1. (a)

$$n_i(T=300K) = 1.66\cdot 10^{15}(300)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot 10^{-23} J/K)(300K)}\right]$$
$$= 2.5\cdot 10^{13}\ cm^{-3}$$

$$n_i(T=600K) = 1.66\cdot 10^{15}(600)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot 10^{-23} J/K)(600K)}\right]$$
$$= 4.15\cdot 10^{16}\ cm^{-3}$$

Comparing these results with those in Example:

$$\frac{n_i(Ge @ 300K)}{n_i(Si @ 300K)} \approx 2315. \qquad \frac{n_i(Ge @ 600K)}{n_i(Si @ 600K)} \approx 27.$$

At higher temperature, the exponential terms approaches one, which implies that $n_i \sim T^{3/2}$, independent of bandgap energy, E_g.

(b) For any doped material, $n\cdot p = n_i^2$. Assuming at $T = 300K$,

$$p = 5\cdot 10^{16} cm^{-3}$$

$$n = [n_i(T=300K)]^2/p = \frac{(2.5\cdot 10^{13} cm^{-3})^2}{5\cdot 10^{16} cm^{-3}} = 1.25\cdot 10^{10} cm^{-3}$$

2. (a) Mobility of electrons in Si = 1350 $cm^2/V\text{-}s$
Mobility of holes in Si = 480 $cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 1.35 \cdot 10^4$ m/s

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 4.8 \cdot 10^3$ m/s

(b) Given $E = 0.1$ V/μm hole current negligible

$\mu_n = 1350$ $cm^2/V\text{-}s$ $\mu_p = 480$ $cm^2/V\text{-}s$

$$J_{tot} = 1\,mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q\mu_n n E$$

$$\therefore n = \frac{J_{tot}}{q\mu_n E} = \frac{1\,mA/\mu m^2}{(1.6\cdot 10^{-19} C)(1350\, cm^2/V\text{-}s)(0.1\, V/\mu m)}$$

$$= 4.6 \cdot 10^{17}\, cm^{-3}$$

3. Given $L = 0.1\ um$ $A = (0.05 um)^2$ $V = 1V$

$\mu_n = 1350\ cm^2/V\text{-}s$ $\mu_p = 480\ cm^2/V\text{-}s$

$n = 10^{17}\ cm^{-3}$ (assuming n-type dopant)

(a) $$n_i(T=300K) = 5.2\cdot10^{15}(300)^{3/2} \exp\left[\frac{-1.12eV}{2(1.38\cdot10^{-23} J/K)(300K)}\right]$$

$$= 1.08\cdot10^{10}\ cm^{-3}$$

$p = n_i^2/n = 1.17\cdot10^{3}\ cm^{-3}$ $E = V/L = 10\ V/um$

$$\therefore I_{tot} = A\cdot J_{tot} = A\cdot q[\mu_n n + \mu_p p]E$$

$$= A\cdot q[\mu_n n + \mu_p (n_i^2/n)]E$$

$$= (0.05um)^2(1.6\cdot10^{-19}C)\left[1350\frac{cm^2}{V\text{-}s}(10^{17}cm^{-3}) + 480\frac{cm^2}{V\text{-}s}(1.17\cdot10^{3}cm^{-3})\right]$$

$$\cdot(10\ V/um)$$

$\Rightarrow I_{tot} \approx 0.054\ mA$

(b) @ 400K : $n_i = 3.7 \cdot 10^{12}\ cm^{-3}$

$p = n_i^2/n = 1.4 \cdot 10^{8}\ cm^{-3}$

$E = 10\ V/um$

$$\therefore I_{tot} = A \cdot q\left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\,um)^2 (1.6 \cdot 10^{-19} C)\left[1350\ \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 480\ \frac{cm^2}{V\text{-}s}(1.4 \cdot 10^{8} cm^{-3})\right] \cdot (10\ V/um)$$

$$\Rightarrow I_{tot} \approx 0.054\ mA.$$

4. Given $L = 0.1\ um$ $A = (0.05 um)^2$ $V = 1V$

$\mu_n = 3900\ cm^2/V\text{-}s$ $\mu_p = 1900\ cm^2/V\text{-}s$

$n = 10^{17} cm^{-3}$ (assuming n-type dopant)

(a) From previous problem,

@ 300k: $n_i = 2.5 \cdot 10^{13}\ cm^{-3}$ $p = n_i^2/n = 6.3 \cdot 10^9 cm^{-3}$

$E = 10\ V/um$

$$I_{tot} = A \cdot J_{tot} = A\, q\left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05 um)^2 (1.6 \cdot 10^{-19} C)\left[3900 \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s}(6.3 \cdot 10^9 cm^{-3})\right] \cdot (10\ V/um)$$

$\Rightarrow I_{tot} \approx 62.4\ mA$

(b) @ 400 k: $n_i = 2.9 \cdot 10^{15} cm^{-3}$ $p = 8.5 \cdot 10^{13} cm^{-3}$

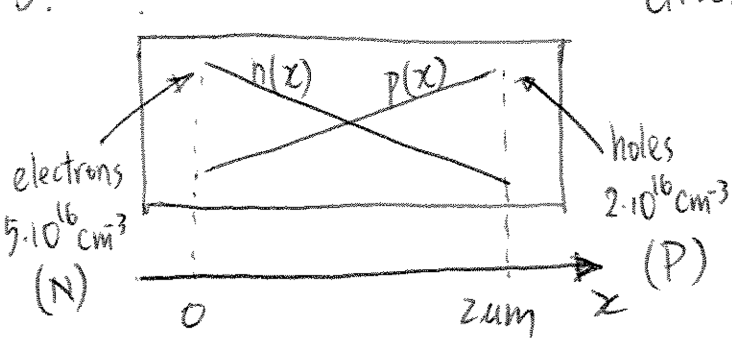
$E = 10 V/um$

$$I_{tot} = A\, q\left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05um)^2 (1.6 \cdot 10^{-19} C)\left[3900 \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s}(8.5 \cdot 10^{13} cm^{-3})\right] \cdot (10\ V/um)$$

$\Rightarrow I_{tot} \approx 62.4\ mA$

5.



Given $D_n = 34\ cm^2/s$
$D_p = 12\ cm^2/s$
$L = 2\ \mu m$
$A = (1\ \mu m)^2$

The injected carriers diffuse from one end to the other.

$$I_{tot} = A \cdot J_{tot} = A \cdot q\left[\frac{dn}{dx}D_n - \frac{dp}{dx}D_p\right]$$

$$= A \cdot q\left[D_n\left(-\frac{N}{L}\right) - D_p\left(\frac{P}{L}\right)\right]$$

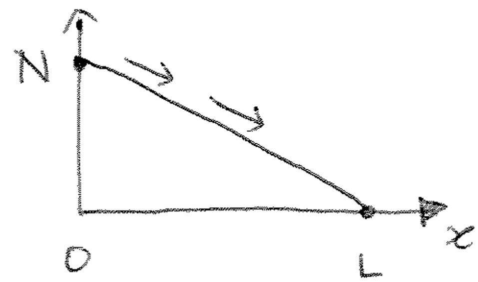
$$= (1\ \mu m)^2(1.6\cdot 10^{-19} C)\left[34\frac{cm^2}{s}\left(-\frac{5\cdot 10^{16} cm^{-3}}{2\ \mu m}\right) - 12\frac{cm^2}{s}\left(\frac{2\cdot 10^{16} cm^{-3}}{2\ \mu m}\right)\right]$$

$$= -15.5\ \mu A.$$

6. Given Area = a

find total e ectrons stored.

$$n(x) = -\frac{N}{L}x + N$$



∴ total electrons stored

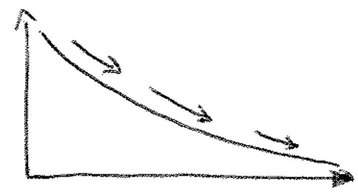
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Bigg|_0^L = \frac{aNL}{2}$$

7. Given Area = a

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



∴ total electrons stored

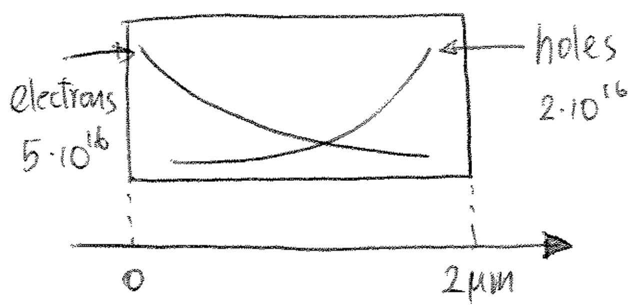
$$= \int a\, n(x)\, dx = \int_0^{\infty} a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN \left(-L_d \cdot \exp\frac{-x}{L_d}\right)\Bigg|_0^{\infty} = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

8.



$$n(x) = N \exp(-x/L_d)$$

$$p(x) = P \exp\left(\frac{x-2}{L_d'}\right)$$

$N = 5\cdot10^{16}\,cm^{-3}$ $\quad P = 2\cdot10^{16}\,cm^{-3}$

total number of electrons $= \int a\cdot n\, dx$

$$= \int_0^2 a\cdot n(x)\, dx = aN\left(-L_d\cdot \exp(-x/L_d)\right|_0^2$$

$$= aNL_d\left[1-\exp(-2/L_d)\right]$$

total number of holes $= \int a\cdot p\, dx$

$$= \int_0^2 a\cdot p(x)\, dx = aP\left(L_d'\cdot \exp\left(\frac{x-2}{L_d'}\right)\right|_0^2$$

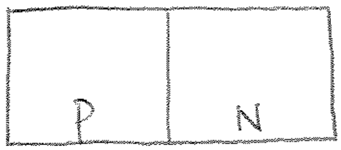
$$= aPL_d'\left[1-\exp\left(-2/L_d'\right)\right]$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

DRIFT		WATER FLOW
electrons	$\longleftrightarrow$	water
electric field	$\longleftrightarrow$	gravitational field.
drift/current	$\longleftrightarrow$	water flow

10. (a)



Assume Si.

$N_A = 4\cdot 10^{16}\,cm^{-3}$ $\quad N_D = 5\cdot 10^{17}\,cm^{-3}$

$$p_p \approx N_A = 4\cdot 10^{16}\,cm^{-3}$$
$$n_p = \frac{n_i^2}{p_p} = \frac{(1.08\cdot 10^{10}\,cm^{-3})^2}{4\cdot 10^{16}\,cm^{-3}} \approx 2.9\cdot 10^{3}\,cm^{-3}$$

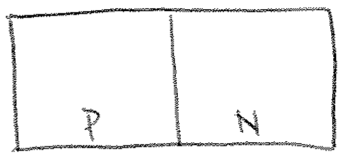
$$n_n \approx N_D = 5\cdot 10^{17}\,cm^{-3}$$
$$p_n = \frac{n_i^2}{n_n} = \frac{(1.08\cdot 10^{10}\,cm^{-3})^2}{5\cdot 10^{17}\,cm^{-3}} \approx 2.3\cdot 10^{2}\,cm^{-3}$$

(b) $V_0 = \frac{kT}{q} \ln\left(\frac{N_A \cdot N_D}{n_i^2}\right)$

@ 250K : $V_0 = 0.905\,V$
@ 300K : $V_0 = 0.848\,V$
@ 350K : $V_0 = 0.789\,V$

Towards higher temperatures, $V_0 \sim T \ln\left(\frac{1}{T^3}\right)$.
That is, overall, V_0 drops with higher T.

11. Given $N_D = 3 \cdot 10^{16}\ cm^{-3}$ $n_i = 1.08 \cdot 10^{10}\ cm^{-3}$



find V_0.

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

$$= \frac{(1.38 \cdot 10^{-23}\ J/K)(300K)}{1.6 \cdot 10^{-19} C} \ln\left(\frac{3 \cdot 10^{16}\ cm^{-3}}{1.08 \cdot 10^{10}\ cm^{-3}}\right)$$

$$= 0.384\ V$$

12. Given $N_D = 3\cdot 10^{16}\ cm^{-3}$ $N_A = 2\cdot 10^{15}\ cm^{-3}$

$V_R = 1.6\ V$ $\epsilon_{si} = 11.7\times 8.85\cdot 10^{-14}\ \frac{F}{cm^2}$

(a) $n_i = 1.08\cdot 10^{10}\ cm^{-3}$

$$V_0 = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \approx (26mV)\ln\left[\frac{3\cdot 10^{16}\times 2\cdot 10^{15}}{(1.08\cdot 10^{10})^2}\right]$$

$$= 0.698\ V$$

$$C_{j0} = \sqrt{\frac{\epsilon_{si}\cdot q}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}$$

$$= \left[\frac{11.7\times 8.85\cdot 10^{-14}\times q}{2}\cdot\frac{3\times 10^{16}\times 2\times 10^{15}}{3\times 10^{16}+2\times 10^{15}}\cdot\frac{1}{V_0}\right]^{\frac{1}{2}}$$

$$= 0.149\ fF/\mu m^2$$

$$\therefore\ C_j(V_R) = [1+1.6/V_0]^{-\frac{1}{2}}\times C_{j0} = 0.082\ fF/\mu m^2$$

(b) Given $C_{j,new} = 2\cdot C_{j,old}$

$$\Rightarrow \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}{1+\frac{V_R}{V_0}}} = \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D'}{N_A+N_D'}\cdot\frac{1}{V_0'}}{1+\frac{V_R}{V_0'}}}\times 2$$

Squaring both sides & simplifying gives:

$$\frac{\left(\frac{N_D}{N_A+N_D}\right)}{V_0+V_R} = 4 \cdot \frac{\left(\frac{N_D'}{N_A+N_D'}\right)}{V_0'+V_R}$$, where N_D' = old value.

Here, there is only one variable, N_D (new value). The solution can be found iteratively by solving this equation. But we can make an assumption that $V_0 + V_R \approx V_0' + V_R$ since $V_R = 1.6$ V, the dominant term. Then we verify V_0 & V_0' afterwards.

$$\Rightarrow \frac{N_D}{N_A+N_D} = 4 \frac{N_D'}{N_A+N_D'}$$

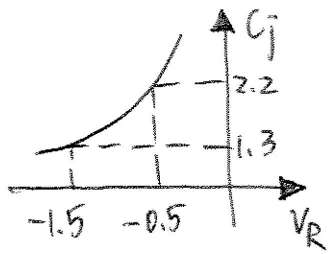
$$\Rightarrow N_D = \frac{4N_D' N_A}{N_A - 3N_D'} = \frac{4(2\cdot 10^{15})(3\cdot 10^{16})}{(3\cdot 10^{16}) - 3\cdot(2\cdot 10^{15})} \approx 1.00 \cdot 10^{16}\, cm^{-3}$$

$$\Rightarrow \frac{N_D}{N_D'} = \frac{1\cdot 10^{16}}{2\cdot 10^{15}} \approx 5$$

Verify: $V_{0,old} = 0.698\,V \Rightarrow V_0 + V_R \approx 2.3\,V$
$V_{0,new} = 0.740\,V \Rightarrow V_0 + V_R \approx 2.3\,V$ (✓)

∴ Increase N_D by 5 times.

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \ ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \ ②$$

① ÷ ② : $\frac{1+1.5/V_0}{1+0.5/V_0} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$$

$$\approx 3.71 \cdot 10^{17}\,cm^{-3}$$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

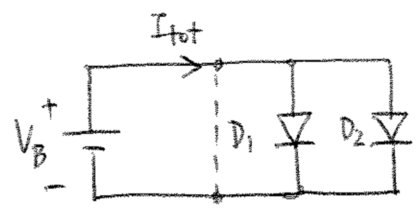
(b) Since $I_S \propto$ Area, doubling area implies doubling I_S. From (a),

$$I_D = 1\ mA = 2 \times I_S e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\ V$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T} - 1) + I_{S_2}(e^{V_B/V_T} - 1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T} - 1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

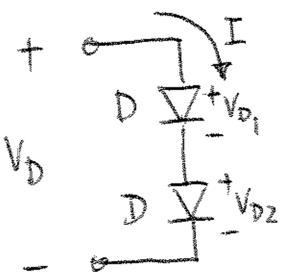
Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

16. (a)



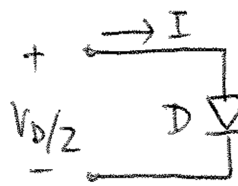
Suppose $I = I_s\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$

$I_{D_2} = I_s\left(e^{\frac{V_{D2}}{V_T}} - 1\right)$

By KCL, $I_{D_1} = I_{D_2} = I$

$$\Rightarrow \left(e^{V_{D1}/V_T} - 1\right) = \left(e^{V_{D2}/V_T} - 1\right) \Rightarrow V_{D_1} = V_{D2} = \frac{V_D}{2}$$

$$\therefore\ I = I_s\left(e^{(V_D/2)/V_T} - 1\right)$$



Therefore, a series combination can be viewed as a single two-terminal device with exponential characteristics.

(b) Suppose V_i = initial V_D. Need 10x increase in I.

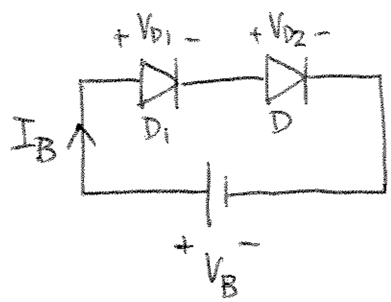
V_f = final V_D

$$\Rightarrow 10 = \frac{I_s\left(e^{V_f/V_T} - 1\right)}{I_s\left(e^{V_i/V_T} - 1\right)} \approx e^{\frac{V_f - V_i}{V_T}}$$

$$\therefore\ \Delta V = V_f - V_i = V_T \ln(10) = (26\,mV)\ln(10)$$

$$\approx 60.\ mV.$$

17.



Find I_B, V_{D_1}, V_{D2} in terms of V_B, $I_{,,}$, I_{S2}

By KVL, $V_B = V_{D_1} + V_{D2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S2}}\right)$$

$$\therefore \; I_B = \sqrt{I_{S_1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

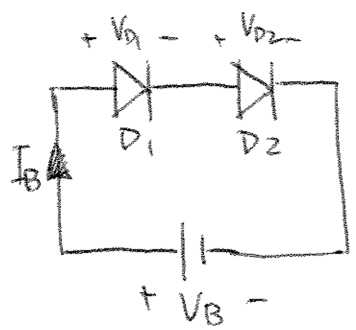
$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S_1}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln\sqrt{\frac{I_{S2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D2} = V_T \ln\left(\frac{I_B}{I_{S2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln\sqrt{\frac{I_{S1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase I_B by 10 times:

$$I_{B,new} = 10\, I_B$$

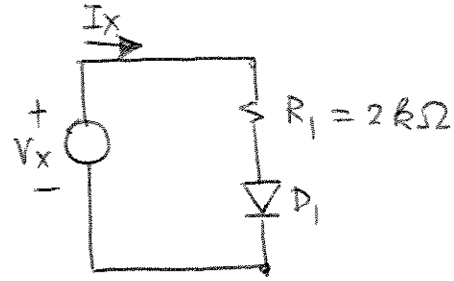
$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\text{ V}$$

$\therefore$ V_B increases by 0.120 V.

19.



$$I_{D_1} = I_S\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

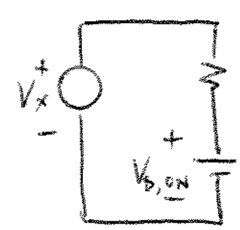
$$I_S = 2 \cdot 10^{-15} A$$

By KVL,
$$V_X = I_X R_1 + V_{D1}$$
$$= I_X R_1 + V_T \ln(I_{D1}/I_S)$$
$$= I_X R_1 + V_T \ln(I_X/I_S)$$

This can be solved directly with special programs or graphing calculators. But this can be solved iteratively, by hand.

$V_X = 0.5V$

We suppose that D_1 is on.
$\Rightarrow$ current flows through D_1.
Assume a $V_{D,ON}$:



$\Rightarrow V_{D_1} = 0.4V$

$$\Rightarrow I_X = \frac{V_X - V_{D1}}{R_1} = \frac{(0.5-0.4)V}{2k\Omega} = 0.05\,mA$$

$$V_{D_1} = V_T \ln\left(\frac{I_X}{I_S}\right) = (0.026V)\ln\left(\frac{0.05mA}{2 \cdot 10^{-15}A}\right) \approx 0.62\,V$$

$\therefore$ Contradiction because V_{D_1} exceeds V_X !!
This means our assumption is incorrect
$\Rightarrow D_1$ is OFF $\Rightarrow V_{D_1} = V_X = 0.5V \quad I_X = 0$

$\boxed{V_X = 0.8\text{ V}}$ Suppose D_1 is on. (This is a reasonable assumption since most diodes turn on at around $V_D = 0.7$ V.)

For startup, use $V_{D_1} = 0.7$ V.

$V_{D_1} = 0.7\text{ V} \Rightarrow I_X = \dfrac{V_X - V_{D_1}}{R_1} = 0.05\text{ mA}$

$\Rightarrow V_{D_1} = V_T \ln(I_X/I_{S_1}) \approx 0.622\text{ V}$

$V_{D_1} = 0.622\text{ V} \Rightarrow I_X = \dfrac{(0.8 - 0.622)\text{V}}{2\text{ k}\Omega} = 0.089\text{ mA}$

$\Rightarrow V_{D_1} = (0.026\text{ V})\ln\left(\dfrac{0.089\text{ mA}}{2\cdot 10^{-15}\text{ A}}\right) \approx 0.637\text{V}$

$V_{D_1} = 0.637\text{ V} \Rightarrow I_X = \dfrac{(0.8 - 0.637)\text{V}}{2\text{ k}\Omega} = 0.082\text{ mA}$

$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\dfrac{0.082\text{ mA}}{2\cdot 10^{-15}\text{ A}}\right) \approx 0.635\text{V}$

$V_{D_1} = 0.635\text{ V} \Rightarrow I_X = \dfrac{(0.8 - 0.635)\text{V}}{2\text{ k}\Omega} = 0.083\text{ mA}$

$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\dfrac{0.083\text{ mA}}{2\cdot 10^{-15}\text{ A}}\right) \approx 0.635\text{V}$

$\therefore$ With an accuracy of three decimal points,

$V_{D_1} \approx 0.635$ V

$I_X \approx 0.082$ mA.

(Of course, more iterations give a more accurate result.)

$V_X = 1\ V$ Suppose, again, that D_1 is on. Use V_{D_1} from previous calculations as starting point.

$$V_{D_1} = 0.635\ V \Rightarrow I_X = \frac{(1-0.635)V}{2\ k\Omega} = 0.18\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.18mA}{2\cdot 10^{-15}A}\right) \approx 0.656\ V$$

$$V_{D_1} = 0.656V \Rightarrow I_X = \frac{(1-0.656)V}{2\ k\Omega} = 0.17\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.17mA}{2\cdot 10^{-15}A}\right) \approx 0.655\ V$$

$$V_{D_1} = 0.655\ V \Rightarrow I_X = \frac{(1-0.655)V}{2\ k\Omega} = 0.17\ mA$$

$$\Rightarrow V_{D_1} = 0.655\ V$$

$\therefore$ $V_{D_1} \approx 0.655\ V$
$I_X \approx 0.17\ mA.$

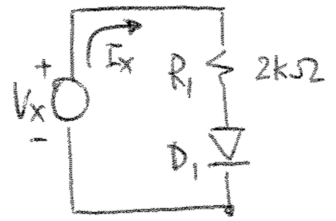
$V_X = 1.2\ V$ Using similar assumptions as those in previous calculations,

$V_{D_1} = 0.655\ V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.667V$
$V_{D_1} = 0.667\ V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.666V$
$V_{D_1} = 0.666V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.666V$

$\therefore$ $I_X \approx 0.27\ mA$ $\quad V_{D_1} = 0.666\ V$

For more than 3x increase in I_x, V_{D_1} only increases by ~30 mV, which is less than 10% of the turn-on voltage of the diode. In other words, once the diode conducts current, its voltage varies marginally (expected due to its exponential characteristic). This also implies that the diode, once on, can allow any amount of current to flow through (until $V_{D_1} \times I_{D_1}$ becomes so large that the diode simply "breaks down".)

20.



Since $I_{S_1} \propto$ Area, I_{D_1} becomes:

$$I_{D_1} = \underbrace{10 \times (2 \cdot 10^{-15} A)}_{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

$V_X = 0.8$ V Suppose D_1 is on. Assume $V_{D_1} = 0.7V$

$$V_{D_1} = 0.7 V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{0.1 V}{2 k\Omega} = 0.05 \ mA$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X / I_{S_1}') = (0.026 V) \ln\left(\frac{0.05 mA}{20 \cdot 10^{-15} A}\right)$$

$$= 0.563 V$$

$$V_{D_1} = 0.563V \Rightarrow I_X = \frac{(0.8 - 0.563) V}{2 k\Omega} = 0.12 \ mA$$

$$\Rightarrow V_{D_1} = (0.026 V) \ln\left(\frac{0.12 mA}{20 \cdot 10^{-15} A}\right) \approx 0.585 V$$

$$V_{D_1} = 0.585 V \Rightarrow I_X = \frac{(0.8 - 0.585) V}{2 k\Omega} = 0.11 \ mA$$

$$\Rightarrow V_{D_1} = (0.026 V) \ln\left(\frac{0.11 mA}{20 \cdot 10^{-15} A}\right) \approx 0.583 V$$

$$V_{D_1} = 0.583 V \Rightarrow I_X = \frac{(0.8 - 0.583) V}{2 k\Omega} = 0.11 \ mA$$

$$\Rightarrow V_{D_1} = 0.583 V$$

$\therefore$ $V_{D_1} \approx 0.583$ V

$I_X \approx 0.11$ mA.

$\boxed{V_X = 1.2\text{ V}}$ Suppose D_1 is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.583)\text{V}}{2\text{ k}\Omega} = 0.31\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.31\text{mA}}{20\times10^{-15}\text{A}}\right) \approx 0.610\text{ V}$$

$$V_{D_1} = 0.610\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.610)\text{V}}{2\text{ k}\Omega} = 0.30\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.30\text{ mA}}{20\cdot10^{-15}\text{A}}\right) \approx 0.609\text{ V}$$

$$V_{D_1} = 0.609\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.609)\text{V}}{2\text{ k}\Omega} = 0.30\text{ mA}$$

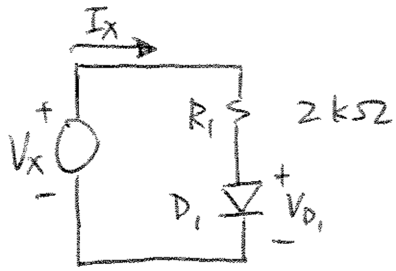
$$\Rightarrow V_{D_1} = 0.609\text{ V}$$

$\therefore V_{D_1} \approx 0.609\text{ V}$

$I_X \approx 0.30\text{ mA}$.

By increasing the cross-section area of D_1, intuitively this means D_1 can conduct same amount of current with less V_{D_1}. The results have shown that in this problem, V_{D_1} is less and I_X is more.

21.



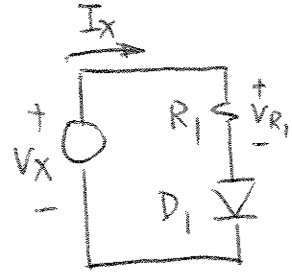
Given: @ $V_X = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.58\ mA$$

$$\therefore I_S = \frac{I_X}{(e^{V_D/V_T} - 1)} \approx I_X \exp[-V_D/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

22.



Given $V_{R_1} = V_X/2$, find V_X.
$I_S = 2 \cdot 10^{-16} A$.

By KCL,

$$\frac{V_{R_1}}{R_1} = I_S\left(e^{V_{D_1}/V_T} - 1\right)$$

Also, $V_{R_1} = V_{D_1} = V_X/2$ (KVL).

$$\therefore \frac{V_X/2}{R_1} = I_S \cdot \left(\exp\left[\frac{V_{D_1}/2}{V_T}\right] - 1\right)$$

This must be solved iteratively. From experience, suppose $V_X = 2V$.

$$V_X = 2V \Rightarrow I_X = \frac{V_X/2}{R_1} = \frac{1V}{2k\Omega} = 5mA$$

$$\Rightarrow V_X = 2 \cdot V_{D_1} = 2V_T \ln(I_X/I_S)$$

$$= 2(0.026V)\ln\left(\frac{5mA}{2 \cdot 10^{-16}A}\right) \approx 1.48 V$$

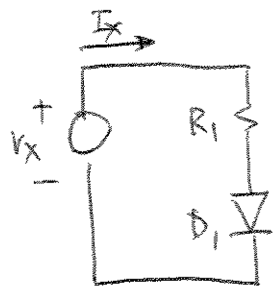
$$V_X = 1.48V \Rightarrow I_X = \frac{1.48/2\,V}{2k\Omega} = 0.37\,mA$$

$$\Rightarrow V_X = 2(0.026V)\ln\left(\frac{0.37mA}{2 \cdot 10^{-16}A}\right) \approx 1.47 V$$

$$V_X = 1.47V \Rightarrow I_X = \frac{(1.47)/2\,V}{2k\Omega} = 0.37\,mA$$

$$\Rightarrow V_X = 1.47 V$$

23.



Given $V_X = 1V \Rightarrow I_X = 0.2mA$

$V_X = 2V \Rightarrow I_X = 0.5mA$

Find R_1 and I_S.

By KVL, $V_{D_1} = V_X - I_X R_1 = V_T \ln\left(\frac{I_X}{I_S}\right)$

$$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_S}\right) \quad \text{—— ①}$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_S}\right) \quad \text{—— ②}$$

② − ① : $1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$

$$\Rightarrow R_1 = \frac{1 - (0.026)\ V}{0.3m\ \ A} = 3.25\ k\Omega$$

Substitute R_1 into ①:

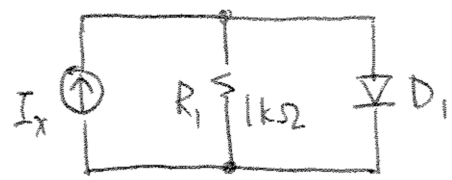
$$I_S = I_X \cdot \exp\left[-\frac{V_X - I_X R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore R_1 \approx 3.25\ k\Omega$

$I_S \approx 2.94 \cdot 10^{-10} A.$

24.



Given $I_S = 3 \cdot 10^{-16} A$, find V_{D_1}.

By KCL, $I_X = \frac{V_{D1}}{R_1} + I_{D_1} = \frac{V_T}{R_1} \ln\left(\frac{I_{D1}}{I_S}\right) + I_{D_1}$

Since I_X, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7\ V$ as starting point.

$I_X = 1mA$

$V_{D_1} = 0.7\ V \Rightarrow I_{D_1} = I_X - V_{D_1}/R_1 = 1mA - \frac{0.7V}{1k\Omega} = 0.3mA$

$$\Rightarrow V_{D_1} = V_T \ln\left(I_X/I_S\right) = (0.026V)\ln\left(\frac{0.3mA}{3 \cdot 10^{-16}A}\right) \approx 0.718\ V$$

$V_{D_1} = 0.718V \Rightarrow I_{D_1} = 1\ mA - \frac{0.718\ V}{1\ k\Omega} = 0.28\ mA$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.28mA}{3 \cdot 10^{-16}A}\right) \approx 0.717\ V$$

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \frac{0.717V}{1k\Omega} = 0.28mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \frac{0.717V}{1k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{1.28mA}{3\cdot 10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \frac{0.756V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{1.24mA}{3\cdot 10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \frac{0.755V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755V$

$\boxed{I_X = 4\,mA}$ Assume $V_{D_1} = 0.755V$ from previous result.

$$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 4mA - \frac{0.755V}{1k\Omega} = 3.25\,mA$$

$$\Rightarrow V_{D_1} = (0.026)V \ln\left(\frac{3.25mA}{3\cdot 10^{-16}A}\right) \approx 0.780\,V$$

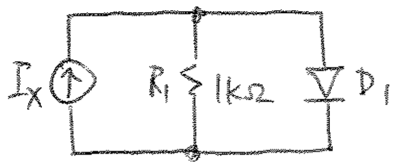
$$V_{D_1} = 0.780\,V \Rightarrow I_{D_1} = 4mA - \frac{0.780V}{1k\Omega} = 3.22\,mA$$

$$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{3.22mA}{3\cdot 10^{-16}A}\right) \approx 0.780V$$

$\therefore\ V_{D_1} \approx 0.780\,V.$

Note: As I_X increases, I_{D_1} increases, while (V_{D_1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

25.



Given $I_{D_1} = 0.5\,mA$ when $I_X = 1.3\,mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1}) R_1$

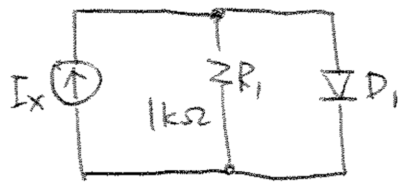
$= (0.8\,mA)\,1\,k\Omega = 0.8\,V$

$$\Rightarrow I_S = I_{D_1} \cdot \exp\left[-V_{D_1}/V_T\right]$$

$$= (0.5\,mA) \exp\left[-0.8\,V / 0.026\,V\right]$$

$$\approx 2.17 \cdot 10^{-17}\,A$$

2b



Given $I_{R_1} = I_x/2$

$I_S = 3 \cdot 10^{-16}$ A

find I_x.

$$V_{D_1} = \frac{I_x}{2} \cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8$ V.

$$V_D = 0.8\text{ V} \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\text{ V}}{1\text{ k}\Omega} = 0.8\text{ mA}$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026\text{V})\ln\left(\frac{0.8\text{ mA}}{3 \cdot 10^{-16}\text{A}}\right)$$

$$\approx 0.744\text{ V}$$

$$V_D = 0.744\text{ V} \Rightarrow I_x/2 = \frac{0.744\text{V}}{1\text{k}\Omega} = 0.744\text{ mA}$$

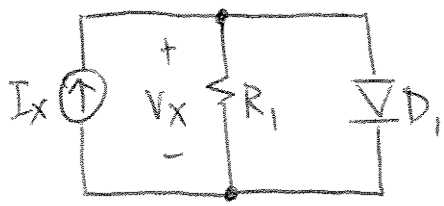
$$\Rightarrow V_D = (0.026\text{V})\ln\left(\frac{0.744\text{ mA}}{3 \cdot 10^{-16}\text{A}}\right) \approx 0.742\text{ V}$$

$V_D = 0.742V \Rightarrow I_x/2 = \dfrac{0.742V}{1k\Omega} = 0.742\ mA$

$\Rightarrow V_D = (0.026V)\ln\left(\dfrac{0.742mA}{3\cdot 10^{-16}A}\right) \approx 0.742V$

$\therefore I_x = 2(0.742mA) = 1.48\ mA$

27.



Given $I_X = 1mA \rightarrow V_X = 1.2V$

$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$I_{D_1} = I_X - V_X/R_1$ (KCL)

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\ V) = (0.026\ V) \ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \quad \text{——— ①}$$

$$(1.8\ V) = (0.026\ V) \ln\left[\frac{(2mA) - (1.8\ V)/R_1}{I_S}\right] \quad \text{——— ②}$$

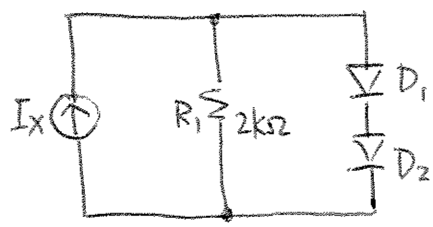
$$② - ①:\ 0.6V = (0.026V) \ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp[0.6/0.026] - 1.8}{1\,mA \cdot \exp[0.6/0.026] - 2\,mA} \approx 1.2\ k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right) \exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\ A.$$

28.



Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$ $= I_X - \frac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7\,V$. Assume $V_{R_1} = 1.4\,V$.

$$V_{R_1} = 1.4\,V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V)\ln\left(\frac{1.3mA}{5 \times 10^{-16} A}\right) \approx 1.49\,V$$

$$V_{R_1} = 1.49\,V \;\Rightarrow\; I_D = 2\,mA - \frac{1.49}{2\,k\Omega} = 1.26\,mA$$

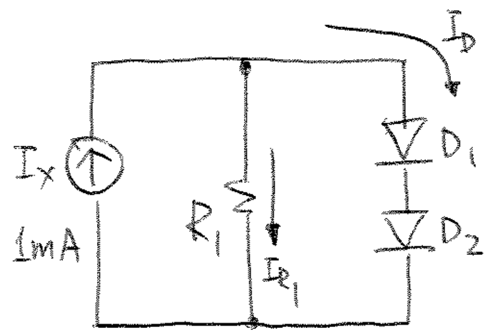
$$\Rightarrow V_{R_1} = 2\,(0.026\,V)\ln\left(\frac{1.26\,mA}{5\cdot 10^{-16}\,A}\right) \approx 1.48\,V$$

$$V_{R_1} = 1.48\,V \;\Rightarrow\; I_D = 2\,mA - \frac{1.48\,V}{2\,k\Omega} = 1.26\,mA$$

$$\Rightarrow V_{R_1} = 1.48\,V$$

$\therefore$ voltage across $R_1 = 1.48\,V$

29.



Given $I_{R_1} = 0.5\,mA$, $I_S = 5\cdot10^{-16}A$ for each diode.

Find R_1.

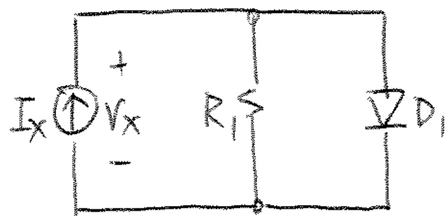
By KCL, $I_D = I_X - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5\cdot10^{-16}A}\right)$$

$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5mA} = 2.87\,k\Omega$$

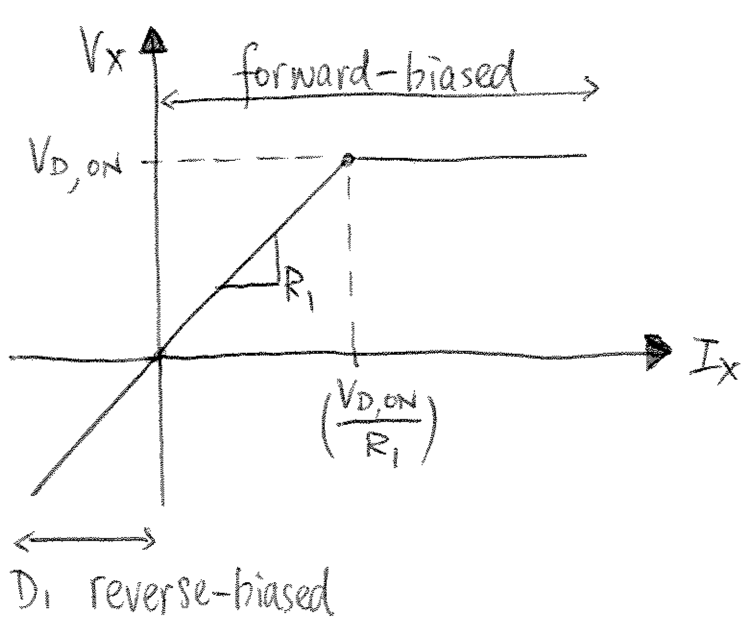
30.



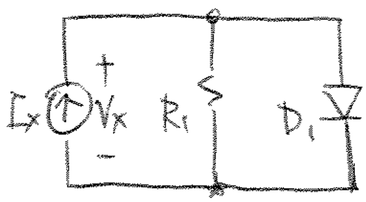
(a) Constant-voltage model:
Consider, first, the extreme cases: when D_1 is off, we have the following:

This implies V_X is linearly proportional to I_X

When D_1 is on, V_X is <u>fixed</u> (by KVL) by D_1 ($= V_{D,ON}$). This implies that any additional current from I_X cannot flow through R_1, which means D_1 will absorb all the currents to satisfy KVL.



(b) exponential model :



Assume I_S negligible.

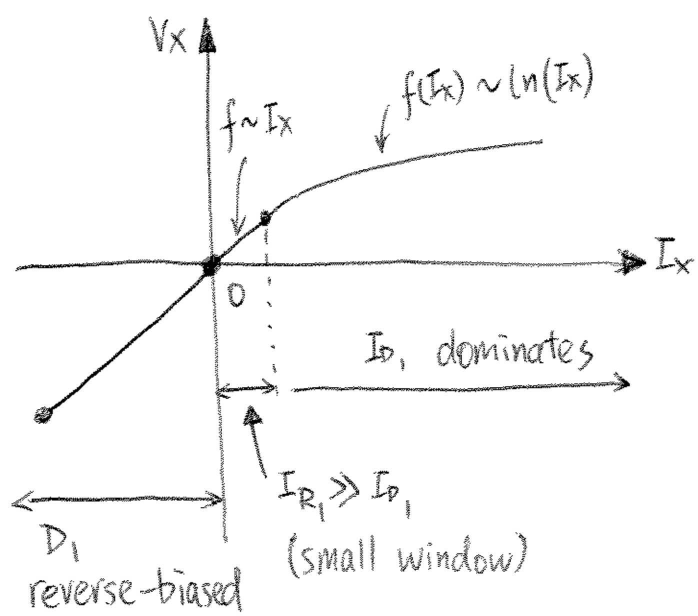
When D_1 is off, most of I_X flows through R_1. When D_1 is on, V_{D_1} ($= V_X$) follows this relationship:

$$V_{D_1} = V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X}{I_S} - \frac{V_X}{I_X R_1}\right)$$

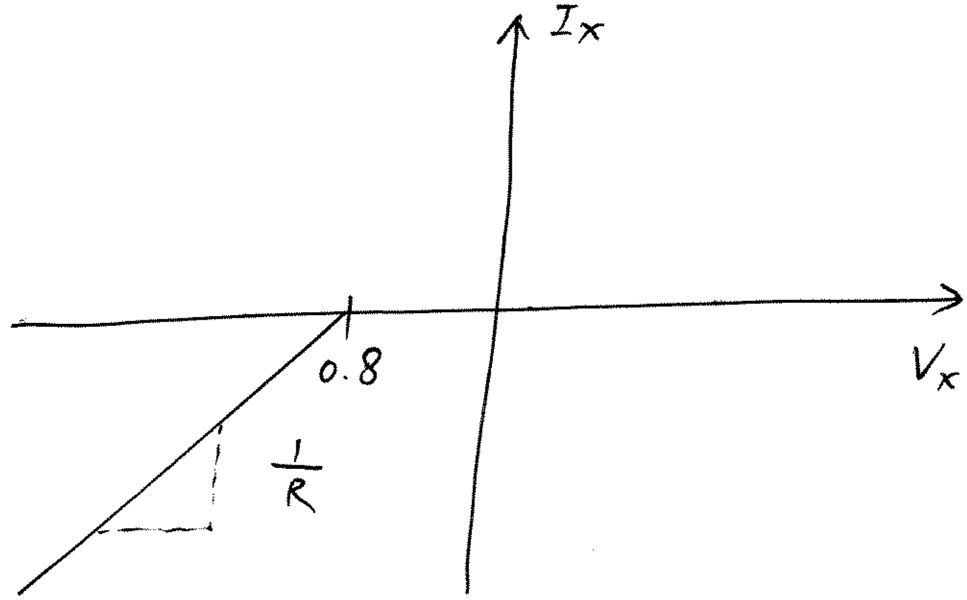
$$\Rightarrow I_X = I_S \exp(V_X/V_T) + V_X/R_1$$

$\approx I_S \exp(V_X/V_T)$ when D_1 is forward-biased ($V_X > V_T$)

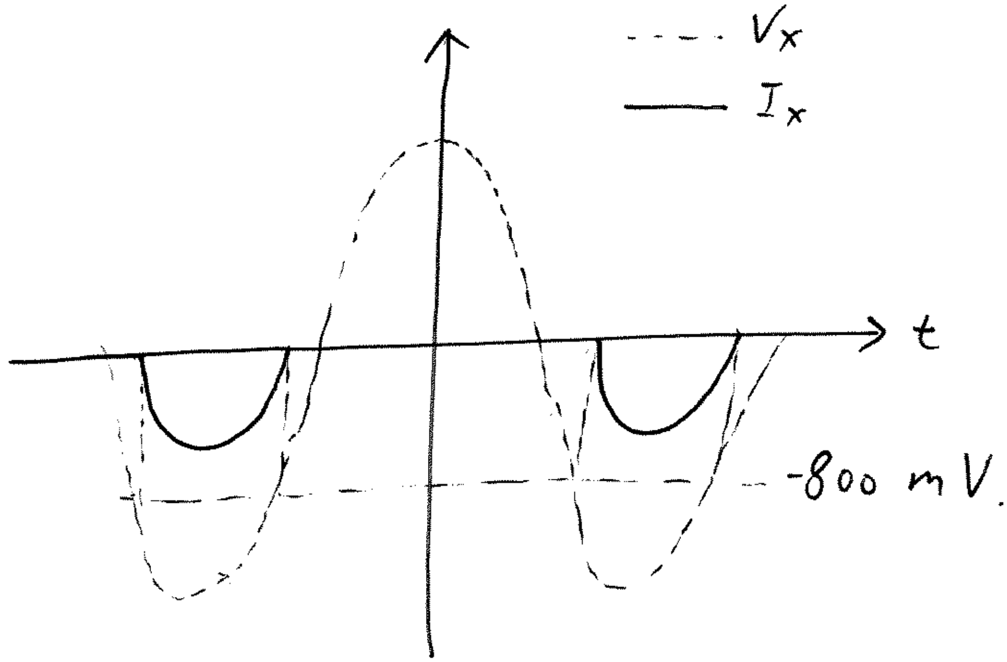
i.e. $V_X \approx V_T \ln(I_X/I_S)$



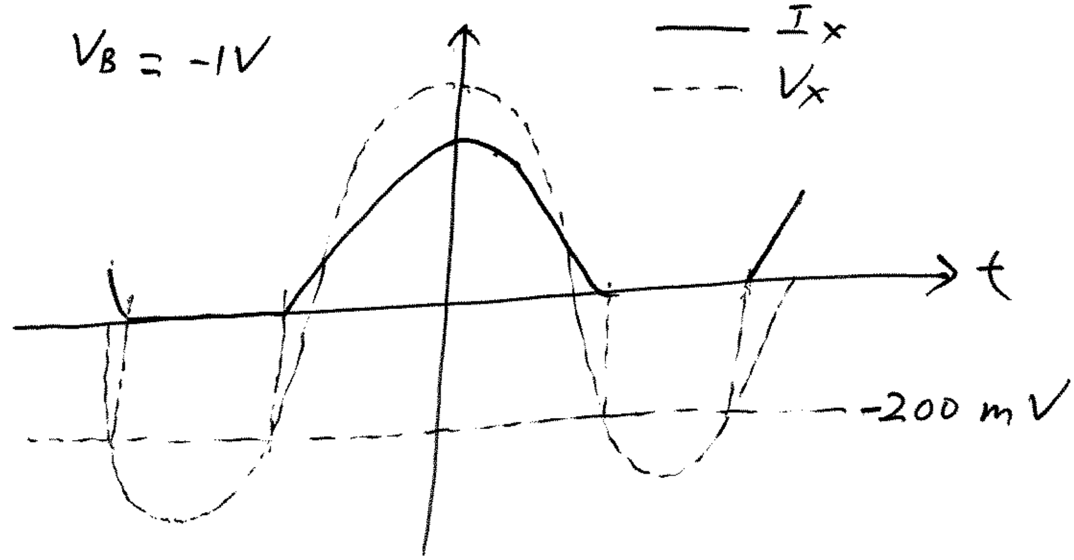
①

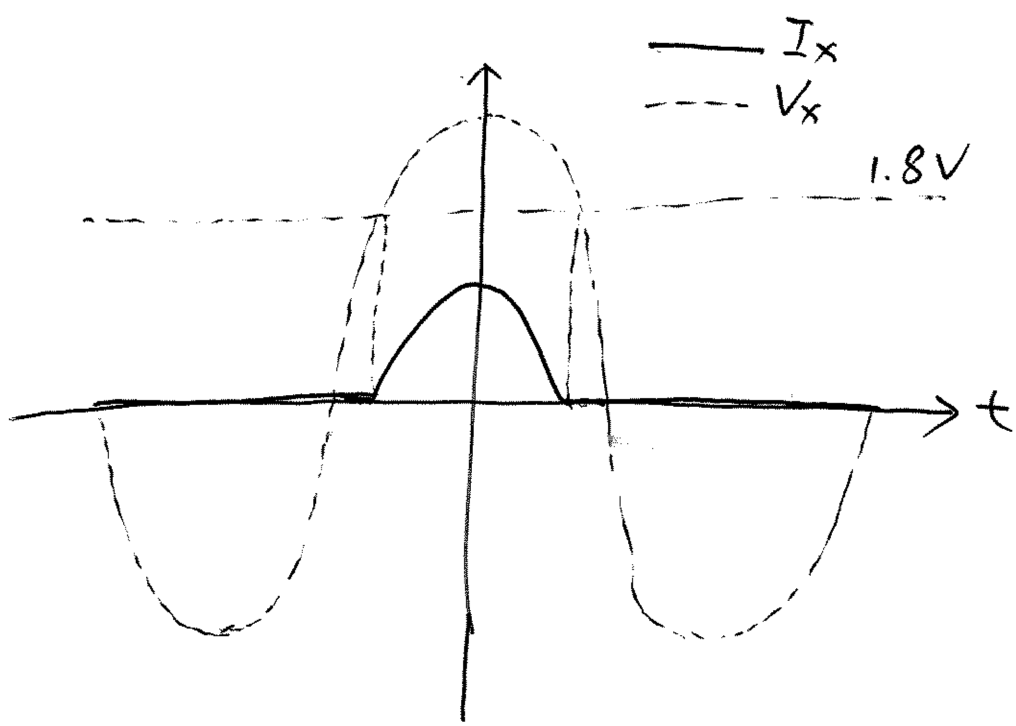


(2)

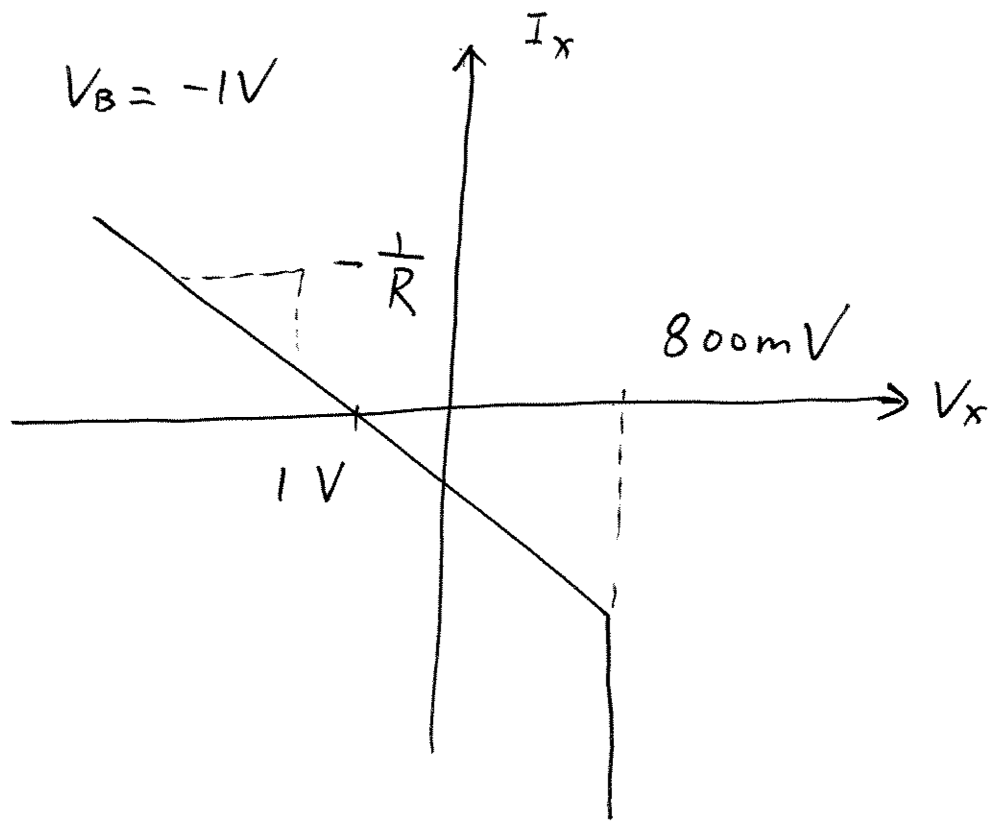


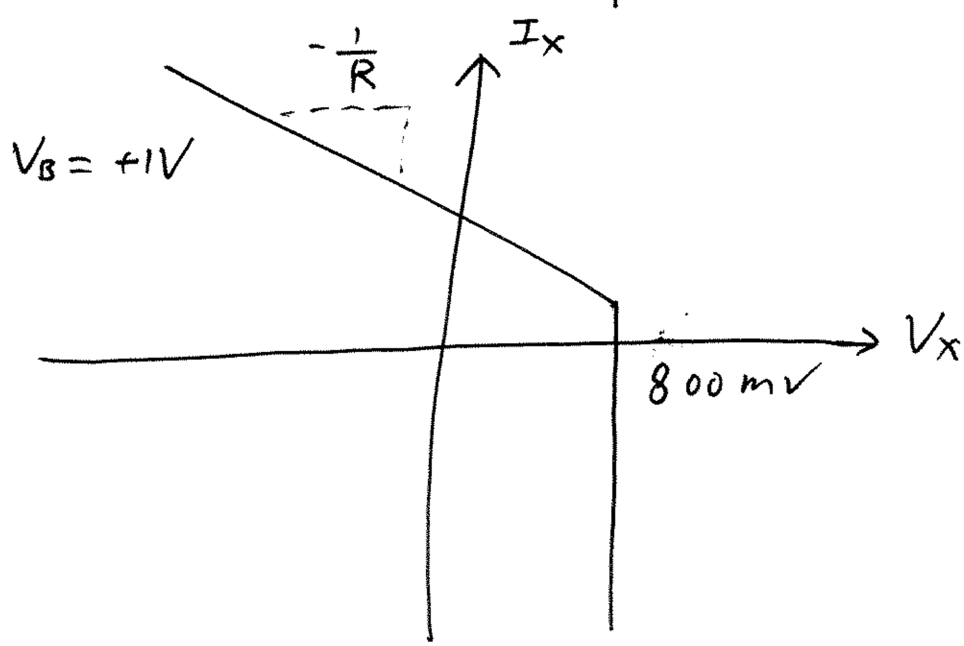
④



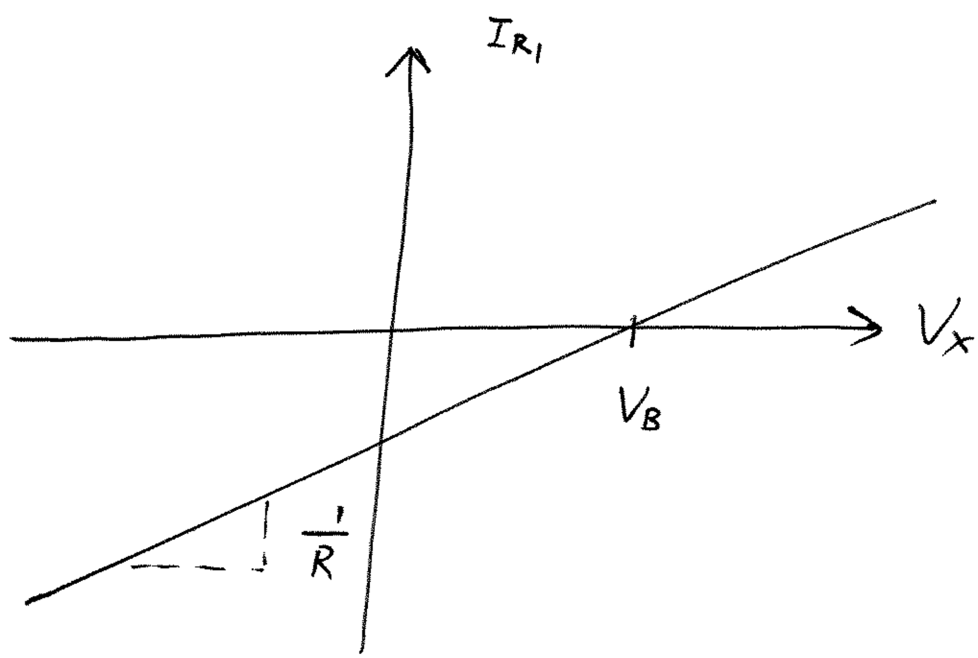


(5)





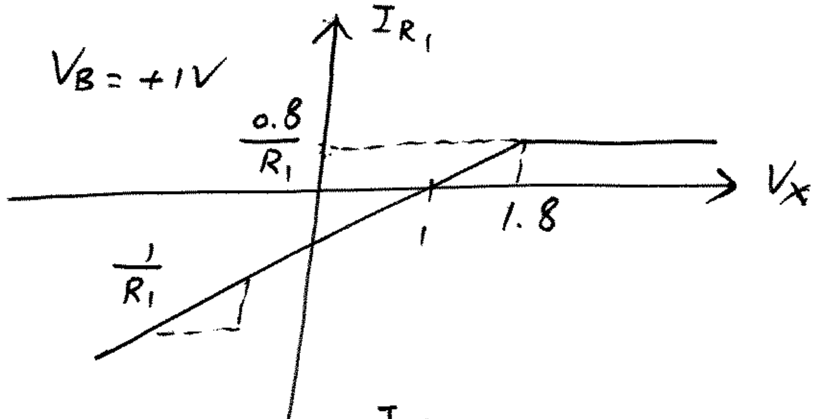
⑥



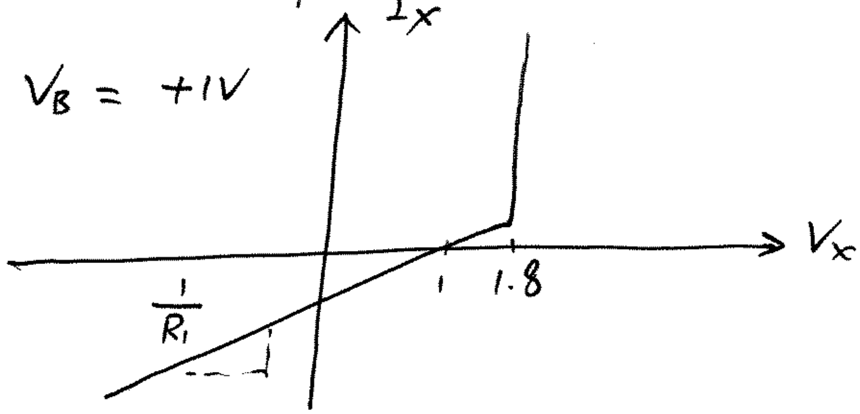
$I_{D1} = 0$ for all V_x

($\because$ $V_B > 0$, D_1 is reverse-biased)

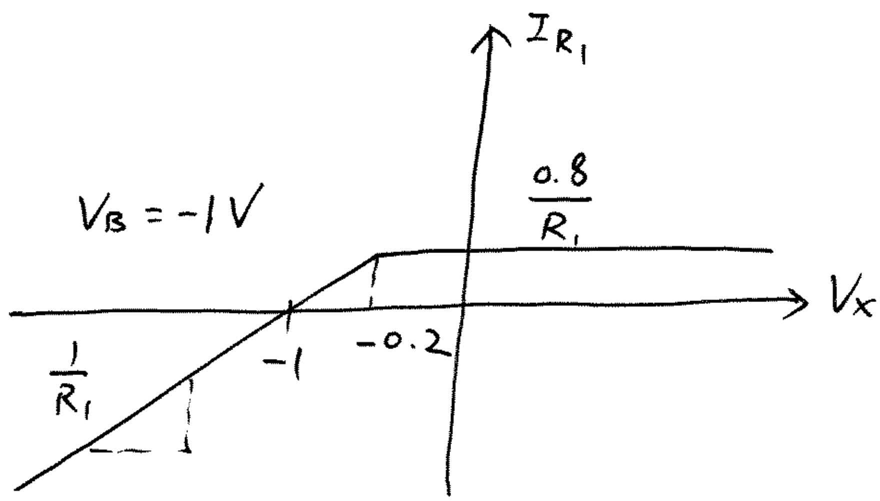
(7)



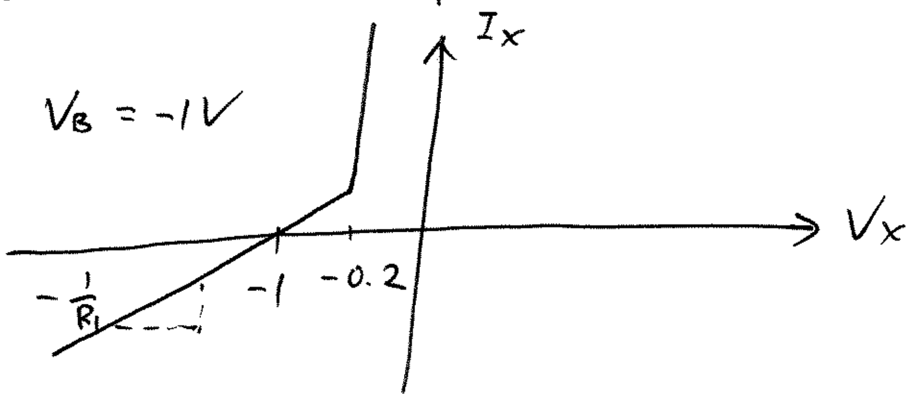
$V_B = +1V$
I_{R_1}
$\frac{0.8}{R_1}$
V_x
1
1.8
$\frac{1}{R_1}$



I_x
$V_B = +1V$
V_x
1
1.8
$\frac{1}{R_1}$

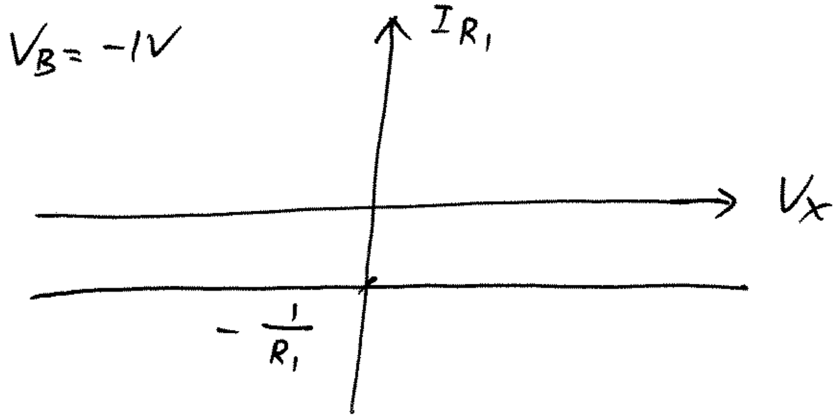


I_{R_1}
$\frac{0.8}{R_1}$
$V_B = -1V$
V_x
-1
-0.2
$\frac{1}{R_1}$

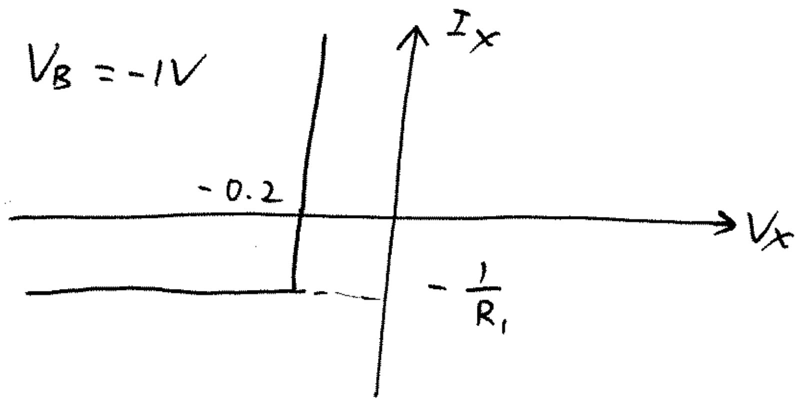


I_x
$V_B = -1V$
V_x
$-\frac{1}{R_1}$
-1
-0.2

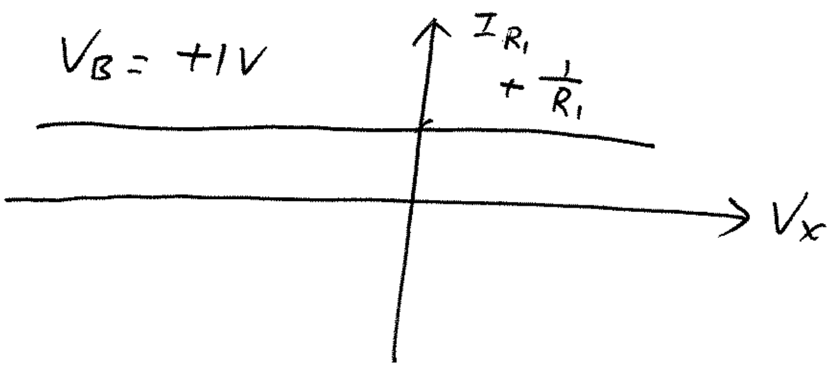
(8)



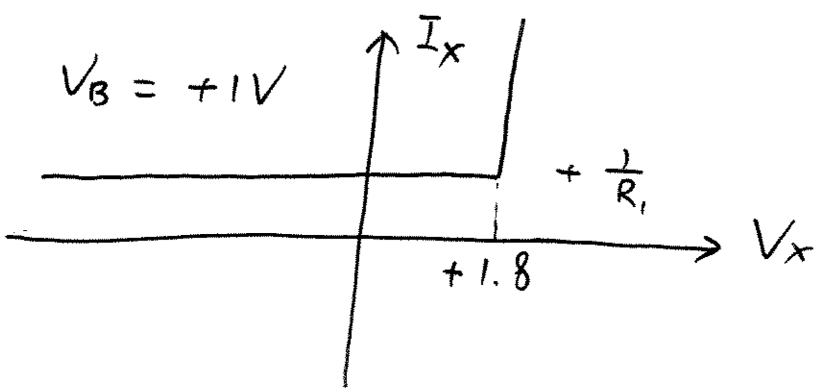
$V_B = -1V$
I_{R_1}
V_x
$-\frac{1}{R_1}$



$V_B = -1V$
I_x
-0.2
V_x
$-\frac{1}{R_1}$



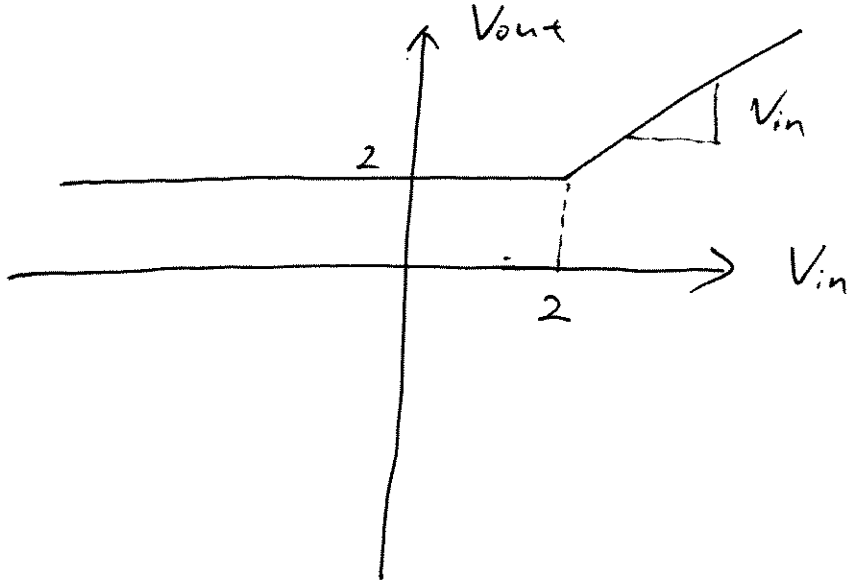
$V_B = +1V$
I_{R_1}
$+\frac{1}{R_1}$
V_x



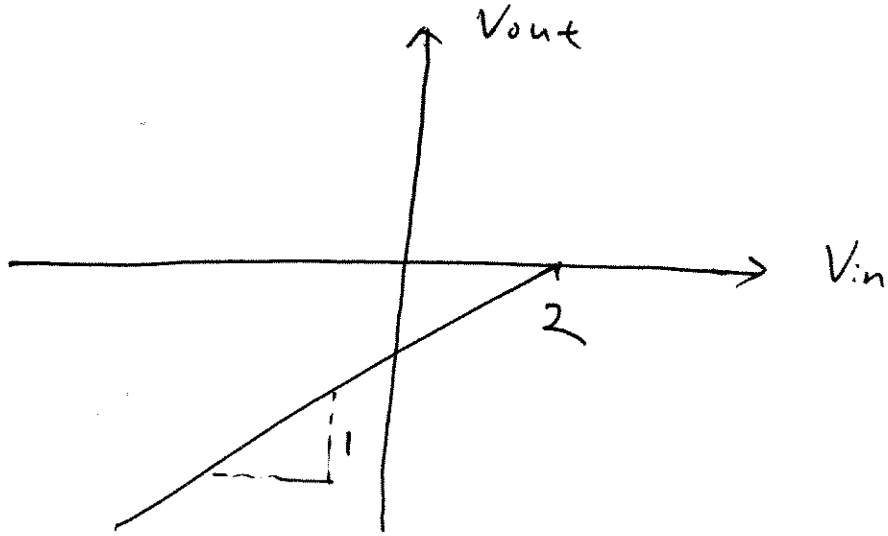
$V_B = +1V$
I_x
$+\frac{1}{R_1}$
V_x
$+1.8$

(9)

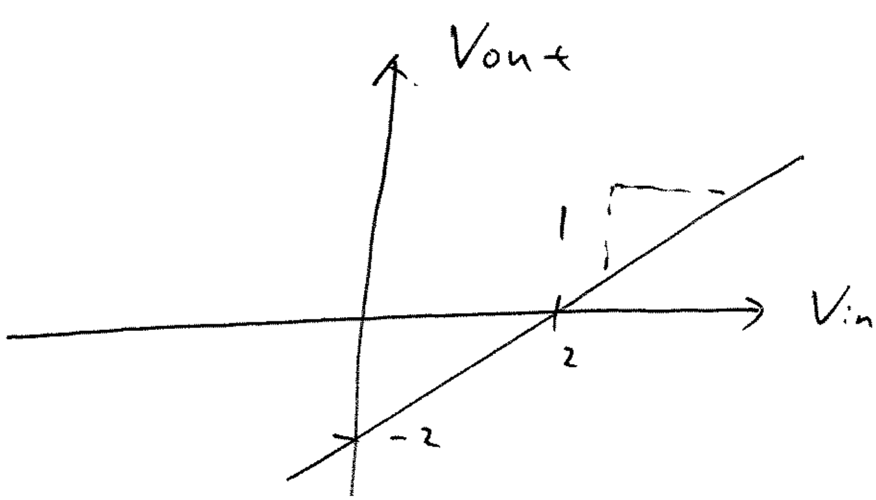
a)



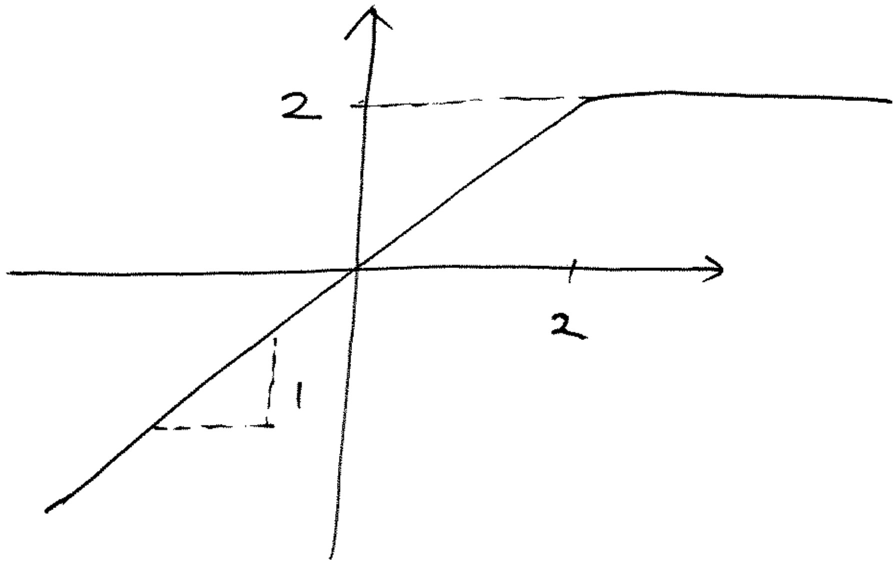
b)



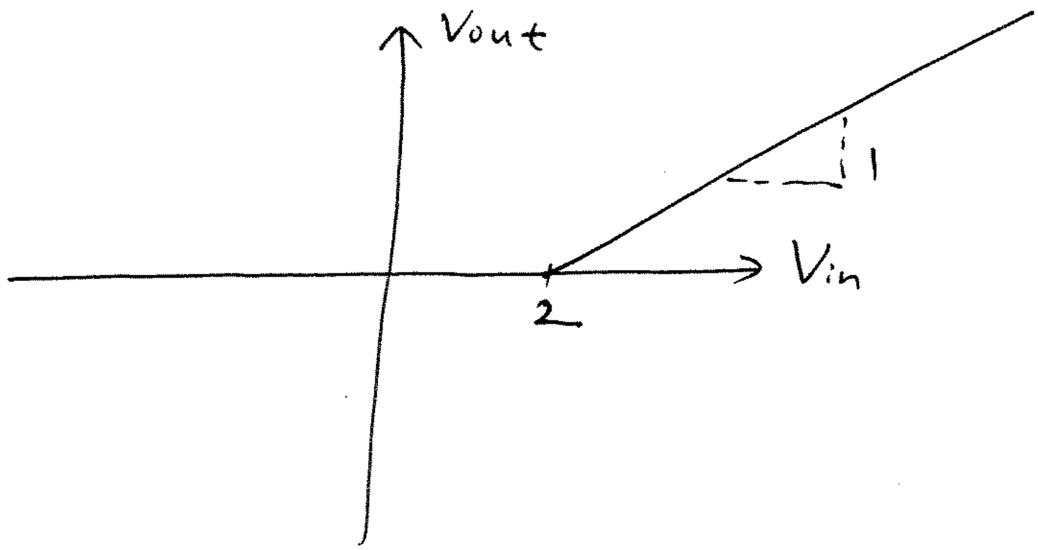
c)



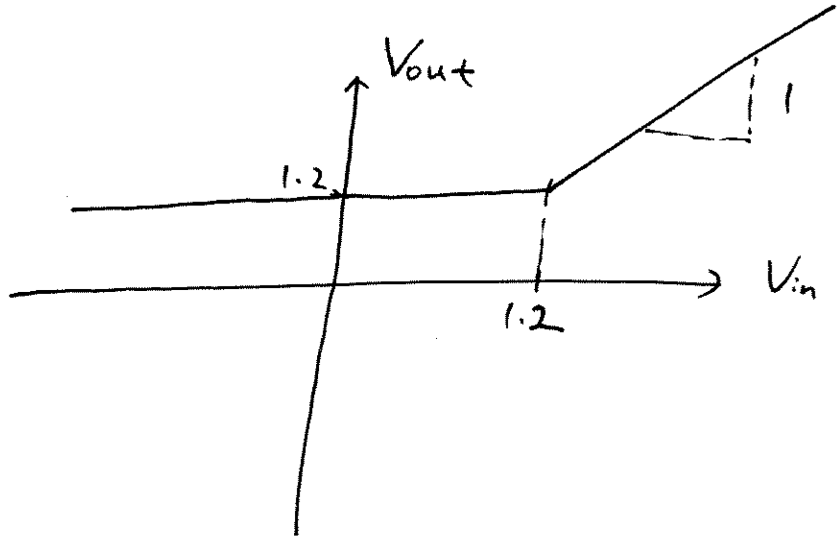
d)



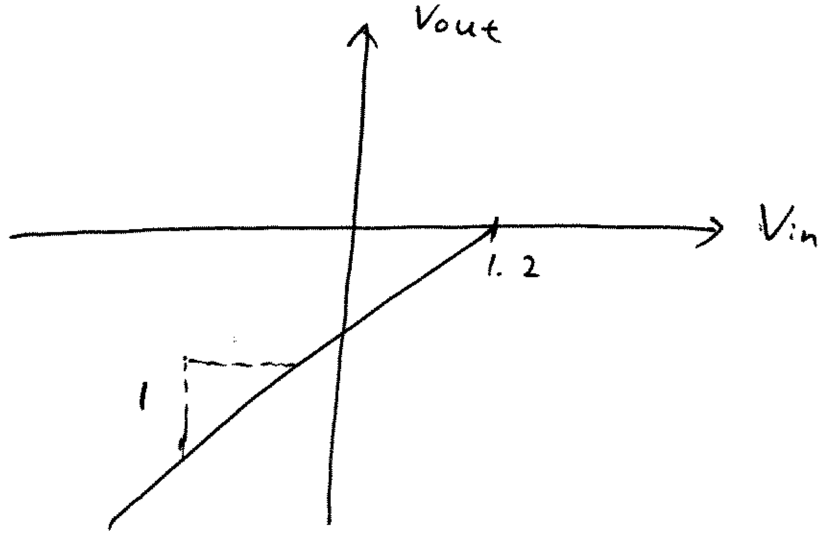
e)



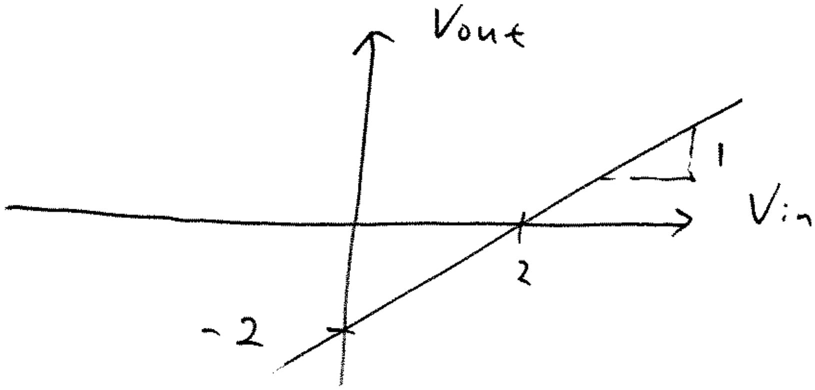
(10) a)



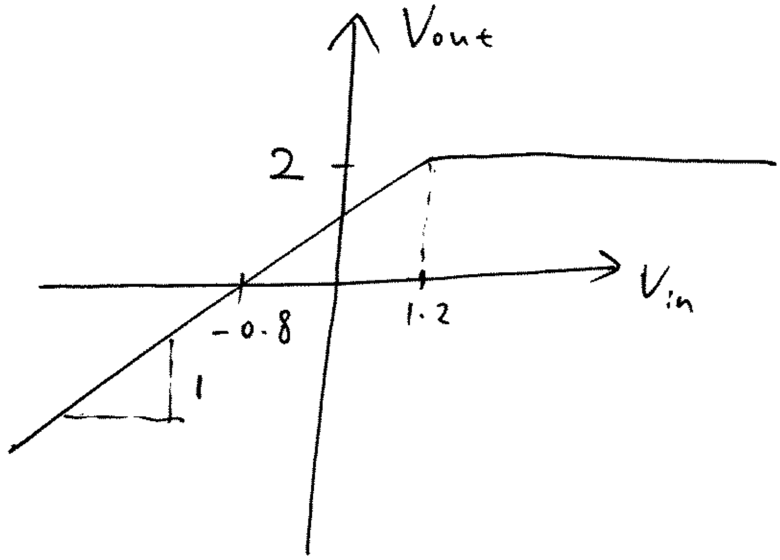
b)



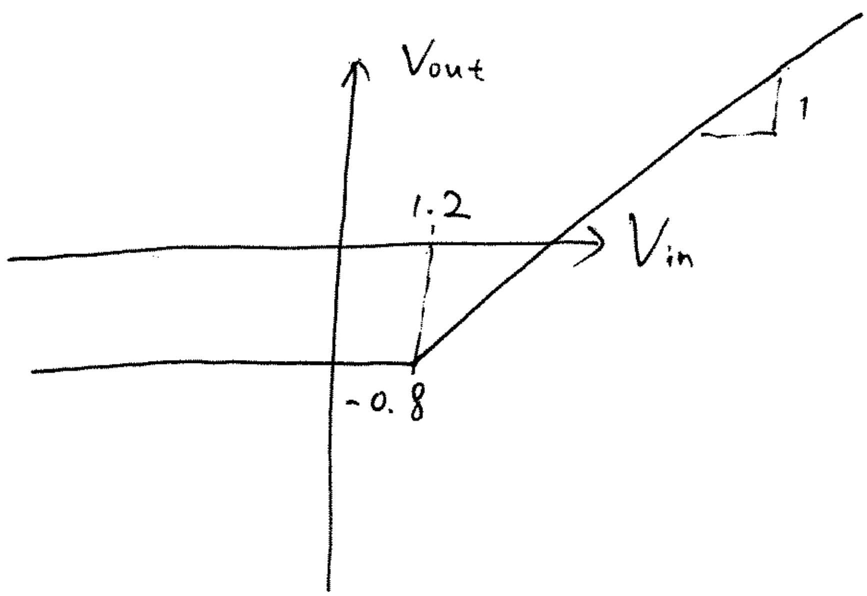
c)



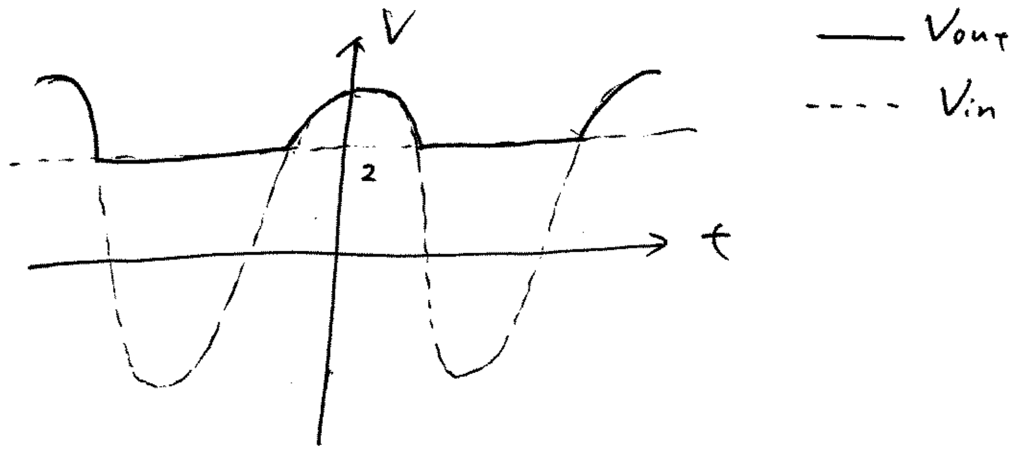
d)



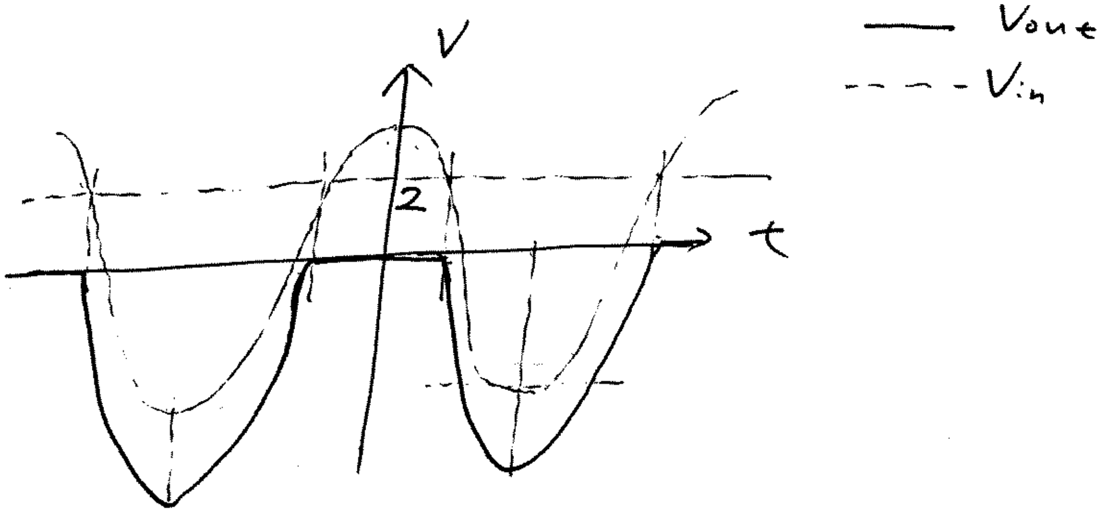
e)



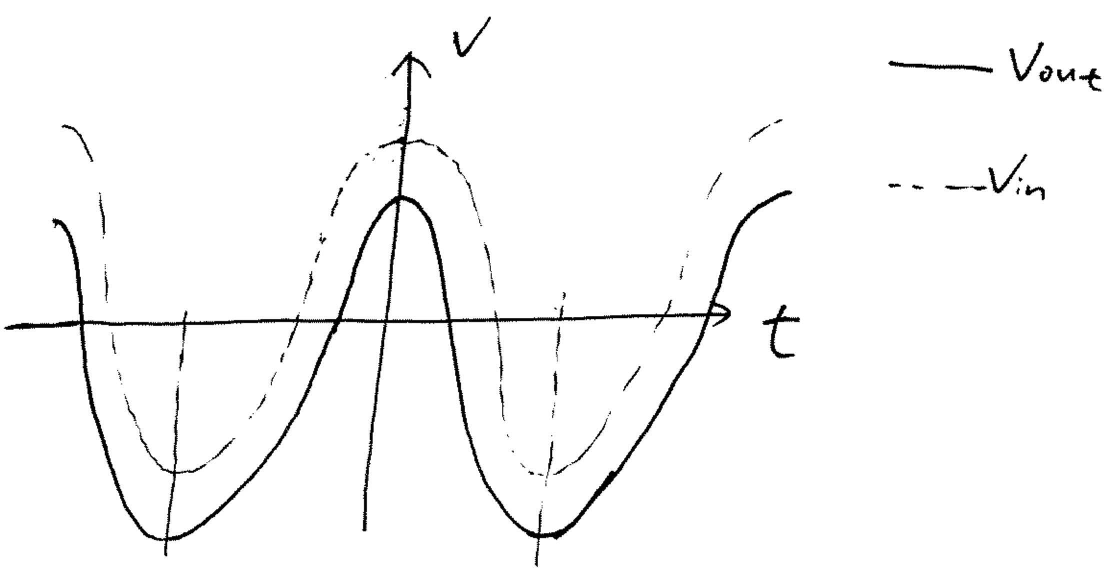
(11) a)



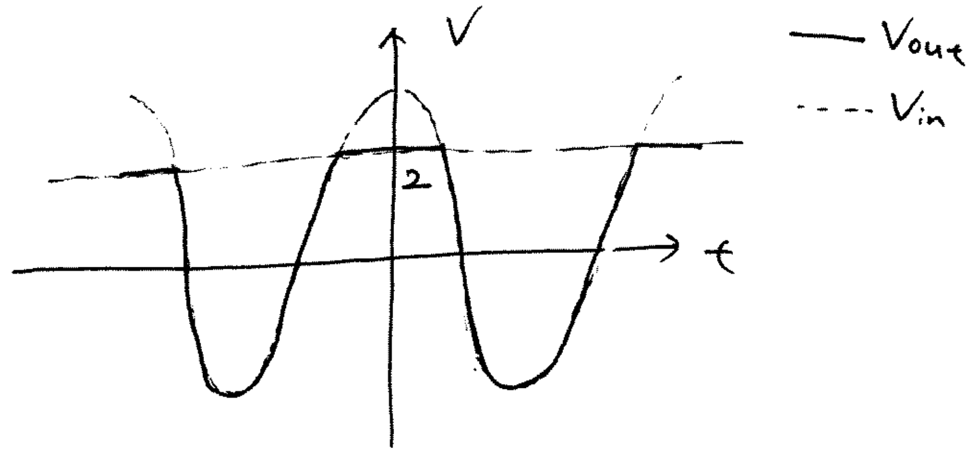
b)



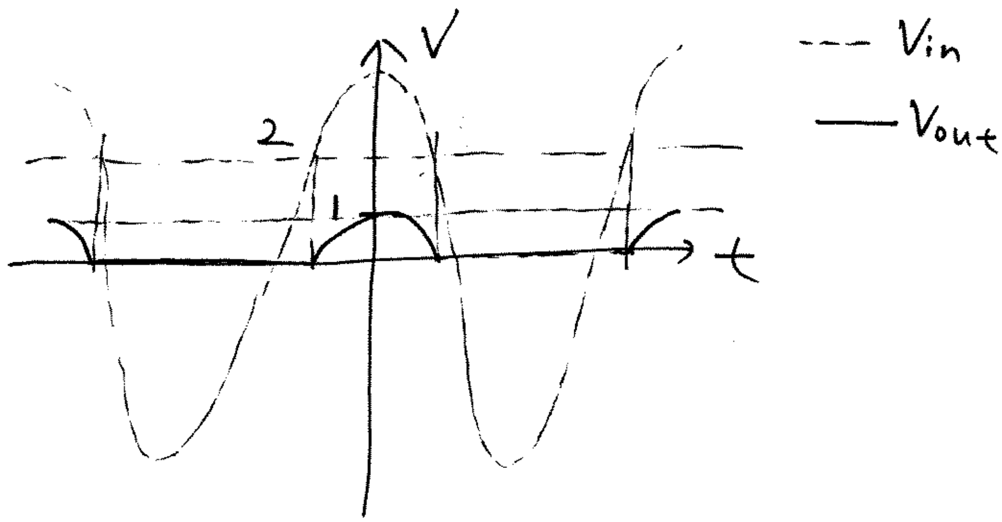
c)



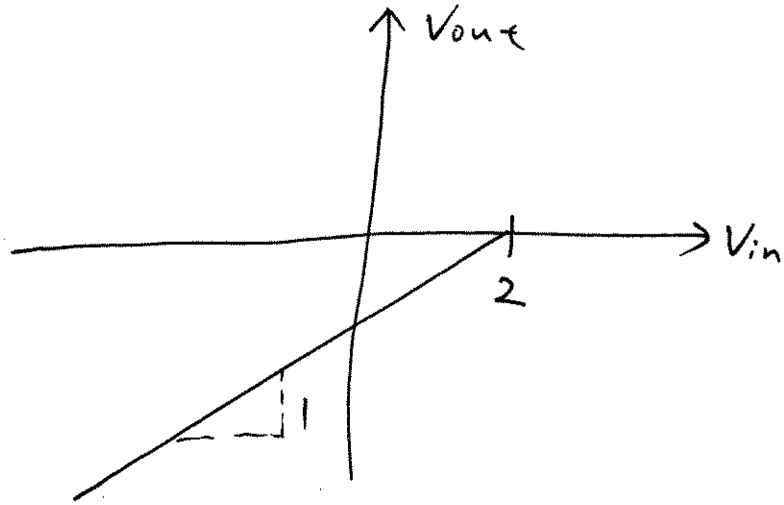
d)



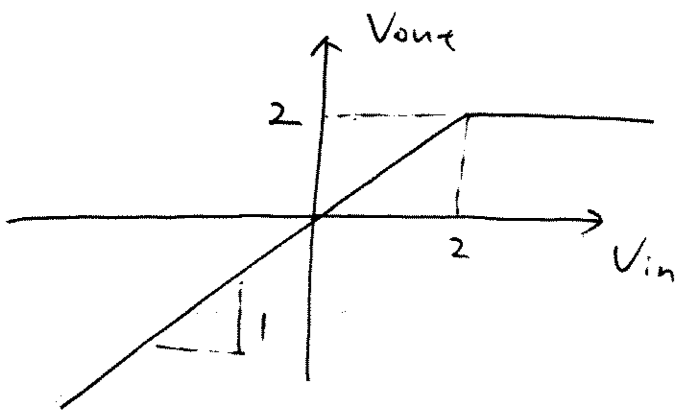
e)



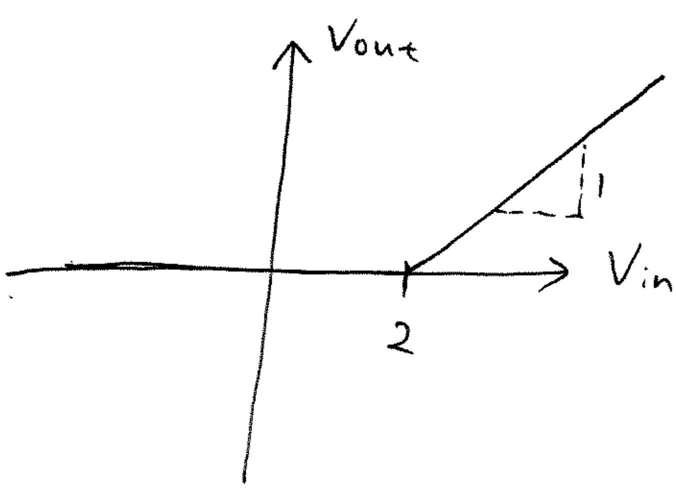
⑫ a)



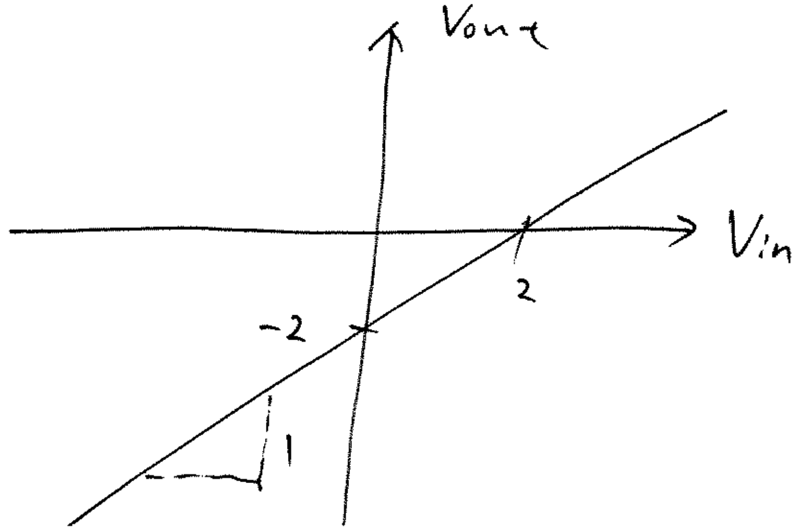
b)



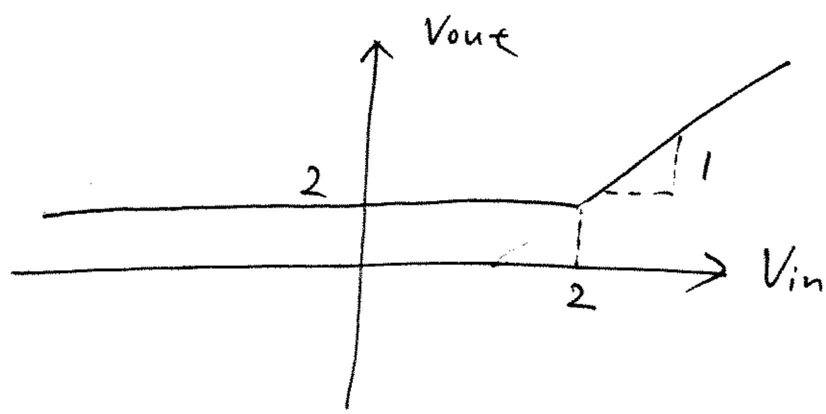
c)



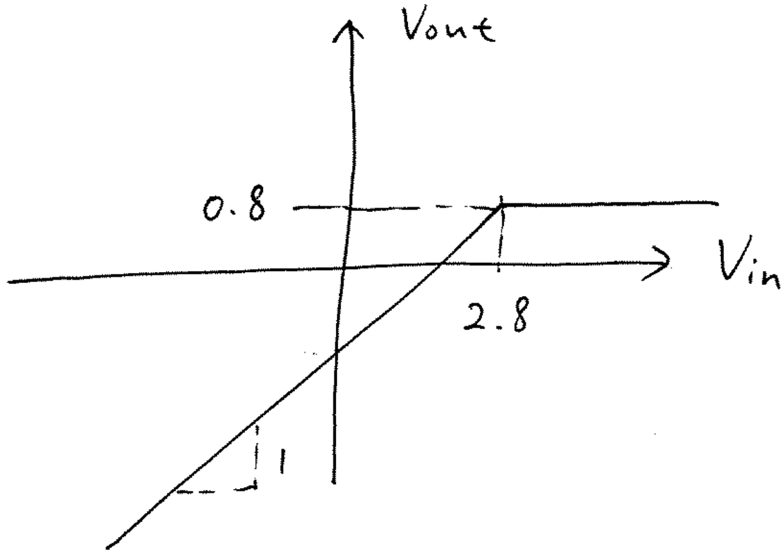
d)



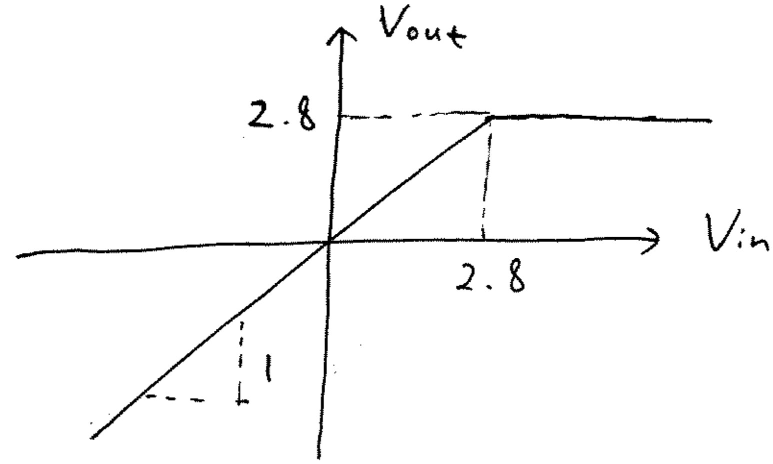
e)



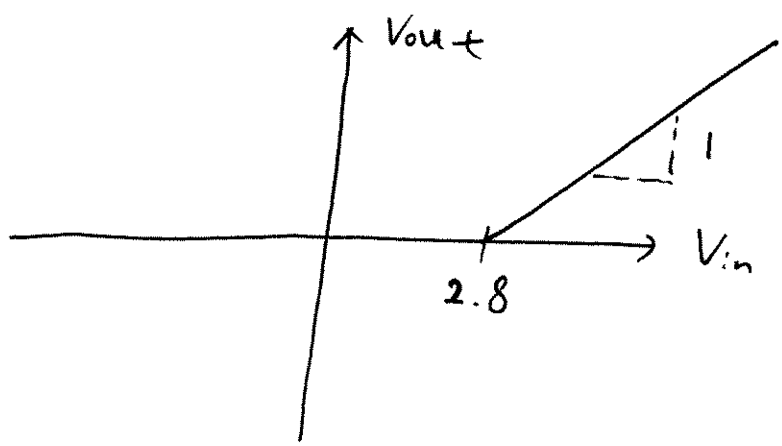
⑬ a)



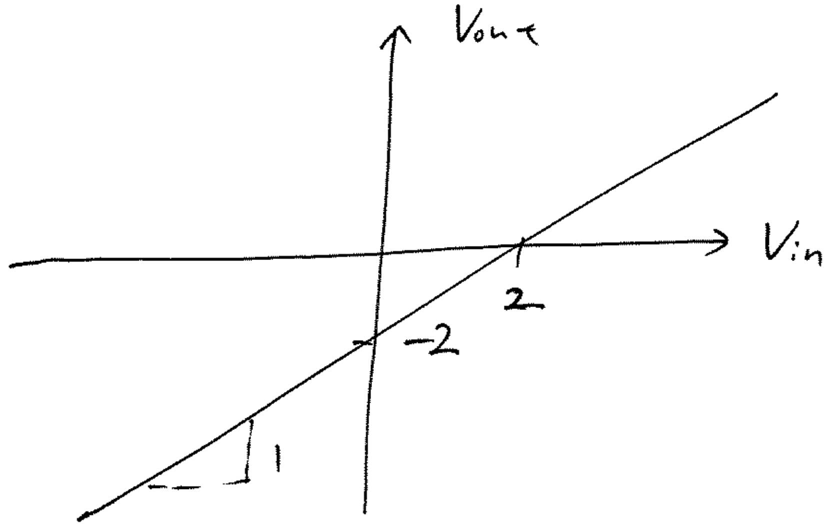
b)



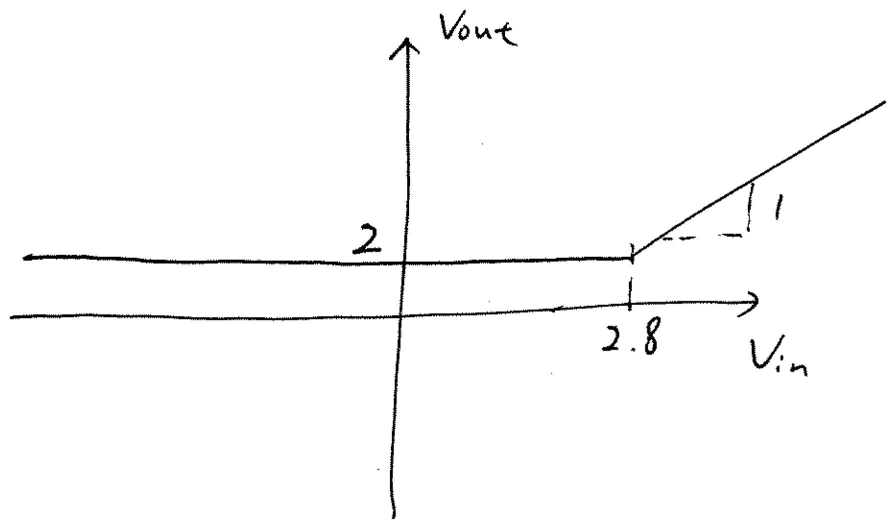
c)



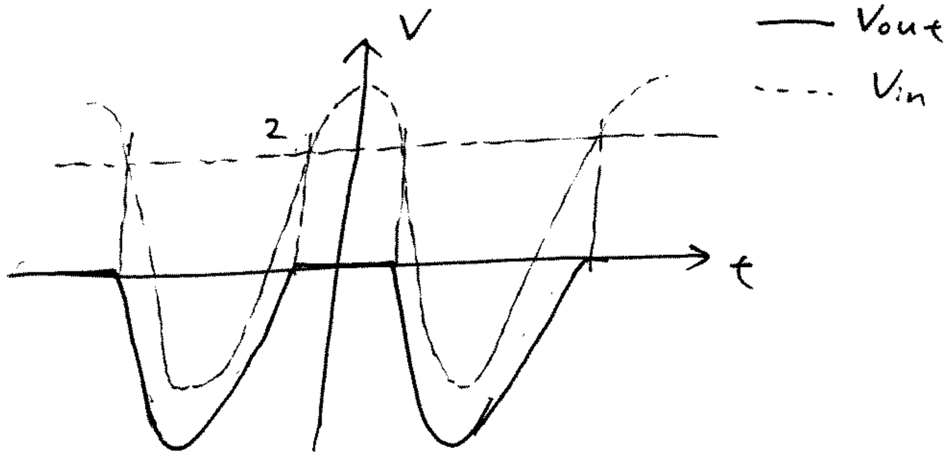
d)



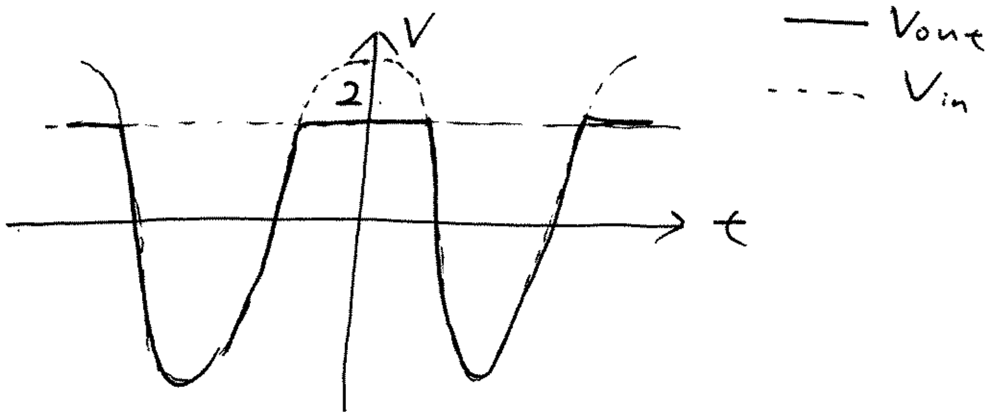
e)



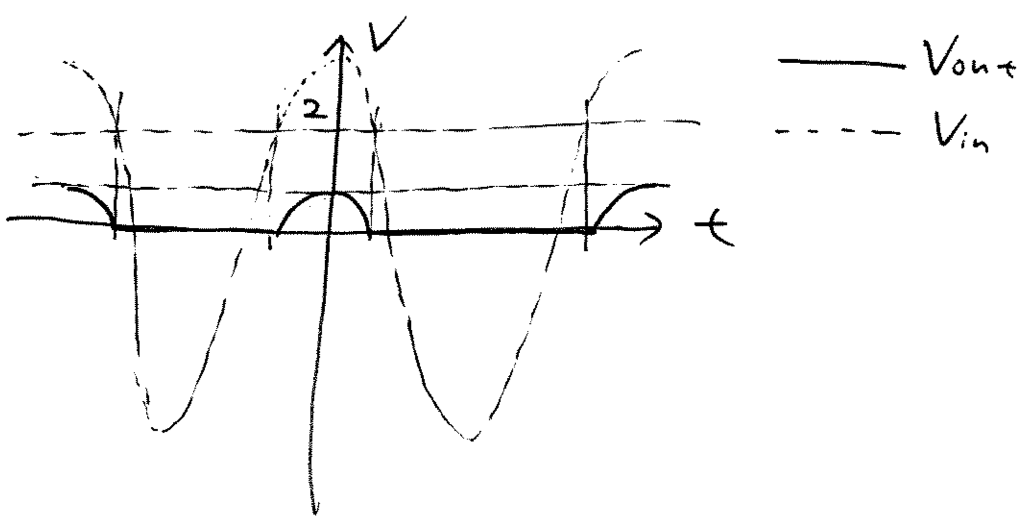
(14) a)



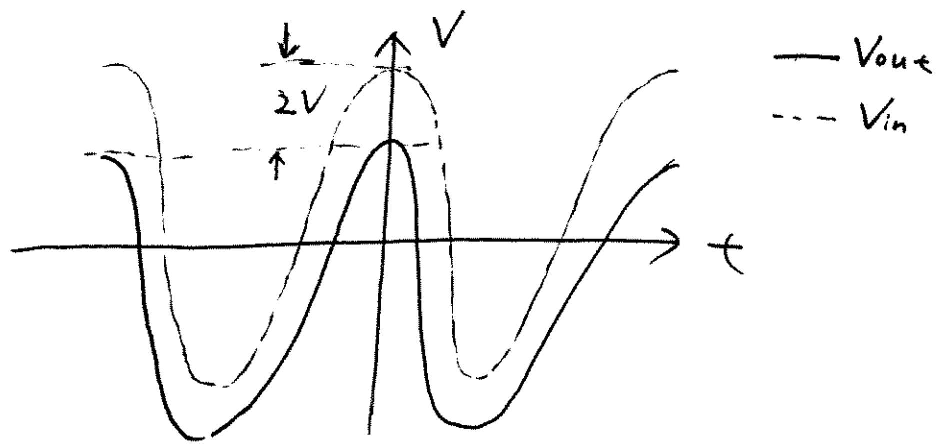
b)



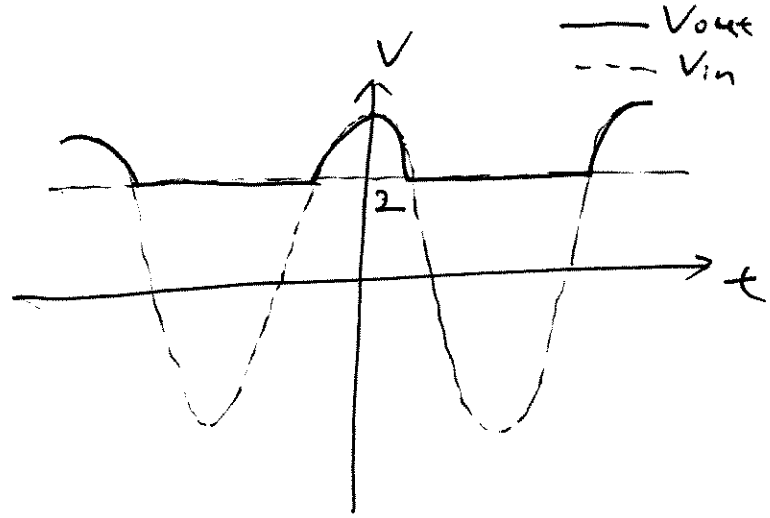
c)



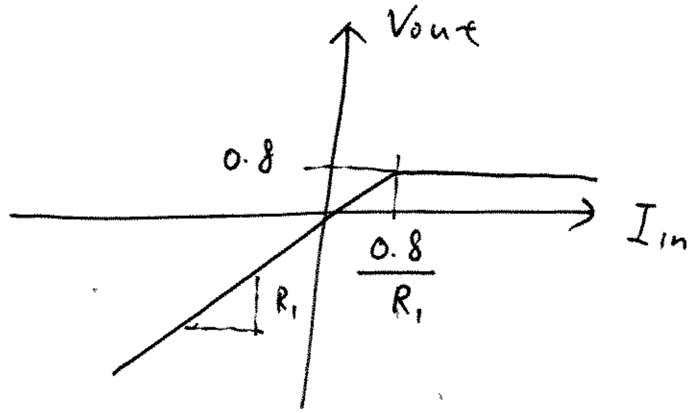
d)



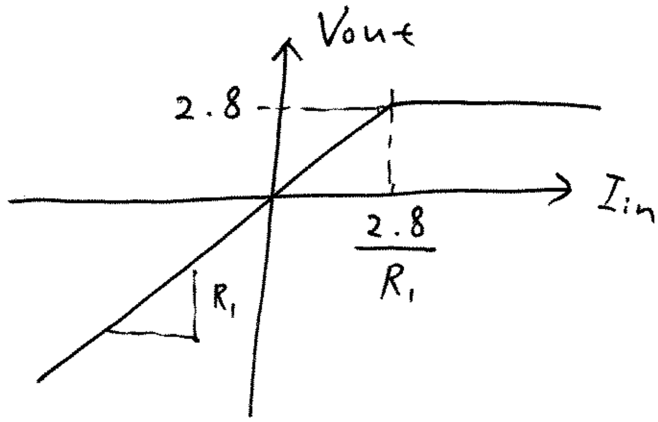
e).



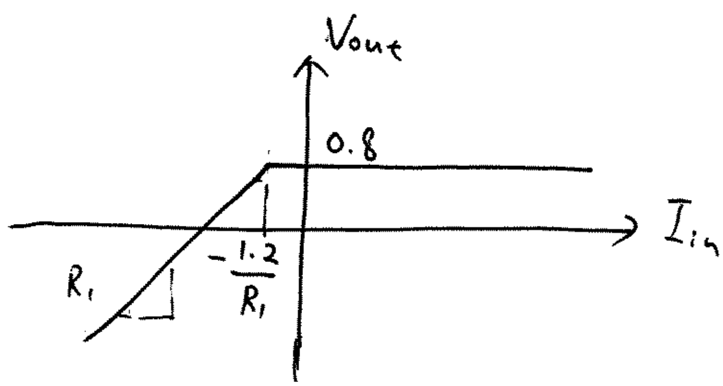
(15) a)


Vout
0.8
Iin
0.8
R1
R1

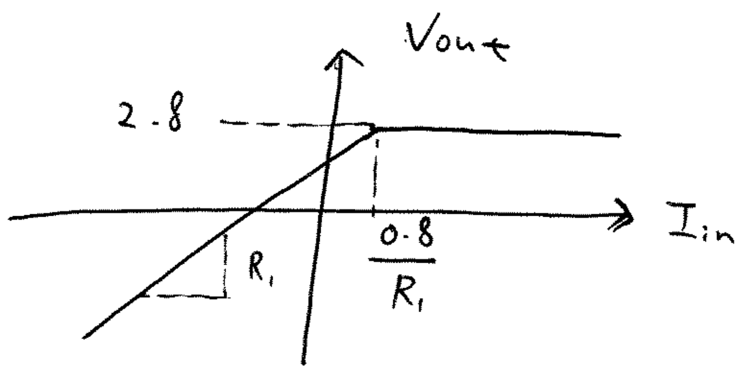
b)


Vout
2.8
Iin
2.8
R1
R1

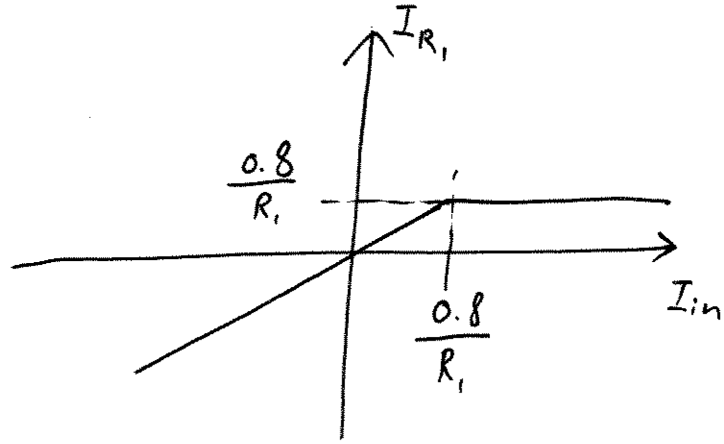
c)


Vout
0.8
Iin
-1.2
R1
R1

d)

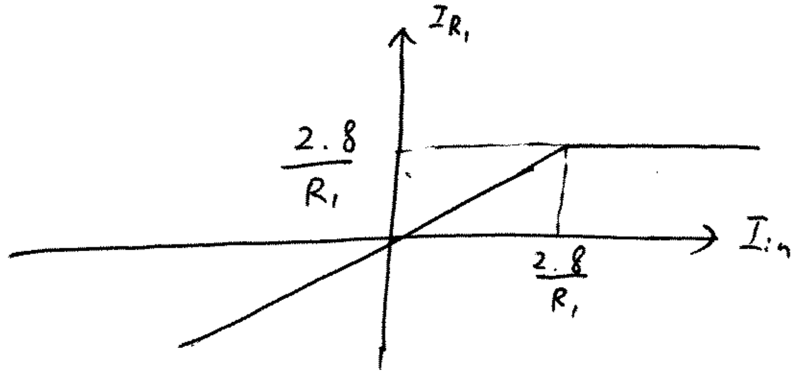

Vout
2.8
Iin
0.8
R1
R1

(16) a)



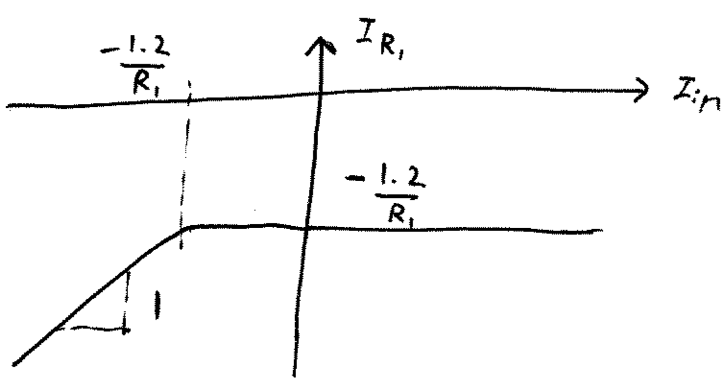
I_{R_1}
$\frac{0.8}{R_1}$
$\frac{0.8}{R_1}$
I_{in}

b)



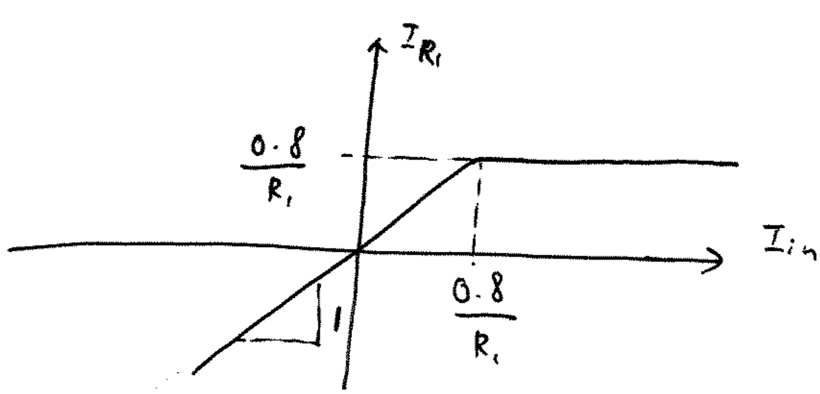
I_{R_1}
$\frac{2.8}{R_1}$
I_{in}
$\frac{2.8}{R_1}$

c)



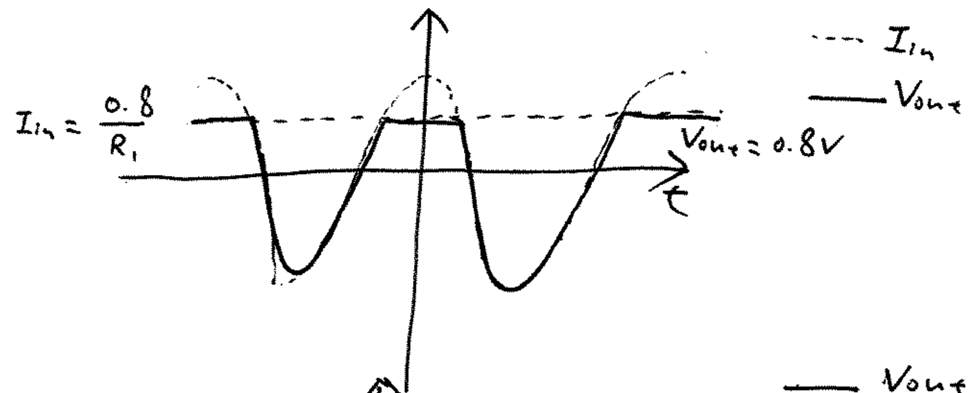
$-\frac{1.2}{R_1}$
I_{R_1}
I_{in}
$-\frac{1.2}{R_1}$
1

d)



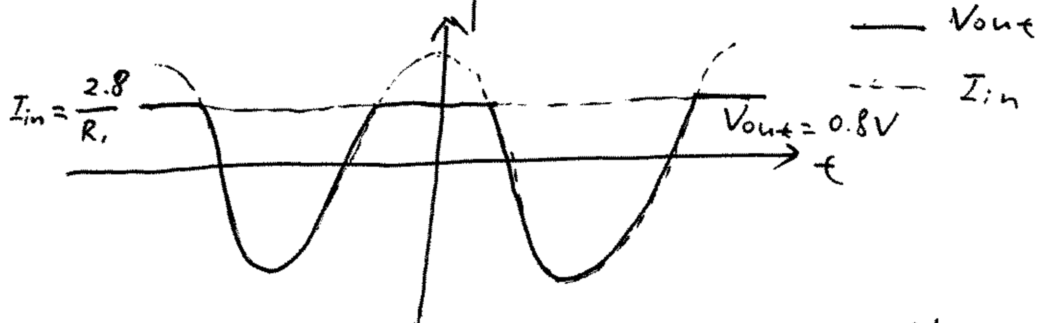
I_{R_1}
$\frac{0.8}{R_1}$
I_{in}
1
$\frac{0.8}{R_1}$

(17) a)



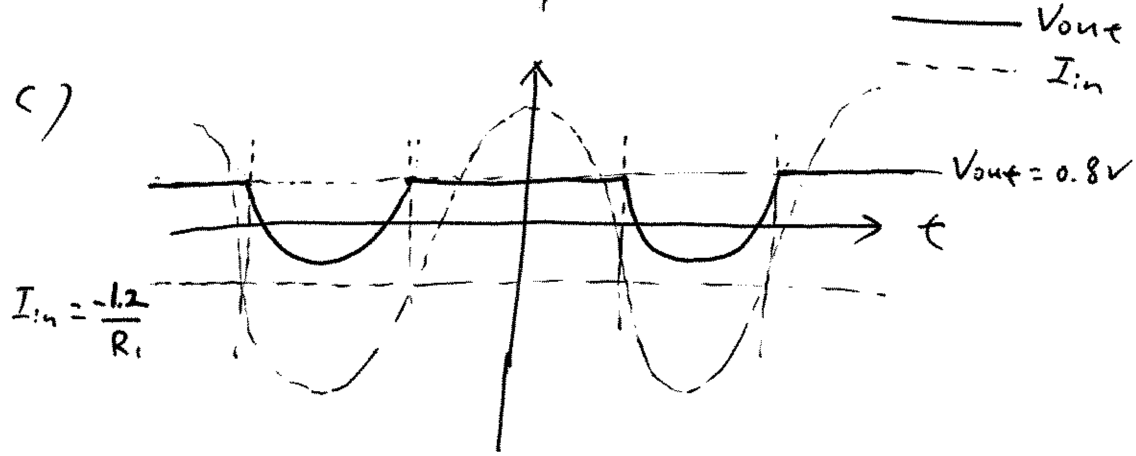
$I_{in} = \frac{0.8}{R_1}$
$V_{out} = 0.8V$
t
I_{in}
V_{out}

b)



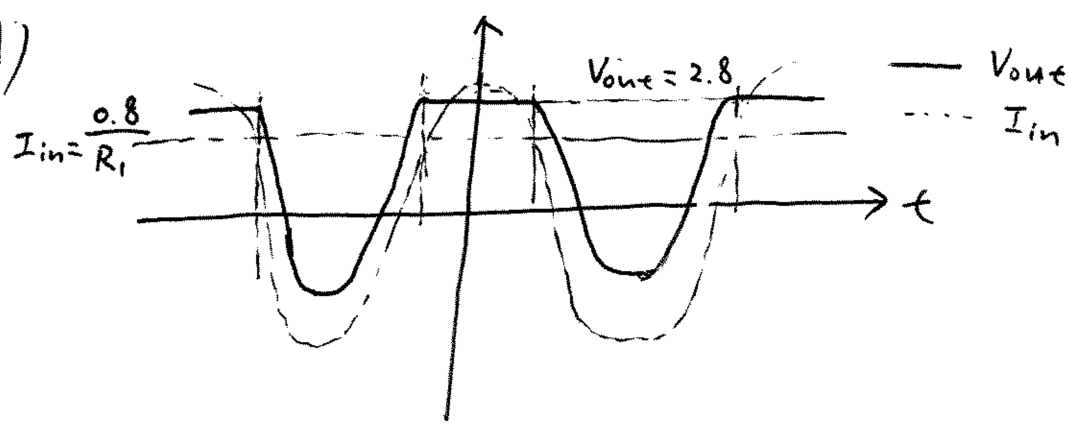
$I_{in} = \frac{2.8}{R_1}$
$V_{out} = 0.8V$
t
V_{out}
I_{in}

c)



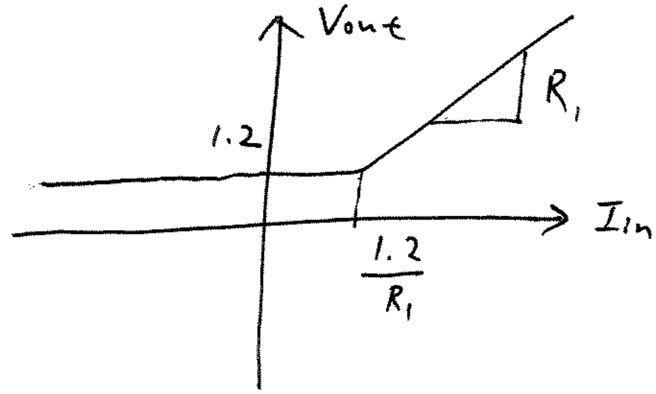
$V_{out} = 0.8V$
t
$I_{in} = \frac{-1.2}{R_1}$
V_{out}
I_{in}

d)

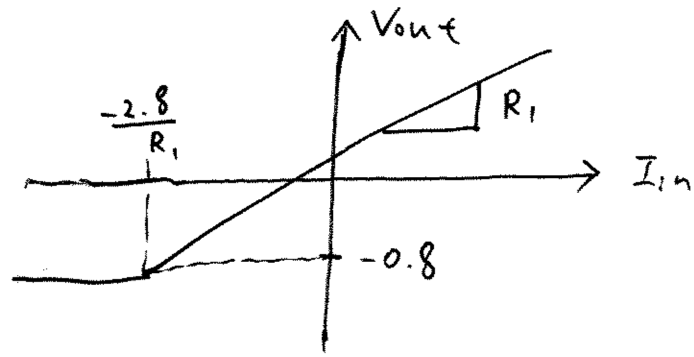


$V_{out} = 2.8$
$I_{in} = \frac{0.8}{R_1}$
t
V_{out}
I_{in}

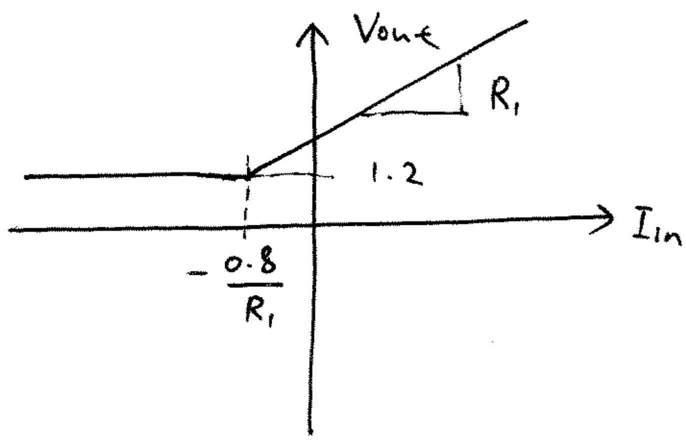
(18) a)



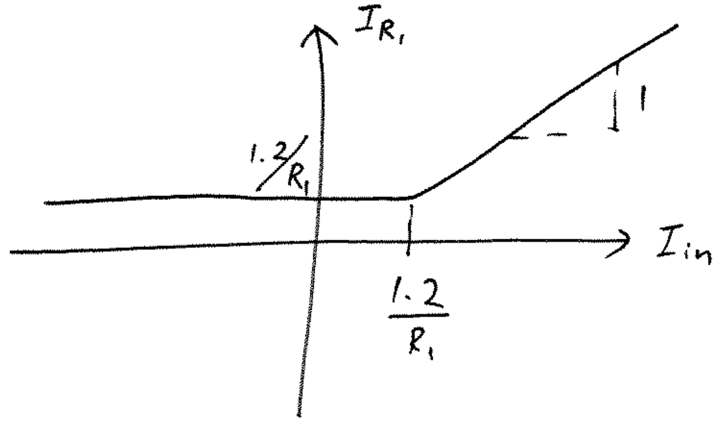
b)



c)

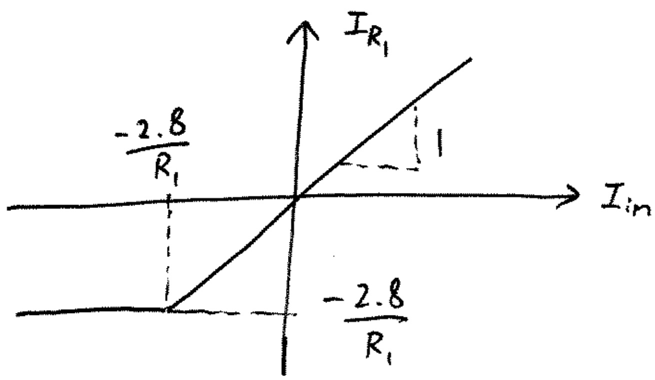


(19) a)



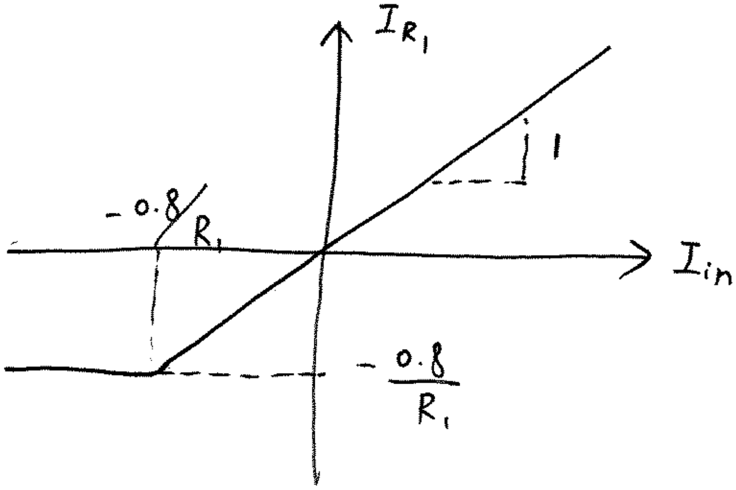
I_{R_1}
1.2/R_1
1
I_{in}
$\frac{1.2}{R_1}$

b)



I_{R_1}
$\frac{-2.8}{R_1}$
1
I_{in}
$-\frac{2.8}{R_1}$

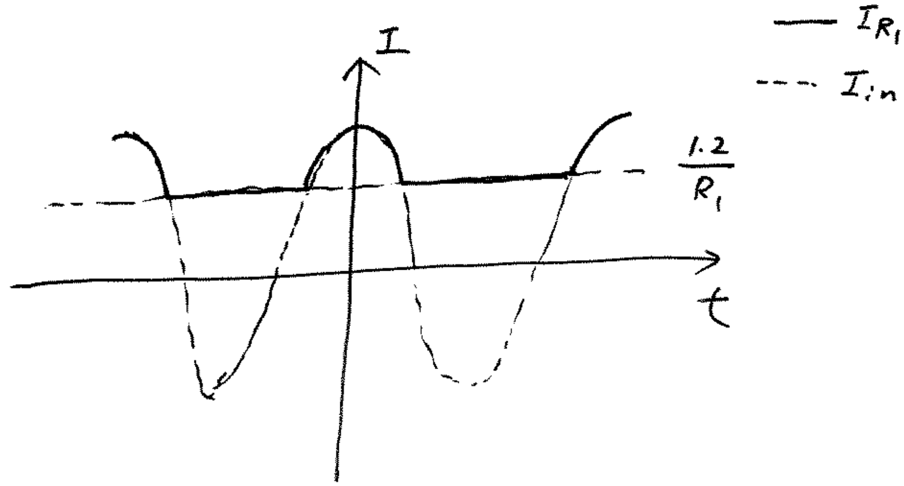
c)



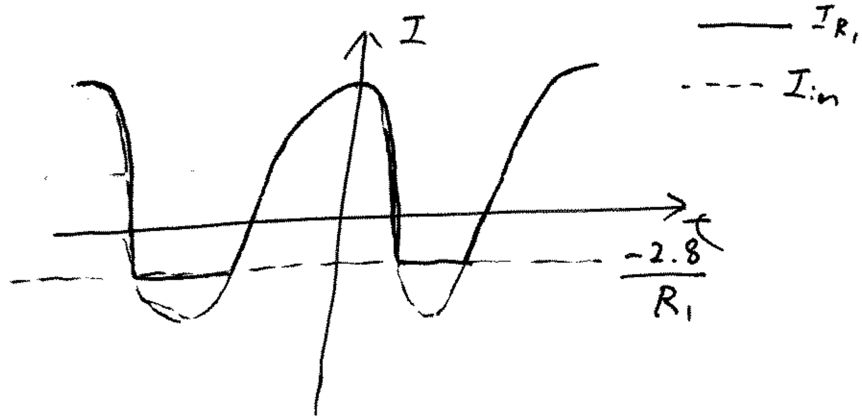
I_{R_1}
1
-0.8/R_1
I_{in}
$-\frac{0.8}{R_1}$

(20)

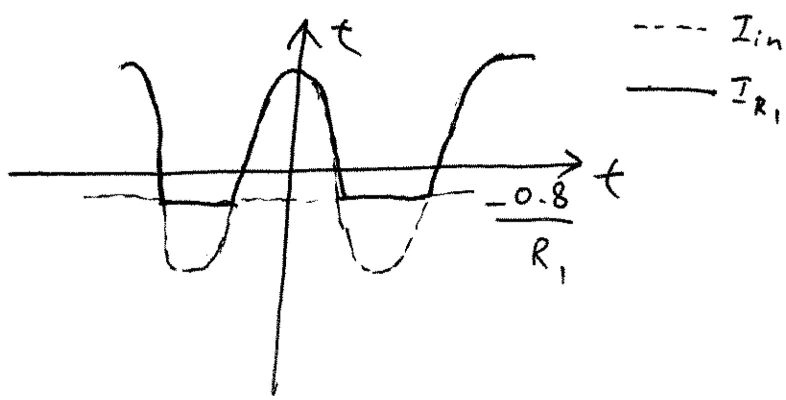
a)



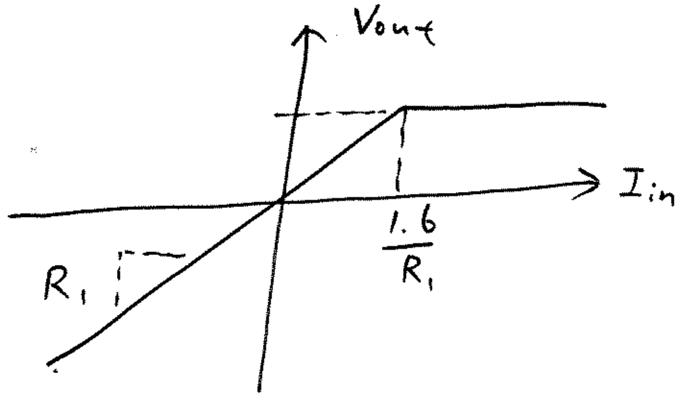
b)



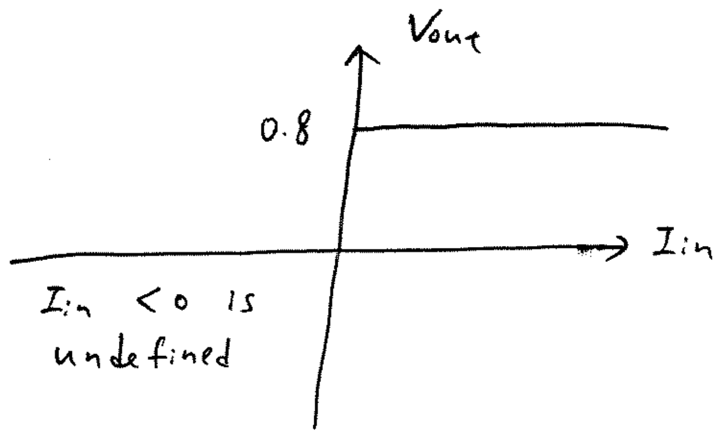
c)



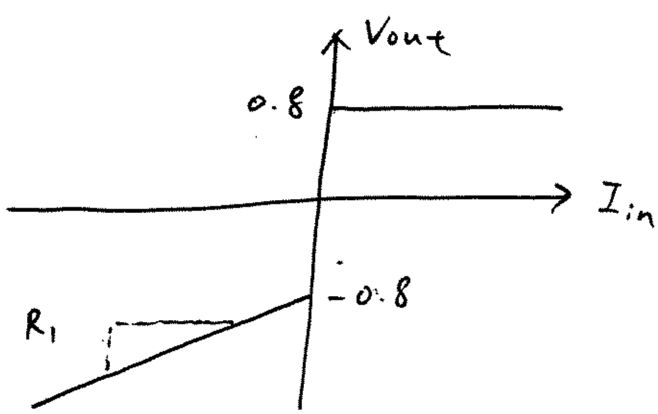
(21) a)


Vout
Iin
1.6 / R1
R1

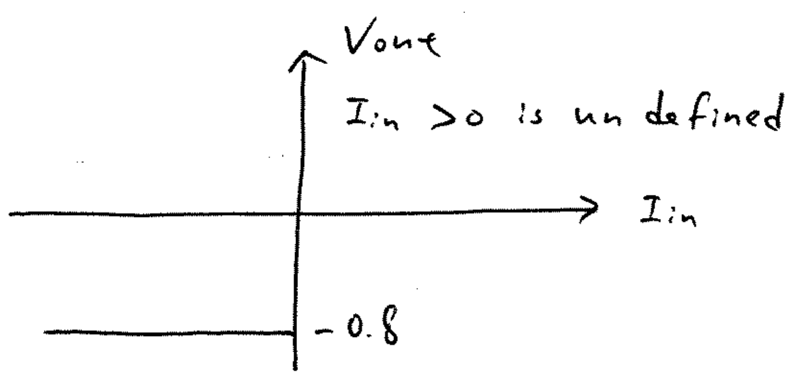
b)


Vout
0.8
Iin
Iin < 0 is undefined

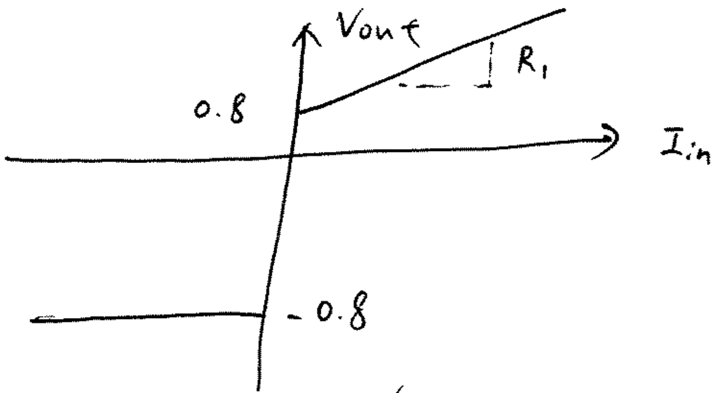
c)


Vout
0.8
Iin
-0.8
R1

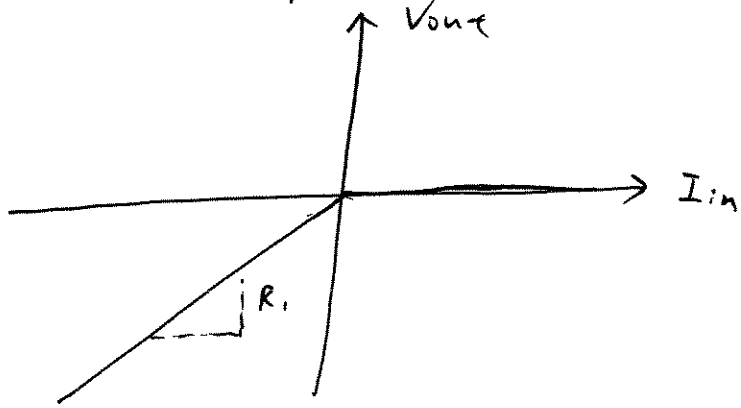
d)


Vout
Iin > 0 is undefined
Iin
-0.8

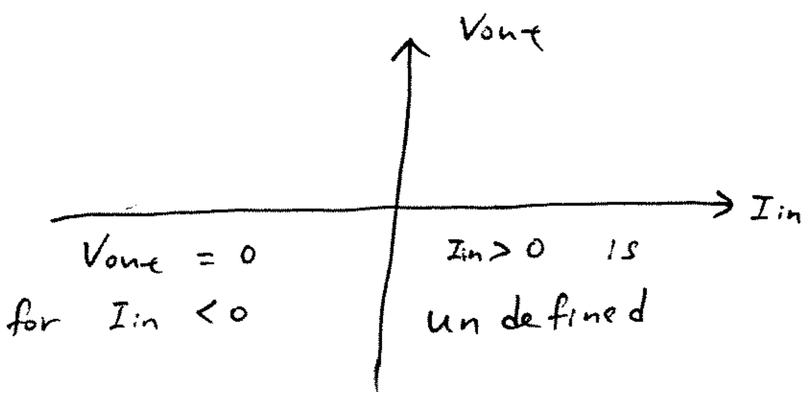
e)


Vout
R1
0.8
Iin
-0.8

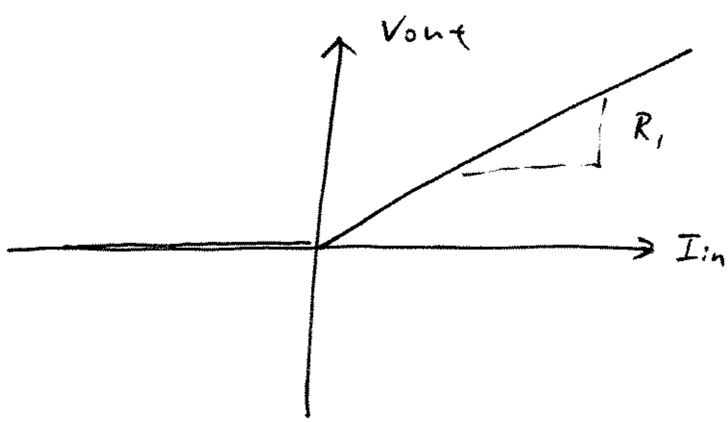
f)


Vout
Iin
R1

g)

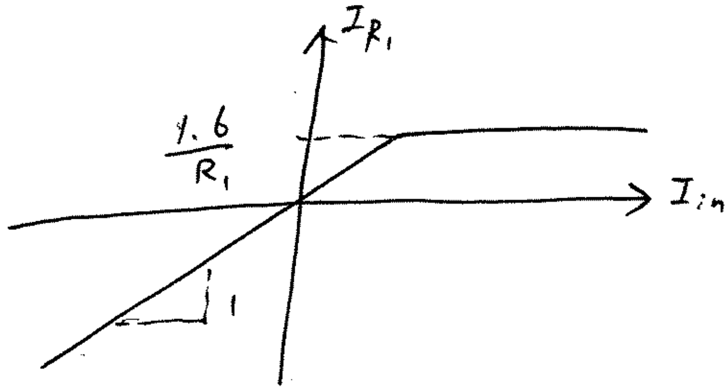

Vout
Iin
Vout = 0
for Iin < 0
Iin > 0 is
undefined

h)


Vout
R1
Iin

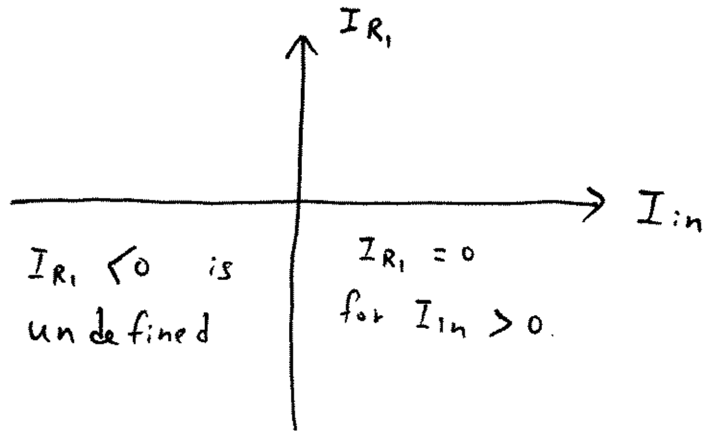
(22)

a)



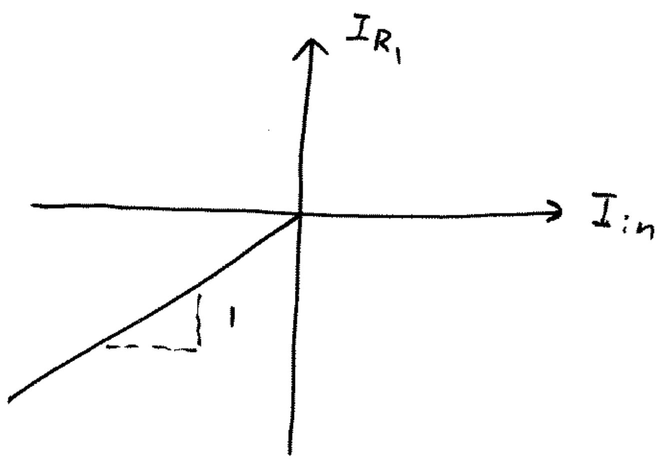
I_{R_1}
$\frac{4.6}{R_1}$
I_{in}
1

b)



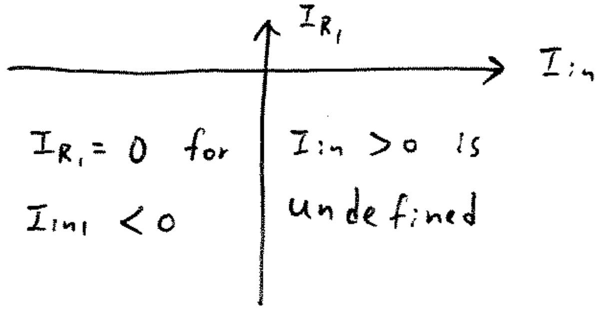
I_{R_1}
I_{in}
$I_{R_1} < 0$ is undefined
$I_{R_1} = 0$ for $I_{in} > 0$

c)



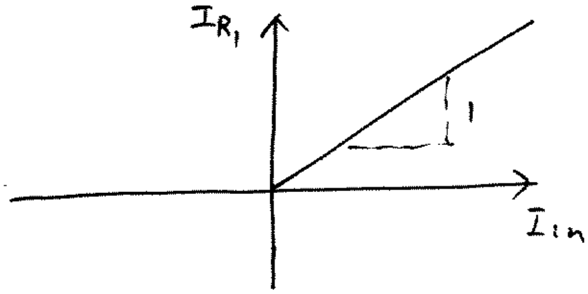
I_{R_1}
I_{in}
1

d)

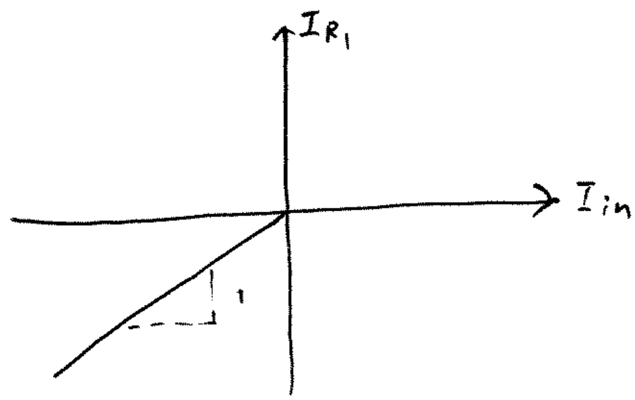


I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in1} < 0$
$I_{in} > 0$ is undefined

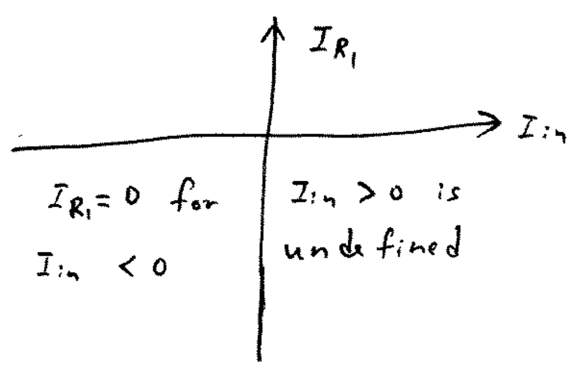
e)


I_{R_1}
1
I_{in}

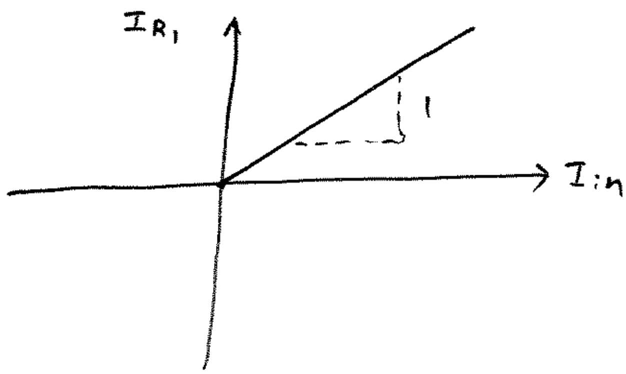
f)


I_{R_1}
I_{in}
1

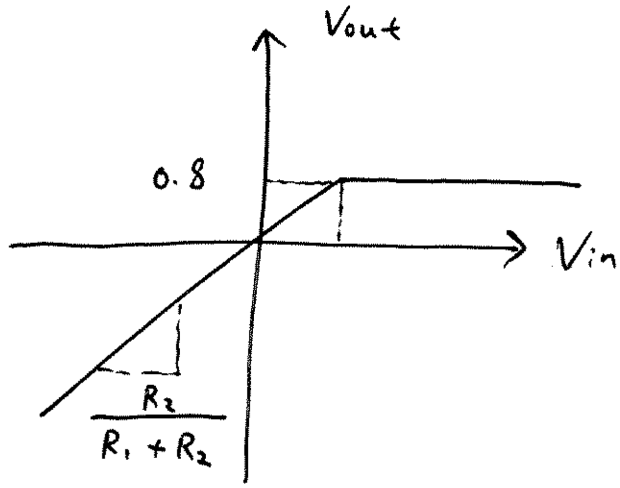
g)


I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in} < 0$
$I_{in} > 0$ is undefined

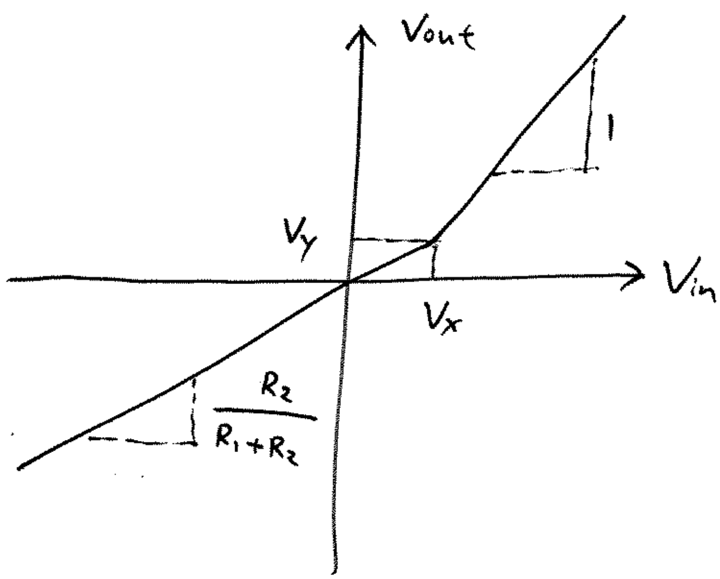
h)


I_{R_1}
1
I_{in}

(23) a)



b)

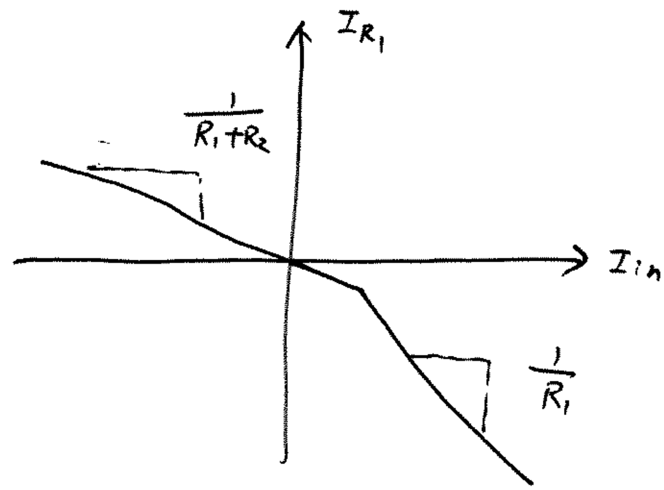


Note: at the turning point when D_1 starts to conduct, V_x, V_y need to satisfy 2 conditions:

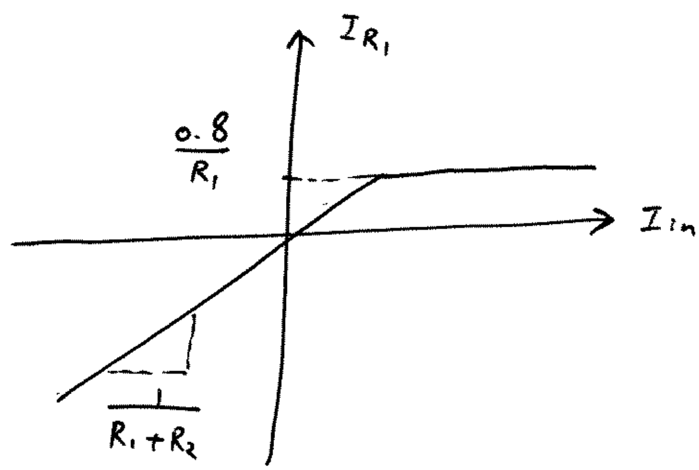
$$V_x - V_y = 0.8 \quad \text{—— (1)}$$

$$V_y = \frac{R_2}{R_1+R_2} V_x \quad \text{—— (2)}$$

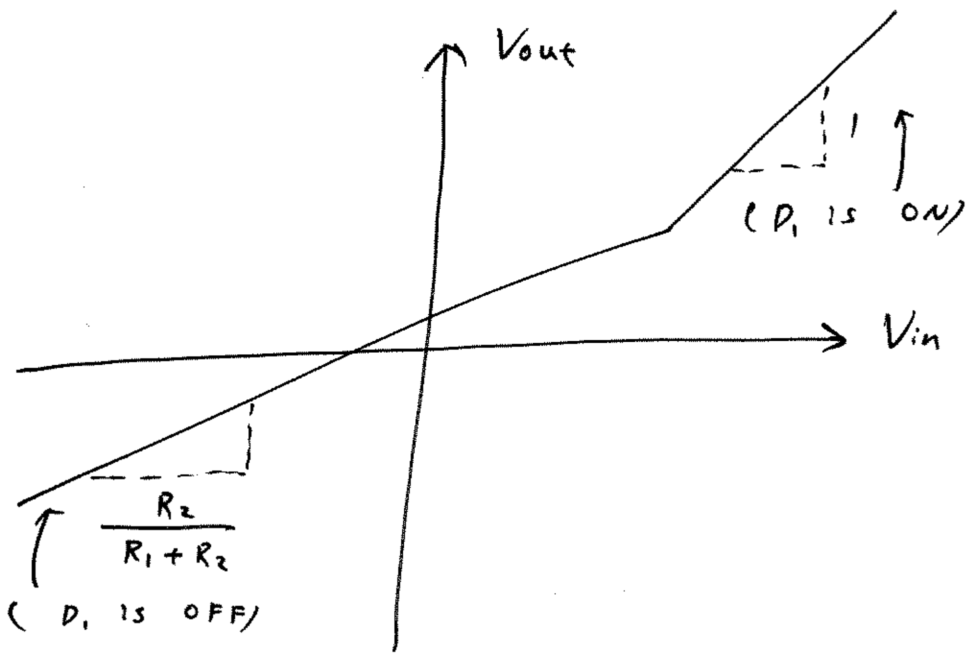
(24) a)



b)

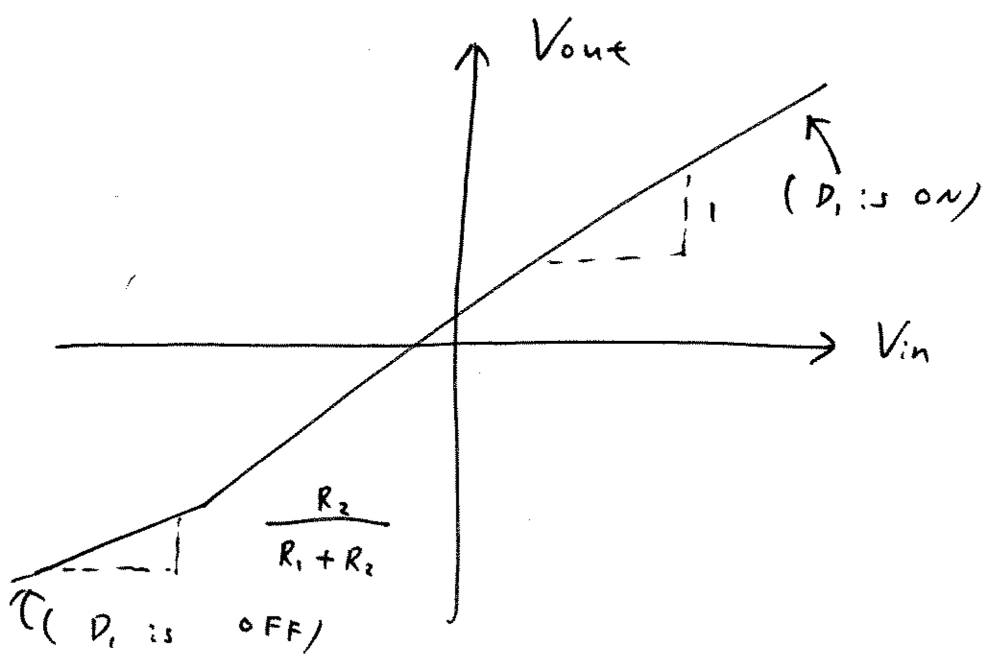


(25) a)



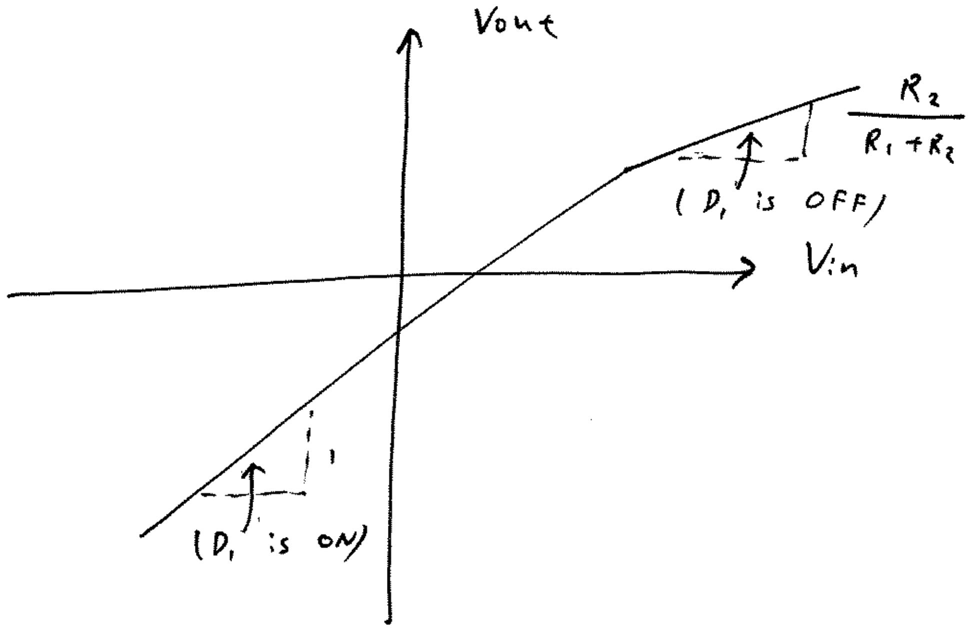
Vout
Vin
1
(D1 is ON)
R2 / (R1 + R2)
(D1 is OFF)

b)

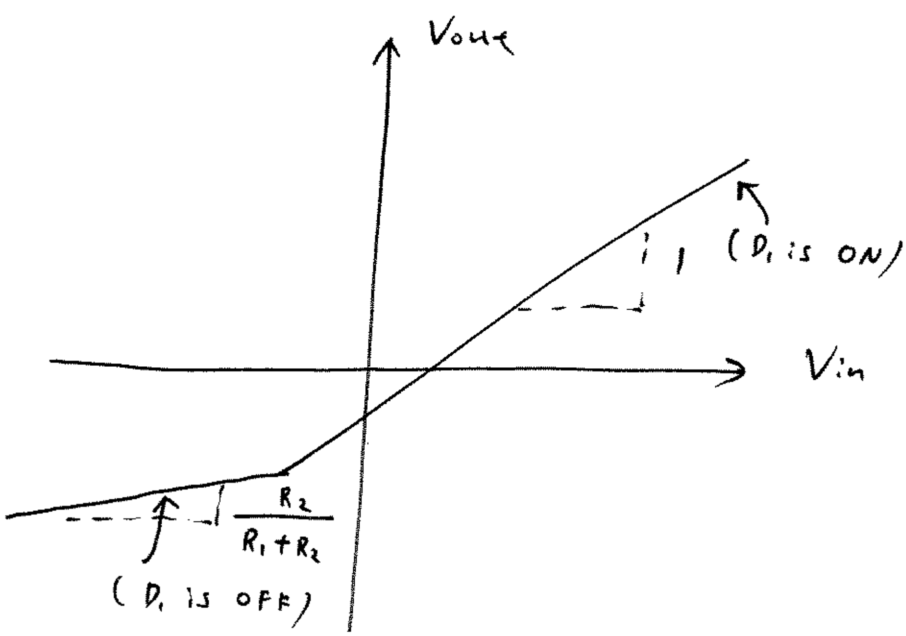


Vout
Vin
1
(D1 is ON)
R2 / (R1 + R2)
(D1 is OFF)

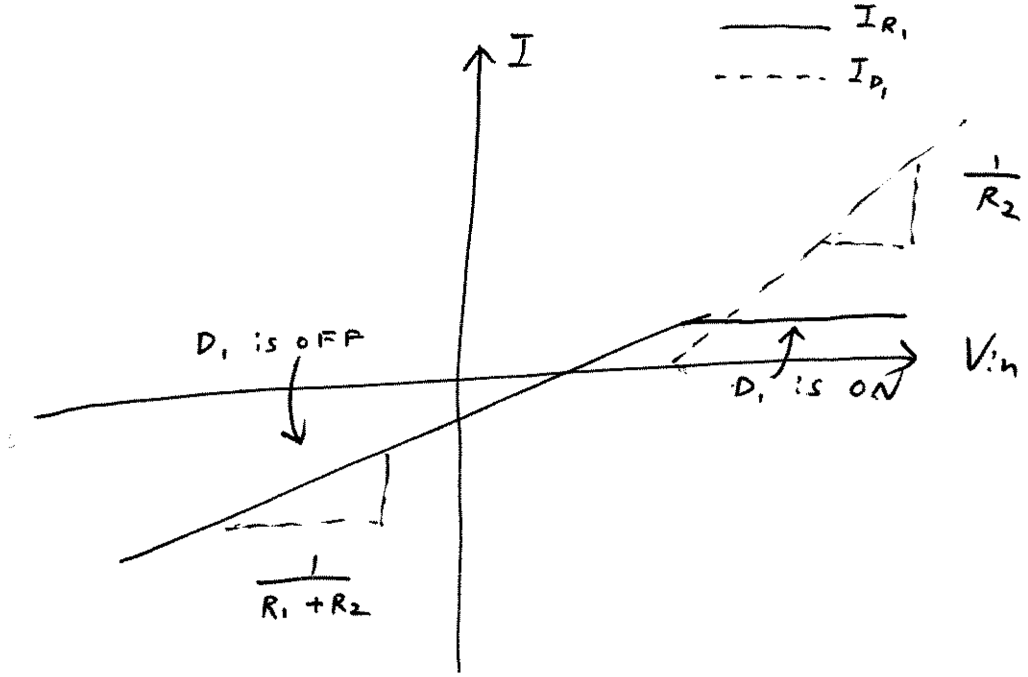
c)


Vout
Vin
$\frac{R_2}{R_1+R_2}$
(D₁ is OFF)
1
(D₁ is ON)

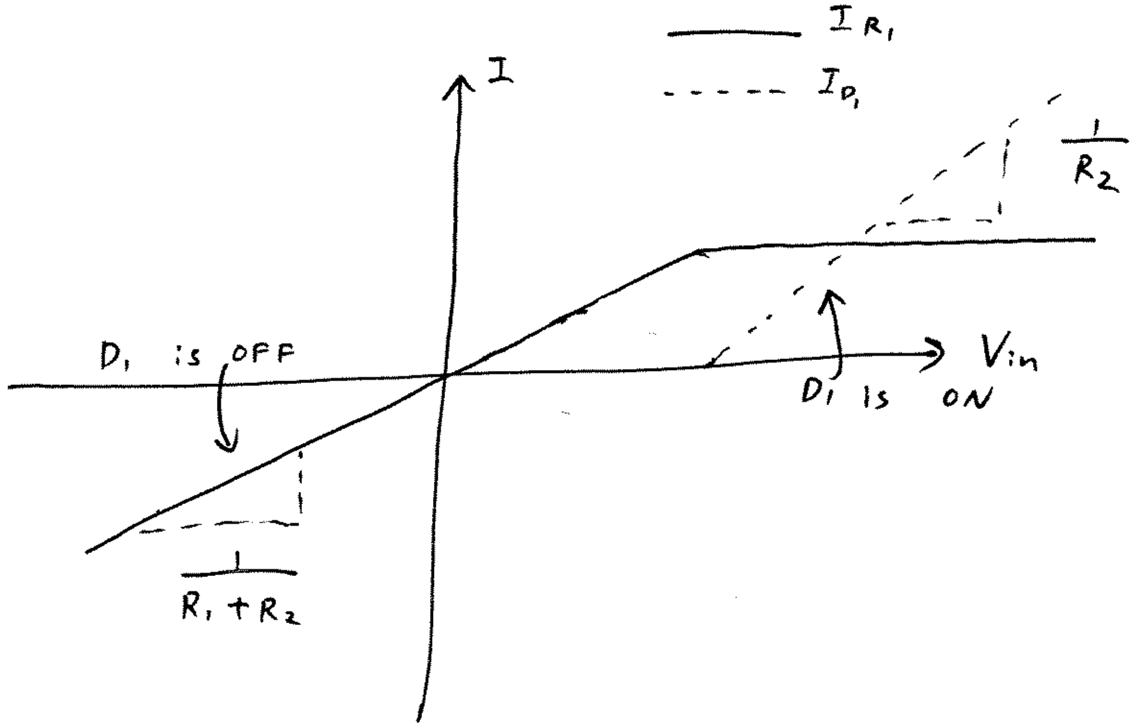
d)


Vout
Vin
1
(D₁ is ON)
$\frac{R_2}{R_1+R_2}$
(D₁ is OFF)

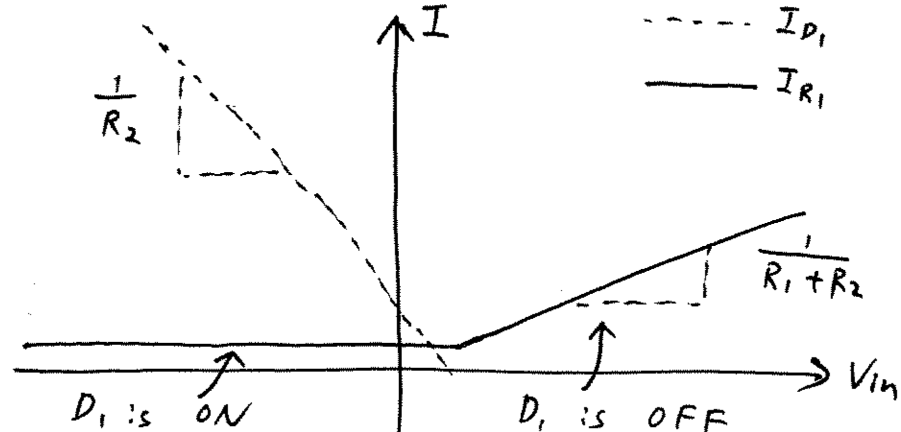
(26) a)


I
I_{R_1}
I_{D_1}
$\frac{1}{R_2}$
D_1 is OFF
D_1 is ON
V_{in}
$\frac{1}{R_1+R_2}$

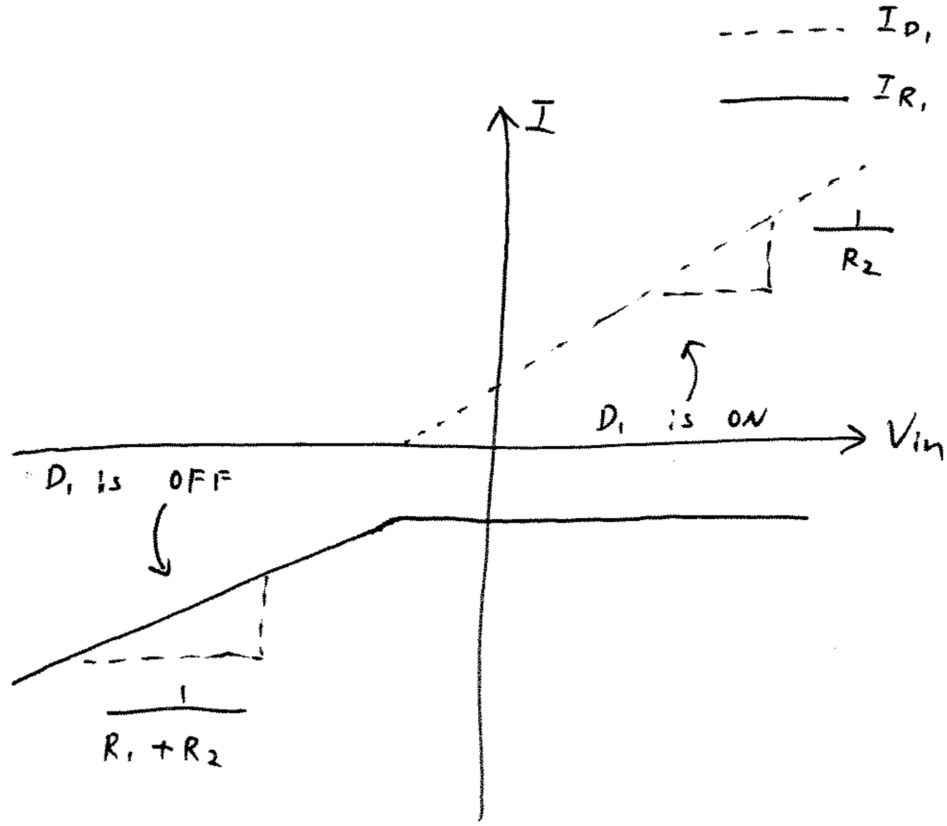
b)


I
I_{R_1}
I_{D_1}
$\frac{1}{R_2}$
D_1 is OFF
D_1 is ON
V_{in}
$\frac{1}{R_1+R_2}$

c/

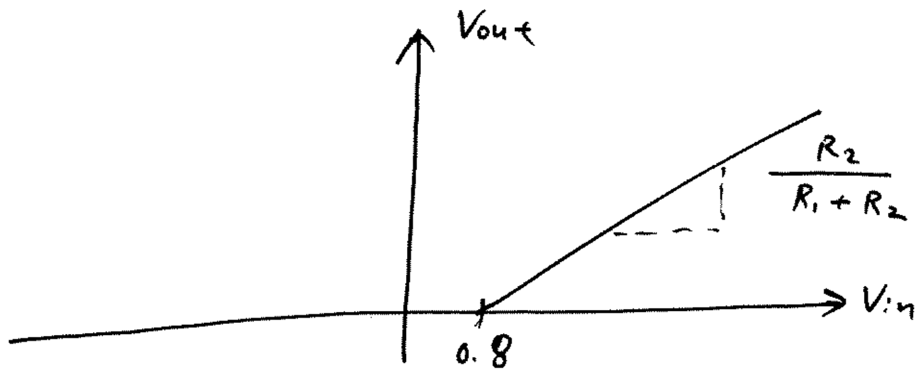

I
I_{D_1}
I_{R_1}
$\frac{1}{R_2}$
$\frac{1}{R_1 + R_2}$
V_{in}
D_1 is ON
D_1 is OFF

d/

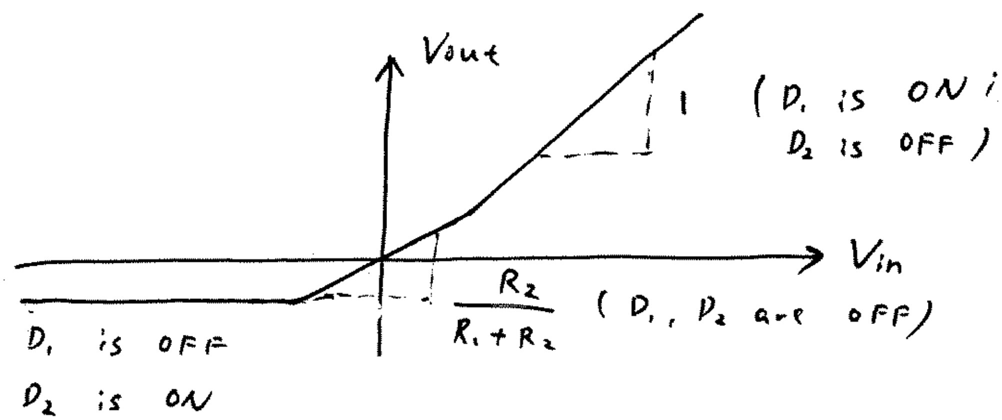

I_{D_1}
I_{R_1}
I
$\frac{1}{R_2}$
D_1 is ON
V_{in}
D_1 is OFF
$\frac{1}{R_1 + R_2}$

(27)

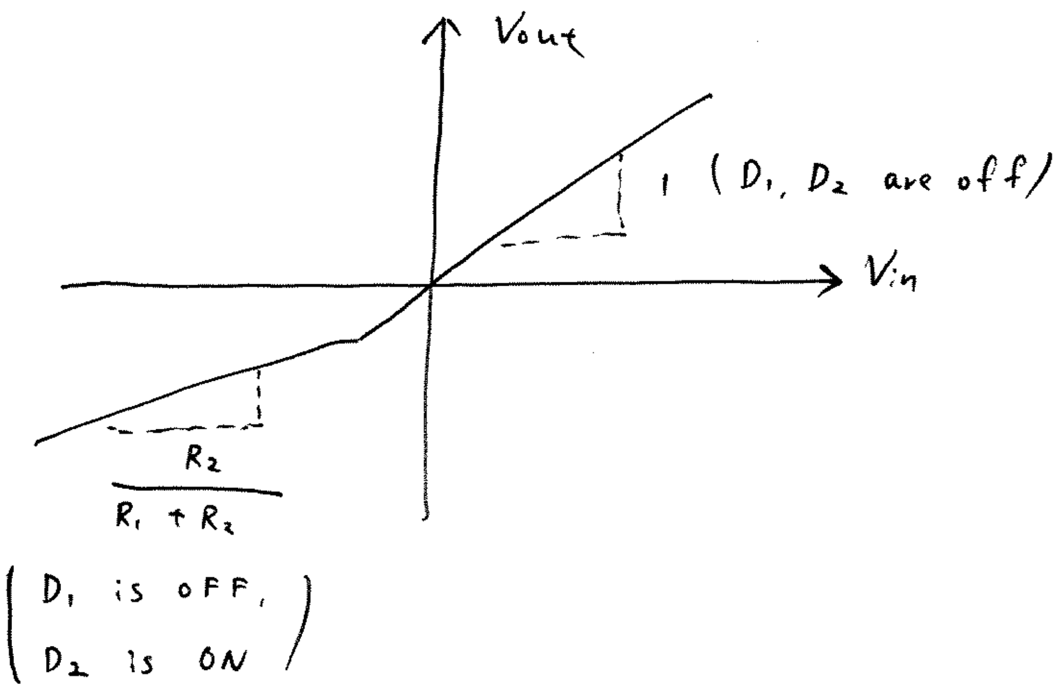
a)


Vout
Vin
0.8
R2
R1 + R2

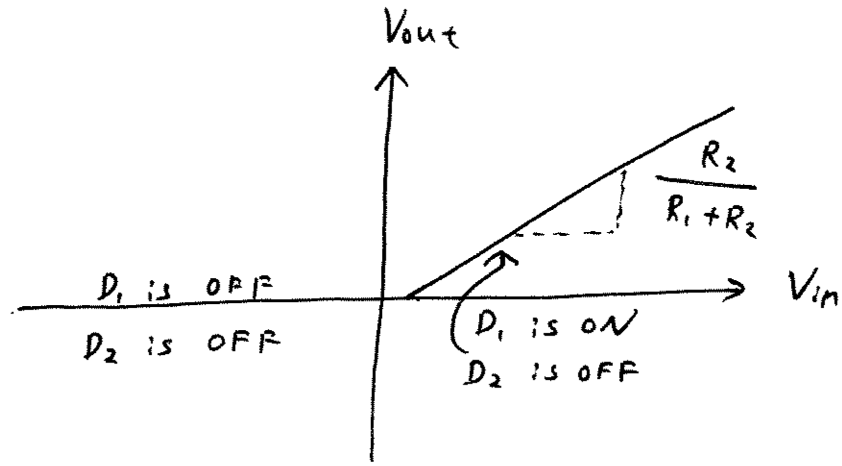
b)


Vout
Vin
1 (D1 is ON;
D2 is OFF)
R2
R1 + R2
(D1, D2 are OFF)
D1 is OFF
D2 is ON

c)

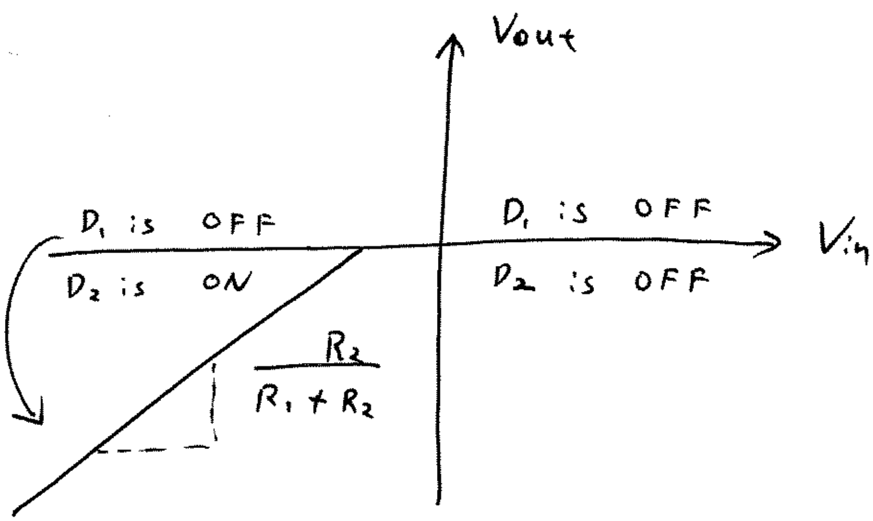

Vout
Vin
1 (D1, D2 are off)
R2
R1 + R2
(D1 is OFF,
D2 is ON)

d)



Vout
Vin
R2 / R1 + R2
D1 is OFF
D2 is OFF
D1 is ON
D2 is OFF

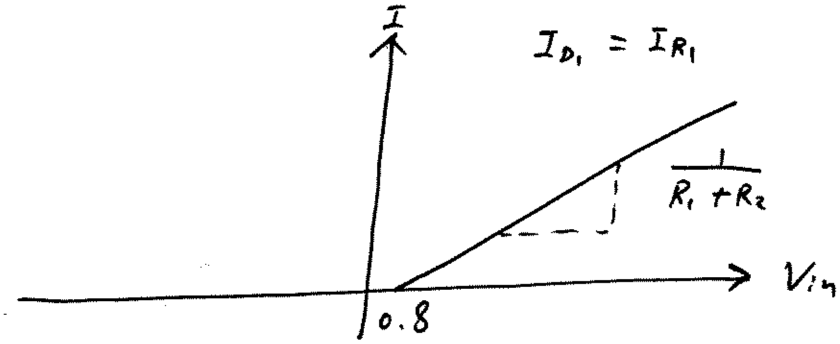
e)



Vout
Vin
D1 is OFF
D2 is ON
D1 is OFF
D2 is OFF
R2 / R1 + R2

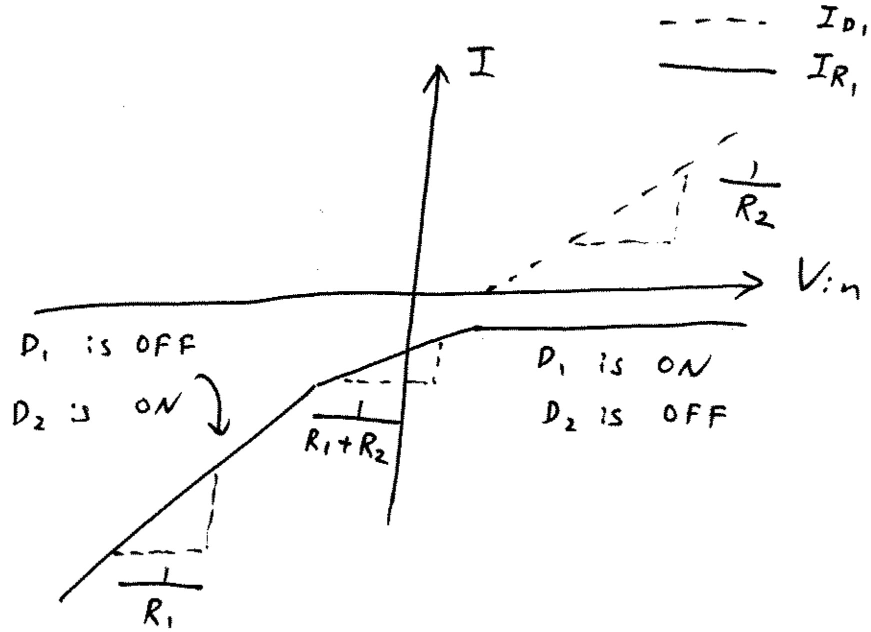
(28)

a)



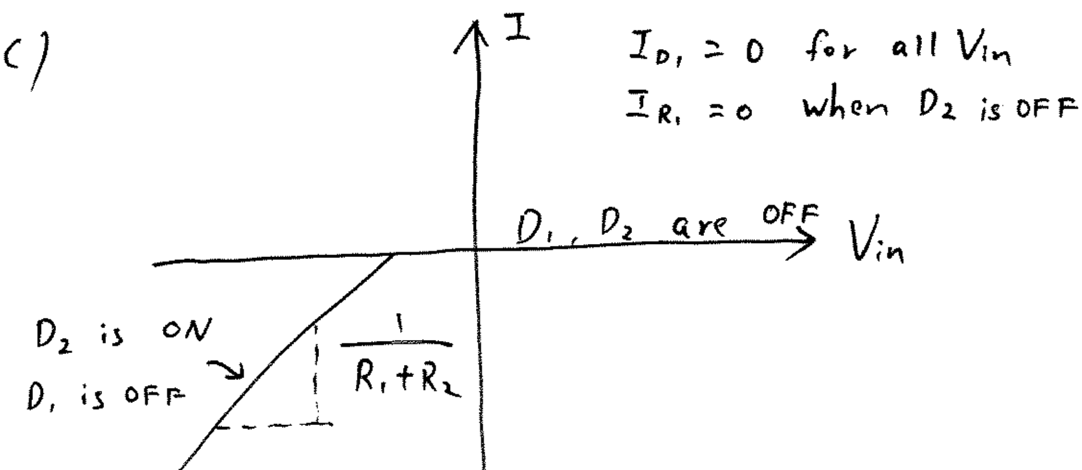
I
$I_{D_1} = I_{R_1}$
$\frac{1}{R_1 + R_2}$
V_{in}
0.8

b)



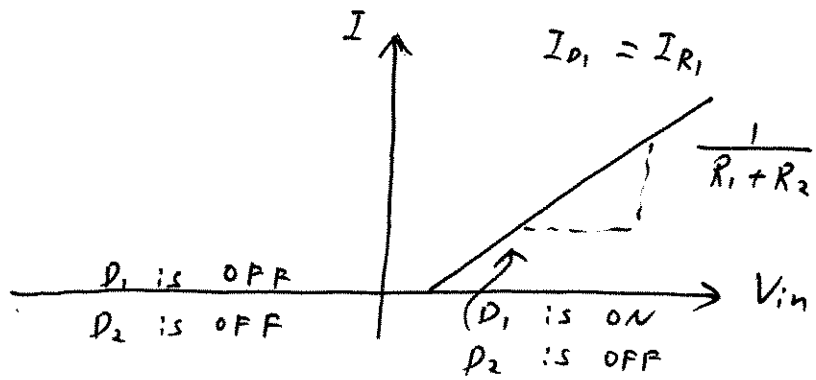
I_{D_1}
I_{R_1}
I
$\frac{1}{R_2}$
V_{in}
D_1 is OFF
D_2 is ON
D_1 is ON
D_2 is OFF
$\frac{1}{R_1+R_2}$
$\frac{1}{R_1}$

c)

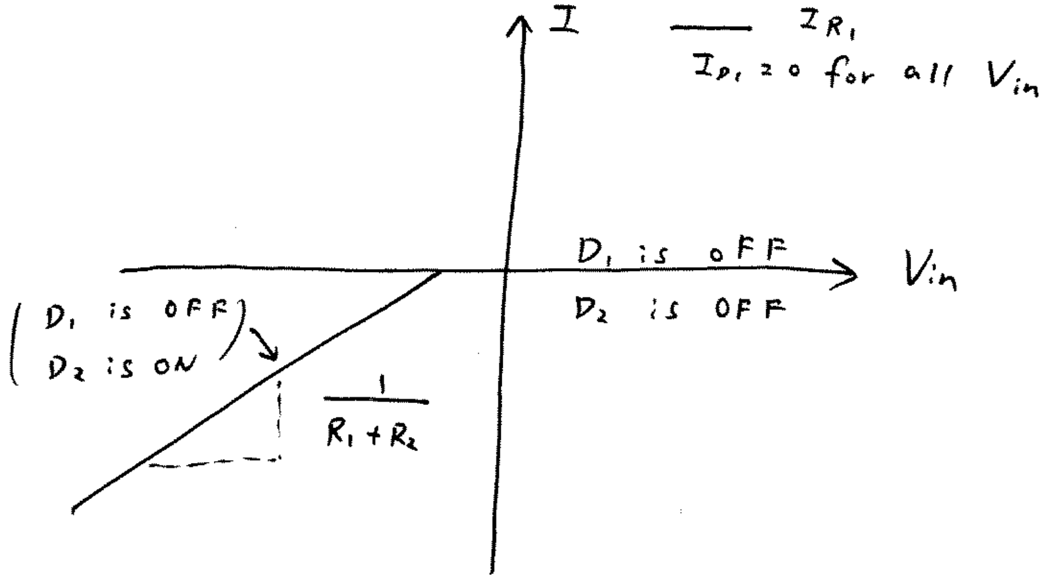


I
$I_{D_1} = 0$ for all V_{in}
$I_{R_1} = 0$ when D_2 is OFF
D_1, D_2 are OFF
V_{in}
D_2 is ON
D_1 is OFF
$\frac{1}{R_1+R_2}$

d)

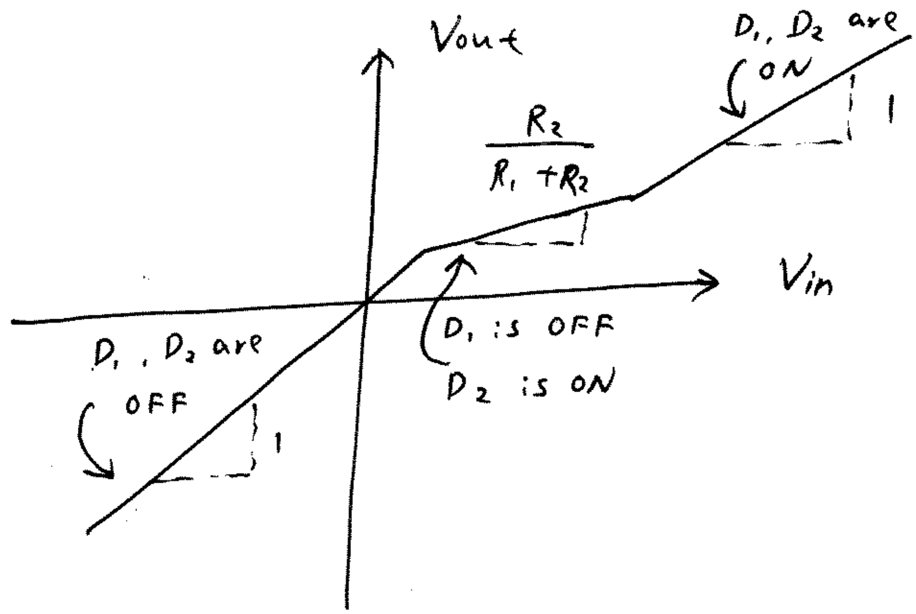

I
$I_{D_1} = I_{R_1}$
$\frac{1}{R_1+R_2}$
D_1 is OFF
D_2 is OFF
V_{in}
D_1 is ON
D_2 is OFF

e)

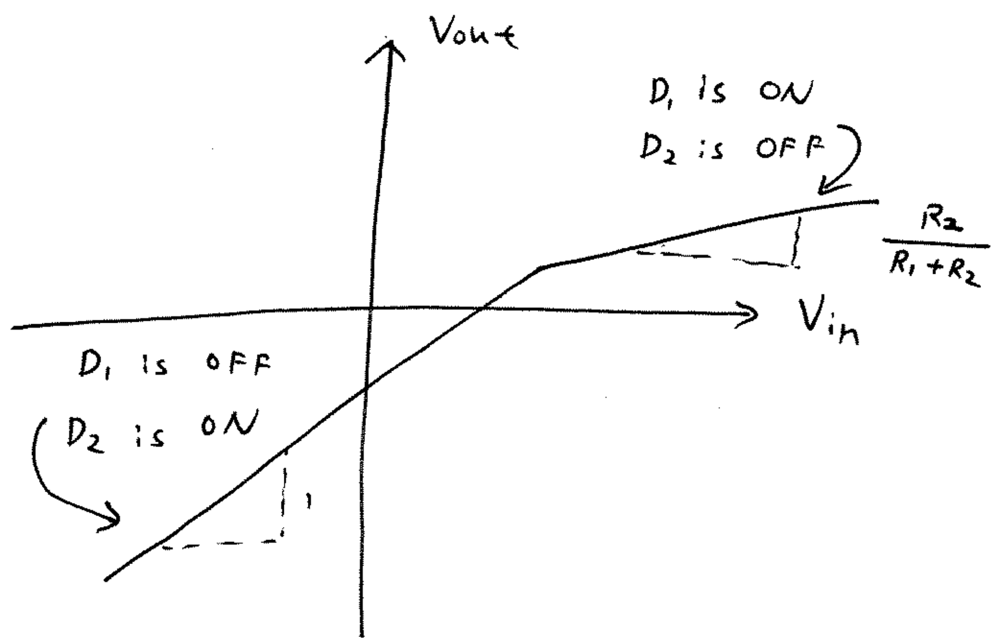

I
I_{R_1}
$I_{D_1} = 0$ for all V_{in}
D_1 is OFF
D_2 is OFF
V_{in}
D_1 is OFF
D_2 is ON
$\frac{1}{R_1+R_2}$

(29)

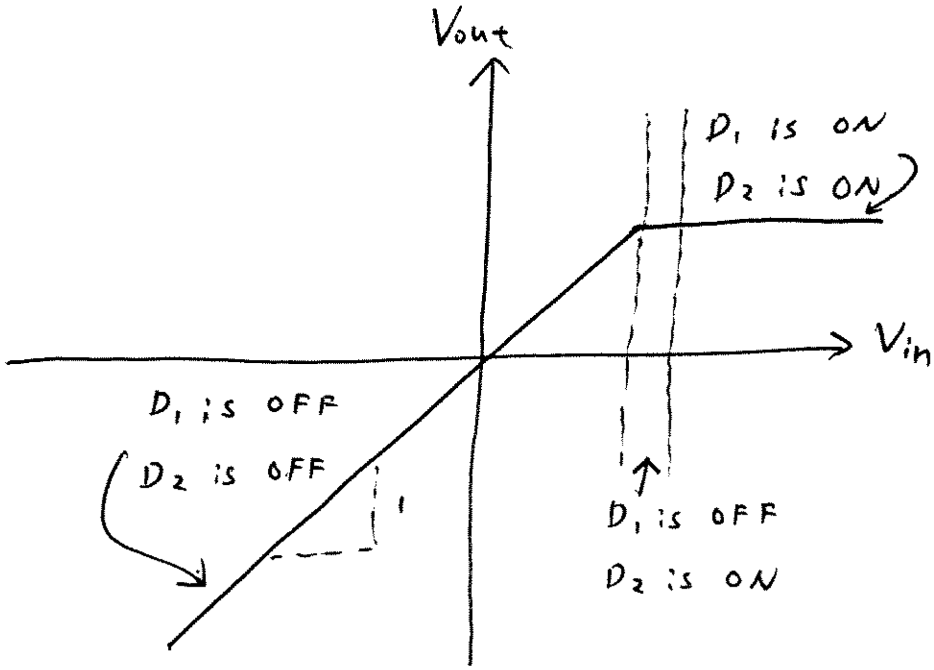
a)


Vout
$\frac{R_2}{R_1+R_2}$
D1, D2 are ON
1
Vin
D1 is OFF
D2 is ON
D1, D2 are OFF
1

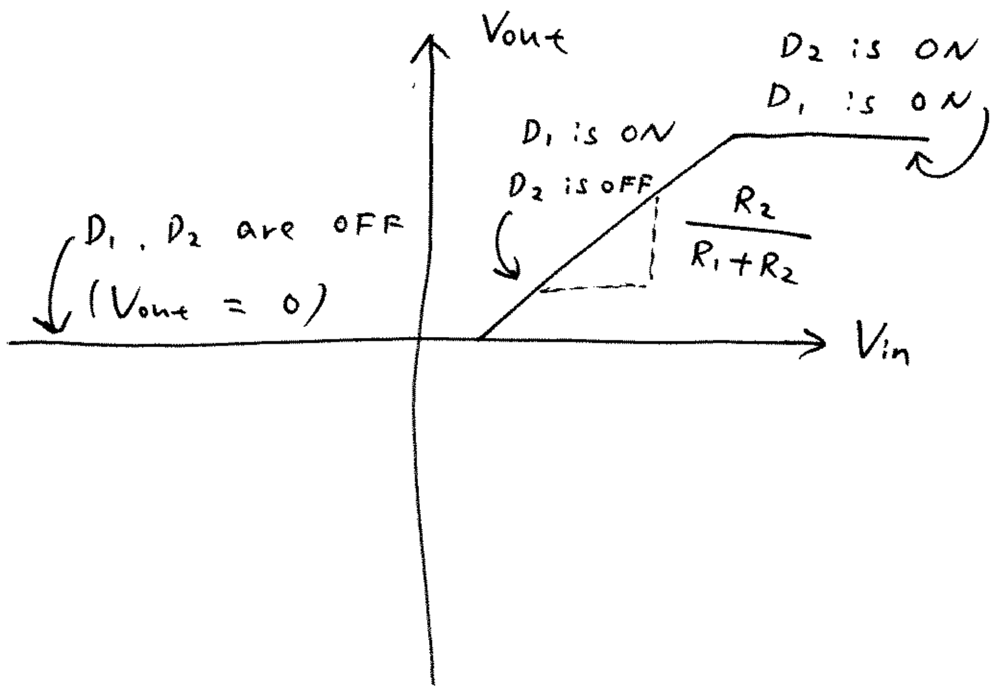
b)


Vout
D1 is ON
D2 is OFF
$\frac{R_2}{R_1+R_2}$
Vin
D1 is OFF
D2 is ON
1

c)

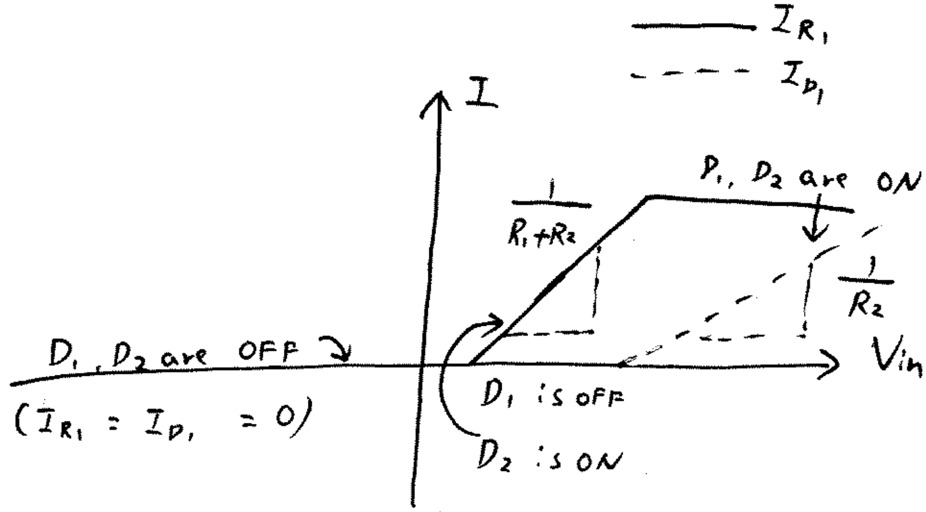

Vout
Vin
D1 is ON
D2 is ON
D1 is OFF
D2 is OFF
1
D1 is OFF
D2 is ON

d)


Vout
Vin
D2 is ON
D1 is ON
D1 is ON
D2 is OFF
R2 / R1+R2
D1 , D2 are OFF
(Vout = 0)

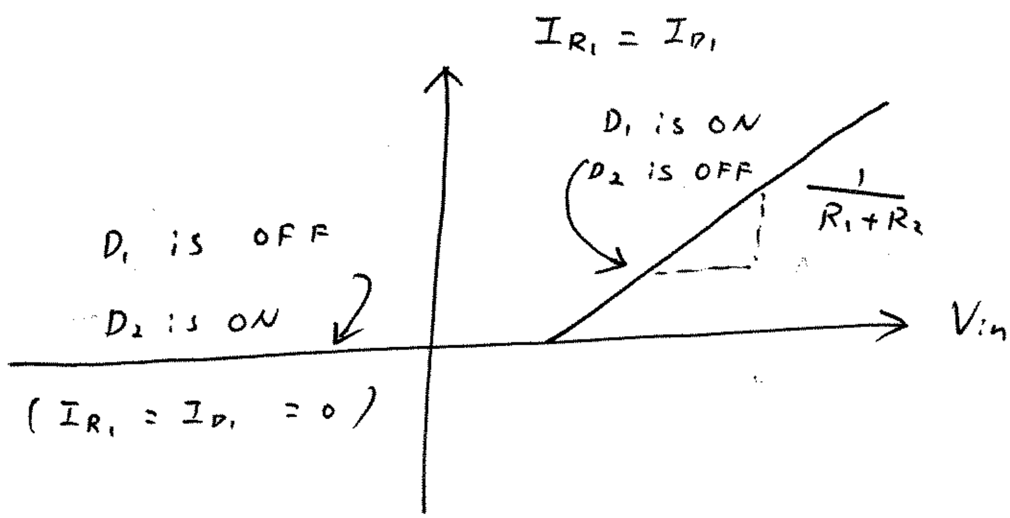
(30)

a)



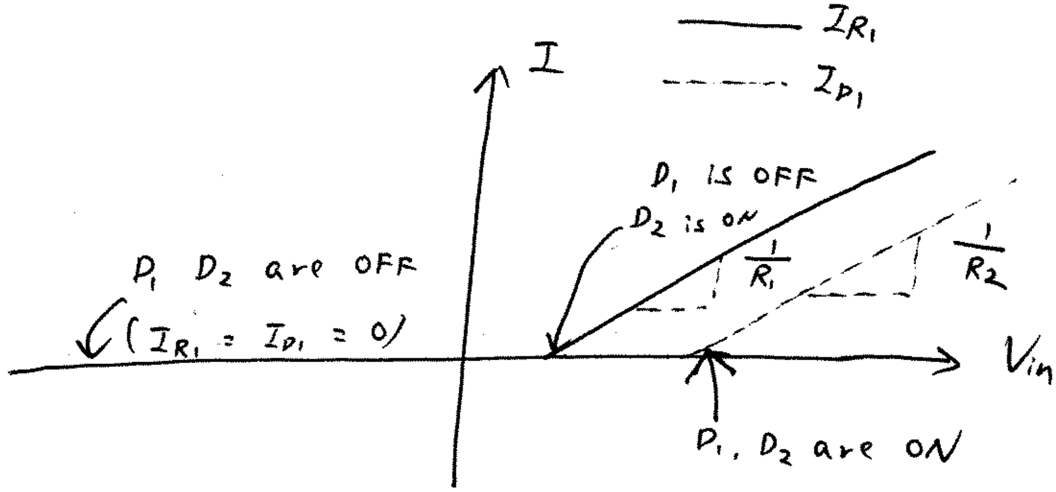
I_{R_1}
I_{D_1}
I
D_1, D_2 are ON
$\frac{1}{R_1+R_2}$
$\frac{1}{R_2}$
D_1, D_2 are OFF
($I_{R_1} = I_{D_1} = 0$)
D_1 is OFF
D_2 is ON
V_{in}

b)



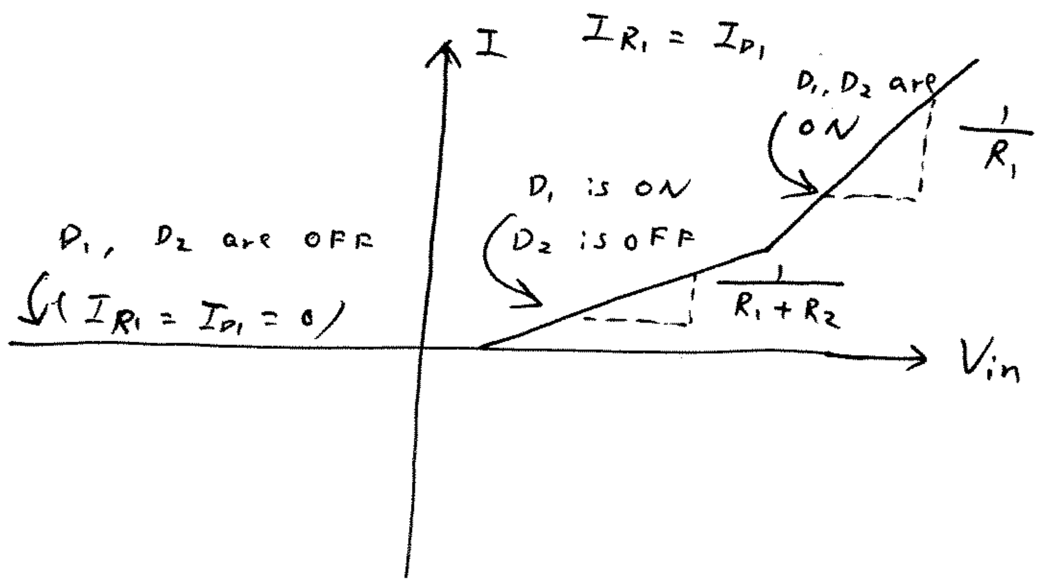
$I_{R_1} = I_{D_1}$
D_1 is ON
D_2 is OFF
$\frac{1}{R_1+R_2}$
D_1 is OFF
D_2 is ON
($I_{R_1} = I_{D_1} = 0$)
V_{in}

c)



I
I_{R_1}
I_{D_1}
D_1 is OFF
D_2 is ON
$\frac{1}{R_1}$
$\frac{1}{R_2}$
D_1 D_2 are OFF
($I_{R_1} = I_{D_1} = 0$)
V_{in}
D_1, D_2 are ON

d)



I
$I_{R_1} = I_{D_1}$
D_1, D_2 are ON
$\frac{1}{R_1}$
D_1 is ON
D_2 is OFF
D_1, D_2 are OFF
($I_{R_1} = I_{D_1} = 0$)
$\frac{1}{R_1 + R_2}$
V_{in}

(31) a) when V_{in} changes from $+2.4V$ to $+2.5V$,

D_1 is ON throughout the change.

$V_{out} \approx V_{in} - 0.8V$,

∴

i.e., V_{out} changes from $+1.6V$ to $+1.7V$.

b) when V_{in} changes from $+2.4V$ to $+2.5V$,

D_1 and D_2 are both ON.

∴ $V_{out} \approx V_{in} - V_{ON,D1}$,

i.e., V_{out} changes from $+1.6V$ to $+1.7V$.

c) when V_{in} changes from $+2.4V$ to $+2.5V$,

D_1 and D_2 are both ON.

$V_{out} \approx V_{ON,D2}$,

i.e., V_{out} stays at $+0.8V$.

d) when V_{in} changes from $+2.4V$ to $+2.5V$,

D_2 is ON.

∴ $V_{out} \approx V_{ON,D2}$,

i.e., V_{out} stays at $+0.8V$

(32) a) $V_{out} = i \times R_1$

$= 0.1\,mA \times 1\,k\Omega$

$= 0.1\,V$

b) $r_{d1} = r_{d2} = \dfrac{26\,mV}{3\,mA.}$ (Eq. 3.58)

$\simeq 8.67\,\Omega.$

$V_{out} = i \times (R_1 + r_{d2})$

$= 0.1\,mA\,(1.00867\,k\Omega)$

$\approx 1.009 \times 10^{-1}\,V$

c) $V_{out} = i \times r_{d2}$

$= 0.1\,mA \times 8.67$ (from (b))

$= 0.867\,mV$

d) $V_{out} = i \times (R_2 \,//\, r_{d2})$

$\approx i \times r_{d2}$ ($\because R_2 >> r_{d2}$)

$= 0.867\,mV$

(33) a) $i_1 = i_{in}$

$= 0.1\ mA$

b) $i_{r_1} = i_{in}$

$= 0.1\ mA$

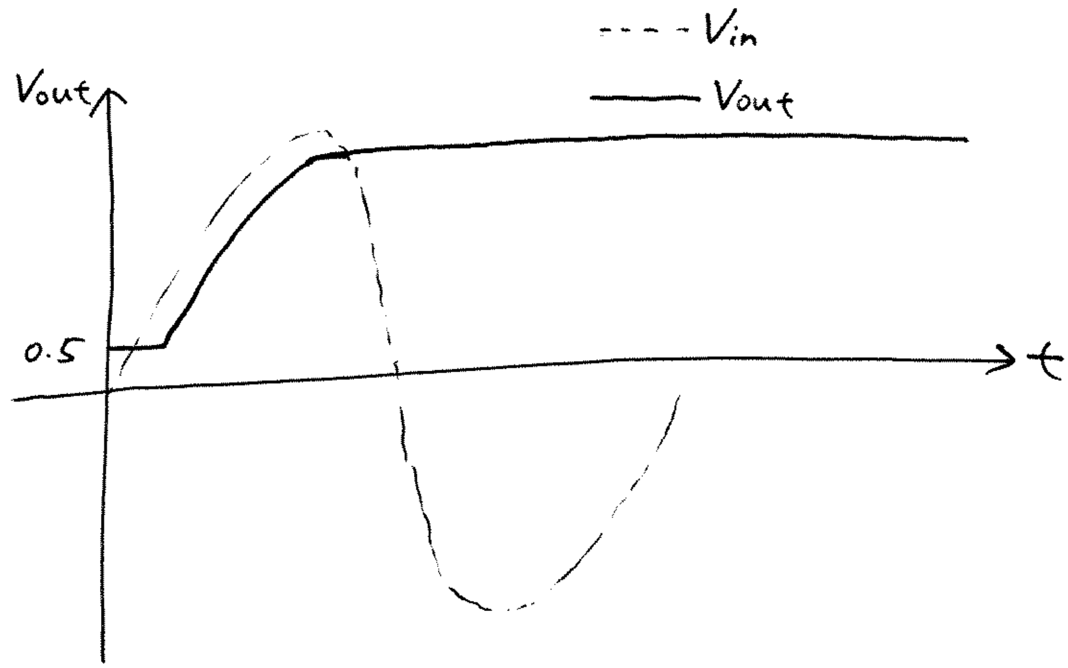
c) $i_{r_1} = i_{in}$

$= 0.1\ mA$

d) $i_{r_1} = i_{in}$

$= 0.1\ mA.$

(34).



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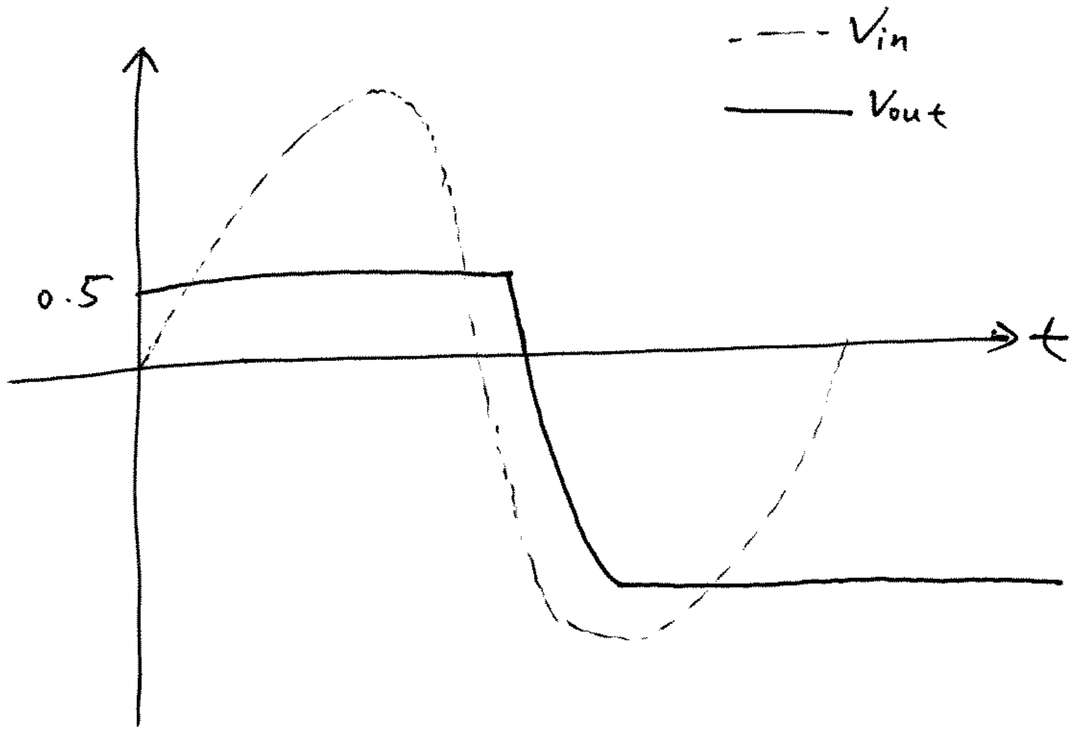


(36) From eq. (3.80),

Ripple amplitude, $V_R \approx \dfrac{V_P - V_{D,on}}{R_L C f_{in}}$

$= \dfrac{3.5 - 0.8}{10 \quad 1000 \times 10^{-6} \times 60}$

$= 0.45V$

(37)

From Eq. (3.83),

$$V_R = \frac{I_L}{C f_{in}}$$

$$\because \quad V_R \leq 300 \text{ mV}$$

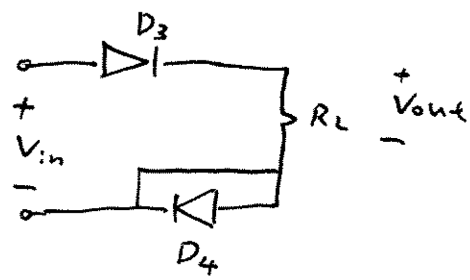
$$\frac{I_L}{C f_{in}} \leq 300 \text{mV}$$

$$\therefore \quad C \geq \frac{I_L}{f_{in} \times 0.3}$$

$$C \geq \frac{0.5}{60 \times 0.3}$$

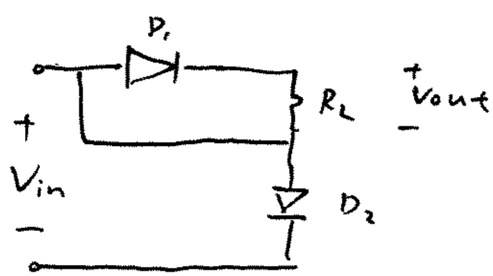
$$\text{i.e.} \quad C \geq 0.278 \text{ F}$$

(38) In the positive half of the cycle, when $V_{in+} > V_{in-}$, the circuit is operating as:



D_4 is shunted, and $D_3 - R_L$ forms a half-wave rectifier.

