarrow \bar{P}_Q = 5.27 \text{ mW}$$

**6.83**

$$I_{BQ} = \frac{10 - 0.70}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$



$$r_o = \frac{100}{0.838} = 119 \text{ k}\Omega$$

Neglecting base currents:

a.

$$R_L = 1 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \parallel 1 \parallel 119} = \frac{-1}{0.902 \text{ k}\Omega}$$

$$\Delta i_C = \frac{-1}{0.902 \text{ k}\Omega} \cdot \Delta V_{ce}$$

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.902)(0.838) = 0.756 \text{ V}$$

$$\overline{P_{RL}} = \frac{1}{2} \frac{(0.756)^2}{1} \Rightarrow \overline{P_{RL}} = 0.286 \text{ mW}$$

b.

$$R_L = 10 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \parallel 10 \parallel 119} = \frac{-1}{4.80}$$

$$\text{For } \Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.838)(4.80) = 4.02$$

$$\text{Max. swing determined by voltage } \overline{P_{RL}} = \frac{1}{2} \frac{(3.16)^2}{10} \Rightarrow \overline{P_{RL}} = 0.499 \text{ mW}$$

**6.84**

a.

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

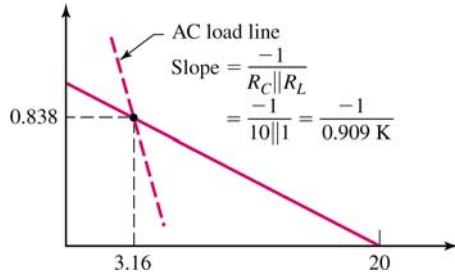
$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$

$$P_Q \cong I_{CQ} V_{CEQ} = (0.838)(3.16) \Rightarrow \underline{P_Q = 2.65 \text{ mW}}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.838)^2 (10) \Rightarrow \underline{P_{RC} = 7.02 \text{ mW}}$$

b.



$$\Delta i_C = \frac{-1}{0.909 \text{ k}\Omega} \cdot \Delta v_{ce}$$

$$\text{For } \Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.909)(0.838) = 0.762 \text{ V}$$

$$\Delta i_0 = \left( \frac{R_C}{R_C + R_L} \right) \Delta i_C = \left( \frac{10}{10+1} \right) \Delta i_C = 0.762 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2} (0.762)^2 (1) \Rightarrow \underline{\overline{P_{RL}} = 0.290 \text{ mW}}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.838 - 0.762)^2 (10) \Rightarrow \underline{\overline{P_{RC}} = 0.0289 \text{ mW}}$$

$$\overline{P_Q} = 2.65 - 0.290 - 0.0289 \Rightarrow \underline{\overline{P_Q} = 2.33 \text{ mW}}$$

## Chapter 7

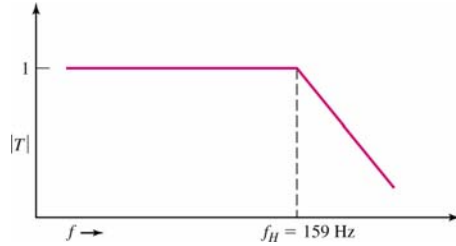
### 7.1

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/(sC_1)}{[1/(sC_1)] + R_1}$$

$$T(s) = \frac{1}{1 + sR_1C_1}$$

b.



$$f_H = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi (10^3)(10^{-6})} \Rightarrow f_H = 159 \text{ Hz}$$

c.

$$V_o(s) = V_i(s) \cdot \frac{1}{1 + sR_1C_1}$$

$$V_i(s) = \frac{1}{s}$$

For a step function

$$V_o(s) = \frac{1}{s} \cdot \frac{1}{1 + sR_1C_1} = \frac{K_1}{s} + \frac{K_2}{1 + sR_1C_1}$$

$$= \frac{K_1(1 + sR_1C_1) + K_2s}{s(1 + sR_1C_1)}$$

$$= \frac{K_1 + s(K_1R_1C_1 + K_2)}{s(1 + sR_1C_1)}$$

$$K_2 = -K_1R_1C_1 \text{ and } K_1 = 1$$

$$V_o(s) = \frac{1}{s} + \frac{-R_1C_1}{1 + sR_1C_1}$$

$$= \frac{1}{s} - \frac{1}{\frac{1}{R_1C_1} + s}$$

$$v_o(t) = 1 - e^{-t/R_1C_1}$$

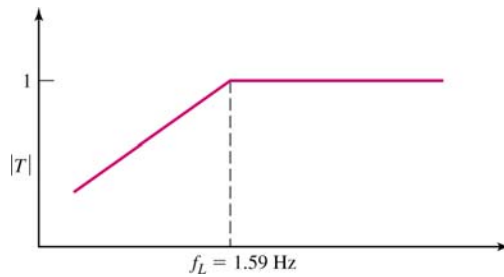
### 7.2

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + [1/(sC_2)]}$$

$$T(s) = \frac{sR_2C_2}{1 + sR_2C_2}$$

b.



$$f_L = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(10^4)(10 \times 10^{-6})} \Rightarrow f_L = 1.59 \text{ Hz}$$

c.

$$V_o(s) = V_i(s) \cdot \frac{sR_2 C_2}{1 + sR_2 C_2}$$

$$V_i(s) = \frac{1}{s}$$

$$V_o(s) = \frac{R_2 C_2}{1 + sR_2 C_2} = \frac{1}{s + \frac{1}{R_2 C_2}}$$

$$v_o(t) = e^{-t/R_2 C_2}$$

### 7.3

$$(a) \quad T(s) = \frac{V_o}{V_i} = \frac{R_2 \parallel \frac{1}{sC_2}}{R_2 \parallel \frac{1}{sC_2} + R_1}$$

$$\text{Now } R_2 \parallel \frac{1}{sC_2} = \frac{R_2 \left( \frac{1}{sC_2} \right)}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2 C_2}$$

$$\text{Then } T(s) = \frac{\frac{R_2}{1 + sR_2 C_2}}{\frac{R_2}{1 + sR_2 C_2} + R_1} = \frac{R_2}{R_1 + R_2 + sR_1 R_2 C_2}$$

$$T(s) = \left( \frac{R_2}{R_1 + R_2} \right) \cdot \frac{1}{1 + s(R_1 \parallel R_2)C_2}$$

$$(b) \quad \tau = (R_1 \parallel R_2)C_2 = (10 \parallel 20) \times 10^3 \times 10 \times 10^{-6} \Rightarrow \tau = 66.7 \text{ ms}$$

$$(c) \quad f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(66.7 \times 10^{-3})} = 2.39 \text{ Hz}$$

**7.4**

a.

$$\tau_s = (R_i + R_p)C_s = (30 + 10) \times 10^3 \times (10 \times 10^{-6}) \Rightarrow \tau_s = 0.40 \text{ s}$$

$$\tau_p = (R_i \parallel R_p)C_p = (30 \parallel 10) \times 10^3 \times (50 \times 10^{-12}) \Rightarrow \tau_p = 0.375 \mu\text{s}$$

b.

$$f_L = \frac{1}{2\pi\tau_s} = \frac{1}{2\pi(0.4)} \Rightarrow f_L = 0.398 \text{ Hz}$$

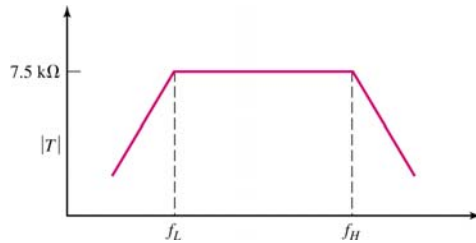
$$f_H = \frac{1}{2\pi\tau_p} = \frac{1}{2\pi(0.375 \times 10^{-6})} \Rightarrow f_H = 424 \text{ kHz}$$

At midband.  $C_s \rightarrow$  short,  $C_p \rightarrow$  open

$$V_o = I_i (R_i \parallel R_p)$$

$$T(s) = R_i \parallel R_p = 30 \parallel 10 \Rightarrow T(s) = 7.5 \text{ k}\Omega$$

c.



**7.5**

(a)  $\frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1} = \frac{20}{20 + 10} = 0.667$

(b)  $\frac{V_o}{V_i} = 1$

(c)  $T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + R_1 \parallel \frac{1}{sC_1}} = \frac{R_2}{R_2 + \frac{R_1}{1 + sR_1C_1}}$

$$T(s) = \frac{R_2(1 + sR_1C_1)}{R_1 + R_2 + sR_1R_2C_1} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot \frac{(1 + sR_1C_1)}{[1 + s(R_1 \parallel R_2)C_1]}$$

We have  $K = \frac{R_2}{R_1 + R_2}$ ,  $\tau_A = R_1C_1$ ,  $\tau_B = (R_1 \parallel R_2)C_1$

7.6

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_p \parallel \frac{1}{sC_p}}{R_p \parallel \frac{1}{sC_p} + \left( R_s + \frac{1}{sC_s} \right)}$$

$$R_p \parallel \frac{1}{sC_p} = \frac{R_p \cdot \frac{1}{sC_p}}{R_p + \frac{1}{sC_p}} = \frac{R_p}{1 + sR_p C_p}$$

Then

$$T(s) = \frac{R_p}{R_p + \left( R_s + \frac{1}{sC_s} \right) (1 + sR_p C_p)}$$

$$= \frac{R_p}{R_p + R_s + \frac{R_p C_p}{C_s} + \frac{1}{sC_s} + sR_s R_p C_p}$$

$$T(s) = \left( \frac{R_p}{R_p + R_s} \right) \times \left( 1 / \left[ 1 + \frac{R_p}{R_p + R_s} \cdot \frac{C_p}{C_s} + \frac{1}{s(R_s + R_p)C_s} + \frac{sR_p R_s}{R_s + R_p} \cdot C_p \right] \right)$$

b.

$$T(s) = \left( \frac{10}{10+10} \right) \times \left( 1 / \left[ 1 + \frac{10}{20} \cdot \frac{10^{-11}}{10^{-6}} + \frac{1}{s(2 \times 10^4) \cdot 10^{-6}} + s(5 \times 10^3) \cdot 10^{-11} \right] \right)$$

$$\cong \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{s(0.02)} + s(5 \times 10^{-8})}$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[ \omega(5 \times 10^{-8}) - \frac{1}{\omega(0.02)} \right]}$$

$$\text{For } \omega_L = \frac{1}{(R_s + R_p)C_s} = \frac{1}{(2 \times 10^4)(10^{-6})} = 50$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[ (50)(5 \times 10^{-8}) - \frac{1}{(50)(0.02)} \right]}$$

$$\approx \frac{1}{2} \cdot \frac{1}{1-j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

For

$$\omega_H = \frac{1}{(R_S \parallel R_P)C_P} = \frac{1}{(5 \times 10^3)(10^{-11})} = 2 \times 10^7$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[ (2 \times 10^7)(5 \times 10^{-8}) - \frac{1}{(2 \times 10^7)(0.02)} \right]}$$

$$T(j\omega) \cong \frac{1}{2} \cdot \frac{1}{1 + j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

In each case,  $|T(j\omega)| = \frac{1}{\sqrt{2}} \cdot \frac{R_P}{R_S + R_P}$

c.

$$R_S = R_P = 10 \text{ k}\Omega, \quad C_S = C_P = 0.1 \text{ }\mu\text{F}$$

$$T(s) = \frac{1}{2} \cdot \left[ 1 / \left[ 1 + \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{s(2 \times 10^4)(10^{-7})} + s(5 \times 10^3)(10^{-7}) \right] \right]$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{2} + j \left[ \omega(5 \times 10^{-4}) - \frac{1}{\omega(2 \times 10^{-3})} \right]}$$

$$\text{For } \omega = \frac{1}{(2 \times 10^4)(10^{-7})} = 500$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1.5 + j \left[ (500)(5 \times 10^{-4}) - \frac{1}{(500)(2 \times 10^{-3})} \right]}$$

$$= \frac{1}{2} \cdot \frac{1}{1.5 - j(0.75)} \Rightarrow |T(j\omega)| = 0.298$$

$$\text{For } \omega = \frac{1}{(5 \times 10^3)(10^{-7})} = 2 \times 10^3$$

$$T(j\omega) = \frac{1}{2} \cdot \left\{ 1 / \left[ 1.5 + j \left[ (2 \times 10^3)(5 \times 10^{-4}) - \frac{1}{(2 \times 10^3)(2 \times 10^{-3})} \right] \right] \right\}$$

$$= \frac{1}{2} \cdot \frac{1}{1.5 + j(0.75)} \Rightarrow |T(j\omega)| = 0.298$$

In each case,  $|T(j\omega)| < \frac{1}{\sqrt{2}} \cdot \frac{R_P}{R_P + R_S}$

### 7.7

$$(a) |T| = \frac{1}{\left[ \sqrt{1 + \left( \frac{f}{f_T} \right)^2} \right]^3}$$

$$\text{At } f = f_T, \quad |T| = \frac{1}{(\sqrt{2})^3} = 0.35355$$



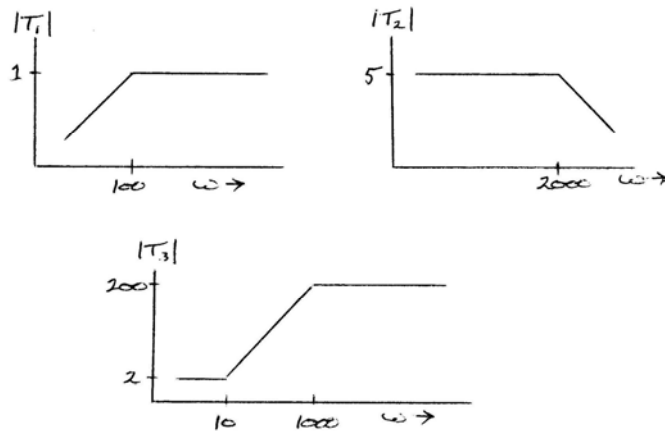
Or  $|T|_{dB} = 20 \log_{10}(0.35355) = -9.03 \text{ dB}$

$$\phi = -\tan^{-1}\left(\frac{f}{f_F}\right)^3 = -3 \tan^{-1}\left(\frac{f}{f_T}\right) = -3 \tan^{-1}(1) = -135^\circ$$

(b) Slope =  $3(-6) = -18 \text{ dB/octave} = -60 \text{ dB/decade}$

$$\phi = 3(-90) = -270^\circ$$

7.8



7.9

(a) (ii)  $\omega_1 = 1 \text{ rad/s}$ ;  $\omega_2 = 10 \text{ rad/s}$ ;  $\omega_3 = 100 \text{ rad/s}$ ;  $\omega_4 = 1000 \text{ rad/s}$

(iii)  $|T(0)| = 10$

(iv)  $|T(\infty)| = 10$

(b) (ii)  $\omega = 5 \text{ rad/s}$

(iii)  $|T(0)| = 0$

(iv)  $|T(\infty)| = \frac{8}{(0.2)^2} = 200$

7.10

$$(a) T(j\omega) = 5 \left( \frac{j\frac{\omega}{10^2}}{1 + j\frac{\omega}{10^2}} \right) \left( \frac{1}{1 + j\frac{\omega}{5 \times 10^4}} \right)$$

or  $T(j\omega) = 2.5 \times 10^5 \left( \frac{j\omega}{10^2 + j\omega} \right) \left( \frac{1}{5 \times 10^4 + j\omega} \right)$

$$(b) |T| = \frac{5\left(\frac{\omega}{10^2}\right)}{\sqrt{1+\left(\frac{\omega}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\omega}{5 \times 10^4}\right)^2}}$$

(i) At  $\omega = 50$  rad/s

$$|T| = \frac{5\left(\frac{50}{100}\right)}{\sqrt{1+\left(\frac{50}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{50}{5 \times 10^4}\right)^2}} = 2.236$$

(ii) At  $\omega = 150$  rad/s

$$|T| = \frac{5\left(\frac{150}{100}\right)}{\sqrt{1+\left(\frac{150}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{150}{5 \times 10^4}\right)^2}} = 4.16$$

(iii) At  $\omega = 10^5$

$$|T| = \frac{5\left(\frac{10^5}{10^2}\right)}{\sqrt{1+\left(\frac{10^5}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{10^5}{5 \times 10^4}\right)^2}} = 2.236$$

### 7.11

a.

$$V_o = -g_m V_\pi R_L \quad V_\pi = \left( \frac{r_\pi}{r_\pi + R_S} \right) V_i$$

$$|T| = g_m R_L \left( \frac{r_\pi}{r_\pi + R_S} \right) = (29)(6) \left( \frac{5.2}{5.2 + 0.5} \right)$$

$$\underline{|T_{\text{midband}}| = 159}$$

b.

$$\tau_S = (R_S + r_\pi) C_C$$

$$f_L = \frac{1}{2\pi\tau_S} \Rightarrow \tau_S = \frac{1}{2\pi f_L} = \frac{1}{2\pi(30)} \Rightarrow \tau_S = 5.31 \text{ ms, Open-Circuit}$$

$$\tau_P = \frac{1}{2\pi f_H} = \frac{1}{2\pi(480 \times 10^3)} \Rightarrow \tau_P = 0.332 \mu\text{s, Short-Circuit}$$

c.

$$C_C = \frac{\tau_S}{(R_S + r_\pi)} = \frac{5.31 \times 10^{-3}}{(0.5 + 5.2) \times 10^3} \Rightarrow C_C = 0.932 \mu\text{F}$$

$$\tau_P = R_L C_L$$

$$C_L = \frac{\tau_P}{R_L} = \frac{0.332 \times 10^{-6}}{6 \times 10^3} \Rightarrow C_L = 55.3 \text{ pF}$$

**7.12**

$$(a) \frac{V_o}{V_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} = \frac{10 + 40}{10 + 10 + 40} = 0.833$$

$$(b) \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2} = \frac{10}{10 + 10} = 0.50$$

$$(c) R_3 \parallel \frac{1}{sC} = \frac{R_3 \left( \frac{1}{sC} \right)}{R_3 + \frac{1}{sC}} = \frac{R_3}{1 + sR_3C}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{R_3}{1 + sR_3C}}{R_1 + R_2 + \frac{R_3}{1 + sR_3C}} = \frac{R_2 + R_3 + sR_2R_3C}{R_1 + R_2 + R_3 + s(R_1 + R_2)R_3C}$$

$$\text{or } T(s) = \left( \frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) \cdot \frac{[1 + s(R_2 \parallel R_3)C]}{[1 + s((R_1 + R_2) \parallel R_3)C]}$$

$$\text{where } K = \frac{R_2 + R_3}{R_1 + R_2 + R_3}, \tau_A = (R_2 \parallel R_3)C, \tau_B = ((R_1 + R_2) \parallel R_3)C$$

**7.13 Computer Analysis**

**7.14**

$$(a) |A_v|_{\max} = g_m R_D, \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.4)(0.8)} = 1.131 \text{ mA/V}$$

$$|A_v|_{\max} = (1.131)(1) = 1.13$$

$$(b) f_H = \frac{1}{2\pi R_D C_L} = \frac{1}{2\pi(10^3)(10^{-12})}$$

$$f_H = BW = 159 \text{ MHz}$$

**7.15**

$$(a) f_H = \frac{1}{2\pi R_C C_L} \Rightarrow R_C = \frac{1}{2\pi f_H C_L} = \frac{1}{2\pi(800 \times 10^6)(0.08 \times 10^{-12})}$$

$$\text{or } R_C = 2.49 \text{ k}\Omega$$

$$(b) I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} = \frac{2.5 - 1.25}{2.487} = 0.503 \text{ mA}$$

$$(c) |A_v|_{\max} = g_m R_C, \quad g_m = \frac{0.5026}{0.026} = 19.33 \text{ mA/V}$$

$$|A_v|_{\max} = (19.33)(2.487) = 48.1$$

**7.16**

$$(a) \quad T(s) = \frac{V_o(s)}{V_i(s)} = -g_m \left( r_o \parallel \frac{1}{sC_L} \right) = -g_m \left( \frac{r_o \cdot \frac{1}{sC_L}}{r_o + \frac{1}{sC_L}} \right) = -g_m r_o \left( \frac{1}{1 + sr_o C_L} \right)$$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.05)(0.1)} = 0.1414 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$|A_v|_{\max} = g_m r_o = (0.1414)(1000) = 141.4$$

$$(c) \quad f_H = BW = \frac{1}{2\pi r_o C_L} = \frac{1}{2\pi(10^6)(0.5 \times 10^{-12})} \Rightarrow f_H = 318 \text{ kHz}$$

**7.17**

a.

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 1.5 = 1.304 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{1.5}{1.5 + 10} \right) (12) = 1.565 \text{ V}$$

$$I_{BQ} = \frac{1.565 - 0.7}{1.304 + (101)(0.1)} = 0.0759 \text{ mA}$$

$$I_{CQ} = 7.585 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{7.59} = 0.343 \text{ k}\Omega$$

$$g_m = \frac{7.59}{0.026} = 292 \text{ mA/V}$$

$$R_i = R_1 \parallel R_2 \parallel [r_\pi + (1 + \beta)R_E]$$

$$= 10 \parallel 1.5 \parallel [0.343 + (101)(0.1)]$$

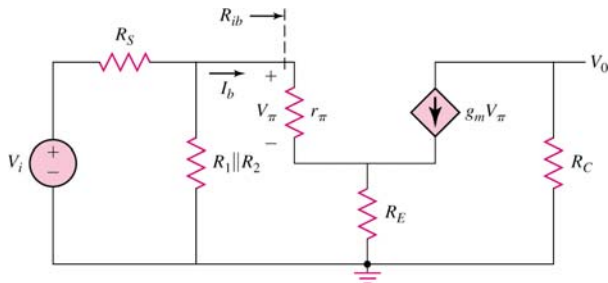
$$= 1.304 \parallel 10.44 \Rightarrow R_i = 1.159 \text{ k}\Omega$$

$$\tau = (R_S + R_i)C_C = (0.5 + 1.16) \times 10^3 \times (0.1 \times 10^{-6})$$

$$\tau = 1.659 \times 10^{-4} \text{ s}$$

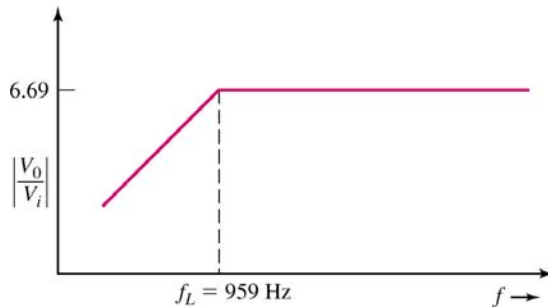
$$f_L = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.66 \times 10^{-4})} \Rightarrow f_L = 959 \text{ Hz}$$

b.



$$\begin{aligned}
 V_o &= -(\beta I_b) R_C \\
 R_{ib} &= r_\pi + (1 + \beta) R_E \\
 &= 0.343 + (101)(0.1) = 10.44 \text{ k}\Omega \\
 I_b &= \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) I_i \\
 &= \left( \frac{1.30}{1.30 + 10.4} \right) I_i = (0.111) I_i \\
 I_i &= \frac{V_i}{R_s + R_1 \parallel R_2 \parallel R_{ib}} \\
 &= \frac{V_i}{0.5 + (1.3) \parallel (10.44)} \\
 I_i &= \frac{V_i}{1.659} \\
 \left| \frac{V_o}{V_i} \right| &= \frac{\beta R_C (0.111)}{1.659} \Rightarrow \left| \frac{V_o}{V_i} \right|_{\text{midband}} = \frac{(100)(1)(0.111)}{1.659} \Rightarrow \left| \frac{V_o}{V_i} \right|_{\text{midband}} = 6.69
 \end{aligned}$$

c.



### 7.18

$$\begin{aligned}
 \text{(a)} \quad V_{DSQ} &= V_{DD} - I_{DQ}(R_D + R_S) \\
 3.2 &= 9 - (0.8)(R_D + 0.5) \Rightarrow R_D = 6.75 \text{ k}\Omega \\
 I_{DQ} &= K_n (V_{GSQ} - V_{TN})^2 \\
 0.8 &= 0.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.465 \text{ V} \\
 V_G &= (0.8)(0.5) + 2.465 = 2.865 \text{ V} \\
 V_G &= \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \Rightarrow 2.865 = \frac{1}{R_1} (160)(9)
 \end{aligned}$$

which yields  $R_1 = 503 \text{ k}\Omega$  and  $R_2 = 235 \text{ k}\Omega$

$$\begin{aligned}
 \text{(b)} \quad g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.8)} = 1.265 \text{ mA/V} \\
 A_v &= \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.265)(6.75)}{1 + (1.265)(0.5)} = -5.23
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad f_L &= \frac{1}{2\pi R_{in} C_C} \Rightarrow C_C = \frac{1}{2\pi f_L R_{in}} = \frac{1}{2\pi(16)(160 \times 10^3)} \Rightarrow C_C = 0.06217 \mu\text{F} \\
 \tau_s &= R_{in} C_C = (160 \times 10^3)(0.06217 \times 10^{-6}) = 9.947 \times 10^{-3} \text{ s}
 \end{aligned}$$

$$|A_v| = 5.23 \left| \frac{s\tau_s}{1 + s\tau_s} \right| = (5.23) \left[ \frac{\left( \frac{f}{f_L} \right)}{\sqrt{1 + \left( \frac{f}{f_L} \right)^2}} \right]$$

(i) For  $f = 5$  Hz,

$$|A_v| = (5.23) \left[ \frac{\frac{5}{16}}{\sqrt{1 + \left( \frac{5}{16} \right)^2}} \right] = 1.56$$

(ii) For  $f = 14$  Hz,

$$|A_v| = (5.23) \left[ \frac{\frac{14}{16}}{\sqrt{1 + \left( \frac{14}{16} \right)^2}} \right] = 3.44$$

(iii) For  $f = 25$  Hz,

$$|A_v| = (5.23) \left[ \frac{\frac{25}{16}}{\sqrt{1 + \left( \frac{25}{16} \right)^2}} \right] = 4.405$$

**7.19**

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow V_{GS} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.5}} + 1 = 2.414 \text{ V}$$

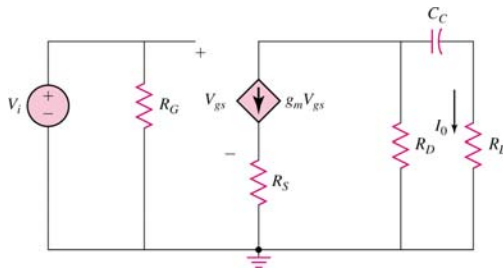
$$V_S = -2.414 \text{ V}$$

$$R_S = \frac{-2.414 - (-5)}{1} \Rightarrow R_S = 2.59 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 3 - 2.414 = 0.586 \text{ V}$$

$$R_D = \frac{5 - 0.59}{1} \Rightarrow R_D = 4.41 \text{ k}\Omega$$

b.



$$I_0 = -(g_m V_{gs}) \left( \frac{R_D}{R_D + R_L + \frac{1}{sC_C}} \right)$$

$$V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$\frac{I_0(s)}{V_i(s)} = \frac{-g_m}{1 + g_m R_S} \cdot R_D \left[ \frac{sC_C}{1 + s(R_D + R_L)C_C} \right]$$

$$\begin{aligned} T(s) &= \frac{I_0(s)}{V_i(s)} \\ &= \frac{-g_m R_D}{1 + g_m R_S} \cdot \frac{1}{R_D + R_L} \cdot \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C} \end{aligned}$$

c.

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow \tau_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(10)} \Rightarrow \tau_L = 15.92 \text{ ms}$$

$$\tau_L = (R_D + R_L)C_C \Rightarrow C_C = \frac{\tau_L}{R_D + R_L} = \frac{15.9 \times 10^{-3}}{(4.41 + 4) \times 10^3} \Rightarrow C_C = 1.89 \mu\text{F}$$

### 7.20

a.

$$\frac{9 - V_{SG}}{R_S} = I_D = K_P (V_{SG} + V_{TP})^2$$

$$9 - V_{SG} = (0.5)(12)(V_{SG}^2 - 4V_{SG} + 4)$$

$$6V_{SG}^2 - 23V_{SG} + 15 = 0$$

$$V_{SG} = \frac{23 \pm \sqrt{(23)^2 - 4(6)(15)}}{2(6)} \Rightarrow V_{SG} = 3 \text{ V}$$

$$g_m = 2K_P (V_{SG} + V_{TP}) = 2(0.5)(3 - 2) \Rightarrow g_m = 1 \text{ mA/V}$$

$$R_o = \frac{1}{g_m} \parallel R_S = 1 \parallel 12 \Rightarrow R_o = 0.923 \text{ k}\Omega$$

b.  $\tau = (R_o + R_L)C_C$

$$f_L = \frac{1}{2\pi\tau} \Rightarrow \tau = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} \Rightarrow \tau = 7.96 \text{ ms}$$

c.  $C_C = \frac{\tau}{R_o + R_L} = \frac{7.96 \times 10^{-3}}{(0.923 + 10) \times 10^3} \Rightarrow C_C = 0.729 \mu\text{F}$

### 7.21

(a)  $R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(4) = 48.4 \text{ k}\Omega$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta} = \frac{1.5}{121} = 0.012397 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$\text{so } \frac{1}{R_1}(48.4)(12) = (0.012397)(48.4) + 0.7 + (1.5)(4)$$

which yields  $R_1 = 79.6 \text{ k}\Omega$  and  $R_2 = 124 \text{ k}\Omega$

$$(b) I_{CQ} = \left(\frac{120}{121}\right)(1.5) = 1.488 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{1.488} = 2.097 \text{ k}\Omega, \quad r_o = \frac{50}{1.488} = 33.6 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(r_o \parallel R_E \parallel R_L)}{r_\pi + (1 + \beta)(r_o \parallel R_E \parallel R_L)}$$

$$\text{Now } r_o \parallel R_E \parallel R_L = 33.6 \parallel 4 \parallel 4 = 1.888 \text{ k}\Omega$$

$$A_v = \frac{(121)(1.888)}{2.097 + (121)(1.888)} = 0.991$$

$$(c) R_o = R_E \parallel r_o \parallel \frac{r_\pi}{1 + \beta} = 4 \parallel 33.6 \parallel \frac{2.097}{121} \Rightarrow R_o = 17.25 \Omega$$

$$(d) f_L = \frac{1}{2\pi(R_o + R_L)C_{C2}} = \frac{1}{2\pi(17.25 + 4000)(2 \times 10^{-6})}$$

$$f_L = 19.8 \text{ Hz}$$

### 7.22

$$(a) V_o(s) = -g_m \left( r_o \parallel R_D \parallel \frac{1}{sC_L} \right) \cdot V_{gs} = -g_m \left[ \frac{(r_o \parallel R_D) \left( \frac{1}{sC_L} \right)}{r_o \parallel R_D + \frac{1}{sC_L}} \right] \cdot V_{gs} = -g_m \left[ \frac{r_o \parallel R_D}{1 + s(r_o \parallel R_D)C_L} \right] \cdot V_{gs}$$

$$V_{gs} = \frac{\left( \frac{1}{sC_i} \right) (V_i(s))}{\frac{1}{sC_i} + R_{Si}} = \frac{V_i(s)}{1 + sR_{Si}C_i}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = -g_m \left( \frac{1}{1 + sR_{Si}C_i} \right) \left( \frac{r_o \parallel R_D}{1 + s(r_o \parallel R_D)C_L} \right)$$

$$(b) \tau = R_{Si}C_i$$

$$(c) \tau = (r_o \parallel R_D)C_L$$

### 7.23

$$(a) \frac{V_{gs}}{V_i} = \frac{-\left( \frac{1}{g_m} \parallel \frac{1}{sC_i} \right)}{\left( \frac{1}{g_m} \parallel \frac{1}{sC_i} \right) + R_S}$$



$$\text{Now } \left( \frac{1}{g_m} \parallel \frac{1}{sC_i} \right) = \frac{\left( \frac{1}{g_m} \right) \left( \frac{1}{sC_i} \right)}{\frac{1}{g_m} + \frac{1}{sC_i}} = \frac{\frac{1}{g_m}}{1 + s \left( \frac{1}{g_m} \right) C_i}$$

$$\text{So } \frac{V_{gs}}{V_i} = \frac{-\frac{1}{g_m}}{\frac{1}{g_m} + R_S \left( 1 + s \left( \frac{1}{g_m} \right) C_i \right)} = \left( \frac{-\frac{1}{g_m}}{\frac{1}{g_m} + R_S} \right) \cdot \frac{1}{\left[ 1 + s \left( \frac{1}{g_m} \parallel R_S \right) C_i \right]}$$

We have

$$V_o = -g_m V_{gs} \left[ \frac{R_D}{R_D + R_L + \frac{1}{sC_C}} \right] \cdot R_L = -g_m V_{gs} \left[ \frac{R_D R_L (sC_C)}{1 + s(R_D + R_L)C_C} \right]$$

$$V_o = -g_m V_{gs} \left( \frac{R_D R_L}{R_D + R_L} \right) \left[ \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C} \right]$$

$$\text{Then } T(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_S} \cdot \frac{1}{\left[ 1 + s \left( \frac{1}{g_m} \parallel R_S \right) C_i \right]} \cdot \left[ \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C} \right]$$

$$(b) \quad \tau = \left( \frac{1}{g_m} \parallel R_S \right) C_i$$

$$(c) \quad \tau = (R_D + R_L) C_C$$

### 7.24

(a)

$$I_{EQ} = \frac{5 - 0.7}{4} = 1.075 \text{ mA} \quad I_{CQ} = 1.064 \text{ mA}$$

$$V_{CEQ} = 10 - (1.064)(2) - (1.075)(4)$$

$$V_{CEQ} = 3.57 \text{ V}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.064}{0.026} = 40.92 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.064} = 2.44 \text{ K}$$

(b)

$$\text{For } C_{C1}; R_{eq1} = R_S + R_E \parallel \frac{r_\pi}{1 + \beta} = 200 + 4000 \parallel \frac{2440}{101}$$

$$R_{eq1} = 224.0 \Omega; \quad \tau_1 = R_{eq1} C_{C1} = 1.053 \text{ ms}$$

$$\text{For } C_{C2}; R_{eq2} = R_C + R_L = 2 + 47 = 49 \text{ k}\Omega$$

$$\tau_2 = R_{eq2} C_{C2} = 49 \text{ ms}$$

$$(c) \quad f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi(1.053 \times 10^{-3})} \Rightarrow \underline{f_1 = 151 \text{ Hz}}$$

**7.25**

(a)

$$\tau_H = (R_C \parallel R_L) C_L = (2 \parallel 47) \times 10^3 \times 10 \times 10^{-12} \\ = 1.918 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi(1.918 \times 10^{-8})} \Rightarrow \underline{f_H = 8.30 \text{ MHz}}$$

(b)

$$\frac{1}{\sqrt{1 + (2\pi\tau_H f)^2}} = 0.1$$

$$\left(\frac{1}{0.1}\right)^2 = 100 = 1 + (2\pi\tau_H f)^2$$

$$f = \frac{\sqrt{99}}{2\pi\tau_H} = \frac{\sqrt{99}}{2\pi(1.918 \times 10^{-8})}$$

$$f = 82.6 \text{ MHz}$$

**7.26**

(a)

$$\frac{5 - V_{SG}}{R_1} = K_P (V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(1.2)(V_{SG} - 1.5)^2 = (1.2)(V_{SG}^2 - 3V_{SG} + 2.25)$$

$$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0 \Rightarrow \underline{V_{SG} = 2.84 \text{ V}}$$

$$\underline{I_{DQ} = 1.8 \text{ mA}}$$

$$V_{SDQ} = 10 - (1.8)(1.2 + 1.2) \Rightarrow \underline{V_{SDQ} = 5.68 \text{ V}}$$

$$g_m = 2\sqrt{K_P I_{DQ}} = 2\sqrt{(1)(1.8)} = 2.683 \text{ mA/V}$$

$$r_o = \infty$$

(b)

$$R_{is} = \frac{1}{g_m} = \frac{1}{2.68} = 0.3727 \text{ k}\Omega$$

$$R_i = 1.2 \parallel 0.373 = 0.284 \text{ k}\Omega$$

For  $C_{C1}$ ,  $\tau_{s1} = (284 + 200)(4.7 \times 10^{-6}) = 2.27 \text{ ms}$

For  $C_{C2}$ ,  $\tau_{s2} = (1.2 \times 10^3 + 50 \times 10^3)(10^{-6}) = 51.2 \text{ ms}$

(c)

$C_{C2}$  dominates,

$$f_{3-dB} = \frac{1}{2\pi\tau_{s2}} = \frac{1}{2\pi(51.2 \times 10^{-3})} = 3.1 \text{ Hz}$$

**7.27**

Assume  $V_{TN} = 1V$ ,  $k'_n = 80\mu A/V^2$ ,  $\lambda = 0$

Neglecting  $R_{Si} = 200\Omega$ , Midband gain is:

$$|A_v| = g_m R_D$$

Let  $I_{DQ} = 0.2 \text{ mA}$ ,  $V_{DSQ} = 5V$

$$\text{Then } R_D = \frac{9-5}{0.2} \Rightarrow R_D = 20 \text{ k}\Omega$$

$$\text{We need } g_m = \frac{|A_v|}{R_D} = \frac{10}{20} = 0.5 \text{ mA/V and } g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$\text{or } 0.5 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.2)} \Rightarrow \underline{\underline{\frac{W}{L} = 7.81}}$$

Let

$$R_1 + R_2 = \frac{9}{(0.2)I_{DQ}} = \frac{9}{(0.2)(0.2)} = 225 \text{ k}\Omega$$

$$I_{DQ} = 0.2 = \left(\frac{0.080}{2}\right)(7.81)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.80 = \left(\frac{R_2}{R_1 + R_2}\right)(9) = \left(\frac{R_2}{225}\right)(9) \Rightarrow$$

$$\underline{R_2 = 45 \text{ k}\Omega, R_1 = 180 \text{ k}\Omega}$$

$$R_{TH} = R_1 \parallel R_2 = 180 \parallel 45 = 36 \text{ k}\Omega$$

$$\tau_1 = \frac{1}{2\pi f_1} = \frac{1}{2\pi(200)} = 7.958 \times 10^{-4} \text{ s} = (R_{Si} + R_{TH})C_C \text{ or } C_C = \frac{7.96 \times 10^{-4}}{(200 + 36 \times 10^3)} \Rightarrow$$

$$C_C = 0.022 \mu F$$

$$\tau_2 = \frac{1}{2\pi f_2} = \frac{1}{2\pi(3 \times 10^3)} = 5.305 \times 10^{-5} \text{ s} = R_D C_L \text{ or } C_L = \frac{5.31 \times 10^{-5}}{20 \times 10^3} \Rightarrow \underline{C_L = 2.65 \text{ nF}}$$

**7.28**

$$I_{BQ} = \frac{10 - 0.7}{430 + (201)(2.5)} = 0.00997 \text{ mA}$$

$$I_{CQ} = (200)I_{BQ} = 1.995 \text{ mA}$$

$$r_{\pi} = \frac{(200)(0.026)}{1.99} = 2.61 \text{ k}\Omega$$

$$R_{ib} = 2.61 + (201)(2.5) = 505 \text{ k}\Omega$$

$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(15)} = 0.0106 \text{ s}$$

$$= R_{eq}C_C = (0.5 + 505 \parallel 430) \times 10^3 C_C = 232.7 \times 10^3 C_C$$

Or  $C_C = 4.55 \times 10^{-8} \text{ F} \Rightarrow 45.5 \text{ nF}$

### 7.29

(a)  $V^+ = V_{CEQ} + I_{EQ}R_E$

$$3.3 = 1.8 + (0.25)R_E \Rightarrow R_E = 6 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.25}{121} = 0.002066 \text{ mA}$$

$$V^+ = I_{BQ}R_B + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$3.3 = (0.002066)(R_B) + 0.7 + (0.25)(6) \Rightarrow R_B = 532 \text{ k}\Omega$$

(b)  $I_{CQ} = \left(\frac{120}{121}\right)(0.25) = 0.2479 \text{ mA}$ ,  $r_{\pi} = \frac{(120)(0.026)}{0.2479} = 12.59 \text{ k}\Omega$

$$R_{ib} = r_{\pi} + (1 + \beta)R_E = 12.59 + (121)(6) = 738.6 \text{ k}\Omega$$

$$R_i = R_B \parallel R_{ib} = 532 \parallel 738.6 = 309.25 \text{ k}\Omega$$

$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 0.007958 = (R_s + R_i)C_C$$

so  $C_C = \frac{0.007958}{(0.1 + 309.25) \times 10^3} \Rightarrow C_C = 0.0257 \mu\text{F}$

(c) For  $R_s \ll R_B$ ,

$$A_v \cong \frac{(1 + \beta)R_E}{r_{\pi} + (1 + \beta)R_E} = \frac{(121)(6)}{12.59 + (121)(6)} = 0.983$$

### 7.30

$$R_{TH} = R_1 \parallel R_2 = 1.2 \parallel 1.2 = 0.6 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left( \frac{1.2}{1.2 + 1.2} \right) (5) = 2.5 \text{ V}$$

$$I_{BQ} = \frac{2.5 - 0.7}{0.6 + (101)(0.05)} = 0.319 \text{ mA}$$

$$I_{CQ} = 31.9 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{31.9} = 0.0815 \text{ k}\Omega$$

$$\tau_{C_{C1}} \gg \tau_{C_{C2}} \text{ and } f = \frac{1}{2\pi\tau} \text{ so that } f_{3-dB}(C_{C1}) \ll f_{3-dB}(C_{C2})$$

Then, for  $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$  acts as an open and for  $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$  acts as a short circuit.

$$f_{3-dB}(C_{C2}) = 25 \text{ Hz} = \frac{1}{2\pi\tau_2}, \text{ so that } \tau_2 = \frac{1}{2\pi(25)} = 0.006366 \text{ s} = R_{eq} C_{C2}$$

where

$$R_{eq} = R_L + R_E \parallel \left( \frac{r_\pi + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right)$$

$$= 10 + 50 \parallel \left( \frac{81.5 + 600 \parallel 300}{101} \right) = 10 + 50 \parallel 2.787 \Rightarrow$$

$$R_{eq} = 12.64 \text{ }\Omega \Rightarrow C_{C2} = \frac{0.00637}{12.6} \Rightarrow \underline{C_{C2} = 504 \text{ }\mu\text{F}}$$

$$R_{ib} = r_\pi + (1 + \beta)R_E \text{ Assume } C_{C2} \text{ an open}$$

$$R_{ib} = 81.5 + (101)(50) = 5132 \text{ }\Omega$$

$$\tau_1 = (100)\tau_2 = (100)(0.006366) = 0.6366 \text{ s} = R_{eq1} C_{C1}$$

$$R_{eq1} = R_S + R_{TH} \parallel R_{ib} = 300 + 600 \parallel 5132 = 837.2 \text{ }\Omega$$

$$\text{So } C_{C1} = \frac{0.6366}{837.2} \Rightarrow \underline{C_{C1} = 760 \text{ }\mu\text{F}}$$

### 7.31

From Problem 7.30  $R_{TH} = 0.6 \text{ K}$ ,  $I_{CQ} = 31.9 \text{ mA}$ ,  $r_\pi = 81.5 \text{ }\Omega$

$$\tau_{C_{C2}} \gg \tau_{C_{C1}} \text{ and } f = \frac{1}{2\pi\tau} \text{ so } f_{3-dB}(C_{C2}) \ll f_{3-dB}(C_{C1})$$

Then  $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$  acts as an open circuit and for  $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$  acts as a short circuit.

$$f_{3-dB}(C_{C1}) = 20 \text{ Hz} = \frac{1}{2\pi\tau_{C1}} \Rightarrow \tau_{C1} = 0.007958 \text{ s}$$

$$R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel R_L) = 81.5 + (101)(50 \parallel 10) = 923.2 \Omega$$

$$\tau_{C1} \Rightarrow R_{eq1} = R_S + R_{TH} \parallel R_{ib} = 300 + 600 \parallel 923.2 = 663.7 \Omega$$

$$C_{C1} = \frac{0.007958}{663.7} \Rightarrow C_{C1} = 12 \mu\text{F}$$

$$\tau_{C2} = 100\tau_{C1} = 0.7958 \text{ s}$$

$$R_{eq2} = R_L + R_E \parallel \left( \frac{r_{\pi} + R_{TH}}{1 + \beta} \right) = 10 + 50 \parallel \left( \frac{81.5 + 600}{101} \right)$$

$$R_{eq2} = 10 + 50 \parallel 6.748 = 15.95 \Omega$$

$$C_{C2} = \frac{0.7958}{15.95} \Rightarrow C_{C2} = 0.050 \text{ F}$$

### 7.32

(a)  $I_{EQ} = 0.2 \text{ mA}$ ,  $I_{BQ} = \frac{0.2}{121} = 0.001653 \text{ mA}$ ,  $I_{CQ} = \left( \frac{120}{121} \right)(0.2) = 0.1983 \text{ mA}$

$$V_E = -(I_{BQ}R_i + V_{BE}(on)) = -[(0.001653)(10) + 0.7] = -0.7165 \text{ V}$$

$$V_C = V_E + V_{CEQ} = -0.7165 + 2.2 = 1.483 \text{ V}$$

$$R_C = \frac{3 - 1.483}{0.1983} = 7.65 \text{ k}\Omega$$

(b)  $r_{\pi} = \frac{(120)(0.026)}{0.1983} = 15.73 \text{ k}\Omega$ ,  $g_m = \frac{0.1983}{0.026} = 7.627 \text{ mA/V}$

$$A_v = -g_m(R_C \parallel R_L) \left( \frac{r_{\pi}}{r_{\pi} + R_i} \right) = -(7.627)(7.65 \parallel 20) \left( \frac{15.73}{15.73 + 10} \right) = -25.8$$

(c) For  $C_C$ :  $\tau_C = (R_C + R_L)C_C$

$$f_C = \frac{1}{2\pi\tau_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$

For  $C_E$ :  $\tau_E = \left( \frac{r_{\pi} + R_i}{1 + \beta} \right) C_E \Rightarrow f_E = \frac{1}{2\pi\tau_E}$

(d)  $f_E = 10 = \frac{1}{2\pi\tau_E} \Rightarrow \tau_E = 0.015915 \text{ s}$

$$0.015915 = \left( \frac{15.73 + 10}{121} \right) \times 10^3 \times C_E \Rightarrow C_E = 74.8 \mu\text{F}$$

$$f_C = 50 = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = 0.003183 \text{ s}$$

$$0.003183 = (7.65 + 20) \times 10^3 \times C_C \Rightarrow C_C = 0.115 \mu\text{F}$$

**7.33**

a.

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.5}{0.5}} + 0.8 = 1.8 \text{ V}$$

$$R_S = \frac{-V_{GS} - (-5)}{0.5} = \frac{5 - 1.8}{0.5} \Rightarrow R_S = 6.4 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 4 - 1.8 = 2.2 \text{ V}$$

$$R_D = \frac{5 - 2.2}{0.5} \Rightarrow R_D = 5.6 \text{ k}\Omega$$

(b)

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$\tau_A = R_S C_S = (6.4 \times 10^3)(5 \times 10^{-6})$$

$$= 3.2 \times 10^{-2} \text{ s}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(3.2 \times 10^{-2})} \Rightarrow f_A = 4.97 \text{ Hz}$$

$$\tau_B = \left( \frac{R_S}{1 + g_m R_S} \right) C_S = \left[ \frac{6.4 \times 10^3}{1 + (1)(6.4)} \right] (5 \times 10^{-6})$$

$$= 4.32 \times 10^{-3} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(4.32 \times 10^{-3})} \Rightarrow f_B = 36.8 \text{ Hz}$$

c.

$$|A_v| = \frac{g_m R_D (1 + sR_S C_S)}{(1 + g_m R_S) \left[ 1 + s \left( \frac{R_S}{1 + g_m R_S} \right) C_S \right]}$$

As  $R_S$  becomes large

$$|A_v| \rightarrow \frac{g_m R_D (sR_S C_S)}{(g_m R_S) \left[ 1 + s \left( \frac{R_S}{g_m R_S} \right) C_S \right]}$$

$$A_v = \frac{(g_m R_D) \left[ s \left( \frac{1}{g_m} \right) C_S \right]}{1 + s \left( \frac{1}{g_m} \right) C_S}$$

The corner frequency  $f_B = \frac{1}{2\pi(1/g_m)C_S}$  and the corresponding  $f_A \rightarrow 0$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$f_B = \frac{1}{2\pi \left( \frac{1}{10^{-3}} \right) (5 \times 10^{-6})} \Rightarrow f_B = 31.8 \text{ Hz}$$

**7.34**

$$(a) \quad (i) \quad T_1(s) = \frac{V_{o1}}{V_i} = -g_{m1} \left( r_o \parallel R_D \parallel \frac{1}{sC_L} \right) = -g_{m1} \left[ \frac{(r_o \parallel R_D) \left( \frac{1}{sC_L} \right)}{(r_o \parallel R_D) + \left( \frac{1}{sC_L} \right)} \right]$$

$$T_1(s) = -g_{m1} (r_o \parallel R_D) \cdot \frac{1}{[1 + s(r_o \parallel R_D)C_L]}$$

$$(ii) \quad T_2(s) = \frac{V_o}{V_{o1}} = -g_{m2} (r_o \parallel R_D) \cdot \frac{1}{[1 + s(r_o \parallel R_D)C_L]}$$

$$(iii) \quad T(s) = \frac{V_o}{V_i} = g_{m1} g_{m2} (r_o \parallel R_D)^2 \cdot \frac{1}{[1 + s(r_o \parallel R_D)C_L]^2}$$

$$(b) \quad (i) \quad f_{3-dB} = \frac{1}{2\pi(r_o \parallel R_D)C_L}$$

$$\text{Now } r_o = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega, \quad r_o \parallel R_D = 100 \parallel 5 = 4.762 \text{ k}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(4.762 \times 10^3)(12 \times 10^{-12})} \Rightarrow f_{3-dB} = 2.785 \text{ MHz}$$

$$(ii) \quad f_{3-dB} = 2.785 \text{ MHz}$$

$$(iii) \quad \text{Want } \left\{ \frac{1}{\sqrt{1 + [(2\pi f)(r_o \parallel R_D)C_L]^2}} \right\}^2 = \frac{1}{\sqrt{2}}$$

$$\text{So } \frac{1}{1 + \left( \frac{f}{2.785 \times 10^6} \right)^2} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\left( \frac{f}{2.785 \times 10^6} \right)^2 = \frac{1}{0.7071} - 1 = 0.4142$$

which yields  $f = 1.792 \text{ MHz}$

### 7.35

- a. Expression for the voltage gain is the same as Equation (7.59) with  $R_s = 0$ .  
b.

$$\tau_A = R_E C_E$$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta)R_E}$$



**7.36**

(a)  $I_{EQ} = \left(\frac{91}{90}\right)(0.15) = 0.1517 \text{ mA}$

$$R_E = \frac{3 - 0.7}{0.1517} = 15.16 \text{ k}\Omega$$

$$V_C = 0.7 - V_{ECQ} = 0.7 - 2.2 = -1.5 \text{ V}$$

$$R_C = \frac{-1.5 - (-3)}{0.15} = 10 \text{ k}\Omega$$

(b)  $g_m = \frac{0.15}{0.026} = 5.769 \text{ mA/V}$

$$A_v = -g_m R_C = -(5.769)(10) = -57.7$$

(c)  $\tau_A = R_E C_E = (15.16 \times 10^3)(3 \times 10^{-6}) = 4.548 \times 10^{-2} \text{ s}$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(4.548 \times 10^{-2})} = 3.5 \text{ Hz}$$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta)R_E}, \text{ where } r_\pi = \frac{(90)(0.026)}{0.15} = 15.6 \text{ k}\Omega$$

$$\tau_B = \frac{(15.16 \times 10^3)(15.6 \times 10^3)(3 \times 10^{-6})}{15.6 \times 10^3 + (91)(15.16 \times 10^3)} = 5.085 \times 10^{-4} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(5.085 \times 10^{-4})} = 313 \text{ Hz}$$

**7.37**

(a)  $I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}, I_{CQ} = \left(\frac{90}{91}\right)(0.93) = 0.9198 \text{ mA}$

$$g_m = \frac{0.9198}{0.026} = 35.38 \text{ mA/V}$$

$$A_v = g_m (R_C \parallel R_L) = (35.38)(5 \parallel 10) = 118$$

(b)  $f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(R_C \parallel R_L)C_L} = \frac{1}{2\pi(5 \parallel 10) \times 10^3 \times (3 \times 10^{-12})}$

$$f = 15.9 \text{ MHz}$$

**7.38**

(a)  $I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$

$$0.2 = 0.1(V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 2.014 \text{ V}$$

$$R_S = \frac{3 - 2.014}{0.2} = 4.93 \text{ k}\Omega$$

$$V_D = V_{SGQ} - V_{SDQ} = 2.014 - 1.9 = 0.114 \text{ V}$$

$$R_D = \frac{0.114 - (-3)}{0.2} = 15.6 \text{ k}\Omega$$

(b)  $f_H = \frac{1}{2\pi(R_D \parallel R_L)C_L}$   
 or  $C_L = \frac{1}{2\pi(15.6 \parallel 20) \times 10^3 \times 4 \times 10^6} \Rightarrow C_L = 4.54 \text{ pF}$

**7.39**

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{166}{166 + 234} \right) (10) = 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$4.15 - V_{GS} = (0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.5)(3.55 - 2)$$

$$g_m = 1.55 \text{ mA/V}$$

$$R_0 = R_S \parallel \frac{1}{g_m} = 0.5 \parallel \frac{1}{1.55} = 0.5 \parallel 0.645$$

$$R_0 = 0.282 \text{ k}\Omega$$

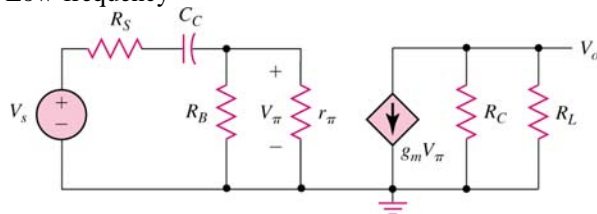
$$\tau = (R_0 \parallel R_L)C_L \quad \text{and} \quad f_H = \frac{1}{2\pi\tau}$$

$$\text{BW} \cong f_H = 5 \text{ MHz} \Rightarrow \tau = \frac{1}{2\pi(5 \times 10^6)} = 3.18 \times 10^{-8} \text{ s}$$

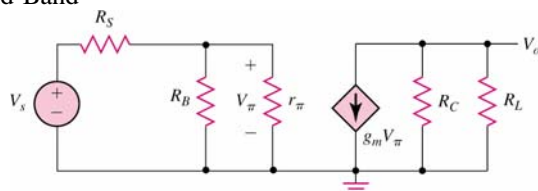
$$C_L = \frac{\tau}{R_0 \parallel R_L} = \frac{3.18 \times 10^{-8}}{(0.282 \parallel 4) \times 10^3} \Rightarrow C_L = 121 \text{ pF}$$

**7.40**

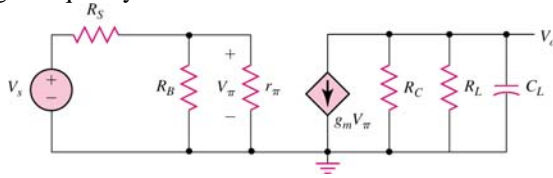
(a) Low-frequency



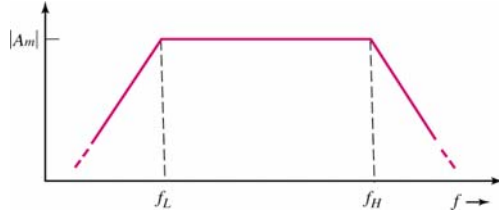
Mid-Band



High-frequency



(b)



(c)

$$I_{BQ} = \frac{12 - 0.7}{1 \text{ M}\Omega} = 11.3 \mu\text{A}$$

$$I_{CQ} = 1.13 \text{ mA}$$

$$r_{\pi} = \frac{(100)(0.026)}{1.13} = 2.3 \text{ k}\Omega$$

$$g_m = \frac{1.13}{0.026} = 43.46 \text{ mA/V}$$

$$A_m = \frac{V_o}{V_s} (\text{midband}) = -g_m (R_C \parallel R_L) \left( \frac{R_B \parallel r_{\pi}}{R_B \parallel r_{\pi} + R_s} \right)$$

$$= -(43.46)(5.1 \parallel 500) \left( \frac{1000 \parallel 2.3}{1000 \parallel 2.3 + 1} \right)$$

$$= -(43.46)(5.05) \left( \frac{2.29}{2.29 + 1} \right) \Rightarrow |A_m| = 153$$

$$|A_m|_{dB} = 43.7 \text{ dB}$$

$$f_L = \frac{1}{2\pi\tau_L}, \quad \tau_L = (R_s + R_B \parallel r_{\pi})C_C = (1 + 1000 \parallel 2.3) \times 10^3 \times (10 \times 10^{-6})$$

$$\Rightarrow \tau_L = 3.29 \times 10^{-2} \text{ s}, \quad f_L = 4.83 \text{ Hz}$$

$$f_H = \frac{1}{2\pi\tau_H}, \quad \tau_H = (R_C \parallel R_L)C_L = (5.1 \parallel 500) \times 10^3 \times (10 \times 10^{-12})$$

$$\Rightarrow \tau_H = 5.05 \times 10^{-8} \text{ s}, \quad f_H = 3.15 \text{ MHz}$$

7.41

$$(a) \quad A_v = -g_m \left( R_D \parallel R_L \parallel \frac{1}{sC_L} \right) = -g_m \left[ \frac{(R_D \parallel R_L) \cdot \frac{1}{sC_L}}{(R_D \parallel R_L) + \frac{1}{sC_L}} \right]$$

$$A_v = -g_m (R_D \parallel R_L) \left[ \frac{1}{1 + s(R_D \parallel R_L)C_L} \right]$$

$$(b) \quad \tau = (R_D \parallel R_L)C_L$$

$$(c) \quad 5 = I_D R_S + V_{SG} = K_p R_S (V_{SG} + V_{TP})^2 + V_{SG}$$

$$5 = (0.25)(3.2)(V_{SG} - 2)^2 + V_{SG}$$

We find  $0.8V_{SG}^2 - 2.2V_{SG} - 1.8 = 0 \Rightarrow V_{SG} = 3.41 \text{ V}$

$$I_{DQ} = (0.25)(3.41 - 2)^2 = 0.497 \text{ mA}$$

$$\tau = (10 \parallel 20) \times 10^3 \times 10 \times 10^{-12} = 6.67 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(6.67 \times 10^{-8})} \Rightarrow f_H = 2.39 \text{ MHz}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.497)} = 0.705 \text{ mA/V}$$

$$A_v = -g_m (R_D \parallel R_L) = -(0.705)(10 \parallel 20) = -4.7$$

#### 7.42 Computer Analysis

#### 7.43 Computer Analysis

#### 7.44 Computer Analysis

#### 7.45

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$f_\beta = \frac{f_T}{\beta_o} = \frac{4 \times 10^9}{120} \Rightarrow f_\beta = 33.3 \text{ MHz}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{9.615 \times 10^{-3}}{2\pi(4 \times 10^9)}$$

or  $C_\pi + C_\mu = 0.3826 \text{ pF}$

Then  $C_\pi = 0.3826 - 0.08 = 0.303 \text{ pF}$

#### 7.46

$$(a) \quad f_\beta = \frac{f_T}{\beta_o} = \frac{2 \times 10^9}{120} \Rightarrow f_\beta = 16.67 \text{ MHz}$$

$$g_m = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{15.38 \times 10^{-3}}{2\pi(2 \times 10^9)}$$

$$C_\pi + C_\mu = 1.224 \text{ pF}, \quad C_\pi = 1.224 - 0.075 = 1.15 \text{ pF}$$

$$(b) \quad |h_{fe}| = \frac{\beta_o}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

$$(i) \text{At } f = 10 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{10}{16.67}\right)^2}} = 103$$

$$(ii) \text{At } f = 20 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{20}{16.67}\right)^2}} = 76.8$$

$$(iii) \text{At } f = 50 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{50}{16.67}\right)^2}} = 38.0$$

**7.47**

$$(a) f_{\beta} = \frac{f_T}{\beta_o} = \frac{540 \times 10^6}{120} \Rightarrow f_{\beta} = 4.5 \text{ MHz}$$

$$g_m = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$C_{\pi} + C_{\mu} = \frac{g_m}{2\pi f_T} = \frac{7.692 \times 10^{-3}}{2\pi(540 \times 10^6)} \Rightarrow C_{\pi} + C_{\mu} = 2.267 \text{ pF}$$

$$C_{\pi} = 2.267 - 0.4 = 1.87 \text{ pF}$$

$$(b) g_m = \frac{0.8}{0.026} = 30.77 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} = \frac{30.77 \times 10^{-3}}{2\pi(2.267 \times 10^{-12})} \Rightarrow f_T = 2.16 \text{ GHz}$$

$$f_{\beta} = \frac{2.16 \times 10^9}{120} \Rightarrow f_{\beta} = 18.0 \text{ MHz}$$

**7.48**

(a)

$$V_o = -g_m V_{\pi} R_L \text{ where}$$

$$V_{\pi} = \frac{r_{\pi} \parallel \frac{1}{sC_1}}{r_{\pi} \parallel \frac{1}{sC_1} + r_b} \cdot V_i = \frac{\frac{r_{\pi}}{1 + sr_{\pi}C_1}}{\frac{r_{\pi}}{1 + sr_{\pi}C_1} + r_b} \cdot V_i$$

$$= \frac{r_{\pi}}{r_{\pi} + r_b + sr_b r_{\pi} C_1} \cdot V_i = \left( \frac{r_{\pi}}{r_{\pi} + r_b} \right) \left( \frac{1}{1 + s(r_b \parallel r_{\pi})C_1} \right) \cdot V_i$$

So

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m R_L \left( \frac{r_{\pi}}{r_{\pi} + r_b} \right) \left( \frac{1}{1 + s(r_b \parallel r_{\pi})C_1} \right)$$

(b) Midband gain:  $r_\pi = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$ ,  $g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$

For  $r_b = 100 \text{ }\Omega$

(i)  $A_{v1} = -(38.46)(4) \left( \frac{2.6}{2.6 + 0.1} \right) \Rightarrow \underline{A_{v1} = -148.1}$

For  $r_b = 500 \text{ }\Omega$

(ii)  $A_{v2} = -(38.46)(4) \left( \frac{2.6}{2.6 + 0.5} \right) \Rightarrow \underline{A_{v2} = -129.0}$

(c)  $f_{3-dB} = \frac{1}{2\pi\tau}$ ,  $\tau = (r_b \parallel r_\pi) C_1$

For  $r_b = 100 \text{ }\Omega$

(i)  $\tau_1 = (0.1 \parallel 2.6) \times 10^3 (2.2 \times 10^{-12}) = 2.12 \times 10^{-10} \text{ s} \Rightarrow \underline{f_{3-dB} = 751 \text{ MHz}}$

For  $r_b = 500 \text{ }\Omega$

(ii)  $\tau_2 = (0.5 \parallel 2.6) \times 10^3 (2.2 \times 10^{-12}) = 9.23 \times 10^{-10} \text{ s} \Rightarrow \underline{f_{3-dB} = 173 \text{ MHz}}$

**7.49**

$f = 10 \text{ kHz} = 10^4$

(b)  $Z_i = 200 + \frac{2500(1 - j(10^4)(1.333 \times 10^{-6}))}{1 + (10^4)^2 (1.333 \times 10^{-6})^2}$   
 $= 200 + 2500 - j333.3 = 2700 - j333.3$

$f = 100 \text{ kHz} = 10^5$

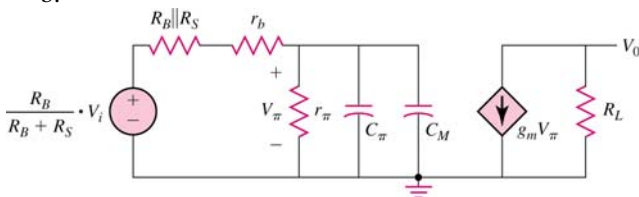
(c)  $Z_i = 200 + \frac{2500(1 - j(10^5)(1.333 \times 10^{-6}))}{1 + (10^5)^2 (1.333 \times 10^{-6})^2}$   
 $Z_i = 200 + 2456 - j327 = 2656 - j327$

$f = 1 \text{ MHz} = 10^6$

(d)  $Z_i = 200 + \frac{2500(1 - j(10^6)(1.333 \times 10^{-6}))}{1 + (10^6)^2 (1.333 \times 10^{-6})^2}$   
 $Z_i = 200 + 900 - j1200 = 1100 - j1200$

**7.50**

- a.  $C_M = C_\mu (1 + g_m R_L)$   
 b.



$$V_o = -g_m V_\pi R_L \quad \text{Let } C_\pi + C_\mu = C_i$$

$$V_\pi = \frac{r_\pi \parallel \frac{1}{sC_i}}{r_\pi \parallel \frac{1}{sC_i} + R_B \parallel R_S + r_b} \cdot \left( \frac{R_B}{R_B + R_S} \right) V_i$$

$$A_v(s) = \frac{V_o(s)}{V_i(s)}$$

$$= -g_m R_L \left( \frac{R_B}{R_B + R_S} \right) \frac{\left[ \frac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}} \right]}{\left[ \frac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}} + R_B \parallel R_S + r_b \right]}$$

$$= -g_m R_L \left( \frac{R_B}{R_B + R_S} \right) \cdot \left[ \frac{r_\pi}{r_\pi + (1 + sr_\pi C_i)(R_B \parallel R_S + r_b)} \right]$$

Let  $R_{eq} = (R_B \parallel R_S + r_b)$

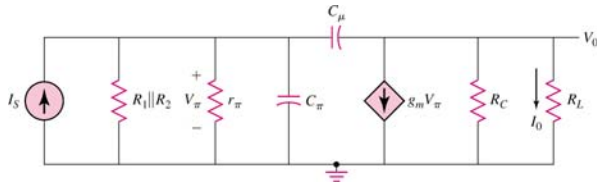
$$A_v(s) = -\beta R_L \left( \frac{R_B}{R_B + R_S} \right) \times \left[ \frac{1}{(r_\pi + R_{eq}) [1 + s(r_\pi \parallel R_{eq}) C_i]} \right]$$

$$A_v(s) = \frac{-\beta R_L}{r_\pi + R_{eq}} \cdot \left( \frac{R_B}{R_B + R_S} \right) \cdot \frac{1}{1 + s(r_\pi \parallel R_{eq}) C_i}$$

c.  $f_H = \frac{1}{2\pi(r_\pi \parallel R_{eq}) C_i}$

**7.51**

High Freq.  $\Rightarrow C_{C1}, C_{C2}, C_E \rightarrow$  short circuits



$$g_m = \frac{I_{CQ}}{V_T} = \frac{5}{0.026} = 192.3 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 250 \times 10^6 = \frac{192 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 122.4 \text{ pF} \Rightarrow C_\mu = 5 \text{ pF}, C_\pi = 117.4 \text{ pF}$$

$$\begin{aligned}
 C_M &= C_\mu (1 + g_m (R_C \parallel R_L)) \\
 &= 5 [1 + (192.3)(1 \parallel 1)] \Rightarrow C_M = 485.8 \text{ pF} \\
 C_i &= C_\pi + C_M = 117 + 485 = 603 \text{ pF} \\
 r_\pi &= \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega \\
 R_{eq} &= R_1 \parallel R_2 \parallel r_\pi = 5 \parallel 1.04 = 0.861 \text{ k}\Omega \\
 \tau &= R_{eq} \cdot C_i = (0.861 \times 10^3) (603 \times 10^{-12}) \\
 &= 5.19 \times 10^{-7} \text{ s} \\
 f &= \frac{1}{2\pi\tau} = \frac{1}{2\pi(5.19 \times 10^{-7})} \Rightarrow f = 307 \text{ kHz}
 \end{aligned}$$

**7.52**

$$\begin{aligned}
 R_{TH} &= R_1 \parallel R_2 = 60 \parallel 5.5 = 5.04 \text{ k}\Omega \\
 V_{TH} &= \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{5.5}{5.5 + 60} \right) (15) = 1.26 \text{ V} \\
 I_{BQ} &= \frac{1.26 - 0.7}{5.04 + (101)(0.2)} = 0.0222 \text{ mA} \\
 I_{CQ} &= 2.22 \text{ mA} \\
 r_\pi &= \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega \\
 g_m &= \frac{2.22}{0.026} = 85.4 \text{ mA/V} \\
 \text{Lower 3 - dB frequency:} \\
 \tau_L &= R_{eq} \cdot C_{C1} \\
 R_{eq} &= R_S + R_1 \parallel R_2 \parallel r_\pi \\
 &= 2 + 60 \parallel 5.5 \parallel 1.17 = 2.95 \text{ k}\Omega \\
 \tau_L &= (2.95 \times 10^3) (0.1 \times 10^{-6}) = 2.95 \times 10^{-4} \text{ s} \\
 f_L &= \frac{1}{2\pi\tau_L} = \frac{1}{2\pi(2.95 \times 10^{-4})} \Rightarrow f_L = 540 \text{ Hz} \\
 \text{Upper 3 - dB frequency:} \\
 f_T &= \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 400 \times 10^6 = \frac{85.4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)} \\
 C_\pi + C_\mu &= 34 \text{ pF}; \quad C_\mu = 2 \text{ pF}; \quad C_\pi = 32 \text{ pF} \\
 C_M &= C_\mu (1 + g_m R_C) = 2 [1 + (85.4)(4)] \Rightarrow C_M = 685 \text{ pF} \\
 C_i &= C_\pi + C_M = 32 + 685 = 717 \text{ pF} \\
 R_{eq} &= R_S \parallel R_1 \parallel R_2 \parallel r_\pi = 2 \parallel 60 \parallel 5.5 \parallel 1.17 \Rightarrow R_{eq} = 0.644 \text{ k}\Omega \\
 \tau &= R_{eq} \cdot C_i = (0.644 \times 10^3) (717 \times 10^{-12}) \\
 &= 4.62 \times 10^{-7} \text{ s} \\
 f_H &= \frac{1}{2\pi\tau} \Rightarrow f_H = 344 \text{ kHz}
 \end{aligned}$$



7.53

$$R_{TH} = R_1 \parallel R_2 = 600 \parallel 55 = 50.38 \text{ K}$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (15) = \left( \frac{55}{600 + 55} \right) (15) = 1.2595 \text{ V}$$

$$I_{BQ} = \frac{1.26 - 0.7}{50.4 + (101)(2)} = 0.00222 \text{ mA}$$

$$I_{CQ} = 0.2217 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.222} = 11.73 \text{ K}$$

$$g_m = \frac{0.2217}{0.026} = 8.527 \text{ mA/V}$$

Lower – 3dB Freq

$$\begin{aligned} \tau_L = R_{eq1} C_{c1}; R_{eq1} &= R_S + R_{TH} \parallel r_\pi \\ &= 0.50 + 50.38 \parallel 11.73 = 10.0 \text{ K} \end{aligned}$$

$$\tau_L = (10 \times 10^3) (0.1 \times 10^{-6}) = 10^{-3} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} = \frac{1}{2\pi(10^{-3})} \Rightarrow f_L = 159 \text{ Hz}$$

Upper – 3dB Freq

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{8.527 \times 10^{-3}}{2\pi(C_\pi + 2) \times 10^{-12}} = 400 \times 10^6$$

$$C_\pi + C_\mu = 3.393 \text{ pF} \Rightarrow C_\pi = 1.393 \text{ pF}$$

$$C_M = C_\mu (1 + g_m R_C) = 2 [1 + (8.527)(40)] = 684 \text{ pF}$$

$$C_T = C_\pi + C_M = 1.393 + 684 = 685.4 \text{ pF}$$

$$\begin{aligned} R_{eq2} = R_S \parallel R_{TH} \parallel r_\pi &= 0.5 \parallel 50.38 \parallel 11.73 \\ &= 50.38 \parallel 0.480 = 0.4750 \text{ K} \end{aligned}$$

$$\begin{aligned} \tau_H = R_{eq2} C_T &= (0.4750 \times 10^3) (685.4 \times 10^{-12}) \\ &= 3.256 \times 10^{-7} \text{ s} \end{aligned}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi(3.256 \times 10^{-7})} \Rightarrow f_H = 489 \text{ KHz}$$

**7.54**

(a)  $R_{TH} = R_1 \parallel R_2 = 33 \parallel 22 = 13.2 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{22}{22 + 33} \right) (5) = 2 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH} + (1 + \beta)R_E} = \frac{2 - 0.7}{13.2 + (151)(4)} = 0.002106 \text{ mA}$$

$$I_{CQ} = 0.3159 \text{ mA}, \quad I_{EQ} = 0.3180 \text{ mA}$$

$$V_{CEQ} = 5 - (0.3159)(5) - (0.3180)(4) = 2.15 \text{ V}$$

(b)  $f_\beta = \frac{f_T}{\beta_o} = \frac{800 \times 10^6}{150} \Rightarrow f_\beta = 5.33 \text{ MHz}$

$$g_m = \frac{0.3159}{0.026} = 12.15 \text{ mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.15 \times 10^{-3}}{2\pi(800 \times 10^6)} \Rightarrow C_\pi + C_\mu = 2.417 \text{ pF}$$

$$C_\pi = 2.417 - 0.45 = 1.97 \text{ pF}$$

$$C_M = C_\mu [1 + g_m R_C] = (0.45)[1 + (12.15)(5)] = 27.79 \text{ pF}$$

(c)  $r_\pi = \frac{(150)(0.026)}{0.3159} = 12.35 \text{ k}\Omega$ ,  $R_{TH} \parallel r_\pi = 13.2 \parallel 12.35 = 6.38 \text{ k}\Omega$

$$f_{3-dB} = \frac{1}{2\pi(R_{TH} \parallel r_\pi)(C_\pi + C_M)} = \frac{1}{2\pi(6.38 \times 10^3)(1.97 + 27.79) \times 10^{-12}}$$

$$f_{3-dB} = 838 \text{ kHz}$$

**7.55**

$$g_{m1} = 2 \sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left( \frac{0.08}{2} \right) \left( \frac{4}{0.8} \right) (0.6)} = 0.6928 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.6928 \times 10^{-3}}{2\pi(50 + 10) \times 10^{-15}} \Rightarrow f_T = 1.84 \text{ GHz}$$

**7.56**

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$0.12 = K_n (0.2)^2 \Rightarrow K_n = 3 \text{ mA/V}^2$$

$$g_m = 2 \sqrt{K_n I_{DQ}} = 2 \sqrt{(3)(0.12)} = 1.2 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{1.2 \times 10^{-3}}{2\pi(40 + 10) \times 10^{-15}} \Rightarrow f_T = 3.82 \text{ GHz}$$

**7.57**

(a)  $g_m = 2 \sqrt{(1.5)(0.05)} = 0.5477 \text{ mA/V}$

$$f_T = \frac{0.5477 \times 10^{-3}}{2\pi(60 + 10) \times 10^{-15}} \Rightarrow f_T = 1.25 \text{ GHz}$$

(b)  $g_m = 2\sqrt{(1.5)(0.3)} = 1.342 \text{ mA/V}$

$$f_T = \frac{1.342 \times 10^{-3}}{2\pi(60+10) \times 10^{-15}} \Rightarrow f_T = 3.05 \text{ GHz}$$

(c)  $3 \times 10^9 = \frac{g_m}{2\pi(60+10) \times 10^{-15}} \Rightarrow g_m = 1.319 \text{ mA/V}$

$$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow I_{DQ} = \frac{1}{K_n} \left( \frac{g_m}{2} \right)^2 = \frac{1}{1.5} \left( \frac{1.319}{2} \right)^2 = 0.29 \text{ mA}$$

(d)  $g_m = 2\sqrt{(1.5)(0.25)} = 1.225 \text{ mA/V}$

$$2.5 \times 10^9 = \frac{1.225 \times 10^{-3}}{2\pi(C_{gs} + 8) \times 10^{-15}} \Rightarrow C_{gs} = 70 \text{ fF}$$

### 7.58

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$C_{gs} + C_{gd} = WLC_{ox}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2\left(\frac{W}{L}\right)\left(\frac{\mu_n C_{ox}}{2}\right)(V_{GS} - V_{TN})$$

$$\text{Then } f_T = \frac{\left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TN})}{2\pi WLC_{ox}}$$

$$f_T = \frac{\mu_n(V_{GS} - V_{TN})}{2\pi L^2}$$

(a)  $f_T = \frac{450(0.5)}{2\pi(1.2 \times 10^{-4})^2} \Rightarrow f_T = 2.49 \text{ GHz}$

(b)  $f_T = \frac{450(0.5)}{2\pi(0.18 \times 10^{-4})^2} \Rightarrow f_T = 111 \text{ GHz}$

### 7.59

(a)  $C_M = C_{gd} [1 + g_m(r_o \parallel R_D)] = (12)[1 + (3)(120 \parallel 10)] = 344.3 \text{ fF}$

(b)  $f_{3-dB} = \frac{1}{2\pi\tau}$

$$\tau = r_i(C_{gs} + C_M) = (10^4)(80 + 344.3) \times 10^{-15} = 4.243 \times 10^{-9} \text{ s}$$

$$f_{3-dB} = \frac{1}{2\pi(4.243 \times 10^{-9})} \Rightarrow f_{3-dB} = 37.5 \text{ MHz}$$

**7.60**

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_{gdT})} \quad (\text{Eq. (7.97)})$$

Let  $C_{gdT} = 0$  and  $C_{gsT} = \left(\frac{2}{3}\right)(WLC_{ox})$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$$

$$\text{So } f_T = \frac{2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right) I_D}}{2\pi\left(\frac{2}{3}\right)(WLC_{ox})}$$

$$= \frac{3}{2\pi L} \cdot \frac{\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right) I_D}}{WC_{ox}}$$

$$f_T = \frac{3}{2\pi L} \cdot \sqrt{\frac{\mu_n I_D}{2WC_{ox}L}}$$

**7.61**

(a)  $K_n = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) = \frac{(400)(6.9 \times 10^{-9})}{2}(8) \Rightarrow K_n = 1.104 \text{ mA/V}^2$

$$I_D = K_n (V_{GS} - V_{TN})^2 = (1.104)(3 - 0.4)^2 = 7.463 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(1.104)(7.463)} = 5.741 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_s}$$

$$(0.8)g_m = \frac{g_m}{1 + g_m r_s} \Rightarrow g_m r_s = \frac{1}{0.8} - 1 = 0.25$$

$$r_s = \frac{0.25}{5.741} \Rightarrow r_s = 43.5 \Omega$$

(b)  $I_D = (1.104)(1 - 0.4)^2 = 0.3974 \text{ mA}$

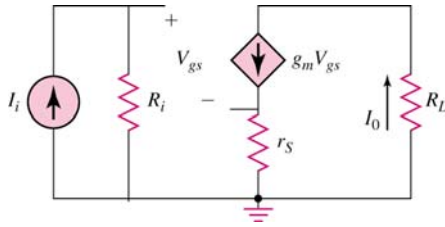
$$g_m = 2\sqrt{(1.104)(0.3974)} = 1.325 \text{ mA/V}$$

$$g'_m = \frac{1.325}{1 + (1.325)(0.04355)} = 1.253 \text{ mA/V}$$

$$\frac{g'_m}{g_m} \Rightarrow 94.5\%$$

7.62

a.

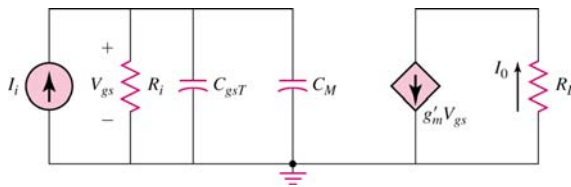


$$I_0 = g_m V_{gs} \text{ and } V_{gs} = I_i R_i - g_m V_{gs} r_s \text{ so } V_{gs} = \frac{I_i R_i}{1 + g_m r_s}$$

$$A_i = \frac{I_0}{I_i} = \frac{g_m R_i}{1 + g_m r_s}$$

Then

b. As an approximation, consider



In this case

$$A_i = \frac{I_0}{I_i} = g'_m R_i \cdot \frac{1}{1 + s R_i (C_{gsT} + C_M)} \text{ where } C_M = C_{gdT} (1 + g'_m R_L) \text{ and } g'_m = \frac{g_m}{1 + g_m r_s}$$

c. As  $r_s$  increases,  $C_M$  decreases, so the bandwidth increases, but the current gain magnitude decreases.

7.63

$$(b) V_{GS} = \left( \frac{225}{225 + 500} \right) (10) = 3.103 \text{ V}$$

$$I_{DQ} = (1)(3.103 - 2)^2 = 1.218 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.218)} = 2.207 \text{ mA/V}$$

$$C_M = C_{gd} (1 + g_m R_D) = (8)[1 + (2.207)(5)] = 96.28 \text{ fF}$$

$$(c) f_{3-dB} = \frac{1}{2\pi\tau}, \quad \tau = (R_i \parallel R_1 \parallel R_2)(C_{gs} + C_M)$$

$$\text{Now } R_i \parallel R_1 \parallel R_2 = 1 \parallel 500 \parallel 225 = 0.9936 \text{ k}\Omega$$

$$\tau = (0.9936 \times 10^3)(50 + 96.28) \times 10^{-15} = 1.453 \times 10^{-10} \text{ s}$$

$$f_{3-dB} = \frac{1}{2\pi(1.453 \times 10^{-10})} \Rightarrow f_{3-dB} = 1.095 \text{ GHz}$$

$$A_v = -g_m R_D \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_i} \right) = -(2.207)(5) \left( \frac{155.2}{155.2 + 1} \right) = -10.96$$

7.64

(a)  $C_M = C_{gd}(1 + |A_v|) = (0.04)(1 + 15) = 0.64 \text{ pF}$

(b)  $f_H = \frac{1}{2\pi\tau}$ ,  $\Rightarrow \tau = \frac{1}{2\pi f} = \frac{1}{2\pi(5 \times 10^6)} = 3.183 \times 10^{-8} \text{ s}$

$$\tau = R_{eq}(C_{gs} + C_M)$$

or  $R_{eq} = \frac{\tau}{(C_{gs} + C_M)} = \frac{3.183 \times 10^{-8}}{(0.2 + 0.64) \times 10^{-12}} \Rightarrow R_{eq} = 37.9 \text{ k}\Omega$

**7.65**

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 22 = 13.2 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) (5) = \left( \frac{22}{22 + 33} \right) (5) = 2 \text{ V}$$

$$I_{BQ} = \frac{2 - 0.7}{13.2 + (121)(4)} = 0.00261 \text{ mA}$$

$$I_{CQ} = 0.3138$$

$$r_\pi = \frac{(120)(0.026)}{0.3138} = 9.94 \text{ k}\Omega$$

$$g_m = \frac{0.3138}{0.026} = 12.07 \text{ mA/V}$$

$$r_o = \frac{100}{0.3138} = 318 \text{ k}\Omega$$

a.

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.07 \times 10^{-3}}{2\pi(600 \times 10^6)}$$

$$C_\pi + C_\mu = 3.20 \text{ pF}; C_\mu = 1 \text{ pF} \Rightarrow C_\pi = 2.20 \text{ pF}$$

$$C_M = C_\mu [1 + g_m(r_o \parallel R_C \parallel R_L)]$$

$$= (1) [1 + (12.07)(318 \parallel 4 \parallel 5)]$$

$$C_M = 27.6 \text{ pF}$$

b.

$$\begin{aligned} \tau &= R_{eq} (C_{\pi} + C_{\mu}) \\ R_{eq} &= R_1 \parallel R_2 \parallel R_S \parallel r_{\pi} = 33 \parallel 22 \parallel 2 \parallel r_{\pi} \\ &= 1.74 \parallel 9.94 \Rightarrow R_{eq} = 1.48 \text{ k}\Omega \\ \tau &= (1.48 \times 10^3)(2.20 + 27.6) \times 10^{-12} \\ \tau &= 4.41 \times 10^{-8} \text{ s} \\ f_H &= \frac{1}{2\pi\tau} = \frac{1}{2\pi(4.41 \times 10^{-8})} \Rightarrow f_H = 3.61 \text{ MHz} \\ V_o &= -g_m V_{\pi} (r_o \parallel R_C \parallel R_L) \\ V_{\pi} &= \frac{R_1 \parallel R_2 \parallel r_{\pi}}{R_1 \parallel R_2 \parallel r_{\pi} + R_S} \cdot V_i \\ R_1 \parallel R_2 \parallel r_{\pi} &= 33 \parallel 22 \parallel 9.94 = 5.67 \text{ k}\Omega \\ V_{\pi} &= \frac{5.67}{5.67 + 2} \cdot V_i = (0.739)V_i \\ r_o \parallel R_C \parallel R_L &= 318 \parallel 4 \parallel 5 = 2.18 \text{ k}\Omega \\ A_v &= -(12.07)(0.739)(2.18) \\ A_v &= -19.7 \end{aligned}$$

**7.66**

$$\begin{aligned} R_{TH} &= R_1 \parallel R_2 = 40 \parallel 5 = 4.44 \text{ k}\Omega \\ V_{TH} &= \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{5}{5 + 40} \right) (10) = 1.111 \text{ V} \\ I_{BQ} &= \frac{1.111 - 0.7}{4.44 + (121)(0.5)} = 0.00633 \text{ mA} \\ I_{CQ} &= 0.760 \text{ mA} \\ r_{\pi} &= \frac{(120)(0.026)}{0.760} = 4.11 \text{ k}\Omega \\ g_m &= \frac{0.760}{0.026} = 29.23 \text{ mA/V} \\ r_o &= \infty \\ f_T &= \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} \\ C_{\pi} + C_{\mu} &= \frac{g_m}{2\pi f_T} = \frac{29.23 \times 10^{-3}}{2\pi(250 \times 10^6)} \\ C_{\pi} + C_{\mu} &= 18.6 \text{ pF}; C_{\mu} = 3 \text{ pF} \Rightarrow C_{\pi} = 15.6 \text{ pF} \\ \text{a.} \\ C_M &= C_{\mu} [1 + g_m (R_C \parallel R_L)] \\ C_M &= 3 [1 + (29.2)(5 \parallel 2.5)] \Rightarrow C_M = 149 \text{ pF} \end{aligned}$$

For upper frequency:

$$\tau_H = R_{eq} (C_\pi + C_M)$$

$$R_{eq} = r_\pi \parallel R_1 \parallel R_2 \parallel R_S = 4.11 \parallel 40 \parallel 5 \parallel 0.5$$

$$R_{eq} = 0.405 \text{ k}\Omega$$

$$\tau_H = (0.405 \times 10^3)(15.6 + 149) \times 10^{-12}$$

$$= 6.67 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \Rightarrow f_H = 2.39 \text{ MHz}$$

For lower frequency:

$$\tau_L = R_{eq} C_{C1}$$

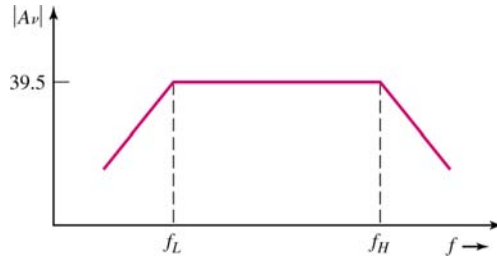
$$R_{eq} = R_S + R_1 \parallel R_2 \parallel r_\pi = 0.5 + 40 \parallel 5 \parallel 4.11$$

$$R_{eq} = 2.64 \text{ k}\Omega$$

$$\tau_L = (2.64 \times 10^3)(4.7 \times 10^{-6}) = 1.24 \times 10^{-2} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow f_L = 12.8 \text{ Hz}$$

b.



$$V_o = -g_m V_\pi (R_C \parallel R_L)$$

$$V_\pi = \left( \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) V_i$$

$$V_\pi = \left( \frac{2.135}{2.135 + 0.5} \right) V_i = 0.8102 V_i$$

$$|A_v| = (29.23)(0.8102)(5 \parallel 2.5)$$

$$\underline{|A_v| = 39.5}$$



**7.67**

$$I_D = K_p (V_{SG} + V_{TP})^2 = \frac{9 - V_{SG}}{R_S}$$

$$(2)(1.2)(V_{SG}^2 - 4V_{SG} + 4) = 9 - V_{SG}$$

$$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$$

$$V_{SG} = \frac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$$

$$V_{SG} = 3.512 \text{ V}$$

$$g_m = 2K_p (V_{SG} + V_{TP}) = 2(2)(3.512 - 2)$$

$$g_m = 6.049 \text{ mA/V}$$

$$I_D = (2)(3.512 - 2)^2 = 4.572 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_o} = \frac{1}{(0.01)(4.56)} \Rightarrow r_o = 21.9 \text{ k}\Omega$$

$$C_M = C_{gdT} (1 + g_m (r_o \parallel R_D))$$

$$C_M = (1) [1 + (6.04)(21.9 \parallel 1)] \Rightarrow C_M = 6.785 \text{ pF}$$

a.

$$b. \tau_H = (R_i \parallel R_G) (C_{gsT} + C_M)$$

$$\tau_H = (2 \parallel 100) \times 10^3 \times (10 + 6.78) \times 10^{-12}$$

$$\tau_H = 3.29 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \Rightarrow f_H = 4.84 \text{ MHz}$$

$$V_o = -g_m (r_o \parallel R_D) \cdot V_{gs}$$

$$V_{gs} = \left( \frac{R_G}{R_G + R_i} \right) \cdot V_i = \left( \frac{100}{102} \right) \cdot V_i$$

$$A_v = -(6.04) \left( \frac{100}{102} \right) (21.9 \parallel 1)$$

$$A_v = -5.67$$

**7.68**

$$(a) I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.5 = 0.5(V_{SGQ} - 0.5)^2 \Rightarrow V_{SGQ} = 1.5 \text{ V}$$

$$R_S = \frac{3 - 1.5}{0.5} = 3 \text{ k}\Omega$$

$$V_D = 1.5 - 2 = -0.5 \text{ V}$$

$$R_D = \frac{-0.5 - (-3)}{0.5} = 5 \text{ k}\Omega$$

$$(b) g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$A_v = -g_m R_D \left( \frac{R_G}{R_G + R_i} \right) = -(1)(5) \left( \frac{200}{200 + 4} \right) = -4.90$$

$$(c) \quad C_M = C_{gd} (1 + g_m R_D) = (0.08)[1 + (1)(5)] = 0.48 \text{ pF}$$

$$(d) \quad f_{3-dB} = \frac{1}{2\pi\tau}$$

$$\text{where } \tau = R_{eq} \cdot C_{eq} = (R_i \parallel R_G)(C_{gs} + C_M) = (4 \parallel 200) \times 10^3 \times (0.8 + 0.48) \times 10^{-12}$$

$$\text{which yields } \tau = 5.02 \times 10^{-9} \text{ s}$$

$$\text{Then } f_{3-dB} = \frac{1}{2\pi(5.02 \times 10^{-9})} \Rightarrow f_{3-dB} = 31.7 \text{ MHz}$$

### 7.69

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left( \frac{22}{22 + 8} \right) (20) - 10$$

$$V_G = 4.67 \text{ V}$$

$$I_D = \frac{10 - V_{SG} - 4.67}{R_S} = K_p (V_{SG} + V_{TP})^2$$

$$5.33 - V_{SG} = (1)(0.5)(V_{SG}^2 - 4V_{SG} + 4)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1 \pm \sqrt{1 + 4(0.5)(3.33)}}{2(0.5)} \Rightarrow V_{SG} = 3.77 \text{ V}$$

$$g_m = 2K_p (V_{SG} + V_{TP}) = 2(1)(3.77 - 2)$$

$$g_m = 3.54 \text{ mA/V}$$

b.

$$C_M = C_{gdT} (1 + g_m (R_D \parallel R_L))$$

$$C_M = (3)[1 + (3.54)(2 \parallel 5)] \Rightarrow C_M = 18.2 \text{ pF}$$

a.

$$\tau = R_{eq} (C_{gsT} + C_M)$$

$$R_{eq} = R_i \parallel R_1 \parallel R_2 = 0.5 \parallel 8 \parallel 22 = 0.461 \text{ k}\Omega$$

$$\tau = (0.461 \times 10^3)(15 + 18.2) \times 10^{-12}$$

$$= 1.53 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 10.4 \text{ MHz}$$

c.

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

$$V_{gs} = \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_i} \right) \cdot V_i = \left( \frac{5.87}{5.87 + 0.5} \right) \cdot V_i = (0.9215)V_i$$

$$A_v = -(3.54)(0.9215)(2 \parallel 5) \Rightarrow A_v = -4.66$$

7.70

$$I_E = 0.5 \text{ mA} \Rightarrow I_{CQ} = \left(\frac{100}{101}\right)(0.5) = 0.495 \text{ mA}$$

$$g_m = \frac{0.495}{0.026} = 19.0 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{0.495} = 5.25 \text{ k}\Omega$$

a. Input: From Eq. (7.114(b))

$$\begin{aligned} \tau_{P\pi} &= \left[ \frac{r_\pi}{1 + \beta} \parallel R_E \parallel R_S \right] C_\pi \\ &= \left[ \frac{5.25}{101} \parallel 0.5 \parallel 0.05 \right] \times 10^3 \times (10 \times 10^{-12}) \\ &= 2.43 \times 10^{-10} \text{ s} \end{aligned}$$

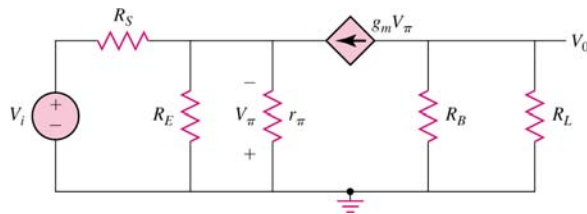
$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{H\pi} = 656 \text{ MHz}$$

Output: From Eq. (7.115(b))

$$\begin{aligned} \tau_{P\mu} &= (R_B \parallel R_L) C_\mu = (100 \parallel 1) \times 10^3 \times (10^{-12}) \\ &= 9.90 \times 10^{-10} \text{ s} \end{aligned}$$

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{H\mu} = 161 \text{ MHz}$$

b.



$$V_o = -g_m V_\pi (R_B \parallel R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left[ g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right] = -\frac{V_i}{R_S}$$

$$V_\pi \left[ 19 + \frac{1}{5.25} + \frac{1}{0.5} + \frac{1}{0.05} \right] = \frac{-V_i}{0.05}$$

$$V_\pi (41.19) = -V_i (20)$$

$$V_\pi = -(0.4856)V_i$$

$$\frac{V_o}{V_i} = -(19)(-0.4856)(100 \parallel 1)$$

$$A_v = 9.14$$

c.

$$\tau = (R_B \parallel R_L) C_L = (100 \parallel 1) \times 10^3 \times (15 \times 10^{-12})$$

$$\tau = 1.485 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 10.7 \text{ MHz}$$

Since  $f < f_{H\mu} \Rightarrow 3\text{-dB}$  frequency dominated by  $C_L$

**7.71**

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

$$I_{CQ} = \left(\frac{100}{101}\right)(1.93) = 1.91 \text{ mA}$$

$$g_m = \frac{1.91}{0.026} = 73.5 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.91} = 1.36 \text{ k}\Omega$$

a. Input:

$$\tau_{P\pi} = \left[ \frac{r_\pi}{1 + \beta} \parallel R_E \parallel R_S \right] C_\pi$$

$$= \left[ \frac{1.36}{101} \parallel 10 \parallel 1 \right] \times 10^3 \times (10 \times 10^{-12})$$

$$\tau_{P\pi} = 1.327 \times 10^{-10} \text{ s}$$

$$f_{P\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{P\pi} = 1.20 \text{ GHz}$$

Output:

$$\tau_{P\mu} = (R_C \parallel R_L) C_\mu = (6.5 \parallel 5) \times 10^3 \times (10^{-12})$$

$$\tau_{P\mu} = 2.826 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 56.3 \text{ MHz}$$

b.

$$V_o = -g_m V_\pi (R_C \parallel R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left( g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) = -\frac{V_i}{R_S}$$

$$V_\pi \left( 73.5 + \frac{1}{1.36} + \frac{1}{10} + \frac{1}{1} \right) = \frac{-V_i}{1}$$

$$V_\pi (75.34) = -V_i \Rightarrow V_\pi = -(0.01327)V_i$$

$$V_o = -(73.5)(-0.01327)(6.5 \parallel 5)V_i$$

$$A_v = 2.76$$

c.

$$\tau = (R_C \parallel R_L) C_L = (6.5 \parallel 5) \times 10^3 \times (15 \times 10^{-12})$$

$$\tau = 4.24 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 3.75 \text{ MHz}$$

Since  $f < f_{P\mu}$ , 3-dB frequency is dominated by  $C_L$

**7.72**

$$V_{GS} + I_D R_S = 5$$

$$I_D = \frac{5 - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$5 - V_{GS} = (3)(10)(V_{GS}^2 - 2V_{GS} + 1)$$

$$30V_{GS}^2 - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.349 \text{ V}$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(3)(1.35 - 1)$$

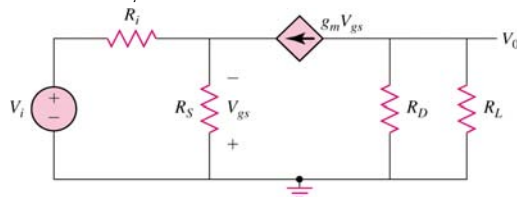
$$g_m = 2.093 \text{ mA/V}$$

On the output:

$$\tau_{P\mu} = (R_D \parallel R_L) C_{gdT} = (5 \parallel 4) \times 10^3 \times (4 \times 10^{-12})$$

$$\tau_{P\mu} = 8.89 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 17.9 \text{ MHz}$$



$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

$$g_m V_{gs} + \frac{V_{gs}}{R_S} + \frac{V_i - (-V_{gs})}{R_i} = 0$$

$$V_{gs} \left( g_m + \frac{1}{R_S} + \frac{1}{R_i} \right) = -\frac{V_i}{R_i}$$

$$V_{gs} \left( 2.093 + \frac{1}{10} + \frac{1}{2} \right) = -\frac{V_i}{2}$$

$$V_{gs} = (0.1857) V_i$$

$$A_v = \frac{V_o}{V_i} = (2.093)(0.1857)(5 \parallel 4)$$

$$A_v = 0.864$$

**7.73**

dc analysis

$$I_D = \frac{V^+ - V_{SG}}{R_S} = K_P (V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

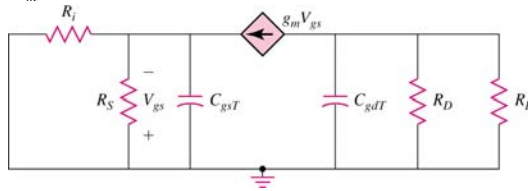
$$= 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(4)(2.44)}}{2(4)} = 1.707$$

$$g_m = 2K_P (V_{SG} + V_{TP}) = 2(1)(1.707 - 0.8)$$

$$g_m = 1.81 \text{ mA/V}$$



$$3\text{-dB frequency due to } C_{gsT} : R_{eq} = \frac{1}{g_m} \parallel R_S \parallel R_i$$

$$f_A = \frac{1}{2\pi R_{eq} \cdot C_{gsT}}$$

$$R_{eq} = \frac{1}{1.81} \parallel 4 \parallel 0.5 = 0.246 \text{ k}\Omega$$

$$f_A = \frac{1}{2\pi(246)(4 \times 10^{-12})} = 162 \text{ MHz}$$

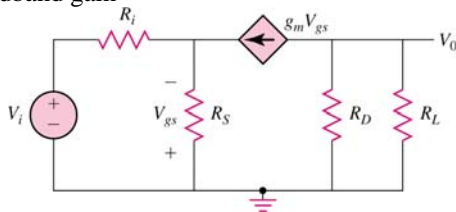
3-dB frequency due to  $C_{gdT}$

$$f_B = \frac{1}{2\pi(R_D \parallel R_L)C_{gdT}}$$

$$= \frac{1}{2\pi(2 \parallel 4) \times 10^3 \times 10^{-12}}$$

$$f = 119 \text{ MHz}$$

Midband gain



$$V_{gs} = \frac{-\frac{1}{g_m} \parallel R_s}{\frac{1}{g_m} \parallel R_s + R_i} \cdot V_i = \frac{-\frac{1}{1.81} \parallel 4}{\frac{1}{1.81} \parallel 4 + 0.5} \cdot V_i$$

$$= -0.492V_i$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

$$A_v = (0.492)(1.81)(4 \parallel 2) \Rightarrow \underline{A_v = 1.19}$$

**7.74**

$$r_\pi = \frac{(120)(0.026)}{1.02} = 3.059 \text{ k}\Omega$$

$$g_m = 39.23 \text{ mA/V}$$

a.

$$\text{Input: } f_{H\pi} = \frac{1}{2\pi\tau_\pi}$$

$$\tau_\pi = (R_s \parallel R_2 \parallel R_3 \parallel r_\pi)(C_\pi + 2C_\mu)$$

$$R_{eq} = 0.1 \parallel 20.5 \parallel 28.3 \parallel 3.06 = 0.096 \text{ k}\Omega$$

$$\tau_\pi = (96)[12 + 2(2)] \times 10^{-12} = 1.537 \times 10^{-9} \text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi(1.537 \times 10^{-9})} \Rightarrow f_{H\pi} = 103.6 \text{ MHz}$$

$$\text{Output: } f_{H\mu} = \frac{1}{2\pi\tau_\mu}$$

$$\tau_\mu = (R_C \parallel R_L)C_\mu$$

$$= (15 \parallel 10) \times 10^3 \times (2 \times 10^{-12})$$

$$\tau_\mu = 6.67 \times 10^{-9} \text{ s}$$

$$f_{H\mu} = \frac{1}{2\pi(6.67 \times 10^{-9})} \Rightarrow f_{H\mu} = 23.9 \text{ MHz}$$

b.

$$A_v = g_m (R_C \parallel R_L) \left( \frac{R_2 \parallel R_3 \parallel r_\pi}{R_2 \parallel R_3 \parallel r_\pi + R_s} \right)$$

$$R_2 \parallel R_3 \parallel r_\pi = 20.5 \parallel 28.3 \parallel 3.059 = 2.433 \text{ k}\Omega$$

$$A_v = (39.23)(5 \parallel 10) \left( \frac{2.433}{2.433 + 0.1} \right) \Rightarrow A_v = 125.6$$

c.  $C_L = 15 \text{ pF} > C_\mu \Rightarrow C_L$  dominates frequency response.

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## Chapter 8

### 8.1

(b) (i)  $R_D = \frac{24}{4} = 6 \Omega$   
 $P_{D,\max} = (12)(2) = 24 \text{ W}$   
(ii)  $P_{D,\max} = 30 = (20)I_{DQ} \Rightarrow I_{DQ} = 1.5 \text{ A}$   
 $I_{D,\max} = 2(1.5) = 3 \text{ A}$   
 $R_D = \frac{40}{3} = 13.3 \Omega$

(c) (i)  $I_{D,\max} = 4 \text{ A}$   
(ii)  $I_{D,\max} = 3 \text{ A}$

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### 8.2

(a)  $P_{Q,\max} = V_{CEQ} \cdot I_{CQ}$   
 $25 = \left(\frac{24}{2}\right) \cdot I_{CQ} \Rightarrow I_{CQ} = 2.083 \text{ A}$   
 $R_L = \frac{24 - 12}{2.083} = 5.76 \Omega$   
 $I_{BQ} = \frac{2.083}{60} = 0.03472 \text{ A}$   
 $R_B = \frac{24 - 0.7}{0.03472} = 671 \Omega$

(b)  $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(60)(0.026)}{2.083} = 0.7489 \Omega$   
 $I_b = \frac{V_p}{r_\pi} = \frac{12 \text{ mV}}{0.7489} = 16.02 \text{ mA}$   
 $I_c = \beta I_b = (60)(0.01602) = 0.9614 \text{ A}$   
 $\bar{P}_{\text{avg}} = \frac{1}{2} I_c^2 R_C = \frac{1}{2} (0.9614)^2 (5.76) = 2.66 \text{ W}$

For the transistor,  
 $P_Q = 25 - 2.66 = 22.34 \text{ W}$

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### 8.3

(a)  $R_L = \frac{30}{1.2} = 25 \Omega$   
 $I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.6}{80} = 0.0075 \text{ A}$   
 $R_B = \frac{V_{CC} - V_{BE}(\text{on})}{I_{BQ}} = \frac{30 - 0.7}{0.0075} \Rightarrow R_B = 3.91 \text{ k}\Omega$   
 $P_{Q,\max} = I_{CQ} V_{CEQ} = (0.6)(15) = 9 \text{ W}$



(b)  $P_{Q,\max} = I_{CQ} V_{CEQ} = \left( \frac{V_{CC} - V_{CEQ}}{R_L} \right) \cdot V_{CEQ}$

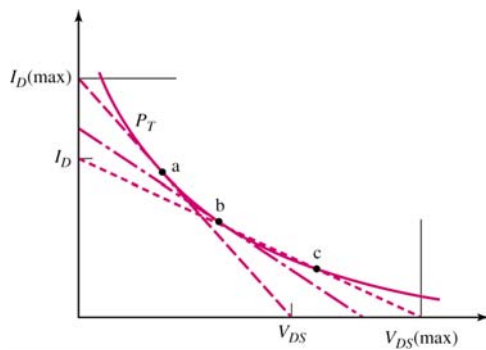
$V_{CEQ} = \frac{1}{2} V_{CC}$

$5 = \frac{1}{25} \left( \frac{V_{CC}}{2} \right) \left( \frac{V_{CC}}{2} \right) \Rightarrow V_{CC} = 22.36 \text{ V}; I_{C,\max} = \frac{22.36}{25} = 0.8944 \text{ A}$

(c) For (a):  $\Delta I_C = 0.6 \text{ A}; \bar{P}_L = \frac{1}{2} (\Delta I_C)^2 R_L = \frac{1}{2} (0.6)^2 (25) = 4.5 \text{ W}$

For (b):  $\Delta I_C = I_{CQ} = \frac{0.8944}{2} = 0.4472 \text{ A}; \bar{P}_L = \frac{1}{2} (0.4472)^2 (25) = 2.5 \text{ W}$

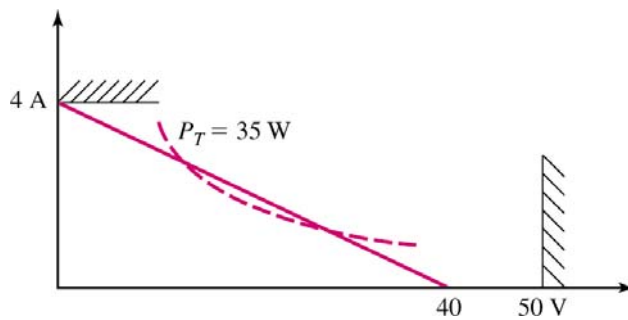
8.4



Point (b): Maximum power delivered to load.  
 Point (a): Will obtain maximum signal current output.  
 Point (c): Will obtain maximum signal voltage output.

8.5

a.



b.

$$V_{GG} = 5 \text{ V}, I_D = 0.25(5-4)^2 = 0.25 \text{ A}, V_{DS} = 37.5 \text{ V}, P = 9.375 \text{ W}$$

$$V_{GG} = 6 \text{ V}, I_D = 0.25(6-4)^2 = 1.0 \text{ A}, V_{DS} = 30 \text{ V}, P = 30 \text{ W}$$

$$V_{GG} = 7 \text{ V}, I_D = 0.25(7-4)^2 = 2.25 \text{ A}, V_{DS} = 17.5 \text{ V}, P = 39.375 \text{ W}$$

$$V_{GG} = 8 \text{ V}, I_D = 0.25[2(8-4)V_{DS} - V_{DS}^2]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 2.92$$

$$I_D = 3.71 \text{ A}, P = 10.8 \text{ W}$$

$$V_{GG} = 9 \text{ V}, I_D = 0.25[2(9-4)V_{DS} - V_{DS}^2]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 1.88 \text{ V}$$

$$I_D = 3.81 \text{ A}, P = 7.16 \text{ W}$$

c. Yes, at  $V_{GG} = 7 \text{ V}, P = 39.375 \text{ W} > P_{D,\max} = 35 \text{ W}$

## 8.6

a.

$$\text{Set } V_{DSQ} = \frac{V_{DD}}{2} = 25 \text{ V}$$

$$I_{DQ} = \frac{50 - 25}{20} = 1.25 \text{ A}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$\sqrt{\frac{1.25}{0.2}} + 4 = V_{GS} = 6.5 \text{ V}$$

$$V_{GS} = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$\text{Let } R_1 + R_2 = 100 \text{ k}\Omega$$

$$6.5 = \left( \frac{R_2}{100} \right) (50) \Rightarrow R_2 = 13 \text{ k}\Omega$$

$$R_1 = 87 \text{ k}\Omega$$

b.

$$P_D = I_{DQ} V_{DSQ} = (1.25)(25) \Rightarrow P_D = 31.25 \text{ W}$$

c.

$$I_{D,\max} = 2I_{DQ} \Rightarrow I_{D,\max} = 2.5 \text{ A}$$

$$V_{DS,\max} = V_{DD} \Rightarrow V_{DS,\max} = 50 \text{ V}$$

$$P_{D,\max} = 31.25 \text{ W}$$

d.

$$\left| \frac{V_0}{V_i} \right| = g_m R_L$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(1.25)} = 1 \text{ A/V}$$

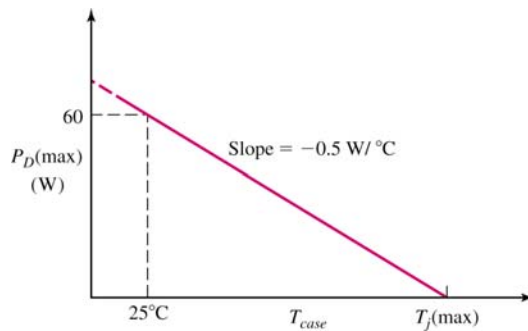
$$|V_0| = (1)(20)(0.5) = 10 \text{ V}$$

$$\overline{P_L} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{1}{2} \cdot \frac{(10)^2}{20} \Rightarrow \overline{P_L} = 2.5 \text{ W}$$

$$\overline{P_Q} = 31.25 - 2.5 \Rightarrow \overline{P_Q} = 28.75 \text{ W}$$

8.7

(a)



(b)  $P_D = P_{D,\max} - (\text{Slope})(T_j - 25)$

At  $P_D = 0$ ,  $T_{j,\max} = \frac{60}{0.5} + 25 \Rightarrow T_{j,\max} = 145^\circ\text{C}$

(c)  $P_{D,\max} = \frac{T_{j,\max} - T_{case}}{\theta_{dev-amb}}$  or  $\theta_{dev-amb} = \frac{145 - 25}{60} \Rightarrow \theta_{dev-amb} = 2^\circ\text{C/W}$

8.8

$$P_{D,\text{rated}} = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case}}$$

or  $\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,\text{rated}}}$

$$= \frac{150 - 25}{50} = 2.5^\circ\text{C/W}$$

Then  $T_{dev} - T_{amb} = P_D (\theta_{dev-case} + \theta_{case-amb})$

$$150 - 25 = P_D (2.5 + \theta_{case-amb}) \Rightarrow 125 = P_D (2.5 + \theta_{case-amb})$$

8.9

(a)  $T_{j,\max} - T_{amb} = P_T (\theta_{dev-case} + \theta_{snk-amb} + \theta_{case-snk})$

$$120 - 25 = P_T (1.5 + 2.8 + 0.6) \Rightarrow P_T = 19.39 \text{ W}$$

- (b)  $T_{case} = 25 + (19.39)(0.6 + 2.8) = 90.9^\circ C$   
 (c)  $T_{shk} = 25 + (19.39)(2.8) = 79.3^\circ C$

**8.10**

- (a)  $T_{j,max} - T_{amb} = P(\theta_{dev-case} + \theta_{case-amb})$   
 $150 - 25 = 30(2.8 + \theta_{case-amb}) \Rightarrow \theta_{case-amb} = 1.37^\circ C / W$   
 (b)  $T_{j,max} = 25 + 20(2.8 + 1.37) = 108^\circ C$

**8.11**

- (a)  $150 - 25 = P_T(3.8 + 1.5 + 4) \Rightarrow P_T = 13.4 \text{ W}$   
 (b)  $P = I_{CQ} V_{CEQ}$   
 $13.4 = (3) \cdot V_{CEQ} \Rightarrow V_{CEQ} = 4.48 \text{ V}$   
 $V_{CC} = V_{CE,max} = 2V_{CEQ} = 8.96 \text{ V}$

**8.12**

$$\eta = \frac{\overline{P}_L}{\overline{P}_S}$$

$$\overline{P}_S = V_{CC} \cdot I_Q$$

$$\overline{P}_L = V_P \cdot I_P = \left( \frac{V_{CC}}{2} \right) (I_Q)$$

$$\eta = \frac{\frac{1}{2} \cdot V_{CC} \cdot I_Q}{V_{CC} \cdot I_Q} \Rightarrow \underline{\eta = 50\%}$$

**8.13**

(a)  $A_v = \frac{(1 + \beta)R_L}{r_\pi + (1 + \beta)R_L} = \frac{(1 + \beta)R_L}{\frac{\beta V_T}{I_C} + (1 + \beta)R_L}$

We have  $(1 + \beta) \cong \beta$

$$A_v = \frac{I_C R_L}{I_C R_L + V_T} = \frac{R_L}{R_L + \frac{V_T}{I_C}} = \frac{R_L}{R_L + \frac{1}{g_m}}$$

- (b) (i)  $0.9 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 1.125 \text{ A/V}, I_C = 29.25 \text{ mA}$   
 (ii)  $0.95 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 2.375 \text{ A/V}, I_C = 61.75 \text{ mA}$   
 (iii)  $0.997 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 41.54 \text{ A/V}, I_C = 108 \text{ mA}$

**8.14**

$$(a) \quad \bar{P}_L = \frac{1}{2} \cdot \frac{V_p^2}{R_L} \Rightarrow 0.5 = \frac{1}{2} \cdot \frac{V_p^2}{8} \Rightarrow V_p = 2.828 \text{ V}$$

$$I_p = \frac{V_p}{R_L} = \frac{2.828}{8} = 0.3536 \text{ A}$$

$$(b) \quad \text{For } V_o = -V_p = -2.828 \text{ V}$$

$$|I_L| = 0.3536 = (0.9)I_o \Rightarrow I_o = 0.393 \text{ A}$$

**8.15**

$$(a) \quad V_o = 1.6 \text{ V}, \quad I_L = \frac{1.6}{8} = 0.20 \text{ A}$$

$$I_C \cong 0.2 + 0.25 = 0.45 \text{ A}$$

$$g_m = \frac{0.45}{0.026} = 17.31 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{17.31}} = 0.9928$$

$$(b) \quad V_o = 0, \quad I_L = 0, \quad \text{so } I_C = 0.25 \text{ A}$$

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{9.615}} = 0.9872$$

$$(c) \quad V_o = -1.6 \text{ V}, \quad I_L = -0.2 \text{ A}, \quad I_C \cong 0.25 - 0.2 = 0.05 \text{ A}$$

$$g_m = \frac{0.05}{0.026} = 1.923 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{1.923}} = 0.939$$

**8.16**

$$v_o(\text{max}) = 4.8 \text{ V}$$

$$i_{C3} = i_{C2} = \frac{-0.7 - (-5)}{1} = 4.3 \text{ mA}$$

$$v_i = v_o + 0.7 \quad i_L(\text{max}) = -4.3 \text{ mA} = \frac{v_s(\text{min})}{1}$$

$$\text{so } -3.6 \leq v_i \leq 5.5 \text{ V} \quad \underline{v_o(\text{min}) = -4.3 \text{ V}}$$

**8.17**

$$I_{D3} = K(V_{GS3} - V_{TN})^2 = \frac{0 - V_{GS3} - (-5)}{R}$$

$$12(V_{GS3} - 0.5)^2 = 5 - V_{GS3}$$

$$2V_{GS3}^2 - 11V_{GS3} - 2 = 0$$

$$V_{GS3} = \frac{11 \pm \sqrt{(11)^2 + 4(12)(2)}}{2(12)}$$

$$V_{GS3} = V_{GS2} = 1.072 \text{ V}$$

$$I_{D3} = I_{D2} = 12(1.072 - 0.5)^2 = 3.93 \text{ mA}$$

$$V_{DS2}(\text{sat}) = V_{GS2} - V_{TN} = 1.072 - 0.5 = 0.572 \text{ V}$$

$$v_o(\text{min}): i_2(\text{max}) = -3.93 = \frac{V_o(\text{min})}{1} \Rightarrow \underline{V_o(\text{min}) = -3.93 \text{ V}}$$

$$v_i(\text{min}) = v_o(\text{min}) + V_{TN} = -3.93 + 0.5$$

$$\underline{v_i(\text{min}) = -3.43 \text{ V}}$$

$$v_o(\text{max}) = 5 - V_{DS}(\text{sat}) = 5 - 0.572$$

$$v_o(\text{max}) = 4.43 \text{ V}$$

$$I_{D1}(\text{max}) = 3.93 + \frac{4.43}{1} = 8.36 \text{ mA}$$

$$I_{D1} = 8.36 = 12(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.33 \text{ V}$$

$$v_i(\text{max}) = v_o + V_{GS1} = 4.43 + 1.33 \Rightarrow \underline{v_i(\text{max}) = 5.76 \text{ V}}$$

**8.18**

(a) For  $v_o = -12 + 0.7 = -11.7 \text{ V}$ ,  $I_Q = \frac{11.3}{0.02} + 50 = 615 \text{ mA}$

$$I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_Q = \left(1 + \frac{2}{40}\right)(615) = 645.75 \text{ mA}$$

$$R = \frac{0 - 0.7 - (-12)}{0.6475} \Rightarrow R = 17.5 \Omega$$

For  $v_o = 12 - 0.7 = +11.3 \text{ V}$ ,  $i_L = \frac{11.3}{0.02} = 565 \text{ mA}$

$$i_{E1}(\text{max}) = I_Q + i_L = 615 + 565 \Rightarrow i_{E1}(\text{max}) = 1.18 \text{ A}$$

(b)  $\bar{P}_L = \frac{1}{2} \cdot \frac{V_o^2}{R_L} = \frac{1}{2} \cdot \frac{(11.3)^2}{20} = 3.19 \text{ W}$

$$P_S = I_Q(24) = (0.615)(24) = 14.76 \text{ W}$$

Define  $\eta = \frac{\bar{P}_L}{P_S} = \frac{3.19}{14.76} \times 100\% = 21.6\%$

**8.19**

- (a)  $V_O = -20 \text{ V}$ ,  $i_L = \frac{-20}{200} = -0.10 \text{ A}$   
 $I_Q = 0.10 + i_{E1}(\text{min}) = 0.10 + 0.02 = 0.12 \text{ A}$   
 $I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_Q = \left(1 + \frac{2}{50}\right)(0.12) = 0.1248 \text{ A}$   
 $R = \frac{0 - 0.7 - (-24)}{0.1248} = 187 \Omega$
- (b)  $P_{Q1} = I_Q V_{CE1} = (0.12)(24) = 2.88 \text{ W}$   
 $P = I_Q V_{CE2} + I_{REF}(24) = (0.12)(24) + (0.1248)(24) = 5.88 \text{ W}$
- (c)  $\overline{P}_L = \frac{1}{2} \cdot \frac{V_O^2}{R_L} = \frac{1}{2} \cdot \frac{(20)^2}{200} = 1 \text{ W}$   
 $\eta = \frac{1}{2.88 + 5.88} \times 100\% = 11.4\%$

**8.20**

- $I_{D1} = K_n (V_{GS} - V_{TN})^2 = 12(0 - (-1.8))^2$   
 $I_{D1} = 38.9 \text{ mA}$
- (a) For  $R_L = \infty$   
 $v_o(\text{max}) = 4.8 \text{ V}$   
 $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.8 \text{ V}$   
 $v_o(\text{min}) = -5 + 1.8 = -3.2 \text{ V}$   
 $v_i = v_o + 0.7 \Rightarrow -2.5 \leq v_i \leq 5.5 \text{ V}$
- (b) For  $R_L = 500 \Omega$   $v_o(\text{max}) = 4.8 \text{ V}$   
 $v_o < 0$ ,  $v_o(\text{min}) = -3.2 \text{ V}$   $I_2' = \frac{v_o}{R_L} = \frac{-3.2}{0.5} = -6.4 \text{ mA}$   
 For  $-2.5 \leq v_i \leq 5.5 \text{ V}$
- (c) For  $v_o = -2 \text{ V}$ ,  $I_2'(\text{max}) = -38.9 \text{ mA}$   
 $R_2(\text{min}) = \frac{-2}{-38.9} \Rightarrow R_L(\text{min}) = 51.4 \Omega$   
 $\overline{P}_L = \frac{1}{2} \cdot \frac{v_o^2}{R_L} = \frac{1}{2} \cdot \frac{(2)^2}{51.4} \Rightarrow \overline{P}_L = 38.9 \text{ mW}$   
 $P_L = 10(38.9) = 389 \text{ mW} \quad \% = \frac{38.9}{389} = 10\%$

**8.21**

$$\overline{P}_L = \frac{V_p^2}{R_L} = \frac{(V^+)^2}{R_L}$$

$$\overline{P}_S = \frac{1}{2} \cdot \frac{(V^+)^2}{R_L} + \frac{1}{2} \cdot \frac{(V^-)^2}{R_L}, \quad V^- = -V^+$$

$$\text{So } \overline{P}_S = \frac{(V^+)^2}{R_L}$$

$$\eta = \frac{\overline{P}_L}{\overline{P}_S} \Rightarrow \eta = 100\%$$

**8.22**

(a)

As maximum conversion efficiency

$$\eta = \frac{\pi}{4}, \quad \frac{V_p}{V_{CC}} = 0.785$$

$$\text{So } V_p(\text{max}) = (0.785)(5) \left( \frac{4}{\pi} \right)$$

$$\underline{V_p(\text{max}) = 5 \text{ V}}$$

$$V_p = \frac{2V_{CC}}{\pi} = \frac{2(5)}{\pi} = 3.183 \text{ V}$$

(b) Maximum power dissipation occurs when

$$\overline{P}_\theta(\text{max}) = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$2 = \frac{(5)^2}{\pi^2 R_L} \Rightarrow \underline{R_L = 1.27 \Omega}$$

(c)

**8.23**

$$\overline{P} = \frac{1}{2} \cdot \frac{V_p^2}{R_L}$$

(a)  $50 = \frac{1}{2} \cdot \frac{V_p^2}{24} \Rightarrow V_p = 49 \text{ V} \Rightarrow \underline{V^+ = 52 \text{ V}, V^- = -52 \text{ V}}$

(b)  $I_p = \frac{V_p}{R_L} = \frac{49}{24} = \underline{2.04 \text{ A}}$

(c)  $\eta = \frac{\pi}{4} \cdot \frac{V_p}{V_{CC}} = \frac{\pi}{4} \left( \frac{49}{52} \right)$

(c)  $\underline{\eta = 74.0\%}$

**8.24**



(a)

$$V_{DS} \geq V_{DS}(\text{sat}) = V_{GS} - V_{TN} = V_{GS}$$

$$V_{DS} = 10 - V_o(\text{max}) \text{ and } I_D = I_L = K_n (V_{GS})^2$$

$$\frac{V_o(\text{max})}{R_L} = K_n (V_{GS})^2$$

$$V_{GS} = \sqrt{\frac{V_o(\text{max})}{R_L \cdot K_n}}$$

$$\text{So } 10 - V_o(\text{max}) = \sqrt{\frac{V_o(\text{max})}{R_L \cdot K_n}} = \sqrt{(5)(0.4)}$$

$$[10 - V_o(\text{max})]^2 = \frac{V_o(\text{max})}{2}$$

$$100 - 20V_o(\text{max}) + V_o^2(\text{max}) = \frac{V_o(\text{max})}{2}$$

$$V_o^2(\text{max}) - 20.5V_o(\text{max}) + 100 = 0$$

$$V_o(\text{max}) = \frac{20.5 \pm \sqrt{(20.5)^2 - 4(100)}}{2} \Rightarrow \underline{V_o(\text{max}) = 8 \text{ V}}$$

$$i_L = \frac{8}{5} \Rightarrow i_L = 1.6 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} = \sqrt{\frac{1.6}{0.4}} = 2 \text{ V} \Rightarrow \underline{V_I = 10 \text{ V}}$$

b.

$$\overline{P_L} = \frac{1}{2} \cdot \frac{(8)^2}{5} = 6.4 \text{ mW}$$

$$\overline{P_S} = \frac{20(1.6)}{\pi} = 10.2 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{6.4}{10.2} \Rightarrow \underline{\eta = 62.7\%}$$

### 8.25

(b)  $v_I = v_{GSn} + v_o$

Also  $v_o = i_L R_L = i_{dn} R_L = KR_L (v_{GSn})^2$ , but  $v_{GSn} = v_I - v_o$

So  $v_o = KR_L (v_I - v_o)^2$

$$\frac{dv_o}{dv_I} = 2KR_L (v_I - v_o) \left( 1 - \frac{dv_o}{dv_I} \right)$$

$$\frac{dv_o}{dv_I} = \frac{2KR_L (v_I - v_o)}{1 + 2KR_L (v_I - v_o)}, \text{ also } v_I - v_o = \sqrt{\frac{v_o}{KR_L}}$$

Then  $\frac{dv_o}{dv_I} = A_v = \frac{2\sqrt{KR_L} \cdot \sqrt{v_o}}{1 + 2\sqrt{KR_L} \cdot \sqrt{v_o}}$

We find  $\sqrt{KR_L} = \sqrt{(4)(0.05)} = 0.4472$

(i) For  $v_o = 0$ ,  $A_v = 0$

(ii) For  $v_o = 1$  V,  $A_v = \frac{2(0.4472)(1)}{1 + 2(0.4472)(1)} = 0.472$

(iii) For  $v_o = 10$  V,  $A_v = \frac{2(0.4472)\sqrt{10}}{1 + 2(0.4472)\sqrt{10}} = 0.739$

### 8.26

(a)  $i_{Cn} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{i_{Cn}}{I_S}\right) = (0.026) \ln\left(\frac{10^{-3}}{2 \times 10^{-15}}\right) = 0.7004$  V

$$V_{BB} = 2V_{BE} = 1.40077$$
 V

$$P_Q = i_C \cdot v_{CE} = (1)(5) = 5$$
 mW

(b) For  $v_o = -3.5$  V,  $i_{Cp} \cong \frac{3.5}{1} = 3.5$  mA  $\cong i_L$

$$v_{EB} = (0.026) \ln\left(\frac{3.5 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.732957$$
 V

$$v_{BE} = 1.40077 - 0.732957 = 0.66781$$
 V

$$i_{Cn} = (2 \times 10^{-15}) \exp\left(\frac{0.66781}{0.026}\right) \Rightarrow i_{Cn} = 0.2857$$
 mA

Then  $i_{Cp} \cong 0.2857 + 3.5 = 3.7857$  mA

$$v_{EB} = (0.026) \ln\left(\frac{3.7857 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.734997$$
 V

$$v_{BE} = 1.40077 - 0.734997 = 0.66577$$
 V

$$i_{Cn} = (2 \times 10^{-15}) \exp\left(\frac{0.66577}{0.026}\right) \Rightarrow i_{Cn} = 0.2642$$
 mA

$$i_{Cp} = 3.5 + 0.2642 = 3.764$$
 mA

$$v_I = v_o - v_{EB} + \frac{V_{BB}}{2} = -3.5 - 0.735 + 0.7004 = -3.535$$
 V

For  $R_L$ :  $P_{RL} = i_L^2 R_L = (3.5)^2 (1) = 12.25$  mW

For  $Q_n$ :  $P_{Qn} = i_{Cn} v_{CEn} = (0.2642)[5 - (-3.5)] = 2.25$  mW

For  $Q_p$ :  $P_{Qp} = i_{Cp} v_{ECp} = (3.764)[-3.5 - (-5)] = 5.65$  mW

### 8.27

(a) (i)  $i_{Dn} = K_n (v_{GSn} - V_{TN})^2$

$$1 = 4(v_{GSn} - 1.5)^2 \Rightarrow v_{GSn} = \frac{V_{BB}}{2} = 2$$
 V,  $\Rightarrow V_{BB} = 4$  V

(ii)  $P = i_{Dn} \cdot v_{DSn} = (1)(12) = 12$  mW

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(b) (i)  $i_D = K[v_{DS}(sat)]^2 = \frac{v_o}{R_L} = \frac{V_{DD} - v_{DS}(sat)}{R_L}$   
 $KR_L[v_{DS}(sat)]^2 + v_{DS}(sat) - V_{DD} = 0$   
 Now  $KR_L = (4)(1) = 4$   
 We have  $4[v_{DS}(sat)]^2 + v_{DS}(sat) - 12 = 0 \Rightarrow v_{DS}(sat) = 1.612 \text{ V}$   
 $v_o(\text{max}) = 12 - 1.612 = 10.39 \text{ V}$   
 (ii)  $i_{Dn} = i_L = 10.39 \text{ mA}$   
 $i_{Dp} = 0$   
 $v_{GSn} = 3.112 \text{ V}$   
 $v_I = \frac{-V_{BB}}{2} + v_{GSn} + v_o = -2 + 3.112 + 10.39 = 11.5 \text{ V}$   
 (iii) For  $R_L$ :  $P_{RL} = i_L^2 R_L = (10.39)^2 (1) = 108 \text{ mW}$   
 For  $M_n$ :  $P_{Mn} = i_{Dn} v_{DSn} = (10.39)[12 - 10.39] = 16.7 \text{ mW}$   
 For  $M_p$ :  $P_{Mp} = i_{Dp} v_{SDp} = 0$

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**8.28**

a.

$$v_0 = 24 \text{ V} \Rightarrow i_L = \frac{24}{8} \Rightarrow \underline{i_L \approx i_N = 3 \text{ A}}$$

$$i_{Bn} = \frac{3}{41} \Rightarrow i_{Bn} = 73.2 \text{ mA}$$

For  $i_D = 25 \text{ mA} \Rightarrow i_{R1} = 25 + 73.2 = 98.2 \text{ mA}$

$$V_{BE} = V_T \ln\left(\frac{i_N}{I_S}\right) = (0.026) \ln\left(\frac{3}{6 \times 10^{-12}}\right)$$

$$= 0.7004 \text{ V}$$

Then  $98.2 = \frac{30 - (24 + 0.7)}{R_1} \Rightarrow R_1 = \frac{5.3}{98.2} \Rightarrow \underline{R_1 = 53.97 \ \Omega}$

$$V_D = (0.026) \ln\left(\frac{25 \times 10^{-3}}{6 \times 10^{-12}}\right) = 0.5759 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 2(0.5759) - 0.7004$$

$$= 0.4514 \text{ V}$$

$$i_p = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = (6 \times 10^{-12}) \exp\left(\frac{0.4514}{0.026}\right) \Rightarrow \underline{i_p = 0.208 \text{ mA}}$$

b. Neglecting base current

$$i_D \approx \frac{30 - 0.6}{R_1} = \frac{30 - 0.6}{53.97} \Rightarrow \underline{i_D \approx 545 \text{ mA}}$$

$$V_D = (0.026) \ln\left(\frac{0.545}{6 \times 10^{-12}}\right) = 0.656 \text{ V}$$

Approximation for  $i_D$  is okay.

