Instructor's Guide to Accompany

Digital Signal Processing: Fundamentals and Applications

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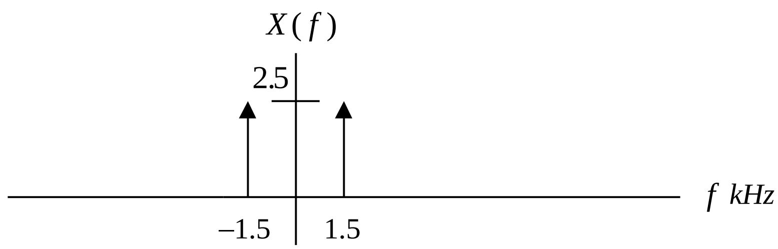
Chapter 2

2.1

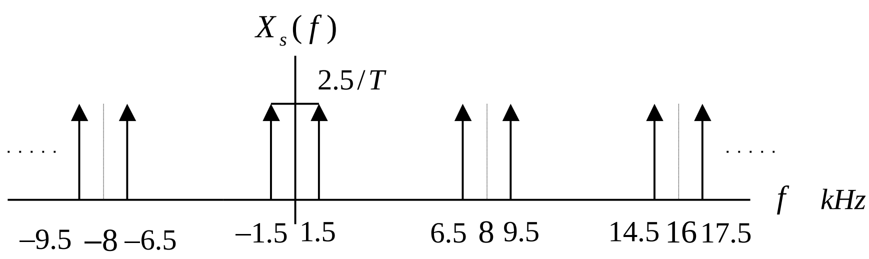
$$5\cos(2\pi\times1000t)=5\cdot\left(\frac{e^{j2\pi\times1500t}+e^{-j2\pi\times1000t}}{2}\right)=2.5e^{j2\pi\times1500t}+2.5e^{-j2\pi\times1500t}\qquad c_1=2.5 \text{ and}$$

$c_{-1}=2.5$

a.



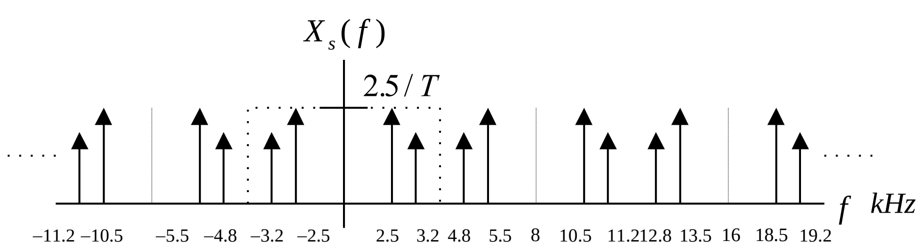
b.



2.2

$$x(t)=e^{-j2\pi\cdot3200t}+2.5e^{-j2\pi\cdot2500t}+2.5e^{j2\pi\cdot2500t}+e^{j2\pi\cdot3200t}$$

a.



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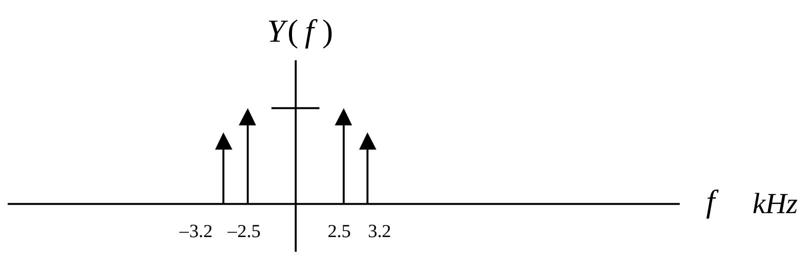
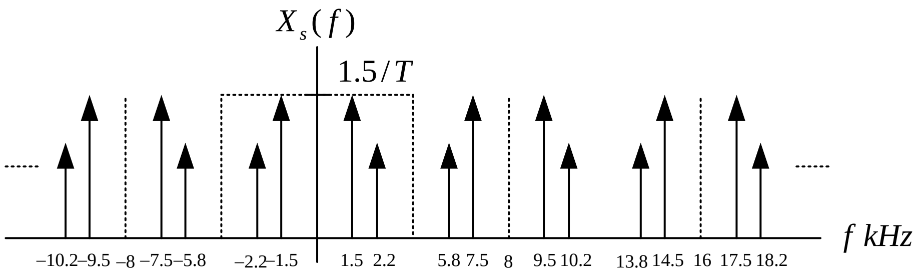
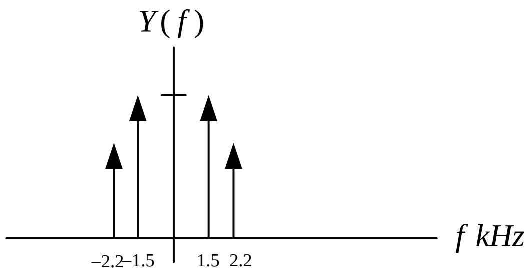


2.3 $x(t) = e^{-j2\pi\cdot 2200t} + 1.5e^{-j2\pi\cdot 1500t} + 1.5e^{j2\pi\cdot 1500t} + e^{j2\pi\cdot 2200t}$

a.

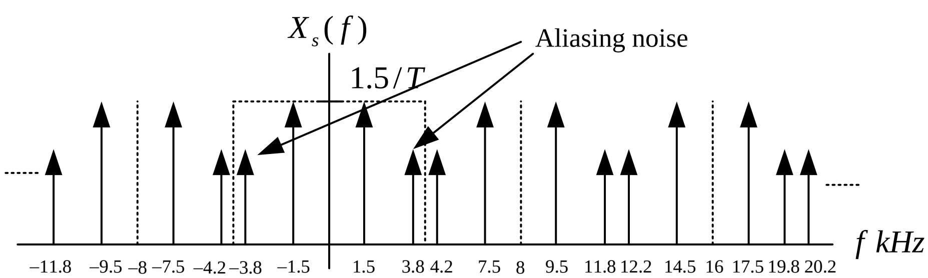


b.

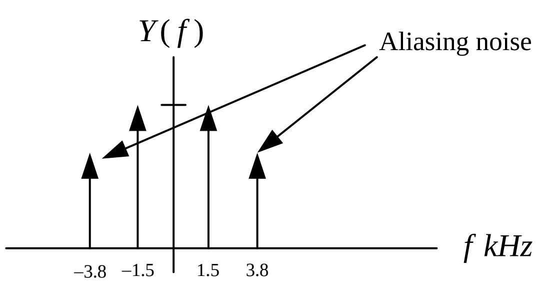


2.4 $x(t) = e^{-j2\pi\cdot 4200t} + 1.5e^{-j2\pi\cdot 1500t} + 1.5e^{j2\pi\cdot 1500t} + e^{j2\pi\cdot 4200t}$

a.



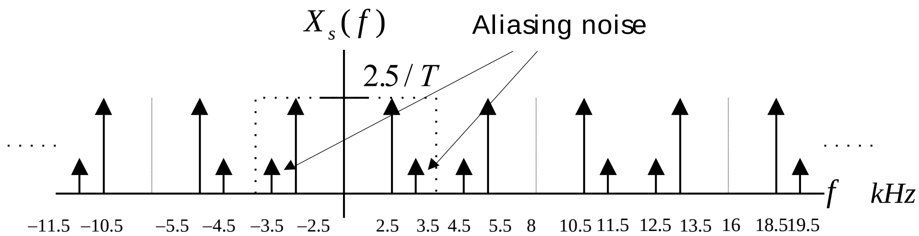
b.



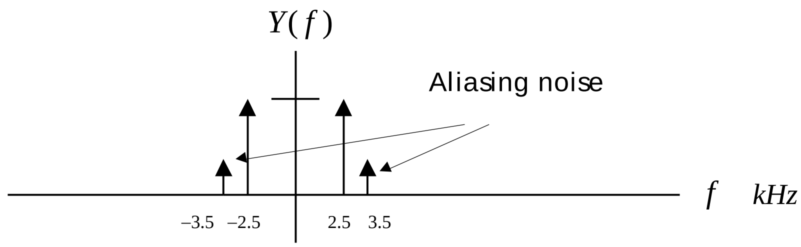
2.5

$$x(t) = e^{-j2\pi \cdot 4500t} + 2.5e^{-j2\pi \cdot 2500t} + 2.5e^{j2\pi \cdot 2500t} + e^{j2\pi \cdot 4500t}$$

a.



b.

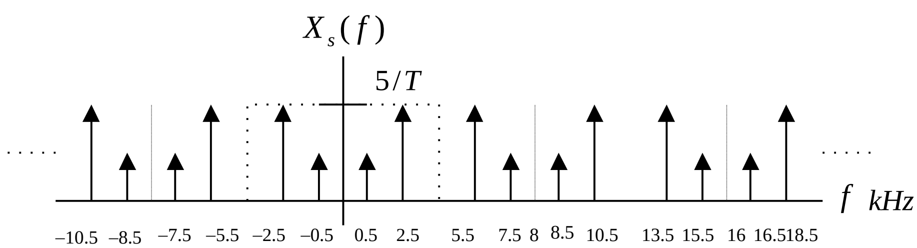


c. The aliasing frequency = 3.5 kHz

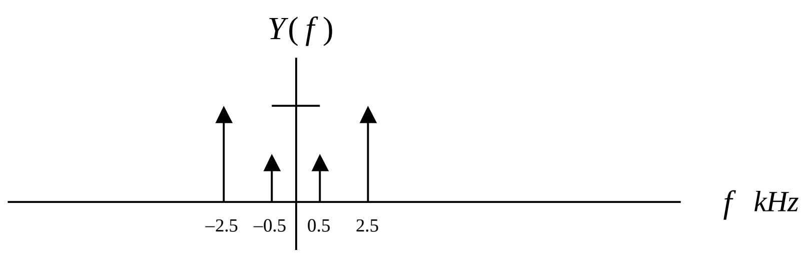
2.6

$$x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 4500t} + 5e^{-j2\pi \cdot 5500t} + 5e^{j2\pi \cdot 5500t} + \frac{2.5}{j} e^{j2\pi \cdot 7500t}$$

a.



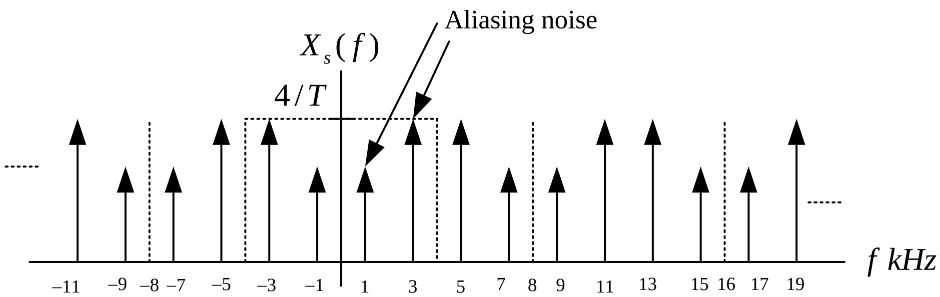
b.



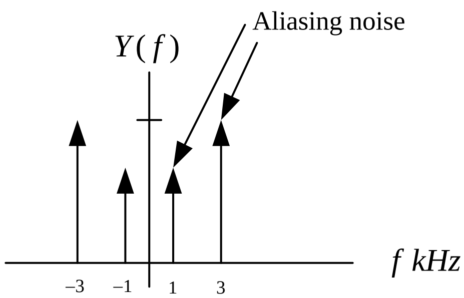
c. The aliasing frequencies: 0.5 kHz and 2.5 kHz.

2.7 $$x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 7000t} + 4e^{-j2\pi \cdot 5000t} + 4e^{j2\pi \cdot 5000t} + \frac{2.5}{j} e^{j2\pi \cdot 7000t}$$

a.



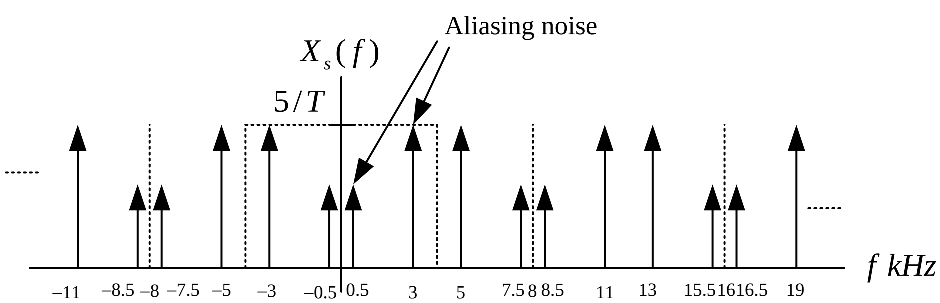
b.



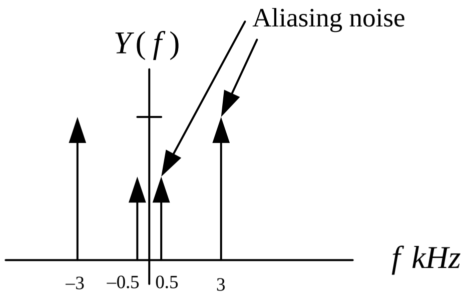
c. The aliasing frequencies: 1 kHz and 3 kHz.

2.8 $$x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 7500t} + 5e^{-j2\pi \cdot 5000t} + 5e^{j2\pi \cdot 5000t} + \frac{2.5}{j} e^{j2\pi \cdot 7500t}$$

a.



b.



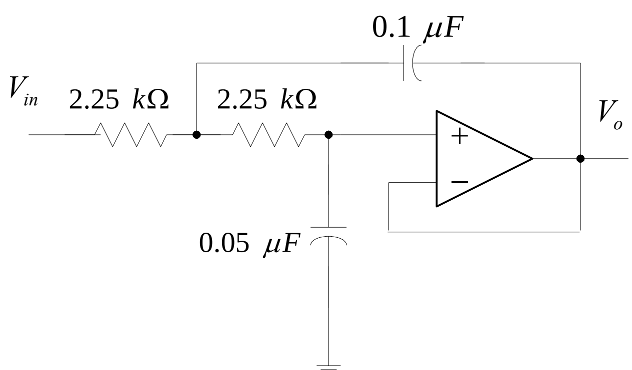
c. The aliasing frequencies: 0.5 kHz and 3 kHz.

2.9

Choose $C_2 = 0.1\ \mu F$

$$R_1 = R_2 = \frac{1.4142}{C_2\left(2\pi f_c\right)} = \frac{1.4142}{0.1\times 10^{-6}\times 2\pi\times 1000} = 2251\ \Omega$$

$$C_1 = \frac{1}{R_1 R_2\left(2\pi f_c\right)^2} = \frac{1}{2251\times 2251\times 0.1\times 10^{-6}\times\left(2\pi\times 1000\right)^2} = 0.05\ \mu F$$



2.10

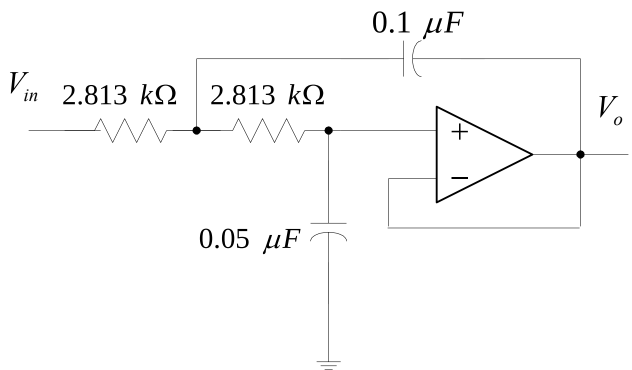
$$\%\ \text{aliasing level} = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{500}{1000}\right)^{4}}}{\sqrt{1+\left(\frac{4000-500}{1000}\right)^{4}}} = 8.39\%$$

2.11

Choose $C_2 = 0.1\ \mu F$

$$R_1 = R_2 = \frac{1.4142}{C_2(2\pi f_c)} = \frac{1.4142}{0.1\times 10^{-6}\times 2\pi\times 800} = 2813\ \Omega$$

$$C_1 = \frac{1}{R_1 R_2 (2\pi f_c)^2} = \frac{1}{2813\times 2813\times 0.1\times 10^{-6}\times(2\pi\times 800)^2} = 0.05\ \mu F$$



2.12

$$\%\ \text{aliasing level} = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{400}{800}\right)^{4}}}{\sqrt{1+\left(\frac{4000-400}{800}\right)^{4}}} = 6.43\%$$

2.13

a. $$\%\ \text{aliasing level} = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{3200}{3200}\right)^{4}}}{\sqrt{1+\left(\frac{8000-3200}{3200}\right)^{4}}} = 57.44\%$$

b. $$\%\ \text{aliasing level} = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{1000}{3200}\right)^{4}}}{\sqrt{1+\left(\frac{8000-1000}{3200}\right)^{4}}} = 20.55\%$$

2.14

When $n = 7$

$$\% \text{ aliasing level } = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{3200}{3200}\right)^{2\times 7}}}{\sqrt{1+\left(\frac{8000-3200}{3200}\right)^{2\times 7}}} = 8.26\%$$

The order of the anti-aliasing filter should be seven (7).

2.15

a. $$\% \text{ aliasing level } = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{3100}{3100}\right)^{4}}}{\sqrt{1+\left(\frac{8000-3100}{3100}\right)^{4}}} = 52.55\%$$

b. $$\% \text{ aliasing level } = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{900}{3100}\right)^{4}}}{\sqrt{1+\left(\frac{8000-900}{3100}\right)^{4}}} = 18.79\%$$

2.16

When $n = 6$

$$\% \text{ aliasing level } = \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1+\left(\frac{3100}{3100}\right)^{2\times 6}}}{\sqrt{1+\left(\frac{8000-3100}{3100}\right)^{2\times 6}}} = 9.05\%$$

The order of the anti-aliasing filter should be six (6).

2.17

a. $fT = 3200/8000 = 0.4$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT}\right) \times 100\% = \left(1 - \frac{\sin(0.4\pi)}{0.4\pi}\right) \times 100\% = 24.32\%$$

b. $fT = 1500/8000 = 0.1875$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT}\right) \times 100\% = \left(1 - \frac{\sin(0.1875\pi)}{0.1875\pi}\right) \times 100\% = 5.68\%$$

2.18

$f = 4000\ \text{Hz},\ \ fT = 4000 \times 1/20000 = 0.2$, and $\ \text{gain} = \dfrac{\sin(0.2\pi)}{0.2\pi} = 0.9355\ (-0.58\ \text{dB})$

$f = 16000\ \text{Hz},\ \ fT = 16000 \times 1/20000 = 0.8$, and $\text{gain} = \dfrac{\sin(0.8\pi)}{0.8\pi} = 0.2339\ (-12.62\ \text{dB})$

Maximum allowable gain variation from 0 to 4000 Hz = 2-0.58=1.42dB
40 -12.62 = 27.38 dB rejection at frequency 16000 Hz.

$$20\log\left(1+\left(4000/f_c\right)^{2n}\right)^{1/2} \le 1.42$$

$$20\log\left(1+\left(16000/f_c\right)^{2n}\right)^{1/2} \ge 27.38$$

Then

$$n = \frac{1}{2}\log\left((10^{2.738}-1)/(10^{0.142}-1)\right)/\log\left(16000/4000\right) = 2.6158 \approx 3$$

$$f_c = \frac{4000}{\left(10^{0.142}-1\right)^{1/6}} = 4686\ \text{Hz}$$

2.19

a. $fT = 3000/8000 = 0.375$

$$\%\ \text{distortion} = \left(1-\frac{\sin(fT\pi)}{\pi fT}\right)\times 100\% = \left(1-\frac{\sin(0.375\pi)}{0.375\pi}\right)\times 100\% = 21.58\%$$

b. $fT = 1600/8000 = 0.2$

$$\%\ \text{distortion} = \left(1-\frac{\sin(fT\pi)}{\pi fT}\right)\times 100\% = \left(1-\frac{\sin(0.2\pi)}{0.2\pi}\right)\times 100\% = 6.45\%$$

2.20

$f = 4000\ \text{Hz},\ \ fT = 4000 \times 1/22000 = 0.18182$, and $\ \text{gain} = \dfrac{\sin(0.18182\pi)}{0.18182\pi} = 0.9465\ (-0.48\ \text{dB})$

$f = 16000\ \text{Hz},\ \ fT = 18000 \times 1/22000 = 0.818182$, and

$\text{gain} = \dfrac{\sin(0.81812\pi)}{0.81812\pi} = 0.2104\ (-13.54\ \text{dB})$

Maximum allowable gain variation from 0 to 4000 Hz = 2-0.48=1.52dB
40 -13.54 = 26.46 dB rejection at frequency 18000 Hz.

$$20\log\left(1+\left(4000/f_c\right)^{2n}\right)^{1/2} \le 1.52$$

$$20\log\left(1+\left(16000/f_c\right)^{2n}\right)^{1/2} \ge 26.46$$

Then

$$n = \frac{1}{2}\log\left((10^{2.646}-1)/(10^{0.152}-1)\right)/\log\left(18000/4000\right) = 2.3138 \approx 3$$

$$f_c = \frac{4000}{\left(10^{0.152}-1\right)^{1/6}} = 4624\ \text{Hz}$$

2.21
b1b0=01

2.22
$$V_0 = V_R\left(\frac{1}{2}b_1 + \frac{1}{4}b_0\right) = 5\times\left(\frac{1}{2}\times 0 + \frac{1}{4}\times 1\right) = 1.25 \text{ Volts}$$

2.23
b1b0=10

2.24
For b1b0=11, $V_0 = V_R\left(\frac{1}{2}b_1 + \frac{1}{4}b_0\right) = 5\times\left(\frac{1}{2}\times 1 + \frac{1}{4}\times 1\right) = 3.75 \text{ Volts}$

For b1b0=10, $V_0 = V_R\left(\frac{1}{2}b_1 + \frac{1}{4}b_0\right) = 5\times\left(\frac{1}{2}\times 1 + \frac{1}{4}\times 0\right) = 2.5 \text{ Volts}$

2.25
a. $L = 2^4 = 16$ levels
b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{16} = 0.3125$
c. $x_q = 0 + 10\times 0.3125 = 3.125$
d. $\frac{3.2}{0.3125}\Delta = 10.24\Delta$, $i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round\left(\frac{10.24\Delta - 0}{\Delta}\right) = round(10.24) = 10$ binary code =1010
e. $e_q = -0.075$

2.26
a. $L = 2^5 = 32$ levels
b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{4}{32} = 0.125$
c. $x_q = 0 + 10\times 0.125 = 1.25$
d. $\frac{1.2}{0.125}\Delta = 9.6\Delta$, $i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round\left(\frac{9.6\Delta - 0}{\Delta}\right) = round(9.6) = 10$ binary code =1010
e. $e_q = 0.05$

2.27

a. $L = 2^3 = 8$ levels

b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{8} = 0.625$

c. $x_q = -2.5 + 2 \times 0.625 = -1.25$

d. $x = \frac{-1.2}{0.625}\Delta = -1.92\Delta$,

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round\left(\frac{-1.92\Delta - (-4\Delta)}{\Delta}\right) = round\left(2.08\right) = 2$$

binary code =010

e. $e_q = -0.05$

2.28

a. $L = 2^8 = 256$ levels

b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{256} = 0.01953125$

c. $x_q = -2.5 + 205 \times 0.01953125 = 1.5039$

d. $x = \frac{1.5}{0.01953125}\Delta = 76.8\Delta$,

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round\left(\frac{76.8\Delta - (-128\Delta)}{\Delta}\right) = round\left(204.8\right) = 205$$

binary code =11001101

e. $e_q = 0.0039$

2.29

a. $L = 2^6 = 64$ levels b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{20}{64} = 0.3125$

c. $SNR_{dB} = 1.76 + 6.02 \times 6 = 37.88$ dB

2.30

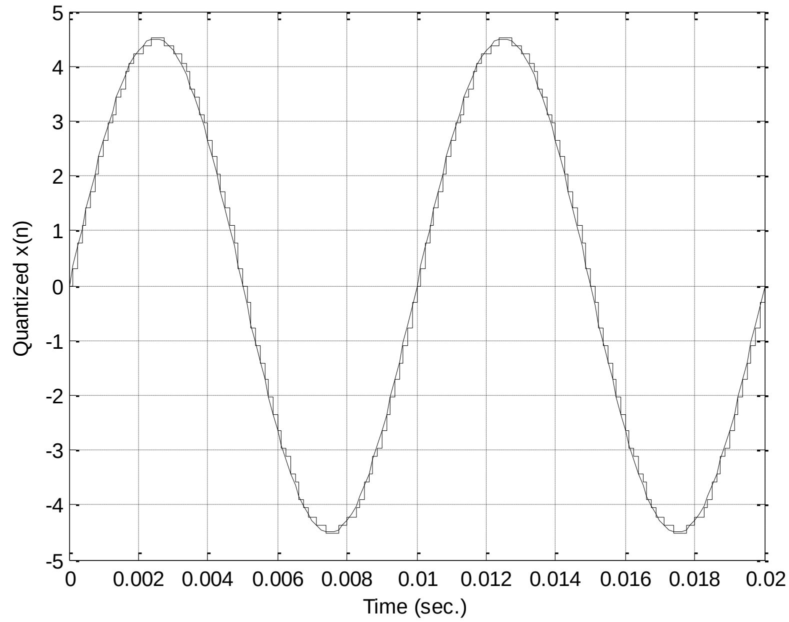
a. $L = 2^6 = 64$ levels b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{64} = 0.078125$

c. $$\begin{aligned} SNR_{dB} &= 4.77 + 20 \times \log\left(x_{rms} / |x|_{\max}\right) + 6.02 \times 6 \\ &= 4.77 + 20 \times \log\left(0.25\right) + 36.12 = 28.85 \end{aligned}$$ dB

2.31

a. See Program 2.1

b.

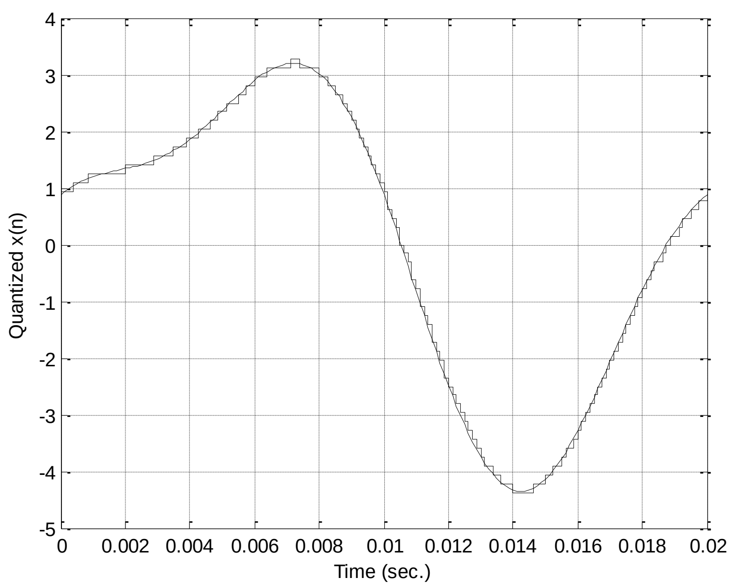


c. SNR = 37 dB

2.32

a. Use Program 2.1

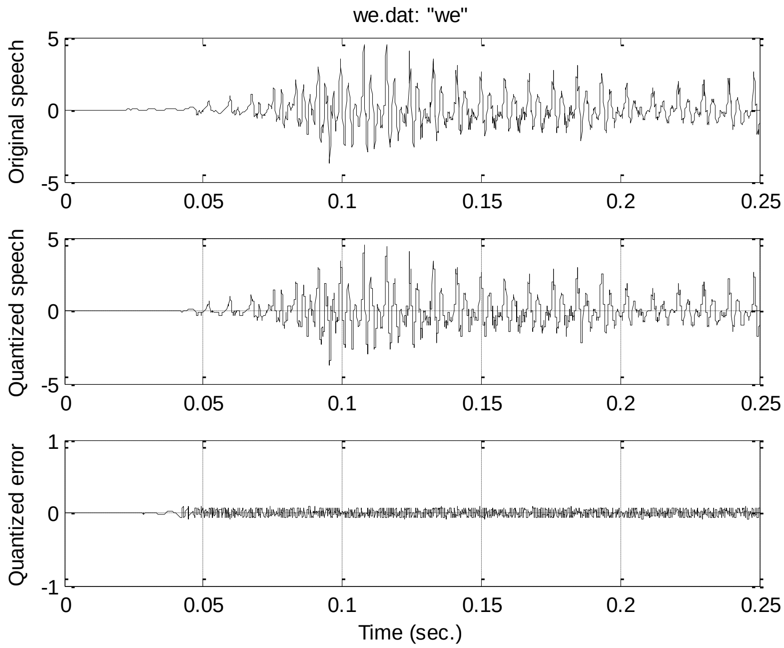
b.



c. SNR = 35 dB

2.33

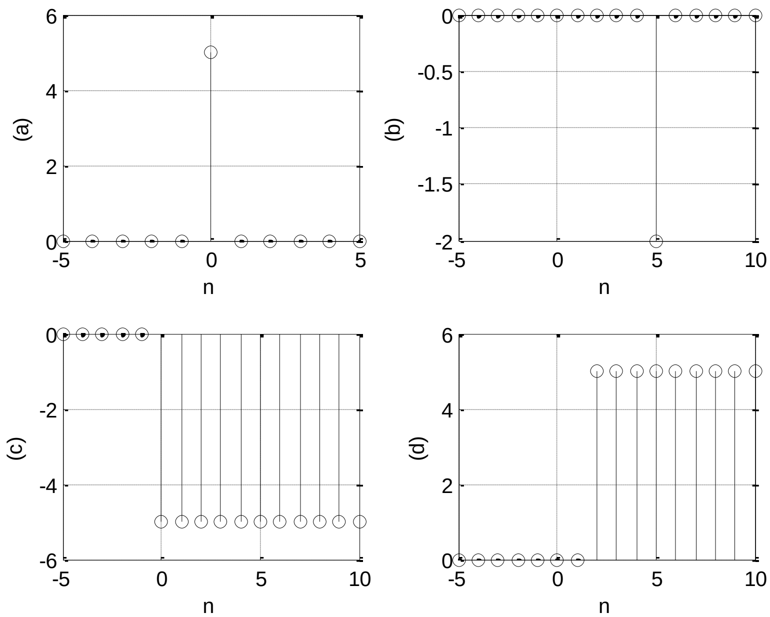
a. See Program 2.2
b.



c. SNR = 26.9 dB

Chapter 3

3.1



3.2

a.

n	0	1	2	3	4	5	6	7
$x(n)$	1.000	0.5000	0.2500	0.1250	0.0625	0.0313	0.0156	0.0078

b.

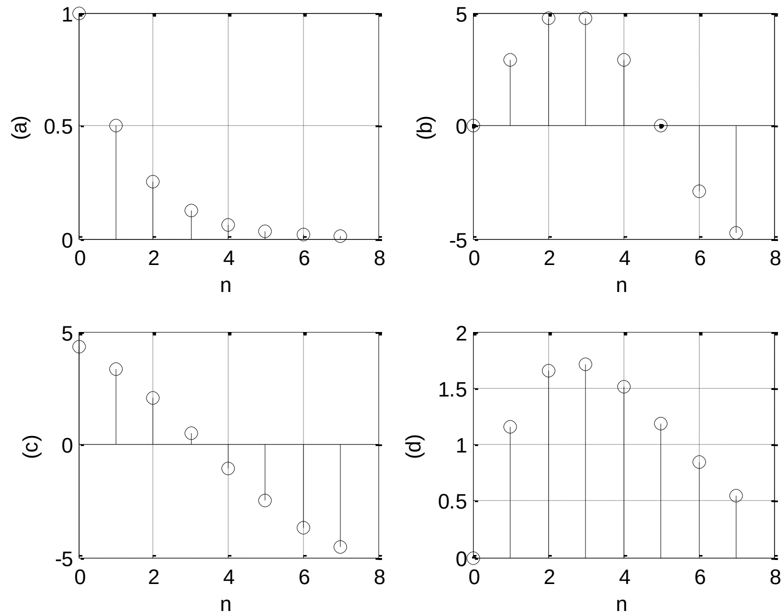
n	0	1	2	3	4	5	6	7
$x(n)$	0.0000	2.9389	4.7553	4.7553	2.9389	0.0000	-2.9389	-4.7553

c.

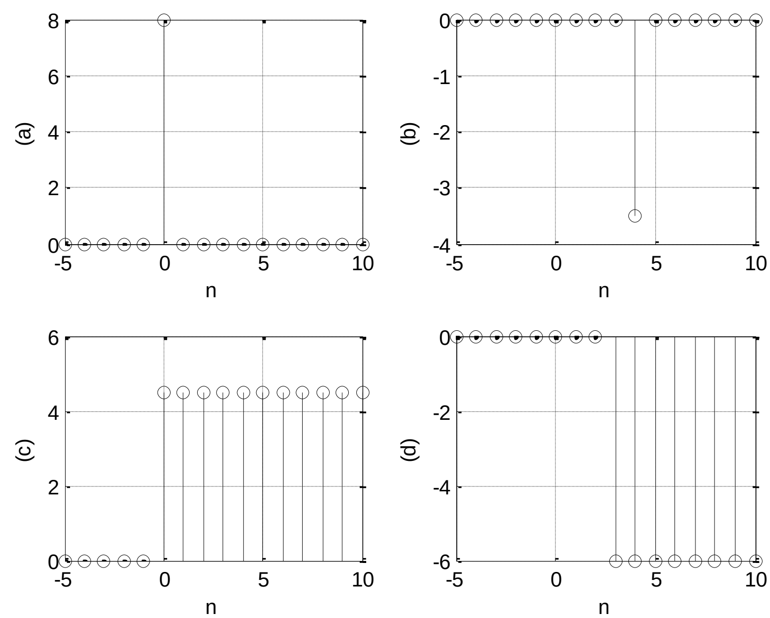
n	0	1	2	3	4	5	6	7
$x(n)$	4.3301	3.3457	2.0377	0.5226	-1.0396	-2.5000	-3.7157	-4.5677

d.

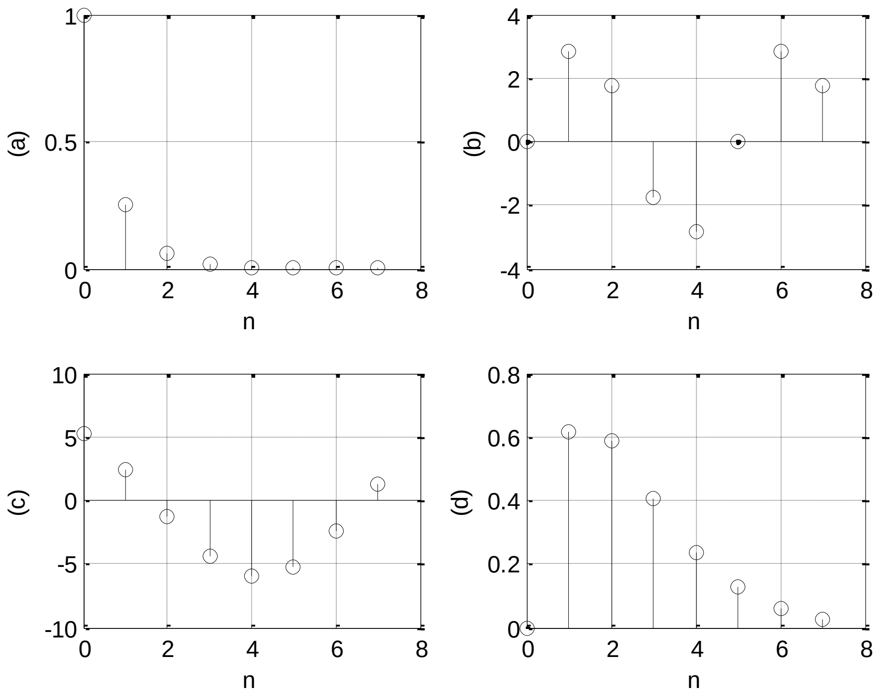
n	0	1	2	3	4	5	6	7
$x(n)$	0.0000	1.1588	1.6531	1.7065	1.5064	1.1865	0.8463	0.5400



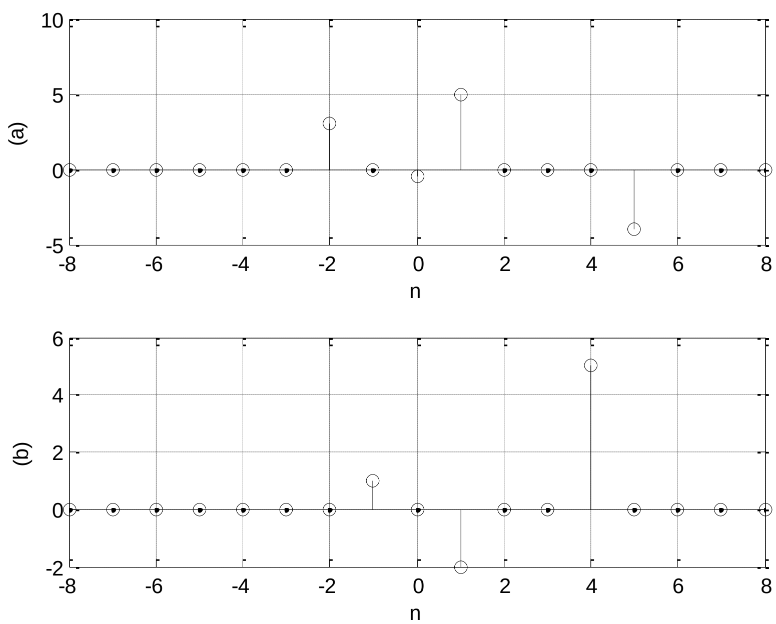
3.3



3.4
xa =[1.0000 0.2500 0.0625 0.0156 0.0039 0.0010 0.0002 0.0001]
xb =[0 2.8532 1.7634 -1.7634 -2.8532 -0.0000 2.8532 1.7634]
xc =[5.1962 2.4404 -1.2475 -4.4589 -5.9671 -5.1962 -2.4404 1.2475]
xd =[0 0.6180 0.5878 0.4045 0.2378 0.1250 0.0594 0.0253]



3.5

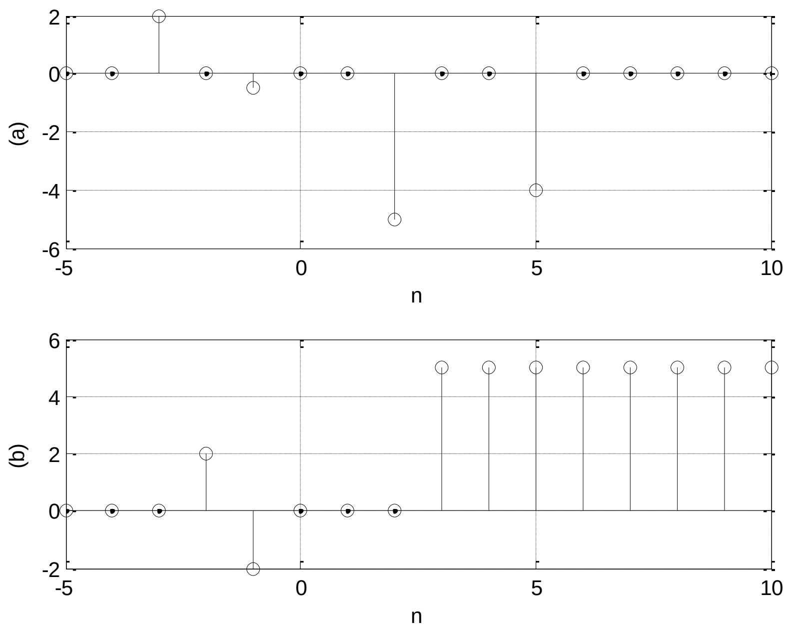


3.6

a. $x(n) = 3\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3) + \delta(n-5)$

b. $x(n) = \delta(n-1) - \delta(n-2) + \delta(n-4) - \delta(n-5)$

3.7



3.8

a. $x(n) = \delta(n+1) + 0.5\delta(n-1) + 3\delta(n) + 2.5\delta(n-2) + \delta(n-3) + 0.5\delta(n-4) + \delta(n-5)$

b. $x(n) = -0.5\delta(n+1) + 0.5\delta(n) + \delta(n-1) - 0.5\delta(n-2) + \delta(n-4) - \delta(n-5)$

3.9

a. $x(n) = e^{-0.5n}u(n) = (0.6065)^n u(n)$

b. $x(n) = 5\sin(0.2\pi n)u(n)$

c. $x(n) = 10\cos(0.4\pi n + \pi/6)u(n)$

d. $x(n) = 10e^{-n}\sin(0.15\pi n)u(n) = 10(0.3679)^n \sin(0.15\pi n)u(n)$

3.10

a. Let $y_1(n) = 5x_1(n) + 2x_1^2(n)$, $y_2(n) = 5x_2(n) + 2x_2^2(n)$

$$y_1(n) + y_2(n) = 5x_1(n) + 2x_1^2(n) + 5x_2(n) + 2x_2^2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 5x(n) + 2x^2(n) = 5\left(x_1(n) + x_2(n)\right) + 2\left(x_1(n) + x_2(n)\right)^2$$

$$= 5x_1(n) + 5x_2(n) + 2x_1^2(n) + 2x_2^2(n) + 4x_1(n)x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

b. Let $y_1(n) = x_1(n-1) + 4x_1(n)$, $y_2(n) = x_2(n-1) + 4x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = x(n-1) + 4x(n) = \left(x_1(n-1) + x_2(n-1)\right) + 4\left(x_1(n) + x_2(n)\right)$$
$$= x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

c. Let $y_1(n) = 4x_1^3(n) - 2x_1(n)$, $y_2(n) = 4x_2^3(n) - 2x_2(n)$

$$y_1(n) + y_2(n) = 4x_1^3(n) - 2x_1(n) + 4x_2^3(n) - 2x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 4x^3(n) - 2x(n) = 4\left(x_1(n) + x_2(n)\right)^3 - 2\left(x_1(n) + x_2(n)\right)$$
$$= 4x_1^3(n) + 8x_1^2(n)x_2(n) + 8x_1(n)x_2^2(n) + 4x_2^3(n) - 2x_1(n) - 2x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

3.11

a. $x(n) = e^{-0.5n}u(n) = (0.6065)^n u(n)$

b. $x(n) = 4\sin(0.3\pi n)u(n)$

c. $x(n) = 7.5\cos(0.1\pi n + \pi/3)u(n)$

d. $x(n) = 20e^{-n}\sin(0.3\pi n)u(n) = 20(0.3679)^n \sin(0.3\pi n)u(n)$

3.12

a. Let $y_1(n) = 4x_1(n) + 8x_1^3(n)$, $y_2(n) = 4x_2(n) + 8x_2^3(n)$

$$y_1(n) + y_2(n) = 4x_1(n) + 8x_1^3(n) + 4x_2(n) + 8x_2^3(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 4x(n) + 8x^3(n) = 4\left(x_1(n) + x_2(n)\right) + 8\left(x_1(n) + x_2(n)\right)^3$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

b. Let $y_1(n) = x_1(n-3) + 3x_1(n)$, $y_2(n) = x_2(n-3) + 3x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-3) + x_2(n-3) + 3x_1(n) + 3x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = x(n-3) + 3x(n) = \left(x_1(n-3) + x_2(n-3)\right) + 3\left(x_1(n) + x_2(n)\right)$$
$$= x_1(n-1) + x_2(n-1) + 3x_1(n) + 3x_2(n)$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

c. Let $y_1(n) = 5x_1^2(n-1) - 3x_1(n)$, $y_2(n) = 5x_2^2(n) - 3x_2(n)$

$$y_1(n) + y_2(n) = 5x_1^2(n-1) - 3x_1(n) + 5x_2^2(n-1) - 3x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 5x^2(n-1) + 3x(n) = 5\left(x_1(n-1) + x_2(n-1)\right)^2 - 3\left(x_1(n) + x_2(n)\right)$$
$$= 5x_1^2(n-1) + 10x_1(n-1)x_2(n-1) + 5x_2^2(n-1) - 3x_1(n) - 3x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

3.13

a. For $x_1(n) = x(n-n_0)$, $y_1(n) = -5x_1(n-10) = -5x(n-10-n_0)$

Since $y(n-n_0) = -5x\left((n-n_0)-10\right) = -5x(n-10-n_0) = y_1(n)$

The system is time invariant.

b. For $x_2(n) = x(n-n_0)$ so that $x_2(n^2) = x(n^2-n_0)$, $y_2(n) = 4x_2(n^2) = 4x_2(n^2-n_0)$

Since shifting $y(n-n_0) = 4x\left((n-n_0)^2\right) = 4x(n^2-2nn_0+n_0^2) \neq y_2(n)$

The system is time invariant.

3.14

a. Since the output is depending on the current input and past inputs, the system is causal.

b. Since the output is depending on the future input $x(n+4)$, the system is a non-causal system.

3.15

a. causal system, since the system output depends on the current input and past inputs.

b. noncausal system, since the system output depends on a future input.

c. causal system, since the equation can be rewritten as

$$y(n) = -2x(n-3) + 2y(n-2)$$

The system output is depending on the past input and the past output which is in turn based on the past outputs.

3.16

a. $h(n) = 0.5\delta(n) - 0.5\delta(n-2)$

b. $h(n) = (0.75)^n$; $n \geq 0$

c. $h(n) = 1.25\delta(n) - 1.25(-0.8)^n$; $n \geq 0$

3.17

a. causal system, since the system output depends on the current input and past input and output.

b. noncausal system, since the system output depends on a future input $x(n+2)$.

c. causal system, since the equation can be rewritten as

$$y(n) = -0.2x(n-1) + 2y(n-2)$$

The system output is depending on the past input and the past output which is in turn based on the past outputs.

3.18

a. $h(n) = 0.2\delta(n) - 0.3\delta(n-2)$

b. $h(n) = 0.5(0.5)^n$; $n \geq 0$

c. $h(n) = -(5/3)\delta(n) + (5/3)(-0.6)^n$; $n \geq 0$

3.19

a. $h(n) = 5\delta(n-10)$

b. $h(n) = \delta(n) + 0.5\delta(n-1)$

3.20

Since $h(n) = 0.5\delta(n) + 100\delta(n-2) - 20\delta(n-10)$

and $S = 0.5 + 100 + 20 = 120.5$ = finite number, the system is stable.

3.21

a. $h(n) = 2.5\delta(n-5)$

b. $h(n) = 2\delta(n) + 1.2\delta(n-1)$

3.22

Since $h(n) = 5\delta(n) + 30\delta(n-3) - 10\delta(n-20)$

and $S = 5 + 30 + 10 = 45$ = finite number, the system is stable.

3.23

a. $h(n) = (0.75)^n u(n)$, $S = \sum_{k=0}^{\infty} (0.75)^k = 1/\left(1 - 0.75\right) = 4$ = finite, the system is stable.

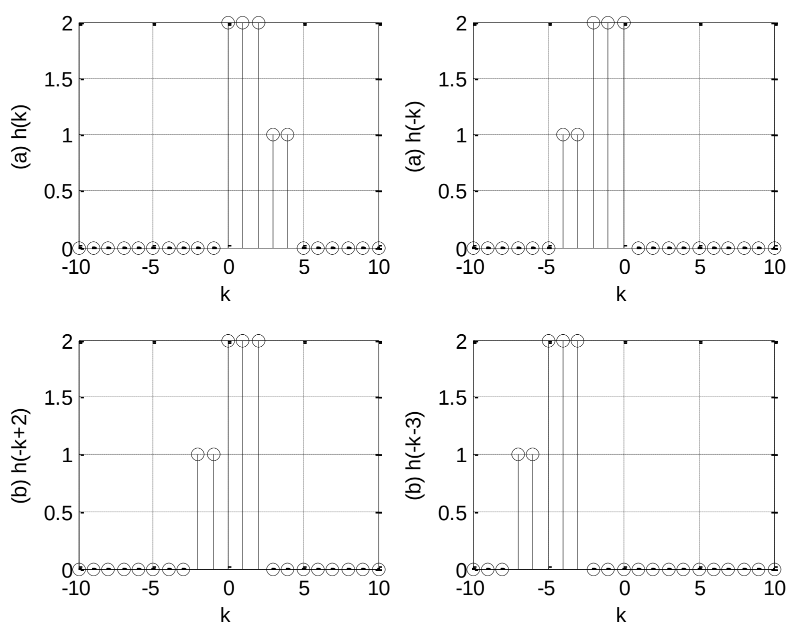
b. $h(n) = (2)^n u(n)$, $S = \sum_{k=0}^{\infty} (2)^k = 1 + 2 + 2^2 + \cdots = \infty$ = infinite, the system is unstable.

3.24

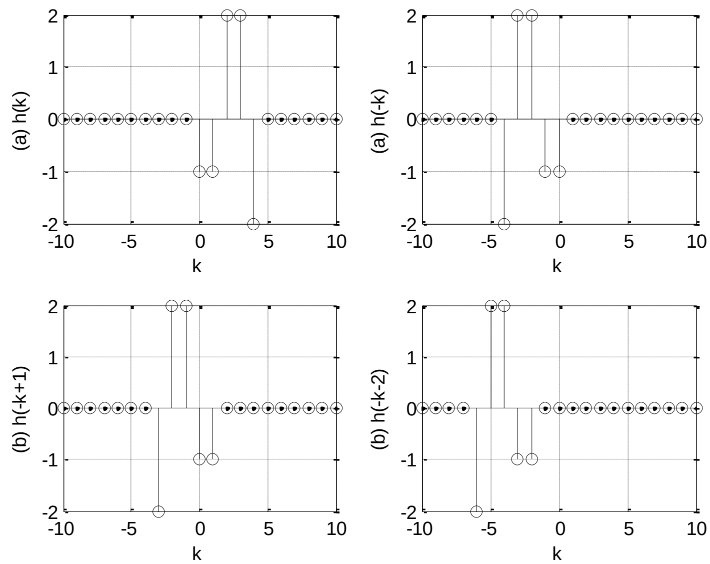
a. $h(n) = (-1.5)^n u(n)$, $S = \sum_{k=0}^{\infty} (-1.5)^k = \lim_{k->\infty} \frac{1-(-1.5)^{k+1}}{1-(-1.5)} = \infty$ = infinite, the system is unstable.

b. $h(n) = (-0.5)^n u(n)$, $S = \sum_{k=0}^{\infty} (-0.5)^k = 1/(1-(-0.5)) = 2/3$ = finite, the system is stable.

3.25



3.26



3.27

$y(0)=4$, $y(1)=6$, $y(2)=8$, $y(3)=6$, $y(4)=5$, $y(5)=2$, $y(6)=1$,

$y(n)=0$ for $n \ge 7$

3.28

$y(0)=-4$, $y(1)=-2$, $y(2)=0$, $y(3)=6$, $y(4)=3$, $y(5)=-2$, $y(6)=-1$

$y(n)=0$ for $n \ge 7$

3.29

$y(0)=0$, $y(1)=1$, $y(2)=2$, $y(3)=1$, $y(4)=0$,

$y(n)=0$ for $n \ge 4$

Chapter 4

4.1
$X(0)=1$, $X(1)=2-j$, $X(2)=-1$, $X(3)=2+j$

4.2

$X(0)=10$, $X(1)=2-2j$, $X(2)=2$, $X(3)=2+2j$

4.3
X =[0.2000 0.4000 - 0.2000i -0.2000 0.4000 + 0.2000i]

4.4
X =[2.0000 0.4000 - 0.4000i 0.4000 0.4000 + 0.4000i]

4.5
From 4.2: $X(0)=10$, $X(1)=2-2j$, $X(2)=2$, $X(3)=2+2j$
$x(0)=4$, $x(1)=3$, $x(2)=2$, $x(3)=1$

4.6
$X(0)=10$, $X(1)=3.5-4.3301j$, $X(2)=2.5-0.8660j$, $X(3)=2$, $X(4)=2.5+0.8660j$,
$X(5)=3.5+4.3301j$

4.7
x =[0.8000 0.6000 0.4000 0.2000]

4.8
X =[2.00 0.70 - 0.866i 0.50 - 0.1732i 0.40 0.50+ 0.1732i 0.70 + 0.866i]

4.9
From 4.4: $X(0)=10$, $X(1)=3.5-4.3301j$, $X(2)=2.5-0.8660j$, $X(3)=2$,
$X(4)=2.5+0.8660j$, $X(5)=3.5+4.3301j$
$\bar{x}(0)=4$, $\bar{x}(4)=0$

4.10
$\Delta f=2.5$ Hz and $f_{\max}=10$ kHz

4.11
$\bar{x}(0)=0.8$, $\bar{x}(4)=0$

4.12
$N=4000$, $\Delta f=4$ Hz, $f_{\max}=f_s/2=8$ kHz

4.13

$N = 4096$, $\Delta f = 0.488$ Hz

4.14

$X(0) = 6$, $X(1) = 2 + 2j$, $X(2) = 6$, $X(3) = 2 - 2j$

f Hz	0	25	50	75
A_k	1.5	0.707	1.5	0.707
P_k	2.25	0.5	2.25	0.5
φ_k degree	0	45	0	-45

4.15

a. w = [0.0800 0.2532 0.6424 0.9544 0.9544 0.6424 0.2532 0.0800]

b. w=[0 0.1883 0.6113 0.9505 0.9505 0.6113 0.1883 0]

4.16

a. xw=[0 0.4000 0 -0.8000 0 0]

b. xw=[0 0.3979 0 -0.9121 0 0.0800]

c. xw=[0 0.3455 0 -0.9045 0 0]

4.17

a.

w=[0.0800 0.1876 0.4601 0.7700 0.9723 0.9723 0.7700 0.4601 0.1876 0.0800]

b.

w=[0 0.1170 0.4132 0.7500 0.9698 0.9698 0.7500 0.4132 0.1170 0]

4.18

a. xw =[0 0.0800 0 -0.1600 0 0]

b. xw =[0 0.0796 0 -0.1824 0 0.0160]

c. xw =[0 0.0691 0 -0.1809 0 0]

4.19

a. $A_0 = 0.1667$, $A_1 = 0.3727$, $A_2 = 0.5$, $A_3 = 0.3727$

$\varphi_0 = 0^0$, $\varphi_1 = 154.43^0$, $\varphi_2 = 0^0$, $\varphi_3 = -154.43^0$

$P_0 = 0.0278$, $P_1 = 0.1389$, $P_2 = 0.25$, $P_3 = 0.1389$

b. $A_0 = 0.2925$, $A_1 = 0.3717$, $A_2 = 0.6375$, $A_3 = 0.3717$

$\varphi_0 = 0^0$, $\varphi_1 = 145.13^0$, $\varphi_2 = 0^0$, $\varphi_3 = -145.13^0$

$P_0 = 0.0586$, $P_1 = 0.1382$, $P_2 = 0.4064$, $P_3 = 0.1382$

c. $A_0 = 0.6580$, $A_1 = 0.3302$, $A_2 = 0.9375$, $A_3 = 0.3302$

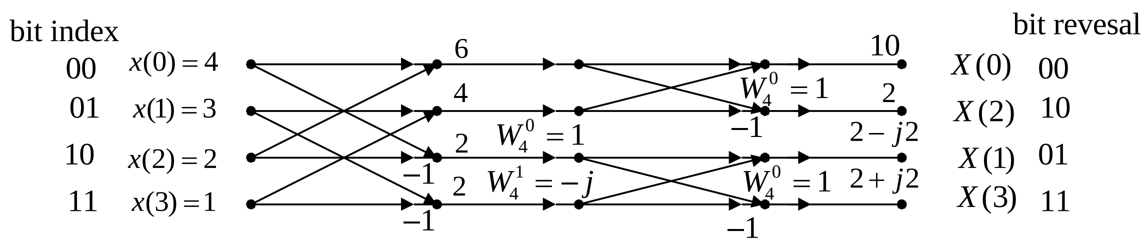
$\varphi_0 = 0^0$, $\varphi_1 = 108.86^0$, $\varphi_2 = 0^0$, $\varphi_3 = -108.86^0$

$P_0 = 0.4330$, $P_1 = 0.1091$, $P_2 = 0.8789$, $P_3 = 0.1091$

4.20

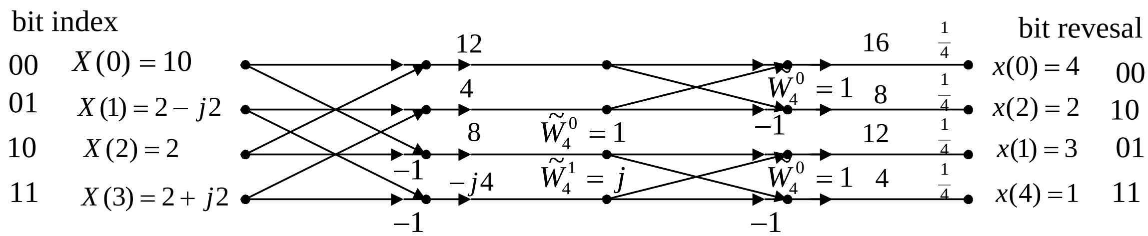
a. $\Delta f = 8000/100 = 80$ Hz, b. Sine one cycles $= (1/2000)/(1/8000) = 4$ samples, and 100 samples /4 samples= 25, which is multiple of the cycles, there is no spectral leakage.
b. $\Delta f = 8000/73 = 109.59$ Hz, 4) Since 73 samples/ 4 samples is not multiple of the cycles, spectral leakage occurs significantly without using the window function.