In the negative half of the cycle, when $V_{in-} > V_{in+}$, the circuit becomes:



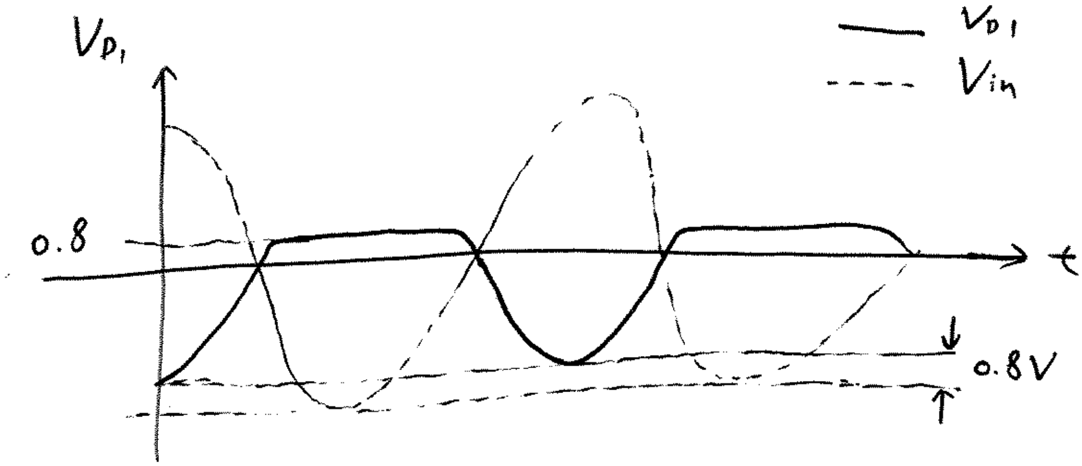
D_1 is shunted and is off.

Thus, $V_{out} = 0$.

Shunting the resistor load with a capacitor has no effect in the above two cases.

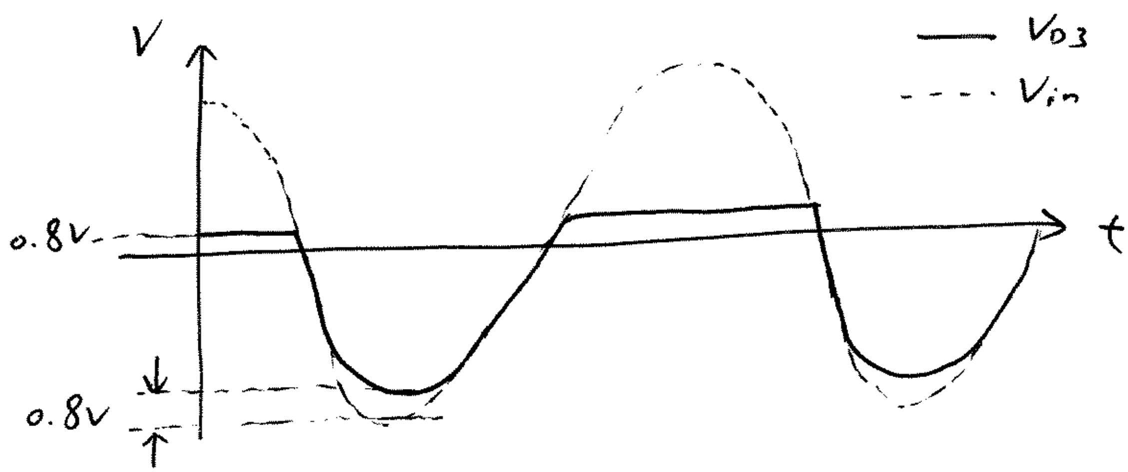
39

(i)

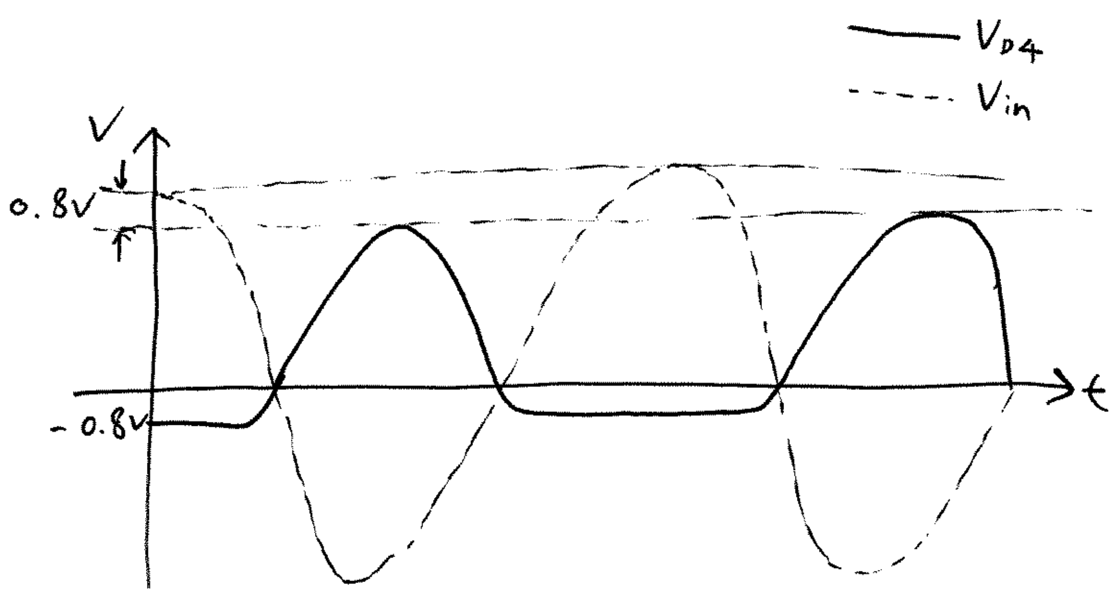


(ii) V_{D2} is same as V_{D1} (above)

(iii)



(iv)



(40) - This circuit would fail to function as a full-wave rectifier.

- It only rectifies for $V_{in-} > V_{in+}$ (Current flows through D_1 and D_2)
- But for $V_{in+} > V_{in-}$, there is no conduction path through the load.
- Thus, this circuit behave like a half-wave rectifier

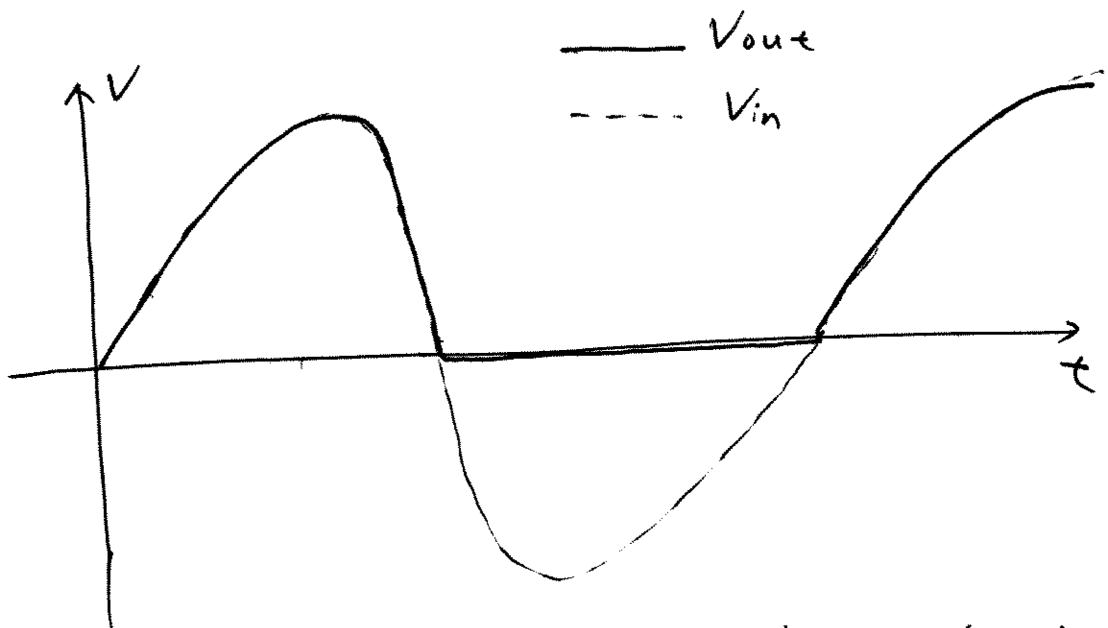
(41) Using Eq. (3.94),

$$V_R \approx \frac{1}{2} \cdot \frac{V_p - 2V_{D,on}}{R_L C_1 f_{in}}$$

$$= \frac{1}{2} \cdot \frac{3 - 2 \times 0.8}{30 \times 1000 \times 10^{-6} \times 60}$$

$$= 0.389V$$

(42)



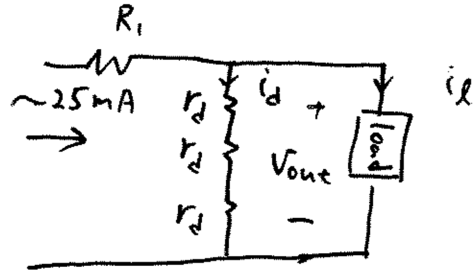
- With the two negative terminals shorted together, the circuit behaves like a half-wave rectifier.

- when $V_{in+} > V_{in-}$, D_3 and D_4 conduct as usual. There will be an additional path that bypasses D_4, since V_{in-} and V_{out-} are shorted. But this additional path causes no change to the V_{out} waveform.

- when $V_{in-} > V_{in+}$, both V_{out+} and V_{out-} track V_{in-}. V_{out+} connects to V_{in-} through D_1; V_{out-} connect to V_{in-} through the additional shorted path.

- Thus $(V_{out+}) - (V_{out-}) = 0$, i.e. $V_{out} = 0$

(43) The circuit can be simplified as:



First, find r_d:

$$r_d = \frac{V_T}{I_D} \quad \text{(from eq. 3.60)}$$

$$= \frac{26\,mV}{5\,mA}$$

$$= 5.2\,\Omega$$

since $i_\ell = +1\,mA$.

$i_d = -1\,mA$.

∴ change in V_{out},

ie. $V_{out} = (-1\,mA)(3 \times 5.2)$

$$= -15.6\,mV$$

(44) a) From Eq. (3.94),

the ripple amplitude, $V_R = \frac{1}{2} \cdot \frac{V_p - 2V_{D,on}}{R_L C_1 f_{in}}$

$= \frac{1}{2} \cdot \frac{5 - 2 \times 0.8}{1000 \times 100 \times 10^{-6} \times 60}$

$= 0.283\ V$

b) The ripple across the load,

$v_\ell = i \times 3 r_d$,

where i is the change in current flowing through R_1, in series with the 3 diodes.

$\because r_d = \frac{V_T}{I_D}$

$\approx \frac{26mV}{5/R_1} = 5.2\ \Omega$

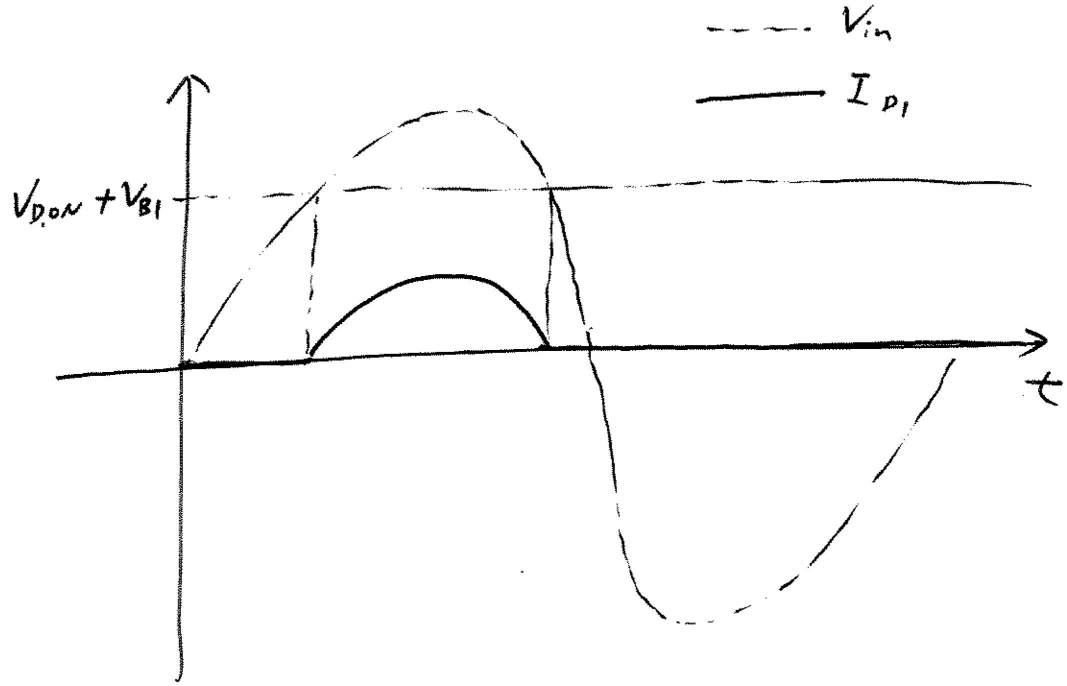
$i \approx \frac{V_R}{R_1 + 3 r_d}$

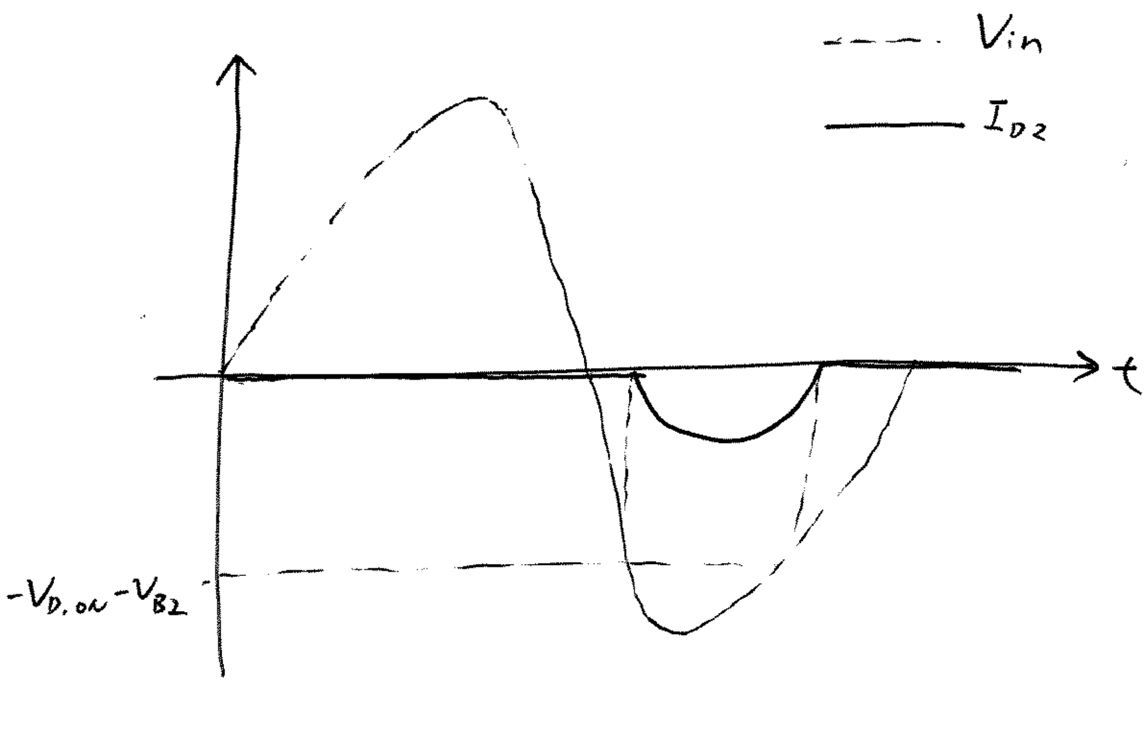
$= 0.279\ mA$

$\therefore v_\ell = 0.279mA \times 3 \times 5.2$

$= 4.35\ mV$

(45)





(46) With positive theshold = + 2.2V,

$$V_{B1} = 2.2 - 0.8$$

$$= +1.4V$$

with negative threshold = −1.9V,

$$-V_{B2} = -1.9 + 0.8$$

$$= -1.1V.$$

$$V_{B2} = 1.1V$$

To meet the maximum current criterion,

Since $I_{R1} = I_{D1}$ or I_{D2},

I_{D1} or I_{D2} is at max when

I_{R1} is at max.

I_{R1} is at max when $|V_R|$ is max,

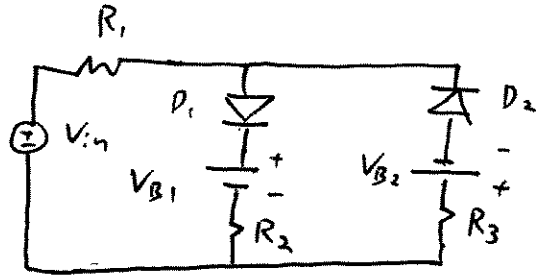
ie. $|V_R| = 5 - 1.9$

$$= 3.1V.$$

Since $I_{R1} \leq 2$ mA.

$R_1 \geq \frac{3.1}{2mA}$, ie. $R_1 \geq 1550\Omega$

(47) The required circuit is:



Similar to Example 3.34,

$$V_{B1} = V_{B2} = (2 - 0.8)V$$
$$= 1.2V$$

To find R_2,

For $V_{in} > 2V$, $\frac{V_{out}}{V_{in}}$ has a slope of 0.5.

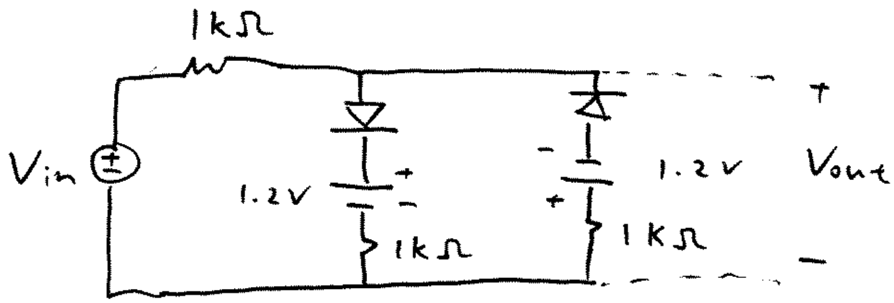
This implies $R_2 = R_1$

(R_1 and R_2 forms a volt. divider).

Similarly, $R_3 = R_1$

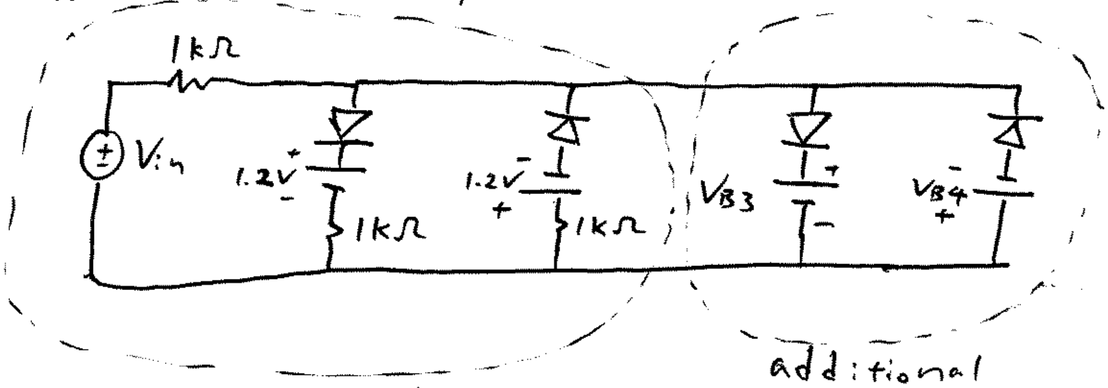
Thus, set $R_1 = R_2 = R_3 = 1\,k\Omega$.

The resulting circuit is:



(48) For $|V_{in}| < 4V$, the $V_{out} - V_{in}$ characteristic is similar to prob. (47).

To get voltage limiting characteristic for $V_{in} > 4V$, and $V_{in} < -4V$, we can shunt the circuit used in prob(47) with two antiparallel diodes as below:



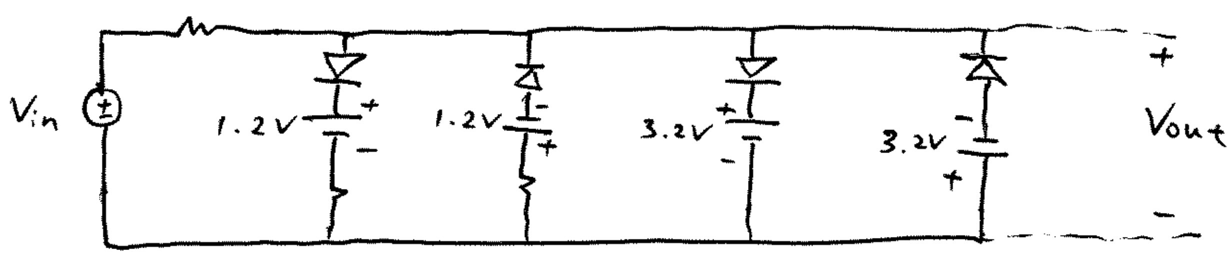
ckt used in prob. 47

additional antiparallel diodes

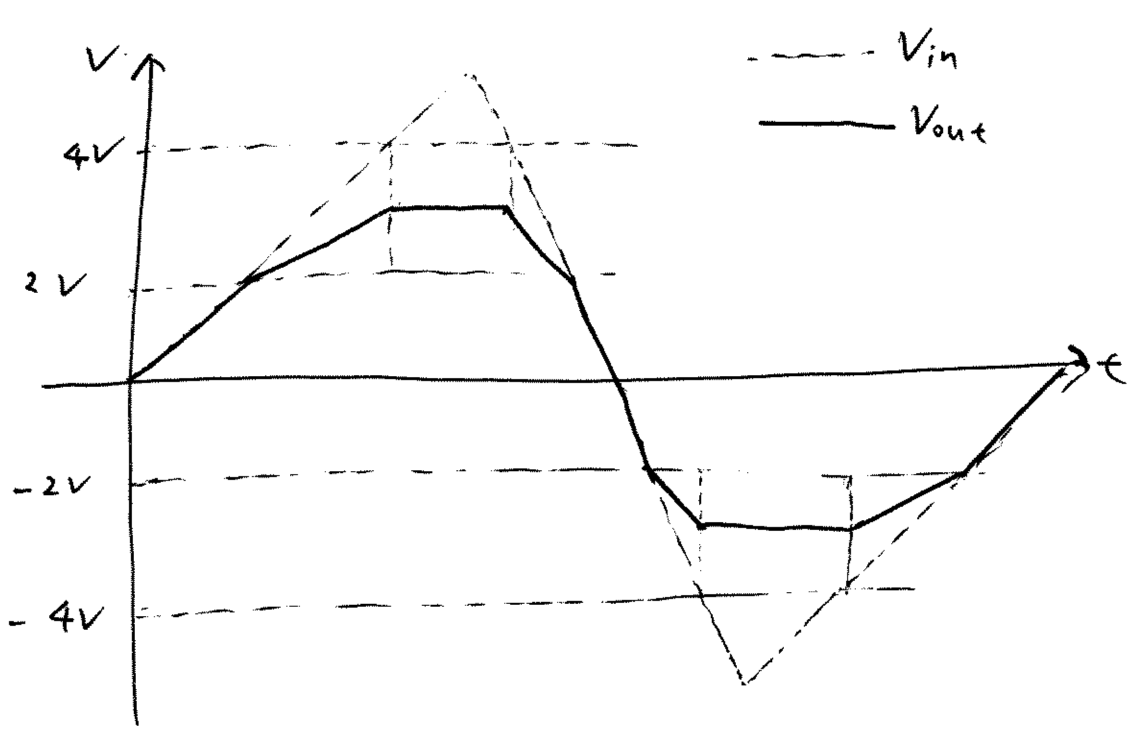
$$V_{B3} = V_{B4} = 4 - 0.8$$

$$= 3.2V$$

Resulting circuit is:

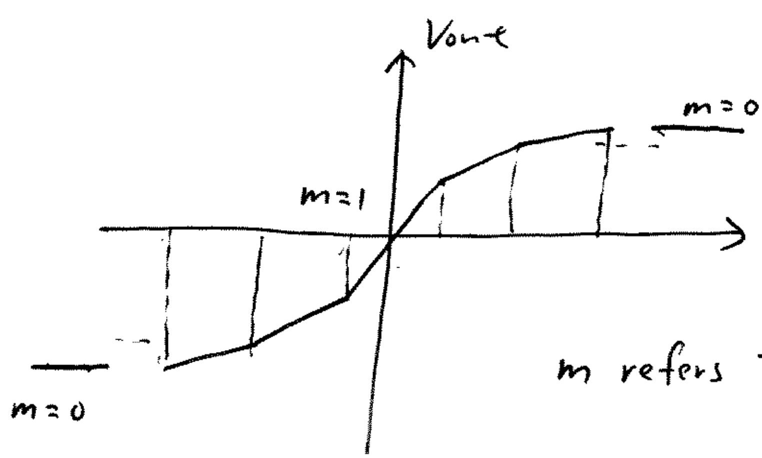


(49)



To get a better approximate of a sinusoid, the slope of the input-output characteristic should decrease <u>more gradually</u> from 1 to 0 through more sections.

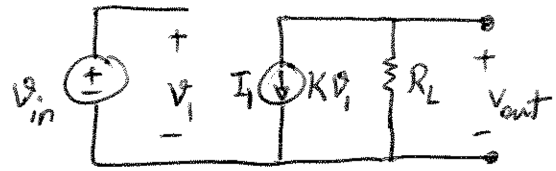
<u>eg</u>:



m refers to slope

Chapter 4

(4.1)



$K = 20\ mA/V$

$\left|\frac{V_{out}}{V_{in}}\right| = 15 \qquad V_{in} = V_1$

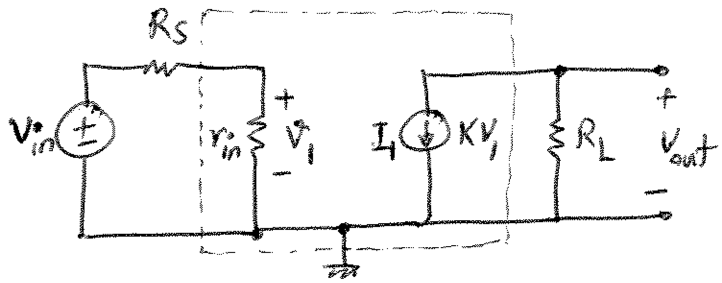
$$V_{out} = -I_1 R_L = -K R_L V_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -K R_L \Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = K R_L$$

$$\Rightarrow K R_L = 15 \Rightarrow R_L = \frac{15}{20\ mA/V} = 750\ \Omega$$

$$\boxed{R_L = 750\ \Omega}$$

4.2



$$\frac{V_{out}}{V_{in}} = ?$$

$$V_1 = \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$I_1 = K V_1$$

$$V_{out} = -R_L I_1$$

$$\Rightarrow V_{out} = -K R_L V_1$$

$$\Rightarrow V_{out} = -K R_L \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$\Rightarrow \boxed{A_V = \frac{V_{out}}{V_{in}} = -K R_L \frac{r_{in}}{r_{in} + R_S}}$$

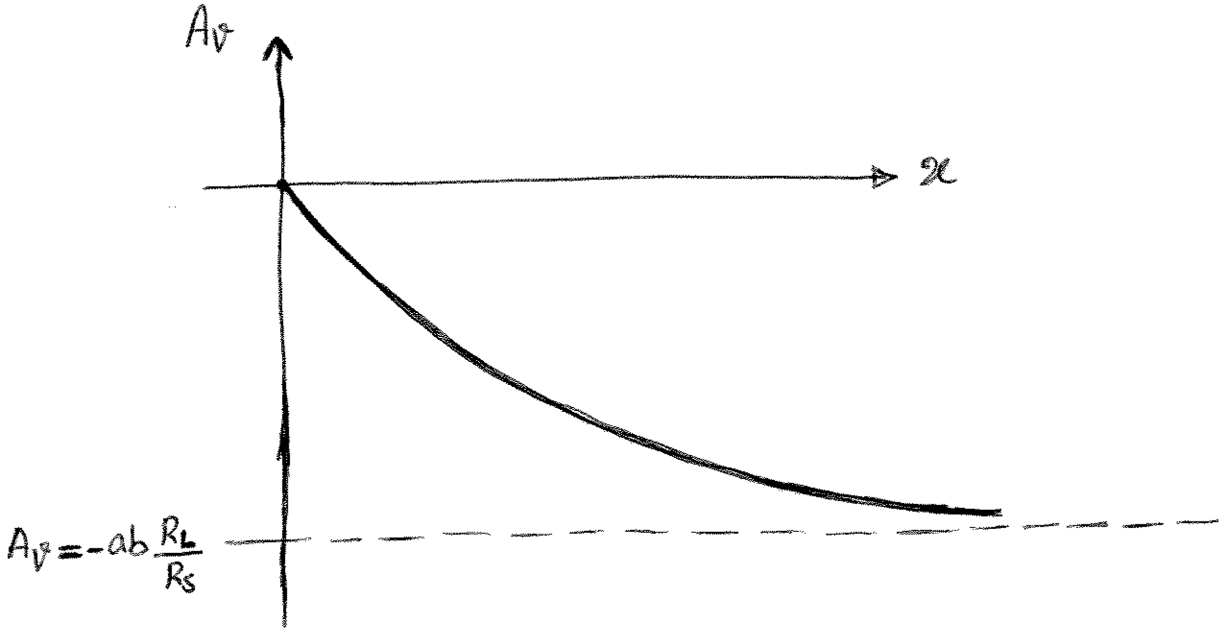
(4.3) From solution for problem 4.2,

$a > 0$
$b > 0$
$x \geq 0$

$$A_v = -KR_L \frac{r_{in}}{r_{in} + R_S}$$

$$\xrightarrow[K = bx]{r_{in} = a/x} A_v = -bx R_L \frac{a/x}{a/x + R_S} = -bR_L \frac{a}{\frac{a}{x} + R_S}$$

$$\Longrightarrow \boxed{A_v = -bR_L \left(\frac{x}{1 + \frac{R_S}{a} x} \right)}$$



(4.4) From equation (4.8) page 136,

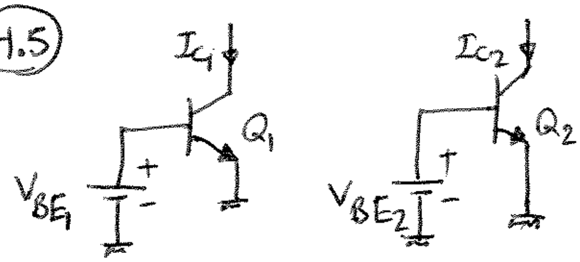
$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}}$$

$W_B \equiv$ width of the Base

if $W_B \uparrow 2 \Rightarrow I_C \downarrow 2$

Collector current decreases by a factor of two

(4.5)



$V_T = 26\,mV$

$I_{C_1} = I_{C_2}$

$V_{BE_1} - V_{BE_2} = 20\,mV$

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$ equation (4.8) page 136

$$\Rightarrow I_C \simeq \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}}$$ $A_E \equiv$ Cross Section

if $I_{C_1} = I_{C_2}$

$$\Rightarrow A_{E_1} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_1}}{V_T}} = A_{E_2} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_2}}{V_T}}$$

$$\Rightarrow \frac{A_{E2}}{A_{E_1}} = \frac{e^{V_{BE1}/V_T}}{e^{V_{BE2}/V_T}}$$

$$\Rightarrow \frac{A_{E2}}{A_{E_1}} = e^{(V_{BE_1} - V_{BE2})/V_T} = e^{20\,mV/26\,mV}$$

$$\Rightarrow \boxed{\frac{A_{E2}}{A_{E_1}} = e^{\frac{20}{26}} \simeq 2.16}$$

(6a) $I_X = 1^{mA} \Rightarrow I_{Q_1} = I_{Q_2} = 0.5^{mA}$

$$I_{Q_1} = I_{S_1} e^{\frac{V_{BE_1}}{V_T}} \Rightarrow 5\times10^{-4} = 3\times10^{-16} e^{\frac{V_B}{26^{mV}}}$$

$$\Rightarrow V_B = 26^{mV} \ln\left(\frac{5}{3}\times10^{12}\right) \Rightarrow \boxed{V_B \simeq 731.7^{mV}}$$

(6b) $I_Y = I_{S_3} e^{\frac{V_B}{V_T}}$

$$\Rightarrow I_{S_3} = I_Y e^{-\frac{V_B}{V_T}} = 2.5\times10^{-3} \times e^{-\frac{V_B}{26^{mV}}} = 2.5\times10^{-3} \times \frac{1}{\frac{5}{3}\times10^{12}}$$

$$\Rightarrow \boxed{I_{S_3} = 1.5\times10^{-15} A}$$

(7a) $I_X = I_1 + I_2$

$$\Rightarrow I_X = I_{S_1} e^{\frac{V_B}{V_T}} + I_{S_2} e^{\frac{V_B}{V_T}} \Longrightarrow I_X = (I_{S_1} + I_{S_2}) e^{\frac{V_B}{V_T}}$$

$$\Rightarrow V_B = V_T \ln\left(\frac{I_X}{I_{S_1} + I_{S_2}}\right) \xrightarrow{I_{S_1} = 2I_{S_2}} \boxed{V_B = V_T \ln\left(\frac{I_X}{\frac{3}{2} I_{S_1}}\right)}$$

$$V_B = 26\times10^{-3} \ln\left(\frac{1.2\times10^{-3}}{\frac{3}{2}\times5\times10^{-16}}\right) \Longrightarrow \boxed{V_B \simeq 730.6\ mV}$$

(7b) Transistors at the edge of the active mode $\Rightarrow V_C = V_B$

applying KVL, we have:

$$V_{CC} = R_C I_X + V_B \Longrightarrow \boxed{R_C = \frac{V_{CC} - V_B}{I_X}}$$

$$\Rightarrow R_C = \frac{2.5 - 0.73}{1.2\times10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 1475\ \Omega}$$

(8a) Same as 7a,

$$\boxed{V_B \simeq 730.6\,mV}$$

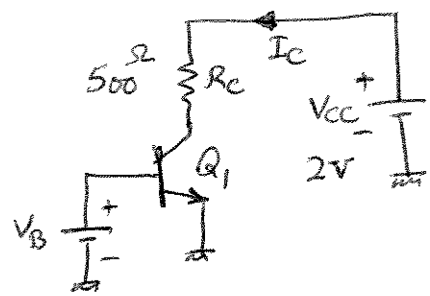
(8b) According to 7b,

$$R_C = \frac{V_{CC} - V_B}{I_X} = \frac{1.5 - 0.73}{1.2 \times 10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 642\ \Omega}$$

(9) Q_1 is at the edge of the active region $\Rightarrow V_C = V_B$

applying KVL, we have:



$$V_{CC} = R_C I_C + V_C$$

$$\xrightarrow{V_C = V_B} V_{CC} = R_C I_C + V_B$$

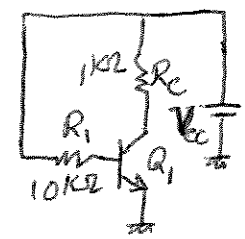
$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\Rightarrow 500\Omega \times 5\times10^{-16} e^{\frac{V_B}{26\,mV}} + V_B = 2V$$

Using numerical methods or simply, trial & error:

$$\boxed{V_B \simeq 760\ mV}$$

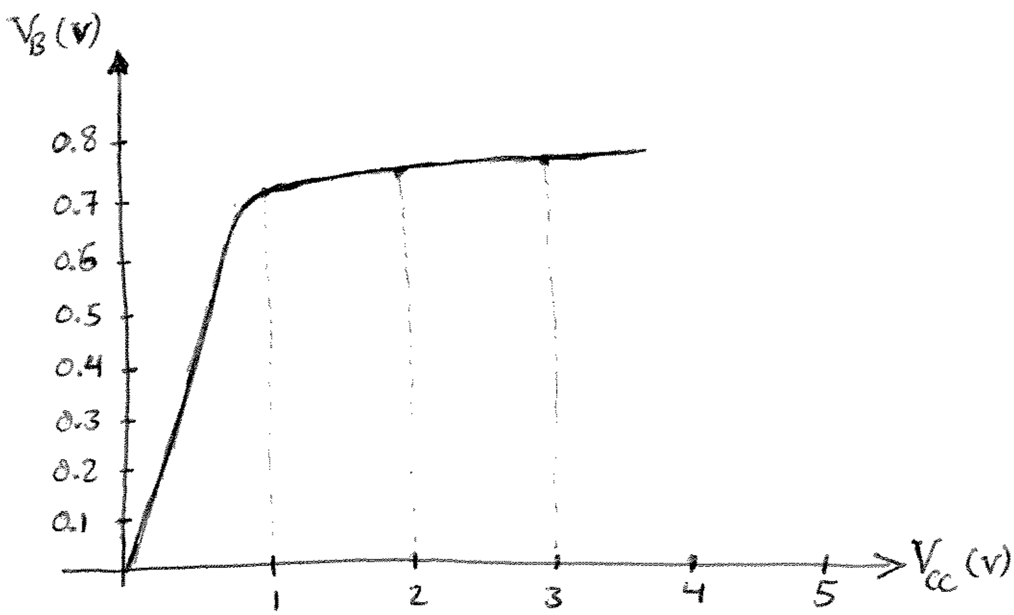
(10) Q_1 at the edge of saturation $\Rightarrow V_C = V_B$



Hence: $V_{CC} = R_C I_C + V_B$

$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\xrightarrow{I_S = 3\times10^{-16}A} V_{CC} = 3\times10^{-13} e^{V_B/V_T} + V_B \xrightarrow[V_{CC}=2^V]{\text{with}} \boxed{V_B \simeq 755\ mV}$$



(11) Assuming $I_E \simeq I_C$, we can write:

Applying KVL: $1.5^V = V_{BE} + V_X$ where $V_X = 1^{k\Omega} \times I_E$

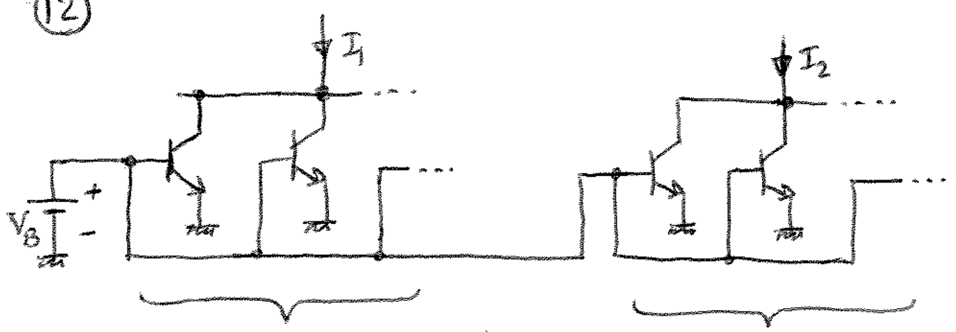
Hence: $1.5 = V_{BE} + 1^{k\Omega} \times I_C$

$$\Rightarrow 1.5 = V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}}$$

$$\xrightarrow[V_T = 26mV]{I_S = 6\times10^{-16}A} 1.5 = V_{BE} + 6\times10^{-13} e^{V_{BE}/26^{mV}} \Rightarrow \boxed{V_{BE} \simeq 724.5^{mV}}$$

$$V_X = 1.5 - V_{BE} \Rightarrow \boxed{V_X \simeq 775.5\ mV}$$

(12)



$$\left.\begin{aligned} I_1 &= n\,I_C = n\,I_S\,e^{V_B/V_T} \\ I_2 &= m\,I_C = m\,I_S\,e^{V_B/V_T} \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{n}{m}$$

$$\Rightarrow \boxed{\frac{n}{m} = \frac{1^{mA}}{1.5^{mA}} = \frac{2}{3}} \xrightarrow{\text{choose}} \boxed{\begin{aligned} n &= 2 \\ m &= 3 \end{aligned}}$$

$$I_1 = n I_C = n I_S\, e^{\frac{V_B}{V_T}}$$

$$\Rightarrow I_1 = n \times 3\times10^{-16}\, e^{\frac{V_B}{26^m}} = 1^{mA} \overset{n=2}{\Longrightarrow} \boxed{V_B \simeq 750^{mV}}$$

(13) Using the same technique as in problem 12, we have:

$$\frac{n_1}{I_1} = \frac{n_2}{I_2} = \frac{n_3}{I_3}$$

$$\Rightarrow \frac{n_1}{0.2} = \frac{n_2}{0.3} = \frac{n_3}{0.45} \Rightarrow \boxed{\frac{n_1}{4} = \frac{n_2}{6} = \frac{n_3}{9}}$$

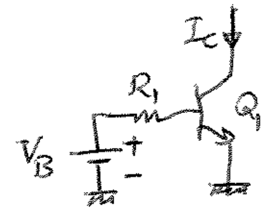
So lets choose $\boxed{\begin{cases} n_1 = 4 \\ n_2 = 6 \\ n_3 = 9 \end{cases}}$

Hence,

$$I_1 = n_1 I_S e^{V_B/V_T} \Rightarrow 0.2 \times 10^{-3} = 4 \times 3 \times 10^{-16} e^{\frac{V_B}{26mV}}$$

$$\Rightarrow \boxed{V_B \simeq 672 \text{ mV}}$$

(14) From KVL,



$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$I_B = \frac{I_C}{\beta} = \frac{1^{mA}}{100} \Rightarrow \boxed{I_B = 10^{-5} A}$$

$$V_{BE_{Q_1}} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26\times10^{-3} \ln\left(\frac{10^{-3}}{7\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE_{Q_1}} \simeq 727.7\ mV}$$

Therefore,

$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$\simeq 10^{K\Omega} \times 10^{-5} A + 728\times10^{-3}$$

$$\Rightarrow V_B \simeq 0.1 + 0.728 \Rightarrow \boxed{V_B \simeq 0.828\ V}$$

(15) According to the solution for problem 14, we have:

Applying KVL: $V_B = R_B I_B + V_{BE}$

$$\Rightarrow V_B = R_B \frac{I_C}{\beta} + V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$\Rightarrow 0.8 = 10^4 \times \frac{I_C}{100} + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

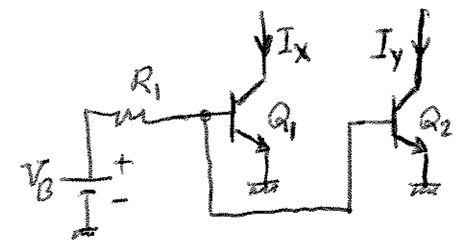
$$\Rightarrow 0.8 = 100\, I_C + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

using trial & error or numerical methods,

$$\boxed{I_C \simeq 7.85\times10^{-4}\,A = 785\,\mu A}$$

(16)

$$\begin{cases} I_X = I_{S_1} \exp\left(\frac{V_{BE_1}}{V_T}\right) \\ I_Y = I_{S_2} \exp\left(\frac{V_{BE_2}}{V_T}\right) \\ V_{BE_1} = V_{BE_2} = V_{BE} \end{cases}$$



$$\Rightarrow \frac{I_X}{I_Y} = \frac{I_{S_1}}{I_{S_2}} = \frac{2 \cancel{I_{S_2}}}{\cancel{I_{S_2}}} \Rightarrow \boxed{\frac{I_X}{I_Y} = 2}$$

$$\begin{cases} I_X = \beta_1 I_{B_1} \\ I_Y = \beta_2 I_{B_2} \\ \beta_1 = \beta_2 \end{cases}$$

$$\Rightarrow \boxed{\frac{I_{B_1}}{I_{B_2}} = \frac{I_X}{I_Y} = 2}$$

Applying KVL:

$$V_B = R_1 (I_{B_1} + I_{B_2}) + V_{BE}$$

$$V_{BE} = V_{BE_1} = V_T \ln\left(\frac{I_X}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{1^{mA}}{4\times10^{-16}}\right) \simeq 742\, mV$$

$$I_{B_1} = \frac{I_X}{\beta} \xrightarrow{\beta=100} I_{B_1} = \frac{1^{mA}}{100} = 10\mu A$$

$$\frac{I_{B_1}}{I_{B_2}} = 2 \longrightarrow I_{B_2} = \frac{I_{B_1}}{2} = \frac{10^{\mu A}}{2} \Rightarrow I_{B_2} = 5\mu A$$

Hence: $V_B = 5\times10^3 \Omega \times (10\mu A + 5\mu A) + 0.742^V$

$$= 0.075 + 0.742 \Longrightarrow \boxed{V_B \simeq 0.817^V}$$

(17) Applying KVL:

$$V_B = R_1 (I_{B_1} + I_{B_2}) + V_{BE} \xrightarrow{\beta_1=\beta_2=\beta} \frac{R_1}{\beta}(I_{C_1} + I_{C_2}) + V_{BE}$$

$$\Rightarrow V_B = \frac{R_1}{\beta}(I_{S_1} + I_{S_2}) \exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE}$$

$$\xRightarrow{\beta=100} 0.8^V = \frac{5000^\Omega}{100}(3\times10^{-16} + 5\times10^{-16}) \exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

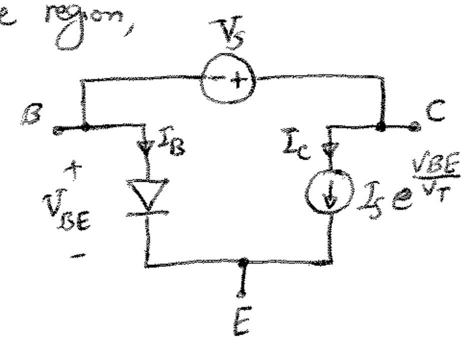
$$\Rightarrow 0.8^V = 4\times10^{-14} \cdot \exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

Numerical methods or Trial & error $\Rightarrow$ $\boxed{V_{BE} \simeq 732\ mV}$

$$I_X = I_{S_1} \exp\left(\frac{V_{BE}}{V_T}\right) = 3\times10^{-16}\left[\exp\left(\frac{732}{26}\right)\right] \Rightarrow \boxed{I_X \simeq 506\ \mu A}$$

$$I_y = I_{S_2} \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-16} \exp\left(\frac{732}{26}\right) \Rightarrow \boxed{I_y \simeq 843\ \mu A}$$

(18) Since Transistor is in Forward active region,



No change across V_{BE}

⇓

No change in I_B

⇓

No change in I_C

(19) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow g_m = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T} \Rightarrow \boxed{V_{BE} = V_T \ln\left(\frac{g_m V_T}{I_S}\right)}$$

$$\xrightarrow[g_m = \frac{1}{13\Omega}]{I_S = 6\times10^{-16}A} V_{BE} = 26^{mV} \cdot \ln\left(\frac{\frac{1}{13\Omega} \times 26\times10^{-3}}{6\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 750\ mV}$$

(20) $g_m = \dfrac{I_C}{V_T}$

$$\Delta g_m = \frac{\Delta I_C}{V_T} = \frac{1}{V_T}\Delta\left(I_S e^{V_{BE}/V_T}\right) \simeq \frac{I_S}{V_T^2} e^{\frac{V_{BE}}{V_T}} \Delta V_{BE}$$

$$\Rightarrow \boxed{\Delta g_m \simeq \frac{I_C}{V_T^2} \Delta V_{BE}}$$

$$\Rightarrow \Delta g_m \simeq \frac{g_m}{V_T} \Delta V_{BE}$$

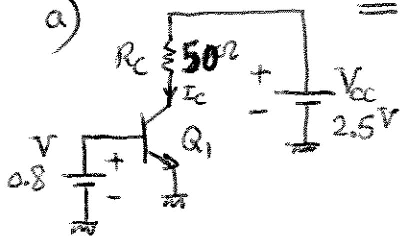
$$\Rightarrow \boxed{\frac{\Delta g_m}{g_m} \simeq \frac{1}{V_T} \Delta V_{BE}}$$

$$\left.\frac{\Delta g_m}{g_m}\right|_{I_C = 1mA} \overset{max}{=\!=} 0.1 \quad \Rightarrow \Delta V_{BE_{max}} = 0.1\, V_T$$

$$\Rightarrow \boxed{\Delta V_{BE} \leqslant 2.6 \text{ mV}}$$

(2) $V_A = \infty \Rightarrow r_o = \infty$, $I_S = 8\times10^{-16}A$, $\beta = 100$

a)



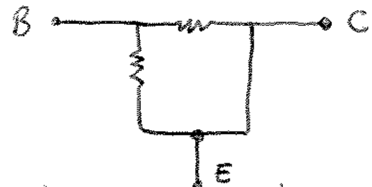
If in Forward active region,

$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$

$= 8\times10^{-16}\exp\left(\frac{800}{26}\right)$

$\Rightarrow \boxed{I_C \simeq 18.5\,mA}$

Small signal model



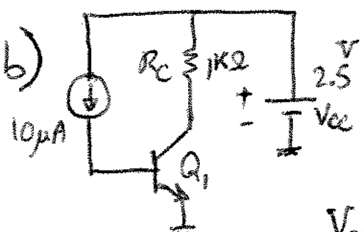
$\Rightarrow \boxed{V_{CE} \simeq 1.575\ V}$

Hence Transistor should be in Forward Active

$I_{C_Q} = \frac{V_{CC} - V_{CE}}{R_C} = \frac{2.5 - 1.575}{50\Omega} = \frac{0.925}{50} \Rightarrow I_{C_Q} \simeq 18.5\,mA$

which matches with I_C

b)



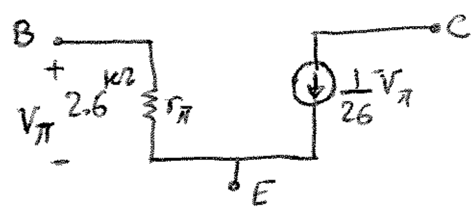
Assuming in Forward active region,

$I_C = \beta I_B = 100\times10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$

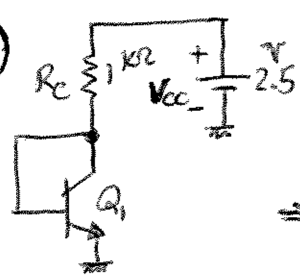
$V_{CE} = V_{CC} - R_C I_C = 2.5 - 1^{k\Omega}\times 1^{mA} \Rightarrow \boxed{V_{CE} = 1.5^V}$

$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26}}\ \Omega^{-1}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi \simeq 2600^{\Omega}}$



c)

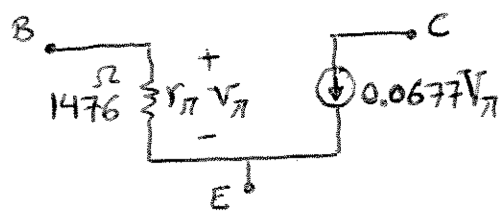


Applying KVL,

$V_{CC} \simeq R_C I_C + V_{BE}$

$\Rightarrow 2.5^V \simeq 1^{k\Omega} \times I_C + V_{BE}$

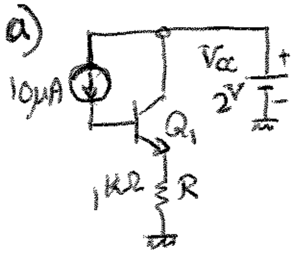
$\Rightarrow 2.5^V \simeq 8\times10^{-13}\cdot\exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE} \Rightarrow \boxed{V_{BE} \simeq 739^{mV}}$



$g_m = \frac{I_C}{V_T} = \frac{\frac{V_{CC} - V_{BE}}{R_C}}{V_T} \Rightarrow g_m = \frac{2.5 - 0.739}{1^{k\Omega}\times 0.026} \Rightarrow \boxed{g_m \simeq 67.7\ mS}$, $\boxed{I_C = 1.76^{mA}}$

(22) $V_A = \infty \Rightarrow r_o = \infty$, $I_S = 8\times10^{-16}A$, $\beta = 100$

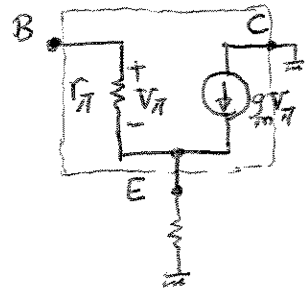
a)



$I_C = \beta I_B = 100\times10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \times \ln\left(\frac{10^{-3}}{8\times10^{-16}}\right)$

$\Rightarrow \boxed{V_{BE} \simeq 724\, mV}$

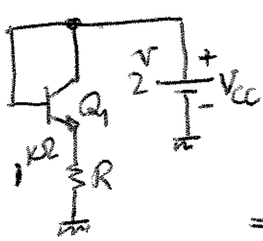


$V_{CE} = V_{CC} - RI_E \simeq V_{CC} - RI_C = 2 - 1^{k\Omega}\times 1^{mA} \Rightarrow \boxed{V_{CE} = 1^V}$

$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi = 2.6^{k\Omega}}$

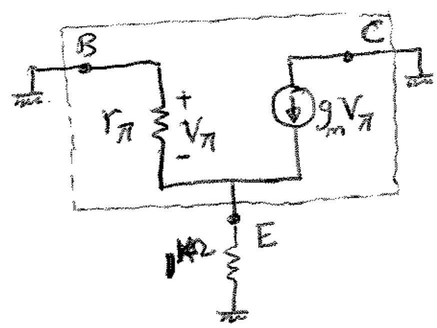
b)



Applying KVL,

$V_{CC} = V_{BE} + RI_E$

$\Rightarrow V_{CC} \simeq V_{BE} + RI_C$



$\Rightarrow V_{CC} \simeq V_{BE} + RI_S \exp\left(\frac{V_{BE}}{V_T}\right)$

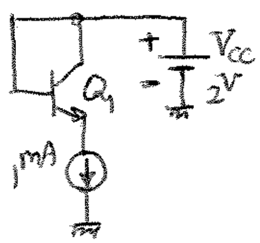
$\Rightarrow 2^V \simeq V_{BE} + 8\times10^{-13}\exp\left(\frac{V_{BE}}{26^{mV}}\right) \Rightarrow \boxed{V_{BE} \simeq 730\, mV}$

$\boxed{V_{CE} = V_{BE} = 730^{mV}}$ $I_C = 8\times10^{-16}\exp\left(\frac{730}{26}\right) \Rightarrow \boxed{I_C \simeq 1.2mA}$

$g_m = \frac{I_C}{V_T} = \frac{1.2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 46\, mS}$ $r_\pi = \frac{\beta}{g_m} \xrightarrow{\beta=100} \boxed{r_\pi \simeq 2167^{\Omega}}$

(22) Continued ...

c)

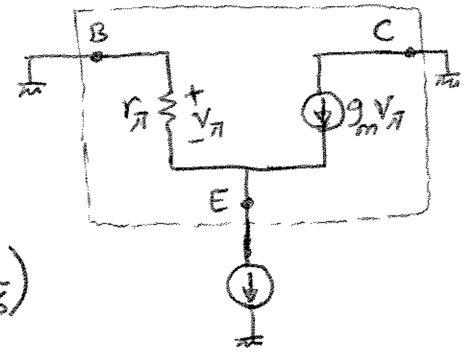


$$\boxed{I_C \simeq I_E = 1^{mA}}$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$= 26^{mV} \ln\left(\frac{1^{mA}}{8\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 724\ mV}$$

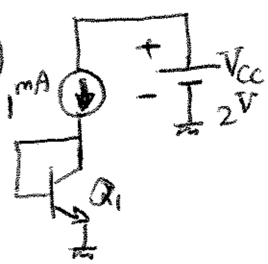


$$\boxed{V_{CE} = V_{BE} = 724\ mV}$$

$$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{1}{26}} \Rightarrow \boxed{r_\pi \simeq 2.6\ k\Omega}$$

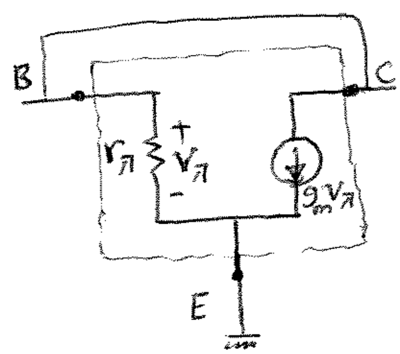
d)



As in part c, we have,

$$\boxed{I_C \simeq 1mA}$$

$$\boxed{V_{CE} \simeq 724\ mV}$$

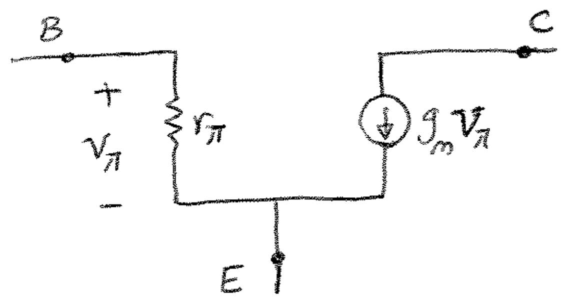


$$\boxed{g_m = \frac{1}{26\Omega}} \qquad \boxed{r_\pi = 2.6\ k\Omega}$$

(23) $I_C = I_S \exp\left(\frac{V_{BE}}{nV_T}\right)$ $\qquad I_C = \beta I_B$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{nV_T} I_S \exp\left(\frac{V_{BE}}{nV_T}\right) = \frac{I_C}{nV_T}$$

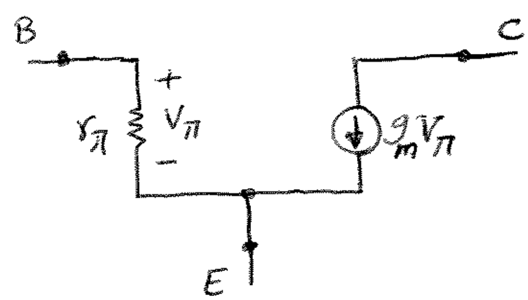
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{\beta}\partial I_C} = \frac{\beta}{g_m} = \frac{n\beta V_T}{I_C}$$



(24) $I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$, $I_C = \alpha I_B^2 \Rightarrow \frac{\partial I_B}{\partial I_C} = \frac{1}{2\sqrt{\alpha I_C}}$

$$g_m = \frac{\partial I_C}{\partial v_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{v_{BE}}{V_T}\right) = \boxed{\frac{I_C}{V_T}}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{2\sqrt{\alpha I_C}}\partial I_C} = \frac{2\sqrt{\alpha I_C}}{g_m} = \frac{2\sqrt{\alpha I_C}}{I_C/V_T} = \boxed{2V_T\sqrt{\frac{\alpha}{I_C}}}$$



(25) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$ V_{BE} is Constant

$$\Delta I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \frac{\Delta I_C}{I_C} = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\cdot \Delta V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]} = \frac{\Delta V_{CE}}{V_A + V_{CE}}$$

$$\frac{\Delta I_C}{I_{C_{min}}} < 0.05 \Rightarrow \frac{\Delta V_{CE}}{V_A + V_{CE_{min}}} < 0.05$$

$$\Rightarrow 20\,\Delta V_{CE} < V_A + V_{CE_{min}}$$

$$\left.\begin{array}{l}\Delta V_{CE} = 2^V \\ V_{CE_{min}} = 1^V\end{array}\right\} \Rightarrow 40 < V_A + 1 \Rightarrow \boxed{V_A > 39^V}$$

(26)

a) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-17} \exp\left(\frac{800^{mV}}{26^{mV}}\right) \simeq 1.15mA$

$V_X = V_{CC} - R_C I_C = 2.5^V - 1^{k\Omega} \times 1.15^{mA}$

$V_X = 1.35V$ Transistor is in Forward Active Region

b) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$\Rightarrow I_C = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5^V}\right]$ equation 1

Also we know: $V_X = V_{CC} - R_C I_C \Rightarrow I_C = \frac{V_{CC} - V_X}{R_C}$ equation 2

equations 1, 2 $\Rightarrow \frac{V_{CC} - V_X}{R_C} = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right]$

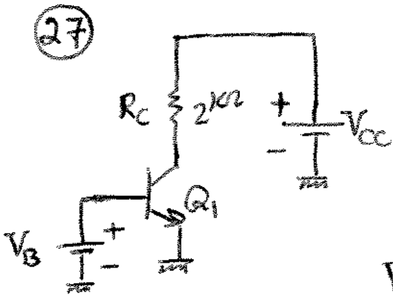
$\Rightarrow V_X + 5\times10^{-14} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right] = 2.5$

$\Rightarrow 1.2306\ V_X \simeq 1.347$

$\Rightarrow V_X \simeq 1.095\ V \xrightarrow{\text{equation 1}} I_C \simeq 1.406\ mA$

Transistor is in Forward Active Region

(27)



$I_S = 1 \times 10^{-17} A \qquad V_A = 5^V$

Applying KVL:

$$V_{CC} = R_C I_C + V_{CE}$$

$$\Rightarrow V_{CC} = R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] + V_{CE}$$

$\xrightarrow{V_{BE}\ \text{Constant}}$ $$\boxed{\Delta V_{CC} = \left[R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A} + 1\right] \cdot \Delta V_{CE}}$$ equation 1

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}\left[1 + \frac{V_{CE}}{V_A}\right] \Rightarrow \Delta I_C = I_S e^{V_{BE}/V_T} \times \frac{1}{V_A} \Delta V_{CE}$$

$$\Rightarrow \boxed{\Delta V_{CE} = \frac{1}{I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}} \cdot \Delta I_C}$$ equation 2

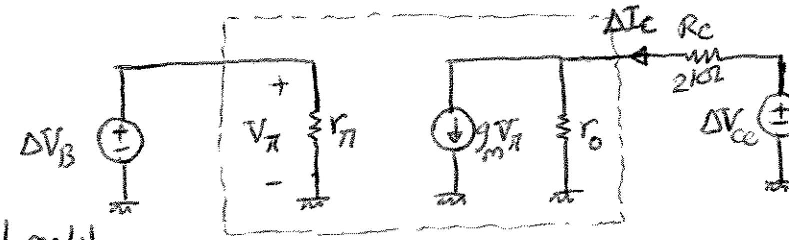
equations 1, 2 $$\Rightarrow \Delta I_C = \frac{I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}}{1 + R_C I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}} \cdot \Delta V_{CC}$$

$$\Rightarrow \boxed{\Delta I_C = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A + R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)} \cdot \Delta V_{CC}} = \frac{1}{r_o + R_C} \cdot \Delta V_{CC}$$

Could also be obtained using small signal model

$$\Rightarrow \Delta I_C = \frac{2.31 \times 10^{-4}}{5 + 0.4613} \times 0.5 \Rightarrow \Delta I_C \simeq 0.021 mA$$

(28)



We use small signal model,
Assuming that the required ΔV_B is small enough.

Applying superposition,

$$\Delta I_C = \left(\frac{1}{r_o + R_C}\right)\Delta V_{CC} + \left(\frac{g_m r_o}{r_o + R_C}\right)\Delta V_B$$

$$\Delta I_C = 0 \implies \boxed{\Delta V_B = -\frac{1}{g_m r_o}\Delta V_{CC}}$$

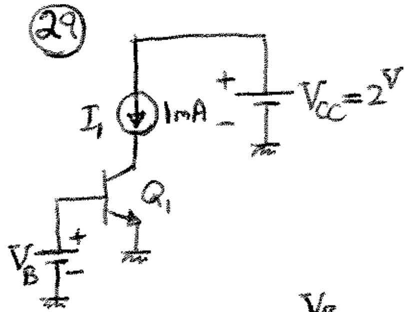
$$\Delta V_B = -\frac{1}{\frac{I_C}{V_T}\cdot\frac{V_A}{I_C}}\Delta V_{CC} \implies \Delta V_B = -\frac{V_T}{V_A}\Delta V_{CC}$$

$$\implies \Delta V_B = -\frac{26\times 10^{-3}}{5}\times(3-2.5)$$

$$\implies \boxed{\Delta V_B = -2.6\ mV}$$

which is small enough
for small signal model

(29)



$I_S = 3 \times 10^{-17} A$

a) $I_C = I_S e^{\frac{V_B}{V_T}} \implies V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{10^{-3}}{3 \times 10^{-17}}\right)$

$\implies \boxed{V_B \simeq 809.6 \text{ mV}}$

b) $I_C = I_S e^{\frac{V_B}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right)$

$10^{-3} = 3 \times 10^{-17} e^{V_B/V_T}\left(1 + \frac{1.5}{5}\right) \implies e^{V_B/V_T} = \frac{10^{14}}{3.9}$

$\implies V_B = 26^{mV} \ln\left(\frac{10^{14}}{3.9}\right) \implies \boxed{V_B \simeq 802.8 \text{ mV}}$

(30) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$$r_o^{-1} = \frac{dI_C}{dV_{CE}} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \simeq \frac{I_C}{V_A} \Rightarrow r_o \simeq \frac{V_A}{I_C}$$

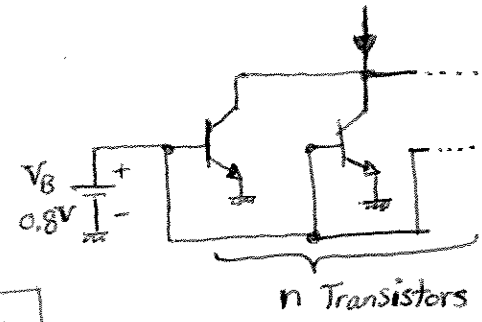
$$r_o > 10^{k\Omega} \Rightarrow \frac{V_A}{I_C} > 10^{k\Omega}$$

$$\Rightarrow V_A > 10^{k\Omega} \times 2^{mA}$$

$$\Rightarrow \boxed{V_A > 20\,V}$$

(31) $I_S = 5\times10^{-16}\,A$, $V_A = 8\,V$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$$



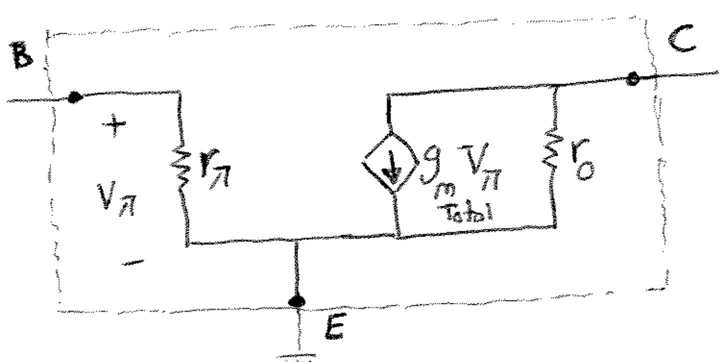
$$g_{m_{Total}} = \frac{I_{C_{Total}}}{V_T} \simeq \frac{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T}$$

$$\Rightarrow g_{m_{Total}} \simeq \frac{n \times 5\times10^{-16} \exp\left(\frac{800}{26}\right)}{26\,mV} \Rightarrow g_{m_{Total}} \simeq 0.4435\,n$$

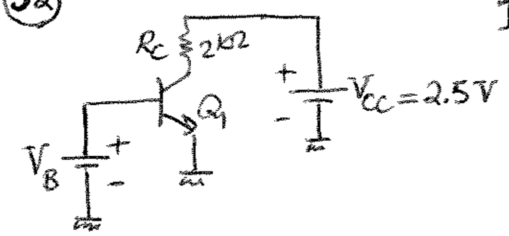
$$r_o^{-1} = \frac{\partial I_{C_{Total}}}{\partial V_{CE}} = \frac{\partial}{\partial V_{CE}}\left[n I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]\right]$$

$$\Rightarrow r_o = \frac{V_A}{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}$$

$$r_\pi = \frac{\beta}{g_{m_{Total}}} \overset{\beta=100}{\simeq} \frac{225.5}{n}$$



(32)



$I_S = 6\times10^{-16} A$, $V_A = \infty$

a) Q_1 at the edge of the active region $\Rightarrow V_{CE} = V_{BE}$

applying KVL, $V_{CC} = R_C I_C + V_{CE}$

$$\xrightarrow[\text{edge}]{\text{at the}} V_{CC} = R_C I_C + V_{BE} \Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow 2^{k\Omega} \times 6\times10^{-16} A \; e^{V_B/26mV} + V_B = 2.5 \Rightarrow \boxed{V_B \simeq 728.5^{mV}}$$

b) Applying KVL, $V_{CC} = R_C I_C + V_{CE}$

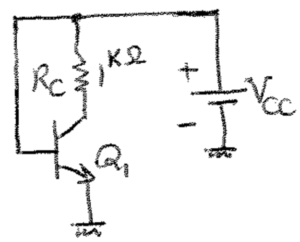
Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V$

$$\Rightarrow V_{CC} = R_C I_C + V_{BE} - 0.2^V$$

$$\Rightarrow 2^{k\Omega} \times 6\times10^{-16} e^{V_B/26mV} + V_B = 2.7V$$

$\Rightarrow \boxed{V_B \simeq 731.5mV}$ So V_B Can increase by 3mV

(33)



$I_S = 7\times 10^{-16} A$, $V_A = \infty$

$\Downarrow$

$r_o = \infty$

Applying KVL,

$$V_{CC} = R_C I_C + V_{CE} \xrightarrow{V_{CE} = V_{BE} - 0.2^V} R_C I_C + V_{BE} - 0.2^V = V_{CC}$$

$$\Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} - 0.2^V = V_{CC}$$

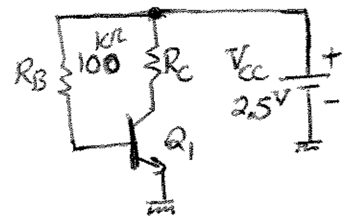
$$\xrightarrow{V_{BE} = V_{CC}} R_C I_S e^{\frac{V_{CC}}{V_T}} + \cancel{V_{CC}} - 0.2 = \cancel{V_{CC}}$$

$$\Rightarrow R_C I_S e^{V_{CC}/V_T} = 0.2^V$$

$$\Rightarrow 1^{K\Omega} \times 7\times 10^{-16} e^{V_{CC}/26^{mV}} = 0.2^V$$

$$\Rightarrow \boxed{V_{CC} \simeq 686 \ mV}$$

(34) $I_S = 2\times10^{-17}A$, $V_A = \infty$ $\beta = 100$



$$\begin{cases} V_{CC} = R_C I_C + V_{CE}, \quad V_{CE} = V_{BE} - 0.2^V \\ V_{CC} = R_B I_B + V_{BE} \Rightarrow V_{CC} = R_B \frac{I_C}{\beta} + V_{BE} \end{cases}$$

$$R_B \frac{I_C}{\beta} + V_{BE} = V_{CC} \Rightarrow \frac{R_B}{\beta} I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow \frac{100^{k\Omega}}{100} \times 2\times10^{-17} e^{\frac{V_{BE}}{26^{mV}}} + V_{BE} = 2.5^V$$

$$\Rightarrow \boxed{V_{BE} \simeq 833.5\ mV}$$

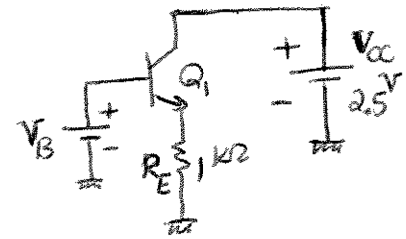
Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V \Rightarrow \boxed{V_{CE} = 692.5^{mV}}$

$$V_{CC} = R_C I_C + V_{CE} \Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$\Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{2.5 - 0.6925}{2\times10^{-17} \exp\left(\frac{892.5}{26}\right)}$$

$$\Rightarrow \boxed{R_C \simeq 112\ \Omega}$$

(35) $I_S = 5\times10^{-16} A$, $V_A = \infty \Rightarrow r_o = \infty$



Soft saturation $\Rightarrow V_{BC} = 200 mV$

$$\Rightarrow V_B = V_C + 0.2^V \Rightarrow \boxed{V_B = 2.7 V}$$

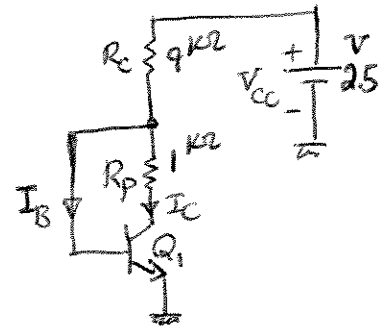
Applying KVL $\Rightarrow V_B = V_{BE} + R_E I_E \xrightarrow{I_E \simeq I_C} V_B = V_{BE} + R_E I_C$

$$\Rightarrow V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}} = 2.7^V$$

$$\Rightarrow V_{BE} + 5\times10^{-13} e^{\frac{V_{BE}}{V_T}} = 2.7^V \Rightarrow \boxed{V_{BE} \simeq 754 mV}$$

$$I_C = I_S e^{V_{BE}/V_T} = 5\times10^{-16} e^{\frac{0.754}{0.026}} \Rightarrow \boxed{I_C \simeq 2 mA}$$

(36) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$



$V_{BC} = 0.2^V \Rightarrow R_P I_C = 0.2^V$

$$\Rightarrow \boxed{I_C = \frac{0.2^V}{R_P}}$$

$$V_{BE} = V_{CC} - R_C(I_B + I_C)$$

$$\overset{\beta=100}{\Longrightarrow} V_{BE} = V_{CC} - \frac{\beta+1}{\beta} R_C I_C \Rightarrow \boxed{V_{BE} = V_{CC} - \frac{\beta+1}{\beta}\frac{R_C}{R_P} \times 0.2}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow \boxed{I_S = \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T} \cdot \frac{\beta+1}{\beta} \cdot \frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\overset{\beta=100}{\Longrightarrow} \boxed{I_S \simeq \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T}\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\Rightarrow \boxed{I_S \simeq 4.06 \times 10^{-16} A}$$

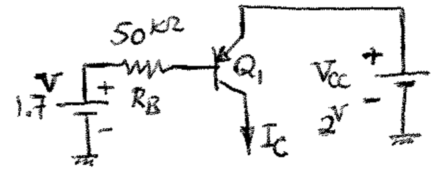
(37) $I_{S_1} = 3I_{S_2} = 6\times10^{-16}\,A$

$$I_1 = I_{S_1}\exp\left(\frac{V_{EB_1}}{V_T}\right) = 6\times10^{-16}\exp\left(\frac{300}{26}\right) \Rightarrow I_1 \simeq 6.155\times10^{-11}\,A$$

$$I_2 = I_{S_2}\exp\left(\frac{V_{EB_2}}{V_T}\right) = 2\times10^{-16}\exp\left(\frac{820}{26}\right) \Rightarrow I_2 \simeq 10\,mA$$

$$I_X = I_1 + I_2 \Rightarrow \boxed{I_X \simeq 10\,mA}$$

(38) $I_S = 2\times10^{-17}A \qquad \beta = 100$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B + 1.7^V$$

$$\Rightarrow 2^V = V_{EB} + R_B \frac{I_C}{\beta} + 1.7^V$$

$$\Rightarrow 0.3^V = V_{EB} + \frac{50^{k\Omega}}{100} I_C$$

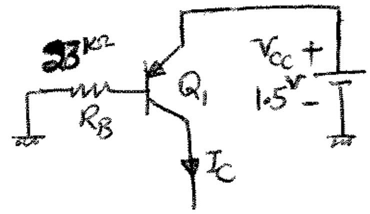
$$\Rightarrow 0.3^V = V_{EB} + 500 \times I_S e^{\frac{V_{EB}}{V_T}}$$

$$\Rightarrow 0.3^V = V_{EB} + 10^{-14} e^{V_{EB}/26mV} \Rightarrow \boxed{V_{EB} \simeq 0.3\ V}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} \Rightarrow I_C = 2\times10^{-17} e^{\frac{300}{26}}$$

$$\Rightarrow \boxed{I_C \simeq 2.05 \times 10^{-12} A}$$

(39) $I_C = 3mA$, $\beta = 100$, $R_B = 23 k\Omega$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B \Rightarrow V_{CC} = V_{EB} + R_B \frac{I_C}{\beta}$$

$$\Rightarrow -I_C \frac{R_B}{\beta} + V_{CC} = V_{EB}$$

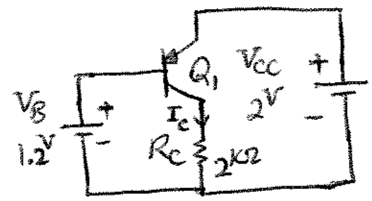
$$I_C = I_S e^{V_{EB}/V_T}$$

$$\Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}}$$

$$\Rightarrow \boxed{I_S = I_C e^{\frac{1}{V_T}\left(\frac{R_B I_C}{\beta} - V_{CC}\right)}}$$

$$\Rightarrow \boxed{I_S \simeq 8.85 \times 10^{-17} A}$$

(40) At the edge of active $\Rightarrow V_{BC} = 0$



$$I_C = \frac{V_B - V_{BC}}{R_C} = \frac{V_B}{R_C}$$

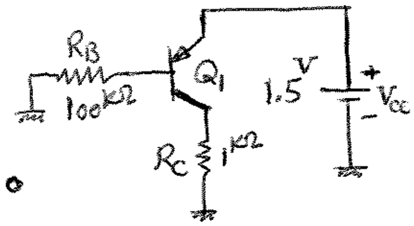
$$\Rightarrow I_C = \frac{1.2^V}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 0.6\ mA}$$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{EB}}{V_T}\right)$$

$$\Rightarrow I_S = 0.6 \times 10^{-3} \exp\left(-\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_S \simeq 2.6 \times 10^{-17}\ A}$$

(41) $I_S = 8\times10^{-16}\,A$



At the edge of the active mode $\Rightarrow V_{BC}=0$

$\Rightarrow V_{EB} = V_{EC}$

Applying KVL,

$$V_{CC} = V_{EC} + R_C I_C \xrightarrow{V_{EB}=V_{EC}} V_{CC} = V_{EB} + R_C I_C$$

$$\Rightarrow V_{EB} + R_C I_S e^{\frac{V_{EB}}{V_T}} = V_{CC}$$

$$\Rightarrow V_{EB} + 8\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} = 1.5 \Rightarrow \boxed{V_{EB} \simeq 718\,mV}$$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow \boxed{I_C \simeq 0.788\,mA}$$

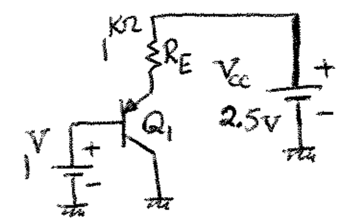
Applying KVL,

$$V_{BC}=0 \Rightarrow V_B = V_C \Rightarrow R_B I_B = R_C I_C$$

$$\Rightarrow R_B \frac{I_C}{\beta} = R_C I_C \Rightarrow \boxed{\beta = \frac{R_B}{R_C}}$$

$$\Rightarrow \beta = \frac{100^{k\Omega}}{1^{k\Omega}} \Rightarrow \boxed{\beta = 100}$$

(42) $I_S = 3\times10^{-17}\,A$



Applying KVL,

$$V_{CC} = R_E I_E + V_{EB} + 1^V \xrightarrow{I_E \simeq I_C} V_{CC} = R_E I_C + V_{EB} + 1^V$$

$$\Rightarrow 2.5 = 1^{k\Omega} \times 3\times10^{-17} e^{V_{EB}/26^{mV}} + V_{EB} + 1$$

$$\Rightarrow V_{EB} + 3\times10^{-14} e^{V_{EB}/26^{mV}} = 1.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 800.5\ mV}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} = 3\times10^{-17} e^{\frac{800.5}{26}} \Rightarrow \boxed{I_C \simeq 0.705\ mA}$$

(43) $I_S = 3\times10^{-17}A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

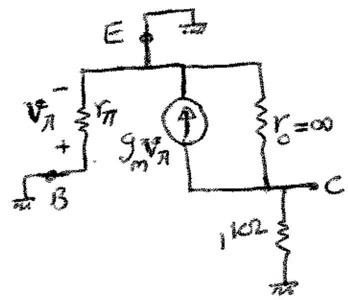
a) $V_{EB} = 2.5 - 1.7 = 0.8\ V$

$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = 3\times10^{-17}\exp\left(\frac{800}{26}\right) \Rightarrow \boxed{I_C \simeq 0.692^{mA}}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 1^{k\Omega}\times 0.692^{mA} \Rightarrow \boxed{V_{EC} \simeq 1.808^{V}}$

$g_m = \frac{I_C}{V_T} = \frac{0.692^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 26.6\,mS}$

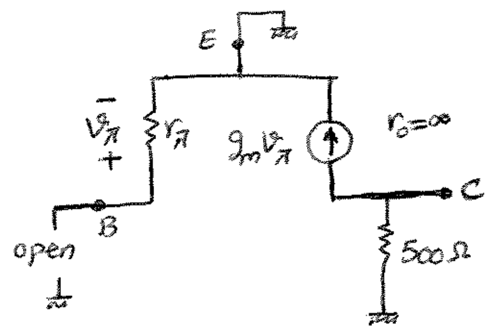
$r_\pi = \frac{\beta}{g_m} = \frac{100}{26.6\times10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3.76\ k\Omega}$



b) $V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) \Rightarrow V_{EB} = V_T \ln\left(\frac{\beta I_B}{I_S}\right)$

$\Rightarrow V_{EB} = 26^{mV}\times \ln\left(\frac{100\times20\times10^{-6}}{3\times10^{-17}}\right)$

$\Rightarrow \boxed{V_{EB} \simeq 827.6\,mV}$



$I_C = \beta I_B \Rightarrow \boxed{I_C = 2mA}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 0.5^{k\Omega}\times 2^{mA} \Rightarrow \boxed{V_{EC} = 1.5\ V}$

$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 77\,mS}$ $\quad r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi \simeq 1.3\,k\Omega}$

(43) Continued .....

c) Applying KVL,

$$V_{cc} = V_{EB} + (I_C + I_B) \times 2^{k\Omega} \simeq V_{EB} + 2^{k\Omega} \times I_C$$

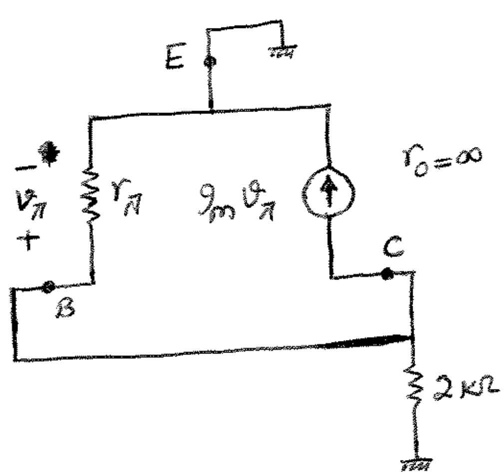
$$\Rightarrow \quad V_{EB} + 2^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} = V_{cc}$$

$$\Rightarrow \quad V_{EB} + 6 \times 10^{-14} e^{V_{EB}/26^{mV}} = 2.5^V \quad \Rightarrow \quad \boxed{V_{EB} \simeq 805 mV}$$

$$I_C \simeq \frac{V_{cc} - V_{EB}}{R} = \frac{2.5 - 0.805}{2^{k\Omega}} \quad \Rightarrow \quad \boxed{I_C \simeq 847.5 \mu A}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8475 \times 10^{-3}}{0.026} \quad \Rightarrow \quad \boxed{g_m \simeq 32.6 \text{ mS}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.6 \times 10^{-3}} \quad \Rightarrow \quad \boxed{r_\pi \simeq 3068 \ \Omega}$$

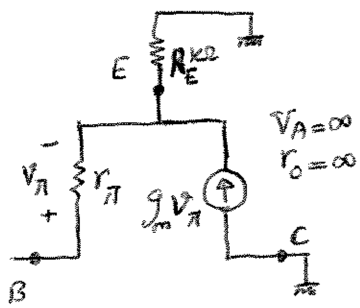


(44) $I_S = 3\times10^{-17}A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

a) Applying KVL,

$$V_{CC} = R_E I_E + V_{EC} \overset{I_E \simeq I_C}{\Longrightarrow} V_{EC} = V_{CC} - R_E I_C \qquad \boxed{I_C = \beta I_B = 0.2^{mA}}$$

$$\Rightarrow \boxed{V_{EC} = V_{CC} - \beta R_E I_B} \xrightarrow[R_E = 2^{k\Omega},\ V_{CC} = 2.5^{V}]{I_B = 2^{\mu A}} \boxed{V_{EC} = 2.1\ V}$$



Transistor is in Forward Active Region

b) Applying KVL,

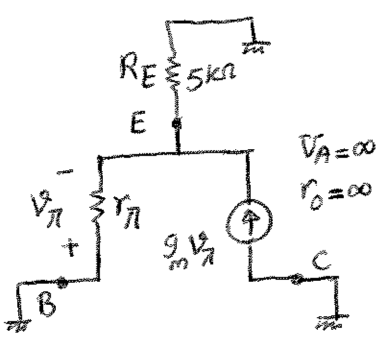
Forward Active Region

$$V_{CC} = R_E I_E + V_{EB} \Rightarrow V_{CC} \simeq R_E I_C + V_{EB}$$

$$\Rightarrow 2.5 = 5^{k\Omega} \times 3\times10^{-17} e^{V_{EB}/V_T} + V_{EB}$$

$$\Rightarrow \boxed{V_{EB} \simeq 781.9\ mV}$$

From Circuit $V_{EC} = V_{EB}$



$$g_m = \frac{I_C}{V_T} = \frac{3\times10^{-17} e^{\frac{781.9}{26}}}{0.026} \Rightarrow \boxed{g_m \simeq 0.0133\ S}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0133} \Rightarrow \boxed{r_\pi \simeq 7538\ \Omega}$$

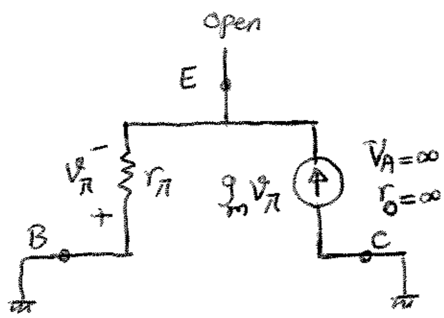
(44) Continued.....

c) $I_E = 0.5mA \Rightarrow \boxed{I_C \simeq 0.5mA}$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow 0.5mA = 3\times10^{-17} e^{\frac{V_{EB}}{26mV}} \Rightarrow \boxed{V_{EB} \simeq 791.6\,mV}$$

In the given circuit : $V_{EC} = V_{EB}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 19.2mS}$$

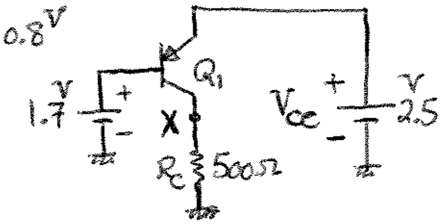


$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0192} \Rightarrow \boxed{r_\pi \simeq 5.2k\Omega}$$

Forward Active Region

(45) $I_S = 5\times10^{-17}\,A$

$V_{EB} = 0.8\,V$



a) $V_A = \infty \Rightarrow r_o = \infty$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow I_C = 5\times10^{-17} e^{\frac{800}{26}} \Rightarrow \boxed{I_C \simeq 1.15\,mA}$$

$$V_X = R_C I_C = 0.5^{k\Omega} \times 1.15^{mA} \Rightarrow \boxed{V_X \simeq 0.58^{V}}$$

b) $V_A = 6\,V$

$$I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{EC}}{V_A}\right), \quad V_{EC} = V_{CC} - R_C I_C$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC} - R_C I_C}{V_A}\right)$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right) - \frac{I_S R_C}{V_A} e^{\frac{V_{EB}}{V_T}} I_C$$

$$\Rightarrow \boxed{I_C = \frac{I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right)}{1 + \frac{I_S R_C}{V_A} e^{V_{EB}/V_T}}} = \frac{5\times10^{-17} e^{\frac{800}{26}}\left(1 + \frac{2.5}{6}\right)}{1 + \frac{5\times10^{-17}\times0.5^{k\Omega}}{6} e^{800/26}}$$

$$\Rightarrow \boxed{I_C = 1.49\,mA} \qquad V_X = R_C I_C = 500 \times 1.49 \times 10^{-3} \Rightarrow \boxed{V_X = .745^{V}}$$

0.745 V

(46) $r_o = 60\,k\Omega$, $I_C = 2\,mA$

$$r_o = \frac{V_A}{I_C} \Rightarrow 60\times10^{3}\,\Omega = \frac{V_A}{2\times10^{-3}\,A} \Rightarrow \boxed{V_A = 120\,V}$$

(47) $r_o = 60\ k\Omega$, $I_C = 1\ mA$

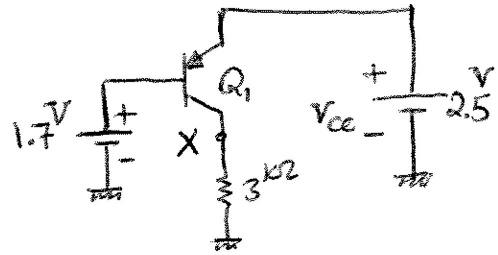
$$r_o = \frac{V_A}{I_C} \Rightarrow \boxed{V_A = r_o \cdot I_C} \Rightarrow V_A \propto I_C$$

$$\Rightarrow V_A = 60^{k\Omega} \times 1^{mA}$$

$$\Rightarrow \boxed{V_A = 60\ V}$$

V_A is half the value in problem 46 as V_A is proportional to I_C.

(48) $V_A = 5\,V$



a) At the edge of active mode

$$\Rightarrow \boxed{V_X = V_B = 1.7\,V}$$

$$I_C = \frac{V_X}{R_C} = \frac{1.7\,V}{3\,k\Omega} \Rightarrow \boxed{I_C \simeq 0.567\,mA}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \Rightarrow \boxed{I_S = \frac{I_C\, e^{-\frac{V_{EB}}{V_T}}}{1 + \frac{V_{EC}}{V_A}}}$$

$$I_S = \frac{0.567 \times 10^{-3} \times e^{-\frac{800}{26}}}{1 + \frac{2.5 - 1.7}{5}} \Rightarrow \boxed{I_S \simeq 2.118 \times 10^{-17}\,A}$$

b) $V_A = \infty$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow \boxed{I_S = I_C\, e^{-\frac{V_{EB}}{V_T}}}$$

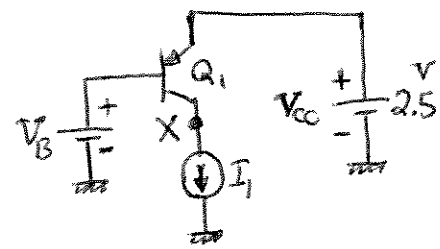
$$I_S = 0.567 \times 10^{-3}\, e^{-\frac{800}{26}} \Rightarrow \boxed{I_S \simeq 2.457 \times 10^{-17}\,A}$$

I_S increases

(49) The direction of Currents in large-signal model shows how Currents would flow when the pnp Transistor is properly DC biased.

The direction of currents in small-signal model shows how the ac Currents flow when ac voltage across Base-Emitter increases.

(50) $I_S = 6\times10^{-16}A$, $V_A = 5V$, $I_1 = 2mA$



a) $$I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right)$$

$$\Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{EC}}{V_A}\right)}\right)$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_B = V_{CC} - V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{CC}-V_X}{V_A}\right)}\right)}$$

$$\Rightarrow V_B = 2.5 - 0.026\ln\left(\frac{2\times10^{-3}}{6\times10^{-16}\left(1+\frac{2.5-1}{5}\right)}\right) \Rightarrow \boxed{V_B \simeq 1.757\,V}$$

b) $$I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right) \Rightarrow 1+\frac{V_{EC}}{V_A} = \frac{I_C}{I_S} e^{-V_{EB}/V_T}$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_X = V_{CC} - V_A\left(\frac{I_C}{I_S} e^{-V_{EB}/V_T} - 1\right)}$$

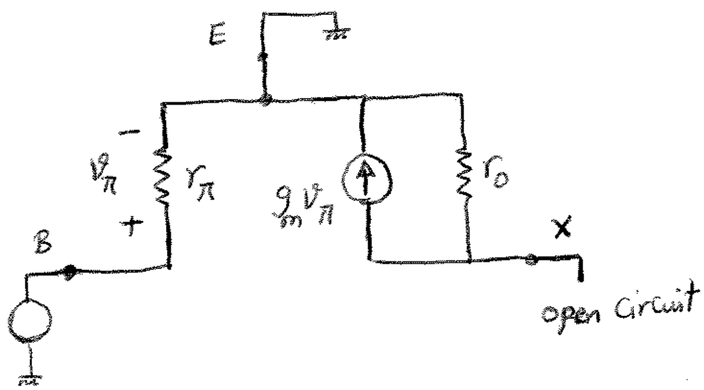
$$\Delta V_X \simeq \frac{dV_X}{dV_{EB}}\Delta V_{EB} \Rightarrow \Delta V_X \simeq \frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_{EB}$$

$$\xrightarrow{\Delta V_{EB} = -\Delta V_B} \boxed{\Delta V_X \simeq -\frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_B}$$

$$\Rightarrow \Delta V_X \simeq -\frac{5}{0.026}\times\frac{2\times10^{-3}}{6\times10^{-16}}\exp\left(-\frac{2.5-1.757}{0.026}\right)\times0.1\times10^{-3} \Rightarrow \boxed{\Delta V_X \simeq -24.9\,mV}$$

(50) Continued .....

c)



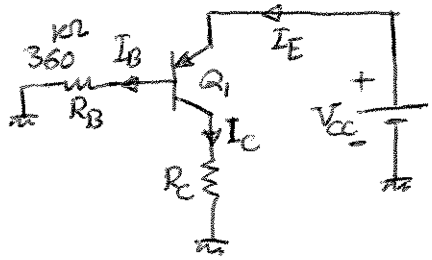
$$r_o = \frac{V_A}{I_C} = \frac{5^V}{2^{mA}} \Rightarrow \boxed{r_o \simeq 2.5\ k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{0.026\ V} \Rightarrow \boxed{g_m \simeq 76.9\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_\pi = 1.3\ k\Omega}$$

(51) $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

$R_B = 360\,k\Omega$



a) given: $V_C = V_B + 0.2^V$

$$\Rightarrow R_C I_C = R_B I_B + 0.2^V$$

$$\Rightarrow R_C I_C = R_B \frac{I_C}{\beta} + 0.2^V \Rightarrow \boxed{I_C = \frac{0.2^V}{R_C - \frac{R_B}{\beta}}} \Rightarrow \boxed{I_C = 0.5^{mA}}$$

$$I_C = I_S e^{+\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\left(\frac{V_{CC} - R_B I_B}{V_T}\right)}$$

$$\Rightarrow \boxed{I_S = \left(\frac{0.2}{R_C - \frac{R_B}{\beta}}\right) \exp\left[-\frac{1}{V_T}\left(V_{CC} - R_B \times \frac{0.2^V}{\beta\left(R_C - \frac{R_B}{\beta}\right)}\right)\right]}$$

$$\Rightarrow \boxed{I_S \simeq 10^{-15} A = 1\ fA}$$

b) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow \boxed{g_m = \frac{0.2^V}{V_T\left(R_C - \frac{R_B}{\beta}\right)}} \Rightarrow \boxed{g_m \simeq 19.23\ mS}$$

(52) $I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{EB} = 0 \Rightarrow Q_1$ is off $\quad I_C \cong 0$

b) $I_B = 0 \Rightarrow Q_1$ is off

c) Applying KVL : $V_{CC} \cong V_{EB} + 1^{k\Omega} \times I_C$

$$\Rightarrow V_{EB} + 1^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} \simeq V_{CC} \Rightarrow V_{EB} + 5\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} \simeq 2.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 751\,mV} \qquad I_C = 5\times10^{-16} e^{\frac{0.751}{0.026}} \Rightarrow \boxed{I_C \simeq 1.8mA}$$

with this current, Transistor is Saturated. Note $V_B < V_C$ Always

(d) $V_{BC} = 0 \Rightarrow$ Transistor is at the edge of Saturation

e) $I_C \cong 0.5mA \Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{0.5mA}{5\times10^{-16}}\right)$

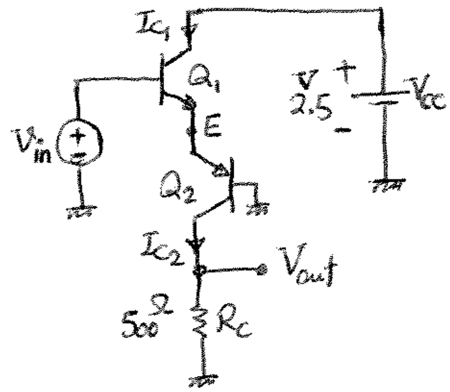
$$\Rightarrow \boxed{V_{EB} \simeq 718\,mV}$$

$V_{Collector} = 500^{\Omega} \times I_C \Rightarrow \boxed{V_C = 0.25^V}$

As $V_B = 0$, $V_C = 0.25^V \Rightarrow$ Transistor is soft saturated

(53) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{B_2} = 0^{V} \xrightarrow[\text{Bias by } 200^{mV}_{max}]{BC\ Q_2 \text{ Forward}} V_{C_{2\,max}} = 0.2\ V$



$$\Rightarrow I_{C_{2\,max}} = \frac{V_{C_{2max}}}{R_C} = \frac{0.2^V}{500^{\Omega}}$$

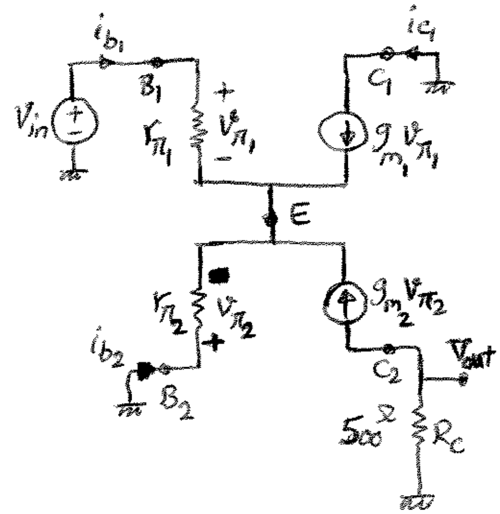
$$\Rightarrow \boxed{I_{C_{2\,max}} = 0.4\ mA}$$

As shown $I_{C_1} = I_{C_2}$

$$V_{in_{max}} = V_{BE_{1\,max}} + V_{EB_{2\,max}} = V_T \ln \frac{I_{C_{1max}}}{I_{S_1}} + V_T \ln \frac{I_{C_{2max}}}{I_{S_2}}$$

$$\Rightarrow V_{in_{max}} = 26^{mV} \cdot \left[\ln \frac{0.4\times10^{-3}}{5\times10^{-16}} + \ln \frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}} \right] \Rightarrow \boxed{V_{in_{max}} \simeq 1.454^V}$$

b) $g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{0.4^{mA}}{26\ mV}$



$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$

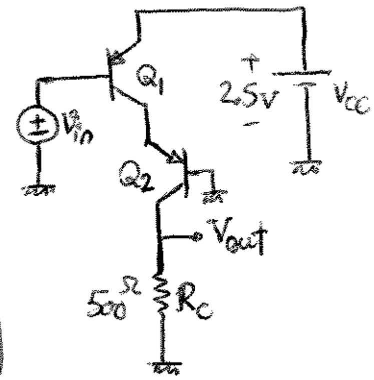
$$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$$

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5\ k\Omega}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{50}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_2} = 3.25\ k\Omega}$$

$V_A = \infty \Rightarrow \boxed{r_o = \infty}$

(54) $I_{S_1} = 3 I_{S_2} = 5\times10^{-16}\,A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$



a) $V_{B_2} = \phi \xrightarrow[\text{Forward biased by 200mV}]{Q_2 \text{ Base-Collector}} V_{C_2} = 0.2^V$

$\Rightarrow I_{C_{2\,max}} = \frac{V_{C_2 max}}{R_C} = \frac{0.2^V}{500^\Omega} \Rightarrow \boxed{I_{C_{2\,max}} = 0.4\,mA}$

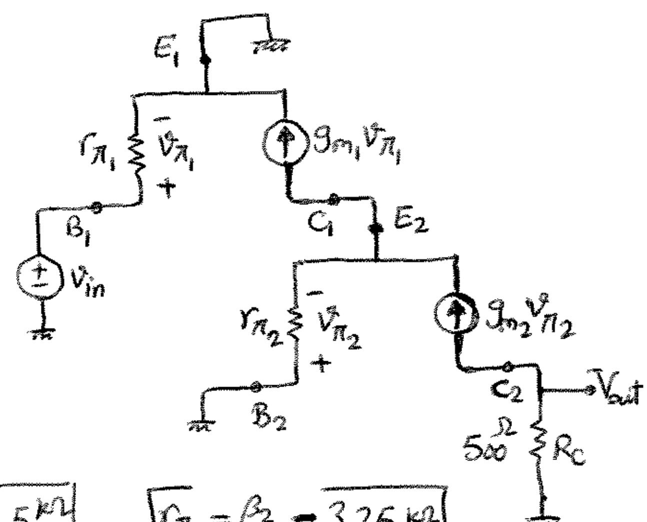
As shown: $I_{C_1} \simeq I_{C_2}$ (Note: $I_{C_1} = I_{E_2} = \frac{\beta_2+1}{\beta_2} I_{C_2}$ precisely)

$I_{C_1} = I_{S_1} e^{V_{EB_1}/V_T} \Rightarrow V_{EB_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \Rightarrow V_{CC} - V_{in} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)$

$\Rightarrow \boxed{V_{in} = V_{CC} - V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)} \Rightarrow V_{in} = 2.5 - 0.026 \ln\left(\frac{4\times10^{-4}}{5\times10^{-16}}\right)$

$\Rightarrow \boxed{V_{in} \simeq 1.787\ V}$ This is minimum acceptable V_{in}

b) $g_{m_1} = \frac{I_{C_1}}{V_T} \simeq \frac{0.4^{mA}}{26^{mV}}$



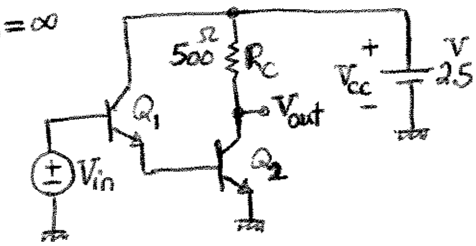
$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$

$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$

$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5^{k\Omega}}$ $\boxed{r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 3.25\ k\Omega}$

$V_{EB_2} = V_T \ln\left(\frac{I_{C_2}}{I_{S_2}}\right) = 26^{mV} \ln\left(\frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}}\right) \Rightarrow \boxed{V_{EB_2} \simeq 741\ mV} \Rightarrow \boxed{V_{EC_1} \simeq 1.759\ V}$ (Q_1 in active mode)

(55) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A,\ \beta_1=100,\ \beta_2=50,\ V_A=\infty$



a) Q_2 is softly saturated $\Rightarrow V_{BC_2} = 0.2^V$

$$V_{BC_2} = 0.2^V \Rightarrow V_{B_2} - V_{C_2} = 0.2^V \Rightarrow V_{BE_2} - (V_{CC} - R_C I_{C_2}) = 0.2^V$$

$$\Rightarrow V_{BE_2} + R_C I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = V_{CC} + 0.2$$

$$\Rightarrow V_{BE_2} + 500 \times \tfrac{5}{3}\times10^{-16} e^{\frac{V_{BE_2}}{V_T}} = 2.5 + 0.2 \Rightarrow \boxed{V_{BE_2} \simeq 800\,mV}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \qquad I_{C_2} = I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = \tfrac{5}{3}\times10^{-16} e^{\frac{800}{26}} \Rightarrow \boxed{I_{C_2} \simeq 3.8^{mA}}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_2}/\beta_2}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{\frac{3.8\times10^{-3}}{50}}{5\times10^{-16}}\right) \Rightarrow \boxed{V_{BE_1} \simeq 669.4\,mV}$$

$$V_{in} = V_{BE_1} + V_{BE_2} \Rightarrow \boxed{V_{in} \simeq 1.469\ V}$$ Maximum allowable value for V_{in}

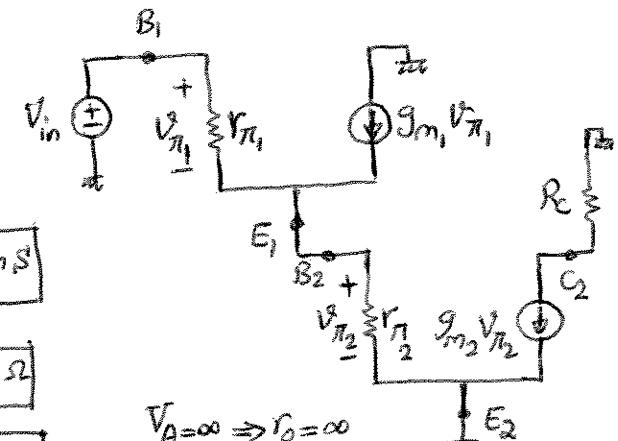
b)
$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{I_{C_2}/\beta_2}{V_T} = \frac{\frac{3.8^{mA}}{50}}{26^{mV}}$$

$$\Rightarrow \boxed{g_{m_1} \simeq 2.9\ mS}$$

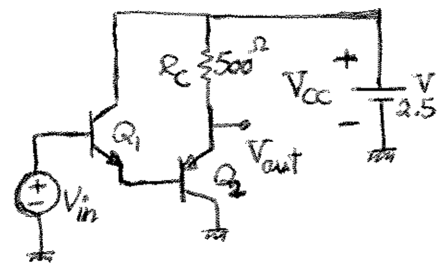
$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{3.8^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 146\ mS}$$

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{2.9\times10^{-3}} \Rightarrow \boxed{r_{\pi_1} \simeq 34121\,\Omega}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{50}{146\times10^{-3}} \Rightarrow \boxed{r_{\pi_2} \simeq 342\,\Omega}$$



(56) $I_{S_1} = 2I_{S_2} = 6\times10^{-17}A$, $\beta_1 = 80$, $\beta_2 = 100$



a) $I_{C_2} = 2^{mA}$

$$V_{EB_2} = V_T \ln\frac{I_{C_2}}{I_{S_2}} = 26^{mV} \ln\left(\frac{2\times10^{-3}}{3\times10^{-17}}\right) \simeq 827.6^{mV}$$

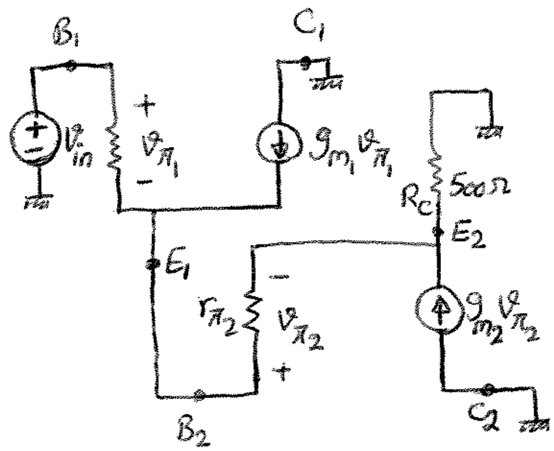
$$V_{BE_1} = V_T \ln\frac{I_{C_1}}{I_{S_1}} = 26^{mV} \ln\left(\frac{\frac{2\times10^{-3}}{100}}{6\times10^{-17}}\right) \simeq 689.9 \text{ mV}$$

$$\boxed{V_{in} = V_{CC} - R_C I_{C_2} - V_{EB_2} + V_{BE_1}} = 2.5 - 0.5^{k\Omega}\times 2^{mA} - 0.8276 + 0.6899$$

$$\Rightarrow \boxed{V_{in} \simeq 1.362 \text{ V}}$$

b) $g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 76.9 \text{ mS}}$

$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{\frac{2^{mA}}{100}}{26^{mV}} \Rightarrow \boxed{g_{m_1} \simeq 769 \mu S}$



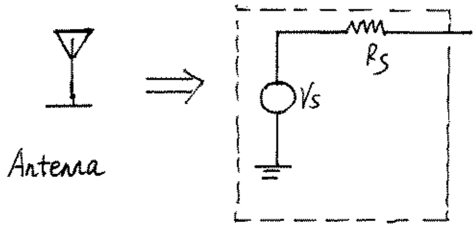
$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{80}{\frac{1}{1300}}$

$\Rightarrow \boxed{r_{\pi_1} = 104 \text{ k}\Omega}$

$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{100}{2/26} \Rightarrow \boxed{r_{\pi_2} = 1300\Omega}$

$V_A = \infty \Rightarrow r_o = \infty$

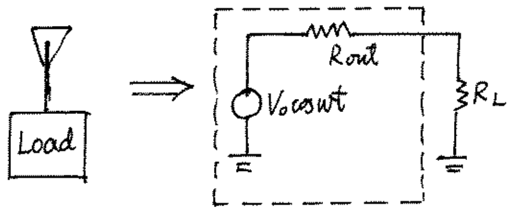
1)



Thevenin Equivalent:

$V_S = V_0 \cos \omega t$

$R_S = R_{out}$



Average power delivered to load $= (I_{RMS})^2 R_L$,

$I_{RMS} = \frac{V_{RMS}}{R_{out} + R_L}$, $\quad V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow I_{RMS} = \frac{V_0}{\sqrt{2}(R_{out}+R_L)}$

$$\text{Average power} = (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out}+R_L)^2} \qquad \text{(Eq. 1)}$$

Plot of Average Power

When R_L is small, Eq. 1 is small.

When R_L is large, Eq. 1 is also small.

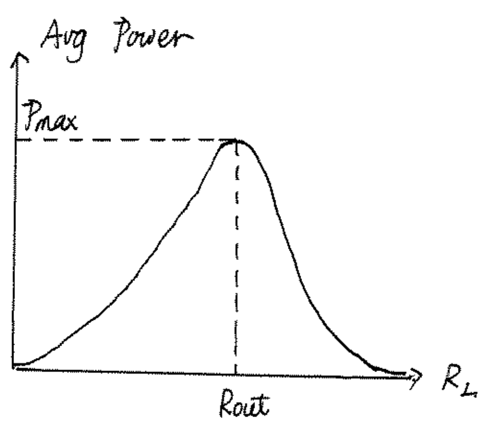
So for some R_L between zero and infinity, the average power will reach its peak. Lets take the derivative of Eq. 1 with respect to R_L to find the optimum R_L.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out}+R_L)^2}\right] = \frac{V_0^2}{2(R_{out}+R_L)^2} - \frac{V_0^2 R_L}{(R_{out}+R_L)^3}$$

Setting it to zero and solve for R_L

$$\frac{V_0^2}{2(R_{out}+R_L)^2} = \frac{V_0^2 R_L}{(R_{out}+R_L)^3} \Rightarrow \frac{(R_{out}+R_L)}{2} = R_L$$

$$\Rightarrow R_{out} + R_L = 2R_L \Rightarrow R_L = R_{out}$$

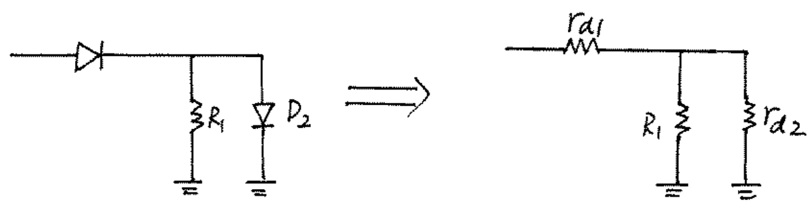


2) In small signal operation, a diode can be replaced by a linear resistor if changes are small.

a).

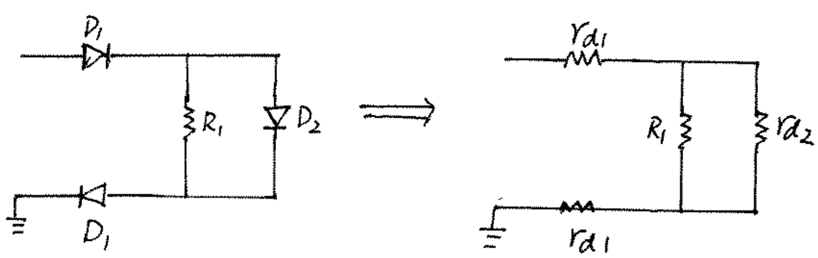
$R_{in} = r_{d1} + R_1$

b).



$R_{in} = r_{d1} + R_1 // r_{d2}$ (// means in parallel)

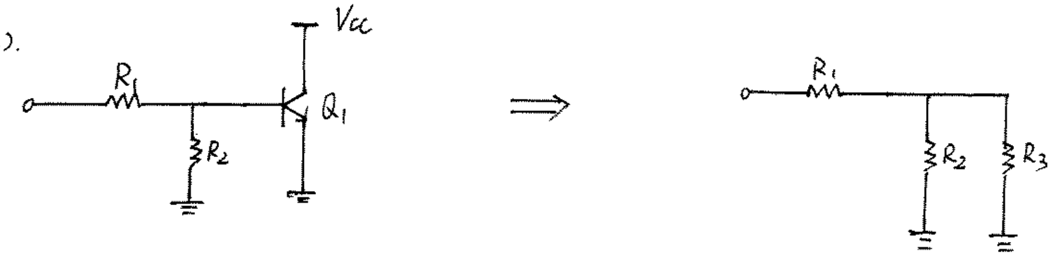
c).



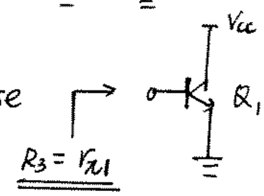
$R_{in} = 2r_{d1} + R_1 // r_{d2}$

3). When $V_A = \infty$, $V_o = \infty$.

a).

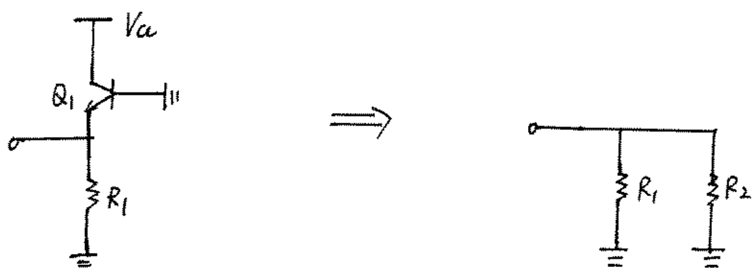


Replacing Q_1 by its equivalent resistance seen at base

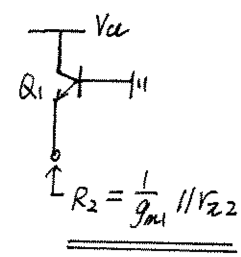


So $R_{in} = R_1 + R_2 // R_3 = R_1 + R_2 // r_{\pi 1}$

b).

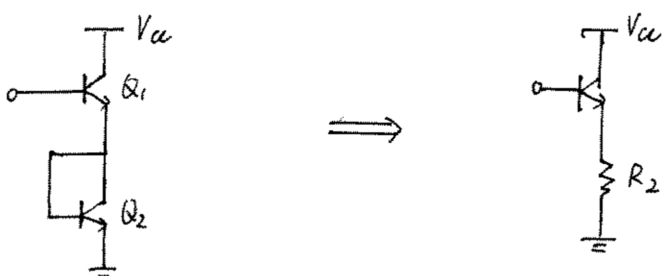


Replacing Q_1 by its equivalent resistance see at emitter

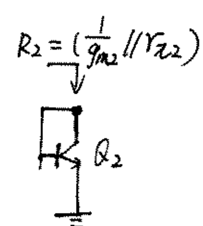


So $R_{in} = R_1 // R_2 = R_{in} // (\frac{1}{g_{m1}} // r_{\pi 1})$

c).



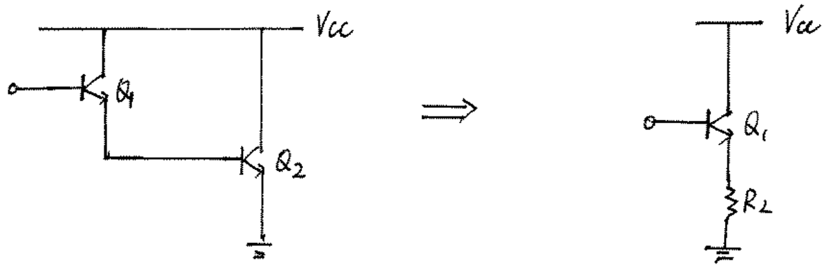
Replacing Q_2 by its equivalent diode-connected resistante



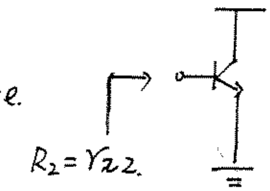
So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta)(\frac{1}{g_{m2}} // r_{\pi 2})$

3).

d).



Replacing Q2 by its equivalent resistance seen at Base.



So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta) r_{\pi 2}$.

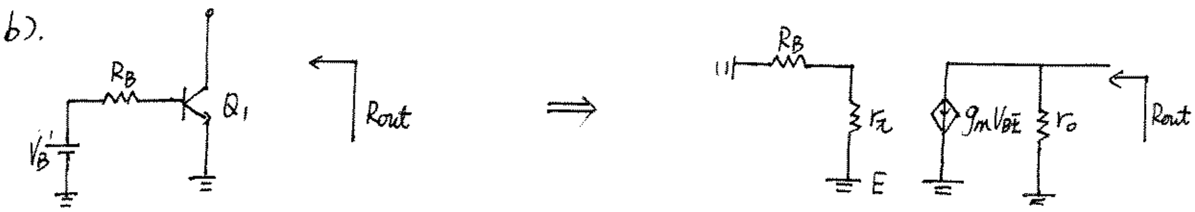
(Please refer to the textbook for all the equivalent resistances)

4). Since the problem doesn't say $V_A = \infty$, r_o must be considered in derivation.

a). Short V_B since it's a DC source, and replace Q_1 with an ideal transistor with it's output resistance.

So $R_{out} = R_1 // r_o // \infty = R_1 // r_o$

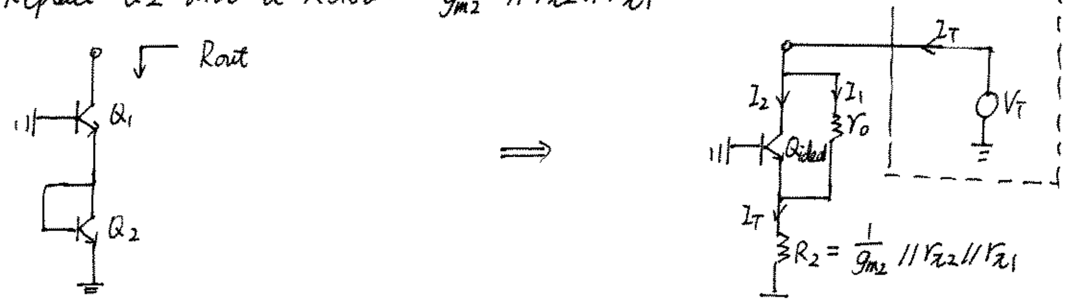
b).



By drawing the small-signal model, it's easy to tell $V_{BE} = 0$ and $R_{out} = r_o$

c). Replace Q_1 with an ideal transistor and an output impedance r_{o1}.

Replace Q_2 with a resistor $\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}$



Here, $r_{\pi 1}$ is included in R_2 because it is also connected from emitter to ground and it accounts for the base current of Q_1.

4) $I_1 = \frac{V_T - I_T R_2}{r_o}$, $I_2 = g_{m1}(0 - I_T R_2)$

$$I_T = I_1 + I_2 = \frac{V_T - I_T R_2}{r_o} - g_{m1} I_T R_2$$

$$\Rightarrow I_T + \frac{I_T R_2}{r_o} + g_m I_T R_2 = \frac{V_T}{r_o}$$

$$\Rightarrow \frac{V_T}{I_T} = r_o \left(1 + \frac{R_2}{r_o} + g_{m1} R_2\right)$$

$$\Rightarrow R_{out} = \frac{V_T}{I_T} = r_o(1 + g_{m1} R_2) + R_2$$

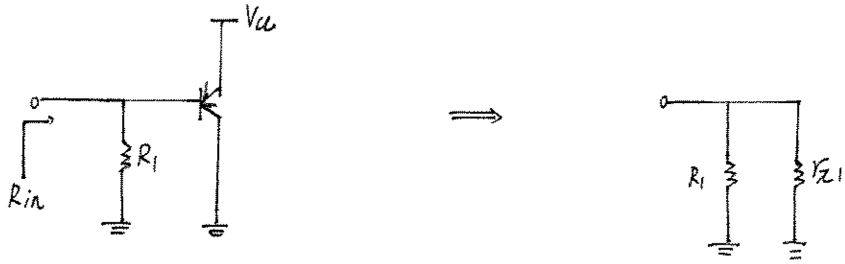
$$= r_o\left[1 + g_{m1}\left(\frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{\pi 1}\right)\right] + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{\pi 1}$$

Usually $\frac{1}{g_m} << r_\pi$, and if $Q_1 = Q_2$

$$R_{out} \approx \frac{1}{g_m} + 2r_o$$

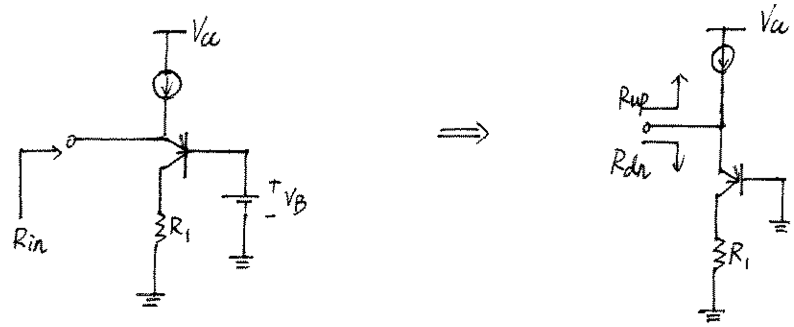
5). $V_A = \infty$, $r_o = \infty$

a).



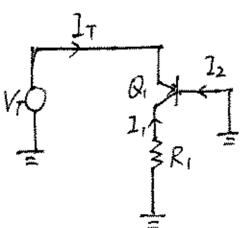
$R_{in} = R_1 // r_{\pi 1}$

b).



$R_{in} = R_{up} // R_{dn}$. $R_{up} = \infty$, since a DC current source is open.

Finding R_{dn}:



$$I_T = -(I_1 + I_2)$$

$$I_1 = g_m(0 - V_T) = -g_m V_T$$

$$I_2 = \frac{I_1}{\beta}.$$

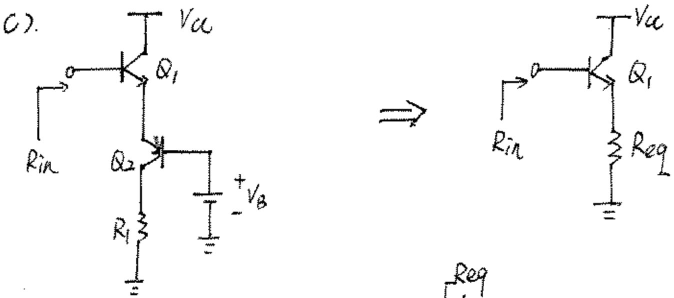
So $$I_T = -\left(-g_m V_T - \frac{g_m V_T}{\beta}\right) = \left(g_m + \frac{g_m}{\beta}\right) V_T$$

$$\frac{V_T}{I_T} = \frac{1}{\left(g_m + \frac{g_m}{\beta}\right)} = \frac{1}{g_m} // r_\pi$$

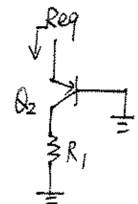
$$R_{dn} = \frac{V_T}{I_T} = \frac{1}{g_m} // r_\pi$$

So $R_{in} = R_{up} // R_{dn} = \infty // \frac{1}{g_m} // r_\pi = \frac{1}{g_m} // r_\pi$

5). c).



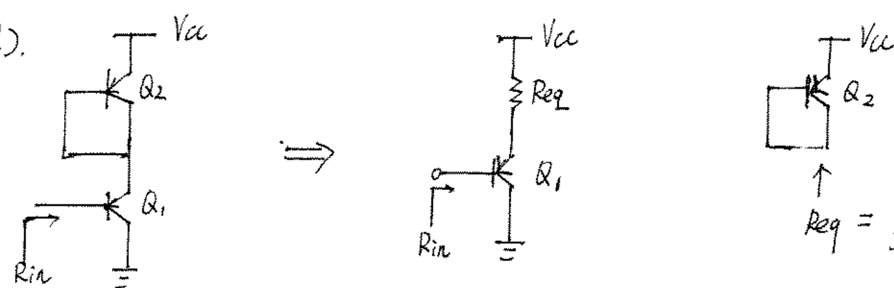
From b), we know that



$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$.

So $R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$.

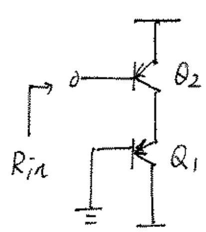
d).



$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$

e)

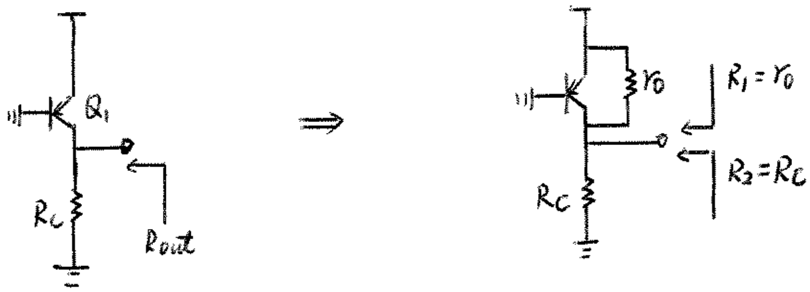


$R_{in} = r_{\pi 2}$. Q_1 plays no role here since it's connected to the collector of Q_2.

It can not be seen from the Base of Q_2.

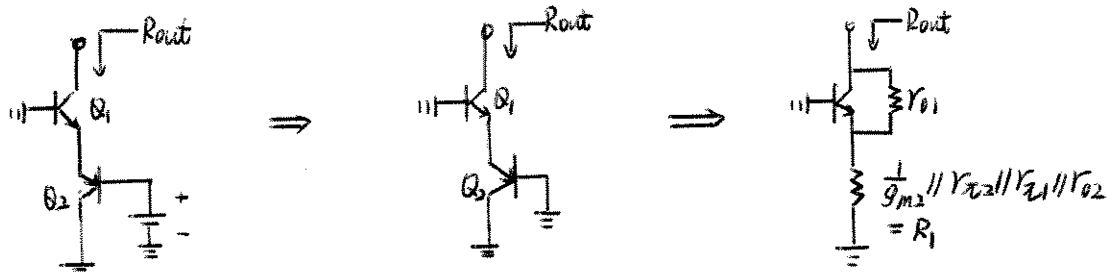
b) Since the problem doesn't state $V_A = \infty$, r_o is not ∞.

a)



$$R_{out} = R_1 // R_2 = R_C // r_o$$

b)

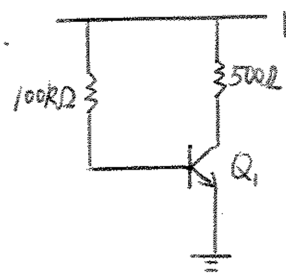


As shown is problem 4). c).

$$R_{out} = R_1 + r_{o1} + g_{m2} r_{o1} R_1 = r_{o1} + (1 + g_{m1} r_{o1}) R_1$$

$$= r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1} // r_{o2} \right)$$

7) a).



$$I_B = \frac{V_{cc} - V_{BE}}{100\ k\Omega}, \quad I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.7V$,

$$I = \beta\left(\frac{V_{cc} - V_{BE}}{100k\Omega}\right) = 1.8mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V$$, not 0.7V, reiterate

$V_{BE} = 0.747V$, $I_c = 1.753mA$

Verify V_{BE}, $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.746V$, converged

$$V_{CE} = 2.5 - (1.753)(0.5k) = 1.62V$$

$V_{CE} > V_{BE}$, Q_1 in forward active region.

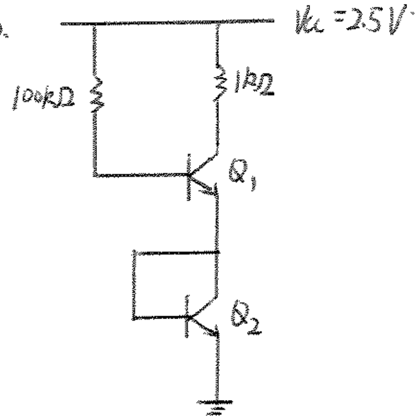
$I_c = 1.754mA$ $\quad$ $V_{CE} = 1.62V$

$I_B = 17.54\mu A$ $\quad$ $V_{BE} = 0.746V$

↑

operating point

7). b).



$$I_{B1} = \frac{2.5 - (V_{BE1} + V_{BE2})}{100K\Omega}$$

$$I_{C1} = \beta I_{B1}$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right), \quad V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

Assume $V_{BE1} = V_{BE2} = 0.8V$

$$I_{C1} = \beta\left(\frac{2.5 - 1.6}{100K}\right) = 0.9mA$$

$$V_{BE1} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.728V$$, not 0.8V, reiterate

$I_{C2} = 0.9mA$, since β's are the same

$V_{BE2} = 0.728V$

$$I_{C1} = \beta\left(\frac{2.5 - (2)(0.728)}{100K\Omega}\right) = 1.042mA = I_{C2}$$

$$V_{BE1} = V_{BE2} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733V$$, iterate once more

$$I_{C1} = I_{C2} = \beta\left(\frac{2.5 - (2)(0.733)}{100K\Omega}\right) = 1.034mA$$

7) b)

$$I_{C_1} = I_2 = 1.034\ mA$$

$$V_{BE_1} = V_{BE_2} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733,$$ converges.

$$V_{CE_1} = 2.5 - 0.733 - (1.034)(1K\Omega) = 0.733V$$

$V_{CE} = V_{BE}$, Q_1 at the edge of active region.

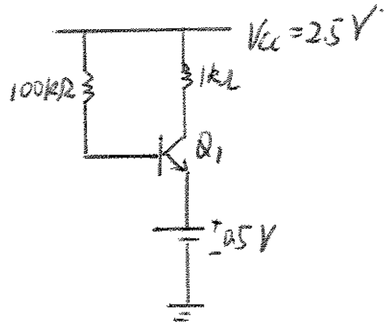
$$V_{BE_2} = V_{CE_2} = 0.733V$$

Operating Point:

I_{C_1}	$= 1.034mA$	I_{C2}	$= 1.034mA$
I_{B_1}	$= 0.01\ mA$	I_{B_2}	$= 0.01\ mA$
V_{BE_1}	$= 0.733\ V$	V_{BE_2}	$= 0.733\ V$
V_{CE_1}	$= 0.733V$	V_{CE_2}	$= 0.733V$

Although, for Q_2 $V_{BE} = V_{CE}$, it is at the edge of active region, the situation is not as severe as Q_1's. Since Q_2's configuration will always render $V_{BE} = V_{CE}$, whereas for Q_1, V_{CE} may drop below V_{BE}.

7) c).



$$I_B = \frac{V_{cc} - (V_{BE} + 0.5)}{100K}$$

$$I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.8V$,

$$I_c = \beta\left(\frac{2.5 - 1.3}{100K}\right) = 1.2\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$$, not 0.8, reiterate

$$V_{BE} = 0.736V, \quad I_c = \beta\left(\frac{2.5 - (0.736 + 0.5)}{100K\Omega}\right) = 1.26\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$, converges.

$$V_{CE} = (2.5 - 0.5) - (1.26)(1K\Omega) = 0.74$$

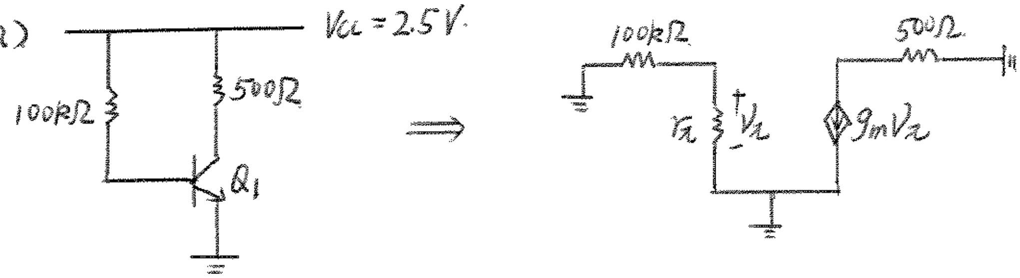
$V_{CE} > V_{BE}$, Q_1 in forward active region

Operating point

$I_c = 1.26mA$ $\quad V_{BE} = 0.738V$

$I_B = 0.0126mA$ $\quad V_{CE} = 0.74V$

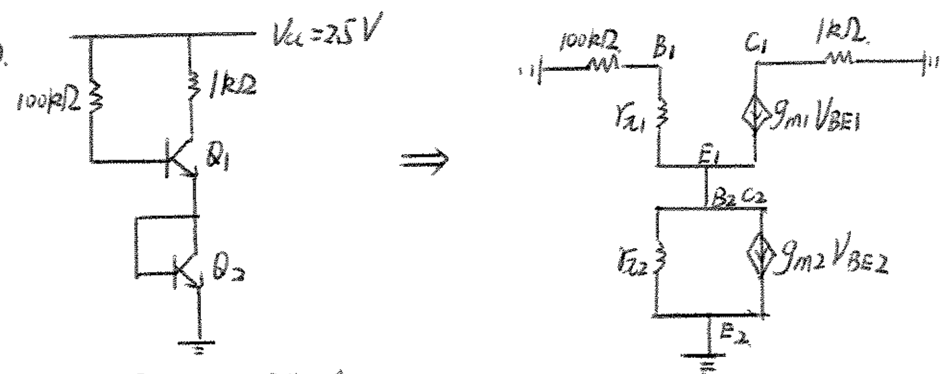
8). a)



$$g_m = \frac{I_C}{V_T} = \frac{1.754mA}{26mV} = 0.0675\ S$$

$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{100}{0.0675}\ \Omega = 1482.3\ \Omega$$

b).



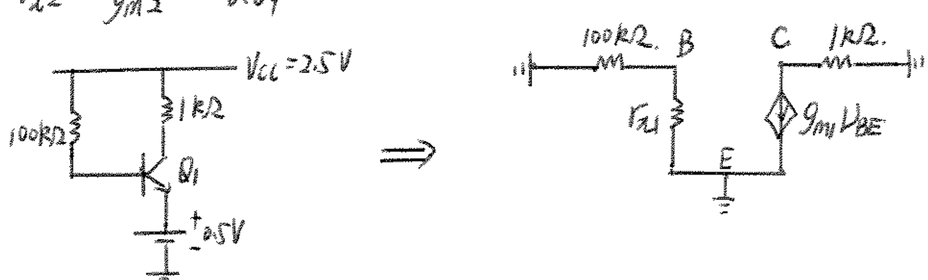
$$g_{m1} = \frac{I_C}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega$$

$$g_{m2} = \frac{I_C}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega$$

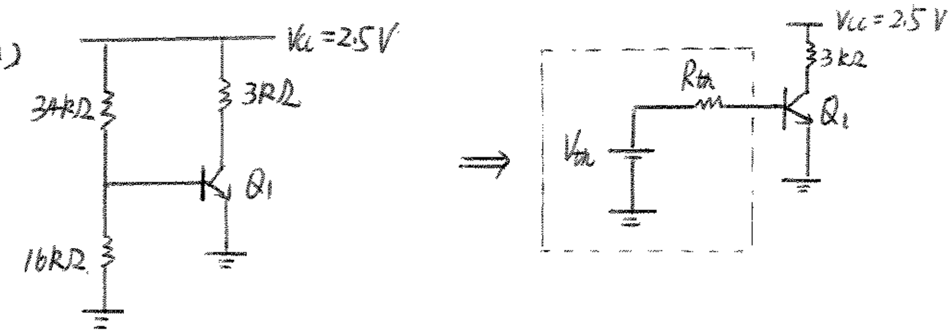
c).



$$g_{m1} = \frac{I_C}{V_T} = \frac{1.26mA}{26mV} = 0.048\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.048}\ \Omega = 2083\ \Omega$$

9). a)



Thevenin Equivalent

$$R_{th} = \frac{34 \times 16}{34+16} k\Omega = 10.88\, k\Omega$$

$$V_{th} = \frac{2.5V \times 16}{34+16} = 0.8\, V.$$

$$I_c = \beta\left(\frac{0.8 - V_{BE}}{10.88K}\right), \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Assume $V_{BE} = 0.7$,

$$I_c = \beta\left(\frac{0.8 - 0.7}{10.88K\Omega}\right) = 0.92\, mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta\left(\frac{0.8 - 0.734}{10.88K\Omega}\right) = 0.61\, mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.724\, V$$

Iterate, $V_{BE} = 0.724\, V$

$$I_c = \beta\left(\frac{0.8 - 0.724}{10.88\, K\Omega}\right) = 0.699\, mA$$

9)

a) $I_c = 0.699 mA$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$$

Iterate, $V_{BE} = 0.727V$

$$I_c = \beta\left(\frac{0.8-0.727}{10.88K\Omega}\right) = 0.67mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726V, \text{ Converged !!}$$

$V_{CE} = 2.5 - (0.67)(3K\Omega) = 0.49$

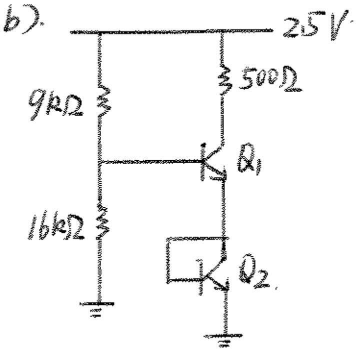
$V_{BE} - V_{CE} = 0.236V$, Soft-saturation, still ok.

Operating point:

$I_c = 0.67mA$ $V_{BE} = 0.726V$

$I_B = 6.7\mu A$ $V_{CE} = 0.49V$

9) b).

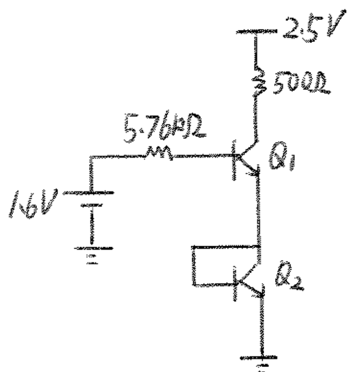


$\Longrightarrow$

$$R_{th} = \frac{9 \times 16}{9+16} k\Omega = 5.76 k\Omega$$

$$V_{th} = 2.5V \times \frac{16}{9+16} = 1.6V$$

$\Downarrow$



$$I_{C1} = \beta \left(\frac{1.6 - (V_{BE1} + V_{BE2})}{5.76 k\Omega} \right)$$

$$V_{BE} = V_{BE1} = V_{BE2} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

$$I_{C1} = I_{C2} = I_c$$

Guess $V_{BE1} = V_{BE2} = 0.7V$

$$I_c = \beta \left(\frac{1.6 - 1.4}{5.76 k\Omega} \right) = 3.47 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.769V$$

Iterate, $V_{BE} = 0.769V$

$$I_c = \beta \left(\frac{1.6 - (2)(0.769)}{5.76 k\Omega} \right) = 1.08 mA$$

9)

b)

$$I_c = 1.08mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.6 - 2(0.738)}{5.76K\Omega}\right) = 2.15mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.756V$$

Iterate, $V_{BE} = 0.756V$

$$I_c = \beta\left(\frac{1.6 - 2(0.756)}{5.76K\Omega}\right) = 1.53mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V$$

Iterate ... (For 3 more times)

$V_{BE} = 0.75V$, $I_c = 1.74mA$ converged

$V_{CE} = 2.5 - 0.75 - (1.74)(0.5) = 0.88V$

Operating Point

$I_{c_1} = 1.74mA$

$I_{B_1} = 17.4\mu A$

$V_{BE} = 0.75V$ (Forward active)

$V_{CE} = 0.88V$

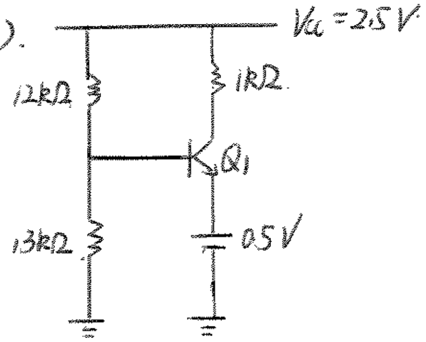
$I_{c_2} = 1.74mA$

$I_{B_2} = 17.4\mu A$

$V_{BE_2} = 0.75V$ (Edge of forward active)

$V_{CE_2} = 0.75V$

9). c).

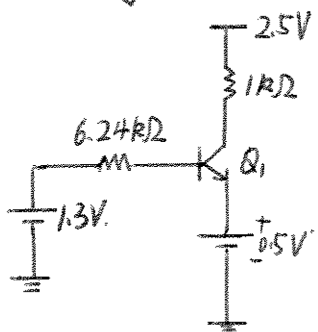


$\Rightarrow$

$$V_{th} = 2.5V \times \frac{13}{12+13} = 1.3V$$

$$R_{th} = \frac{12 \times 13}{12+13} k\Omega = 6.24k\Omega$$

$\Downarrow$



$$I_c = \beta \left(\frac{1.3 - (V_{BE} + 0.5)}{6.24K\Omega} \right)$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.743V$

$$I_c = \beta \left(\frac{1.3 - (0.743 + 0.5)}{6.24K\Omega} \right) = 0.913 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta \left(\frac{1.3 - (0.734 + 0.5)}{6.24K\Omega} \right) = 1.06mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$

9)

c)

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.3-(0.738+0.5)}{6.24K\Omega}\right) = 0.99mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$$

$$V_{CE} = 2.5-0.5-(0.99)(1K\Omega) = 1.01V$$

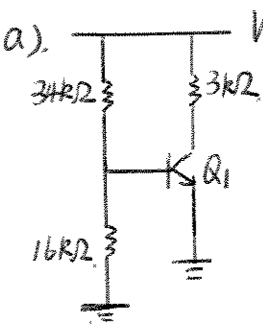
$V_{CE} > V_{BE} \Rightarrow$ Forward Active Region

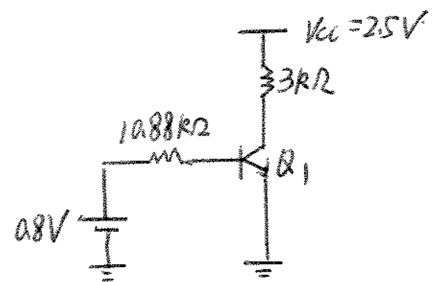
Operating Point

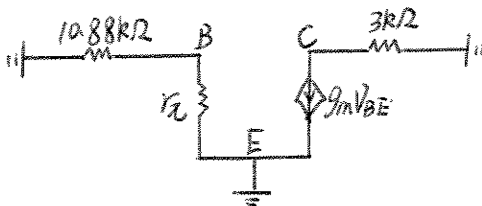
$I_c = 0.99mA$ $V_{BE} = 0.736V$

$I_B = 9.9\mu A$ $V_{CE} = 1.01V$

10). a).



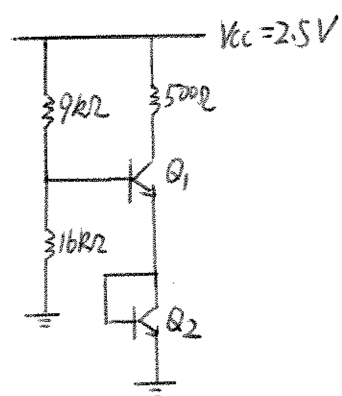


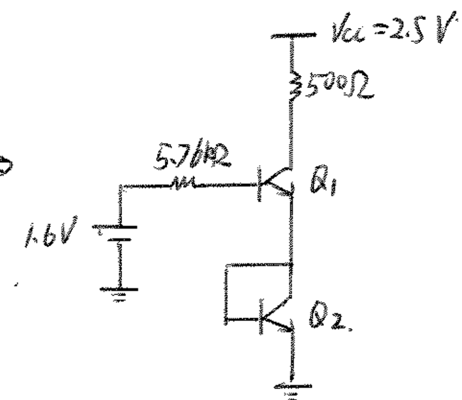


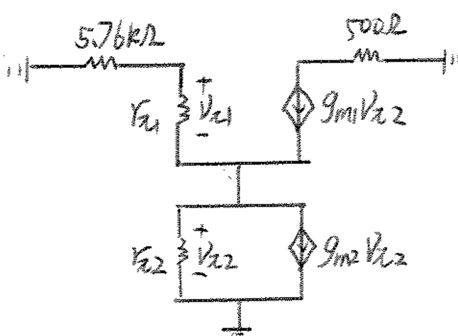
$$g_m = \frac{I_C}{V_T} = \frac{0.67mA}{26mV} = 0.026S$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.026}\Omega = 3846\Omega$$

b).







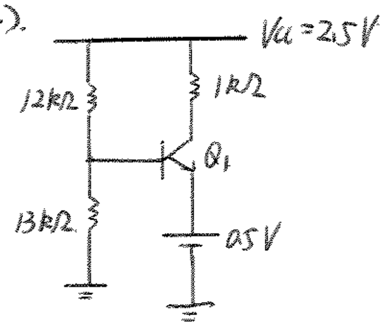
$$g_{m1} = \frac{1.74mA}{26mV} = 0.067S$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 1494.3\Omega$$

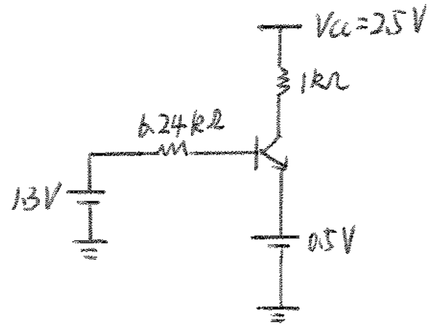
$$g_{m2} = 0.067S$$

$$r_{\pi 2} = 1494.3\Omega.$$

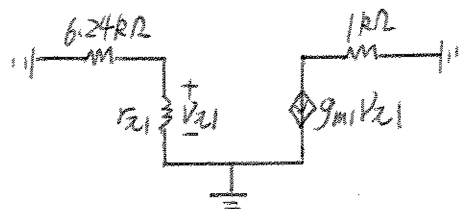
10) c).



⟹



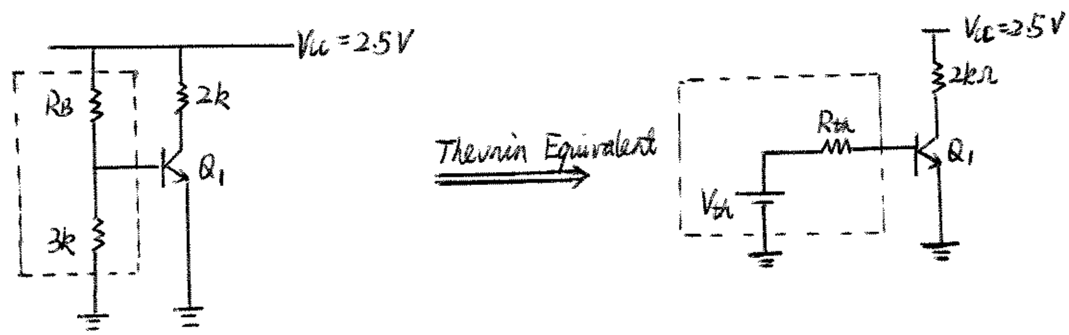
⟹



$$g_{m1} = \frac{I_c}{V_T} = \frac{0.99\,mA}{26\,mV} = 0.038\,S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.038}\,\Omega = 2632\,\Omega.$$

11) a). Find the minimum R_B that guarantees forward active region.



$$R_{th} = \frac{R_B \times 3}{R_B + 3}, \qquad V_{th} = \frac{2.5 \times 3}{R_B + 3}$$

To maintain Q_1 in forward-active region, $V_{CE} \geq V_{BE}$ ⊛

$$V_{CE} = V_{CC} - I_C \cdot 2k, \quad I_C = \beta I_B, \quad I_B = \frac{V_{th} - V_{BE}}{R_{th}}$$

So $V_{CE} = V_{CC} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k$

From ⊛

$$V_{CC} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k \geq V_{BE} \quad ①$$

And $V_{BE} = V_T \ln(I_C / I_S) = V_T \ln[\beta (V_{th} - V_{BE}) / R_{th} / I_S]$ ②

Find the minimum R_B by iteration. Guess $V_{BE} = 0.8$ as initial condition.

Use $V_{BE} = 0.8$, and subsitute R_{th} and V_{th} into ①, it can be calculated

$$R_B \geq 6.178\,k$$

Check the validity of V_{BE}. With $R_B \geq 6.178k$, from ②

$$V_{BE} = 0.727V.$$

So the initial guess of V_{BE} is not accurate.

Reiterate with $V_{BE} = 0.727$, it can be calculated from ①

$$R_B \geq 7.058\,k.$$

ii) With $R_B \geq 7.06k$, from ②

$$V_{BE} = 0.728$$

It's very close to 0.727. So the results have converged. (Satisfy both ① & ②)

The final answer is

$$R_B \geq 7.06k$$

b). β changes from 100 to 200, so $\partial\beta$ is 100

$$V_{CB} = 2.5 - I_C(2k) - V_{BE} = 2.5 - \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)\beta\cdot(2k) - V_{BE}$$

$$\frac{\partial V_{CB}}{\partial \beta} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right)\cdot 2k$$

$$\partial V_{CB} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right)\cdot 2k\cdot(\partial\beta) = -1.6627$$

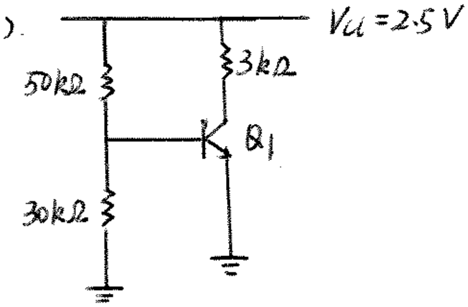
(Forward bias sustained during β's rising : 1.663V)

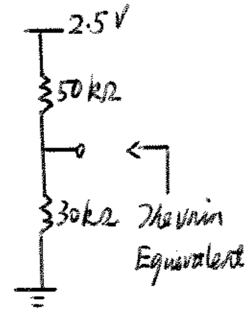
Original $V_{CB} = 0.01428$

Total net forward bias after β has rose to 200 :

$$1 - 1.6627 + 0.01428 = 1.648\ (V)$$

12). a).





$V_{th} = 2.5 \times \frac{30}{50+30}$

$= 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k$

$= 18.75 k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005 mA$.

$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$

$I_C = I_S \, e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta (\frac{V_{th} - V_{BE}}{R_{th}})$

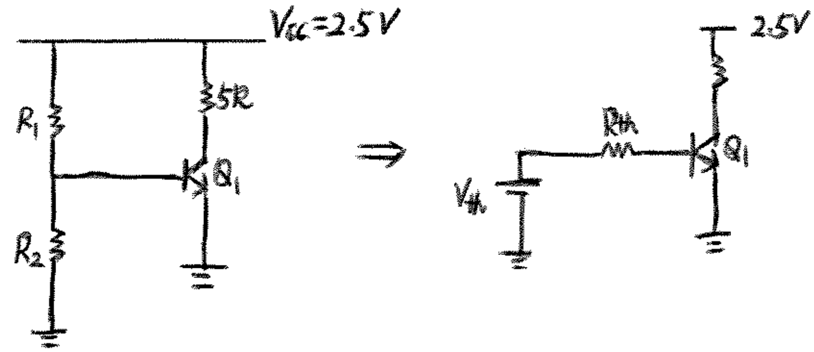
so $V_{BE} = 2.5 - \beta (\frac{V_{th} - V_{BE}}{R_{th}}) \cdot (3k)$

Solve this equation:

$V_{BE} = 0.83$.

$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$

13)



$$V_{th} = 2.5 \times \frac{R_2}{R_1+R_2}$$

$$R_{th} = \frac{R_1 R_2}{R_1+R_2}$$

$$R_{in} = R_{th} \parallel r_\pi = R_{th} \parallel \frac{\beta}{g_m} = R_{th} \parallel V_T\beta/I_C > 10k\Omega$$

$$g_m \geq \frac{1}{260\Omega} = 0.0038S$$

Let's choose g_m to be 0.0038S

$$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 0.104(mA)$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.761(V)$$

Let $R_{in} = 10\ k\Omega$

$$R_{in} = R_{th} \parallel V_T\beta/I_C \Rightarrow R_{th} = 16.13K\Omega \qquad ①$$

$$I_B = \frac{I_C}{\beta} = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{th} = V_{BE} + \frac{I_C \cdot (R_{th})}{\beta} = 0.78V \qquad ②$$

$$② \Rightarrow 2.5 \times \frac{R_2}{R_1+R_2} = 0.78V$$

$$① \Rightarrow \frac{R_1 R_2}{R_1+R_2} = 16.13K\Omega$$

It can be solved that $R_1 = 51.7K\Omega$, $R_2 = 23.44K\Omega$

This is only one possible solution set. The thought process is more important.

14). If g_m at least $\frac{1}{26}$ $= 0.03848\ (\Omega^{-1})$

Let $g_m = 0.03848 = \frac{I_c}{V_T} \Rightarrow I_c = 0.99996\ (mA)$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.82\ V$

$V_{CE} = V_{CC} - I_C \cdot 5k = -2.5$

No solution exists because the transistor is in saturation mode where g_m is essentially zero.

Whereas for problem 13),

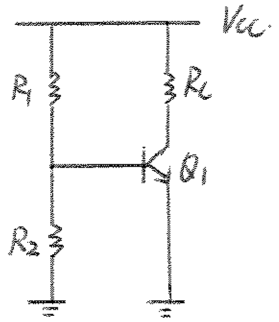
$V_{CE} = V_{CC} - I_C \cdot 5k = 2.5 - 0.104 \times 5 = 1.98\ V$

$V_{BE} = 0.761$

$V_{CE} > V_{BE}$

So Q_1 is still in forward-active region.

15).



Gain $= A_0$

$R_{out} = R_0 = R_C$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \,//\, r_\pi \quad , \quad r_\pi = \frac{\beta}{g_m}$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \,//\, \frac{\beta}{g_m}$$

$$\text{Gain} = A_0 = g_m R_0 \Rightarrow g_m = \frac{A_0}{R_0} = \frac{I_c}{V_T} \Rightarrow I_c = \frac{A_0}{R_0} V_T$$

(I_c is set)

Bias point analysis:

$$\frac{\dfrac{V_{cc} R_2}{R_1 + R_2} - V_{BE}}{\dfrac{R_1 R_2}{R_1 + R_2}} = \frac{A_0}{R_0} V_T$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_s}\right)$$

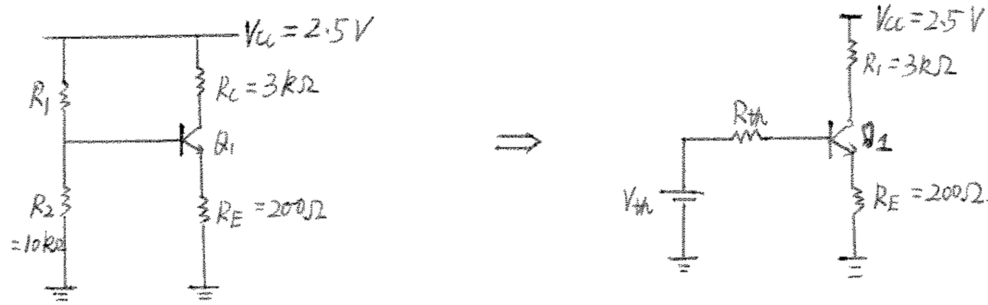
15)
$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{R_1 R_2}{R_1+R_2}} = \frac{A_0}{R_0} V_T$$

$$\frac{R_1 R_2}{R_1+R_2} = \frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{A_0}{R_0} V_T}$$

Max R_{in}:

$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{A_0}{R_0} V_T} \,//\, \beta \frac{R_0}{A_0}$$

1b).



a) $V_{th} = V_{cc} \cdot \frac{R_2}{R_1 + R_2} = 2.5 \times \frac{10k}{10k + R_1}$,

$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \cdot (10k)}{R_1 + 10k}$

$V_{th} = I_B R_{th} + V_{BE} + I_E R_E$ ⊛

Since $I_C = 0.25mA$, $I_B = 0.0025mA$, $I_E = \frac{0.25mA}{99} = 0.2525mA$.

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.696V$

⊛ becomes

$$2.5 \times \frac{10k}{10k + R_1} = 0.0025m \times \frac{R_1 \cdot (10k)}{R_1 + 10k} + 0.696 + 0.2525 \times 0.2$$

So

$$R_1 = 22.73k.$$

b) If R_E deviates by 5%, changes in R_E is 10Ω.

$$I_B = \frac{V_{th} - (V_{BE} + I_E R_E)}{R_{th}} \Rightarrow \frac{I_C}{\beta} = \frac{V_{th} - \left(V_{BE} + \frac{I_C}{\alpha} R_E\right)}{R_{th}}$$

$$\Rightarrow I_C = \frac{\beta\alpha\,(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E}$$

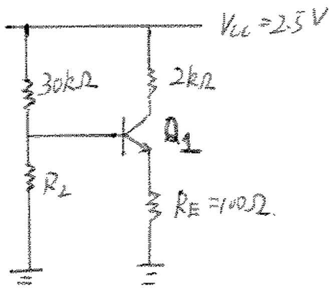
$$\Rightarrow \partial I_C = -\frac{\beta^2 \alpha\,(V_{th} - V_{BE})}{(\alpha R_{th} + \beta R_E)^2}\,\partial R_E$$

16) $\partial R_E = 10$, $V_{Th} = 0.764$, $V_{BE} = 0.7465$, $R_{Th} = 6.94k$, $\alpha = 0.99$, $\beta = 100$

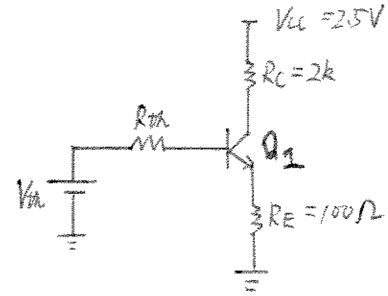
So $\partial I_C = -0.0024$ (mA)

The error is $\frac{0.0024}{0.25} \times 100\% = 0.96\%$ in I_C.

17).



$\Longrightarrow$



$$V_{th} = \frac{R_2 \times 2.5}{30k + R_2}, \quad R_{th} = \frac{30k \times R_2}{30k + R_2}$$

$V_{CE} \geq V_{BE}$ (To be guaranteed in active mode, soft saturation is not allowed.)

$$V_{CE} = V_{CC} - (I_C \cdot 2k + I_E \cdot 100)$$

$$I_C = \frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \quad \left(\because I_C = \frac{V_{th} - (V_{BE} + \frac{I_C}{\alpha} R_E)}{R_{th}} \right)$$

$$\text{So } V_{CE} = V_{CC} - \left[\frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \cdot 2k + \frac{\beta(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \times 100 \right]$$

$V_{CE} \geq V_{BE}$ means

$$2.5 - \left[\frac{99\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 2k + \frac{100\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 100 \right] \geq V_{BE} \quad ①$$

And

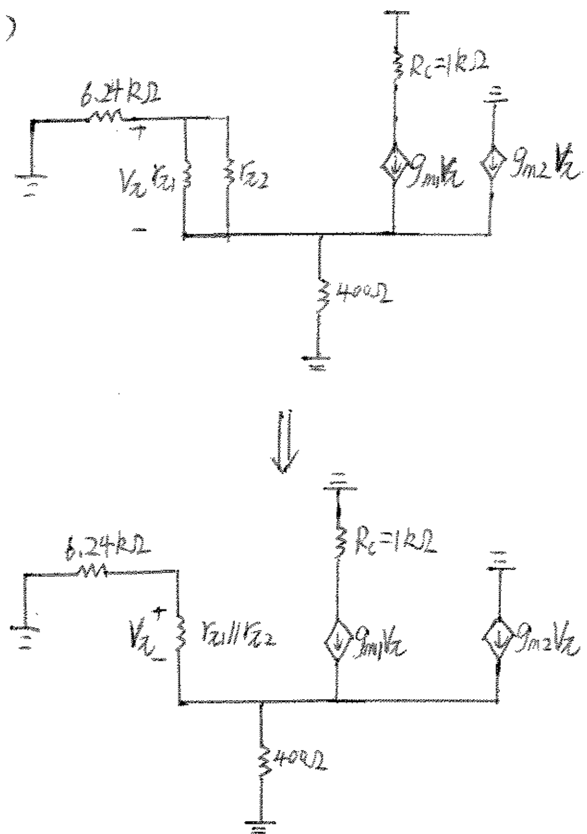
$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = V_T \ln\left[\frac{\beta\alpha(V_{th} - V_{BE})}{I_S(\alpha R_{th} + \beta R_E)}\right] \quad ②$$

There are two unknowns (R_2 and V_{BE}) and two equations (① and ②)

Since ② is a nonlinear equation, the problem can be solved by iteration.

Maximum $R_2 = 20.343k$.

18b)



<u>Small - Signal</u>

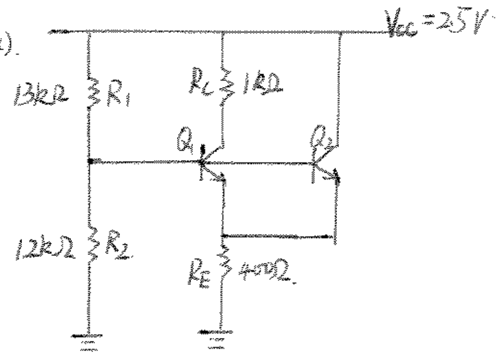
$g_{m1} = \frac{I_C}{V_T} = 0.02855$ (S)

$g_{m2} = 0.0142$ (S)

$r_{\pi1} = 3571.4$ (Ω)

$r_{\pi2} = 7042.3$ (Ω)

18) a).



$I_{S1} = 2 I_{S2} = 5\times10^{-16}$ A

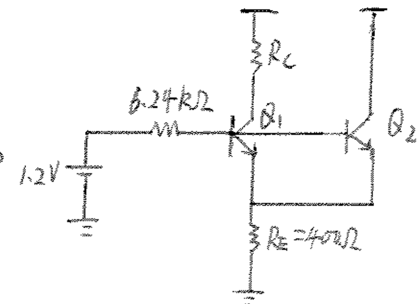
$\beta_1 = \beta_2 = 100$

$I_{S2} = 2.5\times10^{-16}$ A

$V_{th} = V_{cc} \times \frac{R_2}{R_1+R_2} = 1.2\ V$

$R_{th} = R_1 // R_2 = 6.24k\Omega$

$\Rightarrow$



$I_{B2} = \frac{1.2 - (V_{BE} + 3I_{E2}\cdot R_E)}{6.24k}$, and $I_{B2} = \frac{I_{C2}}{\beta}$

so $\frac{I_{C2}}{\beta} = \frac{1.2 - (V_{BE} + 3I_{C2}/\alpha \cdot 0.4k)}{6.24k}$

$\beta = 100,\ \alpha = 0.99$

$\Rightarrow I_{C2} = \frac{(1.2 - V_{BE})\cdot(\beta\alpha)}{(\alpha\cdot 6.24k + 3\beta\cdot 0.4k)} = \frac{(1.2 - V_{BE})(99)}{126.1776}$ (mA).

Guess $V_{BE} = 0.8$, $I_{C2} = 0.314$ mA

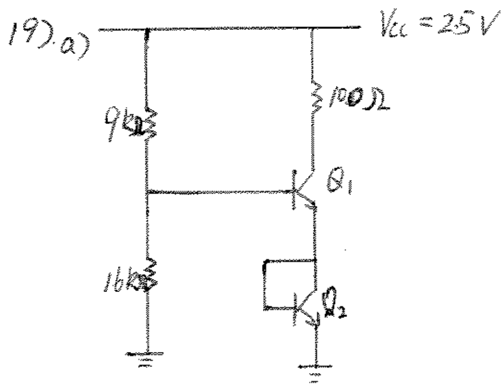
$V_{BE} = V_T \ln\left(\frac{I_C}{I_{S2}}\right) = 0.724$, not 0.8, so reiterate.

$I_{C2} = \frac{(1.2 - 0.724)(99)}{126.1776} = 0.3735$

$V_{BE} = 26 \ln\left(\frac{0.3735}{2.5\times10^{-13}}\right) = 0.728$, close. iterate again

$\Rightarrow V_{BE} \approx 0.729$ (V), $I_{C2} = 0.371$ (mA), $I_{C1} = 0.74$ (mA)

19). a)



$I_{S1} = I_{S2} = 4 \times 10^{-16} A$

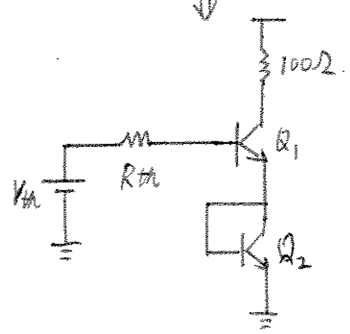
$\beta_1 = \beta_2 = 100$

$V_A = \infty$

$$V_{th} = \frac{(2.5)(16k)}{9k + 16k} = 1.6 (V)$$

$$R_{th} = 9k \,//\, 16k = 5.76k (\Omega)$$

⇓



$$I_{B1} = \frac{V_{th} - 2(V_{BE})}{R_{th}}, \quad I_{C1} = \beta I_{B1} = \beta \frac{V_{th} - 2(V_{BE})}{R_{th}} \qquad ①$$

$$V_{BE} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right). \qquad ②$$

Guess $V_{BE} = 0.7$,

① ⇒ $I_{C1} = 100 \times \frac{1.6 - 2 \times 0.7}{5.76} = 3.47 (mA)$

② ⇒ $V_{BE} = V_T \ln\left(\frac{3.47}{4 \times 10^{-13}}\right) = 0.7746$, not 0.7, reiterate

① ⇒ $I_{C1} = 0.8819$

② ⇒ $V_{BE} = 0.739$, not 0.7746, reiterate ...

After several iterations, V_{BE} converges to $\underline{\underline{0.755}}$

19) a)

$V_{BE} = 0.755\ (V)$

$$I_{B1} = \frac{V_{th} - 2V_{BE}}{R_{th}} = 0.0156\ (mA)$$

$I_{C2} = \beta I_{B1} = 1.56(mA)$

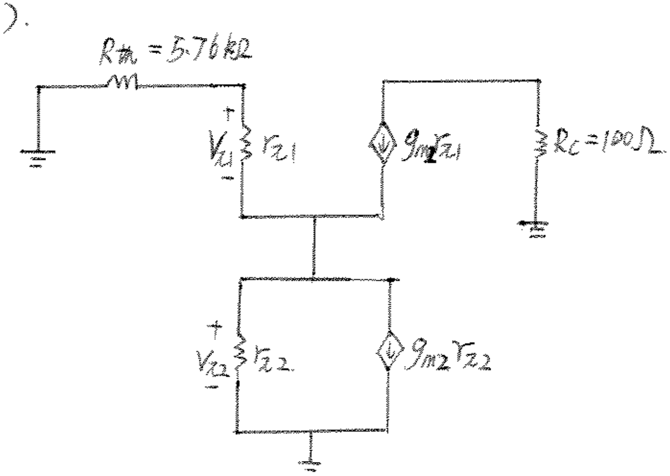
$V_{CE} = V_{CC} - [I_C \cdot (0.1) + V_{BE}] = 1.589\ (V)$

$I_{C2} = 1.56(mA)$

$I_{B2} = (1/\beta) I_{C2} = 0.0156(mA)$

$V_{CE2} = V_{BE} = 0.755(\quad V)$

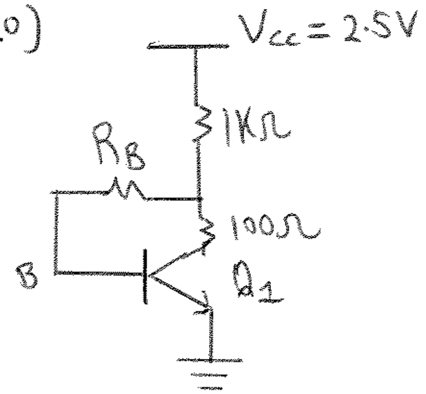
b).



$$g_{m1} = g_{m2} = \frac{I_C}{V_T} = 0.065\ (S)$$

$$r_{\pi1} = r_{\pi2} = \frac{\beta}{g_m} = 1666.7\ (\Omega)$$

20)



$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

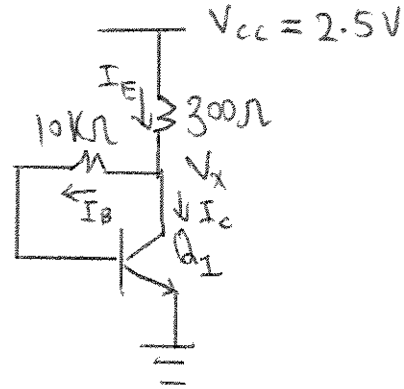
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)



$V_x = 1.1V$

$\beta = 100$

$I_s = ?$

$$I_E = I_B + I_c$$

$$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$$

$$I_B = \frac{I_c}{\beta}$$

$$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$$

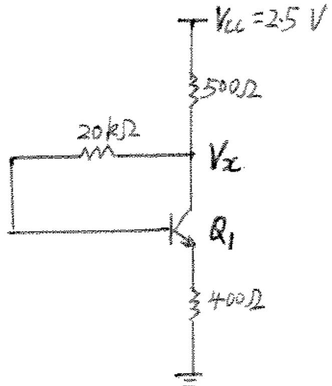
$$I_c = 4.624\,mA$$

$$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}} \quad , \quad V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$$

$$I_s = 1.035 \times 10^{-10}\,mA$$

$$I_s = 1.035 \times 10^{-13}\,A$$

22.



$I_S = 6 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$$\frac{V_{cc} - V_x}{0.5k} = I_C + I_B = I_C\left(1 + \frac{1}{\beta}\right) \Rightarrow V_x = 2.5 - 0.5k \cdot \frac{I_C}{\alpha} \quad ①$$

$$\frac{V_x - (V_{BE} + I_E \cdot 0.4k)}{20k} = \frac{I_C}{\beta} \Rightarrow V_x = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha} \cdot (0.4k) \quad ②$$

Equating V_x in ① and ②

$$2.5 - (0.5k)\left(\frac{I_C}{\alpha}\right) = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha}(0.4k)$$

$$I_C = \frac{2.5 - V_{BE}}{\frac{0.9k}{\alpha} + \frac{20k}{\beta}} = \frac{2.5 - V_{BE}}{1.11k} \quad ③$$

First iteration $V_{BE} = 0.8$.

③ $\Rightarrow$ $I_C = 1.53\,(mA)$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.743$, not 0.8, reiterate

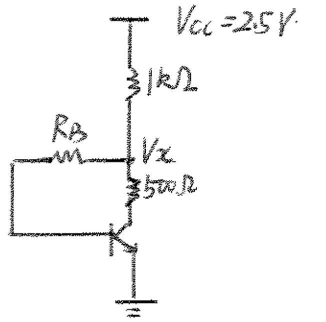
③ $\Rightarrow$ $I_C = \frac{2.5 - 0.743}{1.11} = 1.583\,(mA)$

$V_{BE} = V_T \ln\left(\frac{1.583}{I_S}\right) = 0.744$., converged.

So. $V_{BE} = 0.74(V)$ $\quad I_C = 1.58(mA)$, $I_B = I_C/\beta = 0.0158(mA)$

$V_C = 2.5 - \frac{1.583}{0.99} \times 0.5 = 1.7\,(V)$, $V_E = V_C - (I_B \cdot 20k + V_{BE}) = 0.644\,(V)$, $V_{CE} = V_C - V_E = 1.056\,(V)$

23).



$$I_c = \beta\left(\frac{2.5 - I_E(1K) - V_{BE}}{R_B}\right)$$

$$\frac{I_c R_B}{\beta} = 2.5 - I_E(1K) - V_{BE}$$

$$\left(I_E = \frac{I_c}{\alpha}\right)$$

$$I_c = \frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}} \quad (1)$$

$$V_{BC} \leq 0.2V$$

$$(V_x - I_B R_B) - (V_x - I_c 0.5) \leq 0.2V$$

$$I_c\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V$$

$$\left(\frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}}\right)\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V \quad (2)$$

Guess $V_{BE} = 0.75V \Rightarrow R_B \geq 34.513K\Omega$ (From (2))

Let $R_B = 34.513K\Omega$

$I_c = 1.291mA$, (From (1))

$V_{BE} = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.7564V$, not 0.75, reiterate

$V_{BE} = 0.7564V. \Rightarrow R_B \geq 34.461K\Omega$

23)

Let $R_B = 34.461\ k\Omega$

$I_c = 1.287\ mA$ (from (1))

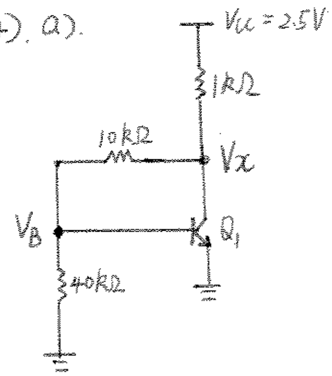
$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7563\ V$, converged !!

So $I_c = 1.287 mA$, $R_B = 34.46\ k\Omega$

Check V_{BC} : $V_{BC} = (1.287)(0.5) - \left(\frac{1.287}{100}\right)(34.46)$

$V_{BC} = 0.1999998$, less than $0.2V$

24). a).



$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_C}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_C}{\beta}\right) 10k + V_B$

Equating V_x $\Rightarrow$ $2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_C}{\alpha} \cdot 1k + \frac{I_C}{\beta} \cdot 10k$

$\Rightarrow I_C = \dfrac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$I_C = \dfrac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33\,m(A)$

Then

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.732$, not 0.8.

Reiterate

$I_C = \dfrac{1.5667}{1.11} = 1.4113\,(mA)$

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733$

So V_B converges to 0.73V

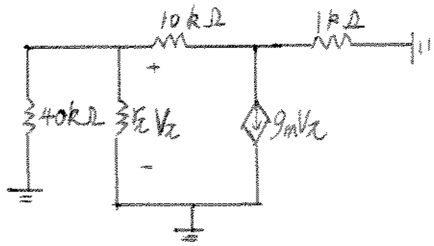
$I_C = 1.41\,mA$

$I_B = 14.1\,uA$

$V_{CE} = 2.5V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal

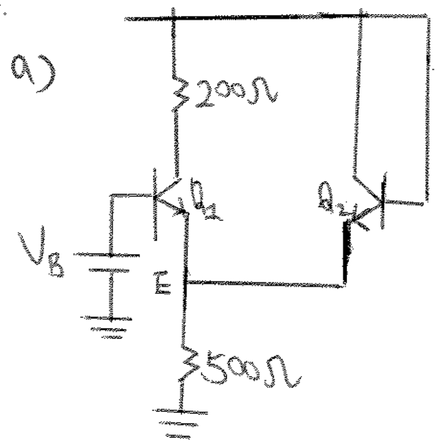


$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

25).

a)



$I_{C_1} = 1mA$, $I_{E_1} = 1.01mA$

$I_{S_1} = I_{S_2} = 3\times10^{-16}A$

$V_A = \infty$

$\beta = 100$

$V_E = (I_{E_1} + I_{E_2})0.5K$, $V_{BE_1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.75V$

$V_E = 2.5 - V_{BE2}$

$V_B - (1.01 + I_{E_2})0.5 = 0.75V.$

Guess $V_{BE_2} = 0.7V$

$V_E = 1.8 \Rightarrow I_{E_1} + I_{E_2} = 3.6mA \Rightarrow I_{E_2} = 2.59mA$

$I_{C_2} = 2.5641mA \Rightarrow V_{BE_2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.774V$

Reiterate

$V_E = 1.726 \Rightarrow I_{E_2} = 2.442mA, I_{C2} = 2.4176mA$

$V_{BE_2} = V_T \ln\left(\frac{I_{C_2}}{I_S}\right) = 0.773$, converged !!

$V_{BE} = 0.773V$, $I_{C_2} = 2.42mA$, $I_{E_2} = 2.44mA$

$V_B = 0.75 + (1.01 + 2.44)0.5 = 2.475V$

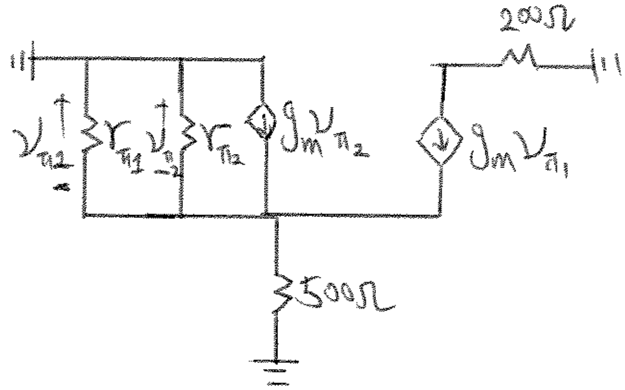
$V_C = 2.5 - (1)(0.2) = 2.3$

Q_1 in soft-saturation region.

25

b)

Small Signal Model



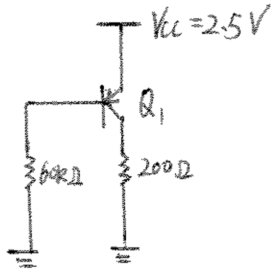
$$g_{m_1} = \frac{1mA}{26mV} = 0.0385 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_1} = \frac{100}{0.0385} = 2.6 K\Omega$$

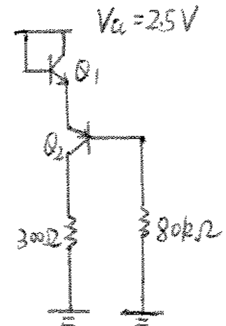
$$g_{m_2} = \frac{2.42mA}{26\,mV} = 0.0931 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_2} = \frac{100}{0.0931} = 1.07 K\Omega$$

26). $\beta_{npn} = 2\beta_{pnp} = 100$ $I_S = 9\times10^{-16}$ A $V_A = \infty$



(a)



(b)

a) $I_C = \frac{2.5 - |V_{BE}|}{60k}\beta_{pnp}$, $V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$

Guess $|V_{BE}| = 0.8 \Rightarrow I_C = 1.42m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\frac{1.42}{9\times10^{-13}}\right) = 0.730(V)$, not 0.8

Reiterate, $I_C = \frac{2.5-0.73}{60k}\times 50 = 1.475m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\frac{1.42}{9\times10^{-13}}\right) = 0.731(V)$

Reiterate, $I_C = \frac{2.5-0.731}{60k}\times 50 = 1.474m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\frac{1.474}{9\times10^{-13}}\right) = 0.731(V)$, converged.

Q_1 : $|V_{BE}| = 0.731V$, $I_C = 1.47mA$, $I_B = 29.4\mu A$.

$|V_{CE}| = 2.206V$.

In forward active region.

2b)

b). $I_{C2} = \frac{2.5 - (V_{BE1} + |V_{BE2}|)}{80k}$ ①

$I_{C2} \cdot \frac{\beta_{pnp}+1}{\beta_{pnp}} = \frac{I_{C1}(\beta_{npn}+1)}{\beta_{npn}}$

$\beta_{npn} = 2\beta_{pnp} = 100$

$\Rightarrow I_{C1} = \frac{2(\beta_{pnp}+1)}{2\beta_{pnp}+1} I_{C2} = 1.0099\, I_{C2}$ ②

$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$ ③

$V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$ ④

Four unknowns: I_{C1}, I_{C2}, V_{BE1}, V_{BE2}, Four equations: ①.②.③.④

Solve by iteration since ③ and ④ are exponential equations.

Guess $V_{BE2} = V_{BE1} = 0.8$

① $\Rightarrow$ $I_{C2} = 50 \times \left(\frac{2.5-1.6}{80k}\right)A = 0.5625\,mA$

② $\Rightarrow$ $I_{C1} = 0.568\,mA$

③ $\Rightarrow$ $V_{BE1} = V_T \ln\left(\frac{0.568}{9\times10^{-13}}\right)V = 0.706\,V$

④ $\Rightarrow$ $V_{BE2} = V_T \ln\left(\frac{0.5625}{9\times10^{-13}}\right)V = 0.706\,V$

Reiterate,

$I_{C2} = 0.68\,mA$, $I_{C1} = 0.6867\,mA$, $V_{BE1} = 0.711\,V$, $V_{BE2} = 0.711\,V$.

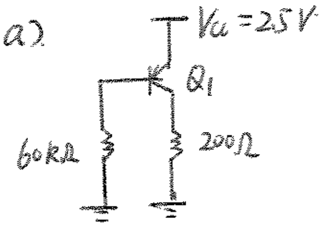
Reiterate,

$I_{C2} = 0.674\,mA$, $I_{C1} = 0.680\,mA$, $V_{BE1} = 0.711\,V$, $V_{BE2} = 0.711\,V$

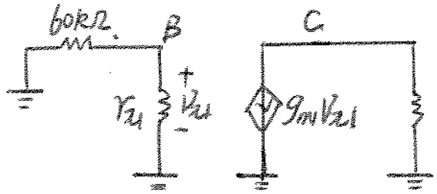
So.

$I_{C1} = 0.680\,mA$	$I_{C2} = 0.674\,mA$
$I_{B1} = 6.8\,uA$	$I_{B2} = 13.48\,uA$
$V_{BE1} = 0.711\,V$	$\lvert V_{BE2}\rvert = 0.711\,V$
$V_{CE1} = 0.711\,V$	$\lvert V_{CE2}\rvert = 2.5\,V - 0.711\,V - (0.674)(0.3)\,V = 1.5868\,V$

27. a).



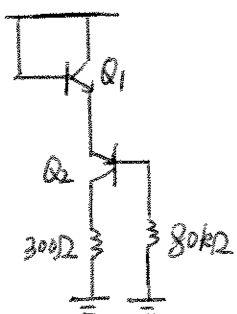
⟹



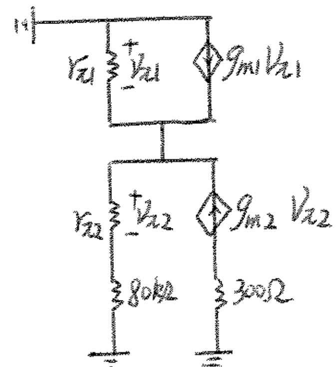
$$g_{m1} = \frac{I_c}{V_T} = \frac{1.47mA}{26mV} = 0.0565\,S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = 884\Omega$$

b).



⟹



$$g_{m1} = \frac{I_{c1}}{V_T} = 0.02615\,S$$

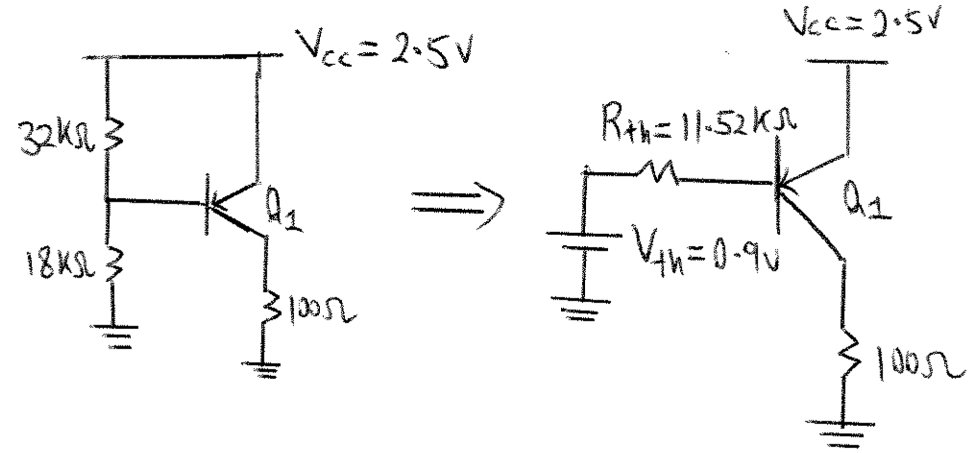
$$r_{\pi 1} = 3823.5\,\Omega$$

$$g_{m2} = \frac{I_{c2}}{V_T} = 0.0259\,S$$

$$r_{\pi 2} = 1928.8\,\Omega$$

28)

a)



$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\ V$, $I_c = 3.91mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757\ V$

Reiterate, $|V_{BE}| = 0.757V$, $I_c = 3.66mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$

Reiterate, $|V_{BE}| = 0.755V$, $I_c = 3.67mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$, Converged!!

$V_c = (3.67mA)(0.1K\Omega) = 0.367V$, $V_B \approx 2.5 - 0.755 = 1.745V$

Q_1 in forward active.

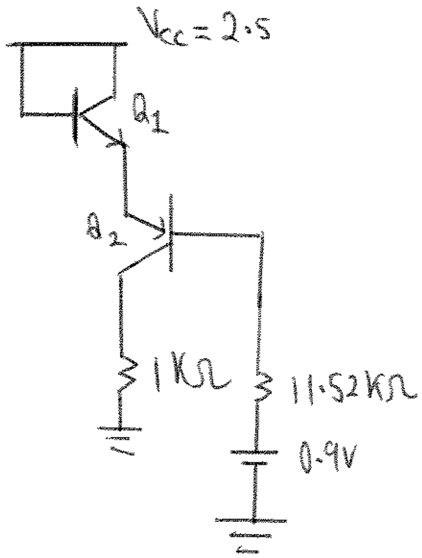
Bias point:

$I_c = 3.67mA$ $|V_{BE}| = 0.755$

$I_B = 73.4\ \mu A$ $|V_{CE}| = 2.5 - 0.367 = 2.133V$

28)

b)



$$I_{c2} = \left(\frac{2.5 - (V_{BE_1} + V_{BE_2}) - 0.9}{11.52K}\right) 50$$

$$I_{c_1} = I_{c_2}(1.0099)$$

(From β relation)

$$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right)$$

$$|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right)$$

Guess, $V_{BE_1} = V_{BE_2} = 0.7V$

$I_{c2} = 0.868\,mA$, $I_{c_1} = 0.877\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right) = 0.718V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right) = 0.717$

Reiterate, $V_{BE_1} = 0.718V$, $|V_{BE_2}| = 0.717V$

$I_{c_2} = 0.716\,mA$, $I_{c_1} = 0.723\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right) = 0.713V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right) = 0.712V$

Reiterate, $V_{BE_1} = 0.713V$, $|V_{BE_2}| = 0.712V$

$I_{c_2} = 0.760\,mA$, $I_{c_1} = 0.767\,mA$

$V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

$I_{C_2} = 0.747mA$, $I_{C_1} = 0.754mA$

$V_{BE_1} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.714V,$

$|V_{BE_2}| = 0.714\ V$

$$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\ V$$

$$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\ V$$

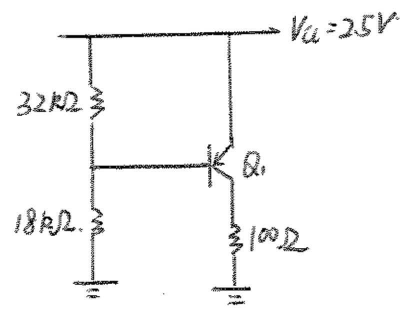
Q_2 is in forward-active region. Q_1 is always in forward-active region.

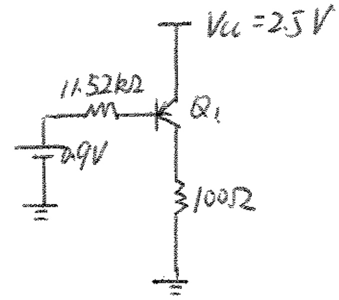
Bias point:

$V_{BE_1} = 0.714V$	$\lvert V_{BE_2}\rvert = 0.714V$
$I_{C_1} = 0.754\ mA$	$I_{C2} = 0.747mA$
$I_{B_1} = 7.54\mu A$	$I_{B2} = 14.94\ \mu A$
$V_{CE_1} = 0.714V$	$\lvert V_{CE_2}\rvert = 2.5 - 0.714 - 0.747 = 1.039V$

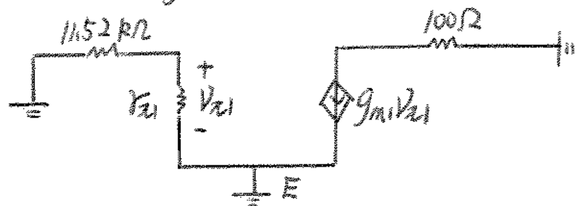
29)

a).





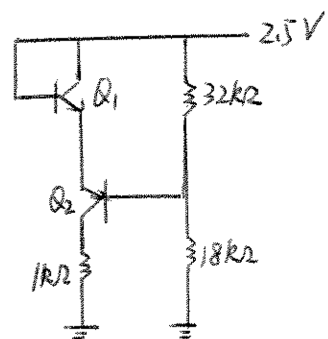
Small Signal:

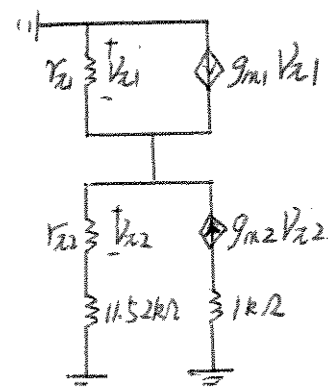


$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_\pi = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).





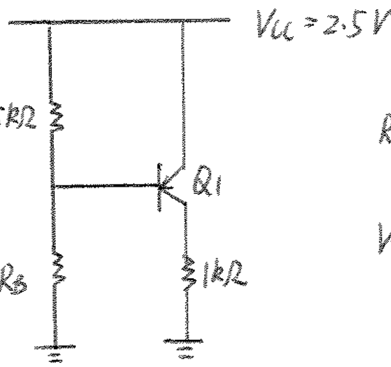
$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

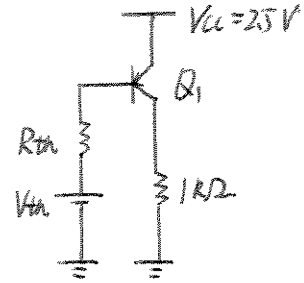
$r_{\pi 2} = 1740.3\ \Omega$.

30)



$$R_{th} = \frac{(R_B)(5k\Omega)}{R_B + 5k\Omega}$$

$$V_{th} = \frac{R_B}{R_B + 5k\Omega} \cdot 2.5V$$



$\beta = 50$, $I_s = 8\times10^{-16}A$, $V_A = \infty$

Edge of saturation: $|V_{BE}| = |V_{CE}|$

$$I_c = \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}, \quad |V_{CE}| = 2.5 - I_c \, 1K\Omega = |V_{BE}|$$

$$2.5 - \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}(1K\Omega) = |V_{BE}|$$

Subsitute in R_{th} and V_{th} and rearrange:

$$12.5R_B + 50|V_{BE}|R_B - |V_{BE}|(5K)R_B = 625 - |V_{BE}|250 \quad (1)$$

Guess $|V_{BE}| = 0.7V$, $(1) \Rightarrow 44R_B = 450 \Rightarrow R_B = 10.23K\Omega$

$V_{th} = 1.68V$ $\quad I_c = 1.7857\,mA$

$R_{th} = 3.36K\Omega$ $\quad |V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.739V$, not 0.7V, reiterate

$|V_{BE}| = 0.739V$, $(1) \Rightarrow 45.755R_B = 440.25$

$R_B = 9.62K\Omega$

$V_{th} = 1.645V$ $\quad I_c = 1.763mA$

$R_{th} = 3.29K\Omega$ $\quad |V_{BE}| = 0.739V$, Converged

3°)

So ±5% of 9.62 KΩ.

+5% Case:

$9.62K\Omega + 5\% = 10.101 K\Omega$

$V_{th} = 1.67\ V,\quad R_{th} = 3.345\ K\Omega$

$$I_c = \frac{(2.5 - 0.74 - 1.67)}{3.345} 50 = 1.3455\ mA$$

(Assume $|V_{BE}| = 0.74V$)

Check for $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.732v$, iterate once

$I_c = 1.4651\ mA,\ |V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$, converged

$|V_{BE}| \approx 0.734,\quad |V_{CE}| = 2.5 - 1.4651(1k\Omega) = 1.0349V$

$V_{BC} = 0.3009\ V$ (Reverse bias)

-5% case:

$9.62k\Omega - 5\% = 9.139\ K\Omega$

$V_{th} = 1.616V,\ R_{th} = 3.23184K\Omega$

$$I_c = \frac{(2.5 - 0.74 - 1.616)}{(3.23184)} 50 = 2.228mA$$

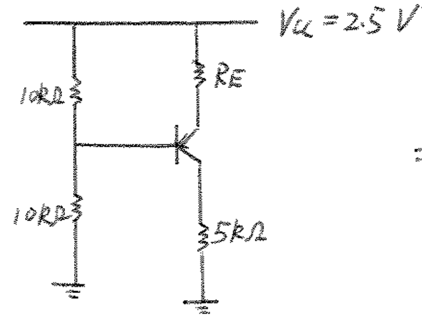
$(|V_{BE}| = 0.7)$, $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.745V$

reiterate: $|V_{BE}| = 0.745V,\ I_c = 2.150\ mA,$

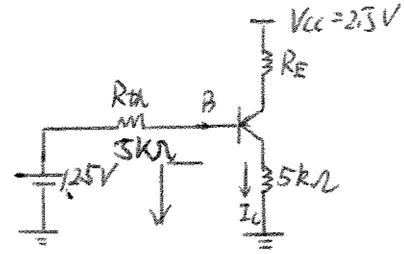
Verify V_{BE}, $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.744V$, Converged

$|V_{CE}| = 2.5 - 2.150(1K\Omega) = 0.35,\ |V_{BE}| = 0.744V,\ V_{BC} = -0.394V$ (Forward Bias)

31)



$\Rightarrow$



$$V_{BC} = 1.25 + I_B R_{th} - I_c 5K = 0.3$$

$$1.25 + \frac{I_c}{\beta}5 - I_c 5K = 0.3$$

$$\beta = 50 \Rightarrow I_c = 0.1939\,mA$$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.682\,V$$

$$I_B = \frac{\left(2.5 - \frac{I_c}{\alpha}R_E - |V_{BE}|\right) - 1.25}{5K}$$

, $\alpha = 0.9804$
$I_B = 0.003878\,mA$
$|V_{BE}| = 0.682\,V$
$I_c = 0.1939\,mA$

$$R_E = 2.89\,k\Omega$$

If R_E is halved $\Rightarrow$ $R_E = 1.44\,k\Omega$

$$I_c = \beta\left(\frac{2.5 - |V_{BE}| - 1.25 - \alpha I_c R_E}{5k}\right)$$

$$I_c = \frac{62.5 - 50|V_{BE}|}{78.44}$$

, Guess $|V_{BE}| = 0.682\,V$

31)

$I_c = 0.3621\ mA$

Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.698V$, not $0.682V$ reiterate

$$I_c = \frac{62.5 - 50(0.698)}{78.44} = 0.352\ mA$$

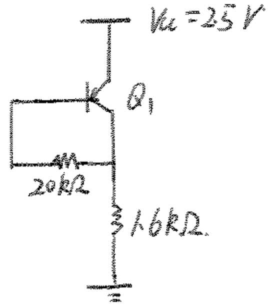
Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.697V$, converged

so $I_c = 0.352\ mA$, which is 1.82 times of $0.1939\ mA$

$$V_{BC} = 1.25 + \frac{(0.352)(5k\Omega)}{50} - (0.352)(5k\Omega) = -0.4748V$$

which drive Q_1 into saturation.

32)



$\beta = 80$

$V_A = \infty$

$V_B = (I_B)(20k\Omega) + I_E \cdot (1.6k\Omega)$

$I_C = 1mA$

$I_B = \frac{1}{80} mA, \quad I_E = \frac{1}{0.98765} mA = 1.0125 mA$

$V_B = (\frac{1}{80})(20)V + (1.0125)(1.6)V$

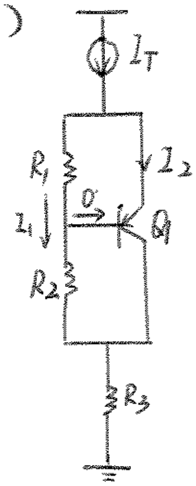
$= 1.87V$

$|V_{BE}| = 2.5V - 1.87V = 0.63V$

$I_C = I_S e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(\frac{V_{BE}}{V_T})}} = \frac{1mA}{e^{(\frac{0.63}{0.026})}}$

$I_S = 3 \times 10^{-11} mA$

33)



If Base current is neglected, $I_C = I_E$

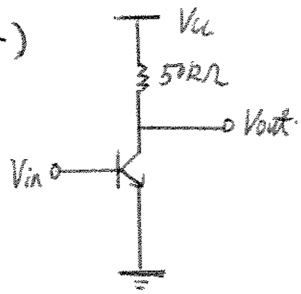
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

So $$\frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)



$$A_V = g_m R_C = 20$$

$$\frac{I_c R_c}{V_T} = 20 \Rightarrow I_c = \frac{20 V_T}{R_c}$$

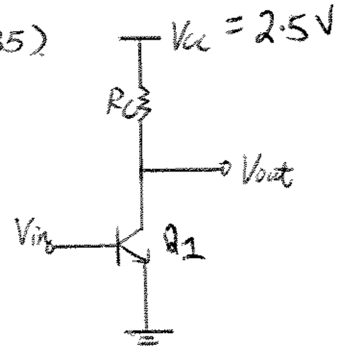
$$I_C = 0.0104\,mA$$

$$V_{cc} - (50k\Omega)(0.0104\,mA) = V_{BE}$$

$$\Rightarrow V_{cc} - 50 \times 0.0104\,V = 0.8\,V$$

$$\Rightarrow V_{cc} = 1.32\,V$$

35)



$V_A = 10V$, $r_o = \frac{V_A}{I_C}$, $g_m = \frac{I_c}{V_T}$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m (R_C // r_o) = g_m \left(\frac{R_C r_o}{R_C + r_o}\right) = \frac{R_C V_A}{V_T \left(R_C + \frac{V_A}{I_C}\right)}$$

As the equation above shows, a large gain means a large I_C. However, a large I_C will drive Q_1 into saturation. So a tradeoff must be made. The maximum limit for I_c is when it drives Q_1 into the edge of saturation, namely, $V_{BE} = V_{CE}$.

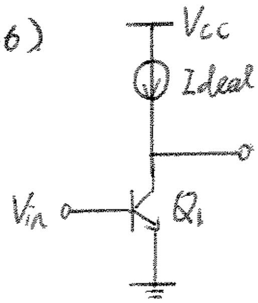
$$V_{CE} = V_{cc} - I_c (1K)$$

$V_{BE} = 0.8V$, $V_{cc} = 2.5V$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7mA$$

36)



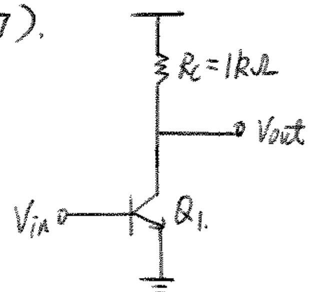
$A_v = 50$

$R_{out} = r_o = 10k\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50\left(\frac{V_T}{R_{out}}\right) = 0.13mA$$

37).



$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

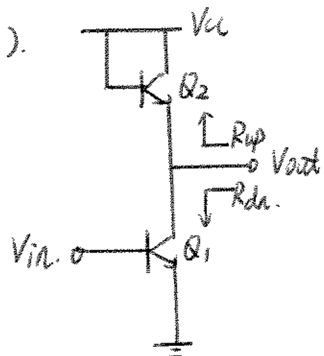
$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

38). (Find Av, Rin, Rout)

a).



$V_A = \infty$

$R_{out} = R_{up} // R_{dn}$

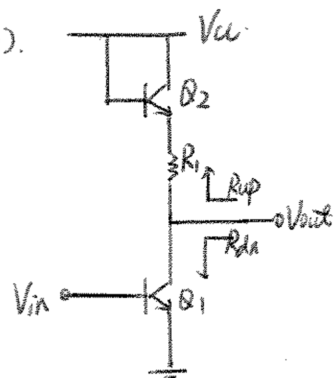
$R_{up} = \frac{1}{g_{m2}} // r_{\pi 2}$, $R_{dn} = \infty$

$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1}$

$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$

b).



$V_A = \infty$

$R_{up} = R_1 + \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{dn} = \infty$

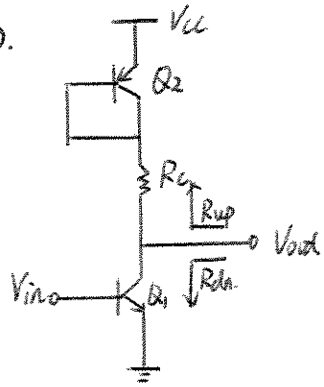
$R_{out} = R_{up} // R_{dn} = R_1 + \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1}$

$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(R_1 + \frac{1}{g_{m2}} // r_{\pi 2}\right)$

38

c).



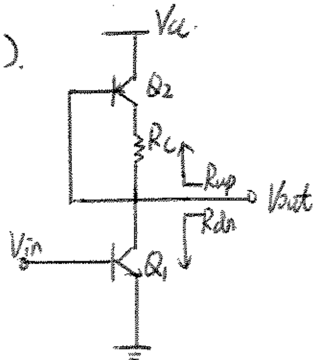
$V_A = \infty$

$R_{up} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi_2}\right)$, $R_{dn} = \infty$

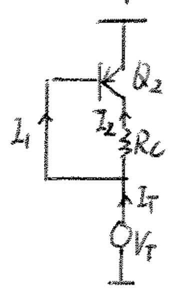
$R_{out} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi 2}\right)$, $R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(R_c + \left(\frac{1}{g_{m2}} // r_{\pi 2}\right)\right)$

d).



Find Rup:



$I_T = I_1 + I_2 = \frac{I_2}{\beta} + I_2$

$I_2 = g_m V_T$, $I_T = \frac{g_m V_T}{\beta} + g_m V_T$

$\frac{V_T}{I_T} = R_{up} = \frac{1}{\left(\frac{g_m}{\beta} + g_m\right)} = r_{\pi_2} // \frac{1}{g_{m_2}}$

$R_{dn} = \infty$

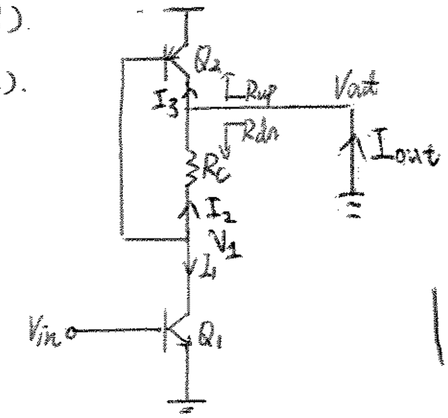
$R_{out} = R_{up} // R_{dn} = r_{\pi 2} // \frac{1}{g_{m2}}$

$R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(r_{\pi 2} // \frac{1}{g_{m2}}\right)$

38).

e).



$$R_{up} = \infty$$

$$R_{dn} = R_c + r_{\pi_2}$$

$$R_{out} = R_c + r_{\pi_2}$$

$$|A_v| = G_m R_{out}$$

where $G_m = \left| \dfrac{I_{out}}{V_{in}} \right|$

$$I_{out} + I_2 = I_3 \ , \quad I_2 = \frac{V_1}{R_c}$$

$$I_3 = V_1 g_{m2}$$

$$V_1 = -I_1 (R_c // r_{\pi_2}) = -g_{m_1} V_{in} (R_c // r_{\pi_2})$$

$$I_{out} = I_3 - I_2 = V_1 \left(g_{m2} - \frac{1}{R_c} \right)$$

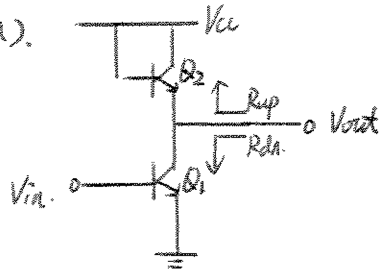
$$I_{out} = -g_{m_1} V_{in} (R_c // r_{\pi_2}) \left(g_{m2} - \frac{1}{R_c} \right)$$

$$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m_1} (R_c // r_{\pi_2}) \left(g_{m2} - \frac{1}{R_c} \right)$$

$$|A_v| = g_{m1} (R_c // r_{\pi_2}) \left(g_{m_2} - \frac{1}{R_c} \right) (R_c + r_{\pi_2})$$

39). $V_A < \infty$, find A_v, R_{in}, R_{out}

a).



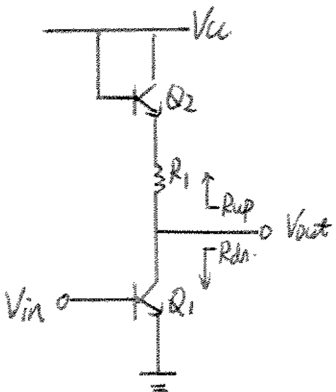
$R_{up} = \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2}$, $R_{dn} = r_{o1}$

$R_{out} = \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o1} \| r_{o2}$

$R_{in} = r_{\pi 1}$

$|A_v| = g_{m1} \left(\frac{1}{g_{m2}} \| r_{\pi 1} \| r_{o1} \| r_{o2} \right)$

b)



$R_{up} = R_1 + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2}$

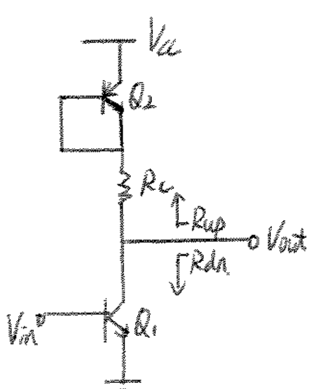
$R_{dn} = r_{o1}$

$R_{out} = r_{o1} \| \left(R_1 + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2} \right)$

$R_{in} = r_{\pi 1}$

$|A_v| = g_{m1} \left[r_{o1} \| \left(R_1 + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2} \right) \right]$

c).



$R_{up} = R_C + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2}$

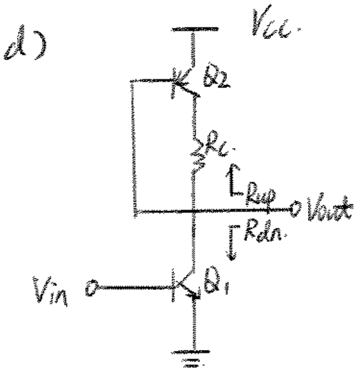
$R_{dn} = r_{o1}$

$R_{in} = r_{\pi 1}$

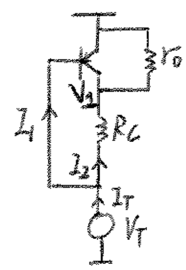
$R_{out} = r_{o1} \| \left(R_C + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2} \right)$

$|A_v| = g_{m1} \left[r_{o1} \| \left(R_C + \frac{1}{g_{m2}} \| r_{\pi 2} \| r_{o2} \right) \right]$

39 d)



Find R_{up}:



$$I_T = I_1 + I_2$$

$$I_2 = \frac{V_T - V_1}{R_C}, \quad I_1 = \frac{g_m V_T}{\beta}$$

$$V_1 = (I_2 - g_m V_T) r_o$$

$$I_2 = \frac{V_T - (I_2 - g_m V_T) r_o}{R_C}$$

$$I_2 = \frac{(1 + g_m r_o)}{R_c + r_o} V_T$$

$$I_T = \frac{g_m V_T}{\beta} + \frac{(1 + g_m r_o)}{R_c + r_o} V_T$$

$$\frac{V_T}{I_T} = R_{up} = \frac{1}{\frac{g_m}{\beta} + \frac{(1 + g_m r_o)}{R_c + r_o}}$$

$$R_{up} = r_\pi \parallel \frac{R_c + r_o}{(1 + g_m r_o)}$$

$$R_{up} = r_{\pi_2} \parallel \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}}$$

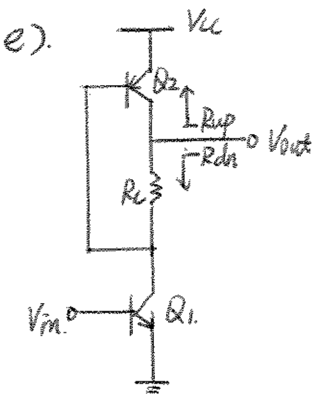
$$R_{dn} = r_{o1}$$

$$R_{out} = r_{\pi_2} \parallel \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} \parallel r_{o1}$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = \left| \frac{V_{out}}{V_{in}} \right| = g_{m1} \left(r_{\pi_2} \parallel \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} \parallel r_{o1} \right)$$

39 e).



$R_{up} = r_{o2}$

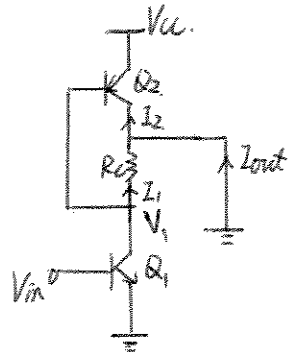
$R_{dn} = R_C + (r_{o1} \,//\, r_{\pi 2})$

$R_{in} = r_{\pi 1}$

$R_{out} = r_{o2} \,//\, [R_C + (r_{o1} \,//\, r_{\pi 2})]$

$|A_v| = G_m R_{out}$

Finding G_m:



$G_m = \left| \frac{I_{out}}{V_{in}} \right|$

$I_{out} + I_1 = I_2$

$I_{out} = I_2 - I_1$

$I_1 = \frac{V_1}{R_C} \qquad I_2 = V_1 g_{m2}$

$V_1 = -(g_{m1} V_{in})(r_{o1} \,//\, R_C \,//\, r_{\pi 2})$

$$I_{out} = V_1 \left(g_{m2} - \frac{1}{R_C}\right) = -g_{m1} V_{in} (r_{o1} \,//\, R_C \,//\, r_{\pi 2}) \left(g_{m2} - \frac{1}{R_C}\right)$$

$$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m1} (r_{o1} \,//\, R_C \,//\, r_{\pi 2}) \left(g_{m2} - \frac{1}{R_C}\right)$$

$$|A_v| = g_{m1} (r_{o1} \,//\, R_C \,//\, r_{\pi 2}) \left(g_{m2} - \frac{1}{R_C}\right) \{ r_{o2} \,//\, [R_C + (r_{o1} \,//\, r_{\pi 2})] \}$$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_V}{\partial I_C} = R_C \left(\frac{g_m R_E}{(1+R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_V}{\partial I_C} = R_C(-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_V = -R_C I_C (0.24)$$

$$\text{Relative change in gain} = \frac{\partial A_V}{A_V} = \frac{-0.24(R_C I_C)}{-\frac{R_C I_C}{V_T(1+R_E g_m)}} = 2.5\%$$

40)

b) $g_m R_E = 7$

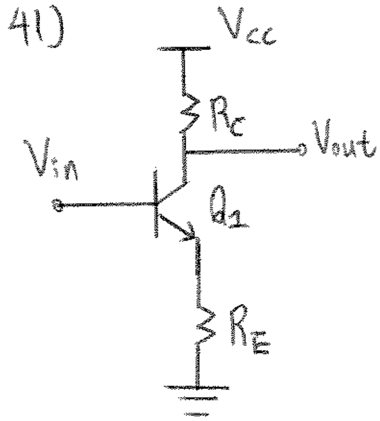
$$\frac{\partial A_v}{\partial I_c} = -R_c \, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06\,(R_c I_c)}{\frac{-R_c I_c}{V_T(1+R_E g_m)}} = 1.25\%$$

41)



$V_A = \infty$

$R_C I_C = 20 V_T$

$R_E I_C = 5 V_T$

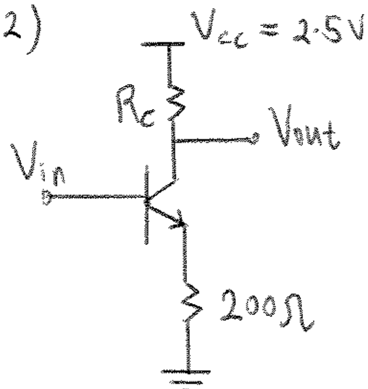
$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume β is large, so $I_C = I_E$.