Diodes and transistors matched  $\Rightarrow \underline{i_N = i_p = 545 \text{ mA}}$

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**8.29**

(a)

$$I_{D1} = K_1 (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = \sqrt{\frac{5}{5}} + 1 = 2 \text{ V}$$

$$I_{D3} = K_3 (V_{GS3} - V_{TN})^2$$

$$200 = K_3 (2 - 1)^2 \Rightarrow K_{n3} = K_{p4} = 200 \mu\text{A}/\text{V}^2$$

(b)

$$v_i + V_{SG4} + V_{GS3} - V_{GS1} = v_o$$

For  $v_o$  large,  $i_L = i_1 = K_{n1} (V_{GS1} - V_{TN})^2$

$$V_{GS1} = \sqrt{\frac{i_L}{K_{n1}}} + V_{TN} = \sqrt{\frac{v_o}{R_L K_{n1}}} + V_{TN}$$

So  $v_i + 2 + 2 - \left( \sqrt{\frac{v_o}{(0.5)(5)}} + 1 \right) = v_o$

$$v_i = v_o + \sqrt{\frac{v_o}{2.5}} - 3$$

$$\frac{dv_i}{dv_i} = 1 = \frac{dv_o}{dv_i} + \frac{1}{2} \cdot \frac{1}{\sqrt{2.5v_o}} \cdot \frac{dv_o}{dv_i}$$

$$1 = \frac{dv_o}{dv_i} \left[ 1 + \frac{1}{2\sqrt{2.5v_o}} \right]$$

For  $v_o = 5 \text{ V}$ :

$$1 = \frac{dv_o}{dv_i} \left[ 1 + \frac{1}{2\sqrt{2.5(5)}} \right] = \frac{dv_o}{dv_i} (1.1414) \Rightarrow \frac{dv_o}{dv_i} = 0.876$$

**8.30**

$$v_o = v_i + \frac{V_{BB}}{2} - V_{GS} \text{ and } V_{GS} = \sqrt{\frac{I_{Dn}}{K_n}} + V_{TN}$$

For  $v_o \approx 0$ ,  $I_{Dn} = I_{DQ} + i_L = I_{DQ} + \frac{v_o}{R_L}$

Then

$$v_o = v_i + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ} + (v_o/R_L)}{K_n}} \quad \text{or} \quad v_o = v_i + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{v_o}{I_{DQ}R_L}}$$

For  $v_o$  small,

$$v_o \cong v_i + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{1}{2} \cdot \frac{v_o}{I_{DQ}R_L}}$$

$$v_o \left[ 1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L} \right] = v_i + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}$$

Now

$$\frac{dv_o}{dv_i} = \frac{1}{\left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ} R_L}\right]} = 0.95$$

$$\text{So } \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ} R_L} = \frac{1}{0.95} - 1 = 0.0526$$

$$\text{For } R_L = 0.1 \text{ k}\Omega, \text{ then } \frac{1}{\sqrt{K_n I_{DQ}}} = 0.01052$$

$$\text{Or } \sqrt{K_n I_{DQ}} = 95.1$$

$$\text{We can write } g_m = 2\sqrt{K_n I_{DQ}} = 190 \text{ mA/V}$$

This is the required transconductance for the output transistor. This implies a very large transistor.

### 8.31

$$(a) R_{TH} = R_1 \parallel R_2 = 14 \parallel 10 = 5.833 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{10}{10 + 14}\right)(12) = 5 \text{ V}$$

$$I_{BQ} = \frac{5 - 0.7}{5.833 + (91)(0.02)} = 0.5619 \text{ mA}, \quad I_{CQ} = 50.57 \text{ mA}$$

$$(b) R_L = \frac{V_{CC}}{I_{CQ}} = \frac{12}{50.57} \Rightarrow R_L = 237 \Omega$$

$$(c) \bar{P}_L(\text{max}) = \frac{1}{2} \cdot \frac{(11)^2}{0.237} = 255 \text{ mW}$$

$$(d) \bar{P}_s = I_{CQ} V_{CC} = (50.57)(12) = 607 \text{ mW}$$

$$\eta = \frac{255}{607} \times 100\% = 42\%$$

**8.32**

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{15}{1} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$\overline{P}_L(\text{max}) = \frac{1}{2} \cdot \frac{V_{CC}^2}{R_L} = \frac{(15)^2}{2(1)} \Rightarrow \overline{P}_L(\text{max}) = 112.5 \text{ mW}$$

Let  $R_{TH} = 10 \text{ k}\Omega$

$$\begin{aligned} V_{TH} &= I_{BQ}R_{TH} + V_{BE} + (1 + \beta)I_{BQ}R_E \\ &= (0.15)(10) + 0.7 + (101)(0.15)(0.1) \end{aligned}$$

$$V_{TH} = 3.715 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} \cdot (10)(15)$$

$$\underline{R_1 = 40.4 \text{ k}\Omega}$$

$$\underline{R_2 = 13.3 \text{ k}\Omega}$$

**8.33**

(a)  $R_{TH} = R_1 \parallel R_2 = 2.3 \parallel 1.75 = 0.9938 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{1.75}{1.75 + 2.3} \right) (12) = 5.185 \text{ V}$$

$$I_{BQ} = \frac{5.185 - 0.7}{0.9938 + (41)(0.02)} = 2.473 \text{ mA}, \quad I_{CQ} = 98.91 \text{ mA}$$

(b) Want  $R'_L = \frac{V_{CC}}{I_{CQ}} = \frac{12}{0.09891} = 121.3 \Omega = a^2 R_L = a^2 (8) \Rightarrow a = 3.89$

(c)  $\overline{P}_L = \frac{1}{2} \cdot \frac{(9)^2}{(0.1213)} = 333.9 \text{ mW}$

$$\overline{P}_S = I_{CQ} V_{CC} = (98.91)(12) \Rightarrow 1.187 \text{ W}$$

(d)  $\eta = \frac{0.3339}{1.187} \times 100\% = 28.1\%$

**8.34**

a. Assuming the maximum power is being delivered, then

$$V'_o(\text{peak}) = 36 \text{ V} \Rightarrow V_o = \frac{36}{4} = 9 \text{ V} \Rightarrow V_{\text{rms}} = \frac{9}{\sqrt{2}} \Rightarrow \underline{V_{\text{rms}} = 6.36 \text{ V}}$$

b.  $V_o = \frac{36}{\sqrt{2}} \Rightarrow \underline{V_o = 25.5 \text{ V}}$

c. Secondary  $I_{\text{rms}} = \frac{\overline{P}_L}{V_{\text{rms}}} = \frac{2}{6.36} \Rightarrow \underline{I_{\text{rms}} = 0.314 \text{ A}}$

Primary  $I_P = \frac{0.314}{4} \Rightarrow \underline{I_P = 78.6 \text{ mA}}$

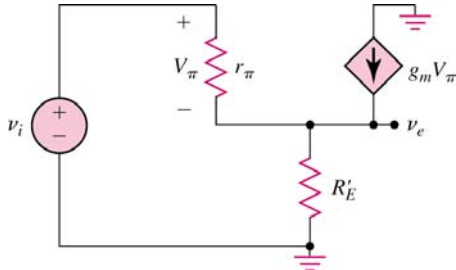
d.

$$\overline{P}_s = I_{CQ} \cdot V_{CC} = (0.15)(36) = 5.4 \text{ W}$$

$$\eta = \frac{2}{5.4} \Rightarrow \underline{\eta = 37\%}$$

**8.35**

a.



$$v_e = \left( \frac{V_\pi}{r_\pi} + g_m V_\pi \right) R'_E = V_\pi \left( \frac{1}{r_\pi} + g_m \right) R'_E$$

$$= V_\pi \left( \frac{1 + \beta}{r_\pi} \right) R'_E$$

$$v_i = V_\pi + v_e \Rightarrow V_\pi = v_i - v_e$$

$$v_e = (v_i - v_e) \left( \frac{1 + \beta}{r_\pi} \right) R'_E$$

$$\frac{v_e}{v_i} = \frac{\frac{1 + \beta}{r_\pi} \cdot R'_E}{1 + \frac{1 + \beta}{r_\pi} \cdot R'_E} = \frac{(1 + \beta) R'_E}{r_\pi + (1 + \beta) R'_E} = \frac{v_e}{v_i} \text{ where } R'_E = \left( \frac{n_1}{n_2} \right)^2 R_L$$

$$v_0 = \frac{v_e}{\left( \frac{n_1}{n_2} \right)} \text{ so } v_e = v_0 \left( \frac{n_1}{n_2} \right)$$

$$\text{so } \frac{v_0}{v_i} = \frac{1}{\left( \frac{n_1}{n_2} \right)} \cdot \frac{(1 + \beta) R'_E}{r_\pi + (1 + \beta) R'_E}$$

b.

$$\overline{P}_L = \frac{1}{2} \cdot I_p^2 R_L, a = \frac{n_1}{n_2}, I_{CQ} = \frac{I_p}{a} \text{ so } \overline{P}_L = \frac{1}{2} \cdot a^2 I_{CQ}^2 R_L$$

$$\overline{P}_s = I_{CQ} \cdot V_{CC}$$

For  $\eta = 50\%$ :

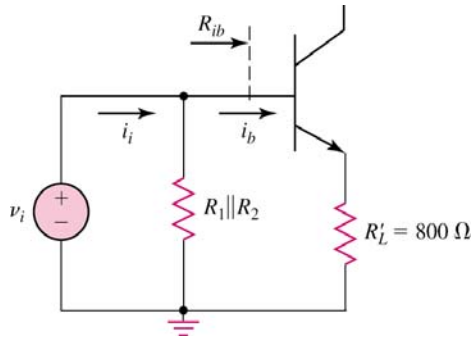
$$\frac{\overline{P}_L}{\overline{P}_s} = 0.5 = \frac{\frac{1}{2} \cdot a^2 I_{CQ}^2 R_L}{I_{CQ} \cdot V_{CC}} = \frac{a^2 I_{CQ} R_L}{2 V_{CC}} \text{ so } a^2 = \frac{V_{CC}}{I_{CQ} \cdot R_L} = \frac{V_{CC}}{(0.1)(50)} \Rightarrow a^2 = \underline{\underline{\frac{V_{CC}}{5}}}$$

c.

$$R_0 = \frac{r_\pi}{1 + \beta} = \frac{\beta V_T}{(1 + \beta) I_{CQ}} = \frac{49(0.026)}{(50)(0.1)} \Rightarrow \underline{\underline{R_0 = 0.255 \Omega}}$$

**8.36**

- a. With a 10:1 transformer ratio, we need a current gain of 8 through the transistor.



$$i_e = (1 + \beta)i_b \text{ and } i_b = \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) i_i \quad \text{so we need } \frac{i_e}{i_i} = 8 = (1 + \beta) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) \text{ where}$$

$$R_{ib} = r_\pi + (1 + \beta)R'_L \approx (1 + \beta)R'_L = (101)(0.8) = 80.8$$

$$\text{Then } 8 = (101) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 80.8} \right)$$

$$\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 80.8} = 0.0792 \text{ or } R_1 \parallel R_2 = 6.95 \text{ k}\Omega$$

$$\text{Set } \frac{2V_{CC}}{2I_{CQ}} = R'_L \Rightarrow I_{CQ} = \frac{V_{CC}}{R'_L} = \frac{12}{0.8} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}$$

$$\frac{1}{R_1}(6.95)(12) = (0.15)(6.95) + 0.7 \Rightarrow \underline{R_1 = 47.9 \text{ k}\Omega} \text{ then } \underline{R_2 = 8.13 \text{ k}\Omega}$$

- b.

$$I_e = 0.9I_{CQ} = 13.5 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 135 \text{ mA}$$

$$\overline{P_L} = \frac{1}{2}(0.135)^2(8) \Rightarrow \underline{\overline{P_L} = 72.9 \text{ mW}}$$

$$\overline{P_S} = V_{CC}I_{CQ} = (12)(15) \Rightarrow \underline{\overline{P_S} = 180 \text{ mW}}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \underline{\eta = 40.5\%}$$



**8.37**

a.

$$V_p = \sqrt{2R_L P_L}$$

$$V_p = \sqrt{2(8)(2)} = 5.66 \text{ V} = \text{peak output voltage}$$

$$I_p = \frac{V_p}{R_L} = \frac{5.66}{8} = 0.708 \text{ A} = \text{peak output current}$$

Set  $V_e = 0.9V_{CC} = aV_p$  to minimize distortion

$$\text{Then } a = \frac{(0.9)(18)}{5.66} \Rightarrow \underline{a = 2.86}$$

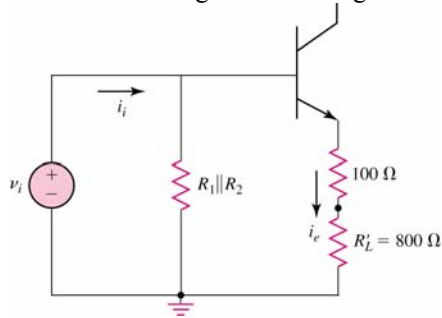
b.

$$\text{Now } I_{CQ} = \frac{1}{0.9} \left( \frac{I_p}{a} \right) = \frac{1}{0.9} \left( \frac{0.708}{2.86} \right) \Rightarrow I_{CQ} = 0.275 \text{ A}$$

$$\text{Then } P_Q = V_{CC} I_Q = (18)(0.275) \Rightarrow \underline{P_Q = 4.95 \text{ W}} \text{ Power rating of transistor}$$

**8.38**

a. Need a current gain of 8 through the transistor.



$$\frac{i_b}{i_i} = 8 = (1 + \beta) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) \text{ where } R_{ib} \approx (1 + \beta)(0.9) = 90.9 \text{ k}\Omega$$

$$\frac{8}{101} = \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 90.9} \right) = 0.0792 \text{ or } R_1 \parallel R_2 = 7.82 \text{ k}\Omega$$

$$\text{Set } \frac{2V_{CC}}{2I_{CQ}} = 0.9 \text{ k}\Omega \Rightarrow I_{CQ} = \frac{12}{0.9} = 13.3 \text{ mA}$$

$$I_{BQ} = \frac{13.3}{100} = 0.133 \text{ mA}$$

$$\text{Then } \frac{1}{R_1} (7.82)(12) = (0.133)(7.82) + 0.7 \Rightarrow \underline{R_1 = 53.9 \text{ k}\Omega} \text{ and } \underline{R_2 = 9.15 \text{ k}\Omega}$$

b.

$$I_e = (0.9)I_{CQ} = 12 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 120 \text{ mA}$$

$$\overline{P}_L = \frac{1}{2}(0.12)^2 (8) \Rightarrow \overline{P}_L = 57.6 \text{ mW}$$

$$\overline{P}_S = V_{CC}I_{CQ} = (12)(13.3) \Rightarrow \overline{P}_S = 159.6 \text{ mW}$$

$$\eta = \frac{\overline{P}_L}{\overline{P}_S} = \frac{57.6}{159.6} \Rightarrow \eta = 36.1\%$$

**8.39**

$$(a) V_{BB} = 2V_T \ln\left(\frac{I_{Bias}}{I_{SD}}\right) = 2(0.026)\ln\left(\frac{10^{-3}}{5 \times 10^{-16}}\right) = 1.473 \text{ V}$$

$$(b) I_{CQ} = I_{SQ} \exp\left(\frac{1.473/2}{0.026}\right) \Rightarrow I_{CQ} = 14 \text{ mA}$$

**8.40**

$$(a) V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_{SQ}}\right) = (0.026)\ln\left(\frac{4 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.73643 = V_{BE_n} = V_{EB_p} = V_D$$

$$I_{Bias} = I_{SD} \exp\left(\frac{V_D}{V_T}\right) = (4 \times 10^{-16})\exp\left(\frac{0.73643}{0.026}\right) \Rightarrow I_{Bias} = 0.8 \text{ mA}$$

$$(b) V_{BB} = 2V_D = 1.473 \text{ V}$$

$$(c) v_{I_1} = -V_{EB_p} = -0.7364 \text{ V}$$

**8.41**

$$(a) V_{D1} = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{10^{-16}}\right) = 0.76025 \text{ V}$$

$$V_{D2} = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{4 \times 10^{-16}}\right) = 0.72421 \text{ V}$$

$$V_{BB} = V_{D1} + V_{D2} = 1.48446 \text{ V}$$

$$(b) i_{C_n} = i_{C_p} = I_{SQ_n} \exp\left(\frac{v_{BE_n}}{V_T}\right) = I_{SQ_p} \exp\left(\frac{v_{EB_p}}{V_T}\right)$$

$$\frac{I_{SQ_p}}{I_{SQ_n}} = \frac{\exp\left(\frac{v_{BE_n}}{V_T}\right)}{\exp\left(\frac{v_{EB_p}}{V_T}\right)} = \frac{\exp\left(\frac{v_{BE_n}}{V_T}\right)}{\exp\left(\frac{V_{BB} - v_{BE_n}}{V_T}\right)} = \exp\left(\frac{2v_{BE_n} - V_{BB}}{V_T}\right)$$

$$\frac{2v_{BE_n} - V_{BB}}{V_T} = \ln\left(\frac{I_{SQ_p}}{I_{SQ_n}}\right)$$

$$v_{BE_n} = \frac{1}{2} \left[ V_{BB} + V_T \ln\left(\frac{I_{SQ_p}}{I_{SQ_n}}\right) \right] = \frac{1}{2} \left[ 1.48446 + (0.026)\ln\left(\frac{1.6 \times 10^{-15}}{8 \times 10^{-16}}\right) \right] = 0.75124 \text{ V}$$

$$v_{EBp} = V_{BB} - v_{BE_n} = 0.73322 \text{ V}$$

$$(c) \quad I_{CQ} = I_{SQ_n} \exp\left(\frac{v_{BE_n}}{V_T}\right) = (8 \times 10^{-16}) \exp\left(\frac{0.75124}{0.026}\right)$$

$$\Rightarrow i_{C_n} = i_{C_p} = 2.828 \text{ mA}$$

$$(d) \quad v_I = -v_{EBp} = -0.73322 \text{ V}$$

**8.42**

a. All transistors are matched.

$$3 \text{ mA} = i_{E1} + i_{B3} = \left(\frac{1+\beta}{\beta}\right) i_C + \frac{i_C}{\beta}$$

$$3 = \left(\frac{61}{60} + \frac{1}{60}\right) i_C \Rightarrow \underline{i_C = 2.90 \text{ mA}}$$

b.

For  $v_o = 6 \text{ V}$ , let  $\underline{R_L = 200 \Omega}$

$$i_o = \frac{6}{200} = 0.03 \text{ A} = 30 \text{ mA} \cong i_{E3}$$

$$i_{B3} = \frac{30}{61} = 0.492 \text{ mA}$$

$$i_{E1} = 3 - 0.492 = 2.508 \text{ mA}$$

$$i_{B1} = \frac{2.508}{61} \Rightarrow \underline{i_{B1} = 41.11 \mu\text{A}}$$

$$i_{E2} \cong 3 \text{ mA} \Rightarrow i_{B2} = \frac{3}{61} \Rightarrow 49.18 \mu\text{A}$$

$$i_I = i_{B2} - i_{B1} = 49.18 - 41.11 \Rightarrow \underline{i_I = 8.07 \mu\text{A}}$$

Current gain

$$A_i = \frac{30 \times 10^{-3}}{8.07 \times 10^{-6}} \Rightarrow \underline{A_i = 3.72 \times 10^3}$$

$$V_{BE3} = V_T \ln\left(\frac{i_{E3}}{I_S}\right) = (0.026) \ln\left(\frac{30 \times 10^{-3}}{5 \times 10^{-13}}\right)$$

$$V_{BE3} = 0.6453 \text{ V}$$

$$V_{EB1} = V_T \ln\left(\frac{i_{E1}}{I_S}\right) = (0.026) \ln\left(\frac{2.508 \times 10^{-3}}{5 \times 10^{-13}}\right)$$

$$V_{EB1} = 0.5807 \text{ V}$$

$$v_I = v_o + V_{BE3} - V_{EB1} = 6 + 0.6453 - 0.5807$$

$$v_I = 6.0646 \text{ V}$$

Voltage gain

$$A_v = \frac{v_o}{v_I} = \frac{6}{6.0646} \Rightarrow \underline{A_v = 0.989}$$

**8.43**

a. For  $i_0 = 1 \text{ A}$ ,  $I_{B3} \cong \frac{1}{50} \Rightarrow 20 \text{ mA}$

$$\frac{10 - V_{EB1}}{R_1} = 2 \left[ \frac{10 - (v_{0,\max} + V_{BE3})}{R_1} - 20 \right]$$

We can then write

If, for simplicity, we assume  $V_{EB1} = V_{BE3} = 0.7 \text{ V}$ , then  $\frac{10 - V_{BE}}{R_1} = \frac{2v_{0,\max}}{R_1} + 40$

If we assume  $v_{0,\max} = 4 \text{ V}$ , then  $\frac{9.3}{R_1} = \frac{2(4)}{R_1} + 40$  which yields  $R_1 = R_2 = 32.5 \Omega$

b. For  $v_i = 0$ ,  $I_{E1} = \frac{9.3}{32.5} \Rightarrow I_{E1} = 0.286 \text{ A} = I_{E2}$

Since  $I_{S3,4} = 10I_{S1,2}$ , then  $I_{E3} = I_{E4} = 2.86 \text{ A}$

c.

We can write

$$R_0 = \frac{1}{2} \left\{ \frac{r_{\pi 3} + R_1 \parallel \frac{r_{\pi 1}}{1 + \beta_1}}{1 + \beta_3} \right\}$$

Now  $r_{\pi 3} = \frac{\beta_3 V_T}{I_{C3}} = \frac{(50)(0.026)}{2.86} = 0.4545 \Omega$

$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C1}} = \frac{(120)(0.026)}{0.286} = 10.91 \Omega$$

So

$$R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 32.5 \parallel \frac{10.91}{121}}{51} \right\}$$

$$32.5 \parallel \frac{10.91}{121} = 32.5 \parallel 0.0902 = 0.0900$$

Then  $R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 0.0900}{51} \right\}$  or  $R_0 = 0.00534 \Omega$

**8.44**

$$R_i = \frac{1}{2} \left\{ r_{\pi 1} + (1 + \beta) \left[ R_1 \parallel (r_{\pi 3} + (1 + \beta) 2R_L) \right] \right\}$$

$$i_{C1} \approx 7.2 \text{ mA and } i_{C3} \approx 7.2 \text{ mA}$$

$$\text{Then } r_{\pi} = \frac{(60)(0.026)}{7.2} = 0.217 \text{ k}\Omega$$

$$\begin{aligned} \text{So } R_i &= \frac{1}{2} \left\{ 0.217 + (61) \left[ 2 \parallel (0.217 + (61)(0.2)) \right] \right\} \\ &= \frac{1}{2} \left\{ 0.217 + 61 \left[ 2 \parallel 12.4 \right] \right\} \text{ or } R_i = 52.6 \text{ k}\Omega \end{aligned}$$

**8.45**

(b) For  $M_{n3}$ ;  $v_{GSn3} = \sqrt{\frac{I_{DQ}}{K_{n3}}} + V_{TN} = \sqrt{\frac{5}{5}} + 1 = 2 \text{ V}$

$$v_{SGp1} = v_{GSn3} = 2 \text{ V}$$

$$I_{DQ1} = 2(2-1)^2 = 2 \text{ mA}$$

$$R_1 = R_2 = \frac{10-2}{2} = 4 \text{ k}\Omega$$

(c)  $I_{DQ1} = I_{DQ2} = 2 \text{ mA}$

(d) For  $v_o = 3.5 \text{ V}$ , Assume  $M_{p4}$  cutoff, so  $I_{DQ4} = 0$

$$I_{Dn3} = i_o = \frac{v_o}{R_L} = \frac{3.5}{0.15} = 23.33 \text{ mA}$$

$$v_{GSn3} = \sqrt{\frac{23.33}{5}} + 1 = 3.160 \text{ V}$$

$$I_{Dp1} = I_{R1} = \frac{10 - (v_o + v_{GSn3})}{R_1} = \frac{10 - (3.16 + 3.5)}{4} = 0.835 \text{ mA}$$

$$v_{SGp1} = \sqrt{\frac{0.835}{2}} + 1 = 1.646 \text{ V}$$

$$v_I = v_o + v_{GSn3} - v_{SGp1} = 3.5 + 3.160 - 1.646 = 5.014 \text{ V}$$

$$v_I = v_{GSn2} + K_{n2} R_2 (v_{GSn2} - V_{TN})^2 + V^-$$

$$15.014 = v_{GSn2} + 8(v_{GSn2}^2 - 2v_{GSn2} + 1)$$

or  $8v_{GSn2}^2 - 15v_{GSn2} - 7.014 = 0 \Rightarrow v_{GSn2} = 2.2625 \text{ V}$

$$I_{Dn2} = 2(2.2625-1)^2 = 3.188 \text{ mA}$$

$$v_{G4} = v_I - v_{GSn2} = 5.014 - 2.2625 = 2.75 \text{ V}$$

$$v_{SGp4} = v_o - v_{G4} = 3.5 - 2.75 = 0.75 \text{ V, } \Rightarrow M_{p4} \text{ cutoff}$$

$$\bar{P}_L = \frac{v_o^2}{R_L} = \frac{(3.5)^2}{0.15} = 81.7 \text{ mW}$$

**8.46**

For  $v_I = -1.5 \text{ V}$  and  $v_O = 0$ ,  $\Rightarrow v_{SG2} = 1.5 \text{ V} = v_{GS1}$

$$i_{D1} = i_{D2} = 0.5 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 20.4$$

$$0.5 = \left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_2 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 51.0$$

$$v_{SG4} = 1.5 \text{ V}, \quad 0.2 = \left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_4 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 20.4$$

$$0.2 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_3 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 8.16$$

**8.47**

For  $v_0 = 0$

$$I_Q = I_{C3} + I_{C2} + I_{E1}$$

$$I_{B3} = I_{E2} = \left(\frac{1 + \beta_n}{\beta_n}\right) I_{C2} = \frac{I_{C3}}{\beta_n}$$

$$I_{C3} = (1 + \beta_n) I_{C2}$$

$$I_{B2} = I_{C1} = \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E1} = \frac{I_{C2}}{\beta_n}$$

$$I_{C2} = \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E1}$$

$$I_{C3} = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E1}$$

$$I_Q = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E1} + \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E1} + I_{E1}$$

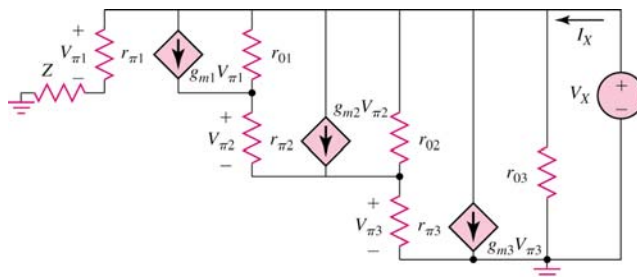
$$= (51)(50)\left(\frac{10}{11}\right) I_{E1} + (50)\left(\frac{10}{11}\right) I_{E1} + I_{E1}$$

$$I_Q = 2318.18 I_{E1} + 45.45 I_{E1} + I_{E1}$$

$$I_{E1} = 1.692 \mu\text{A} \Rightarrow \underline{I_{C1} = 1.534 \mu\text{A}}$$

$$I_{C2} = (50)\left(\frac{10}{11}\right)(1.692) \Rightarrow I_{C2} = 76.9 \mu\text{A}$$

$$I_{C3} = (51)(50)\left(\frac{10}{11}\right)(1.692) \Rightarrow \underline{I_{C3} = 3.92 \text{ mA}}$$



Because of  $r_{\pi 1}$  and  $Z$ , neglect effect of  $r_0$ . Then neglecting  $r_{01}$ ,  $r_{02}$  and  $r_{03}$ , we find

$$I_X = g_{m3}V_{\pi 3} + g_{m2}V_{\pi 2} + g_{m1}V_{\pi 1} + \frac{V_X}{r_{\pi 1} + Z}$$

Now

$$V_{\pi 1} = \left( \frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) V_X, \quad V_{\pi 2} \cong g_{m1}V_{\pi 1}r_{\pi 2}$$

and

$$V_{\pi 3} = (g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})r_{\pi 3} \\ = [g_{m1}V_{\pi 1} + g_{m2}(g_{m1}V_{\pi 1}r_{\pi 2})]r_{\pi 3}$$

$$V_{\pi 3} = \left( \frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) [g_{m1} + g_{m1}g_{m2}r_{\pi 2}]r_{\pi 3} \cdot V_X$$

$$V_{\pi 3} = \frac{(\beta_1 + \beta_1\beta_2)r_{\pi 3}}{r_{\pi 1} + Z} \cdot V_X$$

$$\text{and } V_{\pi 2} = g_{m1} \left( \frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) r_{\pi 2} V_X = \left( \frac{\beta_1 r_{\pi 2}}{r_{\pi 1} + Z} \right) V_X$$

$$\text{Then } I_X = \frac{(\beta_1 + \beta_1\beta_2)\beta_3}{r_{\pi 1} + Z} \cdot V_X + \frac{\beta_1\beta_2}{r_{\pi 1} + Z} \cdot V_X + \frac{\beta_1}{r_{\pi 1} + Z} \cdot V_X + \frac{V_X}{r_{\pi 1} + Z}$$

Then

$$R_0 = \frac{V_X}{I_X} = \frac{r_{\pi 1} + Z}{1 + \beta_1 + \beta_1\beta_2 + (\beta_1 + \beta_1\beta_2)\beta_3}$$

$$r_{\pi 1} = \frac{(10)(0.026)}{1.534} = 0.169 \text{ M}\Omega$$

$$Z = 25 \text{ k}\Omega$$

Then

$$R_0 = \frac{169 + 25}{1 + (10) + (10)(50) + [10 + (10)(50)](50)}$$

$$R_0 = \frac{194}{26,011} = 0.00746 \text{ k}\Omega \text{ or } \underline{R_o = 7.46 \Omega}$$

### 8.48

a Neglect base currents.

$$V_{BB} = 2V_D = 2V_T \ln \left( \frac{I_{\text{Bias}}}{I_S} \right) \\ = 2(0.026) \ln \left( \frac{5 \times 10^{-3}}{10^{-13}} \right) \Rightarrow \underline{V_{BB} = 1.281 \text{ V}}$$

$$V_{BE1} + V_{EB3} = V_{BB}$$

$$I_{E1} = I_{E3} + I_{C2}$$

$$I_{B2} = I_{C3} = \left( \frac{\beta_P}{1 + \beta_P} \right) I_{E3}$$

$$I_{C2} = \beta_n I_{B2} = \beta_n \left( \frac{\beta_P}{1 + \beta_P} \right) I_{E3}$$

$$I_{E1} = I_{E3} + \beta_n \left( \frac{\beta_P}{1 + \beta_P} \right) I_{E3}$$

$$I_{E1} = I_{E3} \left[ 1 + \beta_n \left( \frac{\beta_P}{1 + \beta_P} \right) \right]$$

$$\left( \frac{1 + \beta_n}{\beta_n} \right) I_{C1} = \left( \frac{1 + \beta_P}{\beta_P} \right) I_{C3} \left[ 1 + \beta_n \left( \frac{\beta_P}{1 + \beta_P} \right) \right]$$

$$V_{BE1} = V_T \ln \left[ \frac{I_{C1}}{I_S} \right], \quad V_{EB3} = V_T \ln \left[ \frac{I_{C3}}{I_S} \right]$$

$$(1.01) I_{C1} = \left( \frac{21}{20} \right) I_{C3} \left[ 1 + (100) \left( \frac{20}{21} \right) \right]$$

$$= I_{C3} \left[ \frac{21}{20} + 100 \right] = 101.05 I_{C3}$$

$$I_{C1} = 100.05 I_{C3}$$

$$V_T \ln \left( \frac{100.05 I_{C3}}{I_S} \right) + V_T \ln \left( \frac{I_{C3}}{I_S} \right) = V_{BB}$$

$$V_T \ln \left( \frac{100.05 I_{C3}^2}{I_S^2} \right) = V_{BB}$$

$$\frac{100.05 I_{C3}^2}{I_S^2} = \exp \left( \frac{V_{BB}}{V_T} \right)$$

$$I_{C3} = \frac{I_S}{\sqrt{100.05}} \sqrt{\exp \left( \frac{V_{BB}}{V_T} \right)} = \underline{0.4995 \text{ mA} = I_{C3}}$$

Then  $I_{E3} = 0.5245 \text{ mA}$

Now  $I_{C1} = 100.05 I_{C3} = \underline{49.97 \text{ mA} = I_{C1}}$

$$I_{C2} = (100) \left( \frac{20}{21} \right) (0.5245) = \underline{49.95 \text{ mA} = I_{C2}}$$

$$V_{BE1} = V_T \ln \left( \frac{I_{C1}}{I_S} \right) = 0.026 \ln \left( \frac{49.97 \times 10^{-3}}{10^{-13}} \right)$$

$$= 0.70037$$

$$V_{EB3} = V_T \ln \left( \frac{I_{C3}}{I_S} \right) = 0.026 \ln \left( \frac{0.4995 \times 10^{-3}}{10^{-13}} \right)$$

$$= 0.58062$$

Note:  $V_{BE1} + V_{EB3} = 0.70037 + 0.58062 = 1.28099$   
 $= V_{BB}$



b.

$$v_0 = 10 \text{ V} \Rightarrow i_{E1} \approx \frac{10}{100} = 0.10 \text{ A} = i_{C1}$$

$$i_{B1} = \frac{100}{100} = 1 \text{ mA}$$

$$V_{BB} = 2(0.026) \ln\left(\frac{4 \times 10^{-3}}{10^{-13}}\right) = 1.2694 \text{ V}$$

$$V_{BE1} = (0.026) \ln\left(\frac{0.1}{10^{-13}}\right) = 0.7184$$

$$V_{EB3} = 1.2694 - 0.7184 = 0.55099 \text{ V}$$

$$I_{C3} = 10^{-13} \exp\left(\frac{0.55099}{0.026}\right) = 0.1598 \text{ mA}$$

$$\overline{P}_L = \frac{V_0^2}{R_L} = \frac{(10)^2}{100} \Rightarrow \overline{P}_L = 1 \text{ W}$$

$$P_{Q1} = i_{C1} \cdot v_{CE1} = (0.1)(12 - 10) \Rightarrow \underline{P_{Q1}} = 0.2 \text{ W}$$

$$P_{Q3} = i_{C3} \cdot v_{EC3} = (0.1598)(10 - [0.7 - 12]) \Rightarrow \underline{P_{Q3}} = 3.40 \text{ mW}$$

$$i_{C2} = (100)(i_{C3}) = (100)(0.1598) = 15.98 \text{ mA}$$

$$P_{Q2} = i_{C2} \cdot v_{CE2} = (15.98)(10 - [-12]) \Rightarrow \underline{P_{Q2}} = 0.352 \text{ W}$$

**8.49**

a.

$$V_{BB} = 3(0.026) \ln \left( \frac{10 \times 10^{-3}}{2 \times 10^{-12}} \right) \Rightarrow \underline{V_{BB} = 1.74195 \text{ V}}$$

$$V_{BE1} + V_{BE2} + V_{EB3} = V_{BB}$$

$$I_{C1} \approx \frac{I_{C2}}{\beta_n}, I_{C3} \approx \frac{I_{C2}}{\beta_n^2}$$

$$V_T \ln \left( \frac{I_{C1}}{I_S} \right) + V_T \ln \left( \frac{I_{C2}}{I_S} \right) + V_T \ln \left( \frac{I_{C3}}{I_S} \right) = V_{BB}$$

$$V_T \ln \left[ \frac{I_{C2}^3}{\beta_n^3 I_S^3} \right] = V_{BB}$$

$$I_{C2} = \beta_n I_S \sqrt[3]{\exp \left( \frac{V_{BB}}{V_T} \right)}$$

$$= (20)(20 \times 10^{-12}) \sqrt[3]{\exp \left( \frac{1.74195}{0.026} \right)}$$

$$I_{C2} = 0.20 \text{ A}, I_{C1} \approx 10 \text{ mA}, I_{C3} \approx 0.5 \text{ mA}$$

$$V_{BE1} = (0.026) \ln \left( \frac{10 \times 10^{-3}}{2 \times 10^{-12}} \right) \Rightarrow \underline{V_{BE1} = 0.58065 \text{ V}}$$

$$V_{BE2} = (0.026) \ln \left( \frac{0.2}{2 \times 10^{-12}} \right) \Rightarrow \underline{V_{BE2} = 0.6585 \text{ V}}$$

$$V_{EB3} = (0.026) \ln \left( \frac{0.5 \times 10^{-3}}{2 \times 10^{-12}} \right) \Rightarrow \underline{V_{EB3} = 0.50276 \text{ V}}$$

b.

$$\overline{P_L} = 10 \text{ W} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{1}{2} \cdot \frac{V_0^2}{20} \Rightarrow V_0(\text{max}) = 20 \text{ V}$$

For  $v_0(\text{max})$ :

$$P_L = \frac{v_0^2}{R_L} = \frac{(20)^2}{20} \Rightarrow \underline{P_L = 20 \text{ W}}$$

$$i_0(\text{max}) = -\frac{20}{20} = -1 \text{ A}$$

$$i_{C5} + i_{C4} + i_{E3} = -i_o(\text{max}) = 1 \text{ A}$$

$$i_{C5} + \frac{i_{C5}}{\beta_n} \cdot \left( \frac{\beta_n}{1 + \beta_n} \right) + \frac{i_{C4}}{\beta_n} \left( \frac{1 + \beta_p}{\beta_p} \right) = 1$$

$$i_{C5} + \frac{i_{C5}}{\beta_n} \left( \frac{\beta_n}{1 + \beta_n} \right) + \frac{i_{C5}}{\beta_n} \left( \frac{\beta_n}{1 + \beta_n} \right) \left[ \frac{1}{\beta_n} \left( \frac{1 + \beta_p}{\beta_p} \right) \right] = 1$$

$$i_{C5} \left[ 1 + \frac{1}{20} \left( \frac{20}{21} \right) \right] + \left( \frac{1}{21} \right) \left[ \frac{1}{20} \left( \frac{6}{5} \right) \right] = 1$$

---

$$i_{C5} (1.05048) = 1 \quad \underline{i_{C5} = 0.952 \text{ A}}$$

$$\underline{i_{C4} = 0.0453 \text{ A}}$$

$$i_{E3} = 0.00272 \text{ A}$$

$$i_{C3} = 0.00272 \left( \frac{5}{6} \right)$$

$$\underline{= 0.002267 \text{ A}}$$

$$V_{EB3} = (0.026) \ln \left( \frac{2.267 \times 10^{-3}}{2 \times 10^{-12}} \right) = 0.54206 \text{ V}$$

$$V_{BE1} + V_{BE2} = 1.74195 - 0.54206 = 1.19989$$

$$V_T \ln \left( \frac{I_{C2}}{\beta_n I_S} \right) + V_T \ln \left( \frac{I_{C2}}{I_S} \right) = 1.19989$$

$$i_{C2} = \sqrt{\beta_n} \cdot I_S \sqrt{\exp \left( \frac{1.19989}{0.026} \right)}$$

$$= \sqrt{20} (18.83) \text{ mA}$$

$$\underline{i_{C2} = 93.9 \text{ mA}}$$

$$i_{C1} = \frac{i_{C2}}{\beta_n} \left( \frac{\beta_n}{1 + \beta_n} \right) = \frac{93.9}{21} = \underline{4.47 \text{ mA}}$$

$$P_{Q2} = I_{C2} (24 - (-20)) = (0.0939)(44) = \underline{4.13 \text{ W}}$$

$$P_{Q5} = (0.952)(-10 - (-24)) = \underline{13.3 \text{ W}}$$

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## Chapter 9

### 9.1

(a)

$$v_o = A_d(v_2 - v_1)$$
$$1 = A_d(10^{-3} - (-10^{-3})) \Rightarrow A_d = 500$$

(b)

$$1 = 500(v_2 - 10^{-3}) = 1 + 0.5 = 500v_2$$
$$\underline{v_2 = 3 \text{ mV}}$$

(c)

$$5 = 500(1 - v_1) \Rightarrow 500v_1 = 495$$
$$\underline{v_1 = 0.990 \text{ V}}$$

(d)  $v_o = 0$

(e)

$$-3 = 500(v_2 - (-0.5))$$
$$-250 - 3 = 500v_2$$
$$\underline{v_2 = -0.506 \text{ V}}$$

### 9.2

(a)  $v_2 = \frac{v_o}{A_{od}} = \frac{-2}{10^4} = -2 \times 10^{-4} \text{ V}$

$$v_2 = \left( \frac{1}{1 + 2000} \right) \cdot v_i$$

$$-2 \times 10^{-4} = \left( \frac{1}{2001} \right) \cdot v_i \Rightarrow v_i = -0.4002 \text{ V}$$

(b)  $v_2 = \left( \frac{1}{1 + 2000} \right) \cdot v_i$

$$v_2 = \left( \frac{1}{2001} \right) (2) = 0.9995 \times 10^{-3} \text{ V}$$

$$v_o = 1 = A_{od} v_2 = A_{od} (0.9995 \times 10^{-3}) \Rightarrow A_{od} = 1000.5$$

### 9.3

(a)  $v_o = A_{od}(v_2 - v_1) = (5 \times 10^3)(2.0000 - 2.0010) = -5 \text{ V}$

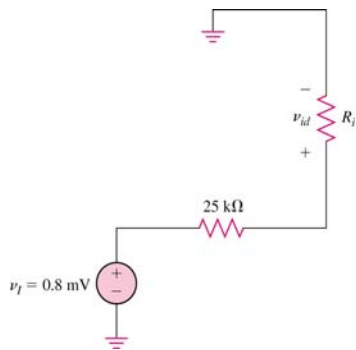
(b)  $v_o = A_{od}(v_2 - v_1)$

$$-3.000 = (2 \times 10^4)(3.0025 - v_1) \Rightarrow v_1 = 3.00265 \text{ V}$$

(c)  $v_o = A_{od}(v_2 - v_1)$

$$1.80 = A_{od}(0.01 - (-0.01)) \times 10^{-3} \Rightarrow A_{od} = 9 \times 10^4$$

9.4



$$v_{id} = \left( \frac{R_i}{R_i + 25} \right) v_I$$

$$0.790 = \left( \frac{R_i}{R_i + 25} \right) (0.80)$$

$$0.9875(R_i + 25) = R_i$$

$$24.6875 = 0.0125R_i$$

$$\underline{R_i = 1975 \text{ K}}$$

9.5

(a)  $A_v = \frac{-R_2}{R_1} = \frac{-200}{20} = -10$

(b)  $A_v = \frac{-120}{40} = -3$

(c)  $A_v = \frac{-40}{40} = -1$

9.6

$$\left. \begin{array}{l} A_v = -\frac{200}{20} = -10 \\ \text{and} \\ R_i = 20 \text{ k}\Omega \end{array} \right\} \text{for each case}$$

9.7

a.

$$A_v = -\frac{100}{10} = -10$$

$$R_i = R_1 = 10 \text{ k}\Omega$$

b.

$$A_v = -\frac{100 \parallel 100}{10} = -5$$

$$R_i = R_1 = 10 \text{ k}\Omega$$

c.

$$A_v = -\frac{100}{10+10} = -5$$

$$R_i = 10+10 = 20 \text{ K}$$

9.8

$$(a) |i_2| = \left| \frac{0-v_o}{R_2} \right| \Rightarrow R_2 = \frac{4}{20 \times 10^{-6}} \Rightarrow R_2 = 200 \text{ k}\Omega$$

$$A_v = \frac{-R_2}{R_1} \Rightarrow -12 = \frac{-200}{R_1} \Rightarrow R_1 = 16.67 \text{ k}\Omega$$

$$(b) i_2 = \frac{0-v_o}{R_2} = \frac{0-1.5}{200 \times 10^3} \Rightarrow i_1 = i_2 = -7.5 \mu\text{A}$$

$$v_I = \frac{v_o}{A_v} = \frac{+1.5}{-12} \Rightarrow v_I = -0.125 \text{ V}$$

9.9

$$A_v = -\frac{R_2}{R_1}$$

$$(a) A_v = -10$$

$$(b) A_v = -1$$

$$(c) A_v = -0.20$$

$$(d) A_v = -10$$

$$(e) A_v = -2$$

$$(f) A_v = -1$$

9.10

$$(a) -3 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 66.67 \text{ k}\Omega$$

$$(b) -8 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 25 \text{ k}\Omega$$

$$(c) -20 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 10 \text{ k}\Omega$$

$$(d) -0.5 = \frac{-R_2}{R_1} = \frac{-R_2}{200} \Rightarrow R_2 = 100 \text{ k}\Omega, R_1 = 200 \text{ k}\Omega$$

9.11

$$(a) |i_1| = \frac{v_I}{R_1} \Rightarrow R_1 = \frac{0.25}{50 \times 10^{-6}} \Rightarrow R_1 = 5 \text{ k}\Omega$$

$$A_v = \frac{-R_2}{R_1} \Rightarrow -6.5 = \frac{-R_2}{5} \Rightarrow R_2 = 32.5 \text{ k}\Omega$$

---

$$(b) \ v_I = \frac{v_o}{A_v} = \frac{-4}{-6.5} = 0.6154 \text{ V}$$

$$i_1 = i_2 = \frac{4}{32.5} = 0.123 \text{ mA}$$

---

**9.12**

$$(a) \ A_v = \frac{-R_2}{R_1}$$

$$-20 = \frac{-R_2}{25} \Rightarrow R_1 = 25 \text{ k}\Omega, \ R_2 = 500 \text{ k}\Omega$$

$$(b) \ -20 = \frac{-1000}{R_1} \Rightarrow R_1 = 50 \text{ k}\Omega, \ R_2 = 1 \text{ M}\Omega$$

$$(c) \ \text{For (a), } i_1 = \frac{v_I}{R_1} = \frac{-0.2}{25} \Rightarrow i_1 = -8 \mu\text{A}$$

$$\text{For (b), } i_1 = \frac{v_I}{R_1} = \frac{-0.2}{50} \Rightarrow i_1 = -4 \mu\text{A}$$

---

**9.13**

a.

$$A_v = \frac{R_2}{R_1} \Rightarrow \frac{1.05R_2}{0.95R_1} = 1.105 \left( \frac{R_2}{R_1} \right)$$

$$\frac{0.95R_2}{1.05R_1} = 0.905 \left( \frac{R_2}{R_1} \right)$$

Deviation in gain is +10.5% and -9.5%

b.

$$A_v \Rightarrow \frac{1.01R_2}{0.99R_1} = 1.02 \left( \frac{R_2}{R_1} \right) \Rightarrow \frac{0.99R_2}{1.01R_1} = 0.98 \left( \frac{R_2}{R_1} \right)$$

Deviation in gain =  $\pm 2\%$

---

**9.14**

$$(a) \ (i) \ v_o = \frac{-R_2}{R_1} \cdot v_I = \frac{-15}{1} (-0.20) = 3 \text{ V}$$

$$(ii) \ i_2 = \frac{0 - v_o}{R_2} = \frac{-3}{15} = -0.20 \text{ mA}$$

$$i_L = \frac{v_o}{R_L} = \frac{3}{4} = 0.75 \text{ mA}$$

$$i_o + i_2 = i_L \Rightarrow i_o = 0.75 - (-0.20) = 0.95 \text{ mA}$$

(b) (i)  $v_o = \frac{-15}{1}(0.05) = -0.75 \text{ V}$   
(ii)  $i_2 = \frac{0 - (-0.75)}{15} = 0.05 \text{ mA}$   
 $i_L = \frac{-0.75}{4} = -0.1875 \text{ mA}$   
 $i_o = i_L - i_2 = -0.1875 - 0.05 = -0.2375 \text{ mA}$

(c) (i)  $v_o = \frac{-15}{1}(8 \sin \omega t) \text{ (mV)} \Rightarrow v_o = -0.12 \sin \omega t \text{ (V)}$   
(ii)  $i_2 = \frac{0.12 \sin \omega t}{15} \Rightarrow i_2 = 8 \sin \omega t \text{ (}\mu\text{ A)}$   
 $i_L = \frac{-0.12 \sin \omega t}{4} \Rightarrow i_L = -30 \sin \omega t \text{ (}\mu\text{ A)}$   
 $i_o = i_L - i_2 = -38 \sin \omega t \text{ (}\mu\text{ A)}$

**9.15**

$$A_v = -\frac{R_2}{R_1 + R_2}$$

$$A_v = -30 \pm 2.5\% \Rightarrow 29.25 \leq |A_v| \leq 30.75$$

So  $\frac{R_2}{R_1 + 2} = 29.25$  and  $\frac{R_2}{R_1 + 1} = 30.75$

We have  $29.25(R_1 + 2) = 30.75(R_1 + 1)$   
Which yields  $R_1 = 18.5 \text{ k}\Omega$  and  $R_2 = 599.6 \text{ k}\Omega$   
For  $v_i = 25 \text{ mV}$ , then  $0.731 \leq |v_o| \leq 0.769 \text{ V}$

**9.16**

$$v_{o1} = \frac{-R_2}{R_1} \cdot v_i = \frac{-80}{10}(-0.15) = 1.2 \text{ V}$$

$$v_o = \frac{-R_4}{R_3} \cdot v_{o1} = \frac{-100}{20}(1.2) = -6 \text{ V}$$

$$i_1 = i_2 = \frac{v_i}{R_1} = \frac{-(0.15)}{10} \Rightarrow i_1 = i_2 = -15 \mu\text{ A}$$

$$i_3 = i_4 = \frac{v_{o1}}{R_3} = \frac{1.2}{20} \Rightarrow i_3 = i_4 = 60 \mu\text{ A}$$

At  $v_{o1}$ :  $i_2 + i_{o1} = i_3 \Rightarrow i_{o1} = 60 - (-15) = 75 \mu\text{ A}$ ; Out of Op-Amp  
At  $v_o$ :  $i_{o2} = i_4 = 60 \mu\text{ A}$ ; Into Op-Amp

**9.17**

$$\frac{v_o}{v_i} = \left( \frac{-R_2}{R_1} \right) \left( \frac{-R_4}{R_3} \right) = 100$$

For  $v_i = 50 \text{ mV}$ ,  $v_o = (100)(0.05) = 5 \text{ V}$



If  $|i_4| = 50 \mu\text{A}$ ,  $R_4 = \frac{5}{50 \times 10^{-6}} \Rightarrow R_4 = 100 \text{ k}\Omega$

Set  $R_1 = 10 \text{ k}\Omega$

Then  $\frac{v_o}{v_I} = 100 = \left(\frac{R_2}{10}\right)\left(\frac{100}{R_3}\right) \Rightarrow \frac{R_2}{R_3} = 10$

Set  $R_2 = 100 \text{ k}\Omega$ ,  $R_3 = 10 \text{ k}\Omega$

**9.18**

$$A_v = \left(\frac{-R_2}{R_1}\right)\left(\frac{-R_4}{R_3}\right)\left(\frac{-R_6}{R_5}\right) = -300$$

For  $v_o = 6 \text{ V}$ , set  $|i_6| = \frac{v_o}{R_6} \Rightarrow 60 \mu\text{A} = \frac{6}{R_6} \Rightarrow R_6 = 100 \text{ k}\Omega$

Set  $R_6 = 200 \text{ k}\Omega$  so that  $|i_6| = 30 \mu\text{A}$

Set  $R_1 = 20 \text{ k}\Omega$

Now  $300 = \left(\frac{R_2}{20}\right)\left(\frac{R_4}{R_3}\right)\left(\frac{200}{R_5}\right)$

For example, set  $R_2 = 100 \text{ k}\Omega$  and  $R_5 = 20 \text{ k}\Omega$

Then  $300 = \left(\frac{100}{20}\right)\left(\frac{R_4}{R_3}\right)\left(\frac{200}{20}\right) = 50\left(\frac{R_4}{R_3}\right)$

Or  $\frac{R_4}{R_3} = 6$ , set  $R_3 = 20 \text{ k}\Omega$  and  $R_4 = 120 \text{ k}\Omega$

**9.19**

(a)  $v_o = \frac{-R_2}{R_1} \cdot v_I = -\left(\frac{22}{1}\right)(-0.40) = 8.8 \text{ V}$

(b)  $A_v = \frac{-R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]} = -(22) \cdot \frac{1}{\left[1 + \frac{1}{5 \times 10^3}(23)\right]} = -21.8993$

$v_o = (-21.8993)(-0.40) = 8.7597 \text{ V}$

(c)  $A_v = -(0.998)(22) = -21.956$

$-21.956 = -(22) \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}(23)\right]} \Rightarrow A_{od} = 1.1477 \times 10^4$

**9.20**

(a)

$$A_v = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1}\right)\right]}$$

$$= -\frac{100}{25} \cdot \frac{1}{\left[1 + \frac{1}{5 \times 10^3} (5)\right]}$$

$$A_v = -3.9960$$

(b)  $v_o = -3.9960(1.00) \Rightarrow v_o = -3.9960 \text{ V}$

$$\frac{4 - 3.9960}{4} \times 100\% = \underline{0.10\%}$$

(c)

(d)

$$v_o = A_{od}(v_2 - v_1) = -A_{od}v_1$$

$$v_1 = -\frac{v_o}{A_{od}} = \frac{-(-3.9960)}{5 \times 10^3}$$

$$v_1 = \underline{0.7992 \text{ mV}}$$

**9.21**

(a)  $A_v = \frac{-100}{10} \cdot \frac{1}{\left[1 + \frac{1}{7 \times 10^3} \left(1 + \frac{100}{10}\right)\right]} = -9.98431$

$$v_I = \frac{v_o}{A_v} = \frac{7}{-9.9843} = -0.7011 \text{ V}$$

$$v_1 = \frac{-v_o}{A_v} = \frac{-7}{7 \times 10^3} \Rightarrow v_1 = -1 \text{ mV}$$

(b)  $A_{od} = \frac{-v_o}{v_1} = \frac{-(-5)}{0.2 \times 10^{-3}} = 2.5 \times 10^4$

$$A_v = (-10) \cdot \frac{1}{\left[1 + \frac{1}{2.5 \times 10^4} (11)\right]} = -9.9956$$

$$v_I = \frac{v_o}{A_v} = \frac{-5}{-9.9956} = 0.50022 \text{ V}$$

**9.22**

(a)  $A_v = \frac{-R_2}{R_1} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right) = \frac{-50}{10} \left(1 + \frac{50}{5} + \frac{50}{50}\right) = -60$

(b) (i)  $-100 = -5 \left(1 + \frac{50}{R_4} + \frac{50}{50}\right) \Rightarrow R_4 = 2.78 \text{ k}\Omega$

(ii)  $-150 = -5 \left(1 + \frac{50}{R_4} + \frac{50}{50}\right) \Rightarrow R_4 = 1.79 \text{ k}\Omega$

**9.23**

a.

$$A_v = -\frac{R_2}{R_1} \left( 1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$R_1 = 500 \text{ k}\Omega$$

$$80 = \frac{R_2}{500} \left( 1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$\text{Set } R_2 = R_3 = 500 \text{ k}\Omega$$

$$80 = 1 \left( 1 + \frac{500}{R_4} + 1 \right) = 2 + \frac{500}{R_4} \Rightarrow R_4 = 6.41 \text{ k}\Omega$$

b.

$$\text{For } v_I = -0.05 \text{ V}$$

$$i_1 = i_2 = \frac{-0.05}{500 \text{ k}\Omega} \Rightarrow i_1 = i_2 = -0.1 \mu\text{A}$$

$$v_X = -i_2 R_2 = -(-0.1 \times 10^{-6})(500 \times 10^3) = 0.05$$

$$i_4 = -\frac{v_X}{R_4} = -\frac{0.05}{6.41} \Rightarrow i_4 = -7.80 \mu\text{A}$$

$$i_3 = i_2 + i_4 = -0.1 - 7.80 \Rightarrow i_3 = -7.90 \mu\text{A}$$

**9.24**

(a)

$$A_v = -1000 = \frac{-R_2}{R_1} = \frac{-500}{R_1}$$

$$R_1 = 0.5 \text{ K}$$

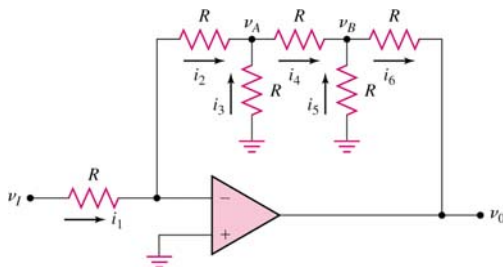
(b)

$$A_v = \frac{-R_2}{R_1} \left( 1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$-1000 = \frac{-250}{R_1} \left( 1 + \frac{500}{250} + \frac{500}{250} \right) = \frac{-1250}{R_1}$$

$$R_1 = 1.25 \text{ K}$$

**9.25**



$$i_1 = \frac{v_I}{R} = i_2$$

$$v_A = -i_2 R = -\left(\frac{v_I}{R}\right)R = -v_I$$

$$i_3 = -\frac{v_A}{R} = \frac{v_I}{R}$$

$$i_4 = i_2 + i_3 = -\frac{v_I}{R} - \frac{v_I}{R} = -\frac{2v_I}{R} = \frac{2v_I}{R}$$

$$v_B = v_A - i_4 R = -v_I - \left(\frac{2v_I}{R}\right)(R) = -3v_I$$

$$i_5 = -\frac{v_B}{R} = -\frac{(-3v_I)}{R} = \frac{3v_I}{R}$$

$$i_6 = i_4 + i_5 = \frac{2v_I}{R} + \frac{3v_I}{R} = \frac{5v_I}{R}$$

$$v_0 = v_B - i_6 R = -3v_I - \left(\frac{5v_I}{R}\right)R \Rightarrow \frac{v_0}{v_I} = -8$$

From Figure 9.12  $\Rightarrow A_v = -3$

### 9.26

$$(a) A_v = \frac{-R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1}\right)\right]} = \frac{-200}{20} \cdot \frac{1}{\left[1 + \frac{1}{5 \times 10^4} \left(1 + \frac{200}{20}\right)\right]} = -9.9978$$

$$(b) v_I = \frac{v_O}{A_v} = \frac{-4.80}{-9.9978} = 0.4801056 \text{ V}$$

$$(c) v_1 = \frac{-v_O}{A_{od}} = \frac{-(-4.80)}{5 \times 10^4} \Rightarrow v_1 = 96 \mu\text{V}$$

$$(d) \frac{4.801056 - 4.8}{4.8} \times 100\% = 0.022\%$$

### 9.27

$$(a) A_v = \frac{-R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1}\right)\right]} = (-1) \cdot \frac{1}{\left[1 + \frac{1}{2.5 \times 10^3} (2)\right]} = -0.9992$$

$$(i) v_O = (-0.9992)(-0.8) = 0.7993605 \text{ V}$$

$$(ii) \frac{0.8 - 0.7993605}{0.8} \times 100\% \cong 0.08\%$$

$$(b) A_v = (-1) \cdot \frac{1}{\left[1 + \frac{1}{2 \times 10^2} (2)\right]} = -0.990099$$

$$(i) v_O = (-0.990099)(-0.8) = 0.79208 \text{ V}$$

$$(ii) \frac{0.8 - 0.79208}{0.9} \times 100\% = 0.99\%$$

**9.28**

$$(a) \quad i_i = \frac{v_i}{R_1} = i_2 = -\frac{v_o}{R_2} \Rightarrow \frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

(b)

$$i_2 = i_1 = \frac{v_i}{R_1} = i_3 + \frac{v_o}{R_L} = i_3 + \frac{1}{R_L} \left( -\frac{R_2}{R_1} \cdot v_i \right)$$

$$\text{Then } i_3 = \frac{v_i}{R_1} \left( 1 + \frac{R_2}{R_L} \right)$$

**9.29**

$$V_{x.\max} = \left( \frac{R_3 \parallel R_1}{R_3 \parallel R_1 + R_4} \right) \cdot V^+ = \left( \frac{0.1 \parallel 1}{0.1 \parallel 1 + 10} \right) (10) \Rightarrow V_{x.\max} = 0.09008 \text{ V}$$

$$|v_o| = \frac{R_2}{R_1} \cdot V_{x.\max}$$

$$10 = \frac{R_2}{R_1} (0.09008) \Rightarrow \frac{R_2}{R_1} = 111$$

$$\text{So } \underline{R_2 = 111 \text{ k}\Omega}$$

**9.30**

$$v_o = -\frac{R_F}{R_1} \cdot v_{i1} - \frac{R_F}{R_2} \cdot v_{i2} - \frac{R_F}{R_3} \cdot v_{i3} = -\frac{120}{40} \cdot v_{i1} - \frac{120}{20} \cdot v_{i2} - \frac{120}{60} \cdot v_{i3}$$

$$v_o = -3v_{i1} - 6v_{i2} - 2v_{i3}$$

$$(a) \quad v_o = -3(-0.25) - 6(0.10) - 2(1.5) = -2.85 \text{ V}$$

$$(b) \quad 0.5 = -3(v_{i1}) - 6(0.25) - 2(-1.2) \Rightarrow v_{i1} = 0.133 \text{ V}$$

**9.31**

$$(a) \quad v_o = -2.5(1.2v_{i1} + 2.5v_{i2} + 0.25v_{i3}) = -3v_{i1} - 6.25v_{i2} - 0.625v_{i3}$$

$$\text{Then } \frac{R_F}{R_1} = 3, \quad \frac{R_F}{R_2} = 6.25, \quad \frac{R_F}{R_3} = 0.625$$

$R_3$  is the largest resistor, so set  $R_3 = 400 \text{ k}\Omega$

Then  $R_F = 250 \text{ k}\Omega$ ,  $R_1 = 83.3 \text{ k}\Omega$ ,  $R_2 = 40 \text{ k}\Omega$

$$(b) \quad v_o = -3(-1) - 6.25(0.25) - 0.625(2) = 0.1875 \text{ V}$$

$$|i_F| = \frac{v_o}{R_F} = \frac{0.1875}{250} \Rightarrow |i_F| = 0.75 \mu\text{A}$$

**9.32**

$$v_o = -2(v_{i1} + 3v_{i2}) = -2v_{i1} - 6v_{i2}$$

$$\text{Then } \frac{R_F}{R_1} = 2, \quad \frac{R_F}{R_2} = 6$$

For  $v_{i1} = -1 \text{ V}$ ,  $v_{i2} = -0.5 \text{ V}$

Then  $v_o = -2(-1) - 6(-0.5) = 5 \text{ V}$

Set  $|i_F| = 80 \mu\text{A} = \frac{v_o}{R_F} = \frac{5}{R_F} \Rightarrow R_F = 62.5 \text{ k}\Omega$

Then  $R_1 = 31.25 \text{ k}\Omega$ ,  $R_2 = 10.42 \text{ k}\Omega$

**9.33**

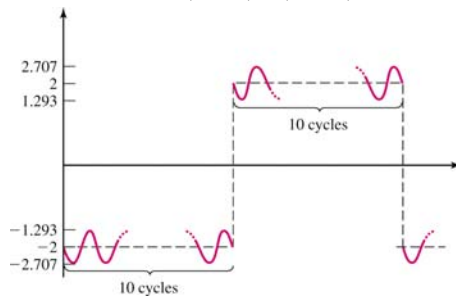
$v_{i1} = (0.05)\sqrt{2} \sin(2\pi ft) = 0.0707 \sin(2\pi ft)$

$f = 1 \text{ kHz} \Rightarrow T = \frac{1}{10^3} \Rightarrow 1 \text{ ms}$      $v_{i2} \Rightarrow T_2 = \frac{1}{100} \Rightarrow 10 \text{ ms}$

$v_o = -\frac{R_F}{R_1} \cdot v_{i1} - \frac{R_F}{R_2} \cdot v_{i2} = -\frac{10}{1} \cdot v_{i1} - \frac{10}{5} \cdot v_{i2}$

$v_o = -(10)(0.0707 \sin(2\pi ft)) - (2)(\pm 1 \text{ V})$

$v_o = -0.707 \sin(2\pi ft) - (\pm 2 \text{ V})$



**9.34**

$v_o = -\frac{R_F}{R_1} \cdot v_{i1} - \frac{R_F}{R_2} \cdot v_{i2}$

$-0.5 \sin \omega t = -\frac{100}{R_1}(0.004 + 0.125 \sin \omega t) - \frac{100}{R_2}(-0.006)$

Set  $-0.5 \sin \omega t = -\frac{100}{R_1}(0.125 \sin \omega t) \Rightarrow R_1 = 25 \text{ k}\Omega$

We have  $0 = -\frac{100}{R_1}(0.004) - \frac{100}{R_2}(-0.006)$

$0 = -\frac{0.4}{25} + \frac{0.6}{R_2} \Rightarrow R_2 = 37.5 \text{ k}\Omega$

**9.35**

(a)  $v_o = -2 \left[ \frac{v_{i1}}{4} + 2v_{i2} + v_{i3} \right] = -\frac{1}{2}v_{i1} - 4v_{i2} - 2v_{i3}$

Then  $\frac{R_F}{R_1} = \frac{1}{2}$ ,  $\frac{R_F}{R_2} = 4$ ,  $\frac{R_F}{R_3} = 2$

Set  $R_1 = 250 \text{ k}\Omega$ , Then  $R_F = 125 \text{ k}\Omega$ ,  $R_2 = 31.25 \text{ k}\Omega$ ,  $R_3 = 62.5 \text{ k}\Omega$

(b) For  $v_{I1} = -2 \text{ V}$ ,  $v_{I2} = 0$ ,  $v_{I3} = -1 \text{ V}$

$$v_o = -\frac{1}{2}(-2) - 4(0) - 2(-1) = 3 \text{ V}$$

For  $v_{I1} = 2 \text{ V}$ ,  $v_{I2} = 0.5 \text{ V}$ ,  $v_{I3} = 0$

$$v_o = -\frac{1}{2}(2) - 4(0.5) - 2(0) = -3 \text{ V}$$

Then  $-3 \leq v_o \leq +3 \text{ V}$

$$|i_F|_{\max} = \frac{|v_o|_{\max}}{R_F} = \frac{3}{125} \Rightarrow |i_F|_{\max} = 24 \mu \text{ A}$$

### 9.36

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} - \frac{R_F}{R_3} \cdot v_{I3}$$

$$-6 \sin \omega t = -\frac{R_F}{R_1}(2 + 2 \sin \omega t) - \frac{R_F}{R_2}(0.5 \sin \omega t) - \frac{R_F}{R_3}(-4)$$

$$\text{We have } -\frac{R_F}{R_1}(2) + \frac{R_F}{R_3}(4) = 0 \Rightarrow R_3 = 2R_1$$

$$\text{Also } -6 = -\frac{R_F}{R_1}(2) - \frac{R_F}{R_2}(0.5)$$

$$\text{For } v_o = 6 \text{ V}, |i_F|_{\max} = 120 \mu \text{ A} = \frac{6}{R_F} \Rightarrow R_F = 50 \text{ k}\Omega$$

$$\text{For } |v_{I1}|_{\max} = 4 \text{ V}, |i_1|_{\max} = 120 \mu \text{ A} = \frac{4}{R_1} \Rightarrow R_1 = 33.33 \text{ k}\Omega \text{ and } R_3 = 2R_1 = 66.66 \text{ k}\Omega$$

$$\text{Now } 6 = \frac{(50)(2)}{R_1} + \frac{(50)(0.5)}{R_2} = \frac{100}{R_1} + \frac{25}{R_2} = \frac{100}{33.33} + \frac{25}{R_2} \Rightarrow R_2 = 8.33 \text{ k}\Omega$$

### 9.37

a.

$$v_o = -\frac{R_F}{R_3} \cdot a_3(-5) - \frac{R_F}{R_2} \cdot a_2(-5) - \frac{R_F}{R_1} \cdot a_1(-5) - \frac{R_F}{R_0} \cdot a_0(-5)$$

$$\text{So } v_o = \frac{R_F}{10} \left[ \frac{a_3}{2} + \frac{a_2}{4} + \frac{a_1}{8} + \frac{a_0}{16} \right] (5)$$

$$v_o = 2.5 = \frac{R_F}{10} \cdot \frac{1}{2} \cdot 5 \Rightarrow \underline{R_F = 10 \text{ k}\Omega}$$

b.

c.

i.  $v_o = \frac{10}{10} \cdot \frac{1}{16} \cdot 5 \Rightarrow \underline{v_o = 0.3125 \text{ V}}$

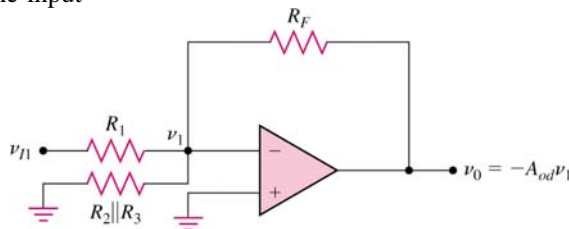
ii.  $v_o = \frac{10}{10} \left[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] (5) \Rightarrow \underline{v_o = 4.6875 \text{ V}}$

**9.38**

- (a)  $v_o = \left(\frac{-10}{1}\right)\left(\frac{-20}{1}\right) \cdot v_{I1} - \left(\frac{20}{1}\right) \cdot v_{I2} = 200v_{I1} - 20v_{I2}$
- (b)  $v_o = (200)(5) - (20)(-25 - 50 \sin \omega t) = 1000 + 500 + 1000 \sin \omega t$  (mV)  
 $v_o = 1.5 + 1.0 \sin \omega t$  (V)
- (c) For the 20 k $\Omega$  resistor:  
 $|i|_{\max} = \frac{2.5}{20} \Rightarrow |i|_{\max} = 0.125$  mA
- For the 10 k $\Omega$  resistor:  
 $|v_{O1}| = \left(\frac{10}{1}\right)(5) = 50$  mV,  $|i|_{\max} = \frac{50 \text{ mV}}{10 \text{ k}\Omega} \Rightarrow |i_{\max}| = 5$   $\mu$  A

**9.39**

For one-input



$$v_1 = -\frac{v_o}{A_{od}}$$

$$\frac{v_{I1} - v_1}{R_1} = \frac{v_1}{R_2 \parallel R_3} + \frac{v_1 - v_o}{R_F}$$

$$\frac{v_{I1}}{R_1} = v_1 \left[ \frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} + \frac{1}{R_F} \right] - \frac{v_o}{R_F}$$

$$= -\frac{v_o}{A_{od}} \left[ \frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} + \frac{1}{R_F} \right] - \frac{v_o}{R_F}$$

$$= -v_o \left[ \frac{1}{A_{od} R_F} + \frac{1}{R_F} + \frac{1}{A_{od}} \left( \frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} \right) \right]$$

$$= -\frac{v_o}{R_F} \left[ \frac{1}{A_{od}} + 1 + \frac{1}{A_{od}} \cdot \frac{R_F}{R_1 \parallel R_2 \parallel R_3} \right]$$

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} \cdot \frac{1}{\left[ 1 + \frac{1}{A_{od}} \left( 1 + \frac{R_F}{R_P} \right) \right]} \quad \text{where } R_P = R_1 \parallel R_2 \parallel R_3$$

Therefore, for three-inputs  $v_o = \frac{-1}{1 + \frac{1}{A_{od}} \left( 1 + \frac{R_F}{R_P} \right)} \cdot \left( \frac{R_F}{R_1} \cdot v_{I1} + \frac{R_F}{R_2} \cdot v_{I2} + \frac{R_F}{R_3} \cdot v_{I3} \right)$



**9.40**

$$(a) A_v = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{150}{15}\right) = 11$$

$$(b) A_v = \left(1 + \frac{150}{50}\right) = 4$$

$$(c) A_v = \left(1 + \frac{20}{50}\right) = 1.4$$

$$(d) A_v = \left(1 + \frac{20}{20}\right) = 2$$

**9.41**

$$(a) A_v = 15 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 14$$

$$\text{For } v_o = -7.5 \text{ V} \Rightarrow v_i = -0.5 \text{ V}$$

$$|i| = 120 \mu\text{A} = \frac{7.5 - 0.5}{R_2} \Rightarrow R_2 = 58.33 \text{ k}\Omega, R_1 = 4.17 \text{ k}\Omega$$

$$(b) v_o = (15)(0.25) = 3.75 \text{ V}$$

$$i_1 = i_2 = \frac{0.25}{4.17} \Rightarrow i_1 = i_2 = 60 \mu\text{A}$$

**9.42**

$$(a) A_v = \left(1 + \frac{R_2}{R_1}\right)$$

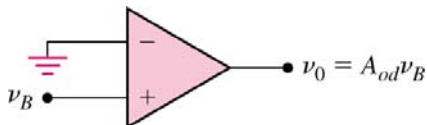
$$3 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 2, \text{ Set } R_2 = 290 \text{ k}\Omega, R_1 = 145 \text{ k}\Omega$$

$$(b) 9 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 8, \text{ Set } R_2 = 290 \text{ k}\Omega, R_1 = 36.25 \text{ k}\Omega$$

$$(c) 30 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 29, \text{ Set } R_2 = 290 \text{ k}\Omega, R_1 = 10 \text{ k}\Omega$$

$$(d) 1 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 0, \text{ Set } R_2 = 0, R_1 = 290 \text{ k}\Omega$$

**9.43**



$$v_B = \left(\frac{1}{1 + 500}\right)v_i \quad v_o = A_{od} \left(\frac{1}{501}\right)v_i$$

- 
- a.  $2.5 = A_{od} \left( \frac{1}{501} \right) (5) \Rightarrow \underline{A_{od} = 250.5}$
- b.  $v_o = 5000 \left( \frac{1}{501} \right) (5) \Rightarrow \underline{v_o = 49.9 \text{ V}}$
- 

**9.44**

$$v_o = \left( 1 + \frac{50}{50} \right) \left[ \left( \frac{20}{20+40} \right) v_{I2} + \left( \frac{40}{20+40} \right) v_{I1} \right]$$
$$\underline{v_o = 1.33v_{I1} + 0.667v_{I2}}$$

---

**9.45**

- (a)
- $$\frac{v_{I1} - v_2}{20} + \frac{v_{I2} - v_2}{40} = \frac{v_2}{10}$$
- $$v_o = \left( 1 + \frac{100}{50} \right) v_2 = 3v_2$$
- Now  $2v_{I1} - 2v_2 + v_{I2} - v_2 = 4v_2$
- $$2v_{I1} + v_{I2} = 7v_2 = 7 \left( \frac{v_o}{3} \right)$$
- So  $v_o = \frac{6}{7} \cdot v_{I1} + \frac{3}{7} \cdot v_{I2}$
- (b)  $v_o = \frac{6}{7} (0.2) + \left( \frac{3}{7} \right) (0.3) \Rightarrow \underline{v_o = 0.3 \text{ V}}$
- (c)  $v_o = \left( \frac{6}{7} \right) (0.25) + \left( \frac{3}{7} \right) (-0.4) \Rightarrow \underline{v_o = 42.86 \text{ mV}}$
- 

**9.46**

- (a)  $v_2 = \left( \frac{R_4}{R_3 + R_4} \right) \cdot v_I$
- $$v_o = \left( 1 + \frac{R_2}{R_1} \right) \cdot v_2 = \left( 1 + \frac{R_2}{R_1} \right) \cdot \frac{1}{\left( 1 + \frac{R_3}{R_4} \right)} \cdot v_I$$
- (b)  $6 = \left( 1 + \frac{R_2}{R_1} \right) \cdot \frac{1}{\left( 1 + \frac{25}{50} \right)} \Rightarrow \left( 1 + \frac{R_2}{R_1} \right) = 9 \Rightarrow \frac{R_2}{R_1} = 8$
- Set  $R_2 = 200 \text{ k}\Omega$ ,  $R_1 = 25 \text{ k}\Omega$
-

**9.47**

(a)

$$\frac{v_o}{v_i} = \left( 1 + \frac{50x}{(1-x)50} \right)$$

$$\frac{v_o}{v_i} = \left( 1 + \frac{x}{1-x} \right) = \frac{1-x+x}{1-x}$$

$$A_v = \frac{v_o}{v_i} = \frac{1}{1-x}$$

(b)  $1 \leq A_v \leq \infty$

(c) If  $x = 1$ , gain goes to infinity.

**9.48**

(a)  $v_x = \left( \frac{v_i}{R} \right) (2R) + v_i = 3v_i$

$$\frac{v_x - v_i}{2R} + \frac{v_x}{R} + \frac{v_x - v_o}{2R} = 0$$

$$v_x \left( \frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} \right) - \frac{v_i}{2R} = \frac{v_o}{2R}$$

$$3v_i \left( \frac{2}{R} \right) - \frac{v_i}{2R} = \frac{v_o}{2R}$$

so  $\frac{v_o}{v_i} = 11$

(b) For  $v_i = 0.25 \text{ V}$ ,  $\Rightarrow v_o = 2.75 \text{ V}$

(c)  $R = 30 \text{ k}\Omega$ ,  $v_i = -0.15 \text{ V}$

For  $R_1$ :  $|i| = \frac{0.15}{30} \Rightarrow 5 \mu\text{ A}$

For  $R_2$ :  $|i| = 5 \mu\text{ A}$

$$v_x = 3v_i = -0.45 \text{ V}$$

For  $R_4$ :  $|i| = \frac{0.45}{30} \Rightarrow 15 \mu\text{ A}$

$$v_o = (11)(-0.15) = -1.65 \text{ V}$$

For  $R_3$ :  $|i| = \frac{1.65 - 0.45}{60} \Rightarrow 20 \mu\text{ A}$

**9.49**

(a)  $\frac{v_O}{v_I} = 1$

(b) From Exercise TYU9.7

$$\frac{v_O}{v_I} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1}\right)\right]}$$

But  $R_2 = 0$ ,  $R_1 = \infty$

$$\frac{v_O}{v_I} = \frac{1}{1 + \frac{1}{A_{od}}} = \frac{1}{1 + \frac{1}{1.5 \times 10^5}} \Rightarrow \frac{v_O}{v_I} = 0.999993$$

Want  $\frac{v_O}{v_I} = 0.990 = \frac{1}{1 + \frac{1}{A_{od}}} \Rightarrow A_{od} = 99$

(b)

**9.50**

(a)  $v_O = A_{od}(v_2 - v_1) = A_{od}(v_I - v_O)$

$$v_O(1 + A_{od}) = A_{od}v_I$$

$$A_v = \frac{v_O}{v_I} = \frac{1}{1 + \frac{1}{A_{od}}} = \frac{1}{1 + \frac{1}{20}} = 0.9524$$

(b)  $A_v = \frac{1}{1 + \frac{1}{200}} = 0.995$

(c)  $A_v = \frac{1}{1 + \frac{1}{2000}} = 0.9995$

(d)  $A_v = \frac{1}{1 + \frac{1}{20000}} = 0.99995$

**9.51**

(a)  $A_{v1} = \frac{v_{O1}}{v_I} = \left(1 + \frac{R_2}{R_1}\right)$

$$A_{v2} = \frac{v_{O2}}{v_I} = -\left(1 + \frac{R_2}{R_1}\right)$$

(b)  $v_{O1} = \left(1 + \frac{60}{20}\right)(-0.5) = -2 \text{ V}$

$$v_{O2} = -\left(1 + \frac{60}{20}\right)(-0.5) = 2 \text{ V}$$

$$(c) \quad v_{o1} - v_{o2} = 2 \left( 1 + \frac{R_2}{R_1} \right) \cdot v_I = 2 \left( 1 + \frac{60}{20} \right) (0.8) = 6.4 \text{ V}$$

**9.52**

$$(a) \quad i_L = \frac{v_I}{R_1}$$

(b)

$$v_{o1} = i_L R_L + v_I = i_L R_L + i_L R_1$$

$$v_{o1} (\text{max}) \cong 10 \text{ V} = i_L (1+9) = 10i_L$$

$$\text{So } i_L (\text{max}) \cong 1 \text{ mA}$$

$$\text{Then } v_I (\text{max}) \cong i_L R_1 = (1)(9) \Rightarrow \underline{v_I (\text{max}) \cong 9 \text{ V}}$$

**9.53**

$$(a) \quad v_o = \left( \frac{20}{20+40} \right) \cdot v_I = (0.3333)v_I$$

$$(i) \quad v_o = 1 \text{ V}$$

$$(ii) \quad v_o = -1.67 \text{ V}$$

$$(b) \quad v_o = \left( \frac{20}{20+40} \right) \cdot v_I = (0.3333)v_I$$

$$(i) \quad v_o = 1 \text{ V}$$

$$(ii) \quad v_o = -1.67 \text{ V}$$

$$(c) \quad v_o = \left( 1 + \frac{10}{10} \right) \left( \frac{6}{6+48} \right) \cdot v_I = (0.2222)v_I$$

$$(i) \quad v_o = 0.667 \text{ V}$$

$$(ii) \quad v_o = -1.111 \text{ V}$$

**9.54**

a.

$$R_{in} = \frac{v_1}{i_1} \text{ and } \frac{v_1 - v_o}{R_F} = i_1 \text{ and } v_o = -A_{od} v_1$$

$$\text{So } i_1 = \frac{v_1 - (-A_{od} v_1)}{R_F} = \frac{v_1 (1 + A_{od})}{R_F}$$

$$\text{Then } R_{in} = \frac{v_1}{i_1} = \frac{R_F}{1 + A_{od}}$$

b.

$$i_1 = \left( \frac{R_S}{R_S + R_{in}} \right) i_S \text{ and } v_0 = -A_{od} \cdot \frac{R_F}{1 + A_{od}} \cdot i_1$$

$$\text{So } v_0 = -R_F \left( \frac{A_{od}}{1 + A_{od}} \right) \left( \frac{R_S}{R_S + R_{in}} \right) i_S$$

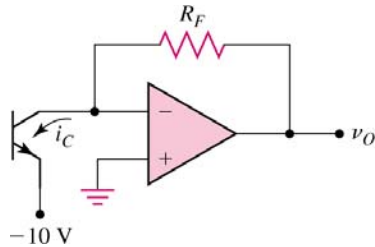
$$R_{in} = \frac{R_F}{1 + A_{od}} = \frac{10}{1001} = 0.009990$$

$$v_0 = -R_F \left( \frac{1000}{1001} \right) \left( \frac{R_S}{R_S + 0.009990} \right) i_S$$

$$\text{Want } \left( \frac{1000}{1001} \right) \left( \frac{R_S}{R_S + 0.009990} \right) \leq 0.990$$

which yields  $R_S \geq 1.099 \text{ k}\Omega$

9.55



$$v_0 = i_C R_F, \quad 0 \leq i_C \leq 8 \text{ mA}$$

$$\text{For } v_0(\text{max}) = 8 \text{ V, Then } R_F = 1 \text{ k}\Omega$$

9.56

$$i = \frac{v_I}{R} \text{ so } 1 = \frac{10}{R} \Rightarrow R = 10 \text{ k}\Omega$$

In the ideal op-amp,  $R_1$  has no influence.

$$\text{Output voltage: } v_0 = \left( 1 + \frac{R_2}{R} \right) v_I$$

$v_0$  must remain within the bias voltages of the op-amp; the larger the  $R_2$ , the smaller the range of input voltage  $v_I$  in which the output is valid.

9.57

$$(a) \quad i_L = \frac{-v_I}{R_2}; \quad R_2 = \frac{-(-5)}{5} = 1 \text{ k}\Omega$$

$$\text{Set } \frac{R_F}{R_1 R_3} = \frac{1}{R_2}; \text{ For example, set } R_1 = R_F = 10 \text{ k}\Omega, \quad R_3 = 1 \text{ k}\Omega$$

$$(b) \quad v_L = (5)(0.2) = 1 \text{ V} = v_1$$

$$i_1 = i_2 = \frac{v_I - v_1}{R_1} = \frac{-5 - 1}{10} = -0.6 \text{ mA}$$

$$v_o = v_1 - i_2 R_F = 1 - (-0.6)(10) = 7 \text{ V}$$

$$i_3 = \frac{v_o - v_L}{R_3} = \frac{7 - 1}{1} = 6 \text{ mA}$$

$$i_4 = \frac{v_L}{R_2} = \frac{1}{1} = 1 \text{ mA}$$

For the op-amp:  $i_o + i_2 = i_3 \Rightarrow i_o = i_3 - i_2 = 6 - (-0.6) = 6.6 \text{ mA}$

**9.58**

(a)

$$i_1 = i_2 \text{ and } i_2 = \frac{v_x}{R_2} + i_D, \quad v_x = -i_2 R_F$$

$$\text{Then } i_1 = -i_1 \left( \frac{R_F}{R_2} \right) + i_D$$

$$\text{Or } i_D = i_1 \left( 1 + \frac{R_F}{R_2} \right)$$

(b)

$$R_1 = \frac{v_I}{i_1} = \frac{5}{1} \Rightarrow R_1 = 5 \text{ k}\Omega$$

$$12 = (1) \left( 1 + \frac{R_F}{R_2} \right) \Rightarrow \frac{R_F}{R_2} = 11$$

For example,  $R_2 = 5 \text{ k}\Omega, \quad R_F = 55 \text{ k}\Omega$

**9.59**

$$(1) \quad I_x = \frac{V_x}{R_2} + \frac{V_x - v_o}{R_3}$$

$$(2) \quad \frac{V_x}{R_1} + \frac{V_x - v_o}{R_F} = 0$$

$$\text{From (2) } v_o = V_x \left( 1 + \frac{R_F}{R_1} \right)$$

$$\text{Then (1) } I_x = V_x \left( \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{1}{R_3} \cdot V_x \left( 1 + \frac{R_F}{R_1} \right)$$

$$\begin{aligned} \frac{I_x}{V_x} &= \frac{1}{R_0} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3} - \frac{R_F}{R_1 R_3} = \frac{1}{R_2} - \frac{R_F}{R_1 R_3} \\ &= \frac{R_1 R_3 - R_2 R_F}{R_1 R_2 R_3} \end{aligned}$$

$$\text{or } R_o = \frac{R_1 R_2 R_3}{R_1 R_3 - R_2 R_F}$$

Note: If  $\frac{R_F}{R_1 R_3} = \frac{1}{R_2} \Rightarrow R_2 R_F = R_1 R_3$  then  $R_o = \infty$ , which corresponds to an ideal current source.

**9.60**

(a)  $R_{id} = R_1 + R_3 = 30 \text{ k}\Omega$ ;  $R_1 = R_3 = 15 \text{ k}\Omega$

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} = 15 \Rightarrow R_2 = R_4 = 225 \text{ k}\Omega$$

(b)  $v_o = i_L R_L = (0.25)(10) = 2.5 \text{ V}$

$$v_{I2} - v_{I1} = \frac{v_o}{A_d} = \frac{2.5}{15} = 0.1667 \text{ V}$$

(c)  $v_o = A_d (v_{I2} - v_{I1}) = 15(1.2 - 1.5) = -4.5 \text{ V}$

$$i_L = \frac{v_o}{R_L} = \frac{-4.5}{10} = -0.45 \text{ mA}$$

(d)  $v_o = (0.5)(10) = 5 \text{ V}$

$$v_{I2} - v_{I1} = \frac{v_o}{A_d} = \frac{5}{15} = 0.333 \text{ V}$$

$$v_{I1} = 2 - 0.333 = 1.667 \text{ V}$$

**9.61**

(a)  $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 40$ ; Set  $R_2 = R_4 = 250 \text{ k}\Omega$ ,  $R_1 = R_3 = 6.25 \text{ k}\Omega$

(b)  $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 25$ ; Set  $R_2 = R_4 = 250 \text{ k}\Omega$ ,  $R_1 = R_3 = 10 \text{ k}\Omega$

(c)  $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 5$ ; Set  $R_2 = R_4 = 250 \text{ k}\Omega$ ,  $R_1 = R_3 = 50 \text{ k}\Omega$

(d)  $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 0.5$ ; Set  $R_2 = R_4 = 125 \text{ k}\Omega$ ,  $R_1 = R_3 = 250 \text{ k}\Omega$

**9.62**

We have

$$v_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4/R_3}{1 + R_4/R_3}\right) v_{I2} - \left(\frac{R_2}{R_1}\right) v_{I1} \text{ or } v_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{1 + R_3/R_4}\right) v_{I2} - \left(\frac{R_2}{R_1}\right) v_{I1}$$

$$\text{Set } R_2 = 50(1+x), R_1 = 50(1-x)$$

$$R_3 = 50(1-x), R_4 = 50(1+x)$$

$$v_o = \left[1 + \left(\frac{1+x}{1-x}\right)\right] \left[\frac{1}{1 + \left(\frac{1-x}{1+x}\right)}\right] v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1}$$

$$\begin{aligned} v_o &= \left[\frac{1-x+(1+x)}{1-x}\right] \cdot \left[\frac{1+x}{1+x+(1-x)}\right] v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1} \\ &= \left(\frac{1+x}{1-x}\right) v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1} \end{aligned}$$



For  $v_{I1} = v_{I2} \Rightarrow v_O = 0$

$$\text{Set } R_2 = 50(1+x) \quad R_1 = 50(1-x)$$

$$R_3 = 50(1+x) \quad R_4 = 50(1-x)$$

$$\begin{aligned} v_O &= \left(1 + \frac{1+x}{1-x}\right) \left(\frac{1}{1 + \frac{1+x}{1-x}}\right) v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1} \\ &= v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1} \end{aligned}$$

$$v_{I1} = v_{I2} = v_{cm}$$

$$\frac{v_O}{v_{cm}} = 1 - \frac{1+x}{1-x} = \frac{1-x-(1+x)}{1-x} = \frac{-2x}{1-x}$$

$$\text{Set } R_2 = 50(1-x) \quad R_1 = 50(1+x)$$

$$R_3 = 50(1-x) \quad R_4 = 50(1+x)$$

$$\begin{aligned} v_O &= \left(1 + \frac{1-x}{1+x}\right) \left(\frac{1}{1 + \frac{1-x}{1+x}}\right) v_{I2} - \left(\frac{1-x}{1+x}\right) v_{I1} \\ &= \left(1 - \frac{1-x}{1+x}\right) v_{cm} \end{aligned}$$

$$A_{cm} = \frac{1+x-(1-x)}{1+x} = \frac{2x}{1+x}$$

Worst common-mode gain

$$A_{cm} = \frac{-2x}{1-x}$$

(b)

$$\text{For } x = 0.01, \quad A_{cm} = \frac{-2x}{1-x} = \frac{-2(0.01)}{1-0.01} = -0.0202$$

$$\text{For } x = 0.02, \quad A_{cm} = \frac{-2(0.02)}{1-0.02} = -0.04082$$

$$\text{For } x = 0.05, \quad A_{cm} = \frac{-2(0.05)}{1-0.05} = -0.1053$$

For this condition, set  $v_{I2} = +\frac{1}{2}$ ,  $v_{I1} = -\frac{1}{2} \Rightarrow v_d = 1 \text{ V}$

$$A_d = \frac{1}{2} \left[ 1 + \left(\frac{1+x}{1-x}\right) \right] = \frac{1}{2} \left[ \frac{1-x+(1+x)}{1-x} \right] = \frac{1}{2} \cdot \frac{2}{1-x} = \frac{1}{1-x}$$

$$\text{For } x = 0.01 \quad A_d = 1.010 \quad C M R R_{dB} = 20 \log_{10} \left| \frac{1.010}{0.0202} \right| = 33.98 \text{ dB}$$

$$\text{For } x = 0.02, \quad A_d = \frac{1}{0.98} = 1.020 \quad C M R R_{dB} = 20 \log_{10} \left| \frac{1.020}{0.04082} \right| = 27.96 \text{ dB}$$

$$\text{For } x = 0.05 \quad A_d = \frac{1}{0.95} = 1.0526 \quad C M R R_{dB} = 20 \log_{10} \left| \frac{1.0526}{0.1053} \right| \cong 20 \text{ dB}$$

**9.63**

(a)  $v_o = 10(v_2 - v_1) = 10(1.4 - 1.8) = -4 \text{ V}$

$$i_3 = i_4 = \frac{v_2}{R + 10R} = \frac{1.4}{(11)(10)} = 0.0127 \text{ mA}$$

$$v_x = v_y = \left(\frac{10}{11}\right) \cdot v_2 = \left(\frac{10}{11}\right)(1.4) = 1.273 \text{ V}$$

$$i_1 = i_2 = \frac{v_1 - v_x}{R} = \frac{1.8 - 1.273}{10} = 0.0527 \text{ mA}$$

(b)  $v_o = 10(v_2 - v_1) = 10(3.6 - 3.2) = 4 \text{ V}$

$$v_x = v_y = \left(\frac{10}{11}\right)(3.6) = 3.273 \text{ V}$$

$$i_3 = i_4 = \frac{v_2}{(11)(10)} = \frac{3.6}{110} = 0.0327 \text{ mA}$$

$$i_1 = i_2 = \frac{3.2 - 3.273}{10} = -0.00727 \text{ mA}$$

(c)  $v_o = 10(-1.35 - (-1.20)) = -1.5 \text{ V}$

$$i_3 = i_4 = \frac{-1.35}{(11)(10)} = -0.0123 \text{ mA}$$

$$v_x = v_y = \left(\frac{10}{11}\right)(-1.35) = -1.227 \text{ V}$$

$$i_1 = i_2 = \frac{-1.2 - (-1.227)}{10} = 0.00273 \text{ mA}$$

**9.64**

(a)  $I_E = (1 + \beta) \cdot I_B = (76)(1.2) = 91.2 \text{ mA}$

$$R = \frac{10}{91.2} \Rightarrow R = 109.6 \Omega$$

(b)  $I_E = (101)(0.2) = 20.2 \text{ mA}$

$$R = \frac{10}{20.2} = 0.495 \text{ k}\Omega$$

(c)  $I_E = \frac{6}{0.1096} = 54.74 \text{ mA}$

$$I_O = \frac{54.74}{76} = 0.72 \text{ mA}$$

(d)  $I_E = \frac{4}{0.495} = 8.08 \text{ mA}$

$$I_O = \frac{8.08}{101} = 0.080 \text{ mA}$$

9.65

$$(a) \quad v_{O1} = \frac{-R_2}{R_1} \cdot v_{CM}$$

$$v_{O2} = \left( \frac{R_4}{R_3 + R_4} \right) \cdot v_{CM} \cdot \left( 1 + \frac{R_2}{R_1} \right)$$

$$v_O = v_{O1} + v_{O2} = \left[ \frac{\frac{R_4}{R_3}}{1 + \frac{R_4}{R_3}} \cdot \left( 1 + \frac{R_2}{R_1} \right) - \frac{R_2}{R_1} \right] \cdot v_{CM}$$

$$A_{CM} = \frac{v_O}{v_{CM}} = \frac{\left( \frac{R_4}{R_3} \right) \left( 1 + \frac{R_2}{R_1} \right) - \left( \frac{R_2}{R_1} \right) \left( 1 + \frac{R_4}{R_3} \right)}{\left( 1 + \frac{R_4}{R_3} \right)} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1 + \frac{R_4}{R_3}}$$

$$(b) \quad A_{CM} = \frac{\frac{86.4}{9.6} - \frac{62.4}{10.4}}{1 + \frac{86.4}{9.6}} = \frac{9 - 6}{10} = 0.3$$

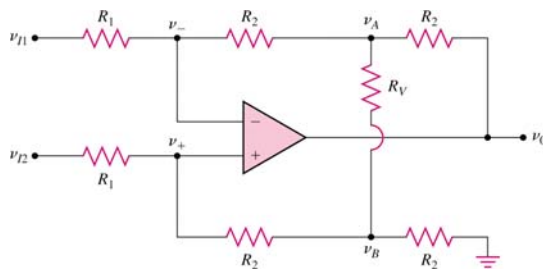
$$(c) \quad A_{CM} = \frac{\frac{80.8}{19.8} - \frac{79.2}{20.2}}{1 + \frac{80.8}{19.8}} = 0.03149$$

or

$$A_{CM} = \frac{\frac{79.2}{20.2} - \frac{80.8}{19.8}}{1 + \frac{79.2}{20.2}} = -0.0325$$

$$\Rightarrow |A_{CM}|_{\max} = 0.0325$$

9.66



$$\frac{v_{I1} - v_A}{R_1 + R_2} = \frac{v_A - v_B}{R_v} + \frac{v_A - v_0}{R_2} \quad (1)$$

$$\frac{v_{I2} - v_B}{R_1 + R_2} = \frac{v_B - v_A}{R_v} + \frac{v_B}{R_2} \quad (2)$$

$$v_- = \left( \frac{R_1}{R_1 + R_2} \right) v_A + \left( \frac{R_2}{R_1 + R_2} \right) v_{I1} \quad (3)$$

$$v_+ = \left( \frac{R_1}{R_1 + R_2} \right) v_B + \left( \frac{R_2}{R_1 + R_2} \right) v_{I2} \quad (4)$$

Now  $v_- = v_+ \Rightarrow R_1 v_A + R_2 v_{I1} = R_1 v_B + R_2 v_{I2}$

So that  $v_A = v_B + \frac{R_2}{R_1} (v_{I2} - v_{I1})$

$$\frac{v_{I1}}{R_1 + R_2} = v_A \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) - \frac{v_B}{R_v} - \frac{v_0}{R_2} \quad (1)$$

$$\frac{v_{I2}}{R_1 + R_2} = v_B \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) - \frac{v_A}{R_v} \quad (2)$$

Then

$$\frac{v_{I1}}{R_1 + R_2} = v_B \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) - \frac{v_B}{R_v} - \frac{v_0}{R_2} + \left( \frac{R_2}{R_1} \right) \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) (v_{I2} - v_{I1}) \quad (1)$$

$$\frac{v_{I2}}{R_1 + R_2} = v_B \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) - \frac{1}{R_v} \left[ v_B + \frac{R_2}{R_1} (v_{I2} - v_{I1}) \right] \quad (2)$$

Subtract (2) from (1)

$$\frac{1}{R_1 + R_2} (v_{I1} - v_{I2}) = \left( \frac{R_2}{R_1} \right) \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) (v_{I2} - v_{I1}) - \frac{v_0}{R_2} + \frac{1}{R_v} \cdot \frac{R_2}{R_1} (v_{I2} - v_{I1})$$

$$\frac{v_0}{R_2} = (v_{I2} - v_{I1}) \left\{ \left( \frac{R_2}{R_1} \right) \left( \frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2} \right) + \frac{1}{R_1 + R_2} + \frac{1}{R_v} \cdot \frac{R_2}{R_1} \right\}$$

$$v_0 = (v_{I2} - v_{I1}) \left( \frac{R_2}{R_1} \right) \left\{ \frac{R_2}{R_1 + R_2} + \frac{R_2}{R_v} + 1 + \frac{R_1}{R_1 + R_2} + \frac{R_2}{R_v} \right\}$$

$$v_0 = \frac{2R_2}{R_1} \left( 1 + \frac{R_2}{R_v} \right) (v_{I2} - v_{I1})$$

**9.67**

(a)  $i_1 = \frac{v_{I1} - v_{I2}}{R_1} = \frac{(1.2 - 0.08 \sin \omega t) - (1.2 + 0.08 \sin \omega t)}{10} \Rightarrow i_1 = -16 \sin \omega t \text{ (}\mu\text{A)}$

$$v_{o1} = (1.2 - 0.08 \sin \omega t) - (0.016 \sin \omega t)(40) = 1.2 - 0.72 \sin \omega t \text{ (V)}$$

$$v_{o2} = (1.2 + 0.08 \sin \omega t) - (-0.016 \sin \omega t)(40) = 1.2 + 0.72 \sin \omega t \text{ (V)}$$

$$v_o = \frac{R_4}{R_3} (v_{o2} - v_{o1}) = \left( \frac{120}{40} \right) (2)(0.72 \sin \omega t) = 4.32 \sin \omega t \text{ (V)}$$

$$(b) \quad i_i = \frac{(-0.65 + 0.05 \sin \omega t) - (-0.60 - 0.05 \sin \omega t)}{10}$$

$$i_i = -5 + 10 \sin \omega t \quad (\mu A)$$

$$v_{o1} = -0.65 + 0.05 \sin \omega t + (40)(-0.005 + 0.010 \sin \omega t) = -0.85 + 0.45 \sin \omega t \quad (V)$$

$$v_{o2} = -0.60 - 0.05 \sin \omega t - (40)(-0.005 + 0.01 \sin \omega t) = -0.40 - 0.45 \sin \omega t \quad (V)$$

$$v_o = \left(\frac{120}{40}\right)[(-0.40 - 0.45 \sin \omega t) - (-0.85 + 0.45 \sin \omega t)] = 1.35 - 2.7 \sin \omega t \quad (V)$$

**9.68**

$$(a) \quad v_{OB} = \left(1 + \frac{40}{12}\right)v_I = 2.1667 \sin \omega t$$

$$(b) \quad v_{OC} = -\frac{30}{12}v_I = -1.25 \sin \omega t$$

$$(c) \quad v_o = v_{OB} - v_{OC} = 2.1667 \sin \omega t - (-1.25 \sin \omega t)$$

$$v_o = 3.417 \sin \omega t$$

$$(d) \quad \frac{v_o}{v_I} = \frac{3.417}{0.5} = 6.83$$

**9.69**

$$(a) \quad i_o = \frac{v_{I1} - v_{I2}}{R}$$

$$(b) \quad R = \frac{0.25 - (-0.25)}{5} \Rightarrow R = 100 \Omega$$

$$(c) \quad v_{o1} = v_{I1} + i_o R_L = 0.25 + (5)(1) = 5.25 \text{ V}$$

$$v_{o2} = v_{I2} = -0.25 \text{ V}$$

$$(d) \quad i_o = \frac{v_{I1} - v_{I2}}{R} = \frac{1.25 - 1.75}{0.5} = -1 \text{ mA}$$

$$v_{o1} = v_{I1} + i_o R_L = 1.25 - (1)(3) = -1.75 \text{ V}$$

$$v_{o2} = v_{I2} = 1.75 \text{ V}$$

**9.70**

$$A_d = \frac{v_o}{v_{I2} - v_{I1}} = \frac{R_d}{R_3} \left(1 + \frac{2R_2}{R_1}\right)$$

$$|v_o| = \frac{200}{50} \left(1 + \frac{2(115)}{R_1}\right) (0.06 \sin \omega t)$$

$$\text{For } |v_o| = 0.5 \quad \frac{230}{R_1} = 1.0833 \Rightarrow R_1 = 212.3 \text{ K}$$

$$|v_o| = 8 \text{ V} \quad \frac{230}{R_1} = 32.33 \Rightarrow R_1 = 7.11 \text{ K} \Rightarrow R_{1f} = 7.11 \text{ K}, R_1(\text{potentiometer}) = 205.2 \text{ K}$$

**9.71**

For  $v_o = 10$  V,  $A_d = 200 \Rightarrow v_{i2} - v_{i1} = \frac{10}{200} = 0.05$  V

$$R_1(\text{fixed}) = \frac{0.05}{50 \times 10^{-6}} \Rightarrow R_1(\text{fixed}) = 1 \text{ k}\Omega$$

$$A_d = \frac{R_4}{R_3} \left( 1 + \frac{2R_2}{R_1} \right)$$

$$200 = (2.5) \left( 1 + \frac{2R_2}{1} \right) \Rightarrow R_2 = 39.5 \text{ k}\Omega$$

For  $A_d = 5$

$$5 = (2.5) \left[ 1 + \frac{2(39.5)}{R_1} \right] \Rightarrow R_1 = 79 = R_1(\text{var}) + R_1(\text{fixed}) = R_1(\text{var}) + 1$$

$$R_1(\text{var}) = 78 \text{ k}\Omega$$

**9.72**

$$v_o(v_{o1}) = -\frac{R_4}{R'_3} \cdot v_{o1}$$

$$v_o(v_{o2}) = \left( \frac{R_4}{R_3 + R_4} \right) \left( 1 + \frac{R_4}{R'_3} \right) \cdot v_{o2}$$

$$v_o = v_o(v_{o1}) + v_o(v_{o2}) \quad \text{and} \quad v_{o1} = v_{o2} \equiv v_{CM}$$

Then

$$A_{CM} = \frac{v_o}{v_{CM}} = \left( \frac{R_4}{R_3 + R_4} \right) \left( 1 + \frac{R_4}{R'_3} \right) - \left( \frac{R_4}{R'_3} \right)$$

$$R_4 = 2R_3 = 60 \text{ k}\Omega, \quad R_3 = 30 \text{ k}\Omega, \quad R'_3 = 30 \text{ k}\Omega \pm 5\%$$

For  $R'_3 = 30 \text{ k}\Omega - 5\% = 28.5 \text{ k}\Omega$

$$A_{CM} = \left( \frac{60}{60 + 30} \right) \left( 1 + \frac{60}{28.5} \right) - \left( \frac{60}{28.5} \right) = -0.03509$$

For  $R'_3 = 30 \text{ k}\Omega + 5\% = 31.5 \text{ k}\Omega$

$$A_{CM} = \left( \frac{60}{60 + 30} \right) \left( 1 + \frac{60}{31.5} \right) - \left( \frac{60}{31.5} \right) = +0.03175$$

Then  $-0.03509 \leq A_{CM} \leq +0.03175$

**9.73**

(a)  $R_1 C_2 = (20 \times 10^3)(0.02 \times 10^{-6}) = 4 \times 10^{-4}$  s

$$v_o = \frac{-1}{4 \times 10^{-4}} \int (0.25) \cos \omega t dt = \frac{-0.25}{(4 \times 10^{-4})\omega} \cdot \sin \omega t$$

For  $|v_o| = 0.25 \Rightarrow (4 \times 10^{-4})(2\pi f) = 1 \Rightarrow f = 398$  Hz

Phase =  $90^\circ$

$$(b) \text{ (i) } |v_o| = 1.5 = \frac{0.25}{2\pi f(4 \times 10^{-4})} \Rightarrow f = 66.3 \text{ Hz}$$

$$\text{(ii) } |v_o| = 0.15 = \frac{0.25}{2\pi f(4 \times 10^{-4})} \Rightarrow f = 663 \text{ Hz}$$

**9.74**

$$(a) \ v_o = \frac{-1}{R_1 C_2} \int_0^t v_I(t') dt' = \frac{-1}{R_1 C_2} (0.25)t' \Big|_0^{1.2}$$

$$-5 = \frac{-(0.25)(1.2)}{R_1 C_2} \Rightarrow R_1 C_2 = 60 \text{ ms}$$

$$(b) \text{ (i) } 0 = -5 + \frac{(0.10)}{0.06} \cdot t'' \Rightarrow t'' = 3 \text{ s}, \ t = 4.2 \text{ s}$$

$$\text{(ii) } 5 = -5 + \frac{(0.10)}{0.06} \cdot t'' \Rightarrow t'' = 6 \text{ s}, \ t = 7.2 \text{ s}$$

**9.75**

$$(a) \ A_v = \frac{-Z_2}{R_1}, \text{ where } Z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2 \left( \frac{1}{j\omega C_2} \right)}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$A_v = \frac{-R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C_2}$$

$$(b) \text{ At } \omega = 0, \ A_v(0) = \frac{-R_2}{R_1}$$

$$(c) \ |A_v| = \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_2 C_2)^2}}$$

$$\text{Set } 1 + (\omega R_2 C_2)^2 = 2 \Rightarrow \omega = \frac{1}{R_2 C_2} \Rightarrow f = \frac{1}{2\pi R_2 C_2}$$

**9.76**

$$(a) \ R_1 = 20 \text{ k}\Omega$$

$$\frac{R_2}{R_1} = 15 \Rightarrow R_2 = 300 \text{ k}\Omega$$

$$\omega = \frac{1}{R_2 C_2} = 2\pi f$$

$$C_2 = \frac{1}{2\pi f R_2} = \frac{1}{2\pi (5 \times 10^3) (300 \times 10^3)} \Rightarrow C_2 = 106 \text{ pF}$$

(b)  $R_1 = 15 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 375 \text{ k}\Omega$$

$$C_2 = \frac{1}{2\pi(15 \times 10^3)(375 \times 10^3)} \Rightarrow C_2 = 28.3 \text{ pF}$$

**9.77**

(a)  $A_v = \frac{-R_2}{Z_1}$ , where  $Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$

$$A_v = \frac{-j\omega R_2 C_1}{1 + j\omega R_1 C_1} = \frac{-R_2}{R_1} \cdot \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$$

(b) As  $\omega \Rightarrow \infty$ ,  $A_v = \frac{-R_2}{R_1}$

(c)  $|A_v| = \frac{R_2}{R_1} \cdot \frac{\omega R_1 C_1}{\sqrt{1 + (\omega R_1 C_1)^2}}$

$$\text{Set } \frac{\omega R_1 C_1}{\sqrt{1 + (\omega R_1 C_1)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega = \frac{1}{R_1 C_1} \Rightarrow f = \frac{1}{2\pi R_1 C_1}$$

**9.78**

(a) Set  $R_2 = 350 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 15 \Rightarrow R_1 = 23.33 \text{ k}\Omega$$

$$2\pi f = \frac{1}{R_1 C_1} \Rightarrow C_1 = \frac{1}{2\pi R_1 f} = \frac{1}{2\pi(23.33 \times 10^3)(20 \times 10^3)} \Rightarrow C_1 = 341 \text{ pF}$$

(b) Set  $R_1 = 20 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 500 \text{ k}\Omega$$

$$C_1 = \frac{1}{2\pi(20 \times 10^3)(35 \times 10^3)} \Rightarrow C_1 = 227 \text{ pF}$$

**9.79**

Assuming the Zener diode is in breakdown,

$$v_o = -\frac{R_2}{R_1} \cdot V_z = -\frac{1}{1}(6.8) \Rightarrow v_o = -6.8 \text{ V}$$

$$i_2 = \frac{0 - v_o}{R_2} = \frac{0 - (-6.8)}{1} \Rightarrow i_2 = 6.8 \text{ mA}$$

$$i_z = \frac{10 - V_z}{R_3} - i_2 = \frac{10 - 6.8}{5.6} - 6.8 \Rightarrow i_z = -6.2 \text{ mA!!!}$$

Circuit is not in breakdown. Now



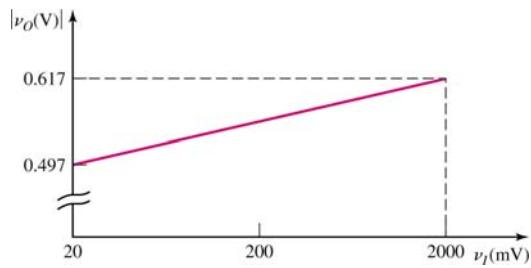
$$\frac{10-0}{R_s + R_1} = i_2 = \frac{10}{5.6+1} \Rightarrow i_2 = 1.52 \text{ mA}$$
$$v_o = -i_2 R_2 = -(1.52)(1) \Rightarrow v_o = -1.52 \text{ V}$$
$$i_z = 0$$

**9.80**

$$v_o = -V_T \ln\left(\frac{v_i}{I_s R_1}\right) = -(0.026) \ln\left[\frac{v_i}{(10^{-14})(10^4)}\right] \Rightarrow v_o = -0.026 \ln\left(\frac{v_i}{10^{-10}}\right)$$

For  $v_i = 20 \text{ mV}$ ,  $|v_o| = 0.497 \text{ V}$

For  $v_i = 2 \text{ V}$ ,  $|v_o| = 0.617 \text{ V}$



**9.81**

$$v_0 = \left(\frac{333}{20}\right)(v_{01} - v_{02}) = 16.65(v_{01} - v_{02})$$

$$v_{01} = -v_{BE1} = -V_T \ln\left(\frac{i_{C1}}{I_S}\right)$$

$$v_{02} = -v_{BE2} = -V_T \ln\left(\frac{i_{C2}}{I_S}\right)$$

$$v_{01} - v_{02} = -V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right) = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right)$$

$$i_{C2} = \frac{v_2}{R_2}, \quad i_{C1} = \frac{v_1}{R_1}$$

$$\text{So } v_{01} - v_{02} = V_T \ln\left(\frac{v_2}{R_2} \cdot \frac{R_1}{v_1}\right)$$

Then

$$v_0 = (16.65)(0.026) \ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$v_0 = 0.4329 \ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$\begin{aligned} \ln(x) &= \log_e(x) = [\log_{10}(x)] \cdot [\log_e(10)] \\ &= 2.3026 \log_{10}(x) \end{aligned}$$

$$\text{Then } v_0 \cong \underline{\underline{(1.0) \log_{10}\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)}}$$

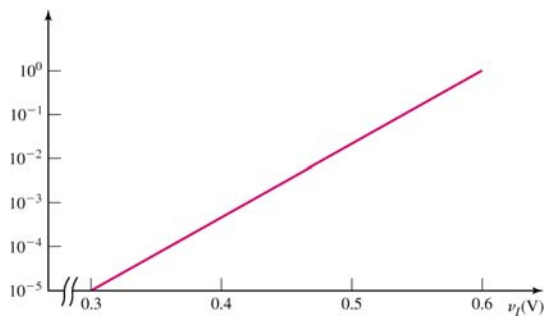
**9.82**

$$v_o = -I_s R (e^{v_i/V_T}) = -(10^{-14})(10^4) e^{v_i/V_T}$$

$$|v_o| = (10^{-10}) e^{v_i/0.026}$$

$$\text{For } v_i = 0.30 \text{ V, } |v_o| = 1.03 \times 10^{-5} \text{ V}$$

$$\text{For } v_i = 0.60 \text{ V, } |v_o| = 1.05 \text{ V}$$



**9.83**

From Figure 9.40

$$v_O = -\frac{R_F}{R_1} \cdot v_{I2} - \frac{R_F}{R_2} \cdot v_{I4} + \left(1 + \frac{R_F}{R_N}\right) \left[ \frac{R_P}{R_A} \cdot v_{I1} + \frac{R_P}{R_B} \cdot v_{I3} \right]$$

$$= -10v_{I2} - v_{I4} + 2v_{I1} + 3v_{I3}$$

Then  $\frac{R_F}{R_1} = 10$ ,  $\frac{R_F}{R_2} = 1$ ,  $\left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_A}\right) = 2$ ,  $\left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_B}\right) = 3$

Set  $R_F = 500 \text{ k}\Omega$ ,  $R_1 = 50 \text{ k}\Omega$ ,  $R_2 = 500 \text{ k}\Omega$

Now  $R_N = R_1 \parallel R_2 = 50 \parallel 500 = 45.45 \text{ k}\Omega$

Then  $\left(1 + \frac{500}{45.45}\right) \left(\frac{R_P}{R_A}\right) = 12 \left(\frac{R_P}{R_A}\right) = 2$ , Also  $\left(1 + \frac{500}{45.45}\right) \left(\frac{R_P}{R_B}\right) = 12 \left(\frac{R_P}{R_B}\right) = 3$

Let  $R_A = 500 \text{ k}\Omega$ , then  $R_B = \frac{2}{3} R_A = 333.3 \text{ k}\Omega$

Then  $R_P = 83.33 \text{ k}\Omega = R_A \parallel R_B \parallel R_C$

We find  $R_A \parallel R_B = 500 \parallel 333.3 = 200 \text{ k}\Omega$

So  $200 \parallel R_C = 83.33 \Rightarrow R_C = 142.8 \text{ k}\Omega$

**9.84**

$$v_O = \left(1 + \frac{R_F}{R_N}\right) \left[ \frac{R_P}{R_A} \cdot v_{I1} + \frac{R_P}{R_B} \cdot v_{I2} + \frac{R_P}{R_C} \cdot v_{I3} \right] - \frac{R_F}{R_1} \cdot v_{I4} - \frac{R_F}{R_2} \cdot v_{I5}$$

$$= 3v_{I1} + 1.5v_{I2} + 2v_{I3} - 4v_{I4} - 6v_{I5}$$

We have  $\frac{R_F}{R_1} = 4$ ,  $\frac{R_F}{R_2} = 6$ ; Set  $R_F = 250 \text{ k}\Omega$ ,  $R_1 = 62.5 \text{ k}\Omega$ ,  $R_2 = 41.67 \text{ k}\Omega$

Now  $R_N = R_1 \parallel R_2 = 62.5 \parallel 41.67 = 25 \text{ k}\Omega$

Also  $\left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{250}{25}\right) = 11$

Now  $\frac{(11)R_P}{R_A} = 3$ ,  $\frac{(11)R_P}{R_B} = 1.5$ ,  $\frac{(11)R_P}{R_C} = 2 \Rightarrow \frac{R_A}{R_B} = \frac{1}{2}$ ,  $\frac{R_A}{R_C} = \frac{2}{3}$

Set  $R_B = 250 \text{ k}\Omega$ ,  $R_A = 125 \text{ k}\Omega$ ,  $R_C = 187.5 \text{ k}\Omega$

This yields  $R_P = 34.09 \text{ k}\Omega$ , We have  $R_P = R_A \parallel R_B \parallel R_C \parallel R_D$

We find  $R_A \parallel R_B \parallel R_C = 125 \parallel 250 \parallel 187.5 = 57.69 \text{ k}\Omega$

Then  $57.69 \parallel R_D = 34.09 \Rightarrow R_D = 83.3 \text{ k}\Omega$

**9.85**

$$\frac{V_O}{V_Z} = \left(1 + \frac{R_2}{R_1}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_1} = 1.143$$

$$I_F = \frac{V_O - V_Z}{R_F}; \text{ Set } I_F = I_Z(\text{min}) = 1.2 \text{ mA}$$

Then  $R_F = \frac{12 - 5.6}{1.2} = 5.33 \text{ k}\Omega$

Set  $I_{D1} = 0.15 \text{ mA}$

$$V'_Z = V_Z + V_\gamma = 5.6 + 0.7 = 6.3 \text{ V}$$

$$\text{Let } I_4 = 0.2 \text{ mA, } \Rightarrow R_4 = \frac{6.3}{0.2} = 31.5 \text{ k}\Omega$$

$$\text{Then } I_3 = 0.2 + 0.15 = 0.35 \text{ mA}$$

$$\text{So } R_3 = \frac{V_S - V'_Z}{I_3} = \frac{10 - 6.3}{0.35} = 10.6 \text{ k}\Omega$$

**9.86**

$$R_1 = \frac{V_O - V_Z}{I_Z} = \frac{12 - 5.6}{2} = 3.2 \text{ k}\Omega$$

$$\frac{V_O}{V_Z} = \left(1 + \frac{R_2}{R_3}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_3} = 1.143$$

$$\text{Let } I_R = 2 \text{ mA, } \Rightarrow R_2 + R_3 = \frac{V_O}{I_R} = \frac{12}{2} = 6 \text{ k}\Omega$$

$$\text{Then } 1.143R_3 + R_3 = 6, \Rightarrow R_3 = 2.8 \text{ k}\Omega \text{ and } R_2 = 3.2 \text{ k}\Omega$$

$$\text{Let } I_{R4} = 4 \text{ mA, } R_4 = \frac{V_{IN} - V_O}{I_{R4}} = \frac{15 - 12}{4} = 0.75 \text{ k}\Omega$$

**9.87**

$$\text{Let } R_1 = R_2 = R_3 = 20 \text{ k}\Omega$$

$$\text{Let } R_T = 20(1 + \delta) \text{ k}\Omega$$

$$\text{Now } v_{O1} = v_A = \left(\frac{R_3}{R_3 + R_1}\right) \cdot V^+ = \frac{1}{2}(10) = 5 \text{ V}$$

$$v_{O2} = v_B = \left(\frac{R_T}{R_T + R_2}\right) \cdot V^+ = \left[\frac{20(1 + \delta)}{20(1 + \delta) + 20}\right](10) = \frac{10(1 + \delta)}{2 + \delta}$$

$$\text{So } v_{OA} = v_A - v_B = 5 - \frac{10(1 + \delta)}{2 + \delta} = \frac{5(2 + \delta) - 10(1 + \delta)}{2 + \delta} \cong -\frac{5\delta}{2} = -2.5\delta$$

$$\text{We have } \delta = y \left(\frac{T - 300}{300}\right); \text{ At } T = 350, R_T = 21 \text{ k}\Omega, \Rightarrow 21 = 20(1 + \delta) \Rightarrow \delta = 0.05$$

$$\text{Then } 0.05 = y \left(\frac{350 - 300}{300}\right) \Rightarrow y = 0.30$$

$$\text{For } \delta = 0.05, |v_{OA}| = (2.5)(0.05) = 0.125 \text{ V}$$

$$\text{For the instrumentation amplifier, } v_o = 5 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right) (0.125)$$

$$\text{For example, set } \frac{R_4}{R_3} = 4 \text{ and } \frac{R_2}{R_1} = 4.5$$

**9.88**

$$(a) \quad v_A = \left( \frac{R - \Delta R}{R - \Delta R + R + \Delta R} \right) \cdot V^+ = \left( \frac{R - \Delta R}{2R} \right) \cdot V^+$$

$$v_B = \left( \frac{R + \Delta R}{R + \Delta R + R - \Delta R} \right) \cdot V^+ = \left( \frac{R + \Delta R}{2R} \right) \cdot V^+$$

$$v_{o1} = v_A - v_B = \left[ \frac{R - \Delta R}{2R} - \frac{R + \Delta R}{2R} \right] \cdot V^+ = -\frac{\Delta R}{R} \cdot V^+ = -\Delta R \left( \frac{9}{20 \times 10^3} \right)$$

$$\text{or } v_{o1} = -(4.5 \times 10^{-4}) (\Delta R)$$

(b) For an instrumentation amplifier,

$$v_o = \frac{R_4}{R_3} \left( 1 + \frac{2R_2}{R_1} \right) \cdot v_{o1}$$

$$\text{For } \Delta R = 200 \Omega, \quad v_o = -5 \text{ V}$$

$$-5 = \frac{R_4}{R_3} \left( 1 + \frac{2R_2}{R_1} \right) (-4.5 \times 10^{-4}) (200)$$

$$\text{or } 55.55 = \frac{R_4}{R_3} \left( 1 + \frac{2R_2}{R_1} \right)$$

$$\text{For example, set } \frac{R_4}{R_3} = 6 \quad \text{and} \quad \frac{R_2}{R_1} = 4.13$$

## Chapter 10

### 10.1

$$I_1 = I_2 = \frac{0 - 2V_\gamma - V^-}{R_1 + R_2}$$

$$2V_\gamma + I_2 R_2 = V_{BE} + I_C R_3$$

$$2V_\gamma + \frac{R_2}{R_1 + R_2}(-2V_\gamma - V^-) = V_{BE} + I_C R_3$$

$$I_C = \frac{1}{R_3} \left\{ 2V_\gamma - (2V_\gamma + V^-) \left( \frac{R_2}{R_1 + R_2} \right) - V_{BE} \right\}$$

a.

$$V_\gamma = V_{BE} \text{ and } R_1 = R_2$$

$$I_C = \frac{1}{R_3} \left\{ 2V_\gamma - \frac{1}{2}(2V_\gamma + V^-) - V_{BE} \right\}$$

$$\text{or } I_C = \frac{-V^-}{2R_3}$$

b.

$$I_C = 2 \text{ mA} = \frac{-(-10)}{2R_3} \Rightarrow R_3 = 2.5 \text{ k}\Omega$$

$$I_1 = I_2 = 2 \text{ mA} = \frac{-2(0.7) - (-10)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 4.3 \text{ k}\Omega \Rightarrow \underline{R_1 = R_2 = 2.15 \text{ k}\Omega}$$

c.

### 10.2

(a) (i)  $I_O = 50 \mu\text{A}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{50 \times 10^{-6}}{10^{-16}} \right) = 0.7004 \text{ V}$

(ii)  $I_O = 150 \mu\text{A}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{150 \times 10^{-6}}{10^{-16}} \right) = 0.7289 \text{ V}$

(iii)  $I_O = 1.5 \text{ mA}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{1.5 \times 10^{-3}}{10^{-16}} \right) = 0.7888 \text{ V}$

(b) (i)  $I_O = \frac{50}{1 + \frac{2}{50}} = 48.08 \mu\text{A}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{48.0769 \times 10^{-6}}{10^{-16}} \right) = 0.6994 \text{ V}$

(ii)  $I_O = \frac{150}{1 + \frac{2}{50}} = 144.23 \mu\text{A}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{144.23 \times 10^{-6}}{10^{-16}} \right) = 0.7279 \text{ V}$

(iii)  $I_O = \frac{1.5}{1 + \frac{2}{50}} = 1.4423 \text{ mA}$ ,  $V_{BE1} = (0.026) \ln \left( \frac{1.4423 \times 10^{-3}}{10^{-16}} \right) = 0.7878 \text{ V}$

**10.3**

$$I_{C1} \cong \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{200}{1 + \frac{2}{80}} = 195.12 \mu\text{ A}$$

$$V_{BE1} = V_{BE2} = (0.026) \ln \left( \frac{195.12 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.6341 \text{ V}$$

$$I_O = I_{S2} \exp \left( \frac{V_{BE2}}{V_T} \right) = (2 \times 10^{-15}) \exp \left( \frac{0.6341}{0.026} \right) \Rightarrow I_O = 78.05 \mu\text{ A}$$

**10.4**

$$I_{C1} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{150}{1 + \frac{2}{120}} = 147.54 \mu\text{ A}$$

$$V_{BE1} = V_{BE2} = (0.026) \ln \left( \frac{147.54 \times 10^{-6}}{10^{-16}} \right) = 0.7285 \text{ V}$$

$$I_O = I_{S2} \exp \left( \frac{V_{BE2}}{V_T} \right) = (3 \times 10^{-16}) \exp \left( \frac{0.7285}{0.026} \right) \Rightarrow I_O = 0.4426 \text{ mA}$$

**10.5**

Approximation:  $I_{C1} \cong \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{200}{1 + \frac{2}{40}} = 190.48 \mu\text{ A}$

$$V_{BE1} = (0.026) \ln \left( \frac{190.48 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.63345 \text{ V}$$

$$I_{B2} \cong \frac{I_{C1}}{\beta} = \frac{190.48}{40} = 4.762 \mu\text{ A}$$

$$V_{BE2} = V_{BE1} - I_{B2} R = 0.63345 - (0.004762)(2) = 0.62393 \text{ V}$$

$$I_O = (5 \times 10^{-15}) \exp \left( \frac{0.62393}{0.026} \right) \Rightarrow I_O = 132.07 \mu\text{ A}$$

**10.6**

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} \Rightarrow 0.250 = \frac{3 - 0.7 - (-3)}{R_1}$$

$$\underline{R_1 = 21.2 \text{ K}}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.250}{1 + \frac{2}{60}} \Rightarrow I_{C1} = I_{C2} = 0.2419 \text{ mA}$$

$$I_{B1} = I_{B2} = 4.03 \mu\text{ A}$$

**10.7**

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{18.3}$$

$$I_{REF} = 0.5082 \text{ mA}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.5082}{1 + \frac{2}{80}} \Rightarrow I_{C1} = I_{C2} = 0.4958 \text{ mA}$$

$$I_{B1} = I_{B2} = (6.198 \text{ } \mu\text{A})$$

**10.8**

$$(a) P = (I_O + I_{REF})(V^+ - V^-)$$

$$1.8 = (0.25 + I_{REF})(5) \Rightarrow I_{REF} = 0.11 \text{ mA}$$

$$V_{BE1} = V_{BE2} = (0.026) \ln\left(\frac{0.25 \times 10^{-3}}{10^{-15}}\right) = 0.68236 \text{ V}$$

$$(b) 0.11 \times 10^{-3} = I_{S1} \exp\left(\frac{0.68236}{0.026}\right) \Rightarrow I_{S1} = 4.4 \times 10^{-16} \text{ A}$$

$$(c) R_1 = \frac{5 - 0.68236}{0.11} = 39.25 \text{ k}\Omega$$

**10.9**

$$(a) I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_O = \left(1 + \frac{2}{120}\right)(0.5) = 0.5083 \text{ mA}$$

$$R_1 = \frac{5 - 0.7 - (-5)}{0.5083} = 18.3 \text{ k}\Omega$$

$$(b) R_o = \frac{V_A}{I_O} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$\Delta I_O = \frac{\Delta V_{CE2}}{R_o} = \frac{7 - 0.7}{200} = 0.0315 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.0315}{0.5} \Rightarrow 6.3\%$$

**10.10**

$$I_0 = nI_{C1}$$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_0}{\beta}$$

$$I_{REF} = I_{C1} \left(1 + \frac{1}{\beta} + \frac{n}{\beta}\right) = I_{C1} \left(1 + \frac{1+n}{\beta}\right)$$

$$= \frac{I_0}{n} \left(1 + \frac{1+n}{\beta}\right) \text{ or } I_0 = \frac{nI_{REF}}{\left(1 + \frac{1+n}{\beta}\right)}$$



**10.11**

$$I_O = \frac{I_{REF}}{1 + \frac{2}{\beta}} \Rightarrow I_{REF} = (0.20) \left( 1 + \frac{2}{40} \right) = 0.210 \text{ mA}$$

$$R_1 = \frac{5 - 0.7}{I_{REF}} = \frac{4.3}{0.21} \Rightarrow \underline{R_1 = 20.5 \text{ K}}$$

**10.12**

(a)  $I_{REF} = \left( 1 + \frac{2}{\beta} \right) \cdot I_O = \left( 1 + \frac{2}{80} \right) (0.12) = 0.123 \text{ mA}$

$$R_1 = \frac{5 - 0.7}{0.123} = 34.96 \text{ k}\Omega$$

(b)  $r_o = \frac{V_A}{I_O} = \frac{80}{0.12} = 666.7 \text{ k}\Omega$

(i)  $\Delta I_O = \frac{\Delta V_{EC}}{r_o} = \frac{2 - 0.7}{666.7} \Rightarrow \Delta I_O = 1.95 \mu\text{A}$

(ii)  $\Delta I_O = \frac{4 - 0.7}{666.7} \Rightarrow \Delta I_O = 4.95 \mu\text{A}$

**10.13**

a.  $I_{REF} = 1 = \frac{5 - 0.7 - (-5)}{R_1} \Rightarrow \underline{R_1 = 9.3 \text{ k}\Omega}$

b.  $I_0 = 2I_{REF} \Rightarrow \underline{I_0 = 2 \text{ mA}}$

c. For  $V_{EC2}(\text{min}) = 0.7 \Rightarrow R_{C2} = \frac{5 - 0.7}{2} \Rightarrow \underline{R_{C2} = 2.15 \text{ k}\Omega}$

**10.14**

$$I_O = 0.50 \text{ mA} \Rightarrow I_{OA} = I_{OB} = 0.25 \text{ mA}$$

$$I_{REF} = I_{OA} \left( 1 + \frac{3}{\beta} \right) = 0.25 \left( 1 + \frac{3}{60} \right)$$

$$I_{REF} = 0.2625 \text{ mA}$$

$$R_1 = \frac{2.5 - 0.7}{0.2625} \Rightarrow \underline{R_1 = 6.86 \text{ K}}$$

**10.15**

Similar to Figure P10.14 biased at  $V^+$  and  $V^-$ .

$$R_1 = \frac{2.5 - 0.7 - (-2.5)}{0.2} = 21.5 \text{ k}\Omega$$

**10.16**

$$I_2 = 2I_1 \text{ and } I_3 = 3I_1$$

(a)  $I_2 = 1.0 \text{ mA}, I_3 = 1.5 \text{ mA}$

(b)  $I_1 = 0.25 \text{ mA}, I_3 = 0.75 \text{ mA}$

(c)  $I_1 = 0.167 \text{ mA}, I_2 = 0.333 \text{ mA}$

**10.17**

a.

$$I_0 = I_{C1} \text{ and } I_{REF} = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{1 + \beta}$$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1 + \beta)} + \frac{V_{BE}}{(1 + \beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2} = I_0 \left( 1 + \frac{2}{\beta(1 + \beta)} \right)$$

$$I_0 = \frac{I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2}}{\left( 1 + \frac{2}{\beta(1 + \beta)} \right)}$$

$$I_{REF} = (0.70) \left( 1 + \frac{2}{(80)(81)} \right) + \frac{0.7}{(81)(10)}$$

$$I_{REF} = 0.700216 + 0.000864$$

$$\underline{I_{REF} = 0.7011 \text{ mA}} = \frac{10 - 2(0.7)}{R_1} \Rightarrow \underline{R_1 = 12.27 \text{ k}\Omega}$$

b.