4.21



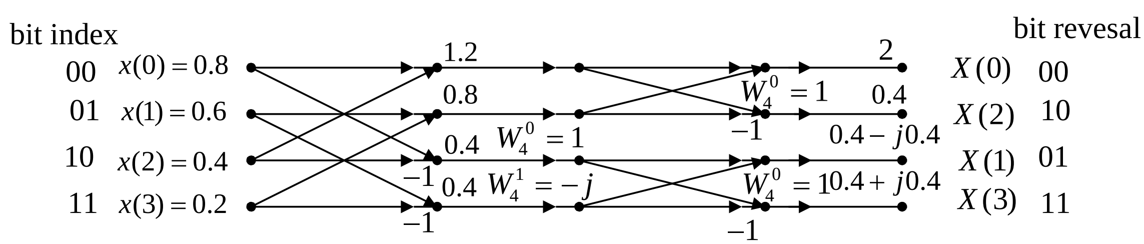
$X(0)=10$, $X(1)=2-2j$, $X(2)=2$, $X(3)=2+2j$, 4 complex multiplications

4.22



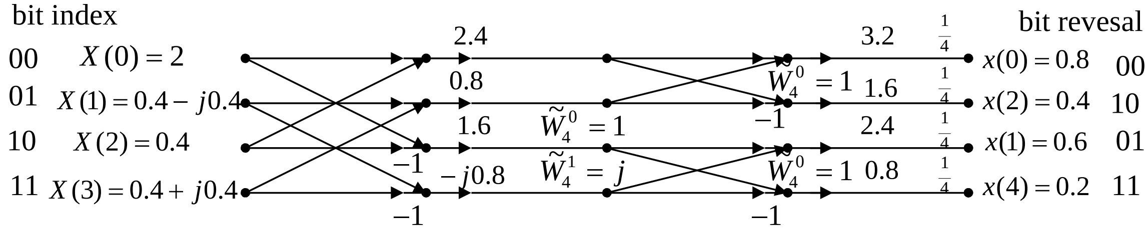
$x(0)=4$, $x(1)=3$, $x(2)=2$, $x(3)=1$, 4 complex multiplications

4.23



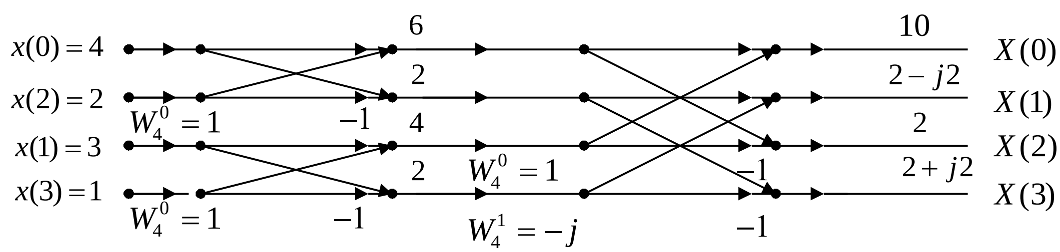
$X(0)=2$, $X(1)=0.4-0.4j$, $X(2)=0.4$, $X(3)=0.4+0.4j$, 4 complex multiplications

4.24



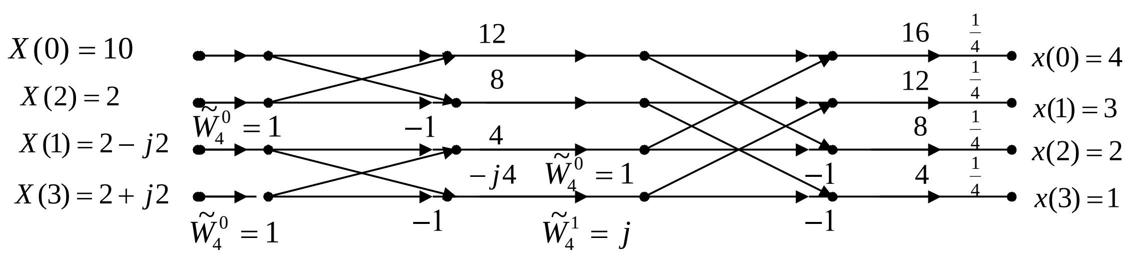
$x(0)=0.8$, $x(1)=0.6$, $x(2)=0.4$, $x(3)=0.2$, 4 complex multiplications

4.25



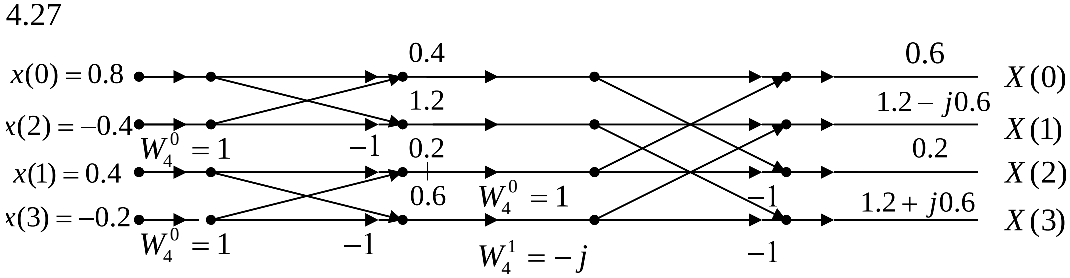
$X(0)=10$, $X(1)=2-2j$, $X(2)=2$, $X(3)=2+2j$, 4 complex multiplications

4.26



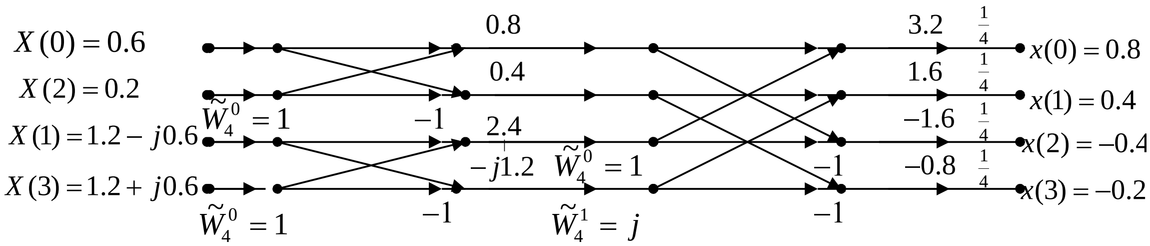
$x(0)=4$, $x(1)=3$, $x(2)=2$, $x(3)=1$, 4 complex multiplications

4.27



$X(0)=0.6$, $X(1)=1.2-j0.6$, $X(2)=0.2$, $X(3)=1.2+j0.6$, 4 complex multiplications

4.28



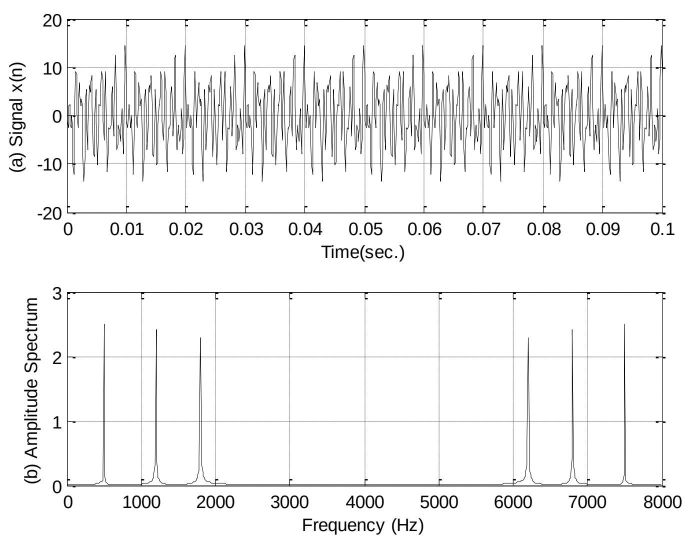
$x(0) = 0.8$, $x(1) = 0.4$, $x(2) = -0.4$, $x(3) = -0.2$, 4 complex multiplications

4.29

a.

```
------------------------------------------------------------------------------------------------------
% generate the sine wave sequence
fs=8000; T=1/fs;                    % Sampling rate and sampling period
t=0:T:0.1;
x1=5*cos(2*pi*500*t);
x2=5*cos(2*pi*1200*t+0.25*pi);
x3=5*cos(2*pi*1800*t+0.5*pi);
x=x1+x2+x3;
% apply the FFT algorithm
N=length(x);
  index_t=[0:1:N-1];
f=[0:1:N-1]*fs/N;       %Map frequency bin to frequency (Hz)
Ak=abs(fft(x))/N;       %Calculate amplitude spectrum
subplot(2,1,1);plot(t,x);grid
xlabel('Time(sec.)');ylabel('(a) Signal x(n)');
subplot(2,1,2);plot(f,Ak);grid
xlabel('Frequency (Hz)');ylabel('(b) Amplitude Spectrum');
--------------------------------------------------------------------------------------------------------------
```

b.



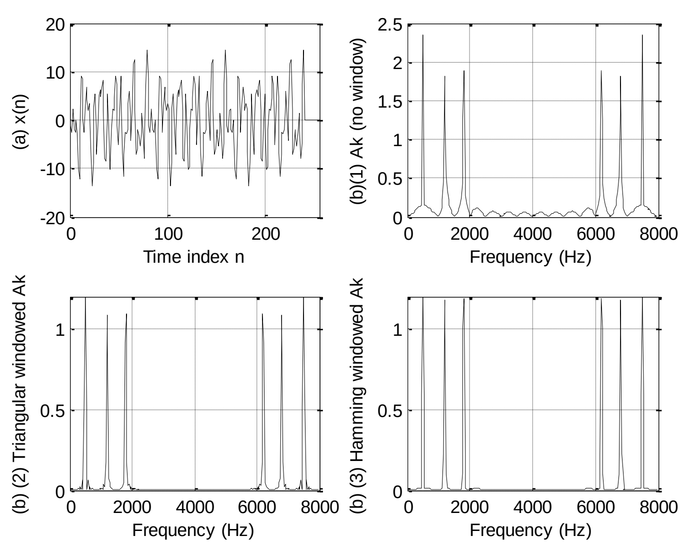
4.30

a.

```
close all;clear all
% generate the sine wave sequence
fs=8000; T=1/fs;                    % Sampling rate and sampling period
N=240;
t=0:T:(N-1)*T;
x1=5*cos(2*pi*500*t);
x2=5*cos(2*pi*1200*t+0.25*pi);
x3=5*cos(2*pi*1800*t+0.5*pi);
x=x1+x2+x3;
% apply the FFT algorithm with zero padding
x=[x,zeros(1,16)];
N=length(x);
index_t=[0:1:N-1];
f=[0:1:N-1]*fs/N;      %Map frequency bin to frequency (Hz)
xf=abs(fft(x))/N;      %Calculate amplitude spectrum
%using Bartlett window
x_b=x.*bartlett(N)';       %Apply triangular window function
xf_b=abs(fft(x_b))/N;   %Calculate amplitude spectrum
subplot(2,2,1);plot(index_t,x);grid
xlabel('Time index n'); ylabel('(a) x(n)');axis([ 0 255 -20 20]);
subplot(2,2,2);plot(f,xf);grid;axis([0 8000 0 2.5]);
xlabel('Frequency (Hz)'); ylabel('(b)(1) Ak (no window)');
subplot(2,2,3); plot(f,xf_b);grid; axis([0 8000 0 1.2]);
xlabel('Frequency (Hz)'); ylabel('(b) (2) Triangular windowed Ak');
%using Hamming window
x_hm=x.*hamming(N)';       %Apply Hamming window function
xf_hm=abs(fft(x_hm))/N;    %Calculate amplitude spectrum
subplot(2,2,4); plot(f,xf_hm);grid;axis([0 fs 0 1.2]);
xlabel('Frequency (Hz)'); ylabel('(b) (3) Hamming windowed Ak');
```

----

b.



c. Hamming window has least spectral leakage.

Chapter 5

5.1

a. $X(z)=\dfrac{4z}{z-1}$, b. $X(z)=\dfrac{z}{z+0.7}$

c. $X(z)=\dfrac{4z}{z-e^{-2}}=\dfrac{4z}{z-0.1353}$,

d. $X(z)=\dfrac{4z[z-0.8\times\cos(0.1\pi)]}{z^2-[2\times 0.8z\cos(0.1\pi)]+0.8^2}=\dfrac{4z(z-0.7608)}{z^2-1.5217z+0.64}$

e. $X(z)=\dfrac{4e^{-3}\sin(0.1\pi)z}{z^2-2e^{-3}z\cos(0.1\pi)+e^{-6}}=\dfrac{0.06154z}{z^2-0.0947z+0.00248}$

5.2

a. $X(z)=\dfrac{z}{z-1}+\dfrac{z}{z-0.5}$

b. $X(z)=\dfrac{z^{-4}z[z-e^{-3}\cos(0.1\pi)]}{z^2-[2e^{-3}\cos(0.1\pi)]z+e^{-6}}=\dfrac{z^{-3}(z-0.0474)}{z^2-0.0948z+0.0025}$

5.3

a. $X(z)=\dfrac{3z^{-3}}{z-1}$, b. $X(z)=\dfrac{2z}{z+0.5}$

c. $X(z)=\dfrac{5z^{-2}}{z-e^{-2}}$, d. $X(z)=\dfrac{6z[z-0.6\cos(0.2\pi)]}{z^2-1.2\cos(0.2\pi)z+0.6^2}=\dfrac{6z[z-0.4854]}{z^2-0.9708z+0.36}$

e. $X(z)=\dfrac{4e^{-3}\sin(0.2\pi)}{z^2-2e^{-3}\cos(0.2\pi)z+e^{-6}}=\dfrac{0.1171}{z^2-0.0806z+0.0025}$

5.4

a. $X(z)=\dfrac{-z}{z-1}+\dfrac{z}{z-0.75}$

b. x $X(z)=\dfrac{e^{-2}\sin(0.3\pi)z}{z^2-2e^{-2}\cos(0.3\pi)z+e^{-2\times 2}}z^{-3}=\dfrac{e^{-2}\sin(0.3\pi)z^{-2}}{z^2-2e^{-2}\cos(0.3\pi)z+e^{-4}}$

5.5

a. $X(z)=15z^{-3}-6z^{-5}$

b. $x(n)=15\delta(n-3)-6\delta(n-5)$

5.6

a. $x(n)=4\delta(n)-10u(n)-(-0.5)^n u(n)$ b. $x(n)=-5u(n)+10nu(n)+2.5n(0.8)^n u(n)$

c. $x(n)=1.25\sin(126.87^0 n)u(n)$

d. $x(n)=4u(n-5)+(n-2)u(n-2)+\delta(n-8)+(0.5)^{n-6}u(n-6)$

5.7

a. $X(z) = X_1(z)X_2(z) = (-2 + 5z^{-2})(4z^{-4}) = -8z^{-4} + 20z^{-6}$

b. $x(n) = -8\delta(n-2) + 20\delta(n-6)$

5.8

a. $x(n) = 5\delta(n) - 7(-1)^n u(n) - 3(0.5)^n u(n)$

b. $x(n) = -3(0.5)^n u(n) + 8(0.8)^n u(n) + 2.5n(0.8)^n u(n)$

c. $x(n) = 4.2433\sin(135^0 n)u(n)$

d. $x(n) = 5u(n-6) + (n-3)u(n-3) + \delta(n-10) + (0.75)^{n-4}u(n-4)$

5.9

a. $X(z) = -25 + \frac{5z}{z-0.4} + \frac{20z}{z+0.1}$, $x(n) = -25\delta(n) + 5(0.4)^n u(n) + 20(-0.1)^n u(n)$

b. $X(z) = \frac{1.6667z}{z-0.2} - \frac{1.6667z}{z+0.4}$, $x(n) = 1.6667(0.2)^n u(n) - 1.6667(-0.4)^n u(n)$

c. $X(z) = \frac{1.3514z}{z+0.2} + \frac{Az}{z-P} + \frac{A^* z}{z-P^*}$

where $P = 0.5 + 0.5j = 0.707\angle 45^0$, and $A = 1.1625\angle -125.54^0$

$x(n) = 1.3514(-0.2)^n u(n) + 2.325(0.707)^n \cos(45^0 \times n - 125.54^0)$

d. $X(z) = \frac{4.4z}{z-0.6} + \frac{-0.4z}{z-0.1} + \frac{-1.2z}{(z-0.1)^2}$, $x(n) = 4.4(0.6)^n u(n) - 0.4(0.1)^n u(n) - 12n(0.1)^n u(n)$

5.10

$Y(z) = \frac{-4.3333z}{z-0.5} + \frac{5.333z}{z-0.8}$, $y(n) = -4.3333(0.5)^n u(n) + 5.3333(0.8)^n u(n)$

5.11

a. $X(z) = 5 + \frac{Az}{z-P} + \frac{A^* z}{z-P^*}$

where $P = -0.1 + 0.4359j = 0.4472\angle 102.92^0$, and $A = -2.5 + j0.5735 = 2.5649\angle 167.08^0$

$x(n) = 5\delta(n) + 5.1298(0.4472)^n \cos(102.92^0 \times n + 167.08^0)$

b. $X(z) = \frac{-1.25z}{z+0.3} + \frac{1.25z}{z-0.5}$, $x(n) = -1.25(-0.3)^n u(n) + 1.25(0.5)^n u(n)$

c. $X(z) = \frac{16z}{z-0.75} + \frac{Az}{z-P} + \frac{A^* z}{z-P^*}$

where $P = 0.5 + j0.5 = 0.7071\angle 45^0$, and $A = -8 + j4 = 8.9443\angle 153.43^0$

$x(n) = 16(0.75)^n u(n) + 17.8886(0.7071)^n \cos(45^0 \times n + 153.43^0)$

d. $X(z) = \frac{-2.4z}{z+0.8} + \frac{2.4z}{z-0.2} + \frac{-0.4z}{(z-0.2)^2}$, $x(n) = -2.4(-0.8)^n u(n) + 2.4(0.2)^n u(n) - 2n(0.2)^n u(n)$

5.12

$$Y(z)=\frac{3.8z^2+0.06z}{(z-0.3)(z+0.2)}=\frac{2.4}{z-0.3}+\frac{1.4}{z+0.2}$$

$$y(n)=2.4(0.3)^n u(n)+1.4(-0.2)^n u(n)$$

5.13

$$Y(z)=\frac{9.84z}{z-0.2}+\frac{-29.46z}{z-0.3}+\frac{20z}{z-0.4}$$

$$y(n)=9.84(0.2)^n u(n)-29.46(0.3)^n u(n)+20(0.4)^n u(n)$$

5.14

a. $Y(z)=\frac{-4z}{z-0.2}+\frac{5z}{z-0.5}$, $y(n)=-4(0.2)^n u(n)+5(0.5)^n u(n)$

b. $Y(z)=\frac{5z}{z-1}+\frac{-5z}{z-0.5}+\frac{z}{z-0.2}$,

$y(n)=5u(n)-5(0.5)^n u(n)+(0.2)^n u(n)$

5.15

$$Y(z)=\frac{1.12z^3+0.28z^2+0.08z}{(z^2-0.6z+0.08)(z-0.5)}=\frac{16.6667z}{z-0.6}+\frac{-18.5600z}{z-0.4}+\frac{3.0133z}{z-0.2}$$

$$y(n)=16.6667(0.5)^n u(n)-18.5600(0.4)^n u(n)+3.0133(0.2)^n u(n)$$

5.16

a. $Y(z)=\frac{z^2+z}{z^2-0.6z+0.25}=\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$

where $P=0.3+j0.4=0.5\angle 53.13^0$, and $A=0.5-j1.625=1.7002\angle -72.90^0$

$y(n)=3.4004(0.5)^n\cos(53.13^0\times n-72.90^0)$

b. $Y(z)=\frac{z^3+z^2}{(z-1)(z^2-0.6z+0.25)}=\frac{3.0768z}{z-1}+\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$

where $P=0.3+j0.4=0.5\angle 53.13^0$, and $A=-1.0385-j0.1827=1.0544\angle -170.22^0$

$y(n)=3.0768u(n)+2.1088(0.5)^n\cos(53.13^0\times n-170.22^0)$

5.17

a. $Y(z)=\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$, $P=0.2+0.5j=0.5385\angle 68.20^0$, $A=0.8602\angle -54.46^0$

$y(n)=1.7204\left(0.5382\right)^n\cos(n\times 68.20^0-54.46^0)$

b. $Y(z)=\frac{1.6854z}{z-1}+\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$, where $P = 0.2 + 0.5j = 0.5385\angle 68.20^0$, $A = 0.4910\angle -136.25^0$

$y(n) = 1.6845u(n) + 0.982\left(0.5382\right)^n \cos(n \times 68.20^0 - 136.25^0)$

5.18

a. $Y(z) = \dfrac{z^2 + 0.3z}{z^2 - 0.2z + 0.17} = \dfrac{Az}{z - P} + \dfrac{A^* z}{z - P^*}$

where $P = 0.1 + j0.4 = 0.4123\angle 75.96^0$, and $A = 0.5 - j0.5 = 0.7071\angle -45.00^0$

$y(n) = 1.4142(0.4123)^n \cos(75.96^0 \times n - 45.00^0)$

b. $Y(z) = \dfrac{z^3 + 0.3z^2}{(z-1)(z^2 - 0.2z + 0.17)} = \dfrac{1.3402z}{z - 1} + \dfrac{Az}{z - P} + \dfrac{A^* z}{z - P^*}$

where $P = 0.3 + j0.4 = 0.5\angle 53.13^0$, and $A = -0.1701 - j0.2423 = 0.2960\angle -125.07^0$

$y(n) = 1.3402u(n) + 0.5920(0.5)^n \cos(53.13^0 \times n - 125.07^0)$

Chapter 6

6.1

a. $y(0) = 0.5$, $y(1) = 0.25$, $y(2) = 0.125$, $y(3) = 0.0625$, $y(4) = 0.03125$

b. $y(0) = 1$, $y(1) = 0$, $y(2) = 0.25$, $y(3) = 0$, $y(4) = 0.0625$

6.2

a. y = [0.1000 0.5600 0.5860 0.4766 0.3485]

b. y=[0 0.5000 0.5500 0.4550 0.3355]

6.3

a. $y(0) = -2$, $y(1) = 2.3750$, $y(2) = -1.0312$, $y(3) = 0.7266$, $y(4) = -0.2910$

b. $y(0) = 0$, $y(1) = 1$, $y(2) = -0.2500$, $y(3) = 0.3152$, $y(4) = -0.0781$

6.4

a. $H(z) = 0.5 + 0.5z^{-1}$, b. $y(n) = 2\delta(n) + 2\delta(n-1)$, c. $y(n) = -5\delta(n) + 10u(n)$

6.5

a. $H(z) = \dfrac{1}{1 + 0.5z^{-1}}$, b. $y(n) = (-0.5)^n u(n)$, c. $y(n) = 0.6667u(n) + 0.3333(-0.5)^n u(n)$

6.6

$H(z) = \dfrac{1 - 0.25z^{-2}}{1 + 1.1z^{-1} + 0.28z^{-2}}$, $B(z) = 1 - 0.25z^{-2}$, $A(z) = 1 + 1.1z^{-1} + 0.28z^{-2}$

6.7

$H(z) = \dfrac{0.5 + 0.5z^{-1}}{1 + 0.6z^{-2}}$, $B(z) = 0.5 + 0.5z^{-1}$, $A(z) = 1 + 0.6z^{-2}$

6.8

$H(z) = \dfrac{0.25z^{-2}}{1 - 0.5z^{-1} + 0.2z^{-2}}$, $B(z) = 0.25z^{-2}$, $A(z) = 1 - 0.5z^{-1} + 0.2z^{-2}$

6.9

$H(z) = 1 - 0.3z^{-1} + 0.28z^{-2}$, $A(z) = 1$, $B(z) = 1 - 0.3z^{-1} + 0.28z^{-2}$

6.10

a. $y(n) = 0.5x(n) + 0.5x(n-1)$

b. $y(n) = 0.5x(n) + 0.3y(n-1)$

6.11

a. $y(n) = 0.1x(n) + 0.2x(n-1) + 0.3x(n-2)$

b. $y(n) = 0.5x(n) - 0.5x(n-2) + 0.3y(n-1) - 0.8y(n-2)$

6.12

a. $y(n) = x(n) - 0.25x(n-2) - 1.1y(n-1) - 0.18y(n-2)$

b. $y(n) = x(n-1) - 0.1x(n-2) + 0.3x(n-3)$

6.13

a. $H(z) = \dfrac{(z+1)(z+1)}{(z+2)(z+3)}$

b. $H(z) = \dfrac{(z+0.4)(z-0.4)}{(z+0.2)(z+0.5)}$

c. $H(z) = \dfrac{(z+2-j)(z++j)}{z(z+1-j2.2361)(z+1+j2.2361)}$

6.14

a. Impulse response: $H(z) = \dfrac{13.3333}{z} - \dfrac{3.3333}{z+0.75}$, $y(n) = 13.3333u(n) - 3.3333(-0.75)^n u(n)$

Step response: $H(z) = \dfrac{11.4286}{z-1} - \dfrac{1.4286}{z+0.75}$, $y(n) = 11.4286u(n) - 1.4286(-0.75)^n u(n)$

c. $H(z) = \dfrac{12.5}{z-1} - \dfrac{2.5}{z+0.75}$, $y(n) = 12.5u(n) - 2.5(-0.75)^n u(n)$

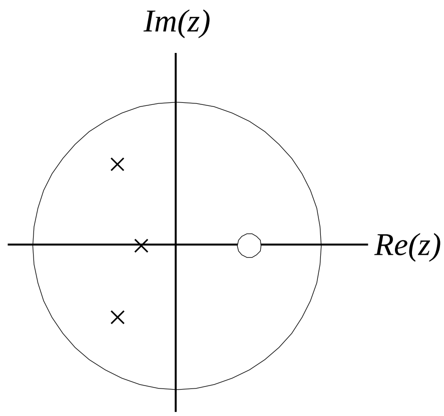
6.15

a. zero: $z = 0.5$, poles: $z = -0.25$ ($|z| = 0.25$), $z = -0.5 \pm 0.7416j$ ($|z| = 0.8944$), stable

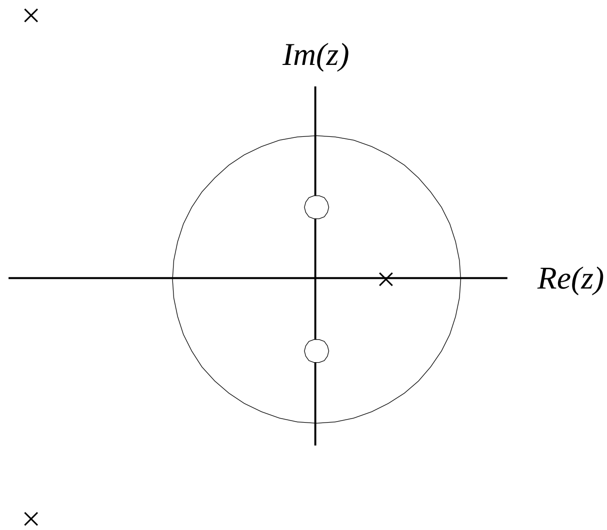
b. zeros: $z = \pm 0.5j$, poles: $z = 0.5$ ($|z| = 0.5$), $z = -2 \pm 1.7321j$ ($|z| = 2.6458$), unstable

c. zero: $z = -0.95$, poles: $z = 0.2$ ($|z| = 0.2$), $z = -0.7071 \pm 0.7071j$ ($|z| = 1$), marginally stable

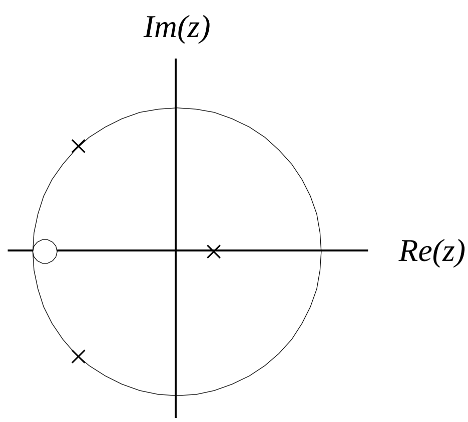
d. zeros: $z = -0.5$, $z = -0.5$, poles: $z = 1$ ($|z| = 1$), $z = -1$, $z = -1$ ($|z| = 1$), $z = 0.36$ ($|z| = 0.36$), unstable



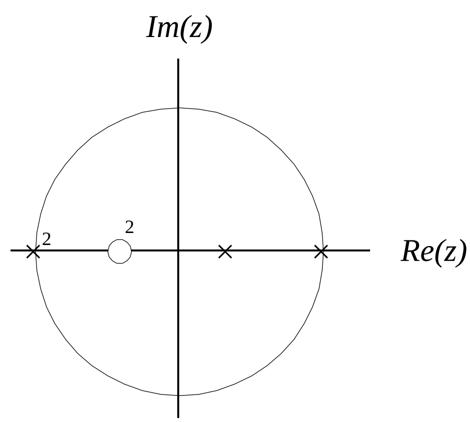
(a)



(b)



(c)



(d)

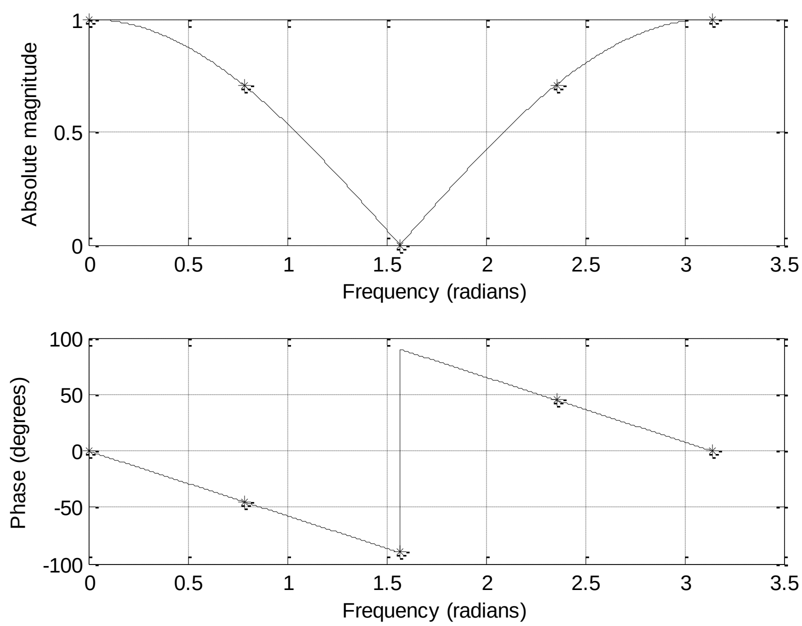
6.16

a. $H(z)=0.5+0.5z^{-2}$, $H(e^{j\Omega})=0.5+0.5e^{-j2\Omega}$

$$\left|H(e^{j\Omega})\right|=0.5\sqrt{(1+\cos 2\Omega)^2+(\sin 2\Omega)^2}\ ,\ \angle H(e^{j\Omega})=\tan^{-1}\left(\frac{-\sin 2\Omega}{1+\cos 2\Omega}\right)$$

b.

Ω (radians)	$f=\frac{\Omega}{2\pi}f_s$ (Hz)	$\left\|H(e^{j\Omega})\right\|$	$\left\|H(e^{j\Omega})\right\|_{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00^0
0.25π	1000	0.707	-3.0102 dB	-45.00^0
0.50π	2000	0.0	$-\infty$ dB	-90.00^0
0.75π	3000	0.707	-3.0102 dB	45.00^0
1.00π	4000	1.000	0 dB	0.00^0



c. Bandstop filter

6.17

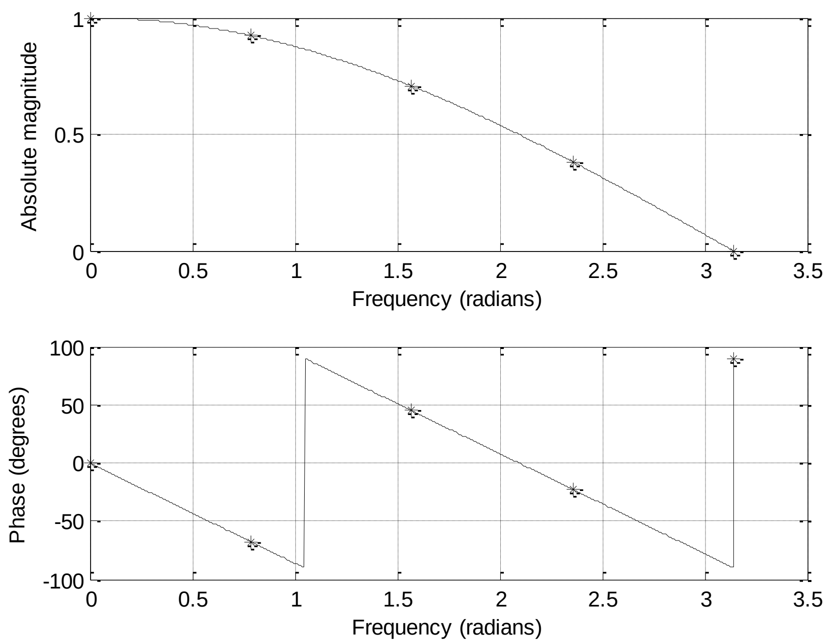
a. $H(z)=0.5z^{-1}+0.5z^{-2}$, $H(e^{j\Omega})=0.5e^{-j\Omega}+0.5e^{-j2\Omega}$

$$\left|H(e^{j\Omega})\right|=0.5\sqrt{(1+\cos\Omega)^2+(\sin\Omega)^2}\ ,\ \angle H(e^{j\Omega})=\tan^{-1}\left(\frac{-\sin\Omega-\sin 2\Omega}{\cos\Omega+\cos 2\Omega}\right)$$

b.

Ω (radians)	$f=\frac{\Omega}{2\pi}f_s$ (Hz)	$\left\|H(e^{j\Omega})\right\|$	$\left\|H(e^{j\Omega})\right\|_{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00^0
0.25π	1000	0.924	-0.6877 dB	-67.50^0

0.50π	2000	0.707	-3.0102 dB	45.00^0
0.75π	3000	0.383	-8.3432 dB	-22.50^0
1.00π	4000	0.000	$-\infty$ dB	90.00^0



c. Lowpass

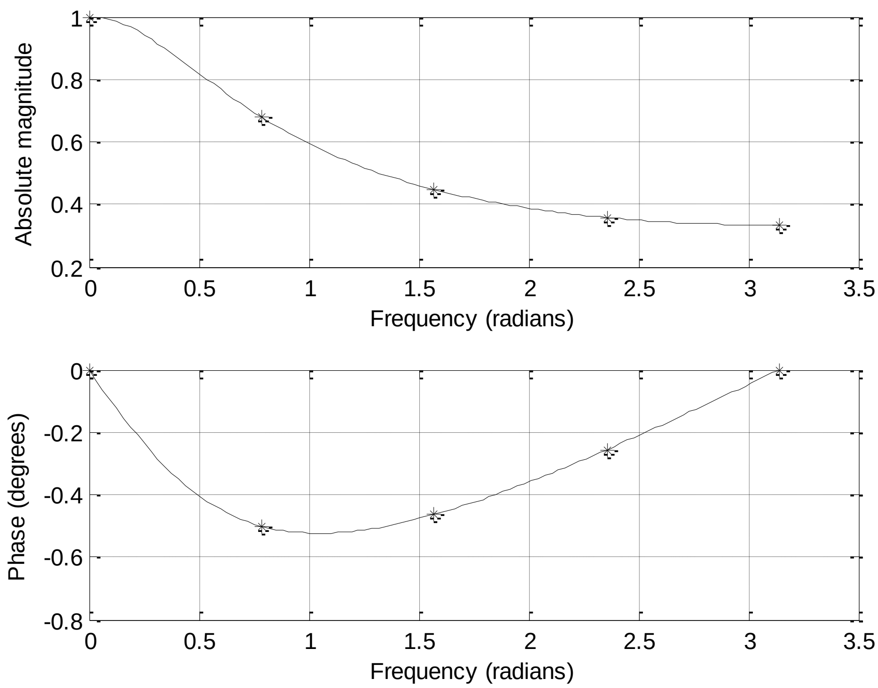
6.18

a. $H(z) = \dfrac{0.5}{1-0.5z^{-1}}$, $H(e^{j\Omega}) = \dfrac{0.5}{1-0.5e^{-j\Omega}}$

$$\left|H(e^{j\Omega})\right| = \frac{0.5}{\sqrt{(1-0.5\cos(\Omega))^2 + (0.5\sin\Omega)^2}},\ \angle H(e^{j\Omega}) = -\tan^{-1}\left(\frac{0.5\sin\Omega}{1-\cos\Omega}\right)$$

b.

Ω (radians)	$f = \dfrac{\Omega}{2\pi} f_s$ (Hz)	$\left\|H(e^{j\Omega})\right\|$	$\left\|H(e^{j\Omega})\right\|_{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00^0
0.25π	1000	0.6786	-3.3677 dB	-28.68^0
0.50π	2000	0.4472	-6.9897 dB	-26.57^0
0.75π	3000	0.3574	-8.9367 dB	-14.64^0
1.00π	4000	0.3333	-9.5424 dB	0.00^0



c. Lowpass

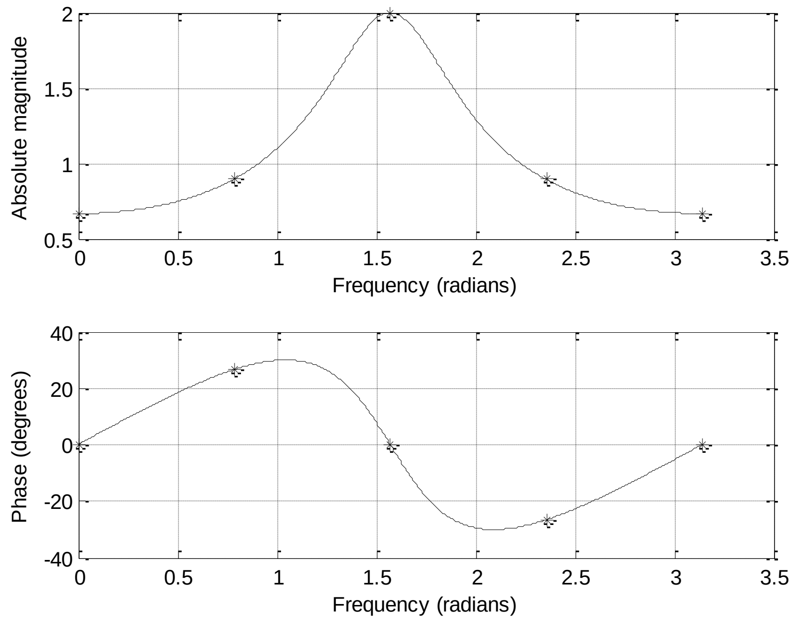
6.19

a. $H(z)=\dfrac{1}{1+0.5z^{-2}}$, $H(e^{j\Omega})=\dfrac{1}{1+0.5e^{-j2\Omega}}$

$\left|H(e^{j\Omega})\right|=\dfrac{1}{\sqrt{(1+0.5\cos 2\Omega)^2+(0.5\sin 2\Omega)^2}}$, $\angle H(e^{j\Omega})=-\tan^{-1}\left(\dfrac{-0.5\sin 2\Omega}{1+0.5\cos 2\Omega}\right)$

b.

Ω (radians)	$f=\dfrac{\Omega}{2\pi}f_s$ (Hz)	$\left\|H(e^{j\Omega})\right\|$	$\left\|H(e^{j\Omega})\right\|_{dB}$	$\angle H(e^{j\Omega})$
0	0	0.6667	0 dB	0.00^0
0.25π	1000	0.8944	-3.0102 dB	26.57^0
0.50π	2000	2.000	$-\infty$ dB	0.00^0
0.75π	3000	0.8944	-3.0102 dB	-26.57^0
1.00π	4000	0.6667	0 dB	0.00^0

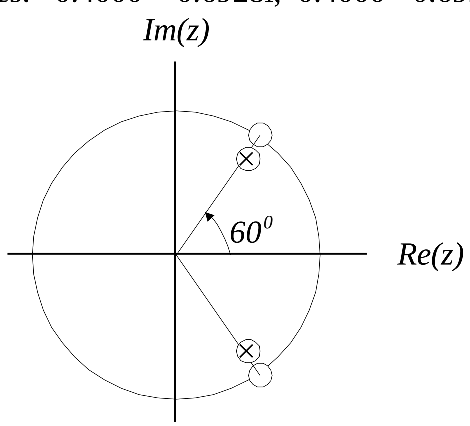


c. Bandpass filter

6.20

a. $H(z)=\dfrac{1-z^{-1}+z^{-2}}{1-0.8z^{-1}+0.64z^{-2}}$

b. Zeros: 0.5000 + 0.8660i , 0.5000 - 0.8660i,
Poles: 0.4000 + 0.6928i, 0.4000 - 0.6928i



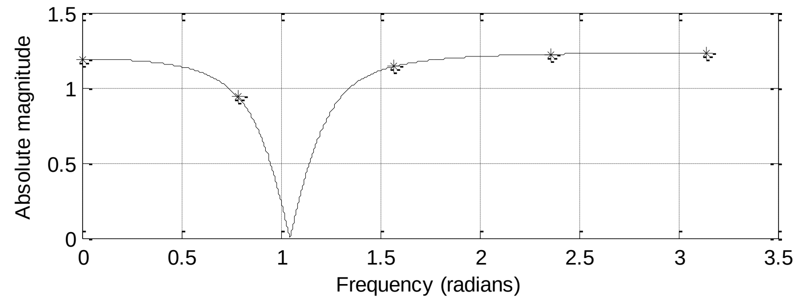
c. Stable since poles are inside the unit cycle.

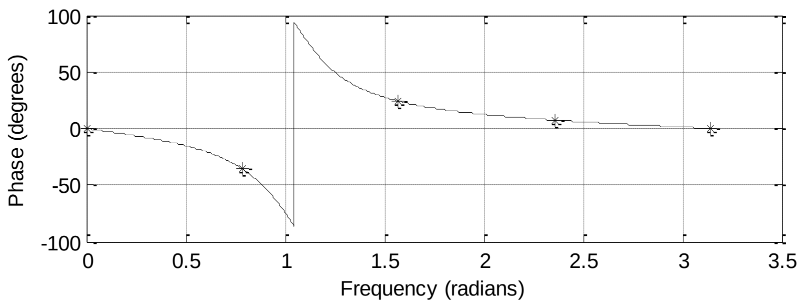
d. and e.

$$\left|H(e^{j\Omega})\right|=\frac{\sqrt{(1-\cos\Omega+\cos 2\Omega)^2+(\sin\Omega+\sin 2\Omega)^2}}{\sqrt{(1-0.8\cos\Omega+0.64\cos 2\Omega)^2+(0.8\sin\Omega+0.64\sin 2\Omega)^2}}$$

$$\angle H(e^{j\Omega})=\tan^{-1}\left(\frac{-\sin\Omega-\sin 2\Omega}{1-\cos\Omega+\cos 2\Omega}\right)-\tan^{-1}\left(\frac{-0.8\sin\Omega-0.64\sin 2\Omega}{1-0.8\cos\Omega+0.64\cos 2\Omega}\right)$$

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$\lvert H(e^{j\Omega}) \rvert$	$\lvert H(e^{j\Omega}) \rvert_{dB}$	$\angle H(e^{j\Omega})$
0	0	1.1905	1.5144 dB	0.00^0
0.25π	1000	0.9401	-0.5369 dB	-35.29^0
0.50π	2000	1.1399	1.1373 dB	24.23^0
0.75π	3000	1.2217	1.1792 dB	7.40^0
1.00π	4000	1.2295	1.7946 dB	0.00^0





6.21

(1) a. $H(z) = 0.5 + 0.5z^{-1}$, $H(e^{j\Omega}) = 0.5 + 0.5e^{-j\Omega}$

$$\lvert H(e^{j\Omega}) \rvert = \sqrt{(0.5 + 0.5\cos\Omega)^2 + (0.5\sin\Omega)^2}, \ \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-0.5\sin\Omega}{0.5 + 0.5\cos\Omega}\right)$$

b. See table and plot
c. Lowpass filter

(2) a. $H(z) = 0.5 - 0.5z^{-1}$, $H(e^{j\Omega}) = 0.5 - 0.5e^{-j\Omega}$

$$\lvert H(e^{j\Omega}) \rvert = \sqrt{(0.5 - 0.5\cos\Omega)^2 + (0.5\sin\Omega)^2}, \ \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{0.5\sin\Omega}{0.5 - 0.5\cos\Omega}\right)$$

b. See table and plot
c. Highpass filter

(3) a. $H(z) = 0.5 + 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5 + 0.5e^{-j2\Omega}$

$$\lvert H(e^{j\Omega}) \rvert = \sqrt{(0.5 + 0.5\cos 2\Omega)^2 + (0.5\sin 2\Omega)^2}, \ \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-0.5\sin 2\Omega}{0.5 + 0.5\cos 2\Omega}\right)$$

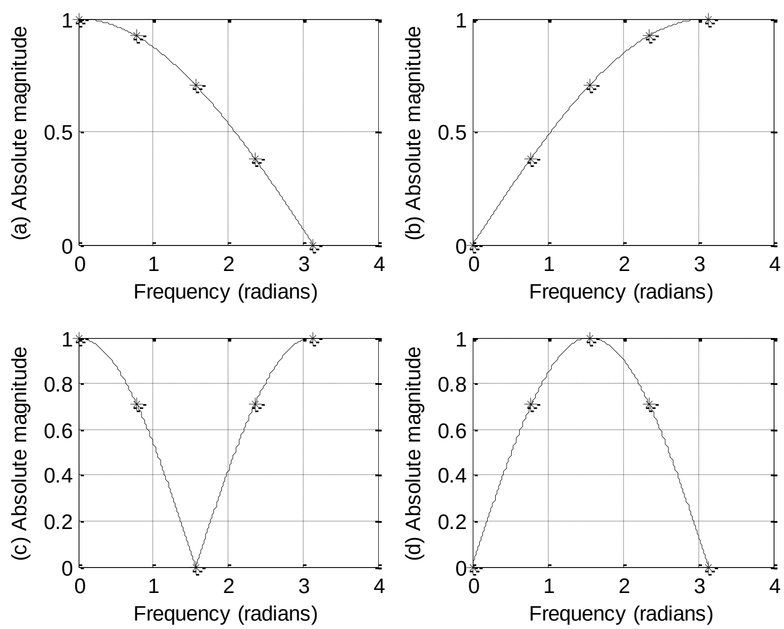
b. See table and plot
c. Bandstop filter

(4) a. $H(z) = 0.5 - 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5 - 0.5e^{-j2\Omega}$

$$\left|H(e^{j\Omega})\right| = \sqrt{(0.5 - 0.5\cos 2\Omega)^2 + (0.5\sin 2\Omega)^2}, \ \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{0.5\sin 2\Omega}{0.5 - 0.5\cos 2\Omega}\right)$$

b. See table and plot
c. Bandpass filter

Ω (radians)	(a) $\left\|H(e^{j\Omega})\right\|$	(b) $\left\|H(e^{j\Omega})\right\|$	(c) $\left\|H(e^{j\Omega})\right\|$	(d) $\left\|H(e^{j\Omega})\right\|$
0	1.0000	0.0000	1.0000	0.0000
0.25π	0.9239	0.3827	0.7071	0.7071
0.50π	0.7071	0.7071	0.0000	1.0000
0.75π	0.3827	0.9239	0.7071	0.7071
1.00π	0.0000	1.0000	1.0000	0.0000



6.22

a. $H(z) = \dfrac{0.5}{1 - 0.2z^{-1}}$

b. $Y(z) = \dfrac{-0.3333z}{z - 0.2} + \dfrac{0.8333z}{z - 0.5}$, $y(n) = -0.3333(0.2)^n u(n) + 0.8333(0.5)^n u(n)$

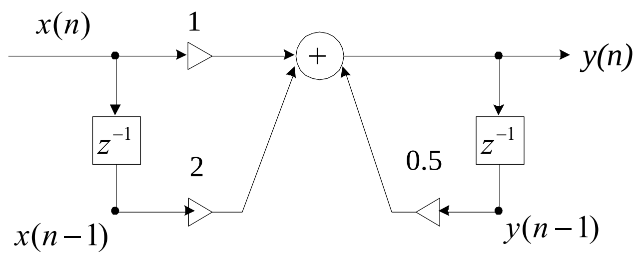
6.23

a. $H(z) = \dfrac{0.5}{1 + 0.7z^{-1} + 0.1z^{-2}}$

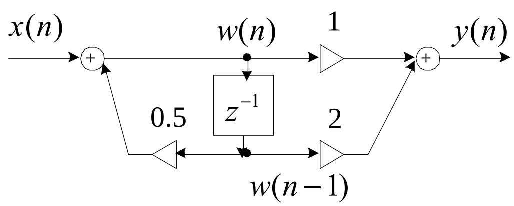
b. $y(n) = 0.5556u(n) - 0.111(-0.2)^n u(n) + 0.5556(-0.5)^n u(n)$

6.24

a. $y(n) = x(n) + 2x(n-1) + 0.5y(n-1)$

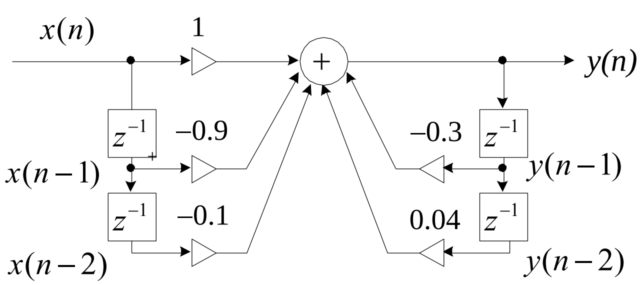


b.
$$w(n) = x(n) + 0.5w(n-1)$$
$$y(n) = w(n) + 2w(n-1)$$

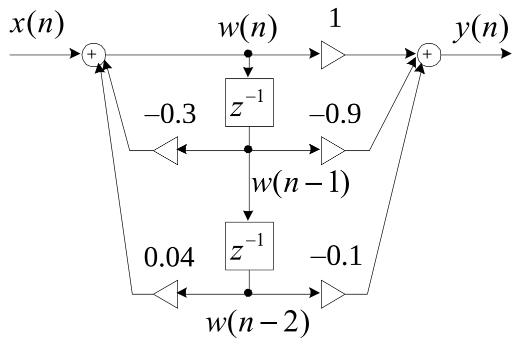


6.25

a. $y(n) = x(n) - 0.9x(n-1) - 0.1x(n-2) - 0.3y(n-1) + 0.04y(n-2)$

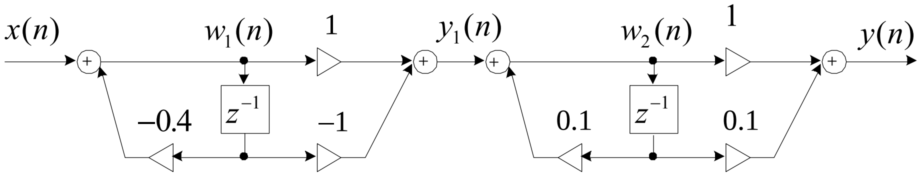


b. $w(n) = x(n) - 0.3w(n-1) + 0.04w(n-2)$
$y(n) = w(n) - 0.9w(n-1) - 0.1w(n-2)$



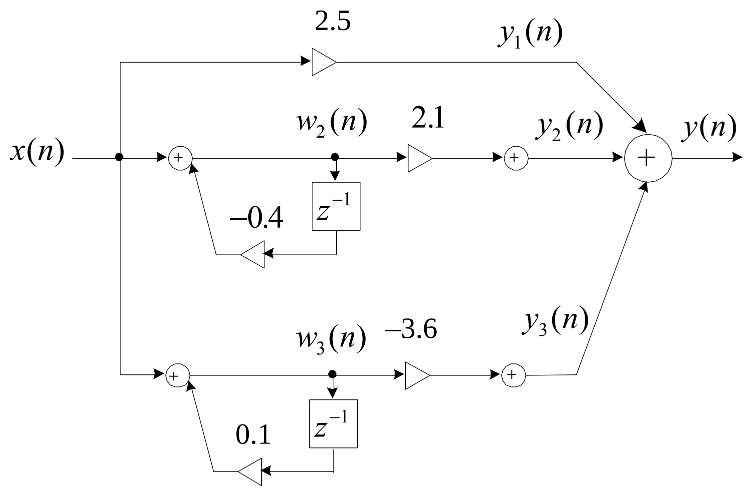
c. $H(z) = \dfrac{(z-1)(z+0.1)}{(z+0.4)(z-0.1)} = \dfrac{1-z^{-1}}{1+0.4z^{-1}} \times \dfrac{1-0.1z^{-1}}{1-0.1z^{-1}}$

$$w_1(n) = x(n) - 0.4w_1(n-1)$$
$$y_1(n) = w_1(n) - w_1(n-1)$$
$$w_2(n) = y_1(n) + 0.1w_2(n-1)$$
$$y(n) = w_2(n) + 0.1w_2(n-1)$$



d. $H(z) = 2.5 + \dfrac{2.1z}{z+0.4} - \dfrac{3.6z}{z-0.1} = 2.5 + \dfrac{2.1}{1+0.4z^{-1}} + \dfrac{-3.6}{1-0.1z^{-1}}$

$$y_1(n) = 2.5x(n)$$
$$w_2(n) = x(n) - 0.4w_2(n-1)$$
$$y_2(n) = 2.1w_2(n)$$
$$w_3(n) = x(n) + 0.1w_3(n-1)$$
$$y_3(n) = -3.6w_3(n)$$
$$y(n) = y_1(n) + y_2(n) + y_3(n)$$



6.26

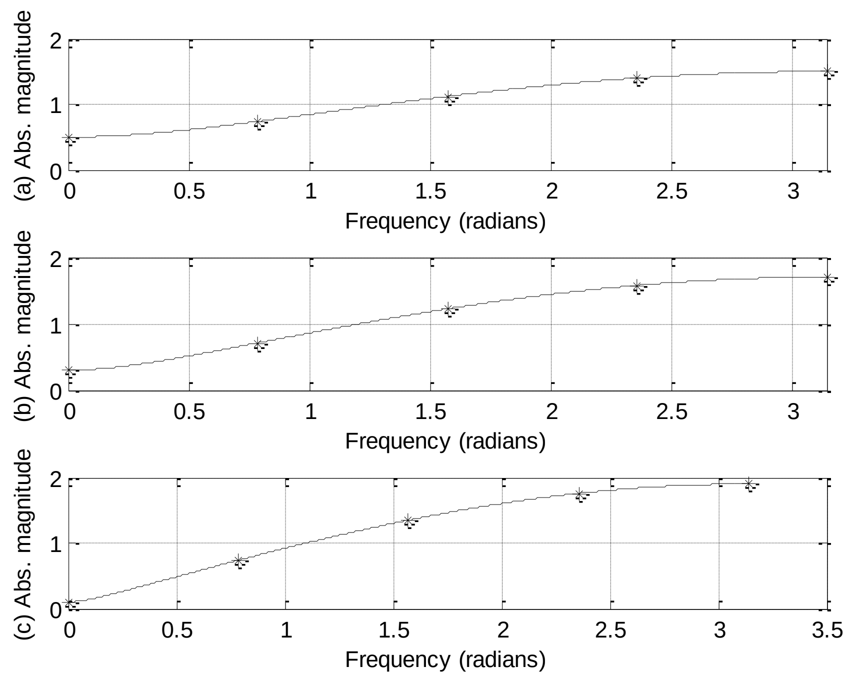
a. $y(n) = x(n) - 0.5x(n-1)$

$y(n) = x(n) - 0.7x(n-1)$

$y(n) = x(n) - 0.9x(n-1)$

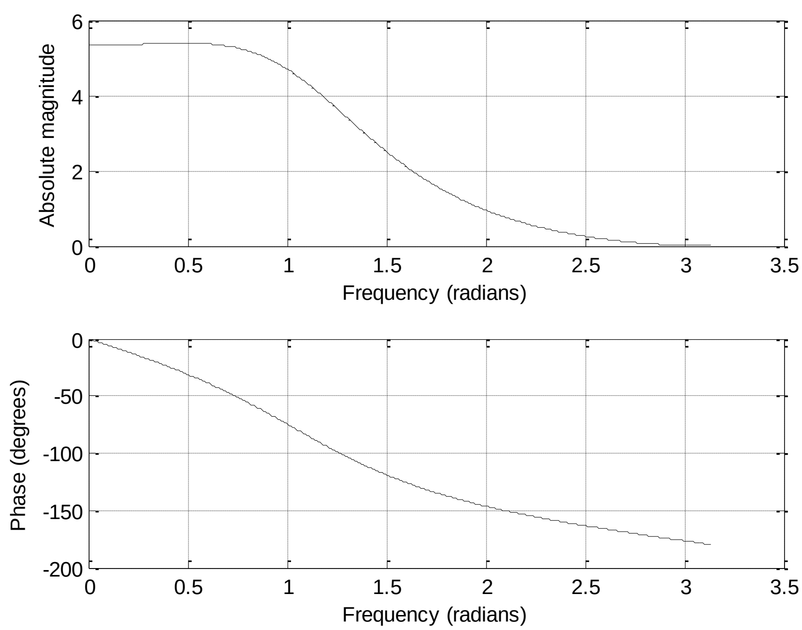
b. From frequency responses, the filter (c) emphasizes high frequencies most.