$R_C I_C = 20 V_T, \quad R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42)



$$|A_v| = \frac{R_C I_C}{R_E I_C + V_T} = 10$$

Edge of Saturation

$V_{CE} = V_{BE} = 2.5 - I_C(R_C + R_E)$

$V_{BE} = 0.8V \Rightarrow I_C R_C = 1.7 - I_C 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_C I_C = 10(R_E I_C + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$1.7 - 0.2I_C = 2I_C + 0.26 \Rightarrow I_C = 0.655mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.725$, not 0.8, Reiterate

$I_C R_C = 1.775 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_C = 0.689mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.727V$, iterate 1 more time

$I_C R_C = 1.773 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow$ $I_c = 0.688\ mA$

Check for V_{BE} $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688mA$

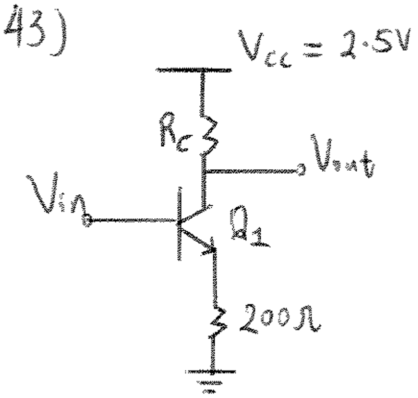
$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0K\Omega$$

43)



$|A_v| = 100$
(Voltage gain)

$$|A_v| = 100 \Rightarrow R_c I_c = 100(R_E I_c + V_T)$$

$$R_c I_c = 20 I_c + 2.6 \quad (1)$$

$$R_c I_c = 1.7 - I_c 0.2 \quad (2) \quad (\text{Assume } V_{BE} = 0.8)$$

Equating (1) and (2) yield

$$1.7 - I_c 0.2 = 20 I_c + 2.6 \Rightarrow I_c = -0.04455\,mA$$

A negative I_c in forward active region is impossible, therefore, a solution does not exist. The reason is because $R_c I_c$ is too large to produce a gain of 100 that drive Q_1 into saturation region.

Maximum gain achievable:

$$\frac{R_c I_c}{R_E I_c + V_T} = |A_v| \quad (\text{Gain Equation})$$

$$2.5 = R_c I_c + V_{CE} + R_E I_c \quad (\text{Operating Point Equation})$$

Let A = Maximum gain

43)

$$AI_c 0.2 + A0.026 = 1.7 - I_c 0.2$$

$$\Rightarrow I_c = \frac{1.7 - A0.026}{A0.2 + 0.2}$$

Since I_c cannot be zero, set

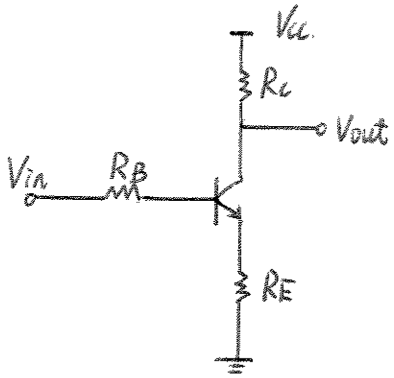
$$\frac{1.7 - A0.026}{A0.2 + 0.2} > 0$$

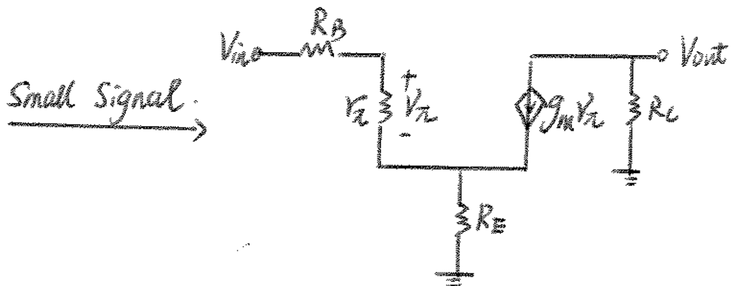
$$1.7 - A0.026 > 0$$

$$1.7 > A0.026$$

$$A < \frac{1.7}{0.026} = 65.4$$ (Maximum gain achievable)

44) $V_A = \infty$





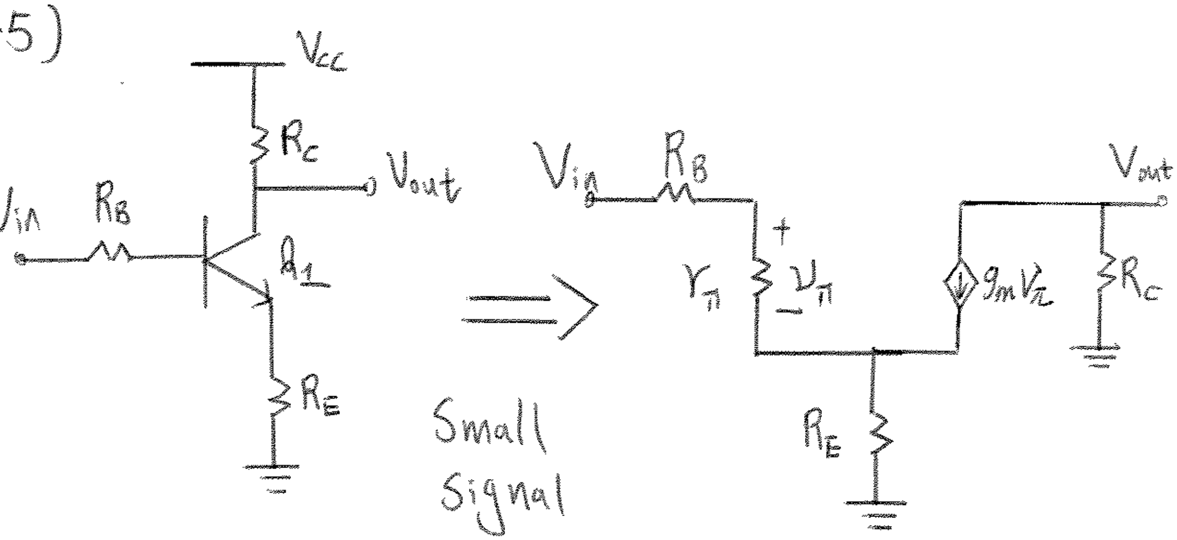
$$V_{out} = -g_m V_\pi R_C$$

$$V_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

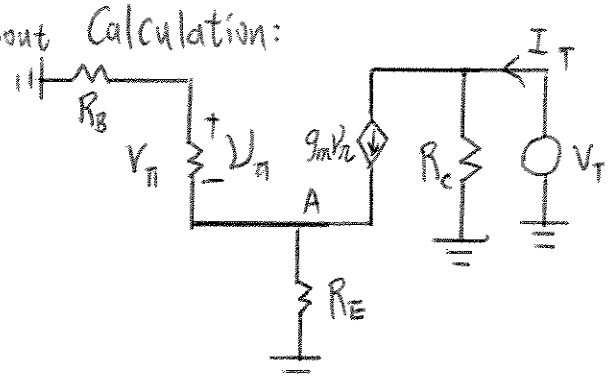
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

45)



R_{out} Calculation:



$$V_A = g_m v_\pi (R_E \,//\, R_B + r_\pi) \qquad (1)$$

$$v_\pi = -\frac{V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = \frac{-v_\pi (r_\pi + R_B)}{r_\pi} \qquad (2)$$

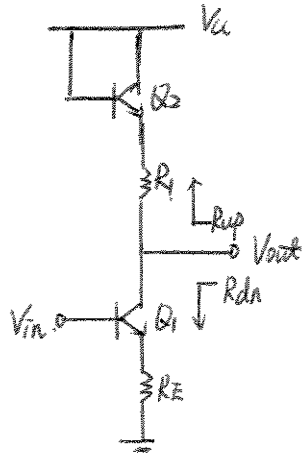
The only possible solution for 1) and 2) is $v_\pi = V_A = 0$, since 1) is positive and 2) is negative.

$$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \frac{V_T}{I_T} = R_C$$

Therefore, $R_{out} = R_C$

46) $V_A = \infty$

a)



$$R_{up} = R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}$$

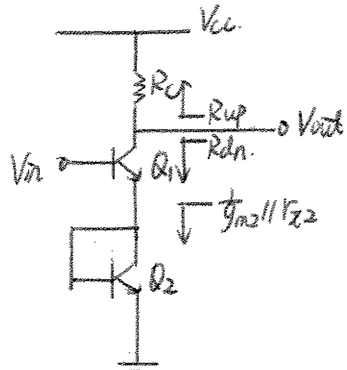
$$R_{dn} = \infty$$

$$R_{out} = R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}$$

$$R_{in} = r_\pi + (1+\beta) R_E$$

$$|A_v| = \frac{R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

b).



$$R_{up} = R_C$$

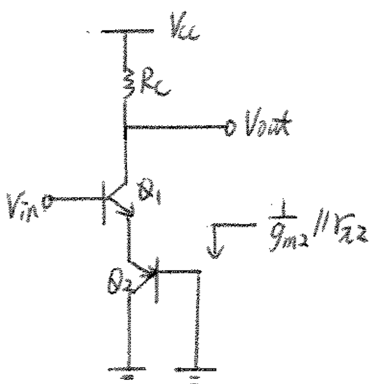
$$R_{dn} = \infty$$

$$R_{out} = R_C$$

$$R_{in} = r_{\pi 1} + (\beta + 1)\left(\frac{1}{g_{m2}} \,//\, r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \,//\, r_{\pi 2} + \frac{1}{g_{m1}}}$$

c)



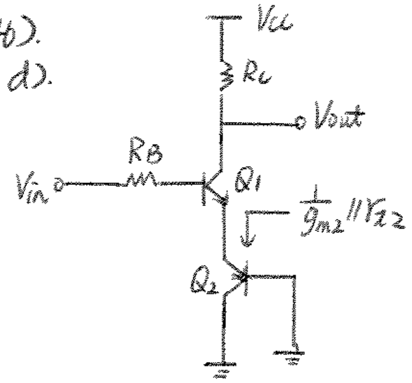
$$R_{out} = R_C$$

$$R_{in} = r_{\pi 1} + (\beta + 1)\left(\frac{1}{g_{m2}} \,//\, r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \,//\, r_{\pi 2} + \frac{1}{g_{m1}}}$$

4b).

d).

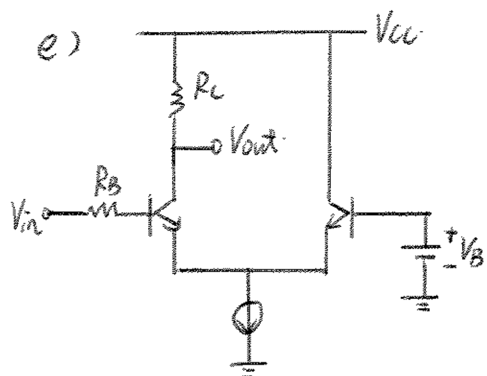


$R_{out} = R_C$

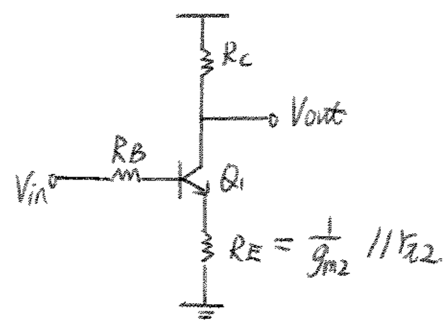
$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

$|A_v| = \dfrac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$

e)



$\Rightarrow$



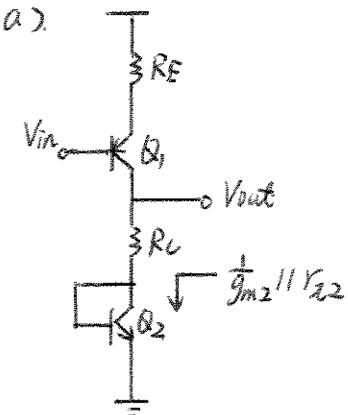
$R_{out} = R_C$

$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

$|A_v| = \dfrac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$

47). $V_A = \infty$.

a).



$$R_{out} = R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

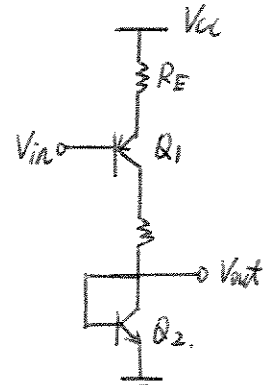
$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

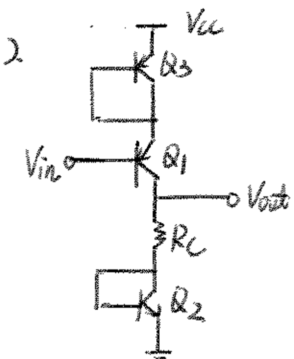
b). $R_{out} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$

$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$



c).



$$R_{out} = R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

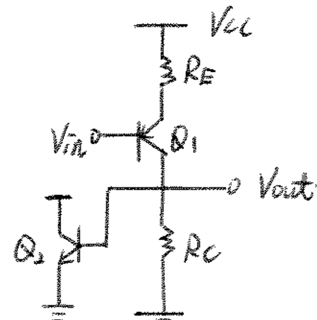
$$R_{in} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}} \parallel r_{\pi 3}}$$

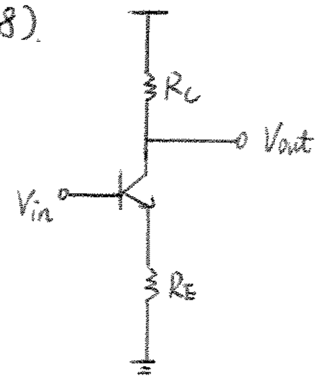
d) $R_{out} = R_C \parallel r_{\pi 2}$

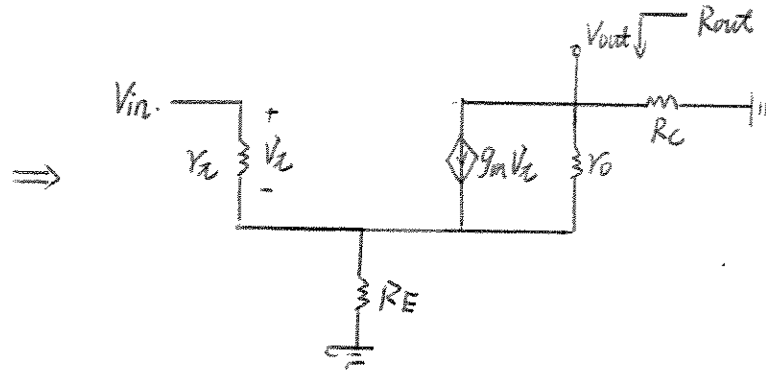
$$R_{in} = r_{\pi 1} + (\beta+1) R_E$$

$$|A_v| = \frac{R_C \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$



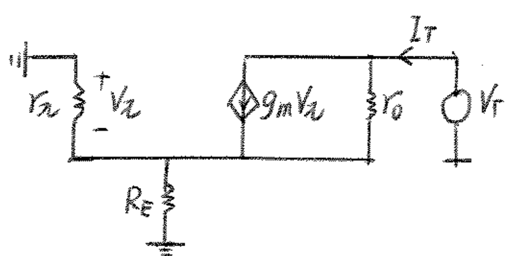
48)





$R_{out} = R_C \,//\, R_{eq}$

Solve for R_{eq}.



$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T\,(r_\pi \,//\, R_E)$$

$$I_T = -g_m I_T\,(r_\pi \,//\, R_E) + \frac{(V_T - I_T(r_\pi \,//\, R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o\left(1 + \frac{(r_\pi \,//\, R_E)}{r_o}\right) + g_m\,(r_\pi \,//\, R_E)\Big)$$

$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

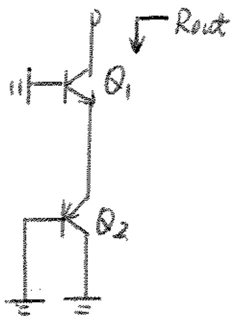
$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

$$R_{out} = R_C \,//\, r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

$$R_{out} \approx R_C \,//\, r_o(1 + g_m(r_\pi \,//\, R_E)) \quad \text{since } g_m r_o \gg 1$$

49). $\beta \gg 1$ and $V_A < \infty$ to have meaningful result.

a).

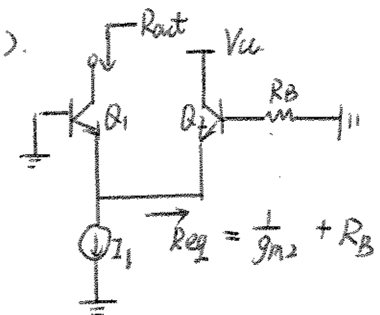


$\frac{1}{g_{m2}} \parallel r_{\pi 2} \approx \frac{1}{g_{m2}}$, since $\beta \gg 1$

$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

$= r_{o1} (1 + g_{m1}/g_{m2})$

b).

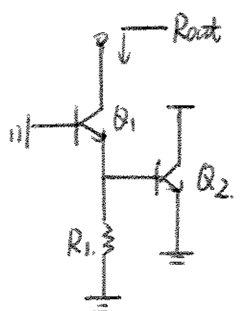


$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left[\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta+1}\right) \parallel r_{\pi 1}\right]$

$\approx r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta}\right)$

$\approx r_{o1} \left[1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta}\right)\right]$.

c).



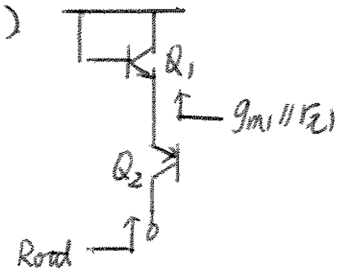
$R_{out} = r_o + (1 + g_{m1} r_o)(R_1 \parallel r_{\pi 2} \parallel r_{\pi 1})$

$R_1 \parallel r_{\pi 1} \parallel r_{\pi 2} \approx R_1$, since $\beta \gg 1$.

$R_{out} \approx r_o (1 + g_{m1} R_1)$

50) $\beta >> 1$, $r_A \to \infty$, for meaningful results

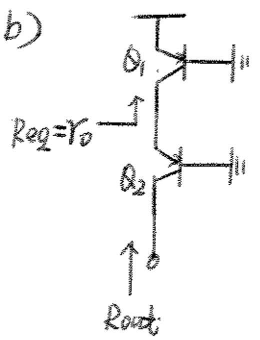
a)



$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2})\left(\frac{1}{g_{m1}} // r_{\pi 2}\right)$$

$$\approx r_{o2}(1 + g_{m2}/g_{m1})$$

b)



$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} // r_{\pi 2})$$

$$\approx r_{o2}\left[1 + g_{m2}(r_{o1} // r_{\pi 2})\right]$$

The output impedance in b) is larger than a) because Q_2's connected for a high impedance load, whereas in a) it's connected to a low impedance load.

51). $r_\pi = \beta V_T / I_C$.

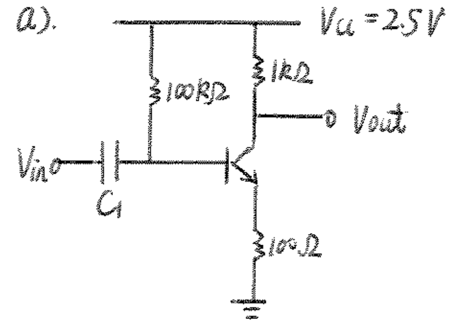
$$R_{in} = r_\pi // R_B = \frac{\frac{\beta V_T}{I_C} R_B}{\frac{\beta V_T}{I_C} + R_B} = \frac{V_T R_B}{V_T + \frac{I_C}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_C}{\beta}} = \frac{\beta V_T}{I_C} \approx r_\pi$

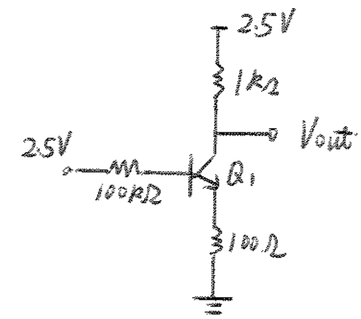
So $R_{in} = r_\pi // R_B \approx r_\pi$.

52). $I_S = 8\times10^{-6}A$, $\beta = 100$, $V_A = \infty$

a).



DC Analysis →



$$I_c = \frac{\beta\left(2.5 - \left(V_{BE} + \frac{I_c}{\alpha}0.1\right)\right)}{100K} \Rightarrow I_c = \frac{100\,(2.5 - V_{BE})}{100K + 10.1K}$$

Guess $V_{BE} = 0.75V$, $I_c = 1.59mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, not $0.75V$, reiterate

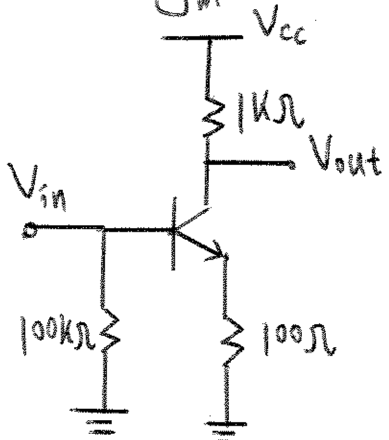
$V_{BE} = 0.736V$, $I_c = 1.60\,mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, converged !!

$I_c = 1.60mA$

$$g_m = \frac{I_c}{V_T} = \frac{1.60mA}{26mV} = 0.0615\left(\frac{1}{\Omega}\right)S$$

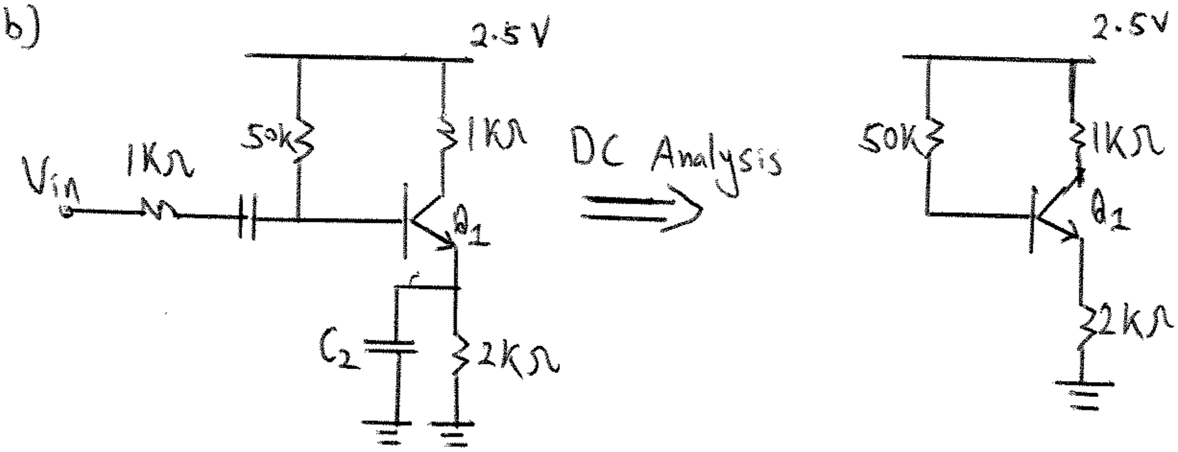
$$r_\pi = \frac{\beta}{g_m} = 1.63\,K\Omega$$



$$A_V = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1K}{0.1 + \frac{1}{g_m}} = 8.6$$

52)

b)



$$I_c = \beta\left(\frac{2.5-(V_{BE}+I_E 2K)}{50K}\right) \Longrightarrow I_c = \frac{100(2.5-V_{BE})}{50K + 202K}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.714\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7155V$, reiterate

$V_{BE} = 0.7155V$, $I_c = 0.708\ mA$

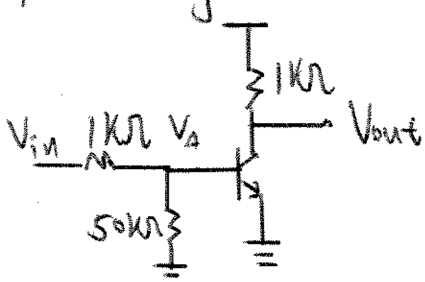
Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$, converged!!

$I_c = 0.708mA$

$g_m = \frac{I_c}{V_T} = 0.02723\left(\frac{1}{\Omega}\right)$ S

$V_{BE} = 0.715V$

AC Analysis:



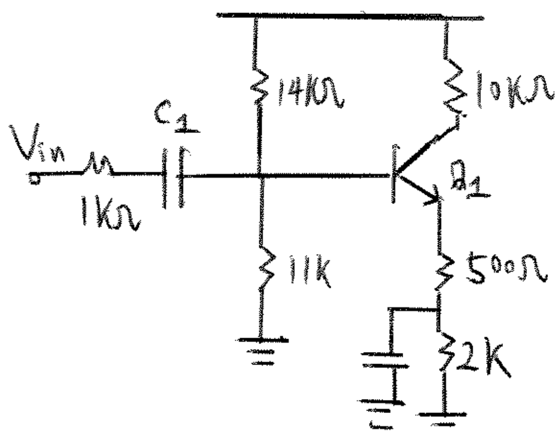
$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right| = 21.1$$

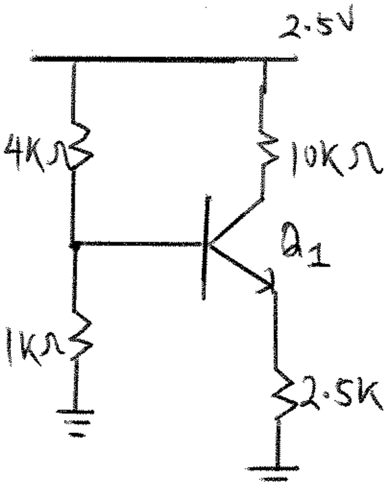
$$\left|\frac{V_{out}}{V_A}\right| = g_m\, 1K\Omega = 27.23$$

$$\left|\frac{V_A}{V_{in}}\right| = \frac{50K // V_\pi}{50K // V_\pi + 1K} = 0.77$$

52)

c)





$$I_c = \beta \left(\frac{1.1 - (V_{BE} + \frac{I_c 2.5}{\alpha})}{14K\Omega // 11K\Omega} \right) \Rightarrow I_c = \frac{100\,(1.1 - V_{BE})}{6.16 + 252.53}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.1546\ mA$

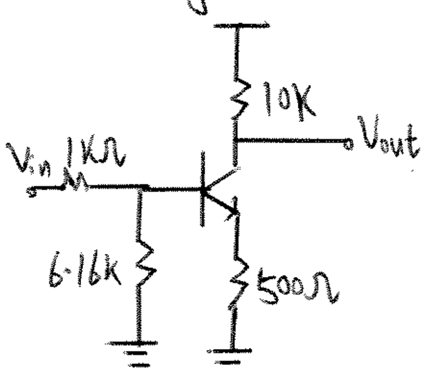
Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.676\ V$, not $0.7V$, reiterate

$V_{BE} = 0.676\ V$, $I_c = 0.164\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.677V$, converged !!

$I_c = 0.164\ mA$, $V_{BE} = 0.677V$, $g_m = 0.00631\ (\frac{1}{\Omega})\ S$, $r_\pi = 15.85k$

AC Analysis:

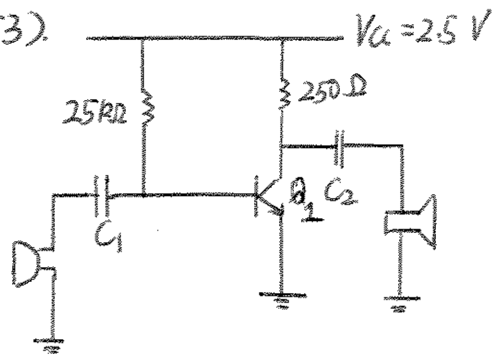


$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right| \left|\frac{V_A}{V_{in}}\right| = 12.9$$

$$\left|\frac{V_{out}}{V_A}\right| = \frac{10K}{\frac{1}{g_m} + 0.5} = 15.2$$

$$\left|\frac{V_A}{V_{in}}\right| = \frac{6.16K // (15.85K + 101(0.5))}{6.16K // (15.85K + 101(0.5)) + 1K} = 0.85$$

53).



$R_B = 25\,K\Omega$

$R_C = 250\Omega$

$I_S = 5 \times 10^{-17} A$

$V_A = \infty$

DC Analysis: Assume collector bias voltage is still 1.5 V. So 1 V across $R_C \Rightarrow I_C = 4mA$.

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.832$$

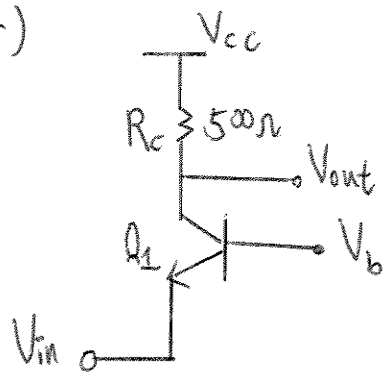
$$I_B = \frac{2.5 - V_{BE}}{25K} = 0.06673\,mA$$

$$\beta = \frac{I_C}{I_B} = \frac{0.832\,mA}{0.06673\,mA} = 60$$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m (250\Omega // 8\Omega) = 1.2, \text{(Greater than unity)}$$

$$g_m = \frac{4mA}{26mV} = 0.1538 \left(\frac{1}{\Omega}\right) S$$

54)



$I_c = 2mA$

$V_A = \infty$

$$g_m = \frac{I_c}{V_T} = 0.0769 \left(\frac{1}{\Omega}\right) S, \quad \frac{1}{g_m} = 13\Omega$$

a)

$$|A_v| = g_m R_c = \frac{0.5}{0.013} = 38.5$$

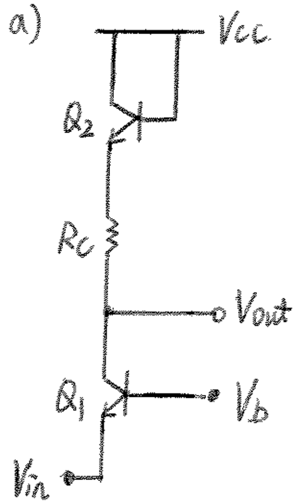
$$R_{in} = \frac{1}{g_m} // r_\pi \approx \frac{1}{g_m} = 13\Omega$$ (Since β is usually large)

$$R_{out} = 500\Omega$$

b) Since $|A_v| = g_m R_c$, and g_m is fixed by I_c. The only way to maximize $|A_v|$ is to maximize R_c. However a large R_c will push Q_1 into saturation, losing its gain altogether. Therefore, V_B has to be as small as possible to provide enough room for V_c to drop $\Rightarrow$ large R_c $\Rightarrow$ large gain.

55) $V_A = \infty$

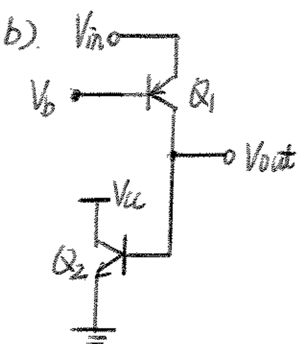
a)



$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}}}$$

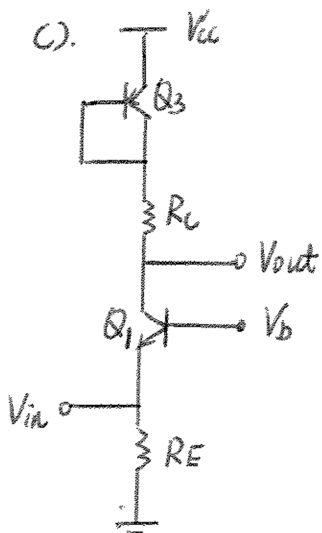
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$$|A_v| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

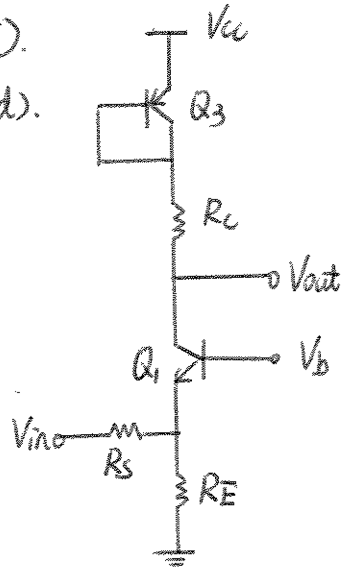


$$|A_v| = \frac{R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

55).

d).

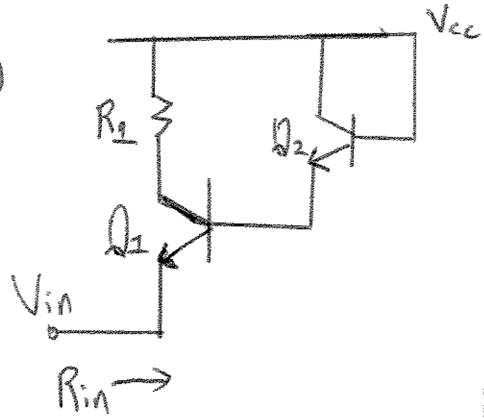


$$|A_v| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right|$$

$$= \left[g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)\right]\left(\frac{R_E \parallel \frac{1}{g_{m1}}}{R_E \parallel \frac{1}{g_{m1}} + R_S}\right)$$

56) $V_A = \infty$

a)

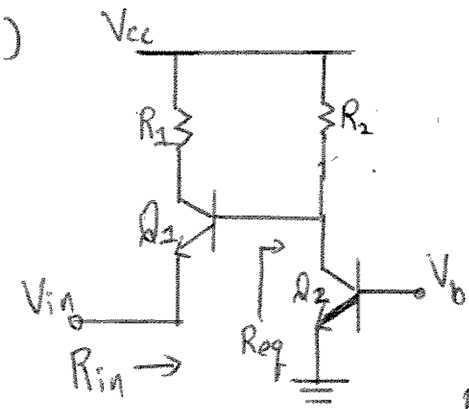


$$R_{in} = \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{\frac{1}{g_{m2}} \,//\, r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{1}{g_{m2}(\beta_1 + 1)}$$

b)



$$R_{eq} = R_2 \,//\, \infty = R_2$$

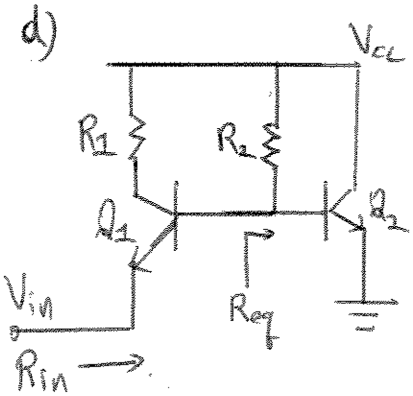
$$R_{in} = \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{R_2}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} = \frac{1}{g_{m1}} + \frac{R_2}{\beta_1 + 1}$$

* Note, part c) and d) have swapped places.

d)



$$R_{eq} = R_2 \,//\, r_{\pi 2}$$

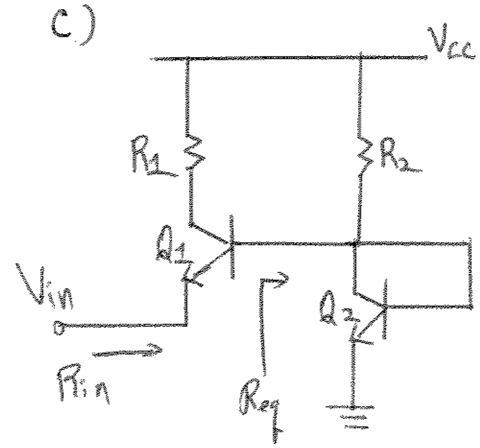
$$R_{in} = \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{R_2 \,//\, r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{R_2 \,//\, r_{\pi 2}}{\beta_1 + 1}$$

5b) * Note, part c) and d) have swapped places

c)



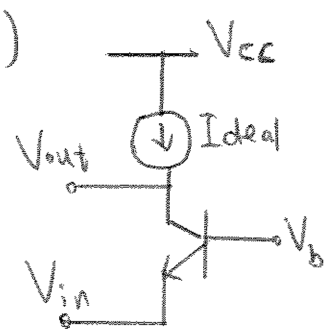
$$R_{eq} = R_2 // \frac{1}{g_{m2}} // r_{\pi 2}$$

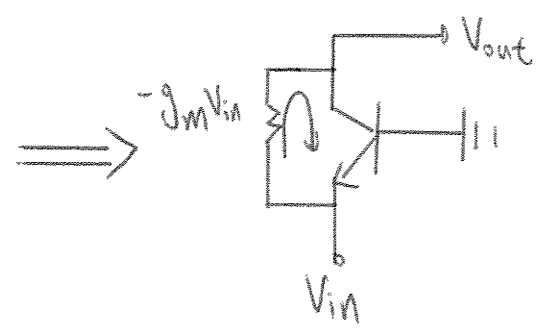
$$R_{in} = \frac{1}{g_{m1}} // r_{\pi 1} + \frac{R_2 // \frac{1}{g_{m2}} // r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{R_2 // \frac{1}{g_{m2}}}{\beta_1 + 1}$$

57)





Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but r_o.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

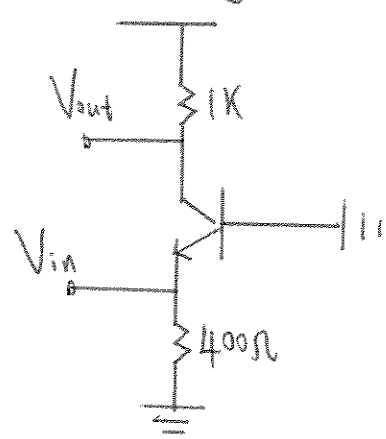
$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$$

58)

b) AC Analysis



$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m 1K$$

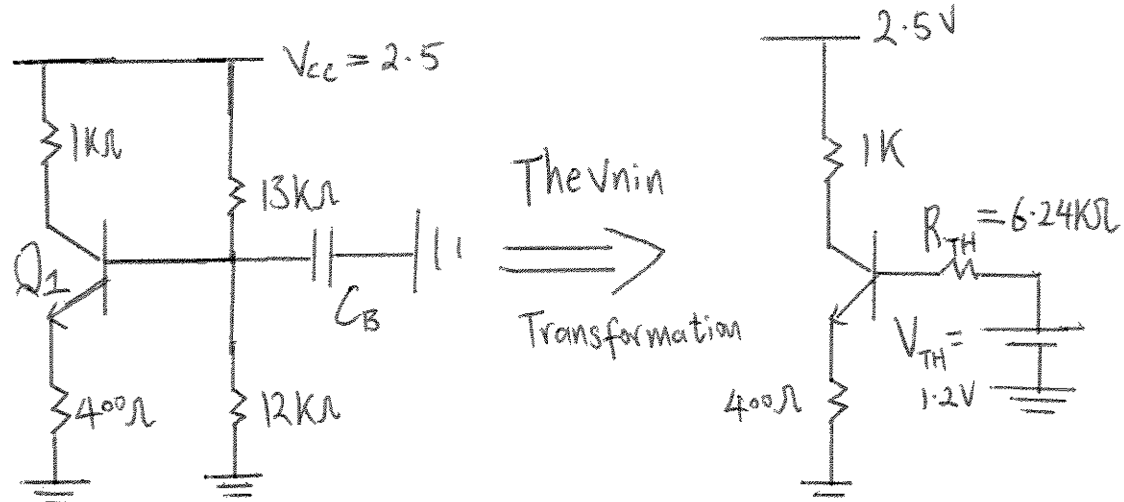
$$g_m = 0.0391 \left(\frac{1}{\Omega}\right) S$$

$$A_v = 39.1$$

$$R_{in} = 400\Omega \,//\, \frac{1}{g_m} = 400\Omega \,//\, 25.583\Omega = 24.0\Omega$$

$$R_{out} = 1K$$

58)



$\beta = 100$, $I_s = 8\times10^{-16}A$, $V_A = \infty$, C_B = Very large

a) DC Analysis:

$$I_c = \beta\left(\frac{1.2 - (V_{BE} + I_E 0.4)}{6.24}\right) \Rightarrow \frac{\beta(1.2 - V_{BE})}{6.24 + \frac{0.4\beta}{\alpha}}$$

Guess $V_{BE} = 0.7 \Rightarrow I_c = 1.072\ mA$

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726\ V$, not 0.7V, reiterate.

$V_{BE} = 0.726\ V$; $I_c = 1.0163\ mA$

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.725V$, Converged!!

$V_{BE} = 0.725$, $V_{CE} = 2.5 - \left[(1.0163)(1K) + 0.4\left(\frac{1.0163}{0.99}\right)\right]$

$V_{CE} = 1.07$

$I_c = 1.0163mA$, $I_B = 10.163\ \mu A$

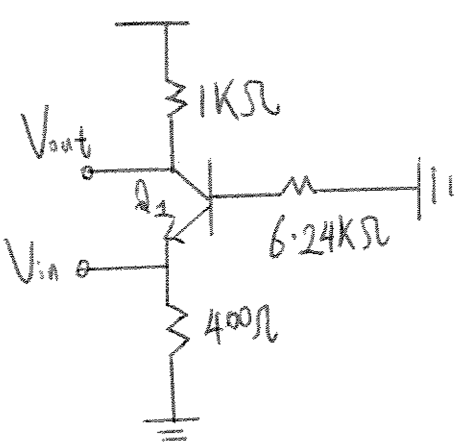
59)

$C_B = 0$

a) Since C_B was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

$V_{BE} = 0.725\text{ V}$
$I_c = 1.0163\text{ mA}$
$I_B = 10.163\ \mu A$
$V_{CE} = 1.07\text{ V}$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.



$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}} = 11.4$$

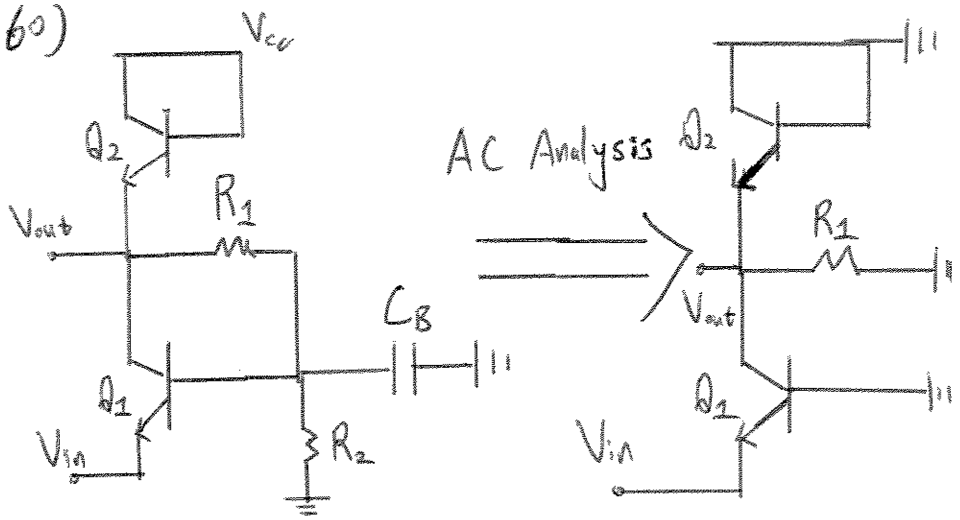
$$R_{in} = 400\Omega \,//\left(\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

$$R_{out} = 1K\Omega$$

Note: 6.24 KΩ is R_{THEV} of 13 KΩ and 12 KΩ combination.

60)

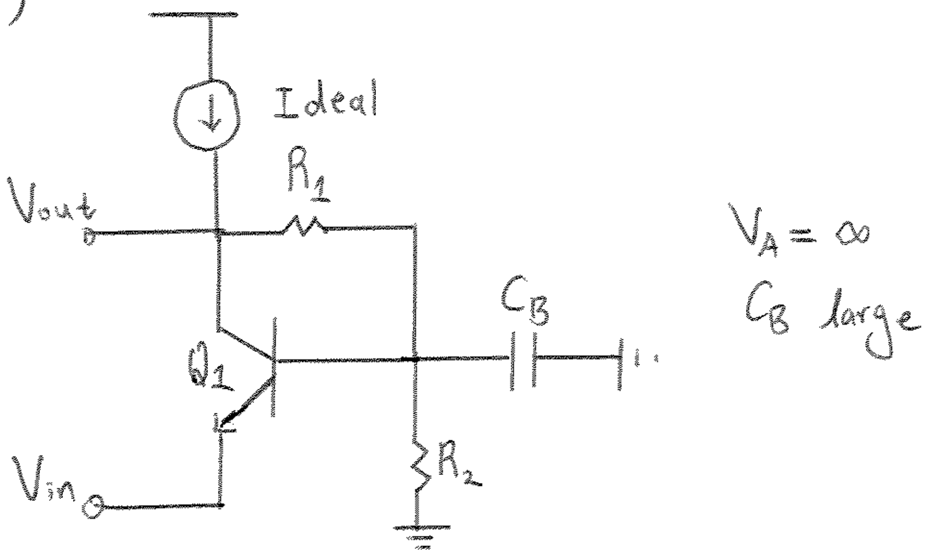


$$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2} // R_1 \approx \frac{1}{g_{m2}} // R_1$$

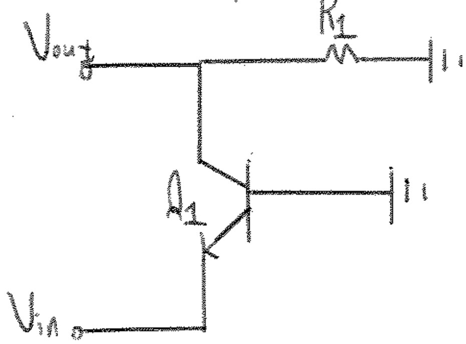
$$A_v = \left| \frac{V_{out}}{V_{in}} \right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // R_1 \right) \approx g_{m1}\left(\frac{1}{g_{m2}} // R_1 \right)$$

$$R_{in} = \frac{1}{g_m} // r_{\pi 1} \approx \frac{1}{g_{m1}}$$

61)



⇓ AC Analysis



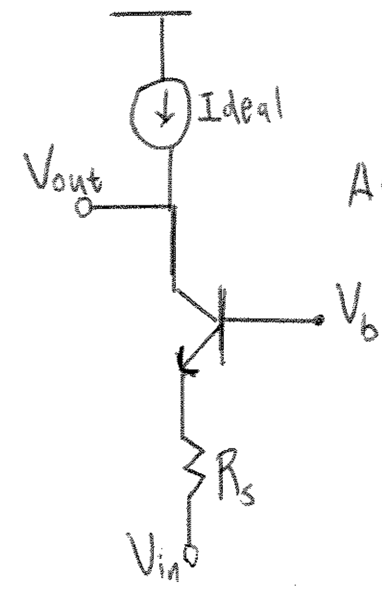
(R_2 shorted out)

$R_{out} = R_1$

$R_{in} = \frac{1}{g_{m_1}} // r_{\pi_1} \approx \frac{1}{g_{m_1}}$

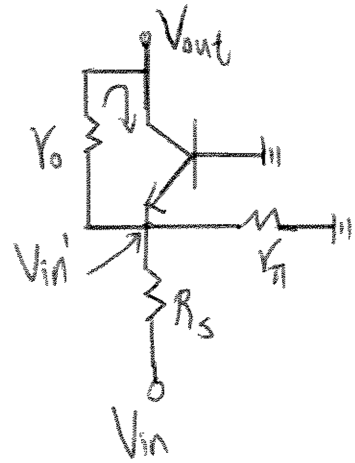
$|A_v| = g_{m_1} R_1$

62)



$V_A < \infty$

AC Analysis $\Longrightarrow$



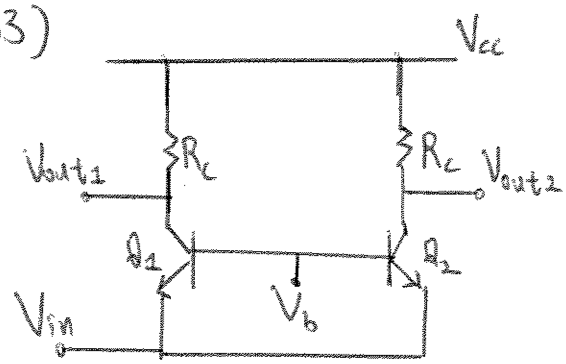
$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since V_{out} is float, so looking at emitter and r_o, we will see an infinite impedance.

$$\frac{V_{out}}{V_{in}'} \Rightarrow -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

63)



$V_A = \infty$

$I_{S1} = 2I_{S2}$

$$\left|\frac{V_{out1}}{V_{in}}\right| = g_{m1} R_c \quad , \quad \left|\frac{V_{out2}}{V_{in}}\right| = g_{m2} R_c$$

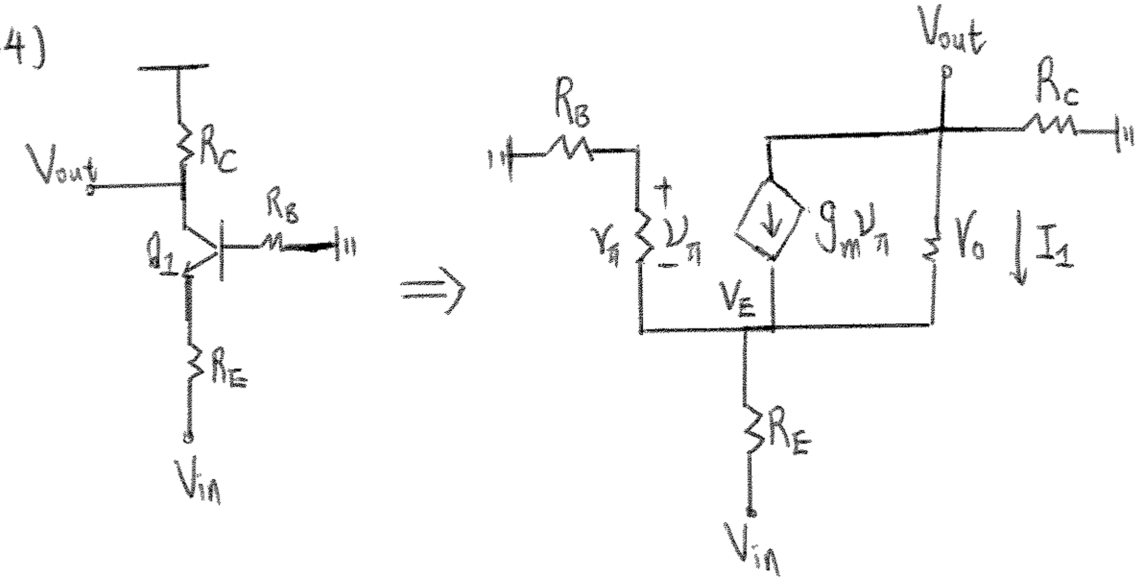
$$g_{m1} = \frac{V_T}{I_{c1}} = \frac{V_T}{2I_{S2} e^{(V_{BE}/V_T)}} \quad , \quad \text{Since } I_{S1} = 2I_{S2}$$

$$g_{m2} = \frac{V_T}{I_{c2}} = \frac{V_T}{I_{S2} e^{(V_{BE}/V_T)}}$$

$(V_{BE1} = V_{BE2} = V_{BE})$

$$\Rightarrow \quad g_{m1} = \frac{g_{m2}}{2} \Rightarrow \left|\frac{V_{out1}}{V_{in}}\right| = \frac{1}{2}\left|\frac{V_{out2}}{V_{in}}\right|$$

64)



$$V_{out} = -(I_1 + g_m v_\pi) R_c \quad , \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right) R_c \; , \; V_E = \frac{-g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi (r_\pi + R_B)}{\beta}}{r_o} + g_m v_\pi\right) R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right) g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right) R_E = V_{in} \qquad (1)$$

64)

Writing V_E in terms of V_π, and V_π in terms of Vout

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

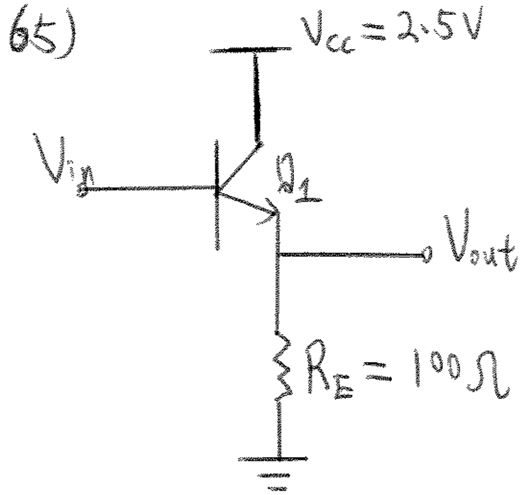
Solving Vout / Vin =>

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

subsituting A into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C}{g_m\left(1+\frac{R_C}{r_o}\right)(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m \left(1+\frac{R_C}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C\right)}$$

65)



$R_E = 100\Omega$

$V_A = \infty$

$|A_v| = 0.8$

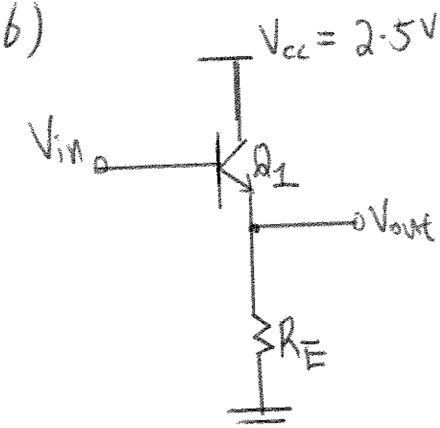
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_c}{R_E I_c + V_T} = 0.8$$

$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T)$, $R_E = 100\Omega$

$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208 \Rightarrow 0.02 I_c = 0.0208$

$\Rightarrow I_c = 1.04mA$

6b)



$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$\Rightarrow R_E I_c > 9V_T = 234\,mV$, Let $R_E I_c = 240\,mV$

$R_{in} = r_\pi + (1+\beta) R_E > 10K \Rightarrow 100 V_T + (101) R_E I_c > 10K\Omega I_c$

Subsituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

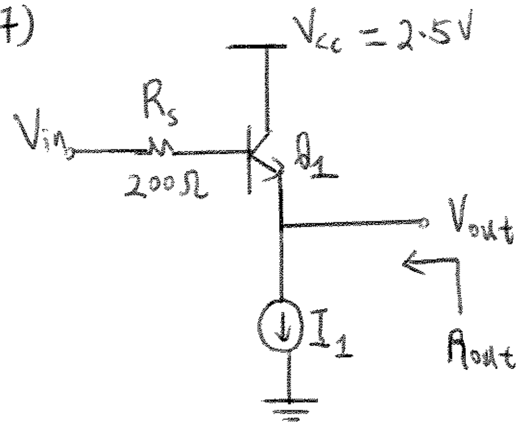
Choose I_c to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

67)



$\beta = 100$

$V_A = \infty$

$$R_{out} = \frac{1}{g_m} + \frac{R_s}{(\beta+1)} \leq 5\,\Omega \quad (\text{Assuming } r_\pi >> \frac{1}{g_m})$$

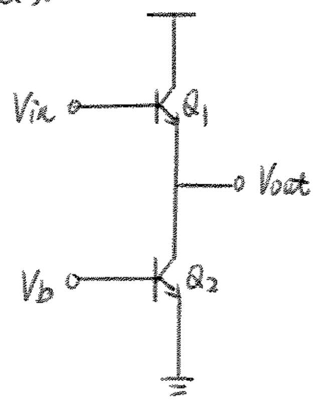
$$R_{out} = 0.026 + \frac{200\Omega I_c}{101} \leq 5\Omega I_c$$

$$\Rightarrow I_c \geq 0.0086A$$

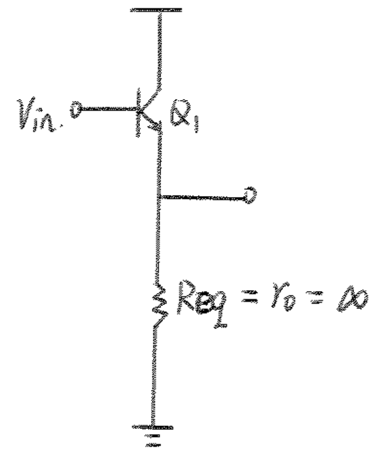
$$\text{Pick } I_c = 0.009A$$

$$R_{out} = \frac{0.026V}{0.009A} + \frac{200}{101} = 4.87\Omega$$

68). a). $V_A = \infty$



$\Rightarrow$



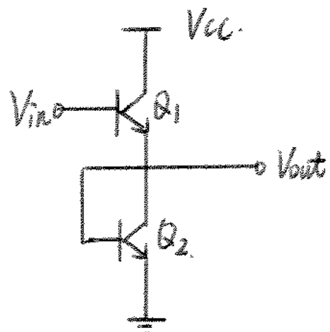
$$|A_v| = \frac{r_o}{r_o + \frac{1}{g_m}}, \quad \text{since } r_o = \infty$$

$$|A_v| = 1.$$

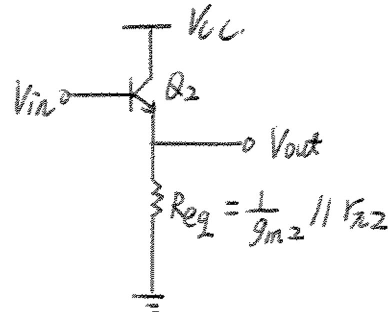
$$R_{in} = \infty \quad (\text{since } r_o = \infty)$$

$$R_{out} = \infty \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} = \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

b).



$\Rightarrow$



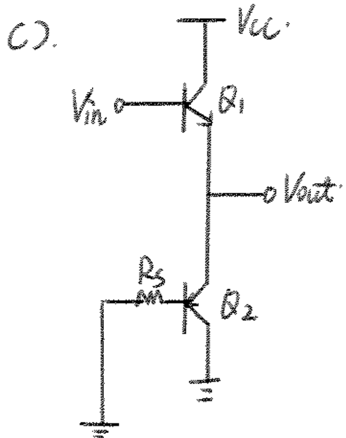
$$|A_v| = \frac{\frac{1}{g_{m2}} \,//\, r_{\pi 2}}{\frac{1}{g_{m2}} \,//\, r_{\pi 2} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_{\pi 1} + (1+\beta)\frac{1}{g_{m2}} \,//\, r_{\pi 2}$$

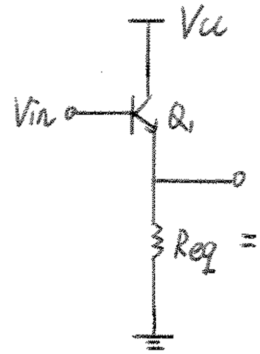
$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

(If $I_{S1} = I_{S2}$, $g_{m1} = g_{m2} = g_m$, $r_{\pi 1} = r_{\pi 2} = r_\pi$, $R_{out} = \frac{1}{2g_m} \,//\, \frac{r_\pi}{2}$)

c).



$\Longrightarrow$



$$R_{eq} = \frac{1}{g_{m2}} + \frac{R_S}{\beta+1}$$

(Assuming $r_\pi >> \frac{1}{g_m}$)

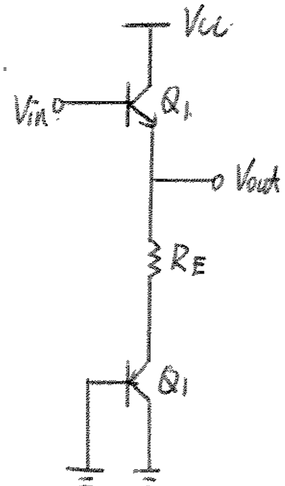
$$|A_v| = \frac{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}}{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right)$$

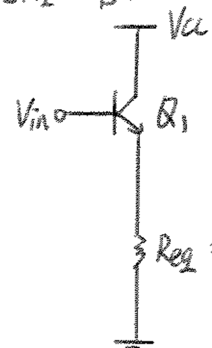
$$R_{out} = \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right).$$

$$R_{out} \approx \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \frac{1}{g_{m1}}$$

d).



$\Longrightarrow$



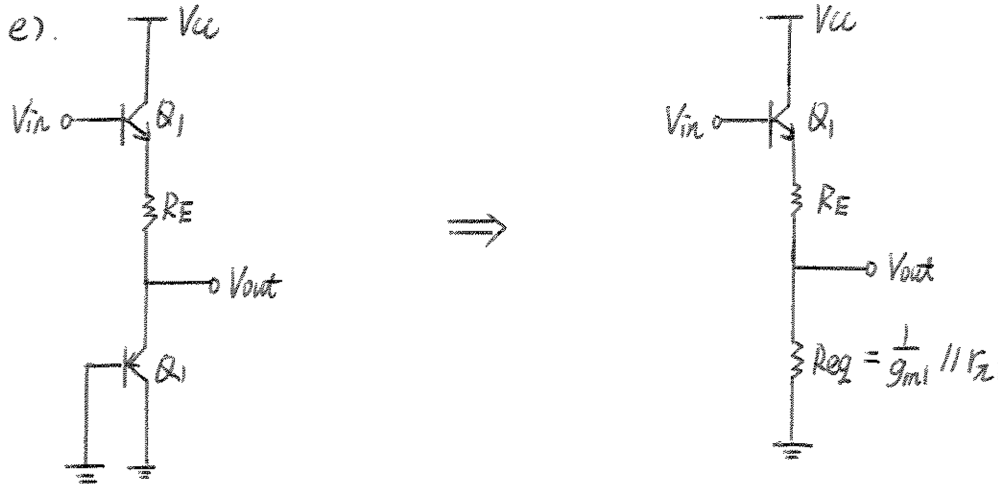
$$R_{eq} = R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

$$|A_v| = \frac{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)}{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right) + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right]$$

$$R_{out} = \left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right] // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

68). e).

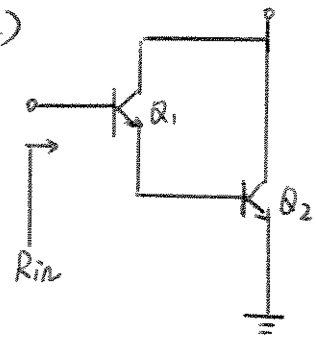


$$|A_v| = \frac{\frac{1}{g_{m1}} \,//\, r_{\pi 1}}{\frac{1}{g_{m1}} \,//\, r_{\pi 1} + R_E + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left[R_E + \frac{1}{g_{m1}} \,//\, r_{\pi 1}\right]$$

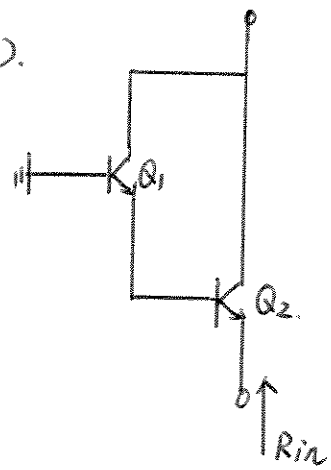
$$R_{out} = \left(\frac{1}{g_{m1}} \,//\, r_{\pi 1}\right) // \left(R_E + \frac{1}{g_{m1}} \,//\, r_{\pi 1}\right).$$

69 a)



$$R_{in} = r_{\pi 1} + (1+\beta) r_{\pi 2}$$

b).



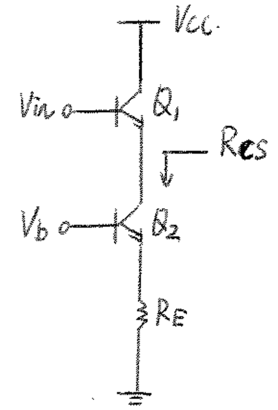
$$R_{in} = \frac{1}{g_{m2}} + \frac{1/g_{m1}}{(\beta+1)} \quad \left(\text{Assume } r_\pi \gg \frac{1}{g_m}\right)$$

c) $$\text{Current Gain} = \frac{(I_{C1} + I_{C2})}{I_{B1}} = \beta + \frac{I_{C2}}{I_{B1}} = \beta + \frac{\beta I_{B2}}{I_{B1}}$$

Since $I_{B2} = I_{C1} = \beta I_{B1}$

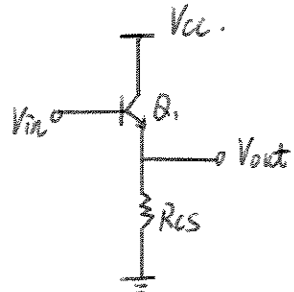
$$\text{Current Gain} = \beta + \beta^2 = \beta(\beta+1), \quad (\text{Assuming } \beta_1 = \beta_2 = \beta)$$

70).



a) $R_{cs} = r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})$

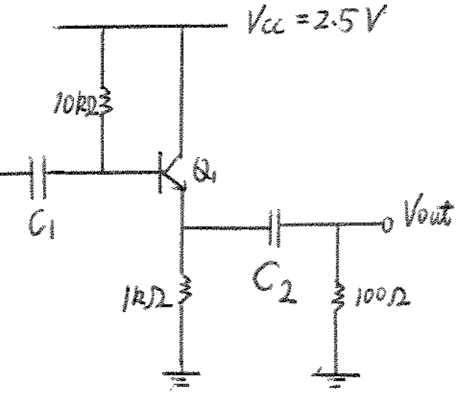
b).



$$A_v = \frac{R_{cs} // r_{o1}}{R_{cs} // r_{o1} + \frac{1}{g_{m1}}}$$

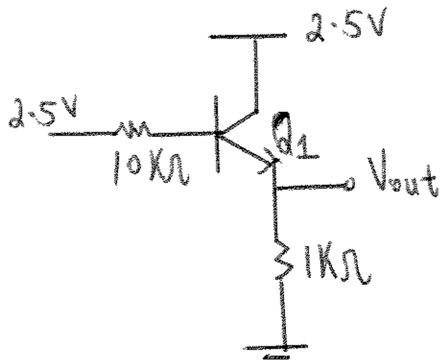
$$A_v = \frac{(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})) // r_{o1}}{(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})) // r_{o1} + \frac{1}{g_{m1}}}$$

71).



$I_s = 7 \times 10^{-16} A$
$\beta = 100$
$V_A = 5V$

<u>DC Analysis:</u> (Ignore r_o's effect)



$$I_c = \beta \left(\frac{2.5 - \left(V_{BE} + \frac{I_c}{\alpha} 1K\Omega\right)}{10K\Omega} \right)$$

rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621mA$

check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

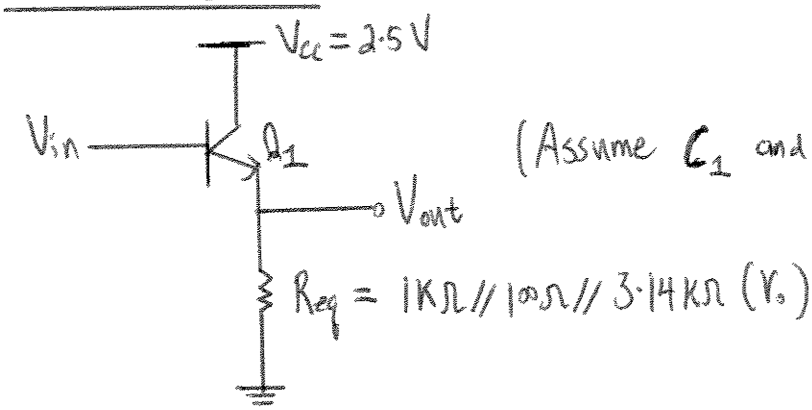
$V_{BE} = 0.740V$, $I_c = 1.59mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, Converged.

So $I_c = 1.59mA$, $g_m = 0.0612 \left(\frac{1}{\Omega}\right) S$, $\frac{1}{g_m} = 16.34\Omega$,
$r_o = 3.14K\Omega$

71)

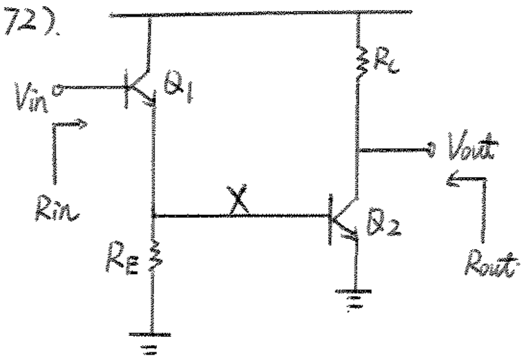
AC Analysis: (Include r_o)



(Assume C_1 and C_2 are large)

$$A_v = \frac{(1K\Omega \,//\, 100\Omega \,//\, 3.14K\Omega)}{16.34\Omega + (1K\Omega \,//\, 100\Omega / 3.14K\Omega)} = 0.84$$

72).



$V_A < \infty$

a) $R_{in} = r_{\pi 1} + (1+\beta)(R_E \,//\, r_{\pi 2} \,//\, r_{o1})$

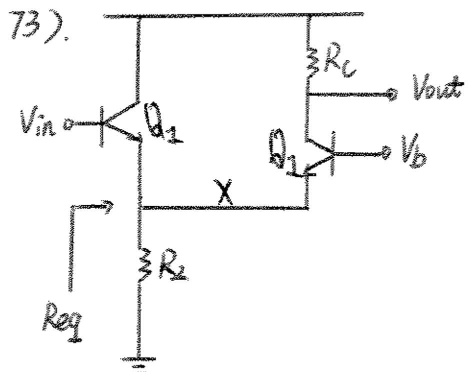
$R_{out} = R_c \,//\, r_{o2}$

b) $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_x}{V_{in}}\right| \left|\frac{V_{out}}{V_x}\right|$

$\left|\frac{V_x}{V_{in}}\right| = \frac{(R_E \,//\, r_{\pi 2} \,//\, r_{o1})}{\frac{1}{g_{m1}} + R_E \,//\, r_{\pi 2} \,//\, r_{o1}}$, $\left|\frac{V_{out}}{V_x}\right| = g_{m2} R_c$

$\left|\frac{V_{out}}{V_{in}}\right| = (g_{m2} R_c)\left[\frac{R_E \,//\, r_{\pi 2} \,//\, r_{o1}}{\frac{1}{g_{m1}} + R_E \,//\, r_{\pi 2} \,//\, r_{o1}}\right]$

73).



$V_A = \infty$

a) $R_{eq} = R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}$

$R_{in} = r_{\pi 1} + (1+\beta)\left[R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}\right]$

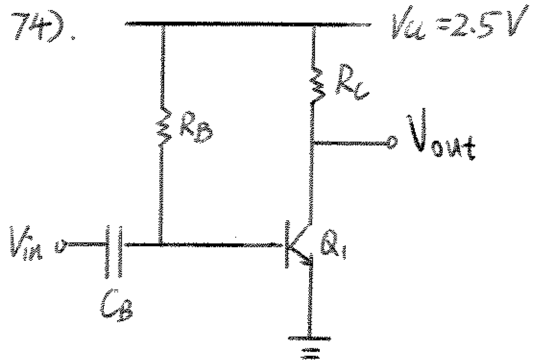
$R_{out} = R_C$

b) $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_X}{V_{in}}\right|\left|\frac{V_{out}}{V_X}\right|$

$\left|\frac{V_X}{V_{in}}\right| = \dfrac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{\frac{1}{g_{m1}} + R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}$, $\left|\frac{V_{out}}{V_X}\right| = g_{m1} R_C$

$$\left|\frac{V_{out}}{V_{in}}\right| = (g_{m1} R_C)\left(\frac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{1}{g_{m1}}}\right)$$

74).



$A_V = 10$

$R_{in} > 5K\Omega$

$R_{out} = 1K$, $R_c = 1K\Omega$

$$A_V = \frac{R_c}{\frac{1}{g_m}} = 10 = \frac{I_c R_c}{V_T} \Rightarrow I_c = 0.26mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.697V.$$

$$I_c = 100\left(\frac{2.5 - 0.697}{R_B}\right) \Rightarrow R_B = 693K\Omega, \quad r_\pi = \frac{\beta V_T}{I_c} = 10K\Omega$$

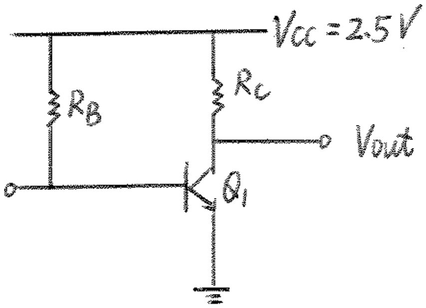
$$R_{in} = 693K // 10K = 9.86K\Omega$$

$$\frac{1}{2\pi(200)C_B} = \frac{1}{10}\frac{1}{g_m} = 10\Omega \Rightarrow C_B = 80\mu f$$

(To avoid gain degradation).

$R_c = 1K\Omega$	$\Rightarrow$	$A_V = 10$
$R_B = 693K$		$R_{out} = 1K\Omega$
$C_B = 80\mu f$		$R_{in} = 9.86K\Omega$

75).



A_V = Maximum
$R_{out} \leq 500\Omega$
$V_{BC} \leq 400$ mV

$A_v = g_m R_c = \frac{I_c R_c}{V_T}$, gain is maximized by maximize $I_c R_c$

$R_{out} = R_c \leq 500\Omega$, Choose $R_c = 450\Omega$, $R_{out} = 450\Omega$
$V_{BC} = V_{BE} - (2.5 - I_c R_c) \leq 400$ mV
Guess $V_{BE} = 0.7$, and let $V_{BC} = 400$ mV to maximize $I_c R_c$
$0.7 - (2.5 - I_c \, 0.450) = 0.4$
$I_c = 4.89$mA, $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.773$

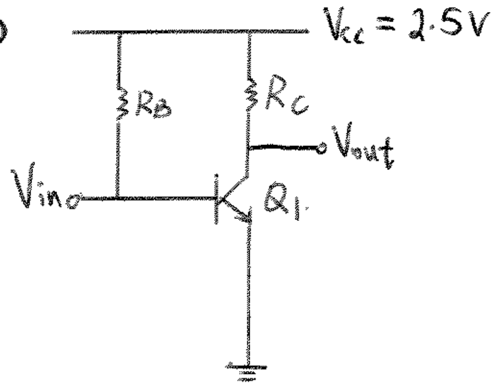
Not 0.7, reiterate.
$0.773 - 2.5 + I_c \, 0.450 = 0.4$
$I_c = 4.73$mA, $V_{BE} = 0.772$ converged!!

$A_v = \left(\frac{I_c}{V_T}\right)\left(R_c\right) = \left(\frac{4.73}{26}\right)\left(450\right) = 81.9$

$R_B = 100\left(\frac{2.5 - 0.772}{4.73}\right) = 36.5K$

$R_B = 36.5K$
$R_c = 450\Omega$
$\Rightarrow$
$A_v = 81.9$
$V_{BC} = 0.4V$
$R_{out} = 450\Omega$

76)



R_{in} : Maximum

$A_v \geq 20$

$R_{out} = R_c = 1K$

$$A_v = g_m R_c = \frac{I_c R_c}{V_T} \geq 20 \Rightarrow I_c \geq 0.52\,mA$$

$$R_{in} = R_B // r_\pi = \frac{\beta R_B V_T}{R_B I_c + V_T \beta} \quad 1), \quad I_c = \beta\left(\frac{2.5 - V_{BE}}{R_B}\right) \quad 2)$$

As we can see from 1), higher I_c means lower R_{in}.
So set I_c as low as possible, $I_c = 0.52mA$.

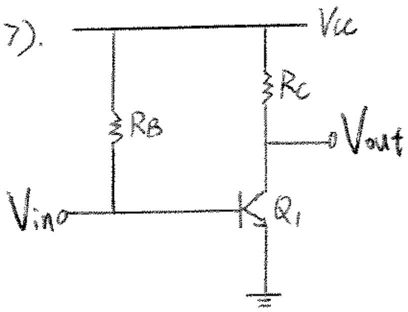
$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$$

From 2), $R_B = \frac{100(2.5 - 0.715)}{0.52} = 343.3K\Omega$, $r_\pi = 5K\Omega$

$R_{in} = 4.93K\Omega$

$R_c = 1K\Omega$
$R_B = 343.3K\Omega$
$\Rightarrow$
$A_v = 20$
$R_{in} = 4.93K\Omega$
$R_{out} = 1K\Omega$

77).



Minimum Supply

$A_v = 15$

$R_{out} = 2K\Omega$, $R_c = 2K\Omega$

$V_{BC} \le 0.4V$

$$A_v = g_m R_c = \frac{I_c}{V_T} R_c = 15 \Rightarrow I_c = 0.195mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.689$$

$$V_{BC} = V_{BE} - (V_{cc} - I_c R_c) \le 0.4V \ , \ I_c R_c = 0.39V$$

$$V_{cc} \ge 0.689 + 0.39 - 0.4 = 0.679V$$

Since the problem is concerned with minimum power supply, let $V_{cc} = 0.69V$, Since $V_{BE} = 0.679V$ ($V_{cc} > V_{BE}$)

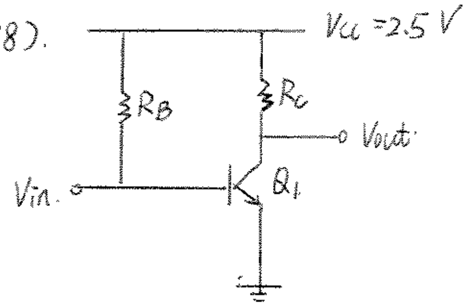
$$I_c = \beta\left(\frac{V_{cc} - 0.689}{R_B}\right) \Rightarrow R_B = 100\left(\frac{0.69 - 0.689}{0.195}\right) = 512.8\Omega$$

$R_c = 2K\Omega$

$R_B = 512.8\Omega$ $\Rightarrow$ $A_v = 15$

$V_{cc} = 0.69V$ $\qquad R_{out} = 2K\Omega$

78).



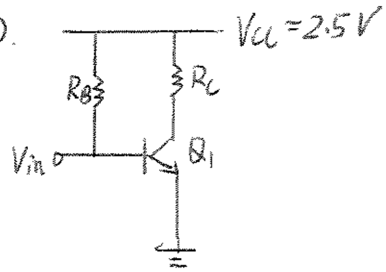
$$A_o = g_m R_C$$

$$A_o = \frac{I_C R_C}{V_T}, \quad \text{Power Dissipation} = I_C V_{CC}$$

$$R_{out} = R_C = \frac{A_o V_T}{I_C}$$

For large R_{out}, I_C has to be small, which decreases power. So small power dissipation and small output impedance cannot be satisfied simultaneously.

79).



Power Budget = 1mW

$A_V = 20$

$A_V = g_m R_c = \frac{I_c R_c}{V_T} = 20$, $V_{cc} I_c = 1mW$

$I_c = 0.4mA$, $R_c = 1.3K$

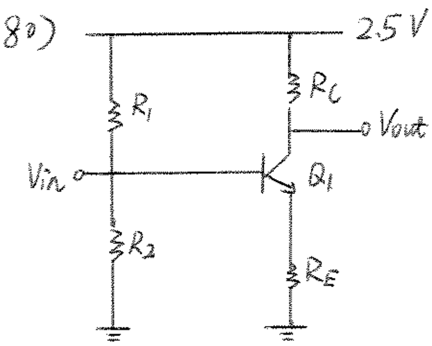
$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.708V$, $I_c = \beta\left(\frac{V_{cc} - V_{BE}}{R_B}\right) = 100\left(\frac{2.5 - 0.708}{R_B}\right)$

$\Rightarrow R_B = 448K$, $R_{in} = 448 // (100)\left(\frac{26}{0.4}\right) = 6.4K$

$R_B = 448K$
$R_c = 1.3K$ $\Rightarrow$

$A_v = 20$
Power Budget = 1mW
$R_{out} = 1.3K\Omega$
$R_{in} = 6.4K$

80)



$A_V = 5$

$R_{out} = R_c = 500\Omega$

$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300+26} \Rightarrow \begin{array}{l} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300mV \Rightarrow R_E = 92\Omega \end{array}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624$$

$$10 I_B = 0.326\, mA$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

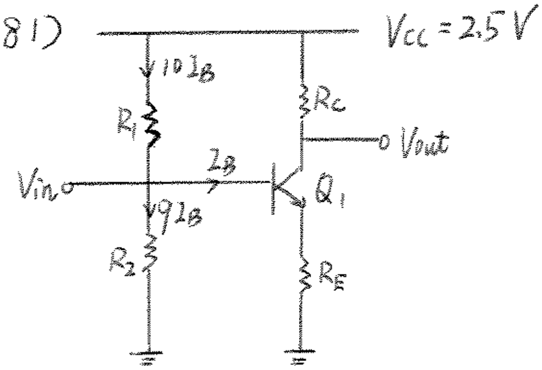
$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62\,K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57, \quad V_{BE} = 0.7624.$

Q_1 is in soft saturation region, so active region characteristics still apply.

$$\begin{array}{l} R_c = 500\Omega \\ R_1 = 4.41K\Omega \\ R_2 = 3.62K\Omega \\ R_E = 92\Omega \end{array} \Rightarrow \begin{array}{l} A_V = 5 \\ R_{out} = 500\Omega \end{array}$$

81)



A_V = Maximum

$R_{out} = R_c \leq 1K\Omega$

$V_{BC} = 0.4V$

$R_E I_c \approx 200\,mV$

$$V_{BC} = (V_{BE} + 0.2) - (2.5 - I_c R_c) = 0.4 \quad \text{i)}$$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{0.226}$$

Rearrange, i) becomes $I_c R_c = 0.4 + 2.5 - (V_{BE} + 0.2)$

Guess $V_{BE} = 0.7 \Rightarrow I_c R_c = 2V$

Let $R_c = 1K \Rightarrow I_c = 2mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, not 0.7, reiterate

$V_{BE} = 0.75 \Rightarrow I_c R_c = 1.95\,V$

$R_c = 1K \Rightarrow I_c = 1.95\,mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, Converged!!

$I_c = 1.95\,mA$, $R_E I_c = 200\,mA \Rightarrow R_E = 103\Omega$

$I_B = 0.0195\,mA$

$$R_1 = \frac{2.5 - (0.750 + 0.2)}{(10)(0.0195)} = 7.95K$$

$$R_2 = \frac{(0.750 + 0.2)}{(9)(0.0195)} = 5.41K$$

81)

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{1.95}{0.226} = 8.63$$

This is the maximum gain we would get when R_{out} is $1K\Omega$ and V_{BC} is at 0.4 V. Since anything larger will violate either requirement.

$R_c = 1K\Omega$

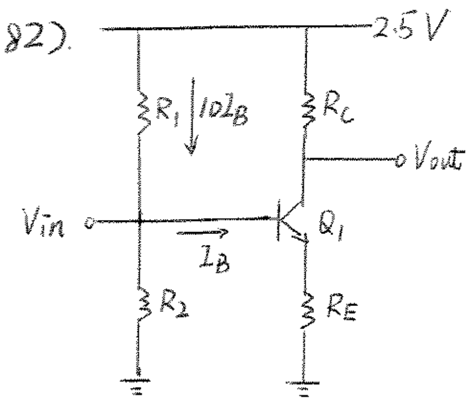
$R_E = 103\Omega$

$R_1 = 7.95K\Omega$

$R_2 = 5.41K\Omega$

$\Longrightarrow$ $A_V = 8.63$, $R_{out} = 1K\Omega$

82).



Power budget = 5 mW

$A_V = 5$

$R_E I_c \approx 200\,mV$

$$V_{cc}\left(I_c + \frac{I_c}{10}\right) = 5\,mW \Rightarrow I_c = 1.82\,mA, \quad I_B = 0.0182\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{0.226} = 5 \Rightarrow R_c I_c = 1.13V \Rightarrow R_c = 621\,\Omega$$

$$R_1 = \frac{2.5-(0.747+0.2)}{(10)(0.0182)} = 8.53K\Omega$$

$$R_2 = \frac{(0.747+0.2)}{(9)(0.0182)} = 5.78K\Omega$$

$$R_E I_c \approx 200\,mV \Rightarrow R_E = 110\,\Omega$$

$R_c = 621\,\Omega$

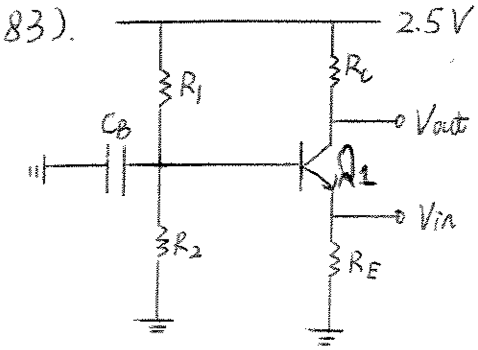
$R_E = 110\,\Omega$

$R_1 = 8.53\,K\Omega$

$R_2 = 5.78\,K\Omega$

$\Longrightarrow$ $A_V = 5$, Power Budget = 5mW

83).



$A_V = 20$
$R_{in} = 50\,\Omega$
$R_E I_C \approx 10 V_T = 260\text{ mV}$

$R_{in} = \dfrac{1}{g_m} = 50\,\Omega$, since R_E doesn't affect input impedance.

$$\frac{V_T}{I_C} = 50\,\Omega \Rightarrow I_C = \frac{V_T}{50\,\Omega} = 0.52\text{ mA}, \quad I_B = 0.0052\text{ mA}$$

$$A_V = \frac{R_C}{1/g_m} = \frac{I_C R_C}{V_T} = 20 \Rightarrow R_C = 1\text{K}\Omega$$

$$R_1 = \frac{2.5 - (0.715 + 0.260)}{(10)(0.0052)} = 29.3\text{K}$$

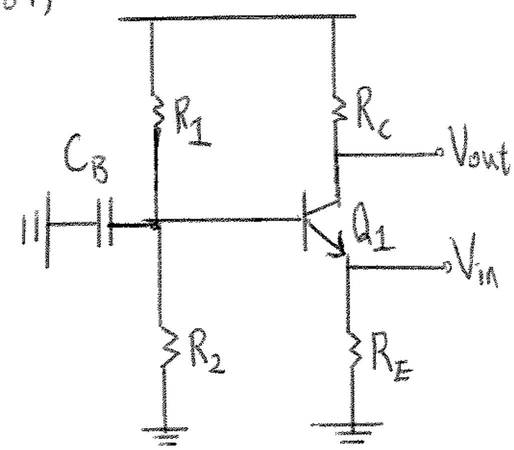
$$R_2 = \frac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83\text{K}$$

$$R_E I_C \approx 260\text{ mV} \Rightarrow R_E \approx 500\,\Omega$$

$$\frac{1}{C_B(2\pi)(200)} = \frac{1}{10}\frac{1}{g_m} = 5\,\Omega \Rightarrow C_B = 159.1\,\mu f$$

$R_C = 1\text{K}\Omega$, $R_E = 500\,\Omega$, $R_1 = 29.3\text{K}\Omega$, $R_2 = 20.83\text{K}$, $C_B = 159.1\,\mu f$
$\Rightarrow A_V = 20$, $R_{in} = 50\,\Omega$

84)



$A_v = 8$

$R_{out} = 500\Omega$

$R_{out} = R_c = 500\Omega$

$A_v = \frac{I_c}{V_T} R_c = 8 \Rightarrow I_c = 0.416\ mA,\ I_B = 0.00416\ mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.709$

$R_E \approx \frac{260\ mV}{I_c} = 625\Omega$, $R_1 = \frac{2.5 - (0.709 + 0.260)}{(10)(0.00416)} = 36.8K\Omega$

$R_2 = \frac{(0.709 + 0.260)}{(9)(0.00416)} = 25.9K\Omega$

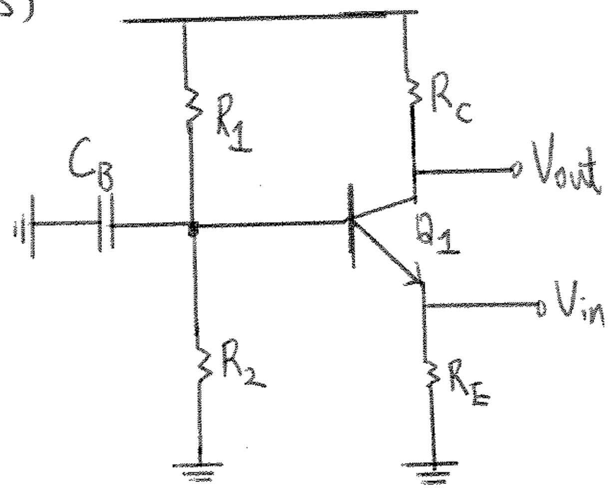
$\frac{1}{g_m} = \frac{V_T}{I_c} = 62.5\Omega$, $\frac{1}{C_B\, 200\,(2\pi)} = \frac{62.5}{10} \Rightarrow C_B = 127.3\,\mu f$

$C_B = 127.3\,\mu f$
$R_1 = 36.8\ K$
$R_2 = 25.9\ K$
$R_c = 500\Omega$
$R_E = 625\Omega$

$\Longrightarrow$ $A_v = 8$

$R_{out} = 500\Omega$

85)



$A_V = 20$
$R_c = 200\Omega$
$(R_c = R_{out})$

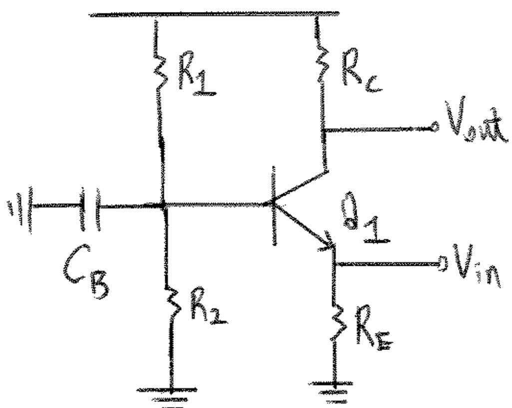
$$A_V = \frac{I_c R_c}{V_T} = 20 \implies I_c = 2.6\,mA$$

$I_B = 0.026\,mA$, $10 I_B = 0.26\,mA$

$$\text{Power} = V_{cc}(I_c + 10 I_B) = 2.5\,(0.26\,mA + 2.6\,mA) = 7.15\,mW$$

This is the minimum power dissipation, since anything lower will lower the voltage gain.

86)



Power = 5mW

$A_v = 10$

$V_{cc} I_c + V_{cc} \frac{I_c}{10} = 5mW$, $V_{cc} I_c 1.1 = 5mW$, $I_c = 1.82\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$

$A_v = g_m R_c = \frac{I_c R_c}{V_T} = 10 \Rightarrow R_c = 0.143K\Omega$

$I_c R_E \approx 260\,mV$, $R_E \approx 142.9\Omega$

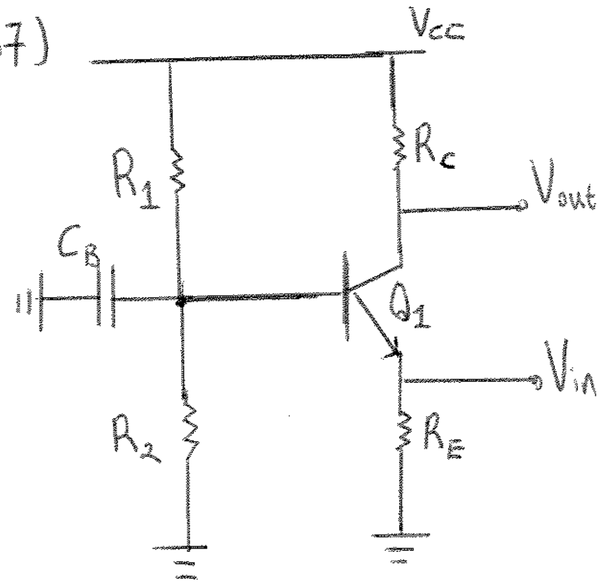
$R_1 = \frac{2.5 - (0.747 + 0.260)}{(10)(0.0182)} = 8.2K$, $R_2 = \frac{(0.747 + 0.260)}{(9)(0.0182)} = 6.15K$

$\frac{1}{g_m} = \frac{V_T}{I_c} = 14.3\Omega$, $\frac{1}{C_B 2\pi (200)} = \frac{14.3}{10} \Rightarrow C_B = 556.5\,\mu f$

$R_c = 143\Omega$, $R_E = 143\Omega$, $R_1 = 8.2K$, $R_2 = 6.15K$, $C_B = 556.5\mu f$.

$\Rightarrow$ $A_v = 10$, Power = 5mW

87)



$R_{in} = 50\Omega$
$A_v = 20$

Assume R_E doesn't affect R_{in} significantly,
$R_{in} \approx \frac{1}{g_m} = 50\Omega$

$A_v = \frac{R_c}{1/g_m} = 20 \Rightarrow R_c = 1K\Omega$, $\frac{1}{g_m} = \frac{V_T}{I_c} \Rightarrow I_c = \frac{26mV}{50\Omega} = 0.52$ mA

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$, $R_E I_c = 260\,mV \Rightarrow R_E = 500\Omega$

$V_{cc} = I_c R_c + V_{CE} + I_c R_E = 0.52 + V_{CE} + 0.260$

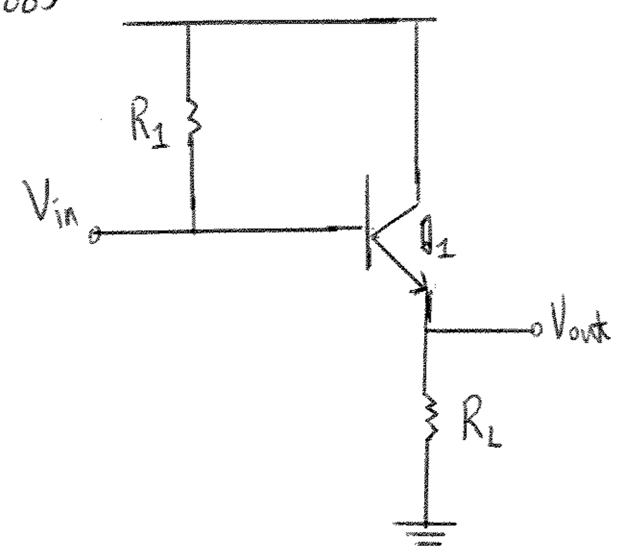
V_{BC} is forward biased to $0.4V$, $V_{CE} = V_{BE} - 0.4 = 0.315\,V$
$V_{cc} = 0.52 + 0.315 + 0.260 = 1.1V$. (Minimum supply Voltage)

$R_1 = \frac{1.1 - (0.715 + 0.260)}{0.052} = 2.4K$, $R_2 = \frac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83K$

$\frac{1}{C_B 2\pi 200} = \frac{1}{10}\frac{1}{g_m} = 5 \Rightarrow C_B = 159.2\,\mu f$

$V_{cc} = 1.1V$, $R_1 = 2.4K$, $R_2 = 20.83K$, $R_c = 1K$, $R_E = 500\Omega$, $C_B = 159.2\mu f$
$\Rightarrow R_{in} = 50\Omega$, $A_v = 20$

88)



$A_v = 0.85$
$R_{in} > 10k\Omega$
$R_L = 200\Omega$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\,mV}{35.294\Omega} = 0.737\,mA, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-13}}\right) = 0.724V$$

$$R_{in} = R_1 \,//\, (r_\pi + (1+\beta)(200\Omega))$$

$$R_{in} = R_1 \,//\, 23.73K$$

$$R_{in} = \frac{R_1\, 23.73K}{R_1 + 23.73K} > 10K \Rightarrow R_1 > 17.28K \text{ (Input Impedance requirement)}$$

To support an I_c of 0.737, R_1 must be determined.

88)

$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737/100}$$

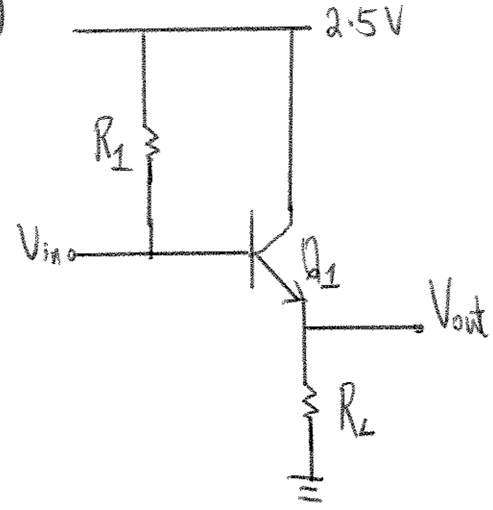
$$R_1 = 220.77K\Omega$$

$R_1 = 220.77K\Omega \Rightarrow R_{in} = 220.77K\Omega // 23.73K\Omega$

$R_{in} = 21.43K\Omega > 10K\Omega$

$R_1 = 220.77K\Omega$
$R_L = 200\Omega$ $\Rightarrow$ $A_v = 0.85$
$R_{in} = 21.43K\Omega$

89)



Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

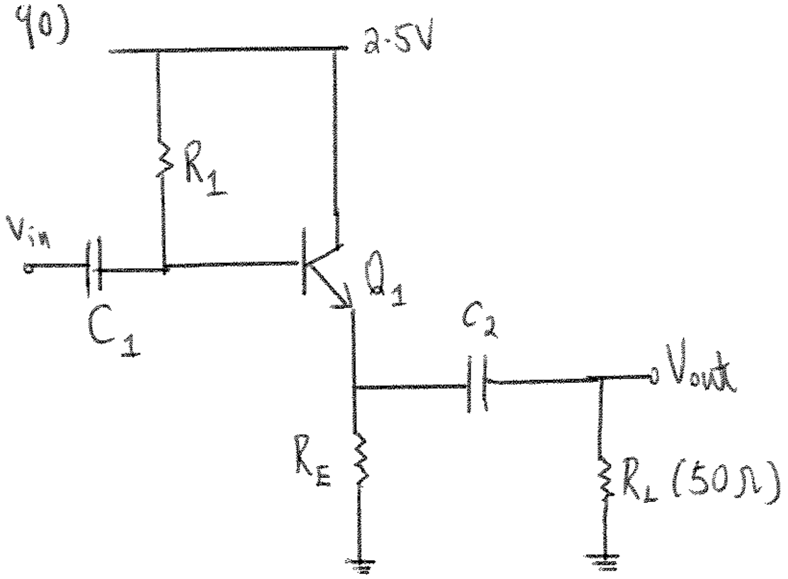
$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,mA$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26mV}{1.98mA} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

4o)



$A_v = 0.8$

Since R_E doesn't affect Voltage gain significantly.

$$A_v \approx \frac{R_L}{R_L + \frac{1}{g_m}} = 0.8$$

$$R_L = 0.8\left(R_L + \frac{1}{g_m}\right)$$

$$0.2R_L = 0.8\frac{1}{g_m}$$

$$R_L = 4\frac{1}{g_m} \Rightarrow \frac{R_L}{4} = \frac{1}{g_m} = 12.5 = \frac{V_T}{I_c}$$

$$I_c = \frac{26mV}{12.5\Omega} = 2.08\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.751\,V$$

Let $R_E I_c = 20V_T$ (0.52V), $R_E = \frac{20}{g_m} = 250\Omega$

$$R_1 = \frac{2.5 - (0.751 + 0.52)}{0.0208\,mA} = 59.1K$$

9c)

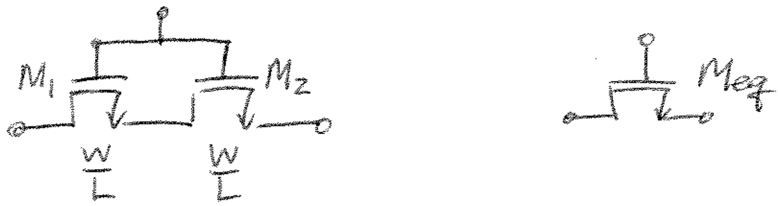
$$\frac{1}{(2\pi)(100\times10^6)C_1} = \frac{1}{10}\frac{1}{g_m} \Rightarrow C_1 = 1.27nf$$

$$\frac{1}{(2\pi)(100\times10^6)C_2} = \frac{1}{10}50 \Rightarrow C_2 = 0.32nf$$

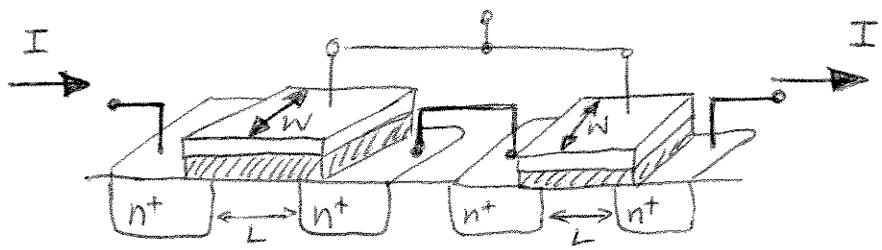
(So C_2 will not load Q_1).

$C_1 = 1.27\ nf$
$C_2 = 0.32\ nf$
$R_1 = 59.1\ K\Omega$ $\Rightarrow$ $A_v = 0.8$
$R_E = 250\ \Omega$
$R_L = 50\Omega$

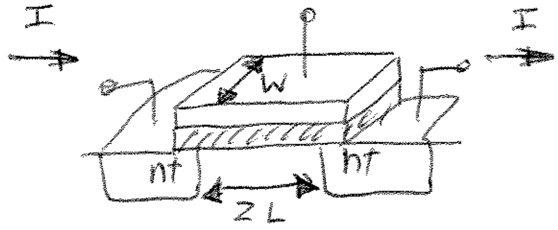
1.



Intuitively, this is similar to having twice of the original channel length:



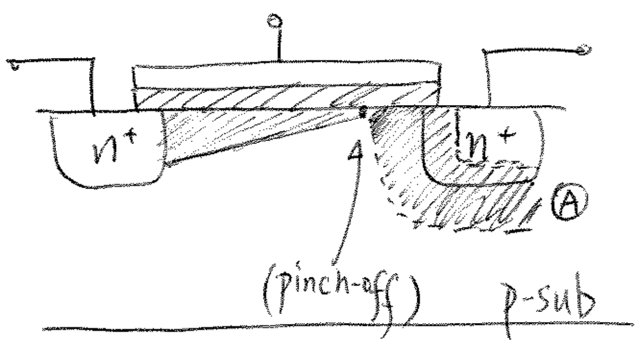
Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:



This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that v is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:



The shaded region, Ⓐ, represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which ≠ 0.

Pinch-off implies that the depletion region created no longer has free carriers. The depletion still sweeps all electrons from inversion channel to drain.

3. Given: $C_{ox} = 10\ fF/\mu m^2$ $\quad W = 5\mu m$ $\quad L = 0.1\mu m$

$V_{GS} - V_{TH} = 1V$ $\quad V_{DS} = 0$

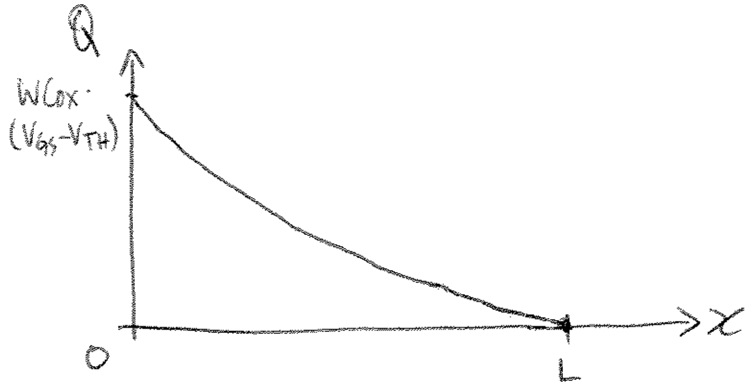
Find: total charge stored in channel, Q_{tot}

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

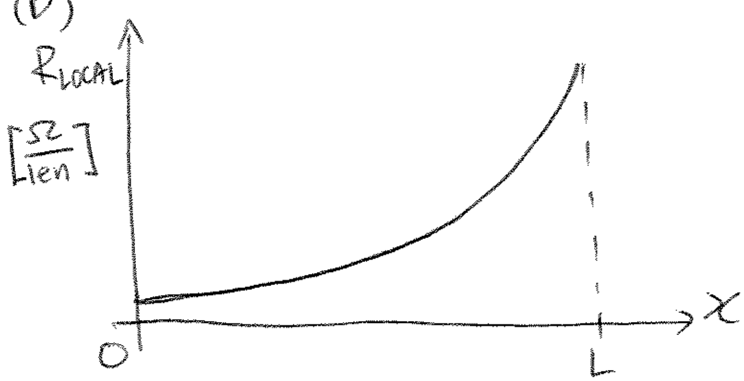
$$= (5\mu m)(10\ fF/\mu m^2)(1V)(0.1\mu m) = 5\ fC$$

4. (a) $Q = WC_{ox}(V_{GS} - V_{TH} - V(x))$

$= -WC_{ox} \cdot V(x) + WC_{ox}(V_{GS} - V_{TH})$



(b)



$R \propto \frac{1}{\mu Q}$

↑ mobility of charge.

5. $I_D = W C_{ox} [V_{GS} - V(x) - V_{TH}] \mu_n \frac{dV(x)}{dx}$

Define: $A = \frac{I_D}{W C_{ox} \mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \frac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

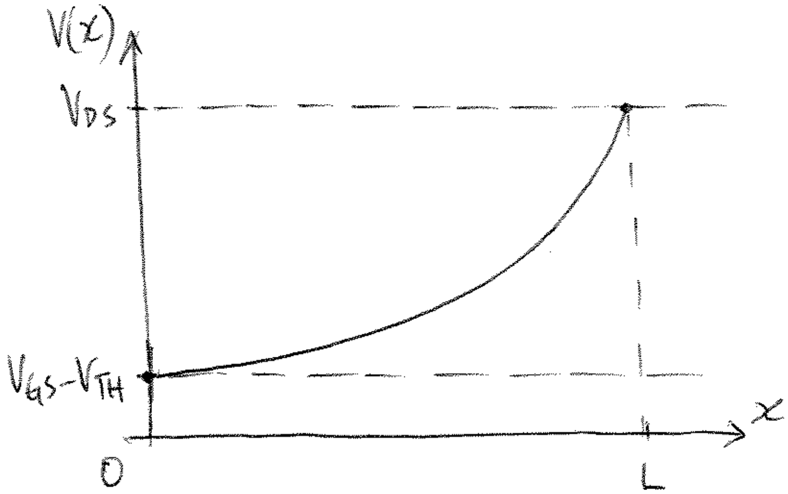
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4 \cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\frac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2 \cdot \frac{I_D}{W C_{ox} \mu_n (V_{GS} - V_{TH})^2}\right] x}\right\}$$

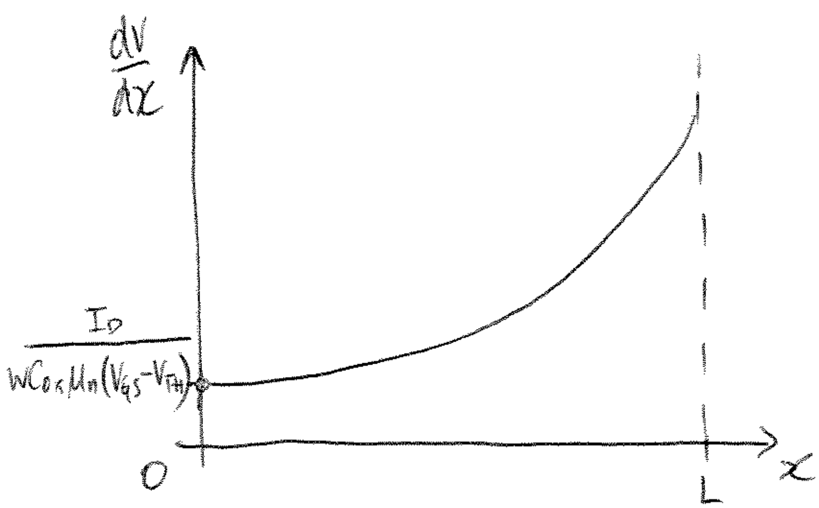
We know that $0 \le V(x) \le V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is > 0. Therefore, we take V_- as the solution.

i.e. $V(x) = (V_{GS}-V_{TH}) \left\{ 1 - \sqrt{1 - \left[\frac{2I_D}{W C_{ox} \mu_n (V_{GS}-V_{TH})^2} \right] x} \right\}$



$\because I_D \propto W$
$\Rightarrow V(x)$ is independent of W.

$$\frac{dV}{dx} = \frac{I_D}{W C_{ox} \mu_n (V_{GS}-V_{TH})} \cdot \left[1 - \frac{2I_D \cdot x}{W C_{ox} \mu_n (V_{GS}-V_{TH})^2} \right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS} - V_{TH}$ & V_{DS}, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given: NMOS $I_D = 1mA$ $V_{GS} - V_{TH} = 0.6V$

$I_D = 1.6mA$ $V_{GS} - V_{TH} = 0.8V$

(triode region) $\mu_n C_{ox} = 200 \frac{\mu A}{V^2}$

Find V_{DS} & W/L.

$$1mA = \mu_n C_{ox} \frac{W}{L} \left[(0.6) V_{DS} - V_{DS}^2/2 \right] \quad \text{——} ①$$

$$1.6mA = \mu_n C_{ox} \frac{W}{L} \left[(0.8) V_{DS} - V_{DS}^2/2 \right] \quad \text{——} ②$$

② ÷ ①: $$1.6 = \frac{0.8 V_{DS} - V_{DS}^2/2}{0.6 V_{DS} - V_{DS}^2/2} = \frac{1.6 - V_{DS}}{1.2 - V_{DS}}$$

$$\Rightarrow V_{DS} = \frac{1.6(0.2)}{0.6} \approx 0.533 \ V$$

$$\Rightarrow \frac{W}{L} = \frac{I_D}{\mu_n C_{ox} \left[(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2/2 \right]}$$

$$= \frac{1 mA}{200 \frac{\mu A}{V^2} \left[(0.6V)(0.533V) - (0.533V)^2/2 \right]}$$

$$\approx 28.$$

8. $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right]$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \cdot 2 V_{DS} = \mu C_{ox} \frac{W}{L} V_{DS}$$

$$g_m \big|_{V_{DS}=0} = 0.$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no I_D ($\because V_{DS} = 0$), it does not matter if there is an incremental change in V_{GS} (i.e. ∂V_{GS}). Since varying V_{GS} gives no change in I_D, $g_m \big|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $\quad \frac{W}{L} = 20 \quad \mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

$V_{TH} = 0.4\ V$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)\ (1.8 - 0.4)V} = 179.\ \Omega$$

10. $$500 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$$

$$400 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$$\Rightarrow \quad 500(1 - V_{TH}) \overset{?}{=} 400(1.5 - V_{TH})$$

$$500(0.6) \neq 400(1.1)$$

$\therefore$ This is not possible.

11. $$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \left[\frac{\partial}{\partial V_{DS}}\left\{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]\right\}\right]^{-1}$$

$$= \left[\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) - \mu C_{ox}\frac{W}{L}V_{DS}\right]^{-1}$$

$$= \frac{1}{\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}-V_{DS})}$$

12. When MOS operates as a resistor,

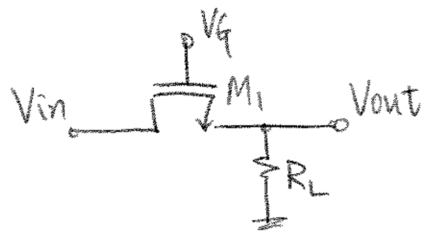
$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{W L C_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,

1) use minimum channel length, and
2) maximize overdrive voltage.

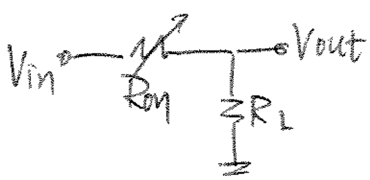
13.



Given $V_{in} \approx 0$
$V_G = 1.8\ V$
$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate M_1 as a linear resistance controlled by V_G. Therefore, the equivalent circuit becomes a resistive divider:



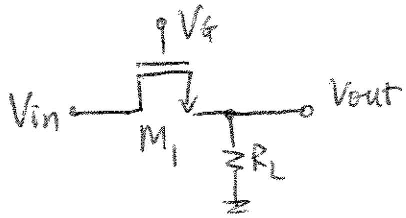
$$V_{out} = 0.95\, V_{in} = \frac{R_L}{R_{on} + R_L} V_{in}$$

$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox}(V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200\frac{\mu A}{V^2}(1.8-0.4)(5.3\Omega)}$$

$$= 674.$$

14.



$V_0 \sim$ few mV.

(a) $V_{in} = V_0 \cos\omega t$ $\qquad V_{out} = 0.95(V_0 \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON} + R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON} + R_L} = 0.95 V_0$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_0}{1-0.95V_0}\right)} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_0/(1-0.95V_0)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\, V_{in} = 0.95\,(V_0 \cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

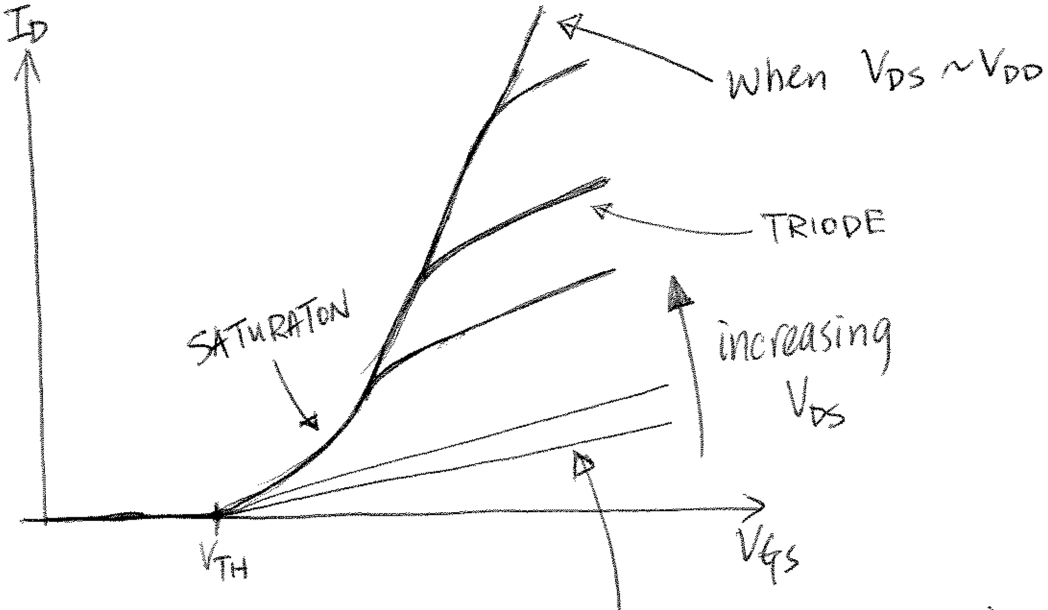
($\because V_0$ is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

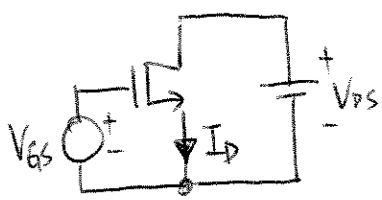
$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the R_{ON} varies with changing sinewave. With a DC bias voltage, R_{ON} becomes more stable (independent of V_o).

15.



I_D
when $V_{DS} \sim V_{DD}$
TRIODE
SATURATON
increasing V_{DS}
V_{TH}
V_{GS}
TRIODE (once $V_{GS} > V_{TH}$)
(V_{DS} so small that it never reaches saturation.)



V_{GS}
+
−
I_D
+
V_{DS}
−

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) I_D cannot be increased further by increasing V_{DS}. Since this curve must be continuous, the peak I_D must originate from the peak of the parabola.

17. $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^{\alpha}$, $\alpha < 3$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot \alpha (V_{GS}-V_{TH})^{\alpha-1}$$

$$= \frac{\alpha}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^{\alpha-1}$$

$$= \frac{\alpha I_D}{(V_{GS}-V_{TH})}$$

18. $I_D = W C_{ox} (V_{GS} - V_{TH}) v_{SAT}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} v_{SAT}$$

19. (a) OFF $\because V_{GS} = 0$

(b) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER: MOSFET is symmetric)

(e) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because V_{GS} = 0$

(g) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

20. (a) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

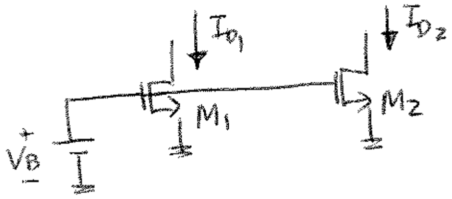
(b) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &

$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

21.



$0.99 I_{D2} < I_{D1} < 1.01 I_{D2}$

Since M_1 & M_2 are treated as current sources, they are assumed to be in saturation.

Evaluate λ at boundaries:

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda V_{DS1}) \quad \text{——— ①}$$

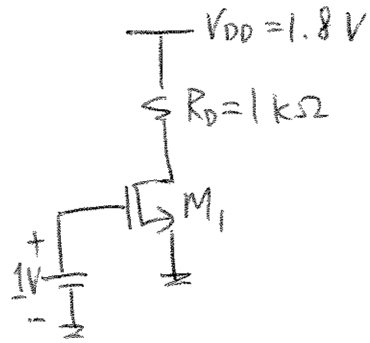
$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda V_{DS2}) \quad \text{——— ②}$$

① ÷ ② : $$\frac{I_{D1}}{I_{D2}} = \frac{0.99\, I_{D2}}{I_{D2}} = \frac{1+\lambda V_{DS1}}{1+\lambda V_{DS2}}$$

$$\therefore \lambda = \frac{0.01}{0.99\, V_{DS2} - V_{DS1}} = \frac{0.01}{0.99(1V) - (0.5V)} = 0.02\ V^{-1}$$

Maximum tolerable $\lambda = 0.02 V^{-1}$

22.



$\lambda = 0$, $V_{TH} = 0.4\,V$

$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

M_1 sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$\Rightarrow V_{DS,edge} = (1 - 0.4)\,V = 0.6\,V$

By KCL, $I_{D_1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$\therefore I_{D_1} = 1.2\,mA = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_{D_1}}{\mu_n C_{ox} (V_{GS} - V_{TH})^2} = \frac{2(1.2\,mA)}{\left(200\,\frac{\mu A}{V^2}\right)\left(1 - 0.4\right)^2 V^2}$$

$\cong 33.$

23. If gate oxide thickness, t_{ox}, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow$ $\mu_n C_{ox}$ is also halved

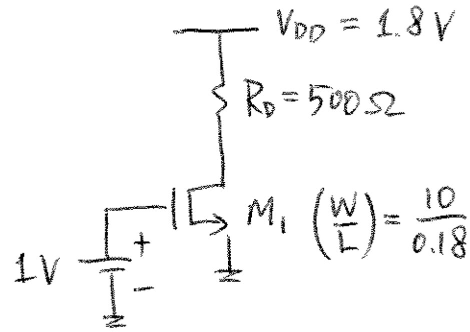
$\Rightarrow$ I_{D_1} is halved $\Rightarrow$ V_{DS} increases

$\Rightarrow$ M_1 stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2mA}{2} = 0.6\ mA$$

$$\Rightarrow V_{DS} = (1.8V) - (0.6mA)(1\ k\Omega) = 1.2\ V$$

24.



$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow$ $V_{DS} \geq 1 - 0.4 = 0.6\,V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$$

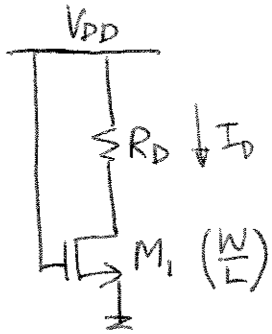
$$= \frac{1}{2}\left(200\,\frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\,mA$$

By KCL, $\dfrac{V_{DD} - V_{DS}}{R_D} = 2\,mA$

$\therefore$ $V_{DD} = (2mA)(500\Omega) + 0.6\,V = 1.6\,V$

Minimum $V_{DD} = 1.6\,V$

25.



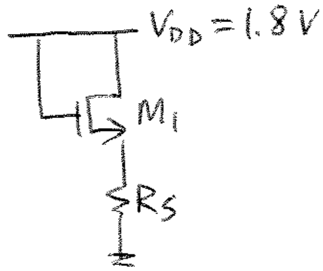
$\lambda = 0$

When M_1 operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{DD} - V_{TH})^2}_{I_D}$$

26.



$\lambda = 0$

Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

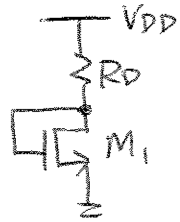
Since $V_{DS} = V_{GS}$ for M_1, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - I_1 R_S - V_{TH}\right)^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2 I_1}{\mu_n C_{ox}\left(V_{DD} - I_1 R_S - V_{TH}\right)^2}$$

27.



Calculate I_1 if $\lambda = 0$.
Assume $V_{GS} > V_{TH}$

By KCL, $I_{R_D} = I_1$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{\triangleq B}(V_{GS}-V_{TH})^2 = \frac{V_{DD}-V_{GS}}{R_D}$$

Re-arrange this to quadratic form:

$$V_{GS}^2(BR_D) - V_{GS}(2BR_DV_{TH}-1) + (V_{TH}^2\cdot BR_D - V_{DD}) = 0$$

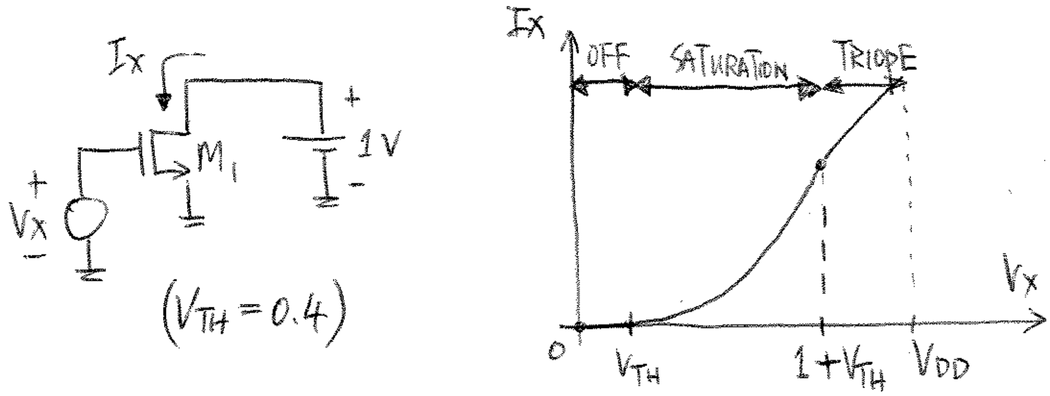
$$\Rightarrow V_{GS_{1,2}} = \frac{(BR_DV_{TH}-1) \pm \sqrt{BR_D(V_{DD}-V_{TH})+1}}{BR_D}$$

$$= \frac{\left(\frac{1}{2}\mu_nC_{ox}\left(\frac{W}{L}\right)\cdot R_D\cdot V_{TH}-1\right) \pm \sqrt{\frac{1}{2}\mu_nC_{ox}\left(\frac{W}{L}\right)R_D(V_{DD}-V_{TH})+1}}{\frac{1}{2}\mu_nC_{ox}\left(\frac{W}{L}\right)}$$

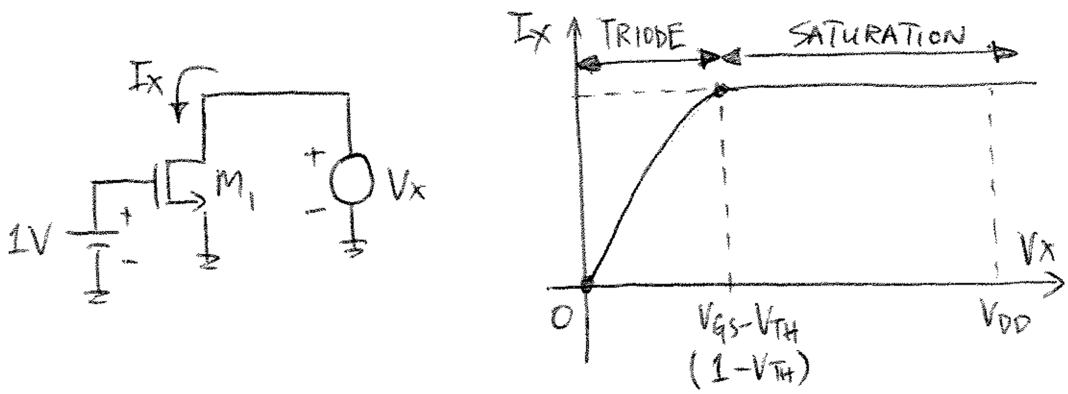
Whether the answer is V_{GS_1} or V_{GS_2} depends on other parameters. Also note that since M_1 is diode-connected, it never goes into triode (i.e either OFF or SATURATION.) This helps in eliminating one of the solutions.

After solving V_{GS}, $I_D = I_1 = \frac{V_{DD} - V_{GS}}{R_D}$

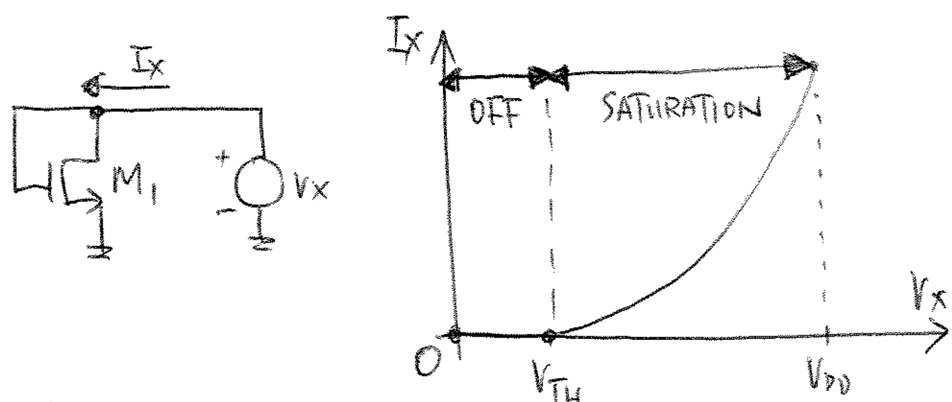
28. (a)


Ix
M1
1V
+
-
Vx
(VTH = 0.4)
Ix
OFF
SATURATION
TRIODE
Vx
0
VTH
1+VTH
VDD

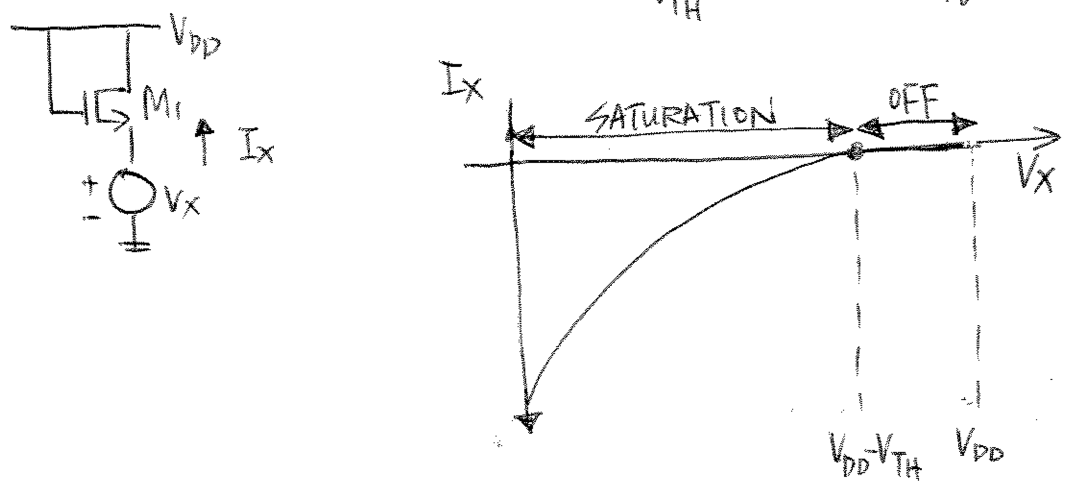
(b)


Ix
M1
1V
Vx
Ix
TRIODE
SATURATION
Vx
0
VGS-VTH
(1-VTH)
VDD

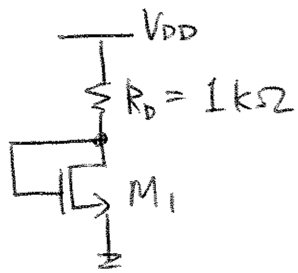
(c)


Ix
M1
Vx
Ix
OFF
SATURATION
Vx
0
VTH
VDD

(d)


VDD
M1
Ix
Vx
Ix
SATURATION
OFF
Vx
VDD-VTH
VDD

29.



$\left(\frac{W}{L}\right) = \frac{10}{0.18}$, $\lambda = 0.1\ V^{-1}$

Find I_{D1}

Since M_1 is diode-connected, it operates in saturation.

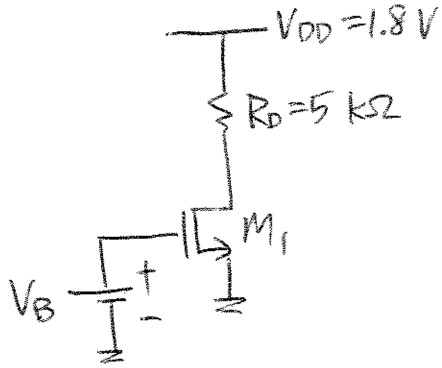
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_G - V_{TH})^2 (1 + \lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding V_G.

Using any method gives $V_G \approx 0.807$ V

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1$ mA

30.



$\frac{W}{L} = \frac{20}{0.18}$, $\lambda = 0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1} = \frac{V_{DD} - (V_B - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda(V_B - V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\ mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50 \ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5$ mA, find W/L.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)^2}{2 \left(200 \frac{\mu A}{V^2}\right)(0.5 mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5V$, find W/L.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)}{\left(200 \frac{\mu A}{V^2}\right)(0.5 V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5V$, find I_D.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)(0.5V)}{2} \approx 5 mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (I_D constant)

Doubling (W/L) implies a $\sqrt{2}$ times increase in g_m: $g_{m,NEW} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$ (I_D constant)

Doubling $(V_{GS} - V_{TH})$ decreases g_m by half:
$g_{m,NEW} = \frac{2 I_D}{2(V_{GS} - V_{TH})} = \frac{1}{2} g_m$

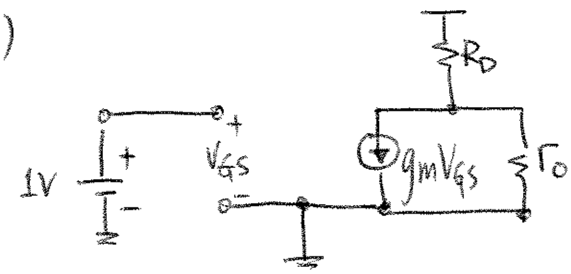
(c) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (W/L constant)

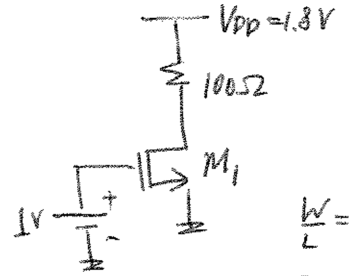
Doubling I_D increases g_m by $\sqrt{2}$ times.

(d) $g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$ ($V_{GS} - V_{TH}$ constant)

Doubling I_D increases g_m by 2 times.

33. (a)





$\frac{W}{L} = \frac{20}{0.18}$

$\lambda = 0.1V^{-1}$

First, verify M_1 is in saturation:

$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - R_D \cdot \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$= 1.8 - 100 \cdot \frac{1}{2} 200 \frac{\mu A}{V^2} \left(\frac{20}{0.18}\right) (1 - 0.4)^2 (1 + \lambda V_{DS})$$

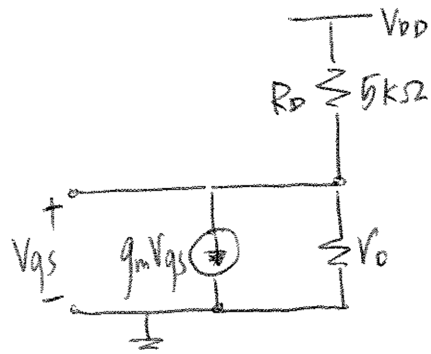
Solving this gives $V_{DS} \approx 1.35V > V_{GS} - V_{TH}$.

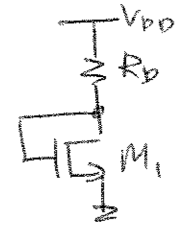
$$\therefore g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = \left(200 \frac{\mu A}{V^2}\right) \left(\frac{20}{0.18}\right) (1 - 0.4V)$$

$$\approx 0.013 \ ^1/_\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda \left(\frac{V_{DD} - V_{DS}}{R_D}\right)} = \frac{1}{0.1V^{-1} \left(\frac{0.45V}{100\Omega}\right)} \approx 2222.\Omega$$

(b)





By KCL, $\dfrac{V_{DD}-V_{GS}}{R_D} = \dfrac{1}{2}\mu_n C_{ox}\dfrac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{GS})$

Solving this yields $V_{GS} \approx 0.546\,V > V_{TH}$

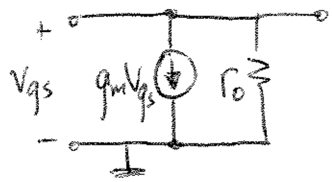
$$\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) = \frac{200\mu A}{V^2}\left(\frac{20}{0.18}\right)(0.146V)$$

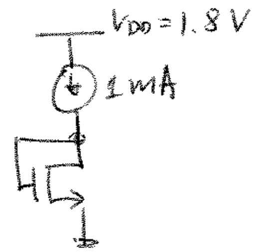
$$\approx 0.00324\ 1/\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda\left(\frac{V_{DD}-V_{GS}}{R_D}\right)} = \frac{1}{0.1V^{-1}\left(\frac{1.8-0.546}{5k}\right)}$$

$$\approx 40.\ k\Omega$$

(c)





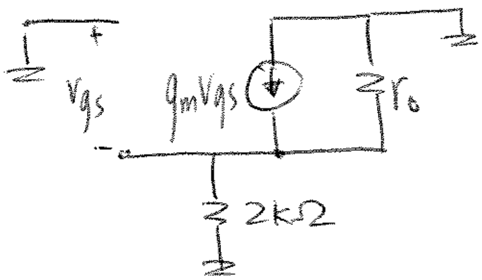
(Note: ideal current source is open in small signal)

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2\left(200 \frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1\,mA)}$$

$$\approx 0.0067 \ ^1/_{\Omega}$$

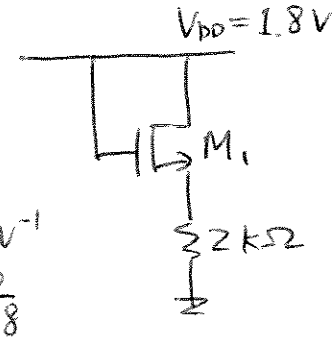
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\,V^{-1})(1\,mA)} = 10\,k\Omega$$

(d)



$\lambda = 0.1\ V^{-1}$

$\frac{W}{L} = \frac{20}{0.18}$



(Note: ideal voltage source is shorted; to GND in this problem because V_{DD} is si -ended.)

By KCL, $I_D = I_R$

$$\Rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD}-V_S)-V_{TH}\right]^2\left[1+\lambda(V_{DD}-V_S)\right] = V_S/2k\Omega.$$

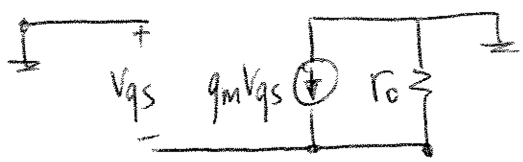
Solving this (analytically or numerically) gives $V_S \approx 1.18\ V$

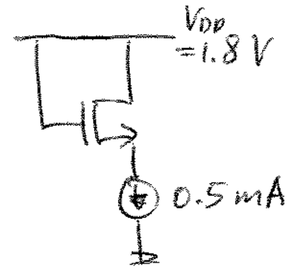
$$\Rightarrow I_D = V_S/2k\Omega \approx 0.59\ mA.$$

$$g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(0.59mA)}{(1.8-1.18-0.4)V} \approx 0.0054\ 1/\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.59mA)} \approx 16.9\ k\Omega$$

(e)





$$0.5\ mA = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_S) - V_{TH} \right]^2 \left[1 + \lambda (V_{DD} - V_S) \right]$$

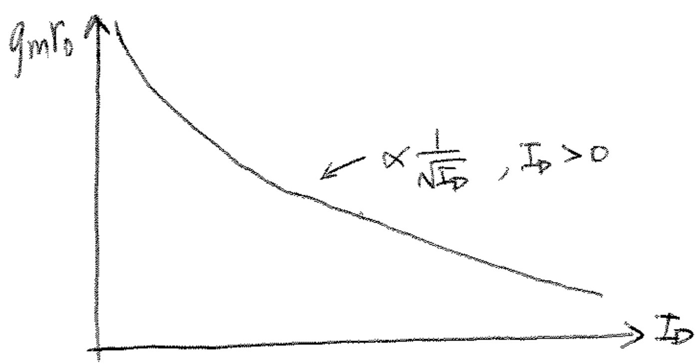
Solving this equation gives $V_S \approx 1.19\ V$

$$\Rightarrow g_m = \frac{2 I_D}{V_{GS} - V_{TH}} = \frac{2(0.5mA)}{(1.8 - 1.19 - 0.4)V} \approx 0.0048 \frac{1}{\Omega}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5mA)} = 20\ k\Omega.$$

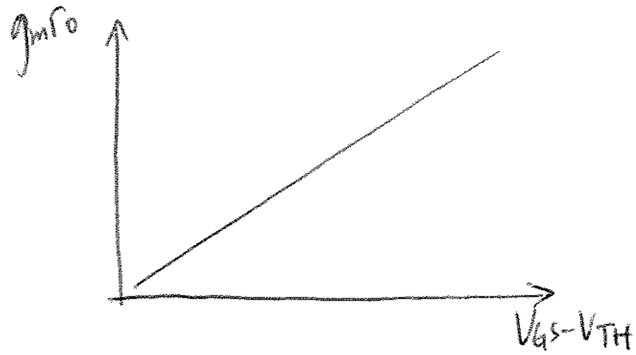
34. $g_m = \sqrt{2\mu C_{ox}\frac{W}{L} I_D}$ $r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}(W/L) I_D}}{\lambda I_D} = \frac{1}{\lambda} \sqrt{\frac{2\mu C_{ox}(W/L)}{I_D}}$$

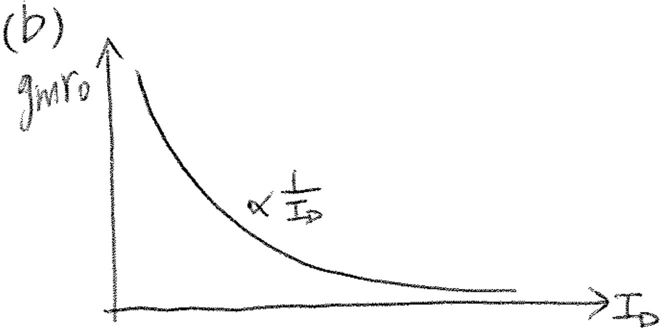


35 (a) $g_m = \mu C_{ox} \frac{W}{L}(V_{GS}-V_{TH})$ $r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox}(W/L)(V_{GS}-V_{TH})}{\lambda I_D}$$



(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$ $V_{DS} = 1.5\ V$

determine W/L if $I_D = 0.5\ mA$.

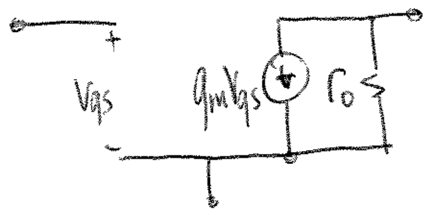
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5\ mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\ k\Omega} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\ k\Omega}\right)^2 \frac{1}{2 \mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\ k\Omega}\right)^2 \frac{1}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 5.$$

37.



Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

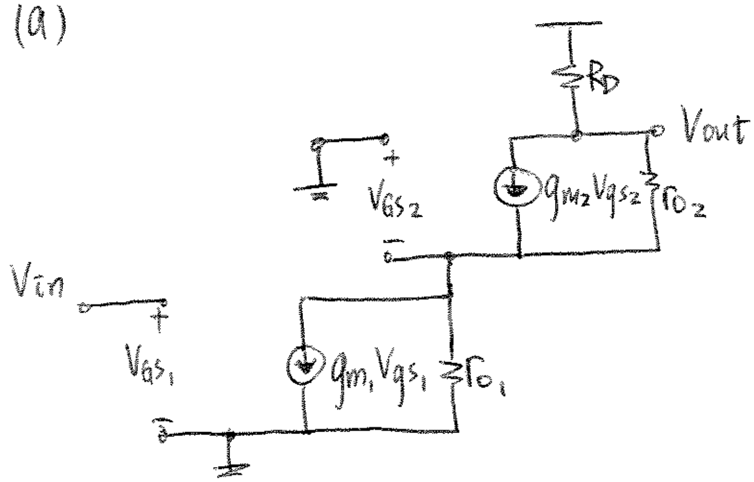
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2V^{-1})(0.5\,mA)$$
$$= 0.002\ 1/\Omega$$

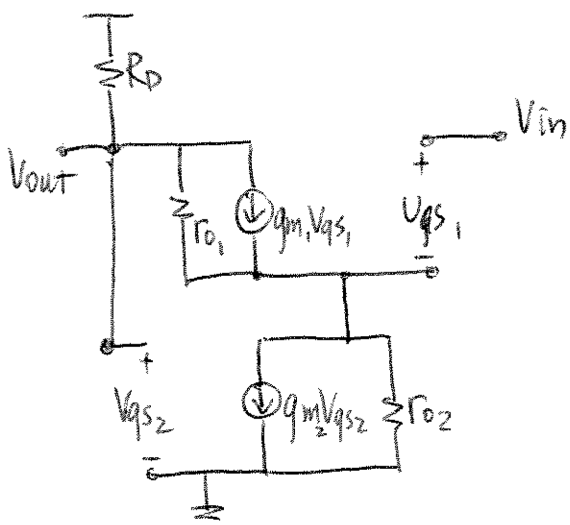
$$\Rightarrow\ g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore\ \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{(0.0002\ 1/\Omega)^2}{2\left(200\frac{\mu A}{V^2}\right)(0.5\,mA)} = 20$$

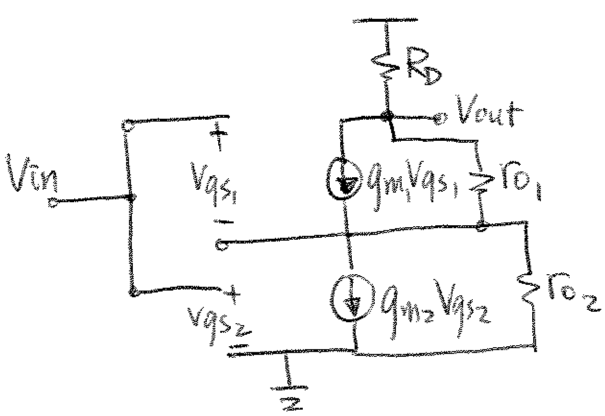
38. (a)


RD
Vout
VGS2
gm2Vgs2
ro2
Vin
Vgs1
gm1Vgs1
ro1

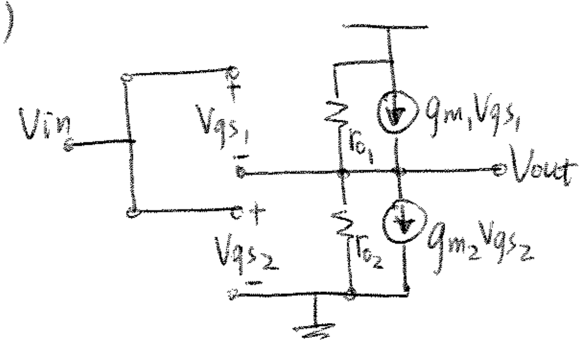
(b)


RD
Vout
Vin
ro1
gm1Vgs1
Vgs1
Vgs2
gm2Vgs2
ro2

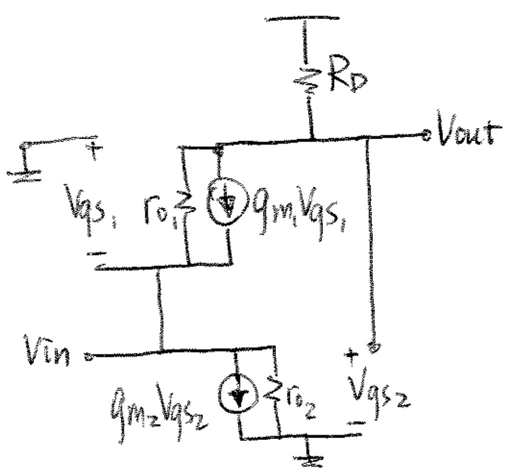
(c)


RD
Vout
Vin
Vgs1
gm1Vgs1
ro1
Vgs2
gm2Vgs2
ro2

(d)


Vin
Vgs1
Vgs2
ro1
ro2
gm1Vgs1
gm2Vgs2
Vout

(e)


RD
Vout
Vgs1
ro1
gm1Vgs1
Vin
gm2Vgs2
ro2
Vgs2

39. (a) OFF $\because |V_{SG}| = 0$

(b) OFF $\because |V_{SG}| < |V_{TH}| = 0.4V$

(c) SATURATION $\because |V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because V_{SG} < |V_{TH}|$

40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

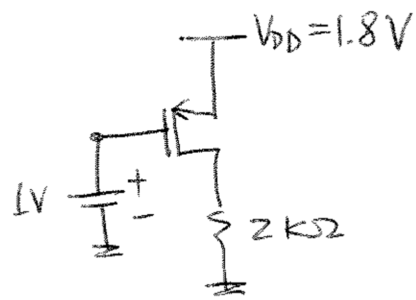
(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} < V_{SG} - |V_{TH}|$

41.



$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$
$\Rightarrow V_D = 1.4\,V.$

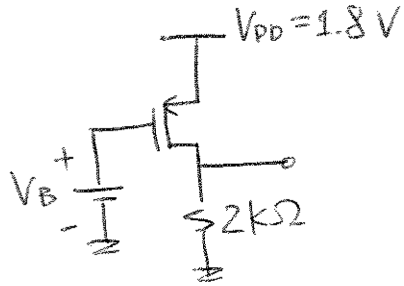
By KCL, $I_{D_1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2 = \frac{V_D}{2k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2k\Omega}\cdot\frac{2}{\mu_p C_{ox}(V_{SG} - |V_{TH}|)^2}$$

$$= \frac{1.4\,V}{2k\Omega}\cdot\frac{2}{100\,\frac{\mu A}{V^2}\,(0.8\,V - 0.4\,V)^2} \approx 87.5$$

42.



$\lambda = 0$

When $V_B = 1V$, $W/L = 87.5$

When $V_B = 0.8V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

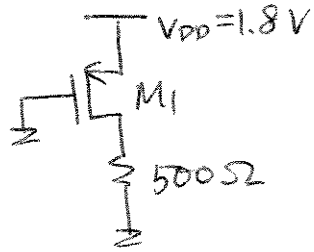
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2K\Omega} = \frac{(1.8 - 0.18)V}{2K\Omega} \approx 0.81\ mA$$

43 (a)



Assume M_1 in triode (since V_{SG} is large). Note: if assumption is incorrect, results will show that.

By KCL, $I_D = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox} \frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = \frac{V_{DD} - V_{SD}}{500\Omega}$$

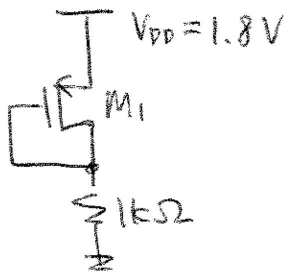
This is a quadratic relation on V_{SD}. Solving it yields $V_{SD} \approx 0.42\,V$

Verify assumption: $V_{SD} \overset{?}{<} V_{SG} - |V_{TH}|$

$0.42 < 1.8 - 0.4 = 1.4$ (✓)

$$I_D = \frac{V_{DD} - V_{SD}}{500\Omega} = \frac{(1.8 - 0.42)V}{500\Omega} \approx 2.8\,mA$$

(b)



$\frac{W}{L} = \frac{10}{0.18}$ $\lambda = 0$

$\mu_p C_{ox} = 100\ \frac{\mu A}{V^2}$

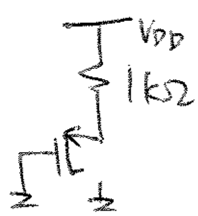
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{(V_{DD} - V_{SG})}{1k\Omega}$$

Solving this quadratic equation gives $V_{SG} \approx 0.61\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} = \frac{(1.8 - 0.61)V}{1k\Omega} \approx 1.2\ mA$$

(c)



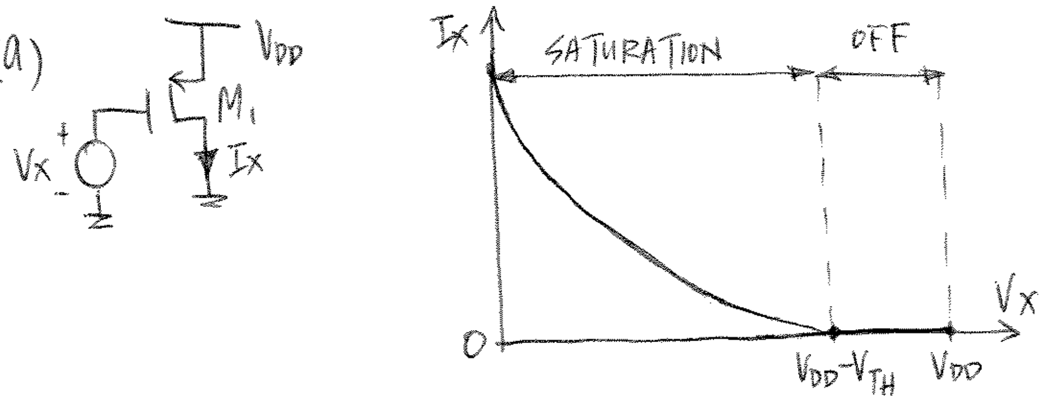
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{V_{DD} - V_{SG}}{1k\Omega}$$

Solving this gives $V_{SG} \approx 0.61\ V$

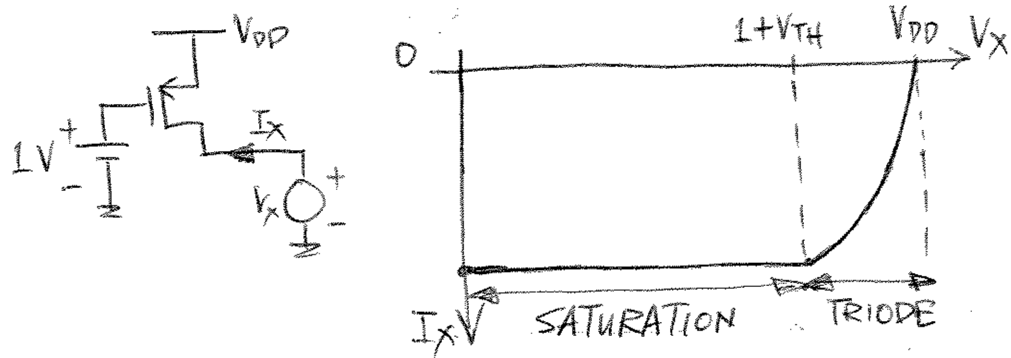
$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} \approx 1.2\ mA$$

44. (a)



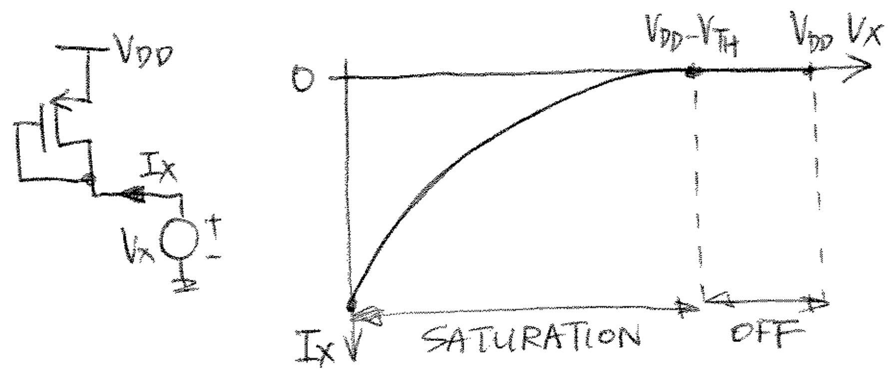
VDD
M1
Vx
Ix
SATURATION
OFF
0
VDD-VTH
VDD

(b)



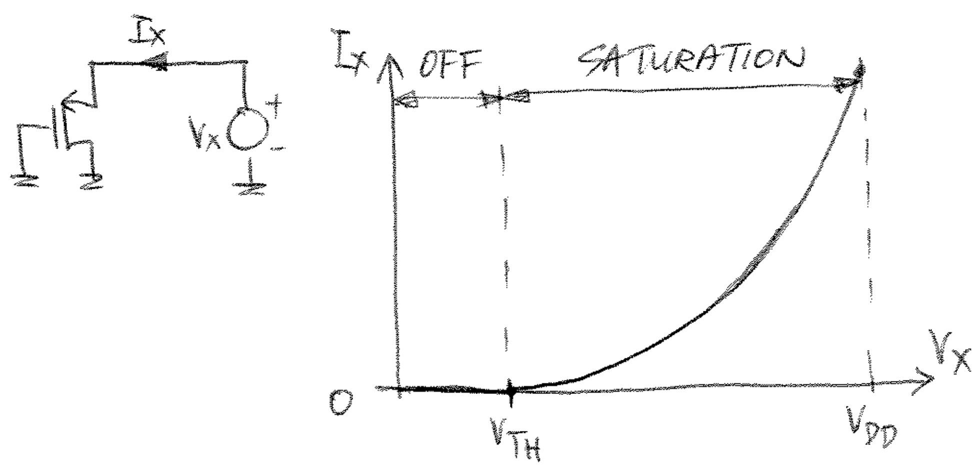
VDD
1V
Ix
Vx
0
1+VTH
SATURATION
TRIODE

(c)



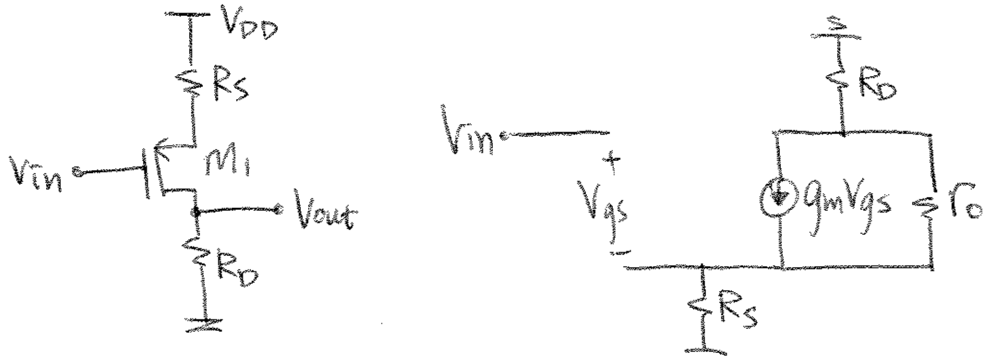
VDD
Ix
Vx
0
VDD-VTH
SATURATION
OFF

(d)

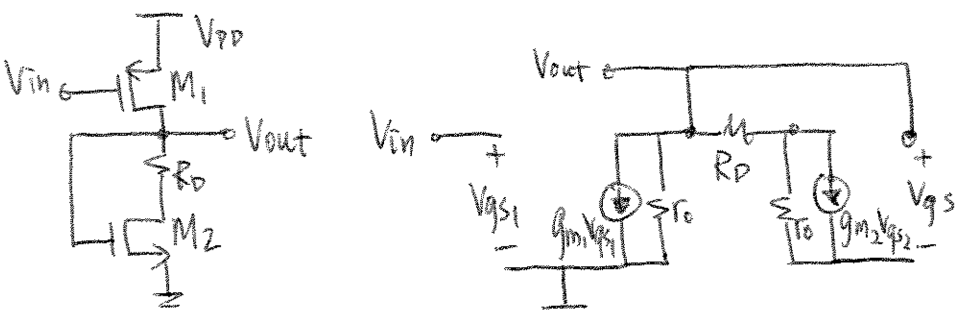


Ix
Vx
OFF
SATURATION
0
VTH
VDD

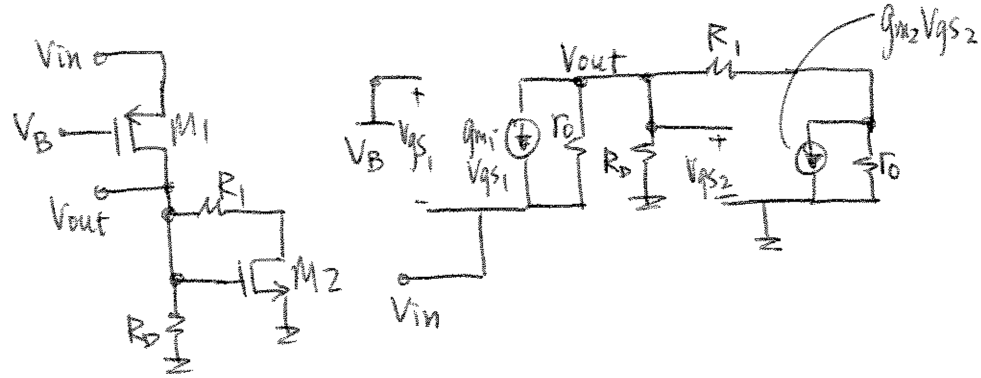
45. (a)


VDD
RS
M1
Vin
Vout
RD
Vin
+
Vgs
−
gmVgs
ro
RD
RS

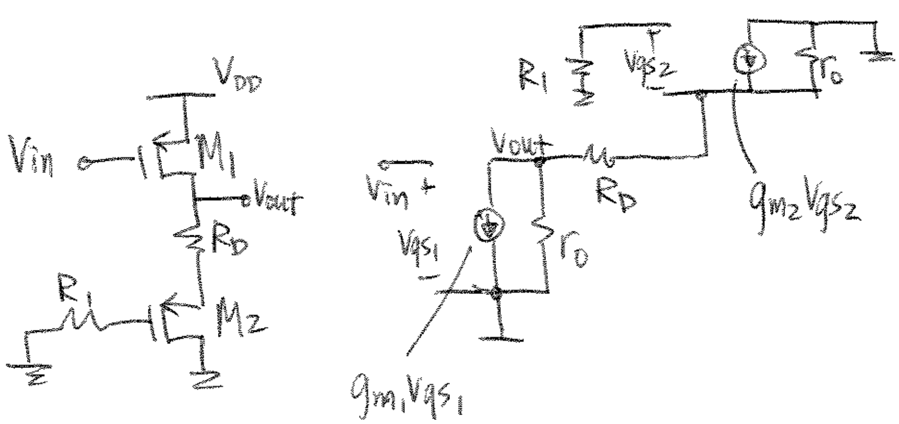
(b)


VDD
Vin
M1
Vout
RD
M2
Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

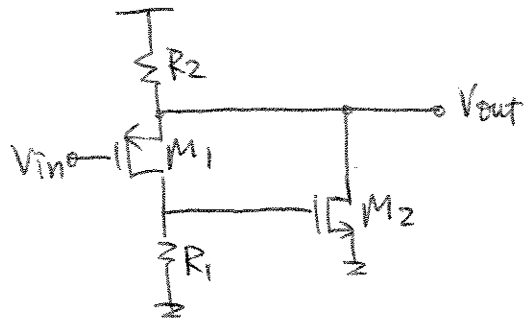
(c)


Vin
VB
M1
Vout
R1
M2
RD
Vout
R1
gm2Vgs2
+
VB
Vgs1
−
gm1
Vgs1
ro
RD
+
Vgs2
−
ro
Vin

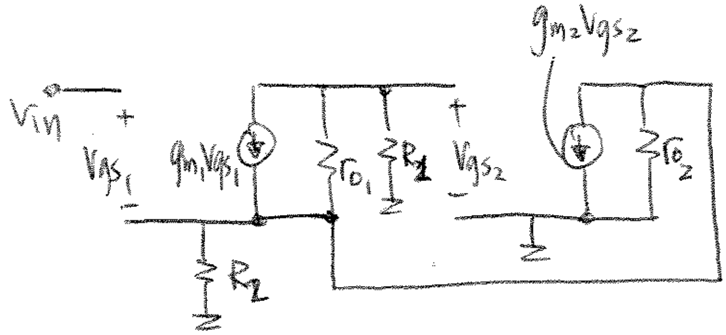
(d)


VDD
Vin
M1
Vout
RD
R1
M2
R1
+
Vgs2
−
ro
Vout
RD
Vin +
Vgs1
−
ro
gm2Vgs2
gm1Vgs1

(e)

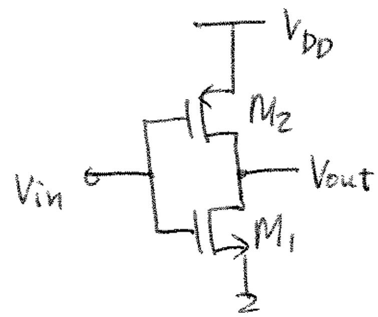


R2
Vout
Vin
M1
M2
R1



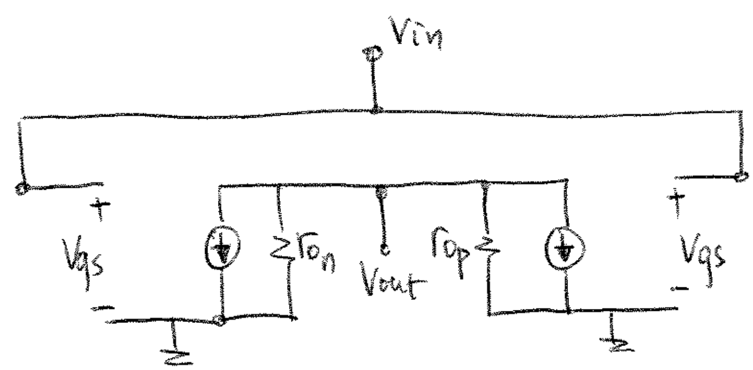
gm2Vgs2
Vin
+
Vgs1
−
gm1Vgs1
ro1
R1
+
Vgs2
−
ro2
R2

46.



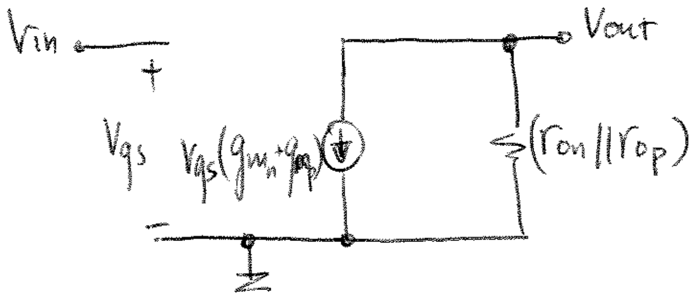
Assume λ_n & λ_p.

(a)



They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both M_1 & M_2 are in saturation, we can combine r_o's & g_m's :



$$\therefore \quad \frac{V_{out}}{V_{in}} = -(g_{mn}+g_{mp})(r_{on} \parallel r_{op})$$

(1) For M_1 to stay in saturation,

$$V_{DS} > V_{GS} - V_{TH},$$

i.e. $$V_{DS} > V_{DD} - V_{TH}$$

$$V_{DS} > 1.4$$

$\because$ $$V_{DS} = V_{DD} - I_{DS}(R_L)_1$$

where $R_L = 1k\Omega$.

and $$I_{DS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$= \frac{1}{2} \times 200 \times 10^{-6} \left(\frac{W}{L}\right)_1 (1.4)^2$$

$\therefore$ $$V_{DS} = V_{DD} - 10^{-4}\left(\frac{W}{L}\right)_1 (1.96) \times 1000$$

i.e. $$1.8 - 1.96 \times 10^{-4}\left(\frac{W}{L}\right)_1 > 1.4$$

$$\frac{0.4}{1.96 \times 10^{-1}} > \left(\frac{W}{L}\right)_1$$

Maximum allowable $\left(\frac{W}{L}\right)_1$ is 2

② To get $I_{DS} = 1\ mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1 \times 10^{-3}\ A.$$

$$\frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

Since $$V_{GS} = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7\ R_1 = .\ R_2,$$

$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \quad \text{———} ①$$

To get input impedance $\geq 20k$.

$$R_1 // R_2 \geq 20\ k\Omega. \quad \text{———} ②$$

By inspection, setting $R_1 = 55\ k\Omega$ and $R_2 = 35\ k\Omega$ Will satisfy both ① and ②.

(3) $V_G = 1.8\,V$

$V_S = I_{DS}(100)$

$V_D = 1.8 - 1000\,I_{DS}$

For M_1 to be in saturation,

$$V_{DS} \geq V_{GS} - V_{TH}$$

$$\therefore V_D - V_S \geq V_G - V_S - V_{TH}$$

$$V_D \geq V_G - V_{TH}$$

$$V_D \geq 1.4\,V$$

$$\therefore I_{DS,\,max} = \frac{1.8 - 1.4}{1000} = 0.4\,mA.$$

and $\therefore V_S = (0.4 \times 10^{-3})(100)$

$$= 0.04\,V$$

$$V_{GS} = 1.76\,V$$

$$g_{m,max} = \frac{2\,I_{DS,\,max}}{(1.76 - 0.4)}$$

$$= 0.588\,mS$$

④ a). $\because$ $V_{RS} = 200\,mV$,

$\therefore$ $I_{DS} R_S = 200\,mV$

$I_{DS} = \frac{0.2}{100}$

$I_{DS} = 2\,mA$.

For M_1 to stay in saturation,

$V_{DS} \geqslant V_{GS} - V_{TH}$.

$\because$ $V_{DS} = V_D - V_S$

$= [1.8 - (2\times10^{-3})\times500] - 0.2$

$= 0.6$,

$\therefore$ $V_{GS} - V_{TH} \leqslant 0.6$,

Since $I_{DS} = \frac{1}{2}(\mu_n C_{ox})(\frac{W}{L})(V_{GS} - V_{TH})^2$,

$(\frac{W}{L})$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $(\frac{W}{L})_1$ is when $(V_{GS} - V_{TH}) = 0.6\,V$,

$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})(\frac{W}{L})_1(0.6)^2$

$\therefore$ min. $(\frac{W}{L})_1 \approx 56$

b) With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

i.e. $1.8 \times \dfrac{R_2}{R_1 + R_2} = 1.2V,$

$$\frac{R_2}{R_1} = 2 \quad \text{——①}$$

Input impedance $= R_2 // R_1$,

i.e. $R_2 // R_1 \geqslant 30k\Omega$ ——②

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$ will satisfy both ① & ②.

(5)

$$V_S = V_{RS}$$

$$= I_{D1}\ (200) = 0.1V$$

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$0.5mA = \frac{1}{2}(200\times 10^{-6})\left(\frac{20}{0.18}\right)(V_{GS} - V_{TH})^2$$

$$\therefore V_{GS} = 0.612\ V$$

$$\therefore V_G = 0.612 + 0.1$$

$$= 0.712.$$

$$\because V_G = V_{DD} - I_{R1}\times R_1$$

$$R_1 = \underline{\underline{21.76\ k\Omega}}.$$

and $V_{GS} = I_{R2} \times R_2.$

$$\therefore R_2 = \frac{0.712}{0.05\times 10^{-3}}$$

$$= \underline{\underline{14.24\,k\Omega}}$$

⑥. $g_m = \sqrt{2\beta I_{DS}} = \frac{1}{100}$,

$\because I_{DS} = 1mA, \quad \beta = 0.05$,

and $I_{DS} = \frac{1}{2}\beta(V_{GS} - V_{TH})^2$,

where $\beta = \mu_n C_{ox}\left(\frac{W}{L}\right)$,

$\therefore 1mA = \frac{1}{2}(0.05)(V_{GS} - V_{TH})^2$.

$V_{GS} = 0.6$.

$\because V_{GS} = V_{DS} = V_{DD} - I_{DS}R_D$,

$0.6 = 1.8 - (0.5\times10^{-3})R_D$,

$R_D = \underline{\underline{2.4\ k\Omega}}$

(7)

$$I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2$$

$$0.5 \times 10^{-3} = (100 \times 10^{-6})\left(\frac{50}{0.18}\right)(V_{GS} - V_{TH})^2$$

$$\therefore V_{GS} = 0.534V$$

$$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$$

$$R_2 = \underline{\underline{10.68k\Omega}}$$

$$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2k\Omega) = 0.1 I_{DS} (R_1 + R_2),$$

$$\therefore 14k\Omega = R_1 + 10.68k\Omega.$$

$$\therefore R_1 = \underline{\underline{3320\Omega}}$$

(8) Without defect,

$$V_{GS} = V_{DS}, \quad (\text{i.e. } V_G = V_D)$$

$$\therefore \frac{20k}{10k+20k} \times 1.8 = 1.8 - I_{DS}(1k\Omega)$$

$$I_{DS} = 0.6\ mA.$$

$$\because \quad V_{GS} = V_G - V_S$$
$$= 1.2 - (0.6\times10^{-3})(200)$$
$$= 1.08.$$

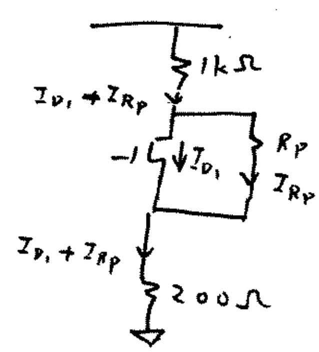
and. $$0.6\ mA = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(V_{GS}-0.4)^2.$$

$$\frac{W}{L} \approx 13 //$$

with R_P

$$V_{GS} = V_{DS} + V_{TH},$$
$$1.2 = V_{DS} + 0.4,$$
$$\therefore V_{DS} = 0.8\ V.$$

$$\because V_{DS} = V_{DD} - V_{1k\Omega} - V_{200\Omega},$$



$$\therefore I_{D_1} + I_{Rp} = \frac{1\ V}{1k\Omega + 200\Omega}$$
$$= 0.833\ mA$$

$$\because I_{Rp} = \frac{V_{DS}}{R_P} = \frac{0.8}{R_P}$$

and $I_{D_1} = 0.6\ mA$ (from above)

$$\therefore \quad \frac{0.8V}{R_p} + 0.6\ mA = 0.833\ mA$$

$$R_p \approx 3430\Omega$$

(a) Without defects,

$$V_{GS} = 1.8V$$

i.e. $$V_{DS} = (1.8 - 0.1)V$$

$$= 1.7V$$

$$I_{DS} = \frac{(1.8-1.7)V}{2000\,\Omega} = 0.05\,mA.$$

$$\therefore 0.05\,mA = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1.8-0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) = 0.255$$

b) With defects,

$$V_{GS} = V_{DS} + 50\,mV$$

$$\therefore V_{Rp} = 50\,mV$$

$$I_{Rp} = \frac{50\,mV}{R_p}$$

$$V_{GS} = 1.8\,V - \frac{0.05V}{R_p} \times 30\,k\Omega \quad \text{—— (1)}$$

$$\because V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{DS}$$

$$V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{GS} - 50\,mV \quad \text{—— (2)}$$

$\because \quad I_{DS} = \frac{1}{2}\left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})^2$

$= \frac{1}{2}(0.255)(200 \times 10^{-6})(V_{GS} - 0.4)^2$

$= 2.55 \times 10^{-5}(V_{GS} - 0.4)^2$

$\therefore$ From ②,

$$1.8 - \left[2.55 \times 10^{-5}(V_{GS} - 0.4)^2 - \frac{0.2}{R_p}\right]2000$$

$$= V_{GS} - 0.05.$$

From ①, $\frac{0.05}{R_p} = \frac{1.8 - V_{GS}}{30\ 000}$

$$\therefore 1.8 - \left[0.051(V_{GS} - 0.4)^2 - \frac{1.8 - V_{GS}}{15}\right] = V_{GS} - 0.3$$

$$1.85 - 0.051 V_{GS}^2 + 0.0408 V_{GS} - 0.00816 + \frac{1.8 - V_{GS}}{15} = V_{GS}$$

$$29.4276 - 15.388 V_{GS} - 0.765 V_{GS}^2 = 0$$

$$\therefore V_{GS} = 1.76\ V$$

$$R_p = \frac{0.05 \times 30\ 000}{1.8 - 1.76}$$

$$\approx 36.3\ k\Omega$$

(10) For M_1,

$$I_x = \frac{1}{2}(200 \times 100^{-6})\left(\frac{W_1}{0.25}\right)(0.8-0.4)^2 \times (1+0.1(0.8))$$

$$10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_1}{0.25}\right)(1.08)$$

$$\therefore W_1 \approx 14.5\,\mu$$

For M_2,

$$0.5 \times 10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_2}{0.25}\right)(1.08)$$

$$\therefore W_2 = 7.25\,\mu$$

Output resistance $= r_o$

$$= \frac{1}{\lambda} \times \frac{1}{I_D}.$$

$$\therefore r_{o1} = \left(\frac{1}{0.1}\right)\left(\frac{1}{10^{-3}}\right)$$

$$= 10\,k\Omega$$

$$r_{o2} = \left(\frac{1}{0.1}\right)\left(\frac{1}{0.5 \times 10^{-3}}\right)$$

$$= 20\,k\Omega$$

(11)

$$R_{out} = \frac{1}{\lambda}\left(\frac{1}{I_D}\right)$$

$$= \frac{1}{0.5\times10^{-3}\,\lambda} = 20k\Omega$$

$$\therefore \lambda = 0.1\ V^{-1}$$

(12) For M_1,

$$0.5\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W_1}{0.25}\right)(1-0.4)^2$$

$$W_1 \approx 3.47\ \mu m$$

For M_2,

$$0.5\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W_2}{0.25}\right)(1.2-0.4)^2$$

$$W_2 \approx 1.95\ \mu m$$

$$\frac{r_{o1}}{r_{o2}} = \frac{\frac{1}{\lambda I_x}}{\frac{1}{\lambda I_y}}$$

$$= 1 \quad (\because I_x = I_y)$$

$$\therefore r_{o1} = r_{o2}$$

(13) Impedance at source of M_1, $Z_S = \frac{1}{g_{mp}}$

$$g_{mp} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right) I_D (1+\lambda V_{DS})}$$

$$= \sqrt{2 \times 100 \times 10^{-6} \left(\frac{10}{0.25}\right)(1+0.1 \times 1.2) I_D}$$

$$= \sqrt{0.0896 I_D}$$

$$I_D = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{10}{0.25}\right)(V_{B1} - V_x + 0.4)^2 \times (1+0.1 \times 1.2)$$

$$\approx 0.806 \text{ mA}$$

$$\therefore g_{mp} = \sqrt{0.0896 \times 0.806 \times 10^{-3}}$$

$$\approx 8.50 \text{ mS}$$

$$\therefore Z_S \approx 118 \Omega$$

(14)

$$I_X = \frac{1}{2}(100\times10^{-6})\left(\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 0.64\,mA$$

$$I_Y = \frac{1}{2}(100\times10^{-6})\left(2\times\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 1.28\,mA$$

$$\because r_o \propto \frac{1}{I}$$

and $I_Y = 2I_X$

$$\therefore r_{out,M_1} = 2\,r_{out,M_2}$$

(15) $|I_{DS1}| = |I_{DS2}|$,

$$\frac{1}{2}(200\times10^{-6})\left(\frac{10}{0.18}\right)(V_B-0.4)^2(1+0.1\times0.9)$$
$$=\frac{1}{2}(100\times10^{-6})(1.8-V_B-0.4)^2(1+0.1\times0.9)\times\left(\frac{30}{0.18}\right)$$

$$2(V_B-0.4)^2 = 3(1.4-V_B)^2$$

$$\sqrt{\frac{2}{3}}(V_B-0.4) = (1.4-V_B)$$

$$1.816\,V_B = 1.7264$$

$$V_B = 0.95$$

(16) a) For M_1,

$$I_{D1} = \frac{1}{2}(200\times10^{-6})\left(\frac{5}{0.18}\right)(V_B - 0.4)^2 (1 + 0.1\times 0.9)$$

$\therefore V_B \approx 0.806\ V$

b) There are 3 regions of operation:

For $V_X < V_B - V_{TH1}$, M_1 is in triode.

and $|I_{DS2}| > |I_{DS1}|$

For $|V_X - V_{DD}| > |V_B - V_{DD} - V_{TH2}|$, M_2 is in triode

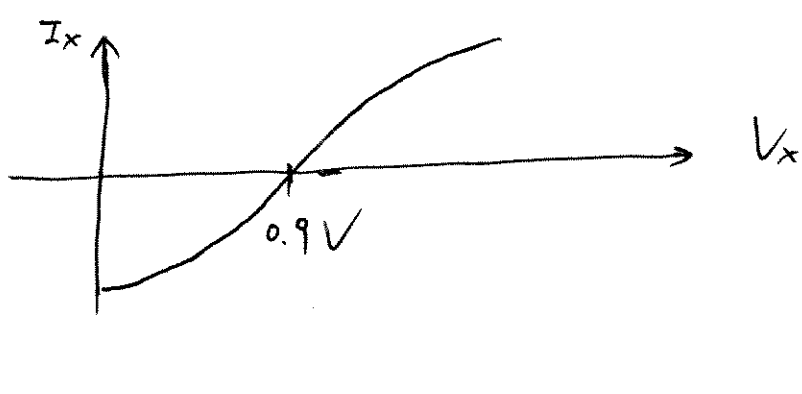
and $I_{DS1} > |I_{DS2}|$

For $V_B - V_{TH} < V_X$ and $|V_X - V_{DD}| < |V_B - V\ \ - V_{TH2}|$

M_1 and M_2 are in saturation.

and $I_{DS1} = |I_{DS2}| = 0.5\ mA$ at $V_X = 0.9\ V$

In all cases, $I_X = I_{DS1} - |I_{DS2}|$



(17)

a) $0.5\times10^{-3} = \frac{1}{2}(200\times10^{-6})(\frac{30}{0.18})(V_{GS}-V_{TH})^2$

$\therefore V_{GS} = 0.573\,V$

$V_{DS} = 1.8 - 0.5\times10^{-3}\times2000$

$= 0.8.$

$\because V_{DS} > V_{GS} - V_{TH},$

M_1 is in saturation.

b) $\because \lambda=0, \quad r_o=\infty.$

$\therefore A_v = g_m R_D$

$= \sqrt{2\times(200\times10^{-6})\times\frac{30}{0.18}\times0.5mA}\times2000$

$= 11.55$

(18)

a) $0.25 \times 10^{-3} = \frac{1}{2} \times (200 \times 10^{-6})\left(\frac{20}{0.18}\right)(V_{GS} - 0.4)^2$

$\therefore V_{GS} = 0.55$

b) $V_{DS,min} = V_{GS} - V_{TH}$

$= 0.15V.$

with $V_{DS} = 0.15V$,

$I_{DS,max} = \frac{1.8 - 0.15}{2000}$

$= 0.825mA.$

$\therefore \frac{0.825 \times 10^{-3}}{0.25 \times 10^{-3}} = \frac{\left(\frac{W}{L}\right)'}{\left(\frac{W}{L}\right)}$,

where $\left(\frac{W}{L}\right)'$ is the new $\left(\frac{W}{L}\right)$.

$\therefore \left(\frac{W}{L}\right)$ can be increased by 3.3 times.

$\because A_v \propto g_m$

$\propto \sqrt{\beta I}$

$\therefore A_v$ is also increased by 3.3 times.

(since both β & I increase by 3.3 times)

(19) Voltage gain (A_v) = 5,

i.e. $g_m R_D = 5$.

Power $(P) = I_{DS} \times V_{DD}$,

$\because P \leq 1\,mW$,

$I_{DS} \times 1.8 \leq 1\,mW$.

$I_{DS} \leq 0.556\,mA$.

$g_m = \sqrt{2 \beta I_{DS}}$

$\therefore g_{m,max} = \sqrt{2 \times 200 \times 10^{-6} \times \left(\frac{20}{0.18}\right) \times 0.556\,mA}$

$= 0.00497\ \Omega^{-1}$

$\therefore R_D = \dfrac{5}{0.00497}$

$\approx 1006\ \Omega$

$\therefore$ This is minimum value required for R_D.

(20)

$$|A_v| = g_{m1} (r_{o1} \parallel r_{o2}) = 10.$$

$$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1 \times 0.5 \times 10^{-3}}$$

$$= 20 k\Omega.$$

$$r_{o2} = \frac{1}{\lambda_2 I_1} = \frac{1}{0.15 \times 0.5 \times 10^{-3}}$$

$$= 13.3 k\Omega.$$

$$\therefore g_{m1} = \frac{10}{20k \parallel 13.3k}$$

$$= 0.00138 \Omega^{-1}$$

$$\because g_{m1} = \sqrt{2 \beta_1 I_{DS}}$$

$$\beta_1 = 0.00192$$

$$200 \times 10^{-6} \left(\frac{W}{L}\right)_1 = 0.00192$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 9.6$$

b)

$$0.5 \times 10^{-3} = \frac{1}{2} (100 \times 10^{-6}) (1.8 - V_B - 0.4)^2 \left(\frac{20}{0.18}\right)$$

$$\therefore V_B \approx 1.1 V$$

(21) $|A_v| = g_{m1} (r_{o1} // r_{o2})$

$g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{20}{0.18}\right) \times (0.001)}$

(Since V_{ds1} is not given, assume $(1+\lambda_1 V_{DS1})$ has minimal effect on g_{m1})

$= 6.67 \text{ mS} \quad (S = \Omega^{-1})$

$r_{o1} = \frac{1}{\lambda_1 \times I_{D1}}$

$= \frac{1}{0.1 \times 1mA}$

$= 10 \text{ k}\Omega.$

$r_{o2} \simeq \infty$

$(\because \lambda_2 << \lambda_1)$

$\therefore |A_v| = 6.67 \times 10^{-3} \times 10^3 \times 10$

$= 66.7$

(22) a) $A_v = g_{m1} (r_1 // r_{o2})$

when length of M_1 and M_2 double,

r_o doubles ($\because r_o \propto \quad \propto L$)

$$r_{o1} // r_{o2} = \frac{r_{o1} \, r_{o2}}{r_{o1} + r_{o2}}$$

$$\therefore (r_{o1} // r_{o2}) \propto \frac{L^2}{L},$$

$$\text{i.e. } (r_{o1} // r_{o2}) \propto L.$$

g_{m1} is constant because both $\left(\frac{W}{L}\right)_{1,2}$ and I_{DS} are constant.

$\therefore$ Voltage gain is doubled.

b) When both length and bias current double, r_o remains the same.

$\because g_{m1} \propto \sqrt{I_{DS}}$.

$\therefore$ Voltage gain increased by $\sqrt{2}$.

(23). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "g_m" than (b),

$\because g_{m1} > g_{m2}$.

(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} // r_{o2})$ is the same for both cases.

(24)

$$A_v = g_{m2} (r_{o1} // r_{o2})$$

$$r_{o1} = \frac{1}{0.15 \times 0.5mA}$$

$$= 13.3 \ k\Omega.$$

$$r_{o2} = \frac{1}{0.05 \times 0.5mA}$$

$$= 40 \ k\Omega.$$

$$\therefore r_{o1} // r_{o2} = 10 \ k\Omega.$$

$$\therefore 15 = \left[\sqrt{2 \times (100 \times 10^{-6}) \left(\frac{W}{L}\right)_2 0.5mA}\right] \cdot (10 \ k\Omega)$$

$$\left(\frac{W}{L}\right)_2 = 22.5$$

(25) From Eq (7.57)

$$3 = \sqrt{\frac{20/0.18}{(W/L)_2}}$$

$\therefore (W/L)_2 \approx 12.3$ //.

(26)

a) For M_1,

$$0.5 \times 10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{10}{0.18}\right)(V_{GS_1} - 0.4)^2$$

$$\therefore V_{GS1} = 0.7\ V$$

$$\therefore V_{DS1,\ min} = V_{GS1} - V_{TH}$$

$$= 0.3\ V$$

For M_2,

$$V_{S,\ min} = V_{DS1,\ min}$$

$$= 0.3\ V.$$

$$V_{GS,\ max} = 1.8 - 0.3$$

$$= 1.5\ V$$

$$\therefore I_{DS2} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)_2 (1.5 - 0.4)^2$$

$$= 0.5\ mA.$$

$$\left(\frac{W}{L}\right)_2 = 4.13$$

b) volt. gain

$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$\approx 3.67$$

c) Because with M_1 at the edge of saturation, V_{GS} of M_2 is at maximum (V_S of M_2 is at minimum). Thus, a minimum (W/L) is required to set up the same bias current.

With minimum (W/L), g_{m2} is at minimum.

Since $A_v \propto \frac{1}{g_{m2}}$, A_v is at maximum.

(27) a) $A_v = \sqrt{\dfrac{(W/L)_1}{(W/L)_2}}$

$\therefore 5 = \sqrt{\dfrac{(W/L)_1}{(2/0.18)}}$

$\therefore (W/L)_1 \approx 277.8$

b) For M_2,

$I_{DS2} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{2}{0.18}\right)(1.8 - V_{S2} - 0.4)^2.$

$I_{DS2} = (0.00111)(1.4 - V_{S2})^2.$

$\because V_{S2} =$ $\quad I_{DS2} = (0.00111)(1.4 - V_{DS1})^2.$

For M_1,

$I_{DS1} = \frac{1}{2}(200 \times 10^{-6})(277.8)(V_{GS1} - 0.4)^2$

$= 0.02778(V_{GS1} - 0.4)^2$

$\because I_{DS1} = I_{DS2}$

$\therefore (0.02778)(V_{GS1} - 0.4)^2 = (0.00111)(1.4 - V_{DS1})^2$

$5(V_{GS1} - 0.4) = (1.4 - V_{DS1})$

At edge of saturation,

$$V_{DS1} = V_{GS1} - 0.4,$$

Let $m = V_{DS1} = V_{GS1} - 0.4.$

$$\therefore 5m = 1.4 - m$$

$$m = 0.233$$

$$\because I_{DS1} = 0.02778(V_{GS1} - 0.4)^2$$

$$= I_{Bias}$$

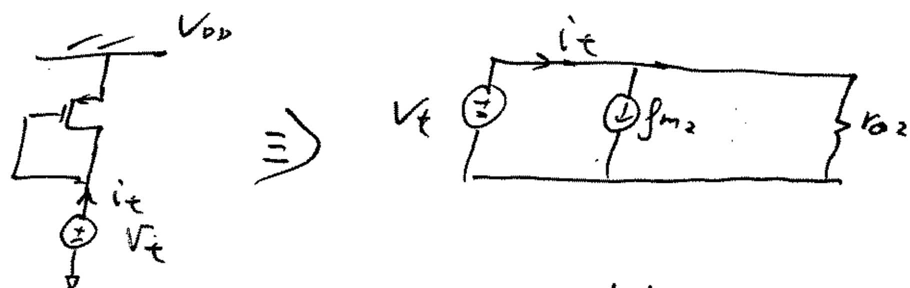
$$\therefore I_{Bias} = 0.02778(0.233)^2$$

$$= 6.48 \text{ mA}$$

(28) a) $A_v = -g_{m1} r_{o1} // Z_2$,

where Z_2 is the impedance presented by M_2.

To find Z_2, apply a test voltage (V_T) at the drain of M_2:



From the small-signal model,

$$i_t = g_{m2} V_t + \frac{V_t}{r_{o2}},$$

$$Z_2 = \frac{V_t}{i_t} = r_{o2} // \frac{1}{g_{m2}}.$$

$$\therefore A_v = -g_{m1} \left(r_{o1} // r_{o2} // \frac{1}{g_{m2}} \right)$$

b) $A_v = -g_{m1} (r_{o1} // Z_2 // Z_3)$

where Z_2 and Z_3 are impedances presented by M_2 and M_3 respectively.

From (a) $Z_3 = r_{o3} // \frac{1}{g_{m3}}$

By inspection, $Z_2 = r_{o2}$

$$\therefore A_v = -g_{m1} \left(r_{o1} // r_{o2} // r_{o3} // \frac{1}{g_{m3}} \right)$$

c) $A_v = -g_{m1}\, r_{o1} \,//\, Z_2 \,//\, Z_3$.

Similar to (b),

$Z_2 = r_{o2}$,

and $Z_3 = r_{o3} \,//\, \frac{1}{g_{m3}}$

(the small signal model of M_3 in this case is equivalent to that of M_2 in (a))

$\therefore A_v = -g_{m1} \left(r_{o1} \,//\, r_{o2} \,//\, r_{o3} \,//\, \frac{1}{g_{m3}} \right)$

d). M_2 is in CS arrangement. (similar to (c)).

$A_v = g_{m2}\, r_{o2} \,//\, Z_1 \,//\, Z_3$.

$Z_3 = \frac{1}{g_{m3}} \,//\, r_{o3}$

$Z_1 = r_{o1}$

$\therefore A_v = g_{m2} \left(r_{o2} \,//\, r_{o1} \,//\, \frac{1}{g_{m3}} \,//\, r_{o3} \right)$

e). $A_v = g_{m2} \left(r_{o2} \,//\, Z_1 \,//\, Z_3 \right)$

$Z_1 = r_{o1}$

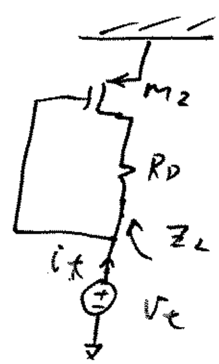
$Z_3 = \frac{1}{g_{m3}} \,//\, r_{o3}$

(recall: impedance looking into source $= \frac{1}{g_m}$)

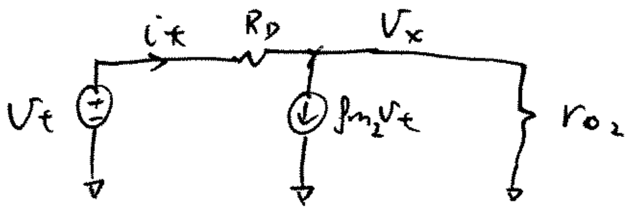
$\therefore A_v = g_{m2} \left(r_{o2} \,//\, r_{o1} \,//\, \frac{1}{g_{m3}} \,//\, r_{o3} \right)$

(28) f) $A_v = -g_{m1}(r_{o1} \parallel Z_L)$,

where Z_L is the impedance depicted as follows:



The equivalent small-signal model is:



$$i_t = g_{m2} v_t + \frac{V_x}{r_{o2}},$$

$$V_x = V_t - R_D i_t$$

$$\therefore i_t = g_{m2} v_t + \frac{V_t}{r_{o2}} - \frac{R_D i_t}{r_{o2}}$$

$$i_T\left(1 + \frac{R_D}{r_{o2}}\right) = v_t\left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$\frac{V_t}{i_t} = \frac{r_{o2} + R_D}{g_{m2} r_{o2} + 1}$$

$$\therefore A_v = -g_{m1}\left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2} r_{o2}}\right)$$

(30) a) From Eq. (7.67)

$$|A_v| = \frac{R_D}{\frac{1}{g_m} + R_S},$$

$$4 = \frac{1000}{\frac{1}{g_m} + \frac{0.2V}{1mA}}$$

$$\frac{4}{g_m} + 800 = 1000$$

$$\therefore g_m = 20 \text{ mS.}$$

$$\therefore 20 \times 10^{-3} = \sqrt{2 \times (200 \times 10^{-6})(\tfrac{W}{L})(1 \times 10^{-3})}$$

$$\therefore W/L = 1000$$

To check if M_1 is in saturation:

$$V_{DS} = V_D - V_S$$
$$= [1.8 - (10^{-3} \times 1k)] - 0.2$$
$$= 0.6 \text{ V}$$

and $10^{-3} = \frac{1}{2}(200 \times 10^{-6})(1000)(V_{GS} - 0.4)^2$

$$V_{GS} = 0.5$$

$$\therefore V_{DS} > V_{GS} - V_t,$$

ie. transistor is in operation.

b) $g_m = \sqrt{2 \times (200 \times 10^{-6}) \times \left(\frac{50}{0.18}\right) \times 10^{-3}}$

$\approx 10.5 \text{ mS}$

$$|A_v| = \frac{R_D}{\frac{1}{g_m} + R_S},$$

$$4 = \frac{R_D}{\frac{1}{10.5 \times 10^{-3}} + 200}$$

$\therefore R_D \approx 1179\,\Omega.$

To check if M_1 is in saturation:

$$V_{DS} = [1.8 - (1179 \times 10^{-3})] - 0.2$$

$$= 0.421$$

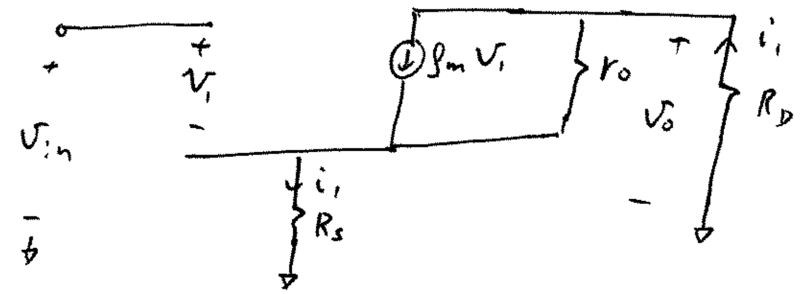
and $10^{-3} = \frac{1}{2} (V_{GS} - 0.4)^2 (200 \times 10^{-6}) \left(\frac{50}{0.18}\right)$

$V_{GS} \approx 0.590$

$\because V_{DS} > V_{GS} - V_t,$

Transistor is in saturation.

(31) The small signal model is:



$$v_o = -i_1 R_D \qquad \text{--- (1)}$$

$$i_1 = g_m v_1 + \frac{v_o - v_1}{r_o}$$

$$= \frac{(g_m r_o - 1) v_1 + v_o}{r_o}$$

$$i_1 \approx g_m v_1 + \frac{v_o}{r_o}$$

$$\therefore -\frac{v_o}{R_D} = g_m v_1 + \frac{v_o}{r_o} \qquad \text{--- (2)}$$

$$v_{in} = v_1 + i_1 R_S$$

$$\therefore v_1 = v_{in} + \frac{v_o}{R_D} R_S \qquad \text{--- (3)}$$

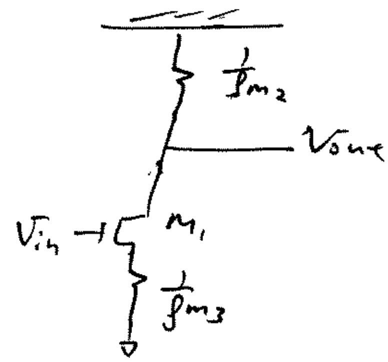
(2) combines with (3):

$$-\frac{v_o}{R_D} = g_m v_{in} + g_m v_o \frac{R_S}{R_D} + \frac{v_o}{r_o}$$

$$-v_o \left[\frac{1}{R_D} + g_m \frac{R_S}{R_D} + \frac{1}{r_o} \right] = g_m v_{in}$$

$$\therefore \text{Volt. gain} = \frac{v_o}{v_{in}} = -\left[\frac{g_m}{r_o + g_m R_S r_o + R_D}\right] (r_o R_D)$$

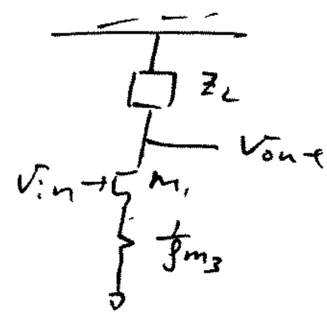
(32) a) Equivalent circuit is:



$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

b) Similar to Prob. 28 (f),

Equivalent circuit is:

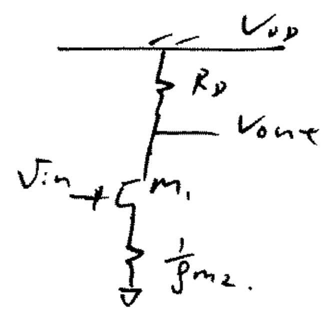


From Prob. 28 (f),

$$Z_L = \frac{1}{g_{m2}} \quad (\text{as } r_{o2} \to \infty)$$

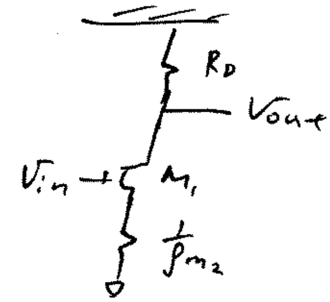
$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

c) Equivalent circuit is:



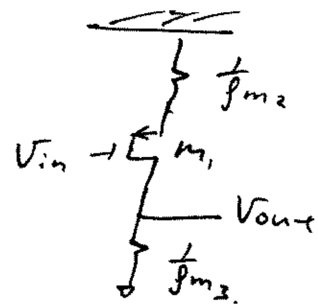
$$\therefore A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(d). Equivalent circuit is



$$A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(e) Equivalent circuit is



$$A_v = \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(33)

a) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

b) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

c) From Eq. (7.71),

$$R_{out} = (1 + g_{m2} r_{o2}) \left(r_{o1} \parallel \frac{1}{g_{m3}}\right) + r_{o2}$$

d) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \left(r_{o2} \parallel \frac{1}{g_{m3}}\right) + r_{o1}$$

(34) To find $\left(\frac{W}{L}\right)$,

$$10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2 \times (1+0.1\,V_{DS}),$$

where $V_{DS} = 1.8 - 1k\Omega \times 1mA$

$= 0.8V$,

$\therefore \left(\frac{W}{L}\right) \simeq 25.7$ //

Voltage gain $(A_v) = -g_{m1}(r_{o1} \,//\, R_D)$

$$g_{m1} = \sqrt{2(200\times10^{-6})(25.7)\times10^{-3}\times(1+0.1\times0.8)}$$

$= 3.33\ mS.$

$$r_{o1} = \frac{1}{0.1\times10^{-3}}$$

$= 10k\Omega.$

$\therefore A_v = (-3.33\times10^{-3})(10k\Omega \,//\, 1k\Omega)$

$= -3.03$ //

(35) with $\lambda = 0$,

$$10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1 - 0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) \doteq 27.8$$

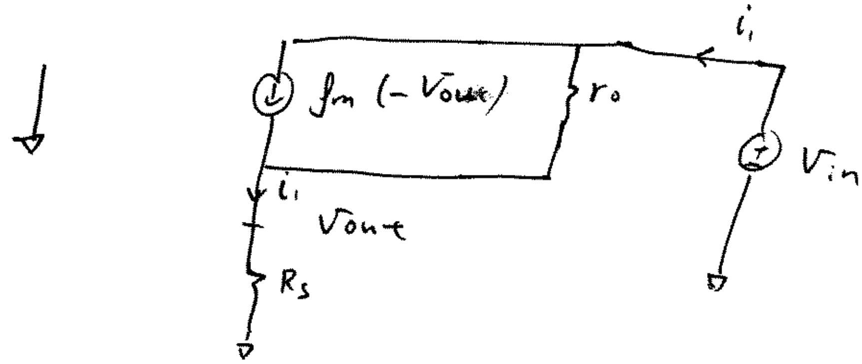
$$A_v = -g_{m1} R_D$$

$$= -\sqrt{2(200 \times 10^{-6})(27.8) \times 10^{-3}} \times 1000$$

$$= -3.33$$

Without r_o, gain increases due mainly to increase in load resistance.

(36) The small-signal circuit is:



$$i_1 = \frac{V_{out}}{R_s} \quad \text{——— (1)}$$

$$i_1 = g_m(-V_{out}) + \frac{V_{in} - V_{out}}{r_o} \quad \text{——— (2)}$$

$$\therefore \frac{V_{out}}{R_s} = -g_m V_{out} + \frac{V_{in}}{r_o} - \frac{V_{out}}{r_o}$$

$$V_{out}\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) = \frac{V_{in}}{r_o}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{r_o}\left(\frac{R_s r_o}{r_o + g_m r_o R_s + R_s}\right)$$

$$= \frac{R_s}{g_m r_o R_s + r_o + R_s}$$

Since $(g_m r_o R_s + r_o) > 0$, the voltage gain < 1.

This is expected: Any variation in V_{in} causes minimal change in the bias current.

$\because V_{out}$ is determined largely by the amount of bias current ($\because V_{out}$ is set by V_{GS1})

$\therefore$ There is almost no variation in V_{out}. (i.e. $\frac{V_{out}}{V_{in}} \ll 1$)

(37) a) $|\text{Voltage gain}| = g_m R_D$

$$= 5$$

$$\therefore g_m = \frac{5}{500}$$

$$= 10\,mS.$$

$$= \sqrt{2(200\times10^{-6})\left(\frac{W}{L}\right)\times 10^{-3}}$$

$$\therefore \frac{W}{L} = 250$$

b) $V_D = 1.8 - 500\times 10^{-3}$

$$= 1.3V$$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$$V_D \geq V_G - 0.2$$

$$\therefore V_G \leq 1.5$$

Also, $I_{R_1+R_2} = 0.1\times10^{-3}A.$

$$\therefore R_1 + R_2 = \frac{1.8}{0.1\times10^{-3}}$$

$$= 18\,k\Omega.$$

choose $R_2 = 15\,k\Omega$ & $R_1 = 3\,k\Omega$

c) With twice of (W/L), M_1 will go further away from triode. As (W/L) doubles, & I_{bias} is fixed by the current source, V_{GS} is forced to decrease (so M_1 will have same I_{DS}). Thus, $(V_{GS} - V_{TH})$ decreases, and V_{DS} can be allowed to drop more before M_1 goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(38) a) $V_G = 1.8V.$

$\therefore V_{D,min} = 1.8 - 0.4$ (for M_1 stays in saturation)

$= 1.4V$

$\therefore R_{D,max} = \frac{1.4V}{1mA}$

$= 1.4k\Omega$ //

b) $|\text{Voltage gain}| = g_m R_D$

$= 5.$

$\therefore g_m = \frac{5}{R_D}$

$= 3.57mS.$

$= \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) \times 10^{-3}}$

$\therefore (\frac{W}{L}) = 31.9$ //

(38) To get $R_{in} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

volt gain $(A_v) = g_m R_D$

$$= 4,$$

$$\therefore R_D = \frac{4}{0.02}$$

$$R_D = 200\,\Omega$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) \times 0.5 \times 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 2000$$

(40) To get $R_{in} = 50\Omega$,

$$g_m = \frac{1}{50}$$

$$= 20\,mS.$$

Voltage gain $(A_v) = g_m R_D$

$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) \times 0.5 \times 10^{-3}}$$

$$\therefore \frac{W}{L} = 2000.$$

$\because$ $V_G = V_B = 1V$,

$$V_{D,min} = V_G - V_{TH}$$

$$= 0.6V$$

$$\therefore R_{D,max} = \frac{1.8 - 0.6}{0.5 \times 10^{-3}}$$

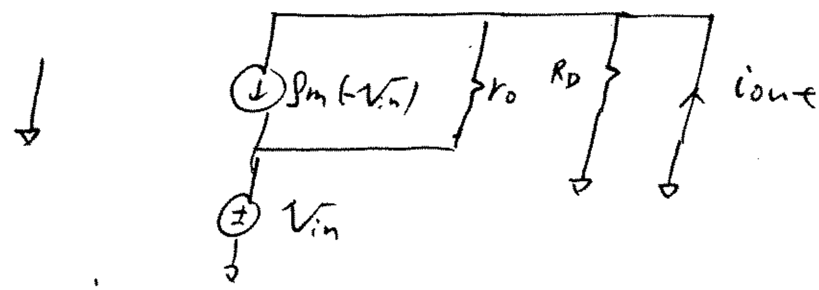
$$= 2400$$

$\therefore$ max. Voltage gain $= 0.02 \times 2400$

$$= 48$$

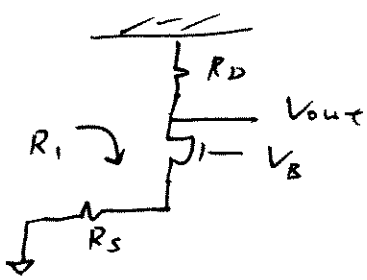
(41) Voltage gain $(A_v) = G_m R_{out}$,
where G_m and R_{out} are the transconductance and output resistance of the circuit respectively.

To find G_m:



$$G_m = \frac{i_{out}}{V_{in}} = g_m + \frac{1}{r_o}$$
$$\simeq g_m \quad (\because g_m r_o \gg 1)$$

To find R_{out}:



$$R_{out} = R_D \,//\, R_1$$
$$= R_D \,//\, [(1 + g_m r_o) R_S + r_o]$$
(from Eq. (7.110))
$$\simeq R_D \,//\, (g_m r_o R_S + r_o) \quad (\because g_m r_o \gg 1)$$
$$= \frac{g_m r_o R_S R_D + r_o R_D}{R_D + g_m r_o R_S + r_o}$$

$$\therefore \text{Voltage gain} = g_m \left[\frac{g_m r_o R_D R_S + r_o R_D}{R_D + g_m r_o R_S + r_o} \right]$$

(42) a) To get $R_{in} = 50\,\Omega$,

$$g_m = \frac{1}{50}$$
$$= 20\,mS.$$

To get $R_{out} = 500\,\Omega$,

$$R_D = 500\,\Omega. \quad (\because r_o = \infty)$$

$\because \quad V_{D,min} = 1.8 - 0.4 = 1.4\,V$

$$\therefore \quad I_{D,max} = \frac{1.8 - 1.4}{500}$$
$$= 0.8\,mA.$$

b) $$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) \times 0.8 \times 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 1250$$

c) Voltage gain $= 0.02 \times 500$

$$= 10$$

(43) a/ To place M_1 100mV away from triode,

$$V_{D,min} = V_G - V_{TH} + 0.1V$$
$$= 1.5V.$$

$$\therefore R_D = \frac{(1.8 - 1.5)V}{1mA}$$

$$= 300\,\Omega$$

b/ Voltage gain $= g_m R_D$

$$\therefore g_m = \frac{5}{300}$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) \approx 694$$

(44) a) Voltage gain $(A_v) = \left[\dfrac{\frac{1}{g_{m1}}}{R_S + \frac{1}{g_{m1}}}\right] \dfrac{g_{m1}}{g_{m2}}$

$$= \frac{g_{m1}/g_{m2}}{1 + g_{m1}R_S}$$

b) Voltage gain $(A_v) = g_{m1} Z_2$

(similar to prob. 32(b))

$$= \frac{g_{m1}}{g_{m2}}$$

c) Voltage gain $= \left[\dfrac{\frac{1}{g_{m1}} // R_1}{R_S + \frac{1}{g_{m1}} // R_1}\right] \dfrac{g_{m1}}{g_{m2}}$

d) Voltage gain $= g_{m1}\left[R_D + r_{o3} // \frac{1}{g_{m2}}\right]$,

$\because r_{o3} = \infty$,

gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$

e) voltage gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$

(45) a) $\frac{V_x}{V_{in}} = -g_{m1}\left[R_{D1} \,//\, \frac{1}{g_{m2}}\right]$

$$\frac{V_{out}}{V_x} = g_{m2} R_{D2}$$

$$\therefore \frac{V_{out}}{V_{in}} = -(g_{m2} R_{D2})\left[g_{m1}\left(R_{D1} \,//\, \frac{1}{g_{m2}}\right)\right]$$

b) if $R_{D1} \to \infty$,

$$\frac{V_{out}}{V_{in}} = (-g_{m2} R_{D2})\left(\frac{g_{m1}}{g_{m2}}\right)$$

$$= -g_{m1} R_{D2}$$

This is expected, because the circuit reduces to a cascode stage.

(∴ gain is the same as that of a cascode stage.)

(46) $$\frac{v_x}{v_{in}} = \left(R_{D1} \parallel \frac{1}{g_{m2}}\right) g_{m1}$$

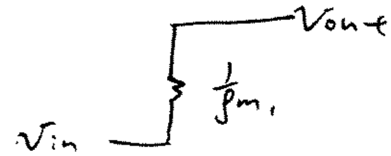
$$\frac{v_{out}}{v_x} = g_{m2} R_{D2}.$$

$$\therefore \frac{v_{out}}{v_{in}} = g_{m1} g_{m2} R_{D2} \left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$

Similar to prob. (45), voltage gain approaches that of cascode stage as R_{D1} approaches infinity. The gain is $g_{m1} R_{D2}$.

(47) with $\lambda = 0$, M_1 appears as a diode-connected device.

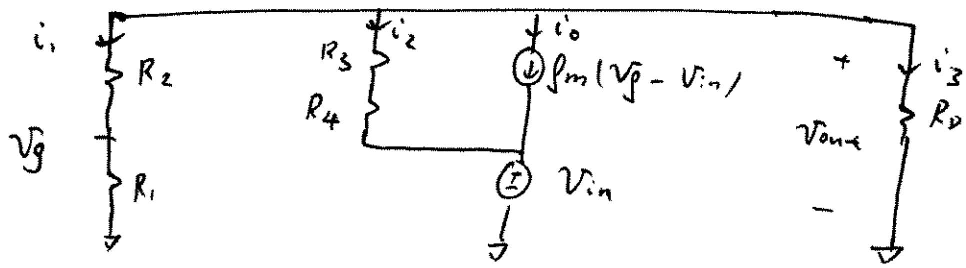
$\therefore$ the circuit becomes :



i.e. $\frac{V_{out}}{V_{in}} = 1$

This is not a common-gate (CG) amplifier, because the gate is not fixed. (i.e. gate is not at an "a.c. ground")

(48) The small-signal model is :



$$\because \; -i_0 = i_1 + i_2 + i_3$$

$$-g_m(V_g - V_{in}) = \frac{V_{out}}{R_2 + R_1} + \frac{V_{out} - V_{in}}{R_3 + R_4} + \frac{V_{out}}{R_D}$$

$$g_m\left(V_{in} - \frac{R_1}{R_1 + R_2} V_{out}\right) = V_{out}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right) - \frac{V_{in}}{R_3+R_4}$$

$$V_{in}\left(g_m + \frac{1}{R_3+R_4}\right) = V_{out}\left(\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(g_m + \frac{1}{R_3+R_4}\right)}{\frac{1}{R_D} + \frac{1}{R_3 + R_4} + \frac{g_m R_1 + 1}{R_1 + R_2}}$$

(49) Voltage gain(A_v) $= \dfrac{r_o // R_s}{\frac{1}{g_m} + r_o // R_s}$

To find I_{DS},

$$I_{DS} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)(1.8 - V_s - 0.4)^2$$

$$= 0.0111 \; (1.4 - I_{DS} \times 1000)^2.$$

$$11100 I_{DS}^2 - 32.08 I_{DS} + 0.021756 = 0$$

$$\therefore I_{DS} = 1.80\,mA \text{ or } 1.08\,mA.$$

Reject $I_{DS} = 1.80\,mA.$

($\because V_s = 1.80V > V_{DD}$)

$$\therefore g_m = \sqrt{2 \times (200 \times 10^{-6}) \times 1.08 \times 10^{-3}}$$

(ignore channel-length modulation)

$$g_m = 0.659\,mS$$

$$r_o = \frac{1}{0.1 \times 1.08 \times 10^{-3}} \approx 9260\,\Omega$$

$$\therefore A_v = \frac{9260\Omega // 1000\Omega}{\frac{1}{0.659mS} + 9260\Omega // 1000\Omega}$$

$$\approx 0.372$$

(50)

$$A_v = \frac{R}{\frac{1}{g_m} + R}$$

$$= 0.8$$

$$\therefore 0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\frac{0.8}{g_m} + 400 = 500$$

$$\therefore g_m = 8\,mS.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) \times \left(\frac{30}{0.18}\right) I_{DS}}$$

$$\therefore I_{DS} = 0.96\,mA$$

$$\therefore V_S = 0.96 \times 10^{-3} \times 500$$

$$= 480\,mV$$

To find V_G:

$$0.96 \times 10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{30}{0.18}\right)(V_G - 0.48 - 0.4)^2$$

$$\therefore V_G = 1.12\,V$$

(51)

$$A_v = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$$= 0.8$$

$$0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\therefore g_m = 8\,mS.$$

$$\because I_{ds} = \frac{1}{2} \beta (V_{GS} - V_t)^2,$$

where $\beta = \left(\frac{W}{L}\right) \mu_n C_{ox}$

and $g_m = \beta (V_{GS} - V_t)$.

