Chapter 21: Electrostatics

Concept Checks

21.1. d **21.2.** a **21.3.** e **21.4.** e **21.5.** c **21.6.** b **21.7.** a **21.8.** a **21.9.** c **21.10.** b **21.11.** a

Multiple-Choice Questions

21.1. b **21.2.** b **21.3.** b **21.4.** d **21.5.** b **21.6.** b **21.7.** a **21.8.** a **21.9.** c **21.10.** b **21.11.** a **21.12.** b **21.13.** a **21.14.** e

Conceptual Questions

21.15. The given quantities are the charge of the two particles, $Q_1 = Q$ and $Q_2 = Q$. They are separated by a distance d. The Coulomb force between the changed particles is $F = k\dfrac{Q_1Q_2}{d^2} = k\dfrac{Q^2}{d^2}$. If the charge on each particle is doubled so that $Q_1' = 2Q = Q_2'$ and the separation distance is $d' = 2d$ the then the Coulomb Force is given by: $F' = k\dfrac{4Q^2}{4d^2} = k\dfrac{Q^2}{d^2}$ so the force is the same as it was in the initial situation.

21.16. The gravitational force between the Sun and the Earth is $F_\text{g} = G\dfrac{M_\text{S}M_\text{E}}{r^2}$ where G is the gravitational constant and is equal to $6.67\cdot 10^{-11}\text{ N m}^2/\text{kg}^2$, M_S is the mass of the Sun ($1.989\cdot 10^{30}$ kg) and M_E is the mass of the Earth ($5.974\cdot 10^{24}$ kg). The Coulomb force is given by the equation $F_\text{C} = k\dfrac{Q_1Q_2}{r^2}$ where k is Coulomb's constant ($k = 8.99\cdot 10^9\text{ N m}^2/\text{C}^2$). In this question $Q_1 = Q_2 = Q$ and is the charge given to the Earth and Sun to cancel out the gravitational force.

$$F_\text{C} = F_\text{g} \;\Rightarrow\; \frac{kQ^2}{r^2} = \frac{GM_\text{S}M_\text{E}}{r^2} \;\Rightarrow\; Q = \sqrt{\frac{GM_\text{S}M_\text{E}}{k}}$$

Therefore,

$$Q = \sqrt{\frac{\left(6.67\cdot 10^{-11}\text{ N m}^2/\text{kg}^2\right)(1.989\cdot 10^{30}\text{ kg})(5.974\cdot 10^{24}\text{ kg})}{8.99\cdot 10^9\text{ N m}^2/\text{C}^2}} = 2.97\cdot 10^{17}\text{ C.}$$

I can get the number of elementary charges, n, by dividing Q by $1.602\cdot 10^{-19}$ C (the charge of one electron):

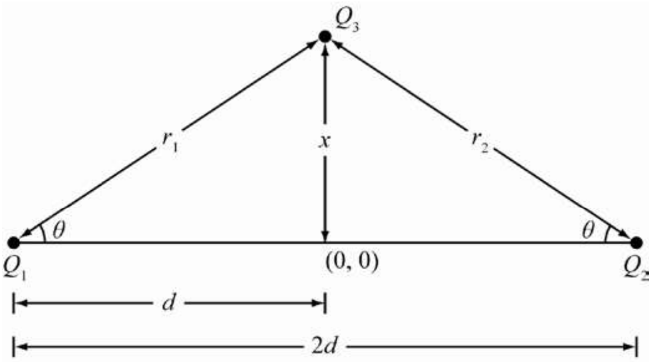
$$n = \frac{2.97\cdot 10^{17}\text{ C}}{1.602\cdot 10^{-19}\text{ C}} = 1.85\cdot 10^{36}.$$

To estimate the number of elementary change of either sign for the Earth I can assume the mass of the Earth is due to the mass of the protons, neutrons and electrons of which it is primarily composed. If I assume that the Earth's mass is due to the proton and neutron masses primarily (became an electrons mass is much smaller than a protons) and I assume that there are an equal number of protons and neutrons than I can get the number of protons by dividing the Earth's mass by two times the mass of a proton. The mass of a proton is $m_\text{p} \approx 1.6726\cdot 10^{-27}$ kg, so you can estimate the number of elementary charges on the Earth, n_E by: $n_\text{E} = \dfrac{m_\text{E}}{m_\text{p}} = \dfrac{5.97\cdot 10^{24}\text{ kg}}{1.67\cdot 10^{-27}\text{ kg}} = 3.57\cdot 10^{51}$. So the percentage of the Earth's changes that would be required to cancel out the gravitational force is $\left(n/n_\text{E}\right)\cdot 100\% = 5.18\cdot 10^{-14}\%$, a very small percentage.

21.17. One reason that it took such a long time to understand the electrostatic force may have been because it was not observed as frequently as the gravitational force. All massive objects are acted on by the gravitational force; however, only objects with a net charge will experience an electrostatic force.

21.18. The accumulation of static charge gives the individual hairs a charge. Since like charges repel and because the electrostatic force is inversely proportional to the charges separation distance squared, the hairs arrange themselves in a manner in which they are as far away from each other as possible. In this case that configuration is when the hairs are standing on end.

21.19. The given quantities are the charge which is $Q_1 = Q_2 = Q$ and the separation distance of $2d$. The third charge is $Q_3 = -0.2Q$ and it is positioned at d. Charge Q_3 is then displaced a distance x perpendicular to the line connecting the positive charges. The displacement $x \ll d$. The question asks for the force, F, on charge Q_3. For $x \ll d$ the question also asks for the approximate motion of the negative charge.



$\vec{F} = \vec{F}_{13} + \vec{F}_{23}$, where $\vec{F}_{13}$ is the force Q_3 feels due to Q_1 and $\vec{F}_{23}$ is the force Q_3 feels due to charge Q_2. Because Q_1 and Q_2 have the same sign and are of equal charge there is no net force in the horizontal direction. The forces due to Q_1 and Q_2 in the vertical direction are given by:

$$F_{13} = k\frac{Q_1Q_3}{r_1^2}\sin\theta \text{ and } F_{23} = k\frac{Q_2Q_3}{r_2^2}\sin\theta,$$

where $r_1 = \sqrt{d^2 + x^2}$ and $r_2 = \sqrt{d^2 + x^2}$. To simplify we can substitute $\sin\theta_1 = x / r_1$ and $\sin\theta_2 = x / r_2$ into force equations. So we can write the force equation as:

$$F = F_{13} + F_{23} = \frac{kQ_1Q_3}{\left(d^2 + x^2\right)}\left(\frac{x}{\sqrt{d^2 + x^2}}\right) + \frac{kQ_2Q_3}{\left(d^2 + x^2\right)}\left(\frac{x}{\sqrt{d^2 + x^2}}\right) = \left(Q_1 + Q_2\right)\frac{kxQ_3}{\left(d^2 + x^2\right)^{3/2}}$$

Substituting $Q_1 = Q_2 = Q$ and $Q_3 = -0.2Q$ gives:

$$F = \left(Q + Q\right)\frac{kx\left(-0.2Q\right)}{\left(d^2 + x^2\right)^{3/2}} = -\frac{2k\left(0.2Q^2\right)x}{\left(d^2 + x^2\right)^{3/2}} = -\frac{0.4kQ^2x}{\left(d^2 + x^2\right)^{3/2}}$$

The negative sign indicates that the force is downward. Since $x \ll d$, it is reasonable to use the approximation $(d^2 + x^2)^{3/2} = (d^2)^{3/2} = d^3$. Hence, $F \approx -\frac{0.4kQ^2x}{d^3}$. This solution is similar in form to Hooke's law which describes the restoring force due to the compression or expansion of a spring, $F_{\text{spring}} = -kx$ where k is the spring constant. The motion of the negative charge can therefore be approximated using simple harmonic motion.

21.20. As the garment is dried it acquires a charge from tumbling in the dryer and rubbing against other clothing. When I put the charged garment on it causes a redistribution of the charge on my skin and this causes the attractive electric force between the garment and my skin.

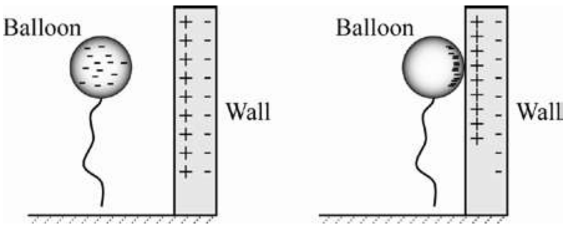
21.21. The initial separation of the spheres is x_1. The magnitude of the force on each of the spheres at separation x_1 is $F_1 = k\frac{Q_1Q_2}{x_1^2}$. The force after the distance change is $F_2 = k\frac{Q_1Q_2}{x_2^2}$, where the new distance is x_2. Because the charge is conserved I can equate the forces F_1 and F_2. $F_1 = k\frac{Q_1Q_2}{x_1^2}$ and $F_2 = k\frac{Q_1Q_2}{x_2^2}$, so $kQ_1Q_2 = F_1x_1^2 = F_2x_2^2$, or $x_2^2 = (F_1/F_2)x_1^2$. Substituting $F_2 = 9F_1$ into the equation gives: $x_2^2 = \frac{F_1}{9F_1}x_1^2 \Rightarrow x_2 = \sqrt{\frac{1}{9}x_1^2} = \frac{1}{3}x_1$. Therefore the distance would have to decrease to a factor of a third of its original value to achieve nine times the original force.

21.22. An electrically neutral atom can exert electrostatic force on another electrically neutral atom if they do not have symmetric charge distribution. In the case of two atoms where one atoms electron or electrons were closer to the proton of the other atom. This type of situation can occur when atoms undergo polar bonding to form a molecule.

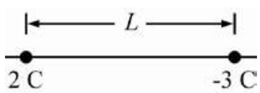
21.23. The scientist could convince themselves that the electrostatic force was not a variant of the gravitational force in various ways. One distinction is that gravitating objects attract but in the electric force like charged objects repel. For Earth bound experiments the scientists may observe that massive objects are pulled towards the ground by the gravitational force at a constant acceleration. If they performed careful experiments with objects of the same charge they would observe that the gravitational force downward on one of the charged objects could be diminished or balanced by the electrostatic force that object felt due to the second like charged object that was placed underneath it.

21.24. The electrostatic force is an inverse square force, of the same form as the Newtonian gravitational force. As long as the bodies are not moving too rapidly (i.e., not at speeds near the speed of light), the problem of determining their motion is the same as the Kepler problem. The motion of the two particles decomposes into a center of mass motion with constant velocity, and a relative motion which traces out a trajectory which can be either a portion of a straight line (for zero angular momentum, i.e., head on collisions) or a Keplerian ellipse (including a circle), parabola, or hyperbola, in the case of opposite charges. For charges of the same sign, for which the force is repulsive, the relative motion must be either a straight line or a hyperbola, an open orbit.

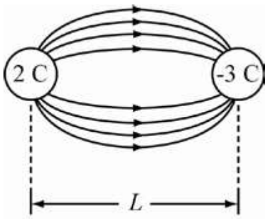
21.25. The wall does not have to be positively charged. The negatively charged balloon induces charges on the wall. The repulsive force between electrons in the balloon and those in the wall cause the electrons in the wall to redistribute. This leaves the portion of the wall that is closest to the balloon with a positive charge. The negatively charged balloon will be attached to the positively charged region of the wall even though the net charge of the wall is neutral.



21.26.

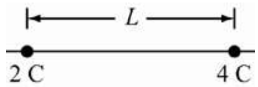


The electric lines flow from the positive charge to the negative charge as is shown in the sketch below.



There is nowhere on the line between the charged particles that I could place a test charge without it moving. This is due to the electric charges on the line having opposite charge, so a test charge (of either sign) that is placed between these two charges would be attracted by one and repelled by the other.

21.27.



In order for the test charge to feel no net force it would have to be at a location where the force it felt due to the charge $Q_2 = 4\text{ C}$ is equal and opposite to the force felt due to the charge $Q_1 = 2\text{ C}$. For convenience I can say that the charge $Q_1 = 2\text{ C}$ is located at $x_1 = 0$, and charge $Q_2 = 4\text{ C}$ is located at $x_2 = L$ and charge Q_3 is located at a position, x_3 which is between 0 and L. I can equate the expressions for the electric force on Q_3 due to Q_1 and the electric force on Q_3 due to Q_2 to solve for x_3 as these forces would have to balance for the charge Q_3 to feel no net force.

$$F_{13} = F_{23}$$
$$\frac{kQ_1Q_3}{x_3^2} = \frac{kQ_2Q_3}{(L-x_3)^2}$$
$$Q_1(L-x_3)^2 = Q_2x_3^2$$
$$Q_1(x_3^2 - 2x_3L + L^2) - Q_2x_3^2 = 0$$
$$(Q_1 - Q_2)x_3^2 - 2Q_1x_3L + Q_1L^2 = 0$$

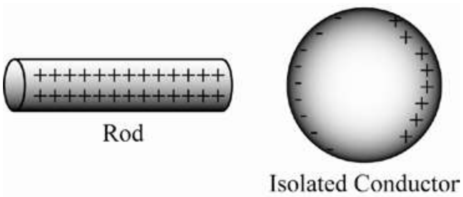
Note that in the second step of the calculation above, it is shown that the sign and magnitude of Q_3 will not impact the answer. I can solve using the quadratic equation:

$$x_3 = \frac{2Q_1L \pm \sqrt{4Q_1^2L^2 - 4(Q_1 - Q_2)(Q_1L^2)}}{2(Q_1 - Q_2)} = \frac{2(2\text{ C})L \pm \sqrt{4(2\text{ C})^2L^2 + 4(4\text{C}^2L^2)}}{-4\text{C}} = 0.414L, -2.414L$$

The correct answer is $x_3 = 0.414L$ because this point is between Q_1 and Q_2. One can also see from the second step of the algebraic manipulation that the magnitude and charge of Q_3 is irrelevant to the position of x_3, as it drops out of the equation. Intuitively, this makes sense, since whatever magnitude and charge of Q_3 is placed between the two existing charges, it will experience opposite forces from Q_1 and Q_2, since they have the same sign.

21.28. When a positively charged rod is brought near to an isolated neutral conductor without touching it the rod will experience an attractive force. The electric charge on the rod induces a redistribution of charge in the conductor.

The net effect of this distribution is that electrons move to the side of the conductor nearest to the rod. The positively charged rod is attracted to this region.



21.29. Using a metal key to touch a metal surface before exiting the car will discharge any charge I carry. When I begin to fuel a car, I can touch the gas pump and the car before pumping the gas, discharging myself. If I get back into the car, I can re-charge myself, and when I again get out of the car and touch the fuel nozzle without grounding myself first, I can get a spark, which might ignite the gasoline.

Exercises

21.30. Since charge is quantized, the number of electrons, when summed, yields the given charge: $n \cdot e = Q$. The charge of each electron is $1.602 \cdot 10^{-19}$ C. The total number n of electrons required to give a total charge of 1.00 C is obtained by dividing the total charge by the charge per electron:

$$n = \frac{Q}{e} = \frac{(1.00 \text{ C})}{(1.602 \cdot 10^{-19} \text{ C/electron})} = 6.18 \cdot 10^{18} \text{ electrons.}$$

21.31. The number of atoms or molecules in one mole of a substance is given by Avogadro's number, $N_{\text{A}} = 6.022 \cdot 10^{23} \text{ mol}^{-1}$. The faraday unit is $F = N_{\text{A}}e$, where e is the elementary charge of an electron or proton and is equal to $1.602 \cdot 10^{-19}$ C. To calculate the number of coulombs in 1.000 faraday you can multiply N_{A} by the elementary charge:

$$1.000 \text{ F} = N_{\text{A}}e = (6.022 \cdot 10^{23} \text{ atoms/mol})(1.602 \cdot 10^{-19} \text{ C}) = 96470 \text{ C.}$$

21.32. $1 \text{ dyne} = 1 \text{ g cm/s}^2 = 1 \cdot 10^{-5}$ N and it is a unit of force. An electrostatic unit or esu is defined as follows: Two point charges, each of 1 esu and separated by one centimeter exert a force of exactly one dyne on each other. Coulomb's law gives the magnitude of the force on one charge due to another, which is $F = k|q_1q_2|/r^2$ (where $k = 8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2$, q_1 and q_2 are electric charges and r is the separation distance between charges.)

