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Quaternionic Analysis and Elliptic Boundary Value Problems

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Prefatory Notes

It is well known that complex methods may be advantageously used for the treatment of boundary value problems of partial differential equations in the plane. Moreover it is very important to transfer results of the classical function theory to function theories over domains in R^n. A comprehensive description of hypercomplex function theories is being made by the research group of R.DELANGHE (Gent) with their book "Clifford Analysis" ([BDS]). The application to solving boundary value problems by the help of hypercomplex function theories is not developed in the same extent.

The main aim of this book consists in the statement of a new strategy for solving linear and nonlinear boundary value problems of partial differential equations of mathematical physics by the help of hypercomplex analysis. In our opinion, it is the first summarizing presentation of a complete hypercomplex solution theory including analytical and numerical investigations in only one closed theory.

Using a special operator calculus and a hypercomplex function theory, the authors study questions of the existence, uniqueness, representation and regularity of solutions of above mentioned problems in a unified form. For the sake of simplicity, the authors restrict their investigations to the case of quaternionic calculus. Sometimes, if it seems to be necessary, it is referred to general results in CLIFFORD algebras. Furthermore suitable numerical approaches which are well-adapted to the quaternionic calculus are included too. The authors not only give an insight into boundary collocation methods but also introduce a new collocation procedure. Occurring for the first time, a discrete model of the quaternionic function theory was developed and applied to constructing and investigating of finite difference methods.

The first chapter makes the reader familiar with a basic knowledge in the field of quaternionic analysis. Most of the results are also valid in more general algebras.

In Chapter 2 the authors have studied algebraic and functional analytical properties of generating operators F_Γ, T_G, and D, which denote a CAUCHY-type operator, a quaternionic analogue to the complex T-operator and a generalized CAUCHY-

RIEMANN operator, respectively.
The third chapter only contains an orthogonal decomposition of the space $L_{2,H}(G)$ of quaternionic-valued functions into the subspaces $\ker D \cap L_{2,H}(G)$ and $D(\mathring{W}^{1}_{2,H}(G))$. This decomposition is an essential methodological instrument throughout the following explanations.
In Chapter 4 a series of linear and nonlinear elliptic boundary value problems of mathematical physics has been investigated by the help of a unified method in a rather complete manner.
Starting with some fundamental functional analytic theorems, a quaternionic version of the boundary collocation method is treated in Chapter 5.
Finally, in Chapter 6 a discrete quaternionic function theory is introduced. These results are used in order to find a well-adapted numerical approach to the analytical theory given in Chapter 4. The line of action is demonstrated by considering the discrete NAVIER-STOKES problem.
The book finishes with an Appendix. It is intended to give a short survey about other questions in the hypercomplex theory which have been investigated recently. The authors apologize in advance that in this summary not all important ideas and papers can be mentioned.
The monograph is suitable for mathematicians, physicists and engineers in research institutes. It has the character of a textbook. All the necessary mathematical preparations are made available. The structure of the method presented is very simple and makes possible a formal use for practical computations. Suitably chosen examples make the reader familiar with the topics and methods of quaternionic analysis. Other special branches such as approximation theory, theory of right-invertible operators, boundary collocation methods, finite difference methods and equations of mathematical physics will be mentioned in the book. Knowledge in numerical mathematics is desirable and facilitates the understanding.
We have to thank Prof. B.SILBERMANN, Prof. H.JÄCKEL (Karl-Marx-Stadt University of Technology) for suggesting the writing of this book Thanks are also due to Prof. P.LOUNESTO (Helsinki University of Technology), Prof. R.DELANGHE (Gent State University), Dr. V.SOUCEK (Prague Charles University),

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