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_t)$$

$$= \frac{1}{2} g_m (1.8 - I_{ds}(500) - 0.4)$$

$$I_{ds} = 4 \times 10^{-3} (1.4 - 500 I_{ds})$$

$$\therefore I_{ds} = 1.87\,mA.$$

$$\because g_m = \sqrt{2 (200 \times 10^{-6}) \frac{W}{L} \times 1.87 \times 10^{-3}}$$

$$\therefore \frac{W}{L} \approx 85.7$$

(52). To get $R_{out} = 100\,\Omega$,

$$\frac{1}{g_m} = 100$$

$$\therefore g_m = 10\,mS.$$

$$\because \quad I_{ds} = \frac{1}{2}\beta(V_{GS} - V_{TH})^2.$$

where $\beta = \mu_n C_{ox} \frac{W}{L}$.

and $g_m = \beta\,(V_{GS} - V_{TH})$

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$= \frac{1}{2}(10\times10^{-3})(0.9-0.4)$$

$$\therefore I_{ds} = 2.5\,mA.$$

$$\because \quad g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)(2.5\times10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 100$$

(53). To get $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 2 \times 10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6})\left(\frac{W}{L}\right)(1.11mA)}$$

$$\therefore \frac{W}{L} = 900$$

(54)

$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$\therefore 0.8 = \frac{50}{\frac{1}{g_m} + 50}$$

$$g_m = 80 \text{ mS}$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 3 \text{ mW}$$

$$\therefore I_{DS} = 1.67 \text{ mA}$$

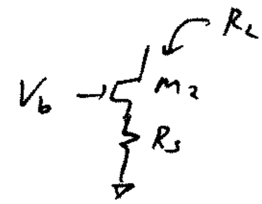
$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (1.67 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 9600$$

(55) a) $$A_v = \frac{r_{o1} \,//\, (R_s + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \,//\, (R_s + r_{o2})}$$

b) $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



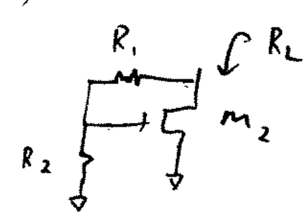
$$R_L = (1 + g_{m2} r_{o2}) R_s + r_{o2} \quad \text{Eq. (7.110)}$$

$$\therefore A_v = \frac{r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}{\frac{1}{g_{m1}} + r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}$$

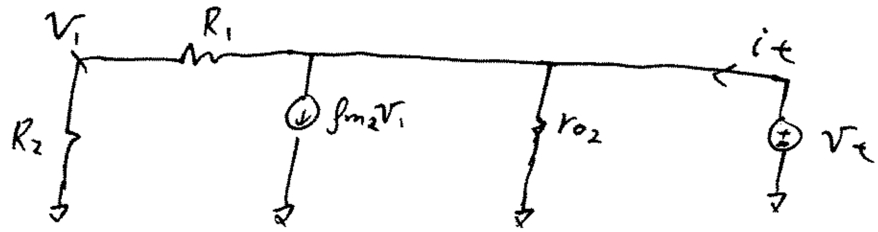
c) $$A_v = \frac{r_{o1} \,//\, \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + (r_{o1} \,//\, \frac{1}{g_{m2}})}$$

d) $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



(c) Finding R_L with small-signal model:
(cont'd)



$$R_L = \frac{v_t}{i_t},$$

where
$$i_t = \frac{v_t}{r_{o2}} + g_{m2} v_1 + \frac{v_t}{R_1 + R_2}$$

$$= \frac{v_t}{r_{o2}} + \frac{g_{m2} R_2 v_t}{R_1 + R_2} + \frac{v_t}{R_1 + R_2}$$

$$\therefore R_L = \frac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}$$

$$\therefore A_v = \frac{r_{o1} \,//\, \dfrac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}{\dfrac{1}{g_{m1}} + r_{o1} \,//\, \dfrac{r_{o2}(R_2 + R_1)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}$$

e) $$A_v = \frac{r_{o2} \,//\, \left(\frac{1}{g_{m1}} \,//\, r_{o3}\right)}{\frac{1}{g_{m2}} + r_{o2}\left(\frac{1}{g_{m1}} \,//\, r_{o3}\right)}$$

f) $$A_v = \frac{r_{o1} \,//\, \left[(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}\right]}{\frac{1}{g_{m1}} + \left\{r_{o1} \,//\, \left[(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}\right]\right\}}$$

(5b)

$$\frac{v_x}{v_{in}} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}.$$

$$\frac{v_{out}}{v_x} = g_{m2} R_D$$

$$\therefore \frac{v_{out}}{v_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

b) if $g_{m1} = g_{m2}$,

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1} R_D}{2}.$$

(57)

$\because R_{out} = 1k\Omega.$

$\therefore R_D = 1k\Omega.$

$\because A_v = 5$

$= g_{m1} R_D$

$\therefore g_{m1}(1000) = 5$

$g_{m1} = 5mS.$

$\because M_1$ is 00 mV away from triode,

$V_D = (V_G - V_{TH}) + 0.1.$

$V_D = (1.8 - 0.4) + 0.1$

$V_D = 1.5V$

$$\therefore I_{DS} = \frac{1.8 - 1.5}{R_D} = \frac{0.3}{R_D}$$

$= 0.3mA$

$$\because g_{m1} = \sqrt{2 \times (200 \times 10^{-6})(\frac{W}{L}) I_{DS1}}$$

$\therefore (\frac{W}{L}) \approx 208$

$\therefore R_D = 1k\Omega$, $R_G = 10k\Omega$, $(\frac{W}{L}) = 208$

(58) $\because$ Power (P) $= 2\,mW$,

$\therefore I_{DS} = \frac{2\times10^{-3}}{1.8}$

$= 1.11\ mA.$

$\because R_D I = 1$

$\therefore R_D = 900\,\Omega.$

$\because |\text{gain } (A_v)| = 5,$

$g_m R_D = 5$

$g_m = 5.56\ mS.$

$\because g_m = \sqrt{2(200\times10^{-6})(\frac{W}{L})(1.11\times10^{-3})}$

$\frac{W}{L} \approx 69.4$

(58) $|A_v| = g_m R_L$.

$\therefore$ To achieve maximum gain, use maximum R_L

i.e. set $R_D = 500\,\Omega$.

For maximum g_m, use maximum I_{DS}

(while keeping M_1 in saturation),

i.e. $V_D \geq V_G - V_{TH}$

$1.8 - (I_{DS})(500) \geq 1.8 - 0.4$,

$\therefore \quad I_{DS} \leq \frac{0.4}{500}$

$I_{DS,max} = 0.8\,mA$.

<u>Note</u>: Setting a large R_D in this case would force $I_{DS,max}$ to be lower (in order to keep M_1 in saturation).

But since $A_v \propto R_D$, while $A_v \propto \sqrt{I_{DS}}$, sacrificing I_{DS} to get higher R_D would yield a higher gain.

(60) $\because$ Power (P) = 2 mW,

$$\therefore I_{DS} = (0.95)\left(\frac{2\times10^{-3}}{1.8}\right)$$

(assuming $(R_1 + R_2)$ consumes 5% of total power)

$$I_{DS} = 1.06\,mA$$

$$\therefore R_S = \frac{0.2\,V}{1.06\,mA}$$

$$\approx 189\,\Omega$$

$$\because g_m = \beta V_{eff}$$

(where $\beta = \mu_n C_{ox}\left(\frac{W}{L}\right)$; $V_{eff} = V_{GS} - V_{TH}$)

and $I_{DS} = \frac{1}{2}\beta V_{eff}^2$

$$\therefore I_{DS} = \frac{1}{2} g_m V_{eff}.$$

Set $V_{eff} = 0.1\,V$ (< maximum allowable overdrive)

$$1.06\times10^{-3} = \frac{1}{2} g_m (0.1)$$

$$g_m = 21.2\ mS.$$

$$\because |A_V| = \frac{g_m R_D}{1 + g_m R_S} = 4.$$

$$\therefore \frac{21.2\times10^{-3}\times R_D}{1+(21.2\times10^{-3})\times189} = 4$$

$$R_D \approx 947\ \Omega.$$

With $V_{GS} - V_{TH} = 0.1\,V$,

$$V_{GS} = 0.1 + 0.4\,V$$
$$= 0.5\,V$$
$$= V_G - V_S$$

$\therefore V_G - 0.2\,V = 0.5\,V$

$\therefore V_G = 0.7\,V$

To find R_1 & R_2,

$\because I_{R_1+R_2} = (0.05)\left(\frac{2\times10^{-3}}{1.8}\right)$

$= 5.56\times10^{-5}\,A$

$\therefore R_1 + R_2 = \frac{1.8\,V}{5.56\times10^{-5}\,A}$

$= 32.4\,k\Omega$

$V_G = \frac{R_2}{R_1+R_2} \times 1.8 = 0.7\,V$

$\therefore R_2 = 12.6\,k\Omega$,

$R_1 = (32.4 - 12.6)\,k\Omega = 19.8\,k\Omega$.

To find $\left(\frac{W}{L}\right)$,

$g_m = \sqrt{2 \times 200\times10^{-6} \times \left(\frac{W}{L}\right) \times 1.06\times10^{-3}} = 21.2\,mS$

$\therefore \left(\frac{W}{L}\right) = 1060$

$\therefore R_1 = 19.8\,k$, $R_2 = 12.6\,k$, $R_S = 189\,\Omega$, $R_D = 947\,\Omega$

$\left(\frac{W}{L}\right) = 1060$. $I_{DS} = 1.06\,mA$

(61) Power $(P) = 6\,mW$

$$\therefore I_{DS} = (0.95)\left(\frac{6\times10^{-3}}{1.8}\right) = 3.17\,mA$$

gain $(A_v) = 5,$

$$\therefore \frac{g_m R_D}{1+g_m R_S} = 5$$

$$5 = (R_D - 5R_S)\,g_m$$

for g_m to be positive,

$R_D > 5R_S$, i.e. $R_s < 50\,\Omega$

choose $R_s = 30\,\Omega$

$\because V_{ov}$ (over drive voltage) $= V_{RS}$

$$\therefore V_{ov} = 3.17\times10^{-3}\times30$$
$$= 95.1\,mV.$$

From $A_v = \frac{g_m R_D}{1+g_m R_s} = 5,$

$$g_m = 100\,mS$$

$$\because g_m = (\mu_n C_{ox})\left(\frac{W}{L}\right)V_{ov}$$

$$\therefore \left(\frac{W}{L}\right) \approx 5260$$

To find R_1 and R_2,

$$I_{R_1+R_2} = (0.05)\left(\frac{6\times10^{-3}}{1.8}\right) = 0.167\,mA$$

$$\therefore R_1+R_2 = \frac{1.8}{0.167\times10^{-3}} = 10.8\,k\Omega$$

$\because \quad V_{GS} - V_{TH} = V_{ov} = 85.1\,mV,$

and $\quad V_S = 85.1\,mV,$

$\therefore (V_G - 85.1\,mV) - 0.4 = 85.1\,mV$

$$V_G = 0.5802$$

$$\because \quad V_G = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$\therefore \quad R_2 = 3.54\,k\Omega$$

$$\text{and } R_1 = 10.8\,k\Omega - 3.54\,k\Omega$$

$$= 7.26\,k\Omega$$

$\therefore R_1 = 7.26\,k\Omega, \quad R_2 = 3.54\,k\Omega, \quad R_S = 30\,\Omega$

$\left(\frac{W}{L}\right) = 5260 \qquad I_{DS} = 3.17\,mA.$

(62) Power $(P) = 2mW$.

$\therefore I_{DS} = \frac{2mW}{1.8V}$

$= 1.11\ mA$.

$\because M_1$ operates 200 mV away from triode

$V_{DS} = (V_{GS} - V_{TH}) + 0.2$

$\therefore V_D = 1.6V$

$R_D = \frac{V_{RD}}{1.11\times10^{-3}A} = \frac{(1.8-1.6)V}{1.11\times10^{-3}A}$

$= 180\ \Omega$

$\because$ gain $(A_v) = \frac{g_m R_D}{1+g_m R_S} = 6$,

$\therefore 6 = (R_D - 6R_S)\ g_m$,

for $g_m > 0$, $R_D - R_S > 0$, i.e. $R_S < 30\Omega$.

Set $R_S = 20\Omega$,

$g_m = \frac{6}{180-6\times20} = 100mS$.

$\because g_m = (\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})$

$0.1 = 200\times10^{-6}\left(\frac{W}{L}\right)(1.8 - 1.11\times10^{-3}\times20 - 0.4)$

$\therefore \frac{W}{L} \approx 363$

$R_{in} = \frac{1}{sC_1} + R_1$

$\because \frac{1}{sC_1}$ is negligible,

$R_{in} = R_1 = 20\,k\Omega$.

To make $\frac{1}{sC_1}$ negligible,

$\frac{1}{sC_1} \ll R_1$

$\frac{1}{2\pi(10^6)C_1} \ll$

$\therefore C_1 \ll 7.96\,pF$

Set $C_1 = 0.796\,pF$

To make $\frac{1}{sC_s}$ negligible,

$\frac{1}{sC_s} \ll R_s // \frac{1}{g_m}$,

$\frac{1}{2\pi(10^6)C_s} \ll 20 // \frac{1}{100\,ms}$

$C_s \ll 23.9\,nF$

Set $C_s = 2.39\,nF$

$\therefore R_D = 180\,\Omega$, $R_s = 20\,\Omega$, $R_1 = 20\,k\Omega$, $\frac{W}{L} = 363$

$C_1 = 0.796\,pF$, $C_s = 2.39\,nF$

(63). Power $(P) = 2mW$,

$\therefore I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$r_{o1} = r_{o2} = \frac{1}{\lambda I_{DS}}$

$= \frac{1}{0.1 \times 1.11 \times 10^{-3}}$

$= 9000 \Omega$.

Gain $(A_v) = g_{m1} (r_{o1} // r_{o2}) = 20$,

$g_{m1} \left(\frac{9000}{2} \right) = 20$.

$\therefore g_{m1} = 4.44$ mS.

Set V_{DS1} (i.e. V_{out}) $= 1.2V$

(which is $< 1.5V$)

$\therefore V_{IN} = V_{GS1} \leq 1.2 + V_{TH}$

(for M_1 to stay in saturation)

Set $V_{GS1} = 1.2V$

$\because g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})$

$\left(\frac{W}{L}\right)_1 = 27.75$

For M_2, $\because M_2$ must be in saturation

for $V_{out} \leq 1.5V$.

$\therefore V_{DD} - V_B \leq V_{DD} - 1.5V + V_{TH}$.

$\therefore V_B \geq 1.1V$

Set $V_B = 1.2V$

$$|I_{DS2}| = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (|V_{GS2}| - V_{TH})^2 (1 + \lambda |V_{DS2}|)$$

$$1.11 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_2 (0.6 - 0.2)^2 (1 + 0.1 \times (1.8 - 1.5))$$

(assuming $V_{out} = 1.5V$)

$$\therefore \left(\frac{W}{L}\right)_2 \approx 135$$

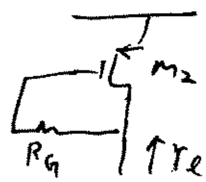
$\therefore \left(\frac{W}{L}\right)_1 = 27.75 \quad \left(\frac{W}{L}\right)_2 = 135$

$V_{IN} = 1.2 \quad V_b = 1.1$

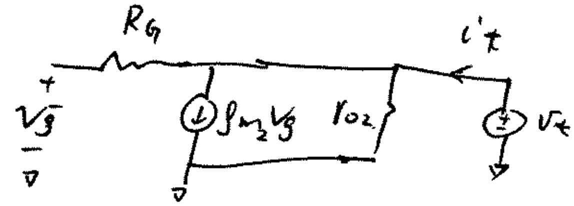
$I_{DS1} = I_{DS2} = 1.11mA$

(64) a) gain $(A_v) = -g_{m1}\, r_{o1} \,//\, r_\ell$,

where R_L is:



The small-signal model is:



$$r_\ell = \frac{v_t}{i_t} = r_{o2} \,//\, \frac{1}{g_{m2}}$$

$$\therefore\ A_v = -g_{m1}\ r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}$$

b) Power $= 3mW$

$$\therefore\ I_{DS1} = |I_{DS2}| = \frac{3mW}{1.8V}$$

$$= 1.67mA.$$

$$V_{OUT} = V_{G2} = \frac{V_{DD}}{2}$$

$$\therefore\ V_{GS2} = -0.9V$$

$$I_{DS2} = \frac{1}{2}(100\times 10^{-6})\left(\frac{W}{L}\right)_2 \times (|-0.9| - V_{TH})^2 \times \left(1 + 0.1 \times \frac{V_{DD}}{2}\right)$$

$$\therefore\ \left(\frac{W}{L}\right)_2 \approx 122.$$

$$g_{m2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left(V_{GS2} - V_{TH}\right)$$

$$= 6.1\,mS.$$

From (a), $|A_v| = \frac{1}{g_{m1}} \times \left(r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right)$

$$\because r_{o1} = \frac{1}{0.1 \times 1.67 \times 10^{-3}} = 6000\,\Omega$$

$$r_{o2} = \frac{1}{0.2 \times 1.67 \times 10^{-3}} = 3000\,\Omega$$

$$\therefore 15 = g_{m1} \left(6000 \,//\, 3000 \,//\, \frac{1}{6.1mS}\right)$$

$$g_{m1} = 99\ mS$$

$$g_{m1} = \sqrt{2\left(\frac{W}{L}\right)_1 \mu_n C_{ox} I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 = 14672$$

$\therefore \left(\frac{W}{L}\right)_1 = 14672.\ \left(\frac{W}{L}\right)_2 = 122,\ I_{DS1} = |I_{DS2}| = 1.67mA.$

(65) a) Impedance looking into drain of M_2

$$= (1 + g_{m2} r_{o2}) R_S + r_{o2}$$

$$= 10\, r_{o1}$$

Assume $g_{m2} r_{o2} \gg 1$,

$\therefore g_{m2} r_{o2} R_S + r_{o2} \approx 10 r_{o1}$

$\because r_{o1} = r_{o2}$ ($\lambda_1 = \lambda_2$ and $I_{DS1} = |I_{DS2}|$)

$\therefore g_{m2} R_S + 1 = 10$

$g_{m2} R_S = 9$ ——— ①

Given $V_B = 1V$,

set $|V_{GS2}| = 0.6V$, (ie. $V_{GS2} - V_{TH} = 0.2V$)

$\therefore V_{S2} = 1.6V$,

$\therefore V_{R_S} = 1.8V - 1.6V = 0.2V$

$\because$ Power $= 2mW$

$I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$\therefore R_S = \frac{V_{R_S}}{1.11 \times 10^{-3}} \approx 180\Omega$ //

From ①, $g_{m2} = \frac{9}{180} = 50\,mS$.

$\because g_{m2} = (\frac{W}{L})_2 (100 \times 10^{-6}) (V_{GS2} - V_{TH})$

$\therefore (\frac{W}{L})_2 = 2500$ //

b). $|\text{gain } (A_v)| = g_{m1} (r_{o1} // 10 r_{o1})$

$$30 = g_{m1} (0.909 \, r_{o1})$$

$$r_{o1} = \frac{1}{0.1 \times 1.11 \times 10^{-3}}$$

$$= 9009 \, \Omega$$

$$\therefore g_{m1} = 3.66 \text{ mS.}$$

$$\because g_{m1} = \sqrt{2 (\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 \times I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 30.2$$

(66) Power = 1mW

$\therefore I_{DS1} = I_{DS2} = \frac{1mW}{1.8V} = 0.556\,mA.$

Volt. gain $(A_v) = -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}$

$= -4$

set $V_{GS1} = V_{GS2} = \frac{V_{DD}}{3}$.

$\because I_{DS1} = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})^2$

$\therefore \left(\frac{W}{L}\right)_1 = 139$

$\therefore \left(\frac{W}{L}\right)_2 = \frac{139}{16}$

$\approx 8.69.$

and $V_{IN} = \frac{V_{DD}}{3} = 0.6V$

(67) $R_{in} = \frac{1}{g_{m1}} = 50\Omega$

$\therefore g_{m1} = 20mS$

Volt. gain $(A_v) = g_m R_D = 5$

$\therefore R_D = 250\Omega$

Power $= 3mW$

$\therefore I_{DS1} = \frac{3mW}{1.8V}$

$= 1.67mA.$

$\because g_{m1} = \sqrt{2 \times \mu_n C_{ox} \times (\frac{W}{L})_1 I_{DS1}}$

$(\frac{W}{L})_1 = 600$

$\therefore R_D = 250\Omega, \quad (\frac{W}{L})_1 = 600 \quad I_{DS} = 1.67mA.$

(68) Power $(P) = 2\,mW$

$$I_{DS_1} = \frac{2\,mW}{1.8\,V} = 1.11\,mA.$$

$\because$ M_1 operates 100mV away from triode,

$$V_{DS} = V_{GS} - V_{TH} + 0.1$$

$$V_D = 1.8 - 0.4 + 0.1 = 1.5\,V$$

$$\therefore R_D = \frac{1.8 - 1.5}{1.11 \times 10^{-3}} \approx 270\,\Omega$$

Volt. gain $(A_v) = g_{m_1} R_D = 4$

$$\therefore g_{m_1} = 14.8\,mS$$

$$\because I_{DS} = \frac{1}{2} g_{m_1} \times (V_{GS_1} - V_{TH})$$

$$V_{GS} \approx 0.550\,V$$

Set $V_G = 0.9\,V$. $\therefore V_S = (0.9 - 0.55)\,V = 0.35\,V$

$$R_S = \frac{0.35}{1.11 \times 10^{-3}} \approx 315\,\Omega$$

To find $\left(\frac{W}{L}\right)_1$: $g_{m_1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH})$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 135$$

$\therefore \left(\frac{W}{L}\right)_1 = 135$, $V_{IN} = 0.9\,V$. $R_S = 315\,\Omega$, $R_D = 270\,\Omega$

$I_{DS} = 1.11\,mA$

(69)

Power $= 5\,mW$

$\therefore I_{DS_1} = \frac{5\times10^{-3}}{1.8} = 2.78\,mA$

Gain $(A_v) = g_m R_D = 5$

$V_{G_1} = V_{OUT} = 1.8 - IR_D$

$V_{S_1} = I R_S$

Let $R_S = \frac{10}{g_{m_1}}$

$\therefore V_{S_1} = \frac{10I}{g_{m_1}}$

$\therefore V_{GS_1} = 1.8 - IR_D - \frac{10I}{g_{m_1}}$

$\because I_{DS} = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$2.78\times10^{-3} = \frac{g_m}{2}\left(1.8 - 2.78\times10^{-3} R_D - \frac{2.78\times10^{-2}}{g_m}\right)$

$= 0.9\,g_m - 1.39\times10^{-3} g_m R_D - 1.39\times10^{-2}$

$\because g_m R_D = A_v = 5$

$2.78\times10^{-3} = 0.9\,g_m - 6.95\times10^{-3} - 1.39\times10^{-2}$

$\therefore g_m \approx 26.3\,mS$

and $R_D = \frac{5}{26.3\times10^{-3}} \approx 190\,\Omega$

$R_S = \frac{10}{26.3\times10^{-3}} = 380\,\Omega$

$\because g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{DS_1}} \Rightarrow \left(\frac{W}{L}\right) \approx 622$

(70) $\because R_s \approx \frac{10}{g_m}$,

$\therefore R_{in} \approx \frac{1}{g_m} = 50\,\Omega$

i.e. $g_m = 20\,mS$ //

$$|\text{Gain } (A_v)| = \frac{g_m R_D}{1 + g_m R_s} = 4$$

$$g_m R_D = 4 + 4 g_m R_s$$

$$R_D = \frac{4 + 0.08 R_s}{0.02} = 200 + 4R_s \quad \text{—— (1)}$$

$\because R_s I_D + V_{GS} - V_{TH} + 0.25 = 1.8 - I_D R_D$ (given)

and $I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

i.e. $V_{GS} - V_{TH} = 100 I_D$

$\therefore R_s I_D + 100 I_D + 0.25 = 1.8 - I_D R_D$

From (1):

$$R_s I_D + 100 I_D + 0.25 = 1.8 - 200 I_D - 4 I_D R_s$$

$$5 R_s I_D + 300 I_D = 1.55$$

Set $R_s = \frac{10}{g_m} = 500\,\Omega$

$\therefore 2500 I_D + 300 I_D = 1.55$

$\therefore I_D = 0.554\,mA$ //

$\because I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$$0.554 \times 10^{-3} = \frac{1}{2} \times 20 \times 10^{-3} (V_{GS} - 0.4)$$

$\therefore V_{GS} = 0.455\,V$

To find $(\frac{W}{L})$:

$$g_m = \sqrt{2 (\frac{W}{L}) \mu_n C_{ox} I_{DS}}$$

$$\therefore (\frac{W}{L}) \approx 1805$$

To find R_D:

$$\because R_D = 200 + 4R_S \quad \text{(from (1))}$$

$$R_D = 2200$$

To find R_1 and R_2,

$$\because R_1 + R_2 = 20\,k\Omega$$

and $$V_{GS} = V_G - I_D R_S = 0.455V$$

i.e. $$V_G = 0.732V$$

$$V_G = \frac{R_2}{R_1 + R_2} \times V_{DD}$$

$$\therefore R_1 = 8133\,\Omega$$

$$R_2 \approx 11.9\,k\Omega.$$

$\therefore R_1 = 8133\,\Omega$, $R_2 = 11.9\,k\Omega$, $R_D = 2200\,\Omega$ $R_S = 500\,\Omega$

$(\frac{W}{L}) = 1805$ $I_{DS} = 0.554\,mA.$

(7)

$R_{in} = R_G = 10\,k\Omega$

$Power = 2mW$

$\therefore I_{DS} = \dfrac{2mW}{1.8V} = 1.11mA$

$A_v = \dfrac{R_S}{\frac{1}{g_m} + R_S} = 0.8$

$\therefore R_S = \dfrac{4}{g_m}$ ——— ①.

$\because V_{OUT} = \dfrac{V_{DD}}{2} = I_{DS} R_S$

$I_{DS} R_S = 0.9$ ——— ②

$\because V_G = 1.8\,V$ and $V_S = 0.9$

$\therefore V_{GS} = 0.9\,V.$

From ②, $\because I_{DS} = 1.11mA$

$R_S = \dfrac{0.9V}{1.11mA} \approx 810\Omega$

From ①, $g_m = \dfrac{4}{810\Omega} \approx 4.94\,mS.$

$\because g_m = \left(\dfrac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})$

$\dfrac{W}{L} \approx 49.4$

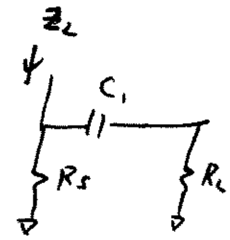
(72) $R_{in} = R_G = 20k\Omega$

$\because$ Power = 3mW

$\therefore I_{DS} = \frac{3mW}{1.8V} = 1.67mA$

$V_{X, at\ DC} = I_{DS} R_S = 0.9V$

$\therefore R_S = 540\Omega$



Load impedance, $Z_L = R_S // (\frac{1}{sC_1} + R_L)$
(at 100 MHz)

$= 540 // (\frac{1}{2\pi \times 10^8 C_1} + 50)$

Voltage gain $(A_v) = \frac{Z_L}{\frac{1}{g_m} + Z_L}$

$g_m = \frac{2 I_{DS}}{V_{GS} - V_{TH}}$

$= \frac{2 \times 1.67 \times 10^{-3}}{(1.8 - 0.9) - 0.4}$

$= 6.67 mS.$

$\therefore A_v = \frac{Z_L}{+ Z_L} = 0.8$

$Z_L = 120 + Z_L (0.8)$

$\therefore Z_L = 150$

$$\therefore 150 = 540 \,//\, \left(\frac{1}{2\pi \times 10^8 C_1} + 50\right)$$

$$= 540 \,//\, \left[\frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}\right]$$

$$= \frac{540 \times \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}{540 + \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}$$

$$\therefore \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1} \approx 208$$

$$\therefore C_1 \approx 10.1\,pF$$

To find $\left(\frac{W}{L}\right)$:

$$\because g_m = \left(\frac{W}{L}\right) \mu_n C_{ox} (V_{GS} - V_{TH})$$

$$\frac{W}{L} = 66.7$$

$$\therefore \frac{W}{L} = 66.7, \quad C_1 = 10.1\,pF, \quad R_S = 540\,\Omega.$$

(73) $\text{Power} = 3\,mW$

$$\therefore I_{DS_{1,2}} = \frac{3\,mW}{1.8\,V} = 1.67\,mA.$$

$$r_{o2} = \frac{1}{\lambda I_{DS2}}$$

$$= \frac{1}{0.1 \times 1.67 \times 10^{-3}} \approx 5990\,\Omega.$$

$$= r_{o1}$$

$$\because A_v = \frac{r_{o2} \parallel r_{o1}}{\frac{1}{g_{m1}} + r_{o2} \parallel r_{o1}} = 0.9$$

$$\therefore 0.9 = \frac{2995}{\frac{1}{g_{m1}} + 2995}$$

$$g_{m1} \approx 3\,mS$$

$\because V_{DS2} \geq 0.3\,V$ (for M_2 to be in saturation)

set V_{out} (i.e. V_{DS2}, nominal) $= 0.3\,V$

$$\because g_{m1} = \frac{2 I_{DS1}}{V_{GS1} - V_{TH}}$$

$$3 \times 10^{-3} = \frac{2 \times 1.67 \times 10^{-3}}{V_G - 0.9 - 0.4}$$

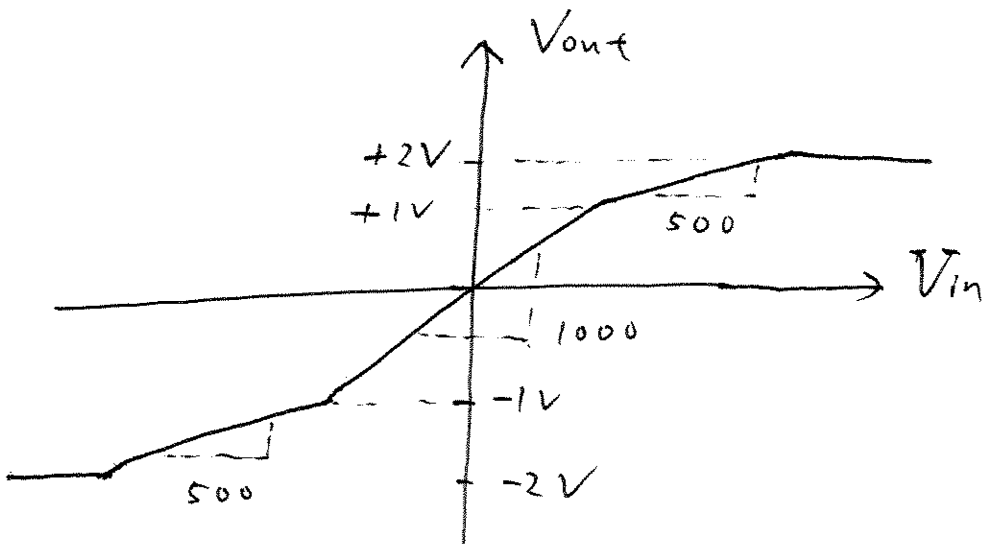
$$\therefore V_{IN} = V_G \approx 1.81\,V$$

$$g_m = \sqrt{2\left(\frac{W}{L}\right)\mu_n C_{ox} \quad I_{DS}}$$

$$\therefore \frac{W}{L} \approx 13.5$$

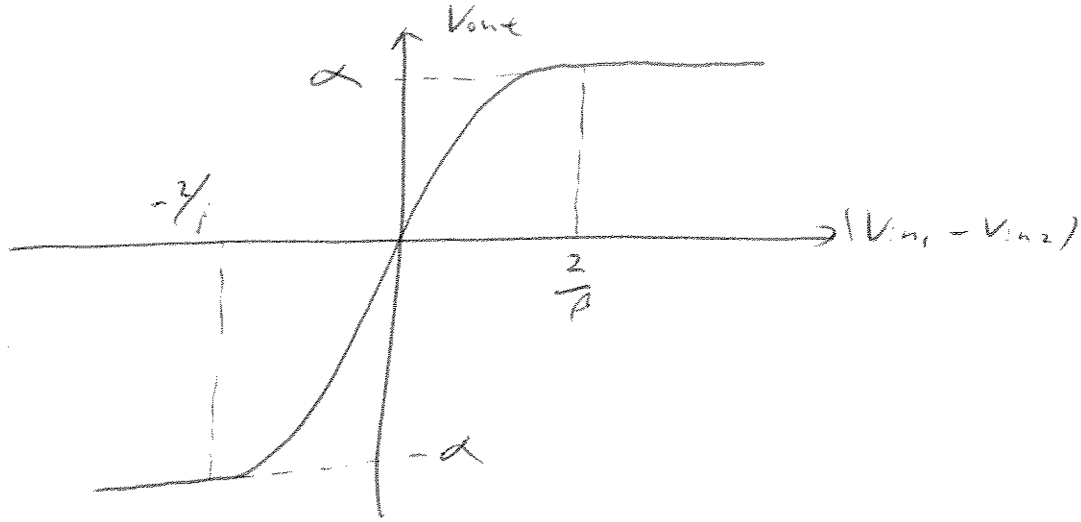
$$\therefore \frac{W}{L} = 13.5, \quad V_{IN} = 1.81V, \quad I_{DS} = 1.67mA.$$

① a)



b) The largest input swing is ± 1mV, because gain is constant at 1000 over this range of input.

② $V_{out} = \alpha \tanh[\beta(V_{in1} - V_{in2})]$



To find small-signal gain,

$\because \tanh z = z - \frac{1}{3}z^3 + \frac{2}{15}z^5 + \cdots$

$\therefore$ for $\beta(V_{in1} - V_{in2}) \simeq 0$,

$$\frac{d V_{out}}{d(V_{in1} - V_{in2})} \simeq \frac{d}{d(V_{in1} - V_{in2})} \alpha\beta(V_{in1} - V_{in2})$$

$$= \underline{\underline{\alpha\beta}}$$

③ Closed-loop gain $= \left(1 + \frac{R_1}{R_2}\right)$

$= 8$

Gain error $= \left(1 + \frac{R_1}{R_2}\right)(A_0)^{-1}$

$= \frac{8}{2000}$

$= \underline{\underline{0.4\%}}$

④ Closed loop gain $= \left(1+\frac{R_1}{R_2}\right)$

$= 4$

Gain error $= \left(1+\frac{R_1}{R_2}\right)\left(\frac{1}{A_0}\right)$

$= 0.1\%$

$\therefore \quad 4/A_0 = 0.1\%$

$A_0 = \underline{\underline{4000}}$

⑤ Let $G_o = \left(1 + \frac{R_1}{R_2}\right)$

Desired gain $= \alpha_1$

$$= \frac{A_o}{1 + \frac{R_2}{R_1 + R_2} A_o}$$

$$\therefore \quad \alpha_1 = \frac{A_o}{1 + \frac{A_o}{G_o}}$$

$$1 + \frac{A_o}{G_o} = \frac{A_o}{\alpha_1}$$

$$\frac{1}{G_o} = \frac{1}{\alpha_1} - \frac{1}{A_o}$$

$$G_o = \frac{A_o \alpha_1}{A_o - \alpha_1}$$

$$\therefore \quad \frac{R_2}{R_1 + R_2} = \frac{1}{G_o} = \frac{1}{\alpha_1} - \frac{1}{A_o}$$ //

b) if A_o drops to $0.6 A_o$,

$$\text{Actual gain} = \frac{0.6 A_o}{1 + \left(\frac{1}{\alpha_1} - \frac{1}{A_o}\right) 0.6 A_o}$$

$$= \frac{0.6 A_o}{0.4 + \frac{0.6 A_o}{\alpha_1}}$$

(5) b) (cont'd)

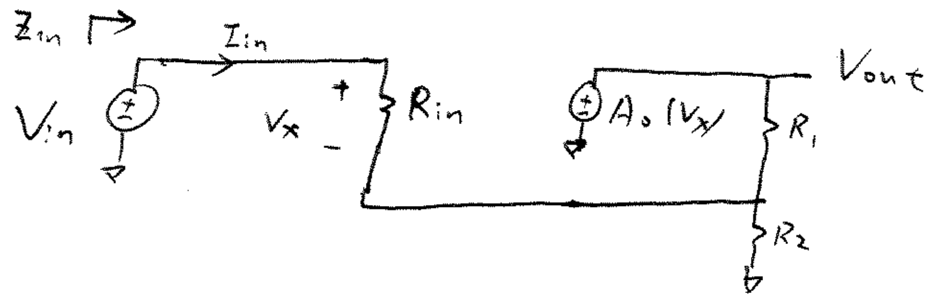
$$\text{Actual gain} = \frac{\alpha_1}{1 + \frac{0.4}{0.6}\frac{\alpha_1}{A_0}}$$

$$\simeq \alpha_1 \left(1 - \frac{0.4}{0.6} \frac{\alpha_1}{A_0}\right)$$

$$\therefore \text{the gain error} \simeq \frac{0.4}{0.6} \frac{\alpha_1^2}{A_0}$$

$$= \frac{2}{3} \frac{\alpha_1^2}{A_0}$$

⑥ Using the model in Fig. 8.44,



$$V_x = V_{in} - V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = A_0 V_x$$

$$= A_0 \left(V_{in} - V_{out} \frac{R_1}{R_1 + R_2} \right)$$

$$A_0 V_{in} = V_{out} \left(1 + A_0 \frac{R_1}{R_1 + R_2} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + A_0 \frac{R_1}{R_1 + R_2}} \quad \text{——— (1)}$$

To find input impedance (Z_{in}),

$$I_{in} = \frac{V_x}{R_{in}}$$

$$= \frac{1}{R_{in}} \left(V_{in} - V_{out} \frac{R_1}{R_1 + R_2} \right)$$

$$= \frac{V_{in}}{R_{in}} \left(1 - \frac{V_{out}}{V_{in}} \frac{R_1}{R_1 + R_2} \right)$$

⑥ (cont'd)

$$I_{in} = \frac{V_{in}}{R_{in}}\left(1 - \frac{A_0}{1 + A_0 \frac{R_1}{R_1+R_2}} \cdot \frac{R_1}{R_1+R_2}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(1 - \frac{1}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(\frac{\frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$\therefore Z_{in} = \frac{V_{in}}{I_{in}} = R_{in}\left[\frac{1 + \frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1}}\right] \quad \text{—②}$$

As $A_0 \to \infty$,

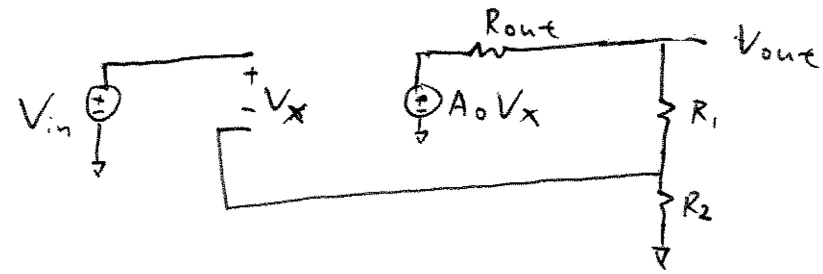
$$\text{gain} = \left.\frac{V_{out}}{V_{in}}\right|_{A_0\to\infty} \quad [\text{From ①}]$$

$$= 1 + \frac{R_2}{R_1}$$

$$Z_{in} = \left.\frac{V_{in}}{I_{in}}\right|_{A_0\to\infty} \quad [\text{From ②}]$$

$$= \infty$$

(7)



Similar to Prob. (6),

$$\text{gain} = \frac{V_{out}}{V_{in}}$$

$$V_x = V_{in} - V_{out}\frac{R_2}{R_1 + R_2}$$

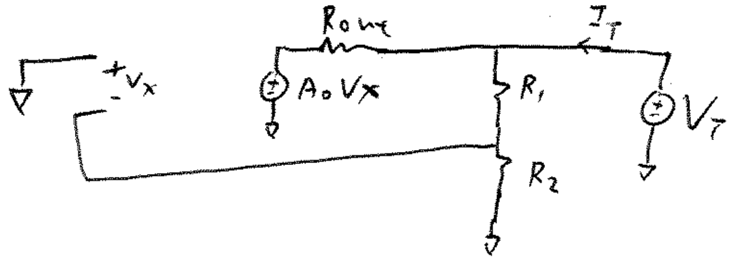
$$V_{out} = A_0 V_x \frac{R_1 + R_2}{R_{out} + R_1 + R_2}$$

$$= A_0\left(V_{in} - V_{out}\frac{R_2}{R_1 + R_2}\right)\frac{R_1 + R_2}{R_{out} + R_1 + R_2}$$

$$V_{in} A_0 \frac{R_1 + R_2}{R_{out} + R_1 + R_2} = V_{out}\left(1 + \frac{A_0 R_2}{R_{out} + R_1 + R_2}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0 \frac{R_1 + R_2}{R_{out} + R_1 + R_2}}{1 + \frac{A_0 R_2}{R_{out} + R_1 + R_2}}$$

To find output impedance (Z_{out})



(7) (cont'd) $V_x = \frac{R_2}{R_1+R_2} V_T$

$$I_T = \frac{V_T}{R_1+R_2} + \frac{V_T - A_0 V_x}{R_{out}}$$

$$= V_T \left[\frac{R_{out} + R_1 + R_2 - A_0 R_2}{(R_{out})(R_1+R_2)} \right]$$

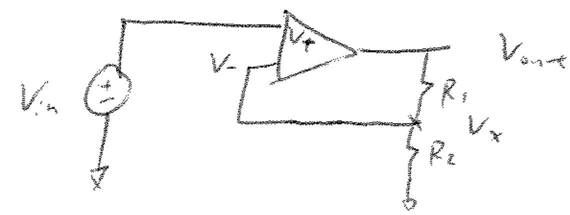
$$Z_{out} = \frac{V_T}{I_T} = \frac{(R_{out})(R_1+R_2)}{R_{out} + R_1 + R_2 - A_0 R_2}$$

As $A_0 \to \infty$,

$$\text{gain} = 1 + \frac{R_1}{R_2}$$

$$Z_{out} = 0$$

(8)



ΔR for now.

$$V_{out} = A_0 (V_x)$$

$$V_x = V_{in} - \frac{R_2}{R_1 + R_2} V_{out}$$

$$\therefore \frac{-V_{out}}{A_0} = V_{in} - \frac{R_1}{R_1 + R_2} V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0 (R_1 + R_2)}{A_0 R_1 - 1} = \text{nominal gain}$$

if $R_2' = \Delta R + R_2$

$$\left(\frac{V_{out}}{V_{in}}\right)' = \frac{A_0 (R_1 + \Delta R + R_2)}{A_0 R_1 - 1}$$

$$\therefore \text{gain error} = \frac{\left(\frac{V_{out}}{V_{in}}\right)' - \left(\frac{V_{out}}{V_{in}}\right)}{\frac{V_{out}}{V_{in}}}$$

$$= \frac{\Delta R}{A_0 R_1 - 1} \times \frac{A_0 R_1 - 1}{A_0 (R_1 + R_2)}$$

$$= \frac{\Delta R}{A_0 (R_1 + R_2)} \quad //$$

(9) Closed-loop gain $\approx \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$

$$= 5\left[1-\frac{5}{A_0}\right]$$

$\therefore$ As A_0 decreases to $0.8A_0$, closed-loop gain decreases along (deviating more from the norminal

A_0 drops to $0.8A_0$ when $|V_{in1}-V_{in2}| = 2mV$.

$$\because \quad V_{in2} = V_{out}\left(\frac{R_2}{R_1+R_2}\right)$$

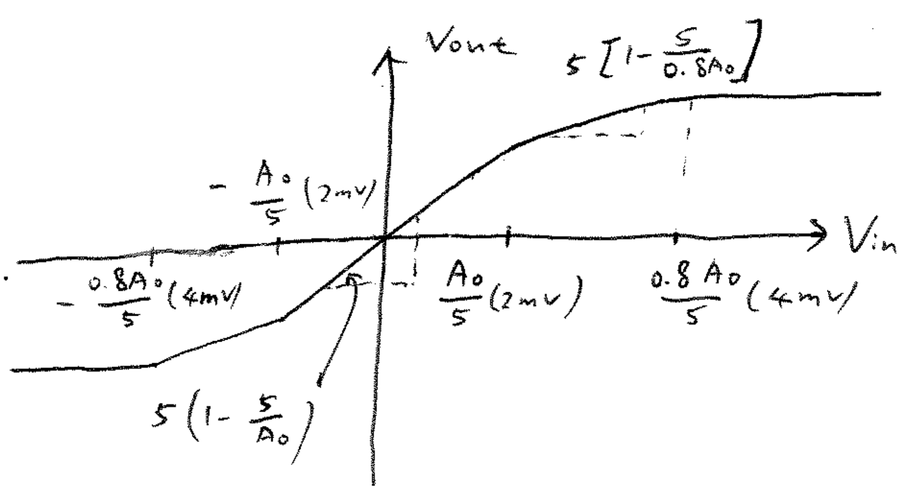
$$\text{and} \quad V_{out} = 5\left(1-\frac{5}{A_0}\right)V_{in1}$$

$$\therefore \quad V_{in2} = 5\left(1-\frac{5}{A_0}\right)\left(\frac{1}{5}\right)V_{in1}$$

$$V_{in1} - V_{in2} = \frac{5}{A_0}V_{in1}$$

At $V_{in1} - V_{in2} = 2mV$,

$$V_{in1} = \frac{A_0}{5}(2mV)$$



(10)

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_s}$$

$\because V_{in} = 1V,$

$$V_{out} = 1 + \frac{R_1}{R_0 + \alpha W}$$

$$\frac{dV_{out}}{dW} = -R_1 \alpha \left(R_0 + \alpha W \right)^{-2}$$

$$= \frac{-R_1 \alpha}{(R_0 + \alpha W)^2}$$

(11) if $A_0 = \infty$,

$$V_+ = V_- = V_{in}$$

$$V_- = \left(\frac{R_2}{R_2+R_3}\right)\left[\frac{R_4 // (R_2 + R_3)}{R_1 + R_4 // (R_2+R_3)}\right] V_{out}$$

$\therefore$ closed-loop gain $\dfrac{V_{out}}{V_{in}}$

$$= \frac{(R_2+R_3)\left[R_1 + R_4 // (R_2+R_3)\right]}{R_2\left[R_4 // (R_2+R_3)\right]}$$

if $R_1 = 0$,

$$G\big|_{R_1=0} = 1 + \frac{R_3}{R_2}$$

if $R_3 = 0$,

$$G\big|_{R_3=0} = \frac{R_2\left[R_1 + R_4 // R_2\right]}{R_2\left[R_4 // R_2\right]}$$

$$= 1 + \frac{R_1}{R_4 // R_2}$$

(12) Gain Error $= \frac{1}{A_0}\left(1 + \frac{R_1}{R_2}\right)$

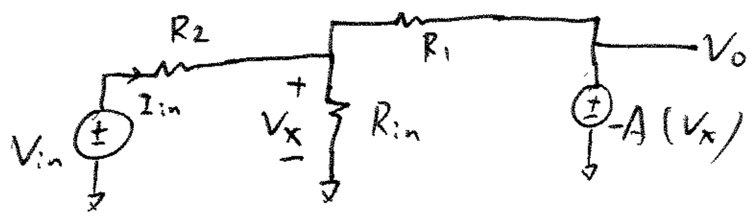
$= \frac{1}{A_0}(1+8)$

$= 0.2\%$

$\therefore \frac{1}{A_0}(9) = 0.2\%$

$A_0 = 4500$

(13)



$$V_o = -A V_x \qquad \text{—— (1)}$$

$$\frac{V_{in} - V_x}{R_2} + \frac{V_o - V_x}{R_1} = \frac{V_x}{R_{in}} \qquad \text{—— (2)}$$

Combining (1) and (2),

$$\frac{V_{in}}{R_2} = -\frac{V_o}{R_1} + \frac{V_o}{(-A)}\left(\frac{1}{R_{in}} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{V_{in}}{R_2} = V_o\left[\frac{A R_{in} R_2 + R_1 R_2 + R_{in} R_2 + R_{in} R_1}{(-A) R_{in} \; R_1 \; R_2}\right]$$

$$\frac{V_o}{V_{in}} = -\frac{A R_{in} R_1}{R_1 R_2 + R_{in} R_2 + R_{in} R_1 + A R_{in} R_2}$$

Input impedance $(Z_{in}) = \dfrac{V_{in}}{I_{in}}$

$$I_{in} - \frac{V_x}{R_{in}} + \frac{(-A)V_x - V_x}{R_1} = 0$$

$$I_{in} = V_x\left[\frac{1}{R_{in}} + \frac{A+1}{R_1}\right]$$

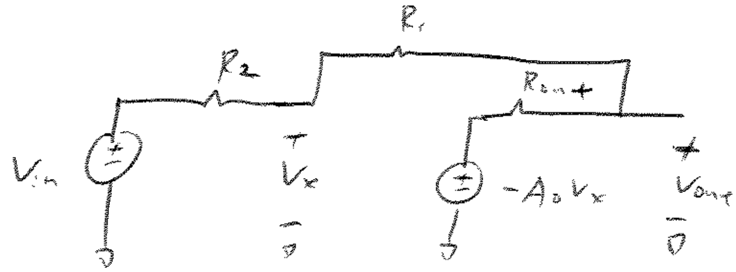
$$\because \quad V_x = V_{in} - I_{in} R_2$$

$$I_{in} = \left[V_{in} - I_{in} R_2 \right] \left[\frac{1}{R_{in}} + \frac{A+1}{R_1} \right]$$

$$I_{in} \left[1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1) \right] = V_{in} \left(\frac{1}{R_{in}} + \frac{A+1}{R_1} \right)$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1)}{\frac{1}{R_{in}} + \frac{A+1}{R_1}}$$

(14)



By KCL,

$$\frac{V_{in} - V_x}{R_2} = - \frac{-A_0 V_x - V_x}{R_1 + R_{out}}$$

$$\because \quad V_x = - \frac{V_{out}}{A_0}$$

$$\frac{V_{in}}{R_2} = - \frac{V_{out}}{A_0 R_2} - \frac{A_0 + 1}{A_0} \frac{V_{out}}{R_1 + R_{out}}$$

$$\because \quad \frac{A_0 + 1}{A_0} \approx 1$$

$$\therefore \quad \frac{V_{in}}{R_2} \approx - \frac{V_{out}}{A_0 R_2} - \frac{V_{out}}{R_1 + R_{out}}$$

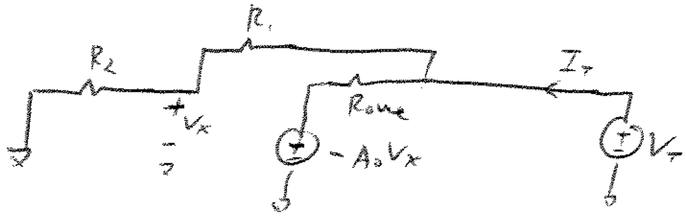
$$\frac{V_{in}}{R_2} \approx - V_{out} \left(\frac{R_1 + R_{out} + A_0 R_2}{A_0 R_2 (R_1 + R_{out})} \right)$$

$$\therefore \quad \frac{V_{out}}{V_{in}} \approx - \frac{A_0 (R_1 + R_{out})}{R_1 + R_{out} + A_0 R_2}$$

//

(14) cont'd:

To find output impedance (Z_{out})



$$Z_{out} = \frac{V_T}{I_T}$$

$$V_x = \frac{R_2}{R_1 + R_2} V_T \quad \text{———} \; ①$$

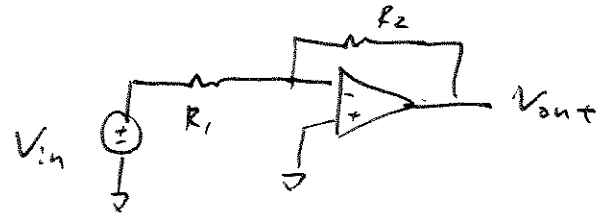
$$I_T = \frac{V_T}{R_1 + R_2} + \frac{V_T + A_0 V_x}{R_{out}} \quad \text{———} \; ②$$

$$I_T = V_T \left[\frac{1}{R_1 + R_2} + \frac{1 + \frac{A_0 R_2}{R_1 + R_2}}{R_{out}} \right]$$

$$\frac{V_T}{I_T} = \frac{R_{out}(R_1 + R_2)}{R_{out} + R_1 + (A_0 + 1) R_2}$$

//

(15)



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_2}{R_1} = 4 \quad \text{——— ①}$$

$\therefore\ R_2 = 4R_1$

$$Z_{in} \approx R_1 = 10\,k\Omega \quad \text{——— ②}$$

$\therefore\ R_2 = 40\,k\Omega$

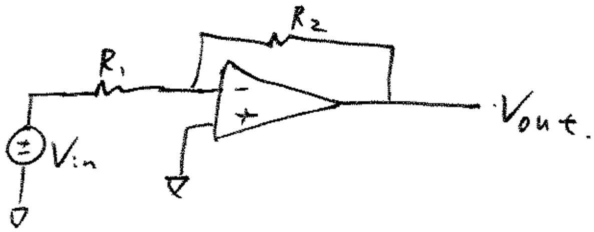
$$A_0 = 1000 \quad \text{——— ③}$$

$$\text{gain error} = \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{1}{1000}(1 + 4)$$

$$= 0.5\,\%$$

(16)



Nominal gain $= \frac{R_2}{R_1} = 8$ —— ①

$R_2 = 8R_1$

Input impedance $\approx R_1 = 1000\ \Omega$ —— ②

$\therefore R_2 = 8000\ \Omega$.

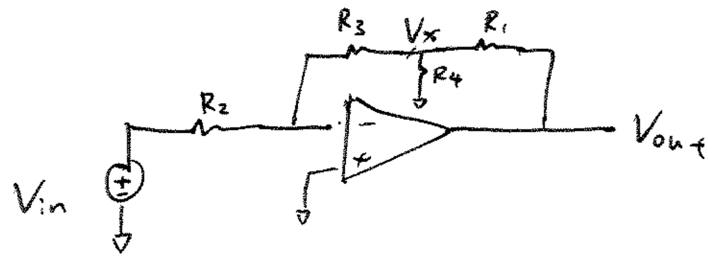
gain error $= 0.1\%$ —— ③

$\therefore \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right) = 0.1\%$

$\frac{1}{A_0}(9) = \frac{0.1}{100}$

$\therefore A_0 = 9000$ //

(17)



$V_- = V_+ = 0 \quad (\because A = \infty)$

$$\frac{V_{in}}{R_2} = -\frac{V_x}{R_3} \longrightarrow ①$$

$$V_x = \frac{R_3 // R_4}{R_1 + R_3 // R_4} V_{out} \longrightarrow ②$$

Combining ① and ②,

$$\frac{V_{in}}{R_2} = -\frac{R_3 // R_4}{R_3 (R_1 + R_3 // R_4)} V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2} \frac{(R_1 + R_3 // R_4)}{R_3 // R_4} \quad //$$

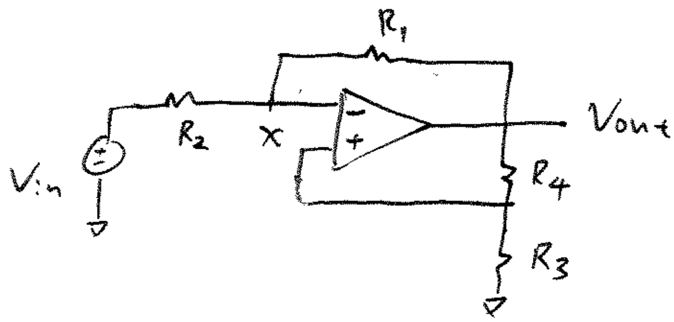
if $R_1 \longrightarrow 0$,

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2} \quad //$$ (typical inverting amplifier)

if $R_3 \longrightarrow 0$,

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_2} \quad //$$ (typical inverting amplifier)

(18)



$\because A = \infty$,

$$V_- = V_+$$

$$\therefore \quad V_x = \frac{R_3}{R_3 + R_4} V_{out}$$

$$\frac{V_{in} - V_x}{R_2} = - \frac{V_{out} - V_x}{R_1}$$

$$\frac{V_{in}}{R_2} = V_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{V_{out}}{R_1}$$

$$= \left[\left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{R_1} \right] V_{out}$$

$$\therefore \quad \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_2}}{\left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{R_1}}$$ //

(19) From eq. (8.31),

$$V_{out} = -\frac{1}{R_1 C_1}\int V_{in}\, dt$$

$$= -\frac{1}{R_1 C_1}\int V_0 \sin \omega t \, dt$$

$$= \frac{V_0}{R_1 C_1 \omega}\cos \omega t$$

$\therefore$ Amplitude of output $= \dfrac{V_0}{R_1 C_1 \omega}$ //

(20) From prob. (19)

Amplification of the integrator $= \frac{1}{R_1 C_1 \omega}$

$$\therefore \quad \frac{1}{R_1 C_1 \omega} = 10$$

$$\frac{1}{\omega} = 10 \times 10\,ns$$

$$\therefore \omega = 10\,MHz.$$

$\therefore$ The frequency of the sinusoid is 10MHz.

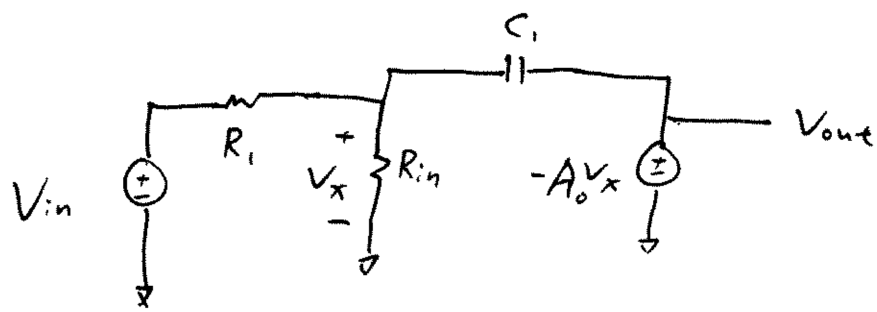
(21) From Eq. (8.37)

$$S_p = \frac{-1}{2\pi (A_0 + 1) R_1 C_1} \leq -1 \text{ Hz}.$$

$$\therefore \quad 2\pi (A_0 + 1)(10\text{ k}\Omega)(1\text{ nF}) \geq 1$$

$$A_0 \geq 15915$$

(22)



$$\frac{V_{in}-V_x}{R_1}+\frac{V_{out}-V_x}{\frac{1}{sC_1}}=\frac{V_x}{R_{in}},$$

where $s = jW$

$\because \quad V_{out} = -A_o V_x$

$$\therefore \quad \frac{V_{in}}{R_1} = (sC_1)\left[-\frac{V_{out}}{A_o} - V_{out}\right] - \frac{V_{out}}{A_o}\left(\frac{1}{R_{in}}+\frac{1}{R_1}\right)$$

$$= -V_{out}\left[\frac{sC_1}{A_o} + sC_1 + \frac{1}{A_oR_{in}} + \frac{1}{A_oR_1}\right]$$

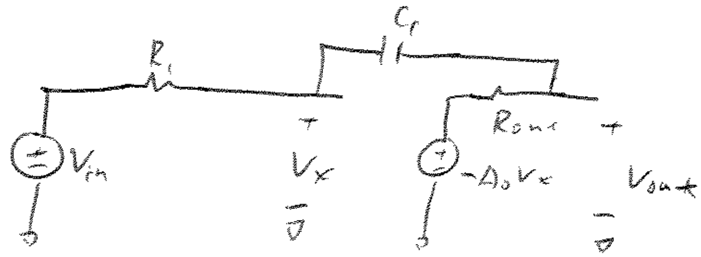
$$\frac{V_{out}}{V_{in}} = \frac{-1}{\left(\frac{1}{A_o}+\frac{R_1}{A_o R_{in}}\right)+\left(1+\frac{1}{A_o}\right)sR_1C_1}$$

To find the pole, equate denominator to zero,

$$s_p = \frac{-1}{(A_o+1)R_1C_1}\left(1+\frac{R_1}{R_{in}}\right)$$

[$\therefore$ pole shifted out by $\left(1+\frac{R_1}{R_{in}}\right)$]

(23)



By KCL,

$$\frac{V_{in} - V_x}{R_1} = -\frac{-A_0 V_x - V_x}{R_{out} + \frac{1}{sC_1}}$$

$$\because \; V_x = -\frac{V_{out}}{A_0}$$

$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{A_0 R_1} - \frac{(A_0+1)}{A_0}\frac{V_{out}}{R_{out} + \frac{1}{sC_1}}$$

$$\simeq -V_{out}\left[\frac{1}{A_0 R_1} + \frac{1}{R_{out} + \frac{1}{sC_1}}\right]$$

$$\left(\because \frac{A_0+1}{A_0} \simeq 1\right)$$

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{A_0 \times \left(R_{out} + \frac{1}{sC_1}\right)}{\left(A_0 R_1 + R_{out} + \frac{1}{sC_1}\right)}$$

$$\text{pole} = -\frac{1}{C_1\,(R_{out} + A_0 R_1)}$$

(24) $\because A_0 = \infty$

$$|A_v| = \frac{R_1}{\frac{1}{\omega C_1}}$$

$$= \omega R_1 C_1$$

$$= 5$$

$$\therefore R_1 C_1 = \frac{5}{\omega}$$

$$= \frac{5}{2\pi \times 10^6}$$

$$= 7.958 \times 10^{-7}$$

(25) From eqn (8.55)

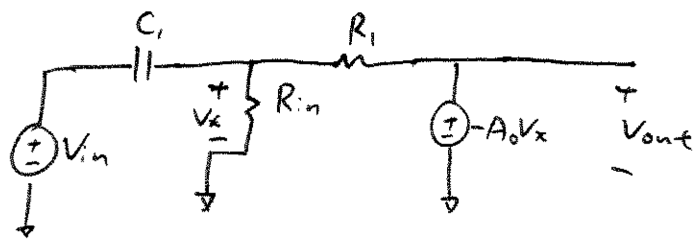
$$S_p = -\frac{A_0+1}{R_1C_1}$$

$$2\pi \times 100 \times 10^6 = \frac{A_0+1}{1000 \times 10^{-9}}$$

(ie. R_1 and C_1 are chosen at minimum)

$$A_0 \approx 627$$

(26)



By KCL,

$$(V_{in} - V_x)\, sC_1 = \frac{V_x}{R_{in}} + \frac{(V_x + A_0 V_x)}{R_1}$$

$$(V_{in})\, sC_1 = V_x \left[sC_1 + \frac{1}{R_{in}} + (A_0 + 1)\frac{1}{R_1} \right]$$

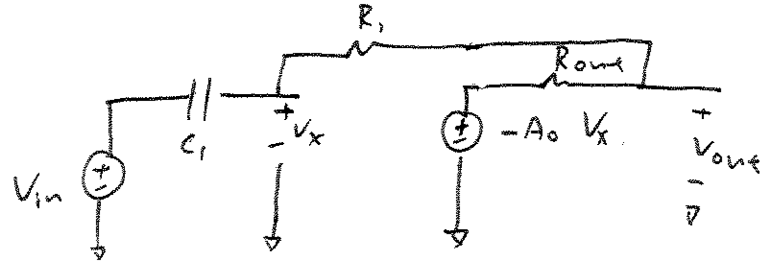
$$\frac{sC_1}{sC_1 + \frac{1}{R_{in}} + (A_0+1)\frac{1}{R_1}} = \frac{V_x}{V_{in}}$$

$\because$ $V_{out} = -A_0 V_x$,

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{A_0\, sC_1}{sC_1 + \frac{1}{R_{in}} + (A_0+1)\frac{1}{R_1}}$$ //

As $A_0 \to \infty$, $\frac{V_{out}}{V_{in}} \to -R_1 C_1 s$. [8.42]

(27)



By KCL,

$$(V_{in} - V_x) SC_1 = (V_x + A_0 V_x) \frac{1}{R_1 + R_{out}}$$

$$(V_{in}) SC_1 = V_x \left[SC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}} \right]$$

$$\frac{V_x}{V_{in}} = \frac{SC_1}{SC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}}}$$

$$V_{out} = (-A_0 V_x - V_x) \frac{R_1}{R_1 + R_{out}}$$

(resistive divider)

$$= -V_x \frac{(A_0 + 1) R_1}{R_1 + R_{out}}$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{\frac{(A_0 + 1) R_1 SC_1}{R_1 + R_{out}}}{SC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}}}$$

$$\frac{V_{out}}{V_{in}} = -R_1 SC_1 \quad (\text{as } A_0 \rightarrow \infty)$$

[8.42]

(28) $\because A_o = \infty$,

$V_+ = V_- = 0$

By KCL,

$$\frac{V_{in}}{R_1 \,//\, \frac{1}{sC_1}} = - \frac{V_{out}}{R_2 \,//\, \frac{1}{sC_2}}.$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{R_2 \,//\, \frac{1}{sC_2}}{R_1 \,//\, \frac{1}{sC_1}}$$

$$= - \frac{R_2}{R_1} \times \frac{1 + R_1 C_1 s}{1 + R_2 C_2 s}.$$

For $\left| \frac{V_{out}}{V_{in}} \right| = 1$,

$$R_2 \,//\, \frac{1}{sC_2} = R_1 \,//\, \frac{1}{sC_1}$$

That is, choose the components such that the impedance of $R_2 // \frac{1}{sC_2}$ is equal to $R_1 // \frac{1}{sC_1}$ at the specific frequency.

(29) if $A_0 < \infty$,

Let V_- be the voltage at the negative input terminal of the opamp.

By KCL,

$$\frac{V_{in} - V_-}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} - V_-}{R_2 // \frac{1}{sC_2}}$$

$$V_{out} = -A_0 V_-,$$

$$\frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}$$

$$V_{in} = -\left[R_1 // \frac{1}{sC_1}\right]\left[\frac{\left(R_2 // \frac{1}{sC_2}\right)\frac{V_{out}}{A_0} + V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}\right.$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{R_2 // \frac{1}{sC_2}}{R_1 // \frac{1}{sC_1}}\left[\frac{A_0}{(A_0+1) + \left(R_2 // \frac{1}{sC_2}\right)}\right]$$

To set $\left|\frac{V_{out}}{V_{in}}\right| = 1$,

Let $x = R_1 // \frac{1}{sC_1}$ and $y = R_2 // \frac{1}{sC_2}$.

$\therefore$ For $\left|\frac{V_{out}}{V_{in}}\right| = 1$, $\quad yA_0 = x\left[(A_0+1) + y\right]$

$$y(A_0-1) = x(A_0+1)$$

(29) Cont'd

$$\therefore \quad \frac{X}{Y} = \frac{A_0+1}{A_0-1},$$

ie. we need to set $\dfrac{R_1 \,//\, \frac{1}{sC_1}}{R_2 \,//\, \frac{1}{sC_1}} = \dfrac{A_0+1}{A_0-1}$.

Since A_0 is generally rather large,

$\dfrac{A_0+1}{A_0-1}$ is a rational fraction, in which the numerator and the denominator are large, and differ by a small amount.

(e.g. if $A_0 = 1000$, $\dfrac{A_0+1}{A_0-1} = \dfrac{1001}{999}$)

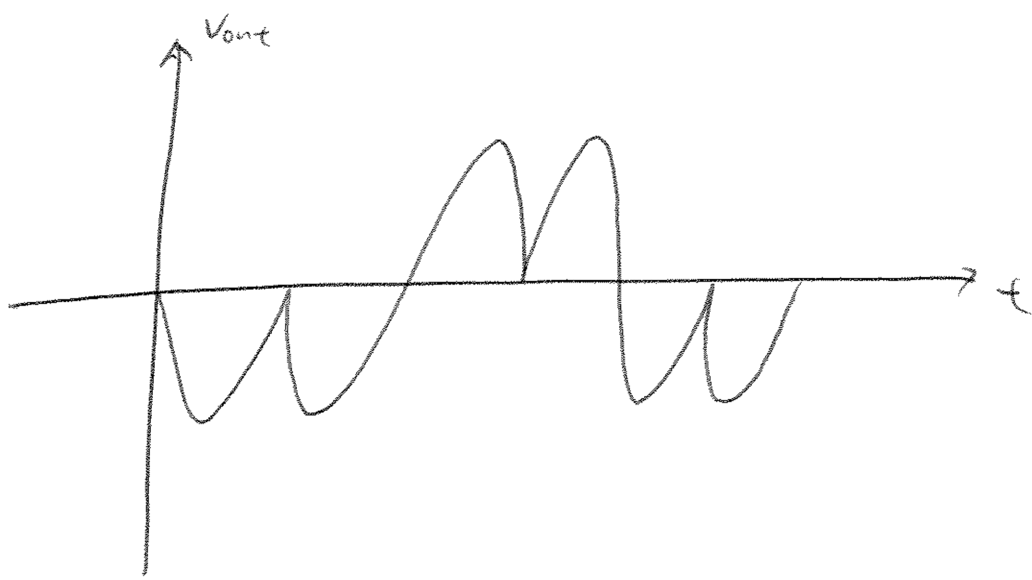
Hence, setting $\left|\dfrac{V_{out}}{V_{in}}\right|$ to unity is possible in principle, although it would be rather difficult to precisely control A_0.

(30) From eqn (8.63),

$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

$\because R_1 = R_2$,

$$V_{out} = -\frac{R_F}{R_1}(V_1 + V_2)$$



(31) By KCL,

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = - \frac{V_{out} - V_x}{R_F}$$

$\because$ $V_{out} = -A_0 V_x$

$$V_x = -\frac{V_{out}}{A_0},$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) + \frac{V_{out}}{A_0}\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F}\right) = -\frac{V_{out}}{R_F}$$

$$-\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) = V_{out}\left[\frac{1}{R_F} + \frac{1}{A_0}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right)\right]$$

$$\therefore \quad V_{out} = -\left(\frac{1}{R_F} + \frac{1}{A_0}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right)\right)^{-1} \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

(32) For $A_0 = \infty$,

$V_+ = V_- = 0$,

$\therefore$ No current flows through R_P,

i.e. No effect due to R_P

$$V_{out} = -R_F \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

For $A_0 \neq \infty$,

By KCL

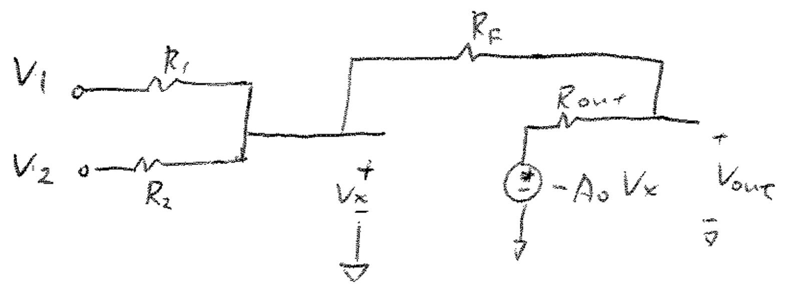
$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} - \frac{V_x}{R_P} = -\frac{V_{out} - V_x}{R_F}$$

$$\because \quad V_x = -\frac{V_{out}}{A_0},$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) + \frac{V_{out}}{A_0} \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_P} + \frac{1}{R_F} \right. = -\frac{V_{out}}{R_F}$$

$$V_{out} = -\left[\frac{1}{R_F} + \frac{1}{A_0} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_P} + \frac{1}{R_F} \right) \right]^{-1} \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

(33)



By KCL,

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = \frac{V_x (A_0 + 1)}{R_F + R_{out}}$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) = V_x \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right]$$

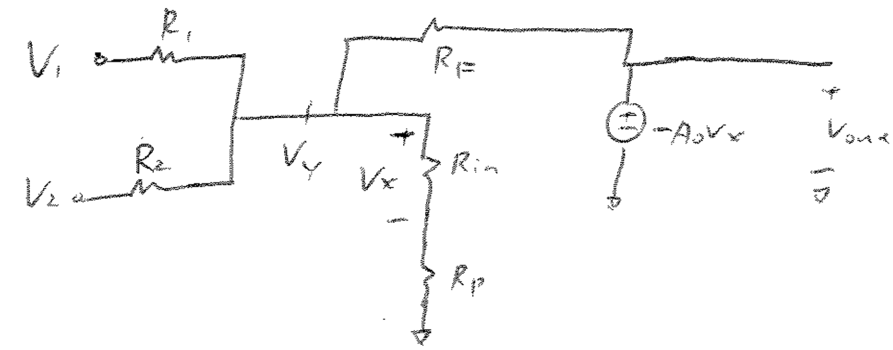
$$\because V_{out} = (-A_0 V_x - V_x) \frac{R_F}{R_F + R_{out}}$$

$$= -V_x (1 + A_0) \frac{R_F}{R_F + R_{out}}$$

$$\therefore \frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{R_F + R_{out}}{R_F (A_0 + 1)} \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right] V_{out}$$

$$V_{out} = -\frac{R_F (A_0 + 1)}{R_F + R_{out}} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right)^{-1} \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

(34)



By KCL,

$$\frac{V_1 - V_y}{R_1} + \frac{V_2 - V_y}{R_2} = \frac{V_y + A_0 V_x}{R_F} + \frac{V_y}{R_{in} + R_p}$$

Using voltage divider,

$$V_x = V_y \frac{R_{in}}{R_{in} + R_p}$$

$$V_y = \frac{R_{in} + R_p}{R_p} V_x$$

$$\therefore \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) = V_y\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_0 V_x}{R_F}$$

$$= \left[\left(\frac{R_{in} + R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_0}{R_F}\right] \times V_x$$

$$\because V_{out} = -A_0 V_x$$

$$\therefore \frac{V_1}{R_1} + \frac{V_2}{R_2} = -\left(\frac{V_{out}}{A_0}\right)\left[\left(\frac{R_{in}+R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p} + \frac{A_0}{R_F}\right]$$

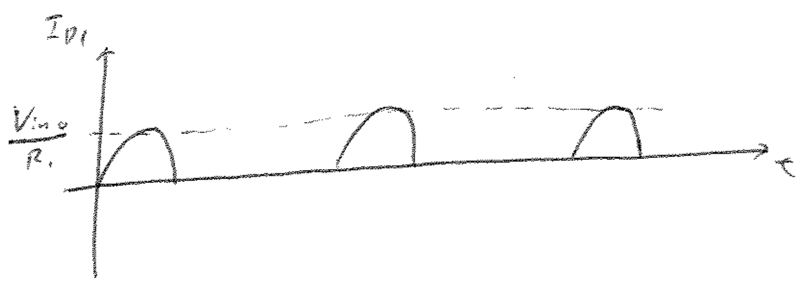
$$V_{out} = -A_0\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) \times \left[\left(\frac{R_{in}+R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p} + \frac{A_0}{R_F}\right)\right]^{-1}$$

(35) When D_1 is on, (i.e. when $V_{in} > 0$)

$$V_{out} = V_{in} = I_{D_1} R_1$$

$$\therefore I_{D_1} = \frac{V_{in}}{R_1}$$

when D_1 is off, $I_{D_1} = 0$.

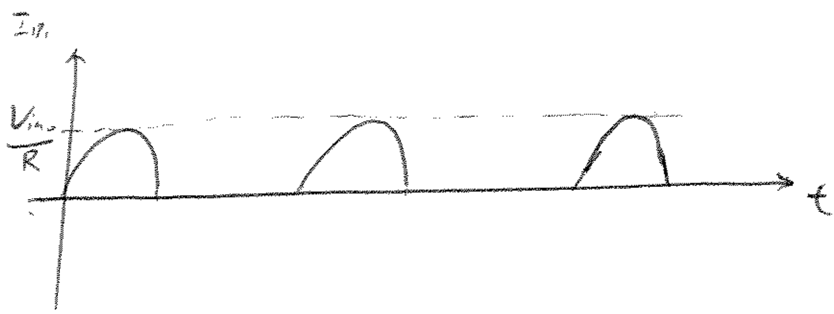


(36) D_1 is on when $V_{in} > 0$,

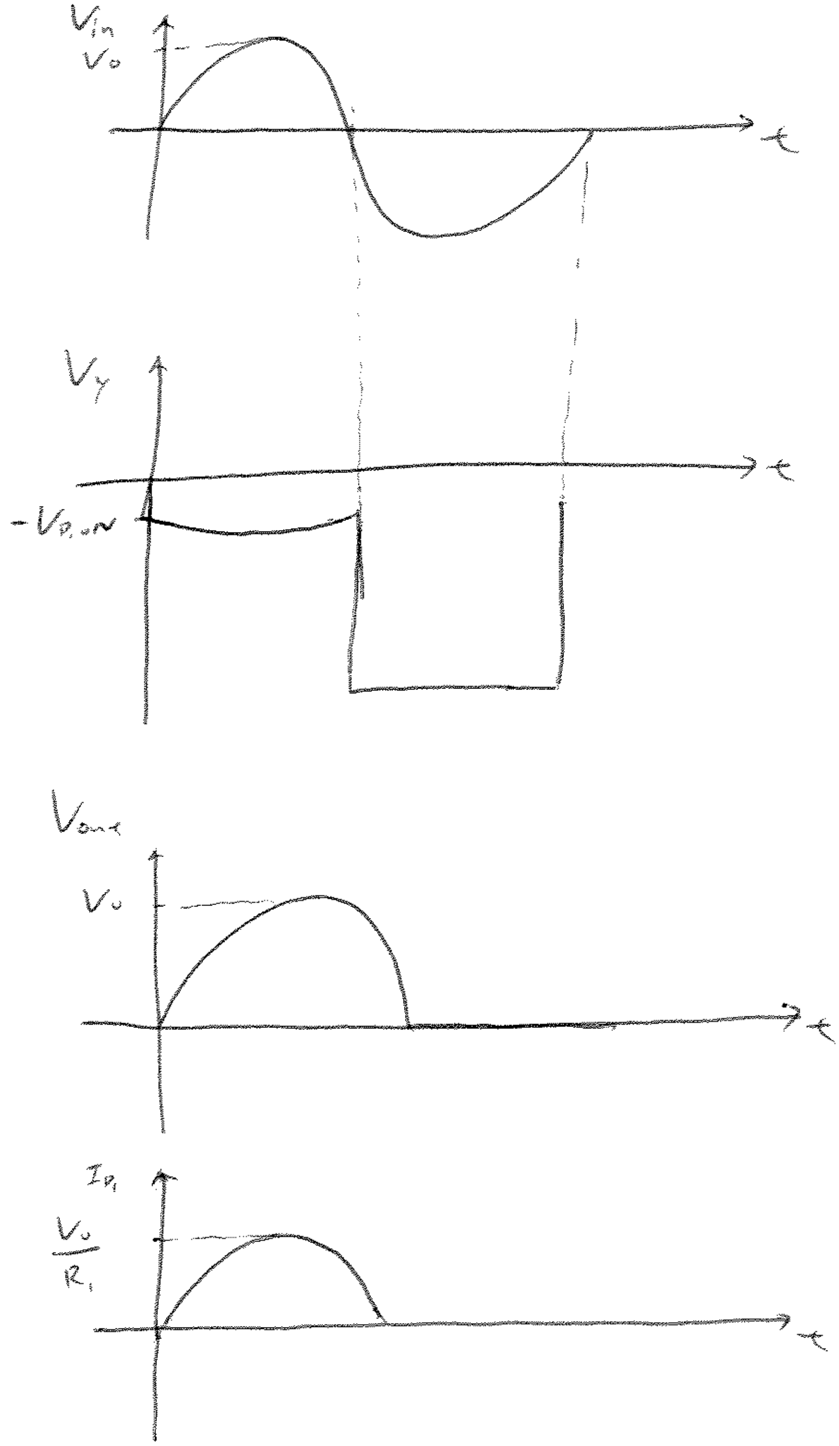
$$V_x = 0$$

By KCL ;

$$I_{D1} = \frac{V_{in}}{R_1}$$



(37)



(38) $\because R_{D_1,on} \ll R_p$

$\therefore$ when diode is on, R_p has no effect.

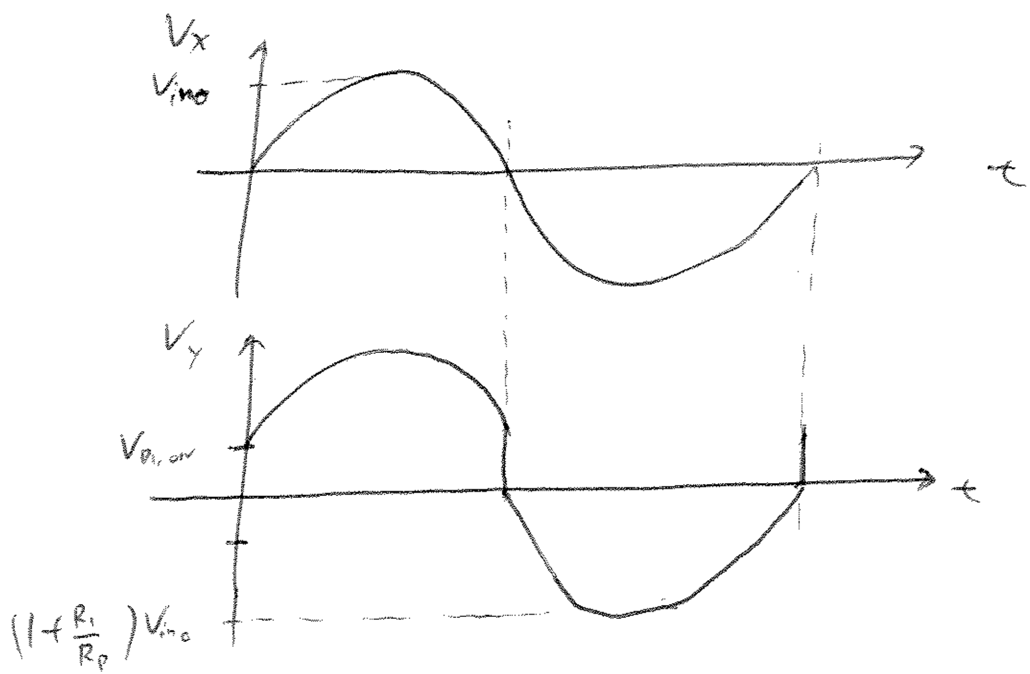
(diode "shorts" nodes X and Y)

when diode is off, R_p functions as a feedback resistor,

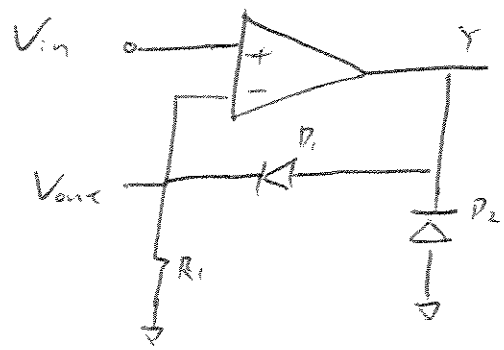
$$\therefore \frac{V_Y}{V_{in}} = 1 + \frac{R_1}{R_p}$$

and $V_{in} = V_{out}$.

$\therefore V_X = V_{in}$ for both D_1 is on and off.

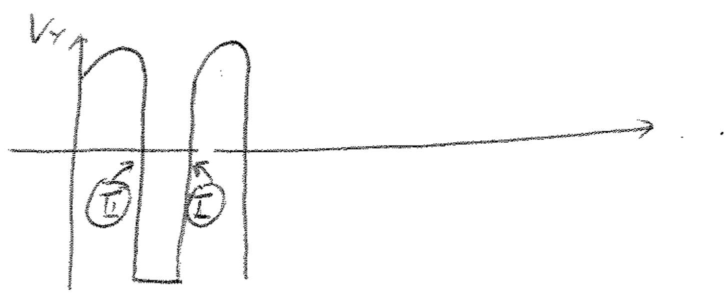


(39) Connecting a diode as below:



D_2 allows the parasitic capacitance to charge up faster, right before D_1 conducts.

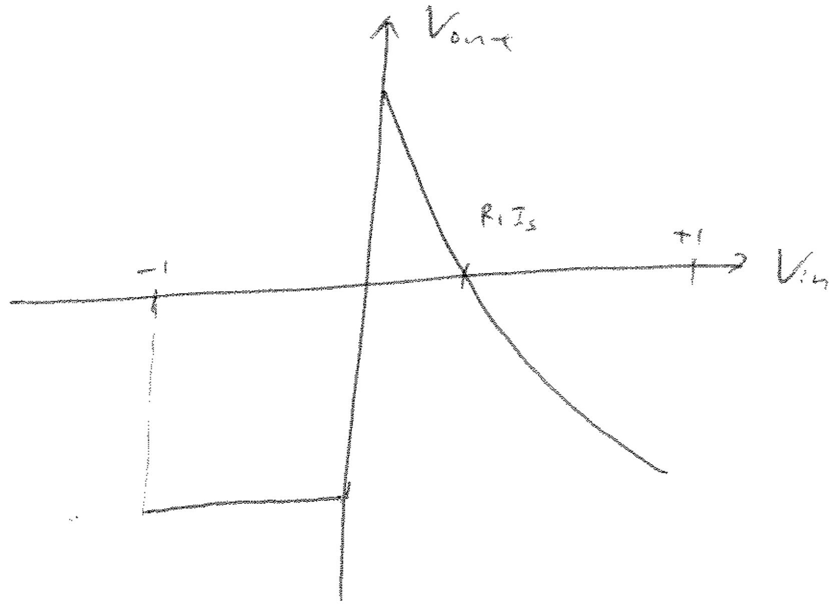
This corresponds to sharpening the transition (I) of V_Y, as shown below



But it will not speed up transition (II).

(which is not critical)

40



(41) By KCL,

$$\frac{V_{in} - V_x}{R_1} = I_{R_1}$$

$$\therefore \quad V_{BE} = V_T \ln \frac{\frac{V_{in} - V_x}{R_1}}{I_s}$$

$$= -V_{out}$$

$$\because -A_0 V_x = V_{out}$$

$$V_x = -\frac{V_{out}}{A_0}$$

$$\therefore \quad V_{out} = -V_T \ln \frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 I_s}$$

//

(42). This circuit will not function as a noninverting opamp:

assuming $A_0 = \infty$,

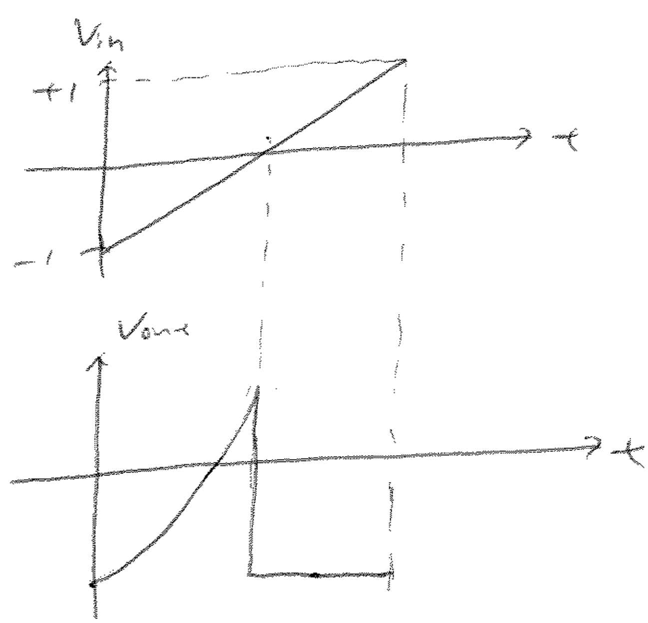
$$V_+ = V_- = V_{in}$$

$$\therefore V_{BE} = V_T \ln \frac{\frac{-V_{in}}{R_1}}{I_s}$$

$$\therefore V_{out} = -V_{BE}$$

$$V_{out} = -V_T \ln \frac{-V_{in}}{R_1 I_s}$$

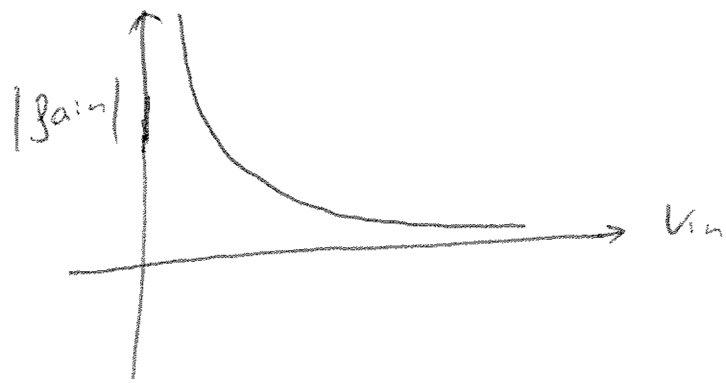
For example, as V_{in} varies from -1V to +1V:



(43)

$$V_{out} = -V_T \ln \frac{V_{in}}{R_1 I_s}$$

$$\frac{dV_{out}}{dV_{in}} = -\frac{V_T}{V_{in}}$$



the gain is compressive, because as Vin increases the magnitude of the gain decreases.

(44) Set $V_{out} = -0.5V$ when $V_{in} = 1V$

$$-0.5 = -V_T \ln \frac{1}{R_1 I_s}$$

$$\therefore R_1 I_s = 2.0612 \times 10^{-9}$$

when $V_{in} = 10V$,

$$V_{out} = -V_T \ln \frac{10}{2.0612 \times 10^{-9}}$$

$$= -0.558V \quad > -1V.$$

$\therefore$ setting $R_1 I_s = 2.0612 \times 10^{-9}$ meets the specification.

choose $I_s = 1 \times 10^{-16} A$.

$R_1 = 20.61 M\Omega$ //

(45) Assume $A_0 = \infty$,

$$I_{R_1} = \frac{V_{in} - V_{TH}}{R_1}$$

$$= \frac{1}{2} k' (V_{GS} - V_{TH})^2,$$

where $k' = \frac{W}{L} C_{ox}' \mu_n$

$\because V_{GS} = - V_{out}$,

$$\therefore \frac{1}{2} k' (-V_{out} - V_{TH})^2 = \frac{V_{in} - V_{TH}}{R_1}$$

$$(-V_{out} - V_{TH})^2 = \frac{2(V_{in} - V_{TH})}{k' R_1}$$

$$(-V_{out} - V_{TH}) = \sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}}$$

$$V_{out} = - \sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}} - V_{TH}$$

$$\text{small signal gain} = - \frac{d}{dV_{in}} \sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}}$$

$$= \frac{1}{k' R} \sqrt{\frac{k' R}{2(V_{in} - V_{TH})}}$$

$$= \sqrt{\frac{1}{2 k' R (V_{in} - V_{TH})}}$$

(46) By KCL,

$$\frac{V_X - V_{in}}{R_1} = I_{SD,M_1}$$

Assume $A_0 = \infty$, $\therefore V_X = V_+ = 0V$.

$$\therefore -\frac{V_{in}}{R_1} = \frac{1}{2} k' (V_{GS} - |V_{TH}|)^2,$$

where $K' = \mu_p \frac{W}{L} C_{ox}'$.

$$\because V = -V_{out}$$

$$\therefore -\frac{V_{in}}{R_1} = \frac{1}{2} K' (-V_{out} - |V_{TH}|)^2$$

$$-\frac{2V_{in}}{R_1 k'} = (V_{out} + |V_{TH}|)^2$$

$$\therefore V_{out} = \sqrt{-\frac{2V_{in}}{R_1 k'}} - |V_{TH}|$$

//

(47) Assume $A_0 = \infty$,

$\therefore V_+ = V_- = V_{in}$

Using voltage divider:

$$V_{in} + V_{os} = V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right)(V_{in} + V_{os})$$

(48) In Fig. (8.25),

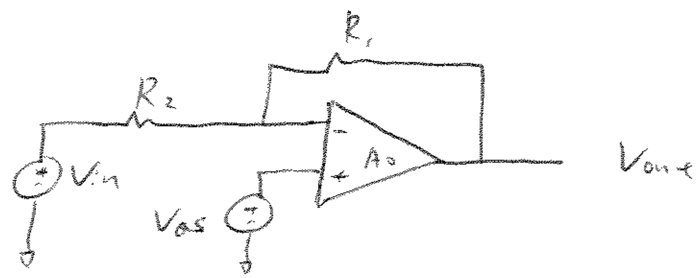
Assuming input is zero,

$$V_x = 10 \times V_{OS,A_1}$$
$$= 30mV$$

$$\therefore V_{out} = 10 \times (V_{OS,A_2} + V_x)$$
$$= 330mV$$

Thus, the maximum offset error is 330mV.

(49)



By KCL,

$$\frac{V_{in} - V_{os}}{R_2} = -\frac{V_{out} - V_{os}}{R_1}$$

$$V_{out} = -\frac{R_1}{R_2}\left(V_{in} - V_{os}\right) + V_{os}$$

(50) By eqn (8.72)

$$V_{out} = V_{os}\left(1 + \frac{R_2}{R_1}\right)$$

$$\therefore\ 20\,mV = 3\,mV\left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{17}{3} = \frac{R_2}{R_1} \qquad \text{——— (1)}$$

$$\because\ \frac{1}{R_2 C_1} << 2\pi(1000)$$

and setting $C_1 = 100\,pF$,

$$\frac{1}{R_2 \times 100 \times 10^{-12}} << 2\pi(1000)$$

$$\frac{1}{R_2} << 6.283 \times 10^{-7}$$

$$\therefore\ R_2 >> 1.59\,M\Omega$$

choose $R_2 = 17\,M\Omega$.

$R_1 = 3\,M\Omega$. (From (1))

(51) From eqn (8.44),

$$V_{out} \propto \frac{d V_{in}}{dt}$$

(proportional)

Since offset is static (invariant with time)

i.e. $\frac{d V_{os}}{dt} = 0$.

$\therefore$ offset has no effect to V_{out}.

(5-2) From eqn (8.6),

with the presence of offset (V_{os}),

$$V_{out} = -V_T \ln \frac{V_{in} + V_{os}}{R_1 I_s}$$

The effect of offset to Vout is very small, because Vout is proportional to the log. of ($V_{in} + V_{os}$)

Thus, Vout is very insensitive to the magnitude of the offset.

(53). From eq$^{\underline{n}}$ (8.76),

$$V_{out} = R_1 I_{B2}$$

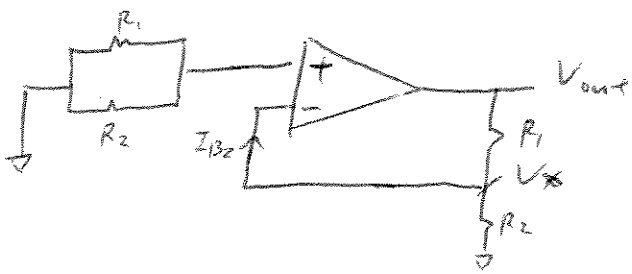
∴ V_{out} is independent of I_{B1}

Also I_{B1} will not affect $\frac{V_{out}}{V_{in}}$.

Thus, the small offset (ΔI) in the input bias currents has no effect on V_{out}.

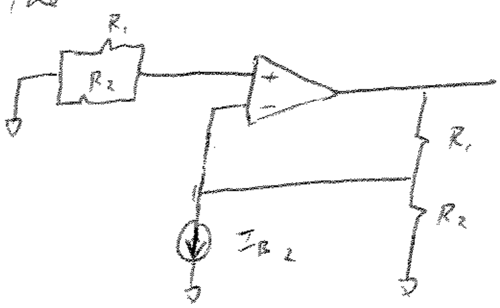
(54) Using superposition:

(I) turn off I_{B1}:



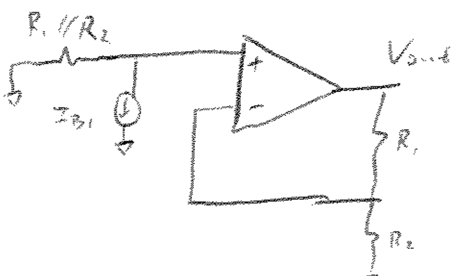
$\because V_+ = V_- = 0$, $\therefore V_x = 0$.

The circuit becomes:

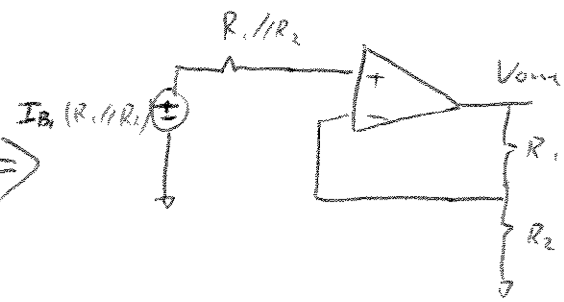


$\therefore$ From eq$\underline{n}$ (8.76), $V_{out,2} = -R_1 I_{B2}$

(II) turn off I_{B2}:



$\Rightarrow$



$$\therefore V_{out, I_{B1}} = I_{B1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) \times \left(1 + \frac{R_1}{R_2}\right)$$

$$= I_{B1} R_1$$

(54) Cont'd

$\therefore$ given $I_{B1} - I_{B2} = \Delta I$, and $V_{out} \; I_{B1} + I_{B2}$,

$$I_{B1} R_1 - I_{B2} R_1 < \Delta V$$

$$\Delta I \; R_1 < \Delta V$$

$$\therefore R_1 < \frac{\Delta V}{\Delta I}$$

There is no dependence of output error on R_2.

(55) Using eqn. (8.84)

$$\text{Gain} = \frac{A_0}{1 + \frac{s}{\omega_1}}$$

For opamp (a): At 100 MHz:

$$\text{Gain}_{(a)} = \frac{1000}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 50}}$$

$$\approx 5 \times 10^{-4}$$

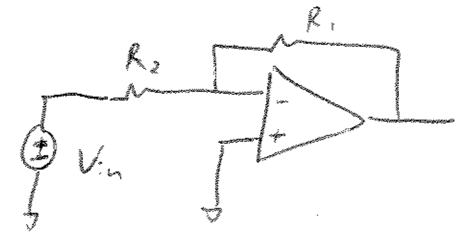
For opamp (b) at 100 MHz,

$$\text{Gain}_{(b)} = \frac{500}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 10}}$$

$$\approx 4.95 \quad > 4$$

$\therefore$ opamp (b) is a possible candidate

(5-6)



Using eqn (8.20),

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\frac{R_2}{R_1} + \frac{1}{A_0}\left(1+\frac{R_2}{R_1}\right)}$$

Here, A_0 becomes $\frac{A_0}{1+\frac{s}{\omega_1}}$.

$$\therefore \frac{V_{out}}{V_{in}} = \frac{-1}{\frac{R_2}{R_1} + \frac{A_0}{1+\frac{s}{\omega_1}}\left(1+\frac{R_2}{R_1}\right)}$$

$$= \frac{-\left(1+\frac{s}{\omega_1}\right)}{\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right)}$$

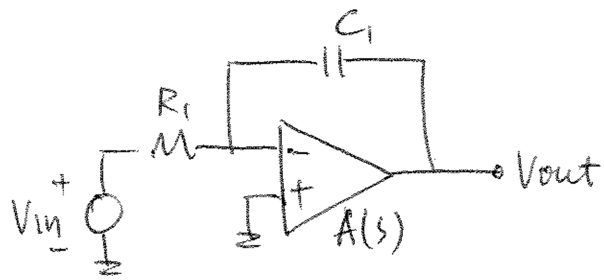
To find the pole, equate denominator to 0.

i.e. $\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right) = 0$

$$\left(1+\frac{s}{\omega_1}\right) = -\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)$$

$$\therefore |\omega_{p,closed}| = \left(1+\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)\right)\omega_1$$

57.



$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

$$\omega_0 \gg \frac{1}{R_1 C_1}$$

$$\frac{V_{in} - V_{(-)}}{R_1} = (V_{(-)} - V_{out})\, sC_1$$

$$-V_{(-)} \times A(s) = V_{out}$$

Substitute ② into ①:

$$\frac{V_{out}}{V_{in}}(s) = -\left[\frac{sC_1R_1 + 1}{A(s)} + sC_1R_1\right]^{-1}$$

$$= -\left[\frac{(sC_1R_1+1)(1+s/\omega_0)}{A_0} + sC_1R_1\right]^{-1}$$

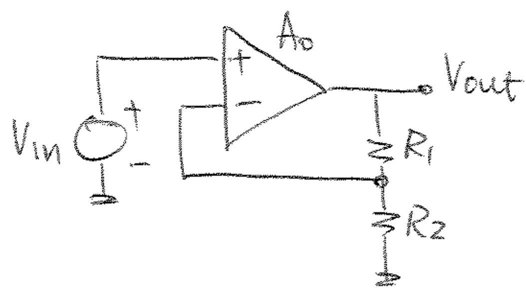
$$\simeq -\left\{\frac{1}{A_0\omega_0}\left[s\omega_0C_1R_1 + s^2C_1R_1 + s\right] + sCR\right\}^{-1}$$

$$\simeq -\left[s\left(C_1R_1 + \frac{1}{A_0\omega_0}\right) + s^2\left(\frac{C_1R_1}{A_0\omega_0}\right)\right]^{-1}$$

$$\approx -\left[sC_1R_1 + s^2\frac{C_1R_1}{A_0\omega_0}\right]^{-1}$$

$$= \frac{-1}{\left(1+\frac{s}{A_0\omega_0}\right)\frac{s}{\left(\frac{1}{R_1C_1}\right)}}$$

58.



Nominal Gain = 4
Slew Rate = 1V/ns
I/p = 0.5 V

$$V_{in}(t) = 0.5 \sin \omega t \Rightarrow V_{out} = 0.5 \times \underbrace{\left(1 + \frac{R_1}{R_2}\right)}_{=4} \sin \omega t.$$

$$\frac{dV_{out}}{dt} = 0.5\left(1 + \frac{R_1}{R_2}\right)\omega \cdot \cos \omega t.$$

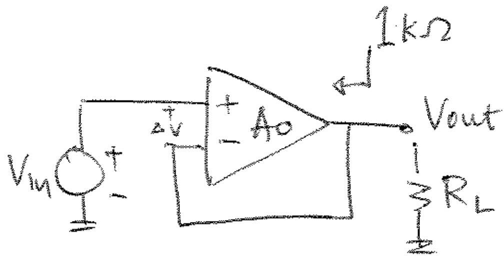
= maximum when $\cos \omega t = 1$

$$\Rightarrow \left.\frac{dV_{out}}{dt}\right|_{max} = 0.5\omega\left(1 + \frac{R_1}{R_2}\right) = 2\omega$$

∴ Highest frequency ⇒ $2\omega = 1\,V/ns$

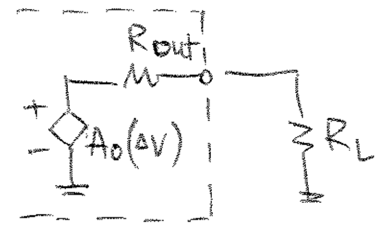
$$\Rightarrow \omega = 0.5\ rad/ns \Rightarrow f_{MAX} \simeq 79.6\ MHz$$

59.



$R_L = 100\,\Omega$

Gain Error $= 0.5\%$



$$(V_{in} - V_{out}) A_0 \times \frac{R_L}{R_{out} + R_L} = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_{out} + R_L}{A_0 R_L}} \simeq 1 - \underbrace{\frac{R_{out} + R_L}{A_0 R_L}}_{= \epsilon}$$

$$\therefore \epsilon = \frac{R_{out} + R_L}{A_0 R_L} \Rightarrow A_0 = \frac{R_{out} + R_L}{\epsilon R_L} = \frac{1000 + 100}{0.5\% \times 100}$$

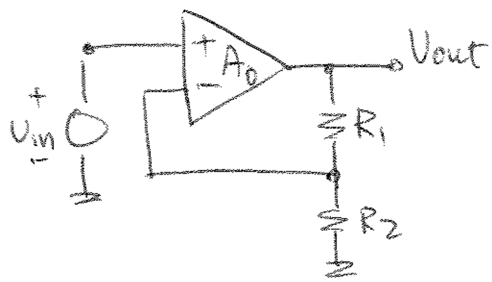
$$\approx 2200.$$

60.

Nominal Gain $= 4$

Gain Error $= 0.2\%$

$R_1 + R_2 = 20\,k\Omega$



$$\left[V_{in} - \frac{R_2}{R_1+R_2} \times V_{out}\right] A_0 = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \simeq \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$$

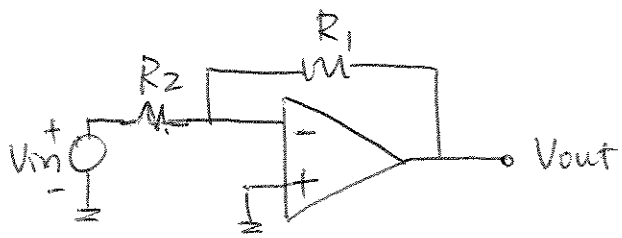
$(1 + R_1/R_2) = 4$ & $(R_1 + R_2) = 20\,k\Omega$

$\Rightarrow R_1 = 15\,k\Omega$, $R_2 = 5\,k\Omega$.

$$0.2\% = \left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0} \Rightarrow A_0 = \left(1+\frac{R_1}{R_2}\right) \times \frac{1}{0.2\%}$$

$$= 2000$$

61.



Nominal Gain = 8
Gain Error = 0.1%
Rout = 0.1%

$$V_x = V_{in} + (V_{out} - V_{in})\frac{R_2}{R_1+R_2} \quad \text{—— (1)}$$

$$\frac{V_{out} - V_{in}}{R_1+R_2} = \frac{-A_0 V_x - V_{out}}{R_{out}} \quad \text{—— (2)}$$

Substitute (2) into (1) gives:

$$\frac{V_{out}}{V_{in}} = \left(-\frac{R_1}{R_2}\right)\underbrace{\frac{A_0 - R_{out}/R_1}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}}_{(1-\varepsilon)}$$

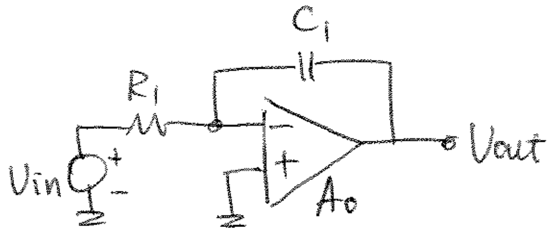
$$\Rightarrow \quad 8 = R_1/R_2$$

$$0.1\% = 1 - \frac{A_0 - 100/R_1}{1 + \frac{100}{R_2} + A_0 + (8)}$$

$\Rightarrow$ Choose $R_1 = 8\,k\Omega$, $R_2 = 1\,k\Omega$

$\Rightarrow$ $A_0 \approx 9100$

62.



$= 100\ kHz$

pole $= 100\ Hz$

$C_{MAX} = 50\ pF.$

$$\frac{V_{in} - V_{(-)}}{R_1} = (V_{(-)} - V_{out})\, sC_1 \quad \text{—— ①}$$

$$V_{(-)} \cdot (-A_0) = V_{out} \quad \text{—— ②}$$

Substitute ② into ① :

$$\frac{V_{out}}{V_{in}} = \frac{-1}{\frac{1}{A_0} + \left(1 + \frac{1}{A_0}\right) R_1 C_1 s}$$

$$\Rightarrow S_p = \frac{-1}{(A_0 + 1) R_1 C_1} = -100\ Hz \quad \text{—— ①}$$

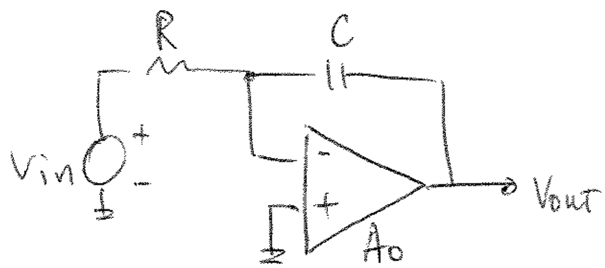
Attenuation above 100 kHz $\Rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{100kHz} = 1$

$$\Rightarrow \left.\frac{A_0}{\sqrt{1 + [(A_0+1) R_1 C_1 \omega_z]^2}}\right|_{100kHz} = 1 \quad \text{—— ②}$$

Substitute ① into ② :

$\Rightarrow A_0 \simeq 1000.$ Choose $C = 50pF \Rightarrow R \simeq 200\ k\Omega.$

63.



$V(t) = \alpha t$

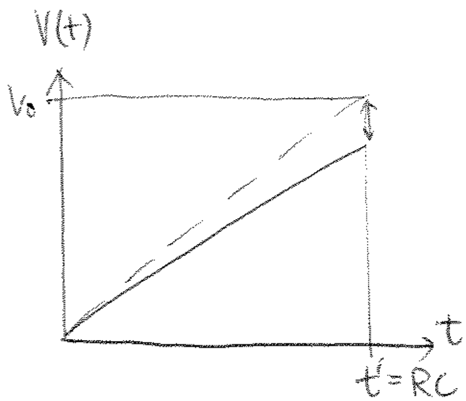
$0 < V(t) < V_0$

where $\alpha = 10 V/\mu s$

$V_0 = 1 V$

$C_{max} = 20 pF$

Error $< 0.1\%$



$V_{out}(t) = -\frac{V_0}{RC} t \quad , \quad t \in [0, RC]$

$V(t) = -\alpha t$

At $t = RC$, $\frac{\Delta V}{V_0} = \frac{V_{out}(t) - V(t)}{V_0} \Big|_{t=RC} < 0.1\%$

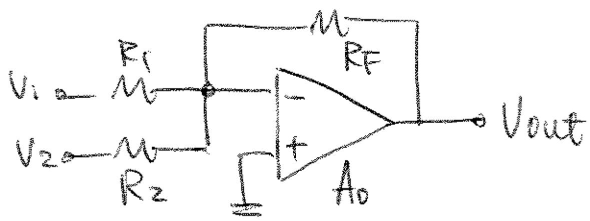
$\Rightarrow \Delta V = V_0 \times 0.1\% = 0.001 V$

$\Rightarrow -\frac{V_0}{RC} \times t + \alpha t \Big|_{t=RC} = 0.00 \ V \ (= \Delta V)$

Choose $C = 20 pF$

$\therefore R = \frac{V_0 - \Delta V}{\alpha C} = \frac{1V - 0.001 V}{10 V/\mu s \times 20 pF} = 4995 \Omega$

64.



$V_{out} = \alpha_1 V_1 + \alpha_2 V_2$, with $\alpha_1 = 0.5$, $\alpha_2 = 1.5$

Error of $\alpha \le 0.5\%$

$r_{in} \ge 10\,k\Omega$.

$$\frac{V_1 - V_{(-)}}{R_1} + \frac{V_2 - V_{(-)}}{R_2} = \frac{V_{(-)} - V_{out}}{R_F} \qquad \text{— ①}$$

$$V(-A_0) = V_{out} \qquad \text{— ②}$$

Substitute ② into ① & solve for V_{out}:

$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)\cdot\left[\frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right) + 1\right]^{-1}$$

$$\simeq -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)\cdot\left[1 - \frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right)\right]$$

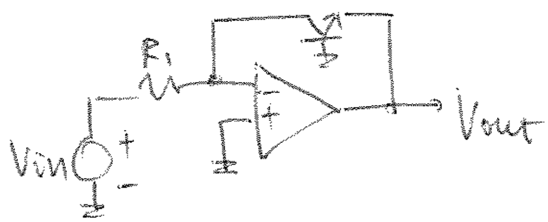
Choose $R_{in,v2}\ (\simeq R_2) = 10\,k\Omega \Rightarrow R_F = \alpha_2 \times R_2 = 15\,k\Omega$

$\Rightarrow R_1 = R_F/\alpha_1 = 30\,k\Omega \simeq R_{in,v_1}$

$$\Rightarrow \epsilon = 0.5\% = \frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right)$$

$$\Rightarrow A_0 = \frac{1}{0.5\%}\left(0.5 + 1.5 + 1\right) = 600 \quad \text{(or larger)}$$

65.



$[0.1, 2]V \longmapsto [-0.5, -1]V$

$$V_{out} = -V_T \ln \frac{V_{in}}{I_s R_1}$$

$$-0.5V = -V_T \ln \left[\frac{(0.1)}{I_s R_1}\right] \Rightarrow I_s R_1 = 4.45 \cdot 10^{-10} V \quad —①$$

$$\Rightarrow -V_T \ln \frac{(2)}{I_s R_1} = -0.026V \ln\left(\frac{2}{4.45 \cdot 10^{-10}}\right) \approx -0.58V$$

∴ input range of $0.1 \leftrightarrow 2$ V corresponds to output range of $-0.5 \leftrightarrow -0.58$ V

Choose $I_s = 1 \times 10^{-16} A \Rightarrow R_1 = 4.45 M\Omega$.

(66.) No, this is not possible to . . . requirements.

$$\frac{dV_{out}}{dV_{in}} = \frac{V_T}{V_{in}}$$

Assuming temperature is fixed, V_T is a fixed quantity that is both process and design independent.

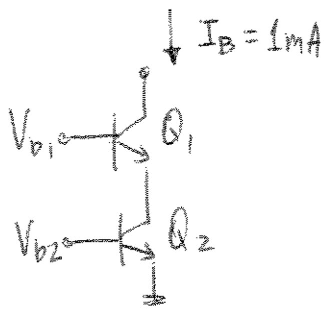
At 25°C, $V_T \approx 25mV$.

$$\therefore \left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=1V} = 25\,mV/V$$

$$\left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=2V} = 12.5\,mV/V$$

Ch9

1.



$I_S = 6 \cdot 10^{-17} A$ $\quad \beta = 100$

$V_A \rightarrow \infty$ $\quad V_T = \frac{kT}{q}$

$\alpha = \frac{\beta}{\beta + 1}$

(a) $V_{b2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V) \ln\left(\frac{1.02mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792\ V$

(b) From the configuration,

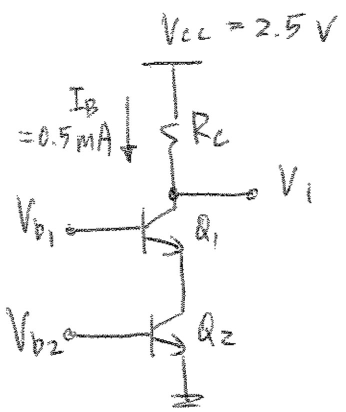
$V_{b1} = V_{CE2} + V_{BE1} = (V_{BE2} - 300mV) + V_{BE1}$

$V_{BE1} = V_T \ln(I_B/I_S) = (0.026V) \ln\left(\frac{1mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792\ V$

$\therefore V_{b1} = (0.792 - 0.3) + 0.79 = 1.28\ V$

2.



(a) $V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V)\ln\left(\frac{0.51\,mA}{6\cdot 10^{-17}A}\right)$

$\approx 0.774\ V$

$V_{BE1} = V_{b1} - V_{C2} = V_{b1} - (V_{b2} - 300mV)$

$\Rightarrow V_{b1} = V_{BE1} + V_{b2} - 0.3V$

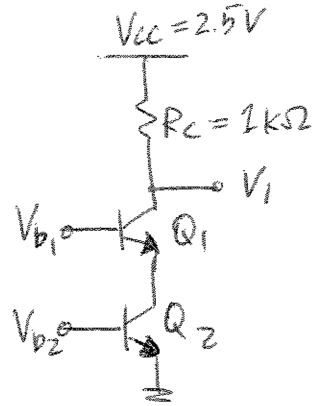
$= (0.026V)\ln\left(\frac{0.5mA}{6\cdot 10^{-17}A}\right) + (0.774V) - (0.3V)$

$\approx 1.25\ V$

(b) $V_1 = V_{b1} - 0.3V = 0.95V$

$\therefore R_C = \frac{V_{CC} - V_1}{I_B} = \frac{(2.5 - 0.95)V}{0.5mA} \approx 3.1\ k\Omega$

3. From previous experience, assume both V_{BE1} & $V_{BE2} = 0.8\,V$



$\Rightarrow V_1 = V_{CE_1} + V_{CE_2}$

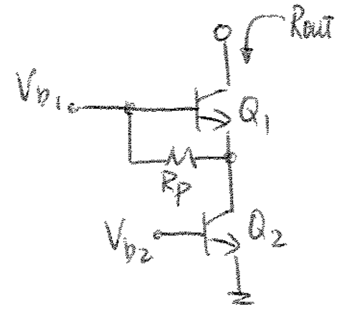
$= (V_{BE_1} - 200mV) + (V_{b2} - 200mV)$

$= 1.2\ V$

• By KCL, maximum bias current

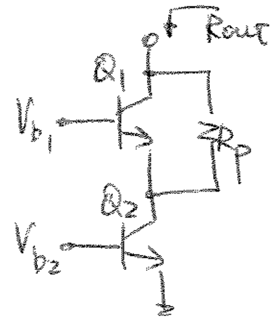
$$\approx \frac{V_{cc} - V_1}{R_c} = \frac{(2.5 - 1.2)V}{1k\Omega} = 1.3\ mA.$$

4. (a) R_P appears in parallel with $r_{\pi 1}$



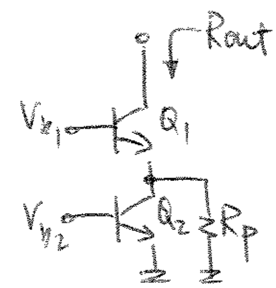
$$\therefore R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1} \| R_P)] r_{o1} + (r_{o2} \| r_{\pi 1} \| R_P)$$

(b) R_P appears in parallel with r_{o1}



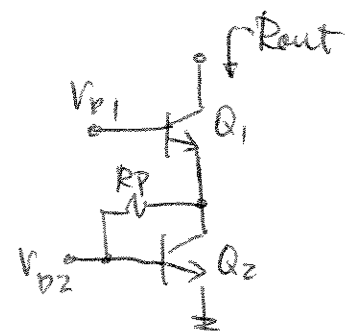
$$\therefore R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1})](r_{o1} \| R_P) + (r_{o2} \| r_{\pi 1})$$

(c) R_P appears in parallel with r_{o2}



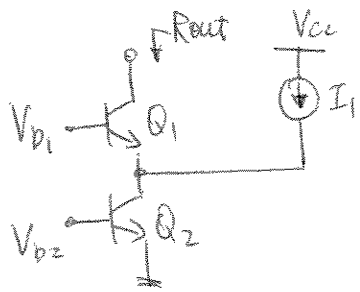
$$\therefore R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1} \| R_P)] r_{o1} + (r_{o2} \| r_{\pi 1} \| R_P)$$

(d) R_P appears in parallel with r_{o2} (in small-signal) $\because V_{b2}$ is AC GND.



$$\therefore R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1} \| R_P)] r_{o1} + (r_{o2} \| r_{\pi 1} \| R_P)$$

5.



$I_1 = 0.5\,mA$

$I_{C1} = 0.5\,mA \qquad I_{C2} = 1\,mA.$

$= 2 I_{C1}$

$\beta = 100 \qquad V_A = 5V$

$$R_{out} = g_{m1} r_{o1} (r_{o2} \parallel r_{\pi 1})$$

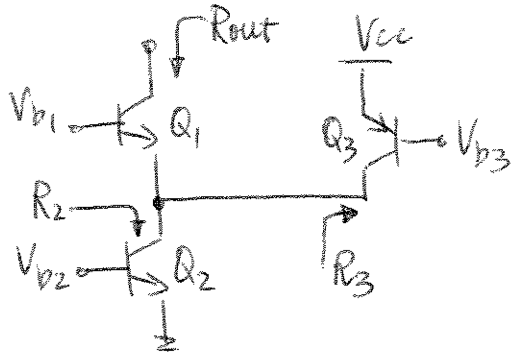
$$= \frac{I_{C1}}{V_T} \cdot \frac{V_A}{I_{C1}} \cdot \frac{V_{A2}/I_{C2} \cdot \beta V_T / I_{C1}}{V_{A2}/I_{C2} + \beta V_T / I_{C1}}$$

$$= \frac{V_A}{V_T} \cdot \frac{\frac{V_{A2}/2}{I_{C1}} \cdot \beta V_T / I_{C1}}{\frac{V_{A2}/2}{I_{C1}} + \beta V_T / I_{C1}} \approx \frac{1}{I_{C1}} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + 2\beta V_T}$$

$$= \frac{1}{0.5mA} \cdot \frac{5V}{0.026V} \cdot \frac{100(5V)(0.026V)}{(5V) + 2(100)(0.026V)}$$

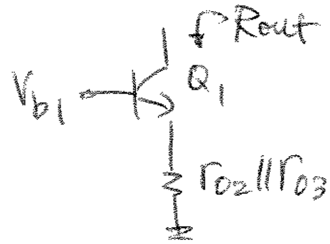
$\therefore$ Rout $\approx 490.\ k\Omega.$

6.



$R_3 = r_{o3}$ (V_{cc} & V_{b3} are AC GND)

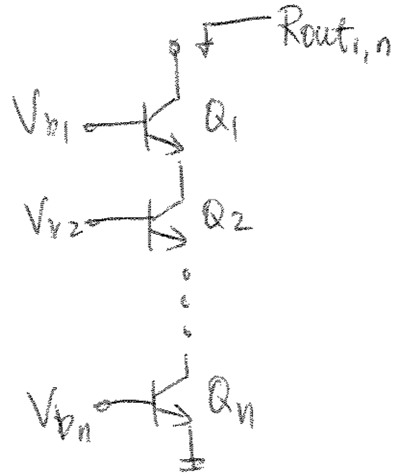
$R_2 = r_{o2}$ (V_{b2} is AC GND)



$$\therefore R_{out} = [(1+g_{m1}(r_{o2}\|r_{o3}\|r_{\pi 1})]r_{o1} + (r_{o2}\|r_{o3}\|r_{\pi 1})$$

$$\approx g_{m1} r_{o1} (r_{o2}\|r_{o3}\|r_{\pi 1})$$

7.



Suppose $Rout_{i,j}$ is the output impedance of the cascode circuit with BJTs $Q_i, Q_{i+1}, Q_{i+2}, \ldots Q_{j-1}, Q_j$.

$$Rout_{n-1,n} = [1 + g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}})] r_{o_{n-1}} + (r_{o_n} \| r_{\pi_{n-1}})$$

$$\approx g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}}) r_{o_{n-1}} \approx g_{m_{n-1}} r_{\pi_{n-1}} r_{o_{n-1}}$$

$$= \beta r_o \qquad (\text{usually, } r_\pi \ll r_o)$$

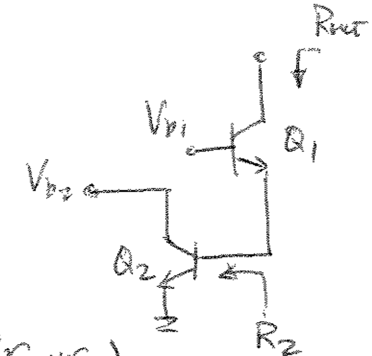
$$Rout_{n-2,n} = [1 + g_{m_{n-2}}(\beta r_o \| r_{\pi_{n-2}})] r_o + (\beta r_o \| r_{\pi_{n-2}})$$

$$\approx g_m r_\pi r_o + r_\pi \cong \beta r_o$$

This means that $Rout \approx \beta r_o$ even if an extra BJT is employed in the cascode configuration.

i.e. $Rout_{MAX} \approx \beta r_o$

8. (a) $R_2 = (r_{\pi_2} \parallel r_{\pi_1})$



$$\therefore R_{out} = [1 + g_{m1} R_2] r_{o1} + R_2$$

$$= [1 + g_{m1}(r_{\pi_1} \parallel r_{\pi_2})] r_{o1} + (r_{\pi_1} \parallel r_{\pi_2})$$

(b) In part (a), $I_{C2} = \beta I_{C1}$ $(= I_{B_2})$

$$\therefore R_{out(a)} = \left[1 + g_{m1}\left(\frac{\beta V_T}{I_{C1}} \parallel \frac{V_T}{I_{C1}}\right)\right] r_{o1} + (r_{\pi_1} \parallel r_{\pi_2})$$

$$\approx \left(1 + g_{m1}\frac{V_T}{I_{C1}}\right) r_{o1} + \frac{V_T}{I_{C1}}$$

$$= 2r_{o1} + V_T / I_{C1}$$

$$R_{out,cascode} = [1 + g_{m1}(r_{o2} \parallel r_{\pi_1})] r_{o1} + (r_{o2} \parallel r_{\pi_1})$$

$$\approx [1 + g_{m1} r_{\pi_1}] r_{o1} + r_{\pi_1}$$

$$\approx \beta r_{o1} + r_{\pi_1} = \beta r_{o1} + V_A / I_{C1}$$

Compare term-by-term:

$$\left.\begin{array}{l} 2r_{o1} \ll \beta r_{o1} \\ V_T \ll V_A \end{array}\right\} \Rightarrow R_{out(a)} \ll R_{out,cascode}$$

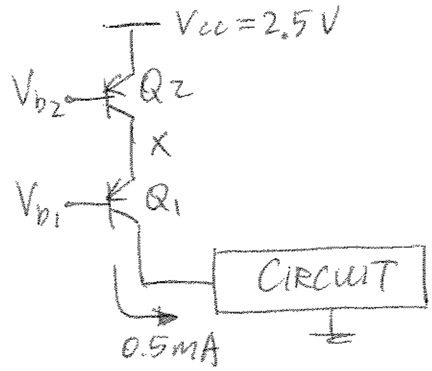
i.e. using (a) reduces the effect of having a cascode configuration.

9. $$R_{out} = \frac{1}{I_C} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + \beta V_T}$$

$$\approx \frac{1}{I_C} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A} = \beta \frac{V_A}{I_C} = \beta r_o$$

$$= R_{out,max}$$

This means that $R_{out,max}$ is often achieved with 2-BJT cascode.

10. 

$I_S = 10^{-16}$ A $\beta = 100$

$I_{BIAS} = 0.5$ mA

(a) $I_{BIAS} \approx I_{C2} = 0.5$ mA

$$\therefore V_{b2} = V_{CC} - |V_{BE2}|$$
$$= V_{CC} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (2.5\text{V}) - (0.026\text{V})\ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.74\text{V}$$

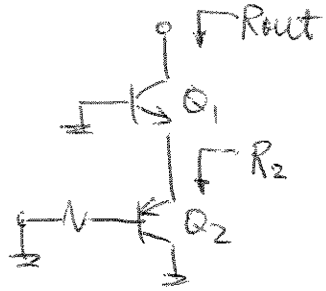
(b) $|V_{CB2}| = V_X - V_{b2} = 200$mV

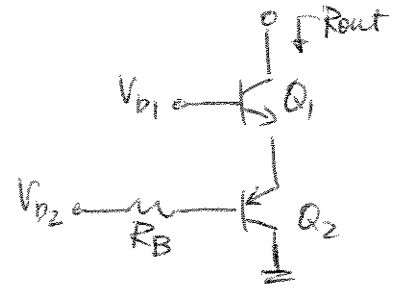
$\Rightarrow V_{C2} = V_{b2} + |V_{CB2}| = 1.94$V

$$\therefore V_{b1} = V_{C2} - |V_{BE1}| = V_{C2} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (1.94\text{V}) - (0.026\text{V})\ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.18\text{V}$$

$\Rightarrow$ Maximum allowable $V_{b1} = 1.18$V

11. (a)





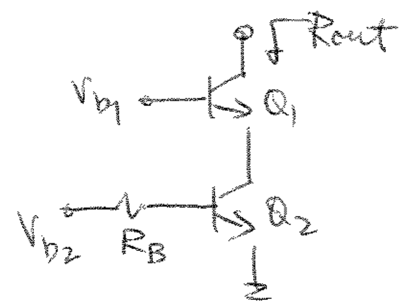
(Ac-small signal)

Looking into emitter of Q2,

$$R_2 = \frac{1}{\left(\frac{\beta+1}{R_B + r_{\pi_2}} + \frac{1}{r_{o_2}}\right)}$$

$$\Rightarrow R_{out} = [1 + g_{m_1}(R_2 \parallel r_{\pi_1})] r_{o_1} + (R_2 \parallel r_{\pi_1})$$

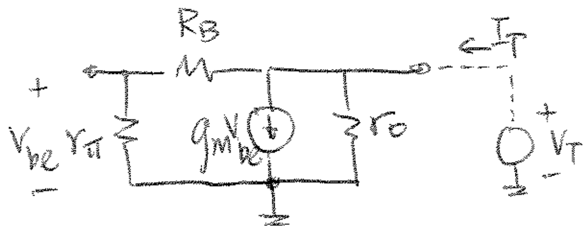
(b) R_B does not affect Q_2 in small-signal R_{out}:

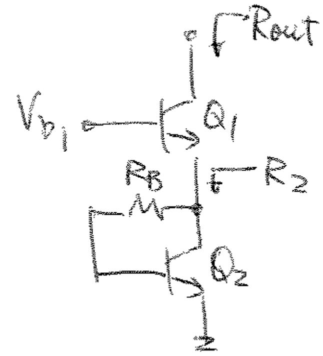


$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})] r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

This is a cascode stage.

(c) Use small-signal analysis:



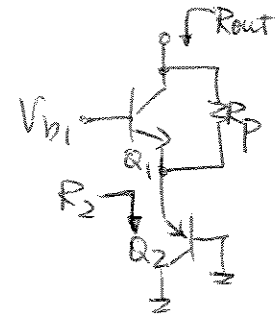


By KCL, $I_T = \dfrac{V_T}{R_B + r_{\pi_2}} + g_{m_2}\dfrac{V_T\, r_{\pi_2}}{r_{\pi_2} + R_B} + \dfrac{V_T}{r_{o_2}}$

$$\Rightarrow \quad R_2 = \frac{V_T}{I_T} = \frac{1}{\left(\dfrac{\beta+1}{R_B + r_{\pi_2}} + \dfrac{1}{r_o}\right)} \approx \frac{1}{\beta/(R_B + r_{\pi_2}) + 1/r_{o_2}}$$

$$\therefore R_{out} = \left[1 + g_{m_1}(R_2 \| r_{\pi_1})\right] r_{o_1} + (R_2 \| r_{\pi_1})$$
$$\approx g_{m_1} r_{o_1} (R_2 \| r_{\pi_1})$$

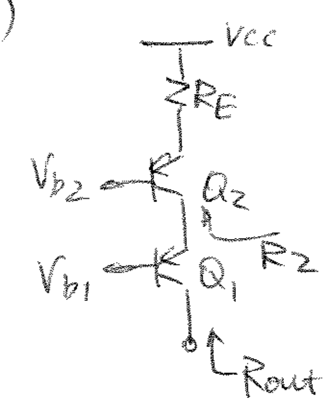
(d) R_p appears in parallel with r_{o1}.



$$R_2 = r_{o2} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}$$
$$\approx r_{o2} \parallel \frac{r_{\pi 2}}{\beta} \parallel r_{\pi 2}$$
$$\approx r_{o2} \parallel (r_{\pi 2}/\beta) \approx r_{\pi 2}/\beta$$

$$\therefore R_{out} = [1 + g_{m1}(R_2 \parallel r_{\pi 1})](r_{o1} \parallel R_p) + (R_2 \parallel r_{\pi 1})$$
$$\approx g_{m1}(r_{o1} \parallel R_p)(r_{\pi 1} \parallel R_2)$$

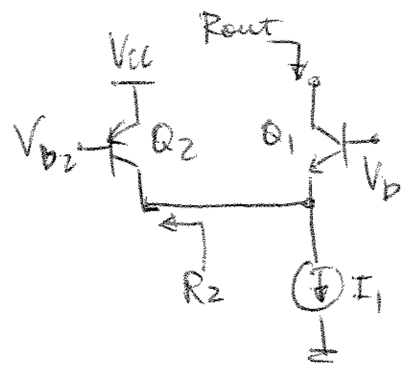
(e) $$R_2 = [1 + g_{m2}(R_E \parallel r_{\pi 2})] r_{o2} + (R_E \parallel r_{\pi 2})$$
$$\approx g_{m2}(R_E \parallel r_{\pi 2}) r_{o2}$$



$$\therefore R_{out} = [1 + g_{m1}(R_2 \parallel r_{\pi 1})] r_{o1} + (R_2 \parallel r_{\pi 1})$$
$$\approx g_{m1}(R_2 \parallel r_{\pi 1}) r_{o1}$$
$$= g_{m1} [g_{m2} r_{o2} (R_E \parallel r_{\pi 2}) \parallel r_{\pi 1}] r_{o1}$$

This is a cascode stage.

(f) $R_2 = r_{o2}$



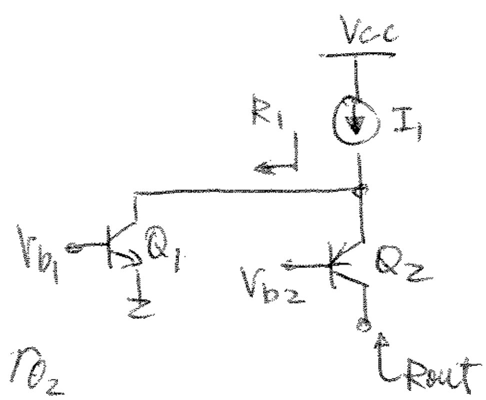
$$\therefore R_{out} = [1 + g_{m1}(R_2 \| r_{\pi 1})] r_{o1} + (R_2 \| r_{\pi 1})$$

$$\approx g_{m1} r_{o1} (R_2 \| r_{\pi 1})$$

$$= g_{m1} r_{o1} (r_{\pi 1} \| r_{o2})$$

(g) $R_1 = r_{o1}$

(output impedance of a common-emitter)

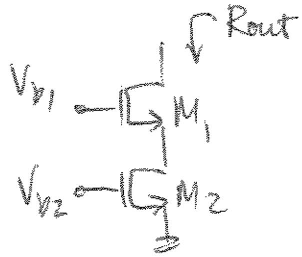


$$\therefore R_{out} = [1 + g_{m2}(R_1 \| r_{\pi 2})] r_{o2}$$

$$+ (R_1 \| r_{\pi 2})$$

$$\approx g_{m2} r_{o2} (r_{o1} \| r_{\pi 2})$$

12.



$I_D = 0.5mA$ $R_{out} \geq 50k\Omega$

$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$ $\frac{W}{L} = \frac{20}{0.18}$

Calculate max λ.

Assume M_1 & M_2 in saturation.

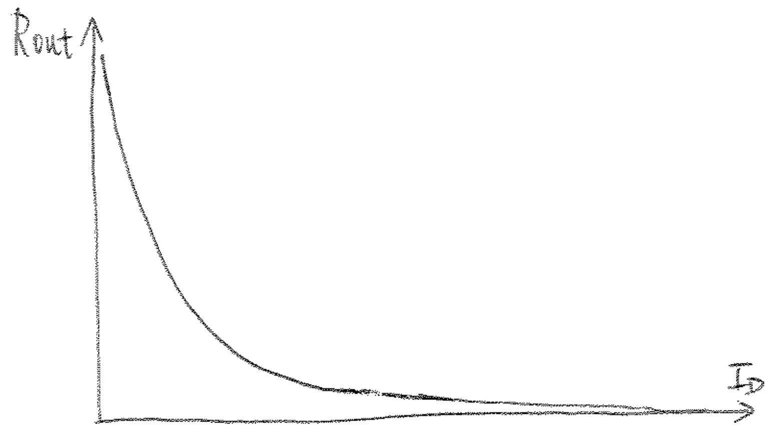
$$\Rightarrow R_{out} \approx g_{m1} r_{o1} r_{o2}$$
$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \times \frac{1}{\lambda I_D} \times \frac{1}{\lambda I_D} \geq 50k\Omega.$$

(All quantities are known).
Solve for λ:

$$\lambda_{max} \approx 0.51 \ V^{-1}$$

13. (a) $R_{out} = g_{m2} r_{o1} r_{o2} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$

$= 2 \mu_n C_{ox} (W/L) \cdot (I_D)^{-3/2}$

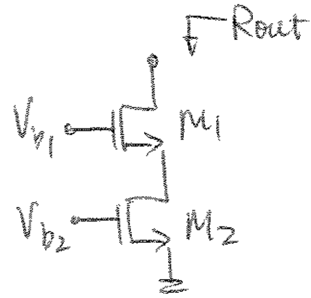


(b) $R_{out}\ (BJT) \propto I_B^{-1}$

$R_{out}\ (MOS) \propto I_B^{-3/2}$

∴ MOS cascode is a stronger function of I in terms of Rout.

14.



$\left(\frac{W}{L}\right)_1 = 30/0.18 \qquad \left(\frac{W}{L}\right)_2 = 20/0.18$

$I_{BIAS} = 0.5\,mA$

$\mu_n C_{ox} = 100\,\frac{\mu A}{V^2} \qquad V_{TH} = 0.4\,V$

(a) $I_{D2} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + V_{TH}$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(20/0.18)}} + 0.4V \approx 0.7\,V$$

M_2 operates in saturation as long as

$V_{GS2} - V_{TH} \le V_{DS2} \Rightarrow V_{DS2} \ge 0.3\,V.$

Observe that $V_{GS1} = V_{b1} - V_{DS2}$

$$I_{D1} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{DS2} - V_{TH})^2$$

$$\Rightarrow V_{b1} \ge \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + 0.4V + 0.3V$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(30/0.18)}} + 0.7V \approx 0.95\,V.$$

$\therefore$ Minimum $V_{b1} = 0.95V.$

(b) $R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$

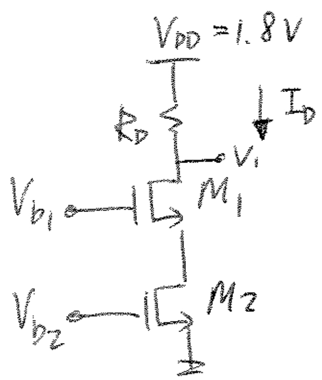
$$= \left(1 + \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{BIAS}} \cdot \frac{1}{\lambda I_{BIAS}}\right) \cdot \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$= \left[1 + \sqrt{2\left(100 \frac{\mu A}{V^2}\right)\left(\frac{30}{0.18}\right)(0.5 mA)} \cdot \frac{1}{(0.1)(0.5m)}\right] \cdot \frac{1}{(0.1)(0.5m)}$$

$$+ \frac{1}{(0.1)(0.5 mA)}$$

$$\approx 1.67 \ M\Omega$$

15.



$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\left(\frac{W}{L}\right)_2 = \frac{40}{0.18}$

$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$ $V_{TH} = 0.4V.$

$I_D = 1 mA$ $R_D = 500\Omega$

(a) Both M_1 & M_2 must stay in saturation.

$\Rightarrow V_1 = 1.8 - I_D R_D = 1.8 - (1mA)(500\Omega) = 1.3V$

Want this value equal to that which makes M_1 operates at the edge of saturation.

$\therefore V_{b1} = V_1 + V_{TH} = 1.3 + 0.4 = 1.7 V.$

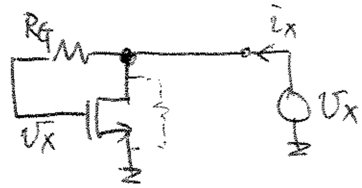
(b) $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot \left[(V_{b1} - V_X) - V_{TH}\right]^2 = 1mA.$

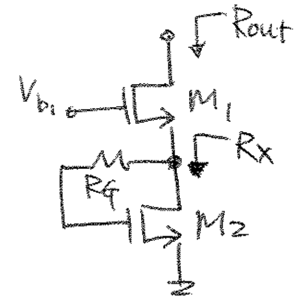
$$\Rightarrow V_X = V_{b1} - V_{TH} - \sqrt{\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}$$

$$= (1.7V) - (0.4V) - \sqrt{\frac{2(1mA)}{(100\, \mu A/V^2)(20/0.18)}}$$

$\approx 1.276 V.$

1b. (a) First compute R_x:

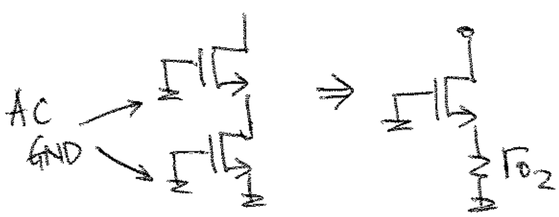


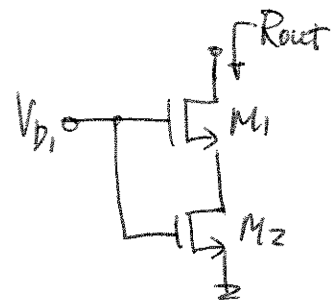


$$I_x = g_{m2} v_x + v_x / r_{o2} \Rightarrow R_x = v_x / i_x = \frac{1}{g_{m2} + 1/r_{o2}}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_x = \frac{g_{m1} r_{o1}}{g_{m2} + 1/r_{o2}}$$

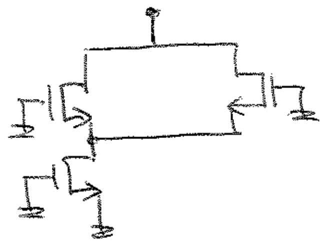
(b) Equivalent circuit:

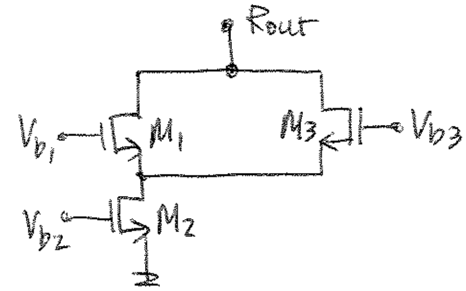


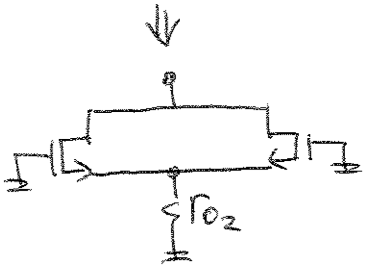


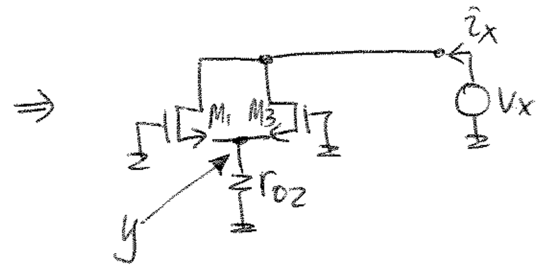
$$\therefore R_{out} = g_{m1} r_{o1} r_{o2}$$

(c) Equivalent Circuit:









By KCL, $V_y = i_x \cdot r_{o2}$ ——— ①

$$i_x = g_{m1}(-V_y) + g_{m3}(-V_y) + (v_x - V_y)\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right) \quad ②$$

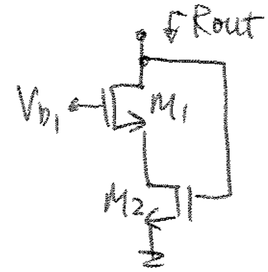
Substitute ① into ② and re-arrange:

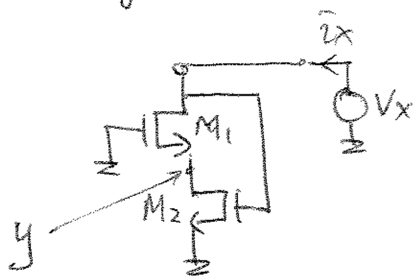
$$R_{out} = \frac{v_x}{i_x} = (r_{o1} \parallel r_{o3}) + r_{o2}(r_{o1} \parallel r_{o3})(g_{m1} + g_{m3}) + r_{o2}$$

$$\approx r_{o2}(r_{o1} \parallel r_{o3})(g_{m1} + g_{m3})$$

(Intuitively this makes sense because we have 2 NMOSs in parallel — ⊕ = $g_m v_{gs}$ adds up, and r_o's are splitting total current, i_x. This is as if an equivalent NMOS replacing M_1 & M_3 with $g_m = (g_{m1} + g_{m3})$ & $r_o = (r_{o1} \parallel r_{o3})$.)

(d) Examine the equivalent circuit with a test voltage:





By observation, i_x must flow through both M_1 & M_2.

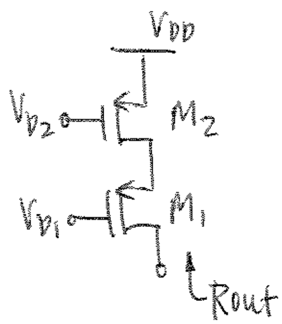
By KCL,
$$i_x = g_{m2} v_x + v_y / r_{o2}$$
$$i_x = g_{m1}(-v_y) + (v_x - v_y)/r_{o1}$$

Substitute ① into ② and re-arrange:

$$R_{out} = \frac{v_x}{i_x} = \frac{g_{m1} r_{o2} + \frac{r_{o2}}{r_{o1}} + 1}{g_{m1} g_{m2} r_{o2} + (g_{m2} r_{o2} + 1)(\frac{1}{r_{o1}})}$$

$$\approx \frac{r_{o2}(g_{m1} + \frac{1}{r_{o1}})}{g_{m2} r_{o2}(g_{m1} + \frac{1}{r_{o1}})} \approx \frac{1}{g_{m2}}$$

17.



$I_{BIAS} = 0.5\ mA$

$R_{out} = 40\ k\Omega$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda = 0.2\ V^{-1}$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$$R_{out} = 40\ k\Omega = (g_{m1} r_{o2} + 1) r_{o1} + r_{o2}$$

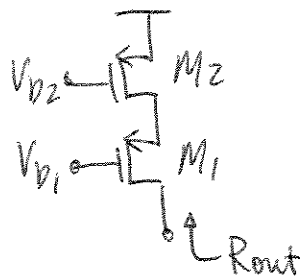
$$\Rightarrow g_{m1} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_{BIAS}} = \left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \cdot \frac{1}{r_{o2}}$$

$$\therefore \left(\frac{W}{L}\right)_1 = \left[\left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \frac{1}{r_{o2}}\right]^2 \cdot \frac{1}{2 \mu_p C_{ox} I_{BIAS}}$$

$$= \left\{\left[\frac{(40k\Omega) - [(0.2)(0.5m)]^{-1}}{[(0.2)(0.5m)]^{-1}} - 1\right] \cdot [0.2 \cdot 0.5m]\right\}^2 \cdot \frac{1}{2\left(50 \frac{\mu A}{V^2}\right)(0.5mA)}$$

$$\approx 0.8$$

18.



$$R_{out} = g_{m1} r_{o1} r_{o2} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

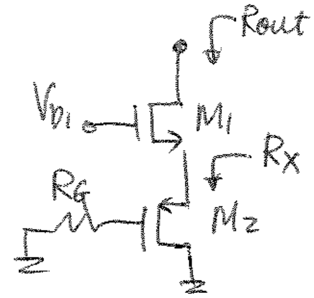
If W_1 & W_2 increase by N times and L_1, L_2, and I_D remain unchanged:

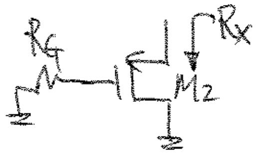
$$R_{out}(\text{New}) = \sqrt{2\mu_p C_{ox} \left(\frac{NW}{L}\right) I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= \sqrt{N} \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} \left(\frac{1}{\lambda I_D}\right)^2 = \sqrt{N}\, R_{out}$$

$\therefore$ Rout is increased by $\sqrt{N}$ times.

19. (a) R_X is the input impedence of a common-gate configuration:





"Looking into" the source of M_2,

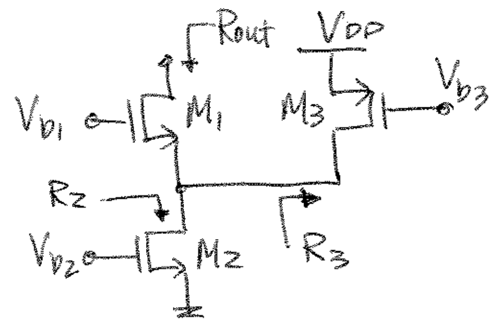
$$R_X = \frac{1}{g_{m2}} \parallel r_{o2}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_X = g_{m1} r_{o1} \left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$$

(b) From observation,

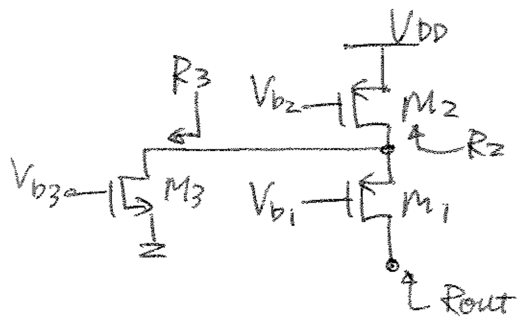
→ $R_3 = r_{o3}$ ($\because V_{SG} = 0$ in AC)

→ $R_2 = r_{o2}$ ($\because V_{SG} = 0$ in AC)



$$\therefore R_{out} = g_{m1} r_{o1} (R_2 \parallel R_3) = g_{m1} r_{o1} (r_{o2} \parallel r_{o3})$$

(c) By observation,

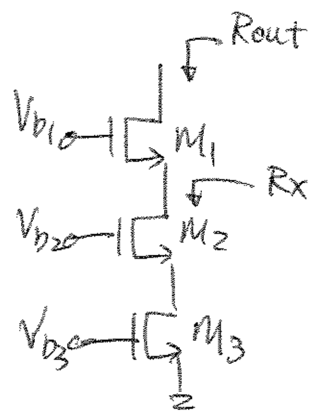


$R_2 = r_{o2}$ ($V_S = V_G = AC\ GND$)

$R_3 = r_{o3}$ ($V_S = V_G = AC\ GND$)

$\therefore R_{out} = g_{m1} r_{o1} (R_2 \| R_3) = g_{m1} r_{o1} (r_{o2} \| r_{o3})$

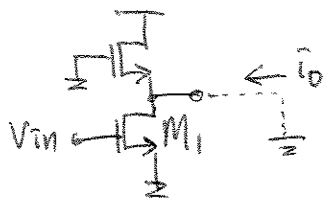
(d) $R_X = g_{m2} r_{o2} r_{o3}$

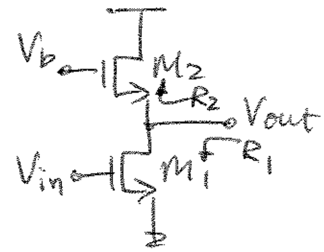


$\Rightarrow R_{out} = g_{m1} r_{o1} R_X$

$= g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}$

20.(a) Equivalent circuit :



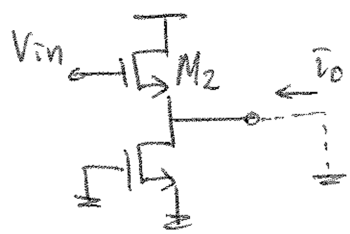


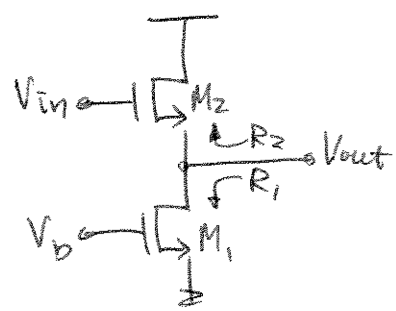
$$i_o = G_m v_{in} \Rightarrow G_m = \frac{i_o}{v_{in}} = g_{m1}$$

$$R_1 = r_{o1} \; ; \; R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(r_{o1} \,//\, 1/g_{m2})$$

(b) Equivalent circuit :



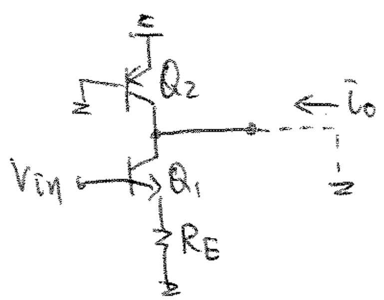


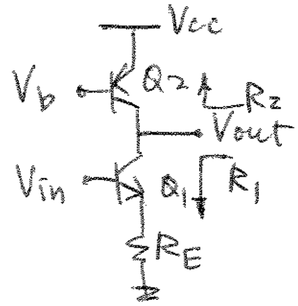
$$-i_o = g_{m2} V_{in} \Rightarrow G_m = \frac{i_o}{V_{in}} = -g_{m2}$$

$$R_1 = r_{o1} \; ; \; R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = g_{m2}(r_{o1} \,//\, 1/g_{m2})$$

(c) Equivalent circuit:





With output node shorted, this is a common-emitter stage with degeneration.

$$\Rightarrow G_m = \frac{g_{m1}}{g_{m1}(R_E \| r_{o1}) + 1}$$

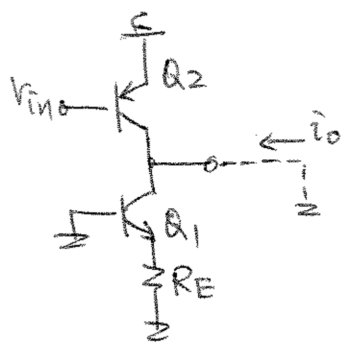
$$R_1 = [1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})$$

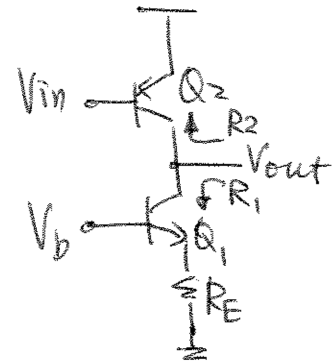
$$R_2 = r_{o2}$$

$$\Rightarrow R_{out} = R_1 \| R_2$$

$$\therefore A_v = -G_m R_{out} = -\frac{g_{m1}\left(\{[1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})\} \| r_{o2}\right)}{g_{m1}(R_E \| r_{o1}) + 1}$$

(d) Equivalent circuit:





With output shorted to AC GND, circuit becomes a simple common-emitter stage:

$$\Rightarrow G_m = g_{m2}$$

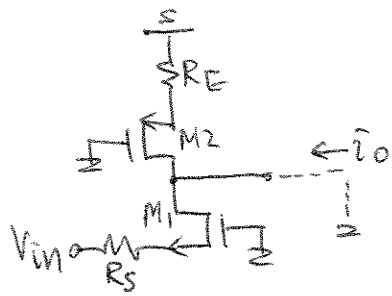
$$R_1 = [1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})$$

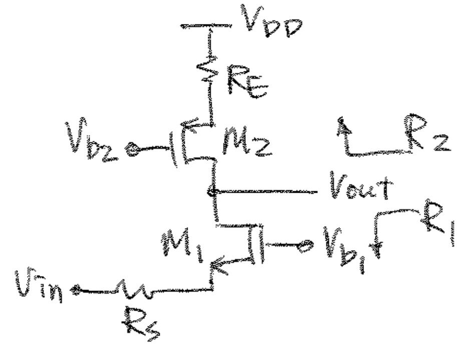
$$R_2 = r_{o2}$$

$$\Rightarrow R_{out} = R_1 \| R_2$$

$$\therefore A_v = -G_m R_{out} = -g_{m2}\left(\{[1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})\} \| r_{o2}\right)$$

(e) Equivalent circuit:





Observing that i_o must flow through M_1 only:

$$i_o = g_{m1}\left(-\left(\underbrace{v_{in} + i_o R_S}_{\text{gate voltage of } M_1}\right)\right)$$

$$\Rightarrow G_m = \frac{i_o}{v_{in}} = \frac{-g_{m1}}{1 + g_{m1} R_S}$$

$$R_1 = (1 + g_{m1} R_S) r_{o1} + R_S$$

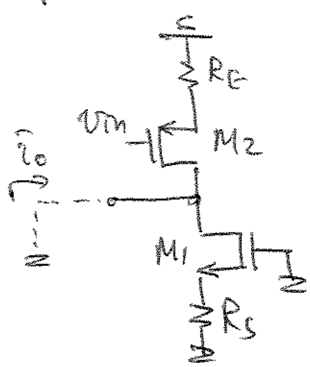
$$R_2 = (1 + g_{m2} R_E) r_{o2} + R_E$$

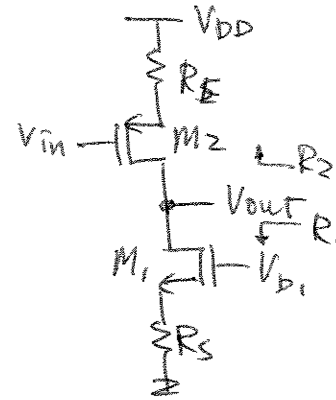
$$R_{out} = R_1 \parallel R_2$$

$$\therefore A_v = -G_m R_{out}$$

$$= \frac{g_{m1}}{1 + g_{m1} R_S}\left\{\left[(1 + g_{m1} R_S) r_{o1} + R_S\right] \parallel \left[(1 + g_{m2} R_E) r_{o2} + R_E\right]\right\}$$

(f) Equivalent circuit:





This is a common-source stage with degeneration:

$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}R_E}$$

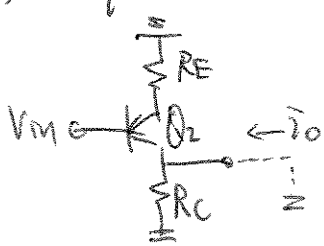
$$R_1 = (1+g_{m1}R_S)r_{o1} + R_S$$

$$R_2 = (1+g_{m2}R_E)r_{o2} + R_E$$

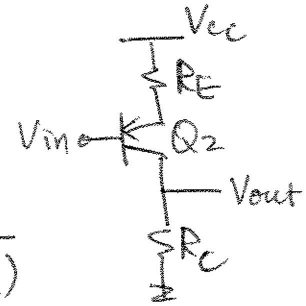
$$\therefore A_v = -G_m(R_1 \| R_2)$$

$$= \frac{g_{m2}}{1+g_{m2}R_E}\left\{\left[(1+g_{m1}R_S)r_{o1}+R_S\right] \| \left[(1+g_{m2}R_E)r_{o2}+R_E\right]\right\}$$

(g) Equivalent circuit:



$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}(R_E \| r_{\pi 2})}$$



$$R_{out} = \left\{\left[1+g_{m2}(R_E \| r_{\pi 2})\right]r_{o2} + (R_E \| r_{\pi 2})\right\} \| R_C$$

$$\Rightarrow A_v = -G_m R_{out} = \frac{g_{m2}R_{out}}{1+g_{m2}(R_E \| r_{\pi 2})}$$

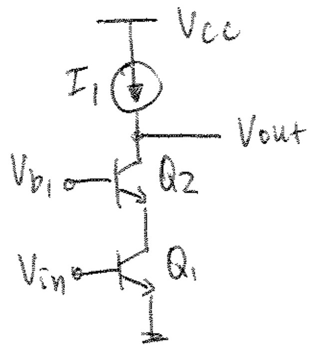
21. $A_v = -g_{m1} r_{o1} g_{m1} (r_{o1} \| r_{\pi 2})$

$$= -\frac{I_{C1}}{V_T} \cdot \frac{V_{A1}}{I_{C1}} \cdot \frac{I_{C1}}{V_T} \cdot \frac{1}{\frac{I_{C1}}{V_{A1}} + \frac{I_{C2}}{\beta V_T}}$$

Since $I_{C1} \approx I_{C2}$,

$$A_v \approx -\frac{V_{A1}/V_T^2}{\frac{1}{V_{A1}} + \frac{1}{\beta V_T}} = -\frac{\beta V_A^2}{V_T(V_A + \beta V_T)}$$

22.

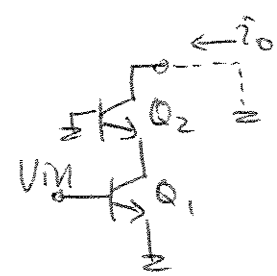


$A_v = 500$ $\quad I_1 = 1mA$

$\beta_1 = \beta_2 = 100$

Determine minimum $V_{A_1} = V_{A_2}$.

Using small-signal analysis,



$$G_M = \frac{i_o}{v_{in}} = g_{m_1}\left(\frac{\beta+1}{\beta}\right) = \frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right)$$

$$R_{out} = \left[1 + g_{m_2}(r_{o_1} \| r_{\pi 2})\right] r_{o2} + (r_{o_1} \| r_{\pi 2})$$

$$\approx g_{m2}(r_{o_1} \| r_{\pi_2}) r_{o2} = \frac{\beta V_A^2}{I_C(V_A + \beta V_T)}$$

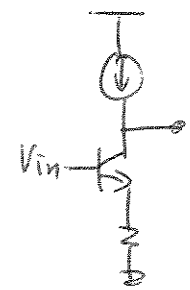
$$\Rightarrow A_v = -G_m R_{out}$$

$$= -\frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right) \cdot \frac{\beta V_A^2}{I_C(V_A + \beta V_T)} = 500$$

$\Rightarrow$ All values are given. V_A is solved using the quadratic formula:

$\therefore V_A \approx 0.65$ V

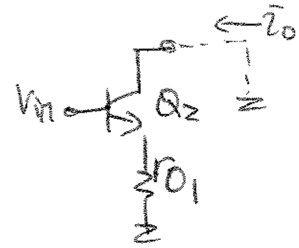
23. (a) Even though Rout is independent of where Vin is applied, Gm changes:



The circuit is a commom-emitter with degeneration, which always has Gm ≤ Gm of commom-emitter stage without degeneration.

Alternatively, this circuit has less gain because it only has one amplifier stage.

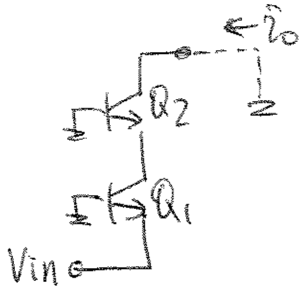
(b) $G_m = \dfrac{i_o}{v_{in}} = \dfrac{g_{m2}}{1 + g_{m2}(r_{o1} \| r_{o2})}$



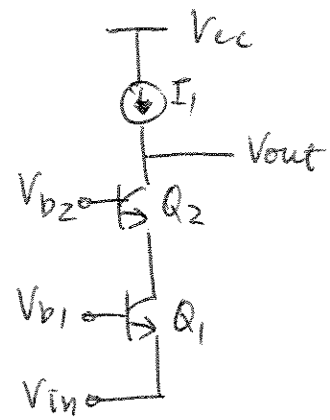
$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}$

$\Rightarrow A_v = -G_m R_{out} = -\dfrac{g_{m2}\{[1 + g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}\}}{1 + g_{m2}(r_{o1} \| r_{o2})}$

24. Equivalent circuit:



$\beta \to \infty.$

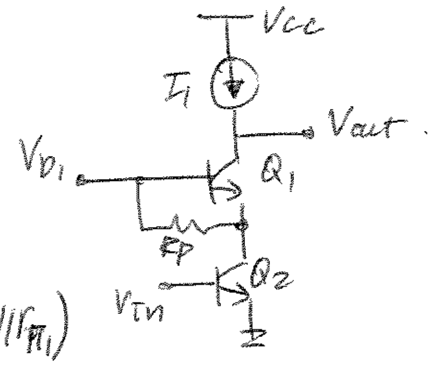


$$G_m = \frac{i_o}{V_{in}} \approx -g_{m1}$$

$$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})]\, r_o + (r_{o1} \| r_{\pi 2})$$

$$\Rightarrow A_v = -G_m R_{out} = g_{m1}\left[\{1 + g_{m2}(r_{o1} \| r_{\pi 2})\} r_{o2} + (r_{o1} \| r_{\pi 2})\right)$$

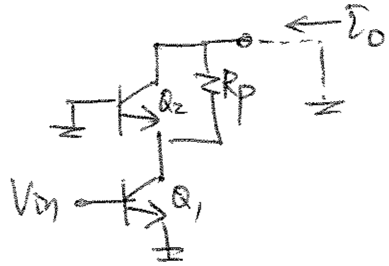
25. (a) From lecture, we know that the voltage gain of a BJT cascode circuit $\approx -g_{m2} r_{O2}\, g_{m2} (r_{O2} \| r_{\pi_1})$

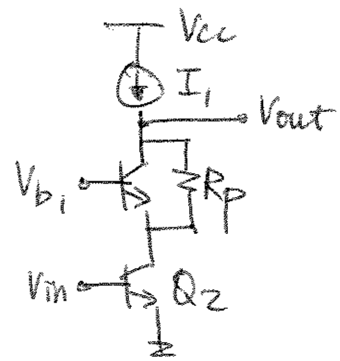


This circuit resembles such, and the only difference is that r_{π_1} now becomes $(r_{\pi_1} \| R_p)$

$$\therefore A_v \approx -g_{m2}^2 r_{O2} (r_{O2} \| r_{\pi_1} \| R_p)$$

(b) Equivalent circuit:



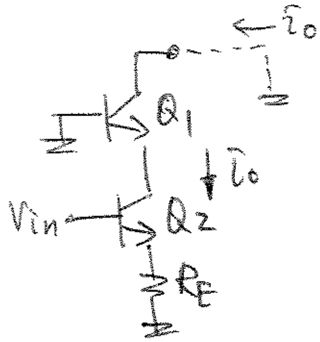


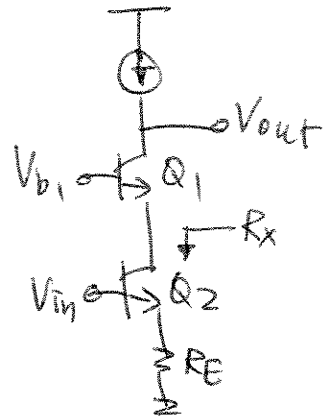
$$G_m = \frac{i_o}{V_{in}} = \frac{\beta+1}{\beta} g_{m_1} \approx g_{m_1}$$

$$R_{out} = [1 + g_{m2}(r_{O_1} \| r_{\pi_2})](r_{O2} \| R_p) + (r_{O_1} \| r_{\pi_2})$$

$$\therefore A_v = -G_m R_{out}$$
$$= -g_{m_1} \{[1 + g_{m2}(r_{O_1} \| r_{\pi_2})](r_{O2} \| R_p) + (r_{O_1} \| r_{\pi_2})\}$$

(c) Equivalent circuit:





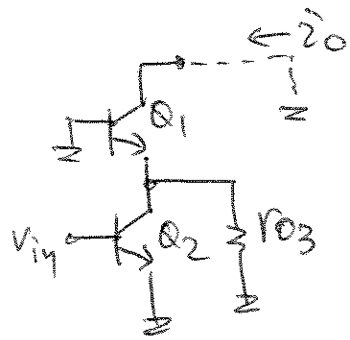
$$G_m = \frac{i_o}{v_{in}} = \frac{g_{m2}}{1 + g_{m2}R_E}$$

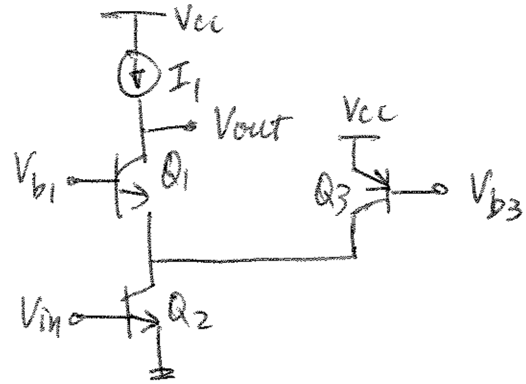
(small-signal analysis)

$$R_{out} = (1 + g_{m1} R_X) r_{o1} + R_X$$
$$= [1 + g_{m1}[(1 + g_{m2}R_E)r_{o2} + R_E]] r_{o1} + [(1 + g_{m2}R_E)r_{o2} + R_E]$$

$$\therefore A_v = -G_m R_{out}$$
$$= \frac{g_{m2}\{[1+g_{m1}[(1+g_{m2}R_E)r_{o2}+R_E]]r_{o1}+[(1+g_{m2}R_E)r_{o2}+R_E]\}}{1+g_{m2}R_E}$$

(d) Equivalent circuit:

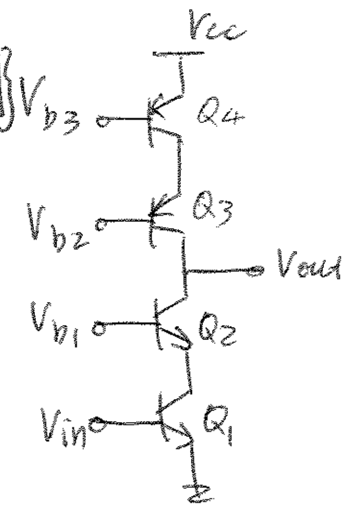




This resembles the BJT cascode topology, only now r_{o2} becomes $(r_{o2} \| r_{o3})$

$$\Rightarrow A_v \approx -g_{m2}^2 (r_{o2} \| r_{o3})(r_{o2} \| r_{o3} \| r_{\pi 1})$$

2b. $$A_v = -g_{m_1}\Big\{\underbrace{\left[g_{m_2} r_{o2}(r_{o1} \| r_{\pi 2})\right]}_{R_{on}} \,\|\, \underbrace{\left[g_{m_3} r_{o3}(r_{o4} \| r_{\pi 3})\right]}_{R_{op}}\Big\}$$



$$R_{on} = \frac{(V_{An}/V_T)}{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}$$

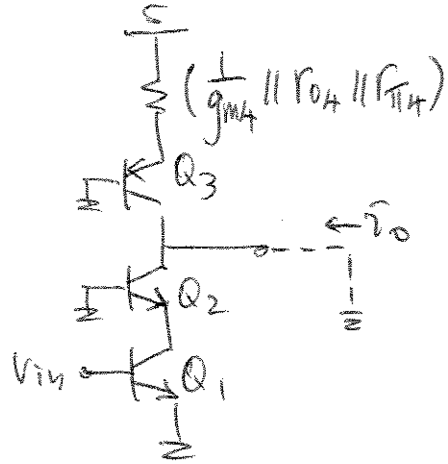
$$R_{op} = \frac{(V_{AP}/V_T)}{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C} \qquad g_{m_1} = \frac{I_C}{V_T}$$

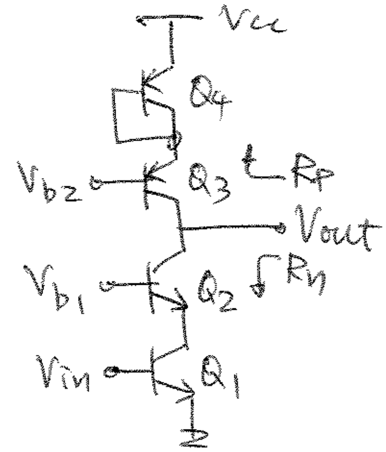
$$\therefore A_v = \frac{-(I_C/V_T)}{\dfrac{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}{V_{An}/V_T} + \dfrac{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C}{V_{AP}/V_T}}$$

$$= \frac{V_{AN} \cdot V_{AP}}{V_T^2 \left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_p V_T}\right)}$$

$\therefore$ A_v is independent of bias current, I_C.

27. Equivalent circuit.





$$G_m = g_{m1} = \frac{i_o}{v_{in}} = \frac{i_{c1}}{v_{in}}$$

$$R_{out} = R_p \parallel R_n$$

$$R_p = \left[1 + g_{m3}\left(\frac{1}{g_{m4}} \parallel r_{o4} \parallel r_{\pi 4} \parallel r_{\pi 3}\right)\right] r_{o3} + \left[\frac{1}{g_{m4}} \parallel r_{o4} \parallel r_{\pi 4} \parallel r_{\pi 3}\right]$$

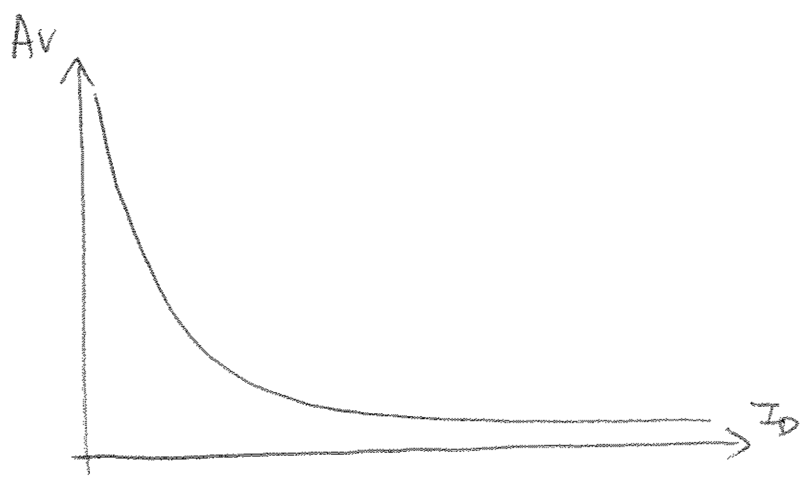
$$R_n = \left[1 + g_{m2}(r_{o1} \parallel r_{\pi 2})\right] r_{o2} + (r_{o1} \parallel r_{\pi 2})$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(R_p \parallel R_n)$$

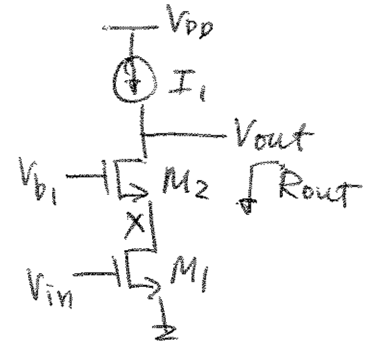
28. $|A_v| = g_{m1} r_{o1} g_{m2} r_{o2}$

$$= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D}$$

$$= 2\mu_n C_{ox} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \cdot \frac{1}{\lambda^2 I_D}$$



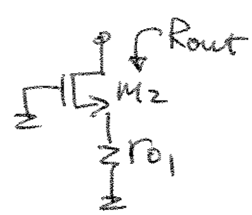
29. $|A_v| = 200$



$\mu_n C_{ox} = 100 \, \frac{\mu A}{V^2}$ $\lambda = 0.1 \, V^{-1}$

Determine $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$R_{out} = (1 + g_{m2} r_{o1}) r_{o2} + r_{o1}$



$G_m \cong g_{m1}$ (short-circuit current flows through both M_1 & M_2)

$$|A_v| = G_m R_{out} = g_{m1}\left[(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}\right]$$

$$\approx g_{m1} g_{m2} r_{o1} r_{o2} = (g_m r_o)^2 = 200$$

$\left(\because \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \text{ and } I_{D1} = I_{D2}\right)$

$$(g_m r_o)^2 = \left(\frac{2 I_D}{V_{GS} - V_{TH}} \cdot \frac{1}{\lambda I_D}\right)^2 = 200$$

$$\Rightarrow V_{GS} - V_{TH} = \left(\sqrt{200} \cdot \lambda/2\right)^{-1} = \left[\sqrt{200} \cdot (0.05 \, V^{-1})\right]^{-1}$$

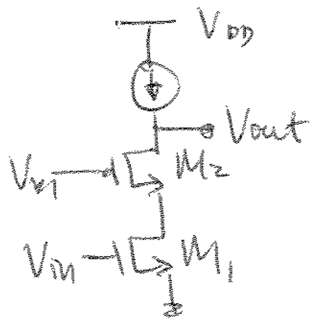
$$\approx 1.41 \, V$$

$$\Rightarrow \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2$$

$$\therefore \left(\frac{W}{L}\right) = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

$$= \frac{2(1 \text{ mA})}{100 \frac{\mu A}{V^2} (1.41V)^2} \approx 10$$

30.



$$\left(\frac{W}{L}\right)_{1,new} = N\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = N\left(\frac{W}{L}\right)_2$$

$$\lambda_{n,1} = \lambda_{n,2}$$

$$A_{v_{new}} \approx -g_{m1} g_{m2} r_{o1} r_{o2}$$

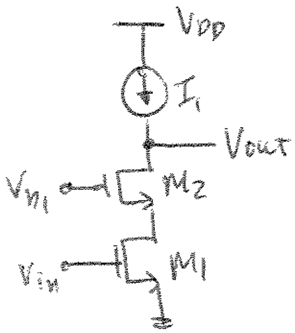
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} I_D} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -\sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -N\left(g_{m1} g_{m2} r_{o1} r_{o2}\right) = -N \cdot A_{v,old}.$$

Gain is N times of original value.

31.



$$\left(\frac{W}{L}\right)_{1,new} = \frac{1}{N}\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = \frac{1}{N}\left(\frac{W}{L}\right)_2$$

Assume $\lambda_{n,1} = \lambda_{n,2}$

$$A_{v_{new}} \approx -g_{m1}\left(g_{m2}\left(r_{o1} r_{o2}\right)\right)$$

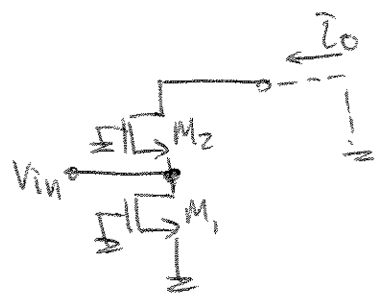
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} \cdot I_{D1}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_{D1}} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

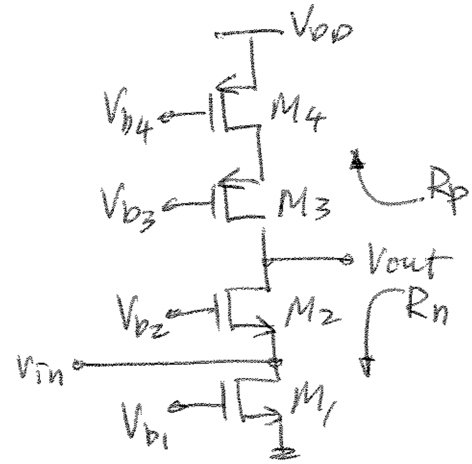
$$= -\sqrt{\frac{1}{N}}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{\frac{1}{N}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= -\frac{1}{N} g_{m1} g_{m2} r_{o1} r_{o2} = -\frac{1}{N}\left(A_{v,old}\right)$$

Gain is $1/N$ of original value.

32.





By KCL,

$$\frac{i_o}{V_{in}} = -\left(g_{m2} + \frac{1}{r_{o1} \| r_{o2}}\right) = G_m$$

$$R_n = r_{o2} \qquad R_p \approx g_{m3} r_{o3} r_{o4}$$

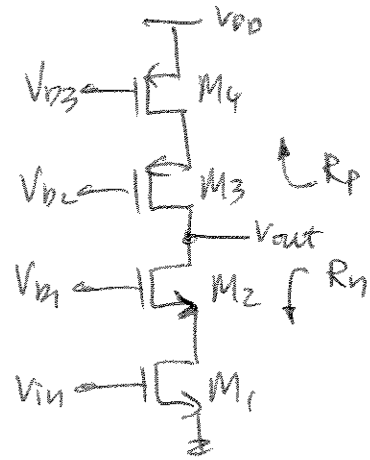
$$\therefore A_v = -G_m (R_n \| R_p) = \left(g_m + \frac{1}{r_{o1} \| r_{o2}}\right)(r_{o2} \| g_{m3} r_{o3} r_{o4})$$

33. $\left(\frac{W}{L}\right) = 20/0.18$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$ $\lambda_p = 0.15\ V^{-1}$



Calculate I_{BIAS} such as $A_v = 500$.

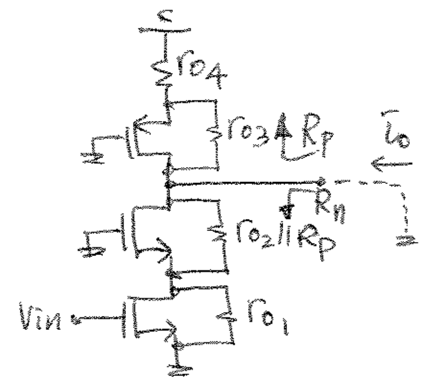
The cascode circuit has gain $\approx -g_{m1} \cdot [g_{m2} r_{o1} r_{o2} \,\|\, g_{m3} r_{o3} r_{o4}]$

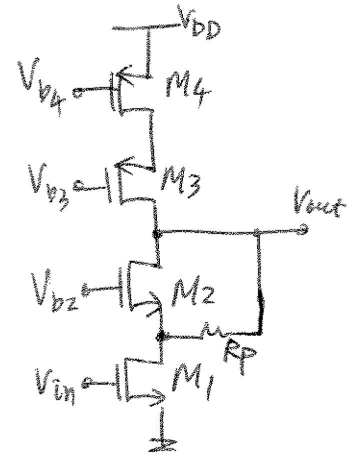
$$\Rightarrow 500 = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}\left(\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_n)^2 I_D^{3/2}} \,\Big\|\, \frac{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_p)^2 I_D^{3/2}}\right)$$

All quantities are known. Solving I_D gives:

$I_D = I_{BIAS} \approx 1.06\ mA$.

34 (a) Equivalent circuit:



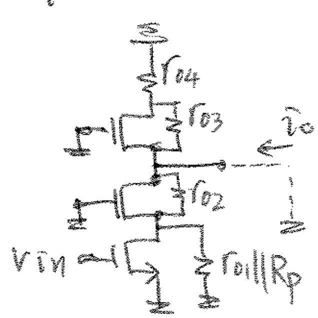


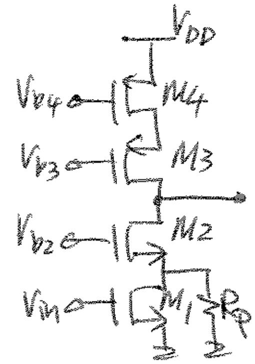
$$G_m = \frac{i_o}{v_{in}} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o2} \| R_p) r_{o1}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o2} \| R_p) r_{o1}\right]$$

(b) Equivalent circuit



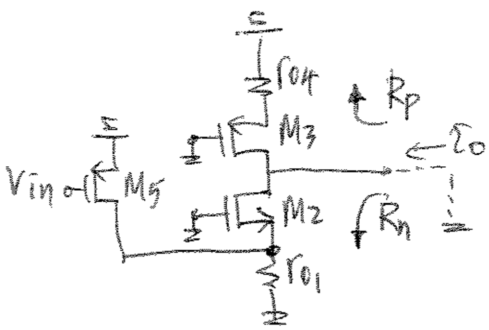


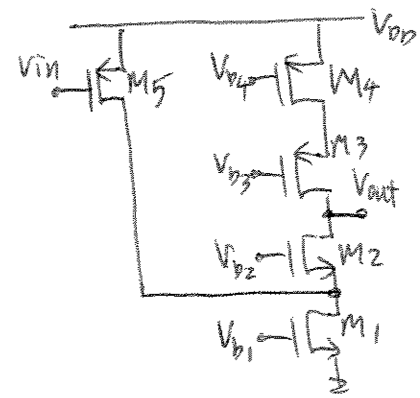
$$G_m = i_o / v_{in} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o1} \| R_p) r_{o2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o1} \| R_p) r_{o2}\right]$$

(c) Equivalent circuit:





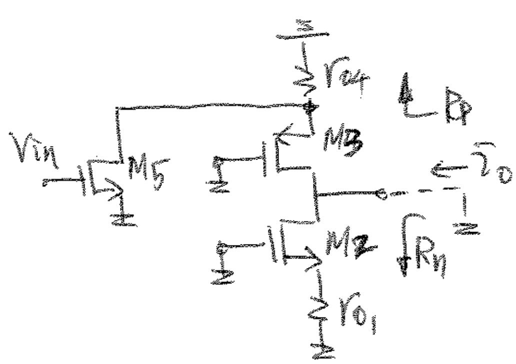
(Realize that r_{o1} & r_{o5} are in parallel.)

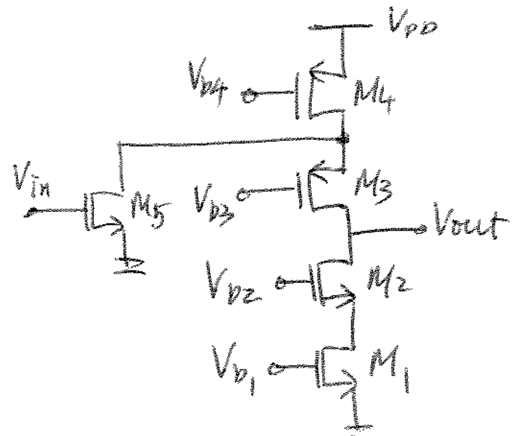
$$G_m = \frac{i_o}{v_{in}} \approx -g_{m5} \qquad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2} r_{o2} (r_{o1} \| r_{o5})$$

$$\therefore A_v = -G_m R_{out} = g_{m5} [g_{m3} r_{o3} r_{o4} \| g_{m2} r_{o2} (r_{o1} \| r_{o5})]$$

(d) Equivalent circuit:





$$G_m = \frac{i_o}{v_{in}} \approx g_{m5} \qquad R_p = g_{m3} r_{o3} (r_{o4} \| r_{o5})$$

$$R_n = g_{m2} r_{o2} r_{o1}$$

$$\therefore A_v = -G_m R_{out} = g_{m5} [g_{m3} r_{o3} (r_{o4} \| r_{o5}) \| g_{m2} r_{o2} r_{o1}]$$

35. $\frac{R_2}{R_1 + R_2} V_{CC} = V_T \ln\left(\frac{I_1}{I_S}\right)$

$$\Rightarrow I_1 = I_S \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$\frac{\partial I_1}{\partial V_{CC}} = \frac{I_S}{V_T} \cdot \frac{R_2}{R_1+R_2} \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$= \frac{I_1}{V_T} \cdot \frac{R_2}{R_1+R_2} = g_m\left(\frac{R_2}{R_1+R_2}\right)$$

Intuitively, we know that an exponential relationship exists between I_C & V_{BE}. Its transconductance is also a function (linear) of I_C. Since V_{BE} comes from a voltage divider (which is also linear), we expect a linear relationship between I_C & V_{CC}.

36. $$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)^2$$

$$\frac{\partial I_1}{\partial V_{DD}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot 2\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)\cdot\frac{R_2}{R_1+R_2}$$

$$= \mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}\right)\left(\frac{R_2\cdot V_{DD}}{R_1+R_2} - V_{TH}\right).$$

$$= g_m\cdot\frac{R_2}{R_1+R_2}$$

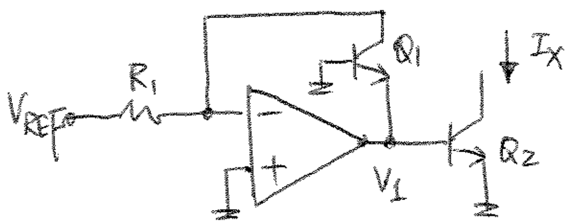
Intuitively, the voltage divider gives a linear relationship between V_{DD} & V_{GS1}. Since g_m of MOS is linearly proportional to $(V_{GS1} - V_{TH})$, we expect the same relationship between V_{DD} & $\frac{\partial I_1}{\partial V_{DD}}$

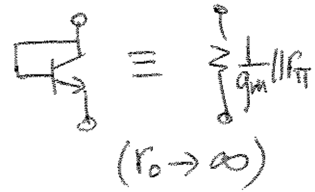
37. $I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)^2$

$$\frac{\partial I_1}{\partial V_{TH}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right) \cdot (-1)$$

$$= -\mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)$$

38.





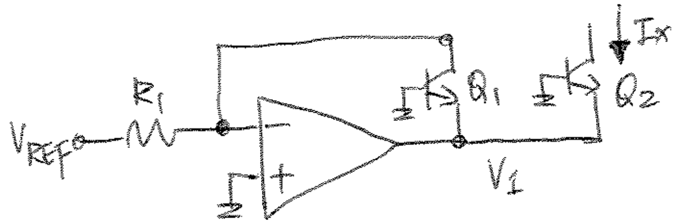
This is a negative feedback circuit.

The inverting input (−) of the op-amp is virtual ground. (∵ of feedback) in DC. ⇒ Q_1 becomes diode-connected.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{0 - V_1}{\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)} \Rightarrow V_1 = -\frac{V_{REF}\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)}{R_1} < 0$$

This implies $V_{BE2} < 0 \Rightarrow I_X = 0$!

39.



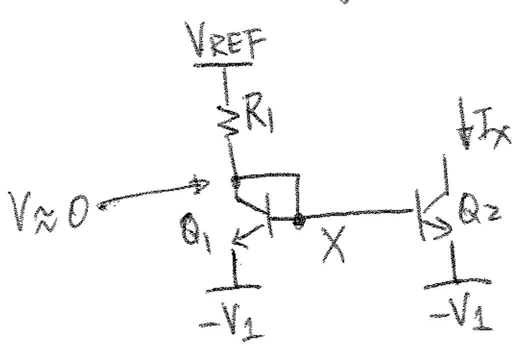
This is a negative feedback circuit.

The inverting input (−) is virtual ground, as a result. Q_1 then becomes diode-connected, and its resistance $= (\frac{1}{g_m} \| r_\pi)$, assuming $r_o \to \infty$.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{-V_1}{(\frac{1}{g_{m1}} \| r_{\pi 1})} \Rightarrow V_1 = -\frac{V_{REF}(\frac{1}{g_{m1}} \| r_{\pi 1})}{R_1}$$

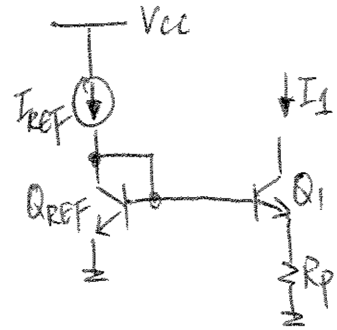
$$\Rightarrow V_{BE1} = V_{BE2} = -V_1$$

This circuit will work if the negative supply voltage of the op-amp allows value of $-V_1$ or lower.



– An equivalent circuit. (without op-amp). The op-amp guarantees a stable voltage at node X. (i.e. inverting input.)

40.



$Q_{REF} = Q_1$

$\beta \to \infty$.

$I_1 = \frac{I_{REF}}{2}$

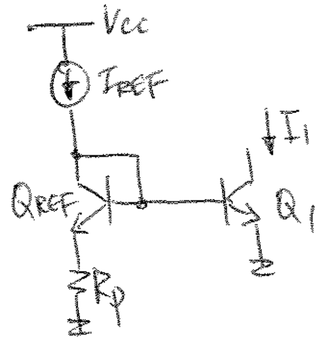
By KVL, $V_{BE,REF} = V_{BE_1} + I_1 R_p$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) = V_T \ln\left(\frac{I_{REF}/2}{I_{S,1}}\right) + \frac{I_{REF}}{2} R_p$$

$$V_T \ln(2) = \frac{I_{REF}}{2} R_p$$

$$R_p = 2 \cdot \ln(2) \cdot (V_T / I_{REF})$$

41.



$Q_{REF} = Q_1$
$\beta \to \infty$

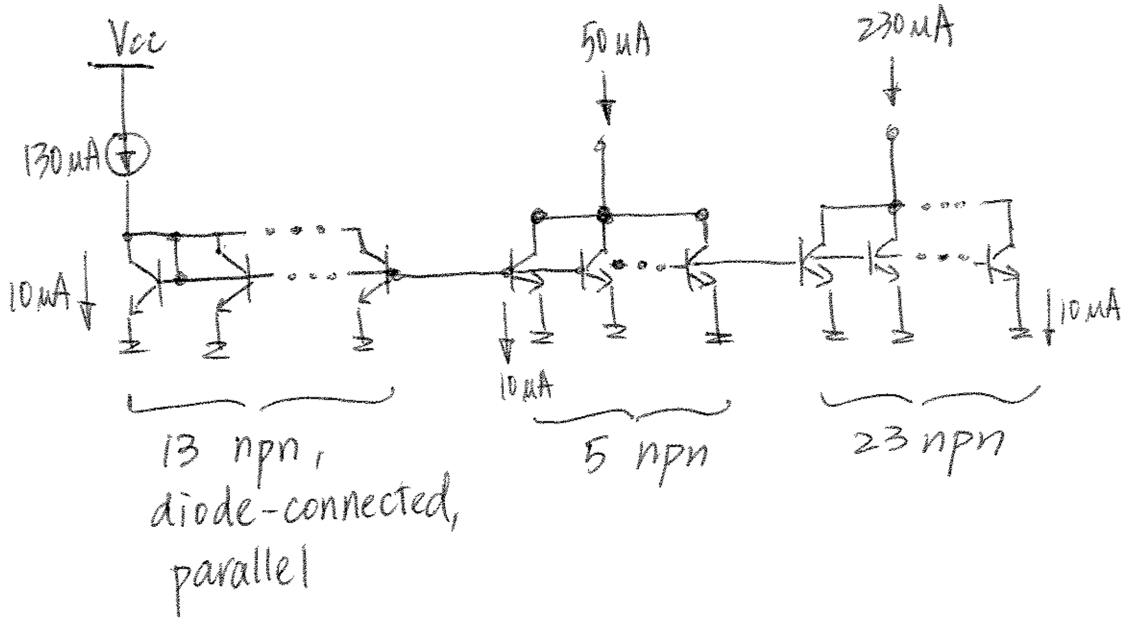
By KVL, $V_{BE,REF} + I_{REF} R_p = V_{BE,1}$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) + I_{REF} R_p = V_T \ln\left(\frac{2 I_{REF}}{I_{S,1}}\right)$$

$$I_{REF} R_p = V_T \ln(2)$$

$$R_p = \frac{V_T}{I_{REF}} \ln(2)$$

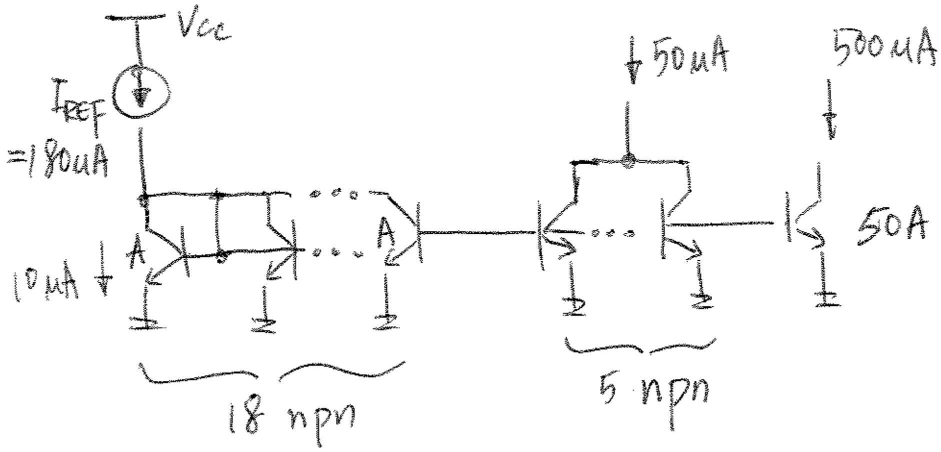
42.

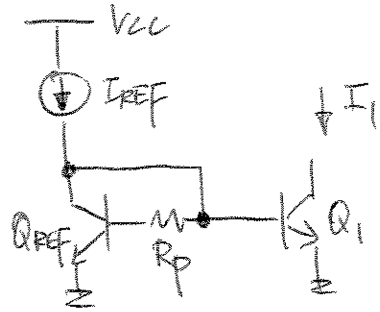


All the bases are the same node.

If the area of BJT is flexible, the 5-npn group can be replaced by one BJT that is 5 times as big in area. Similar concept applies to 23-npn grouping.

43.



44. 

$Q_{REF} = Q_1$

I_1 10% larger. ($I_1 = 1.1\, I_{C,REF}$)
Solve for R_P.

By KVL,

$$V_{BE_{REF}} + \frac{I_{C,REF}}{\beta} \cdot R_P = V_{BE_1}$$

$$\Rightarrow V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{C,REF}}{I_S}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

$$V_T \ln\left(\frac{I_1}{I_{C,REF}}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

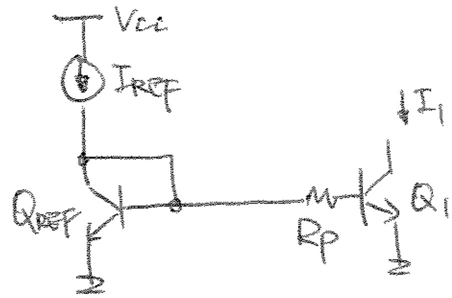
$$\Rightarrow V_T \ln(1.1) = \frac{I_{C,REF}}{\beta} R_P \quad \Rightarrow I_{C,REF} = \frac{\beta V_T \ln(1.1)}{R_P}$$

By KCL,
$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$= \frac{\beta}{R_P} V_T \ln(1.1) \cdot \left(1 + 1/\beta\right) + I_1/\beta$$

$$\therefore R_P = \frac{(\beta+1)\, V_T \ln(1.1)}{I_{REF} - I_1/\beta}$$

45.



$$I_1 = 0.9 I_{C,REF}$$

By KVL, $V_{BE_{REF}} = \frac{I_1}{\beta} R_P + V_{BE_1}$

$$\Rightarrow V_T \ln\left(\frac{I_{C,REF}}{I_1}\right) = \frac{I_1}{\beta} R_P$$

$$V_T \ln\left(\frac{1}{0.9}\right) = 0.9\ I_{C,REF}\ \frac{R_P}{\beta}$$

$$\Rightarrow I_{C,REF} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)$$

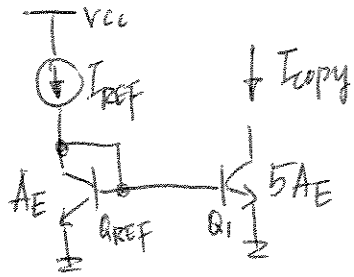
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$\therefore I_{REF} - \frac{I_1}{\beta} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)\left(1 + \frac{1}{\beta}\right)$$

$$\Rightarrow R_P = \frac{(\beta + 1)\, V_T \ln\left(10/9\right)}{0.9\left(I_{REF} - I_1/\beta\right)}$$

4b (a)



Q_1 has I_S 5 times as that of Q_{REF}

$\Rightarrow I_{C_{REF}} = I_{copy} / 5$

By KCL,

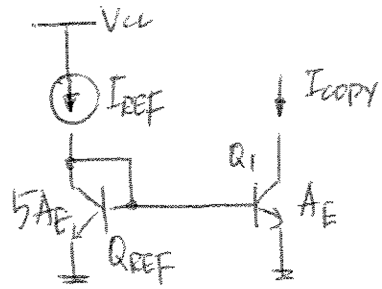
$$I_{REF} = I_{C_{REF}} + \frac{I_{C_{REF}}}{\beta} + \frac{I_{copy}}{\beta}$$

$$= \frac{I_{copy}}{5}\left(1 + \frac{1}{\beta}\right) + \frac{I_{copy}}{\beta}$$

$$\therefore \; I_{copy} = I_{REF}\left(\frac{5\beta}{\beta+6}\right)$$

$$\therefore \; error = \frac{I_{copy}}{I_{REF}} = \frac{5\beta}{\beta+6}$$

(b)



Q_1 & Q_{REF} have the same V_{BE}, but area of Q_{REF} is 5 times larger

$\Rightarrow I_{C,REF} = 5 \cdot I_{copy}$

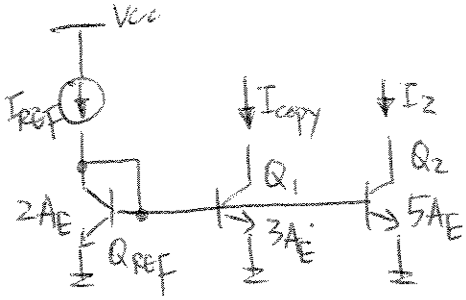
By KCL,

$$I_{REF} = I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta}$$

$$= I_{copy} \cdot 5 + \left(1 + \frac{1}{\beta}\right) + I_{copy}\left(\frac{1}{\beta}\right)$$

$$\therefore I_{copy} = I_{REF}\left(\frac{\beta}{5\beta + 6}\right)$$

(c)



Q_1 & Q_{REF} have identical V_{BE}, but area of Q_1 is 1.5 times larger.

$\Rightarrow 3 I_{C,REF} = 2 I_{copy}$

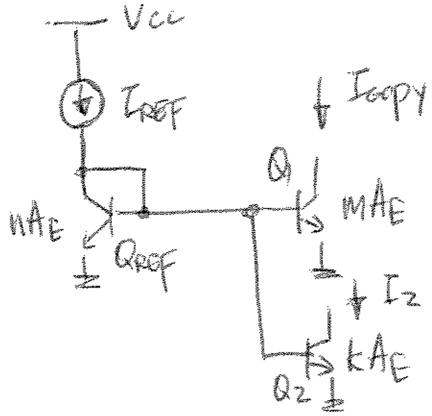
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_{copy}/\beta + I_2/\beta$$

$$= I_{copy}\left(\frac{2}{3}\right)\left[\left(1 + 1/\beta\right) + 1/\beta + 5/3\left(\frac{1}{\beta}\right)\right]$$

$$\rightarrow I_{copy} = \frac{9\beta}{6\beta + 22} I_{REF}$$

47.



By observing the areas of the BJTs,

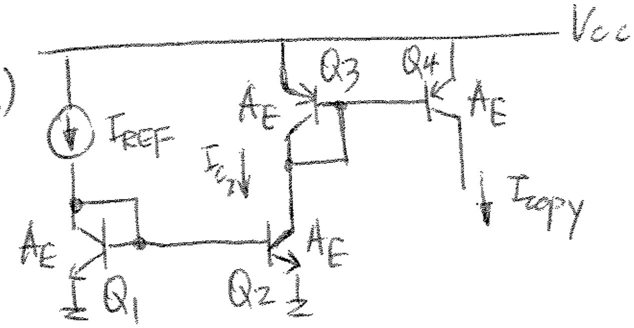
$$I_{C,REF} = \left(\frac{n}{m}\right) I_{copy} = \left(\frac{n}{k}\right) I_2$$

By KCL, $I_{C,REF} = I_{REF} - \frac{I_{C,REF}}{\beta} - \frac{I_{copy}}{\beta} - \frac{I_2}{\beta}$

$$\Rightarrow \frac{n}{m} I_{copy} = I_{REF} - \frac{\left(\frac{n}{m}\right) I_{copy}}{\beta} - \frac{I_{copy}}{\beta} - \frac{\left(\frac{k}{m}\right) I_{copy}}{\beta}$$

$$\therefore I_{copy} = I_{REF}\left[\frac{\beta m}{(\beta+1)n + k + m}\right]$$

48. (a)



$V_{BE_1} = V_{BE_2}$
$\Rightarrow I_{C_1} = I_{C2}$

$V_{BE_3} = V_{BE_4}$
$\Rightarrow I_{C_3} = I_{C_4}$

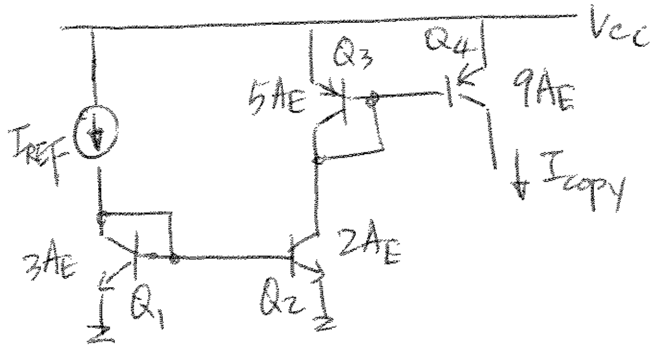
First compute $I_{C_{1,2}}$:

$$I_{C1} = I_{REF} - \frac{I_{C1}}{\beta} - \frac{I_{C2}}{\beta} \Rightarrow I_{C_2} = \frac{\beta}{\beta+2} \cdot I_{REF}.$$

View I_{C2} as the "I_{REF}" for the Q_3–Q_4 current mirror and apply the equation derived.

$$\Rightarrow I_{copy} = \frac{\beta}{\beta+2}\left[\frac{\beta}{\beta+2} \cdot I_{REF}\right] = I_{REF}\left(\frac{\beta}{\beta+2}\right)^2$$

(b)



$V_{BE_1} = V_{BE_2}$:

$\Rightarrow I_{C_1} = \frac{3}{2} I_{C_2}$

$V_{BE_3} = V_{BE_4}$:

$\Rightarrow I_{copy} = \frac{9}{5} I_{C_3}$

- By KCL,

$$I_{REF} = I_{C_1} + \frac{I_{C_1}}{\beta} + \frac{I_{C_2}}{\beta}$$

$$\Rightarrow I_{C_2} = \frac{2\beta}{3\beta+5} I_{REF} \qquad ①$$

- By KCL,

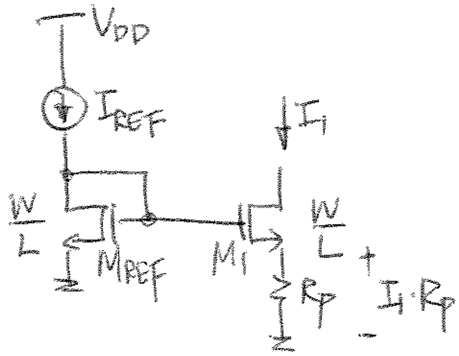
$$I_{C_2} = I_{C_3} + \frac{I_{C_3}}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = \frac{9\beta}{5\beta+14} I_{C_2}$$

Substitute ① into Icopy :

$$\therefore I_{copy} = \frac{9\beta}{5\beta+14} \cdot \frac{2\beta}{3\beta+5} \cdot I_{REF}.$$

49.



Determine R_p such that $I_1 = \frac{I_{REF}}{2}$.

First calculate $V_{GS,REF}$:

$$V_{GS,REF} = \sqrt{\frac{2\,I_{REF}}{\mu_n C_{ox}\, W/L}} + V_{TH} \qquad \text{———} \quad ①$$

Assuming M_1 in saturation:

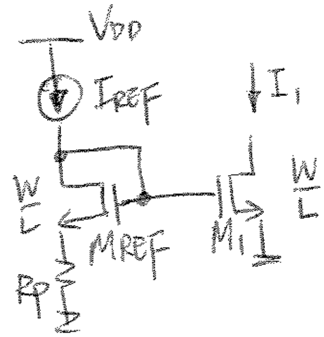
$$I_1 = \frac{I_{REF}}{2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_{GS_1} - V_{TH}\right]^2$$

$$\Rightarrow \quad \frac{I_{REF}}{2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_{GS,REF} - \frac{I_{REF}}{2}(R_p) - V_{TH}\right]^2$$

Rearrange, substitute ① into equation above and solve for R_p:

$$\therefore R_p = \frac{2(\sqrt{2}-1)}{\sqrt{I_{REF}\cdot \mu_n C_{ox}\frac{W}{L}}}$$

50.



Determine R_P such that $I_1 = 2I_{REF}$.

First calculate $V_{GS,1}$:

$$V_{GS,1} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} = 2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} \quad -(1)$$

Assuming I_1 is in saturation:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS,REF} - V_{TH}\right)^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS,1} - I_{REF}R_P - V_{TH}\right]^2$$

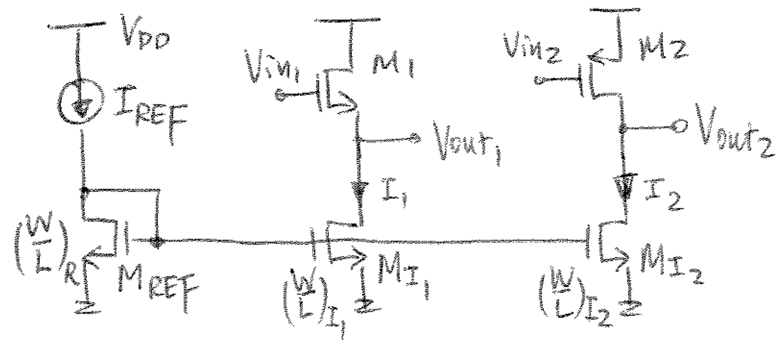
Substitute (1) into I_{REF}:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} - I_{REF}R_P\right]^2 \quad -(2)$$

Solve for R_P: $R_P = \dfrac{(2-\sqrt{2})}{\sqrt{I_{REF}\cdot\mu_n C_{ox}\left(\frac{W}{L}\right)}}$

From (2), we find that R_P is <u>independent</u> of any change in V_{TH}, ΔV!!

51.



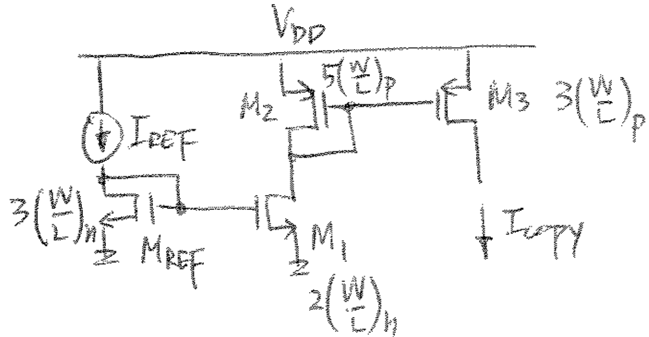
This figure implies that $V_{GS,REF} = V_{GS,I_1} = V_{GS,I_2}$.
Assuming all devices operate in saturation, with $(V_{GS} - V_{TH})$ fixed, $I_D \propto (\frac{W}{L})$

$\Rightarrow$ we have $(\frac{W}{L})_R = 7(\frac{W}{L})$

$$\left(\frac{W}{L}\right)_{I_1} = 4\left(\frac{W}{L}\right)$$

$$\left(\frac{W}{L}\right)_{I_2} = 10\left(\frac{W}{L}\right)$$

52. (a)

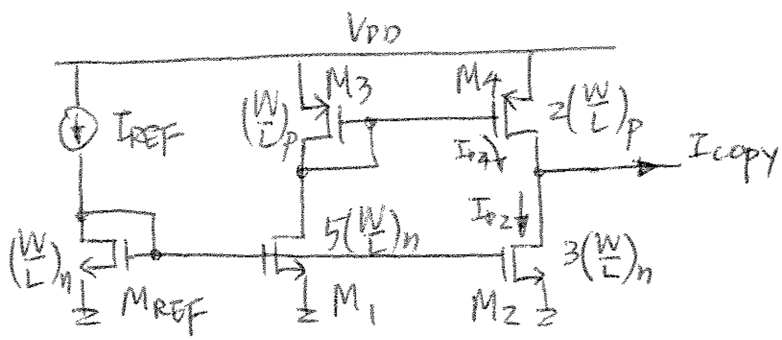


$V_{GS,REF} = V_{GS,1}$: $\Rightarrow I_{D,1} = \frac{2}{3} I_{REF}$

$V_{GS,2} = V_{GS,3}$: $\Rightarrow I_{copy} = \frac{3}{5} I_{D,2} = \frac{3}{5} I_{D,1}$

$= \frac{3}{5} \cdot \left(\frac{2}{3} I_{REF}\right) = \frac{2}{5} I_{REF}$

(b)



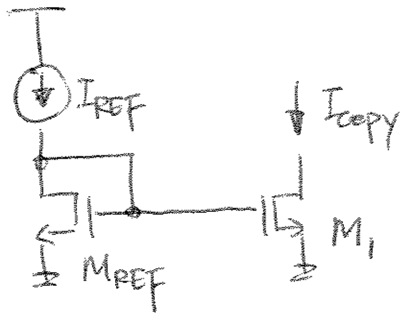
$V_{GS,REF} = V_{GS,1}$: $I_{D,1} = 5 I_{REF}$

$V_{GS,3} = V_{GS,4}$: $I_{D4} = 2 I_{D3} = 2 I_{D,1} = 10 I_{REF}$

$V_{GS,REF} = V_{GS,2}$: $I_{D2} = 3 I_{REF}$

$\therefore I_{copy} = I_{D4} - I_{D2} = 7 I_{REF}.$

53.



$V_{GS,REF} = V_{GS,1} = V_{GS}$

$\lambda \neq 0$

(a) $$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{GS})$$

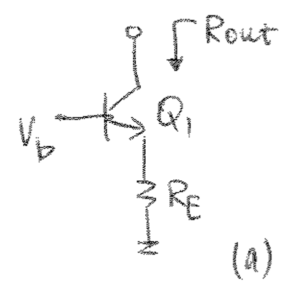
$$I_{copy} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{DS,1})$$

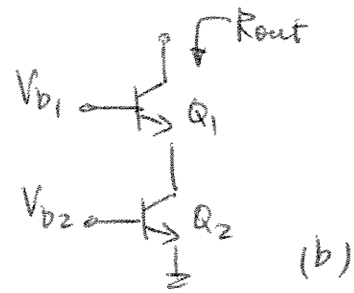
For $I_{REF} = I_{copy} \Rightarrow V_{DS,1} = V_{GS}$

(b) $$\frac{I_{REF}}{I_{copy}} = \frac{1+\lambda V_{GS}}{1+\lambda(V_{GS}-V_{TH})}$$

$$\Rightarrow I_{copy} = I_{REF}\left(1 - \frac{\lambda V_{TH}}{1+\lambda V_{GS}}\right)$$

54.