(a) By substituting the values given in the question into Coulomb's law, the relationship between the esu and the Coulomb can be determined:

$$1 \cdot 10^{-5} \text{ N} = \frac{k(1 \text{ esu})^2}{(0.01 \text{ m})^2} \Rightarrow 1 \text{ esu} = \sqrt{\frac{(0.01 \text{ m})^2(1 \cdot 10^{-5} \text{ N})}{8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2}} = 3.34 \cdot 10^{-10} \text{ C}$$

(b) The result of part (a) shows that $1 \text{ esu} = 3.34 \cdot 10^{-10}$ C. The elementary charge on an electron or proton is $e = 1.602 \cdot 10^{-19}$ C. To get the relationship between the esu and elementary charge, divide 1 esu by the charge per electron (or proton).

$$1 \text{ esu} = \frac{3.34 \cdot 10^{-10} \text{ C}}{1.602 \cdot 10^{-19} \text{ C}/e} = 2.08 \cdot 10^9 e$$

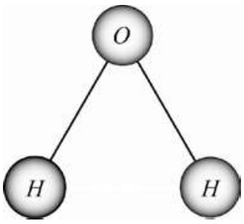
21.33. The given quantities are the current, $I = 5.00 \cdot 10^{-3}$ A and the exposure time, $t = 10.0$ s. One coulomb is equal to 1 A s. To calculate the number of electrons that flow through your skin at this current and during this time, multiply I by t to yield the quantity of charge in coulombs. Then divide by the elementary charge per electron, which is $1.602 \cdot 10^{-19} C$.

$$I\cdot t=\left(5.00\cdot 10^{-3}\text{ A}\right)\left(10.0\text{ s}\right)=0.0500\text{ A s}=0.0500\text{ C};$$

$$\frac{0.0500\text{ C}}{1.602\cdot 10^{-19}\text{ C}/e}=3.12\cdot 10^{17}\text{ electrons.}$$

21.34. **THINK:** Consider a mass, $m=1.00$ kg of water. To calculate how many electrons are in this mass, a relationship must be found between mass, the number of water atoms presents and their charge. Let η denote the number of electrons.

SKETCH:



RESEARCH: The molecular mass of water (H_2O), $m_{\text{W}}=18.015$ g/mol. The number of moles of water can be found by dividing the mass of water by its molecular mass. The number of electrons present in the water can be found from the atomic numbers, Z, for hydrogen and oxygen ($Z=1$ and $Z=8$ respectively). The total number of water molecules can be found by multiplying the number of moles of water present by Avogadro's number, $N_{\text{A}}=6.022\cdot 10^{23}\text{ mol}^{-1}$.

SIMPLIFY: $\eta=\dfrac{m}{m_{\text{W}}}\cdot N_{\text{A}}\cdot\dfrac{10\text{ electrons}}{\text{H}_2\text{O atom}}$

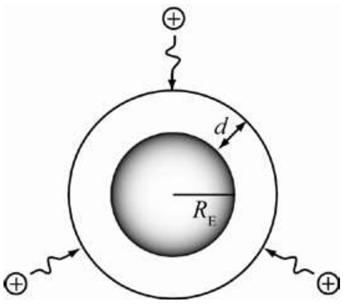
CALCULATE: $\eta=\left(\dfrac{1.00\cdot 10^{3}\text{ g}}{18.015\text{ g/mol}}\right)\left(6.022\cdot 10^{23}\text{ mol}^{-1}\right)\left(10\text{ electrons}\right)=3.34277\cdot 10^{26}\text{ electrons}$

ROUND: The values in the question were provided to 3 significant figures, so the answer is $3.34\cdot 10^{26}$ electrons.

DOUBLE-CHECK: Considering that there are approximately 55 moles of H_2O per kilogram of water and there are 10 electrons per H_2O atom, it makes sense that the answer is approximately 550 times greater than Avogadro's number.

21.35. **THINK:** Protons are incident on the Earth from all directions at a rate of $n=1245.0\text{ protons}/\left(\text{m}^2\text{ s}\right)$. Assuming that the depth of the atmosphere is $d=120\text{ km}=120{,}000\text{ m}$ and that the radius of the Earth is $r=6378\text{ km}=6{,}378{,}000\text{ m},$ I want to determine the total charge incident upon the Earth's atmosphere in 5.00 minutes.

SKETCH:



RESEARCH: Modeling the Earth like a sphere, the surface area A can be approximated as $A=4\pi r^2$. The total number of protons incident on the Earth in the time t can be found by multiplying the rate, n

by the surface area of the Earth and the time, t. The total charge Q can be found by multiplying the total number of protons, P by the charge per proton. The elementary charge of a proton is $1.602\cdot 10^{-19}$ C.

SIMPLIFY: $P = nAT = n4\pi r^2 t,\quad Q = P\left(1.602\cdot 10^{-19}\text{ C}/P\right)$

CALCULATE:

$P = 1245.0\text{ protons}/(\text{m}^2\text{s})\left[4\pi(6{,}378{,}000\text{ m}+120{,}000\text{ m})^2\right](300.\text{ s}) = 1.981800\cdot 10^{20}\text{ protons},$

$Q = 1.981800\cdot 10^{20}\text{ protons}\cdot\left(1.602\cdot 10^{-19}\text{ C}/\text{protons}\right) = 31.74844\text{ C}$

ROUND: To three significant figures 31.7 C

DOUBLE-CHECK: The calculated answer has the correct units of charge. The value seems reasonable considering the values that were provided in the question.

21.36. The charges obtained by the student performing the experiment are listed here: $3.26\cdot 10^{-19}$ C, $6.39\cdot 10^{-19}$ C, $5.09\cdot 10^{-19}$ C, $4.66\cdot 10^{-19}$ C, $1.53\cdot 10^{-19}$ C. Dividing the above values by the smallest measured value will give the number of electrons, n_e found in each measurement.

Observed charge	n_e	Integer value	Observed charge (integer value)
$3.26\cdot 10^{-19}$ C	2.13	2	$1.63\cdot 10^{-19}$ C
$6.39\cdot 10^{-19}$ C	4.17	4	$1.60\cdot 10^{-19}$ C
$5.09\cdot 10^{-19}$ C	3.32	3	$1.69\cdot 10^{-19}$ C
$4.66\cdot 10^{-19}$ C	3.04	3	$1.55\cdot 10^{-19}$ C
$1.53\cdot 10^{-19}$ C	1	1	$1.53\cdot 10^{-19}$ C

The number of electrons, n_e, must be rounded to their closest integer value because charge is quantized. Dividing the observed charge by the integer number of electrons gives the charge per electron. Taking the average of the observed charge/integer value data the average charge on an electron is calculated to be $(1.60\pm 0.03)\cdot 10^{-19}$ C.

Optional Error analysis: Given a set of n measured values a_i, there exists a mean value, μ. Then the standard deviation σ of the data is given by the relation:

$$\sigma^2 = \frac{\sum_{i=1}^{n} a_i^2}{n} - \frac{n}{n-1}\mu^2$$

$$\sigma = \sqrt{\frac{\left(1.63^2+1.60^2+1.69^2+1.55^2+1.53^2\right)}{5} - \frac{5}{4}\left(\frac{1.63+1.60+1.69+1.55+1.53}{5}\right)^2} = 0.06403$$

The error in a repeated measurement of the same quantity is:

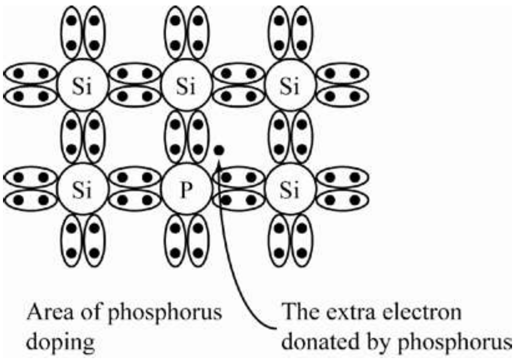
$$\text{Error} = \frac{\text{standard deviation}}{\sqrt{\text{number of measurements}}} = \frac{\sigma}{\sqrt{n}}.$$

$$\text{Error} = \frac{0.06403}{\sqrt{5}} = 0.0286$$

The measurements have an error of 2.8%.

21.37. **THINK:** An intrinsic silicon sample is doped with phosphorous. The level of doping is 1 phosphorous atom per one million silicon atoms. The density of silicon is $\rho_{\text{S}} = 2.33 \text{ g/cm}^3$ and its atomic mass is $m_{\text{S}} = 28.09 \text{ g/mol}$. The phosphorous atoms act as electron donors. The density of copper is $\rho_{\text{C}} = 8.96 \text{ g/cm}^3$ and its atomic mass is $m_{\text{C}} = 63.54 \text{ g/mol}$.

SKETCH:



RESEARCH: Avogadro's number is $N_{\text{A}} = 6.022 \cdot 10^{23} \text{ mol}^{-1}$. It gives the number of atoms or molecules per mole of a substance. Density, $\rho = m/V$, where $m = \text{mass}$ and $V = \text{volume}$.

SIMPLIFY:

(a) There will be 1 conduction electron per $1.00 \cdot 10^6$ silicon atoms. The number of silicon atoms per cm^3 is $n_{\text{S}} = \left(\rho_{\text{S}} / m_{\text{S}}\right) \cdot N_{\text{A}}$. The number of conduction electrons per cm^3 is $n_{\text{e}} = n_{\text{S}} / (1.00 \cdot 10^6)$.

(b) The number of copper atoms is $n_{\text{C}} = \left(\rho_{\text{C}} / m_{\text{C}}\right) \cdot N_{\text{A}}$. The number of conduction electrons in the copper is n_{C}. The ratio of conduction electrons in silicon to conduction electrons in copper is $n_{\text{e}} / n_{\text{C}}$.

CALCULATE:

(a) $n_{\text{C}} = \left(\dfrac{2.33 \text{ g/cm}^3}{28.09 \text{ g/mol}}\right) 6.022 \cdot 10^{23} \text{ mol}^{-1} = 4.995 \cdot 10^{22} \text{ /cm}^3$

$$n_{\text{e}} = \frac{4.995 \cdot 10^{22}}{1.00 \cdot 10^{6}} = 4.995 \cdot 10^{16} \text{ conduction electrons / cm}^3$$

(b) $n_{\text{C}} = \left(\dfrac{8.96 \text{ g/cm}^3}{63.54 \text{ g/mol}}\right) 6.022 \cdot 10^{23} \text{ mol}^{-1} = 8.4918 \cdot 10^{22} \text{ /cm}^3$

$$\frac{n_{\text{e}}}{n_{\text{C}}} = \frac{4.995 \cdot 10^{16}}{8.4918 \cdot 10^{22}} = 5.88215 \cdot 10^{-7}$$

ROUND: There were three significant figures provided in the question so the answers should be:

(a) $n_{\text{e}} = 5.00 \cdot 10^{16}$ conduction electrons / cm^3

(b) There are $5.88 \cdot 10^{-7}$ conduction electrons in the doped silicon sample for every conduction electron in the copper sample.

DOUBLE-CHECK: It is reasonable that there are approximately $5 \cdot 10^{-7}$ less conduction electrons in the doped silicon sample compared to the copper sample.

21.38. The force between the two charged spheres is $F_1 = k\dfrac{q_a q_b}{d_1^2}$ initially. After the spheres are moved the force is $F_2 = k\dfrac{q_a q_b}{d_2^2}$. Taking the ratio of the force after to the force before gives: $F_2 / F_1 = \left(k\dfrac{q_a q_b}{d_2^2}\right) / \left(k\dfrac{q_a q_b}{d_1^2}\right) = d_1^2 / d_2^2 = 4$. The new distance is then $d_2 = \sqrt{d_1^2 / 4} = d_1 / 2 = 4 \text{ cm}$.

21.39. The charge on each particle is q. When the separation distance is $d = 1.00$ m, the electrostatic force is $F = 1.00$ N. The charge q is found from $F = kq_1q_2 / d^2 = kq^2 / d^2$. Then,

$$q = \sqrt{\frac{Fd^2}{k}} = \sqrt{\frac{(1.00 \text{ N})(1.00 \text{ m})^2}{8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2}} = 1.05 \cdot 10^{-5} \text{ C.}$$

The sign does not matter, so long as each particle has a charge of the same sign, so that they repel.

21.40. In order for two electrons to experience an electrical force between them equal to the weight of one of the electrons, the distance d separating them must be such that. $F_\text{g} = F_\text{Coulomb} \Rightarrow m_e g = ke^2 / d^2$. Then,

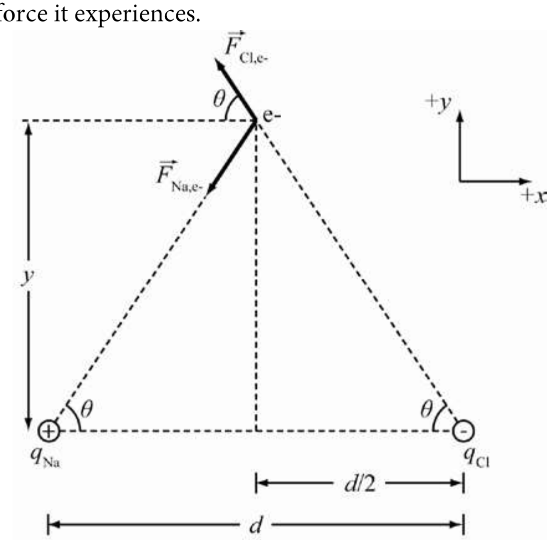
$$d = \sqrt{\frac{ke^2}{m_e g}} = \sqrt{\frac{\left(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2\right)\left(1.602 \cdot 10^{-19} \text{ C}\right)^2}{(9.109 \cdot 10^{-31} \text{ kg})(9.81 \text{ m/s}^2)}} = 5.08 \text{ m}$$

21.41. In solid sodium chloride, chloride ions have a charge $q_\text{Cl} = -e = -1.602 \cdot 10^{-19}$ C, while sodium ions have a charge $q_\text{Na} = e = 1.602 \cdot 10^{-19}$ C. These ions are separated by about $d = 0.28$ nm. The Coulomb force between the ions is

$$F = \frac{kq_\text{Cl}q_\text{Na}}{d^2} = \frac{-\left(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2\right)(1.602 \cdot 10^{-19} \text{ C})^2}{(0.28 \cdot 10^{-9} \text{ m})^2} = -2.94285 \cdot 10^{-9} \text{ N} \approx -2.9 \cdot 10^{-9} \text{ N.}$$

The negative sign indicates that the force is attractive.

21.42. In gaseous sodium chloride, chloride ions have a charge $q_\text{Cl} = -e = -1.602 \cdot 10^{-19}$ C, while sodium ions have a charge $q_\text{Na} = e = 1.602 \cdot 10^{-19}$ C. These ions are separated by about $d = 0.24$ nm. Another electron is located $y = 0.48$ nm above the midpoint of the sodium chloride molecule. Find the magnitude and the direction of the Coulomb force it experiences.



The x-component of the force is

$$F_x = F_{\text{Cl, e}^-x} + F_{\text{Na, e}^-x}$$
$$= \frac{-ke^2\cos\theta}{\left(\frac{d}{2}\right)^2 + y^2} - \frac{ke^2\cos\theta}{\frac{d^2}{4}+y^2} = -\frac{2ke^2\cos\theta}{\frac{d^2}{4}+y^2} = -\frac{2ke^2}{\frac{d^2}{4}+y^2}\cdot\frac{d/2}{\sqrt{\frac{d^2}{4}+y^2}} = -\frac{ke^2 d}{\left(\frac{d^2}{4}+y^2\right)^{3/2}}$$
$$= -\frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(1.602\cdot 10^{-19}\text{ C})^2(0.24\cdot 10^{-9}\text{ m})}{\left[\frac{(0.24\cdot 10^{-9}\text{ m})^2}{4}+(0.48\cdot 10^{-9}\text{ m})^2\right]^{3/2}}$$
$$= -4.5717\cdot 10^{-10}\text{ N} \approx -4.6\cdot 10^{-10}\text{ N}$$

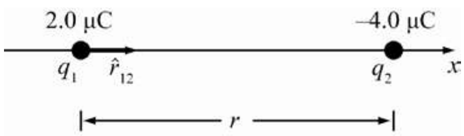
By symmetry, the y-components cancel; that is $F_{\text{Cl, e}^-y} = F_{\text{Na, e}^-y}$. The magnitude is therefore $F = 4.6\cdot 10^{-10}\text{ N}$; The electron is pulled in the $-\hat{x}$ direction (in this coordinate system).