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$	(c) $\lvert H(e^{j\Omega})\rvert$
0	0.5000	0.3000	0.1000
0.25π	0.6368	0.7071	0.7329
0.50π	1.1189	1.2207	1.3454
0.75π	1.3990	1.5748	1.7558
1.00π	1.5000	1.7000	1.9000



6.27

```
>> [H W]=freqz([1 2 1],[1 -0.5 0.25],512);
>> subplot(2,1,1);plot(W,abs(H),'k');grid;
>> xlabel('Frequency (radians)');ylable('Absolute magnitude');
>> xlabel('Frequency (radians)');ylabel('Absolute magnitude');
>> subplot(2,1,2);plot(W,angle(H)*180/pi,'k');grid;
>> xlabel('Frequency (radians)');ylabel('Phase (degrees)');
```



6.28

a.

```
>> x=[1 0.5 0.25 0.125 0.0625];
>> Xi=[-1 0]; Yi=[1 2];
>> Zi=filtic([0 1],[1 0.75 0.125],Yi,Xi);
>> y=filter([0 1],[1 0.75 0.125],x,Zi)
y = -2.0000    2.3750   -1.0313    0.7266   -0.2910
```

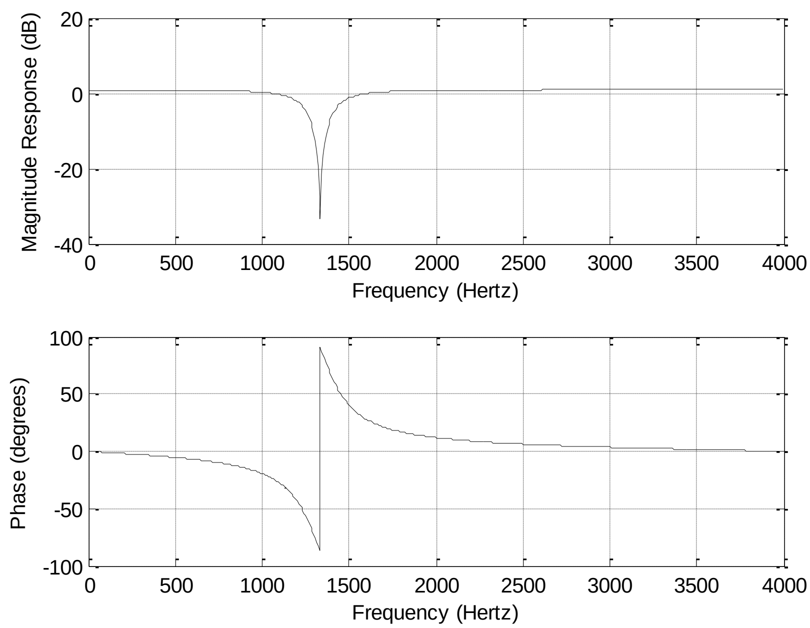
b.

```
>> x=[1 0.5 0.25 0.125 0.0625];
>> y=filter([0 1],[1 0.75 0.125],x)
y = 0    1.0000   -0.2500    0.3125   -0.0781
```

6.29

a.

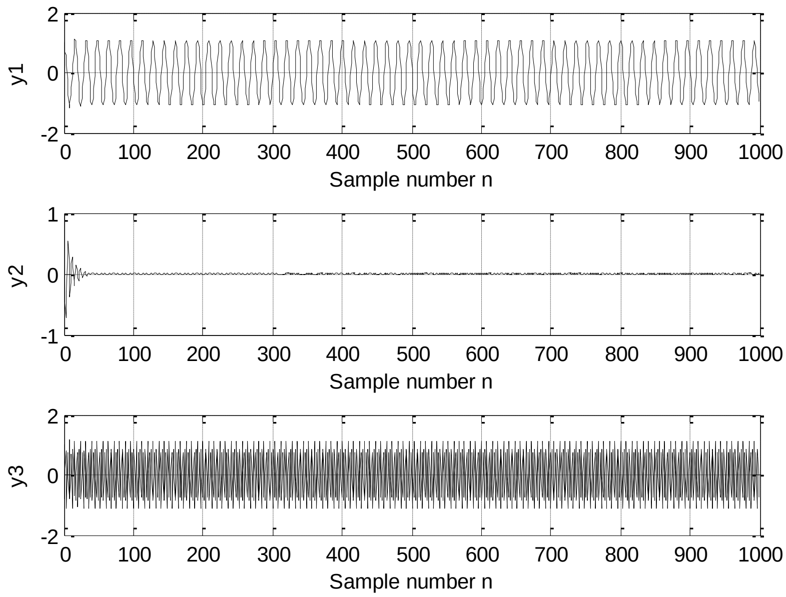
```
>> freqz([1 -1 1],[1 -0.9 0.81],512,8000)
```



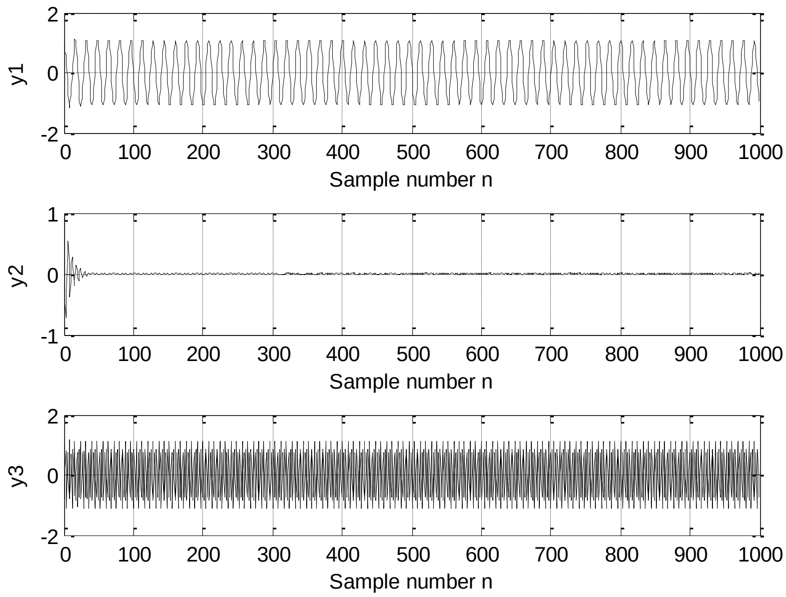
b. Notch filter

c. $y(n) = x(n) - x(n-1) + x(n-2) + 0.9y(n-1) - 0.81y(n-2)$

d.



```
>>n=0:1:999;
>>x1=cos(pi*1000*n/8000);x1=[0 0 x1];
>>x2=cos((8/3)*pi*1000*n/8000); x2=[0 0 x2];
>>x3=cos(6*pi*1000*n/8000); x3=[0 0 x3];
>>w1=zeros(1,1002); w2=w1; w3=w1;y1=w1;y2=w2;y3=w3;
 >>for nn=3:1:1002
        w1(nn)=x1(nn)+0.9*w1(nn-1)-0.81*w1(nn-2);
        y1(nn)=w1(nn)-w1(nn-1)+w1(nn-2);
        w2(nn)=x2(nn)+0.9*w2(nn-1)-0.81*w2(nn-2);
         y2(nn)=w2(nn)-w2(nn-1)+w2(nn-2);
        w3(nn)=x3(nn)+0.9*w3(nn-1)-0.81*w3(nn-2);
         y3(nn)=w3(nn)-w3(nn-1)+w3(nn-2);
>>end
>>y1=y1(3:1002);y2=y2(3:1002);y3=y3(3:1002);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');
```



e.

```
>> n=0:1:999;
>> x1=cos(pi*1000*n/8000);
>> x2=cos((8/3)*pi*1000*n/8000);
>> x3=cos(6*pi*1000*n/8000);
>> y1=filter([1 -1 1],[1 -0.9 0.81],x1);
>> y2=filter([1 -1 1],[1 -0.9 0.81],x2);
>> y3=filter([1 -1 1],[1 -0.9 0.81],x3);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');
```

See figure in (d)

6.30

```
>>n=0:1:999;
>>x1=cos(pi*1000*n/8000);x1=[0 0 x1];
>>x2=cos((8/3)*pi*1000*n/8000); x2=[0 0 x2];
>>x3=cos(6*pi*1000*n/8000); x3=[0 0 x3];
>>y1=zeros(1,1002); y2=y1; y3=y1;
 >>for nn=3:1:1002
        y1(nn)=x1(nn)-x1(nn-1)+x1(nn-2)+0.9*y1(nn-1)-0.81*y1(nn-2);
        y2(nn)=x2(nn)-x2(nn-1)+x2(nn-2)+0.9*y2(nn-1)-0.81*y2(nn-2);
        y3(nn)=x3(nn)-x3(nn-1)+x3(nn-2)+0.9*y3(nn-1)-0.81*y3(nn-2);
>>end
>>y1=y1(3:1002);y2=y2(3:1002);y3=y3(3:1002);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');
```

See plots in Problem 6.29

Chapter 7

7.1

a. $H(z) = 0.2941 + 0.3750z^{-1} + 0.2941z^{-2}$

$y(n) = 0.2941x(n) + 0.3750x(n-1) + 0.2941x(n-2)$

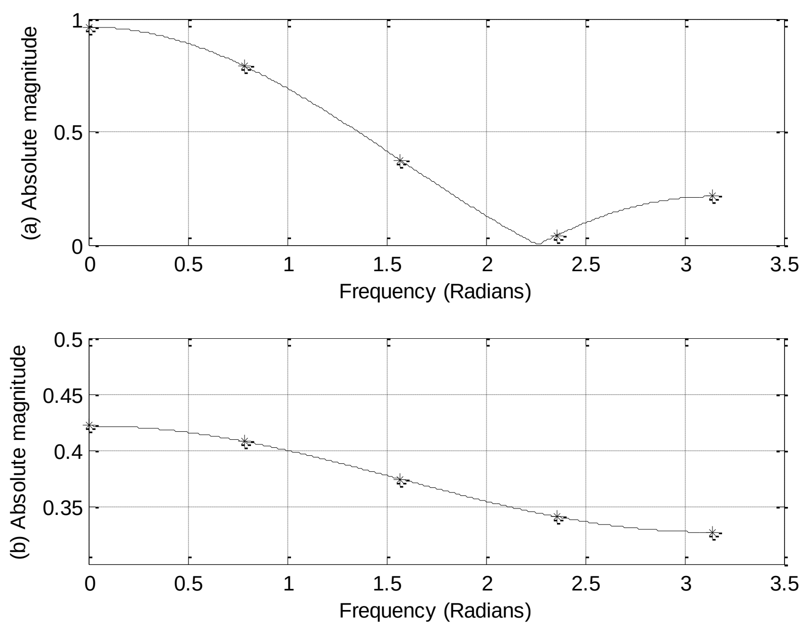
$H(e^{j\Omega}) = e^{-j\Omega}\left(0.3750 + 0.5882\cos(\Omega)\right)$

b. $H(z) = 0.0235 + 0.3750z^{-1} + 0.0235z^{-2}$

$y(n) = 0.0235x(n) + 0.3750x(n-1) + 0.0235x(n-2)$

$H(e^{j\Omega}) = e^{-j\Omega}\left(0.3750 + 0.0470\cos(\Omega)\right)$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	0.9623	0.4220
0.25π	0.7909	0.4082
0.50π	0.3750	0.3750
0.75π	0.0409	0.3418
1.00π	0.2132	0.3280



7.2

a. $H(z) = -0.3027 + 0.6000z^{-1} - 0.3027z^{-2}$

$y(n) = -0.3027x(n) + 0.6000x(n-1) - 0.3027x(n-2)$

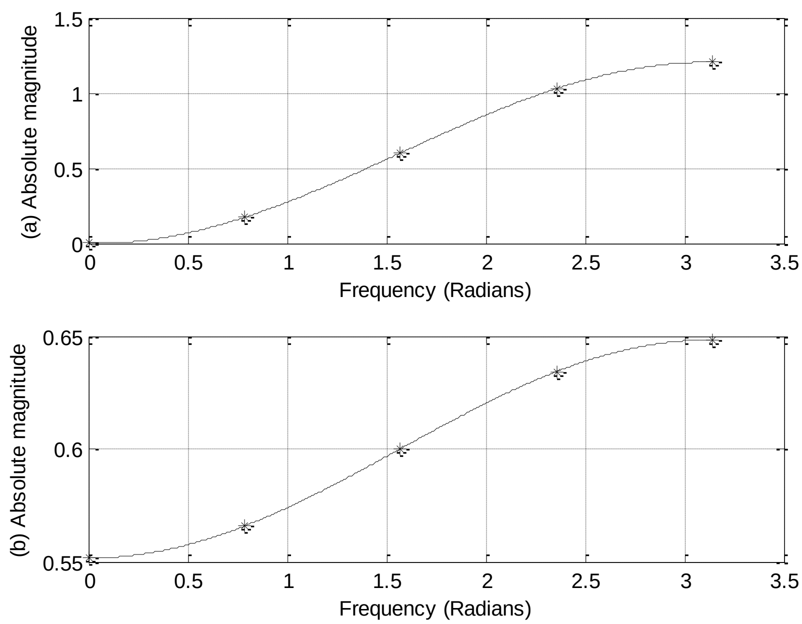
$H(e^{j\Omega}) = e^{-j\Omega}\left(0.6000 - 0.6054\cos(\Omega)\right)$

b. $H(z) = -0.0242 + 0.6000z^{-1} - 0.0242z^{-2}$

$y(n) = -0.0242x(n) + 0.6000x(n-1) - 0.0242x(n-2)$

$H(e^{j\Omega}) = e^{-j\Omega}\left(0.6000 - 0.0484\cos(\Omega)\right)$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	0.0054	0.5520
0.25π	0.1719	0.5661
0.50π	0.6000	0.6000
0.75π	1.0281	0.6339
1.00π	1.2054	0.6480



7.3

a. $H(z) = 0.1514 + 0.1871z^{-1} + 0.2000z^{-2} + 0.1871z^{-3} + 0.1514z^{-4}$

$y(n) = 0.1514x(n) + 0.1871x(n-1) + 0.2x(n-2) + 0.1871x(n-3) + 0.1514x(n-4)$

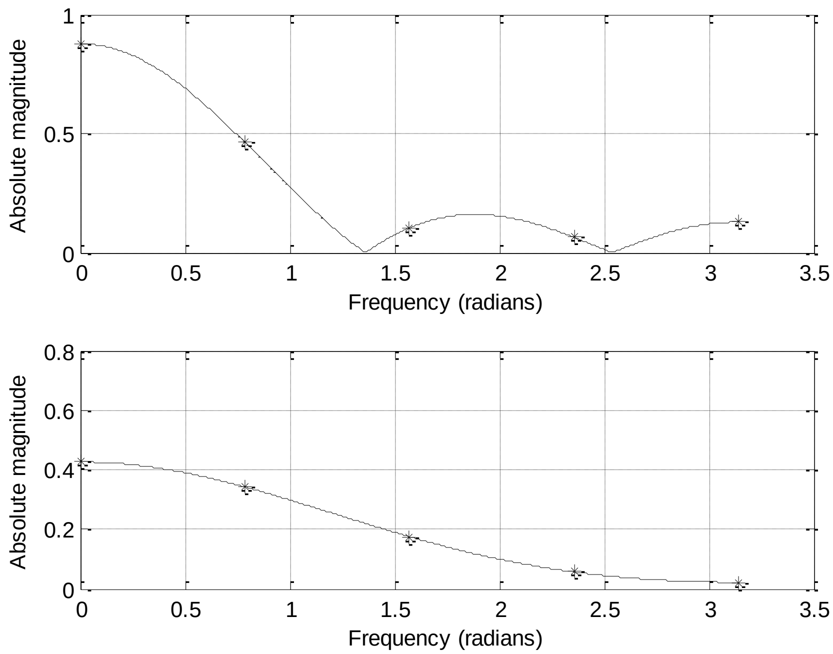
$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.2000 + 0.3562\cos(\Omega) + 0.3028\cos(2\Omega)\right)$

b. $H(z) = 0.0121 + 0.1010z^{-1} + 0.2000z^{-2} + 0.1010z^{-3} + 0.0121z^{-4}$

$y(n) = 0.0121x(n) + 0.1010x(n-1) + 0.2x(n-2) + 0.1010x(n-3) + 0.0121x(n-4)$

$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.2000 + 0.2020\cos(\Omega) + 0.0242\cos(2\Omega)\right)$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	0.8770	0.4262
0.25π	0.4646	0.3428
0.50π	0.1028	0.1758
0.75π	0.0646	0.0572
1.00π	0.1286	0.0222



7.4

a. $H(z) = -0.0000 - 0.3183z^{-1} + 0.5000z^{-2} - 0.3183z^{-3} - 0.0000z^{-4}$

$y(n) = -0.3183x(n-1) + 0.5x(n-2) - 0.3183x(n-3)$

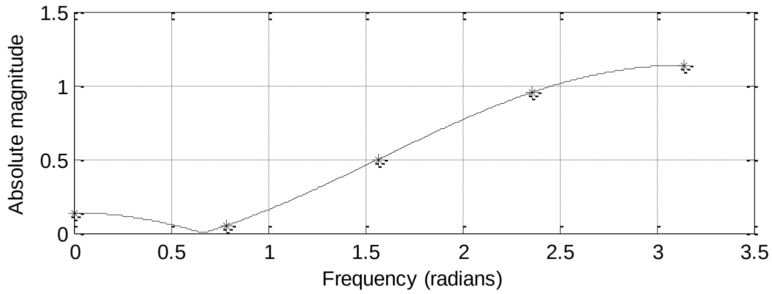
$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.5000 - 0.6366\cos(\Omega)\right)$

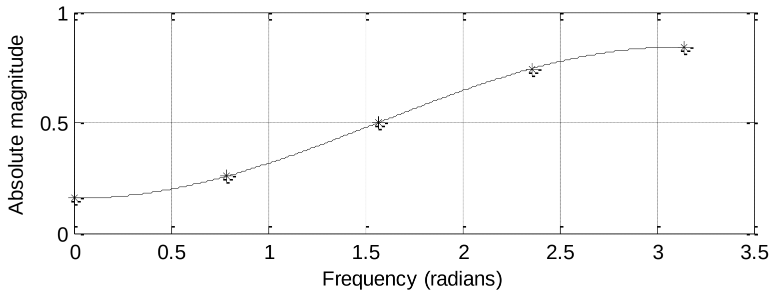
b. $H(z) = -0.0 - 0.1719z^{-1} + 0.5000z^{-2} - 0.1719z^{-3} - 0.0z^{-4}$

$y(n) = -0.1719x(n-1) + 0.5x(n-2) - 0.1719x(n-3)$

$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.5000 - 0.3438\cos(\Omega)\right)$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	0.1366	0.1562
0.25π	0.0499	0.2569
0.50π	0.5000	0.5000
0.75π	0.9501	0.7431
1.00π	1.1366	0.8438





7.5

a. $H(z) = -0.0444 + 0.0117z^{-1} + 0.0500z^{-2} + 0.0117z^{-3} - 0.0444z^{-4}$

$y(n) = -0.0444x(n) + 0.0117x(n-1) + 0.0500x(n-2) + 0.0117x(n-3) - 0.0444x(n-4)$

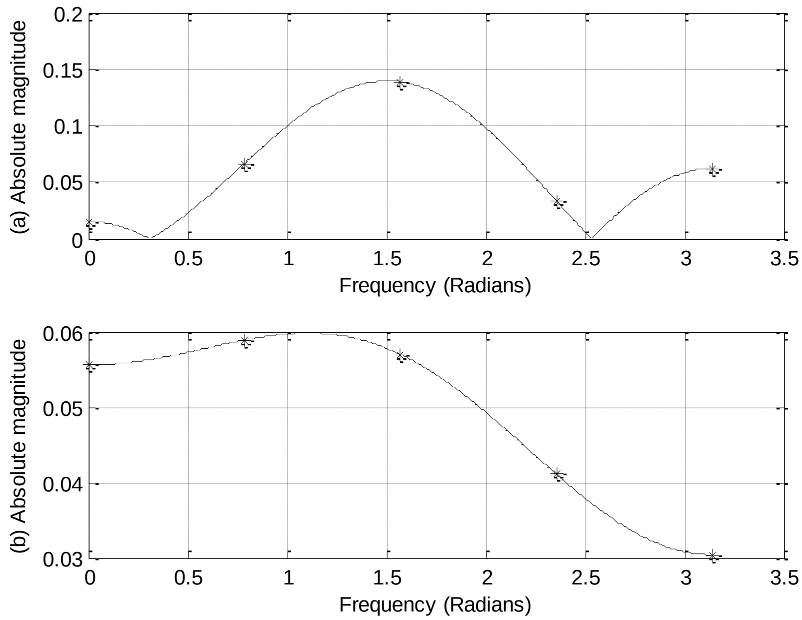
$$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.0500 + 0.0234\cos\Omega - 0.0888\cos 2\Omega\right)$$

b. $H(z) = -0.0035 + 0.0063z^{-1} + 0.0500z^{-2} + 0.0063z^{-3} - 0.0035z^{-4}$

$y(n) = -0.0035x(n) + 0.0063x(n-1) + 0.0500x(n-2) + 0.0063x(n-3) - 0.0035x(n-4)$

$$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.0500 + 0.0126\cos\Omega - 0.0070\cos 2\Omega\right)$$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	0.0154	0.0556
0.25π	0.0665	0.0589
0.50π	0.1388	0.0570
0.75π	0.0335	0.0411
1.00π	0.0622	0.2132



7.6

a. $H(z) = 0.0444 - 0.0117z^{-1} + 0.9500z^{-2} - 0.0117z^{-3} + 0.0444z^{-4}$

$y(n) = 0.0444x(n) - 0.0117x(n-1) + 0.9500x(n-2) - 0.0117x(n-3) + 0.0444x(n-4)$

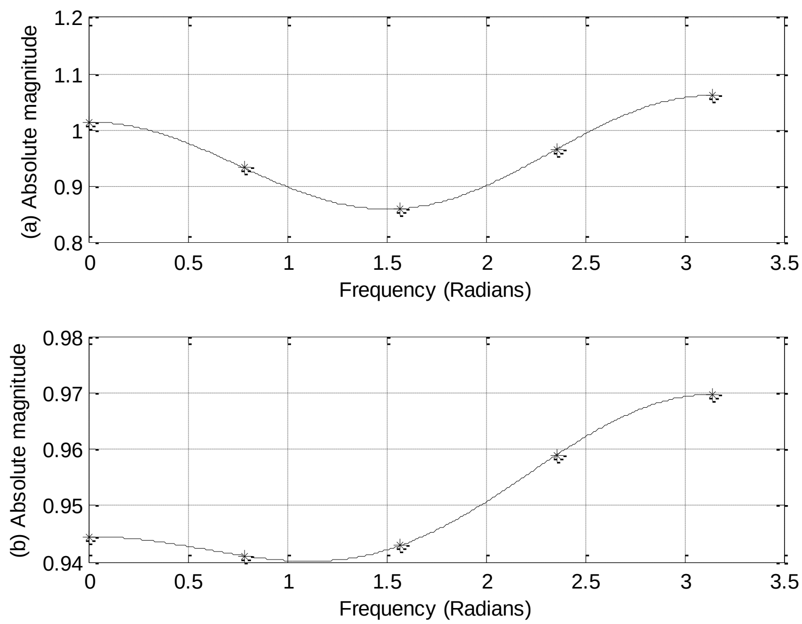
$$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.9500 - 0.0234\cos\Omega + 0.0888\cos 2\Omega\right)$$

b. $H(z) = 0.0035 - 0.0063z^{-1} + 0.9500z^{-2} - 0.0063z^{-3} + 0.0035z^{-4}$

$y(n) = 0.0035x(n) - 0.0063x(n-1) + 0.9500x(n-2) - 0.0063x(n-3) + 0.0035x(n-4)$

$$H(e^{j\Omega}) = e^{-j2\Omega}\left(0.9500 - 0.0126\cos\Omega + 0.0070\cos 2\Omega\right)$$

Ω (radians)	(a) $\lvert H(e^{j\Omega})\rvert$	(b) $\lvert H(e^{j\Omega})\rvert$
0	1.0154	0.9444
0.25π	0.9335	0.9411
0.50π	0.8612	0.9430
0.75π	0.9665	0.9589
1.00π	1.0622	0.9696



7.7

a. Hanning window, b. filter length =63, c. cut-off frequency = 1000 Hz

7.8

a. Blackman window, b. filter length =89, c. cut-off frequency = 1750 Hz

7.9

a. Hamming window

b. filter length =45

c. lower cut-off frequency = 1500 Hz, upper cut-off frequency =2300 Hz

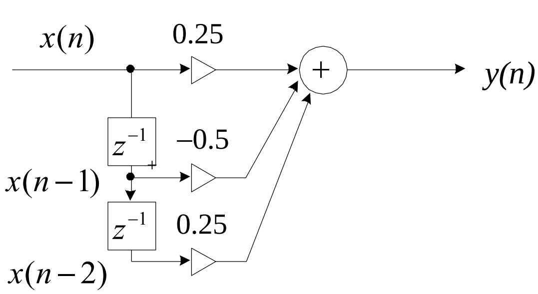
7.10

a. Blackman window, b. filter length =111

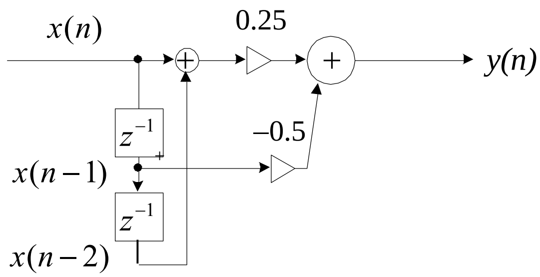
c. lower cut-off frequency = 1400 Hz, upper cut-off frequency =2200 Hz

7.11

a. $y(n) = 0.25x(n) - 0.5x(n-1) + 0.25x(n-2)$

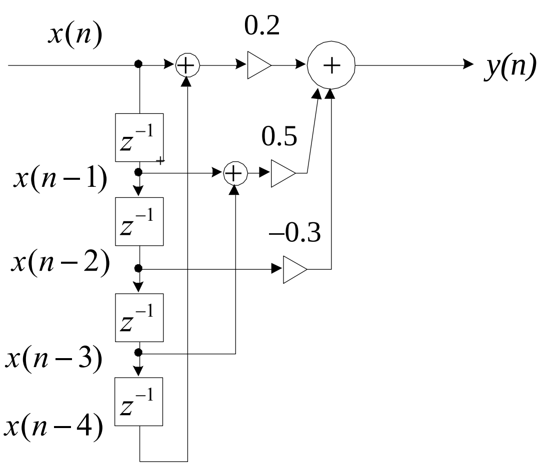


b. $y(n) = 0.25\,x(n) + x(n-2) \; - 0.5x(n-1)$



7.12

$$y(n) = 0.2\big(x(n) + x(n-4)\big) + 0.5\big(x(n-1) + x(n-3)\big) - 0.3x(n-3)$$



7.13

$N = 3$, $\Omega_c = 3\pi/10$, $\Omega_0 = 0$, $H_0 = 1$, $\Omega_1 = 2\pi/3$, $H_1 = 0$

$H(z) = 0.3333 + 0.3333z^{-1} + 0.3333z^{-2}$

7.14

$N = 3$, $\Omega_c = \pi/2$, $\Omega_0 = 0$, $H_0 = 0$, $\Omega_1 = 2\pi/3$, $H_1 = 1$

$H(z) = -0.3333 + 0.6667z^{-1} - 0.3333z^{-2}$

7.15

$N = 5$, $\Omega_0 = 0$, $H_0 = 1$, $\Omega_1 = 2\pi/5$, $H_1 = 1$, $\Omega_2 = 4\pi/5$, $H_2 = 0$

$H(z) = -0.1236 + 0.3236z^{-1} + 0.6z^{-2} + 0.3236z^{-3} - 0.1236z^{-4}$

7.16

$N = 5$, $\Omega_0 = 0$, $H_0 = 0$, $\Omega_1 = 2\pi/5$, $H_1 = 0$, $\Omega_2 = 4\pi/5$, $H_2 = 1$

$H(z) = 0.1236 - 0.3236z^{-1} + 0.4z^{-2} - 0.3236z^{-3} + 0.1236z^{-4}$

7.17

$N = 7$, $\Omega_0 = 0$, $H_0 = 0$, $\Omega_1 = 2\pi/7$, $H_1 = 0$,

$\Omega_2 = 4\pi/7$, $H_2 = 1$, $\Omega_3 = 6\pi/7$, $H_3 = 0$

$H(z) = 0.1718 - 0.2574z^{-1} - 0.0636z^{-2} + 0.2857z^{-3} - 0.0636z^{-4} - 0.2574z^{-5} + 0.1781z^{-6}$

7.18

$N = 7$, $\Omega_0 = 0$, $H_0 = 1$, $\Omega_1 = 2\pi/7$, $H_1 = 1$,

$\Omega_2 = 4\pi/7$, $H_2 = 0$, $\Omega_3 = 6\pi/7$, $H_3 = 1$

$H(z) = -0.1718 + 0.2574z^{-1} + 0.0636z^{-2} + 0.7143z^{-3} + 0.0636z^{-4} + 0.2574z^{-5} - 0.1781z^{-6}$

7.19 $\delta_p = 10^{(1/20)} - 1 = 0.1221$, $\delta_s = 10^{(-40/20)} = 0.01$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.01} \approx \frac{12}{1} = \frac{W_s}{W_p}$$

$W_p = 1$, $W_s = 12$

7.20 $\delta_p = 10^{(1/20)} - 1 = 0.1221$, $\delta_s = 10^{(-30/20)} = 0.0316$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.0316} \approx \frac{39}{10} = \frac{W_s}{W_p}$$

$W_p = 10$, $W_s = 39$

7.21 $\delta_p = 10^{(1/20)} - 1 = 0.1221$, $\delta_s = 10^{(-60/20)} = 0.001$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.001} \approx \frac{122}{1} = \frac{W_s}{W_p}$$

$W_p = 1$, $W_s = 122$

7.22 $\delta_p = 10^{(1/20)} - 1 = 0.1221$, $\delta_s = 10^{(-25/20)} = 0.0562$

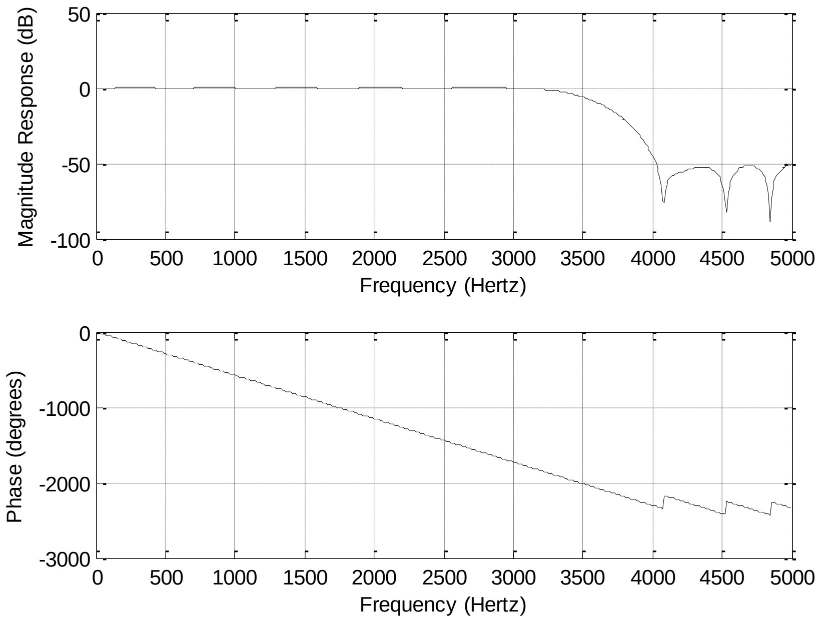
$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.0562} \approx \frac{22}{10} = \frac{W_s}{W_p}$$

$W_p = 10$, $W_s = 22$

7.23

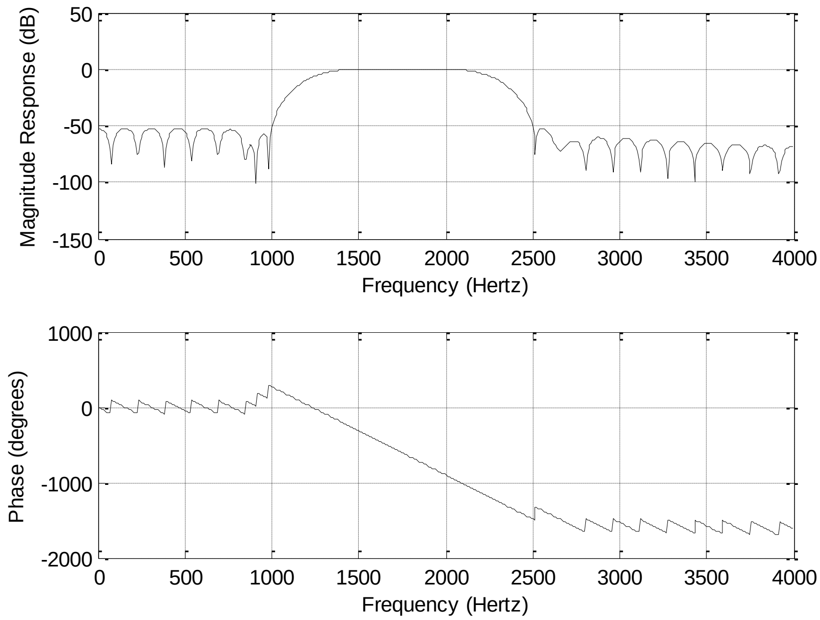
Hamming window, filter length= 33, lower cut-off frequency =3500 Hz

```
>> B=firwd(33,1,3500*2*pi/10000,0,4);
B =  -0.0009    0.0019   -0.0015   -0.0012    0.0054   -0.0067    0.0000    0.0129   -0.0204
      0.0088    0.0223   -0.0506    0.0405    0.0302   -0.1461    0.2552    0.7000    0.2552
     -0.1461    0.0302    0.0405   -0.0506    0.0223    0.0088   -0.0204    0.0129    0.0000
     -0.0067    0.0054   -0.0012   -0.0015    0.0019   -0.0009
>> freqz(B,1,512,10000)
```



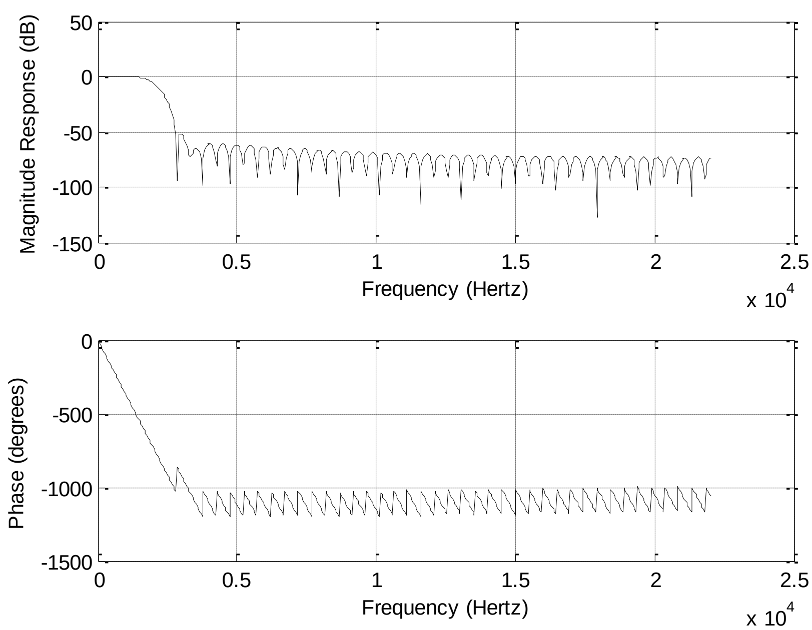
7.24
Hamming window, filter length=53,
lower cut-off frequency=1250 Hz, upper cut-off frequency=2250 Hz

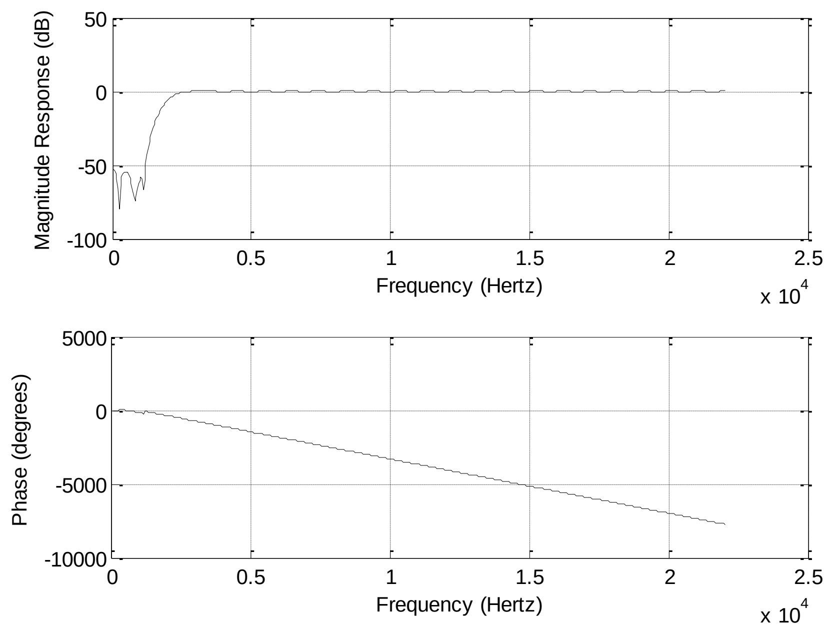
```
>> B=firwd(53,3,1250*2*pi/8000,2250*2*pi/8000,4);
B = 0.0005    0.0008         0    0.0011    0.0010   -0.0038   -0.0044    0.0040    0.0064
   -0.0009    0.0000    0.0014   -0.0144   -0.0136    0.0223    0.0289   -0.0121   -0.0200
         0   -0.0288   -0.0254    0.0898    0.1066   -0.1057   -0.2052    0.0474    0.2500
    0.0474   -0.2052   -0.1057    0.1066    0.0898   -0.0254   -0.0288         0   -0.0200
   -0.0121    0.0289    0.0223   -0.0136   -0.0144    0.0014    0.0000   -0.0009    0.0064
    0.0040   -0.0044   -0.0038    0.0010    0.0011         0    0.0008    0.0005
>> freqz(B,1,512,8000)
```



7.25
Lowpass filter: Hamming window, filter length=91, cut-off frequency =2000 Hz
High filter: Hamming window, filter length=91, cut-off frequency =2000 Hz

```
>> BL=firwd(91,1,2000*2*pi/44100,0,4);
>> freqz(BL,1,512,44100)
>> BH=firwd(91,2,0,2000*2*pi/44100,4);
>> freqz(BH,1,512,44100)
```

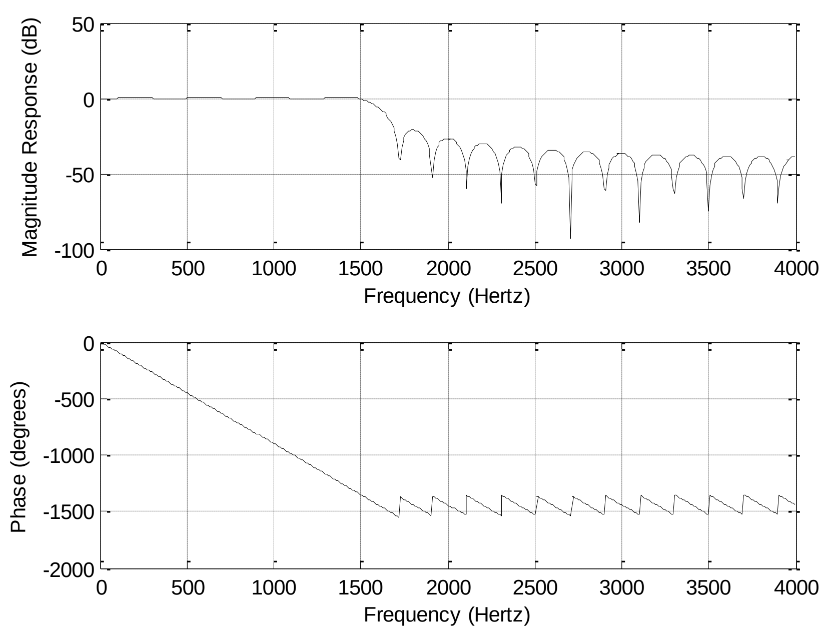




7.26

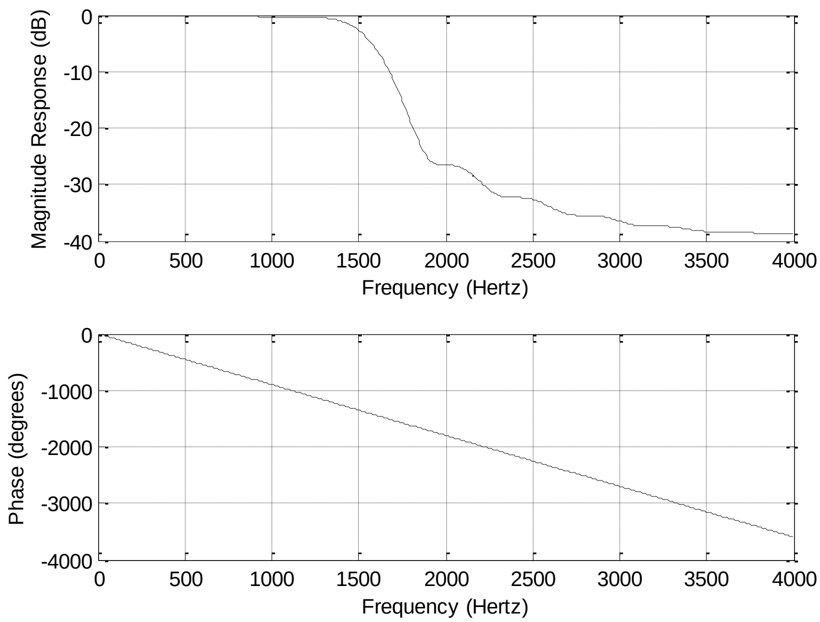
a.

```
>> B1=firwd(41,1,1600*2*pi/8000,0,1);
B =  -0.0000   -0.0159   -0.0104    0.0110    0.0189   -0.0000   -0.0216   -0.0144    0.0156
 0.0275   -0.0000   -0.0336   -0.0234    0.0267    0.0505   -0.0000   -0.0757   -0.0624
 0.0935    0.3027    0.4000    0.3027    0.0935   -0.0624   -0.0757   -0.0000    0.0505
  0.0267   -0.0234   -0.0336   -0.0000    0.0275    0.0156   -0.0144   -0.0216   -0.0000
  0.0189    0.0110   -0.0104   -0.0159   -0.0000
>> freqz(B1,1,512,8000)
```



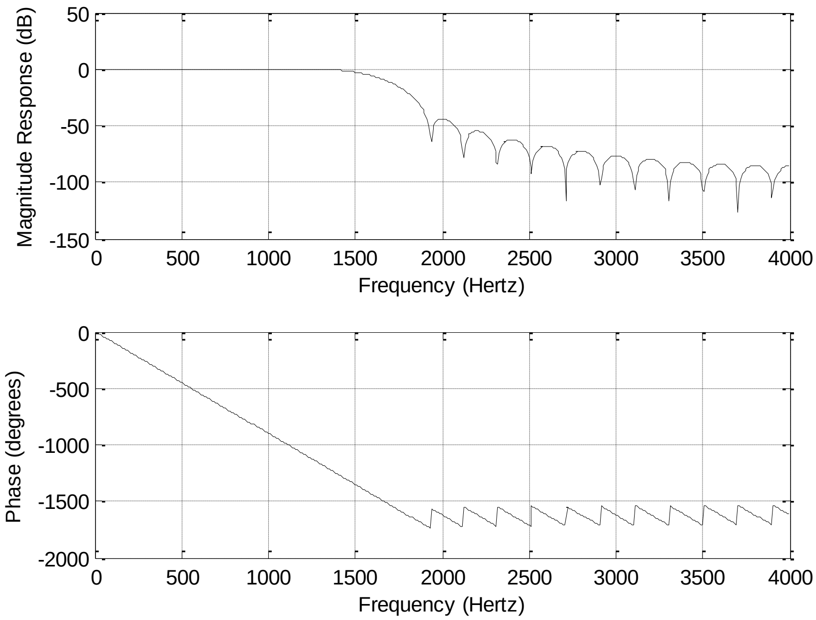
b.

```
>> B2=firwd(41,1,1600*2*pi/8000,0,2);
B = 0   -0.0008   -0.0010    0.0017    0.0038   -0.0000   -0.0065   -0.0050    0.0062
    0.0124   -0.0000   -0.0185   -0.0140    0.0174    0.0353   -0.0000   -0.0605   -0.0530
    0.0842    0.2876    0.4000    0.2876    0.0842   -0.0530   -0.0605   -0.0000    0.0353
    0.0174   -0.0140   -0.0185   -0.0000    0.0124    0.0062   -0.0050   -0.0065   -0.0000
    0.0038    0.0017   -0.0010   -0.0008         0
>> freqz(B2,1,512,8000)
```



c.

```
>> B3=firwd(41,1,1600*2*pi/8000,0,3);
B = 0   -0.0001   -0.0003    0.0006    0.0018   -0.0000   -0.0045   -0.0039    0.0054
    0.0116   -0.0000   -0.0194   -0.0153    0.0194    0.0401   -0.0000   -0.0685   -0.0590
    0.0913    0.3009    0.4000    0.3009    0.0913   -0.0590   -0.0685   -0.0000    0.0401
    0.0194   -0.0153   -0.0194   -0.0000    0.0116    0.0054   -0.0039   -0.0045   -0.0000
    0.0018    0.0006   -0.0003   -0.0001         0
>> freqz(B3,1,512,8000)
```

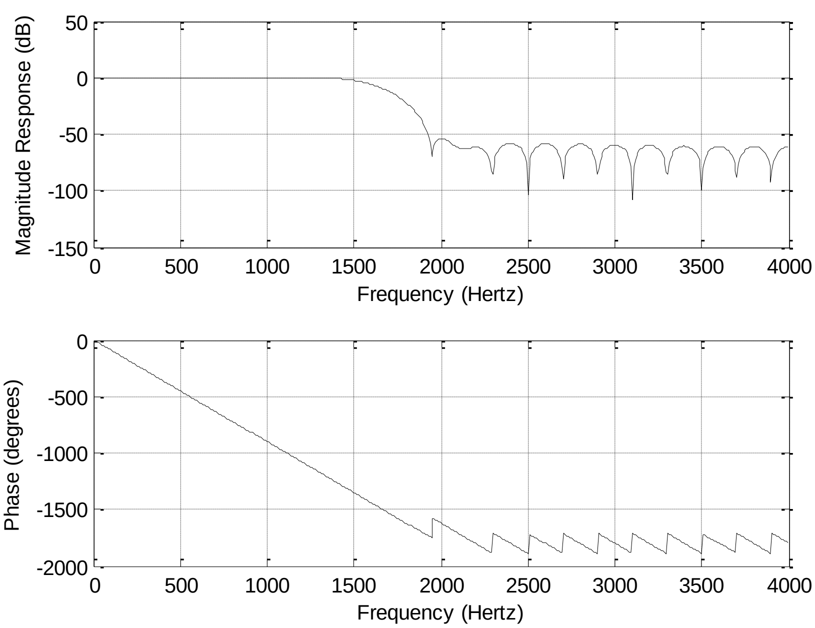


d.

\>> B4=firwd(41,1,1600*2*pi/8000,0,4);

B = -0.0000 -0.0014 -0.0011 0.0014 0.0032 -0.0000 -0.0058 -0.0048 0.0062
0.0129 -0.0000 -0.0206 -0.0160 0.0200 0.0409 -0.0000 -0.0690 -0.0592
0.0914 0.3010 0.4000 0.3010 0.0914 -0.0592 -0.0690 -0.0000 0.0409
0.0200 -0.0160 -0.0206 -0.0000 0.0129 0.0062 -0.0048 -0.0058 -0.0000
0.0032 0.0014 -0.0011 -0.0014 -0.0000

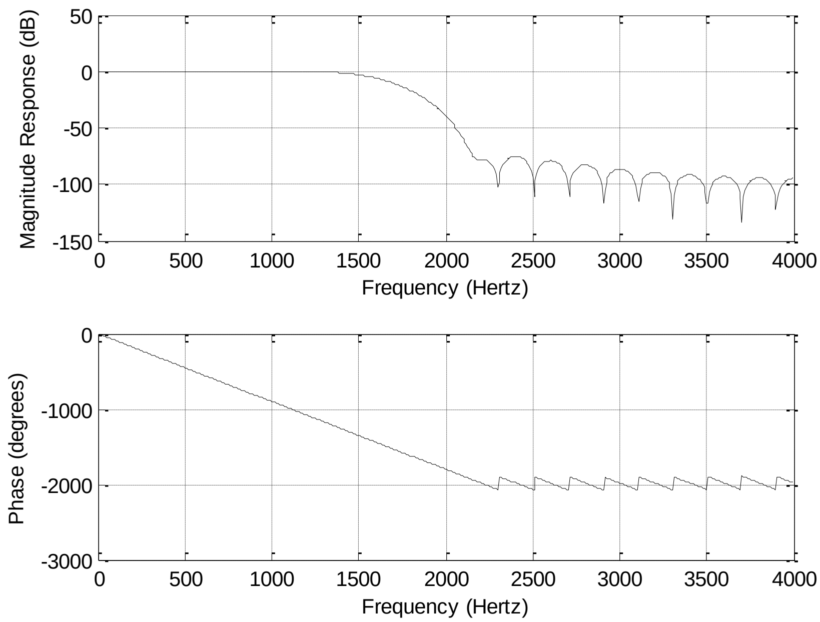
\>> freqz(B4,1,512,8000)



e.

\>> B5=firwd(41,1,1600*2*pi/8000,0,5);

B = 0.0000 -0.0000 -0.0001 0.0002 0.0008 -0.0000 -0.0022 -0.0021 0.0031
0.0073 -0.0000 -0.0142 -0.0119 0.0160 0.0348 -0.0000 -0.0643 -0.0569
0.0898 0.2997 0.4000 0.2997 0.0898 -0.0569 -0.0643 -0.0000 0.0348
0.0160 -0.0119 -0.0142 -0.0000 0.0073 0.0031 -0.0021 -0.0022 -0.0000
0.0008 0.0002 -0.0001 -0.0000 0.0000
>> freqz(B5,1,512,8000)



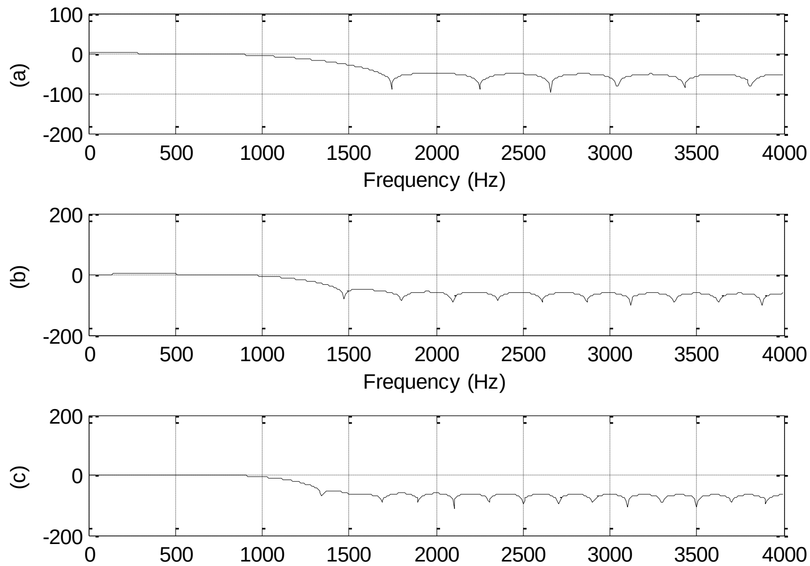
7.27
a.
>> B1=firwd(21,1,1000*2*pi/8000,0,4);
B = 0.0025 0.0026 -0.0000 -0.0087 -0.0211 -0.0243 0.0000 0.0608 0.1452
0.2200 0.2500 0.2200 0.1452 0.0608 0.0000 -0.0243 -0.0211 -0.0087
-0.0000 0.0026 0.0025
>> [h1 f]=freqz(B1,1,512,8000);

b.
>> B2=firwd(31,1,1000*2*pi/8000,0,4);
B = -0.0012 -0.0020 -0.0021 0.0000 0.0048 0.0099 0.0099 -0.0000 -0.0189
-0.0362 -0.0347 0.0000 0.0684 0.1528 0.2228 0.2500 0.2228 0.1528
0.0684 0.0000 -0.0347 -0.0362 -0.0189 -0.0000 0.0099 0.0099 0.0048
0.0000 -0.0021 -0.0020 -0.0012
>> [h2 f]=freqz(B2,1,512,8000);

c.
>> B3=firwd(41,1,1000*2*pi/8000,0,4);

B = 0.0000 0.0010 0.0018 0.0017 -0.0000 -0.0032 -0.0061 -0.0057 0.0000
0.0096 0.0172 0.0153 -0.0000 -0.0241 -0.0430 -0.0390 0.0000 0.0713
0.1556 0.2238 0.2500 0.2238 0.1556 0.0713 0.0000 -0.0390 -0.0430

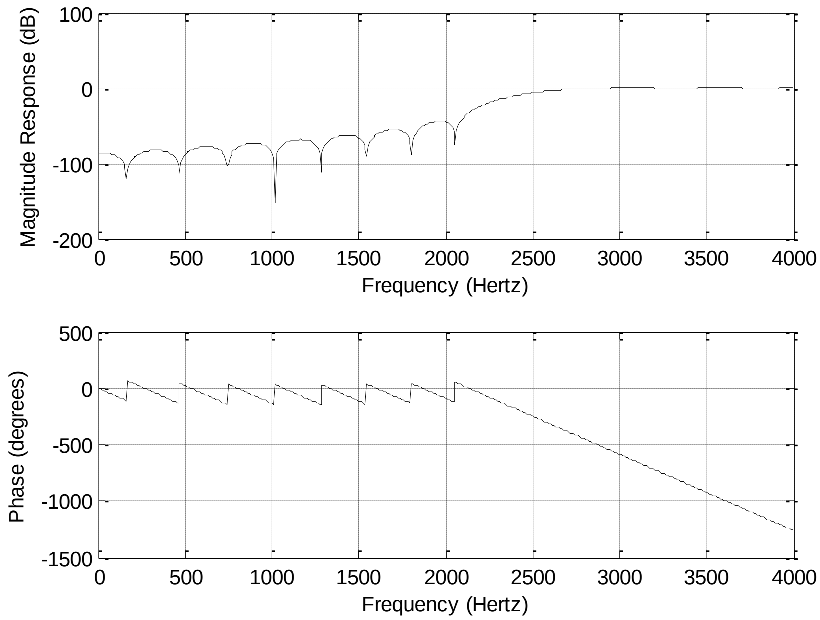
```
  -0.0241   -0.0000    0.0153    0.0172    0.0096    0.0000   -0.0057   -0.0061   -0.0032
  -0.0000    0.0017    0.0018    0.0010    0.0000
>> [h3 f]=freqz(B3,1,512,8000);

>> subplot(3,1,1),plot(f,20*log10(abs(h1)));grid
>> subplot(3,1,2),plot(f,20*log10(abs(h2)));grid
>> subplot(3,1,3),plot(f,20*log10(abs(h3)));grid
```



7.28

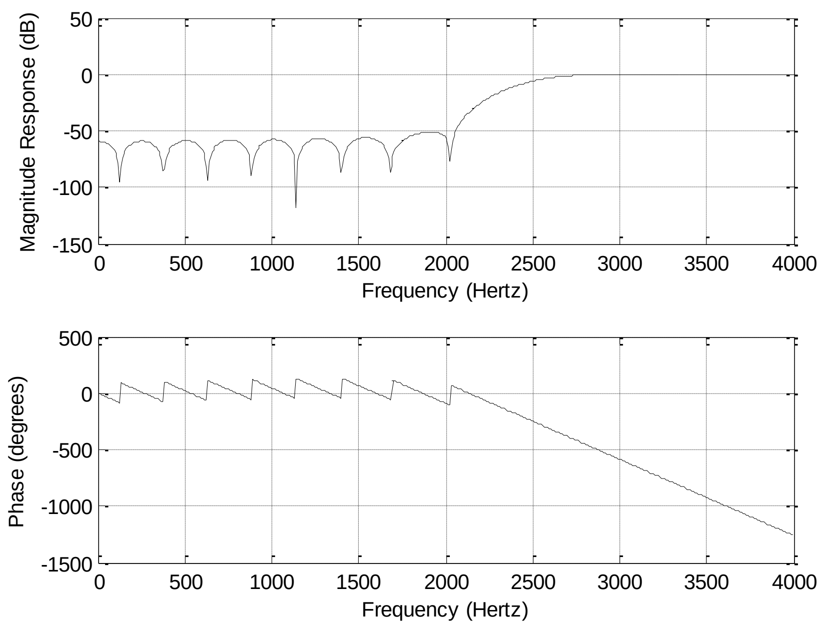
a.

```
 >> B1=firwd(31,2,0,2500*2*pi/8000,3);
B =         0   -0.0002   -0.0004    0.0025   -0.0018   -0.0056    0.0113   -0.0000   -0.0232
    0.0246    0.0183   -0.0664    0.0367    0.1077   -0.2909    0.3750   -0.2909    0.1077
    0.0367   -0.0664    0.0183    0.0246   -0.0232   -0.0000    0.0113   -0.0056   -0.0018
    0.0025   -0.0004   -0.0002         0
>> freqz(B1,1,512,8000)
```



b.