Given $I_{BIAS} = 1mA$, $V_{RE} \approx V_{CE,2} \approx 0.5V$, design the circuit.

R_E can be readily calculated:

$$R_E = \frac{V_{RE}}{I_{BIAS}/\alpha} = \frac{0.5V}{1mA/0.909} = 505\,\Omega$$

$$V_{be_1} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V)\ln\left(\frac{1mA}{6\cdot 10^{-16}A}\right) \approx 0.732V$$

$$\Rightarrow V_b = V_{be_1} + V_{RE} = 0.732V + 0.5V = 1.232V$$

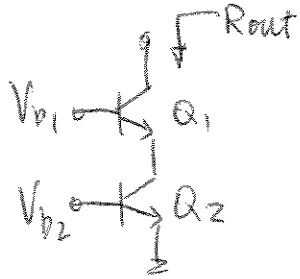
$$R_{out(a)} = [1 + g_{m_1}(R_E \parallel r_{\pi_1})]r_{o_1} + (R_E \parallel r_{\pi_1})$$

$$R_{out(b)} = [1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})]r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

In most cases $r_o > r_\pi > R_E$

$\therefore$ $R_{out(b)}$ is relatively larger than $R_{out(a)}$

55.



$I_{BIAS} = 1mA$

$\beta = 100$

Given $R_{out} = 50\,k\Omega$, $V_{BC_2} = 100\,mV$, determine V_{b1}.

$$R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1})] r_{o1} + (r_{o2} \| r_{\pi 1})$$
$$\approx g_{m1}(r_{o2} \| r_{\pi 1}) r_{o1}$$
$$= \frac{\beta V_A^2}{(V_A + \beta V_T) I_{BIAS}}$$

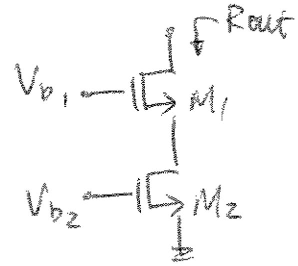
$$\Rightarrow I_{BIAS} = \left[\frac{R_{out}(V_A + \beta V_T)}{\beta V_A^2}\right]^{-1} = \left[\frac{(50k\Omega)(5V + 100 \cdot 0.026V)}{100(5V)^2}\right]^{-1}$$
$$\approx 6.6\,mA.$$

$$V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_{BIAS}}{I_S}\right) = (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right)$$
$$\approx 0.78\,V$$

$$\Rightarrow V_{C2} = V_{BE2} - 100mV = 0.68V$$

$$\therefore V_{b1} = V_{C2} + V_{BE1} = V_{C2} + V_T \ln\left(\frac{I_{BIAS}}{I_S}\right)$$
$$= 0.68V + (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right) \approx 1.46\,V.$$

5b. Given $R_{out} = 200k\Omega$
$I_{BIAS} = 0.5mA$



(a) Determine $(W/L)_1 = (W/L)_2$ with $\lambda = 0.1\,V^{-1}$

$$R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$$
$$= \left[1 + \sqrt{2 I_{BIAS} \mu_n C_{ox} \left(\frac{W}{L}\right)_1} \cdot \frac{1}{\lambda I_{BIAS}}\right] \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \cong \frac{\left[\left(R_{out} - \frac{1}{\lambda I_{BIAS}}\right)(\lambda I_{BIAS})^2\right]^2}{2 I_{BIAS} \mu_n C_{ox}}$$

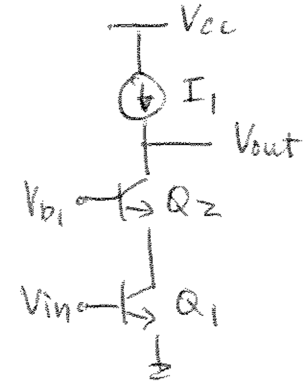
$$= \frac{\left\{\left[(200k\Omega) - \frac{1}{(0.1V^{-1})(0.5mA)}\right](0.1V^{-1})^2(0.5mA)^2\right\}^2}{2(0.5mA)(100\ \frac{\mu A}{V^2})}$$

$$\approx 2.0$$

(b) $I_{BIAS} = 0.5\ mA = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b2} - V_{TH,n})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox} (W/L)_1}} + V_{TH}$$
$$= \sqrt{\frac{2(0.5mA)}{(100\ \frac{\mu A}{V^2})(2.0)}} + (0.4V) \approx 2.62\ V$$

57. Given $|A_v| = 500$
$\beta = 100$



(a) $A_v = -g_{m1} r_{o1} g_{m2} (r_{o1} \| r_{\pi 2})$

$$= -\frac{V_A}{V_T} \times \frac{I_{C2}}{V_T}\left(\frac{V_A}{I_{C1}} \| \frac{\beta}{g_{m2}}\right)$$

Assume $I_{C1} \approx I_{C2}$. After expanding $(r_{o1} \| r_{\pi 2})$,

$$A_v \approx -\frac{V_A/V_T}{\frac{V_T}{V_A} + \frac{1}{\beta}} \Rightarrow V_A^2 + V_A\left(\frac{V_T A_v}{\beta}\right) + (A_v V_T^2) = 0$$

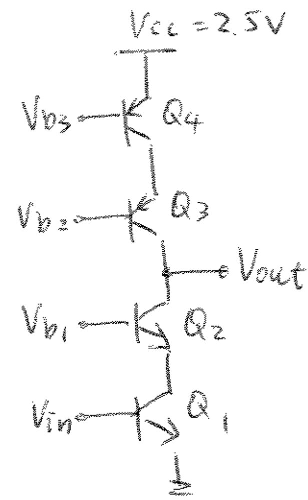
$\Rightarrow V_A \approx 0.65 \text{ V}$

(b) $V_{in} = V_T \ln\left(\frac{I_1}{I_S}\right) = (0.026 \text{ V}) \ln\left(\frac{0.5 \text{ mA}}{6 \cdot 10^{-16} \text{ A}}\right)$

$\approx 0.71 \text{ V}$

(c) $V_{b1} = V_{BE2} + 500 \text{ mV}$
$= V_T \ln\left(\frac{I_1}{I_S}\right) + 0.5 \text{ V}$
$= 0.71 \text{ V} + 0.5 \text{ V} = 1.21 \text{ V}$

58. Given power budget = 2mW
$V_{BC_1} = V_{CB_4} = 200\ mV$,
calculate voltage gain.



$$\alpha_p = \frac{50}{50+1} \approx 0.98$$

$$\alpha_n = \frac{100}{100+1} \approx 0.99$$

$\therefore$ we assume $I_{c,p} \approx I_{e,p}$ & $I_{c,n} \approx I_{e,n}$

This implies that $I_{BIAS} = \frac{Power}{V_{cc}} = \frac{2mW}{2.5V}$

$\approx 0.8mA$.

$$\Rightarrow V_{BE_1} = V_{in} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V) \cdot \ln\left(\frac{0.8mA}{6 \cdot 10^{-16}A}\right) \approx 0.726V$$

$$V_{c,1} = V_{BE_1} - V_{BC_1} = 0.726V - 0.2V = 0.526V$$

$$\therefore V_{b_1} = V_{c_1} + V_{BE_2} = (0.526V) + (0.026V)\ln\left(\frac{0.8mA}{6 \cdot 10^{-6}A}\right)$$

$\approx 1.252V$

$$\Rightarrow V_{EB_4} = V_{cc} - V_{b_3} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,4}}\right) = 0.026V \cdot \ln\left(\frac{0.8mA}{6 \cdot 10^{-16}A}\right)$$

$\approx 0.726V$

$V_{D3} = V_{CC} - 0.726V = 1.774\ V$

$V_{C4} = V_{D3} + V_{CB4} = 1.774V + 0.2V = 1.974\ V$

$\therefore V_{D2} = V_{C4} - V_{EB3} = (1.974V) - (0.026)\ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right)$

$\approx 1.248\ V$

$$A_V = -g_{m1}\{[g_{m2}r_{o2}(r_{o1}\| r_{\pi 2})] \| [g_{m3}r_{o3}(r_{o4}\| r_{\pi 3})]\}$$

After simplifying, A_V is independent of I_{BIAS}:

$$A_V \approx \frac{V_{AN}\cdot V_{AP}}{V_T^2\left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$$= \frac{5\cdot 5}{(0.026V)^2\left(\frac{5}{5} + \frac{5}{100\cdot 0.026} + \frac{5}{5} + \frac{5}{50\cdot 0.026}\right)}$$

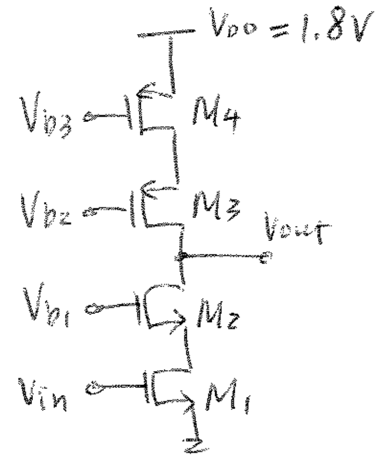
≈ 4760

59. Given $A_v = 200$

power budget = 2mW

all $\left(\frac{W}{L}\right) = \frac{20}{0.18}$

$V_{b_1} = V_{b_2} = 0.9$ V



$\lambda_n = 0.1\ V^{-1}$

$\lambda_p = 0.2\ V^{-1}$

calculate V_{in} & V_{b3}

$$A_v \cong -g_{m_1}(g_{m_2} r_{o_1} r_{o_2} \parallel g_{m_3} r_{o_3} r_{o_4}) = 200$$

$$\text{power} = V_{DD} \times I_{BIAS} \Rightarrow I_{BIAS} = \frac{\text{power}}{V_{DD}} = \frac{2mW}{1.8V} \approx 1.11\ mA$$

$$g_{m_2} r_{o_1} r_{o_2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{BIAS}} \cdot \left(\frac{1}{\lambda_n I_{BIAS}}\right)^2$$

$$= \sqrt{2 \cdot 100\ \mu A \cdot \frac{20}{0.18} \cdot 1.11\ mA} \cdot \left[\frac{1}{(0.1V^{-1})(1.11mA)}\right]^2$$

$$\approx 403\ k\Omega$$

$$g_{m_3} r_{o_3} r_{o_4} \approx 71.\ k\Omega$$

We know that $\frac{|A_v|}{(g_{m_2} r_{o_1} r_{o_2} \parallel g_{m_3} r_{o_3} r_{o_4})} = g_{m_1} = \frac{2I_D}{V_{GS_1} - V_{TH}}$

$$\therefore V_{in} = V_{GS_1} = V_{TH} + 2I_D \cdot \frac{g_{m_2} r_{o_1} r_{o_2} \parallel g_{m_3} r_{o_3} r_{o_4}}{A_v}$$

$$= (0.4\,V) + 2(1.11\,mA)\,\frac{(403\,k\Omega \parallel 71.\,k\Omega)}{200}$$

$$\approx 1.07\ V$$

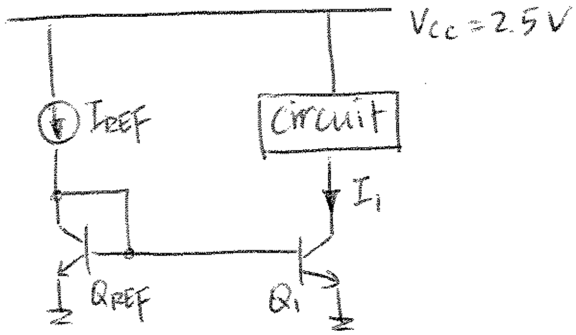
$$g_{m4} = \frac{2I_D}{V_{DD} - V_{D3} - |V_{THp}|} = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D}$$

$$\therefore\ V_{D3} = V_{DD} - |V_{THp}| - \frac{2I_D}{\sqrt{2\mu_p C_{ox}\frac{W}{L} I_D}}$$

$$= (1.8\,V) - (0.5\,V) - \frac{2(1.22\,mA)}{\sqrt{2\cdot\left(50\,\frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1.22\,mA)}}$$

$$\approx 0.67\ V$$

60.



$I_1 = 0.5mA$

power = 2mW

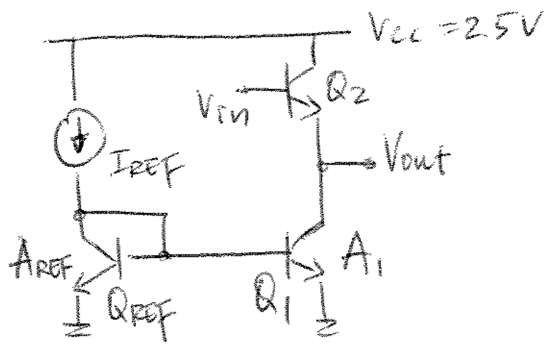
$$\text{Power} = V_{cc}(I_{REF} + I_1)$$

$$\Rightarrow I_{REF} = \frac{\text{Power}}{V_{cc}} - I_1 = \frac{2mW}{2.5V} - 0.5mA = 0.3mA$$

Therefore, if Q_{REF} has area A_E, then Q_1 has area $\frac{5}{3}A_E$ for the currents specified.

i.e. $\frac{A_{REF}}{A_1} = \frac{3}{5}$

61.



power = 3mW
$R_{out} = 50\Omega$

For an emitter follower, $R_{out} = r_{\pi 2} \| \frac{1}{g_{m2}}$

$$\Rightarrow R_{out} = 50\Omega = \frac{1}{\frac{I_{C2}}{V_T}\left(1+\frac{1}{\beta}\right)}$$

$$\therefore I_{C2} = \frac{V_T}{R_{out}} \cdot \frac{1}{1+1/\beta} = \frac{0.026}{50} \frac{1}{1+0.01} \approx 0.51\ mA$$

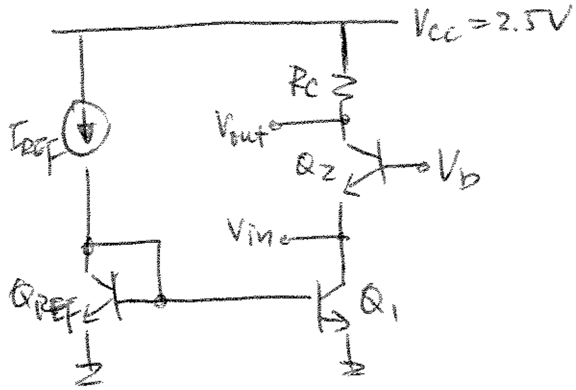
Realize that Vcc is providing current through I_{REF} & I_{C2}, and we are given

$$power = V_{CC}(I_{REF} + I_{C2}) = 3mW$$

$$\Rightarrow I_{REF} = \frac{power}{V_{CC}} - I_{C2} = \frac{3mW}{2.5V} - 0.51mA \approx 0.69mA$$

$$\Rightarrow \frac{I_{C2}}{I_{REF}} = \frac{A_1}{A_{REF}} = \frac{0.51}{0.69} \approx \frac{5}{7}$$

b2.



$R_{out} = 50\Omega$
$A_v = 20$
power $= 1.5\,mW$
$\beta >> 1$, $V_A \to \infty$

$$R_{out} = R_C \Rightarrow R_C = 50\Omega$$

$$A_v = g_m R_C = 20 \Rightarrow g_m = \frac{A_v}{R_C} = \frac{I_{C2}}{V_T}$$

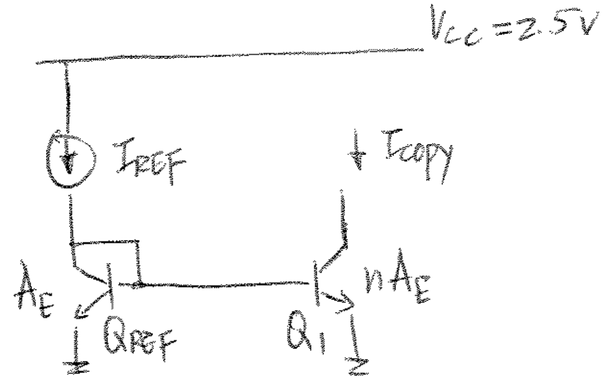
$$\Rightarrow I_{C2} = \frac{A_v V_T}{R_C} = \frac{20\,(0.026V)}{50\Omega} \approx 10.4\,mA$$

Realize that V_{cc} is providing current through I_{REF} & I_{C2}:

$$\text{power} = V_{cc}\,(I_{REF} + I_{C2})$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{cc}} - I_{C2} = \frac{1.5mW}{2.5V} - 10.4mA$$

63.



Given $I_{copy} = 0.5mA$

By KCL, $I_{REF} = I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta}$

$$= \frac{I_{copy}}{n} + \frac{I_{copy}/n}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = I_{REF} \cdot \frac{n}{1 + \frac{1}{\beta}(n+1)} = 0.5 \text{ mA}$$

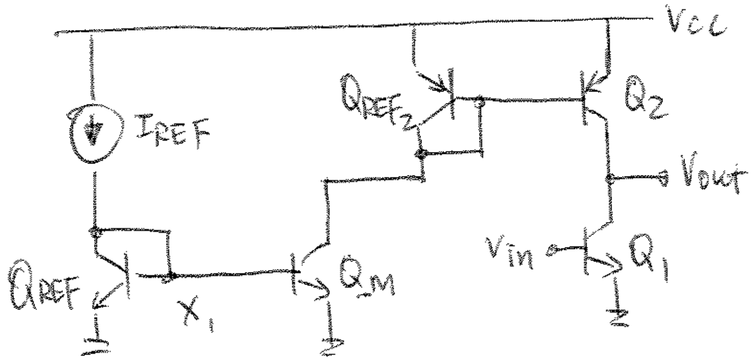
Within 1% implies that:

$$\Rightarrow I_{REF} \geq \frac{0.5mA}{0.99} \approx 0.505 \text{ mA.}$$

- For given n and β, $I_{copy} \leq n I_{REF}$. Since the error term causes $I_{copy} < n I_{REF}$ (strictly less than), one needs to increase I_{REF} in order to maintain the desired I_{copy}. This, however, means an increase of power (i.e. $\Delta p = V_{CC} \cdot \Delta I_{REF}$)

$\Rightarrow$ Trade off between accuracy & power dissipation.

64.



$$I_{C2} = I_{REF} \underbrace{\frac{(A_M/A_{REF})}{1+\frac{1}{\beta_n}\left(A_M/A_{REF}+1\right)}}_{X} \cdot \frac{(A_2/A_{REF_2})}{1+\frac{1}{\beta_p}\left(A_2/A_{REF_2}+1\right)}$$

Given $I_{C,M} \geq 0.98\, I_{REF}$ (less than 2% error)

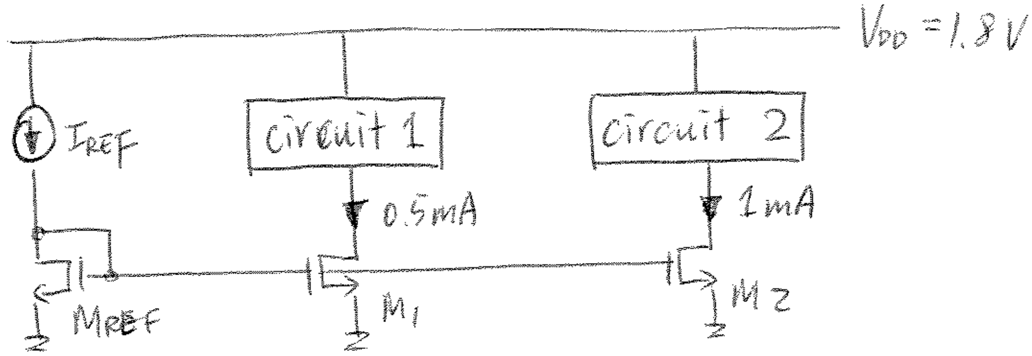
$$I_{C2} = 1\,mA = 0.98\, I_{REF} \cdot \frac{A_2/A_{REF_2}}{1+\frac{1}{50}\left(A_2/A_{REF_2}+1\right)}$$

Suppose $X = 0.98$ & $I_{REF} = 2\,mA$.

$$\Rightarrow \frac{A_2}{A_{REF_2}} \approx 0.5$$

Solution is not unique because no power constraint is present (i.e. I_{REF} is arbitrary.)

65.



power budget = 3 mW.

$$\text{power} = V_{DD}(I_{REF} + 0.5mA + 1mA)$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - 0.5mA - 1mA \approx 0.17mA$$

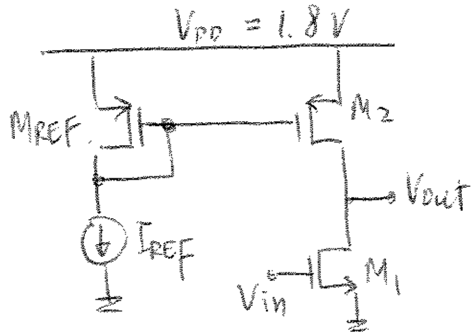
Assuming M_1 & M_2 operate in saturation,

If M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_1}{(W/L)_{REF}} = \frac{I_1}{I_{REF}} = \frac{50}{17}$$

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_2}{I_{REF}} = \frac{100}{17}$$

66.



$A_v = -20$

power = 2mW

$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\quad \lambda_n = 0.1V^{-1}$

$\lambda_p = 0.2V^{-1}$

$$R_{out} = r_{o2} \parallel r_{o1} = \frac{1}{\lambda_n I_{D1} + \lambda_p I_{D1}}$$

$$\Rightarrow A_v = -G_m R_{out} = \frac{-g_{m1}}{\lambda_n I_{D1} + \lambda_p I_{D1}} = -\frac{2I_{D1}/(V_{GS1}-V_{TH})}{I_{D1}(\lambda_n+\lambda_p)}$$

$$\Rightarrow -20 = -\frac{2}{(V_{GS1}-V_{TH})(\lambda_n+\lambda_p)}$$

$$\Rightarrow V_{GS1} = \frac{1}{10(\lambda_n+\lambda_p)} + V_{THn}$$

$$= \frac{1}{10(0.1+0.2)V^{-1}} + 0.4V \approx 0.73\ V$$

$$\Rightarrow I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{THn})^2$$

$$= \frac{1}{2}\left(100\ \frac{uA}{V^2}\right)\left(\frac{20}{0.18}\right)(0.33V)^2 \approx 0.61\ mA$$

$\because$ power $= V_{DD}(I_{REF} + I_{D1})$

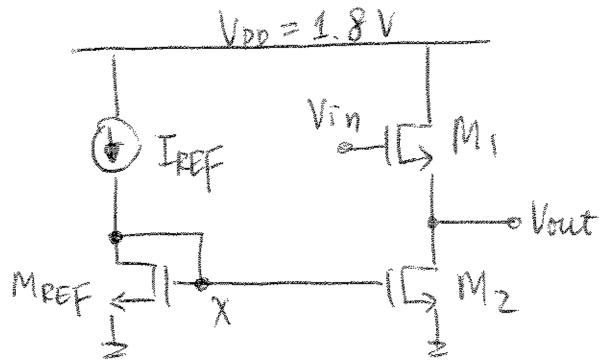
$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - I_{D1} = \frac{2mW}{1.8V} - 0.61mA$$

$$\approx 0.5\ mA$$

$\therefore$ if M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_{D2}}{I_{REF}} = \frac{61}{50} \approx 1.2$$

67.



Given:

$A_v = 0.85$

$R_{out} = 100\,\Omega$

$(W/L)_2 = 10/0.18$

$\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$$R_{out} = r_{o2} \,\|\, \left(\frac{1}{g_{m1}} \,\|\, r_{o1}\right) = \frac{1}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 100$$

For source follower,

$$A_v = \frac{g_{m1}}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 0.85$$

$$\Rightarrow g_{m1} = \frac{0.85}{100} = 8.5 \cdot 10^{-3}\,S$$

$$R_{out} = \frac{1}{g_{m1} + 2/r_o} = 100$$

$$\Rightarrow r_o = \frac{200}{1 - 100 g_{m1}} = \frac{200}{1 - 100(8.5 \cdot 10^{-3})}$$

$$\approx 1333\,\Omega$$

$$\Rightarrow I_{D1} = \frac{1}{\lambda_n r_{o1}} = 7.5\,mA.$$

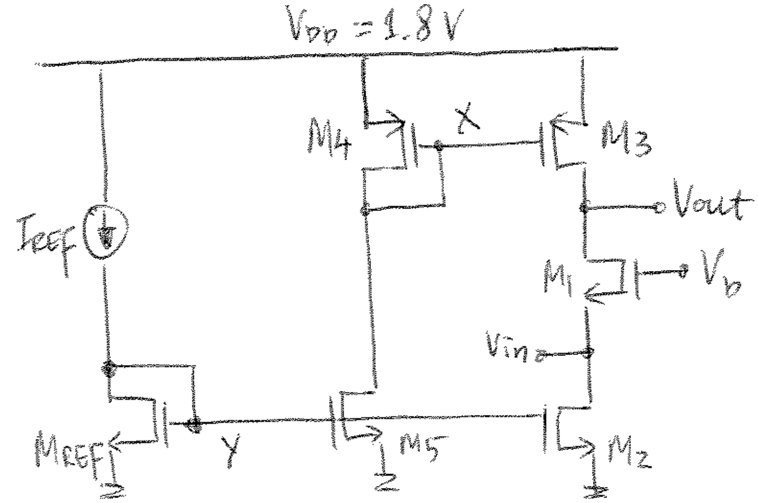
Assume $V_X \approx 1\,V$

$$\left(\frac{W}{L}\right)_2 = \frac{2 I_{D1}}{\mu_n C_{ox} (V_X - V_{TH})^2} \approx 416$$

Set $I_{REF} \approx 0.75$ mA.

$$\Rightarrow \left(\frac{W}{L}\right)_{REF} = \left(\frac{W}{L}\right)_{2} \frac{I_{REF}}{I_{D2}} \approx 42.$$

68.



$$\left(\frac{W}{L}\right)_3 = 20/0.18, \quad \lambda_n = 0.1\ V^{-1}, \quad \lambda_p = 0.2\ V^{-1}$$

$$A_v = 20, \quad R_{in} = 50\,\Omega$$

$$R_{in} = 50\,\Omega = r_{O2} \parallel 1/g_{m1} = \frac{1}{\lambda_n I_{D1} + g_{m1}} \quad \text{---} \ (1)$$

$$R_{out} = r_{O3}$$

$$A_v = g_{m1} r_{O3} = \frac{g_{m1}}{\lambda_p I_{D1}} \quad \text{---} \ (2)$$

Solve for g_{m1} in (2) and substitute it into (1):

$$50 = \frac{1}{\lambda_n I_{D1} + A_v \lambda_p I_{D1}}$$

$$\Rightarrow I_{D1} = \frac{1}{(\lambda_n + A_v \lambda_p)(50\,\Omega)} = \frac{1}{(0.1 + 20(0.2))(50\,\Omega)} \approx 4.88\text{ mA}$$

$$|V_{GS3}| = \sqrt{\frac{2 I_{D1}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} + |V_{THp}| \approx 1.44 \text{ V}$$

$$g_{m1} = A_v \lambda_p I_{D1} \Rightarrow \left(\frac{W}{L}\right)_1 = \left[\frac{A_v \lambda_p I_{D1}}{\sqrt{2 \mu_n C_{ox} I_{D1}}}\right]^2$$

$$\approx 390.$$

Since $V_X \approx 0.4$ V, size up other transistors to allow them to operate in saturation.

Suppose $I_{D4} = 1.2$ mA $\Rightarrow \left(\frac{W}{L}\right)_4 = \dfrac{2 I_{D4}}{\mu_p C_{ox} (|V_{GS3}| - |V_{THp}|)^2}$

$$\approx 10/0.18$$

$$I_{D5} = I_{D4} \Rightarrow \left(\frac{W}{L}\right)_5 = \frac{2 I_{D5}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{100}{0.18}$$

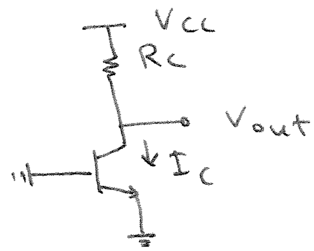
(Assume $V_y = 0.6$; this is arbitrary, but must ensure M_5 in saturation.)

Set $I_{REF} = I_{D5} \Rightarrow \left(\frac{W}{L}\right)_{REF} \approx \dfrac{100}{0.18}$

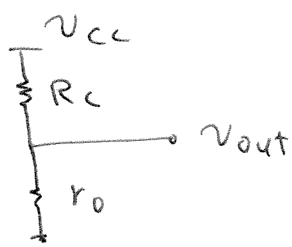
$$I_{D2} \approx I_{D3} \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{2 I_{D2}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{45}{0.18}$$

Total power $= V_{DD}(I_{REF} + I_{D4} + I_{D3})$
$= 1.8(7.3)$ mW ≈ 13 mW

(1)

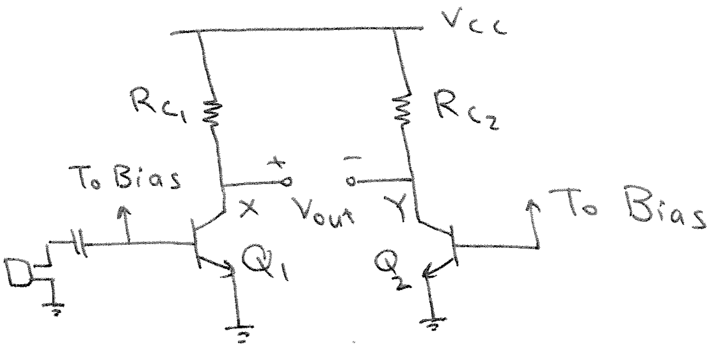


the small signal model is as follow:



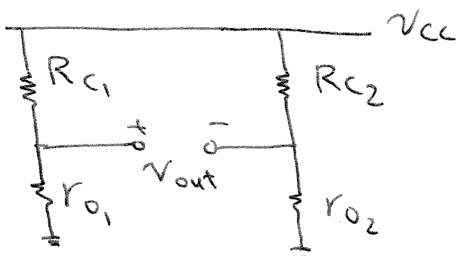
$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{\frac{V_A}{I_C}}{\frac{V_A}{I_C} + R_C} = \frac{V_A}{V_A + R_C I_C}$$

(2)



$R_{C_1} = R_{C_2}$

The small signal model is:

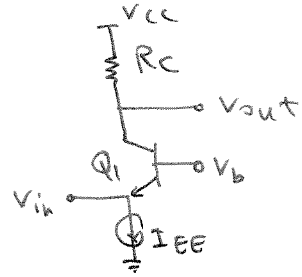


$r_{o_1} = r_{o_2}$
$R_{C_1} = R_{C_2}$

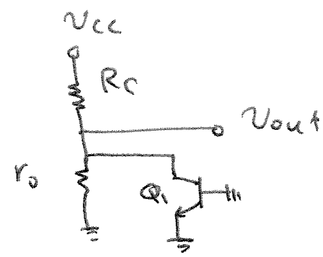
$$\frac{v_{out}}{v_{cc}} = \frac{1}{v_{cc}}\left(\frac{r_{o_1}}{R_{C_1}+r_{o_1}} - \frac{r_{o_2}}{R_{C_2}+r_{o_2}}\right)v_{cc} = 0$$

(3)

(a)

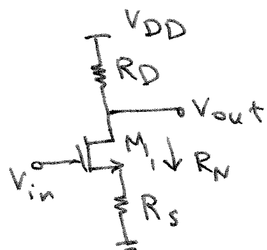


The small signal model is:

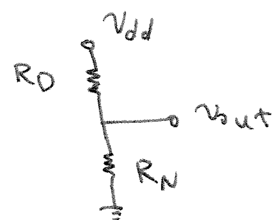


$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{V_A}{V_A + R_C I_{EE}}$$

(b)

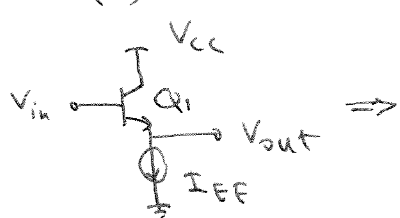


$\Rightarrow$

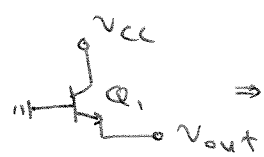


$$R_N = g_{m1} r_{o1} R_S + r_{o1} + R_S \quad , \quad \frac{v_{out}}{v_{dd}} = \frac{R_N}{R_N + R_D}$$

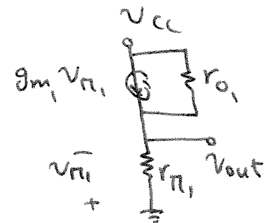
(c)



$\Rightarrow$



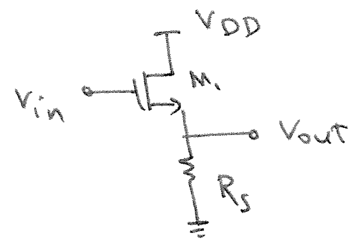
$\Rightarrow$



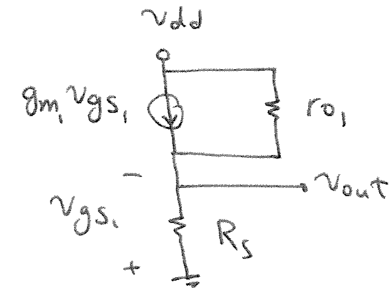
$$\frac{v_{out}}{r_{\pi 1}} + \frac{v_{out} - v_{cc}}{r_{o1}} + g_{m1} v_{out} = 0 \rightarrow (r_{\pi 1} + r_{o1} + g_{m1} r_{o1} r_{\pi 1}) v_{out} = r_{\pi 1} v_{cc}$$

$$\Rightarrow \quad \frac{v_{out}}{v_{cc}} = \frac{r_{\pi 1}}{\beta r_{o1} + r_{o1} + r_{\pi 1}}$$

(3) (d)



$\Rightarrow$

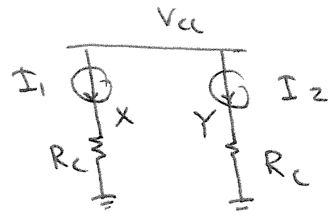


$$\frac{v_{out}}{R_S} + \frac{v_{out} - v_{dd}}{r_{o_1}} + g_{m_1} v_{out} = 0$$

$$\Rightarrow (r_{o_1} + R_S + g_{m_1} r_{o_1} R_S) v_{out} = R_S \, v_{dd}$$

$$\Rightarrow \frac{v_{out}}{v_{dd}} = \frac{R_S}{g_{m_1} r_{o_1} R_S + R_S + r_{o_1}}$$

④

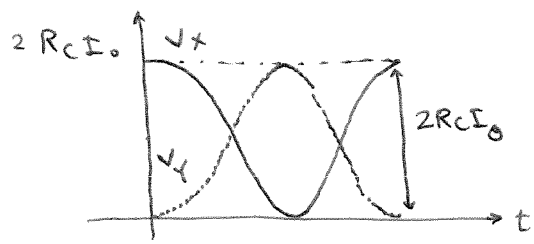


$I_1 = I_o \cos\omega t + I_o$

$I_2 = -I_o \cos\omega t + I_o$

$V_X = R_C I_1 = R_C I_o (1 + \cos\omega t)$

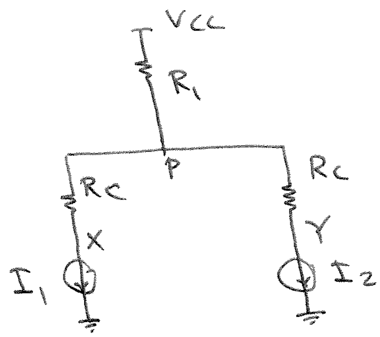
$V_Y = R_C I_2 = R_C I_o (1 - \cos\omega t)$



$V_{X_{p-p}} = V_{Y_{p-p}} = 2R_C I_o$

$V_{X,CM} = V_{Y,CM} = R_C I_o$

(5)



$I_1 = I_0 \cos\omega t + I_0$

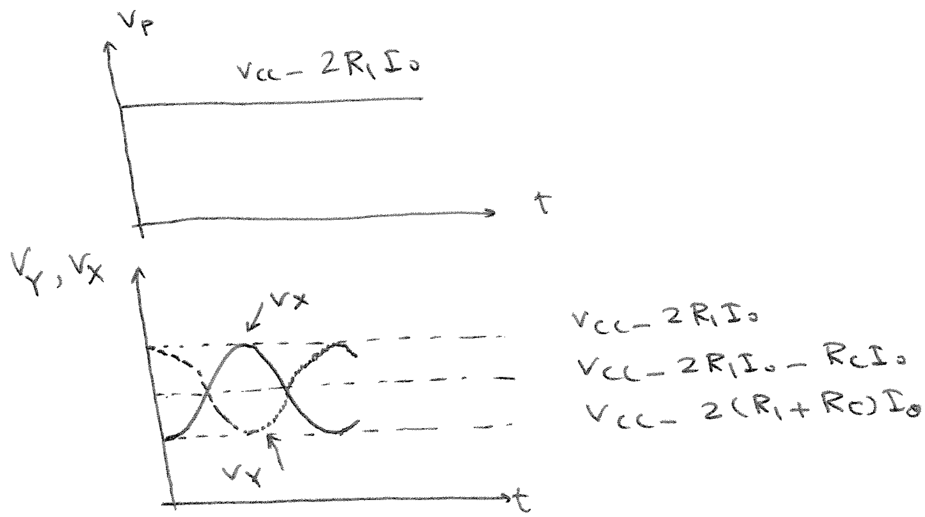
$I_2 = -I_0 \cos\omega t + I_0$

$V_P = V_{CC} - R_1 (I_1 + I_2) = V_{CC} - 2 R_1 I_0$

$V_X = V_P - R_C I_1 = V_{CC} - 2R_1 I_0 - R_C I_0 - R_C I_0 \cos\omega t$

$\Rightarrow V_X = V_{CC} - (2R_1 + R_C) I_0 - R_C I_0 \cos\omega t$

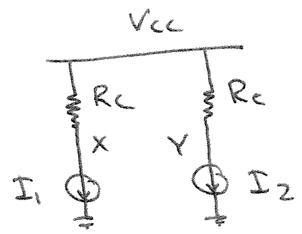
$V_Y = V_P - R_C I_2 = V_{CC} - (2R_1 + R_C) I_0 + R_C I_0 \cos\omega t$



$V_{X,CM} = V_{Y,CM} = V_{CC} - (2R_1 + R_C) I_0$

$V_{X,P\text{-}P} = V_{Y,P\text{-}P} = 2 R_C I_0$

(6)

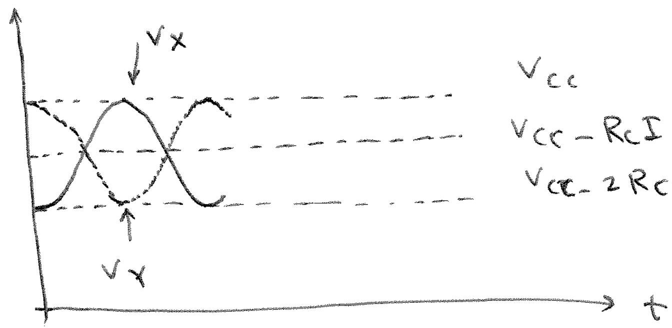


$$I_1 = I_0 \cos\omega t + I_0$$

$$I_2 = -I_0 \cos\omega t + I_0$$

$$V_X = V_{CC} - R_C I_1 = V_{CC} - R_C I_0 (1 + \cos\omega t)$$

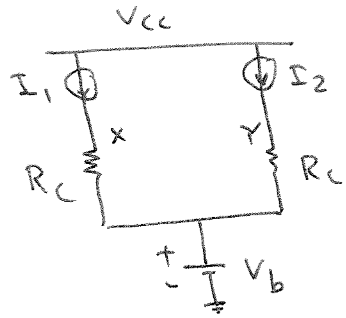
$$V_Y = V_{CC} - R_C I_2 = V_{CC} - R_C I_0 (1 - \cos\omega t)$$



$$V_{X,CM} = V_{Y,CM} = V_{CC} - R_C I_0$$

$$V_{X,p\text{-}p} = V_{Y,p\text{-}p} = 2 R_C I_0$$

⑦

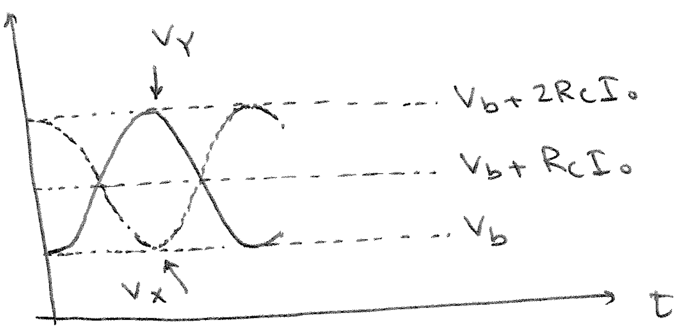


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = R_C I_1 + V_b = R_C I_0 (1 + \cos\omega t) + V_b$

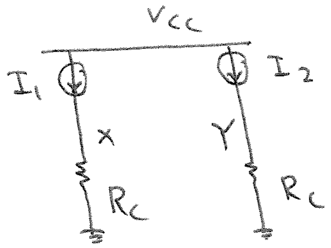
$V_Y = R_C I_2 + V_b = R_C I_0 (1 - \cos\omega t) + V_b$



$V_{X,CM} = V_{Y,CM} = V_b + R_C I_0$

$V_{X,P\text{-}P} = V_{Y,P\text{-}P} = 2 R_C I_0$

(8)

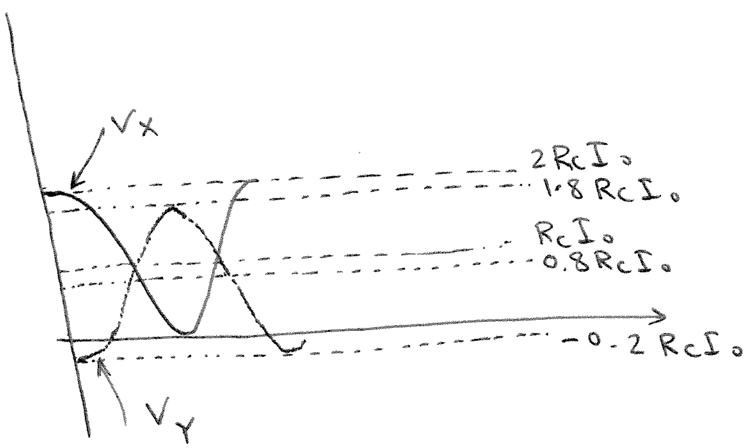


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + 0.8 I_0$

$V_X = R_C I_1 = R_C I_0 (1 + \cos\omega t)$

$V_Y = R_C I_2 = R_C I_0 (0.8 - \cos\omega t)$



$V_{X,CM} = R_C I_0$ $\quad V_{Y,CM} = 0.8 R_C I_0$

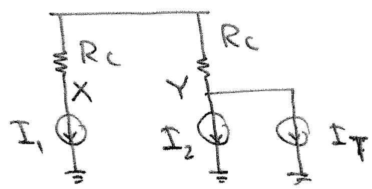
$V_{X,p\text{-}p} = 2 R_C I_0$ $\quad V_{Y,p\text{-}p} = 2 R_C I_0$

X and Y are true differential signals.

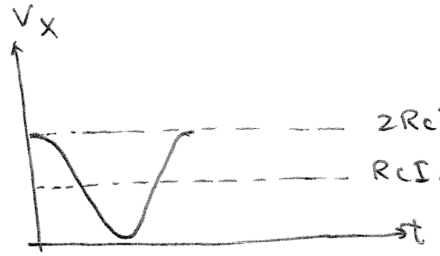
(9)

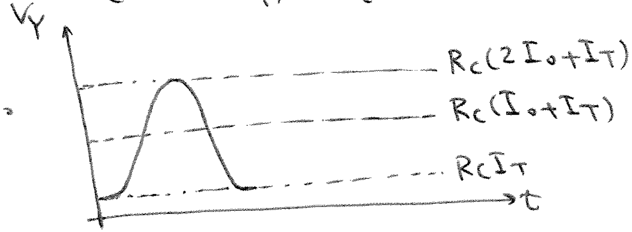
$$I_1 = I_o \cos\omega t + I_o$$
$$I_2 = -I_o \cos\omega t + I_o$$

(a)

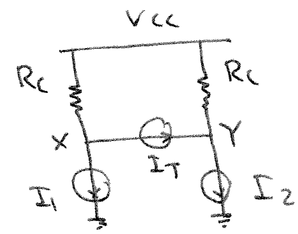


$$V_X = R_C I_1 = R_C I_o (1 + \cos\omega t)$$
$$V_Y = R_C (I_2 + I_T) =$$
$$R_C (I_o + I_T) - R_C I_o \cos\omega t$$

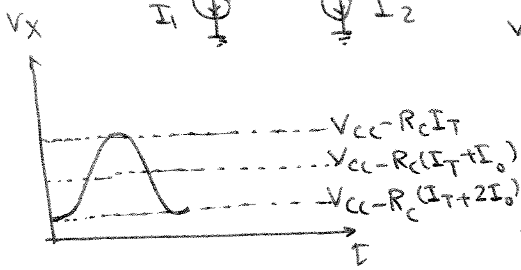


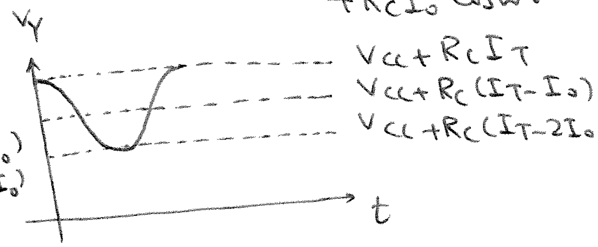


(b)

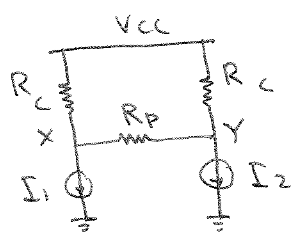


$$V_X = V_{CC} - R_C(I_1 + I_T) = V_{CC} - R_C(I_o + I_T) - R_C I_o \cos\omega t$$
$$V_Y = V_{CC} - R_C(I_2 - I_T) = V_{CC} - R_C(I_o - I_T) + R_C I_o \cos\omega t$$



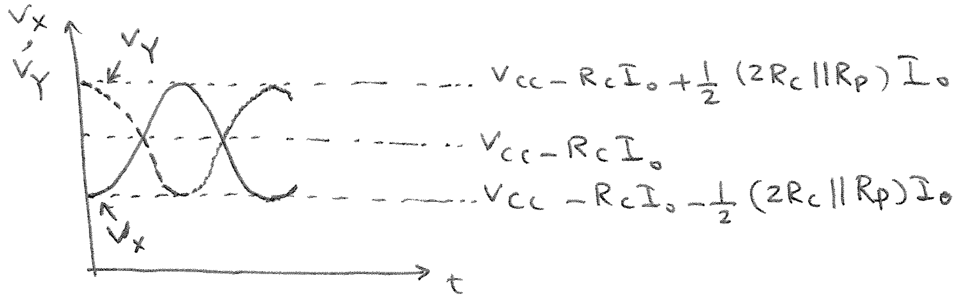


(c)

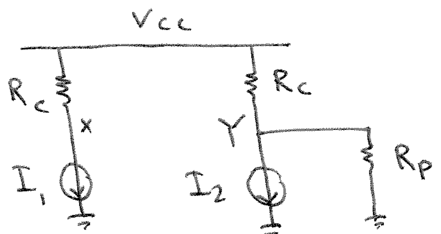


$$\begin{cases} \dfrac{V_X - V_{CC}}{R_C} + \dfrac{V_X - V_Y}{R_P} + I_1 = 0 \\ \dfrac{V_Y - V_{CC}}{R_C} + \dfrac{V_Y - V_X}{R_P} + I_2 = 0 \end{cases} \Rightarrow$$

$$V_X = V_{CC} - R_C I_o - \frac{R_C R_P}{2R_C + R_P} I_o \cos\omega t$$
$$V_Y = V_{CC} - R_C I_o + \frac{R_C R_P}{2R_C + R_P} I_o \cos\omega t$$



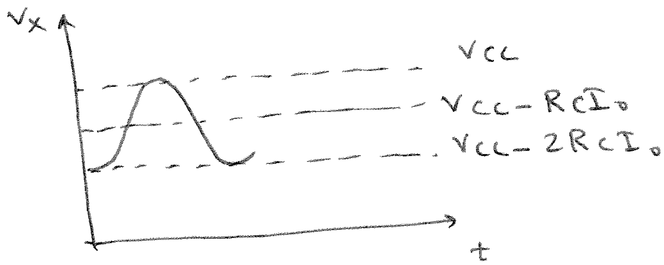
(d)

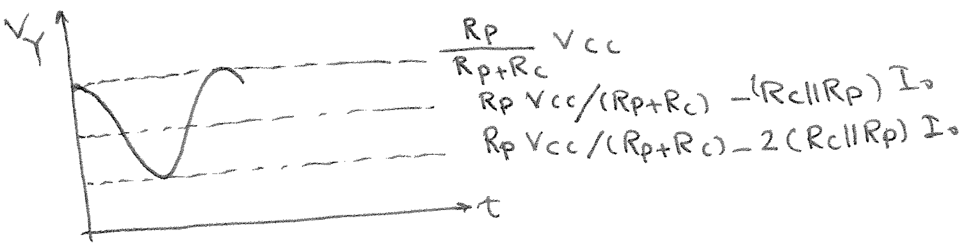


$$v_X = V_{CC} - R_C I_1 = V_{CC} - R_C I_0 (1+\cos\omega t)$$

$$v_Y = \frac{R_P}{R_P+R_C} V_{CC} - (R_C \| R_P) I_2 =$$

$$\frac{R_P}{R_P+R_C} V_{CC} - (R_C \| R_P) I_0 (1-\cos\omega t)$$



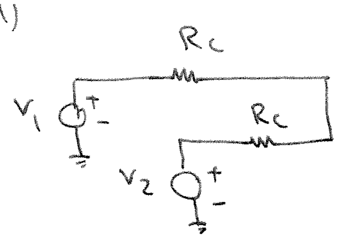


10)

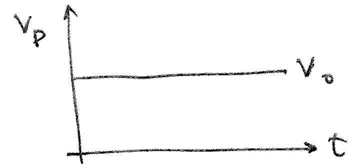
$V_1 = V_0 \cos\omega t + V_0$

$V_2 = -V_0 \cos\omega t + V_0$

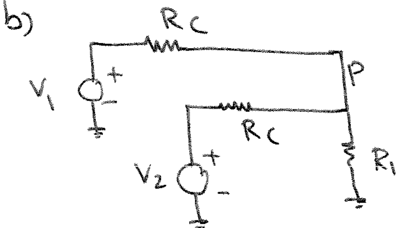
(a)



$V_p = \frac{V_1 + V_2}{2} = V_0$



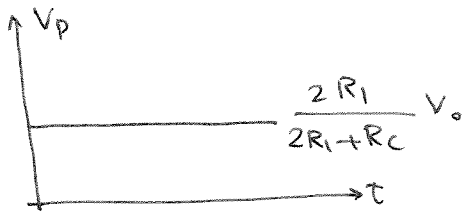
(b)



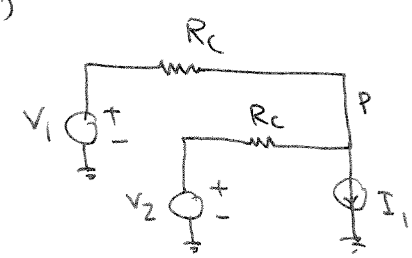
$\frac{2V_p - V_1 - V_2}{R_c} + \frac{V_p}{R_1} = 0 \Rightarrow$

$\frac{2V_p - 2V_0}{R_c} + \frac{V_p}{R_1} = 0 \Rightarrow (2R_1 + R_c)V_p = 2V_0 R_1$

$\rightarrow V_p = \frac{2R_1}{2R_1 + R_c} V_0$



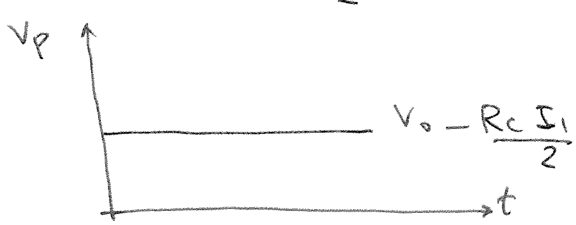
(c)



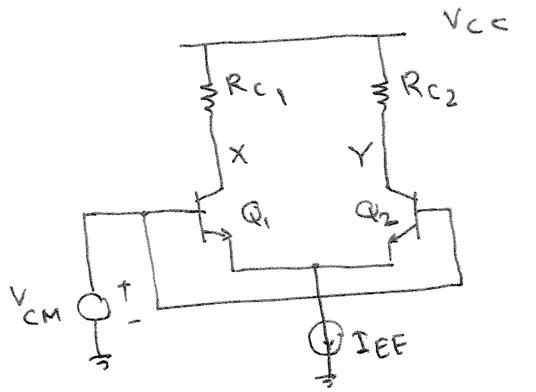
$\frac{2V_p - V_1 - V_2}{R_c} + I_1 = 0 \Rightarrow$

$\frac{2V_p - 2V_0}{R_c} = -I_1 \Rightarrow$

$V_p = V_0 - \frac{R_c I_1}{2}$



(11)



$R_{C1} = R_{C2} = R_C$

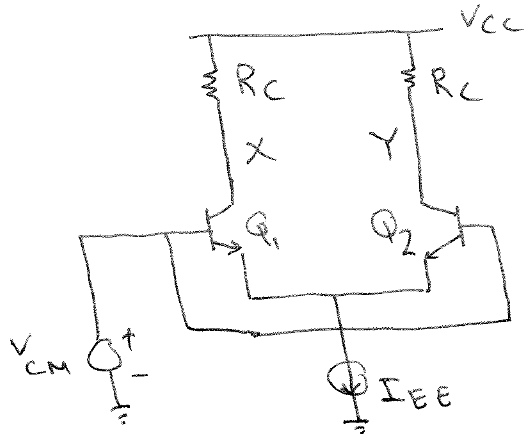
if V_{CC} changes by ΔV then:

$\Delta V_X = \Delta V$, $\Delta V_Y = \Delta V$ $\Rightarrow$

$\Delta (V_X - V_Y) = 0$

Since both V_X and V_Y response to the supply changes similarly, the circuit rejects supply noise.

12

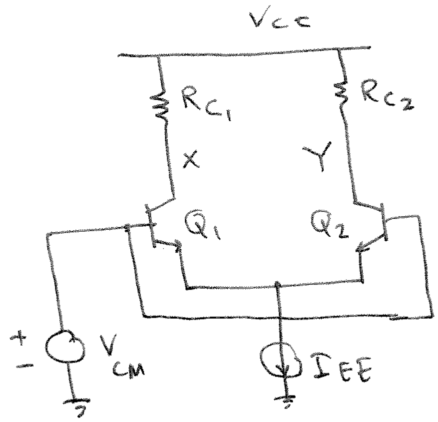


$$\Delta V_X = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Delta V_Y = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Rightarrow \Delta(V_X - V_Y) = 0$$

(13)



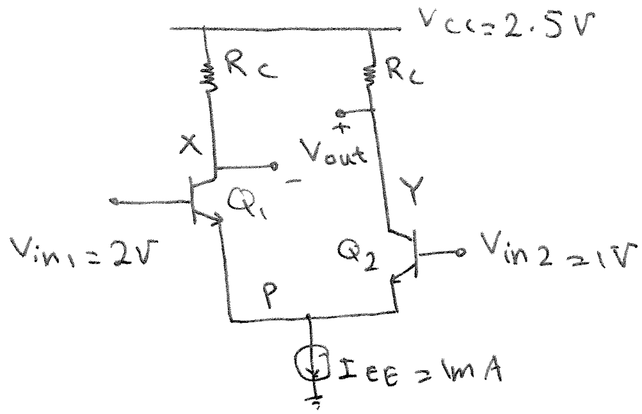
$R_{C1} = R_{C2} + \Delta R$

$$\Delta V_X = -\frac{R_{C1}\,\Delta I}{2} = -\frac{(R_{C2} + \Delta R)\,\Delta I}{2}$$

$$\Delta V_Y = -\frac{R_{C2}\,\Delta I}{2} \Rightarrow$$

$$\Delta(V_X - V_Y) = -\frac{\Delta R\,\Delta I}{2}$$

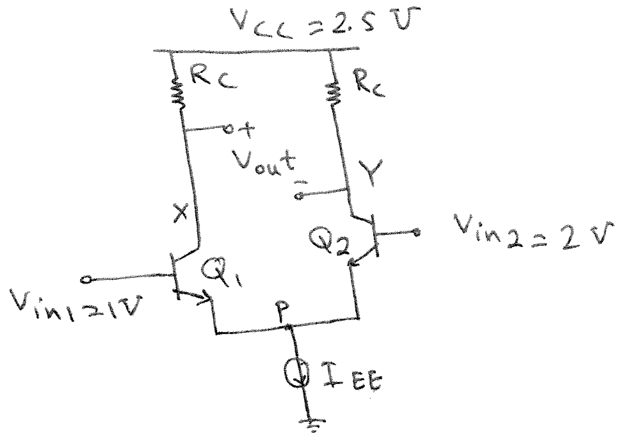
(14)



$$V_X \geq V_{in_1} \Rightarrow V_X \geq 2 \Rightarrow V_{CC} - R_C I_{EE} \geq 2$$

$$\Rightarrow 2.5 - R_C^{(K\Omega)} \geq 2 \Rightarrow R_C \leq 0.5\ K\Omega$$

15

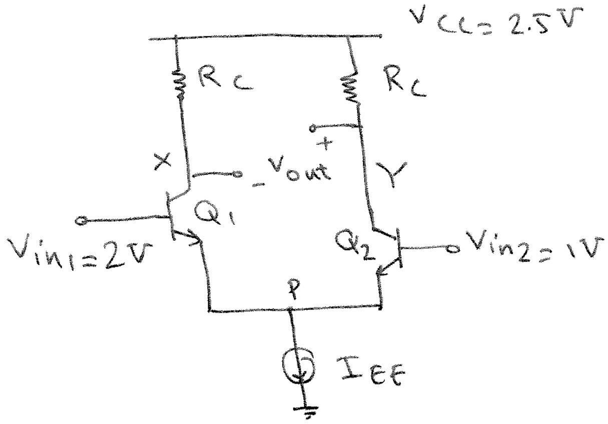


$R_C = 500\Omega$

$$V_Y \geq V_{in2} \Rightarrow \quad V_{CC} - R_C I_{EE} \geq 2 \Rightarrow$$

$$2.5 - 500 I_{EE} \geq 2 \Rightarrow \quad I_{EE} \leq 1mA$$

16



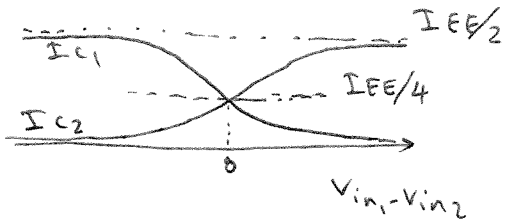
IEE = 1mA
Rc = 800 Ω

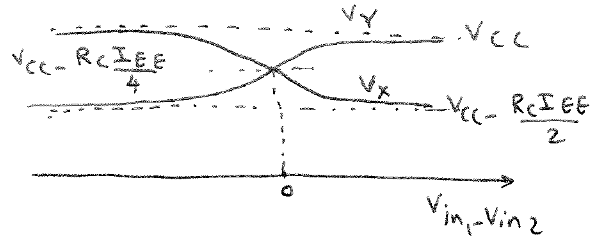
$$V_X = V_{CC} - R_C I_{EE} = 2.5 - 0.8 = 1.7\ V$$

$\Rightarrow$ $V_X < V_{in1}$ $\Rightarrow$ Q_1 is in saturation region.

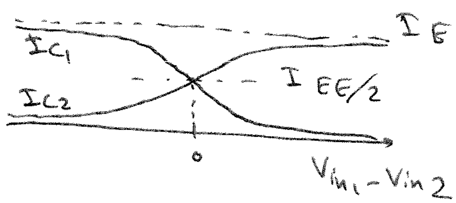
(17)

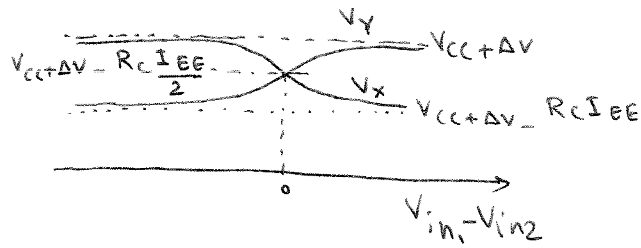
(a)


$I_{EE}/2$
I_{C1}
$I_{EE}/4$
I_{C2}
0
$V_{in1}-V_{in2}$

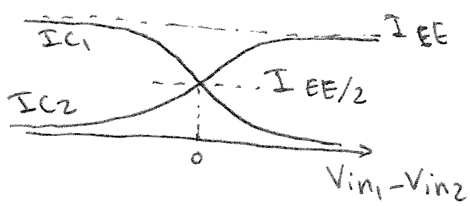

V_Y
V_{CC}
$V_{CC}-R_C\frac{I_{EE}}{4}$
V_X
$V_{CC}-\frac{R_C I_{EE}}{2}$
0
$V_{in1}-V_{in2}$

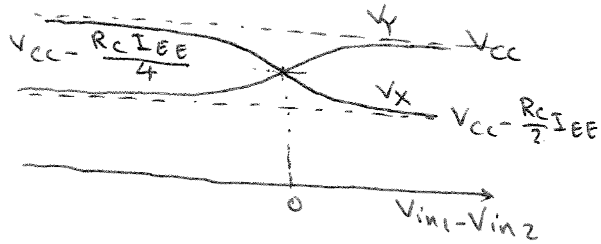
(b)


I_{EE}
I_{C1}
$I_{EE}/2$
I_{C2}
0
$V_{in1}-V_{in2}$

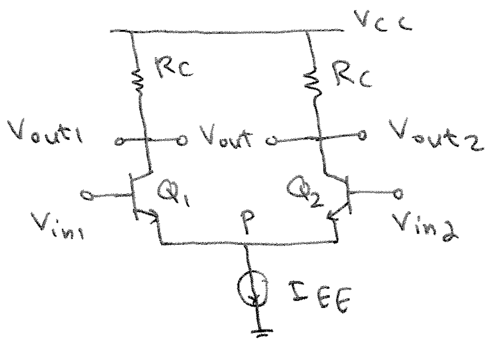

V_Y
$V_{CC}+\Delta V$
$V_{CC}+\Delta V-R_C\frac{I_{EE}}{2}$
V_X
$V_{CC}+\Delta V-R_C I_{EE}$
0
$V_{in1}-V_{in2}$

(c)


I_{EE}
I_{C1}
$I_{EE}/2$
I_{C2}
0
$V_{in1}-V_{in2}$


V_Y
V_{CC}
$V_{CC}-\frac{R_C I_{EE}}{4}$
V_X
$V_{CC}-\frac{R_C}{2}I_{EE}$
0
$V_{in1}-V_{in2}$

18



$$\frac{I_{C1}}{I_{C2}} = 5$$

$$V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 0.026 \ln 5 = 41.845\ mV$$

at 27°, $V_T = 26mV$ $\Rightarrow$ at 100°C,

$$V_T = \frac{(273+100)}{273+27}\ 26^{mV} = 32.33\ mV$$

$$\Rightarrow \quad 41.845\ mV = 32.33\ mV \ln \frac{I_{C1}}{I_{C2}} \Rightarrow \frac{I_{C1}}{I_{C2}} = 3.65$$

(19)

$I_{C_2} = I_{C_1} = \frac{I_{EE}}{2}$

if I_{C_2} changes by 10% then

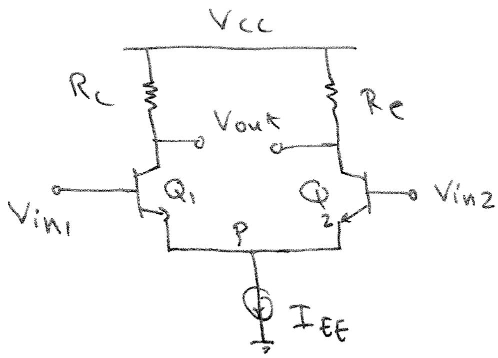
$$1.1 \times I_{C_2 \, bias} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$1.1 \times \frac{I_{EE}}{2} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$V_{in1} - V_{in2} = V_T \ln \frac{0.9}{1.1} = -0.2 \, V_T = -5.217 \, mV$$

So the input differential voltage should change by no more than 5.2 mV.

(20)



$$I_{C2} = \frac{I_{EE}}{1+\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}$$

$$I_{C2,bias} = \frac{I_{EE}}{2}$$

if the transconductance of Q_2 drops by a factor of 2, then $I_{C2} = \frac{I_{EE}}{4}$

$$\Rightarrow \quad \frac{I_{EE}}{4} = \frac{I_{EE}}{1+\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)} \Rightarrow$$

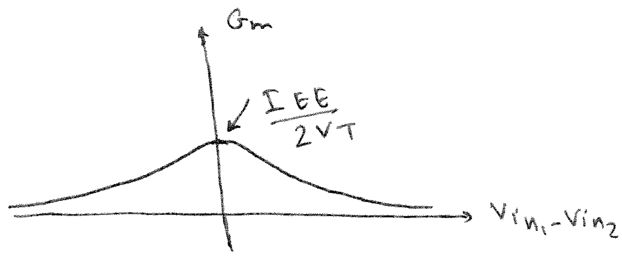
$$V_{in1} - V_{in2} = V_T \ln 3 = 1.0986\, V_T = 28.564\ mV$$

(21) $V_{in1} - V_{in2} = \Delta V_{in}$

$$I_{C1} - I_{C2} = \frac{I_{EE} \exp \frac{\Delta V_{in}}{V_T}}{1 + \exp \frac{\Delta V_{in}}{V_T}} - \frac{I_{EE}}{1 + \exp \frac{\Delta V_{in}}{V_T}}$$

$$\Rightarrow \frac{\partial (I_{C1} - I_{C2})}{\partial (\Delta V_{in})} = I_{EE} \left[\frac{\frac{1}{V_T} \exp(\frac{\Delta V_{in}}{V_T})(1 + \exp \frac{\Delta V_{in}}{V_T}) - \frac{(\exp \frac{\Delta V_{in}}{V_T})^2}{V_T}}{(1 + \exp \frac{\Delta V_{in}}{V_T})^2} + \frac{\frac{1}{V_T} \exp \frac{\Delta V_{in}}{V_T}}{(1 + \exp \frac{\Delta V_{in}}{V_T})^2} \right]$$

$$= \frac{2 I_{EE}}{V_T} \frac{\exp(\frac{V_{in1} - V_{in2}}{V_T})}{(1 + \exp(\frac{V_{in1} - V_{in2}}{V_T}))^2}$$



$$\begin{cases} \max G_m = \frac{I_{EE}}{2V_T} \\ \text{At } V_{in1} - V_{in2} = 0 \end{cases}$$

if $G_m = \frac{1}{2} G_{m_{max}} = \frac{I_{EE}}{4V_T} \Rightarrow$

$$\frac{\exp(\frac{V_{in1} - V_{in2}}{V_T})}{(1 + \exp(\frac{V_{in1} - V_{in2}}{V_T}))^2} = \frac{1}{8} \Rightarrow V_{in1} - V_{in2} = \pm 1.763 V_T$$

$$= \pm 45.838 \text{ mV}$$

(22)

$$V_{out1} - V_{out2} = -R_C I_{EE} \tanh \frac{V_{in1} - V_{in2}}{2V_T}$$

$$A_V = \frac{\partial (V_{out1} - V_{out2})}{\partial (V_{in1} - V_{in2})} = -\frac{2R_C I_{EE}}{V_T} \frac{\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}{\left[1+\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)\right]^2}$$

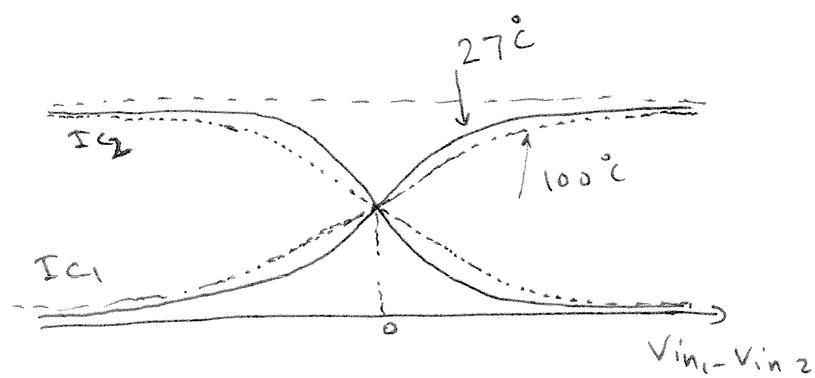
if $V_{in1} - V_{in2} = 30mV \Rightarrow$

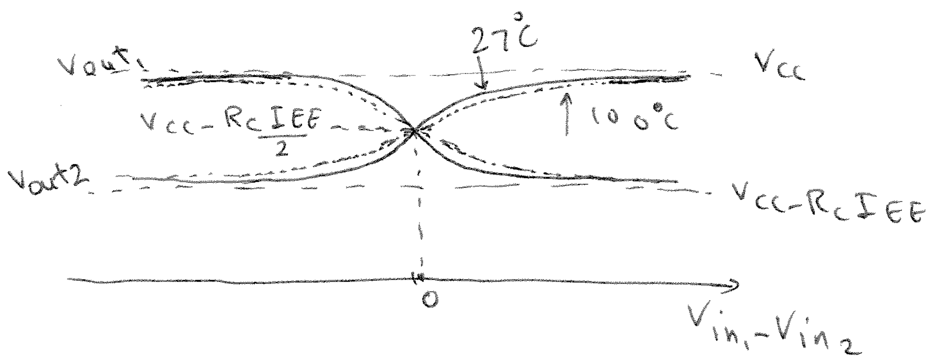
$$A_V = -14.02 \; R_C I_{EE}$$

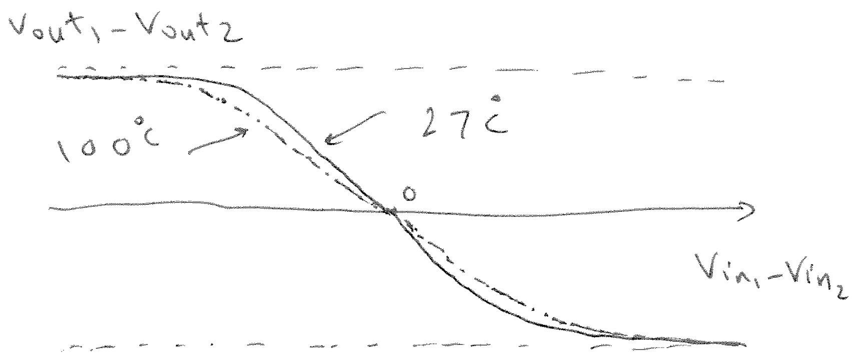
(23)

if the ambient temperature goes from 27°C to 100°C, V_T will change from 26mV to 32.33mV.

Therefor the curves stretch out as shown below to the sides:

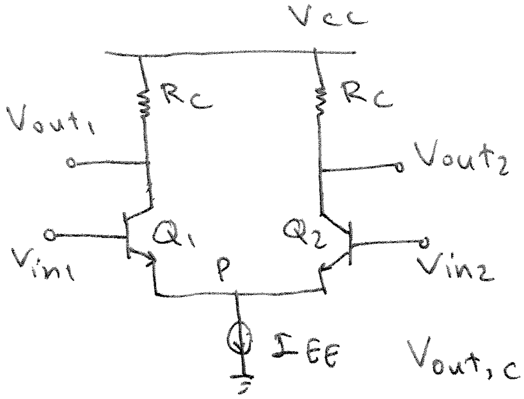






(24) $R_C = 500\Omega$, $I_{EE} = 1mA$, $V_{CC} = 2.5V$

$V_{in_1} = V_0 \sin \omega t + V_{CM}$ $V_{in_2} = -V_0 \sin \omega t + V_{CM}$, $V_{CM} = 1V$

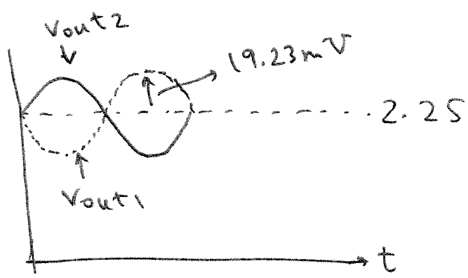


$A_v = -g_m R_C = -\frac{I_{EE}}{2V_T} R_C = -\frac{10^{-3} \times 500}{2 \times 0.026} = -9.615$

$V_{out,CM} = V_{CC} - R_C \frac{I_{EE}}{2} = 2.5 - 0.5 \times 0.5 \Rightarrow$

$V_{out,CM} = 2.25$

(a) $|V_{out}| = |A_v V_{in}| = 9.615 \times 2m = 19.23mV$

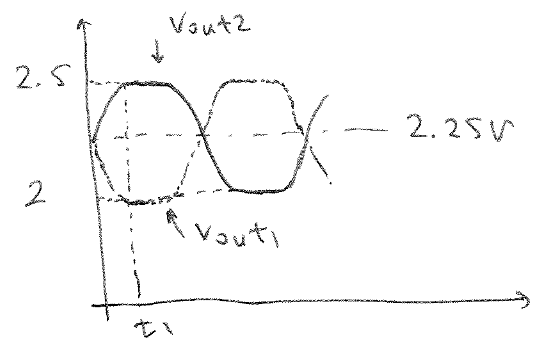


(b) $I_{C_1} = 0.95 I_{EE}$, $I_{C_2} = 0.05 I_{EE}$, $\frac{I_{C_1}}{I_{C_2}} = 19$

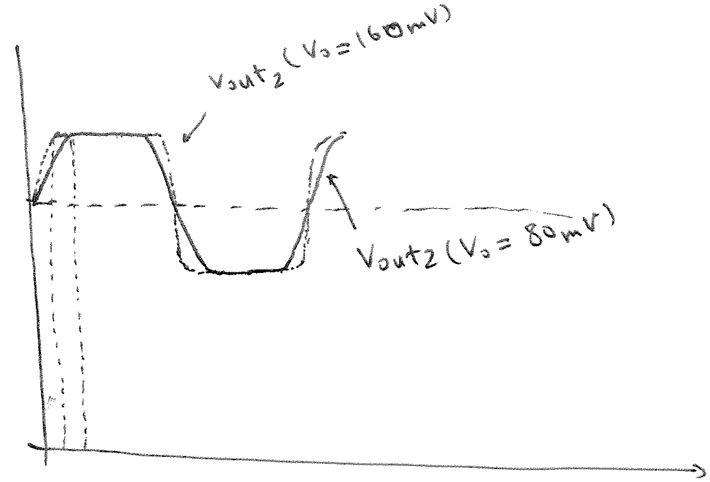
$V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555 mV$

$\frac{V_{in_1} - V_{in_2}}{2} = 50mV \sin \omega t_1 \Rightarrow 38.278mV = 50mV \sin \omega t_1 \Rightarrow$

$t_1 = \frac{0.872}{\omega}$



(25)



The time at which one transistor takes 95% of the tail current source is achievable through:

$$\frac{I_{C_1}}{I_{C_2}} = 19 \Rightarrow V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555\text{ mV}$$

$$\frac{V_{in_1} - V_{in_2}}{2} = V_o \sin \omega t_1 \Rightarrow t_1 = \frac{\text{Arc Sin } \frac{38.278}{V_o}}{\omega}$$

evidently as V_o increases, t_1 decreases and the output waveform becomes sharper.

$$t_1 (V_o = 80\text{ mV}) = \frac{0.499}{\omega}$$

$$t_1 (V_o = 160\text{ mV}) = \frac{0.242}{\omega}$$

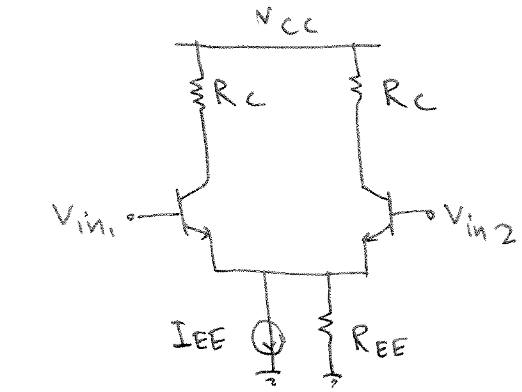
(26) $\omega = 2\pi \times (100\,MHz)$

$$\text{Slope} \approx \frac{V_{cc} - V_{cm}}{t_1} = \frac{0.25\,\omega}{\text{ArcSin}\left(\frac{38.278}{V_o\,(mV)}\right)}$$

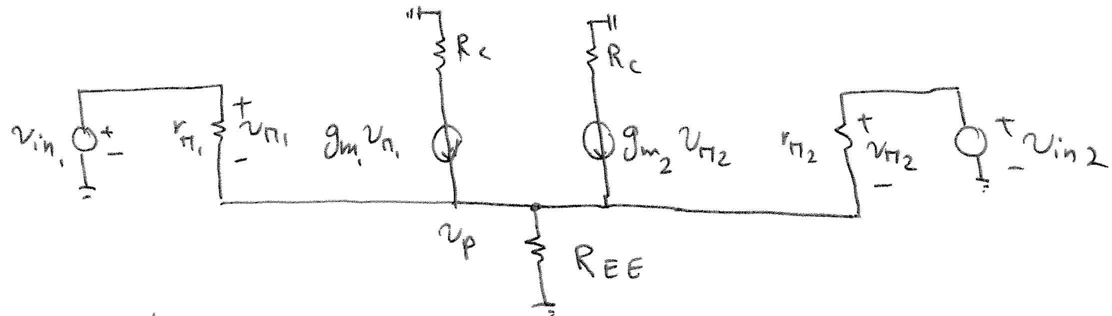
$\Rightarrow$ if $V_o = 80\,mV \Rightarrow \text{Slope} = 3.148 \times 10^{8}\ V/s$

if $V_o = 160\,mV \Rightarrow \text{slope} = 6.491 \times 10^{8}\ V/s$

(27)



The small signal model is:

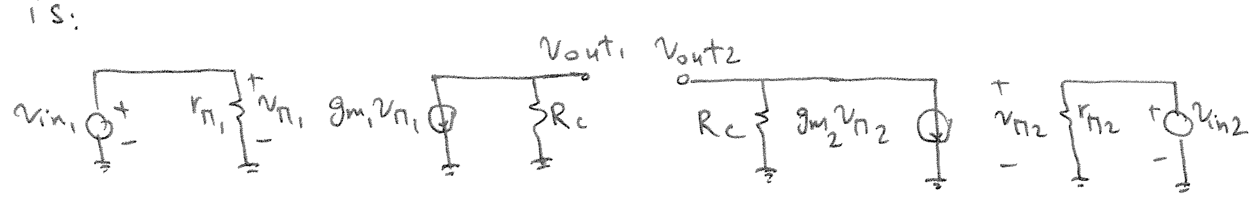


writing the node equation at P we have:

$$\frac{v_p}{R_{EE}} + \frac{v_p - v_{in_1}}{r_{\pi_1}} + g_{m_1}(v_p - v_{in_1}) + \frac{v_p - v_{in2}}{r_{\pi_2}} + g_{m_2}(v_p - v_{in_2}) = 0$$

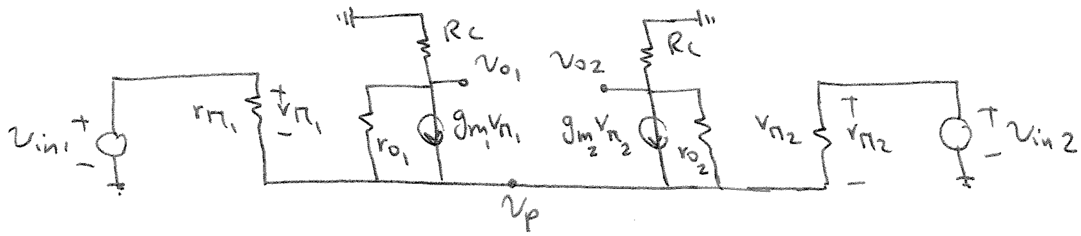
Since $v_{in1} = -v_{in2}$ and $\begin{cases} r_{\pi_1} = r_{\pi_2} \\ g_{m_1} = g_{m_2} \end{cases}$, the above equation simplifies to:

$$\frac{v_p}{R_{EE}} + \frac{2v_p}{r_{\pi_1}} + 2g_{m_1}v_p = 0 \Rightarrow v_p = 0 \Rightarrow$$ the small signal model is:



$$A_v = \frac{v_{out_1} - v_{out_2}}{v_{in1} - v_{in2}} = \frac{-g_{m_1}v_{in_1} R_C + g_{m_2}v_{in_2}R_C}{v_{in_1} - v_{in_2}} = -g_{m_1}R_C$$

(28)



$v_{in_1} = -v_{in2} \rightarrow v_{in1} + v_{in2} = 0$

$g_{m1} = g_{m2}\,,\ r_{\pi_1} = r_{\pi 2}\,,\ r_{o1} = r_{o2}$

Writing the node equation at v_p:

$$\frac{v_p - v_{in1}}{r_{\pi_1}} + \frac{v_p - v_{o1}}{r_{o1}} + g_{m_1}(v_p - v_{in1}) + g_{m_2}(v_p - v_{in2}) +$$

$$\frac{v_p - v_{in2}}{r_{\pi 2}} + \frac{v_p - v_{o2}}{r_{o2}} = 0 \Rightarrow 2g_{m_1} v_p + \frac{2v_p - v_{o1} - v_{o2}}{r_{o1}} + \frac{v_p \times 2}{r_{\pi_1}} = 0 \quad ①$$

Now the node equations at nodes v_{o1} and v_{o2} leads to:

$$\begin{cases} \dfrac{v_{o1}}{R_C} + \dfrac{v_{o1} - v_p}{r_{o1}} + g_{m_1}(v_{in_1} - v_p) = 0 & ② \\ \dfrac{v_{o2}}{R_C} + \dfrac{v_{o2} - v_p}{r_{o2}} + g_{m_2}(v_{in_2} - v_p) = 0 & ③ \end{cases} \Rightarrow ② + ③ =$$

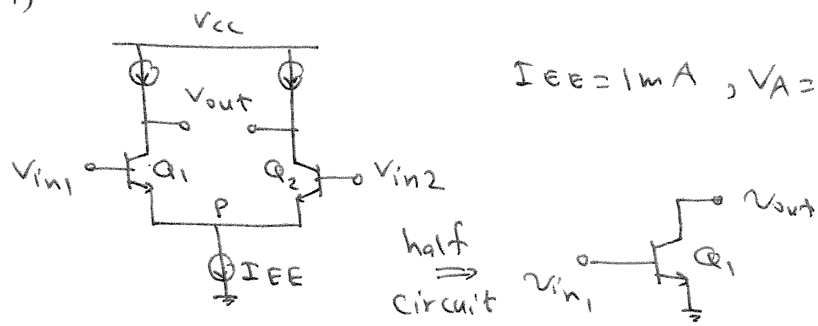
$$(v_{o1} + v_{o2})\left(\frac{1}{R_C} + \frac{1}{r_{o1}}\right) = \frac{2v_p}{r_{o1}} + 2g_{m_1} v_p \quad ④$$

placing 4 in ① $\Rightarrow$

$$2g_{m_1} v_p + \frac{1}{r_{o1}}\left(2v_p - \frac{1}{\frac{1}{R_C} + \frac{1}{r_{o1}}}\left(\frac{2v_p}{r_{o1}} + 2g_{m_1} v_p\right)\right) + \frac{2v_p}{r_\pi} = 0$$

$$\Rightarrow \underline{v_p = 0}$$

(29)

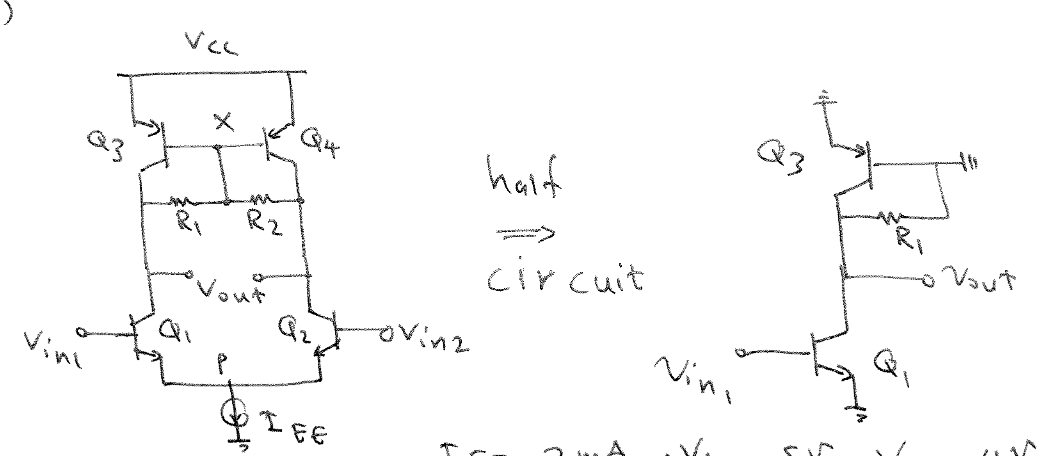


$I_{EE} = 1mA$, $V_A = 5V$

$$A_v = -g_{m1} \cdot r_{o1} = -\frac{I_{EE}}{2V_T} \frac{V_A}{\frac{I_{EE}}{2}} = -\frac{V_A}{V_T} = \frac{-5}{0.026}$$

$\rightarrow A_v = -192.31$

(30)



$I_{EE} = 2mA$, $V_{A,n} = 5V$, $V_{A,p} = 4V$

$$A_v = -g_{m_1} \left(r_{o_1} \| r_{o_3} \| R_1 \right)$$

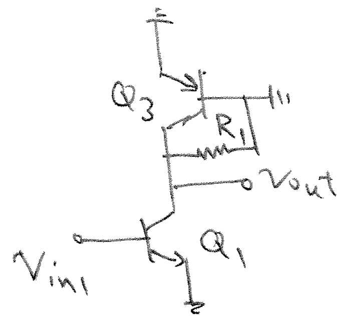
$$\Rightarrow 50 = \frac{I_{EE}}{2V_T} \left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \| \frac{V_{A,p}}{\frac{I_{EE}}{2}} \| R_1 \right) \Rightarrow$$

$$50 = \frac{2}{2 \times 26} \left(\frac{5}{10^{-3}} \| \frac{4}{10^{-3}} \| R_1 \right) \rightarrow$$

$$R_1 = 3132.53\Omega$$

(31)

The half circuit is:



$$A_v = -g_{m_1}(r_{o_1} \| r_{o_3} \| R_1)$$

$\Rightarrow$

$$50 = \frac{I_{EE}}{2 \times 0.026} \left(\frac{5}{\frac{I_{EE}}{2}} \| \frac{4}{\frac{I_{EE}}{2}} \| 5^K \right) \Rightarrow$$

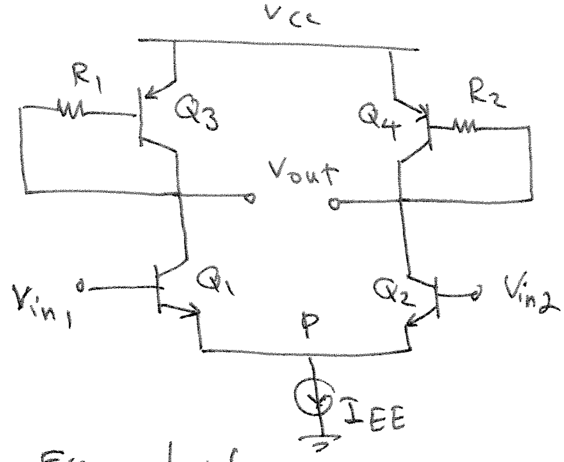
$$50 = \frac{1}{0.052} \left(10 \| 8 \| 5 I_{EE} \right) \Rightarrow$$

(5 → kΩ, I_{EE} → mA)

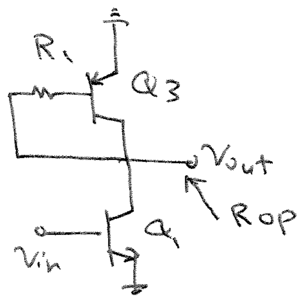
$$I_{EE} = 1.253 \text{ mA}$$

(32)

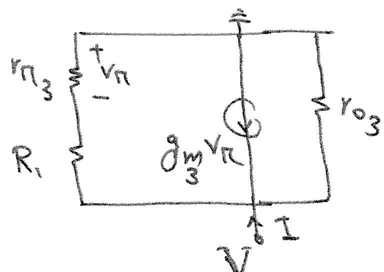
(a)



From half circuit concept we have: $A_v = -g_{m_1}(r_{o_1} \| R_{op})$



To calculate R_{op}, from small signal model we have:



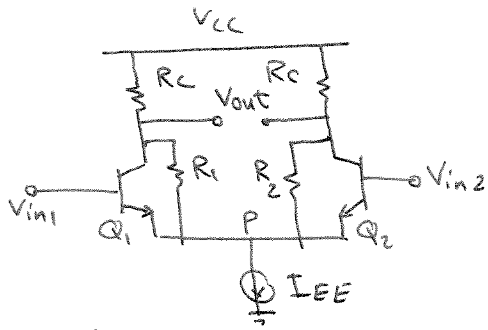
$$I = \frac{V}{r_{o_3}} - g_{m_3} V_\pi + \frac{V}{R_1 + r_{\pi_3}} = V\left[\frac{1}{r_{o_3}} + \frac{1}{R_1 + r_{\pi_3}}\right] + g_{m_3}\frac{r_{\pi_3}}{R_1 + r_{\pi_3}} V$$

$$\rightarrow R_{op} = \frac{V}{I} = r_{o_3} \| (R_1 + r_{\pi_3}) \| \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)$$

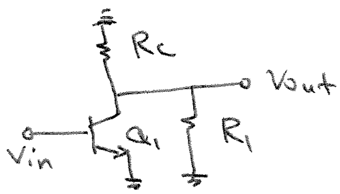
$$\rightarrow A_v = -g_{m_1}\left[r_{o_1} \| r_{o_3} \| (R_1 + r_{\pi_3}) \| \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)\right]$$

(32)

(b)

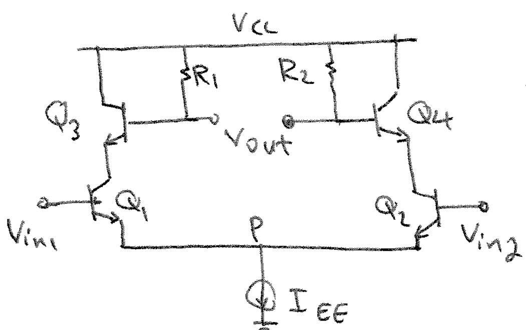


From half circuit concept:

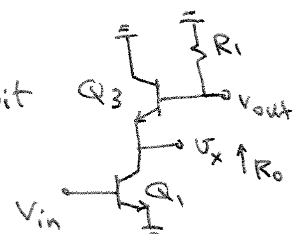


$\rightarrow A_v = -g_{m_1}(R_1 \| R_C \| r_{o_1})$

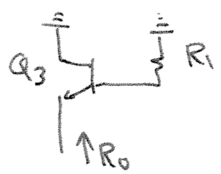
(C)

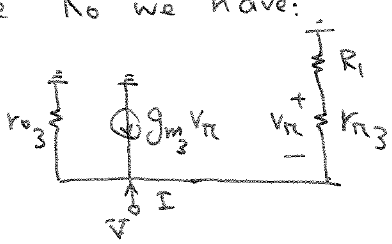


half circuit $\Longrightarrow$



To calculate R_o we have:





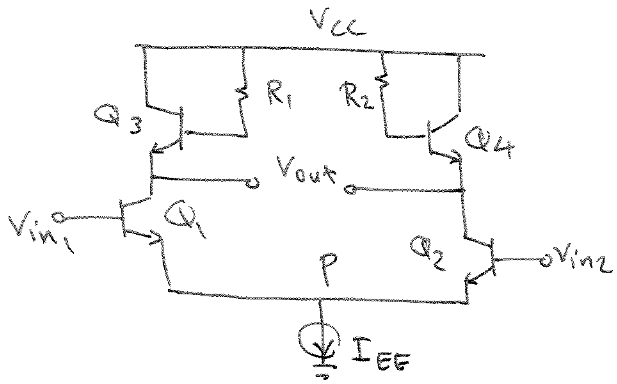
$$R_o = \frac{V}{I}$$

$$I = \frac{V}{r_{o_3}} + \frac{V}{R_1 + r_{\pi 3}} + g_{m_3}\frac{r_{\pi 3}}{r_{\pi 3} + R_1} V$$

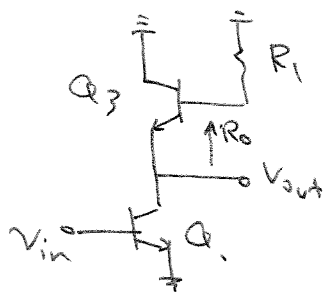
$$\Longrightarrow R_o = r_{o3} \| (R_1 + r_{\pi 3}) \| \left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m_3}}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{v_x}{v_{in}} \frac{v_{out}}{v_x} = -g_{m_1}(r_{o_1} \| R_o)\frac{R_1}{R_1 + r_{\pi 3}}$$

(32) (d)



From half circuit concept :



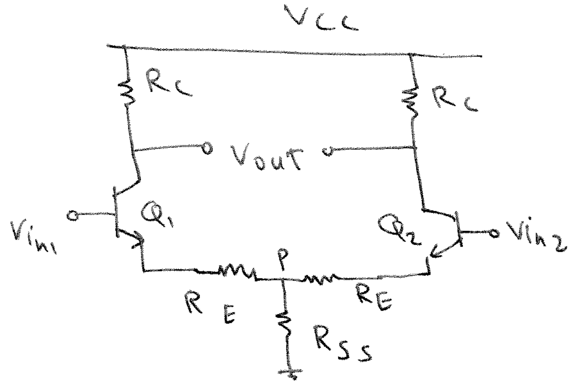
we already proved in part (c) that

$$R_o = r_{o3} \,\|\, (R_1 + r_{\pi 3}) \,\|\, \left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m3}}$$

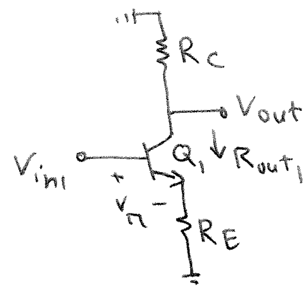
$$\rightarrow A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} \,\|\, R_o)$$

(33)

(a)

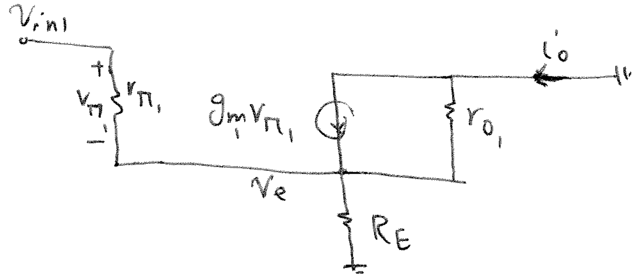


half $\Longrightarrow$ circuit



$$R_{out} = R_C \| R_{out_1} = R_C \| \left(g_{m_1} r_{o_1} (R_E \| r_{\pi_1}) + r_{o_1} + (R_E \| r_{\pi_1}) \right)$$

To calculate G_m, the small signal model is:



writing node equation at node v_e:

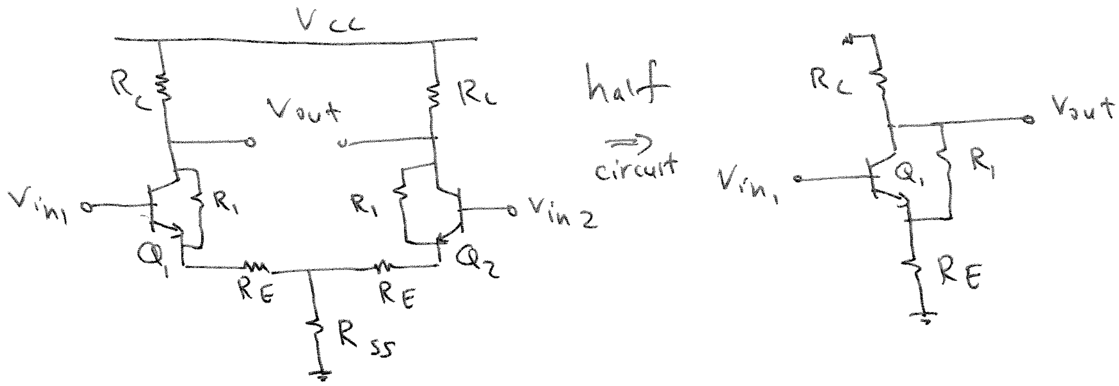
$$\frac{v_e}{R_E \| r_{o_1}} = \left(g_{m_1} + \frac{1}{r_{\pi_1}}\right) v_{\pi_1} = \underbrace{\left(g_{m_1} + \frac{1}{r_{\pi_1}}\right)}_{\approx g_{m_1}} (v_{in_1} - v_e)$$

$$\Rightarrow v_e = \frac{g_{m_1}}{g_{m_1} + \frac{1}{R_E \| r_{o_1}}} v_{in_1} \Rightarrow i_o = \frac{v_e}{r_{o_1}} - g_{m_1} v_{\pi_1}$$

$$= \frac{v_e}{r_{o_1}} + g_{m_1}(v_e - v_{in_1}) \Rightarrow G_m = \frac{i_o}{v_{in_1}} = + \frac{g_{m_1} r_{o_1}}{g_{m_1} r_{o_1} R_E + r_{o_1} + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

(33) (b)



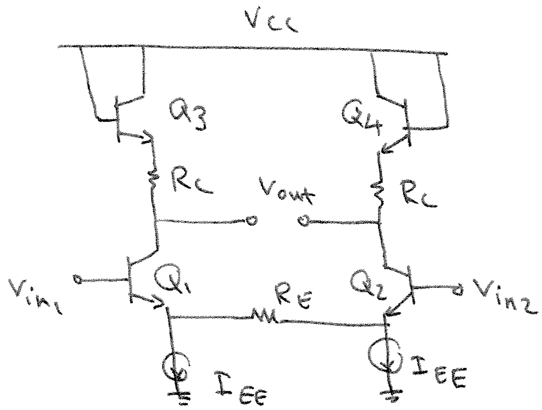
$$R_{out} = R_C \,||\, \big(g_{m1}(r_{o1}||R_1)(R_E||r_{\pi 1}) + (r_{o1}||R_1) + (R_E||r_{\pi 1}) \big)$$

Similar to the approach in part (a)

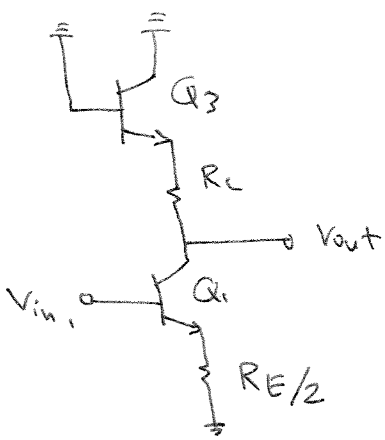
$$G_m = + \frac{g_{m1}(r_{o1}||R_1)}{g_{m1}(r_{o1}||R_1)R_E + (r_{o1}||R_1) + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

(34)



The half circuit is shown as:

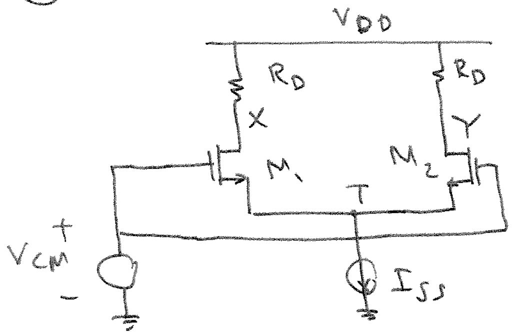


a) $$A_v = \frac{v_{out}}{v_{in1}} = -\frac{R_C + 1/g_{m3}}{R_E/2 + 1/g_{m1}}$$

b) if $\frac{R_C}{R_E/2} = A$, then if $\frac{1/g_{m3}}{1/g_{m1}} = A$

we conclude $A_v = -A$. So the circuit is very linear.

(35)



$$V_T = V_{CM} - V_{GS1} = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(a) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{2W}{L}}}$$

The tail voltage increases

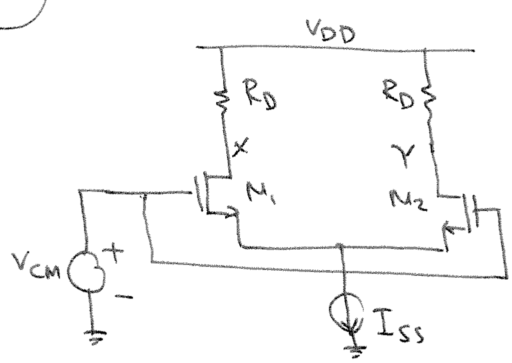
(b) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

The tail voltage decreases

(c) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n \frac{C_{ox}}{2} \frac{W}{L}}}$$

The tail voltage decreases

(36)



$V_{CM} = 1V$
$I_{SS} = 1mA$
$R_D = 1k\Omega$

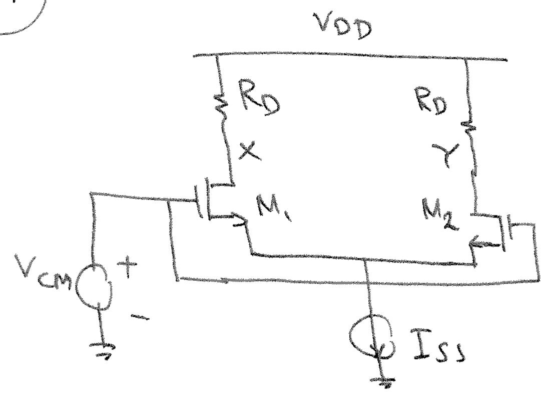
$$V_X = V_{DD} - R_D \frac{I_{SS}}{2} = V_{DD} - 0.5$$

To ensure that the devices work in saturation

$$V_{CM} \leq V_X - V_{TH} \rightarrow V_{DD} - 0.5 - V_{TH} \geq 1$$

if $V_{TH} = 0.5 \rightarrow V_{DD} > 2$

(37)



$V_{GS} - V_{TH} = 200\,mV$

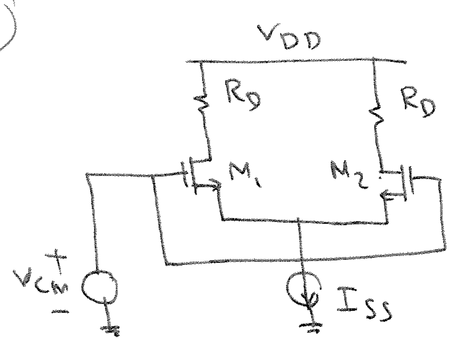
$\mu_n C_{ox} = 100\ \mu A/V^2$

$\frac{W}{L} = 20/0.18$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \rightarrow$$

$$0.2 = \sqrt{\frac{I_{SS}}{10^{-4} \times \frac{20}{0.18}}} \rightarrow I_{SS} = 0.44\,mA$$

(38)



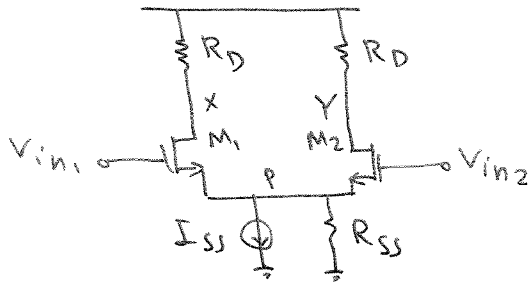
$$\frac{I_{D1}}{W} = \frac{I_{SS}}{2W} = J$$

current density

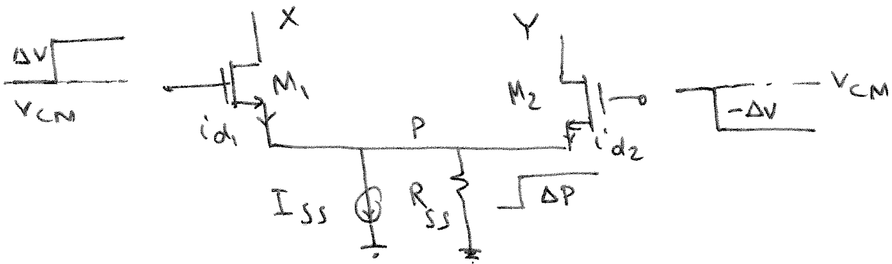
$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} =$$

$$\sqrt{\frac{2J}{\mu_n C_{ox} \frac{1}{L}}}$$

(39)



$g_{m_1} = g_{m_2} = g_m$



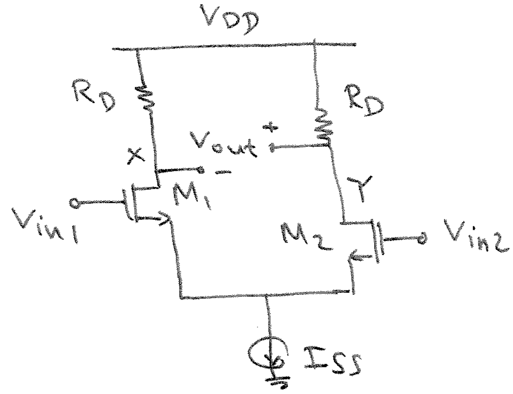
if V_{in_1} and V_{in_2} chang by ΔV and $-\Delta V$ we have:

node equation at P: $i_{d_1} + i_{d_2} + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$

$$g_m(\Delta V - \Delta P) + g_m(-\Delta V - \Delta P) + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$$

$$-g_m \Delta P - g_m \Delta P + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow \Delta P = 0$$

(40)



$V_{in1} = 1.5$

$V_{in2} = 0.3$

$$V_X - V_{TH} > V_{in1} \rightarrow V_{DD} - R_D I_{SS} - V_{TH} > 1.5$$

(41)

$P = 2mW$

$V_{DD} = 2V \rightarrow I_{SS} = 1mA$

$V_{CM,out} = V_{DD} - R_D \frac{I_{SS}}{2} = 2 - 0.5 R_D \rightarrow$

$$R_D = \frac{2 - V_{CM,out}}{0.5} \quad (k\Omega)$$

$$g_m R_D = 5 \rightarrow \sqrt{2\mu_n C_{ox} \frac{W}{L} \frac{I_{SS}}{2}} \; \overset{\downarrow k\Omega}{\frac{2 - V_{CM,out}}{0.5}} = 5$$

$$\rightarrow \sqrt{2 \times 10^{-4} \frac{W}{L} \frac{10^{-3}}{2}} \; 2 \times 10^{3} (2 - V_{CM,out}) = 5$$

if $V_{CM,out} = 1.6 \rightarrow \frac{W}{L} = 390.625$

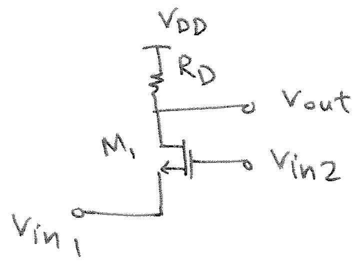
To formulate the trade off we have:

$$I_{SS} = \frac{2}{V_{DD}} \, mA \Rightarrow R_D = \frac{V_{DD} - V_{CM,out}}{1} \times V_{DD} \quad (k\Omega)$$

$$\Rightarrow |g_m R_D| = 5 \Rightarrow$$

$$\sqrt{2 \times 10^{-4} \left(\frac{W}{L}\right) \frac{1}{V_{DD}} \times 10^{-3}} \; (V_{DD} - V_{CM,out}) V_{DD} = 5$$

(42)



$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{in2} - V_{in1} - V_{TH}\right)^2$$

(1) The current is not an odd function of $(V_{in2} - V_{in1})$. Therefore it is not symmetric around $V_{in1} = V_{in2}$ $\left[(V_{in1} - V_{in2}) = 0\right]$.

(2) The input impedance seen at V_{in1} and V_{in2} are different

(3) The circuit cannot suppress the supply noise. because there is no differential output available.

(43)

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1} I_{D2}})$$

(a)

$I_{D1} = 0 \Rightarrow$

$$(V_{in1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = -\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum differential input voltage to turn M_1 off.

(b) $I_{D1} = \frac{I_{SS}}{2} \Rightarrow I_{D2} = \frac{I_{SS}}{2}$

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - I_{SS}) = 0 \rightarrow V_{in1} - V_{in2} = 0$$

This is the equilibrium input case.

(c) $I_{D1} = I_{SS} \rightarrow I_{D2} = 0$

$$(V_{in1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = +\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum input differential voltage to turn M_2 off.

(44)

$$I_{D_1} = \frac{I_{SS}}{2} - \frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]}$$

The analyses which led to the above equation assume that the transistors work in saturation region.

So:

$$-(V_{in_1}-V_{in_2})_{max} \le V_{in_1}-V_{in_2} \le (V_{in_1}-V_{in_2})_{max}$$

$$(V_{in_1}-V_{in_2})_{max} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow$$

$$\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 \le 2I_{SS} \Rightarrow$$

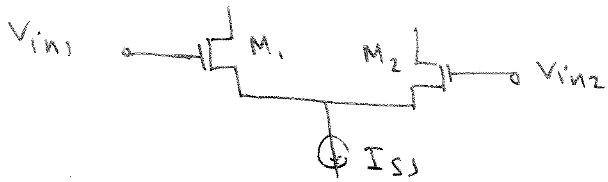
$$-\left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right] \ge 0 \Rightarrow$$

$$\frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]} \ge \frac{1}{2}I_{SS}$$

$$\Rightarrow \quad I_{D_1} < 0$$

(45)

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$



The equilibrium overdrive voltage is:

$$(V_{GS1} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{ov} \Rightarrow$$

$$\mu_n C_{ox} \frac{W}{L} = \frac{I_{SS}}{V_{ov}^2} \quad \text{therefore}$$

$$I_{D1} - I_{D2} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{ov}^2} \sqrt{\frac{4 I_{SS}}{\frac{I_{SS}}{V_{ov}^2}} - (V_{in1} - V_{in2})^2} \Rightarrow$$

$$I_{D1} - I_{D2} = I_{SS} \Rightarrow$$

$$I_{SS} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{ov}^2} \sqrt{4 V_{ov}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow (V_{in1} - V_{in2})^4 - 4 V_{ov}^2 (V_{in1} - V_{in2})^2 + 4 V_{ov}^4 = 0$$

$$\Rightarrow \left((V_{in1} - V_{in2})^2 - 2 V_{ov}^2 \right)^2 = 0 \Rightarrow$$

$$V_{in1} - V_{in2} = \sqrt{2}\, V_{ov} = \sqrt{2} \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(46)

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1}-V_{in2})^2}$$

$$V_{ov} = (V_{GS}-V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow \mu_n C_{ox}\frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$

$$\Rightarrow I_{D1} - I_{D2} = \frac{I_{SS}}{2}\,\frac{(V_{in1}-V_{in2})}{V_{ov}^2}\sqrt{4V_{ov}^2 - (V_{in1}-V_{in2})^2}$$

$$\Rightarrow G_m = \frac{\partial(I_{D1}-I_{D2})}{\partial(V_{in1}-V_{in2})} =$$

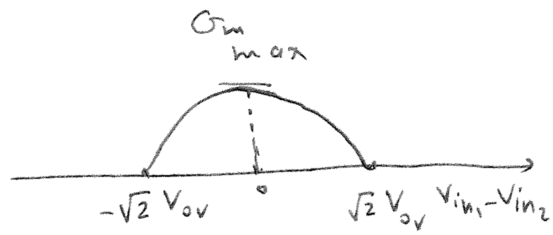
$$\frac{I_{SS}}{2V_{ov}^2}\left[\sqrt{4V_{ov}^2-(V_{in1}-V_{in2})^2} - \frac{(V_{in1}-V_{in2})^2}{\sqrt{4V_{ov}^2-(V_{in1}-V_{in2})^2}}\right] =$$

$$\frac{I_{SS}}{2V_{ov}^2}\;\frac{4V_{ov}^2 - 2(V_{in1}-V_{in2})^2}{\sqrt{4V_{ov}^2-(V_{in1}-V_{in2})^2}} =$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\;\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2(V_{in1}-V_{in2})^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1}-V_{in2})^2}}$$

$V_{in1} - V_{in2} = 0 \Rightarrow$

$$G_{m_{max}} = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}}$$



(G_m plot)

47

From problem 46:

$$G_{m_{max}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{\frac{I_{SS}}{V_{ov}^2} I_{SS}} = \frac{I_{SS}}{V_{ov}}$$

$\Rightarrow$ if $G_m = \frac{1}{2} \frac{I_{SS}}{V_{ov}}$ we have

$$\frac{1}{2} \frac{I_{SS}}{V_{ov}} = \frac{I_{SS}}{2 V_{ov}^2} \frac{4V_{ov}^2 - 2(V_{in1}-V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1}-V_{in2})^2}} \Rightarrow$$

$$V_{ov} = \frac{4V_{ov}^2 - 2(V_{in1}-V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1}-V_{in2})^2}} \Rightarrow$$

$$(4V_{ov}^2 - (V_{in1}-V_{in2})^2) V_{ov}^2 = 16 V_{ov}^4 + 4(V_{in1}-V_{in2})^4 - 16 V_{ov}^2 (V_{in1}-V_{in2})^2$$

$$\Rightarrow 4(V_{in1}-V_{in2})^4 - 15 V_{ov}^2 (V_{in1}-V_{in2})^2 + 12 V_{ov}^4 = 0$$

$$\Rightarrow (V_{in1}-V_{in2})^2 = \frac{15 V_{ov}^2 \pm \sqrt{225 V_{ov}^4 - 192 V_{ov}^4}}{8} =$$

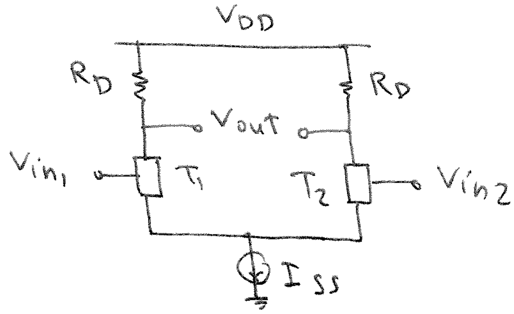
$$\frac{15 V_{ov}^2 \pm \sqrt{33} V_{ov}^2}{8}$$ positive sign is not accepted

because $(V_{in1}-V_{in2})^2 \leq 2V_{ov}^2 \Rightarrow$

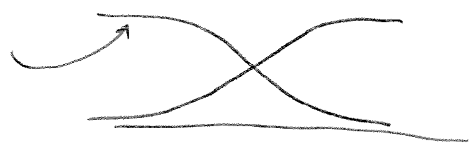
$$V_{in1} - V_{in2} = \pm \sqrt{\frac{15-\sqrt{33}}{8}} V_{ov} = \pm 1.0756 V_{ov}$$

48-

$$I_D = \gamma (V_{GS} - V_{TH})^3$$



(a) The characteristic of $I_{D1} - I_{D2}$ V.S. $V_{in1} - V_{in2}$ is similar to the standard CMOS differential pair. because it has saturation part.



(b) $$I_D = \frac{I_{SS}}{2} = \gamma (V_{GS} - V_{TH})^3 \Rightarrow$$

$$(V_{GS} - V_{TH})_{equil} = \sqrt[3]{\frac{I_{SS}}{2\gamma}}$$

(c) $$I_{D_1} = I_{SS} = \gamma (V_{GS_1} - V_{TH})^3 \Rightarrow V_{GS_1} - V_{TH} = \sqrt[3]{\frac{I_{SS}}{\gamma}}$$

$$I_{D2} = 0 = \gamma (V_{GS2} - V_{TH})^3 \Rightarrow V_{GS2} - V_{TH} = 0$$

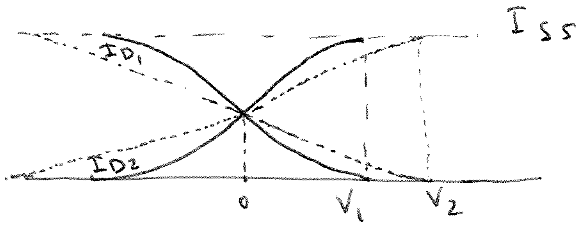
$$\Rightarrow V_{GS_1} - V_{GS_2} = V_{in_1} - V_{in_2} = \sqrt[3]{\frac{I_{SS}}{\gamma}} =$$

$$\sqrt[3]{2}\,(V_{GS} - V_{TH})_{equil}$$

(49)

(a)

gate oxide thickness is doubled ⟹ C_{ox} is halved ⟹ $(V_{in_1} - V_{in_2})_{max}$ scales up by $\sqrt{2}$.

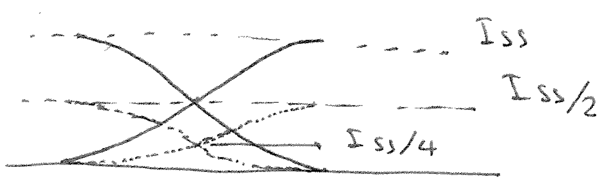


$V_2 = \sqrt{2}\, V_1$

so all the curves stretch out to the sides by $\sqrt{2}$ times.

(b) if threshold voltage is halved, nothing will change in the curves. The reason is that the curves depend on $V_{in_1} - V_{in_2}$.

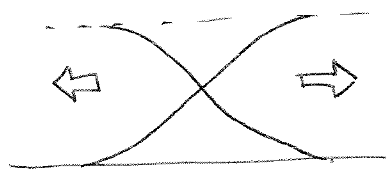
(c) In this case, $(V_{in_1} - V_{in_2})_{max}$ does not change so, all the curves scale half downward because Iss is halved.



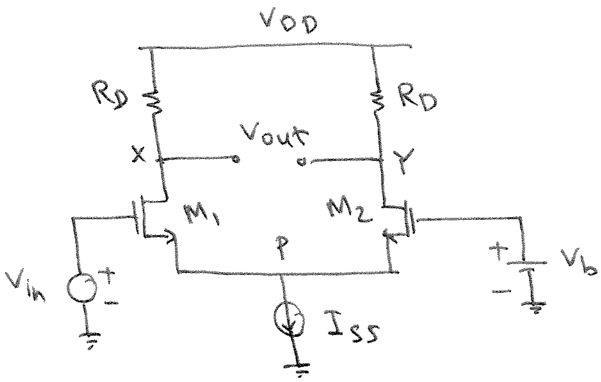
(50)

if mobility falls then $(V_{in1}-V_{in2})_{max}$ will increase because $(V_{in1}-V_{in2})_{max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

So the curves stretch out to the sides.

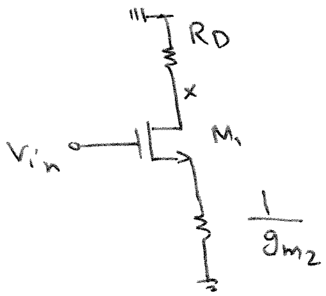


(51)



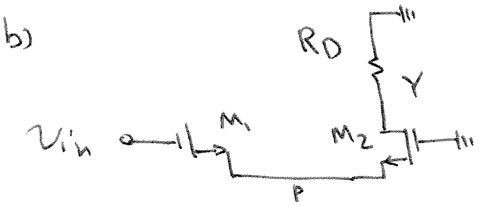
$g_{m1} = g_{m2} = g_m$

(a)



$$v_x = -g_{m1} v_{gs1} R_D = -g_{m1} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in} R_D = -\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in} = -\frac{g_m}{2} R_D v_{in}$$

(b)



$$v_p = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in}$$

$$\Rightarrow v_p = \frac{g_{m1}}{g_{m1} + g_{m2}} v_{in} \Rightarrow$$

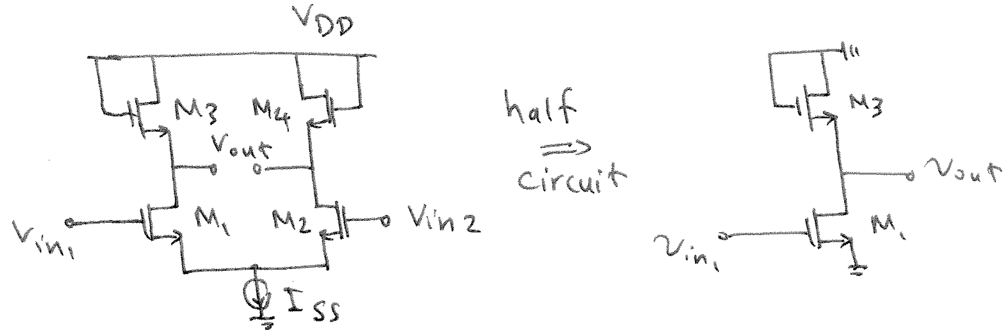
$$v_y = -g_{m2} v_{gs2} R_D = g_{m2} v_p R_D = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in}$$

$$\rightarrow v_y = \frac{g_m}{2} R_D v_{in}$$

(c) $\frac{v_x - v_y}{v_{in}} = -g_m R_D$ This value is equal to the gain of the differential amplifier.

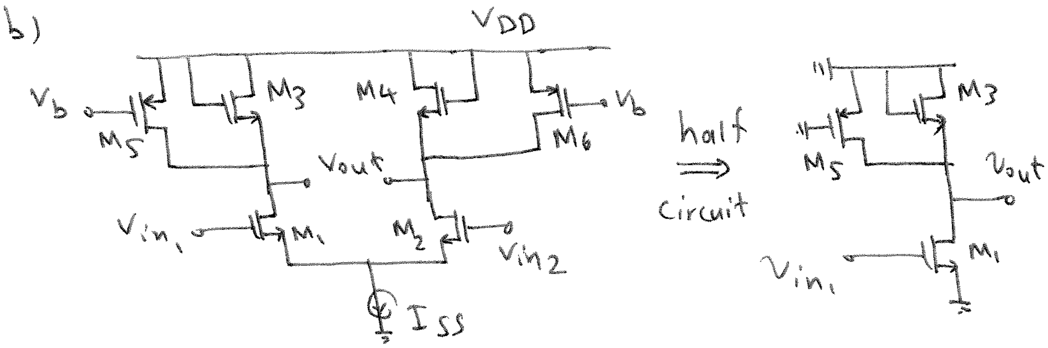
(52)

(a)



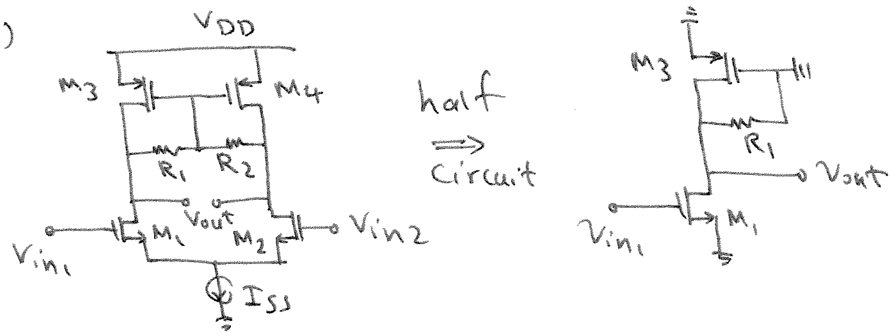
$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| \frac{1}{g_{m_3}}\right)$$

(b)



$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_5} \| \frac{1}{g_{m_3}} \| r_{o_3}\right)$$

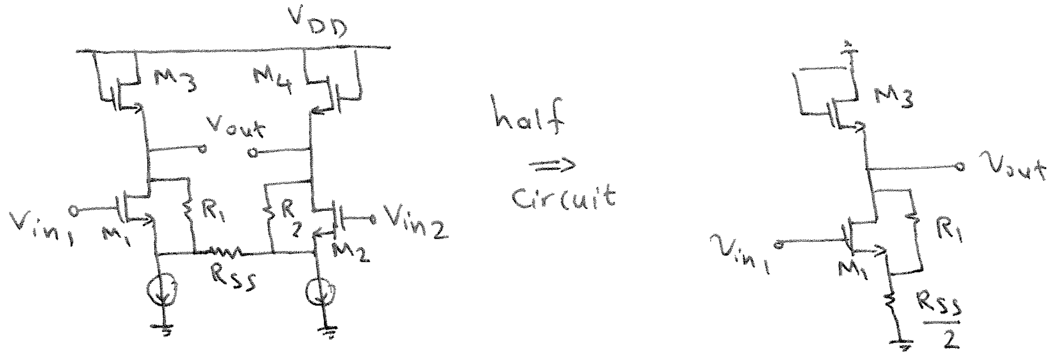
(C)



$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| R_1\right)$$

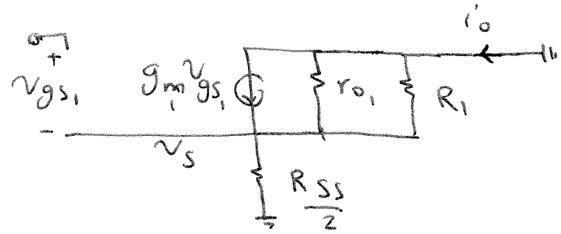
(53)

(a)



$$R_{out} = \left(r_{o3} \,\|\, \frac{1}{g_{m3}}\right) \,\|\, \left(g_{m1} (R_1 \| r_{o1}) \frac{R_{SS}}{2} + \frac{R_{SS}}{2} + R_1 \| r_{o1} \right)$$

To calculate G_m:



$$v_{gs1} = v_{in} - v_s$$

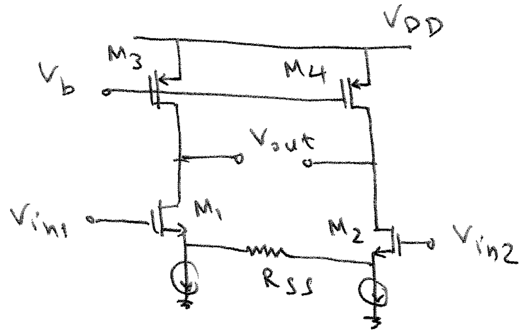
$$\frac{v_s}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}} + g_{m1} v_s = g_{m1} v_{in} \Rightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

$$i_o = + \frac{v_s}{\frac{R_{SS}}{2}} = + \frac{1}{\frac{R_{SS}}{2}} \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}} \Rightarrow$$

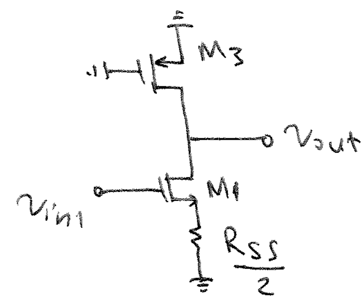
$$G_m = \frac{i_o}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

$$A_v = -G_m R_{out}$$

53 (b)

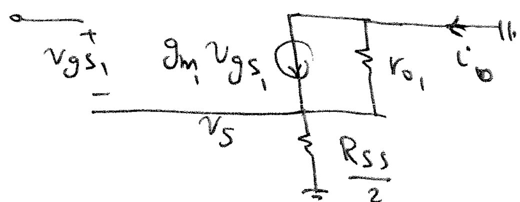


half ⇒ circuit



$$R_{out} = r_{o3} \parallel \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

To calculate G_m:

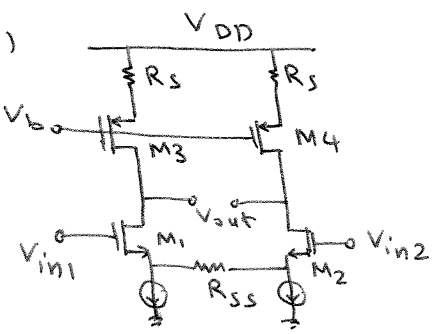


$$\frac{v_S}{r_{o_1} \parallel \frac{R_{SS}}{2}} + g_{m_1} v_S = g_{m_1} v_{in} \Rightarrow v_S = \frac{g_{m_1} v_{in}}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o_1}}}$$

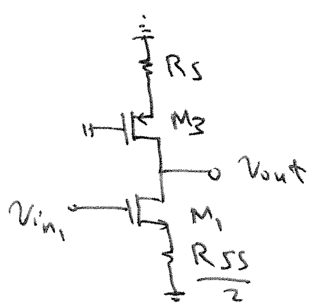
$$G_m = \frac{i_o}{v_{in}} = + \frac{v_S}{\frac{R_{SS}}{2}} \frac{1}{v_{in}} = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o_1}}}$$

$\rightarrow A_v = -G_m R_{out}$

(c)



half ⇒ circuit

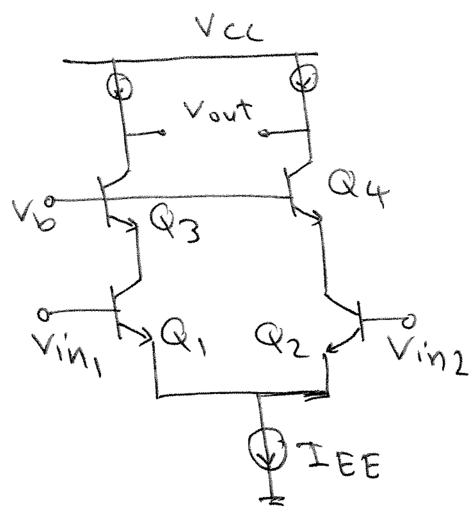


$$R_{out} = (g_{m_3} r_{o_3} R_S + r_{o_3} + R_S) \parallel \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

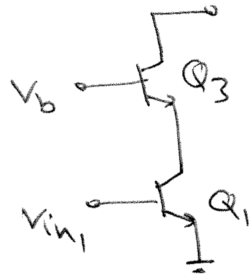
G_m for this circuit is equal to the one for part (b) so:

$$G_m = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o_1}}} \Rightarrow A_v = -G_m R_{out}$$

(54)



half ⟹ Circuit



$A_v = 4000$

$\beta = 100$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1} \| r_{\pi_3}) r_{o_3} + r_{o_3} + r_{o_1} \| r_{\pi_3}\right]$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} \qquad r_{o_{1-4}} = \frac{2V_A}{I_{EE}} \qquad r_{\pi_3} = \frac{2V_T\beta}{I_{EE}}$$

$$4000 = \frac{I_{EE}}{2V_T}\left[\frac{I_{EE}}{2V_T}\left(\frac{2V_A}{I_{EE}} \| \frac{2V_T\beta}{I_{EE}}\right)\frac{2V_A}{I_{EE}} + \frac{2V_A}{I_{EE}} + \left(\frac{2V_A}{I_{EE}} \| \frac{2V_T\beta}{I_{EE}}\right)\right]$$

$$\Rightarrow 4000 = \frac{1}{2V_T}\left[\frac{V_A}{V_T}(2V_A \| 2V_T\beta) + 2V_A + (2V_A \| 2V_T\beta)\right] \Rightarrow$$

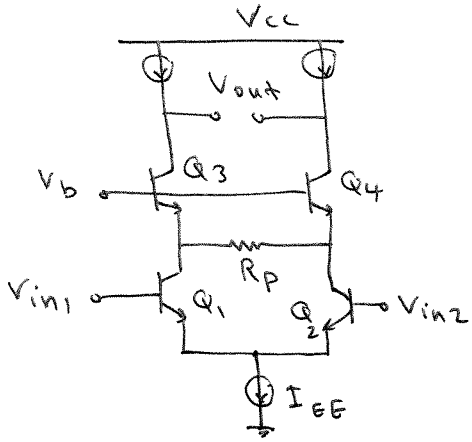
$$4000 = \frac{1}{V_T}\left[\frac{V_A}{V_T}(V_A \| \beta V_T) + V_A + (V_A \| \beta V_T)\right] \Rightarrow$$

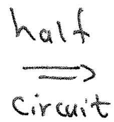
$$4000 = \frac{1}{V_T}\left[\frac{\beta V_A^2}{\beta V_T + V_A} + V_A + \frac{\beta V_A V_T}{\beta V_T + V_A}\right] \Rightarrow$$

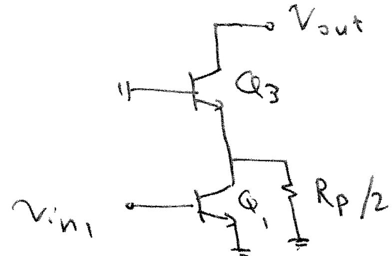
$$4000 = \frac{1}{0.026}\left[\frac{100 V_A^2}{2.6 + V_A} + V_A + \frac{2.6 V_A}{2.6 + V_A}\right] \Rightarrow$$

$V_A = 2.197$

(55)





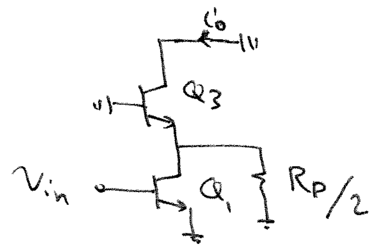


$$R_{out} = g_{m_3} r_{o3} \left(\frac{R_p}{2} \| r_{o1} \| r_{\pi 3}\right) + r_{o3} + \frac{R_p}{2} \| r_{o1} \| r_{\pi 3}$$

To calculate G_m

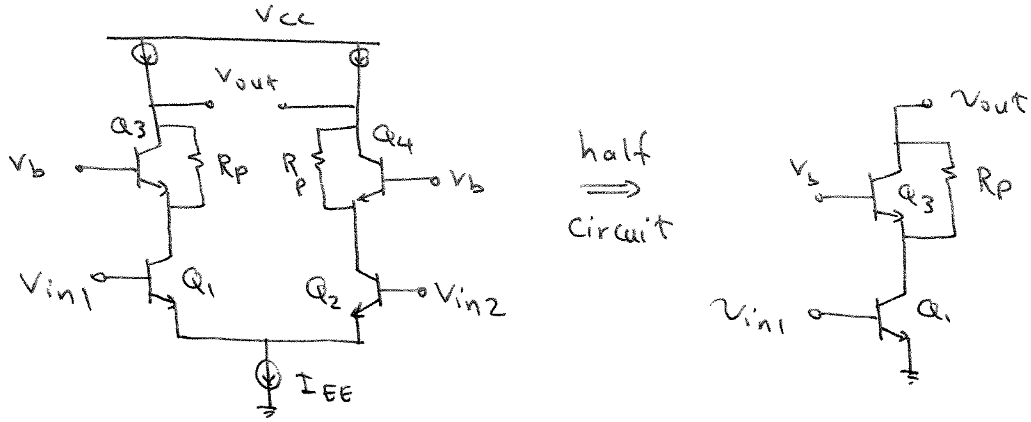
$$G_m = \frac{i_o'}{v_{in}}$$

$$\frac{i_o'}{v_{in}} = +g_{m_1} \frac{g_{m_3} \left(\frac{R_p}{2} \| r_{o1}\right)}{g_{m_3} \left(\frac{R_p}{2} \| r_{o1}\right) + 1}$$



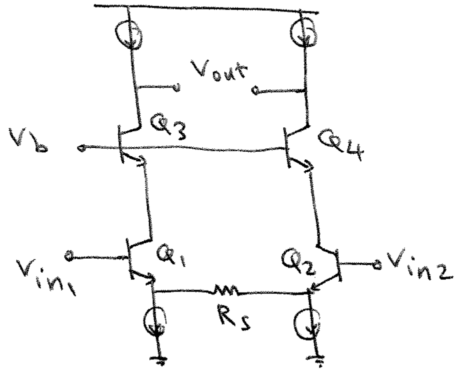
$$A_v = -G_m R_{out}$$

(56)

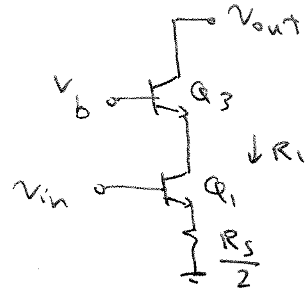


$$A_v = -g_{m_1}\left(g_{m_3}(r_{o3} \| R_P)(r_{o1} \| r_{\pi 3}) + (r_{o3} \| R_P) + (r_{o1} \| r_{\pi 3}) \right)$$

(57)



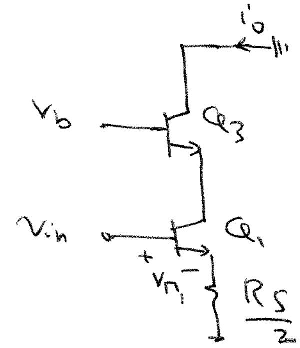
half
⟹
circuit



$$R_1 = g_{m_1} r_{o_1} \left(\frac{R_S}{2} \parallel r_{\pi_1}\right) + r_{o_1} + \frac{R_S}{2} \parallel r_{\pi_1}$$

$$R_{out} = g_{m_3} r_{o_3} (R_1 \parallel r_{\pi_3}) + r_{o_3} + (R_1 \parallel r_{\pi_3})$$

To calculate G_m:



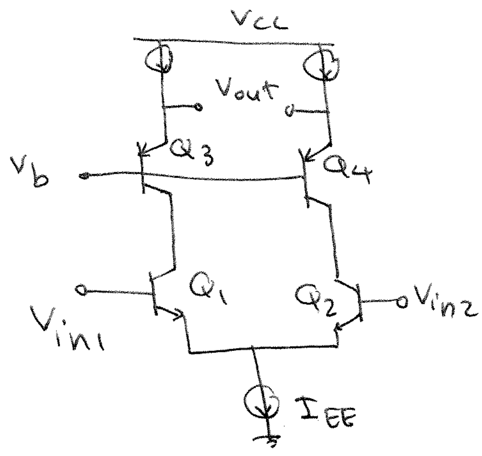
$$v_{\pi_1} \approx \frac{\frac{1}{g_{m_1}}}{\frac{1}{g_{m_1}} + \frac{R_S}{2}} v_{in}$$

$$= \frac{1}{1 + g_{m_1} \frac{R_S}{2}} v_{in}$$

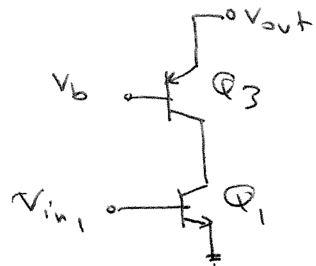
$$G_m = \frac{i_o}{v_{in}} = \frac{+g_{m_1} v_{\pi}}{v_{in}} = \frac{+g_{m_1}}{1 + g_{m_1} \frac{R_S}{2}}$$

$$A_v = -G_m R_{out}$$

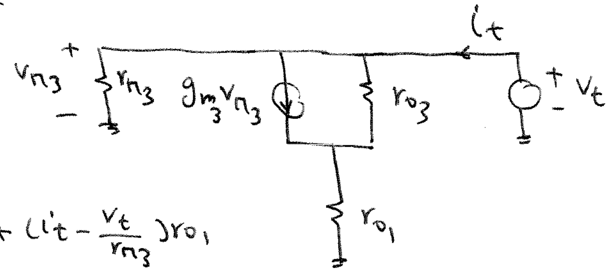
(58)



half ⟹ Circuit



To calculate R_{out}



$v_{\pi 3} = v_t$

$$v_t = \left(i_t - g_{m_3} v_t - \frac{v_t}{r_{\pi 3}}\right) r_{o3} + \left(i_t - \frac{v_t}{r_{\pi 3}}\right) r_{o1}$$

$$\rightarrow \quad v_t\left(1 + g_{m_3} r_{o3} + \frac{r_{o3}}{r_{\pi 3}} + \frac{r_{o1}}{r_{\pi 3}}\right) = i_t (r_{o1} + r_{o3})$$

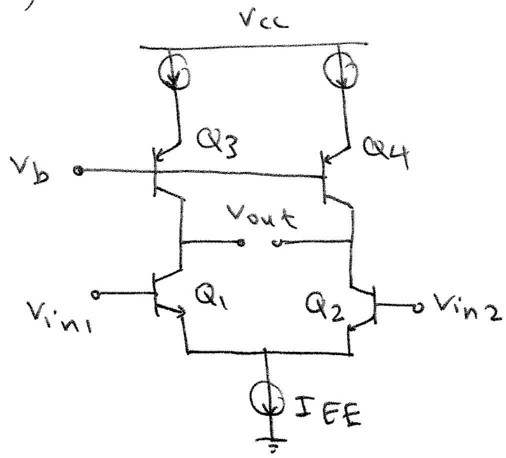
$$\rightarrow \quad \frac{v_t}{i_t} = \frac{r_{o1} + r_{o3}}{1 + g_{m_3} r_{o3} + \frac{g_{m3} r_{o3}}{\beta_3} + \frac{g_{m_3} r_{o1}}{\beta_3}} = R_{out} \rightarrow$$

$$R_{out} \approx \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

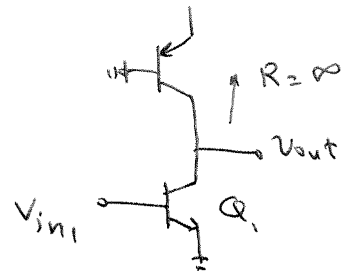
$$G_m = +g_{m_1} \quad \Rightarrow$$

$$A_v = -G_m R_{out} = -g_{m_1} \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

(59)

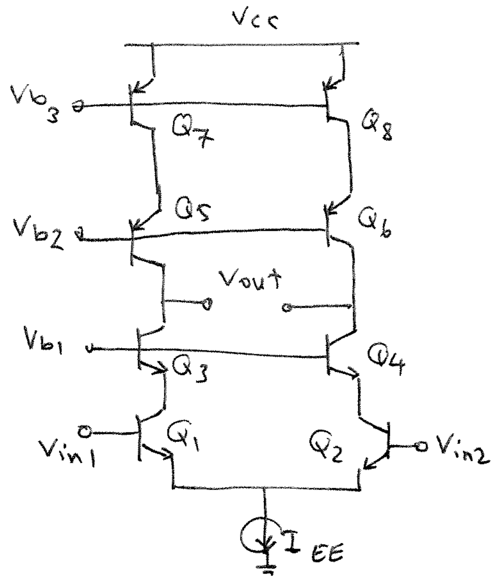


half ⟹ circuit

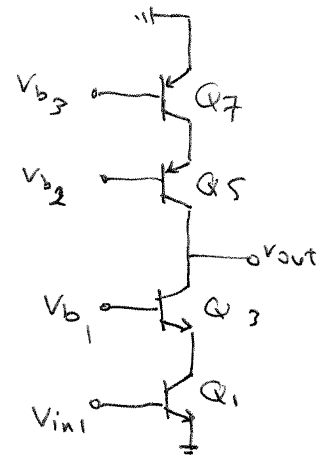


$A_v = -g_{m1} r_{o1}$

(60)



half circuit $\Longrightarrow$



$A_v = 800$, $\beta_n = 2\beta_p = 100$, $V_{A,n} = 2V_{A,p}$

$$A_v \approx -g_{m_1}\left[g_{m_3} r_{o_3}(r_{o_1} \| r_{\pi_3})\right] \| \left[g_{m_5} r_{o_5}(r_{o_7} \| r_{\pi_5})\right] =$$

$$-\frac{I_{EE}}{2V_T}\left[\frac{I_{EE}}{2V_T}\frac{V_{A,n}}{\frac{I_{EE}}{2}}\left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \| \frac{\beta_n V_T}{\frac{I_{EE}}{2}}\right)\right] \| \left[\frac{I_{EE}}{2V_T}\frac{V_{A,p}}{\frac{I_{EE}}{2}}\left(\frac{V_{A,p}}{\frac{I_{EE}}{2}} \| \frac{\beta_p V_T}{\frac{I_{EE}}{2}}\right)\right]$$

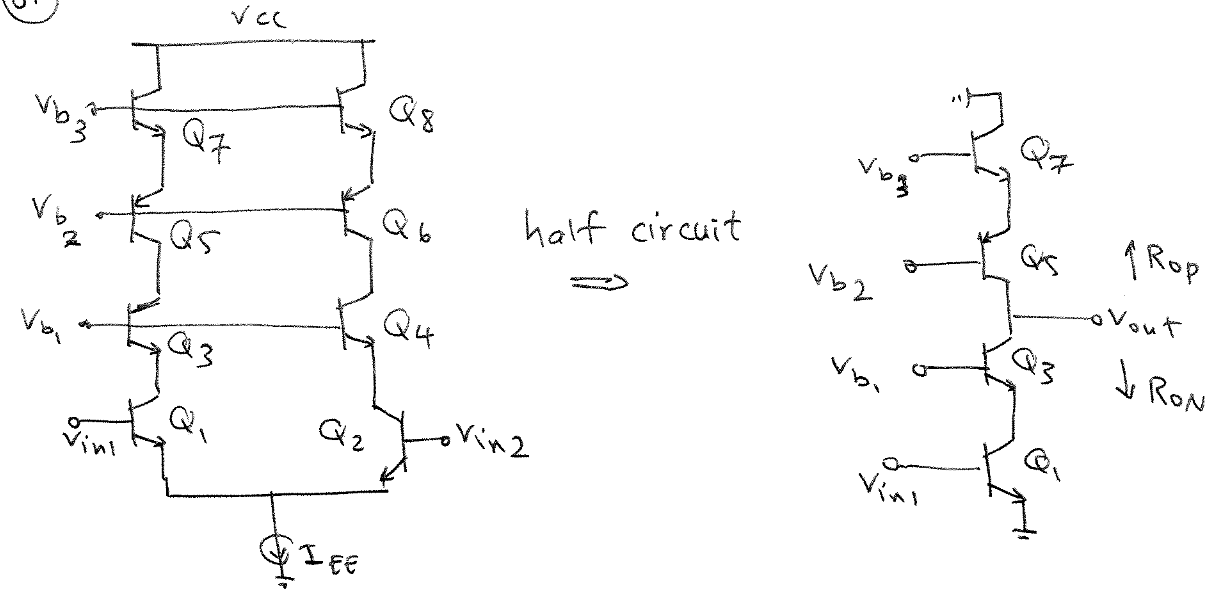
$$= -\frac{1}{V_T^2}\left[V_{A,n}(V_{A,n} \| \beta_n V_T)\right] \| \left[V_{A,p}(V_{A,p} \| \beta_p V_T)\right] =$$

$$-\frac{1}{V_T^2}\left[V_{A,n}(V_{A,n} \| \beta_n V_T)\right] \| \left[\frac{V_{A,n}}{2}\left(\frac{V_{A,n}}{2} \| \frac{\beta_n}{2} V_T\right)\right] \Rightarrow$$

$$800 = \frac{1}{5V_T^2}\left(V_{A,n}(V_{A,n} \| 2.6)\right) \Rightarrow V_{A,n} = 2.245 \text{ V}$$

$$\Rightarrow V_{A,p} = 1.225 \text{ V}$$

(65)



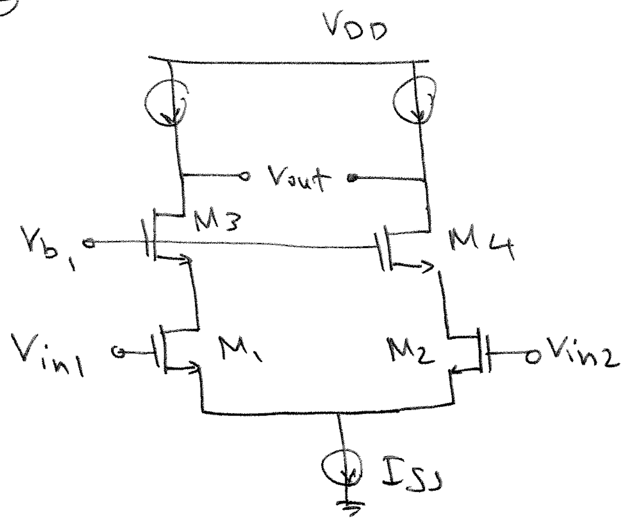
$$R_{oN} = (r_{o1} \| r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (r_{o1} \| r_{\pi 3})$$

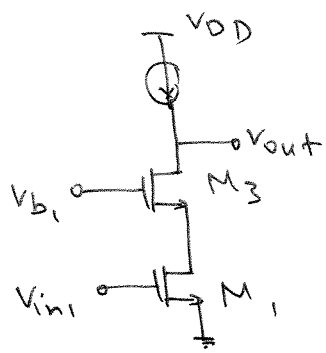
$$R_{op} = \frac{1}{g_{m7}} \| r_{o7} \| r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \| r_{o7} \| r_{\pi 5}\right) \approx$$

$$\frac{1}{g_{m7}} \| r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \| r_{\pi 5}\right)$$

$$A_v = -g_{m1} (R_{oN} \| R_{op})$$

(62)





half circuit

$A_v = 300$, $W/L = \frac{20}{0.18}$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$\lambda = 0.1\,V^{-1}$

$$A_v \approx -g_{m_3} r_{o_3} g_{m_1} r_{o_1}$$

$$g_{m_1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \frac{I_{SS}}{2}} \qquad g_{m_3} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_3 \frac{I_{SS}}{2}}$$

$$\rightarrow g_{m_1} = g_{m_3} = \sqrt{10^{-4}\ \frac{20}{0.18}\ I_{SS}}$$

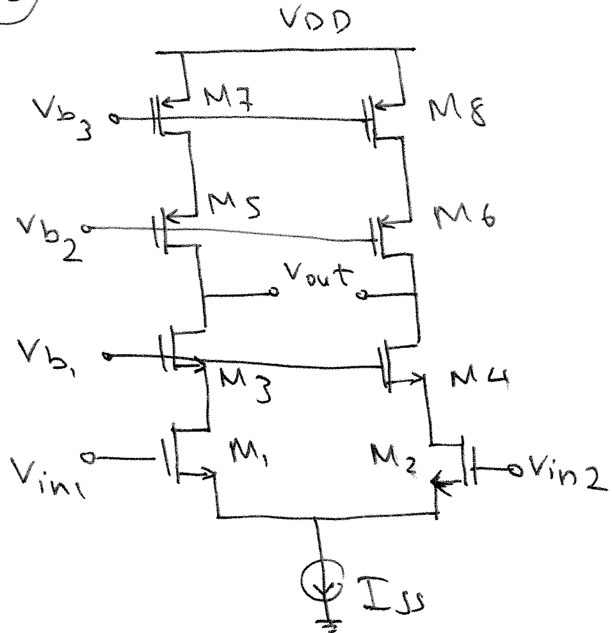
$$r_{o_1} = \frac{1}{\lambda \frac{I_{SS}}{2}} \ , \ r_{o_3} = \frac{1}{\lambda \frac{I_{SS}}{2}} \rightarrow r_{o_1} = r_{o_3} = \frac{20}{I_{SS}}$$

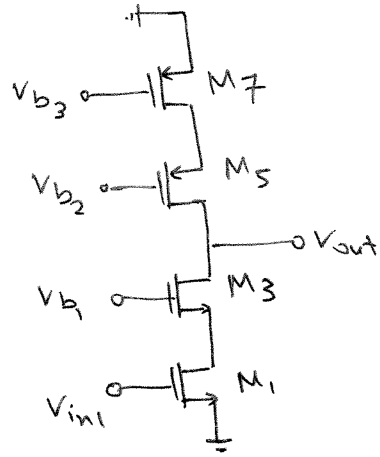
So:

$$300 = \left(10^{-4}\ \frac{20}{0.18}\ I_{SS}\right) \frac{\frac{400}{2}}{I_{SS}} \Rightarrow$$

$I_{SS} = 14.815\ mA$

(63)





$A_v = 200$, $I_{SS} = 1mA$, $\mu_n C_{ox} = 100\,\mu A/V^2$
$\mu_p C_{ox} = 50\,\mu A/V^2$, $\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$
$(\frac{W}{L})_1 = \cdots = (W/L)_8 = ?$

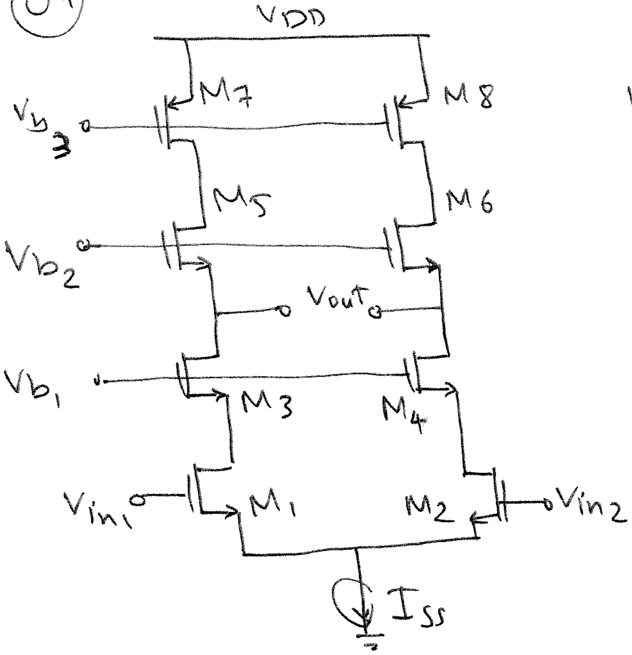
$$A_v \approx -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] \Rightarrow$$

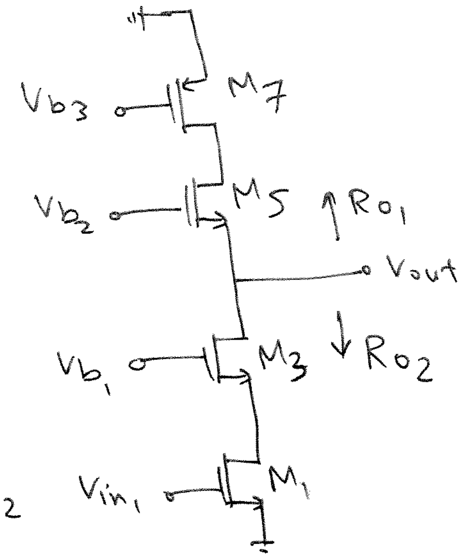
$$200 = \sqrt{\mu_n C_{ox} (\tfrac{W}{L})_1 I_{SS}} \left[\left(\sqrt{\mu_n C_{ox} (\tfrac{W}{L})_3 I_{SS}} \left(\frac{2}{\lambda_n I_{SS}}\right)^2\right) \| \left(\sqrt{\mu_p C_{ox} (\tfrac{W}{L})_5 I_{SS}} \left(\frac{2}{\lambda_p I_{SS}}\right)^2\right) \right]$$

$$\Rightarrow 200 = \sqrt{10^{-4} (\tfrac{W}{L})_1 10^{-3}} \left[\left(\sqrt{10^{-4} (\tfrac{W}{L}) 10^{-3}} \left(\frac{20}{10^{-3}}\right)^2\right) \| \left(\sqrt{0.5\times 10^{-4} (\tfrac{W}{L}) 10^{-3}} \left(\frac{10}{10^{-3}}\right)^2\right) \right]$$

$$\Rightarrow \frac{W}{L} = 33.28$$

(64)

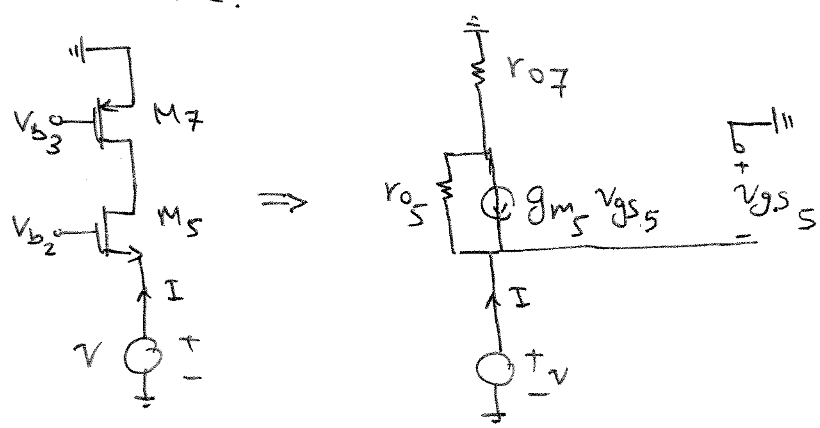




half circuit

$$R_{o2} = g_{m3} r_{o3} r_{o1} + r_{o1} + r_{o3}$$

To calculate R_{o1}, using the small signal model we have:

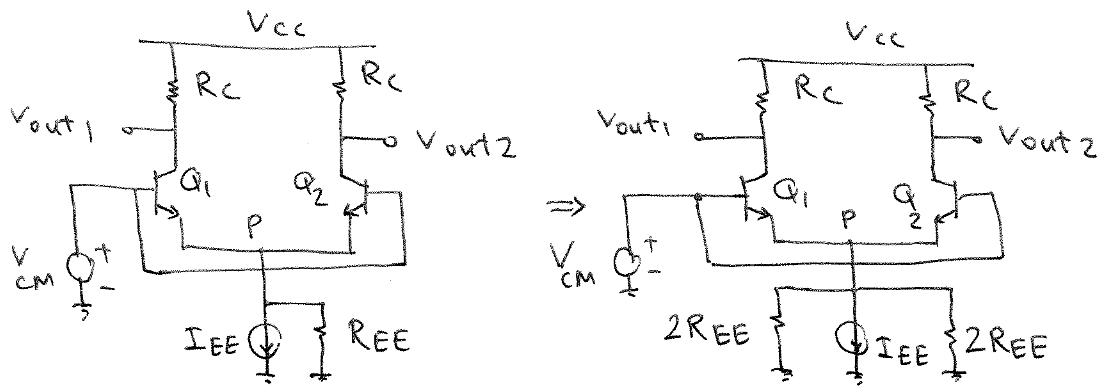


$$v_{gs_5} = -v \rightarrow g_{m_5} v_{gs_5} = -g_{m_5} v$$

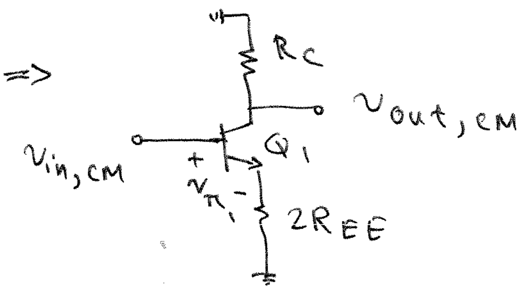
From KVL: $v = r_{o5}(I - g_{m_5} v) + r_{o7} I$

$$\rightarrow \frac{v}{I} = R_{o1} = \frac{r_{o5} + r_{o7}}{1 + g_{m_5} r_{o5}} \Rightarrow A_v = -g_{m_1}(R_{o1} \| R_{o2})$$

(65)



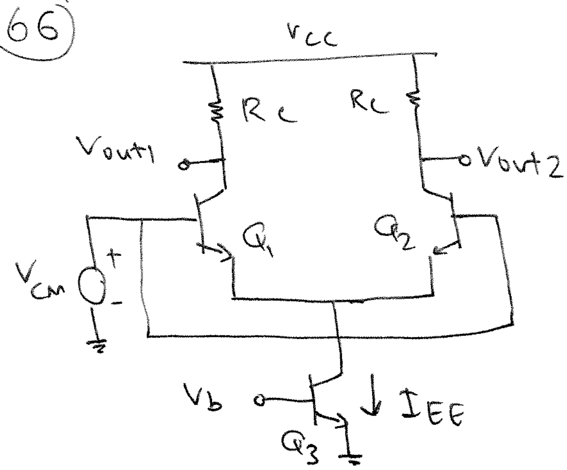
⇒



$$v_{out,cm} = -g_{m1} v_{\pi1} R_C = -g_{m1} R_C \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + 2R_{EE}} v_{in,cm}$$

$$\Rightarrow \frac{v_{out,cm}}{v_{in,cm}} = -\frac{g_{m1} R_C}{1 + 2R_{EE} g_{m1}} = -\frac{\frac{R_C}{2}}{R_{EE} + \frac{1}{2g_{m1}}}$$

(66)

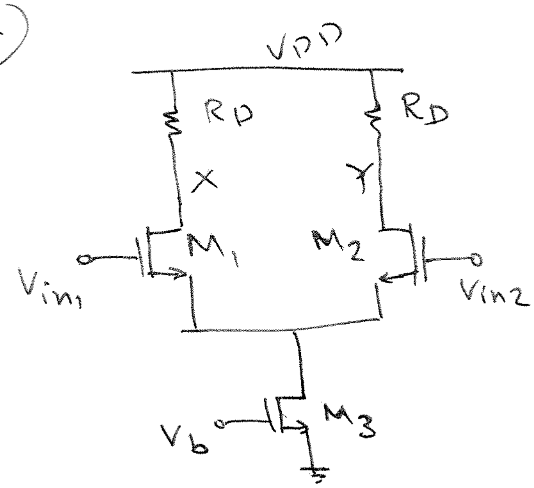


$$A_{CM} = \frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_C/2}{\frac{1}{2g_m} + r_{o3}} \Rightarrow$$

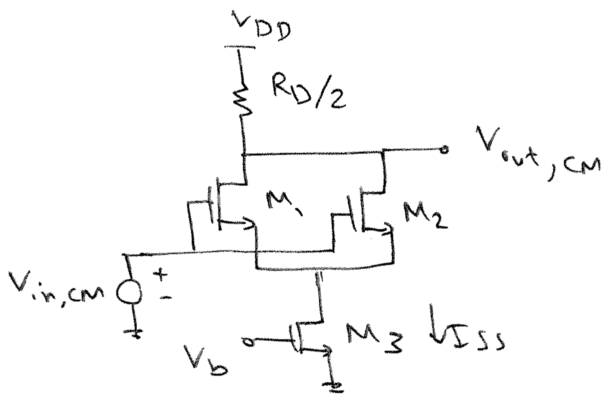
$$A_{CM} \leq 0.01 \Rightarrow \frac{R_C/2}{\frac{1}{2\frac{I_{EE}}{2V_T}} + \frac{V_A}{I_{EE}}} < 0.01 \Rightarrow$$

$$\frac{R_C I_{EE}}{2(V_A + V_T)} < 0.01 \Rightarrow R_C I_{EE} < 0.02(V_A + V_T)$$

(67)



The same value for the inputs common-mode leads to the following circuit:



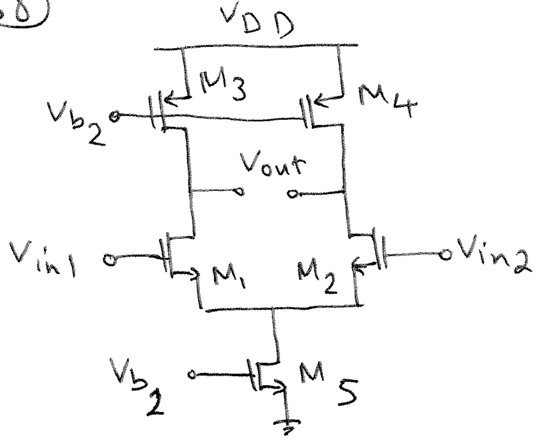
$$g_{m_1} = g_{m_2} = \frac{2\, I_{SS}/2}{(V_{GS}-V_{TH})_{eq}} \Rightarrow = \frac{I_{SS}}{(V_{GS}-V_{TH})_{eq}}$$

$$\frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_D/2}{\frac{1}{2g_{m_1}} + r_{o3}}$$

$$= \frac{-R_D}{\frac{1}{g_{m_1}} + 2r_{o3}} = \frac{-R_D}{\frac{(V_{GS}-V_{TH})_{eq}}{I_{SS}} + \frac{2}{\lambda I_{SS}}} \Rightarrow$$

$$A_{CM} = -\frac{R_D I_{SS}}{\frac{2}{\lambda} + (V_{GS}-V_{TH})_{eq}}$$

(68)



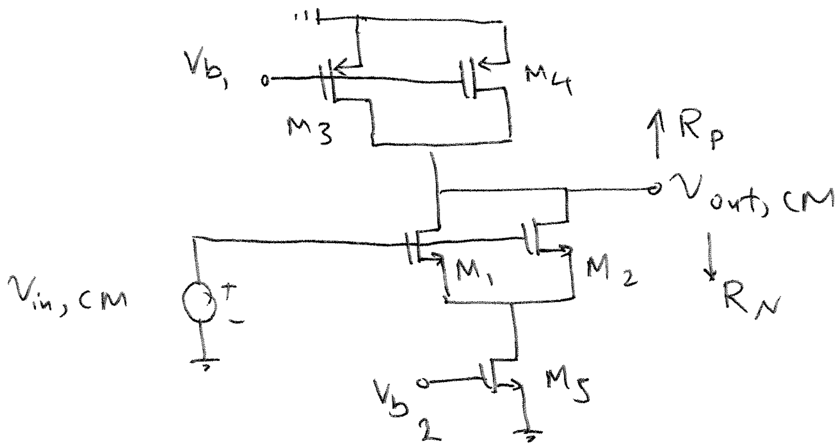
$\lambda > 0, \; g_m r_o \gg 1$

$r_{o3} = r_{o4}$

$r_{o1} = r_{o2}$

$g_{m1} = g_{m2}$

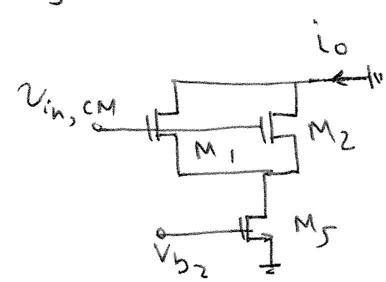
For the common mode input we have:



$$R_P = r_{o3} \parallel r_{o4} = \frac{r_{o3}}{2} = \frac{r_{o4}}{2}$$

$$R_N = r_{o5} + \frac{r_{o1}}{2} + 2g_{m1}\frac{r_{o1}}{2}r_{o5} =$$

$$g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2}$$



$$\frac{i_o}{v_{in,CM}} = G_m = \frac{2g_{m1} v_{gs1}}{v_{in,CM}} = 2g_{m1} \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o5}}$$

$$\rightarrow G_m = \frac{2g_{m1}}{1 + 2g_{m1} r_{o5}} \approx \frac{1}{r_{o5}}$$

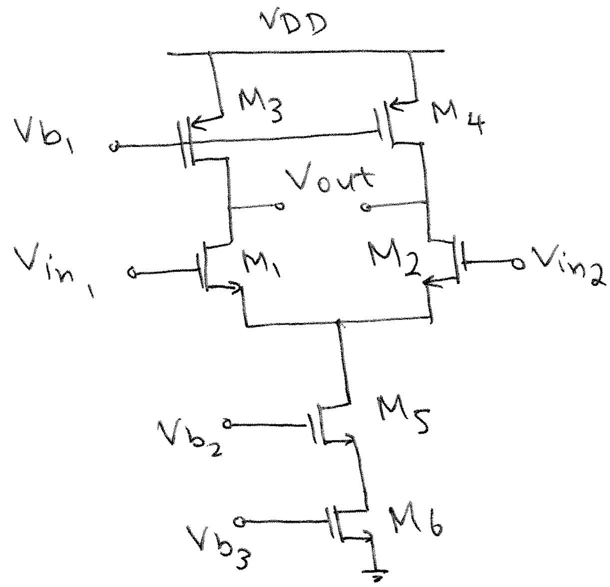
$$\Rightarrow A_{CM} = -G_m R_o$$

$$R_o = R_P \| R_N = \frac{r_{o4}}{2} \| \left(g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2} \right)$$

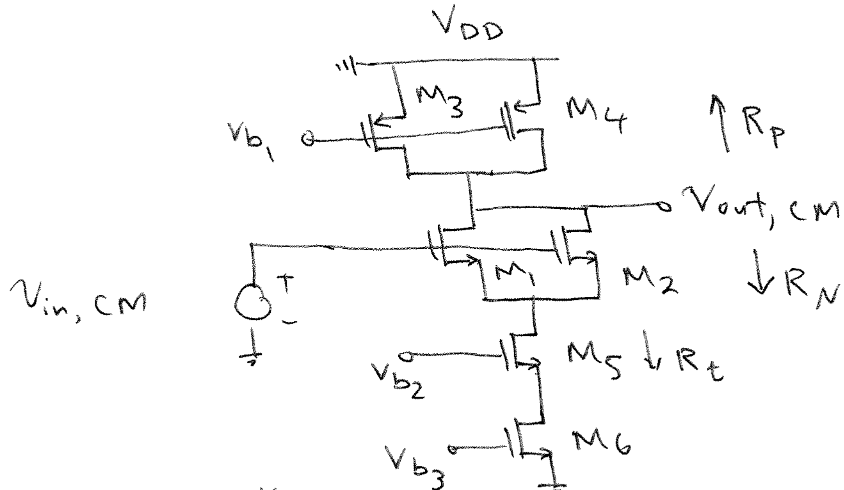
$$\approx \frac{r_{o4}}{2} \| g_{m1} r_{o1} r_{o5} \approx \frac{r_{o4}}{2}$$

$$\rightarrow A_{CM} = -\frac{1}{r_{o5}} \frac{r_{o4}}{2} = -\frac{r_{o4}}{2 r_{o5}}$$

(69) (a)



For the common mode input:



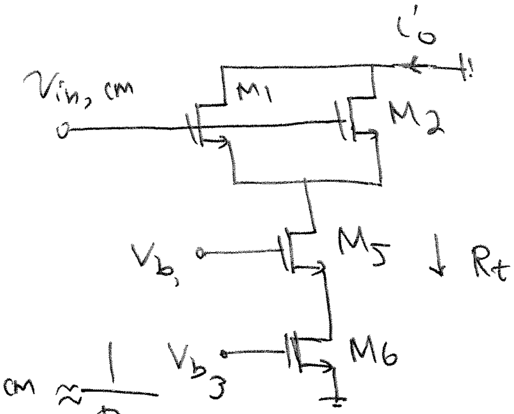
$$R_P = r_{o3} \| r_{o4} = \frac{r_{o3}}{2}$$

$$R_N = \frac{r_{o1}}{2} + R_t + 2g_{m1}\frac{r_{o1}}{2}R_t \simeq g_{m1} r_{o1} R_t \approx$$

$$g_{m1} r_{o1} g_{m5} r_{o5} r_{o6}$$

$$R_{out} = R_P \| R_N = \frac{r_{o3}}{2} \| g_{m1} g_{m5} r_{o1} r_{o5} r_{o6} \approx \frac{r_{o3}}{2}$$

To calculate G_m:



$$G_m = \frac{i_o}{v_{in,CM}} = \frac{2g_{m_1} v_{gs_1}}{v_{in,CM}}$$

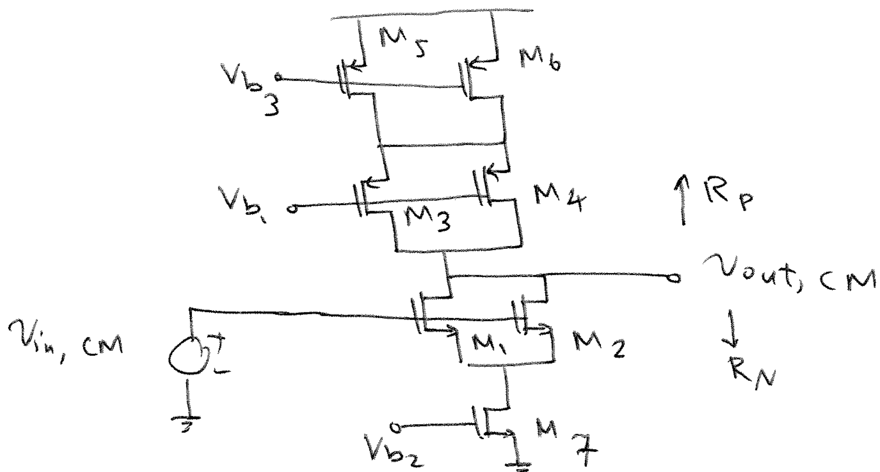
$$= \frac{2g_{m_1}}{v_{in,cm}} \frac{\frac{1}{2g_{m_1}}}{\frac{1}{2g_{m_1}} + R_t} v_{in,cm} \approx \frac{1}{R_t}$$

$$\rightarrow A_{CM} = -G_m R_{out} = -\frac{r_{o_3}}{2R_t} =$$

$$-\frac{r_{o_3}}{2 g_{m_5} r_{o_5} r_{o_6}}$$

(69) (b)

For the common mode input, the circuit is:

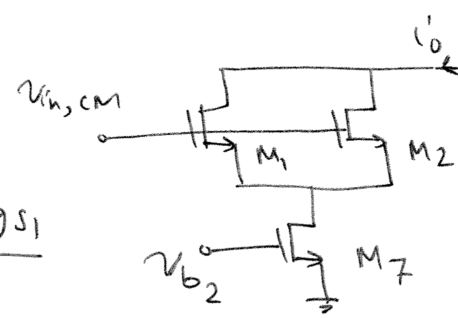


$$R_P = 2g_{m_3}\frac{r_{o3}}{2}\frac{r_{o5}}{2} + \frac{r_{o3}}{2} + \frac{r_{o5}}{2} \approx \frac{g_{m_3} r_{o3} r_{o5}}{2}$$

$$R_N = 2g_{m_1}\frac{r_{o1}}{2} r_{o7} + \frac{r_{o1}}{2} + r_{o7} \approx g_{m_1} r_{o1} r_{o7}$$

$$R_{out} = R_N \| R_p$$

To calculate G_m:



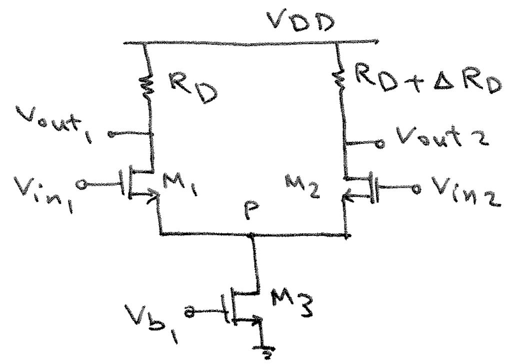
$$G_m = \frac{i'_o}{v_{in,CM}} = \frac{2g_{m_1} v_{gs_1}}{v_{in,CM}}$$

$$= \frac{2g_{m_1}}{v_{in,CM}} \frac{\frac{1}{2g_{m_1}}}{\frac{1}{2g_{m_1}} + r_{o7}} v_{in,CM} \approx \frac{1}{r_{o7}}$$

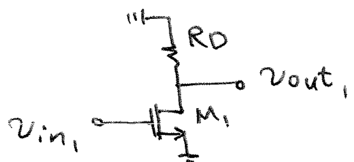
$$\rightarrow A_{CM} = -G_m R_{out}$$

(70)

(a)

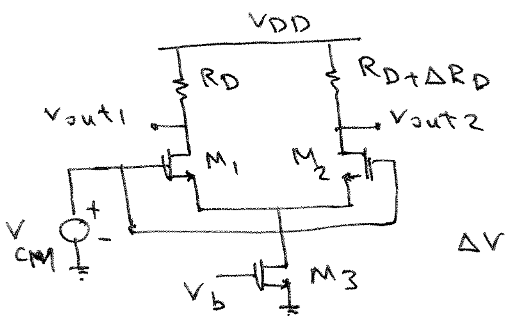


To calculate A_{DM}, using the half circuit:



$$A_{DM} = \frac{v_{out}}{v_{in1}} = -g_{m1} R_D$$

To calculate, A_{CM-DM} we have:



$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3} \Rightarrow$$

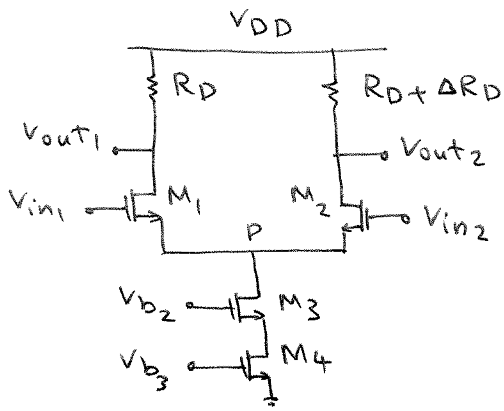
$$\Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m1}} + 2r_{o3}\right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D \, \Delta I_D \Rightarrow$$

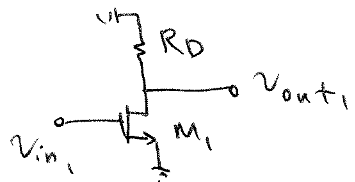
$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}} \Rightarrow$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}$$

(70) (b)



To calculate A_{DM}, using the half circuit, we have



$$A_{DM} = \frac{v_{out1}}{v_{in1}} = -g_{m1} R_D$$

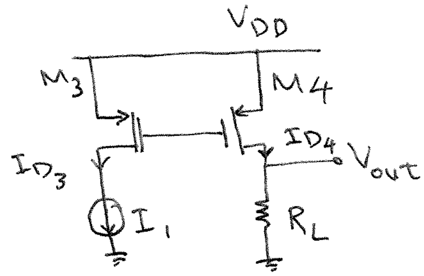
Similar to part (a) we have:

$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM-DM}} = \left(1 + 2g_{m1}\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]\right)\frac{R_D}{\Delta R_D}$$

Notice that CMMR of part (b) is much higher than the one for part (a).

(71)



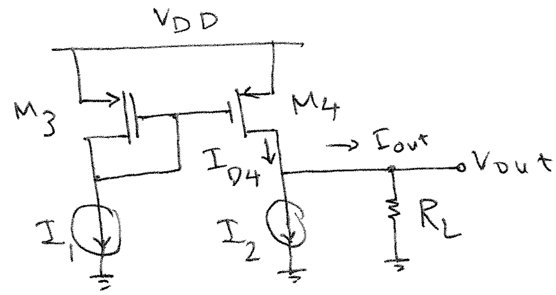
$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4$

$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4 \Rightarrow I_{D3} = N\, I_{D4} \Rightarrow \underbrace{i_{d3} = N\, i_{d4}}_{\text{Small signal}}$

$$\begin{cases} i_{d3} = i_1 \\ v_{out} = R_L\, i_{d4} = \frac{R_L}{N} i_{d3} = \frac{R_L}{N} i_1 \Rightarrow \end{cases}$$

$$\frac{v_{out}}{i_1} = \frac{R_L}{N}$$

(72)



(a) $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$

if $I_1 = I_2 = I_0 \Rightarrow \begin{cases} V_{out} = I_{out} \times R_L = 0 & \text{because} \downarrow \\ I_{out} = I_{D_4} - I_2 = I_1 - I_2 = 0 \end{cases}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = I_{D3} = I_1 = I_0 + \Delta I$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = 2\Delta I$

$V_{out} = I_{out} R_L = 2 R_L \Delta I$

(b) $\left(\frac{W}{L}\right)_3 = 2\left(\frac{W}{L}\right)_4$

$\Rightarrow I_{D3} = 2 I_{D4}$

if $I_1 = I_2 = I_0$ then $I_{D3} = I_1 = I_0 \Rightarrow I_{D4} = \frac{I_{D3}}{2}$

$\Rightarrow I_{D4} = \frac{I_0}{2} \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2}$

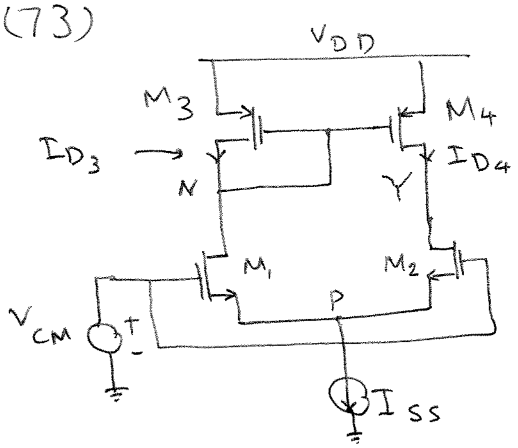
$\Rightarrow V_{out} = R_L I_{out} = -\frac{R_L I_0}{2}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = \frac{I_{D3}}{2} = \frac{I_1}{2} = \frac{I_0 + \Delta I}{2}$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2} + \frac{3\Delta I}{2}$

$\Rightarrow V_{out} = R_L I_{out} = + R_L\left(\frac{-I_0}{2} + \frac{3\Delta I}{2}\right)$

(73)



$V_{TH3} = |V_{TH,p}|$

(a) $I_{D3} = \frac{I_{ss}}{2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 (V_{SG3} - V_{TH3})^2 \Rightarrow$

$V_{SG3} = V_{TH3} + \sqrt{\frac{I_{ss}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$

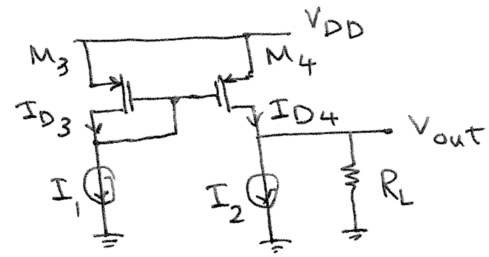
$V_N = V_{DD} - V_{SG3} = V_{DD} - V_{TH} - \sqrt{\frac{I_{ss}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$

(b) Since the circuit is symmetric:

$\begin{cases} I_{D3} = I_{D4} \\ V_{SG3} = V_{SG4} \end{cases} \Rightarrow V_{SD3} = V_{SD4} \Rightarrow V_Y = V_N$

(c) if V_{DD} changes by a small amount ΔV, both V_N and V_Y will chang by ΔV.

(74)

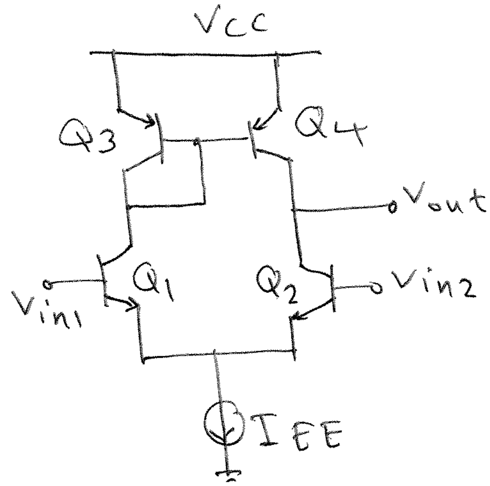


$$I_{D3} = I_{D4} = I_1$$

$$V_{out} = (I_{D4} - I_2) R_L = (I_1 - I_2) R_L$$

small $\Rightarrow$ signal $\quad \dfrac{v_{out}}{i_1} = R_L \quad , \quad \dfrac{v_{out}}{i_2} = -R_L$

(75)



$V_{A,n} = 5\,V$

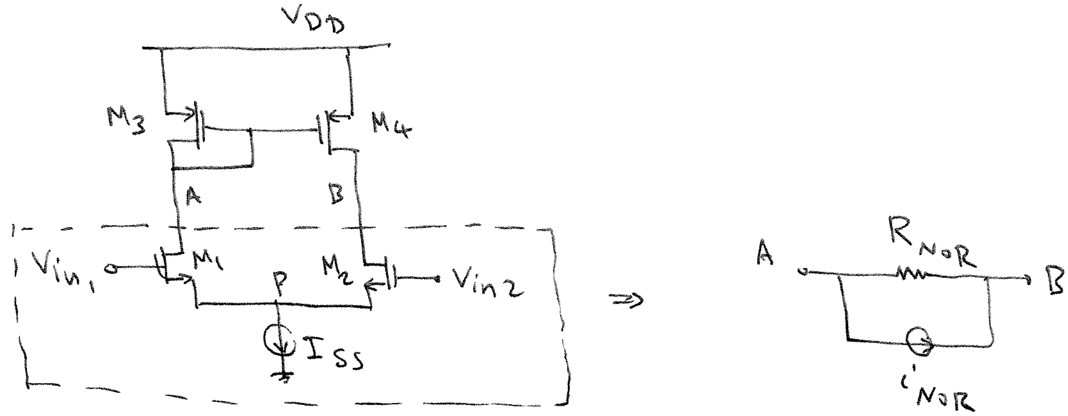
$A_v = 100$

$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN}(r_{oN} \| r_{oP}) =$$

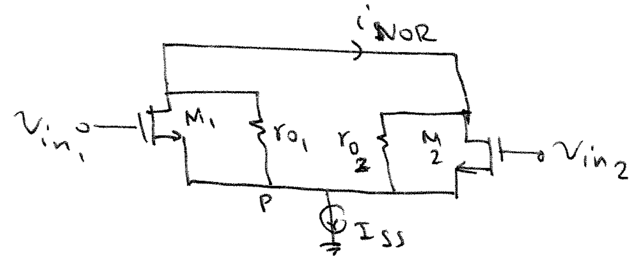
$$\frac{I_{EE}/2}{V_T}\left(\frac{V_{A,n}}{I_{EE}/2} \Big\| \frac{V_{A,p}}{I_{EE}/2}\right) = \frac{V_{A,n}\,V_{A,p}}{(V_{A,n}+V_{A,p})\,V_T}$$

$$\Rightarrow 100 = \frac{5\,V_{A,p}}{(5+V_{A,p})\,0.026} \Rightarrow V_{A,p} = 5.417\,V$$

(76)



To calculate i_{NOR} we have:

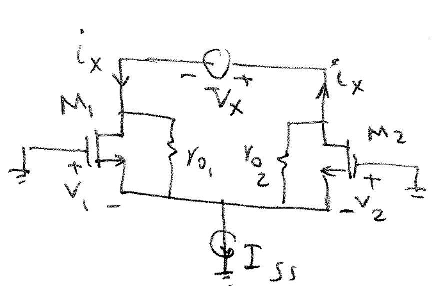


$$r_{o1}(i_{NOR} + g_{m1} v_{in1}) + r_{o2}(i_{NOR} - g_{m2} v_{in2}) = 0$$

$$\rightarrow 2 r_{oN} i_{NOR} = -g_{m1} r_{oN} (v_{in1} - v_{in2}) \Rightarrow$$

$$i_{NOR} = -\frac{g_{mN}}{2}(v_{in1} - v_{in2})$$

To calculate R_{NOR}:

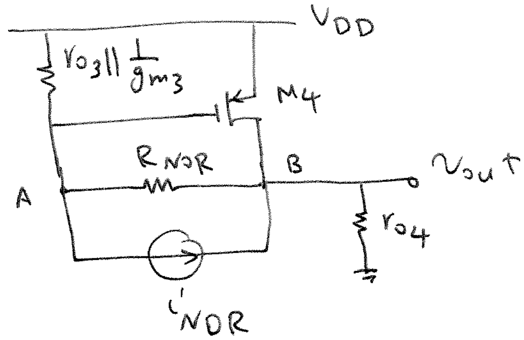


$$v_1 = v_2$$

$$(i_x - g_{m1} v_1) r_{o1} + (i_x + g_{m2} v_2) r_{o2} = v_x$$

$$\Rightarrow R_{NOR} = \frac{v_x}{i_x} = 2 r_{oN}$$

Therefore, utilizing the Norton model we have:



$$\begin{cases} \dfrac{v_A - v_B}{R_{NOR}} + \dfrac{v_A}{r_{o3} \| \frac{1}{g_{m3}}} + i_{NOR} = 0 \Rightarrow v_A = \dfrac{\frac{v_B}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} \\ \dfrac{v_B - v_A}{R_{NOR}} + \dfrac{v_B}{r_{o4}} - i_{NOR} + g_{m4} v_A = 0 \end{cases} , \quad v_B = v_{out}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{o4}}\right) + \left(g_{m4} - \frac{1}{R_{NOR}}\right) \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} = i_{NOR}$$

$$\frac{1}{g_{m3}} \ll r_{o3}, \quad \frac{1}{g_{m3}} \ll R_{NOR}, \quad g_{m3} = g_{m4} = g_m, \quad r_{o3} = r_{o4} = r_{op}$$

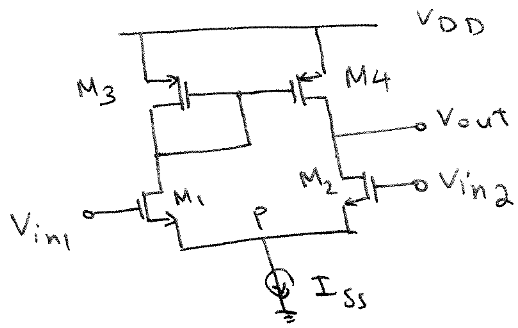
$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + g_{m4} \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{g_{m3}} = i_{NOR}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + \frac{v_{out}}{R_{NOR}} = 2 i_{NOR} \Rightarrow$$

$$\frac{2 v_{out}}{R_{NOR}} + \frac{v_{out}}{r_{op}} = 2 i_{NOR} \Rightarrow v_{out}\left(\frac{1}{r_{oN}} + \frac{1}{r_{op}}\right) = -g_{mN}(v_{in1} - v_{in2})$$

$$\Rightarrow \frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mN} \left(r_{oN} \| r_{op}\right)$$

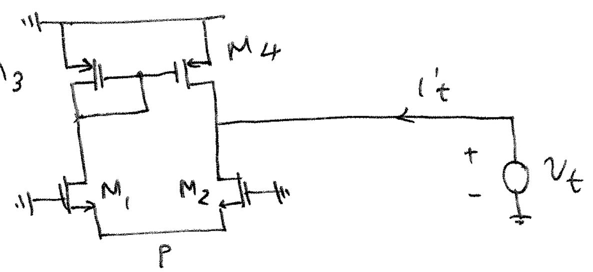
(77)



$g_m r_o \gg 1$

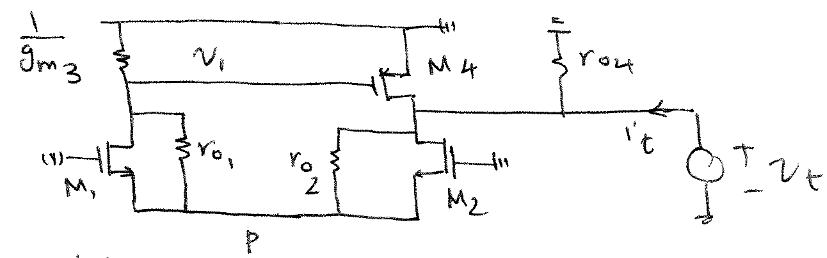
$g_{m1} = g_{m2}$

To calculate the output impedance we have the following circuit:



$R_{out} = \frac{v_t}{i_t'}$

⇓ neglecting r_{o3} ($r_{o3} \gg \frac{1}{g_{m3}}$)

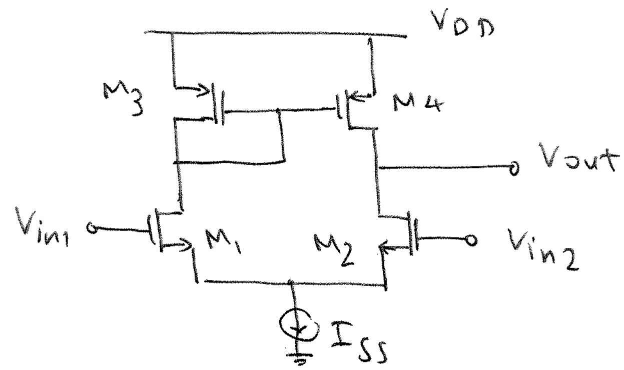


writing node equations of v_1 and v_p:

$$\begin{cases} g_{m3} v_1 + \frac{v_1 - v_p}{r_{o1}} - g_{m1} v_p = 0 \\ 2 g_{m1} v_p + \frac{v_p - v_1}{r_{o1}} + \frac{v_p - v_t}{r_{o2}} = 0 \end{cases} \xrightarrow{g_m r_o \gg 1} \begin{cases} g_{m3} v_1 \approx g_{m1} v_p \\ 2 g_{m1} v_p \approx 0 \end{cases}$$

$$\Rightarrow v_p \approx v_1 = 0 \Rightarrow R_{out} = \frac{v_t}{i_t} = r_{o4} \| r_{o2} = r_{oN} \| r_{oP}$$

(78)

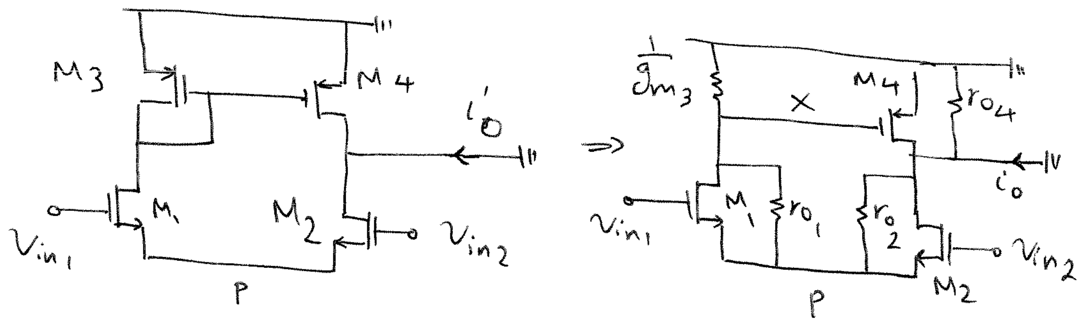


$g_m r_o >> 1$

$g_{m_1} = g_{m_2}$

$g_{m_3} = g_{m_4}$

To calculate G_m we have from small signal model:



writing node equations of nodes P and X we have:

$$\begin{cases} g_{m_1}(v_P - v_{in_1}) + g_{m_2}(v_P - v_{in_2}) + \dfrac{v_P - v_X}{r_{o_1}} + \dfrac{v_P}{r_{o_2}} = 0 \\ g_{m_3} v_X + g_{m_1}(v_{in_1} - v_P) + \dfrac{v_X - v_P}{r_{o_1}} = 0 \end{cases}$$

Since $g_m r_o >> 1$ we have

$$\begin{cases} g_{m_1}(v_P - v_{in_1}) + g_{m_2}(v_P - v_{in_2}) = 0 \Rightarrow v_P = \dfrac{v_{in_1} + v_{in_2}}{2} \\ v_X = -\dfrac{g_{m_1}}{g_{m_3}}(v_{in_1} - v_P) = -\dfrac{g_{m_1}}{g_{m_3}}\left(\dfrac{v_{in_1} - v_{in_2}}{2}\right) \end{cases}$$

$$i'_o = -\frac{v_P}{r_{o_2}} + g_{m_2}(v_{in_2} - v_P) - g_{m_4}(-v_X)$$

$$\Rightarrow \quad i_o \approx -g_{m4}(-v_x) + g_{m2}\left(v_{in2} - \frac{v_{in1}+v_{in2}}{2}\right)$$

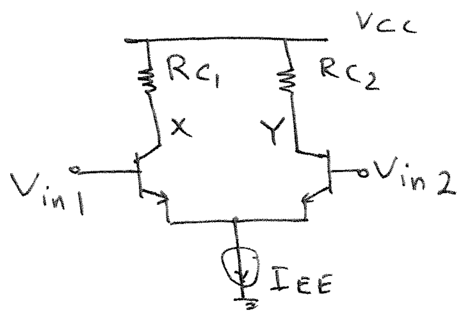
$$= -\left[g_{m4}\ \frac{g_{m1}}{g_{m3}}\left(\frac{v_{in1}-v_{in2}}{2}\right) + g_{m2}\left(\frac{v_{in1}-v_{in2}}{2}\right)\right]$$

$$= -g_{m1}\left(v_{in1} - v_{in2}\right)$$

$$G_m = \frac{i_o}{v_{in1}-v_{in2}} = -g_{m1} = -g_{mN}$$

$$\rightarrow A_v = -G_m R_{out} = g_{mN}\left(r_{oN} \| r_{oP}\right)$$

(79)



$A_v = 10$,

$P = 2\,mW$

$V_{cc} = 2.5\,V$

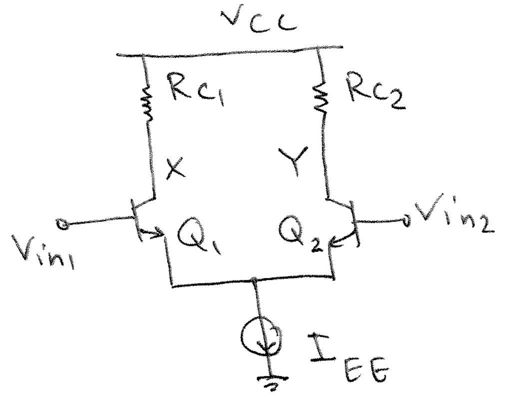
$V_A = \infty$

$$P = V_{cc} I_{EE} \Rightarrow 2\times10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 0.8\,mA$$

$$A_v = \frac{v_{XY}}{v_{in_1} - v_{in_2}} = -g_m R_C = -\frac{I_{EE}/2}{V_T} R_C$$

$$\Rightarrow 10 = \frac{0.4\times10^{-3}}{0.026} R_C \Rightarrow R_C = 650\,\Omega$$

(80)



$V_{in,CM} = 1.2\ V$

$P = 3\ mW$

$V_{CC} = 2.5\ V$

$$P = I_{EE} V_{CC} \Rightarrow 3\times10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 1.2\, mA$$

$$A_v = -g_{m_1} R_C = -\frac{I_{EE}/2}{V_T} R_C = -\frac{R_C I_{EE}}{2V_T}$$

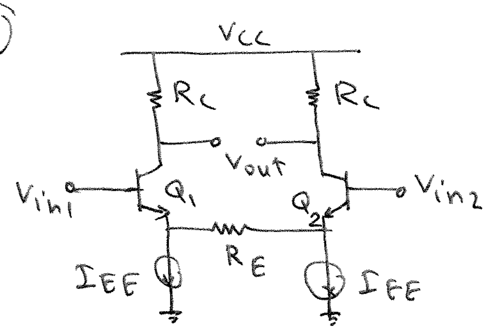
To maximize gain, $R_C I_{EE}$ and therefore R_C should be maximum. However, the upper bound of R_C value is limited by the voltage value of X. because:

$$V_{in,CM} \leq V_X \Rightarrow 1.2 \leq V_{CC} - R_C I_{EE}/2 \Rightarrow$$

$$R_C \leq 2\frac{V_{CC}-1.2}{I_{EE}} \Rightarrow R_C \leq 2\,\frac{2.5-1.2}{1.2\times10^{-3}} \Rightarrow$$

$$R_C \leq 2.167\, k\Omega \Rightarrow R_C = 2.167\, k\Omega$$

(81)



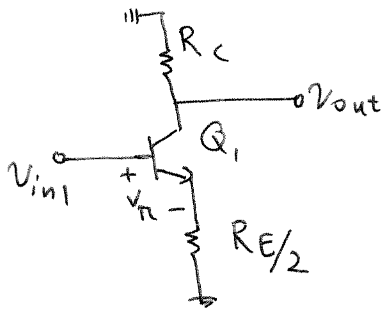
$A_v = 5$

$P = 4\,mW$

$V_{CC} = 2.5\,V$

$V_A = \infty$

The half circuit is:



$$A_v = \frac{v_{out}}{v_{in1}} \simeq \frac{-g_{m1} v_\pi R_C}{v_{in1}} = -\frac{g_{m1} R_C}{v_{in1}} \cdot \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_E}{2}} v_{in1}$$

$$= -\frac{R_C}{\frac{R_E}{2} + \frac{1}{g_{m1}}}$$

$P = 4\,mW = 2 I_{EE} V_{CC} = 5 I_{EE} \Rightarrow I_{EE} = 0.8\,mA$

$g_m = \frac{I_{EE}}{V_T} = 0.03077$

$$A_v = 5 \Rightarrow \frac{R_C}{\frac{R_E}{2} + 32.5} = 5 \quad ①$$

if I_{EE} increases by 10%, the gain will be:

$$A_v = \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} \Rightarrow 5 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} < 5 \times 1.02 \quad ②$$

if I_{EE} decreases by 10% then:

$$5 \times 0.98 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} < 5 \quad ③$$

The worse case is:

$$\begin{cases} \dfrac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} = 5 \times 1.02 & (4) \\[2ex] \dfrac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} = 5 \times 0.98 & (5) \end{cases}$$

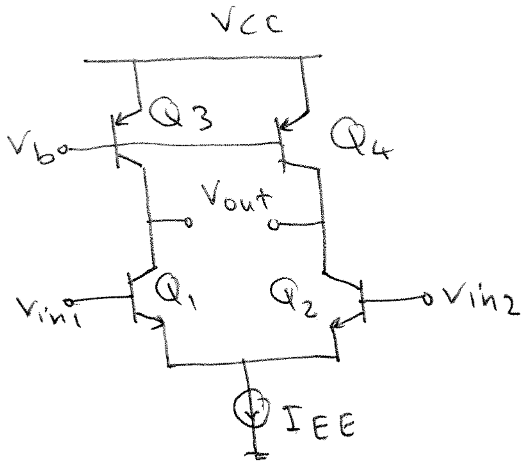
dividing (4) and (5) to (1) leads to:

$$\begin{cases} \dfrac{\frac{R_E}{2} + 32.5}{\frac{R_E}{2} + \frac{32.5}{1.1}} = 1.02 \Rightarrow R_E = 236.36\,\Omega \\[2ex] \dfrac{\frac{R_E}{2} + 32.5}{\frac{R_E}{2} + \frac{32.5}{0.9}} = 0.98 \Rightarrow R_E = 288.89\,\Omega \end{cases}$$

To ensure less than 2% gain variation for 10% current variation $R_E = 288.89\,\Omega$

From (1) $R_C = 5\left(\frac{R_E}{2} + 32.5\right) = 884.72\,\Omega$

(82)



$A_v = 100$

$P = 1mW$

$V_{A,n} = 6$

$V_{CC} = 2.5\ V$

$$P = 1mW = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4mA$$

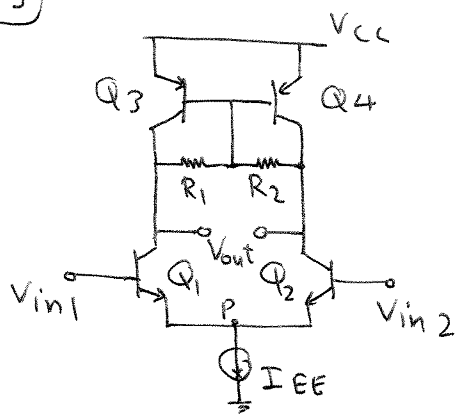
$$r_{o_N} = \frac{V_{A_{N}}}{I_{EE}/2} = \frac{6}{0.2 \times 10^{-3}} = 30k\Omega,\ g_{m_N} = \frac{I_{EE}/2}{V_T} = \frac{0.2}{26}\ S$$

$$A_v = -g_{m_N} (r_{o_N} \| r_{op}) \Rightarrow$$

$$100 = \frac{0.2}{26} (30 \times 10^3 \| r_{op}) \Rightarrow r_{op} = 22.94k\Omega$$

$$\Rightarrow V_{A,p} = r_{op} \frac{I_{EE}}{2} = 4.588\ V$$

(83)



$A_v = 50$

$P = 1mW$

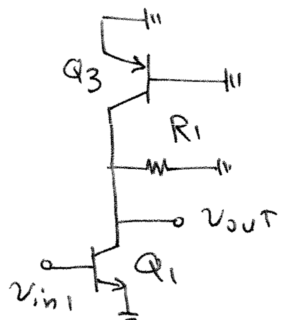
$V_{A,n} = 10\ V$

$V_{A,p} = 5\ V$

$V_{CC} = 2.5\ V$

$R_1 = R_2$

The half circuit is:



$$A_v = -g_{m_1}(R_1 \| r_{o_1} \| r_{o_3})$$

$$g_{m_1} = \frac{I_{EE}}{2V_T} \quad r_{o_1} = \frac{2V_{A,n}}{I_{EE}} \quad r_{o_3} = \frac{2V_{A,p}}{I_{EE}}$$

$$\Rightarrow 50 = \frac{1}{2V_T}\left((R_1 I_{EE}) \| \underbrace{(2V_{A,n})}_{20} \| \underbrace{(2V_{A,p})}_{10}\right) \Rightarrow$$

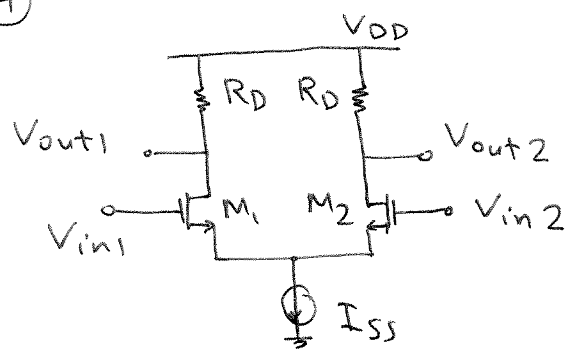
$R_1 I_{EE} = 4.262$

$P = 1mW = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4\,mA$

$\rightarrow R_1 = \frac{4.262}{I_{EE}} = 10.655\ K\Omega$

Notice that larger value of R_1, requires smaller I_{EE} and saves power.