**10.18**

a.

$$I_{0i} = I_{CR} \text{ and } I_{REF} = I_{CR} + I_{BS} = I_{CR} + \frac{I_{ES}}{1 + \beta}$$

$$\begin{aligned} I_{ES} &= I_{BR} + I_{B1} + I_{B2} + \dots + I_{BN} = (1 + N)I_{BR} \\ &= \frac{(1 + N)I_{CR}}{\beta} \end{aligned}$$

$$\text{Then } I_{REF} = I_{CR} + \frac{(1 + N)I_{CR}}{\beta(1 + \beta)}$$

$$\text{or } \underline{I_{0i} = \frac{I_{REF}}{\left( 1 + \frac{(1 + N)}{\beta(1 + \beta)} \right)}}$$

$$I_{REF} = (0.5) \left[ 1 + \frac{6}{(50)(51)} \right] = 0.5012 \text{ mA}$$

b.

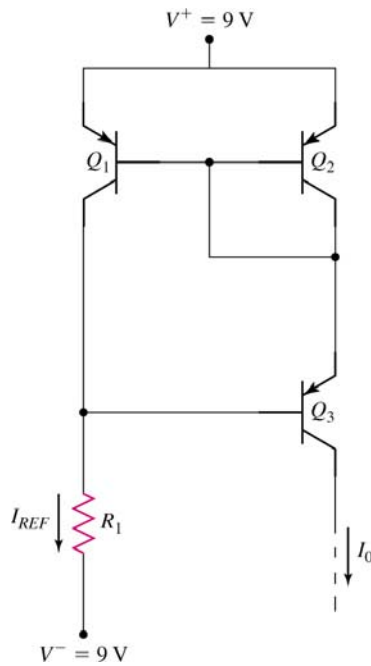
$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5012} \Rightarrow \underline{R_1 = 17.16 \text{ k}\Omega}$$

**10.19**

$$I_{REF} = I_O \left[ 1 + \frac{2}{\beta(1+\beta)} \right] = (0.15) \left[ 1 + \frac{2}{(40)(41)} \right] = 0.15018 \text{ mA}$$

$$R_1 = \frac{3 - 0.7 - 0.7 - (-3)}{0.15018} = 30.63 \text{ k}\Omega$$

**10.20**



$$I_O = I_{REF} \cdot \frac{1}{\left( 1 + \frac{2}{\beta(2+\beta)} \right)}$$

For  $I_O = 0.8 \text{ mA}$

$$I_{REF} = (0.8) \left( 1 + \frac{2}{25(27)} \right) \Rightarrow \underline{I_{REF} = 0.8024 \text{ mA}}$$

$$R_1 = \frac{18 - 2(0.7)}{0.8024} \Rightarrow \underline{R_1 = 20.69 \text{ k}\Omega}$$

**10.21**

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2 + \beta)}\right)}$$

**10.22**

(a)  $I_O = 0.5 \text{ mA}$ ,  $I_{B2} = \frac{0.5}{90} = 0.005556 \text{ mA}$

$$I_{B1} = \frac{I_{B2}}{2} = 0.002778 \text{ mA}$$

$$I_{E3} = I_{B1} + I_{B2} = 0.008334 \text{ mA}$$

$$I_{B3} = \frac{0.008334}{61} = 0.0001366 \text{ mA}$$

$$I_{C1} = \frac{I_O}{2} = 0.25 \text{ mA}$$

$$I_{REF} = I_{C1} + I_{B3} = 0.2501366 \text{ mA}$$

$$R_1 = \frac{5 - 0.6 - 0.7 - (-5)}{0.2501366} = 34.78 \text{ k}\Omega$$

(b)  $I_{B1} = 0.002778 \text{ mA}$

$$I_{B2} = 0.005556 \text{ mA}$$

$$I_{E3} = 0.008334 \text{ mA}$$

$$I_{B3} = 0.0001366 \text{ mA}$$

**10.23**

(a)

Assuming  $R_O \approx \frac{\beta r_{o3}}{2}$

$$r_{o3} = \frac{V_A}{I_O} = \frac{V_A}{I_{REF}} = \frac{100}{0.25} = 400 \text{ K}$$

$$R_O = \frac{(100)(400)}{2} \Rightarrow \underline{R_O = 20 \text{ M}\Omega}$$

(b)

$$R_O = \frac{\Delta V}{\Delta I_O} \Rightarrow \Delta I_O = \frac{\Delta V}{20 \text{ M}\Omega} = \frac{5}{20 \text{ M}\Omega}$$

$$\underline{\Delta I_O = 0.25 \mu\text{A}}$$

**10.24**

$$I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_1} = \frac{5 - 0.7}{9.3}$$

$$I_{REF} = 0.4624 \text{ mA}$$

$$I_O = \frac{V_T}{R_E} \ln\left(\frac{I_{REF}}{I_O}\right) = \frac{0.026}{1.5} \ln\left(\frac{0.4624}{I_O}\right)$$

$$I_O = 0.01733 \ln\left(\frac{0.4624}{I_O}\right)$$

By trial and error

$$I_O = 41.7 \mu\text{A}$$

$$V_{BE2} = 0.7 - I_O R_E = 0.7 - (0.0417)(1.5)$$

$$V_{BE2} = 0.6375 \text{ V}$$

**10.25**

$$(a) V_{BE1} = (0.026) \ln\left(\frac{200 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6347 \text{ V}$$

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

$$I_O(0.5) = (0.026) \ln\left(\frac{0.2}{I_O}\right)$$

$$I_O \cong 61.4 \mu\text{A}$$

$$V_{BE2} = (0.026) \ln\left(\frac{61.4 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6040 \text{ V}$$

$$(b) V_{BE1} = (0.026) \ln\left(\frac{200 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6347 \text{ V}$$

$$V_{BE1} - V_{BE2} = I_O R_E$$

$$V_T \left[ \ln\left(\frac{I_{REF}}{I_{S1}}\right) - \ln\left(\frac{I_O}{I_{S2}}\right) \right] = I_O R_E$$

$$V_T \ln\left(\frac{I_{REF}}{I_O} \cdot \frac{I_{S2}}{I_{S1}}\right) = I_O R_E$$

$$(0.026) \ln\left[\left(\frac{0.2}{I_O}\right)\left(\frac{7}{5}\right)\right] = I_O(0.5)$$

$$I_O \cong 71.2 \mu\text{A}$$

$$V_{BE2} = (0.026) \ln\left(\frac{71.2 \times 10^{-6}}{7 \times 10^{-15}}\right) = 0.5991 \text{ V}$$

**10.26**

(a)  $V_{BE1} = (0.026) \ln \left( \frac{100 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.61669 \text{ V}$   
 $V_{BE2} = V_{BE1} + I_{REF} R_E = 0.61669 + (0.1)(0.7) = 0.68669 \text{ V}$   
 $I_O = (5 \times 10^{-15}) \exp \left( \frac{0.68669}{0.026} \right) \Rightarrow I_O = 1.477 \text{ mA}$

(b)  $V_{BE1} = 0.61669 \text{ V}$   
 $V_{BE2} = 0.68669 \text{ V}$   
 $I_O = (2 \times 10^{-15}) \exp \left( \frac{0.68669}{0.026} \right) \Rightarrow I_O = 0.5906 \text{ mA}$

**10.27**

(a)  $I_{REF} = \frac{5 - 0.7 - (-5)}{50} = 0.186 \text{ mA}$   
 $I_O R_E = V_T \ln \left( \frac{I_{REF}}{I_O} \right)$   
 $I_O (3) = (0.026) \ln \left( \frac{0.186}{I_O} \right)$   
 $I_O \cong 19.53 \mu \text{ A}$   
 $V_{BE2} = V_{BE1} - I_O R_E = 0.7 - (0.01953)(3) = 0.6414 \text{ V}$

(b)  $r_{o2} = \frac{V_A}{I_O} = \frac{80}{0.01953} \Rightarrow r_{o2} = 4.096 \text{ M}\Omega$   
 $g_{m2} = \frac{0.01953}{0.026} = 0.7512 \text{ mA/V}$   
 $r_{\pi 2} = \frac{(120)(0.026)}{0.01953} = 159.8 \text{ k}\Omega$   
 $R_E \parallel r_{\pi 2} = 3 \parallel 159.8 = 2.945 \text{ k}\Omega$   
 $R_o = (4.096) [1 + (0.7512)(2.945)] = 13.16 \text{ M}\Omega$

**10.28**

$R_o = r_{o2} [1 + g_{m2} (R_E \parallel r_{\pi 2})]$   
 From 10.27,  $I_O = 19.53 \mu \text{ A}$   
 $g_{m2} = \frac{I_O}{V_T} = \frac{0.01953}{0.026} = 0.7512 \text{ mA/V}$   
 $r_{\pi 2} = \frac{\beta V_T}{I_O} = \frac{(80)(0.026)}{0.01953} = 106.5 \text{ k}\Omega$   
 $r_{o2} = \frac{V_A}{I_O} = \frac{80}{0.01953} \Rightarrow 4.096 \text{ M}\Omega$   
 Then  $R_o = (4.096) [1 + (0.7512)(3 \parallel 106.5)] = 13.07 \text{ M}\Omega$   
 $\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{5}{13.07} = 0.382 \mu \text{ A}$

**10.29**

$$I_{REF} = \frac{5 - 0.7 - (-5)}{R_1} = 0.50$$

$$R_1 = 18.6 \text{ K}$$

$$I_O R_E = V_T \ln \left( \frac{I_{REF}}{I_O} \right)$$

$$R_E = \frac{0.026}{0.050} \ln \left( \frac{0.50}{0.050} \right)$$

(a)  $R_E = 1.20 \text{ K}$

$$R_O = r_{c2} [1 + R'_E g_{m2}]$$

$$R'_E = R_E \parallel r_{\pi 2}$$

$$r_{\pi 2} = \frac{(75)(0.026)}{0.050} = 39 \text{ K} \quad g_{m2} = \frac{0.050}{0.026} = 1.923 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega \quad R'_E = 1.20 \parallel 39 = 1.164 \text{ K}$$

(b)  $R_O = 2 [1 + (1.164)(1.923)] \Rightarrow R_O = (6.477) \text{ M}\Omega$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{5}{6.477} = 0.772 \text{ }\mu\text{A}$$

(c)  $\frac{\Delta I_O}{I_O} \times 100\% = \frac{0.772}{50} \times 100 = \underline{1.54\%}$

**10.30**

Let  $R_1 = 10 \text{ k}\Omega$

Then  $I_{REF} = \frac{3 - 0.7 - (-3)}{10} = 0.53 \text{ mA}$

$$R_E = \frac{V_T}{I_O} \ln \left( \frac{I_{REF}}{I_O} \right) = \frac{(0.026)}{(0.05)} \ln \left( \frac{0.53}{0.05} \right) \Rightarrow R_E = 1.228 \text{ k}\Omega$$

**10.31**

$$V_{BE} = V_T \ln \left( \frac{I_{REF}}{I_S} \right)$$

$$0.7 = (0.026) \ln \left( \frac{10^{-3}}{I_S} \right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

$$\begin{aligned} \text{At } 2 \text{ mA, } V_{BE} &= (0.026) \ln \left( \frac{2 \times 10^{-3}}{2.03 \times 10^{-15}} \right) \\ &= 0.718 \text{ V} \end{aligned}$$

$$R_1 = \frac{15 - 0.718}{2} \Rightarrow R_1 = 7.14 \text{ k}\Omega$$

$$R_E = \frac{V_T}{I_0} \ln \left( \frac{I_{REF}}{I_0} \right) = \frac{0.026}{0.050} \cdot \ln \left( \frac{2}{0.050} \right) \Rightarrow R_E = 1.92 \text{ k}\Omega$$

**10.32**

$$(a) \quad I_{REF} = \frac{3 - 0.7 - (-3)}{20} = 0.265 \text{ mA}$$

$$(b) \quad R_E = \frac{V_T}{I_O} \ln\left(\frac{I_{REF}}{I_O}\right) = \frac{(0.026)}{(0.1)} \ln\left(\frac{0.265}{0.1}\right) \Rightarrow R_E = 253 \Omega$$

**10.33**

$$I_{REF} \approx \frac{10 - 0.7 - (-10)}{40} = 0.4825 \text{ mA}$$

$$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026) \ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Now

$$V_{BE} = (0.026) \ln\left(\frac{0.4825 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.681 \text{ V}$$

$$\underline{V_{BE1} = 0.681 \text{ V}}$$

So

$$I_{REF} \cong \frac{10 - 0.681 - (-10)}{40} \Rightarrow \underline{I_{REF} = 0.483 \text{ mA}}$$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0 (12) = (0.026) \ln\left(\frac{0.483}{I_0}\right)$$

By trial and error.

$$\Rightarrow \underline{I_0 \cong 8.7 \mu\text{A}}$$

$$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12) \Rightarrow \underline{V_{BE2} = 0.5766 \text{ V}}$$

**10.34**

$$V_{BE1} + I_{REF} R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF} R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$\text{Then } \underline{V_T \ln\left(\frac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF} R_{E1}}$$

Output resistance looking into the collector of  $Q_2$  is increased.



**10.35**

$$(a) \quad V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.70038 \text{ V}$$

$$V^+ = I_{REF} R_1 + V_{BE1} + I_{REF} R_{E1} + V^-$$

$$\text{Then } R_1 = \frac{3 - 0.70038 - (0.5)(0.5) - (-3)}{0.5} = 10.1 \text{ k}\Omega$$

$$V_{BE2} = (0.026) \ln\left(\frac{0.2 \times 10^{-3}}{10^{-15}}\right) = 0.67656 \text{ V}$$

$$V_{BE1} + I_{REF} R_{E1} = V_{BE2} + I_O R_{E2}$$

$$\text{Then } R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.67656}{0.2} = 1.37 \text{ k}\Omega$$

$$(b) \quad R_1 = 10.1 \text{ k}\Omega$$

$$V_{BE2} = (0.026) \ln\left(\frac{0.2 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.65854 \text{ V}$$

$$R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.65854}{0.2} = 1.46 \text{ k}\Omega$$

**10.36**

Assume all transistors are matched.

a.

$$2V_{BE1} = V_{BE3} + I_0 R_E$$

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE3} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$2V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = I_0 R_E$$

$$V_T \left[ \ln\left(\frac{I_{REF}}{I_S}\right)^2 - \ln\left(\frac{I_0}{I_S}\right) \right] = I_0 R_E$$

$$\underline{V_T \ln\left(\frac{I_{REF}^2}{I_0 I_S}\right) = I_0 R_E}$$

b.

$$V_{BE} = 0.7 \text{ V at } 1 \text{ mA} \Rightarrow 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right) \text{ or } I_S = 2.03 \times 10^{-15} \text{ A}$$

$$V_{BE} \text{ at } 0.1 \text{ mA} \Rightarrow V_{BE} = (0.026) \ln\left(\frac{0.1 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.640 \text{ V}$$

$$\text{Since } I_0 = I_{REF}, \text{ then } V_{BE} = I_0 R_E \Rightarrow R_E = \frac{0.640}{0.1} \text{ or } \underline{R_E = 6.4 \text{ k}\Omega}$$

**10.37**

$$(a) \quad I_{REF} = \frac{5 - 0.7 - (-5)}{10} = 0.93 \text{ mA}$$

$$I_{O2} R_{E2} = V_T \ln \left( \frac{I_{REF}}{I_{O2}} \right)$$

$$I_{O2}(1) = (0.026) \ln \left( \frac{0.93}{I_{O2}} \right)$$

$$I_{O2} \cong 68 \mu \text{ A}$$

$$I_{O3}(2) = (0.026) \ln \left( \frac{0.93}{I_{O3}} \right)$$

$$I_{O3} \cong 40.7 \mu \text{ A}$$

$$(b) \quad R_{E2} = \frac{V_T}{I_{O2}} \ln \left( \frac{I_{REF}}{I_{O2}} \right) = \frac{(0.026)}{(0.02)} \ln \left( \frac{0.93}{0.02} \right) \Rightarrow R_{E2} = 4.99 \text{ k}\Omega$$

$$R_{E3} = \frac{(0.026)}{(0.08)} \ln \left( \frac{0.93}{0.08} \right) \Rightarrow R_{E3} = 0.797 \text{ k}\Omega$$

**10.38**

(a)

$$V_{BE1} = V_{BE2}$$

$$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$$

Now

$$2V_{BE1} + I_{REF} R_2 = V_{BE3} + I_O R_E$$

or

$$\underline{I_O R_E = 2V_{BE1} - V_{BE3} + I_{REF} R_2}$$

We have

$$V_{BE1} = V_T \ln \left( \frac{I_{REF}}{I_S} \right) \text{ and } V_{BE3} = V_T \ln \left( \frac{I_O}{I_S} \right)$$

(b)

$$\text{Let } R_1 = R_2 \text{ and } I_O = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$$

Then

$$V_{BE} = I_O R_E - I_{REF} R_2 = I_O (R_E - R_2)$$

so

$$\begin{aligned} I_{REF} = I_O &= \frac{V^+ - V^- - 2I_O (R_E - R_2)}{2R_2} \\ &= \frac{V^+ - V^-}{2R_2} - I_O \left( \frac{R_E}{R_2} \right) + I_O \end{aligned}$$

Then

$$I_O = \frac{V^+ - V^-}{2R_E}$$

(c)

Want  $I_O = 0.5 \text{ mA}$

$$\text{So } R_E = \frac{5 - (-5)}{2(0.5)} \Rightarrow R_E = 10 \text{ k}\Omega$$

$$2R_2 = \frac{5 - 2(0.7) - (-5)}{0.5} = 17.2 \text{ k}\Omega$$

Then  $R_1 = R_2 = 8.6 \text{ k}\Omega$

### 10.39

a.

$$I_{REF} = \frac{20 - 0.7 - 0.7}{12} = 1.55 \text{ mA}$$

$$I_{O1} = 2I_{REF} = 3.1 \text{ mA}$$

$$I_{O2} = I_{REF} = 1.55 \text{ mA}$$

$$I_{O3} = 3I_{REF} = 4.65 \text{ mA}$$

b.

$$V_{CE1} = -I_{O1}R_{C1} - (-10) = -(3.1)(2) + 10 \Rightarrow V_{CE1} = 3.8 \text{ V}$$

$$V_{EC2} = 10 - I_{O2}R_{C2} = 10 - (1.55)(3) \Rightarrow V_{EC2} = 5.35 \text{ V}$$

$$V_{EC3} = 10 - I_{O3}R_{C3} = 10 - (4.65)(1) \Rightarrow V_{EC3} = 5.35 \text{ V}$$

### 10.40

$$(a) \quad I_{REF} = \frac{10 - 0.7 - 0.7 - (-10)}{24} = 0.775 \text{ mA}$$

$$I_{O1} = 2I_{REF} = 1.55 \text{ mA}$$

$$I_{O2} = I_{REF} = 0.775 \text{ mA}$$

$$I_{O3} = 3I_{REF} = 2.325 \text{ mA}$$

$$(b) \quad R_{C1} = \frac{0 - 0.7 - (-10)}{1.55} = 6 \text{ k}\Omega$$

$$R_{C2} = \frac{10 - 0.7}{0.775} = 12 \text{ k}\Omega$$

$$R_{C3} = \frac{10 - 0.7}{2.325} = 4 \text{ k}\Omega$$

### 10.41

$$I_{C1} = I_{C2} = \frac{10 - 0.7 - 0.7 - (-10)}{10} = 1.86 \text{ mA}$$

$$I_{C3} = I_{C4} = 1.86 \text{ mA}$$

$$I_{C5}(0.5) = 0.026 \ln\left(\frac{1.86}{I_{C5}}\right)$$

By Trial and error.

$$\Rightarrow I_{C5} = 0.136 \text{ mA} = I_{C6} = I_{C7}$$

$$2I_{C3}(0.8) + V_{CE3} = 10 \Rightarrow V_{CE3} = 10 - 2(1.86)(0.8)$$

$$V_{CE3} = 7.02 \text{ V}$$

$$5 = V_{EB6} + V_{CE5} + I_{C5}(0.5) - 10$$

$$V_{CE5} = 5 + 10 - 0.7 - (0.136)(0.5)$$

$$V_{CE5} = 14.2 \text{ V}$$

$$5 = V_{EC7} + I_{C7}(0.8)$$

$$V_{EC7} = 5 - (0.136)(0.8)$$

$$V_{EC7} = 4.89 \text{ V}$$

**10.42**

$$I_{C1} = I_{C2} = \frac{10 - 0.7 - 0.7 - (-10)}{10} \Rightarrow I_{C1} = I_{C2} = 1.86 \text{ mA}$$

$$I_{C4} = I_{C5} = 1.86 \text{ mA}$$

$$I_{C3}R_{E1} = V_T \ln\left(\frac{I_{C1}}{I_{C3}}\right) \Rightarrow I_{C3}(0.3) = 0.026 \ln\left(\frac{1.86}{I_{C3}}\right)$$

By trial and error  $I_{C3} = 0.195 \text{ mA}$

$$I_{C6}R_{E2} = V_T \ln\left(\frac{I_{C5}}{I_{C6}}\right) \Rightarrow I_{C6}(0.5) = 0.026 \ln\left(\frac{1.86}{I_{C6}}\right)$$

By trial and error  $I_{C6} = 0.136 \text{ mA}$

**10.43**

$$I_{REF} = \frac{10 - 0.7}{6.3 + 3} = 1 \text{ mA}$$

$$V_{BE}(Q_R) = 0.7 \text{ V as assumed}$$

$$V_{RER} = I_{REF} \cdot R_{ER} = (1)(3) = 3 \text{ V}$$

$$V_{RE1} = 3 \text{ V} \Rightarrow R_{E1} = \frac{V_{RE1}}{I_{01}} = \frac{3}{1} \Rightarrow R_{E1} = 3 \text{ k}\Omega$$

$$V_{RE2} = 3 \text{ V} \Rightarrow R_{E2} = \frac{V_{RE2}}{I_{02}} = \frac{3}{2} \Rightarrow R_{E2} = 1.5 \text{ k}\Omega$$

$$V_{RE3} = 3 \text{ V} \Rightarrow R_{E3} = \frac{V_{RE3}}{I_{03}} = \frac{3}{4} \Rightarrow R_{E3} = 0.75 \text{ k}\Omega$$

$$I_{01} = 1 \text{ mA}$$

$$I_{02} = 2 \text{ mA}$$

$$I_{03} = 4 \text{ mA}$$

**10.44**

$$I_{REF} = \frac{2.5 - V_{GS}}{15} = \left(\frac{0.08}{2}\right)(6)(V_{GS} - 0.5)^2$$

$$2.5 - V_{GS} = 3.6(V_{GS}^2 - V_{GS} + 0.25)$$

$$3.6V_{GS}^2 - 2.6V_{GS} - 1.6 = 0$$

$$V_{GS} = \frac{2.6 \pm \sqrt{6.76 + 23.04}}{2(3.6)}$$

$$V_{GS} = 1.12 \text{ V} (1.1193)$$

$$I_{REF} = \frac{2.5 - 1.1193}{15} \Rightarrow I_{REF} = 92.0 \mu\text{A} (92.05)$$

$$I_o = 92.0 \mu\text{A}$$

$$V_{DS2}(\text{sat}) = V_{GS} - V_{TN} = 1.1193 - 0.5$$

$$V_{DS2}(\text{sat}) = 0.619 \text{ V}$$

**10.45**

(a)

$$I_{REF} = 50 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS} - 0.5)^2$$

$$I_{REF} = 0.050 = \frac{2.0 - V_{GS}}{R}$$

Design such that  $V_{DS2}(\text{sat}) = 0.25 = V_{GS} - 0.5$

$$V_{GS} = 0.75 \text{ V}$$

$$0.050 = \frac{2 - 0.75}{R} \Rightarrow R = 25 \text{ K}$$

So

$$50 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_1 (0.75 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 20$$

$$\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} = \frac{I_{REF}}{I_o} \Rightarrow \frac{20}{\left(\frac{W}{L}\right)_2} = \frac{50}{100} \Rightarrow \left(\frac{W}{L}\right)_2 = 40$$

(b)  $R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.015)(0.1)} \Rightarrow R_o = 667 \text{ K}$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{1}{666} \Rightarrow 1.5 \mu\text{A}$$

(c)  $\frac{\Delta I_o}{I_o} \times 100\% = \left(\frac{1.5}{100}\right) \times 100\% \Rightarrow 1.5\%$

**10.46**

$$I_{REF} = 250 = \left(\frac{80}{2}\right)(3)(V_{GS} - 1)^2$$

$$V_{GS} = 2.44 \text{ V}$$

$$I_o = 250 \mu\text{A at } V_{DS2} = V_{GS} = 2.44 \text{ V}$$

$$(a) \quad R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.25)} = 200 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 2.44}{200} \Rightarrow 2.8 \mu\text{A}$$

$$(i) \quad \underline{I_o = 252.8 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{4.5 - 2.44}{200} \Rightarrow 10.3 \mu\text{A}$$

$$(ii) \quad \underline{I_o = 260.3 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{6 - 2.44}{200} \Rightarrow 17.8 \mu\text{A}$$

$$(iii) \quad \underline{I_o = 267.8 \mu\text{A}}$$

$$I_o = \frac{4.5}{3}(250) = 375 \mu\text{A at } V_{DS} = 2.44 \text{ V}$$

$$(b) \quad R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.375)} = 133.3 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 2.44}{133.3} \Rightarrow 4.20 \mu\text{A}$$

$$(i) \quad \underline{I_o = 379.2 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{4.5 - 2.44}{133.3} \Rightarrow 15.5 \mu\text{A}$$

$$(ii) \quad \underline{I_o = 390.5 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{6 - 2.44}{133.3} \Rightarrow 26.7 \mu\text{A}$$

$$(iii) \quad \underline{I_o = 401.7 \mu\text{A}}$$

**10.47**

$$(a) \quad I_{REF} = K_{n1}(V_{GS1} - V_{TN1})^2$$

$$0.2 = 0.2(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.4 \text{ V}$$

$$I_o = K_{n2}(V_{GS2} - V_{TN2})^2$$

$$\text{Now } I_o = (0.2 - 0.01)(1.4 - 0.4)^2 = 0.19 \text{ mA}$$

$$I_o = (0.2 + 0.01)(1.4 - 0.4)^2 = 0.21 \text{ mA}$$

$$\text{So } 0.19 \leq I_o \leq 0.21 \text{ mA}$$

$$(b) \quad I_o = (0.2)[1.4 - (0.4 - 0.02)]^2 = 0.2081 \text{ mA}$$

$$I_o = (0.2)[1.4 - (0.4 + 0.02)]^2 = 0.1921 \text{ mA}$$

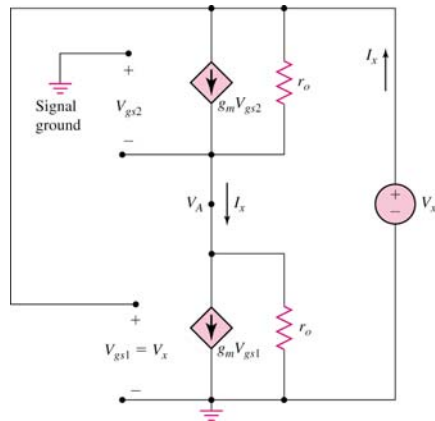
So  $0.1921 \leq I_o \leq 0.2081 \text{ mA}$

**10.48**

(a)  $I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2$   
 $0.2 = 0.2(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.5 \text{ V}$   
 $V_{GS1} = V_{GS2} + I_o R_S = V_{GS2} + K_{n2} R_S (V_{GS2} - V_{TN})^2$   
 $1.5 = V_{GS2} + (0.2)(10)(V_{GS2}^2 - V_{GS2} + 0.25)$   
 or  $2V_{GS2}^2 - V_{GS2} - 1 = 0 \Rightarrow V_{GS2} = 1.0 \text{ V}$   
 $I_o = \frac{V_{GS1} - V_{GS2}}{R_S} = \frac{1.5 - 1.0}{10} \Rightarrow I_o = 50 \mu\text{A}$

(b)  $I_o = 0.5 I_{REF} = 0.1 \text{ mA}$   
 $0.1 = 0.2(V_{GS2} - 0.5)^2 \Rightarrow V_{GS2} = 1.207 \text{ V}$   
 $R_S = \frac{V_{GS1} - V_{GS2}}{I_o} = \frac{1.5 - 1.207}{0.1} = 2.93 \text{ k}\Omega$

**10.49**



$$(1) \quad I_x = \frac{V_x - V_A}{r_o} + g_m V_{gs2}$$

$$I_x = \frac{V_A}{r_o} + g_m V_{gs1}$$

$$(2) \quad V_{gs1} = V_x, \quad V_{gs2} = -V_A$$

So

$$(1) \quad I_x = \frac{V_x}{r_o} - V_A \left( \frac{1}{r_o} + g_m \right)$$

$$(2) \quad I_x = \frac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o [I_x - g_m V_x]$$

Then

$$I_x = \frac{V_x}{r_o} - r_o (I_x - g_m V_x) \left( \frac{1}{r_o} + g_m \right)$$

$$I_x = \frac{V_x}{r_o} - r_o \left[ \frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o} \cdot V_x - g_m^2 V_x \right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x [2 + g_m r_o] = V_x \left[ \frac{1}{r_o} + g_m + g_m^2 r_o \right]$$

Since  $g_m \gg \frac{1}{r_o}$

$$I_x [2 + g_m r_o] \cong V_x (g_m) (1 + g_m r_o)$$

Then  $\frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m (1 + g_m r_o)}$

Usually,  $g_m r_o \gg 2$ , so that  $R_o \cong \frac{1}{g_m}$

### 10.50

$$V_{GS2} = V_{DS2}(\text{sat}) + V_{TN} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_O = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$0.15 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_2 (1.5 - 0.5)^2 \Rightarrow \left( \frac{W}{L} \right)_2 = 3.75$$

$$I_{REF} = 0.5 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_1 (1.5 - 0.5)^2 \Rightarrow \left( \frac{W}{L} \right)_1 = 12.5$$

$$V_{GS3} = (V^+ - V^-) - V_{GS1} = 1.8 - (-1.8) - 1.5 = 2.1 \text{ V}$$

$$I_{REF} = 0.5 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_3 (2.1 - 0.5)^2 \Rightarrow \left( \frac{W}{L} \right)_3 = 4.88$$



**10.51**

(a)

$$I_{REF} = \left(\frac{60}{2}\right)(20)(V_{GS1} - 0.7)^2 = \left(\frac{60}{2}\right)(3)(V_{GS3} - 0.7)^2$$

$$V_{GS1} + V_{GS3} = 5$$

$$\sqrt{\frac{20}{3}}(V_{GS1} - 0.7) = 5 - V_{GS1} - 0.7$$

$$3.582V_{GS1} = 6.107 \Rightarrow V_{GS1} = V_{GS2} = 1.705 \text{ V}$$

$$I_o = \left(\frac{60}{2}\right)(12)(1.705 - 0.7)^2 = 363.6 \text{ } \mu\text{A at } V_{DS2} = 1.705 \text{ V}$$

$$I_{REF} = \left(\frac{60}{2}\right)(20)(1.705 - 0.7)^2 = 606 \text{ } \mu\text{A}$$

$$R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.015)(0.3636)} = 183.4 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{1.5 - 1.705}{183.4} \Rightarrow -1.12 \text{ } \mu\text{A}$$

(b)  $I_o = 362.5 \text{ } \mu\text{A}$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 1.705}{183.4} \Rightarrow 7.06 \text{ } \mu\text{A}$$

(c)  $I_o = 370.7 \text{ } \mu\text{A}$

**10.52**

$$I_{REF} = \left(\frac{50}{2}\right)(15)(V_{SG1} - 0.5)^2 = \left(\frac{50}{2}\right)(3)(V_{SG3} - 0.5)^2$$

$$V_{SG1} + V_{SG3} = 10 \Rightarrow V_{SG3} = 10 - V_{SG1}$$

$$\sqrt{\frac{15}{3}}(V_{SG1} - 0.5) = 10 - V_{SG1} - 0.5$$

$$3.236V_{SG1} = 10.618 \Rightarrow V_{SG1} = 3.28 \text{ V}$$

$$I_{REF} = \left(\frac{50}{2}\right)(15)(3.28 - 0.5)^2 \Rightarrow I_{REF} = 2.90 \text{ mA}$$

$$I_o = I_{REF} = 2.90 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 3.28 - 0.5 \Rightarrow \underline{V_{SD2}(\text{sat}) = 2.78 \text{ V}}$$

**10.53**

$$V_{SD2}(\text{sat}) = 1.2 = V_{SG2} - 0.35 \Rightarrow V_{SG2} = 1.55 \text{ V}$$

$$I_O = 100 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_2 (1.55 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.78$$

$$\frac{I_{REF}}{I_O} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \Rightarrow \frac{200}{100} = \frac{\left(\frac{W}{L}\right)_1}{2.78} \Rightarrow \left(\frac{W}{L}\right)_1 = 5.56$$

$$V_{SG1} + V_{SG3} = 4 \Rightarrow V_{SG3} = 2.45 \text{ V}$$

$$I_{REF} = 200 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_3 (2.45 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 1.81$$

**10.54**

$$I_{REF} = \left(\frac{80}{2}\right)(25)(V_{SG1} - 1.2)^2 = \left(\frac{80}{2}\right)(4)(V_{SG3} - 1.2)^2$$

$$V_{SG1} + 2V_{SG3} = 10 \Rightarrow V_{SG3} = \frac{10 - V_{SG1}}{2}$$

Then  $\sqrt{\frac{25}{4}}(V_{SG1} - 1.2) = \frac{10 - V_{SG1}}{2} - 1.2$

$$3V_{SG1} = 6.8 \Rightarrow V_{SG1} = 2.27 \text{ V}$$

$$I_{REF} = \left(\frac{80}{2}\right)(25)(2.267 - 1.2)^2 \Rightarrow I_{REF} = I_O = 1.14 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 2.27 - 1.2 \Rightarrow \underline{V_{SD2}(\text{sat}) = 1.07 \text{ V}}$$

**10.55**

$$V_{SG2} = V_{SD2}(\text{sat}) - V_{TP} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_O = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$$

$$80 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_2 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.67$$

$$I_{REF} = 250 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_1 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 8.33$$

$$V_{SG3} = V_{SG4}$$

Then  $2V_{SG3} = V^+ - V^- - V_{SG1} = 3 - (-3) - 1.5 = 4.5 \text{ V}$

So  $V_{SG3} = 2.25 \text{ V}$

$$250 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_{3,4} (2.25 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 2.72$$

**10.56**

$$(a) \quad I_{REF} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_1 (V_{SG1} + V_{TP})^2 = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$V_{SG3} = 3 - V_{SG1}$$

$$\sqrt{25}(V_{SG1} - 0.4) = \sqrt{5}(3 - V_{SG1} - 0.4)$$

$$3.236V_{SG1} = 3.4944 \Rightarrow V_{SG1} = 1.08 \text{ V and } V_{SG3} = 1.92 \text{ V}$$

$$I_{REF} = \left(\frac{60}{2}\right)(25)(1.08 - 0.4)^2 \Rightarrow I_{REF} = 0.347 \text{ mA}$$

$$I_O = \left(\frac{60}{2}\right)(15)(1.08 - 0.4)^2 \Rightarrow I_O = 0.208 \text{ mA}$$

$$(b) \quad V_{SD2}(sat) = V_{SG2} + V_{TP} = 1.08 - 0.4 = 0.68 \text{ V}$$

$$R = \frac{3 - 0.68}{0.208} = 11.15 \text{ k}\Omega$$

**10.57**

$$V_{SD2}(sat) = 0.35 = V_{SG2} + V_{TP} = V_{SG2} - 0.4 \Rightarrow V_{SG2} = 0.75 \text{ V}$$

$$I_{REF} = 220 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_1 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 59.9$$

$$I_O = 80 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_2 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 21.8$$

$$V_{SG3} = 3 - 0.75 = 2.25 \text{ V}$$

$$220 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_3 (2.25 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 2.14$$

**10.58**

$$(a) \quad I_{REF} = 100 = 100(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.5 \text{ V}$$

$$\text{For } V_{D4} = -2 \text{ V, } V_{DS4} + V_{DS2} = 3 \text{ V} = V_{DS3} + V_{DS1}$$

$$\text{Then } I_O = 100 \mu\text{A}$$

$$(b) \quad R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$$

$$g_m = 2\sqrt{K_n I_O} = 2\sqrt{(0.1)(0.1)} = 0.2 \text{ mA/V}$$

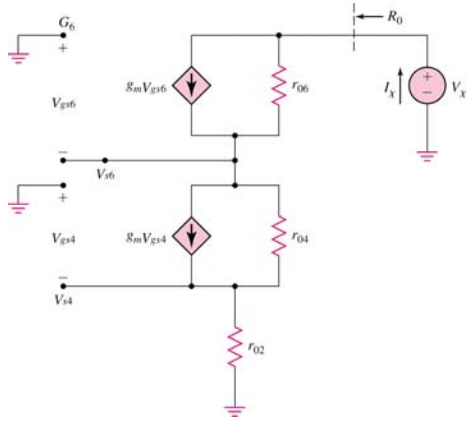
$$r_{o2} = r_{o4} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_o = 500 + 500[1 + (0.2)(500)] \Rightarrow R_o = 51 \text{ M}\Omega$$

$$\Delta I_O = \frac{\Delta V_{D4}}{R_o} = \frac{4}{51} = 0.07843 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} \times 100\% = \left(\frac{0.07843}{100}\right) \times 100\% = 0.07843\%$$

10.59



$$\begin{aligned}
 V_{gs4} &= -I_X r_{02} \\
 V_{S6} &= (I_X - g_m V_{gs4}) r_{04} + I_X r_{02} \\
 &= (I_X + g_m I_X r_{02}) r_{04} + I_X r_{02} \\
 V_{S6} &= I_X [r_{02} + (1 + g_m r_{02}) r_{04}] = -V_{gs6} \\
 I_X &= g_m V_{gs6} + \frac{V_X - V_{S6}}{r_{06}} = \frac{V_X}{r_{06}} - V_{S6} \left( g_m + \frac{1}{r_{06}} \right) \\
 I_X &= \frac{V_X}{r_{06}} - I_X \left( g_m + \frac{1}{r_{06}} \right) [r_{02} + (1 + g_m r_{02}) r_{04}] \\
 I_X \left\{ 1 + \left( g_m + \frac{1}{r_{06}} \right) [r_{02} + (1 + g_m r_{02}) r_{04}] \right\} &= \frac{V_X}{r_{06}} \\
 \frac{V_X}{I_X} = R_0 = r_{06} + (1 + g_m r_{06}) [r_{02} + (1 + g_m r_{02}) r_{04}] \\
 I_0 \approx I_{REF} = 0.2 \text{ mA} = 0.2 (V_{GS} - 1)^2 \\
 V_{GS} &= 2 \text{ V} \\
 g_m = 2K_n (V_{GS} - V_{TN}) &= 2(0.2)(2 - 1) = 0.4 \text{ mA/V} \\
 r_{02} = r_{04} = r_{06} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.2)} &= 250 \text{ k}\Omega \\
 R_0 = 250 + [1 + (0.4)(250)] \times \{ 250 + [1 + (0.4)(250)](250) \} \\
 R_0 = 2575750 \text{ k}\Omega \Rightarrow \underline{R_0 = 2.58 \times 10^9 \Omega}
 \end{aligned}$$

10.60

$$\begin{aligned}
 I_{REF} &= \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2 = \left( \frac{k'_p}{2} \right) \left( \frac{W}{L} \right)_4 (V_{SG4} + V_{TP})^2 \\
 \left( \frac{100}{2} \right) (5)(V_{GS3} - 0.4)^2 &= \left( \frac{60}{2} \right) (10)(V_{SG4} - 0.4)^2 \\
 \text{We find } V_{SG4} &= 0.91287V_{GS3} + 0.03485 \\
 I_{REF} &= \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2
 \end{aligned}$$

$$\sqrt{20}(V_{GS1} - 0.4) = \sqrt{5}(V_{GS3} - 0.4)$$

Now  $V_{SG4} + V_{GS3} + V_{GS1} = 6$

$$0.91287V_{GS3} + 0.03485 + V_{GS3} + V_{GS1} = 6$$

Then  $V_{GS3} = 3.1184 - 0.52277V_{GS1}$

And  $2(V_{GS1} - 0.4) = 3.1184 - 0.52277V_{GS1} - 0.4$

So  $V_{GS1} = 1.395 \text{ V}$

$$V_{GS3} = 2.389 \text{ V}$$

$$V_{SG4} = 2.216 \text{ V}$$

$$I_{REF} = \left(\frac{0.1}{2}\right)(20)(1.395 - 0.4)^2 = 0.99 \text{ mA}$$

$$I_O = \left(\frac{0.1}{2}\right)(20)(1.395 - 0.4)^2 = 0.99 \text{ mA}$$

$$V_{DS2}(sat) = 1.395 - 0.4 = 0.995 \text{ V}$$

### 10.61

$$V_{DS2}(sat) = 0.5 \text{ V} = V_{GS2} - V_{TN} = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.9 \text{ V}$$

$$I_O = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$50 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_2 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 4$$

$$I_{REF} = 500 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_1 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 40$$

$$V_{GS3} = V_{SG4}$$

$$6 = V_{GS1} + V_{GS3} + V_{SG4} = 0.9 + 2V_{GS3} \Rightarrow V_{GS3} = V_{SG4} = 2.55 \text{ V}$$

$$500 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_3 (2.55 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 2.16$$

$$500 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_4 (2.55 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 3.61$$

### 10.62

- a. As a first approximation

$$I_{REF} = 80 = 80(V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 2 \text{ V}$$

Then  $V_{DS1} \cong 2(2) = 4 \text{ V}$

The second approximation

$$80 = 80(V_{GS1} - 1)^2 [1 + (0.02)(4)]$$

Or  $\frac{80}{86.4} = (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.962$

Then

$$I_O = K_n (V_{GS1} - V_{TN})^2 (1 + \lambda_n V_{GS1})$$

$$= 80(1.962 - 1)^2 [1 + (0.02)(1.962)]$$

Or  $I_0 = 76.94 \mu\text{A}$

- b. From a PSpice analysis,  $I_0 = 77.09 \mu\text{A}$  for  $V_{D3} = -1 \text{ V}$  and  $I_0 = 77.14 \mu\text{A}$  for  $V_{D3} = 3 \text{ V}$ .  
The change is  $\Delta I_0 \approx 0.05 \mu\text{A}$  or 0.065%.

### 10.63

- a. For a first approximation,

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 \Rightarrow V_{GS4} = 2 \text{ V}$$

As a second approximation

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 [1 + (0.02)(2)]$$

Or  $V_{GS4} = 1.98 \text{ V} = V_{GS1}$

$$I_O = K_n (V_{GS2} - V_{TN})^2 (1 + \lambda V_{GS2})$$

To a very good approximation  $I_0 = 80 \mu\text{A}$

- b. From a PSpice analysis,  $I_0 = 80.00 \mu\text{A}$  for  $V_{D3} = -1 \text{ V}$  and the output resistance is  $R_0 = 76.9 \text{ M}\Omega$ .

Then

For  $V_D = +3 \text{ V}$

$$\Delta I_0 = \frac{1}{R_0} \cdot V_{D3} = \frac{4}{76.9} = 0.052 \mu\text{A}$$

$I_0 = 80.05 \mu\text{A}$

### 10.64

$$V_{DS3}(\text{sat}) = V_{GS3} - V_{TN} \text{ or } V_{GS3} = V_{DS3}(\text{sat}) + V_{TN} = 0.2 + 0.8 = 1.0$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L}\right) (V_{GS3} - V_{TN})^2$$

$$50 = 48 \left(\frac{W}{L}\right) (0.2)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 26$$

(a)

$$V_{GS5} - V_{TN} = 2(V_{GS3} - V_{TN})$$

(b)  $V_{GS5} = 0.8 + 2(0.2) \Rightarrow V_{GS5} = 1.2 \text{ V}$

(c)  $V_{D1}(\text{min}) = 2V_{DS}(\text{sat}) = 2(0.2) \Rightarrow V_{D1}(\text{min}) = 0.4 \text{ V}$

**10.65**

(a)

$$K_{n1} = \frac{k'_n}{2} \left( \frac{W}{L} \right)_1 = 50(5) = 250 \mu\text{A}/\text{V}^2$$

$$R = \frac{1}{\sqrt{K_{n1} I_{D1}}} \left( 1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}} \right)$$

$$= \frac{1}{\sqrt{(0.25)(0.05)}} \left( 1 - \sqrt{\frac{5}{50}} \right) = (8.944)(0.6838)$$

$$R = 6.12 \text{ k}\Omega$$

(b)

$$V^+ - V^- = V_{SD3}(\text{sat}) + V_{GS1}$$

$$V_{SD3}(\text{sat}) = V_{SG3} + V_{TP}$$

$$I_{D1} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow V_{SG3} = 1.207 \text{ V}$$

$$\text{Then } V_{SD3}(\text{sat}) = 1.21 - 0.5 = 0.707 \text{ V}$$

$$\text{Also } I_{D1} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 0.9472 \text{ V}$$

$$\text{Then } \underline{(V^+ - V^-)_{\min} = 0.71 + 0.947 = 1.66 \text{ V}}$$

(c)

$$I_{O1} = 25 = 50 \left( \frac{W}{L} \right)_5 (0.947 - 0.5)^2 \Rightarrow \underline{\left( \frac{W}{L} \right)_5 = 2.5}$$

$$I_{O2} = 75 = 20 \left( \frac{W}{L} \right)_6 (1.207 - 0.5)^2 \Rightarrow \underline{\left( \frac{W}{L} \right)_6 = 7.5}$$

**10.66**

$$V_{GS3} = V_{GS4} = V_{GS5} \Rightarrow V_{GS3} = \frac{5}{3} \text{ V}$$

$$I_{REF} = 0.1 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_3 \left( \frac{5}{3} - 0.7 \right)^2 \Rightarrow \underline{\left( \frac{W}{L} \right)_{3,4,5} = 2.68}$$

$$I_{O1} = 0.2 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_1 \left( \frac{5}{3} - 0.7 \right)^2 \Rightarrow \underline{\left( \frac{W}{L} \right)_1 = 5.35}$$

$$I_{O2} = 0.3 = \left( \frac{0.08}{2} \right) \left( \frac{W}{L} \right)_2 \left( \frac{5}{3} - 0.7 \right)^2 \Rightarrow \underline{\left( \frac{W}{L} \right)_2 = 8.03}$$

**10.67**

$$P = (I_{REF} + I_{O1} + I_{O2} + I_{O3})(V^+ - V^-)$$

$$5 = (I_{REF} + 0.1 + 0.2 + 0.4)[1.8 - (-1.8)] \Rightarrow I_{REF} = 0.689 \text{ mA}$$

$$V_{DS2}(\text{sat}) = 0.4 = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.8 \text{ V} = V_{GS}$$

$$I_{REF} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2$$

$$0.689 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 86.1$$

$$I_{O1} = 0.1 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_2 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 12.5$$

$$I_{O2} = 0.2 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_3 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 25$$

$$I_{O3} = 0.4 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_4 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 50$$

**10.68**

$$24 = I_{REF} R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\frac{I_{REF}}{\left(\frac{k'_p}{2}\right)(1)}} - V_{TP} = 5.7735\sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\left(\frac{k'_n}{2}\right)(1)}} + V_{TN} = 4.472\sqrt{I_{REF}} + 0.8$$

$$\text{So } 24 = I_{REF}(100) + 5.7735\sqrt{I_{REF}} + 0.8 + 4.4721\sqrt{I_{REF}} + 0.8$$

$$\text{Let } \sqrt{I_{REF}} = x$$

$$\text{Then } 100x^2 + 10.2456x - 22.4 = 0 \Rightarrow x = 0.4248 \Rightarrow x^2 = I_{REF} = 0.1805 \text{ mA}$$

$$I_1 = (0.2)I_{REF} = 0.0361 \text{ mA}$$

$$I_2 = (1.25)I_{REF} = 0.2256 \text{ mA}$$

$$I_3 = (0.8)I_{REF} = 0.1444 \text{ mA}$$

$$I_4 = (4)I_{REF} = 0.722 \text{ mA}$$

**10.69**

$$10 = I_{REF} R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\frac{I_{REF}}{\left(\frac{0.06}{2}\right)(1)}} + 0.8 = 5.7735\sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\left(\frac{0.1}{2}\right)(1)}} + 0.8 = 4.472\sqrt{I_{REF}} + 0.8$$

$$8.4 = 100I_{REF} + 10.2456\sqrt{I_{REF}}$$

$$\text{Let } \sqrt{I_{REF}} = x$$

$$\text{Then } 100x^2 + 10.2456x - 8.4 = 0 \Rightarrow x = 0.2431 \Rightarrow x^2 = I_{REF} = 59.09 \mu \text{ A}$$

$$I_1 = (0.2)I_{REF} = 11.82 \mu \text{ A}, \quad I_2 = (1.25)I_{REF} = 73.87 \mu \text{ A}$$

$$I_3 = (0.8)I_{REF} = 47.27 \mu \text{ A}, \quad I_4 = (4)I_{REF} = 236.4 \mu \text{ A}$$



**10.70**

$$I_{D2} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \cdot I_{REF} = \frac{9}{15}(200) \Rightarrow \underline{I_{D2} = 120 \mu\text{A}}$$

$$I_O = \frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_3} \cdot I_{D2} = \left(\frac{20}{9}\right)(120) \Rightarrow \underline{I_O = 267 \mu\text{A}}$$

$$I_O = 266.7 = \left(\frac{40}{2}\right)(20)(V_{SG4} - 0.6)^2$$

$$V_{SG4} = 1.416 \text{ V}$$

$$V_{SD4}(\text{sat}) = 1.416 - 0.6 \Rightarrow \underline{V_{SD4}(\text{sat}) = 0.816 \text{ V}}$$

**10.71**

$$\text{For } M_1: I_{REF} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$100 = \left(\frac{100}{2}\right)(4)(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.107 \text{ V}$$

$$I_{D2} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 = \left(\frac{100}{2}\right)(2.5)(1.107 - 0.4)^2 \Rightarrow I_{D2} = 62.5 \mu\text{A}$$

$$\text{Also } I_{D2} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$62.5 = \left(\frac{60}{2}\right)(6)(V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = V_{SG4} = 0.9893 \text{ V}$$

$$I_O = \left(\frac{60}{2}\right)(4)(0.9893 - 0.4)^2 = 41.67 \mu\text{A}$$

**10.72**

$$I_{REF} = 50 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_1 (V_{SG1} - 0.6)^2$$

$$I_{REF} = \frac{1.75 - V_{SG1}}{R} = 50$$

$$V_{SD2}(\text{sat}) = 0.35 = V_{SG2} - 0.6 \Rightarrow V_{SG2} = 0.95 \text{ V}$$

$$R = \frac{1.75 - 0.95}{0.05} \Rightarrow \underline{R = 16 \text{ K}}$$

$$50 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_1 (0.95 - 0.6)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 20.4}$$

$$\frac{I_{O1}}{I_{REF}} = \frac{120}{50} = \frac{\left(\frac{W}{L}\right)_2}{(20.4)} \Rightarrow \left(\frac{W}{L}\right)_2 = 49$$

$$\frac{I_{D3}}{I_{REF}} = \frac{25}{50} = \frac{\left(\frac{W}{L}\right)_3}{(20.4)} \Rightarrow \left(\frac{W}{L}\right)_3 = 10.2$$

$$V_{DS5}(\text{sat}) = 0.35 = V_{GS5} - 0.4 \Rightarrow V_{GS5} = 0.75 \text{ V}$$

$$I_{O2} = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_5 (0.75 - 0.4)^2 = 150 \Rightarrow \left(\frac{W}{L}\right)_5 = 24.5$$

$$\frac{I_{D4}}{I_{O2}} = \frac{I_{D3}}{I_{O2}} = \frac{25}{150} = \frac{\left(\frac{W}{L}\right)_4}{24.5} \Rightarrow \left(\frac{W}{L}\right)_4 = 4.08$$

### 10.73

For  $v_{GS} = 0$ ,  $i_D = I_{DSS}(1 + \lambda v_{DS})$

$$V_D = -5 \text{ V}, \quad v_{DS} = 5$$

a.  $i_D = (2)[1 + (0.05)(5)] \Rightarrow \underline{i_D = 2.5 \text{ mA}}$

$$V_D = 0, \quad v_{DS} = 10$$

b.  $i_D = (2)[1 + (0.05)(10)] \Rightarrow \underline{i_D = 3 \text{ mA}}$

$$V_D = 5 \text{ V}, \quad v_{DS} = 15 \text{ V}$$

c.  $i_D = (2)[1 + (0.05)(15)] \Rightarrow \underline{i_D = 3.5 \text{ mA}}$

### 10.74

$$I_0 = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 4 \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{V_P} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

$$\text{So } V_{GS} = (0.293)(-4) = -1.17 \text{ V}$$

$$\text{Then } I_0 = \frac{V_S}{R} \text{ and } V_S = -V_{GS}$$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow \underline{R = 0.586 \text{ k}\Omega}$$

$$\text{Need } v_{DS} \geq v_{DS}(\text{sat}) = v_{GS} - V_P$$

$$= -1.17 - (-4)$$

$$v_{DS} \geq 2.83 \text{ V}$$

$$\text{So } V_D \geq v_{DS}(\text{sat}) + V_S = 2.83 + 1.17 \Rightarrow \underline{V_D \geq 4 \text{ V}}$$

**10.75**

$$(a) A_v = \frac{-\frac{1}{V_T}}{\frac{1}{V_{AN}} + \frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125} \Rightarrow A_v = -1846$$

$$(b) I_{CQ} = I_{S1} \left[ \exp\left(\frac{V_I}{V_T}\right) \right] \left( 1 + \frac{V_{CE1}}{V_{AN}} \right)$$

$$200 \times 10^{-6} = (5 \times 10^{-16}) \left[ \exp\left(\frac{V_I}{V_T}\right) \right] \left( 1 + \frac{1.25}{120} \right)$$

$$\text{or } V_I = (0.026) \ln \left[ \frac{200 \times 10^{-6}}{(5 \times 10^{-16}) (1 + 1.25/120)} \right] = 0.6943 \text{ V}$$

$$(c) 200 \times 10^{-6} = (10^{-15}) \left[ \exp\left(\frac{V_{EB}}{V_T}\right) \right] \left( 1 + \frac{1.25}{80} \right)$$

$$V_{EB} = (0.026) \ln \left[ \frac{200 \times 10^{-6}}{(10^{-15}) (1 + 1.25/80)} \right] = 0.6762 \text{ V}$$

$$V_{EB} = 0.6762 = V^+ - V_B \Rightarrow V_B = 1.824 \text{ V}$$

**10.76**

$$(a) A_v = -g_{m1} (r_{o1} \| r_{o2})$$

$$r_{o1} = \frac{1}{\lambda_1 I_{DQ}} = \frac{1}{(0.02)(0.2)} = 250 \text{ k}\Omega, \quad r_{o2} = \frac{1}{\lambda_2 I_{DQ}} = \frac{1}{(0.03)(0.2)} = 166.7 \text{ k}\Omega$$

$$-100 = -g_{m1} (250 \| 166.7) \Rightarrow g_{m1} = 1 \text{ mA/V}$$

$$g_{m1} = 1 = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_1 (0.2)} \Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$(b) I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 (V_I - V_{TN})^2 (1 + \lambda_1 V_{DS1})$$

$$0.2 = \left(\frac{0.1}{2}\right) (25) (V_I - 0.5)^2 [1 + (0.02)(1.25)] \Rightarrow V_I = 0.895 \text{ V}$$

$$(c) K_{n1} = K_{p2} \Rightarrow \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2$$

$$\text{Then } \left(\frac{W}{L}\right)_2 = \left(\frac{100}{60}\right) (25) = 41.67$$

$$I_{DQ} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2 (1 + \lambda_2 V_{SD2})$$

$$0.2 = \left(\frac{0.06}{2}\right) (41.67) (V_{SG2} - 0.5)^2 [1 + (0.03)(1.25)] \Rightarrow V_{SG2} = 0.8927 \text{ V}$$

$$V_{SG2} = 0.8927 = V^+ - V_G \Rightarrow V_G = 1.607 \text{ V}$$

**10.77**

$$I_{REF} = I_{S1} \exp\left(\frac{V_{EB1}}{V_T}\right)$$

a. or  $V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-13}}\right) \Rightarrow \underline{V_{EB1} = 0.5568}$

b.  $R_1 = \frac{5 - 0.5568}{1} \Rightarrow \underline{R_1 = 4.44 \text{ k}\Omega}$

c. From Equations (10.79) and (10.80) and letting  $V_{CE0} = V_{EC2} = 2.5 \text{ V}$

$$10^{-12} \exp\left(\frac{V_I}{V_T}\right) \left[1 + \frac{2.5}{120}\right] = 10^{-3} \left(\frac{1 + \frac{2.5}{80}}{1 + \frac{0.5568}{80}}\right)$$

$$1.0208333 \times 10^{-12} \exp\left(\frac{V_I}{V_T}\right) = (10^{-3}) \left(\frac{1.03125}{1.00696}\right)$$

Then  $V_I = 0.026 \ln(1.003222 \times 10^9)$

So  $V_I = 0.5389 \text{ V}$

$$A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

d.  $\underline{A_v = -1846}$

**10.78**

a. 
$$V_{BE} = V_T \ln \left( \frac{I_{REF}}{I_{S1}} \right) = (0.026) \ln \left( \frac{0.5 \times 10^{-3}}{10^{-12}} \right) \Rightarrow \underline{V_{BE} = 0.5208}$$

b. 
$$R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow \underline{R_1 = 8.96 \text{ k}\Omega}$$

c. Modify Eqs. 10.79 and 10.80 to apply to pnp and npn, and set the two equation equal to each other.

$$I_{CO} = I_{SO} \exp \left( \frac{V_{EBO}}{V_T} \right) \left( 1 + \frac{V_{ECO}}{V_{AP}} \right) = I_{C2} = I_{S2} \exp \left( \frac{V_{BE}}{V_T} \right) \left( 1 + \frac{V_{CE2}}{V_{AN}} \right)$$

$$5 \times 10^{-13} \exp \left( \frac{V_{EBO}}{V_T} \right) \left( 1 + \frac{2.5}{80} \right) = 10^{-12} \exp \left( \frac{V_{BE}}{V_T} \right) \left( 1 + \frac{2.5}{120} \right)$$

$$5.15625 \times 10^{-13} \exp \left( \frac{V_{EBO}}{V_T} \right) = 1.020833 \times 10^{-12} \exp \left( \frac{V_{BE}}{V_T} \right)$$

$$\frac{\exp \left( \frac{V_{EBO}}{V_T} \right)}{\exp \left( \frac{V_{BE}}{V_T} \right)} = 1.9798 = \exp \left( \frac{V_{EBO} - V_{BE}}{V_T} \right)$$

$$V_{EBO} = V_{BE} + V_T \ln(1.9798) = 0.5208 + (0.026) \ln(1.9798)$$

$$V_{EBO} = 0.5386 \Rightarrow V_I = 5 - 0.5386 \Rightarrow \underline{V_I = 4.461 \text{ V}}$$

$$A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

d. 
$$\underline{A_v = -1846}$$

**10.79**

(a) For  $M_o$ : 
$$I_{DQ} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_o (V_I - V_{TN})^2 (1 + \lambda_n V_{DSO})$$

$$100 = \left( \frac{100}{2} \right) \left( \frac{W}{L} \right)_o (1.2 - 0.5)^2 [1 + (0.02)(1.5)] \Rightarrow \left( \frac{W}{L} \right)_o = 3.96$$

For  $M_1, M_2$ : For  $I_{REF} = I_o \Rightarrow V_{SD2} = V_{SD1} = V_{SG} = 1.5 \text{ V}$

$$100 = \left( \frac{60}{2} \right) \left( \frac{W}{L} \right)_2 (1.5 - 0.5)^2 [1 + (0.02)(1.5)] \Rightarrow \left( \frac{W}{L} \right)_2 = \left( \frac{W}{L} \right)_1 = 3.24$$

For  $M_3$ :  $V_{SG3} = 3 - 1.5 = 1.5 \text{ V} = V_{SD3} \Rightarrow \left( \frac{W}{L} \right)_3 = 3.24$

$$(b) A_v = -g_{mO}(r_{oO} \| r_{o2})$$

$$g_{mO} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_O I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(3.96)(0.1)} = 0.2814 \text{ mA/V}$$

$$r_{oO} = r_{o2} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$A_v = -(0.2814)(500 \| 500) = -70.35$$

### 10.80

$$(a) I_{REF} = K_p (V_{SG} + V_{TP})^2 (1 + \lambda_p V_{SD1})$$

$$0.080 = 0.1(V_{SG} - 0.5)^2 [1 + (0.02)V_{SG}]$$

$$\text{Approximation, for } \lambda = 0 \Rightarrow V_{SG} = 1.394 \text{ V}$$

$$(b) I_O = I_2$$

$$K_n (V_I - V_{TN})^2 (1 + \lambda_n V_{DSO}) = K_p (V_{SG} + V_{TP})^2 (1 + \lambda_p V_{SD2})$$

$$K_n = K_p, \quad V_{DSO} = V_{SD2}, \quad \lambda_n = \lambda_p$$

$$\text{So } V_I = 1.394 \text{ V}$$

$$(c) I_O \cong I_{REF} = 80 \mu\text{A}$$

$$A_v = -g_{mO}(r_{oO} \| r_{o2})$$

$$g_{mO} = 2\sqrt{K_n I_O} = 2\sqrt{(0.1)(0.08)} = 0.1789 \text{ mA/V}$$

$$r_{oO} = r_{o2} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.08)} = 625 \text{ k}\Omega$$

$$A_v = -(0.1789)(625 \| 625) = -55.9$$

### 10.81

$$(a) I_{REF} = I_{CO} = \frac{3 - 0.6}{47} \Rightarrow I_{CO} = 51.06 \mu\text{A}$$

$$r_{oO} = \frac{V_{AN}}{I_{CO}} = \frac{120}{0.05106} \Rightarrow r_{oO} = 2.35 \text{ M}\Omega$$

$$r_{o2} = \frac{V_{AP}}{I_{CO}} = \frac{90}{0.05106} \Rightarrow r_{o2} = 1.763 \text{ M}\Omega$$

$$g_m = \frac{I_{CO}}{V_T} = \frac{0.05106}{0.026} = 1.964 \text{ mA/V}$$

$$A_v = -g_m (r_{oO} \| r_{o2}) = -(1.964)(2350 \| 1763) = -1978$$

$$(b) A_v = -g_m (r_{oO} \| r_{o2} \| R_L) = -(1.964)(2350 \| 1763 \| 300) = -454$$

$$(c) A_v = -(1.964)(2350 \| 1763 \| 150) = -256$$

**10.82**

$$I_{REF} = \frac{5 - 0.6}{35} = 0.1257 \text{ mA}$$

Then

$$I_{CO} = 2I_{REF} = 0.2514 \text{ mA}$$

From Eq. 10.96

$$A_v = \frac{\frac{-0.2514}{0.026}}{\frac{0.2514}{120} + \frac{0.2514}{80} + \frac{1}{R_L}} = \frac{-9.6692}{0.002095 + 0.0031425 + \frac{1}{R_L}}$$

$$A_v = \frac{-9.6692}{0.0052375 + \frac{1}{R_L}}$$

(a)  $R_L = \infty$ ,  $A_v = -1846$

(b)  $R_L = 250 \text{ K}$ ,  $A_v = -1047$

**10.83**

(a) To a good approximation, output resistance is the same as the widlar current source.

$$R_0 = r_{02} [1 + g_{m2} (r_{\pi 2} \parallel R_E)]$$

(b)  $A_v = -g_{m0} (r_0 \parallel R_L \parallel R_0)$

**10.84**

Output resistance of Wilson source

$$R_0 \cong \frac{\beta r_{03}}{2}$$

Then

$$A_v = -g_m (r_0 \parallel R_0)$$

$$r_{03} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$r_0 = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_v = -7.69 \left[ 600 \parallel \left( \frac{(80)(400)}{2} \right) \right] = -7.69 [600 \parallel 16,000] \Rightarrow \underline{A_v = -4448}$$

**10.85**

(a)  $I_{D2} = I_{D0} = I_{REF} = 200 \mu\text{A}$

For  $M_2$ :  $r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.2)} = 250 \text{ K}$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{\left(\frac{0.04}{2}\right)(35)(0.2)}$$

$$g_{m2} = 0.748 \text{ mA/V}$$

For  $M_0$ :  $r_{oo} = \frac{1}{\lambda_n I_{D0}} = \frac{1}{(0.015)(0.2)} = 333 \text{ K}$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)(20)(0.2)} \Rightarrow g_{mo} = 0.80 \text{ mA/V}$$

$$A_v = -g_{mo}(r_{o2} \parallel r_{oo}) = -(0.80)(250 \parallel 333)$$

(b)  $A_v = -114.3$

Want  $A_v = -57.15 = -0.80(142.8 \parallel R_L)$

$$142.8 \parallel R_L = 71.375 = \frac{142.8 R_L}{142.8 + R_L} \Rightarrow R_L = 143 \text{ K}$$

(c)

**10.86**

Assume  $M_1, M_2$  matched

$$I_{REF} = I_{D2} = I_{D0} = 200 \mu\text{A}$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.2)} = 250 \text{ K}$$

$$r_{oo} = \frac{1}{\lambda_n I_{D0}} = \frac{1}{(0.015)(0.2)} = 333 \text{ K}$$

$$A_v = -g_{mo}(r_{o2} \parallel r_{oo})$$

$$-100 = -g_{mo}(250 \parallel 333) \Rightarrow g_{mo} = 0.70 \text{ mA/V}$$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_0 (0.2)} = 0.70$$

$$\left(\frac{W}{L}\right)_0 = 15.3$$

Now  $\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_0 = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_2$

$$\left(\frac{80}{2}\right)(15.3) = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_2$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 30.6$$



**10.87**

$$g_{m1} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_n I_{D1}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = r_{o2} + r_{o3}(1 + g_{m2}r_{o2})$$

$$g_{m2} = 2\sqrt{\left(\frac{0.06}{2}\right)(80)(0.1)} = 0.9798 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = 500 + 500[1 + (0.9798)(500)] = 245,949 \text{ k}\Omega$$

$$A_v = -g_{m1}(r_{o1} \parallel R_{o2}) = -(0.6325)(500 \parallel 245,949) = -316$$

**10.88**

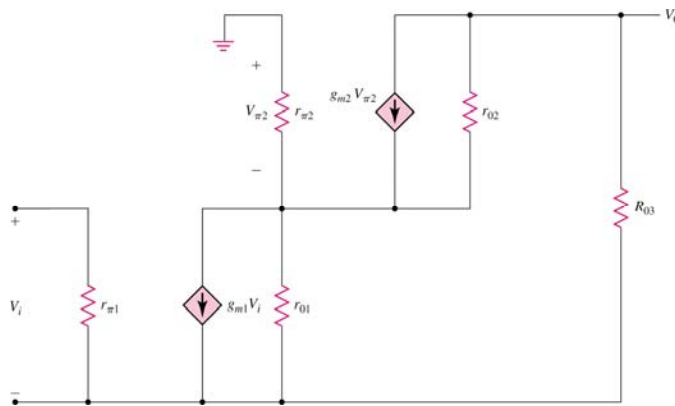
$$A_v = \frac{-g_m^2}{\frac{1}{r_{o3}r_{o4}} + \frac{1}{r_{o1}r_{o2}}}$$

$$g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(0.08)} = 0.6325 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5 \text{ k}\Omega$$

$$A_v = \frac{-(0.6325)^2}{\frac{1}{(312.5)^2} + \frac{1}{(312.5)^2}} = \frac{-0.40}{2(0.00001024)} = -19,531$$

**10.89**



$$(1) \quad g_{m1}V_i = \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2}}{r_{o1}} + g_{m2}V_{\pi 2} + \frac{V_o - (-V_{\pi 2})}{r_{o2}}$$

$$(2) \quad \frac{V_o}{R_{o3}} + \frac{V_o - (-V_{\pi 2})}{r_{o2}} + g_{m2}V_{\pi 2} = 0$$

$$(1) \quad g_{m1}V_i = V_{\pi 2} \left( \frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) + \frac{V_o}{r_{o2}}$$

$$(2) \quad V_o \left( \frac{1}{R_{O3}} + \frac{1}{r_{o2}} \right) + V_{\pi 2} \left( \frac{1}{r_{o2}} + g_{m2} \right) = 0$$

$$g_m \gg \frac{1}{r_o}$$

$$(1) \quad g_{m1}V_i = V_{\pi 2} \left( \frac{1+\beta}{r_{\pi 2}} \right) + \frac{V_o}{r_{o2}}$$

$$(2) \quad V_o \left( \frac{1}{R_{O3}} + \frac{1}{r_{o2}} \right) + V_{\pi 2} \cdot g_{m2} = 0$$

$$V_{\pi 2} = -\frac{V_o}{g_{m2}} \left( \frac{1}{R_{O3}} + \frac{1}{r_{o2}} \right)$$

Then

$$(1) \quad g_{m1}V_i = -\frac{V_o}{g_{m2}} \left( \frac{1}{R_{O3}} + \frac{1}{r_{o2}} \right) \left( \frac{1+\beta}{r_{\pi 2}} \right) + \frac{V_o}{r_{o2}}$$

$$= -V_o \left( \frac{1}{R_{O3}} + \frac{1}{r_{o2}} \right) \left( \frac{1+\beta}{\beta} \right) + \frac{V_o}{r_{o2}}$$

$$\approx -\frac{V_o}{R_{O3}} \left( \frac{1+\beta}{\beta} \right)$$

$$\frac{V_o}{V_i} = -g_{m1}R_{O3} \left( \frac{\beta}{1+\beta} \right)$$

From Equation (10.20)  $R_{O3} \approx \beta r_{o3}$

So

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o3}\beta^2}{1+\beta} \quad g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{o3} = \frac{80}{0.25} = 320 \text{ K}$$

$$A_v = \frac{-(9.615)(320)(120)^2}{121} = -366,165$$

### 10.90 Design Problem

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**10.91**

Let  $(W/L)_{1,2} = 50$  and  $(W/L) = 25$  for all other transistors

Let  $I_{REF} = 80 \mu\text{ A}$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5 \text{ k}\Omega$$

$$g_m = 2\sqrt{\left(\frac{0.06}{2}\right)(50)(0.08)} = 0.6928 \text{ mA/V}$$

$$A_v = \frac{-g_m^2}{\frac{1}{r_{o3}r_{o4}} + \frac{1}{r_{o1}r_{o2}}} = \frac{-(0.6928)^2}{\frac{1}{(312.5)^2} + \frac{1}{(312.5)^2}} = \frac{-0.480}{2(0.00001024)}$$

$$A_v = -23,438$$

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## Chapter 11

### 11.1

- (a)  $CMRR_{dB} = \infty \Rightarrow v_o = A_d v_d = (250)(1.5 \sin \omega t)$  (mV)  
 $v_o = 0.375 \sin \omega t$  (V)
- (b)  $CMRR_{dB} = 80 \text{ dB} \Rightarrow CMRR = 10^4 = \frac{250}{A_{cm}} \Rightarrow A_{cm} = 0.025$   
 $v_o = (250)(0.0015 \sin \omega t) + (0.025)(3 \sin \omega t)$   
 $v_o = 0.45 \sin \omega t$  (V)
- (c)  $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2 = \frac{250}{A_{cm}} \Rightarrow A_{cm} = 0.791$   
 $v_o = (250)(0.0015 \sin \omega t) + (0.791)(3 \sin \omega t)$   
 $v_o = 2.75 \sin \omega t$  (V)

### 11.2

- (a) (i)  $v_{o1} = -g_m R v_1 = -(1)(5)(0.7 + 0.1 \sin \omega t)$   
 $v_{o1} = -3.5 - 0.5 \sin \omega t$  (V)
- (ii)  $v_{o2} = -g_m R v_2 = -(1)(5)(0.7 - 0.1 \sin \omega t)$   
 $v_{o2} = -3.5 + 0.5 \sin \omega t$  (V)
- (iii)  $v_{o1} - v_{o2} = -1.0 \sin \omega t$  (V)
- (b)  $\Delta(v_1 - v_2) = (0.7 + 0.1 \sin \omega t) - (0.7 - 0.1 \sin \omega t) = 0.2 \sin \omega t$
- (c) (i)  $A_{d1} = \frac{-0.5}{0.2} = -2.5$
- (ii)  $A_{d2} = \frac{+0.5}{0.2} = +2.5$
- (iii)  $A_{d3} = \frac{-1}{0.2} = -5$

### 11.3

- (a) Neglect dc base currents  
 $I_E = I_{C1} + I_{C2} = 0.2 \text{ mA} = \frac{-0.7 - (-3)}{R_E} \Rightarrow R_E = 11.5 \text{ k}\Omega$
- $v_{o1} = 1.2 \text{ V}, \Rightarrow R_C = \frac{3 - 1.2}{0.1} = 18 \text{ k}\Omega$
- (c) For  $V_{CB} = 0 \Rightarrow V_{CE} = 0.7 \text{ V}$   
 $3 = I_C(18) + 0.7 + 2I_C(11.5) - 3$   
 So  $I_C = 0.1293 \text{ mA}$   
 $v_{o1} = v_{cm+} = 3 - (0.1293)(18) = 0.673 \text{ V}$   
 $v_{cm-} = -2.3 \text{ V}$   
 So  $-2.3 \leq v_{cm} \leq 0.673 \text{ V}$

### 11.4

a.

$$I_1 = \frac{10 - 2(0.7)}{8.5} \Rightarrow \underline{I_1 = 1.01 \text{ mA}}$$

$$I_{C2} = \frac{I_1}{1 + \frac{2}{\beta(1+\beta)}} = \frac{1.01}{1 + \frac{2}{(100)(101)}} \Rightarrow \underline{I_{C2} \cong 1.01 \text{ mA}}$$

$$I_{C4} = \left(\frac{100}{101}\right)\left(\frac{1.01}{2}\right) \Rightarrow \underline{I_{C4} \cong 0.50 \text{ mA}}$$

$$V_{CE2} = (0 - 0.7) - (-5) \Rightarrow \underline{V_{CE2} = 4.3 \text{ V}}$$

$$V_{CE4} = [5 - (0.5)(2)] - (-0.7) \Rightarrow \underline{V_{CE4} = 4.7 \text{ V}}$$

b.

$$\text{For } V_{CE4} = 2.5 \text{ V} \Rightarrow V_{C4} = -0.7 + 2.5 = 1.8 \text{ V}$$

$$I_{C4} = \frac{5 - 1.8}{2} \Rightarrow \underline{I_{C4} = 1.6 \text{ mA}}$$

$$I_{C2} + \left(\frac{1+\beta}{\beta}\right)(2I_{C4}) = \left(\frac{101}{100}\right)(2)(1.6) \Rightarrow \underline{I_{C2} = 3.23 \text{ mA}}$$

$$\underline{I_1 \approx I_{C2} = 3.23 \text{ mA}}$$

$$R_1 = \frac{10 - 2(0.7)}{3.23} \Rightarrow \underline{R_1 = 2.66 \text{ k}\Omega}$$

### 11.5

a. Neglecting base currents

$$I_1 = I_3 = 400 \mu\text{A} \Rightarrow R_1 = \frac{30 - 0.7}{0.4} \Rightarrow \underline{R_1 = 73.25 \text{ k}\Omega}$$

$$V_{CE1} = 10 \text{ V} \Rightarrow V_{C1} = 9.3 \text{ V}$$

$$R_C = \frac{15 - 9.3}{0.2} \Rightarrow \underline{R_C = 28.5 \text{ k}\Omega}$$

b.

$$r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$r_0(Q_3) = \frac{50}{0.4} = 125 \text{ k}\Omega$$

We have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13 + 10)} \Rightarrow \underline{A_d = 62}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left\{ \frac{1}{1 + \frac{2r_0(1+\beta)}{r_\pi + R_B}} \right\}$$

$$= -\frac{(100)(28.5)}{13 + 10} \left\{ \frac{1}{1 + \frac{2(125)(101)}{13 + 10}} \right\} \Rightarrow \underline{A_{cm} = -0.113}$$

$$CMRR_{dB} = 20 \log_{10} \left( \frac{62}{0.113} \right) \Rightarrow \underline{CMRR_{dB} = 54.8 \text{ dB}}$$

c.

$$R_{id} = 2(r_{\pi} + R_B) = 2(13 + 10) \Rightarrow \underline{R_{id} = 46 \text{ k}\Omega}$$

$$R_{icm} = \frac{1}{2} [r_{\pi} + R_B + 2(1 + \beta)r_o]$$

$$= \frac{1}{2} [13 + 10 + 2(101)(125)] \Rightarrow \underline{R_{icm} = 12.6 \text{ M}\Omega}$$

### 11.6

(a)  $v_{CM}(\text{max}) \Rightarrow V_{CB} = 0$  so that  $v_{CM}(\text{max}) = 5 - \frac{I_Q}{2}(R_C) = 5 - \frac{(0.5)}{2}(8)$

$$\underline{v_{CM}(\text{max}) = 3 \text{ V}}$$

(b)

$$\Delta I = g_m \cdot \frac{V_d}{2} = \left( \frac{I_{CQ}}{V_T} \right) \cdot \frac{V_d}{2} = \left( \frac{0.25}{0.026} \right) \left( \frac{0.018}{2} \right) = 0.08654 \text{ mA}$$

$$\Delta V_{C2} = \Delta I \cdot R_C = (0.08654)(8) = 0.692 \text{ V}$$

(c)

$$\Delta I = \left( \frac{0.25}{0.026} \right) \left( \frac{0.010}{2} \right) = 0.04808 \text{ mA}$$

$$\Delta V_{C2} = (0.04808)(8) = 0.385 \text{ V}$$

### 11.7

$$P = (I_1 + I_{C4})(V^+ - V^-)$$

$$I_1 \cong I_{C4} \text{ so } 1.2 = 2I_1(6) \Rightarrow \underline{I_1 = I_{C4} = 0.1 \text{ mA}}$$

$$R_1 = \frac{3 - 0.7 - (-3)}{0.1} \Rightarrow \underline{R_1 = 53 \text{ k}\Omega}$$

$$\text{For } v_{CM} = +1\text{V} \Rightarrow V_{C1} = V_{C2} = 1 \text{ V} \Rightarrow R_C = \frac{3 - 1}{0.05} \Rightarrow \underline{R_C = 40 \text{ k}\Omega}$$

One-sided output

$$A_d = \frac{1}{2} g_m R_C \text{ where } g_m = \frac{0.05}{0.026} = 1.923 \text{ mA/V}$$

Then

$$A_d = \frac{1}{2} (1.923)(40) \Rightarrow \underline{A_d = 38.5}$$

### 11.8

a.

$$0 = 0.7 + \frac{I_E}{2}(2) + I_E(85) - 5$$

$$I_E = \frac{5 - 0.7}{85 + 1} \Rightarrow \underline{I_E = 0.050 \text{ mA}}$$

$$I_{C1} = I_{C2} = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{I_E}{2} \right) = \left( \frac{100}{101} \right) \left( \frac{0.050}{2} \right)$$

$$\text{Or } \underline{I_{C1} = I_{C2} = 0.0248 \text{ mA}}$$

$$V_{CE1} = V_{CE2} = [5 - I_{C1}(100)] - (-0.7)$$

$$\text{So } \underline{V_{CE1} = V_{CE2} = 3.22 \text{ V}}$$

b.  $v_{cm}(\max)$  for  $V_{CB} = 0$  and  $V_C = 5 - I_{C1}(100) = 2.52 \text{ V}$

So  $v_{cm}(\max) = 2.52 \text{ V}$

$v_{cm}(\min)$  for  $Q_1$  and  $Q_2$  at the edge of cutoff  $\Rightarrow v_{cm}(\min) = -4.3 \text{ V}$

(c) Differential-mode half circuits

$$-\frac{v_d}{2} = V_\pi + \left( \frac{V_\pi}{r_\pi} + g_m V_\pi \right) R'_E$$

$$= V_\pi \left[ 1 + \frac{(1+\beta) R'_E}{r_\pi} \right]$$

Then

$$V_\pi = \frac{-(v_d/2)}{\left[ 1 + \frac{(1+\beta) R'_E}{r_\pi} \right]}$$

$$v_o = -g_m V_\pi R_C \Rightarrow A_d = \frac{1}{2} \cdot \frac{\beta R_C}{r_\pi + (1+\beta) R'_E}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105 \text{ k}\Omega \quad R'_E = 2 \text{ k}\Omega$$

Then

$$A_d = \frac{1}{2} \cdot \frac{(100)(100)}{105 + (101)(2)} \Rightarrow A_d = 16.3$$

### 11.9

a. For  $v_1 = v_2 = 0$  and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow R_E = 62 \text{ k}\Omega$$

b.

$$A_d = \frac{v_{o2}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow A_d = 71.0$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left[ \frac{1}{1 + \frac{2R_E(1+\beta)}{r_\pi + R_B}} \right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5} \left[ \frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}} \right] \Rightarrow A_{cm} = -0.398$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{71.0}{0.398} \right| \Rightarrow CMRR_{dB} = 45.0 \text{ dB}$$

c.

$$R_{id} = 2(r_{\pi} + R_B)$$

$$R_{id} = 2(34.7 + 0.5) \Rightarrow \underline{R_{id} = 70.4 \text{ k}\Omega}$$

Common-mode input resistance

$$R_{icm} = \frac{1}{2}[r_{\pi} + R_B + 2(1 + \beta)R_E]$$

$$= \frac{1}{2}[34.7 + 0.5 + 2(101)(62)] \Rightarrow \underline{R_{icm} = 6.28 \text{ M}\Omega}$$

### 11.10

(a)  $I_{E1} = I_{E2} = \left(\frac{81}{80}\right)(50) = 50.625 \Rightarrow I_E = 101.25 \mu\text{ A}$

$$R_E = \frac{3 - 0.6}{0.10125} = 23.7 \text{ k}\Omega$$

$$R_C = \frac{-1.5 - (-3)}{0.05} = 30 \text{ k}\Omega$$

(b) (i)  $I_E = \frac{3 - (1 + 0.6)}{23.7} \Rightarrow I_E = 59.07 \mu\text{ A}$

$$I_{C1} = I_{C2} = \frac{1}{2}\left(\frac{80}{81}\right)(59.07) = 29.17 \mu\text{ A}$$

$$v_{C1} = v_{C2} = I_C R_C - 3 = (0.02917)(30) - 3 = -2.125 \text{ V}$$

(ii)  $g_m = \frac{0.02917}{0.026} = 1.122 \text{ mA/V}$

$$v_d = 12 \text{ mV}, \quad \frac{v_d}{2} = 6 \text{ mV}$$

$$v_{C2} = -2.125 - (1.122)(30)(0.006) = -2.327 \text{ V}$$

$$v_{C1} = -2.125 + (1.122)(30)(0.006) = -1.923 \text{ V}$$

### 11.11

(a)

$$v_1 = v_2 = 0$$

$$I_{E1} = I_{E2} \cong 6 \mu\text{ A}$$

$$\beta = 60$$

$$I_{C1} = I_{C2} = 5.90 \mu\text{ A}$$

$$v_{C1} = v_{C2} = (5.90)(0.360) - 3 \\ = -0.875 \text{ V}$$

$$V_{EC1} = V_{EC2} = +0.6 - (-0.875) \\ = 1.475 \text{ V}$$

(b)

(i)

$$g_m = \frac{5.90}{0.026} \Rightarrow 0.227 \text{ mA/V}$$

$$A_d = g_m R_C = (0.227)(360) = 81.7$$

$$A_{cm} = 0$$



(ii)

$$A_d = \frac{g_m R_C}{2} = 40.8 \quad r_\pi = \frac{(60)(0.026)}{0.0059} = 264 \text{ K}$$

$$A_{cm} = \frac{-(0.227)(360)}{1 + \frac{2(61)(4000)}{264}} = -0.0442$$

### 11.12

$$I_Q = 0.2 \text{ mA}, \Rightarrow I_{C1} = I_{C2} = \frac{1}{2} \left( \frac{80}{81} \right) (0.2) = 0.09877 \text{ mA}$$

(a) For  $V_{CB} = 0$ ,  $v_{C1} = v_{C2} = v_{CM} = -2.5 = I_C R_C - 5$

$$\text{So } R_C = \frac{5 - 2.5}{0.09877} = 25.3 \text{ k}\Omega$$

(b)  $A_d = g_m R_C$ ,  $g_m = \frac{0.09877}{0.026} = 3.799 \text{ mA/V}$

$$A_d = (3.799)(25.3) = 96.1$$

(c)  $v_d = 14 \text{ mV}$ ,  $\frac{v_d}{2} = 7 \text{ mV}$

$$v_{C1} = -2.5 - g_m R_C \cdot \frac{v_d}{2} = -2.5 - (3.799)(25.3)(0.007) = -3.173 \text{ V}$$

$$v_{C2} = -2.5 + (3.799)(25.3)(0.007) = -1.827 \text{ V}$$

(d)  $CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$

$$1000 = \frac{1}{2} \left[ 1 + \frac{(81)(0.2)R_o}{(0.026)(80)} \right] \Rightarrow R_o = 257 \text{ k}\Omega$$

### 11.13

(a) Neglect dc base currents

$$I_E = I_{C1} + I_{C2} = 240 \mu\text{A}$$

$$R_E = \frac{-0.7 - (-5)}{0.24} = 17.9 \text{ k}\Omega$$

$$R_C = \frac{5 - 3}{0.12} = 16.7 \text{ k}\Omega$$

(b)  $g_m = \frac{0.12}{0.026} = 4.615 \text{ mA/V}$

$$A_d = g_m R_C = (4.615)(16.667) = 76.9$$

(c)  $A_{cm} \cong \frac{\Delta R_C}{R_E} = \frac{0.5}{17.9} = 0.0279$

$$CMRR_{dB} = 20 \log_{10} \left( \frac{76.9}{0.0279} \right) = 68.8 \text{ dB}$$

11.14

$$v_1 = v_2 = 0$$

$$I_E = \frac{-0.7 - (-10)}{75}$$

$$= 0.124 \text{ mA}$$

$$I_{C1} = I_{C2} = 0.0615 \text{ mA}$$

$$g_m = \frac{0.0615}{0.026} = 2.365 \text{ mA/V}$$

$$\frac{\Delta g_m}{g_m} = 0.01$$

$$g_{m1} = 2.377 \text{ mA/V}$$

$$g_{m2} = 2.353 \text{ mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{0.0615} = 50.7 \text{ K}$$

$$\Delta I = g_m \frac{V_d}{2}$$

$$\Delta v_{C1} = -g_{m1} \frac{V_d}{2} R_C$$

$$\Delta v_{C2} = +g_{m2} \frac{V_d}{2} R_C$$

$$v_o = \Delta v_{C1} - \Delta v_{C2} = -g_{m1} \frac{V_d}{2} R_C - g_{m2} \frac{V_d}{2} R_C$$

$$= -\frac{V_d}{2} R_C (g_{m1} + g_{m2})$$

$$A_d = -\frac{R_C}{2} (g_{m1} + g_{m2}) = \frac{-50}{2} (2.377 + 2.353) \Rightarrow A_d = -118.25$$

Common-Mode

$$\Delta v_{C1} = \frac{-g_{m1} R_C v_{cm}}{1 + \left( \frac{1 + \beta}{r_\pi} \right) (2R_E)} \quad \Delta v_{C2} = \frac{-g_{m2} R_C v_{cm}}{1 + \left( \frac{1 + \beta}{r_\pi} \right) (2R_E)}$$

$$\frac{v_o}{v_{cm}} = A_{cm} = \frac{-(g_{m1} - g_{m2}) R_C}{1 + \left( \frac{1 + \beta}{r_\pi} \right) (2R_E)} = \frac{-(2.377 - 2.353)(50)}{1 + \left( \frac{121}{50.7} \right) (2)(75)}$$

$$= \frac{-1.2}{358.99} \Rightarrow A_{cm} = -0.003343$$

$$CMRR \int_{dB} = \underline{91 \text{ dB}}$$

**11.15**

(a)

$$v_1 = v_2 = 0$$

$$v_E = +0.7 \text{ V}$$

$$I_E = \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

$$I_{C1} = I_{C2} = 2.132 \text{ mA}$$

$$v_{C1} = v_{C2} = (2.132)(1) - 5 \\ = -2.87 \text{ V}$$

$$v_1 = 0.5, v_2 = 0 \quad Q_2 \text{ on}$$

$Q_1$  off

(b)

$$I_{C1} = 0, I_{C2} = 4.3 \left( \frac{120}{121} \right) \text{ mA} = 4.264 \text{ mA}$$

$$v_{C1} = -5 \text{ V} \quad v_{C2} = (4.264)(1) - 5$$

$$v_{C2} = -0.736 \text{ V}$$

(c)

$$v_E \approx 0.7 \text{ V} \quad g_m = \frac{2.132}{0.026} = 82.0 \text{ mA/V}$$

$$\Delta I = g_m \frac{v_d}{2} \quad \Delta v_c = \Delta I \cdot R_C = g_m \frac{V_d}{2} \cdot R_C = \frac{(82.0)}{2} \cdot V_d (1) = 41.0 V_d$$

$$V_d = 0.015 \Rightarrow \Delta v_c = 0.615 \text{ V}$$

$$v_{C2} \downarrow \quad v_{C1} \uparrow$$

$$v_{C1} = -2.87 + 0.615 = -2.255 \text{ V}$$

$$v_{C2} = -2.87 - 0.615 = -3.485 \text{ V}$$

**11.16**

(a)

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{0.01} = 100$$

$$A_d = g_m R_C$$

$$100 = 38.46 R_C$$

$$R_C = 2.6 \text{ K}$$

(b)

With  $v_1 = v_2 = 0$

$$v_{C1} = v_{C2} = 10 - (1)(2.6) = 7.4 \text{ V} \Rightarrow v_{cm}(\text{max}) = 7.4 \text{ V}$$

**11.17**

(a)  $I_O = I_{E1} + I_{E2} \Rightarrow I_{E1} = I_{E2} = 0.4 \text{ mA}$

(i)  $v_{O1} - v_{O2} = [V^+ - I_{E1}R_{C1}] - [V^+ - I_{E2}R_{C2}] = I_{E2}R_{C2} - I_{E1}R_{C1}$   
 $v_{O1} - v_{O2} = 0$

(ii)  $v_{O1} - v_{O2} = (0.4)(7.6 - 7.4) = 0.08 \text{ V}$

(b)  $V_{BE1} = V_{BE2}$

$$0.8 = (2.9 \times 10^{-15} + 3.1 \times 10^{-15}) \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow \exp\left(\frac{V_{BE}}{V_T}\right) = 1.333 \times 10^{14}$$

Then  $I_{E1} = (2.9 \times 10^{-15})(1.333 \times 10^{14}) = 0.3867 \text{ mA}$

$I_{E2} = (3.1 \times 10^{-15})(1.333 \times 10^{14}) = 0.4133 \text{ mA}$

(i)  $v_{O1} - v_{O2} = (0.4133 - 0.3867)(7.5) = 0.1995 \text{ V}$

(ii)  $v_{O1} - v_{O2} = (0.4133)(7.6) - (0.3867)(7.4) = 0.2795 \text{ V}$

**11.18**

(a)  $\frac{i_{C1}}{I_Q} = 0.20 = \frac{1}{1 + \exp\left(\frac{-v_d}{V_T}\right)} \Rightarrow \exp\left(\frac{-v_d}{V_T}\right) = 4$

so that  $-v_d = (0.026)\ln(4) \Rightarrow v_d = -36.0 \text{ mV}$

(b)  $\frac{i_{C2}}{I_Q} = 0.90 = \frac{1}{1 + \exp\left(\frac{+v_d}{V_T}\right)} \Rightarrow \exp\left(\frac{+v_d}{V_T}\right) = 0.1111$

so that  $v_d = (0.026)\ln(0.1111) = -57.1 \text{ mV}$

**11.19**

(a) 
$$\frac{\left[0.5I_Q + \left(\frac{I_Q}{4V_T}\right) \cdot v_d(\text{max})\right] - \frac{I_Q}{1 + \exp(-v_d(\text{max})/V_T)}}{\left[0.5I_Q + \left(\frac{I_Q}{4V_T}\right) \cdot v_d(\text{max})\right]} = 0.005$$

$$(0.995) \left[0.5 + \left(\frac{1}{4V_T}\right) \cdot v_d(\text{max})\right] = \frac{1}{1 + \exp(-v_d(\text{max})/V_T)}$$

By trial and error,  $v_d(\text{max}) \cong 14 \text{ mV}$

(b) 
$$(0.985) \left[0.5 + \left(\frac{1}{4V_T}\right) \cdot v_d(\text{max})\right] = \frac{1}{1 + \exp(-v_d(\text{max})/V_T)}$$

By trial and error,  $v_d(\text{max}) \cong 21.2 \text{ mV}$

**11.20**

For  $v_{CM} = 3.5\text{ V}$  and a maximum peak-to-peak swing in the output voltage of  $2\text{ V}$ , we need the quiescent collector voltage to be

$$V_C = 3.5 + 1 = 4.5\text{ V}$$

Assume the bias is  $\pm 10\text{ V}$ , and  $I_Q = 0.5\text{ mA}$ .

Then  $I_C = 0.25\text{ mA}$

$$\text{Now } R_C = \frac{10 - 4.5}{0.25} \Rightarrow R_C = 22\text{ k}\Omega$$

$$\text{In this case, } r_\pi = \frac{(100)(0.026)}{0.25} = 10.4\text{ k}\Omega$$

Then

$$A_d = \frac{(100)(22)}{2(10.4 + 0.5)} = 101 \quad \text{So gain specification is met.}$$

For  $CMRR_{dB} = 80\text{ dB} \Rightarrow$

$$CMRR = 10^4 = \frac{1}{2} \left[ 1 + \frac{(1 + \beta)I_Q R_o}{V_T \beta} \right] = \frac{1}{2} \left[ 1 + \frac{(101)(0.5)R_o}{(0.026)(100)} \right] \Rightarrow R_o = 1.03\text{ M}\Omega$$

Need to use a Modified Widlar current source.

$$R_o = r_o [1 + g_m (R_{E1} \parallel r_\pi)]$$

$$\text{If } V_A = 100\text{ V, then } r_o = \frac{100}{0.5} = 200\text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\text{ k}\Omega$$

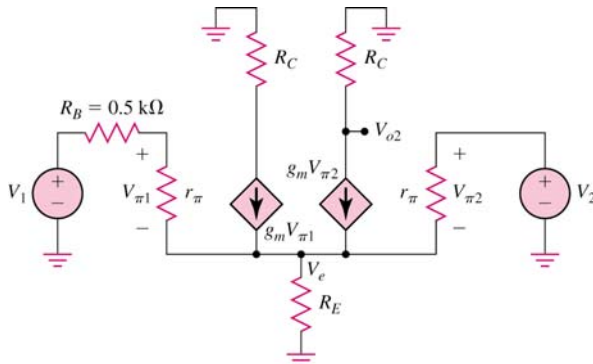
$$g_m = \frac{0.5}{0.026} = 19.23\text{ mA/V}$$

$$\text{Then } 1030 = 200 [1 + (19.23)(R_{E1} \parallel r_\pi)] \Rightarrow R_{E1} \parallel r_\pi = 0.216\text{ k}\Omega \Rightarrow R_{E1} \parallel 5.2 = 0.216$$

So,  $R_{E1} = 225\ \Omega$ ; also  $I_{REF} \cong 0.5\text{ mA}$

**11.21**

(a) 
$$R_E = \frac{-0.7 - (-10)}{0.25} \Rightarrow R_E = 37.2\text{ k}\Omega$$



(b)

$$\frac{V_{\pi 1}}{r_{\pi}} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi}} + g_m V_{\pi 2} = \frac{V_e}{R_E} \text{ or } (1) \left( \frac{1+\beta}{r_{\pi}} \right) (V_{\pi 1} + V_{\pi 2}) = \frac{V_e}{R_E}$$

$$\frac{V_{\pi 1}}{r_{\pi}} = \frac{V_1 - V_e}{R_B + r_{\pi}} \Rightarrow V_{\pi 1} = \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right) (V_1 - V_e)$$

$$V_{\pi 2} = V_2 - V_e$$

Then

$$(1) \left( \frac{1+\beta}{r_{\pi}} \right) \left[ \frac{r_{\pi}}{r_{\pi} + R_B} (V_1 - V_e) + (V_2 - V_e) \right] = \frac{V_e}{R_E}$$

From this, we find

$$V_e = \frac{V_1 + \frac{r_{\pi} + R_B}{r_{\pi}} \cdot V_2}{\left[ \frac{r_{\pi} + R_B}{R_E (1+\beta)} + 1 + \frac{r_{\pi} + R_B}{r_{\pi}} \right]}$$

Now

$$V_o = -g_m V_{\pi 2} R_C = -g_m R_C (V_2 - V_e)$$

We have

$$r_{\pi} = \frac{(120)(0.026)}{0.125} \cong 25 \text{ k}\Omega, \quad g_m = \frac{0.125}{0.026} = 4.81 \text{ mA/V}$$

(i)

$$\text{Set } V_1 = \frac{V_d}{2} \text{ and } V_2 = -\frac{V_d}{2}$$

Then

$$V_e = \frac{\frac{V_d}{2} \left( 1 - \left( \frac{25+0.5}{25} \right) \right)}{\left[ \frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25} \right]} = \frac{\frac{V_d}{2} (-0.02)}{2.026}$$

So

$$V_e = -0.00494 V_d$$

Now

$$V_o = -(4.81)(50) \left( -\frac{V_d}{2} - (-0.00494)V_d \right) \Rightarrow A_d = \frac{V_o}{V_d} = 119$$

(ii)

$$\text{Set } V_1 = V_2 = V_{cm}$$

Then

$$V_e = \frac{V_{cm} \left( 1 + \frac{25+0.5}{25} \right)}{\left[ \frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25} \right]} = \frac{V_{cm} (-2.02)}{2.02567}$$

$$V_e = V_{cm} (0.9972)$$

Then

$$V_o = -(4.81)(50) [V_{cm} - V_{cm} (0.9972)]$$

$$\text{or } A_{cm} = \frac{V_o}{V_{cm}} = -0.673$$

**11.22**

(a)  $A_d = g_m R_C$

$$A_d = \frac{1.2}{0.016} = 75$$

$$g_m = \frac{0.125}{0.026} = 4.808 \text{ mA/V}$$

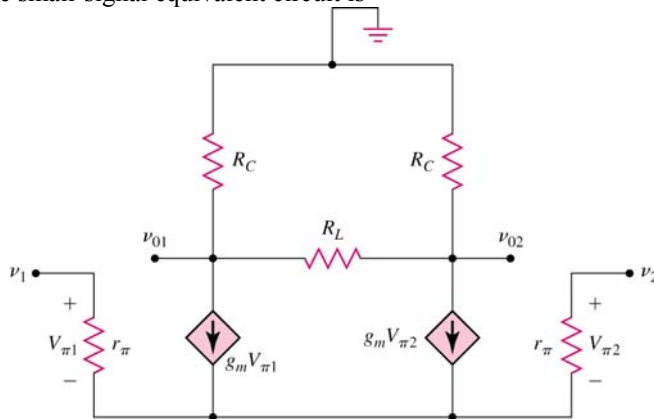
Then  $R_C = \frac{75}{4.808} = 15.6 \text{ k}\Omega$

(b) For  $V_{CB} = 0 \Rightarrow v_{c1} = v_{c2} = v_{CM} = 3 - (0.125)(15.6) = 1.05 \text{ V}$

(c)  $CMRR = \frac{1}{2} \left[ 1 + \frac{(0.25)(4000)}{(0.026)} \right] = 19,231 \Rightarrow CMRR_{dB} = 85.7 \text{ dB}$

**11.23**

The small-signal equivalent circuit is



A KVL equation:  $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

A KCL equation

$$\frac{V_{\pi 1}}{r_{\pi}} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi}} + g_m V_{\pi 2} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left( \frac{1}{r_{\pi}} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then  $v_1 - v_2 = 2V_{\pi 1} \Rightarrow V_{\pi 1} = \frac{1}{2}(v_1 - v_2)$  and  $V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$

At the  $v_{01}$  node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi 1} = 0$$

$$v_{01} \left( \frac{1}{R_C} + \frac{1}{R_L} \right) - v_{02} \left( \frac{1}{R_L} \right) = \frac{1}{2} g_m (v_2 - v_1) \quad (1)$$

At the  $v_{02}$  node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi 2} = 0$$

$$v_{02} \left( \frac{1}{R_C} + \frac{1}{R_L} \right) - v_{01} \left( \frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2) \quad (2)$$

From (1):

$$v_{02} = v_{01} \left( 1 + \frac{R_L}{R_C} \right) - \frac{1}{2} g_m R_L (v_2 - v_1)$$

Substituting into (2)

$$v_{01} \left( 1 + \frac{R_L}{R_C} \right) \left( \frac{1}{R_C} + \frac{1}{R_L} \right) - \frac{1}{2} g_m R_L (v_2 - v_1) \left( \frac{1}{R_C} + \frac{1}{R_L} \right) - v_{01} \left( \frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2)$$

$$v_{01} \left( \frac{1}{R_C} + \frac{R_L}{R_C^2} + \frac{1}{R_C} \right) = \frac{1}{2} g_m (v_1 - v_2) \left[ 1 - \left( \frac{R_L}{R_C} + 1 \right) \right]$$

$$\frac{v_{01}}{R_C} \left( 2 + \frac{R_L}{R_C} \right) = -\frac{1}{2} g_m \left( \frac{R_L}{R_C} \right) (v_1 - v_2)$$

For  $v_1 - v_2 = v_d$

$$A_{v1} = \frac{v_{01}}{v_d} = \frac{-\frac{1}{2} g_m R_L}{\left( 2 + \frac{R_L}{R_C} \right)}$$

$$A_{v2} = \frac{v_{02}}{v_d} = \frac{\frac{1}{2} g_m R_L}{\left( 2 + \frac{R_L}{R_C} \right)}$$

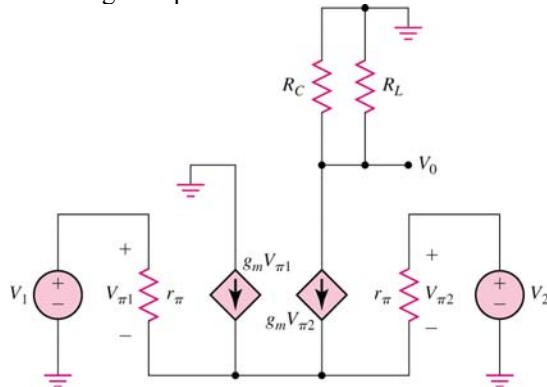
From symmetry:

$$A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left( 2 + \frac{R_L}{R_C} \right)}$$

Then

### 11.24

The small-signal equivalent circuit is



KVL equation:  $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$  or  $v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$



KCL equation:

$$\frac{V_{\pi 1}}{r_{\pi}} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi}} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left( \frac{1}{r_{\pi}} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then  $v_1 - v_2 = -2V_{\pi 2}$ , or  $V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$

Now  $v_o = -g_m V_{\pi 2} (R_C \parallel R_L)$

$$= \frac{1}{2} g_m (R_C \parallel R_L) (v_1 - v_2)$$

For  $v_1 - v_2 \equiv v_d \Rightarrow A_d = \frac{v_o}{v_d} = \frac{1}{2} g_m (R_C \parallel R_L)$

### 11.25

We have  $V_{C2} = -g_m V_{\pi 2} R_C = -g_m (V_{b2} - V_e) R_C$

and

$$V_{C1} = -g_m V_{\pi 1} R_C = -g_m (V_{b1} - V_e) R_C$$

Then

$$V_0 = V_{C2} - V_{C1}$$

$$= -g_m (V_{b2} - V_e) R_C - [-g_m (V_{b1} - V_e) R_C]$$

$$= g_m R_C (V_{b1} - V_{b2})$$

Differential gain  $A_d = \frac{V_0}{V_{b1} - V_{b2}} = g_m R_C$

Common-mode gain  $A_{cm} = 0$

### 11.26

(a)

$$v_{cm} = 3 \text{ V} \Rightarrow V_{C1} = V_{C2} = 3 \text{ V}$$

$$\text{Then } R_C = \frac{10 - 3}{0.1} \Rightarrow R_C = 70 \text{ k}\Omega$$

(b)

$$CMRR_{dB} = 75 \text{ dB} \Rightarrow CMRR = 5623$$

Now

$$CMRR = \frac{1}{2} \left[ 1 + \frac{(1 + \beta) I_Q R_o}{\beta V_T} \right]$$

$$5623 = \frac{1}{2} \left[ 1 + \frac{(151)(0.2) R_o}{(150)(0.026)} \right] \Rightarrow R_o = 1.45 \text{ M}\Omega$$

Use a Widlar current source.

$$R_o = r_o [1 + g_m R'_E]$$

Let  $V^A$  of current source transistor be 100 V.

$$\text{Now } r_o = \frac{100}{0.2} = 500 \text{ k}\Omega, \quad g_m = \frac{0.2}{0.026} = 7.69 \text{ mA/V}$$

$$r_\pi = \frac{(150)(0.026)}{0.2} = 19.5 \text{ k}\Omega$$

$$\text{So } 1450 = 500[1 + (7.69)R'_E] \Rightarrow R'_E = 0.247 \text{ k}\Omega$$

$$\text{Now } R'_E = R_E \parallel r_\pi \Rightarrow 0.247 = R_E \parallel 19.5 \Rightarrow R_E = 250 \Omega$$

$$\text{We have } I_Q R_E = V_T \ln\left(\frac{I_{REF}}{I_Q}\right)$$

$$(0.2)(0.250) = (0.026) \ln\left[\frac{I_{REF}}{(0.2)}\right] \Rightarrow I_{REF} = 1.37 \text{ mA}$$

$$\text{Then } R_1 = \frac{10 - 0.7 - (-10)}{1.37} \Rightarrow R_1 = 14.1 \text{ k}\Omega$$

### 11.27

$$v_A = \left(\frac{R_A}{R_A + R}\right) \cdot V^+ = \left[\frac{R(1 + \delta)}{R(1 + \delta) + R}\right] \cdot V^+ = \left(\frac{1 + \delta}{2 + \delta}\right) \cdot V^+$$

$$v_B = \frac{1}{2} V^+ = 2.5 \text{ V}$$

$$\text{For } \delta = +0.01, \quad v_A = \left(\frac{1.01}{2.01}\right)(5) = 2.5124 \text{ V}$$

$$\text{For } \delta = -0.01, \quad v_A = \left(\frac{0.99}{1.99}\right)(5) = 2.4874 \text{ V}$$

$$v_d \cong 12.5 \text{ mV}$$

$$R_B \cong R \parallel R = 20 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.1} = 31.2 \text{ k}\Omega$$

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(120)(15)}{2(31.2 + 20)} = 17.58$$

$$v_o = A_d \cdot v_d = (17.58)(0.0125) = 0.22 \text{ V}$$

$$-0.22 \leq v_{o2} \leq +0.22 \text{ V}$$

### 11.28

$$(a) \quad R_{id} = 2r_\pi = \frac{2\beta V_T}{I_{CQ}} = \frac{2(120)(0.026)}{0.125} = 49.9 \text{ k}\Omega$$

$$(b) \quad R_{icm} = \frac{1}{2} [r_\pi + (1 + \beta)(2R_o)] = \frac{1}{2} \left[ \frac{(120)(0.026)}{0.125} + (121)(2)(8000) \right]$$

$$R_{icm} = 968 \text{ M}\Omega$$

**11.29**

(a)

$$I_1 = \frac{10 - 0.7 - (-10)}{R_1} = 0.5 \Rightarrow R_1 = 38.6 \text{ K}$$

$$R_2 = \frac{0.026}{0.14} \ln\left(\frac{0.5}{0.14}\right) \Rightarrow R_2 = 236 \text{ } \Omega$$

(b)

$$R_{icm} \approx (1 + \beta) R_o$$

$$R_o = r_{o4} (1 + g_{m4} R'_E) \quad g_{m4} = \frac{0.14}{0.026} = 5.385 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(180)(0.026)}{0.14} = 33.4 \text{ K}$$

$$R'_E = 33.4 \parallel 0.236 = 0.234 \text{ K}$$

$$r_{o4} = \frac{100}{0.14} = 714 \text{ K}$$

$$R_o = 714 [1 + (5.385)(0.234)] \\ = 1614 \text{ K}$$

$$R_{icm} = (181)(1614) \approx 292 \text{ M}\Omega$$

(c)

$$A_{cm} = \frac{-g_{m1} R_C}{1 + \frac{2(1 + \beta) R_o}{r_{\pi 1}}} \quad g_{m1} = \frac{0.07}{0.026} = 2.692 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(180)(0.026)}{0.07} = 66.86 \text{ K}$$

$$A_{cm} = \frac{-(2.692)(40)}{1 + \frac{2(181)(1614)}{66.86}}$$

$$A_{cm} = -0.0123$$

**11.30**

$$A_{d1} = g_{m1} (R_1 \parallel r_{\pi 3})$$

$$g_{m1} = \frac{I_{Q1} / 2}{V_T} = 19.23 I_{Q1}$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{Q2} / 2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3} R_2}{2}, \quad g_{m3} = \frac{I_{Q2} / 2}{V_T} = 19.23 I_{Q2}$$

$$\text{Then } 30 = \frac{(19.23) I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2} R_2 = 3.12 \text{ V}$$

Maximum  $v_{o2} - v_{o1} = \pm 18 \text{ mV}$  for linearity

$$v_{o3}(\text{max}) = (\pm 18)(30) \text{ mV} \Rightarrow \pm 0.54 \text{ V}$$

so  $I_{Q2} R_2 = 3.12 \text{ V}$  is OK.

From  $A_{d1}$  :

$$20 = 19.23I_{Q1}(R_1 \parallel r_{\pi 3}) = 19.23I_{Q1} \left[ \frac{R_1 \left( \frac{5.2}{I_{Q2}} \right)}{R_1 + \left( \frac{5.2}{I_{Q2}} \right)} \right]$$

$$20 = \frac{19.23I_{Q1}R_1(5.2)}{I_{Q2}R_1 + 5.2}$$

Let  $\frac{I_{Q1}}{2} \cdot R_1 = 5 \text{ V}$ ,  $\Rightarrow I_{Q1}R_1 = 10 \text{ V}$

Then  $20 = \frac{19.23(10)(5.2)}{I_{Q2}R_1 + 5.2} \Rightarrow I_{Q2}R_1 = 44.8 \text{ V}$

Now  $I_{Q1}R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$

So  $I_{Q2} \left( \frac{10}{I_{Q1}} \right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$

Let  $I_{Q1} = 100 \mu\text{A}$ , then  $I_{Q2} = 448 \mu\text{A}$

Then

$$I_{Q2}R_2 = 3.12 \Rightarrow R_2 = 6.96 \text{ k}\Omega$$

$$I_{Q1}R_1 = 10 \Rightarrow R_1 = 100 \text{ k}\Omega$$

### 11.31

a.

$$I_1 = \frac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$$

$$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$$

$$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)} \Rightarrow \underline{V_{GS3} = 3.16 \text{ V}}$$

$$I_1 = \frac{20 - 3.16}{50} \Rightarrow \underline{I_1 = I_Q = 0.337 \text{ mA}}$$

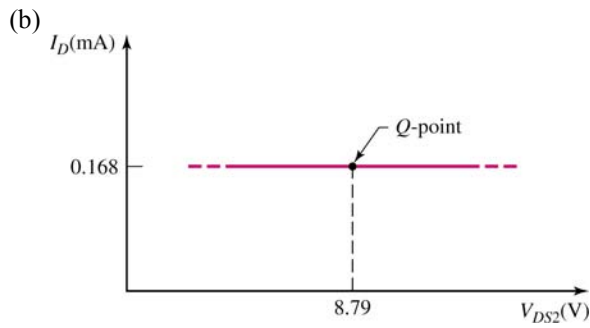
$$I_{D1} = \frac{I_Q}{2} \Rightarrow \underline{I_{D1} = 0.168 \text{ mA}}$$

$$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow \underline{V_{GS1} = 2.82 \text{ V}}$$

$$V_{DS4} = -2.82 - (-10) \Rightarrow \underline{V_{DS4} = 7.18 \text{ V}}$$

$$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_{DS1} = 5.97 - (-2.82) \Rightarrow \underline{V_{DS1} = 8.79 \text{ V}}$$



(c)

$$\text{Max } v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(\text{sat}) = V_{GS1} - V_{TN}$$

$$2.82 - 2 = 0.82 \text{ V}$$

Now  $V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$

$$V_S(\text{max}) = 5.97 - V_{DS1}(\text{sat}) = 5.97 - 0.82$$

$$V_S(\text{max}) = 5.15 \text{ V}$$

$$v_{CM}(\text{max}) = V_S(\text{max}) + V_{GS1} = 5.15 + 2.82$$

$$\underline{v_{CM}(\text{max}) = 7.97 \text{ V}}$$

$$v_{CM}(\text{min}) = V^- + V_{DS4}(\text{sat}) + V_{GS1}$$

$$V_{DS4}(\text{sat}) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16 \text{ V}$$

Then  $v_{CM}(\text{min}) = -10 + 1.16 + 2.82 \Rightarrow \underline{v_{CM}(\text{min}) = -6.02 \text{ V}}$

### 11.32

(a)  $I_{D1} = K_{n1}(V_{GS1} - V_{TN})^2$

$$60 = 100(V_{GS1} - 0.3)^2 \Rightarrow V_{GS1} = 1.075 \text{ V}$$

$$V_{D1} = V_{DS1} - V_{GS1} + V_1 = 4 - 1.075 - 1.15 = 1.775 \text{ V}$$

$$R_D = \frac{3 - 1.775}{0.06} = 20.4 \text{ k}\Omega$$

(i)  $I_Q = I_1 = I_{D1} + I_{D2} = 120 \mu\text{A}$

$$I_1 = K_3(V_{GS3} - V_{TN})^2$$

$$120 = 200(V_{GS3} - 0.3)^2 \Rightarrow V_{GS3} = 1.075 \text{ V}$$

(ii)  $R_1 = \frac{3 - 1.075 - (-3)}{0.12} = 41 \text{ k}\Omega$

(iii)  $V_{GS1} = V_{GS4} = 1.075 \text{ V}$

(b)  $r_o = \frac{1}{\lambda_4 I_Q} = \frac{1}{(0.01)(0.12)} = 833.3 \text{ k}\Omega$

$$\Delta I_Q = \frac{\Delta V_{DS4}}{r_o} = \frac{2.3}{833.3} \Rightarrow \Delta I_Q = 2.76 \mu\text{A}$$

**11.33**

(a)

$$I_Q = 160 \mu\text{A}$$

$$I_D = \frac{k'_n}{2} \left( \frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$80 = \frac{80}{2} (4) (V_{OS} - 0.5)^2$$

$$80 = 160 (V_{OS} - 0.5)^2$$

$$V_{GS} = \sqrt{\frac{80}{160}} + 0.5 = 1.207 \text{ V}$$

$$R_D = \frac{5 - 2}{0.08} = 37.5 \text{ K} \quad V_{DS} = 2 - (-1.207) = 3.21 \text{ V}$$

(c)

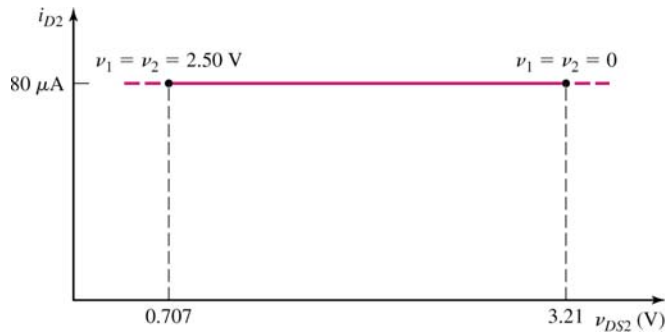
$$V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.207 - 0.5 = 0.707 \text{ V}$$

$$\text{Then } V_S = V_{O2} - V_{DS}(\text{sat}) = 2 - 0.707 = +1.29 \text{ V}$$

$$\text{And } v_1 = v_2 = v_{cm} = V_{GS} + V_S = 1.207 + 1.29$$

$$\underline{v_{cm} = 2.50 \text{ V}}$$

(b)



**11.34**

(a)  $v_{CM}(\text{max}) = v_{D2} - v_{DS2}(\text{sat}) + v_{GS2}$

$$i_{D2} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (v_{GS2} - V_{TN})^2$$

$$0.09 = \left( \frac{0.1}{2} \right) (4) (v_{GS2} - 0.35)^2 \Rightarrow v_{GS2} = 1.021 \text{ V}$$

$$v_{DS2}(\text{sat}) = 1.0208 - 0.35 = 0.6708 \text{ V}$$

$$\text{Then } 2.25 = v_{D2} - 0.6708 + 1.021 \Rightarrow v_{D2} = 1.90 \text{ V}$$

$$R_D = \frac{3 - 1.90}{0.09} = 12.2 \text{ k}\Omega$$

(b) (i)  $v_{D2} = 1.90 \text{ V}$

(ii)  $g_m = 2 \sqrt{\left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left( \frac{0.1}{2} \right) (4) (0.09)} = 0.2683 \text{ mA/V}$

$$v_{D2} = 1.90 + g_m \cdot \frac{v_d}{2} \cdot R_D = 1.90 + (0.2683) \left( \frac{0.12}{2} \right) (12.2) = 2.096 \text{ V}$$

$$(iii) v_{D2} = 1.90 - g_m \cdot \frac{v_d}{2} \cdot R_D = 1.90 - (0.2683) \left( \frac{0.05}{2} \right) (12.2) = 1.818 \text{ V}$$

**11.35**

- (a)  $I_Q = I_1 = K_{n3} (V_{GS3} - V_{TN})^2$   
 $0.25 = 0.20 (V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 1.518 \text{ V}$   
 $R_1 = \frac{5 - 1.518 - (-5)}{0.25} = 33.9 \text{ k}\Omega$
- (b)  $I_{D1} = I_{D2} = \frac{0.25}{2} = 0.125 \text{ mA}$   
 $v_{O1} - v_{O2} = (V^+ - I_{D1} R_{D1}) - (V^+ - I_{D2} R_{D2}) = I_{D2} R_{D2} - I_{D1} R_{D1}$   
 (i)  $v_{O1} - v_{O2} = 0$   
 (ii)  $v_{O1} - v_{O2} = (0.125)(15.5 - 14.5) = 0.125 \text{ V}$
- (c)  $I_Q = I_{D1} + I_{D2} = (K_{n1} + K_{n2})(V_{GS} - V_{TN})^2$   
 $0.25 = (0.125 + 0.115)(V_{GS} - V_{TN})^2 \Rightarrow (V_{GS} - V_{TN})^2 = 1.04166$   
 $I_{D1} = (0.125)(1.04166) = 0.1302 \text{ mA}$   
 $I_{D2} = (0.115)(1.04166) = 0.1198 \text{ mA}$   
 (i)  $v_{O1} - v_{O2} = (15)(0.1198 - 0.1302) = -0.156 \text{ V}$   
 (ii)  $v_{O1} - v_{O2} = (0.1198)(15.5) - (0.1302)(14.5) = -0.031 \text{ V}$

**11.36**

- (a)  $\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left( \frac{K_n}{2I_Q} \right) \cdot v_d^2}$   
 $0.20 = 0.50 + \sqrt{\frac{0.20}{2(0.15)}} \cdot v_d \sqrt{1 - \left( \frac{0.20}{2(0.15)} \right) \cdot v_d^2}$   
 $-0.3674 = v_d \sqrt{1 - (0.6667)v_d^2}$   
 $0.135 = v_d^2 [1 - (0.6667)v_d^2]$   
 $0.6667v_d^4 - v_d^2 + 0.135 = 0 \Rightarrow v_d^2 = 0.15 \Rightarrow v_d = -0.3873 \text{ V}$
- (b)  $0.80 = 0.50 - \sqrt{\frac{0.20}{2(0.15)}} \cdot v_d \sqrt{1 - \left( \frac{0.20}{2(0.15)} \right) \cdot v_d^2}$   
 $0.3674 = -v_d \sqrt{1 - (0.6667)v_d^2} \Rightarrow v_d = -0.3873 \text{ V}$
- (c)  $v_d = v_{d,\max} = \sqrt{\frac{I_Q}{K_n}} = \sqrt{\frac{0.15}{0.20}} = +0.866 \text{ V}$

**11.37**

$$\frac{i_{D1}}{I_Q} \Big|_{Lin} = 0.5 + \frac{g_f v_d}{I_Q}, \text{ Now } g_f = \sqrt{\frac{K_n I_Q}{2}} = \sqrt{\frac{(0.2)(0.10)}{2}} = 0.10 \text{ mA/V}$$

$$= 0.5 + \frac{(0.1)}{(0.1)} \cdot v_d = 0.5 + v_d$$

We have  $\frac{K_n}{2I_Q} = \frac{(0.2)}{2(0.1)} = 1$

(a)  $\frac{0.5 + v_d - [0.5 + v_d \sqrt{1 - v_d^2}]}{0.5 + v_d} = 0.005$

$$0.0025 = v_d [0.995 - \sqrt{1 - v_d^2}], \Rightarrow v_d \cong 0.19 \text{ V}$$

(b)  $\frac{0.5 + v_d - [0.5 + v_d \sqrt{1 - v_d^2}]}{0.5 + v_d} = 0.015$

$$0.0075 = v_d [0.985 - \sqrt{1 - v_d^2}], \Rightarrow v_d \cong 0.285 \text{ V}$$

**11.38**

(b)

$$g_m = 2\sqrt{K_p I_D} = 2\sqrt{(0.05)(0.008696)} \\ = 0.0417 \text{ mA/V}$$

$$\Delta I = g_m \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063$$

$$v_{D2} \uparrow \Rightarrow v_{D2} = 1.063 - 4.565 = -3.502 \text{ V}$$

$$v_{D1} = -1.063 - 4.565 = -5.628 \text{ V}$$

$$9 = I_S R_S + V_{SG} + 1$$

$$I_S = 2I_D$$

$$8 = 2K_p R_S (V_{SG} + V_{TP})^2 + V_{SG}$$

$$8 = (2)(0.05)(390)(V_{SG} - 0.8)^2 + V_{SG}$$

$$8 = 39(V_{SG}^2 - 1.6V_{SG} + 0.64) + V_{SG}$$

$$39V_{SG}^2 - 61.4V_{SG} + 16.96 = 0$$

$$V_{SG} = \frac{61.4 \pm \sqrt{3769.96 - 4(39)(16.96)}}{2(39)}$$

$$= 1.217 \text{ V } V_S = 2.217$$

$$I_S = 0.01739 \text{ mA } I_{D1} = I_{D2} \Rightarrow 8.696 \mu\text{A}$$

$$v_{D1} = v_{D2} = (8.696)(0.510) - 9 = -4.565 \text{ V}$$

(b)

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.05)(0.008696)} = 0.0417 \text{ mA/V}$$

$$\Delta v_D = \Delta I_D \cdot R_D \quad \Delta I_D = g_m \cdot \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063 \text{ V}$$



$$v_1 \uparrow, I_{D1} \downarrow, v_{D1} \downarrow$$

$$v_{D1} = -4.565 - 1.063 = -5.628 \text{ V}$$

$$v_{D2} = -4.565 + 1.063 = -3.502 \text{ V}$$

**11.39**

(a)

$$v_1 = v_2 = 0$$

$$I_D = K_n (V_{SG} + V_{TP})^2$$

$$I_D = 6 \mu\text{A}$$

$$\sqrt{\frac{6}{30}} + 0.4 = V_{SG}$$

$$V_{SG} = 0.847 \text{ V}$$

$$V_S = +0.847 \text{ V}$$

$$v_D = I_D R_D - 3$$

$$= (6)(0.36) - 3 = -0.84 \text{ V}$$

$$V_{SD} = V_S - v_D = 0.847 - (-0.84)$$

$$v_{SD} = 1.69 \text{ V}$$

(b)

(i)

$$A_d = g_m R_D \quad g_m = 2\sqrt{K_n I_D}$$

$$= 2\sqrt{(30)(6)} = 26.83 \mu\text{A/V}$$

$$A_d = (26.83)(0.36) \Rightarrow A_d = 9.66$$

$$A_{cm} = 0$$

(ii)

$$A_d = \frac{g_m R_D}{2} = \frac{(26.83)(0.36)}{2} \Rightarrow A_d = 4.83$$

$$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_O} = \frac{-(26.83)(0.36)}{1 + 2(26.83)(4)} = -0.0448$$

**11.40**

(a)  $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.12)(0.075)} = 0.1897 \text{ mA/V}$

(i)  $v_{D1} - v_{D2} = g_m R_D v_d = (0.1897)(30)(0.1) = 0.5692 \text{ V}$

(ii)  $v_{D1} - v_{D2} = g_m R_D v_d = (0.1897)(30)(0.2) = 1.138 \text{ V}$

(b) (i)  $\Delta v_{D2} = g_m R_D \cdot \frac{v_d}{2} = (0.1897)(30)(0.1) = 0.5692 \text{ V}$

(ii)  $A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_o} = \frac{-(0.1897)(30)}{1 + 2(0.1897)(4000)} = -0.003748$

$$A_d = \frac{g_m R_D}{2} = \frac{(0.1897)(30)}{2} = 2.846$$

$$\Delta v_{D2} = A_d v_d + A_{cm} v_{cm}, \quad \text{let } v_{cm} = 1 \text{ V}, v_d = 0.2 \text{ V}$$

$$\text{Then } \Delta v_{D2} = (2.846)(0.2) - (0.003748)(1) = 0.5655 \text{ V}$$

**11.41**

For  $v_1 = v_2 = 0$

$$0 = V_{GS} + 2I_D R_S - 10$$

$$10 = V_{GS} + 2K_n R_S (V_{GS} - V_{TN})^2$$

$$= V_{GS} + 2(0.15)(75)(V_{GS} - 1)^2$$

$$22.5V_{GS}^2 - 44V_{GS} + 12.5 = 0$$

So  $V_{GS} = 1.61 \text{ V}$  and  $I_D = (0.15)(1.61 - 1)^2 \Rightarrow 55.9 \mu\text{A}$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.15)(0.0559)}$$

$$g_m = 0.1831 \text{ mA/V}$$

Use Half-circuits – Differential gain

$$v_{D1} = -g_m \left( \frac{V_d}{2} \right) \left( R_D + \frac{\Delta R}{2} \right)$$

$$v_{D2} = g_m \left( \frac{V_d}{2} \right) \left( R_D - \frac{\Delta R}{2} \right)$$

$$v_o = v_{D1} - v_{D2} = -g_m V_d R_D$$

$$A_d = \frac{v_o}{V_d} = -g_m R_D$$

Now – Common-Mode Gain

$$V_i = V_{gs} + g_m V_{gs} (2R_S) = V_{cm}$$

$$V_{gs} = \frac{V_{cm}}{1 + g_m (2R_S)}$$

$$v_{D1} = \frac{-g_m \left( R_D + \frac{\Delta R_D}{2} \right) V_{cm}}{1 + g_m (2R_S)}$$

$$v_{D2} = \frac{-g_m \left( R_D - \frac{\Delta R_D}{2} \right) V_{cm}}{1 + g_m (2R_S)}$$

$$v_o = v_{D1} - v_{D2}$$

$$\text{So } v_o = \frac{-g_m (\Delta R_D) V_{cm}}{1 + g_m (2R_D)}$$

$$A_{cm} = \frac{v_o}{V_{cm}} = \frac{-g_m (\Delta R_D)}{1 + g_m (2R_S)}$$

Then

$$A_d = -(0.1831)(50) = -9.16$$

$$A_{cm} = \frac{-(0.1831)(0.5)}{1 + (0.1831)(2)(75)} = -0.003216$$

$$CMRR \Big|_{dB} = 69.1 \text{ dB}$$

**11.42**

From 11.41

$$I_{D1} = I_{D2} = 55.9 \mu\text{A}$$

$$g_m = 0.183 \text{ mA/V}$$

$$A_d : \Delta v_{D1} = -g_{m1} \frac{V_d}{2} \cdot R_D \quad \Delta v_{D2} = +g_{m2} \left( \frac{+V_d}{2} \right) R_D$$

$$v_o = \Delta v_{D1} - \Delta v_{D2} = -g_{m1} \frac{V_d}{2} R_D - g_{m2} \frac{V_d}{2} R_D$$

$$v_o = \frac{-V_d}{2} \cdot R_D (g_{m2} + g_{m1}) = \frac{-V_d}{2} \cdot R_D \left( g_m - \frac{\Delta g_m}{2} + \left( g_m - \frac{\Delta g_m}{2} \right) \right)$$

$$A_d = -g_m R_D = -(0.183)(50) = \underline{\underline{-9.15}}$$

$$A_{CM} : v_o = \Delta v_{D1} - \Delta v_{D2} = \frac{-\left( g_m + \frac{\Delta g_m}{2} \right) R_D v_{cm}}{1 + g_m (2R_S)} + \frac{\left( g_m - \frac{\Delta g_m}{2} \right) R_D v_{cm}}{1 + g_m (2R_S)}$$

$$A_{cm} = \frac{v_o}{v_{cm}} = \frac{-\Delta g_m R_D}{1 + g_m (2R_S)} \quad \Delta g_m = (0.01)(0.183) = 0.00183$$

$$A_{cm} = \frac{-(0.00183)(50)}{1 + (0.183)(2)(75)} = -0.003216$$

$$CMRR \int_{dB} = 69.1 \text{ dB}$$

**11.43**

(a)  $5 = I_S R_S + V_{SG} = 2K_p (V_{SG} + V_{TP})^2 R_S + V_{SG}$

$$5 = 2(1.2)(2)(V_{SG}^2 - 1.2V_{SG} + 0.36) + V_{SG}$$

$$4.8V_{SG}^2 - 4.76V_{SG} - 3.272 = 0 \Rightarrow V_{SG} = 1.459 \text{ V} = v_S$$

$$I_S = 2(1.2)(1.459 - 0.6)^2 = 1.77 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.885 \text{ mA}$$

$$v_{D1} = v_{D2} = I_D R_D - 5 = (0.885)(1) - 5 = -4.115 \text{ V}$$

(b)  $5 = 2(1.2)(2)(V_{SG}^2 - 1.2V_{SG} + 0.36) + V_{SG} + 1$

$$4.8V_{SG}^2 - 4.76V_{SG} - 2.272 = 0 \Rightarrow V_{SG} = 1.344 \text{ V}$$

$$v_S = V_{SG} + 1 = 2.344 \text{ V}$$

$$I_{D1} = I_{D2} = (1.2)(1.344 - 0.6)^2 = 0.664 \text{ mA}$$

$$v_{D1} = v_{D2} = (0.664)(1) - 5 = -4.336 \text{ V}$$

(c) Using part (a),  $g_m = 2\sqrt{(1.2)(0.885)} = 2.061 \text{ mA/V}$

$$v_{D1} = -4.115 + g_m R_D \cdot \frac{v_d}{2} = -4.115 + (2.061)(1)(0.1) = -3.909 \text{ V}$$

$$v_{D2} = -4.115 - (2.061)(1)(0.1) = -4.321 \text{ V}$$

(d) Using part (b),  $g_m = 2\sqrt{(1.2)(0.664)} = 1.785 \text{ mA/V}$

$$v_{D1} = -4.336 + g_m R_D \cdot \frac{v_d}{2} = -4.336 + (1.785)(1)(0.1) = -4.158 \text{ V}$$

$$v_{D2} = -4.336 - (1.785)(1)(0.1) = -4.515 \text{ V}$$

**11.44**

(a)

$$g_m = 2\sqrt{K_n I_D}$$

$$= 2\sqrt{(0.4)(1)}$$

$$g_m = 1.265 \text{ mA/V}$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{0.1} = 10$$

$$A_d = g_m R_D$$

$$10 = (1.265)R_D$$

$$\underline{R_D = 7.91 \text{ K}}$$

(b)

Quiescent  $v_1 = v_2 = 0$

$$v_{D1} = v_{D2} = 10 - (1)(7.91) = 2.09 \text{ V}$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.4}} + 0.8 = 2.38 \text{ V}$$

$$V_{DS}(\text{sat}) = 2.38 - 0.8 = 1.58$$

So  $v_{cm} = v_D - V_{DS}(\text{sat}) + V_{GS}$

$$= 2.09 - 1.58 + 2.38$$

$$\underline{v_{cm} = 2.89 \text{ V}}$$

**11.45**

$$A_d = \frac{g_m R_D}{2}$$

For  $v_{CM} = 2.5 \text{ V}$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25 \text{ mA}$$

Let  $V_{D1} = V_{D2} = 3 \text{ V}$ , then  $R_D = \frac{10-3}{0.25} \Rightarrow \underline{R_D = 28 \text{ k}\Omega}$

Then  $100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14 \text{ mA/V}$

And  $g_m = 2\sqrt{\frac{k'_n}{2}\left(\frac{W}{L}\right)I_D}$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 1274 \text{ (Extremely large transistors to meet the gain requirement.)}$$

Need  $|A_{CM}| = 0.10$

From Eq. (11.82(b))

$$|A_{CM}| = \frac{g_m R_D}{1 + 2g_m R_o}$$

So  $0.10 = \frac{(7.14)(28)}{1 + 2(7.14)R_o} \Rightarrow R_o = 140 \text{ k}\Omega$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

This current source is adequate to meet common-mode gain requirement.