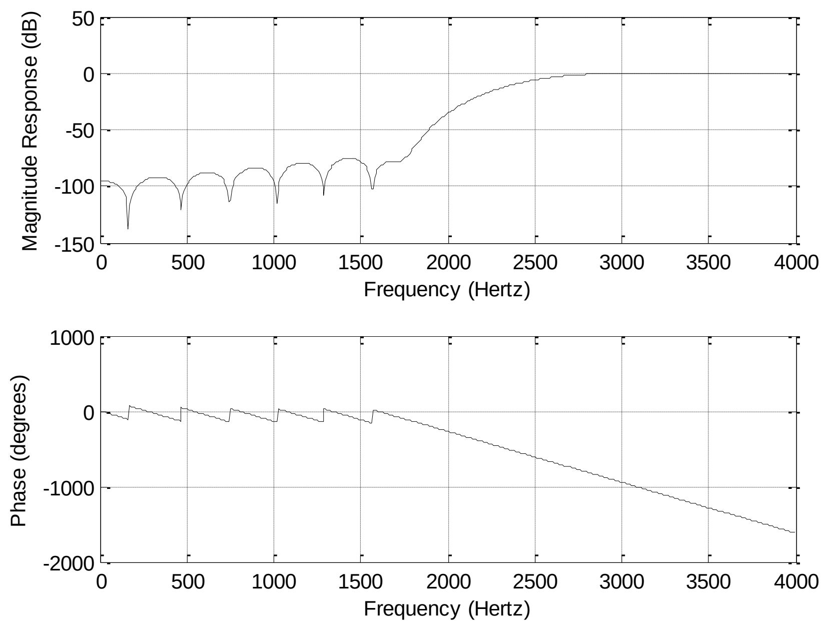
```
>> B2=firwd(31,2,0,2500*2*pi/8000,4);
B =    0.0016   -0.0014   -0.0011    0.0045   -0.0026   -0.0070    0.0130   -0.0000   -0.0247
    0.0256    0.0188   -0.0675    0.0370    0.1081   -0.2911    0.3750   -0.2911    0.1081
    0.0370   -0.0675    0.0188    0.0256   -0.0247   -0.0000    0.0130   -0.0070   -0.0026
    0.0045   -0.0011   -0.0014    0.0016
>> freqz(B2,1,512,8000)
```



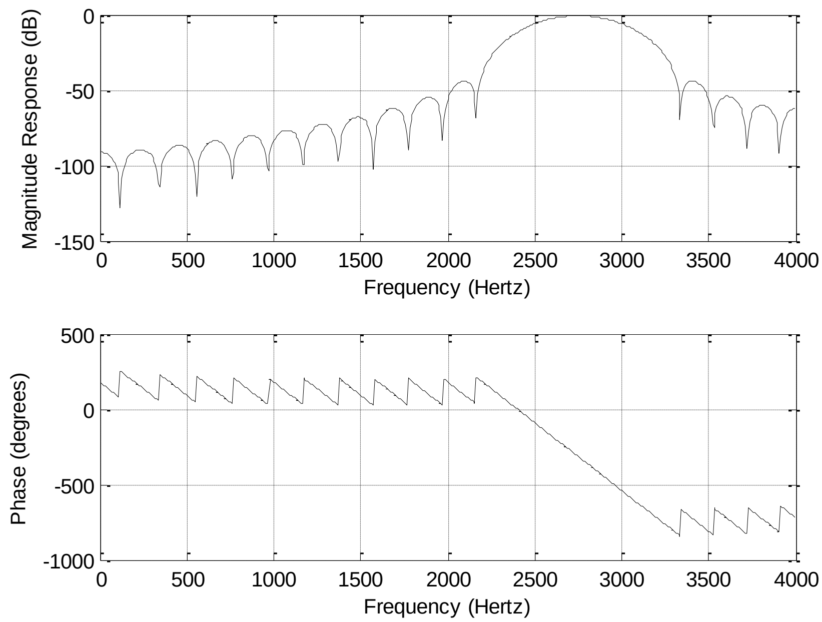
c.

```
>> B3=firwd(31,2,0,2500*2*pi/8000,5);
```

B = -0.0000 -0.0001 -0.0002 0.0011 -0.0009 -0.0029 0.0066 -0.0000 -0.0166
0.0191 0.0153 -0.0594 0.0345 0.1047 -0.2888 0.3750 -0.2888 0.1047
0.0345 -0.0594 0.0153 0.0191 -0.0166 -0.0000 0.0066 -0.0029 -0.0009
0.0011 -0.0002 -0.0001 -0.0000
>> freqz(B3,1,512,8000)

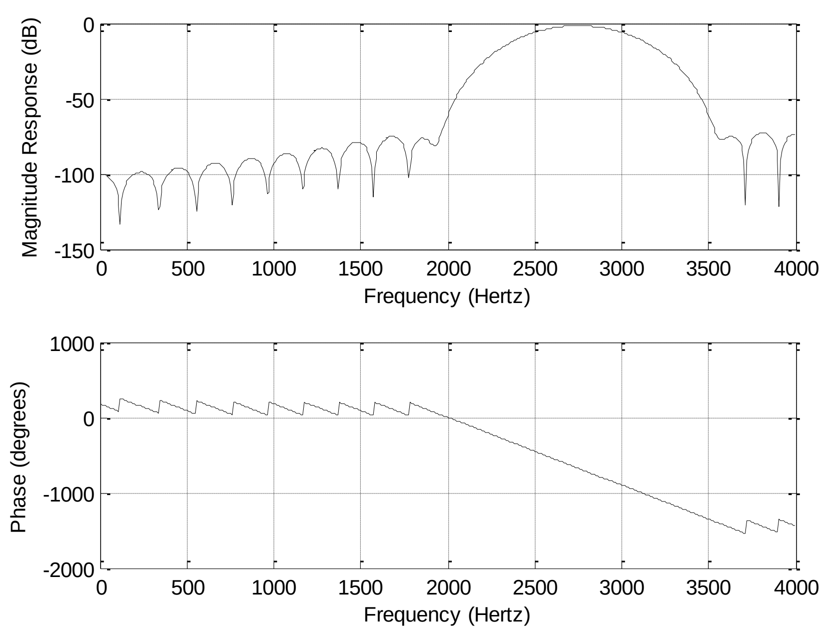


7.29
a.
>> B1=firwd(41,3,2500*2*pi/8000,3000*2*pi/8000,3);
B = 0 0.0001 -0.0001 -0.0002 -0.0000 0.0007 0.0014 -0.0073 0.0092
0.0040 -0.0272 0.0334 -0.0000 -0.0539 0.0719 -0.0176 -0.0720 0.1093
-0.0455 -0.0686 0.1250 -0.0686 -0.0455 0.1093 -0.0720 -0.0176 0.0719
-0.0539 -0.0000 0.0334 -0.0272 0.0040 0.0092 -0.0073 0.0014 0.0007
-0.0000 -0.0002 -0.0001 0.0001 0
>> freqz(B1,1,512,8000)



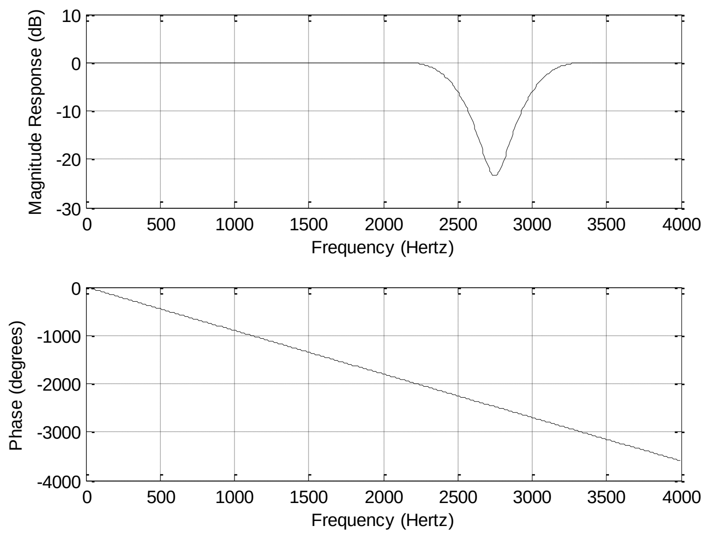
b.

```
>> B2=firwd(41,3,2500*2*pi/8000,3000*2*pi/8000,5);
B =    0.0000    0.0000   -0.0000   -0.0001   -0.0000    0.0003    0.0007   -0.0039    0.0053
    0.0025   -0.0185    0.0244   -0.0000   -0.0445    0.0624   -0.0160   -0.0676    0.1055
   -0.0448   -0.0683    0.1250   -0.0683   -0.0448    0.1055   -0.0676   -0.0160    0.0624
   -0.0445   -0.0000    0.0244   -0.0185    0.0025    0.0053   -0.0039    0.0007    0.0003
   -0.0000   -0.0001   -0.0000    0.0000    0.0000
>> freqz(B2,1,512,8000)
```



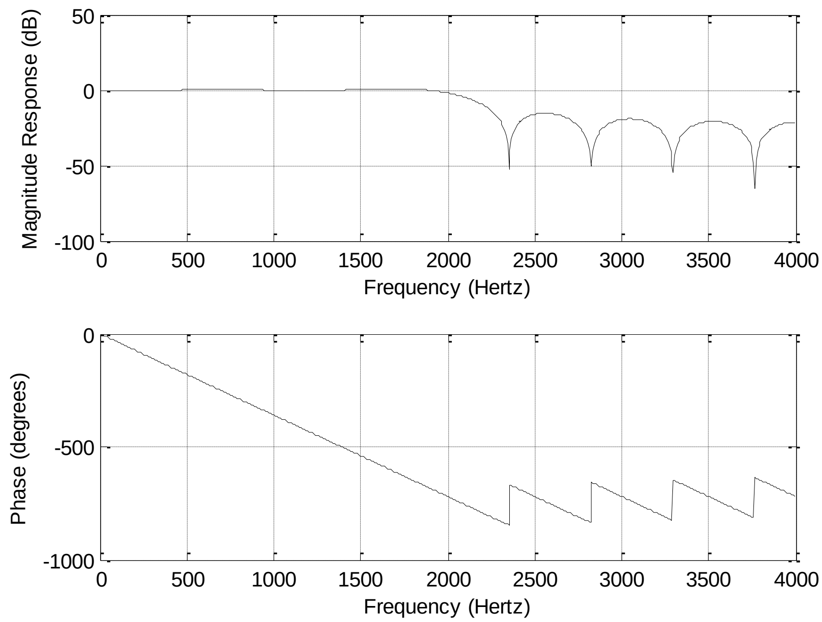
7.30

```
>> B=firwd(41,4,2500*2*pi/8000,3000*2*pi/8000,4);
B =    0.0013   -0.0016    0.0005    0.0005    0.0000   -0.0010   -0.0018    0.0088   -0.0106
   -0.0044    0.0293   -0.0353    0.0000    0.0555   -0.0734    0.0179    0.0726   -0.1098
    0.0456    0.0686    0.8750    0.0686    0.0456   -0.1098    0.0726    0.0179   -0.0734
    0.0555    0.0000   -0.0353    0.0293   -0.0044   -0.0106    0.0088   -0.0018   -0.0010
    0.0000    0.0005    0.0005   -0.0016    0.0013
>> freqz1(B,1,512,8000)
```



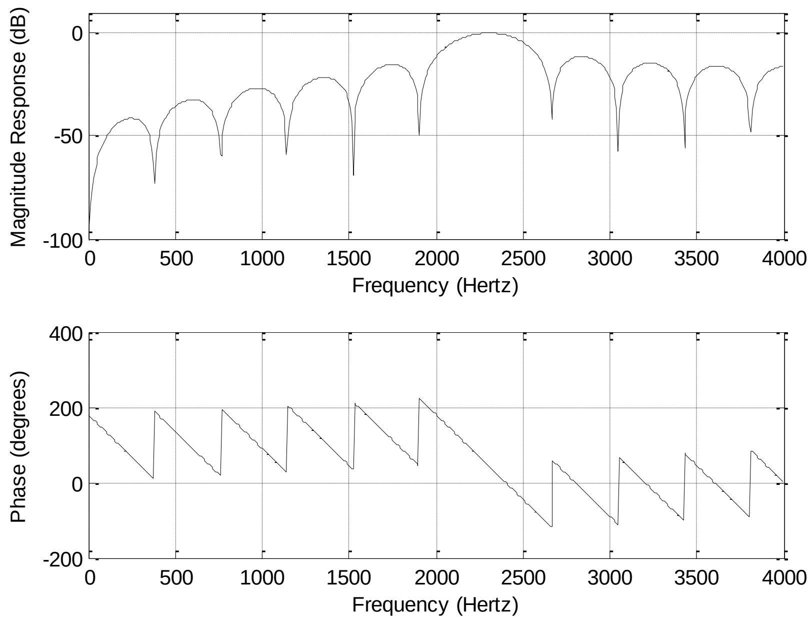
7.31

```
>> B=firfs(17,[1 1 1 1 1 0 0 0 0])
B =    0.0398   -0.0488   -0.0346    0.0660    0.0315   -0.1075   -0.0299    0.3188    0.5294
    0.3188   -0.0299   -0.1075    0.0315    0.0660   -0.0346   -0.0488    0.0398
>> freqz(B,1,512,8000)
```



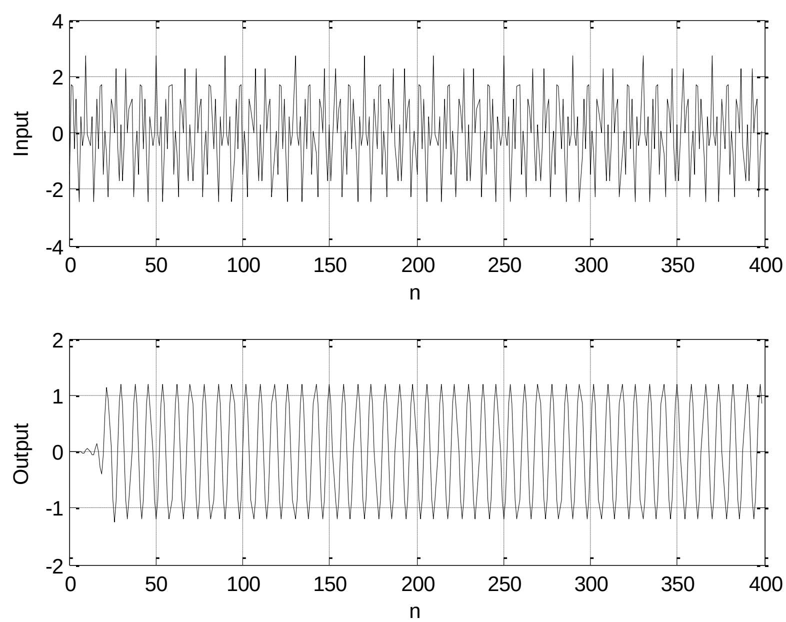
7.32

```
>> B=firfs(21,[0 0 0 0 0 0 1 0 0 0 0])
B =    0.0594   -0.0858   -0.0212    0.0952   -0.0212   -0.0858    0.0594    0.0594   -0.0858
   -0.0212    0.0952   -0.0212   -0.0858    0.0594    0.0594   -0.0858   -0.0212    0.0952
   -0.0212   -0.0858    0.0594
>> freqz(B,1,512,8000)
```



7.33

```
>> B=firwd(41,1,1600*2*pi/8000,0,4);
>> x=1.2*sin(2*pi*1000*[0:1:399]/8000)-1.5*cos(2*pi*2800*[0:1:399]/8000);
>> y=filter(B,1,x);
>> n=0:1:399;
>> x=1.2*sin(2*pi*1000*n/8000)-1.5*cos(2*pi*2800*n/8000);
>> y=filter(B,1,x);
>> subplot(2,1,1),plot(n,x);xlabel('n');ylabel('Input');grid
>> subplot(2,1,2),plot(n,y);xlabel('n');ylabel('Output');grid
```

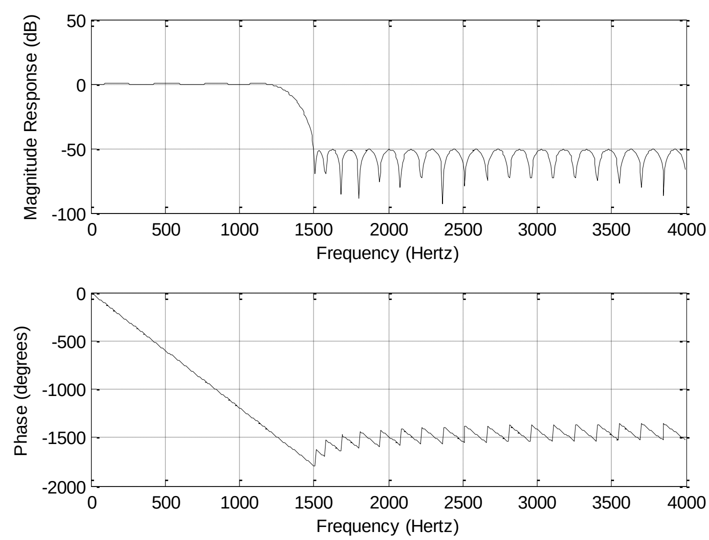


7.34

$\delta_p = 10^{\left(\frac{1}{20}\right)} - 1 = 0.1220$, $\delta_s = 10^{\left(\frac{-40}{20}\right)} = 0.01$, $W_s = 12$ and $W_p = 1$

```
>> fs=8000;
>> f=[0 0.3 0.375 1];
>> m=[1  1   0   0];
>> W=[1 12];
>> b=remez(53,f,m,W);
>> freqz(b,1,512,fs)
```

B =[0.0020 -0.0003 -0.0043 -0.0086 -0.0089 -0.0032 0.0049 0.0083 0.0029
-0.0073 -0.0121 -0.0048 0.0098 0.0173 0.0075 -0.0134 -0.0252 -0.0120
0.0190 0.0387 0.0204 -0.0300 -0.0684 -0.0420 0.0656 0.2114 0.3156
0.3156 0.2114 0.0656 -0.0420 -0.0684 -0.0300 0.0204 0.0387 0.0190
-0.0120 -0.0252 -0.0134 0.0075 0.0173 0.0098 -0.0048 -0.0121 -0.0073
0.0029 0.0083 0.0049 -0.0032 -0.0089 -0.0086 -0.0043 -0.0003 0.0020]



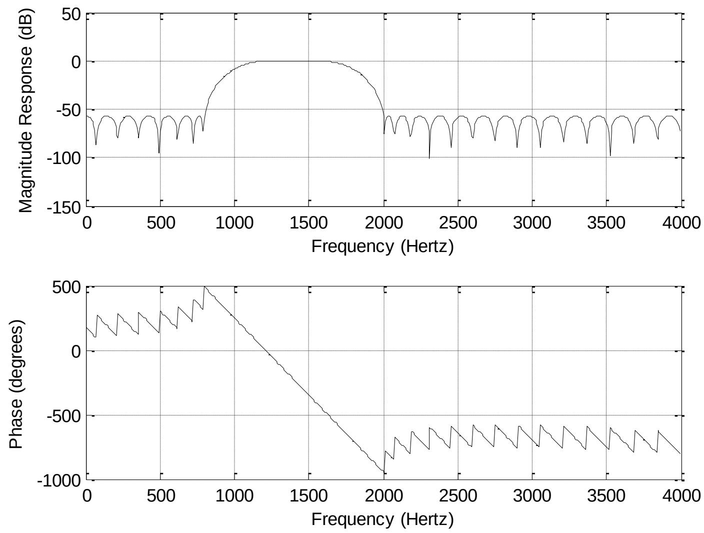
7.35

$\delta_p = 10^{\left(\frac{1}{20}\right)} - 1 = 0.1220$, $\delta_s = 10^{\left(\frac{-40}{20}\right)} = 0.01$, $W_s = 12$ and $W_p = 1$

```
>> fs=8000;
>> f=[0 0.2 0.3 0.4 0.5 1];
>> m=[0 0   1    1   0  0];
>> W=[12 1 12];
>> b=remez(53,f,m,W);
>> freqz1(b,1,512,fs)
```

B =[-0.0004 -0.0013 -0.0000 0.0003 -0.0006 -0.0002 0.0041 0.0068 -0.0010
-0.0140 -0.0140 0.0049 0.0207 0.0126 -0.0057 -0.0066 0.0027 -0.0108

-0.0393	-0.0232	0.0545	0.1041	0.0301	-0.1138	-0.1533	-0.0139	0.1565
0.1565	-0.0139	-0.1533	-0.1138	0.0301	0.1041	0.0545	-0.0232	-0.0393
-0.0108	0.0027	-0.0066	-0.0057	0.0126	0.0207	0.0049	-0.0140	-0.0140
-0.0010	0.0068	0.0041	-0.0002	-0.0006	0.0003	-0.0000	-0.0013	-0.0004



Chapter 8

8.1

$$H(z) = \frac{0.3333 + 0.3333z^{-1}}{1 - 0.3333z^{-1}}$$

$$y(n) = 0.3333x(n) + 0.3333x(n-1) + 0.3333y(n-1)$$

8.2

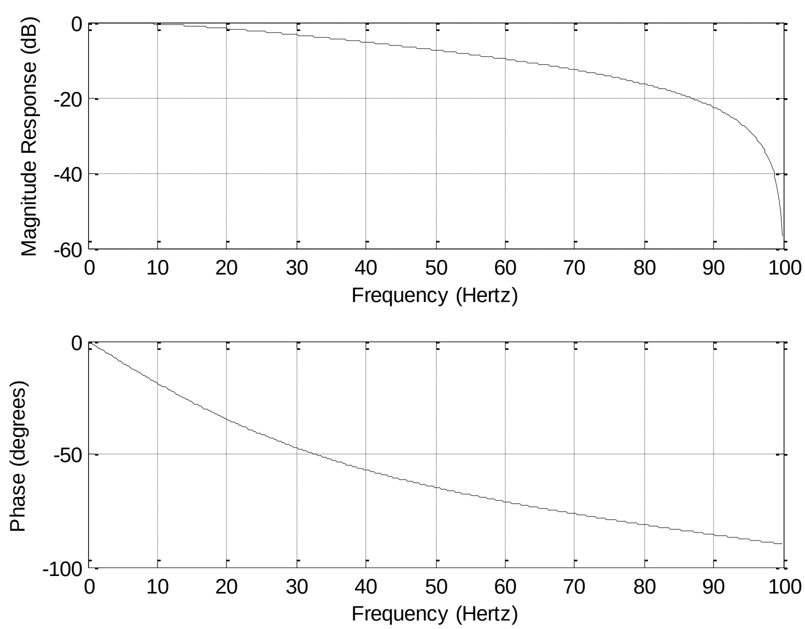
a.

$$H(z) = \frac{0.3375 - 0.3375z^{-1}}{1 - 0.3249z^{-1}}$$

$$y(n) = 0.3375x(n) + 0.3375x(n-1) + 0.3249y(n-1)$$

b.

freqz([0.3375 0.3375],[1 -0.3249],512,200)



8.3

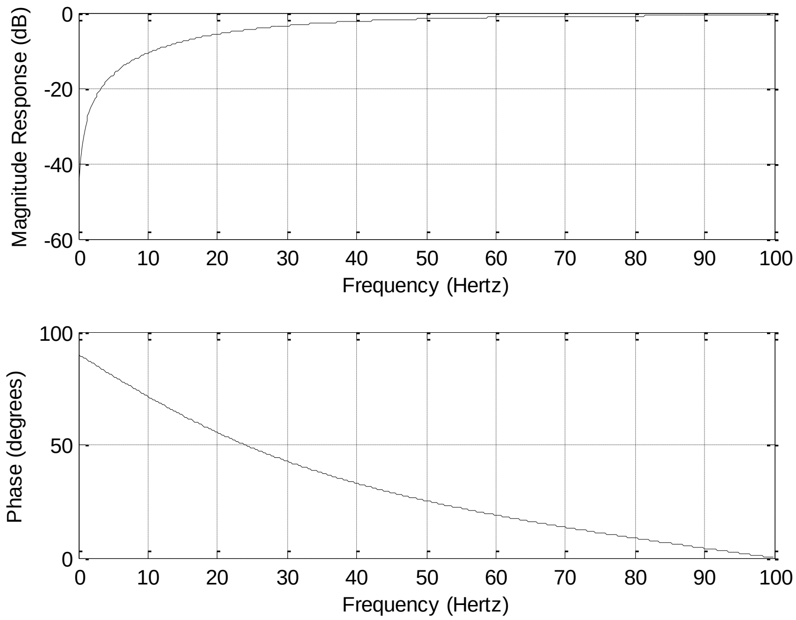
a.

$$H(z) = \frac{0.6625 - 0.6625z^{-1}}{1 - 0.3249z^{-1}}$$

$$y(n) = 0.6225x(n) - 0.6225x(n-1) + 0.3249y(n-1)$$

b.

>> freqz([0.6225 -0.6225],[1 -0.3594],512,200)



8.4

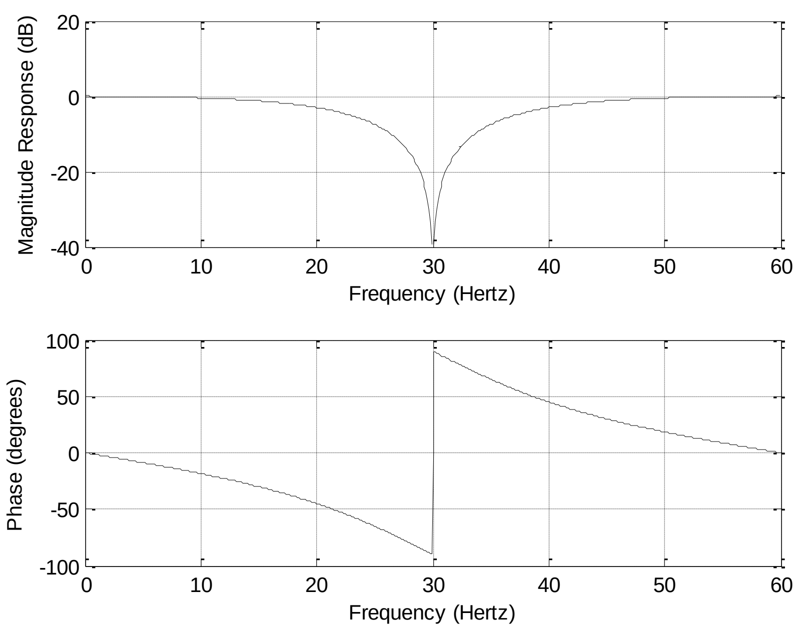
a.

$$H(z) = \frac{0.6340 - 0.6340z^{-2}}{1 + 0.2679z^{-2}} \text{ b =}$$

$$y(n) = 0.6340x(n) - 0.6340x(n-2) - 0.2679y(n-2)$$

b.

>> freqz([0.6340 0 0.6340],[1 0 0.2679],512,120)



8.5

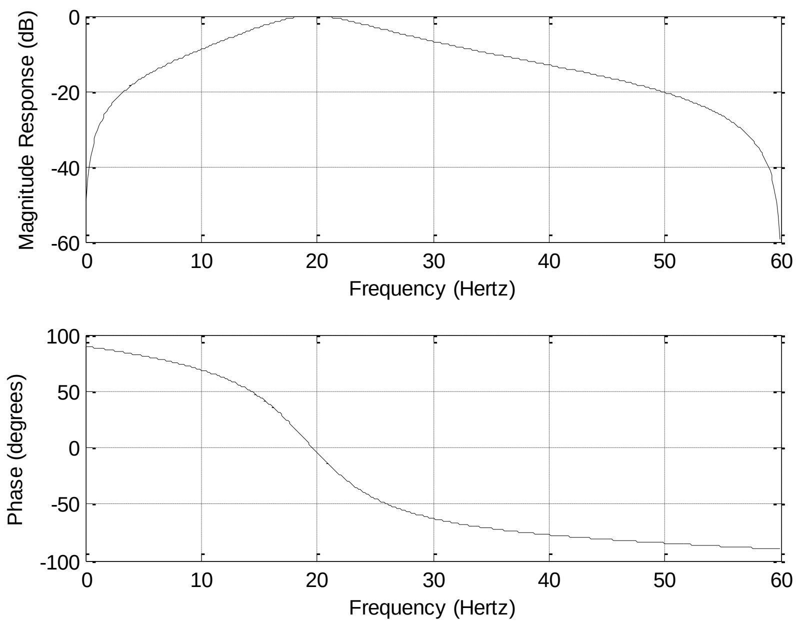
a.

$$H(z)=\frac{0.2113-0.2113z^{-2}}{1-0.8165z^{-1}+0.5774z^{-2}}$$

$$y(n)=0.2113x(n)-0.2113x(n-2)+08165y(n-1)-0.5774y(n-2)$$

b.

```
>> freqz([0.2113 0 -0.2113],[1 -0.8165 0.5774],512,120)
```



8.6

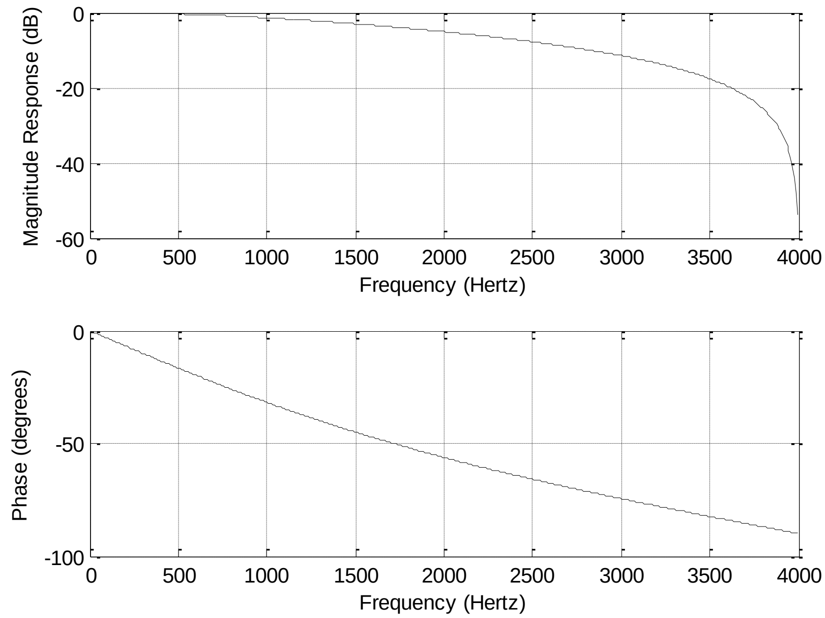
a.

$$H(z)=\frac{0.4005+0.4005z^{-1}}{1-0.1989z^{-1}}$$

$$y(n)=0.4005x(n)+0.4005x(n-1)+0.1989y(n-1)$$

b.

```
>> freqz([0.4005 0.4005],[1 -0.1989],512,8000)
```



8.7

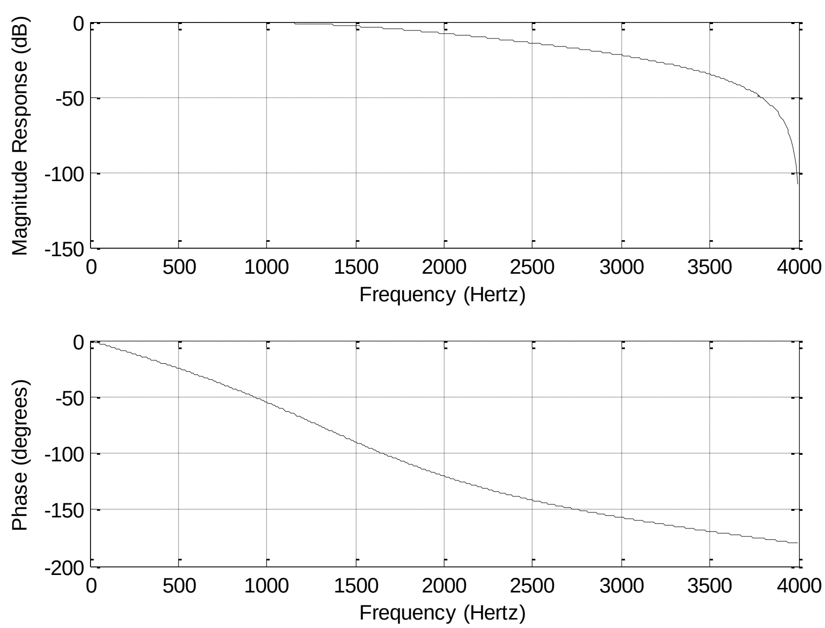
a.

$$H(z)=\frac{0.1867+0.3734z^{-1}+0.1867z^{-2}}{1-0.4629z^{-1}+0.2097z^{-2}}$$

$$y(n)=0.1867x(n)+0.3734x(n-1)+0.1867x(n-2)+0.4629y(n-1)-0.2097y(n-2)$$

b.

```
>> freqz([0.1867 0.3734 0.1867],[1 -0.4629 0.2097],512,8000)
```

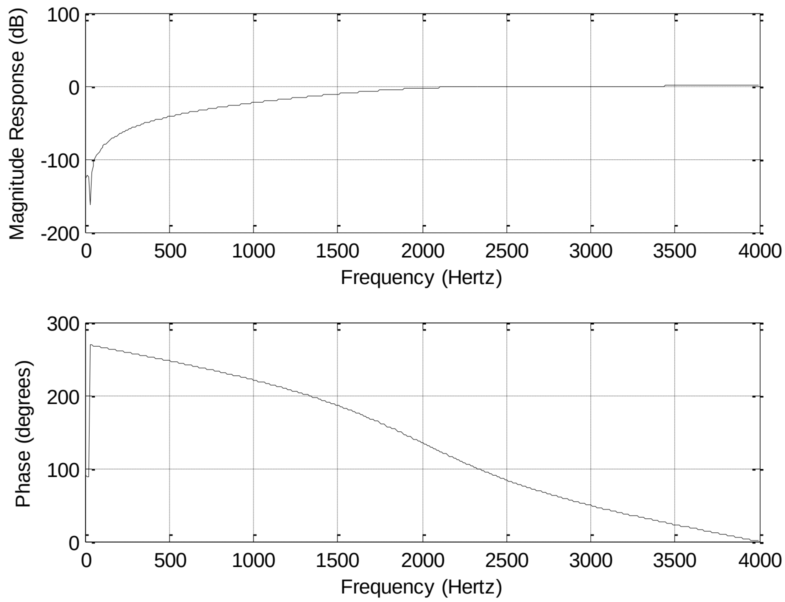


8.8
a.

$$H(z) = \frac{0.1667 - 0.5000z^{-1} + 0.5000z^{-2} - 0.1667z^{-3}}{1 + 0.3333z^{-2}}$$

$$y(n) = 0.1667x(n) - 0.5x(n-1) + 0.5x(n-2) - 0.1667x(n-3) - 0.3333y(n-2)$$

b.
>> freqz([0.1667 -0.5 0.5 -0.1667],[1 0 0.3333 0],512,8000)

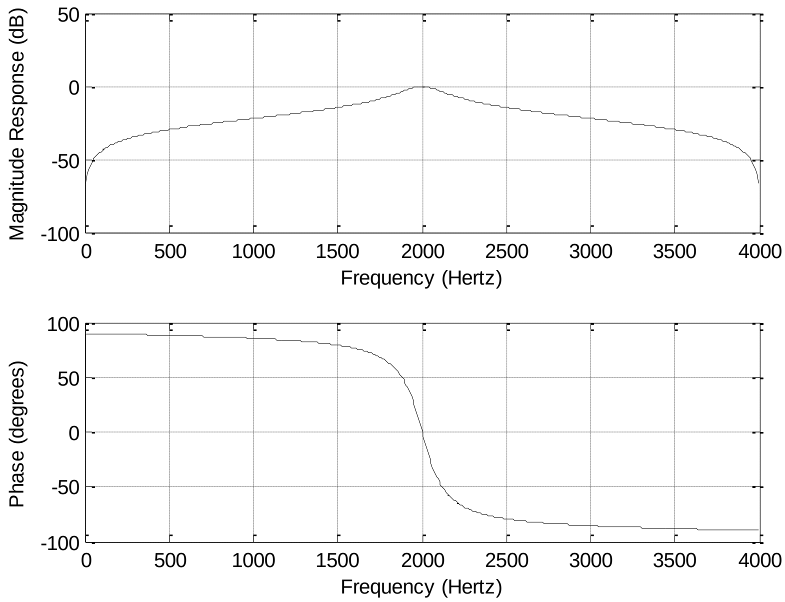


8.9
a.

$$H(z) = \frac{0.0730 - 0.0730z^{-2}}{1 + 0.8541z^{-2}}$$

$$y(n) = 0.0730x(n) - 0.0730x(n-2) - 0.8541y(n-2)$$

b.
>> freqz([0.0730 0 -0.0730],[1 0 0.8541],512,8000)



8.10

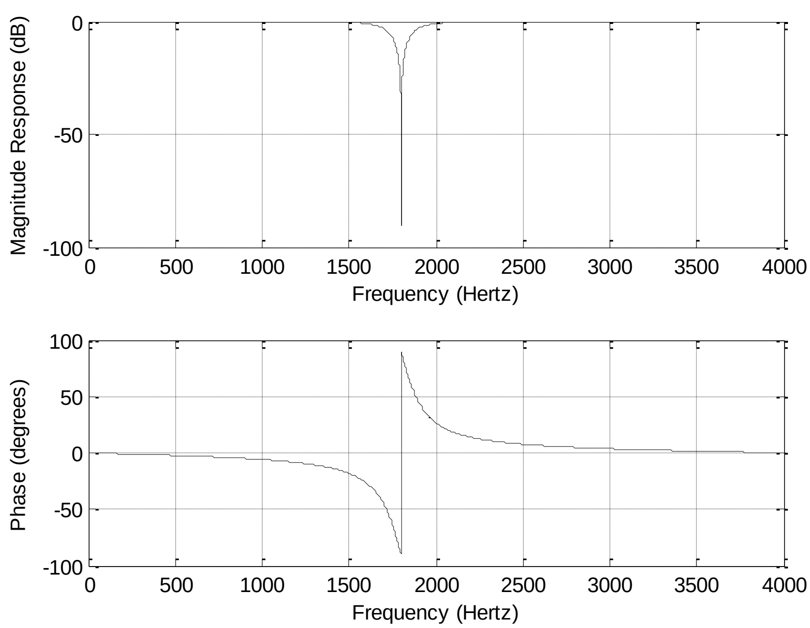
a.

$$H(z)=\frac{0.9266-0.2899z^{-1}+0.9266z^{-2}}{1-0.2899+0.8532z^{-2}}$$

$$y(n)=0.9266x(n)-0.2899x(n-1)+0.9266x(n-2)$$
$$+0.2899y(n-1)-0.8532y(n-2)$$

b.

```
>> freqz([0.9266 -0.2899 0.9266],[1 -0.2899 0.8532],8000,8000)
```



8.11

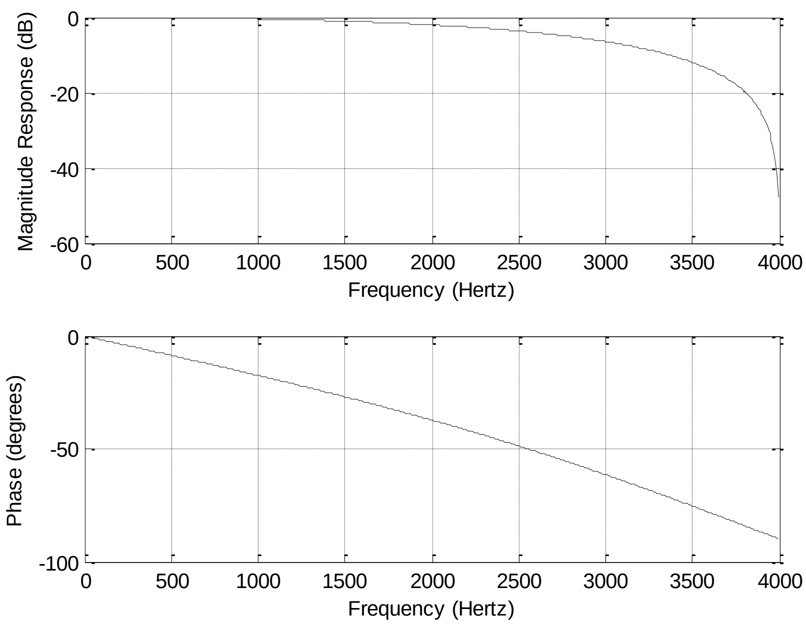
a.

$$H(z) = \frac{0.5677 + 0.5677z^{-1}}{1 + 0.1354z^{-1}}$$

$$y(n) = 0.5677x(n) + 0.5677x(n-1) - 0.1354y(n-1)$$

b.

>> freqz([0.5667 0.5667],[1 0.1354],512,8000)



8.12

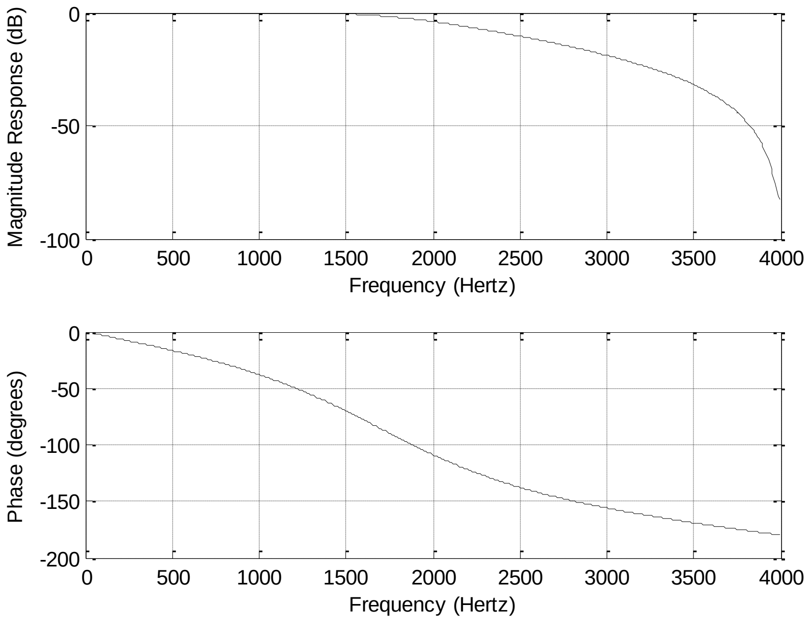
a.

$$H(z) = \frac{0.2430 + 0.4861z^{-1} + 0.2430z^{-2}}{1 - 0.2457 + 0.2755z^{-2}}$$

$$y(n) = 0.2430x(n) + 0.4861x(n-1) + 0.2430x(n-2)$$
$$+ 0.2457y(n-1) - 0.2755y(n-2)$$

b.

>> freqz([0.2430 0.4861 0.2430],[1 -0.2457 0.2755],512,8000)



8.13

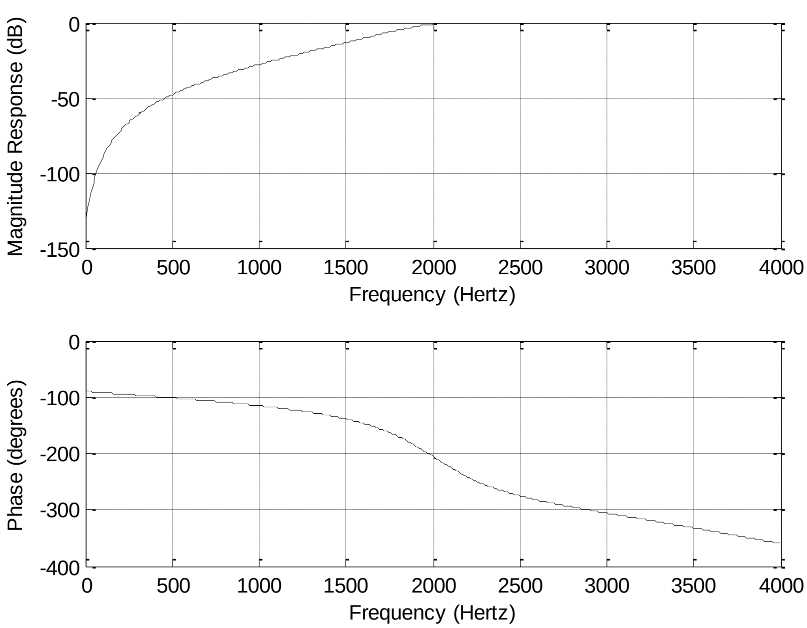
a.

$$H(z) = \frac{0.1321 - 0.3964z^{-1} + 0.3964z^{-2} - 0.1321z^{-3}}{1 + 0.3432z^{-1} + 0.6044z^{-2} + 0.2041z^{-3}}$$

$$y(n) = 0.1321x(n) - 0.3964x(n-1) + 0.3964x(n-2) - 0.1321x(n-3)$$
$$- 0.3432y(n-1) - 0.6044y(n-2) - 0.2041y(n-3)$$

b.

```
>> freqz([0.1321 -0.3964 0.3964 -0.1321],[1 0.3432 0.6044 0.2041],512,8000)
```



8.14

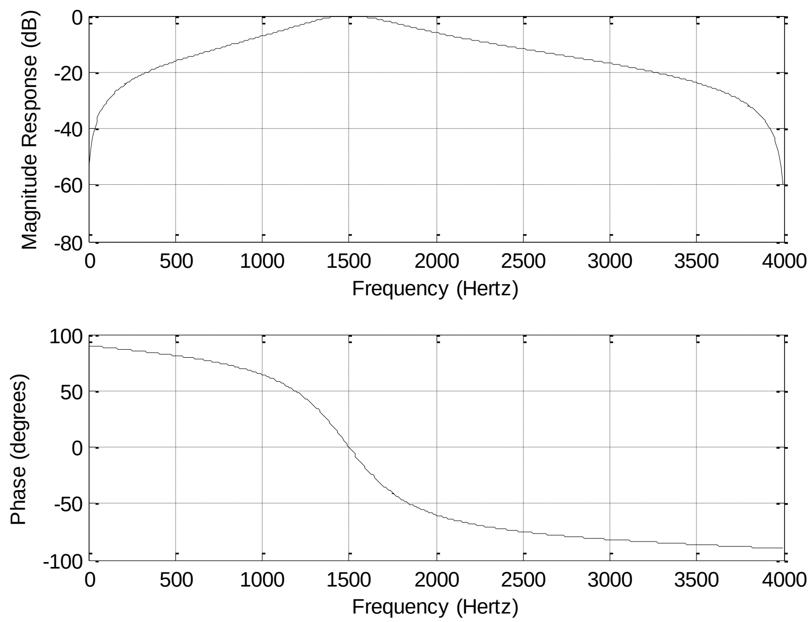
a.

$$H(z) = \frac{0.1815 - 0.1815z^{-2}}{1 - 0.6265z^{-1} + 0.6370z^{-2}}$$

$$y(n) = 0.1815x(n) - 0.1815x(n-2) + 0.6265y(n-1) - 0.6370y(n-2)$$

b.

```
>> freqz([0.1815 0 -0.1815],[1 -0.6265 0.6370],512,8000)
```



8.15

a.