(84)



$\Delta V_{in,max} = 0.3\,V$

$P = 3\,mW$

$R_D = 500\,\Omega$

$\lambda = 0$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$V_{DD} = 1.8\,V$

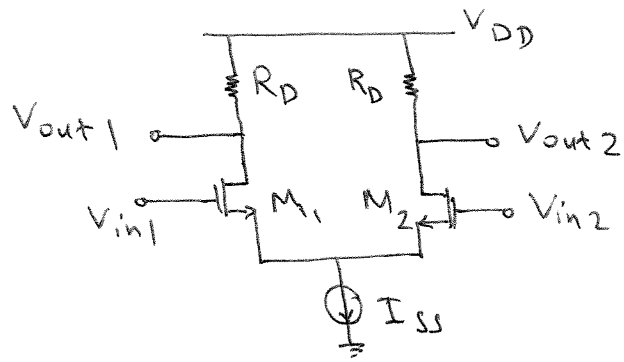
$$P = V_{DD} I_{SS} \Rightarrow 3\times10^{-3} = 1.8\, I_{SS} \Rightarrow$$

$$I_{SS} = 1.67\ mA$$

$$\Delta V_{in,max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.3 = \sqrt{\frac{2\times1.67\times10^{-3}}{10^{-4}\times\frac{W}{L}}} \Rightarrow \frac{W}{L} = 370.37$$

(85)



$P = 2mW$

overdrive $= 100 mV$

$V_{CM} = 1V$

$\lambda = 0$, $\mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8V$

$V_{TH,n} = 0.5$

$$P = I_{SS} V_{DD} \Rightarrow 2\times 10^{-3} = 1.8\, I_{SS} \Rightarrow I_{SS} = 1.11\ mA$$

$$V_{GS_1} - V_{TH} = \sqrt{\frac{2 I_{D_1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.1^2 = \frac{1.11\times 10^{-3}}{10^{-4} \times \frac{W}{L}} \Rightarrow \frac{W}{L} = 1111.11$$

$$g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{10^{-4} \times 1111.11 \times 1.11\times 10^{-3}} = 0.011$$

To place the transistor at the edge of triode region:

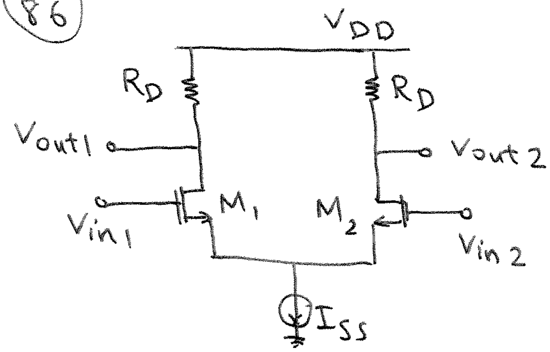
$$V_{in,CM} = V_{out1} + V_{TH,n}$$

$$1 = V_{DD} - R_D \frac{I_{SS}}{2} + 0.5 \Rightarrow$$

$$1 = 1.8 - R_D \frac{1.11\times 10^{-3}}{2} + 0.5 \Rightarrow R_D = 2.34\ k\Omega$$

$$A_V = -g_m R_D = -25.74$$

(86)



$A_v = 5$

$P = 1mW$

$(V_{GS} - V_{TH})_{equil} = 150 mV$

$\lambda = 0, \mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8 V$

$P = 1mw = V_{DD} I_{SS} = 1.8 I_{SS} \Rightarrow I_{SS} = 0.556 mA$

$$g_{m_1} = \frac{2 I_{D1}}{(V_{GS} - V_{TH})_{equil}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{equil}} = \frac{0.556 \times 10^{-3}}{0.15}$$

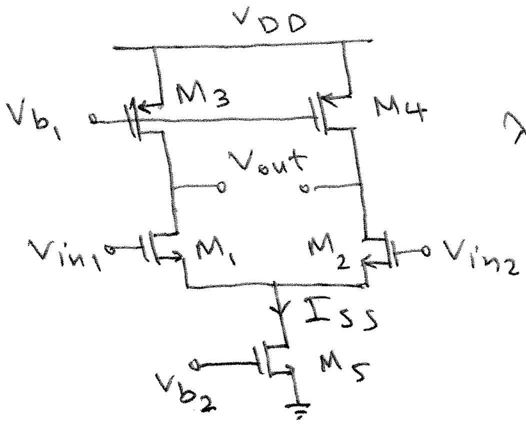
$= 3.704 \; mS$

$A_v = -g_{m_1} R_D \Rightarrow \quad 5 = 3.704 \times 10^{-3} \times R_D \Rightarrow R_D = 1.35 k\Omega$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.15 = \sqrt{\frac{0.556 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 246.91$$

(87)



$A_V = 40$

$(V_{GS} - V_{TH})_{equil} = ?$

$\lambda_n = 0.1 V^{-1}$ $\lambda_p = 0.2 V^{-1}$

$\mu_n C_{ox} = 100 \mu A/V^2$

$\mu_p C_{ox} = 50 \mu A/V^2$

$V_{DD} = 1.8$

$P = 2mW$

$$A_V = -g_{m_N}(r_{o_p} \| r_{o_N}) = -\frac{I_{SS}}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\frac{I_{SS}}{2}\lambda_n} \| \frac{1}{\frac{I_{SS}}{2}\lambda_p}\right)$$

$$= -\frac{2}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\lambda_n} \| \frac{1}{\lambda_p}\right) \Rightarrow$$

$$\frac{2}{(V_{GS_1} - V_{TH})_{equil}}(10 \| 5) = 40 \Rightarrow (V_{GS_1} - V_{TH})_{equil} = 166.67\ mV$$

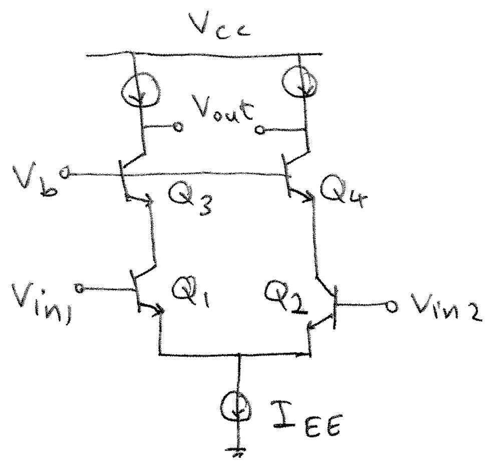
$$P = 2\times10^{-3} = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\ mA$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox}(V_{GS_1} - V_{TH})^2_{equil}} = \frac{1.11\times10^{-3}}{10^{-4}\times(0.16667)^2} = 400$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{1.11\times10^{-3}}{0.5\times10^{-4}\times(0.16667)^2} = 800$$

$$\left(\frac{W}{L}\right)_5 = \frac{2 I_{SS}}{\mu_n C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{2\times 1.11\times10^{-3}}{10^{-4}\times(0.16667)^2} = 800$$

(88)



$A_v = 4000$

$\beta = 100$

$V_{CC} = 2.5\,V$

$P = 1\,mW$

$$P = I_{EE} V_{CC} = 10^{-3} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4\,mA$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.2}{26} = 7.692\,mS$$

$$r_{\pi_{1-4}} = \frac{\beta}{g_{m_1}} = 13\,K\Omega$$

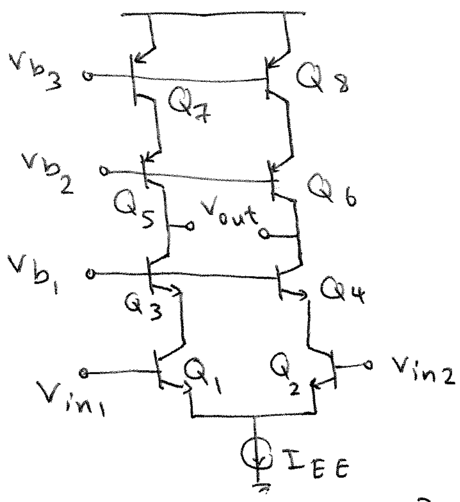
$$r_{o_{1-4}} = \frac{V_A}{\frac{I_{EE}}{2}} = 5\times10^3\,V_A$$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1}\|r_{\pi_3})r_{o_3} + (r_{o_1}\|r_{\pi_3}) + r_{o_3}\right] \Rightarrow$$

$$4000 = \frac{0.2}{26}\left[\frac{0.2}{26}(5\times10^3 V_A \| 13\times10^3)5\times10^3 V_A + (5\times10^3 V_A)\|13\times10^3 + 5\times10^3 V_A\right]$$

$$\Rightarrow V_A = 2.197$$

(89)



$A_v = 2000$

$\beta_n = 100$

$\beta_p = 50$

$V_{A,n} = 5V$

$V_{CC} = 2.5V$

$P = 2mW$

$$P = I_{EE} V_{CC} = 2\times10^{-3} \Rightarrow I_{EE} = \frac{2\times10^{-3}}{2.5} = 0.8mA$$

$$g_{m_{1-8}} = \frac{I_{EE}}{2V_T} = \frac{0.4}{26} = 0.0154 \Rightarrow$$

$$r_{\pi_{1-4}} = \frac{\beta_n}{g_{m_1}} = 6.5K\Omega \qquad r_{\pi_{5-8}} = \frac{\beta_p}{g_{m_5}} = 3.25K\Omega$$

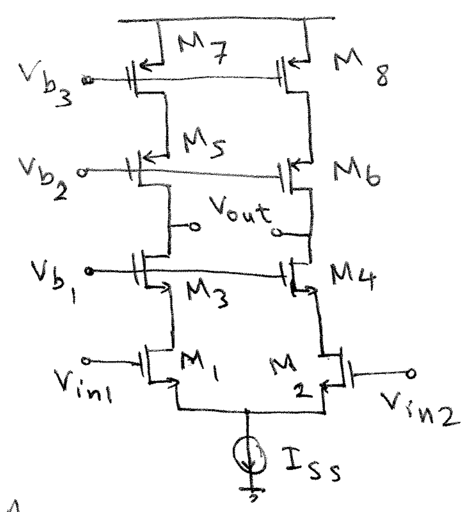
$$r_{o_{1-4}} = \frac{V_{A,n}}{I_{EE}/2} = 12.5K\Omega \qquad r_{o_{5-8}} = \frac{V_{A,P}}{I_{EE}/2}$$

$$A_v \approx -g_{m_1}\left[g_{m_3} r_{o_3}(r_{o_1} \| r_{\pi_3})\right] \| \left[g_{m_5} r_{o_5}(r_{o_7} \| r_{\pi_5})\right]$$

$$\Rightarrow \frac{0.4}{26}\left[\frac{0.4}{26}\times 12.5\times10^{+3}(12.5\times10^{3} \| 6.5\times10^{3})\right] \| \left[\frac{0.4}{26}\ \frac{V_{A,P}}{I_{EE}/2}\left(\frac{V_{A,P}}{\frac{I_{EE}}{2}} \| 3250\right)\right] = 2000$$

$$\Rightarrow V_{A,P} = 2.027\ V$$

(90)



$A_V = 600$

$P = 4\,mW$

$(V_{GS}-V_{TH})_{NMOS} = 100\,mV$

$(V_{GS}-V_{TH})_{PMOS} = 150\,mV$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$

$$A_V \simeq -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] = -600$$

$$P = 4mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{4\times10^{-3}}{1.8} = 2.22\,mA$$

$$g_{m1-4} = \frac{2I_{D1}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{2.22\times10^{-3}}{0.1} = 22.22\,mS$$

$$g_{m5-8} = \frac{2I_{D5}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{2.22\times10^{-3}}{0.15} = 14.815\,mS$$

$$r_{o1-4} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1\times\frac{2.22}{2}\times10^{-3}} = 9K\Omega$$

$$r_{o5-8} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = \frac{1}{\lambda_p\times\frac{2.22}{2}\times10^{-3}} = \frac{0.9\times10^{3}}{\lambda_p}$$

$\Rightarrow$ in A_V equation

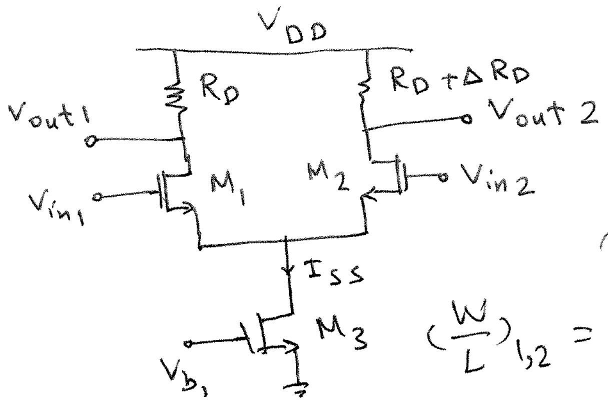
$$22.22\times10^{-3}\left[(22.22\times10^{-3}\times81\times10^{6}) \| (14.815\times10^{-3}\times\frac{0.81\times10^{6}}{\lambda_p^2})\right] = 600 \Rightarrow$$

$\lambda_p = 0.66\ V^{-1}$

$$\left(\frac{W}{L}\right)_{NMOS} = I_{SS}/(\mu_n C_{ox}(V_{GS}-V_{TH})^2_{NMOS}) = 2222.2$$

$$\left(\frac{W}{L}\right)_{PMOS} = I_{SS}/(\mu_p C_{ox}(V_{GS}-V_{TH})^2_{PMOS}) = 1975.31$$

(91)



CMMR = 60dB

$P = 2mW$

$A_{DM} = 5$

$\lambda(M_3) = ?$

$(\frac{W}{L})_{1,2} = \frac{10}{0.18}$, $\mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8$, $\frac{\Delta R_D}{R_D} = 2\%$

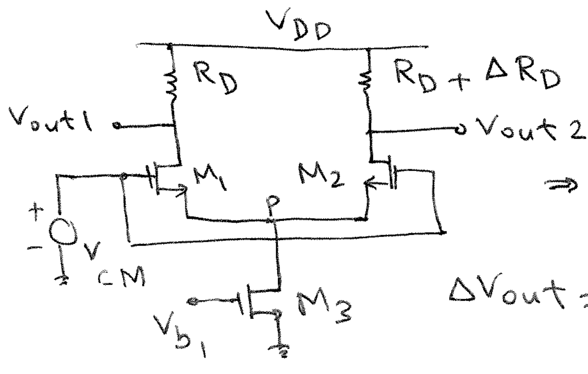
$$P = 2mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11 \, mA$$

$$A_{DM} = -g_{m_1} R_D$$

$$g_{m_1} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_1 I_{SS}} = \sqrt{10^{-4} \times \frac{10}{0.18} \times 1.11\times10^{-3}} = 2.4845 \, ms$$

$$\Rightarrow R_D = \frac{|A_{DM}|}{g_{m_1}} = \frac{5}{2.4845\times10^{-3}} = 2.012 \, k\Omega$$

To calculate $A_{CM,DM}$ we have:



$$\Delta V_{CM} = \Delta V_{GS} + 2 \Delta I_D r_{o3}$$

$$\Rightarrow \Delta V_{CM} = \Delta I_D (\frac{1}{g_{m_1}} + r_{o3})$$

$$\Delta V_{out} = \Delta V_{out_1} - \Delta V_{out_2} \cong -\Delta R_D I_D$$

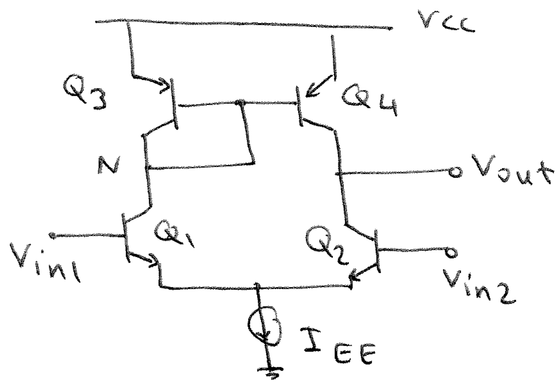
$$\Rightarrow A_{CM,DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = -\frac{\Delta R_D / 2}{\frac{1}{2 g_{m_1}} + r_{o3}}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM,DM}} = (1 + 2 g_{m_1} r_{o3}) \frac{R_D}{\Delta R_D} \, , \quad r_{o3} = \frac{1}{\lambda_3 I_{SS}}$$

$$\Rightarrow \quad CMMR = 60\,dB = 10^3 = \left(1 + 2 \times 2.4845 \times 10^{-3} \frac{1}{\lambda_3 \times 1.11 \times 10^{-3}}\right) 50$$

$$\Rightarrow \quad \lambda_3 = 0.2354$$

(92)



$A_v = 200$

$P = 3mW$

$V_{cc} = 2.5V$

$V_{A,n} = 2V_{A,p}$

$$P = V_{cc} I_{EE} \Rightarrow I_{EE} = \frac{3\times10^{-3}}{2.5} = 1.2\ mA$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.6}{26} = 23.077\ mS$$

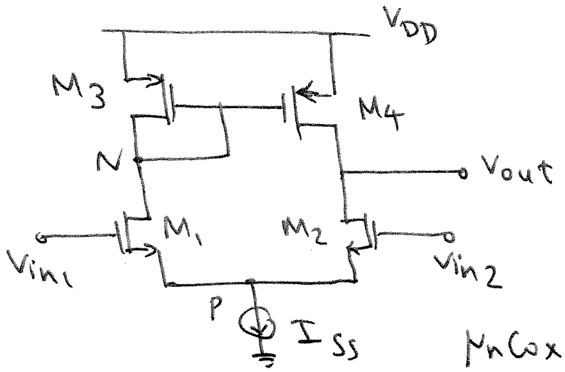
$$r_{o_1} = r_{o_2} = \frac{V_{A,n}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{0.6\times10^{-3}}$$

$$r_{o_3} = r_{o_4} = \frac{V_{A,p}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{1.2\times10^{-3}}$$

$$A_v = g_{m_1}(r_{o_2} \| r_{o_4}) \Rightarrow \frac{0.6}{26}\left(\frac{V_{A,n}}{0.6\times10^{-3}} \| \frac{V_{A,n}}{1.2\times10^{-3}}\right) = 200$$

$$\Rightarrow V_{A,n} = 15.6\ V$$

(93)



$A_v = 20$

$P = 1mW$

$V_{DD} = 1.8\,V$

$V_{in,CM} = 1\,V$

$\mu_n C_{ox} = 2\mu_p C_{ox} = 100\mu A/V^2$

$V_{TH,n} = 0.5V,\; V_{TH,p} = -0.4\,V$

$\lambda_n = \dfrac{\lambda_p}{2} = 0.1\,V^{-1}$

$$P = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{10^{-3}}{1.8} = 0.556\ mA$$

$$A_v = + g_{m_N} (r_{o_N} \| r_{o_P}) = 20$$

$$r_{o_N} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1 \quad \frac{0.556 \times 10^{-3}}{2}} = 36\,k\Omega$$

$$r_{o_P} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = 18\,k\Omega$$

$$g_{m_N} (36^k \| 18^k) = 20 \Rightarrow g_{m_N} = 1.667m\,S$$

$$\Rightarrow g_{m_N} = \frac{2 I_{D_N}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{NMOS}} \Rightarrow$$

$$(V_{GS} - V_{TH})_{NMOS} = 0.333\ V$$

$$(V_{GS} - V_{TH})_{NMOS} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} (\frac{W}{L})_{NMOS}}} \Rightarrow (\frac{W}{L})_{1,2} = 50$$

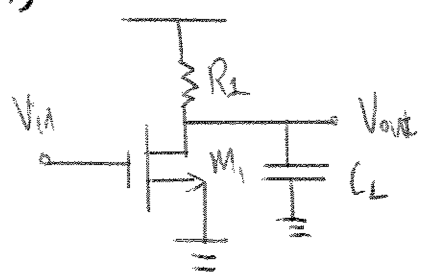
$$V_N = V_{in,CM} - V_{TH,n} = 1 - 0.5 = 0.5\ V$$

$$\rightarrow \quad |V_{GS_3}| - |V_{TH,p}| = 1.3 - 0.4 = 0.9\ V$$

$$\rightarrow \quad 0.9 = \sqrt{\frac{I_{SS}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{PMOS}}} \quad \Rightarrow$$

$$\left(\frac{W}{L}\right)_{3,4} = 13.717$$

1)



$R_1 = 1\,K\Omega$

$C_L = 1pF$

$$V_{out} = -g_{m_1} R_1 // \frac{1}{C_L S} V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = -g_m R_1 // \frac{1}{C_L S}$$

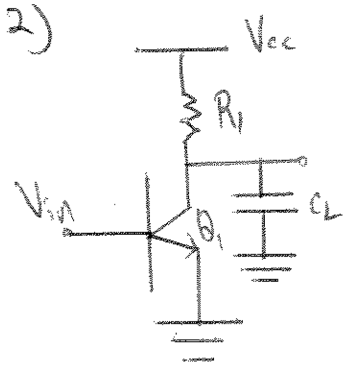
$$\frac{V_{out}}{V_{in}} = -g_{m_1}\left(\frac{R_1}{R_1 C_L S + 1}\right), \quad S \to j\omega, \quad \frac{V_{out}}{V_{in}}(j\omega) = -g_{m_1}\left(\frac{R_1}{R_1 C_L j\omega + 1}\right)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}}, \quad \text{Fall by } 10\% = \left|\frac{V_{out}}{V_{in}}\right| = g_m R_1\, 0.9$$

$$\Rightarrow \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}} = g_m R_1\, 0.9 \Rightarrow \omega_{-10\%} = 4.84 \times 10^9 \text{ rad/s}$$

$$2\pi f = 4.84 \times 10^9 \Rightarrow f_{-10\%} = 7.708 \times 10^8 \text{ Hz}$$

2)



-3dB bandwidth = 1 GHz

$C_L = 2pF$

Power = 2mW

Low freq gain ?

Power = $2.5V\, I_c$, $I_c = 0.8\, mA$

Dominant Pole at the output = $\dfrac{1}{R_1 C_L} = 2\pi(1GHz)$

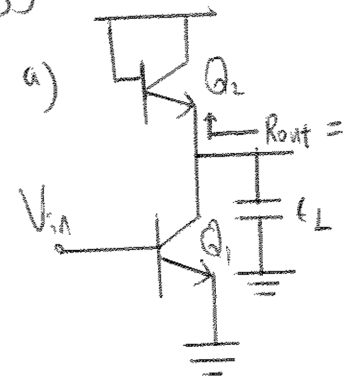
$R_1 = 79.58$ Ohm.

Low freq gain : $-g_m R_1 = \dfrac{-I_c R_1}{V_T} = \dfrac{(79.58)(0.8)}{26}$

$A_v \big|_{low\,freq} = -2.45$

3)

a)

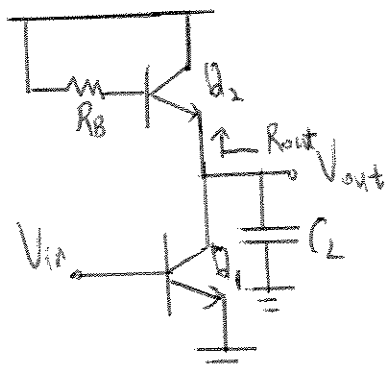


$R_{out} = \frac{1}{g_{m2}}$

Assume $\beta \gg 1$

$$-3dB = \frac{g_{m2}}{C_L}$$

b)

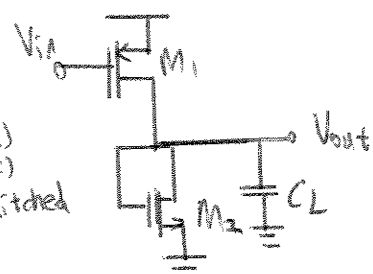


$$R_{out} = \frac{1}{g_{m2}} + \frac{R_B}{\beta+1}$$

$$-3dB = \frac{(\beta+1)\, g_{m2}}{C_L\,[\beta + R_B g_{m2}]}$$

d)

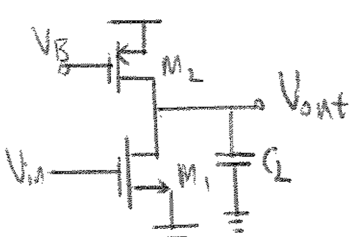
↑ note, parts d) and c) are switched



$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{o2} \,//\, r_{o1} \approx \frac{1}{g_{m2}}$$

$$-3dB = \frac{g_{m2}}{C_L}$$

c)

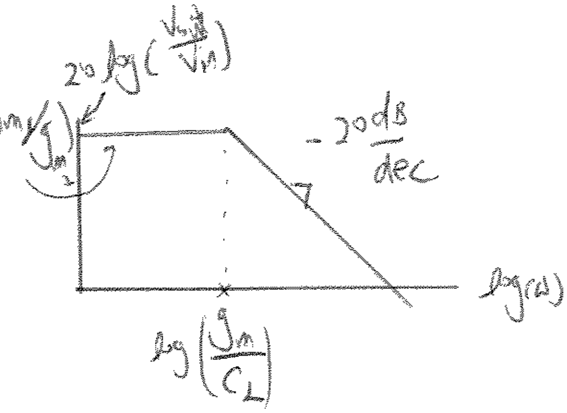


$$R_{out} = r_{o1} // r_{o2}$$

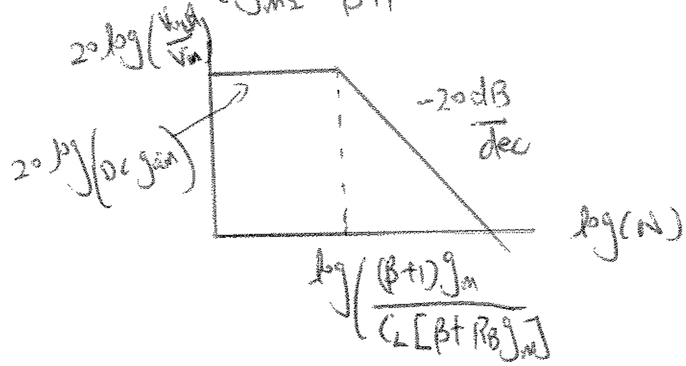
$$-3dB = \frac{1}{(r_{o1} // r_{o2})\, C_L}$$

4)

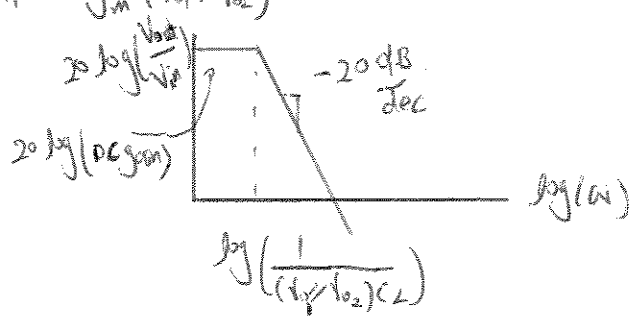
a) $|DC\ gain| = \frac{g_{m1}}{g_{m2}}$



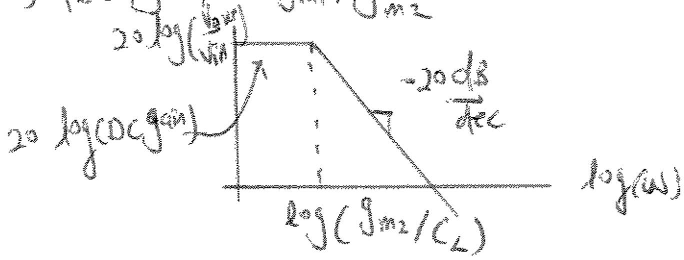
b) $|DC\ gain| = g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta+1}\right)$



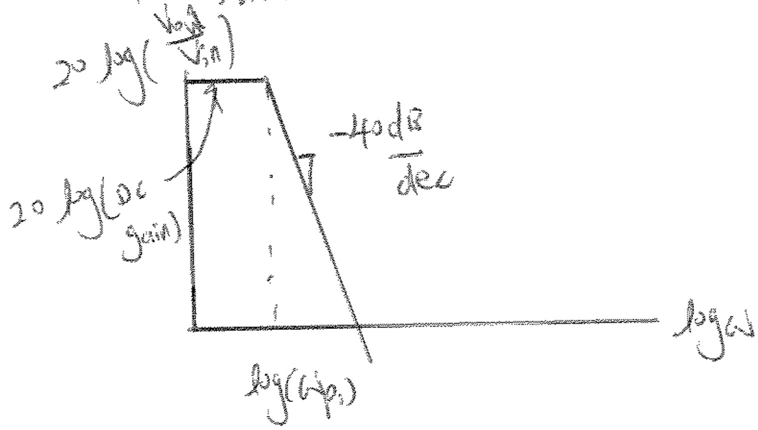
c) $|DC\ gain| = g_m(r_{o1}//r_{o2})$



d) $|DC\ gain| = g_{m1}/g_{m2}$



5) 2 poles at ω_{p1}, means slope is −40 dB/dec



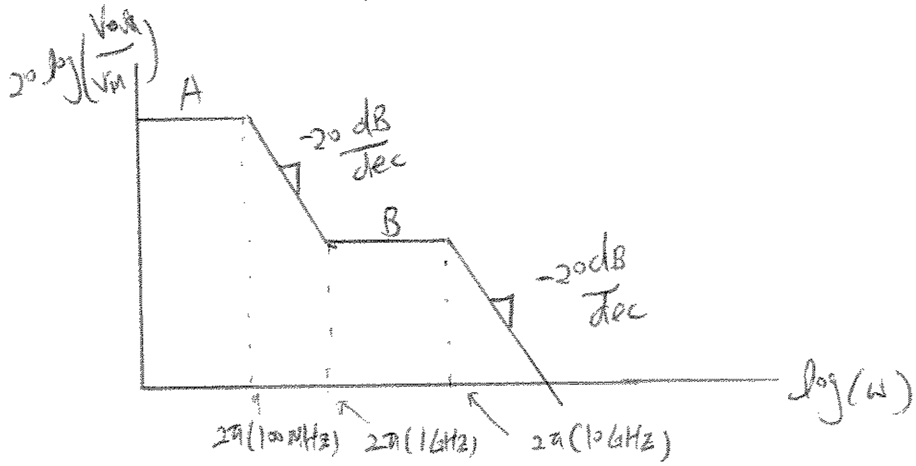
* Assuming transfer function is in the form of

$$\frac{A}{\left(\sqrt{\left(\frac{\omega}{\omega_{p1}}\right)^2+1}\right)^2}$$

6)

Poles at 100 MHz, 10 GHz

Zero at 1 GHz.

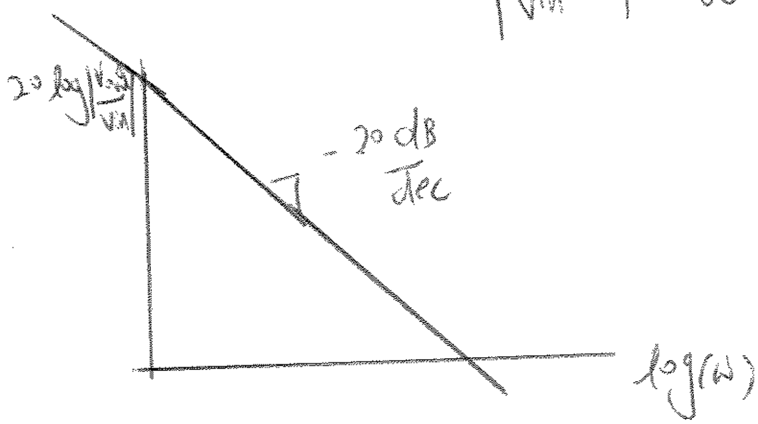


A (100 MHz) = B (1GHz)

B = 0.1A

7) Ideal Integrator : $\frac{V_{out}}{V_{in}}(s) = \frac{1}{s}$

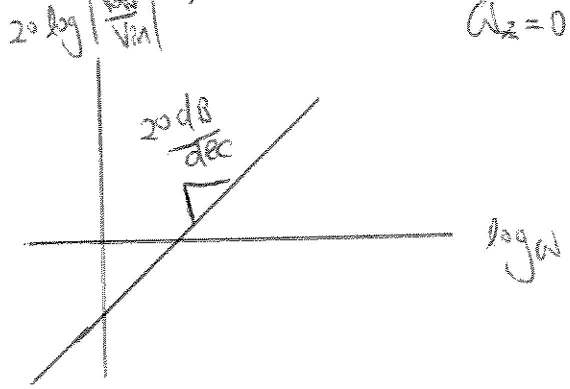
$$\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \frac{1}{\omega}$$



For an integrator, the gain at arbitrary low freq approaches infinity.

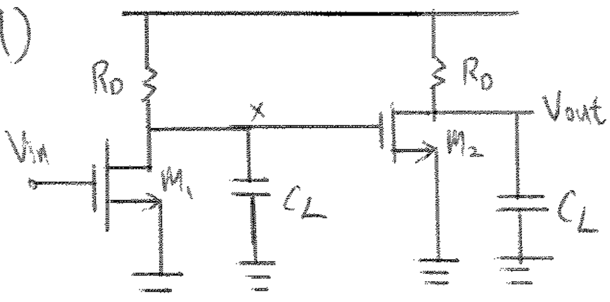
8) Ideal differentiator : $S = \frac{V_{out}}{V_{in}}$, $\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \omega$

$\omega_z = 0$



For an ideal differentiator, gain at arbitrary high freq approaches infinity.

9)



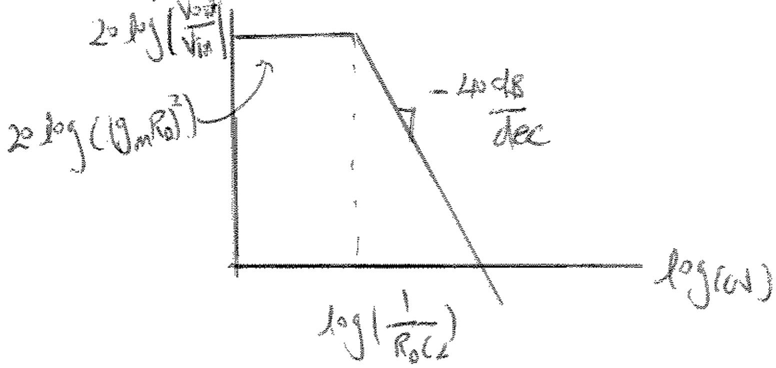
$\lambda = 0$, & neglect other caps.

DC gain: $\frac{V_x}{V_{in}} = -g_m R_D$, $\frac{V_{out}}{V_x} = -g_m R_D$

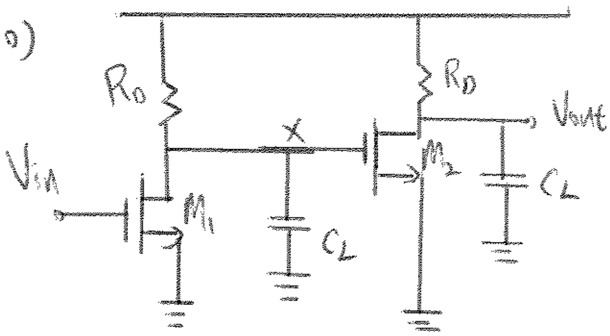
$$\frac{V_{out}}{V_{in}} = (g_m R_D)^2 \quad \text{(At DC)}$$

2 poles at $\frac{1}{R_D C_L}$

Bode plot:



10)



$$\frac{V_x}{V_{in}}(s) = -g_m \left(R_D \parallel \frac{1}{C_L s}\right), \quad \frac{V_{out}}{V_x}(s) = -g_m \left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$= -g_m \left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$H(s) = \frac{V_x}{V_{in}}(s)\,\frac{V_{out}}{V_x}(s) = \left(\frac{g_m R_D}{R_D C_L s + 1}\right)^2$$

$$s \to \omega j, \quad H(j\omega) = \left(\frac{g_m R_D}{1 + R_D C_L j\omega}\right)^2$$

$$|H(j\omega)| = \frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2}$$

−3dB Band Width:

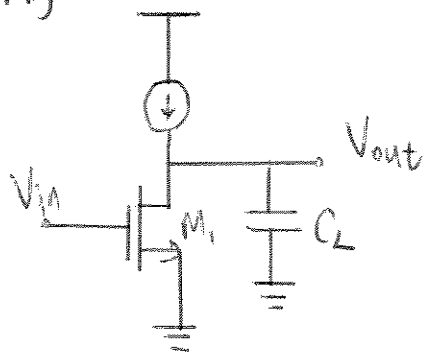
$$\frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2} = \frac{(g_m R_D)^2}{\sqrt{2}}$$

$$\Rightarrow (R_D C_L \omega)^2 + 1 = \sqrt{2}$$

$$\Rightarrow \omega = \frac{\sqrt{\sqrt{2} - 1}}{R_D C_L} = \frac{0.6436}{R_D C_L} \ (\text{rad/s})$$

$$2\pi f = \frac{0.6436}{R_D C_L} \Rightarrow f = \frac{0.10243}{R_D C_L} \ (\text{Hz})$$

1 a)



$\lambda > 0$

Since $\lambda > 0$, and we have an ideal current source, the impedance looking from out to ground is $r_o // \frac{1}{C_L s}$

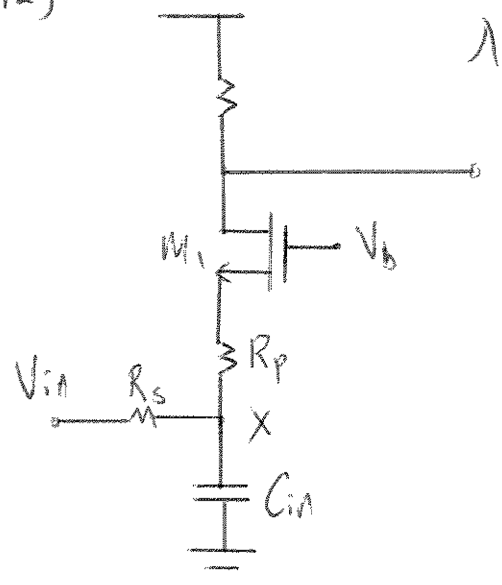
So, $V_{out} = -g_m V_{in} \left(r_o // \frac{1}{C_L s} \right)$

$H(s) = -g_m \left(\frac{r_o}{r_o C_L s + 1} \right)$, $|H(j\omega)| = \frac{g_m r_o}{\sqrt{(r_o C_L \omega)^2 + 1}}$

For $\lambda \to 0$, $r_o \to \infty \Rightarrow H(s) \to \frac{-g_m r_o}{r_o C_L s}$

$H(s) = \frac{-g_m}{C_L s}$. A pole at origin, thus operating as an ideal integrator.

12)



$\lambda = 0$

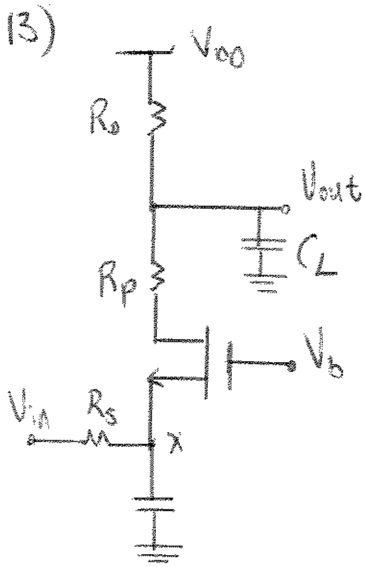
To find input pole, let $V_{in} = 0$ and find the equivalent resistance and capacitance from node X to ground.

$$R_X = R_S \parallel \left(R_P + \frac{1}{g_{m_1}} \right), \quad C_X = C_{in}$$

$$\omega_{p,in} = \frac{1}{C_{in}\left[R_S \parallel \left(R_P + \frac{1}{g_m}\right)\right]}$$

$$\omega_{p,out} = \frac{1}{R_D C_L}$$

13)



$\lambda = 0$, neglect all other caps.

$$R_X = R_S // \frac{1}{g_m}$$

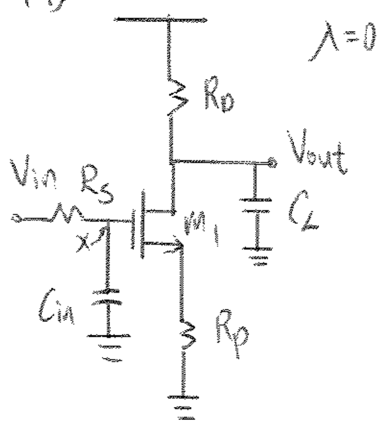
$$C_X = C_{in}$$

$$R_{out} = R_D \quad (\text{since } r_o = \infty)$$

$$C_{out} = C_L$$

$$\omega_{pin} = \frac{1}{(R_S // \frac{1}{g_m})\, C_{in}}, \qquad \omega_{pout} = \frac{1}{R_D C_L}$$

14)

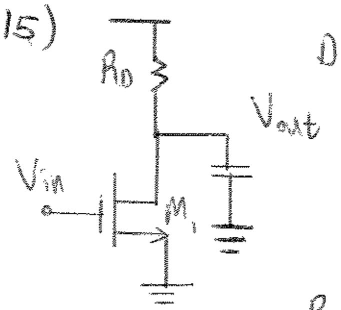


$R_X = R_S$, $R_{out} = R_D$

$C_x = C_{in}$, $C_{out} = C_L$

$$\omega_{pin} = \frac{1}{R_S C_{in}} , \quad \omega_{pout} = \frac{1}{R_D C_L}$$

15)



DC Gain: $g_m R_D = \frac{2 I_D R_D}{V_{eff}}$

where $V_{eff} = V_{GS} - V_{th}$

Band Width: $\frac{1}{R_D C_L}$

Power Consumption: $V_{DD} I_D$

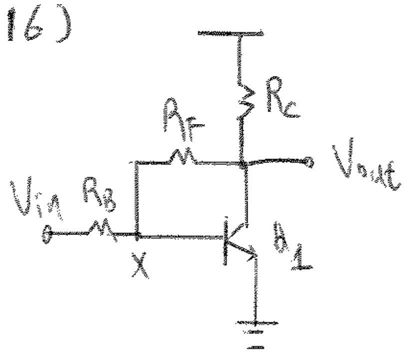
$$\text{F.O.M.} \quad (11.5) = \frac{\text{Gain} \times \text{Band Width}}{\text{Power Consumption}}$$

$$= \frac{\left(\frac{2 I_D R_D}{V_{eff}}\right)\left(\frac{1}{R_D C_L}\right)}{V_{DD} I_D}$$

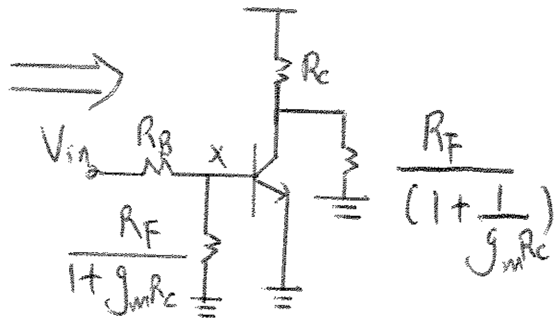
$$= \frac{2}{V_{eff} V_{DD} C_L}$$

For practical design, $V_{eff} > V_t$, thus bipolar has a larger F.O.M. than MOS.

16)



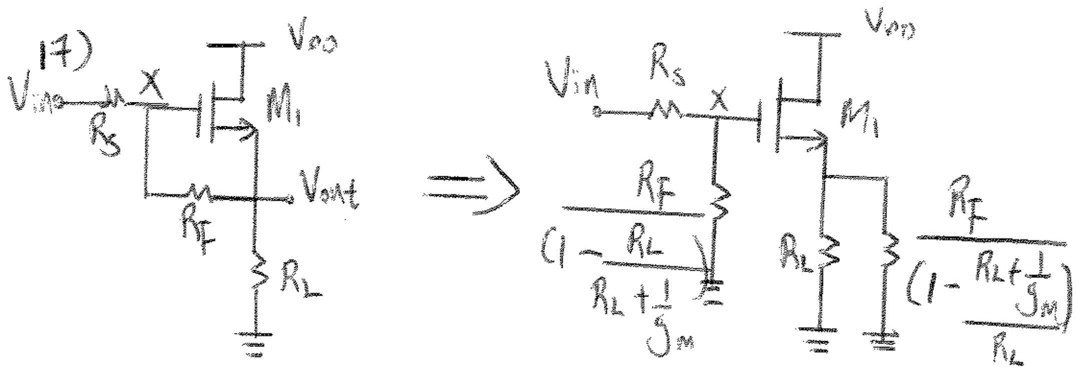
$V_A = \infty$, R_F is large



$$R_X = R_B \parallel \left(\frac{R_F}{1+g_m R_C} \right), \quad R_{out} = R_C \parallel \left(\frac{R_F}{1+\frac{1}{g_m R_C}} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_{out}}{\frac{1}{g_m} + \frac{R_X}{\beta+1}} = \frac{-R_C \parallel \left(\frac{R_F}{1+1/g_m R_C} \right)}{\frac{1}{g_m} + \frac{R_B \parallel (R_F/(1+g_m R_C))}{\beta+1}}$$

17)



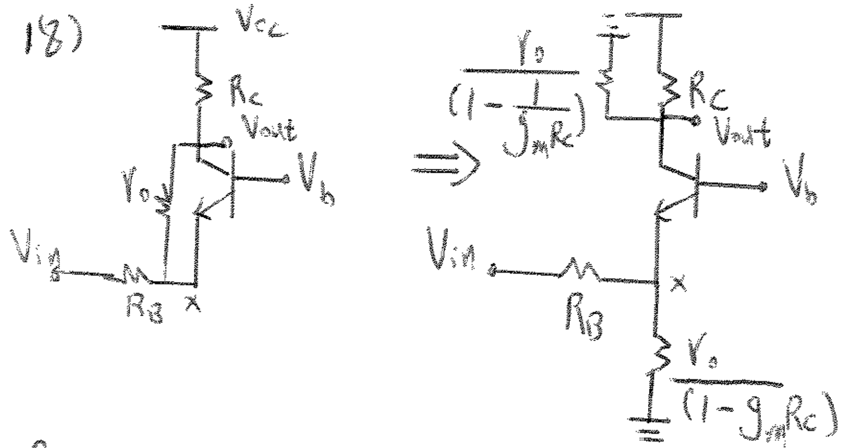
$$R_{out}: R_L \parallel \frac{R_F}{1-\frac{R_L+\frac{1}{g_m}}{R_L}} = R_L \parallel \frac{R_F}{-\frac{1}{g_m R_L}}$$

$$R_{out}: R_L \parallel -R_F g_m R_L$$

(note that R_{out} may be negative)

$$\frac{V_{out}}{V_{in}} = \frac{R_L \parallel -R_F g_m R_L}{R_L \parallel (-R_F R_L g_m) + \frac{1}{g_m}}$$

18)

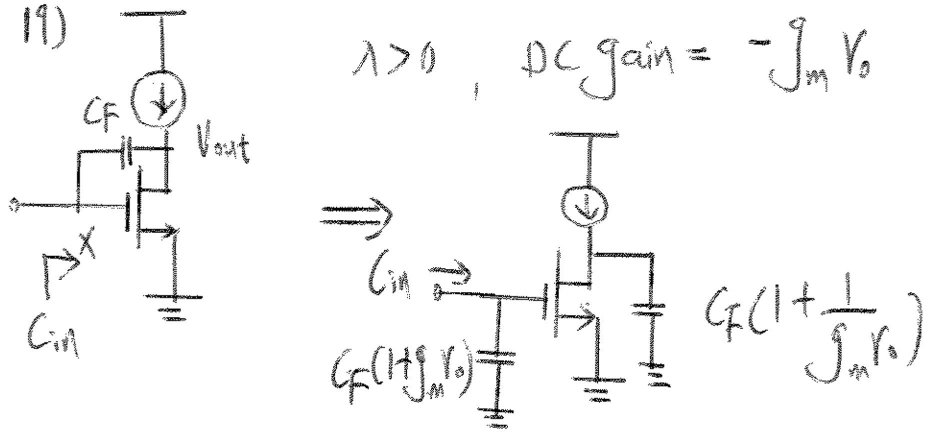


$$R_{out} = R_c \,//\left(\frac{r_o}{1-\frac{1}{g_m R_c}}\right)$$

$$R_x = R_B \,//\left(\frac{r_o}{1-g_m R_c}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{R_{out}}{R_x + \frac{1}{g_m}} = \frac{R_c \,//\left(\frac{r_o}{1-\frac{1}{g_m R_c}}\right)}{R_B \,//\left(\frac{r_o}{1-g_m R_c}\right) + \frac{1}{g_m}}$$

19)



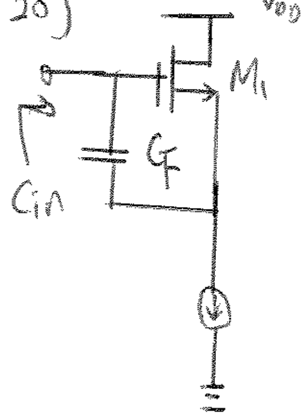
$\lambda > 0$, DC gain $= -g_m r_o$

$C_{in} = C_F(1+g_m r_o)$, neglecting other caps.

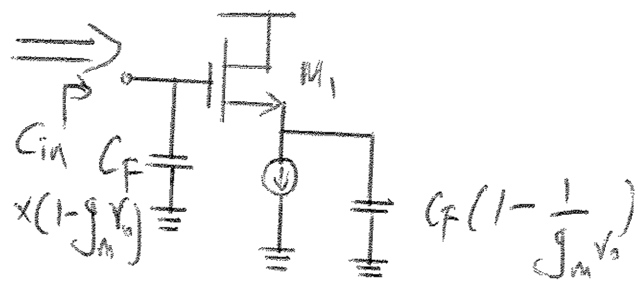
As $\lambda \to 0$, $r_o \to \infty$, DC gain $\to \infty$,

$C_{in} \to \infty$, this bandwidth will $\to 0$.

20)



$\lambda > 0$, DC gain $= g_m r_o$

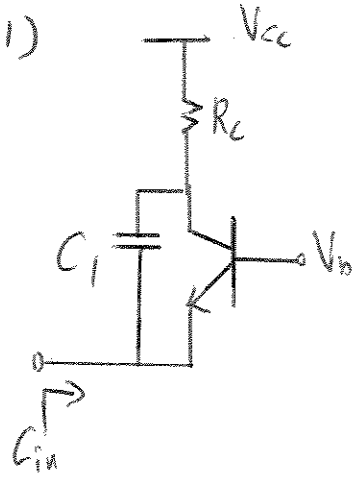


$$C_{in} = C_F(1 - g_m r_o)$$

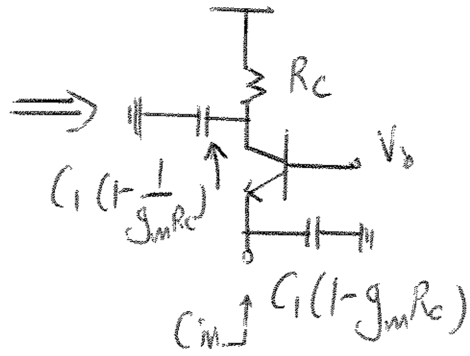
As $\lambda \to 0$, $r_o \to \infty$, $g_m r_o \to \infty$, $C_{in} = -\infty$

When $C \to$ negative in value, we have inductive activity. So right here, we have an effective infinite inductor.

21)



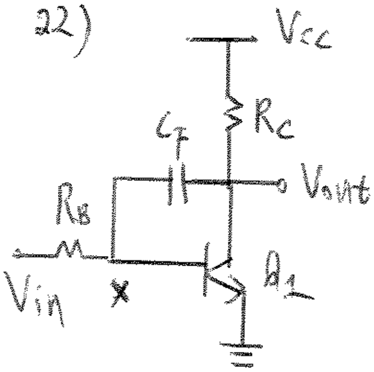
DC gain: $g_m R_c$



$$C_{in} = C_1 (1 - g_m R_c).$$

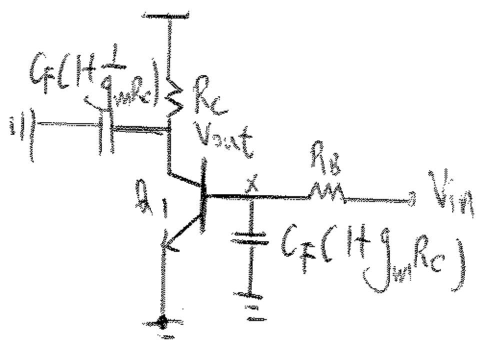
If $g_m R_c$ is designed to be larger than 1, as it normally would, we will have inductive action.

22)



DC gain (from x to out): $-g_m R_C$

⇓



$C_{in} = C_F(1+g_mR_C)$

$R_{in} = R_B // r_\pi$

$C_{out} = C_F(1+\frac{1}{g_mR_C})$

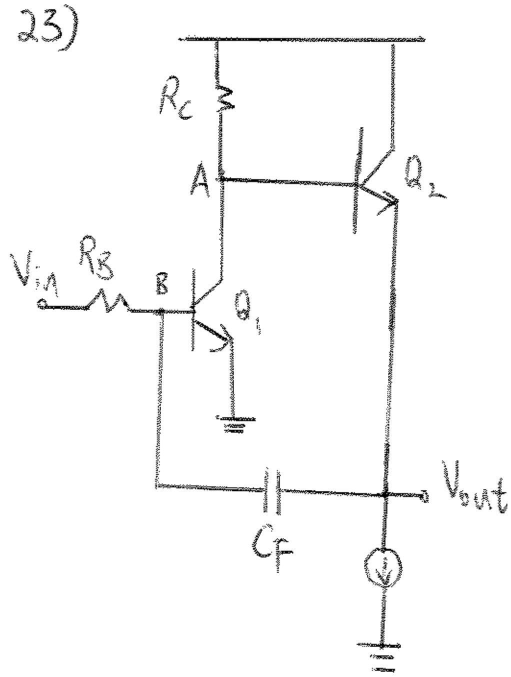
$R_{out} = R_C$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1+g_mR_C)\right]}$$

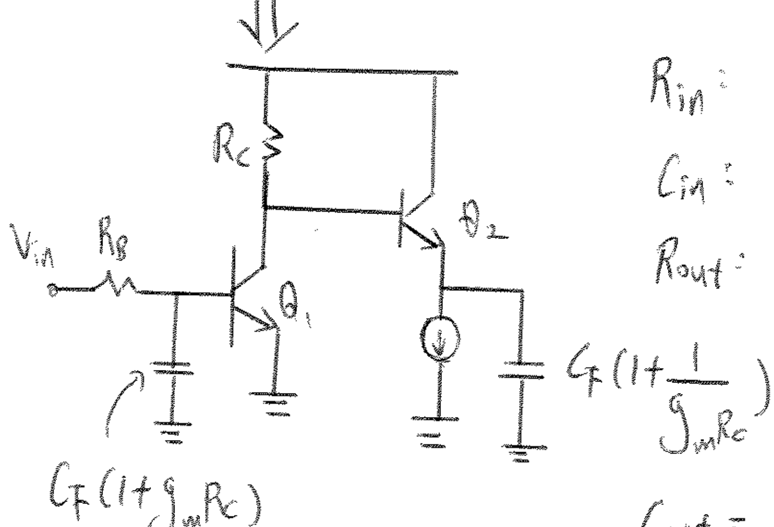
$$\omega_{pout} = \frac{1}{R_C C_F(1+\frac{1}{g_mR_C})} \approx \frac{1}{R_C C_F}$$

(If $g_mR_C \gg 1$)

23)



The gain from B to A is $-g_m R_C$, from A to out is 1 (since we have an ideal current source). So the gain from B to out is $-g_m R_C$.



R_{in}: $R_B // r_\pi$

C_{in}: $C_F(1+g_m R_C)$

R_{out}: $\frac{1}{g_m} + \frac{R_C}{\beta+1}$

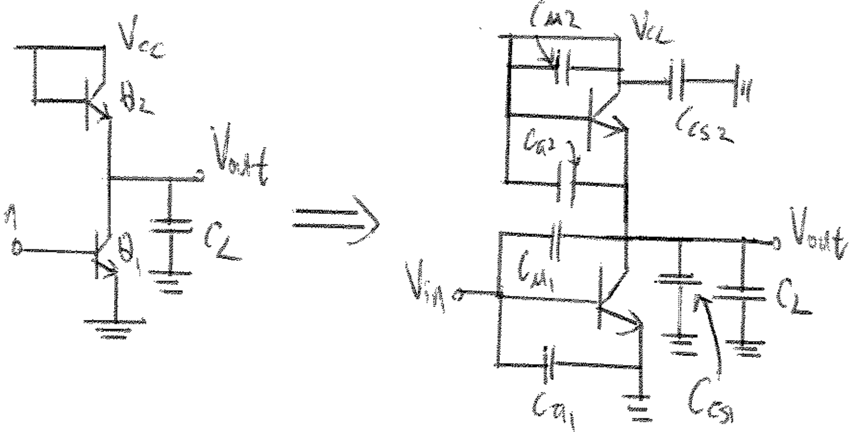
$C_{out} = C_F\left(1+\frac{1}{g_m R_C}\right)$

$$\omega_{pin} = \frac{1}{R_B // r_\pi [C_F(1+g_m R_C)]}$$

$$\omega_{pout} = \frac{1}{\left(\frac{1}{g_m}+\frac{R_C}{\beta+1}\right) C_F\left(1+\frac{1}{g_m R_C}\right)} \approx \frac{1}{\left(\frac{1}{g_m}+\frac{R_C}{\beta+1}\right) C_F}, \quad (g_m R_C \gg 1)$$

24)

a)

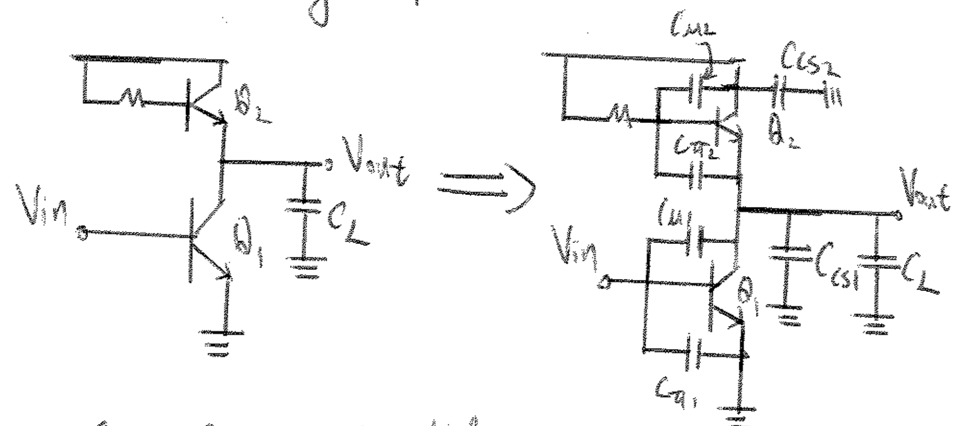


$C_{\pi2}$, C_{cs1}, C_L are in parallel

$C_{\mu2}$, C_{cs2} are grounded on both ends.

(and technically in parallel as well)

b)

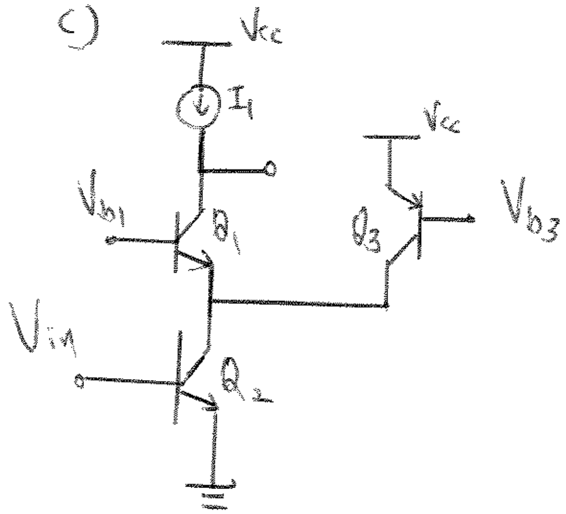


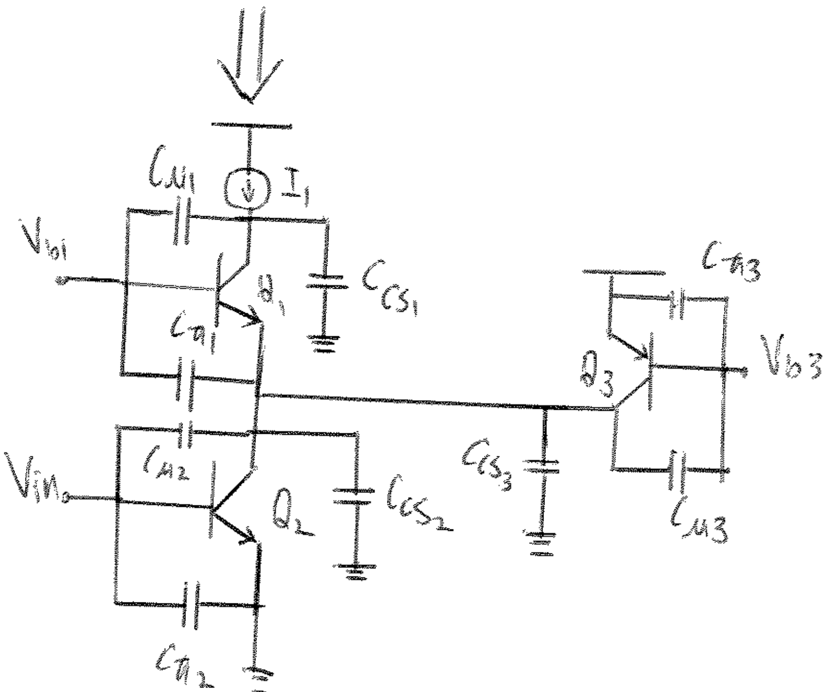
C_{cs1}, C_L are in parallel

C_{cs2} is grounded on both ends

24)

c)



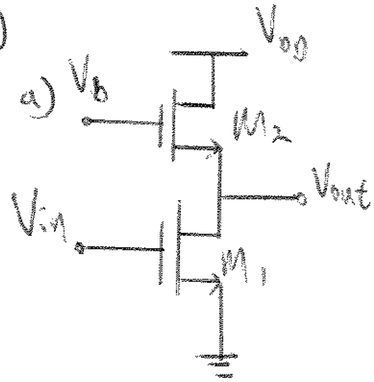


$C_{\pi 1}$, C_{CS_2}, C_{CS_3}, $C_{\mu 3}$ are in parallel
$C_{\mu 1}$, C_{CS_1} are also in parallel

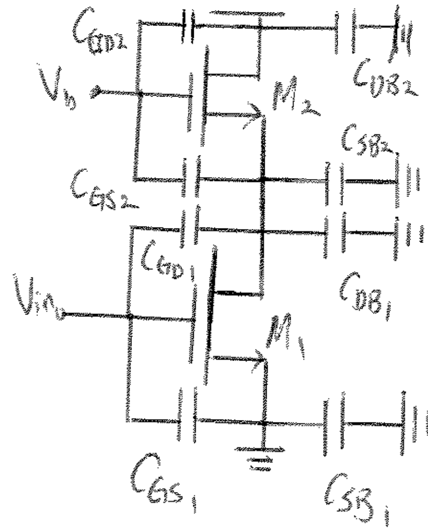
$C_{\pi 3}$ is grounded on both ends

25)

a)

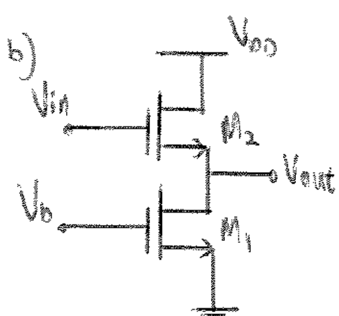


⟹

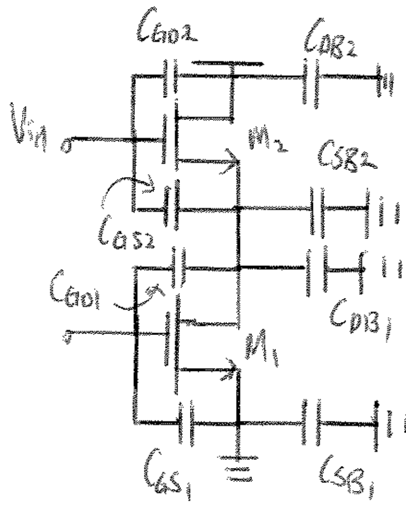


C_{GS2}, C_{SB2}, C_{DB1} are in parallel
C_{GD2}, C_{DB2} are in parallel and grounded on both ends
C_{SB1} is grounded on both ends.

b)



⟹



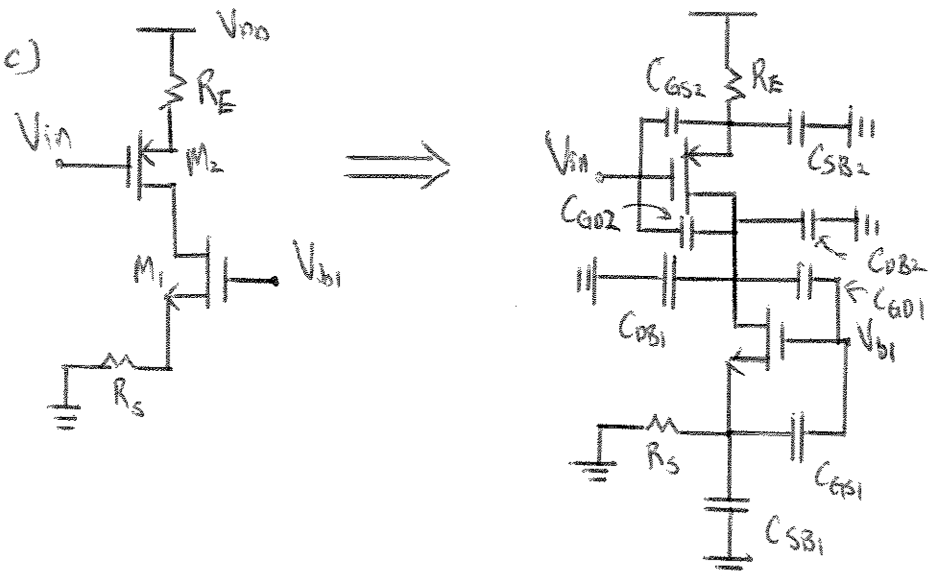
C_{GD1}, C_{DB1}, C_{SB2} are in parallel

C_{GS1}, C_{SB1} are in parallel and grounded on both ends

C_{DB2} is grounded on both ends.

25)

c)

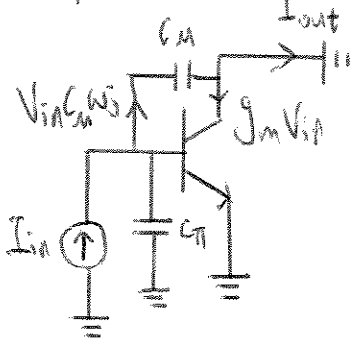


C_{DB2}, C_{GD1}, C_{DB1}, are in parallel

C_{SB1}, C_{GS1} are also in parallel.

2b)

Bipolar



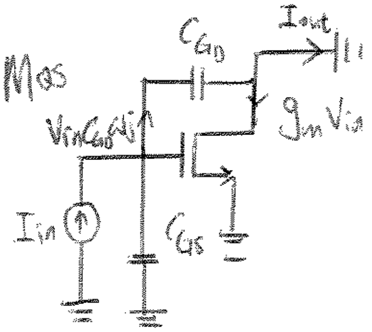
$$V_{in} = (I_{in})\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$ (Assuming we are at freq, and r_π can be neglected)

$$I_{out} = V_{in} C_\mu \omega j - g_m I_{in}\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{C_\mu \omega j - g_m}{[C_\mu + C_\pi]\omega j} \Rightarrow \left|\frac{I_{out}}{I_{in}}\right| = \frac{\sqrt{(g_m)^2 + (C_\mu \omega_T)^2}}{[C_\mu + C_\pi]\omega_T} = 1$$

$$\omega_T^2 = \frac{g_m^2}{2C_\mu C_\pi + C_\pi^2} \Rightarrow \omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_\mu C_\pi + C_\pi^2}}$$

Mos



Similarly for Mos, with C_μ and C_π replaced by C_{GD} and C_{GS} respectively.

$$\omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_{GD} C_{GS} + C_{GS}^2}}$$

27)

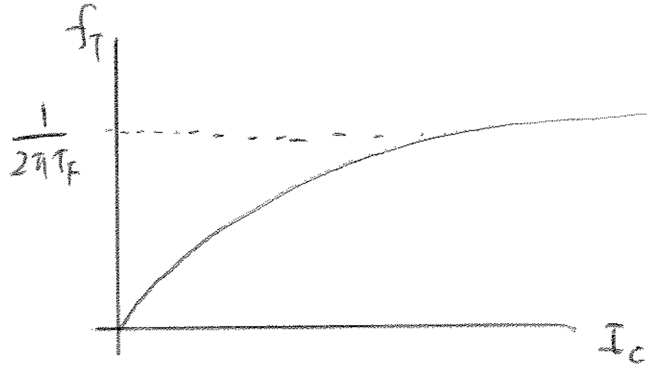
$$C_\pi = g_m \tau_F + C_{je}$$

$$2\pi f_T = \frac{g_m}{C_\pi} = \frac{g_m}{g_m \tau_F + C_{je}}$$

Assume C_{je} to be independent of I_C.

a) $$2\pi f_T = \frac{\frac{I_C}{V_T}}{\frac{I_C}{V_T}\tau_F + C_{je}} \Rightarrow f_T = \frac{I_C}{2\pi(I_C \tau_F + V_T C_{je})}$$

b)



As $I_C \to \infty$, $f_T \to \frac{1}{2\pi\tau_F}$

28)

$$C_{GS} \approx \left(\frac{2}{3}\right) W L C_{ox}$$

$$2\pi f_T = \frac{g_m}{C_{GS}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})}{\frac{2}{3} W L\ C_{ox}}$$

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(\ V_{GS} - V_{TH})$$

29)

$$2\pi f_T = \frac{3}{2}\frac{2I_D}{WLC_{ox}}\frac{1}{(V_{GS}-V_{TH})}$$

Apparently, f_T decreases with the overdrive. However, when we look closely, I_D is actually proportional to $(V_{GS}-V_{TH})^2$ (In saturation), so f_T is proportional to $(V_{GS}-V_{TH})$.

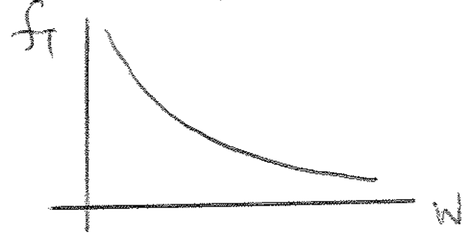
3c)

a) As W ↑, $(V_{GS} - V_{TH})$ has to ↓ by

$\frac{1}{\sqrt{W}}$ in order to maintain I_D constant

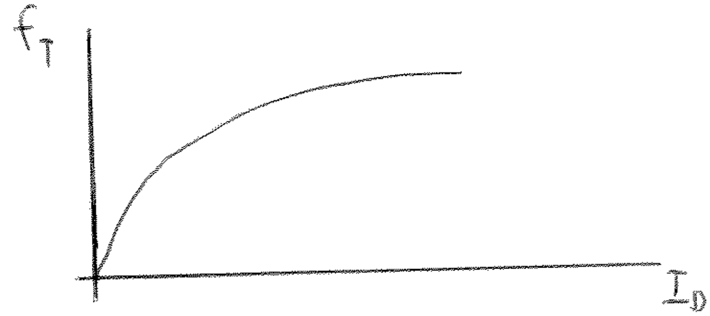
using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

$2\pi f_T \propto \frac{1}{\sqrt{W}}$



b) I_D ↑, W constant it means $V_{GS} - V_{TH}$ ↑ with $\sqrt{I_D}$. Using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

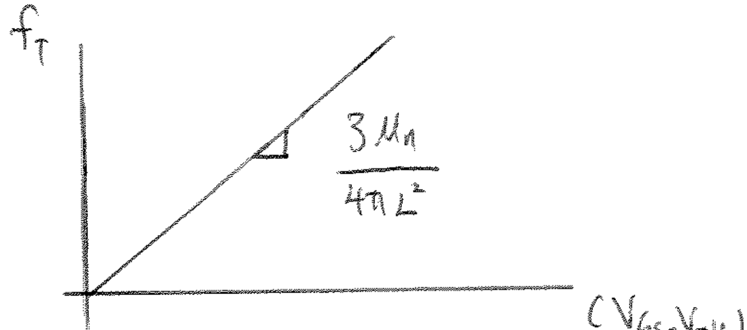
$2\pi f \propto \sqrt{I_D}$



31)

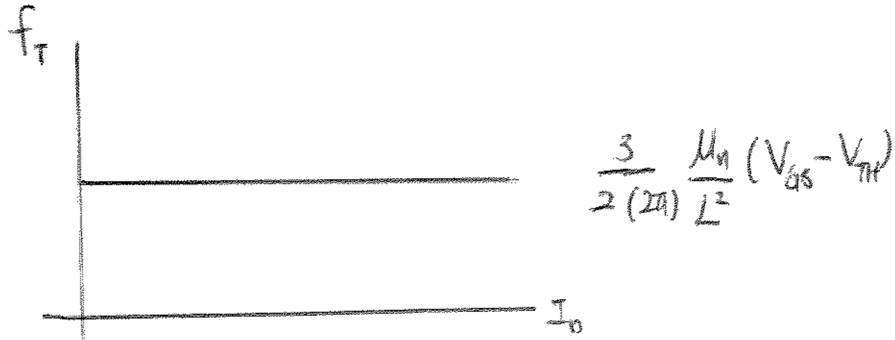
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

a) $2\pi f_T \propto (V_{GS}-V_{TH})$



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

$2\pi f$ = constant for all I_D.

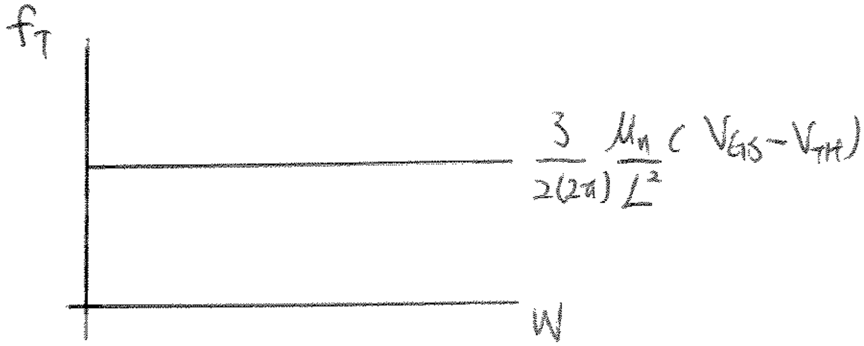


32)

a)

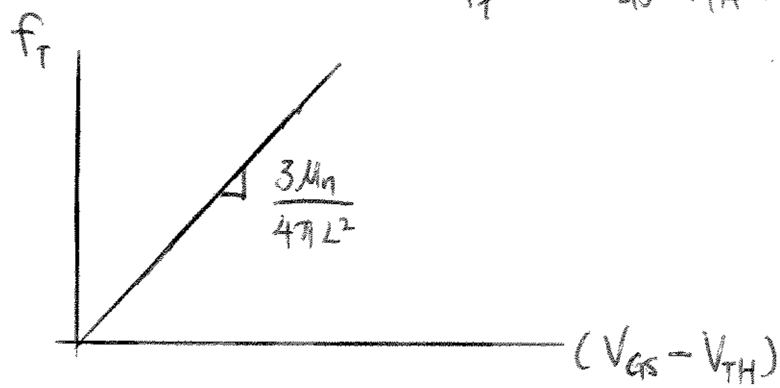
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$

we Know that $2\pi f_T$ is constant for all W.



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$.

we Know that $2\pi f_T \propto (V_{GS} - V_{TH})$.



33)

a) $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{TH})^2$

As $L\uparrow$, to maintain the same current and overdrive voltage, $W\uparrow$ as well.

So W also 2X.

b) Since $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$, and

L 2X while $(V_{GS} - V_{TH})$ is constant,

$f_T \downarrow$ by $\frac{3}{4}$ or $f_{T_{new}} = \frac{1}{4} f_{T_{old}}$.

34)

a) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

Constant I_D and $W\uparrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

b) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

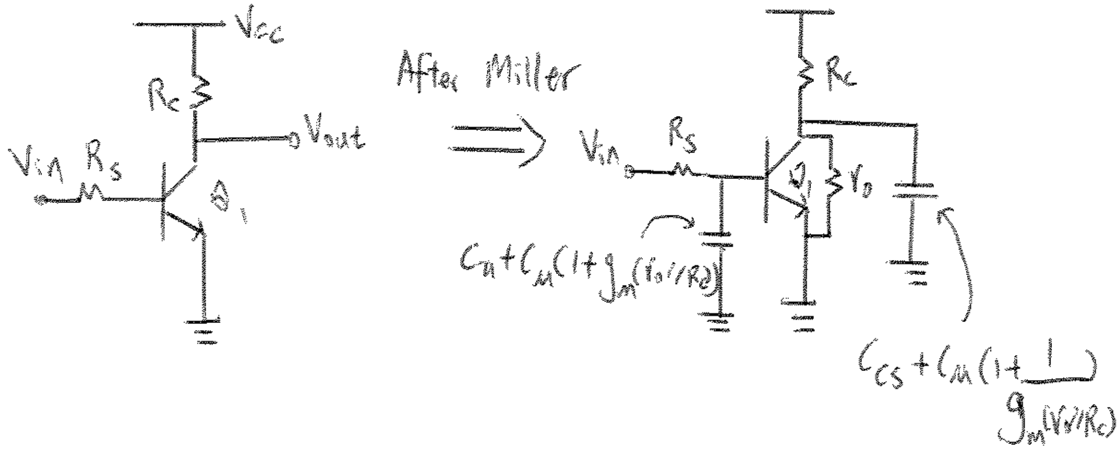
Constant W and $I_D\downarrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

35)

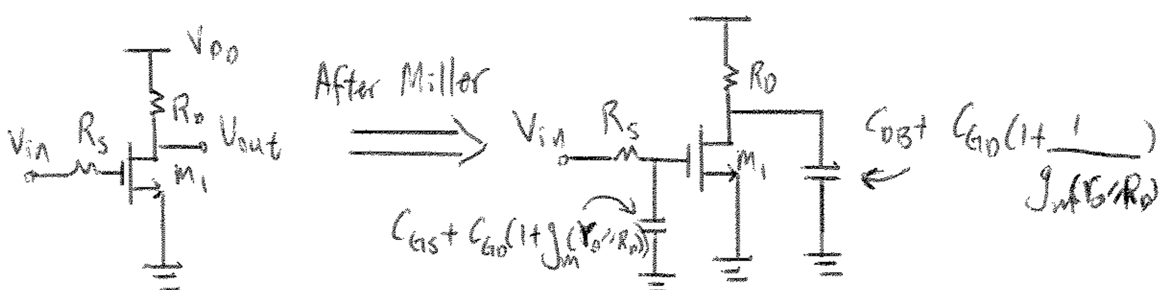
Bipolar CE Stage



$$\omega_{p_{in}} : \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m(r_o // R_C))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_C // r_o)[C_{CS} + C_\mu(1+1/g_m(r_o // R_C))]}$$

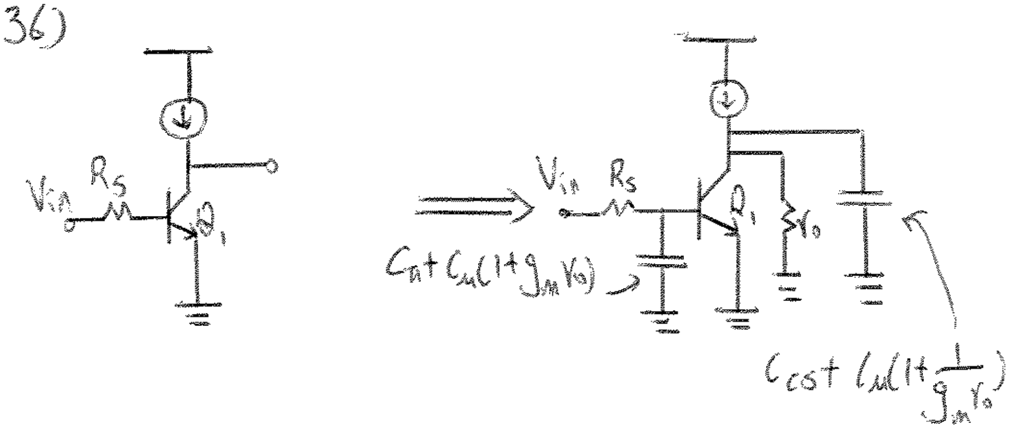
MOS CS Stage



$$\omega_{p_{in}} : \frac{1}{R_S[C_{GS} + C_{GD}(1+g_m(r_o // R_D))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_D // r_o)[C_{DB} + C_{GD}(1+1/g_m(r_o // R_D))]}$$

36)



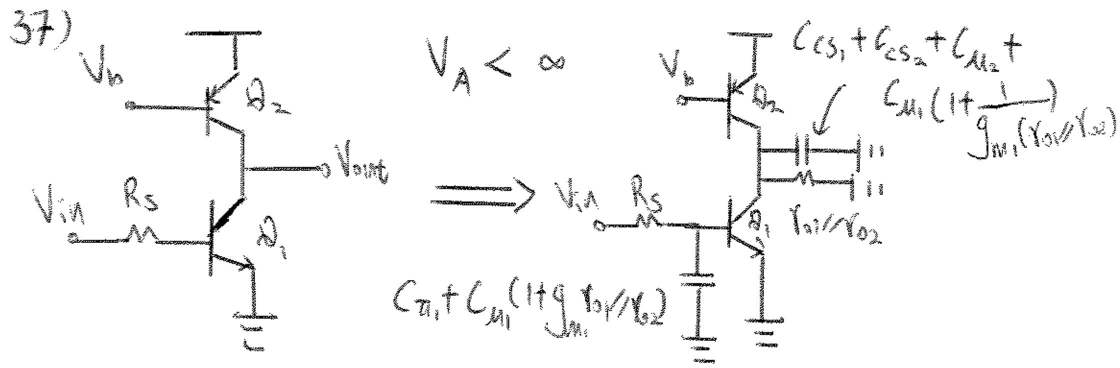
$$\omega_{p,in} = \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}$$

$$\omega_{p,out} = \frac{1}{r_o[C_{cs} + C_\mu(1+1/g_m r_o)]}$$

$$H(s) = \frac{\text{DC gain}}{\left(1+\frac{s}{\omega_{p,in}}\right)\left(1+\frac{s}{\omega_{p,out}}\right)}$$

$$H(s) = \frac{g_m r_o (r_\pi/(r_\pi+R_S))}{\left(1+\frac{s}{1/(R_S // r_\pi)[C_\pi+C_\mu(1+g_m r_o)]}\right)\left(1+\frac{s}{1/(r_o[C_{cs}+C_\mu(1+\frac{1}{g_m r_o})])}\right)}$$

37)



$$\omega_{pin} = \frac{1}{(R_S // r_{\pi_1})\left[C_{\pi_1} + C_{\mu}(1 + g_{m_1}(r_{o1} // r_{o2}))\right]}$$

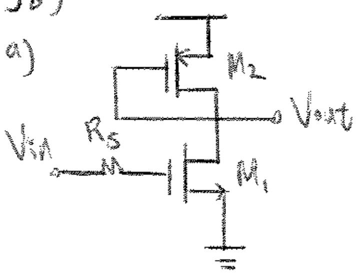
$$\omega_{pout} = \frac{1}{(r_{o1} // r_{o2})\left[C_{cs_1} + C_{cs_2} + C_{\mu_2} + C_{\mu_1}(1 + 1/(g_{m_1} r_{o1} // r_{o2}))\right]}$$

$$g_m r_o >> 1$$

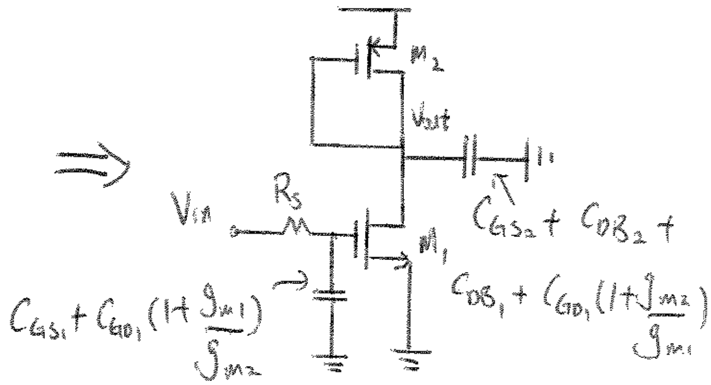
$$H(s) = \frac{g_m(r_{o1} // r_{o2})(r_\pi / r_\pi + R_S)}{(1 + R_S[C_{\pi_1} + C_{\mu}(g_m(r_{o1} // r_{o2}))]s)(1 + (r_{o1} // r_{o2})[C_{cs1} + C_{cs2} + C_{\mu_2} + C_{\mu_1}]s)}$$

38)

a)



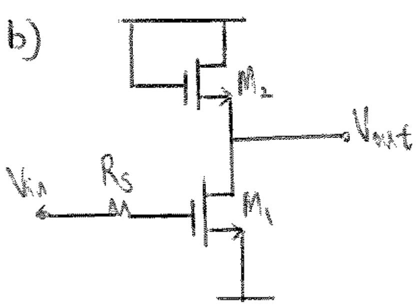
$\Longrightarrow$



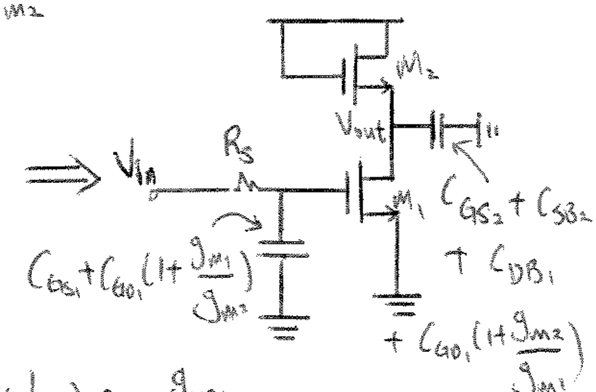
$$\text{DC gain} = -g_{m1}\left(r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{p\,in} = \frac{1}{R_S\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)} \qquad \omega_{p\,out} = \frac{g_{m2}}{\left(C_{GS_2} + C_{DB_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)\right)}$$

b)



$\Longrightarrow$



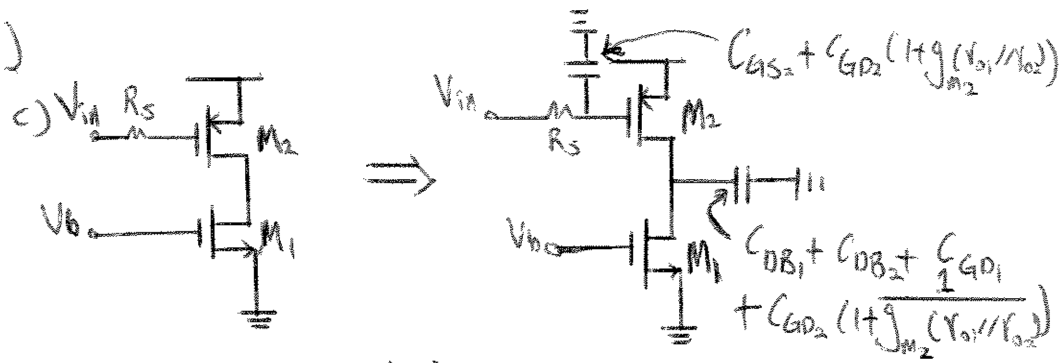
$$\text{DC gain} = -g_{m1}\left(r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{p\,in} = \frac{1}{R_S\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)}$$

$$\omega_{p\,out} = \frac{g_{m2}}{C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)}$$

38)

c)



DC gain: $-g_{m2}(r_{o1}//r_{o2})$

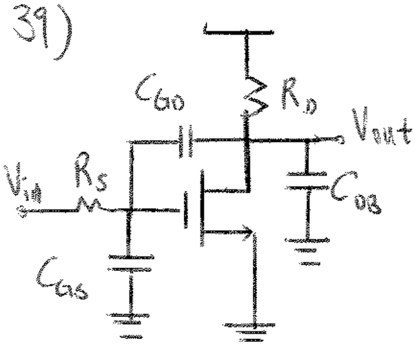
$$\omega_{pin} = \frac{1}{R_s\left(C_{GS_2} + C_{GD_2}(1+g_{m_2}(r_{o1}//r_{o2}))\right)}$$

$$\omega_{pout} = \frac{1}{(r_{o1}//r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\left(1+\frac{1}{g_{m_2}(r_{o1}//r_{o2})}\right)\right]}$$

$$\omega_{pout} \approx \frac{1}{(r_{o1}//r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\right]}$$

Since $g_{m2}(r_{o1}//r_{o2}) >> 1$

39)



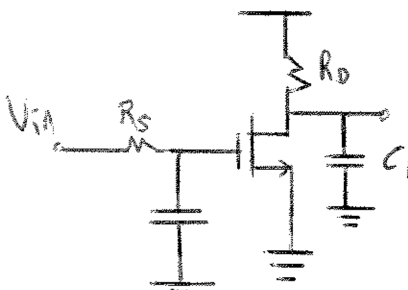
$R_S = 200\Omega \quad C_{GS} = 50fF$

$R_D = 1K\Omega \quad C_{GD} = 10fF$

$I_D = 1mA \quad C_{DB} = 15fF$

a) Miller's approximation:

$$-g_m R_D = -\frac{(1)(2)(1)}{(0.2)} = -10$$



$C_{DB} + C_{GD}\left(1 + \frac{1}{g_m R_D}\right)$

$C_{GS} + (1 + g_m R_D) C_{GD}$

$$\omega_{p\,in} = \frac{1}{R_S(C_{GS} + (1 + g_m R_D) C_{GD})} = \frac{1}{200(50fF + (11)(10fF))} = 31.25GHz$$

$$\omega_{p\,out} = \frac{1}{R_D\left(C_{DB} + (1 + \frac{1}{10}) C_{GD}\right)} = \frac{1}{1000(15fF + (1 + 0.1)10fF)} = 38.46GHz$$

3q)

b) Equation $\frac{V_{out}}{V_{thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$

$a = R_{thev} R_L (C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$

$R_{Thev} = R_S$, $C_{in} = C_{GS}$, $C_{out} = C_{DB}$, $R_L = R_O$, $C_{XY} = C_{GD}$

$a = (200)(1000)[(50)(10^{-15})(10)(10^{-15}) + (15)(10^{-15})(10)(10^{-15}) + (50)(10^{-15})(15)(10^{-15})] = 2.8 \times 10^{-22}$

$b = (1 + g_m R_L) C_{XY} R_{thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})$

$b = (1+10)(10)(10^{-15})(200) + (200)(50 \times 10^{-15}) + (1000)(10)(10^{-15}) + 1000(15)(10^{-15})$

$b = 5.7 \times 10^{-11}$

So denominator $= (2.8 \times 10^{-22} s^2 + 5.7 \times 10^{-11} s + 1)$

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$S = -1.93909 \times 10^{10}$, -1.84181×10^{11}

$\omega_{p_1} = 19.39$ GHz , $\omega_{p_2} = 184.2$ GHz

Which one is ω_{pin}, and which one is ω_{pout}?

This can be seen from inspection, at output and high frequency C_{GD} starts to become a short and thus the output resistance collapses to $1/g_m$, and pushes the output pole out. Whereas at the input the pole location does not change too much because R_S is small and C_{GS} is large. Therefore, we conclude that when solving the transfer function directly, the ω_{pin} is 19.39 GHz (on the same order as

39)

b). that obtained from Miller's approximation), while ω_{pout} is pushed out significantly, 184.2 GHz (when compared to that obtained from Miller's approximation).

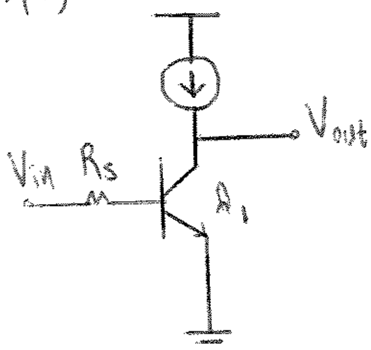
<u>Miller Approximation</u>

ω_{pin} = 31.25 GHz
ω_{pout} = 38.46 GHz

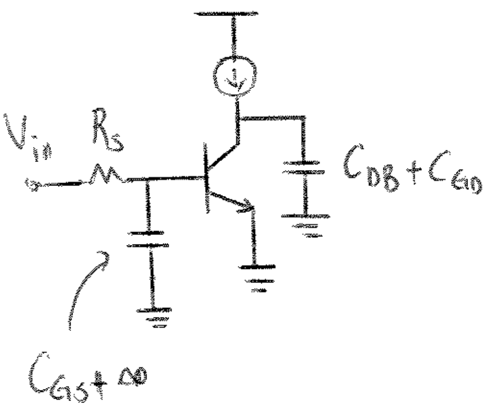
<u>Transfer Function</u>

ω_{pin} = 19.39 GHz
ω_{pout} = 184.2 GHz

40)



a) Miller's Approximation: DC gain: $-\infty$



$$\omega_{pin} = \frac{1}{R_S(\infty)} = 0, \quad \omega_{pout} = \frac{1}{\infty(C_{DB}+C_{GD})} = 0$$

b) Transfer Function:

$$\frac{V_{out}}{V_{thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1+g_mR_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out})$$

40)

b) $R_L \to \infty$

$$\frac{V_{out}}{V_{thev}} = \frac{C_{XY} s - g_m}{s\left[R_{thev}(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})s + g_m C_{XY} R_{thev} + (C_{XY} + C_{out})\right]}$$

So $\omega_{p_1} = 0$

$$\omega_{p_2} = \frac{(g_m C_{XY} R_{thev} + (C_{XY} + C_{out}))}{R_{thev}[C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}]}$$

$$\omega_{p_2} = \frac{g_m C_\mu R_S /\!/ R_\pi + C_\mu + C_{cs}}{R_S /\!/ r_\pi [C_\pi C_\mu + C_{cs} C_\mu + C_\pi C_{cs}]}$$

$\omega_{p_1} = \omega_{p_{in}}$, $\omega_{p_2} = \omega_{p_{out}}$.

<u>Miller:</u>

$\omega_{p_{in}} = 0$, $\omega_{p_{out}} = 0$

Again, the output pole predicted by the transfer function is pushed out, and the input poles are similar. (In fact, they are the same this time.)
This shows one of the short-comings of Miller's approximation.

4i) <u>Dominant-Pole approximation:</u>

$$\omega_{p1} = \frac{1}{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L(C_{XY} + C_{out})}$$

$$\omega_{p1} = 0 \qquad (\text{Since } R_L = \infty)$$

$$\omega_{p2} = \frac{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L(C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

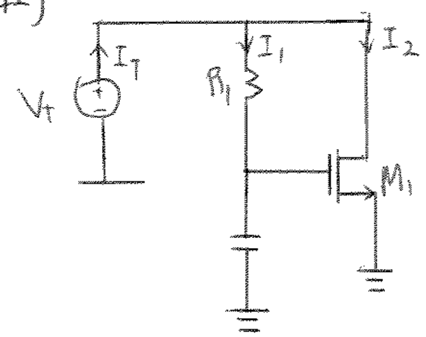
Since $R_L = \infty$

$$\omega_{p2} = \frac{g_m C_{XY} R_{Thev} + C_{XY} + C_{out}}{R_{Thev} (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

$$\omega_{p2} = \frac{g_m C_\mu R_S // R_\pi + C_\mu + C_{CS}}{R_s (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}$$

Dominant-Pole approximation gives the same result as the transfer function method.

42)



$\lambda = 0$, and neglect other capacitances.

$I_T = I_1 + I_2$

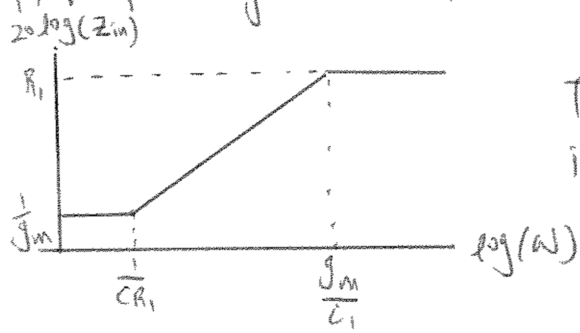
$$I_1 = \frac{V_T}{\left(R_1 + \frac{1}{C_1 S}\right)}, \quad I_2 = \frac{g_{m_1} V_T}{C_1 R_1 S + 1}$$

$$I_T = \frac{C_1 S V_T}{C_1 R_1 S + 1} + \frac{g_{m_1} V_T}{C_1 R_1 S + 1} \Rightarrow \frac{V_T}{I_T} = \frac{C_1 R_1 S + 1}{C_1 S + g_{m_1}}$$

$$S \to j\omega \Rightarrow \frac{C_1 R_1 (j\omega) + 1}{C_1 j\omega + g_{m_1}} = Z_T (j\omega)$$

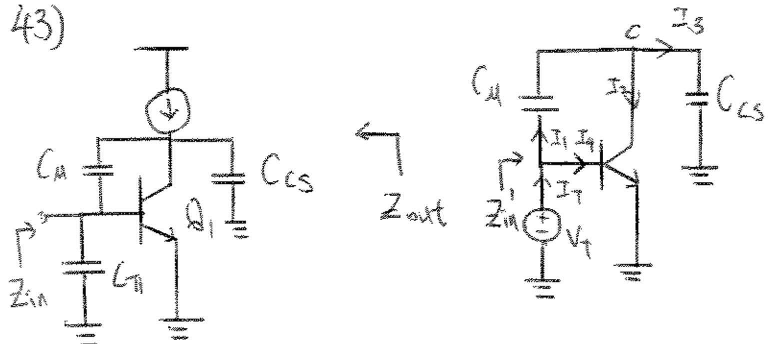
$$|Z_T| = |Z_{in}| = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{\sqrt{(C_1 \omega)^2 + g_{m_1}^2}} = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{g_m \sqrt{\left(\frac{C_1 \omega}{g_{m_1}}\right)^2 + 1}}$$

At $\omega = \frac{1}{C_1 R_1}$, we have a zero, at $\omega = \frac{g_{m_1}}{C_1}$, we have a pole. If $R_1 > \frac{1}{g_{m_1}}$, the zero is at a lower frequency than the pole, and the bode-plot for magnitude would look like the following.



The bode-plot shows an impedance that increases with frequency, an inductive behavior.

43)



$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s}, \quad I_T = I_1 + I_4 = C_\mu s V_{bc} + \frac{g_m V_T}{\beta}$$

$$V_{bc} = V_T - V_c, \quad V_c = (I_1 - g_m V_T)\frac{1}{C_{cs} s}$$

$$I_1 = \left[V_T - (I_1 - g_m V_T)\frac{1}{C_{cs} s}\right] C_\mu s$$

$$I_1 = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] / \left(1 + \frac{C_\mu}{C_{cs}}\right)$$

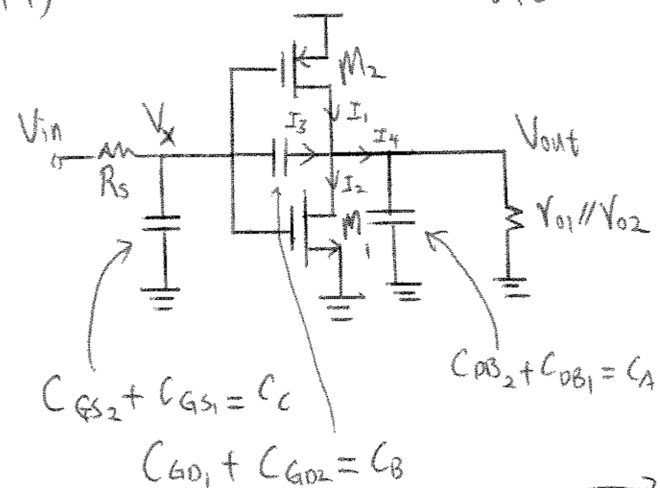
$$I_T = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] / \left(1 + \frac{C_\mu}{C_{cs}}\right) + \frac{g_m V_T}{\beta}$$

$$Z_{in}' = \frac{V_T}{I_T} = \frac{1}{\dfrac{g_m}{\beta} + \dfrac{C_\mu s}{\left(1 + \frac{C_\mu}{C_{cs}}\right)} + \dfrac{g_m \frac{C_\mu}{C_{cs}}}{\left(1 + \frac{C_\mu}{C_{cs}}\right)}}$$

$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} = r_\pi \,//\, \frac{1}{\dfrac{C_{cs} C_\mu s}{C_{cs} + C_\mu}} \,//\, \frac{1}{C_\pi s} \,//\, \frac{C_{cs} + C_\mu}{g_m C_\mu}$$

$$Z_{out} = \frac{1}{(C_\mu + C_{cs})s}$$

44) $\lambda > 0$



$$V_{out} = I_4 \left(r_{o1} // r_{o2} // \frac{1}{[C_{DB2} + C_{DB1}]s} \right) \rightarrow Z_{out}$$

$$I_4 = I_1 + I_3 - I_2$$

$$I_1 = (0 - V_x) g_{m2}$$

$$I_2 = V_x g_{m1}$$

$$I_3 = (V_x - V_{out})\left((C_{GD1} + C_{GD2}) s \right)$$

$$I_4 = -V_x g_{m2} + (V_x - V_{out}) C_B s - V_x g_{m1}$$

$$V_{out} = Z_{out} \left[-V_x (g_{m2} + g_{m1}) + (V_x - V_{out}) C_B s \right]$$

Writing a node equation at X.

$$\frac{V_x - V_{in}}{R_s} + V_x C_C s + (V_x - V_{out}) C_B s = 0$$

$$V_x = \frac{V_{out} C_B s + V_{in}/R_s}{(1/R_s + C_C s + C_B s)}$$

44)

Subsitute everything and we get

$$V_{out} = Z_{out}\left[-(g_{m1}+g_{m2})\left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s}\right) + \left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s} - V_{out}\right)C_B s\right]$$

collect all the Vout's on one-side and likewise for Vin's, we will get

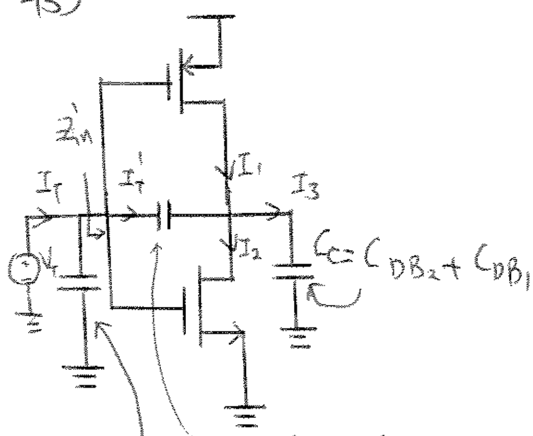
$$\frac{V_{out}}{V_{in}} = \frac{\frac{Z_{out}}{R_s}\left(C_B s - (g_{m1}+g_{m2})\right)}{1/R_s + (C_c+C_B)s + Z_{out}C_B s(g_{m1}+g_{m2}) + Z_{out}C_B s\left(\frac{1}{R_s}+(C_c+C_B)s\right) - Z_{out}C_B^2 s^2}$$

where $Z_{out} = r_{o1}//r_{o2}//\frac{1}{[C_{DB1}+C_{DB2}]s}$

$C_B = C_{GD1} + C_{GD2}$

$C_c = C_{GS1} + C_{GS2}$

45)



$C_A = C_{GD1} + C_{GD2}$

$C_B = C_{GS1} + C_{GS2}$

$$Z_{in} = \frac{V_T}{I_T} = \frac{1}{C_B} \,//\, Z_{in}'$$

$$Z_{in}' = \frac{V_T}{I_T'}$$

$$I_T' = \left[V_T - \left(I_3 \frac{1}{C_C s}\right)\right] C_A s$$

$$I_3 = I_T' - V_T g_{m2} - g_{m1} V_T$$

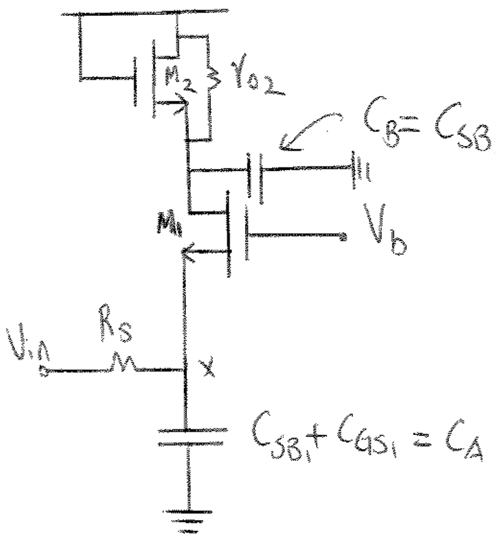
We get $\Rightarrow$ $I_T'\left(1 + \frac{C_A}{C_C}\right) = V_T\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]$

$$Z_{in}' = \frac{V_T}{I_T'} = \frac{\left(1 + \frac{C_A}{C_C}\right)}{\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]}$$

$$Z_{in} = \frac{1}{[C_{GS1} + C_{GS2}]s} \,//\, \frac{\left(1 + \frac{C_{GD1} + C_{GD2}}{C_{DB1} + C_{DB2}}\right)}{\left[(C_{GD1} + C_{GD2})s + (g_{m1} + g_{m2})\frac{C_{GD1} + C_{GD2}}{C_{DB2} + C_{DB1}}\right]}$$

46)

a)



$$V_{out} = -(0 - V_x)g_{m_1}\left[\frac{1}{g_{m_2}} // \frac{1}{C_B s}\right] = V_x g_{m_1}\left[\frac{1}{g_{m_2}} // \frac{1}{C_B s}\right]$$

Node equation at X, $\dfrac{V_x - V_{in}}{R_s} + V_x C_A S - g_m(0 - V_x) = 0$

$$V_x\left(\frac{1}{R_s} + C_A S + g_m\right) = \frac{V_{in}}{R_s} \Rightarrow V_x = \frac{V_{in}}{(1 + R_s C_A S + R_s g_m)}$$

subsitute in V_x and solving for V_{out}/V_{in} $\Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}\left[1/g_{m_2} // \frac{1}{C_B S}\right]}{(1 + R_s C_A S + R_s g_m)}$$

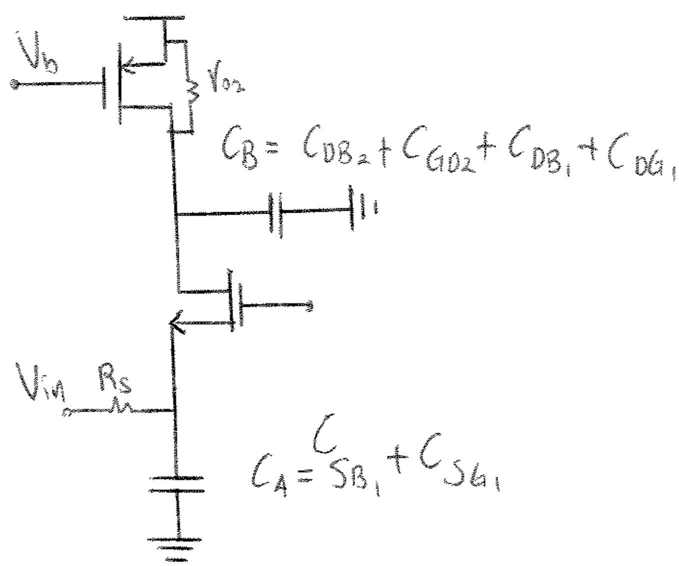
$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{(C_B(1/g_{m_2})S + 1)(1 + R_s C_A S + R_s g_{m_1})}$$

Where $C_B = C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{GS_1}$

4b)

b)



Similar to part a), with $\frac{1}{g_{m_2}}$ replaced by r_{o2},

and different C_B

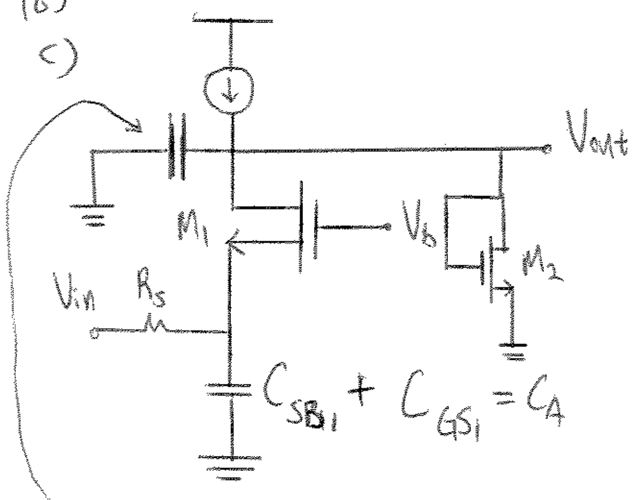
So $$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} r_{o2}}{(C_B r_o S + 1)(1 + R_S C_A S + R_S g_{m_1})}$$

Where $C_B = C_{DB_2} + C_{GD_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{SG_1}$

4b)

c)



$C_B = C_{DB1} + C_{GD1} + C_{DB2} + C_{GS2}$

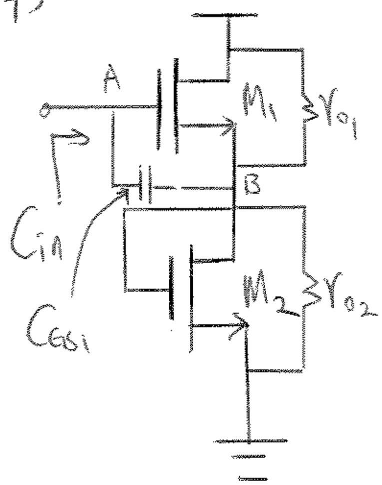
AC-wise, this circuit is very similar to part a). Its transfer function is the same as part a), except for C_B.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}(1/g_{m2})}{(C_B(1/g_{m2})s+1)(1+R_S C_A s + R_S g_{m1})}$$

where $C_B = C_{DB1} + C_{GD1} + C_{DB2} + C_{GS2}$

$C_A = C_{SB1} + C_{GS1}$

47)



DC gain from A to B:

$$A_V = \frac{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2}}{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2} + \frac{1}{g_{m1}}}$$

$$A_V \approx \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

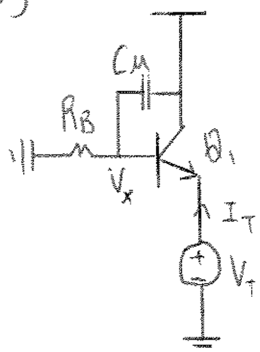
since $g_m r_o >> 1$

Using Miller's Capacitance:

$$C_{in} = C_{GS1}(1 - A_V) = C_{GS1}\left(1 - \frac{g_{m1}}{g_{m1} + g_{m2}}\right)$$

$$C_{in} = C_{GS1}\left(\frac{g_{m2}}{g_{m2} + g_{m1}}\right)$$

48)



$V_A = \infty$,

$\frac{\beta}{\beta+1} \approx 1$, if $\beta \gg 1$

$$I_T = -(V_x - V_T)\, g_m \alpha \approx -(V_x - V_T)\, g_m$$

$$V_x = \frac{I_T}{\beta}\left(R_B \,//\, \frac{1}{C_\mu s}\right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta}\left(R_B \,//\, \frac{1}{C_\mu s}\right)\right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta}\left(R_B \,//\, \frac{1}{C_\mu s}\right) I_T$$

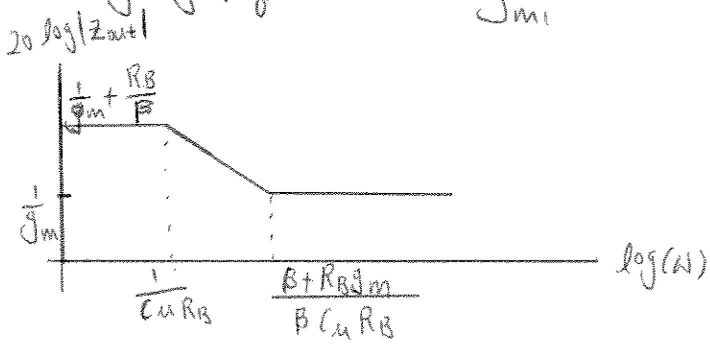
$$I_T\left(1 + \frac{g_m}{\beta}\left(R_B \,//\, \frac{1}{C_\mu s}\right)\right) = g_m V_T$$

$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B \,//\, \frac{1}{C_\mu s}}{\beta} = \frac{\beta C_\mu R_B \left(s + \frac{\beta + R_B g_m}{\beta C_\mu R_B}\right)}{g_m \beta (1 + C_\mu R_B s)}$$

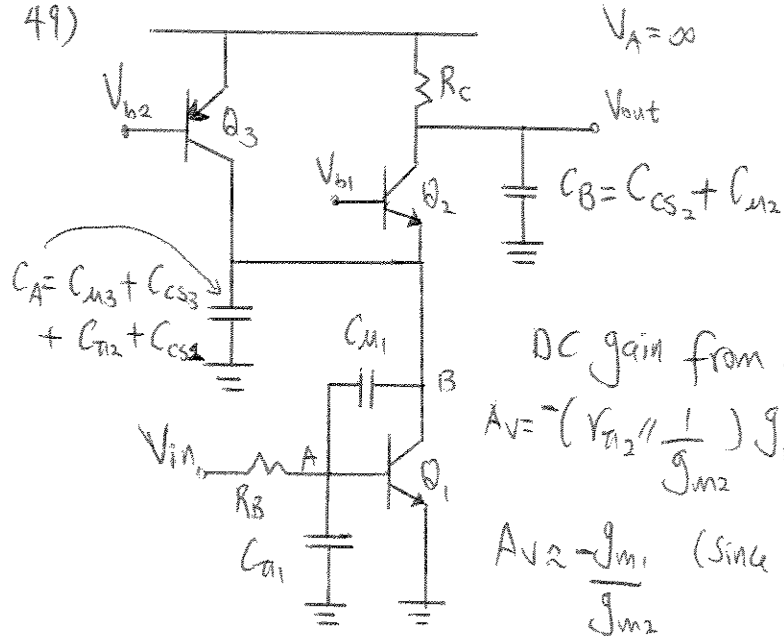
Zero: $\frac{\beta + R_B g_m}{\beta C_\mu R_B}$, Pole: $\frac{1}{C_\mu R_B}$

At DC, $|Z_{out}| = \frac{1}{g_{m1}} + \frac{R_B}{\beta}$

At very high freq: $|Z_{out}| = \frac{1}{g_{m1}}$



49)



DC gain from A to B:

$A_V = -\left(r_{\pi 2} \,//\, \frac{1}{g_{m2}}\right) g_{m1}$

$A_V \simeq -\frac{g_{m1}}{g_{m2}}$ (Since $\beta \gg 1$)

We have $I_{C2} = 0.25 I_{C1}$, $g_m = \frac{I_C}{V_T} \Rightarrow g_{m2} = 0.25 g_{m1}$

$A_V = -\frac{g_{m1}}{g_{m2}} = -4$. (If $I_{C2} = I_{C1}$, $A_V = -1$)

Applying Miller's Theroem: $C_{in} = C_{\pi 1} + C_{\mu 1}(1+4) = C_{\pi 1} + 5C_{\mu 1}$

$C_B = C_A + C_\mu\left(1+\frac{1}{4}\right) = C_A + C_\mu\left(\frac{5}{4}\right)$

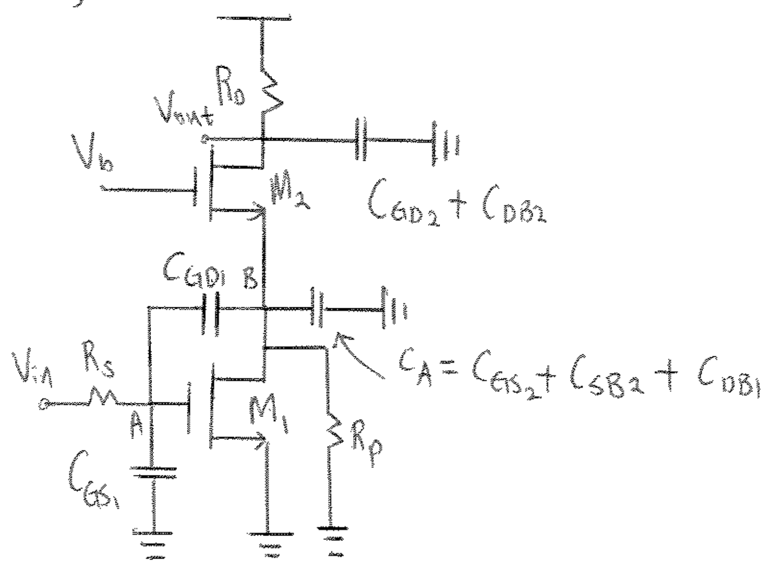
$\omega_{p\,in}\ (\omega_{pA}) = \frac{1}{(R_B // r_\pi)[C_{\pi 1} + 5C_\mu]}$, $\omega_{pB} = \frac{0.25\, g_{m1}}{[C_A + \frac{5}{4} C_{\mu 1}]}$

$\omega_{pB} = \frac{g_{m2}}{[C_A + \frac{5}{4} C_{\mu 1}]}$, $\omega_{p\,out} = \frac{1}{R_C [C_{CS2} + C_{\mu 2}]}$

Where $C_A = C_{\mu 3} + C_{CS3} + C_{\pi 2} + C_{CS1}$.

Since the DC gain is increased, Miller effect is more significant.
(In magnitude)

50)



DC gain from A to B is $-g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)$

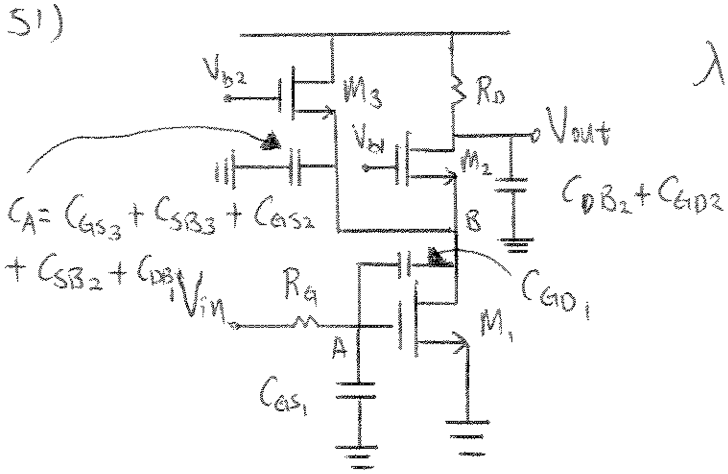
Applying Miller's Theorem:

$$\omega_{pin}\,(\omega_{pA}) = \frac{1}{R_S\left(C_{GS1} + C_{GD1}\left(1 + g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)\right)\right)}$$

$$\omega_{pB} = \frac{1}{R_P \parallel \frac{1}{g_{m2}}\left[C_{GS2} + C_{SB2} + C_{DB1} + C_{GD1}\left(1 + 1/g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)\right)\right]}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{GD2} + C_{DB2}\right)}$$

5')



$\lambda = 0$

DC gain from A to B: $-g_{m_1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{m2}}\right) = -g_{m_1}\left(\frac{1}{g_{m2}+g_{m3}}\right)$

Applying Miller's Theorem:

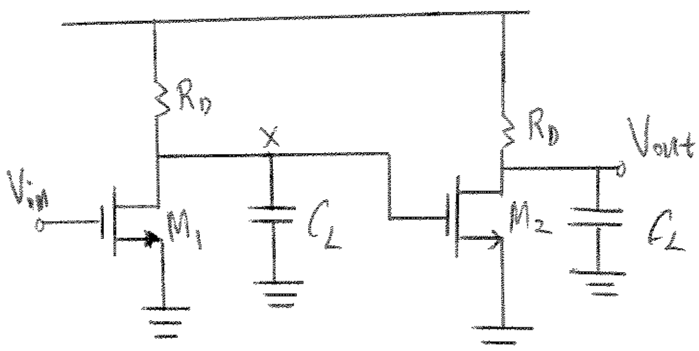
$$\omega_{pin}\,(\omega_{pa}) = \frac{1}{R_G\left(C_{GS_1} + C_{GD_1}\left(\frac{g_{m_1}+g_{m2}+g_{m3}}{g_{m2}+g_{m3}}\right)\right)}$$

$$\omega_{pB} = \frac{g_{m3}+g_{m2}}{\left(C_A + C_{GD_1}\left(\frac{g_{m_1}+g_{m2}+g_{m3}}{g_{m_1}}\right)\right)}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{DB2} + C_{GD2}\right)}$$

Where $C_A = C_{GS3} + C_{SB3} + C_{GS2} + C_{SB2} + C_{DB_1}$

52)



Bias Current = 1mA (each stage)

$C_L = 50 fF$

$\mu_n C_{ox} = 100 \mu A/V^2$, $A_V = 20$, $-3dB$: 1GHz

DC gain: $(g_m R_D)^2 = 20$

$-3dB$ bandwidth: $0.10243/(R_D C_L) = 1 GHz$

Since $C_L = 50fF$, $R_D = 2048.6 \Omega$

$(g_m R_D)^2 = 20 \Rightarrow g_m = 0.002183 = \frac{2I_D}{V_{eff}} \Rightarrow V_{eff} = 0.916V$

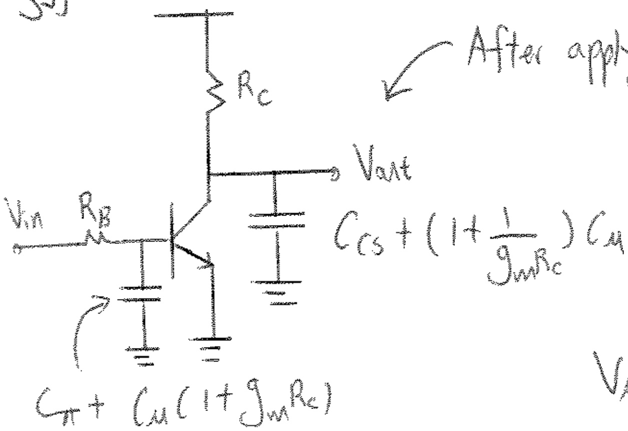
$V_{eff} = V_{GS} - V_{th} = 0.916V$

$g_m = \mu_n C_{ox} \frac{W}{L}(V_{eff}) \Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{eff})} = 23.83$

So $R_D = 2.05K$, $C_L = 50fF$

$V_{GS} - V_{th} = 0.916V$, $W/L = 23.83$

53)



$\omega_{pin} = (2\pi)(500MHz)$

$\omega_{pout} = (2\pi)(2G)$

$I_C = 1mA$, $C_\pi = 20fF$,

$C_\mu = 5fF$, $C_{CS} = 10fF$

$V_A = \infty$.

Low frequency Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pin} = \frac{1}{(R_B // r_\pi)(C_\pi + C_\mu(1+g_m R_C))} = (2\pi)(500MHz)$$

$$\omega_{pout} = \frac{1}{R_C[C_{CS} + (1+1/(g_m R_C))C_\mu]} = (2\pi)(2G)$$

$$\Rightarrow g_m = 2\pi(2G)[g_m R_C C_{CS} + g_m R_C C_\mu + C_\mu]$$

$$\Rightarrow R_C = \left(\frac{g_m}{(2\pi)(2G)} - C_\mu\right) / (g_m(C_{CS} + C_\mu))$$

$$g_m = \frac{I_C}{V_T} = 0.0386 \frac{1}{\Omega}, \quad R_C = 5296.53\,\Omega$$

53)

In order to maximize low frequency gain V_{out}/V_{in}, R_B should be as small as possible (restricted by the input pole location). So $R_B // r_\pi \approx R_B$.

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_m R_C))} = (2\pi)(500\times10^6)$$

$$g_m R_C = 204.446$$

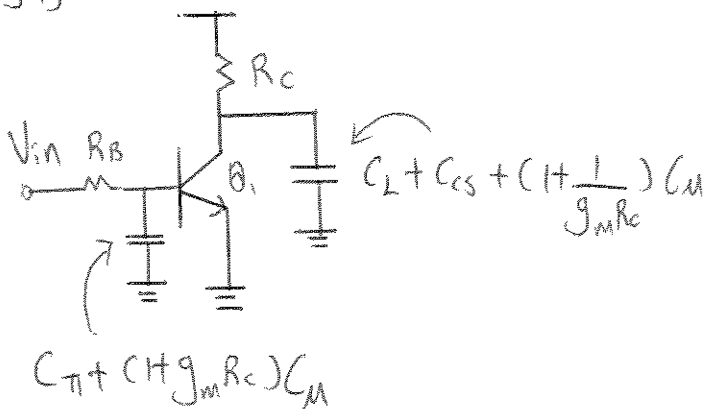
$$R_B = \frac{1}{\omega_{pin}(C_\pi + C_\mu(1+g_m R_C))} \approx 303.95\,\Omega$$

So

$R_B = 303.95\,\Omega$

$R_C = 5296.53\,\Omega$

54)



Low freq voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{p,out} = \frac{1}{R_C [C_L + C_{CS} + (1 + \frac{1}{g_m R_C}) C_\mu]} = (2\pi)(2\,GHz)$$

$$g_m = \frac{I_C}{V_T} = 0.0386 \frac{1}{\Omega}$$

$$g_m = (2\pi)(2\,GHz)[g_m R_C [C_L + C_{CS}] + g_m R_C C_\mu + C_\mu]$$

$$R_C = \left[\frac{g_m}{(2\pi)(2G)} - C_\mu\right] / (g_m [C_L + C_{CS} + C_\mu])$$

$R_C = 2269.94\,\Omega \approx 2.27\,K\Omega$

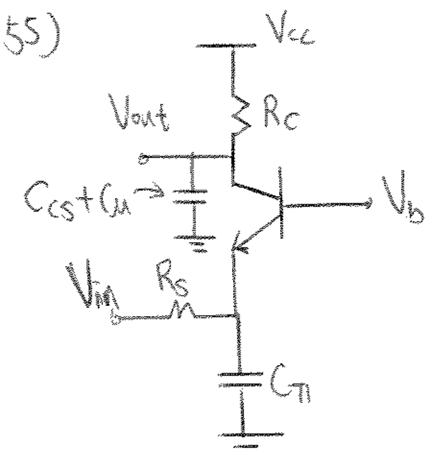
Again, to maximize low freq gain, R_B should be as small as possible, so $R_B // r_\pi \approx R_B$

$$\omega_{p,in} \approx \frac{1}{R_B (C_\pi + C_\mu (1 + g_m R_C))} = (2\pi)(500 \times 10^6), \quad g_m R_C = 87.62$$

$R_B = 687.35\,\Omega$

So, $R_C = 2.27\,K\Omega$, $R_B = 687.35\,\Omega$

55)



$V_A = \infty$, $I_C = 1mA$, $R_S = 50\Omega$,
$C_\pi = 20fF$, $C_{CS} = 20fF$, $C_\mu = 5fF$

-3dB bandwidth: 10GHz

Since the output node sees a larger capacitance and resistance than the input, (R_C usually large for large gain), dominant pole and thus -3dB bandwidth occurs at the output.

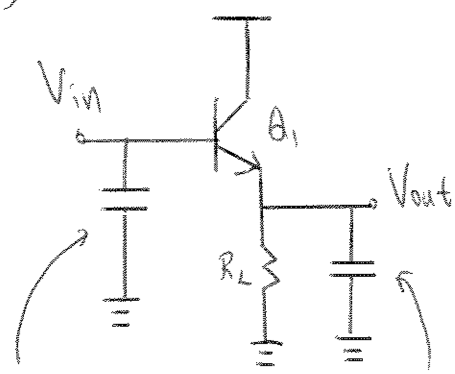
$$\omega_{pout} = \frac{1}{R_C[C_\mu + C_{CS}]} = (2\pi)(10GHz)$$

$R_C = 636.62\Omega$, $\frac{1}{g_m} = \frac{25.9mV}{1mA}$

Maximum achievable gain: $\frac{R_C}{R_S + \frac{1}{g_m}} = 8.4$

Here we have a tradeoff between gain and bandwidth.

56)



$$C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) \qquad C_\pi\left(1 - \frac{R_L + 1/g_m}{R_L}\right)$$

DC gain: $\dfrac{R_L}{R_L + 1/g_m}$

$V_A = \infty$

$C_\mu = 10\,fF,\; C_\pi = 100\,fF$

$$C_{in} < 50\,fF \Rightarrow C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50\,fF$$

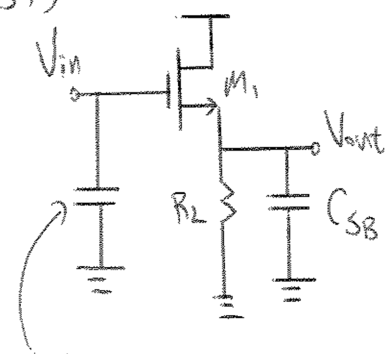
$$10\,fF + 100\,fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50\,fF$$

$$100\,fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 40\,fF$$

$$\left(\frac{\frac{1}{g_m}}{R_L + 1/g_m}\right) < 0.4$$

$$R_L > \frac{3}{2g_m} = 38.85\,\Omega$$

57)



$R_L = 100\Omega$, $I_D = 1mA$

$A_V = \frac{V_{out}}{V_{in}} = 0.8$ $\mu_n C_{ox} = 100\mu A/V^2$

$L = 0.18\mu m$, $\lambda = 0$, $C_{GD} \approx 0$,

$C_{SB} \approx 0$, $C_{GS} = (2/3) WL C_{ox}$

$C_{ox} = 12 fF/(\mu m)^2$

$C_{in} = C_{GD} + C_{GS}(0.2)$, $C_{in} = C_{GS}(0.2) = C_{in(min)}$

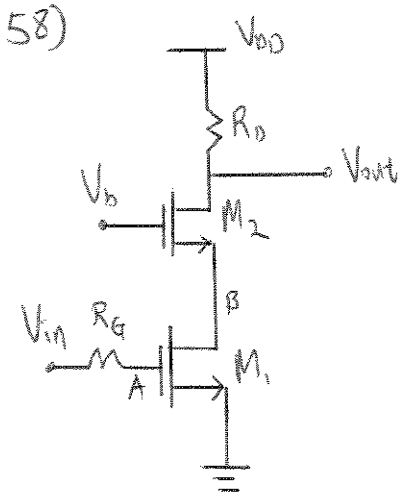
$A_V = \frac{R_L}{R_L + 1/g_m} = 0.8$, $\frac{1}{g_m} = 25 = \frac{V_{eff}}{2I_D}$

$V_{eff} = 50mV$, $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 \Rightarrow W = 1440$

$C_{in,min} = 0.2\, C_{GS} = 0.2\left(\frac{2}{3}\right) WL\, C_{ox} = 414.72 fF$

or $C_{in,min} = 0.415 pF$

58)



$\omega_{pin} = 5\ GHz$, $\omega_{pout} = 10\ GHz$

$V_{eff} = 200\ mV\ (V_{GS} - V_{th})$, $I_D = 0.5\ mA$

$\lambda = 0$, $C_{GS} = (2/3)\ WL\ C_{ox}$,

$L = 0.18\ \mu m$, $\mu_n C_{ox} = 100\ \mu A/V^2$

$C_{GD} = W C_o$, $C_o = 0.2\ fF/\mu m$

$C_{ox} = 12\ fF/\mu m^2$

DC gain from A to B: $\dfrac{-g_{m1}}{g_{m2}} = 1$

$C_{in} = C_{GS} + C_{GD}(1 + g_{m1}/g_{m2}) = C_{GS} + 2C_{GD}$

$I_D = \dfrac{1}{2}\dfrac{W}{L}\mu_n C_{ox}(V_{eff})^2 = 0.5\ mA \Rightarrow \dfrac{W}{L} = 250$

$L = 0.18\mu$, $W = 45\mu$

$$\omega_{pin} = (2\pi)(5\times10^9) = \frac{1}{R_G\left[\frac{2}{3}(45)(0.18)(12 fF/\mu m^2) + (0.2)(45)(2)\right]}$$

$R_G = 384.43\ \Omega$

$$\omega_{pout} = \frac{1}{R_D[0.2W]} = (10\times10^9)(2\pi),\ W = 45\ \mu m$$

$\Rightarrow R_D = 1.8\ K\Omega$ $(1768.4\ \Omega)$ exact value

$$\text{Gain} = |g_m R_D| = \frac{2I_D}{V_{eff}} R_D = 8.842$$

59)

$W_2 = 4W_1$, $V_{eff_2} = \frac{V_{eff_1}}{2}$ (To maintain the current constant)

$V_{eff_1} = 200\,mV$, $V_{eff_2} = 100\,mV$ (Assume V_{eff_1} is not changed)

DC gain: $-\frac{g_{m_1}}{g_{m_2}} = \frac{-g_{m_1}}{2g_{m_1}} = -\frac{1}{2}$

$$\omega_{pin} = \frac{1}{R_G\left[\frac{2}{3}WLC_{ox} + (0.2)W\left(\frac{1}{2}\right)\right]} = (5\times10^9)(2\pi)$$

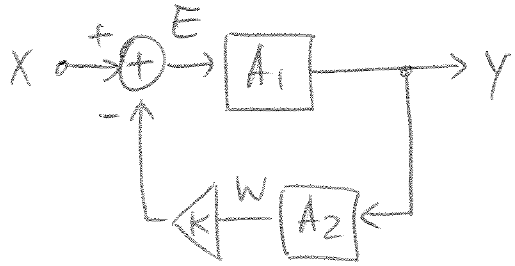
$W = 45\mu$

$\Rightarrow R_G = 459.32\,\Omega$

$$R_o = \frac{1}{(10\times10^9)(2\pi)(0.2)(4)(45)} = 442.097\,\Omega$$

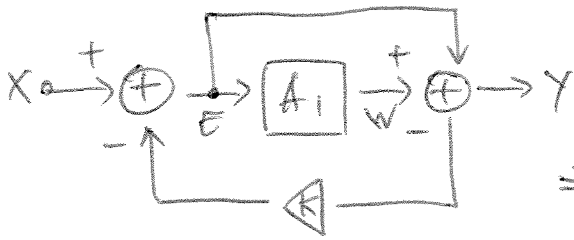
DC gain: $|g_{m_1}R_o| = \frac{2I_D}{V_{eff_1}}R_o = 2.2105$

1\. (a)



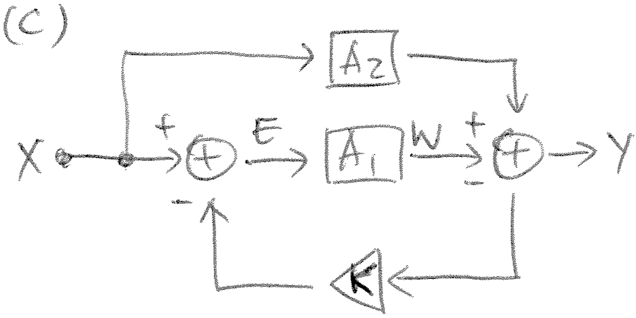
$$Y = A_1 E_1$$
$$= A_1 [X - Y A_2 k]$$
$$\Rightarrow \frac{Y}{X} = \frac{A_1}{1 + k A_1 A_2}$$

(b)



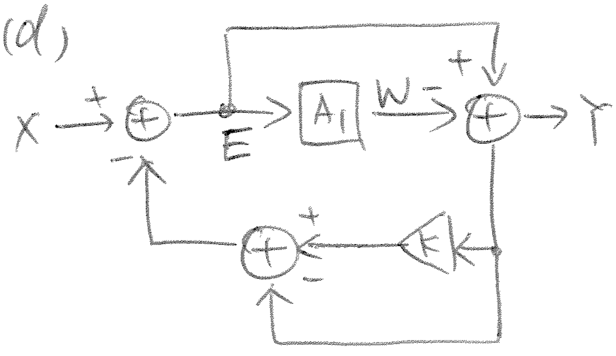
$$Y = E - A_1 W$$
$$= (X - kY) - A_1 (X - kY)$$
$$\Rightarrow \frac{Y}{X} = \frac{1 - A_1}{1 + (1 - A_1)k}$$

(c)



$$Y = X A_2 - W$$
$$= X A_2 - A_1 (X - YK)$$
$$\Rightarrow \frac{Y}{X} = \frac{A_2 - A_1}{1 - A_1 Y}$$

(d)



$$Y = E - W$$
$$= (X - (kY - Y)) -$$
$$A_1 [X - (kY - Y)]$$
$$\Rightarrow \frac{Y}{X} = \frac{(1 - A_1)}{1 + (k - 1)(1 - A_1)}$$

2. (a) $W = A_2 Y = A_2 [(X - KW) A_1]$

$$\Rightarrow \frac{W}{X} = \frac{A_1 A_2}{1 + A_1 A_2 K}$$

(b) $W = A_1 \times E = A_1 [X - K(\frac{W}{A_1} - W)]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + K(1 - A_1)}$$

(c) $W = A_1 E = A_1 [X - (A_2 X - W) K]$

$$\Rightarrow \frac{W}{X} = \frac{A_1 (1 - A_2 K)}{(1 - A_1 K)}$$

(d) $W = A_1 E = A_1 [X - \{(\frac{W}{A_1} - W) K - (\frac{W}{A_1} - W)\}]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + (K-1)(1 - A_1)}$$

3. (a) $E = X - kA_2A_1E$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + kA_2A_1}$$

(b) $E = X - k[E - A_1E]$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + k(1 - A_1)}$$

(c) $E = X - k[A_2X - A_1E]$

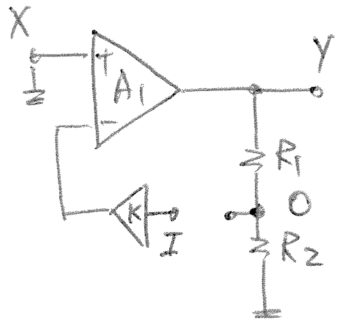
$$\Rightarrow \frac{E}{X} = \frac{1 - A_2k}{1 - A_1k}$$

(d) $E = X - \{k[E - A_1E] - [E - A_1E]\}$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + (k-1)(1 - A_1)}$$

4.

(a)

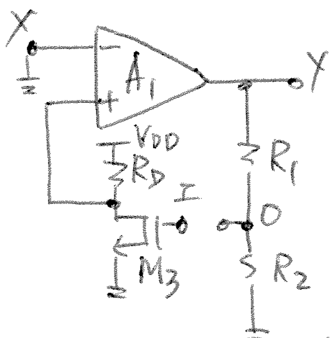


(X is grounded in loop-gain calculation)

$$O = Y\frac{R_2}{R_1+R_2} = (-IK)A_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +KA_1\left(\frac{R_2}{R_1+R_2}\right)$$

(b)

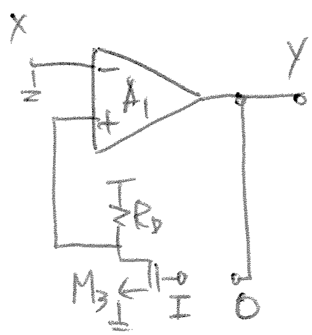


(X is grounded)

$$O = Y\left(\frac{R_2}{R_1+R_2}\right) = -Ig_{m3}R_DA_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_DA_1\left(\frac{R_2}{R_1+R_2}\right)$$

(c)

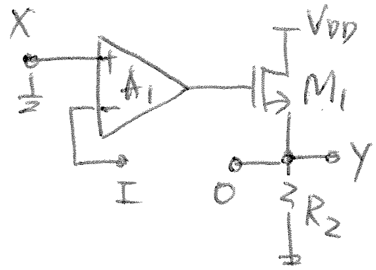


(X is grounded)

$$O = Y = -Ig_{m3}R_DA_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_DA_1$$

(d)



(X is grounded)

$$O = Y = -I \times \frac{g_{m1} R_2}{1 + g_{m1} R_2} \times A_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain}$$

$$= +A_1 \frac{g_{m1} R_2}{1 + g_{m1} R_2}$$

5\. (a) $$\frac{Y}{X} = \frac{A_{O.L.}}{1 + \text{Loop Gain}} = \frac{A_1}{1 + A_1 K\left(\frac{R_2}{R_1 + R_2}\right)}$$

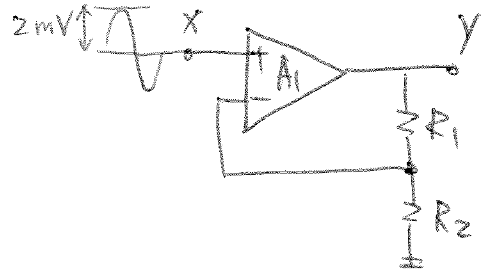
(b) $$\frac{Y}{X} = \frac{A_1}{1 + g_{m3} R_D A_1 \left(\frac{R_2}{R_1 + R_2}\right)}$$

(c) $$\frac{Y}{X} = \frac{A_1}{1 + g_{m3} R_D A_1}$$

(d) $$\frac{Y}{X} = \frac{A_1}{1 + A_1 \left(\frac{g_{m1} R_2}{1 + g_{m1} R_2}\right)}$$

6. $A_1 = 500$

$R_1/R_2 = 7$.



$$\frac{Y}{X} \approx 1 + \frac{R_1}{R_2} = 8.$$

$$\Rightarrow \frac{R_2}{R_1+R_2} = \frac{1}{8} = K$$

$$E = \frac{X}{1+KA_1} = \frac{2\,mV}{1+500/8} \approx 0.031\,mV$$

∴ Amplitude of feedback waveform

$= X - E \approx 1.969\,mV$

Amplitude of output waveform

$$= X\frac{A_1}{1+KA_1} \approx 15.75\,mV$$

7. $A_{CL} = \dfrac{A_1}{1+A_1K}$

$$\frac{dA_{CL}}{dA_1} = \frac{1}{(1+A_1K)^2} \Rightarrow dA_{CL} = \frac{dA_1}{(1+A_1K)^2}$$

$$\Rightarrow \frac{dA_{CL}}{A_{CL}} = \frac{dA_{CL}}{\left(\frac{A_1}{1+A_1K}\right)} = dA_1\left(\frac{1+A_1K}{A_1}\right)\left(\frac{1}{(1+A_1K)^2}\right)$$

$$= \frac{(dA_1/A_1)}{(1+A_1K)}$$

This equation implies that for a fractional change in A_{CL}, it is reduced by $(1+A_1K)$ compared to a fractional change in A_1.

$$\Rightarrow 0.01 > \frac{0.2}{1+A_1K} \Rightarrow A_1K > 19$$

8. $A_{OL} = -g_{m1} r_{o1}$ (max gain)

$$= -\frac{2}{\lambda V_{eff}}$$

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL} K}$$

$$\frac{dA_{CL}}{d\lambda} = \frac{d}{d\lambda}\left(\frac{A_{OL}}{1+A_{OL}K}\right) = \frac{\frac{2}{\lambda^2 V_{eff}}}{(1+A_{OL}K)^2} = \frac{-A_{OL} \times \frac{1}{\lambda}}{(1+A_{OL}K)^2}$$

$$\Rightarrow \frac{dA_{CL}}{A_{CL}} = \frac{-A_{OL} \times \frac{d\lambda}{\lambda}}{(1+A_{OL}K)^2} \times \frac{(1+A_{OL}K)}{A_{OL}} = \frac{-\frac{d\lambda}{\lambda}}{(1+A_{OL}K)}$$

$\Rightarrow$ This implies a change in λ $\left(\frac{d\lambda}{\lambda}\right)$ leads to a change in A_{CL} $\left(\frac{dA_{CL}}{A_{CL}} = \frac{d\lambda}{\lambda} \cdot \frac{1}{(1+\text{Loop Gain})}\right)$ in the <u>opposite</u> direction (i.e. the sign)

$$5\% > \frac{20\%}{1+A_{OL}K} \Rightarrow \text{Loop Gain} = A_{OL}K > 3$$

9. From the question,

$(1-10\%)A_0 = |A(j\omega')|$ where ω' = -1-dB bandwidth frequency

$$0.9A_0 = \frac{A_0}{\left|1+j\frac{\omega'}{\omega_0}\right|} = \frac{A_0}{\sqrt{1+\left(\frac{\omega'}{\omega_0}\right)^2}}$$

$\Rightarrow \omega' \cong 0.48\,\omega_0$

$\Rightarrow$ This is the open-loop -1dB bandwidth.

Similarly,

$0.9\,\frac{A_0}{1+L.G.} = \left|\frac{Y}{X}(j\omega'')\right|$ where ω'' = -1dB bandwidth frequency

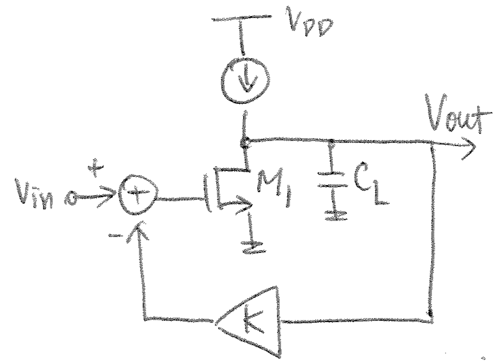
L.G = Loop Gain

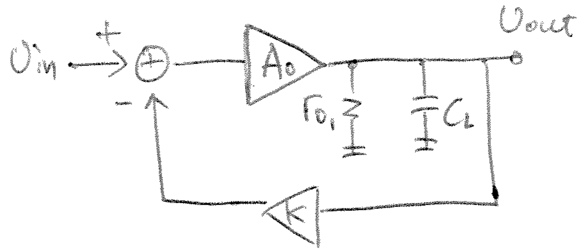
$$0.9\,\frac{A_0}{1+LG} = \frac{\frac{A_0}{1+L.G.}}{\left|1+j\frac{\omega''}{\omega_0(1+LG)}\right|} = \frac{\frac{A_0}{1+LG}}{\sqrt{1+\left[\frac{\omega''}{\omega_0(1+LG)}\right]^2}}$$

$\Rightarrow \omega'' \cong 0.48\,\omega_0\,(1+LG)$

$\therefore$ -1-dB bandwidth is boosted (expected) by $(1+LG)$ in closed-loop measurement.

10. An equivalent circuit is shown below:





Open-loop transfer function (without feedback):

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}} = \frac{-g_{m1} r_{o1}}{1 + \frac{s}{r_{o1} C_L}}$$

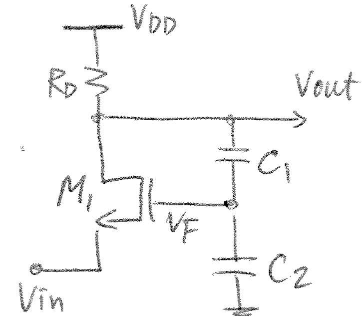
$$\text{Loop Gain} = A_0 K = g_{m1} r_{o1} K$$

$\Rightarrow$ Closed-loop -3dB bandwidth

$$= B = (r_{o1} C_L)(1 + A_0 K) = r_{o1} C_L (1 + g_{m1} r_{o1} K)$$

$$\therefore\ K = \left(\frac{B}{r_{o1} C_L} - 1\right) \times \frac{1}{g_{m1} r_{o1}}$$

11.



Feedforward System

M_1 & R_D (common-gate stage)

Sense Mechanism (Feedback):

$C_1 + C_2$ (capacitive divider)

$$\Rightarrow V_F = \frac{C_1}{C_2 + C_1} \times V_{out}$$

Comparison Mechanism: M_1 itself.

$$\text{Open-loop gain} = A_0 = g_m R_D$$

$$\text{Closed-loop gain} = \frac{v_{out}}{v_{in}} = \frac{A_0}{1 + KA_0} = \frac{g_m R_D}{1 + \frac{C_1}{C_1 + C_2} g_m R_D}$$

$$\text{Open-loop } r_{in} = \frac{1}{g_m}$$

$$\text{" } r_{out} = R_D$$

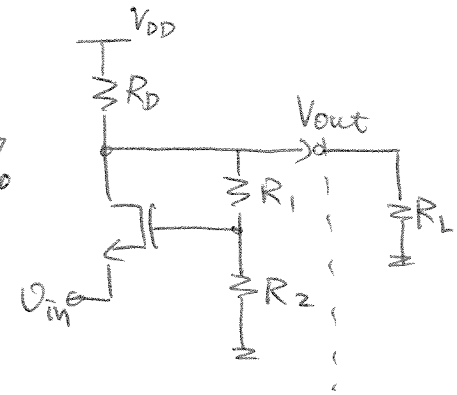
$$\text{Closed-loop } r_{in} = r_{in,ol} \times (1 + KA_0) = \frac{1}{g_m}\left(1 + \frac{C_1}{C_1+C_2} g_m R_D\right)$$

$$\text{" } r_{out} = \frac{r_{out,ol}}{(1+KA_0)} = \frac{R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

12.

Given:

$$\frac{\text{Gain}_{loaded} - \text{Gain}_{loaded}}{\text{Gain}_{unloaded}} = 10\% = 0.1$$



$$\text{Gain}_{unloaded} \cong \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} \quad (\text{assume } R_1 + R_2 \gg R_D)$$

$$\text{Gain}_{loaded} = \frac{g_m (R_D \| R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D \| R_L)}$$

$$\therefore 0.1 = \frac{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} - \dfrac{g_m (R_D \| R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D \| R_L)}}{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}}$$

After solving for R_L:

$$R_L = \frac{9 R_D}{1 + \left(\frac{R_2}{R_1+R_2}\right) g_m R_D}$$

13. Gain at $x_1 = \frac{500}{1+500K}$

Gain at $x_2 = \frac{420}{1+420K}$

$$\Rightarrow \frac{\frac{500}{1+500K} - \frac{420}{1+420K}}{\frac{500}{1+500K}} < 0.05$$

$$\Rightarrow K > \frac{11}{2100}$$

$$A_{x_1} = \frac{500}{1+500(K)} = \frac{2625}{19} \approx 138.16$$

$$A_{x_2} = \frac{420}{1+420(K)} = \frac{525}{4} \cong 131.25$$

14. $y = \alpha_1 x - \alpha_3 x^3$

(a) $\frac{\partial y}{\partial x} = \alpha_1 - 3\alpha_3 x^2$

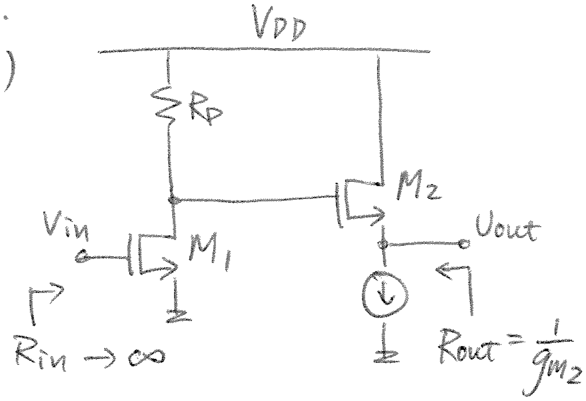
$\left.\frac{\partial y}{\partial x}\right|_{x=0} = \alpha_1$ $\qquad$ $\left.\frac{\partial y}{\partial x}\right|_{x=\Delta x} = \alpha_1$ (around $x=0$)

(b) Closed-loop $|_{x=0} = \frac{\alpha_1}{1+\alpha_1 K}$

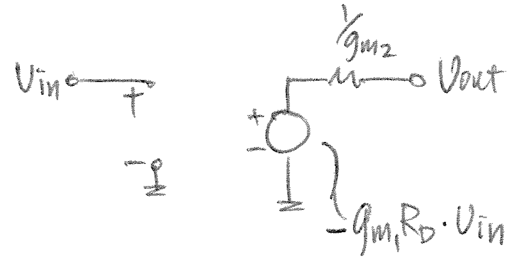
Closed-loop $|_{x=\Delta x} = \frac{\alpha_1}{1+\alpha_1 K}$ (around $x=0$)

15.

(a)

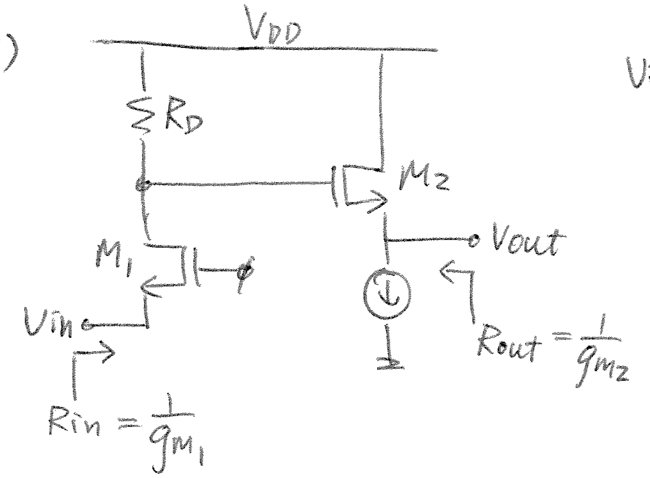


VDD
RD
M2
Vin
M1
Vout
Rin → ∞
Rout = 1/gm2

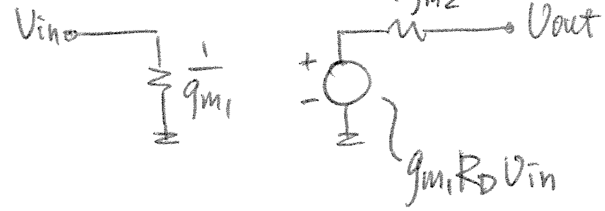


Vin
+
−
1/gm2
Vout
+
−
−gm1RD·Vin

(b)

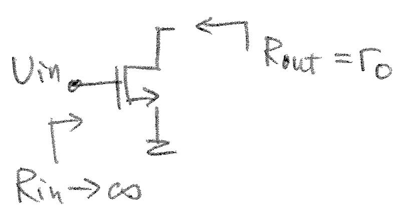


VDD
RD
M2
M1
Vout
Vin
Rout = 1/gm2
Rin = 1/gm1

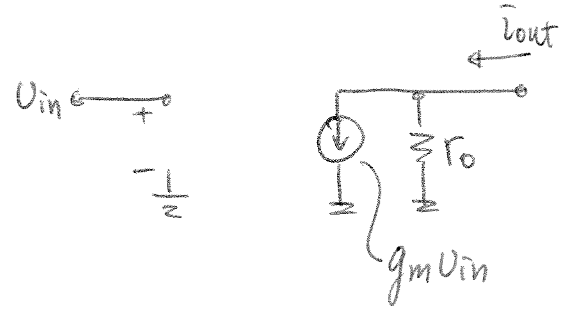


Vin
1/gm1
1/gm2
Vout
+
−
gm1RDVin

(c)

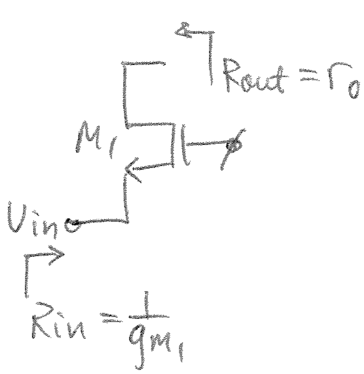


Rout = ro
Vin
Rin→∞

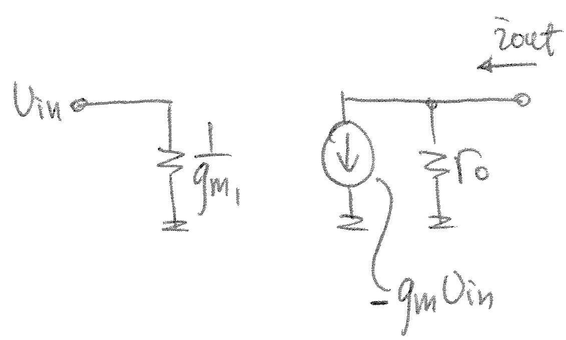


iout
Vin
+
−
ro
gmVin

(d)



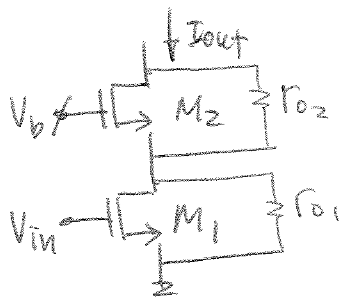
Rout = ro
M1
Vin
Rin = 1/gm1

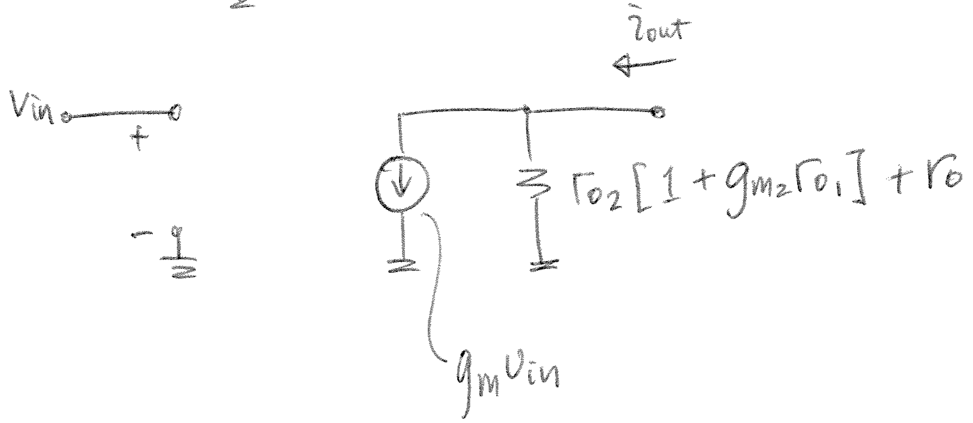


iout
Vin
1/gm1
ro
−gmVin

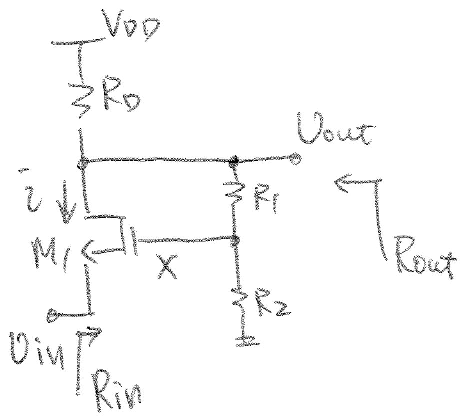
16.

$\lambda > 0$





17.

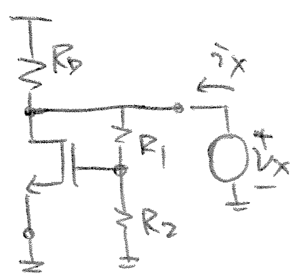


$$-v_{out} = i\,[R_D \parallel (R_1+R_2)]$$

$$i = g_{m1}(v_X - v_{in}) = g_{m1}\left(v_{out}\times\frac{R_2}{R_1+R_2} - v_{in}\right)$$

Combining the equations above yields:

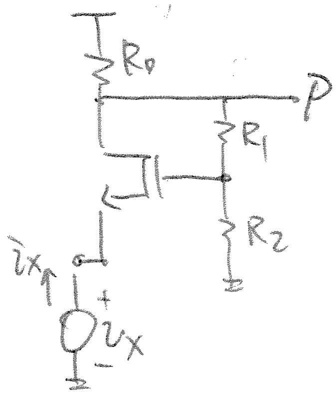
$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}[R_D\parallel(R_1+R_2)]}{1+\frac{R_2}{R_1+R_2}g_{m1}[R_D\parallel(R_1+R_2)]} \triangleq A_v$$



By KCL,

$$i_x = \frac{v_x}{R_1+R_2}+\frac{v_x}{R_D}+g_{m1}\left(v_x\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow \frac{v_x}{i_x} = R_{out} = [(R_1+R_2)\parallel R_D]\left[1+g_{m1}\frac{R_2}{R_1+R_2}(R_D\parallel(R_1+R_2))\right]$$



By KCL,

$$i_x = g_{m1}\left(v_x - v_P \frac{R_2}{R_1+R_2}\right)$$

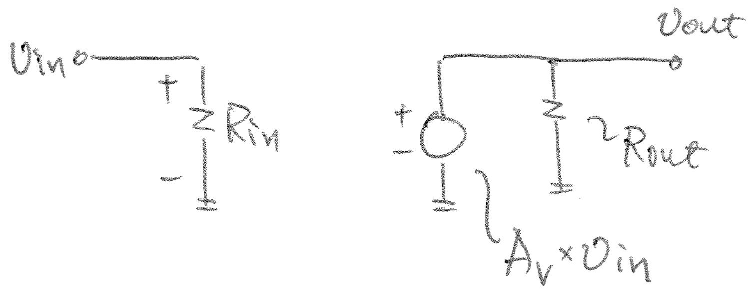
$$\Rightarrow v_P = \left(v_x - \frac{i_x}{g_{m1}}\right)\left(\frac{R_1+R_2}{R_2}\right) \quad \text{—— (1)}$$

$$i_x = \frac{v_P}{R_D \parallel (R_1+R_2)} \quad \text{—— (2)}$$

Substitute (1) into (2) & solve for $\frac{v_x}{i_x}$:

$$\frac{v_x}{i_x} = R_{in} = \frac{1}{g_{m1}}\left[1 + g_{m1}\{R_D \parallel (R_1+R_2)\}\frac{R_2}{R_1+R_2}\right]$$

Model:



18. (a) Sense Mechanism : Current from M_3
Return Mechanism : Voltage to Gate of M_2.

(b) Sense Mechanism: Voltage output (M_3)
Return Mechanism: Voltage to Gate of M_2.

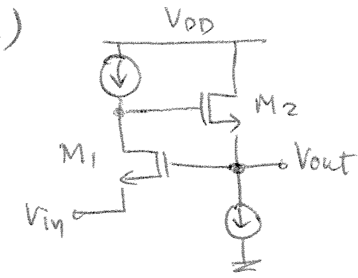
(c) Sense Mechanism: Current from M_3
Return Mechanism : Voltage to Gate of M_2.

(d) Sense Mechanism: Current from M_3
Return Mechanism: Voltage to Gate of M_2.

(e) Sense Mechanism: Voltage Divider $\left(\frac{R_2}{R_1+R_2}\right)$
Return Mechanism : Voltage to Gate of M_2.

(f) Sense Mechanism: From Common Source of M_3
Return Mechanism : Voltage to Gate of M_2.

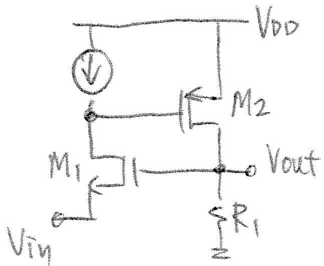
19. (a)



Sense Mechanism:
Voltage sensing at Vout.

Return Mechanism:
Voltage to Gate of M_1.

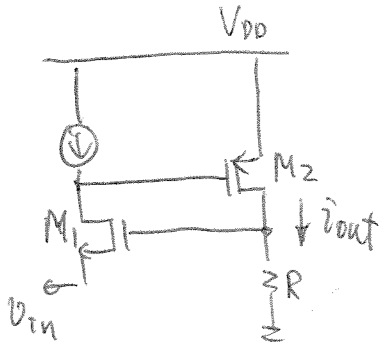
(b)



Sense Mechanism:
Voltage output from M_2.

Return Mechanism:
Voltage to Gate of M_1.

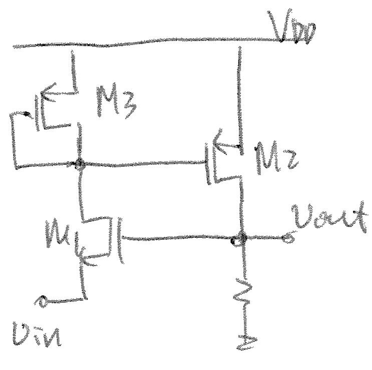
(c)



Sense Mechanism:
R_1

Return Mechanism:
Voltage to Gate of M_1.

(d)

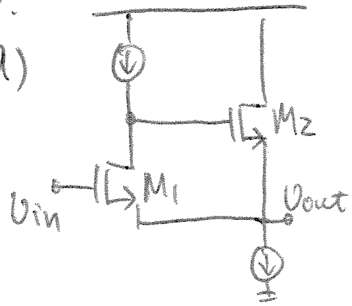


Sense Mechanism:
Voltage output of M_2

Return Mechanism:
Voltage to Gate of M_1

20.

(a)



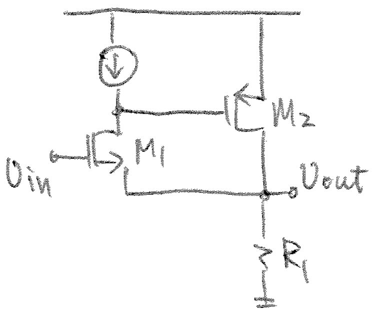
Sense Mechanism:

Voltage output of M_2

Return Mechanism:

Voltage to Source of M_1

(b)



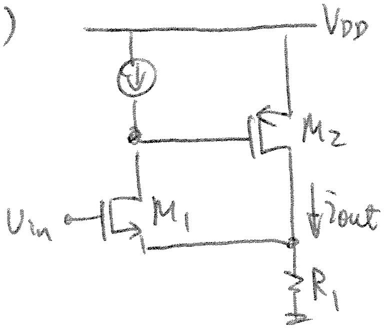
Sense Mechanism:

Voltage output of M_2

Return Mechanism:

Voltage to Source of M_1

(c)



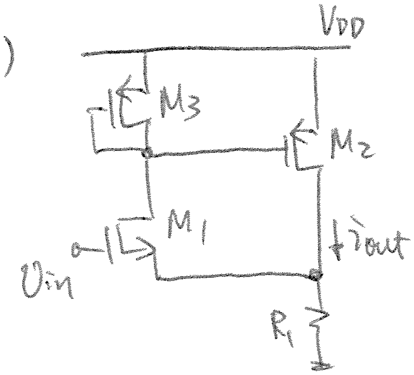
Sense Mechanism:

Current through R_1.

Return Mechanism:

$i_{out} \times R_1$ to Source of M_1.

(d)



Sense Mechanism:

Current through R_1.

Return Mechanism:

$i_{out} \times R_1$ to Source of M_1

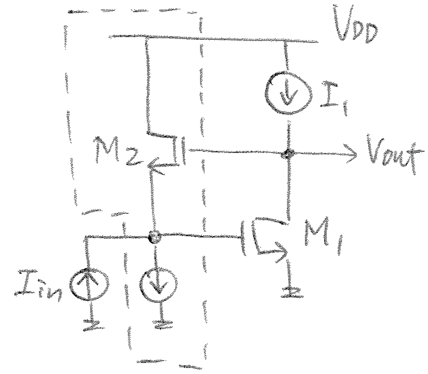
21. (a)

Sense Mechanism:

Gate of M_2 (Voltage)

Return Mechanism:

Current output of M_2.



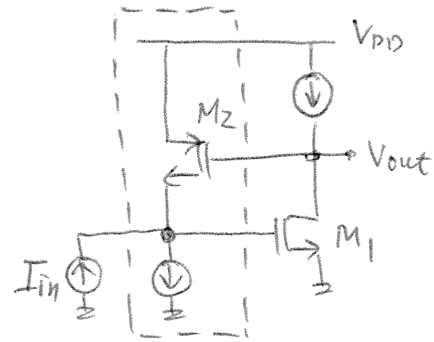
(b)

Sense Mechanism:

Gate of M_2 (Voltage)

Return Mechanism:

Current output of M_2.



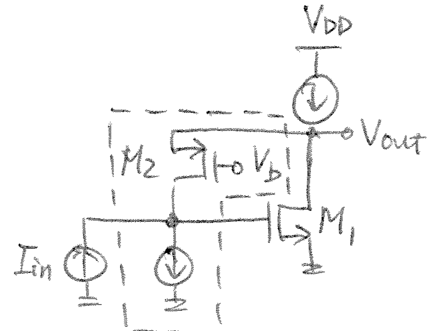
(c)

Sense Mechanism:

Source of M_2 (Voltage)

Return Mechanism:

Current output of M_2

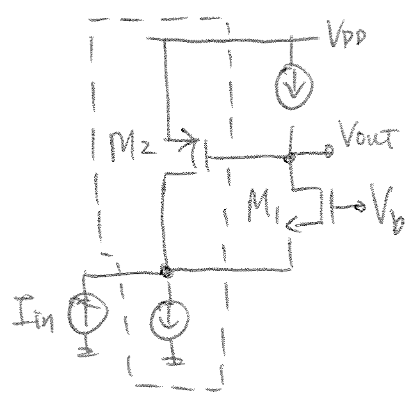


(d) Sense Mechanism:

Gate of M_2. (Voltage)

Return Mechanism:

Current output of M_2



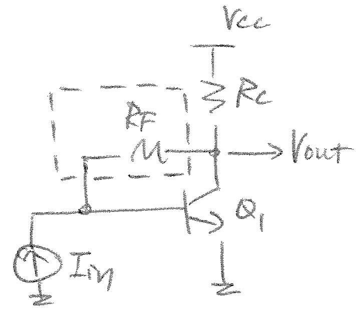
22.

(a) Sense Mechanism:

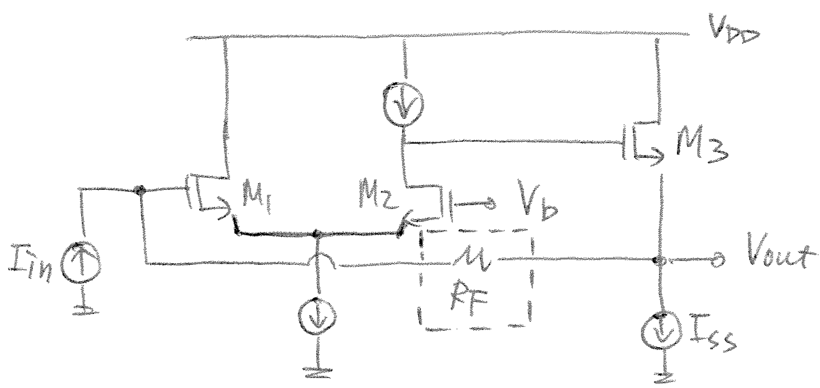
Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F.



(b)



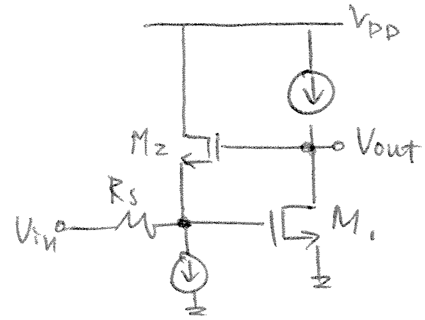
Sense Mechanism:

Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F

23. First, recognize that
(a) both input & output are voltages.



* v_{in} primarily drives the Gate of M_1.

Sequence: Suppose v_{in} increases by Δv_{in}

⇒ v_{out} drops by $+g_{m1}\Delta v_{in} \times r_{o1}$ (Common-Source)

⇒ Source of M_2 decreases by same amount (Source follower)

∴ $v_{in} \uparrow$ ⇒ $V_{M_1,D} \downarrow$ ⇒ $V_{M_1,G} \downarrow$

⇒ effective v_{in} driving $M_{1,G} \downarrow$

⇒ negative feedback

(b) $v_{in} \uparrow$ ⇒ $v_{out} \downarrow$ ⇒ $v_{M2,G} \uparrow$

⇒ effective v_{in} driving $M_{1,G} \uparrow$

⇒ positive feedback.

(c) $v_{in} \uparrow \Rightarrow v_{out} \downarrow \Rightarrow v_{M1,G} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \downarrow$

$\Rightarrow$ negative feedback.

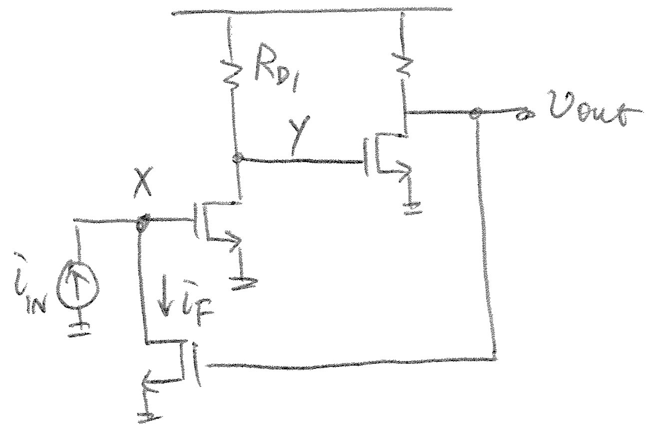
(d) $v_{in} \uparrow \Rightarrow v_{out} \uparrow$ (common-base, M_1)

$\Rightarrow v_{M1,S} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,S} \downarrow$

$\Rightarrow$ negative feedback.

24.



$i_{IN} \uparrow \Rightarrow v_X \uparrow \Rightarrow v_Y \downarrow \Rightarrow v_{out} \uparrow \Rightarrow i_F \uparrow$

$\Rightarrow$ effective $v_X \downarrow$

$\Rightarrow$ negative feedback.

25.

$\uparrow$ = increases by
$\downarrow$ = decreases by

<u>Figure 12.79</u>

(a) $V_{in} \uparrow \Rightarrow V_{G,M3} \uparrow$

$\Rightarrow V_{out} \uparrow$ same amount (Emitter Follower)
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2}) \downarrow$
$\Rightarrow$ Negative feedback.

(b) $V_{in} \uparrow \Rightarrow V_{G,M_3} \uparrow$

$\Rightarrow V_{out} \uparrow$ by $\Delta V_{G,M_3} \times g_{m3}(r_{o3} \parallel R_1)$
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2}) \downarrow$
$\Rightarrow$ Negative feedback.

(c) Same as (b)

(d) Same as (a)

(e) $V_{in} \uparrow \Rightarrow V_{out} \uparrow$

$\Rightarrow V_{G,M_2} \uparrow$ by $\Delta V_{out} \times \dfrac{R_2}{R_1 + R_2}$
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2}) \downarrow$
$\Rightarrow$ Negative feedback.

(f) $V_{in} \uparrow \Rightarrow V_{G,M_3} \uparrow$

$\Rightarrow V_{out} \uparrow$ (Common-Source Stage)

$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2}) \downarrow$

$\Rightarrow$ Negative feedback.

Figure 12.80

(a) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Source Follower)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

(b) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

(c) Same as (b)

(d) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

Figure 12.81

(a) $V_{in} \uparrow \Rightarrow V_{G,M_2} \downarrow$ (Common Source)
$\Rightarrow V_{out} \uparrow$ (Source Follower)
$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \uparrow$
$\Rightarrow$ Positive feedback.

(b) $V_{in} \uparrow \Rightarrow V_{G,M_2} \downarrow$ (Common Source)
$\Rightarrow V_{out} \uparrow$ (Common Source)
$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \downarrow$
$\Rightarrow$ Negative feedback.

(c) Same as (b) $(V_{out} = I_{out} \times R_1)$

(d) $V_{in} \uparrow \Rightarrow V_{G,M_2}$ (Common Source)
$\Rightarrow V_{out} \uparrow$ (Common Source)
$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \downarrow$
$\Rightarrow$ Negative feedback.

Figure 12.82

(a) $i_{in} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1.)

$\Rightarrow i_{D,M2} \downarrow$ (Source Follower, M_2)

$\Rightarrow$ Counteracts the effect of i_{in}

$\Rightarrow$ Negative feedback.

(b) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1)

$\Rightarrow i_{D,M2} \uparrow$ (Common Source, M_2)

$\Rightarrow$ Enhances V_{out} through M_1

$\Rightarrow$ Positive feedback.

(c) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1)

$\Rightarrow i_{D,M2} \downarrow$ (Common Gate)

$\Rightarrow$ Counteracts the effect of i_{IN}

$\Rightarrow$ Negative feedback.

(d) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \uparrow$ (Common Gate)

$\Rightarrow i_{D,M2} \downarrow$ (Common Source)

$\Rightarrow$ Counteracts the effect of i_{IN}

$\Rightarrow$ Negative feedback.

Figure 12.83

(a) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Emitter)

$\Rightarrow$ R_F demands more current momentarily

$\Rightarrow$ effective i_{IN} into $Q_1 \downarrow$

$\Rightarrow$ $V_{out} \uparrow$

$\Rightarrow$ Negative feedback.

(b) $i_{IN} \uparrow \Delta \Rightarrow V_{G, M_3} \uparrow$ (Diff Pair Action)

$\Rightarrow$ $V_{out} \uparrow$ (Source Follower)

$\Rightarrow$ V_{out} "tracks" behavior of i_{IN}.

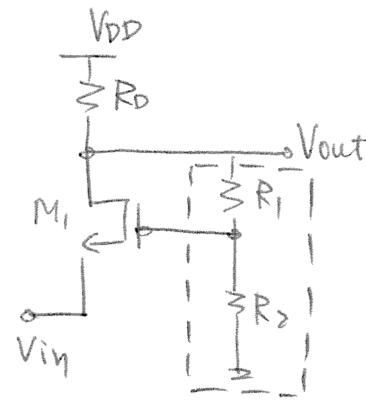
(Current demanded by R_F from $i_{IN} \uparrow$)

$\Rightarrow$ Positive feedback.

26.

(Without feedback)

$$\frac{v_{out}}{v_{in}} = A_{O.L.} = g_m R_D$$



$R_1 + R_2 \gg R_D$

$\lambda = 0$

Feedback factor, k:

$$k = \frac{R_2}{R_1+R_2}$$

$$\Rightarrow A_{C.L.} = \left.\frac{v_{out}}{v_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L} \cdot K} = \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

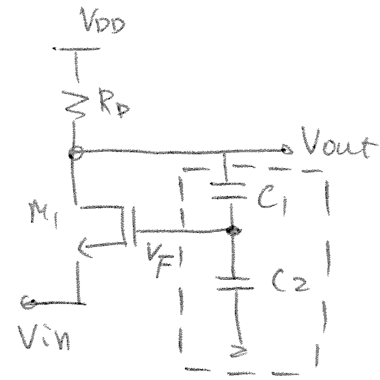
$$R_{in,CLOSED} = \frac{1}{g_{m1}}\left(1 + \frac{R_2}{R_1+R_2} g_m R_D\right)$$

$$R_{out,CLOSED} = \frac{R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

27.

(Without feedback)

$A_{O.L.} = g_m R_D$



$\lambda = 0$

C_1, C_2 small.

Feedback factor, k:

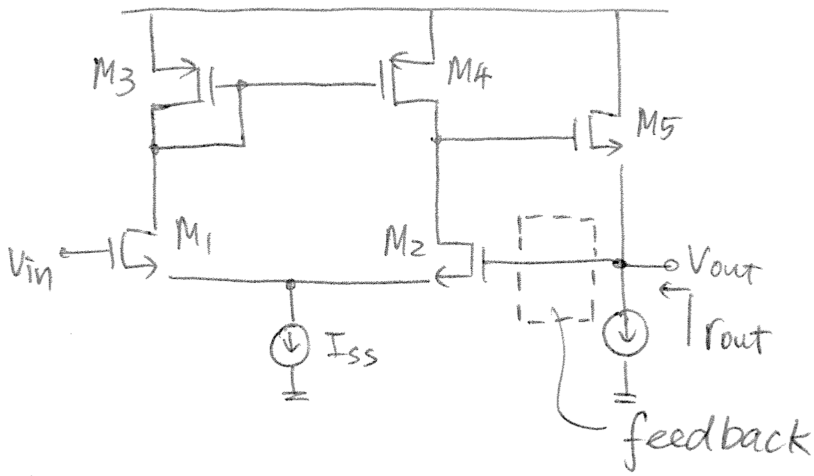
$$k = \frac{C_1}{C_1 + C_2}$$

$$\Rightarrow A_{C.L.} = \left.\frac{V_{out}}{V_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L.} k} = \frac{g_m R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

$$R_{in,CLOSED} = \frac{1}{g_m}\left[1 + \frac{C_1}{C_1+C_2} g_m R_D\right]$$

$$R_{out,CLOSED} = \frac{R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

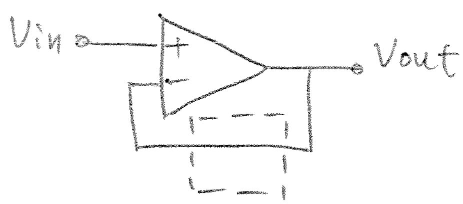
28.



$\lambda > 0$

r_{out} low.

Note that Vout is directly fed back to input :



∴ gain ≈ 1 (a buffer)

$\Rightarrow k = 1$.

$A_{O.L.}$ (i.e. without feedback)

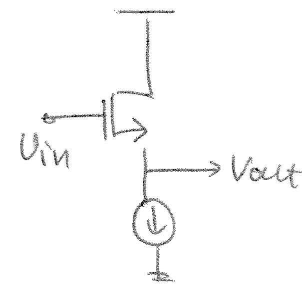
$$= g_{m1}(r_{o2} \| r_{o4}) \times \frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1} \quad \left(\approx g_{m1}(r_{o2} \| r_{o4})\right)$$

$$\Rightarrow A_{C.L.} = \frac{A_{OL}}{1 + A_{OL} \cdot k} = \frac{g_{m1}(r_{o2} \| r_{o4})\left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)}{1 + g_{m1}(r_{o2} \| r_{o4})\left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)}$$

$$r_{out} = \frac{r_{out}\text{(no feedback)}}{1 + A_{O.L.} \times k} = \frac{\left(\frac{1}{g_{m5}} \| r_{o5}\right)}{1 + g_{m1}(r_{o2} \| r_{o4})\left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)}$$

$$\text{Gain} = A = \frac{g_m r_o}{g_m r_o + 1}$$

$$r_{out} = \frac{1}{g_m} \parallel r_o$$

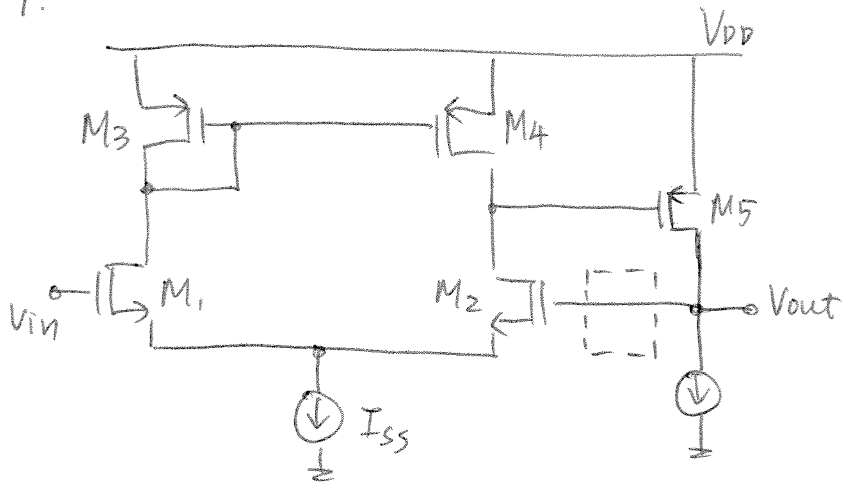


$\lambda > 0$

In comparison, the amplifier's gain is reduced by $\dfrac{g_{m1}(r_{o2} \parallel r_{o4})}{1 + g_{m1}(r_{o2} \parallel r_{o4})\left(\dfrac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)}$ times.

$\left(= \dfrac{A_{C.L.}}{A}\right)$. Output resistance of the amplifier is reduced by $\left[1 + g_{m1}(r_{o2} \parallel r_{o4})\left(\dfrac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)\right]$ times.

29.



(a) By inspection,

$V_{in} \uparrow \Rightarrow V_{G,M5} \uparrow$

$\Rightarrow$ $V_{out} \downarrow$ (Common Source)

$\Rightarrow$ effective V_{in} ($V_{in} - V_{out}$) $\uparrow \Rightarrow V_{G,M5} \uparrow$

$\Rightarrow$ Positive feedback.

(b) $A_{O.L.}$ (without feedback)

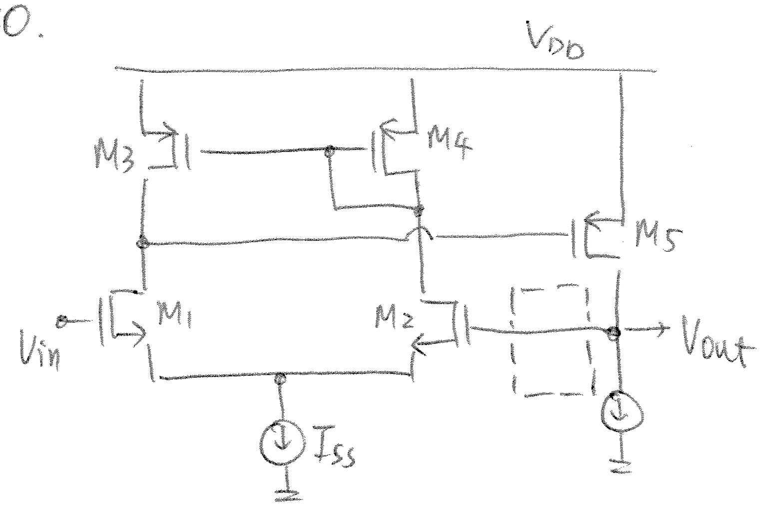
$= g_{m1}(r_{o2} \parallel r_{o4}) \times (-g_{m5} r_{o5})$

K (feedback factor) $= 1$

$\Rightarrow$ loop gain $= A_{O.L.} \times K = -g_{m5} r_{o5} \times g_{m1}(r_{o2} \parallel r_{o4})$

$\Rightarrow$ $A_{C.L.}$ becomes negative $\Rightarrow$ Positive feedback.

30.



$k = 1$.
$\lambda > 0$

$A_{O.L.}$ (without feedback)

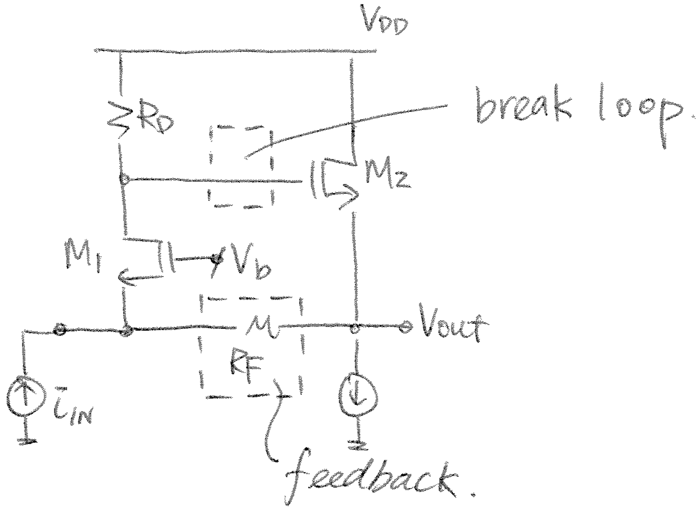
$= -g_{m1}(r_{o1} \parallel r_{o3}) \times (-g_{m5} r_{o5}) = g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}$

$$\Rightarrow A_{C.L.} = \frac{V_{out}}{V_{in}} = \frac{g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}$$

$r_{in} \rightarrow \infty$

$$r_{out} = \frac{r_{out}\,(\text{no feedback})}{1 + A_{O.L.} \times K} = \frac{r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}$$

31.



(a) $i_{IN} \uparrow \Rightarrow V_{G, M_2} \uparrow$ (Common Gate ; i_{IN} mostly flows to M_1 ∵ resistance $= \frac{1}{g_m}$)

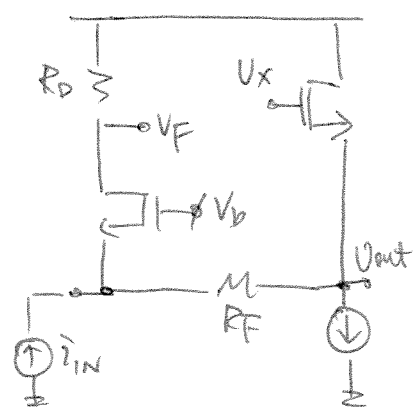
$\Rightarrow V_{out} \uparrow$ (Source Follower)

$\Rightarrow R_F$ momentarily provides <u>more</u> current to source of M_1)

$\Rightarrow V_{G, M_2} \uparrow$

$\Rightarrow$ Positive feedback.

(b)



$$v_F = v_X \frac{g_{m1} R_D}{1 + g_{m2}\left(R_F + \frac{1}{g_{m1}}\right)}$$

$$\Rightarrow -\frac{v_F}{v_X} = \text{Loop Gain} = -\frac{g_{m1} R_D}{1 + g_{m2}\left(R_F + \frac{1}{g_{m1}}\right)}$$

∴ feedback is positive.

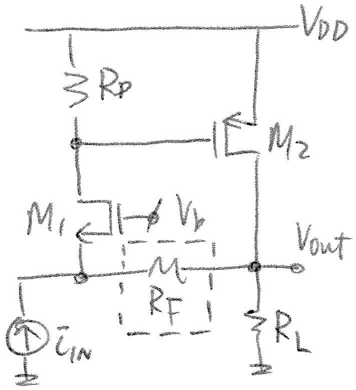
32.

(a) $i_{IN} \uparrow \Delta \Rightarrow \Delta i_{IN}$ mostly flows in $\frac{1}{g_{m1}} \Rightarrow V_{G,M2} \uparrow$ (Common Gate)

$\Rightarrow V_{out} \downarrow$ (Common Source)

$\Rightarrow R_F$ momentarily demands more current from i_{IN}

$\Rightarrow$ Negative feedback.



$\lambda = 0$

$R_F \gg 1.$

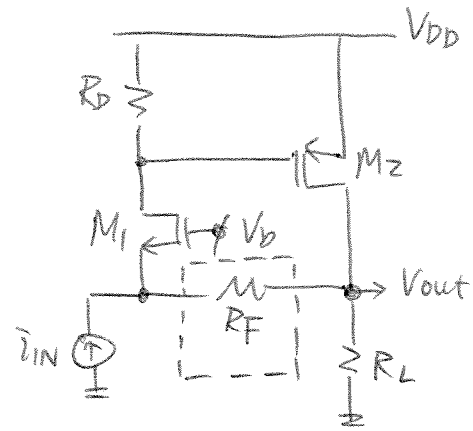
(b) $R_{O.L.} = \left.\frac{V_{out}}{i_{IN}}\right|_{O.L.} = -R_D \times g_{m2} R_L$

(c) K (feedback factor) $= \frac{-1}{R_F}$

$$\Rightarrow R_{C.L.} = \frac{R_{O.L}}{1 + R_{O.L.} \times K} = \frac{-R_D \times g_{m2} R_L}{1 + \frac{R_D}{R_F} g_{m2} R_L}$$

33.

$$R_{c.L.} = \frac{-g_{m2} R_D R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$



$$\text{loop gain} = \frac{g_{m2} R_D R_L}{R_F}$$

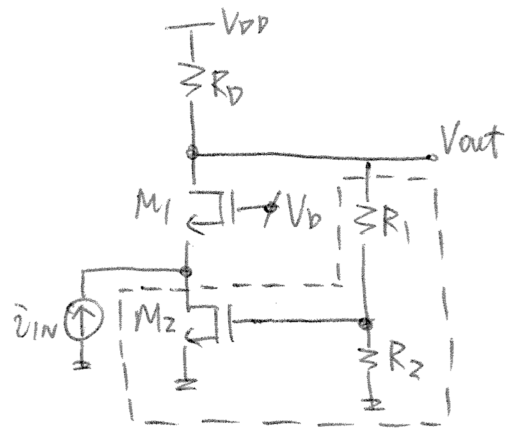
$$r_{in} \approx \frac{1}{g_{m1}} \qquad r_{out} \approx R_L \qquad (R_F \text{ large})$$

$$\Rightarrow r_{in}\Big|_{c.L.} = \frac{1/g_{m1}}{1 + \frac{g_{m2} R_D R_L}{R_F}} \qquad r_{out}\Big|_{c.L.} = \frac{R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$

34.

$$R_{O.L.} = \frac{v_{out}}{i_{IN}} \text{ (no feedback)}$$

$$= R_D$$



$\lambda = 0$

$R_1 + R_2 \gg 1.$

K (feedback factor)

$$= g_{m2} \times \frac{R_2}{R_1+R_2}$$

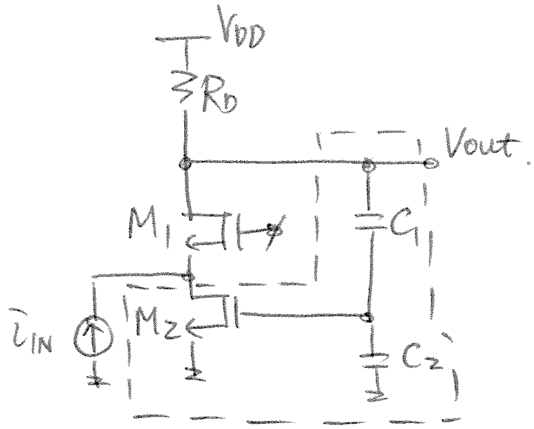
$$\Rightarrow R_{C.L.} = \frac{v_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2} \frac{R_2}{R_1+R_2}}$$

$$r_{in}|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2} \frac{R_2}{R_1+R_2}}$$

$$r_{out}|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2} \frac{R_2}{R_1+R_2}}$$

35.

$$R_{O.L.} = \frac{V_{out}}{i_{IN}} \text{ (no feedback)}$$
$$= R_D$$



K (feedback factor)

$$= g_{m2} \times \frac{C_1}{C_1+C_2}$$

$$\Rightarrow R_{C.L.} = \frac{V_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1+C_2}}$$

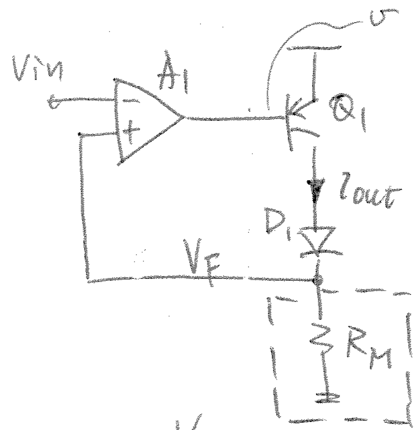
$$r_{in}|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2} \frac{C_1}{C_1+C_2}}$$

$$r_{out}|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1+C_2}}$$

36.

(a) $G_{O.L.} = \frac{i_{out}}{v_{in}} = g_{m1} A_1$

(common emitter)



$V_A \to \infty$

R_M small.

(b) K (feedback factor)

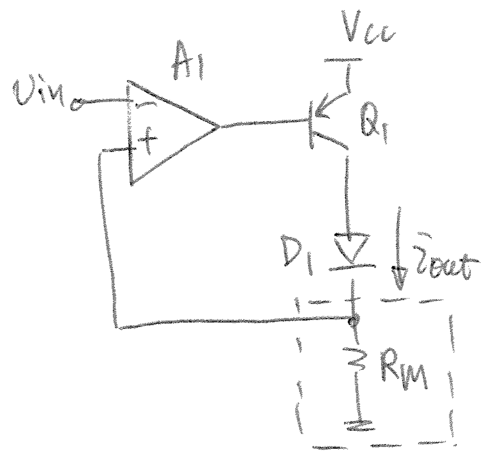
$\Rightarrow V_F = i_{out} \times R_M$

$\Rightarrow K = \frac{V_F}{i_{out}} = R_M$

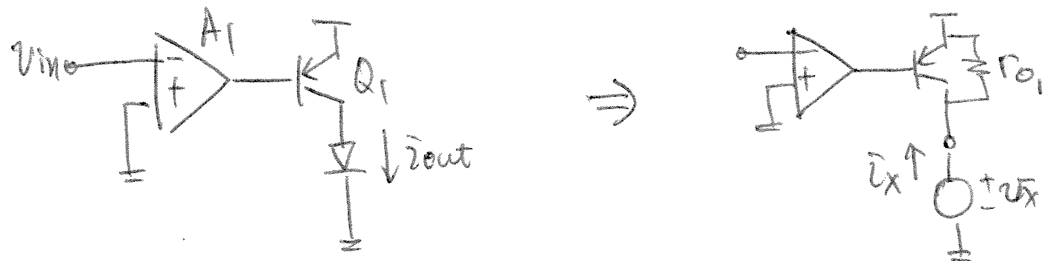
$\therefore$ Loop Gain $= G_{OL}K = g_{m1}A_1R_M$

$$G_{C.L.} = \frac{G_{OL}}{1+G_{OL}K} = \frac{g_{m1}A_1}{1+g_{m1}A_1R_M}$$

37.



Since R_M is small, the following circuit results:

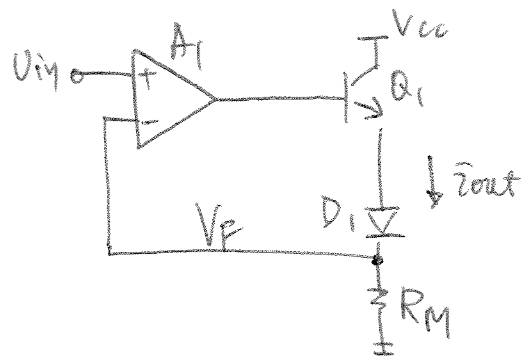


$\therefore$ $R_{out,OPEN} = \frac{v_x}{i_x} = r_{o1}$

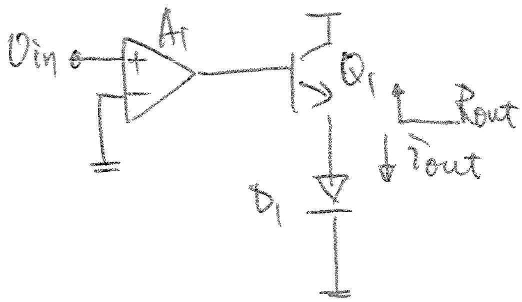
$G_{OL} = \frac{i_{out}}{v_{in}} = A_1 g_{m1}$ $\qquad K = R_M$

$\therefore$ $R_{out,CLOSED} = R_{out,OPEN}(1 + G_{OL}K)$
$= r_{o1}(1 + A_1 g_{m1} R_M)$

38.



Since R_M is small, the open-loop equivalent becomes the following:



$$G_{OL} = \frac{i_{out}}{v_{in}} \approx A_1 g_{m1}$$

$$R_{out} = \frac{r_\pi}{\beta+1} \approx \frac{1}{g_{m1}}$$

$$K = \frac{V_F}{i_{out}} = R_M$$

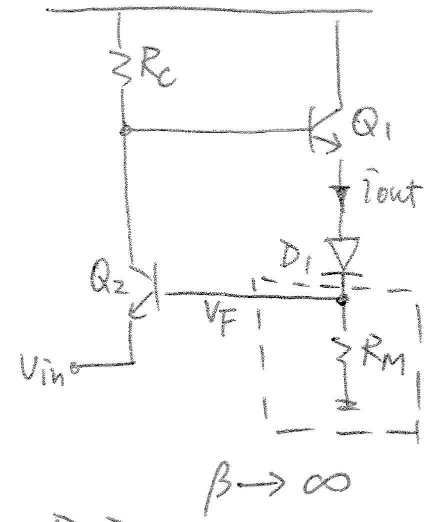
$$\Rightarrow G_{C.L.} = \frac{G_{OL}}{1+G_{OL}K} = \frac{A_1 g_{m1}}{1+g_{m1}A_1R_M}$$

$$\text{Loop Gain} = G_{OL}K = g_{m1}A_1R_M$$

$$R_{out,\,CLOSED} = \frac{1}{g_{m1}}(1+g_{m1}A_1R_M)$$

This circuit provides a much lower output resistance which in general is non-desirable (ideally any current source should have high impedance.)

39.



<u>Using procedure in Ex 12.21</u>

$$G_{O.L.} = \frac{i_{out}}{V_{in}} = g_{m2} R_C \times g_{m1}$$

K (feedback factor)

$$= \frac{V_F}{i_{out}} = R_M.$$

$$\Rightarrow \text{loop gain} = G_{O.L.} \times K = g_{m1} g_{m2} R_C R_M$$

$$\Rightarrow \text{closed-loop gain } G_{CL} = \frac{g_{m1} g_{m2} R_C}{1 + g_{m1} g_{m2} R_C R_M}$$

<u>Using procedure in Ex. 12.22</u>

$$G_{OL} = g_{m1} g_{m2} R_C \qquad K = R_M$$

$$r_{in}\big|_{O.L.} = \frac{1}{g_{m1}} \qquad r_{out}\big|_{O.L.} \cong \frac{1}{g_{m2}}$$

$$r_{in}\big|_{C.L.} = \frac{1}{g_{m1}} (1 + g_{m1} g_{m2} R_C R_M)$$

$$r_{out}\big|_{C.L.} = \frac{1}{g_{m2}} (1 + g_{m1} g_{m2} R_C R_M)$$

40.

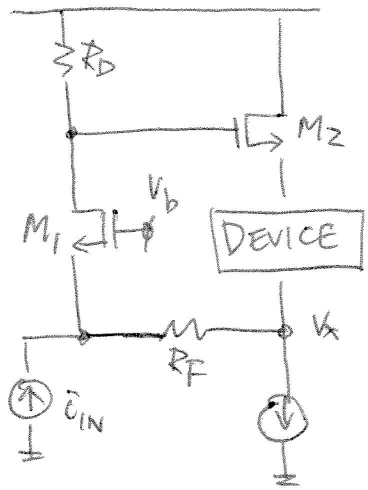
(a) $i_{IN}\uparrow$ Δ ⇒ Most of i_{IN} flows into $1/g_{m1}$

⇒ $V_{G,M2}\uparrow$ (Common Gate)

⇒ $V_x\uparrow$ (Source Follower)

⇒ R_F momentarily provides more current to Source of M_1

⇒ $V_{G,M2}\uparrow$ ⇒ Positive feedback.

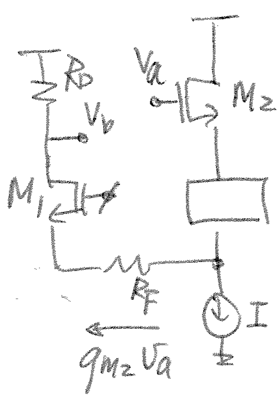


(b)

$v_a \times g_{m2} \times R_D = v_b$

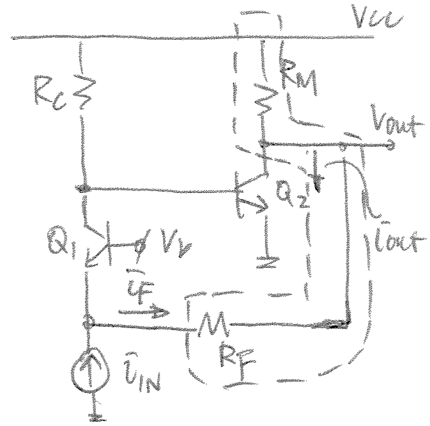
⇒ loop gain $= -\dfrac{v_b}{v_a} = -g_{m2}R_D$.

Since loop gain is negative, feedback is positive.



41.

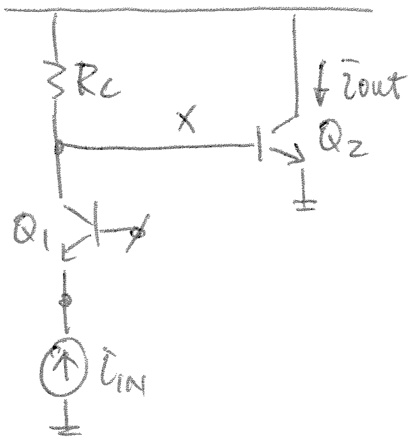
(a) The open-loop equivalent becomes:



$V_A \to \infty$

R_M small

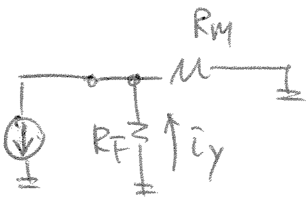
R_F large



$$A_I = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}} \cong g_{m2} \times R_C$$

$$R_{in,OPEN} \cong \frac{1}{g_{m1}} \qquad R_{out,OPEN} \to \infty \quad (\because V_{A_2} \to \infty)$$

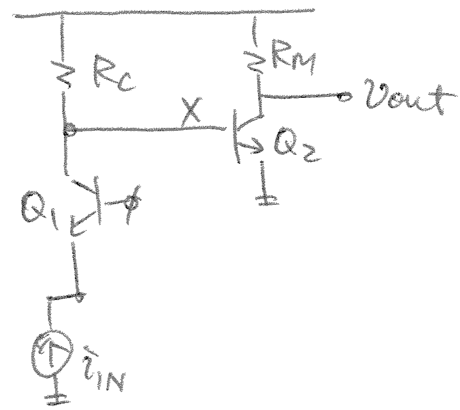
$$k = \frac{i_Y}{i_X} = + \frac{(R_F \| R_M)}{R_F}$$

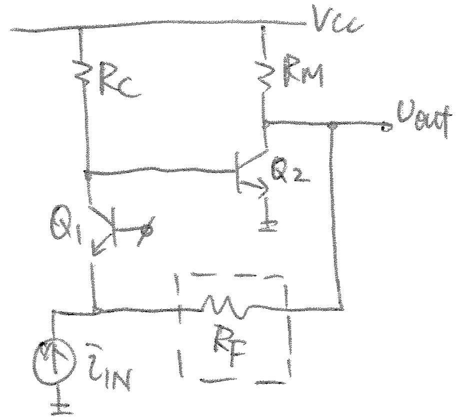


$$A_{CL} = \frac{A_I}{1 + A_I k} = \frac{g_{m2} R_C}{1 + g_{m2} R_C \frac{(R_F \| R_M)}{R_F}}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 + g_{m2} R_C \frac{(R_F \| R_M)}{R_F}} \qquad R_{out,CLOSED} \to \infty$$

(b) The open-loop equivalent becomes:





$V_A \to \infty$

R_F large

$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}} \cong -g_{m2} R_M \times R_C$$

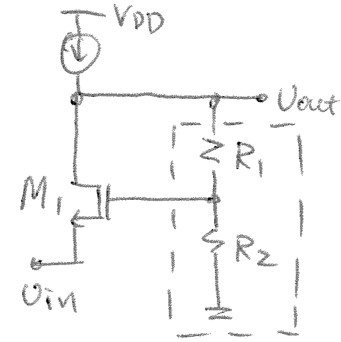
$$R_{in,OPEN} = \frac{1}{g_{m1}} \qquad R_{out,OPEN} = R_M$$

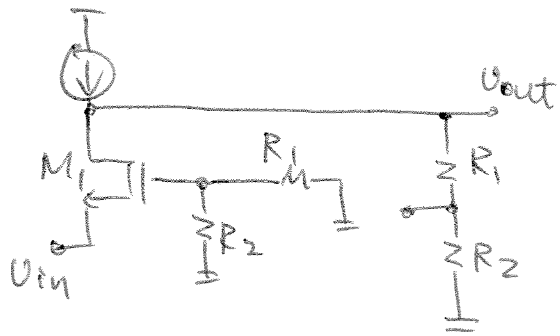
$$K = -\frac{1}{R_M}$$

$$R_{CL} = \frac{R_{OL}}{1 + R_{OL} K} = \frac{-g_{m2} R_M R_C}{1 + g_{m2} R_C}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 + g_{m2} R_C} \qquad R_{out,CLOSED} = \frac{R_M}{1 + g_{m2} R_C}$$

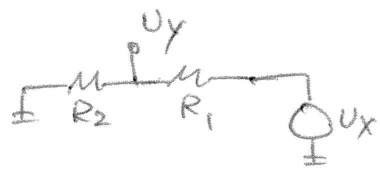
42. Breaking the feedback network results in the following circuit:





Feedback factor

$= K = \frac{V_Y}{V_X} = \frac{R_2}{R_1 + R_2}$



$A_{O.L.} = +g_{m1}(R_1 + R_2)$

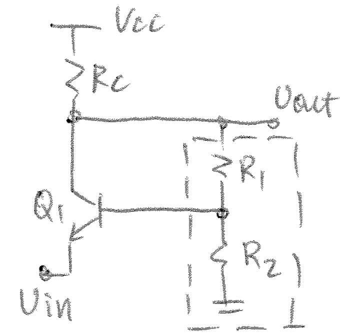
Loop Gain $= A_{O.L.}K = g_{m1}R_2$

$\therefore A_{C.L.} = \frac{A_{OL}}{1 + A_{OL}K} = \frac{g_{m1}(R_1 + R_2)}{1 + g_{m1}R_2}$

$R_{in,CLOSED} = \frac{1}{g_{m1}}(1 + g_{m1}R_2)$

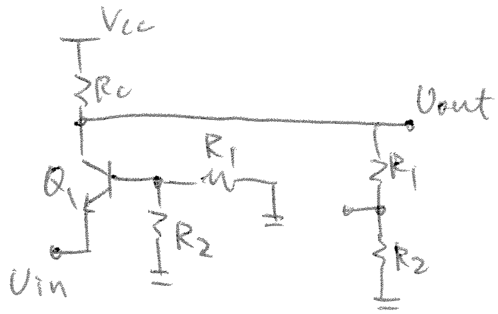
$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + g_{m1}R_2}$

43. Breaking the feedback network results in the following circuit:



$V_A \to \infty$

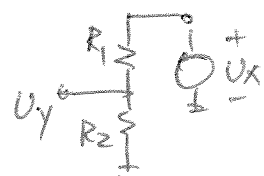
$1 \ll \beta < \infty$



$A_{O.L.} \approx g_{m_1}[R_C \| (R_1 \| R_2)]$ since $i_b \to 0$ $(\beta \gg 1)$

Feedback factor

$= K = \dfrac{v_y}{v_x} = \dfrac{R_2}{R_1 + R_2}$



$R_{in,OPEN} = \dfrac{r_{\pi_1} + (R_1 \| R_2)}{\beta_1 + 1} \approx \dfrac{1}{g_{m_1}} + \dfrac{R_1 \| R_2}{\beta_1 + 1}$

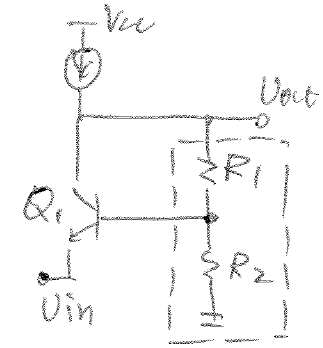
$R_{out,OPEN} = R_C \| (R_1 + R_2)$

$\Rightarrow A_{C.L.} = \dfrac{A_{OL}}{1 + A_{OL}K} = \dfrac{g_{m_1}[R_C \| (R_1 + R_2)]}{1 + g_{m_1}[R_C \| (R_1 + R_2)] \times \dfrac{R_2}{R_1 + R_2}}$

$R_{in,CLOSED} = \left(\dfrac{1}{g_{m_1}} + \dfrac{R_1 \| R_2}{\beta_1 + 1}\right)(1 + A_{OL}K)$

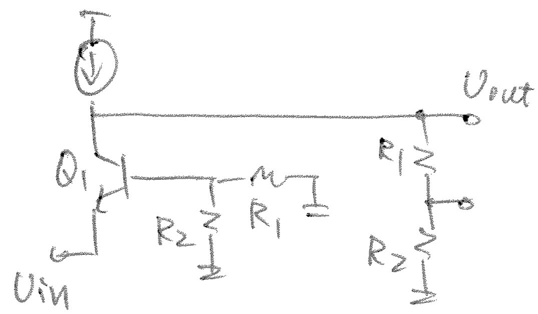
$R_{out,CLOSED} = \dfrac{R_C \| (R_1 + R_2)}{1 + A_{OL}K}$

44. Breaking the feedback network results in the following circuit:



$V_A \to \infty$

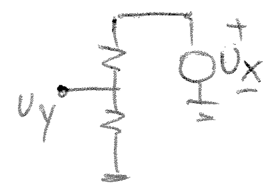
$1 \ll \beta < \infty$



$A_{OL} \cong g_{m1}(R_1 + R_2)$ since $i_b \to 0$ $(\beta \gg 1)$

Feedback factor

$= K = \dfrac{v_Y}{v_X} = \dfrac{R_2}{R_1 + R_2}$



$R_{in,OPEN} \cong \dfrac{1}{g_{m1}} + \dfrac{R_1 \| R_2}{\beta_1 + 1}$

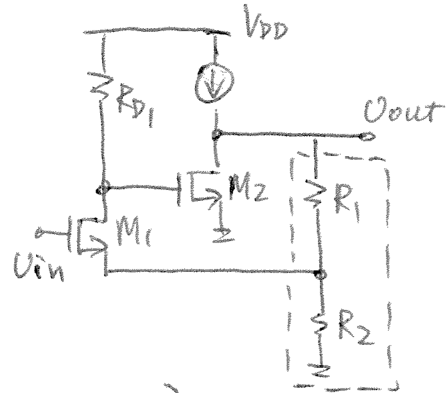
$R_{out,OPEN} = R_1 + R_2$

$\Rightarrow A_{CL} = \dfrac{A_{OL}}{1 + A_{OL}K} = \dfrac{g_{m1}(R_1 + R_2)}{1 + g_{m1}R_2}$

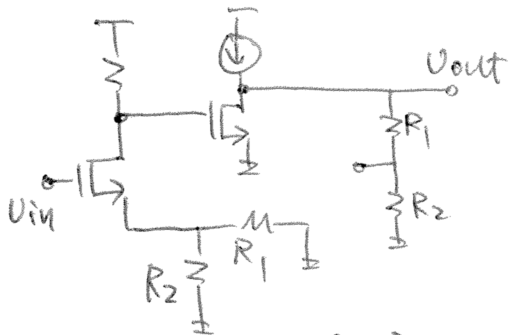
$R_{in,CLOSED} = \left(\dfrac{1}{g_{m1}} + \dfrac{R_1 \| R_2}{\beta_1 + 1}\right)(1 + g_{m1}R_2)$

$R_{out,CLOSED} = \dfrac{R_1 + R_2}{1 + g_{m1}R_2}$

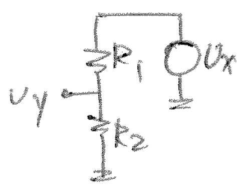
45. Breaking the feedback network results in the following circuit:



$\lambda = 0$



$$A_{OL} = \frac{-g_{m1}R_{D1}}{1 + g_{m1}(R_1 \| R_2)} \times -g_{m2}(R_1 + R_2)$$



$$K = \frac{v_y}{v_x} = \frac{R_2}{R_1 + R_2}$$

$R_{in,OPEN} \to \infty$

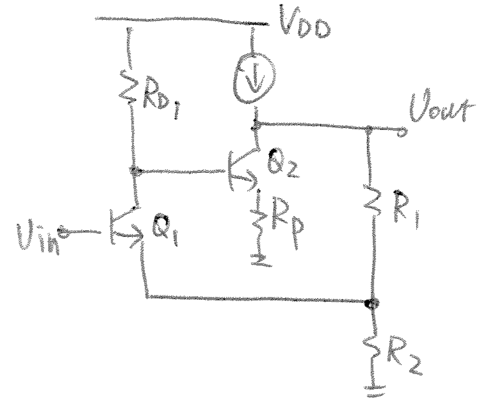
$R_{out,OPEN} = R_1 + R_2$

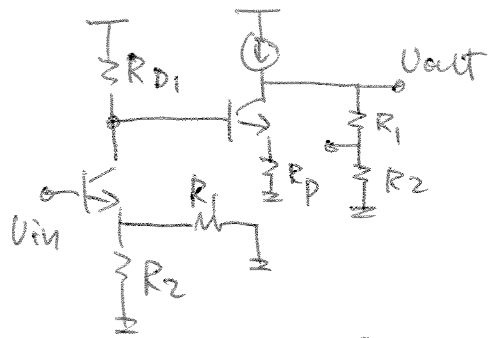
$$\therefore A_{CL} = \frac{A_{OL}}{1 + A_{OL}K} = \frac{\left[\dfrac{g_{m1}g_{m2}R_{D1}(R_1+R_2)}{1 + g_{m1}(R_1 \| R_2)}\right]}{1 + \dfrac{g_{m1}g_{m2}R_{D1}}{1 + g_{m1}(R_1 \| R_2)}}$$

$R_{in,CLOSED} \to \infty$

$$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + A_{OL}K}$$

4b. Breaking the feedback network results in the following circuit:

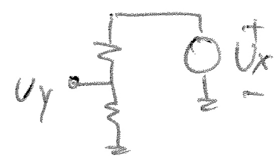




$$A_{OL} \cong \frac{-g_{m1} \times \{R_{D1} \,||\, [r_{\pi 2} + (\beta_2+1)R_p]\}}{1 + g_{m1}(R_1 \,||\, R_2)} \times \frac{-g_{m2}(R_1+R_2)}{1+g_{m2}R_p}$$

$$R_{in,OPEN} = r_{\pi 1} + (R_1 \,||\, R_2) \qquad R_{out,OPEN} = R_1 + R_2$$

$$K = \frac{v_Y}{v_X} = \frac{R_2}{R_1+R_2}$$

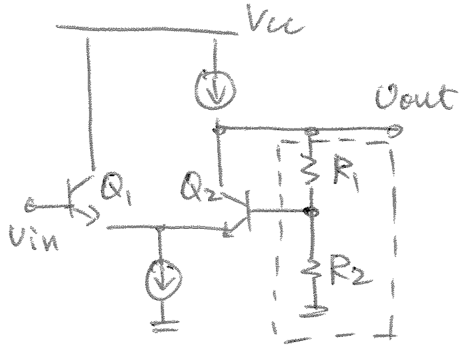


$$\Rightarrow A_{CL} = \frac{A_{OL}}{1 + A_{OL}K}$$

$$R_{in,CLOSED} = [r_{\pi 1} + (R_1 \,||\, R_2)](1 + A_{OL}K)$$

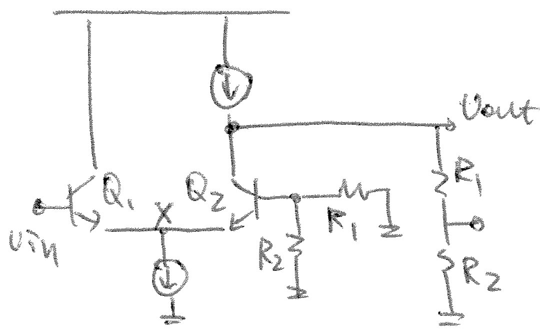
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+A_{OL}K}$$

47. Breaking the feedback network results in the following circuit:



$(V_A \to \infty)$

$$K = \frac{R_2}{R_1 + R_2}$$



$$A_{OL} \cong \underbrace{\frac{g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)}{1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)}}_{\text{emitter follower } (Q_1) \,=\, \frac{V_X}{V_{in}}} \times \underbrace{g_{m2}(R_1 + R_2)}_{\text{common-base stage } (Q_2) \,=\, \frac{V_{out}}{V_X}}$$

$$R_{in,OPEN} = r_{\pi 1} + (\beta_1 + 1)\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)$$

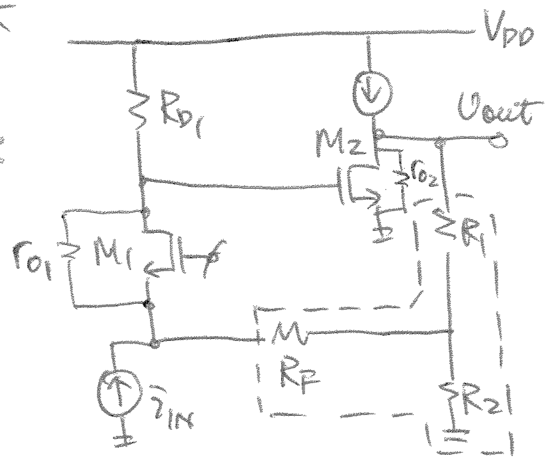
$$R_{out,OPEN} = R_1 + R_2$$

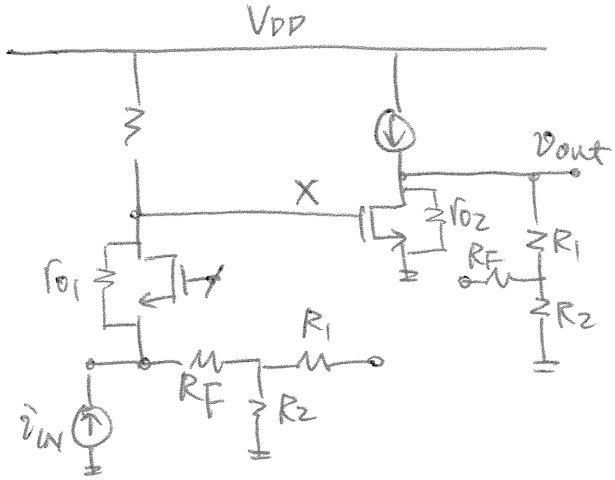
$$\therefore \ A_{CL} = \frac{A_{OL}}{1 + A_{OL} K}$$

$$R_{in,CLOSED} = \left[r_{\pi 1} + (\beta_1 + 1)\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)\right] \times (1 + A_{OL} K)$$

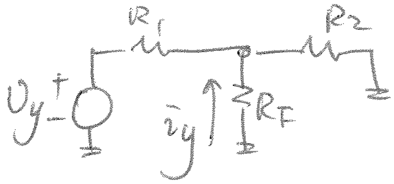
$$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + A_{OL} K}$$

48. Breaking the feedback network results in the following circuit:





$$K = -\frac{(R_F \| R_2)/R_F}{R_F \| R_2 + R_1}$$



$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

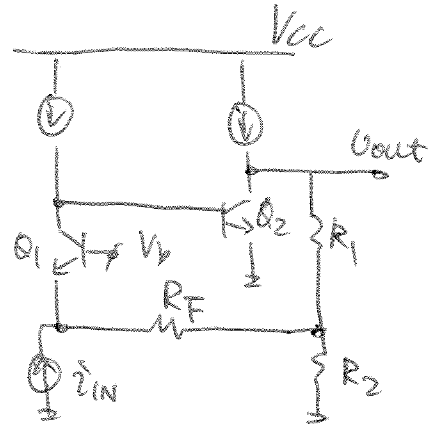
$$= -g_{m2}\left[r_{O2} \| (R_1+R_2)\right] \times \left\{ \frac{\frac{1}{r_{O1}} + \frac{1}{R_{D1}}}{\frac{1}{r_{O1}} + g_{m1}} \times \left[\frac{1}{(R_F+R_2) \| \frac{1}{g_{m1}}} + \frac{1}{r_{O1}}\right] - \frac{1}{r_{O1}} \right\}$$

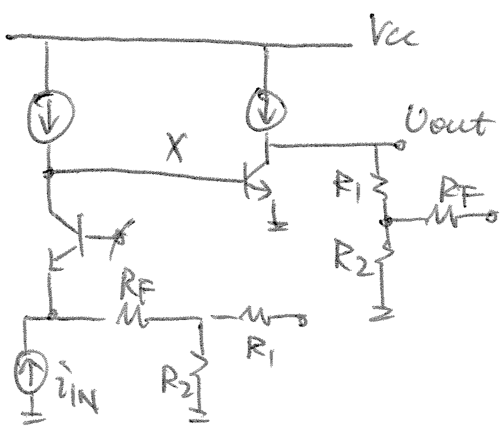
$$R_{in,OPEN} \cong \frac{1}{g_{m1}} \| (R_F+R_2) \| \frac{r_{O1}}{1-g_{m1}R_{D1}} \qquad R_{out,OPEN} = r_{O2} \| (R_1+R_2)$$

$$R_{CL} = \frac{R_{OL}}{1+R_{OL}K} \qquad R_{in,CLOSED} = \frac{R_{in,OPEN}}{1+R_{OL}K}$$

$$R_{out,CLOSED} = \frac{R_{out,OPEN}}{1+R_{OL}K}$$

49. Breaking the feedback network results in the following circuit:

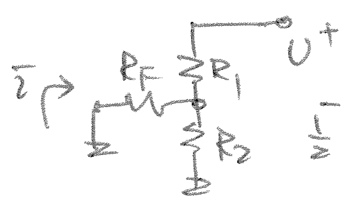




$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}} = \left[-g_{m2}(R_1+R_2)\right] \times \left[g_{m1} r_{\pi 2} \left\{\frac{1}{g_{m1}} \parallel (R_F+R_2)\right\}\right]$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \parallel (R_F+R_2) \qquad R_{out,OPEN} = R_1+R_2$$

$$K = \frac{i}{v} = -\frac{(R_2 \parallel R_F)/R_F}{R_1+(R_2 \parallel R_F)}$$

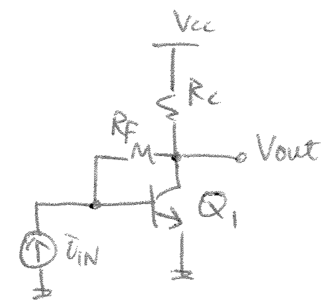


$$\therefore \; R_{CL} = \frac{R_{OL}}{1+R_{OL}K}$$

$$R_{in,CLOSED} = \frac{\frac{1}{g_{m1}} \parallel (R_F+R_2)}{1+R_{OL}K}$$

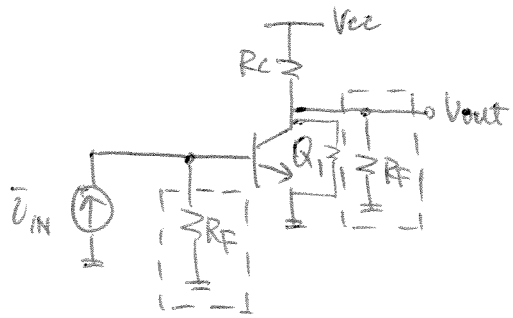
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+R_{OL}K}$$

50. The feedback network consists of R_F.



$(V_A < \infty)$

Using the method discussed in lecture, break the circuit as follows:



This is the open-loop circuit with consideration of I/O loading.