**11.46**

Not in detail, Approximation looks good.

a.

$$I_S = \frac{-V_{GS1} - (-5)}{R_S} \text{ and } I_S = 2I_D = 2K_n (V_{GS1} - V_{TN})^2$$

$$\frac{5 - V_{GS1}}{20} = 2(0.050)(V_{GS1} - 1)^2$$

$$5 - V_{GS1} = 2(V_{GS1}^2 - 2V_{GS1} + 1)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2 + 4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186 \text{ V}$$

$$I_S = \frac{5 - 2.186}{20} \Rightarrow I_S = 0.141 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow I_{D1} = I_{D2} = 0.0704 \text{ mA}$$

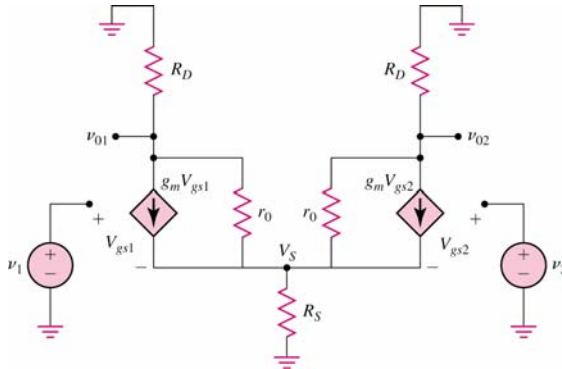
$$v_{02} = 5 - (0.0704)(25) \Rightarrow v_{02} = 3.24 \text{ V}$$

b.

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.05)(2.186 - 1)$$

$$g_m = 0.119 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710 \text{ k}\Omega$$



$$V_{gs1} = v_1 - V_S, \quad V_{gs2} = v_2 - V_S$$

$$\frac{v_{01}}{R_D} + g_m V_{gs1} + \frac{v_{01} - V_S}{r_o} = 0$$

$$v_{01} \left( \frac{1}{R_D} + \frac{1}{r_o} \right) + g_m (v_1 - V_S) - \frac{V_S}{r_o} = 0 \quad (1)$$

$$\frac{v_{02}}{R_D} + g_m V_{gs2} + \frac{v_{02} - V_S}{r_o} = 0$$

$$v_{02} \left( \frac{1}{R_D} + \frac{1}{r_o} \right) + g_m (v_2 - V_S) - \frac{V_S}{r_o} = 0 \quad (2)$$

$$\begin{aligned}
 g_m V_{gs1} + \frac{v_{01} - V_S}{r_0} + \frac{v_{02} - V_S}{r_0} + g_m V_{gs2} &= \frac{V_S}{R_S} \\
 g_m (v_1 - V_S) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} - \frac{2V_S}{r_0} + g_m (v_2 - V_S) &= \frac{V_S}{R_S} \\
 g_m (v_1 + v_2) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} &= V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\}
 \end{aligned} \quad (3)$$

From (1)

$$v_{01} = \frac{V_S \left( g_m + \frac{1}{r_0} \right) - g_m v_1}{\left( \frac{1}{R_D} + \frac{1}{r_0} \right)}$$

Then

$$g_m (v_1 + v_2) + \frac{V_S \left( g_m + \frac{1}{r_0} \right) - g_m v_1}{r_0 \left( \frac{1}{R_D} + \frac{1}{r_0} \right)} + \frac{v_{02}}{r_0} = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \quad (3)$$

$$\begin{aligned}
 g_m (v_1 + v_2) r_0 \left( \frac{1}{R_D} + \frac{1}{r_0} \right) + V_S \left( g_m + \frac{1}{r_0} \right) - g_m v_1 + v_{02} \left( \frac{1}{R_D} + \frac{1}{r_0} \right) &= V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \cdot r_0 \left( \frac{1}{R_D} + \frac{1}{r_0} \right) \\
 g_m (v_1 + v_2) \left( 1 + \frac{r_0}{R_D} \right) - g_m v_1 + v_{02} \left( \frac{1}{R_D} + \frac{1}{r_0} \right) &= V_S \left\{ \left( 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right) \left( 1 + \frac{r_0}{R_D} \right) - \left( g_m + \frac{1}{r_0} \right) \right\} \\
 g_m \left( v_1 \cdot \frac{r_0}{R_D} + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left( \frac{1}{R_D} + \frac{1}{r_0} \right) &= V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} + 2g_m \cdot \frac{r_0}{R_D} + \frac{2}{R_D} + \frac{r_0}{R_S R_D} - g_m - \frac{1}{r_0} \right\} \\
 g_m \left( v_1 \cdot \frac{r_0}{R_D} + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left( \frac{1}{R_D} + \frac{1}{r_0} \right) &= V_S \left\{ 2g_m + \frac{1}{r_0} + \frac{1}{R_S} \left( 1 + \frac{r_0}{R_D} \right) + \frac{2}{R_D} (1 + g_m r_0) \right\}
 \end{aligned} \quad (4)$$

$$v_{02} \left( \frac{1}{R_D} + \frac{1}{r_0} \right) + g_m v_2 = V_S \left( g_m + \frac{1}{r_0} \right)$$

Then substituting into (2),

$$\begin{aligned}
 (0.119) \left[ v_1 \frac{710}{25} + v_2 + v_2 \frac{710}{25} \right] + v_{02} \left[ \frac{1}{25} + \frac{1}{710} \right] \\
 = V_S \left\{ 0.119 + \frac{1}{710} + \frac{1}{20} \left( 1 + \frac{710}{25} \right) + \frac{2}{25} [1 + (0.119)(710)] \right\}
 \end{aligned} \quad (4)$$

Substitute numbers:

$$\begin{aligned}
 (0.119)[28.4v_1 + 29.4v_2] + (0.0414)v_{02} &= V_S \{0.1204 + 1.470 + 6.8392\} \\
 &= V_S (8.4296)
 \end{aligned}$$

$$\text{or } V_S = 0.4010v_1 + 0.4150v_2 + 0.00491v_{02}$$

$$\text{Then } v_{02} \left( \frac{1}{25} + \frac{1}{710} \right) + (0.119)v_2 = V_S \left( 0.119 + \frac{1}{710} \right) \quad (2)$$

$$v_{o2}(0.0414) + v_2(0.119) = (0.1204)[0.401v_1 + 0.4150v_2 + 0.00491v_{o2}]$$

$$v_{o2}(0.0408) = (0.04828)v_1 - (0.0690)v_2$$

$$v_{o2} = (1.183)v_1 - (1.691)v_2$$

$$\text{Now } v_1 = v_{cm} + \frac{v_d}{2}$$

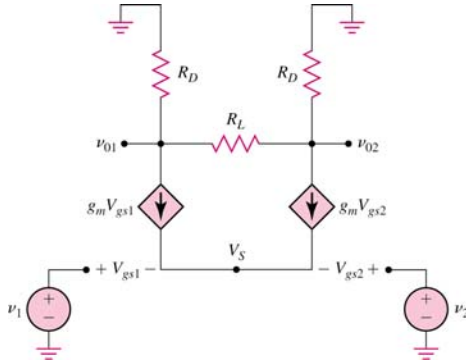
$$v_2 = v_{cm} - \frac{v_d}{2}$$

$$\text{So } v_{o2} = (1.183)\left(v_{cm} + \frac{v_d}{2}\right) - (1.691)\left(v_{cm} - \frac{v_d}{2}\right)$$

$$\text{Or } v_{o2} = 1.437v_d - 0.508v_{cm} \Rightarrow \underline{A_d = 1.437}, \quad \underline{A_{cm} = -0.508}$$

$$\underline{CMRR_{dB}} = 20 \log_{10} \left( \frac{1.437}{0.508} \right) \Rightarrow \underline{CMRR_{dB} = 9.03 \text{ dB}}$$

11.47



KVL:

$$v_1 = V_{gs1} - V_{gs2} + v_2$$

$$\text{So } v_1 - v_2 = V_{gs1} - V_{gs2}$$

KCL:

$$g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$$

$$\text{So } V_{gs1} = \frac{1}{2}(v_1 - v_2), \quad V_{gs2} = -\frac{1}{2}(v_1 - v_2)$$

Now

$$\begin{aligned} \frac{v_{o2}}{R_D} + \frac{v_{o2} - v_{o1}}{R_L} &= -g_m V_{gs2} \\ &= v_{o2} \left( \frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{v_{o1}}{R_L} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{v_{o1}}{R_D} + \frac{v_{o1} - v_{o2}}{R_L} &= -g_m V_{gs1} \\ &= v_{o1} \left( \frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{v_{o2}}{R_L} \end{aligned} \quad (2)$$

From (1):

$$v_{o1} = v_{o2} \left( 1 + \frac{R_L}{R_D} \right) + g_m R_L V_{gs2}$$

Substitute into (2):

$$-g_m V_{gs1} = v_{o2} \left( 1 + \frac{R_L}{R_D} \right) \left( \frac{1}{R_D} + \frac{1}{R_L} \right) + g_m R_L \left( \frac{1}{R_D} + \frac{1}{R_L} \right) V_{gs2} - \frac{v_{o2}}{R_L}$$

$$-g_m (v_1 - v_2) + g_m \left( 1 + \frac{R_L}{R_D} \right) \left( \frac{1}{2} \right) (v_1 - v_2) = v_{o2} \left( \frac{1}{R_D} + \frac{R_L}{R_D^2} + \frac{1}{R_D} \right)$$

$$\frac{1}{2} g_m \left( \frac{R_L}{R_D} \right) (v_1 - v_2) = \frac{v_{o2}}{R_D} \left( 2 + \frac{R_L}{R_D} \right) \Rightarrow A_{d2} = \frac{v_{o2}}{v_1 - v_2} = \frac{\frac{1}{2} \cdot g_m R_L}{\left( 2 + \frac{R_L}{R_D} \right)}$$

$$A_{d1} = \frac{v_{o1}}{v_1 - v_2} = \frac{-\frac{1}{2} \cdot g_m R_L}{\left( 2 + \frac{R_L}{R_D} \right)}$$

From symmetry

$$A_v = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = \frac{g_m R_L}{\left( 2 + \frac{R_L}{R_D} \right)}$$

Then

#### 11.48

$$(a) \ v_o = -g_m (R_D \parallel R_L) \left( \frac{-v_d}{2} \right)$$

$$A_d = \frac{1}{2} g_m (R_D \parallel R_L)$$

$$(b) \ v_o = -g_m (R_D \parallel R_L) v_{gs}$$

$$v_{cm} = v_{gs} + g_m v_{gs} (2R_o)$$

$$v_{gs} = \frac{v_{cm}}{1 + 2g_m R_o}$$

$$A_{cm} = \frac{-g_m (R_D \parallel R_L)}{1 + 2g_m R_o}$$

#### 11.49

$$(a) \ A_d = \frac{\Delta v_o}{\Delta v_d} = \frac{0.5}{0.1} = 5$$

$$A_d = 5 = \frac{g_m R_D}{2}, \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.15)(0.1)} = 0.2449 \text{ mA/V}$$

$$R_D = \frac{2(5)}{0.2449} = 40.8 \text{ k}\Omega$$

$$(b) \ v_{CM}(\text{max}) = v_o - v_{DS}(\text{sat}) + v_{GS}$$

$$i_D = K_n (v_{GS} - V_{TN})^2$$

$$0.1 = 0.15(v_{GS} - 0.4)^2 \Rightarrow v_{GS} = 1.216 \text{ V}$$

$$v_{DS}(\text{sat}) = 1.216 - 0.4 = 0.8165 \text{ V}$$

$$v_o = 5 - (0.1)(40.8) = 0.92 \text{ V}$$

$$v_{CM}(\text{max}) = 0.92 - 0.8165 + 1.216 = 1.32 \text{ V}$$



**11.50**

$$V_{d1} = -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s)$$

$$V_{d2} = -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s)$$

$$\text{Now } V_o = V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s))$$

$$V_o = g_m R_D (V_1 - V_2)$$

Define  $V_1 - V_2 \equiv V_d$

$$\text{Then } A_d = \frac{V_o}{V_d} = g_m R_D \text{ and } A_{cm} = 0$$

**11.51**

(a)  $v_{CM} = v_O - v_{DS}(sat) + v_{GS}$

$$i_D = K_n (v_{GS} - V_{TN})^2$$

$$0.1 = \left(\frac{0.1}{2}\right) (10)(v_{GS} - 0.4)^2 \Rightarrow v_{GS} = 0.8472 \text{ V}$$

$$v_{DS}(sat) = 0.8472 - 0.4 = 0.4472 \text{ V}$$

Then  $v_O = 1.5 + 0.4472 - 0.8472 = 1.1 \text{ V}$

$$v_O = 3 - (0.1)R_D = 1.1 \Rightarrow R_D = 19 \text{ k}\Omega$$

Now  $A_d = \frac{g_m R_D}{2}$ , where  $g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.1)} = 0.4472 \text{ mA/V}$

$$A_d = \frac{(0.4472)(19)}{2} = 4.248$$

$$CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$$

$$CMRR = \frac{1}{2} \left[ 1 + 2\sqrt{K_n I_Q} \cdot R_o \right] \Rightarrow 316.2 = \frac{1}{2} \left[ 1 + 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.2)} \cdot R_o \right] \Rightarrow R_o = 998 \text{ k}\Omega$$

(b) Use cascode current source similar to Figure 10.18 with  $v_{DS2}(sat) = 0.3 \text{ V}$ .

**11.52**

(a) From Problem 11.27,  $v_d \cong 12.5 \text{ mV}$ .

$$A_d = \frac{g_m R_D}{2}, \text{ where } g_m = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$$

$$A_d = \frac{(0.6325)(20)}{2} = 6.325$$

$$v_o = (6.325)(0.0125) = 0.0791 \text{ V}$$

So  $-0.0791 \leq v_o \leq 0.0791 \text{ V}$

**11.53**

From previous results

$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_{m1} R_1 = \sqrt{2K_{n1} I_{Q1}} \cdot R_1 = 20$$

$$\text{and } A_{d2} = \frac{v_{o3}}{v_{o2} - v_{o1}} = \frac{1}{2} g_{m3} R_2 = \frac{1}{2} \sqrt{2K_{n3} I_{Q2}} \cdot R_2 = 30$$

$$\text{Set } \frac{I_{Q1} R_1}{2} = 5 \text{ V and } \frac{I_{Q2} R_2}{2} = 2.5 \text{ V}$$

$$\text{Let } I_{Q1} = I_{Q2} = 0.1 \text{ mA}$$

$$\text{Then } R_1 = 100 \text{ k}\Omega, \quad R_2 = 50 \text{ k}\Omega$$

$$\text{Then } 2 \left( \frac{0.06}{2} \right) \left( \frac{W}{L} \right)_1 (0.1) = \left( \frac{20}{100} \right)^2 \Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 6.67$$

$$\text{and } 2 \left( \frac{0.060}{2} \right) \left( \frac{W}{L} \right)_3 (0.1) = \left( \frac{2(30)}{50} \right)^2 \Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 240$$

**11.54**

a. 
$$i_{D1} = I_{DSS} \left( 1 - \frac{v_{GS1}}{V_P} \right)^2$$

$$i_{D2} = I_{DSS} \left( 1 - \frac{v_{GS2}}{V_P} \right)^2$$

$$\begin{aligned} \sqrt{i_{D1}} - \sqrt{i_{D2}} &= \sqrt{I_{DSS}} \left( 1 - \frac{v_{GS1}}{V_P} \right) - \sqrt{I_{DSS}} \left( 1 - \frac{v_{GS2}}{V_P} \right) \\ &= \frac{\sqrt{I_{DSS}}}{V_P} (v_{GS2} - v_{GS1}) \\ &= -\frac{\sqrt{I_{DSS}}}{V_P} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_P)} \cdot v_d \end{aligned}$$

$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$

$$\left( \sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}} \right)^2 = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

$$i_{D1} - 2\sqrt{i_{D1}(I_Q - i_{D1})} + (I_Q - i_{D1}) = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

$$\text{Then } \sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2} \left[ I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]$$

Square both sides

$$i_{D1}^2 - i_{D1} I_Q + \frac{1}{4} \left[ I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]^2 = 0$$

$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4 \left( \frac{1}{4} \right) \left[ I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]^2}}{2}$$

$$i_{D1} = \frac{I_Q}{2} \pm \frac{1}{2} \sqrt{I_Q^2 - \left[ I_Q^2 - \frac{2I_Q I_{DSS} v_d^2}{(-V_P)^2} + \left( \frac{I_{DSS} v_d^2}{(-V_P)^2} \right)^2 \right]}$$

Use + sign

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \sqrt{\frac{2I_Q I_{DSS}}{(-V_P)^2} \cdot v_d^2 - \left( \frac{I_{DSS}}{(-V_P)} \cdot v_d^2 \right)^2}$$

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \frac{I_Q}{(-V_P)} v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2}$$

Or

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left( \frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2}$$

We had

$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left( \frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2}$$

b.

If  $i_{D1} = I_Q$ , then

$$1 = \frac{1}{2} + \left( \frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2}$$

$$|V_P| = v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2}$$

Square both sides

$$|V_P|^2 = v_d^2 \left[ \frac{2I_{DSS}}{I_Q} - \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{v_d}{V_P} \right)^2 \right]$$

$$\left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{1}{V_P} \right)^2 (v_d^2)^2 - \frac{2I_{DSS}}{I_Q} \cdot v_d^2 + |V_P|^2 = 0$$

$$v_d^2 = \frac{\frac{2I_{DSS}}{I_Q} \pm \sqrt{\left( \frac{2I_{DSS}}{I_Q} \right)^2 - 4 \left( \frac{I_{DSS}}{I_Q} \right)^2 \left( \frac{1}{V_P} \right)^2 (V_P)^2}}{2 \left( \frac{2I_{DSS}}{I_Q} \right)^2 \left( \frac{1}{V_P} \right)^2}$$

$$v_d^2 = (V_P)^2 \left( \frac{I_Q}{I_{DSS}} \right)$$

$$\text{Or } |v_d| = |V_P| \left( \frac{I_Q}{I_{DSS}} \right)^{1/2}$$

c. For  $v_d$  small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2} \cdot \frac{I_Q}{(-V_P)} \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \frac{di_{D1}}{dv_d} \Big|_{v_d \rightarrow 0} = \frac{1}{2} \cdot \frac{I_Q}{(-V_P)} \cdot \sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$\text{Or } \Rightarrow g_f (\text{max}) = \left( \frac{1}{-V_P} \right) \sqrt{\frac{I_Q I_{DSS}}{2}}$$

**11.55**

a.

$$I_Q = I_{D1} + I_{D2} \Rightarrow I_Q = 1 \text{ mA}$$

$$v_o = 7 = 10 - (0.5)R_D \Rightarrow R_D = 6 \text{ k}\Omega$$

b.

$$g_f(\text{max}) = \left( \frac{1}{-V_P} \right) \sqrt{\frac{I_Q \cdot I_{DSS}}{2}}$$

$$g_f(\text{max}) = \left( \frac{1}{4} \right) \sqrt{\frac{(1)(2)}{2}} \Rightarrow g_f(\text{max}) = 0.25 \text{ mA/V}$$

c.

$$A_d = \frac{g_m R_D}{2} = g_f(\text{max}) \cdot R_D$$

$$A_d = (0.25)(6) \Rightarrow A_d = 1.5$$

**11.56**

a.

$$I_S = \frac{-V_{GS} - (-5)}{R_S} = (2)I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 - V_{GS} = (2)(0.8)(20) \left( 1 - \frac{V_{GS}}{(-2)} \right)^2$$

$$5 - V_{GS} = (2)16 \left( 1 + V_{GS} + \frac{1}{4}V_{GS}^2 \right)$$

$$8V_{GS}^2 + 33V_{GS} + 27 = 0$$

$$V_{GS} = \frac{-33 \pm \sqrt{1089 - 4(8)(27)}}{2(8)}$$

$$= -1.125 \text{ V}$$

$$I_S = \frac{5 - (-1.125)}{20}$$

$$= 0.306 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.153 \text{ mA}$$

$$v_{o2} = 1.17 \text{ V}$$

(b)

**11.57**

Equivalent circuit and analysis is identical to that in problem 11.47.

$$A_{d2} = \frac{\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_{d1} = \frac{-\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_v = \frac{v_{o2} - v_{o1}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

**11.58**

(a)  $A_d = g_m (r_{o2} \parallel r_{o4})$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{CQ}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$A_d = (7.692)(600 \parallel 400) = 1846$$

(b)  $R_o = r_{o2} \parallel r_{o4} = 600 \parallel 400 = 240 \text{ k}\Omega$

(c)  $A_d = g_m (r_{o2} \parallel r_{o4} \parallel R_L)$

$$A_d = (0.75)(1846) = 1384.5 = (7.692)(240 \parallel R_L) \Rightarrow R_L = 720 \text{ k}\Omega$$

**11.59**

(a)

$$I_Q = 250 \mu\text{A} \quad I_{REF} = I_Q \left(1 + \frac{2}{\beta}\right)$$

$$= 250 \left(1 + \frac{2}{180}\right) = 252.8 \mu\text{A}$$

$$R_1 = \frac{5 - (0.7) - (-5)}{0.2528} \Rightarrow R_1 = 36.8 \text{ K}$$

(b)

$$A_d = g_m (r_{o2} \parallel r_{o4}) \quad g_m = \frac{0.125}{0.026} = 4.808 \text{ mA/V}$$

$$r_{o2} = \frac{150}{0.125} = 1200 \text{ K}$$

$$A_d = (4.808)(1200 \parallel 800) \quad r_{o4} = \frac{100}{0.125} = 800 \text{ K}$$

$$A_d = 2308$$

(c)

$$R_{id} = 2r_x = \frac{2(180)(0.026)}{0.125} \Rightarrow R_{id} = 74.9 \text{ K}$$

$$R_o = r_{o2} \parallel r_{o4} = 1200 \parallel 800 = 480 \text{ K} = R_o$$

(d)

$$v_{cm}(\text{max}) = 5 - 0.7 = 4.3 \text{ V}$$

$$v_{cm}(\text{min}) = 0.7 + 0.7 - 5 = -3.6 \text{ V}$$

### 11.60

a.

$$I_0 = I_{B3} + I_{B4} \approx 2 \left( \frac{I_Q}{2} \right) \left( \frac{1}{\beta} \right)$$

$$I_0 = \frac{I_Q}{\beta} = \frac{0.2}{100} \Rightarrow I_0 = 2 \mu\text{A}$$

b.

$$r_{o2} = r_{o4} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_d = g_m (r_{o2} \parallel r_{o4}) = (3.846)(1000 \parallel 1000) \Rightarrow A_d = 1923$$

c.

$$A_d = g_m (r_{o2} \parallel r_{o4} \parallel R_L)$$

$$A_d = (3.846)(1000 \parallel 1000 \parallel 250) \Rightarrow A_d = 641$$

### 11.61

$$(a) A_d = \frac{\frac{I_Q}{2V_T}}{\frac{I_Q}{2V_{A2}} + \frac{I_Q}{2V_{A4}} + \frac{1}{R_L}}$$

$$1000 = \frac{\frac{I_Q}{2(0.026)}}{\frac{I_Q}{2(90)} + \frac{I_Q}{2(60)} + \frac{1}{250}}$$

$$1000(I_Q)(0.005556 + 0.008333) + 4 = I_Q(19.23) \Rightarrow I_Q = 0.749 \text{ mA}$$

$$(b) v_{CM}(\text{max}) = V^+ - 2V_{EB}(\text{on}) = 5 - 2(0.6) = 3.8 \text{ V}$$

**11.62**

$$(b) v_{o1} = -g_{m1}(r_{o1} \parallel R_{o3}) \left( \frac{v_d}{2} \right)$$

$$A_{d1} = \frac{v_{o1}}{v_d} = -\frac{1}{2} g_{m1}(r_{o1} \parallel R_{o3})$$

$$g_{m1} = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{CQ}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$R_{o3} = r_{o3} \parallel r_{\pi 3} \parallel \frac{1}{g_{m3}}, \text{ where } g_{m3} = 7.692 \text{ mA/V,}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega, \quad r_{o3} = 400 \text{ k}\Omega$$

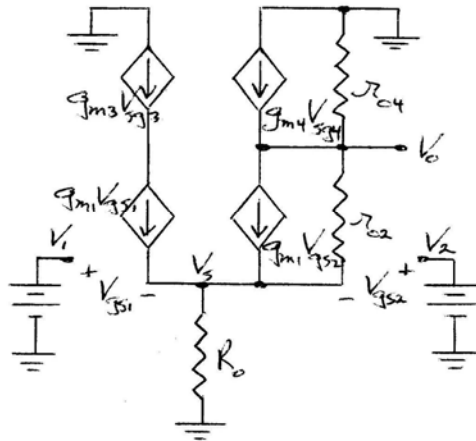
$$\text{Then } R_{o3} = 400 \parallel 15.6 \parallel \frac{1}{7.692} = 15.014 \parallel 0.130 = 0.1289 \text{ k}\Omega$$

$$A_{d1} = -\frac{1}{2} (7.692)(600 \parallel 0.1289) = -0.4956$$

$$(c) A_{d2} = +\frac{1}{2} g_{m2}(r_{o2} \parallel R_{o4}) = +0.4956$$

$$(d) A_{d3} = +g_{m1}(r_{o1} \parallel R_{o3}) = 2(0.4956) = 0.9912$$

**11.63**



$$g_{m4} V_{sg4} = g_{m3} V_{sg3} = g_{m1} V_{gs1}$$

$$V_{gs1} = V_1 - V_s, \text{ and } V_{gs2} = V_2 - V_s$$

$$g_{m1} V_{gs1} + g_{m1} V_{gs2} + \frac{V_o - V_s}{r_{o2}} = \frac{V_s}{R_o}$$

$$g_{m1} (V_1 + V_2 - 2V_s) + \frac{V_o}{r_{o2}} = V_s \left( \frac{1}{R_o} + \frac{1}{r_{o2}} \right)$$

$$g_{m1}(V_1 + V_2) + \frac{V_o}{r_{o2}} = V_s \left( 2g_{m1} + \frac{1}{R_o} + \frac{1}{r_{o2}} \right)$$

Now  $g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.1)} = 0.4472 \text{ mA/V}$

$$r_{o2} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

Then  $(0.4472)(V_1 + V_2) + \frac{V_o}{500} = V_s \left[ 2(0.4472) + \frac{1}{2000} + \frac{1}{500} \right] = V_s (0.8969)$

We have (1)  $V_s = (0.4986)(V_1 + V_2) + V_o(0.0022299)$

Also (2)  $\frac{V_o - V_s}{r_{o2}} + \frac{V_o}{r_{o4}} + g_{m1}V_{gs2} - g_{m1}V_{gs1} = 0$

$$V_o \left( \frac{1}{r_{o2}} + \frac{1}{r_{o4}} \right) - \frac{V_s}{r_{o2}} + g_{m1}(V_2 - V_s - (V_1 - V_s)) = 0$$

$$V_o \left( \frac{1}{r_{o2}} + \frac{1}{r_{o4}} \right) - \frac{V_s}{r_{o2}} + g_{m1}(V_2 - V_1) = 0$$

We find  $r_{o4} = \frac{1}{(0.03)(0.1)} = 333.3 \text{ k}\Omega$

Then  $V_o \left( \frac{1}{500} + \frac{1}{333.3} \right) - \frac{V_s}{500} [(0.4986)(V_1 + V_2) + V_o(0.0022299)] + (0.4472)(V_2 - V_1) = 0$

$$V_o [(0.005) - (0.0000446)] - (0.0009972)(V_1 + V_2) + (0.4472)(V_2 - V_1) = 0$$

(a) Let  $V_1 = V_d$  and  $V_2 = 0$

$$V_o [0.00499554] - V_d (0.0009972) - V_d (0.4472) = 0$$

$$A_d = \frac{V_o}{V_d} = 89.72$$

(b) Let  $V_1 = 0$  and  $V_2 = -V_d$

$$V_o [0.00499554] + V_d (0.0009972) - V_d (0.4472) = 0$$

$$A_d = \frac{V_o}{V_d} = 89.32$$

(c) Let  $V_1 = \frac{V_d}{2}$  and  $V_2 = -\frac{V_d}{2}$

$$V_o [0.00499554] = V_d (0.4472)$$

$$A = \frac{V_o}{V_d} = 89.52$$

### 11.64

- a. From symmetry.



$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

$$\text{Or } V_{DS3} = V_{DS4} = 2 \text{ V}$$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$V_{SD1} = V_{SD2} = V_{SG1} - (V_{DS3} - 10) \\ = 2 - (2 - 10)$$

$$\text{Or } V_{SD1} = V_{SD2} = 10 \text{ V}$$

b.

$$r_{0n} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$r_{0p} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$g_m = 2K_p (V_{SG} + V_{TP}) \\ = 2(0.1)(2 - 1) = 0.2 \text{ mA/V}$$

$$A_d = g_m (r_{on} \parallel r_{op}) = (0.2)(1000 \parallel 667) \Rightarrow \underline{A_d = 80}$$

(c)

$$I_{D2} = I_{D1} = \frac{I_Q}{2} = 0.1 \text{ mA}$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 667 \parallel 1000 = 400 \text{ k}\Omega$$

### 11.65

$$(a) A_d = g_m (r_{o2} \parallel r_{o4})$$

$$g_m = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_n I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2}\right) (8)(0.06)} = 0.3098 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.06)} = 666.7 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.06)} = 416.7 \text{ k}\Omega$$

$$A_d = (0.3098)(666.7 \parallel 416.7) = 79.44$$

$$(b) i_{D3} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p (v_{SG3} + V_{TP})^2$$

$$0.06 = \left(\frac{0.04}{2}\right) (10)(v_{SG3} - 0.3)^2 \Rightarrow v_{SG3} = 0.8477 \text{ V}$$

$$v_o = 1.8 - 0.8477 = 0.9523$$

$$v_{CM}(\text{max}) = v_o - v_{DS}(\text{sat}) + v_{GS}$$

$$0.06 = \left(\frac{0.1}{2}\right) (8)(v_{GS} - 0.3)^2 \Rightarrow v_{GS} = 0.6873 \text{ V}, \quad v_{DS}(\text{sat}) = 0.3873 \text{ V}$$

$$v_{CM}(\max) = 0.9523 - 0.3873 + 0.6873 = 1.25 \text{ V}$$

**11.66**

$$R_{04} = r_{04} [1 + g_{m4} (R \parallel r_{\pi 4})]$$

$$r_{04} = \frac{80}{0.1} = 800 \text{ K}$$

$$g_{m4} = \frac{0.1}{0.026} = 3.846$$

$$r_{\pi 4} = \frac{(100)(0.026)}{0.1} \\ = 26 \text{ K}$$

$$R \parallel r_{\pi 4} = 1 \parallel 26 = 0.963 \text{ K}$$

Assume  $\beta = 100$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_{m3} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$R_{04} = 800 [1 + (3.846)(0.963)] \Rightarrow 3.763 \text{ M}\Omega$$

$$\Rightarrow R_0 = 3.763 \text{ M}\Omega$$

Then

$$A_v = -g_m (r_{02} \parallel R_0)$$

$$r_{02} = \frac{120}{0.1} = 1200 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_v = -(3.846) [1200 \parallel 3763] \Rightarrow \underline{A_v = -3499}$$

b.

For

$$R = 0, \quad r_{04} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

$$A_v = -g_m (r_{02} \parallel r_{04})$$

$$= -(3.846) [1200 \parallel 800] \Rightarrow \underline{A_v = -1846}$$

- (c) For part (a),  $R_o = (3.763 \parallel 1.2) = 0.910 \text{ M}\Omega$   
 For part (b),  $R_o = (1.2 \parallel 0.8) = 0.48 \text{ M}\Omega$

**11.67**

$$I_{B5} = \frac{I_{E5}}{1 + \beta} = \frac{I_{B3} + I_{B4}}{1 + \beta} = \frac{I_{C3} + I_{C4}}{\beta(1 + \beta)}$$

Now  $I_{C3} + I_{C4} \approx I_Q$

So  $I_{B5} \approx \frac{I_Q}{\beta(1 + \beta)}$

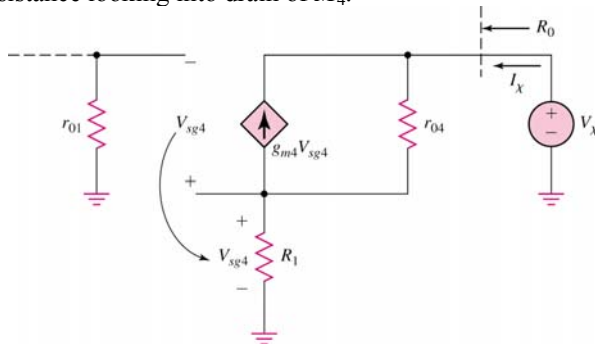
$$I_{B6} = \frac{I_{E6}}{1 + \beta} = \frac{I_{Q1}}{\beta(1 + \beta)}$$

For balance, we want  $I_{B6} = I_{B5}$

So that  $\underline{I_{Q1} = I_Q}$

**11.68**

Resistance looking into drain of  $M_4$ .



$$V_{sg4} \cong I_X R_1$$

$$I_X + g_{m4} V_{sg4} = \frac{V_X - V_{sg4}}{r_{o4}}$$

$$I_X \left[ 1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right] = \frac{V_X}{r_{o4}}$$

Or  $R_o = r_{o4} \left[ 1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right]$

a.

$$A_d = g_{m2} (r_{o2} \parallel R_o)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_o = 500 \left[ 1 + (0.179)(1) + \frac{1}{500} \right] = 590.5 \text{ k}\Omega$$

$$A_d = (0.179) [667 \parallel 590.5] \Rightarrow A_d = 56.06$$

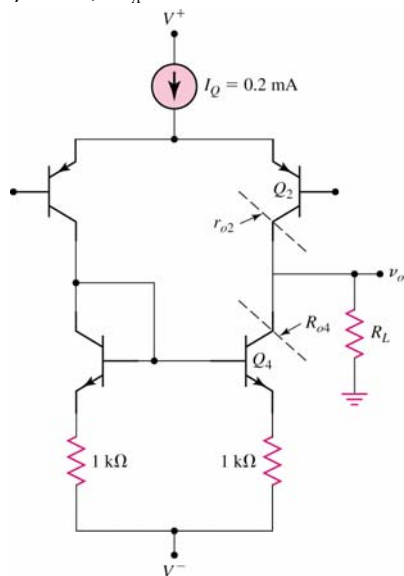
b.

When  $R_i = 0$ ,  $R_o = r_{o4} = 500 \text{ k}\Omega$   
 $A_d = (0.179) [667 \parallel 500] \Rightarrow A_d = 51.15$

(c) For part (a),  $R_o = r_{o2} \parallel R_o = 667 \parallel 590.5 \Rightarrow R_o = 313 \text{ k}\Omega$   
 For part (b),  $R_o = r_{o2} \parallel r_{o4} = 667 \parallel 500 \Rightarrow R_o = 286 \text{ k}\Omega$

**11.69**

Let  $\beta = 100$ ,  $V_A = 100 \text{ V}$



$$r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$R_{o4} = r_{o4} [1 + g_m R'_E] \text{ where } R'_E = r_\pi \parallel R_E$$

Now

$$r_\pi = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

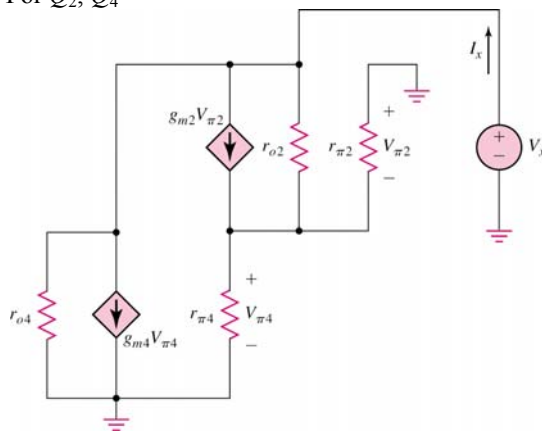
$$R'_E = 26 \parallel 1 = 0.963 \text{ k}\Omega$$

$$\text{Then } R_{o4} = 1000 [1 + (3.846)(0.963)] = 4704 \text{ k}\Omega$$

$$A_d = g_m (r_{o2} \parallel R_{o4}) = 3.846 (1000 \parallel 4704) \Rightarrow \underline{A_d = 3172}$$

**11.70**

(a) For  $Q_2, Q_4$



$$(1) \quad I_x = \frac{V_x - V_{\pi 4}}{r_{o2}} + g_{m2}V_{\pi 2} + g_{m4}V_{\pi 4} + \frac{V_x}{r_{o4}}$$

$$(2) \quad g_{m2}V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{o2}} = \frac{V_{\pi 4}}{r_{\pi 4} \parallel r_{\pi 2}}$$

$$(3) \quad V_{\pi 4} = -V_{\pi 2}$$

$$\text{From (2)} \quad \frac{V_x}{r_{o2}} = V_{\pi 4} \left[ \frac{1}{r_{\pi 4} \parallel r_{\pi 2}} + \frac{1}{r_{o2}} + g_{m2} \right]$$

Now

$$I_{C4} = \left( \frac{\beta}{1+\beta} \right) \left( \frac{I_Q}{2} \right) = \left( \frac{120}{121} \right) (0.5) = 0.496 \text{ mA}$$

$$I_{C2} = \left( \frac{I_Q}{2} \right) \left( \frac{1}{1+\beta} \right) \left( \frac{\beta}{1+\beta} \right) = (0.5) \left( \frac{120}{(121)^2} \right) \Rightarrow I_{C2} = 0.0041 \text{ mA}$$

So

$$r_{\pi 2} = \frac{(120)(0.026)}{0.0041} = 761 \text{ k}\Omega$$

$$g_{m2} = \frac{0.0041}{0.026} = 0.158 \text{ mA/V}$$

$$r_{o2} = \frac{100}{0.0041} \Rightarrow 24.4 \text{ M}\Omega$$

$$r_{\pi 4} = \frac{(120)(0.026)}{0.496} = 6.29 \text{ k}\Omega$$

$$g_{m4} = \frac{0.496}{0.026} = 19.08 \text{ mA/V}$$

$$r_{o4} = \frac{100}{0.496} = 202 \text{ k}\Omega$$

Now

$$\frac{V_x}{r_{o2}} = V_{\pi 4} \left[ \frac{1}{6.29 \parallel 761} + \frac{1}{24400} + 0.158 \right] \Rightarrow \text{which yields } V_{\pi 4} = \frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o4}} + V_{\pi 4} \left( g_{m4} - g_{m2} - \frac{1}{r_{o2}} \right)$$

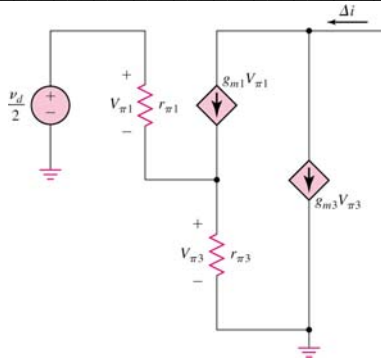
$$\frac{I_x}{V_x} = \left[ \frac{1}{24400} + \frac{1}{202} + \frac{\left( 19.08 - 0.158 - \frac{1}{24400} \right)}{(0.318)(24400)} \right] \text{ which yields } R_{o2} = \frac{V_x}{I_x} = 135 \text{ k}\Omega$$

$$\text{Now } r_{o6} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

$$\text{Then } R_o = R_{o2} \parallel r_{o6} = 135 \parallel 160 \Rightarrow R_o = 73.2 \text{ k}\Omega$$

(b)

$$A_d = g_m^c R_o \text{ where } g_m^c = \frac{\Delta i}{v_d / 2}$$



$$\Delta i = g_{m1} V_{\pi 1} + g_{m3} V_{\pi 3} \text{ and } V_{\pi 1} + V_{\pi 3} = \frac{v_d}{2}$$

$$\text{Also } \left( \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) r_{\pi 3} = V_{\pi 3}$$

$$\text{So } V_{\pi 1} \left( \frac{1 + \beta}{r_{\pi 1}} \right) r_{\pi 3} = V_{\pi 3}$$

$$\text{Or } V_{\pi 1} \left( \frac{121}{761} \right) (6.29) = V_{\pi 3} \cong V_{\pi 1}$$

$$\text{Then } 2V_{\pi 1} = \frac{v_d}{2} \Rightarrow V_{\pi 1} = \frac{v_d}{4}$$

$$\text{So } \Delta i = (g_{m1} + g_{m3}) V_{\pi 1} = (0.158 + 19.08) \left( \frac{v_d}{4} \right) = 9.62 \left( \frac{v_d}{2} \right)$$

$$\text{So } g_m^c = \frac{\Delta i}{v_d / 2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow \underline{A_d = 704}$$

$$\text{Now } R_{id} = 2R_i \text{ where } R_i = r_{\pi 1} + (1 + \beta) r_{\pi 3}$$

$$R_i = 761 + (121)(6.29) = 1522 \text{ k}\Omega$$

$$\text{Then } \underline{R_{id} = 3.044 \text{ M}\Omega}$$

### 11.71 - Design Problem

#### 11.72

$$\text{Input: } -8 \leq V_d \leq 8 \text{ mV}$$

$$\text{Output: } -0.8 \leq V_o \leq 0.8 \text{ V}$$

$$A_d = \frac{V_o}{V_d} = \frac{0.8}{0.008} = 100$$

$$A_d = g_m (r_{o2} \parallel r_{o4})$$

$$\text{Let } I_Q = 0.5 \text{ mA, } I_{DQ} = 0.25 \text{ mA}$$

$$r_{o2} = \frac{1}{(0.025)(0.25)} = 160 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.25)} = 100 \text{ k}\Omega$$

$$100 = g_m (160 \parallel 100) \Rightarrow g_m = 1.625 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n I_{DQ}}$$

$$1.625 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.25)} \Rightarrow \left(\frac{W}{L}\right)_n = 52.8$$

### 11.73

For current source,  $I_{REF} = I_Q$

$$P = (2I_Q)(V^+ - V^-)$$

$$0.8 = (2I_Q)(6) \Rightarrow I_Q \cong 66 \mu\text{A}$$

$$A_d = g_m(r_{o2} \parallel r_{o4})$$

$$r_{o2} = \frac{1}{(0.02)\left(\frac{0.066}{2}\right)} = 1515 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.03)\left(\frac{0.066}{2}\right)} = 1010 \text{ k}\Omega$$

$$240 = g_m(1515 \parallel 1010) \Rightarrow g_m = 0.396 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n I_{DQ}}$$

$$0.396 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.033)} \Rightarrow \left(\frac{W}{L}\right)_n = 23.76$$

### 11.74

$$A_d = g_m(r_{o2} \parallel R_o)$$

$$\approx g_m r_{o2}$$

$$r_{o2} = \frac{1}{\lambda_n I_D}$$

$$= \frac{1}{(0.015)(0.1)} = 666.7 \text{ K}$$

$$A_d = 400 = g_m(666.7)$$

$$g_m = 0.60 \text{ mA/V}$$

$$= 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n I_D}$$

$$0.60 = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_n (0.1)}$$

$$0.090 = 0.004\left(\frac{W}{L}\right)_n$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 22.5$$



**11.75**

$$A_d = g_m (R_{o4} \parallel R_{o6})$$

where

$$R_{o4} = r_{o4} + r_{o2} [1 + g_{m4} r_{o4}]$$

$$R_{o6} = r_{o6} + r_{o8} [1 + g_{m6} r_{o6}]$$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667 \text{ k}\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.02)(0.040)} = 1250 \text{ k}\Omega$$

$$g_{m4} = 2 \sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268 \text{ mA/V}$$

$$g_{m6} = 2 \sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141 \text{ mA/V}$$

Then

$$R_{o4} = 1667 + 1667 [1 + (0.268)(1667)] \Rightarrow 748 \text{ M}\Omega$$

$$R_{o6} = 1250 + 1250 [1 + (0.141)(1250)] \Rightarrow 222.8 \text{ M}\Omega$$

(a)

$$R_o = R_{o4} \parallel R_{o6} = 748 \parallel 222.8 \Rightarrow R_o = 172 \text{ M}\Omega$$

(b)

$$A_d = g_{m4} (R_{o4} \parallel R_{o6}) = (0.268)(172000) \Rightarrow A_d = 46096$$

**11.76**

(a)  $A_d = g_m (r_{o2} \parallel r_{o4})$

$$g_m = 2 \sqrt{K_n I_{DQ}} = 2 \sqrt{(0.2)(0.06)} = 0.2191 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{1}{(0.025)(0.06)} = 666.7 \text{ k}\Omega$$

$$A_d = (0.2191)(666.7 \parallel 666.7) = 73.0$$

(b)  $R_o = r_{o2} \parallel r_{o4} = 333.3 \text{ k}\Omega$

(c)  $i_{D3} = K_p (v_{SG3} + V_{TP})^2$

$$0.06 = 0.2(v_{SG3} - 0.3)^2 \Rightarrow v_{SG3} = 0.8477 \text{ V}$$

$$v_o = V^+ - v_{SG3} = 2.8 - 0.8477 = 1.9523 \text{ V}$$

$$i_{D1} = K_n (v_{GS1} - V_{TN})^2$$

$$0.06 = 0.2(v_{GS1} - 0.3)^2 \Rightarrow v_{GS1} = 0.8477 \text{ V}, \Rightarrow v_{DS1}(\text{sat}) = 0.5477 \text{ V}$$

$$v_{CM}(\text{max}) = v_o - v_{DS1}(\text{sat}) + v_{GS1} = 1.9523 - 0.5477 + 0.8477 = 2.25 \text{ V}$$

**11.77**

(a)  $i_{D3} = K_p (v_{SG3} + V_{TP})^2$   
 $0.25 = 0.25(v_{SG3} - 0.4)^2 \Rightarrow v_{SG3} = 1.4 \text{ V}$   
 Then  $v_{GS1} = 1.4 \text{ V}$ ,  $v_{DS1}(\text{sat}) = 1.0 \text{ V}$   
 $v_{CM} = v_O - v_{DS1}(\text{sat}) + v_{GS1}$   
 $3 = (V^+ - 1.4) - 1.0 + 1.4 \Rightarrow V^+ = 4 \text{ V} = -V^-$

(b)  $A_d = g_m (r_{o2} \parallel r_{o4})$   
 $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.25)(0.25)} = 0.50 \text{ mA/V}$   
 $r_{o2} = r_{o4} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$   
 $A_d = (0.50)(200 \parallel 200) = 50$

**11.78**

(a) For  $v_{cm} = +2 \text{ V} \Rightarrow V^+ = 2.7 \text{ V}$   
 If  $I_Q$  is a 2-transistor current source,  
 $V^- = v_{cm} - 0.7 - 0.7$   
 $V^- = -3.4 \text{ V} \Rightarrow V^+ = -V^- = 3.4 \text{ V}$

(b)  $A_d = g_m (r_{o2} \parallel r_{o4})$   $r_{o2} = \frac{100}{0.1} = 1000 \text{ K}$   
 $r_{o4} = \frac{60}{0.1} = 600 \text{ K}$   
 $g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$   
 $A_d = (3.846)(1000 \parallel 600) \Rightarrow A_d = 1442$

**11.79**

(a)  $V^+ = -V^- = 3.4 \text{ V}$

(b)  $r_{o2} = \frac{75}{0.06} = 1250 \text{ K}$   
 $r_{o4} = \frac{40}{0.06} = 666.7 \text{ K}$   
 $g_m = \frac{0.06}{0.026} = 2.308 \text{ mA/V}$   
 $A_d = (2.308)(1250 \parallel 666.7)$   
 $A_d = 1004$

**11.80**

(a)  $I_{E2} = 0.25 \text{ mA}$ ,  $I_{B2} = 0.001656 \text{ mA}$ ,  $I_{C2} = 0.2483 \text{ mA}$

$$I_{D1} = 0.25 + 0.001656 = 0.25166 \text{ mA}$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.25166)} = 0.4487 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.2483}{0.026} = 9.55 \text{ mA/V}$$

$$r_{\pi_2} = \frac{(150)(0.026)}{0.2483} = 15.71 \text{ k}\Omega$$

$$g_m^C = \frac{(0.4487)[1 + (9.55)(15.71)]}{1 + (0.4487)(15.71)} = 8.42 \text{ mA/V}$$

(b)  $I_{E2} = 0.45 \text{ mA}$ ,  $I_{B2} = 0.002980 \text{ mA}$ ,  $I_{C2} = 0.4470 \text{ mA}$

$$I_{D1} = 0.05 + 0.00298 = 0.05298 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.2)(0.05298)} = 0.2059 \text{ mA/V}$$

$$g_{m2} = \frac{0.4470}{0.026} = 17.19 \text{ mA/V}$$

$$r_{\pi_2} = 8.725 \text{ k}\Omega$$

$$g_m^C = \frac{(0.2059)[1 + (17.19)(8.725)]}{1 + (0.2059)(8.725)} = 11.12 \text{ mA/V}$$

**11.81**

$$r_0(M_2) = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_0(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

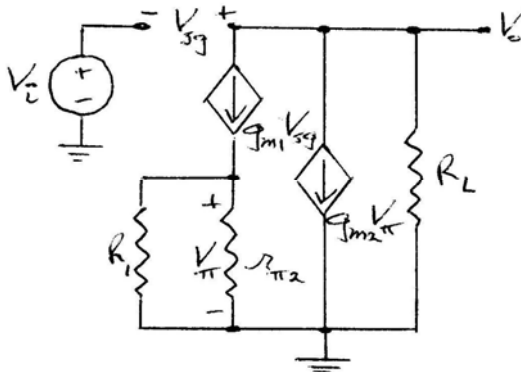
$$g_m(M_2) = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$A_d = g_m(M_2)[r_0(M_2) \parallel r_0(Q_2)] = 0.4[500 \parallel 400] \Rightarrow A_d = 88.9$$

If the  $I_Q$  current source is ideal,  $\frac{A_{cm}}{A_d} = 0$  and  $\frac{CMRR_{dB}}{A_d} = \infty$

11.82

(a)



$$(b) \quad I_{R1} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_Q = 0.5 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.4125 \text{ mA}$$

$$I_{B2} = 0.002279 \text{ mA}, \quad I_{C2} = 0.41022 \text{ mA}$$

$$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.002279 = 0.08978 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.7)(0.08978)} = 0.5014 \text{ mA/V}$$

$$g_{m2} = \frac{0.41022}{0.026} = 15.78 \text{ mA/V}$$

$$r_{\pi 2} = 11.41 \text{ k}\Omega$$

$$(c) \quad V_o = -(g_{m1}V_{sg} + g_{m2}V_{\pi})R_L$$

$$V_{\pi} = (g_{m1}V_{sg})(R_1 \parallel r_{\pi 2})$$

$$V_o = -[g_{m1}V_{sg} + g_{m2}g_{m1}V_{sg}(R_1 \parallel r_{\pi 2})]R_L$$

$$-V_o = V_{sg}[g_{m1} + g_{m2}g_{m1}(R_1 \parallel r_{\pi 2})]R_L$$

Now  $V_{sg} = V_o - V_i$

$$\text{So } V_o[1 + g_{m1}(1 + g_{m2}(R_1 \parallel r_{\pi 2}))R_L] = V_i g_{m1}[1 + g_{m2}(R_1 \parallel r_{\pi 2})]R_L$$

$$A_v = \frac{V_o}{V_i} = \frac{g_{m1}[1 + g_{m2}(R_1 \parallel r_{\pi 2})]R_L}{1 + g_{m1}[1 + g_{m2}(R_1 \parallel r_{\pi 2})]R_L}$$

$$A_v = \frac{(0.5014)[1 + (15.78)(8 \parallel 11.41)]R_L}{1 + (0.5014)[1 + (15.78)(8 \parallel 11.41)]R_L} = \frac{37.71R_L}{1 + 37.71R_L}$$

(i) For  $R_L = 10 \text{ k}\Omega$ ,

$$A_v = \frac{(37.71)(10)}{1 + (37.71)(10)} = 0.99736$$

(ii) For  $R_L = 100 \text{ k}\Omega$ ,

$$A_v = \frac{(37.71)(100)}{1 + (37.71)(100)} = 0.99973$$

**11.83**

(a)  $I_{R1} = \frac{0.7}{8} = 0.0875 \text{ mA}$

$I_Q = 0.8 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.7125 \text{ mA}$

$I_{B2} = 0.003936 \text{ mA}, I_{C2} = 0.7086 \text{ mA}$

$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.003936 = 0.091436 \text{ mA}$

$g_{m1} = 2\sqrt{(0.7)(0.091436)} = 0.506 \text{ mA/V}$

$g_{m2} = \frac{0.7086}{0.026} = 27.25 \text{ mA/V}$

$r_{\pi 2} = 6.605 \text{ k}\Omega$

(b)  $I_x = g_{m2}V_{\pi} + g_{m1}V_{sg}$   
 $V_{\pi} = (g_{m1}V_{sg})(R_1 \parallel r_{\pi 2})$

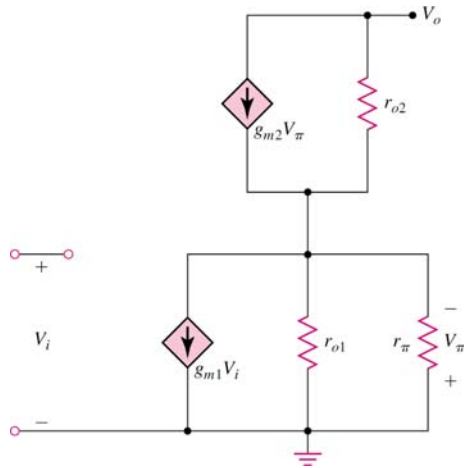
Now  $V_{sg} = V_x$

$\frac{I_x}{V_x} = \frac{1}{R_o} = g_{m1} + g_{m2}g_{m1}(R_1 \parallel r_{\pi 2}) = 0.506 + (0.506)(27.25)(8 \parallel 6.605) = 50.39 \text{ mA/V}$

Or  $R_o = 19.8 \Omega$

**11.84**

(a)



(1)  $g_{m2}V_{\pi} + \frac{V_o - (-V_{\pi})}{r_{o2}} = 0$

(2)  $g_{m2}V_{\pi} + \frac{V_o - (-V_{\pi})}{r_{o2}} = g_{m1}V_i + \frac{-V_{\pi}}{r_{o1}} + \frac{-V_{\pi}}{r_{\pi}}$  or  $0 = g_{m1}V_i - V_{\pi} \left( \frac{1}{r_{o1}} + \frac{1}{r_{\pi}} \right)$

$V_{\pi} = \frac{g_{m1}V_i}{\left( \frac{1}{r_{o1}} + \frac{1}{r_{\pi}} \right)}$

Then

From (1)

$$\left(g_{m2} + \frac{1}{r_{o2}}\right)V_{\pi} + \frac{V_o}{r_{o2}} = 0$$

$$V_o = -r_{o2} \left(g_{m2} + \frac{1}{r_{o2}}\right)V_{\pi} = -r_{o2}g_{m1}V_i \left(\frac{g_{m2} + \frac{1}{r_{o2}}}{\frac{1}{r_{o1}} + \frac{1}{r_{\pi}}}\right)$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o2} \left(g_{m2} + \frac{1}{r_{o2}}\right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_{\pi}}\right)}$$

Now

$$g_{m1} = 2\sqrt{K_n I_Q} = 2\sqrt{(0.25)(0.025)} = 0.158 \text{ mA/V}$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.025}{0.026} = 0.9615 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.025)} = 2000 \text{ k}\Omega$$

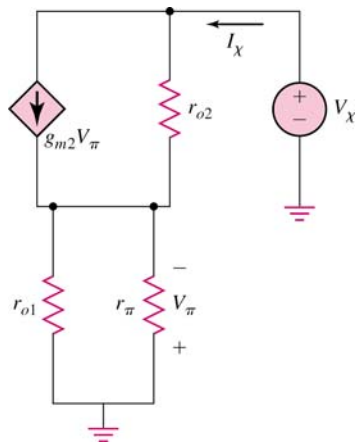
$$r_{o2} = \frac{V_A}{I_Q} = \frac{50}{0.025} = 2000 \text{ k}\Omega$$

$$r_{\pi} = \frac{\beta V_T}{I_Q} = \frac{(100)(0.026)}{0.025} = 104 \text{ k}\Omega$$

Then

$$A_v = \frac{-(0.158)(2000) \left(0.9615 + \frac{1}{2000}\right)}{\left(\frac{1}{2000} + \frac{1}{104}\right)} \Rightarrow A_v = -30039$$

To find  $R_o$ ; set  $V_i = 0 \Rightarrow g_{m1}V_i = 0$



$$I_x = g_{m2}V_{\pi} + \frac{V_x - (-V_{\pi})}{r_{o2}}$$

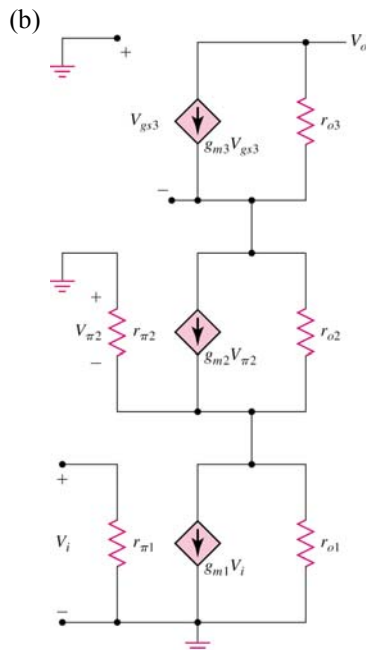
$$V_{\pi} = -I_x(r_{o1} \parallel r_{\pi})$$

Then

$$I_x = \left( g_{m2} + \frac{1}{r_{o2}} \right) (-I_x)(r_{o1} \parallel r_{\pi}) + \frac{V_x}{r_{o2}}$$

Combining terms,

$$\begin{aligned} R_o = \frac{V_x}{I_x} &= r_{o2} \left[ 1 + (r_{o1} \parallel r_{\pi}) \left( g_{m2} + \frac{1}{r_{o2}} \right) \right] \\ &= 2000 \left[ 1 + (2000 \parallel 104) \left( 0.9615 + \frac{1}{2000} \right) \right] \Rightarrow \underline{R_o = 192.2 \text{ M}\Omega} \end{aligned}$$



$$(1) \quad g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = 0$$

$$(2) \quad g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = g_{m2}V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}} \text{ or } 0 = V_{\pi2} \left( g_{m2} + \frac{1}{r_{o2}} \right) - \frac{V_{gs3}}{r_{o2}}$$

$$(3) \quad \frac{V_{\pi2}}{r_{\pi2}} + g_{m2}V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}} = g_{m1}V_i + \frac{(-V_{\pi2})}{r_{o1}}$$

$$V_{\pi2} = \frac{V_{gs3}}{r_{o2} \left( g_{m2} + \frac{1}{r_{o2}} \right)}$$

From (2),

Then

$$(3) \quad V_{\pi2} \left( \frac{1}{r_{\pi2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right) = g_{m1}V_i + \frac{V_{gs3}}{r_{o2}}$$

or

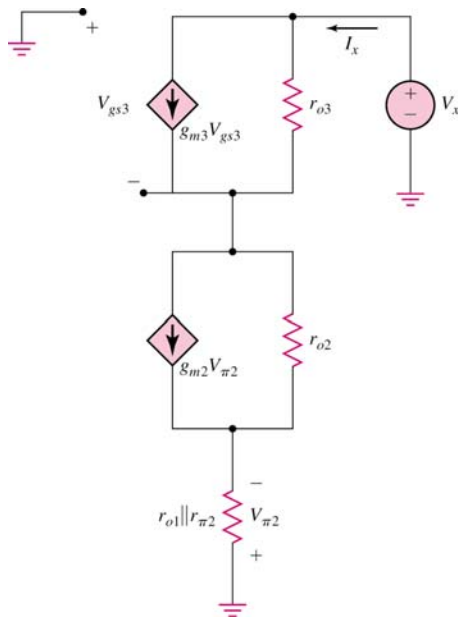
$$\frac{V_{gs3}}{r_{o2} \left( g_{m2} + \frac{1}{r_{o2}} \right)} \left[ \frac{1}{r_{\pi2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right] = g_{m1} V_i + \frac{V_{gs3}}{r_{o2}}$$

$$\frac{V_{gs3}}{2000 \left( 0.9615 + \frac{1}{2000} \right)} \left[ \frac{1}{104} + 0.9615 + \frac{1}{2000} + \frac{1}{2000} \right] = 0.9615 V_i + \frac{V_{gs3}}{2000}$$

Then  $V_{gs3} = 1.83 \times 10^5 V_i$

From (1),  $\left( g_{m3} + \frac{1}{r_{o3}} \right) V_{gs3} = \frac{-V_o}{r_{o3}}$  or  $V_o = -2000 \left( 0.158 + \frac{1}{2000} \right) (1.83 \times 10^5) V_i$

$$A_v = \frac{V_o}{V_i} = -5.80 \times 10^7$$



To find  $R_o$

$$(1) \quad I_x = g_{m3} V_{gs3} + \frac{V_x - (-V_{gs3})}{r_{o3}}$$

$$(2) \quad g_{m3} V_{gs3} + \frac{V_x - (-V_{gs3})}{r_{o3}} = g_{m2} V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}}$$

$$(3) \quad V_{\pi2} = -I_x (r_{o1} \parallel r_{\pi2})$$

From (1)  $I_x = V_{gs3} \left( g_{m3} + \frac{1}{r_{o3}} \right) + \frac{V_x}{r_{o3}}$

$$I_x = V_{gs3} \left( 0.158 + \frac{1}{2000} \right) + \frac{V_x}{2000}$$

So  $V_{gs3} = \frac{I_x - \frac{V_x}{2000}}{0.1585}$



From (2),

$$V_{gs3} \left[ g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o2}} \right] + \frac{V_x}{r_{o3}} = V_{\pi2} \left( g_{m2} + \frac{1}{r_{o2}} \right)$$

$$V_{gs3} \left[ 0.158 + \frac{1}{2000} + \frac{1}{2000} \right] + \frac{V_x}{2000} = V_{\pi2} \left( 0.9615 + \frac{1}{2000} \right)$$

$$\text{Then } \left[ \frac{I_x - V_x / 2000}{0.1585} \right] (0.159) + \frac{V_x}{2000} = -I_x (2000 \parallel 104) (0.962)$$

$$\text{We find } R_o = \frac{V_x}{I_x} = 6.09 \times 10^{10} \Omega$$

### 11.85

Assume emitter of  $Q_1$  is capacitively coupled to signal ground.

$$I_{CQ} = 0.2 \left( \frac{80}{81} \right) = 0.1975 \text{ mA}$$

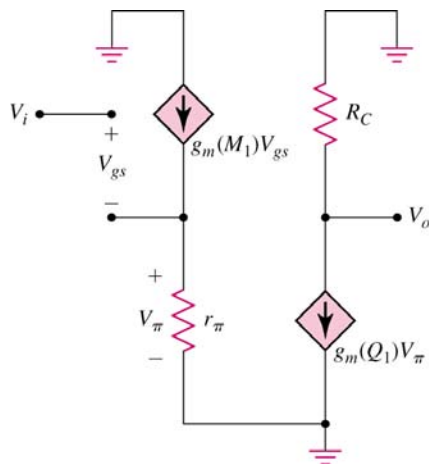
$$I_{DQ} = \frac{0.2}{81} = 0.00247 \text{ mA}$$

$$r_{\pi} = \frac{(80)(0.026)}{0.1975} = 10.5 \text{ k}\Omega$$

$$g_m(Q_1) = \frac{0.1975}{0.026} = 7.60 \text{ mA/V}$$

$$g_m(M_1) = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1) = 0.0445 \text{ mA/V}$$



$$V_i = V_{gs} + V_{\pi} \text{ and } V_{\pi} = g_m(M_1)V_{gs}r_{\pi} \text{ or } V_{gs} = \frac{V_{\pi}}{g_m(M_1)r_{\pi}}$$

$$\text{Then } V_i = V_{\pi} \left( 1 + \frac{1}{g_m(M_1)r_{\pi}} \right) \text{ or } V_{\pi} = \frac{V_i}{\left( 1 + \frac{1}{g_m(M_1)r_{\pi}} \right)}$$

$$V_o = -g_m(Q_1)V_{\pi}R_C \Rightarrow A_v = \frac{V_o}{V_i} = \frac{-g_m(Q_1)R_C}{\left( 1 + \frac{1}{g_m(M_1)r_{\pi}} \right)}$$

$$\text{Then } A_v = \frac{-(7.60)(20)}{\left( 1 + \frac{1}{(0.0445)(10.5)} \right)} \Rightarrow A_v = -48.4$$

**11.86**

$$r_{\pi 11} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega$$

$$R'_E = R_3 \parallel r_{\pi 11} = 0.2 \parallel 15.6 = 0.1975 \text{ k}\Omega$$

$$g_{m11} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o11} = \frac{V_{A11}}{I_{C11}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$R_{C11} = r_{o11} (1 + g_{m11} R'_E) = 600 [1 + (7.692)(0.1975)] = 1512 \text{ k}\Omega$$

$$R_{C7} = r_{o7} = \frac{V_{A7}}{I_{C7}} = \frac{60}{0.2} = 300 \text{ k}\Omega$$

$$Z = R_{C7} \parallel R_{C11} = 300 \parallel 1512 = 250 \text{ k}\Omega$$

$$r_{\pi 8} = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$I_{C9} = \frac{1}{120} \left( \frac{120}{121} \right) = 0.008264 \text{ mA}$$

$$r_{\pi 9} = \frac{(120)(0.026)}{0.008264} = 377.5 \text{ k}\Omega$$

$$\text{Now } R_o = R_4 \parallel \left( \frac{r_{\pi 8} + \left( \frac{r_{\pi 9} + Z}{121} \right)}{121} \right) = 5 \parallel \left( \frac{3.12 + \left( \frac{377.5 + 250}{121} \right)}{121} \right) = 5 \parallel 0.06864$$

$$\text{Or } R_o = 67.7 \Omega$$

11.87

$$R_i = r_{\pi 1} + (1 + \beta)r_{\pi 2}$$

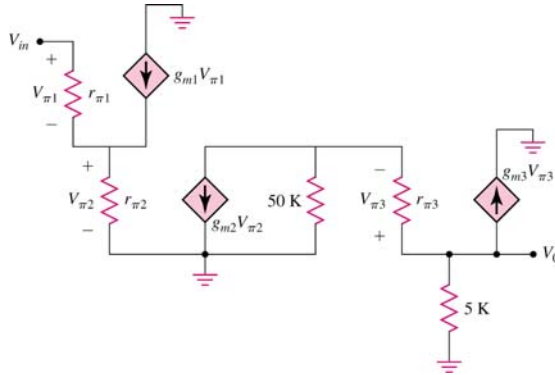
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2(0.026)}{0.5} = 520 \text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow R_i \cong 1.05 \text{ M}\Omega$$

$$R_0 = 5 \parallel \left[ \frac{r_{\pi 3} + 50}{101} \right], \quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$R_0 = 5 \parallel \left[ \frac{2.6 + 50}{101} \right] = 5 \parallel 0.521 \Rightarrow R_0 = 0.472 \text{ k}\Omega$$



$$V_0 = -\left( \frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} \right) (5)$$

$$V_0 = -V_{\pi 3} \left( \frac{1 + \beta}{r_{\pi 3}} \right) (5) \quad (1)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2} V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2} V_{\pi 2} = V_{\pi 3} \left( \frac{1}{r_{\pi 3}} + \frac{1}{50} \right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi 2} = \left( \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) r_{\pi 2}$$

$$= V_{\pi 1} \left( \frac{1 + \beta}{r_{\pi 1}} \right) r_{\pi 2} \quad (3)$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3} \left( \frac{101}{2.6} \right) (5) \Rightarrow V_{\pi 3} = -V_0 (0.005149) \quad (1)$$

And

$$19.23V_{\pi 2} = -V_0(0.005149)\left(\frac{1}{2.6} + \frac{1}{50}\right) - \frac{V_0}{50}$$

$$= -V_0(0.02208)$$

Or  $V_{\pi 2} = -V_0(0.001148)$  (2)

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0(0.001148)$$
 (4)

So

$$-V_0(0.001148) = [V_{in} + V_0(0.001148)]\left(\frac{101}{520}\right) \quad (5.2)$$

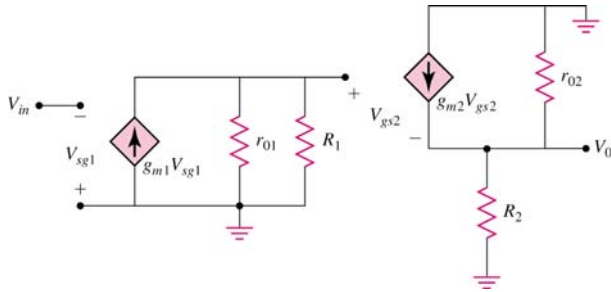
$$-V_0(0.001148) - V_0(0.001159) = V_{in}(1.01) \Rightarrow A_v = \frac{V_0}{V_{in}} = -438$$

**11.88**

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2} V_{gs2})(R_2 \parallel r_{02})$$

$$V_{gs2} = (g_{m1} V_{sg1})(r_{01} \parallel R_1) - V_0 \text{ and } V_{sg1} = -V_{in}$$

$$\text{So } V_{gs2} = -(g_{m1} V_{in})(r_{01} \parallel R_1) - V_0$$

Then

$$V_0 = g_{m2}(R_2 \parallel r_{02})[-(g_{m1} V_{in})(r_{01} \parallel R_1) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2}(R_2 \parallel r_{02})g_{m1}(r_{01} \parallel R_1)}{1 + g_{m2}(R_2 \parallel r_{02})}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

$$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406 \text{ mA/V}$$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2 \parallel r_{02} = 5 \parallel 100 = 4.76 \text{ k}\Omega$$

$$R_1 \parallel r_{01} = 35 \parallel 485 = 32.6 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

$$\text{So } \Rightarrow A_v = -11.5$$

Output Resistance—From the results for a source follower in Chapter 4.

$$R_0 = \frac{1}{g_{m2}} \parallel R_2 \parallel r_{02} = \frac{1}{1.414} \parallel 5 \parallel 100$$

$$= 0.707 \parallel 4.76$$

$$\text{So } R_0 = 0.616 \text{ k}\Omega$$

### 11.89

$$(a) R_2 = \frac{5 - 0}{1} = 5 \text{ k}\Omega$$

$$I_2 = K_p(V_{SG} + V_{TP})^2 \Rightarrow 1 = 1(V_{SG} - 0.8)^2 \Rightarrow V_{SG} = 1.8 \text{ V}$$

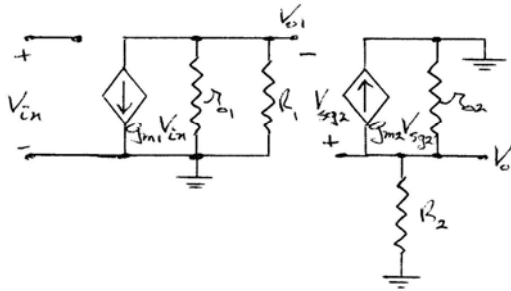
$$R_1 = \frac{5 - (-1.8)}{0.25} = 27.2 \text{ k}\Omega$$

$$(b) g_{m1} = 2\sqrt{(0.5)(0.25)} = 0.7071 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{(1)(1)} = 2 \text{ mA/V}$$

$$r_{o1} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega, \quad r_{o2} = \frac{1}{(0.02)(1)} = 50 \text{ k}\Omega$$

(c)



$$V_{o1} = -g_{m1} V_{in} (r_{o1} \parallel R_1)$$

$$V_{sg2} = V_o - V_{o1}$$

$$\frac{V_o}{R_2} + \frac{V_o}{r_{o2}} + g_{m2} V_{sg2} = 0$$

$$V_o \left( \frac{1}{R_2} + \frac{1}{r_{o2}} \right) + g_{m2} [V_o + g_{m1} V_{in} (r_{o1} \parallel R_1)] = 0$$

$$V_o \left( \frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2} \right) = -g_{m1} g_{m2} (r_{o1} \parallel R_1) V_{in}$$

$$A_v = \frac{V_o}{V_{in}} = \frac{-g_{m1} g_{m2} (r_{o1} \parallel R_1)}{\left( \frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2} \right)} = \frac{-(0.7071)(2)(200 \parallel 27.2)}{\left( \frac{1}{5} + \frac{1}{50} + 2 \right)} = -15.25$$

(d)  $I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{R_2} + g_{m2} V_x$

$$R_o = r_{o2} \parallel R_2 \parallel \frac{1}{g_{m2}} = 50 \parallel 5 \parallel \frac{1}{2} = 4.545 \parallel 0.5$$

$$R_o = 0.450 \text{ k}\Omega$$

**11.90**

(a)  $R = \frac{5-0}{0.25} = 20 \text{ k}\Omega$

$$R_{E1} = \frac{-0.7 - (-5)}{0.25} = 17.2 \text{ k}\Omega$$

$$R_C = \frac{5-0.7}{0.25} = 17.2 \text{ k}\Omega$$

$$R_{E2} = \frac{0 - (-5)}{2} = 2.5 \text{ k}\Omega$$

(b)  $A_{d1} = \frac{v_{o2}}{v_d} = \frac{g_{m1}}{2} (R \parallel r_{\pi3})$

$$g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{d1} = \frac{(9.615)}{2} (20 \parallel 12.48) = 36.94$$

$$A_3 = -g_{m3} (R_C \parallel R_{i4})$$

$$R_{i4} = r_{\pi 4} + (1 + \beta)R_{E2}$$

$$g_{m3} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$$

$$R_{i4} = 1.56 + (121)(2.5) = 304 \text{ k}\Omega$$

$$A_3 = -(9.615)(17.2 \parallel 304) = -156.5$$

$$A_4 = \frac{(1 + \beta)R_{E2}}{r_{\pi 4} + (1 + \beta)R_{E2}} = \frac{(121)(2.5)}{1.56 + (121)(2.5)} = 0.995$$

$$\text{Now } A_d = \frac{v_o}{v_d} = A_{d1} \cdot A_3 \cdot A_4 = (36.94)(-156.5)(0.995) = -5752$$

$$(c) \quad A_{cm1} = \frac{-g_{m1} (R \parallel r_{\pi 3})}{1 + \frac{r_{\pi 1}}{2(1 + \beta)R_o}}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.615)(20 \parallel 12.48)}{1 + \frac{12.48}{2(121)(200)}} = -0.01905$$

$$A_{cm} = A_{cm1} \cdot A_3 \cdot A_4 = (-0.01905)(-156.5)(0.995) = 2.966$$

$$CMRR_{dB} = 20 \log_{10} \left( \frac{5752}{2.966} \right) = 65.8 \text{ dB}$$

### 11.91

a.

$$R_{C1} = \frac{10 - v_{01}}{I_{C1}} = \frac{10 - 2}{0.1} \Rightarrow \underline{R_{C1} = 80 \text{ k}\Omega}$$

$$R_{C2} = \frac{10 - v_{04}}{I_{C4}} = \frac{10 - 6}{0.2} \Rightarrow \underline{R_{C2} = 20 \text{ k}\Omega}$$

b.

$$A_{d1} = \frac{v_{01} - v_{02}}{v_d} = -g_{m1} (R_{C1} \parallel r_{\pi 3})$$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80 \parallel 23.4) \Rightarrow \underline{A_{d1} = -69.6}$$

$$A_{d2} = \frac{v_{04}}{v_{01} - v_{02}} = \frac{1}{2} g_{m4} R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2} (7.692)(20) = 76.9$$

$$\text{Then } A_d = (76.9)(-69.6) \Rightarrow \underline{A_d = -5352}$$

### 11.92

- a. Neglect the effect of  $r_0$  in determining the differential-mode gain.

$$A_{d1} = \frac{v_{02}}{v_d} = \frac{1}{2} g_{m2} (R_C \parallel R_{i3}) \text{ where } R_{i3} = r_{\pi 3} + (1 + \beta) R_E$$

$$A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1 + \beta) R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2} \cdot (1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{I_{C3}}$$

$$v_{02} = 12 - \frac{1}{2} (1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2} (37.3) [8 \parallel 668] = 147.4$$



Then

$$A_d = A_{d1} \cdot A_2 = (147.4)(-1.197) \Rightarrow \underline{A_d = -176}$$

$$R_0 = r_{05} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}(R_C \parallel R_{i3})}{1 + \frac{r_{\pi 2}}{2(1 + \beta)R_0}}$$

$$r_{\pi 2} = \frac{(200)(0.026)}{\frac{1}{2} \cdot (1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8 \parallel 668)}{1 + \frac{5.36}{2(201)(41.2)}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow \underline{A_{cm} = 0.114}$$

b.

$$v_d = v_1 - v_2 = 2.015 \sin \omega t - 1.985 \sin \omega t$$

$$v_d = 0.03 \sin \omega t \text{ (V)}$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2.0 \sin \omega t$$

$$v_{03} = A_d v_d + A_{cm} v_{cm} \\ = (-176)(0.03) + (0.114)(2)$$

$$\text{Or } \underline{v_{03} = -5.052 \sin \omega t}$$

$$\text{Ideal, } A_{cm} = 0$$

So

$$v_{03} = A_d v_d = (-176)(0.03)$$

$$\underline{v_{03} = -5.28 \sin \omega t}$$

c.

$$R_{id} = 2r_{\pi 2} = 2(5.36) \Rightarrow \underline{R_{id} = 10.72 \text{ k}\Omega}$$

$$2R_{icm} \cong 2(1 + \beta)R_0 \parallel (1 + \beta)r_0$$

$$r_0 = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2} \cdot (1.94)} = 82.5 \text{ k}\Omega$$

$$2R_{icm} = [2(201)(41.2)] \parallel [(201)(82.5)] \\ = 16.6 \text{ M}\Omega \parallel 16.6 \text{ M}\Omega$$

$$\text{So } \Rightarrow \underline{R_{icm} = 4.15 \text{ M}\Omega}$$

**11.93**

a.

$$I_1 = \frac{24 - V_{GS4}}{R_1} = k_n (V_{GS4} - V_{Th})^2$$

$$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$$

$$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$$

$$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$v_{02} = 12 - \left(\frac{0.375}{2}\right)(40) = 4.5 \text{ V}$$

$$\frac{v_{02} - V_{GS3}}{R_5} = I_{D3} = k_n (V_{GS3} - V_{Th})^2$$

$$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)}$$

$$= 0.387 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2} g_{m2} R_D = \frac{1}{2} (0.387)(40) \Rightarrow A_{d1} = 7.74$$

$$A_2 = \frac{-g_{m3} R_{D2}}{1 + g_{m3} R_5}$$

$$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)}$$

$$= 0.434 \text{ mA/V}$$

$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$

$$\text{So } A_d = A_{d1} \cdot A_2 = (7.74)(-0.482) \Rightarrow \underline{A_d = -3.73}$$

$$R_0 = r_{o5} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2} R_D}{1 + 2g_{m2} R_0} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)}$$

$$= -0.149$$

$$A_{cm} = (-0.149)(-0.482) \Rightarrow \underline{A_{cm} = 0.0718}$$

b.

$$v_d = v_1 - v_2 = 0.3 \sin \omega t$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin \omega t$$

$$v_{o3} = A_d v_d + A_{cm} v_{cm}$$

$$= (-3.73)(0.3) + (0.0718)(2) \Rightarrow \underline{v_{o3} = -0.975 \sin \omega t \text{ (V)}}$$

Ideal,  $A_{cm} = 0$

$$v_{o3} = A_d v_d = (-3.73)(0.3)$$

Or

$$\Rightarrow \underline{v_{o3} = -1.12 \sin \omega t \text{ (V)}}$$

### 11.94

$$(a) A_d = \frac{\beta R_C}{r_\pi + R_B}$$

$$\text{Assuming } I_{CQ} \cong I_{EQ}, \quad r_\pi = \frac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$$

$$A_d = \frac{(150)(10)}{9.75 + 0.5} = 146$$

(b)  $C_M \cong C_\mu(1 + g_m R_C)$

$$g_m = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$C_M = (0.2)[1 + (15.38)(10)] = 30.96 \text{ pF}$$

(c)  $f_H = \frac{1}{2\pi(r_\pi \| R_B)(C_\pi + C_M)} = \frac{1}{2\pi(9.75 \| 0.5) \times 10^3 (1.2 + 30.96) \times 10^{-12}}$   
 $f_H = 10.4 \text{ MHz}$

**11.95**

(a)  $f_z = \frac{1}{2\pi R_o C_o} = \frac{1}{2\pi(10 \times 10^6)(0.4 \times 10^{-12})} \Rightarrow f_z = 39.8 \text{ kHz}$

(b)  $r_\pi = \frac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$

$$R_{eq} = \frac{R_o \left(1 + \frac{R_B}{r_\pi}\right)}{1 + \frac{R_B}{r_\pi} + \frac{2(1 + \beta)R_o}{r_\pi}} = \frac{(10) \left(1 + \frac{0.5}{9.75}\right)}{1 + \frac{0.5}{9.75} + \frac{2(151)(10,000)}{9.75}}$$

Or  $R_{eq} = 33.94 \Omega$

$$f_p = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{2\pi(33.94)(0.4 \times 10^{-12})}$$

$$f_p = 11.7 \text{ GHz}$$

**11.96**

a. From Equation (7.73),  $f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$\text{Then } 800 \times 10^6 = \frac{38.46 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\text{Or } C_\pi + C_\mu = 7.65 \times 10^{-12} \text{ F} = 7.65 \text{ pF}$$

$$\text{And } C_\pi = 6.65 \text{ pF}$$

$$C_M = C_\mu(1 + g_m R_C) = 1[1 + (38.46)(10)] \\ = 386 \text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi[3.12 \| 1] \times 10^3 \times (6.65 + 386) \times 10^{-12}}$$

$$\text{Or } \underline{f_H = 535 \text{ kHz}}$$

$$\text{b. From Equation (11.140), } f_z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi(10 \times 10^6)(10^{-12})}$$

$$\text{Or } \underline{f_z = 15.9 \text{ kHz}}$$

### 11.97

$$\text{(a) } A_v = \frac{\frac{1}{2} \beta R_C}{r_\pi + (1 + \beta) R_E}$$

$$r_\pi = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_v = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.1)} = 19.5$$

$$\text{(b) } A_v = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.25)} = 11.2$$

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## Chapter 12

### 12.1

$$(a) A_f = \frac{A}{1 + A\beta} \Rightarrow 100 = \frac{5 \times 10^4}{1 + (5 \times 10^4)\beta} \Rightarrow \beta = 9.98 \times 10^{-3}$$

$$(b) 80 = \frac{A}{1 + A(0.012)} \Rightarrow A = 2000$$

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### 12.2

$$(a) A_f = \frac{A}{1 + A\beta} \Rightarrow -80 = \frac{-10^5}{1 + (-10^5)\beta} \Rightarrow \beta = -0.01249$$

$$(b) A_f = \frac{-5 \times 10^4}{1 + (-5 \times 10^4)(-0.015)} = -66.58$$

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### 12.3

$$(a) A_f = \frac{A}{1 + A\beta} \quad \beta = 0.15$$

$$T = A\beta$$

$$(i) T = \infty$$

$$(ii) A = 80 \text{ dB} \Rightarrow A = 10^4 \Rightarrow T = 1.5 \times 10^3$$

$$(iii) T = 15$$

$$(i) A_f = \frac{1}{\beta} = 6.667$$

$$(ii) A_f = 6.662$$

$$(iii) A_f = 6.25$$

(b)

$$(i) T = \infty$$

$$(ii) T = 2.5 \times 10^3$$

$$(iii) T = 25$$

$$(i) A_f = \frac{1}{\beta} = 4.00$$

$$(ii) A_f = 3.9984$$

$$(iii) A_f = 3.846$$

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### 12.4

(a)

$$A_f = \frac{A}{1 + A\beta} \cong \frac{1}{\beta} = 125$$

$$\beta = 0.0080$$

(b)

$$A_f = (125)(0.9975) = 124.6875$$

$$124.6875 = \frac{A}{1 + (0.008)A}$$

$$124.6875[1 + (0.008)A] = A$$

$$124.6875 = A[1 - 0.9975]$$

$$A = 49,875$$

**12.5**

(a)  $A_f = \frac{A}{1 + A\beta} \Rightarrow -80 = \frac{-2 \times 10^4}{1 + (-2 \times 10^4)\beta} \Rightarrow \beta = -0.01245$

(b)  $\frac{dA_f}{A_f} = \frac{1}{(1 + A\beta)} \cdot \frac{dA}{A}$

$$0.01 = \frac{1}{[1 + (-2 \times 10^4)(-0.01245)]} \cdot \frac{dA}{A} \Rightarrow \frac{dA}{A} = 2.5\%$$

**12.6**

$$\frac{dA_f}{A_f} = \frac{1}{1 + \beta A} \cdot \frac{dA}{A}$$

$$0.001 = \frac{1}{1 + \beta A} \cdot (0.10) \Rightarrow 1 + \beta A = 100$$

$$\beta = \frac{100 - 1}{5 \times 10^4} \Rightarrow \beta = 1.98 \times 10^{-3}$$

Now  $A_f = \frac{A}{1 + \beta A} = \frac{5 \times 10^4}{1 + (1.98 \times 10^{-3})(5 \times 10^4)} \Rightarrow A_f = 500$

**12.7**

(a)  $A_{vf} = \left( \frac{A_1}{1 + A_1\beta_1} \right) \left( \frac{A_2}{1 + A_2\beta_1} \right)$

$$50 = \left( \frac{200}{1 + 200\beta_1} \right) \left( \frac{10}{1 + 10\beta_1} \right)$$

$$(1 + 200\beta_1)(1 + 10\beta_1) = \frac{(200)(10)}{50} = 40$$

Then  $2000\beta_1^2 + 210\beta_1 - 39 = 0 \Rightarrow \beta_1 = 0.096685$

$$A_{vf} = \frac{A_1 A_2}{1 + A_1 A_2 \beta_2}$$

$$50 = \frac{2000}{1 + 2000\beta_2} \Rightarrow \beta_2 = 0.0195$$

(b) From circuit (a):

$$A_{vf} = \left( \frac{200}{1 + (200)(0.096685)} \right) \left( \frac{9}{1 + (9)(0.096685)} \right) = 47.33 \Rightarrow (-5.43\%)$$

For circuit (b):

$$A_{vf} = \frac{1800}{1 + (1800)(0.0195)} = 49.86 \Rightarrow (-0.28\%)$$

### 12.8

$$V_o = (-10)(-15)(-20)V_e = -3000V_e$$

(a)  $V_e = \beta V_o + V_s$

So  $V_o = -3000(\beta V_o + V_s)$

We find

$$A_{vf} = \frac{V_o}{V_s} = \frac{-3000}{1 + 3000\beta}$$

For  $A_{vf} = -120 = \frac{-3000}{1 + 3000\beta} \Rightarrow \beta = 0.008$

(b) Now  $V_o = (-9)(-13.5)(-18)V_e = -2187V_e$

Then

$$A_{vf} = \frac{-2187}{1 + 2187\beta} = \frac{-2187}{1 + 2187(0.008)} = -118.24$$

$$\% \text{ change} = \frac{120 - 118.24}{120} \times 100 \Rightarrow \underline{1.47\% \text{ change}}$$

### 12.9

(a)  $A_v f_H = A_{vf} f_C$

$$(5 \times 10^4)(10) = (25)f_C \Rightarrow f_C = 20 \text{ kHz}$$

(b)  $A_{vf} = \frac{A_v f_H}{f_C} = \frac{(10^5)(8)}{20 \times 10^3} = 40$

### 12.10

(a)  $f_C = \frac{(5 \times 10^4)(10)}{50} \Rightarrow f_C = 10 \text{ kHz}$

(b)  $f_C = \frac{(10^4)(10)}{50} \Rightarrow f_C = 2 \text{ kHz}$

### 12.11

(a) (i)  $A_v = \frac{A_{vf} f_C}{f_H} = \frac{(75)(35 \times 10^3)}{5} = 5.25 \times 10^5$

(ii)  $75 = \frac{5.25 \times 10^5}{1 + (5.25 \times 10^5)\beta} \Rightarrow \beta = 0.01333$



$$(b) A = (5.25 \times 10^5)(0.9) = 4.725 \times 10^5$$

$$A_{vf} = \frac{4.725 \times 10^5}{1 + (4.725 \times 10^5)(0.01333)} = 74.99$$

$$f_C = \frac{A_v f_H}{A_{vf}} = \frac{(4.725 \times 10^5)(5)}{74.99} \Rightarrow f_C = 31.5 \text{ kHz}$$

### 12.12

Low freq.  $A_f = \frac{A_0}{1 + A_0 \beta}$

$$100 = \frac{5000}{1 + (5000)\beta} \Rightarrow \beta = 0.0098$$

Freq. response

$$A_f = \frac{A}{1 + A\beta} = \frac{\frac{5000}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)}}{1 + \frac{5000}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)}}$$

$$= \frac{5000}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right) + 49}$$

$$= \frac{5000}{1 + j\frac{f}{f_1} + j\frac{f}{f_2} + \left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right) + 49}$$

$$= \frac{5000}{50 + j\frac{f}{f_1} + j\frac{f}{f_2} + \left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right)}$$

Also

$$A_f = \frac{A_{f0}}{\left(1 + j\frac{f}{f_A}\right)\left(1 + j\frac{f}{f_B}\right)} = \frac{100}{1 + j\frac{f}{f_A} + j\frac{f}{f_B} + \left(j\frac{f}{f_A}\right)\left(j\frac{f}{f_B}\right)}$$

So

$$\frac{100}{1 + j\frac{f}{f_A} + j\frac{f}{f_B} + \left(j\frac{f}{f_A}\right)\left(j\frac{f}{f_B}\right)} = \frac{100}{1 + j\frac{f}{50f_1} + j\frac{f}{50f_2} + \frac{1}{50}\left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right)}$$

Then

$$\frac{1}{f_A} + \frac{1}{f_B} = \frac{1}{50f_1} + \frac{1}{50f_2}$$

and  $\frac{1}{f_A f_B} = \frac{1}{50f_1 f_2}$

$f_1 = 10$  and  $f_2 = 2000$

$$\frac{1}{f_A} + \frac{1}{f_B} = \frac{1}{50(10)} + \frac{1}{50(2000)} = 0.002 + 0.000010 = 0.002010$$

and

$$\frac{1}{f_A f_B} = \frac{1}{(50)(10)(2000)} \Rightarrow \frac{1}{f_A} = \frac{f_B}{10^6}$$

$$\frac{f_B}{10^6} + \frac{1}{f_B} = 0.002010$$

Then

$$10^{-6} f_B^2 + 1 = 2.01 + 10^{-3} f_B$$

$$10^{-6} f_B^2 - 2.01 \times 10^{-3} f_B + 1 = 0$$

$$f_B = \frac{2.01 \times 10^{-3} \pm \sqrt{4.0401 \times 10^{-6} - 4(10^{-6})(1)}}{2(10^{-6})}$$

$$f_B = \frac{2.01 \times 10^{-3} \pm 2.0025 \times 10^{-4}}{2(10^{-6})}$$

+ sign  $f_B = 1.105 \times 10^3$  Hz  
 + sign  $f_A = 9.05 \times 10^2$  Hz

**12.13**

(a) Fig. P12.7(a)

$$A_f = \left[ \frac{\left( \frac{200}{1 + j \frac{f}{f_1}} \right)}{1 + \left( \frac{200}{1 + j \frac{f}{f_1}} \right) (0.1126)} \right] \left[ \frac{10}{1 + (10)(0.1126)} \right]$$

$$= \left[ \frac{200}{\left( 1 + j \frac{f}{f_1} \right) + 22.52} \right] (4.704)$$

$$= \frac{940.73}{23.52 + j \frac{f}{f_1}} = \frac{940.73}{23.52} \cdot \frac{1}{1 + j \frac{f}{(23.52)f_1}}$$

$$= \frac{40}{1 + \frac{jf}{(23.52)f_1}} \quad f_{-3dB} = (23.52)(100) \Rightarrow \underline{2.352 \text{ kHz}}$$

Fig P12.7(b)

$$A_f = \frac{\frac{(200)(10)}{1 + j \frac{f}{f_1}}}{1 + \frac{(0.0245)(200)(10)}{1 + j \frac{f}{f_1}}} = \frac{2000}{1 + j \frac{f}{f_1} + 49}$$

$$= \frac{2000}{50} \cdot \frac{1}{1 + j \frac{f}{(50)f_1}} \quad f_{-3dB} = (50)(100) \Rightarrow \underline{5 \text{ KHz}}$$

(b) Overall feedback  $\Rightarrow$  wider bandwidth.

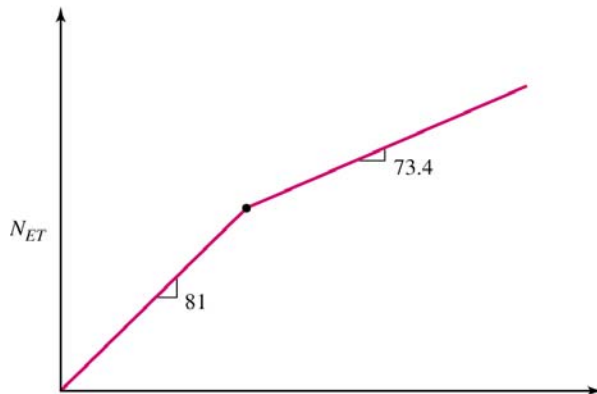
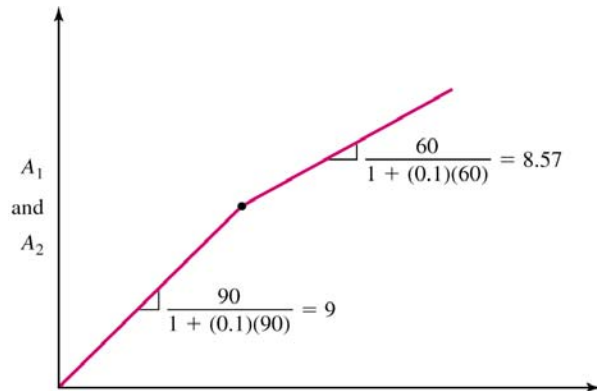
**12.14**

$$v_o = A_1 A_2 v_i + A_1 v_n$$

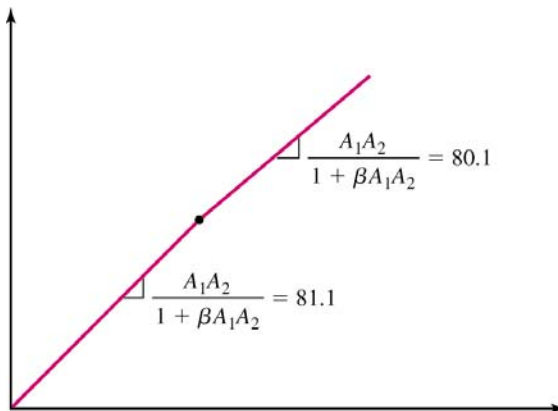
$$v_o = (100)v_i + (1)v_n = (100)(10) + (1)(1) \Rightarrow \frac{S_o}{N_o} = \frac{1000}{1} = 1000$$

12.15

(a)



(b)



Circuit (b) – less distortion

**12.16**

$$A_{vf} = \frac{A_v}{1 + \beta_v A_v} = \frac{5 \times 10^3}{1 + (0.0080)(5 \times 10^3)} \Rightarrow A_{vf} = 121.95$$

$$R_{if} = R_i(1 + \beta_v A_v) = (10)[1 + (0.0080)(5 \times 10^3)] \Rightarrow R_{if} = 410 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + \beta_v A_v} = \frac{1 \times 10^3}{1 + (0.0080)(5 \times 10^3)} \Rightarrow R_{of} = 24.4 \Omega$$

**12.17**

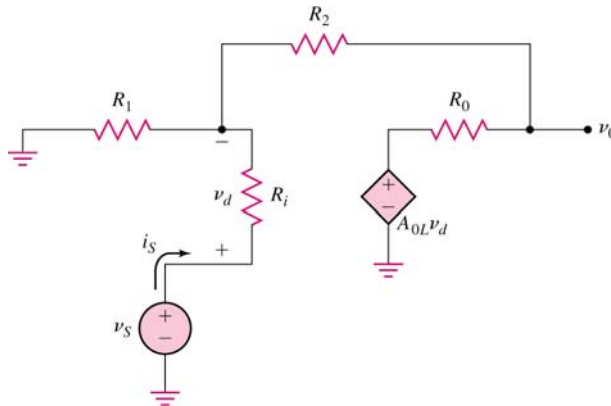
$$V_{fb} = \beta_v V_o = (0.0096)(2.5) \Rightarrow V_{fb} = 24 \text{ mV}$$

$$V_\epsilon = V_i - V_{fb} = 25 - 24 = 1 \text{ mV}$$

$$V_o = A_v V_\epsilon \Rightarrow A_v = \frac{V_o}{V_\epsilon} = \frac{2.5}{0.001} = 2.5 \times 10^3 \text{ V/V}$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{2.5}{0.025} = 100 \text{ V/V}$$

**12.18**



$$A_{vf} \approx \left(1 + \frac{R_2}{R_1}\right) = 20 \Rightarrow \frac{R_2}{R_1} = 19$$

$$v_d = i_s R_i$$

$$i_s = \frac{v_s - v_d}{R_1} + \frac{(v_s - v_d) - v_o}{R_2} \quad (1)$$

$$\frac{v_o - A_0L v_d}{R_0} + \frac{v_o - (v_s - v_d)}{R_2} = 0 \quad (2)$$

$$v_o \left( \frac{1}{R_0} + \frac{1}{R_2} \right) = \frac{A_0L v_d}{R_0} + \frac{(v_s - v_d)}{R_2}$$

$$v_o = \frac{\frac{A_0L v_d}{R_0} + \frac{(v_s - v_d)}{R_2}}{\left( \frac{1}{R_0} + \frac{1}{R_2} \right)}$$

From (1):

$$i_s = \frac{v_s - v_d}{R_1} + \frac{v_s - v_d}{R_2} - \frac{1}{R_2} \left[ \frac{A_{0L} v_d}{R_0} + \frac{(v_s - v_d)}{R_2} \right]$$

$$i_s = v_s \left( \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2} \left( \frac{1}{R_0} + \frac{1}{R_2} \right) \right) - v_d \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{A_{0L} - 1}{R_0} \right)$$

$$v_d = i_s R_i$$

$$i_s \left\{ 1 + \frac{R_i \left[ \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \left( 1 + \frac{R_2}{R_0} \right) + \frac{A_{0L} - 1}{R_0} \right]}{1 + \frac{R_2}{R_0}} \right\} = v_s \left[ \frac{\left( \frac{1}{R_1} + \frac{1}{R_2} \right) \left( 1 + \frac{R_2}{R_0} \right) - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right]$$

$$i_s \left\{ 1 + \frac{R_2}{R_0} + R_i \left[ \frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} + \frac{A_{0L}}{R_0} \right] \right\} = v_s \left[ \frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} \right]$$

$$i_s \left\{ R_0 + R_2 + R_i \left[ \frac{R_0}{R_1} + \left( 1 + \frac{R_2}{R_1} \right) + A_{0L} \right] \right\} = v_s \left[ \frac{R_0}{R_1} + \left( 1 + \frac{R_2}{R_1} \right) \right] \quad (1)$$

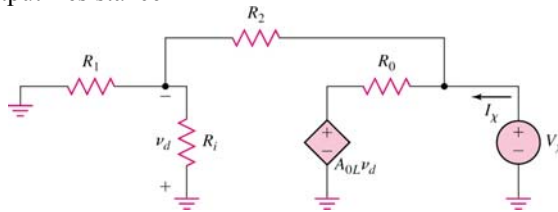
Let  $R_2 = 190 \text{ k}\Omega$ ,  $R_1 = 10 \text{ k}\Omega$

$$i_s \left\{ 0.1 + 190 + 100 \cdot \left[ \frac{0.1}{10} + 20 + 10^5 \right] \right\} = v_s \left[ \frac{0.1}{10} + 20 \right]$$

$$i_s (1.000219 \times 10^7) = v_s (20.01)$$

$$R_{if} = \frac{v_s}{i_s} \cong 5 \times 10^5 \text{ k}\Omega \Rightarrow R_{if} \cong 500 \text{ M}\Omega$$

Output Resistance



$$I_x = \frac{V_x - A_{0L} v_d}{R_0} + \frac{V_x}{R_2 + R_1 \parallel R_i}$$

$$v_d = \frac{-R_1 \parallel R_i}{R_1 \parallel R_i + R_2} \cdot V_x$$

$$\frac{I_x}{V_x} = \frac{1}{R_0} = \frac{1}{R_0} + \frac{A_{0L} \cdot R_1 \parallel R_i}{R_0 (R_1 \parallel R_i + R_2)} + \frac{1}{R_2 + R_1 \parallel R_i}$$

$$R_1 \parallel R_i = 10 \parallel 100 = 9.09$$

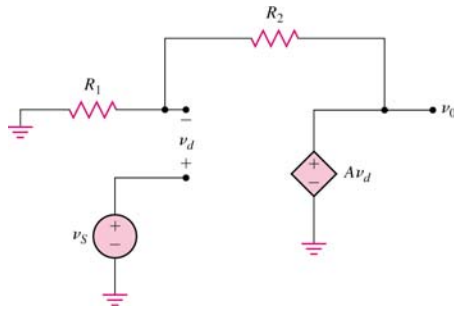
$$\frac{1}{R_{of}} = \frac{1}{0.1} + \frac{10^5}{0.1} \cdot \left( \frac{9.09}{9.09 + 190} \right) + \frac{1}{190 + 9.09}$$

$$= 10 + 4.566 \times 10^4 + 0.00502$$

$$R_{of} = 2.19 \times 10^{-5} \text{ k}\Omega \Rightarrow R_{of} = 0.0219 \text{ }\Omega$$

12.19

a.



$$\frac{v_s - v_d}{R_1} = \frac{v_0 - (v_s - v_d)}{R_2} \quad \text{and} \quad v_d = \frac{v_0}{A}$$

$$\begin{aligned} \frac{v_s}{R_1} + \frac{v_s}{R_2} &= \frac{v_0}{R_2} + v_d \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \\ &= \frac{v_0}{R_2} + \frac{v_0}{A} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \end{aligned}$$

$$v_s \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_0}{R_2} \left[ 1 + \frac{1}{A} \left( 1 + \frac{R_2}{R_1} \right) \right]$$

$$\frac{v_0}{v_s} = \frac{\left( 1 + \frac{R_2}{R_1} \right)}{1 + \frac{1}{A} \left( 1 + \frac{R_2}{R_1} \right)}$$

which can be written as

$$A_f = \frac{v_0}{v_s} = \frac{A}{1 + \left[ A \left( 1 + \frac{R_2}{R_1} \right) \right]}$$

$$\beta = \frac{1}{1 + \frac{R_2}{R_1}}$$

b.

$$20 = \frac{10^5}{1 + (10^5)\beta}$$

c.

$$\text{So } \beta = \frac{\frac{10^5}{20} - 1}{10^5} \Rightarrow \beta = 0.04999$$

$$\text{Then } \frac{R_2}{R_1} = \frac{1}{\beta} - 1 = \frac{1}{0.04999} - 1 \Rightarrow \frac{R_2}{R_1} = 19.004$$

d.  $A \rightarrow 9 \times 10^4$

$$A_f = \frac{9 \times 10^4}{1 + (9 \times 10^4)(0.04999)} = 19.99956$$

$$\frac{\Delta A_f}{A_f} = \frac{-4.444 \times 10^{-4}}{20} = -2.222 \times 10^{-3}\% \Rightarrow \frac{\Delta A_f}{A_f} = -0.005\%$$

**12.20**

$$I_{\epsilon} = I_i - I_{fb} = 20 - 19 = 1 \mu\text{ A}$$

$$I_{fb} = \beta_i I_o \Rightarrow I_o = \frac{19}{0.0095} \Rightarrow I_o = 2 \text{ mA}$$

$$A_i = \frac{I_o}{I_{\epsilon}} = \frac{2 \times 10^{-3}}{1 \times 10^{-6}} = 2 \times 10^3 \text{ A/A}$$

$$A_{if} = \frac{I_o}{I_i} = \frac{2}{0.020} = 100 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + \beta_i A_i} = \frac{500}{1 + (0.0095)(2 \times 10^3)} = 25 \Omega$$

$$R_{of} = (1 + \beta_i A_i) R_o = [1 + (0.0095)(2 \times 10^3)](20 \times 10^3) \Rightarrow R_{of} = 400 \text{ k}\Omega$$

**12.21**

$$I_{fb} = I_i - I_{\epsilon} = 25 - 0.8 = 24.2 \mu\text{ A}$$

$$I_o = A_{if} I_i = (125)(25) \Rightarrow I_o = 3.125 \text{ mA}$$

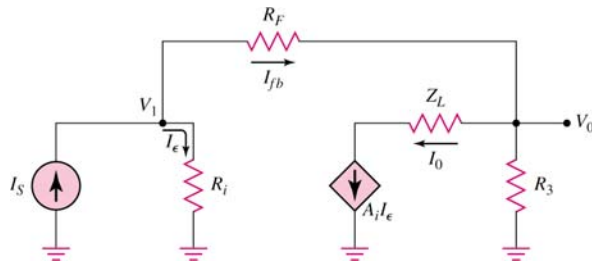
$$\beta_i = \frac{I_{fb}}{I_o} = \frac{24.2}{3125} = 0.007744 \text{ A/A}$$

$$A_{if} = \frac{A_i}{1 + \beta_i A_i}$$

$$125 = \frac{A_i}{1 + (0.007744)A_i} \Rightarrow A_i = 3906 \text{ A/A}$$

**12.22**

a.



Assume that  $V_1$  is at virtual ground.

$$V_0 = -I_{fb} R_F$$

Now

$$I_{fb} = I_o + \frac{V_0}{R_3} = I_o - \frac{I_{fb} R_F}{R_3}$$

$$I_{fb} = I_s - I_{\epsilon}$$

and

$$I_o = A_i I_{\epsilon} = \frac{I_o}{A_i}$$

so

$$I_{fb} = I_S - \frac{I_0}{A_i}$$

From above

$$I_{fb} \left( 1 + \frac{R_F}{R_3} \right) = I_0$$

$$\left( I_S - \frac{I_0}{A_i} \right) \left( 1 + \frac{R_F}{R_3} \right) = I_0$$

$$I_S \left( 1 + \frac{R_F}{R_3} \right) = I_0 \left[ 1 + \frac{1}{A_i} \left( 1 + \frac{R_F}{R_3} \right) \right]$$

or

$$A_{vf} = \frac{I_0}{I_S} = \frac{\left( 1 + \frac{R_F}{R_3} \right)}{\left[ 1 + \frac{1}{A_i} \left( 1 + \frac{R_F}{R_3} \right) \right]}$$

$$= \frac{A_i}{1 + \frac{A_i}{\left( 1 + \frac{R_F}{R_3} \right)}} = A_{vf}$$

b.  $\beta_i = \frac{1}{\left( 1 + \frac{R_F}{R_3} \right)}$

c.  $25 = \frac{10^5}{1 + (10^5)\beta_i}$

so  $\beta_i = \frac{10^5 - 1}{25 \cdot 10^5} \Rightarrow \beta_i = 0.03999$

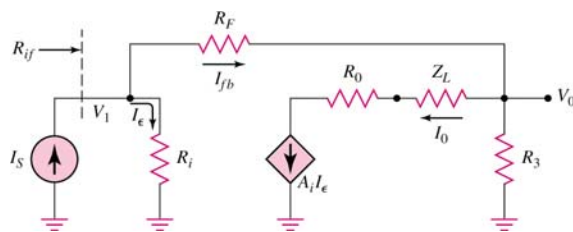
so  $\frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \frac{R_F}{R_3} = 24.0$

d.  $A_i = 10^5 - (0.15)(10^5) = 8.5 \times 10^4$

so  $A_{vf} = \frac{8.5 \times 10^4}{1 + (8.5 \times 10^4)(0.03999)} = 24.9989$

so  $\frac{\Delta A_{vf}}{A_{vf}} = -\frac{1.10 \times 10^{-3}}{25} = -4.41 \times 10^{-5} \Rightarrow -4.41 \times 10^{-3}\%$

12.23





$$I_S = I_e + I_{fb}, V_1 = I_e R_i$$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \quad \text{and} \quad V_0 = V_1 - I_{fb} R_F$$

$$I_0 = A_i I_e \Rightarrow I_e = \frac{I_0}{A_i}$$

Now

$$I_{fb} = A_i I_e + \frac{1}{R_3} (V_1 - I_{fb} R_F)$$

$$I_{fb} \left[ 1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_{fb} = I_S - I_e$$

$$(I_S - I_e) \left[ 1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_S \left[ 1 + \frac{R_F}{R_3} \right] = I_e \left[ \left( 1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{V_1}{R_3}$$

$$I_e = \frac{V_1}{R_i}$$

$$I_S \left[ 1 + \frac{R_F}{R_3} \right] = V_1 \left\{ \frac{1}{R_i} \cdot \left[ \left( 1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}$$

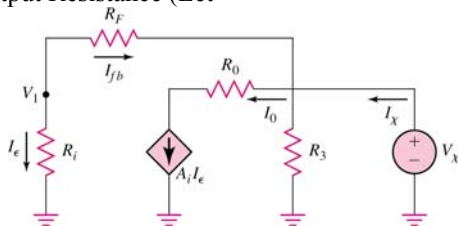
$$R_{if} = \frac{V_1}{I_S} = \frac{\left( 1 + \frac{R_F}{R_3} \right)}{\left\{ \frac{1}{R_i} \cdot \left[ \left( 1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}}$$

The  $1/R_3$  term in the denominator will be negligible. Then

$$R_{if} = \frac{25}{\left\{ \frac{1}{2} [(25) + 10^5] \right\}}$$

$$R_{if} \cong 5 \times 10^{-4} \text{ k}\Omega \Rightarrow R_{if} = 0.5 \Omega$$

Output Resistance (Let  $Z_L = 0$ )



$$I_X = \frac{V_X}{R_3} + A_i I_e + \frac{V_X}{R_F + R_i}$$

$$I_e = \frac{V_X}{R_F + R_i}$$

so

$$\frac{I_X}{V_X} = \frac{1}{R_0} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i} \cdot \frac{R_F}{R_3} = 24$$

Let  $R_F = 240 \text{ k}\Omega$ ,  $R_3 = 10 \text{ k}\Omega$

$$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5 + 1}{240 + 2}$$

$$\text{so } R_{of} \approx \frac{R_F + R_i}{A_i + 1} = \frac{240 + 2}{10^5 + 1} \Rightarrow R_{of} \approx 2.42 \times 10^{-3} \text{ k}\Omega \text{ or } \underline{R_{of} \approx 2.42 \Omega}$$

**12.24**

$$V_\epsilon = V_i - V_{fb} = 0.2 - 0.195 \Rightarrow V_\epsilon = 5 \mu\text{V}$$

$$\beta_z = \frac{V_{fb}}{I_o} = \frac{0.195 \times 10^{-3}}{5 \times 10^{-3}} = 0.039 \text{ V/A}$$

$$A_g = \frac{I_o}{V_\epsilon} = \frac{5 \times 10^{-3}}{5 \times 10^{-6}} = 1000 \text{ A/V}$$

$$A_{gf} = \frac{A_g}{1 + A_g \beta_z} = \frac{1000}{1 + (1000)(0.039)} = 25 \text{ A/V}$$

$$R_{if} = R_i (1 + \beta_z A_g) = (20 \times 10^3) [1 + (0.039)(1000)] \Rightarrow R_{if} = 800 \text{ k}\Omega$$

$$R_{of} = R_o (1 + \beta_z A_g) = (10 \times 10^3) [1 + (0.039)(1000)] \Rightarrow R_{of} = 400 \text{ k}\Omega$$

**12.25**

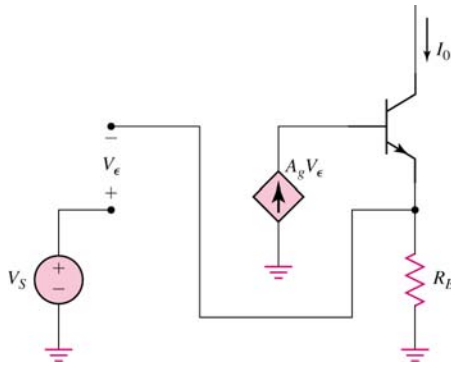
$$A_{gf} = \frac{A_g}{1 + \beta_z A_g} = \frac{2000}{1 + (2000)(0.0245)} = 40 \text{ A/V}$$

$$I_o = A_{gf} V_i = (40)(150 \times 10^{-6}) \Rightarrow I_o = 6 \text{ mA}$$

$$V_{fb} = \beta_z I_o = (0.0245)(6 \times 10^{-3}) \Rightarrow V_{fb} = 147 \mu\text{V}$$

$$V_\epsilon = V_i - V_{fb} = 150 - 147 \Rightarrow V_\epsilon = 3 \mu\text{V}$$

**12.26**



$$I_E = \frac{(1 + h_{FE})}{h_{FE}} \cdot I_o = \frac{V_S - V_\epsilon}{R_E}$$

Also  $I_o = h_{FE} (A_g V_\epsilon)$  so  $V_\epsilon = \frac{I_o}{h_{FE} A_g}$   
Then

$$\frac{1+h_{FE}}{h_{FE}} \cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE}A_gR_E}$$

$$\left[ \frac{1+h_{FE}}{h_{FE}} + \frac{1}{h_{FE}A_gR_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\left[ \frac{A_g(1+h_{FE})R_E+1}{h_{FE}A_gR_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\frac{I_0}{V_S} = \frac{1}{R_E} \cdot \left[ \frac{h_{FE}A_gR_E}{1+A_g(1+h_{FE})R_E} \right] \Rightarrow \frac{I_0}{V_S} \approx \frac{h_{FE}A_g}{1+(h_{FE}A_g)R_E}$$

b.  $\beta_z = R_E$

c.  $10 = \frac{5 \times 10^5}{1+(5 \times 10^5)\beta_z}$

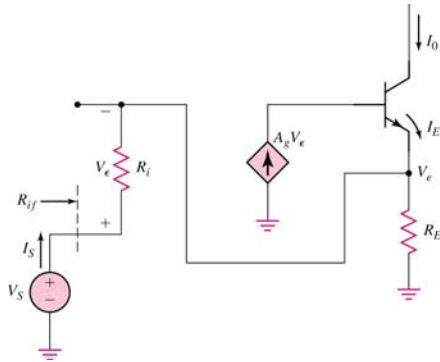
$$\beta_z = \frac{\frac{5 \times 10^5}{10} - 1}{5 \times 10^5} \Rightarrow \beta_z = R_E = 0.099998 \text{ k}\Omega$$

d. If  $A_g \rightarrow 5.5 \times 10^5$  then

$$A_{gf} = \frac{5.5 \times 10^5}{1+(5.5 \times 10^5)(0.099998)} = 10.0000182$$

$$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82 \times 10^{-5}}{10} \Rightarrow 1.82 \times 10^{-4}\%$$

12.27



$$I_E = (1+h_{FE})A_gV_e, \quad I_E = \frac{V_e}{R_E} - I_S \quad \text{and} \quad V_e = I_S R_i, \quad V_e = V_S - V_e = V_S - I_S R_i$$

Now  $(1+h_{FE})A_g I_S R_i = \frac{1}{R_E} \cdot (V_S - I_S R_i) - I_S$

$$\left[ (1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right] I_S = \frac{V_S}{R_E}$$

$$R_{if} = \frac{V_S}{I_S} = R_E \left[ (1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right]$$

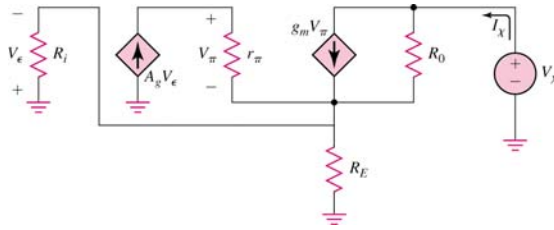
We have

$$(1+h_{FE})A_g \approx h_{FE}A_g = 5 \times 10^5 \text{ mS}$$

$$R_E \approx 0.1 \text{ k}\Omega$$

so  $R_{if} = (0.1) \left[ (5 \times 10^5)(20) + \frac{20}{0.1} + 1 \right]$

or  $R_{if} = 10^6 \text{ k}\Omega$



$$\frac{V_\pi}{r_\pi} = A_g V_\varepsilon$$

$$I_x = g_m V_\pi + \frac{V_x - (-V_\varepsilon)}{R_0} \quad (1)$$

$$V_\varepsilon = -(I_x + A_g V_\varepsilon)(R_E \parallel R_i) \quad (2)$$

or  $V_\varepsilon [1 + A_g (R_E \parallel R_i)] = -I_x (R_E \parallel R_i)$

Now:

$$I_x = g_m A_g r_\pi V_\varepsilon + \frac{V_x}{R_0} + \frac{V_\varepsilon}{R_0} \quad (1)$$

$$I_x = \left( g_m A_g r_\pi + \frac{1}{R_0} \right) \left[ \frac{-I_x (R_E \parallel R_i)}{1 + A_g (R_E \parallel R_i)} \right] + \frac{V_x}{R_0}$$

$$R_{of} = \frac{V_x}{I_x}$$

$$= R_0 \left\{ 1 + \left( g_m A_g r_\pi + \frac{1}{R_0} \right) \left[ \frac{(R_E \parallel R_i)}{1 + A_g (R_E \parallel R_i)} \right] \right\}$$

$$g_m r_\pi A_g = h_{FE} A_g = 5 \times 10^5 \text{ mS}$$

Let  $h_{FE} = 100$  so  $A_g = 5 \times 10^3 \text{ mS}$

$$R_E \parallel R_i = 0.1 \parallel 20 \approx 0.1 \text{ k}\Omega$$

Then

$$R_{of} = 50 \left\{ 1 + \left( 5 \times 10^5 + \frac{1}{50} \right) \left[ \frac{0.1}{1 + (5 \times 10^3)(0.1)} \right] \right\}$$

or  $R_{of} = 5.04 \text{ M}\Omega$

### 12.28

$$A_{zf} = \frac{A_z}{1 + \beta_g A_z}$$

$$0.20 \times 10^6 = \frac{A_z}{1 + (4.25 \times 10^{-6}) A_z} \Rightarrow A_z = 1.333 \text{ V}/\mu\text{A}$$

$$R_{if} = \frac{R_i}{1 + \beta_g A_z} = \frac{500}{1 + (4.25 \times 10^{-6})(1.333 \times 10^6)} \Rightarrow R_{if} = 75 \Omega$$

$$R_{of} = \frac{R_o}{1 + \beta_g A_z} = 75 \Omega$$

**12.29**

$$I_\varepsilon = I_i - I_{fb} = 40 - 38 = 2 \mu\text{A}$$

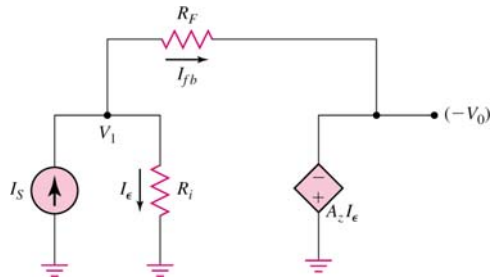
$$A_z = \frac{V_o}{I_\varepsilon} = \frac{8}{2} = 4 \frac{\text{V}}{\mu\text{A}}$$

$$\beta_g = \frac{I_{fb}}{V_o} = \frac{38}{8} = 4.75 \frac{\mu\text{A}}{\text{V}}$$

$$A_{jf} = \frac{V_o}{I_i} = \frac{8}{40} = 0.2 \frac{\text{V}}{\mu\text{A}}$$

**12.30**

a.



Assuming  $V_1$  is at virtual ground

$$(-V_0) = -I_{fb}R_F \quad \text{and} \quad (-V_0) = -A_z I_\varepsilon \Rightarrow I_\varepsilon = \frac{V_0}{A_z}$$

$$I_{fb} = I_S - I_\varepsilon$$

$$V_0 = (I_S - I_\varepsilon)R_F = I_S R_F - \left(\frac{V_0}{A_z}\right)R_F$$

So

$$V_0 \left[1 + \frac{R_F}{A_z}\right] = I_S R_F$$

$$A_{jf} = \frac{V_0}{I_S} = \frac{R_F}{\left[1 + \frac{R_F}{A_z}\right]} = \frac{A_z R_F}{A_z + R_F}$$

so

$$A_{jf} = \frac{A_z}{1 + A_z \left(\frac{1}{R_F}\right)} = \frac{A_z}{1 + A_z \beta_g}$$

or

$$\beta_g = \frac{1}{R_F}$$

b.

$$5 \times 10^4 = \frac{5 \times 10^6}{1 + (5 \times 10^6) \beta_g}$$

c.

$$\beta_g = \frac{\frac{5 \times 10^6}{5 \times 10^4} - 1}{5 \times 10^6} \Rightarrow \beta_g = 1.98 \times 10^{-5}$$

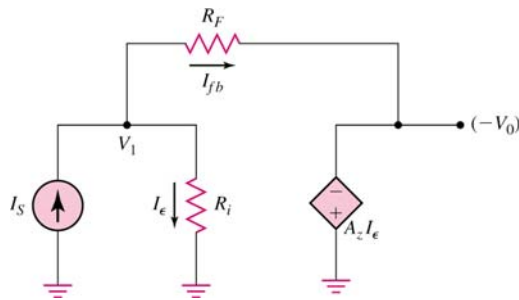
$$R_F = \frac{1}{\beta_g} \Rightarrow R_F = 50.5 \text{ k}\Omega$$

d.  $A_z = (0.9)(5 \times 10^6) = 4.5 \times 10^6$

$$A_{zf} = \frac{4.5 \times 10^6}{1 + (4.5 \times 10^6)(1.98 \times 10^{-5})} = 4.994 \times 10^4$$

$$\frac{\Delta A_{zf}}{A_{zf}} = -\frac{55.4939}{5 \times 10^4} = -1.11 \times 10^{-3} \Rightarrow \underline{-0.111\%}$$

12.31



$$V_1 = I_e R_i, -V_0 = -A_z I_e \Rightarrow V_0 = A_z I_e$$

$$I_{fb} = I_S - I_e \text{ and } -V_0 = V_1 - I_{fb} R_F$$

$$-A_z I_e = V_1 - (I_S - I_e) R_F$$

$$-A_z \left( \frac{V_1}{R_i} \right) = V_1 - I_S R_F + \left( \frac{V_1}{R_i} \right) R_F$$

$$I_S R_F = V_1 \left[ 1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]$$

$$R_{zf} = \frac{V_1}{I_S} = \frac{R_F}{\left[ 1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]}$$

We have

$$R_{zf} = \frac{50.5 \times 10^3}{\left[ 1 + \frac{5 \times 10^6}{10 \times 10^3} + \frac{50.5 \times 10^3}{10 \times 10^3} \right]}$$

$$= \frac{50.5 \times 10^3}{[1 + 500 + 5.05]} \Rightarrow \underline{R_{zf} = 99.79 \Omega}$$

12.32

- (a) Low input  $R \Rightarrow$  Shunt input  
Low output  $R \Rightarrow$  Shunt output  
Or a Shunt-Shunt circuit
- (b) High input  $R \Rightarrow$  Series input  
High output  $R \Rightarrow$  Series output  
Or a series-Series circuit
- (c) Shunt-Series circuit
- (d) Series-Shunt circuit

**12.33**

(a)  $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow R_i(\max) \cong 10^5 \text{ k}\Omega$

$R_i(\min) = \frac{R_i}{1+T} = \frac{10}{1+10^4} \cong 10^{-3} \text{ k}\Omega$

Or  $R_i(\min) = 1\Omega$

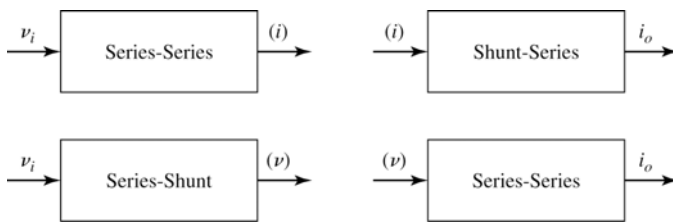
(b)  $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow R_o(\max) \cong 10^4 \text{ k}\Omega$

$R_o(\min) = \frac{R_o}{1+T} = \frac{1}{1+10^4} \cong 10^{-4} \text{ k}\Omega$

Or  $R_o(\min) = 0.1\Omega$

**12.34**

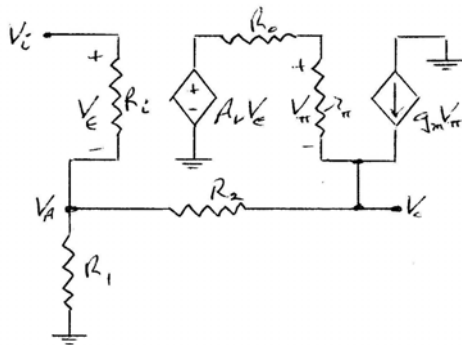
Overall Transconductance Amplifier,  $A_g = \frac{i_o}{v_i}$  Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.