21.43. The two up quarks have identical charge $q = (2/3)e = (2/3)\left(1.602\cdot 10^{-19}\text{ C}\right)$. They are $d = 0.900\cdot 10^{-15}$ m apart. The magnitude of the electrostatic force between them is

$$F = \frac{kq^2}{d^2} = \frac{\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)\left[\frac{2}{3}(1.602\cdot 10^{-19}\text{ C})\right]^2}{(0.900\cdot 10^{-15}\text{ m})^2} = 127\text{ N}.$$

This is large, however the proton does not 'break apart' because of the strength of the strong nuclear force which binds the quarts together to form the proton. A proton is made of 2 up quarks, each with charge $(2/3)e$, and one down quark with charge $-(1/3)e$. The net charge of the proton is e.

21.44. Coulomb's Law can be used to find the force on $q_1 = 2.0\ \mu\text{C}$ due to $q_2 = -4.0\ \mu\text{C}$, where q_2 is $r = 0.200$ m to the right of q_1.

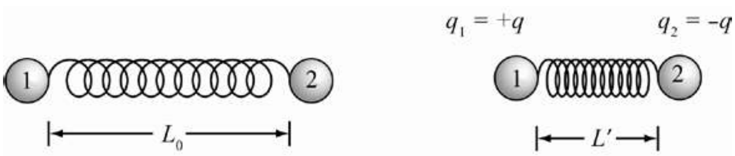


$$\vec{F}_{2\to 1} = -k\frac{q_1q_2}{r^2}\hat{r}_{21} = -k\frac{q_1q_2}{r^2}\hat{x} = -\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)\frac{\left(2.0\ \mu\text{C}\right)\left(-4.0\ \mu\text{C}\right)}{\left(0.200\text{ m}\right)^2}\hat{x} = 1.8\text{ N }\hat{x}$$

The $-4.0\ \mu\text{C}$ charge pulls the $2.0\ \mu\text{C}$ charge to the right.

21.45. **THINK:** The two identical spheres are initially uncharged. They are connected by an insulating spring of equilibrium length $L_0 = 1.00$ m and spring constant $k = 25.0$ N/m. Charges $+q$ and $-q$ are then placed on metal spheres 1 and 2, respectively. Because the spring is insulating, the charges cannot neutralize across the spring. The spring contracts to new length $L' = 0.635$ m, due to the attractive force between the charges spheres. Determine the charge q. If someone coats the spring with metal to make it conducting, find the new length of the spring.

SKETCH:



RESEARCH: The magnitude of the spring force is $F_S = k_S \Delta x$. The magnitude of the electrostatic force is $F = kq_1q_2 / r^2$. For this isolated system, the two forces must be in balance, that is $F_S = F$. From this balance, the charge q can be determined. The spring constant is denoted by k_S to avoid confusion with the Coulomb constant, k.

SIMPLIFY: $F_S = F \Rightarrow k_S \Delta x = \dfrac{kq_1q_2}{r^2} \Rightarrow k_S(L_0 - L') = \dfrac{kq^2}{(L')^2} \Rightarrow q = \sqrt{\dfrac{k_S (L')^2 (L_0 - L')}{k}}$

CALCULATE: $q = \sqrt{\dfrac{(25.0\text{ N/m})(0.635\text{ m})^2(1.00\text{ m} - 0.635\text{ m})}{(8.99 \cdot 10^9\text{ N m}^2/\text{C}^2)}} = 2.02307 \cdot 10^{-5}\text{ C}$

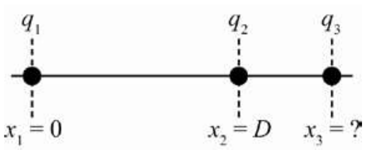
If someone were to coat the spring such that it conducted electricity, the charge on the two spheres would distribute themselves evenly about the system. If the charges are equal in magnitude and opposite in sign, as they are in this case, the net charge in the system would be zero. Then the electrostatic force between the two spheres would be zero, and the spring would return to its equilibrium length of 1.00 m.

ROUND: To three significant figures, $q = 2.02 \cdot 10^{-5}$ C.

DOUBLE-CHECK: Dimensional analysis confirms that the answer is in coulombs, the appropriate unit for charge.

21.46. **THINK:** A point-like charge of $q_1 = +3q$ is located at $x_1 = 0$, and a point-like charge of $q_2 = -q$ is located on the x-axis at $x_2 = D$, where $D = 0.500$ m. Find the location on the x-axis x_3 where will a third charge $q_3 = q_0$ experiences no net force from the other two charges.

SKETCH:



RESEARCH: The magnitude of the electrostatic force is $F = kq_1q_2 / r^2$. The net force on the third charge q_3 is zero when the sum of the forces from the other two charges is zero: $F_{\text{net},3} = F_{13} + F_{23} = 0 \Rightarrow F_{13} = -F_{23}$. The two forces F_{13} and F_{23} must be equal in magnitude, but opposite in direction. Consider the following three possible locations for the charge q_3. Note that this analysis is independent of the charge of q_3. In the case $x_3 < x_1 = 0$, the two forces F_{13} and F_{23} will be opposite in direction but they cannot be equal in magnitude: the charge q_1 at x_1 is greater in magnitude than the charge q_2 at x_2 and x_3 would be closer to x_1. (Remember that the electrostatic force increases as the distance between the charges decreases.) This makes the magnitude of F_{13} greater than that of F_{23}. In the case $0\text{ m} < x_3 < D$, the two forces are in the same direction and therefore cannot balance. In the case $x_3 > x_2 = D$, the two forces are opposite in direction, and in direct opposition to the first situation, the force F_{13} and F_{23} can now be balanced. The solution will have a positive x position, or more accurately, the third charge q_3 must be placed near the smaller fixed charge, q_2, without being between the two fixed charges q_1 and q_2

SIMPLIFY:

Since $x_3 > x_2$, consider only the magnitudes of the forces. Since only the magnitudes of the forces are compared, only the magnitudes of the charges need be considered.

$$|F_{13}| = |F_{23}| \Rightarrow \left|\frac{kq_1q_3}{x_3^2}\right| = \left|\frac{kq_2q_3}{(x_3 - x_2)^2}\right| \Rightarrow |q_1|(x_3 - x_2)^2 = |q_2|x_3^2 \Rightarrow 3q(x_3 - D)^2 = qx_3^2$$

$$3(x_3 - D)^2 - x_3^2 = 0 \Rightarrow 2x_3^2 - 6x_3D + 3D^2 = 0$$

Solving for $x_3: x_3 = \dfrac{6D \pm \sqrt{36D^2 - 4(2)(3D^2)}}{4}$

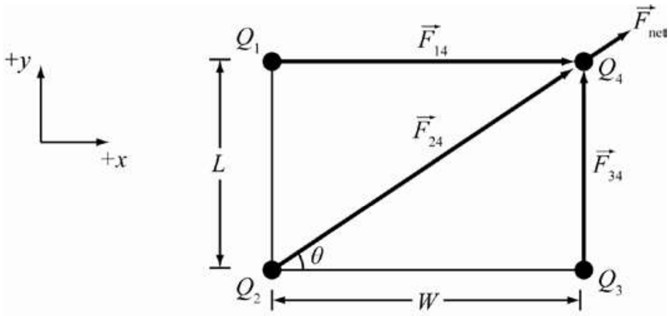
CALCULATE: $x_3 = \dfrac{6(0.500\text{ m}) \pm \sqrt{36(0.500\text{ m})^2 - 24(0.500\text{ m})^2}}{4} = 1.1830\text{ m}, 0.3170\text{ m}$

ROUND: Since $x_3 > x_2$, $x_3 = 1.18$ m.

DOUBLE-CHECK: The solution fits the expected location that was determined above (where $x_3 > x_2$).

21.47. **THINK:** Identical point charges $Q = 32 \cdot 10^{-6} C$ are placed at each of the four corners of a rectangle of dimensions $L = 2.0$ m by $W = 3.0$ m. Find the magnitude of the electrostatic force on any one of the charges. Note that by symmetry the magnitude of the net force on each charge is equal. Choose to compute the net electrostatic force on Q_4.

SKETCH:



RESEARCH: The magnitude of the force between two charges is $\vec{F_{12}} = \left(kq_1q_2 / \left|\vec{r_{21}}\right|^2\right)\hat{r}_{21}$. The total force on a charge is the sum of all the forces acting on that charge. The magnitude of the force is found from $F = \left(F_x^{\,2} + F_y^{\,2}\right)^{1/2}$, where the components F_x and F_y can be considered one at a time.

SIMPLIFY: x-component: $F_x = F_{14,x} + F_{24,x} + F_{34,x} = \dfrac{kQ^2}{W^2} + \dfrac{kQ^2}{W^2 + L^2}\cos\theta + 0 = kQ^2\left(\dfrac{1}{W^2} + \dfrac{W}{\left(W^2 + L^2\right)^{3/2}}\right)$

y-component: $F_y = F_{14,y} + F_{24,y} + F_{34,y} = 0 + \dfrac{kQ^2}{W^2 + L^2}\sin\theta + \dfrac{kQ^2}{L^2} = kQ^2\left(\dfrac{W}{\left(W^2 + L^2\right)^{3/2}} + \dfrac{1}{L^2}\right)$

$F_{\text{net}} = \sqrt{F_x^{\,2} + F_y^{\,2}}$

CALCULATE: $F_x = \left(8.99 \cdot 10^9\text{ N m}^2 / C^2\right)(32 \cdot 10^{-6}\text{ C})^2\left(\dfrac{1}{\left(3.0\text{ m}\right)^2} + \dfrac{3.0\text{ m}}{\left[\left(3.0\text{ m}\right)^2 + \left(2.0\text{ m}\right)^2\right]^{3/2}}\right) = 1.612\text{ N}$

$F_y = \left(8.99 \cdot 10^9\text{ N m}^2 / C^2\right)(32 \cdot 10^{-6}\text{ C})^2\left(\dfrac{2.0\text{ m}}{\left[\left(3.0\text{ m}\right)^2 + \left(2.0\text{ m}\right)^2\right]^{3/2}} + \dfrac{1}{\left(2.0\text{ m}\right)^2}\right) = 2.694\text{ N}$

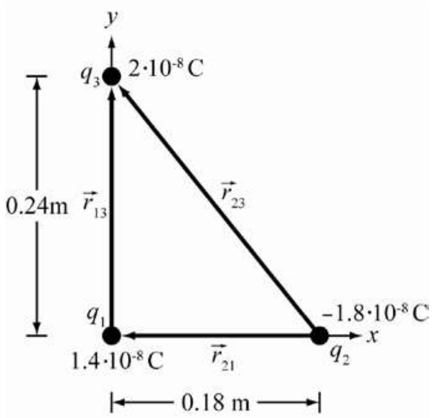
$F_{\text{net}} = \sqrt{\left(1.612\text{ N}\right)^2 + \left(2.694\text{ N}\right)^2} = 3.1397\text{ N}$

ROUND: Since each given value has 2 significant figures, $F_{\text{net}} = 3.1$ N

DOUBLE-CHECK: Since L is less than W, the y-component of F_{net} should be greater than the x-component.

21.48. **THINK:** Charge $q_1 = 1.4\cdot 10^{-8}C$ is at $r_1 = (0,0)$. Charge $q_2 = -1.8\cdot 10^{-8}C$ is at $r_2 = (0.18\text{ m},0\text{ m})$, and charge $q_3 = 2.1\cdot 10^{-8}C$ is at $r_3 = (0\text{ m},0.24\text{ m})$. Determine the net force (magnitude and direction) F_3 on charge q_3.

SKETCH:



RESEARCH: The magnitude of the force between two charges is $\overrightarrow{F_{12}} = kq_1q_2\hat{r}_{12} / \left|\overrightarrow{r_{12}}\right|^2 = kq_1q_2\overrightarrow{r_{12}} / r_{12}^{\ 3}$. The total force on charge q_3 is the sum of all the forces acting on it. The magnitude of F_3 is found from $F_3 = \left(F_1^2 + F_2^2\right)^{1/2}$, and the direction θ is found from $\theta = \tan^{-1}\left(F_y / F_x\right)$.

SIMPLIFY: $\vec{F}_{\text{net},3} = \vec{F}_{13} + \vec{F}_{23}$

$$= \frac{kq_1q_3\overrightarrow{r_{13}}}{r_{13}^{\ 3}} + \frac{kq_2q_3\overrightarrow{r_{23}}}{r_{23}^{\ 3}}$$

$$= \frac{kq_1q_3\left[(x_3-x_1)\hat{x} + (y_3-y_1)\hat{y}\right]}{\left[(x_3-x_1)^2 + (y_3-y_1)^2\right]^{3/2}} + \frac{kq_2q_3\left[(x_3-x_2)\hat{x} + (y_3-y_2)\hat{y}\right]}{\left[(x_3-x_2)^2 + (y_3-y_2)^2\right]^{3/2}}$$

$$= \frac{kq_1q_3}{y_3^{\ 3}}\left(y_3\hat{y}\right) + \frac{kq_2q_3}{(x_2^{\ 2} + y_3^{\ 2})^{3/2}}\left(-x_2\hat{x} + y_3\hat{y}\right)$$

CALCULATE: $$\vec{F}_{\text{net},3} = \frac{\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)(1.4\cdot 10^{-8}\text{ C})(2.1\cdot 10^{-8}\text{ C})\left(0.24\text{ m}\right)}{\left(0.24\text{ m}\right)^3}\hat{y}$$

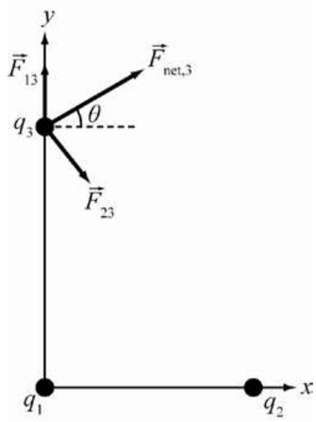
$$+\frac{\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)(-1.8\cdot 10^{-8}\text{ C})(2.1\cdot 10^{-8}\text{ C})\left(-0.18\hat{x}\text{ m} + 0.24\hat{y}\text{ m}\right)}{\left[\left(0.18\text{ m}\right)^2 + \left(0.24\text{ m}\right)^2\right]^{3/2}}$$

$$= (4.5886\cdot 10^{-5}\text{ N})\hat{y} + (2.265\cdot 10^{-5}\text{ N})\hat{x} - (3.0206\cdot 10^{-5}\text{ N})\hat{y}$$

$$= \left(2.265\cdot 10^{-5}\text{ N}\right)\hat{x} + \left(1.568\cdot 10^{-5}\text{ N}\right)\hat{y}$$

$$F_{\text{net},3} = \sqrt{F_x^{\ 2} + F_y^{\ 2}} = \sqrt{(2.265\cdot 10^{-5}\text{ N})^2 + (1.568\cdot 10^{-5}\text{ N})^2} = 2.755\cdot 10^{-5}\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1.568\cdot 10^{-5}\text{ N}}{2.265\cdot 10^{-5}\text{ N}}\right) = 34.69^\circ \text{ above the horizontal}$$

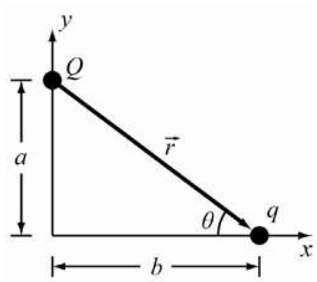


ROUND: With 2 significant figures in each given value, the final answers should be rounded to $\vec{F}_{\text{net},3}=\left(2.265\cdot 10^{-5}\text{ N}\right)\hat{x}+\left(1.568\cdot 10^{-5}\text{ N}\right)\hat{y}=2.8\cdot 10^{-5}\text{ N}$ and $\theta=35°$.