- By inspection,

$$v_{out} = i_c \times (R_C \| R_F \| r_o)$$

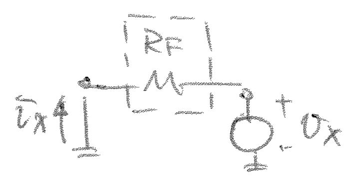
$$= -g_m (i_{IN} \times (R_F \| r_\pi)) \times (R_C \| R_F \| r_o)$$

$$\Rightarrow R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m (R_F \| r_\pi)(R_C \| R_F \| r_o) \quad \text{— (1)}$$

$$R_{in,open} = (R_F \| r_\pi) \qquad R_{out,open} = (R_C \| R_F \| r_o)$$

- Feedback factor k:

$$k = \frac{v_x}{i_x} = -\frac{1}{R_F}$$

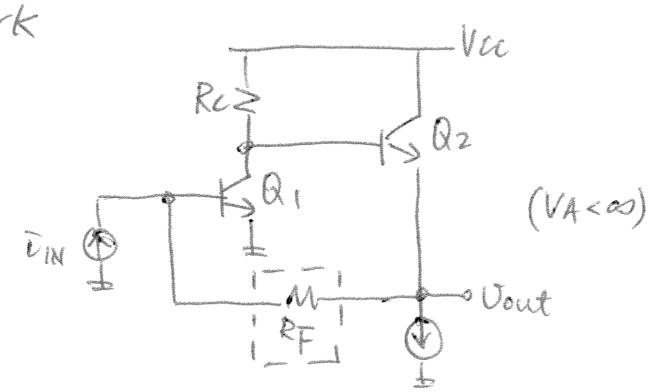


$$\therefore \quad R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{-g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{1 + \frac{g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{R_F}}$$

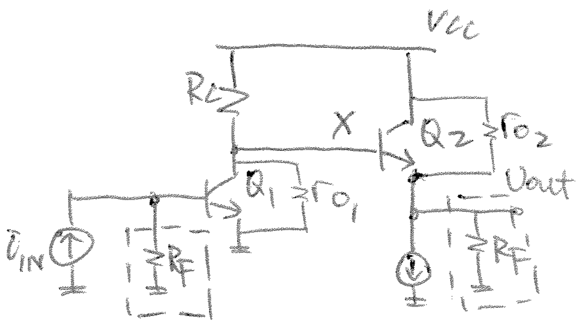
$$R_{in,CLOSED} = \frac{(R_F \| r_\pi)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{(R_C \| R_F \| r_o)}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

51. The feedback network consists of R_F.



Using the method discussed in lecture, break the circuit as follows:



This is the open-loop circuit with consideration of I/O loading.

- Gain of common-emitter stage:

$$\frac{v_X}{i_{IN}} = -g_{m1}(R_F \| r_{\pi 1}) \times \left\{ R_C \| r_{o1} \| \left[r_{\pi 2} + (\beta_2 + 1)(R_F \| r_{o2}) \right] \right\}$$

- Gain of emitter-follower stage:

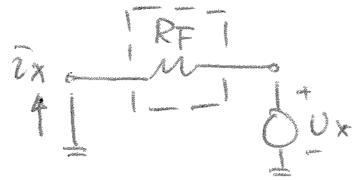
$$\frac{v_{out}}{v_X} = \frac{g_{m2}(R_F \| r_{o2})}{1 + g_{m2}(R_F \| r_{o2})}$$

$$\Rightarrow R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X} \qquad \text{—— (1)}$$

$R_{in,OPEN} = R_F \| r_{\pi 1}$ $\qquad$ $R_{out,OPEN} \cong R_F \| r_{o2} \| \frac{1}{g_{m2}}$

- Feedback factor k:

$$k = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



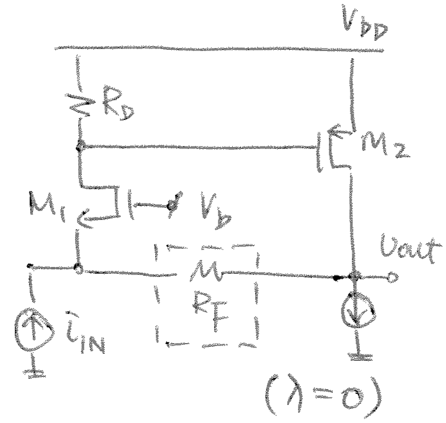
$$\therefore \; R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{R_{O.L.}}{1 - R_{O.L.}/R_F}$$

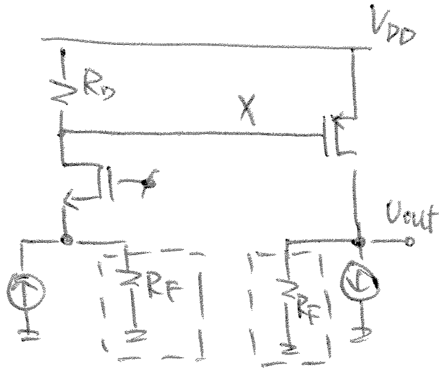
$$R_{in,CLOSED} = \frac{(R_F \,||\, r_{\pi_1})}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{R_F \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

52.

(a) Breaking the feedback loop results in the following circuit:



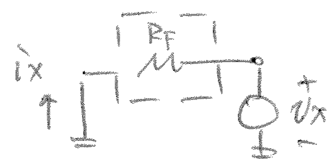


$$R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X}$$

$$= g_{m_1} R_D \left(\frac{1}{g_{m_1}} \| R_F\right) \times (-g_{m_2} R_F)$$

$$R_{in,OPEN} = \frac{1}{g_{m_1}} \| R_F \qquad R_{out,OPEN} = R_F$$

- Feedback factor k:



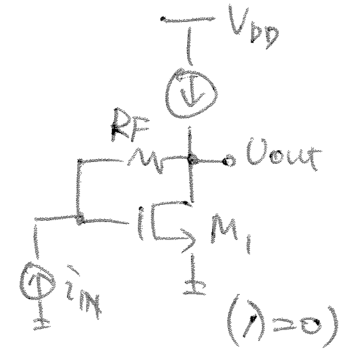
$$k = \frac{v_X}{i_X} = -\frac{1}{R_F}$$

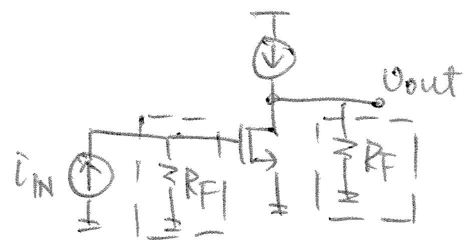
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times k} = \frac{-g_{m_1} g_{m_2} R_D R_F \left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{in,CLOSED} = \frac{\left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{out,CLOSED} = \frac{R_F}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

(b) Breaking the feedback loop results in the following circuit:

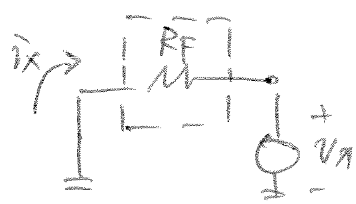




$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m R_F R_F = -g_m R_F^2$$

$R_{in,OPEN} = R_F$ $\quad$ $R_{out,OPEN} = R_F$

- Feedback factor K:



$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$

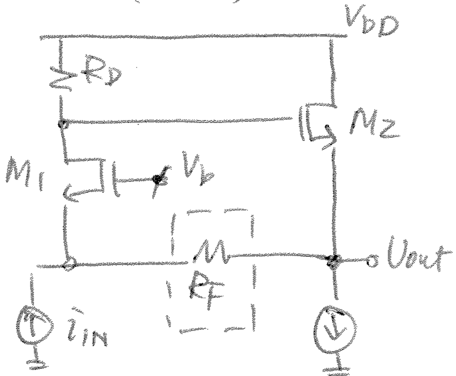
$$\Rightarrow R_{C.L.} = \frac{-g_m R_F^2}{1 + g_m R_F}$$

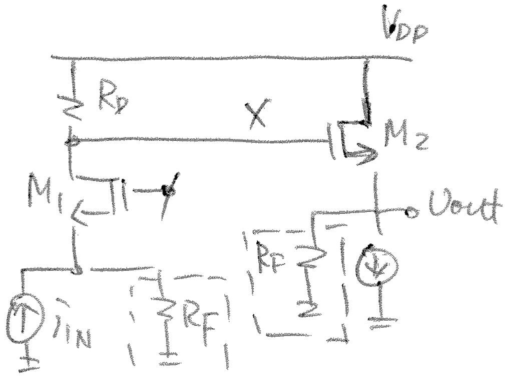
$$R_{in,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

$$R_{out,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

(c) Breaking the feedback loop results in the following circuit:

($\lambda = 0$)





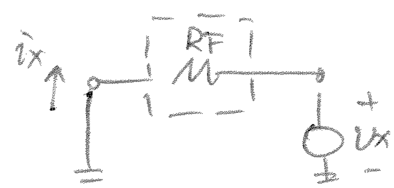
$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = \frac{v_x}{i_{IN}} \cdot \frac{v_{out}}{v_x}$$

$$= g_{m1} R_D \left(\frac{1}{g_{m1}} \| R_F\right) \times g_{m2} \left(R_F \| \frac{1}{g_{m2}}\right)$$

$$R_{in,OPEN} = \left(\frac{1}{g_{m1}} \| R_F\right) \qquad R_{out,OPEN} = \left(R_F \| \frac{1}{g_{m2}}\right)$$

- Feedback factor K:

$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



(Note: Feedback is positive.)

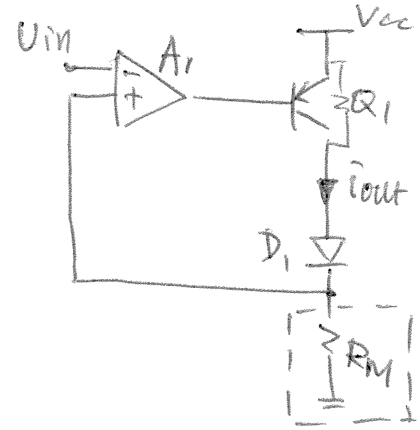
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K}$$

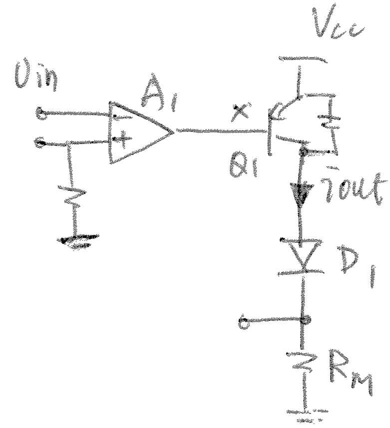
$$= \frac{g_{m1} g_{m2} R_D \left(1/g_{m1} \| R_F\right)\left(1/g_{m2} \| R_F\right)}{1 - g_{m1} g_{m2} \left(\frac{R_D}{R_F}\right)\left(\frac{1}{g_{m1}} \| R_F\right)\left(1/g_{m2} \| R_F\right)}$$

$$R_{IN,CLOSED} = \frac{\left(1/g_{m1} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{\left(1/g_{m2} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}}$$

53. Breaking the feedback network (i.e. R_M) results in the following circuit:

$(V_A < \infty)$



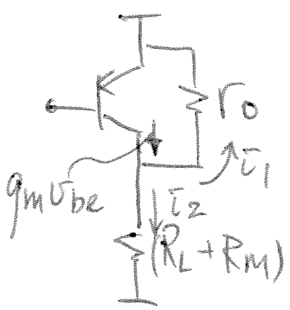


$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \times \frac{v_x}{v_{in}} \quad \text{—(1)}$$

$$= \underbrace{g_{m_1} \times \frac{[(R_L + R_M) \| r_{o_1}]}{(R_L + R_M)}}_{} \times (-A_1)$$

↖ (current division)

Note: current $(g_m v_{be})$ splits between r_o & $[R_L$ (impedance of D_1) $+ R_M]$



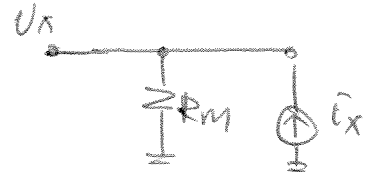
$$g_m v_{be} = i_1 + i_2$$

$R_{in, OPEN} \to \infty$

$R_{out, OPEN} = r_{o_1} + R_M$

- Feedback factor K:

$$K = \frac{v_x}{i_x} = R_M$$



$$\therefore\ G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.} \times K} = \frac{G_{O.L.}}{1 + G_{O.L.} \times R_M}$$

$$R_{in,\,CLOSED} \rightarrow \infty$$

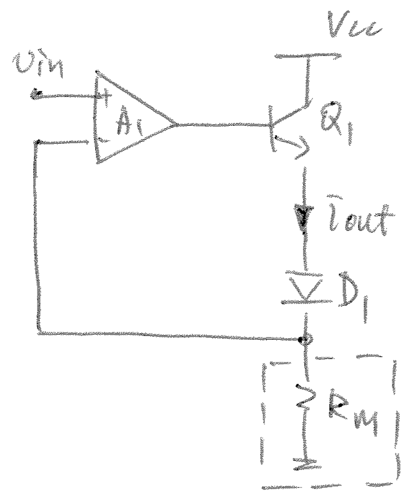
$$R_{out,\,CLOSED} = (r_{o1} + R_M)(1 + G_{O.L.} \times R_M)$$

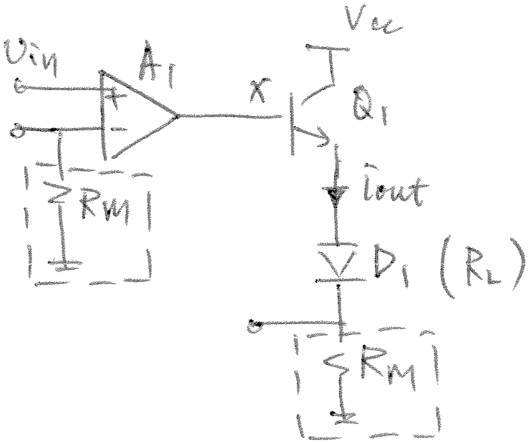
where $G_{O.L.}$ is given by (1)

54.

(a) Breaking the feedback loop results in the following circuit:

$(V_A \to \infty)$

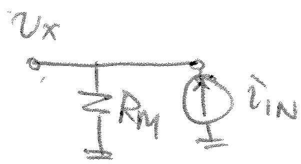




$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \cdot \frac{v_x}{v_{in}} = \frac{g_{m1}(R_L + R_M)}{1 + g_{m1}(R_L + R_M)} \times A_1$$

$R_{in,OPEN} \to \infty$ $\qquad$ $R_{out,OPEN} \approx R_M + \frac{1}{g_{m1}}$

– Feedback factor K:

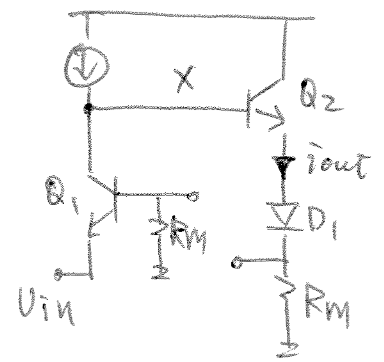


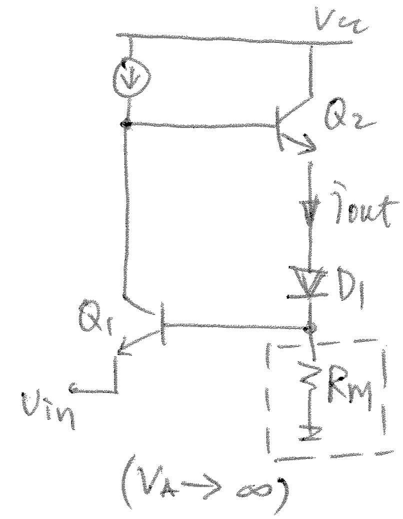
$$K = \frac{v_x}{i_x} = R_M$$

$$G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.}K} = \frac{G_{O.L.}}{1 + G_{O.L.}R_M}$$

$R_{in,CLOSED} \to \infty$ $\qquad$ $R_{out,CLOSED} = (R_M + \frac{1}{g_{m1}}) \times (1 + G_{O.L.}R_M)$

(b) Breaking the feedback loop results in the following circuit:



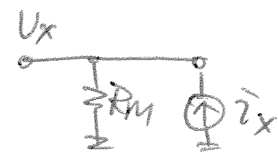


$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \times \frac{v_x}{v_{in}} \cong \frac{g_{m2}(R_L + R_M)}{1 + g_{m2}(R_L + R_M)} \times g_{m1}\left[r_{\pi 2} + (\beta_2 + 1)(R_L + R_M)\right]$$

$$R_{in,OPEN} = \frac{r_{\pi 1} + R_M}{\beta_1 + 1} \approx \frac{1}{g_{m1}} + \frac{R_M}{\beta_1 + 1}$$

$$R_{out,OPEN} = \frac{1}{g_{m2}} + R_M$$

- Feedback Factor K:



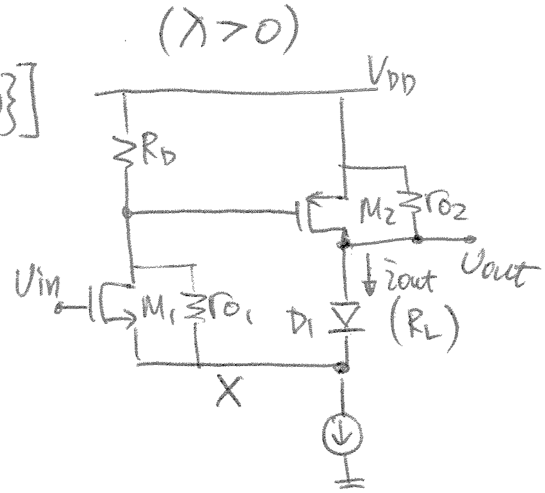
$$K = \frac{v_x}{i_x} = R_M$$

$$\Rightarrow G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.} \times K} = \frac{G_{O.L.}}{1 + G_{O.L.} R_M}$$

$$R_{in,CLOSED} = \left(\frac{1}{g_{m1}} + \frac{R_M}{\beta_1 + 1}\right)\left(1 + G_{O.L.} R_M\right)$$

$$R_{out,CLOSED} = \left(\frac{1}{g_{m2}} + R_M\right)\left(1 + G_{O.L.} R_M\right)$$

55.



$$v_{out} = \left[-g_{m1}(v_{in}-v_x)R_D\right]\cdot\left[-g_{m2}\left\{r_{o2}\|\left(\frac{1}{g_{m1}}+R_L\right)\right\}\right]$$

(ignore r_{o1} for now) — ①

$$\frac{v_{out}-v_x}{R_L} = i_{out}$$

$$\Rightarrow v_{out} = i_{out}R_L + v_x \quad \text{— ②}$$

$$i_{out} = -g_{m1}(v_{in}-v_x) \Rightarrow v_{in} + \frac{i_{out}}{g_{m1}} = v_x \quad \text{— ③}$$

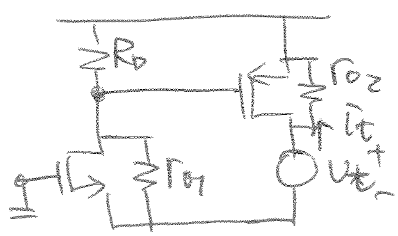
-Substitute ② & ③ into ① and solve for $\frac{i_{out}}{v_{in}}$:

$$\frac{i_{out}}{v_{in}} \cong -\frac{1}{R_L + \frac{1}{g_{m1}} + g_{m2}R_D\left\{r_{o2}\|\left(\frac{1}{g_{m1}}+R_L\right)\right\}} = G_{C.L.}$$

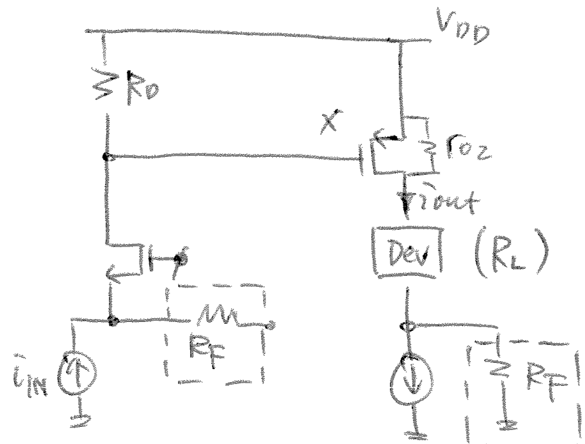
$R_{in,CLOSED} \to \infty$

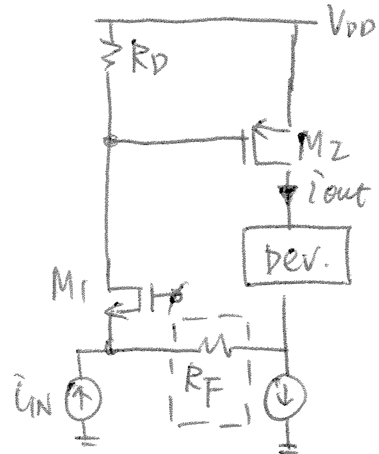
$R_{out,CLOSED}$

$$= r_{o2} + \left(\frac{r_{o1}+R_D}{1+g_{m1}r_{o1}}\right)$$



5b. Breaking the feedback loop results in the following circuit:





$(r_{o2} < \infty)$

$$A_{I,O.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

$$= -g_{m2} \times \frac{(R_L + R_F) \| r_{o2}}{(R_L + R_F)} \times R_D$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \qquad R_{out,OPEN} = r_{o2} + R_F$$

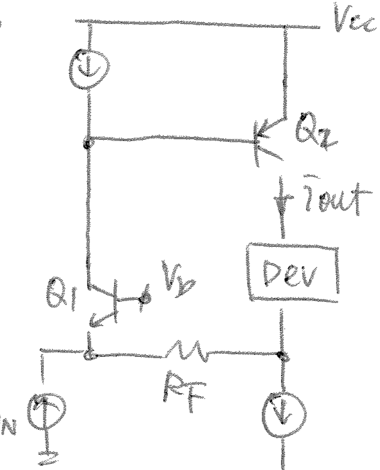
– Feedback factor K:

$$K = \frac{i_Y}{i_X} = -1$$

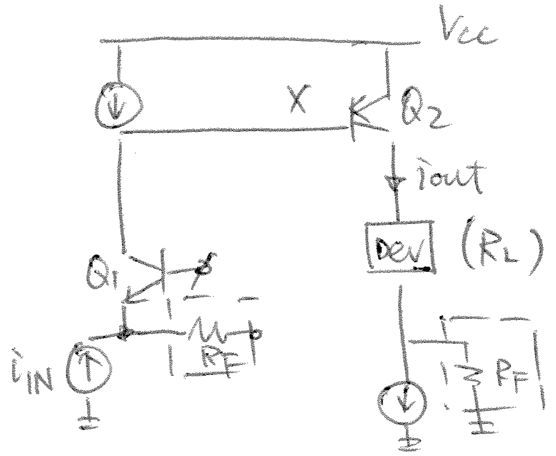
$$\Rightarrow A_{I,C.L.} = \frac{A_{I,O.L.}}{1 + A_{I,OL} \times K} = \frac{A_{I,OL}}{1 - A_{I,OL}}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 - A_{I,OL}} \qquad R_{out,CLOSED} = (r_{o2} + R_F)(1 - A_{I,OL})$$

57. Breaking the feedback loop results in the following circuit:



(VA → ∞)

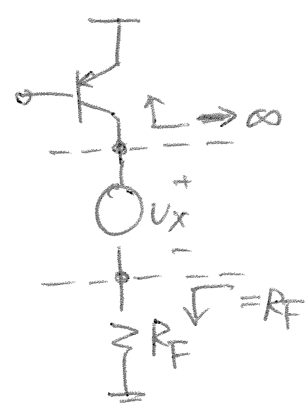


$$A_{I,o.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

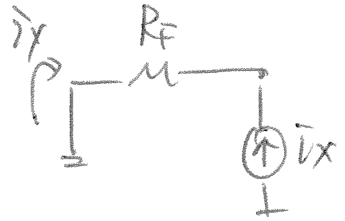
$$\cong -g_{m_2} \times r_{\pi_2} = \beta_2$$

$$R_{in,OPEN} = \frac{r_{\pi_1}}{\beta_1 + 1} \approx \frac{1}{g_{m_1}} \qquad R_{out,OPEN} \rightarrow \infty$$

(Note: $R_{out} \rightarrow \infty$ because there is no current path:)



- Feedback factor K:

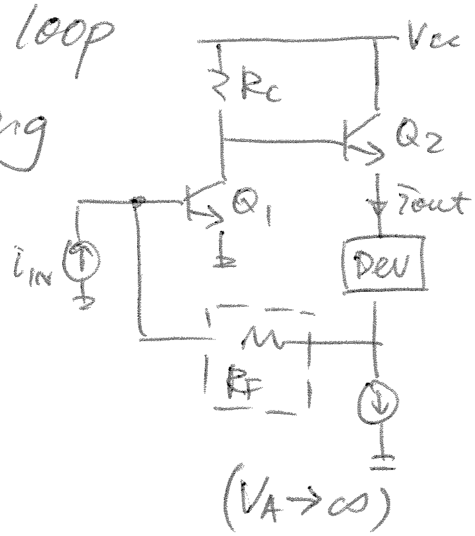


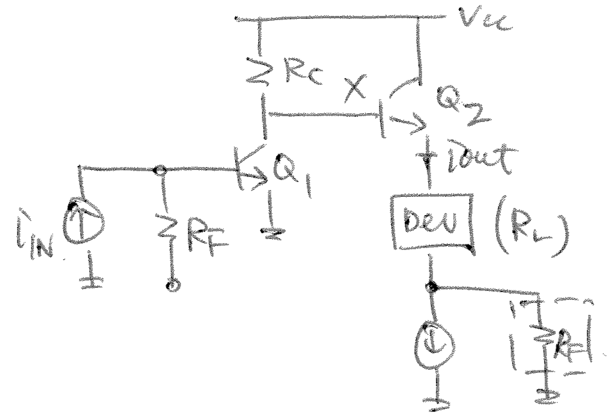
$$K = \frac{i_Y}{i_X} = -1$$

$$\Rightarrow A_{I,C.L.} = \frac{A_{I,O.L.}}{1 + A_{I,OL} \times K} = \frac{\beta_2}{1 - \beta_2}$$

$$R_{in,CLOSED} = \frac{1/g_{m_1}}{1 - \beta_2} \qquad R_{out,CLOSED} \rightarrow \infty$$

58. Breaking the feedback loop results in the following circuit:





$R_{in,open} = r_{\pi 1}$

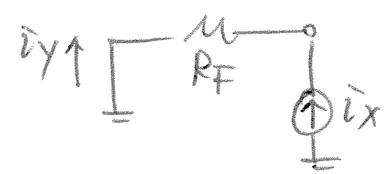
$R_{out,open} \cong \frac{1}{g_{m2}} + R_F$

$$A_{I,O.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

$$= \frac{g_{m2}(R_L + R_F)}{1 + g_{m2}(R_L + R_F)} \times \left[\beta_1 \times \left[R_C \parallel \left\{ r_{\pi 2} + (\beta_2 + 1)(R_L + R_F) \right\} \right] \right]$$

- Feedback factor K:

$$K = \frac{i_Y}{i_X} = -1$$



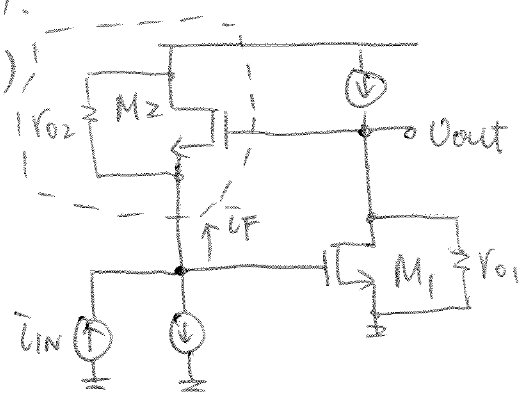
$$\Rightarrow A_{I,C.L.} = \frac{A_{I,OL}}{1 + A_{I,OL} \times K} = \frac{A_{I,OL}}{1 - A_{I,OL}}$$

$$R_{IN,CLOSED} = \frac{r_{\pi 1}}{1 - A_{I,OL}}$$

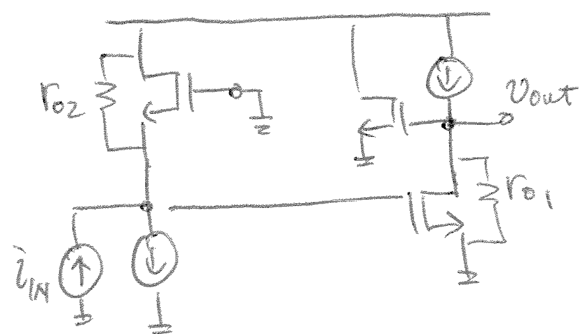
$$R_{out,CLOSED} = \left(\frac{1}{g_{m2}} + R_F\right)(1 - A_{I,OL})$$

59.

(a)



Breaking the feedback network results in the following:



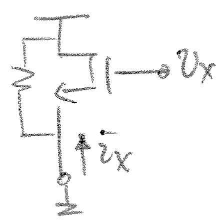
$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$= -\left(r_{o2} \parallel \frac{1}{g_{m2}}\right) g_{m1} r_{o1}$$

$$R_{in,open} = r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{out,open} = r_{o1}$$

K:



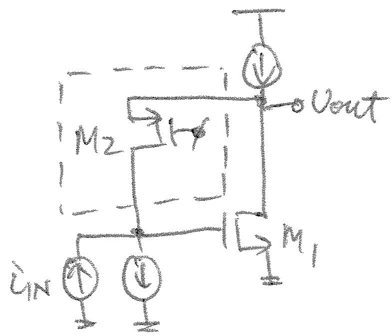
$$K = \frac{i_x}{v_x} = g_{m2}$$

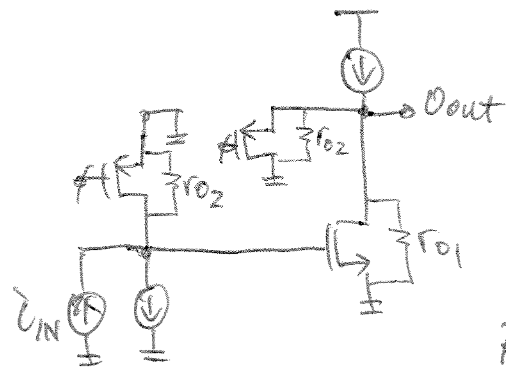
$$\therefore \quad R_{CL} = \frac{R_{OL}}{1 + R_{OL}K} = \frac{\left(r_{o2} \parallel \frac{1}{g_{m2}}\right) g_{m1} r_{o1}}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \parallel \frac{1}{g_{m2}}\right)}$$

$$R_{IN,CLOSED} = \frac{\left(r_{o2} \parallel \frac{1}{g_{m2}}\right)}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \parallel 1/g_{m2}\right)}$$

$$R_{out,CLOSED} = \frac{r_{o1}}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \parallel \frac{1}{g_{m2}}\right)}$$

(b) Breaking the feedback network results in the following:





$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$= -r_{o2} \times g_{m1} \left[r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}} \right]$$

$$R_{IN,\,OPEN} = r_{o2}$$

$$R_{out,\,OPEN} = r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}}$$

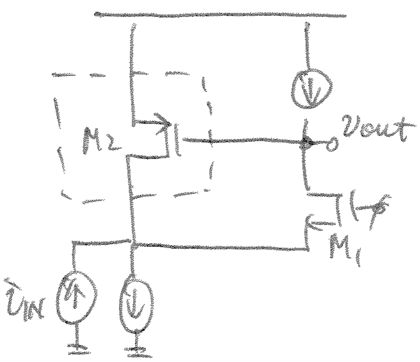
$$K = \frac{i_X}{v_X} = -g_{m2}$$

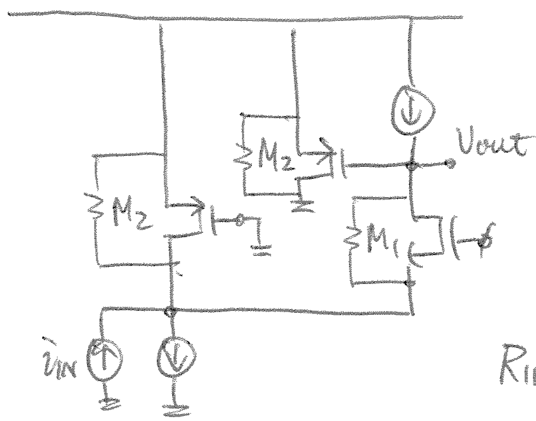
$$\therefore \; R_{CL} = \frac{R_{OL}}{1 + R_{OL}K} = \frac{-g_{m1} r_{o2} \left[r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}} \right]}{1 + g_{m1} g_{m2} r_{o2} \left[r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}} \right]}$$

$$R_{IN,\,CLOSED} = \frac{r_{o2}}{1 + R_{OL}K}$$

$$R_{out,\,CLOSED} = \frac{r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}}}{1 + R_{OL}K}.$$

(c) Breaking the feedback network results in the following:





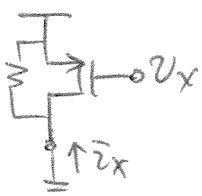
$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$\approx \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})$$

$$R_{in,open} \cong \frac{1}{g_{m1}} \parallel r_{o2}$$

$$R_{out,open} \simeq r_{o2} + r_{o1}(1+g_{m1}r_{o2})$$

K:



$$K = \frac{i_x}{v_x} = g_{m2}$$

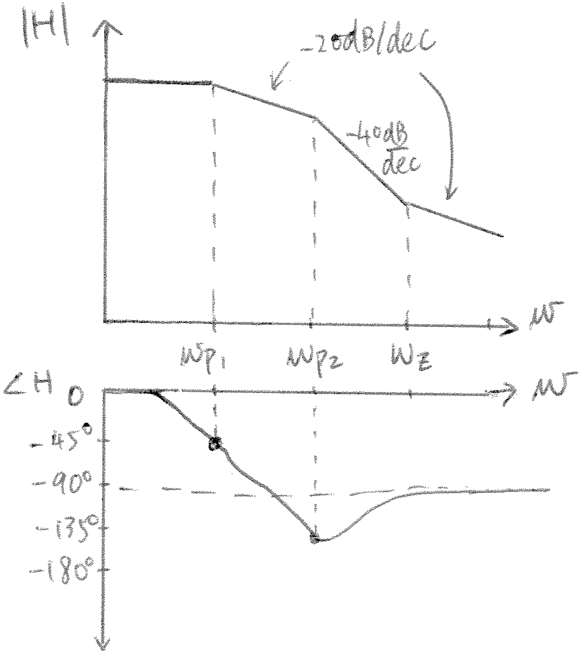
$$\therefore \; R_{CL} = \frac{R_{OL}}{1+R_{OL}K} \simeq \frac{\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})}{1+g_{m2}\left(\frac{1}{g_{m1}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})}$$

$$R_{IN,CLOSED} = \frac{\frac{1}{g_{m1}} \parallel r_{o2}}{1+R_{OL}\cdot K}$$

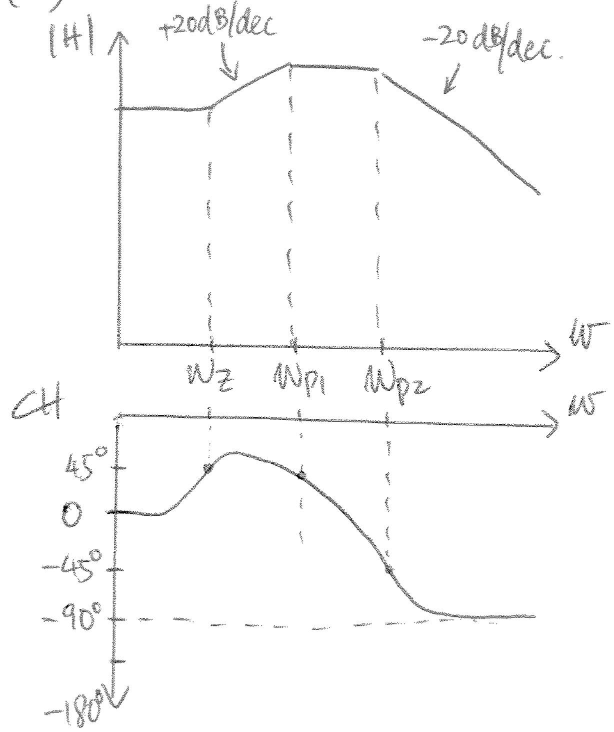
$$R_{out,CLOSED} = \frac{r_{o2}+r_{o1}(1+g_{m1}r_{o2})}{1+R_{OL}\cdot K}$$

60.

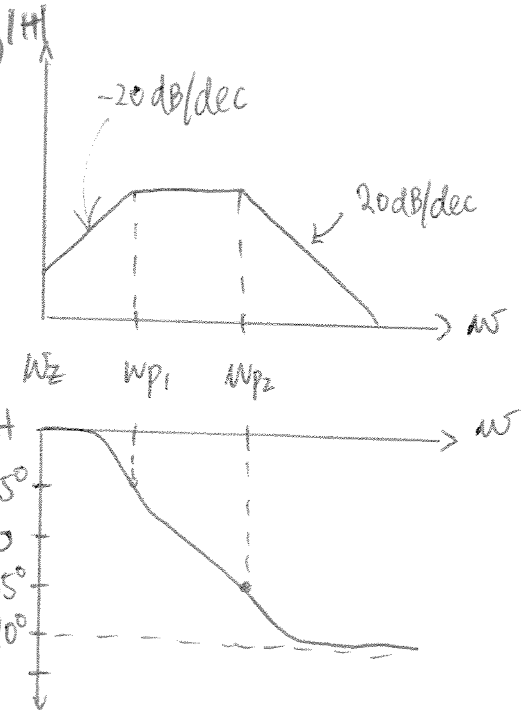
(a)


|H|
-20dB/dec
-40dB/dec
ω
ω_{p1}
ω_{p2}
ω_z
∠H
0
-45°
-90°
-135°
-180°
ω

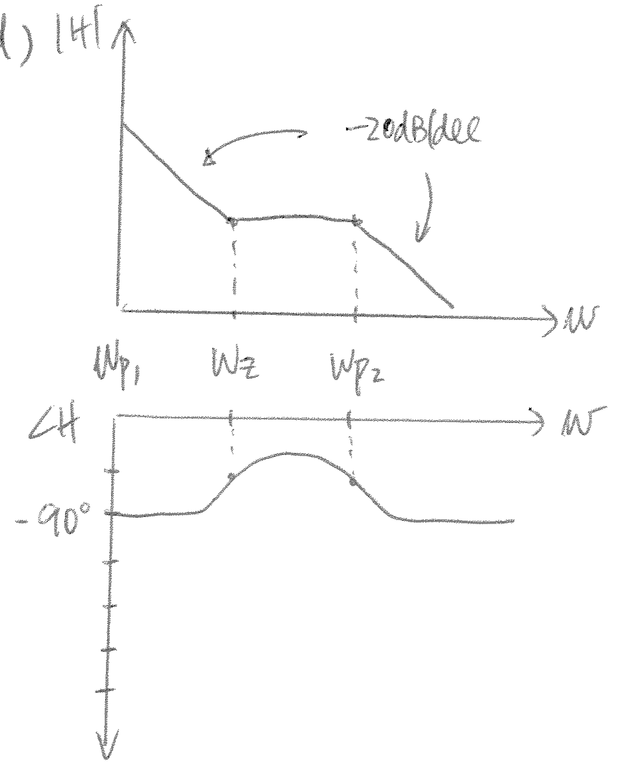
(b)


|H|
+20dB/dec
-20dB/dec.
ω
ω_z
ω_{p1}
ω_{p2}
∠H
ω
45°
0
-45°
-90°
-180°

(c)


|H|
-20dB/dec
20dB/dec
ω
ω_z
ω_{p1}
ω_{p2}
∠H
ω
45°
0
-45°
-90°

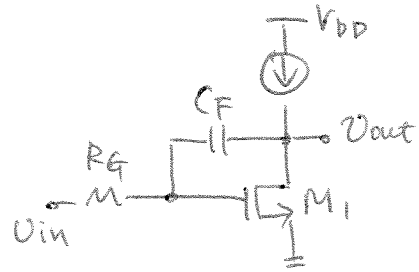
(d)


|H|
-20dB/dec
ω
ω_{p1}
ω_z
ω_{p2}
∠H
ω
-90°

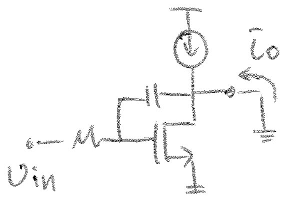
61. As w_z comes closer to w_{p_1} or w_{p_2}, it cancels out the effect (i.e. −20dB/dec decrease) — pole-zero cancellation. It would appear as if nothing occured at that overlapping frequency.

62.

Calculate $G_m(s)$ of circuit:

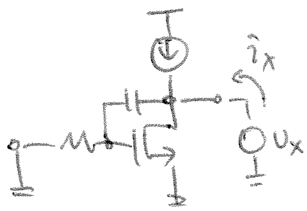


$(\lambda = 0)$



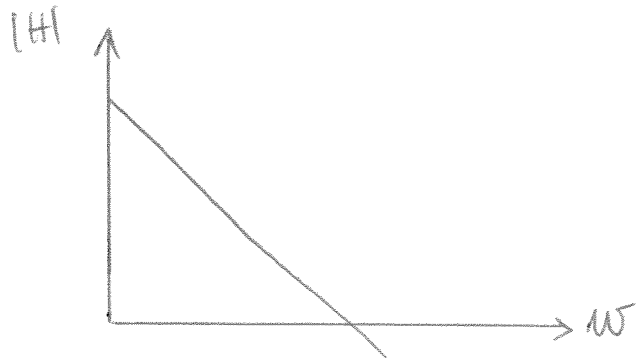
$$G_m = \frac{i_o}{V_{in}}$$

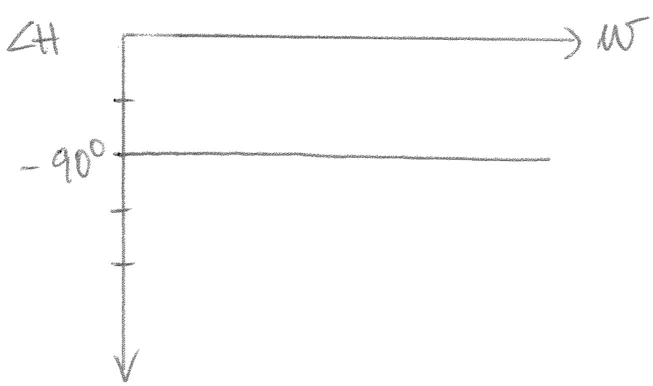
$$= g_{m1} \times \frac{1}{1 + sC_F R_G}$$



$$Z_{out} = \frac{V_x}{i_x} = \frac{1 + sC_F R_G}{sC_F \times (1 + g_{m1} R_G)}$$

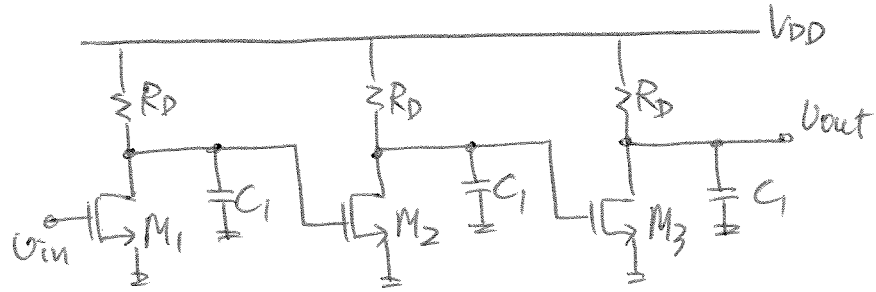
$$\Rightarrow H(s) = G_m \times Z_{out} = \frac{g_{m1}}{g_{m1} R_G + 1} \times \frac{1}{sC_F} = \frac{1}{(g_{m1} R_G + 1)\, s\left(\frac{C_F}{g_{m1}}\right)}$$





63. By Nyquist Criterion, decreasing K $(K>0)$ eventually leads to $|KH| < 1$ at $\angle H = -180°$, which implies stability.

64.



$(M_1 = M_2 = M_3)$

$$H(s) = \frac{v_{out}}{v_{in}}(s) = \frac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3} \qquad \text{where } \omega_p = \frac{1}{R_D C_1}$$

$$\Rightarrow \angle H(j\omega) = \angle(-g_m R_D)^3 - \angle\left(1 + j\frac{\omega}{\omega_p}\right)^3$$

$$= 0 - 3\,\mathrm{TAN}^{-1}\left(\frac{\omega}{\omega_p}\right)$$

$$\therefore \angle H\Big|_{\omega = 0.1\omega_p} = -3\,\mathrm{TAN}^{-1}\left(\frac{0.1\,\omega_p}{\omega_p}\right) \cong -17.1^\circ$$

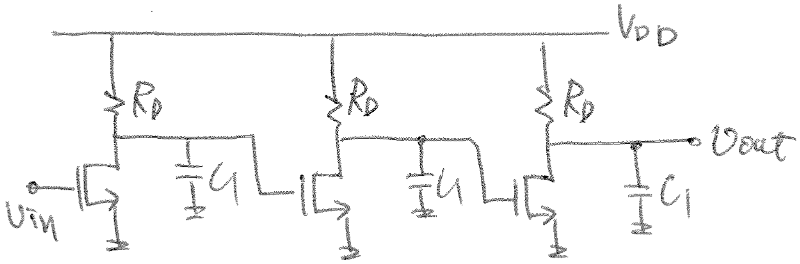
65. $H(s) = \dfrac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3}$ $\qquad (M_1 = M_2 = M_3)$

$$\Rightarrow |H|\Big|_{\omega=\omega_p} = \frac{|g_m R_D{}^3|}{\left|\left(1 + j\frac{\omega_p}{\omega_p}\right)^3\right|} = \frac{(g_m R_D)^3}{\left(\sqrt{1+1}\right)^3} = \frac{(g_m R_D)^3}{\sqrt{8}}$$

$$\Rightarrow 20 \log |H|\Big|_{\omega=\omega_p} = 20 \log (g_m R_D)^3 - 20 \log \sqrt{8}$$
$$\cong 20 \log (g_m R_D)^3 - (9 \text{ dB})$$

$\therefore$ $|H|$ falls by 9 dB due to the three coincident poles.

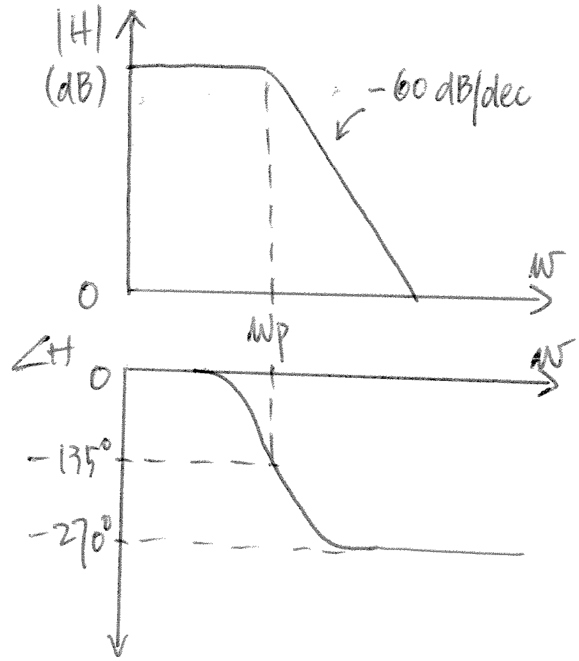
66.



$$\omega_p = \frac{1}{R_D C_1}$$

$$k = 0.1$$

$$H(j\omega) = \frac{(-g_m R_D)^3}{\left(1 + j\frac{\omega}{\omega_p}\right)}$$



$$\angle H = -3\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

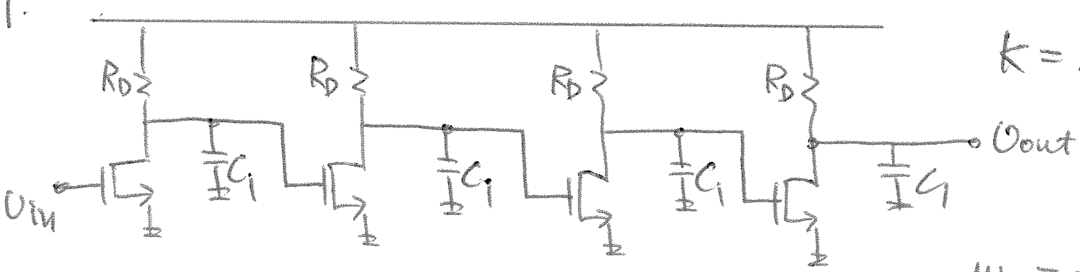
- Need to guarantee that $|kH| < 1$ when $\angle H = -180°$:

$$\angle H = -180° = -3\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \Rightarrow \omega = \sqrt{3}\,\omega_p$$

$$|kH(j\omega)|_{\omega=\omega_p\sqrt{3}} = \frac{(g_m R_D)^3}{(\sqrt{1+3})^3} \times 0.1 < 1$$

$$\Rightarrow g_m R_D < \sqrt[3]{80} \approx 8.94$$

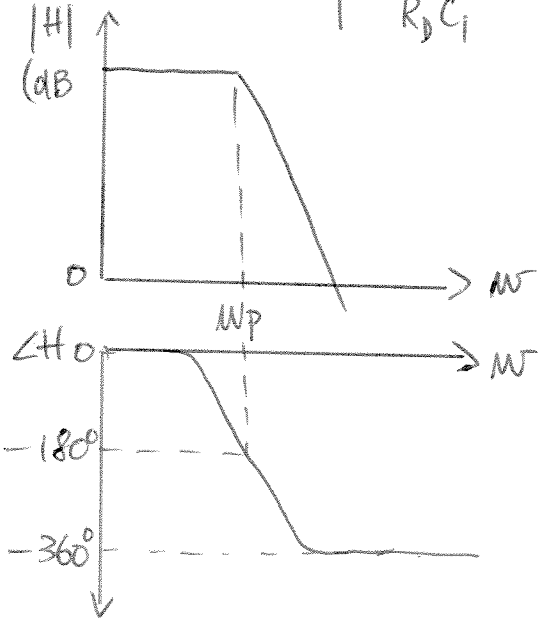
67.



$K = 1$

$\omega_p = \dfrac{1}{R_D C_1}$

$$H(j\omega) = \frac{(g_m R_D)^4}{\left(1 + j\frac{\omega}{\omega_p}\right)^4}$$



$$\angle H = -4 \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

- To guarantee stability,

$|KH| < 1$ when $\angle H = -180°$.

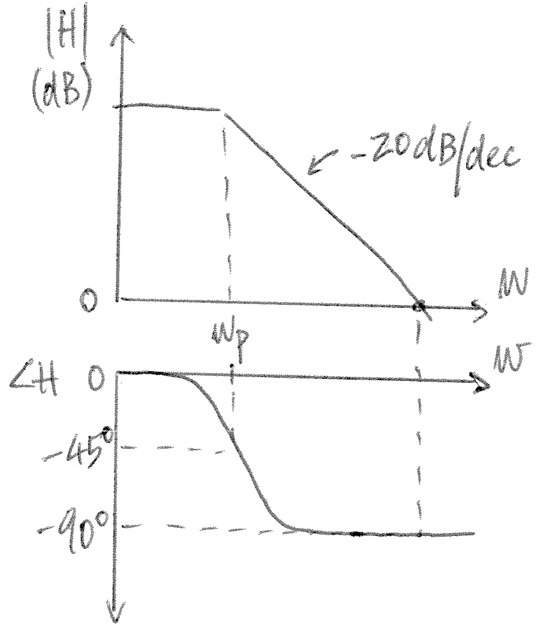
$$\angle H = -180° = -4 \tan^{-1}\left(\frac{\omega}{\omega_p}\right) \Rightarrow \omega = \omega_p$$

$$|KH|\Big|_{\omega=\omega_p} = \frac{(g_m R_D)^4}{(\sqrt{1+1})^4} < 1$$

$$\Rightarrow g_m R_D < \sqrt{2}$$

This four-pole system implies a lower upper-limit ($=\sqrt{2}$) on $g_m R_D$, which makes sense since $|H|$ drops faster here.

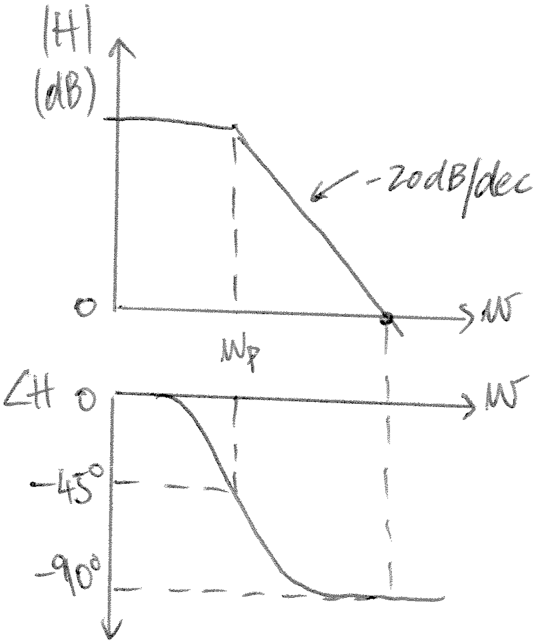
68. $H(s) = \dfrac{A_o}{1 + \dfrac{s}{\omega_p}}$ $K = 1.$



∴ Phase margin
$= 180° - 90° = 90°$

(i.e. system is stable)

69. $H(s) = \dfrac{A_0}{1 + \frac{s}{\omega_p}}$ $K = 0.5$



Phase margin = 90°
(independent of K for one-pole systems.)

70. All three scenarios will become stable eventually (depending on how far w_{Qx} is from w_{Px}, & $w_{Qx} < w_{Px}$.)

71. In the $20 \cdot \log|KH|$ vs. $\log|w|$ plot, the magnitude plot decreases at a rate of 20 dB/dec. between w_{P1} & w_{P2}.

$K=1 \Rightarrow 20\log|H|$

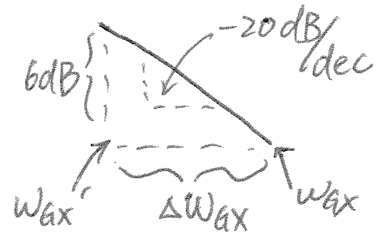
$K=0.5 \Rightarrow 20\log|\frac{H}{2}| = 20\log|H| - 6dB$

$\therefore$ K=0.5 induces a 6dB decrease on the magnitude plot at all frequencies.

$\Rightarrow \Delta w_{Gx}[\log] = \frac{6dB}{20dB} \approx -0.3$

$\frac{w_{Gx}}{w_{Gx}'} = 10^{\frac{6dB}{20dB}} \approx 0.5$

$\Rightarrow w_{Gx}' = \frac{w_{Gx}}{0.5} = 2w_{Gx}$



$\Delta w_{Gx} = \log w_{Gx} - \log w_{Gx}' = \log \frac{w_{Gx}}{w_{Gx}'}$

$\angle H = -\tan^{-1}\left[w\left(\frac{1}{w_{P1}} + \frac{1}{w_{P2}}\right)\right] \approx -\tan^{-1}\left(\frac{w}{w_{P1}}\right) \quad (w_{P2} \gg w_{P1})$

$\Rightarrow$ Phase plot stays the same with $K=0.5$.

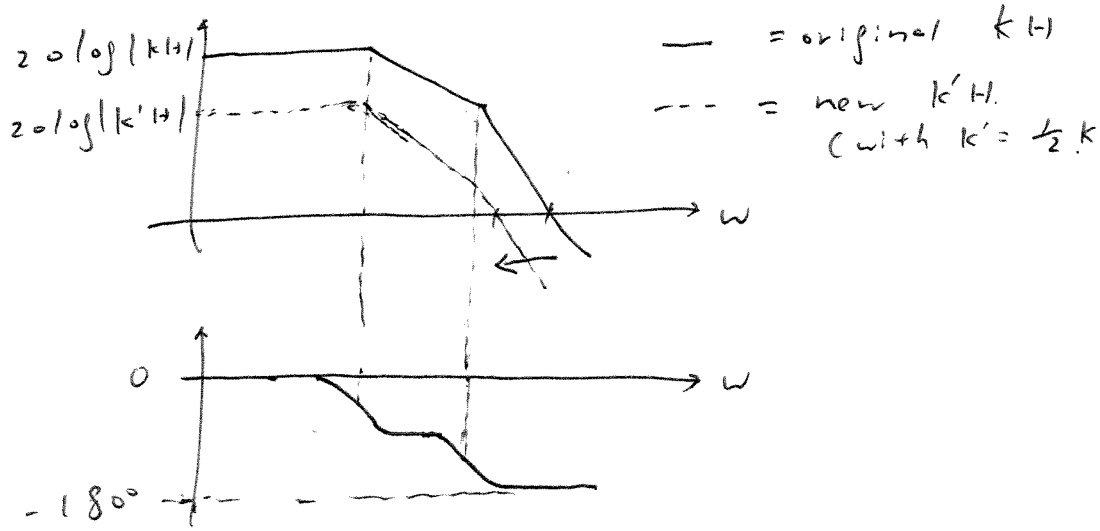
In original example $\angle H(w_{P2}) \cong -\tan^{-1}\left(\frac{w_{Gx}}{w_{P1}}\right) = -135°$

$\angle H(w_{Gx}') = \angle H(2w_{Gx}) \cong -\tan^{-1}\left(\frac{2w_{Gx}}{w_{P1}}\right)$

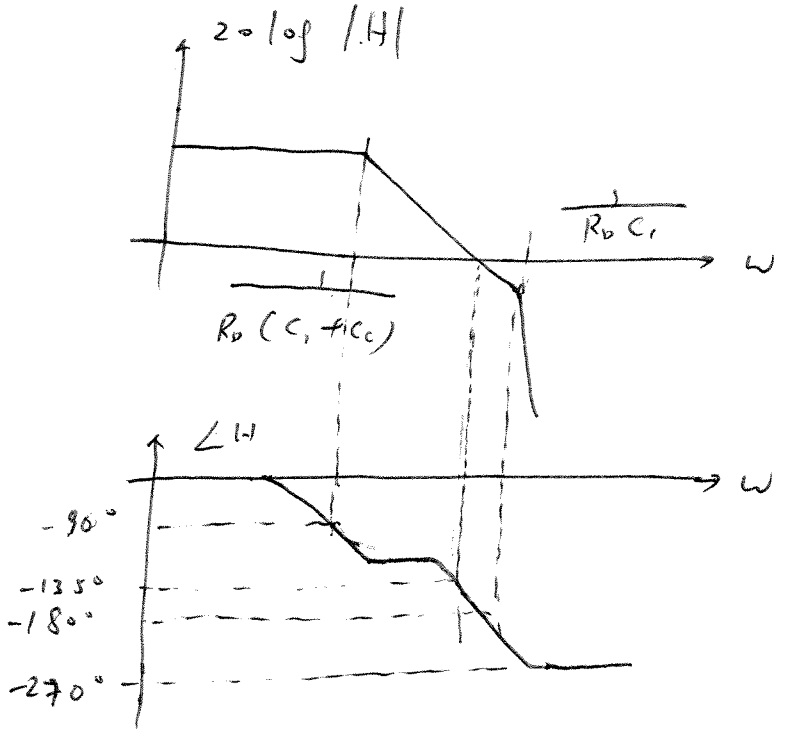
$\Rightarrow \angle H(w_{Gx}') = -\tan^{-1}[2\tan 135°] \cong -116°$

$\Rightarrow$ Phase Margin $= 180° + \angle H(w_{Gx}') \approx 63°$

(72) When k drops by a factor of 2, the phase margin improves. This is because a lower k corresponds to shifting the amplitude part of kH down by 6 dB. (The phase $\angle kH$ remains unchanged, since phase is only depends on pole location and is independent of amplitude of kH.)
Thus, the gain $|kH|$ drops to 0 dB at a lower frequency. This results in a larger phase margin.



(73) With the compensation capacitor C_c in place, the pole associated with C_c becomes dominant (i.e. at a lower frequency than the other 2 poles at $\frac{1}{R_D C_1}$).



(74) Open-loop gain, $A_o = g_{mN}(r_{oN} // r_{op})$

$$= 50$$

Closed-loop gain, $\dfrac{V_{out}}{V_{in}} = \dfrac{g_{mN}(r_{oN} // r_{op})}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}(r_{oN} // r_{op})}$

$$= \frac{50}{1 + \frac{R_2}{R_1 + R_2} \times 50}$$

$$= 4$$

$$\therefore \quad 4 = \frac{50}{1 + \frac{R_2}{R_1 + R_2} \times 50}$$

$$1 + \frac{50 R_2}{R_1 + R_2} = 12.5$$

$$\frac{R_2}{R_1 + R_2} = 0.23$$

Choosing $r_{o2} // r_{o4} = 5\,k\Omega$.

i.e. $g_{mN} = 10\,mS$ //

and $R_1 + R_2 \approx 50\,k\Omega$.

$\therefore \quad R_2 = 0.23 \times 50\,k\Omega$

$= 11.5\,k\Omega$ //

and $R_1 = 38.5\,k\Omega$ //

(75) Open loop gain, $A_o = g_m R_D$

(assuming $R_1 + R_2$ is very large.)

i.e. $g_m R_D = 10$

$$\text{Closed-loop gain} = \frac{g_m R_D}{1 + \left(\frac{R_2}{R_1 + R_2}\right) g_m R_D}$$

$$= 2$$

$$\therefore \frac{10}{1 + \left(\frac{R_2}{R_1 + R_2}\right) \times 10} = 2$$

$$\frac{R_2}{R_1 + R_2} = 0.4$$

$$\text{Closed-loop input impedance} = \frac{1}{g_m}\left[1 + \frac{R_2}{R_1 + R_2} \times 10\right]$$

$$= 50\Omega.$$

$$\therefore \frac{1}{g_m} \times 5 = 50$$

$$g_m = 0.1S$$

$$\therefore R_D = 100\Omega.$$

$$\therefore R_1 + R_2 = 10 \times 100\Omega$$

$$= 1k\Omega.$$

$$\therefore R_2 = 400\Omega$$

$$R_1 = 600\Omega$$

(76) Open-loop gain $= R_{D1}(g_{m2} R_{D2})$

$= 10\,k\Omega$

Closed-loop gain $= \dfrac{10\,k\Omega}{1 + \dfrac{10\,k\Omega}{R_F}}$

$= 1\,k\Omega.$

$$\frac{10}{1 + \dfrac{10\,k\Omega}{R_F}} = 1\,k\Omega$$

$\therefore R_F = 1.11\,k\Omega$ //

Closed-loop input impedance $= \dfrac{1}{g_{m1}}(1+9)^{-1}$

$= 50\Omega.$

$\therefore g_{m1} = 2mS$ //

Closed-loop output impedance $= \dfrac{R_{D2}}{10}$

$= 200\Omega.$

$\therefore R_{D2} = 2000\Omega$ //

$\because R_{D1} = 1\,k\Omega,$

$\therefore g_{m2} = 5\,mS$ //

(77) Assuming R_F is very large,

Open-loop gain $= R_D \left(g_{m2} R_L \right)$

$= 10\,k\Omega$

Closed-loop gain $= \dfrac{10\,k\Omega}{1 + \dfrac{10\,k\Omega}{R_F}}$

$= 1\,k\Omega.$

$\therefore R_F = 1.11\,k\Omega$ //

Closed-loop input impedance $= \frac{1}{g_{m1}} (1+9)^{-1} = 50\Omega.$

$g_{m1} = 2\,mS$ //

Closed-loop output impedance $= \dfrac{R_L}{10}$

$\therefore R_L = 2000\Omega.$ //

$R_D = 1\,k\Omega.$

$\therefore g_{m2} = 5\,mS$ //

(78) a) open-loop gain $= R_c \, (g_{m2} \, R_m)$

$= 20 \text{ k}\Omega.$

$g_{m2} = \frac{I}{V_T}$

$\therefore g_{m2} = \frac{1mA}{26mV} = 38.5 \text{ mS}$

$\therefore R_c \, R_m = \frac{20 \text{ k}\Omega}{38.5 \text{ mS}}$

open-loop output impandance $= R_m$ $(\because r_o = \infty)$

$\therefore R_m = 500\Omega$ //

$R_c = 1040\Omega$ //

---

b) closed-loop gain $= \frac{20 \text{ k}\Omega}{1 + \frac{20 \text{ k}\Omega}{R_F}}$

$= 1 \text{ k}\Omega$

$\therefore R_F = 1053 \Omega$ //

c) Closed-loop input impedance $= \frac{\frac{1}{38.5 \text{ mS}}}{1 + \frac{20k}{1053}}$

$= 1.30\Omega$ //

closed-loop output impedance $\approx (500)(\frac{1}{20})$

$= 25\Omega$ //

(79) Open-loop voltage gain

$$= \frac{R_{D1}}{\frac{1}{g_{m1}} + R_1 // R_2} \cdot [g_{m2} (R_1 // R_2)]$$

$$= 20 \quad \text{——— (1)}$$

closed-loop gain $= \dfrac{20}{1 + 20\left(\frac{R_2}{R_1 + R_2}\right)} = 4$

$$\therefore \frac{R_2}{R_1 + R_2} = 0.2. \quad \text{——— (2)}$$

Open-loop output impedance $= R_1 + R_2$

$$= 2\ k\Omega.$$

$$\therefore R_2 = 0.4\ k\Omega \quad //$$

$$R_1 = 1.6\ k\Omega. \quad //$$

From (1),

$$\frac{R_{D1}}{\frac{1}{g_{m1}} + 2\ k\Omega} [g_{m2} \times 320\ \Omega] = 20$$

$$\frac{g_{m1} R_{D1}}{1 + 2k\, g_{m1}} \times g_{m2} = 0.0625$$

Setting $g_{m1} = g_{m2} = 20\ mS$. //

$$R_{D1} \approx 6400\ \Omega \quad //$$

To minimize power consumption, g_{m1} and g_{m2} should be minimized. (and R_D maximized)

(80) Since $A_{OL} = \dfrac{g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 // R_2}{\beta_2 + 1}\right)}{1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 // R_2}{\beta_2 + 1}\right)} \times g_{m2}(R_1 + R_2)$

$R_{out,closed} = \dfrac{R_1 + R_2}{1 + A_{OL}k}$

$\because A_{CL} = 2$

$\therefore 2 = \dfrac{A_{OL}}{1 + A_{OL}k}$

see $g_{m1} = g_{m2} = 50m℧$, $(R_1 + R_2) = 1k$,

$A_{OL} = \dfrac{50m℧\left(\frac{1}{50m℧} + \frac{R_1 // R_2}{101}\right)}{1 + 50m℧\left(\frac{1}{50m℧} + \frac{R_1 // R_2}{101}\right)} \times 50m℧(1k\Omega)$

$\approx \dfrac{1 + 10^{-5}(R_1 // R_2)}{2 + 10^{-5}(R_1 // R_2)} \times 50$

$\simeq 25$

$\therefore 2 = \dfrac{25}{1 + 25k}$

$\therefore k = 0.46$ $[\therefore R_1 = 460\Omega,\ R_2 = 540\Omega]$

$R_{out,closed} = \dfrac{1k\Omega}{1 + 25 \times 0.46}$

$= 80\Omega$

(81) open-loop gain $(A_{OL}) = -g_{m1}(R_F \,//\, r_{\pi 1}) \times [R_C \,//\, [r_{\pi 2} + (\beta_2 + 1) R_F]]$

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL} \frac{1}{R_F}}$$

$\because$ $A_{CL} = 1000\Omega.$

$$1000 = \frac{A_{OL}}{1 + \frac{A_{OL}}{R_F}}$$

$$R_{in,\,closed} = \frac{R_F \,//\, r_{\pi 1}}{1 + \frac{A_{OL}}{R_F}} = 50\Omega$$

$$R_{out,\,closed} = \frac{R_F}{1 + \frac{A_{OL}}{R_F}} = 50\Omega$$

setting $R_F = 1200\Omega$.

$\therefore$ $A_{OL} = 6000\,\Omega$ //

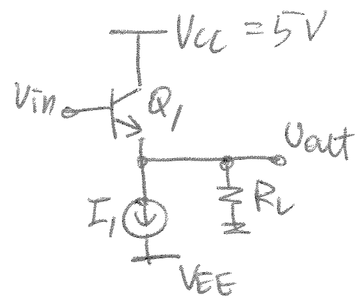
$\therefore$ $r_{\pi 1} = 400$ //

$g_{m2} = 2.5m\,℧$ //

Assume $\beta_2 = 100$, and $g_{m1} = g_{m2}$,

$R_C = 8570\Omega$ //

1. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$



$P_{LOAD} = 0.5\,W$

$R_L = 8\Omega$

(Assume Vout biased at $V_{BE(ON),1} \approx 800\,mV$

(a) $0.8 = \dfrac{g_{m1}(8\Omega)}{1 + g_{m1}(8\Omega)}$

$\Rightarrow g_{m1} = 0.5 = \dfrac{I_C}{V_T} = \dfrac{I_1}{V_T}$

$\therefore I_1 = 13\,mA$

(b) When $V_{in} = V_p = V_{CC}$, $V_{out} \approx V_{CC} - V_{BE(ON),1}$

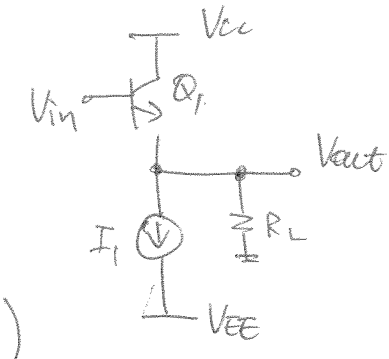
$I_{C1} = I_1 + \dfrac{V_{out}}{R_L} \Rightarrow I_{C1} = I_1 + \dfrac{5 - 0.8}{8} \approx 0.54\,A$

$\Rightarrow g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{0.54A}{0.026V} = 20.8\,S$

$\Rightarrow A_v\Big|_{V_{in}=V_p} = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L} = \dfrac{(20.8\,S)(8\Omega)}{1 + (20.8\,S)(8\Omega)} \approx 0.99$

2.

(a) $I_1 = V_p / R_L \qquad V_p \gg V_T$



$$A_v = \frac{I_C R_L}{I_C R_L + V_T}$$

$$= \frac{\frac{I_C}{I_1} V_p}{\frac{I_C}{I_1} V_p + V_T} = \frac{V_p}{V_p + V_T} \quad (\approx 1)$$

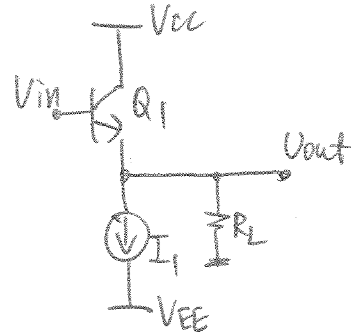
(b) When $V_{out} = V_p$, $I_{C_1} = I_1 + \frac{V_{out}}{R_L} = \frac{V_p}{R_L} + \frac{V_p}{R_L}$

$$= \frac{2V_p}{R_L}$$

$$\therefore A_v = \frac{\left(2\frac{V_p}{R_L}\right) R_L}{\left(\frac{2V_p}{R_L}\right) R_L + V_T} = \frac{2V_p}{2V_p + V_T} \quad \left(\approx \frac{2V_p}{2V_p} = 1.\right)$$

$$\Delta A_V = \frac{\frac{2V_p}{2V_p + V_T} - \frac{V_p}{V_p + V_T}}{\frac{V_p}{V_p + V_T}} = \frac{V_T}{2V_p + V_T} \quad \left(\approx \frac{V_T}{V_p}\right)$$

3. $A_v = 0.7$ $R_L = 4\Omega$



Q_1 shuts off when:

$I_1 = \frac{V_P}{R_L}$

- Suppose $V_{out} = V_p \sin\omega t$. $\left(\omega = \frac{2\pi}{T}\right)$

$$P_{R_L,AVG} = \frac{1}{T}\int_0^T \frac{(V_{out})^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2\omega t}{R_L}\,dt = \frac{1}{2} V_P^2/R_L$$

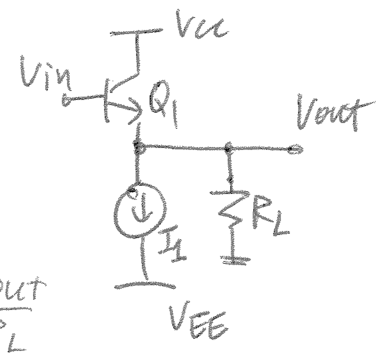
$\therefore$ Largest power (average) $= \frac{1}{2}\frac{(I_1 R_L)^2}{R_L}$

$$A_v = 0.7 = \frac{g_{m1} R_L}{1 + g_{m1} R_L} \Rightarrow g_{m1} = \frac{A_v}{(1-A_v)R_L} = \frac{0.7}{(1-0.7)(4)} = 0.58\,S$$

$$\Rightarrow I_{C1}\ (= I_1) = g_{m1} V_T = 0.015\ A$$

$$\therefore P_{AV,MAX} = \frac{1}{2} I_1^2 R_L = \frac{1}{2}(0.015\,A)^2 (4\Omega) = 0.45\,W$$

4. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$ $\left(g_m = \dfrac{I_C}{V_T}\right)$



- Q_1 shuts off when $I_1 = -\dfrac{V_{out}}{R_L}$

$\Rightarrow V_p = I_1 \times R_L$

$$g_{m1} = \frac{A_v}{(1-A_v)R_L} = \frac{I_{C1}}{V_T} \Rightarrow I_{C1} = \frac{V_T A_v}{R_L(1-A_v)} \; (= I_1)$$

- Power delivered to R_L:

$$P_{R_L} = \frac{1}{T}\int_0^T \frac{V_{out}^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2 \omega t}{R_L}\,dt$$

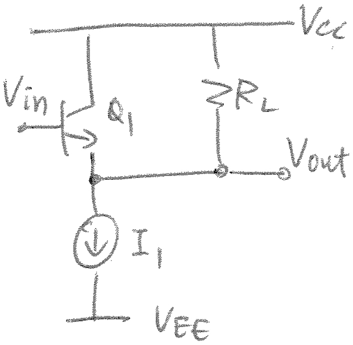
$$= \frac{1}{2}\frac{V_p^2}{R_L}$$

$\therefore$ Maximum power $= \dfrac{1}{2}\dfrac{(I_1 R_L)^2}{R_L}$

$$= \frac{1}{2}\left[\frac{V_T A_v}{(1-A_v)}\right]^2 \cdot \frac{1}{R_L}$$

5.

Assume $V_{CC} = 5V$



$I_{S1} = 5 \cdot 10^{-17} A$

$R_L = 8 \Omega$

$I_1 = 20 mA.$

(a) By KCL,

$$I_1 = I_{S1} \cdot \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) + \frac{V_{CC} - V_{out}}{R_L}$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(\frac{I_1}{I_{S1}} - \frac{V_{CC} - V_{out}}{I_{S1} R_L}\right)$$

$= 0$ (X) — no solution

$\therefore V_{out} \approx 5 - I_1 R_L = 4.84 V$

(i.e. Q_1 is off.)

(b) $(0.01) I_1 = I_1 - \frac{V_{CC} - V_{out}}{R_L}$

$\Rightarrow V_{out} = 4.84 \ V$

$$I_{C1} = (0.01) I_1 = I_{S1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right)$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(0.01 \frac{I_1}{I_S}\right)$$

$$= 4.84 + (0.026) \ln\left(0.01 \ \frac{20mA}{5 \cdot 10^{-17} A}\right)$$

$\approx 5.59 \ V$

(exceeds V_{CC})

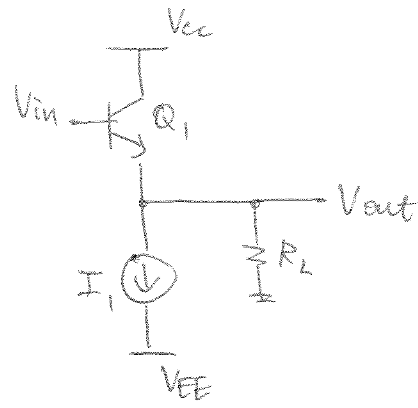
b.

(a) Calculate V_{BE} for

$V_{in} = 1\,V$:

$$I_{C_1} = I_1 + \frac{V_{out}}{R_L}$$

$$\Rightarrow I_{S_1} \cdot \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 + \frac{V_{out}}{R_L}$$



$I_{S_1} = 6 \cdot 10^{-17} A$

$R_L = 8\,\Omega$

$I_1 = 25\,mA$

$V_p = 1\,V$

Solving for Vout gives:

$V_{out} \approx 0.113\,V$

$$\therefore V_{BE}\Big|_{V_{in}=1V} = V_{in} - V_{out} = 1 - 0.113 = 0.887\,V$$

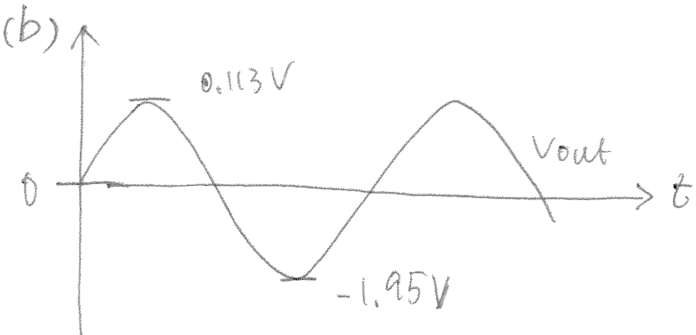
$V_{in} = -1\,V$:

$$I_{C_1} = I_1 + \frac{-V_{out}}{R_L} \Rightarrow I_{S_1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 - \frac{V_{out}}{R_L}$$

Solving for Vout gives: $V_{out} \approx -1.95\,V$

$$\therefore V_{BE}\Big|_{V_{in}=-1V} = V_{in} - V_{out} = -1 - (-1.95V) = 0.95\,V$$

(b)



7. Determine V_p such that

$$V_{BE}\Big|_{V_{in}=+V_p} - V_{BE}\Big|_{V_{in}=-V_p} = 10\,mV$$

$$\Rightarrow \left(V_p^{+} - V_{out,+}\right) - \left(V_p^{-} - V_{out,-}\right) = 10\,mV$$

$$I_s \exp\left(\frac{V_p^{+} - V_{out,+}}{V_T}\right) = I_1 + \frac{V_{out,+}}{R_L} \quad \text{—— (1)}$$

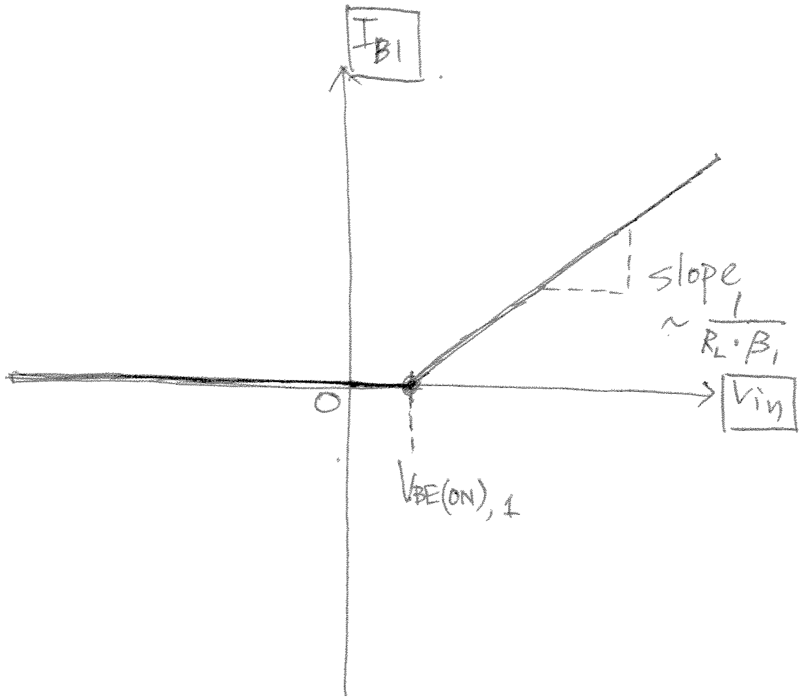
$$I_s \exp\left(\frac{V_p^{-} - V_{out,-}}{V_T}\right) = I_1 - \frac{V_{out,-}}{R_L} \quad \text{—— (2)}$$

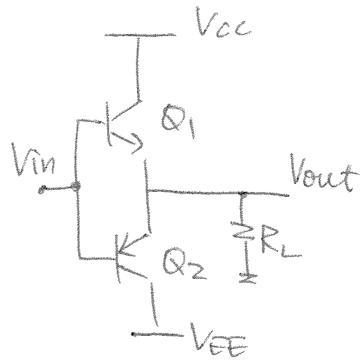
Iterate (1) & (2). This gives:

$V_p \approx 0.7\ V$

$$\Rightarrow \text{Nonlinearity} = \frac{10\,mV}{0.7 \times 2} \approx 0.007.$$

8.





β_1 : current gain of Q_1

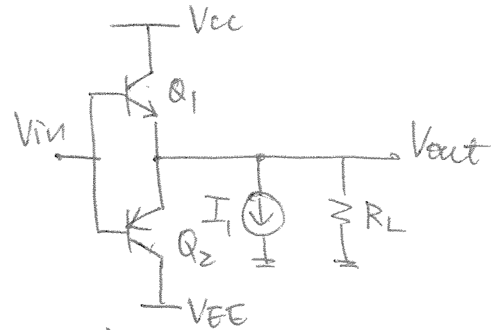
- Q_1 is on whenever $V_{in} \geq V_{BE(ON),1}$. In this region,

$$V_{out} = V_{in} - V_{BE(ON)1} \qquad I_{C_1} = \frac{V_{out}}{R_L}$$

$$\therefore I_{B_1} = \frac{I_{C1}}{\beta} = \frac{V_{out}}{\beta R_L} = \frac{V_{in} - V_{BE(ON)1}}{\beta R_L}.$$

9.

(a) To guarantee Q_1 on,



- $V_{out} \approx V_{in} - V_{BE(ON)1}$
  $= -800\,mV$

$\Rightarrow I_{C_1} = I_1 + \frac{V_{out}}{R_L}$ (Q_2 is off)

$I_{S_2} = 6 \cdot 10^{-17} A$
$R_L = 8\,\Omega$

- $I_{C_1} \geq 0 \Rightarrow I_1 + \frac{V_{out}}{R_L} \geq 0$

$\Rightarrow I_1 + \frac{-800mV}{R_L} \geq 0$

$\therefore I_1 R_L \geq 800\,mV$ ——— ①

(b) When Q_2 turns on,

$$-\frac{V_{out}}{R_L} - I_1 = I_{C_2}$$

$$\Rightarrow -\frac{V_{out}}{R_L} - \left(\frac{800\,mV}{R_L}\right) = I_{S_2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$\Rightarrow V_{out} = -R_L I_{S2} \cdot \exp\left(\frac{V_{BE2}}{V_T}\right) - 0.8$$

$$= -(8\Omega)(6 \times 10^{-17} A) \exp\left(\frac{0.8}{0.026}\right) - 0.8$$

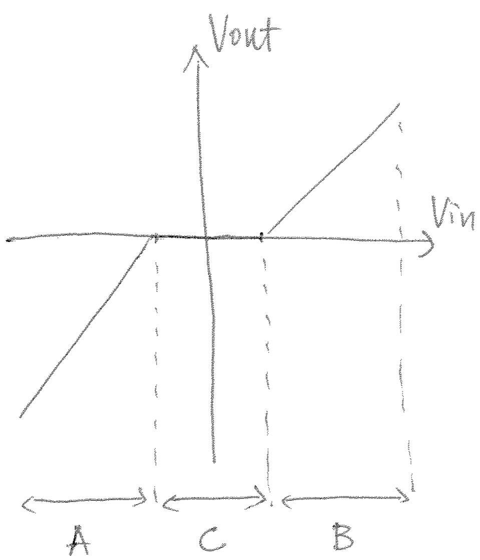
$$\approx -0.81\ V$$

$\therefore V_{in} = V_{out} - |V_{BE(ON)2}| = -0.81 - 0.8 = -1.61\ V$

10. Consider two scenarios:

- In gain regions ($|V_{in}| \geq |V_{BE(ON)}|$), V_{out} tracks V_{in}.
- In dead zone, both transistors shut off.

In both cases, V_{out} has an important role. Current source I_1 affects the input/output characteristic by modulating V_{out}:



I/O characteristic of Push-pull stage.

Consider region A:

$$I_{C2} + I_1 = \frac{-V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE2}| = |V_{out} - V_{in}|$ stays relatively constant.

(Q_2 absorbs/sinks all the currents from I_1 in order to have the same $|V_{BE2}|$)

Consider region B:

$$I_{C1} = I_1 + \frac{V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE1}| = |V_{in} - V_{out}|$ stays relatively constant.

(Q_1 provides/sources current to I_1 in order to have $|V_{BE1}|$ constant.)

Consider region C: (Dead zone).

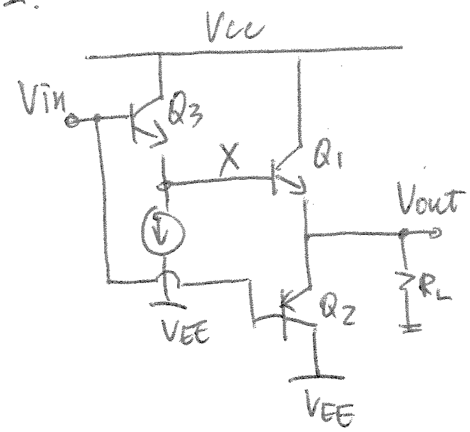
$I_1 = -\frac{V_{out}}{R_L}$ (Both transistors off)

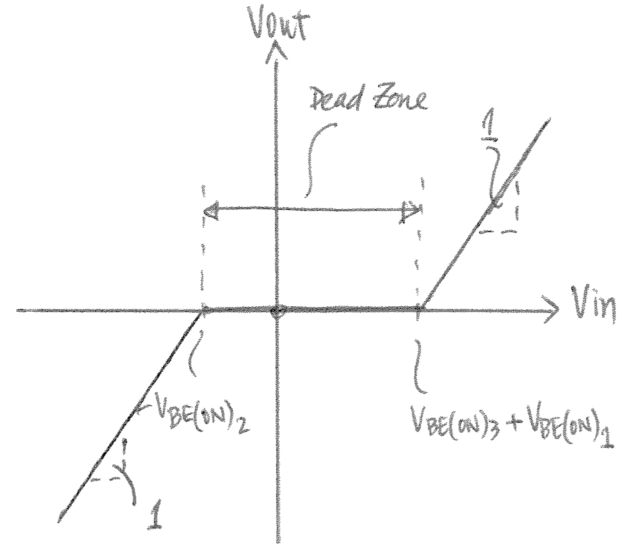
$\therefore I_1 \uparrow \Rightarrow V_{out} \downarrow$

$I_1 \downarrow \Rightarrow V_{out} \uparrow$

i.e. In the dead zone, Vout is predominantly controlled by I_1. One can use this to control Vout and effectively shift the region of dead zone.
($\because V_{out}|_{V_{in}=0} \neq 0$ anymore)

11.





Dead Zone

$= |V_{BE(ON)_2}| + V_{BE(ON)_3} + V_{BE(ON)_1}$

Analysis

- $(0 < V_{in} < V_{BE(ON)_3} + V_{BE(ON)_1})$:

  Q_1 is OFF $(V_{in} < V_{BE(ON)_1})$
  Q_2 is OFF (V_{BE_2} reverse-biased)
  $\Rightarrow V_{out} = 0$

- $(-|V_{BE(ON)_2}| < V_{in} < 0)$:

  Q_1, Q_2 OFF. $V_{out} = 0$

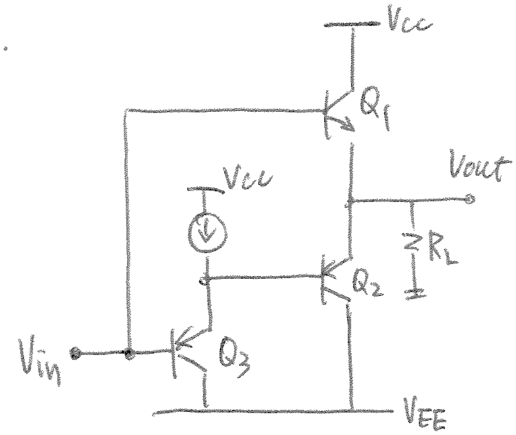
- $(V_{BE(ON)_3} + V_{BE(ON)_1} < V_{in} < V_{cc})$

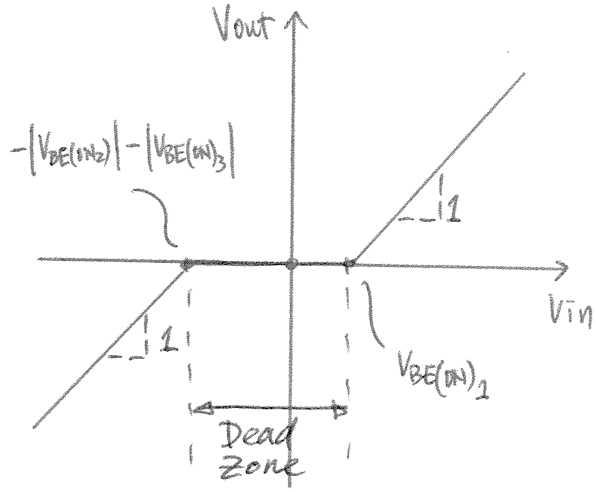
  Q_1 ON
  Q_2 OFF
  $V_{out} = V_{in} - V_{BE(ON)_3} - V_{BE(ON)_1}$

- $(-|V_{EE}| < V_{in} < -|V_{BE(ON)_2}|)$

  Q_2 ON
  Q_1 OFF
  $V_{out} = V_{in} + |V_{BE(ON)_2}|$

12.





$-V_{EE} < V_{in} < -(|V_{BE(ON)2}| + |V_{BE(ON)3}|)$:

$\Rightarrow$ Q_2, Q_3 ON, Q_1 OFF $\Big\}$ $V_{out} = V_{in} + |V_{BE(ON)3}| + |V_{BE(ON)2}|$

$-(|V_{BE(ON)2}| + |V_{BE(ON)3}|) < V_{in} < V_{BE(ON)1}$:

$\Rightarrow$ Q_1, Q_2 OFF $\Rightarrow$ $V_{out} \cong 0$

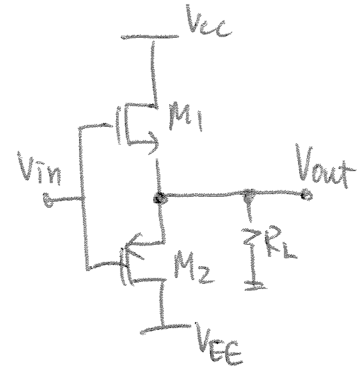
$V_{BE(ON)1} < V_{in} < V_{cc}$:

$\Rightarrow$ Q_1 ON, Q_2, Q_3 OFF $\Big\}$ $V_{out} = V_{in} - V_{BE(ON)1}$

$$\text{Dead Zone} = V_{BE(ON)1} + |V_{BE(ON)2}| + |V_{BE(ON)3}|$$

13.

(a)



Ignore body effect.

$\Rightarrow$ M_1 & M_2 can never on at the same time.

$-|V_{EE}| < V_{in} < -|V_{t,p}|$:

$\Rightarrow$ M_1 OFF, M_2 ON (saturation) } $V_{out} = V_{in} + V_{SG,2}$

$V_{cc} > V_{in} > V_{t,n}$:

$\Rightarrow$ M_1 ON, M_2 OFF } $V_{out} = V_{in} - V_{GS,1}$

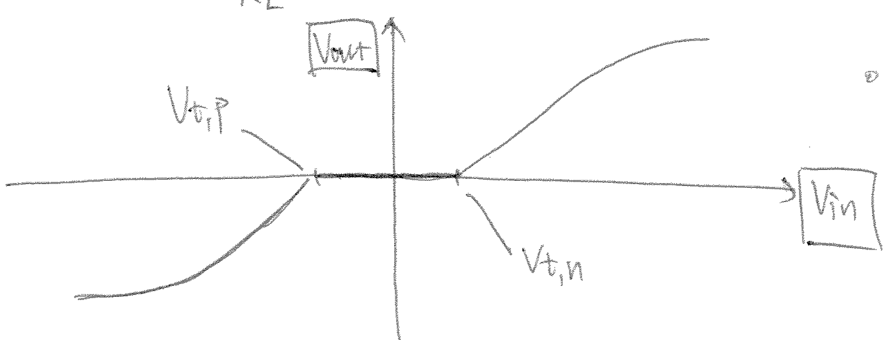
$-|V_{t,p}| < V_{in} < V_{t,n}$:

M_1, M_2 OFF $\Rightarrow$ $V_{out} = 0$

For MOS, $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(|V_{GS}|-|V_t|)^2$ — saturation region.

$\Rightarrow$ M_1 ON: $\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{in}-V_{out}-V_{t,n})^2$, $V_{out} > 0$

M_2 ON: $-\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{out}-V_{in}-V_{t,p})^2$, $V_{out} < 0$

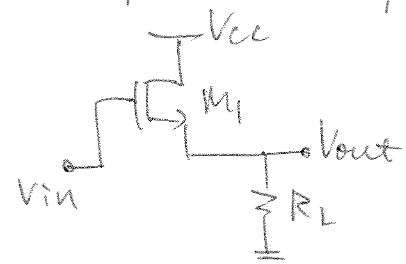


- Solve for Vout in both cases.

(b) Outside dead zone

$\Rightarrow$ either M_1 or M_2 is on.

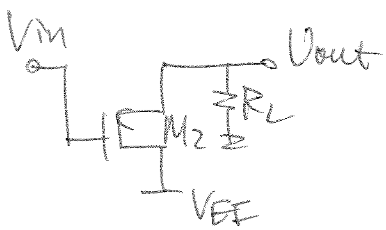
• For positive inputs:



Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m1}}{1+g_{m1}R_L}$$

• For negative inputs:

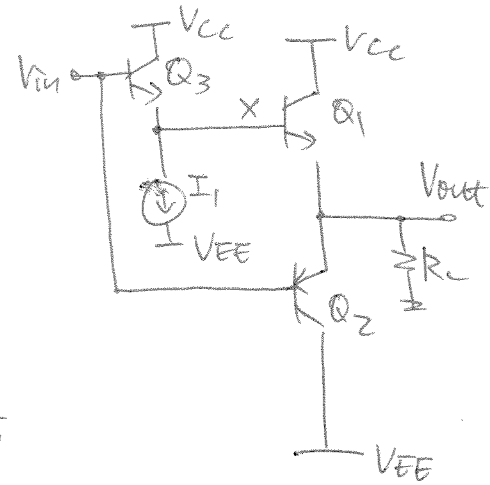


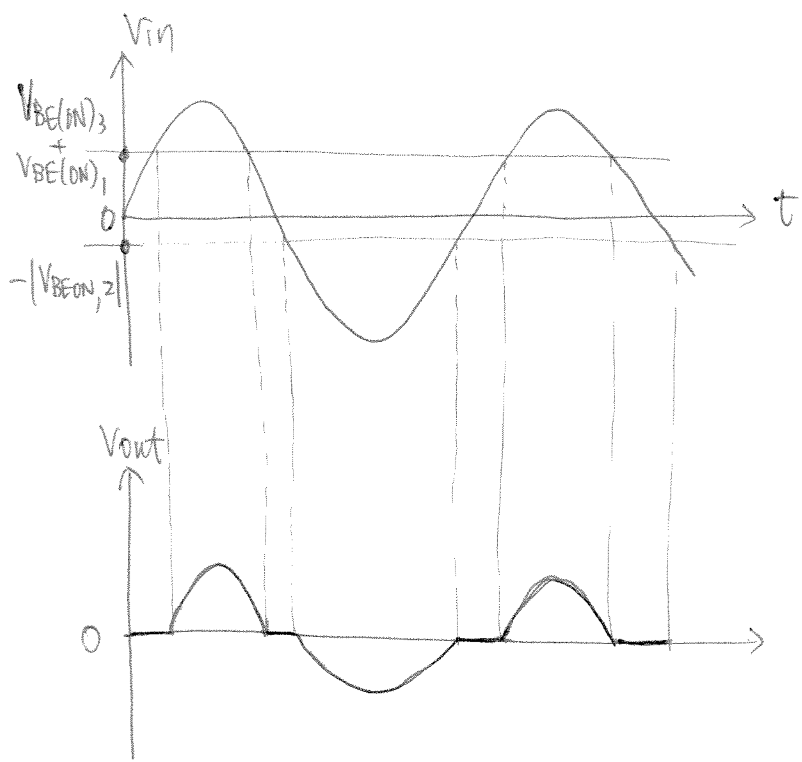
Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m2}}{1+g_{m2}R_L}$$

14. Dead zone :

$$V_{out} \in \left[-|V_{BE(ON)_2}| ,\ V_{BE(ON)_3} + V_{BE(ON)_1} \right]$$

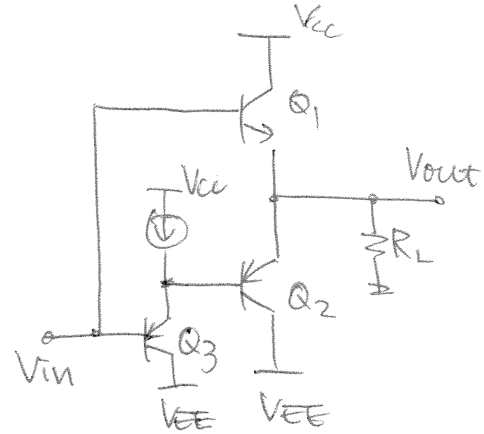


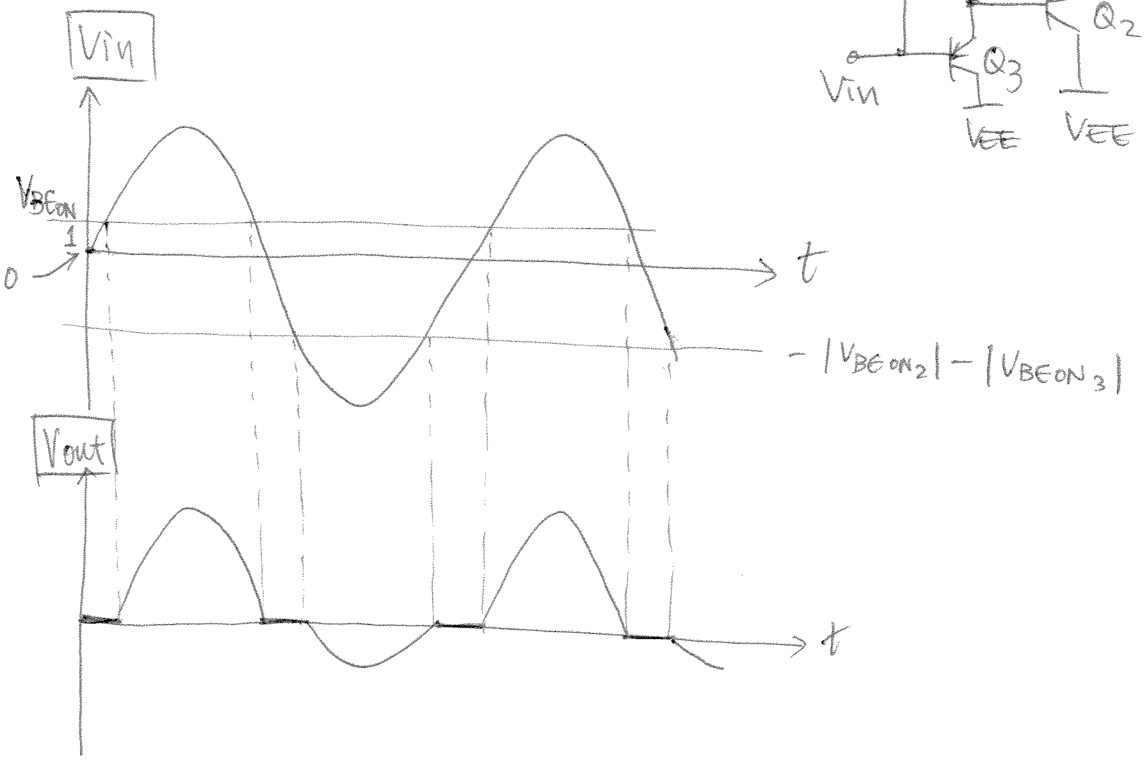


15.

Dead zone:

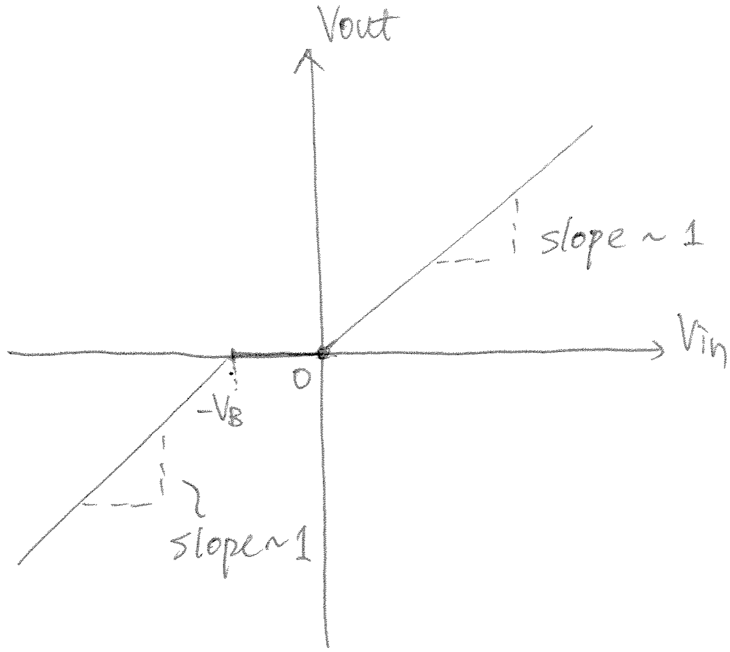
$V_{out} \in \left[-\left(|V_{BE,ON,2}| + |V_{BE,ON,3}| \right), V_{BE,ON,1} \right]$



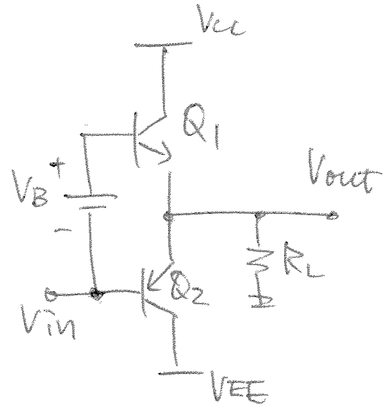


16.

(a)



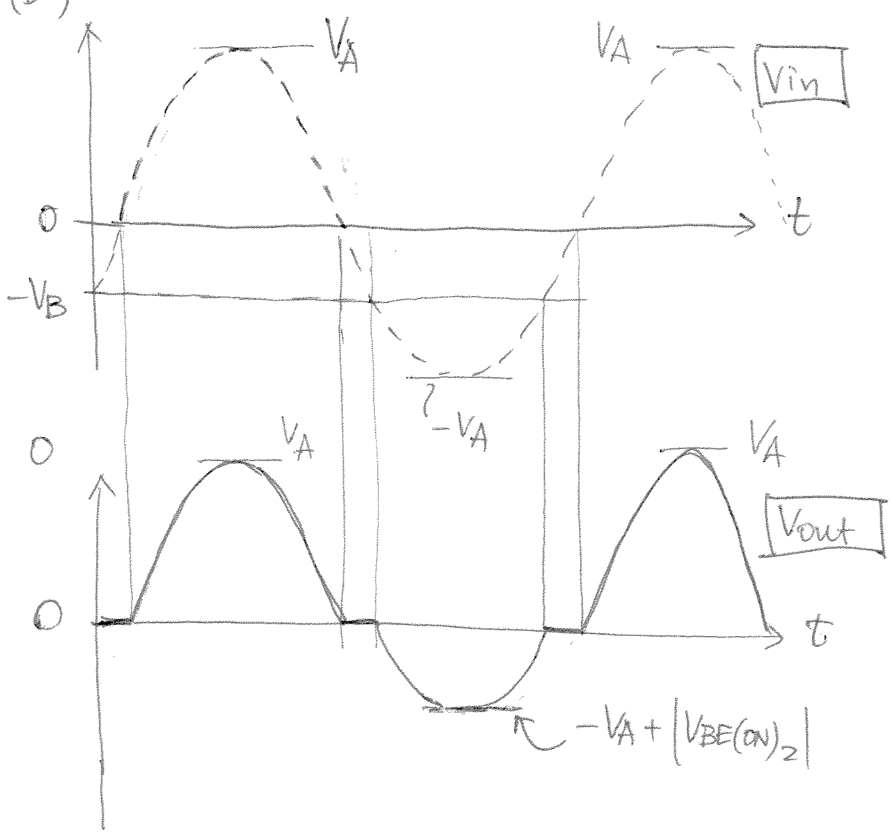
Vout
slope ~ 1
Vin
0
-VB
slope~1



Vcc
Q1
Vout
VB
+
-
RL
Vin
Q2
VEE

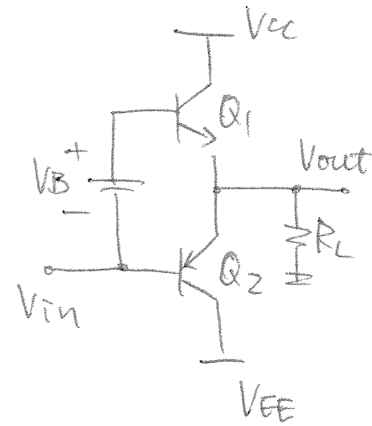
$(V_B = V_{BE})$

(b)



VA
VA
Vin
0
t
-VB
-VA
0
VA
VA
Vout
0
t
-VA + |VBE(ON)2|

17.



• $V_{out} = 0$:

$\Rightarrow I_{C_1} = I_{C_2} = I_{BIAS}$

$$\Rightarrow I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = I_{S_2} \exp\left(\frac{|V_{out}-V_{in}|}{V_T}\right)$$

$$\ln\left(\frac{I_{S1}}{I_{S2}}\right) + \frac{V_{in}+V_B-V_{out}}{V_T} = \frac{V_{out}-V_{in}}{V_T}$$

$I_{S_1} = 5\cdot 10^{-17} A$

$I_{S_2} = 8\cdot 10^{-17} A$

$I_{BIAS} = 5 mA$ $(V_{out} = 0)$

• For $V_{out} = 0$, $V_T = 0.026\ V$:

$$\Rightarrow \ln\left(\frac{5}{8}\right) + \frac{V_{in}+V_B}{0.026} = +\frac{V_{in}}{0.026}$$

• Given $I_{C2} = 5 mA$

$$\Rightarrow I_{S2} \exp\left(\frac{-V_{in}}{0.026}\right) = 5mA \Rightarrow V_{in} = -0.83\ V$$

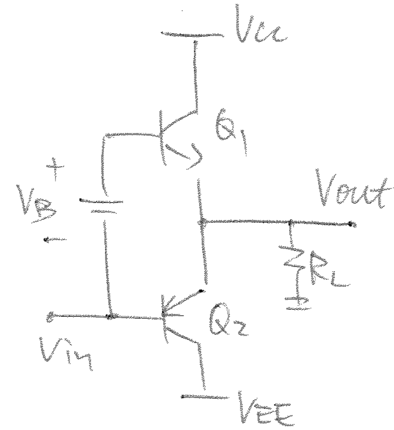
$$I_{C_1} = I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = (5\cdot 10^{-17}A) \exp \frac{-0.83+V_B}{V_T}$$

$$\Rightarrow V_B = 0.83 + 0.026 \ln\left(\frac{5mA}{5\cdot 10^{-17}A}\right)$$

$$\approx 1.67\ V.$$

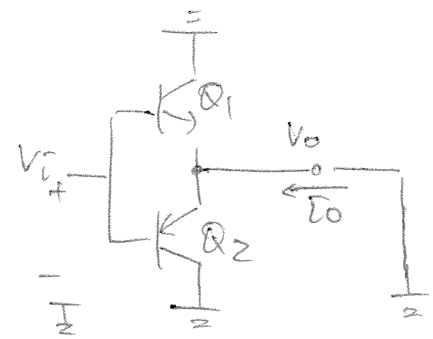
18.

(a) Equivalent circuit (small-signal) around $V_{out} = 0$:



$V_{P_{in}} = 2\,V$

$R_L = 8\,\Omega$



$$i_o = -g_{m1} v_i + (-v_i) g_{m2}$$

$$= -(g_{m1} + g_{m2}) v_i$$

$$\therefore G_m = \frac{i_o}{v_i} = -(g_{m1} + g_{m2})$$

$$\therefore A_V = \frac{v_o}{v_i} = \frac{i_o \times R_L}{v_i} = -(g_{m1} + g_{m2}) R_L$$

(b) $$A_V = -(g_{m1} + g_{m2}) R_L = -\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right) R_L$$

$$= -\left(\frac{5mA}{0.026V} + \frac{5mA}{0.026V}\right)(8\,\Omega) = -3.08$$

$$\Rightarrow |V_o|_p = |v_i A_V|_p = |(2V)(-3.08)| = 6.16\,V$$

(Assume Vcc is large enough)

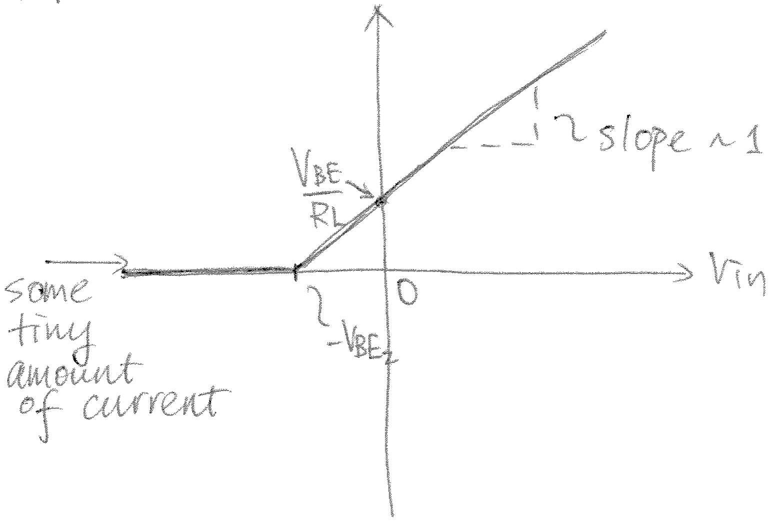
(c) $I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$

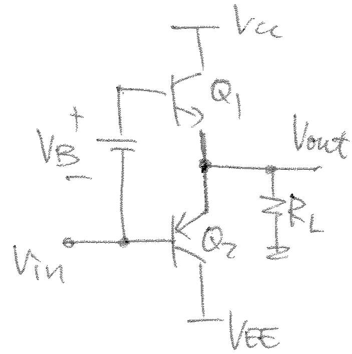
$$I_{C1,peak} = I_{C2} + \frac{V_P}{R_L}$$

$$= 5mA + \frac{6.16V}{8\Omega}$$

$$= 775mA$$

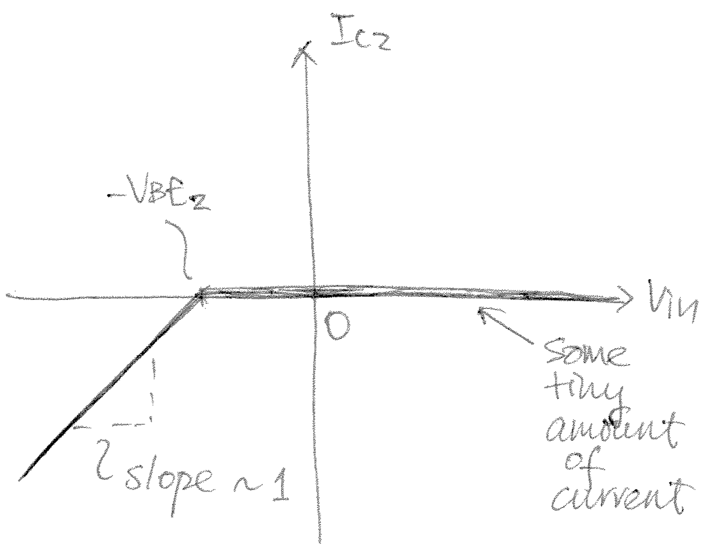
19.



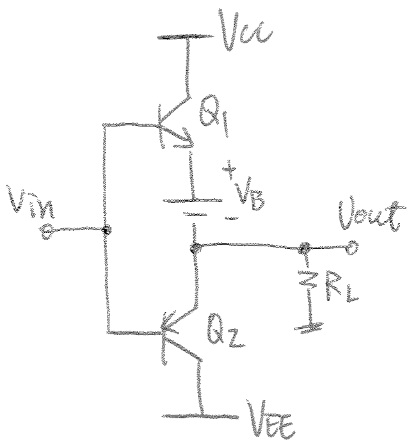


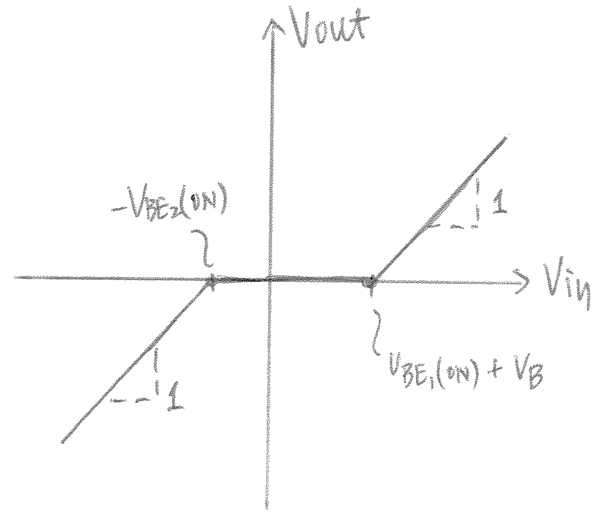
$$V_{out} = V_{in} + |V_{BE2}|$$

$$\Rightarrow I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$$



20.





- To analyze such circuit, assume $V_{out} = 0$:

$\Rightarrow \quad -V_{BE2(ON)} < V_{in} < V_{BE1(ON)} + V_B$.

$(V_{BE1(ON)} + V_B) < V_{in}$: $\quad V_{out} = V_{in} - V_{BE1(ON)} - V_B$

$V_{in} < -V_{BE2(ON)}$: $\quad V_{out} = V_{in} + |V_{BE2(ON)}|$

21. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \left[\ln \frac{I_{C1}}{I_{S_{Q1}}} + \ln \frac{I_{C2}}{I_{S_{Q2}}} \right] = V_T \left[\ln \frac{I_{D_1}}{I_{SD_1}} + \ln \frac{I_{D2}}{I_{SD_2}} \right]$$

$$\Rightarrow \frac{I_{C_1} I_{C2}}{I_{S_{Q_1}} I_{S_{Q_2}}} = \frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}$$

$\therefore$ If $I_{S_{Q_1}} I_{S_{Q_2}} = I_{SD_1} I_{SD_2}$,

then $I_{C_1} I_{C_2} = I_{D_1} I_{D_2}$

22. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right) = V_T \ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———— ①}$$

$I_1 = I_{D_1} = I_{D_2} = 1mA$; $I_{S,Q} = 16 I_{S,D}$

$V_{out} = 0 \Rightarrow I_{C_1} = I_{C_2}$ ———— ②

Substitute all into ①:

$$\frac{I_{C_1} I_{C_1}}{(16 I_{S,D})^2} = \frac{(1mA)^2}{(I_{S,D})^2} \Rightarrow I_{C_1} = I_{C_2} = 16\ mA$$

23. $V_{BE_1} + |V_{BE2}| = V_{D_1} + V_{D2}$

$$\Rightarrow \frac{I_{C_1} I_{C2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$$I_{C_1} = I_{C_2} = 5mA \quad \text{———} \quad (2)$$

$$I_{S,Q} = 8 I_{S,D} \quad \text{———} \quad (3)$$

Substitute all into (1):

$$\frac{(5mA)^2}{(8 I_{S,D})^2} = \frac{I_{D_1} I_{D_2}}{(I_{S,D})^2} \Rightarrow I_1 = I_D = 0.625\ mA$$

24. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{I_{C1} I_{C2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$I_1 = I_D = 2mA$

$I_{S,Q_1} = 8 I_{S,D_1}$; $I_{S,Q_2} = 16 I_{S,D_2}$

Substitute all into (1):

$$\frac{I_{C1} I_{C2}}{(8 I_{S,D_1})(16 I_{S,D_2})} = \frac{(2mA)^2}{I_{S,D_1} I_{S,D_2}}$$

$\Rightarrow I_{C_1} = I_{C_2} \cong 22.6\ mA$

25. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{kT_Q}{q}\left[\ln\left(\frac{I_{C1} I_{C2}}{I_{S,Q_1} I_{S,Q2}}\right)\right] = \frac{kT_D}{q}\left[\ln\left(\frac{I_{D_1} I_{D2}}{I_{S,D_1} I_{S,D2}}\right)\right]$$

Suppose $T_D = (T_Q + \Delta T)$:

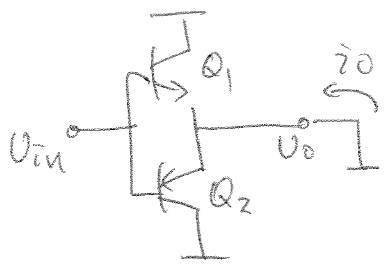
$$\Rightarrow T_Q\left[\ln\frac{I_{C_1} I_{C2}}{I_{SQ_1} I_{SQ2}} - \ln\frac{I_{D_1} I_{D2}}{I_{SD_1} I_{SD2}}\right] = \Delta T \cdot \ln\frac{I_{D_1} I_{D2}}{I_{SD_1} I_{SD2}}$$

$$\Rightarrow I_{C1} \cdot I_{C2} = I_{SQ_1} \cdot I_{SQ_2} \cdot \left(\frac{I_{D_1} I_{D2}}{I_{SD_1} I_{SD2}}\right)^{1+\frac{\Delta T}{T_Q}}$$

Typically, $\frac{I_{D_1} I_{D2}}{I_{SD_1} I_{SD2}} > 1$

$\Rightarrow$ A ΔT introduces a factor $\left(\frac{I_{D_1} I_{D2}}{I_{SD_1} I_{SD2}}\right)^{\frac{\Delta T}{T_Q}} < 0$, implying that the $I_{C_1} I_{C2}$ product drops corresponding to a change (positive) in temperature.

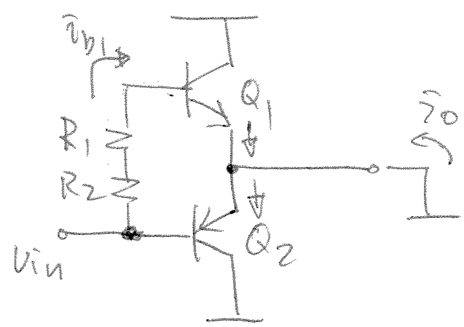
26. Small Signal:



$$G_m = \frac{i_o}{v_{in}} = -(g_{m1} + g_{m2})$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{i_o R_L}{v_{in}} = +(g_{m1} + g_{m2}) R_L$$

27. Small-Signal:



$$i_o = -g_{m1} v_{be1} + g_{m2}|v_{be2}| \qquad (i_o = i_{c2} - i_{c1})$$

$$|v_{be2}| = v_{in}$$

$$v_{be1} = v_{in} - i_{b1}(R_1+R_2) = v_{in} - \frac{i_{c1}}{\beta_1}(R_1+R_2)$$

$$= v_{in} - \frac{i_{c2} - i_o}{\beta_1}(R_1+R_2)$$

$$= v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)$$

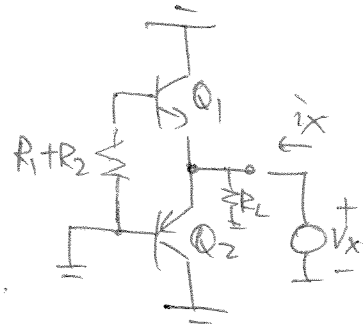
$$\therefore\ g_{m1}\left[v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)\right] + i_o = -g_{m2} v_{in}$$

Solving for $\frac{i_o}{v_{in}}$ gives:

$$G_m = \frac{i_o}{v_{in}} = \frac{-\left[g_{m1} + \frac{g_{m1} g_{m2}}{\beta_1}(R_1+R_2) + g_{m2}\right]}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}$$

Rout:

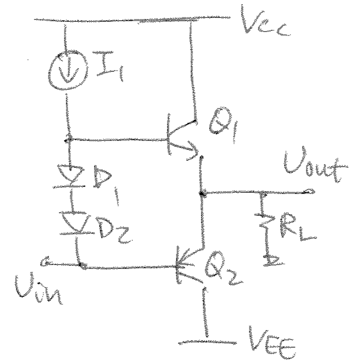
$$\frac{v_x}{i_x} = R_{out} = \left(r_{\pi 2} \,\|\, \frac{1}{g_{m2}}\right) \,\|\, \left[(r_{\pi 1}+R_1+R_2) \,\|\, \frac{1}{g_{m1}}\right] \,\|\, R_L$$



$$\therefore A_v = G_m R_{out}$$

$$= -\left[\frac{g_{m1} + \dfrac{g_{m1} g_{m2}(R_1+R_2)}{\beta_1} + g_{m2}}{1 + \dfrac{g_{m1}(R_1+R_2)}{\beta_1}}\right] \cdot \left\{\left[r_{\pi 2} \,\|\, \frac{1}{g_{m2}}\right] \,\|\, \left[(r_{\pi 1}+R_1+R_2) \,\|\, \frac{1}{g_{m1}}\right] \,\|\, R_L\right\}$$

28. Small signal gain around $V_{out} = 0$:



$R_L = 8\Omega$

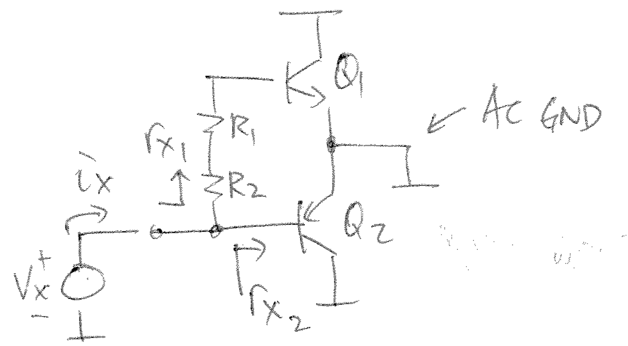
$$A_v = +(g_{m1} + g_{m2}) R_L$$

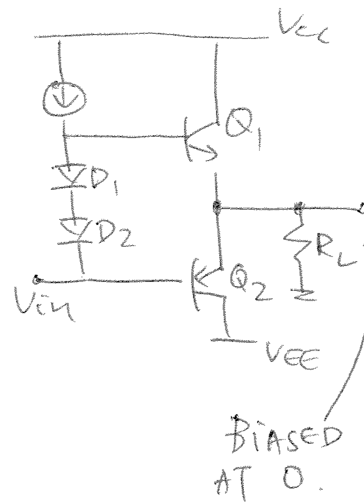
$$0.8 = (I_{C1} + I_{C2}) \frac{R_L}{V_T}$$

If $I_{C1} = I_{C2} = I_{BIAS}$, then

$$I_C = \frac{0.8}{2} \times \frac{V_T}{R_L} = 0.4 \frac{V_T}{R_L} = 0.01 \cdot R_L = 0.08 \text{ A}$$

29. Small-signal equivalent:





$$R_{in} = \frac{v_x}{i_x} = r_{x_1} \parallel r_{x_2}$$

$$= (R_1 + R_2 + r_{\pi_1}) \parallel r_{\pi_2}$$

- R_1 & R_2 can be neglected when $r_{\pi_1} \gg (R_1 + R_2)$

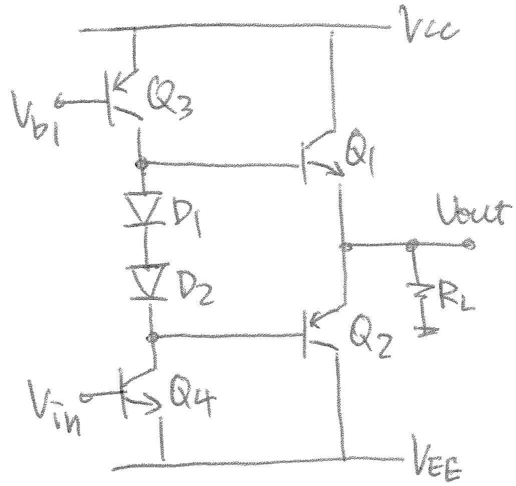
30. $I_{C_1} = I_{C_2} = 10\ mA$

$I_{C_3} = I_{C_4} = 1\ mA$

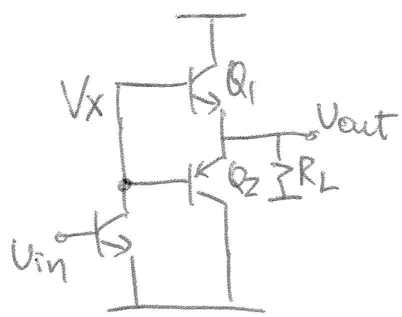
$\beta_1 = 40 \quad \beta_2 = 20$

$R_L = 8\Omega$

$R_{D_1} = R_{D_2} = 0$



Small-signal



$$A_v = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}}$$

$$= -g_{m4}\left[(g_{m1}+g_{m2})(r_{\pi_1} \| r_{\pi_2})R_L + (r_{\pi_1} \| r_{\pi_2})\right] \times \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$= -g_{m4}(r_{\pi_1} \| r_{\pi_2})(g_{m1}+g_{m2})R_L$$

$$\therefore A_v = -\frac{I_{C4}}{V_T}\left(\frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}}\right)\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right)R_L$$

$$= -\frac{1mA}{0.026}\left[35\right] \cdot \left(2 \times \frac{10mA}{V_T}\right)(8)$$

$$\approx -8.3$$

31.

$$\frac{v_{out}}{v_{in}} = -g_{m4}(r_{\pi 1} \parallel r_{\pi 2})(g_{m1}+g_{m2})R_L \qquad \left(r_\pi = \frac{\beta}{g_m}\right)$$

When $g_{m1} \approx g_{m2}$: $(\Rightarrow r_\pi$

$$\frac{v_{out}}{v_{in}} \approx -g_{m4}R_L(2g_{m1})\left(\frac{\beta_1}{g_{m1}} \parallel \frac{\beta_2}{g_{m1}}\right)$$

$$= -g_{m4}R_L(2g_{m1})\left[\frac{1}{g_{m1}} \cdot \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right]$$

$$= -\frac{2\beta_1\beta_2}{\beta_1+\beta_2}g_{m4}R_L$$

32. From lecture, small-signal gain of the output stage is:

$$\left|\frac{v_{out}}{v_{in}}\right| = +g_{m4}(r_{\pi 1} \parallel r_{\pi 2})(g_{m1}+g_{m2})R_L$$

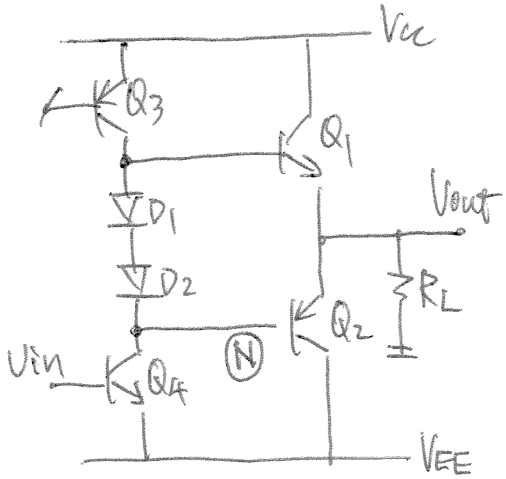
$$\approx +g_{m4}R_L \times \frac{2\beta_1\beta_2}{\beta_1+\beta_2}$$

$$\Rightarrow 4 = +\frac{I_{C4}}{V_T}(8\Omega) \times \frac{2(40)(20)}{40+20}$$

$$\Rightarrow I_{C4} \approx I_{C3}$$

$$= \frac{4V_T}{(8\Omega)} \cdot \frac{40+20}{2(40)(20)}$$

$$= 0.49\ mA$$



$A_V = \frac{v_{out}}{v_{in}} = 4$

$\beta_1 = 40$

$\beta_2 = 20$

$R_L = 8\Omega$

33. From lecture,

$$\frac{v_x}{i_x} = \frac{1}{g_{m1} + g_{m2}} + \frac{r_{o3} \| r_{o4}}{(g_{m1} + g_{m2})(r_{\pi_1} \| r_{\pi_2})}$$

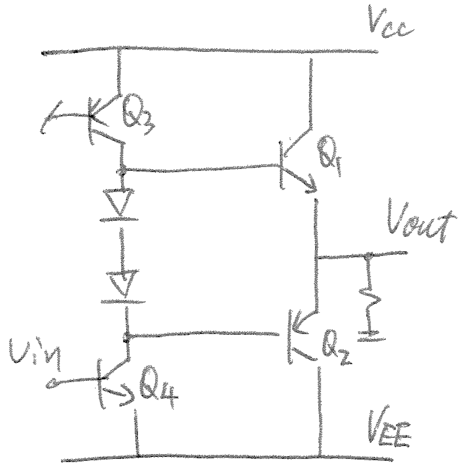
If $g_{m1} \approx g_{m2} = g_m$:

$$\frac{v_x}{i_x} \approx \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2g_m\left(\frac{\beta_1}{g_m} \| \frac{\beta_2}{g_m}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2g_m\left(\frac{1}{g_m} \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2\beta_1\beta_2}(\beta_1+\beta_2)$$

34.



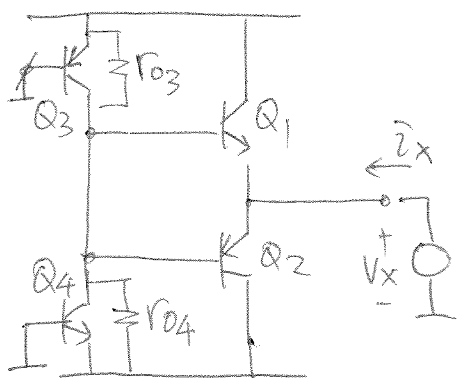
$I_3 = I_4 = 1\,mA$

$I_1 = I_2 = 8\,mA$

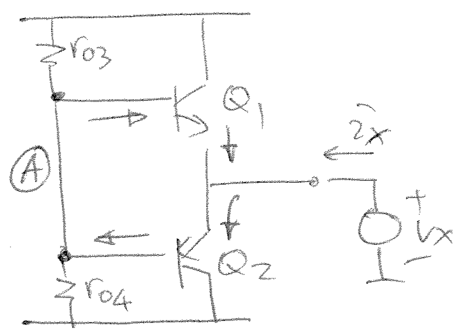
$V_{A3} = 10\,V$

$V_{A4} = 15\,V$

(a) Small-signal equivalent:



$\Rightarrow$



$$v_{eb} = v_x \frac{(r_{\pi_1} \| r_{\pi_2})}{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})}$$

$$v_{be} = v_A - v_x$$

$$i_x + i_{c_1} = i_{c_2} \Rightarrow i_x = i_{c_2} - i_{c_1} = g_{m_2} v_{eb} - g_{m_1} v_{be}$$

$$\therefore i_x = [g_{m_2} + g_{m_1}]\, v_x \frac{(r_{\pi_1} \| r_{\pi_2})}{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})}$$

$$\Rightarrow \frac{v_x}{i_x} = R_{out} = \frac{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})}{[g_{m_1} + g_{m_2}]\,(r_{\pi_1} \| r_{\pi_2})}$$

$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C1}} = 130\,\Omega$ $\qquad$ $r_{\pi 2} = \frac{\beta_2 V_T}{I_{C2}} = 65\,\Omega$

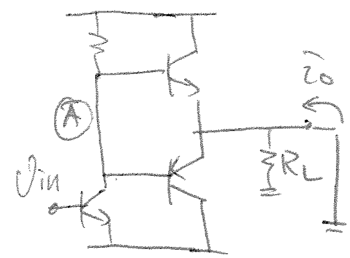
$r_{O3} = \frac{V_{A3}}{I_{C3}} = 10\,k\Omega$ $\qquad$ $r_{O4} = \frac{V_{A4}}{I_{C4}} = 15\,k\Omega$

$g_{m1} = 0.31\,S$ $\qquad$ $g_{m2} = 0.31\,S$

$$\Rightarrow R_{out} = \frac{43.3 + 6000}{(0.62)(43.3)} \approx 6001\,\Omega$$

(b) Effective $R_{out} = R_{out,a} \parallel 8\,\Omega \approx 8\,\Omega$.

$$G_m = \frac{i_o}{v_A} \cdot \frac{v_A}{v_{in}}$$
$$= -g_{m4}(r_{\pi 1} \parallel r_{\pi 2} \parallel r_{O3}) \cdot (g_{m1} + g_{m2})$$

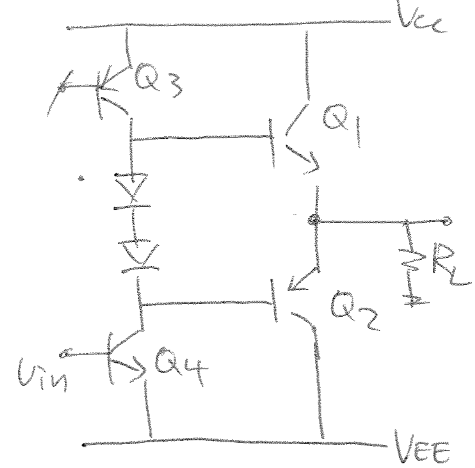


$g_{m4} = \frac{I_{C4}}{V_T} = 0.038\,S$

$$\therefore A_v = G_m R_{out}$$
$$= -g_{m4}(r_{\pi 1} \parallel r_{\pi 2} \parallel r_{O3})(g_{m1} + g_{m2}) R_{out}$$
$$= -0.038[130 \parallel 65 \parallel 10k][0.62](8)$$
$$\approx -8.1$$

35. Max current delivered by $Q_1 = I_{C3} \cdot \beta_1 = 1mA \cdot 40 = 40\,mA$. ($Q_4$ off)

Max current delivered by $Q_2 = I_{C4} \cdot \beta_2 = 1mA \cdot 20 = 20\,mA$. ($Q_3$ off)

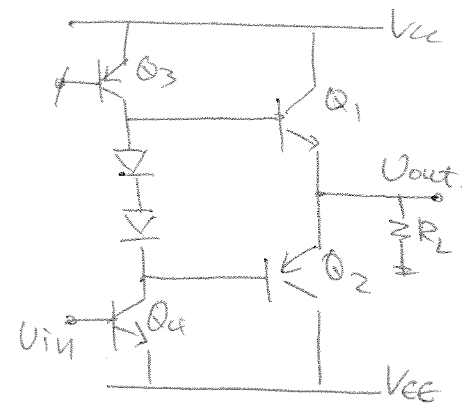


$I_{C3} = I_{C4} = 1mA$

$\beta_1 = 40 \quad \beta_2 = 20$

36. $P = 0.5\,W$ $\quad R_L = 8\,\Omega$

$\beta_1 = 40 \quad \beta_2 = 20.$



$$P_{AVG} = \frac{1}{2}\frac{V_p^2}{R_L} = 0.5$$

$$\Rightarrow V_p^2 = 2(0.5)R_L$$

$$\Rightarrow V_p = \sqrt{R_L} = 2\sqrt{2}$$

At positive V_p, $I_{C_1} = \frac{V_p}{R_L} = \frac{2\sqrt{2}}{8} = 0.35\,A.$

At negative V_p, $I_{C_2} = \frac{V_p}{R_L} \Rightarrow I_{C_2} = 0.35\,A.$

- At $+V_p$, all of I_{C3} supports the base current of Q_1

$$\Rightarrow I_{C3} = I_{B_1} = \frac{I_{C1}}{\beta_1} = \frac{0.35A}{40} = 8.75\,mA$$

- At $-V_p$, all of I_{C4} supports the base current of Q_2

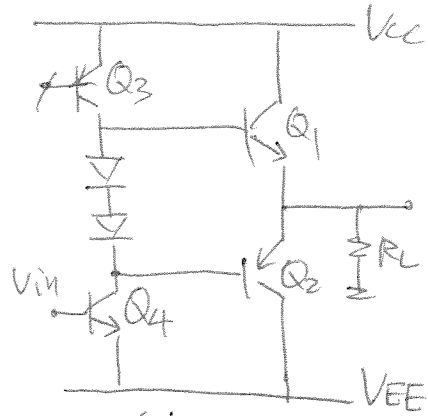
$$\Rightarrow I_{C4} = I_{B_2} = \frac{I_{C2}}{\beta_2} = \frac{0.35A}{20} = 17.5\,mA$$

37. $P_{AVG} = 0.5\,W \qquad R_L = 8\,\Omega$

$V_{CC} = 5V$

$$\Rightarrow 0.5W = \frac{1}{2}\frac{V_p^2}{R_L}$$

$$\Rightarrow V_p = 2\sqrt{2}\ V$$



(Assume negligible currents at Vout=0)

$$P_{Q_1} = \frac{1}{T}\int_0^{T/2} I_{C_1} V_{CE_1}\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left(\frac{V_p \sin\omega t}{R_L}\right)\cdot(V_{CC} - V_p \sin\omega t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left[\frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^2}{2R_L}\right] dt$$

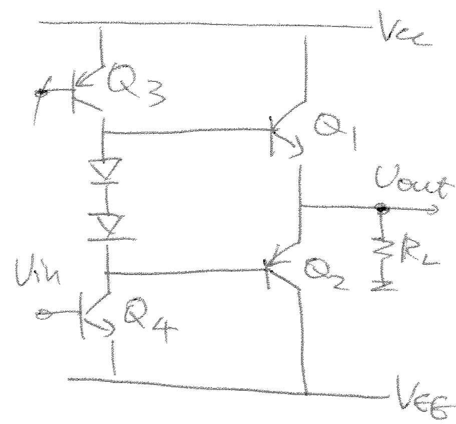
$$= \frac{V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4}\right) = \frac{2\sqrt{2}}{8}\left(\frac{5}{\pi} - \frac{2\sqrt{2}}{4}\right)$$

$$\approx 0.31\ W$$

38. $P_{Q,MAX} = 0.75W$

$R_L = 8\,\Omega$

$V_{CC} = 5V$



• Out of all 4 transistors, Q_1 & Q_2 must sustain the most currents.

$$P_{Q_1,MAX\,(INST)} = V_{CE} \times I_{C_1,MAX} = (V_{CC} - V_{out})\, I_{C_1,MAX}$$

$$\Rightarrow P_{Q_1,MAX\,(AVG)} = \frac{1}{T}\int_0^{T/2} \frac{V_p \sin\omega t}{R_L} \cdot (V_{CC} - V_p \sin\omega t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left(\frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^2}{2R_L}\right) dt$$

$$= \frac{V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4}\right)$$

$$\Rightarrow \frac{dP_Q}{dV_p} = \frac{V_{CC}}{\pi R_L} - \frac{V_p}{2R_L}$$

$$= 0 \quad \text{when} \quad V_p = \frac{2V_{CC}}{\pi} = 3.18\,V$$

$$P_Q \Big|_{V_p = \frac{2V_{CC}}{\pi}} = 0.32\,W$$

$$\therefore P_{R_L,MAX} = \frac{1}{2}\frac{V_p^2}{R_L} = 0.63\,W$$

39. $P_{Q_1,MAX} = \left(\frac{V_{CC}}{\pi} - \frac{2V_{CC}}{4\pi} \right) \cdot \frac{2V_{CC}}{\pi R_L} \leq 0.75\,W$

$\Rightarrow V_{CC}|_{MAX} = 7.7\ V$

$\Rightarrow V_{P,MAX} = \frac{2V_{CC\,MAX}}{\pi} = 4.9\ V$

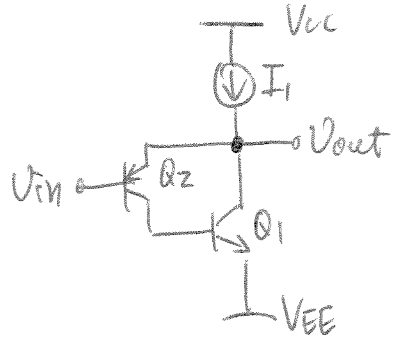
$\Rightarrow P_{R_L,MAX} = \frac{1}{2} \frac{V_{P\,MAX}^2}{R_L} = 1.5\ W$

40. $I_1 = I_{C1} + I_{E2}$

$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{C2}$

$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{B1}$

$= \beta_1 I_{B1} + \frac{\beta_1+1}{\beta_1} I_{B1}$



$I_1 = 5mA$

$\beta_1 = 40$

$\beta_2 = 50.$

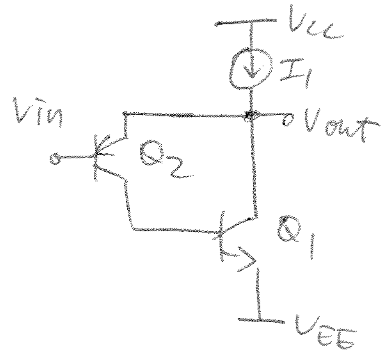
$\Rightarrow I_{B1} = \frac{I_1}{\beta_1 + \frac{\beta_1+1}{\beta_1}} = \frac{0.005}{40 + \frac{41}{40}}$

$\approx 0.12 mA$

$\Rightarrow I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{B1}}{\beta_2} = 0.0024 mA$

41. $V_{in} = 0.5\,V$

$I_{S2} = 6 \cdot 10^{-17}\,A.$



$I_{B1} = I_{C2} = 0.12\,mA$

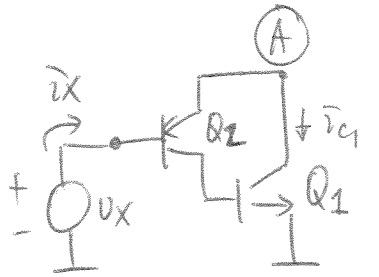
$\Rightarrow\ I_{C2} = I_{S2} \cdot \exp\left(\frac{V_{out} - V_{in}}{V_T}\right)$

$\therefore\ V_{out} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) + V_{in}$

$= 0.026 \ln\left(\frac{0.12\,mA}{6 \cdot 10^{-17}A}\right) + 0.5$

$\approx 1.24\,V$

42.



$i_{c2} = i_x \beta_2$

$i_{c1} = -g_{m2} v_{eb2} = i_{e2} = i_{c2} + i_{b2}$
$= i_x(\beta_2+1)$

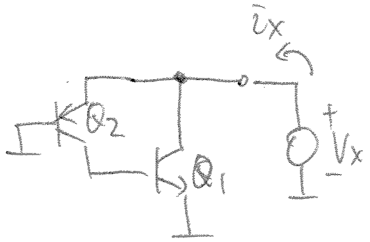
$v_{eb2} = v_A - v_x$

where $v_A = v_x - i_x r_{\pi 2}$

$\therefore\ i_{b1} = -g_{m2}(i_x \cdot r_{\pi 2})$

$i_{c1} = i_{b1} + i_{b1} \cdot \beta_1 = -g_m i_x \cdot r_{\pi 2}(1+\beta_1)$

$\Rightarrow \frac{v_x}{i_x} \to \infty$ (Rin)



$i_x = i_{e2} + i_{c1}$
$= i_{e2} + i_{b1}\beta_1$
$= i_{e2} + i_{c2}\beta_1$
$= i_{c2} + i_{b2} + i_{c2}\beta_1$
$= i_{c2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$
$= v_x g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$

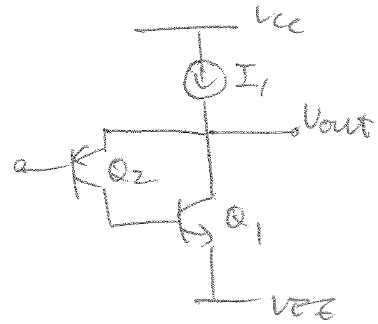
$g_{m2} = \frac{I_{B2}\beta_2}{V_T}$
$= 4.6\ S$

$\Rightarrow R_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$

$= 0.005\ \Omega.$

43. $R_{out} = 1\Omega.$

$\beta_1 = 40 \qquad \beta_2 = 50.$



$$R_{out} = 1 = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$$

$$\Rightarrow g_{m2} = 0.024\ S = \frac{I_{B2}\beta_2}{V_T}$$

$$\Rightarrow I_{B_2} = 0.012\ mA.$$

$$I_1 = I_{C_1} + I_{E2} = I_{B_1}\beta_1 + (I_{C2} + I_{B2})$$

$$= I_{C2}\beta_1 + I_{B2}(\beta_2 + 1)$$

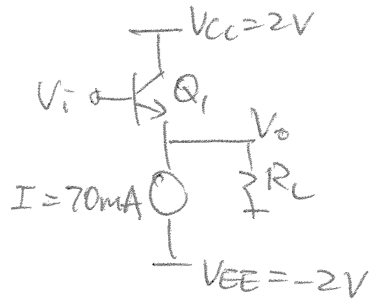
$$= I_{B2}\beta_2\beta_1 + I_{B_2}(\beta_2+1)$$

$$= 0.012\left[50\times40 + 50 + 1\right]$$

$$= 25.6\ mA$$

44. $V_p = 0.5V$

$R_L = 8\Omega$.



$$P_{R_L} = \frac{V_p^2}{2R_L} = \frac{0.25}{16} = 0.0156\ W$$

$$P_I = -I \times V_{EE} = 0.14\ W$$

$$P_{Q_1} = I_1\left(V_{CC} - \frac{V_p}{2}\right) = 0.1225\ W$$

$$\therefore \eta = \frac{P_{RL}}{P_{R_L} + P_I + P_{Q_1}} = \frac{0.0156}{0.2781} = 5.6\%$$

45. $P_{R_L} = \dfrac{V_P^2}{2R_L} = \dfrac{(V_{CC}-V_{BE})^2}{2R_L}$

$P_{Q_1} = I_1\left(V_{CC} - \dfrac{V_{CC}-V_{BE}}{2}\right)$

$P_I = +I_1|V_{EE}|$

• Assume $|V_{CC}| = |V_{EE}|$,

$I_1 = V_P/R_L = \dfrac{V_{CC}-V_{BE}}{R_L}$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L}+P_{Q_1}+P_I} = \frac{\dfrac{(V_{CC}-V_{BE})^2}{2R_L}}{\dfrac{(V_{CC}-V_{BE})^2}{2R_L} + I_1\left[V_{CC} - \dfrac{V_{CC}-V_{BE}}{2} + |V_{EE}|\right]}$$

$$= \frac{\dfrac{1}{2R_L}}{\dfrac{1}{2R_L} + \dfrac{3V_{CC}-V_{BE}}{2R_L(V_{CC}-V_{BE})}}$$

$$= \frac{1}{1 + \dfrac{3V_{CC}-V_{BE}}{V_{CC}-V_{BE}}} \approx \frac{V_{CC}-V_{BE}}{3V_{CC}-V_{BE}}$$

46. $$\eta = \frac{\frac{V_p^2}{2R_L}}{\frac{V_p^2}{2R_L} + \frac{2V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4}\right)}$$

$$= \frac{\pi}{4}\frac{V_p}{V_{CC}}$$

$$\Rightarrow \eta\Big|_{V_p = V_{CC} - V_{BE}} = \frac{\pi}{4} - \frac{\pi}{4}\cdot\frac{V_{BE}}{V_{CC}}$$

47. $$\eta = \frac{\dfrac{(V_p/2)^2}{2R_L}}{\dfrac{(V_p/2)^2}{2R_L} + \dfrac{2(V_p/2)}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p/2}{4}\right)}$$

$$= \frac{V_p^2/8R_L}{\dfrac{V_p^2}{8R_L} + \dfrac{V_p}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p}{8}\right)} = \frac{1/8R_L}{1/8R_L + \dfrac{1}{R_L}\left(\dfrac{V_{cc}}{V_p\pi} - \dfrac{1}{8}\right)}$$

$$= \frac{1}{1 + \left(\dfrac{8V_{cc}}{V_p\pi} - 1\right)} = \frac{\pi}{8}\frac{V_p}{V_{cc}} \approx 39\%.$$

48. $V_{cc} = 3V$ $P_{R_L} = 0.2W$ $R_L = 8\Omega$.

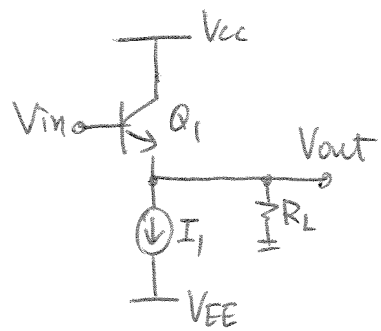
$$P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = \sqrt{2P_{R_L} \times R_L} = 1.8\ V$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)} = \frac{0.2}{0.2 + \frac{3.6}{8}\left(\frac{3}{\pi} - \frac{1.8}{4}\right)}$$

$$\approx 18\%.$$

49. Power = 1 W

$R_L = 8\Omega$



$$P_{LOAD} = \frac{1}{2}\frac{V_P^2}{R_L} = 1W$$

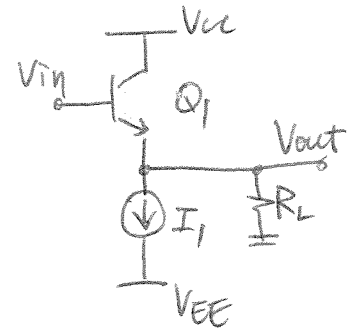
$$\Rightarrow V_P = 4\,V \Rightarrow I_1 = \frac{V_P}{R_L} = 0.5\text{ mA}$$

(Note: the problem does not specify small-signal voltage gain, so choose $V_P = I_1 R_L$)

$$P_{Q_1}\text{ (power rating)} = I_1 (V_{CC})$$
$$= (0.5\text{mA})(5V)$$
$$= 2.5\text{ mW}$$

50. $A_V = 0.8$

$R_L = 4\Omega$



$$A_V = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{4}{4 + \frac{0.026}{I_{C_1}}} = 0.8$$

$\Rightarrow I_{C_1} = 26 \text{ mA}$

$\therefore I_1 = I_{C_1} = 26\text{mA}$ (Vout biased at 0 V.)

Max Output Swing $= I_1 R_L$

$\approx (26\text{mA})(8\Omega)$

$= 0.208\text{V}$

P_{Q_1} (power rating) $= I_1 V_{CC}$ ($V_p = 0$)

$= (26 \text{ mA})(5\text{V}) = 130 \text{ mW}$

51. $A_v = 0.6$

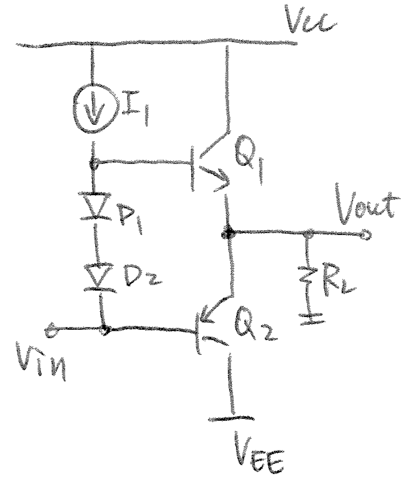
$R_L = 8\ \Omega$

$r_{D_1} = r_{D_2} = 0$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{(8\Omega)}{(8\Omega) + \frac{0.026V}{I_{Q_1}}}$$

$= 0.6$

$\Rightarrow I_{Q_1} = I_{Q_2} = 4.8\ mA$



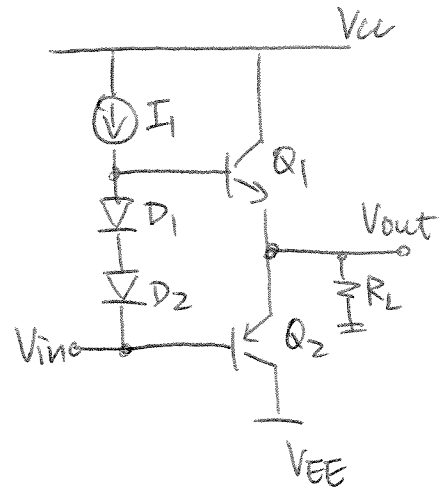
(Vout biased at 0V.)

52. Power = 1 W (to load)

$R_L = 8\Omega$

$|V_{BE}| \approx 0.8$ V

$\beta_1 = 40$



$$P_L = \frac{1}{2}\frac{V_p^2}{R_L} = 1 \text{ W}$$

$$\Rightarrow V_p = 4 \text{ V}$$

$\therefore$ Min allowable supply voltage $= V_p + |V_{BE}| = 4.8$ V

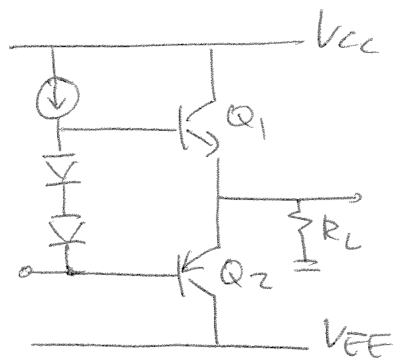
- At $+V_p$, all of I_1 goes to base of Q_1

$$\Rightarrow I_1 = I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{V_p}{R_L}\cdot\frac{1}{\beta_1} \quad (Q_2 \text{ off})$$

$$= \frac{4}{8}\cdot\frac{1}{40} = \frac{1}{80} = 12.5 \text{ mA}.$$

53. $P_{Q,MAX} = 2W$

$R_L = 8\Omega$.



For this circuit,

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \quad \left(V_p = \frac{2V_{CC}}{\pi}\right)$$

$$= 2W$$

$\Rightarrow V_{CC}\Big|_{MAX} = 12.6\ V \quad \Rightarrow V_p\Big|_{max} = \frac{25.2}{\pi} = 8.02\ V$

$$\therefore P_{R_L\,MAX} = \frac{V_{P\,max}^2}{2R_L} = \frac{(8.02)^2}{2 \cdot 8} = 4.02\ W$$

54. For this circuit,

$P_{Q,MAX} = 2W$

$R_L = 4\Omega$

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \qquad \left(V_P = \frac{2V_{CC}}{\pi}\right)$$

$$\Rightarrow V_{CC\,max} = \sqrt{\frac{\pi^2 R_L P_{Q\,MAX}}{1}} = 8.9\ V$$

$$\Rightarrow V_{P_{max}} = \frac{2V_{CC\,max}}{\pi} = 5.6\ V$$

$$\therefore P_{R_L,MAX} = \frac{V_{P,MAX}^2}{2R_L} = \frac{32}{2(4)} = 4W$$

55. $A_V = 4$ $\quad R_L = 8\Omega$ $\quad I_{C1} \approx I_{C2}$

$\beta_1 = 40$ $\quad \beta_2 = 20$

Suppose we want 1st-stage (CE amplifier) to have gain $= 5$ $\Rightarrow$ 2nd stage gain $= 0.8$.

$$\Rightarrow 0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1} + g_{m2}}}$$

$$0.8 = \frac{8}{8 + \frac{1}{2g_{m1}}} \Rightarrow g_{m1} = .5 \Rightarrow I_{C1} = I_{C2} = 6.5\,mA$$

$$r_{\pi 1} \| r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = \frac{40(0.026)}{6.5\,mA} \| \frac{20(0.026)}{6.5\,mA} \approx 133\Omega$$

- $$A_V = 4 = g_{m4}(r_{\pi 1} \| r_{\pi 2})(g_{m1} + g_{m2})R_L$$

$$= \frac{I_{C4}}{V_T}(133)(0.5)\,8$$

$$\Rightarrow I_{C4} = I_{C3} = \frac{4 V_T}{8(133)(0.5)} = 0.195\,mA$$

Max I_{Q1} when all of I_{C3}/I_{C4} supports base current of Q_1

$$\Rightarrow I_{Q1,MAX} = I_{C4} = 0.195\,mA$$

56. $A_v = 4$ $R_L = 4\,\Omega$ $I_{C1} \approx I_{C2}$

$\beta_1 = 40$ $\beta_2 = 20$

1st stage gain = 5 (CE amplifier)

2nd " " = 0.8

- $$0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}} = \frac{4}{4 + \frac{1}{2g_{m1}}}$$

$\Rightarrow g_{m1} = 0.5\ S^{-1} \Rightarrow I_{C1} = I_{C2} = 13\,mA$

$$r_{\pi1} \| r_{\pi2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = 80 \| 40 = 26.7\,\Omega$$

- $$A_v = 4 = g_{m4}(r_{\pi1}\|r_{\pi2})(g_{m1}+g_{m2})R_L$$
$$= \frac{I_{C4}}{V_T}(26.7)(1)(4)$$

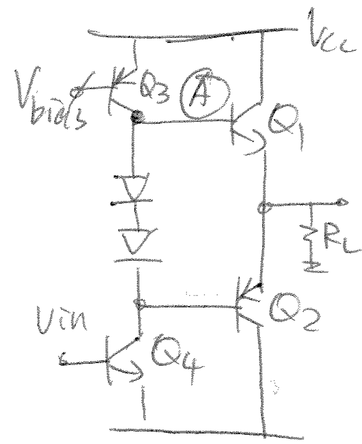
$$\Rightarrow I_{C4} = I_{C3} = \frac{4V_T}{(26.7)(1)(4)} = 0.974\ mA.$$

- Max I_{Q1} ($I_{Q1\,MAX}$) when $I_{C4} = I_{Q\,MAX} = 0.974\,mA$.
- For a reduction of 2x the R_L, we have to provide ~5x current to base of Q_1. $\left(\frac{0.974}{0.195} \approx 5\right)$

57. $P_{R_L} = 2W$ $\beta_1 = 40$

$R_L = 8\,\Omega$ $\beta_2 = 20$

$|V_{BE}| = 0.8V$



(a) $P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p \approx 5.6\ V$

- At $+V_p$, $V_A = V_p + |V_{BE}|$.
- For Q_3 in active region, $V_A \leq V_{bias}$

$\Rightarrow V_{CC} \geq V_{bias} + |V_{BE}| = V_p + 2|V_{BE}|$

$\geq 5.6 + 1.6 = 7.2\ V.$

(b) $I_p = \frac{V_p}{R_L} = 0.7\ A.\ (\approx I_{E_1}),\ (\approx I_{E_2})$

$\Rightarrow I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = 17\ mA.$

$\therefore$ We bias Q_3 & Q_4 with $I_C = 17\ mA.$

(c) $P_{AV} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$

$= \frac{5.6}{8}\left(\frac{5}{\pi} - \frac{5.6}{4}\right) = 3.66\ W$

(d) $P_{IQ3} = 2V_{CC} \times I_{Q3} = 10 \times 17mA = 170mW$

$P_{AV,Q_1} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right) = 3.66\ W$

$P_{R_L} = 2W$

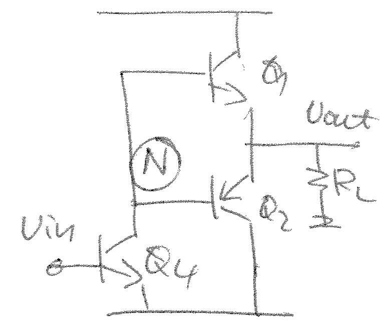
$\Rightarrow \eta = \frac{P_{R_L}}{P_{IQ3} + 2 \cdot P_{AV,Q_1} + P_{R_L}}$

$= \frac{2}{170m + 3.66 \times 2 + 2} = 0.21 = 21\%$

58.

(a) $A_v = 5 \quad R_L = 4\Omega \quad \beta_1 = 40 \quad \beta_2 = 20.$

Assume $I_{C1} \approx I_{C2}$.



$$\frac{v_{out}}{v_N} = \frac{R_L}{(g_{m1}+g_{m2})^{-1} + R_L} = 0.8$$

$$\Rightarrow 2g_{m1}^{-1} = 1 \Rightarrow I_{C1} = 2V_T$$

$$= 0.052\ A.$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = +g_{m4}(r_{\pi1} \parallel r_{\pi2})(g_{m1}+g_{m2})R_L = 5$$

Assume $g_{m1} \approx g_{m2}$:

$$\Rightarrow I_{C4} = V_T \frac{5}{(r_{\pi1} \parallel r_{\pi2})(g_{m1}+g_{m2})R_L}$$

$$= V_T \times \frac{5}{(r_{\pi1} \parallel r_{\pi2})(g_{m1} \times 2)R_L}$$

$$= 0.026 \frac{5}{(6.7\Omega)(2\times 2)(4)}$$

$$\approx 1.2\ mA.$$

$\Rightarrow$ Max I by $Q_1 = \beta_1 \times I_{C4} = 48\ mA$

$\Rightarrow P_{R_L} = \frac{1}{2} I^2 R_L = 24 \times 4\ mW = 96\ mW$, BELOW requirement!

(b) $P = 5\,W = \frac{1}{2}\frac{V_P^2}{R_L} \Rightarrow V_P = 6.3\,V$

$\Rightarrow I_P = \frac{V_P}{R_L} = 1.6\,A$

$\Rightarrow I_{B2,MAX} = \frac{I_P}{\beta_2} = \frac{1.6}{20} = 79\,mA$

$\Rightarrow I_{C4}$ must equal 79 mA to allow max output swing V_P

$\Rightarrow g_{m4} = \frac{I_{C4}}{V_T} = 3.04\,S$

Suppose 2nd stage gain $= 0.8$ $(I_{C1} \simeq I_{C2})$

$\Rightarrow \frac{v_{out}}{v_N} = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}} = 0.8$ $\Rightarrow g_{m1} = 0.5\,S$

$\Rightarrow I_{C1} = I_{C2} = 13\,mA.$

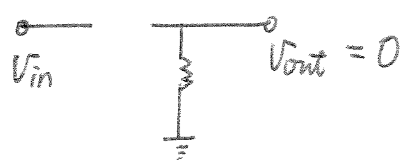
$r_{\pi 1} \,||\, r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \,||\, \frac{\beta_2 V_T}{I_{C2}} = 26.7\,\Omega.$

$\therefore \frac{v_{out}}{v_{in}} = -(3.04)(26.7\Omega)(0.5+0.5)4$

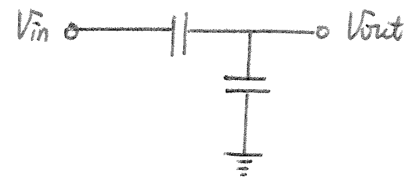
$= -324$!! (Huge! Impractical)

- Even when the 2nd stage gets close to 1, we still need <u>huge</u> gain from first stage.

1. (a).



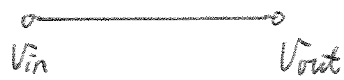
(Low freq.)



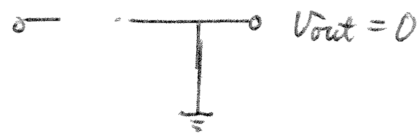
(High freq.)

This is a high pass filter.

(b).



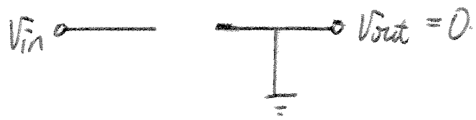
(Low freq.)



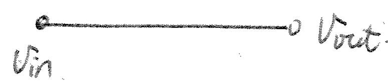
(High freq.)

This is a low pass filter.

(c).



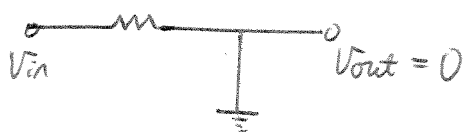
(Low freq)



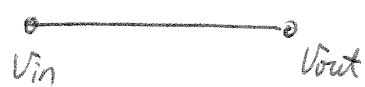
(High freq.)

This is a high pass filter.

(d)



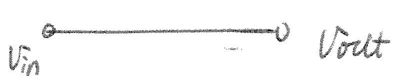
(Low freq.)



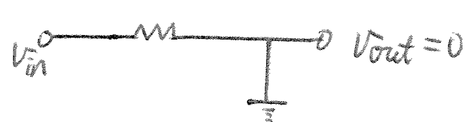
(High freq.)

This is a high pass filter.

(e)

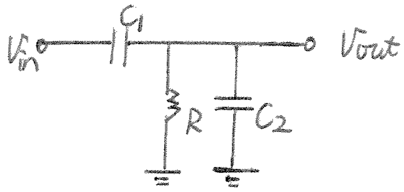


(Low freq.)



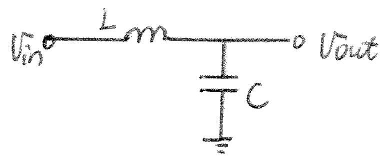
(High freq.)

This is a low pass filter.

2. (a). 

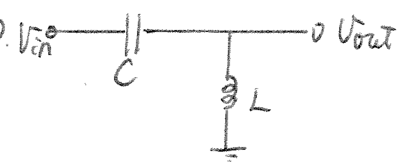
$$V_{out} = \frac{R // \frac{1}{sC_2}}{\frac{1}{sC_1} + R // \frac{1}{sC_2}} V_{in}.$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{C_1}{C_1+C_2} s}{s + \frac{1}{R(C_1+C_2)}}, \quad zero = 0; \quad pole = -\frac{1}{R(C_1+C_2)}$$

(b) 

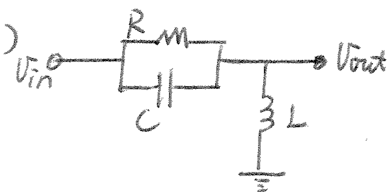
$$V_{out} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/(LC)}{s^2 + \frac{1}{LC}}$$ zero: No finite zero; poles $= \pm i \frac{1}{\sqrt{LC}}$

(c) 

$$V_{out} = \frac{sL}{sL + \frac{1}{sC}} V_{in}$$

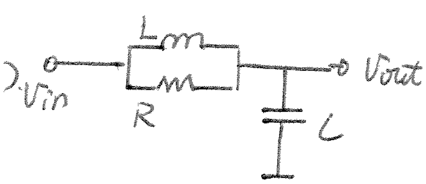
$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{LC}}$$ zeros: Two zeros at 0; poles $= \pm i \frac{1}{\sqrt{LC}}$

(d) 

$$V_{out} = \frac{sL}{sL + R // \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s(s + \frac{1}{RC})(RC)^2}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$ zeros: $0, -\frac{1}{RC}$

poles $= \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$

(e) 

$$V_{out} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL // R} V_{in}$$

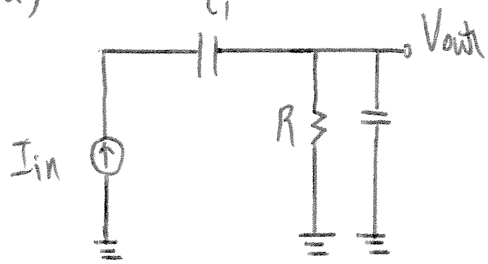
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{RC}(s + \frac{R}{L})}{s^2 + \frac{s}{RC} + \frac{1}{LC}}, \quad zero = -\frac{R}{L}; \quad poles = \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}.$$

3.

Since $\frac{V_{out}}{V_{in}} = \frac{1}{(s+a)(s+b)}$, where a and b are real and positive, the transfer function contains no finite zero and two real poles on the left hand plane. But, after reviewing Problem #2 we discover that NONE of the networks yield this case.

4.

a)

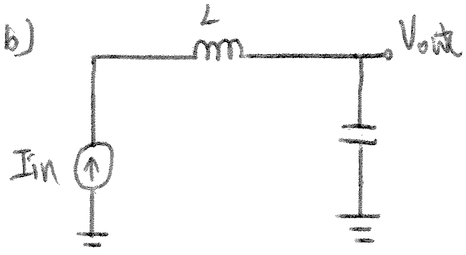


$$\frac{V_{out}}{I_{in}} = \frac{1/C_2}{s + 1/(RC_2)}$$

Zero: No finite zero

Pole: $-1/(RC_2)$

b)

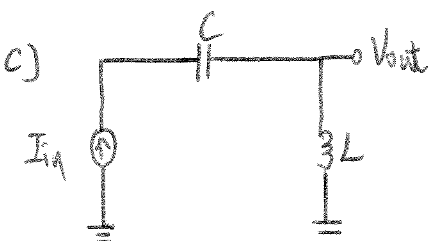


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

c)

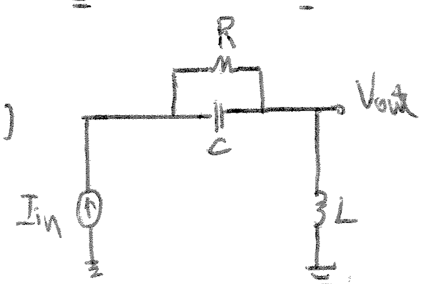


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

d)

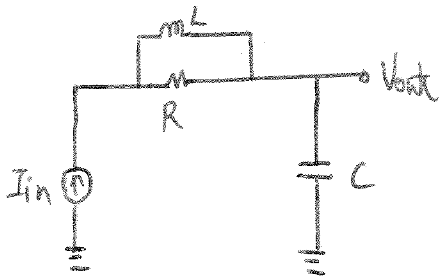


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

e)

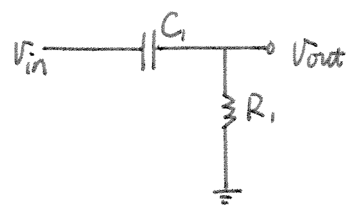


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

5).



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + \frac{1}{C_1 S}} = \frac{S}{S + \frac{1}{R_1 C_1}}$$

zero $= 0$; pole $= -\frac{1}{R_1 C_1}$.

$$\frac{dP}{dC_1} = \frac{1}{(R_1 C_1)^2} \cdot R_1 = \frac{1}{R_1 C_1^2}.$$

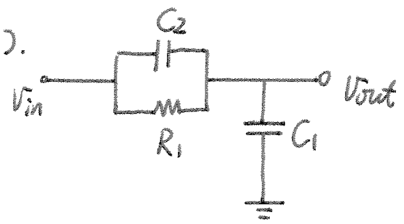
$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{1}{R_1 C_1^2} \cdot C_1 \cdot (R_1 C_1) = -1.$$

Similarly

$$S_{R_1}^{P} = -1.$$

As for the sensitivity of zero, since the zero is at 0, which is indepedant of R_1 and C_1, $S_{R_1}^{z} = S_{C_1}^{z} = 0$.

6).



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{C_1 S}}{\frac{1}{C_1 S} + R_1 // \frac{1}{SC_2}}.$$

$$= \frac{1}{R_1 C_2} \cdot \frac{S + \frac{1}{R_1 C_2}}{R_1(C_1+C_2)\left[S + \frac{1}{R_1(C_1+C_2)}\right]}$$

$$zero = -\frac{1}{R_1 C_2}, \qquad pole = -\frac{1}{R_1(C_1+C_2)}$$

$$\frac{dP}{dR_1} = [R_1(C_1+C_2)]^{-2} \cdot (C_1+C_2) = -\frac{C_1+C_2}{R_1(C_1+C_2)} \cdot P. = -\frac{P}{R_1}$$

$$S_{R_1}^{P} = \frac{\frac{dP}{P}}{\frac{dR_1}{R_1}} = \frac{dP}{dR_1} \cdot \frac{R_1}{P} = -1.$$

$$\frac{dP}{dC_1} = [R_1(C_1+C_2)]^{-2} \cdot R_1 = -\frac{P}{C_1+C_2}$$

$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{C_1}{C_1+C_2}$$

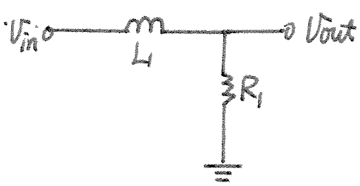
Conversely,

$$S_{C_2}^{P} = -\frac{C_2}{C_1+C_2}.$$

From Problem 5).

$$S_{R_1}^{z} = S_{C_2}^{z} = -1.$$

7)



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + L_1 S} = \frac{R_1/L_1}{S + R_1/L_1}$$

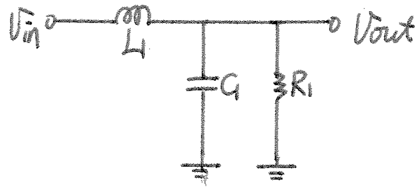
$$\text{pole} = -\frac{R_1}{L_1}$$

$$dp = \frac{\partial p}{\partial R_1} \cdot dR_1 + \frac{\partial p}{\partial L_1} dL_1 = -\frac{1}{L_1} dR_1 + \frac{R}{L^2} dL$$

$$\Rightarrow \frac{dp}{p} = \frac{dR_1}{R_1} - \frac{dL_1}{L_1}$$

$$\left|\frac{dp}{p}\right| \le 5\%, \text{ and } \left|\frac{dR_1}{R_1}\right| \le 3\%$$

$$\Rightarrow \left|\frac{dL_1}{L_1}\right| \le 2\%$$

8). 

a). $V_{out} = V_{in} \cdot \dfrac{R_1 // \frac{1}{C_1 S}}{R_1 // \frac{1}{C_1 S} + SL_1}$

$= V_{in} \cdot \dfrac{\frac{R_1}{R_1 C_1 S + 1}}{\frac{R_1}{R_1 C_1 S + 1} + SL_1}$

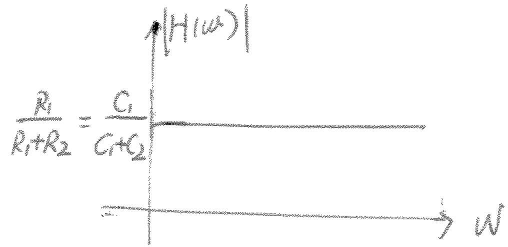
$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 C_1 L_1 S^2 + L_1 S + R_1} = \frac{1}{L_1 C_1} \cdot \frac{1}{S^2 + \frac{1}{R_1 C_1} S + \frac{1}{L_1 C_1}}$$

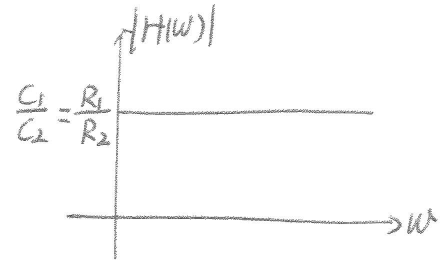
b). $\text{poles} = \dfrac{-\frac{1}{R_1 C_1} \pm \sqrt{\left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1}}}{2}$.

For them to be real $\Rightarrow \left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1} \geq 0$

$\Rightarrow \dfrac{1}{R_1 C_1} \geq \dfrac{2}{\sqrt{L_1 C_1}}$

9). If the zero and pole coincide, they will neutralize each other, and also render the transfer function flat.





10).

$$H(s) = \frac{\alpha s^2 + \beta s + \gamma}{s^2 + \frac{w_n}{Q} s + w_n^2}$$

$$P_{1,2} = -\frac{w_n}{2Q} \pm j w_n \sqrt{1 - \frac{1}{4Q^2}}$$

If $Q = \frac{1}{2}$, then $P_{1,2} = -\frac{w_n}{2Q}$.

11.

$$|H(j\omega)|^2 = \frac{r^2}{(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2}$$

No peaking means no local minimum for $(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2$, which is also known as $D(\omega)$.

A local min exists if $\frac{\partial D(\omega)}{\partial \omega} = 0$.

$$\frac{\partial D(\omega)}{\partial \omega} = \left(\frac{\partial D(\omega)}{\partial \omega^2}\right)\left(\frac{\partial \omega^2}{\partial \omega}\right), \quad \frac{\partial D(\omega)}{\partial \omega^2} = -2(\omega_n^2-\omega^2)+\left(\frac{\omega_n}{Q}\right)^2$$

$$\frac{\partial \omega^2}{\partial \omega} = 2\omega, \text{ so } \frac{\partial D(\omega)}{\partial \omega} = 2\omega\left[2(\omega^2-\omega_n^2)+\left(\frac{\omega_n}{Q}\right)^2\right]=0$$

Solving for ω, we have $\omega = 0, \pm\sqrt{\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2}$

Will bring $D(\omega)$ to its min value.

At $\omega=0$, we have the DC value of the transfer function. However if $Q^2<\frac{1}{2}$ or $Q<\frac{1}{\sqrt{2}}$, $\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2$ becomes negative, which is not physical. Therefore, there is no peaking for $Q<\frac{1}{\sqrt{2}}$. And at $Q=\frac{1}{\sqrt{2}}$, we have $\omega=\pm 0$, which corresponding to the DC value of the transfer function, not peaking. Therefore, the only option left is for $Q>\frac{1}{\sqrt{2}}$, and that is the condition for peaking.

12). $|H(j\omega)|^2 = \dfrac{\gamma^2}{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}$

If $Q > \sqrt{2}/2$, it will peak at $\omega_0 = \omega_n\sqrt{1-1/(2Q)^2}$

$$H(j\omega) = \frac{\gamma}{\sqrt{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}}$$

$$\Rightarrow H(j\omega_0) = \frac{\gamma}{\sqrt{[\omega_n^2 - (\omega_n\sqrt{1-\frac{1}{2Q^2}})^2]^2 + (\frac{\omega_n}{Q}\omega_n\sqrt{1-\frac{1}{2Q^2}})^2}}$$

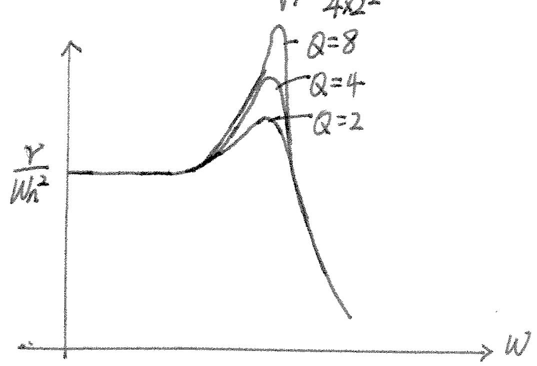
$$= \frac{\gamma}{\sqrt{(\omega_n^2\cdot\frac{1}{2Q^2})^2 + \frac{\omega_n^4}{Q^2}(1-\frac{1}{2Q^2})}}$$

$$= \frac{\gamma}{\sqrt{\frac{\omega_n^4}{4Q^4} + \frac{\omega_n^4}{Q^2} - \frac{\omega_n^4}{2Q^4}}}$$

$$= \frac{Q\gamma}{\omega^2\sqrt{1-\frac{1}{4Q^2}}}$$

Normalize to passband $\Rightarrow \dfrac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$

$Q=2$, peak $= \dfrac{2}{\sqrt{1-\frac{1}{4\times 2^2}}} = 2.07$; $Q=4$, peak $=4.03$; $Q=8$, peak $=8.02$



13).

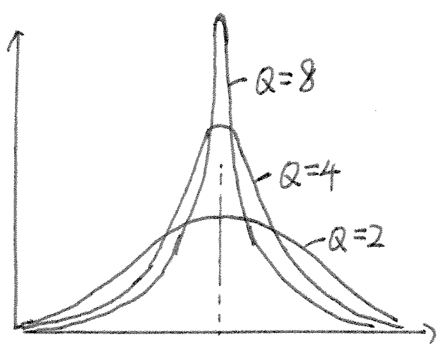
$$H(s) = \frac{\beta s}{s^2 + \frac{w_n}{Q}s + w_n^2} \quad , \quad H(jw) = \frac{j\beta w}{w_n^2 + j\frac{w_n}{Q}w - w^2}$$

$$|H(jw)| = \frac{\beta w}{\sqrt{(w_n^2 - w^2)^2 + (\frac{w_n}{Q}w)^2}}$$

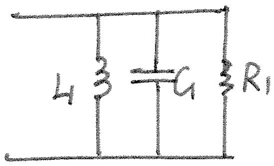
At $w = w_n$,

$$|H(jw_n)| = \frac{\beta w_n}{\frac{w_n}{Q}w} = \frac{Q}{w_n}\beta.$$

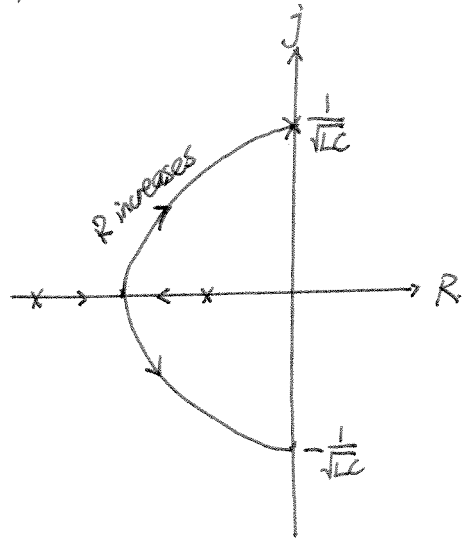
So if we normalize to β, we get $\frac{Q}{w_n}$.



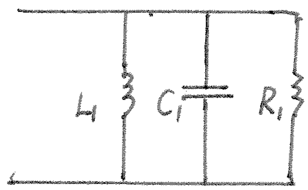
14).

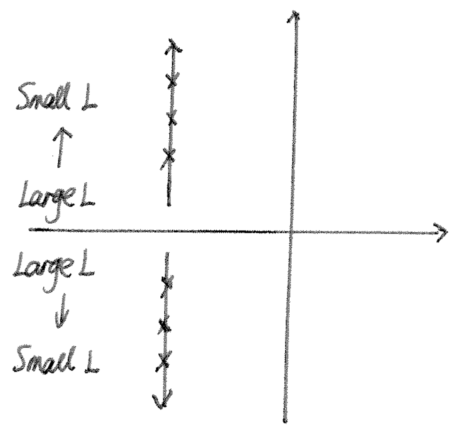


Assume R is never negative



15).





16.

1 dB peaking $\Rightarrow \dfrac{Q^2}{\left(1-\frac{1}{4Q^2}\right)} = (1.1)^2 = 1.21$

$Q^2 = 1.21\left(1-\frac{1}{4Q^2}\right) \Rightarrow 4Q^4 - 4(1.1)^2 Q^2 + (1.1)^2 = 0$

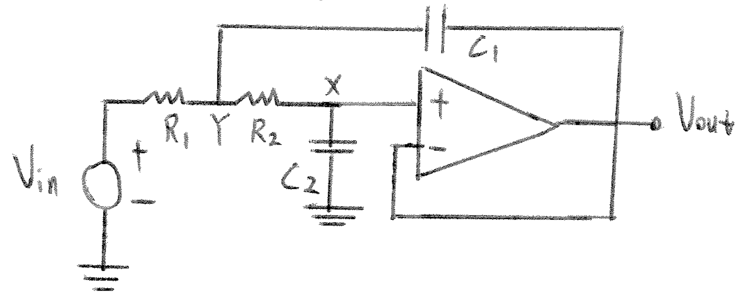
$Q^2 = 0.85704,\ 0.35296,\ Q = 0.925765,\ 0.59410$

$Q = 0.925765$, since $Q > \frac{1}{\sqrt{2}}$ for peaking

$Q = \dfrac{\omega_n}{\beta} = \dfrac{RC}{\sqrt{LC}} = R\sqrt{\dfrac{C}{L}} = 0.925765$

17.

Sallen and Key filter



$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1+R_2)C_2s + 1}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}},$$

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},$$

$$H(s) = \left(s^2 + \frac{(R_1+R_2)C_2}{R_1R_2C_1C_2}s + \frac{1}{R_1R_2C_1C_2}\right)^{-1}$$

$$P_{1,2} = -\frac{(R_1+R_2)}{R_1R_2C_1} \pm \sqrt{\left(\frac{R_1+R_2}{R_1R_2C_1}\right)^2 - \frac{4}{R_1R_2C_1C_2}}$$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1} \pm \sqrt{\left(\frac{1}{(R_1//R_2)C_1}\right)^2 - \frac{4}{R_1C_1R_2C_2}}$$

Assuming $\frac{4}{R_1C_1R_2C_2} > \frac{1}{[(R_1//R_2)C_1]^2}$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1} \pm j2\sqrt{\frac{1}{R_1C_1R_2C_2} - \frac{1}{4[(R_1//R_2)C_1]^2}}$$

17.

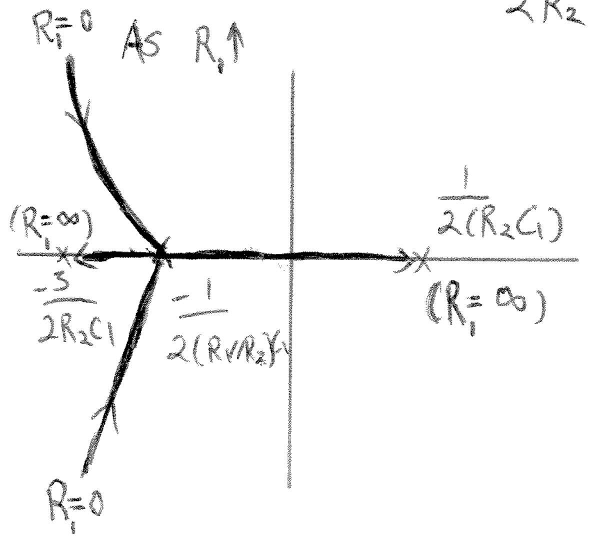
a) R_1: $0 \to \infty$

When $R_1 = 0$, Poles are at $\pm\infty$, so no finite poles. As $R_1 \uparrow$, $\frac{1}{R_1C_1R_2C_2}$ approaches 0, and $\frac{1}{4[(R_1//R_2)C_1]^2}$ approaches $\frac{1}{4[R_2C_1]^2}$. There exists

a R_1 such that $\frac{1}{R_1C_1R_2C_2} = \frac{1}{4[(R_1//R_2)C_1]^2}$ $\Rightarrow$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1}$$

As $R_1 \to \infty$, $P_{1,2} = -\frac{1}{2R_2C_1} \pm \frac{1}{R_2C_1} = -\frac{3}{2R_2C_1}, \frac{1}{2R_2C_1}$



17. b)

R_2 from $0 \to \infty$

when $R_2 = 0$, $P_{1,2}$ are at $\pm\infty$

As $R_2 \uparrow$, $\dfrac{-1}{2(R_1 /\!/ R_2)C_1} \longrightarrow -\dfrac{1}{2R_1C_1}$

$\dfrac{1}{R_1C_1R_2C_2} \to 0$, and $\dfrac{1}{4[(R_1/\!/R_2)C_1]^2} \to \dfrac{1}{4[R_1C_1]^2}$

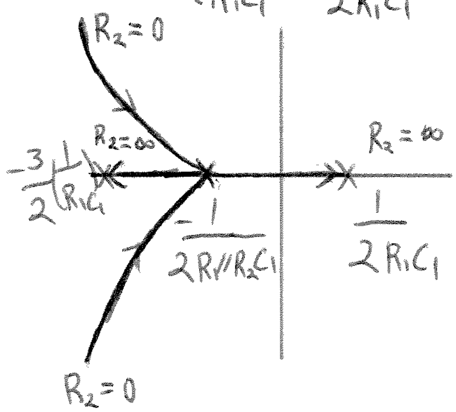
For a certain R_2, $\dfrac{1}{R_1C_1R_2C_2} = \dfrac{1}{4[(R_1/\!/R_2)C_1]^2}$

and $P_{1,2} = \dfrac{-1}{2(R_1/\!/R_2)C_1}$

Finally, when $R_2 = \infty$,

$$P_{1,2} = -\frac{1}{2R_1C_1} \pm 2\sqrt{\frac{1}{4[R_1C_1]^2}} = -\frac{1}{2R_1C_1} \pm \frac{1}{R_1C_1}$$

$$P_{1,2} = -\frac{3}{2}\frac{1}{R_1C_1}, \quad \frac{1}{2R_1C_1}$$

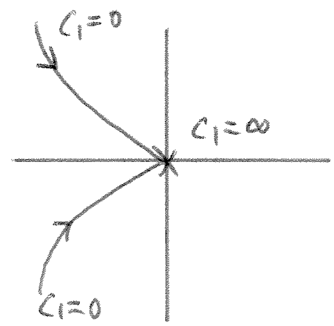


17. c)

As $C_1: 0 \to \infty$

When $C_1 = 0$, Poles are at $\pm\infty$

As $C_1 \uparrow$, Poles approach 0.

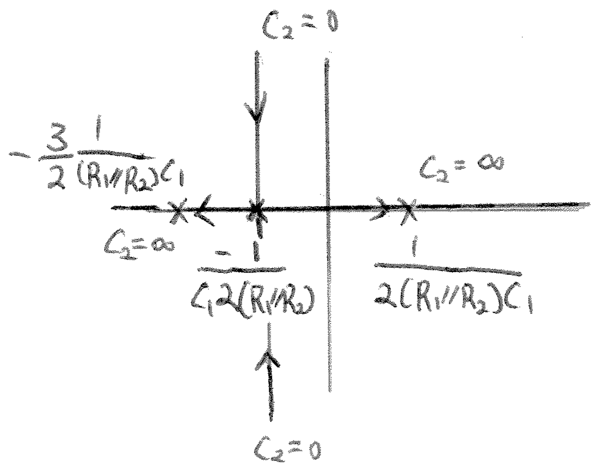


d) As $C_2: 0 \to \infty$

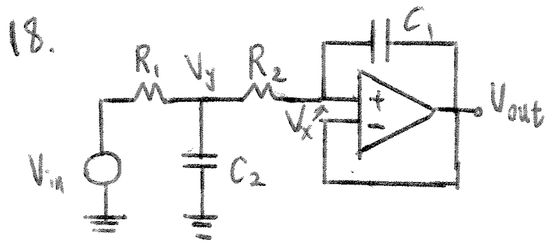
When $C_2 = 0$, Poles are at $\pm\infty$

When $C_2 = \infty$, poles: $-\frac{1}{2(R_1//R_2)C_1} \pm \frac{1}{R_1//R_2 C_1} = -\frac{3}{2}\left(\frac{1}{R_1//R_2C_1}\right)$

$+ \frac{1}{2 R_1//R_2 C_1}$ (note, real part doesn't depend on C_2).



18.

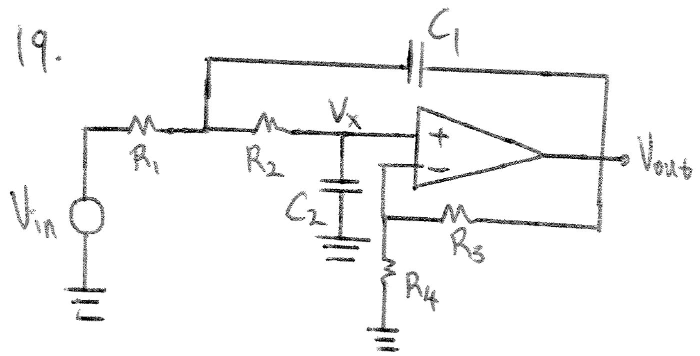


Assuming an ideal Op-amp, $V_x = V_{out}$. Therefore, no current will flow thru C_1. Moreover, since the input impedance of an Op-amp (ideal) is infinite, no current will flow thru R_2 as well, which means $V_y = V_x = V_{out}$.

$$\Rightarrow V_y = V_{out} = \frac{1/C_2 s}{R_1 + 1/C_2 s}$$

Not very useful since it's only a simple single pole lowpass filter. We can implement it with passive components, instead of op-amp.

19.



$K=4, \quad C_1=C_2$

$Q=4$

$$K = 1 + \frac{R_3}{R_4} = 4 \Rightarrow \frac{R_3}{R_4} = 3, \quad \frac{C_1}{C_2} = 1$$

$$\frac{1}{Q} = \sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2}{R_1}\frac{C_2}{C_1}} - \sqrt{\frac{R_1 C_1}{R_2 C_2}}\,\frac{R_3}{R_4}$$

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} - 3\sqrt{\frac{R_1}{R_2}} \Rightarrow \sqrt{\frac{R_2}{R_1}} - 2\sqrt{\frac{R_1}{R_2}} = \frac{1}{Q}$$

$$\frac{1}{Q} = \left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} - 2\left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} \Rightarrow \text{squaring both sides} \Rightarrow$$

$$\frac{1}{Q^2} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_1}{R_2}\right)^{-1} \Rightarrow \frac{1}{16} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_2}{R_1}\right)$$

$$\left(\frac{1}{16}+4\right)\frac{R_1}{R_2} = \frac{R_1}{R_2}\left(4\frac{R_1}{R_2} + \frac{R_2}{R_1}\right) \Rightarrow 4.0625\,\frac{R_1}{R_2} = 4\left(\frac{R_1}{R_2}\right)^2 + 1$$

$$4\left(\frac{R_1}{R_2}\right)^2 - 4.0625\left(\frac{R_1}{R_2}\right) + 1 = 0, \quad \frac{R_1}{R_2} = 0.41908,\ 0.5~~96~~55$$

This leads to a negative Q.

19.

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{0.41908} - \sqrt{1/0.41908} - 3\sqrt{0.41908}\right] 4$$

$$S_{R_1}^{Q} = 5.68$$

20.

$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1+R_2)C_2s + 1}$$

$$Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.1 \Rightarrow Q^2 = (1.1)^2\left(1-\frac{1}{4Q^2}\right)$$

$$\Rightarrow 3.3058Q^4 - 4Q^2 + 1 = 0$$

$Q^2 = 0.85704,\ 0.35296,$

$Q = \pm 0.925765,\ \pm 0.5941$

In order to peak, $Q > \frac{1}{\sqrt{2}} \Rightarrow Q = 0.925765$

$$\Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}} = 0.925765$$

let $\frac{C_1}{C_2} = 1 \Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1R_2} = 0.925765$

$$\Rightarrow R_1R_2 = (0.925765)^2(R_1+R_2)^2$$

$$\frac{R_1R_2}{R_1+R_2} = R_1 /\!/ R_2 = 0.85704(R_1+R_2)$$

$R_1 /\!/ R_2 = 0.85704(R_1+R_2)$

only if $\frac{C_1}{C_2} = 1$

21.

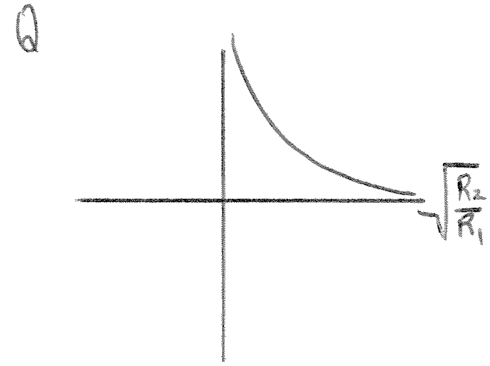
$$S_{R_1}^{Q} = 2, \quad C_2 = C_1, \quad Q = f\left(\sqrt{\frac{R_2}{R_1}}\right)$$

Range of Q and $\sqrt{R_2/R_1}$

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S_{R_1}^{Q} = -\frac{1}{2} + Q\sqrt{\frac{R_2 C_2}{R_1 C_1}} \implies 2 = -\frac{1}{2} + Q\sqrt{\frac{R_2}{R_1}}$$

$$\implies 2.5 = Q\sqrt{\frac{R_2}{R_1}}, \quad Q = \frac{2.5}{\sqrt{R_2/R_1}}$$



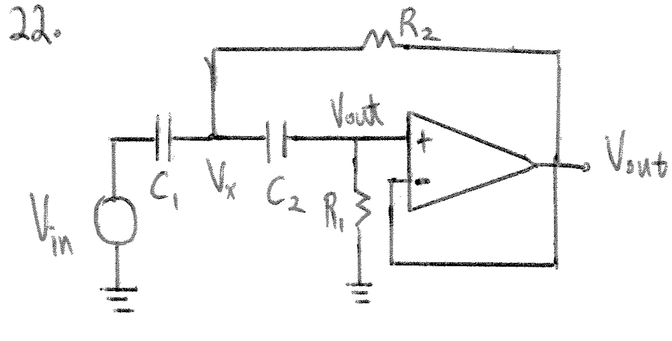
($\frac{R_2}{R_1}$ can't be negative)

Range: $0 < Q < \infty$

$0 < \sqrt{\frac{R_2}{R_1}} < \infty$

If the transfer function does not want to experience peaking, then $0 < Q \le \frac{1}{\sqrt{2}} \implies 3.5356 \le \sqrt{\frac{R_2}{R_1}} < \infty$

22.



Assuming an ideal Op amp.

1) $(V_x - V_{in})C_1 s + (V_x - V_{out})\left(C_2 s + \frac{1}{R_2}\right) = 0$, nodal equation at V_x.

2) $(V_x - V_{out})C_2 s - \frac{V_{out}}{R_1} = 0$, nodal equation at V_{out}.

2) $\Rightarrow$ $V_x = V_{out}\left[\frac{C_2 s + 1/R_1}{C_2 s}\right]$ (A) The stuff in the bracket becomes "A"

1) $\Rightarrow$ $(AV_{out} - V_{in})C_1 s + (AV_{out} - V_{out})\left[C_2 s + \frac{1}{R_2}\right] = 0$

$\Rightarrow AV_{out}C_1 s + V_{out}(A-1)(C_2 s + 1/R_2) = V_{in}C_1 s$

$A - 1 = \frac{1}{R_1 C_2 s}$, $A = \frac{C_2 s + 1/R_1}{C_2 s}$

Subsitute (A-1) and A into 1) $\Rightarrow$

$$\left(\frac{C_2 s + 1/R_1}{C_2 s}\right)C_1 s V_{out} + \left(C_2 s + \frac{1}{R_2}\right)\frac{1}{R_1 C_2 s}V_{out} = V_{in}C_1 s$$

22.

$$\frac{V_{out}}{V_{in}} = \frac{C_1 S}{\frac{C_1}{C_2}\left(C_2 S + \frac{1}{R_1}\right) + \frac{1}{R_1 C_2 S}\left(C_2 S + \frac{1}{R_2}\right)}$$

Rearranging

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{S^2}{S^2 + \left(\frac{C_1 + C_2}{C_2 R_1 C_1}\right) S + \frac{1}{R_2 C_2 R_1 C_1}}$$

$$\omega_n^2 = \frac{1}{R_2 C_2 R_1 C_1}, \quad \frac{\omega_n}{Q} = \frac{C_1 + C_2}{C_2 R_1 C_1}$$

$$\omega_n = \frac{1}{\sqrt{R_2 C_2 R_1 C_1}}, \quad Q = \sqrt{\frac{C_2 C_1 R_1}{R_2}}\left(\frac{1}{C_1 + C_2}\right)$$

23.

$$Q = \frac{1}{C_1+C_2}\sqrt{\frac{C_2C_1R_1}{R_2}} \Rightarrow \frac{1}{Q} = (C_1+C_2)\sqrt{\frac{R_2}{C_2C_1R_1}}$$

1) $$\frac{d}{dQ}\left[\frac{1}{Q}\right] = -\frac{1}{Q^2} \Rightarrow d\left[\frac{1}{Q}\right] = -\frac{1}{Q^2}\,dQ$$

2) $$\frac{d\left[\frac{1}{Q}\right]}{dR_2} = \frac{1}{2}\frac{C_1+C_2}{\sqrt{C_2C_1R_1R_2}} \Rightarrow d\left[\frac{1}{Q}\right] = \frac{1}{2}\frac{C_1+C_2}{\sqrt{C_2C_1R_1R_2}}\,dR_2$$

Equating 1) and 2) and multiple 2) by $\frac{R_2}{R_2}$

$$-\frac{dQ}{Q^2} = \frac{1}{2}\frac{(C_1+C_2)R_2}{\sqrt{C_2C_1R_1R_2}}\frac{dR_2}{R_2}$$

$$\frac{dQ}{Q}\Big/\frac{dR_2}{R_2} = -\frac{Q}{2}(C_1+C_2)\sqrt{\frac{R_2}{C_1C_2R_1}}$$

$$S_{R_2}^{Q} = -\frac{Q}{2}(C_1+C_2)\sqrt{\frac{R_2}{C_1C_2R_1}} = -\frac{1}{2}$$

$$\frac{1}{Q} = (C_1+C_2)\sqrt{\frac{R_2}{C_2C_1R_1}} = C_1\sqrt{\frac{R_2}{C_2C_1R_1}} + C_2\sqrt{\frac{R_2}{C_2C_1R_1}}$$

$$\frac{\partial\left(\frac{1}{Q}\right)}{\partial C_1} = \frac{1}{2}\sqrt{\frac{R_2}{C_2R_1C_1}} - \frac{C_2}{2C_1}\sqrt{\frac{R_2}{C_2R_1C_1}}, \quad \frac{d\left(\frac{1}{Q}\right)}{dQ} = -\frac{1}{Q^2}$$

Rearranging $\Rightarrow$ $$-\frac{\partial Q}{Q^2} = \frac{\partial C_1}{C_1}\left(\frac{C_1-C_2}{2}\sqrt{\frac{R_2}{C_2R_1C_1}}\right)$$

$$\frac{\partial Q}{Q}\Big/\frac{\partial C_1}{C_1} = S_{C_1}^{Q} = -Q\left(\frac{C_1-C_2}{2}\sqrt{\frac{R_2}{C_2R_1C_1}}\right)$$

23. Similarly:

$$S_{C_2}^{Q} = -Q\left(\frac{C_2 - C_1}{2}\sqrt{\frac{R_2}{C_2 R_1 C_1}}\right)$$

$$S_{R_1}^{Q} = Q\left(\frac{C_1 + C_2}{2}\sqrt{\frac{R_2}{C_2 C_1 R_1}}\right) = \frac{1}{2}$$

24.

$$\frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

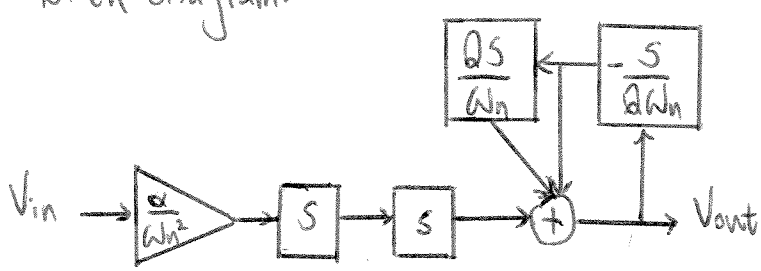
Cross-multiply.

$$V_{out}s^2 + V_{out}\frac{\omega_n}{Q}s + V_{out}\omega_n^2 = V_{in}\alpha s^2$$

Rearranging

$$V_{out} = V_{in}\frac{\alpha}{\omega_n^2}s^2 - V_{out}\frac{s^2}{\omega_n^2} - V_{out}\frac{s}{Q\omega_n}$$

Block diagram:



25.

$Q = 2, \quad \omega_n = (2\pi)(2\times 10^6)$

$R_6 = R_3, \quad R_1 = R_2, \quad C_1 = C_2$

$10\text{pF} < \text{Total } C < 1\text{nF}, \quad 1\text{K}\Omega < \text{Total } R < 50\text{K}\Omega$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right), \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Since $R_6 = R_3 \Rightarrow \omega_n^2 = \left(\frac{1}{R_1C_1}\right)^2 = \left(2\pi\times 2\times 10^6\right)^2$

$$\frac{1}{R_1C_1} = 2\pi\times 2\times 10^6 = \omega_n$$

$$Q = \frac{R_4+R_5}{R_4} = 2 \Rightarrow R_5 = R_4$$

Let $C_1 = C_2 = 100\text{pf}, \quad R_1 = \frac{1}{(2\pi)(2\times 10^6)(100\text{pf})} = 795.77\Omega$

So $R_1 = R_2 = 795.77\Omega, \quad C_1 = C_2 = 100\text{pf}$.

Since R_3, R_4, R_5, R_6 don't affect Q and ω_n,
let them be 500Ω each.

Total R: $(4)(500) + (2)(795.77) = 3.6\text{K}\Omega$

Total C: $100\text{pf} + 100\text{pf} = 200\text{pf}$.

26.

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\,\frac{1}{R_1C_1}, \quad \omega_n^2 = \frac{R_6}{R_3}\cdot\frac{1}{R_1R_2C_1C_2}$$

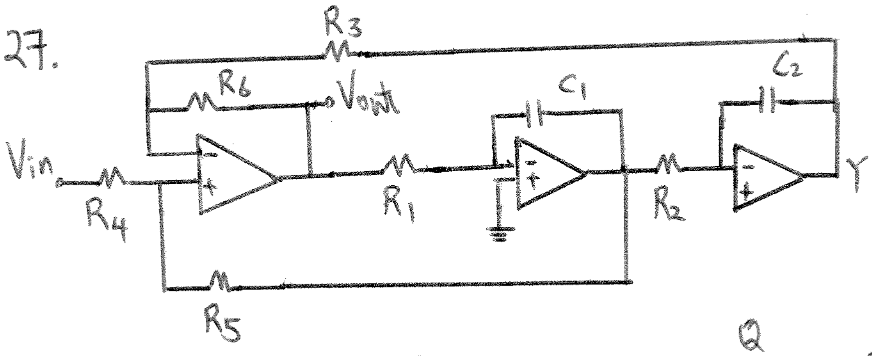
$$Q = \omega_n\left(\frac{R_4+R_5}{R_4}\right)R_1C_1, \quad \omega_n = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}$$

$$Q = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}\left(\frac{R_4+R_5}{R_4}\right)R_1C_1$$

$$Q = \sqrt{\frac{R_6}{R_3}}\sqrt{\frac{R_1C_1}{R_2C_2}}\left(\frac{R_4+R_5}{R_4}\right)$$

If $R_6 = R_3$, Q doesn't depend on R_6 and R_3, hence zero sensitivity.

27.