**12.35**

$I_{CQ} = 0.2 \text{ mA}$ ,  $r_\pi = \frac{(140)(0.026)}{0.2} = 18.2 \text{ k}\Omega$ ,  $g_m = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$

(a)



(1)  $\frac{V_\pi}{r_\pi} + g_m V_\pi = \frac{V_o - V_A}{R_2} = V_\pi \left( \frac{1+h_{FE}}{r_\pi} \right)$

(2)  $\frac{V_\pi}{r_\pi} = \frac{A_v V_\epsilon - V_o}{R_o + r_\pi}$

(3)  $\frac{V_A - V_o}{R_2} + \frac{V_A}{R_1} + \frac{V_A - V_i}{R_i} = 0$

(4)  $V_\epsilon = V_i - V_A$

$$\text{From (1), } \frac{V_o - V_A}{R_2} = \left( \frac{1 + h_{FE}}{r_\pi} \right) \left[ r_\pi \left( \frac{A_v V_\epsilon - V_o}{R_o + r_\pi} \right) \right] = (1 + h_{FE}) \left( \frac{A_v V_\epsilon - V_o}{R_o + r_\pi} \right)$$

$$\text{Using (4), } \frac{V_o - V_i + V_\epsilon}{R_2} = (1 + h_{FE}) \left( \frac{A_v V_\epsilon - V_o}{R_o + r_\pi} \right) \quad (\text{Eq. 5})$$

$$\text{From (3), } V_A \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

$$(V_i - V_\epsilon) \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

$$(V_i - V_\epsilon) \left( \frac{1}{10} + \frac{1}{1} + \frac{1}{30} \right) = \frac{V_o}{10} + \frac{V_i}{30}$$

$$(V_i - V_\epsilon)(1.1333) = V_o(0.10) + V_i(0.0333)$$

$$\text{We find, } V_\epsilon = V_i(0.9706) - V_o(0.08824)$$

From Eq. (5) above,

$$\frac{V_o - V_i}{R_2} + \frac{(1 + h_{FE})V_o}{R_o + r_\pi} = \frac{(1 + h_{FE})}{R_o + r_\pi} \cdot A_v V_\epsilon - \frac{V_\epsilon}{R_2}$$

$$\frac{V_o - V_i}{10} + \frac{141}{0.5 + 18.2} \cdot V_o = \frac{141}{0.5 + 18.2} (10^5) V_\epsilon - \frac{V_\epsilon}{10}$$

$$7.640V_o - V_i(0.1) = 7.540 \times 10^5 V_\epsilon = 7.540 \times 10^5 [V_i(0.9706) - V_o(0.08824)]$$

$$6.653705 \times 10^4 V_o = V_i(7.318235 \times 10^5)$$

$$\text{Then } \frac{V_o}{V_i} = 11.0$$

$$\text{(b) From (2), } V_\pi = r_\pi \left( \frac{A_v V_\epsilon - V_o}{R_o + r_\pi} \right)$$

$$\text{From (1), } V_o = V_A + R_2 V_\pi \left( \frac{1 + h_{FE}}{r_\pi} \right)$$

$$V_o = (V_i - V_\epsilon) + R_2 (1 + h_{FE}) \left( \frac{A_v V_\epsilon - V_o}{R_o + r_\pi} \right)$$

$$V_o \left[ 1 + \frac{R_2(1 + h_{FE})}{R_o + r_\pi} \right] = V_i + V_\epsilon \left[ \frac{R_2(1 + h_{FE})A_v}{R_o + r_\pi} - 1 \right] \quad (\text{Eq. 6})$$

From (3),

$$V_A \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

$$(V_i - V_\epsilon) \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

Then, using Eq. (6),

$$V_i \left( \frac{1}{R_2} + \frac{1}{R_1} \right) - V_\epsilon \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{1}{R_2} \left\{ \frac{V_i + V_\epsilon \left[ \frac{R_2(1 + h_{FE})A_v}{R_o + r_\pi} - 1 \right]}{1 + \frac{R_2(1 + h_{FE})}{R_o + r_\pi}} \right\}$$



$$V_i(1.10) - V_\epsilon(1.1333) = \frac{1}{10} \left\{ \frac{V_i + V_\epsilon(7.540106 \times 10^6)}{75.401} \right\}$$

$$V_i(1.098674) = V_\epsilon(1.1333 + 1.0 \times 10^4)$$

$$\text{So } \frac{V_i}{V_\epsilon} = 9.103 \times 10^3$$

$$\text{Then } R_{if} = R_i \left( \frac{V_i}{V_\epsilon} \right) = (30 \times 10^3)(9.103 \times 10^3) \Rightarrow R_{if} = 273 \text{ M}\Omega$$

$$(c) \quad I_x + g_m V_\pi + \frac{V_\pi}{r_\pi} = \frac{V_x - V_A}{R_2}$$

$$\text{From (2), } V_\pi = r_\pi \left( \frac{A_v V_\epsilon - V_x}{R_o + r_\pi} \right)$$

$$\text{Then, } I_x + \frac{(1 + h_{FE})}{R_o + r_\pi} (A_v V_\epsilon - V_x) = \frac{V_x - V_A}{R_2}$$

$$\text{Now, } V_A = -V_\epsilon \text{ and } V_\epsilon = -V_x(0.08824)$$

$$\text{So, } I_x + \frac{141}{0.5 + 18.2} [10^5(-0.08824V_x) - V_x] = \frac{V_x}{10} - \frac{V_x(0.08824)}{10}$$

$$I_x = V_x(0.091176 + 6.654 \times 10^4)$$

$$R_{of} = \frac{V_x}{I_x} = 15 \mu\Omega$$

### 12.36

a. Neglecting base currents

$$I_{C2} = 0.5 \text{ mA}, V_{C2} = 12 - (0.5)(22.6) = 0.7 \text{ V}$$

$$I_{C1} = 0.5 \text{ mA} \Rightarrow v_0 = 0$$

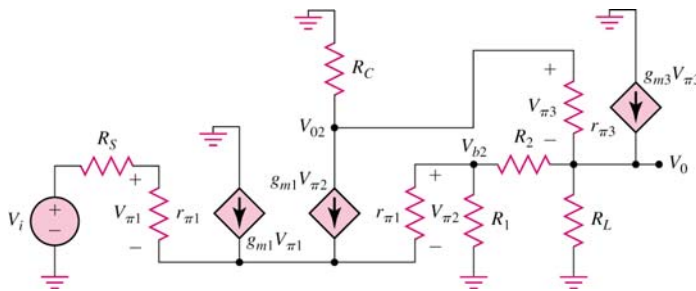
Then  $I_{C3} = 2 \text{ mA}$

$$b. \quad r_{\pi1} = r_{\pi2} = \frac{h_{FE} \cdot V_T}{I_{C1}} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + g_{m1} V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left( \frac{1}{r_{\pi 1}} + g_{m1} \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2} \quad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}} (R_S + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

$$V_i = V_{\pi 1} \left( 1 + \frac{R_S}{r_{\pi 1}} \right) - V_{\pi 2} + V_{b2}$$

or

$$\text{But } V_{\pi 2} = -V_{\pi 1}$$

so

$$V_i = V_{\pi 1} \left( 2 + \frac{R_S}{r_{\pi 1}} \right) + V_{b2} \quad (2)$$

$$\frac{V_{02}}{R_C} + g_{m1} V_{\pi 2} + \frac{V_{02} - V_0}{r_{\pi 3}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_0}{R_L} + \frac{V_0 - V_{b2}}{R_2}$$

$$V_{\pi 3} = V_{02} - V_0$$

so

$$(V_{02} - V_0) \left( \frac{1 + h_{FE}}{r_{\pi 3}} \right) = V_0 \left( \frac{1}{R_L} + \frac{1}{R_2} \right) - \frac{V_{b2}}{R_2} \quad (4)$$

$$\frac{V_{b2} - V_0}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \quad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2} \left( 2 + \frac{1}{5.2} \right) + V_{b2}$$

$$V_i = -V_{\pi 2} (2.192) + V_{b2} \quad (2)$$

$$V_{02} \left( \frac{1}{22.6} + \frac{1}{1.3} \right) + (19.23) V_{\pi 2} - V_0 \left( \frac{1}{1.3} \right) = 0$$

$$V_{02} (0.8135) + (19.23) V_{\pi 2} - (0.7692) V_0 = 0 \quad (3)$$

$$V_{02} \left( \frac{101}{1.3} \right) = V_0 \left( \frac{101}{1.3} + \frac{1}{4} + \frac{1}{50} \right) - V_{b2} \left( \frac{1}{50} \right)$$

$$V_{02} (77.69) = V_0 (77.96) - V_{b2} (0.02) \quad (4)$$

$$V_{b2} \left( \frac{1}{50} + \frac{1}{10} \right) - V_0 \left( \frac{1}{50} \right) + V_{\pi 2} \left( \frac{1}{5.2} \right) = 0$$

$$V_{b2} (0.120) - V_0 (0.020) + V_{\pi 2} (0.1923) = 0 \quad (5)$$

From (2):  $V_{b2} = V_i + V_{\pi 2} (2.192)$ . Substitute in (4) and (5) to obtain:

$$V_{02} (77.69) = V_0 (77.96) - [V_i + V_{\pi 2} (2.192)] (0.02) \quad (4')$$

$$[V_i + V_{\pi 2} (2.192)] (0.120) - V_0 (0.020) + V_{\pi 2} (0.1923) = 0 \quad (5')$$

So we now have the following three equations:

$$V_{02} (0.8135) + (19.23) V_{\pi 2} - (0.7692) V_0 = 0 \quad (3)$$

$$V_{02} (77.69)$$

$$= V_0 (77.96) - V_i (0.02) - V_{\pi 2} (0.04384) \quad (4')$$

$$(0.120) V_i + V_{\pi 2} (0.4553) - V_0 (0.020) = 0 \quad (5')$$

From (3):  $V_{02} = V_0 (0.9455) - V_{\pi 2} (23.64)$ . Substitute for  $V_{02}$  in (4') to obtain:

$$(77.69)[V_o(0.9455) - V_{\pi_2}(23.64)] = V_o(77.96) - V_i(0.02) - V_{\pi_2}(0.04384)$$

or

$$0 = V_o(4.504) - V_i(0.02) + V_{\pi_2}(1836.5)$$

Next, solve (5') for  $V_{\pi_2}$ :

$$(0.120)V_i + V_{\pi_2}(0.4553) - V_o(0.020) = 0$$

$$V_{\pi_2} = V_o(0.04393) - V_i(0.2636)$$

Finally,

$$0 = V_o(4.504) - V_i(0.02) + (1836.5)[V_o(0.04393) - V_i(0.2636)]$$

$$0 = V_o(85.18) - V_i(484.12)$$

So

$$A_{vf} = \frac{V_o}{V_i} = \frac{484.12}{85.18} \Rightarrow \underline{A_{vf} = 5.68}$$

### 12.37

a.  $R_{TH} = R_1 \parallel R_2 = 400 \parallel 75 = 63.2 \text{ k}\Omega$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{75}{75 + 400} \right) (10) = 1.579 \text{ V}$$

$$I_{BQ1} = \frac{1.579 - 0.7}{63.2 + (121)(0.5)} = 0.007106 \text{ mA}$$

$$I_{CQ1} = 0.853 \text{ mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49 \text{ V}$$

$$I_{C2} \approx \frac{2.49 - 0.7}{3.6} = 0.497 \text{ mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54 \text{ V}$$

$$I_{C3} \approx \frac{3.54 - 0.7}{1.4} = 2.03 \text{ mA}$$

Then

$$r_{\pi_1} = \frac{(120)(0.026)}{0.853} = 3.66 \text{ k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81 \text{ mA/V}$$

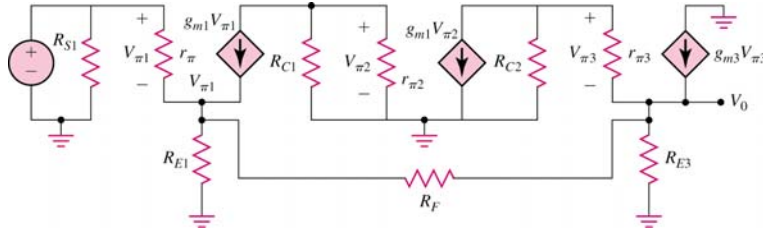
$$r_{\pi_2} = \frac{(120)(0.026)}{0.497} = 6.28 \text{ k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12 \text{ mA/V}$$

$$r_{\pi_3} = \frac{(120)(0.026)}{2.03} = 1.54 \text{ k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08 \text{ mA/V}$$

b.



$$V_i = V_{\pi 1} + V_{\epsilon 1} \Rightarrow V_{\epsilon 1} = V_i - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_{\epsilon 1}}{R_{E1}} + \frac{V_{\epsilon 1} - V_0}{R_F} \quad (2)$$

$$V_{\pi 2} = -(g_{m1} V_{\pi 1})(R_{C1} \parallel r_{\pi 2}) \quad (3)$$

$$g_{m2} V_{\pi 2} + \frac{V_{\pi 3} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{\epsilon 1}}{R_F} \quad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi 1} \left( \frac{1}{3.66} + 32.81 \right) = (V_i - V_{\pi 1}) \left( \frac{1}{0.5} + \frac{1}{10} \right) - \frac{V_0}{10}$$

$$\text{or } V_{\pi 1}(35.18) = V_i(2.10) - V_0(0.10) \quad (2)$$

$$V_{\pi 2} = -(32.81)V_{\pi 1}(88 \parallel 6.28)$$

$$\text{or } V_{\pi 2} = -V_{\pi 1}(120.2) \quad (3)$$

$$(19.12)V_{\pi 2} + \frac{V_{\pi 3}}{13} + \frac{V_0}{13} + \frac{V_{\pi 3}}{1.54} = 0$$

or

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

$$V_{\pi 3} \left( \frac{1}{1.54} + 78.08 \right) = V_0 \left( \frac{1}{1.4} + \frac{1}{10} \right) - \frac{V_i - V_{\pi 1}}{10}$$

or

$$V_{\pi 3}(78.73) = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10) \quad (5)$$

Now substituting  $V_{\pi 2} = -V_{\pi 1}(120.2)$  in (4):

$$(19.12)[-V_{\pi 1}(120.2)] + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

or

$$-V_{\pi 1}(2298.2) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

Then

$$V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$$

Substituting  $V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$  in (5):

$$(78.73)[V_{\pi 1}(3164.3) - V_0(0.1059)] = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10)$$

$$\text{or } V_{\pi 1}(2.49 \times 10^5) - V_0(9.152) = -V_i(0.10)$$

Then

$$V_{\pi 1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

Now substituting  $V_{\pi 1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$

$$-V_i(4.014 \times 10^{-7}) \text{ in (2):}$$

$$(35.18)[V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})]$$

$$= V_i(2.10) - V_0(0.10)$$

or  $V_0(0.1013) = V_i(2.10)$

So  $\frac{V_0}{V_i} = 20.7$

c.  $R_{if} = \frac{V_i}{I_i}$  and  $I_i = I_{RB1} + I_{b1}$

$$I_{RB1} = \frac{V_i}{R_{B1}}$$

$$I_{b1} = \frac{V_{\pi1}}{r_{\pi1}}$$

Now

$$V_{\pi1} = (20.7V_i)(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

$$V_{\pi1} = V_i(7.60 \times 10^{-4})$$

Then

$$R_{if} = \frac{V_i}{\frac{V_i}{63.2} + \frac{V_i(7.60 \times 10^{-4})}{3.66}}$$

$$= \frac{1}{0.01582 + 2.077 \times 10^{-4}}$$

or  $R_{if} = 62.4 \text{ k}\Omega$

d. To determine  $R_{of}$ :

Equation (1) is modified to  $V_{\pi1} + V_{e1} = 0$  ( $V_i = 0$ ) Equation (5) is modified to:

$$V_{\pi3}(78.73) + I_x = V_0(0.8143) + V_{\pi1}(0.10) \quad (5)$$

Now

$$V_{\pi1}(35.18) = -V_0(0.10) \quad (2)$$

$$V_{\pi2} = -V_{\pi1}(120.2) \quad (3)$$

$$V_{\pi2}(19.12) + V_{\pi3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

Now

$$V_{\pi1} = -V_0(0.002843)$$

so

$$V_{\pi2} = -(-V_0)(0.002843)(120.2)$$

$$V_{\pi2} = V_0(0.3417)$$

Then

$$V_0(0.3417)(19.12) + V_{\pi3} + (0.7263) + V_0(0.07692) = 0$$

or  $V_{\pi3} = -V_0(9.101) \quad (4)$

So then

$$-V_0(9.101)(78.73) + I_x$$

$$= V_0(0.8143) + (0.10)(-V_0)(0.002843)$$

or

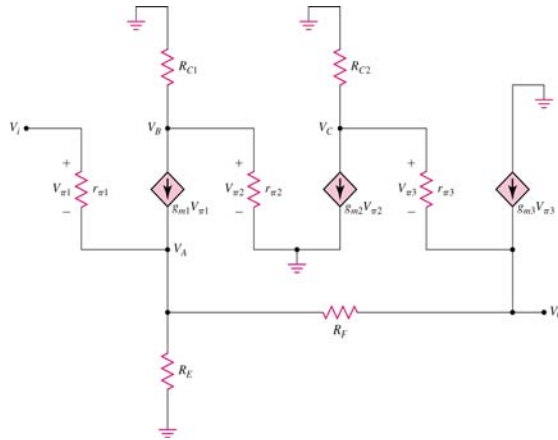
$$I_x = V_0(717.3) \quad (5)$$

or

$$R_{of} = \frac{V_0}{I_x} = 0.00139 \text{ k}\Omega \Rightarrow \underline{R_{of} = 1.39 \text{ }\Omega}$$

12.38

(a)



$$(1) \quad \frac{V_i - V_A}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

$$(2) \quad \frac{V_B}{R_{C1}} + g_{m1} V_{\pi 1} + \frac{V_B}{r_{\pi 2}} = 0$$

$$(3) \quad \frac{V_C}{R_{C2}} + g_{m2} V_{\pi 2} + \frac{V_C - V_O}{r_{\pi 3}} = 0$$

$$(4) \quad g_{m3} V_{\pi 3} + \frac{V_C - V_O}{r_{\pi 3}} = \frac{V_O - V_A}{R_F}$$

$$V_{\pi 1} = V_i - V_A \quad V_{\pi 2} = V_B \quad V_{\pi 3} = V_C - V_O$$

$$(1) \quad (V_i - V_A) \left( \frac{1}{r_{\pi 1}} + g_{m1} \right) = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

$$(2) \quad V_B \left( \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} \right) + g_{m1} (V_i - V_A) = 0$$

$$(3) \quad V_C \left( \frac{1}{R_{C2}} + \frac{1}{r_{\pi 3}} \right) + g_{m2} V_B - \frac{V_O}{r_{\pi 3}} = 0$$

$$(4) \quad (V_C - V_O) \left( g_{m3} + \frac{1}{r_{\pi 3}} \right) = \frac{V_O - V_A}{R_F}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{14.3} = 0.182 \text{ K} \quad g_{m1} = \frac{14.3}{0.026} = 550 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{4.62} = 0.563 \text{ K} \quad g_{m2} = \frac{4.62}{0.026} = 178 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{4.47} = 0.582 \text{ K} \quad g_{m3} = \frac{4.47}{0.026} = 172 \text{ mA/V}$$

$$(1) \quad (V_i - V_A) \left( \frac{1}{0.182} + 550 \right) = \frac{V_A}{0.05} + \frac{V_A - V_O}{1.2}$$

$$(2) \quad V_B \left( \frac{1}{0.3} + \frac{1}{0.563} \right) + (550)(V_i - V_A) = 0$$

$$(3) \quad V_C \left( \frac{1}{0.65} + \frac{1}{0.582} \right) + 178 V_B - \frac{V_O}{0.582} = 0$$

$$(4) \quad (V_C - V_O) \left( 172 + \frac{1}{0.582} \right) = \frac{V_O - V_A}{1.2}$$

$$(1) \quad (V_i - V_A)(555.5) = V_A(20) + (V_A - V_O)(0.8333)$$

$$(2) \quad V_B(5.109) + 550(V_i - V_A) = 0$$

$$(3) \quad V_C(3.257) + 178V_B - V_O(1.718) = 0$$

$$(4) \quad (V_C - V_O)(173.7) = (V_O - V_A)(0.8333)$$

$$(1) \quad V_i(555.5) + V_O(0.8333) = V_A(576.3)$$

$$(2) \quad V_B(5.109) + 550V_i - V_A(550) = 0$$

$$(3) \quad V_C(3.257) + 178V_B - V_O(1.718) = 0$$

$$(4) \quad V_C(173.7) + V_A(0.8333) = V_O(174.5)$$

From (2)  $V_B = V_A(107.7) - V_i(107.7)$

From (4)  $V_C = V_O(1.0046) - V_A(0.004797)$

Substitute into (3)

$$(3.257)[V_O(1.0046) - V_A(0.004797)]$$

$$+ (178)[V_A(107.7) - V_i(107.7)] - V_O(1.718) = 0$$

$$V_O(3.272) - V_A(0.01562) + V_A(19170.6) - V_i(19170.6) - V_O(1.718) = 0$$

$$V_A(19170.6) = V_i(19170.6) - V_O(1.554)$$

$$V_A = V_i(1.00) - V_O(0.00008106)$$

Substitute into (1)

$$V_i(555.5) + V_O(0.8333) = (576.3)[V_i(1.00) - V_O(0.00008106)]$$

$$= V_i(576.3) - V_O(0.0467)$$

$$V_O(0.880) = V_i(20.8)$$

$$\frac{V_O}{V_i} = A_{vf} = 23.6$$

Ideal

$$A_{vf} = \frac{R_F + R_E}{R_E} = \frac{1.2 + 0.05}{0.05} = 25.0$$

$$(b) \quad R_{if} = \frac{V_i}{I_i} \quad \text{and} \quad I_i = \frac{V_{\pi 1}}{r_{\pi 1}} = \frac{V_i - V_A}{r_{\pi 1}}$$

We have

$$V_A = V_i(1.00) - V_O(0.00008106)$$

$$= V_i(1.00) - (23.6)V_i(0.00008106)$$

$$V_A = V_i(0.99809)$$

$$I_i = \frac{V_i(1 - 0.99809)}{0.182} = V_i(0.01051)$$

Then

$$R_{if} = \frac{V_i}{V_i(0.01051)} \Rightarrow R_{if} = 95.1 \text{ K}$$

To find  $R_{of}$ , set  $V_i = 0$

$$I_X + g_{m3}V_{\pi3} + \frac{V_{\pi3}}{r_{\pi3}} = \frac{V_X - V_A}{R_F}$$

$$V_{\pi3} = V_C - V_X$$

$$I_X + (V_C - V_X)\left(g_{m3} + \frac{1}{r_{\pi3}}\right) = \frac{V_X - V_A}{R_F}$$

For  $V_i = 0$ , we have

$$V_C = V_X(1.0046) - V_A(0.004797)$$

$$V_A(576.3) = V_X(0.8333)$$

$$V_A = V_X(0.001446)$$

$$V_C = V_X(1.0046) - V_X(0.001446)(0.004797)$$

$$V_C = V_X(1.0046)$$

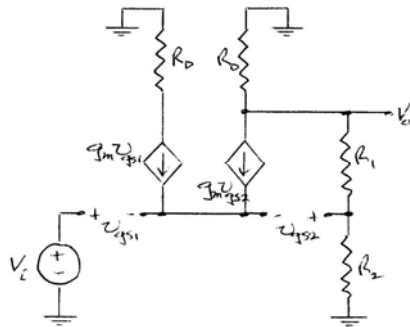
$$I_X + V_X(1.0046 - 1.0)\left(172 + \frac{1}{0.582}\right) = \frac{V_X(1 - 0.004797)}{1.2}$$

$$I_X + V_X(0.7991) = V_X(0.8293)$$

$$I_X = V_X(0.03024)$$

$$R_{of} = \frac{V_X}{I_X} = 33.1 \text{ K}$$

### 12.39



$$g_m(v_{gs1} + v_{gs2}) = 0 \Rightarrow v_{gs2} = -v_{gs1}$$

$$V_i = v_{gs1} - v_{gs2} + \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_o$$

$$V_i = -2v_{gs2} + \frac{1}{2}V_o$$

$$\text{Also } \frac{V_o}{R_1 + R_2} + g_m v_{gs2} + \frac{V_o}{R_D} = 0$$

$$v_{gs2} = -\frac{V_o}{g_m} \left( \frac{1}{R_D} + \frac{1}{R_1 + R_2} \right)$$

$$\text{Then } V_i = \frac{2V_o}{g_m} \left( \frac{1}{R_D} + \frac{1}{R_1 + R_2} \right) + \frac{1}{2}V_o$$

We have  $R_1 + R_2 \gg R_D$



$$V_i \cong V_o \left( \frac{2}{g_m R_D} + \frac{1}{2} \right)$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{1}{\left( \frac{2}{g_m R_D} + \frac{1}{2} \right)} = \frac{g_m R_D}{\left( 2 + \frac{1}{2} g_m R_D \right)}$$

Now  $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1.0 \text{ mA/V}$

$$A_{vf} = \frac{(1)(7)}{\left[ 2 + \frac{1}{2} (1)(7) \right]} = 1.273$$

**12.40**

(a) Neglect base currents

$$\frac{V_{GG} - V_{GS}}{R_L} = I_{D1} + I_{C2}$$

$$I_{D1} = \frac{5 - V_D}{R_{D1}} = \frac{5 - V_D}{5}$$

$$I_{C2} = \frac{5 - (V_D + 0.7)}{R_{E2}} = \frac{4.3 - V_D}{1.6}$$

$$V_D = 5 - 5I_{D1} \Rightarrow I_{C2} = \frac{4.3 - (5 - 5I_{D1})}{1.6} = -0.4375 + 3.125I_{D1}$$

Then  $\frac{2.5 - V_{GS}}{1.2} = I_{D1} + (-0.4375 + 3.125I_{D1})$

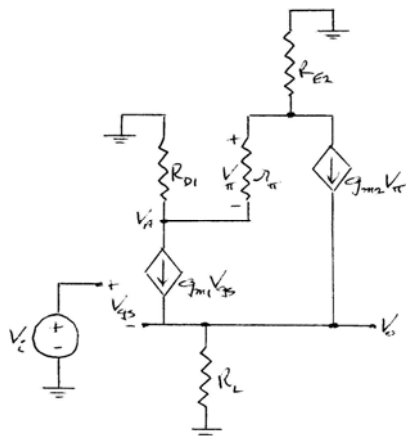
$$3.025 = V_{GS} + 4.95I_{D1} = V_{GS} + 4.95(K_n)(V_{GS} - 0.5)^2$$

We find  $7.425V_{GS}^2 - 6.425V_{GS} - 1.16875 = 0 \Rightarrow V_{GS} = 1.0197 \text{ V}$

Then  $I_{D1} = (1.5)(1.0197 - 0.5)^2 = 0.405 \text{ mA}$

$$I_{C2} = -0.4375 + 3.125(0.405) = 0.828 \text{ mA}$$

(b)



$$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(1.5)(0.405)} = 1.559 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.828}{0.026} = 31.85 \text{ mA/V}$$

$$r_{\pi2} = \frac{\beta V_T}{I_{C2}} = \frac{(120)(0.026)}{0.828} = 3.768 \text{ k}\Omega$$

$$(1) V_o = (g_{m1} V_{gs} + g_{m2} V_{\pi}) R_L$$

$$(2) V_i = V_{gs} + V_o \Rightarrow V_{gs} = V_i - V_o$$

$$(3) \frac{V_A}{R_{D1}} + g_{m1} V_{gs} = \frac{V_{\pi}}{r_{\pi}}$$

$$(4) \frac{V_A + V_{\pi}}{R_{E2}} + \frac{V_{\pi}}{r_{\pi}} + g_{m2} V_{\pi} = 0$$

$$\text{So } \frac{-V_A}{1.6} = V_{\pi} \left( \frac{1}{1.6} + \frac{1}{3.768} + 31.85 \right) = V_{\pi} (32.74)$$

$$V_A = -52.385 V_{\pi}$$

$$(3) \frac{-52.385 V_{\pi}}{5} + 1.559 V_{gs} = \frac{V_{\pi}}{3.768}$$

$$\text{or } 1.559 V_{gs} = V_{\pi} (0.2654 + 10.477)$$

$$\text{so } V_{\pi} = 0.1451 V_{gs} = 0.1451 (V_i - V_o)$$

$$(1) V_o = [1.559 (V_i - V_o) + 31.85 (0.1451 (V_i - V_o))] (1.2)$$

$$\text{Then } A_v = \frac{V_o}{V_i} = 0.8812$$

$$(c) \text{Set } V_i = 0$$

$$I_x + g_{m2} V_{\pi} + g_{m1} V_{gs} = \frac{V_x}{R_L}$$

$$V_{gs} = -V_x$$

$$V_{\pi} = 0.1451 V_{gs} = -0.1451 V_x$$

$$I_x = V_x \left[ \frac{1}{1.2} + (31.85)(0.1451) + (1.559) \right] = V_x (7.014)$$

$$R_o = \frac{V_x}{I_x} = 143 \Omega$$

### 12.41

$$(a) (i) A_{vf} = \frac{g_m R_S}{1 + g_m R_S}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1.5)(1.2)} = 2.683 \text{ mA/V}$$

$$A_{vf} = \frac{(2.683)(1.5)}{1 + (2.683)(1.5)} = 0.801$$

$$(ii) R_{of} = \frac{1}{g_m} \parallel R_S = \frac{1}{2.683} \parallel 1.5 = 0.3727 \parallel 1.5$$

$$R_{of} = 299 \Omega$$

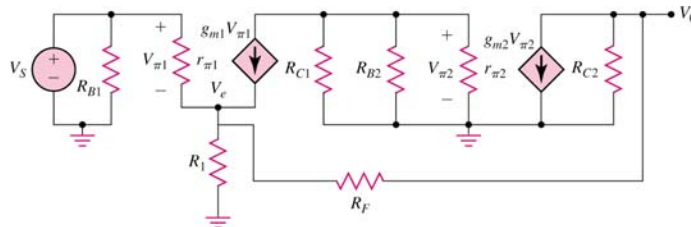
(b) (i)  $g_m = 2\sqrt{(2.25)(1.2)} = 3.286 \text{ mA/V}$   
 $A_{v_f} = \frac{(3.286)(1.5)}{1 + (3.286)(1.5)} = 0.8313 \text{ mA/V}$   
 $\frac{\Delta A_{v_f}}{A_{v_f}} \times 100\% = +3.78\%$

(ii)  $R_{o_f} = \frac{1}{3.286} \parallel 1.5 = 0.3043 \parallel 1.5$   
 $R_{o_f} = 253 \Omega$   
 $\frac{\Delta R_{o_f}}{R_{o_f}} \times 100\% = -15.4\%$

**12.42**

dc analysis:

$R_{TH1} = 150 \parallel 47 = 35.8 \text{ k}\Omega$ ,  
 $V_{TH1} = \left( \frac{47}{47+150} \right) (25) = 5.96 \text{ V}$   
 $R_{TH2} = 33 \parallel 47 = 19.4 \text{ k}\Omega$ ,  
 $V_{TH2} = \left( \frac{33}{33+47} \right) (25) = 10.3 \text{ V}$   
 $I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187 \text{ mA}$   
 $I_{C1} = (50)(0.0187) = 0.935 \text{ mA}$   
 $I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705 \text{ mA}$   
 $I_{C2} = (50)(0.03705) = 1.85 \text{ mA}$   
 $r_{\pi 1} = \frac{(50)(0.026)}{0.935} = 1.39 \text{ k}\Omega$ ;  
 $r_{\pi 2} = \frac{(50)(0.026)}{1.85} = 0.703 \text{ k}\Omega$   
 $g_{m1} = \frac{0.935}{0.026} = 35.96 \text{ mA/V}$   
 $g_{m2} = \frac{1.85}{0.026} = 71.15 \text{ mA/V}$



$V_S = V_{\pi 1} + V_e$  (1)

$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_e}{R_1} + \frac{V_e - V_O}{R_F}$  (2)

$$g_{m1}V_{\pi1} + \frac{V_{\pi2}}{R_{C1}} + \frac{V_{\pi2}}{R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \quad (3)$$

$$g_{m2}V_{\pi2} + \frac{V_0}{R_{C2}} + \frac{V_0 - V_e}{R_F} = 0 \quad (4)$$

Substitute numerical values in (2), (3) and (4):

$$V_e = V_s - V_{\pi1} \quad (1)$$

$$\frac{V_{\pi1}}{1.39} + (35.96)V_{\pi1} = (V_s - V_{\pi1})\left(\frac{1}{0.1} + \frac{1}{4.7}\right) - V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi1}(46.89) = V_s(10.213) - V_0(0.2128) \quad (2)$$

$$(35.96)V_{\pi1} + V_{\pi2}\left(\frac{1}{10} + \frac{1}{19.4} + \frac{1}{0.703}\right) = 0$$

or

$$(35.96)V_{\pi1} + V_{\pi2}(1.574) = 0 \quad (3)$$

$$(71.15)V_{\pi2} + V_0\left(\frac{1}{4.7} + \frac{1}{4.7}\right) - (V_s - V_{\pi1})\left(\frac{1}{4.7}\right) = 0$$

or

$$(71.15)V_{\pi2} + V_0(0.4255) - V_s(0.2128) + V_{\pi1}(0.2128) = 0 \quad (4)$$

From (3):  $V_{\pi2} = -V_{\pi1}(22.85)$

Then substitute in (4):

$$-(71.15)V_{\pi1}(22.85) + V_0(0.4255) - V_s(0.2128) + V_{\pi1}(0.2128) = 0$$

or

$$-V_{\pi1}(1625.6) + V_0(0.4255) - V_s(0.2128) = 0$$

From (2):  $V_{\pi1} = V_s(0.2178) - V_0(0.004538)$

Then

$$-(1625.6)[V_s(0.2178) - V_0(0.004538)] + V_0(0.4255) - V_s(0.2128) = 0$$

or  $-V_s(354.3) + V_0(7.802) = 0$

Finally

$$\Rightarrow \frac{V_0}{V_s} = 45.4$$

### 12.43

For example, use a 2-stage amplifier. Each stage is shown in Fig. 12.29.

### 12.44

$$I_{DQ1} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

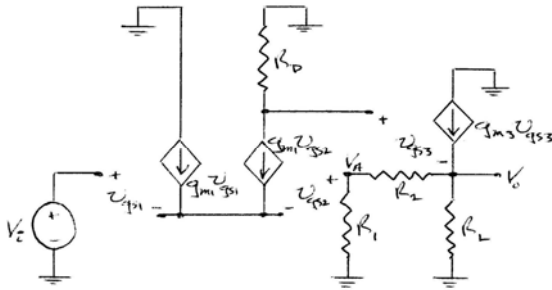
$$\text{Let all } \left(\frac{W}{L}\right) = 20$$

$$0.5 = \left(\frac{0.1}{2}\right)(20)(V_{GS} - 1.5)^2 \Rightarrow V_{GS1,2} = 2.207 \text{ V}$$

$$I_{DQ3} = 2 = \left(\frac{0.1}{2}\right)(20)(V_{GS3} - 1.5)^2 \Rightarrow V_{GS3} = 2.914 \text{ V}$$

Want  $V_{G3} = 2.914 \text{ V}$ , then

$$R_D = \frac{12 - 2.914}{0.5} = 18.2 \text{ k}\Omega$$



$$g_{m1} v_{gs1} + g_{m1} v_{gs2} = 0 \Rightarrow v_{gs2} = -v_{gs1}$$

$$V_i = v_{gs1} - v_{gs2} + V_A = -2v_{gs2} + V_A$$

$$V_A = \left( \frac{R_1}{R_1 + R_2} \right) \cdot V_o$$

$$\frac{V_o}{R_L} + \frac{V_o}{R_1 + R_2} = g_{m3} v_{gs3}$$

$$v_{gs3} = -(g_{m1} v_{gs2} R_D) - V_o$$

$$V_o \left( \frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) = g_{m3} (-g_{m1} v_{gs2} R_D - V_o)$$

$$v_{gs2} = \frac{1}{2} (V_A - V_i) = \frac{1}{2} \left( \frac{R_1}{R_1 + R_2} \right) \cdot V_o - \frac{1}{2} \cdot V_i$$

$$V_o \left( \frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) = -g_{m1} g_{m3} R_D \left[ \frac{1}{2} \left( \frac{R_1}{R_1 + R_2} \right) \cdot V_o - \frac{1}{2} \cdot V_i \right] - g_{m3} V_o$$

$$V_o \left[ \frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2} g_{m1} g_{m3} R_D \left( \frac{R_1}{R_1 + R_2} \right) + g_{m3} \right] = \frac{1}{2} g_{m1} g_{m3} R_D V_i$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{\frac{1}{2} g_{m1} g_{m3} R_D}{\left[ \frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2} g_{m1} g_{m3} R_D \left( \frac{R_1}{R_1 + R_2} \right) + g_{m3} \right]}$$

$$g_{m1} = 2 \sqrt{\left( \frac{0.1}{2} \right) (20)(0.5)} = 1.414 \text{ mA/V}$$

$$g_{m3} = 2 \sqrt{\left( \frac{0.1}{2} \right) (20)(2)} = 2.828 \text{ mA/V}$$

$$\text{Then } 8 = \frac{\frac{1}{2} (1.414)(2.828)(18.2)}{\left[ \frac{1}{10} + \frac{1}{R_1 + R_2} + \frac{1}{2} (1.414)(2.828)(18.2) \left( \frac{15}{R_1 + R_2} \right) + 2.828 \right]}$$

$$8 = \frac{36.389}{\left[ 2.928 + \frac{546.8}{R_1 + R_2} \right]}$$

$$\Rightarrow R_1 + R_2 = 337 \text{ k}\Omega \quad \Rightarrow R_2 = 322 \text{ k}\Omega$$

**12.45**

$$\frac{I_o}{I_s} = \frac{5}{0.06} = 83.33 = 1 + \frac{R_1}{R_2} \Rightarrow \frac{R_1}{R_2} = 82.33$$

For example, let  $R_2 = 3 \text{ k}\Omega$ ,  $R_1 = 247 \text{ k}\Omega$

**12.46**

(a) (1)  $V_A = (I_{D1} + I_{D2})R_{D2}$

(2)  $I_{D1}R_{D1} = V_{SG2}$

(3)  $\frac{(V_G - V_{GS1}) - V_A}{R_F} = I_{D1}$

Now  $V_G - V_{GS1} - (I_{D1} + I_{D2})R_{D2} = I_{D1}R_F$

And  $V_{GS2} = \sqrt{\frac{I_{D2}}{K_p}} - V_{TP} = I_{D1}R_{D1}$

$$\sqrt{I_{D2}}(0.3162) + 1 = I_{D1}(0.525)$$

$$\sqrt{I_{D2}} = I_{D1}(1.660) - 3.162$$

$$I_{D2} = [I_{D1}(1.660) - 3.162]^2$$

Then  $V_G - V_{GS1} - [I_{D1}(1.660) - 3.162]^2 R_{D2} = I_{D1}(R_{D2} + R_F)$

$$I_{D1} = K_n (V_{GS1} - V_{TN})^2$$

$$7.6 - V_{GS1} - [I_{D1}(1.660) - 3.162]^2 (0.25) = I_{D1}(0.75)$$

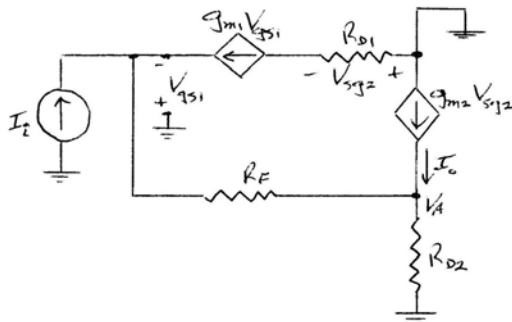
$$I_{D1} = 10(V_{GS1} - 1)^2$$

By trial and error,  $I_{D1} \cong 3.98 \text{ mA}$

Then  $V_{SG2} = I_{D1}R_{D1} = (3.98)(0.525) = 2.0895 \text{ V}$

$$I_{D1} = (10)(2.0895 - 1)^2 = 11.87 \text{ mA}$$

(b)



$$I_o = g_{m2} V_{sg2}$$

$$V_{sg2} = g_{m1} V_{gs1} R_{D1}$$

$$I_o = \frac{V_A}{R_{D2}} + \frac{V_A - (-V_{gs1})}{R_F}$$

$$I_i + g_{m1} V_{gs1} + \frac{V_A + V_{gs1}}{R_F} = 0$$

$$\text{or } V_A = -R_F \left[ I_i + g_{m1} V_{gs1} + \frac{V_{gs1}}{R_F} \right]$$

$$I_o = V_A \left( \frac{1}{R_{D2}} + \frac{1}{R_F} \right) + \frac{V_{gs1}}{R_F}$$

$$I_o = -R_F \left( I_i + g_{m1} V_{gs1} + \frac{V_{gs1}}{R_F} \right) \left( \frac{1}{R_{D2}} + \frac{1}{R_F} \right) + \frac{V_{gs1}}{R_F}$$

$$\text{Now } V_{gs1} = \frac{V_{sg2}}{g_{m1} R_{D1}} = \frac{I_o}{g_{m2}} \cdot \frac{1}{g_{m1} R_{D1}}$$

$$I_o = -R_F I_i \left( \frac{1}{R_{D2}} + \frac{1}{R_F} \right) - R_F V_{gs1} \left[ \left( g_{m1} + \frac{1}{R_F} \right) \left( \frac{1}{R_{D2}} + \frac{1}{R_F} \right) - \frac{1}{R_F^2} \right]$$

$$\text{Then } I_o \left[ 1 + \frac{R_F}{g_{m1} g_{m2} R_{D1}} \left( \frac{g_{m1}}{R_{D2}} + \frac{g_{m1}}{R_F} + \frac{1}{R_F R_{D2}} \right) \right] = -I_i \left( 1 + \frac{R_F}{R_{D2}} \right)$$

$$\text{So } A_i = \frac{I_o}{I_i} = \frac{-g_{m2} R_{D1}}{1 + \frac{1}{g_{m1} (R_F + R_{D2})} + \frac{g_{m2} R_{D1} R_{D2}}{R_F + R_{D2}}}$$

$$(c) \quad g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(10)(3.98)} = 12.62 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(11.87)} = 21.79 \text{ mA/V}$$

$$A_i = \frac{-(21.79)(0.525)}{1 + \frac{1}{(12.62)(0.75)} + \frac{(21.79)(0.525)(0.25)}{0.75}}$$

$$A_i = -2.33$$

### 12.47

$$(a) \quad I_Q = I_{D1} + I_{D2}$$

$$V_{SG2} = I_{D1} R_D$$

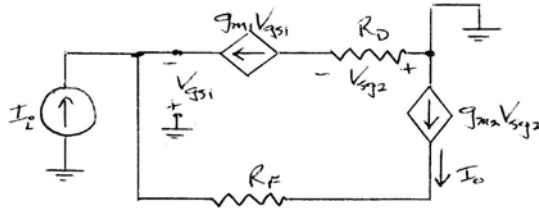
$$I_{D2} = K_p (V_{SG2} + V_{TP})^2 = K_p (I_{D1} R_D + V_{TP})^2$$

$$I_Q = I_{D1} + K_p [I_{D1}^2 R_D^2 - 2I_{D1} R_D + 1]$$

$$\text{We find } 2.756 I_{D1}^2 - 9.5 I_{D1} - 6 = 0$$

$$\Rightarrow I_{D1} = 3.99 \text{ mA and } I_{D2} = 12.01 \text{ mA}$$

(b)



$$I_i + g_{m1} V_{gs1} + I_o = 0$$

$$V_{sg2} = g_{m1} V_{gs1} R_D \Rightarrow V_{gs1} = \frac{V_{sg2}}{g_{m1} R_D}$$

$$I_o = g_{m2} V_{sg2}$$

$$\text{Then } V_{gs1} = \frac{I_o}{g_{m1} g_{m2} R_D}$$

$$I_i + g_{m1} \left( \frac{I_o}{g_{m1} g_{m2} R_D} \right) + I_o = 0$$

$$\text{So that } A_i = \frac{I_o}{I_i} = \frac{-1}{\left( 1 + \frac{1}{g_{m2} R_D} \right)} = \frac{-g_{m2} R_D}{1 + g_{m2} R_D}$$

$$\text{We find } g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(12.01)} = 21.92 \text{ mA/V}$$

$$A_i = \frac{-(21.92)(0.525)}{1 + (21.92)(0.525)} = -0.920$$

### 12.48

(a) Neglect base currents

$$I_Q = I_{C1} + I_{C2}$$

$$V_{EB2} = I_{C1} R_C$$

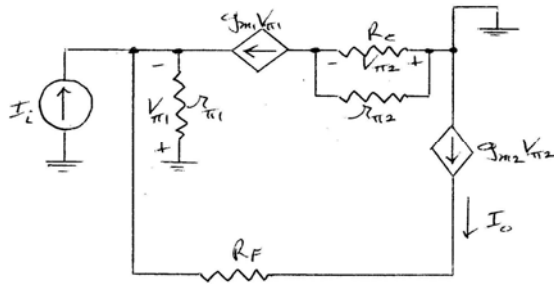
$$I_Q = I_{C1} + I_{S2} \exp\left(\frac{I_{C1} R_C}{V_T}\right)$$

$$16 \times 10^{-3} = I_{C1} + (10^{-15}) \exp\left[\frac{I_{C1}(200)}{0.026}\right]$$

By trial and error,  $I_{C1} = 3.92 \text{ mA}$  and  $I_{C2} = 12.08 \text{ mA}$



(b)



$$I_o = g_{m2} V_{\pi 2}$$

$$V_{\pi 2} = g_{m1} V_{\pi 1} (r_{\pi 2} \parallel R_C)$$

$$\text{Then } I_o = g_{m1} g_{m2} V_{\pi 1} (r_{\pi 2} \parallel R_C)$$

$$I_i + I_o + g_{m1} V_{\pi 1} + \frac{V_{\pi 1}}{r_{\pi 1}} = 0$$

$$I_i + I_o + V_{\pi 1} \left( \frac{1 + \beta_1}{r_{\pi 1}} \right) = 0$$

$$\text{Now } V_{\pi 1} = \frac{I_o}{g_{m1} g_{m2} (r_{\pi 2} \parallel R_C)}$$

$$\text{Then } I_i + I_o \left[ 1 + \frac{1}{g_{m1} g_{m2} (r_{\pi 2} \parallel R_C)} \left( \frac{1 + \beta_1}{r_{\pi 1}} \right) \right] = 0$$

$$\text{So } A_i = \frac{I_o}{I_i} = \frac{-\left( \frac{\beta_1}{1 + \beta_1} \right) (g_{m2}) (r_{\pi 2} \parallel R_C)}{1 + \left( \frac{\beta_1}{1 + \beta_1} \right) (g_{m2}) (r_{\pi 2} \parallel R_C)}$$

$$(c) \quad g_{m2} = \frac{I_{C2}}{V_T} = \frac{12.08}{0.026} = 464.6 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2 V_T}{I_{C2}} = \frac{(180)(0.026)}{12.08} = 0.3874 \text{ k}\Omega$$

$$r_{\pi 2} \parallel R_C = 387.4 \parallel 200 = 131.9 \Omega$$

$$A_i = \frac{-\left( \frac{180}{181} \right) (464.6) (0.1319)}{1 + \left( \frac{180}{181} \right) (464.6) (0.1319)} = -0.984$$

12.49

$$I_{C1} = \left( \frac{h_{FE}}{1+h_{FE}} \right) I_{E1} = \left( \frac{100}{101} \right) (0.2) = 0.198 \text{ mA}$$

$$V_{C1} = 10 - (0.198)(40) = 2.08 \text{ V}$$

$$I_{E2} = \frac{2.08 - 0.7}{1} = 1.38 \text{ mA}$$

$$(a) \quad I_{C2} = \left( \frac{100}{101} \right) (1.38) = 1.37 \text{ mA}$$

For  $Q_1$ :

$$r_{\pi 1} = \frac{(100)(0.026)}{0.198} = 13.1 \text{ k}\Omega$$

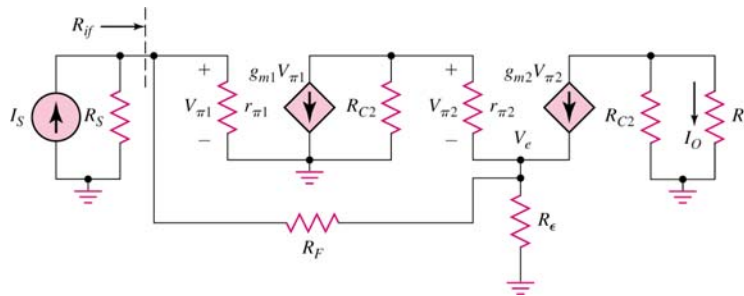
$$g_{m1} = \frac{0.198}{0.026} = 7.62 \text{ mA/V}$$

For  $Q_2$ :

$$r_{\pi 2} = \frac{(100)(0.026)}{1.37} = 1.90 \text{ k}\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7 \text{ mA/V}$$

(b)



$$I_S = \frac{V_{\pi 1}}{R_S} + \frac{V_{\pi 1}}{r_{\pi 1}} + \frac{V_{\pi 1} - V_e}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_{\pi 1}}{R_F} \quad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_S = V_{\pi 1} \left( \frac{1}{10} + \frac{1}{13.1} + \frac{1}{10} \right) - V_e \left( \frac{1}{10} \right)$$

$$I_S = V_{\pi 1} (0.2763) - V_e (0.10) \quad (1)$$

$$(7.62)V_{\pi 1} + V_{\pi 2} \left( \frac{1}{40} + \frac{1}{1.90} \right) + V_e \left( \frac{1}{40} \right) = 0$$

$$(7.62)V_{\pi 1} + V_{\pi 2} (0.5513) + V_e (0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left( \frac{1}{1.90} + 52.7 \right) = V_e \left( \frac{1}{1} + \frac{1}{10} \right) - V_{\pi 1} \left( \frac{1}{10} \right)$$

$$V_{\pi 2} (53.23) = V_e (1.10) - V_{\pi 1} (0.10) \quad (3)$$

From (3),

$$V_e = V_{\pi 2} (48.39) + V_{\pi 1} (0.0909)$$

Substituting into (1),

$$I_S = V_{\pi 1}(0.2763) - (0.10)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)]$$

or

$$I_S = V_{\pi 1}(0.2672) - V_{\pi 2}(4.839) \quad (1')$$

and substituting into (2),

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + (0.025)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)] = 0$$

or

$$(7.622)V_{\pi 1} + V_{\pi 2}(1.761) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}(0.2310) \quad (2')$$

Then substituting (2') into (1'), we obtain

$$I_S = (0.2672)(-V_{\pi 2})(0.2310) - V_{\pi 2}(4.839)$$

or

$$I_S = -V_{\pi 2}(4.901)$$

Now

$$\begin{aligned} I_O &= -g_{m2}V_{\pi 2} \left( \frac{R_{C2}}{R_{C2} + R_L} \right) \\ &= -(52.7) \left( \frac{2}{2+0.5} \right) V_{\pi 2} = -(42.16)V_{\pi 2} \end{aligned}$$

Then

$$I_O = -(42.16) \left( \frac{-I_S}{4.901} \right)$$

or

$$A_{vf} = \frac{I_O}{I_S} = 8.60$$

$$(c) \quad R_i = \frac{V_{\pi 1}}{I_S} \quad \text{and} \quad R_i = R_S \parallel R_{if}$$

We had

$$V_{\pi 1} = -V_{\pi 2}(0.2310) \quad \text{and} \quad I_S = -V_{\pi 2}(4.901)$$

so

$$I_S = - \left( \frac{-V_{\pi 1}}{0.2310} \right) (4.901) = V_{\pi 1}(21.22)$$

Then

$$R_i = \frac{V_{\pi 1}}{I_S} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow R_{if} = 47.4 \, \Omega$$

### 12.50

(a)

$$(1) \quad I_i = \frac{V_{\pi 1}}{R_S \parallel R_{B1} \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F}$$

$$(2) \quad g_{m1}V_{\pi 1} + \frac{V_{C1}}{R_{C1} \parallel R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0$$

$$(3) \quad \frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F}$$

$$(4) \quad I_o = -(g_{m2}V_{\pi2}) \left( \frac{R_{C2}}{R_{C2} + R_L} \right)$$

Now

$$(1)' \quad I_i = \frac{V_{\pi1}}{R_S \parallel R_{B1} \parallel r_{\pi1} \parallel R_F} - \frac{V_{e2}}{R_F}$$

$$V_{e2} = \left( \frac{R_F}{R_S \parallel R_{B1} \parallel r_{\pi1} \parallel R_F} \right) V_{\pi1} - I_i R_F$$

So

Now, from (2)

$$g_{m1}V_{\pi1} + \frac{V_{\pi2} + V_{e2}}{R_{C1} \parallel R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0$$

$$(2)' \quad \left( g_{m1} + \frac{1}{r_{\pi2}} \right) V_{\pi1} + \frac{V_{\pi2}}{R_{C1} \parallel R_{B2} \parallel r_{\pi2}} + \frac{V_{e2}}{R_{C1} \parallel R_{B2}} = 0$$

Also

$$(3)' \quad \left( g_{m2} + \frac{1}{r_{\pi2}} \right) V_{\pi2} + \frac{V_{\pi1}}{R_F} = V_{e2} \left( \frac{1}{R_{E2}} + \frac{1}{R_F} \right)$$

And

$$(4)' \quad V_{\pi2} = - \left( \frac{I_o}{g_{m2}} \right) \left( \frac{R_{C2} + R_L}{R_{C2}} \right)$$

Substitute (1)' into (2)' and (3)'

$$(2)'' \quad \left( g_{m1} + \frac{1}{r_{\pi2}} \right) V_{\pi1} + \frac{V_{\pi2}}{R_{C1} \parallel R_{B2} \parallel r_{\pi2}} + \frac{1}{R_{C1} \parallel R_{B2}} \left[ \frac{R_F}{R_S \parallel R_{B1} \parallel r_{\pi1} \parallel R_F} - V_{\pi1} - I_i R_F \right] = 0$$

$$(3)'' \quad \left( g_{m2} + \frac{1}{r_{\pi2}} \right) V_{\pi2} + \frac{V_{\pi1}}{R_F} = \left( \frac{1}{R_{E2}} + \frac{1}{R_F} \right) \left[ \frac{R_F}{R_S \parallel R_{B1} \parallel r_{\pi1} \parallel R_F} - V_{\pi1} - I_i R_F \right]$$

Solve for  $V_{\pi1}$  from (2)'' and substitute into (3)''. Also use Equation (4)'.

$$(b) \quad R_{B1} = R_1 \parallel R_2 = 20 \parallel 80 = 16 \text{ K}$$

$$V_{TH1} = \left( \frac{20}{100} \right) (10) = 2 \text{ V}$$

$$I_{BQ1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111 \text{ mA}$$

$$I_{CQ1} = 1.11 \text{ mA}$$

$$R_{TH2} = 15 \parallel 85 = 12.75 \text{ K}$$

$$V_{TH2} = \left( \frac{15}{100} \right) (10) = 1.5 \text{ V}$$

$$I_{BQ2} = \frac{1.5 - 0.7}{12.75 + (101)(0.5)} = 0.01265 \text{ mA}$$

$$I_{CQ2} = 1.265 \text{ mA}$$

$$g_{m1} = \frac{1.11}{0.026} = 42.69 \text{ mA/V}$$

$$g_{m2} = \frac{1.265}{0.026} = 48.65 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34 \text{ K}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.265} = 2.06 \text{ K}$$

Now  $R_{C1} \parallel R_{B2} = 2 \parallel 12.75 = 1.729 \text{ K}$

$$R_S \parallel R_{B1} \parallel r_{\pi 1} \parallel R_F \cong R_{B1} \parallel r_{\pi 1} \parallel R_F = 16 \parallel 2.34 \parallel 10 = 1.695 \text{ K}$$

$$R_{C1} \parallel R_{B2} \parallel r_{\pi 2} = 1.729 \parallel 2.06 = 0.940 \text{ K}$$

Now

$$\left( 42.69 + \frac{1}{2.06} \right) V_{\pi 1} + \frac{V_{\pi 2}}{0.940} + \frac{1}{1.729} \left[ \frac{10}{1.695} \cdot V_{\pi 1} - I_i(10) \right]$$

$$(2)'' \quad 46.587V_{\pi 1} + 1.064V_{\pi 2} - 5.784I_i = 0$$

$$\left( \frac{101}{2.06} \right) V_{\pi 2} + \frac{V_{\pi 1}}{10} = \left( \frac{1}{0.5} + \frac{1}{10} \right) \left[ \frac{10}{1.695} \cdot V_{\pi 1} - I_i(10) \right]$$

$$(3)'' \quad 49.03V_{\pi 2} = 12.29V_{\pi 1} - 21I_i$$

From (2)''  $V_{\pi 1} = (0.1242)I_i - (0.02284)V_{\pi 2}$

Then

$$(3)'' \quad 49.03V_{\pi 2} = 12.29[(0.1242)I_i - (0.02284)V_{\pi 2}] - 21I_i$$

$$49.31V_{\pi 2} = -19.47I_i$$

From (4)'  $V_{\pi 2} = -\left( \frac{I_o}{48.65} \right) \left( \frac{4+4}{4} \right) = -(0.0411)I_o$

Then

$$(49.31)[-(0.0411)I_o] = -19.47I_i$$

$$\frac{I_o}{I_i} = A_i = 9.61$$

### 12.51

$$R_{TH} = 13.5 \parallel 38.3 = 9.98 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{13.5}{13.5 + 38.3} \right) (10) = 2.606 \text{ V}$$

a.  $I_{C1} = \frac{(120)(2.606 - 0.7)}{9.98 + (121)(1)} = 1.75 \text{ mA}$

$$V_{C1} = 10 - (1.75)(3) = 4.75 \text{ V}$$

$$I_{C2} \approx \frac{4.75 - 0.7}{8.1} = 0.50 \text{ mA}$$

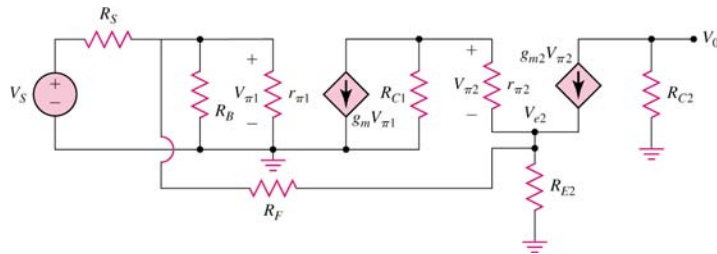
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78 \text{ k}\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24 \text{ k}\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23 \text{ mA/V}$$

b.



$$\frac{V_S - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_{C2} \quad (4)$$

Substitute numerical values in (1), (2), and (3)

$$\frac{V_S}{0.6} = V_{\pi 1} \left[ \frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \parallel 1.78} + \frac{1}{1.2} \right] - \frac{V_{e2}}{1.2}$$

$$V_S (1.67) = V_{\pi 1} (4.011) - V_{e2} (0.8333) \quad (1)$$

$$(67.31) V_{\pi 1} + V_{\pi 2} \left( \frac{1}{3} + \frac{1}{6.24} \right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1} (67.31) + V_{\pi 2} (0.4936) + V_{e2} (0.3333) = 0 \quad (2)$$

$$V_{\pi 1} \left( \frac{1}{6.24} + 19.23 \right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 2}}{1.2}$$

or

$$V_{\pi 2} (19.39) = V_{e2} (0.9568) - V_{\pi 1} (0.8333) \quad (3)$$

From (1)

$$V_{e2} = V_{\pi 1} (4.813) - V_S (2.00)$$

Then

$$V_{\pi 1} (67.31) + V_{\pi 2} (0.4936) + (0.3333) [V_{\pi 1} (4.813) - V_S (2.00)] = 0$$

or

$$V_{\pi 1} (68.91) + V_{\pi 2} (0.4936) - V_S (0.6666) = 0 \quad (2')$$

and

$$V_{\pi 2} (19.39) = (0.9568) [V_{\pi 1} (4.813) - V_S (2.00)] - V_{\pi 1} (0.8333)$$

or

$$V_{\pi 2} (19.39) = V_{\pi 1} (3.772) - V_S (1.914) \quad (3')$$

We find

$$V_{\pi 1} = V_S (0.009673) - V_{\pi 2} (0.007163)$$

Then

$$V_{\pi 2} (19.39) = (3.772) [V_S (0.009673) - V_{\pi 2} (0.007163)] - V_S (1.914)$$

$$V_{\pi 2} (19.42) = V_S (-1.878) \quad \text{or} \quad V_{\pi 2} = -V_S (0.09670)$$

so that

$$V_0 = -(19.23)(4)(-V_S)(0.09670)$$

Then

$$\frac{V_0}{V_S} = 7.44$$

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**12.52**

Using the circuit from Problem 12.51, we have  $R_{if} = \frac{V_{\pi 1}}{I_S}$ .

$$I_S = \frac{V_S - V_{\pi 1}}{R_S}$$

Where

From Problem 12.51

$$\begin{aligned} V_{\pi 1} &= V_S(0.009673) - V_{\pi 2}(0.007163) \\ &= V_S(0.009673) - (0.007163)(-V_S)(0.09670) \\ &= V_S(0.01037) \end{aligned}$$

So

$$R_{if} = \frac{V_S(0.01037) \cdot (0.6)}{V_S - V_S(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$\underline{R_{if} = 6.29 \Omega}$$

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**12.53**

$$R_{TH} = 1.4 \parallel 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{1.4}{1.4 + 17.9} \right) (10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

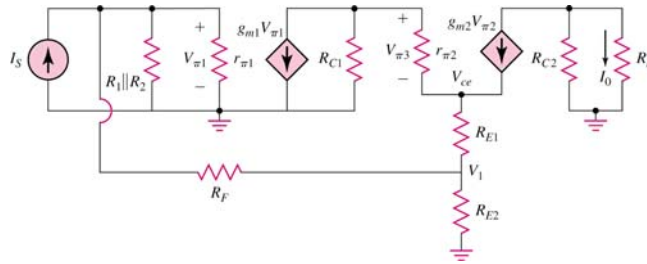
$$I_{C2} = \left( \frac{50}{51} \right) (3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$

$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$

$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_S = \frac{V_{\pi 1}}{R_1 \parallel R_2 \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2} - V_1}{R_{E1}} \quad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E1}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \quad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_S = \frac{V_{\pi 1}}{17.9 \parallel 1.4 \parallel 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_S = V_{\pi 1}(1.722) - V_1(0.20) \quad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{e2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{e2}(4) - V_1(4) \quad (3)$$

$$\frac{V_{e2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{e2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \quad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \quad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)[V_1(1.55) - V_{\pi 1}(0.05)] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_S = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$



or

$$I_S = V_{\pi_1}(35.75) + V_{\pi_2}(2.342)$$

and substituting in (3'):

$$V_{\pi_2}(125.5) = (2.20)[-V_{\pi_1}(170.16) - V_{\pi_2}(11.71)] - V_{\pi_1}(0.20)$$

$$V_{\pi_2}(151.3) = -V_{\pi_1}(374.55)$$

or  $V_{\pi_1} = -V_{\pi_2}(0.4040)$  so that

Then

$$I_S = (35.75)[-V_{\pi_2}(0.4040)] + V_{\pi_2}(2.342)$$

$$I_S = -V_{\pi_2}(12.10)$$

$$\begin{aligned} I_0 &= -(g_{m2}V_{\pi_2})\left(\frac{R_{C2}}{R_{C2} + R_L}\right) \\ &= -(123)\left(\frac{2.2}{2.2 + 2}\right)V_{\pi_2} = -(64.43)V_{\pi_2} \end{aligned}$$

or  $V_{\pi_2} = -(0.01552)I_0$

Then

$$\frac{I_0}{I_S} = \frac{1}{(0.01552)(12.10)} \Rightarrow \frac{I_0}{I_S} = 5.33$$

### 12.54

For example, use the circuit shown in Figure P12.49

### 12.55

$$(a) \quad \frac{V_i - V_L}{R_1} = \frac{V_L - V_O}{R_F}$$

$$\frac{V_O}{R_F} = V_L \left( \frac{1}{R_1} + \frac{1}{R_F} \right) - \frac{V_i}{R_1}$$

$$\text{So (1) } V_O = V_L \left( 1 + \frac{R_F}{R_1} \right) - \left( \frac{R_F}{R_1} \right) V_i$$

$$\frac{V_O - V_L}{R_3} = \frac{V_L}{R_L} + \frac{V_L}{R_2}$$

$$\text{So (2) } V_O = V_L \left( 1 + \frac{R_3}{R_L} + \frac{R_3}{R_2} \right)$$

Then, from (1) = (2)

$$V_L \left( 1 + \frac{R_F}{R_1} \right) - \left( \frac{R_F}{R_1} \right) V_i = V_L \left( 1 + \frac{R_3}{R_L} + \frac{R_3}{R_2} \right)$$

$$V_L \left( 1 + \frac{R_F}{R_1} - 1 - \frac{R_3}{R_L} - \frac{R_3}{R_2} \right) = \left( \frac{R_F}{R_1} \right) V_i$$

Now  $V_L = I_O R_L$

$$I_O R_L \left( \frac{R_F}{R_1} - \frac{R_3}{R_L} - \frac{R_3}{R_2} \right) = \left( \frac{R_F}{R_1} \right) V_i$$

$$A_{gf} = \frac{I_o}{V_i} = \frac{\frac{R_F}{R_1}}{\left(\frac{R_L R_F}{R_1} - R_3 - \frac{R_L R_3}{R_2}\right)}$$

(b) Set  $\frac{R_F}{R_1} = \frac{R_3}{R_2}$

Then  $A_{gf} = \frac{\cancel{R_F}/R_1}{-R_3} = \frac{-R_F}{R_1 R_3}$

Or  $A_{gf} = \frac{I_o}{V_i} = \frac{-1}{R_2}$

(c) For  $\frac{I_o}{V_i} = -0.5 \text{ mA/V} = \frac{-1}{R_2} \Rightarrow R_2 = 2 \text{ k}\Omega$

For  $\frac{R_F}{R_1} = \frac{R_3}{R_2}$ , set  $R_3 = 2 \text{ k}\Omega$  and  $R_1 = R_F = 10 \text{ k}\Omega$

**12.56**

(a)  $V_{D1} = 3 - (1)(1.6) = 1.40 \text{ V}$ ,  $V_{SG3} = (1)(1.6) = 1.6 \text{ V}$   
 $I_{DQ3} = K_p (V_{SG3} + V_{TP})^2 = (10)(1.6 - 0.5)^2 = 12.1 \text{ mA}$   
 $V_G = (12.1)(0.248) - 3 \cong 0$

(b)  $g_{m1} V_{gs1} + g_{m1} V_{gs2} = 0 \Rightarrow V_{gs2} = -V_{gs1}$   
 $V_{sg3} = g_{m1} V_{gs1} R_D$   
 $I_o = g_{m3} V_{sg3} = g_{m1} g_{m3} R_D V_{gs1}$   
 $V_i = V_{gs1} - V_{gs2} + I_o R_L = 2V_{gs1} + I_o R_L$   
 $V_{gs1} = \frac{V_i - I_o R_L}{2}$

$$I_o = g_{m1} g_{m3} R_D \left( \frac{V_i - I_o R_L}{2} \right)$$

$$I_o \left( 1 + \frac{g_{m1} g_{m3} R_D R_L}{2} \right) = \frac{g_{m1} g_{m3} R_D}{2} \cdot V_i$$

$$A_{gf} = \frac{I_o}{V_i} = \frac{g_{m1} g_{m3} R_D}{2 + g_{m1} g_{m3} R_D R_L}$$

(c)  $g_{m1} = 2\sqrt{K_n I_{DQ1}} = 2\sqrt{(2)(1)} = 2.828 \text{ mA/V}$

$$g_{m3} = 2\sqrt{K_p I_{DQ3}} = 2\sqrt{(10)(12.1)} = 22 \text{ mA/V}$$

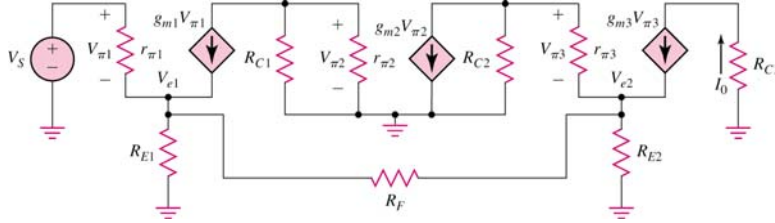
$$A_{gf} = \frac{(2.828)(22)(1.6)}{2 + (2.828)(22)(1.6)(0.248)} = 3.73 \text{ mA/V}$$

12.57

$$r_{\pi 1} = 6.24 \text{ k}\Omega, r_{\pi 2} = 3.12 \text{ k}\Omega, r_{\pi 3} = 1.56 \text{ k}\Omega$$

$$g_{m1} = 19.23 \text{ mA/V}, g_{m2} = 38.46 \text{ mA/V},$$

$$g_{m3} = 76.92 \text{ mA/V}$$



$$V_S = V_{\pi 1} + V_{e1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_{e3}}{R_F} \quad (2)$$

$$V_{\pi 2} = -g_{m1} V_{\pi 1} (R_{C1} \parallel r_{\pi 2}) \quad (3)$$

$$g_{m2} V_{\pi 2} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_{e3}}{R_{E2}} + \frac{V_{e3} - V_{e1}}{R_F} \quad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi 1}}{6.24} + (19.23)V_{\pi 1} = V_{e1} \left( \frac{1}{0.1} + \frac{1}{0.8} \right) - V_{e3} \left( \frac{1}{0.8} \right)$$

or

$$V_{\pi 1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \quad (2)$$

$$V_{\pi 2} = -(19.23)V_{\pi 1}(5 \parallel 3.12) = -(36.94)V_{\pi 1} \quad (3)$$

$$(38.46)V_{\pi 2} + V_{\pi 3} \left( \frac{1}{2} + \frac{1}{1.56} \right) + V_{e3} \left( \frac{1}{2} \right) = 0$$

or

$$V_{\pi 2}(38.46) + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4)$$

$$V_{\pi 3} \left( \frac{1}{1.56} + 76.92 \right) = V_{e3} \left( \frac{1}{0.1} + \frac{1}{0.8} \right) - V_{e3} \left( \frac{1}{0.8} \right)$$

or

$$V_{\pi 3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \quad (5)$$

From (1)  $V_{\pi 1} = V_S - V_{e1}$

Then

$$(V_S - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

$$\text{or } V_S(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \quad (2')$$

$$V_{\pi 2} = -V_S(36.94) + V_{e1}(36.94) \quad (3')$$

$$(38.46)[-V_S(36.94) + V_{e1}(36.94)] + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4')$$

From (5):  $V_{e3} = V_{\pi 3}(6.894) + V_{e1}(0.1111)$

Then

$$V_S(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi 3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_S(19.39) = V_{e1}(30.50) - V_{\pi 3}(8.6175) \quad (2'')$$

and

$$-V_S(1420.7) + V_{e1}(1420.7) + V_{\pi3}(1.141) + (0.5)[V_{\pi3}(6.894) + V_{e1}(0.1111)] = 0$$

or

$$-V_S(1420.7) + V_{e1}(1420.76) + V_{\pi3}(4.588) = 0 \quad (4'')$$

From (2''):

$$V_{e1} = V_S(0.6357) + V_{\pi3}(0.2825)$$

Then substituting in (4''):

$$-V_S(1420.7) + (1420.76)[V_S(0.6357) + V_{\pi3}(0.2825)] + V_{\pi3}(4.588) = 0$$

$$-V_S(517.5) + V_{\pi3}(405.95) = 0$$

Now

$$I_0 = g_{m3}V_{\pi3} = 76.92V_{\pi3} \quad \text{OR} \quad V_{\pi3} = I_0(0.0130)$$

Then  $-V_S(517.5) + I_0(0.0130)(405.95) = 0$

or

$$\frac{I_0}{V_S} = 98.06 \text{ mA/V}$$

### 12.58

Computer Analysis

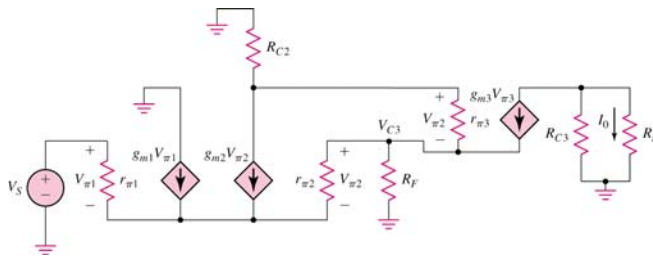
### 12.59

$$r_{\pi1} = r_{\pi2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + g_{m2}V_{\pi2} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \quad (1)$$

Since  $r_{\pi1} = r_{\pi2}$  and  $g_{m1} = g_{m2}$ , then  $V_{\pi1} = -V_{\pi2}$

$$V_S = V_{\pi1} - V_{\pi2} + V_{e3} = -2V_{\pi2} + V_{e3} \quad (2)$$

$$g_{m2}V_{\pi2} + \frac{V_{\pi3}}{r_{\pi3}} + \frac{V_{\pi3} + V_{e3}}{R_{C2}} = 0 \quad (3)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_{e3}}{R_F} + \frac{V_{\pi2}}{r_{\pi2}} \quad (4)$$

$$I_0 = -\left(\frac{R_{C3}}{R_{C3} + R_L}\right)(g_{m3}V_{\pi3}) \quad (5)$$

From (2):  $V_{e3} = V_s + 2V_{\pi2}$

$$(19.23) \quad V_{\pi2} + \frac{V_{\pi3}}{1.3} + \frac{V_{\pi3}}{18.6} + \frac{1}{18.6}(V_s + 2V_{\pi2}) = 0$$

or

$$(19.23)V_{\pi2} + (0.8230)V_{\pi3} + (0.05376)V_s = 0 \quad (3')$$

$$V_{\pi3}\left(\frac{1}{1.3} + 76.92\right) = \left(\frac{1}{10}\right)(V_s + 2V_{\pi2}) + \frac{V_{\pi2}}{5.2}$$

or

$$(77.69)V_{\pi3} = (0.3923)V_{\pi2} + (0.1)V_s \quad (4')$$

$$I_0 = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi3} = -(51.28)V_{\pi3} \quad (5')$$

From (3'):

$$V_{\pi2} = -(0.04255)V_{\pi3} - (0.002780)V_s$$

Then

$$(77.69)V_{\pi3} = (0.3923)[-(0.04255)V_{\pi3} - (0.002780)V_s] + (0.1)V_s$$

$$(77.71)V_{\pi3} = (0.0989)V_s$$

or

$$V_{\pi3} = (0.001273)V_s$$

so that

$$I_0 = -(51.28)(0.001273)V_s$$

or

$$\frac{I_0}{V_s} = -(0.0653) \text{ mA/V}$$

### 12.60

(a)  $A_{gf} = \frac{I_o}{V_i} = \frac{1}{R_E} = 3 \text{ mA/V} \Rightarrow R_E = 333.3 \Omega$

(b)  $A_{gf} = \frac{I_o}{V_i} = \frac{h_{FE}A_g}{1 + (h_{FE})A_gR_E} = \frac{(80)(1000)}{1 + (80)(1000)(0.33333)} = 2.9998875 \text{ mA/V}$

$$I_o = A_{gf}(5) = 14.9994377$$

$$\text{Error} = \frac{14.9994377 - 15.0}{15.0} \times 100\% = -0.00375\%$$

### 12.61

(a)  $5 = (1 + h_{FE})I_{BQ}R_E + V_{EB}(on) + I_{BQ}R_B$

$$I_{BQ} = \frac{5 - 0.7}{100 + (81)(0.5)} = 0.030605 \text{ mA}$$

$$I_{CQ} = 2.448 \text{ mA, and } I_{EQ} = 2.479 \text{ mA}$$

$$V_{ECQ} = 5 - (2.479)(0.5) - (2.448)(1) = 1.31 \text{ V}$$

$$(b) \frac{V_o}{R_C} + \frac{V_o}{r_o} + h_{FE} I_b + \frac{V_o - V_\pi}{R_F} = 0$$

$$V_o \left( \frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) + h_{FE} I_b - \frac{V_\pi}{R_F} = 0$$

$$V_\pi = I_b r_\pi$$

$$I_s = \frac{V_\pi}{R_B} + \frac{V_\pi}{r_\pi} + \frac{V_\pi - V_o}{R_F}$$

$$I_s = I_b \left( 1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right) - \frac{V_o}{R_F}$$

$$I_b = \frac{I_s + \frac{V_o}{R_F}}{\left( 1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right)}$$

$$V_o \left( \frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) + \left( h_{FE} - \frac{r_\pi}{R_F} \right) \left[ \frac{I_s + \frac{V_o}{R_F}}{1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F}} \right] = 0$$

$$V_o \left( \frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) \left( 1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right) + \left( h_{FE} - \frac{r_\pi}{R_F} \right) \cdot I_s + \left( h_{FE} - \frac{r_\pi}{R_F} \right) \cdot \frac{V_o}{R_F} = 0$$

$$\text{Now } r_o = \frac{V_A}{I_{CQ}} = \frac{100}{2.448} = 40.85 \text{ k}\Omega$$

$$r_\pi = \frac{h_{FE} V_T}{I_{CQ}} = \frac{(80)(0.026)}{2.448} = 0.8497 \text{ k}\Omega$$

$$\text{Then } V_o \left( \frac{1}{1} + \frac{1}{40.85} + \frac{1}{10} \right) \left( 1 + \frac{0.8497}{100} + \frac{0.8497}{10} \right) + \left( 80 - \frac{0.8497}{10} \right) \cdot I_s + \left( 80 - \frac{0.8497}{10} \right) \cdot \frac{V_o}{10} = 0$$

$$V_o (9.22114) + I_s (79.915) = 0$$

$$A_{z_f} = \frac{V_o}{I_s} = -8.666 \text{ V/mA}$$

$$(c) R_{i_f} = \frac{V_\pi}{I_s} = \frac{I_b r_\pi}{I_s} = r_\pi \left( \frac{1 + \frac{V_o}{I_s} \cdot \frac{1}{R_F}}{1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F}} \right) = (0.8497) \left( \frac{1 - \frac{8.666}{10}}{1 + \frac{0.8497}{100} + \frac{0.8497}{10}} \right)$$

$$R_{i_f} = 0.1037 \text{ k}\Omega$$

$$(d) I_x = \frac{V_x}{R_C} + \frac{V_x}{r_o} + h_{FE} I_b + \frac{V_x}{R_F + R_B \parallel r_\pi}$$

$$I_b = \frac{V_\pi}{r_\pi}, \text{ and } V_\pi = \left( \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_F} \right) \cdot V_x$$

$$\text{Then } \frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1}{R_C} + \frac{1}{r_o} + \left( \frac{h_{FE}}{r_\pi} \right) \left( \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_F} \right) + \frac{1}{R_B \parallel r_\pi + R_F}$$

$$R_B \parallel r_\pi = 100 \parallel 0.8497 = 0.8425 \text{ k}\Omega$$

$$\frac{1}{R_{of}} = \frac{1}{1} + \frac{1}{40.85} + \left( \frac{80}{0.8497} \right) \left( \frac{0.8425}{0.8425 + 10} \right) + \frac{1}{0.8425 + 10}$$

Then  $R_{of} = 0.1186 \text{ k}\Omega$

### 12.62

(a) (i)  $V^+ = I_{DQ} R_D + V_{GS} = K_n R_D (V_{GS} - V_{TN})^2 + V_{GS}$   
 $3 = (0.5)(5)(V_{GS}^2 - 0.8V_{GS} + 0.16) + V_{GS}$

We find  $2.5V_{GS}^2 - V_{GS} - 2.6 = 0 \Rightarrow V_{GS} = 1.239 \text{ V}$

$$I_{DQ} = (0.5)(1.239 - 0.4)^2 = 0.3522 \text{ mA}$$

(ii)  $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.3522)} = 0.8392 \text{ mA/V}$

(b) (1)  $\frac{V_o}{R_D} + g_m V_{gs} + \frac{V_o - V_{gs}}{R_F} = 0$

(2)  $I_s = \frac{V_{gs} - V_o}{R_F} \Rightarrow V_{gs} = I_s R_F + V_o$

Then  $V_o \left( \frac{1}{R_D} + \frac{1}{R_F} \right) + \left( g_m - \frac{1}{R_F} \right) (I_s R_F + V_o) = 0$

$$V_o \left( \frac{1}{R_D} + g_m \right) + I_s (g_m R_F - 1) = 0$$

$$A_{zf} = \frac{V_o}{I_s} = \frac{1 - g_m R_F}{\frac{1}{R_D} + g_m} = \frac{1 - (0.8392)(25)}{\frac{1}{5} + 0.8392}$$

$$A_{zf} = -19.23 \text{ V/mA}$$

(c)  $I_x = \frac{V_x}{R_D} + g_m V_{gs}$  and  $V_{gs} = V_x$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1}{R_D} + g_m = \frac{1}{5} + 0.8392$$

$$R_{of} = 0.962 \text{ k}\Omega$$

### 12.63

(a)  $A_{zf} = \frac{1 - g_m R_F}{\frac{1}{R_D} + g_m} = \frac{\frac{1}{g_m} - R_F}{\frac{1}{g_m R_D} + 1}$

As  $g_m \rightarrow \infty$ ,  $A_{zf} = -R_F$

$$(b) \quad -0.95R_F = \frac{1 - g_m R_F}{\frac{1}{R_D} + g_m}$$

$$-23.75 = \frac{1 - g_m(25)}{0.2 + g_m}$$

So that  $g_m = 4.6 \text{ mA/V}$

**12.64**

dc analysis

$$R_{TH} = 24 \parallel 150 = 20.7 \text{ k}\Omega$$

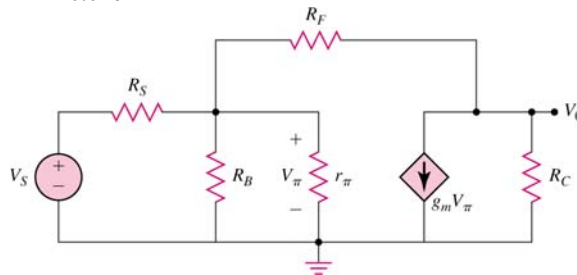
$$V_{TH} = \left( \frac{24}{24 + 150} \right) (12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{1.655 - 0.7}{20.7 + (151)(1)} = 0.00556 \text{ mA}$$

so  $I_{CQ} = 0.834 \text{ mA}$

$$r_\pi = \frac{(150)(0.026)}{0.834} = 4.68 \text{ k}\Omega$$

$$g_m = \frac{0.834}{0.026} = 32.08 \text{ mA/V}$$



$$\frac{V_S - V_\pi}{R_S} = \frac{V_\pi}{R_B \parallel r_\pi} + \frac{V_\pi - V_0}{R_F} \quad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \quad (2)$$

From (1):

$$\frac{V_S}{5} = V_\pi \left[ \frac{1}{5} + \frac{1}{20.7 \parallel 4.68} + \frac{1}{R_F} \right] - \frac{V_0}{R_F}$$

or

$$V_S(0.20) = V_\pi \left( 0.4620 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

From (2):

$$\left( 32.08 - \frac{1}{R_F} \right) V_\pi + V_0 \left( \frac{1}{6} + \frac{1}{R_F} \right) = 0$$

so

$$V_\pi = \frac{-V_0 \left( 0.1667 + \frac{1}{R_F} \right)}{\left( 32.08 - \frac{1}{R_F} \right)} \quad (2)$$



Then

$$V_S(0.20) = \left(0.4620 + \frac{1}{R_F}\right) \left[ \frac{-V_0 \left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)} \right] - \frac{V_0}{R_F}$$

Neglect the  $R_F$  in the denominator term. Now

$$\frac{V_0}{V_S} = -5 \Rightarrow V_S = -\frac{V_0}{5} = -V_0(0.20)$$

$$-V_0(0.20)(0.20)R_F = (0.4620R_F + 1) \left[ \frac{-V_0(0.1667R_F + 1)}{32.08R_F} \right] - V_0$$

$$-1.283R_F^2 = -(0.4620R_F + 1)(0.1667R_F + 1) - 32.08R_F$$

$$1.206R_F^2 - 32.71R_F - 1 = 0$$

$$R_F = \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)}$$

so that

$$\underline{R_F = 27.2 \text{ k}\Omega}$$

### 12.65

dc analysis

$$R_{TH} = 4 \parallel 15 = 3.16 \text{ k}\Omega = R_B$$

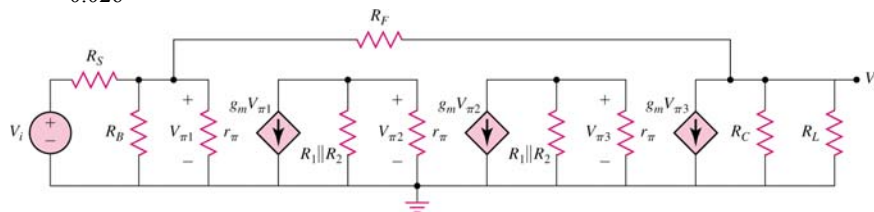
$$V_{TH} = \left(\frac{4}{4+15}\right)12 = 2.526 \text{ V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$$I_{CQ} = 0.452 \text{ mA}$$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4 \text{ k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4 \text{ mA/V}$$



$$\frac{V_i - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \parallel r_\pi} + \frac{V_{\pi 1} - V_0}{R_F} \quad (1)$$

$$g_m V_{\pi 1} + \frac{V_{\pi 2}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (2)$$

$$g_m V_{\pi 2} + \frac{V_{\pi 3}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (3)$$

$$g_m V_{\pi 3} + \frac{V_0}{R_C} + \frac{V_0}{R_L} + \frac{V_0 - V_{\pi 1}}{R_F} = 0 \quad (4)$$

Now

$$R_C \parallel R_B \parallel r_\pi = 8 \parallel 3.16 \parallel 10.4 = 1.86 \text{ k}\Omega$$

$$R_B \parallel r_\pi = 3.16 \parallel 10.4 = 2.42 \text{ k}\Omega$$

Now substituting in (2):

$$(17.4)V_{\pi 1} + \frac{V_{\pi 2}}{1.86} = 0 \quad \text{or} \quad V_{\pi 2} = -(32.36)V_{\pi 1}$$

and substituting in (3):

$$(17.4)V_{\pi 2} + \frac{V_{\pi 3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi 1}] + \frac{V_{\pi 3}}{1.86} = 0$$

$$\text{or } V_{\pi 3} = (1047.3)V_{\pi 1}$$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1} \left( \frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1} \left( 0.513 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0 \left( \frac{1}{8} + \frac{1}{4} + \frac{1}{R_F} \right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1} \left( 1.822 \times 10^4 - \frac{1}{R_F} \right) + V_0 \left( 0.375 + \frac{1}{R_F} \right) = 0$$

$$V_{\pi 1} = \frac{-V_0 \left( 0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}}$$

so that

$$V_i(0.10) = \left( 0.513 + \frac{1}{R_F} \right) \left[ \frac{-V_0 \left( 0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{V_0}{R_F}$$

$$\text{We have } \frac{V_0}{V_i} = -80 \quad \text{or} \quad V_i = -\frac{V_0}{80}$$

$$-\frac{(0.10)}{80} = \left( 0.513 + \frac{1}{R_F} \right) \left[ \frac{-\left( 0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{1}{R_F}$$

Neglect that  $1/R_F$  term in the denominator.

$$-(0.00125R_F) = -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822 \times 10^4 R_F} - 1$$

$$22.775R_F^2 = (0.513R_F + 1)(0.375R_F + 1) + 1.822 \times 10^4 R_F$$

We find

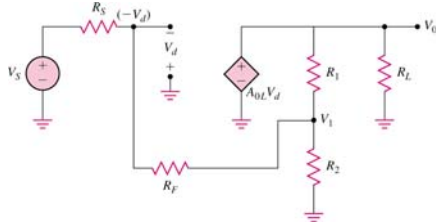
$$22.58R_F^2 - 1.822 \times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822 \times 10^4 \pm \sqrt{(1.822 \times 10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$R_F = 0.807 \text{ M}\Omega$$

12.66



$$\frac{V_S - (-V_d)}{R_S} = \frac{-V_d - V_1}{R_F}$$

a.  
or

$$V_d \left( \frac{1}{R_S} + \frac{1}{R_F} \right) + \frac{V_S}{R_S} + \frac{V_1}{R_F} = 0 \quad (1)$$

$$\frac{V_0 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - (-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F} \right) + \frac{V_d}{R_F} \quad (2)$$

and  $V_0 = A_{0L}V_d$  or  $V_d = \frac{V_0}{A_{0L}}$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4} \cdot \left( \frac{1}{5} + \frac{1}{10} \right) + \frac{V_S}{5} + \frac{V_1}{10} = 0$$

or

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + V_1(0.10) = 0 \quad (1)$$

$$\frac{V_0}{50} = V_1 \left( \frac{1}{50} + \frac{1}{10} + \frac{1}{10} \right) + \frac{V_0}{10^4} \cdot \left( \frac{1}{10} \right)$$

or

$$V_0(0.02 - 10^{-5}) = V_1(0.22) \quad (2)$$

$$V_1 = V_0 \left( \frac{0.02 - 10^{-5}}{0.22} \right)$$

Then  
and

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + (0.10) \left[ V_0 \left( \frac{0.02 - 10^{-5}}{0.22} \right) \right] = 0$$

$$V_0 \left[ 0.3 \times 10^{-4} - 0.4545 \times 10^{-5} + 0.00909 \right] + V_S(0.20) = 0$$

$$\frac{V_0}{V_S} = \frac{-0.20}{9.115 \times 10^{-3}} \Rightarrow \frac{V_0}{V_S} = -21.94$$

Then

$$R_{if} = \frac{-V_d}{\frac{V_S - (-V_d)}{R_S}} = \frac{-V_d \cdot R_S}{V_S + V_d}$$

b.

$$V_d = \frac{V_0}{A_{0L}} = -\frac{21.94V_S}{10^4}$$

Now

$$R_{if} = \frac{(21.94 \times 10^{-4})(5)}{1 - 21.94 \times 10^{-4}}$$

Then

$$R_{if} = 1.099 \times 10^{-2} \text{ k}\Omega \Rightarrow R_{if} = 10.99 \Omega$$

or

c. Because of the  $A_{0L}V_d$  source,

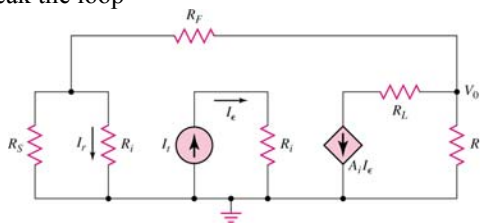
$$R_{0f} = 0$$

**12.67**

For example, use the circuit shown in Figure 12.41

**12.68**

Break the loop



$$I_r = I_e$$

$$A_i I_r + \frac{V_0}{R_i} + \frac{V_0}{R_F + R_S \parallel R_i} = 0$$

Now

$$I_r = \left( \frac{R_S}{R_S + R_i} \right) \cdot \frac{V_0}{R_F + R_S \parallel R_i}$$

$$V_0 = I_r \left( \frac{R_S + R_i}{R_S} \right) \cdot (R_F + R_S \parallel R_i)$$

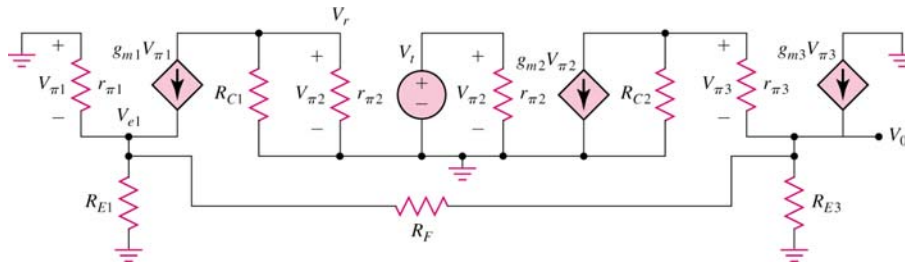
or

Then

$$A_i I_r + \left( \frac{1}{R_i} + \frac{1}{R_F + R_S \parallel R_i} \right) \times \left[ I_r \left( \frac{R_S + R_i}{R_S} \right) (R_F + R_S \parallel R_i) \right] = 0$$

$$T = -\frac{I_r}{I_t} \Rightarrow T = \frac{A_i}{\left[ \frac{1}{R_i} + \frac{1}{R_F + R_S \parallel R_i} \right] \left( \frac{R_S + R_i}{R_S} \right) (R_F + R_S \parallel R_i)}$$

12.69



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_{\varepsilon 1}}{R_{E1}} + \frac{V_{\varepsilon 1} - V_0}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_r}{R_{C1} \parallel r_{\pi 2}} = 0 \Rightarrow V_r = -(g_{m1} V_{\pi 1})(R_{C1} \parallel r_{\pi 2}) \quad (2)$$

$V_{\pi 2} = V_r$  so that

$$g_{m2} V_r + \frac{V_{\pi 3} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{\varepsilon 1}}{R_F} \quad (4)$$

From (4):

$$V_0 \left( \frac{1}{R_{E3}} + \frac{1}{R_F} \right) = V_{\pi 3} \left( \frac{1}{r_{\pi 3}} + g_{m3} \right) + \frac{V_{\varepsilon 1}}{R_F}$$

But  $V_{\varepsilon 1} = -V_{\pi 1}$

$$V_0 = \frac{V_{\pi 3} \left( \frac{1}{r_{\pi 3}} + g_{m3} \right) - \frac{V_{\pi 1}}{R_F}}{\left( \frac{1}{R_{E3}} + \frac{1}{R_F} \right)}$$

so

Then

$$V_{\pi 1} \left[ \left( \frac{1}{r_{\pi 1}} + g_{m1} \right) - \left( \frac{1}{R_{E1}} + \frac{1}{R_F} \right) \right] = \frac{-V_{\pi 3} \left( \frac{1}{r_{\pi 3}} + g_{m3} \right) + \frac{V_{\pi 1}}{R_F}}{R_F \cdot \left( \frac{1}{R_{E3}} + \frac{1}{R_F} \right)} \quad (1')$$

and

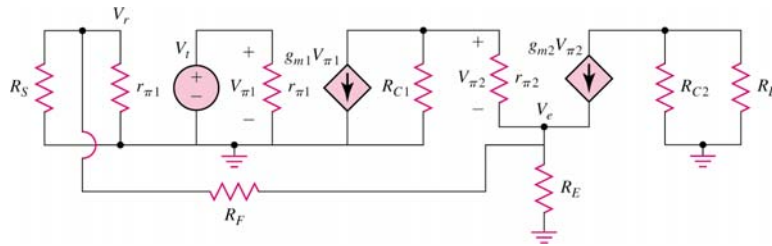
$$g_{m2} V_r + V_{\pi 3} \left( \frac{1}{R_{C2}} + \frac{1}{r_{\pi 3}} \right) + \frac{V_{\pi 3} \left( \frac{1}{r_{\pi 3}} + g_{m3} \right) - \frac{V_{\pi 1}}{R_F}}{R_{C2} \cdot \left( \frac{1}{R_{E3}} + \frac{1}{R_F} \right)} = 0 \quad (3')$$

From (3'), solve for  $V_{\pi 3}$  and substitute into (1'). Then from (1'), solve for  $V_{\pi 1}$  and substitute into (2).

$$T = -\frac{V_r}{V_i}$$

Then

12.70



$$\frac{V_r}{R_S} + \frac{V_r}{r_{\pi 1}} + \frac{V_r - V_e}{R_F} = 0 \quad (1)$$

$$g_{m1} V_r + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_r}{R_F} \quad (3)$$

Assume  $r_{\pi 1} = 15.8 \text{ k}\Omega$ ,  $g_{m1} = 7.62 \text{ mA/V}$ ,  $r_{\pi 2} = 2.28 \text{ k}\Omega$ ,  $g_{m2} = 52.7 \text{ mA/V}$

$$V_r \left( \frac{1}{10} + \frac{1}{15.8} + \frac{1}{10} \right) - \frac{V_e}{10} = 0$$

or

$$V_r (0.2633) = V_e (0.10) \quad (1)$$

$$(7.62)V_r + V_{\pi 2} \left( \frac{1}{40} + \frac{1}{2.28} \right) + \frac{V_e}{40} = 0$$

or

$$V_r (7.62) + V_{\pi 2} (0.4636) + V_e (0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left( \frac{1}{2.28} + 52.7 \right) = V_e \left( \frac{1}{10} + \frac{1}{10} \right) - \frac{V_r}{10}$$

or

$$V_{\pi 2} (53.14) = V_e (1.10) - V_r (0.10)$$

Then

$$V_{\pi 2} = V_e (0.0207) - V_r (0.001882) \quad (3)$$

Substituting in (2):

$$V_r (7.62) + (0.4636)[V_e (0.0207) - V_r (0.001882)] + V_e (0.025) = 0$$

or

$$V_r (7.62) + V_e (0.03460) - V_r (0.0008725) = 0$$

From (1)  $V_e = V_r (2.633)$

Then

$$V_r (7.62) + V_r (2.633)(0.03460) - V_r (0.0008725) = 0$$

$$V_r (7.62) + V_r (0.09023) = 0$$

$$\frac{V_r}{V_i} = -84.45$$

or

Now

$$T = -\frac{V_r}{V_i} \Rightarrow T = 84.45$$

12.71

$$R_{TH} = R_1 \parallel R_2 = 24 \parallel 150 = 20.69 \text{ k}\Omega$$

$$V_{TH} = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left( \frac{24}{24 + 150} \right) (12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + h_{FE})R_E} = \frac{1.655 - 0.7}{20.69 + (51)(1)} = 0.0133 \text{ mA}$$

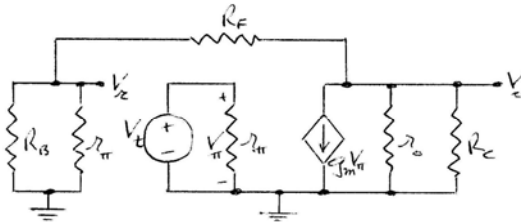
$$I_{CQ} = 0.666 \text{ mA}$$

$$g_m = \frac{0.666}{0.026} = 25.62 \text{ mA/V}$$

$$r_\pi = \frac{(50)(0.026)}{0.666} = 1.951 \text{ k}\Omega$$

$$r_o = \frac{100}{0.666} = 150 \text{ k}\Omega$$

From Problem 12.64, let  $R_F = 27.2 \text{ k}\Omega$



We see that  $V_\pi = V_i$ . Let  $R_B = R_1 \parallel R_2 \parallel R_S$

$$(1) \quad g_m V_t + \frac{V_o}{r_o \parallel R_C} + \frac{V_o}{R_F + r_\pi \parallel R_B} = 0$$

$$(2) \quad V_r = \left( \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_F} \right) \cdot V_o$$

Now  $R_B = 24 \parallel 150 \parallel 5 = 20.69 \parallel 5 = 4.027 \text{ k}\Omega$ ;  $r_\pi \parallel R_B = 1.951 \parallel 4.027 = 1.314 \text{ k}\Omega$

$$r_o \parallel R_C = 150 \parallel 6 = 5.769 \text{ k}\Omega$$

$$(1) \quad 25.62V_t + V_o \left[ \frac{1}{5.769} + \frac{1}{27.2 + 1.314} \right] = 0 \Rightarrow V_o = -122.93V_t$$

$$(2) \quad V_r = \left( \frac{1.314}{1.314 + 27.2} \right) \cdot V_o = (0.04608)(-122.93)V_t$$

$$T = -\frac{V_r}{V_t} = 5.66$$

12.72

$$(a) \quad \phi = -\tan^{-1} \left( \frac{f}{10^3} \right) - 2 \tan^{-1} \left( \frac{f}{5 \times 10^4} \right)$$

At  $f_{180} = 5.1 \times 10^4 \text{ Hz}$ ,  $\phi \cong -180^\circ$

$$(b) |T| = 0.25 = \frac{\beta(5 \times 10^4)}{\sqrt{1 + \left(\frac{5.1 \times 10^4}{10^3}\right)^2} \left[1 + \left(\frac{5.1 \times 10^4}{5 \times 10^4}\right)^2\right]}$$

$$0.25 = \frac{\beta(5 \times 10^4)}{(51.0098)(2.0404)}$$

$$\beta = 5.2 \times 10^{-4}$$

**12.73**

$$(a) \phi = -2 \tan^{-1}\left(\frac{f}{5 \times 10^3}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^5}\right)$$

At  $f_{180} = 7.1 \times 10^4$  Hz,  $\phi \cong -180^\circ$

$$(b) |T| = 1 = \frac{\beta(10^4)}{\left[1 + \left(\frac{7.1 \times 10^4}{5 \times 10^3}\right)^2\right] \sqrt{1 + \left(\frac{7.1 \times 10^4}{5 \times 10^5}\right)^2}}$$

$$1 = \frac{\beta(10^4)}{(202.64)(1.01)} \Rightarrow \beta = 0.0205$$

$$(c) A_{v_f}(0) = \frac{10^4}{1 + (0.0205)(10^4)} = 48.54$$

(d) Smaller

**12.74**

$$\phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right)$$

At  $f = 8.1 \times 10^4$  Hz,  $\phi = -180.28^\circ$

Determine  $|T(f)|$  at this frequency.

$$|T| = \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^4}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{5 \times 10^4}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^5}\right)^2}}$$

$$= \frac{\beta(10^3)}{(8.161)(1.904)(1.287)}$$

a. For  $\beta = 0.005$

$$|T(f)| = 0.250 < 1 \Rightarrow \text{Stable}$$

b. For  $\beta = 0.05$

$$|T(f)| = 2.50 > 1 \Rightarrow \text{Unstable}$$

**12.75**

(b) Phase margin =  $80^\circ \Rightarrow \phi = -100^\circ$

$$\phi = -100 = -2 \tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^4}\right)$$

By trial and error,  $f = 1.16 \times 10^3$  Hz

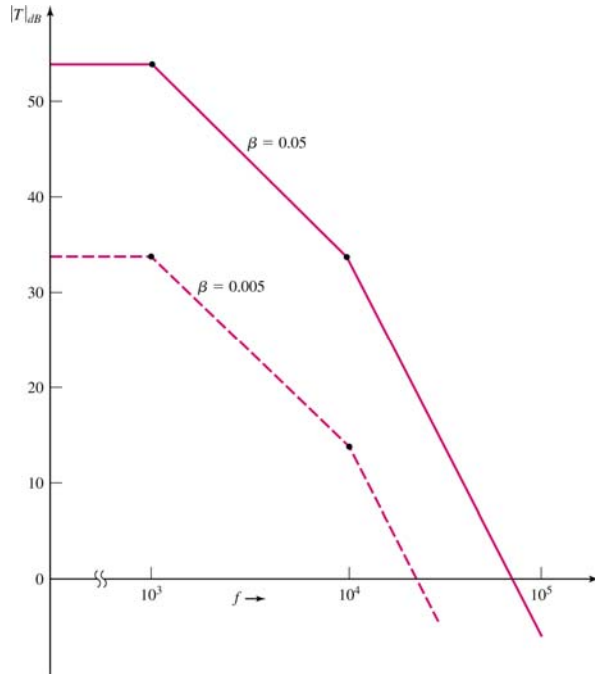
Then



$$|T|=1 = \frac{\beta(5 \times 10^3)}{\left( \sqrt{1 + \left( \frac{1.16 \times 10^3}{10^3} \right)^2} \right)^2 \cdot \sqrt{1 + \left( \frac{1.16 \times 10^3}{5 \times 10^4} \right)^2}}$$

$$= \frac{\beta(5 \times 10^3)}{(2.35)(1.00)} \Rightarrow \beta = 4.7 \times 10^{-4}$$

12.76



- c. For  $\beta = 0.005$ ,  
 $|T(f)| = 1 (0 \text{ dB})$  at  $f \approx 2.10 \times 10^4 \text{ Hz}$

Then

$$\phi = -\tan^{-1}\left(\frac{2.10 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{2.10 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{2.10 \times 10^4}{10^5}\right)$$

$$= -87.27 - 64.54 - 11.86$$

or

$$\phi = -163.7$$

System is stable.

Phase margin =  $16.3^\circ$

For  $\beta = 0.05$ ,

$$|T(f)| = 1 (0 \text{ dB}) \text{ at } f \approx 6.44 \times 10^4 \text{ Hz}$$

Then

$$\phi = -\tan^{-1}\left(\frac{6.44 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{6.44 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{6.44 \times 10^4}{10^5}\right)$$

$$= -89.11 - 81.17 - 32.78$$

or

$$\phi = -203.1^\circ \Rightarrow \text{System is unstable.}$$

**12.77**

$$(a) \quad \phi = -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

$$\text{At } f_{180} = 10^6 \text{ Hz, } \phi \cong -180^\circ$$

$$(b) \quad \phi = -135^\circ = -\tan^{-1}\left(\frac{f_{135}}{10^3}\right) - \tan^{-1}\left(\frac{f_{135}}{10^5}\right) - \tan^{-1}\left(\frac{f_{135}}{10^7}\right)$$

$$f_{135} \cong 10^5 \text{ Hz}$$

$$|T(f_{135})| = 1 = \frac{\beta(5 \times 10^4)}{\sqrt{1 + \left(\frac{10^5}{10^3}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^5}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^7}\right)^2}}$$

$$1 = \frac{\beta(5 \times 10^4)}{(100)(1.414)(1)} \Rightarrow \beta = 2.83 \times 10^{-3}$$

$$(c) \quad A_{v_f}(0) = \frac{5 \times 10^4}{1 + (2.83 \times 10^{-3})(5 \times 10^4)} = 351$$

**12.78**

$$(a) \quad 100 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 9.99 \times 10^{-3}$$

$$|T| = 1 = \beta A_v = \frac{(9.99 \times 10^{-3})(10^5)}{\sqrt{1 + \left(\frac{f}{10^3}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$1 = \frac{999}{\sqrt{1 + \left(\frac{f}{10^3}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$f = 3.08 \times 10^5 \text{ Hz}$$

Phase

$$\phi = -\tan^{-1}\frac{f}{10^3} - \tan^{-1}\frac{f}{10^5}$$

$$= -\tan^{-1}\frac{3.08 \times 10^5}{10^3} - \tan^{-1}\frac{3.08 \times 10^5}{10^5}$$

$$= -89.81 - 72.01$$

$$\phi = -161.8$$

Stable

$$(b) \quad \text{Phase Margin} = 180 - 161.8 = 18.2^\circ$$

**12.79**

$$(a) \quad \phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$$

At  $f_{180} = 3.33 \times 10^5$  Hz,  $\phi \cong -180^\circ$

$$(b) \quad (i) \quad |T(f_{180})| = \frac{(0.019)(10^3)}{\sqrt{1+(33.3)^2} \sqrt{1+(3.33)^2} \sqrt{1+(0.333)^2}}$$

$$|T(f_{180})| = \frac{(0.019)(10^3)}{(33.315)(3.477)(1.054)} = 0.156$$

$$(ii) \quad |T| = 1 = \frac{(0.019)(10^3)}{\sqrt{1+\left(\frac{f}{10^4}\right)^2} \sqrt{1+\left(\frac{f}{10^5}\right)^2} \sqrt{1+\left(\frac{f}{10^6}\right)^2}} \Rightarrow f = 1.2 \times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{1.2 \times 10^5}{10^4}\right) - \tan^{-1}\left(\frac{1.2 \times 10^5}{10^5}\right) - \tan^{-1}\left(\frac{1.2 \times 10^5}{10^6}\right)$$

$$= -85.236^\circ - 50.194^\circ - 6.843^\circ$$

$$\phi = -142.3^\circ$$

$$(c) \quad A_f(0) = \frac{10^3}{1+(0.019)(10^3)} = 50$$

### 12.80

$$\phi = -180 = -\tan^{-1} \frac{f_{180}}{5 \times 10^3} - 2 \tan^{-1} \frac{f_{180}}{10^5}$$

$$(a) \quad f_{180} = 1.05 \times 10^5 \text{ Hz}$$

$$|T| = \frac{(0.0045)(2 \times 10^3)}{\sqrt{1+\left(\frac{1.05 \times 10^5}{5 \times 10^3}\right)^2} \left[1+\left(\frac{1.05 \times 10^5}{10^5}\right)^2\right]}$$

$$= \frac{9}{(21.02)(2.1025)}$$

$$(b) \quad |T| = f_{180}^0 = 0.204$$

System is stable

$$|T| = 1 = \frac{9}{\sqrt{1+\left(\frac{f}{5 \times 10^3}\right)^2} \left[1+\left(\frac{f}{10^5}\right)^2\right]}$$

$$f = 3.88 \times 10^4 \text{ Hz}$$

$$\phi = -\tan^{-1} \frac{3.88 \times 10^4}{5 \times 10^3} - 2 \tan^{-1} \frac{3.88 \times 10^4}{10^5}$$

$$= -82.66 - 42.41$$

$$\phi = -125.1^\circ$$

$$|T| = \frac{(0.15)(2 \times 10^3)}{\sqrt{1 + \left(\frac{1.05 \times 10^5}{5 \times 10^3}\right)^2} \left[ 1 + \left(\frac{1.05 \times 10^5}{10^5}\right)^2 \right]}$$

$$= \frac{300}{(21.02)(2.1025)}$$

(c)  $|T| = 6.79$

System is unstable

$$|T| = 1 = \frac{300}{\sqrt{1 + \left(\frac{f}{5 \times 10^3}\right)^2} \left[ 1 + \left(\frac{f}{10^5}\right)^2 \right]}$$

$$f = 2.33 \times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1} \frac{2.33 \times 10^5}{5 \times 10^3} - 2 \tan^{-1} \frac{2.33 \times 10^5}{10^5}$$

$$= -88.77 - 133.54$$

$$\phi = -222.3^\circ$$

### 12.81

Phase Margin =  $45^\circ \Rightarrow \phi = -135^\circ$   
 $\phi = -135^\circ$

$$= -\tan^{-1} \left( \frac{f}{10^3} \right) - \tan^{-1} \left( \frac{f}{10^4} \right) - \tan^{-1} \left( \frac{f}{10^5} \right) - \tan^{-1} \left( \frac{f}{10^6} \right)$$

At  $f = 10^4 \text{ Hz}$ ,  $\phi = -135.6^\circ$

$$|T| = 1$$

$$= \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^3}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^4}\right)^2}} \times$$

$$\times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^5}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^6}\right)^2}}$$

$$1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\beta = 0.01428$$

### 12.82

$$\phi = -120^\circ = -\tan^{-1} \left( \frac{f_{120}}{f_{PD}} \right) - \tan^{-1} \left( \frac{f_{120}}{4 \times 10^5} \right) - \tan^{-1} \left( \frac{f_{120}}{4 \times 10^6} \right) - \tan^{-1} \left( \frac{f_{120}}{4 \times 10^7} \right)$$

$$f_{120} \cong 2.31 \times 10^5 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1 + \left(\frac{2.31 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^5}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^6}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^7}\right)^2}}$$

$$1 = \frac{4000}{\sqrt{1 + \left(\frac{2.31 \times 10^5}{f_{PD}}\right)^2} (1.1548)(1.00167)(1.0)}$$

$$\frac{2.31 \times 10^5}{f_{PD}} = \frac{4000}{(1.1548)(1.00167)(1.09)}$$

$$f_{PD} = 66.8 \text{ Hz}$$

**12.83**

(a)  $f_{180} \Rightarrow -180^\circ = -2 \tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^6}\right)$

$$f_{180} = 1.42 \times 10^5 \text{ Hz}$$

$$|T(f_{180})| = \frac{10^3}{\left[1 + (14.2)^2\right] \sqrt{1 + (0.142)^2}} = \frac{10^3}{(202.64)(1.01)} = 4.89$$

$|T(f_{180})| > 1 \Rightarrow \text{Unstable}$

(b)  $\phi = -135^\circ = -\tan^{-1}\left(\frac{f_{135}}{f_{PD}}\right) - 2 \tan^{-1}\left(\frac{f_{135}}{10^4}\right) - \tan^{-1}\left(\frac{f_{135}}{10^6}\right)$

$$f_{135} \cong 0.414 \times 10^4 \text{ Hz}$$

$$|T(f_{135})| = 1 = \frac{10^3}{\sqrt{1 + \left(\frac{0.414 \times 10^4}{f_{PD}}\right)^2} \left[1 + \left(\frac{0.414 \times 10^4}{10^4}\right)^2\right] \sqrt{1 + \left(\frac{0.414 \times 10^4}{10^6}\right)^2}}$$

$$\frac{0.414 \times 10^4}{f_{PD}} = \frac{10^3}{(1.171)(1)}$$

$$f_{PD} = 4.85 \text{ Hz}$$

**12.84**

(a)  $\phi = -180 = -\tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{5 \times 10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^5}\right)$

$$f_{180} \cong 8.06 \times 10^4 \text{ Hz}$$

$$|T| = \frac{500}{\sqrt{1 + \left(\frac{8.06 \times 10^4}{10^4}\right)^2} \sqrt{1 + \left(\frac{8.06 \times 10^4}{5 \times 10^4}\right)^2} \sqrt{1 + \left(\frac{8.06 \times 10^4}{10^5}\right)^2}}$$

$$= \frac{500}{(8.122)(1.897)(1.284)}$$

(b)  $|T| = 25.3$

(c)  $T = \frac{500}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{10^4}\right) \left(1 + j \frac{f}{5 \times 10^4}\right) \left(1 + j \frac{f}{10^5}\right)}$

Phase Margin =  $60^\circ \Rightarrow \phi = -120^\circ$

$$-120 = -\tan^{-1} \frac{f}{f_{PD}} - \tan^{-1} \frac{f}{10^4} - \tan^{-1} \frac{f}{5 \times 10^4} - \tan^{-1} \frac{f}{10^5}$$

$$\tan^{-1} \frac{f}{f_{PD}} \cong 90^\circ$$

Assume

Then  $f \cong 4.2 \times 10^3$  Hz

$$|T| = 1 = \frac{500}{\sqrt{1 + \left(\frac{4.2 \times 10^3}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{4.2 \times 10^3}{10^4}\right)^2} \sqrt{1 + \left(\frac{4.2 \times 10^3}{5 \times 10^4}\right)^2} \sqrt{1 + \left(\frac{4.2 \times 10^3}{10^5}\right)^2}}$$

$$1 = \frac{500}{\sqrt{1 + \left(\frac{4.2 \times 10^3}{f_{PD}}\right)^2} (1.085)(1.004)(1.0)}$$

$$\frac{4.2 \times 10^3}{f_{PD}} \cong \frac{500}{(1.0846)(1.0035)(1.0)}$$

$$f_{PD} = 9.14 \text{ Hz}$$

### 12.85

(a)  $50 = \frac{10^4}{1 + (10^4)\beta} \Rightarrow \beta = 0.0199$

$$T = \frac{(0.0199)(10^4)}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{10^5}\right)}$$

Phase margin =  $45^\circ \Rightarrow \phi = -135^\circ$

$$-135 = -\tan^{-1} \frac{f}{f_{PD}} - \tan^{-1} \frac{f}{10^5}$$

$f = 10^5$  Hz

$$|T| = 1 = \frac{(0.0199)(10^4)}{\sqrt{1 + \left(\frac{10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^5}\right)^2}}$$

$$\frac{10^5}{f_{PD}} = \frac{(0.0199)(10^4)}{1.414}$$

$$f_{PD} = 711 \text{ Hz}$$

$$(b) \quad 20 = \frac{10^4}{1 + (10^4)\beta} \Rightarrow \beta = 0.0499$$

$$T = \frac{(0.0499)(10^4)}{\left(1 + j\frac{f}{711}\right)\left(1 + j\frac{f}{10^5}\right)}$$

$$|T| = 1 = \frac{(0.0499)(10^4)}{\sqrt{1 + \left(\frac{f}{711}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$f = 1.76 \times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{1.76 \times 10^5}{711}\right) - \tan^{-1}\left(\frac{1.76 \times 10^5}{10^5}\right)$$

$$= -89.77 - 60.40$$

$$\phi = -150.2$$

$$\text{Phase Margin} = 180 - 150.2 = 29.8^\circ$$

### 12.86

$$(a) \quad A_o = 100 \text{ dB} \Rightarrow A_o = 10^5$$

$$20 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 0.04999$$

$$T = \frac{(0.04999)(10^5)}{\left(1 + j\frac{f}{f_{PD}}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)}$$

$$\text{Phase Margin} = 45^\circ \Rightarrow \phi = -135^\circ$$

$$-135 = -\tan^{-1}\frac{f}{f_{PD}} - \tan^{-1}\frac{f}{10^6} - \tan^{-1}\frac{f}{10^7}$$

$$f \approx 10^6 \text{ Hz}$$

$$|T| = 1 = \frac{(0.04999)(10^5)}{\sqrt{1 + \left(\frac{10^6}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{10^6}{10^6}\right)^2} \sqrt{1 + \left(\frac{10^6}{10^7}\right)^2}}$$

$$1 = \frac{(0.04999)(10^5)}{\sqrt{1 + \left(\frac{10^6}{f_{PD}}\right)^2} (1.414)(1.005)}$$

$$\frac{10^6}{f_{PD}} = \frac{(0.04999)(10^5)}{(1.414)(1.005)}$$

$$f_{PD} = 2.84 \text{ Hz}$$

$$(b) \quad 5 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 0.19999$$

$$|T| = 1 = \frac{(0.19999)(10^5)}{\sqrt{1 + \left(\frac{f}{284}\right)^2} \sqrt{1 + \left(\frac{f}{10^6}\right)^2} \sqrt{1 + \left(\frac{f}{10^7}\right)^2}}$$

$$f = 2.25 \times 10^6 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{2.25 \times 10^6}{284}\right) - \tan^{-1}\left(\frac{2.25 \times 10^6}{10^6}\right) - \tan^{-1}\left(\frac{2.25 \times 10^6}{10^7}\right)$$

$$= -89.99 - 66.04 - 12.68$$

$$\phi = -168.7$$

$$\text{Phase Margin} = 180 - 168.7 = 11.3^\circ$$

### 12.87

a.  $T(0) = 100 \text{ dB} \Rightarrow T(0) = 10^5$

$$T(f) = \frac{10^5}{\left(1 + j\frac{f}{10}\right)\left(1 + j\frac{f}{5 \times 10^6}\right)\left(1 + j\frac{f}{10 \times 10^6}\right)}$$

$$|T| = 1 = 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}}$$

By trial and error

$$f = 0.976 \text{ MHz}$$

$$\phi = -\tan^{-1}\left(\frac{0.976 \times 10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) - \tan^{-1}\left(\frac{0.976}{10}\right)$$

$$= -90^\circ - 11.05^\circ - 5.574^\circ = -106.6^\circ$$

$$\text{Phase Margin} = 180^\circ - 106.6^\circ = 73.4^\circ$$

b.  $f'_{p1} \propto \frac{1}{C_F}$  so  $\frac{10}{f'_{p1}} = \frac{75}{20}$

or

$$f'_{p1} = 2.67 \text{ Hz}$$

Now

$$|T| = 1 = 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}}$$

By trial and error

$$f \approx 2.66 \times 10^5 \text{ Hz}$$

then



$$\phi = -\tan^{-1}\left(\frac{2.66 \times 10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) - \tan^{-1}\left(\frac{0.266}{10}\right)$$

$$= -90^\circ - 3.045^\circ - 1.524^\circ = -94.57^\circ$$

$$\text{Phase Margin} = 180^\circ - 94.57^\circ = \underline{85.4^\circ}$$

**12.88**

$$(a) f_P = \frac{1}{2\pi(R_{o1} \parallel R_{i2})C_i} = \frac{1}{2\pi(2 \parallel 0.75) \times 10^6 \times (1.2 \times 10^{-12})}$$

$$f_P = 243 \text{ kHz}$$

$$(b) f_{PD} = 6 = \frac{1}{2\pi(R_{o1} \parallel R_{i2})(C_i + C_M)} = \frac{1}{2\pi(2 \parallel 0.75) \times 10^6 \times (C_i + C_M)}$$

$$C_i + C_M = 4.863 \times 10^{-8} = 1.2 \times 10^{-12} + C_M$$

$$C_M = 0.0486 \mu\text{F}$$

$$(c) C_M = C_F(1 + |A|)$$

$$0.0486 \times 10^{-6} = C_F(1001)$$

$$C_F = 48.6 \text{ pF}$$

**12.89**

$$\phi = -120^\circ = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{4 \times 10^6}\right) - \tan^{-1}\left(\frac{f_{120}}{4 \times 10^7}\right)$$

$$f_{120} \cong 2.05 \times 10^6 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1 + \left(\frac{2.05 \times 10^6}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.05 \times 10^6}{4 \times 10^6}\right)^2} \sqrt{1 + \left(\frac{2.05 \times 10^6}{4 \times 10^7}\right)^2}}$$

$$\frac{2.05 \times 10^6}{f_{PD}} = \frac{4000}{(1.1237)(1.0013)}$$

$$f_{PD} = 577 \text{ Hz}$$

**12.90**

$$(a) 40 = \frac{5 \times 10^5}{1 + \beta(5 \times 10^5)} \Rightarrow \beta = 0.024998$$

$$(b) \phi = -120^\circ = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{5 \times 10^5}\right) - \tan^{-1}\left(\frac{f_{120}}{10^7}\right)$$

---

$$f_{120} \cong 2.71 \times 10^5 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{(0.024998)(5 \times 10^5)}{\sqrt{1 + \left(\frac{2.71 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.71 \times 10^5}{5 \times 10^5}\right)^2} \sqrt{1 + \left(\frac{2.71 \times 10^5}{10^7}\right)^2}}$$

$$\frac{2.71 \times 10^5}{f_{PD}} = \frac{(0.024998)(5 \times 10^5)}{(1.1374)(1)}$$

$$f_{PD} = 24.66 \text{ Hz}$$

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## Chapter 13

### 13.1

$$(a) R_{D2} = \frac{0 - (-3)}{0.2} = 15 \text{ k}\Omega$$

$$I_{D3} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$0.2 = \left(\frac{0.04}{2}\right)(40)(V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = 0.9 \text{ V}$$

$$R_{D1} = \frac{0.9}{0.1} = 9 \text{ k}\Omega$$

$$(b) (i) A_d = \frac{v_{o1}}{v_d} = \frac{g_{m1}}{2} \cdot R_{D1}$$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$A_d = \left(\frac{0.6325}{2}\right)(9) = 2.846$$

$$(ii) A_2 = \frac{v_o}{v_{o1}} = -g_{m3}R_{D2}$$

$$g_{m3} = 2\sqrt{\left(\frac{0.04}{2}\right)(40)(0.2)} = 0.8 \text{ mA/V}$$

$$A_2 = -(0.8)(15) = -12$$

$$(c) A = A_d \cdot A_2 = (2.846)(-12) = -34.15$$

### 13.2

$$(a) R_{C2} = \frac{3 - 0}{0.4} = 7.5 \text{ k}\Omega$$

$$V_{B3} = 0.7 + (0.4)(0.5) - 3 = -2.1 \text{ V}$$

$$R_{C1} = \frac{-2.1 - (-3)}{0.25} = 3.6 \text{ k}\Omega$$

$$(b) (i) A_d = \frac{v_{o1}}{v_d} = \frac{g_{m1}}{2} (R_{C1} \parallel R_{i3})$$

$$g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}, r_{\pi3} = \frac{(180)(0.026)}{0.4} = 11.7 \text{ k}\Omega$$

$$R_{i3} = r_{\pi3} + (1 + \beta_n)R_E = 11.7 + (181)(0.5) = 102.2 \text{ k}\Omega$$

$$A_d = \left(\frac{9.615}{2}\right)(3.6 \parallel 102.2) = 16.72$$

$$(ii) A_2 = \frac{v_o}{v_{o1}} = \frac{-\beta_n R_{C2}}{r_{\pi3} + (1 + \beta_n)R_E} = \frac{-(180)(7.5)}{11.7 + (181)(0.5)} = -13.21$$

$$(c) A = A_d \cdot A_2 = (16.72)(-13.21) = -220.9$$

### 13.3 Computer Analysis

### 13.4 Computer Analysis

### 13.5

$$(a) \quad A_d = g_{m1} (r_{o2} \| r_{o4} \| R_{i6})$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{20}{0.026} \Rightarrow 0.769 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$R_{i6} = r_{\pi6} + (1 + \beta_n) [R_1 \| r_{\pi7}]$$

$$r_{\pi7} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega$$

$$I_{C6} \cong \frac{V_{BE(on)}}{R_1} = \frac{0.6}{20} = 0.030 \text{ mA}$$

$$r_{\pi6} = \frac{(120)(0.026)}{0.030} = 104 \text{ k}\Omega$$

Then

$$R_{i6} = 104 + (121) [20 \| 15.6] \Rightarrow 1.16 \text{ M}\Omega$$

Then

$$A_d = 769 (4 \| 4 \| 1.16) \Rightarrow \underline{A_d = 565}$$

Now

$$\begin{aligned} V_o &= -I_{c7} r_{o7} = -(\beta_n I_{b7}) r_{o7} = -\beta_n r_{o7} \left( \frac{R_1}{R_1 + r_{\pi7}} \right) I_{c6} \\ &= -\beta_n (1 + \beta_n) r_{o7} \left( \frac{R_1}{R_1 + r_{\pi7}} \right) I_{b6} \text{ and } I_{b6} = \frac{V_{o1}}{R_{i6}} \end{aligned}$$

Then

$$A_{v2} = \frac{V_o}{V_{o1}} = \frac{-\beta_n (1 + \beta_n) r_{o7} \left( \frac{R_1}{R_1 + r_{\pi7}} \right)}{R_{i6}}$$

$$r_{o7} = \frac{V_A}{I_{C7}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

So

$$A_{v2} = \frac{-(120)(121)(400) \left( \frac{20}{20 + 15.6} \right)}{1160} \Rightarrow \underline{A_{v2} = -2813}$$

$$\text{Overall gain} = A_d \cdot A_{v2} = (565)(-2813) \Rightarrow \underline{A = -1.59 \times 10^6}$$

$$(b) \quad R_{id} = 2r_{\pi1} \text{ and } r_{\pi1} = \frac{(80)(0.026)}{0.020} = 104 \text{ k}\Omega$$

$$\underline{R_{id} = 208 \text{ k}\Omega}$$

$$f_{PD} = \frac{1}{2\pi R_{eq} C_M} \text{ and } C_M = (10)(1 + 2813) = 28,140 \text{ pF}$$

$$(c) \quad R_{eq} = r_{o2} \parallel r_{o4} \parallel R_{i6} = 4 \parallel 4 \parallel 1.16 = 0.734 \text{ M}\Omega$$

$$f_{PD} = \frac{1}{2\pi(0.734 \times 10^6)(28,140 \times 10^{-12})} = 7.71 \text{ Hz}$$

$$\text{Gain-Bandwidth Product} = (7.71)(1.59 \times 10^6) \Rightarrow \underline{12.3 \text{ MHz}}$$

### 13.6

- $Q_3$  acts as the protection device.
- Same as part (a).

### 13.7

If we assume  $V_{BE(\text{on})} = 0.7 \text{ V}$ , then  $V_m = 0.7 + 0.7 + 50 + 5$

So breakdown voltage  $\approx \underline{56.4 \text{ V}}$ .