DOUBLE-CHECK: Due to the attraction between q_2 and q_3 and that q_1 is directly underneath q_3, the x component of $\vec{F}_{\text{net},3}$ has to be positive.

21.49. **THINK:** A positive charge Q is on the y-axis at a distance a from the origin and another positive charge q is on the x-axis at a distance b from the origin. (a) Find the value(s) of b for which the x-component of the force on q is a minimum. (b) Find the value(s) of b for which the x-component of the force on q is a maximum.

SKETCH:



RESEARCH: The electrostatic force is $F=kqQr/\left|r\right|^2$. The x-component of this force is $F_x=(kqQ/r^2)\cos\theta$. The values of b for which F_x is a minimum can be determined by inspection; the values of b for which F_x is a maximum can be found by calculating the extrema of F_x, that is, taking the derivative of F_x with respect to b, setting it to zero, and solving for b.

SIMPLIFY: $F_x=\dfrac{kqQ}{r^2}\cos\theta=\dfrac{kqQb}{r^3}=\dfrac{kqQb}{\left(a^2+b^2\right)^{3/2}}$

a) Minima: By inspection, the least possible value of F_x is zero, and this is attained only when $b=0$.

b) Maxima: $\dfrac{dF_x}{db}=0 \;\Rightarrow\; \dfrac{kqQ}{\left(a^2+b^2\right)^{3/2}}-\dfrac{3}{2}kqQ\left(a^2+b^2\right)^{-5/2}2b=0 \;\Rightarrow\; \dfrac{kqQ\left(a^2+b^2\right)-3kqQb^2}{\left(a^2+b^2\right)^{5/2}}=0$

$$\Rightarrow\; \left(a^2+b^2\right)-3b^2=0 \;\Rightarrow\; b=\pm\frac{a}{\sqrt{2}}$$

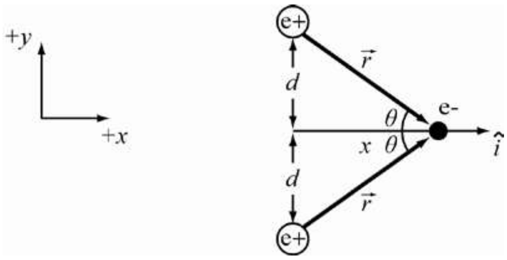
CALCULATE: Reject the negative solution, since distances have to be positive: $b=\dfrac{a}{\sqrt{2}}.$

ROUND: Not applicable

DOUBLE-CHECK: It makes sense that the possible values of b should be symmetrically distributed about the origin (above which lies the charge Q).

21.50. **THINK:** Two protons are placed near one electron as shown in the figure provided. Determine the electrostatic force on the electron. The charge of the electron is $q_e = -e$ and the charge of each proton is $q_p = e$, where $e = 1.602 \cdot 10^{-19}$ C.

SKETCH:



RESEARCH: By symmetry the forces in the vertical direction cancel. The force is therefore due solely to the horizontal contribution $F\cos\theta$ in the $\hat{x}$ direction: the Coulomb force is $F_{21} = kq_1q_2 / r_{21}^2$.

SIMPLIFY: By symmetry, and with the two protons, $\vec{F} = 2F_{pe}\cos\theta\hat{x} = -2\frac{ke^2}{r^2}\frac{x}{r}\hat{x} = -2\frac{ke^2 x}{\left(x^2 + d^2\right)^{3/2}}\hat{x}.$

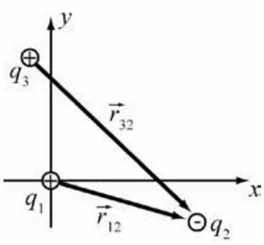
CALCULATE: $\vec{F} = -2\frac{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(1.602 \cdot 10^{-19} \text{ C}\right)^2\left(0.0700 \text{ m}\right)}{\left[\left(0.0700 \text{ m}\right)^2 + \left(0.0500 \text{ m}\right)^2\right]^{3/2}}\hat{x} = (-5.0742 \cdot 10^{-26} \text{ N})\hat{x}$

ROUND: $\vec{F} = \left(-5.07 \cdot 10^{-26} \text{ N}\right)\hat{x}$

DOUBLE-CHECK: This is a reasonable force as the charges are as small as they can possibly be and the separation is large.

21.51. **THINK:** The positions of the three fixed charges are $q_1 = 1.00$ mC at $r_1 = (0,0)$, $q_2 = -2.00$ mC at $r_2 = (17.0 \text{ mm}, -5.00 \text{ mm})$, and $q_3 = +3.00$ mC at $r_3 = (-2.00 \text{ mm}, 11.0 \text{ mm})$. Find the net force on the charge q_2.

SKETCH:



RESEARCH: The magnitude force is $\overrightarrow{F_{12}} = kq_1q_2\hat{r}_{12} / \left|\overrightarrow{r_{12}}\right|^2 = kq_1q_2\overrightarrow{r_{12}} / r_{12}^{\,3}$. The net force on q_2 is the sum of all the forces acting on q_2.

SIMPLIFY: $\vec{F}_{\text{net},2} = \vec{F}_{12} + \vec{F}_{32} = kq_2\left(\frac{q_1\left[(x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y}\right]}{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]^{3/2}} + \frac{q_3\left[(x_2 - x_3)\hat{x} + (y_2 - y_3)\hat{y}\right]}{\left[(x_2 - x_3)^2 + (y_2 - y_3)^2\right]^{3/2}}\right)$

CALCULATE: Without units,

$$\vec{F}_{\text{net},2} = \left(8.99\cdot 10^{9}\right)(-2.00)\left[\frac{(1.00)\left(17.0\hat{x}-5.00\hat{y}\right)}{\left[\left(17.0\right)^{2}+\left(-5.00\right)^{2}\right]^{3/2}}+\frac{(3.00)(19.0\hat{x}-16.0\hat{y})}{\left[\left(19.0\right)^{2}+\left(-16.0\right)^{2}\right]^{3/2}}\right]$$
$$= -1.2181\cdot 10^{8}\,\hat{x}+7.2469\cdot 10^{7}\,\hat{y}.$$

Then, the units of $\vec{F}_{\text{net},2}$ are:

$$\left[\vec{F}_{\text{net},2}\right] = \left(\text{N m}^2/\text{C}^2\right)(\text{mC})\left[\frac{(\text{mC})(\text{mm}-\text{mm})}{\left[\left(\text{mm}\right)^{2}+\left(\text{mm}\right)^{2}\right]^{3/2}}+\frac{(\text{mC})(\text{mm}-\text{mm})}{\left[\left(\text{mm}\right)^{2}+\left(\text{mm}\right)^{2}\right]^{3/2}}\right] = \text{N}$$

Altogether , $\vec{F}_{\text{net},2} = \left(-1.2181\cdot 10^{8}\text{ N}\right)\hat{x}+\left(7.2469\cdot 10^{7}\text{ N}\right)\hat{y}.$ The magnitude of the force is

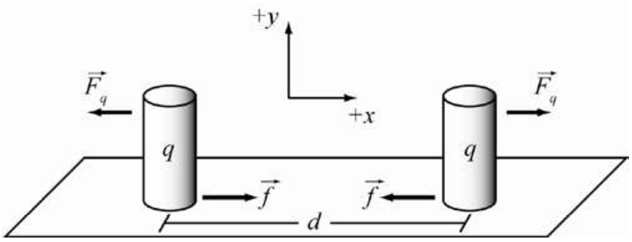
$$F_{\text{net},2} = \sqrt{F_x^{\,2}+F_y^{\,2}} = \sqrt{\left(-1.2181\cdot 10^{8}\text{ N}\right)^{2}+\left(7.2469\cdot 10^{7}\text{ N}\right)^{2}} = 1.4174\cdot 10^{8}\text{ N}$$

ROUND: $\vec{F}_{\text{net},2} = \left(-1.22\cdot 10^{8}\text{ N}\right)\hat{x}+\left(7.25\cdot 10^{7}\text{ N}\right)\hat{y}$ and $\left|\vec{F}_{\text{net},2}\right| = 1.42\cdot 10^{8}$ N.

DOUBLE-CHECK: The charges are large and the separation distance are small, so $F_{\text{net},2}$ should be very strong.

21.52. **THINK:** the masses of the beads are $m = 10.0\text{ mg} = 1.00\cdot 10^{-5}$ kg and they have identical charge. They are a distance $d = 0.0200$ m apart. The coefficient of static friction between the beads and the surface is $\mu = 0.200.$ Find the minimum charge q needed for the beads to start moving.

SKETCH:



RESEARCH: Assume the surface is parallel to the surface of the Earth. The frictional force is $f = \mu N,$ where $N = mg$. The electrostatic force is $F = kq^2/d^2$. The beads will start to move as soon as F is greater than f, enabling one bead to move away from the other. Then the minimum charge q can be found by equating f and F.

SIMPLIFY: $F = f \Rightarrow \dfrac{kq^2}{d^2} = \mu mg \Rightarrow q = \sqrt{d^2\mu mg/k}$

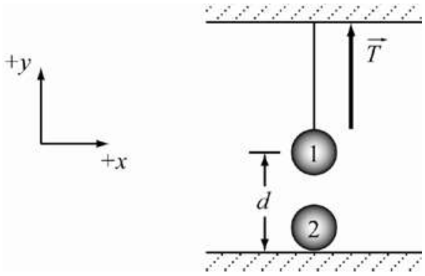
CALCULATE: $q = \sqrt{\dfrac{\left(0.0200\text{ m}\right)^{2}(0.200)(1.00\cdot 10^{-5}\text{ kg})(9.81\text{ m/s}^2)}{\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)}} = 9.3433\cdot 10^{-10}\text{ C}$

ROUND: All of the given values have three significant figures, so $q = 9.34\cdot 10^{-10}$ C.

DOUBLE-CHECK: The units of the solution are those of charge. This is a reasonable charge required to overcome the frictional force.

21.53. **THINK:** The ball's mass is $m_1 = 0.0300$ kg; its charge is $q_1 = -0.200\ \mu C.$ The ball is suspended a distance of $d = 0.0500$ m above an insulating floor. The second small ball has mass $m_2 = 0.0500$ kg and a charge $q_2 = 0.400\ \mu\text{C}.$ Determine if the second ball leaves the floor. Find the tension T in the string when the second ball is directly beneath the first ball. Because the balls are small, we will treat them as point charges with radius zero.

SKETCH:



RESEARCH: The electrostatic force between two charges is $F = kq_1q_2 / r^2$. The force of gravity is $F_g = mg$. The ball will leave the floor if the electrostatic force between the two balls is greater that the force of gravity, that is if $F > F_g$, and if the charges are opposite. The tension in the rope can be found by considering all of the vertical forces acting on the first ball.

SIMPLIFY: The electrostatic force is: $F = kq_1q_2 / d^2$. The gravitational force is: $F_g = m_2(-g)$. The forces acting on m_1 in the y-direction sum to: $0 = T - F_{\text{coulomb}} - m_1g$. So the tension is $T = F_{\text{coulomb}} + m_1g$.

CALCULATE: $F = \dfrac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(-0.200\cdot 10^{-6}\text{ C})(0.400\cdot 10^{-6}\text{ C})}{(0.0500\text{ m})^2} = -0.28768\text{ N},$

$F_g = (0.0500\text{ kg})(-9.81\text{ m/s}^2) = -0.4905\text{ N},\qquad T = -0.28768\text{ N} + (0.0300\text{ kg})(-9.81\text{ m/s}^2) = -0.58198\text{ N}.$

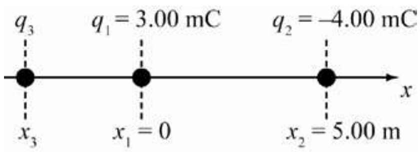
Since $F_g > F$, the second ball does not leave the ground.

ROUND: With all given values containing three significant figures, round the tension to $T = -0.582$ N.

DOUBLE-CHECK: The balls are not quite close enough to overcome the force of gravity, but the magnitude of F_{coulomb} is comparable to F_g, despite the small charges (on the order of 10^{-7} C).

21.54. **THINK:** A $q_1 = +3.00$ mC charge and a $q_2 = -4.00$ mC charge are fixed in position and separated by $d = 5.00$ m. Take the position of q_1 to be at $x_1 = 0$, and position of q_2 to be at $x_2 = 5.00$ m. (a) Find the location, x_3, of a $q_3 = +7.00$ mC charge so that the net force on it is zero. (b) Find the location, x_3', of a $q_3 = -7.00$ mC charge so that the net force on it is zero.

SKETCH:



RESEARCH: The electrostatic force between two charges is $F = kq_1q_2 / r^2$. The net force on a third charge is zero: $F_{\text{net},3} = F_{13} + F_{23} = 0 \Rightarrow F_{13} = -F_{23}$. The two forces must be equal in magnitude, but opposite in direction. Consider the following three possible locations for the charge q_3. Note that this analysis is independent of the charge of q_3: At $x_3 > 5.00$ m, the two forces F_{13} and F_{23} will be opposite in direction but they cannot be equal in magnitude: the charge q_2 at $x_2 = 5.00$ m is greater in magnitude than the charge q_1 at $x_1 = 0$ and x_3 would be closer to x_2. (Remember that the electrostatic force increases as the distance between the charges decreases.) This makes the magnitude of F_{23} greater than that of F_{13}. Next, consider values of x_3 satisfying: $0\text{ m} < x_3 < 5.00$ m. The two forces are in the same direction and therefore cannot balance. At $x_3 < 0$ m, the two forces are opposite in direction, and in direct opposition to the first situation, the force F_{13} and F_{23} can now be balanced. The solution will have a negative position, or more accurately, the third charge q_3 must be placed near the smaller fixed charge, q_1, without being between the two fixed charges q_1 and q_2. This answer is independent of the charge of q_3, therefore the numeric answer to parts a and b is the same.

SIMPLIFY: With $x_3 < 0$, and $\hat{F}_{13}$ opposite in direction to $\hat{F}_{23}$, the force are balanced when

$$F_{13} = -F_{23} \Rightarrow \frac{kq_1q_3}{|x_3|^2} = -\frac{kq_2q_3}{(x_2 - x_3)^2} \Rightarrow q_1(x_2 - x_3)^2 = -q_2x_3^2 \Rightarrow (q_1 + q_2)x_3^2 - 2q_1x_2x_3 + q_1x_2^2 = 0.$$

Solving for x_3:

$$x_3 = \frac{2q_1x_2 \pm \sqrt{4q_1^2x_2^2 - 4(q_1 + q_2)q_1x_2^2}}{2(q_1 + q_2)}.$$

CALCULATE:

$$x_3 = \frac{2(3.00 \text{ mC})(5.00 \text{ m}) \pm \text{mC m}\sqrt{4(3.00)^2(5.00)^2 - 4(3.00 - 4.00)(3.00)(5.00)^2}}{2(3.00 \text{ mC} - 4.00 \text{ mC})} = -32.321 \text{ m}, 2.305 \text{ m}$$

By the convention established in this solution, x_3 is negative. (The second solution places q_3 a between q_1 and q_2, a possibility which has been ruled out.)