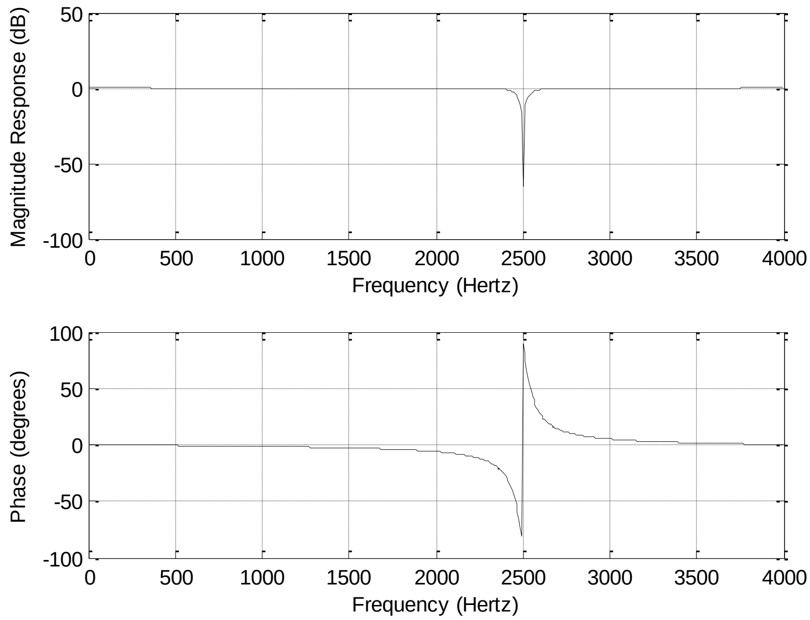
$$H(z) = \frac{0.9609 + 0.7354z^{-1} + 0.9609z^{-2}}{1 + 0.7354z^{-1} + 0.9217z^{-2}}$$

$$y(n) = 0.9609x(n) + 0.7354x(n-1) + 0.9609x(n-2)$$

$$-0.7354y(n-1) - 0.9217y(n-2)$$

b.

```
>> freqz([0.9609 0.7354 0.9609],[1 0.7354 0.9217],512,8000)
```



8.16

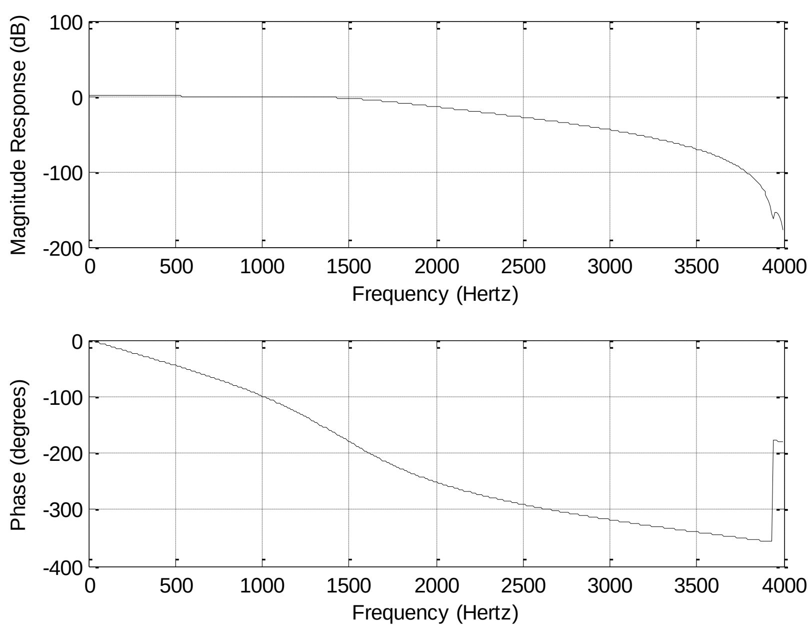
a.

$$H(z)=\frac{0.0380+0.1519z^{-1}+0.2278z^{-2}+0.1519z^{-3}+0.0380z^{-4}}{1-0.9784z^{-1}+0.7901z^{-2}-0.2419z^{-3}+0.0377z^{-4}}$$

$$y(n)=0.0380x(n)+0.1519x(n-1)+0.2278x(n-2)+0.1519x(n-3)+0.0380x(n-4)$$
$$+0.9784y(n-1)-0.7901y(n-2)+0.2419y(n-3)-0.0377y(n-4)$$

b.

```
>> freqz([0.0380 0.1519 0.2278 0.1519 0.0380],[1 -0.9784 0.7901 -0.2419 0.0377],512,8000)
```



8.17

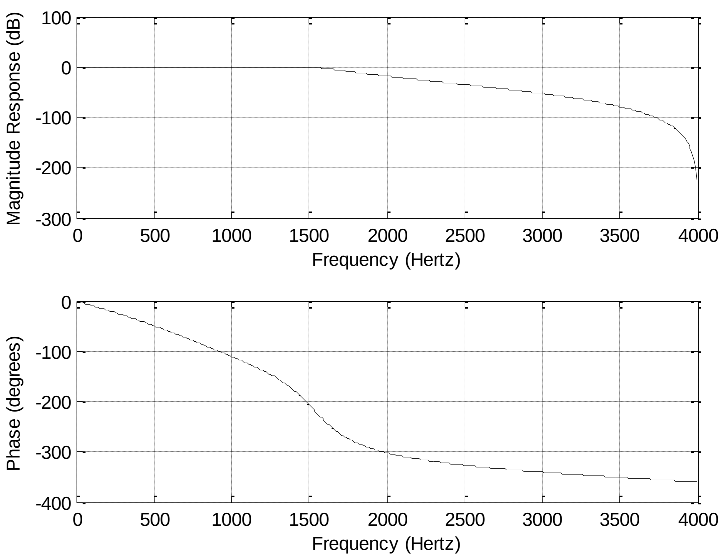
a.

$$H(z)=\frac{0.0242+0.0968z^{-1}+0.1452z^{-2}+0.0968z^{-3}+0.0242z^{-4}}{1-1.5895z^{-1}+1.6690z^{-2}-0.9190z^{-3}+0.2497z^{-4}}$$

$$y(n)=0.0242x(n)+0.0968x(n-1)+0.1452x(n-2)+0.0968x(n-3)+0.0242x(n-4)$$
$$+1.5895y(n-1)-1.6690y(n-2)+0.9190y(n-3)-0.2497y(n-4)$$

b.

```
>> freqz([0.0242 0.0968 0.1452 0.0968 0.0242],[1 -1.5895 1.6690 -0.9190 0.2497],512,8000)
```



8.18

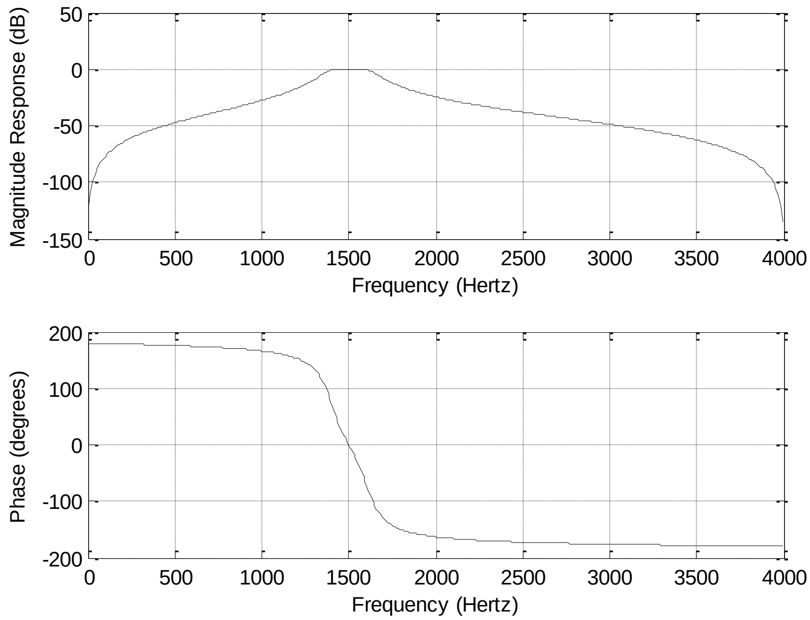
a.

$$H(z)=\frac{0.00767-0.01534z^{-2}+0.00767z^{-2}}{1-1.4428z^{-1}+2.2935z^{-2}-1.2918z^{-3}+0.8027z^{-4}}$$

$$y(n)=0.00767x(n)-0.01534x(n-2)+0.00767x(n-4)$$
$$+1.4428y(n-1)-2.2935y(n-2)+1.2918y(n-3)-0.8027y(n-4)$$

b.

```
>> freqz([0.00767 0 -0.01534 0 0.00767],[1 -1.4428 2.2935 -1.2918 0.8027],512,8000)
```



8.19

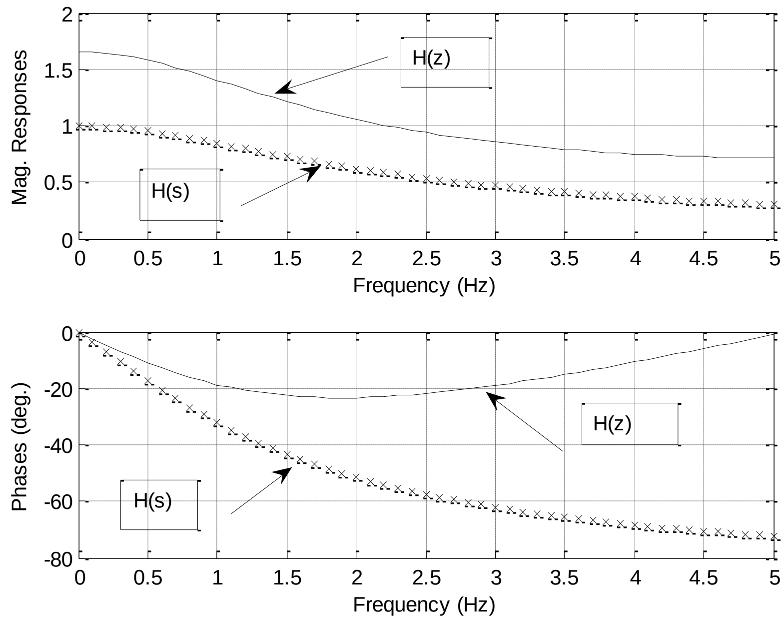
a.

$$H(z) = \frac{1}{1 - 0.3679z^{-1}}$$

$$y(n) = x(n) + 0.3679y(n-1)$$

b. and c.

```
-------------------------------------------------------------------------------------------------
 f=0:0.1:5;T=0.1                        % frequency range and sampling interval
  w=2*pi*f;                              %  frequency range in radians/sec
  hs=freqs([10], [1 10],w);              % analog magnitude frequency response
  phis=180*angle(hs)/pi;
% for the z-transfer function H(z)
  hz=freqz([1],[1 -0.3967],length(w));   % digital magnitude frequency response
   phiz=180*angle(hz)/pi;
%plot magnitude and phase responses.
  subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid; axis([0 5 0 2]);
  xlabel('Frequency (Hz)'); ylabel('Mag. Responses')
  subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
  xlabel('Frequency (Hz)');  ylabel('Phases (deg.)');
---------------------------------------------------------------------------------------
```



8.20

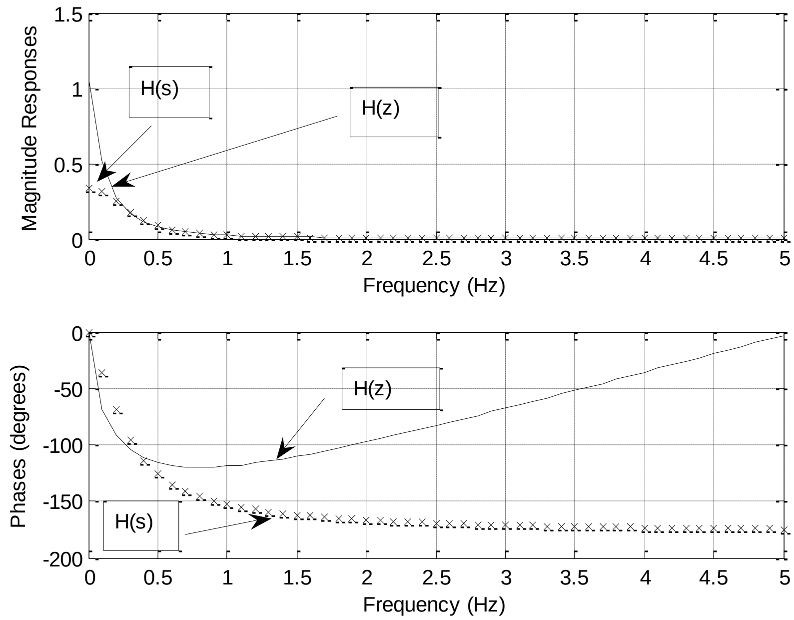
a.

$$H(z)=\frac{0.0086z^{-1}}{1-1.7326z^{-1}+0.7408z^{-2}}$$

$$y(n)=0.0086x(n-1)+1.7326y(n-1)+0.7408y(n-2)$$

b. and c.

```
------------------------------------------------------------------------------------------------
 f=0:0.1:5;T=0.1;  % initialize analog frequency range in Hz and sampling interval
  w=2*pi*f;          % convert the frequency range to radians/second
  hs=freqs([1], [1 3 3],w); % calculate analog filter frequency responses
  phis=180*angle(hs)/pi;
  % for the z-transfer function H(z)
% calculate digital filter frequency responses
  hz=freqz([0.0086],[1 -1.7326 0.7408],length(w));
  phiz=180*angle(hz)/pi;
% plot magnitude and phase responses
  subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid;
  xlabel('Frequency (Hz)'); ylabel('Magnitude Responses')
  subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
  xlabel('Frequency (Hz)'); ylabel('Phases (degrees)')
------------------------------------------------------------------------------------------------
```



8.21

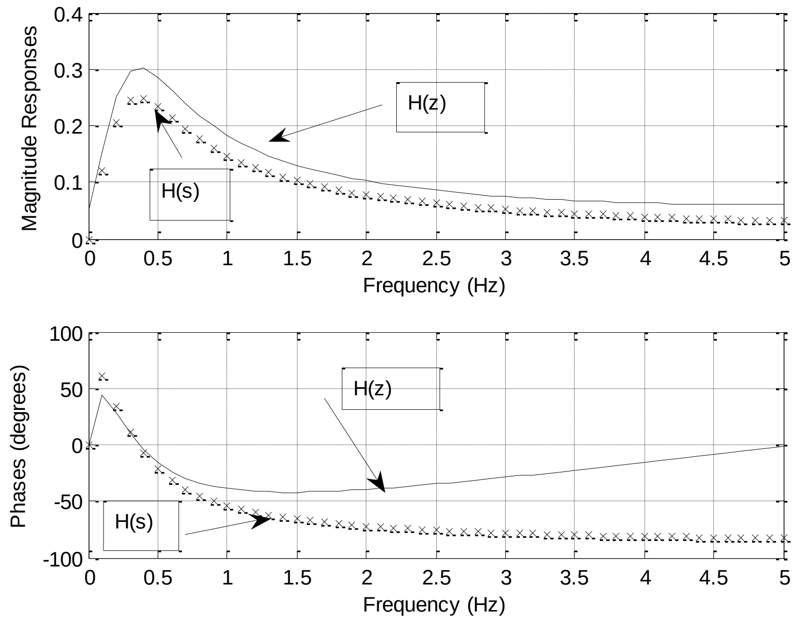
a.

$$H(z)=\frac{0.1-0.09781z^{-1}}{1-1.6293z^{-1}+0.6703z^{-2}}$$

$$y(n)=0.1x(n)-0.0978x(n-1)+1.6293y(n-1)-0.6703y(n-2)$$

b. and c.

```
-----------------------------------------------------------------------------------------------
 f=0:0.1:5;T=0.1;  % initialize analog frequency range in Hz and sampling interval
  w=2*pi*f;          % convert the frequency range to radians/second
  hs=freqs([1 0], [1 4 5],w); % calculate analog filter frequency responses
  phis=180*angle(hs)/pi;
  % for the z-transfer function H(z)
% calculate digital filter frequency responses
  hz=freqz([0.1 -0.09781],[1 -1.6293 0.6703],length(w));
  phiz=180*angle(hz)/pi;
% plot magnitude and phase responses
  subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid;
  xlabel('Frequency (Hz)'); ylabel('Magnitude Responses')
  subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
  xlabel('Frequency (Hz)'); ylabel('Phases (degrees)')
-----------------------------------------------------------------------------------------------
```



8.22

$$H(z) = \frac{0.0385 - 0.0385z^{-2}}{1 + 0.9230z^{-2}}$$
$$y(n) = 0.0385x(n) - 0.0385x(n-2) - 0.9230y(n-2)$$

8.23

$$H(z) = \frac{0.9320 - 1.3180z^{-1} + 0.9320z^{-2}}{1 - 1.3032z^{-1} + 0.8492}$$
$$y(n) = 0.9320x(n) - 1.3180x(n-1) + 0.9329x(n-2) + 1.3032y(n-1) - 0.8492y(n-2)$$

8.24

$$H(z) = \frac{0.0785 + 0.0785z^{-1}}{1 - 0.8429z^{-1}}$$
$$y(n) = 0.0785x(n) + 0.0785x(n-1) - 0.8429y(n-1)$$

8.25

$$H(z) = \frac{0.9215 + 0.9215z^{-1}}{1 + 0.8429z^{-1}}$$
$$y(n) = 0.9215x(n) + 0.9215x(n-1) - 0.8429y(n-1)$$

8.26

$$H(z) = \frac{0.0589 - 0.0589z^{-1}}{1 + 0.8822z^{-1}}$$
$$y(n) = 0.0589x(n) - 0.0589x(n-1) - 0.8822y(n-1)$$

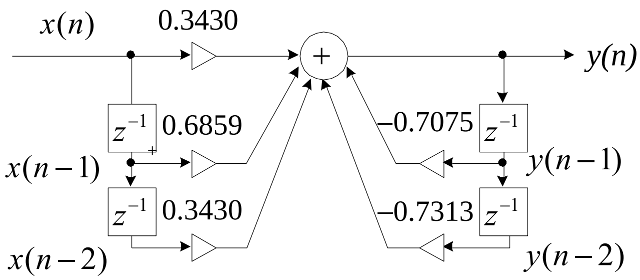
8.27

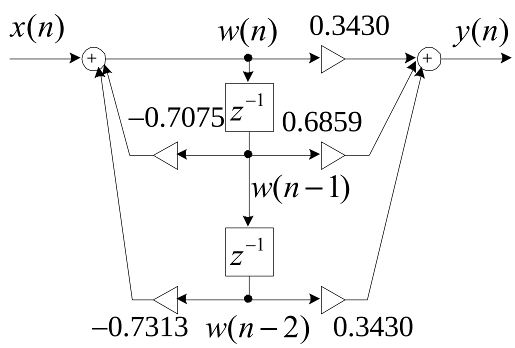
$$H(z) = \frac{0.9607 - 0.9607z^{-1}}{1 - 0.9215z^{-1}}$$

$$y(n) = 0.9607x(n) - 0.9607x(n-1) + 0.9215y(n-1)$$

8.28

a.





b.

Direct-form I:

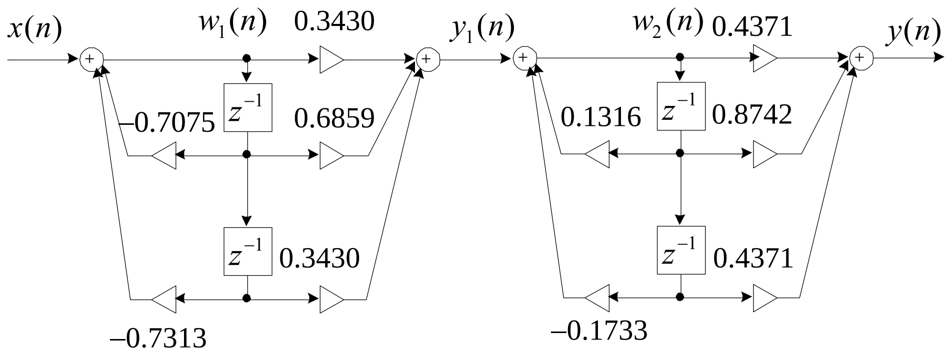
$$y(n) = 0.3430x(n) + 0.6859x(n-1) + 0.3430x(n-2)$$
$$-0.7075y(n-1) - 0.7131y(n-2)$$

Direct-form II:

$$w(n) = x(n) - 0.7075w(n-1) - 0.7313w(n-2)$$
$$y_1(n) = 0.3430w(n) + 0.6859w(n-1) + 0.3430w(n-2)$$

8.29

a.



b. for section 1: $w_1(n) = x(n) - 0.7075w_1(n-1) - 0.7313w_1(n-2)$

$y_1(n) = 0.3430w_1(n) + 0.6859w_1(n-1) + 0.3430w_1(n-2)$

for section 2: $w_2(n) = y_1(n) + 0.1316w_2(n-1) - 0.1733w_2(n-2)$

$y_2(n) = 0.4371w_2(n) + 0.8742w_2(n-1) + 0.4371w_2(n-2)$

8.30

$$\Omega_0 = 2\pi \times 200/1000 = 0.4\pi\ ,\ H(z) = \frac{0.9511z^{-1}}{1.0000 - 0.6180z^{-1} + z^{-2}}$$

$$y(n) = 0.9511x(n-1) + 0.618y(n-1) - y(n-2)$$

8.31

$$\Omega_0 = 2\pi \times 250/8000 = 0.0625\pi\ ,\ H(z) = \frac{0.1951z^{-1}}{1.0000 - 1.9616z^{-1} + z^{-2}}$$

$$y(n) = 0.1951x(n-1) + 1.9616y(n-1) - y(n-2)$$

8.32 a. $H_{852}(z) = \frac{0.6203z^{-1}}{1 - 1.5687z^{-1} + z^{-2}}$, $H_{1477}(z) = \frac{0.9168z^{-1}}{1 - 0.7986z^{-1} + z^{-2}}$

b. $y_{852}(n) = 0.6203x(n-1) + 1.5678y_{852}(n-1) - y_{852}(n-2)$

$y_{1477}(n) = 0.9168x(n-1) + 0.7986y_{1477}(n-1) - y_{1477}(n-2)$

$y_9(n) = y_{1477}(n) + y_{852}(n)$

8.33 a. $H_{697}(z) = \frac{0.5205z^{-1}}{1 - 1.7077z^{-1} + z^{-2}}$, $H_{1477}(z) = \frac{0.9168z^{-1}}{1 - 0.7986z^{-1} + z^{-2}}$

b. $y_{697}(n) = 0.5205x(n-1) + 1.7077y_{697}(n-1) - y_{697}(n-2)$

$y_{1477}(n) = 0.9168x(n-1) + 0.7986y_{1477}(n-1) - y_{1477}(n-2)$

$y_3(n) = y_{1477}(n) + y_{697}(n)$

8.34 $X(0)=2$, $|X(0)|^2=4$, $A_0=0.5$ (single side)

$X(1)=1-j3$, $|X(1)|^2=10$, $A_1=1.5811$ (single side)

8.35 $X(2)=0$, $|X(2)|^2=0$, $A_2=0.0$ (single side)

$X(3)=1+j3$, $|X(3)|^2=10$, $A_3=1.5811$ (single side)

8.36 $A_0=2.5$, $A_2=0.5$

8.37 $A_1=0.7071$, $A_3=0.7071$

8.38
Chebyshev notch filter: order =2

$$H(z)=\frac{0.9915-1.9042z^{-1}+0.9915z^{-2}}{1.0000-1.9042z^{-1}+0.9830z^{-2}}$$

$$y(n)=0.9915x(n)-1.9042x(n-1)+0.9915x(n-2)$$
$$+1.9042y(n-1)-0.9830y(n-2)$$

```
-----------------------------------------------------------------------------------
fs=8000;T=1/fs;
w0=2*pi*360; wa0=(2/T)*tan(w0*T/2);
wL=2*pi*330; waL=(2/T)*tan(wL*T/2);
wH=2*pi*390; waH=(2/T)*tan(wH*T/2);
waaL=wa0*wa0/waH;BW1=waH-waaL
waaH=wa0*wa0/waL;BW2=waaH-waL
[B,A]=lp2bs(2.8628,[1 2.8628],wa0,BW2);
[b,a]=bilinear(B,A,fs)
freqz(b,a,8000,8000);
-----------------------------------------------------------------------------
```

8.39 Chebyshev notch filter 1: order =2

$$H(z)=\frac{0.9915-1.9042z^{-1}+0.9915z^{-2}}{1.0000-1.9042z^{-1}+0.9830z^{-2}}$$

$$y(n)=0.9915x(n)-1.9042x(n-1)+0.9915x(n-2)$$
$$+1.9042y(n-1)-0.9830y(n-2)$$

See Problem 8.38.

Chebyshev notch filter 2: order =2

$$H(z)=\frac{0.9917-1.3117z^{-1}+0.9917z^{-2}}{1.0000-1.3117z^{-1}+0.9835z^{-2}}$$

$$y(n)=0.9917x(n)-1.3117x(n-1)+0.9917x(n-2)$$
$$+1.3117y(n-1)-0.9835y(n-2)$$

```
fs=8000;T=1/fs;
w0=2*pi*1080; wa0=(2/T)*tan(w0*T/2);
wL=2*pi*1050; waL=(2/T)*tan(wL*T/2);
wH=2*pi*1110; waH=(2/T)*tan(wH*T/2);
waaL=wa0*wa0/waH;BW1=waH-waaL
waaH=wa0*wa0/waL;BW2=waaH-waL
[B,A]=lp2bs(2.8628,[1 2.8628],wa0,BW2);
[b,a]=bilinear(B,A,fs)
freqz(b,a,8000,8000);
```

8.40

$$\omega_{dp} = 2\pi(3000) = 6000\pi \quad \omega_{ds} = 2\pi(4000) = 8000\pi$$

$$\omega_{ap} = \frac{2}{T}\tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{6000\pi/10000}{2}\right) = 27528 \quad \text{radians/se.}$$

$$\omega_{as} = \frac{2}{T}\tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{8000\pi/10000}{2}\right) = 61554 \ \text{radians/sec}$$

$$v_s = \omega_{as}/\omega_{ap} = 61554/27528 = 2.24 \ \text{and} \ A_s = 25 \ \text{dB}$$

$$\varepsilon^2 = 10^{0.1\times 3} - 1 = 1$$

$$n = \frac{\log_{10}(10^{0.1\times 25} - 1)}{2\cdot \log_{10}(2.24)} = 3.5669 \quad \text{Choose } n = 4$$

Butterworth filter order = n=4

$$H(z) = \frac{0.1672 + 0.6687z^{-1} + 1.0031z^{-2} + 0.6687z^{-3} + 0.1672z^{-4}}{1.0000 + 0.7821z^{-1} + 0.6800z^{-2} + 0.1827z^{-3} + 0.0301z^{-4}}$$

```
fs=10000;T=1/fs;
wd=2*pi*3000; wa=(2/T)*tan(wd*T/2);
[B,A]=lp2lp(1,[ 1 2.6131 3.4142 2.6131 1],wa);
[b,a]=bilinear(B,A,fs)
freqz(b,a,512,fs);
```

8.41

$$\omega_{dp} = 2\pi f = 2\pi(3000) = 6000\pi \ \text{radians/second,}$$

$$\omega_{ds} = 2\pi f = 2\pi(4000) = 8000\pi \ \text{radians/second, and } T = 1/f_s = 1/8000 \ \text{sec.}$$

$$\omega_{ap} = \frac{2}{T}\tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{6000\pi/10000}{2}\right) = 27528 \quad \text{radians/se.}$$

$$\omega_{as} = \frac{2}{T}\tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{8000\pi/10000}{2}\right) = 61554 \ \text{radians/sec}$$

$$v_s = \omega_{as} / \omega_{ap} = 61554/27528 = 2.24 \text{ and } A_s = 35 \text{ dB}$$

$$\varepsilon^2 = 10^{0.1\times1} - 1 = 0.2589$$

$$n \ge \frac{\cosh^{-1}\left[\left(\frac{10^{0.1\times35}-1}{0.2589}\right)^{0.5}\right]}{\cosh^{-1}(2.24)} = 3.7342$$

Chebyshev filter order = 4;

$$H(z) = \frac{0.1103 + 0.4412z^{-1} + 0.6618z^{-2} + 0.4412z^{-3} + 0.1103z^{-4}}{1.0000 + 0.1509z^{-1} + 0.8041z^{-2} - 0.1619z^{-3} + 0.1872z^{-4}}$$

```
fs=10000;T=1/fs;
wd=2*pi*3000; wa=(2/T)*tan(wd*T/2);
[B,A]=lp2lp(0.2456,[ 1 0.9528 1.4539 0.7426 0.2756],wa);
[b,a]=bilinear(B,A,fs)
freqz(b,a,512,fs);
```

8.42 $r = 1 - (500/8000) \times \pi = 0.8037$ and $\theta = (1750/8000) \times 360^0 = 78.75^0$

$$K = \frac{(1-0.8037)\sqrt{1-2\times0.8037\cos157.5^0 + 0.8037^2}}{2\left|\sin 78.75^0\right|} = 0.1771$$

```
>> r=1-500*pi/8000;
>> theta=1750*2*pi/8000;
>> K=(1-r)*sqrt(1-2*r*cos(2*theta)+r*r)/(2*abs(sin(theta)));
r = 0.8037
K =0.1771
-2*r*cos(theta)
ans = -0.3136
```

$$H(z) = \frac{0.1771 - 0.1771z^{-2}}{1 - 0.3136z^{-1} + 0.6459z^{-2}}$$

8.43

$$\omega_{dpL} = 2\pi(1500) = 3000\pi \quad \omega_{dpH} = 2\pi(2000) = 4000\pi$$

$$\omega_{dsL} = 2\pi(1000) = 2000\pi \quad \omega_{dsH} = 2\pi(2500) = 5000\pi$$

$$\omega_{apL} = 16000 \times \tan\left(\frac{3000\pi/8000}{2}\right) = 10691 \quad \omega_{apH} = 16000 \times \tan\left(\frac{4000\pi/8000}{2}\right) = 16000$$

Fixing $\omega_{asL} = 16000 \times \tan\left(\frac{2000\pi/8000}{2}\right) = 6627$

$$\omega_{asH} = \omega_{apL}\omega_{apH} / \omega_{asL} = 25812 \;\; W_s = 19185$$

Fixing $\omega_{asH} = 16000 \times \tan\left(\frac{5000\pi/8000}{2}\right) = 23946$,

$\omega_{asL} = \omega_{apL}\omega_{apH} / \omega_{asH} = 7143$, $W_s = 16803$

We choose a smaller bandwidth for aggressive design:

$\omega_{asH} = 23946$, $\omega_{asL} = \omega_{apL}\omega_{apH} / \omega_{asH} = 7143$

$v_s = (\omega_{asH} - \omega_{asL})/(\omega_{apH} - \omega_{apL}) = (23946 - 7143)/(16000 - 10691) = 3.17$ and $A_s = 20$ dB

$\varepsilon^2 = 10^{0.1\times 3} - 1 = 1$

$n = \frac{\log_{10}(10^{0.1\times 20} - 1)}{2 \cdot \log_{10}(3.17)} = 1.9914$ Choose $n = 2$

filter order 2n= 4

$$H(z) = \frac{0.0300 - 0.0599z^{-2} + 0.0300z^{-4}}{1.0000 - 0.6871z^{-1} + 1.5741z^{-2} - 0.5176z^{-3} + 0.5741z^{-4}}$$

```
fs=8000;T=1/fs;
wL=2*pi*1500; waL=(2/T)*tan(wL*T/2);
wH=2*pi*2000; waH=(2/T)*tan(wH*T/2);
wa0=sqrt(waL*waH); BW=waH-waL
[B,A]=lp2bp(1,[1 1.4142 1],wa0,BW);
[b,a]=bilinear(B,A,fs)
freqz(b,a,512,8000);
```

8.44

a.

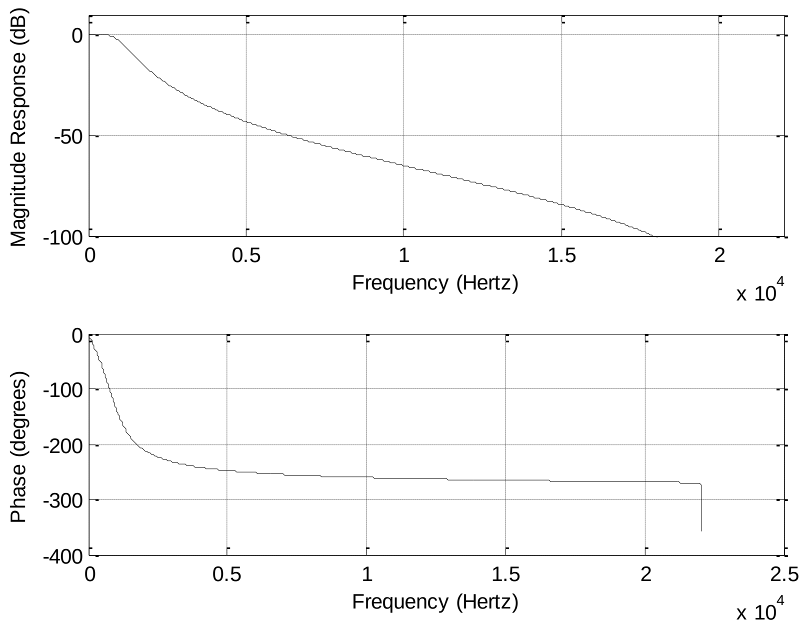
$$H_{LP}(z) = \frac{0.0003151+0.0009452z^{-1} + 0.0009452z^{-2} + 0.0003151z^{-3}}{1.0000 - 2.7153z^{-1} + 2.4697z^{-2} - 0.7519z^{-3}}$$

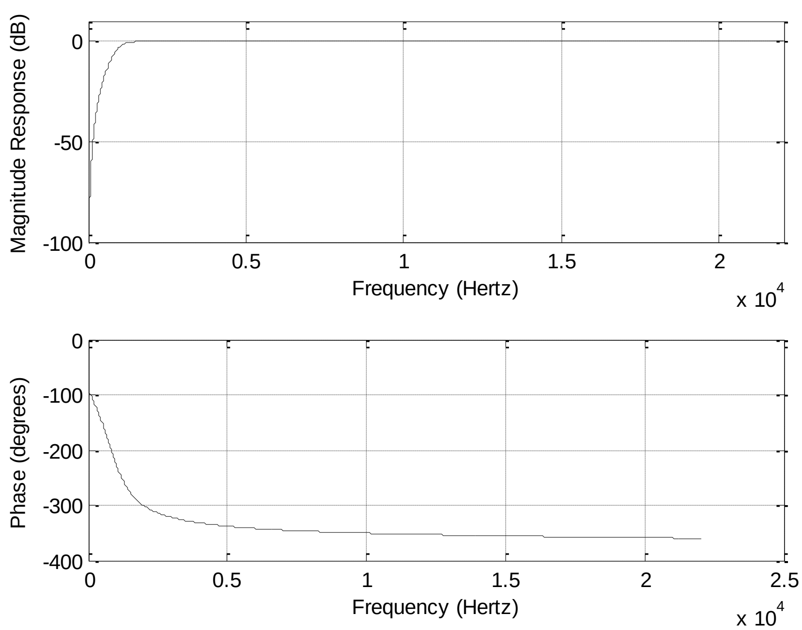
$$y_{LP}(n) = 0.0003151x(n) + 0.0009452x(n-1) + 0.0009452x(n-2) + 0.0003151x(n-3)$$
$$2.7153y_{LP}(n-1) - 2.4697y_{LP}(n-2) + 0.7519y_{LP}(n\text{-}3)$$

$$H_{HP}(z) = \frac{0.8671 - 2.6013z^{-1} + 2.6013z^{-2} - 0.8671z^{-3}}{1.0000 - 2.7153z^{-1} + 2.4697z^{-2} - 0.7519z^{-3}}$$

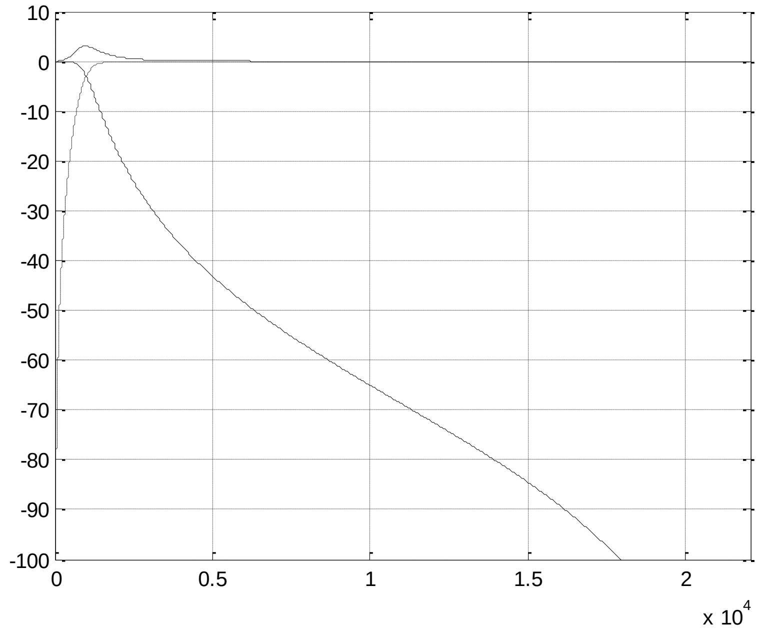
$$y_{HP}(n) = 0.8671x(n) - 2.6013x(n-1) + 2.6013x(n-2) - 0.8671x(n-3)$$
$$2.7153y_{HP}(n-1) - 2.4697y_{HP}(n-2) + 0.7519y_{HP}(n\text{-}3)$$

b.


Magnitude Response (dB)
0
-50
-100
0
0.5
1
1.5
2
Frequency (Hertz)
x 10^4
Phase (degrees)
0
-100
-200
-300
-400
0
0.5
1
1.5
2
2.5
Frequency (Hertz)
x 10^4


Magnitude Response (dB)
0
-50
-100
0
0.5
1
1.5
2
Frequency (Hertz)
x 10^4
Phase (degrees)
0
-100
-200
-300
-400
0
0.5
1
1.5
2
2.5
Frequency (Hertz)
x 10^4

c.



```
fs=44100;T=1/fs;
wd=2*pi*1000; wa=(2/T)*tan(wd*T/2);
[B,A]=lp2lp(1,[ 1 2 2 1],wa);
[bL,aL]=bilinear(B,A,fs)
freqz(bL,aL,512,fs);
[hL,ff]=freqz(bL,aL,512,fs);
figure
[B,A]=lp2hp(1,[ 1 2 2 1],wa);
[bH,aH]=bilinear(B,A,fs)
freqz(bH,aH,512,fs);
[hH,ff]=freqz(bH,aH,512,fs);
figure
H=abs(hL)+abs(hH);
plot(ff,20*log10(abs(hL)),ff,20*log10(abs(hH)),'-.', ff,20*log10(H));
```

8.45

a.

$$H(z)=\frac{0.5878z^{-1}}{1-1.6180z^{-1}+z^{2}}$$

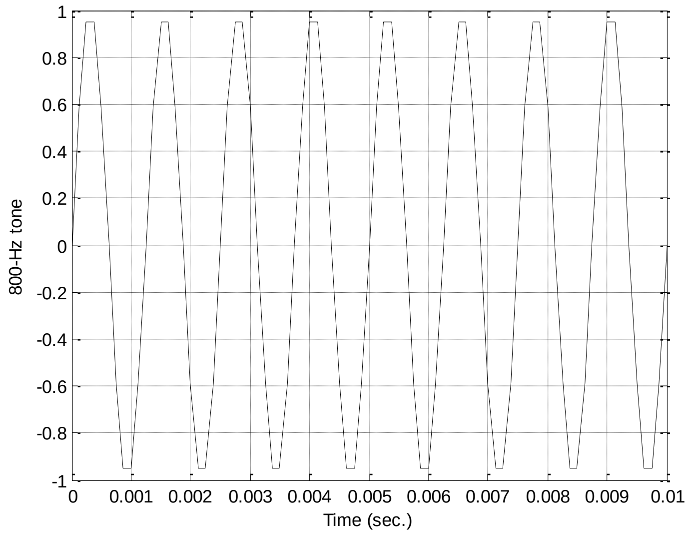
b.

$$y(n)=0.5878x(n-1)+1.6180y(n-1)-y(n-2)$$

c.

```
>> fs=8000; T=1/fs;
>> t=0:T:0.01;
```

```
>> x=zeros(1,length(t)); x(1)=1;
>> y=filter([0 0.5878],[1 -1.6180 1],x);
>> plot(t,y);grid;xlabel('Time (sec.)');ylabel('800-Hz tone');
```



8.46

a.

$$H_1(z)=\frac{0.5686z^{-1}}{1-1.6453z^{-1}+z^{2}} \qquad H_2(z)=\frac{0.2593z^{-1}}{1-1.9316z^{-1}+z^{2}}$$

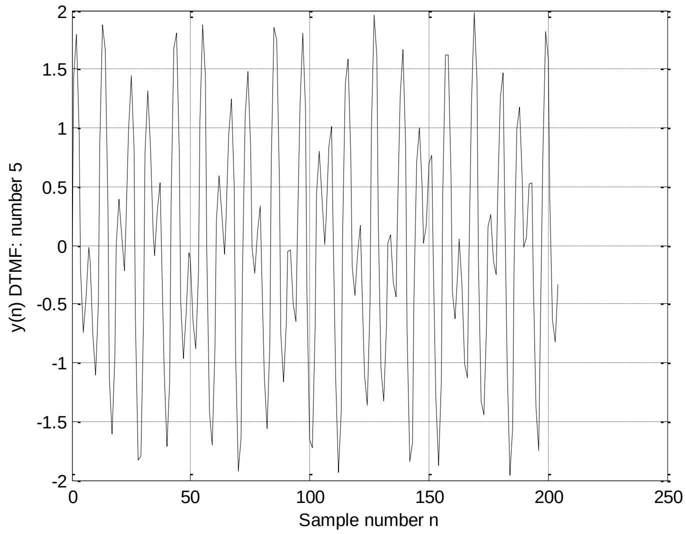
b.

$$y_1(n)=0.5686x(n-1)+1.6453y_1(n-1)-y_1(n-2)$$

$$y_2(n)=0.2593x(n-1)+1.9316y_2(n-1)-y_2(n-2)$$

c.

```
------------------------------------------------------------------------------------------
fs=8000;T=1/fs;
x=zeros(1,205);x(1)=1;
y1=filter([0 sin(2*pi*770/fs)],[1 -2*cos(2*pi*770/fs) 1],x);
y2=filter([0 sin(2*pi*1336/fs) ],[1 -2*cos(2*pi*1336/fs) 1],x);
y=y1+y2;
plot(N,y,'k');grid
ylabel('y(n) DTMF: number 5');xlabel('Sample number n')
------------------------------------------------------------------------------------------
```



8.47

$x(4) = 0$

for $n = 0,1,2,3,4$

$v_0(n) = 2v_0(n-1) - v_0(n-2) + x(n)$

$y_0(n) = v_0(n) - v_0(n-1)$ with $v_0(-2) = 0$, $v_0(-1) = 0$

$X(0) = y_0(4)$

$\left|X(0)\right|^2 = v_0^2(4) + v_0^2(3) - 2v_0(4)v_0(3)$

a. $X(0) = 1$; b. $\left|X(0)\right|^2 = 1$; c. $A_0 = 0.25$

for $n = 0,1,2,3,4$

$v_1(n) = -v_1(n-2) + x(n)$

$y_1(n) = v_1(n) + jv_1(n-1)$ with $v_1(-2) = 0$, $v_1(-1) = 0$

$X(1) = y_1(4)$

$\left|X(1)\right|^2 = v_1^2(4) + v_1^2(3)$

a. $X(1) = 1 - j2$; b. $\left|X(1)\right|^2 = 5$; c. $A_1 = 1.12$ (single side)

8.48

$x(4) = 0$

for $n = 0,1,2,3,4$

$v_2(n) = -2v_2(n-1) - v_2(n-2) + x(n)$

$y_2(n) = v_2(n) + v_2(n-1)$ with $v_2(-2) = 0$, $v_2(-1) = 0$

$X(2) = y_2(4)$

$\left|X(2)\right|^2 = v_2^2(4) + v_2^2(3) + 2v_2(4)v_2(3)$

a. $X(2)=1$; b. $\left|X(2)\right|^2=1$; c. $A_2=0.25$

for $n=0,1,2,3,4$

$v_3(n)=-v_3(n-2)+x(n)$

$y_3(n)=v_3(n)-jv_3(n-1)$ with $v_3(-2)=0$, $v_3(-1)=0$

$X(3)=y_3(4)$

$\left|X(3)\right|^2=v_3^2(4)+v_3^2(3)$

a. $X(3)=1+j2$; b. $\left|X(3)\right|^2=5$; c. $A_3=0.559$

8.49

$$k_L=\frac{770}{8000}\times 205\approx 20, \text{ and } k_H=\frac{1336}{8000}\times 205\approx 34$$

a.

$$H_{20}(z)=\frac{1}{1-1.6359z^{-1}+z^{-2}} \text{ and } H_{34}(z)=\frac{1}{1-1.0088z^{-1}+z^{-2}}$$

b.

$$v_{20}(n)=1.6359v_{20}(n-1)-v_{20}(n-2)+x(n) \text{ with } x(205)=0, \text{ for } n=0,1,\cdots,205$$

$$v_{34}(n)=1.0088v_{34}(n-1)-v_{34}(n-2)+x(n) \text{ with } x(205)=0, \text{ for } n=0,1,\cdots,205$$

c.

$$\left|X(20)\right|^2=\left(v_{20}(205)\right)^2+\left(v_{20}(204)\right)^2-1.6359\left(v_{20}(205)\right)\times\left(v_{20}(204)\right)$$

$$A_{20}=\frac{2\sqrt{\left|X(20)\right|^2}}{205}$$

and

$$\left|X(34)\right|^2=\left(v_{34}(205)\right)^2+\left(v_{34}(204)\right)^2-1.1631\left(v_{34}(205)\right)\times\left(v_{34}(204)\right)$$

$$A_{34}=\frac{2\sqrt{\left|X(34)\right|^2}}{205}$$

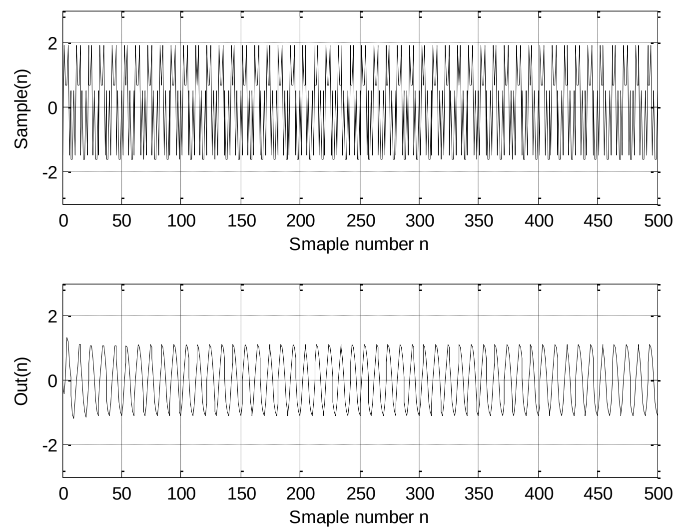
d.

```
------------------------------------------------------------------------------------------
fs=8000;T=1/fs;
x=zeros(1,205);x(1)=1;
y1=filter([0 sin(2*pi*770/fs)],[1 -2*cos(2*pi*770/fs) 1],x);
y2=filter([0 sin(2*pi*1336/fs) ],[1 -2*cos(2*pi*1336/fs) 1],x);
y=y1+y2;
xDTMF=[y 0];
v20=filter(1,[1 -2*cos(2*pi*20/205) 1],xDTMF);
v34=filter(1,[1 -2*cos(2*pi*34/205) 1],xDTMF);
X20=sqrt(v20(206)^2+v20(205)^2-2*cos(2*pi*20/205)*v20(206)*v20(205));
X34=sqrt(v34(206)^2+v34(205)^2-2*cos(2*pi*34/205)*v34(206)*v34(205));
A20=2*X20/205
A34=2*X34/205
---------------------------------------------------------------------------------------------
```

A20 = 0.8818
A34 = 0.9147

8.50
a.

```
-------------------------------------------------------------------------------------------------
n=0:N-1;
sample=1.2*sin(2*pi*1000*n/10000)-1.5*cos(2*pi*4000*n/10000);
%direct-form I implementation
 x=[0 0 0 0 0]; %input buffer [x(n) x(n-1) ..]
 y=[0 0 0 0 0]; %output buffer [y(n) y(n-1) ... ]
 b=[0.1103 0.4412 0.6618 0.4412 0.1603]; %Numerator coefficients [b0 b1 ...]
 a=[1 0.1509 0.9041 -0.1619 0.1872]; %Denominator coefficients [1 a1 ...]
 KKb=length(b); KKa=length(a);
 for n=1:1:length(sample) % loop processing
    for k=KKb:-1:2       % shift input by one sample
       x(k)=x(k-1);
    end
    x(1)=sample(n);     % get new sample
    for k=KKa:-1:2       % shift input by one sample
       y(k)=y(k-1);
    end
    y(1)=0;             % perform IIR filtering
    for k=1:1:KKb
       y(1)=y(1)+b(k)*x(k);
    end
    for k=2:1:KKa
       y(1)=y(1)-a(k)*y(k);
    end
    out(n)=y(1); %send filtered sample to the output array
 end
 subplot(2,1,1);plot(sample);grid;axis([0 500 -3 3]);
 xlabel('Smaple number n');ylabel('Sample(n)');
 subplot(2,1,2);plot(out);grid;axis([0 500 -3 3]);
 xlabel('Smaple number n');ylabel('Out(n)');
-------------------------------------------------------------------------------------------
```



b.

```
------------------------------------------------------------------------------------------
n=0:N-1;
sample=1.2*sin(2*pi*1000*n/10000)-1.5*cos(2*pi*4000*n/10000);
%direct-form II implementation
 w=[0 0 0 0 0]; %filter states [w(n) w(n-1) ..]
 b=[0.1103 0.4412 0.6618 0.4412 0.1603]; %Numerator coefficients [b0 b1 ...]
 a=[1 0.1509 0.9041 -0.1619 0.1872]; %Denominator coefficients [1 a1 ...]
 KKb=length(b); KKa=length(a); KKw=length(w);
 for n=1:1:length(sample) % loop processing
    for k=KKw:-1:2        % shift input by one sample
        w(k)=w(k-1);
    end
    w(1)=sample(n);
    for k=2:1:KKa
        w(1)=w(1)-a(k)*w(k);   %IIR filtering
    end
    sum=0;
    for k=1:1:KKb
        sum=sum+b(k)*w(k);    %FIR filtering
    end
    out(n)=sum; %send filtered sample to the output array
 end
 subplot(2,1,1);plot(sample);grid;axis([0 500 -3 3]);
 xlabel('Smaple number n');ylabel('Sample(n)');
 subplot(2,1,2);plot(out');grid;axis([0 500 -3 3]);
 xlabel('Smaple number n');ylabel('Out(n)');
------------------------------------------------------------
```

Plots are the same as ones in (b).

Chapter 9

9.1

$0.2560123 \times 2^{15} = 8389_{10} = 0010000011000101$

0.2560123 (decimal) = 0. 0 1 0 0 0 0 0 1 1 0 0 0 1 0 1 (Q-15)

9.2

$-0.2160123 \times 2^{15} = -7078_{10} = 1110010001011010$

-0.2160123 (decimal) = 1. 1 1 0 0 1 0 0 0 1 0 1 1 0 1 0 (Q-15)

9.3

$-0.3567921 \times 2^{15} = -11691_{10} = 1101001001010101$

-0.3567921 (decimal) = 1. 1 0 1 0 0 1 0 0 1 0 1 0 1 0 1 (Q-15)

9.4

$0.4798762 \times 2^{15} = 15725_{10} = 0011110101101101$

0.4798762 (decimal) = 0. 0 1 1 1 1 0 1 0 1 1 0 1 1 0 1 (Q-15)

9.5

0. 1 0 1 0 1 0 0 0 1 0 1 1 1 1 0 = 0.6591186

1. 0 1 0 1 0 1 1 1 0 1 0 0 0 1 0 (Q-15)= -0.6591186

9.6

0. 0 0 1 0 0 0 1 1 1 1 0 1 1 1 0 (Q-15) = 0.1400756

9.7

0. 1 1 0 1 0 1 0 0 0 1 0 0 0 1 0 (Q-15) = 0.8291626

9.8

0. 1 1 0 1 1 1 0 1 1 0 1 0 0 0 1 = 0.3657532

1. 1 0 1 0 0 0 1 0 0 1 0 1 1 1 1 (Q-15) = - 0.3657532

9.9

1. 1 0 1 0 1 0 1 1 1 0 0 0 0 0 1 + 0. 0 1 0 0 0 1 1 1 1 0 1 1 0 1 0

= 1. 1 1 1 1 0 0 1 1 0 0 1 1 0 1 1

9.10

0.0 0 1 0 1 0 1 0 1 0 0 0 0 0 1 + 0. 0 1 0 1 0 1 1 1 1 0 1 0 0 1 0

= 0.1 0 0 0 0 0 1 0 0 0 1 0 0 1 1

9.11

1.0 0 1 0 1 0 1 0 1 0 0 0 0 0 1 + 1.0 1 0 1 0 1 1 1 1 0 1 0 0 1 0

= 0.1 0 0 0 0 0 1 0 0 0 1 0 0 1 1 (overflow, since adding two negative numbers results in a positive number)

9.12
0.0 0 1 0 1 0 1 0 1 0 0 0 0 0 1 + 1.0 1 0 1 0 1 1 1 1 0 1 0 0 1 0
= 1.1 0 0 0 0 0 1 0 0 0 1 0 0 1 1

9.13
a. $0.1101235 = 0.880988 \times 2^{-3}$, $-3 = 1101$ (exponent bits)
$0.880988 \times 2^{11} = 1804_{10} = 011100001100$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
1101 011100001100
b. $-10.430527 = -0.651908 \times 2^{4}$, $4 = 0100$ (exponent bits)
$-0.651908 \times 2^{11} = -1335_{10} = 101011001001$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
0100 101011001001

9.14
a. $2.5568921 = 0.6392230 \times 2^{2}$, $2 = 0010$ (exponent bits)
$0.6392230 \times 2^{11} = 1309_{10} = 010100011101$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
0010 010100011101
b. $-0.678903 = -0.678903 \times 2^{0}$, $0 = 0000$ (exponent bits)
$-0.678903 \times 2^{11} = -1390_{10} = 101010010010$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
0000 101010010010
c. $0.0 = 0.0 \times 2^{0}$, $0 = 0000$ (exponent bits)
$0.0 \times 2^{11} = 0_{10} = 000000000000$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
0000 000000000000
d. $-1.0 = -0.5 \times 2^{1}$, $0 = 0001$ (exponent bits)
$-0.5 \times 2^{11} = -1024_{10} = 110000000000$ (Q-11)
Cascading the exponent bits and the mantissa bits yields
0001 110000000000

9.15
1101 011100011011 (floating) = 0.8881835×2^{-3} (decimal)
0100 101111100101 (floating) = -0.5131835×2^{4} (decimal)
0.8881835×2^{-3} (decimal)= 0.0069389×2^{4} (decimal)= 0100 000000001110 (floating)

0100 101111100101 (floating) + 0100 000000001110 (floating)
= 0100 101111110010 (floating) = -8.1016 (decimal)

9.16
0111 110100011011 (floating) = -0.3618164×2^{7} (decimal)

0101 001000100101 (floating) = 0.2680664×2^5 (decimal)
0.2680664×2^5 (decimal)= 0.0670166×2^7 (decimal)= 0111 000010001001 (floating)

0111 110100011101 (floating) + 0111 000010001001 (floating)
= 0111 110110100100 (floating) = -37.75 (decimal)

9.17
0001 000000010011 (floating) = 0.00927734×2^1 (decimal)
0100 001000000101 (floating) = 0.25244141×2^4 (decimal)
0.00927734×2^1 (decimal)= 0.00115967×2^4 (decimal)= 0100 000000000010 (floating)

0100 000000000010 (floating) + 0100 001000000101 (floating)
= 0100 001000000111 (floating) = 4.0546875 (decimal)

9.18
$(-1)^1\times 1.025\times 2^{160-127}=-8.8047\times 10^9$

9.19
$(-1)^0\times 1.625\times 2^{164-127}=2.2334\times 10^{11}$

9.20
$(-1)^0\times 1.625\times 2^{1536-1023}=4.3575\times 10^{154}$

9.21
$(-1)^0\times 1.325\times 2^{1536-1023}=3.5531\times 10^{154}$

9.22
$B=1$
$S=|h(0)|+|h(1)|+|h(2)|=0.2+0.6+0.2=1$

$$x_s(n)=x(n),\; y_s(n)=-0.2x_s(n)+0.6x_s(n-1)+0.2x_s(n-2)\;,\; y(n)=y_s(n)$$

9.23

$$H(z)=\frac{0.6z}{z-0.3},\; h(n)=0.6(0.3)^n u(n),\; S=0.6(1+0.3+0.3^2+\cdots)=0.6\frac{1}{1-0.3}=\frac{6}{7}$$

Select: $S=1, B=1,\; C=1$

Direct form I:

$$x_s(n)=x(n),\; y_s(n)=0.6x_s(n)+0.3y_f(n-1),\; y_f(n)=y_s(n),\; y(n)=y_s(n)$$

9.24

$$A(z)=\frac{1}{1-0.3z^{-1}}=\frac{z}{z-0.3},\; h(n)=(0.3)^n u(n),\; S=(1+0.3+0.3^2+\cdots)=\frac{1}{1-0.3}=\frac{10}{7}$$

Select: $S=2,\; A=1,\; B=1$

Direct form II:

$$x_s(n)=x(n)/2,\; w_s(n)=x_s(n)+0.3w(n-1),\; w(n)=w_s(n)$$

$y_s(n) = 0.6w(n)$, $y(n) = (B \times S) y_s(n) = 2y_s(n)$

9.25

$B = 2$

$S = \frac{1}{2}\left(|h(0)| + |h(1)| + |h(2)|\right) = \frac{1}{2}\left(0.36 + 1.6 + 0.36\right) = 1.16$, choose $S = 2$

$x_s(n) = \frac{x(n)}{2}$, $y_s(n) = -0.18x_s(n) + 0.8x_s(n-1) + 0.18x_s(n-2)$, $y(n) = 4y_s(n)$

9.26

$C = 2$

$H(z) = \frac{1.35}{1 - 0.3z^{-1}} = \frac{1.35z}{z - 0.3}$ $h(n) = 1.35 \times (0.3)^n u(n)$

$S = 0.5 \times \left(1.35(0.3)^0 + 1.35(0.3)^1 + 1.35(0.3)^2 + \cdots\right) = \frac{0.5 \times 1.35 \times 1}{1 - 0.3} = 0.9643$

Choose $S = 1$

$x_s(n) = x(n)$, $y_s(n) = 0.675x_s(n) + 0.15y_f(n-1)$, $y_f(n) = 2y_s(n)$, $y(n) = y_f(n)$

9.27

$A = 1$, $B = 2$

$A(z) = \frac{1}{1 - 0.3z^{-1}} = \frac{z}{z - 0.3}$ $h(n) = (0.3)^n u(n)$

$S = 0.5 \times \left((0.3)^0 + (0.3)^1 + (0.3)^2 + \cdots\right) = \frac{0.5 \times 1}{1 - 0.3} = \frac{5}{7}$

Choose $S = 1$

Direct form II:

$x_s(n) = x(n)$, $w_s(n) = x_s(n) + 0.3w(n-1)$, $w(n) = w_s(n)$

$y_s(n) = 0.675w(n)$, $y(n) = (B \times S) y_s(n) = 2y_s(n)$

9.28

$H(z) = \frac{0.72 + 1.42z^{-1} + 0.72z^{-2}}{1 + 1.35z^{-1} + 0.5z^{-2}}$

Using MATLAB function leads

```
>> h=impz([0.72 1.42 0.72],[1 1.35 0.5]);
>> sf=sum(abs(h))
sf = 1.7876
```

Choose $S = 2$.

Choosing $C = 2$ to scale down the filter coefficients by half.

$x_s(n) = x(n)/2$,

$y_s(n) = 0.36x_s(n) + 0.71x_s(n-1) + 0.36x_s(n-2) - 0.675y_f(n-1) - 0.25y_f(n-2)$, $y_f(n) = 2y_s(n)$,

$y(n) = 2y_f(n)$

9.29

$$A(z) = \frac{1}{1 + 1.35z^{-1} + 0.5z^{-2}}$$

Using MATLAB function leads

```
>> h=impz(1,[1 1.35 0.5]);
>> sf=sum(abs(h))
  sf = 7.0504
```

Choose $S = 8$.

Choosing $A = 2$ to scale down the denominator coefficients by half.