### 13.8

$$(a) \quad V_{EB12} = V_{BE11} = (0.026) \ln \left( \frac{0.5 \times 10^{-3}}{5 \times 10^{-16}} \right) = 0.7184 \text{ V}$$

$$R_5 = \frac{15 - 0.7184 - 0.7184 - (-15)}{0.5} = 57.1 \text{ k}\Omega$$

$$R_4 = \frac{(0.026)}{(0.03)} \ln \left( \frac{0.5}{0.03} \right) = 2.438 \text{ k}\Omega$$

$$V_{BE10} = V_{BE11} - I_{C10} R_4 = 0.7184 - (0.03)(2.438) = 0.6453 \text{ V}$$

$$(b) \quad I_{REF} = \frac{15 - 0.6 - 0.6 - (-15)}{57.1} = 0.5044 \text{ mA}$$

$$I_{C10}(2.438) = (0.026) \ln \left( \frac{0.5044}{I_{C10}} \right) \Rightarrow I_{C10} \cong 30.1 \mu\text{A}$$

$$(c) \quad I_{REF} : \left( \frac{0.5044 - 0.5}{0.5} \right) \times 100\% = 0.88\%$$

$$I_{C10} : \left( \frac{30.1 - 30}{30} \right) \times 100\% = 0.33\%$$

### 13.9

$$(a) \quad R_5 = \frac{5 - 0.7184 - 0.7184 - (-5)}{0.5} = 17.13 \text{ k}\Omega$$

$$R_4 = \frac{(0.026)}{(0.03)} \ln \left( \frac{0.5}{0.03} \right) = 2.438 \text{ k}\Omega$$

$$V_{EB12} = V_{BE11} = 0.7184 \text{ V}$$

$$V_{BE10} = 0.6453 \text{ V}$$

(b) 
$$I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{17.13} = 0.5137 \text{ mA}$$

$$I_{C10}(2.438) = (0.026) \ln\left(\frac{0.5137}{I_{C10}}\right) \Rightarrow I_{C10} \cong 30.22 \mu\text{A}$$

(c) 
$$I_{REF} : \left(\frac{0.5137 - 0.5}{0.5}\right) \times 100\% = 2.74\%$$

$$I_{C10} : \left(\frac{30.22 - 30}{30}\right) \times 100\% = 0.733\%$$

**13.10**

(a) 
$$I_{REF} = \frac{3 - V_{EB2} - V_{BE1} - (-3)}{80}$$

$$V_{EB2} = V_{BE1} = (0.026) \ln\left(\frac{I_{REF}}{3 \times 10^{-15}}\right)$$

By trial and error,  $I_{REF} \cong 59.6 \mu\text{A}$

$$I_3(3.5) = (0.026) \ln\left(\frac{0.0596}{I_3}\right) \Rightarrow I_3 \cong 11.9 \mu\text{A}$$

$$I_4 = \frac{I_{S4}}{I_{S2}} \cdot I_{REF} = \frac{(3 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 35.76 \mu\text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}} \cdot I_{REF} = \frac{(1 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 11.92 \mu\text{A}$$

(b)  $I_{REF} = 59.6 \mu\text{A}$ ,  $I_3 = 11.9 \mu\text{A}$

$$I_4 = \frac{I_{S4}}{I_{S2}} \cdot I_{REF} = \frac{(8 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 95.36 \mu\text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}} \cdot I_{REF} = \frac{(2 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 23.84 \mu\text{A}$$

**13.11**

$$I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{40} \Rightarrow I_{REF} = 0.22 \text{ mA}$$

$$I_{C10} R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026) \ln\left(\frac{0.22}{I_{C10}}\right)$$

By trial and error;

$$I_{C10} \cong 14.2 \mu\text{A}$$

$$I_{C6} \cong \frac{I_{C10}}{2} \Rightarrow I_{C6} = 7.10 \mu\text{A}$$

$$I_{C17} = 0.75 I_{REF} \Rightarrow I_{C17} = 0.165 \text{ mA}$$

$$I_{C13A} = 0.25 I_{REF} \Rightarrow I_{C13A} = 0.055 \text{ mA}$$

**13.12**

$$I_{C8} = I_{C9}$$

$$2I = I_{C8} + \frac{2I_{C9}}{\beta_p} = I_{C9} \left( 1 + \frac{2}{\beta_p} \right)$$

$$I_{E1} = I_{E2} = I \left( \frac{1 + \beta_n}{\beta_n} \right)$$

$$I_{B3} = I_{B4} = I \left( \frac{1 + \beta_n}{\beta_n} \right) \left( \frac{1}{1 + \beta_p} \right)$$

$$I_{C10} = I_{C9} + 2I \left( \frac{1 + \beta_n}{\beta_n} \right) \left( \frac{1}{1 + \beta_p} \right)$$

$$I_{C10} = I_{C9} + I_{C9} \left( 1 + \frac{2}{\beta_p} \right) \left( \frac{1 + \beta_n}{\beta_n} \right) \left( \frac{1}{1 + \beta_p} \right)$$

$$50 = I_{C9} \left[ 1 + \left( 1 + \frac{2}{40} \right) \left( \frac{91}{90} \right) \left( \frac{1}{41} \right) \right] = I_{C9} (1.0259) \Rightarrow I_{C9} = 48.738 \mu\text{A}$$

$$I_{C2} = I = \frac{I_{C9}}{2} \left( 1 + \frac{2}{\beta_p} \right) = \frac{48.738}{2} \left( 1 + \frac{2}{40} \right) = 25.587 \mu\text{A}$$

$$I_{C4} = I_{C2} \left( \frac{\beta_p}{1 + \beta_p} \right) \left( \frac{1 + \beta_n}{\beta_n} \right) = (25.587) \left( \frac{40}{41} \right) \left( \frac{91}{90} \right) = 25.240 \mu\text{A}$$

$$I_{B9} = \frac{I_{C9}}{\beta_p} = \frac{48.738}{40} = 1.218 \mu\text{A}$$

$$I_{B4} = \frac{I_{C4}}{\beta_p} = \frac{25.240}{40} = 0.631 \mu\text{A}$$

**13.13**

$$V_{B5} - V^- = V_{BE}(\text{on}) + I_{C5}(1)$$

$$= 0.6 + (0.0095)(1) = 0.6095$$

$$I_{C7} = \frac{0.6095}{50} \Rightarrow I_{C7} = 12.2 \mu\text{A}$$

$$I_{C8} = I_{C9} = 19 \mu\text{A}$$

$$I_{REF} = 0.72 \text{ mA}$$

$$I_{E13} = I_{REF} = 0.72 \text{ mA}$$

$$I_{C14} = 138 \mu\text{A}$$

$$\text{Power} = (V^+ - V^-) [I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}]$$

$$= 30 [0.0122 + 0.019 + 0.019 + 0.72 + 0.72 + 0.138]$$

$$\Rightarrow \text{Power} = 48.8 \text{ mW}$$

Current supplied by  $V^+$  and  $V^- = I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}$

$$= 1.63 \text{ mA}$$

**13.14**

- (a)  $v_{cm}(\text{min}) = -15 + 0.6 + 0.6 + 0.6 + 0.6 = -12.6 \text{ V}$   
 $v_{cm}(\text{max}) = +15 - .6 = 14.4 \text{ V}$   
 So  $-12.6 \leq v_{cm} \leq 14.4 \text{ V}$
- (b)  $v_{cm}(\text{min}) = -5 + 4(0.6) = -2.6 \text{ V}$   
 $v_{cm}(\text{max}) = 5 - 0.6 = 4.4 \text{ V}$   
 So  $-2.6 \leq v_{cm} \leq 4.4 \text{ V}$

**13.15**

$$I_{C13A} = (0.25)I_{REF} = (0.25)(0.5) = 0.125 \text{ mA}$$

$$I_{R10} \cong \frac{0.6}{50} = 0.012 \text{ mA}$$

$$I_{C19} \cong I_{C13A} - I_{R10} = 0.125 - 0.012 = 0.113 \text{ mA}$$

$$I_{B19} = \frac{I_{C19}}{\beta_n} = \frac{0.113}{200} \Rightarrow I_{B19} = 0.565 \mu \text{ A}$$

$$I_{C18} = I_{R10} + I_{B19} = 12 + 0.565 = 12.565 \mu \text{ A}$$

$$V_{BE18} = (0.026) \ln \left( \frac{12.565 \times 10^{-6}}{10^{-14}} \right) = 0.54474 \text{ V}$$

$$V_{BE19} = (0.026) \ln \left( \frac{0.113 \times 10^{-3}}{10^{-14}} \right) = 0.60185 \text{ V}$$

$$\frac{V_{BB}}{2} = \frac{V_{BE18} + V_{BE19}}{2} = 0.57329 \text{ V}$$

$$I_{C14} = (3 \times 10^{-14}) \exp \left( \frac{0.57329}{0.026} \right) \Rightarrow I_{C14} = 113 \mu \text{ A}$$

**13.16**

- (a)  $V_{BB} = 2V_T \ln \left( \frac{I_{Bias}}{I_{SD}} \right) = 2(0.026) \ln \left( \frac{80 \times 10^{-6}}{5 \times 10^{-15}} \right) = 1.22178 \text{ V}$   
 $I_{CN} = I_{CP} = I_{SQ} \exp \left[ \frac{1.22178/2}{0.026} \right] \Rightarrow I_{CN} = I_{CP} = 128 \mu \text{ A}$
- (b) For  $v_i = 3 \text{ V}$ ,  $v_o \cong 3 \text{ V}$ ,  $i_L \cong \frac{3}{10} = 0.3 \text{ mA}$

First approximation:

$$I_{BN} \cong \frac{0.3}{120} \Rightarrow I_{BN} = 2.5 \mu \text{ A}$$

$$I_D = 80 - 2.5 = 77.5 \mu \text{ A}$$

$$V_{BB} = 2(0.026) \ln \left( \frac{77.5 \times 10^{-6}}{5 \times 10^{-15}} \right) = 1.22013 \text{ V}$$

$$V_{BEN} = (0.026) \ln \left( \frac{0.3 \times 10^{-3}}{8 \times 10^{-15}} \right) = 0.63304 \text{ V}$$

$$V_{EBP} = V_{BB} - V_{BEN} = 0.58709 \text{ V}$$



$$I_{CP} = (8 \times 10^{-15}) \exp\left(\frac{0.58709}{0.026}\right) \Rightarrow I_{CP} = 51.24 \mu\text{A}$$

Second approximation:

$$I_{CN} = \left(\frac{120}{121}\right)(0.3 + 0.05124) = 0.34834 \text{ mA}, \quad I_{BN} = 2.903 \mu\text{A}$$

$$I_D = 80 - 2.903 = 77.1 \mu\text{A}$$

$$V_{BB} = 2(0.026) \ln\left(\frac{77.1 \times 10^{-6}}{5 \times 10^{-15}}\right) = 1.219864 \text{ V}$$

$$V_{BEN} = (0.026) \ln\left(\frac{0.34834 \times 10^{-3}}{8 \times 10^{-15}}\right) = 0.636922 \text{ V}$$

$$V_{EBP} = 1.219864 - 0.636922 = 0.582942 \text{ V}$$

$$I_{CP} = (8 \times 10^{-15}) \exp\left(\frac{0.582942}{0.026}\right) \Rightarrow I_{CP} = 43.7 \mu\text{A}$$

### 13.17

$$R_1 + R_2 = \frac{V_{BB}}{(0.1)I_{Bias}} = \frac{1.160}{(0.1)(0.12)} = 96.67 \text{ k}\Omega$$

$$V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_S}\right) = (0.026) \ln\left(\frac{(0.9)(120 \times 10^{-6})}{5 \times 10^{-15}}\right) = 0.6187 \text{ V}$$

$$V_{BE} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{BB}) \Rightarrow 0.6187 = \left(\frac{R_2}{96.67}\right)(1.16)$$

So  $R_2 = 51.56 \text{ k}\Omega$ ,  $R_1 = 45.11 \text{ k}\Omega$

### 13.18

(a)  $A_d = -g_m (r_{o4} \parallel r_{o6} \parallel R_{i2})$

From example 13.4

$$g_m = \frac{9.5}{0.026} = 365 \mu\text{A/V}, \quad r_{o4} = 5.26 \text{ M}\Omega$$

Now

$$r_{o6} = r_{o4} = 5.26 \text{ M}\Omega$$

Assuming  $R_8 = 0$ , we find

$$\begin{aligned} R_{i2} &= r_{\pi 16} + (1 + \beta_n)R'_E \\ &= 329 + (201)(50 \parallel 9.63) \Rightarrow 1.95 \text{ M}\Omega \end{aligned}$$

Then

$$A_d = -(365)(5.26 \parallel 5.26 \parallel 1.95) \Rightarrow A_d = -409$$

(b) From Equation (13.20),

$$A_{v2} = \frac{-\beta_n(1 + \beta_n)R_9 (R_{act2} \parallel R_{i3} \parallel R_{017})}{R_{i2} \{R_9 + [r_{\pi 17} + (1 + \beta_n)R_g]\}}$$

For  $R_g = 0$ ,  $R_{i2} = 1.95 \text{ M}\Omega$

Using the results of Example 13.5

$$A_{v_2} = \frac{-200(201)(50)(92.6 \parallel 4050 \parallel 92.6)}{(1950)\{50 + 9.63\}} \Rightarrow \underline{A_{v_2} = -792}$$

### 13.19

Let  $I_{C10} = 40 \mu\text{A}$ , then  $I_{C1} = I_{C2} = 20 \mu\text{A}$ . Use the procedure in Example 13.4:

$$R_{i_2} = 4.07 \text{ M}\Omega$$

$$r_{\pi 6} = \frac{(200)(0.026)}{0.020} = 260 \text{ k}\Omega$$

$$g_{m6} = \frac{0.020}{0.026} = 0.769 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$$R_{\text{act}1} = 2.5[1 + (0.769)(1 \parallel 260)] = 4.42 \text{ M}\Omega$$

$$r_{o6} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$$\begin{aligned} A_d &= -\left(\frac{I_{CQ}}{V_T}\right)(r_{o4} \parallel R_{\text{act}1} \parallel R_{i_2}) \\ &= -\left(\frac{20}{0.026}\right)(2.5 \parallel 4.42 \parallel 4.07) \end{aligned}$$

So

$$\underline{A_d = -882}$$

### 13.20

From Problem 13.11

$$I_1 = I_2 = 7.10 \mu\text{A}, I_{C17} = 0.165 \text{ mA}, I_{C13A} = 0.055 \text{ mA}$$

$$\begin{aligned} I_{C16} &\approx I_{B17} + \frac{I_{E17}R_8 + V_{BE17}}{R_9} = \frac{0.165}{200} + \frac{(0.165)(0.1) + 0.6}{50} \\ &= 0.000825 + 0.01233 \end{aligned}$$

$$I_{C16} = 0.0132 \text{ mA}$$

$$r_{\pi 17} = \frac{(200)(0.026)}{0.165} = 31.5 \text{ K}$$

$$\begin{aligned} R_E^1 &= R_9 \parallel [r_{\pi 17} + (1 + \beta)R_8] = 50 \parallel [31.5 + (201)(0.1)] \\ &= 50 \parallel 51.6 = 25.4 \text{ K} \end{aligned}$$

$$r_{\pi 16} = \frac{(200)(0.026)}{0.0132} = 394 \text{ K}$$

Then

$$R_{i_2} = r_{\pi 16} + (1 + \beta)R_E^1 = 394 + (201)(25.4) \Rightarrow 5.50 \text{ M}\Omega$$

Now

$$r_{\pi 6} = \frac{(200)(0.026)}{0.0071} = 732 \text{ K}$$

$$g_{m6} = \frac{0.0071}{0.026} = 0.273 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

$$R_{\text{act1}} = r_{o6}[1 + g_{m6}(R \parallel r_{\pi 6})] \\ = 7.04[1 + (0.273)(1 \parallel 732)] = 8.96 \text{ M}\Omega$$

$$r_{o4} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

Then

$$A_d = -g_{m1}(r_{o4} \parallel R_{\text{act1}} \parallel R_{i2}) \\ = -\left(\frac{7.10}{0.026}\right)(7.04 \parallel 8.96 \parallel 5.5)$$

$$A_d = -627$$

Now  $R_{\text{act2}} = \frac{50}{0.165} \Rightarrow 303 \text{ K}$      $R_{o17} = \frac{50}{0.165} = 303 \text{ K}$

From Eq. (13.20), assuming  $R_{i3} \rightarrow \infty$

$$A_{v2} \cong -\frac{\beta(1+\beta)R_9(R_{\text{act2}} \parallel R_{o17})}{R_{i2}\{R_9 + [r_{\pi 17} + (1+\beta)R_8]\}} \\ = \frac{-(200)(201)(50)(303 \parallel 303)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-3.045 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -545$$

Overall gain  $A_v = (-627)(-545) = 341,715$

### 13.21

Using results from Problem 13.20

$$R_{i2} = 5.50 \text{ M}\Omega, R_{\text{act1}} \left(\frac{100}{0.0071}\right)[1 + (0.273)(1 \parallel 732)] \Rightarrow 17.93 \text{ M}\Omega$$

$$r_{o4} = \frac{100}{0.0071} \Rightarrow 14.08 \text{ M}\Omega$$

$$A_d = -\left(\frac{7.10}{0.026}\right)(14.08 \parallel 17.93 \parallel 5.50)$$

$$A_d = -885$$

Now

$$R_{\text{act2}} = \frac{100}{0.165} = 606 \text{ K} \quad R_{o17} = \frac{100}{0.165} = 606 \text{ K}$$

$$A_{v2} = \frac{-(200)(201)(50)(606 \parallel 606)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-6.09 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -1090$$

Overall gain

$$A_v = (-885)(-1090) = 964,650$$

**13.22**

(a)  $I_{C18} + I_{C19} = I_{Bias}$   
 $(0.25)I_{C19} + I_{C19} = 0.18 \text{ mA}, \Rightarrow I_{C19} = 0.144 \text{ mA}$   
 $I_{C18} = 0.036 \text{ mA}$

$$V_{BE19} = (0.026) \ln \left( \frac{0.144 \times 10^{-3}}{10^{-14}} \right) = 0.60815 \text{ V}$$

$$R_{10} = \frac{0.60815}{0.036} = 16.9 \text{ k}\Omega$$

(b)  $V_{BE19} = 0.60815 \text{ V}$

$$V_{BE18} = (0.026) \ln \left( \frac{0.036 \times 10^{-3}}{10^{-14}} \right) = 0.5721 \text{ V}$$

**13.23**

(b)  $I_{C19} = 0.168 \text{ mA}, g_{m19} = \frac{0.168}{0.026} = 6.462 \text{ mA/V},$

$$r_{\pi19} = \frac{(200)(0.026)}{0.168} = 30.95 \text{ k}\Omega, r_{o19} = \frac{50}{0.168} = 298 \text{ k}\Omega$$

$$I_{C18} = 12.84 \mu\text{A}, g_{m18} = \frac{0.01284}{0.026} = 0.4938 \text{ mA/V}$$

$$r_{\pi18} = \frac{(200)(0.026)}{0.01284} = 405 \text{ k}\Omega, r_{o18} = \frac{50}{0.01284} = 3894 \text{ k}\Omega$$

$$V_x = V_{\pi18} + V_{\pi19}$$

(1)  $I_x = \frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} + g_{m19}V_{\pi19} + \frac{V_x}{r_{o19}}$

(2)  $\frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} = \frac{V_{\pi19}}{R_{10}} + \frac{V_{\pi19}}{r_{\pi19}}$

Then  $V_{\pi18} \left( \frac{1}{405} + 0.4938 + \frac{1}{3894} \right) = V_{\pi19} \left( \frac{1}{50} + \frac{1}{30.95} \right)$

$$V_{\pi18} (0.49653) = V_{\pi19} (0.05231) \Rightarrow V_{\pi18} = V_{\pi19} (0.10535)$$

Then (1)  $I_x = V_{\pi18} (0.49653) + (6.462)V_{\pi19} + \frac{V_x}{298}$

$$I_x = V_{\pi19} (0.10535)(0.49653) + (6.462)V_{\pi19} + (0.003356)V_x$$

$$I_x = V_{\pi19} (6.5143) + V_x (0.003356)$$

Now  $V_x = V_{\pi18} + V_{\pi19} = V_{\pi19} (0.10535) + V_{\pi19} = (1.10535)V_{\pi19}$

Or  $V_{\pi19} = (0.90469)V_x$

Then  $I_x = (0.90469)V_x (6.5143) + V_x (0.003356)$

So  $\frac{I_x}{V_x} = \frac{1}{R_{eq}} = 5.8968 \Rightarrow R_{eq} = 170 \Omega$

**13.24**

Now

$$R_{e14} = \frac{r_{\pi14} + R_{01}}{1 + \beta_p} \quad \text{and} \quad R_0 = R_6 + R_{e14}$$

Assume series resistance of  $Q_{18}$  and  $Q_{19}$  is small. Then

$$R_{01} = r_{013A} \parallel R_{e22}$$

$$R_{e22} = \frac{r_{\pi22} + R_{017} \parallel r_{013B}}{1 + \beta_p}$$

where

$$\text{and } R_{017} = r_{017} [1 + g_{m17} (R_8 \parallel r_{\pi17})]$$

Using results from Example 13.6,

$$r_{\pi17} = 9.63 \text{ k}\Omega \quad r_{\pi22} = 7.22 \text{ k}\Omega$$

$$g_{m17} = 20.8 \text{ mA/V} \quad r_{017} = 92.6 \text{ k}\Omega$$

Then

$$R_{017} = 92.6 [1 + (20.8)(0.1 \parallel 9.63)] = 283 \text{ k}\Omega$$

$$r_{013B} = \frac{50}{0.54} = 92.6 \text{ k}\Omega$$

Then

$$R_{e22} = \frac{7.22 + 283 \parallel 92.6}{51} = 1.51 \text{ k}\Omega$$

$$R_{01} = r_{013A} \parallel R_{e22} = 278 \parallel 1.51 = 1.50 \text{ k}\Omega$$

$$r_{\pi14} = \frac{(50)(0.026)}{2} = 0.65 \text{ k}\Omega$$

Then

$$R_{e14} = \frac{0.65 + 1.50}{51} = 0.0422 \text{ k}\Omega$$

or

$$R_{e14} = 42.2 \text{ }\Omega$$

Then

$$R_0 = 42.2 + 27 \Rightarrow R_0 = \underline{69.2 \text{ }\Omega}$$

**13.25**

$$R_{id} = 2 \left[ r_{\pi1} + (1 + \beta_n) \left( \frac{r_{\pi3}}{1 + \beta_p} \right) \right]$$

$$\beta_n = 200, \quad \beta_p = 10$$

(a)

$$I_{c1} = 9.5 \text{ }\mu\text{A}$$

$$r_{\pi1} = \frac{(200)(0.026)}{0.0095} = 547 \text{ K}$$

$$r_{\pi3} = \frac{(10)(0.026)}{0.0095} = 27.4 \text{ K}$$

Then

$$R_{id} = 2 \left[ 547 + \frac{(201)(27.4)}{11} \right]$$

$$R_{id} \Rightarrow \underline{2.095 \text{ M}\Omega}$$

(b)

$$\begin{aligned}I_{C1} &= 7.10 \mu\text{A} \\r_{\pi 1} &= \frac{(200)(0.026)}{0.0071} = 732 \text{ K} \\r_{\pi 3} &= \frac{(10)(0.026)}{0.0071} = 36.6 \text{ K} \\R_{id} &= 2 \left[ 732 + \frac{(201)(36.6)}{11} \right] \\R_{id} &\Rightarrow 2.80 \text{ M}\Omega\end{aligned}$$

### 13.26

We can write

$$\begin{aligned}A(f) &= \frac{A_0}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{f_1}\right)} \\&= \frac{181,260}{\left(1 + j \frac{f}{10.7}\right) \left(1 + j \frac{f}{f_1}\right)}\end{aligned}$$

Phase:

$$\phi = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

For a Phase margin = 70°,  $\phi = -110^\circ$   
So

$$-110^\circ = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

Assuming  $f \gg 10.7 \text{ Hz}$ , we have

$$\tan^{-1}\left(\frac{f}{f_1}\right) = 20^\circ \Rightarrow \frac{f}{f_1} = 0.364$$

At this frequency,  $|A(f)| = 1$ , so

$$\begin{aligned}1 &= \frac{181,260}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2} \cdot \sqrt{1 + (0.364)^2}} \\&= \frac{170,327}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2}}\end{aligned}$$

or  $\frac{f}{10.7} = 170,327 \Rightarrow f = 1.82 \text{ MHz}$

Then, second pole at

$$f_1 = \frac{f}{0.364} \Rightarrow \underline{f_1 = 5 \text{ MHz}}$$

**13.27**

(a)  $10 \times 10^6 = f_{PD} (10^6) \Rightarrow f_{PD} = 10 \text{ Hz}$

(b)  $f_{PD} = \frac{1}{2\pi R_{eq} C_M} \Rightarrow C_M = \frac{1}{2\pi (1.2 \times 10^6)(10)} = 1.326 \times 10^{-8} \text{ F}$

$C_M = C_F (1 + |A|) \Rightarrow C_F = \frac{1.326 \times 10^{-8}}{1001} \Rightarrow C_F = 13.25 \text{ pF}$

**13.28**

$\phi = -110 = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - 2 \tan^{-1}\left(\frac{f}{f_{2,3}}\right) = -90 - 2 \tan^{-1}\left(\frac{f}{f_{2,3}}\right)$

So  $\left(\frac{f}{f_{2,3}}\right) = 0.1763$

$|A| = 1 = \frac{200,000}{\sqrt{1 + \left(\frac{f}{f_{PD}}\right)^2} \left[1 + \left(\frac{f}{f_{2,3}}\right)^2\right]} \cong \frac{200,000}{\left(\frac{f}{10}\right) [1 + (0.1763)^2]} \Rightarrow f = 1.9397 \text{ MHz}$

Then  $f_{2,3} = \frac{1.9397 \times 10^6}{0.1763} \Rightarrow f_{2,3} = 11.0 \text{ MHz}$

**13.29**

(a)  $I_{D3} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

$150 = \left(\frac{40}{2}\right) (50) (V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = 0.7873 \text{ V}$

$R_{D1} = \frac{0.7873}{0.1} = 7.87 \text{ k}\Omega$

$I_{D4} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_4 (V_{GS4} - V_{TN})^2$

$200 = \left(\frac{100}{2}\right) (40) (V_{GS4} - 0.4)^2 \Rightarrow V_{GS4} = 0.7162 \text{ V}$

$R_{D2} = \frac{0.7162 - (-3)}{0.15} = 24.8 \text{ k}\Omega$

$R_S = \frac{0 - (-3)}{0.15} = 15 \text{ k}\Omega$

(b) (i)  $A_{d1} = \left(\frac{g_{m1}}{2}\right) \cdot R_{D1}$ ,  $g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$

$A_{d1} = \left(\frac{0.6325}{2}\right)(7.87) = 2.49$

(ii)  $A_2 = -g_{m3} R_{D2}$ ,  $g_{m3} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.15)} = 0.7746 \text{ mA/V}$

$A_2 = -(0.7746)(24.8) = -19.21$

$$(iii) A_3 = \frac{g_{m4} R_S}{1 + g_{m4} R_S}, \quad g_{m4} = 2 \sqrt{\left(\frac{0.1}{2}\right)(40)(0.2)} = 1.265 \text{ mA/V}$$

$$A_3 = \frac{(1.265)(15)}{1 + (1.265)(15)} = 0.950$$

$$(c) A = A_{d1} \cdot A_2 \cdot A_3 = (2.49)(-19.21)(0.95) = -45.4$$

### 13.30

$$(a) I_{D3} = 100 = \left(\frac{100}{2}\right)(25)(V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 0.6828 \text{ V}$$

$$R_{D1} = \frac{0.6828}{0.05} = 13.66 \text{ k}\Omega$$

$$I_{D4} = 200 = \left(\frac{40}{2}\right)(100)(V_{SG4} - 0.4)^2 \Rightarrow V_{SG4} = 0.7162 \text{ V}$$

$$R_{D3} = \frac{3 - (-0.7162)}{0.1} = 37.16 \text{ k}\Omega$$

$$R_S = \frac{3 - 0}{0.2} = 15 \text{ k}\Omega$$

$$(b) (i) A_d = \left(\frac{g_{m1}}{2}\right) \cdot R_{D1}, \quad g_{m1} = 2 \sqrt{\left(\frac{0.04}{2}\right)(80)(0.05)} = 0.5657 \text{ mA/V}$$

$$A_d = \left(\frac{0.5657}{2}\right)(13.66) = 3.864$$

$$(ii) A_2 = -g_{m3} R_{D3}, \quad g_{m3} = 2 \sqrt{\left(\frac{0.1}{2}\right)(25)(0.1)} = 0.7071 \text{ mA/V}$$

$$A_2 = -(0.7071)(37.16) = -26.28$$

$$(iii) A_3 = \frac{g_{m4} R_S}{1 + g_{m4} R_S}, \quad g_{m4} = 2 \sqrt{\left(\frac{0.04}{2}\right)(100)(0.2)} = 1.265 \text{ mA/V}$$

$$A_3 = \frac{(1.265)(15)}{1 + (1.265)(15)} = 0.950$$

$$(c) A = A_d \cdot A_2 \cdot A_3 = (3.864)(-26.28)(0.95) = -96.5$$

### 13.31

a. Original  $g_{m1}$  and  $g_{m2}$

$$K_{p1} = K_{p2} = \left(\frac{W}{L}\right) \left(\frac{\mu_p C_{ox}}{2}\right) = (12.5)(10) \\ = 125 \mu\text{A/V}^2$$

So

$$g_{m1} = g_{m2} = 2 \sqrt{K_{p1} \left(\frac{I_Q}{2}\right)} = 2 \sqrt{(0.125)(10)} \\ = 0.09975 \text{ mA/V}$$

If  $\left(\frac{W}{L}\right)$  is increased to 50, then



$$K_{p1} = K_{p2} = (50)(10) = 500 \mu\text{A}/\text{V}^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{(0.5)(0.0199)} = 0.1995 \text{ mA/V}$$

b. Gain of first stage

$$A_d = g_{m1}(r_{o2} \parallel r_{o4}) = (0.1995)(5025 \parallel 5025)$$

or

$$A_d = 501$$

Voltage gain of second stage remains the same, or

$$A_{v2} = 251$$

Then  $A_v = A_d \cdot A_{v2} = (501)(251)$

or

$$A_d = 125,751$$

### 13.32

$$v_{CM}(\text{max}) = V^+ - v_{SD6}(\text{min}) - v_{SG1}$$

$$v_{SD6}(\text{min}) = v_{SD}(\text{sat}) + 0.2 = 0.9022 - 0.5 + 0.2 = 0.6022 \text{ V}$$

$$i_{D1} = K_{p1}(v_{SG1} + V_{TP})^2$$

$$20.2 = 250(v_{SG1} - 0.5)^2 \Rightarrow v_{SG1} = 0.7843 \text{ V}$$

$$v_{CM}(\text{max}) = 5 - 0.6022 - 0.7843 = 3.61 \text{ V}$$

$$v_{CM}(\text{min}) = V^- + v_{GS3} + v_{SD1}(\text{min}) - v_{SG1}$$

$$i_{D3} = K_n(v_{GS3} - V_{TN})^2, \quad K_n = \left(\frac{0.1}{2}\right)(6.25) = 0.3125 \text{ mA/V}^2$$

$$20.2 = 312.5(v_{GS3} - 0.5)^2 \Rightarrow v_{GS3} = 0.7542 \text{ V}$$

$$v_{SD1}(\text{min}) = 0.7843 - 0.5 + 0.2 = 0.4843 \text{ V}$$

$$v_{CM}(\text{min}) = -5 + 0.7542 + 0.4843 - 0.7843 = -4.55 \text{ V}$$

So  $-4.55 \leq v_{CM} \leq 3.61 \text{ V}$

### 13.33

$$(a) \quad K_{p5} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_5 = \left(\frac{0.04}{2}\right)(50) = 1.0 \text{ mA/V}^2$$

$$K_{p5}(V_{SG5} + V_{TP})^2 = \frac{V^+ - V_{SG5} - V^-}{R_{set}}$$

$$(1)(50)(V_{SG5}^2 - 1.4V_{SG5} + 0.49) = 10 - V_{SG5}$$

$$50V_{SG5}^2 - 69V_{SG5} + 14.5 = 0 \Rightarrow V_{SG5} = 1.121 \text{ V}$$

$$I_{set} = I_Q = \frac{10 - 1.121}{50} = 0.1776 \text{ mA} = I_{D7}$$

$$(b) \quad A_d = g_{m1}(r_{o2} \parallel r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)\left(\frac{0.1776}{2}\right)} = 0.5960 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.04)\left(\frac{0.1776}{2}\right)} = 281.5 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.02)\left(\frac{0.1776}{2}\right)} = 563.1 \text{ k}\Omega$$

$$A_d = (0.5960)(281.5 \parallel 563.1) = 111.9$$

$$A_2 = -g_{m7}(r_{o7} \parallel r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.1776)} = 1.032 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.02)(0.1776)} = 281.5 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.04)(0.1776)} = 140.8 \text{ k}\Omega$$

$$A_2 = -(1.032)(281.5 \parallel 140.8) = -96.86$$

$$A = A_d \cdot A_2 = (111.9)(-96.86) = -10,839$$

**13.34**

$$A_d = g_{m1}(r_{o2} \parallel r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.1)} = 400 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.015)(0.1)} = 666.7 \text{ k}\Omega$$

$$A_d = (0.6325)(400 \parallel 666.7) = 158.1$$

$$A_2 = -g_{m7}(r_{o7} \parallel r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.2)} = 1.095 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.015)(0.2)} = 333.3 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.025)(0.2)} = 200 \text{ k}\Omega$$

$$A_2 = -(1.095)(333.3 \parallel 200) = -136.9$$

$$A = A_d \cdot A_2 = (158.1)(-136.9) = -21,644$$

**13.35**

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

where  $R_{eq} = r_{o4} \parallel r_{o2}$  and  $C_i = C_1(1 + |A_{v2}|)$

We can find that  $A_{v2} = 251$  and  $r_{o4} = r_{o2} = 5.025 \text{ M}\Omega$

Now  $R_{eq} = 5.025 \parallel 5.025 = 2.51 \text{ M}\Omega$

and  $C_i = 12(1 + 251) = 3024 \text{ pF}$

So  $f_{PD} = \frac{1}{2\pi(2.51 \times 10^6)(3024 \times 10^{-12})}$

or  $f_{PD} = 21.0 \text{ Hz}$

**13.36**

From Problem 13.33,

$$|A_2| = 96.86, \quad r_{o2} = 281.5 \text{ k}\Omega, \quad r_{o4} = 563.1 \text{ k}\Omega$$

$$f_{PD} = \frac{1}{2\pi(r_{o2} \| r_{o4})C_M}$$

$$\text{Then } C_M = \frac{1}{2\pi(8)(281.5 \| 563.1) \times 10^3} = 1.06 \times 10^{-7} \text{ F}$$

$$C_M = C_1(1 + |A_2|) \Rightarrow 1.06 \times 10^{-7} = C_1(97.86)$$

$$\text{Or } C_1 = 1.08 \times 10^{-9} \text{ F}$$

**13.37**

$$R_0 = r_{o7} \| r_{o8}$$

We can find that

$$r_{o7} = r_{o8} = 2.52 \text{ M}\Omega$$

Then

$$R_0 = 2.52 \| 2.52$$

or

$$R_0 = 1.26 \text{ M}\Omega$$

**13.38**

$$(a) \quad V_{GS2} = \frac{6}{3} = 2 \text{ V}$$

$$I_D = 0.5 = \left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right) (2 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_{2-5} = 3.91$$

$$(b) \quad A_v = \frac{g_{m1}(r_{o1} \| r_{o2})}{1 + g_{m1}(r_{o1} \| r_{o2})}, \quad r_{o1} = r_{o2} = \frac{1}{(0.025)(0.5)} = 80 \text{ k}\Omega, \quad r_{o1} \| r_{o2} = 40 \text{ k}\Omega$$

$$0.98 = \frac{g_{m1}(40)}{1 + g_{m1}(40)} \Rightarrow g_{m1} = 1.225 \text{ mA/V}$$

$$g_{m1} = 1.225 = 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_1 (0.5)} \Rightarrow \left(\frac{W}{L}\right)_1 = 15$$

$$(c) \quad R_o = \frac{1}{g_{m1}} \| r_{o1} \| r_{o2} = \frac{1}{1.225} \| 40 = 0.8163 \| 40$$

$$R_o = 800 \Omega$$

**13.39**

$$I_{Q2} = \left(\frac{80}{2}\right) (20) [1.1737 - 0.7]^2$$

$$(a) \quad I_{Q2} = 180 \mu\text{A}$$

$$I_{D6} = \left(\frac{80}{2}\right)(25)(V_{GS6} - 0.7)^2 = 25 \Rightarrow V_{GS6} = 0.8581 \text{ V}$$

$$(b) \quad I_{D7} = \left(\frac{40}{2}\right)(50)(V_{SG7} - 0.7)^2 = 25 \Rightarrow V_{SG7} = 0.8581 \text{ V}$$

Set

$$V_{SG8P} = V_{GS8N} = 0.8581 \text{ V}$$

$$180 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_{8P} (0.8581 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right)_{8P} = 360$$

$$180 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_{8N} (0.8581 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right)_{8N} = 180$$

### 13.40

$$I_{REF} = I_{Q1} = 0.150 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_{11} (V_{GS11} - 0.5)^2 = \left(\frac{0.1}{2}\right)(20)(V_{GS11} - 0.5)^2 \Rightarrow V_{GS11} = 0.8873 \text{ V}$$

$$V_{GS,REF} = 5 - 0.8873 = 4.1127 \text{ V}$$

For three NMOS transistors in series:

$$V_{GS} = \frac{4.1127}{3} = 1.3709 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(1.3709 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right) = 3.96$$

### 13.41

(a)

$$I_{Q2} = 250 \mu\text{A} = \left(\frac{80}{2}\right)(5)(V_{GS8} - 0.7)^2$$

$$\Rightarrow V_{GS8} = 1.818 \text{ V}$$

$$\Rightarrow V_{GS6} = V_{SG7} = \frac{1.818}{2} = 0.909 \text{ V}$$

$$I_{D6} = I_{D7} = \left(\frac{80}{2}\right)(25)(0.909 - 0.7)^2 = \underline{43.7 \mu\text{A}}$$

(b)

$$g_{m1} = 2\sqrt{\left(\frac{80}{2}\right)(15)\left(\frac{250}{2}\right)} \Rightarrow 0.5477 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.01)(0.125)} = 800 \text{ K}$$

$$r_{o4} = \frac{1}{(0.015)(0.125)} = 533.3 \text{ K}$$

$$A_{d1} = g_{m1}(r_{o2} \parallel r_{o4}) = (0.5477)(800 \parallel 533.3)$$

$$A_{d1} = 175$$

Second stage:

$$A_2 = -g_{m5}(r_{o5} \parallel r_{o9})$$

$$g_{m5} = 2\sqrt{\left(\frac{40}{2}\right)}(80)(250) \Rightarrow 1.265 \text{ mA/V}$$

$$r_{o5} = \frac{1}{(0.015)(0.25)} = 266.7 \text{ K}$$

$$r_{o9} = \frac{1}{(0.01)(0.25)} = 400 \text{ K}$$

$$A_2 = -(1.265)(266.7 \parallel 400)$$

$$A_2 = -202$$

Assume the gain of the output stage  $\approx 1$ , then

$$A_v = A_{d1} \cdot A_2 = (175)(-202)$$

$$A_v = -35,350$$

### 13.42

(a)  $A_d = g_{m1}(R_{o6} \parallel R_{o8})$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu\text{A/V}$$

$$g_{m1} = g_{m8}$$

$$g_{m6} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu\text{A/V}$$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(25)} = 2.67 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.015)(50)} \Rightarrow 1.33 \text{ M}\Omega$$

Now

$$R_{o8} = g_{m8}(r_{o8} r_{o10}) = (224)(2.67)(2.67) = 1597 \text{ M}\Omega$$

$$R_{o6} = g_{m6}(r_{o6})(r_{o4} \parallel r_{o1}) = (224)(2.67)(2.67 \parallel 1.33) \Rightarrow R_{o6} = 531 \text{ M}\Omega$$

Then

$$A_d = (224)(531 \parallel 1597) \Rightarrow A_d = 89,264$$

(b)  $R_o = R_{o6} \parallel R_{o8} = 531 \parallel 1597 \Rightarrow R_o = 398 \text{ M}\Omega$

(c)  $f_{PD} = \frac{1}{2\pi R_o C_L} = \frac{1}{2\pi(398 \times 10^6)(5 \times 10^{-12})} \Rightarrow f_{PD} = 80 \text{ Hz}$

(c)  $GBW = (89,264)(80) \Rightarrow GBW = 7.14 \text{ MHz}$

### 13.43

(a)

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(25)} = 2 \text{ M}\Omega$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(25)} = 2.67 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(50)} = 1.33 \text{ M}\Omega$$

$$g_{m1} = 2 \sqrt{\left(\frac{35}{2}\right) \left(\frac{W}{L}\right)_1} (25) = 41.8 \sqrt{\left(\frac{W}{L}\right)_1} = g_{m8}$$

$$g_{m6} = 2 \sqrt{\left(\frac{80}{2}\right) \left(\frac{W}{L}\right)_6} (25) = 63.2 \sqrt{\left(\frac{W}{L}\right)_6}$$

$$R_o = R_{o6} \parallel R_{o8} = [g_{m6}(r_{o6})(r_{o4} \parallel r_{o1})] \parallel [g_{m8}(r_{o8} r_{o10})]$$

Define  
Then

$$X_1 = \sqrt{\left(\frac{W}{L}\right)_1} \quad \text{and} \quad X_6 = \sqrt{\left(\frac{W}{L}\right)_6}$$

$$\begin{aligned} R_o &= [63.2 X_6 (2.67)(1.33 \parallel 2)] \parallel [41.8 X_1 (2)(2)] \\ &= 134.8 X_6 \parallel 167.2 X_1 = \frac{22,539 X_1 X_6}{134.8 X_6 + 167.2 X_1} \end{aligned}$$

$$\begin{aligned} A_d = g_{m1} R_o &= (41.8 X_1) \left( \frac{22,539 X_1 X_6}{134.8 X_6 + 167.2 X_1} \right) \\ &= 10,000 \end{aligned}$$

Now

$$X_6 = \sqrt{\left(\frac{W}{L}\right)_6} = \sqrt{\frac{1}{2.2} \left(\frac{W}{L}\right)_1} = 0.674 X_1$$

We then find

$$X_1^2 = \left(\frac{W}{L}\right)_1 = 4.06 = \left(\frac{W}{L}\right)_p$$

and

$$\left(\frac{W}{L}\right)_n = 1.85$$

### 13.44

Let  $V^+ = 5V$ ,  $V^- = -5V$

$$P = I_T(10) = 3 \Rightarrow I_T = 0.3 \text{ mA} \Rightarrow I_{REF} = 0.1 \text{ mA} = 100 \mu\text{A}$$

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{(0.02)(50)} = 1 \text{ M}\Omega$$

$$r_{o6} = \frac{1}{(0.015)(50)} = 1.33 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{(0.015)(100)} = 0.667 \text{ M}\Omega$$

$$g_{m1} = 2 \sqrt{\left(\frac{35}{2}\right) \left(\frac{W}{L}\right)_1} (50) = 59.2 X_1 = g_{m8}$$

where

$$X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$$

Assume all width-to-length ratios are the same.

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)}(50) = 89.4X_1$$

Now

$$\begin{aligned} R_o &= R_{o6} \parallel R_{o8} = [g_{m6}(r_{o6})(r_{o4} \parallel r_{o1})] \parallel [g_{m8}(r_{o8}r_{o10})] \\ &= [89.4X_1(1.33)(0.667 \parallel 1)] \parallel [59.2X_1(1)(1)] \\ &= [47.6X_1] \parallel [59.2X_1] = \frac{(47.6X_1)(59.2X_1)}{47.6X_1 + 59.2X_1} \end{aligned}$$

So  $R_o = 26.4X_1$

Now

$$A_d = g_{m1}R_o = (59.2X_1)(26.4X_1) = 25,000$$

So that  $\frac{X_1^2}{L} = \frac{W}{L} = 16$  for all transistors

### 13.45

(a)  $A_d = Bg_{m1}(r_{o6} \parallel r_{o8})$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)}(20)(0.06) = 0.4899 \text{ mA/V}$$

$$r_{o6} = \frac{1}{(0.04)(0.06)(3)} = 138.9 \text{ k}\Omega$$

$$r_{o8} = \frac{1}{(0.02)(0.06)(3)} = 277.8 \text{ k}\Omega$$

$$A_d = (3)(0.4899)(138.9 \parallel 277.8) = 136.1$$

(b)  $R_o = r_{o6} \parallel r_{o8} = 138.9 \parallel 277.8 = 92.6 \text{ k}\Omega$

(c)  $f_{PD} = \frac{1}{2\pi R_o C} = \frac{1}{2\pi(92.6 \times 10^3)(5 \times 10^{-12})} \Rightarrow f_{PD} = 343.7 \text{ kHz}$

$$GBW = (136.1)(343.7 \times 10^3) \Rightarrow GBW = 46.8 \text{ MHz}$$

### 13.46

$$r_{o6} = \frac{1}{(0.02)(2.5)(40)} = 0.5 \text{ M}\Omega$$

$$r_{o8} = \frac{1}{(0.015)(2.5)(40)} = 0.667 \text{ M}\Omega$$

$$A_d = Bg_{m1}(r_{o6} \parallel r_{o8})$$

$$400 = (2.5)g_{m1}(0.5 \parallel 0.667) \Rightarrow g_{m1} = 560 \mu\text{A/V}$$

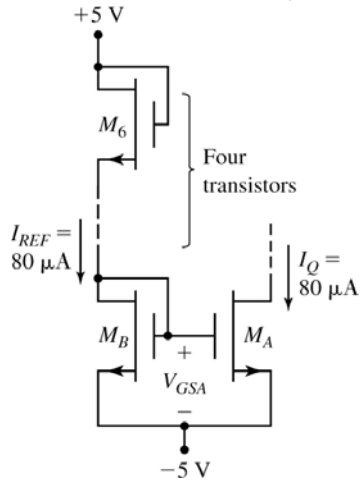
$$g_{m1} = 560 = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)}(40) \Rightarrow \left(\frac{W}{L}\right) = 49$$

(a)

Assume all  $(W/L)$  ratios are the same except for

$$M_5 \text{ and } M_6, \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = 122.5$$

- (b) Assume the bias voltages are  
 $V^+ = 5V, V^- = -5V.$



Assume  $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 49$

$$I_Q = \left(\frac{80}{2}\right)(49)(V_{GSA} - 0.5)^2 = 80 \Rightarrow V_{GSA} = 0.702 \text{ V}$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (V_{GSC} - 0.5)^2$$

For four transistors

$$V_{GSC} = \frac{10 - 0.702}{4} = 2.325 \text{ V}$$

$$80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (2.325 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_C = 0.60$$

(c)  $f_{3-dB} = \frac{1}{2\pi R_o C} \quad R_o = 0.5 \parallel 0.667 = 0.286 \text{ M}\Omega$

$$f_{3-dB} = \frac{1}{2\pi(286 \times 10^3)(3 \times 10^{-12})} = 185 \text{ kHz}$$

$$GBW = (400)(185 \times 10^3) \Rightarrow 74 \text{ MHz}$$

### 13.47

- (a) From previous results, we can write

$$R_{o10} = g_{m10}(r_{o10}r_{o6})$$

$$R_{o12} = g_{m12}(r_{o12}r_{o8})$$

$$A_d = Bg_{m1}(R_{o10} \parallel R_{o12})$$

Now

$$r_{o10} = r_{o6} = \frac{1}{\lambda_p B (I_Q/2)} = \frac{1}{(0.02)(2.5)(40)} = 0.5 \text{ M}\Omega$$

$$r_{o12} = r_{o8} = \frac{1}{\lambda_n B (I_Q/2)} = \frac{1}{(0.015)(2.5)(40)} = 0.667 \text{ M}\Omega$$



Assume all transistors have the same width-to-length ratios except for  $M_5$  and  $M_6$ .

Let  $\left(\frac{W}{L}\right) = X^2$

Then

$$g_{m10} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_{10} (I_{DQ10})} = 2\sqrt{\left(\frac{35}{2}\right)X^2(2.5)(40)}$$

$$= 83.67 X$$

$$g_{m12} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_{12} (I_{DQ12})} = 2\sqrt{\left(\frac{80}{2}\right)X^2(2.5)(40)}$$

$$= 126.5 X$$

$$g_{m1} = 2\sqrt{\left(\frac{80}{2}\right)X^2(40)} = 80 X$$

Then

$$R_{o10} = (83.67 X)(0.5)(0.5) = 20.9 X \text{ M}\Omega$$

$$R_{o12} = (126.5 X)(0.667)(0.667) = 56.3 X \text{ M}\Omega$$

We want

$$20,000 = (2.5)(80 X)[20.9 X \parallel 56.3 X]$$

$$= 200 X \left[ \frac{(20.9 X)(56.3 X)}{20.9 X + 56.3 X} \right] = 3048 X^2$$

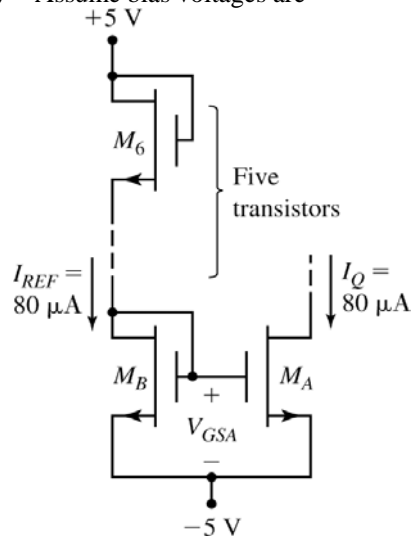
Then

$$X^2 = 6.56 = \left(\frac{W}{L}\right)$$

Then

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = (2.5)(6.56) = 16.4$$

(b) Assume bias voltages are  $V^+ = 5V$ ,  $V^- = -5V$



Assume  $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 6.56$

$$I_Q = 80 = \left(\frac{80}{2}\right)(6.56)(V_{GSA} - 0.5)^2 \Rightarrow V_{GSA} = 1.052 \text{ V}$$

Need 5 transistors in series

$$V_{GSC} = \frac{10 - 1.052}{5} = 1.79 \text{ V}$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (1.79 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_C = 1.20$$

(c)  $f_{3-dB} = \frac{1}{2\pi R_o C}$  where  $R_o = R_{o10} \parallel R_{o12}$

Now

$$R_{o10} = 20.9\sqrt{6.56} = 53.5 \text{ M}\Omega$$

$$R_{o12} = 56.3\sqrt{6.56} = 144 \text{ M}\Omega$$

Then

$$R_o = 53.5 \parallel 144 = 39 \text{ M}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(39 \times 10^6)(3 \times 10^{-12})} = 1.36 \text{ kHz}$$

$$GBW = (20,000)(1.36 \times 10^3) \Rightarrow GBW = 27.2 \text{ MHz}$$

### 13.48

(a)  $\Delta V_{O1} = 0.7 + (0.3)(0.4) = 0.82 \text{ V}$

$$R_D = \frac{0.82}{0.15} = 5.47 \text{ k}\Omega$$

$$R_C = \frac{3 - 0}{0.3} = 10 \text{ k}\Omega$$

(b)  $A_d = \frac{g_{m1}}{2}(R_D \parallel R_{iC})$ ,  $g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.15)} = 0.7746 \text{ mA/V}$

$$R_{iC} = r_\pi + (1 + \beta)R_E, \quad r_\pi = \frac{(120)(0.026)}{0.3} = 10.4 \text{ k}\Omega$$

$$R_{iC} = 10.4 + (121)(0.4) = 58.8 \text{ k}\Omega$$

$$A_d = \left(\frac{0.7746}{2}\right)(5.47 \parallel 58.8) = 1.938$$

(c)  $A_2 = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = \frac{-(120)(10)}{10.4 + (121)(0.4)} = -20.41$

(d)  $A_v = A_d \cdot A_2 = (1.938)(-20.41) = -39.6$

### 13.49

(a)  $R_S = \frac{0 - (-3)}{0.3} = 10 \text{ k}\Omega$

$$I_{D1} = K_n (V_{GS1} - V_{TN})^2$$

$$0.3 = 3(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = 0.7162 \text{ V}$$

$$R_{C2} = \frac{0.7162 - (-3)}{0.3} = 12.39 \text{ k}\Omega$$

$$\Delta V_{O1} = (0.3)(0.5) + 0.7 = 0.85 \text{ V}$$

$$R_{C1} = \frac{0.85}{0.25} = 3.4 \text{ k}\Omega$$

$$(b) \quad g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}, \quad r_{\pi1} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$g_{m3} = \frac{0.3}{0.026} = 11.54 \text{ mA/V}, \quad r_{\pi3} = \frac{(120)(0.026)}{0.3} = 10.4 \text{ k}\Omega$$

$$A_d = \frac{g_{m1}}{2} (R_{C1} \parallel R_{i3})$$

$$R_{i3} = r_{\pi3} + (1 + \beta)R_E = 10.4 + (121)(0.5) = 70.9 \text{ k}\Omega$$

$$A_d = \frac{9.615}{2} (3.4 \parallel 70.9) = 15.60$$

$$(c) \quad A_2 = \frac{-\beta R_{C2}}{r_{\pi3} + (1 + \beta)R_E} = \frac{-(120)(12.39)}{10.4 + (121)(0.5)} = -20.97$$

$$(d) \quad A_3 = \frac{g_{mD} R_S}{1 + g_{mD} R_S}, \quad g_{mD} = 2\sqrt{(3)(0.3)} = 1.897 \text{ mA/V}$$

$$A_3 = \frac{(1.897)(10)}{1 + (1.897)(10)} = 0.950$$

$$(e) \quad A_v = A_d \cdot A_2 \cdot A_3 = (15.6)(-20.97)(0.95) = -310.8$$

### 13.50

(a) For PMOS:

$$g_{m1} = g_{m2} = 2\sqrt{\left(\frac{0.04}{2}\right)(40)(0.125)} = 0.6325 \text{ mA/V}$$

$$r_{oP} = \frac{1}{\lambda I_{D1}} = \frac{1}{(0.035)(0.125)} = 228.6 \text{ k}\Omega$$

For BJT:

$$r_{o2} = \frac{V_A}{I_C} = \frac{150}{0.125} = 1200 \text{ k}\Omega$$

$$(b) \quad A_d = g_{m2} (r_{oP} \parallel r_{o2}) = (0.6325)(228.6 \parallel 1200) = 121.5$$

### 13.51

(a) For NMOS:

$$g_{mN} = 2\sqrt{\left(\frac{0.1}{2}\right)(40)(0.125)} = 1.0 \text{ mA/V}$$

$$r_{oN} = \frac{1}{(0.02)(0.125)} = 400 \text{ k}\Omega$$

For BJT:

$$r_{o2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$(b) \quad A_d = g_{mN} (r_{oN} \parallel r_{o2}) = (1.0)(400 \parallel 800) = 266.7$$

### 13.52

$$I_{REF} = 200 \mu A \quad K_n = K_p = 0.5 \text{ mA/V}^2$$

$$\lambda_n = \lambda_p = 0.015 \text{ V}^{-1}$$

(a)  $A_d = g_{m1} (R_{o6} \parallel R_{o8})$

where

$$R_{o8} = g_{m8} (r_{o8} r_{o10})$$

$$R_{o6} = g_{m6} (r_{o6}) (r_{o4} \parallel r_{o1})$$

Now

$$g_{m8} = 2\sqrt{K_p I_{D8}} = 2\sqrt{(0.5)(0.1)} = 0.447 \text{ mA/V}$$

$$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 667 \text{ k}\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(0.2)} = 333 \text{ k}\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$g_{m1} = 2\sqrt{K_p I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447 \text{ mA/V}$$

So

$$R_{o8} = (0.447)(667)(667) \Rightarrow 198.9 \text{ M}\Omega$$

$$R_{o6} = (3.846)(800)(333 \parallel 667) \Rightarrow 683.4 \text{ M}\Omega$$

Then

$$A_d = 447(198.9 \parallel 683.4) \Rightarrow \underline{A_d = 68,865}$$

### 13.53

Assume biased at  $V^+ = 10V$ ,  $V^- = -10V$ .

$$P = 3I_{REF}(20) = 10 \Rightarrow I_{REF} = 167 \mu A$$

$$A_d = g_{m1} (R_{o6} \parallel R_{o8}) = 25,000$$

$$k'_n = 80 \mu A/V^2, k'_p = 35 \mu A/V^2$$

$$\lambda_n = 0.015 \text{ V}^{-1}, \lambda_p = 0.02 \text{ V}^{-1}$$

Assume  $\left(\frac{W}{L}\right)_p = 2.2 \left(\frac{W}{L}\right)_n$

$$R_{o8} = g_{m8} (r_{o8} r_{o10})$$

$$R_{o6} = g_{m6} (r_{o6}) (r_{o4} \parallel r_{o1})$$

$$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.02)(83.3)} = 0.60 \text{ M}\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 0.60 \text{ M}\Omega$$

$$g_{m8} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_8 I_{D8}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3X$$

$$X^2 = \left(\frac{W}{L}\right)_n$$

where

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{83.3} = 0.960 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(167)} = 0.40 \text{ M}\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.02)(83.3)} = 0.60 \text{ M}\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{83.3}{0.026} = 3204 \text{ }\mu\text{A/V}$$

$$g_{m1} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3X$$

Now

$$R_{o6} = (3204)(0.960)[0.40\|0.60] = 738 \text{ M}\Omega$$

$$R_{o8} = (113.3X)(0.60)(0.60) = 40.8X \text{ M}\Omega$$

Then

$$A_d = 25,000 = (113.3X)[738\|40.8X]$$

$$= (113.3X)\left[\frac{30,110X}{738 + 40.8X}\right]$$

which yields  $X = 2.48$

or

$$X^2 = 6.16 = \left(\frac{W}{L}\right)_n$$

and

$$\left(\frac{W}{L}\right)_p + (2.2)(6.16) = 12.3$$

### 13.54

For  $v_{cm}(\text{max})$ , assume  $V_{CB}(Q_5) = 0$ . Then

$$V_S = 15 - 0.6 - 0.6 = 13.8 \text{ V}$$

$$I_{D9} = I_{D10} = \frac{0.236}{2} = 0.118 \text{ mA}$$

Using parameters given in Example 13.12

$$V_{SG} = \sqrt{\frac{I_{D9}}{K_p}} - V_{TP} = \sqrt{\frac{0.118}{0.20}} + 1.4 = 2.17 \text{ V}$$

Then

$$v_{cm}(\text{max}) = 13.8 - 2.17 \Rightarrow v_{cm}(\text{max}) = 11.6 \text{ V}$$

For

$v_{cm}(\text{min})$ , assume

$$V_{SD}(M_9) = V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2.17 - 1.4 = 0.77 \text{ V}$$

Now

$$V_{D10} = I_{D10}(0.5) + 0.6 + I_{D10}(0.5) - 15$$

$$= 0.118 + 0.6 - 15 \Rightarrow V_{D10} = -14.28 \text{ V}$$

Then

$$v_{cm}(\min) = -14.28 + V_{SD}(\text{sat}) - V_{SG}$$

$$= -14.28 + 0.77 - 2.17 = -15.68 \text{ V}$$

Then, common-mode voltage range

$$\underline{-15.68 \leq v_{cm} \leq 11.6}$$

Or, assuming the input is limited to  $\pm 15 \text{ V}$ , then

$$\underline{-15 \leq v_{cm} \leq 11.6 \text{ V}}$$

### 13.55

$$I_1 = I_2$$

$$K_p (V_{SG} + V_{TP})^2 = \frac{V_{SG} - V_{BE7}}{R_1}$$

$$I_2 = 0.15 = \frac{V_{SG} - 0.6}{8} \Rightarrow V_{SG} = 1.8 \text{ V}$$

$$I_1 = 0.15 = K_p (1.8 - 1)^2 \Rightarrow K_p = 0.234 \text{ mA/V}^2$$

### 13.56

$$(a) \quad K_p (V_{SG} + V_{TP})^2 = \frac{V_{SG} - V_{BE7}}{R_1}$$

$$0.15(V_{SG} - 1.2)^2 = \frac{V_{SG} - 0.6}{8}$$

$$\text{We find } 1.2V_{SG}^2 - 3.88V_{SG} + 2.328 = 0, \Rightarrow V_{SG} = 2.437 \text{ V}$$

$$I_1 = I_2 = \frac{2.437 - 0.6}{8} = 0.2296 \text{ mA}$$

$$(b) \quad V_{C7} = V^+ - 0.6 - 0.6$$

$$V_{C6} = V^- + 2.437$$

$$\text{Set } V_{C6} = V_{C7}$$

$$\text{Then } V_S - 1.2 = -V_S + 2.437 \Rightarrow V_S = 1.82 \text{ V}$$

### 13.57

$$I_{C5} = I_{C4} = 300 \mu\text{A}$$

We have

$$R_{i2} = r_{\pi 13} = \frac{\beta_n V_T}{I_{C13}} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

$$A_d = \sqrt{2K_n I_{Q5}} \cdot (R_{i2}) = \sqrt{2(0.6)(0.3)} \cdot (17.3)$$

or

$$\underline{A_d = 10.38}$$

Now

$$g_{m13} = \frac{I_{C13}}{V_T} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{o13} = \frac{V_A}{I_{C13}} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

Then

$$|A_{v2}| = g_{m13} \cdot r_{o13} = (11.5)(167)$$

or

$$\underline{|A_{v2}| = 1917}$$

Overall gain:

$$\underline{|A_v| = (10.38)(1917) = 19,895}$$

### 13.58

Assuming the resistances looking into  $Q_4$  and into the output stage are very large, we have

$$|A_{v2}| = \frac{\beta R_{o13}}{r_{\pi13} + (1 + \beta)R_{E13}}$$

where  $R_{o13} = r_{o13} [1 + g_{m13} (R_{E13} \parallel r_{\pi13})]$

$$I_{C13} = 300 \mu\text{A}, r_{o13} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

$$g_{m13} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{\pi13} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

So

$$R_{o13} = (167) [1 + (11.5)(1 \parallel 17.3)] \Rightarrow 1.98 \text{ M}\Omega$$

Then

$$|A_{v2}| = \frac{(200)(1980)}{17.3 + (201)(1)} = 1814$$

Now

$$C_i = C_1(1 + |A_{v2}|) = 12[1 + 1814]$$

$$\Rightarrow C_i = 21,780 \text{ pF}$$

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

$$R_{eq} = R_{\tau2} \parallel r_{o12} \parallel r_{o10}$$

Neglecting  $R_3$ ,

$$r_{o10} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

Neglecting  $R_5$ ,

$$r_{o12} = \frac{50}{0.15} = 333 \text{ k}\Omega$$

$$R_{\tau2} = r_{\pi13} + (1 + \beta)R_{E13} = 17.3 + (201)(1)$$

$$= 218 \text{ k}\Omega$$

Then

$$f_{PD} = \frac{1}{2\pi [218 \parallel 333 \parallel 333] \times 10^3 \times (21,780) \times 10^{-12}}$$

or

$$\underline{f_{PD} = 77.4 \text{ Hz}}$$

Unity-Gain Bandwidth

Gain of first stage:

$$\begin{aligned} A_d &= \sqrt{2K_n I_{Q5}} \cdot (R_{12} \parallel r_{\pi 12} \parallel r_{\pi 10}) \\ &= \sqrt{2(0.6)(0.3)} \cdot (218 \parallel 333 \parallel 333) \\ &= (0.6)(218 \parallel 333 \parallel 333) \end{aligned}$$

or  $A_d = 56.6$

Overall gain:

$$A_v = (56.6)(1814) = 102,672$$

Then unity-gain bandwidth =  $(77.4)(102,672)$

$$\Rightarrow \underline{7.95 \text{ MHz}}$$

**13.59**

Since  $V_{GS} = 0$  in  $J_6$ ,  $I_{REF} = I_{DSS}$

$$\Rightarrow \underline{I_{DSS} = 0.8 \text{ mA}}$$

**13.60**

a.  $R_{i2} = r_{\pi 5} + (1 + \beta)[r_{\pi 6} + (1 + \beta)R_E]$

$$r_{\pi 6} = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$I_{C5} \cong \frac{I_{C6}}{\beta} = \frac{200 \mu\text{A}}{100} = 2 \mu\text{A}$$

So

$$r_{\pi 5} = \frac{(100)(0.026)}{0.002} = 1300 \text{ k}\Omega$$

Then

$$R_{i2} = 1300 + (101)[13 + (101)(0.3)]$$

or

$$\underline{R_{i2} = 5.67 \text{ M}\Omega}$$

b.  $A_v = g_{m2} (r_{\pi 2} \parallel r_{\pi 4} \parallel R_{i2})$

$$\begin{aligned} g_{m2} &= \frac{2}{V_P} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{3} \cdot \sqrt{(0.1)(0.2)} \\ &= 0.0943 \text{ mA/V} \end{aligned}$$

$$r_{\pi 2} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{\pi 4} = \frac{V_A}{I_{C4}} = \frac{5.0}{0.1} = 500 \text{ k}\Omega$$

Then

$$A_v = (0.0943)[500 \parallel 500 \parallel 5670]$$

or



$$A_v = 22.6$$

**13.61**

a. Need  $V_{SD}(Q_E) \geq V_{SD}(\text{sat}) = V_P$  For minimum bias  $\pm 3$  V

Set  $V_P = 3$  V and  $V_{ZK} = 3$  V

$$I_{REF2} = \frac{V_{ZK} - V_{D1}}{R_3}$$

$$\text{so that } R_3 = \frac{3 - 0.6}{0.1} \Rightarrow R_3 = 24 \text{ k}\Omega$$

Set bias in  $Q_E = I_{REF2} + I_{Z2} = 0.1 + 0.1 = 0.2$  mA

Therefore,

$$I_{DSS} = 0.2 \text{ mA}$$

b. Neglecting base currents

$$I_{01} = I_{REF1} = 0.5 \text{ mA} = \frac{12 - 0.6}{R_4}$$

so that

$$R_4 = 22.8 \text{ k}\Omega$$

**13.62**

a. We have

$$g_{m2} = \frac{2}{|V_P|} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{4} \cdot \sqrt{(0.5)(1)} = 0.354 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$g_{m4} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

So

$$\begin{aligned} R_{04} &= r_{04} [1 + g_{m4} (r_{\pi 4} \| R_2)] \\ &= 200 [1 + (19.23)(10.4 \| 0.5)] \\ &= 2035 \text{ k}\Omega \end{aligned}$$

$$|A_d| = g_{m2} (r_{02} \| R_{04} \| R_L)$$

$$R_L \rightarrow \infty$$

$$\text{For } |A_d| = 0.354(100 \| 2035) = 33.7$$

With these parameter values, gain can never reach 500.

b. Similarly for this part, gain can never reach 700.



## Chapter 14

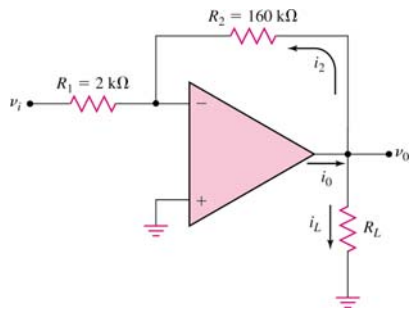
### 14.1

$$A_d = \frac{v_o}{v_i} = -80$$

$$v_o(\text{max}) = 4.5 \Rightarrow v_i(\text{max}) = 56.25 \text{ mV}$$

$$\text{So } v_i(\text{max})_{\text{rms}} = \frac{56.25}{\sqrt{2}} = \underline{\underline{39.77 \text{ mV}}}$$

### 14.2



(a)

$$i_2 = \frac{4.5}{160} = 0.028125 \text{ mA}$$

$$i_L = \frac{4.5}{1} = 4.5 \text{ mA}$$

Output Circuit = 4.528 mA

$$v_i = -\frac{v_o}{A} = \frac{-4.5}{80} \Rightarrow v_i = -0.05625 \text{ V}$$

(b)

$$i_o \approx 15 \text{ mA} = \frac{v_o}{R_L} = \frac{4.5}{R_L}$$

$$\Rightarrow \underline{\underline{R_L(\text{min}) = 300 \Omega}}$$

### 14.3

(1)  $v_o = 2 \text{ V}$

(2)  $v_2 = 12.5 \text{ mV}$

(3)  $A_{oL} = 2 \times 10^4$

(4)  $v_1 = 8 \mu\text{V}$

(5)  $A_{oL} = 1000$

**14.4**

$$(a) \quad A_{CL}(\infty) = -\frac{R_2}{R_1} = -\frac{120}{5.6} = -21.42857$$

$$A_{CL} = \frac{-21.42857}{1 + \frac{22.42857}{10^5}} = -21.42376$$

$$\frac{-21.42376 - (-21.42857)}{-21.42857} \times 100\% = -0.0224\%$$

$$(b) \quad A_{CL}(\infty) = -\frac{R_2}{R_1} = -\frac{120}{8.2} = -14.634146$$

$$A_{CL} = \frac{-14.634146}{1 + \frac{15.634146}{10^5}} = -14.63186$$

$$\frac{-14.63186 - (-14.634146)}{-14.634146} \times 100\% = -0.0156\%$$

**14.5**

$$(a) \quad (i) \quad A_{CL} = \frac{1 + \frac{47}{6.8}}{1 + \frac{\left(1 + \frac{47}{6.8}\right)}{2 \times 10^4}} = \frac{7.91176}{1 + \frac{7.91176}{2 \times 10^4}} = 7.90863$$

$$(ii) \quad \frac{7.90863 - 7.91176}{7.91176} \times 100\% = -0.03956\%$$

$$(b) \quad (i) \quad A_{CL} = \frac{7.91176}{1 + \frac{7.91176}{10^3}} = 7.84966$$

$$(ii) \quad \frac{7.84966 - 7.91176}{7.91176} \times 100\% = -0.785\%$$

**14.6**

$$(a) \quad -15.0 = \frac{-\frac{R_2}{R_1}}{1 + \frac{\left(1 + \frac{R_2}{R_1}\right)}{2 \times 10^3}} = \frac{-\frac{R_2}{R_1}}{1.0005 + 5 \times 10^{-4} \left(\frac{R_2}{R_1}\right)} \Rightarrow \frac{R_2}{R_1} = 15.12091$$

$$(b) \quad A_{CL} = \frac{-15.12091}{1 + \frac{16.12091}{5 \times 10^4}} = -15.1160$$

**14.7**

$$(1 - 0.0001)(90) = \frac{90}{1 + \frac{90}{A_{OL}}} \Rightarrow A_{OL} = 8.9991 \times 10^5$$

**14.8**

$$A_{CL} = (1 - 0.0002)(1) = \frac{1}{1 + \frac{1}{A_{OL}}} \Rightarrow A_{OL} = 4999$$

**14.9**

(a)  $|A| = + \frac{R_2}{R_1} = \frac{210(1 \pm 0.001)}{21(1 \pm 0.001)}$

$$|A|_{\max} = \frac{210.21}{20.979} = 10.02$$

$$|A|_{\min} = \frac{209.79}{21.021} = 9.98$$

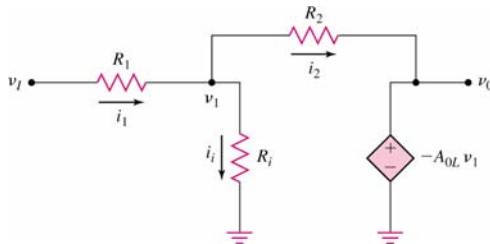
So  $9.98 \leq |A| \leq 10.02$

(b)  $|A|_{\max} = \frac{10.02}{1 + \frac{1}{10^4}} = 10.009$

$$|A|_{\min} = \frac{9.98}{1 + \frac{1}{10^4}} = 9.969$$

So  $9.969 \leq |A| \leq 10.009$

**14.10**



$$\frac{v_I - v_1}{R_1} = \frac{v_1 - v_0}{R_2} + \frac{v_1}{R_i} \text{ and } v_0 = -A_{OL} v_1$$

so that  $v_1 = -\frac{v_0}{A_{OL}}$

$$\frac{v_I + v_0}{R_1} + \frac{v_0}{R_2} = v_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right)$$

So

$$\frac{v_I}{R_1} = -v_0 \left[ \frac{1}{R_2} + \frac{1}{A_{OL}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right) \right]$$

Then

$$\frac{v_0}{v_I} = \frac{-(1/R_1)}{\left[ \frac{1}{R_2} + \frac{1}{A_{OL}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right) \right]} = A_{CL}$$

From Equation (14.20) for  $R_L = \infty$  and  $R_0 = 0$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2} \cdot \frac{(1 + A_{0L})}{1}$$

a. For  $R_i = 1 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[ \frac{1}{100} + \frac{1}{10^3} \left( \frac{1}{20} + \frac{1}{100} + \frac{1}{1} \right) \right]}$$

$$= \frac{-0.05}{[0.01 + 1.06 \times 10^{-3}]}$$

or

$$\Rightarrow A_{CL} = -4.52$$

$$\frac{1}{R_{if}} = \frac{1}{1} + \frac{1 + 10^3}{100} \Rightarrow R_{if} = 90.8 \Omega$$

b. For  $R_i = 10 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[ \frac{1}{100} + \frac{1}{10^3} \left( \frac{1}{20} + \frac{1}{100} + \frac{1}{10} \right) \right]}$$

$$= \frac{-0.05}{[0.01 + 1.6 \times 10^{-4}]}$$

or

$$\Rightarrow A_{CL} = -4.92$$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1 + 10^3}{100} \Rightarrow R_{if} = 98.9 \Omega$$

c. For  $R_i = 100 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[ \frac{1}{100} + \frac{1}{10^3} \left( \frac{1}{20} + \frac{1}{100} + \frac{1}{100} \right) \right]}$$

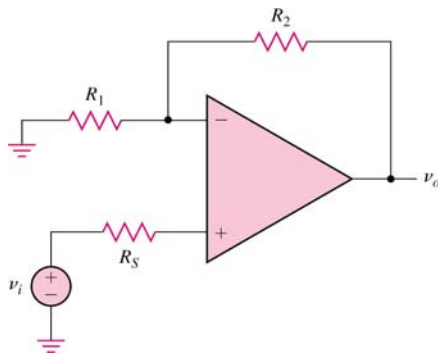
$$= \frac{-0.05}{[0.01 + 7 \times 10^{-5}]}$$

or

$$\Rightarrow A_{CL} = -4.965$$

$$\frac{1}{R_{if}} = \frac{1}{100} + \frac{1 + 10^3}{100} \Rightarrow R_{if} = 99.8 \Omega$$

14.11



$$A_{CL} = \frac{v_o}{v_i} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{OL}} \left(1 + \frac{R_2}{R_1}\right)\right]}$$

For the ideal:

$$\left(1 + \frac{R_2}{R_1}\right) = \frac{0.10}{0.002} = 50$$

$$v_o(\text{actual}) = (0.10)(1 - 0.001) = 0.0999$$

So

$$\frac{0.0999}{0.002} = \frac{50}{1 + \frac{1}{A_{OL}}(50)} = 49.95$$

which yields

$$\underline{A_{OL} = 1000}$$

### 14.12

From Equation (14.18)

$$A_{vfl} = \frac{v_{o1}}{v_i} = \frac{-\left(\frac{A_{OL}}{R_o} - \frac{1}{R_2}\right)}{\left(\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}\right)}$$

Or

$$v_{o1} = \frac{-\left(\frac{5 \times 10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{10} + \frac{1}{1} + \frac{1}{100}\right)} \cdot v_i = \frac{-(4.99999 \times 10^3)}{1.11} \cdot v_i$$

$$v_{o1} = -4.504495 \times 10^3 \cdot v_i$$

Now

$$\frac{i_i}{v_i} = \frac{v_i - v_1}{R_1 v_1} \equiv K$$

Then

$$v_i - v_1 = KR_1 v_1$$

which yields

$$v_1 = \frac{v_i}{KR_1 + 1}$$

Now, from Equation (14.20)

$$\begin{aligned} K &= \frac{1}{10} + \frac{1}{100} \left[ \frac{1 + 5 \times 10^3 + \frac{1}{10}}{1 + \frac{1}{10} + \frac{1}{100}} \right] \\ &= (0.1) + (0.01) \left[ \frac{5.0011 \times 10^3}{1.11} \right] = 45.15495 \end{aligned}$$

Then

$$v_1 = \frac{v_i}{(45.15495)(10) + 1} = \frac{v_i}{452.5495}$$

We find

$$v_{o1} = -4.504495 \times 10^3 \left[ \frac{v_i}{452.5495} \right]$$

Or

$$A_{vf1} = \frac{v_{o1}}{v_i} = -9.9536$$

For the second stage,  $R_L = \infty$

$$v_{o2} = -\frac{\left( \frac{5 \times 10^3}{1} - \frac{1}{100} \right)}{\left( 1 + \frac{1}{100} \right)} \cdot v_1' = -4.950485 \times 10^3 \cdot v_1'$$

$$K \equiv \frac{1}{10} + \frac{1}{100} \left[ \frac{1 + 5 \times 10^3}{1 + \frac{1}{100}} \right] = 49.61485$$

$$v_1' = \frac{v_{o1}}{KR_1 + 1} = \frac{v_{o1}}{(49.61485)(10) + 1} = \frac{v_{o1}}{497.1485}$$

Then

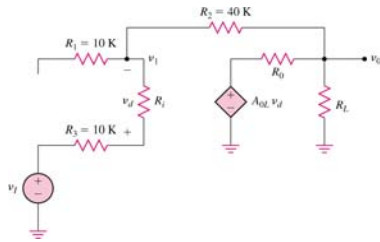
$$\frac{v_{o2}}{v_{o1}} = \frac{-4.950485 \times 10^3}{497.1485} = -9.95776$$

So

$$A_{vf} = \frac{v_{o2}}{v_i} = (-9.9536)(-9.95776) \Rightarrow \underline{A_{vf} = 99.12}$$

### 14.13

a.



$$\frac{v_1 - v_L}{R_3 + R_i} + \frac{v_1}{R_1} + \frac{v_1 - v_o}{R_2} = 0 \quad (1)$$

$$v_1 \left[ \frac{1}{R_3 + R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{v_o}{R_2} + \frac{v_L}{R_3 + R_i}$$

$$\frac{v_o}{R_L} + \frac{v_o - A_{0L} v_d}{R_0} + \frac{v_o - v_1}{R_2} = 0 \quad (2)$$

or

$$v_o \left[ \frac{1}{R_L} + \frac{1}{R_0} + \frac{1}{R_2} \right] = \frac{v_1}{R_2} + \frac{A_{0L} v_d}{R_0}$$

$$v_d = \left( \frac{v_L - v_1}{R_3 + R_i} \right) \cdot R_i \quad (3)$$

So substituting numbers:



$$v_1 \left[ \frac{1}{10+20} + \frac{1}{10} + \frac{1}{40} \right] = \frac{v_0}{40} + \frac{v_I}{10+20} \quad (1)$$

or

$$v_1[0.15833] = v_0[0.025] + v_I[0.03333]$$

$$v_0 \left[ \frac{1}{1} + \frac{1}{0.5} + \frac{1}{40} \right] = \frac{v_1}{40} + \frac{(10^4)v_d}{0.5} \quad (2)$$

or

$$v_0[3.025] = v_1[0.025] + (2 \times 10^4)v_d$$

$$v_d = \left( \frac{v_I - v_1}{10+20} \right) \cdot 20 = 0.6667(v_I - v_1) \quad (3)$$

So

$$v_0[3.025] = v_1[0.025] + (2 \times 10^4)(0.6667)(v_I - v_1) \quad (2)$$

or

$$v_0[3.025] = 1.333 \times 10^4 v_I - 1.333 \times 10^4 v_1$$

From (1):

$$v_1 = v_0(0.1579) + v_I(0.2105)$$

Then

$$v_0[3.025] = 1.333 \times 10^4 v_I - 1.333 \times 10^4 [v_0(0.1579) + v_I(0.2105)]$$

$$v_0[2.1078 \times 10^3] = v_I[1.0524 \times 10^4]$$

or

$$A_{CL} = \frac{v_0}{v_I} = 4.993$$

To find  $R_{if}$ : Use Equation (14.27)

$$i_I \left( 1 + \frac{0.5}{1} + \frac{0.5}{40} \right)$$

$$= v_1 \left\{ \left( \frac{1}{10} + \frac{1}{40} \right) \left( 1 + \frac{0.5}{1} + \frac{0.5}{40} \right) - \frac{0.5}{(40)^2} \right\} - \frac{(10^3)v_d}{40}$$

$$i_I(1.5125) = v_1 \{ (0.125)(1.5125) - 0.0003125 \} - 25v_d$$

or

$$i_I(1.5125) = v_I \{ 0.18875 \} - 25v_d$$

Now

$$v_d = i_I R_i = i_I(20) \quad \text{and} \quad v_1 = v_I - i_I(20)$$

So

$$i_I(1.5125) = [v_I - i_I(20)] \cdot [0.18875] - 25i_I(20)$$

$$i_I[505.3] = v_I(0.18875)$$

or

$$\frac{v_I}{i_I} = 2677 \text{ k}\Omega$$

Now  $R_{if} = 10 + 2677 \Rightarrow R_{if} = 2.687 \text{ M}\Omega$

To determine  $R_{of}$ : Using Equation (14.36)

$$\frac{1}{R'_{of}} = \frac{1}{R_0} \cdot \left[ \frac{A_{oL}}{1 + \frac{R_2}{R_1 \parallel R_i}} \right] = \frac{1}{0.5} \cdot \left[ \frac{10^3}{1 + \frac{40}{10 \parallel 20}} \right]$$

or  $R'_{of} = 3.5 \Omega$

Then  $R_{of} = 1 \text{ k}\Omega \parallel 3.5 \Omega$

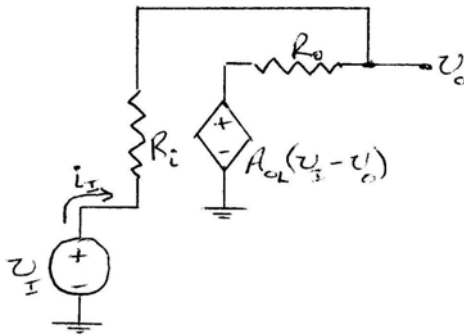
$\Rightarrow R_{of} = 3.49 \Omega$

b. Using Equation (14.16)

$$\frac{dA_{CL}}{A_{CL}} = (-10) \left( \frac{5}{10^3} \right) \Rightarrow \frac{dA_{CL}}{A_{CL}} = -(0.05)\%$$

**14.14**

(a)



(b) (i)  $\frac{v_I - v_o}{R_i} = \frac{v_o - A_{OL}(v_I - v_o)}{R_o}$

$$\frac{v_I}{R_i} + \frac{A_{OL}v_I}{R_o} = v_o \left( \frac{1}{R_i} + \frac{1}{R_o} + \frac{A_{OL}}{R_o} \right)$$

$$v_I \left( \frac{1}{10} + \frac{5 \times 10^3}{1} \right) = v_o \left( \frac{1}{10} + \frac{1}{1} + \frac{5 \times 10^3}{1} \right)$$

$$v_I (5.0001 \times 10^3) = v_o (5.0011 \times 10^3)$$

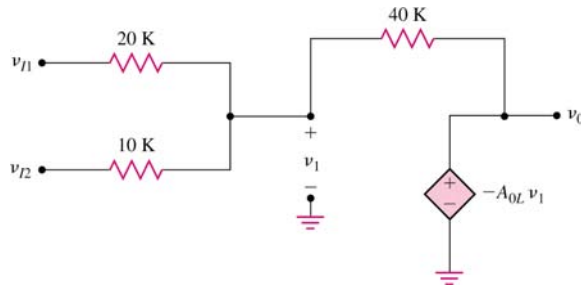
$$\frac{v_o}{v_I} = 0.9998$$

(ii)  $I_x = \frac{V_x - A_{OL}(-V_x)}{R_o} + \frac{V_x}{R_i}$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1 + A_{OL}}{R_o} + \frac{1}{R_i} = \frac{1 + 5 \times 10^3}{1} + \frac{1}{10}$$

$$R_{of} \cong 0.2 \Omega$$

14.15



$$\frac{v_{I1} - v_1}{20} + \frac{v_{I2} - v_1}{10} = \frac{v_1 - v_0}{40}$$

$$\frac{v_{I1}}{20} + \frac{v_{I2}}{10} + \frac{v_0}{40} = v_1 \left[ \frac{1}{20} + \frac{1}{10} + \frac{1}{40} \right]$$

and  $v_0 = -A_{OL} v_1$  so that  $v_1 = -\frac{v_0}{A_{OL}}$

Then

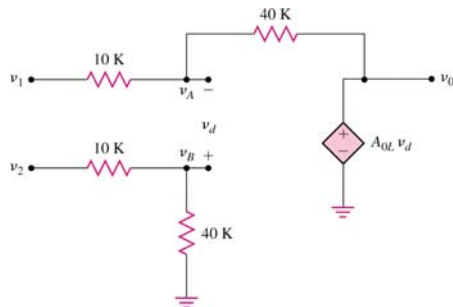
$$v_{I1}(0.05) + v_{I2}(0.10) = -v_0 \left\{ \frac{1}{40} + \frac{1}{2 \times 10^3} \cdot \left( \frac{7}{40} \right) \right\}$$

$$= -v_0 [2.50875 \times 10^{-2}]$$

$$\Rightarrow v_0 = -1.993 v_{I1} - 3.986 v_{I2}$$

$$\frac{\Delta v_0}{v_0} = \frac{2 - 1.993}{2} \Rightarrow \frac{\Delta v_0}{v_0} = 0.35\%$$

14.16



$$v_B = \left( \frac{40}{40 + 10} \right) v_2 = \left( \frac{4}{5} \right) v_2 = 0.8 v_2 \quad (1)$$

$$\frac{v_1 - v_A}{10} = \frac{v_A - v_0}{40}$$

$$\frac{v_1 + v_0}{10} + \frac{v_0}{40} = v_A \left( \frac{1}{10} + \frac{1}{40} \right)$$

$$v_1(0.1) + v_0(0.025) = v_A(0.125) \quad (2)$$

$$v_0 = A_{OL} v_d = A_{OL} (v_B - v_A) \quad (3)$$

or

$$v_0 = A_{0L}[0.8v_2 - v_A]$$

$$\frac{v_0}{A_{0L}} - 0.8v_2 = -v_A$$

$$\Rightarrow v_A = 0.8v_2 - \frac{v_0}{A_{0L}}$$

Then

$$v_1(0.1) + v_0(0.025) = (0.125) \left[ 0.8v_2 - \frac{v_0}{A_{0L}} \right]$$

$$v_1(0.1) - v_2(0.1) = -v_0 \left[ 0.025 + \frac{0.125}{10^3} \right]$$

$$= -v_0 [2.5125 \times 10^{-2}]$$

$$\Rightarrow A_d = \frac{v_0}{v_2 - v_1} = 3.9801$$

$$\Rightarrow \frac{\Delta A_d}{A_d} = \frac{0.0199}{4} \Rightarrow \underline{0.4975\%}$$

#### 14.17

a. Considering the second op-amp and Equation (14.20), we have

$$\frac{1}{R_{if2}} = \frac{1}{10} + \frac{1}{0.1} \cdot \left[ \frac{1+100}{1+\frac{1}{0.1}} \right] = 0.10 + \frac{101}{(0.1)(11)}$$

So  $R_{if2} = 0.0109 \text{ k}\Omega$

The effective load on the first op-amp is then

$$R_{L1} = 0.1 + R_{if2} = 0.1109 \text{ k}\Omega$$

Again using Equation (14.20), we have

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1}{1} \cdot \frac{1+100+\frac{1}{0.1109}}{1+\frac{1}{0.1109}+\frac{1}{1}} = 0.10 + \frac{110.017}{11.017}$$

so that

$$\underline{R_{if} = 99.1 \Omega}$$

b. To determine  $R_{of}$ :

For the first op-amp, we can write, using Equation (14.36)

$$\frac{1}{R_{of1}} = \frac{1}{R_0} \cdot \left[ \frac{A_{0L}}{1+\frac{R_2}{R_1 \parallel R_i}} \right] = \frac{1}{1} \cdot \left[ \frac{100}{1+\frac{40}{1 \parallel 10}} \right]$$

which yields  $R_{of1} = 0.021 \text{ k}\Omega$

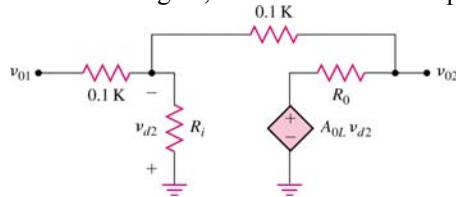
For the second op-amp, then

$$\frac{1}{R_{of}} = \frac{1}{R_0} \cdot \left[ \frac{A_{0L}}{1 + \frac{R_2}{(R_1 + R_{of1}) \parallel R_i}} \right]$$

$$= \frac{1}{1} \cdot \left[ \frac{100}{1 + \frac{0.10}{(0.121) \parallel 10}} \right]$$

or  $R_{of} = 18.4 \Omega$

c. To find the gain, consider the second op-amp.



$$\frac{v_{01} - (-v_{d2})}{0.1} + \frac{v_{d2}}{R_i} = \frac{-v_{d2} - v_{02}}{0.1} \quad (1)$$

$$\frac{v_{01}}{0.1} + v_{d2} \left( \frac{1}{0.1} + \frac{1}{10} + \frac{1}{0.1} \right) = -\frac{v_{02}}{0.1}$$

or

$$v_{01}(10) + v_{d2}(20.1) = -v_{02}(10)$$

$$\frac{v_{02} - A_{0L}v_{d2}}{R_0} + \frac{v_{02} - (-v_{d2})}{0.1} = 0 \quad (2)$$

$$\frac{v_{02}}{1} - v_{d2} \left( \frac{100}{1} - \frac{1}{0.1} \right) + \frac{v_{02}}{0.1} = 0$$

$$v_{02}(11) - v_{d2}(90) = 0$$

or

$$v_{d2} = v_{02}(0.1222)$$

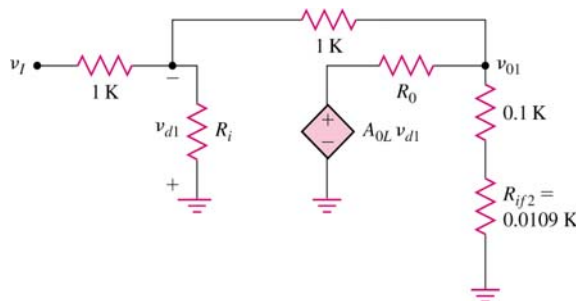
Then Equation (1) becomes

$$v_{01}(10) + v_{02}(0.1222)(20.1) = -v_{02}(10)$$

or

$$v_{01} = -v_{02}(1.246)$$

Now consider the first op-amp.



$$\frac{v_I - (-v_{d1})}{1} + \frac{v_{d1}}{R_i} = \frac{-v_{d1} - v_{01}}{1} \quad (1)$$

$$v_I(1) + v_{d1} \left( \frac{1}{1} + \frac{1}{10} + \frac{1}{1} \right) = -v_{01}(1)$$

or

$$v_I(1) + v_{d1}(2.1) = -v_{01}(1)$$

$$\frac{v_{01}}{0.1109} + \frac{v_{01} - A_{0L}v_{d1}}{R_o} + \frac{v_{01} - (-v_{d1})}{1} = 0 \quad (2)$$

$$v_{01} \left( \frac{1}{0.1109} + \frac{1}{1} + \frac{1}{1} \right) - v_{d1} \left( \frac{100}{1} - \frac{1}{1} \right) = 0$$

$$v_{01}(11.017) - v_{d1}(99) = 0$$

or

$$v_{d1} = v_{01}(0.1113)$$

Then Equation (1) becomes

$$v_I(1) + v_{01}(0.1113)(2.1) = -v_{01}$$

$$\text{or } v_I = -v_{01}(1.234)$$

We had  $v_{01} = -v_{02}(1.246)$

$$\text{So } v_I = v_{02}(1.246)(1.234)$$

$$\frac{v_{02}}{v_I} = 0.650$$

$$\text{or } \frac{v_{02}}{v_I} = 1$$

$$\text{d. Ideal } \frac{v_{02}}{v_I} = 1$$

So ratio of actual to ideal = 0.650.

#### 14.18

(a) For the op-amp.  $A_{0L} \cdot f_{3dB} = 10^6$

$$f_{3dB} = \frac{10^6}{2 \times 10^4} = 50 \text{ Hz}$$

For the closed-loop amplifier.

$$f_{3dB} = \frac{10^6}{25} = 40 \text{ kHz}$$

(b) Open-loop amplifier.

$$A = \frac{2 \times 10^4}{1 + j \frac{f}{f_{3dB}}} \Rightarrow |A| = \frac{2 \times 10^4}{\sqrt{1 + \left( \frac{f}{f_{3dB}} \right)^2}}$$

$$f = 0.25 f_{3dB} \Rightarrow |A| = \frac{2 \times 10^4}{\sqrt{1 + (0.25)^2}} = 1.94 \times 10^4$$

$$f = 5 f_{3dB} \Rightarrow |A| = \frac{2 \times 10^4}{\sqrt{1 + (5)^2}} = 3.92 \times 10^3$$

Closed-loop amplifier

$$f = 0.25 f_{3dB} \Rightarrow |A| = \frac{25}{\sqrt{1 + (0.25)^2}} = 24.25$$

$$f = 5 f_{3dB} \Rightarrow |A| = \frac{25}{\sqrt{1 + (5)^2}} = 4.90$$

**14.19**

$$A_o = 100 \text{ dB}, \Rightarrow A_o = 10^5$$

$$A = 38 \text{ dB}, A = 79.43$$

$$\text{Then } 79.43 = \frac{10^5}{\sqrt{1 + \left(\frac{10^4}{f_{PD}}\right)^2}}$$

$$\frac{10^4}{f_{PD}} \cong \frac{10^5}{79.43} \Rightarrow f_{PD} = 7.94 \text{ Hz}$$

$$GBW = (10^5)(7.94) = 7.94 \times 10^5 \text{ Hz}$$

**14.20**

$$(a) A_{CLO} = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{150}{15}\right) = 11$$

$$f_T = 1.2 \times 10^6 = (11)f_{3-dB} \Rightarrow f_{3-dB} = 109 \text{ kHz}$$

$$(b) A_{CLO} = \left[1 + \frac{(150)(1 \pm 0.05)}{(15)(1 \pm 0.05)}\right]$$

$$A_{CLO}(\text{max}) = 1 + \frac{157.5}{14.25} = 12.05$$

$$A_{CLO}(\text{min}) = 1 + \frac{142.5}{15.75} = 10.05$$

$$\text{Then } 10.05 \leq A_{CLO} \leq 12.05$$

$$f_T = 1.2 \times 10^6 = (12.05)f_{3-dB} \Rightarrow f_{3-dB} = 99.6 \text{ kHz}$$

$$f_T = 1.2 \times 10^6 = (10.05)f_{3-dB} \Rightarrow f_{3-dB} = 119.4 \text{ kHz}$$

$$\text{Then } 99.6 \leq f_{3-dB} \leq 119.4 \text{ kHz}$$

**14.21**

The open loop gain can be written as

$$A_{oL}(f) = \frac{A_o}{\left(1 + j \cdot \frac{f}{f_{PD}}\right) \left(1 + j \cdot \frac{f}{5 \times 10^6}\right)}$$

where  $A_o = 2 \times 10^5$ .

The closed-loop response is

$$A_{CL} = \frac{A_{oL}}{1 + \beta A_{oL}}$$

At low frequency,

$$100 = \frac{2 \times 10^5}{1 + \beta(2 \times 10^5)}$$

So that  $\beta = 9.995 \times 10^{-3}$ .

Assuming the second pole is the same for both the open-loop and closed-loop, then

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

For a phase margin of  $80^\circ$ ,  $\phi = -100^\circ$ .

So

$$-100 = -90 - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

or

$$f = 8.816 \times 10^5 \text{ Hz}$$

Then

$$\begin{aligned} |A_{OL}| &= 1 \\ &= \frac{2 \times 10^5}{\sqrt{1 + \left(\frac{8.816 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{8.816 \times 10^5}{5 \times 10^6}\right)^2}} \end{aligned}$$

or

$$\frac{8.816 \times 10^5}{f_{PD}} \cong 1.9696 \times 10^5$$

or

$$\underline{f_{PD} = 4.48 \text{ Hz}}$$

#### 14.22

(a) 1<sup>st</sup> stage

$$(10)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 100 \text{ kHz}$$

2<sup>nd</sup> stage

$$(50)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 20 \text{ kHz}$$

Bandwidth of overall system  $\cong 20 \text{ kHz}$

(b) If each stage has the same gain, so

$$K^2 = 500 \Rightarrow K = 22.36$$

Then bandwidth of each stage

$$(22.36)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 44.7 \text{ kHz}$$

#### 14.23

$$(a) A_{CLO} = \frac{-\frac{R_2}{R_1}}{1 + \frac{R_2/R_1}{A_O}} = \frac{-10.0}{1 + \frac{11}{5 \times 10^4}} = -9.9978$$

$$f_T = 1.5 \times 10^6 = (9.9978)f_{3-dB} \Rightarrow f_{3-dB} = 150.033 \text{ kHz}$$

$$(b) A_{CLO} = (-9.9978)^3 = -999.34$$

$$\text{At } f_{3-dB}; \Rightarrow |A_{CL}| = \frac{999.34}{\sqrt{2}} = 706.64$$



---

$$\text{Then } 706.64 = \frac{999.34}{\left[ \sqrt{1 + \left( \frac{f_{3-dB}}{150.033 \times 10^3} \right)^2} \right]^3}$$
$$\left[ 1 + \left( \frac{f_{3-dB}}{150.033 \times 10^3} \right)^2 \right]^3 = \left( \frac{999.34}{706.64} \right)^2 \Rightarrow f_{3-dB} = 76.49 \text{ kHz}$$

---

**14.24**

$$(5 \times 10^4) f_{PD} = 10^6 \Rightarrow f_{PD} = 20 \text{ Hz}$$

$$(25) f_{3-dB} 10^6 \Rightarrow f_{3-dB} = 40 \text{ kHz}$$

$$A_v = \frac{A_{vo}}{1 + j \frac{f}{f_{3-dB}}} \Rightarrow |A_v| = \frac{25}{\sqrt{1 + \left( \frac{f}{40 \times 10^3} \right)^2}}$$

At  $f = 0.5 f_{3-dB} = 20 \text{ kHz}$

$$|A_v| = \frac{25}{\sqrt{1 + (0.5)^2}} = 22.36$$

At  $f = 2 f_{3-dB} = 80 \text{ kHz}$

$$|A_v| = \frac{25}{\sqrt{1 + (2)^2}} = 11.18$$

---

**14.25**

$$(20 \times 10^3) \cdot |A_{vf}|_{MAX} = 10^6 \Rightarrow |A_{vf}|_{MAX} = 50$$

---

**14.26**

(a)  $f_{\max} = \frac{SR}{2\pi V_{PO}} = \frac{5 \times 10^6}{2\pi(5)} \Rightarrow f_{\max} = 159 \text{ kHz}$

(b)  $f_{\max} = \frac{5 \times 10^6}{2\pi(1.5)} \Rightarrow f_{\max} = 530.5 \text{ kHz}$

(c)  $f_{\max} = \frac{5 \times 10^6}{2\pi(0.4)} \Rightarrow f_{\max} = 1.99 \text{ MHz}$

---

**14.27**

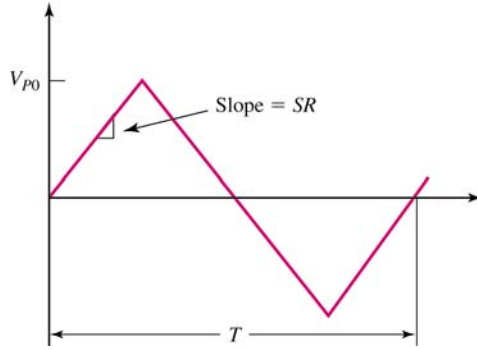
a. Using Equation (14.55),

$$V_{P0} = \frac{8 \times 10^6}{2\pi(250 \times 10^3)}$$

or

$$V_{P0} = 5.09 \text{ V}$$

b.



$$\text{Period } T = \frac{1}{f} = \frac{1}{250 \times 10^3} = 4 \times 10^{-6} \text{ s}$$

$$\text{One-fourth period} = 1 \mu\text{s}$$

$$\text{Slope} = \frac{V_{P0}}{1 \mu\text{s}} = SR = 8 \text{ V}/\mu\text{s}$$

$$\Rightarrow \underline{V_{P0} = 8 \text{ V}}$$

**14.28**

$$f_{\max} = \frac{SR}{2\pi V_{PO}}$$

$$SR = 2\pi(10)(12 \times 10^3) = 7.54 \times 10^5 \text{ V/s}$$

$$\text{Or } SR = 0.754 \text{ V}/\mu\text{s}$$

**14.29**

$$(a) f_{\max} = 20 \times 10^3 = \frac{0.63 \times 10^6}{2\pi V_{PO}} \Rightarrow V_{PO} = 5.0 \text{ V}$$

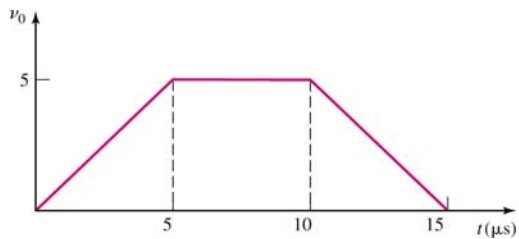
$$(b) V_{PO} = \frac{3 \times 10^6}{2\pi(20 \times 10^3)} = 23.87 \text{ V}$$

**14.30**

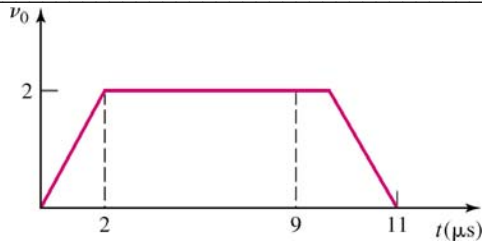
For input (a), maximum output is 5 V.

$$SR = 1 \text{ V}/\mu\text{s}$$

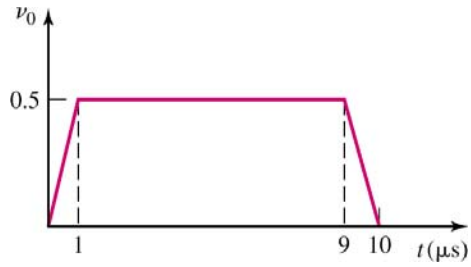
so



For input (b), maximum output is 2 V.

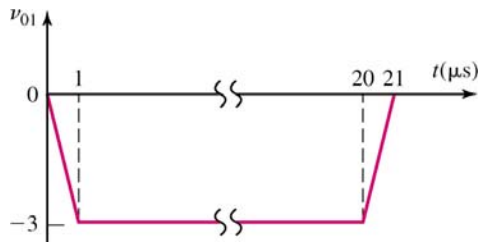


For input (c), maximum output is 0.5 V so the output is

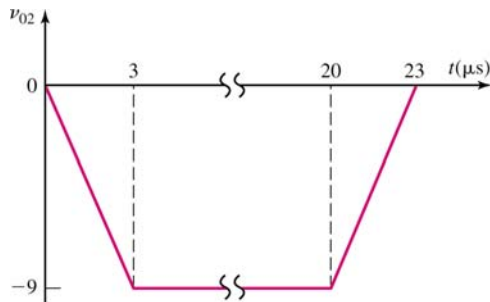


**14.31**

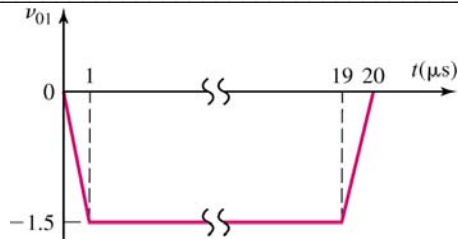
For input (a),  $\max |v_{o1}| = 3$  V.



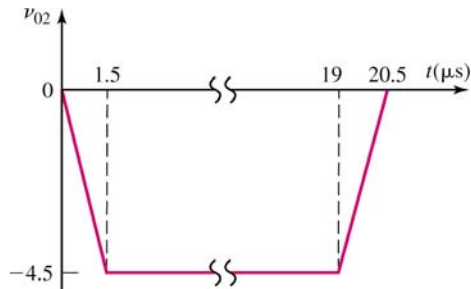
Then  $|v_{o2}|_{\max} = 3(3) = 9$  V



For input (b),  $\max |v_{o1}| = 1.5$  V.



Then  $|v_{02}|_{\max} = 3(1.5) = 4.5 \text{ V}$



#### 14.32

$$I_1 = I_{S1} \exp\left(\frac{V_{BE1}}{V_T}\right), \quad I_2 = I_{S2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

Want  $I_1 = I_2$ , so

$$\begin{aligned} \frac{I_1}{I_2} = 1 &= \frac{5 \times 10^{-14} (1+x) \exp\left(\frac{V_{BE1}}{V_T}\right)}{5 \times 10^{-14} (1-x) \exp\left(\frac{V_{BE2}}{V_T}\right)} \\ &= \frac{(1+x)}{(1-x)} \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right) \end{aligned}$$

Or

$$\begin{aligned} \frac{1+x}{1-x} &= \exp\left(\frac{V_{BE2} - V_{BE1}}{V_T}\right) = \exp\left(\frac{V_{OS}}{V_T}\right) \\ &= \exp\left(\frac{0.0025}{0.026}\right) = 1.10 \end{aligned}$$

Now

$$\begin{aligned} 1+x &= (1-x)(1.10) \Rightarrow \\ x &= 0.0476 \Rightarrow 4.76\% \end{aligned}$$

#### 14.33

- (a) Balanced circuit,  $I_{S4} = 5 \times 10^{-15} \text{ A}$   
 (b) From Eq. (14.62),  $v_{CE1} = 5 \text{ V}$ ,  $v_{CE2} = 5.6 - 1.2 = 4.4 \text{ V}$

$$\frac{1 + \frac{5}{120}}{1 + \frac{0.6}{80}} = \frac{I_{S3}}{I_{S4}} \cdot \frac{\left(1 + \frac{4.4}{120}\right)}{\left(1 + \frac{1.2}{80}\right)}$$

$$\frac{1.041667}{1.0075} = \frac{I_{S3}}{I_{S4}} \cdot \frac{(1.036667)}{(1.015)}$$

$$\frac{I_{S3}}{I_{S4}} = 1.0123 \Rightarrow I_{S4} = 4.939 \times 10^{-15} \text{ A}$$

(c)  $v_{CE1} = 5 \text{ V}$ ,  $v_{CE2} = 5.6 - 2.5 = 3.1 \text{ V}$

$$\frac{1 + \frac{5}{120}}{1 + \frac{0.6}{80}} = \frac{I_{S3}}{I_{S4}} \cdot \frac{\left(1 + \frac{3.1}{120}\right)}{\left(1 + \frac{2.5}{80}\right)}$$

$$\frac{1.041667}{1.0075} = \frac{I_{S3}}{I_{S4}} \cdot \frac{(1.025833)}{(1.03125)}$$

$$\frac{I_{S3}}{I_{S4}} = 1.03937 \Rightarrow I_{S4} = 4.811 \times 10^{-15} \text{ A}$$

**14.34**

$$K_n = 150 \mu \text{ A/V}^2$$

$$\Delta K_n = 150(1+x) - 150(1-x) = 300x \mu \text{ A/V}^2$$

$$|V_{os}| = \frac{1}{2} \sqrt{\frac{I_Q}{2K_n}} \left( \frac{\Delta K_n}{K_n} \right)$$

$$15 \times 10^{-3} = \frac{1}{2} \sqrt{\frac{200}{2(150)}} \left( \frac{300x}{150} \right) = 0.8165x \Rightarrow x = 0.01837$$

**14.35**

(a)  $v_o = -30(10 \pm 2) \times 10^{-3} = (-300 \pm 60) \times 10^{-3} \text{ V}$

So  $-0.360 \leq v_o \leq -0.240 \text{ V}$

(b)  $v_o = -30(100 \pm 2) \times 10^{-3} = -3 \pm 0.06 \text{ V}$

So  $-3.06 \leq v_o \leq -2.94 \text{ V}$

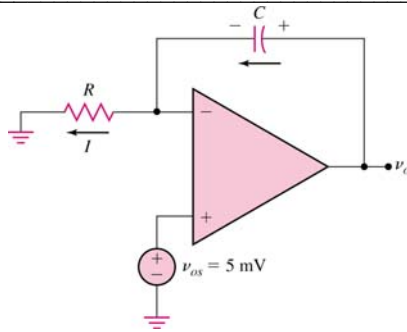
**14.36**

$$v_o = -30(25 \sin \omega t \pm 2) \text{ mV}$$

$$v_o = -0.75 \sin \omega t \pm 0.06 \text{ V}$$

So  $(-0.75 \sin \omega t - 0.06) \leq v_o \leq (-0.75 \sin \omega t + 0.06) \text{ V}$

**14.37**



$$I = \frac{0.5 \times 10^{-3}}{10^4} = 5 \times 10^{-8} \text{ A}$$

Also

$$I = C \frac{dV_o}{dt} \Rightarrow V_o = \frac{1}{C} \int I dt = \frac{I}{C} \cdot t$$

Then

$$5 = \frac{5 \times 10^{-8}}{10 \times 10^{-6}} t \Rightarrow t = 10^3 \text{ s}$$

**14.38**

(a)  $v_{o1} = \left(1 + \frac{100}{10}\right)(\pm 3) \text{ mV}, -33 \leq v_{o1} \leq 33 \text{ mV}$

$$v_{o2} = -\left(\frac{50}{10}\right)(\pm 33 \pm 3) \text{ mV}, -180 \leq v_{o2} \leq 180 \text{ mV}$$

(b)  $v_{o1} = (11)(10 \pm 3) \text{ mV}, \Rightarrow 77 \leq v_{o1} \leq 143 \text{ mV}$

$$v_{o2} = -5(143 + 3) = -730 \text{ mV}$$

$$v_{o2} = -5(77 - 3) = -370 \text{ mV}$$

So  $-0.73 \leq v_{o2} \leq -0.37 \text{ V}$

(c)  $v_{o1} = (11)(100 \pm 3) \text{ mV}$

$$1.067 \leq v_{o1} \leq 1.133 \text{ V}$$

$$v_{o2} = -5(1.133 + 0.003) = -5.68 \text{ V}$$

$$v_{o2} = -5(1.067 - 0.003) = -5.32 \text{ V}$$

So  $-5.68 \leq v_{o2} \leq -5.32 \text{ V}$

**14.39**

$v_o$  due to  $v_t$

$$v_o = (0.5) \left(1 + \frac{1}{1.1}\right) = 0.9545 \text{ V}$$

Wiper arm at  $V^+ = 10 \text{ V}$ , (using superposition)

$$v_1 = \left( \frac{R_1 \parallel R_5}{R_1 \parallel R_5 + R_4} \right) (10) = \left( \frac{0.0909}{0.0909 + 10} \right) (10) = 0.090$$

Then  $v_{01} = -\left( \frac{1}{1} \right) (0.090) = -0.090$

Wiper arm in center,  $v_1 = 0$  and  $v_{02} = 0$

Wiper arm at  $V^- = -10$  V,  $v_1 = -0.090$

So

$$v_{03} = 0.090$$

Finally, total output  $v_o$ : (from superposition)

Wiper arm at  $V^+$ ,

$$\underline{v_o = 0.8645 \text{ V}}$$

Wiper arm in center,

$$\underline{v_o = 0.9545 \text{ V}}$$

Wiper arm at  $V^-$ ,

$$\underline{v_o = 1.0445 \text{ V}}$$

#### 14.40

a.  $R'_1 = R'_2 = 0.5 \parallel 25 = 0.490 \text{ k}\Omega$

or

$$\underline{R'_1 = R'_2 = 490 \Omega}$$

b. From Equation (14.75),

$$(0.026) \ln \left( \frac{125 \times 10^{-6}}{2 \times 10^{-14}} \right) + (0.125)R'_1 = (0.026) \ln \left( \frac{125 \times 10^{-6}}{2.2 \times 10^{-14}} \right) + (0.125)R'_2$$

$$0.586452 + (0.125)R'_1 = 0.583974 + (0.125)R'_2$$

$$0.002478 = (0.125)(R'_2 - R'_1)$$

So  $R'_2 - R'_1 = 0.0198 \text{ k}\Omega \Rightarrow 19.8 \Omega$

Then

$$\frac{R_2(1-x)R_x}{R_2 + (1-x)R_x} - \frac{R_1 \times R_x}{R_1 + xR_x} = 0.0198$$

$$\frac{(0.5)(1-x)(50)}{(0.5) + (1-x)(50)} - \frac{(0.5)(50)x}{(0.5) + x(50)} = 0.0198$$

$$\frac{25(1-x)}{50.5 - 50x} - \frac{25x}{0.5 + 50x} = 0.0198$$

$$\frac{(0.5 + 50x)(25 - 25x) - (25x)(50.5 - 50x)}{(50.5 - 50x)(0.5 + 50x)} = 0.0198$$

$$25\{0.5 - 0.5x + 50x - 50x^2 - 50.5x + 50x^2\} = 0.0198\{25.25 + 2525x - 25x - 2500x^2\}$$

$$25\{0.5 - x\} = 0.0198\{25.25 + 2500x - 2500x^2\}$$

$$0.5 - x = 0.019998 + 1.98x - 1.98x^2$$

$$1.98x^2 - 2.98x + 0.48 = 0$$

$$x = \frac{2.98 \pm \sqrt{(2.98)^2 - 4(1.98)(0.48)}}{2(1.98)}$$

So

$$x = 0.183$$

and

$$1 - x = 0.817$$

#### 14.41

$$R'_1 = R_1 \parallel 15 = 0.5 \parallel 15 = 0.4839 \text{ k}\Omega$$

$$R'_2 = R_2 \parallel 35 = 0.5 \parallel 35 = 0.4930 \text{ k}\Omega$$

From Equation (14.75),

$$(0.026) \ln \left( \frac{i_{C1}}{I_{S3}} \right) + i_{C1} R'_1 = (0.026) \ln \left( \frac{i_{C2}}{I_{S4}} \right) + i_{C2} R'_2$$

$$(0.026) \ln \left( \frac{i_{C1}}{i_{C2}} \right) = i_{C2} R'_2 - i_{C1} R'_1$$

$$(0.026) \ln \left( \frac{i_{C1}}{i_{C2}} \right) = i_{C2} R'_2 \left[ 1 - \frac{i_{C1}}{i_{C2}} \cdot \frac{R'_1}{R'_2} \right]$$

$$(0.026) \ln \left( \frac{i_{C1}}{i_{C2}} \right) = i_{C2} (0.4930) \left[ 1 - (0.9815) \left( \frac{i_{C1}}{i_{C2}} \right) \right]$$

By trial and error:

$$\underline{i_{C1} = 252 \mu\text{A}} \quad \text{and} \quad \underline{i_{C2} = 248 \mu\text{A}}$$

or

$$\underline{\frac{i_{C1}}{i_{C2}} = 1.0155}$$

#### 14.42

$$(a) \quad v_O \Big|_{(1 \mu\text{A})} = (10^{-6}) (200 \times 10^3) = 0.2 \text{ V}$$

Insert resistor  $R_3$

$$v_O \Big|_{(2 \mu\text{A})} = -0.2 = -(2 \times 10^{-6}) R_3 \left( 1 + \frac{200}{20} \right) \Rightarrow R_3 = 9.09 \text{ k}\Omega$$

$$(b) \quad v_O \Big|_{(0.8 \mu\text{A})} = (0.8 \times 10^{-6}) (200 \times 10^3) = 0.16 \text{ V}$$

$$v_O \Big|_{(0.5 \mu\text{A})} = -0.16 = -(0.5 \times 10^{-6}) R_3 \left( 1 + \frac{200}{20} \right) \Rightarrow R_3 = 29.09 \text{ k}\Omega$$



**14.43**

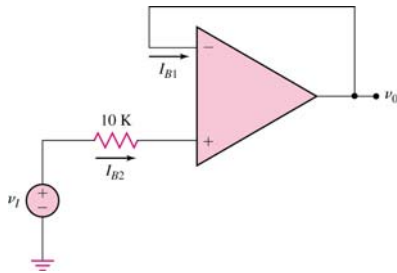
- (a)  $v_o = -I_B R_2 = -(2 \times 10^{-6})(150 \times 10^3) = -0.3 \text{ V}$   
 (b)  $v_o = -\frac{150}{15}(0.02) - 0.3 = -0.5 \text{ V}$   
 (c)  $v_o = -\frac{150}{15}(-0.02) - 0.3 = -0.1 \text{ V}$   
 (d)  $v_o = -\frac{150}{15}(0.1) - 0.3 = -1.3 \text{ V}$

**14.44**

- (a)  $v_o = (0.6 \times 10^{-6})(250 \times 10^3) = 0.15 \text{ V}$   
 (b)  $v_o = (41)(0.008) + 0.15 = 0.478 \text{ V}$   
 (c)  $v_o = (41)(-0.0035) + 0.15 = 0.0065 \text{ V}$   
 (d)  $v_o = (41)(0.005 \sin \omega t) + 0.15 = 0.205 \sin \omega t + 0.15 \text{ (V)}$

**14.45**

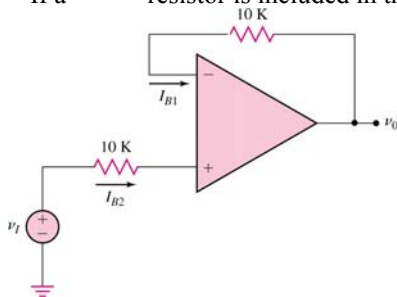
a.



For  $I_{B2} = 1 \mu\text{A}$ , then  $v_o = -(10^{-6})(10^4)$

or  $v_o = -0.010 \text{ V}$

b. If a  $10 \text{ k}\Omega$  resistor is included in the feedback loop



Now  $v_o = -I_{B2}(10) + I_{B1}(10) = 0$

Circuit is compensated if  $I_{B1} = I_{B2}$ .

**14.46**

From Equation (14.83), we have

$$v_0 = R_2 I_{0s}$$

where  $R_2 = 40 \text{ k}\Omega$  and  $I_{0s} = 3 \text{ }\mu\text{A}$ .

Then

$$v_0 = (40 \times 10^3)(3 \times 10^{-6})$$

or

$$\underline{v_0 = 0.12 \text{ V}}$$

**14.47**

- a. Assume all bias currents are in the same direction and into each op-amp.

$$v_{01} = I_{B1}(100 \text{ k}\Omega) = (10^{-6})(10^5) \Rightarrow \underline{v_{01} = 0.1 \text{ V}}$$

Then

$$\begin{aligned} v_{02} &= v_{01}(-5) + I_{B1}(50 \text{ k}\Omega) \\ &= (0.1)(-5) + (10^{-6})(5 \times 10^4) \\ &= -0.5 + 0.05 \end{aligned}$$

or

$$\underline{v_{02} = -0.45 \text{ V}}$$

- b. Connect  $R_3 = 10 \parallel 100 = 9.09 \text{ k}\Omega$  resistor to noninverting terminal of first op-amp, and  $R_3 = 10 \parallel 50 = 8.33 \text{ k}\Omega$  resistor to noninverting terminal of second op-amp.

**14.48**

- a. For a constant current through a capacitor.

$$v_0 = \frac{1}{C} \int_0^t I dt$$

$$\text{or } v_0 = \frac{0.1 \times 10^{-6}}{10^{-6}} \cdot t \Rightarrow \underline{v_0 = (0.1)t}$$

b. At  $t = 10 \text{ s}$ ,  $v_0 = 1 \text{ V}$

c. Then

$$v_0 = \frac{100 \times 10^{-12}}{10^{-6}} \cdot t \Rightarrow v_0 = (10^{-4})t$$

At  $t = 10 \text{ s}$ ,  $\underline{v_0 = 1 \text{ mV}}$

**14.49**

(a)  $v_{01} = (3 \times 10^{-6})(50 \times 10^3) = 0.15 \text{ V}$

$$v_{02} = 0.15 \text{ V}$$

$$v_{03} = -\frac{20}{20}(0.15) + (3 \times 10^{-6})(20 \times 10^3) = -0.09 \text{ V}$$

(b)  $R_A = 10 \parallel 50 = 8.33 \text{ k}\Omega$

$$R_B = 20 \parallel 20 = 10 \text{ k}\Omega$$

(c)  $v_{01} = \pm(50 \times 10^3)(0.3 \times 10^{-6}) = \pm 0.015 \text{ V}$

$$v_{02} = \pm 0.015 \text{ V}$$

$$v_{03} = \pm(20 \times 10^3)(0.3 \times 10^{-6}) \pm 0.015 = \pm 0.021 \text{ V}$$

**14.50**

- a. Using Equation (14.79),  
Circuit (a),

$$v_o = (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(25 \times 10^3) \left(1 + \frac{50}{50}\right)$$

or

$$v_o = 0$$

Circuit (b),

$$\begin{aligned} v_o &= (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(10^3) \left(1 + \frac{50}{50}\right) \\ &= 4 \times 10^{-2} - 1.6 \end{aligned}$$

or

$$v_o = -1.56 \text{ V}$$

- b. Assume  $I_{B1} = 0.7 \mu\text{A}$  and  $I_{B2} = 0.9 \mu\text{A}$ , then using Equation (14.79):  
Circuit (a),

$$\begin{aligned} v_o &= (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(25 \times 10^3) \left(1 + \frac{50}{50}\right) \\ &= 0.035 - 0.045 \end{aligned}$$

or

$$v_o = -0.010 \text{ V}$$

Circuit (b),

$$\begin{aligned} v_o &= (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(10^3) \left(1 + \frac{50}{50}\right) \\ &= 0.035 - 1.8 \end{aligned}$$

or

$$v_o = -1.765 \text{ V}$$

---

### 14.51

(a) For  $V_{OS}$ :  $v_o = \left(1 + \frac{100}{10}\right)(\pm 3) = \pm 33 \text{ mV}$

For  $I_B$ :  $v_o(\text{max}) = (0.43 \times 10^{-6})(100 \times 10^3) = 0.043 \text{ V}$

$$v_o(\text{max}) = (0.37 \times 10^{-6})(100 \times 10^3) = 0.037 \text{ V}$$

So  $4 \leq v_o \leq 76 \text{ mV}$

(b) For  $V_{OS}$ :  $v_o = \pm 33 \text{ mV}$

For  $I_{OS}$ :  $v_o = \pm (0.06 \times 10^{-6})(100 \times 10^3) = \pm 0.006 \text{ V}$

So  $-39 \leq v_o \leq 39 \text{ mV}$

(c)  $v_o = \left(1 + \frac{100}{10}\right)(0.2) \pm 0.039$

So  $2.161 \leq v_o \leq 2.239 \text{ V}$

---

### 14.52

a. 
$$\left(\frac{R_i}{R_i + R_2}\right)(15) = 0.010 \text{ V}$$

$$\frac{15}{15 + R_2} = 0.0006667$$

$$15(1 - 0.0006667) = 0.0006667 R_2$$

Then

$$R_2 = 22.48 \text{ M}\Omega$$

b.  $R_1 = R_i \parallel R_f = 15 \parallel 10 \Rightarrow R_1 = 6 \text{ k}\Omega$

**14.53**

- a. Assume the offset voltage polarities are such as to produce the worst case values, but the bias currents are in the same direction.