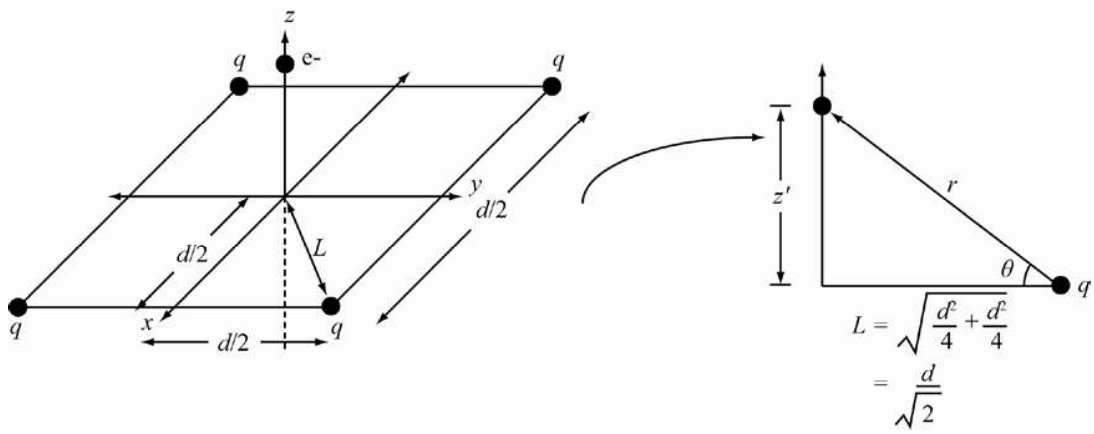
ROUND: All given values have three significant figures, so $x_3 = -32.3$ m.

DOUBLE-CHECK: Inserting the calculated value of x_3 back into the expressions for the Coulomb force:

$$F_{13} = \frac{kq_1q_3}{x_3^2} = \frac{k(3.00 \text{ mC})(7.00 \text{ mC})}{(-32.3 \text{ m})^2} = 181 \text{ N and } F_{23} = \frac{kq_2q_3}{(x_2 - x_3)^2} = \frac{k(-4.00 \text{ mC})(7.00 \text{ mC})}{(5.00 \text{ m} + 32.3 \text{ m})^2} = -181 \text{ N}.$$

21.55. **THINK:** Four point charges, each with charge q, are fixed to the four corners of a square with a sides of length $d = 1.00$ cm. An electron is suspended above a point at which its weight is balanced by the electrostatic force due to the four electrons: $z' = 15.0$ nm above the center of the square. The mass of an electron is $m_e = 9.109 \cdot 10^{-31}$ kg, and the charge is $q_e = -e = -1.602 \cdot 10^{-19}$ C. Find the value of q of the fixed charges, in Coulombs and as a multiple of the electron charge.

SKETCH:



RESEARCH: The electrostatic force between two charges is $F = kq_1q_2 / r^2$. By symmetry, the net force in the horizontal direction is zero, and the problem reduces to a balance of the forces in the vertical direction, with one fixed charge balancing a quarter of the electron's weight. The vertical component of the electrostatic force is $F\sin\theta$. The weight of the electron is $W = m_eg$.

SIMPLIFY: Balancing the forces in the vertical (z) direction yields $F_{\text{coulomb}} = \frac{1}{4}W \Rightarrow \frac{kqq_e}{r^2}\sin\theta = \frac{1}{4}m_eg.$

Solving for q: $q=\frac{1}{4}\frac{m_e gr^2}{kq_e\sin\theta}=\frac{m_e gr^3}{4kq_e z'}=\frac{m_e g(L^2+z'^2)^{3/2}}{4kq_e z'}=\frac{-m_e g\left(\frac{d^2}{2}+z'^2\right)^{3/2}}{4kez'}.$

CALCULATE: $q=\frac{-\left(9.109\cdot 10^{-31}\text{ kg}\right)(9.81\text{ m/s}^2)\left[\frac{(0.100\text{ m})^2}{2}+(15.0\cdot 10^{-9}\text{ m})^2\right]^{3/2}}{4\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)(1.602\cdot 10^{-19}\text{ C})(15.0\cdot 10^{-9}\text{ m})}$

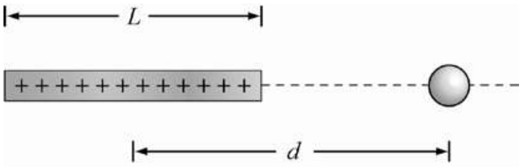
$=-3.6561\cdot 10^{-17}$ C, or $-228.22e$.

ROUND: With three significant figures in z', $q=-3.66\cdot 10^{-17}\text{ C}=-228e$.

DOUBLE-CHECK: The gravitational force on an electron is small. Each charge q needs to be a few hundred electron charges to balance the gravitational force on the electron.

21.56. **THINK:** A uniformly charged thin rod of length L has a total charge Q. Find an expression for the electrostatic force strength acting on an electron, whose magnitude of charge is e, is positioned on the axis of the rod at distance d from the center.

SKETCH:



RESEARCH: The electrostatic force between two charges is $F=kqQ/r^2$. The net electrostatic force acting on a charge q is the sum of all the electrostatic forces acting on q. In the event of a continuous and linear distribution of charge of length L and total charge Q, the force due to an infinitesimal amount of charge dq' from the distribution acting on the charge q is: $dF=kq\,dq'/x^2$, where $dq'=(Q/L)dx=\lambda dx$. (λ is the linear charge density.) In this case, the total force on the electron is then

$$F=\int_{d-L/2}^{d+L/2}\frac{ke\lambda}{x^2}dx,$$

where the integration runs over the length of the rod, starting from the point closest to the electron $\left(d-L/2\right)$ and ending with the point farthest from the electron $\left(d+L/2\right)$.

SIMPLIFY:

$$F=\int_{d-L/2}^{d+L/2}\frac{ke\lambda}{x^2}dx=ke\lambda\int_{d-L/2}^{d+L/2}\frac{1}{x^2}dx=ke\lambda\left(-x\Big|_{d-L/2}^{d+L/2}\right)=2ke\lambda\left(\frac{1}{2d-L}-\frac{1}{2d+L}\right)=\frac{4ke\lambda L}{4d^2-L^2}=\frac{4keQ}{4d^2-L^2}$$

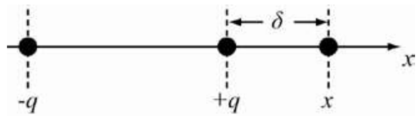
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The answer is in the correct units of force: $\left[F\right]=\frac{\left(\frac{\text{N m}^2}{\text{C}^2}\right)(\text{C})(\text{C})}{\text{m}^2}=\text{N}.$

21.57. **THINK:** A negative charge $-q$ is located and fixed at $(0, 0)$. A positive charge $+q$ is initially at $(x, 0)$. The positive charge will accelerate towards the negative charge. Use the binomial expansion to show that when the positive charge moves a distance $\delta \ll x$ closer to the negative charge, the force on it increases by $\Delta F=2kq^2\delta/x^3$.

SKETCH:



RESEARCH: The Coulomb force is $\overrightarrow{F_{21}} = kq_1q_2\hat{r}_{21} / r_{21}^{\ 2}$, where $\hat{r}_{21}$ is the unit vector that points from charge 2 to charge 1. To first order, the binomial expansion is $(1+x)^n \approx 1+nx$ for $|x| << 1$.

SIMPLIFY: The initial force on $+q$ (when it was at $(x,\ 0)$ was $\vec{F} = -\frac{kq^2}{x^2}\hat{x}$. After moving closer to $-q$ by δ, the new force on $+q$ is $\vec{F}' = -\frac{kq^2}{(x-\delta)^2}\hat{x} = -\frac{kq^2}{x^2\left(1-\frac{\delta}{x}\right)^2}\hat{x} = -\frac{kq^2}{x^2}\left(1-\frac{\delta}{x}\right)^{-2}\hat{x}$. Using the binomial expansion, $\vec{F}' = -\frac{kq^2}{x^2}\left(1-(-2)\frac{\delta}{x}+...\right)\hat{x} \approx -\frac{kq^2}{x^2}\left(1+2\frac{\delta}{x}\right)\hat{x}$ (to first order in δ). Then, $\Delta\vec{F} = \vec{F}' - \vec{F} \approx -\frac{2kq^2\delta}{x^3}\hat{x}$ and $\Delta F = \frac{2kq^2\delta}{x^3}$, as desired.

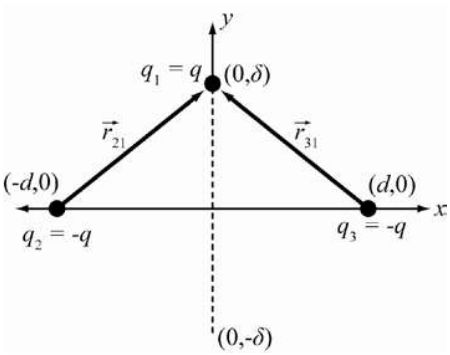
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The charge in force has the correct units for force: $\left[\Delta F\right] = \frac{\frac{\text{N m}^2}{\text{C}^2}\text{C}\frac{\text{C}}{\text{m}}\text{m}}{\text{m}^2} = \text{N}.$

21.58. **THINK:** Two charges, both $-q$, are located and fixed at coordinates $(-d,0)$ and $(d,0)$ in the x-y plane. A positive charge of the same magnitude q and of mass m is placed at coordinate $(0,0)$. The positive charge is then moved a distance $\delta \ll d$ along the +y direction and then released. It will oscillate between coordinates $(0,\delta)$ and $(0,-\delta)$. Find the net force F_{net} acting on the positive charge when it is moved to $(0,\delta)$ and use the binomial expansion to find an expression for the frequency of the resulting oscillation.

SKETCH:



RESEARCH: The Coulomb force is $\overrightarrow{F_{21}} = kq_1q_2\hat{r}_{21} / r_{21}^{\ 2}$, where $\overrightarrow{F_{21}}$ is the force on the charge 1 by charge 2, and $\hat{r}_{21}$ points from charge 2 to charge 1. To first order, the binomial expansion is, in general, $(1+x)^n \approx 1+nx$ for $x \ll 1$. The restoring force of a simple harmonic oscillator obeys Hooke's Law, $F = -\omega^2 mx$, where ω is the characteristic angular frequency, and $f = \omega / (2\pi)$.

SIMPLIFY: $\vec{F}_{\text{net}}=\frac{kq_1q_2\vec{r}_{21}}{r_{21}^{\ 3}}+\frac{kq_1q_3\vec{r}_{31}}{r_{31}^{\ 3}}=\frac{-kq^2}{(d^2+\delta^2)^{3/2}}(d\hat{x}+\delta\hat{y})-\frac{kq^2}{(d^2+\delta^2)^{3/2}}(-d\hat{x}+\delta\hat{y})=\frac{-2kq^2\delta}{(d^2+\delta^2)^{3/2}}\hat{y}$

$$=-\frac{2kq^2\delta}{d^3\left(1+\frac{\delta^2}{d^2}\right)^{3/2}}\hat{y}=-\frac{2kq^2\delta}{d^3}\left(1+\frac{\delta^2}{d^2}\right)^{-3/2}\hat{y}$$

Note the binomial expansion of $\left(1+\delta^2/d^2\right)^{-3/2}\approx1-\left(3/2\right)\left(\delta^2/d^2\right)$. Neglecting the term δ^2/d^2 (keeping only terms linear in δ), the net force is $\vec{F}_{\text{net}}\approx-2\left(kq^2\delta/d^3\right)\hat{y}$. Then from $F=-\omega^2mx$, $\omega=\sqrt{-F/\left(mx\right)}$ with $x=\delta$, the angular frequency is $\omega=\sqrt{2kq^2\delta/\left(md^3\delta\right)}=\sqrt{2kq^2/\left(md^3\right)}=\frac{q}{d}\sqrt{\frac{2k}{md}}$ and the frequency is $f=\frac{q}{2\pi d}\sqrt{\frac{2k}{md}}=\frac{q}{\pi d}\sqrt{\frac{k}{2md}}$.

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The frequency of oscillation should depend directly on the magnitude of the charges and inverse on the distance separating the charges. This lends support to the formulas found above.

21.59. The gravitational force between the Earth and Moon is given by $F_g=GM_{\text{Earth}}m_{\text{Moon}}/r_{\text{EM}}^{\ 2}$. The static electrical force between the Earth and the Moon is $F=kQ^2/r_{\text{EM}}^{\ 2}$, where Q is the magnitude of the charge on each the Earth and the Moon. If the static electrical force is 1.00% that of the force of gravity, then the charge Q would be:

$$F=0.01F_g\ \Rightarrow\ \frac{kQ^2}{r_{\text{EM}}^{\ 2}}=\frac{0.0100GM_{\text{Earth}}m_{\text{Moon}}}{r_{\text{EM}}^{\ 2}}\ \Rightarrow\ Q=\sqrt{\frac{0.0100GM_{\text{Earth}}m_{\text{Moon}}}{k}}.$$

This gives $Q=\sqrt{\frac{0.0100(6.67\cdot10^{-11}\text{ N m}^2/\text{kg})(5.97\cdot10^{24}\text{ kg})(7.36\cdot10^{22}\text{ kg})}{\left(8.99\cdot10^9\text{ N m}^2/\text{C}^2\right)}}=5.71\cdot10^{12}\text{ C}.$

21.60. The gravitational force between the Earth and Moon is given by $F_g=GM_{\text{Earth}}m_{\text{moon}}/r_{\text{EM}}^{\ 2}$. If this is due solely to static electrical force between the Earth and Moon, the magnitude of Q would be:

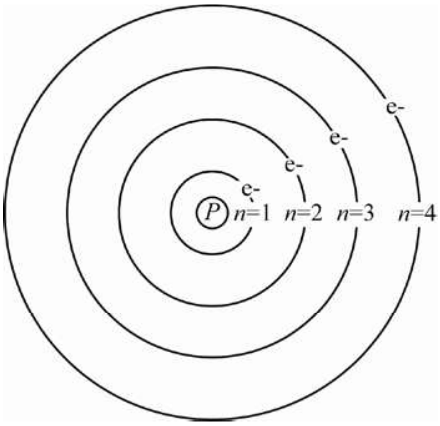
$$F_g=-G\frac{M_{\text{Earth}}m_{\text{Moon}}}{r_{\text{EM}}^{\ 2}}=-k\frac{Q^2}{r_{\text{EM}}^{\ 2}}\ \Rightarrow\ Q=\sqrt{\frac{GM_{\text{Earth}}m_{\text{Moon}}}{k}}.$$

So, $Q=\sqrt{\frac{(6.67\cdot10^{-11}\text{ N m}^2/\text{kg})(5.97\cdot10^{24}\text{ kg})(7.36\cdot10^{22}\text{ kg})}{\left(8.99\cdot10^9\text{ N m}^2/\text{C}^2\right)}}=5.71\cdot10^{13}\text{ C}.$

This is a large amount of charge, on the order of 10^{31} electrons worth of charge. This is equivalent to about 60 million moles of electrons.

21.61. **THINK:** The radii of the electron orbits are $r_n=n^2a_B$, where n is an integer (not 0) and $a_B=5.29\cdot10^{-11}$ m. Calculate the electrostatic force between the electron (charge $-e$ and mass $m_e=9.109\cdot10^{-31}$ kg) and the proton (charge e and mass $m_p=1.673\cdot10^{-27}$ kg) for the first 4 orbits and compare them to the gravitational interaction between the two. Note $e=1.602\cdot10^{-19}C$.

SKETCH:



RESEARCH: The Coulomb force is $F = k|q_1||q_2|/r^2$, or $F_n = ke^2/r_n^2$ in this case. The gravitational force is $F_g = Gm_1m_2/r^2$, or $F_{g,n} = Gm_em_p/r_n^2$.