Since the second adder output after scaling is

$$y_s(n) = \frac{0.72}{B} w(n) + \frac{1.42}{B} w(n-1) + \frac{0.72}{B} w(n-2)$$

$B = 4$

$x_s(n) = x(n)/8$, $w_s(n) = 0.5x_s(n) - 0.675w(n-1) - 0.25w(n-2)$, $w(n) = 2w_s(n)$

$y_s(n) = 0.18w(n) + 0.355w(n-1) + 0.18w(n-2)$, $y(n) = 32y_s(n)$

Chapter 10

10.1

$$\frac{dJ}{dw} = -40 + 10\times 2w = 0\,,\ \ w^* = 2$$

$$J_{\min} = J\,|_{w=w^*} = 50 - 40w + 10w^2\,|_{w=1} = 50 - 40\times 2 + 10\times 2^2 = 10$$

10.2

$$\frac{dJ}{dw} = 20 + 10\times 2w = 0\,,\ \ w^* = -1$$

$$J_{\min} = J\,|_{w=w^*} = 15 + 20w + 10w^2\,|_{w=1} = 15 + 20\times(-1) + 10\times(-1)^2 = 5$$

10.3

$$\frac{dJ}{dw} = 20 + 2\times 2w = 0\,,\ \ w^* = -5$$

$$J_{\min} = J\,|_{w=w^*} = 100 + 20w + 2w^2\,|_{w=1} = 100 + 20\times(-5) + 2\times(-5)^2 = 50$$

10.4

$$\frac{dJ}{dw} = -30 + 2\times 15w = 0\,,\ \ w^* = 1$$

$$J_{\min} = J\,|_{w=w^*} = 10 - 30w + 15w^2\,|_{w=1} = 10 - 30\times 1 + 15\times 1^2 = -5$$

10.5

$$\frac{dJ}{dw} = -40 + 20w_n$$

$$\mu\frac{dJ}{dw} = 0.04\times\left(-40 + 20w_0\right)|_{w_0=0} = -1.6\,,\ \ w_1 = w_0 - \mu\frac{dJ}{dw} = 0 - \left(-1.6\right) = 1.6$$

$$\mu\frac{dJ}{dw} = 0.04\times\left(-40 + 20w_1\right)|_{w_1=1.6} = -0.32\ \ \ w_2 = w_1 - \mu\frac{dJ}{dw} = 1.6 - \left(-0.32\right) = 1.92$$

$$\mu\frac{dJ}{dw} = 0.04\times\left(-40 + 20w_2\right)|_{w_2=1.92} = -0.064\ \ \ w_3 = w_2 - \mu\frac{dJ}{dw} = 1.92 - \left(-0.064\right) = 1.984$$

$$J_{\min} \approx 40 - 20w + 10w^2\,|_{w=0.992} = 50 - 40\times 1.984 + 10\times 1.984^2 = 10.0026$$

$w^* \approx w_2 = 1.984$, and $J_{\min} = 10.0026$

10.6

$$\frac{dJ}{dw} = 20 + 20w_n$$

$$\mu\frac{dJ}{dw} = 0.04\times\left(20 + 20w_0\right)|_{w_0=0} = 0.8\,,\ \ w_1 = w_0 - \mu\frac{dJ}{dw} = 0 - \left(0.8\right) = -0.8$$

$$\mu\frac{dJ}{dw} = 0.04\times\left(20 + 20w_1\right)|_{w_1=-0.8} = 0.16\ \ \ w_2 = w_1 - \mu\frac{dJ}{dw} = -0.8 - \left(0.16\right) = -0.96$$

$$\mu \frac{dJ}{dw} = 0.04 \times \left(20 + 20w_2\right)|_{w_2 = -0.96} = 0.032 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = -0.96 - \left(0.032\right) = -0.992$$

$$J_{\min} \approx 40 - 20w + 10w^2 \,|_{w=0.992} = 15 + 20 \times (-0.992) + 10 \times (-0.992)^2 = 5.00064$$

$$w^* \approx w_3 = -0.992 \text{, and } J_{\min} = 5.00064$$

10.7

$$\frac{dJ}{dw} = 20 + 4w_n$$

$$\mu \frac{dJ}{dw} = 0.2 \times \left(20 + 4w_0\right)|_{w_0 = -4} = 0.8, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = -4 - \left(0.08\right) = -4.8$$

$$\mu \frac{dJ}{dw} = 0.2 \times \left(20 + 4w_1\right)|_{w_1 = -4.8} = 0.16 \quad w_2 = w_1 - \mu \frac{dJ}{dw} = -4.8 - \left(0.16\right) = -4.96$$

$$\mu \frac{dJ}{dw} = 0.2 \times \left(20 + 4w_2\right)|_{w_2 = -4.96} = 0.032 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = -4.96 - \left(0.032\right) = -4.992$$

$$J_{\min} \approx 100 + 20w + 2w^2 \,|_{w=-4.992} = 100 + 20 \times (-4.992) + 2 \times (-4.992)^2 = 5.0001$$

$$w^* \approx w_3 = -4.992 \text{, and } J_{\min} = 5.0001$$

10.8

$$\frac{dJ}{dw} = -30 + 30w_n$$

$$\mu \frac{dJ}{dw} = 0.02 \times \left(-30 + 30w_0\right)|_{w_0 = 2} = 0.6, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = 2 - \left(0.6\right) = 1.4$$

$$\mu \frac{dJ}{dw} = 0.02 \times \left(-30 + 30w_1\right)|_{w_1 = 1.4} = 0.24 \quad w_2 = w_1 - \mu \frac{dJ}{dw} = 1.4 - \left(0.24\right) = 1.16$$

$$\mu \frac{dJ}{dw} = 0.02 \times \left(-30 + 30w_2\right)|_{w_2 = 1.16} = 0.096 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = 1.16 - \left(0.096\right) = 1.064$$

$$J_{\min} \approx 10 - 30w + 15w^2 \,|_{w=1.064} = 10 - 30 \times (1.064) + 15 \times (1.064)^2 = -4.93856$$

$$w^* \approx w_3 = 1.064 \text{, and } J_{\min} = -4.93856$$

10.9

a.

$$y(n) = w(0)x(n) + w(1)x(n-1)$$

$$e(n) = d(n) - y(n)$$

$$w(0) = w(0) + 0.2 \times e(n)x(n)$$

$$w(1) = w(1) + 0.2 \times e(n)x(n-1)$$

b.

for $n = 0$: $y(0) = 0$, $e(0) = 3$, $w(0) = 1.8$, $w(1) = 1$

for $n = 1$: $y(1) = 1.2$, $e(1) = -3.2$, $w(0) = 2.44$, $w(1) = -0.92$

for $n = 2$: $y(2) = 5.8$, $e(2) = -4.8$, $w(0) = 0.52$, $w(1) = 0.04$

10.10

$$d(n) = 0.25x(n) + 0.25x(n-1) + 0.5d(n-1)$$
$$y(n) = w(0)x(n) + w(1)x(n-1) + w(2)x(n-2) + w(3)x(n-3) + w(4)x(n-4)$$
$$e(n) = d(n) - y(n)$$
for $i = 0, \cdots, 4$
$$w(i) = w(i) + 2\mu e(n)x(n-i)$$
or
$$w(0) = w(0) + 2\mu e(n)x(n)$$
$$w(1) = w(1) + 2\mu e(n)x(n-1)$$
$$w(2) = w(2) + 2\mu e(n)x(n-2)$$
$$w(3) = w(3) + 2\mu e(n)x(n-3)$$
$$w(4) = w(4) + 2\mu e(n)x(n-4)$$

10.11

a.
$$y(n) = w(0)x(n) + w(1)x(n-1) + w(2)x(n-2)$$
$$e(n) = d(n) - y(n)$$
$$w(0) = w(0) + 0.2 \times e(n)x(n)$$
$$w(1) = w(1) + 0.2 \times e(n)x(n-1)$$
$$w(2) = w(2) + 0.2 \times e(n)x(n-2)$$

b.

for $n = 0$: $y(0) = 0$, $e(0) = 3$, $w(0) = 1.8$, $w(1) = 0$, $w(2) = 0$

for $n = 1$: $y(1) = 1.2$, $e(1) = -0.2$, $w(0) = 1.84$, $w(1) = -0.12$, $w(2) = 0.0$

for $n = 2$: $y(2) = 3.8$, $e(2) = -2.8$, $w(0) = 0.72$, $w(1) = 0.44$, $w(2) = -1.68$

10.12 $d(n) = 0.2x(n) + 0.3x(n-1) + 0.2x(n-2)$
$$y(n) = w(0)x(n) + w(1)x(n-1) + w(2)x(n-2) + w(3)x(n-3) + w(4)x(n-4)$$
$$e(n) = d(n) - y(n)$$
for $i = 0, \cdots, 4$
$$w(i) = w(i) + 2\mu e(n)x(n-i)$$
or
$$w(0) = w(0) + 2\mu e(n)x(n)$$
$$w(1) = w(1) + 2\mu e(n)x(n-1)$$
$$w(2) = w(2) + 2\mu e(n)x(n-2)$$
$$w(3) = w(3) + 2\mu e(n)x(n-3)$$
$$w(4) = w(4) + 2\mu e(n)x(n-4)$$

10.13

a. $n(n) = 0.5 \cdot x(n-5)$

b. $xx(n) = 5000 \cdot \delta(n)$, $yy(n) = 0.7071xx(n-1) + 1.4141yy(n-1) - yy(n-2)$

c. $d(n) = yy(n) - n(n)$

d. for $i = 0, \cdots, 24$, $w(i) = 0$

$$y(n) = \sum_{i=0}^{24} w(i)x(n-i)$$

$$e(n) = d(n) - y(n)$$

for $i = 0, \cdots, 24$

$$w(i) = w(i) + 2\mu e(n)x(n-i)$$

10.14

a. $w(0) = w(1)$, $\mu = 0.1$

$y(n) = w(0)x(n) + w(1)x(n-1)$

$e(n) = d(n) - y(n)$

$w(0) = w(0) + 0.2e(n)x(n)$

$w(1) = w(1) + 0.2e(n)x(n-1)$

b. for $n = 0$: $y(0) = 0$, $e(0) = 0$, $w(0) = 0$, $w(1) = 0$

for $n = 1$: $y(1) = 0$, $e(1) = 2$, $w(0) = 0.4$, $w(1) = 0.4$

for $n = 2$: $y(2) = 0$, $e(2) = -1$, $w(0) = 0.6$, $w(1) = 0.2$

10.15

a. $w(0) = w(1) = w(2) = 0$, $\mu = 0.1$

$y(n) = w(0)x(n) + w(1)x(n-1) + w(2)x(n-2)$

$e(n) = d(n) - y(n)$

$w(0) = w(0) + 0.2e(n)x(n)$

$w(1) = w(1) + 0.2e(n)x(n-1)$

$w(2) = w(2) + 0.2e(n)x(n-2)$

b. for $n = 0$: $y(0) = 0$, $e(0) = 0$, $w(0) = 0$, $w(1) = 0$, $w(2) = 0$

for $n = 1$: $y(1) = 0$, $e(1) = 2$, $w(0) = 0.4$, $w(1) = 0.4$, $w(2) = 0$

for $n = 2$: $y(2) = 0$, $e(2) = -1$, $w(0) = 0.6$, $w(1) = 0.2$, $w(2) = -0.2$

10.16

a. $w(0) = w(1) = 0$, $\mu = 0.1$

$x(n) = d(n-2)$

$y(n) = w(0)x(n) + w(1)x(n-1)$

$e(n) = d(n) - y(n)$

$w(0) = w(0) + 0.2e(n)x(n)$

$w(1) = w(1) + 0.2e(n)x(n-1)$

b. for $n = 0$: $x(0) = 0$, $y(0) = 0$, $e(0) = -1$, $w(0) = 0$, $w(1) = 0$

for $n = 1$: $x(1) = 0$, $y(1) = 0$, $e(1) = 1$, $w(0) = 0$, $w(1) = 0$

for $n = 2$: $x(2) = -1$, $y(2) = 0$, $e(2) = -1$, $w(0) = 0.2$, $w(1) = 0$

for $n = 3$: $x(3) = 1$, $y(3) = 0.2$, $e(3) = 0.8$, $w(0) = 0.36$, $w(1) = -0.16$

for $n = 4$: $x(4) = -1$, $y(4) = -0.52$, $e(4) = -0.48$, $w(0) = 0.456$, $w(1) = -0.256$

10.17

a. $w(0) = w(1) = 0$, $\mu = 0.1$

$x(n) = d(n-3)$

$y(n) = w(0)x(n) + w(1)x(n-1)$

$e(n) = d(n) - y(n)$

$w(0) = w(0) + 0.2e(n)x(n)$

$w(1) = w(1) + 0.2e(n)x(n-1)$

b. for $n = 0$: $x(0) = 0$, $y(0) = 0$, $e(0) = -1$, $w(0) = 0$, $w(1) = 0$

for $n = 1$: $x(1) = 0$, $y(1) = 0$, $e(1) = 1$, $w(0) = 0$, $w(1) = 0$

for $n = 2$: $x(2) = 0$, $y(2) = 0$, $e(2) = -1$, $w(0) = 0$, $w(1) = 0$

for $n = 3$: $x(3) = -1$, $y(3) = 0$, $e(3) = 1$, $w(0) = -0.2$, $w(1) = 0$

for $n = 4$: $x(4) = 1$, $y(4) = -0.2$, $e(4) = -0.8$, $w(0) = -0.36$, $w(1) = 0.16$

10.18

a. 30 coefficients

b. for $i = 0, \cdots, 29$, $w(i) = 0$

$$y(n) = \sum_{i=0}^{29} w(i)x(n-i-\Delta)$$

$$e(n) = d(n) - y(n)$$

for $i = 0, \cdots, 29$

$$w(i) = w(i) + 2\mu e(n)x(n-i)$$

10.19 10 coefficients

for $i = 0, \cdots, 9$, $w(i) = 0$

$$y(n) = \sum_{i=0}^{9} w(i)x(n-i-\Delta)$$

$$e(n) = d(n) - y(n)$$

for $i = 0, \cdots, 9$

$$w(i) = w(i) + 2\mu e(n)x(n-i)$$

10.20

for $i = 0, \cdots, 19$, $w(i) = 0$

$$y(n) = \sum_{i=0}^{19} w(i)x(n-i)$$

$$e(n) = d(n) - y(n)$$

for $i = 0, \cdots, 19$

$$w(i) = w(i) + 2\mu e(n)x(n-i)$$

10.21

$$y_A(n) = \sum_{i=0}^{3} w(i)x_B(n-i)$$

$$e_A(n) = d_A(n) - y_A(n)$$

for $i = 0, \cdots, 3$

$$w(i) = w(i) + 2\mu e_A(n)x_B(n-i)$$

10.22

a. See Section 10.4.3, b. See Section 10.4.3

c. $$y_A(n) = \sum_{i=0}^{N-1} w(i)x_B(n-i)$$

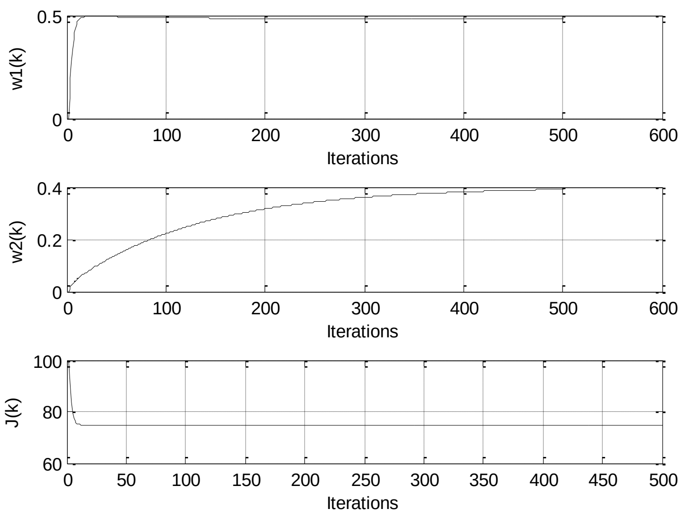
$$e_A(n) = d_A(n) - y_A(n)$$

for $i = 0, \cdots, N-1$

$$w(i) = w(i) + 2\mu e_A(n)x_B(n-i)$$

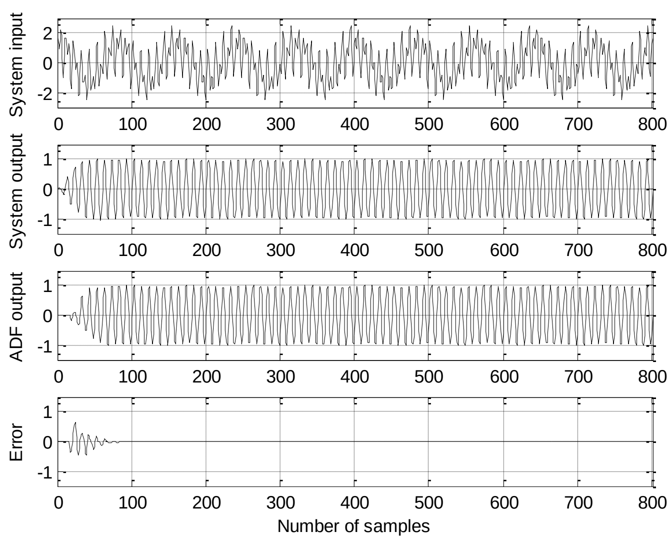
10.23

```
-----------------------------------------------------------------------------------------------
close all; clear all
w1(1)=0;w2(1)=0;mu=0.001;
for k=1:500
   J(k)=100+100*w1(k)^2+4*w2(k)^2-100*w1(k)-8*w2(k)+10*w1(k)*w2(k);
   dj1(k+1)=200*w1(k)-100+10*w2(k);
   dj2(k+1)=8*w2(k)-8+10*w1(k);
   w1(k+1)=w1(k)-mu*dj1(k);
   w2(k+1)=w2(k)-mu*dj2(k);
end
subplot(3,1,1);plot(w1,'k');grid;
xlabel('Iterations');ylabel('w1(k)');
subplot(3,1,2);plot(w2,'k');grid
xlabel('Iterations');ylabel('w2(k)');
subplot(3,1,3);plot(J,'k');grid
xlabel('Iterations');ylabel('J(k)');
---------------------------------------------------------------------
```

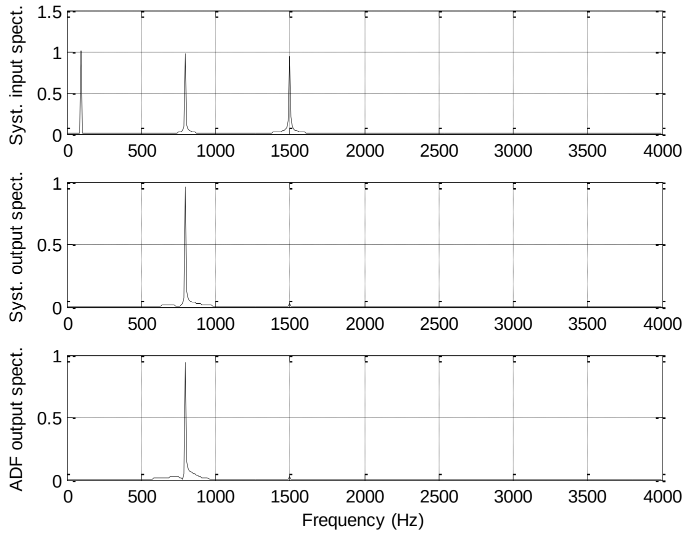


10.24 (a) and (b) Modify Program 10.2 in Section 10.3.2.

c.



d.

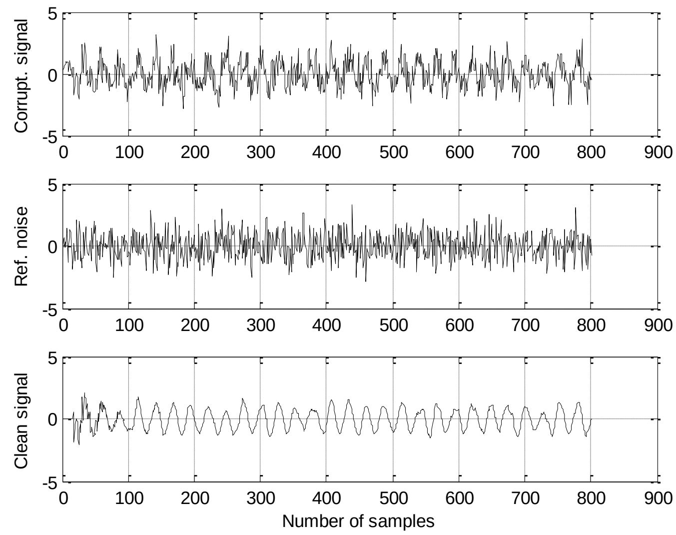


10.25
a.

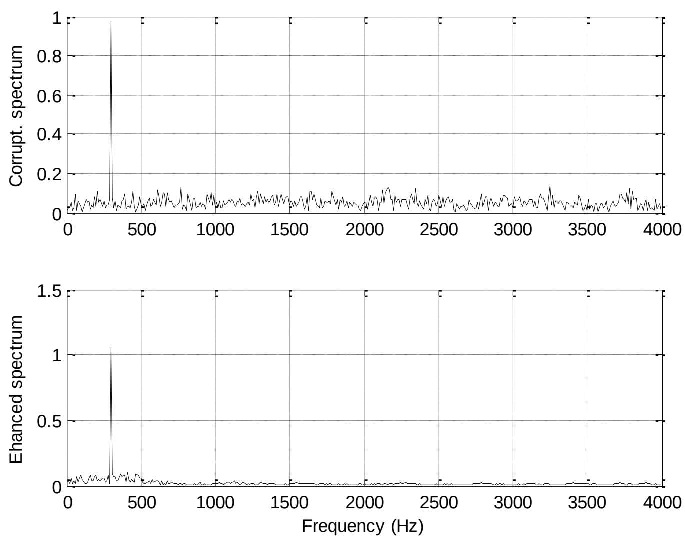
```
close all; clear all
fs=8000;T=1/fs;
t=0:T:0.1;
x=randn(1,length(t));
n=filter([ 0 0 0 0 0 0 0 0 0 0.8 ],1,x);
d=sin(2*pi*300*t)+n;                    % generate signal plus noise
mu=0.01;w=zeros(1,16);y=zeros(1,length(t));
e=y;
 % adaptive filtering using LMS algorithm
for m=17:1:length(t)-1
     sum=0;
    for i=1:1:16
     sum=sum+w(i)*x(m-i);
    end
     y(m)=sum;
     e(m)=d(m)-y(m);
     for i=1:1:16
     w(i)=w(i)+2*mu*e(m)*x(m-i);
    end
end
 D=2*abs(fft(d))/length(d);D(1)=D(1)/2;
 f=[0:1:length(d)/2]*8000/length(d);
 E=2*abs(fft(e))/length(e);E(1)=E(1)/2;
 subplot(3,1,1),plot(d,'k');grid; ylabel('Corrupt. signal')
 subplot(3,1,2),plot(x,'k');grid;ylabel('Ref. noise');
```

```
 subplot(3,1,3),plot(e,'k');grid; ylabel('Clean signal');
 xlabel('Number of samples');
figure
subplot(2,1,1),plot(f,D(1:length(f)),'k');grid; ylabel('Corrupt. spectrum')
subplot(2,1,2),plot(f,E(1:length(f)),'k');grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');
```
----

b.



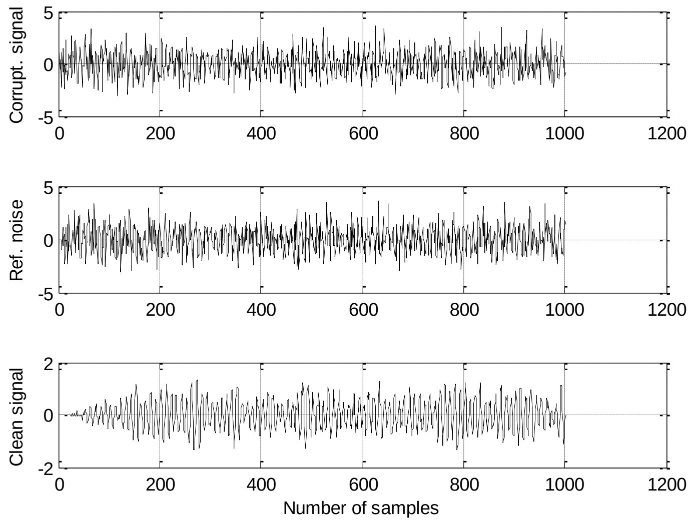
c.

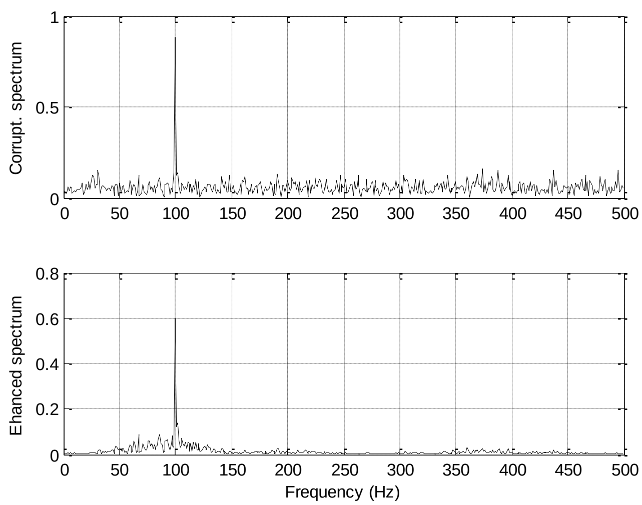


10.26

a.

```
----------------------------------------------------------------------------------------------
close all; clear all;clc
fs=1000;T=1/fs;
t=0:T:1;
n=randn(1,length(t));
d=sin(2*pi*100*t)+n;
x=filter([ 0 0 0 0 0 0 0 1 ],1,d);
mu=0.001;w=zeros(1,16);y=zeros(1,length(t));
e=y;
 % adaptive filtering using LMS algorithm
for m=17:1:length(t)-1
     sum=0;
    for i=1:1:16
     sum=sum+w(i)*x(m-i);
    end
     y(m)=sum;
     e(m)=d(m)-y(m);
     for i=1:1:16
     w(i)=w(i)+2*mu*e(m)*x(m-i);
    end
end
 D=2*abs(fft(d))/length(d);D(1)=D(1)/2;
 f=[0:1:length(d)/2]*1000/length(d);
 Y=2*abs(fft(y))/length(y);Y(1)=Y(1)/2;
 subplot(3,1,1),plot(d,'k');grid; ylabel('Corrupt. signal')
 subplot(3,1,2),plot(x,'k');grid;ylabel('Ref. noise');
 subplot(3,1,3),plot(y,'k');grid; ylabel('Clean signal');
 xlabel('Number of samples');
figure
subplot(2,1,1),plot(f,D(1:length(f)),'k');grid; ylabel('Corrupt. spectrum')
subplot(2,1,2),plot(f,Y(1:length(f)),'k');grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');
----------------------------------------------------------------------------------------------
```





b. Δ is around 8 samples.

Chapter 11

11.1

a. $\Delta = \frac{5}{2^3 - 1} = 0.714$ volts

b. for $x = 1.6 = 2.24\Delta$ volts, binary code=110, $x_q = 1.428$ volts, and $e_q = -0.172$ volts

for $x = -0.2 = -0.28\Delta$ volts, binary code=000, $x_q = 0$ volts, and $e_q = 0.2$ volts

11.2

a. $\Delta = \frac{8}{2^3 - 1} = 1.143$ volts

b. for $x = -2.6 = -2.275\Delta$ volts, binary code=010, $x_q = 2.286$ volts, and $e_q = -0.314$ volts

for $x = 0.1 = 0.0875\Delta$ volts, binary code=100, $x_q = 0$ volts, and $e_q = -0.1$ volts

11.3

a. $\Delta = \frac{10}{2^3 - 1} = 1.4286$ volts

b. for $x = -2.6 = -1.82\Delta$ volts, binary code=010, $x_q = -2.8572$ volts, and $e_q = -0.2572$ volts

for $x = 3.5 = 2.4500\Delta$ volts, binary code=110, $x_q = 2.8572$ volts, and $e_q = -0.6482$ volts

11.4

a. $\Delta = \frac{20}{2^3 - 1} = 2.8571$ volts

b. for $x = -5 = -1.75\Delta$ volts, binary code=010, $x_q = -5.7142$ volts, and $e_q = -0.7142$ volts

for $x = 0 = 0\Delta$ volts, binary code=100, $x_q = 0$ volts, and $e_q = 0$ volts

for $x = 7.2 = 2.52\Delta$ volts, binary code=111, $x_q = 8.5713$ volts, and $e_q = 1.3713$ volts

11.5

for $x = 1.6$ volts, $y = sign(1.6)\frac{\ln\left(1 + 255\frac{|1.6|}{2.5}\right)}{\ln(1 + 255)} = 0.920$

$\Delta = \frac{2}{2^3 - 1} = 0.286$, $y = \frac{0.920}{0.286} = 3.216\Delta$, binary code=111

$y_q = 3\Delta = 0.858 \quad x_q = 2.5 \times sign(0.858)\frac{(1 + 255)^{|0.858|} - 1}{255} = 1.132$

$e_q = x_q - x = 1.132 - (1.6) = -0.468$ volts

for $x = -0.2$, we get

$$y = sign(-0.2)\frac{\ln\left(1+255\frac{|-0.2|}{2.5}\right)}{\ln(1+255)} = -0.552$$

$$y = -\frac{0.552}{0.286}\Delta = -1.93\Delta \text{ and binary code } =010$$

$$y_q = -2\Delta = -0.572,\ x_q = 2.5\times sign(-0.572)\frac{(1+255)^{|-0.572|}-1}{255} = -0.224 \text{ volts}$$

$$e_q = -0.224-(-0.2) = -0.024 \text{ volts}$$

11.6

$$\text{for } x = -2.6 \text{ volts, } y = sign(-2.6)\frac{\ln\left(1+255\frac{|-2.6|}{4}\right)}{\ln(1+255)} = -0.923$$

$$\Delta = \frac{2}{2^3-1} = 0.286,\ y = \frac{-0.923}{0.286} = -3.226\Delta,\ \text{binary code=011}$$

$$y_q = -3\Delta = -0.858,\ x_q = 4\times sign(-0.858)\frac{(1+255)^{|-0.858|}-1}{255} = -1.842$$

$$e_q = x_q - x = -1.842-(-2.6) = 0.758 \text{ volts}$$

for $x = 0.1$, we get

$$y = sign(0.2)\frac{\ln\left(1+255\frac{|0.1|}{4}\right)}{\ln(1+255)} = 0.360$$

$$y = \frac{0.360}{0.286}\Delta = 1.26\Delta \text{ and binary code } =101$$

$$y_q = \Delta = 0.286,\ x_q = 4\times sign(0.286)\frac{(1+255)^{|0.286|}-1}{255} = 0.061 \text{ volts}$$

$$e_q = 0.061-0.1 = -0.039 \text{ volts}$$

11.7

$$\text{for } x = -2.6 \text{ volts, } y = sign(-2.6)\frac{\ln\left(1+255\frac{|2.6|}{5}\right)}{\ln(1+255)} = -0.8827$$

$$\Delta = \frac{2}{2^3-1} = 0.286,\ y = \frac{-0.920}{0.286} = -3.086\Delta,\ \text{binary code=011}$$

$$y_q = -3\Delta = -0.858 \quad x_q = 5\times sign(-0.858)\frac{(1+255)^{|-0.858|}-1}{255} = -2.2644$$

$$e_q = x_q - x = -2.2644 - (-2.6) = 0.3356 \text{ volts}$$

for $x = 3.5$, we get

$$y = sign(3.5)\frac{\ln\left(1+255\frac{|3.5|}{5}\right)}{\ln(1+255)} = 0.9360$$

$$y = \frac{0.9360}{0.286}\Delta = 3.27\Delta \text{ and binary code } =111$$

$$y_q = 3\Delta = 0.858,\ x_q = 5\times sign(0.858)\frac{(1+255)^{|0.858|}-1}{255} = 2.2644 \text{ volts}$$

$$e_q = 2.2644 - 3.5 = -1.2356 \text{ volts}$$

11.8

for $x = -5$ volts, $y = sign(-5)\dfrac{\ln\left(1+255\frac{|-5|}{10}\right)}{\ln(1+255)} = -0.8757$

$$\Delta = \frac{2}{2^3-1} = 0.286,\ y = \frac{-0.8757}{0.286} = -3.0619\Delta, \text{ binary code=011}$$

$$y_q = -3\Delta = -0.858 \quad x_q = 10\times sign(-0.858)\frac{(1+255)^{|-0.858|}-1}{255} = -4.5288$$

$$e_q = x_q - x = -4.5288 - (-5) = 0.4712 \text{ volts}$$

for $x = 0$, we get

$$y = sign(0)\frac{\ln\left(1+255\frac{|0|}{10}\right)}{\ln(1+255)} = 0.0$$

$$y = \frac{0.0}{0.286}\Delta = 0.0\Delta \text{ and binary code } =100$$

$$y_q = 0\Delta = 0.0,\ x_q = 10\times sign(0.0)\frac{(1+255)^{|0.0|}-1}{255} = 0.0 \text{ volts}$$

$$e_q = 0.0 - 0.0 = 0.0 \text{ volts}$$

for $x = 7.2$, we get

$$y = sign(7.2)\frac{\ln\left(1+255\frac{|7.2|}{10}\right)}{\ln(1+255)} = 0.9410$$

$$y = \frac{0.9410}{0.286}\Delta = 3.29\Delta \text{ and binary code } =111$$

$$y_q = 3\Delta = 0.858,\ x_q = 10 \times sign(0.0)\frac{(1+255)^{|0.858|}-1}{255} = 4.5288 \text{ volts}$$

$$e_q = 4.5288 - 7.2 = -2.6712 \text{ volts}$$

11.9

a. 0 0 0 1 0 1 0 1 b. 1 1 1 0 0 1 1 1

11.10

a. 0 0 0 0 0 0 0 0 0 1 1 1 b. 1 0 1 1 0 0 1 1 0 0 0 0

11.11

a. 0 1 1 0 0 1 0 1 b. 1 0 0 0 1 1 0 1

11.12

a. 0 0 0 0 0 0 1 1 1 0 1 1 b. 1 0 0 0 0 0 0 0 0 1 0 1

11.13

$\tilde{x}(0) = \hat{x}(0-1) = 0$, $d(0) = x(0) - \tilde{x}(0) = -6$

$d_q(0) = Q[d(0)] = -5$, binary code= 010, $\hat{x}(0) = \tilde{x}(0) + d_q(0) = -5$

$\tilde{x}(1) = \hat{x}(0) = -5$, $d(1) = x(1) - \tilde{x}(1) = -8 - (-5) = -3$

$d_q(1) = Q[d(1)] = -2$, binary code= 001, $\hat{x}(1) = \tilde{x}(1) + d_q(1) = -5 + (-2) = -7$

$\tilde{x}(2) = \hat{x}(1) = -7$, $d(2) = x(2) - \tilde{x}(2) = -13 - (-7) = -5$

$d_q(2) = Q[d(2)] = -5$, binary code= 010, $\hat{x}(2) = \tilde{x}(2) + d_q(2) = -7 + (-5) = -12$

11.14

for binary code =110

$\tilde{x}(0) = \hat{x}(-1) = 0$, $d_q(0) = 5$, $\hat{x}(0) = \tilde{x}(0) + d_q(0) = 0 + 5 = 5$

for binary code =100

$\tilde{x}(1) = \hat{x}(0) = 5$, $d_q(1) = 0$, $\hat{x}(1) = \tilde{x}(1) + d_q(1) = 5 + 0 = 5$

for binary code =110

$\tilde{x}(2) = \hat{x}(1) = 5$, $d_q(2) = 2$, $\hat{x}(2) = \tilde{x}(2) + d_q(2) = 5 + 2 = 7$

11.15

$\tilde{x}(0) = \hat{x}(0-1) = 0$, $d(0) = x(0) - \tilde{x}(0) = 6$

$d_q(0) = Q[d(0)] = 5$, binary code= 110, $\hat{x}(0) = \tilde{x}(0) + d_q(0) = 5$

$\tilde{x}(1) = \hat{x}(0) = 5$, $d(1) = x(1) - \tilde{x}(1) = 8 - 5 = 3$

$d_q(1) = Q[d(1)] = 2$, binary code= 101, $\hat{x}(1) = \tilde{x}(1) + d_q(1) = 5 + 2 = 7$

$\tilde{x}(2) = \hat{x}(1) = 7$, $d(2) = x(2) - \tilde{x}(2) = 13 - 7 = 5$

$d_q(2) = Q[d(2)] = 5$, binary code= 110, $\hat{x}(2) = \tilde{x}(2) + d_q(2) = 7 + 5 = 12$

11.16

for binary code =010

$\tilde{x}(0)=\hat{x}(-1)=0$, $d_q(0)=-5$, $\hat{x}(0)=\tilde{x}(0)+d_q(0)=0-5=-5$

for binary code =000

$\tilde{x}(1)=\hat{x}(0)=-5$, $d_q(1)=0$, $\hat{x}(1)=\tilde{x}(1)+d_q(1)=-5+0=-5$

for binary code =001

$\tilde{x}(2)=\hat{x}(1)=-5$, $d_q(2)=-2$, $\hat{x}(2)=\tilde{x}(2)+d_q(2)=-5-2=-7$

11.17

a. 1 :1, b. 2:1, c. 4:1

11.18

a. 128 kbps, b. 64 kbps, c. 32 kbps

11.19

a. 1 :1, b. 2:1, c. 4:1

11.20

a. 529.2 kbps, b. 352.8 kbps, c. 176.4 kbps

11.21

a. 12 channels, b. 24 channels, c. 48 channels

11.22

$X_{DCT}(0)=54$, $X_{DCT}(1)=0.5412$, $X_{DCT}(2)=-4$, $X_{DCT}(3)=-1.3066$

11.23

$X_{DCT}(0)=38.8909$, $X_{DCT}(1)=-3.5355$

11.24

$X_{DCT}(0)=41.7193$, $X_{DCT}(2)=-4.2376$, $X_{DCT}(4)=0.7071$, $X_{DCT}(6)=-2.8373$

11.25

$X_{DCT}(1)=33.9730$, $X_{DCT}(3)=-10.4308$, $X_{DCT}(5)=1.2001$, $X_{DCT}(7)=-1.6102$

11.26

a. The inverse DCT: 10.0845 6.3973 13.6027 -2.0845

```
>> idct([14 6 -6 8])
ans =10.0845    6.3973   13.6027   -2.0845
```

b. Quantized DCT coefficients: 16, 8 -8, 8

Recovered inverse DCT: 11.3910 8.9385 15.0615 -3.3910

```
>> idct([16 8 -8 8])
ans = 11.3910    8.9385   15.0615   -3.3910
```

c. Quantization error: 1.3066 2.5412 1.4588 -1.3066

11.27

a. The inverse DCT: 11.4546 5.3128 -1.3128 6.5454

```
>> idct([11 5 7 -3])
ans = 11.4546    5.3128   -1.3128    6.5454
```

b. Quantized DCT coefficients: 8, 8 -8, 0

Recovered inverse DCT: 5.2263 10.1648 5.8352 -5.2263

```
>> idct([8 8 -8 0])
ans =5.2263   10.1648    5.8352   -5.2263
```

c. Quantization error: -6.2284 4.8519 7.1481 -11.7716

11.28

a.

(1)

$$f\left(n+\frac{N}{2}\right)h\left(n+\frac{N}{2}\right)=\sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)\sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)$$
$$=\sin\left(\frac{\pi}{N}(n+0.5)+\frac{\pi}{2}\right)\sin\left(\frac{\pi}{N}(n+0.5)+\frac{\pi}{2}\right)=\cos^2\left(\frac{\pi}{N}(n+0.5)\right)$$

and

$$f(n)h(n)=\sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right)=\sin^2\left(\frac{\pi}{N}(n+0.5)\right)$$

Hence

$$f\left(n+\frac{N}{2}\right)h\left(n+\frac{N}{2}\right)+f(n)h(n)=\cos^2\left(\frac{\pi}{N}(n+0.5)\right)+\sin^2\left(\frac{\pi}{N}(n+0.5)\right)=1$$

(2)

$$f\left(n+\frac{N}{2}\right)h(N-n-1)=\sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)\sin\left(\frac{\pi}{N}(N-n-1+0.5)\right)$$
$$=\sin\left(\frac{\pi}{N}(n+0.5)+\frac{\pi}{2}\right)\sin\left(\pi-\frac{\pi}{N}(n+0.5)\right)=\cos\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right)$$

$$f(n)h(N/2-n-1)=\sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(N/2-n-1+0.5)\right)$$
$$=\sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{N}(n+0.5)\right)=\sin\left(\frac{\pi}{N}(n+0.5)\right)\cos\left(\frac{\pi}{N}(n+0.5)\right)$$

Hence

$$f\left(n+\frac{N}{2}\right)h(N-n-1)=\sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)\sin\left(\frac{\pi}{N}(N-n-1+0.5)\right)-f(n)h(N/2-n-1)$$
$$=\cos\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right)-\sin\left(\frac{\pi}{N}(n+0.5)\right)\cos\left(\frac{\pi}{N}(n+0.5)\right)=0$$

b.

$$\begin{aligned}(-1)^{\frac{N}{2}+1} X_{MDCT}(N-1-k) &= (-1)^{\frac{N}{2}+1} 2\sum_{n=0}^{N-1} x(n)h(n)\cos\left[\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(N-1-k+0.5\right)\right] \\ &= (-1)^{\frac{N}{2}+1} 2\sum_{n=0}^{N-1} x(n)h(n)\cos\left[\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(N-k-0.5\right)\right] \\ &= (-1)^{\frac{N}{2}+1} 2\sum_{n=0}^{N-1} x(n)h(n)\cos\left[2\pi n+\pi+\frac{N\pi}{2}+\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(k+0.5\right)\right] \\ &= (-1)^{\frac{N}{2}+1} 2\sum_{n=0}^{N-1} x(n)h(n)(-1)\cos\left[\frac{N\pi}{2}+\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(k+0.5\right)\right]\end{aligned}$$

for $N =$ even number

$$\begin{aligned}(-1)^{\frac{N}{2}+1} X_{MDCT}(N-1-k) &= (-1)^{\frac{N}{2}+1} 2\sum_{n=0}^{N-1} x(n)h(n)(-1)(-1)^{\frac{N}{2}}\cos\left[\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(k+0.5\right)\right] \\ &= (-1)^{N+2} 2\sum_{n=0}^{N-1} x(n)h(n)\cos\left[\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(k+0.5\right)\right] \\ &= 2\sum_{n=0}^{N-1} x(n)h(n)\cos\left[\frac{2\pi}{N}\left(n+0.5+N/4\right)\left(k+0.5\right)\right] = X_{MDCT}(k)\end{aligned}$$

11.29

a.

First block data: 1 2 3 4
Second block data: 3 4 5 4
Third block data: 5 4 3 2

```
>> wmdct([1 2 3 4])
      ans = -9.0711   -0.5858
>> wmdct([3 4 5 4])
      ans = -13.3137   -0.0000
>> wmdct([5 4 3 2])
      ans =  -7.8995    0.5858
```

b.

```
>> x1=wimdct([-9.0711   -0.5858])
x1 =  -0.5607    1.3536    3.9749    1.6465
>> x2=wimdct([-13.3137   -0.0000 ])
x2 =-0.9749    2.3536    5.6820    2.3536
>> x3=wimdct([-7.8995    0.5858])
x3 =  -0.6820    1.6465    3.2678    1.3536
>> [x1 0 0 0 0]+ [0 0 x2 0 0 ]+ [ 0 0 0 0 x3]
```

-0.5607 1.3536 **3.0000** **4.0000** **5.0000** **4.0000** 3.2678 1.3536

The recovered first two subblocks have values as 3, 4, 5, 4 which are consistent with the input data.

11.30

a.

First block data: 1 2 3 4 5 4

Second block data: 4 5 4 3 2 1

Third block data: 3 2 1 2 3 4

```
>> wmdct([1 2 3 4 5 4])
   ans = -18.1953   -1.7932   -1.2247
>> wmdct([4 5 4 3 2 1])
   ans = -11.1242    2.8284   -0.1895
>> wmdct([3 2 1 2 3 4])
   ans = -9.5713   -3.8637    0.7071
```

b.

```
>> x1=wimdct([-18.1953   -1.7932   -1.2247])
x1 = -0.6830    0.0000    2.5490    4.7321    5.0000    1.2680
>> x2=wimdct([-11.1242    2.8284   -0.1895])
x2 = -0.7320    0.0000    2.7320    3.0490    2.0000    0.8170
>> x3=wimdct([-9.5713   -3.8637    0.7071])
x3 = -0.0490    0.0000    0.1830    2.8660    3.0000    0.7679
>> [x1 0 0 0 0 0 0]+ [0 0 0 x2 0 0 0 ]+ [ 0 0 0 0 0 0 x3]
```

-0.6830 0.0000 2.5490 **4.0000 5.0000 4.0000 3.0000 2.0000 1.0000** 2.8660 3.0000 0.7679

The recovered first two subblocks have values as 4, 5, 4, 3, 2, 1 which are consistent with the input data.

11.31

a. (1) unacceptable, (2) intelligent, (3) good

b. (1) unacceptable, (2) good, (3) excellent

11.32 Excellent

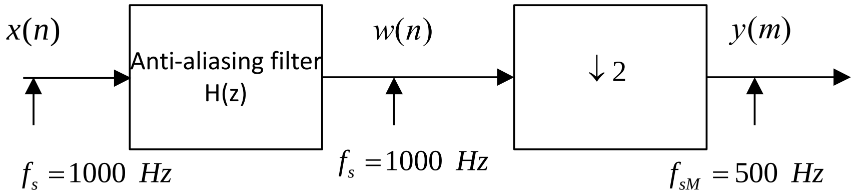
11.33

For (a), (b), (c), and (d), the decoded speech quality using the M-DCT coding is better.

Chapter 12

12.1

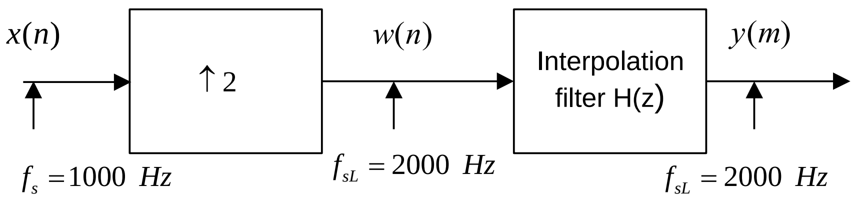
a.



b. Blackman window, $N = 37$, $f_c = 175$ Hz

12.2

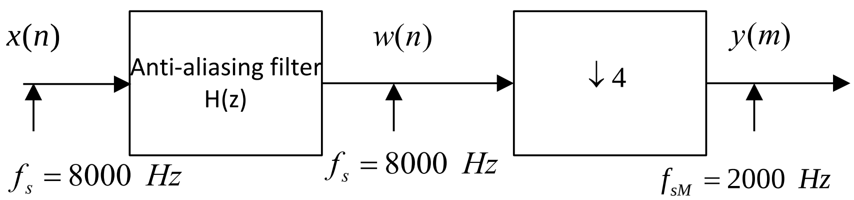
a.



b. Hamming window, $N = 19$, $f_c = 325$ Hz

12.3

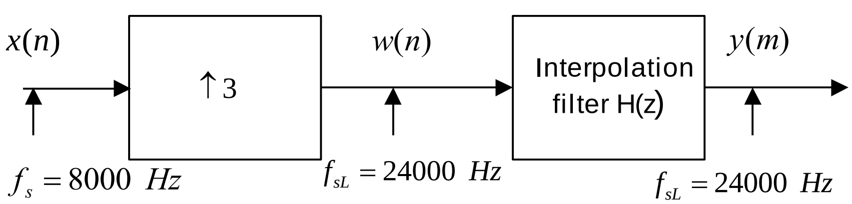
a.



b. Hamming window, $N = 133$, $f_c = 900$ Hz

12.4

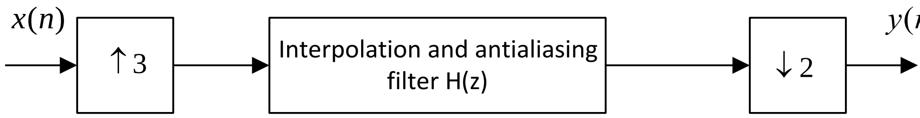
a.



b. Hamming window, $N = 133$, $f_c = 3700$ Hz

12.5

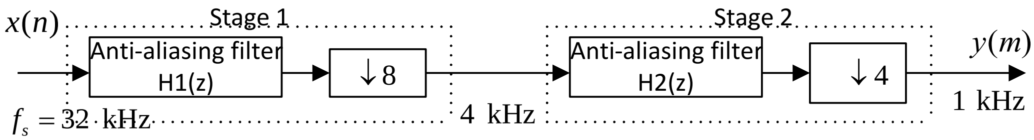
a.



b. Choose interpolation filter $H(z)$ and anti-aliasing requirement is automatically satisfied.
Hamming window, $N = 25$, $f_c = 1200$ Hz

12.6

a.



b. the sampling rate after stage is 4 kHz and the sampling rate after the second stage is 1 kHz.
$M_1 = 8$, and $M_2 = 4$

c.

Filter specification for $H_1(z)$:

Passband frequency range: 0 – 250 Hz
Passband ripples: =0.05/2=0.025 ($\delta_p\ dB = 20\log_{10}(1+\delta_p)$ =0.212 dB)
Stop band frequency range: 2000 Hz – 16000 Hz
Stop band attenuation = 0.005, $\delta_s\ dB = 20\times\log_{10}\left(\delta_s\right) = -46$ dB
Filter type: FIR type, Hamming window

$$f_{stop} = \frac{f_s}{M_1} - \frac{f_s}{2\times M} = \frac{32000}{8} - \frac{32000}{2\times 32} = 3500\ Hz$$

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{3500-250}{32000} \quad N = \frac{3.3}{\Delta f} = 32.49, \text{ choose } N = 33$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{3500+250}{2} = 1875 \text{ Hz}$$

d.

Filter specification for $H_2(z)$:

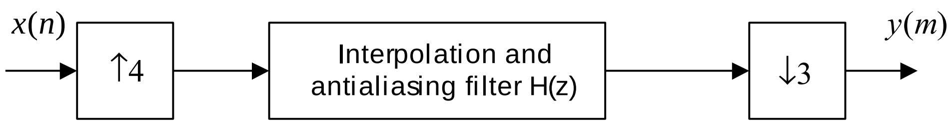
Passband frequency range: 0 – 250 Hz
Passband ripples: =0.05/2=0.025 (0.212 dB)
Stop band frequency range: 500 Hz – 2000 Hz
Stop band attenuation = 0.005, $\delta_s\ dB = -46$ dB
Filter type: FIR type, Hamming window

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sM1}} = \frac{500-250}{4000},\ N = \frac{3.3}{\Delta f} = 52.8, \text{ choose } N = 53,$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{500+250}{2} = 375 \text{ Hz}$$

12.7

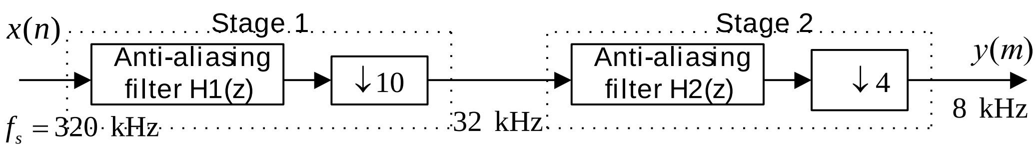
a.



b. Choose interpolation filter $H(z)$ and anti-aliasing requirement is automatically satisfied.
Hamming window, $N = 133$, $f_c = 2700$ Hz

12.8

a.



b. the sampling rate after stage is 32 kHz and the sampling rate after the second stage is 8 kHz. $M_1 = 10$, and $M_2 = 4$

c.

Filter specification for $H_1(z)$:

Passband frequency range: 0 – 3400 Hz

Passband ripples: =0.05/2=0.025 ($\delta_p \ dB = 20\log_{10}(1+\delta_p)$ =0.212 dB)

Stop band frequency range: 28000 Hz – 160000 Hz

Stop band attenuation = 0.005, $\delta_s \ dB = 20 \times \log_{10}(\delta_s) = -46$ dB

Filter type: FIR type, Hamming window

$$f_{stop} = \frac{f_s}{M_1} - \frac{f_s}{2 \times M} = \frac{320000}{10} - \frac{320000}{2 \times 40} = 28000 \ Hz$$

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{28000 - 3400}{320000} \quad N = \frac{3.3}{\Delta f} = 42.9, \text{ choose } N = 43$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{28000+3400}{2} = 15700 \text{ Hz}$$

d.

Filter specification for $H_2(z)$:

Passband frequency range: 0 – 3400 Hz

Passband ripples: =0.05/2=0.025 (0.212 dB)

Stop band frequency range: 4000 Hz – 16000 Hz

Stop band attenuation = 0.005, $\delta_s \ dB = -46$ dB

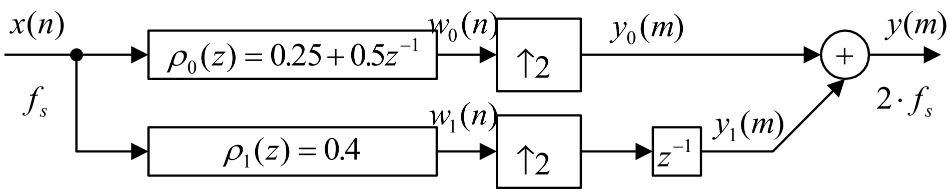
Filter type: FIR type, Hamming window

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sM1}} = \frac{4000-3400}{32000},\ N = \frac{3.3}{\Delta f} = 176,\ \text{choose}\ N = 177,$$

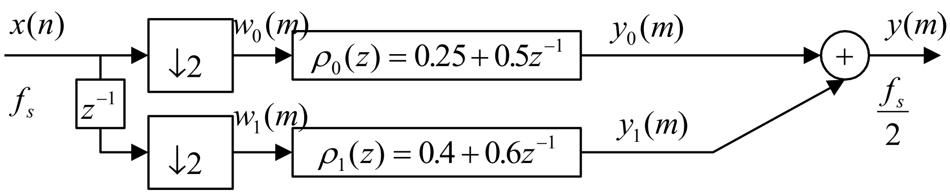
$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{4000+3400}{2} = 3700\ \text{Hz}$$

12.9

a.

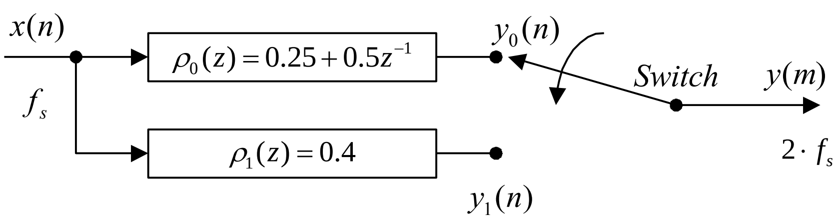


b.

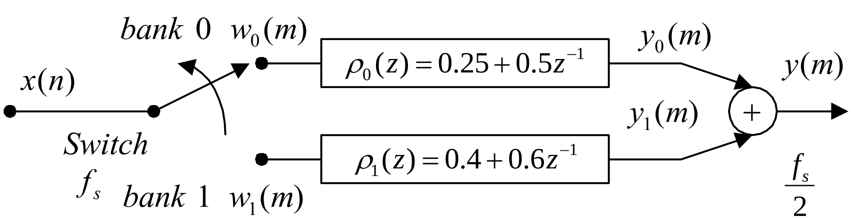


12.10

a.

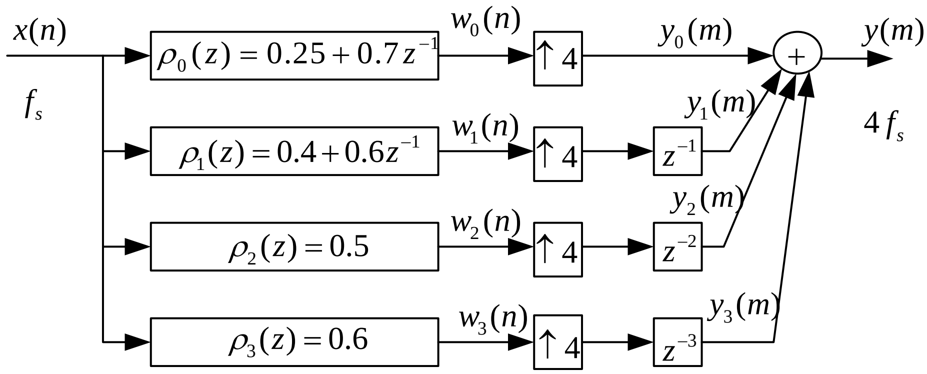


b.