Use superposition:

Offset voltages

$$|v_{01}| = \left(1 + \frac{100}{10}\right)(10) = 110 \text{ mV} = |v_{01}|$$

$$|v_{02}| = (5)(110) + \left(1 + \frac{50}{10}\right)(10)$$

$$\Rightarrow |v_{02}| = 610 \text{ mV}$$

Bias Currents:

$$v_{01} = I_B(100 \text{ k}\Omega) = (2 \times 10^{-6})(100 \times 10^3) = 0.2 \text{ V}$$

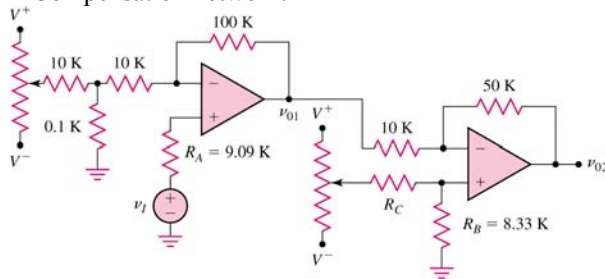
Then

$$v_{02} = (-5)(0.2) + (2 \times 10^{-6})(50 \times 10^3) = -0.9 \text{ V}$$

Worst case:  $v_{01}$  is positive and  $v_{02}$  is negative, then

$$\underline{v_{01} = 0.31 \text{ V}} \text{ and } \underline{v_{02} = -1.51 \text{ V}}$$

- b. Compensation network:



If we want

$$\left(\frac{R_B}{R_B + R_C}\right)V^+ = 20 \text{ mV and } V^+ = 10 \text{ V}$$

$$\left(\frac{8.33}{8.33 + R_C}\right)(10) = 0.020$$

or  $R_C \cong 4.15 \text{ M}\Omega$

**14.54**

(a) Offset voltage:

$$v_{o1} = \left(1 + \frac{50}{10}\right)(\pm 2) = \pm 12 \text{ mV}$$

$$v_{o2} = \pm 12 \pm 2 = \pm 14 \text{ mV}$$

$$v_{o3} = \left(-\frac{20}{20}\right)(\pm 12) + (2)(\pm 2) = \pm 16 \text{ mV}$$

Bias current:

$$v_{o1} = (0.21 \times 10^{-6})(50 \times 10^3) = 0.0105 \text{ V}$$

$$\text{or } v_{o1} = (0.19 \times 10^{-6})(50 \times 10^3) = 0.0095 \text{ V}$$

$$v_{o2} = v_{o1}$$

$$v_{o3} = (-1)(v_{o1}) + (0.21 \times 10^{-6})(20 \times 10^3) = -v_{o1} + 0.0042$$

$$\text{or } v_{o3} = -v_{o1} + (0.19 \times 10^{-6})(20 \times 10^3) = -v_{o1} + 0.0038$$

By superposition

$$-2.5 \leq v_{o1} \leq 22.5 \text{ mV}$$

$$-4.5 \leq v_{o2} \leq 24.5 \text{ mV}$$

$$-22.3 \leq v_{o3} \leq 10.7 \text{ mV}$$

(b) Bias currents:

$$v_{o1} = \pm I_{OS}(50 \times 10^3) = \pm(0.02 \times 10^{-6})(50 \times 10^3) \Rightarrow v_{o1} = \pm 1 \text{ mV}$$

$$v_{o3} = \pm I_{OS}(20 \times 10^3) = \pm(0.02 \times 10^{-6})(20 \times 10^3) \Rightarrow v_{o3} = \pm 0.4 \text{ mV}$$

By superposition:  $v_{o3} = \pm v_{o1} \pm 2(2) \pm 0.4$

$$-13 \leq v_{o1} \leq 13 \text{ mV}$$

$$-15 \leq v_{o2} \leq 15 \text{ mV}$$

$$-17.4 \leq v_{o3} \leq 17.4 \text{ mV}$$

#### 14.55

For circuit (a), effect of bias current:

$$v_o = (50 \times 10^3)(100 \times 10^{-9}) \Rightarrow 5 \text{ mV}$$

Effect of offset voltage

$$v_o = (2) \left(1 + \frac{50}{50}\right) = 4 \text{ mV}$$

So net output voltage is  $\underline{v_o = 9 \text{ mV}}$

For circuit (b), effect of bias current:

Let  $I_{B2} = 550 \text{ nA}$ ,  $I_{B1} = 450 \text{ nA}$ , then from Equation (14.79),

$$\begin{aligned} v_o &= (450 \times 10^{-9})(50 \times 10^3) - (550 \times 10^{-9})(10^6) \left(1 + \frac{50}{50}\right) \\ &= 2.25 \times 10^{-2} - 1.1 \end{aligned}$$

or

$$v_o = -1.0775 \text{ V}$$

If the offset voltage is negative, then

$$v_o = (-2)(2) = -4 \text{ mV}$$

So the net output voltage is

$$\underline{v_o = -1.0815 \text{ V}}$$

#### 14.56

- a. At  $T = 25^\circ\text{C}$ ,  $V_{os} = 2 \text{ mV}$  so the output voltage for each circuit is  

$$v_o = 4 \text{ mV}$$
- b. For  $T = 50^\circ\text{C}$ , the offset voltage for is  

$$V_{os} = 2 \text{ mV} + (0.0067)(25) = 2.1675 \text{ mV}$$
  
 so the output voltage for each circuit is  

$$v_o = 4.335 \text{ mV}$$

**14.57**

- a. At  $T = 25^\circ\text{C}$ ,  $V_{os} = 1 \text{ mV}$ , then  

$$v_{o1} = (1) \left( 1 + \frac{50}{10} \right) \Rightarrow v_{o1} = 6 \text{ mV}$$
  
 and  

$$v_{o2} = v_{o1} \left( 1 + \frac{60}{20} \right) + (1) \left( 1 + \frac{60}{20} \right)$$

$$= 6(4) + (1)(4) \Rightarrow v_{o2} = 28 \text{ mV}$$
- b. At  $T = 50^\circ\text{C}$ ,  $V_{os} = 1 + (0.0033)(25) = 1.0825 \text{ mV}$ , then  

$$v_{o1} = (1.0825)(6) \Rightarrow v_{o1} = 6.495 \text{ mV}$$
  
 and  

$$v_{o2} = (6.495)(4) + (1.0825)(4)$$
  
 or  

$$v_{o2} = 30.31 \text{ mV}$$

**14.58**

- $25^\circ\text{C}; I_B = 500 \text{ nA}, I_{os} = 200 \text{ nA}$   
 $50^\circ\text{C}, I_B = 500 \text{ nA} + (8 \text{ nA}/^\circ\text{C})(25^\circ\text{C}) = 700 \text{ nA}$   
 $I_{os} = 200 \text{ nA} + (2 \text{ nA}/^\circ\text{C})(25^\circ\text{C}) = 250 \text{ nA}$
- a. Circuit (a): For  $I_B$ , bias current cancellation,  $v_o = 0$   
 Circuit (b): For  $I_B$ , Equation (14.79),  

$$v_o = (500 \times 10^{-9})(50 \times 10^3) - (500 \times 10^{-9})(10^6) \left( 1 + \frac{50}{50} \right)$$

$$= 0.025 - 1.00 \Rightarrow v_o = -0.975 \text{ V}$$
- b. Due to offset bias currents.  
 Circuit (a):  

$$v_o = (200 \times 10^{-9})(50 \times 10^3) \Rightarrow v_o = 0.010 \text{ V}$$
  
 Circuit (b):  
 Let  $I_{B2} = 600 \text{ nA}$   
 $I_{B1} = 400 \text{ nA}$   
 Then  

$$v_o = (400 \times 10^{-9})(50 \times 10^3) - (600 \times 10^{-9})(10^6) \left( 1 + \frac{50}{50} \right)$$

$$= 0.020 - 1.20 \Rightarrow v_o = -1.18 \text{ V}$$
- c. Circuit (a): Due to  $I_B$ ,  $v = 0$   
 Circuit (b): Due to  $I_B$ ,

$$v_0 = (700 \times 10^{-9})(50 \times 10^3) - (700 \times 10^{-9})(10^6) \left(1 + \frac{50}{50}\right)$$

$$= 0.035 - 1.40 \Rightarrow v_0 = \underline{-1.365 \text{ V}}$$

Circuit (a): Due to  $I_{os}$ ,

$$v_0 = (250 \times 10^{-9})(50 \times 10^3) \Rightarrow v_0 = \underline{0.0125 \text{ V}}$$

Circuit (b): Due to  $I_{os}$ ,

Let  $I_{B2} = 825 \text{ nA}$

$I_{B1} = 575 \text{ nA}$

Then

$$v_0 = (575 \times 10^{-9})(50 \times 10^3) - (825 \times 10^{-9})(10^6) \left(1 + \frac{50}{50}\right)$$

$$= 0.02875 - 1.65 \Rightarrow v_0 = \underline{-1.62 \text{ V}}$$

### 14.59

$$25^\circ\text{C}; I_B = 2 \mu\text{A}, I_{os} = 0.2 \mu\text{A}$$

$$50^\circ\text{C}; I_B = 2 \mu\text{A} + (0.020 \mu\text{A}/^\circ\text{C})(25^\circ\text{C}) = 2.5 \mu\text{A}$$

$$I_{os} = 0.2 \mu\text{A} + (0.005 \mu\text{A}/^\circ\text{C})(25^\circ\text{C}) = 0.325 \mu\text{A}$$

a. Due to  $I_B$ : (Assume bias currents into op-amp).

$$v_{01} = I_B(50 \text{ k}\Omega) = (2 \times 10^{-6})(50 \times 10^3)$$

$$\Rightarrow v_{01} = \underline{0.10 \text{ V}}$$

$$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_B(60 \text{ k}\Omega) - I_B(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.1)(4) + (2 \times 10^{-6})(60 \times 10^3) - (2 \times 10^{-6})(60 \times 10^3)4$$

$$v_{02} = \underline{0.12 \text{ V}}$$

or

b. Due to  $I_{os}$ :

1st op-amp. Let  $I_{B1} = 2.1 \mu\text{A}$

2nd op-amp. Let  $I_{B1} = 2.1 \mu\text{A}$

$I_{B2} = 1.9 \mu\text{A}$

$$v_{01} = I_{B1}(50 \text{ k}\Omega) = (2.1 \times 10^{-6})(50 \times 10^3)$$

$$\Rightarrow v_{01} = \underline{0.105 \text{ V}}$$

$$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_{B1}(60 \text{ k}\Omega) - I_{B2}(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.105)(4) + (2.1 \times 10^{-6})(60 \times 10^3) - (1.9 \times 10^{-6})(50 \times 10^3)(4)$$

or

$$v_{02} = \underline{0.166 \text{ V}}$$

c. Due to  $I_B$ :

$$v_{01} = (2.5 \times 10^{-6})(50 \times 10^3) \Rightarrow v_{01} = \underline{0.125 \text{ V}}$$

$$v_{01} = v_{02} \left(1 + \frac{60}{20}\right) + I_B(60 \text{ k}\Omega) - I_B(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.125)(4) + (2.5 \times 10^{-6})(60 \times 10^3) - (2.5 \times 10^{-6})(50 \times 10^3)(4)$$

or  $v_{02} = \underline{0.15 \text{ V}}$

Due to  $I_{os}$ :

Let  $I_{B1} = 2.625 \mu\text{A}$

$I_{B2} = 2.3375 \mu\text{A}$

$v_{01} = I_{B1}(50 \text{ k}\Omega) = (2.6625 \times 10^{-6})(50 \times 10^3)$

$\Rightarrow v_{01} = 1.133 \text{ V}$

$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_{B1}(60 \text{ k}\Omega) - I_{B2}(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$

$= (0.133)(4) + (2.6625 \times 10^{-6})(60 \times 10^3) - (2.3375 \times 10^{-6})(50 \times 10^3)(4)$

or

$v_{02} = 0.224 \text{ V}$

### 14.60

(a)  $A_d = \frac{50}{10} = 5.0$

For common-mode,  $v_{i1} = v_{i2}$

From Chapter 9,

$$A_{cm} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left(1 + \frac{R_3}{R_4}\right)} - \frac{R_2}{R_1}$$

If  $R_2 = 50(1.015) = 50.75$ ,  $R_1 = 10(1 - 0.015) = 9.85$

$R_3 = 10(1 - 0.015) = 9.85$ ,  $R_4 = 50(1.015) = 50.75$

Then  $A_{cm} = \frac{1 + \frac{50.75}{9.85}}{1 + \frac{9.85}{50.75}} - \frac{50.75}{9.85} = \frac{6.15228}{1.19409} - 5.15228 = 5.046 \times 10^{-6}$

If  $R_3 = 10(1.015) = 10.15$ ,  $R_4 = 50(1 - 0.015) = 49.25$

Then  $A_{cm} = \frac{1 + \frac{50.75}{9.85}}{1 + \frac{10.15}{49.25}} - \frac{50.75}{9.85} = \frac{6.15228}{1.20609} - 5.15228 = -0.051268$

If  $R_2 = 49.25$ ,  $R_1 = 10.15$

Then  $A_{cm} = \frac{1 + \frac{49.25}{10.15}}{1 + \frac{9.85}{50.75}} - \frac{49.25}{10.15} = \frac{5.85222}{1.19409} - 4.85222 = +0.04877$

Now  $CMRR_{dB}(\text{min}) = 20 \log_{10} \left( \frac{5}{0.051268} \right) = 39.8 \text{ dB}$

(b)  $R_2 = 50(1.03) = 51.5$ ,  $R_1 = 10(0.97) = 9.70$

$R_4 = 50(0.97) = 48.5$ ,  $R_3 = 10(1.03) = 10.3$



$$A_{cm} = \frac{1 + \frac{51.5}{9.70}}{1 + \frac{10.3}{48.5}} - \frac{51.5}{9.70} = \frac{6.30928}{1.21237} - 5.30928 = -0.10519$$

$$CMRR_{dB} = 20 \log_{10} \left( \frac{5}{0.10519} \right) = 33.5 \text{ dB}$$

**14.61**

(a)  $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$

$$|A_{cm}| = \frac{5}{316.2} = 0.01581$$

From Problem 14.60,

$$A_{cm} = -0.01581 = \frac{1 + \frac{50(1+x)}{10(1-x)}}{1 + \frac{10(1+x)}{50(1-x)}} - \frac{50(1+x)}{10(1-x)}$$

$x$  is small, so that  $\frac{1}{1-x} \cong 1+x$

$$\text{Then } -0.01581 = \frac{1 + 5(1+x)(1+x)}{1 + 0.2(1+x)(1+x)} - 5(1+x)(1+x)$$

Neglect  $x^2$ ,

$$-0.01581 \cong \frac{1 + 5(1 + 2x)}{1 + 0.2(1 + 2x)} - 5(1 + 2x)$$

$$(-0.01581)[1 + 0.2(1 + 2x)] = 1 + 5(1 + 2x) - 5(1 + 2x)[1 + 0.2(1 + 2x)]$$

$$\text{We find } (1 + 2x)^2 - 0.003162(1 + 2x) - 1.01581 = 0 \Rightarrow (1 + 2x) = 1.009456$$

$$\text{Then } x = 0.004728 \Rightarrow x = 0.4728\%$$

(b)  $CMRR_{dB} = 75 \text{ dB}, \Rightarrow CMRR = 5623.4$

$$A_{cm} = \frac{5}{5623.4} = 0.00088914$$

$$\text{Then } (-0.00088914)[1 + 0.2(1 + 2x)] = 1 - (1 + 2x)^2$$

$$(1 + 2x)^2 - 0.000177828(1 + 2x) - 1.00088914 = 0 \Rightarrow (1 + 2x) = 1.0005334 \Rightarrow x = 0.0267\%$$

## Chapter 15

### 15.1

(a) Noninverting amplifier

$$8 = 1 + \frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = 7 \Rightarrow R_2 = 210 \text{ k}\Omega, R_1 = 30 \text{ k}\Omega$$

At noninverting terminal

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi(30 \times 10^3)} = 5.305 \times 10^{-6}$$

Let input  $C = 0.001 \mu\text{F}$ , then  $R = 5.305 \text{ k}\Omega$

(b) Set  $R_1 = 15 \text{ k}\Omega$ ,  $R_2 = 300 \text{ k}\Omega$  for inverting amplifier

$$R_1 C = \frac{1}{2\pi f} = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Put  $C$  in series with  $R_1$

$$C = \frac{7.958 \times 10^{-6}}{15 \times 10^3} \Rightarrow C = 530.5 \text{ pF}$$

### 15.2

(a)  $|T| = \frac{1}{\sqrt{1+(1.5)^4}} = 0.4061 \Rightarrow -7.83 \text{ dB}$

(b)  $|T| = \frac{1}{\sqrt{1+(1.5)^6}} = 0.2841 \Rightarrow -10.93 \text{ dB}$

(c)  $|T| = \frac{1}{\sqrt{1+(1.5)^8}} = 0.1938 \Rightarrow -14.25 \text{ dB}$

(d)  $|T| = \frac{1}{\sqrt{1+(1.5)^{10}}} = 0.1306 \Rightarrow -17.68 \text{ dB}$

### 15.3

$|T| = -6 \text{ dB}, \Rightarrow |T| = 0.50$

$$0.50 = \frac{1}{\sqrt{1+\left(\frac{1}{0.9}\right)^{2N}}} = \frac{1}{\sqrt{1+(1.111)^{2N}}}$$

or  $(1.111)^{2N} = \left(\frac{1}{0.5}\right)^2 - 1 = 3$

For  $N=6$ ,  $(1.111)^{(2)(6)} = 3.54$ ,  $\Rightarrow$  6-pole filter

### 15.4

(a) From Figure 15.8(a)

$$RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(25 \times 10^3)} = 6.366 \times 10^{-6}$$

Let  $C = 0.001 \mu\text{F}$ , then  $R = 6.366 \text{ k}\Omega$

And  $R_3 = (0.707)R = 4.50 \text{ k}\Omega$

$R_4 = (1.414)R = 9.0 \text{ k}\Omega$

$$(b) \text{ (i) } |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{22}\right)^4}} = 0.6123 \Rightarrow -4.26 \text{ dB}$$

$$(ii) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{25}\right)^4}} = 0.707 \Rightarrow -3 \text{ dB}$$

$$(iii) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{28}\right)^4}} = 0.7819 \Rightarrow -2.14 \text{ dB}$$

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### 15.5

$$(a) RC = \frac{1}{2\pi f_{3\text{-dB}}} = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Let  $R = 20 \text{ k}\Omega$ , then  $C = 397.9 \text{ pF}$

And  $C_1 = (3.546)C = 1411 \text{ pF}$

$C_2 = (1.392)C = 553.9 \text{ pF}$

$C_3 = (0.2024)C = 80.5 \text{ pF}$

$$(b) \text{ (i) } |T| = \frac{1}{\sqrt{1 + \left(\frac{10}{20}\right)^6}} = 0.9923 \Rightarrow -0.0673 \text{ dB}$$

$$(ii) |T| = \frac{1}{\sqrt{1 + \left(\frac{15}{20}\right)^6}} = 0.9214 \Rightarrow -0.711 \text{ dB}$$

$$(iii) |T| = \frac{1}{\sqrt{1 + \left(\frac{20}{20}\right)^6}} = 0.7071 \Rightarrow -3.0 \text{ dB}$$

$$(iv) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{20}\right)^6}} = 0.4557 \Rightarrow -6.83 \text{ dB}$$

$$(v) |T| = \frac{1}{\sqrt{1 + \left(\frac{30}{20}\right)^6}} = 0.2841 \Rightarrow -10.9 \text{ dB}$$

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**15.6**

From Equation (15.7).

$$T(s) = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4 (Y_1 + Y_2 + Y_3)}$$

For a high-pass filter, let  $Y_1 = Y_2 = sC$ ,

$$Y_3 = \frac{1}{R_3}, \quad \text{and} \quad Y_4 = \frac{1}{R_4}$$

Then

$$\begin{aligned} T(s) &= \frac{s^2 C^2}{s^2 C^2 + \frac{1}{R_4} \left( sC + sC + \frac{1}{R_3} \right)} \\ &= \frac{1}{1 + \frac{1}{sR_4 C} \left( 2 + \frac{1}{sR_3 C} \right)} \end{aligned}$$

Define  $\tau_3 = R_3 C$  and  $\tau_4 = R_4 C$

$$T(s) = \frac{1}{1 + \frac{1}{s\tau_4} \left( 2 + \frac{1}{s\tau_3} \right)}$$

Set  $s = j\omega$

$$\begin{aligned} T(j\omega) &= \frac{1}{1 + \frac{1}{j\omega\tau_4} \left( 2 + \frac{1}{j\omega\tau_3} \right)} \\ &= \frac{1}{1 - \frac{j}{\omega\tau_4} \left( 2 - \frac{j}{\omega\tau_3} \right)} \\ &= \frac{1}{\left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2j}{\omega\tau_4}} \end{aligned}$$

$$|T(j\omega)| = \left\{ \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right)^2 + \frac{4}{\omega^2 \tau_4^2} \right\}^{-1/2}$$

For a maximally flat filter, we want

$$\left. \frac{d|T|}{d\omega} \right|_{\omega \rightarrow \infty} = 0$$

Taking the derivative, we find

$$\frac{d|T(j\omega)|}{d\omega} = -\frac{1}{2} \left\{ \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right)^2 + \frac{4}{\omega^2 \tau_4^2} \right\}^{-3/2} \times \left[ 2 \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) \left( \frac{2}{\omega^3 \tau_3 \tau_4} \right) + \frac{4(-2)}{\omega^3 \tau_4^2} \right]$$

or

$$\begin{aligned} \left. \frac{d|T(j\omega)|}{d\omega} \right|_{\omega \rightarrow \infty} &= 0 \\ &= \left[ \left( \frac{4}{\omega^3 \tau_3 \tau_4} \right) \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{8}{\omega^3 \tau_4^2} \right] \\ &= \frac{4}{\omega^3} \left[ \frac{1}{\tau_3 \tau_4} \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2}{\tau_4^2} \right] \end{aligned}$$

Then

$$\left[ \frac{1}{\tau_3 \tau_4} \left( 1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2}{\tau_4^2} \right]_{\omega \rightarrow \infty} = 0$$

So that  $\frac{1}{\tau_3} = \frac{2}{\tau_4} \Rightarrow 2\tau_3 = \tau_4$

Then the transfer function can be written as:

$$\begin{aligned} |T(j\omega)| &= \left\{ \left[ 1 - \frac{1}{\omega^2 (2\tau_3^2)} \right]^2 + \frac{4}{\omega^2 (4\tau_3^2)} \right\}^{-1/2} \\ &= \left\{ 1 - \frac{1}{\omega^2 \tau_3^2} + \frac{1}{4(\omega^2 \tau_3^2)^2} + \frac{1}{\omega^2 \tau_3^2} \right\}^{-1/2} \\ &= \left\{ 1 + \frac{1}{4(\omega^2 \tau_3^2)^2} \right\}^{-1/2} \end{aligned}$$

3-dB frequency

$$2\omega^2 \tau_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2}(\tau_3)} = \frac{1}{\sqrt{2}(R_3 C)}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$R_3 = \frac{R}{\sqrt{2}}$$

We had  $2\tau_3 = \tau_4$  or  $2(R_3 C) = R_4 C \Rightarrow R_4 = 2R_3$

So that  $R_4 = \sqrt{2} \cdot R$

### 15.7

$$-14 \text{ dB} \Rightarrow |T| = 0.1995$$

$$0.1995 = \frac{1}{\sqrt{1+(1.2)^{2N}}} \Rightarrow (1.2)^{2N} = \left( \frac{1}{0.1995} \right)^2 - 1 = 24.1$$

N=9, 9<sup>th</sup> order filter

**15.8**

$$-12 \text{ dB} \Rightarrow |T| = 0.2512$$

$$0.2512 = \frac{1}{\sqrt{1 + \left(\frac{4}{3}\right)^{2N}}} \Rightarrow (1.333)^{2N} = \left(\frac{1}{0.2512}\right)^2 - 1 = 14.85$$

N=5, 5<sup>th</sup> order filter

**15.9**

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}}$$

At  $f = 12 \text{ kHz}$ ,  $|T| = 0.9$

$$0.9 = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}} = \frac{1}{\sqrt{1 + \left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = \frac{1}{(0.9)^2} - 1 = 0.2346$$

Also

$$0.01 = \frac{1}{\sqrt{1 + \left(\frac{14}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{14}{f_{3dB}}\right)^{2N} = \frac{1}{(0.01)^2} - 1 = 9999$$

$$\frac{\left(\frac{14}{f_{3dB}}\right)^{2N}}{\left(\frac{12}{f_{3dB}}\right)^{2N}} = \left(\frac{14}{12}\right)^{2N} = \frac{9999}{0.2346} = 4.262 \times 10^4$$

$$(1.16667)^{2N} = 4.262 \times 10^4$$

$$N = 35$$

Then

$$0.9 = \frac{1}{\sqrt{1 + \left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = 0.2346$$

$$\left(\frac{12}{f_{3dB}}\right) = (0.2346)^{\left(\frac{1}{2N}\right)} = (0.2346)^{0.014286} = 0.9795$$

So

$$\underline{f_{3dB} = 12.25 \text{ kHz}}$$

**15.10**

$$(a) |T| = \frac{1}{\sqrt{1 + \left(\frac{1}{0.8}\right)^{2N}}} = \frac{1}{\sqrt{1 + (1.25)^{2N}}}$$

$$\text{For } N=3, |T| = \frac{1}{\sqrt{1 + (1.25)^6}} = 0.4557 \Rightarrow -6.83 \text{ dB}$$

$$(b) \text{ For } N=5, |T| = \frac{1}{\sqrt{1 + (1.25)^{10}}} = 0.3114 \Rightarrow -10.1 \text{ dB}$$

$$(c) \text{ For } N=7, |T| = \frac{1}{\sqrt{1 + (1.25)^{14}}} = 0.2053 \Rightarrow -13.8 \text{ dB}$$

**15.11**

$$(a) |T| = \frac{1}{\sqrt{1 + (1.4)^{2N}}}$$

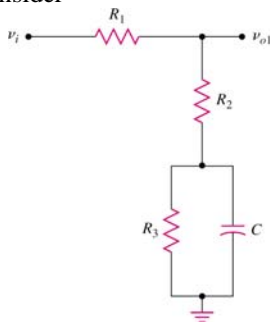
$$\text{For } N=3, |T| = \frac{1}{\sqrt{1 + (1.4)^6}} = 0.3424 \Rightarrow -9.31 \text{ dB}$$

$$(b) \text{ For } N=5, |T| = \frac{1}{\sqrt{1 + (1.4)^{10}}} = 0.1828 \Rightarrow -14.8 \text{ dB}$$

$$(c) \text{ For } N=7, |T| = \frac{1}{\sqrt{1 + (1.4)^{14}}} = 0.0944 \Rightarrow -20.5 \text{ dB}$$

**15.12**

Consider

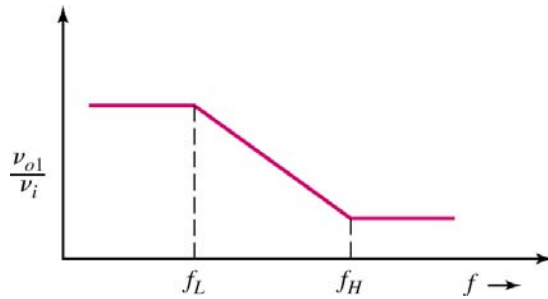


$$\text{For low-frequency: } \frac{v_o}{v_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

$$\text{For high-frequency: } \frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2}$$

So we need

$$\frac{R_2 + R_3}{R_1 + R_2 + R_3} = 25 \left( \frac{R_2}{R_1 + R_2} \right)$$

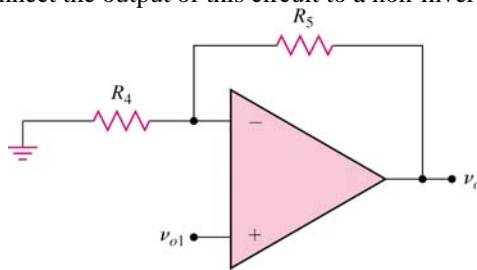


Let  $R_1 + R_2 = 50 \text{ k}\Omega$  and  $R_2 = 1.5 \text{ k}\Omega \Rightarrow R_1 = 48.5 \text{ k}\Omega$

Then

$$\frac{1.5 + R_3}{50 + R_3} = 25 \left( \frac{1.5}{50} \right) \Rightarrow R_3 = 144 \text{ k}\Omega$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$v_{o1} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot v_i = \frac{1.5 + 144}{48.5 + 1.5 + 144} \cdot v_i = 0.75v_i$$

Need to have  $v_o = 25$ .

$$v_o = 25 = \left( 1 + \frac{R_5}{R_4} \right) \cdot v_{o1} = \left( 1 + \frac{R_5}{R_4} \right) (0.75)v_i \Rightarrow \frac{R_5}{R_4} = 32.3$$

To check at high-frequency.

$$v_{o1} = \frac{R_2}{R_1 + R_2} v_i = \frac{1.5}{1.5 + 48.5} v_i = 0.03v_i$$

$$v_o = (1 + 32.3)v_{o1} = (33.3)(0.03)v_i = (1.0)v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i} = \frac{R_2 + R_3 \parallel \frac{1}{sC}}{R_1 + R_2 + R_3 \parallel \frac{1}{sC}}$$

Now

$$R_3 \parallel \frac{1}{sC} = \frac{R_3}{1 + sR_3C}$$



Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_3 + R_2(1 + sR_3C)}{R_3 + (R_1 + R_2)(1 + sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2 + R_3)(1 + s(R_2 \parallel R_3)C)}{(R_1 + R_2 + R_3)(1 + s(R_3 \parallel (R_1 + R_2))C)}$$

So

$$f_L \cong \frac{1}{2\pi(R_2 \parallel R_3)C} = \frac{1}{2\pi(1.5 \parallel 144) \times 10^3 C} = \frac{1}{(9.33 \times 10^3)C}$$

$$f_H \cong \frac{1}{2\pi(R_3 \parallel (R_1 + R_2))C} = \frac{1}{2\pi(144 \parallel 50) \times 10^3 C}$$

$$= \frac{1}{(2.33 \times 10^5)C}$$

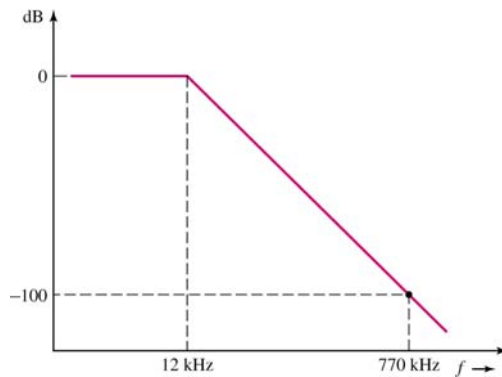
Set

$$25 \text{ kHz} = \frac{f_L + f_H}{2} = \frac{1}{2} \left[ \frac{1}{(9.33 \times 10^3)C} + \frac{1}{(2.33 \times 10^5)C} \right]$$

Which yields

$$C = \underline{2.23 \text{ nF}}$$

### 15.13



$$|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^{2N}}} - 100 \text{ dB} \Rightarrow 10^{-5}$$

So

$$10^{-5} = \frac{1}{\sqrt{1 + \left(\frac{770}{12}\right)^{2N}}}$$

or

$$1 + (64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

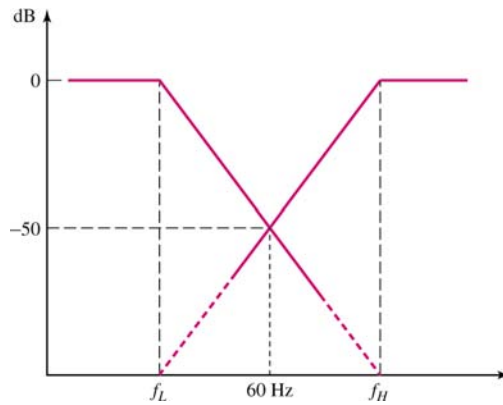
$$(64.2)^{2N} \cong 10^{10}$$

Now

$N$	<i>Left Side</i>
1	$4.112 \times 10^3$
2	$1.7 \times 10^7$
3	$7 \times 10^{10}$

So, we need a 3<sup>rd</sup> order filter.

**15.14**



Low-pass:  $-50 \text{ dB} \Rightarrow 3.16 \times 10^{-3}$

Then

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{60}{f_L}\right)^4}}$$

We find  $f_L = 3.37 \text{ Hz}$

High Pass:

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{60}\right)^4}}$$

We find  $f_H = 1067 \text{ Hz}$

Bandwidth:  $BW = f_H - f_L = 1067 - 3.37 \Rightarrow$

$$\underline{BW \cong 1064 \text{ Hz}}$$

**15.15**

a.

$$\frac{v_I}{R_4} = -\frac{v_{02}}{R_3} - \frac{v_0}{R_1 \parallel \left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{v_0}{R_2} = -\frac{v_{01}}{\left(\frac{1}{sC}\right)} \quad (2)$$

$$\frac{v_{01}}{R_5} = -\frac{v_{02}}{R_5} \Rightarrow v_{01} = -v_{02} \quad (3)$$

Then

$$\frac{v_0}{R_2} = +\frac{v_{02}}{\left(\frac{1}{sC}\right)} \quad \text{or} \quad v_{02} = v_0 \left(\frac{1}{sR_2C}\right) \quad (2)$$

And

$$\begin{aligned} \frac{v_I}{R_4} &= -\frac{v_0}{R_3} \cdot \left(\frac{1}{sR_2C}\right) - \frac{v_0}{R_1 \parallel \left(\frac{1}{sC}\right)} \\ &= -v_0 \left[ \frac{1}{R_3(sR_2C)} + \frac{1}{\frac{R_1 \cdot (1/sC)}{R_1 + (1/sC)}} \right] \quad (1) \\ &= -v_0 \left[ \frac{1}{R_3(sR_2C)} + \frac{1+sR_1C}{R_1} \right] \\ &= -v_0 \left[ \frac{R_1 + (1+sR_1C)(sR_2R_3C)}{(sC)R_1R_2R_3} \right] \end{aligned}$$

Then

$$\frac{v_0}{v_I} = -\frac{1}{R_4} \left[ \frac{(sC)(R_1R_2R_3)}{R_1 + sR_2R_3C + s^2R_1R_2R_3C^2} \right]$$

or

$$A_v(s) = \frac{v_0}{v_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + sC + \frac{1}{sCR_2R_3}}$$

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\omega C + \frac{1}{j\omega CR_2R_3}}$$

b.

or

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\left[\omega C - \frac{1}{\omega CR_2R_3}\right]}$$

$$= -\frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_1C - \frac{R_1}{\omega CR_2R_3}\right]\right\}}$$

$$|A_v(j\omega)| = \frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_1C - \frac{R_1}{\omega CR_2R_3}\right]^2\right\}^{-1/2}}$$

$$|A_v|_{\max} \text{ when } \left[\omega R_1C - \frac{R_1}{\omega CR_2R_3}\right] = 0$$

Then

$$|A_v|_{\max} = \frac{R_1}{R_4} = \frac{85}{3} \Rightarrow |A_v|_{\max} = 28.3$$

Now

$$\omega R_1C \left[1 - \frac{1}{\omega^2 C^2 R_2 R_3}\right] = 0 \quad \text{or} \quad \omega = \frac{1}{C\sqrt{R_2 R_3}}$$

Then

$$f = \frac{1}{2\pi C\sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1 \times 10^{-6})\sqrt{(300)^2}}$$

So

$$f = 5.305 \text{ kHz}$$

To find the two  $3\text{-dB}$  frequencies,

$$\left[\omega R_1C - \frac{R_1}{\omega CR_2R_3}\right] = \pm 1$$

$$\omega^2 R_1 R_2 R_3 C^2 - R_1 = \pm \omega R_2 R_3 C$$

$$\omega^2 (85 \times 10^3)(300)^2 (0.1 \times 10^{-6})^2 - 85 \times 10^3 = \pm \omega (300)^2 (0.1 \times 10^{-6})$$

$$\omega^2 (7.65 \times 10^{-5}) - 85 \times 10^3 = \pm \omega (9 \times 10^{-3})$$

$$\omega^2 (7.65 \times 10^{-5}) \pm \omega (9 \times 10^{-3}) - 85 \times 10^3 = 0$$

$$\omega = \frac{\pm(9 \times 10^{-3}) \pm \sqrt{(9 \times 10^{-3})^2 + 4(7.65 \times 10^{-5})(85 \times 10^3)}}{2(7.65 \times 10^{-5})}$$

We find  $f = 5.315 \text{ kHz}$  and  $f = 5.296 \text{ kHz}$

**15.16**

a.

$$\frac{v_I - v_A}{R} = \frac{v_A}{\left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{v_I - v_B}{R} = \frac{v_B - v_0}{R} \quad (2)$$

and  $v_A = v_B$

So

$$\frac{v_I}{R} = v_A \left( \frac{1}{R} + sC \right) = v_A \left( \frac{1 + sRC}{R} \right) \quad (1)$$

or

$$v_A = \frac{v_I}{1 + sRC}$$

Then

$$v_I + v_0 = 2v_B = 2v_A = \frac{2v_I}{1 + sRC} \quad (2)$$

$$v_0 = v_I \left[ \frac{2}{1 + sRC} - 1 \right] = v_I \left[ \frac{1 - sRC}{1 + sRC} \right]$$

Now

$$\frac{v_0}{v_I} = A(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$|A| = \frac{\sqrt{1 + \omega^2 R^2 C^2}}{\sqrt{1 + \omega^2 R^2 C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2 \tan^{-1}(\omega RC)$$

b.  $RC = (10^4)(15.9 \times 10^{-9}) = 1.59 \times 10^{-4}$

$f$	$\phi$
0	0
$10^2$	-11.4
$5 \times 10^3$	-53.1
$1/2\pi RC = 10^3$ Hz	-90°
$5 \times 10^3$	-157
$10^4$	-169

**15.17**

a.

$$\frac{V_i}{R_1} + \frac{V_i - V_0}{R_2 \parallel (1/sC)} = 0$$

$$\frac{V_i}{R_1} + \frac{V_i - V_0}{\left[ \frac{R_2}{1 + sR_2C} \right]} = 0$$

$$\frac{R_2}{R_1} \cdot \frac{1}{1 + sR_2C} (V_i) + V_i = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1(1 + sR_2C)}{R_1(1 + sR_2C)} = \frac{(R_2 + R_1)[1 + s(R_1 \parallel R_2)C]}{R_1(1 + sR_2C)}$$

$$\Rightarrow \frac{V_0}{V_i} = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{1 + s(R_1 \parallel R_2)C}{(1 + sR_2C)} \right]$$

$$\Rightarrow f_{3dB_1} = \frac{1}{2\pi R_2 C}$$

$$\Rightarrow f_{3dB_2} = \frac{1}{2\pi(R_1 \parallel R_2)C}$$

b.

$$\frac{V_i}{R_1 \parallel (1/sC)} + \frac{V_i - V_0}{R_2} = 0$$

$$\frac{V_i}{\left( \frac{R_1}{1 + sR_1C} \right)} + \frac{V_i}{R_2} = \frac{V_0}{R_2}$$

$$V_i \left[ \frac{R_2}{R_1} \cdot (1 + sR_1C) + 1 \right] = V_0$$

$$\frac{V_i}{R_1} \cdot [R_2 + R_1 + sR_1R_2C] = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1}{R_1} \cdot [1 + s(R_1 \parallel R_2)C] \Rightarrow \frac{V_0}{V_i} = \left( 1 + \frac{R_2}{R_1} \right) [1 + s(R_1 \parallel R_2)C] \Rightarrow f_{3dB} = \frac{1}{2\pi(R_1 \parallel R_2)C}$$

**15.18**

a.

$$\frac{V_i}{R_1 + (1/sC_1)} = \frac{-V_0}{R_2 \parallel (1/sC_2)}$$

$$V_i \left( \frac{sC_1}{1 + sR_1C_1} \right) = -V_0 \left( \frac{1 + sR_2C_2}{sC_2} \right)$$

$$\frac{V_0}{V_i} = \frac{-sR_2C_1}{(1 + sR_1C_1)(1 + sR_2C_2)} = \frac{-sR_2C_1}{1 + sR_1C_1 + sR_2C_2 + s^2R_1R_2C_1C_2}$$

$$\frac{V_0}{V_i} = -\frac{R_2}{R_1} \times \left[ \frac{sC_1}{\frac{1}{R_1} + sC_1 \left( 1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right) + s^2R_2C_1C_2} \right]$$

or

$$T(s) = \frac{V_o}{V_i} = -\frac{R_2}{R_1} \cdot \left[ \frac{1}{\frac{1}{sR_1C_1} + \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right) + sR_2C_2} \right]$$

b.

$$|T(j\omega)| = -\frac{R_2}{R_1} \times \frac{1}{\left\{ \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)^2 + \left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1}\right)^2 \right\}^{1/2}}$$

when  $\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1}\right) = 0$ , we want

$$|T(j\omega)| = 50 = \frac{R_2}{R_1} \cdot \frac{1}{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)}$$

At the 3-dB frequencies, we want

$$\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1}\right) = \pm \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)$$

For  $f = 5$  kHz, use + sign and for  $f = 200$  Hz, use - sign.

$$\omega_1 = 2\pi(200) = 1257$$

$$\omega_2 = 2\pi(5 \times 10^3) = 3.142 \times 10^4$$

Define  $\tau_2 = R_2 C_2$  and  $\tau_1 = R_1 C_1$

Then

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{\tau_2}{\tau_1}} \quad (1)$$

$$\left(\omega_2 \tau_2 - \frac{1}{\omega_2 \tau_1}\right) = + \left(1 + \frac{\tau_2}{\tau_1}\right) \quad (2)$$

$$\left(\omega_1 \tau_2 - \frac{1}{\omega_1 \tau_1}\right) = - \left(1 + \frac{\tau_2}{\tau_1}\right) \quad (3)$$

From (2)

$$\frac{\omega_2^2 \tau_1 \tau_2 - 1}{\omega_2 \tau_1} = \frac{\tau_1 + \tau_2}{\tau_1}$$

or

$$\omega_2 \tau_1 \tau_2 - \frac{1}{\omega_2} = \tau_1 + \tau_2$$

$$\tau_1 (\omega_2 \tau_2 - 1) = \frac{1}{\omega_2} + \tau_2$$

So

$$\tau_1 = \frac{\frac{1}{\omega_2} + \tau_2}{\omega_2 \tau_2 - 1}$$

Substituting into (3), we find

$$\omega_1 \tau_2 - \frac{1}{\omega_1 \cdot \left( \frac{1}{\omega_2} + \tau_2 \right)} = - \left[ 1 + \frac{\tau_2 (\omega_2 \tau_2 - 1)}{\frac{1}{\omega_2} + \tau_2} \right]$$

$$\omega_1 \tau_2 \left[ \frac{1}{\omega_2} + \tau_2 \right] - \frac{1}{\omega_1} (\omega_2 \tau_2 - 1) = - \left[ \left( \frac{1}{\omega_2} + \tau_2 \right) + \tau_2 (\omega_2 \tau_2 - 1) \right]$$

$$\frac{\omega_1}{\omega_2} \cdot \tau_2 + \omega_1 \tau_2^2 - \frac{\omega_2}{\omega_1} \cdot \tau_2 + \frac{1}{\omega_1} = - \frac{1}{\omega_2} - \tau_2 - \omega_2 \tau_2^2 + \tau_2$$

$$(\omega_1 + \omega_2) \tau_2^2 + \left( \frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} \right) \tau_2 + \left( \frac{1}{\omega_1} + \frac{1}{\omega_2} \right) = 0$$

$$(3.2677 \times 10^4) \tau_2^2 - 24.96 \tau_2 + 8.273 \times 10^{-4} = 0$$

$$\tau_2 = \frac{24.96 \pm \sqrt{(24.96)^2 - 4(3.2677 \times 10^4)(8.273 \times 10^{-4})}}{2(3.2677 \times 10^4)}$$

Since  $\omega_2$  is large,  $\tau_2$  should be small so use minus sign:

$$\tau_2 = 3.47 \times 10^{-5} \text{ s}$$

Then

$$\tau_1 = \frac{3.18 \times 10^{-5} + 3.47 \times 10^{-5}}{9.09 \times 10^{-2}} \Rightarrow \tau_1 = 7.32 \times 10^{-4} \text{ s}$$

Now

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{3.47 \times 10^{-5}}{7.32 \times 10^{-4}}}$$

Then

$$\frac{R_2}{R_1} = 52.37 \quad \text{or} \quad \underline{R_2 = 524 \text{ k}\Omega}$$

Also

$$\tau_1 = R_1 C_1 \text{ so that } C_1 = 0.0732 \mu\text{F}$$

$$\tau_2 = R_2 C_2 \text{ so that } C_2 = 66.3 \text{ pF}$$

### 15.19

Two noninverting amplifiers,

$$\left( 1 + \frac{R_2}{R_1} \right)^2 = 30 \text{ dB} \Rightarrow 31.62$$

which gives  $\frac{R_2}{R_1} = 4.62$ , then  $R_2 = 250 \text{ k}\Omega$ ,  $R_1 = 54.1 \text{ k}\Omega$

For high-pass filter:

$$RC = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Set  $R = 250 \text{ k}\Omega$ , then  $C = 31.8 \text{ pF}$

For low-pass filter:



$$RC = \frac{1}{2\pi(150)} = 1.061 \times 10^{-3}$$

Set  $R = 250 \text{ k}\Omega$ , then  $C = 0.00424 \text{ }\mu\text{F}$

**15.20**

(a)  $R_{eq} = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-12})} \Rightarrow R_{eq} = 40 \text{ M}\Omega$

(b)  $R_{eq} = \frac{1}{(50 \times 10^3)(2 \times 10^{-12})} \Rightarrow R_{eq} = 10 \text{ M}\Omega$

(c)  $R_{eq} = \frac{1}{(50 \times 10^3)(10 \times 10^{-12})} \Rightarrow R_{eq} = 2 \text{ M}\Omega$

**15.21**

a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}} \cdot T_C$$

and  $f_C = 100 \text{ kHz}$  so that  $T_C = \frac{1}{100 \times 10^3} \Rightarrow 10 \text{ }\mu\text{s}$

Now

$$R_{eq} = \frac{1}{f_C C} = \frac{1}{(100 \times 10^3)(10 \times 10^{-12})} \Rightarrow 1 \text{ M}\Omega$$

So

$$Q = \frac{(2-1)(10 \times 10^{-6})}{10^6} = 10 \times 10^{-12} \text{ C}$$

or

$$Q = 10 \text{ pC}$$

b.  $I_{eq} = \frac{Q}{T_C} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}} \text{ or } I_{eq} = 1 \text{ }\mu\text{A}$

c.  $Q = CV$  so find the time that  $V_0$  reaches 99% of its full value.

$$V_o = V_1(1 - e^{-t/\tau}) \text{ where } \tau = RC$$

Then  $0.99 = 1 - e^{-t/\tau}$  or  $e^{-t/\tau} = 0.01$

or  $t = \tau \ln(100)$

$$\tau = RC = (10^3)(10 \times 10^{-12}) = 10^{-8} \text{ s}$$

Then

$$t = 4.61 \times 10^{-8} \text{ s}$$

**15.22**

$$\text{gain} = -10 \Rightarrow \frac{C_1}{C_2} = 10$$

Low frequency

$$f_{3dB} = 10 \times 10^3 \text{ Hz} = \frac{f_C C_2}{2\pi C_F}$$

Set

$$f_c = 10f_{3dB} = 100 \text{ kHz}$$

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10 \times 10^3)}{100 \times 10^3} = 0.628$$

The largest capacitor is  $C_1$ , so let

$$C_1 = 30 \text{ pF}$$

Then

$$C_2 = 3 \text{ pF}$$

and

$$C_F = 4.78 \text{ pF}$$

---

### 15.23

a. Time constant  $= \tau = R_{eq}C_F$  where

$$R_{eq} = \frac{1}{f_c C_1} = \frac{1}{(100 \times 10^3)(5 \times 10^{-12})} = 2 \times 10^6 \Omega$$

Then

$$\tau = (2 \times 10^6)(30 \times 10^{-12})$$

or

$$\tau = 60 \mu\text{s}$$

b.  $v_o = -\frac{1}{\tau} \int v_i dt$

or

$$\Delta v_o = \frac{(1)(T_c)}{\tau} \text{ and } T_c = \frac{1}{f_c}$$

So

$$\Delta v_o = \frac{1}{(60 \times 10^{-6})(100 \times 10^3)}$$

or

$$\Delta v_o = 0.167 \text{ V}$$

c. Now  $\Delta v_o = 13 = N(0.167)$

or

$$N = 78 \text{ clock pulses}$$

---

### 15.24

(a)  $f_o = \frac{1}{2\pi\sqrt{3}RC} = \frac{1}{2\pi\sqrt{3}(20 \times 10^3)(0.001 \times 10^{-6})}$   
 $f_o = 4.59 \text{ kHz}, R_2 = 8R = 160 \text{ k}\Omega$

(b)  $C = \frac{1}{2\pi\sqrt{3}Rf_o} = \frac{1}{2\pi\sqrt{3}(20 \times 10^3)(25 \times 10^3)}$   
 $C = 184 \text{ pF}, R_2 = 160 \text{ k}\Omega$

---

**15.25**

a.

$$v_1 = \frac{R}{R + (1/sC_V)} \cdot v_0 = \left( \frac{sRC_V}{1 + sRC_V} \right) \cdot v_0$$

$$v_2 = \frac{R}{R + \frac{1}{sC}} \cdot v_1 = \left( \frac{sRC}{1 + sRC} \right) \cdot v_1$$

$$v_3 = \frac{R}{R + \frac{1}{sC}} \cdot v_2 = \left( \frac{sRC}{1 + sRC} \right) \cdot v_2$$

$$v_0 = -\frac{R_2}{R} \cdot v_3$$

Then

$$v_0 = -\frac{R_2}{R} \left( \frac{sRC}{1 + sRC} \right)^2 \left( \frac{sRC_V}{1 + sRC_V} \right) v_0$$

Set  $s = j\omega$

$$1 = -\frac{R_2}{R} \left( \frac{-\omega^2 R^2 C^2}{1 + 2j\omega RC - \omega^2 R^2 C^2} \right) \left( \frac{j\omega RC_V}{1 + j\omega RC_V} \right)$$

The real part of the denominator must be zero.

$$1 - \omega^2 R^2 C^2 - 2\omega^2 R^2 CC_V = 0$$

so

$$\omega_0 = \frac{1}{R\sqrt{C(C + 2C_V)}}$$

b.

$$f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[10^{-11}]})}$$

$$f_{0,\max} = 919 \text{ kHz}$$

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[50 \times 10^{-12}])}}$$

$$f_{0,\min} = 480 \text{ kHz}$$

**15.26**

$$(a) f_o = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(12 \times 10^3)(150 \times 10^{-12})}$$

$$f_o = 36.1 \text{ kHz}, \quad R_2 = 29R = 348 \text{ k}\Omega$$

$$(b) R = \frac{1}{2\pi\sqrt{6}Cf_o} = \frac{1}{2\pi\sqrt{6}(0.001 \times 10^{-6})(22 \times 10^3)}$$

$$R = 2.95 \text{ k}\Omega, \quad R_2 = 29R = 85.6 \text{ k}\Omega$$

15.27

$$\frac{v_0 - v_1}{\frac{1}{sC}} = \frac{v_1}{R} + \frac{v_1 - v_2}{\frac{1}{sC}} \quad (1)$$

$$\text{or } (v_0 - v_1)sC = \frac{v_1}{R} + (v_1 - v_2)sC$$

$$\frac{v_1 - v_2}{\frac{1}{sC}} = \frac{v_2}{R} + \frac{v_2}{\frac{1}{sC} + R} \quad (2)$$

$$\text{or } (v_1 - v_2)sC = \frac{v_2}{R} + \frac{v_2(sC)}{1 + sRC}$$

$$\frac{v_2}{\frac{1}{sC} + R} = -\frac{v_0}{R_2} \quad (3)$$

$$\text{or } \frac{v_2 sC}{1 + sRC} = -\frac{v_0}{R_2}$$

so

$$v_2 = \frac{-v_0}{sR_2 C} (1 + sRC)$$

From (2)

$$v_1(sC) = v_2 \left[ sC + \frac{1}{R} + \frac{sC}{1 + sRC} \right]$$

or

$$v_1 = -\frac{v_0(1 + sRC)}{sR_2 C} \cdot \left[ 1 + \frac{1}{sRC} + \frac{1}{1 + sRC} \right]$$

From (1)

$$v_0(sC) = v_1 \left[ sC + \frac{1}{R} + sC \right] - v_2(sC)$$

Then

$$v_0 = \left[ 2 + \frac{1}{sRC} \right] \left[ \frac{-v_0(1 + sRC)}{sR_2 C} \right] \times \left[ \frac{1 + sRC}{sRC} + \frac{1}{1 + sRC} \right] + \frac{v_0}{sR_2 C} \cdot (1 + sRC)$$

$$-1 = \left[ \frac{1 + 2sRC}{sRC} \right] \left[ \frac{1 + sRC}{sR_2 C} \right] \left[ \frac{(1 + sRC)^2 + sRC}{(sRC)(1 + sRC)} \right] - \frac{1 + sRC}{sR_2 C}$$

$$-1 = \frac{(1 + 2sRC)(1 + 2sRC + s^2 R^2 C^2 + sRC)}{(sRC)^2 (sR_2 C)} - \frac{(1 + sRC)(sRC)^2}{(sRC)^2 (sR_2 C)}$$

$$\text{Set } s = j\omega, \text{ then } -1 = \frac{(1 + 2j\omega RC)(1 + 3j\omega RC + \omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)} - \frac{(1 + j\omega RC)(-\omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)}$$

The real part of the numerator must be zero.

$$1 - \omega^2 R^2 C^2 - 6\omega^2 R^2 C^2 + \omega^2 R^2 C^2 = 0$$

$$6\omega^2 R^2 C^2 = 1$$

so that

$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

Condition for oscillation:

$$-1 = \frac{2j\omega RC + 3j\omega RC - 2j\omega^3 R^3 C^3 + j\omega^3 R^3 C^3}{(-\omega^3 R^2 C^2)(j\omega R_2 C)}$$

$$1 = \frac{5 - \omega^2 R^2 C^2}{(\omega RC)(\omega R_2 C)}$$

But

$$\omega = \omega_0 = \frac{1}{\sqrt{6RC}}$$

Then

$$1 = \frac{5 - \frac{1}{6}}{\frac{(RC)(R_2 C)}{6R^2 C^2}} = \frac{\left(5 - \frac{1}{6}\right)(6R^2 C^2)}{RR_2 C^2}$$

$$1 = \frac{\left(\frac{29}{6}\right)(6R)}{R_2} \text{ or } \frac{R_2}{R} = 29$$

### 15.28

Let  $R_{F1} = R_{F2} = R_{F3} \equiv R_F$

$$v_{o1} = \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC}\right) \cdot v_o$$

$$v_{o2} = \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC}\right) \cdot v_{o1}$$

$$\frac{v_{o3} - v_{o2}}{R} + \frac{v_{o3}}{1/sC} + \frac{v_{o3}}{R} = 0$$

$$v_{o3} \left(\frac{2}{R} + sC\right) = \frac{v_{o2}}{R}$$

$$v_{o3} = \left(\frac{1}{2 + sRC}\right) \cdot v_{o2}$$

$$v_o = -\frac{R_F}{R} \cdot v_{o3}$$

$$v_o = -\frac{R_F}{R} \left(\frac{1}{2 + sRC}\right) \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC}\right) \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC}\right) \cdot v_o$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left(\frac{1}{2 + sRC}\right) \left(\frac{1}{1 + sRC}\right) \left(\frac{1}{1 + sRC}\right)$$

Let  $s = j\omega$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left(\frac{1}{2 + j\omega RC}\right) \left(\frac{1}{1 + j\omega RC}\right) \left(\frac{1}{1 + j\omega RC}\right)$$

$$= -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left(\frac{1}{2 + j\omega RC}\right) \left(\frac{1}{1 + 2j\omega RC - \omega^2 R^2 C^2}\right)$$

$$= -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left(\frac{1}{2 + 4j\omega RC - 2\omega^2 R^2 C^2 + j\omega RC - 2\omega^2 R^2 C^2 - j\omega^3 R^3 C^3}\right)$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[ \frac{1}{2 - 4\omega^2 R^2 C^2 + 5j\omega RC - j\omega^3 R^3 C^3} \right]$$

Imaginary Term must be zero

$$5j\omega_0 RC - j\omega_0^3 R^3 C^3 = 0$$

$$5 - j\omega_0^2 R^2 C^2 = 0$$

$$\omega_0 = \frac{\sqrt{5}}{RC}$$

Then

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[ \frac{1}{2 - \frac{4R^2 C^2 - 5}{R^2 C^2}} \right]$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[ \frac{1}{2 - 20} \right] = \frac{1}{18} \cdot \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2$$

$$18 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \Rightarrow \underline{\underline{\frac{R_F}{R} = 2}}$$

### 15.29

(a) 1<sup>st</sup> stage:

$$v_{o1} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_V} \cdot \left(1 + \frac{R_F}{R}\right) \cdot v_o = \left( \frac{1}{1 + j\omega R_V C} \right) \left(1 + \frac{R_F}{R}\right) \cdot v_o$$

2<sup>nd</sup> stage:

$$v_{o2} = \left( \frac{1}{1 + j\omega RC} \right) \left(1 + \frac{R_F}{R}\right) \cdot v_{o1}$$

At node of  $C_3$ :

$$\frac{v_A - v_{o2}}{R} + v_A (j\omega C) + \frac{v_A}{R} = 0 \Rightarrow v_A = \frac{v_{o2}}{2 + j\omega RC}$$

3<sup>rd</sup> stage:

$$v_o = -\frac{R_F}{R} \cdot v_A = -\frac{R_F}{R} \cdot \frac{v_{o2}}{(2 + j\omega RC)}$$

Now

$$v_o = -\frac{R_F}{R} \cdot \frac{1}{(2 + j\omega RC)} \left( \frac{1}{1 + j\omega RC} \right) \left(1 + \frac{R_F}{R}\right) \left( \frac{1}{1 + j\omega R_V C} \right) \left(1 + \frac{R_F}{R}\right) \cdot v_o$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{1}{(2 + j\omega RC)} \cdot \frac{1}{(1 + j\omega RC)} \cdot \frac{1}{(1 + j\omega R_V C)}$$

From the denominator, we have:

$$\begin{aligned} (2 + j\omega RC)(1 + j\omega RC)(1 + j\omega R_V C) &= (2 + 3j\omega RC - \omega^2 R^2 C^2)(1 + j\omega R_V C) \\ &= [(2 - \omega^2 R^2 C^2) + 3j\omega RC](1 + j\omega R_V C) \\ &= (2 - \omega^2 R^2 C^2) + 3j\omega RC + j\omega R_V C(2 - \omega^2 R^2 C^2) - 3\omega^2 R R_V C^2 \end{aligned}$$

For oscillation, imaginary part must be zero.

$$3\omega_o RC + \omega_o R_V C(2 - \omega_o^2 R^2 C^2) = 0$$

$$3RC + 2R_V C - \omega_o^2 R_V R^2 C^3 = 0$$

$$\text{So that } \omega_o = \frac{1}{RC} \sqrt{\frac{3R + 2R_V}{R_V}}$$

$$(b) \quad 1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{1}{(2 - \omega_o^2 R^2 C^2 - 3\omega_o^2 R R_V C^2)}$$

Consider the term:

$$\begin{aligned} 2 - \omega_o^2 (R^2 C^2 + 3R R_V C^2) &= 2 - \frac{1}{R^2 C^2} \left(\frac{3R + 2R_V}{R_V}\right) (R^2 C^2 + 3R R_V C^2) \\ &= 2 - \left(\frac{3R + 2R_V}{R_V}\right) \left(1 + \frac{3R_V}{R}\right) = \frac{2R R_V - (3R + 2R_V)(R + 3R_V)}{R R_V} \\ &= \frac{2R R_V - (3R^2 + 9R R_V + 2R R_V + 6R_V^2)}{R R_V} = -\frac{(3R^2 + 9R R_V + 6R_V^2)}{R R_V} \end{aligned}$$

Then

$$1 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{R R_V}{(3R^2 + 9R R_V + 6R_V^2)}$$

For  $R = R_V$

$$1 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{R^2}{18R^2}$$

$$\text{Or } 18 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \Rightarrow \frac{R_F}{R} = 2$$

(c) For  $R_V = 15 \text{ k}\Omega$ ,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{3(25) + 2(15)}{15}} \Rightarrow f_o = 16.84 \text{ kHz}$$

For  $R_V = 30 \text{ k}\Omega$ ,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{3(25) + 2(30)}{30}} \Rightarrow f_o = 13.5 \text{ kHz}$$

So  $13.5 \leq f_o \leq 16.84 \text{ kHz}$

### 15.30

$$(a) \quad \frac{v_{o1} - v_o}{R} + v_{o1}(j\omega C) + \frac{v_{o1} - v_{o2}}{R} = 0$$

$$v_{o1} \left(\frac{2}{R} + j\omega C\right) = \frac{v_o}{R} + \frac{v_{o2}}{R}$$

$$\text{Then (1) } v_{o1}(2 + j\omega RC) = v_o + v_{o2}$$

$$\frac{v_{o2} - v_{o1}}{R} + v_{o2}(j\omega C) + \frac{v_{o2} - v_{o3}}{R} = 0$$

$$\text{So (2) } v_{o2}(2 + j\omega RC) = v_{o1} + v_{o3}$$

$$\frac{v_{o3} - v_{o2}}{R} + v_{o3}(j\omega C) + \frac{v_{o3}}{R} = 0$$

And (3)  $v_{o3}(2 + j\omega RC) = v_{o2}$

Also (4)  $v_o = -\frac{R_F}{R} \cdot v_{o3}$

From (1)  $v_{o1}(2 + j\omega RC) = v_o + v_{o3}(2 + j\omega RC) \Rightarrow v_{o1} = \frac{v_o}{(2 + j\omega RC)} + v_{o3}$

From (2)  $v_{o3}(2 + j\omega RC)^2 = v_{o1} + v_{o3} \Rightarrow v_{o3}(2 + j\omega RC)^2 = 2v_{o3} + \frac{v_o}{(2 + j\omega RC)}$

$$v_{o3}(4 + 4j\omega RC - \omega^2 R^2 C^2 - 2) = \frac{v_o}{(2 + j\omega RC)}$$

$$v_{o3} = \frac{v_o}{(2 + j\omega RC)(2 - \omega^2 R^2 C^2 + 4j\omega RC)}$$

Using (4)  $v_o = -\frac{R_F}{R} \cdot \frac{v_o}{(2 + j\omega RC)(2 - \omega^2 R^2 C^2 + 4j\omega RC)}$

$$1 = -\frac{R_F}{R} \cdot \frac{1}{[(2)(2 - \omega^2 R^2 C^2) + 8j\omega RC + j\omega RC(2 - \omega^2 R^2 C^2) - 4\omega^2 R^2 C^2]}$$

For oscillation, denominator must be real, so

$$8\omega_o RC + \omega_o RC(2 - \omega_o^2 R^2 C^2) = 0 \Rightarrow \omega_o = \frac{\sqrt{10}}{RC}$$

(b)  $1 = -\frac{R_F}{R} \cdot \frac{1}{[(2)(2 - \omega_o^2 R^2 C^2) - 4\omega_o^2 R^2 C^2]}$

$$1 = -\frac{R_F}{R} \cdot \frac{1}{[4 - 2\omega_o^2 R^2 C^2 - 4\omega_o^2 R^2 C^2]} = -\frac{R_F}{R} \cdot \frac{1}{\left[4 - 6\left(\frac{10}{R^2 C^2}\right)R^2 C^2\right]} = -\frac{R_F}{R} \cdot \frac{1}{(-56)}$$

Or  $\frac{R_F}{R} = 56$

(c)  $C = \frac{\sqrt{10}}{2\pi f_o R} = \frac{\sqrt{10}}{2\pi(22 \times 10^3)(20 \times 10^3)} \Rightarrow C = 0.00114 \mu\text{F}$

$$R_F = (56)R \Rightarrow R_F = 1.12 \text{ M}\Omega$$

### 15.31

(a)  $\frac{v_{o1} - v_o}{R_V} + v_{o1}(j\omega C) + \frac{v_{o1} - v_{o2}}{R} = 0$

$$v_{o1} \left( \frac{1}{R_V} + \frac{1}{R} + j\omega C \right) = \frac{v_o}{R_V} + \frac{v_{o2}}{R}$$

$$v_{o1} \left( \frac{1}{R \parallel R_V} + j\omega C \right) = \frac{v_o}{R_V} + \frac{v_{o2}}{R}$$

Then (1)  $v_{o1}(1 + j\omega(R \parallel R_V)C) = (R \parallel R_V) \left( \frac{v_o}{R_V} + \frac{v_{o2}}{R} \right)$

From Problem 15.30,

(2)  $v_{o2}(2 + j\omega RC) = v_{o1} + v_{o3}$

(3)  $v_{o3}(2 + j\omega RC) = v_{o2}$



$$(4) v_o = -\frac{R_F}{R} \cdot v_{o3}$$

$$\text{From (2), } v_{o3}(2 + j\omega RC)^2 = v_{o1} + v_{o3} \Rightarrow v_{o1} = v_{o3}[(2 + j\omega RC)^2 - 1]$$

$$\text{Then (1) } v_{o3}[(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] = (R\|R_V) \left[ \frac{v_o}{R_V} + \frac{v_{o3}}{R}(2 + j\omega RC) \right]$$

$$v_{o3} \left\{ [(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R}(2 + j\omega RC) \right\} = (R\|R_V) \cdot \frac{v_o}{R_V}$$

$$\text{Then, } v_o = -\frac{R_F}{R} \cdot \frac{(R\|R_V)}{R_V} \cdot v_o \cdot \frac{1}{\left\{ [(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R}(2 + j\omega RC) \right\}}$$

Consider the denominator:

$$\left[ 4 + 4j\omega RC - \omega^2 R^2 C^2 - 1 \right] [1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R}(2 + j\omega RC)$$

$$(3 - \omega^2 R^2 C^2 + 4j\omega RC) [1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R}(2 + j\omega RC)$$

For oscillation, the denominator must be real, so

$$4j\omega_o RC + j\omega_o(R\|R_V)C[3 - \omega_o^2 R^2 C^2] - \frac{R\|R_V}{R} \cdot j\omega_o RC = 0$$

$$4R + (R\|R_V)[3 - \omega_o^2 R^2 C^2] - R\|R_V = 0$$

$$4R + 2(R\|R_V) = \omega_o^2 (R\|R_V) R^2 C^2$$

$$\text{So that } \omega_o = \frac{1}{RC} \sqrt{4 \left( \frac{R}{R\|R_V} \right) + 2}$$

(b) For  $R_V = 15 \text{ k}\Omega$ ,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{4(25)}{25\|15} + 2} \Rightarrow f_o = 22.66 \text{ kHz}$$

For  $R_V = 30 \text{ k}\Omega$ ,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{4(25)}{25\|30} + 2} \Rightarrow f_o = 19.45 \text{ kHz}$$

So  $19.45 \leq f_o \leq 22.66 \text{ kHz}$

### 15.32

a. We can write

$$v_A = \left( \frac{R_1}{R_1 + R_2} \right) v_o \quad \text{and} \quad v_B = \left( \frac{Z_p}{Z_p + Z_s} \right) v_o$$

$$\text{where } Z_p = R_B \parallel \frac{1}{sC_B} = \frac{R_B}{1 + sR_B C_B}$$

$$\text{and } Z_s = R_A + \frac{1}{sC_A} = \frac{1 + sR_A C_A}{sC_A}$$

Setting  $v_A = v_B$ , we have

$$\frac{R_1}{R_1 + R_2} = \frac{\frac{R_B}{1 + sR_B C_B}}{\frac{R_B}{1 + sR_B C_B} + \frac{1 + sR_A C_A}{sC_A}}$$

$$\frac{R_1}{R_1 + R_2} = \frac{sR_B C_A}{sR_B C_A + (1 + sR_A C_A)(1 + sR_B C_B)} \quad (1)$$

To find the frequency of oscillation, set  $s = j\omega$  and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_B C_A + (1 + j\omega R_A C_A)(1 + j\omega R_B C_B)$$

or

$$j\omega R_B C_A + 1 + j\omega R_A C_A + j\omega R_B C_B - \omega^2 R_A R_B C_A C_B \quad (2)$$

Then from (2), we must have

$$1 - \omega_0^2 R_A R_B C_A C_B = 0$$

or

$$f_0 = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega R_B C_A}{j\omega R_B C_A + j\omega R_A C_A + j\omega R_B C_B}$$

or

$$1 + \frac{R_2}{R_1} = 1 + \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

Then

$$\frac{R_2}{R_1} = \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

### 15.33

a. We can write

$$v_A = \left( \frac{R_1}{R_1 + R_2} \right) v_0$$

and

$$v_B = \left( \frac{R \parallel sL}{R \parallel sL + R + sL} \right) v_0$$

Setting  $v_A = v_B$ , we have

$$\frac{R_1}{R_1 + R_2} = \left[ \frac{\frac{sRL}{R + sL}}{\frac{sRL}{R + sL} + R + sL} \right] \cdot v_0$$

$$\frac{R_1}{R_1 + R_2} = \frac{sRL}{sRL + (R + sL)^2} \quad (1)$$

To find the frequency of oscillation, set  $s = j\omega$  and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \quad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{R}{L}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\frac{R_2}{R_1} = 2$$

### 15.34

$$f_o = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_o} = \frac{1}{2\pi(35 \times 10^3)} = 4.547 \times 10^{-6}$$

Let  $C = 0.001 \mu\text{F}$ , then  $R = 4.55 \text{ k}\Omega$

$$\text{Set } \frac{R_2}{R_1} = 2$$

### 15.35

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.7)(0.8)} = 1.497 \text{ mA/V}$$

$$\frac{C_2}{C_1} = g_m R = (1.497)(2) = 2.993$$

$$C_2 \cong 3(0.02) = 0.06 \mu\text{F}$$

$$2\pi f_o = \frac{1}{\sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}}$$

$$\text{Then } L \left[ \frac{(0.02)(0.06) \times 10^{-6}}{0.02 + 0.06} \right] = \left( \frac{1}{2\pi(350 \times 10^3)} \right)^2 \Rightarrow L = 13.8 \mu\text{H}$$

15.36

$$V_{\pi} = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_{\pi} = -g_m V_0$$

$$V_0 \left[ sC_2 + sC_1 + \frac{1}{R_L} + g_m \right] = V_1 (sC_1) \quad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_{\pi} = 0 \quad (2)$$

$$V_1 \left( \frac{1}{sL} + sC_1 \right) = V_0 (sC_1 + g_m)$$

$$V_1 = \frac{V_0 (sC_1 + g_m)}{\left( \frac{1}{sL} + sC_1 \right)}$$

Then

$$V_0 \left[ s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] = \frac{V_0 (sC_1)(sC_1 + g_m)}{\left( \frac{1}{sL} + sC_1 \right)}$$

$$\left[ s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] \left( \frac{1}{sL} + sC_1 \right) = sC_1 (sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2 C_1 (C_1 + C_2) + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + sg_m C_1 + \frac{g_m}{sL} = s^2 C_1^2 + sg_m C_1$$

$$\frac{C_1 + C_2}{L} + s^2 C_1 C_2 + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set  $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2 C_1 C_2 + \frac{1}{j\omega R_L L} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1 C_2 L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_L L} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_L L} = \frac{(C_1 + C_2) C_1}{C_1 C_2 L R_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2 R_L}$$

$$g_m R_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \frac{C_1}{C_2} = g_m R_L$$

**15.37**

a.

$$\frac{V_0}{sL_1} + \frac{V_0}{R} + g_m V_\pi + \frac{V_0}{\frac{1}{sC} + sL_2} = 0 \quad (1)$$

$$V_\pi = \left( \frac{sL_2}{\frac{1}{sC} + sL_2} \right) V_0 \quad (2)$$

Then

$$V_0 \left\{ \frac{1}{sL_1} + \frac{1}{R} + \frac{sC}{1 + s^2 L_2 C} + \frac{g_m (s^2 L_2 C)}{1 + s^2 L_2 C} \right\} = 0$$

$$\left\{ \frac{R(1 + s^2 L_2 C) + (sL_1)(1 + s^2 L_2 C)}{(sRL_1)(1 + s^2 L_2 C)} + \frac{s^2 RL_1 C + g_m (sRL_1)(s^2 L_2 C)}{(sRL_1)(1 + s^2 L_2 C)} \right\} = 0$$

Set  $s = j\omega$ . Both real and imaginary parts of the numerator must be zero.

$$R(1 - \omega^2 L_2 C) + j\omega L_1(1 - \omega^2 L_2 C) - \omega^2 RL_1 C + (j\omega g_m RL_1)(-\omega^2 L_2 C) = 0$$

Real part:

$$R(1 - \omega^2 L_2 C) - \omega^2 RL_1 C = 0$$

$$R = \omega^2 RC(L_1 + L_2)$$

or

$$\omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

b. Imaginary part:

$$j\omega L_1(1 - \omega^2 L_2 C) - j\omega g_m RL_1(\omega^2 L_2 C) = 0$$

$$L_1 = \omega^2 L_1 L_2 C + g_m RL_1(\omega^2 L_2 C)$$

$$\text{Now } \omega^2 = \frac{1}{(L_1 + L_2)}$$

$$1 = \frac{1}{C(L_1 + L_2)} [L_2 C + g_m RL_2 C]$$

$$1 = \frac{L_2}{L_1 + L_2} (1 + g_m R) \Rightarrow \frac{L_1}{L_2} = (1 + g_m R)$$

or

$$\frac{L_1}{L_2} = g_m R$$

**15.38**

$$(a) (1) \quad g_m V_\pi + \frac{V_C}{R} + \frac{V_C}{j\omega L_1} + (V_C - V_\pi)(j\omega C) = 0$$

$$(2) \quad V_\pi = (V_C - V_\pi)(j\omega C)(j\omega L_2)$$

$$V_\pi(1 - \omega^2 CL_2) = -V_C(\omega^2 CL_2)$$

$$V_{\pi} = \frac{V_C (\omega^2 L_2 C)}{\omega^2 L_2 C - 1}$$

Then (1)  $V_{\pi} (g_m - j\omega C) + V_C \left( \frac{1}{R} + \frac{1}{j\omega L_1} + j\omega C \right) = 0$

$$V_C \cdot \frac{(\omega^2 L_2 C)(g_m - j\omega C)}{\omega^2 L_2 C - 1} + V_C \cdot \left( \frac{1}{R} - \frac{j}{\omega L_1} + j\omega C \right) = 0$$

$$\frac{g_m \omega^2 L_2 C}{\omega^2 L_2 C - 1} + \frac{1}{R} + j \left[ \omega C - \frac{1}{\omega L_1} - \frac{\omega C (\omega^2 L_2 C)}{\omega^2 L_2 C - 1} \right] = 0$$

Set the imaginary part equal to zero.

$$\frac{\omega^2 L_1 C - 1}{\omega L_1} = \frac{\omega C (\omega^2 L_2 C)}{\omega^2 L_2 C - 1}$$

$$(\omega^2 L_1 C - 1)(\omega^2 L_2 C - 1) = (\omega^2 L_1 C)(\omega^2 L_2 C)$$

$$\omega^4 L_1 L_2 C^2 - \omega^2 L_2 C - \omega^2 L_1 C + 1 = \omega^4 L_1 L_2 C^2$$

$$\omega_o^2 (L_1 + L_2) C = 1$$

So  $\omega_o = \frac{1}{\sqrt{(L_1 + L_2) C}}$

(b) Set the real part equal to zero,

$$\frac{g_m \omega_o^2 L_2 C}{\omega_o^2 L_2 C - 1} + \frac{1}{R} = 0$$

$$g_m R \omega_o^2 L_2 C = 1 - \omega_o^2 L_2 C$$

$$\omega_o^2 L_2 C (g_m R + 1) = 1$$

Then  $\frac{L_2 C (g_m R + 1)}{(L_1 + L_2) C} = 1 \Rightarrow g_m R + 1 = \frac{(L_1 + L_2) C}{L_2 C}$

We find  $g_m R = \frac{L_1}{L_2}$

(c)  $C = \frac{1}{(2\pi f_o)^2 (L_1 + L_2)} = \frac{1}{[2\pi(750 \times 10^3)]^2 (100 \times 10^{-6})} \Rightarrow C = 450 \text{ pF}$

$$g_m R = \frac{L_1}{L_2} = 1 \Rightarrow R = \frac{1}{g_m} = \frac{1}{30} \Rightarrow R = 33.3 \Omega$$

15.39

$$\frac{v_0 - v_1}{\left(\frac{1}{sC}\right)} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

and

$$\frac{v_B}{\left(\frac{1}{sC}\right)} + \frac{v_B - v_1}{R} = 0 \quad (2)$$

or

$$v_B \left( sC + \frac{1}{R} \right) = \frac{v_1}{R} \Rightarrow v_1 = v_B (1 + sRC)$$

From (1)

$$v_0(sC) = v_1 \left( sC + \frac{2}{R} \right) - \frac{v_B}{R}$$

or

$$v_0(sRC) = v_B(1 + sRC)(2 + sRC) - v_B = v_B [(1 + sRC)(2 + sRC) - 1]$$

Now

$$T(s) = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{sRC}{(1 + sRC)(2 + sRC) - 1} \right] = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{sRC}{2 + 3sRC + s^2 R^2 C^2 - 1} \right]$$

or

$$T(s) = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{sRC}{s^2 R^2 C^2 + 3sRC + 1} \right]$$

$$T(j\omega) = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC} \right]$$

Frequency of oscillation:

$$f_0 = \frac{1}{2\pi RC}$$

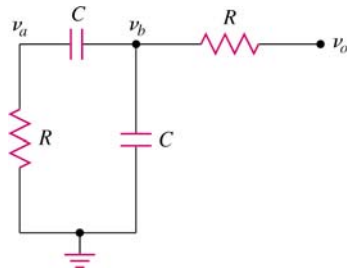
Condition for oscillation:

$$1 = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{j\omega RC}{3j\omega RC} \right]$$

or

$$\frac{R_2}{R_1} = 2$$

15.40



$$\frac{v_b - v_o}{R} + \frac{v_b}{sC} + \frac{v_b - v_a}{sC} = 0$$

$$(1) \quad \frac{v_b - v_o}{R} + 2v_b \cdot sC - v_a \cdot sC = 0$$

$$\frac{V_a - V_b}{sC} + \frac{V_a}{R} = 0$$

$$(2) \quad V_a \left( sC + \frac{1}{R} \right) = v_b \cdot sC \Rightarrow v_b = v_a \left( \frac{1 + sRC}{sRC} \right)$$

From (1)

$$v_b \left( \frac{1}{R} + 2sC \right) = \frac{v_o}{R} + v_a \cdot sC$$

Substitute (2) into (1)

$$v_a \left( \frac{1 + sRC}{sRC} \right) \left( \frac{1 + 2sRC}{R} \right) = \frac{v_o}{R} + v_a \cdot sC$$

$$v_a \left[ \frac{(1 + sRC)(1 + 2sRC)}{(sRC) \cdot R} - sC \right] = \frac{v_o}{R}$$

$$v_a \left[ \frac{(1 + sRC)(1 + 2sRC)}{sRC} - sRC \right] = v_o$$

$$\frac{v_o}{v_a} = \frac{(1 + sRC)(1 + 2sRC) - s^2 R^2 C^2}{sRC}$$

$$\frac{v_o}{v_a} = \frac{sRC}{1 + 3sRC + 2(sRC)^2 - s^2 R^2 C^2}$$

$$\frac{v_a}{v_o} = \frac{sRC}{1 + 3sRC + (sRC)^2}$$

$$T(s) = \left( 1 + \frac{R_2}{R_1} \right) \cdot \frac{sRC}{1 + 3sRC + (sRC)^2}$$

$$T(j\omega) = \left( 1 + \frac{R_2}{R_1} \right) \left[ \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC} \right]$$

So  $1 - \omega_0^2 R^2 C^2 = 0$

So  $f_o = \frac{1}{2\pi RC}$

Also  $1 = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{1}{3} \right)$

So  $\frac{R_2}{R_1} = 2$



**15.41**

$$\frac{v_0 - v_1}{sL} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

$$v_B = \left( \frac{sL}{R + sL} \right) v_1 \quad (2)$$

or

$$v_1 = \left( \frac{R + sL}{sL} \right) v_B$$

Then

$$\frac{v_0}{sL} = v_1 \left( \frac{1}{sL} + \frac{2}{R} \right) - \frac{v_B}{R}$$

or

$$\begin{aligned} \frac{v_0}{sL} &= \left( \frac{R + sL}{sL} \right) \left( \frac{1}{sL} + \frac{2}{R} \right) v_B - \frac{v_B}{R} \\ &= v_B \left\{ \left( \frac{R + sL}{sL} \right) \left( \frac{R + 2sL}{sRL} \right) - \frac{1}{R} \right\} \end{aligned} \quad (1)$$

Then

$$v_B = \frac{v_0}{sL} \cdot \frac{1}{\left\{ \frac{(R + sL)(R + 2sL) - (sL)^2}{(sL)(sRL)} \right\}}$$

Now

$$T(s) = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2} \right)$$

or

$$T(s) = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{sRL}{s^2L^2 + 3sRL + R^2} \right)$$

And

$$T(j\omega) = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{j\omega RL}{R^2 - \omega^2L^2 + 3j\omega RL} \right)$$

Frequency of oscillation:  $f_0 = \frac{R}{2\pi L}$

Condition for oscillation:

$$1 = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{1}{3} \right)$$

or

$$\frac{R_2}{R_1} = 2$$

**15.42**

$$v_I = -\frac{R_2}{R_1} \cdot V_{REF} = -\frac{(R_{2f} + R_{2v})}{R_1} \cdot V_{REF}$$

Set  $R_{2f} = 20 \text{ k}\Omega$

For  $v_I = -2 \text{ V}$ , set  $R_{2v} = 0$

$$-2 = -\frac{20}{R_1}(5) \Rightarrow R_1 = 50 \text{ k}\Omega$$

Then

$$-4 = -\frac{(20 + R_{2v})}{50}(5) \Rightarrow R_{2v} = 20 \text{ k}\Omega$$

**15.43**

For  $v_o = 10 \text{ V}$ ,  $v_I = -5 \text{ V}$

$$i = 0.2 \text{ mA} = \frac{10 - (-5)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 75 \text{ k}\Omega$$

$$(V_{TH} - V_{TL}) = \frac{R_1}{R_2}(V_H - V_L)$$

$$0.4 = \frac{R_1}{R_2}[10 - (-10)] \Rightarrow R_2 = 50R_1$$

$$R_1 + R_2 = R_1 + 50R_1 = 75 \Rightarrow R_1 = 1.47 \text{ k}\Omega, \quad R_2 = 73.53 \text{ k}\Omega$$

**15.44**

$$(a) \quad V_{TH} - V_{TL} = \left( \frac{R_1}{R_1 + R_2} \right) (V_H - V_L)$$

$$0.2 = \left( \frac{R_1}{R_1 + R_2} \right) [9 - (-9)] = 18 \left( \frac{R_1}{R_1 + R_2} \right)$$

Set  $R_1 = 2 \text{ k}\Omega$

$$\text{Then } 2 + R_2 = \frac{(18)(2)}{0.2} \Rightarrow R_2 = 178 \text{ k}\Omega$$

$$(b) \quad i = \frac{9}{R_1 + R_2} = \frac{9}{2 + 178} \Rightarrow i = 50 \mu\text{A}$$

**15.45**

$$(a) \quad V_{TH} = \left( \frac{R_1}{R_1 + R_2} \right) \cdot V_H = \left( \frac{2}{2 + 48} \right) (10) = 0.4 \text{ V}$$

$$V_{TL} = \left( \frac{R_1}{R_1 + R_2} \right) \cdot V_L = \left( \frac{2}{2 + 48} \right) (-10) = -0.4 \text{ V}$$

(b) For  $33.33 \leq t \leq 41.67 \text{ ms}$ ,  $\sin[2\pi(60)t] \Rightarrow$  positive half cycle

At  $v_I = 0$ ,  $v_o = +10 \text{ V}$

At  $v_I = +0.4 = 10 \sin[2\pi(60)t_1] \Rightarrow t_1 = +0.106 + 33.333 = 33.439 \text{ ms}$

So, for  $33.333 \leq t \leq 33.439 \text{ ms}$ ,  $v_o = +10 \text{ V}$

$$33.439 \leq t \leq 41.77 \text{ ms, } v_o = -10 \text{ V}$$

$$41.77 \leq t \leq 50 \text{ ms, } v_o = +10 \text{ V}$$

**15.46**

a. Upper crossover voltage when  $v_o = +V_P$ ,

Now

$$v_B = \left( \frac{R_1}{R_1 + R_2} \right) (+V_P)$$

and

$$v_A = \left( \frac{R_A}{R_A + R_B} \right) V_{REF} + \left( \frac{R_B}{R_A + R_B} \right) V_{TH}$$

$v_A = v_B$  so that

$$\left( \frac{R_1}{R_1 + R_2} \right) V_P = \left( \frac{R_A}{R_A + R_B} \right) V_{REF} + \left( \frac{R_B}{R_A + R_B} \right) V_{TH}$$

or

$$V_{TH} = \left( \frac{R_A + R_B}{R_1 + R_2} \right) \left( \frac{R_1}{R_B} \right) V_P - \left( \frac{R_A}{R_B} \right) V_{REF}$$

Lower crossover voltage when  $v_o = -V_P$

So

$$V_{TL} = - \left( \frac{R_A + R_B}{R_1 + R_2} \right) \left( \frac{R_1}{R_B} \right) V_P - \left( \frac{R_A}{R_B} \right) V_{REF}$$

b.

$$V_{TH} = \left( \frac{10 + 20}{5 + 20} \right) \left( \frac{5}{20} \right) (10) - \left( \frac{10}{20} \right) (2)$$

or

$$\underline{V_{TH} = 2 \text{ V}}$$

and

$$V_{TL} = - \left( \frac{10 + 20}{5 + 20} \right) \left( \frac{5}{20} \right) (10) - 1 \Rightarrow \underline{V_{TL} = -4 \text{ V}}$$

**15.47**

a.

$$\frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_o - v_B}{R_2}$$

$$v_B \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{REF}}{R_3} + \frac{v_o}{R_2}$$

$$V_{TH} = v_B \text{ when } v_o = +V_P \text{ and } V_{TL} = v_B \text{ when } v_o = -V_P$$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

b.

$$V_S = \frac{V_{REF}}{R_3 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$-5 = \frac{-10}{10 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10}\right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So  $\underline{R_2 = 600 \text{ k}\Omega}$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$

$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow \underline{R_1 = 10.17 \text{ k}\Omega}$$

c.

$$V_{TH} = -5 + 0.1 = -4.9$$

$$V_{TL} = -5 - 0.1 = -5.1$$

### 15.48

- a. If the saturated output voltage is  $|V_P| < 6.2 \text{ V}$ , then the circuit behaves as a comparator where  $|v_0| < 6.2 \text{ V}$ .

If the saturated output voltage is  $|V_P| > 6.2 \text{ V}$ , the output will flip to either  $+V_P$  or  $-V_P$  and the input has no control.

- b. Same as part (a) except the curve at  $v_I \approx 0$  will have a finite slope.  
c. Circuit works as a comparator as long as  $v_{01} < 8.7 \text{ V}$  and  $v_{02} > -3.7 \text{ V}$ . Otherwise the input has no control.

**15.49**

(a) When  $v_o = 0$ ,  $v_+ = V_S = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{REF}$

When  $v_o = V_H$ ,  $v_+ = V_{TH}$

$$\frac{V_{REF} - V_{TH}}{R_1} = \frac{V_{TH} - V_H}{R_2}$$

$$\frac{V_{REF}}{R_1} + \frac{V_H}{R_2} = V_{TH} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = V_{TH} \left( \frac{R_1 + R_2}{R_1 R_2} \right)$$

$$V_{TH} = V_{REF} \left( \frac{R_2}{R_1 + R_2} \right) + V_H \left( \frac{R_1}{R_1 + R_2} \right) = V_S + V_H \left( \frac{R_1}{R_1 + R_2} \right)$$

$$V_{TL} = V_S + V_L \left( \frac{R_1}{R_1 + R_2} \right)$$

(b)  $V_S = -1.75 \text{ V}$ ,  $R_1 = 4 \text{ k}\Omega$

$$V_{TH} = -1.5 = -1.75 + (12) \left( \frac{4}{4 + R_2} \right) \Rightarrow R_2 = 188 \text{ k}\Omega$$

$$-1.75 = \left( \frac{188}{188 + 4} \right) \cdot V_{REF} \Rightarrow V_{REF} = -1.787 \text{ V}$$

(c) (i) For  $v_o = 12 \text{ V}$ ,  $|i| = \frac{12 - (-1.787)}{R_1 + R_2} = \frac{13.787}{4 + 188} \Rightarrow |i| = 71.8 \mu\text{A}$

(ii) For  $v_o = -12 \text{ V}$ ,  $|i| = \frac{12 - 1.787}{192} \Rightarrow |i| = 53.2 \mu\text{A}$

**15.50**

a. Switching point when  $v_o = 0$ .

Now

$$v_+ = V_{REF} = \left( \frac{R_2}{R_1 + R_2} \right) v_I \quad \text{where } v_I = V_S.$$

Then

$$V_S = \left( \frac{R_1 + R_2}{R_2} \right) V_{REF} = \left( 1 + \frac{R_1}{R_2} \right) V_{REF}$$

Now upper crossover voltage for  $v_1$  occurs when  $v_o = V_L$  and  $v_+ = V_{REF}$ . Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$

$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF} \left( 1 + \frac{R_1}{R_2} \right)$$

$$\text{or } V_{TH} = V_S - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for  $v_I$  occurs when  $v_o = V_H$  and  $v_+ = V_{REF}$ . Then

$$\frac{V_H - V_{REF}}{R_2} = \frac{V_{REF} - V_{TL}}{R_1}$$

$$\text{or } V_{TL} = -\frac{R_1}{R_2} \cdot V_H + V_{REF} \left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H$$

b. For  $V_{TH} = -1$  and  $V_{TL} = -2$ ,  $V_S = -1.5$  V. Then  $V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20}$  (12)

so that  $\underline{R_1 = 0.833 \text{ k}\Omega}$

Now

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

$$-1.5 = \left(1 + \frac{0.833}{20}\right) V_{REF}$$

which gives

$$\underline{V_{REF} = -1.44 \text{ V}}$$

### 15.51

(a)  $V_{TH} = \left(\frac{R_1}{R_1 + R_3}\right) \cdot V_\gamma = \left(\frac{25}{25 + 75}\right)(0.7) = 0.175 \text{ V}$

$$V_{TL} = \left(\frac{R_1}{R_1 + R_3}\right)(-V_\gamma) = -0.175 \text{ V}$$

(c) (i)  $v_I = 2 \text{ V}$ ,  $v_O = -0.7 \text{ V}$

$$I_{D1} = 0$$

$$I_{R2} = -\left(\frac{-0.7 - (-10)}{20}\right) = -0.465 \text{ mA}$$

$$I_{R3} = +\left(\frac{-0.175 - (-0.7)}{75}\right) \Rightarrow I_{R3} = 7 \mu\text{A}$$

$$I_{R3} + I_{R2} + I_{D2} = 0 \Rightarrow I_{D2} = -I_{R3} - I_{R2} = -0.007 - (-0.465) = 0.458 \text{ mA}$$

(ii)  $v_I = -2 \text{ V}$ ,  $v_O = +0.7 \text{ V}$

$$I_{D2} = 0$$

$$I_{R2} = \left(\frac{10 - 0.7}{20}\right) = 0.465 \text{ mA}$$

$$I_{R3} = \left(\frac{0.175 - (0.7)}{75}\right) \Rightarrow I_{R3} = -7 \mu\text{A}$$

$$I_{D1} = I_{R2} + I_{R3} = 0.465 - 0.007 = 0.458 \text{ mA}$$

**15.52**

(a)  $V_\gamma + V_Z = 0.7 + 5.6 = 6.3 \text{ V}$

$$\frac{V_{TH}}{R_1} = \frac{0 - (-6.3)}{R_2} \Rightarrow V_{TH} = \frac{R_1}{R_2} (6.3)$$

$$V_{TL} = -\frac{R_1}{R_2} (6.3)$$

$$V_{TH} - V_{TL} = 0.6 = \frac{R_1}{R_2} [6.3 - (-6.3)] = (12.6) \frac{R_1}{R_2}$$

Then  $\frac{R_2}{R_1} = 21$ , Set  $R_1 = 4 \text{ k}\Omega$ , then  $R_2 = 84 \text{ k}\Omega$

(b) Maximum current in  $R_2$ ,

$$i_{R_2} = \frac{6.3}{R_2} = \frac{6.3}{84} = 0.075 \text{ mA}$$

$$i_R = \frac{10 - 6.3}{R} = 0.8 + 0.075 = 0.875 \text{ mA}$$

$$R = \frac{10 - 6.3}{0.875} = 4.23 \text{ k}\Omega$$

**15.53**

a.

$$v_0 = V_{REF} + 2V_\gamma$$

$$5 = V_{REF} + 2(0.7)$$

or

$$\underline{V_{REF} = 3.6 \text{ V}}$$

b.

$$V_{TH} = \left( \frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

$$0.5 = \left( \frac{R_1}{R_1 + R_2} \right) (5)$$

$$\text{or } 1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$$

For example, let  $R_2 = 90 \text{ k}\Omega$  and  $R_1 = 10 \text{ k}\Omega$

c. For  $v_1 = 10 \text{ V}$ , and  $v_0$  is in its low state.  $D_1$  is on and  $D_2$  is off.

$$\frac{v_1 - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_0}{1}$$

For  $v_1 = -0.7$ , then

$$\frac{10 - 0}{100} + \frac{3.6 - (-0.7)}{1} = \frac{-0.7 - v_0}{1}$$

or

$$\underline{v_0 = -5.1 \text{ V}}$$

**15.54**

For  $v_0 = \text{High} = (V_{REF} + 2V_\gamma)$ . Then switching point is when.

$$v_I = v_B = \left( \frac{R_1}{R_1 + R_2} \right) v_0$$

$$\text{or } V_{TH} = \left( \frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

Lower switching point is when

$$v_I = v_B = \left( \frac{R_1}{R_1 + R_2} \right) v_0 \quad \text{and} \quad v_0 = -(V_{REF} + 2V_\gamma)$$

so

$$V_{TL} = - \left( \frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

**15.55**

By symmetry, inverting terminal switches about zero.

Now, for  $v_0$  low, upper diode is on.

$$V_{REF} - v_1 = v_1 - v_0$$

$$v_0 = 2v_1 - V_{REF} \quad \text{where} \quad v_1 = -V_\gamma$$

so

$$v_0 = -(V_{REF} + 2V_\gamma)$$

Similarly, in the high state

$$v_0 = (V_{REF} + 2V_\gamma)$$

Switching occurs when non-inverting terminal is zero.

So for  $v_0$  low.

$$\frac{V_{TH} - 0}{R_1} = \frac{0 - [-(V_{REF} + 2V_\gamma)]}{R_2}$$

$$\text{or } V_{TH} = \frac{R_1}{R_2} \cdot (V_{REF} + 2V_\gamma)$$

By symmetry

$$V_{TL} = - \frac{R_1}{R_2} \cdot (V_{REF} + 2V_\gamma)$$

**15.56**

$$(a) \quad v_+ = \left( \frac{R_1}{R_1 + R_2} \right) \cdot v_o = \left( \frac{10}{10 + 20} \right) (5) = 1.667 \text{ V}$$

$$v_x = 5 + (-1.667 - 5) \exp\left(-\frac{t}{\tau_x}\right) = 5 - 6.667 \exp\left(-\frac{t}{\tau_x}\right), \quad \text{for } 0 < t < t_1$$

$$v_x = -5 + (1.667 - (-5)) \exp\left(-\left(\frac{t-t_1}{\tau_x}\right)\right) = -5 + 6.667 \exp\left(-\left(\frac{t-t_1}{\tau_x}\right)\right), \quad \text{for } t_1 < t < T$$



$$(b) \quad 1.667 = 5 - 6.667 \exp\left(-\frac{t_1}{\tau_x}\right)$$

$$6.667 \exp\left(-\frac{t_1}{\tau_x}\right) = 3.333$$

$$\exp\left(+\frac{t_1}{\tau_x}\right) = 2 \Rightarrow t_1 = \tau_x \ln(2)$$

By symmetry,  $T - t_1 = \tau_x \ln(2)$

Then  $T = 2\tau_x \ln(2)$

$$f = \frac{1}{T} = \frac{1}{2\tau_x \ln(2)}, \quad \tau_x = R_x C_x = (40 \times 10^3)(0.02 \times 10^{-6}) = 8 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{2(8 \times 10^{-4}) \ln(2)} = 902 \text{ Hz}$$

Duty cycle = 50%

### 15.57

$$(a) \quad v_+(+) = \left(\frac{R_1}{R_1 + R_2}\right)(5) = \left(\frac{10}{10 + 20}\right)(5) = 1.667 \text{ V}$$

$$v_+(-) = \left(\frac{R_1}{R_1 + R_2}\right)(-10) = \left(\frac{10}{10 + 20}\right)(-10) = -3.333 \text{ V}$$

$$v_x = 5 + (-3.333 - 5) \exp\left(-\frac{t}{\tau_x}\right) = 5 - 8.333 \exp\left(-\frac{t}{\tau_x}\right), \quad \text{for } 0 < t < t_1$$

$$v_x = -10 + (1.667 - (-10)) \exp\left(-\frac{(t-t_1)}{\tau_x}\right) = -10 + 11.667 \exp\left(-\frac{(t-t_1)}{\tau_x}\right), \quad \text{for } t_1 < t < T$$

$$(b) \quad 1.667 = 5 - 8.333 \exp\left(-\frac{t_1}{\tau_x}\right)$$

$$8.333 \exp\left(-\frac{t_1}{\tau_x}\right) = 3.333 \Rightarrow \exp\left(+\frac{t_1}{\tau_x}\right) = 2.5$$

$$t_1 = \tau_x \ln(2.5)$$

$$\text{Also } -3.333 = -10 + 11.667 \exp\left(-\frac{(T-t_1)}{\tau_x}\right)$$

$$11.667 \exp\left(-\frac{(T-t_1)}{\tau_x}\right) = 6.667 \Rightarrow \exp\left(+\frac{(T-t_1)}{\tau_x}\right) = 1.75$$

$$T - t_1 = \tau_x \ln(1.75)$$

$$\text{Now } T = \tau_x [\ln(2.5) + \ln(1.75)] = \tau_x \ln[(2.5)(1.75)]$$

$$f = \frac{1}{T}, \quad \tau_x = R_x C_x = 8 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{(8 \times 10^{-4}) \ln[(2.5)(1.75)]} = 847 \text{ Hz}$$

**15.58**

$$f = \frac{1}{2.2R_X C_X}$$

$$R_X C_X = \frac{1}{2.2f} = \frac{1}{(2.2)(12 \times 10^3)}$$

$$R_X C_X = 3,788 \times 10^{-5}$$

$$R_X = 56 \text{ K}$$

For example, Let  $C_X = 680 \text{ pF}$

Within  $\frac{1}{2}$  of 1% of design specification.

**15.59**

$$t_1 = 1.1R_X C_X = (1.1)(10^4)(0.1 \times 10^{-6}) \Rightarrow t_1 = 1.1 \text{ ms}$$

$$0 < t < t_1, \quad v_Y = 10(1 - e^{-t/\tau_Y})$$

$$\begin{aligned} \tau_Y = R_Y C_Y &= (2 \times 10^3)(0.02 \times 10^{-6}) \\ &= 4 \times 10^{-5} \text{ s} \end{aligned}$$

Now  $\frac{t_1}{\tau_Y} = 2.75$

$\Rightarrow C_Y$  completely charges during each cycle.

**15.60**

a. Switching voltage

$$v_X = \left( \frac{R_1 + R_3}{R_1 + R_3 + R_2} \right) \cdot V_P = \left( \frac{10 + 10}{10 + 10 + 10} \right) (\pm 10)$$

So  $v_X = \pm 6.667 \text{ V}$

Using Equation (15.83(a))

$$v_X = V_P + \left( -\frac{2}{3}V_P - V_P \right) e^{-t_1/\tau_X} = \frac{2}{3}V_P$$

Then  $1 - \frac{5}{3} \cdot e^{-t_1/\tau_X} = \frac{2}{3}$

$$\frac{1}{3} = \frac{5}{3} \cdot e^{-t_1/\tau_X} \quad \text{or} \quad t_1 = \tau_X \ln(5)$$

$$t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2(500)} \Rightarrow t_1 = 0.001 \text{ s}$$

$$10^{-3} = \tau_X \ln(5) \Rightarrow \tau_X = 6.21 \times 10^{-4} = R_X (0.01 \times 10^{-6})$$

So  $R_X = 62.1 \text{ k}\Omega$

b. Switching voltage

$$v_X = \left( \frac{R_1}{R_1 + R_3 + R_2} \right) (\pm V_P)$$

$$= \left( \frac{10}{10 + 10 + 10} \right) (\pm V_P) = \frac{1}{3} \cdot (\pm V_P)$$

Using Equation (15.83(a))

$$v_X = V_P + \left(-\frac{1}{3}V_P - V_P\right)e^{-t_1/\tau_X} = \frac{1}{3}V_P$$

Then  $1 - \frac{4}{3}e^{-t_1/\tau_X} = \frac{1}{3}$

$$\frac{2}{3} = \frac{4}{3} \cdot e^{-t_1/\tau_X}$$

$$t_1 = \tau_X \ln(2) = (6.21 \times 10^{-4}) \ln(2) = 4.30 \times 10^{-4} \text{ s}$$

$$T = 2t_1 = 8.6 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{T} \Rightarrow f = 1.16 \text{ kHz}$$

**15.61**

(a)  $\beta = \frac{R_1}{R_1 + R_2} = \frac{20}{20 + 12} = 0.625$

$$T = 250 \times 10^{-6} = \tau_X \ln \left[ \frac{1 + \frac{0.7}{10}}{1 - 0.625} \right] = \tau_X (1.0485)$$

So  $\tau_X = 2.384 \times 10^{-4} = R_X C_X$

Set  $C_X = 0.01 \mu\text{F}$ , then  $R_X = 23.84 \text{ k}\Omega$

(b) For  $t < 0$ ,  $v_X = 0.7 \text{ V}$

So  $v_Y$  must be  $< v_X \Rightarrow v_I < 0$

(c)  $v_X = V_Y = 10 + [-\beta(10) - 10] \exp\left(-\frac{(T' - T)}{\tau_X}\right)$

$$0.7 = 10 - 16.25 \exp\left(-\frac{(T' - T)}{\tau_X}\right) \Rightarrow 16.26 \exp\left(-\frac{(T' - T)}{\tau_X}\right) = 9.3$$

$$\exp\left(+\frac{(T' - T)}{\tau_X}\right) = 1.747$$

$$T' - T = \tau_X \ln(1.747) = 2.384 \times 10^{-4} \ln(1.747)$$

$$T' - T = 133 \mu\text{s}$$

**15.62**

(a)  $T = \tau_X \ln \left[ \frac{1 + \frac{V_Y}{V_P}}{1 - \beta} \right]$ ,  $V_Y = 0.7 \text{ V}$ ,  $V_P = 5 \text{ V}$ ,  $\beta = 0.5$

$$\tau_X = R_X C_X = (20 \times 10^3)(1.2 \times 10^{-6}) = 2.4 \times 10^{-2} \text{ s}$$

$$T = (2.4 \times 10^{-2}) \ln \left[ \frac{1 + \frac{0.7}{5}}{1 - 0.5} \right] \Rightarrow T = 19.78 \text{ ms}$$

(b)  $T' - T \cong 0.4\tau_x = 9.6 \text{ ms}$

**15.63**

a. From Equation (15.95)

$$T = 1.1 RC$$

For  $T = 60 \text{ s} = 1.1 RC$

then  $RC = 54.55 \text{ s}$

For example, let

$$C = 50 \text{ } \mu\text{F} \text{ and } R = 1.09 \text{ M}\Omega$$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

If  $V^+ = 5 \text{ V}$ , then  $I_B \cong \frac{5-0.7}{100} = 0.043 \text{ mA}$

If  $\beta = 100$ ,  $I_C = 4.3 \text{ mA}$

$$V_C = \frac{1}{C} \int I_C dt = \frac{I_C}{C} \cdot t$$

Capacitor has charged to  $\frac{2}{3} \cdot V^+ = 3.33 \text{ V}$

So that  $t = \frac{V_C \cdot C}{I_C} = \frac{(3.33)(50 \times 10^{-6})}{4.3 \times 10^{-3}}$

So recovery time  $t \approx 38.7 \text{ ms}$

**15.64**

$$T = 1.1 RC$$

$$5 \times 10^{-6} = 1.1 RC$$

so  $RC = 4.545 \times 10^{-6} \text{ s}$

For example, let

$$C = 100 \text{ pF} \text{ and } R = 45.5 \text{ k}\Omega$$

From Problem (15.63), recovery time

$$t \cong \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100 \times 10^{-12})}{4.3 \times 10^{-3}}$$

or

$$t = 77.4 \text{ ns}$$

**15.65**

$$\text{Duty cycle} = 60\% = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$0.60 = \frac{25 + R_B}{25 + 2R_B} \Rightarrow R_B = 50 \text{ k}\Omega$$

$$f = 80 \times 10^3 = \frac{1}{(0.693)(25 + 2(50)) \times 10^3 C} \Rightarrow C = 144.3 \text{ pF}$$

**15.66**

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$$R_A = R_1 = 10 \text{ k}\Omega, R_B = R_2 + xR_3$$

$$\text{So } 10 \text{ k}\Omega \leq R_B \leq 110 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(10 + 2(110)) \times 10^3 \times (0.01 \times 10^{-6})} = 627 \text{ kHz}$$

$$f_{\max} = \frac{1}{(0.693)(10 + 2(10)) \times 10^3 \times (0.01 \times 10^{-6})} = 4.81 \text{ kHz}$$

$$\text{So } 627 \text{ Hz} \leq f \leq 4.81 \text{ kHz}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

Now

$$\frac{10 + 10}{10 + 2(10)} \times 100\% = \underline{66.7\%}$$

and

$$\frac{10 + 110}{10 + 2(110)} \times 100\% = \underline{52.2\%}$$

$$\text{So } 52.2 \leq \text{Duty cycle} \leq 66.7\%$$

**15.67**

$$1 \text{ k}\Omega \leq R_A \leq 51 \text{ k}\Omega$$

$$1 \text{ k}\Omega \leq R_B \leq 51 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(1 + 2(51)) \times 10^3 \times (0.01 \times 10^{-6})} = 1.40 \text{ Hz}$$

$$f_{\max} = \frac{1}{(0.693)(51 + 2(1)) \times 10^3 \times (0.01 \times 10^{-6})} = 2.72 \text{ kHz}$$

$$\text{or } 1.40 \text{ kHz} \leq f \leq 2.72 \text{ kHz}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$\frac{1 + 51}{1 + 2(51)} \times 100\% = 50.5\%$$

or

$$\frac{51 + 1}{51 + 2(1)} \times 100\% = \underline{98.1\%}$$

or

$$50.5\% \leq \text{Duty cycle} \leq 98.1\%$$

**15.68**

a.

$$I_{E3} = I_{E4} = \frac{V^+ - 3V_{EB}}{R_{1A} + R_{1B}}$$

Assume  $V_{EB} = 0.7$

$$I_{E3} = I_{E4} = \frac{22 - 3(0.7)}{25 + 25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$

$$\underline{I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}}$$

$$I_{C1} = I_{C2} = \frac{0.398}{21} \left(\frac{20}{21}\right) \Rightarrow \underline{I_{C1} = I_{C2} = 0.018 \text{ mA}}$$

b.  $\underline{I_D = 0.398 \text{ mA}}$ , current in  $D_1$  and  $D_2$

$$V_{BB} = 2V_D = 2V_T \ln \left(\frac{I_D}{I_S}\right) = 2(0.026) \ln \left(\frac{0.398 \times 10^{-3}}{10^{-13}}\right)$$

or

$$V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$

$$I_{C4} = 0.379 \text{ mA}$$

$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right)I_{E8}$$

So

$$I_{E8} = 1.05I_{B9} = 1.05 \left(\frac{I_{C8}}{100}\right)$$

$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right)I_{E8} + I_{E8} = I_{C4} + (96.24) \left(\frac{21}{20}\right)I_{C8}$$

So  $I_{C7} = 0.379 \text{ mA} + 101I_{C8}$

and

$$V_{BE7} = V_T \ln \left(\frac{I_{C7}}{I_S}\right); V_{EB8} = V_T \ln \left(\frac{I_{C8}}{I_S}\right)$$

Then

$$1.149 = 0.026 \left[ \ln \left( \frac{I_{C7}}{I_S} \right) + \ln \left( \frac{I_{C8}}{I_S} \right) \right]$$

$$44.19 = \ln \left[ \frac{I_{C8}(0.379 \times 10^{-3}) + 101I_{C8}}{(10^{-13})^2} \right]$$

$$(10^{-13})^2 \exp(44.19) = 101I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$

$$1.554 \times 10^{-7} = 101I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$

$$I_{C8} = \frac{-3.79 \times 10^{-4}}{2(101)} \pm \frac{\sqrt{(3.79 \times 10^{-4})^2 + 4(101)(1.554 \times 10^{-7})}}{2(101)}$$

$$\underline{I_{C8} = 37.4 \mu\text{A}}$$

$$I_{C7} = 0.379 + 101(0.0374) \Rightarrow \underline{I_{C7} = 4.16 \text{ mA}}$$

$$I_{C9} = 4.16 - 0.379 - 0.0374 \left( \frac{21}{20} \right)$$

$$\underline{I_{C9} = 3.74 \text{ mA}}$$

c.  $P = (0.398 + 0.398 + 4.16)(22) \Rightarrow \underline{P = 109 \text{ mW}}$

### 15.69

a. From Figure 15.44, 3.7 W to the load

b.  $\underline{V^+ \approx 19 \text{ V}}$

$$\bar{P} = \frac{1}{2} \frac{V_P^2}{R_L}$$

c.  
or

$$V_P = \sqrt{2R_L \bar{P}} = \sqrt{2(10)(3.7)} \Rightarrow \underline{V_P = 8.6 \text{ V}}$$

### 15.70

(a)  $\left( 1 + \frac{R_2}{R_1} \right) = 12 \Rightarrow \frac{R_2}{R_1} = 11$ , and  $\frac{R_4}{R_3} = 12$

(b)  $\bar{P}_L = \frac{V_L^2}{2R_L} \Rightarrow V_L = \sqrt{2R_L \bar{P}_L} = \sqrt{2(12)(15)} = 18.97 \text{ V}$

$$|v_{O1}|_{\max} = |v_{O2}|_{\max} = 9.49 \text{ V}$$

$$I_L = \frac{V_L}{R_L} = \frac{18.97}{12} = 1.58 \text{ A}$$

(c) (i)  $\bar{P}_L = \left( \frac{I_L}{\sqrt{2}} \right) \left( \frac{V_L}{\sqrt{2}} \right) = \left( \frac{0.8}{\sqrt{2}} \right) \left( \frac{24}{\sqrt{2}} \right) = 9.6 \text{ W}$

(ii)  $R_L = \frac{V_L}{I_L} = \frac{24}{0.8} = 30 \Omega$

**15.71**

a.  $v_{01} = iR_2 + v_I$  where  $i = \frac{v_I}{R_1}$

Then

$$v_{01} = v_I \left( 1 + \frac{R_2}{R_1} \right)$$

Now

$$v_{02} = -iR_3 = -v_I \left( \frac{R_3}{R_1} \right)$$

So

$$v_L = v_{01} - v_{02} = v_I \left( 1 + \frac{R_2}{R_1} \right) - \left[ -v_I \left( \frac{R_3}{R_1} \right) \right]$$

$$A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}$$

b. Want  $A_v = 10 \Rightarrow \frac{R_2}{R_1} + \frac{R_3}{R_1} = 9$

Also want  $\left( 1 + \frac{R_2}{R_1} \right) = \frac{R_3}{R_1}$

Then  $\frac{R_2}{R_1} + \left( 1 + \frac{R_2}{R_1} \right) = 9$  so  $\frac{R_2}{R_1} = 4$

For  $R_1 = 50 \text{ k}\Omega$ ,  $R_2 = 200 \text{ k}\Omega$

and

$$\frac{R_3}{R_1} = 5 \quad \text{so} \quad R_3 = 250 \text{ k}\Omega$$

$$\bar{P} = \frac{1}{2} \frac{V_p^2}{R_L}$$

c.

or

$$V_p = \sqrt{2R_L \bar{P}} = \sqrt{2(20)(10)} = 20 \text{ V}$$

So peak values of output voltages are

$$|v_{01}| = |v_{02}| = 10 \text{ V}$$

$$\text{Peak load current} = \frac{20}{20} = 1 \text{ A}$$

**15.72**

(a)  $v_{O1} = \left( 1 + \frac{R_2}{R_1} \right) \cdot v_I$

$$v_{O2} = -\frac{R_4}{R_3} \cdot v_{O1} = -\frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) \cdot v_I$$

$$v_L = v_{O1} - v_{O2}$$

$$A_v = \frac{v_L}{v_I} = \left( 1 + \frac{R_2}{R_1} \right) \left( 1 + \frac{R_4}{R_3} \right)$$



$$(b) \left(1 + \frac{R_2}{R_1}\right) = 12 \Rightarrow \frac{R_2}{R_1} = 11, \text{ Set } R_2 = 120 \text{ k}\Omega, \text{ then } R_1 = 10.9 \text{ k}\Omega$$

$$\frac{R_4}{R_3} = 1, \text{ Set } R_3 = R_4 = 120 \text{ k}\Omega$$

$$(c) (i) \bar{P}_L = \frac{V_L^2}{2R_L} = \frac{(16)^2}{2(25)} = 5.12 \text{ W}$$

$$(ii) I_L = \frac{V_L}{R_L} = \frac{16}{25} = 0.64 \text{ A}$$

### 15.73

(a) From Problem 15.72

$$v_{o2} = -\frac{R_4}{R_3} \cdot v_{o1}, \text{ Set } \frac{R_4}{R_3} = 1$$

$$A_v = \left(1 + \frac{R_2}{R_1}\right) \left(1 + \frac{R_4}{R_3}\right)$$

$$25 = \left(1 + \frac{R_2}{R_1}\right) (2) \Rightarrow \frac{R_2}{R_1} = 11.5$$

Set  $R_2 = R_3 = R_4 = 100 \text{ k}\Omega$ , then  $R_1 = 8.69 \text{ k}\Omega$

$$(b) (i) \bar{P}_L = \left(\frac{V_L}{\sqrt{2}}\right) \left(\frac{I_L}{\sqrt{2}}\right) = \left(\frac{24}{\sqrt{2}}\right) \left(\frac{1.2}{\sqrt{2}}\right) = 14.4 \text{ W}$$

$$(ii) R_L = \frac{V_L}{I_L} = \frac{24}{1.2} = 20 \Omega$$

$$(c) \bar{P}_L = \frac{V_L^2}{2R'_L} = \frac{(24)^2}{2(40)} = 7.2 \text{ W}$$

### 15.74

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta I = \frac{\Delta V^+}{R_1} \text{ and } \Delta V_z = r_z \cdot \Delta I \text{ and } \Delta V_0 = 10\Delta V_z$$

So

$$\Delta V_0 = 10 \cdot r_z \cdot \frac{\Delta V^+}{R_1}$$

So

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+} = \frac{10(15)}{9300} \Rightarrow 1.61\%$$

**15.75**

$$(a) R_{of} = \frac{|\Delta V_o|}{|\Delta I_o|} = \frac{8}{2} \Rightarrow R_{of} = 4 \text{ m}\Omega$$

$$(b) \Delta V_o = R_{of} \cdot \Delta I_o = (10)(1.2) \Rightarrow \Delta V_o = 12 \text{ mV}$$

**15.76**

For  $V_o = 8 \text{ V}$

$$V^+(\text{min}) = V_o + I_o(\text{max})R_{11} + V_{BE11} + V_{BE10} + V_{EB5}$$

This assumes  $V_{BC5} = 0$ .

Then

$$V^+(\text{min}) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$$\underline{V^+(\text{min}) = 9.99 \text{ V}}$$

**15.77**

a.

$$I_{C3} = I_{C5} = \frac{V_Z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571 \text{ mA}$$

$$I_{C8} = \frac{1}{2} \left( \frac{0.6}{2.84} \right) = 0.106 \text{ mA}$$

Neglecting current in  $Q_9$ , total collector current and emitter current in  $Q_5$  is  
 $0.571 + 0.106 = 0.677$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB5}$$

$$V_{EB4} = V_T \ln \left( \frac{I_{Z2}}{I_5} \right)$$

$$V_{EB5} = V_T \ln \left( \frac{I_{C5}}{2I_5} \right)$$

Then  $I_{Z2}R_4 = V_T \ln \left( \frac{I_{C5}}{2I_{Z2}} \right)$

$$R_4 = \frac{0.026}{0.25} \cdot \ln \left( \frac{0.677}{2(0.25)} \right)$$

or

$$\underline{R_4 = 31.5 \Omega}$$

b. From Example 15.16,  $V_{B7} = 3.43 \text{ V}$ . Then

$$\left( \frac{R_{13}}{R_{12} + R_{13}} \right) V_0 = V_{B8} = V_{B7}$$

or

$$\left( \frac{2.23}{2.23 + R_{12}} \right) (12) = 3.43$$

$$3.43(2.23 + R_{12}) = (2.23)(12)$$

which yields

$$\underline{R_{12} = 5.57 \text{ k}\Omega}$$

### 15.78

Line regulation =  $\frac{\Delta V_0}{\Delta V^+}$

Now

$$\Delta V_{B7} = \Delta I_{C3} \cdot R_1$$

and  $\left( \frac{R_{13}}{R_{12} + R_{13}} \right) (\Delta V_0) = \Delta V_{B7} = \Delta I_{C3} R_1$

and  $\Delta I_{C3} = \frac{\Delta V_Z}{R_1 + R_2 + R_3} = \frac{\Delta I_Z \cdot r_z}{R_1 + R_2 + R_3}$

and  $\Delta I_Z = \frac{\Delta V^+}{r_0}$  where  $r_0 = \frac{V_A}{I_Z}$

Then

$$(0.4288)(\Delta V_0) = \Delta I_{C3}(3.9) = (3.9)\Delta I_Z \left( \frac{0.015}{7.876} \right)$$

$$r_0 = \frac{50}{0.571} = 87.6 \text{ k}\Omega$$

Then

$$(0.4288)(\Delta V_0) = (0.00743) \left( \frac{\Delta V^+}{87.6} \right)$$

So

$$\underline{\frac{\Delta V_0}{\Delta V^+} = 0.0198\%}$$

### 15.79

(a)  $R_1 + r_z = \frac{V^+}{I_Z} = \frac{25}{12} = 2.0833 \text{ k}\Omega$

$$R_1 = 2.0833 - 0.012 = 2.0713 \text{ k}\Omega$$

(b)  $V^+ = 5.6 + (0.012)(12) = 5.744 \text{ V}$

For  $x = 0$ ,

$$5.744 = \left( \frac{R_3 + R_4}{R_2 + R_3 + R_4} \right) \cdot V_o \Rightarrow V_o = \left( \frac{1+2+1}{2+1} \right) (5.744) = 7.659 \text{ V}$$

For  $x=1$ ,

$$5.744 = \left( \frac{R_4}{R_2 + R_3 + R_4} \right) \cdot V_o \Rightarrow V_o = \left( \frac{1+2+1}{1} \right) (5.744) = 22.976 \text{ V}$$

So  $7.659 \leq V_o \leq 22.976 \text{ V}$

(c) For  $x=1$ ,  $V_1 = \frac{1}{4} V_o$

$$v_d = V_{REF} - V_1 = V_{REF} - \frac{1}{4} V_o$$

$$V_o = A_{OL} v_d - V_{BE} = A_{OL} v_d - V_T \ln \left( \frac{I_o}{I_S} \right)$$

$$V_o = A_{OL} \left( V_{REF} - \frac{1}{4} V_o \right) - V_T \ln \left( \frac{I_o}{I_S} \right)$$

$$V_o \left( 1 + \frac{1}{4} A_{OL} \right) = A_{OL} V_{REF} - V_T \ln \left( \frac{I_o}{I_S} \right)$$

$$V_o = \frac{A_{OL} V_{REF} - V_T \ln \left( \frac{I_o}{I_S} \right)}{1 + \frac{1}{4} A_{OL}}$$

$$\begin{aligned} \text{Load regulation} &= \frac{V_o(NL) - V_o(FL)}{V_o(NL)} \\ &= \frac{\frac{A_{OL} V_{REF} - V_T \ln [I_o(NL)/I_S]}{1 + (1/4)A_{OL}} - \left[ \frac{A_{OL} V_{REF} - V_T \ln [I_o(FL)/I_S]}{1 + (1/4)A_{OL}} \right]}{\frac{A_{OL} V_{REF} - V_T \ln [I_o(NL)/I_S]}{1 + (1/4)A_{OL}}} \\ &= \frac{V_T \left[ \ln \left( \frac{I_o(FL)}{I_S} \right) - \ln \left( \frac{I_o(NL)}{I_S} \right) \right]}{A_{OL} V_{REF} - V_T \ln \left( \frac{I_o(NL)}{I_S} \right)} = \frac{V_T \ln \left( \frac{I_o(FL)}{I_o(NL)} \right)}{A_{OL} V_{REF} - V_T \ln \left( \frac{I_o(NL)}{I_S} \right)} \end{aligned}$$

Let  $I_o(FL) = 5 \text{ A}$ ,  $I_o(NL) = 1\% = 0.05 \text{ A}$

Let  $V_T \ln \left( \frac{I_o(NL)}{I_S} \right) \cong 0.7 \text{ V}$

$$\text{Load regulation} = \frac{(0.026) \ln \left( \frac{5}{0.05} \right)}{(5 \times 10^3)(5.744) - 0.7} = \frac{0.11973}{2.8719 \times 10^4}$$

Load regulation =  $4.169 \times 10^{-4} \%$

**15.80**

$$(a) \quad I_E = \frac{V_Z}{R_2} = \frac{6.8}{5} = 1.36 \text{ mA}$$

$$I_O = \left( \frac{\beta}{1+\beta} \right) \cdot I_E = \left( \frac{80}{81} \right) (1.36) = 1.343 \text{ mA}$$

$$\text{For } V_{BC} = 0, \quad V_O = 20 - V_Z - V_{EB} = 20 - 6.8 - 0.6 = 12.6 \text{ V}$$

$$R_L(\text{max}) = \frac{V_O}{I_O} = \frac{12.6}{1.343} = 9.38 \text{ k}\Omega$$

$$\text{So } 0 \leq R_L \leq 9.38 \text{ k}\Omega$$

$$(b) \quad V_+ = \left( \frac{R_1}{R_1 + r_Z} \right) (V^+ - V_{ZO}) = \left( \frac{10^4}{10^4 + 20} \right) (V^+ - 6.8)$$

$$\text{For } V^+ = 20 \text{ V}, \quad V_+ = \left( \frac{10^4}{10^4 + 20} \right) (13.2) = 13.17365 \text{ V}$$

$$I_O = \left( \frac{80}{81} \right) \left( \frac{20 - 13.17365}{5} \right) = 1.3484 \text{ mA}$$

$$\text{For } V^+ = 16 \text{ V}, \quad V_+ = \left( \frac{10^4}{10^4 + 20} \right) (16 - 6.8) = 9.181637 \text{ V}$$

$$I_O = \left( \frac{80}{81} \right) \left( \frac{16 - 9.181637}{5} \right) = 1.3468 \text{ mA}$$

$$\text{So } 1.3468 \leq I_O \leq 1.3484 \text{ mA}$$

## Chapter 16

### 16.1

$$(a) \quad v_o = V_{DD} - \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_D[2(v_i - V_{TN})v_o - v_o^2]$$

$$0.1 = 3.3 - \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(40)[2(3.3 - 0.5)(0.1) - (0.1)^2]$$

$$0.1 = 3.3 - \left(\frac{W}{L}\right)(1.1) \Rightarrow \left(\frac{W}{L}\right) = 2.91$$

$$(b) \quad \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_D v_{o_i}^2 + v_{o_i} - V_{DD} = 0$$

$$\left(\frac{0.1}{2}\right)(2.91)(40)v_{o_i}^2 + v_{o_i} - 3.3 = 0$$

$$5.82v_{o_i}^2 + v_{o_i} - 3.3 = 0 \Rightarrow v_{o_i} = 0.672 \text{ V}$$

$$v_{o_i} = v_{i_i} - V_{TN} \Rightarrow v_{i_i} = 1.172 \text{ V}$$

$$(c) \quad i_{D,\max} = \frac{3.3 - 0.1}{40} \Rightarrow i_{D,\max} = 80 \mu\text{A}$$

$$P_{D,\max} = (80)(3.3) = 264 \mu\text{W}$$

### 16.2

$$(a) \quad (i) \quad K_n R_D v_{o_i}^2 + v_{o_i} - V_{DD} = 0$$

$$(0.05)(100)v_{o_i}^2 + v_{o_i} - 3.3 = 0 \Rightarrow v_{o_i} = 0.7185 \text{ V}$$

$$\Rightarrow v_{i_i} = 1.219 \text{ V}$$

$$(ii) \quad v_o = 3.3 - (0.05)(100)[2(3.3 - 0.5)v_o - v_o^2]$$

We find  $5v_o^2 - 29v_o + 3.3 = 0 \Rightarrow v_o = 0.116 \text{ V}$

$$(b) \quad (i) \quad (0.05)(30)v_{o_i}^2 + v_{o_i} - 3.3 = 0 \Rightarrow v_{o_i} = 1.187 \text{ V}$$

$$\Rightarrow v_{i_i} = 1.687 \text{ V}$$

$$(ii) \quad v_o = 3.3 - (0.05)(30)[2(3.3 - 0.5)v_o - v_o^2]$$

Or  $1.5v_o^2 - 9.4v_o + 3.3 = 0 \Rightarrow v_o = 0.373 \text{ V}$

$$(c) \quad (i) \quad (0.05)(5)v_{o_i}^2 + v_{o_i} - 3.3 = 0 \Rightarrow v_{o_i} = 2.147 \text{ V}$$

$$\Rightarrow v_{i_i} = 2.647 \text{ V}$$

$$(ii) \quad v_o = 3.3 - (0.05)(5)[2(3.3 - 0.5)v_o - v_o^2]$$

Or  $0.25v_o^2 - 2.4v_o + 3.3 = 0 \Rightarrow v_o = 1.663 \text{ V}$

**16.3**

(a)  $P = IV$

$$0.25 = I(3.3) \Rightarrow I = 75.76 \mu\text{A}$$

$$R = \frac{3.3 - 0.15}{0.07576} = 41.6 \text{ k}\Omega$$

$$I = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$$

$$75.76 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)\left[2(3.3 - 0.5)(0.15) - (0.15)^2\right] \Rightarrow \left(\frac{W}{L}\right) = 1.85$$

(b)  $I_D = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2 = \frac{V_{DD} - V_{DS}(\text{sat})}{R}$

$$\left(\frac{0.1}{2}\right)(1.85)V_{DS}^2(\text{sat}) = \frac{3.3 - V_{DS}(\text{sat})}{41.6}$$

Or  $3.848V_{DS}^2(\text{sat}) + V_{DS}(\text{sat}) - 3.3 = 0 \Rightarrow V_{DS}(\text{sat}) = 0.805 \text{ V}$

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN}$$

$$0.805 = V_{GS} - 0.5 \Rightarrow V_{GS} = 1.305 \text{ V}$$

Then  $0.5 \leq V_{GS} \leq 1.305 \text{ V}$

**16.4**

(a) From Equation (16.21)

$$\begin{aligned} \frac{K_D}{K_L} &= \frac{(W/L)_D}{(W/L)_L} = \frac{(V_{DD} - v_O - V_{TNL})^2}{2(v_I - V_{TND})v_O - v_O^2} \\ &= \frac{(1.8 - 0.08 - 0.4)^2}{2(1.4 - 0.4)(0.08) - (0.08)^2} = \frac{1.7424}{0.1536} \end{aligned}$$

$$\frac{(W/L)_D}{(W/L)_L} = 11.34$$

$$P_{D,\text{max}} = i_{D,\text{max}} \cdot V_{DD}$$

$$0.3 = i_{D,\text{max}}(1.8) \Rightarrow i_{D,\text{max}} = 0.1667 \text{ mA}$$

$$i_{D,\text{max}} = 0.1667 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_D \left[2(1.4 - 0.4)(0.08) - (0.08)^2\right]$$

Which yields  $\left(\frac{W}{L}\right)_D = 21.7$  and  $\left(\frac{W}{L}\right)_L = 1.91$

(b)  $V_{It} = \frac{1.8 - 0.4 + (0.4)(1 + \sqrt{11.34})}{1 + \sqrt{11.34}} = 0.7206 \text{ V}$

$0.4 \leq V_{It} \leq 0.7206 \text{ V}$

**16.5**

(a)  $\frac{K_D}{K_L} \left[2(3 - 0.5)(0.25) - (0.25)^2\right] = [3 - 0.25 - 0.5]^2$

$$\frac{K_D}{K_L}(1.1875) = (5.0625) \Rightarrow \frac{K_D}{K_L} = 4.26$$

$$(b) \frac{K_D}{K_L} [2(2.5 - 0.5)(0.25) - (0.25)^2] = [3 - 0.25 - 0.5]^2$$

$$\frac{K_D}{K_L} (0.9375) = (5.0625) \Rightarrow \frac{K_D}{K_L} = 5.4$$

$$(c) i_D = K_L (V_{GS} - V_{TN})^2 = \left(\frac{0.1}{2}\right) (1)(3 - 0.25 - 0.5)^2 = 0.253 \text{ mA}$$

$$P = i_D \cdot V_{DD} = (0.253)(3) = 0.759 \text{ mW}$$

### 16.6

$$(a) \frac{K_D}{K_L} = \frac{(V_{DD} - v_O - V_{TNL})^2}{2(v_I - V_{TND})v_O - v_O^2}$$

$$v_I = \text{Logic 1} = V_{DD} - V_{TNL} = 3 - 0.5 = 2.5 \text{ V}$$

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = \frac{(3 - 0.1 - 0.5)^2}{2(2.5 - 0.5)(0.1) - (0.1)^2} = \frac{5.76}{0.39}$$

$$\frac{(W/L)_D}{(W/L)_L} = 14.77$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$0.4 = i_{D,\max} (3) \Rightarrow i_{D,\max} = 0.1333 \text{ mA}$$

$$i_{D,\max} = 0.1333 = \left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_D [2(2.5 - 0.5)(0.1) - (0.1)^2]$$

Which yields  $\left(\frac{W}{L}\right)_D = 6.84$  and  $\left(\frac{W}{L}\right)_L = 0.463$

$$(b) V_{It} = \frac{3 - 0.5 + (0.5)(1 + \sqrt{14.77})}{1 + \sqrt{14.77}} = 1.016 \text{ V}$$

$$V_{Ot} = 1.016 - 0.5 = 0.516 \text{ V}$$

### 16.7

We have

$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (V_{DD} - v_O - V_{TNL})^2$$

$$\frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - V_{TN} - V_{TN})(0.08V_{DD}) - (0.08V_{DD})^2] = (V_{DD} - 0.08V_{DD} - V_{TN})^2$$

$$\frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - 2(0.2)V_{DD})(0.08V_{DD}) - 0.0064V_{DD}^2] = [(0.92 - 0.2)V_{DD}]^2 = 0.5184V_{DD}^2$$

$$\frac{(W/L)_D}{(W/L)_L} [0.096] = 0.5184 \Rightarrow \frac{(W/L)_D}{(W/L)_L} = 5.4$$



**16.8**

$$V_{OH} = V_B - V_{TN} = \text{Logic 1}$$

So

(a)  $V_B = 4\text{ V} \Rightarrow V_{OH} = 3\text{ V}$

(b)  $V_B = 5\text{ V} \Rightarrow V_{OH} = 4\text{ V}$

(c)  $V_B = 6\text{ V} \Rightarrow V_{OH} = 5\text{ V}$

(d)  $V_B = 7\text{ V} \Rightarrow V_{OH} = 5\text{ V}$ , since  $V_{DS} = 0$

For  $v_i = V_{OH}$

$$K_D [2(v_i - V_T)v_o - v_o^2] = K_L [V_B - v_o - V_T]^2$$

Then

(a)  $(1)[2(3-1)V_{OL} - V_{OL}^2] = (0.4)[4 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.657\text{ V}$

(b)  $(1)[2(4-1)V_{OL} - V_{OL}^2] = (0.4)[5 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.791\text{ V}$

(c)  $(1)[2(5-1)V_{OL} - V_{OL}^2] = (0.4)[6 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.935\text{ V}$

(d) Load in non-sat region

$$i_{DD} = i_{OL}$$

(1)  $[2(5-1)V_{OL} - V_{OL}^2] = (0.4)[2(7 - V_{OL} - 1)(5 - V_{OL}) - (5 - V_{OL})^2]$

$$8V_{OL} - V_{OL}^2 = (0.4)[2(6 - V_{OL})(5 - V_{OL}) - (25 - 10V_{OL} + V_{OL}^2)]$$

$$= (0.4)[2(30 - 11V_{OL} + V_{OL}^2) - 25 + 10V_{OL} - V_{OL}^2]$$

$$= (0.4)[60 - 22V_{OL} + 2V_{OL}^2 - 25 + 10V_{OL} - V_{OL}^2]$$

$$8V_{OL} - V_{OL}^2 = 14 - 4.8V_{OL} + 0.4V_{OL}^2$$

$$1.4V_{OL}^2 - 12.8V_{OL} + 14 = 0$$

$$V_{OL} = \frac{12.8 \pm \sqrt{163.84 - 4(1.4)(14)}}{2(1.4)}$$

$$V_{OL} = 1.27\text{ V}$$

For load

$$V_{DS}(\text{sat}) = 7 - 1.27 - 1 = 4.73\text{ V}$$

$$V_{DS} = 5 - 1.27 = 3.73\text{ non-sat}$$

**16.9**

(a)  $\sqrt{\frac{K_D}{K_L}}(V_{It} - V_{TND}) = -V_{TNL}$

$$\sqrt{\frac{500}{100}}(V_{It} - 0.5) = -(-0.8) \Rightarrow V_{It} = 0.8578\text{ V}$$

For Driver:  $V_{Oit} = V_{It} - V_{TND} = 0.8578 - 0.5 = 0.3578\text{ V}$

For Load:  $V_{Oit} = V_{DD} + V_{TNL} = 3.3 + (-0.8) = 2.5\text{ V}$

(b)  $\frac{K_D}{K_L} [2(v_i - V_{TND})v_o - v_o^2] = (-V_{TNL})^2$

$$\left(\frac{500}{100}\right) [2(3.3 - 0.5)v_o - v_o^2] = [ -(-0.8) ]^2$$

We find  $5v_o^2 - 28v_o + 0.64 = 0 \Rightarrow v_o = 0.0230\text{ V}$

$$(c) \quad i_{D,\max} = K_L (-V_{TNL})^2 = 100[-(-0.8)]^2 = 64 \mu\text{ A}$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD} = (64)(3.3) = 211 \mu\text{ W}$$

**16.10**

$$\left(\frac{500}{50}\right) [2(3-0.5)(0.1) - (0.1)^2] = (-V_{TNL})^2$$

So

$$(-V_{TNL})^2 = 4.9 \Rightarrow V_{TNL} = -2.21\text{ V}$$

**16.11**

$$(a) \quad P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$80 = i_{D,\max} (1.8) \Rightarrow i_{D,\max} = 44.44 \mu\text{ A}$$

$$i_{D,\max} = 44.44 = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_L [-(-0.6)]^2 \Rightarrow \left(\frac{W}{L}\right)_L = 2.47$$

$$\frac{K_D}{K_L} [2(1.8-0.3)(0.06) - (0.06)^2] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L} (0.1764) = (0.36) \Rightarrow \frac{K_D}{K_L} = 2.04 = \frac{(W/L)_D}{(W/L)_L}$$

$$\text{Then } \left(\frac{W}{L}\right)_D = 5.04$$

$$(b) \quad \sqrt{\frac{K_D}{K_L}} (V_{It} - V_{TND}) = -V_{TNL}$$

$$\sqrt{\frac{5.04}{2.47}} (V_{It} - 0.3) = [-(-0.6)] \Rightarrow V_{It} = 0.720\text{ V}$$

$$\text{For Driver: } V_{Ot} = 0.720 - 0.3 = 0.420\text{ V}$$

$$\text{For Load: } V_{Ot} = 1.8 - 0.6 = 1.2\text{ V}$$

$$(c) \quad P_{D,\max} = 80 \mu\text{ W}$$

$$2\left(\frac{5.04}{2.47}\right) [2(1.8-0.3)v_o - v_o^2] = [-(-0.6)]^2$$

$$\text{We find } 4.08v_o^2 - 12.24v_o + 0.36 = 0 \Rightarrow v_o = 0.0297\text{ V}$$

**16.12**

a. From Equation (16.27(b)):

$$\left(\frac{W}{L}\right)_D [2(2.5-0.5)(0.05) - (0.05)^2] = \left(\frac{W}{L}\right)_L [(-1)]^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

$$\text{Then } \left(\frac{W}{L}\right)_D = 5.06$$

b.  $i_D = \left(\frac{80}{2}\right)(1)[-(-1)]^2$   
 or  $i_D = 40 \mu\text{A}$   
 $P = i_D \cdot V_{DD} = (40)(2.5) \Rightarrow P = 100 \mu\text{W}$

**16.13**

a. i.  $v_I = 0.5 \text{ V} \Rightarrow i_D = 0 \Rightarrow P = 0$   
 ii.  $v_I = 5 \text{ V}$ , From Equation (16.11),  
 $v_0 = 5 - (0.1)(20)[2(5 - 1.5)v_0 - v_0^2]$   
 $2v_0^2 - 15v_0 + 5 = 0$   
 $v_0 = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow v_0 = 0.35 \text{ V}$   
 $i_D = \frac{5 - 0.35}{20} = 0.2325 \text{ mA}$   
 $P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow P = 1.16 \text{ mW}$

b. i.  $v_I = 0.25 \text{ V} \Rightarrow i_D = 0 \Rightarrow P = 0$   
 ii.  $v_I = 4.3 \text{ V}$ , From Equation (16.21),  
 $100[2(4.3 - 0.7)v_0 - v_0^2] = 10[5 - v_0 - 0.7]^2$   
 $10[7.2v_0 - v_0^2] = 18.49 - 8.6v_0 + v_0^2$

Then

$$11v_0^2 - 80.6v_0 + 18.49 = 0$$

$$v_0 = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)} \Rightarrow v_0 = 0.237 \text{ V}$$

Then

$$i_D = 10[5 - 0.237 - 0.7]^2 = 165 \mu\text{A}$$

$$P = i_D \cdot V_{DD} = (165)(5) \Rightarrow P = 825 \mu\text{W}$$

c. i.  $v_I = 0.03 \text{ V} \Rightarrow i_D = 0 \Rightarrow P = 0$   
 $v_I = 5 \text{ V}$   
 $i_D = K_L(-V_{TNL})^2 = (10)[-(-2)]^2 = 40 \mu\text{A}$   
 ii.  $P = i_D \cdot V_{DD} = (40)(5) \Rightarrow P = 200 \mu\text{W}$

**16.14**

(a)  $v_I = V_{DD} - V_{TNL} = 5 - 0.5 = 4.5 \text{ V}$   
 $K_D[2(v_I - V_{TND})v_{O1} - v_{O1}^2] = K_L[V_{DD} - v_{O1} - V_{TNL}]^2$   
 $10[2(4.5 - 0.5)v_{O1} - v_{O1}^2] = (1)[5 - v_{O1} - 0.5]^2$   
 We find  $11v_{O1}^2 - 89v_{O1} + 20.25 = 0 \Rightarrow v_{O1} = 0.234 \text{ V}$   
 $v_{O2} = V_{DD} - V_{TNL} = 4.5 \text{ V}$   
 (b)  $v_I = 0.234 \text{ V} \Rightarrow v_{O1} = 4.5 \text{ V}$   
 From part (a),  $v_{O2} = 0.234 \text{ V}$

**16.15**

$$(a) \quad \frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\left(\frac{4}{1}\right) [2(5 - 0.8)v_{O1} - v_{O1}^2] = [-(-1.2)]^2$$

$$\text{We find } 4v_{O1}^2 - 33.6v_{O1} + 1.44 = 0 \Rightarrow v_{O1} = 0.0431 \text{ V} \\ \Rightarrow v_{O2} = 5 \text{ V}$$

$$(b) \text{ For } v_I = 0.0431 \text{ V, } v_{O1} = 5 \text{ V}$$

$$\text{From part (a), } v_{O2} = 0.0431 \text{ V}$$

**16.16**

$$(a) \quad v_O = V_{DD} - V_{TNLO} = 2.5 - 0.5 = 2.0 \text{ V}$$

$$(b) \quad v_O = V_{DD} - \left[ V_{TNLO} + \gamma \left( \sqrt{2\phi_{fp} + v_O} - \sqrt{2\phi_{fp}^-} \right) \right]$$

$$v_O = 2.5 - \left[ 0.5 + 0.25 \left( \sqrt{0.7 + v_O} - \sqrt{0.7} \right) \right]$$

$$v_O - 2.209 = -0.25\sqrt{0.7 + v_O}$$

$$v_O^2 - 4.418v_O + 4.88 = 0.0625(0.7 + v_O)$$

$$v_O^2 - 4.4805v_O + 4.836 = 0 \Rightarrow v_O = 1.81 \text{ V}$$

**16.17**

$$(a) \quad \sqrt{\frac{K_D}{K_L}} (v_I - V_{TND}) = (-V_{TNL})$$

$$\text{Or } v_I = [-(V_{TNL})] \sqrt{\frac{K_L}{K_D}} + V_{TND} = [-(-0.6)] \sqrt{\frac{20}{100}} + 0.4 = 0.6683 \text{ V}$$

$$(b) \quad V_{TNL} = V_{TNLO} + \gamma \left( \sqrt{2\phi_{fp} + v_O} - \sqrt{2\phi_{fp}^-} \right)$$

$$= -0.6 + 0.25 \left( \sqrt{0.7 + 1.25} - \sqrt{0.7} \right) = -0.460 \text{ V}$$

$$v_I = [-(-0.460)] \sqrt{\frac{20}{100}} + 0.4 = 0.6057 \text{ V}$$

**6.18**

$$(a) \quad \frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\frac{K_D}{K_L} [2(1.8 - 0.4)(0.1) - (0.1)^2] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L} (0.27) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.33$$

$$(b) \quad 2\left(\frac{K_D}{K_L}\right)\left[2(v_I - V_{TND})v_o - v_o^2\right] = [-V_{TNL}]^2$$

$$2(1.333)\left[2(1.8 - 0.4)v_o - v_o^2\right] = [-(-0.6)]^2$$

$$\text{We find } 2.667v_o^2 - 7.467v_o + 0.36 = 0 \Rightarrow v_o = 49.1 \text{ mV}$$

$$(c) \quad i_{D,\max} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \left(\frac{0.1}{2}\right)(1)[-(-0.6)]^2 \Rightarrow i_{D,\max} = 18 \mu\text{A}$$

$$P = i_{D,\max} \cdot V_{DD} = (18)(1.8) = 32.4 \mu\text{W}$$

### 6.19

(a) One input high,

$$\frac{K_D}{K_L} \left[2(3 - 0.5)(0.1) - (0.1)^2\right] = [-(-1)]^2 \Rightarrow \frac{K_D}{K_L} = 2.04$$

(b)  $P = i_D \cdot V_{DD}$

$$0.1 = i_D(3) \Rightarrow i_D = 33.33 \mu\text{A}$$

$$i_D = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L [-V_{TNL}]^2$$

$$33.33 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L (1) \Rightarrow \left(\frac{W}{L}\right)_L = 0.667$$

$$\text{Then } \left(\frac{W}{L}\right)_D = (2.04)(0.667) = 1.36$$

$$(c) \quad 3(2.04)\left[2(3 - 0.5)v_o - v_o^2\right] = [-(-1)]^2$$

$$6.12v_o^2 - 30.6v_o + 1 = 0 \Rightarrow v_o = 0.0329 \text{ V}$$

### 16.20

$$(a) \quad \frac{K_D}{K_L} \left[2(2.5 - 0.4)(0.05) - (0.05)^2\right] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L} (0.2075) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.735$$

(b)  $P = i_{D,\max} \cdot V_{DD}$

$$50 = i_{D,\max}(2.5) \Rightarrow i_{D,\max} = 20 \mu\text{A}$$

$$i_{D,\max} = 20 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L [-(-0.6)]^2$$

$$\text{We find } \left(\frac{W}{L}\right)_L = 1.11 \text{ and } \left(\frac{W}{L}\right)_D = 1.93$$

$$(c) \quad (i) \quad 2(1.735)\left[2(2.5 - 0.4)v_o - v_o^2\right] = 0.36$$

$$3.47v_o^2 - 14.574v_o + 0.36 = 0 \Rightarrow v_o = 24.9 \text{ mV}$$

$$(ii) \quad 3(1.735)\left[2(2.5 - 0.4)v_o - v_o^2\right] = 0.36$$

$$5.205v_o^2 - 21.861v_o + 0.36 = 0 \Rightarrow v_o = 16.5 \text{ mV}$$

$$\begin{aligned} \text{(iii)} \quad & 4(1.735)[2(2.5 - 0.4)v_o - v_o^2] = 0.36 \\ & 6.94v_o^2 - 29.148v_o + 0.36 = 0 \Rightarrow v_o = 12.4 \text{ mV} \end{aligned}$$

**16.21**

a.

$$P = i_D \cdot V_{DD}$$

$$250 = i_D (5) \Rightarrow i_D = 50 \mu\text{A}$$

$$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_{ML1} [-V_{TNL1}]^2$$

$$50 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_{ML1} [ -(-2) ]^2$$

$$\left(\frac{W}{L}\right)_{ML1} = 0.417$$

So that

$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_o - v_o^2] = [-V_{TNL}]^2$$

$$\frac{K_D}{K_L} [2(5 - 0.8)(0.15) - (0.15)^2] = [ -(-2) ]^2$$

$$\frac{K_D}{K_L} = 3.23 \Rightarrow \left(\frac{W}{L}\right)_{MD1} = 1.35$$

or

b. For  $v_x = v_y = 0 \Rightarrow v_{01} = 5$  and  $v_{03} = 4.2$

Then

$$K_{D2} [2(v_{01} - V_{TND})v_{02} - v_{02}^2] + K_{D3} [2(v_{03} - V_{TND})v_{02} - v_{02}^2] = K_{L2} [-V_{TNL2}]^2$$

$$K_{D2} \propto 8, K_{D3} \propto 8, K_{L2} \propto 1$$

$$8[2(5 - 0.8)v_{02} - v_{02}^2] + 8[2(4.2 - 0.8)v_{02} - v_{02}^2] = (1)[-(-2)]^2$$

$$67.2v_{02} - 8v_{02}^2 + 54.4v_{02} - 8v_{02}^2 = 4$$

Then

$$16v_{02}^2 - 121.6v_{02} + 4 = 0$$

$$v_{02} = \frac{121.6 \pm \sqrt{(121.6)^2 - 4(16)(4)}}{2(16)}$$

So  $v_{02} = 0.0330 \text{ V}$

**16.22**

$$\text{(a)} \quad i_{D,\max} = \left(\frac{100}{2}\right)(1)[V_{DD} - v_o - V_{TN}]^2 = 50[3.3 - v_o - 0.4]^2 = 50[2.9 - v_o]^2$$

The output is small, so neglect  $v_o^2$ .

$$\text{Then } i_{D,\max} \cong 50[8.41 - 5.8v_o] \quad (\text{Eq. 1})$$

Also,

$$\begin{aligned} i_{D,\max} &= \left(\frac{100}{2}\right)(12)[2(v_{GSX} - V_{TN})v_{DSX} - v_{DSX}^2] \\ &= 600[2(2.9 - 0.4)v_{DSX} - v_{DSX}^2] \cong 3000v_{DSX} \quad (\text{Eq. 2}) \end{aligned}$$

And

$$i_{D,\max} = \left(\frac{100}{2}\right)(12)\left[2(v_{GSY} - V_{TN})v_{DSY} - v_{DSY}^2\right]$$

We have  $v_{GSY} = v_Y - v_{DSX}$ ;  $v_{DSY} = v_O - v_{DSX}$

$$i_{D,\max} = 600\left[2(2.9 - v_{DSX} - 0.4)(v_O - v_{DSX}) - (v_O - v_{DSX})^2\right]$$

$$\cong 600\left[2(2.5 - v_{DSX})(v_O - v_{DSX})\right] \quad (\text{Eq. 3})$$

Now  $3000v_{DSX} = 50[8.41 - 5.8v_O]$

$$v_{DSX} = 0.016667[8.41 - 5.8v_O] \quad (\text{Eq. 4})$$

Also  $3000v_{DSX} = 600[2(2.5 - v_{DSX})(v_O - v_{DSX})] \cong 1200[2.5v_O - 2.5v_{DSX}]$

$$2.5v_{DSX} = 2.5v_O - 2.5v_{DSX}$$

Or  $v_{DSX} \cong 0.5v_O$

Then from Eq. 4,

$$0.5v_O = 0.140 - 0.09667v_O \Rightarrow v_O \cong 0.2346 \text{ V}$$

(b)  $v_{DSX} \cong 0.5v_O \cong 0.1173 \text{ V}$

$$v_{GSX} = 2.9 \text{ V}$$

$$v_{GSY} \cong 2.9 - 0.1173 \cong 2.783 \text{ V}$$

$$v_{DSY} = v_O - v_{DSX} \cong 0.1173 \text{ V}$$

### 16.23

a. We can write

$$K_x \left[ 2(v_x - V_{TNX})v_{DSX} - v_{DSX}^2 \right] = K_y \left[ 2(v_y - v_{DSX} - V_{TNY})v_{DSY} - v_{DSY}^2 \right] = K_L \left[ -V_{TNL} \right]^2$$

From the first and third terms, (neglect  $v_{DSX}^2$ ),

$$4 \left[ 2(5 - 0.8)v_{DSX} \right] = (1) \left[ -(-1.5) \right]^2$$

or  $v_{DSX} = 0.067 \text{ V}$

From the second and third terms, (neglect  $v_{DSY}^2$ ),

$$4 \left[ 2(5 - 0.067 - 0.8)v_{DSY} \right] = (1) \left[ -(-1.5) \right]^2$$

or  $v_{DSY} = 0.068 \text{ V}$

Now

$$v_{GSX} = 5, v_{GSY} = 5 - 0.067 \Rightarrow v_{GSY} = 4.933 \text{ V}$$

and  $v_0 = v_{DSX} + v_{DSY} \Rightarrow v_0 = 0.135 \text{ V}$

Since  $v_0$  is close to ground potential, the body-effect has little effect on the results.