SIMPLIFY: $n=1:\ F_1 = \frac{ke^2}{a_B^2};\ F_{g,1} = \frac{Gm_em_p}{a_B^2}, n=2:\ F_2 = \frac{ke^2}{(4a_B)^2};\ F_{g,2} = \frac{Gm_em_p}{(4a_B)^2}$

$n=3:\ F_3 = \frac{ke^2}{(9a_B)^2};\ F_{g,3} = \frac{Gm_em_p}{(9a_B)^2}, n=4:\ F_4 = \frac{ke^2}{(16a_B)^2};\ F_{g,4} = \frac{Gm_em_p}{(16a_B)^2}$

CALCULATE: Note that: $\frac{ke^2}{a_B^2} = \frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(1.602\cdot 10^{-19} \text{ C})^2}{\left(5.29\cdot 10^{-11} \text{ m}\right)^2} = 8.2465\cdot 10^{-8}\text{N}$ and

$\frac{Gm_em_p}{a_B^2} = \frac{(6.67\cdot 10^{-11} \text{ N m}^2/\text{kg})(9.109\cdot 10^{-31} \text{ kg})(1.673\cdot 10^{-27} \text{ kg})}{\left(5.29\cdot 10^{-11} \text{ m}\right)^2} = 3.632\cdot 10^{-47}\text{N}.$

Then for $n=1:\ F_1 = \frac{ke^2}{a_B^2} = 8.2465\cdot 10^{-8}\text{N};\ F_{g,1} = \frac{Gm_em_p}{a_B^2} = 3.6342\cdot 10^{-47}\text{N}$

$n=2:\ F_2 = \frac{ke^2}{\left(4a_B\right)^2} = 5.1515\cdot 10^{-9}\text{N};\ F_{g,2} = \frac{Gm_em_p}{\left(4a_B\right)^2} = 2.2712\cdot 10^{-48}\text{N}$

$n=3:\ F_3 = \frac{ke^2}{\left(9a_B\right)^2} = 1.1081\cdot 10^{-9}\text{N};\ F_{g,3} = \frac{Gm_em_p}{\left(9a_B\right)^2} = 4.4863\cdot 10^{-49}\text{N}$

$n=4:\ F_4 = \frac{ke^2}{\left(16a_B\right)^2} = 3.2213\cdot 10^{-10}\text{N};\ F_{g,4} = \frac{Gm_em_p}{\left(16a_B\right)^2} = 1.4195\cdot 10^{-49}\text{N}$

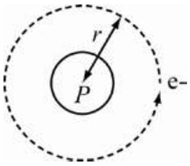
ROUND: Since a_B has three significant figures, $F_1 = 8.25\cdot 10^{-8}\text{N}$, $F_{g,1} = 3.63\cdot 10^{-47}\text{N}$, $F_2 = 5.15\cdot 10^{-9}\text{N}$, $F_{g,2} = 2.27\cdot 10^{-48}\text{N}$, $F_3 = 1.12\cdot 10^{-9}\text{N}$, $F_{g,3} = 4.49\cdot 10^{-49}\text{N}$, $F_4 = 3.22\cdot 10^{-10}\text{N}$, and $F_{g,4} = 1.42\cdot 10^{-49}\text{N}$. In every case the gravitational force between the proton and the electron is almost forty orders of magnitude smaller than the electrostatic force between them.

DOUBLE-CHECK: As n increases, the distance between the proton and the electron increases. Since each force follows an inverse-square law with respect to the distance, the forces decrease as n increases

21.62. **THINK:** The net force on the orbiting electron is the centripetal force, F_C. This is due to the electrostatic force between the electron and the proton, F. The radius of the hydrogen atom is $r = 5.29\cdot 10^{-11}$ m. The charge of an electron is $q_e = -e = -1.602\cdot 10^{-19}$ C, and the charge of a proton is $q_p = e = 1.602\cdot 10^{-19}$ C.

Find the velocity v and the kinetic energy K of the electron orbital. The mass of an electron is $m_{\text{e}} = 9.109 \cdot 10^{-31}$ kg.

SKETCH:



RESEARCH: The centripetal force is $F_{\text{C}} = m_{\text{e}} v^2 / r$. The electrostatic force is $F = k|q_1||q_2| / r^2$. The kinetic energy is $K = mv^2 / 2$.

SIMPLIFY: Solve for v^2:

$$F_{\text{C}} = F \;\Rightarrow\; \frac{m_{\text{e}} v^2}{r} = \frac{k|q_1||q_2|}{r^2} = \frac{ke^2}{r^2} \Rightarrow v^2 = \frac{ke^2}{rm_{\text{e}}}$$

Find K: $K = \frac{1}{2} m_{\text{e}} v^2 = \frac{1}{2} m_{\text{e}} \left(\frac{ke^2}{rm_{\text{e}}} \right) = \frac{ke^2}{2r}$.

CALCULATE: $K = \dfrac{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)(1.602 \cdot 10^{-19} \text{ C})^2}{2(5.29 \cdot 10^{-11} \text{ m})} = 2.1807 \cdot 10^{-18} \text{ J} = 13.6077 \text{ eV}$

ROUND: $K = 13.6$ eV

DOUBLE-CHECK: Because the electron has very little mass, it is capable of approaching speeds on the order of $0.01c$ or $0.1c$ (where c is the speed of light). For the same reason, its kinetic energy is small (on the order of a few electron volts, in the case of the hydrogen atom).

21.63. For the atom described in the previous question, the ratio of the gravitational force between the electron and proton to the electrostatic force is:

$$F_{\text{g}} / F = \frac{\dfrac{Gm_{\text{e}} m_{\text{p}}}{r^2}}{\dfrac{k|q_1||q_2|}{r^2}} = \frac{Gm_{\text{e}} m_{\text{p}}}{ke^2}$$

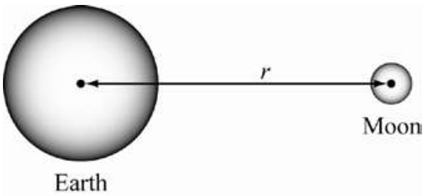
$$= \frac{\left(6.6742 \cdot 10^{-11} \text{m}^3 / (\text{kg s}^2)\right)(9.109 \cdot 10^{-31} \text{ kg})(1.673 \cdot 10^{-27} \text{ kg})}{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)(1.602 \cdot 10^{-19} \text{ C})^2}$$

$$= 4.41 \cdot 10^{-40}$$

This value is independent of the radius; if this radius is doubled, the ratio does not change.

21.64. **THINK:** The Earth and the Moon each have a charge $q = -1.00 \cdot 10^6$ C. Their masses are $m_{\text{E}} = 5.97 \cdot 10^{24}$ kg and $m_{\text{M}} = 7.36 \cdot 10^{22}$ kg, respectively. The distance between them is $r = 384{,}403$ km, center-to-center. (a) Compare their electrostatic repulsion, F, with their gravitational attraction, F_{g}. (b) Discuss the effects of the electrostatic force on the size, shape and stability of the Moon's orbit around the Earth.

SKETCH:



RESEARCH: Treat each object as a point particle. The electrostatic force is $F = k|q_1||q_2|/r^2$, and the gravitational force is $F_g = GMm/r^2$.

SIMPLIFY:

(a) $F = kq^2/r^2$; $F_g = GM_E m_M/r^2$

(b) Not applicable.

CALCULATE:

(a) $F = \dfrac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(-1.00\cdot 10^6 \text{ C})^2}{\left(3.84403\cdot 10^8 \text{ m}\right)^2} = 60839.6 \text{ N}$

$$F_g = \frac{\left(6.6742\cdot 10^{-11} \text{ m}^3/\left(\text{kg s}^2\right)\right)\left(5.9742\cdot 10^{24} \text{ kg}\right)\left(7.36\cdot 10^{22} \text{ kg}\right)}{\left(3.84403\cdot 10^8 \text{ m}\right)^2} = 1.986\cdot 10^{20} \text{ N}$$

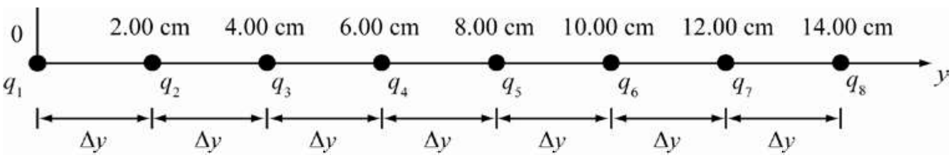
(b) The force of gravity is about 16 orders of magnitude greater than the electrostatic repulsion. The electrostatic force is an inverse-square central force. It therefore has no effect on the shape or stability of the Moon's orbit. It could only affect the size of the orbit, but given the orders of magnitude in difference between this and F_g, the effect is probably undetectable.

ROUND:

(a) $F = 6.08\cdot 10^4$ N and $F_g = 1.99\cdot 10^{20}$ N

DOUBLE-CHECK: F_g should greater than F, otherwise the Moon would not remain in the Earth's orbit.

21.65. Eight +1.00-μC charges are aligned on the y-axis with a distance $\Delta y = 2.00$ cm between each closest pair:



The force on the charge at $y = 4.00$ cm, q_3, is:

$$\vec{F}_{\text{tot},3} = \sum_{n=1,\, n\neq 3}^{8} \vec{F}_{n,3} = \vec{F}_{13} + \vec{F}_{23} + \vec{F}_{43} + \vec{F}_{53} + \vec{F}_{63} + \vec{F}_{73} + \vec{F}_{83} = (F_{13} + F_{23} - F_{43} - F_{53} - F_{63} - F_{73} - F_{83})\hat{y}$$

All terms have in common the factor $k|q_3|$. Then,

$$F_{\text{tot},3} = k|q_3|\left(\frac{|q_1|}{\left|\vec{y}_1 - \vec{y}_3\right|^2} + \frac{|q_2|}{\left|\vec{y}_2 - \vec{y}_3\right|^2} - \frac{|q_4|}{\left|\vec{y}_4 - \vec{y}_3\right|^2} - \frac{|q_5|}{\left|\vec{y}_5 - \vec{y}_3\right|^2} - \frac{|q_6|}{\left|\vec{y}_6 - \vec{y}_3\right|^2} - \frac{|q_7|}{\left|\vec{y}_7 - \vec{y}_3\right|^2} - \frac{|q_8|}{\left|\vec{y}_8 - \vec{y}_3\right|^2}\right)$$

Since $q_1 = q_2 = \ldots = q_8 = q$,

$$F_{\text{tot},3} = kq^2\left(\frac{1}{(2\Delta y)^2} + \frac{1}{(\Delta y)^2} - \frac{1}{(\Delta y)^2} - \frac{1}{(2\Delta y)^2} - \frac{1}{(3\Delta y)^2} - \frac{1}{(4\Delta y)^2} - \frac{1}{(5\Delta y)^2}\right)$$

$$\vec{F}_{\text{tot},3} = \frac{kq^2}{(\Delta y)^2}\left(\frac{1}{2^2} + 1 - 1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{5^2}\right)\hat{y}$$

$$= \frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(1.00\cdot 10^{-6} \text{ C}\right)^2}{\left(0.0200 \text{ m}\right)^2}\left(-\frac{769}{3600}\right)\hat{y}$$

$$= -4.80 \text{ N } \hat{y}$$

21.66. The distance between the electron (charge $q_e = -e$) and the proton (charge $q_p = e$) is $r = 5.29 \cdot 10^{-11}$ m. The net force on the electron is the centripetal force, $F_c = m_e a_c = m_e v^2 / r$. This is due to the Coulomb force, $F = k|q_1||q_2|/r^2$. That is, $F_c = F \Rightarrow m_e v^2 / r = k|q_1||q_2|/r^2$. The speed of the electron is:

$$m_e v^2 = \frac{ke^2}{r} \Rightarrow v = \sqrt{\frac{ke^2}{m_e r}} = \sqrt{\frac{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)(1.602 \cdot 10^{-19} \text{ C})^2}{(9.109 \cdot 10^{-31} \text{ kg})(5.29 \cdot 10^{-11} \text{ m})}} = 2.18816 \cdot 10^6 \text{ m/s} \approx 2.19 \cdot 10^6 \text{ m/s}.$$

21.67. The radius of the nucleus of ^{14}C is $r_0 = 1.505$ fm. The nucleus has charge $q_0 = +6e$.

(a) A proton (charge $q = e$) is placed $d = 3.00$ fm from the surface of the nucleus. Treating the nucleus as a point charge, the distance between the proton and the charge of the nucleus is $r = d + r_0$. The force is repulsive due to the like charges. The magnitude of this force is

$$F = \frac{k|q||q_0|}{r^2} = \frac{k6e^2}{(d+r_0)^2} = \frac{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)6(1.602 \cdot 10^{-19} \text{ C})^2}{\left(3.00 \cdot 10^{-15} \text{ m} + 1.505 \cdot 10^{-15} \text{ m}\right)^2} = 68.2097 \text{ N} \approx 68.2 \text{ N}$$

(b) The proton's acceleration is:

$$F = m_p a \Rightarrow a = \frac{F}{m_p} = \frac{68.210 \text{ N}}{1.673 \cdot 10^{-27} \text{ kg}} = 4.077 \cdot 10^{28} \text{ m/s}^2 \approx 4.08 \cdot 10^{28} \text{ m/s}^2$$

21.68. The original force is $F = k|q_1||q_2|/r^2 = 0.100$ N. Now q_1 becomes $(1/2)q_1$, while r becomes $2r$. The new force is:

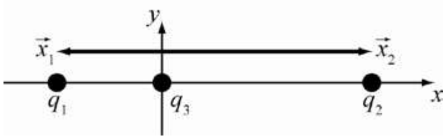
$$F' = \frac{k\left|\frac{1}{2}q_1\right||q_2|}{(2r)^2} = \frac{1}{8}\frac{k|q_1||q_2|}{r^2} = \frac{1}{8}F = \frac{1}{8}(0.100 \text{ N}) = 0.0125 \text{ N}$$

21.69. The charge and position of three point charges on the x-axis are:

$q_1 = +19.0$ μC; $\vec{x}_1 = -10.0$ cm

$q_2 = -57.0$ μC; $\vec{x}_2 = +20.0$ cm

$q_3 = -3.80$ μC; $\vec{x}_3 = 0$



The magnitude of the total electrostatic force on q_3 is:

$$F_{\text{tot},3} = \left|\vec{F}_{13} + \vec{F}_{23}\right| = \left|-F_{13} - F_{23}\right| = \left(F_{13} + F_{23}\right) = \frac{k|q_3||q_1|}{\left|\vec{x}_1 - \vec{x}_3\right|^2} + \frac{k|q_3||q_2|}{\left|\vec{x}_2 - \vec{x}_3\right|^2} = k|q_3|\left(\frac{|q_1|}{(x_1)^2} + \frac{|q_2|}{(x_2)^2}\right)$$

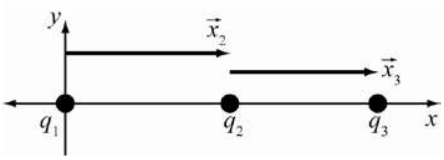
$$= \left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(|-3.80 \ \mu\text{C}|\right)\left(\frac{|19.0 \ \mu\text{C}|}{(0.100 \text{ m})^2} + \frac{|-57.0 \ \mu\text{C}|}{(0.200 \text{ m})^2}\right) = 113.59 \text{ N} \approx 114 \text{ N}$$

21.70. The charge and position of three point charges on the x-axis are:

$q_1 = +64.0$ μC; $\vec{x}_1 = 0.00$ cm

$q_2 = +80.0$ μC; $\vec{x}_2 = 25.0$ cm

$q_3 = -160.0$ μC; $\vec{x}_3 = 50.0$ cm



The magnitude of the total electrostatic force on q_1 is:

$$F_{\text{tot, 1}} = \left|\vec{F}_{21} + \vec{F}_{31}\right| = \left|-F_{21} + F_{31}\right| = \left|\frac{-k|q_2||q_1|}{\left|\vec{x}_2 - \vec{x}_1\right|^2} + \frac{k|q_3||q_1|}{\left|\vec{x}_3 - \vec{x}_1\right|^2}\right| = k|q_1|\left|\frac{|q_3|}{(x_3)^2} - \frac{|q_2|}{(x_2)^2}\right|$$

$$= \left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(|64.0 \ \mu\text{C}|\right)\left(\left|\frac{160.0 \ \mu\text{C}}{(0.500 \text{ m})^2} - \frac{80.0 \ \mu\text{C}}{(0.250 \text{ m})^2}\right|\right) = 368.23 \text{ N} \approx 368 \text{ N}$$

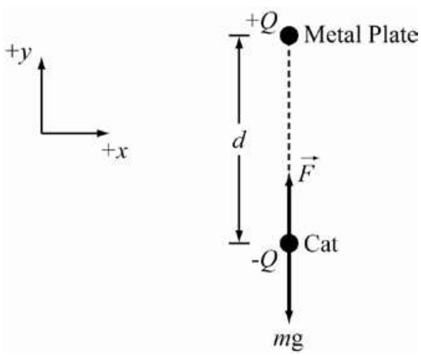
21.71. The charge of the Earth is $Q = -6.8\cdot 10^5$ C. The mass of the object is $m = 1.0$ g. For this object to be levitated near the Earth's surface ($r_{\text{E}} = 6378$ km), the Coulomb force and the force of gravity must be the same. The charge q of the object can be found from balancing these forces:

$$F_{\text{g}} = F_{\text{Coulomb}} \Rightarrow mg = \frac{k|Qq|}{r_{\text{E}}^{\ 2}} \Rightarrow |q| = \frac{mgr_{\text{E}}^{\ 2}}{k|Q|}$$

$$|q| = \frac{(0.0010 \text{ kg})(9.81 \text{ m/s}^2)(6.378\cdot 10^6 \text{ m})^2}{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left|-6.8\cdot 10^5 \text{ C}\right|} = 6.5278\cdot 10^{-5} \text{ C} \approx 65 \ \mu\text{C}$$

Since Q is negative, and the object is levitated by the repulsion of like charges, it must be that $q \approx -65 \ \mu\text{C}$.