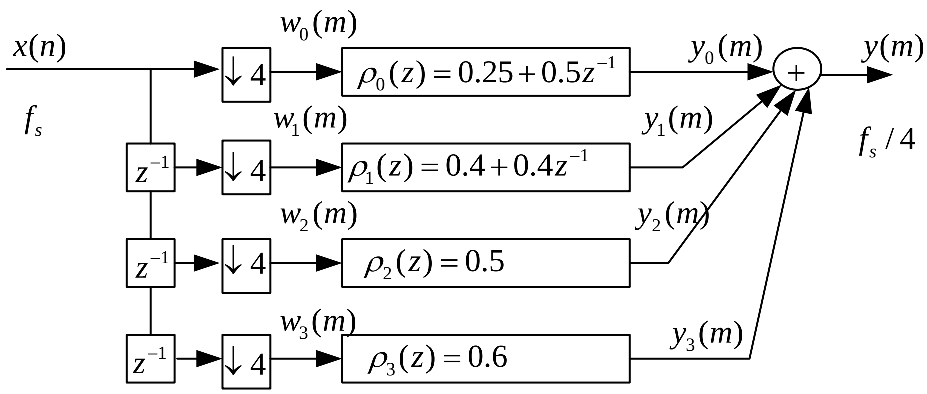


12.11

a.

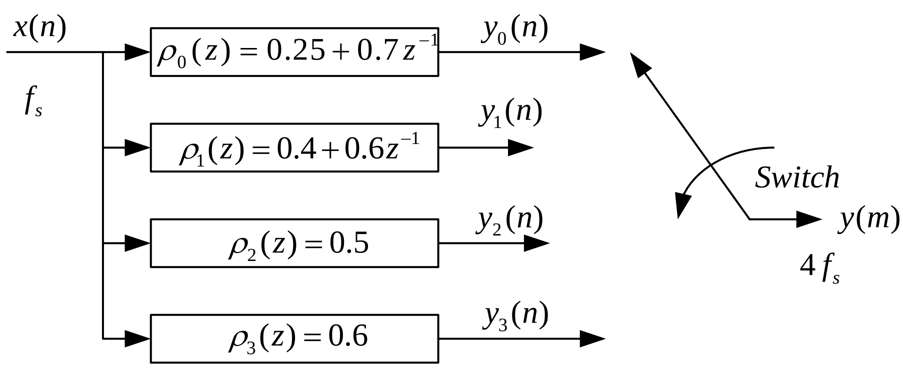


b.

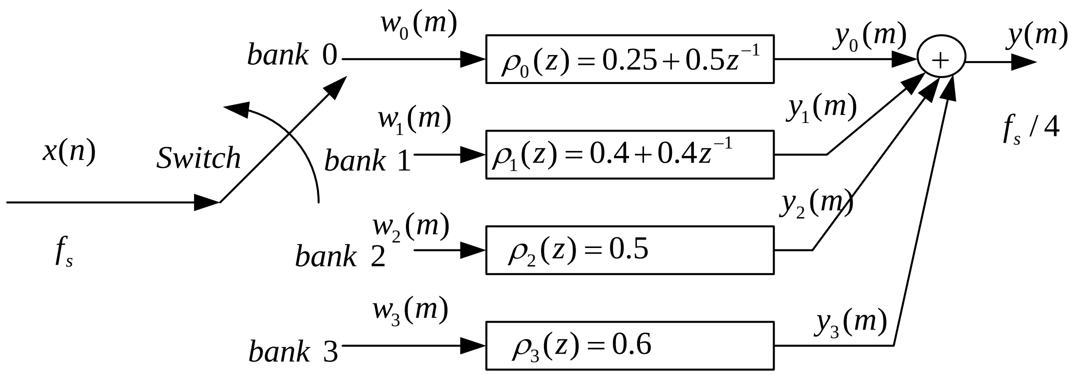


12.12

a.



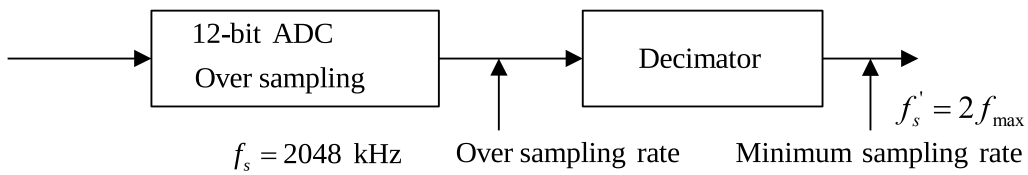
b.



12.13

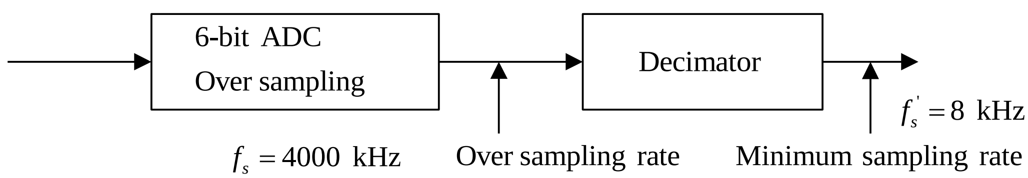
a. $f_s = 2 f_{\max} 2^{2(n-m)} = 2 \times 4 \times 2^{2\times(16-12)} = 2048 \text{ kHz}$

b.



12.14

a.



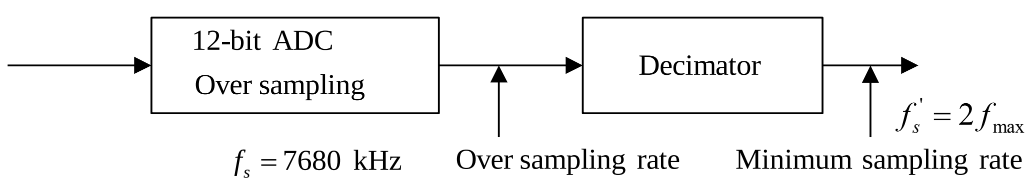
b.

$$n = m + 0.5 \times \log_2\left(\frac{f_s}{2 f_{\max}}\right) = 6 + 0.5 \times \log_2\left(\frac{4000 \text{ kHz}}{2 \times 4 \text{ kHz}}\right) = 10.48 \approx 10 \text{ bits}$$

12.15

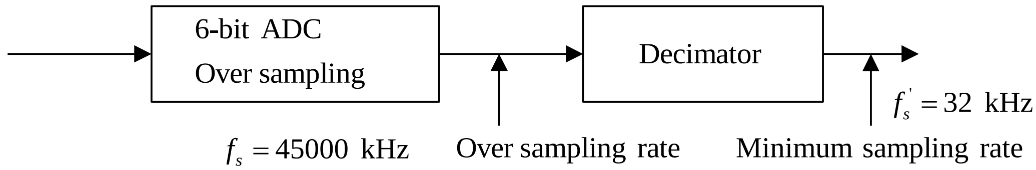
a. $f_s = 2 f_{\max} 2^{2(n-m)} = 2 \times 15 \times 2^{2\times(16-12)} = 7680 \text{ kHz}$

b.



12.16

a.

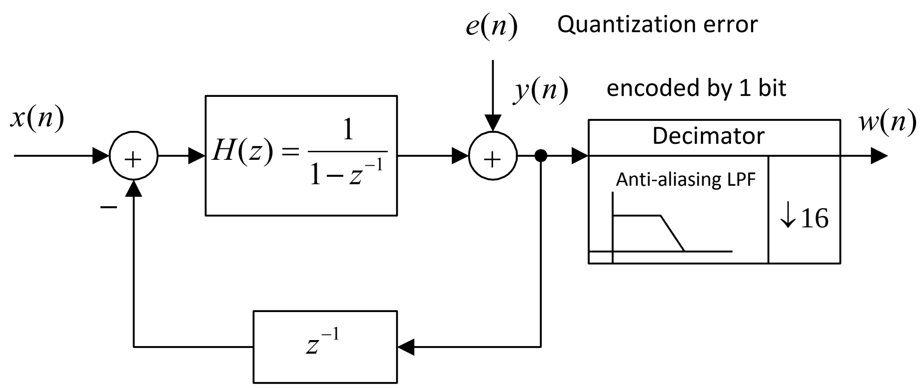


b.

$$n = m + 0.5 \times \log_2\left(\frac{f_s}{2f_{\max}}\right) = 6 + 0.5 \times \log_2\left(\frac{45000 \text{ kHz}}{2 \times 15 \text{ kHz}}\right) = 11.28 \approx 11 \text{ bits}$$

12.17

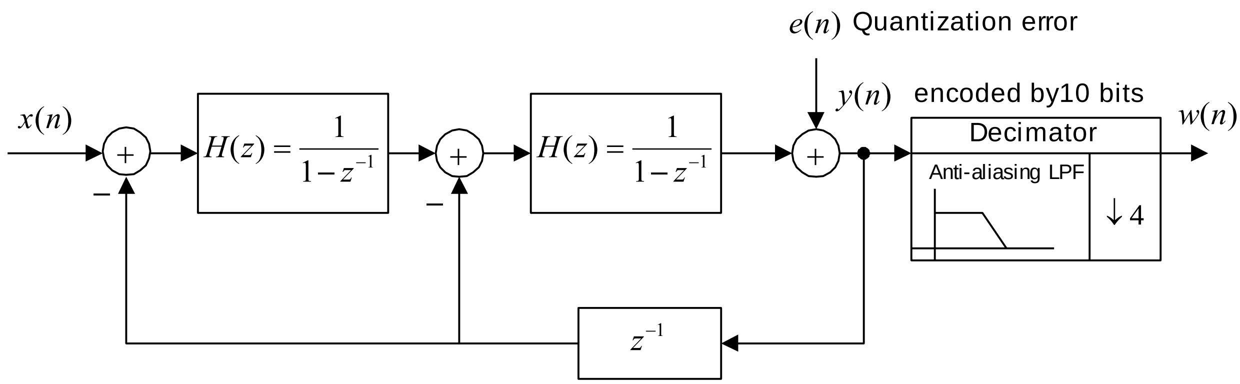
a.



b. $n = 1 + 1.5 \times \log_2\left(\frac{128}{2 \times 4}\right) - 0.86 \approx 6$ bits

12.18

a.

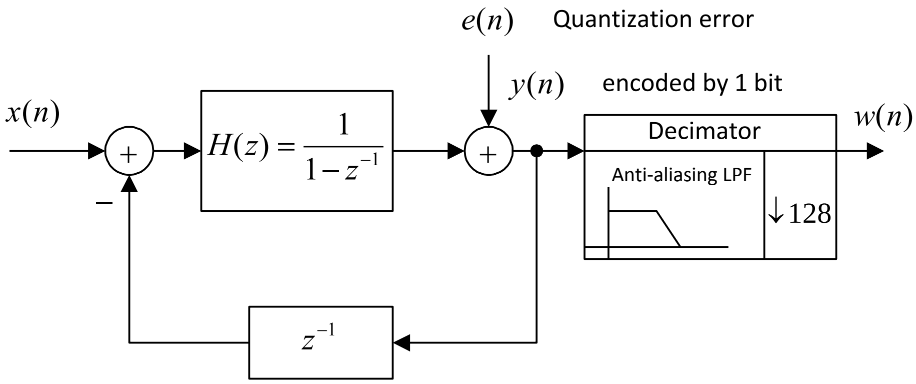


b.

$$n = m + 2.5 \times \log_2\left(\frac{f_s}{2f_{\max}}\right) - 2.14 = 10 + 2.5 \times \log_2\left(\frac{160}{2 \times 20}\right) - 2.14 = 12.86 \approx 13 \text{ bits}$$

12.19

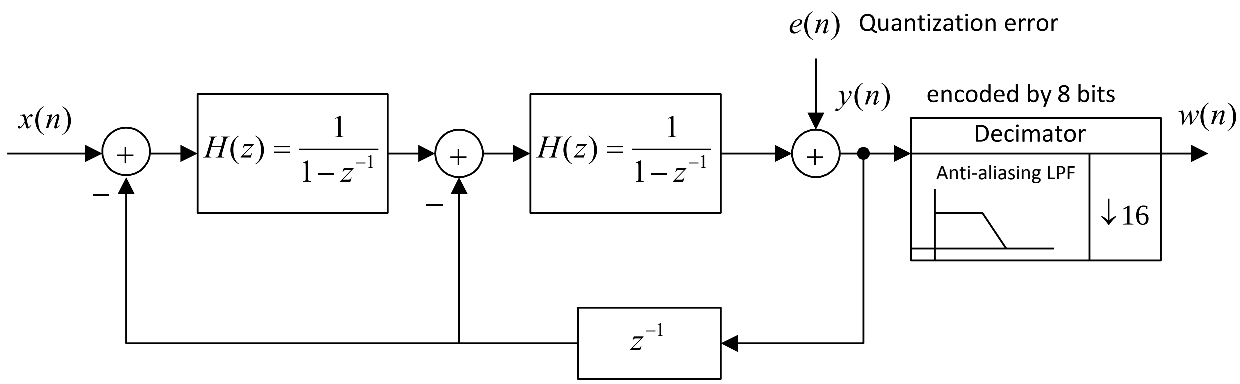
a.



b. $n = 1 + 1.5 \times \log_2 \left(\frac{128}{2 \times 0.5} \right) - 0.86 \approx 10$ bits

12.20

a.

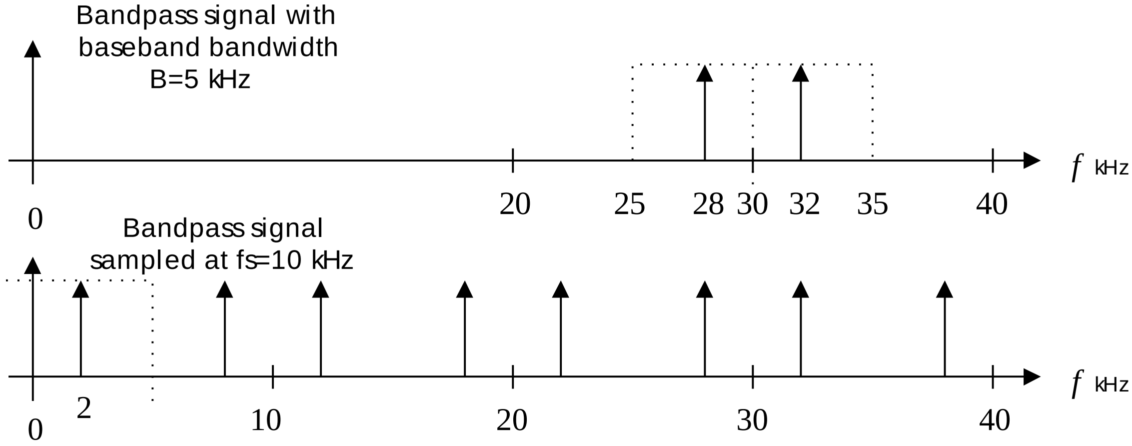


b.

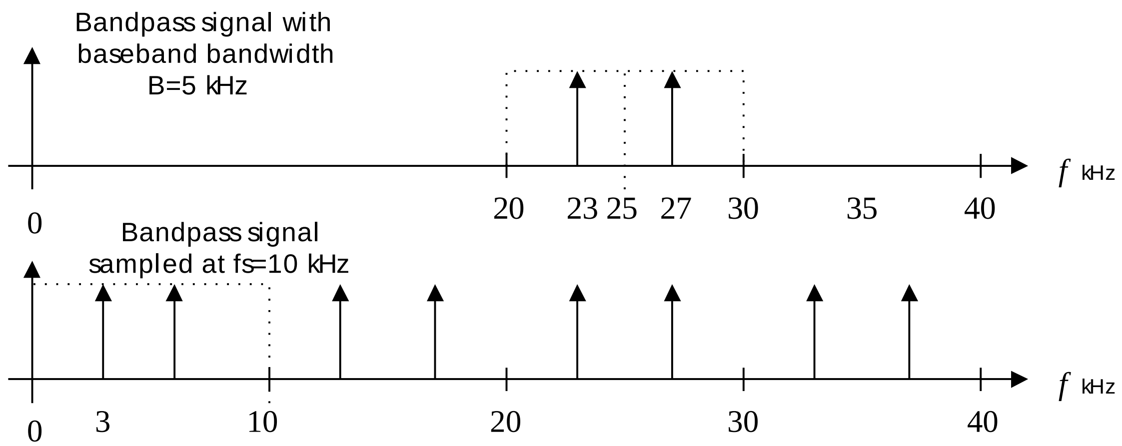
$$n = m + 2.5 \times \log_2 \left(\frac{f_s}{2 f_{\max}} \right) - 2.14 = 8 + 2.5 \times \log_2 \left(\frac{16}{2 \times 0.5} \right) - 2.14 = 15.86 \approx 15 \text{ bits}$$

12.21

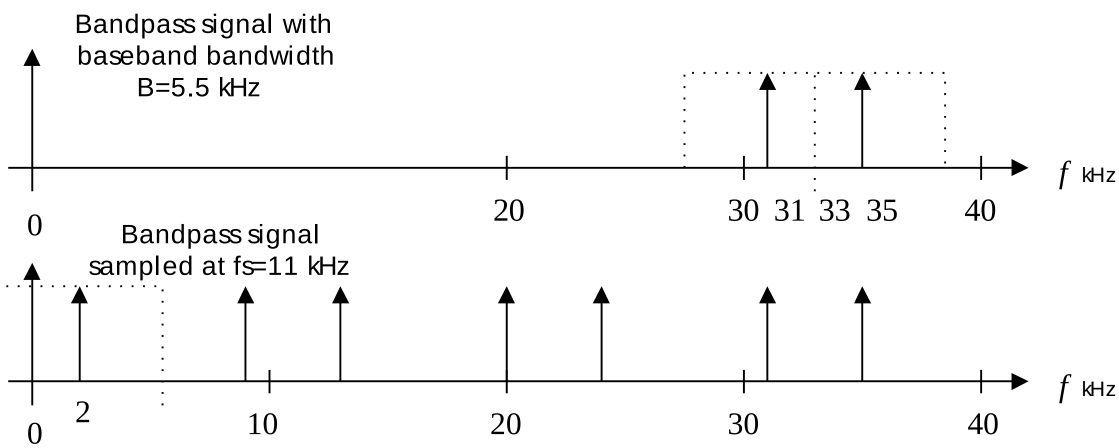
a. $f_c / B = 6$ is an even number, which is the case 1, we select $f_s = 10$ kHz



b. Since $f_c / B = 5$ is an odd number, we select $f_s = 10$ kHz

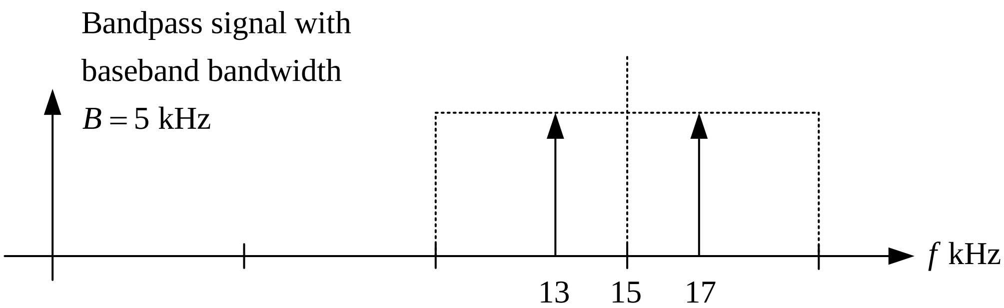


c. Now, $f_c / B = 6.6$ which is a non integer. We extend the band width $\bar{B} = 5.5$ kHz, so $f_c / \bar{B} = 6$ and $f_s = 2\bar{B} = 11$ kHz.

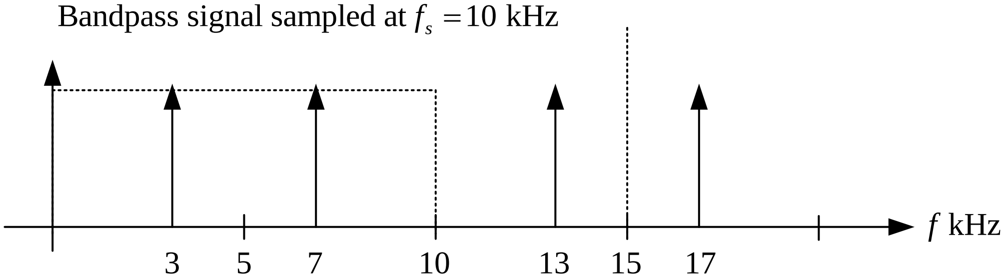


12. 22

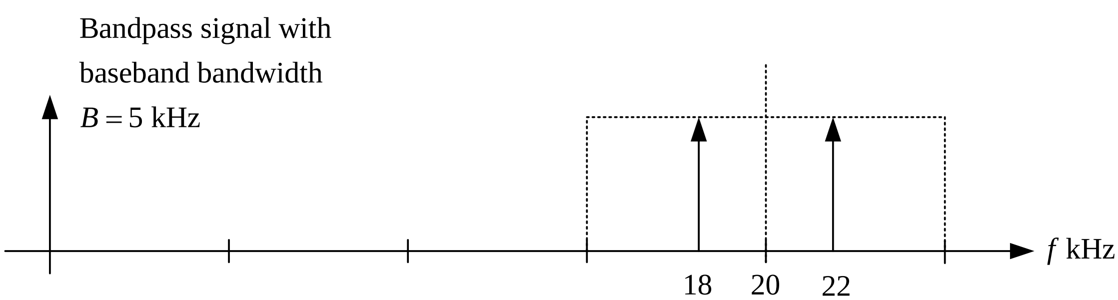
a. Since $f_c / B = 3$ is an odd number, we select $f_s = 10$ kHz



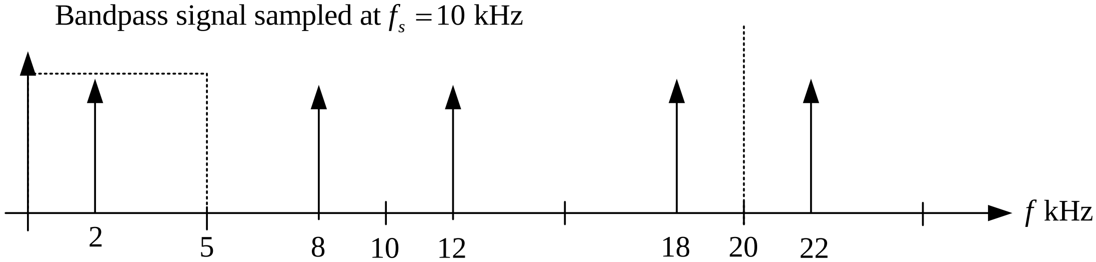
Bandpass signal sampled at $f_s = 10$ kHz



b. Since $f_c / B = 4$ is an even number, we select $f_s = 10$ kHz

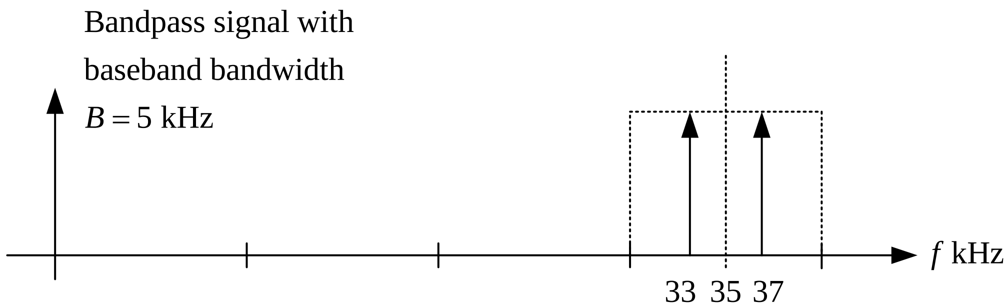


Bandpass signal sampled at $f_s = 10$ kHz

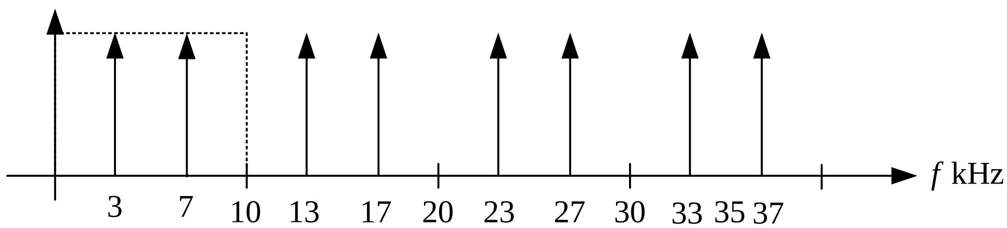


12.23

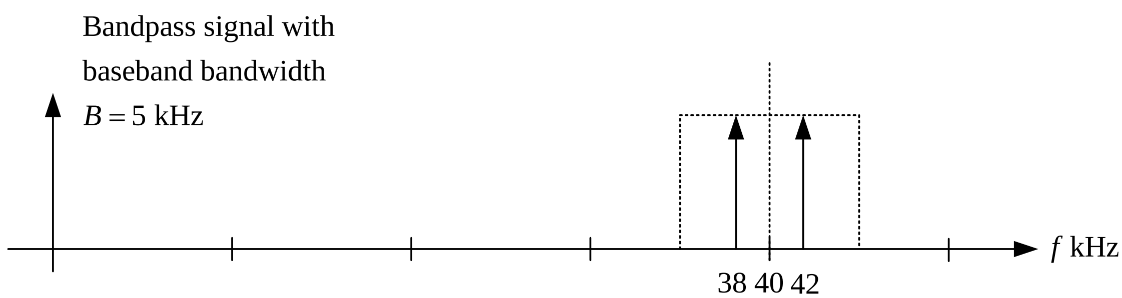
a. Since $f_c / B = 7$ is an odd number, we select $f_s = 10$ kHz



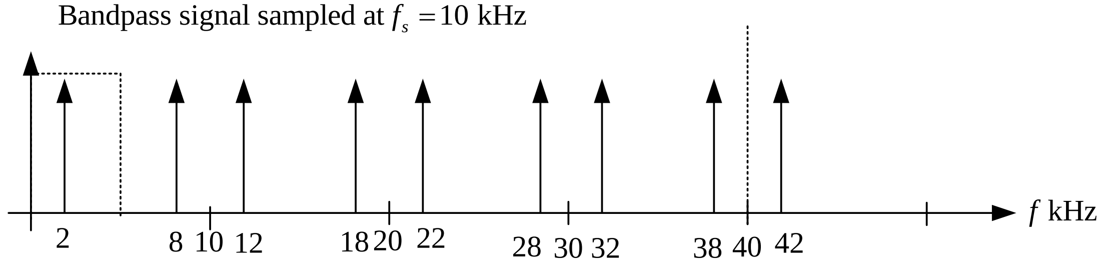
Bandpass signal sampled at $f_s = 10$ kHz



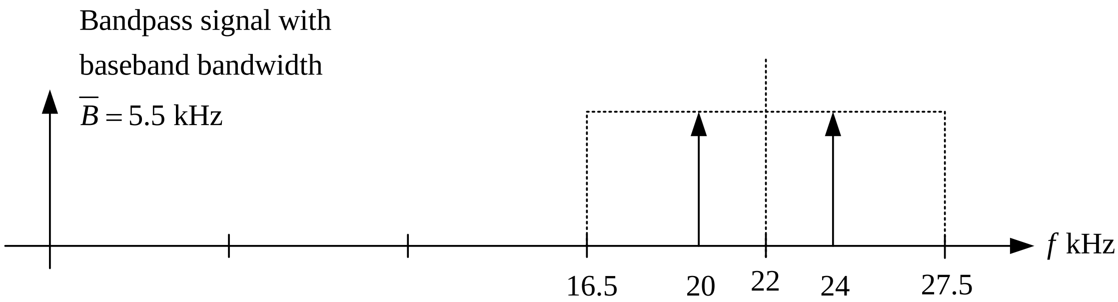
b. Since $f_c / B = 8$ is an even number, we select $f_s = 10$ kHz



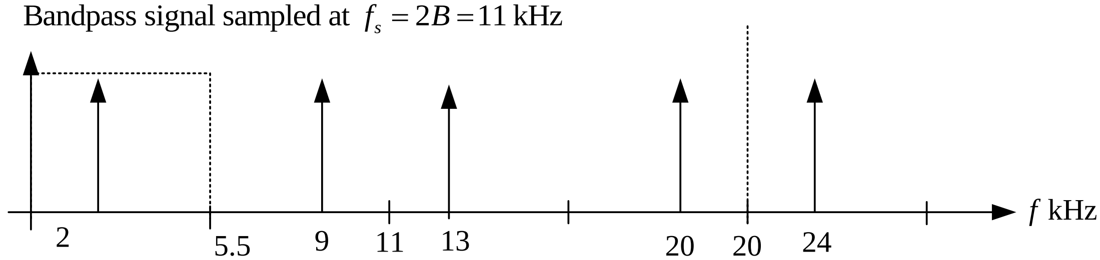
Bandpass signal sampled at $f_s = 10$ kHz



c. Now, $f_c / B = 22/5 = 4.4$ which is a non integer. We extend the bandwidth $\bar{B} = 5.5$ kHz, so $f_c / \bar{B} = 4$ and $f_s = 2\bar{B} = 11$ kHz.

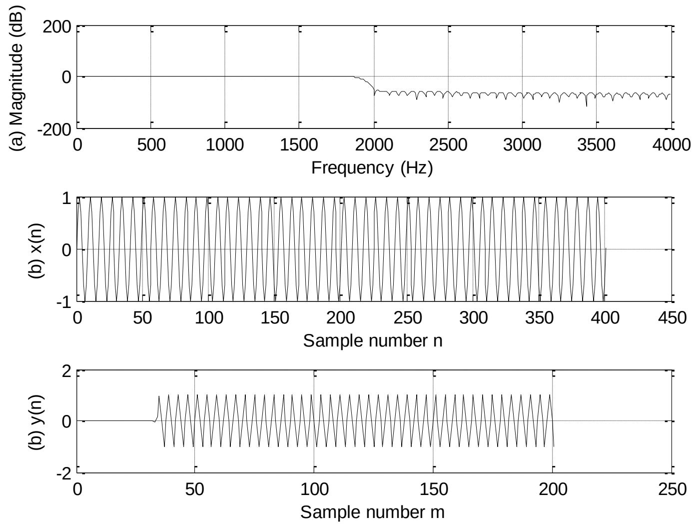


Bandpass signal sampled at $f_s = 2B = 11$ kHz



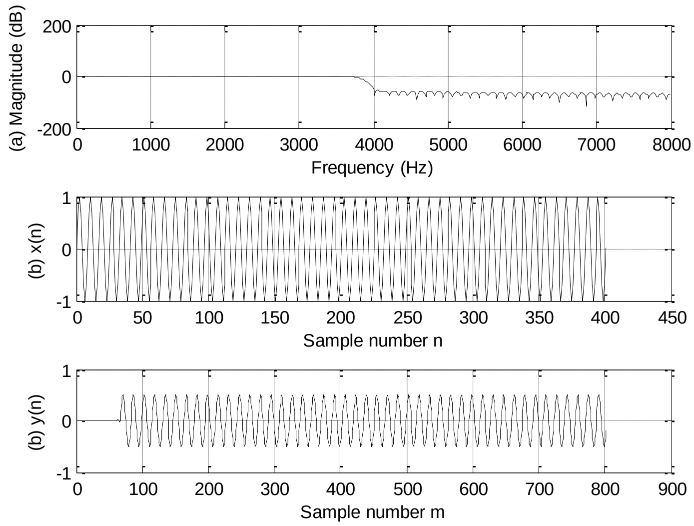
12.24

```
fs=8000; T=1/fs;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*1900/fs,0,4);
[h,f]=freqz(b,1,512,fs);
figure(2)
w=filter(b,1,x);
y=w(1:2:length(w));
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');
```



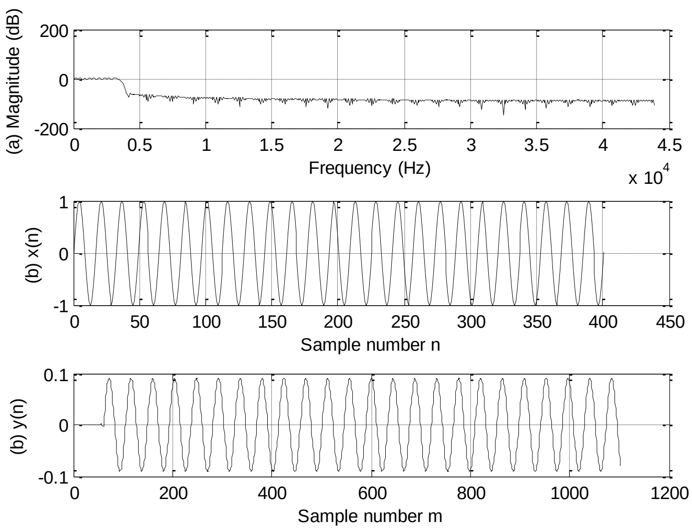
12.25

```
---------------------------------------------------------------------------
fs=8000; T=1/fs;fsL=16000;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*3800/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
figure(2)
w=zeros(1,2*length(x));
w(1:2:length(w))=x;
y=filter(b,1,w);
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');
-----------------------------------------------------------
```



12.26

```
---------------------------------------------------------------------
fs=8000; T=1/fs;fsL=88000;fsM=22000;
t=0:T:0.05;
x=sin(2*pi*500*t);
b=firwd(485,1,2*pi*3600/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
figure(2)
w=zeros(1,11*length(x));
w(1:11:length(w))=x;
y1=filter(b,1,w);
y=y1(1:4:length(y1));
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');
---------------------------------------------------
```



12.27

```
-------------------------------------------------------------------------------
fs=8000; T=1/fs;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*1900/fs,0,4);
[h,f]=freqz(b,1,512,fs);
%polyphase decimator
p0=b(1:2:length(b)); p1=b(2:2:length(b));
x_delay=filter([0 1],1,x);
w0=x(1:2:length(x)); w1=x_delay(1:2:length(x_delay));
y0=filter(p0,1,w0); y1=filter(p1,1,w1);
y=y0+y1;
-------------------------------------------------------------------------------
```

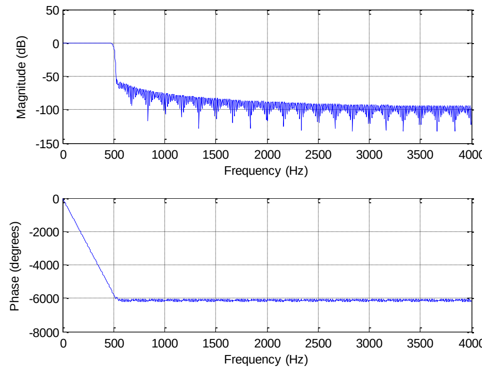
12.28

```
-------------------------------------------------------------------------------
fs=8000; T=1/fs;fsL=16000;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*3800/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
%plophase interpolator
p0=b(1:2:length(b));p1=b(2:2:length(b));
w0=filter(p0,1,x);w1=filter(p1,1,x);
y0=zeros(1,2*length(x)); y1=y0;
y0(1:2:length(y0))=w0; y1(1:2:length(y1))=w1;
y1=filter([0 1],1,y1);
y=y0+y1;
-----------------------------------------------------------------------------------------------
```

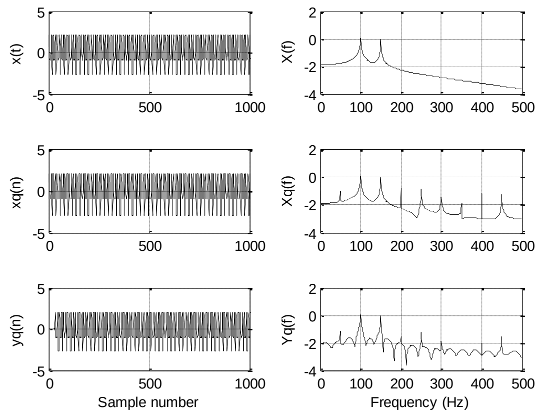
12.29

a. and b. for L=8

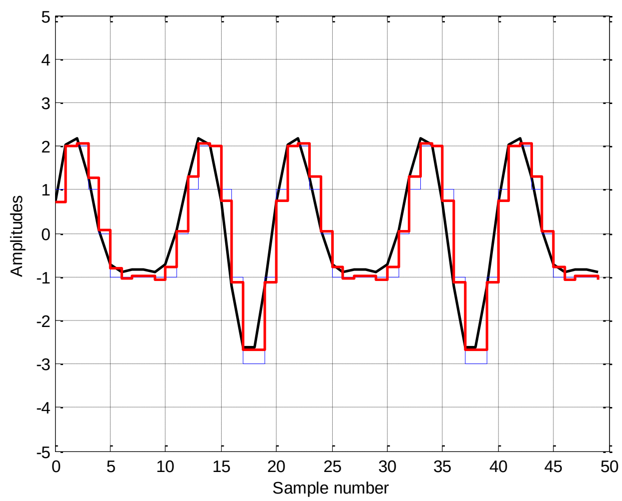
```
clear all; close all,clc
ntotal=1024;
n=0:ntotal; % Number of samples
L=8; % over saampling factor
nL=0:ntotal*L; % Number of samples for over sampling
numb=3; % Number of bits
A=2^(numb-1)-1; %Peak value
f1=100;C1=1.8;f2=150;C2=1; %Frequencies and amplitides
fmax=500;fs=1000;T=1/fs; % Maximum frequency, sampling rate, sampling period
fsL=L*fs;TL=1/fsL;%Over sampling rate, and over sampling period
% Sampling at fs=1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Qunrized signal at the minimum sampling rate
NN=length(n);
f=[0:ntotal-1]*fs/NN;
M=32*L;nd=M/L; %Number of delay in samples due to anti-aliasing filtering
B=firwd(2*M+1,1,2*pi*fmax/fsL,0,4); % anti-aliasing filter design (ensure 5%
normalized transition BW)
figure(1);freqz(B,1,1000,fsL)
% Oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
xxq=round(xx); % Quantized signal
% down sampling
y=filter(B,1,xxq);%Anti-aliasing filtering
yd=y(1:L:length(y));% down sample
figure (2)
subplot(3,2,1);plot(n,x,'k');grid;axis([0 1000 -5 5]);ylabel('x(t)')
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2
subplot(3,2,2);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid;ylabel('X(f)');axis
([0 500 -4 2])
subplot(3,2,3);plot(n,xq,'k');grid;axis([0 1000 -5 5]);ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2
subplot(3,2,4);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid;ylabel('Xq(f)');axi
s([0 500 -4 2])
subplot(3,2,5);plot(n,yd,'k');grid;axis([0 1000 -5 5]);ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2
subplot(3,2,6);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid;ylabel('Yq(f)');axi
s([0 500 -4 2])
xlabel('Frequency (Hz)');
figure (3)
plot(n(1:50),x(1:50),'k','LineWidth',2); hold % plot of first 50 samples
stairs(n(1:50),xq(1:50),'b');
stairs(n(1:50),yd(1+nd:50+nd),'r','LineWidth',2);grid
axis([0 50 -5 5]);xlabel('Sample number');ylabel('Amplitudes')
snr(x,xq);
snr(x(1:ntotal-nd),yd(1+nd:ntotal));
```



Anti-aliasing filter



Signal and spectra continuous case, quantization with regular sampling, and quantization with oversampling L=8



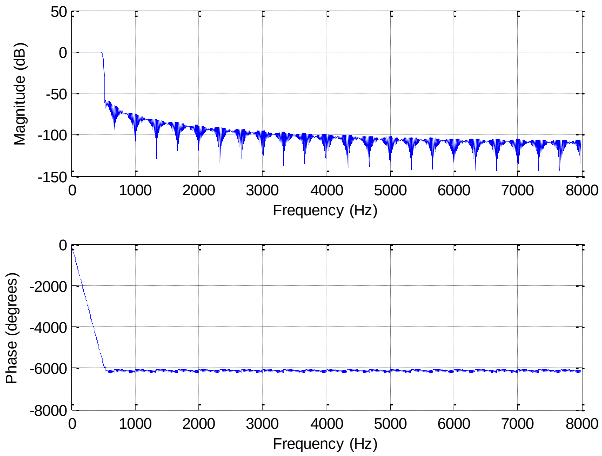
Detailed comparisons, continuous (black), regular (blue, SQNR=16 dB), oversampling (red, SQNR=24 dB)

c. SQNR> 23 dB for L>= 8

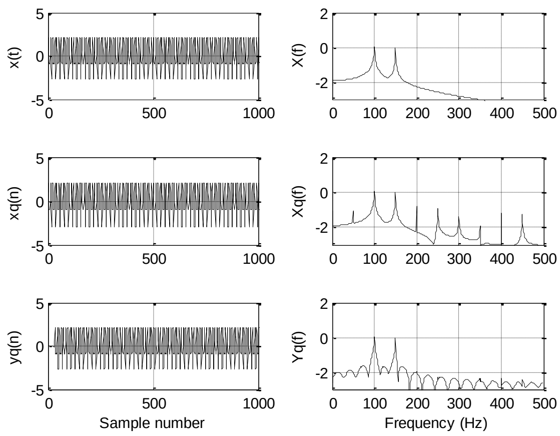
12.30

a. and b. (use the model from Figure 12.30)

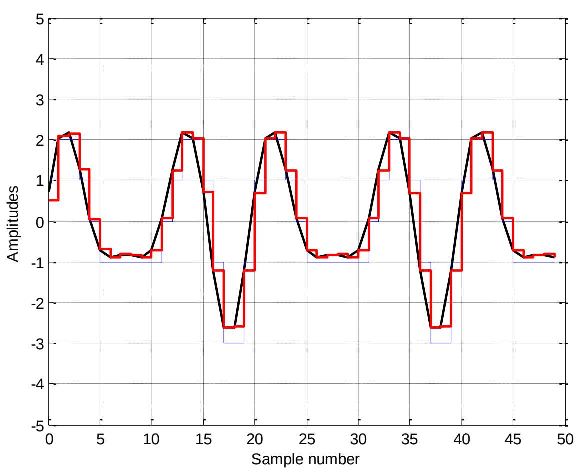
```
clear all; close all;clc
ntotal=1024; %Number of samples
n=0:ntotal;
L=16; %Oversampling factor
nL=0:ntotal*L;numb=3;A=2^(numb-1)-1; %Peak value
f1=100;C1=1.8;f2=150;C2=1;%Frequencies and amplitudes
fmax=500;fs=1000; T=1/fs% Sampling rte and sampling period
fsL=L*fs;TL=1/fsL; % Oversampling rate and sampling period
% Sampling at fs-1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Quantization
NN=length(n);
M=32*L;nd=M/L; %Delay in terms of samples for anti-filtering
B=firwd(2*M+1,1,2*pi*fmax/fsL,0,4);% Deisgn of anti-aliasing filter
figure(1)
freqz(B,1,1000,fsL);
% oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
% the first-order SDM processing
   yq=zeros(1,ntotal*L+1+1); %Initializign the buffer
   y=yq;
for i=1:ntotal*L
    y(i+1)=(xx(i+1)-yq(i))+y(i);
    yq(i+1)=round(y(i+1));
end
xxq=yq(1:ntotal*L+1); %Signal Quantization
% down sampling
y=filter(B,1,xxq);
yd=y(1:L:length(y));
f=[0:ntotal-1]*fs/NN;
figure (2)
subplot(3,2,1);plot(n,x,'k');grid;axis([0 1000 -5 5]);ylabel('x(t)');
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,2);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid;
axis([0 500 -3 2]);ylabel('X(f)');
subplot(3,2,3);plot(n,xq,'k');grid;axis([0 1000 -5 5]);ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,4);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid
axis([0 500 -3 2]);ylabel('Xq(f)');
subplot(3,2,5);plot(n,yd,'k');grid;axis([0 1000 -5 5]);ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,6);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid
axis([0 500 -3 2]);ylabel('Yq(f)');xlabel('Frequency (Hz)');
figure (3)
plot(n(1:50),x(1:50),'k','LineWidth',2); hold
stairs(n(1:50),xq(1:50),'b');
stairs(n(1:50),yd(1+nd:50+nd),'r','LineWidth',2);
axis([0 50 -5 5]);grid;xlabel('Sample number');ylabel('Amplitudes');
snr(x,xq);
snr(x(1:ntotal-nd),yd(1+nd:ntotal));
```



Anti-alising filter



Continuous, regular sampling, oversampling SDM with L=16;



Continuous (black), regular (blue, SQNR=16 dB), oversampling SDM with L=16 (red, SQNR=43.6 dB)

c. (use the model from Figure 12.32)

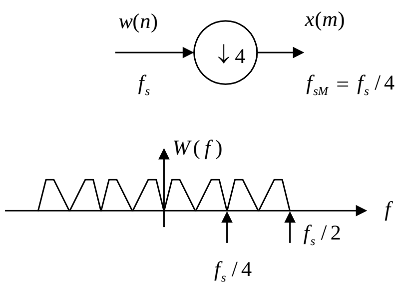
```
------------------------------------------------------------------------
clear all; close all;clc
ntotal=1024; %Number of samples
n=0:ntotal;
L=16; %Oversampling factor
nL=0:ntotal*L;numb=3;A=2^(numb-1)-1; %Peak value
f1=100;C1=1.8;f2=150;C2=1;%Frequencies and amplitudes
fmax=500;fs=1000; T=1/fs% Sampling rte and sampling period
fsL=L*fs;TL=1/fsL; % Oversampling rate and sampling period
% Sampling at fs-1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Quantization
NN=length(n);
M=32*L;nd=M/L; %Delay in terms of samples for anti-filtering
B=firwd(2*M+1,1,2*pi*fmax/fsL,0,4);% Deisgn of anti-aliasing filter
figure(1)
freqz(B,1,1000,fsL);
% oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
% the second-order SDM processing
   yq=zeros(1,ntotal*L+1+1); %Initilizign the buffer
   y=yq; y1=y;
for i=1:ntotal*L
    y1(i+1)=(xx(i+1)-yq(i))+y1(i); %first-stage SDM
    y(i+1)=(y1(i+1)-yq(i))+y(i); %second-stage SDM
    yq(i+1)=round(y(i+1));
end
xxq=yq(1:ntotal*L+1); %Signal Quantization
% down sampling
y=filter(B,1,xxq);
yd=y(1:L:length(y));
f=[0:ntotal-1]*fs/NN;
figure (2)
subplot(3,2,1);plot(n,x,'k');grid;axis([0 1000 -5 5]);ylabel('x(t)');
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,2);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid;
axis([0 500 -3 2]);ylabel('X(f)');
subplot(3,2,3);plot(n,xq,'k');grid;axis([0 1000 -5 5]);ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,4);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid
axis([0 500 -3 2]);ylabel('Xq(f)');
subplot(3,2,5);plot(n,yd,'k');grid;axis([0 1000 -5 5]);ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,6);plot(f(1:NN/2),log10(Ak(1:NN/2)),'k');grid
axis([0 500 -3 2]);ylabel('Yq(f)');xlabel('Frequency (Hz)');
figure (3)
plot(n(1:50),x(1:50),'k','LineWidth',2); hold
stairs(n(1:50),xq(1:50),'b');
stairs(n(1:50),yd(1+nd:50+nd),'r','LineWidth',2);
axis([0 50 -5 5]);grid;xlabel('Sample number');ylabel('Amplitudes');
snr(x,xq);
snr(x(1:ntotal-nd),yd(1+nd:ntotal));
------------------------------------------------------------------------
```

SQNR =47.9 dB is better than SQNR=43.6 in (b)

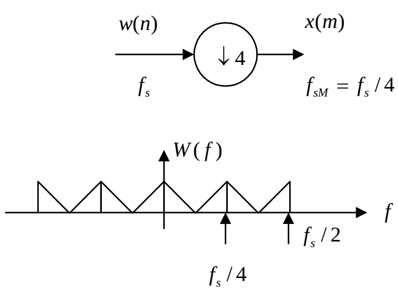
Chapter 13

13.1

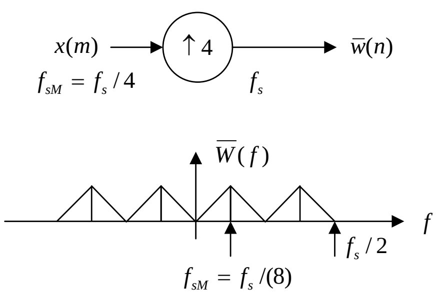
a. $B = f_{sM}/2 = f_s/(2M)$ $f_c = 2(f_s/(2M)) = 2B$, $f_c/B = 2 = \text{even}$

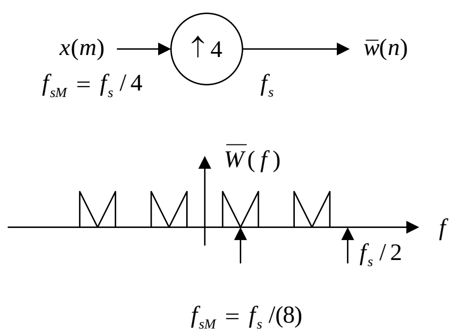


b. $B = f_{sM}/2 = f_s/(2M)$ $f_c = f_s/(2M) = B$, $f_c/B = 1 = \text{odd}$



13.2



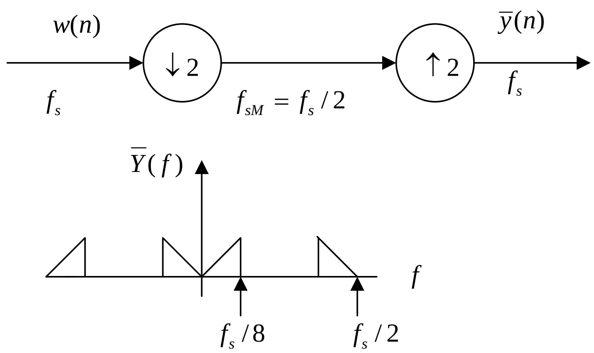


13.3

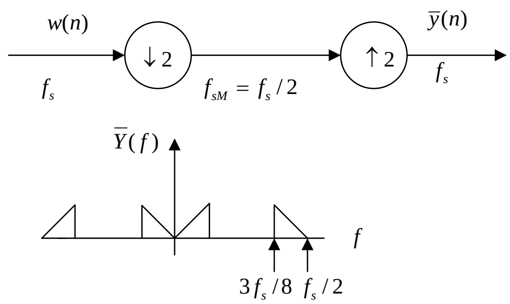
From Equation (13.7), $\bar{Y}(z)=\frac{1}{2}(W(z)+W(e^{-j\pi}z))$,

$\bar{Y}(e^{j\Omega})=\frac{1}{2}(W(e^{j\Omega})+W(e^{j(\Omega-\pi)}))$, $W(e^{j(\Omega-\pi)})$ is the shifted version of $W(e^{j\Omega})$ by $f_s/2$.

a.



b.

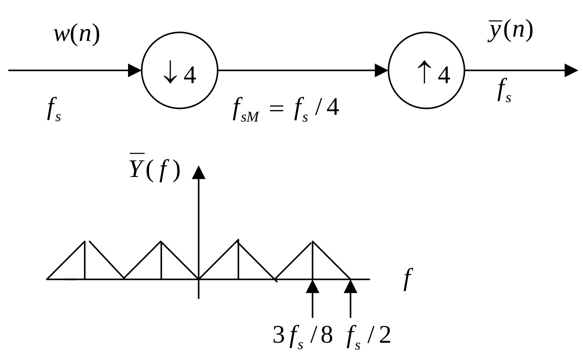


13.4

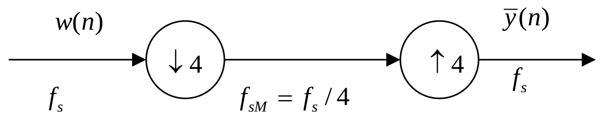
From Equation (13.7), $\bar{Y}(z)=\frac{1}{4}(W(z)+W(e^{-j\pi/2}z)+W(e^{-j\pi}z)+W(e^{-j3\pi/2}z))$,

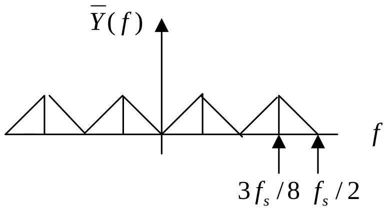
$\bar{Y}(e^{j\Omega})=\frac{1}{2}(W(e^{j\Omega})+W(e^{j(\Omega-\pi/2)})+W(e^{j(\Omega-\pi)})+W(e^{j(\Omega-3\pi/2)}))$

a.



b.