### 16.24

(a)  $\frac{1}{4} \left( \frac{K_D}{K_L} \right) \left[ 2(v_I - V_{TND})v_O - v_O^2 \right] = (-V_{TNL})^2$

$$\frac{1}{4} \left( \frac{K_D}{K_L} \right) \left[ 2(3.3 - 0.4)(0.1) - (0.1)^2 \right] = [-(-0.6)]^2$$

$$\left( \frac{K_D}{K_L} \right) (0.1425) = 0.36 \Rightarrow \left( \frac{K_D}{K_L} \right) = 2.53$$

(b)  $P = i_{D,\max} \cdot V_{DD}$   
 $100 = i_{D,\max} (3.3) \Rightarrow i_{D,\max} = 30.3 \mu\text{ A}$   
 $i_{D,\max} = 30.3 = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_L [ -(-0.6) ]^2$   
 Which yields  $\left(\frac{W}{L}\right)_L = 1.68$  and  $\left(\frac{W}{L}\right)_D = 4.26$

**16.25**

$\bar{Y} = [A \text{ OR } (B \text{ AND } C)] \text{ AND } D$

**16.26**

Considering a truth table, we find

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

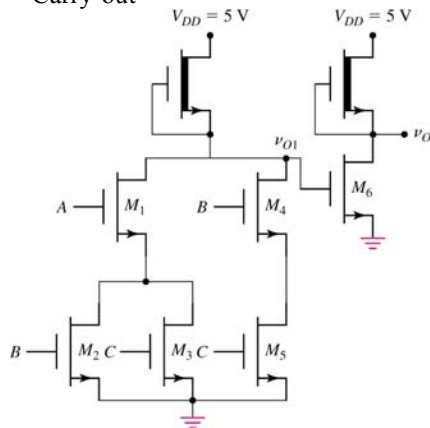
which shows that the circuit performs the exclusive-OR function.

**16.27**

$(A+B)(C+D)$

**16.28**

(a) Carry-out =  $A \bullet (B + C) + B \bullet C$



(b) For  $v_{O1} = \text{Low} = 0.2\text{ V}$   
 $\frac{K_D}{K_L} [2(5-0.8)(0.2) - (0.2)^2] = [ -(-1.5) ]^2 \Rightarrow$   
 For  $\left(\frac{W}{L}\right)_L = 1$ , then  $\left(\frac{W}{L}\right)_D = 1.37$   
 So, for  $M_6 : \left(\frac{W}{L}\right)_6 = 1.37$

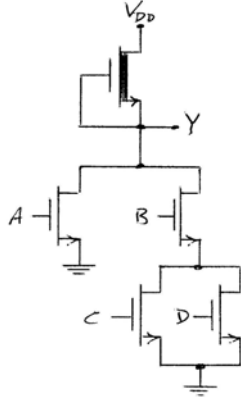


To achieve the required composite conduction parameter,

$$\text{For } M_1 - M_5 : \left(\frac{W}{L}\right)_{1-5} = 2.74$$

**16.29**

(a)



$$\begin{aligned} \text{(b) } \frac{(W/L)_A}{1} [2(2.5 - 0.4)(0.05) - (0.05)^2] &= [-(-0.6)]^2 \\ \left(\frac{W}{L}\right)_A (0.2075) &= 0.36 \Rightarrow \left(\frac{W}{L}\right)_A = 1.735 \\ \Rightarrow \left(\frac{W}{L}\right)_{B,C,D} &= 2 \left(\frac{W}{L}\right)_A = 3.47 \end{aligned}$$

**16.30** Design Problem

**16.31**

(a) By symmetry,  $V_{It} = \frac{V_{DD}}{2} = \frac{2.5}{2} = 1.25 \text{ V}$

$$V_{OPt} = 1.25 - (-0.4) = 1.65 \text{ V}$$

$$V_{ONt} = 1.25 - 0.4 = 0.85 \text{ V}$$

(c) For  $v_i = 1.1 \text{ V}$ , NMOS in saturation, PMOS in nonsaturation

$$K_n [v_i - V_{TN}]^2 = K_p [2(V_{DD} - v_i + V_{TP})(V_{DD} - v_o) - (V_{DD} - v_o)^2]$$

$$(1.1 - 0.4)^2 = 2(2.5 - 1.1 - 0.4)(V_{DD} - v_o) - (V_{DD} - v_o)^2$$

$$(V_{DD} - v_o)^2 - 2(V_{DD} - v_o) + 0.49 = 0 \Rightarrow (V_{DD} - v_o) = 0.2859 \text{ V}$$

Or  $v_o = 2.5 - 0.2859 = 2.214 \text{ V}$

For  $v_i = 1.4 \text{ V}$ , NMOS in nonsaturation, PMOS in saturation

$$K_n [2(v_i - V_{TN})v_o - v_o^2] = K_p (V_{DD} - v_i + V_{TP})^2$$

$$2(1.4 - 0.4)v_o - v_o^2 = (2.5 - 1.4 - 0.4)^2$$

$$v_o^2 - 2v_o + 0.49 = 0 \Rightarrow v_o = 0.286 \text{ V}$$

**16.32**

(a)  $K_n = \left(\frac{100}{2}\right)(2) = 100 \mu\text{A/V}^2$  ;  $K_p = \left(\frac{40}{2}\right)(5) = 100 \mu\text{A/V}^2$

(i) By symmetry

$$V_{It} = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65 \text{ V}$$

$$\Rightarrow V_{OpI} = 1.65 - (-0.4) = 2.05 \text{ V}$$

$$\Rightarrow V_{ONt} = 1.65 - 0.4 = 1.25 \text{ V}$$

(iii) For  $v_o = 0.25 \text{ V}$ ; NMOS in nonsaturation, PMOS in saturation

$$2(v_I - V_{TN})v_o - v_o^2 = (V_{DD} - v_I + V_{TP})^2$$

$$2(v_I - 0.4)(0.25) - (0.25)^2 = (3.3 - v_I - 0.4)^2$$

$$0.5v_I - 0.2 - 0.0625 = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 6.3v_I + 8.6725 = 0 \Rightarrow v_I = 2.03 \text{ V}$$

For  $v_o = 3.05 \text{ V}$ ; NMOS in saturation, PMOS in nonsaturation

$$(v_I - V_{TN})^2 = 2(V_{DD} - v_I + V_{TP})(V_{DD} - v_o) - (V_{DD} - v_o)^2$$

$$(v_I - 0.4)^2 = 2(3.3 - v_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2$$

$$v_I^2 - 0.8v_I + 0.16 = 0.5(2.9 - v_I) - 0.0625$$

$$v_I^2 - 0.3v_I - 1.2275 = 0 \Rightarrow v_I = 1.27 \text{ V}$$

(b)  $K_n = \left(\frac{100}{2}\right)(4) = 200 \mu\text{A/V}^2$  ;  $K_p = \left(\frac{40}{2}\right)(5) = 100 \mu\text{A/V}^2$

(i)  $V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{100}}(0.4)}{1 + \sqrt{\frac{200}{100}}} = 1.436 \text{ V}$

$$\Rightarrow V_{OpI} = 1.436 + 0.4 = 1.836 \text{ V}$$

$$\Rightarrow V_{ONt} = 1.436 - 0.4 = 1.036 \text{ V}$$

(iii) For  $v_o = 0.25 \text{ V}$ ; NMOS in nonsaturation, PMOS in saturation

$$200[2(v_I - 0.4)(0.25) - (0.25)^2] = 100(3.3 - v_I - 0.4)^2$$

$$2(0.5v_I - 0.2 - 0.0625) = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 6.8v_I + 8.935 = 0 \Rightarrow v_I = 1.78 \text{ V}$$

For  $v_o = 3.05 \text{ V}$ ; NMOS in saturation, PMOS in nonsaturation

$$200(v_I - 0.4)^2 = 100[2(3.3 - v_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2]$$

$$2(v_I^2 - 0.8v_I + 0.16) = 0.5(2.9 - v_I) - 0.0625$$

$$2v_I^2 - 1.1v_I - 1.0675 = 0 \Rightarrow v_I = 1.06 \text{ V}$$

**16.33**

(a)  $K_n = \left(\frac{100}{2}\right)(4) = 200 \mu\text{A/V}^2$  ;  $K_p = \left(\frac{40}{2}\right)(12) = 240 \mu\text{A/V}^2$

$$(i) \quad V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{240}}(0.4)}{1 + \sqrt{\frac{200}{240}}} = 1.707 \text{ V}$$

$$\Rightarrow V_{OPt} = 1.707 + 0.4 = 2.107 \text{ V}$$

$$\Rightarrow V_{ONt} = 1.707 - 0.4 = 1.307 \text{ V}$$

(ii) For  $v_o = 3.1 \text{ V}$ ; NMOS in saturation, PMOS in nonsaturation

$$200(v_I - 0.4)^2 = 240[2(3.3 - v_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2]$$

$$v_I^2 - 0.8v_I + 0.16 = 1.2[0.4(2.9 - v_I) - 0.04]$$

$$v_I^2 - 0.32v_I - 1.184 = 0 \Rightarrow v_I = 1.26 \text{ V}$$

(iii) For  $v_o = 0.2 \text{ V}$ ; NMOS in nonsaturation, PMOS in saturation

$$200[2(v_I - 0.4)(0.2) - (0.2)^2] = 240(3.3 - v_I - 0.4)^2$$

$$0.4v_I - 0.2 = 1.2(8.41 - 5.8v_I + v_I^2)$$

$$1.2v_I^2 - 7.36v_I + 10.292 = 0 \Rightarrow v_I = 2.157 \text{ V}$$

$$(b) \quad K_n = \left(\frac{100}{2}\right)(6) = 300 \mu \text{ A/V}^2; \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu \text{ A/V}^2$$

$$(i) \quad V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{300}{80}}(0.4)}{1 + \sqrt{\frac{300}{80}}} = 1.25 \text{ V}$$

(ii) For  $v_o = 3.1 \text{ V}$ ; NMOS in saturation, PMOS in nonsaturation

$$300(v_I - 0.4)^2 = 80[2(3.3 - v_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2]$$

$$3.75(v_I^2 - 0.8v_I + 0.16) = 0.4(2.9 - v_I) - 0.04$$

$$3.75v_I^2 - 2.6v_I - 0.52 = 0 \Rightarrow v_I = 0.855 \text{ V}$$

(iii) For  $v_o = 0.2 \text{ V}$ ; NMOS in nonsaturation, PMOS in saturation

$$300[2(v_I - 0.4)(0.2) - (0.2)^2] = 80(3.3 - v_I - 0.4)^2$$

$$3.75(0.4v_I - 0.2) = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 7.3v_I + 9.16 = 0 \Rightarrow v_I = 1.61 \text{ V}$$

### 16.34

a. For  $v_{o1} = 0.6 < V_{TN} \Rightarrow v_{o2} = 5 \text{ V}$

$N_1$  in nonsaturation and  $P_1$  in saturation. From Equation (16.43),

$$[2(v_I - 0.8)(0.6) - (0.6)^2] = [5 - v_I - 0.8]^2$$

$$1.2v_I - 1.32 = 17.64 - 8.4v_I + v_I^2$$

or

$$v_I^2 - 9.6v_I + 18.96 = 0$$

$$v_I = \frac{9.6 \pm \sqrt{(9.6)^2 - 4(1)(18.96)}}{2}$$

or

$$v_I = 2.78 \text{ V}$$

b.  $V_{0N1} \leq v_{02} \leq V_{0P1}$

From symmetry,  $V_{I1} = 2.5 \text{ V}$

$$V_{0P1} = 2.5 + 0.8 = 3.3 \text{ V}$$

and  $V_{0N1} = 2.5 - 0.8 = 1.7 \text{ V}$

So  $\underline{1.7 \leq v_{02} \leq 3.3 \text{ V}}$

**16.35**

a.  $V_{0N1} \leq v_{01} \leq V_{0P1}$

By symmetry,  $V_{I1} = 2.5 \text{ V}$

$$V_{0P1} = 2.5 + 0.8 = 3.3 \text{ V}$$

and  $V_{0N1} = 2.5 - 0.8 = 1.7 \text{ V}$

So  $\underline{1.7 \leq v_{01} \leq 3.3 \text{ V}}$

b. For  $v_{O2} = 0.6 < V_{TN} \Rightarrow v_{O3} = 5 \text{ V}$

$N_2$  in nonsaturation and  $P_2$  in saturation. From Equation (16.43),

$$\left[ 2(v_{I2} - 0.8)(0.6) - (0.6)^2 \right] = [5 - v_{I2} - 0.8]^2$$

$$1.2v_{I2} - 1.32 = 17.64 - 8.4v_{I2} + v_{I2}^2$$

or

$$v_{I2}^2 - 9.6v_{I2} + 18.96 = 0$$

So  $v_{I2} = v_{01} = \underline{2.78 \text{ V}}$

For  $v_{01} = 2.78$ , both  $N_1$  and  $P_1$  in saturation. Then

$$\underline{v_I = 2.5 \text{ V}}$$

**16.36**

(a)  $V_{I1} = \frac{2.5}{2} = 1.25 \text{ V}$

For  $0.4 \leq v_I \leq 1.25 \text{ V}$ ;  $i_D = 120(v_I - 0.4)^2 \mu \text{ A}$

$$i_{D,peak} = 120(1.25 - 0.4)^2 = 86.7 \mu \text{ A}$$

For  $1.25 \leq v_I \leq 2.1 \text{ V}$ ;  $i_D = 120(2.5 - v_I - 0.4)^2 \mu \text{ A}$

(b)  $V_{I1} = \left( \frac{1.8}{2} \right) = 0.9 \text{ V}$

For  $0.4 \leq v_I \leq 0.9 \text{ V}$ ;  $i_D = 120(v_I - 0.4)^2 \mu \text{ A}$

$$i_{D,peak} = 120(0.9 - 0.4)^2 = 30 \mu \text{ A}$$

For  $0.9 \leq v_I \leq 1.4 \text{ V}$ ;  $i_D = 120(1.8 - v_I - 0.4)^2 \mu \text{ A}$

**16.37**

(a)  $K_n = \left( \frac{80}{2} \right) (2) = 80 \mu \text{ A/V}^2$ ,  $K_p = \left( \frac{40}{2} \right) (4) = 80 \mu \text{ A/V}^2$

$$V_{It} = \frac{1.8}{2} = 0.9 \text{ V}$$

$$i_{D,peak} = 80(0.9 - 0.35)^2 = 24.2 \mu\text{ A}$$

$$(b) \quad K_n = \left(\frac{80}{2}\right)(2) = 80 \mu\text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(6) = 120 \mu\text{ A/V}^2$$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{80}{120}}(0.35)}{1 + \sqrt{\frac{80}{120}}} = 0.9556 \text{ V}$$

$$i_{D,peak} = 80(0.9556 - 0.35)^2 = 29.34 \mu\text{ A}$$

$$(c) \quad K_n = \left(\frac{80}{2}\right)(4) = 160 \mu\text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu\text{ A/V}^2$$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{160}{80}}(0.35)}{1 + \sqrt{\frac{160}{80}}} = 0.8057 \text{ V}$$

$$i_{D,peak} = 160(0.8057 - 0.35)^2 = 33.23 \mu\text{ A}$$

**16.38**

$$(a) \quad K_n = \left(\frac{100}{2}\right)(3) = 150 \mu\text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(7.5) = 150 \mu\text{ A/V}^2$$

$$V_{It} = \frac{3.3}{2} = 1.65 \text{ V}$$

$$i_{D,peak} = 150(1.65 - 0.4)^2 = 234 \mu\text{ A}$$

$$(b) \quad K_n = \left(\frac{100}{2}\right)(4) = 200 \mu\text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu\text{ A/V}^2$$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{80}}(0.4)}{1 + \sqrt{\frac{200}{80}}} = 1.369 \text{ V}$$

$$i_{D,peak} = 200(1.369 - 0.4)^2 = 188 \mu\text{ A}$$

$$(c) \quad K_n = \left(\frac{100}{2}\right)(3) = 150 \mu\text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(12) = 240 \mu\text{ A/V}^2$$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{150}{240}}(0.4)}{1 + \sqrt{\frac{150}{240}}} = 1.796 \text{ V}$$

$$i_{D,peak} = 150(1.796 - 0.4)^2 = 292 \mu\text{ A}$$

**16.39**

(a)  $P = fC_L V_{DD}^2 = (10^7)(0.2 \times 10^{-12})(2.5)^2 \Rightarrow P = 12.5 \mu\text{ W}$   
 (b)  $P = fC_L V_{DD}^2 = (10^7)(0.2 \times 10^{-12})(1.8)^2 \Rightarrow P = 6.48 \mu\text{ W}$

**16.40**

(a)  $P = (4 \times 10^6)(150 \times 10^6)(0.12 \times 10^{-12})(1.8)^2 \Rightarrow P = 233 \text{ W}$   
 (b)  $P = 233.28 = (4 \times 10^6)(300 \times 10^6)(0.12 \times 10^{-12})V_{DD}^2 \Rightarrow V_{DD} = 1.27 \text{ V}$

**16.41**

(a)  $P = \frac{3}{10^7} = 3 \times 10^{-7} \text{ W}$   
 (b)  $P = fC_L V_{DD}^2 \Rightarrow C_L = \frac{P}{fV_{DD}^2}$   
 (i)  $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(5)^2} \Rightarrow C_L = 0.0024 \text{ pF}$   
 (ii)  $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(3.3)^2} \Rightarrow C_L = 0.00551 \text{ pF}$   
 (iii)  $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(1.5)^2} \Rightarrow C_L = 0.0267 \text{ pF}$

**16.42**

(a)  $P = \frac{10}{5 \times 10^6} = 2 \times 10^{-6} \text{ W}$   
 (b)  $C_L = \frac{P}{fV_{DD}^2}$   
 (i)  $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(5)^2} \Rightarrow C_L = 0.01 \text{ pF}$   
 (ii)  $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(3.3)^2} \Rightarrow C_L = 0.023 \text{ pF}$   
 (iii)  $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(1.5)^2} \Rightarrow C_L = 0.111 \text{ pF}$

**16.43**

(a) For  $v_I \cong V_{DD}$ , NMOS in nonsaturation  
 $i_D = K_n [2(v_I - V_{TN})v_{DS} - v_{DS}^2]$  and  $v_{DS}$  very small  
 So  $\frac{1}{r_{ds}} = \frac{di_D}{dv_{DS}} \cong K_n [2(v_I - V_{TN})]$

$$\text{Or } r_{ds} \cong \frac{1}{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n [2(V_{DD} - V_{TN})]} = \frac{1}{k'_n \left(\frac{W}{L}\right)_n (V_{DD} - V_{TN})}$$

For  $v_I \cong 0$ , PMOS in nonsaturation

$$i_D = K_p [2(V_{DD} - v_I + V_{TP})v_{SD} - v_{SD}^2] \text{ and } v_{SD} \text{ very small}$$

$$\text{So } \frac{1}{r_{sd}} = \frac{di_D}{dv_{SD}} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_p [2(V_{DD} - v_I + V_{TP})] \cong k'_p \left(\frac{W}{L}\right)_p (V_{DD} + V_{TP})$$

$$\text{Or } r_{sd} = \frac{1}{k'_p \left(\frac{W}{L}\right)_p (V_{DD} + V_{TP})}$$

(b) Let  $\left(\frac{W}{L}\right)_n = 2$ ,  $\left(\frac{W}{L}\right)_p = 4$ , and  $V_{DD} = 5 \text{ V}$

For NMOS:

$$r_{ds} = \frac{1}{(0.1)(2)(5-0.5)} \Rightarrow r_{ds} = 1.11 \text{ k}\Omega$$

$$i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{1.11} = 0.45 \text{ mA}$$

For PMOS:

$$r_{sd} = \frac{1}{(0.04)(4)(5-0.5)} \Rightarrow r_{sd} = 1.39 \text{ k}\Omega$$

$$i_d = \frac{v_{sd}}{r_{sd}} = \frac{0.5}{1.39} = 0.36 \text{ mA}$$

#### 16.44

(a)  $V_{IL} = V_{TN} + \frac{3}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \frac{3}{8}(3.3 - 0.5 - 0.5)$   
 $\Rightarrow V_{IL} = 1.3625 \text{ V}$

$V_{IH} = V_{TN} + \frac{5}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \frac{5}{8}(3.3 - 0.5 - 0.5)$   
 $\Rightarrow V_{IH} = 1.9375 \text{ V}$

(b)  $V_{OHU} = \frac{1}{2}[2(1.3625) + 3.3 - 0.5 + 0.5] = 3.0125 \text{ V}$

$V_{OLU} = \frac{1}{2}[2(1.9375) - 3.3 - 0.5 + 0.5] = 0.2875 \text{ V}$

Then  $NM_L = 1.3625 - 0.2875 = 1.075 \text{ V}$

$NM_H = 3.0125 - 1.9375 = 1.075 \text{ V}$

#### 16.45

(a)  $\frac{K_n}{K_p} = \frac{100}{50} = 2$

$$V_{IL} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)} \left[ 2\sqrt{\frac{2}{2+3}} - 1 \right]$$

$$\Rightarrow V_{IL} = 0.8268 \text{ V}$$

$$V_{IH} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)} \left[ \frac{2(2)}{\sqrt{3(2)+1}} - 1 \right]$$

$$\Rightarrow V_{IH} = 1.2713 \text{ V}$$

$$(b) V_{OHU} = \frac{1}{2} [(1+2)(0.8268) + 2.5 - (2)(0.35) + 0.35]$$

$$\Rightarrow V_{OHU} = 2.3152 \text{ V}$$

$$V_{OLU} = \frac{(1.2713)(1+2) - 2.5 - (2)(0.35) + 0.35}{2(2)}$$

$$\Rightarrow V_{OLU} = 0.2410 \text{ V}$$

$$\text{Then } NM_L = 0.8268 - 0.2410 = 0.5858 \text{ V}$$

$$NM_H = 2.3152 - 1.2713 = 1.0439 \text{ V}$$

### 16.46

$$(a) K_n = \left(\frac{100}{2}\right)(2) = 100 \mu\text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(5) = 100 \mu\text{A/V}^2$$

$$V_{IL} = 0.4 + \frac{3}{8}(3.3 - 0.4 - 0.4) = 1.3375 \text{ V}$$

$$V_{IH} = 0.4 + \frac{5}{8}(3.3 - 0.4 - 0.4) = 1.9625 \text{ V}$$

$$V_{OHU} = \frac{1}{2} [2(1.3375) + 3.3 - 0.4 + 0.4] = 2.9875 \text{ V}$$

$$V_{OLU} = \frac{1}{2} [2(1.9625) - 3.3 - 0.4 + 0.4] = 0.3125 \text{ V}$$

$$\text{Then } NM_L = 1.3375 - 0.3125 = 1.025 \text{ V}$$

$$NM_H = 2.9875 - 1.9625 = 1.025 \text{ V}$$

$$(b) K_n = \left(\frac{100}{2}\right)(4) = 200 \mu\text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(12) = 240 \mu\text{A/V}^2$$

$$\frac{K_n}{K_p} = \frac{200}{240} = 0.8333$$

$$V_{IL} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)} \left[ 2\sqrt{\frac{0.8333}{3.8333}} - 1 \right] = 1.4127 \text{ V}$$

$$V_{IH} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)} \left[ \frac{2(0.8333)}{\sqrt{3(0.8333)+1}} - 1 \right] = 2.0370 \text{ V}$$

$$\text{Now } V_{OHU} = \frac{1}{2} [(1 + 0.8333)(1.4127) + 3.3 - (0.8333)(0.4) + 0.4]$$

$$\Rightarrow V_{OHU} = 2.9783 \text{ V}$$



$$V_{OLU} = \frac{2.0370(1 + 0.8333) - 3.3 - (0.8333)(0.4) + 0.4}{2(0.8333)}$$

$$\Rightarrow V_{OLU} = 0.3007 \text{ V}$$

$$\text{Then } NM_L = 1.4127 - 0.3007 = 1.1112 \text{ V}$$

$$NM_H = 2.9783 - 2.0370 = 0.9413 \text{ V}$$

**16.47**

a.  $v_A = v_B = 5 \text{ V}$

$N_1$  and  $N_2$  on, so  $v_{DS1} \approx v_{DS2} \approx 0 \text{ V}$

$P_1$  and  $P_2$  off

So we have a  $P_3 - N_3$  CMOS inverter. By symmetry,  $v_C = 2.5 \text{ V}$  (Transition Point).

b. For  $v_A = v_B = v_C \equiv v_i$

Want  $K_{n,eff} = K_{p,eff}$

$$\frac{k'_n}{2} \cdot \left(\frac{W}{3L}\right)_n = \frac{k'_p}{2} \cdot \left(\frac{3W}{L}\right)_p$$

With  $k'_n = 2k'_p$ , then

$$\frac{2}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L}\right)_n = \frac{1}{2} \cdot 3 \cdot \left(\frac{W}{L}\right)_p$$

$$\left(\frac{W}{L}\right)_n = \frac{9}{2} \cdot \left(\frac{W}{L}\right)_p$$

Or

c. We have

$$K_n = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_n = \left(\frac{2k'_p}{2}\right) \left(\frac{9}{2}\right) \left(\frac{W}{L}\right)_p$$

$$K_p = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p$$

Then from Equation (16.41)

$$V_h = \frac{5 + (-0.8) + \sqrt{\frac{K_n}{K_p}} \cdot (0.8)}{1 + \sqrt{\frac{K_n}{K_p}}}$$

Now

$$\frac{K_n}{K_p} = (2) \left(\frac{9}{2}\right) = 9$$

Then

$$V_h = \frac{5 + (-0.8) + 3(0.8)}{1 + 3} \Rightarrow \underline{V_h = 1.65 \text{ V}}$$

**16.48**

By definition, NMOS is on if gate voltage is 5 V and is off if gate voltage is 0 V.

State	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$	$v_0$
1	off	on	off	on	on	0

---

2	off	off	on	on	off	0
3	on	on	off	off	on	5
4	on	on	off	on	on	0

---

Logic function  $(v_x \text{ OR } v_y) \otimes (v_x \text{ AND } v_z)$   
 Exclusive OR of  $(v_x \text{ OR } v_y)$  with  $(v_x \text{ AND } v_z)$

---

**16.49**

NMOS in Parallel  $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

4-PMOS in series  $\Rightarrow \left(\frac{W}{L}\right)_p = 4(4) = 16$

(b)  $C_L$  doubles  $\Rightarrow$  current must double to maintain switching speed.

$\Rightarrow \left(\frac{W}{L}\right)_n = 4$

$\left(\frac{W}{L}\right)_p = 32$

---

**16.50**

4-NMOS in series  $\left(\frac{W}{L}\right)_n = 4(2) = 8$

4-PMOS in parallel  $\left(\frac{W}{L}\right)_p = 4$

$\left(\frac{W}{L}\right)_n = 16$

(b)  $\left(\frac{W}{L}\right)_p = 8$

---

**16.51**

(a) NMOS in parallel  $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

3-PMOS in series  $\Rightarrow \left(\frac{W}{L}\right)_p = 3(4) = 12$

$\left(\frac{W}{L}\right)_n = 4$

(b)  $\left(\frac{W}{L}\right)_p = 24$

---

**16.52**

(a) 3-NMOS in series  $\left(\frac{W}{L}\right)_n = 3(2) = 6$

3-PMOS in parallel  $\left(\frac{W}{L}\right)_p = 4$

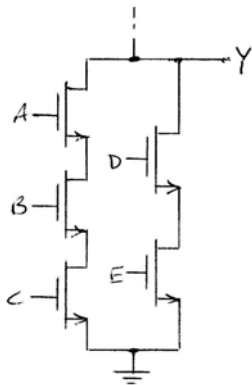
$\left(\frac{W}{L}\right)_n = 12$

(b)  $\left(\frac{W}{L}\right)_p = 8$

**16.53**

(a)  $Y = \overline{ABC + DE}$

(b)

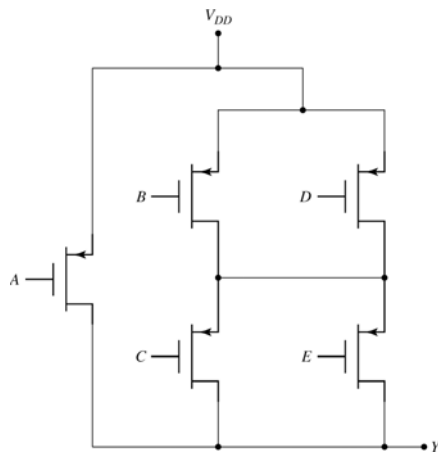


(c)  $(W/L)_{A_n, B_n, C_n} = 6$ ,  $(W/L)_{D_n, E_n} = 4$   
 All  $(W/L)_p = 8$

**16.54**

(a)  $Y = \overline{A(BD + CE)}$

(b)



(c) NMOS: 3 transistors in series for pull down mode.

$$\left(\frac{W}{L}\right)_n = 2(3)(2) = 12$$

For twice the speed:

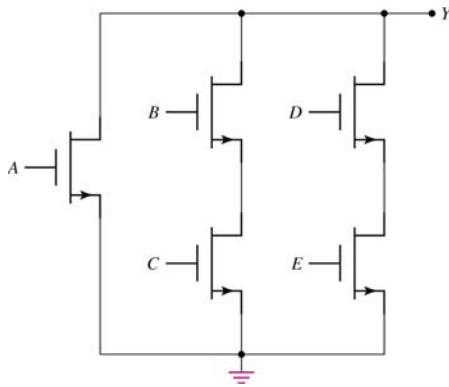
$$\text{PMOS: } \left(\frac{W}{L}\right)_{p,A} = 2(4) = 8$$

$$\left(\frac{W}{L}\right)_{p,B,C,D,E} = 2(2)(4) = 16$$

**16.55**

(a)  $Y = \overline{A + BC + DE}$

(b)



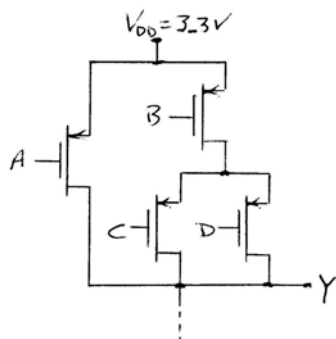
(c) NMOS:  $\left(\frac{W}{L}\right)_{n,A} = 2$        $\left(\frac{W}{L}\right)_{n,B,C,D,E} = 4$

PMOS: 3 transistors in series for the pull-up mode

$$\left(\frac{W}{L}\right)_p = 3(4) = 12$$

**16.56**

(a)  $Y = A[B + CD]$



(b)

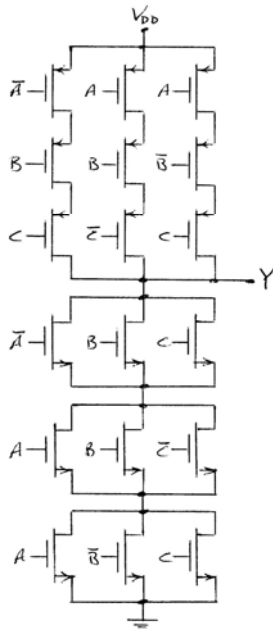
(c)  $(W/L)_{A_n, B_n} = 4$ ,  $(W/L)_{C_n, D_n} = 8$

$$(W/L)_{A_p} = 4, (W/L)_{B_p, C_p, D_p} = 8$$

**16.57**

(a) For  $Y = \overline{A\overline{B}\overline{C}} + \overline{A\overline{B}C} + \overline{A\overline{B}\overline{C}}$

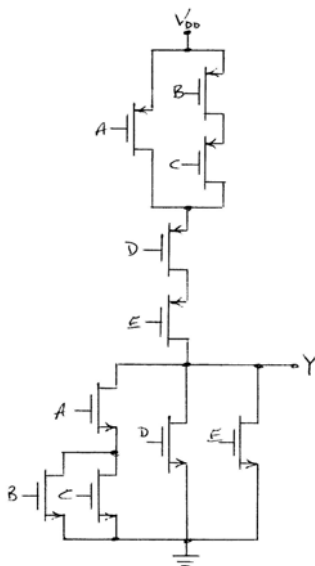
We have  $\overline{Y} = \overline{A\overline{B}\overline{C} + \overline{A\overline{B}C} + \overline{A\overline{B}\overline{C}}} = (\overline{A + B + C})(\overline{A + B + \overline{C}})(\overline{A + \overline{B} + C})$



(b) All  $(W/L)_n = 3$   
 All  $(W/L)_p = 6$

**16.58**

(a)



(b)  $(W/L)_{A_n, B_n, C_n} = 2$ ,  $(W/L)_{D_n, E_n} = 1$   
 $(W/L)_{A_p, D_p, E_p} = 6$ ,  $(W/L)_{B_p, C_p} = 12$

**16.59**

- (a)  $\bar{Y} = C(A + B)$   
 (b) All  $(W/L)_n = 4$   
 $(W/L)_{Cp} = 4$ ,  $(W/L)_{Ap, Bp} = 8$

**16.60**

- (a) All  $(W/L)_n = 2$   
 All  $(W/L)_p = 20$   
 (b) All  $(W/L)_n = 10$   
 All  $(W/L)_p = 4$

**16.61**

By definition:

- NMOS off if gate voltage = 0  
 NMOS on if gate voltage = 5 V  
 PMOS off if gate voltage = 5 V  
 PMOS on if gate voltage = 0

State	$N_1$	$P_1$	$N_A$	$N_B$	$N_C$	$v_{01}$	$N_2$	$P_2$	$v_{02}$
1	off	on	off	off	off	5	on	off	0
2	on	off	on	off	off	5	on	off	0
3	off	on	off	off	off	5	on	off	0
4	on	off	off	off	on	5	on	off	0
5	off	on	off	off	off	5	on	off	0
6	on	off	off	on	on	0	off	on	5

Logic function is

$$v_{02} = (v_A \text{ OR } v_B) \text{ AND } v_C$$

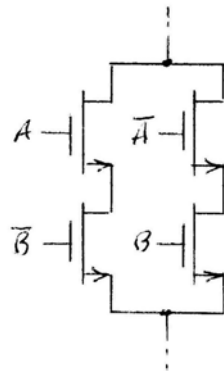
**16.62**

State	$v_{01}$	$v_{02}$	$v_{03}$
1	5	5	0
2	0	0	5
3	5	5	0
4	5	0	5
5	5	5	0
6	0	5	0

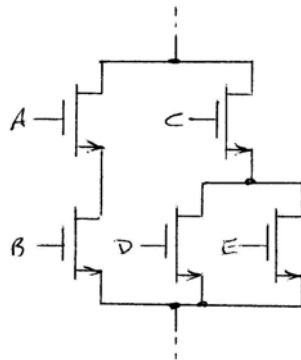
Logic function:

$$v_{03} = (v_X \text{ OR } v_Z) \text{ AND } v_Y$$

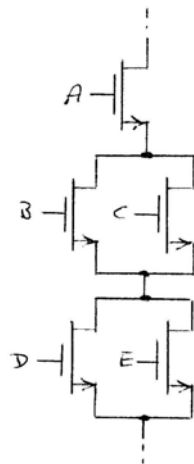
16.63



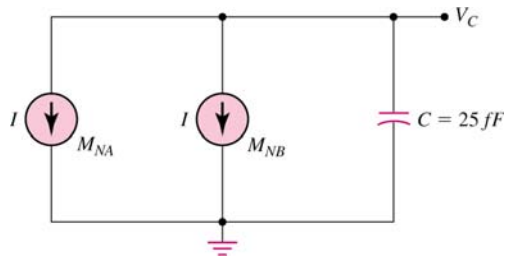
16.64



16.65



**16.66**



$$2I = -C \frac{dV_C}{dt}$$

So

$$\Delta V_C = -\frac{1}{C}(2I) \cdot t$$

For  $\Delta V_C = -0.5 \text{ V}$

$$-0.5 = -\frac{2(2 \times 10^{-12}) \cdot t}{25 \times 10^{-15}} \Rightarrow t = \underline{3.125 \text{ ms}}$$

**16.67**

- (a) (i)  $v_O = 0$   
 (ii)  $v_O = \phi - V_{TN} = 3.3 - 0.4 = 2.9 \text{ V}$   
 (iii)  $v_O = 2.5 \text{ V}$
- (b) (i)  $v_O = 0$   
 (ii)  $v_O = \phi - V_{TN} = 1.8 - 0.4 = 1.4 \text{ V}$   
 (iii)  $v_O = \phi - V_{TN} = 1.8 - 0.4 = 1.4 \text{ V}$

**16.68**

- (a) (i)  $v_O = 0$   
 (ii)  $v_O = \phi - V_{TN} = 2.5 - 0.5 = 2 \text{ V}$   
 (iii)  $v_O = 1.8 \text{ V}$
- (b) (i)  $v_O = 0$   
 (ii)  $v_O = \phi - V_{TN} = 2 - 0.5 = 1.5 \text{ V}$   
 (iii)  $v_O = \phi - V_{TN} = 2 - 0.5 = 1.5 \text{ V}$

**16.69**

- (a)  $v_{O1} = 2.5 - 0.4 = 2.1 \text{ V}$   
 $v_{O2} = 2.5 \text{ V}$
- (b)  $v'_{I1} = 2.5 - 0.4 = 2.1 \text{ V}$



$$\frac{(W/L)_1}{(W/L)_2} [2(2.1 - 0.4)(0.1) - (0.1)^2] = (2.5 - 0.1 - 0.4)^2$$

$$(W/L)_1 [0.33] = 4 \Rightarrow \left(\frac{W}{L}\right)_1 = 12.1$$

$$v'_{i3} = 2.1 \text{ V}$$

$$\frac{(W/L)_3}{(W/L)_4} [2(2.1 - 0.4)(0.1) - (0.1)^2] = [-(-0.6)]^2$$

$$(W/L)_3 [0.33] = 0.36 \Rightarrow \left(\frac{W}{L}\right)_3 = 1.09$$

**16.70**

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0.1

⇒ indeterminate

Without the top transistor, the circuit performs the exclusive-NOR function.

**16.71**

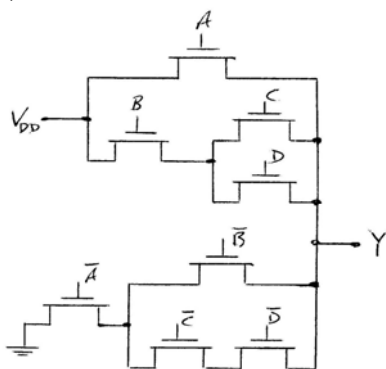
A	$\bar{A}$	B	$\bar{B}$	Y	Z
0	1	0	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0

$$Y = A + \bar{A}B = A + B$$

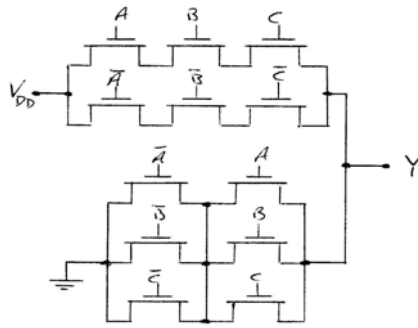
$$Z = \bar{Y} \text{ or } Z = \bar{A}\bar{B}$$

**16.72**

(a)



(b)



**16.73**

- (a) (i)  $Y = 0$   
 (ii)  $Y = 2.5 \text{ V}$   
 (iii)  $Y = 0$   
 (iv)  $Y = 2.5 \text{ V}$
- (b) (i)  $Y = 0$   
 (ii)  $Y = 0$   
 (iii)  $Y = 2.5 \text{ V}$   
 (iv)  $Y = 2.5 \text{ V}$
- (c) For  $\phi = 1, \bar{\phi} = 0$ ; then  $Y = B$   
 For  $\phi = 0, \bar{\phi} = 1$ ; then  $Y = A$   
 A multiplexer

**16.74**

$$Y = AC + B\bar{C}$$

**16.75**

- (a) (i)  $Y = 0$   
 (ii)  $Y = 2.5 \text{ V}$   
 (iii)  $Y = 2.5 \text{ V}$   
 (iv)  $Y = 0$
- (b)  $Y = A\bar{B} + \bar{A}B = A \otimes B$

**16.76**

<u>A</u>	<u>B</u>	<u>Y</u>
0	0	0
1	0	1
0	1	1
1	1	0

Exclusive-OR function.

**16.77**

This circuit is referred to as a two-phase ratioed circuit. The same width-to-length ratios between the driver and load transistors must be maintained as discussed previously with the enhancement load inverter.

When  $\phi_1$  is high,  $v_{o1}$  becomes the complement of  $v_i$ . When  $\phi_2$  goes high, then  $v_o$  becomes the complement of  $v_{o1}$  or is the same as  $v_i$ . The circuit is a shift register.

**16.78**

Want  $\bar{Q}$  to be the transition point of  $M_5 - M_6$ .

From Equation (16.26(b)),

$$\sqrt{\frac{K_5}{K_6}}(V_{It} - V_{TN5}) = -V_{TN6}$$

$$\sqrt{\frac{100}{40}}(V_{It} - 0.4) = [-(-0.6)] \Rightarrow V_{It} = \bar{Q} = 0.7795 \text{ V}$$

This is region where  $M_1$  and  $M_3$  are biased in the saturation region.

$$\sqrt{\frac{K_1}{K_3}}(V_{It} - V_{TN1}) = -V_{TN3}$$

$$\sqrt{\frac{150}{40}}(V_{It} - 0.4) = [-(-0.6)] \Rightarrow V_{It} = S = 0.7098 \text{ V}$$

This analysis neglected the effect of  $M_2$  starting to conduct.

**16.79**

$$v_{It} = \frac{3.3 + (-0.4) + 0.5}{1 + 1} = 1.7 \text{ V}$$

$v_i = 1.5 \text{ V}$  NMOS Sat; PMOS Non Sat

$$(v_i - 0.5)^2 = [2(3.3 - v_i - 0.4)(3.3 - v_{o1}) - (3.3 - v_{o1})^2] \Rightarrow v_{o1} = 2.88 \text{ V}$$

$$v_i = 1.6 \text{ V} \quad v_{o1} = 2.693 \text{ V}$$

$v_i = 1.7 \text{ V}$   $v_{o1} = \text{variable}$  (switching region)

$v_i = 1.8 \text{ V}$  NMOS Non Sat; PMOS Sat

$$(3.3 - v_i - 0.4)^2 = [2(v_i - 0.5)v_{o1} - v_{o1}^2] \Rightarrow v_{o1} = 0.607 \text{ V}$$

Now

$$v_i = 1.5 \text{ V}, v_{o1} = 2.88 \text{ V} \Rightarrow v_o \approx 0 \text{ V}$$

$$v_i = 1.6 \text{ V}, v_{o1} = 2.693 \text{ V}$$

NMOS Non Sat; PMOS Sat

$$(3.3 - v_{o1} - 0.4)^2 = [2(v_{o1} - 0.5)v_o - v_o^2]$$

$$v_o = 0.00979 \text{ V}$$

$v_i = 1.7 \text{ V}$ ,  $v_{o1} = \text{Switching Mode} \Rightarrow v_o = \text{Switching Mode}$ .

$v_i = 1.8 \text{ V}$ ,  $v_{o1} = 0.607 \text{ V}$  NMOS Sat; PMOS Non Sat

$$(v_{o1} - 0.5)^2 = [2(3.3 - v_{o1} - 0.4)(3.3 - v_o) - (3.3 - v_o)^2] \Rightarrow v_o = 3.298 \text{ V}$$

---

**16.80**

For  $R = \phi = V_{DD}$  and  $S = 0 \Rightarrow Q = 0, \bar{Q} = 1$

For  $S = \phi = V_{DD}$  and  $R = 0 \Rightarrow Q = 1, \bar{Q} = 0$

The signal  $\phi$  is a clock signal.

For  $\phi = 0$ , The output signals will remain in their previous state.

---

**16.81**

- Positive edge triggered flip-flop when  $\text{CLK} = 1$ , output of first inverter is  $\bar{D}$  and then  $Q = \bar{\bar{D}} = D$ .
  - For example, put a CMOS transmission gate between the output and the gate of  $M_1$  driven by a  $\overline{\text{CLK}}$  pulse.
- 

**16.82**

For  $J = 1, K = 0$ , and  $\text{CLK} = 1$ ; this makes  $Q = 1$  and  $\bar{Q} = 0$ .

For  $J = 0, K = 1$ , and  $\text{CLK} = 1$ , and if  $Q = 1$ , then the circuit is driven so that  $Q = 0$  and  $\bar{Q} = 1$ .

If initially,  $Q = 0$ , then the circuit is driven so that there is no change and  $Q = 0$  and  $\bar{Q} = 1$ .

$J = 1, K = 1$ , and  $\text{CLK} = 1$ , and if  $Q = 1$ , then the circuit is driven so that  $Q = 0$ .

If initially,  $Q = 0$ , then the circuit is driven so that  $Q = 1$ .

So if  $J = K = 1$ , the output changes state.

---

**16.83**

For  $J = v_x = 1, K = v_y = 0$ , and  $\text{CLK} = v_z = 1$ , then  $v_0 = 0$ .

For  $J = v_x = 0, K = v_y = 1$ , and  $\text{CLK} = v_z = 1$ , then  $v_0 = 1$ .

Now consider  $J = K = \text{CLK} = 1$ . With  $v_x = v_z = 1$ , the output is always  $v_0 = 0$ . So the output does not change state when  $J = K = \text{CLK} = 1$ . This is not actually a  $J-K$  flip-flop.

---

**16.84**

(a)  $256\text{K} \Rightarrow 262,144 \text{ cells} \Rightarrow 512 \times 512$

Each decoder  $\Rightarrow 9$  inputs

(b) (i) Row 52, address = 000110011

(ii) Row 129, address = 010000000

(iii) Row 241, address = 011110000

(c) (i) Column 24, address = 000010111

(ii) Column 165, address = 010100100

(iii) Column 203, address = 011001010

---

**16.85**

1-Megabit memory  $\Rightarrow$

(a)  $= 1,048,576 \Rightarrow 1024 \times 1024$

Number of input row and column decodes lines necessary  $\stackrel{=10}{=}$

(b)  $250\text{K} \times 4 \text{ bits} \Rightarrow 262,144 \times 4 \text{ bits} \Rightarrow 512 \times 512$

For 512 lines  $\Rightarrow 9$  row and column decoder lines necessary.

---

**16.86**

32 rows  $\times$  16 columns  
Each column contains 8 bits

**16.87**

Assume the address line is initially uncharged, then

$$I = C \frac{dV_C}{dt} \quad \text{or} \quad V_C = \frac{1}{C} \int Idt = \frac{I}{C} \cdot t$$

$$\text{Then } t = \frac{V_C \cdot C}{I} = \frac{(2.7)(5.8 \times 10^{-12})}{250 \times 10^{-6}} \Rightarrow$$

$$t = 6.26 \times 10^{-8} \text{ s} \Rightarrow 62.6 \text{ ns}$$

**16.88**

$$(a) \quad \frac{V_{DD} - Q}{R} = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_D \left[ 2(V_{GSD} - V_{TN})Q - Q^2 \right]$$

$$\frac{2.5 - 0.02}{1} = \left( \frac{80}{2} \right) \left( \frac{W}{L} \right)_D \left[ 2(2.5 - 0.4)(0.02) - (0.02)^2 \right]$$

$$0.062 = \left( \frac{W}{L} \right)_D (0.0836) \Rightarrow \left( \frac{W}{L} \right)_D = 0.74$$

(b) 16K  $\Rightarrow$  16,384 cells

$$i_D \cong \frac{V_{DD}}{2} = \frac{1.2}{1} = 1.2 \mu\text{A}$$

$$P = (1.2)(1.2)(16,384) \Rightarrow P = 23.6 \text{ mW}$$

**16.89**

16 K  $\Rightarrow$  16,384 cells

$$P_T = 200 \text{ mW} \Rightarrow \text{Power per cell} = \frac{200}{16,384} \Rightarrow 12.2 \mu\text{W}$$

$$i_D = \frac{P}{V_{DD}} = \frac{12.2}{2.5} = 4.88 \mu\text{A} \cong \frac{V_{DD}}{R} = \frac{2.5}{R} \Rightarrow R = 0.512 \text{ M}\Omega$$

If we want  $v_o = 0.1 \text{ V}$  for a logic 0, then

$$i_D = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) \left[ 2(V_{DD} - V_{TN})v_o - v_o^2 \right]$$

$$4.88 = \left( \frac{35}{2} \right) \left( \frac{W}{L} \right) \left[ 2(2.5 - 0.7)(0.1) - (0.1)^2 \right]$$

$$\text{So } \left( \frac{W}{L} \right) = 0.797$$

**16.90**

$$\bar{D} = V_{DD} = 2.5 \text{ V}$$

Assume  $M_{P3}$  in saturation;  $M_{NA}$ ,  $M_{N1}$  in nonsaturation

$$I_{DP3} = I_{DNA} = I_{DN1}$$

$$I_{DP3} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2 = \left(\frac{35}{2}\right)(1)(2.5 - 0.4)^2 = 77.175 \mu\text{A}$$

$$I_{DP3} = I_{DN1}$$

$$77.175 = \left(\frac{80}{2}\right)(2)[2(2.5 - 0.4)Q - Q^2]$$

We find  $Q^2 - 4.2Q + 0.9647 = 0 \Rightarrow Q = 0.244 \text{ V}$

$$I_{DP3} = I_{DNA}$$

$$77.175 = \left(\frac{80}{2}\right)(1)[2(2.5 - 0.244 - 0.4)(D - 0.244) - (D - 0.244)^2]$$

$$1.9294 = [3.712(D - 0.244) - (D - 0.244)^2]$$

$$= 3.712D - 0.9057 - D^2 + 0.488D - 0.05954$$

We find  $D^2 - 4.2D + 2.895 = 0 \Rightarrow D = 0.869 \text{ V}$

2<sup>nd</sup> approximation,  $M_{p3}$  in nonsaturation

Assume  $V_{DS} = V_{DD} - D \cong 2.5 - 0.869 = 1.631 \text{ V}$

$$I_{DP3} = \left(\frac{35}{2}\right)(1)[2(2.5 - 0.4)(1.631) - (1.631)^2] = 73.33 \mu\text{A}$$

$$I_{DP3} = I_{DN1}$$

$$73.33 = \left(\frac{80}{2}\right)(2)[2(2.5 - 0.4)Q - Q^2]$$

$$Q^2 - 4.2Q + 0.9166 = 0 \Rightarrow Q = 0.231 \text{ V}$$

$$I_{DP3} = I_{DNA}$$

$$73.33 = \left(\frac{80}{2}\right)(1)[2(2.5 - 0.231 - 0.4)(D - 0.231) - (D - 0.231)^2]$$

$$1.833 = [3.738(D - 0.231) - (D^2 - 0.462D + 0.05336)]$$

We find  $D^2 - 4.2D + 2.75 = 0 \Rightarrow D = 0.812 \text{ V}$

### 16.91

Approximation,  $M_{N2}$  cutoff

$$I_{DP2} = I_{DB}, \text{ assume } \bar{D} = 0$$

Both  $M_{p2}$  and  $M_B$  in nonsaturation

$$\left(\frac{35}{2}\right)(4)[2(2.5 - 0.4)(2.5 - \bar{Q}) - (2.5 - \bar{Q})^2]$$

$$= \left(\frac{80}{2}\right)(1)[2(2.5 - 0.4)\bar{Q} - \bar{Q}^2]$$

$$70[4.2(2.5 - \bar{Q}) - (6.25 - 5\bar{Q} + \bar{Q}^2)] = 40[4.2\bar{Q} - \bar{Q}^2]$$

We find  $0.75\bar{Q}^2 + 2.8\bar{Q} - 7.4375 = 0 \Rightarrow \bar{Q} = 1.794 \text{ V}$

Approximation,  $M_{p1}$  cutoff; assume  $D = 2.5 \text{ V}$

$$I_{DNA} = I_{DN1}$$

Both  $M_A$  and  $M_{N1}$  in nonsaturation

$$\begin{aligned} & \left(\frac{80}{2}\right)(1)[2(2.5-Q-0.4)(2.5-Q)-(2.5-Q)^2] \\ & = \left(\frac{80}{2}\right)(2)[2(2.5-0.4)Q-Q^2] \end{aligned}$$

$$\begin{aligned} 2(2.1-Q)(2.5-Q)-(2.5-Q)^2 &= 2(4.2Q-Q^2) \\ 2(5.25-2.1Q-2.5Q+Q^2)-(6.25-5Q+Q^2) &= 8.4Q-2Q^2 \end{aligned}$$

We find  $3Q^2 - 12.6Q + 4.25 = 0 \Rightarrow Q = 0.370 \text{ V}$

**16.92**

For Logic 1,  $v_1$ :

$$(5)(0.05) + (4)(1) = (1 + 0.05)v_1 \Rightarrow v_1 = \underline{4.0476 \text{ V}}$$

$v_2$ :

$$(5)(0.025) + (4)(1) = (1 + 1.025)v_2 \Rightarrow v_2 = \underline{4.0244 \text{ V}}$$

For Logic 0,  $v_1$ :

$$(0)(0.05) + (4)(1) = (1 + 0.05)v_1 \Rightarrow v_1 = \underline{3.8095 \text{ V}}$$

$v_2$ :

$$(0)(0.025) + (4)(1) = (1 + 0.025)v_2 \Rightarrow v_2 = \underline{3.9024 \text{ V}}$$

**16.93**

Design Problem

**16.94**

Design Problem

**16.95**

Design Problem

**16.96**

(a) Quantization error  $= \frac{1}{2} LSB \leq 1\% \leq 0.05 \text{ V}$   
 Or  $LSB \leq 0.10 \text{ V}$

For a 6-bit word,  $LSB = \frac{5}{64} = 0.078125 \text{ V}$

(b)  $1 - LSB = \frac{5}{64} = 0.078125 \text{ V}$

(c)  $\frac{3.5424}{5} \times 64 = 45.34 \Rightarrow n = 45$

Digital Output = 101101

$$\frac{45 \times 5}{64} = 3.515625$$

$$\Delta = 3.5424 - 3.515625 = 0.026775 < \frac{1}{2} LSB.$$

**16.97**

(a) Quantization error =  $\frac{1}{2}$  LSB  $\leq 0.5\% \leq 0.0165$  V

1-LSB = 0.033 V

For a 7-bit word, LSB =  $\frac{3.3}{128} = 0.0258$  V

(b) 1-LSB = 0.02578125 V

(c)  $\frac{2.5321}{3.3} \times 128 = 98.2 \Rightarrow n = 98$

Digital output = 1100010

Now  $\frac{(98)(3.3)}{128} = 2.5265625$  V

$\Delta = 2.5321 - 2.5265625 = 0.0055375$  V  $< \frac{1}{2}$  LSB

**16.98**

(a)  $v_o = \left( \frac{0}{2} + \frac{1}{4} + \frac{1}{8} + \frac{0}{16} \right) (5) = 1.875$  V

(b)  $v_o = \left( \frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16} \right) (5) = 2.8125$  V

**16.99**

LSB =  $\left( \frac{1}{16} \right) (5) = 0.3125$  V

(a)  $\frac{1}{2}$  LSB = 0.15625 V

Now  $v_o = \left( \frac{10}{20 + \Delta R_1} \right) (5)$

For  $v_o = 2.5 + 0.15625 = 2.65625$  V

$20 + \Delta R_1 = \frac{(10)(5)}{2.65625} \Rightarrow \Delta R_1 = -1.176$  K

For  $v_o = 2.5 - 0.15625 = 2.34375$  V

$20 + \Delta R_1 = \frac{(10)(5)}{2.34375}$

$\Delta R_1 = +1.333$  K

For  $|\Delta R_1| = 1.176$  K  $\Rightarrow \Delta R_1 = 5.88\%$

(b) For  $R_4: v_o = \left( \frac{10}{160 + \Delta R_4} \right) (5)$



$$v_o = 0.3125 + 0.15625 = 0.46875 \text{ V}$$

$$160 + \Delta R_4 = \frac{(10)(5)}{0.46875} \Rightarrow \Delta R_4 = -53.33 \text{ K}$$

$$\text{Or } v_o = 0.3125 - 0.15625 = 0.15625 \text{ V}$$

$$160 + \Delta R_4 = \frac{(10)(5)}{0.15625} \Rightarrow \Delta R_4 = 160 \text{ K}$$

$$\text{For } |\Delta R_4| = 53.33 \text{ K} \Rightarrow \underline{\Delta R_4 = 33.33\%}$$

**16.100**

$$R_5 = 320 \text{ k}\Omega$$

$$R_6 = 640 \text{ k}\Omega$$

$$R_7 = 1280 \text{ k}\Omega$$

(a)  $R_8 = 2560 \text{ k}\Omega$

(b)  $v_o = \left(\frac{10}{2560}\right)(5) = 0.01953125 \text{ V}$

**16.101**

(a)

$$I_1 = \frac{V_{REF}}{2R} = \frac{-5}{10} \Rightarrow I_1 = -0.50 \text{ mA}$$

$$I_2 = \frac{I_1}{2} = -0.25 \text{ mA}$$

$$I_3 = \frac{I_2}{2} = -0.125 \text{ mA}$$

$$I_4 = \frac{I_3}{2} = -0.0625 \text{ mA}$$

$$I_5 = \frac{I_4}{2} = -0.03125 \text{ mA}$$

$$I_6 = \frac{I_5}{2} = -0.015625 \text{ mA}$$

$$\Delta v_o = |I_6| R_F = (0.015625)(5)$$

(b)  $\Delta v_o = 0.078125 \text{ V}$

$$v_o = -[I_2 + I_5 + I_6] R_F = [0.25 + 0.03125 + 0.015625](5)$$

(c)  $\underline{v_o = 1.484375 \text{ V}}$

(d) For 101010;  $v_o = (0.50 + 0.125 + 0.03125)(5) = 3.28125 \text{ V}$

For 010101;  $v_o = (0.25 + 0.0625 + 0.015625)(5) = 1.640625 \text{ V}$

$$\underline{\Delta v_o = 1.640625 \text{ V}}$$

**16.102**

$$\frac{1}{2} \text{ LSB} = \left(\frac{V_{REF}}{8R}\right)\left(\frac{R}{2}\right) = \frac{3.3}{16} = 0.20625 \text{ V}$$

Ideal

$$v_A \text{ for } 101 \Rightarrow \left(\frac{5V_{REF}}{8R}\right)(R) = \frac{5}{8}(3.3) = 2.0625 \text{ V}$$

---

$$\text{Range of } v_A = 2.0625 \pm \frac{1}{2} \text{ LSB}$$
$$1.85625 \leq v_A \leq 2.26875 \text{ V}$$

---

**16.103**

$$6\text{-bits} \Rightarrow 2^6 = 64 \text{ resistors}$$
$$2^6 - 1 = 63 \text{ comparators}$$

---

**16.104**

(a) 10-bit output  $\Rightarrow 1024$  clock periods

$$= \frac{1}{f} = \frac{1}{10^6} = 1 \mu\text{S}$$

1 clock period

$$\text{Maximum conversion time} = 1024 \mu\text{S} = \underline{1.024 \text{ mS}}$$

(b)

$$\frac{1}{2} \text{ LSB} = \frac{1}{2} \left( \frac{5}{1024} \right) = 0.002441406 \text{ V}$$

$$v'_A = (128 + 16 + 2) \left( \frac{5}{1024} \right) = 0.712890625 \text{ V}$$

So range of  $v_A = v'_A \pm \frac{1}{2} \text{ LSB}$

$$0.710449219 \leq v_A \leq 0.715332031 \text{ V}$$

(c) 0100100100  $\Rightarrow 256 + 32 + 4 = \underline{292 \text{ clock pulses}}$

---

**16.105**

(a)  $3.125 = \frac{N \times 5}{1024} \Rightarrow N = 640 \Rightarrow 512 + 128$

Output = 1010000000

(b)

$$1.8613 = \frac{N \times 5}{1024} \Rightarrow N = 381.19 \Rightarrow N = 381 \Rightarrow 256 + 64 + 32 + 16 + 8 + 4 + 1$$

Output = 0101111101

---

## Chapter 17

### 17.1

- (a)  $R_C = \frac{0 - (-0.2)}{0.1} = 2 \text{ k}\Omega$
- (b) (i)  $v_1 = -1 \text{ V}$ ,  $Q_1$  off,  $Q_2$  on  
 $v_{O2} = 0 - (0.2)(2) = -0.4 \text{ V}$   
 $v_{O1} = 0$
- (ii)  $v_1 = -0.4 \text{ V}$ ,  $Q_1$  on,  $Q_2$  off  
 $v_{O1} = 0 - (0.2)(2) = -0.4 \text{ V}$   
 $v_{O2} = 0$
- (c) For (i) and (ii)  
 $P = (I_Q)(0 - V^-) = (0.2)(1.8) = 0.36 \text{ mW}$

### 17.2

- (a)  $i_E = \frac{-1 - 0.7 - (-2.5)}{R_E} = 0.08 \text{ mA}$ ,  $\Rightarrow R_E = 10 \text{ k}\Omega$   
 $R_C = \frac{0 - (-0.25)}{0.04} = 6.25 \text{ k}\Omega$
- (b) (i)  $v_1 = -1.3 \text{ V}$ ,  $Q_1$  off,  $Q_2$  on  
 $i_E = \frac{-1 - 0.7 - (-2.5)}{10} = 0.08 \text{ mA}$   
 $v_{O2} = 0 - (0.08)(6.25) = -0.50 \text{ V}$   
 $v_{O1} = 0$
- (ii)  $v_1 = -0.7 \text{ V}$ ,  $Q_1$  on,  $Q_2$  off  
 $i_E = \frac{-0.7 - 0.7 - (-2.5)}{10} = 0.11 \text{ mA}$   
 $v_{O1} = 0 - (0.11)(6.25) = -0.6875 \text{ V}$   
 $v_{O2} = 0$
- (c) (i)  $i_E = 0.08 \text{ mA}$ ,  $P = (0.08)(2.5) = 0.2 \text{ mW}$   
(ii)  $i_E = 0.11 \text{ mA}$ ,  $P = (0.11)(2.5) = 0.275 \text{ mW}$

### 17.3

- (a)  $i_{C2} = I_Q = 0.5 = \frac{3 - 0}{R_{C2}} \Rightarrow R_{C2} = 6 \text{ K}$
- (b)  $i_{C1} = I_Q = 0.5 = \frac{3 - 1}{R_{C1}} \Rightarrow R_{C1} = 4 \text{ K}$

$$\begin{aligned} \frac{i_{C1}}{I_Q} &= \frac{I_S \exp\left(\frac{V_{BE1}}{V_T}\right)}{I_S \left[ \exp\left(\frac{V_{BE1}}{V_T}\right) + \exp\left(\frac{V_{BE2}}{V_T}\right) \right]} \\ &= \frac{1}{1 + \exp\left(\frac{V_{BE2} - V_{BE1}}{V_T}\right)} \end{aligned}$$

(c)  $v_I = V_{BE1} - V_{BE2}$

So

$$\begin{aligned} \frac{i_{C1}}{I_Q} &= \frac{1}{1 + \exp\left(\frac{-v_I}{V_T}\right)} \\ \frac{0.1}{0.5} &= \frac{1}{1 + \exp\left(\frac{-v_I}{V_T}\right)} = 0.2 \\ \exp\left(\frac{-v_I}{V_T}\right) &= \frac{1}{0.2} - 1 = 4 \\ (-v_I) &= (0.026) \ln(4) \\ \underline{v_I} &= \underline{-0.0360 \text{ V}} \end{aligned}$$

#### 17.4

- (a)  $v_I = 0.5 \text{ V}$ ,  $Q_1$  on,  $Q_2$  off  $\Rightarrow v_{O2} = 3 \text{ V}$   
 $v_{O1} = 3 - (1)(0.5) = 2.5 \text{ V}$
- (b)  $v_I = -0.5 \text{ V}$ ,  $Q_1$  off,  $Q_2$  on  $\Rightarrow v_{O1} = 3 \text{ V}$   
 $v_{O2} = 3 - (1)(0.5) = 2.5 \text{ V}$

#### 17.5

- (a)  $Q_2$  on,  $v_E = -1.2 - 0.7 = -1.9 \text{ V}$   
 $i_E = i_{C2} = \frac{-1.9 - (-5.2)}{2.5} = 1.32 \text{ mA}$   
 $v_2 = -1 \text{ V} = -i_{C2} R_{C2} = -(1.32)(R_{C2})$   
 $\underline{R_{C2} = 0.758 \text{ k}\Omega}$
- (b)  $Q_1$  on,  $v_E = -0.7 - 0.7 = -1.40 \text{ V}$   
 $i_E = i_{C1} = \frac{-1.4 - (-5.2)}{2.5} = 1.52 \text{ mA}$   
 $v_1 = -1 \text{ V} = -i_{C1} R_{C1} = -(1.52)(R_{C1})$   
 $\underline{R_{C1} = 0.658 \text{ k}\Omega}$
- (c) For  $v_{in} = -0.7 \text{ V}$ ,  $Q_1$  on,  $Q_2$  off  
 $\Rightarrow v_{O1} = -0.70 \text{ V}$   
 $v_{O2} = -1 - 0.7 \Rightarrow \underline{v_{O2} = -1.7 \text{ V}}$
- For  $v_{in} = -1.7 \text{ V}$ ,  $Q_1$  off,  $Q_2$  on  
 $\Rightarrow v_{O2} = -0.7 \text{ V}$   
 $v_{O1} = -1 - 0.7 \Rightarrow \underline{v_{O1} = -1.7 \text{ V}}$

(d) (i) For  $v_{in} = -0.7V$ ,  $i_E = 1.52 mA$

$$i_{C4} = \frac{-1.7 - (-5.2)}{3} = 1.17 mA$$

$$i_{C3} = \frac{-0.7 - (-5.2)}{3} = 1.5 mA$$

$$P = (i_E + i_{C4} + i_{C3})(5.2) = (1.52 + 1.17 + 1.5)(5.2)$$

or  $P = 21.8 mW$

(ii) For  $v_{in} = -1.7V$ ,  $i_E = 1.32 mA$

$$i_{C4} = \frac{-0.7 - (-5.2)}{3} = 1.5 mA$$

$$i_{C3} = \frac{-1.7 - (-5.2)}{3} = 1.17 mA$$

$$P = (1.32 + 1.5 + 1.17)(5.2)$$

or  $P = 20.7 mW$

### 17.6

a.  $I_3 = \frac{3.7 - 0.7}{0.67 + 1.33} = 1.5 mA$

$$V_R = I_3 R_4 + V_Y = (1.5)(1.33) + 0.7$$

or

$$V_R = 2.70 V$$

b. logic 1 level  $= 3.7 - 0.7 \Rightarrow 3.0 V$

For  $v_X = v_Y = \text{logic 1}$ .

$$i_E = \frac{3 - 0.7}{0.8} = 2.875 mA = i_{RC1}$$

$$v_{B3} = 3.7 - (2.875)(0.21) = 3.10 V$$

$$\Rightarrow v_{01}(\text{logic 0}) = 2.4 V$$

For  $v_X = v_Y = \text{logic 0}$ ,  $Q_R$  on

$$i_E = \frac{2.7 - 0.7}{0.8} = 2.5 mA = i_{RC2}$$

$$v_{B4} = 3.7 - (2.5)(0.24) = 3.1 V$$

$$\Rightarrow v_{02}(\text{logic 0}) = 2.4 V$$

### 17.7

(a)  $R_1 = \frac{0.7 - 0.7 - (-2.1)}{0.20} = 10.5 k\Omega$

(b)  $R_5 = R_6 = \frac{0 - (-2.1)}{0.12} = 17.5 k\Omega$

(c)  $I_Q = I_{REF} = 0.20 mA$

$$v_{O1} = -0.7 V, v_{C2} = 0$$

$$R_{C1} = \frac{0.7 - 0}{0.2} = 3.5 k\Omega$$

---

(d)  $I_Q = I_{REF} = 0.20 \text{ mA}$   
 $v_{O2} = -0.7 \text{ V}, v_{CR} = 0$   
 $R_{C2} = \frac{0.7 - 0}{0.20} = 3.5 \text{ k}\Omega$

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**17.8**

$$V_R = -0.5 \text{ V}$$
$$i_E = \frac{-0.5 - 0.7 - (-3)}{R_E} = 0.4 \text{ mA}, R_E = 4.5 \text{ k}\Omega$$
$$R_5 = \frac{-0.5 - (-3)}{0.4} = 6.25 \text{ k}\Omega$$
$$V_{B2} = -0.5 + 0.7 = 0.2 \text{ V}, R_1 = \frac{0.7 - 0.2}{0.4} = 1.25 \text{ k}\Omega$$
$$R_2 = \frac{0.2 - 0.7 - 0.7 - (-3)}{0.4} = 4.5 \text{ k}\Omega$$
$$R_3 = R_4 = \frac{0 - (-3)}{0.8} = 3.75 \text{ k}\Omega$$
$$v_{OR} = -1 \text{ V}, \Rightarrow v_{CR} = -0.3 \text{ V}$$
$$R_{C2} = \frac{0.7 - (-0.3)}{0.4} = 2.5 \text{ k}\Omega$$
$$v_I = 0, i_E = \frac{-0.7 - (-3)}{4.5} = 0.511 \text{ mA}$$
$$v_{C1} = -1 + 0.7 = -0.3 \text{ V}$$
$$R_{C1} = \frac{0.7 - (-0.3)}{0.511} = 1.957 \text{ k}\Omega$$

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**17.9**

$$v_O = \text{logic 1} = 1.8 \text{ V}, \text{ logic 0} = 1.2 \text{ V}$$

For  $v_I = \text{logic 1} = 1.8 \text{ V}$

$$i_E = 0.8 = \frac{1.8 - 0.7}{R_E} \Rightarrow R_E = 1.375 \text{ k}\Omega$$
$$v_{C1} = 1.2 + 0.7 = 1.9 \text{ V}, \Rightarrow R_{C1} = \frac{2.5 - 1.9}{0.8} = 0.75 \text{ k}\Omega$$

For  $v_I = \text{logic 0}, Q_R \text{ on}; i_E = \frac{1.5 - 0.7}{R_E} = \frac{0.8}{1.375} = 0.5818 \text{ mA}$

$$v_{CR} = 1.2 + 0.7 = 1.9 \text{ V}$$
$$R_{C2} = \frac{2.5 - 1.9}{0.5818} = 1.031 \text{ k}\Omega$$
$$R_2 = R_3 = \frac{1.8}{0.8} = 2.25 \text{ k}\Omega$$

---

**17.10**

Neglecting base currents:

- (a)  $I_{E1} = 0, I_{E3} = 0$   
 $I_{E5} = \frac{5-0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$   
 $Y = 0.7 \text{ V}$   
 $I_{E1} = \frac{5-0.7}{18} \Rightarrow I_{E1} = 0.239 \text{ mA}$   
 $I_{E3} = 0$   
 $I_{E5} = \frac{5-0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$
- (b)  $Y = 0.7 \text{ V}$
- (c)  $I_{E1} = I_{E3} = \frac{5-0.7}{18} \Rightarrow I_{E1} = I_{E3} = 0.239 \text{ mA}$   
 $I_{E5} = 0, Y = 5 \text{ V}$
- (d) Same as (c).

**17.11**

- (a)  $V_R = -(1)(1) - 0.7 \Rightarrow V_R = -1.7 \text{ V}$
- (b)  $Q_R$  off, then  $v_{O1} = \text{Logic 1} = -0.7 \text{ V}$   
 $Q_R$  on, then  $v_{O1} = -(1)(2) - 0.7 \Rightarrow$   
 $v_{O1} = \text{Logic 0} = -2.7 \text{ V}$   
 $Q_A / Q_B$  off, then  $v_{O2} = \text{Logic 1} = -0.7 \text{ V}$   
 $Q_A / Q_B$  on, then  $v_{O2} = -(1)(2) - 0.7 \Rightarrow$   
 $v_{O2} = \text{Logic 0} = -2.7 \text{ V}$   
 $A = B = \text{Logic 0} = -2.7 \text{ V}, Q_R$  on,
- (c)  $V_E = -1.7 - 0.7 \Rightarrow V_E = -2.4 \text{ V}$   
 $A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A / Q_B$  on,  
 $V_E = -0.7 - 0.7 \Rightarrow V_E = -1.4 \text{ V}$
- (d)  $A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A / Q_B$  on,  
 $i_{C3} = \frac{-2.7 - (-5.2)}{1.5} = 1.67 \text{ mA}$   
 $i_{C2} = \frac{-0.7 - (-5.2)}{1.5} = 3 \text{ mA}$   
 $P = (1.67 + 1 + 1 + 1 + 3)(5.2) \Rightarrow P = 39.9 \text{ mW}$   
 $A = B = \text{Logic 0} = -2.7 \text{ V}$   
 $i_{C3} = 3 \text{ mA}, i_{C2} = 1.67 \text{ mA}$   
 $P = 39.9 \text{ mW}$

**17.12**

a. AND logic function

b. logic 0 = 0 V

$$Q_3 \text{ on, } i = \frac{5 - (1.6 + 0.7)}{1.2} = 2.25 \text{ mA}$$

$$V_2 = (2.25)(0.8) \Rightarrow \underline{\text{logic 1} = 1.8 \text{ V}}$$

$$i_{E1} = \frac{5 - 0.7}{2.6} \Rightarrow \underline{i_{E1} = 1.65 \text{ mA}}$$

$$i_{E2} = \frac{5 - (0.7 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 3 \text{ mA}}$$

$$\underline{i_{C3} = 0, \quad i_{C2} = i_{E2} = 3 \text{ mA}}$$

$$\underline{V_2 = 0}$$

c.

$$i_{E1} = \frac{5 - (1.8 + 0.7)}{2.6} \Rightarrow \underline{i_{E1} = 0.962 \text{ mA}}$$

$$i_{E2} = \frac{5 - (1.6 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 2.25 \text{ mA}}$$

$$\underline{i_{C2} = 0, \quad i_{C3} = i_{E2} = 2.25 \text{ mA}}$$

$$\underline{V_2 = 1.8 \text{ V}}$$

d.

**17.13**

(a)  $v_R = (\text{logic 0} + \text{logic 1})/2 - 0.7 = \frac{3.5 + 3.1}{2} - 0.7 = 2.6 \text{ V}$

(b) For  $v_X = v_Y = \text{logic 1} = 3.5 \text{ V}$

$$v_{E1} = 3.5 - 0.7 - 0.7 = 2.1 \text{ V}$$

$$i_{E1} = \frac{2.1 - 0}{12} = 0.175 \text{ mA}$$

$$i_{RC1} = \frac{1}{3} \cdot i_{E1} = 0.05833 \text{ mA} = \frac{0.4}{R_{C1}} \Rightarrow R_{C1} = 6.86 \text{ k}\Omega$$

(c) For  $v_X = v_Y = \text{logic 0} = 3.1 \text{ V}$

$$v_{E1} = v_R - 0.7 = 2.6 - 0.7 = 1.9 \text{ V}$$

$$i_{E2} = \frac{1.9}{12} = 0.1583 \text{ mA}$$

$$i_{RC2} = \frac{1}{3} \cdot i_{E2} = 0.05277 \text{ mA} = \frac{0.4}{R_{C2}} \Rightarrow R_{C2} = 7.58 \text{ k}\Omega$$

(d) For  $v_X = v_Y = \text{logic 0} = 3.1 \text{ V}$

$$i_E = 0.1583 \text{ mA}$$

$$i_{R1} = \frac{3.1 - 0.7}{8} = 0.3 \text{ mA}$$

$$P = (0.1583 + 0.3)(3.5) = 1.60 \text{ mW}$$



**17.14**

Assume  $V_\gamma = 0.4 \text{ V}$

(a) Logic 1 = 0.2 V, Logic 0 = -0.2 V

$$(b) \quad i_E = \frac{0 - 0.7 - (-3.10)}{R_E} = 0.25 \Rightarrow R_E = 9.6 \text{ k}\Omega$$

$$(c) \quad i_{D1} + i_{R1} = i_E$$

$$2i_{R1} + i_{R1} = 3i_{R1} = 0.25 \Rightarrow i_{R1} = 0.08333 \text{ mA}$$

$$R_1 = \frac{0.4}{0.08333} = 4.8 \text{ k}\Omega$$

$$(d) \quad i_E = \frac{0.2 - 0.7 - (-3.10)}{9.6} = 0.2708 \text{ mA}$$

$$i_{R2} = \frac{0.4}{R_2} = \frac{0.4}{4.8} = 0.0833 \text{ mA}$$

$$i_{D2} = i_E - i_{R2} = 0.2708 - 0.0833 = 0.1875 \text{ mA}$$

(e)  $i_E = 0.2708 \text{ mA}$

$$i_{R4} = \frac{-0.2 - (-3.10)}{3.3} = 0.8788 \text{ mA}$$

$$i_{R3} = \frac{0.2 - (-3.10)}{3.3} = 1.0 \text{ mA}$$

$$P = (i_E + i_{R3} + i_{R4})[0.9 - (-3.10)] = (0.2708 + 1.0 + 0.8788)(4) = 8.6 \text{ mW}$$

**17.15**

a.

$$i_1 = \frac{(-0.9 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.4 \text{ mA}$$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.14 \text{ mA}$$

$$i_4 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_4 = 0.14 \text{ mA}$$

$$i_2 + i_D = i_1 + i_3 = 1.4 + 0.14 = 1.54 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow i_2 = 0.8 \text{ mA}$$

$$i_D = 0.74 \text{ mA}$$

$$v_0 = -0.4 \text{ V}$$

b.  $i_1 = 1.4 \text{ mA}$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.153 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.153 \text{ mA}$$

$$i_2 + i_D = i_4 \Rightarrow i_2 = 0.153 \text{ mA}$$

$$i_D = 0$$

$$v_0 = -(0.153)(0.5) \Rightarrow v_0 = -0.0765 \text{ V}$$

c.

$$i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.6 \text{ mA}}$$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.14 \text{ mA}}$$

$$i_4 = i_3 \Rightarrow \underline{i_4 = 0.14 \text{ mA}}$$

$$i_2 + i_D = i_3 \Rightarrow \underline{i_2 = 0.14 \text{ mA}}$$

$$\underline{i_D = 0.0}$$

$$v_0 = -(0.14)(0.5) \Rightarrow \underline{v_0 = -0.07 \text{ V}}$$

d.

$$i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.6 \text{ mA}}$$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.153 \text{ mA}}$$

$$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153 \text{ mA}}$$

$$i_2 + i_D = i_1 + i_4 = 1.6 + 0.153 = 1.753 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow \underline{i_2 = 0.8 \text{ mA}}$$

$$\underline{i_D = 0.953 \text{ mA}}$$

$$\underline{v_0 = -0.40 \text{ V}}$$

### 17.16

(a) (i)  $A = B = C = D = 0 \text{ V}$ ,  $Q_1, Q_2, Q_3, Q_4$  off;  $Q_5, Q_6$  on

$$2.5 = i_{R1}(2) + 0.7 + \frac{1}{2} \left( \frac{i_{R1}}{91} \right) (15)$$

$$1.8 = i_{R1}(2 + 0.0824) \Rightarrow i_{R1} = 0.8644 \text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.771 \text{ V}$$

(ii)  $A = B = 0 \text{ V}$ ,  $C = D = 2.5 \text{ V}$ ;  $Q_1, Q_2, Q_6$  off;  $Q_3, Q_4, Q_5$  on

$$2.5 = i_{R1}(2) + 0.7 + \left( \frac{i_{R1}}{91} \right) (15)$$

$$1.8 = i_{R1}(2 + 0.1648) \Rightarrow i_{R1} = 0.8315 \text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.837 \text{ V}$$

(iii)  $A = C = 2.5 \text{ V}$ ,  $B = D = 0 \text{ V}$ ;  $Q_1, Q_3$  on,  $Q_5, Q_6$  off

$$i_{R1} = 0 \Rightarrow Y = 2.5 \text{ V}$$

(b)  $Y = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

(c) (i)  $P = i_{R1}(2.5) = (0.8644)(2.5) = 2.16 \text{ mW}$

(ii)  $i_{R3} = \frac{2.5 - 0.7}{15} = 0.12 \text{ mA}$

$$P = (i_{R1} + i_{R3})(2.5) = (0.8315 + 0.12)(2.5) = 2.38 \text{ mW}$$

(iii)  $i_{R2} = i_{R3} = \frac{2.5 - 0.7}{15} = 0.12 \text{ mA}$

$$P = (i_{R2} + i_{R3})(2.5) = (0.24)(2.5) = 0.60 \text{ mW}$$

**17.17**

- a. logic 1 = 0 V  
 logic 0 = -0.4 V
- b.  $v_{01} = \overline{A \text{ OR } B}$   
 $v_{02} = \overline{C \text{ OR } D}$   
 $v_{03} = v_{01} \text{ OR } v_{02}$
- or
- $v_{03} = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

**17.18**

a. For CLOCK = high,  $I_{DC}$  flows through the left side of the circuit. If  $D$  is high,  $I_{DC}$  flows through the left  $R$  resistor pulling  $\bar{Q}$  low. If  $D$  is low,  $I_{DC}$  flows through the right  $R$  resistor pulling  $Q$  low.

For CLOCK = low,  $I_{DC}$  flows through the right side of the circuit maintaining  $Q$  and  $\bar{Q}$  in their previous state.

- b.  $P = (I_{DC} + 0.5I_{DC} + 0.1I_{DC} + 0.1I_{DC})(3)$   
 $P = 1.7I_{DC}(3) = (1.7)(50)(3) \Rightarrow P = 255 \mu\text{W}$

**17.19**

- (a) (i) For  $v_I = 0.1 \text{ V}$   
 $v_1 = 0.8 \text{ V}$   
 $i_1 = \frac{2.5 - 0.8}{12} = 0.1417 \text{ mA}$   
 $i_2 = i_3 = 0, v_O = 2.5 \text{ V}$
- (ii) For  $v_I = 2.5 \text{ V}$   
 $v_1 = 0.7 + 0.8 = 1.5 \text{ V}$   
 $i_1 = i_2 = \frac{2.5 - 1.5}{12} = 0.0833 \text{ mA}$   
 $v_O = 0.1 \text{ V}$   
 $i_3 = \frac{2.5 - 0.1}{12} = 0.20 \text{ mA}$
- (b) (i)  $v_1 = 0.7 + 0.7 = 1.4 \text{ V}$   
 $v_I = v_1 - 0.7 = 0.7 \text{ V}$
- (ii)  $v_1 = 0.7 + 0.8 = 1.5 \text{ V}$   
 $v_I = v_1 - 0.7 = 0.8 \text{ V}$

**17.20**

(a)

$$v_I = 0 \Rightarrow V_1 = 0.7 \text{ V}$$

$$i_1 = \frac{3.3 - 0.7}{6} = 0.433 \text{ mA}$$

$$i_B = i_C = 0$$

$$v_o = 3.3 \text{ V}$$

(b)

$$v_I = 3.3 \text{ V} \quad v_1 = 0.7 + 0.8 = 1.5 \text{ V}$$

$$i_1 = \frac{3.3 - 1.5}{6} = 0.3 \text{ mA}$$

$$i_R = \frac{0.8}{20} = 0.04 \text{ mA}$$

$$i_B = 0.3 - 0.04 = 0.26 \text{ mA}$$

$$i_C = \frac{3.3 - 0.1}{4} = 0.8 \text{ mA}$$

$$v_o = 0.1 \text{ V}$$

**17.21**

i. For  $v_X = v_Y = 0.1 \text{ V} \Rightarrow v' = 0.8 \text{ V}$

$$i_1 = \frac{5 - 0.8}{8} \Rightarrow i_1 = 0.525 \text{ mA}$$

$$i_3 = i_4 = 0$$

ii. For  $v_X = v_Y = 5 \text{ V}$ ,

$$v' = 0.8 + 0.7 + 0.7 \Rightarrow v' = 2.2 \text{ V}$$

$$i_1 = \frac{5 - 2.2}{8} \Rightarrow i_1 = 0.35 \text{ mA}$$

$$i_4 = i_1 - \frac{0.8}{15} \Rightarrow i_4 = 0.297 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.4} \Rightarrow i_3 = 2.04 \text{ mA}$$

**17.22**

(i)  $v_X = v_Y = 0.1 \text{ V}$

$$v' = 0.8 \text{ V}$$

$$i_1 = \frac{3.3 - 0.8}{8} = 0.3125 \text{ mA}$$

$$i_3 = i_4 = 0$$

(ii)  $v_X = v_Y = 3.3 \text{ V}$

$$v' = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$$

$$i_1 = \frac{3.3 - 2.2}{8} = 0.1375 \text{ mA}$$

$$i_4 = 0.1375 - \frac{0.8}{15} = 0.08417 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{2.4} = 1.333 \text{ mA}$$

**17.23**

- a. For  $v_X = v_Y = 5 \text{ V}$ , both  $Q_1$  and  $Q_2$  driven into saturation.

$$v_1 = 0.8 + 0.7 + 0.8 \Rightarrow v_1 = 2.3 \text{ V}$$

$$i_1 = \frac{5 - 2.3}{4} \Rightarrow i_1 = i_{B1} = 0.675 \text{ mA}$$

$$i_2 = \frac{5 - (0.8 + 0.7 + 0.1)}{2} \Rightarrow i_2 = 1.7 \text{ mA}$$

$$i_4 = i_{B1} + i_2 \Rightarrow i_4 = 2.375 \text{ mA}$$

$$i_5 = \frac{0.8}{10} \Rightarrow i_5 = 0.08 \text{ mA}$$

$$i_{B2} = i_4 - i_5 \Rightarrow i_{B2} = 2.295 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{4} \Rightarrow i_3 = 1.225 \text{ mA}$$

$$v_0 = 0.1 \text{ V}$$

$$i'_L = \frac{5 - (0.1 + 0.7)}{4} = 1.05 \text{ mA}$$

- b.  $i_C(\text{max}) = \beta i_{B2} = N i'_L + i_3$

$$(20)(2.295) = N(1.05) + 1.225$$

So

$$N = 42$$

**17.24**

- (a)  $v_1 = 0.8 + 0.7 + 0.8 = 2.3 \text{ V}$

$$i_1 = \frac{3.3 - 2.3}{4} = 0.25 \text{ mA} = i_{B1}$$

$$v_{C1} = 0.8 + 0.7 + 0.1 = 1.6 \text{ V}$$

$$i_2 = \frac{3.3 - 1.6}{2} = 0.85 \text{ mA}$$

$$i_4 = i_{B1} + i_2 = 0.25 + 0.85 = 1.10 \text{ mA}$$

$$i_5 = \frac{0.8}{10} = 0.08 \text{ mA}$$

$$i_{B2} = i_4 - i_5 = 1.10 - 0.08 = 1.02 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{4} = 0.80 \text{ mA}$$

- (b)  $i_{Co}(\text{max}) = \beta i_{B2} = i_3 + N i'_L$

$$i'_L = \frac{3.3 - (0.1 + 0.7)}{4} = 0.625 \text{ mA}$$

$$(20)(1.02) = 0.8 + N(0.625) \Rightarrow N = 31.36 \Rightarrow N = 31$$

**17.25**

$D_x$  and  $D_y$  off,  $Q_1$  forward active mode

$$v_1 = 0.8 + 0.7 + 0.7 = 2.2V$$

$$5 = i_1 R_1 + i_2 R_2 + v_1 \text{ and } i_1 = (1 + \beta) i_2$$

So  $5 - 2.2 = i_2 [(1 + \beta) R_1 + R_2]$

Assume  $\beta = 25$

$$i_2 = \frac{5 - 2.2}{(26)(1.75) + 2} \Rightarrow i_2 = 0.0589 \text{ mA}$$

$$i_1 = (1 + \beta) i_2 = (26)(0.05895) \Rightarrow i_1 = 1.53 \text{ mA}$$

$$i_3 = \beta i_2 \Rightarrow i_3 = 1.47 \text{ mA}$$

$$i_{Bo} = i_2 + i_3 - \frac{0.8}{5} = 0.0589 + 1.47 - 0.16 \Rightarrow$$

$$i_{Bo} = 1.37 \text{ mA}$$

$Q_o$  in saturation

$$i_{Co} = \frac{5 - 0.1}{6} \Rightarrow i_{Co} = 0.817 \text{ mA}$$

**17.26**

(a)  $v_l = 0 \text{ V}$ ,  $Q_1$  forward active

$$i_B = \frac{5 - 0.7}{6} = 0.717 \text{ mA}$$

$$i_C = (25)(0.71667) = 17.9 \text{ mA}$$

$$i_E = (26)(0.71667) = 18.6 \text{ mA}$$

(b)  $V_l = 0.8 \text{ V}$

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

Because of the relative doping levels of the Emitter and collector, and because of the difference in B-C and B-E areas, we have  $-i_C \approx i_B = 0.583 \text{ mA}$  and  $i_E = \text{small value}$ .

(c)  $v_l = 3.6 \text{ V}$ ,  $Q_1$  inverse active.

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

$$i_E = -\beta_R i_B = -(0.5)(0.583) = -0.292 \text{ mA}$$

$$i_C = -i_B - i_E = -0.583 - 0.292 \Rightarrow i_C = -0.875 \text{ mA}$$

**17.27**

(a) (i) For  $v_I = 0.1$  V,  $v_1 = 0.1 + 0.8 = 0.9$  V, and  $v_O = 2.5$  V

$$i_1 = \frac{2.5 - 0.9}{12} = 0.1333 \text{ mA}$$

$$i_2 = i_3 = 0$$

(ii) For  $v_I = 2.5$  V,  $v_1 = 0.8 + 0.7 = 1.5$  V, and  $v_O = 0.1$  V

$$i_1 = \frac{2.5 - 1.5}{12} = 0.0833 \text{ mA}$$

$$i_2 = i_1(1 + 0.1) = (0.0833)(1.1) = 0.09167 \text{ mA}$$

$$i_3 = \frac{2.5 - 0.1}{12} = 0.20 \text{ mA}$$

(b) (i)  $v_1 = 0.7 + 0.7 = 1.4$  V

$$v_I = 1.4 - 0.8 = 0.6 \text{ V}$$

(ii)  $v_1 = 0.8 + 0.7 = 1.5$  V

$$v_I = 1.5 - 0.8 = 0.7 \text{ V}$$

**17.28**

a. i.  $v_X = v_Y = 0.1$  V, so  $Q_1$  in saturation.

$$i_1 = \frac{5 - (0.1 + 0.8)}{6} \Rightarrow i_1 = 0.683 \text{ mA}$$

$$\Rightarrow i_{B2} = i_2 = i_4 = i_{B3} = i_3 = 0$$

ii.  $v_X = v_Y = 5$  V, so  $Q_1$  in inverse active mode.

Assume  $Q_2$  and  $Q_3$  in saturation.

$$i_1 = \frac{5 - (0.8 + 0.8 + 0.7)}{6} \Rightarrow i_1 = i_{B2} = 0.45 \text{ mA}$$

$$i_2 = \frac{5 - (0.8 + 0.1)}{2} \Rightarrow i_2 = 2.05 \text{ mA}$$

$$i_4 = \frac{0.8}{1.5} \Rightarrow i_4 = 0.533 \text{ mA}$$

$$i_{B3} = (i_{B2} + i_2) - i_4 = 0.45 + 2.05 - 0.533$$

or

$$i_{B3} = 1.97 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.2} \Rightarrow i_3 = 2.23 \text{ mA}$$

b. For  $Q_3$ :

$$\frac{i_3}{i_{B3}} = \frac{2.23}{1.97} = 1.13 < \beta$$

For  $Q_2$ :

$$\frac{i_2}{i_{B2}} = \frac{2.05}{0.45} = 4.56 < \beta$$

Since  $(I_C / I_B) < \beta$ , then each transistor is in saturation.

**17.29**

(a) (i)  $v_X = 0.1 \text{ V}$ ,  $v_Y = 3.3 \text{ V}$

$$v' = 0.1 + 0.7 = 0.8 \text{ V}$$

$$i_i = \frac{3.3 - 0.8}{16} = 0.156 \text{ mA}$$

$$i_3 = i_4 = 0$$

(ii)  $v_X = v_Y = 3.3 \text{ V}$

$$v' = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$$

$$i_1 = \frac{3.3 - 2.2}{16} = 0.06875 \text{ mA}$$

$$i_4 = 0.06875 - \frac{0.8}{20} = 0.02875 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{6} = 0.5333 \text{ mA}$$

(b)  $i_{C1}(\text{max}) = \beta i_4 = i_3 + N i'_L$

$$i'_L = \frac{3.3 - (0.1 + 0.7)}{16} = 0.15625 \text{ mA}$$

$$(50)(0.02875) = 0.5333 + N(0.15625) \Rightarrow N = 5.8 \Rightarrow N = 5$$

(c)  $i_{C1}(\text{max}) = \beta i_4 = (50)(0.02875) = 1.44 \text{ mA} < 5 \text{ mA}$ ,  $\Rightarrow N = 5$

**17.30**

a. For  $v_X = v_Y = 5 \text{ V}$ ,  $Q$ , in inverse active mode.

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{6} = 0.45 \text{ mA}$$

$$i_{B2} = i_{B1} + 2\beta_R i_{B1} = 0.45(1 + 2[0.1]) = 0.54 \text{ mA}$$

$$i_{C2} = \frac{5 - (0.8 + 0.1)}{2} = 2.05 \text{ mA}$$

$$i_{B3} = (i_{B2} + i_{C2}) - \frac{0.8}{1.5} = 0.54 + 2.05 - 0.533$$

or

$$\underline{i_{B3} = 2.06 \text{ mA}}$$

Now

$$i'_L = \frac{5 - (0.1 + 0.8)}{6} = 0.683 \text{ mA}$$

Then

$$i_{C3}(\text{max}) = \beta_F i_{B3} = N i'_L$$

$$\text{or } (20)(2.06) = N(0.683)$$

$$\Rightarrow \underline{N = 60}$$

b. From above, for  $v_0$  high,  $I'_L = (0.1)(0.45) = 0.045 \text{ mA}$ . Now

$$I'_L(\text{max}) = (1 + \beta_F) \left( \frac{5 - 4.9}{R_2} \right) = \frac{(21)(0.1)}{2} = 1.05 \text{ mA}$$

So

$$I_L(\text{max}) = N I'_L$$

$$\text{or } 1.05 = N(0.045)$$

$$\Rightarrow \underline{N = 23}$$



**17.31**

(a) (i)  $V_{in} = 0.1 \text{ V}$

$$i_{RB} = \frac{5 - (0.1 + 0.8)}{4} = 1.025 \text{ mA}$$

$$i_{RCP} = i_{Bo} = 0, \quad V_{out} = 5 \text{ V}$$

(ii)  $V_{in} = 5 \text{ V}$

$$i_{RB} = \frac{5 - (0.7 + 0.8 + 0.7)}{4} = 0.70 \text{ mA}$$

$$V_{out} = 0.7 + 0.1 = 0.8 \text{ V}$$

$$i_{RCP} = \frac{5 - 0.8}{1} = 4.2 \text{ mA}$$

$$i_{BS} = (1.1)(0.7) = 0.77 \text{ mA}$$

$$i_{Co} = \beta i_{Bo}, \quad i_{CS} = 4.2 - \beta i_{Bo}, \quad i_{ES} = 0.77 + (4.2 - \beta i_{Bo})$$

$$i_{Bo} = i_{ES} - \frac{0.7}{1}$$

$$i_{Bo} = 0.77 + 4.2 - \beta i_{Bo} - 0.7$$

$$(1 + \beta)i_{Bo} = 4.27 \Rightarrow i_{Bo} = \frac{4.27}{51} = 0.0837 \text{ mA}$$

(b) (i)  $V_{in} = 0.1 \text{ V}, \quad V_{out} = \text{High},$

$$i'_L = 5\beta_R i_{RB} = 5(0.1)(0.7) = 0.35 \text{ mA}$$

$$V_{out} = 5 - (0.35)(1) = 4.65 \text{ V}$$

$$P = (5 - 0.1)(1.025) + (0.35)(5 - 4.65) = 5.145 \text{ mW}$$

(ii)  $i_L = 5(1.025) = 5.125 \text{ mA}$

$$P = (0.77 + 4.2)(5) + (5.125)(0.1) = 25.4 \text{ mW}$$

**17.32**

a.  $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{3.9} \Rightarrow \underline{i_{B1} = 1.05 \text{ mA}}$$

Then

$$\underline{i_{C1} = i_{B2} = i_{C2} = i_{B3} = i_{C3} = 0}$$

b.  $v_X = v_Y = v_Z = 5 \text{ V}$

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{3.9} \Rightarrow \underline{i_{B1} = 0.692 \text{ mA}}$$

Then

$$i_{C1} = i_{B2} = i_{B1}(1 + 3\beta_R) = (0.692)(1 + 3[0.5])$$

$$\Rightarrow \underline{i_{C1} = i_{B2} = 1.73 \text{ mA}}$$

$$i_{C2} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow \underline{i_{C2} = 2.05 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{0.8} = 1.73 + 2.05 - 1.0$$

$$\Rightarrow \underline{i_{B3} = 2.78 \text{ mA}}$$

$$i_{R3} = \frac{5-0.1}{2.4} = 2.04 \text{ mA}$$
$$i'_L = \frac{5-(0.1+0.8)}{3.9} = 1.05 \text{ mA}$$
$$i_{C3} = i_{R3} + 5i'_L = 2.04 + (5)(1.05)$$
$$\Rightarrow \underline{i_{C3} = 7.29 \text{ mA}}$$

**17.33**

(a) (i)  $I'_L = 5 \mu\text{A}$ ,  $i_{B4} = \frac{5}{51} = 0.098 \mu\text{A}$

$$V_{B5} = 5 - (0.000098)(2) \cong 5 \text{ V}$$

$$v_O = 3.6 \text{ V}$$

(ii)  $I'_L = 5 \text{ mA}$ ,  $i_{B4} = \frac{5}{51} = 0.098 \text{ mA}$

$$V_{B5} = 5 - (0.098)(2) = 4.804 \text{ V}$$

$$v_O = 4.804 - 1.4 = 3.404 \text{ V}$$

(iii)  $Q_4$  in saturation

$$I_B = \frac{5 - V_B}{2} = \frac{5 - (V_E + 0.8)}{2}$$

$$I_C = \frac{5 - V_C}{0.13} = \frac{5 - (V_E + 0.1)}{0.13}$$

$$I_B + I_C = I_E = I_L = 25 = \frac{5 - (V_E + 0.7)}{2} + \frac{5 - (V_E + 0.1)}{0.13}$$

$$25 = 2.10 - V_E(0.5) + 37.69 - V_E(7.692) \Rightarrow V_E = 1.81 \text{ V}$$

$$v_O = 1.81 - 0.7 = 1.11 \text{ V}$$

(b)  $V_{B4} = 0.7 + 0.8 = 1.5 \text{ V}$

$$I_{B4} = \frac{5 - 1.5}{2} = 1.75 \text{ mA}$$

$$V_{C4} = 0.7 + 0.1 = 0.8 \text{ V}, \quad I_{C4} = \frac{5 - 0.8}{0.13} = 32.31 \text{ mA}$$

$$I_L = 1.75 + 32.31 = 34.06 \text{ mA}$$

**17.34**

a.  $v_X = v_Y = v_Z = 2.8 \text{ V}$ ,  $Q_1$  biased in the inverse active mode.

$$i_{B1} = \frac{2.8 - (0.8 + 0.8 + 0.7)}{2} \Rightarrow \underline{i_{B1} = 0.25 \text{ mA}}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = 0.25(1 + 3[0.3]) \\ \Rightarrow \underline{i_{B2} = 0.475 \text{ mA}}$$

$$v_{C2} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_{B4} = \frac{0.9 - (0.7 + 0.1)}{(1 + \beta_F)(0.5)} = \frac{0.1}{(101)(0.5)} \\ = 0.00198 \text{ mA (Negligible)}$$

$$i_{R2} = \frac{5 - 0.9}{0.9} = 4.56 \text{ mA} \\ \Rightarrow \underline{i_{C2} = 4.56 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{1} = 0.475 + 4.56 - 0.8 \\ \Rightarrow \underline{i_{B3} = 4.235 \text{ mA}}$$

b.  $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow \underline{i_{B1} = 2.05 \text{ mA}}$$

From part (a),

$$i'_L = \beta_R \cdot i_{B1} = (0.3)(2.05) = 0.075 \text{ mA}$$

Then

$$i_{B4} = \frac{5i'_L}{1 + \beta_F} = \frac{5(0.075)}{101} \Rightarrow \underline{i_{B4} = 0.00371 \text{ mA}}$$

**17.35**

a.  $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{2 - (0.1 + 0.8)}{R_{B1}} + i_{B3}$$

where

$$i_{B3} = \frac{(2 - 0.7) - (0.9)}{R_{B2}} = \frac{0.4}{1} \\ \Rightarrow \underline{i_{B3} = 0.4 \text{ mA}}$$

Then

$$i_{B1} = \frac{1.1}{1} + 0.4 \Rightarrow \underline{i_{B1} = 1.5 \text{ mA}} \\ \underline{i_{B2} = 0 = i_{C2}}$$

$Q_3$  in saturation  $i_{C3} = 5i'_L$  For  $v_0$  high,

$$v'_{B1} = 0.8 + 0.7 = 1.5 \text{ V} \Rightarrow Q'_3 \text{ off}$$

$$i'_{B1} = \frac{2 - 1.5}{1} = 0.5 \text{ mA}$$

$$i'_L = \beta_R i'_{B1} = (0.2)(0.5) = 0.1 \text{ mA}$$

Then

$$\underline{i_{C3} = 0.5 \text{ mA}}$$

b.  $v_X = v_Y = v_Z = 2 \text{ V}$

From part (a),

$$\Rightarrow \underline{i_{B1} = 0.5 \text{ mA}}$$

$$\underline{i_{B3} = 0 = i_{C3}}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = (0.5)(1 + 3[0.2])$$

$$\underline{i_{B2} = 0.8 \text{ mA}}$$

$$i_{C2} = 5i'_L, \text{ and from part (a), } i'_L = 1.5 \text{ mA}$$

So

$$\underline{i_{C2} = 7.5 \text{ mA}}$$

### 17.36

(a)  $I_B + I_D = \frac{5.8 - 0.7}{10} = 0.51 \text{ mA}$

$$I_C - I_D = \frac{5 - (0.7 - 0.3)}{1} = 4.6 \text{ mA}$$

Now

$$I_D = 0.51 - I_B = 0.51 - \frac{I_C}{\beta} = 0.51 - \frac{I_C}{50}$$

Then

$$I_C - I_D = I_C - \left(0.51 - \frac{I_C}{50}\right) = I_C \left(1 + \frac{1}{50}\right) - 0.51 = 4.6$$

So  $\underline{I_C = 5.01 \text{ mA}}$

$$I_B = \frac{I_C}{\beta} = \frac{5.01}{50} \Rightarrow \underline{I_B = 0.1002 \text{ mA}}$$

$$I_D = 0.51 - 0.1002 \Rightarrow \underline{I_D = 0.4098 \text{ mA}}$$

$$\underline{V_{CE} = 0.4 \text{ V}}$$

(b)  $\underline{I_D = 0, V_{CE} = V_{CE}(\text{sat}) = 0.1 \text{ V}}$

$$I_B = \frac{5.8 - 0.8}{10} \Rightarrow \underline{I_B = 0.5 \text{ mA}}$$

$$I_C = \frac{5 - 0.1}{1} \Rightarrow \underline{I_C = 4.9 \text{ mA}}$$

### 17.37

(a) (i)  $v_I = 0, v_1 = 0.3 \text{ V}$

$$i_1 = \frac{1.5 - 0.3}{1} = 1.2 \text{ mA}$$

$$i_B = i_C = 0, v_O = 1.5 \text{ V}$$

(ii)  $v_I = 1.5 \text{ V}, v_1 = 0.7 + 0.3 = 1.0 \text{ V}$

$$i_1 = \frac{1.5 - 1.0}{1} = 0.5 \text{ mA}$$

$$i_B = 0.5 - \frac{0.7}{20} = 0.465 \text{ mA}$$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}, \quad v_O = 0.4 \text{ V}$$

(b) (i)  $v_1 = 0.7 + 0.3 = 1.0 \text{ V}, \quad v_I = 0.7 \text{ V}$

$$i_B = i_C = 0$$

(ii)  $v_1 = 1.0 \text{ V}, \quad v_I = 0.7 \text{ V}$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}$$

$$i_B = \frac{i_C}{\beta} = \frac{0.9167}{25} = 0.03667 \text{ mA}$$

(c)  $i_{C0}(\text{max}) = \beta i_B = i_C + N i'_L$

$$i'_L = \frac{1.5 - (0.4 + 0.3)}{1} - \frac{0.4}{20} = 0.78 \text{ mA}$$

$$(25)(0.465) = (0.9167) + N(0.78) \Rightarrow N = 13.7 \Rightarrow N = 13$$

### 17.38

a.  $v_X = v_Y = 0.4 \text{ V}$

$$v_{B1} = 0.4 + 0.7 \Rightarrow v_{B1} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.1}{2.8} \Rightarrow i_{B1} = 1.39 \text{ mA}$$

$$v_{B2} = 0.4 + 0.4 \Rightarrow v_{B2} = 0.8 \text{ V}$$

$$i_{B2} = i_{C2} = i_{B0} = i_{C0} = i_{B5} = i_{C5} \\ = i_{B3} = i_{C3} = 0 \text{ (No load)}$$

$$5 = i_{B4} R_2 + V_{BE} + (1 + \beta) i_{B4} R_4$$

$$i_{B4} = \frac{5 - 0.7}{0.76 + (31)(3.5)} \Rightarrow i_{B4} = 0.0394 \text{ mA}$$

$$i_{C4} = \beta_F i_{B4} \Rightarrow i_{C4} = 1.18 \text{ mA}$$

$$v_{B4} = 5 - (0.0394)(0.76) \Rightarrow v_{B4} = 4.97 \text{ V}$$

b.  $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 0.7 + 0.7 + 0.3 \Rightarrow v_{B1} = 1.7 \text{ V}$$

$$v_{B2} = 1.4 \text{ V}$$

$$v_{B0} = 0.7 \text{ V}$$

$$v_{C2} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.7}{2.8} \Rightarrow i_{B1} = 1.1786 \text{ mA}$$

$$i_{B2} = i_{B1}(1 + 2\beta_R) = 1.18(1 + 2[0.1])$$

$$i_{B2} = 1.41 \text{ mA}$$

$$i_{B4} = \frac{1.1 - 0.7}{(31)(3.5)} \Rightarrow i_{B4} = 0.00369 \text{ mA}$$

$$i_{R2} = \frac{5 - 1.1}{0.76} = 5.13 \text{ mA} \Rightarrow i_{C2} \approx 5.13 \text{ mA}$$

$$i_{B0} \approx i_{B2} + i_{C2}$$

$$i_{B0} = 6.54 \text{ mA}$$

**17.39**

(a) For the load,  $i_{RB1} = \frac{2.5 - (0.4 + 0.3)}{R_{B1}} = \frac{1}{2}(0.2) \Rightarrow R_{B1} = 18 \text{ k}\Omega$

For  $v_X = v_Y = v_Z = \text{logic 1}$

$$i_{RB1} = \frac{2.5 - (0.7 + 0.8)}{18} = 0.05556 \text{ mA}$$

$$i_{RC1} = \frac{2.5 - (0.7 + 0.1)}{R_{C1}}$$

$$i_{B2} = 0.1 = 0.05556 + \frac{1.7}{R_{C1}} - \frac{0.7}{0.7} \Rightarrow R_{C1} = 1.63 \text{ k}\Omega$$

(b)  $v_X = 0.4 \text{ V}$ ,  $v_{B1} = 0.7 \text{ V}$ ,  $v_{B2} = 0$

$$v_O \cong 2.5 - 0.7 = 1.8 \text{ V}$$

$$\text{All } i_B = 0, \text{ All } i_C = 0$$

(c)  $v_{B1} = 1.5 \text{ V}$ ,  $v_{B2} = 0.7 \text{ V}$

$$i_{B1} = \frac{2.5 - 1.5}{18} = 0.0556 \text{ mA}$$

$$i_{C1} = \frac{2.5 - (0.7 + 0.1)}{1.63} = 1.043 \text{ mA}$$

$$i_{B2} = 0.10 \text{ mA}$$

$$i_{C2} = \frac{4[2.5 - (0.4 + 0.3)]}{18} = 0.40 \text{ mA}$$

$$v_O = 0.4 \text{ V}$$

(d)  $i_{C2}(\text{max}) = \beta i_{B2} = N i'_L$

$$i'_L = 0.1 \text{ mA}$$

$$(20)(0.1) = N(0.1) \Rightarrow N = 20$$

**17.40**

a. For  $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 3(0.7) = 2.1 \Rightarrow i_{B1} = \frac{5 - 2.1}{10} = 0.29 \text{ mA}$$

$$v_{C1} = 0.7 + 0.7 + 0.4 = 1.8 \text{ V} \Rightarrow i_{C1} = \frac{5 - 1.8}{10} = 0.32 \text{ mA}$$

$$i_{B2} = i_{B1} + i_{C1} - \frac{1.4}{15} = 0.29 + 0.32 - 0.0933$$

So

$$i_{B2} = 0.517 \text{ mA}$$

$$v_{C2} = 0.7 + 0.4 = 1.1 \text{ V}$$

$$i_{C2} = \frac{5 - 1.1}{4.1} = 0.951 \text{ mA}$$

$$i_{B5} = i_{B2} + i_{C2} - \frac{0.7}{4} = 0.517 + 0.951 - 0.175$$

or  $i_{B5} = 1.293 \text{ mA}$

For  $v_0 = 0.4 \text{ V}$ ,  $v'_{B1} = 0.4 + 0.7 = 1.1 \text{ V}$

Then

$$i'_{B1} = \frac{1.1 - 0.7}{(31)(15)} = 0.00086 \text{ mA}$$

$$i'_L = \frac{5 - 1.1}{10} - 0.00086 \text{ or } i'_L \approx 0.39 \text{ mA}$$

So  $i_{C5}(\text{max}) = \beta i_{B5} = N i'_L$

$$(30)(1.293) = N(0.39) \Rightarrow \underline{N = 99}$$

b.  $P = (0.29 + 0.32 + 0.951)(5) + (99)(0.39)(0.4)$

$$P = 7.805 + 15.444 \text{ or } \underline{P = 23.2 \text{ mW}}$$

(Assuming 99 load circuits which is unreasonably large.)

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### 17.41

a. Assume no load. For  $v_X = \text{logic } 0 = 0.4 \text{ V}$

$$i_{E1} = \frac{5 - (0.4 + 0.7)}{40} = 0.0975 \text{ mA}$$

Essentially all of this current goes to ground from  $V_{CC}$ .

$$P = i_{E1} \cdot V_{CC} = (0.0975)(5) \Rightarrow \underline{P = 0.4875 \text{ mW}}$$

b.  $i_{R1} = \frac{5 - (3)(0.7)}{40} = 0.0725 \text{ mA}$

$$i_{R2} = \frac{5 - (0.7 + 0.7 + 0.4)}{50} = 0.064 \text{ mA}$$

$$i_{R3} = \frac{5 - (0.7 + 0.4)}{15} = 0.26 \text{ mA}$$

$$P = (0.0725 + 0.064 + 0.26)(5)$$

$$\underline{P = 1.98 \text{ mW}}$$

c. For  $v_0 = 0$ ,  $v_{C7} = 0.7 + 0.4 = 1.1 \text{ V}$

$$i_{R7} = \frac{5 - 1.1}{0.050} \Rightarrow i_{R7} = \underline{78 \text{ mA} \approx i_{SC}}$$

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### 17.42

(a)  $i_E = \frac{3 - (0.7 + 0.3)}{2} = 1.0 \text{ mA}$

(b)  $i_E = \frac{2.4 - 0.7}{2} = 0.85 \text{ mA}$

$$R_C = \frac{3 - 2.4}{0.85} = 0.706 \text{ k}\Omega$$

(c) (i)  $P = (1.0)(3) = 3.0 \text{ mW}$

(ii)  $P = (0.85)(3) = 2.55 \text{ mW}$

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**17.43**

(a)  $v_I = v_O = 2.5 \text{ V}$ ; A transient situation

$$v_{DS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V}$$

$$v_{GS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V} \Rightarrow M_N \text{ in saturation}$$

$$v_{SD}(M_P) = 5 - (2.5 + 0.7) = 1.8 \text{ V}$$

$$v_{SG}(M_P) = 5 - 2.5 = 2.5 \text{ V} \Rightarrow M_P \text{ in saturation}$$

$$i_{DN} = K_n(v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2 \Rightarrow i_{DN} = 0.1 \text{ mA}$$

$$i_{DP} = K_p(v_{SGP} + V_{TP})^2 = (0.1)(2.5 - 0.8)^2 \Rightarrow i_{DP} = 0.289 \text{ mA}$$

$$i_{C1} = \beta i_{DP} = (50)(0.289) \Rightarrow i_{C1} = 14.45 \text{ mA}$$

$$i_{C2} = \beta i_{DN} = (50)(0.1) \Rightarrow i_{C2} = 5 \text{ mA}$$

Difference between  $i_{E1}$  and  $i_{DN} + i_{C2}$  is a load current.

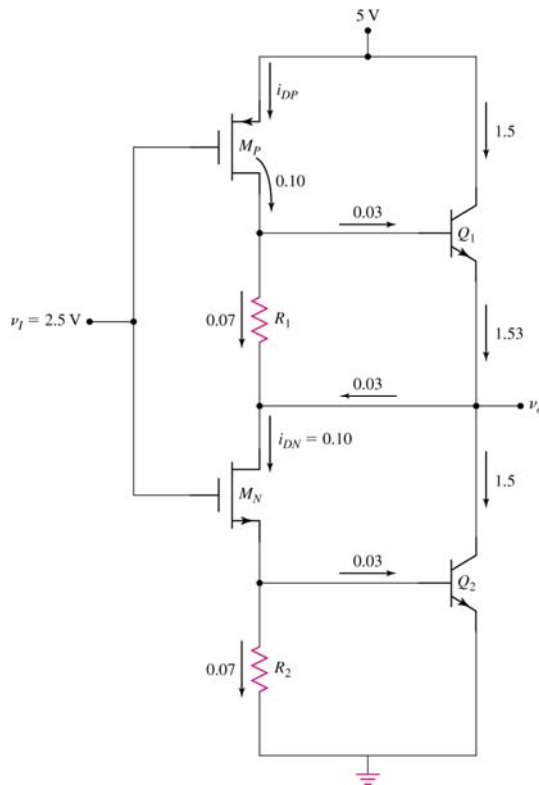
(b) Assume  $i_{C1} = 14.45 \text{ mA}$  is a constant

$$V_C = \frac{1}{C} \int i_{C1} dt = \frac{i_{C1} \cdot t}{C} \Rightarrow t = \frac{(V_C)(C)}{i_{C1}}$$

$$t = \frac{(5)(15 \times 10^{-12})}{14.45 \times 10^{-3}} \Rightarrow t = 5.19 \text{ ns}$$

(c)  $t = \frac{(5)(15 \times 10^{-12})}{0.289 \times 10^{-3}} \Rightarrow t = 260 \text{ ns}$

**17.44**





(a) Assume  $R_1 = R_2 = 10 \text{ k}\Omega$ ;  $\beta = 50$

$$\text{Then } i_{R1} = i_{R2} = \frac{0.7}{10} = 0.07 \text{ mA}$$

NMOS in saturation region;  $v_{GSN} = 2.5 - 0.7 = 1.8 \text{ V}$

$$i_{DN} = K_n (v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2$$

$$i_{DN} = 0.10 \text{ mA}$$

Then  $i_{B2} = 0.03 \Rightarrow i_{C2} = (50)(0.03) = 1.5 \text{ mA}$

$$i_{E1} = 1.53 \text{ mA} \Rightarrow i_{B1} = 0.03 \text{ mA} \Rightarrow i_{C1} = 1.5 \text{ mA}$$

So  $i_{DP} = 0.10 \text{ mA}$

Now,  $M_P$  biased in non-saturation region

$$v_{SGP} = 2.5 \text{ V}$$

$$i_{DP} = 0.10 = 0.10 [2(2.5 - 0.8)v_{SD} - v_{SD}^2]$$

$$0.10v_{SD}^2 - 0.34v_{SD} + 0.10 = 0$$

$$v_{SD} = \frac{0.34 \pm \sqrt{(0.34)^2 - 4(0.10)(0.10)}}{2(0.10)}$$

Or

$$v_{SD} = 0.325 \text{ V}$$

Then  $v_o = 5 - 0.325 - 0.7$

$$v_o = 3.975 \text{ V}$$

$$(b) \quad v = \frac{1}{C} \int i dt = \frac{i \cdot t}{C}$$

$$t = \frac{Cv}{i} = \frac{(15 \times 10^{-12})(5)}{1.53 \times 10^{-3}}$$

$$t = 49 \text{ ns}$$

(c)

$$t = \frac{Cv}{i} = \frac{(15 \times 10^{-12})(5)}{0.1 \times 10^{-3}}$$

$$t = 0.75 \mu\text{s}$$