21.72. The mass of the cat is 7.00 kg. The distance between the cat and the metal plate is 2.00 m. The cat is suspended due to attractive electric force between the cat and the metal plate.



The attractive force between the cat and the metal plate is $F = kQQ/d^2$. Since the cat is suspended in the air, this means that $F = mg$. Therefore $mg = kQ^2/d^2$. Solving for Q gives $Q = \sqrt{mgd^2/k} = d\sqrt{mg/k}$. Substituting $m = 7.00$ kg , $g = 9.81 \text{ m/s}^2$, $k = 8.99\cdot 10^9 \text{ N m}^2/C^2$ and $d = 2.00$ m yields

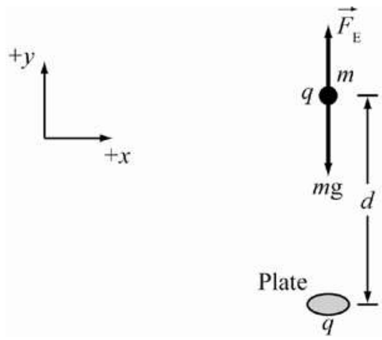
$$Q = 2.00 \text{ m}\sqrt{\frac{7.00 \text{ kg}\cdot 9.81 \text{ m/s}^2}{8.99\cdot 10^9 \text{ N m}^2/\text{C}^2}} = 1.748\cdot 10^{-4}\text{C}.$$

The number of electrons that must be extracted is

$$N = \frac{Q}{q_{\text{e}}} = \frac{1.748\cdot 10^{-4} \text{ C}}{1.602\cdot 10^{-19} \text{ C}} = 1.09\cdot 10^{15} \text{ electrons.}$$

21.73. **THINK:** A 10.0 g mass is suspended 5.00 cm above a non-conducting flat plate. The mass and the plate have the same charge q. The gravitational force on the mass is balanced by the electrostatic force.

SKETCH:



RESEARCH: The electrostatic force on the mass m is $F_E = kq^2/d^2$. This force is balanced by the gravitational force $F_g = mg$. Therefore, $F_E = F_g$ or $kq^2/d^2 = mg$.

SIMPLIFY: The charge on the mass m that satisfies the balanced condition is $q = d\sqrt{mg/k}$.

CALCULATE: Putting in the numerical values gives:

$$q = 0.0500 \text{ m}\sqrt{\frac{\left(10.0\cdot 10^{-3}\text{ kg}\right)\left(9.81\text{ m/s}^2\right)}{8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2}} = 1.6517\cdot 10^{-7}\text{C}.$$

The number of electrons on the mass m is:

$$N = \frac{q}{e} = \frac{1.6517\cdot 10^{-7}\text{C}}{1.602\cdot 10^{-19}\text{C / electron}} = 1.0310\cdot 10^{12}\text{ electrons.}$$

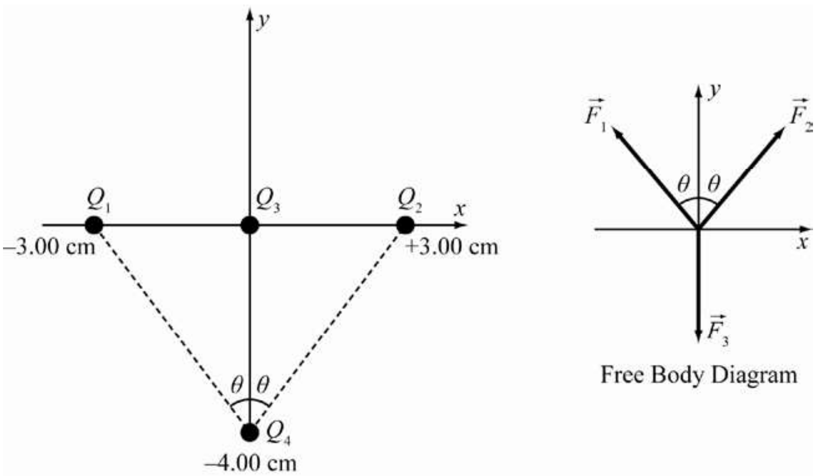
The additional mass of electrons is $\Delta m = \left(1.0310\cdot 10^{12}\right)\left(9.11\cdot 10^{-31}\text{ kg}\right) = 9.39263\cdot 10^{-19}$ kg.

ROUND: Rounding to three significant figures gives $q = 1.65\cdot 10^{-7} e$, and $\Delta m = 9.39\cdot 10^{-19}$ kg.

DOUBLE-CHECK: It is expected that Δm is negligible since the mass of electron is very small.

21.74. **THINK:** This problem involves superposition of forces. Since there are three forces on Q_4, the net force is the vector sum of three forces.

SKETCH:



RESEARCH: The magnitude of the forces between two charges, q_1 and q_2, is $F = kq_1q_2/r^2$. The forces on Q_4 are

$$\vec{F}_1 = k\frac{Q_1Q_4}{r_{14}^{\ 2}}\left(-\sin\theta\hat{x} + \cos\theta\hat{y}\right),\quad \vec{F}_2 = k\frac{Q_2Q_4}{r_{24}^{\ 2}}\left(\sin\theta\hat{x} + \cos\theta\hat{y}\right),\quad \text{and}\quad \vec{F}_3 = k\frac{Q_3Q_4}{r_{34}^{\ 2}}\left(-\hat{y}\right).$$

SIMPLIFY: By symmetry, the horizontal components of F_1 and F_2 cancel, and F_3 has no horizontal component. The net force is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = kQ_4\left[\left(\frac{Q_1}{r_{14}^{\ 2}} + \frac{Q_2}{r_{24}^{\ 2}}\right)\cos\theta - \frac{Q_3}{r_{34}^{\ 2}}\right]\hat{y}.$$

Since $Q_1 = Q_2$ and $r_{14} = r_{24}$, the above equation simplifies to

$$\vec{F} = kQ_4\left[\frac{2Q_1\cos\theta}{r_{14}^{\ 2}} - \frac{Q_3}{r_{34}^{\ 2}}\right]\hat{y}.$$

CALCULATE: The distance r_{14} and r_{34} are $r_{14} = \sqrt{(3\text{ cm})^2 + (4\text{ cm})^2} = 5\text{ cm};\ r_{34} = 4\text{ cm}$. Therefore $\cos\theta = 4/5$. Substituting the numerical values yields:

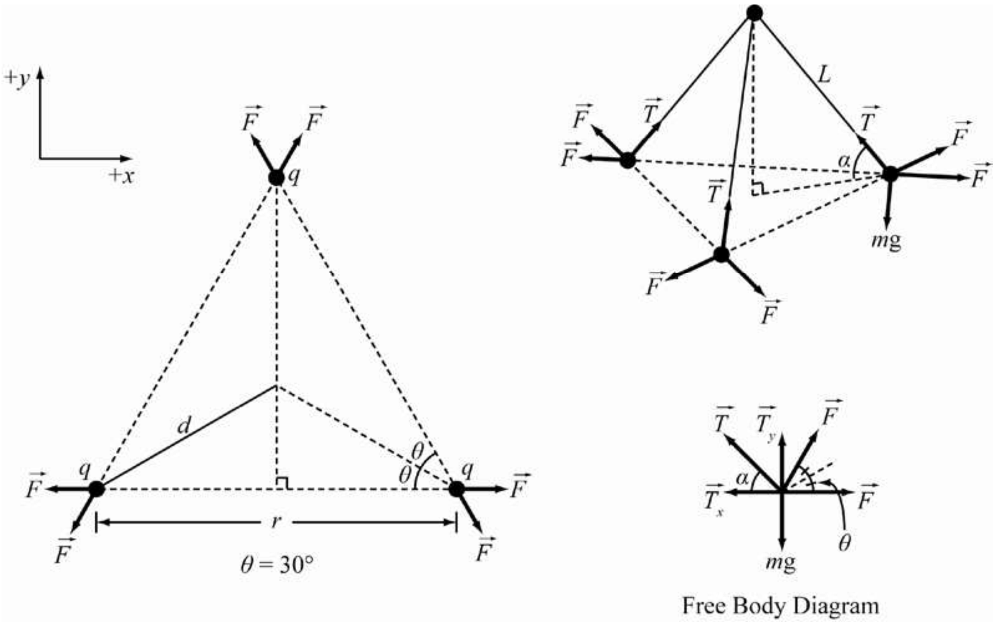
$$F = \left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)\left(2\cdot 10^{-3}\text{ C}\right)\left[\left(\frac{2\cdot 1\cdot 10^{-3}\text{ C}}{\left(5\cdot 10^{-2}\text{ m}\right)^2}\right)\left(\frac{4}{5}\right) - \frac{1.024\cdot 10^{-3}\text{ C}}{\left(4\cdot 10^{-2}\text{ m}\right)^2}\right] = 0\text{ N}.$$

ROUND: Not needed

DOUBLE-CHECK: It is clear from the symmetry of the problem that this is a reasonable outcome.

21.75. **THINK:** Three 5.00-g Styrofoam balls of radius 2.00 cm are tied to 1.00 m long threads and suspended freely from a common point. The charge of each ball is q and the balls form an equilateral triangle with sides of 25.0 cm.

SKETCH:



RESEARCH: The magnitude of the force between two charges, q_1 and q_2, is $F_{12} = kq_1q_2/r^2$. The magnitude of F in the above figure is $F = kq^2/r^2$. Using Newton's Second Law, it is found that $T_y = T\sin\alpha = mg$ and $T_x = T\cos\alpha = 2F\cos\theta$.

SIMPLIFY: Eliminating T in the above equations yields $\tan\alpha = mg/\left(2F\cos\theta\right)$. Rearranging gives, $F = mg/\left(2\tan\alpha\cos\theta\right) = kq^2/r^2$. Therefore, the charge q is

$$q = \sqrt{\frac{mgr^2}{2k\tan\alpha\cos\theta}}.$$

From the sketch, it is clear that the distance of the ball to the center of the triangle is $d = r/\left(2\cos\theta\right)$. Therefore $\tan\theta = \sqrt{L^2 - d^2}/d$.

CALCULATE: Substituting the numerical values, $r = 0.250$ m, $m = 5.00\cdot 10^{-3}$ kg, $g = 9.81$ m/s^2, $L = 1.00$ m and $\theta = 30°$ (exact) gives

$$d = \frac{0.250\text{ m}}{2\cos(30°)} = 0.1443\text{ m}$$

$$\tan\alpha = \frac{\sqrt{\left(1.00\text{ m}\right)^2 - \left(0.1443\text{ m}\right)^2}}{0.1443\text{ m}} = 6.856$$

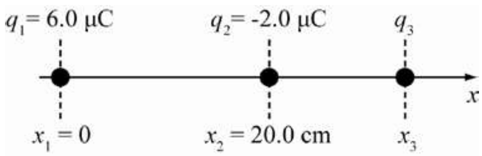
$$q = \sqrt{\frac{5.00\cdot 10^{-3}\text{ kg}\left(9.81\text{ m/s}^2\right)\left(0.250\text{ m}\right)^2}{2\cdot\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)6.856\cos(30°)}} = 1.69463\cdot 10^{-7}\text{ C}$$

ROUND: $q = 0.169\ \mu\text{C}$

DOUBLE-CHECK: This charge is approximately 11 orders of magnitude larger than the elementary charge *e*. The charge required to deflect 5.00 g balls by a distance of 25.0 cm would need to be fairly large.

21.76. **THINK:** Two point charges lie on the *x*-axis. A third point charge needs to be placed on the *x*-axis such that it is in equilibrium. This means that the net force on the third charge due to the other charges is zero.

SKETCH:



RESEARCH: In order for the third charge to be in equilibrium, the force on it due to q_1, $\vec{F}_1$, must be equal in magnitude and opposite in direction to $\vec{F}_2$ the force due to q_2. Note that the sign of the third charge is irrelevant, so I can arbitrarily assume it is positive. Since $|q_1| > |q_2|$, the third charge must be closer to q_2 than to q_1. Also, since q_1 and q_2 are oppositely charged, the forces on a particle between them will be in the same direction and hence cannot cancel. The third charge must be in the region $x > 20.0$ cm. The net force on q_3 is $F_{\text{net}} = \frac{k|q_1||q_3|}{x_3^2} - \frac{k|q_2||q_3|}{\left(x_3 - x_2\right)^2}$.

SIMPLIFY: Solving $F_{\text{net}} = 0$ for x_3 yields $|q_2|x_3^2 = |q_1|\left(x_3 - x_2\right)^2$ or $\sqrt{|q_2|}\left(x_3 - x_2\right)$. Therefore the position of q_3 is $x_3 = \frac{\sqrt{q_1}x_2}{\sqrt{|q_1|} - \sqrt{|q_2|}}$.

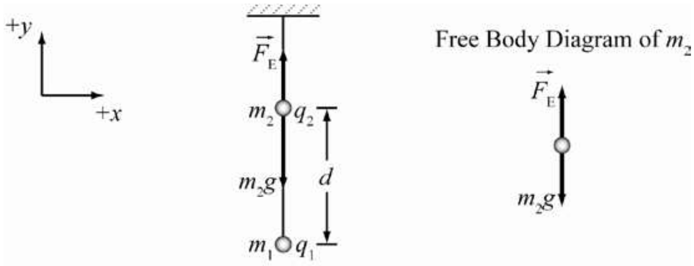
CALCULATE: Putting in the numerical values yields $x_3 = \frac{\left(\sqrt{6.0\ \mu\text{C}}\right)\left(20.0\text{ cm}\right)}{\sqrt{6.0\ \mu\text{C}} - \sqrt{2.0\ \mu\text{C}}} = 47.32$ cm.

ROUND: Using only two significant digits, the position x_3 is $x_3 = 47$ cm

DOUBLE-CHECK: This is correct since $x_3 > x_2$.

21.77. **THINK:** In this problem, a gravitational force on an object is balanced by an electrostatic force on the object.

SKETCH:



RESEARCH: The electric force on q_2 is given by $F_E = kq_1q_2 / d^2$. The gravitational force on m_2 is $F_g = m_2 g$.

SIMPLIFY: $\frac{kq_1q_2}{d^2} \rightarrow \frac{kq^2}{d^2} = m_2 g \Rightarrow m_2 = \frac{kq^2}{gd^2}$.

CALCULATE: Substituting the numerical values, $q_1 = q_2 = +2.67$ μe, $d = 0.360$ m produces

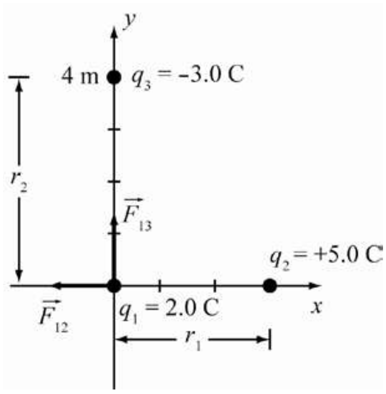
$$m_2 = \frac{\left(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2\right)\left(2.67 \cdot 10^{-6} \text{ C}\right)^2}{\left(9.81 \text{ m/s}^2\right)\left(0.360 \text{ m}\right)^2} = 0.05041 \text{ kg}.$$

ROUND: Keeping only three significant digits gives $m_2 = 50.4$ g.