Low pass, low freq gain of 2. $S^{Q}_{R_3,R_6} = 0$

$$\frac{V_Y}{V_{in}} = \left(\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}\right)\left(\frac{1}{R_1R_2C_1C_2 s^2}\right)$$ Low pass transfer function

Low freq gain: $\dfrac{\alpha}{\omega_n^2 R_1R_2C_1C_2}$, where $\alpha = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)$

and $\omega_n^2 = \dfrac{R_6}{R_3}\left(\dfrac{1}{R_1R_2C_1C_2}\right)$

Therefore, Low freq gain: $\dfrac{\alpha}{\frac{R_6}{R_3}} = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)\left(\dfrac{R_3}{R_6}\right)$

$$S^{Q}_{R_3,R_6} = \frac{Q}{2}\,\frac{|R_3-R_6|}{1+R_5/R_4}\sqrt{\frac{R_2C_2}{R_3R_6R_1C_1}}$$

To obtain $S^{Q}_{R_3,R_6} = 0$, $R_3 = R_6$, however this makes the low freq gain: $2\left(\dfrac{R_5}{R_4+R_5}\right) \neq 2$.

Therefore, it's impossible a low freq gain of 2 if $S^{Q}_{R_3,R_6} = 0$.

28.

Peaking : 1 dB, $R_3 = R_6$

Normalized Peak Value: $\dfrac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1$

solving for Q^2: 0.8570, ~~0.3830~~ (not possible for peaking)

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1 C_1}\right), \quad \left(\frac{\omega_n}{Q}\right)^2 = \left(\frac{R_4}{R_4+R_5}\right)^2\left(\frac{1}{R_1 C_1}\right)^2$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1 R_2 C_1 C_2}\right) = \frac{1}{R_1 R_2 C_1 C_2} \quad (\text{since } R_6 = R_3)$$

Therefore, $Q^2 = \left(\dfrac{R_1 C_1}{R_2 C_2}\right)\left(\dfrac{R_4+R_5}{R_4}\right)^2 = 0.8570$

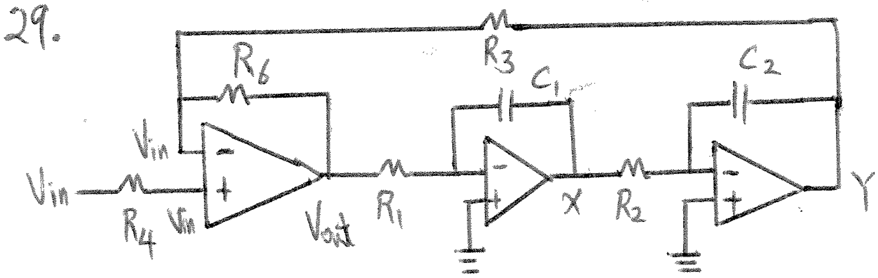
Low pass gain: $2\dfrac{R_5}{R_4+R_5} = \alpha$ (since $R_6 = R_3$)

So $\dfrac{\alpha}{2} = \dfrac{R_5}{R_4+R_5}, \quad \dfrac{R_4}{R_4+R_5} = 1-\dfrac{\alpha}{2} \Rightarrow \dfrac{R_4+R_5}{R_4} = \left(1-\dfrac{\alpha}{2}\right)^{-1}$

So $\left(\dfrac{R_1 C_1}{R_2 C_2}\right)\left(1-\dfrac{\alpha}{2}\right)^{-2} = 0.8570$, if $\alpha = 1 \Rightarrow \dfrac{R_1 C_1}{R_2 C_2} = 0.214$.

However, can't go down any further without knowing more information

29.



$$V_X = -\frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right), \quad V_Y = -\frac{V_X}{R_2}\left(\frac{1}{C_2 S}\right), \quad V_{out} = V_{in} - \frac{(V_Y - V_{in})R_6}{R_3}$$

Subsituting V_x into $V_y \Rightarrow V_y = \frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right)\left(\frac{1}{R_2 C_2 S}\right)$

Subsituting V_y into V_{out} and rearranging:

$$\frac{V_{out}}{V_{in}} = \frac{(R_1C_1)(R_2C_2)S^2\left(1+\frac{R_6}{R_3}\right)}{(R_1C_1)(R_2C_2)S^2 + \frac{R_6}{R_3}}$$

simplifying

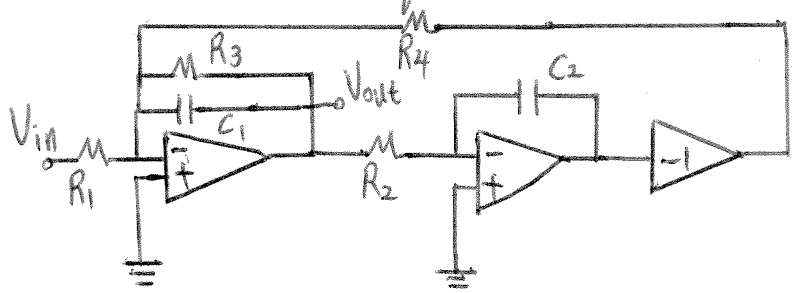
$$\frac{V_{out}}{V_{in}} = \frac{S^2\left(1+\frac{R_6}{R_3}\right)}{S^2 + \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right)}$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right), \quad Q = \infty$$

$$\alpha = \left(1 + \frac{R_6}{R_3}\right)$$

30.

Tow-Thomas Biquad:



$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}} \quad , \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

$$\frac{\partial \omega_n}{\partial R_2} = -\frac{1}{2}\,\frac{1}{R_2\sqrt{R_2 R_4 C_1 C_2}} = -\frac{1}{2}\,\frac{\omega_n}{R_2}$$

$$\frac{\partial \omega_n}{\omega_n} \Big/ \frac{\partial R_2}{R_2} = S_{R_2}^{\omega_n} = -\frac{1}{2}$$

Since R_2, R_4, C_1, C_2 are equivalent in ω_n's definition, all of their sensitivities $= -\frac{1}{2}$

Sensitivities of Q:

$$\frac{\partial Q}{\partial R_3} = \sqrt{\frac{C_1}{R_2 R_4 C_2}}\left(\frac{R_3}{R_3}\right) \Rightarrow \frac{\partial Q}{Q} = \frac{\partial R_3}{R_3} \Rightarrow S_{R_3}^{Q} = 1$$

$$\frac{\partial Q}{\partial C_1} = \frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}} \left(\frac{1}{R_2 R_4 C_2}\right) \frac{C_1}{C_1} \Rightarrow \frac{\partial Q}{Q} = \frac{1}{2}\,\frac{\partial C_1}{C_1} \Rightarrow S_{C_1}^{Q} = \frac{1}{2}$$

$$\frac{\partial Q}{\partial R_2} = -\frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}} \frac{C_1}{R_4 C_2}\left(\frac{1}{R_2^2}\right) \Rightarrow \frac{\partial Q}{Q} = -\frac{1}{2}\,\frac{\partial R_2}{R_2} \Rightarrow S_{R_2}^{Q} = -\frac{1}{2}$$

30.

Since R_2, R_4 and C_2 are equivalent in the expression

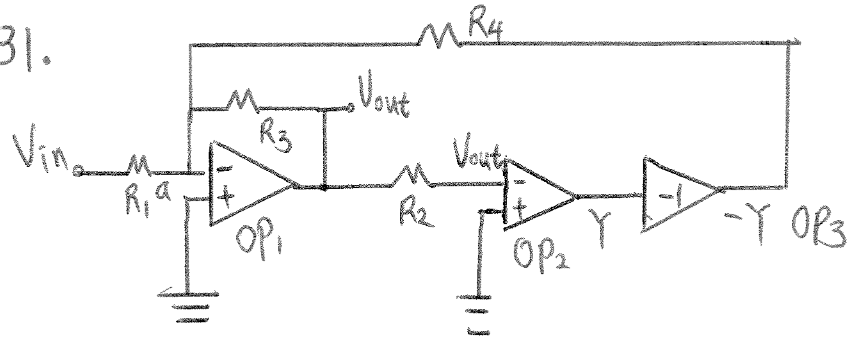
$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}$$

So, $S^{\omega_n}_{R_2,R_4,C_1,C_2} = -\frac{1}{2}$, $S^{\omega_n}_{R_1,R_3} = 0$

$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}, \quad S^{Q}_{C_1} = \frac{1}{2}, \quad S^{Q}_{R_3} = 1$$

$$S^{Q}_{R_1} = 0$$

31.



(Low freq)

Vout equals zero because of OP2's negative feedback.
Likewise, Va equals to zero as well.
So, summing all the currents thru R_3, we have

$$-\left(\frac{0-V_Y}{R_4}+\frac{V_{in}}{R_1}\right)R_3 = V_{out} = 0$$

$$\Rightarrow \frac{V_{in}}{R_1} = \frac{V_Y}{R_4} \Rightarrow \frac{V_Y}{V_{in}} = \frac{R_4}{R_1}$$

32.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3}\right)$$

$\omega_n = (2\pi)(10\ MHz)$, $R_3 = 1K$, $R_2 = R_4$, $C_1 = C_2$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

Peaking: 1dB

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1, \quad Q^2 = 0.8570$$

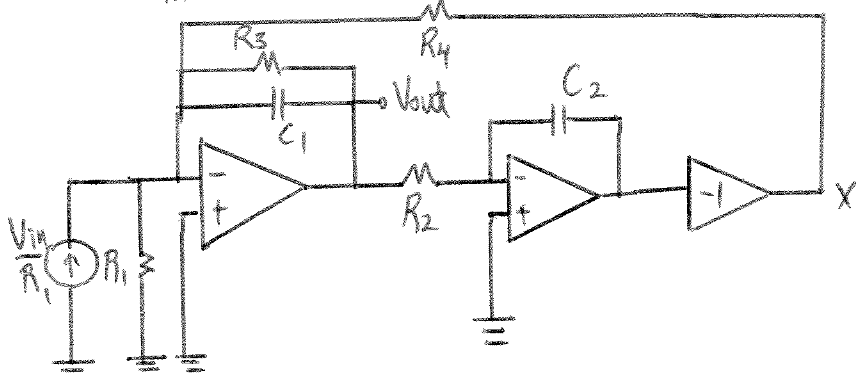
$$\omega_n = \frac{1}{\sqrt{(R_2 C_1)^2}} = (2\pi)(10\times10^6) \Rightarrow \frac{1}{R_2 C_1} = (2\pi)(10\times10^6)$$

$$\frac{1}{Q} = \frac{1}{1000}\sqrt{R_2^2} = \frac{1}{Q} = \frac{R_2}{1000} \Rightarrow \begin{array}{l} R_2 = 1166.86\ Ohm \\ R_2 = 1.2\ K\Omega \end{array}$$

Solving for C_1 we have: $C_1 = 13.64\ pf.$

33.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2 R_3 R_4 C_1 C_2 s^2 + R_2 R_4 C_2 s + R_3}\right)$$



When R_1 and V_{in} are replaced with its Norton equivalent, we see that the "upper" terminal of R_1 is at virtual ground. Since R_1's two terminals are at the same potential, no current will flow throw it, therefore it can be seen as an open. So R_1 is not in the signal path, and therefore will not affect the frequency response. However, since its magnitude is embedded in the Norton current source, it will affect the DC gain.

34.

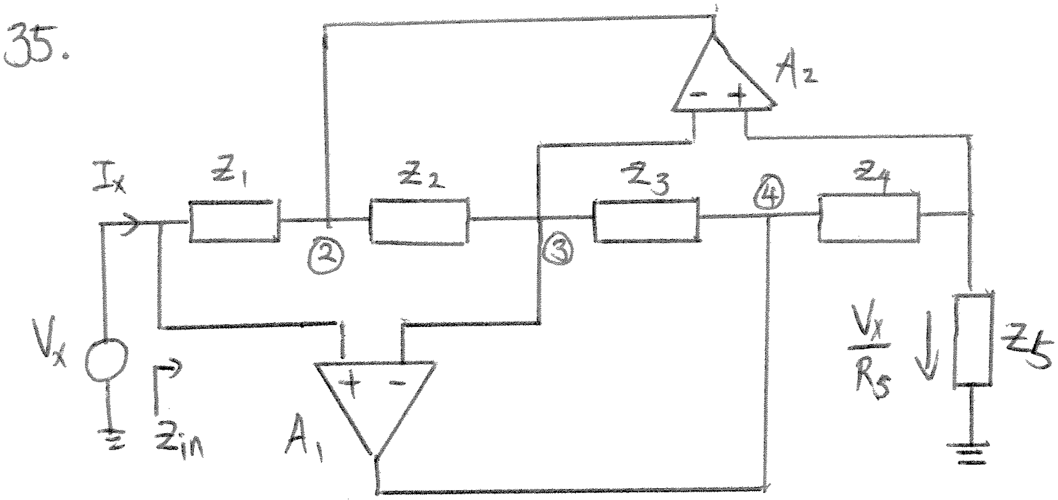
$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$ (for circuit diagram, please refer to Problem # 35)

For Z_{in} to be inductive, the following combinations will work.

1	2	3
$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = C$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$
$Z_2 = C$	$Z_2 = C$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = R$

Any other combination will result in DC path blockage at a node. Moreover, in #2 it's assumed that the input can provide a DC bias.

35.



$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

For Z_{in} to be Capacitive the following combinations can be used.

1	2	3	4	5
$Z_5 = C$	$Z_5 = C$	$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$	$Z_3 = C$	$Z_3 = C$
$Z_2 = R$	$Z_2 = C$	$Z_2 = R$	$Z_2 = R$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = C$	$Z_1 = R$	$Z_1 = C$

Any other combination results in a DC path blockage at a node. Moreover, in # 2, 3, 5, it is assumed that the input node will produce a DC bias.

36.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$$Z_5 = R_x + \frac{1}{CS}, \quad Z_4 = R_x, \quad Z_3 = R_x, \quad Z_2 = R_x,$$

$$Z_1 = \frac{1}{CS}$$

$$Z_{in} = \frac{\frac{R_x}{CS}\left(R_x + \frac{1}{CS}\right)}{R_x^2} = \frac{1}{CSR_x}\left(R_x + \frac{1}{CS}\right)$$

$$Z_{in} = \frac{1}{CS} + \frac{1}{C^2 S^2 R_x}$$

$$V_{out} = \frac{V_{in}\left[S^2[C^2 R_x] + CS\right]}{\left[S^2[C^2 R_x] + SC + R_1[S^3 C^3 R_x]\right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{SCR_x + 1}{S^2 R_1 R_x C^2 + SCR_x + 1}$$

37. $Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$ (for circuit diagram, please refer to problem # 35)

Let Z_5 be a capacitor, Z_2 and Z_4 be large resistors and Z_1 and Z_3 be small resistors compared to Z_2 and Z_4.

For example, let Z_1 and Z_3 equal 50Ω and Z_2 and Z_4 equal $5K\Omega$. Then there's a $(100)^2 = 10000$ multiplication factor onto C_5.

38.

Butterworth filter: roll-off of 1dB @ $\omega = 0.9\,\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}} = 0.9 \Rightarrow 2n = \frac{\log(0.2345679)}{\log(0.9)}$$

$n = 6.88$

So we need a 7th order.

39.

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}}$$

$n=0 \Rightarrow -3\,dB$

$n=1 \Rightarrow -2.577\,dB$

$n=3 \Rightarrow -1.851\,dB$

$n=5 \Rightarrow -1.299\,dB$

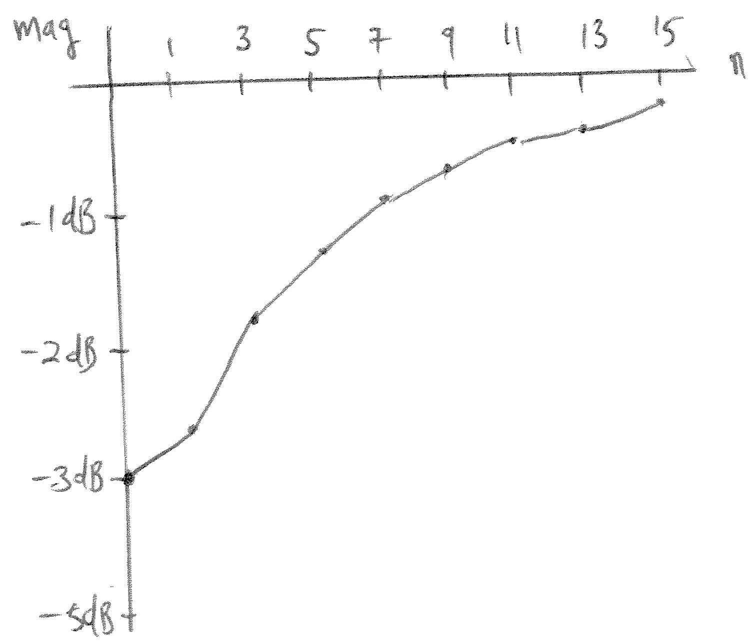
$n=7 \Rightarrow -0.895\,dB$

$n=9 \Rightarrow -0.607\,dB$

$n=11 \Rightarrow -0.408\,dB$

$n=13 \Rightarrow -0.272\,dB$

$n=15 \Rightarrow -0.180\,dB$



40.

$1.1\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(1.1)^{2n}}} = 0.1 \Rightarrow 2n = \frac{\log(99)}{\log(1.1)}$$

$n = 24.106$ so needs $n = 25$.

$n=0 \Rightarrow -3$ dB

$n=1 \Rightarrow -3.4439$ dB

$n=3 \Rightarrow -4.427$ dB

$n=5 \Rightarrow -5.555$ dB

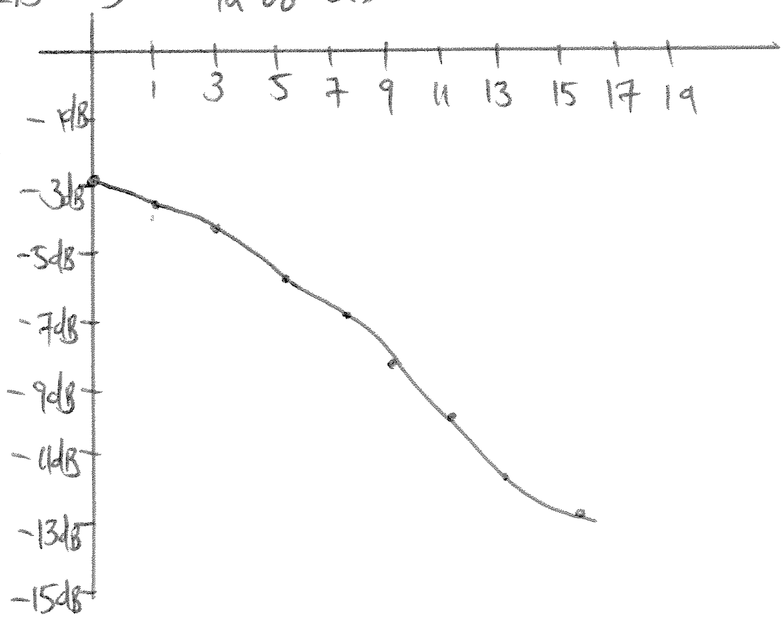
$n=7 \Rightarrow -6.810$ dB

$n=9 \Rightarrow -8.169$ dB

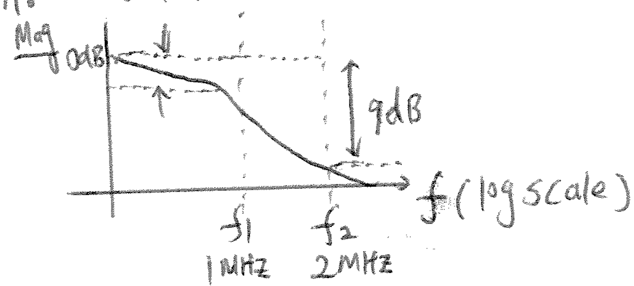
$n=11 \Rightarrow -9.61$ dB

$n=13 \Rightarrow -11.112$ dB

$n=15 \Rightarrow -12.66$ dB



41.



$$|H(f)| = \frac{1}{\left(1 + \left(\frac{2\pi f}{\omega_0}\right)^6\right)^{\frac{1}{2}}}$$

$$|H(5\text{MHz})| = 0.02438$$

Suppression: $20 \log(0.02438) = -32.26$ dB

42.

Low-pass Butterworth: Passband flatness of 0.5 dB
$f_1 = 1\,MHz$, $f_2 = 2\,MHz$, order < 5

$$-0.5\,dB = 20\log(x) \Rightarrow x = 0.944$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}} = 0.944 \Rightarrow \frac{1}{\left(1+\left(\frac{1}{f_0}\right)^{2n}\right)} = (0.944)^2$$
@ f = 1MHz

$$\Rightarrow 1 + \frac{1}{(f_0)^{2n}} = \frac{1}{(0.944)^2} \Rightarrow f_0 = 10^{\frac{0.91306}{2n}}$$

for $n=1$, $f_0 \approx 2.86$ MHz

for $n=5$, $f_0 \approx 1.234$ MHz
Therefore, for greatest attenuation $n=5$

So $$H(2MHz) = \frac{1}{\sqrt{1+\left(\frac{2}{1.234}\right)^{10}}} = 0.089$$

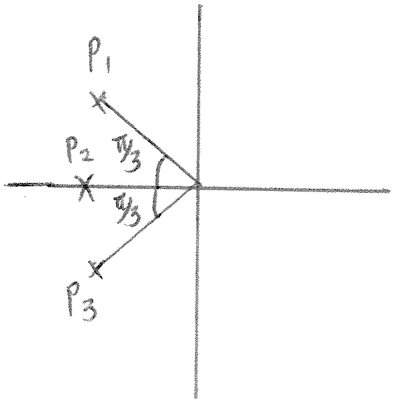
$$20\log(0.089) = -21.0\,dB \text{ at } n=5$$

43.

$$P_K = \omega_0 \exp\left(\frac{j\pi}{2}\right) \exp\left(j\,\frac{2K-1}{2n}\pi\right), \quad K = 1, 2, \cdots n$$

The poles lie on a circle because all of their magnitude, which is the distance from the origin to the poles, are the same (ω_0) with each K; only the phase, which is the angle the poles make with the positive real axis, differ. Therefore, a circle is formed.

44.



$$P_1 = 2\pi(1.45\,\text{MHz})\left[\cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$P_2 = (2\pi)(1.45\,\text{MHz})$$

$$P_3 = (2\pi)(1.45\,\text{MHz})\left[\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$H(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[2\pi(1.45\,\text{MHz})]^2}{s^2 - [4\pi(1.45\,\text{MHz})\cos(2\pi/3)]s + [2\pi(1.45\,\text{MHz})]^2}$$

KHN Low pass Transfer function:

$$\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{1}{R_1R_2C_1C_2s^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\frac{\alpha}{R_1R_2C_1C_2} = (2\pi \times 1.45\times10^6)^2, \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right) = [2\pi \times 1.45\times10^6]^2$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right) = -(4\pi \times 1.45\times10^6 \times \cos(2\pi/3))$$

$$\frac{\alpha}{R_1R_2C_1C_2} = \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Let $R_6 = R_3$, $R_2 = 4R_1$, $C_1 = C_2$

$$\omega_n^2 = \left(\frac{1}{4R_1C_1}\right)^2 = (2\pi\times1.45\times10^6)^2 \Rightarrow \frac{1}{2R_1C_1} = 2\pi\times1.45\times10^6$$

44.

Let $R_1 = 5K \Rightarrow C_1 = 10.98pf$, $R_2 = 20K$, $C_2 = 10.98pf$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5}\left(\frac{1}{R_1 C_1}\right) = 9110618.7$$

$$\Rightarrow \frac{R_4}{R_4 + R_5} = \frac{1}{2}$$

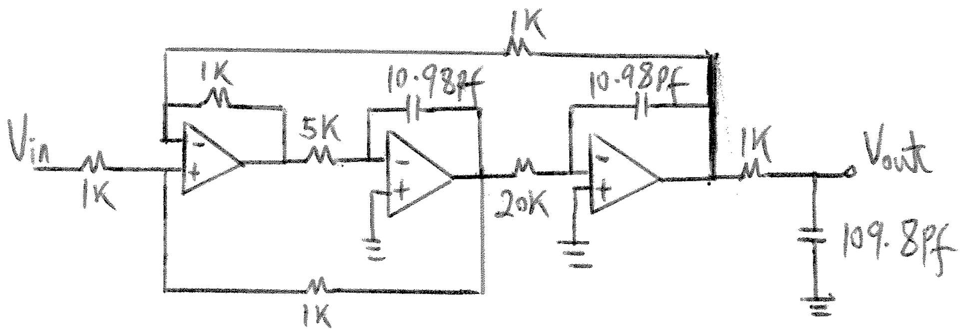
$$\frac{R_5}{R_4 + R_5}\left(1 + \frac{R_6}{R_3}\right)\left(\frac{1}{R_1 R_2 C_1 C_2}\right) = \left(\frac{1}{2}\right)(2)\left(2\pi \times 1.45 \times 10^6\right)^2 = \left(2\pi \times 1.45 \times 10^6\right)^2$$

let R_5 and R_4 be 1K apiece.

So $R_5 = R_4 = R_6 = R_3 = 1K$

$R_1 = 5K$, $R_2 = 20K$

$C_1 = C_2 = 10.98pf$



45.

Tow-Thomas Biquad

$$\frac{V_Y}{V_{in}} = \frac{R_3R_4}{R_1}\left(\frac{1}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}\right)$$

$$\frac{V_Y}{V_{in}} = \frac{1/(R_1R_2C_1C_2)}{s^2 + 1/(R_3C_1)s + 1/(R_2R_4C_1C_2)}$$

$$\frac{V_Y}{V_{in}} = \frac{(2\pi \times 1.45\times 10^6)^2}{s^2 - (4\pi\times 1.45\times 10^6 \times \cos(2\pi/3))s + (2\pi\times 1.45\times 10^6)^2}$$

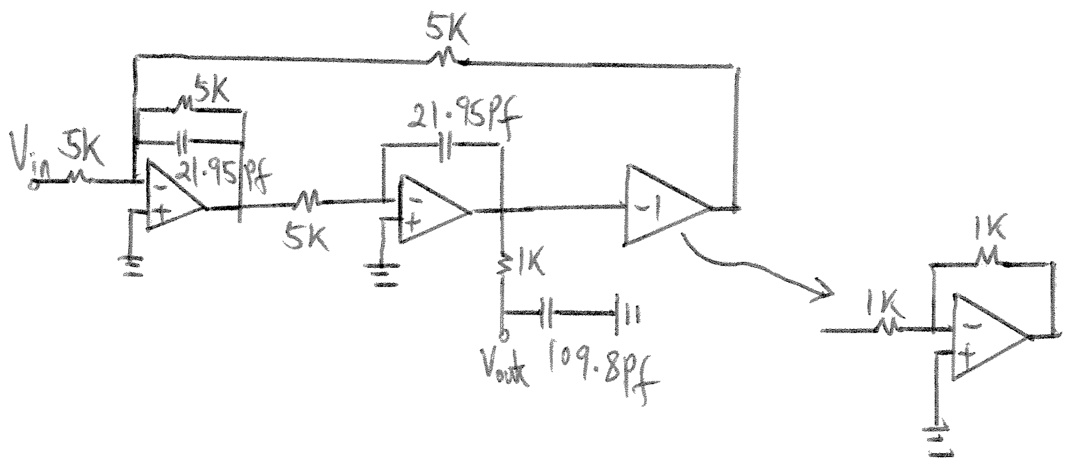
$$\frac{1}{R_1R_2C_1C_2} = (2\pi\times 1.45\times 10^6)^2, \quad \frac{1}{R_2R_4C_1C_2} = (2\pi\times 1.45\times 10^6)^2$$

$$\frac{1}{R_3C_1} = 2\pi\times 1.45\times 10^6$$

Let $R_1 = R_2 = R_3 = R_4$, $C_1 = C_2$

let $R_3 = 5K \Rightarrow C_1 = 21.95pf$

So $R_1 = R_2 = R_3 = R_4 = 5K$, and $C_1 = C_2 = 21.95pf$



46.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}} \qquad n=4, \quad \epsilon = 0.2$$

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \cos\left(4\cos^{-1}\frac{\omega}{\omega_0}\right)$$

$$C_n^2\left(\frac{\omega}{\omega_0}\right) = \cos^2\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \frac{1}{2}\left(1+\cos\left(2n\cos^{-1}\frac{\omega}{\omega_0}\right)\right)$$

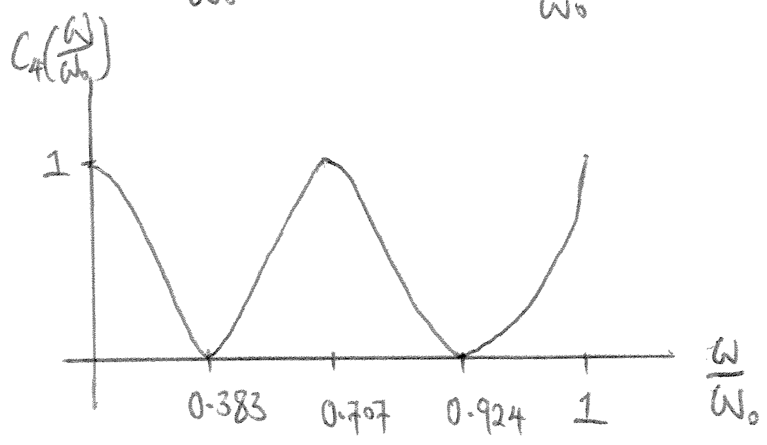
$$2n\cos^{-1}\frac{\omega}{\omega_0} = \pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.924$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 3\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.383$$

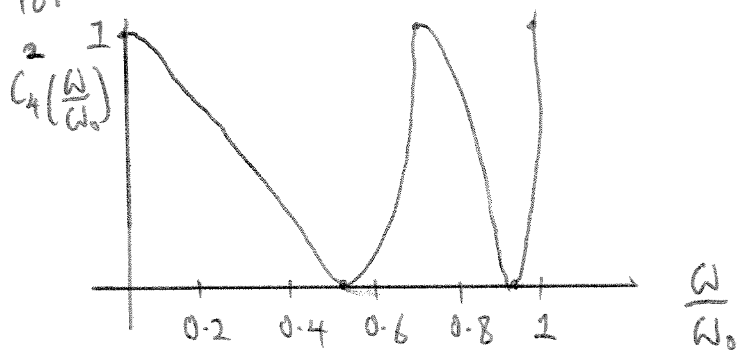
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 0, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 1$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 2\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.707$$

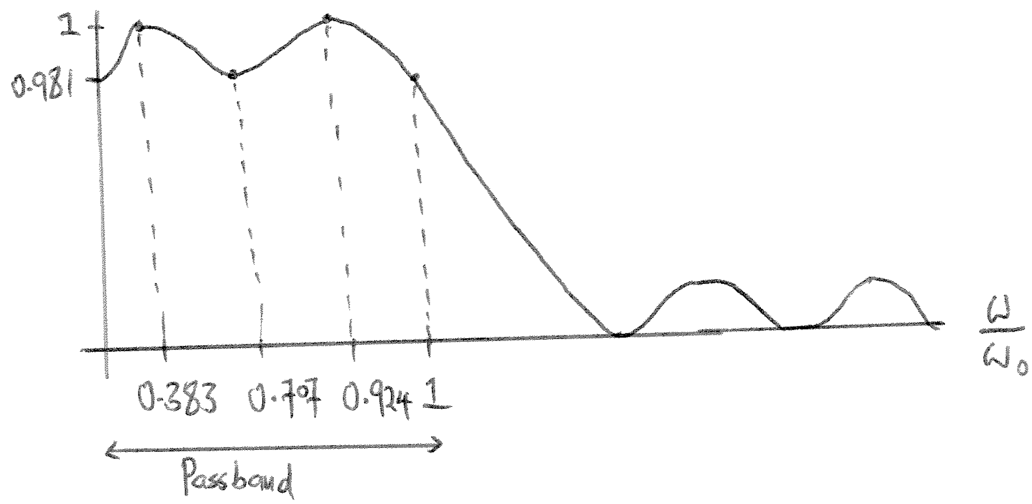
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 4\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0$$



46.



$$H(j\omega) = \frac{1}{\sqrt{1 + (0.2)^2\left[C_4^2\left(\frac{\omega}{\omega_0}\right)\right]}}$$



47. Chebyshev: 25 dB at 5 MHz.

$n=5,\quad W_0 = 2\text{MHz},\quad \frac{W}{W_0} = \frac{5}{2}.$

$$\frac{1}{\sqrt{1+\epsilon^2\cosh^2(n\cosh^{-1}\frac{5}{2})}} = -25\,dB = 0.056234$$

$$\Rightarrow \frac{1}{1+\epsilon^2(1.5939\times10^6)} = 0.0031622771$$

$$\Rightarrow \epsilon^2 = 1.9777\times10^{-4}$$

$\Rightarrow$ Minimum Ripple

$$\frac{1}{\sqrt{1+(1.9777\times10^{-4})}} = 0.99990 = -8.6\times10^{-4}\,dB.$$

48. $n = 6$

$$\cosh^2\left(6\cos^{-1}\left(\frac{5}{2}\right)\right) = 36590401$$

$$\frac{1}{\sqrt{1+\epsilon^2(36590401)}} = 0.056234$$

$$\epsilon^2 = 8.615 \times 10^{-6}$$

$$\text{Minimum Ripple} = \frac{1}{\sqrt{1+8.615\times10^{-6}}} = -3.74\times10^{-5}\ dB$$

Smaller than when $n=5$.

49. $\epsilon = 0.509$, $n = 4$

$$P_{1,4} = -0.140 w_0 \pm 0.983 j w_0$$

$$P_{2,3} = -0.337 w_0 \pm 0.407 j w_0$$

$$H_{1,4}(w) = \frac{0.986 w_0^2}{s^2 + 0.28 w_0 s + 0.986 w_0^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{w_n}{Q}s + w_n^2}$$

$$Q = 3.55$$

$$w_n = (2\pi)(4.965\text{ MHz})$$

$$w_n^2 = [(2\pi)(4.965\times10^6)]^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\frac{w_n}{Q} = (0.28)(5\text{MHz})(2\pi) = (1.4\text{MHz})(2\pi) = \frac{R_4}{R_4+R_5}\cdot\left(\frac{1}{R_1C_1}\right)$$

$$\frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right) = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\Rightarrow \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right) = \frac{R_6}{R_3}$$

$$\Rightarrow \frac{R_5}{R_4+R_5} = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}$$

$$\Rightarrow 1-\alpha = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}, \quad \alpha = \frac{R_4}{R_4+R_5}$$

$$\Rightarrow \frac{R_6}{R_3} = \frac{1-\alpha}{\alpha}$$

Let $\alpha = 0.5 \Rightarrow \frac{R_6}{R_3} = 1$

49.

$$\Rightarrow W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right) = \frac{1}{R_1C_1R_2C_2} \quad (*)$$

Since $\frac{W_n}{Q} = \alpha\left(\frac{1}{R_1C_1}\right) = 1.4\times10^6 \times 2\pi$, $\alpha = 0.5$

$$\Rightarrow \frac{1}{R_1C_1} = 1.76\times10^7 \quad (1)$$

Consider (*)

$$\Rightarrow \frac{1}{R_2C_2} = W_n^2 \cdot R_1C_2 = 5.53\times10^7 \quad (2)$$

$R_6 = R_3 = R_5 = R_4 = 1K$. According to (1).(2), choose

$R_1 = 5K$. $C_1 = 11.368p$

$R_2 = 5K$, $C_2 = 3.62p$

For $H_{2,3}(S) = \frac{0.279W_0^2}{S^2 + 0.674W_0S + 0.279W_0^2}$

$W_n = (2\pi)(2.64\times10^6)$

$\frac{W_n}{Q} = (2\pi)(0.674\times5\times10^6) = (2\pi)(3.37\times10^6)$

Let $\alpha = 0.5$

49. $\frac{W_n}{Q} = (\alpha)\left(\frac{1}{R_1C_1}\right)$

$\Rightarrow \frac{1}{R_1C_1} = \frac{W_n}{Q}\cdot\frac{1}{\alpha} = 4.23\times10^7$ ③

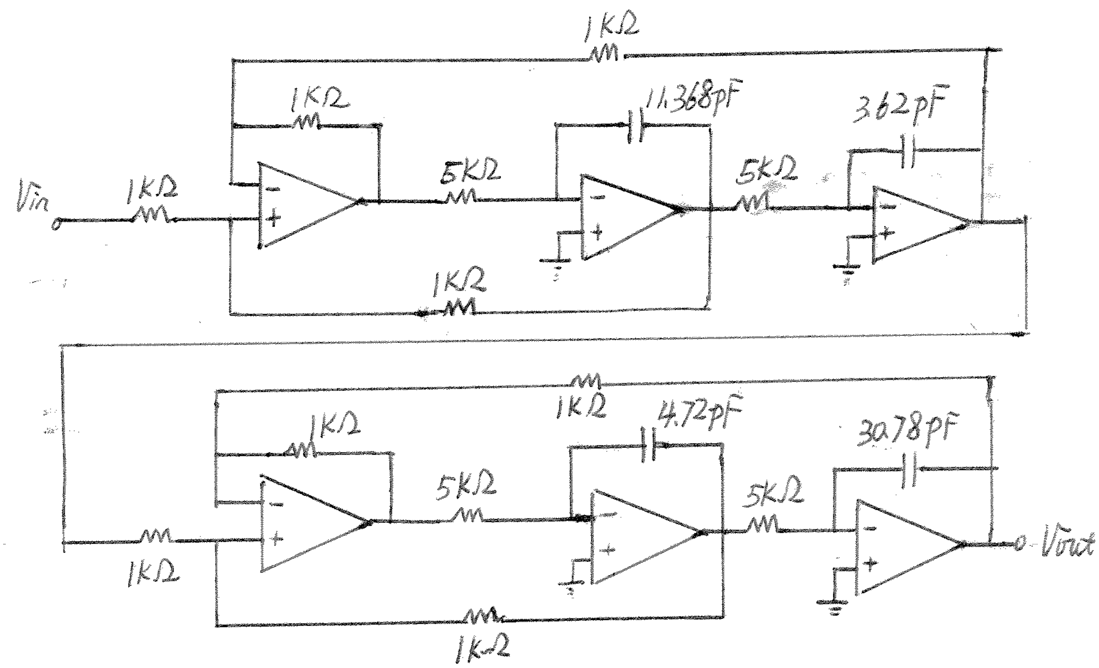
$\frac{R_6}{R_3} = 1, \quad W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right) = \frac{1}{R_1C_1R_2C_2}$

$\Rightarrow \frac{1}{R_2C_2} = W_n^2 \cdot R_1C_1 = 6.50\times10^6$ ④

Consider ③ ④, choose

$R_1 = 5K, \quad C_1 = 4.72p; \quad R_2 = 5k, \quad C_2 = 30.78p$

$R_6 = R_3 = R_5 = R_4 = 1K.$



50. Tow Thomas

Low Pass Transfer Function

$$P_{1,4} = \frac{V_4}{V_{in}} = \frac{\frac{1}{R_1R_2C_1C_2}}{S^2 + \frac{1}{R_3C_1}S + \frac{1}{R_2R_4C_1C_2}} = \frac{0.986\,W_o^2}{S^2 + 0.28\,W_oS + 0.986\,W_o^2}$$

$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(4.965\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = 2\pi\times1.4\times10^6$$

Let $R_3 = 5K$, $C_1 = 22.736P$

Let $C_1 = C_2$, $C_2 = 22.736P$

$R_1 = R_2 \overset{(*)}{\Rightarrow} R_1 = R_2 = R_4 = 1.4K$ } for $P_{1,4}$

$R_3 = 5K$, $C_1 = C_2 = 22.736P$ }

$$P_{2,3} = \frac{0.279\,W_o^2}{S^2 + 0.674W_oS + 0.279W_o^2} = \frac{\frac{1}{R_1R_2C_1C_2}}{S^2 + \frac{1}{R_3C_1}S + \frac{1}{R_2R_4C_1C_2}}$$

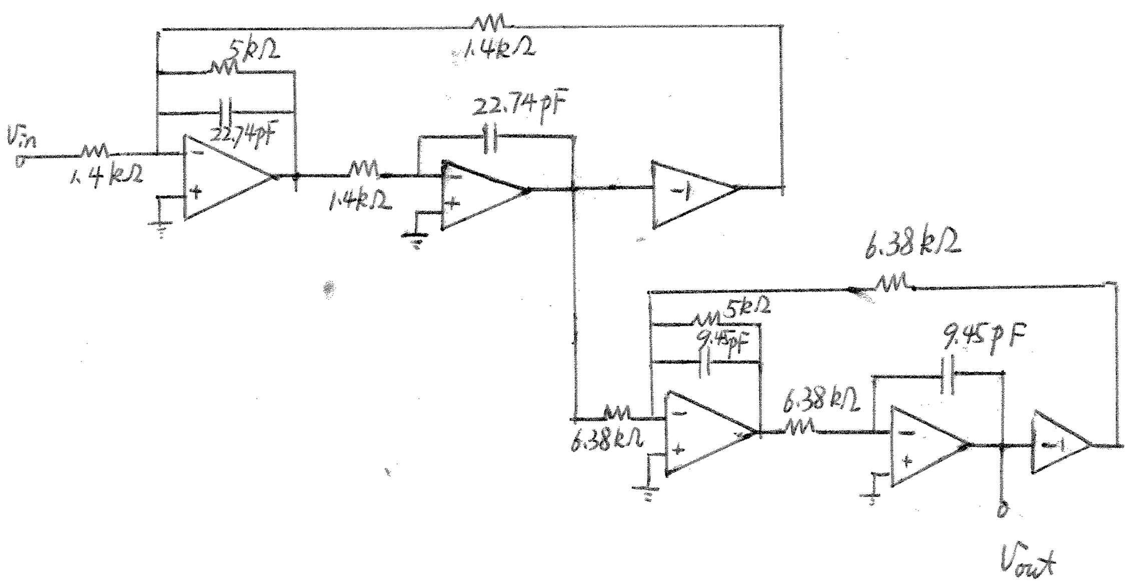
$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(2.64\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = (2\pi)(3.37\times10^6)$$

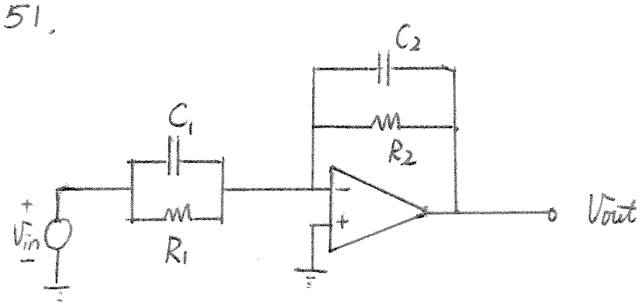
50. Let $R_3 = 5K$, $C_1 = 9.45p$

Let $C_1 = C_2 = 9.45p$, $R_1 = R_2 \overset{(*)'}{\Longrightarrow} R_1 = R_2 = R_4 = 6.38K$.

For $P_{1,4}$. $\begin{cases} R_1 = R_2 = R_4 = 6.38K, \quad R_3 = 5K \\ C_1 = C_2 = 9.45\,p. \end{cases}$

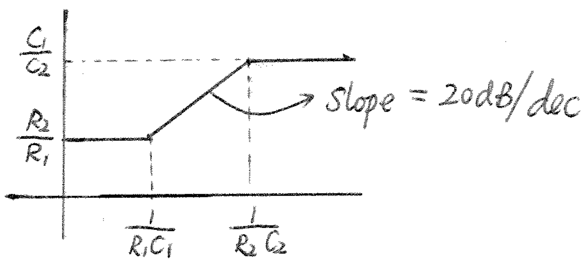


51.



High pass, 1 MHz ⇒ 10 dB atten

$f > 5\,MHz$, gain = 1



$$\frac{1}{R_2C_2} = (5\,MHz)(2\pi)$$

Let $\frac{C_1}{C_2} = 1$, $\frac{R_2}{R_1} = -10\,dB = 0.316$

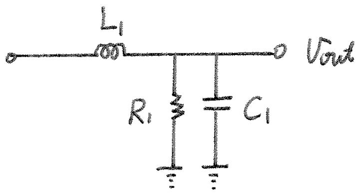
So $\frac{1}{0.316} = 3.1623$, $\frac{5\,MHz}{3.1623} = 1.58\,MHz$

$$\Rightarrow \frac{1}{R_1C_1} = (1.58\,MHz)(2\pi)$$

Choose $C_2 = 31.83\,pF \Rightarrow R_2 = 1\,k\Omega$

$C_1 = 31.83\,pF \Rightarrow R_1 = 3.16\,k\Omega$

52.



Peaking : 1dB

bandwidth : 100MHz

$L_1 < 100nH$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{L_1C_1}}{S^2 + \frac{1}{R_1C_1}S + \frac{1}{L_1C_1}} = \frac{\gamma}{S^2 + \frac{W_n}{Q}S + W_n^2}\Bigg|_{S=jw} = \frac{\gamma}{(jw)^2 + \frac{W_n}{Q}(jw) + W_n^2}$$

$$H(jw) = \frac{\gamma}{(W_n^2 - W^2) + \frac{W_n}{Q}Wj}$$

$$|H(jw)| = \frac{\gamma}{\sqrt{(W_n^2 - W^2)^2 + (\frac{W_n}{Q}W)^2}}$$

At W_1, $$|H(jW_1)| = \frac{\gamma}{\sqrt{(W_n^2 - W_1^2)^2 + (\frac{W_n}{Q}W_1)^2}} = \frac{\gamma}{W_n^2\sqrt{2}}$$

$$\Rightarrow \sqrt{\frac{(W_n^2 - W_1)^2 + (\frac{W_n}{Q}W_1)^2}{W_n^4}} = \sqrt{2}.$$

$$\Rightarrow (W_n^2 - W_1)^2 + (\frac{W_n}{Q}W_1)^2 = 2W_n^4 \qquad (*).$$

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1 \Rightarrow Q = 0.9258,\ 0.5941\ (< \tfrac{1}{\sqrt{2}},\ \text{can't produce peaking})$$

So $Q = 0.9258$.

Solve (*) gives $W = \sqrt{1.5}\,W_n$.

$$W = \sqrt{1.5}\,\frac{1}{\sqrt{L_1C_1}} = (2\pi)(100 \times 10^6) \qquad (1)$$

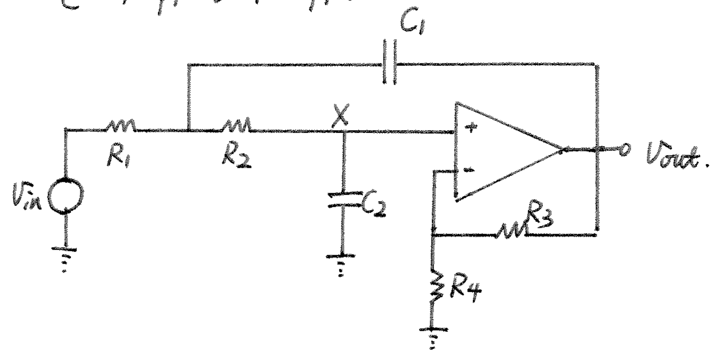
52.

$$\frac{\omega_n}{Q} = \frac{1}{R_1 C_1} \Rightarrow Q = R_1 C_1 \frac{1}{\sqrt{L_1 C_1}} = R_1 \sqrt{\frac{C_1}{L_1}} = 0.9258 \quad (2)$$

Let $L_1 = 90nH \overset{(1)}{\Longrightarrow} C_1 = 42.22pF \overset{(2)}{\Longrightarrow} R_1 = 42.74\Omega$.

53. $W_n = (2\pi)(50MHz)$, $Q = 1.5$, Low fequency gain = 2.

$C = 10pF$ to $100pF$.



$$\frac{V_{out}}{V_{in}} = \frac{1+\frac{R_3}{R_4}}{R_1R_2C_1C_2S^2 + (R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1)S + 1}$$

$$= \frac{(1+\frac{R_3}{R_4})/(R_1R_2C_1C_2)}{S^2 + \frac{(R_1C_2 + R_2C_2 - \frac{R_1R_3}{R_4}C_1)S}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

$$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad \frac{W_n}{Q} = \frac{R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1}{R_1R_2C_1C_2}$$

Low fequency gain $(1+\frac{R_3}{R_4}) = 2$, Let $\frac{R_3}{R_4} = 1$.

$$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = (2\pi)(50\times10^6) \quad ①$$

$$R_1C_2 + R_2C_2 - R_1C_1 = \frac{W_n}{Q}(R_1R_2C_1C_2) = \frac{1}{(1.5)(2\pi)(50\times10^6)} \quad ②$$

Let $C_1 = C_2 = 10pF$

$$② \Rightarrow R_2 = \frac{1}{(1.5)(2\pi)(50\times10^6)} \cdot \frac{1}{C_2} = 212.2\ (\Omega)$$

$$① \Rightarrow R_1 = 1/[(2\pi)^2(50\times10^6)^2 \cdot R_2C_1C_2] = 477.5\ (\Omega)$$

Let $R_3 = R_4 = 1k\Omega$.

54. $W_{3dB} = (30\times10^6)(2\pi)$, gain $= 2$, sensitivities no greater than 1.

$$H(s) = \frac{K w_n^2}{s^2 + \frac{w_n}{Q} s + w_n^2} \quad , \quad s = jw \Rightarrow$$

$$H(jw) = \frac{K w_n^2}{w_n^2 - w^2 + \frac{w_n}{Q} w j}$$

$$|H(jw)| = \frac{K w_n^2}{\sqrt{(w_n^2 - w^2)^2 + (\frac{w_n}{Q} w)^2}}$$

$$|H(jw)| = \frac{K}{\sqrt{2}} \Rightarrow \frac{w_n^2}{\sqrt{(w_n^2 - w^2)^2 + (\frac{w_n}{Q} w)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (w_n^2 - w^2)^2 + (\frac{w_n}{Q} w)^2 = 2 w_n^4$$

$$\Rightarrow w_n^4 (1 - \frac{w^2}{w_n^2})^2 + w_n^4 \left[(\frac{1}{Q})^2 \cdot (\frac{w}{w_n})^2 \right] = 2 w_n^4$$

$$\Rightarrow \left[1 - (\frac{w}{w_n})^2 \right]^2 + (\frac{1}{Q})^2 (\frac{w}{w_n})^2 = 2$$

$$\Rightarrow (\frac{w}{w_n})^4 + \left[(\frac{1}{Q})^2 - 2 \right] (\frac{w}{w_n})^2 - 1 = 0$$

$S^{w_n}_{R_2, C_1, C_2, R_1} = -\frac{1}{2}$ (sensitivities of w_n all < 1)

$$S^{Q}_{R_1} = -S^{Q}_{R_2} = -\frac{1}{2} + Q\sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = -\frac{1}{2} + Q \left(\sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} \right) = \frac{1}{2} + Q\sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

$$S^{Q}_{K} = QK\sqrt{\frac{R_1 C_1}{R_2 C_2}} = 2Q\sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

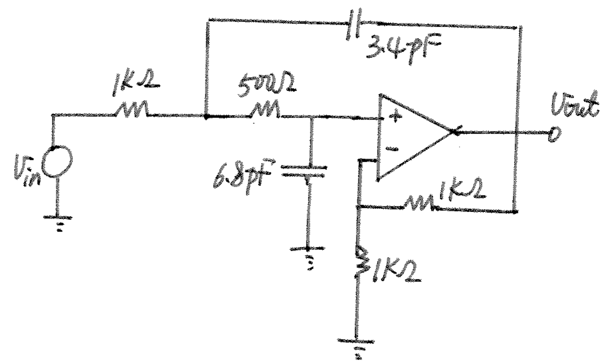
Let $\sqrt{\frac{R_1 C_1}{R_2 C_2}} = 1$, and $Q = \frac{1}{2}$,

$S_K^Q = 2 \cdot (\frac{1}{2}) = 1$, $\quad S_{C_1}^Q = \frac{1}{2} + \frac{1}{2} = 1$

$S_{C_2}^Q = -1$, $\quad S_{R_1}^Q = -\frac{1}{2} + \frac{1}{2} = 0$, $\quad S_{R_2}^Q = 0$

Since $Q = \frac{1}{2}$.,

$$\left(\frac{w}{w_n}\right)^4 + 2\left(\frac{w}{w_n}\right)^2 - 1 = 0$$



$$\Rightarrow \left(\frac{w}{w_n}\right)^2 = 0.4142$$

$$\Rightarrow w = \sqrt{0.4142}\, w_n.$$

Since $R_1 C_1 = R_2 C_2$,

$$w_n = \frac{1}{\sqrt{(R_1 C_1)^2}} = \frac{1}{R_1 C_1}$$

$$\Rightarrow \sqrt{0.4142}\, w_n = \frac{\sqrt{0.4142}}{R_1 C_1} = (2\pi)(30 \times 10^6) \quad ①$$

Also $\frac{1}{Q w_n} = R_1 C_2 + R_2 C_2 - R_1 C_1 = R_1 C_2 \quad ②$

$$\Rightarrow \frac{R_1 C_1}{R_1 C_2} = \frac{\frac{1}{w_n}}{\frac{1}{Q w_n}} = Q = \frac{1}{2}.$$

$$\Rightarrow \frac{C_1}{C_2} = \frac{1}{2}.$$

Let $R_1 = 1K\Omega \overset{①}{\Rightarrow} C_1 = \frac{\sqrt{0.4142}}{(2\pi)(30 \times 10^6) \cdot R_1} = 3.4pF.$

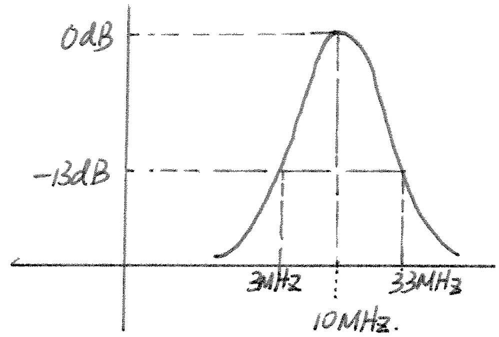
$\Rightarrow C_2 = 2C_1 = 6.8pF.$

$\Rightarrow R_2 = R_1 \frac{C_1}{C_2} = 500\Omega.$

And as before, $R_3 = R_4 = 1K\Omega$.

55. 10MHz, Gain = 1 (peak), $R_6 = R_3$, −13 dB @ 3MHz, 33MHz.

$$\frac{V_x}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q}S + W_n^2} \cdot \frac{-1}{R_1 C_1 S}$$



$$1 = \left(\frac{\alpha}{R_1C_1}\right) \cdot \frac{Q}{W_n}, \quad \alpha = \frac{R_5}{R_4+R_5}\left(1 + \frac{R_6}{R_3}\right), \quad \frac{W_n}{Q} = \frac{R_4}{R_4+R_5} \cdot \frac{1}{R_1C_1}.$$

since $R_6 = R_3$, $\alpha = 2\frac{R_5}{R_4+R_5} \Rightarrow \frac{\alpha}{2} = \frac{R_5}{R_4+R_5}$

$$\Rightarrow \frac{R_4}{R_4+R_5} = 1 - \frac{\alpha}{2} \Rightarrow \frac{Q}{W_n} = \frac{R_1C_1}{1-\frac{\alpha}{2}}.$$

$$\Rightarrow \left(\frac{\alpha}{R_1C_1}\right)\cdot\left(\frac{R_1C_1}{1-\frac{\alpha}{2}}\right) = 1 \Rightarrow \alpha = 1 - \frac{\alpha}{2}.$$

$$\Rightarrow \alpha = \frac{2}{3}, \quad \frac{R_5}{R_4+R_5} = \frac{1}{3}$$

$$\Rightarrow R_5 = \frac{1}{2}R_4.$$

$$\left.\begin{aligned} \frac{W_n}{Q} &= \left(\frac{2}{3}\right)\left(\frac{1}{R_1C_1}\right) \\ W_n &= \frac{1}{\sqrt{R_1R_2C_1C_2}} \end{aligned}\right\} \Rightarrow \frac{3}{2}\frac{R_1C_1}{\sqrt{R_1R_2C_1C_2}} = Q$$

$$\Rightarrow \frac{3}{2}\sqrt{\frac{R_1C_1}{R_2C_2}} = Q$$

Let $R_1C_1 = R_2C_2 \Rightarrow Q = \frac{3}{2}, \quad W_n = \frac{1}{R_1C_1}.$

$$H(j\omega) = \frac{\frac{2}{3}\omega^2}{\frac{\omega}{\omega_n}\sqrt{(\omega_n^2-\omega^2)^2+(\frac{2}{3}\omega_n\omega)^2}} = \frac{\frac{2}{3}\omega^2}{\frac{\omega}{\omega_n}\sqrt{\omega_n^4-\frac{14}{9}(\omega_n\omega)^2+\omega^4}}$$

$$H(j\omega) = 1$$

$$\Rightarrow \frac{4}{9}\omega^2 = \omega_n^2 - \frac{14}{9}\omega^2 + \frac{\omega^4}{\omega_n^2}$$

$$\Rightarrow \omega_n^4 - 2\omega^2\omega_n^2 + \omega^4 = 0$$

$$\Rightarrow \omega_n^2 = \omega^2 = [(2\pi)(10MHz)]^2$$

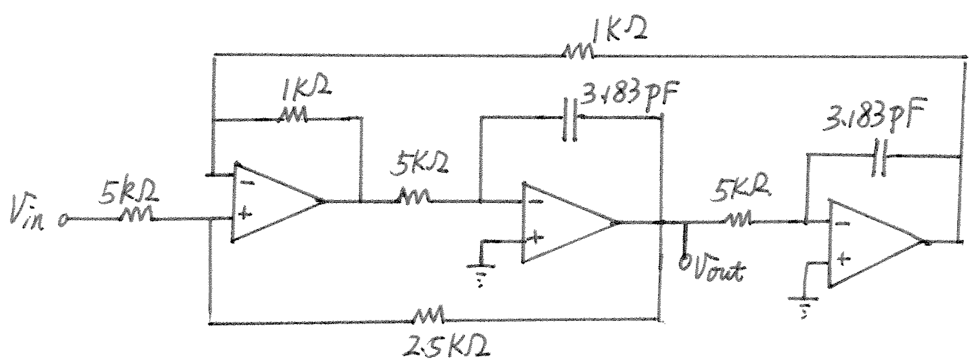
As derived, $\omega_n = \frac{1}{R_1C_1}$

Let $R_1 = 5K\Omega \Rightarrow C_1 = 3.183pF$.

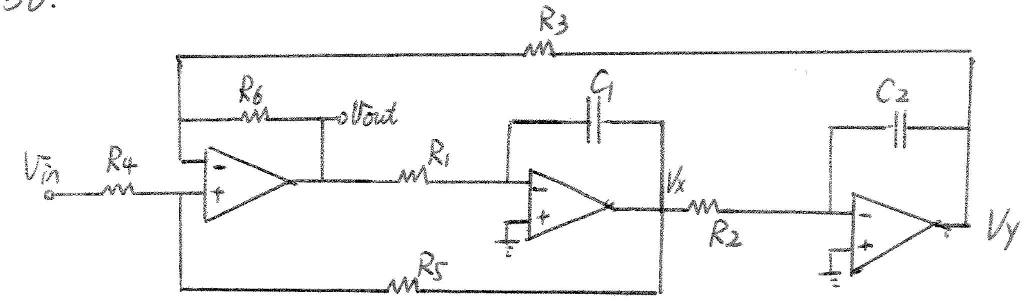
Let $R_2 = R_1 = 5k\Omega \Rightarrow C_2 = C_1 = 3.183pF$

Let $R_4 = 5K\Omega \Rightarrow R_5 = \frac{1}{2}R_4 = 2.5k\Omega$.

Let $R_3 = R_6 = 1K\Omega$.



56.



Low pass, $\frac{V_y}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q} S + W_n^2} \cdot \frac{1}{R_1 R_2 C_1 C_2 S^2}$

$$= \frac{\alpha}{(S^2 + \frac{W_n}{Q} S + W_n^2)(R_1 R_2 C_1 C_2)}$$

$$H(S) = \frac{\alpha / (R_1 R_2 C_1 C_2)}{S^2 + \frac{W_n}{Q} S + W_n^2}$$

$$H(jW) = \frac{\alpha / (R_1 R_2 C_1 C_2)}{(W_n^2 - W^2) + j \frac{W_n}{Q} W}$$

$$|H(jW)| = \frac{\alpha / (R_1 R_2 C_1 C_2)}{\sqrt{(W_n^2 - W^2)^2 + (\frac{W_n W}{Q})^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha / (R_1 R_2 C_1 C_2)}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_n W_{3dB}}{Q})^2}} = \frac{\alpha}{W_n^2 (R_1 R_2 C_1 C_2)\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_n W_{3dB}}{Q})^2}}{W_n^2} = \sqrt{2}$$

$$\Rightarrow 1 - 2\left(\frac{W_{3dB}}{W_n}\right)^2 + \left(\frac{W_{3dB}}{W_n}\right)^4 + \frac{1}{Q^2}\left(\frac{W_{3dB}}{W_n}\right)^2 = 2$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$Q = 1.5 \Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 - 1.556\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^2 = 2.0446, \quad -0.4891 \text{ (impossible)}$$

$$\Rightarrow W_{3dB} = 1.43\, W_n$$

$\Rightarrow$ Low pass corner $= 14.3$ MHz.

High pass:

$$\frac{V_{out}}{V_{in}}(S) = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q}S + W_n^2}$$

$$|H(jw)| = \frac{\alpha w^2}{\sqrt{(W_n^2 - W^2)^2 + \left(\frac{WW_n}{Q}\right)^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha W_{3dB}^2}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + \left(\frac{W_{3dB}W_n}{Q}\right)^2}} = \frac{\alpha}{\sqrt{2}}$$

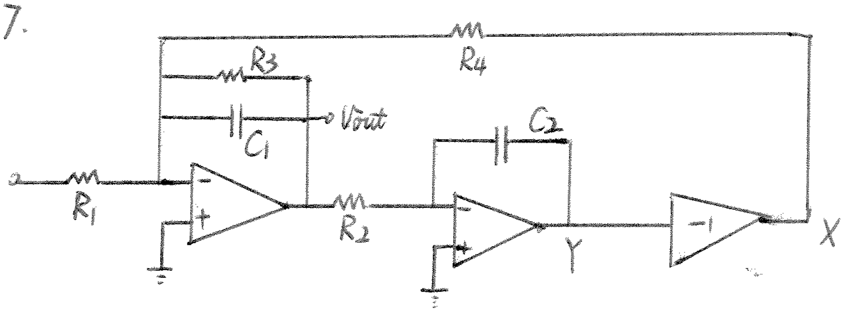
$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_n}{W_{3dB}}\right)^2 - 1 = 0$$

Since $Q = 1.5$

$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^2 = 2.0446 \Rightarrow W_{3dB} = \frac{W_n}{1.43}$$

$\Rightarrow W_{3dB} = 7$ MHz. (high pass corner)

57.



$W_n = 10\,MHz$,

$-13dB = 3MHz$,

$33MHz$.

$$\frac{V_{out}}{V_{in}} = -\frac{R_2R_3R_4}{R_1}\left(\frac{C_2S}{R_2R_3R_4C_1C_2S^2 + R_2R_4C_2S + R_3}\right)$$

Same as in #55, $Q = \frac{10}{6.684} = 1.5$.

$$\frac{V_{out}}{V_{in}} = -\frac{\frac{1}{R_1C_1}S}{S^2 + \frac{1}{R_3C_1}S + \frac{1}{R_2R_4C_1C_2}}$$

$$\frac{W_n}{Q} = \frac{1}{R_3C_1}, \quad W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}.$$

$$Q = \frac{R_3C_1}{\sqrt{R_2R_4C_1C_2}} = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\beta S}{S^2 + \frac{W_n}{Q}S + W_n^2};$$

At $W = W_n \Rightarrow |H(jW_n)| = 1 = \frac{\beta Q}{W_n}$.

$$\frac{\beta Q}{W_n} = \left(\frac{1}{R_1C_1}\right)(R_3C_1) = \frac{R_3}{R_1} = 1$$

$Q = R_3\sqrt{\frac{C_1}{R_2 R_4 C_2}} = 1.5$, $\quad W_n = \frac{1}{\sqrt{R_2 R_4 C_2 C_1}} = (10\times10^6)(2\pi)$

Let $R_2 = R_4 = 1K\Omega$.

$$\frac{1}{\sqrt{10^6 \times C_1 C_2}} = (10\times10^6)(2\pi). \Rightarrow C_1 C_2 = 2.533\times10^{-22}$$

Let $C_1 = C_2 = 15.9pF$

$$R_3\sqrt{\frac{1}{1000\times1000}} = 1.5$$

$\Rightarrow R_3 = 1.5\,k\Omega = R_1$

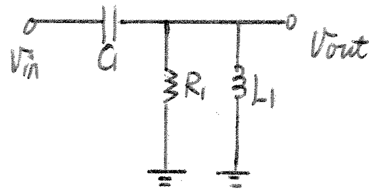
So : $\quad R_1 = 1.5\,K\Omega$, $R_2 = 1K\Omega$, $R_3 = 1.5K\Omega$, $R_4 = 1K\Omega$.

$\quad C_1 = C_2 = 15.9pF$.

58. Peaking : 1dB @ 7MHz.

Corner : 3.69 MHz

−13.6dB @ 2MHz.



$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{L_1C_1}}$$

Peaking 1dB ⇒ $Q = 0.926$.

$$\frac{\omega_n}{\sqrt{1 - 1/(2Q^2)}} = (2\pi)(7MHz) \Rightarrow \omega_n = (2\pi)(4.52MHz)$$

$$\frac{\omega_n}{Q} = \frac{(2\pi)(4.52MHz)}{0.926} = (2\pi)(4.88MHz) = \frac{1}{R_1C_1}$$

$$\omega_n^2 = [(2\pi)(4.52MHz)]^2 = \frac{1}{L_1C_1}$$

Let $C_1 = 100pF$, $L_1 = 12.4\mu H$, $R_1 = 326.1\Omega$.

With simulated inductor

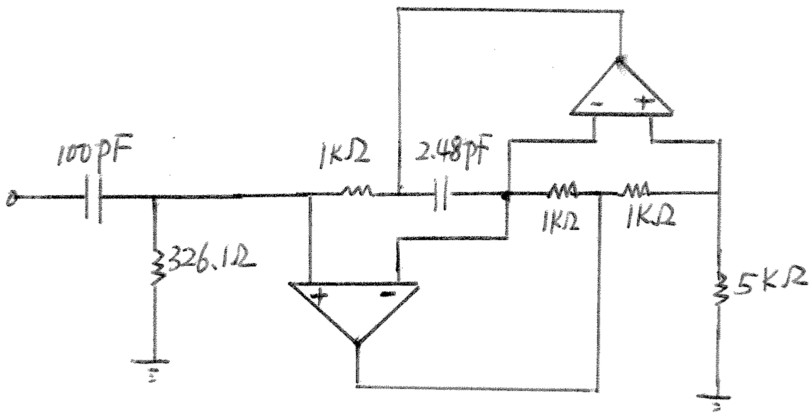
$$Z_{in} = \left(\frac{Z_1 Z_3}{Z_2 Z_4}\right) Z_5 = R_Y R_X C_s$$

Let $Z_1 = Z_3 = Z_4 = R_Y$, $Z_5 = R_X$, $Z_2 = C_s^{-1}$

Let $R_Y = 1K\Omega$, $R_X = 5K\Omega$.

$$12.4 \times 10^{-6} = (1000)(5000)\, C.$$

⇒ $C = 2.48pF$ to simulate an L of $12.4\mu H$.


100pF
326.1Ω
1KΩ
2.48pF
1KΩ
1KΩ
5KΩ

59. Corner @ 16.38 MHz, Peaking 0.5 dB @ 8 MHz.

5.9 dB ≈ 6 dB attenuation @ 20 MHz.

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_1 R_x C^2 S^2 + R_1 C S + 1} = \frac{1/(R_1 R_x C^2)}{S^2 + \frac{S}{R_x C} + \frac{1}{R_1 R_x C^2}}$$

0.5 dB ⟺ 1.05292.

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.05292 \Rightarrow Q = 0.8636$$

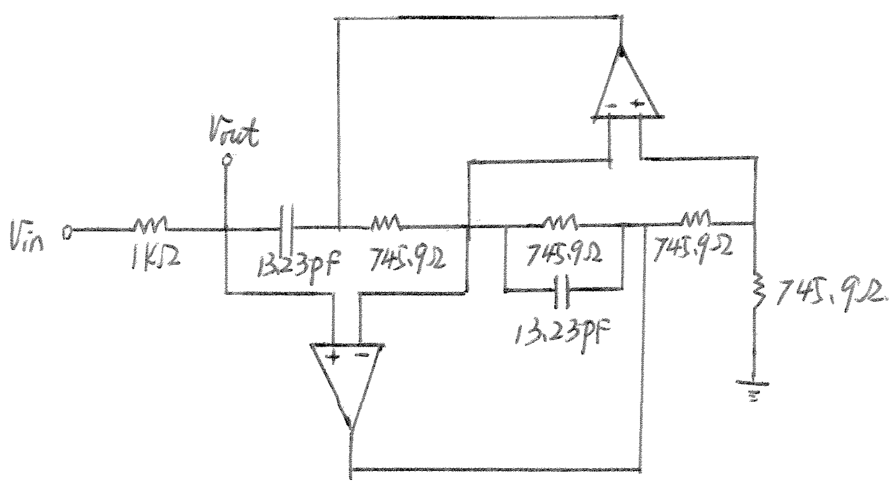
$$W_n \sqrt{1-\frac{1}{2Q^2}} = (2\pi)(8\times10^6)$$

$$\Rightarrow W_n = (2\pi)(13.934\times10^6)$$

$$\frac{1}{R_1 R_x C^2} = W_n^2, \quad \frac{1}{R_x C} = \frac{W_n}{Q} = (2\pi)(16.134\times10^6)$$

$$\Rightarrow \frac{1}{R_1 C} = 7.56\times10^7$$

Let $R_1 = 1k\Omega$, $C = 13.23pF$, $R_x = 745.9\Omega$.



60. Butterworth

a) Passband 0.5dB @ 1MHz , $-0.5dB \Leftrightarrow 0.944$

Attenuation 12dB @ 2.5MHz , $-12dB \Leftrightarrow 0.2512$

$$|H(j\omega)|^2_{1MHz} = \frac{1}{1+\left[\frac{(2\pi)(10^6)}{W_0}\right]^{2n}} = 0.944^2 \qquad ①$$

$$|H(j\omega)|^2_{2.5MHz} = \frac{1}{1+\left(\frac{2\pi\times 2.5\times 10^6}{W_0}\right)^{2n}} = 0.2512^2 \qquad ②$$

$$① \Rightarrow 1 = (0.944)^2\left[\left(\frac{2\pi\times 10^6}{W_0}\right)^{2n}+1\right]$$

$$\Rightarrow W_0^{2n} = 8.186\times(2\pi\times 10^6)^{2n} \qquad ③$$

$$② \Rightarrow 1 = (0.2512)^2\left[\frac{(2\pi\times 2.5\times 10^6)^{2n}}{8.186\times(2\pi\times 10^6)^{2n}}+1\right]$$

$$\Rightarrow n = 2.62$$

So choose $n=3$. $\overset{③}{\Rightarrow}$ $W_0 = 2\pi\times 1.42MHz$.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{W}{2\pi\times 1.42\times 10^6}\right)^6}}$$

b). Passband: 0.1dB @ 1MHz

$$\frac{1}{1+\left(\frac{2\pi \times 10^6}{W_0}\right)^{2n}} = (0.98855)^2 \qquad (1)$$

Stopband attenuation: 12dB @ 2.5MHz

$$\frac{1}{1+\left(\frac{2\pi \times 2.5 \times 10^6}{W_0}\right)^{2n}} = (0.2512)^2 \qquad (2)$$

$$(1) \Rightarrow W_0^{2n} = 42.931 \times (2\pi \times 10^6)^{2n}$$

$$(2) \Rightarrow n = 3.52$$

Choose $n=4 \overset{(3)}{\Longrightarrow} W_0 = 2\pi \times 1.6 MHz$.

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{W}{2\pi \times 1.6 \times 10^6}\right)^8}}$$

c). Passband 1dB @ 1MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi \times 10^6}{W_0}\right)^{2n}} = (0.90)^2 \qquad (4)$

Attenuation 18dB @ 2.5MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi \times 2.5 \times 10^6}{W_0}\right)^{2n}} = (0.1259)^2 \qquad (5)$

$$(4) \Rightarrow W_0^{2n} = 4.263 \times (2\pi \times 10^6)^{2n} \qquad (6)$$

$$(5) \Rightarrow n = 3.0$$

Choose $n=3 \overset{(6)}{\Longrightarrow} W_0 = 2\pi \times 1.27 MHz$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{W}{2\pi \times 1.27 \times 10^6}\right)^6}}$$

d) Passband: 0.5dB @ 1MHz $\Rightarrow \dfrac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = 0.944^2$ ①

Attenuation: 18dB @ 2.5MHz $\Rightarrow \dfrac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = 0.1259^2$ ②

① $\Rightarrow W_0^{2n} = 8.186\times(2\pi\times10^6)^{2n}$ ③

② $\Rightarrow n = 3.4$

Choose $n=4 \overset{③}{\Longrightarrow} W_0 = 2\pi\times1.3\text{MHz}$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{2\pi\times1.3\times10^6}\right)^8}}$$

Chebyshev

a) Passband 0.5dB @ 1MHz $\Rightarrow 0.5 = 20\log(\sqrt{1+\epsilon^2})$

$\Rightarrow \epsilon = 0.3493$, $W_0 = 1\text{MHz}$.

Attenuation 12dB @ 2.5MHz

$\Rightarrow \dfrac{1}{\sqrt{1+\epsilon^2\cosh^2[n\cosh^{-1}(\frac{w}{W_0})]}} = 0.2512$, when $w = 2.5\times10^6\times2\pi$.

Since w, w_0, ϵ, known

$\Rightarrow n = 1.9733$

Choose $n=2$, $|H(jw)| = \dfrac{1}{\sqrt{1+0.3493^2\, C_2^2(\frac{w}{W_0})}}$, $W_0 = 2\pi\times1\text{MHz}$

b). Passband 0.1 dB @ 1MHz , w_o = 1MHz

$$\Rightarrow 0.1 = 20\log(\sqrt{1+\epsilon^2}) \Rightarrow \epsilon = 0.1526.$$

Attenuation 12dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.1526^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.2512^2$$

$$\Rightarrow n = 2.5$$

Choose n=3, $|H(jw)| = \frac{1}{\sqrt{1+0.1526^2 C_3^2(\frac{w}{w_o})}}$, $w_o=(2\pi)(1MHz)$

c). Passband 1dB @ 1MHz, w_o = 1MHz

$$\Rightarrow 1 = 20\log\sqrt{1+\epsilon^2} \Rightarrow \epsilon = 0.5089.$$

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.5089^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.19$$

Choose n = 3, $|H(jw)| = \frac{1}{\sqrt{1+0.5089^2 C_3^2(\frac{w}{w_o})}}$, $w_o=(2\pi)(1MHz)$

d). Passband 0.5dB @ 1MHz $\Rightarrow \epsilon = 0.3493$

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.3493^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.43$$

Choose n=3, $|H(jw)| = \frac{1}{\sqrt{1+0.3493^2 C_3^2(\frac{w}{w_o})}}$, $w_o=(2\pi)(1MHz)$

61. a) Butterworth in Sallen and Key

$n=3,\ W_0=(2\pi)(1.42MHz)$

$P_K = W_0 \cdot \exp(\frac{j\pi}{2})\ \exp(j\frac{2K-1}{2n}\pi),\quad K=1,2,3.$

$P_1 = W_0 \exp(j\frac{2\pi}{3}) = (2\pi)(1.42MHz)\times(\cos\frac{2\pi}{3}+j\sin\frac{2\pi}{3})$

$P_2 = W_0 \exp(j\pi) = -(2\pi)(1.42MHz)$

$P_3 = W_0 \exp(j\frac{4\pi}{3}) = (2\pi)(1.42MHz)\times(\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3})$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)}$$

$$= \frac{[2\pi\times(1.42MHz)]^2}{S^2-[4\pi\times(1.42MHz)\cos\frac{2\pi}{3}]S+[2\pi\times(1.42MHz)]^2}$$

$W_n = 2\pi\times 1.42MHz\ \left(= \frac{1}{\sqrt{R_1R_2C_1C_2}}\right)$

$\frac{W_n}{Q} = 2\pi\times 1.42MHz\cdot\cos(\frac{2\pi}{3}) \Rightarrow Q = \frac{-1}{2\cos(\frac{2\pi}{3})}\left(= \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}\right)$

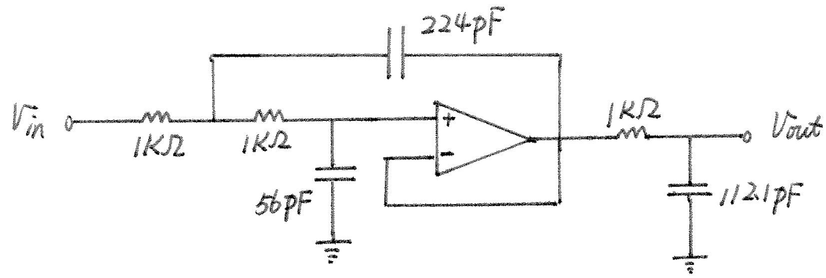
Let $C_1=4C_2,\ R_1=R_2$, so that it satisfies $Q=\frac{-1}{2\cos(\frac{2\pi}{3})}=1$

Also $\frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi\times 1.42MHz$

Let $R_1=R_2=1K\Omega \Rightarrow C_1=224pF,\ C_2=56pF$

$P_2 = -W_0, \quad \frac{1}{R_3 C_3} = (2\pi)(1.42\,MHz)$

Let $R_3 = 1K\Omega \Rightarrow C_3 = 1121\,pF$.



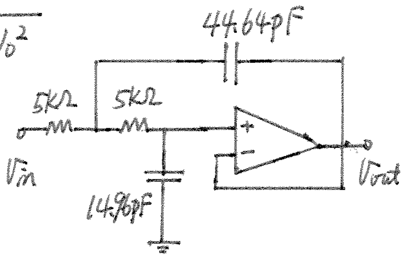
Chebyshev in Sallen and Key

$$P_k = -W_0 \sin\frac{(2K-1)\pi}{2n}\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) + jW_0\cos\frac{(2K-1)\pi}{2n}\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$n=2, \quad W_0 = (2\pi)(1\,MHz), \quad \epsilon = 0.3493, \quad K=1,2.$

$P_{1,2} = -0.7128W_0 \pm j1.0041W_0$

$$H_{SK}(s) = \frac{(-P_1)(-P_2)}{(S-P_1)(S-P_2)} = \frac{(1.2314)^2 W_0^2}{S^2 + 1.4256W_0 S + (1.2314)^2 W_0^2}$$



$W_n = 1.2314W_0 = (2\pi)(1.2314\,MHz)$

$\frac{W_n}{Q} = 1.4256W_0 \Rightarrow Q = \frac{1.2314}{1.4256} = 0.8638$.

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

Let $R_1 = R_2 \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 2.9844$

$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = 2\pi(1.2314\,MHz), \Rightarrow \frac{1}{R_1C_2\cdot\sqrt{2.9844}} = 2\pi(1.2314\,MHz)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 14.96\,pF, \quad C_1 = 44.64\,pF$

b). Butterworth with SK

$n=4,\quad w_0 = (2\pi)(1.6\,MHz)$

$P_K = w_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi), \quad K=1,2,3,4.$

$P_1 = w_0 \exp(j\frac{5\pi}{8}),\ P_2 = w_0 \exp(j\frac{7\pi}{8}),\ P_3 = w_0 \exp(-j\frac{5\pi}{8}),\ P_4 = w_0 \exp(-j\frac{7\pi}{8})$

$$H_{SK1,4}(S) = \frac{(-P_1)(-P_4)}{(S-P_1)(S-P_4)} = \frac{[(2\pi)(1.6\times10^6)]^2}{S^2 - [4\pi\times(1.6\times10^6)\cos(\frac{5\pi}{8})]S + [2\pi\times(1.6\times10^6)]^2}$$

$W_n = 2\pi \times 1.6\times10^6$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

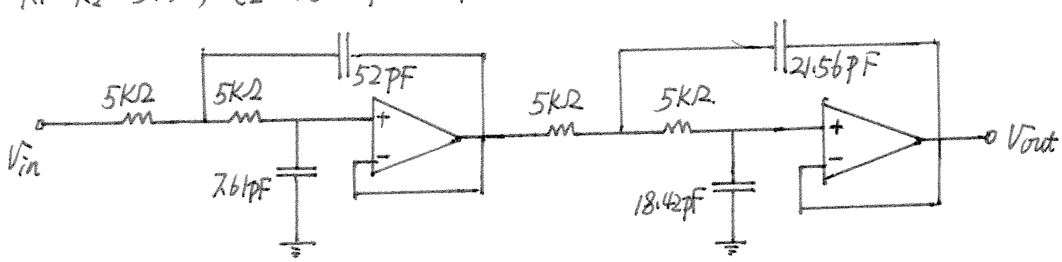
Let $R_1 = R_2$, $Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.83.$

$W_n = \frac{1}{\sqrt{6.83}\,R_1C_2} = 2\pi\times1.6\times10^6$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 7.61\,pF,\ C_1 = 52\,pF.$

Similarly, $H_{SK2,3}(S) = \frac{(-P_2)(-P_3)}{(S-P_2)(S-P_3)}$, it can be derived for $H_{SK2,3}$.

$R_1 = R_2 = 5K\Omega,\ C_2 = 18.42\,pF,\ C_1 = 21.55\,pF.$



b). Chebyshev in SK.

$n=3, \quad W_0 = (2\pi)(1\times10^6), \quad \epsilon = 0.1526$

$P_1 = -W_0(0.9694)\sin(\frac{1}{6}\pi) + jW_0(1.3927)\cos(\frac{\pi}{6}) = -0.4847W_0 + j1.2061W_0$

$P_2 = -W_0(0.9694)\sin(\frac{3}{6}\pi) + jW_0(1.3927)\cos(\frac{3\pi}{6}) = -0.9496W_0$

$P_3 = -W_0(0.9694)\sin(\frac{5}{6}\pi) + jW_0(1.3927)\sin(\frac{5\pi}{6}) = -0.4847W_0 - j1.2061W_0$

$$H_{SK}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{1.3^2 W_0^2}{s^2 + 0.9694W_0 s + (1.3)^2 W_0^2}$$

$W_n = 1.3W_0$

$\frac{W_n}{Q} = 0.9694W_0 \Rightarrow Q = 1.3410.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

Let $R_1 = R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = (2Q)^2 = 7.1931.$

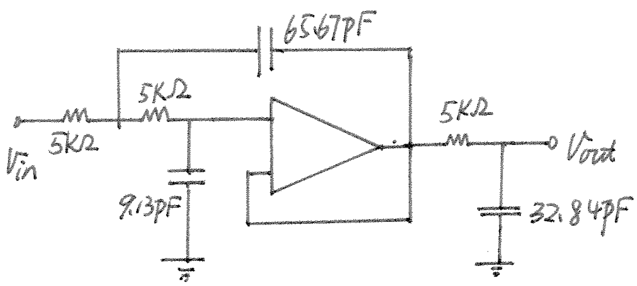
$W_n = \frac{1}{\sqrt{7.1931}\,R_1C_2} = (1.3)(2\pi)(1\times10^6)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 9.13pF \Rightarrow C_1 = 65.67pF$

$P_2 = (2\pi)(0.9694\times10^6)$, and

$\frac{1}{R_3C_3} = P_2$

Let $R_3 = 5K\Omega \Rightarrow C_3 = 32.84pF$



c). Butterworth in SK.

$n=3, \quad w_0 = (2\pi)(1.27\times10^6)$

$P_K = w_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi), \quad K=1,2,3.$

$H_{P_{1,3}}(s) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)} = \frac{[(2\pi)(1.27\times10^6)]^2}{S^2-[4\pi\times(1.27\times10^6)\cos(\frac{2\pi}{3})]S+[2\pi(1.27\times10^6)]^2}$

$W_n = (2\pi)(1.27\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3}) \Rightarrow Q=1.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

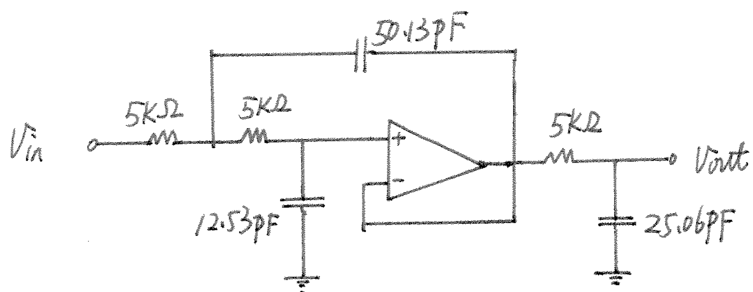
Let $C_1 = 4C_2, R_1 = R_2$

$W_n = \frac{1}{2R_1C_2} = (2\pi)(1.27\times10^6)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 12.53pF, \; C_1 = 50.13pF.$

$P_2 = -W_0 = (2\pi)(1.27\times10^6) = \frac{1}{R_3C_3}$

Let $R_3 = 5K\Omega \Rightarrow C_3 = 25.06pF$



C). Chebyshev in SK.

$n=3,\ \epsilon \doteq 0.5089,\ W_0=(2\pi)(10^6).$

$P_1 = -0.2470\,W_0 + j\,0.9660\,W_0$

$P_2 = -0.4941\,W_0$

$P_3 = -0.2470\,W_0 - j\,0.9660\,W_0$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)} \doteq \frac{[(2\pi)(0.9971\times10^6)]^2}{S^2+(0.4940)(2\pi)(10^6)S+(2\pi\times0.9971\times10^6)^2}$$

$W_n = (2\pi)(0.9971\times10^6)$

$\frac{W_n}{Q} = (0.4940)(2\pi\times10^6) \Rightarrow Q = 2.02.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},\quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

Let $R_1=R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 16.296.$

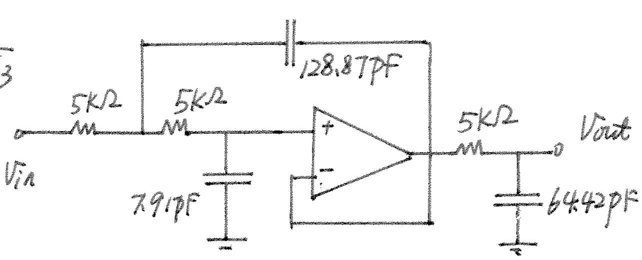
$W_n = \frac{1}{\sqrt{16.296}\,R_1C_2} = (2\pi)(0.9971\times10^6)$

Let $R_1 = R_2 = 5\,K\Omega \Rightarrow C_2 = 7.91\,pF \Rightarrow C_1 = 128.87\,pF$

$P_2 = 2\pi\times0.4941\times10^6 = \frac{1}{R_3C_3}$

Let $R_3 = 5\,K\Omega$

$\Rightarrow C_3 = 64.42\,pF$



d). Butterworth in SK.

$n=4, \quad w_0=(2\pi)(1.3\times10^6)$

$P_k = w_0 \exp(j\frac{\pi}{2})\exp(j\frac{2k-1}{2n}\pi), \quad k=1,2,3,4.$

$$H_{SK1,4}(s) = \frac{(-P_1)(-P_4)}{(s-P_1)(s-P_4)} = \frac{w_0^2}{s^2-[2w_0\cos(\frac{5\pi}{8})]s+w_0^2}$$

$w_n = w_0 = (2\pi)(1.3\times10^6)$

$\frac{w_n}{Q} = 4\pi(1.3\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$

$w_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

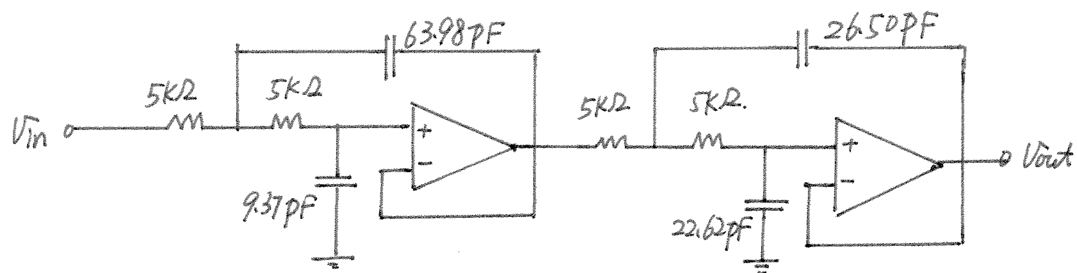
Let $R_1=R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.828$

$w_n = \frac{1}{\sqrt{6.828}\,R_1C_2} = (2\pi)(1.3\times10^6)$

Let $R_1=R_2=5K\Omega \Rightarrow C_2 = 9.37pF \Rightarrow C_1 = 63.98\,pF$

Similarly, $H_{SK2,3}(s) = \frac{(-P_2)(-P_3)}{(s-P_2)(s-P_3)}$. It can be derived that

$R_1=R_2=5K\Omega, \quad C_2=22.62pF, \quad C_1=26.50pF.$



d). Chebyshev in SK.

$n=3$, $\epsilon=0.3493$, $w_0=(2\pi)(1\times10^6)$

$p_1 = -w_0 0.6265 \sin(\frac{1}{6}\pi) + jw_0(1.11800)\cos(\frac{1}{6}\pi)$

$p_2 = -w_0 0.6265$

$p_3 = -w_0 0.6265 \sin(\frac{5}{6}\pi) + jw_0(1.1800)\cos(\frac{5}{6}\pi)$

$$H_{p_{1,3}} = \frac{(-p_1)(-p_3)}{(s-p_1)(s-p_3)} = \frac{[2\pi\times1.069\times10^6]^2}{s^2+(0.6265)(2\pi\times10^6)s+(2\pi\times1.069\times10^6)^2}$$

$w_n=(2\pi)(1.069\times10^6)$

$\frac{w_n}{Q}=(0.6265)(2\pi\times10^6) \Rightarrow Q=1.7063$

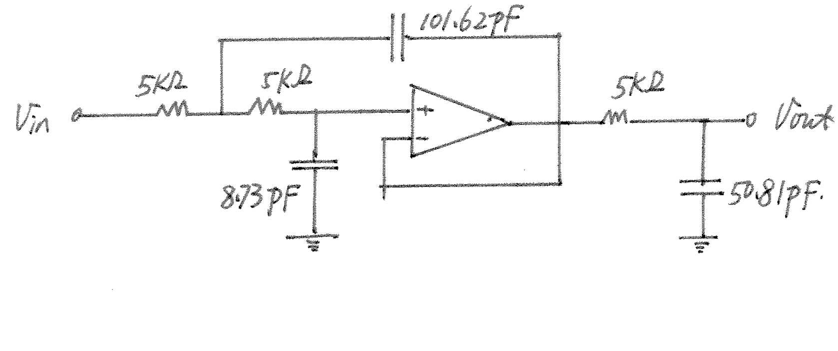
Let $R_1=R_2 \Rightarrow Q=\frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2}=4Q^2=11.6459$.

$w_n=\frac{1}{\sqrt{R_1R_2C_1C_2}}=\frac{1}{\sqrt{11.6459}\,R_1C_2}=(2\pi)(1.069\times10^6)$

Let $R_1=R_2=5k\Omega \Rightarrow C_2=8.73pF \Rightarrow C_1=101.62pF$.

$-p_2=(0.6265)(2\pi\times10^6)=\frac{1}{R_3C_3}$

Let $R_3=5k\Omega \Rightarrow C_3=50.81pF$.



62) a). Butterworth TT

$n=3$, $w_0 = (2\pi)(1.42\times10^6)$

$P_1 = w_0 \exp(j\frac{2\pi}{3})$, $P_2 = -w_0$, $P_3 = w_0 \exp(-j\frac{2\pi}{3})$.

$$H_{P_{1,3}} = \frac{[2\pi\times(1.42\times10^6)]^2}{s^2 - [4\pi\times(1.42\times10^6)\cos(\frac{2\pi}{3})]s + [2\pi\times1.42\times10^6)^2}$$

$w_n = (2\pi)(1.42\times10^6)$.

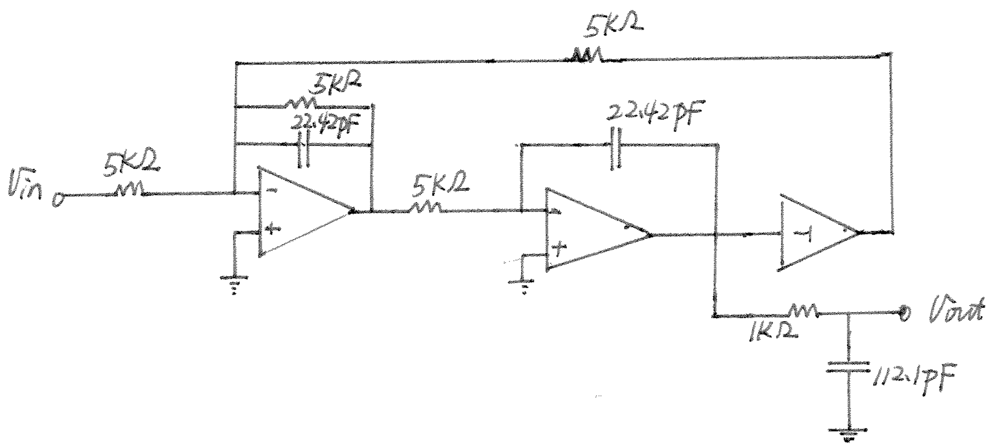
$$Q = \frac{-1}{2\cos(\frac{2\pi}{3})} = 1$$

$w_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{C_2R_2R_4}}$.

Let $R_2 = R_4 = R_3 = 5K\Omega$, $C_1 = C_2 = 22.42pF$

$-P_2 = +(2\pi)(1.42\times10^6) = \frac{1}{R_3C_3}$.

Let $R_3 = 1K\Omega \Rightarrow C_3 = 112.1pF$



$R_1 = R_2 = R_4$, to match low frequency gain requirement.

a) Chebyshev II

$n=2,\ w_0=(2\pi)(1MHz),\ \epsilon=0.3493$

$$H_{P_{1,2}} = \frac{(1.2314)^2 w_0^2}{s^2+1.4256w_0 s+(1.2314)^2 w_0^2}$$

$$w_n=(2\pi)(1.2314\times10^6)$$

$$Q=\frac{1.2314}{1.4256}=0.8638$$

$$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}},\qquad w_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$$

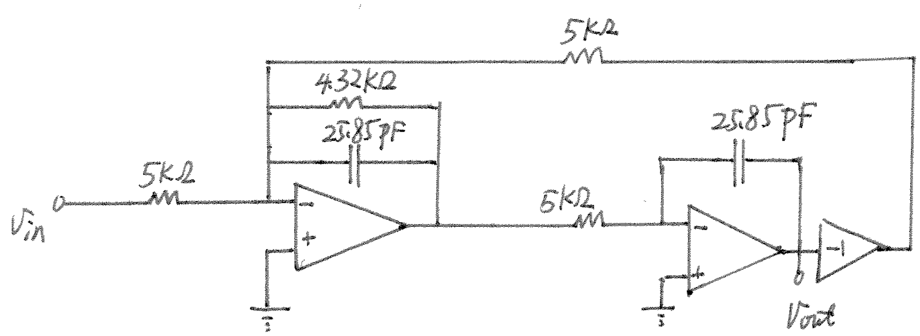
Let $C_1=C_2,\ R_2=R_4$

$$w_n=\frac{1}{R_2C_1}=(2\pi)(1.2314\times10^6)$$

Let $R_2=R_4=5K\Omega,\ C_1=C_2=25.85pF$

$R_1=R_2=R_4=5K\Omega$, to match low frequency gain of unity.

$$Q=\frac{R_3}{R_2}\Rightarrow R_3=4.32K\Omega.$$



b). Butterworth with TT

$n=4,\ w_0=(2\pi)(1.6\times10^6)$

$P_1 = w_0 \exp(j\frac{5\pi}{8})$, $P_4 = w_0 \exp(-j\frac{5\pi}{8})$

$P_2 = w_0 \exp(j\frac{7\pi}{8})$, $P_3 = w_0 \exp(-j\frac{7\pi}{8})$

$$H_{P_{1,4}} = \frac{w_0^2}{s^2 - [2w_0\cos(\frac{5\pi}{8})]s + w_0^2}$$

$W_n = W_0 = (2\pi)(1.6\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

Let $R_2 = R_4$, $C_1 = C_2 \Rightarrow W_n = \frac{1}{R_2C_1} = (2\pi)(1.6\times10^6)$

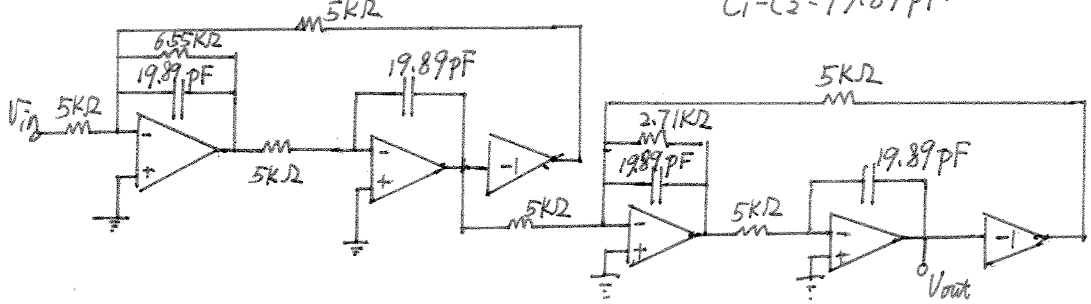
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 19.89pF$.

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$Q = \frac{R_3}{R_2} = 1.31 \Rightarrow R_3 = 1.31R_2 = 6.55K\Omega$.

Similarly for $H_{P_{2,3}}$, it can be derived that $R_1=R_2=R_4=5K\Omega, R_3=2.7K\Omega$

$C_1=C_2=19.89pF$.



b). Chebyshev with TT

$n=3$, $\epsilon=0.1526$, $W_0=(2\pi)(1\times10^6)$

$P_{1,3}=0.4847W_0 \pm j1.2061W_0$

$P_2=-0.9694W_0$

$$H_{P_{1,3}}(S)=\frac{(1.3)^2W_0^2}{S^2+0.9694W_0S+(1.3)^2W_0^2}$$

$W_n=1.3W_0$, $\frac{W_n}{Q}=0.9694W_0$

$$Q=\frac{1.3}{0.9694}=1.3410$$

$$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}},\quad W_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$$

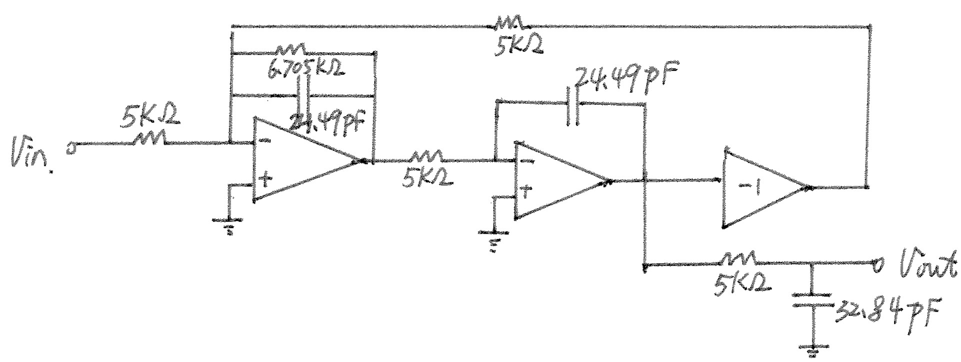
Let $R_2=R_4$, $C_1=C_2$, $\frac{1}{R_2C_1}=(1.3)(2\pi)(10^6)$

Let $R_2=R_4=5K\Omega$, $\Rightarrow C_1=C_2=24.49pF$.

$R_1=R_2=R_4=5K\Omega$, to obtain Low-frequency gain of unity.

$R_3=QR_2=6.705K\Omega$.

$-P_2=(2\pi)(0.9694\times10^6)=\frac{1}{R_5C_5}$, Let $R_5=5K\Omega \Rightarrow C_5=32.84pF$.



c) Butterworth with TT

$n=3$, $w_0 = (2\pi)(1.27\times10^6)$

$P_1 = w_0 \exp(j\frac{2\pi}{3})$, $P_3 = w_0 \exp(-j\frac{2\pi}{3})$, $P_2 = -w_0$.

$$H_{P_{1,3}}(s) = \frac{[(2\pi)(1.27\times10^6)]^2}{s^2 - [(4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})]s + [(2\pi)(1.27\times10^6)]^2}$$

$W_n = (2\pi)(1.27\times10^6)$, $\frac{W_n}{Q} = (4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})$

$$Q = -\frac{1}{2\cos(\frac{2\pi}{3})} = 1.$$

$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$, $W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$

Let $R_2 = R_4$, $C_1 = C_2$, $\frac{1}{R_2C_1} = (2\pi)(1.27\times10^6)$

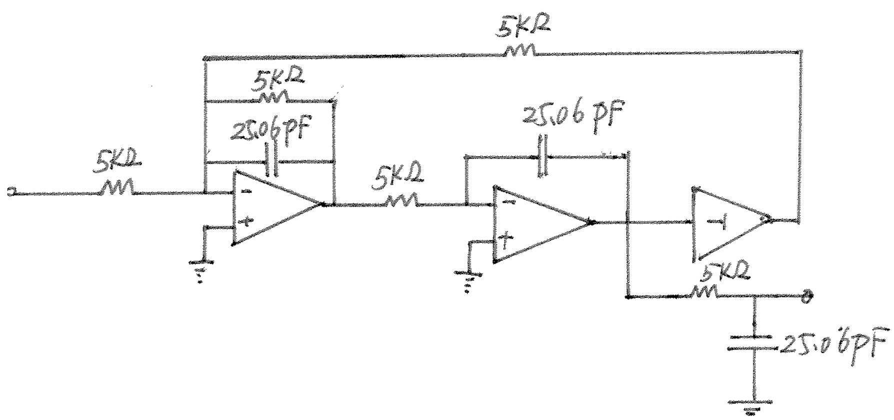
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 25.06\ pF$.

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain Low-fequency gain of unity.

$R_3 = Q R_2 = 5K\Omega$.

$-P_2 = W_0 \Rightarrow \frac{1}{R_5C_5} = (2\pi)(1.27\times10^6)$

Let $R_5 = 5K\Omega \Rightarrow C_5 = 25.06pF$



c) Chebyshev TT

$n=3,\ \epsilon=0.5089,\ w_0=(2\pi)(1\times10^6)$

$P_{1,3}=-0.2470\,w_0 \pm j\,0.9660\,w_0\ ,\ P_2=-0.4941\,w_0$

$$H_{P_{1,3}}(s)=\frac{[(2\pi)(0.9971\times10^6)]^2}{s^2+(0.4940)(2\pi\times10^6)s+[2\pi\times0.9971\times10^6]^2}$$

$W_n=(2\pi)(0.9971\times10^6)$

$\frac{W_n}{Q}=(0.4940)(2\pi\times10^6) \Rightarrow Q=2.02$

$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}}\ ,\quad W_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$

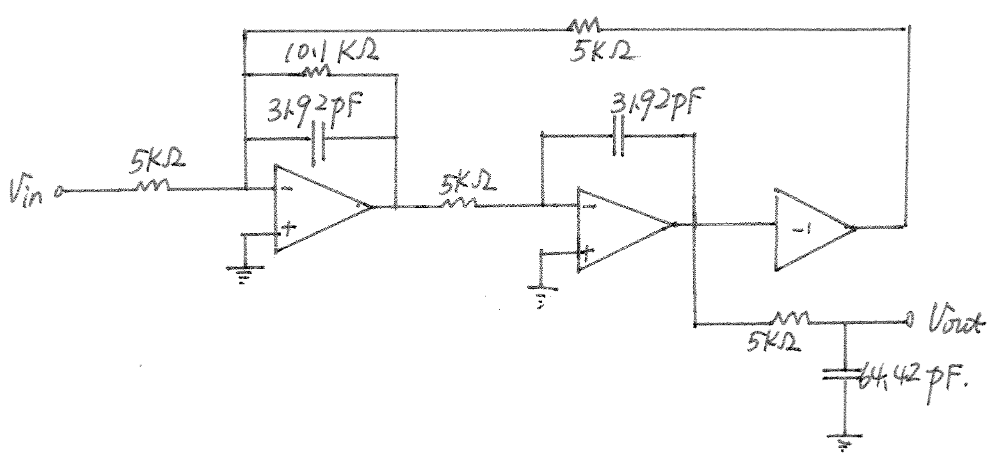
Let $R_2=R_4,\ C_1=C_2 \Rightarrow \frac{1}{R_2C_1}=(2\pi)(0.9971\times10^6)$

Let $R_2=R_4=5K\Omega \Rightarrow C_1=C_2=31.92pF$

$R_1=R_2=R_4=5K\Omega$, to obtain Low-frequency gain of unity.

$R_3=QR_2=10.1\ K\Omega$.

$-P_2=(2\pi)(0.4941\times10^6)=\frac{1}{R_5C_5}$. Let $R_5=5K\Omega \Rightarrow C_5=64.42pF$.



d). Butterworth in TT

$n=4,\ W_0 = (2\pi)(1.3\times10^6)$

$P_{1,4} = W_0 \exp(\pm j\frac{5\pi}{8}),\quad P_{2,3} = W_0 \exp(\pm j\frac{7\pi}{8})$

$$H_{P_{1,4}}(S) = \frac{W_0^2}{S^2 - [2W_0\cos(\frac{5\pi}{8})]S + W_0^2}$$

$W_n = (2\pi)(1.3\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.3\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = -\frac{1}{2\cos(\frac{5\pi}{8})} = 1.31.$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}},\quad Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

Let $R_2 = R_4,\ C_1 = C_2,\ W_n = \frac{1}{\sqrt{R_2^2C_1^2}} = \frac{1}{R_2C_1} = (2\pi)(1.3\times10^6)$

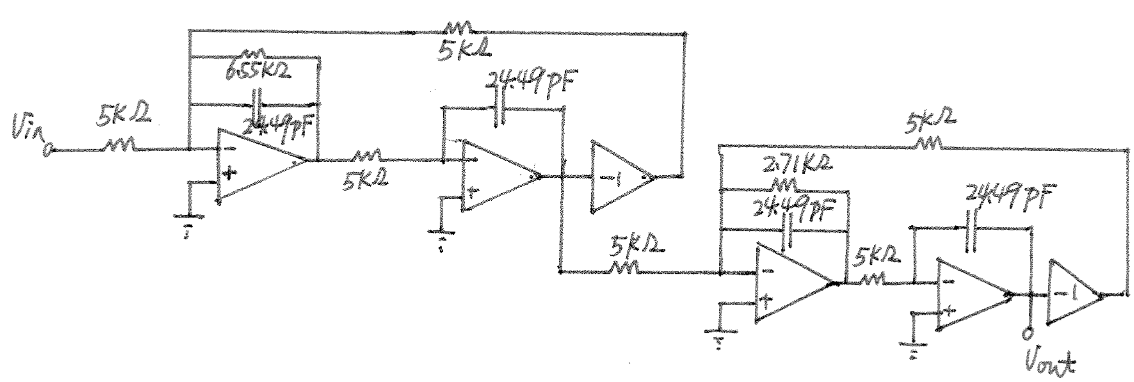
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 24.49\,pF.$

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$R_3 = Q R_2 = 6.55K\Omega.$

Similarly, $H_{P_{2,3}}(S) = \frac{W_0^2}{S^2 - [2W_0\cos(\frac{7\pi}{8})]S + W_0^2}$

It can be obtained that $R_1=R_2=R_4=5K\Omega$, $R_3 = 2.71K\Omega$, $C_1=C_2=24.49\,pF$.



d). Chebyshev TT

$n=3, \quad \epsilon = 0.3493, \quad W_0 = (2\pi)(1\times10^6)$

$P_{1,3} = -0.3133\, W_0 \pm j1.022 W_0, \quad P_2 = -0.6265 W_0.$

$$H_{P_{1,3}} = \frac{[2\pi \times 1.069\times10^6]^2}{s^2 + (0.6265)(2\pi\times10^6)s + (2\pi\times1.069\times10^6)^2}$$

$W_n = (2\pi)(1.069\times10^6)$

$\frac{W_n}{Q} = (0.6265)(2\pi\times10^6) \Rightarrow Q = \frac{1.069}{0.6265} = 1.7063$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}, \quad Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

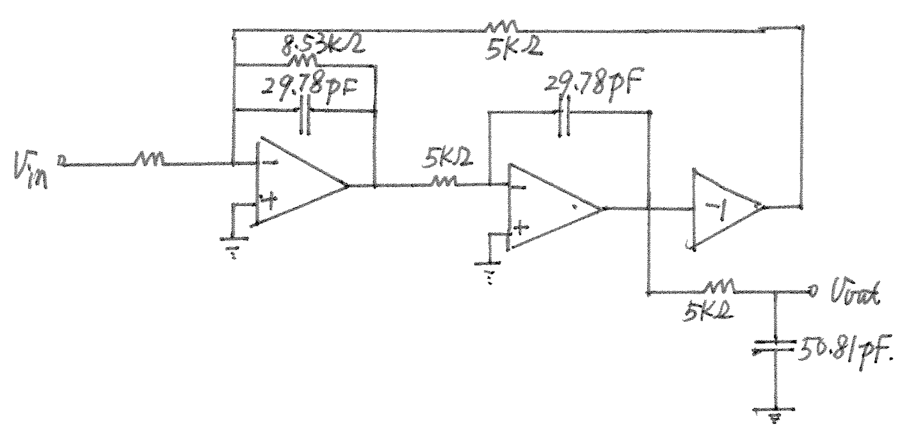
Let $R_2 = R_4$, $C_1 = C_2$, $\frac{1}{R_2C_1} = (2\pi)(1.069\times10^6)$

Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 29.78\,pF$.

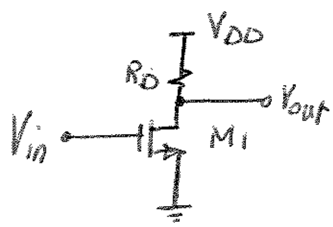
$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$R_3 = QR_2 = 8.53K\Omega$.

$-P_2 = 0.6265\times(2\pi\times10^6) = \frac{1}{R_5C_5}$, Let $R_5 = 5K\Omega \Rightarrow C_5 = 50.81\,pF$



1.



M_1 operates in the triode region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out} = V_{DD} - R_D I_D$$

$R_D = 10K$

$\left(W/L\right)_1 = 3/0.18$

$V_{out,min} = ?$ when $V_{in} = V_{DD}$

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]\times R_D$$

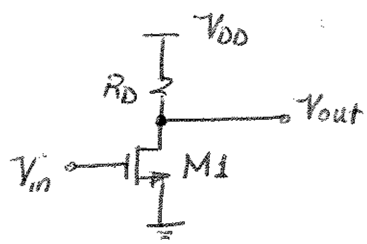
If the second term in the square brackets is neglected, then

$$V_{out,min} \simeq \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH})\times R_D}$$

$$= \frac{1.8}{1+100\times10^{-6}\times 3/0.18 \times(1.8-0.4)\times 10^5}$$

$$\boxed{V_{out,min} \simeq 74\ mV}$$

2.



$V_{out,min} \leq 100\,mV$

$R_D = 5K\Omega$

$(W/L)_{1,min} = ?$

Output low level establishes for $V_{in} = V_{DD}$, driving M_1 into the triode region.

$$I_{D\,max} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out,min} = V_{DD} - R_D \times I_{D,max}$$

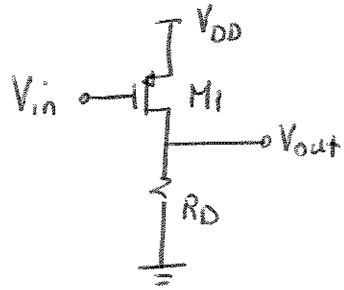
$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]\times R_D$$

$$\left(\frac{W}{L}\right)_1 = \frac{V_{DD} - V_{out,min}}{\frac{1}{2}\mu_n C_{ox}\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]\times R_D}$$

$$\left(\frac{W}{L}\right)_{1,min} = \frac{1.8 - 100\times10^{-3}}{\frac{1}{2}\times100\times10^{-6}\left[2(1.8-0.4)100\times10^{-3} - (100\times10^{-3})^2\right]\times 5\times10^{3}}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 25}$$

3.



$\left(\frac{W}{L}\right)_1 = 20/0.18$, $R_D = 5K$

V_{OL} , V_{OH} = ?

(1) $V_{in} = V_{DD} \longrightarrow M_1$ off $\longrightarrow I_D = 0 \longrightarrow \boxed{V_{out} = V_{OL} = 0}$

(2) $V_{in} = 0 \longrightarrow M_1$ operates in the triode region

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{SG} - |V_{THP}|)V_{SD} - V_{SD}^2\right]$$

$$I_{D,max} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD} - |V_{THP}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] \quad (1)$$

$$I_{D,max} = \frac{V_{out}}{R_D} \quad (2)$$

Equating (1) and (2) and neglecting the second order term in the brackets

$$\frac{V_{out}}{R_D} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \times 2(V_{DD} - |V_{THP}|)(-V_{out} + V_{DD})$$

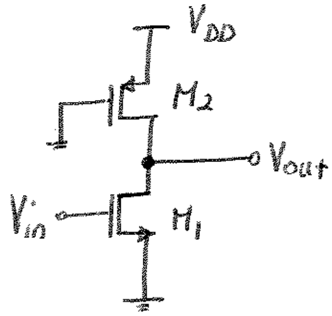
$$V_{out}\left[\frac{1}{R_D} + \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|) \times V_{DD}$$

$$V_{out} = \frac{R_D}{R_D + \dfrac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)}} V_{DD}$$

$$V_{out} = \frac{5000}{5000 + \frac{1}{50\times10^{-6} \times \left(\frac{20}{0.18}\right) \times (1.8 - 0.5)}} \times 1.8$$

$$\boxed{V_{out} = V_{OH} = 1.75\ V}$$

4.



$\left(\frac{W}{L}\right)_1 = 3/0.18 \quad \left(W/L\right)_2 = 2/0.18$

(a) if $V_{in} = V_{DD}$, M_2 Saturated $\rightarrow V_{OL} = ?$

(b) if $V_{in} = V_{out} \longrightarrow V_{in} = ?$

(a) $I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{THP}|\right)^2$

$I_{D2} = \frac{1}{2} \times 50\times10^{-6} \times \left(\frac{2}{0.18}\right)\left(1.8 - 0.5\right)^2$, Note that $V_{SG} = V_{DD}$

$I_{D2} = 4.7\times10^{-4}$ A

$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS} - V_{THN})V_{DS} - V_{DS}^2\right]$

$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{THN})V_{OL} - V_{OL}^2\right]$

However $I_{D1} = I_{D2}$

$4.7\times10^{-4} = \frac{1}{2}\times 100\times10^{-6} \times \left(\frac{3}{0.18}\right)\left[2(1.8-0.4)V_{OL} - V_{OL}^2\right]$

Neglecting the second-order term yields:

$\boxed{V_{OL} = 0.2\ V}$

As $(V_{in} - V_{THN}) = (V_{DD} - V_{THN}) = (1.8 - 0.4) = 1.4 > V_{DS_1} = V_{OL} = 0.2\,V$

the assumption of M_1 being in Triode region is correct

We define, $V_X = V_{in} - V_{TH,N} \rightarrow V_{in} = V_X + V_{TH,N}$

$$\frac{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2} V_X^2 = 2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{TH,N} - V_X) - (V_{DD} - V_{TH,N} - V_X)^2$$

$$\frac{100}{50} \times \frac{3/0.18}{2/0.18} V_X^2 = 2(1.8-0.5)(1.8-0.4-V_X) - (1.8-0.4-V_X)^2$$

$$3V_X^2 = 2.6(1.4 - V_X) - (1.4 - V_X)^2$$

$$3V_X^2 = 3.64 - 2.6V_X - 1.96 + 2.8V_X - V_X^2$$

$$4V_X^2 - 0.2V_X - 1.68 = 0$$

$$V_X = \frac{0.2 \pm \sqrt{0.2^2 + 4 \times 4 \times 1.68}}{8} \longrightarrow \boxed{V_X = 0.67\ V}$$

$$V_{in} = V_X + V_{TH,N} = 0.67 + 0.4 \rightarrow \boxed{V_{in} = V_{out} = 1\ V}$$

This value of V_{out} guarantees that M_2 operates in the triode region.

Now, let's investigate the region of operation of M_2

$$V_{SD2} = V_{DD} - V_{out}$$

$$= 1.8 - 0.2$$

$$V_{SD2} = 1.6\ V$$

$$V_{SG2} - |V_{THP}| = V_{DD} - |V_{THP}|$$

$$= 1.8 - 0.5$$

$$V_{SG2} - |V_{THP}| = 1.3$$

As $V_{SD2} > V_{SG2} - |V_{THP}|$, M_2 operates in the saturation region and the initial assumption is valid.

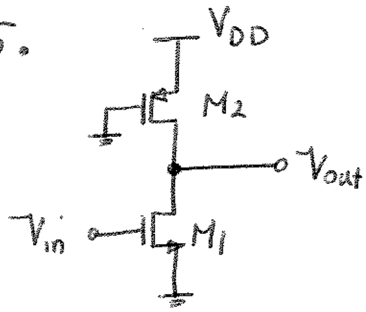
(b) As $V_{in} = V_{out} \rightarrow M_1$ is saturated.

We assume that M_2 is in the triode region and check the validity of this assumption

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{THP}|) \times (V_{DD} - V_{in}) - (V_{DD} - V_{in})^2\right]$$

5.



$V_{OL} \leq 100\ mV$

$(W/L)_2 = 3/0.18$

$\left(\frac{W}{L}\right)_{1,min} = ?$

$V_{in} = V_{DD} \rightarrow$ M_1 operates in The triode region and M_2 in the saturation.

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{TH,p}|\right)^2$$

$$= \frac{1}{2} \times 50\times10^{-6} \times \left(\frac{3}{0.18}\right) \times (1.8 - 0.5)^2$$

$$I_{D2} = 7.041 \times 10^{-4}\ A$$

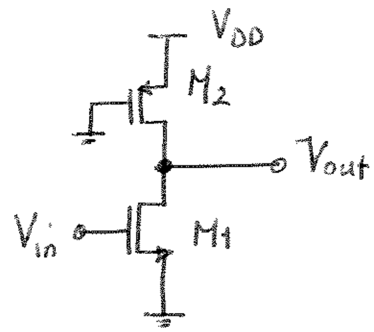
$$I_{D1} = I_{D2} = 7.041\times10^{-4}\ A$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{GS} - V_{TH,N}\right)V_{DS} - V_{DS}^2\right]$$

$$7.041\times10^{-4} = \frac{1}{2}\times 100\times10^{-6}\left(\frac{W}{L}\right)_1 \left[2(1.8-0.4)\,0.1 - (0.1)^2\right]$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 52.16}$$

6.



$V_{OL} \leq 80\ mV$

$\left(\frac{W}{L}\right)_1 = 2/0.18$

$\left(\frac{W}{L}\right)_{2,max}$

$V_{in} = V_{DD} \rightarrow M_1$ operates in the triode region and M_2 in the saturation

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$I_{D1} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right) \times \left[2(1.8 - 0.4)\,0.08 - 0.08^2\right]$$

$$I_{D1} = 1.2 \times 10^{-4}\ A$$

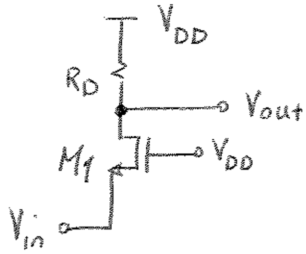
$$I_{D2} = I_{D1} = 1.2 \times 10^{-4}\ A$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH,P}|)^2$$

$$1.2 \times 10^{-4} = \frac{1}{2} \times 50 \times 10^{-6}\left(\frac{W}{L}\right)_2 (1.8 - 0.5)^2$$

$$\boxed{\left(\frac{W}{L}\right)_{2,max} = 2.86}$$

7.



(a) If $V_{in} = 0$, V_{DD}, $V_{out} = ?$

If $V_{in} = 0 \longrightarrow M_1$ operates in the triode region.

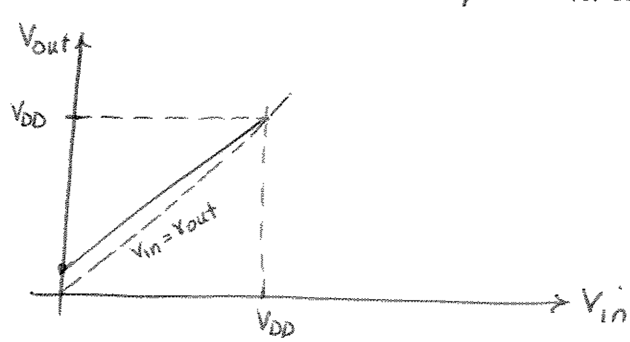
$$R_{on1} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N})}$$

$$V_{out} \cong \frac{R_{on1}}{R_{on1} + R_D} \times V_{DD} \longrightarrow V_{out} \cong \frac{1}{1 + \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N}) R_D} \times V_{DD}$$

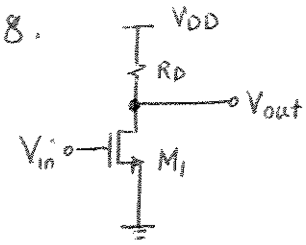
If $V_{in} = V_{DD} \longrightarrow V_{out} = V_{DD}$

No, this circuit does not invert.

(b) A trip point cannot be found for this circuit because $V_{out} = V_{in}$ line does not intersect the transfer characteristic of this buffer.



8.



$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$R_D = 2K\Omega$$

$$NM_L,\ NM_H = ?$$

Small signal gain of the circuit is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1 \quad , \quad V_{GS} = V_{IL}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH} = \frac{1}{100\times10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$$\boxed{V_{IL} = 0.58\ V}$$

To determine NM_H, we note that V_{in} drives M_1 into the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH,N}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH,N}) \frac{\partial V_{out}}{\partial V_{in}} - 2V_{out} \frac{\partial V_{out}}{\partial V_{in}} + 2V_{out}\right] R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @\ V_{IH}$$

$$-1 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-2(V_{in} - V_{TH,N}) + 4V_{out}\right] R_D$$

$$1 = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-V_{in} + V_{TH,N} + 2V_{out} \right] R_D$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} = -(V_{in} - V_{TH,N}) + 2V_{out}$$

$$V_{out} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2} \longrightarrow V_{out} = 0.5V_{in} - 0.11$$

Substituting this in (1) yields:

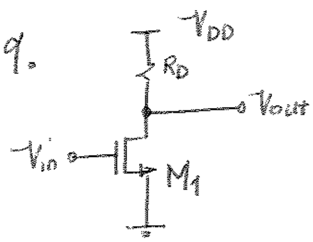
$$0.5V_{in} - 0.11 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2(V_{in} - 0.4)(0.5V_{in} - 0.11) - (0.5V_{in} - 0.11)^2 \right]$$

$$0.75V_{in}^2 - 0.33V_{in} - 0.6117 = \emptyset$$

$$V_{in} = V_{IH} = 1.15$$

$$NM_H = V_{DD} - V_{IH} = 1.8 - 1.15 \longrightarrow \boxed{NM_H = 0.65\ V}$$

9.



Small signal gain of the inverter is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1 \,, \quad V_{GS} = V_{IL}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1 \longrightarrow V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N}$$

If we double the value of $\left(\frac{W}{L}\right)_1$ or R_D

$$V_{IL} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2} + 0.4 \longrightarrow \boxed{V_{IL} = 0.49}$$

To determin NM_H, we note that V_{in} drives M_1 into the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2 \right] R_D. \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}} \right]$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @ \ V_{IH}$$

$$V_{out} = \frac{1}{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2}$$

Doubling $\left(\frac{W}{L}\right)_1$ or R_D leads to $\boxed{V_{out} = 0.5V_{in} - 0.155}$

Substituting in (1) yields:

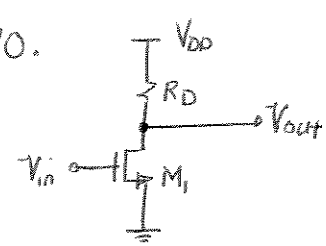
$$0.5V_{in} - 0.155 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2\left[2(V_{in} - 0.4)(0.5V_{in} - 0.155) - (0.5V_{in} - 0.155)^2\right]$$

$$0.75V_{in}^2 - 0.465V_{in} - 0.251925 = 0$$

$$V_{in} = 0.967\,V \longrightarrow NM_H = 1.8 - 0.967$$

$$\boxed{NM_H = 0.833\,V}$$

10.



$$\left(\frac{W}{L}\right)_1 = \frac{5}{0.18}$$

$R_D = 2K$

NM_L and $NM_H = ?$ if $\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ instead of -1

Small signal gain of the inverter is equal to "$-g_m R_D$"

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 0.5$$

$$V_{IL} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N} = \frac{1}{2 \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$\boxed{V_{IL} = 0.49}$ which is less than 0.58 obtained in problem 8.

To determine NM_H, note that M_1 operates in the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right] R_D$$

$\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ @ V_{IH}

$$-0.5 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-(V_{in} - V_{TH,N}) + 3V_{out}\right] R_D$$

$$V_{out} = \frac{1}{3\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{THN}}{3} \longrightarrow V_{out} = -73.33 \times 10^{-3} + 0.33 V_{in}^{\circ}$$

or $V_{out} = -\frac{0.22}{3} + \frac{V_{in}}{3}$

Substituting in (1) yields:

$$-\frac{0.22}{3} + \frac{V_{in}}{3} = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2\left(V_{in} - 0.4\right)\left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right) - \left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right)^2 \right]$$

$$5V_{in}^2 - 2.2V_{in} - 5.59 = 0$$

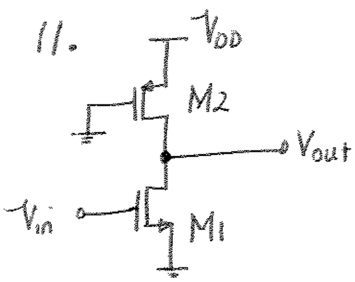
$$V_{in} = V_{IH} = 1.3$$

$$NM_H = 1.8 - 1.3$$

$$\boxed{NM_H = 0.5\,V}$$

less than 0.65 V obtained in problem 8 because V_{IH} is now further pushed up toward V_{DD}.

11.



$$\left(\frac{W}{L}\right)_1 = \frac{4}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{9}{0.18}$$

To calculate V_{IL}, we assume that M_1 and M_2 operate in saturation and triode region respectively.

$$I_{D1} = I_{D2} \quad (1)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right) - 2(V_{DD} - V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

By substituting $\frac{\partial V_{out}}{\partial V_{in}}$ with "-1" in the above relationship:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|) - 2(V_{DD} - V_{out})\right]$$

$$V_{out} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}(V_{in} - V_{THN}) + |V_{THP}| = \frac{100\times10^{-6}\times 4/0.18}{50\times10^{-6}\times 9/0.18}(V_{in} - 0.4) + 0.5$$

$$V_{out} = 0.144 + 0.88\,V_{in}^{\circ} \quad \text{or} \quad \boxed{V_{out} = \frac{8}{9}V_{in}^{\circ} + \frac{1.3}{9}}$$

Substituting V_{out} in (1) by the derivation versus V_{in} gives:

$$136V_{in}^2 - 108.8\,V_{in}^{\circ} - 115.13 = 0$$

$$\boxed{V_{in}^{\circ} = V_{IL} = NM_L = 1.4V}$$

To Calculate V_{IH}, we assume that M_1 and M_2 operate in the triode and saturation region respectively.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{THP}|\right)^2 \quad (2)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] = \emptyset$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right] = \emptyset$$

$V_{out} = \dfrac{V_{in}-V_{THN}}{2}$ supstituted in (2) yields:

$$V_{in} = \sqrt{\frac{3}{2}}\left(V_{DD}-|V_{THP}|\right)+V_{THN}$$

$V_{in} = 2 \longrightarrow V_{out} = 0.8$ This value of V_{out} puts M_2 into the triode region so our initial assumption is not correct

Now we assume that both M_1 and M_2 operate in the triode region.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-|V_{THP}|\right)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (3)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)-2(V_{DD}-V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)-2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}(V_{in}-V_{THN})-|V_{THP}|}{2\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}-1}$$

$$V_{out}=\frac{8}{7}V_{in}-1.1$$

After substituting in (3) it leads to:

$$2.1769\,V_{in}^2-4.19\,V_{in}+0.576=\emptyset$$

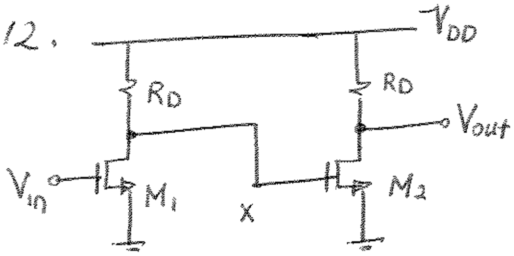
$$\boxed{V_{in}=1.77\,V}$$

$V_{out}=0.93\,V$ → The assumption is correct

$V_{IH}=1.77\,V$ → $NM_H=1.8-1.77$

$$\boxed{NM_H=0.03\,V}$$

12.



The small signal gain of the circuit is equal to $-g_m R_D$ and since

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{THN})$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{THN}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{THN} = \frac{2}{S} + 0.4 \quad ; \left(\frac{W}{L}\right)_{1,2} = S$$

Now we calculate the output of M_1 for $V_{in} = V_{DD}$:

$$V_{DD} - R_D I_D = V_{out}$$

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{THN}) V_{out} - V_{out}^2\right] R_D = V_{out} \quad ; \left(\frac{W}{L}\right)_{1,2} = S$$

$$1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times S \left[2(1.8 - 0.4)\left(\frac{2}{S} + 0.4\right) - \left(\frac{2}{S} + 0.4\right)^2\right] \times 5000 = \left(\frac{2}{S} + 0.4\right)$$

$$1.8 - 0.25 \times \left[2.8(2 + 0.4S) - S\left(\frac{2}{S} + 0.4\right)^2\right] = \frac{2}{S} + 0.4$$

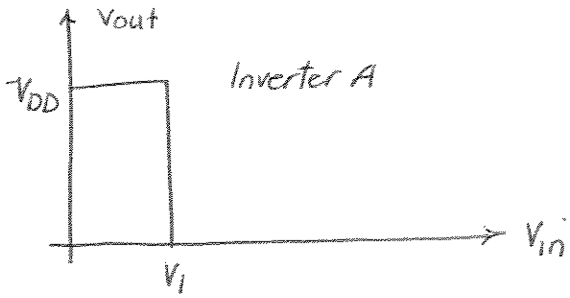
$$1.8S - 0.25 \times \left[2.8(2S + 0.4S^2) - S^2\left(\frac{2}{S} + 0.4\right)^2\right] = 2 + 0.4S$$

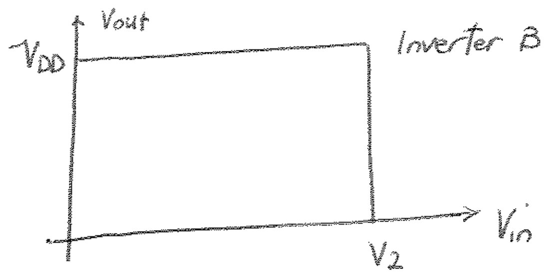
$$1.8S - 0.25 \times \left[5.6S + 1.12S^2 - 4 - 1.6S - 0.16S^2\right] = 2 + 0.4S$$

$$0.24S^2 - 0.4S + 1 = \emptyset$$

$$\Delta < \emptyset !$$

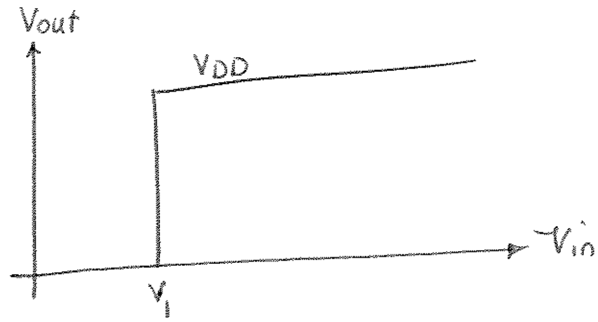
13.


Vout
VDD
Inverter A
Vin
V1

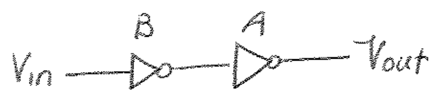

Vout
VDD
Inverter B
Vin
V2

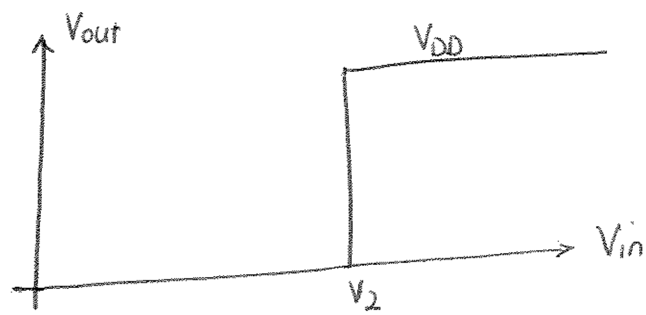
(a)


Vin
A
B
Vout

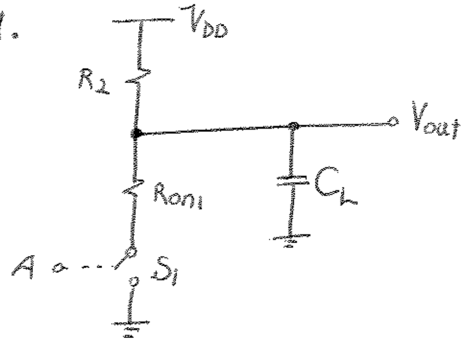

Vout
VDD
Vin
V1

(b)


Vin
B
A
Vout


Vout
VDD
Vin
V2

14.



$R_{on1} \ll R_2 \longrightarrow V_{out,min} \approx 0$

(a) $V_{out}(t) = V_{out}(0^-) + [V_{DD} - V_{out}(0^-)]\left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$

Note that $V_{out}(0^-) = 0$, $V_{out}(\infty) = V_{DD}$

$V_{out}(t) = V_{DD} \times \left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$

$0.95\, V_{DD} = V_{DD} \times \left(1 - \exp\frac{-T_{95\%}}{R_2 C_L}\right)$

$$\boxed{T_{95\%} = 3 R_2 C_L}$$

(b) $V_{out}(t) = V_{out}(0^-) + [V_{out}(\infty) - V_{out}(0^-)] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$

$V_{out}(t) = V_{DD} + [0 - V_{DD}] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$

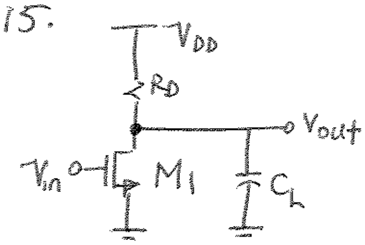
$V_{out}(t) = V_{DD} \exp\frac{-t}{R_2 C_L}$

$0.05\, V_{DD} = V_{DD} \exp\frac{-T_{0.05}}{R_2 C_L}$

$$\boxed{T_{5\%} = 3 R_2 C_L}$$

If $R_{on1} \ll R_2$, inverter exhibits equal rise and fall time (or low-to-high and high-to-low delay) at the output.

15.



$C_L = 50\,fF$

$T_R = 100\,pS$

$T_R = 3\,\tau_{out}$

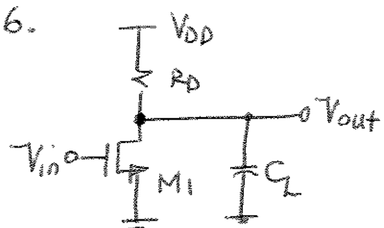
$R_{D,max} = ?$

$T_R = 3 R_D C_L = 100\,pS$

$R_D \leqslant \frac{100\,pS}{3 \times 50\,fF}$

$\boxed{R_D \leqslant 666.67\,\Omega}$

16.



$C_L = 100\,fF$

$V_{out,min} = 50\,mV$

$T_R = 200\,pS$

$R_D,\ \left(\frac{W}{L}\right)_1 = ?$

$T_R = 3\tau_{out}$

$T_R = 3R_DC_L$

$200\times10^{-12} = 3\times R_D\times 100\times10^{-15}$

$$R_D = 666.667\ \Omega$$

$V_{in} = V_{DD}$ places M_1 in the triode region

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D\left[2(V_{DD}-V_{THN})V_{out,min} - V_{out,min}^2\right]$$

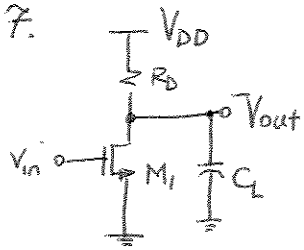
Neglecting the 2nd-order term in the square brackets yields:

$$V_{out,min} = \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D(V_{DD}-V_{TH})}$$

$$50\times10^{-3} = \frac{1.8}{1+100\times10^{-6}\times\left(\frac{W}{L}\right)_1\times 666.7\times(1.8-0.4)}$$

$$\left(\frac{W}{L}\right)_1 = 375$$

17.



$C_L = 100\,fF$ $\quad V_{out,min} \approx 0$

$I_{D_{max}} \leqslant 1mA$

$T_{R,min} = ?$

$$I_{D,max} = \frac{V_{DD} - V_{out,min}}{R_D}$$

$$10^{-3} = \frac{1.8 - 0}{R_D}$$

$$\boxed{R_D = 1.8\,K\Omega}$$

$$V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{out,min} + \left[V_{DD} - V_{out,min}\right] \times \left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{DD}\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

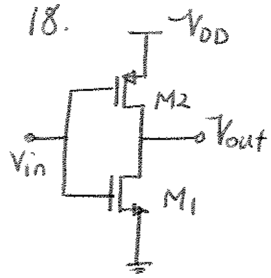
$$0.1 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{10\%}}{R_D C_L}\right) \longrightarrow T_{10\%} = 0.105\,R_D C_L$$

$$0.9 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{90\%}}{R_D C_L}\right) \longrightarrow T_{90\%} = 2.3\,R_D C_L$$

$$T_R = T_{90\%} - T_{10\%} = 2.197\,R_D C_L = 2.197 \times 1.8 \times 10^{3} \times 100 \times 10^{-15}$$

$$\boxed{T_R = 395.5\,pS}$$

18.



$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$I_{D1} = I_{D2}$

At the trip point $V_{in} = V_{out}$; therefore, both M_1 and M_2 operate in the Saturation region.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}^o - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in}^o - |V_{THP}|)^2$$

$$V_{in}^o = \frac{V_{DD} - |V_{THP}| + \sqrt{\frac{\mu_n C_{ox}(W/L)_1}{\mu_p C_{ox}(W/L)_2}} \times V_{THN}}{1 + \sqrt{\frac{\mu_n C_{ox}(W/L)_1}{\mu_p C_{ox}(W/L)_2}}}$$

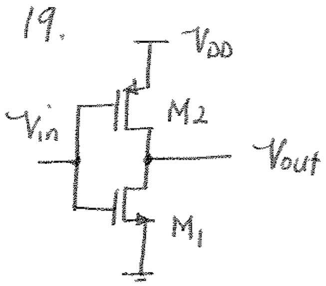
$$V_{in}^o = \frac{1.8 - 0.5 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2} \times 0.4}{1 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2}}$$

$$\boxed{V_{in} = V_{out} = 0.82\,V}$$

$$I_{D1} = I_{D2} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)(0.82 - 0.4)^2$$

$$\boxed{I_{D1} = I_{D2} = 97\,\mu A}$$

19.



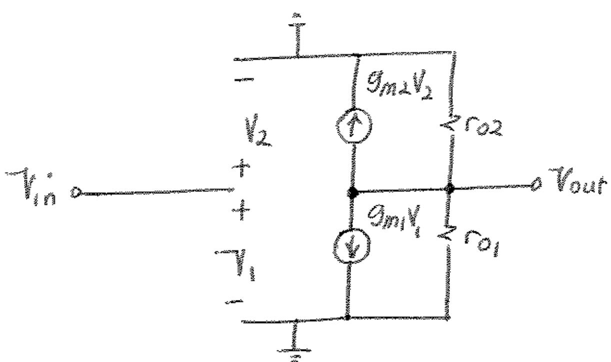
$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$$\lambda_N = 0.1\,V^{-1}$$

$$\lambda_P = 0.2\,V^{-1}$$

Replacing M_1 and M_2 with their small-signal model in the saturation region yields:



$$V_{out} = (-g_{m1}V_1 - g_{m2}V_2)(r_{o1} \| r_{o2})$$

$$V_1 = V_2 = V_{in}$$

$$V_{out} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})V_{in}$$

$$\frac{V_{out}}{V_{in}} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2$$

$$g_{m1} = \frac{\partial I_{D1}}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L_1}\right)_1 (V_{in} - V_{THN}) = \frac{2I_{D1}}{V_{in} - V_{THN}}$$

$$g_{m1} = \frac{2 \times 9.7 \times 10^{-5}}{(0.817 - 0.4)} \rightarrow \boxed{g_{m1} = 4.641 \times 10^{-4}\ \mho}$$

$$g_{m2} = \frac{2 I_{D2}}{(V_{SG} - |V_{THP}|)} = \frac{2 \times 9.7 \times 10^{-5}}{(1.8 - 0.817 - 0.5)} \rightarrow \boxed{g_{m2} = 4.02 \times 10^{-4}\ \mho}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2 \lambda \simeq \lambda I_D$$

$$r_o \cong \frac{1}{\lambda I_D}$$

$$r_{oN} \simeq \frac{1}{0.1 \times 9.7 \times 10^{-5}} = 103.17\,K\Omega$$

$$r_{oP} \simeq \frac{1}{0.2 \times 9.7 \times 10^{-5}} = 51.58\,K\Omega$$

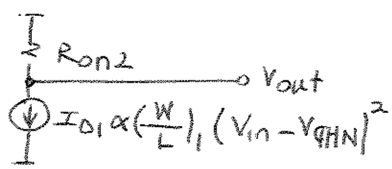
$$Gain = \frac{V_{out}}{V_{in}} = -(4.641 \times 10^{-4} + 4.02 \times 10^{-4})(51.58K \| 103.17K)$$

$$\boxed{Gain = -29.8}$$

20.

(a) Length of M_1 is increased

Let's assume that $V_{in} < V_{TH1}$, as a result M_1 is off and M_2 is on operating in the triode region. As V_{in} increases beyond V_{TH1}, M_1 starts pulling current (Conducting) in the saturation region while M_2 is still in the triode region. operating as a resistor; therefore, CMOS inverter can be modelled as follows:

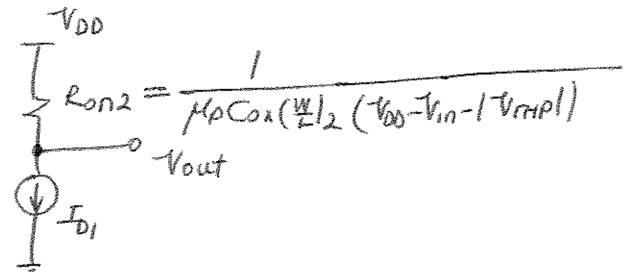


By increasing L_1, I_{D1} is weakened due to the inverse proportionality; as a result, an excess V_{in} is required to drop V_{out} to the point where $V_{out} = V_{in} + |V_{TH2}|$ and M_2 is placed at the edge of saturation. Therefore characteristic is shifted to the right and it will be steeper at the gain region where both M_1 and M_2 are in saturation region.

(b) Length of M_2 is increased

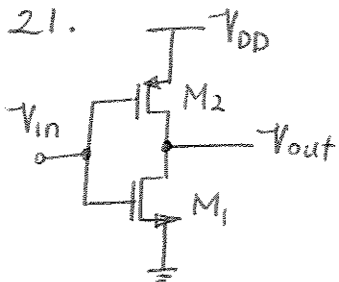
Again if we assume that $V_{in} < V_{TH1}$, M_1 is off and M_2 is operating in the triode region with no current. By increasing

V_{in} above V_{TH1}, M_1 conducts in the saturation region while M_2 is operating in the triode region. Using the same models as used in part (a) yields:



By increasing L_2, R_{on2} becomes larger; as a result, lower value of I_{D1} causes comparable voltage drop at the output. This will drive M_2 into the saturation with lower current (I_{D1}) and; hence, lower value of V_{in}. Therefore, characteristic is shifted to the left and small signal gain will be higher.

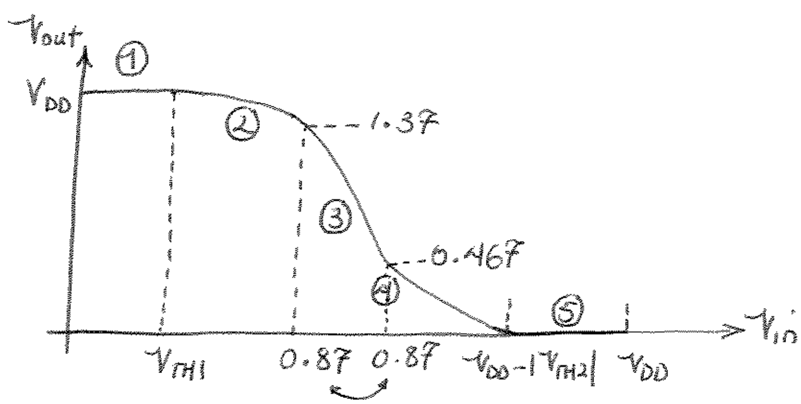
21.



$$\left(\frac{W}{L}\right)_1 = \frac{3}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{7}{0.18}$$

VTC looks like the following figure



① M_1 off, M_2 in triode region

$$I_{D1} = 0$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{TH2}|\right)V_{SD} - V_{SD}^2\right] = 0$$

$$V_{SD} = 0 \rightarrow V_{out} = V_{DD} \quad (1)$$

② M_1 in saturation, M_2 in triode region

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{THP}|\right)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}\times(V_{in}-0.4)^2=\frac{1}{2}\times 50\times 10^{-6}\times\frac{7}{0.18}\times\left[2(1.8-V_{in}-0.5)\times(1.8-V_{out})-(1.8-V_{out})^2\right]$$

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{out})-(1.8-V_{out})^2\right]\quad(2)$$

If V_{out} falls significantly, M_2 enters saturation. That is $V_{out}=V_{in}+|V_{TH2}|$. then M_2 is about to exit the triode region.

Replacing V_{out} by $V_{in}+|V_{TH2}|$ in (2) leads to:

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{in}-0.5)-(1.8-V_{in}-0.5)^2\right]$$

$$6(V_{in}-0.4)^2=7(1.3-V_{in})^2\rightarrow\sqrt{\frac{6}{7}}(V_{in}-0.4)=(1.3-V_{in})$$

$V_{in}=0.867\,V$, $V_{out}=1.37$

(3) $$\frac{1}{2}\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2(1+\lambda_1V_{out})=\frac{1}{2}\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2\times\left[1+\lambda_2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2-\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}{\lambda_2\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2+\lambda_1\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}$$

$$V_{out}=\frac{7(1.3-V_{in})^2-6(V_{in}-0.4)^2}{7\lambda_2(1.3-V_{in})^2+6\lambda_1(V_{in}-0.4)^2}\quad(3)$$

in region (3) M_1 and M_2 are both in saturation.

④ M_1 in triode region, M_2 in saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times (V_{DD}-V_{in}-|V_{TH2}|)^2 \quad (4)$$

$$6\left[2(V_{in}-0.4)V_{out}-V_{out}^2\right]=7(1.3-V_{in})^2$$

If V_{out} falls sufficiently, M_1 enters the triode region. That is, if $V_{in}=V_{out}+V_{TH1}$. Then M_1 is about to enter the triode region.

By substituting V_{in} with $V_{out}+0.4$ in (4), we have:

$$6\left[2V_{out}^2-V_{out}^2\right]=7(0.9-V_{out})^2$$

$V_{out}=0.467$, $V_{in}=0.867$

As channel length modulation has been neglected in this calculation tha value of input voltage that makes CMOS inverter transition from region ② to ③ is the same as that which makes inverter transition from region ③ to ④.

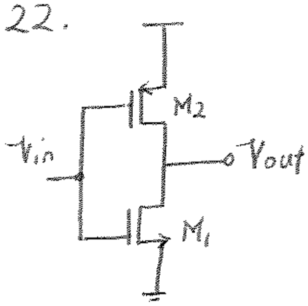
The slope in region ③ is infinit; however, we assume a finite slope in that region to emphasize the behavior of inverter as to producing a high gain.

⑤ M_1 in triode region, M_2 off

$I_{D2}=0$; $I_{D1}=0$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=0 \longrightarrow V_{out}=0$$

22.



$V_{in} = V_{out} = 0.5\ V$

M_1 and M_2 are both in saturation region

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.5 - 0.4)^2 = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.5 - 0.5)^2$$

$$\left(\frac{W}{L}\right)_1 / \left(\frac{W}{L}\right)_2 = 32$$

23. The value of the trip point has to be larger than the threshold voltage of NMOS transistor, 0.4 V. Therefore, 0.3 V Cannot be the trip point of such an inverter.

24.

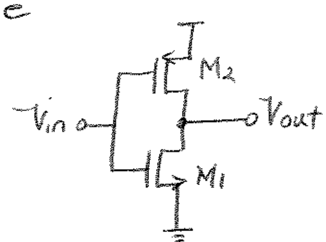
(a) If the inverter exhibits a very high voltage gain around the trip point, the range of input voltage values which guarantees that M_1 and M_2 are in saturation region is very narrow. Therefore this range can be fairly approximated with only one value of input voltage.

(b) $(W/L)_1 = 3/0.18$ and $(W/L)_2 = 7/0.18$

To calculate the minimum input voltage at which both transistors operate in saturation we assume

M_1 saturation

M_2 triode



$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]$$

$V_{out} = V_{in} + |V_{TH2}|$ places M_2 at the edge of saturation

$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times \left[2(1.8 - V_{in} - 0.5)(1.8 - V_{in} - 0.5) - (1.8 - V_{in} - 0.5)^2\right]$$

$$\boxed{V_{in,min} = 0.867}$$

To calculate $V_{in,max}$, we assume that M_1 and M_2 are in triode and saturation region respectively

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH1})V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

When M_1 is just going to leave the saturation and enters the triode region

$$V_{in} = V_{out} + 0.4^{(V_{TH1})}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times \left[2(V_{out} + V_{TH1} - V_{TH1}) V_{out} - V_{out}^2 \right] = \frac{1}{2} \times 50 \times 10^{-6} \times \frac{7}{0.18} \times (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (V_{DD} - V_{out} - V_{TH1} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (0.9 - V_{out})^2$$

$$V_{out} = 0.467\,V, \quad \boxed{V_{in,max} = 0.867}$$

To find the trip point, M_1 and M_2 are assumed to be in saturation.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

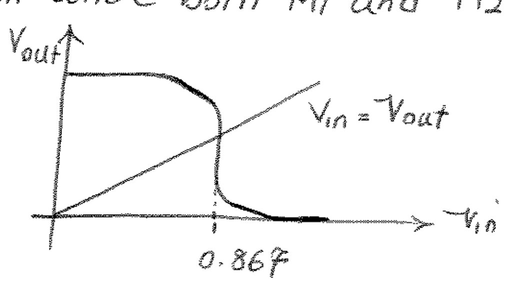
$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times (1.8 - V_{in} - 0.5)^2$$

$$\boxed{V_{in}^{o} = 0.867 \text{ @ trip point}}$$

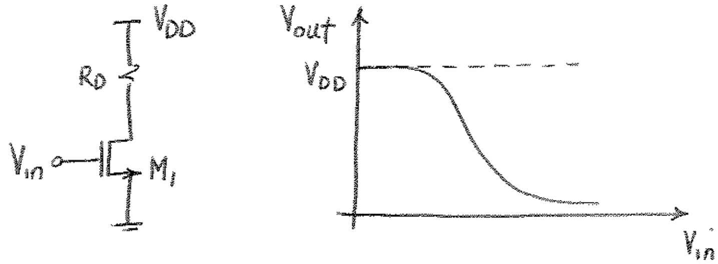
$$V_{in,trip} - V_{in,min} = \emptyset$$

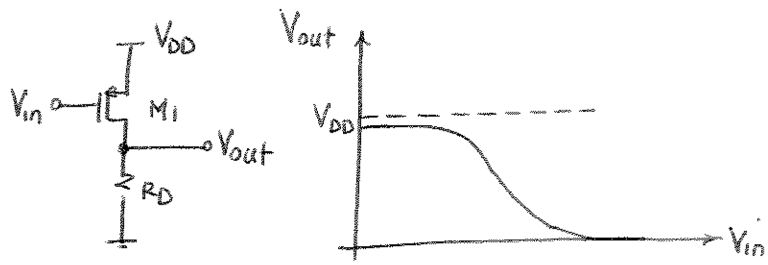
$$V_{in,max} - V_{in,trip} = \emptyset$$

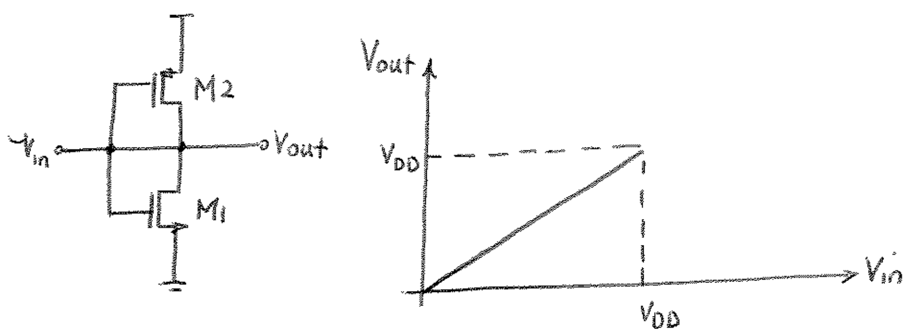
This result is not surprising because VTC of inverter has infinite slope at the region where both M_1 and M_2 are in saturation region



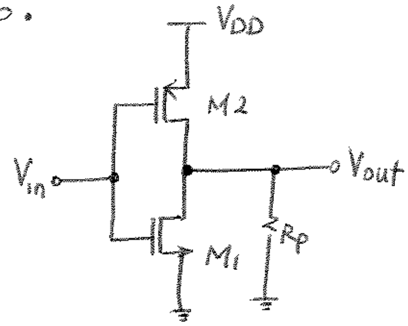
25.


VDD
RD
Vin
M1
Vout
VDD
Vin


VDD
Vin
M1
Vout
RD
Vout
VDD
Vin


M2
Vin
Vout
M1
Vout
VDD
Vin
VDD

26.



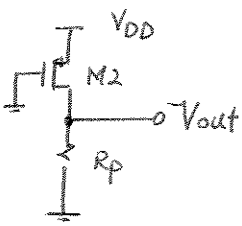
$R_p = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{OL}, V_{OH}, V_{in,trip} = ?$

To calculate V_{OH}, V_{in} is assumed to be $0V$



$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$I_{D2} = \frac{V_{out}}{R_p}$$

$$\frac{V_{out}}{R_p} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$V_{out}^2 - 0.28V_{out} - 1.44 = 0$$

$$\boxed{V_{out} = V_{OH} = 1.348\,V}$$

$\boxed{V_{OL} = 0}$ because M_2 is off for $V_{in} = V_{DD}$

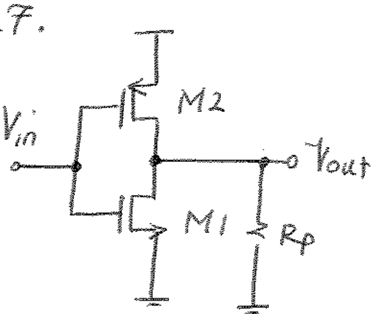
(a) trip point $V_{in} = V_{out}$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD} - V_{in} - |V_{TH2}|)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{in} - V_{TH1})^2 + \frac{V_{out}}{2000}$$

$$0.05V_{out}^2 + 0.59V_{out} - 0.3745 = 0 \longrightarrow \boxed{V_{in} = V_{out} = 0.6\,V}$$

27.



$R_P = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{in} = V_{out} = 0.6V$ @ trip point

With R_P

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P}$$

$$\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in} - V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05 V_{out}^2 + 0.59 V_{out} - 0.3745 = 0$$

$$V_{in} = V_{out} = 0.6V$$

$$I_{D1} = \frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times (0.6 - 0.4)^2$$

$$\boxed{I_{D1} = 3.33 \times 10^{-5} A}$$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P} = 3.33 \times 10^{-5} + \frac{0.6}{2000}$$

$$\boxed{I_{D2} = 3.35 \times 10^{-4} A}$$

$$g_{m1} = \frac{2I_{D1}}{V_{eff1}} = \frac{2 \times 3.33 \times 10^{-5}}{(0.6 - 0.4)} \rightarrow \boxed{g_{m1} = 3.3 \times 10^{-3} \mho}$$

$$g_{m2} = \frac{2I_{D2}}{V_{eff2}} = \frac{2 \times 3.35 \times 10^{-4}}{(1.8 - 0.6 - 0.5)} \rightarrow \boxed{g_{m2} = 9.4 \times 10^{-3}}$$

$A_V = -(g_{m1} + g_{m2}) \times R_p$

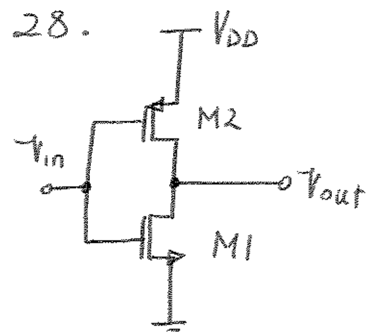
$A_V = -(3.3 \times 10^{-3} + 9.4 \times 10^{-3}) \times 2000$

$\boxed{A_V = -25.4}$

Without R_p

$A_V \Rightarrow -\infty$

28.



$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$\left(\frac{W}{L}\right)_2 = 11/0.18$$

NM_L and NM_H = ?

To calculate NM_L, M_1 and M_2 are assumed to operate in the saturation and triode region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\times\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right] \quad (2)$$

Obtaining V_{OH} from (2) and substituting in (1) yields:

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2} = \frac{100}{50}\times\frac{5}{11} = \frac{10}{11}$$

$$V_{IL} = \frac{2\sqrt{10/11}\,(1.8-0.4-0.5)}{(10/11-1)\sqrt{10/11+3}} - \frac{1.8-(10/11)\times 0.4-0.5}{10/11-1}$$

$$\boxed{V_{IL} = 0.7516\ V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{TH2}|-V_{in}\right)^2 \quad (3)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times \left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$\frac{\partial V_{out}}{\partial V_{in}}=-1$ yields

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[V_{out}-V_{in}+V_{TH1}+V_{out}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$$100\times 5\times\left[2V_{out}-(V_{in}-0.4)\right]=-50\times 11\times(1.8-0.5-V_{in})$$

$$V_{out}=1.05V_{in}-0.915 \quad (4)$$

Substituting (4) in (3) yields an equation versus V_{in} as follows:

$$10\times\left[2(V_{in}-0.4)(1.05V_{in}-0.915)-(1.05V_{in}-0.915)^2\right]=11(1.3-V_{in})^2$$

$$1.025V_{in}^2-21.115V_{in}+19.64225=0$$

$$V_{in}=V_{IH}=0.9765\ V$$

$$NM_H=V_{DD}-V_{IH}$$

$NM_H=0.823\ V$

29. $NM_L = 0.6\,V$

$$(W/L)_1 / (W/L)_2 = ?$$

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}$$

$$0.6 = \frac{2\sqrt{a}\,(1.8 - 0.4 - 0.5)}{(a-1)\sqrt{a+3}} - \frac{1.8 - a\,0.4 - 0.5}{a-1}$$

$$a = 3\sqrt{\frac{a}{a+3}} - \frac{1.3 - 0.4a}{0.6} + 1$$

$$a = \frac{a+3}{9} \times \left[a - 1 + \frac{1.3 - 0.4a}{0.6}\right]^2$$

$$\boxed{a = 1}$$

$$(W/L)_1 / (W/L)_2 = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} = \frac{1}{2}$$

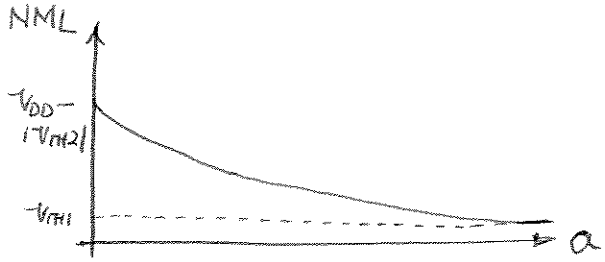
$$\boxed{(W/L)_1 / (W/L)_2 = \frac{1}{2}}$$

30. $$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

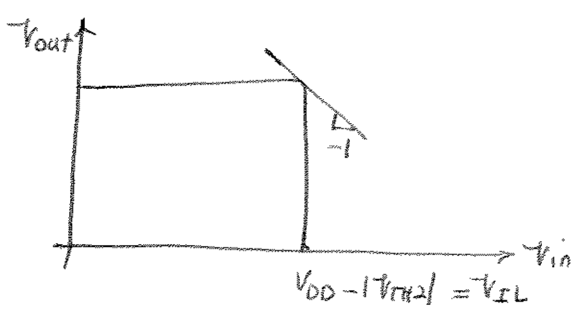
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad V_{IL} = V_{DD} - |V_{TH2}|$

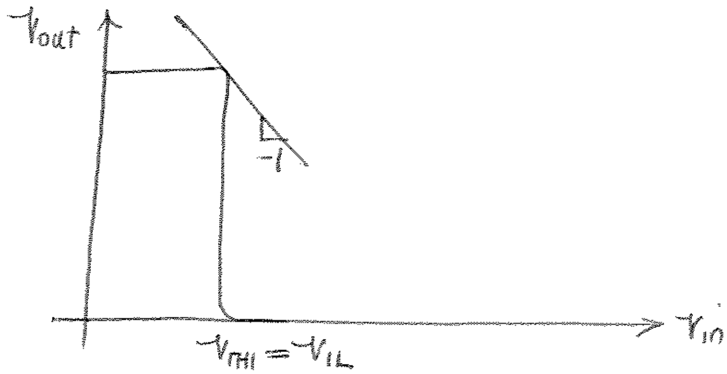
$a \to \infty \quad V_{IL} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is extremely stronger than NMOS. Therefor, as V_{in} increases from 0V, the output of inverter stays at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cut off and V_{out} sharply drops to 0V.



When $a \to \infty$, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.


Vout
-1
Vin
VTH = VIL

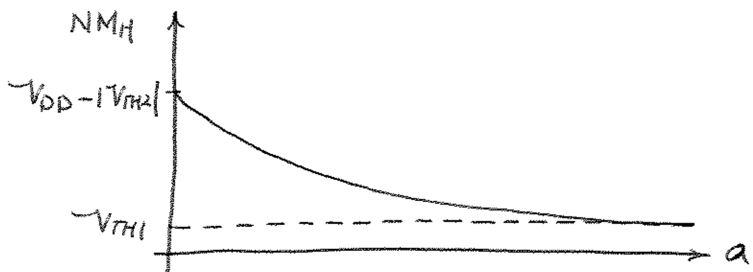
31.

$$NM_H = V_{DD} - \frac{2a\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{1+3a}} + \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

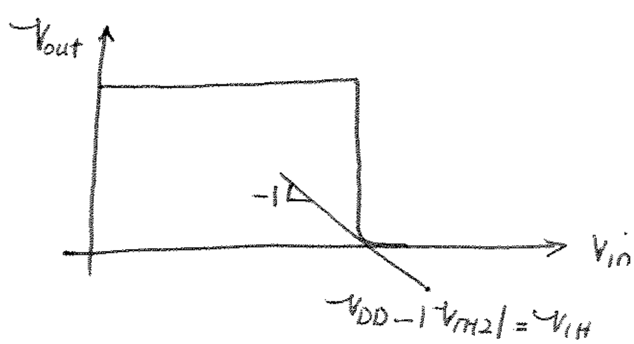
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad NM_H = |V_{TH2}| \,,\quad V_{IH} = V_{DD} - |V_{TH2}|$

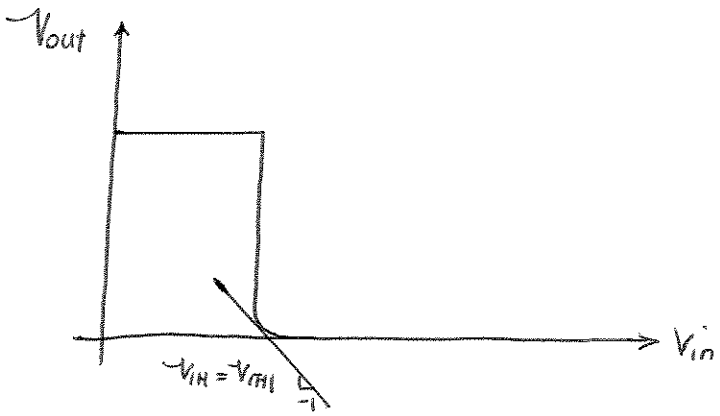
$a \to \infty \quad NM_H = V_{DD} - V_{TH1} \,,\quad V_{IH} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is much stronger than NMOS. Therefor, as V_{in} increases from 0 V, the output of inverter remains at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cutoff and V_{out} sharply drops to 0 V.

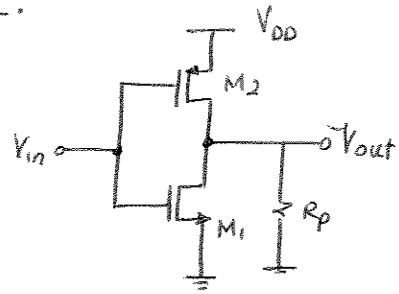


When "a" approaches infinity, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0 V.



Note that the separation between V_{IH} and V_{IL} depends on the slope of VTC in the transition region. If "a" approaches either "0" or infinity, VTC exhibits infinite gain in its transition region. Therefore V_{IL} and V_{IH} coincide.

32.



$R_p = 2K$

$(W/L)_1 = 3/0.18$

$(W/L)_2 = 5/0.18$

$NM_L,\ NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to be in the saturation and triode region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IL})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] =$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 + \frac{V_{out}}{R_p} \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[-2(V_{DD} - V_{out}) - 2(V_{DD} - V_{in} - |V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD} - V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right] =$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1}) + \frac{1}{R_p}\frac{\partial V_{out}}{\partial V_{in}}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH1}) - \frac{1}{R_p} = \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right] \quad (2)$$

$$\boxed{V_{OH} = 1.01\,V_{IL} + 0.73}$$

Replacing V_{out} in (1) with its equivalent versus V_{IL} obtained from (2) yields:

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{IL}-|V_{TH2}|)(V_{DD}-1.1V_{IL}-0.73)-(V_{DD}-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{IL}-V_{TH1})^2+\frac{1.1V_{IL}+0.73}{R_p}$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\left[2(1.8-V_{IL}-0.5)(1.8-1.1V_{IL}-0.73)-(1.8-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}(V_{IL}-0.4)^2+\frac{1.1V_{IL}+0.73}{2000}$$

$$-52.5\times 10^{-3}V_{IL}-0.6195V_{IL}+0.229875=0$$

$$\boxed{V_{IL}=NM_L=0.36\,V} < V_{TH1} \quad \text{Not Acceptable !}$$

This is less than threshold voltage of M_1; therefore, this answer is not acceptable. It means that M_1 is off and should be left out in this calculation.

$$I_{D1}=0,\quad I_{D2}=\frac{V_{out}}{R_p}$$

$$\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right]=-\frac{1}{R_p}$$

$$50\times 10^{-6}\times\frac{5}{0.18}\times\left[2V_{OH}-V_{IL}-0.5-1.8\right]=-\frac{1}{2000}$$

$$\boxed{V_{OH}=V_{out}=0.5V_{IL}+0.97} \quad (3)$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\times\left[2(1.8-V_{IL}-0.5)(1.8-0.5V_{IL}-0.97)-(1.8-0.5V_{IL}-0.97)^2\right]=$$

$$\frac{0.5V_{IL}+0.97}{2000}$$

$$0.1875V_{IL}^2-0.6225V_{IL}+0.192675=0$$

$$\boxed{V_{IL}=NM_L=0.345\,V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IH})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH1})V_{out} - V_{out}^2\right] + \frac{V_{out}}{R_p} \quad (4)$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH1})\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] + \frac{\partial V_{out}}{\partial V_{in}}\frac{1}{R_p}$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|) = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{out} - V_{in} + V_{TH1} + V_{out}\right] - \frac{1}{R_p}$$

$$-50\times10^{-6}\times\frac{5}{0.18}\times(1.8 - V_{in} - 0.5) = 100\times10^{-6}\times\frac{3}{0.18}\times(2V_{out} - V_{in} + 0.4) - \frac{1}{2000}$$

$$\boxed{V_{out} = \frac{0.1V_{in} + 0.59}{1.2}} \quad (5)$$

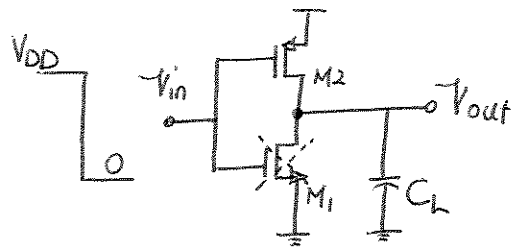
Combining equs (4) and (5) yields:

$$\frac{1}{2}\times50\times10^{-6}\times\frac{5}{0.18}(1.8 - V_{in} - 0.5)^2 = \frac{1}{2}\times100\times10^{-6}\times\frac{3}{0.18}\left[2(V_{in} - 0.4)\frac{0.1V_{in}+0.59}{1.2} - \frac{(0.1V_{in}+0.59)^2}{1.2^2}\right] + \frac{0.1V_{in}+0.59}{1.2\times2000}$$

$$-0.291V_{in}^2 + 1.3182V_{in} - 0.75531 = 0$$

$$V_{in} = V_{IH} = 0.673\text{ V} \longrightarrow \boxed{NM_H = V_{DD} - V_{IH} = 1.127\text{ V}}$$

33.



$V_{out}(t=0)=0$

$(W/L)_2 = 6/0.18$

$C_L = 50fF$

$0 < V_{out} < |V_{TH2}|$; M_2 in the saturation

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 = \frac{1}{2}\times 50\times 10^{-6}\times \frac{6}{0.18}(1.8-0.5)^2$$

$$= 1.4\times 10^{-3} A$$

$$V_{out}(t) = \frac{I_{D2}}{C_L}\times t$$

$$|V_{TH2}| = \frac{I_{D2}}{C_L}\cdot T_1 \longrightarrow T_1 = \frac{C_L \times |V_{TH2}|}{I_{D2}} = 50\times 10^{-15}\times (1.4\times 10^{-3})^{-1}\times 0.5$$

$T_1 = 17.75\ pS$

$|V_{TH2}| < V_{out} < V_{DD}/2$, M_2 in Triode

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(-V_{out}+V_{DD}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{1}{(V_{DD}-V_{out})\left[2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})\right]} = \frac{1}{2(V_{DD}-|V_{TH2}|)}\left[\frac{1}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})} + \frac{1}{V_{DD}-V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-|V_{TH2}|)}\left[\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}+\frac{dV_{out}}{V_{DD}-V_{out}}\right]=\frac{1}{\cancel{2}}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$Ln\,\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t+C$$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=K.\exp\left[\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t\right]$$

Time origin is assumed to be at $t=T_1=17.75\,pS$

$V_{out}(t=0)=|V_{TH2}| \longrightarrow K=1$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=e^{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out}=V_{DD}/2$ $\quad T_2=\dfrac{Ln(3-4|V_{TH2}|/V_{DD})}{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)}$

$$=\frac{Ln(3-4\times0.5/1.8)}{50\times10^{-6}\times\frac{1}{50\times10^{-15}}\times\frac{6}{0.18}\times(1.8-0.5)}$$

$$\boxed{T_2=1.467\times10^{-11}}$$

$T_{0\to V_{DD}/2}=T_1+T_2=17.75+14.67$

$$\boxed{T_{0\to V_{DD}/2}=32.43\,pS}$$

34. $|V_{TH2}| < V_{out} < 0.95 V_{DD}$ M_2 in Triode

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}} = e^{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out} = 0.95 V_{DD}$, $T_2 = \dfrac{\ln(39-40|V_{TH2}|/V_{DD})}{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)}$

$$= \frac{\ln(39-40\times 0.5/(1.8))}{50\times10^{-6}\times\frac{1}{50\times10^{-15}}\times\frac{6}{0.18}\times(1.8-0.5)}$$

$$\boxed{T_2 = 7.68\times10^{-11}}$$

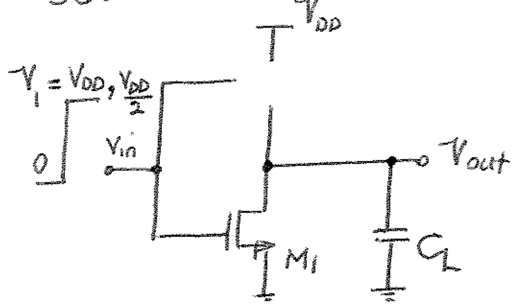
$T_1 = 17.75$ ps from previous problem

$T_{0\to 0.95V_{DD}} = T_1 + T_2 = 17.75 + 76.8$

$$\boxed{T_{0\to 0.95V_{DD}} = 94.55 \text{ ps}}$$

$$\boxed{(T_{0\to 0.95V_{DD}})/(T_{0\to V_{DD}/2}) \approx 3}$$

35.



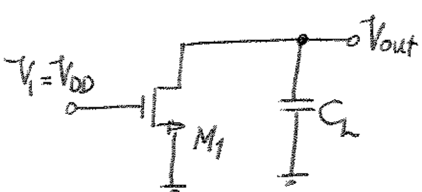
$V_{out}(t=0) = V_{DD}$ $\qquad C_L = 30fF$

$(W/L)_1 = 1/0.18$ $\qquad T_{V_{DD}\to V_{DD}/2} = ?$

$(W/L)_2 =$ Not necessary $\qquad V_1 = V_{DD}$ & $V_{DD}/2$

(a) $V_1 = V_{DD}$

$V_{DD} - V_{TH1} \leqslant V_{out} \leqslant V_{DD}$ $\quad M_1$ saturation



$\frac{V_{DD}}{2} \leqslant V_{out} \leqslant V_{DD} - V_{TH1}$ $\quad M_1$ Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2 =$$
$$5.44 \times 10^{-4} A$$

$$dV_{out} = -\frac{I_{D1}}{C_L} \cdot dt$$

$$V_{out}(t) - V_{DD} = -\frac{I_{D1}}{C_L}t \longrightarrow V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L}t$$

$$T_{V_{DD}\to V_{DD}-V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - V_{TH1})V_{out} - V_{out}^2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$\frac{1}{\left[2(V_{DD} - V_{TH1}) - V_{out}\right]V_{out}} = \frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{1}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{1}{V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD}-V_{TH1})-V_{out}}+\frac{dV_{out}}{V_{out}}\right]=-\frac{1}{\cancel{2}}\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$-Ln\left[2(V_{DD}-V_{TH1})-V_{out}\right]+Ln\,V_{out}=-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t+C$$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=K\cdot\exp\left[-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t\right]$$

$V_{out}(t=0)=V_{DD}-V_{TH1}$ Note that time origin is assumed to be 2.2×10^{-11}

$K=1 \longrightarrow$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t}$$

$$V_{out}=V_{DD}/2 \longrightarrow \frac{V_{DD}/2}{2(V_{DD}-V_{TH1})-V_{DD}/2}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})T_{(V_{DD}-V_{TH1})\to V_{DD}/2}}$$

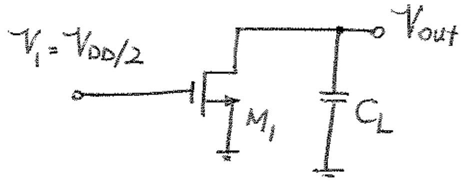
$$T_{(V_{DD}-V_{TH1})\to V_{DD}/2}=\frac{Ln\left(3-\frac{4V_{TH1}}{V_{DD}}\right)}{\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})}$$

$$=\frac{Ln\left(3-4\times0.4/1.8\right)}{100\times10^{-6}\times\frac{1}{30\times10^{-15}}\times\frac{1}{0.18}\times(1.8-0.4)}=2.88\times10^{-11}\,S$$

$$T_{V_{DD}\to V_{DD}/2}=T_{V_{DD}\to V_{DD}-V_{TH1}}+T_{(V_{DD}-V_{TH1})\to V_{DD}/2}$$

$$\boxed{T_{V_{DD}\to V_{DD}/2}=5\times10^{-11}=50.86\ pS}$$

(b) $V_1 = V_{DD}/2$



$V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$V_{DD}/2 < V_{out} < V_{DD}$ M_1 in Saturation

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(0.9-0.4)^2$$

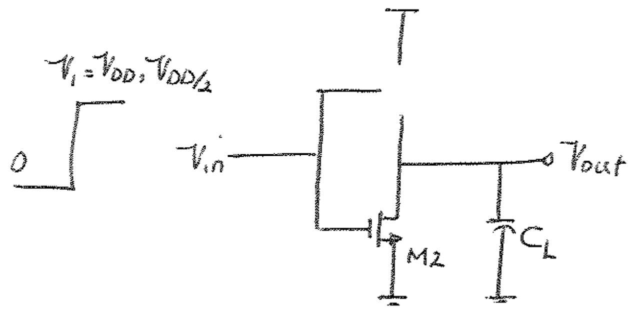
$$= 6.944 \times 10^{-5}$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/2 = V_{DD} - \frac{I_{D1}}{C_L} \times T_{(V_{DD} \rightarrow V_{DD}/2)} \rightarrow T_{(V_{DD} \rightarrow V_{DD}/2)} = \frac{(V_{DD}/2) \times C_L}{I_{D1}}$$

$$\boxed{T_{(V_{DD} \rightarrow V_{DD}/2)} = 3.888 \times 10^{-10}}$$

36.



$V_{out}(0) = V_{DD}$

$\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}$

$C_L = 30fF$

$T_{V_{DD} \longrightarrow 0.05V_{DD}} = ?$

(a) $V_1 = V_{DD}$ $\qquad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ M_1 in saturation

$0.05V_{DD} < V_{out} < V_{DD} - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2$$

$$= 5.44 \times 10^{-4} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{dV_{out}}{V_{out}}\right] = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$V_{out}(t = 0) = V_{DD} - V_{TH1}$ Note that time origin is assumed to be $2.2 \times 10^{-11} S$

$$\frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) t}$$

$V_{out} = 0.05 V_{DD}$

$$\frac{0.05 V_{DD}}{2(V_{DD} - V_{TH1}) - 0.05 V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) T_{(V_{DD} - V_{TH1}) \to 0.05 V_{DD}}}$$

$$T_{(V_{DD} - V_{TH1}) \to 0.05 V_{DD}} = \frac{\ln(39 - 40 V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}$$

$$= \frac{\ln(39 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18} (1.8 - 0.4)}$$

$$T_{(V_{DD} - V_{TH1}) \to 0.05 V_{DD}} = 131.33\ pS$$

$$T_{(V_{DD} \to 0.05 V_{DD})} = T_{(V_{DD} \to V_{DD} - V_{TH1})} - T_{(V_{DD} - V_{TH1} \to 0.05 V_{DD})}$$

$$= 2.2 \times 10^{-11} + 1.3133 \times 10^{-10}$$

$$\boxed{T_{(V_{DD} \to 0.05 V_{DD})} = 153.33\ pS}$$

(b) $V_1 = V_{DD}/2$ $\quad V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$0.05 V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18} (0.9 - 0.4)^2$$

$$= 6.944 \times 10^{-5} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/2 - V_{TH1} = V_{DD} - \frac{I_{D1}}{C_L} T_{(V_{DD} \to V_{DD}/2 - V_{TH1})}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = \frac{(V_{DD}/2 + V_{TH1}) \times C_L}{I_{D1}}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = 5.616 \times 10^{-10}$$

<u>for $0.05V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$</u>

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}/2 - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{V_{out}}{2(V_{DD}/2 - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1})t}$$

$$V_{out} = 0.05V_{DD} \to \frac{0.05V_{DD}}{2(V_{DD}/2 - V_{TH1}) - 0.05V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1}) \times T}$$

$$T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})} = \frac{Ln(19 - 40V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1})}$$

$$= \frac{Ln(19 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18}(0.9 - 0.4)}$$

$$= 2.5 \times 10^{-10}$$

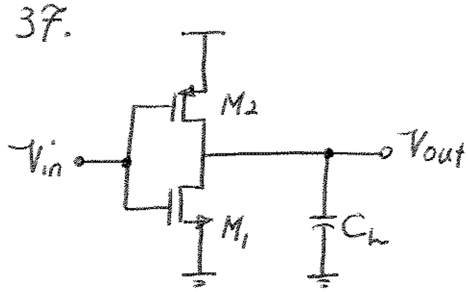
$$T_{(V_{DD} \to 0.05V_{DD})} = T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} + T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})}$$

$$\boxed{T_{(V_{DD} \to 0.05V_{DD})} = 5.616 \times 10^{-10} + 2.5 \times 10^{-10} = 811.5\,ps}$$

By decreasing V_{in} from V_{DD} to $V_{DD}/2$, the time it takes the output to reach $0.05V_{DD}$ will be 5.3 time larger!

$$\frac{T(V_{DD} \rightarrow 0.05V_{DD})(V_{in} = V_{DD})}{T(V_{DD} \rightarrow 0.05V_{DD})(V_{in} = V_{DD}/2)} = \frac{811.5p}{153.33p} \approx 5.3$$

37.



$$\left(\frac{W}{L}\right)_1 = 1/0.18$$

$$\left(\frac{W}{L}\right)_2 = 3/0.18$$

$$C_L = 80fF$$

$$T_{PHL},\ T_{PLH} = ?$$

To calculate T_{PLH}

$0 < V_{out} < |V_{TH2}|$ M_2 in Saturation

$|V_{TH2}| < V_{out} < V_{DD}/2$ M_2 in Triode

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2 t.$$

$$V_{out}(T_{PLH1}) = |V_{TH2}|$$

$$T_{PLH1} = \frac{|V_{TH2}| \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2}.$$

for M_2 operating in Triode region

$$C_L \frac{dV_{out}}{dt} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2\left(V_{DD} - |V_{TH2}|\right)\left(V_{DD} - V_{out}\right) - \left(V_{DD} - V_{out}\right)^2\right]$$

$$\frac{dV_{out}}{2\left(V_{DD} - |V_{TH2}|\right)\left(V_{DD} - V_{out}\right) - \left(V_{DD} - V_{out}\right)^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt.$$

Defining $V_{DD} - V_{out} = u$ and noting that $\int \frac{du}{au - u^2} = \frac{1}{a} \ln \frac{u}{a-u}$,

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}} \Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_p \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$T_{PLH} = \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + \ln\left(3 - 4\frac{0.5}{1.8}\right)\right]$$

$$\boxed{T_{PLH} = 1.0377 \times 10^{-10}}$$

To calculate T_{PHL} $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ $\quad M_1$ in Saturation

$V_{DD}/2 < V_{out} < V_{DD} - V_{TH1}$ $\quad M_1$ in Triode

$$T_{PHL1} = \frac{-\Delta V_{out} \times C_L}{I_{D1}} = \frac{V_{TH1} \times C_L}{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

after this point in time.

$$C_L \frac{dV_{out}}{dt} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2\right]$$

$$V_{out}(t=0) = V_{DD} - V_{TH1}$$

$$\frac{1}{2(V_{DD}-V_{TH1})} Ln \frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}} \Bigg|_{V_{out}=V_{DD}-V_{TH1}}^{V_{out}=V_{DD}/2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 T_{HL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$T_{PHL} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}(1.8-0.4)} \times \left[\frac{2\times0.4}{1.8-0.4} + Ln\left(3-4\frac{0.4}{1.8}\right)\right]$$

$$\boxed{T_{PHL} = 1.3563\times10^{-10}}$$

38. $V_{DD} = 1.8 + 1.8 \times 0.1 = 1.98$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)}\left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} \times (1.98 - 0.5)} \times \left[\frac{2 \times 0.5}{1.98 - 0.5} + \ln\left(3 - 4 \times \frac{0.5}{1.98}\right)\right]$$

$$\boxed{T_{PLH} = 8.846 \times 10^{-11}}$$

$$\text{Decrease in } T_{PLH} = \left|\frac{8.846 \times 10^{-11} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 14.75\%$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (1.98 - 0.4)}\left[\frac{2 \times 0.4}{1.98 - 0.4} + \ln\left(3 - 4\frac{0.4}{1.98}\right)\right]$$

$$\boxed{T_{PHL} = 1.1767 \times 10^{-10}}$$

$$\text{Decrease in } T_{PHL} = \left|\frac{1.1767 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 13.24\%$$

39. $V_{DD} = 0.9\,V$

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$T_{PLH} = \frac{\Delta V_{out} \times C_L}{I_{D2}} = \frac{(V_{DD}/2) \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}$$

$$= \frac{0.45 \times 80 \times 10^{-15}}{\frac{1}{2} \times 50 \times 10^{-6} \times \frac{3}{0.18} \times (0.9 - 0.5)^2}$$

$$\boxed{T_{PLH} = 5.4 \times 10^{-10} = 540\ pS}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (0.9 - 0.4)} \times \left[\frac{2 \times 0.4}{0.9 - 0.4} + Ln\left(3 - 4\frac{0.4}{0.9}\right)\right]$$

$$\boxed{T_{PHL} = 5.186 \times 10^{-10} = 518.6\ pS}$$

$$\text{Increase in } T_{PLH} = \left|\frac{5.4 \times 10^{-10} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 420.38\,\%$$

$$\text{Increase in } T_{PHL} = \left|\frac{5.186 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 282.36\,\%$$

40. $\tau_{PLH} = \tau_{PHL} = 80\ pS$

$C_L = 50\ fF$

$\left(W/L\right)_1\ ,\ \left(W/L\right)_2 = ?$

$$\tau_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)}\left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{50\times10^{-6}\times(1.8-0.5)\times\left(\frac{W}{L}\right)_2}\times\left[\frac{2\times0.5}{1.8-0.5} + \ln\left(3-4\times\frac{0.5}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{2.4}{0.18}}$$

$$\tau_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD} - V_{TH1}\right)}\times\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{100\times10^{-6}\times(1.8-0.4)\times\left(\frac{W}{L}\right)_1}\times\left[\frac{2\times0.4}{1.8-0.4} + \ln\left(3-4\times\frac{0.4}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}}$$

41.

$$\tau_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$V_{TH1} = 0.4$

$$\frac{2V_{TH1}}{V_{DD} - V_{TH1}} = Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1 + \frac{2}{Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 1.57}$

$$\frac{2V_{TH1}}{V_{DD} - V_{TH1}} = 0.1 \times Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1 + \frac{20}{Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 8.16}$

42. $\left(\frac{W}{L}\right)_1 = 1/0.18$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$T_{PHL} = 100\,pS$

$C_L = 80\,fF$

$V_{DD} = ?$

$$100\times10^{-12} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}\times(V_{DD} - 0.4)}\times\left[\frac{2\times0.4}{V_{DD} - 0.4} + \ln\left(3 - 4\frac{0.4}{V_{DD}}\right)\right]$$

$$V_{DD} = 0.4 + 1.44\left[\frac{0.8}{V_{DD} - 0.4} + \ln\left(3 - \frac{1.6}{V_{DD}}\right)\right]$$

$$\boxed{V_{DD} = 2.22}$$

43. $T_{PHL} = 120\ ps$ $(W/L)_1 = ?$

$C_L = 90\ fF$ $V_{TH1} = ?$

$V_{DD} = 1.8$

$T_{PHL} = 160\ ps$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$V_{DD} = 1.5\ V$

$C_L = 90\ fF$

$$120 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6}\left(\frac{W}{L}\right)_1 (1.8 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)\right] \quad (1)$$

$$160 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6}\left(\frac{W}{L}\right)_1 (1.5 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] \quad (2)$$

Dividing Equations (1) and (2) yields:

$$0.75 = \frac{1.5 - V_{TH1}}{1.8 - V_{TH1}} \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

$$V_{TH1} = 1.8 - \left(\frac{1.5 - V_{TH1}}{0.75}\right) \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

This equation does not lead to a real Value for V_{TH1} so we use another derivation

$$V_{TH1} = 0.45 \times \left\{3 - e^{\left[0.75\frac{1.8 - V_{TH1}}{1.5 - V_{TH1}} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] - \frac{2V_{TH1}}{1.8 - V_{TH1}}\right]}\right\}$$

$$V_{TH1} = 0.39$$

$$\left(\frac{W}{L}\right)_1 = \frac{1.26}{0.18}$$

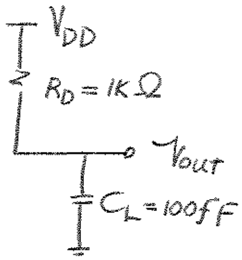
44.

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + \ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$\ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$ is meaningless if $V_{DD} < 4V_{TH1}/3$.

Let's consider the case where $V_{DD} = \frac{4}{3}V_{TH1}$; then, T_{PHL} is the time it takes for the output to drop from $V_{DD} = \frac{4}{3}V_{TH1}$ to $\frac{V_{DD}}{2} = \frac{2}{3}V_{TH1}$. However, $\left(V_{in}^{0} = V_{DD} = \frac{4}{3}V_{TH1}\right) - \left(V_{out} = \frac{2}{3}V_{TH1}\right) = \frac{2}{3}V_{TH1} < V_{TH1}$. In other words, M_1 never enters the triode region in the region where T_{PHL} is calculated. The logrithmic term is derived from equation in which M_1 was assumed to be in Triode region. Therefore the logarithmic term is meaningless for $V_{DD} < \frac{4}{3}V_{TH1}$.

45.



$$V_{R_D} = (V_{DD} - V_{out})$$

$$I_{R_D} = C_L \frac{dV_{out}}{dt}$$

$$P_{R_D}(t) = V_{R_D} \cdot I_{R_D} = C_L (V_{DD} - V_{out}) \frac{dV_{out}}{dt}$$

$$E_{R_D} = \int_{t=0}^{\infty} P_{R_D}(t)\, dt = \int_{V_{out}=0}^{V_{DD}} (V_{DD} - V_{out})\, dV_{out} = \frac{1}{2} C_L V_{DD}^2$$

$$= \frac{1}{2} \times 100 \times 10^{-15} \times (1.8)^2$$

$$\boxed{E_{R_D} = 0.162\ pJ}$$

46. 10^6 Gates

$f = 2\,GHz$

20% of gates switch in every clock cycle

$C_L = 20fF$ for each gate

$P_{av} = ?$

$$P_{av,\,gate} = f_{in} C_L V_{DD}^2$$

$$P_{av,\,total} = 0.2 \times 10^6 \times f_{in} C_L V_{DD}^2$$

$$= 0.2 \times 10^6 \times 2 \times 10^9 \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av,\,total} = 25.92\ W}$$

47. $f = 2\,GHz$

5×10^{6} Transistors with $W = 1\,\mu m$, $L = 0.18\,\mu m$, $C_{ox} = 10\,fF/\mu m^2$

$C_{gate} = WLC_{ox}$

$C_{Load} = 5\times10^{6}\,C_{gate}$

$= 5\times10^{6} \times WLC_{ox}$

$= 5\times10^{6} \times 1\,\mu m \times 0.18\,\mu m \times 10\,fF/\mu m^2$

$C_{Load} = 9\,pF$

$P_{av} = f_{in} C_L V_{DD}^2$

$= 2\times10^{9} \times 9\times10^{-9} \times (1.8)^2$

$P_{av} = 58.32\,W$

48.

$$V_{DD} = V_{DD} + 0.1\,V_{DD} = 1.98$$

$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{4}{0.18}$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(\frac{V_{DD}}{2} - V_{TH1}\right)^2\left(1 + \lambda_1\frac{V_{DD}}{2}\right)$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.9 - 0.4\right)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = 1.388 \times 10^{-4}$$

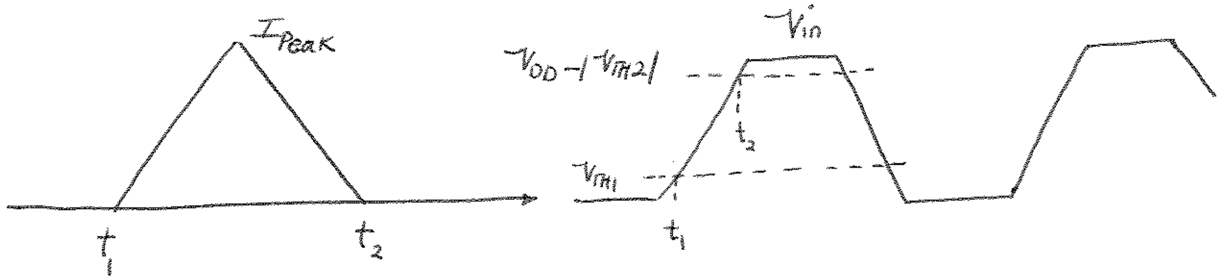
$$I_{Peak}\Big|_{V_{DD}=1.98} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.99 - 0.4\right)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.98} = 1.9338 \times 10^{-4}$$

$$\text{Change in Crowbar Current} = \frac{1.9338 \times 10^{-4} - 1.388 \times 10^{-4}}{1.388 \times 10^{-4}}$$

Change in Crowbar Current = 39.24%

49.



Total Energy drawn from V_{DD} during the interval $[t_1, t_2]$ is:

$$E = V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

In a periode the total energy is:

$$E_{tot} = 2 \times V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

$$P_{av} = V_{DD} I_{Peak} (t_2 - t_1) f_1$$

$$\text{Slope of input voltage} = \frac{0.9V_{DD} - 0.1V_{DD}}{t_r} = \frac{0.8V_{DD}}{t_r}$$

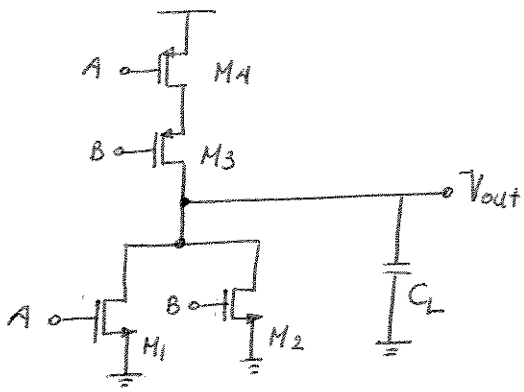
$$(t_2 - t_1) = \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} \times t_r$$

$$P_{av} = V_{DD} \times \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 \times \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} t_r \times f_1$$

$$P_{av} = \frac{1}{1.6} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 (V_{DD} - V_{TH1} - |V_{TH2}|) f_1 \cdot t_r$$

$$\boxed{P_{av} = 1.4 \times 10^{-5} \left(\frac{W}{L}\right)_1 \times t_r \times f_1}$$

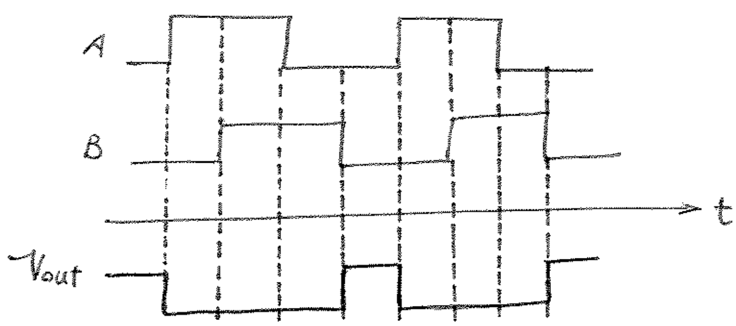
50.



$C_L = 20\,fF$

$f_1 = 500\,MHz$

$P_{av} = ?$

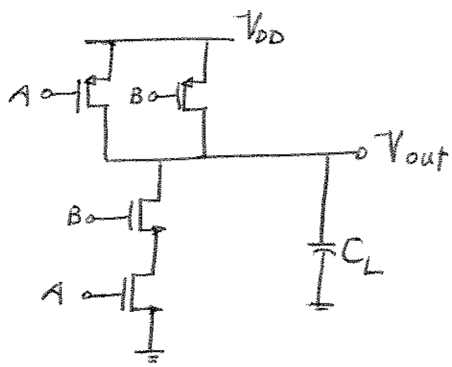


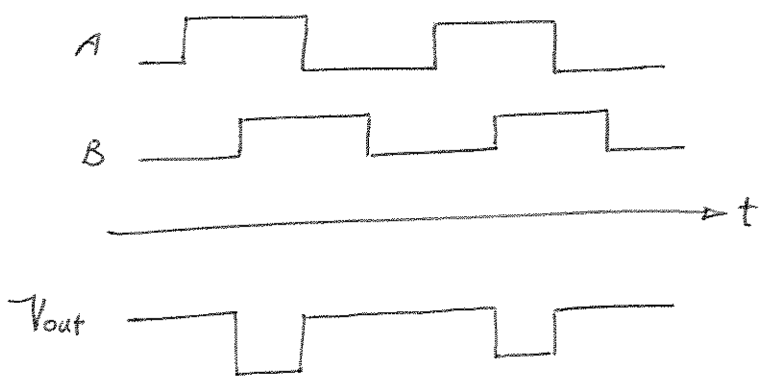
$$P_{av} = f_{in} C_L V_{DD}^2$$

$$= 500\times10^{6} \times 20\times10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

51.



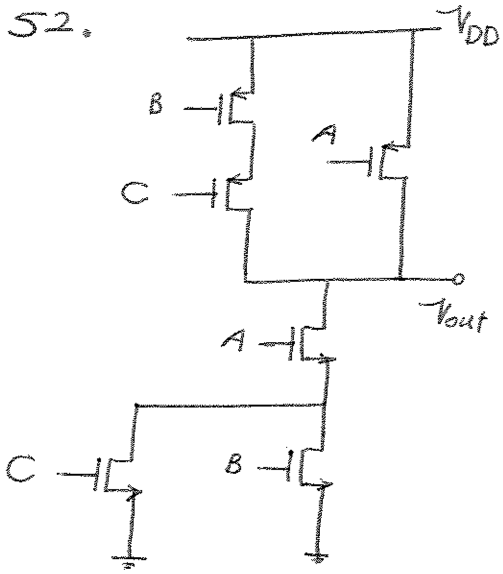


$$P_{av} = f_{in} C_L V_{DD}^2$$

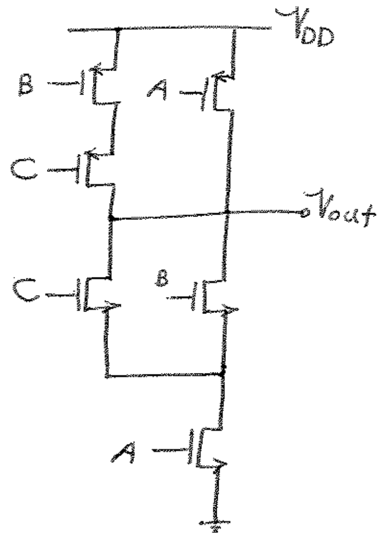
$$= 500 \times 10^{6} \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24 \times 10^{-5}\,W}$$

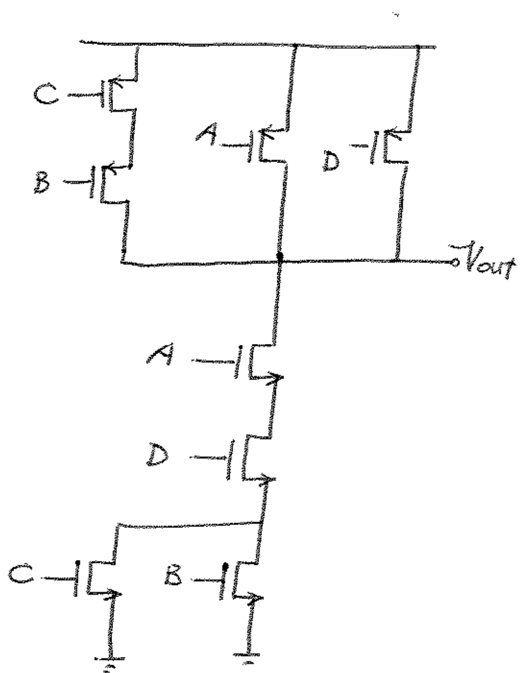
52.



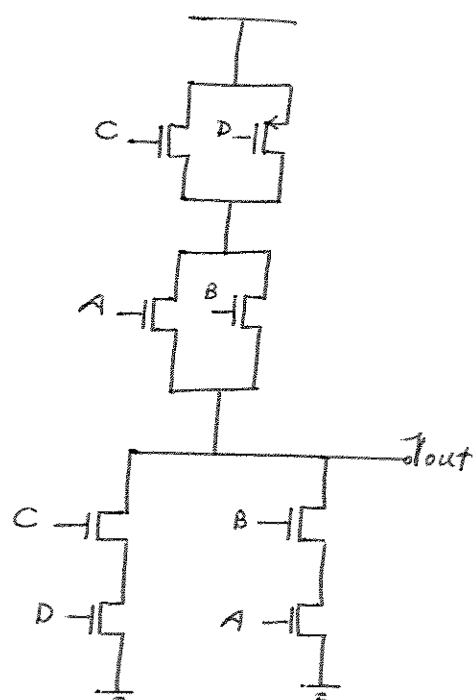
$V_{out} = \overline{(B+C)A}$



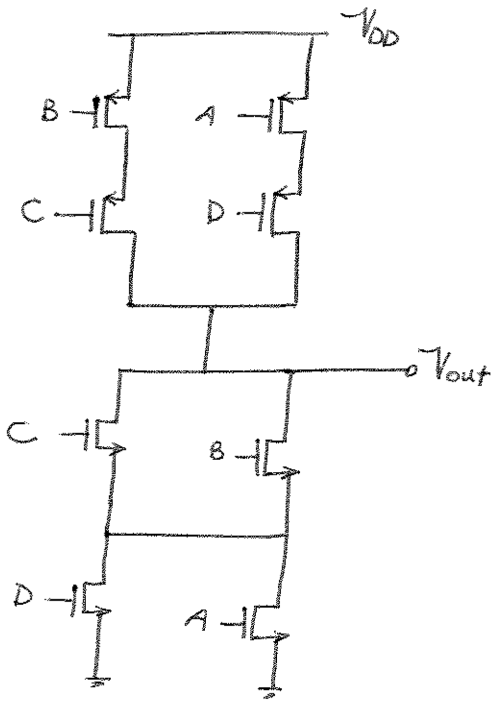
$V_{out} = \overline{(B+C)\cdot A}$



$V_{out} = \overline{(B+C)\,D\cdot A}$

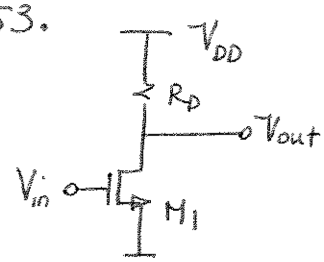


$V_{out} = \overline{A\cdot B + C\cdot D}$



$$V_{out} = \overline{(A+D)\cdot(B+C)}$$

53.



$P_{static} = 0.5\,mW$

$V_{OL} = 100\,mV$

$$\frac{(V_{DD} - V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD} - V_{OL}}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{(1.8 - 0.1)^2}{R_D} + 0.1 \times \frac{1.8 - 0.1}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{1}{R_D} \times 3.06 = 0.5 \times 10^{-3}$$

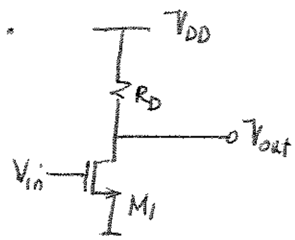
$$\boxed{R_D = 6120\,\Omega}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times \left[2(1.8 - 0.4)0.1 - 0.1^2\right] = \frac{1.8 - 0.1}{6120}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{3.7}{0.18}}$$

54.



$P_{Static} = 0.25\ mW$

$NM_L = 600\ mV$

Small signal gain $= -g_m R_D$

$$g_m = \mu_n C_{ox} (W/L) (V_{GS} - V_{TH})$$

$$\mu_n C_{ox} \frac{W}{L} (V_{IL} - V_{TH1}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$NM_L = V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} = (NM_L - V_{TH1}) \rightarrow \left(\frac{W}{L}\right) R_D = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) \left[2 (V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2\right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \mu_n C_{ox} \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})} \times \left[2 (V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2\right] = (V_{DD} - V_{OL})$$

$$2 (V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 = 2 (NM_L - V_{TH1}) (V_{DD} - V_{OL})$$

$$V_{OL}^2 - 2 (V_{DD} - V_{TH1}) V_{OL} - 2 (NM_L - V_{TH1}) V_{OL} + 2 (NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 2 (V_{DD} + NM_L - 2 V_{TH1}) V_{OL} + 2 (NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 3.2 V_{OL} + 0.72 = 0$$

$$\boxed{V_{OL} = 0.2435}$$

$$\frac{(V_{DD} - V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD} - V_{OL}}{R_D} = 0.25 \times 10^{-3}$$

$$\frac{(1.8 - 0.24)^2 + 0.24 \times (1.8 - 0.24)}{R_D} = 0.25 \times 10^{-3}$$

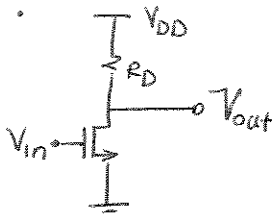
$$\boxed{R_D = 11206.55\ \Omega}$$

$$\left(\frac{W}{L}\right) = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1}) R_D}$$

$$\left(\frac{W}{L}\right) = \frac{1}{100 \times 10^{-6} (0.6 - 0.4)\, 11206.55}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{0.8}{0.18}}$$

55.



$V_{OL} = 100mV$

$P_{av} = 0.25mW$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + \frac{V_{OL}(V_{DD}-V_{OL})}{R_D} = P_{av}$$

$$\frac{(1.8-0.1)^2 + 0.1 \times (1.8-0.1)}{0.25\times 10^{-3}} = R_D$$

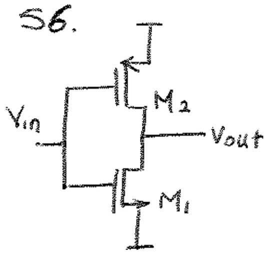
$$\boxed{R_D = 12240}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{DD}-V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\left(\frac{W}{L}\right)\times\left[2(1.8-0.4)\times 0.1 - 0.1^2\right] = \frac{1.8-0.1}{12240}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{1.85}{0.18}}$$

56.



$V_{in} = V_{out} = 0.8V$, $I_{D1} = I_{D2} = 0.5 mA$

$\lambda_n = 0.1 V^{-1}$

$\lambda_p = 0.2 V^{-1}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 (1 + \lambda_N V_{out}) = I_{D1}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.8 - 0.4)^2 (1 + 0.1 \times 0.8) = 0.5 \times 10^{-3}$$

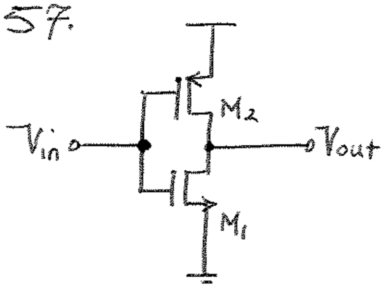
$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{10.4}{0.18}}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_{out})\right] = I_{D2}$$

$$\frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.8 - 0.5)^2 \left[1 + 0.2 \times (1.8 - 0.8)\right] = 0.5 \times 10^{-3}$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{12}{0.18}}$$

57.



$NM_L = NM_H = 0.7\,V$

NM_L : M_1 in Saturation and M_2 in triode

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

Differentiating both sides with respect to V_{in}

$$2\mu_n\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

(a) $V_{in} = V_{IL}$, $\frac{\partial V_{out}}{\partial V_{in}} = -1$

$$\mu_n\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 (2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}) \quad (2)$$

obtaining V_{OH} from (2), substituting in (1), we arrive at

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n\left(\frac{W}{L}\right)_1}{\mu_p\left(\frac{W}{L}\right)_2}$$

NM_H , M_1 in triode and M_2 in Saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)^2$$

Differentiating both sides with respect to V_{in}:

$$\mu_n\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=2\mu_p\left(\frac{W}{L}\right)_2\times\left(V_{in}-V_{DD}-|V_{TH2}|\right)$$

Assuming $\frac{\partial V_{out}}{\partial V_{in}}=-1$, $V_{in}=V_{IH}$, and $V_{out}=V_{OL}$ obtaining

$$V_{IH}=\frac{2a\left(V_{DD}-V_{TH1}-|V_{TH2}|\right)}{(a-1)\sqrt{1+3a}}-\frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$V_{IL}=NM_L=0.7$$

$$V_{IH}=V_{DD}-NM_H=1.8-0.7=1.1$$

$$0.7=\frac{2\sqrt{a}\,(1.8-0.4-0.5)}{(a-1)\sqrt{a+3}}-\frac{1.8-0.4a-0.5}{a-1}$$

$$0.7(a-1)=\frac{1.8\sqrt{a}}{\sqrt{a+3}}-\frac{1.3-0.4a}{1}$$

$$0.7a-0.7+1.3-0.4a=\sqrt{\frac{a}{a+3}}\times 1.8$$

$$\frac{0.6+0.3a}{1.8}=\sqrt{\frac{a}{a+3}}\longrightarrow a^3+7a^2-20a+12=0$$

$$a=\begin{cases}-9.3\\1.3\\1\end{cases}\longrightarrow \boxed{a=1.3}$$

$$1.1 = \frac{2a(1.8-0.4-0.5)}{(a-1)\sqrt{1+3a}} - \frac{1.8-0.4a-0.5}{a-1}$$

$$1.1(a-1) = \frac{1.8a}{\sqrt{1+3a}} - 1.3 + 0.4a$$

$$1.1a - 1.1 + 1.3 - 0.4a = \frac{1.8a}{\sqrt{1+3a}}$$

$$0.2 + 0.7a = \frac{1.8a}{\sqrt{1+3a}}$$

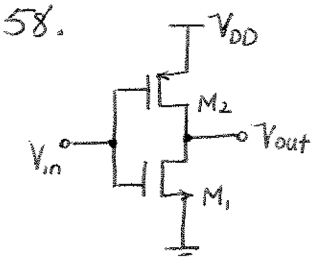
$$147a^3 - 191a^2 + 40a + 4 = 0 \longrightarrow \begin{cases} a_1 = 1 \\ a_2 = 0.37 \\ a_3 = -0.073 \end{cases} \longrightarrow \boxed{a = 0.37}$$

No it is not possible to design a CMOS inverter with $NM_L = NM_H = 0.7$.

The reason is that each value of $a = \frac{\mu_n C_{ox}(W/L)_1}{\mu_p C_{ox}(W/L)_2}$ specifies a unique set of noise margins (NM_L, NM_H).

Remember, the relative strength of NMOS and PMOS determines the noise margins interdependently.

58.



$T_{PLH} = T_{PHL} = 100\,ps$

$C_L = 50\,fF$

T_{PLH}

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$T_{PLH1} = \frac{2|V_{TH2}|C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

$$|I_{D2}| = C_L \frac{dV_{out}}{dt}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] = C_L \frac{dV_{out}}{dt}$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}} \Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2}$$

$$= \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{50 \times 10^{-6}\left(\frac{W}{L}\right)_2 (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + \ln\left(3 - 4\frac{0.5}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{1.9}{0.18}}$$

T_{PHL}

$$T_{PHL1} = \frac{2V_{TH1}C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right] = -C_L \frac{dV_{out}}{dt}$$

$$\frac{-1}{2(V_{DD} - V_{TH1})} \ln \frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} \Bigg|_{V_{out} = V_{DD} - V_{TH1}}^{V_{out} = V_{DD}/2} = \frac{1}{2}\mu_n C_{ox}/C_L \left(\frac{W}{L}\right)_1 T_{PHL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times (1.8 - 0.4)} \times \left[\frac{2 \times 0.4}{1.8 - 0.4} + \ln\left(3 - 4 \times \frac{0.4}{1.8}\right)\right], \quad \boxed{\left(\frac{W}{L}\right)_1 = \frac{0.85}{0.18}}$$