13.5

Since $H_0(z)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}z^{-1}$, $N=2$

$$H_1(z)=z^{-(N-1)}H_0(-z^{-1})=z^{-(2-1)}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}(-(z^{-1})^{-1}\right)=-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}z^{-1}$$

$$G_0(z)=-H_1(-z)=-\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}(-z)^{-1}\right)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}z^{-1}$$

$$G_1(z)=H_0(-z)=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}(-z)^{-1}\right)=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}z^{-1}$$

13.6

a. $h_0(k)=[0.707\ 0.707]$, $\rho(2n)=\sum_{k=0}^{N-1}h_0(k)h_0(k+2n)$

$n=0$

$$\rho(0)=\sum_{k=0}^{2-1}h_0(k)h_0(k)=h_0(0)h_0(0)+h_0(1)h_0(1)=0.707\times 0.707+0.707\times 0.707=1$$

$n=1$

$$\rho(2)=\sum_{k=0}^{2-1}h_0(k)h_0(k+2)=h_0(0)h_0(2)+h_0(1)h_0(3)=0.707\times 0+0.707\times 0=0$$

Similarly, for $n=2,3,\ldots$, $\rho(n)=0$

$n=-1$

$$\rho(-2)=\sum_{k=0}^{2-1}h_0(k)h_0(k-2)=h_0(0)h_0(-2)+h_0(1)h_0(-3)=0.707\times 0+0.707\times 0=0$$

Similarly, for $n=-2,-3,\ldots$, $\rho(n)=0$.

b.

$$H_0(z)=0.707+0.707z^{-1}$$

$$R(z)=H_0(z)H_0(1/z)=(0.707+0.707z^{-1})(0.707+0.707z)=0.5z^{-1}+1+0.5z^{-1}$$

$$R(-z) = -0.5z^{-1} + 1 - 0.5z^{-1}$$
$$R(z) + R(-z) = 2$$

13.7

$$H_0(-z^{-1}) = 0.483 - 0.837z + 0.224z^2 + 0.129z^3$$
$$H_1(z) = z^{-(4-1)}H_0(-z^{-1}) = 0.129 + 0.224z^{-1} - 0.837z^{-2} + 0.483z^{-3}$$
$$G_0(z) = -H_1(-z) = -0.129 + 0.224z^{-1} + 0.837z^{-2} + 0.483z^{-3}$$
$$G_1(z) = H_0(-z) = 0.483 - 0.837z^{-1} + 0.224z^{-2} + 0.129z^{-3}$$

13.8

$$\rho(n) = \sum_{k=0}^{3} h_0(k)h_0(k+n) = h_0(0)h_0(n) + h_0(1)h_0(1+n) + h_0(2)h_0(2+n) + h_0(3)h_0(3+n)$$

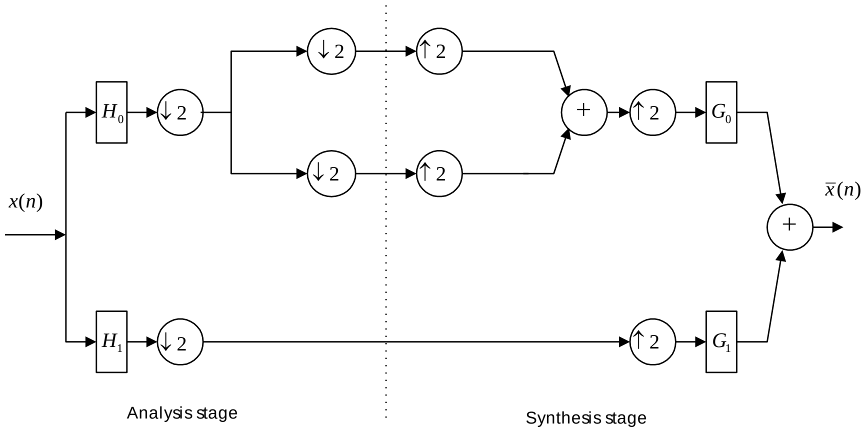
$$\rho(0) = h_0(0)h_0(0) + h_0(1)h_0(1) + h_0(2)h_0(2) + h_0(3)h_0(3)$$
$$= 0.483^2 + 0.837^2 + 0.224^2 + (-0.129)^2 = 1.00$$
$$\rho(2) = h_0(0)h_0(2) + h_0(1)h_0(3) + h_0(2)h_0(4) + h_0(3)h_0(5)$$
$$= 0.483 \times 0.224 + 0.837 \times (-0.129) + 0.224 \times 0 + (-0.129) \times 0 = 0.000219 \approx 0$$
$$\rho(-2) = h_0(0)h_0(-2) + h_0(1)h_0(-1) + h_0(2)h_0(0) + h_0(3)h_0(1)$$
$$= 0.483 \times 0 + 0.837 \times 0 + 0.224 \times 0.483 + (-0.129) \times 0.837 \approx 0$$
$$\rho(4) = h_0(0)h_0(4) + h_0(1)h_0(5) + h_0(2)h_0(6) + h_0(3)h_0(7) = 0$$
$$\rho(-4) = h_0(0)h_0(-4) + h_0(1)h_0(-3) + h_0(2)h_0(-2) + h_0(3)h_0(-1) = 0$$

Similarly, $\rho(\pm 6) = \rho(\pm 8) = \cdots = 0$

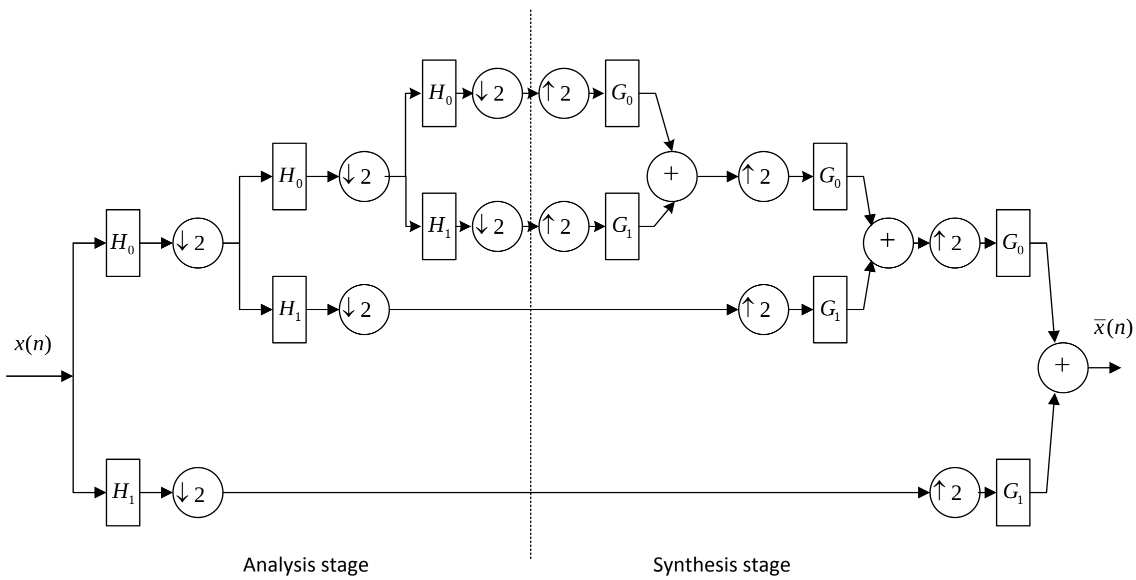
$$H_0(z) = 0.483 + 0.837z^{-1} + 0.224z^{-2} - 0.129z^{-3}, H_0(z^{-1}) = 0.483 + 0.837z + 0.224z^2 - 0.129z^3$$
$$R(z) = H_0(z)H_0(z^{-1})$$
$$= (0.483 + 0.837z^{-1} + 0.224z^{-2} - 0.129z^{-3})(0.483 + 0.837z + 0.224z^2 - 0.129z^3)$$
$$R(z) = -0.0623z^{-3} + 0.5629z^{-1} + 1 + 0.5269z - 0.0623z^3$$
$$R(-z) = 0.0623z^{-3} - 0.5629z^{-1} + 1 - 0.5269z + 0.0623z^3$$

Then $R(z) + R(-z) = 2$

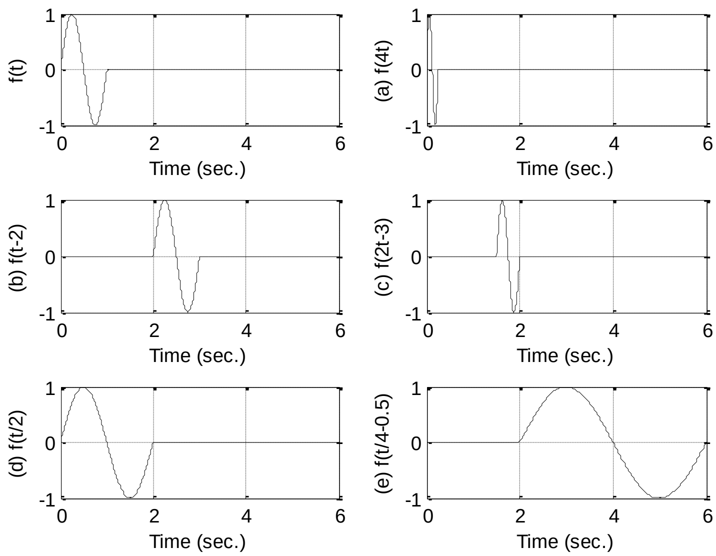
13.9


x(n)
H0
H1
↓2
↑2
+
G0
G1
x̄(n)
Analysis stage
Synthesis stage

3.10

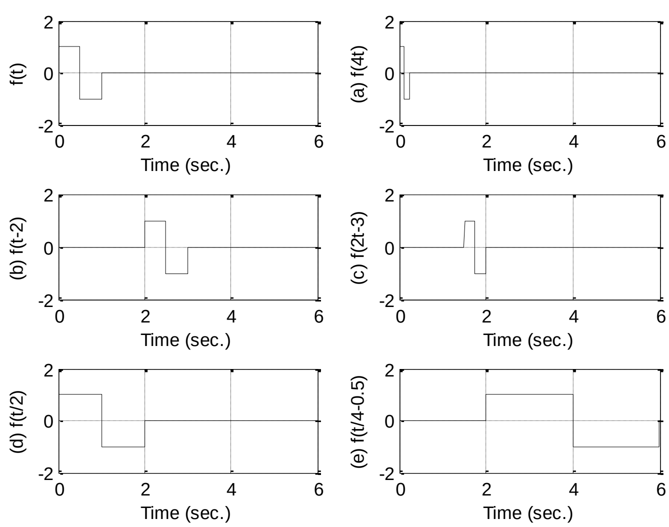

x(n)
H0
H1
↓2
↑2
+
G0
G1
x̄(n)
Analysis stage
Synthesis stage

13.11

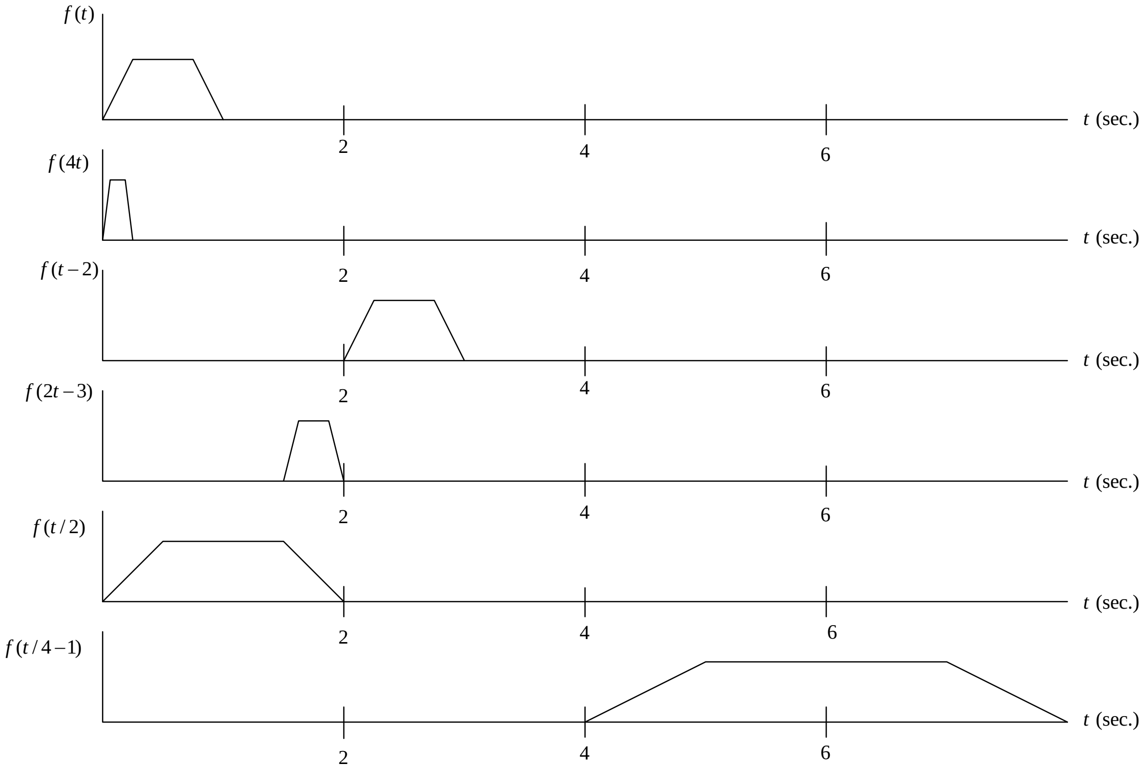


13.12
For Figure 13.52 (b), $a = 0.25$, $b = 1.5$
For Figure 13.52 (c), $a = 0.125$, $b = 2$

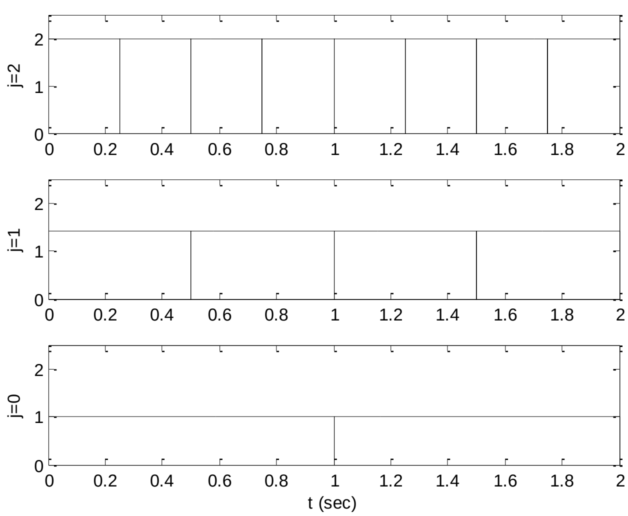
13.13



13.14



13.15



13.16

Repeat Figure 13.7 for one more second.

13.17

a. $f(t)=\sum_{k=-\infty}^{\infty} c_1(k)2^{1/2}\phi\left(2t-k\right)$

$$c_1(k)=\left\langle f(t)\phi_{1k}(t)\right\rangle=\int f(t)2^{1/2}\phi(2t-k)dt$$

$$c_1(0)=\left\langle f(t)\phi_{10}(t)\right\rangle=\int_0^{1/2} f(t)2^{1/2}\phi(2t)dt=2\times 2^{1/2}$$

$$c_1(1)=\left\langle f(t)\phi_{11}(t)\right\rangle=\int_{1/2}^{1} f(t)2^{1/2}\phi(2t-1)dt=-2^{1/2}$$

$$f(t)=c_1(0)2^{1/2}\phi(2t)+c_1(1)2^{1/2}\phi(2t-1)=4\phi(2t)-2\phi(2t-1)$$

b. $f(t)=\sum_{k=-\infty}^{\infty} c_0(k)\phi_{0k}(t)+\sum_{k=-\infty}^{\infty} d_0(k)\psi_{0k}(t)$

$$c_0(0)=\left\langle f(t)\phi_{00}(t)\right\rangle=\int_0^{1} f(t)\phi(t)dt=1$$

$$d_0(0)=\left\langle f(t)\psi_{00}(t)\right\rangle=\int_0^{1} f(t)\psi(t)dt=3$$

$$f(t)=c_0(0)\phi(t)+d_0(0)\psi(t)=\phi(t)+3\psi(t)$$

13.18

a. $f(t)=\sum_{k=-\infty}^{\infty} c_2(k)2\phi\left(4t-k\right)$

$$c_2(k)=\left\langle f(t)\phi_{2k}(t)\right\rangle=\int f(t)2\phi(4t-k)dt$$

$$c_2(0)=\left\langle f(t)\phi_{20}(t)\right\rangle=\int_0^{1/4} f(t)2\phi(4t)dt=\int_0^{1/4}8dt=2$$

$$c_2(1)=\left\langle f(t)\phi_{21}(t)\right\rangle=\int f(t)2\phi(4t-1)dt=\int_{1/4}^{2/4}(-2)2dt=-1$$

$$c_2(2)=\left\langle f(t)\phi_{22}(t)\right\rangle=\int f(t)2\phi(4t-2)dt=\int_{2/4}^{3/4}2\times 2dt=1$$

$$c_2(3)=\left\langle f(t)\phi_{23}(t)\right\rangle=\int f(t)2\phi(4t-3)dt=\int_{3/4}^{1}(-1)2dt=-0.5$$

$$\begin{aligned}f(t)&=c_2(0)2\phi(4t)+c_2(1)2\phi(4t-1)+c_2(2)2\phi(4t-2)+c_2(3)2\phi(4t-3)\\&=4\phi(4t)-2\phi(4t-1)+2\phi(4t-2)-1\phi(4t-3)\end{aligned}$$

b. From Equation (13.46), $f(t)=\sum_{k=-\infty}^{\infty} c_1(k)2^{1/2}\phi(2t-k)+\sum_{k=-\infty}^{\infty} d_1(k)2^{1/2}\psi(2t-k)$

$$c_1(0)=\left\langle f(t)\phi_{10}(t)\right\rangle=\int f(t)2^{1/2}\phi(2t)dt=\int_0^{1/4}4\times 2^{1/2}dt+\int_{1/4}^{1/2}(-2)\times 2^{1/2}dt=0.5\times 2^{1/2}$$

$$c_1(1)=\left\langle f(t)\phi_{11}(t)\right\rangle=\int f(t)2^{1/2}\phi(2t-1)dt=\int_{1/2}^{3/4}2\times 2^{1/2}dt+\int_{1/4}^{1/2}(-1)\times 2^{1/2}dt=0.25\times 2^{1/2}$$

$$d_1(0)=\langle f(t)\psi_{10}(t)\rangle=\int f(t)2^{1/2}\psi(2t)dt=\int_0^{1/4}4\times 2^{1/2}dt+\int_{1/4}^{1/2}(-2)\times 2^{1/2}(-1)dt=1.5\times 2^{1/2}$$

$$d_1(1)=\langle f(t)\psi_{11}(t)\rangle=\int f(t)2^{1/2}\psi(2t-1)dt=\int_{1/2}^{3/4}2\times 2^{1/2}dt+\int_{1/4}^{1/2}(-1)\times 2^{1/2}(-1)dt=0.75\times 2^{1/2}$$

$$f(t)=c_1(0)2^{1/2}\phi(2t)+c_1(1)2^{1/2}\phi(2t-1)+d_1(0)2^{1/2}\psi(2t)+d_1(1)2^{1/2}\psi(2t-1)$$
$$=\phi(2t)+0.5\phi(2t-1)+3\psi(2t)+1.5\psi(2t-1)$$

c. From Equation (13.47),

$$f(t)=\sum_{k=-\infty}^{\infty}c_0(k)\phi(t-k)+\sum_{j=0}^{1}\sum_{k=-\infty}^{\infty}d_j(k)2^{j/2}\psi(2^jt-k)$$
$$=c_0(0)\phi(t)+d_0(0)\psi(t)+d_1(0)2^{1/2}\psi(2t)+d_1(1)2^{1/2}\psi(2t-1)$$

$$c_0(0)=\langle f(t)\phi_{00}(t)\rangle=\int f(t)\phi(t)dt=0.75$$

$$d_0(0)=\langle f(t)\psi_{00}(t)\rangle=\int f(t)\psi(t)dt=0.25$$

$$d_1(0)=\langle f(t)\psi_{10}(t)\rangle=\int f(t)2^{1/2}\psi(2t)dt=\int_0^{1/4}4\times 2^{1/2}dt+\int_{1/4}^{1/2}(-2)\times 2^{1/2}(-1)dt=1.5\times 2^{1/2}$$

$$d_1(1)=\langle f(t)\psi_{11}(t)\rangle=\int f(t)2^{1/2}\psi(2t-1)dt=\int_{1/2}^{3/4}2\times 2^{1/2}dt+\int_{1/4}^{1/2}(-1)\times 2^{1/2}(-1)dt=0.75\times 2^{1/2}$$

$$f(t)=0.75\phi(t)+0.25\psi(t)+3\psi(2t)+1.5\psi(2t-1)$$

13.19

a. $f(t)=\sum_{k=-\infty}^{\infty}c_1(k)2^{1/2}\phi\left(2t-k\right)$

$$c_1(k)=\langle f(t)\phi_{1k}(t)\rangle=\int f(t)2^{1/2}\phi(2t-k)dt$$

$$c_1(0)=\langle f(t)\phi_{10}(t)\rangle=\int_0^{1/2}\sin(2\pi t)2^{1/2}\phi(2t)dt=(1/\pi)\times 2^{1/2}$$

$$c_1(1)=\langle f(t)\phi_{11}(t)\rangle=\int_{1/2}^{1}\sin(2\pi t)2^{1/2}\phi(2t-1)dt=(-1/\pi)2^{1/2}$$

$$f(t)=c_1(0)2^{1/2}\phi(2t)+c_1(1)2^{1/2}\phi(2t-1)=(2/\pi)\phi(2t)-(2/\pi)\phi(2t-1)$$

b. $f(t)=\sum_{k=-\infty}^{\infty}c_0(k)\phi_{0k}(t)+\sum_{k=-\infty}^{\infty}d_0(k)\psi_{0k}(t)$

$$c_0(0)=\langle f(t)\phi_{00}(t)\rangle=\int_0^{1}\sin(2\pi t)\phi(t)dt=0$$

$$d_0(0)=\langle f(t)\psi_{00}(t)\rangle=\int_0^{1}\sin(2\pi t)\psi(t)dt=2/\pi$$

$$f(t)=c_0(0)\phi(t)+d_0(0)\psi(t)=(2/\pi)\psi(t)$$

13.20

a. $f(t)=\sum_{k=-\infty}^{\infty}c_1(k)2^{1/2}\phi\left(2t-k\right)$

$$c_1(k)=\langle f(t)\phi_{1k}(t)\rangle=\int f(t)2^{1/2}\phi(2t-k)dt$$

$$c_1(0)=\langle f(t)\phi_{10}(t)\rangle=\int_0^{1/2}e^{-5t}2^{1/2}\phi(2t)dt=\frac{1}{5}(1-e^{-2.5})\times 2^{1/2}$$

$$c_1(1) = \langle f(t)\phi_{11}(t)\rangle = \int_{1/2}^{1} e^{-5t} 2^{1/2}\phi(2t-1)dt = \frac{1}{5}(e^{-2.5} - e^{-5})2^{1/2}$$

$$f(t) = c_1(0)2^{1/2}\phi(2t) + c_1(1)2^{1/2}\phi(2t-1) = (2/5)(1-e^{-2.5})\phi(2t) + (2/5)(e^{-2.5} - e^{-5})\phi(2t-1)$$

b. $f(t) = \sum_{k=-\infty}^{\infty} c_0(k)\phi_{0k}(t) + \sum_{k=-\infty}^{\infty} d_0(k)\psi_{0k}(t)$

$$c_0(0) = \langle f(t)\phi_{00}(t)\rangle = \int_0^1 e^{-5t}\phi(t)dt = (1/5)(1-e^{-5})$$

$$d_0(0) = \langle f(t)\psi_{00}(t)\rangle = \int_0^1 e^{-5t}\psi(t)dt = \int_0^{1/2} e^{-5t}dt + \int_{1/2}^{1} e^{-5t}(-1)dt = (1/5)(1-2e^{-2.5} + e^{-5})$$

$$f(t) = c_0(0)\phi(t) + d_0(0)\psi(t) = (1/5)(1-e^{-5})\phi(t) + (1/5)(1-2e^{-2.5} + e^{-5})\psi(t)$$

13.21

$h_0(k) = [0.707\ 0.707]$ and $h_1(k) = [0.707\ \ -0.707]$

a. $$\sum_{k=-\infty}^{\infty} \sqrt{2}h_0(k)\phi(4t-k) = \sqrt{2}h_0(0)\phi(4t) + \sqrt{2}h_0(1)\phi(4t-1)$$

$$= \sqrt{2}\times 0.707\phi(4t) + \sqrt{2}\times 0.707\phi(4t-1) = \phi(4t) + \phi(4t-1) = \phi(2t)$$

b. $$\sum_{k=-\infty}^{\infty} \sqrt{2}h_1(k)\phi(4t-k) = \sqrt{2}h_1(0)\phi(4t) + \sqrt{2}h_1(1)\phi(4t-1)$$

$$= \sqrt{2}\times 0.707\phi(4t) + \sqrt{2}(-0.707)\phi(4t-1) = \phi(4t) - \phi(4t-1) = \psi(2t)$$

13.22

$h_0(k) = [0.483\ \ 0.837\ \ 0.224\ \ \text{-}0.129]$

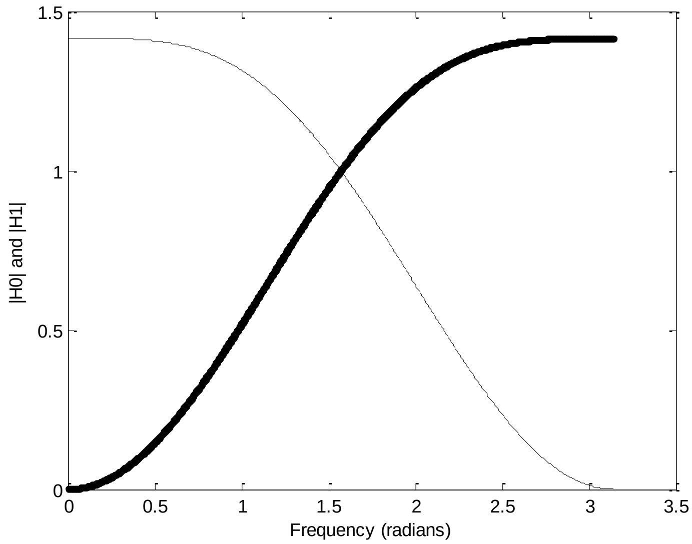
$h_1(k) = (-1)^k h_0(4-1-k)$, $h_1(0) = (-1)^0 h_0(4-1-0) = h_1(3) = -0.129$

$h_1(1) = (-1)^1 h_0(4-1-1) = -h_1(2) = -0.224$, $h_1(2) = (-1)^2 h_0(4-1-2) = h_1(1) = 0.837$

$h_1(3) = (-1)^3 h_0(4-1-3) = -h_1(0) = -0.483$

$h_1(k) = [\text{-}0.129\ \text{-}0.224\ 0.837\ \text{-}0.483\ ]$

```
>> h0=[0.483  0.837  0.224  -0.129]
>> h1= [-0.129 -0.224 0.837 -0.483 ]
>> [H0,w]=freqz(h0,1,1000);
>> [H1,w]=freqz(h1,1,1000);
>> plot(w,abs(H0),'k',w,abs(H1),'k.');
>> xlabel('Frequency (radians)');ylabel('|H0| and |H1|')
```



13.23

$w(k) = [5.5000 \quad 0.5000 \quad 7.0711 \quad 2.1213]$

13.24

$w(k) = [3.5355 \quad 5.6569 \quad -0.5000 \quad 0.5000 \quad 7.0711 \quad 0.7071 \quad 0.7071 \quad -1.4142]$

13.25

$c(k) = [2.2929 \quad 3.7071 \quad 2.4142 \quad -0.4142]$

13.26

$c(k) = [\ 1.6213 \quad 1.6213 \quad 2.6213 \quad 2.6213 \quad 1.7071 \quad 1.7071 \quad -0.2929 \quad -0.2929]$

13.27

$c(k) = [2.1213 \quad 3.5355 \quad 2.8284 \quad 0]$

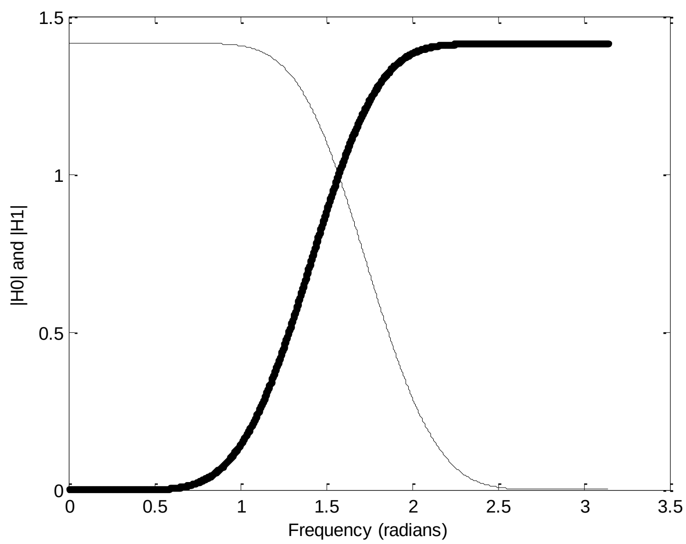
13.28

a. W=[100 20 16 -5 -3 4 2 -6]
b. W=[100 20 16 -5]
c. W=[100 20]
d. W=[100]

13.29

MATLAB program

```
------------------------------------------------------------------
clc, clear all; close all;
%Problem 13.29
h0=[0.054415842243144   0.312871590914520   0.675630736297712  ...
    0.585354683654425  -0.015829105256675  -0.284015542962009  ...
    0.000472484573805   0.128747426620538  -0.017369301001845  ...
    -0.044088253930837   0.013981027917411   0.008746094047413 ...
    -0.004870352993456  -0.000391740373377   0.000675449406451 ...
    -0.000117476784125];
for k=0:15
    h1(k+1)=(-1)^(k)*h0(16-k);
end
disp('Output p(2n)=>');
p=xcorr(h0,h0);
p(2:2:length(p))
disp('output R(z)=>');
p
disp('Output R(-z)=');
pp=xcorr(h1,h1)
disp('Output R(z)+R(-z)=');
p+pp
[H0,w]=freqz(h0,1,1000);
[H1,w]=freqz(h1,1,1000);
plot(w,abs(H0),'k',w,abs(H1),'k.');
xlabel('Frequency (radians)');ylabel('|H0| and |H1|')
------------------------------------------------------------------
```



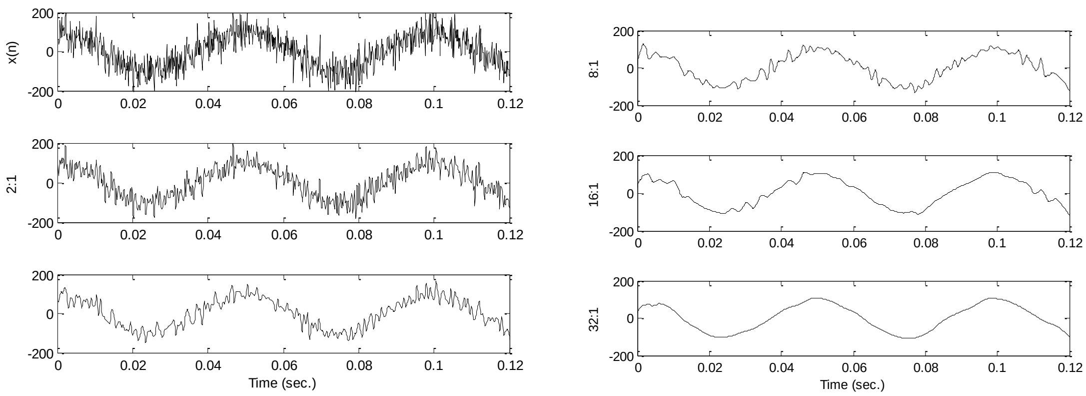
13.30

See answers in problema 13.23-13.27.

13.31
MATLAB program

```
close all; clear all;clc
t=0:1:1023;t=t/8000;
x=100*cos(20*2*pi*t)+50*randn(1,1024);
h0=[0.230377813308896   0.714846570552915   0.630880767929859 ...
    -0.027983769416859  -0.187034811719092   0.030841381835561 ....
    0.032883011666885  -0.010597401785069];
N=1024; nofseg=1
rec_sig=[]; rec_sig2t1=[]; rec_sig4t1=[]; rec_sig8t1=[]; rec_sig16t1=[];
rec_sig32t1=[];
for i=1:nofseg
    sp=x((i-1)*1024+1:i*1024);
    w=dwt(h0,sp,10);
% Quantization
    wmax=round(max(abs(w)));
    wcode=round(2^15*w/wmax); % 16-bit code for storage
    w=wcode*wmax/2^15; % Recovered wavelet coefficients
    w(513:1024)=zeros(1,512); % 2:1 compression ratio
    sig_rec2t1=idwt(h0,w,10);
    rec_sig2t1=[rec_sig2t1 sig_rec2t1'];
    w(257:1024)=0; % 4:1 compression ratio
    sig_rec4t1=idwt(h0,w,10);
    rec_sig4t1=[rec_sig4t1 sig_rec4t1'];
    w(129:1024)=0; % 8:1 compression ratio
    sig_rec8t1=idwt(h0,w,10);
    rec_sig8t1=[rec_sig8t1 sig_rec8t1'];
    w(65:1024)=0; % 16:1 compression ratio
    sig_rec16t1=idwt(h0,w,10);
    rec_sig16t1=[rec_sig16t1 sig_rec16t1'];
    w(33:1024)=0; % 32:1 compression ratio
    sig_rec32t1=idwt(h0,w,10);
    rec_sig32t1=[rec_sig32t1 sig_rec32t1'];
end
subplot(3,1,1),plot(t,x,'k'); axis([0 0.12 -200 200]);ylabel('x(n)');
subplot(3,1,2),plot(t,rec_sig2t1,'k'); axis([0 0.12 -200 200]);ylabel('2:1');
subplot(3,1,3),plot(t,rec_sig4t1,'k'); axis([0 0.12 -200 200]);ylabel(4:1);
xlabel('Time (sec.)')
NN=min(length(x),length(rec_sig2t1)); axis([0 0.12 -200 200]);
figure (2)
subplot(3,1,1),plot(t,rec_sig8t1,'k'); axis([0 0.12 -200 200]);ylabel('8:1');
subplot(3,1,2),plot(t,rec_sig16t1,'k'); axis([0 0.12 -200
200]);ylabel('16:1');
subplot(3,1,3),plot(t,rec_sig32t1,'k'); axis([0 0.12 -200
200]);ylabel('32:1');
xlabel('Time (sec.)')
NN=min(length(x),length(rec_sig2t1)); axis([0 0.12 -200 200]);
err=rec_sig2t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (2:1)=>');
SNR=10*log10(SNR)
err=rec_sig4t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (4:1)=>');
```

```
SNR=10*log10(SNR)
err=rec_sig8t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (8:1)=>');
SNR=10*log10(SNR)
err=rec_sig16t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (16:1)=>');
SNR=10*log10(SNR)
err=rec_sig32t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (32:1)=>');
SNR=10*log10(SNR)
```



Reconstruction SNR dB (2:1)=>
SNR =8.0710

Reconstruction SNR dB (4:1)=>
SNR = 6.1910

Reconstruction SNR dB (8:1)=>

SNR =5.4304
Reconstruction SNR dB (16:1)=>

SNR = 5.0725
Reconstruction SNR dB (32:1)=>

SNR = 4.7970

Chapter 14

14.1
a. 76.8 Kbytes; b. 921.6 Kbytes; c. 1920.768 Kbytes.

14.2
a. $2^{16} = 65536$ b. $2^{24} = 16777216 = 16.777 \times 10^6$

14.3

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} 200 \\ 10 \\ 100 \end{bmatrix} = \begin{bmatrix} 142 \\ 54 \\ 11 \end{bmatrix}$$

$Y = 142$, $I = 54$, $Q = 11$

14.4

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.000 & 0.956 & 0.621 \\ 1.000 & -0.272 & -0.647 \\ 1.000 & -1.106 & 1.703 \end{bmatrix} \begin{bmatrix} 141 \\ 46 \\ 5 \end{bmatrix} = \begin{bmatrix} 188 \\ 125 \\ 99 \end{bmatrix}$$

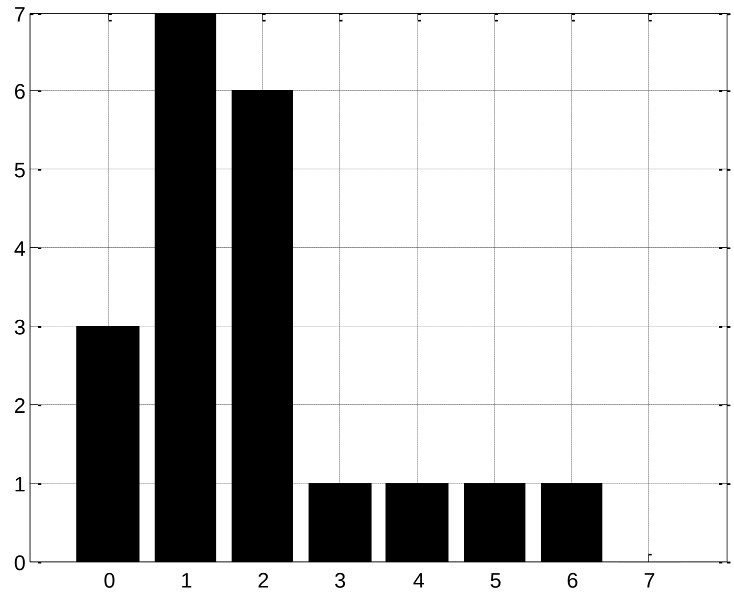
$R = 188$, $G = 125$, $B = 99$

14.5

$$Y = 0.299 \times \begin{bmatrix} 100 & 50 \\ 100 & 150 \end{bmatrix} + 0.587 \times \begin{bmatrix} 20 & 40 \\ 10 & 30 \end{bmatrix} + 0.114 \times \begin{bmatrix} 100 & 50 \\ 200 & 150 \end{bmatrix} = \begin{bmatrix} 53 & 44 \\ 59 & 50 \end{bmatrix}$$

14.6

Pixel $p(m,n)$ level	Number of pixels
0	3
1	7
2	6
3	1
4	1
5	1
6	1
7	0

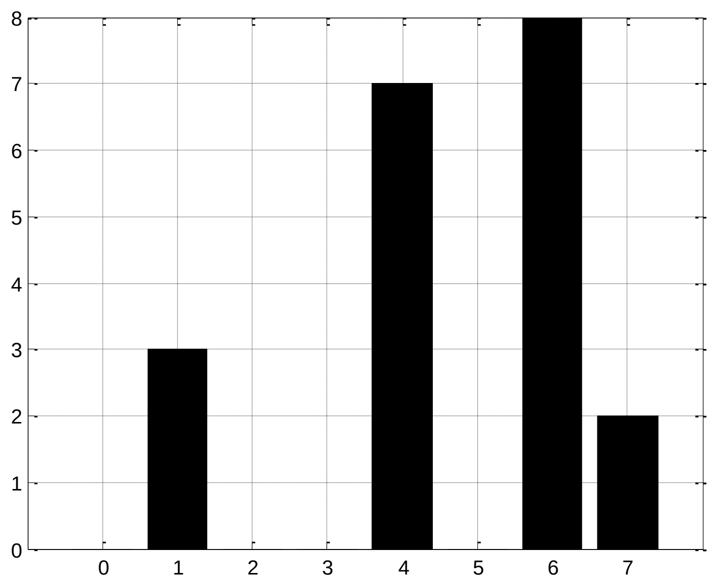


14.7

Pixel $p(m,n)$ level	Number of pixels	Number of pixels $\le p(m,n)$	Equalized pixel level
0	3	3	1
1	7	10	4
2	6	16	6
3	1	17	6
4	1	18	6
5	1	19	7
6	1	20	7
7	0	20	7

Equalized image $\begin{bmatrix} 1 & 4 & 6 & 6 & 1 \\ 6 & 4 & 4 & 6 & 4 \\ 4 & 4 & 6 & 6 & 6 \\ 1 & 6 & 7 & 7 & 4 \end{bmatrix}$

Pixel $p(m,n)$ level	Number of pixels
0	0
1	3
2	0
3	0
4	7
5	0
6	8
7	2



14.8

Full range: $H=4$, $L=2$, $\text{Top}=2^3-1=7$, Bottom=0

Shift up: $H=4$, $L=2$, $\text{Top}=7$, Bottom=3

Shift-down: $H=4$, $L=2$, , $\text{Top}=3$, Bottom=0

Pixel $p(m,n)$ level	Full range	Range [3 7]	Range [0-3]
2	0	3	0
3	4	5	2
4	7	7	3

Full range: $\begin{bmatrix} 0 & 7 & 7 & 0 \\ 0 & 4 & 4 & 4 \\ 7 & 7 & 7 & 0 \\ 4 & 0 & 4 & 7 \end{bmatrix}$, Shift up: $\begin{bmatrix} 3 & 7 & 7 & 3 \\ 3 & 5 & 5 & 5 \\ 7 & 7 & 7 & 3 \\ 5 & 3 & 5 & 7 \end{bmatrix}$, Shift down: $\begin{bmatrix} 0 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 \\ 3 & 3 & 3 & 0 \\ 2 & 0 & 2 & 3 \end{bmatrix}$

14.9

Padding zeros for boundaries:

93	116	109	108	0
92	107	103	108	0
84	107	86	107	0
87	113	106	99	0
0	0	0	0	0

4x4 Enhanced image: $\begin{bmatrix} 102 & 109 & 104 & 51 \\ 98 & 101 & 101 & 54 \\ 98 & 103 & 100 & 51 \\ 50 & 55 & 51 & 25 \end{bmatrix}$ 4x4 original image: $\begin{bmatrix} 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \end{bmatrix}$

14.10

padding zeros for boundaries:

0	0	0	0	0	0
0	100	255	100	100	0
0	0	255	255	100	0
0	100	0	100	0	0
0	100	255	100	100	0
0	0	0	0	0	0

4x4 filtered image: $\begin{bmatrix} 0 & 100 & 100 & 0 \\ 0 & 100 & 100 & 100 \\ 0 & 100 & 100 & 100 \\ 0 & 100 & 0 & 0 \end{bmatrix}$ 4x4 original image: $\begin{bmatrix} 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \end{bmatrix}$

14.11

a.

Vertical Sobel detector: $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ Processed image $\begin{bmatrix} 225 & 125 & 130 & 33 \\ 249 & 119 & 136 & 6 \\ 249 & 119 & 136 & 6 \\ 255 & 125 & 130 & 0 \\ 255 & 128 & 128 & 30 \end{bmatrix}$

b. Laplacian edge detector: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ Processed image: $\begin{bmatrix} 0 & 106 & 106 & 0 \\ 106 & 255 & 255 & 106 \\ 106 & 255 & 255 & 106 \\ 117 & 223 & 223 & 117 \\ 0 & 117 & 117 & 0 \end{bmatrix}$

14.12 The green color is dominant in the area pointed by the arrow; the blue color is dominant in the background

14.13 The blue color is dominant in the area pointed by the arrow; the red color is dominant in the background.

14.14

$$X(1,2) = 90.00 - j51.96 \quad \text{and} \quad A(1,2) = 11.5470$$

14.15

$$X(u,v) = \begin{bmatrix} 460 & -40 \\ -240 & -140 \end{bmatrix} \text{ and } A(u,v) = \begin{bmatrix} 115 & 10 \\ 60 & 35 \end{bmatrix}$$

14.16

Forward DCT: $F(u,v) = \begin{bmatrix} 230 & -20 \\ -120 & -70 \end{bmatrix}$

14.17

Inverse DCT: $p(i,j) = \begin{bmatrix} 110 & 100 \\ 100 & 90 \end{bmatrix}$

14.18

$$d(n) = DC(n) - DC(n-1)$$

DPCM sequence: 400, -10, -40, 10, 10

DPCM sequence using the Huffman table:

(9, 110010000), (4,0101), (6,010111), (4,1010), (4, 1010)

14.19

a. (0,-2) (3,4), (2,-3), (0,7), (4, -2), (0,0)

b. (0000, 0010, 01), (0011, 0011, 100), (0010, 0010, 00),
(0000, 0011, 111), (0100, 0010, 01), (0000, 0000)

14.20

```
w =  230.0000  -20.0000;
    -120.0000  -70.0000
```

14.21

```
f = 110.0000  100.0000;
    100.0000   90.0000
```

14.22

```
W=  210.0000  130.0000  -30.0000  -20.0000
    185.0000  155.0000   15.0000  -15.0000
     40.0000   10.0000  -20.0000         0
     45.0000   25.0000  -25.0000   35.0000
```

14.23

```
f = 115.0000  145.0000   25.0000   45.0000
    105.0000  135.0000    5.0000   25.0000
     30.0000   20.0000    7.5000   27.5000
     10.0000   -0.0000   -7.5000   12.5000
```

14.24

The trace jumping from the end of horizontal scan line to the beginning of the next horizontal line scan in the same frame is called the horizontal retrace. The trace from the end of the bottom line to the beginning of the top line in the next frame is called the vertical retrace. The vertical retrace takes longer time. See Section 13.9.1

14.25

The interlaced scanning contains the odd field and even field per frame. The purpose of using interlaced scanning is to transmit a full frame quickly to reduce flicker.
See Section 14.9.1

14.26

NTSC bandwidth = 6 MHz, luminance Y bandwidth = 4.2 MHz, I bandwidth= 1.6 MHz, nd Q bandwidth = 0.6 MHz. See section 14.9.1

14.27

Frequency modulated (FM) using a peak frequency deviation of 25 kHz. Assuming the audio baseband bandwidth is 15 kHz, the stereo FM audio requires a transmission bandwidth of 80 kHz with an audio carrier located at 4.5 MHz relative to the picture carrier. FM (frequency modulation). See Section 14.9.1

14.28

$$Composite \times 2\sin(2\pi f_{sc}t) = Y \times 2\sin(2\pi f_{sc}t) + I\cos(2\pi f_{sc}t) \times 2\sin(2\pi f_{sc}t) + Q \times 2\sin^2(2\pi f_{sc}t)$$

$$= Y \times 2\sin(2\pi f_{sc}t) + I\sin(2 \times 2\pi f_{sc}t) + Q - Q\cos(2 \times 2\pi f_{sc}t)$$

Then apply lowpass filtering

14.29

The back porch of the blanking contains the color sub-carrier burst for the color demodulation. The color burst carrier is centered at 3.58 MHz above the picture carrier and has the 8 cycles. See Section 14.9.1

14.30

See Table 14.9.1.

14.31

The scan line rate 525 lines per frame x 30 frames per second = 15.75 kHz

The vertical synchronizing pulse rate (used with equalization pulses to provide timing) = 31.5 kHz. See Section 14.9.1

14.32

The method (c) since the chroma is down sampled both horizontally and vertically by a factor of 2. See Section 14.9.2

14.33

The progressive scanning traces a whole picture which is called the frame via row-wise, the interlaced scanning retraces the odd field and even field in each frame alternatively. CIF uses the progressive scan. See Section 14.9.2

14.34

b. takes more computations since finding the motion vectors requires using the sequential search algorithm to obtain the best matched microblock in the reference frame. See Section 14.10

14.35

$$\frac{80 \times 80}{16 \times 16}\left(16^2 \times 32^2 \times 3\right) = 19.661 \times 10^6 \text{ operations}$$

14.36

Refer to Program 14.1.

14.37

Refer to Program 14.2.

The bigger the SIGMA, the more blurred the image is.

The bigger the kernel size, the more blurred the image is

14.38 Refer to Program 14.2.
The bigger the kernel size of the median filter, the blurred the image is

14.39
Refer to Program 14.2.

14.40
Refer to Program 14.3

14.41

```
-------------------------------------------------------------------------------------------
close all; clear all; clc
% This test program is a part of simulation for Grayscale JPEG image
% compression.
% image size: 240x320
X=imread('cruiseorg','TIFF');            %note the image size= 240x320
Y=rgb2gray(X);                        %Y =grayscale image
size(Y)
Q=[ 16 11 10 16 24 40 51 61;           %quantization quality factor
   12 12 14 19 26 58 60 55;
   14 13 16 24 40 57 69 56;
   14 17 22 29 51 87 80 62;
   18 22 37 56 68 109 103 77;
   24 35 55 64 81 104 113 92;
   49 64 78 87 103 121 120 101;
   72 92 95 98 112 100 103 99];
factor=1;
Q=factor*Q;
count=0;     % number of image blocks; total number of blocks =30x40=1200
for i=1:30
   for j=1:40
      count=count+1;
      P(1:8,1:8,count)=Y((i-1)*8+1:i*8,(j-1)*8+1:j*8);
   end
end
for m=1:1200
   F(:,:,m)=round(dct2(P(:,:,m)));   % 2-D DCT transform
   FF(:,:,m)=round(F(:,:,m) ./Q);    % quantization of DCT coefficients
end
%Coding and lossless compression. (not included)
%decoding
for m=1:1200
   FC(:,:,m)=FF(:,:,m) .*Q;         % recover 2-D DCT coefficients
   PP(:,:,m)=round(idct2(FC(:,:,m)));  % recover the 8x8 image image
   err(:,:,m)=single(PP(:,:,m))-single(P(:,:,m));  % quantization vector
```

```
end
count=0;
for i=1:30
   for j=1:40
       count=count+1;
       YY((i-1)*8+1:i*8,(j-1)*8+1:j*8)=PP(1:8,1:8,count); % recover image
   end
end
YY=uint8(YY); % level adjustment
figure
subplot(1,2,1),imshow(Y);  % original image
title('Orignal image');
subplot(1,2,2),imshow(YY)  % JPEG image
title('JPEG compressed image');
-------------------------------------------------------------------------------------------------
```

I. Excellent image, II. Begin to see the block effects.
III. Image degradation with the block effects become severe. The image quality is unacceptable.

14.42

(a), (b), and (c)

```
----------------------------------------------------------------------
close all; clear all; clc
X=imread('cruiseorg','TIFF');
Y=rgb2gray(X);
h0=[ 0.230377813308896   0.714846570552915   0.630880767929859 ...
    -0.027983769416859  -0.187034811719092   0.030841381835561 ...
    0.032883011666885  -0.010597401785069];
M= length(h0);
h1(1:2:M-1) = h0(M:-2:2);h1(2:2:M) = -h0(M-1:-2:1);%Obtain QMF highpass
filter
[m n]=size(Y);
%level-1 transform
[m n]=size(Y);
for i=1:m
    W1(i,:)=dwt(h0,double(Y(i,:)),1)';
end
for i=1:n
    W1(:,i)=dwt(h0,W1(:,i),1); % Wavelet coefficents at level-1
end
% fnished
%level-2 transform
Y1=W1(1:m/2,1:n/2); %Obtain LL subband
[m n]=size(Y1);
for i=1:m
    W2(i,:)=dwt(h0,Y1(i,:),1)';
end
for i=1:n
    W2(:,i)=dwt(h0,W2(:,i),1);
end
```

```
% fnished
W22=W1;
W22(1:m,1:n)=W2; % wavelet coefficients at level-2 transform
wmax=max(max(abs(W22)));
% 8-bit Quantization
W22=round(W22*2^7/wmax);
W22=double(W22)*wmax/2^7;
figure(1), imshow(uint8(W22));
% Reconstruction with various methods
 [m, n]=size(W22);
 method=input('Reconstuction methods: 1, 2, 3, 4, 5=>');
% Method I: use all the coefficnets
if method ==1
end
% Method II: only use LL2
if method ==2
 WW=zeros(m,n);
 WW(1:m/4,1:n/4)=W22(1:m/4,1:n/4);
 W22=WW;%Discard HL2,LH2, HH2, HL1, LH1, HH1 subbands
end
% Method III: Use LL2, HL2, LH2, and HH2
if method ==3
 WW=zeros(m,n);
 WW(1:m/2,1:n/2)=W22(1:m/2,1:n/2);
 W22=WW;
end
% Method IV: LL2, HL2, LH2
if method ==4
 WW=zeros(m,n);
 WW(1:m/2,1:n/2)=W22(1:m/2,1:n/2);
 W22=WW;
 WW22(m/4+1:m/2:n/4+1:n/2)=0; % set HH2 to be zeros
end
 % Method V: LL2, HL2, LH2, HH2, LH1, HL1
if method == 5
   W22(m/2+1:m,n/2+1:n)=0;
end
   % decoding from Level-2 transform
[m,n]=size(W22);
Wd2=W22(1:m/2,1:n/2);

%level-2
[m n]=size(Wd2);
for i=1:n
    Wd1(:,i)=idwt(h0,double(Wd2(:,i)),1);
end
for i=1:m
    Wd1(i,:)=idwt(h0,double(Wd1(i,:))',1);
end
%level-1
[m, n]=size(W22);Yd11=W22;
Yd11(1:m/2,1:n/2)=Wd1;
for i=1:n
    Yd(:,i)=idwt(h0,Yd11(:,i),1);
end
for i=1:m
    Yd(i,:)=idwt(h0,double(Yd(i,:)),1)';
```

```
end
% fnished
figure (2),imshow(Y);
 Y11=uint8(Yd);
figure (3),imshow(Y11);
------------------------------------------------------------
```

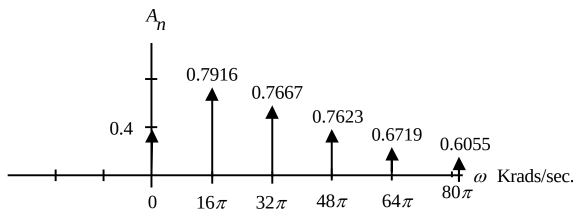
d.
Quality in decreasing order: I, V, III, IV, II

Appendix B

B.1

a. $A_0 = 0.4$, $A_1 = 0.7916$, $A_2 = 0.7667$, $A_3 = 0.7263$, $A_4 = 0.6719$, $A_5 = 0.6055$

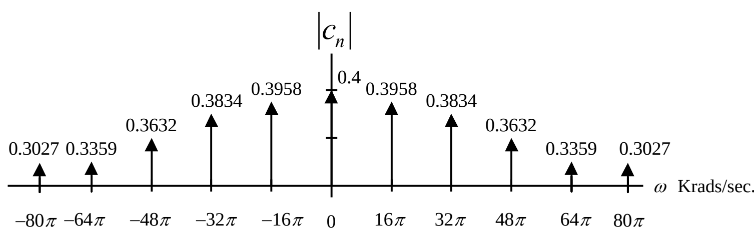
$\omega_0 = 2\pi \times 8000 = 16000\pi$



b.

$|c_0| = 0.4$, $|c_1| = |c_{-1}| = 0.3958$,

$|c_2| = |c_{-2}| = 0.3834$, $|c_3| = |c_{-3}| = 0.3632$, $|c_4| = |c_{-4}| = 0.3359$ $|c_5| = |c_{-5}| = 0.3027$



B.2

a.

$$x(t) = \frac{4A}{\pi}\left(\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \frac{1}{7}\cos 7\omega_0 t + \cdots\right)$$

$A = 10$, $T_0 = 1$ ms, $f_0 = 1000$ Hz, $\omega_0 = 2000\pi$ radians/second

$$x(t) = \frac{40}{\pi}\cos 2000\pi t - \frac{40}{3\pi}\cos 3000\pi t + \frac{40}{5\pi}\cos 10000\pi t - \frac{40}{7\pi}\cos 14000\pi t + \cdots$$

$$x(t) = 12.73\cos 2000\pi t + 4.24\cos(3000\pi t + 180^0)$$
$$+ 2.55\cos 10000\pi t + 1.82\cos(14000\pi t + 180^0) + \cdots$$

b.

$f_3 = 3000$ Hz $A_3 = 4.24$ volts

B.3

a.

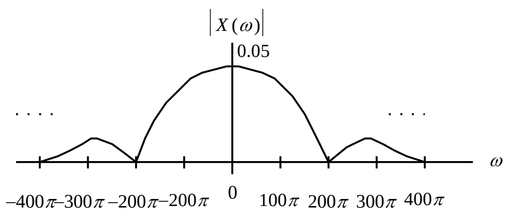
$$x(t) = 2 + 3.7420 \times \cos(2000\pi t) + 3.0273 \times \cos(4000\pi t)$$
$$+2.0182 \times \cos(6000\pi t) + 0.9355 \times \cos(8000\pi t) + \cdots$$

b.

$$f_2 = 2000 \text{ Hz}, \; A_2 = 3.0273$$

B.4

$$X(f) = 0.05\left(\frac{\sin 0.01\pi f}{0.01\pi f}\right)$$



B.5

$$X(f) = 5\left(\frac{\sin \pi f}{\pi f}\right)^2$$

B.6

$$X(f) = 5\left(\frac{\sin \pi f}{\pi f}\right)$$

B.7

a. $X(s) = 10$, b. $X(s) = -100/s^2$, c. $X(s) = \dfrac{10}{s+2}$

d. $X(s) = \dfrac{2e^{-5s}}{s}$, e. $X(s) = \dfrac{10s}{s^2+9}$, f. $X(s) = \dfrac{14.14 + 7.07s}{s^2+9}$

g. $X(s) = \dfrac{3(s+2)}{(s+2)^2+9}$, h. $X(s) = \dfrac{12000}{s^6}$

B.8

a. $x(t) = 10e^{-2t}u(t)$, b. $x(t) = 100e^{-2t}u(t) - 100e^{-3t}u(t)$

c. $x(t) = -66.67e^{-2t}u(t) + 166.67e^{-5t}u(t)$, d. $x(t) = 5e^{-2t}\sin(5t)u(t)$

B.9

a. $X(s) = \dfrac{7.5}{s(s+1.5)}$, b. $x(t) = 5u(t) - 5e^{-1.5t}u(t)$

B.10

a. $X(s)=\dfrac{10}{s(s^2+3s+2)}$

b. $X(s)=\dfrac{10}{s(s+1)(s+2)}=\dfrac{5}{s}+\dfrac{-10}{s+1}+\dfrac{5}{s+2}$, $x(t)=5u(t)-10e^{-1t}u(t)+5e^{-2t}u(t)$

B.11

a. zero: $s=3$, poles: $s=-2$, $s=-2$, stable

b. zeros: $s=0$, $s=\pm 2.236j$, poles: $s=\pm 3j$, $s=-1\pm 1.732j$, marginally stable

c. zeros: $s=\pm j$, $s=-1$, poles: $s=0$, $s=-3$, $s=-4$, $s=-8$, $s=1$, unstable

B.12

a. $h(t)=5e^{-5t}u(t)$ b. $Y(s)=\dfrac{5}{s\left(s+5\right)}$ c. $y(t)=u(t)-e^{-5t}u(t)$

B.13

a. $H(j\omega)=\dfrac{1}{\dfrac{j\omega}{5}+1}$, b. $A(\omega)=\dfrac{1}{\sqrt{1+\left(\dfrac{\omega}{5}\right)^2}}$ $\beta(\omega)=\angle-\tan\left(\dfrac{\omega}{5}\right)$

c. $Y(j2)=4.6424\angle-21.80^0$ that is, $y_{ss}(t)=4.6424\sin(2t-21.80^0)u(t)$

B.14

$$x(t)=u(t),\ h(t)=5e^{-5t}u(t)$$

$$y(t)=h(t)*u(t)=\int_0^{\infty}h(\tau)u(t-\tau)d\tau$$

$$=\int_0^t 5e^{-5\tau}d\tau=-e^{-5\tau}\Big|_0^t=-e^{-5t}-\left(-e^{-5\times 0}\right)$$

$$=1-e^{-5t}$$