DOUBLE-CHECK: This makes sense since F_E is small.

21.78. **THINK:** Because this is a two-dimensional problem, the directions of forces are important for determining a net force.

SKETCH:



RESEARCH: The magnitude of the force between two charges is $F = k|q_1||q_2| / r^2$. The net force on q_1 is $\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} = -\frac{k|q_1||q_2|}{r_1^2}\hat{x} - \frac{k|q_1||q_3|}{r_2^2}\hat{y}$. The direction of the net force is $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$.

SIMPLIFY: Not needed

CALCULATE: $\vec{F}_{\text{net}} = \frac{-\left(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2\right)(2.00 \text{ C})(5.00 \text{ C})}{\left(3.00 \text{ m}\right)^2}\hat{x} + \frac{\left(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2\right)(2.00 \text{ C})(3.00 \text{ C})}{\left(4.00 \text{ m}\right)^2}\hat{y}$

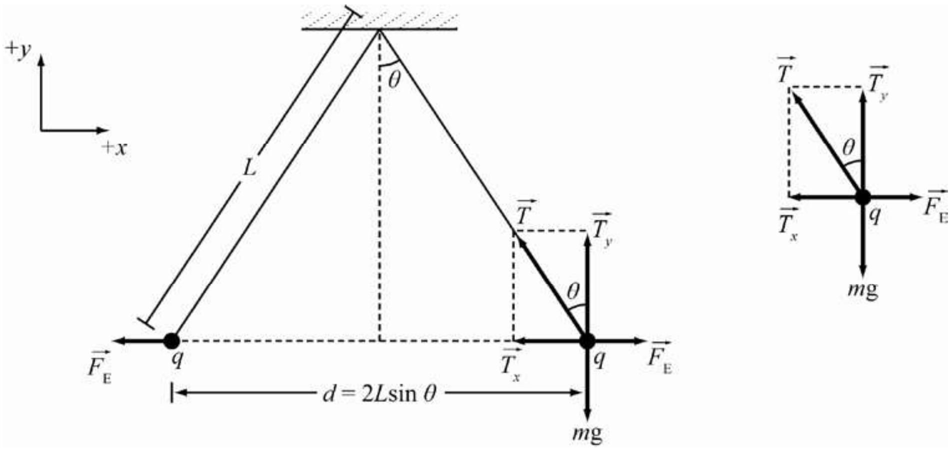
$$= \left(-9.998 \cdot 10^9 \text{ N}\right)\hat{x} + \left(3.371 \cdot 10^9 \text{ N}\right)\hat{y}$$

The magnitude of $\vec{F}_{\text{net}}$ is $\left|F_{\text{net}}\right| = \sqrt{9.99^2 + 3.37^2} \cdot 10^9 \text{ N} = 10.551 \cdot 10^9 \text{ N}$. The direction of $\vec{F}_{\text{net}}$ is $\theta = \tan^{-1}\left(\frac{3.37 \cdot 10^9 \text{ N}}{-9.99 \cdot 10^9 \text{ N}}\right) = 161.36°$ with respect to the positive x-axis, or $18.64°$ above the negative x-axis (the net force points up and to the left, in quadrant II).

ROUND: Keeping only three significant digits yields $F_{\text{net}} = \left(-1.00 \cdot 10^8 \text{ N}\right)\hat{x} + \left(3.4 \cdot 10^9 \text{ N}\right)\hat{y}$ and $\left|F_{\text{net}}\right| = 10.6 \cdot 10^9 \text{ N}$ at $18.6°$ above the negative x-axis.

21.79. **THINK:** To solve this problem, the force due to the charges and the tension in the string must balance the gravitational force on the spheres.

SKETCH:



RESEARCH: The force due to electrostatic repulsion of the two spheres is $F_{\text{E}} = kq_1q_2 / d^2 = kq^2 / d^2$. Applying Newton's Second Law yields (I) $T_x = T\sin\theta = F_{\text{E}}$ and (II) $T_y = T\cos\theta = mg$. $L = 0.45 \text{ m}$, $m = 2.33 \cdot 10^{-3} \text{ kg}$, $\theta = 10.0°$.

SIMPLIFY: Dividing (I) by (II) gives $\tan\theta = F_{\text{E}} / (mg) = kq^2 / \left(d^2 mg\right)$. After simple manipulation, it is found that the charge on each sphere is $q = \sqrt{d^2 mg \tan\theta / k} = 2L\sin\theta\sqrt{mg\tan\theta / k}$ using $d = 2L\sin\theta$.

CALCULATE: Substituting the numerical values, it is found that

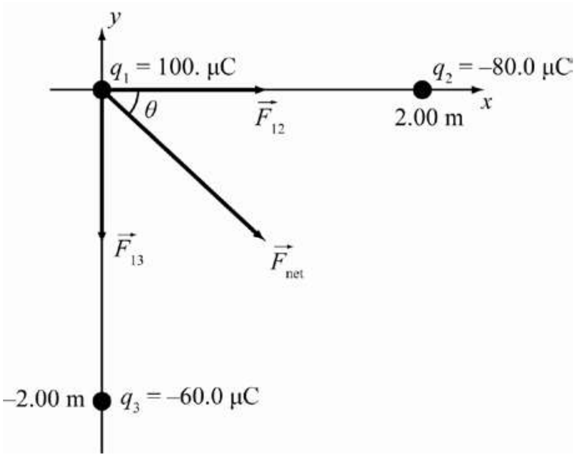
$$q = (2)(0.450 \text{ m})(\sin 10.0°)\sqrt{\frac{2.33 \cdot 10^{-3} \text{ kg}\left(9.81 \text{ m/s}^2\right)\tan(10.0°)}{8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2}} = 1.0464 \cdot 10^{-7} \text{ C}.$$

ROUND: Keeping only three significant digits gives $q = 0.105\ \mu\text{C}$.

DOUBLE-CHECK: This is reasonable. The relatively small spheres and small distance will mean the charge is small.

21.80. **THINK:** I want to find the magnitude and direction of the net force on a point charge q_1 due to point charges q_2 and q_3. The charges q_1, q_2, and q_3 are located at (0,0), (2.0,0.0), and (0,−2.00), respectively.

SKETCH:



RESEARCH: The magnitude of the force between two charges is $F = k|q_1||q_2|/r^2$. The net force on q_1 is

$$\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} = -\frac{k|q_1||q_2|}{r_1^2}\hat{x} - \frac{k|q_1||q_3|}{r_2^2}\hat{y}.$$

SIMPLIFY: Not needed

CALCULATE: Putting in the numerical values yields

$$\vec{F}_{\text{net}} = \frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(100.\cdot 10^{-9} \text{ C})(80.0\cdot 10^{-9} \text{ C})}{\left(2.00 \text{ m}\right)^2}\hat{x} - \frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)(100.\cdot 10^{-9} \text{ C})(60.0\cdot 10^{-9} \text{ C})}{\left(2.00 \text{ m}\right)^2}\hat{y}$$

$$= 1.798\cdot 10^{-5} \text{ N}\hat{x} - 1.348\cdot 10^{-5} \text{ N}\hat{y}$$

The magnitude of $\vec{F}_{\text{net}}$ is $\left|\vec{F}_{\text{net}}\right| = \sqrt{1.798^2 + 1.348^2}\cdot 10^{-5} \text{ N} = 2.247\cdot 10^{-5}$ N. The direction of $\vec{F}_{\text{net}}$ is

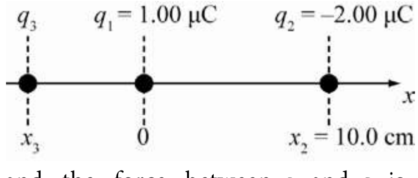
$$\theta = \tan^{-1}\left(\frac{1.348}{1.798}\right) = -36.860°.$$

ROUND: Rounding to three significant digits, it is found that $\left|\vec{F}_{\text{net}}\right| = 2.25\cdot 10^{-5}$ N and $\theta = 36.9°$ below the horizontal.

DOUBLE-CHECK: Since both forces acting on q_1 are attractive, it is expected that the direction of the net force would be between the two contributing force vectors.

21.81. **THINK:** If it is assumed that the third charge is positive, then the third charge experiences a repulsive force with q_1 and an attractive force with q_2.

SKETCH:



RESEARCH: Because $|q_1| > |q_2|$ and the force between q_1 and q_3 is attractive, the possible region where q_3 can experience zero net force is in the region $x < 0$. The net force on q_3 is

$$F_{\text{net}} = -\frac{k|q_1||q_3|}{\left(0 - x_3\right)^2} + \frac{k|q_2||q_3|}{\left(x_2 - x_3\right)^2}.$$

SIMPLIFY: Solving $F_{\text{net}} = 0$ for x_3 yields $x_3^2|q_2| = |q_1|(x_2 - x_3)^2$ implies:

$$\text{(I)} \quad x_3\sqrt{|q_2|} = \sqrt{|q_1|}(x_2 - x_3) \quad \text{or} \quad \text{(II)} \quad -x_3\sqrt{|q_2|} = \sqrt{|q_1|}(x_2 - x_3)$$

Equation (I) gives $x_3 > 0$ and equation (II) gives $x_3 < 0$. Therefore the correct solution is the solution of Equation (II). Solving (II) yields $x_3 = \dfrac{-\sqrt{|q_1|}x_2}{\sqrt{|q_2|} - \sqrt{|q_1|}}$.

CALCULATE: Substituting $q_1 = 1.00\ \mu\text{C}$, $q_2 = -2.00\ \mu\text{C}$ and $x_2 = 10.0\text{ cm}$ into above equation gives

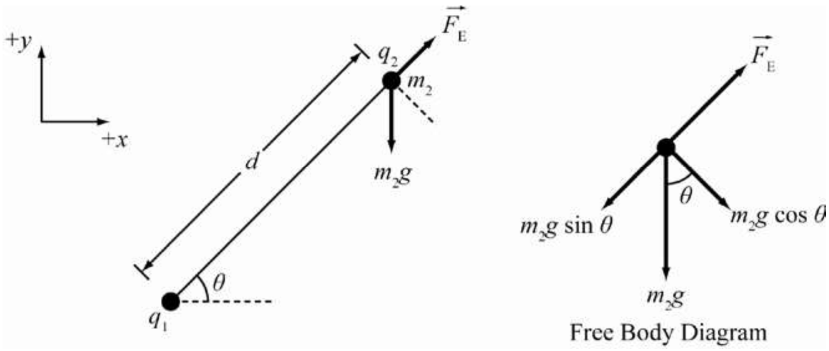
$$x_3 = \frac{-\sqrt{1.00\ \mu\text{C}} \cdot 10.0\text{ cm}}{\sqrt{2.00\ \mu\text{C}} - \sqrt{1.00\ \mu\text{C}}} = -24.142\text{ cm}$$

ROUND: To three significant figures: $x_3 = -24.1$ cm.

DOUBLE-CHECK: The negative value of x indicates that q_3 is located in the region $x < 0$, as expected.

21.82. **THINK:** The electrostatic force on a bead is balanced by its gravitational weight.

SKETCH:



RESEARCH: The repulsive force between two charged beads is $F_E = k\dfrac{q_1q_2}{d^2}$. Using Newton's Second Law,

$$F_E = k\frac{q_1q_2}{d^2} = m_2 g\sin\theta .$$

SIMPLIFY: Therefore the distance d is $d = \sqrt{\dfrac{kq_1q_2}{m_2 g\sin\theta}}$.

CALCULATE: Substituting the numerical values into the above equation gives

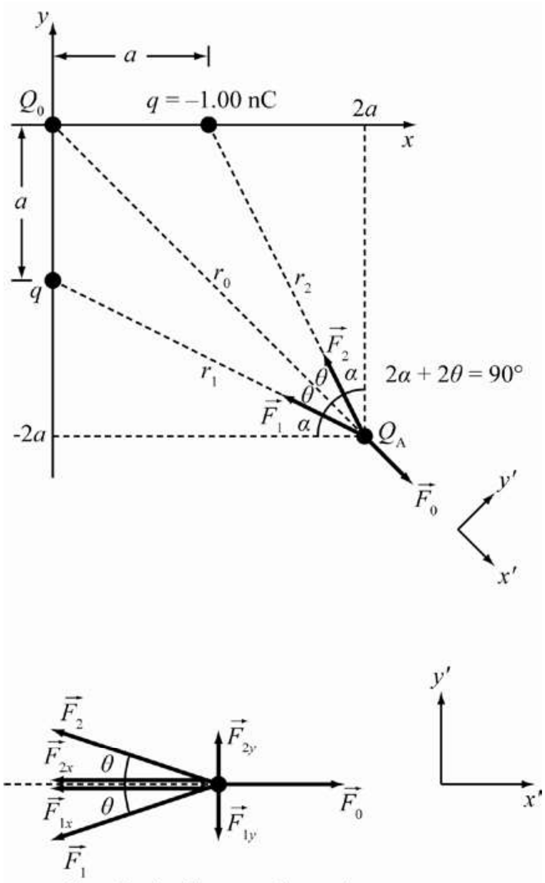
$$d = \sqrt{\frac{\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)(1.27\cdot 10^{-6}\text{ C})(6.79\cdot 10^{-6}\text{ C})}{3.77\cdot 10^{-3}\text{ kg}\left(9.81\text{ m/s}^2\right)\sin\left(51.3^\circ\right)}} = 1.638\text{ m.}$$

ROUND: Keeping only three significant digits gives $d = 1.64$ m.

DOUBLE-CHECK: The beads are very light, so a small charge is sufficient to cause a relatively large separation.

21.83. **THINK:** Since this is a two dimensional problem, electrostatic forces are added as vectors. It is assumed that Q_A is a positive charge.

SKETCH:



Free Body diagram along x′

RESEARCH: To balance the forces F_1 and F_2, the charge on Q_0 must be positive. The electrostatic forces on Q_A are $F_1 = \frac{k|q|Q_A}{r_1^2}$, $F_2 = \frac{k|q|Q_A}{r_2^2}$, and $F_0 = \frac{kQ_0Q_A}{r_0^2}$. Applying Newton's Second Law, it is found that $F_0 = F_{1x} + F_{2x}$ or $kQ_0Q_A / r_0^2 = F_1\cos\theta + F_2\cos\theta$. Using $r_1 = r_2$ this becomes $\frac{kQ_0Q_A}{r_0^2} = \frac{k|q|Q_A}{r_1^2}2\cos\theta$.

SIMPLIFY: Solving the above equation for Q_0 gives the charge Q_0, $Q_0 = (r_0/r_1)^2|q|2\cos\theta$. From the above figure, it is noted that $r_0 = \sqrt{(2a)^2 + (2a)^2} = 2a\sqrt{2}$, $r_1 = \sqrt{(2a)^2 + a^2} = a\sqrt{5}$, and

$$\cos\theta = \cos(45° - \alpha) = \cos 45°\cos\alpha + \sin 45°\sin\alpha \Rightarrow \cos\theta = \frac{\sqrt{2}}{2}\frac{2a}{a\sqrt{5}} + \frac{\sqrt{2}}{2}\frac{a}{a\sqrt{5}} = \frac{3}{2}\sqrt{\frac{2}{5}} = \frac{3}{10}\sqrt{10}.$$

Therefore the magnitude of charge Q_0 is $|Q_0| = 2|q|\frac{8a^2}{5a^2}\frac{3}{10}\sqrt{10} = \frac{48}{50}\sqrt{10}|q|$.

CALCULATE: Substituting $q = -1.00$ nC yields $|Q_0| = \frac{48}{50}\sqrt{10}\cdot|-1.00 \text{ nC}| = 3.036$ nC.

ROUND: Rounding to three significant figures gives $|Q_0| = 3.04$ nC.

DOUBLE-CHECK: Since r_0 is larger than r_1, it is expected that Q_0 is larger than $2|q| = 2$ nC.