UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

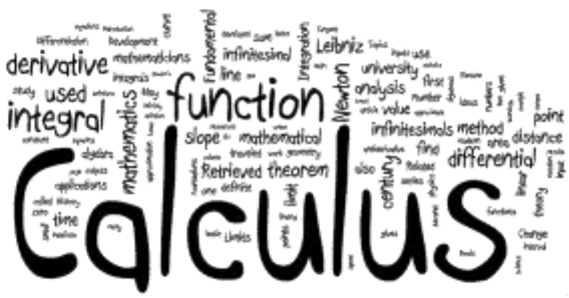
الإمارات العربية المتحدة
وزارة التربية والتعليم

Advanced Math 2019-2020

Grade 12 MATHS

Term 1

CHAPTER 1 Preliminaries of Calculus



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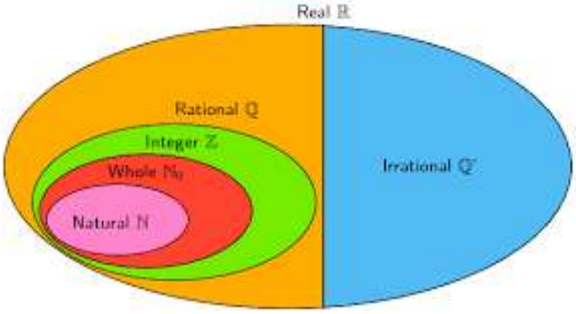
LESSON 1-1 Polynomials and Rational Functions



The Real Number System

$N = \{1, 2, 3, \ldots\}$ Natural Number

$N_{\circ} = \{0, 1, 2, 3, \ldots\}$ Whole Number

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ Integer Number

$\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\right\}$ Rational Number

$I = \overline{\mathbb{Q}} =$ Irrational Number

$\mathbb{R} = \mathbb{Q} \cup I = \mathbb{Q} \cup \overline{\mathbb{Q}} =$ Real Number $= (\infty, -\infty)$

Intervals

For real numbers a and b, where $a < b$, we define the **closed interval** $[a, b]$ to be the set of numbers between a and b, including a and b (the **endpoints**). That is,

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\},$$

Similarly, the **open interval** (a, b) is the set of numbers between a and b, but *not* including the endpoints a and b, that is,

$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\},$$

Interval Notation	Number Line Sketch	Set-builder Notation
(a, b)		$\{x \mid a < x < b\}$
$(a, b]$		$\{x \mid a < x \leq b\}$
$[a, b)$		$\{x \mid a \leq x < b\}$
$[a, b]$		$\{x \mid a \leq x \leq b\}$

(a, ∞)		$\{x \mid x > a\}$
$(-\infty, b)$		$\{x \mid x < b\}$
$[a, \infty)$		$\{x \mid x \geq a\}$
$(-\infty, b]$		$\{x \mid x \leq b\}$
$(-\infty, \infty)$		R

Solving Inequalities:

• $2x+5<13$	• $3-2x<7$
• $6<1-3x\leq 10$	$-2<3x+4\leq 10$

• $\dfrac{x-1}{x+2}\geq 0$

• $\dfrac{x+2}{x-4}\leq 0$

• $x^2+x-6<0$

• $x^2+2x-3<0$

DEFINITION 1.1

The **absolute value** of a real number x is $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$.

Notice that for any real numbers a and b,

$$|a \cdot b| = |a| \cdot |b|,$$

although

$$|a + b| \neq |a| + |b|,$$

$$|a + b| \leq |a| + |b|.$$

Solving an Inequality Containing an Absolute Value

• $\lvert x-2\rvert < 5$	• $\lvert x+4\rvert \leq 7$
• $\lvert x-3\rvert \geq 2$	• $\lvert 2x+8\rvert > 6$

DEFINITION 1.2

For $x_1 \neq x_2$, the **slope** of the straight line through the points (x_1, y_1) and (x_2, y_2) is the number

$$m = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1.5}$$

When $x_1 = x_2$ and $y_1 \neq y_2$, the line through (x_1, y_1) and (x_2, y_2) is **vertical** and the slope is undefined.

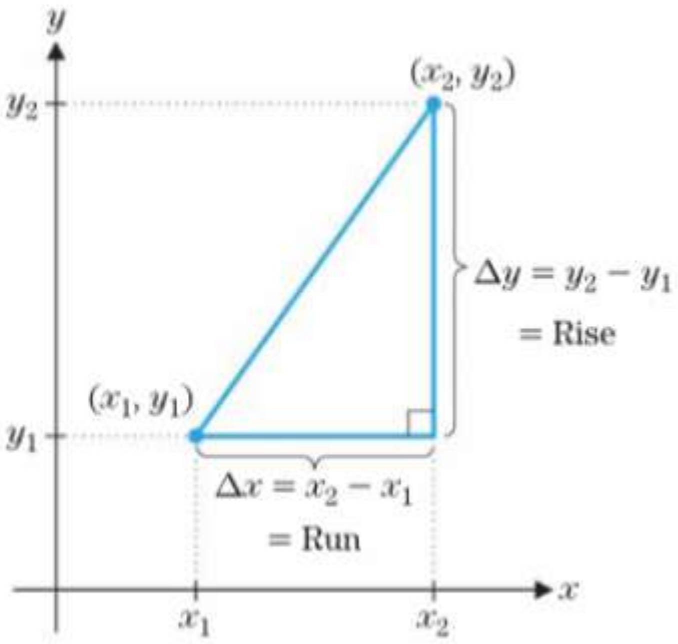


FIGURE 1.12a
Slope

POINT-SLOPE FORM OF A LINE

$$y = m(x - x_0) + y_0. \tag{1.7}$$

THEOREM 1.2

Two (nonvertical) lines are **parallel** if they have the same slope. Further, any two vertical lines are parallel. Two (nonvertical) lines of slope m_1 and m_2 are **perpendicular** whenever the product of their slopes is -1 (i.e., $m_1 \cdot m_2 = -1$). Also, any vertical line and any horizontal line are perpendicular.

Find the Equation of the line

1)Passing through the points (2,-3) , (5,9) .

2)

Find an equation of the line parallel to $y = 3x - 2$ and through the point $(-1, 3)$.

3)

Find an equation of the line perpendicular to $y = -2x + 4$ and intersecting the line at the point $(1, 2)$.

In exercises 11–14, determine if the points are colinear.

11. (2, 1), (0, 2), (4, 0)

12. (3, 1), (4, 4), (5, 8)

13. (4, 1), (3, 2), (1, 3)

14. (1, 2), (2, 5), (4, 8)

In exercises 23–28, determine if the lines are parallel, perpendicular, or neither.

23. $y = 3(x-1)+2$ and $y = 3(x+4)-1$

24. $y = 2(x-3)+1$ and $y = 4(x-3)+1$

25. $y = -2(x+1)-1$ and $y = \frac{1}{2}(x-2)+3$

26. $y = 2x-1$ and $y = -2x+2$

27. $y = 3x+1$ and $y = -\frac{1}{3}x+2$

28. $x+2y = 1$ and $2x+4y = 3$

The Distance between two points (x_1, y_1) , $(x_2 . y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In exercises 15–18, find (a) the distance between the points, (b) the slope of the line through the given points, and (c) an equation of the line through the points.

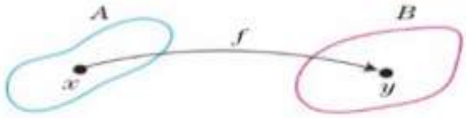
15. (1, 2), (3, 6)

16. (1, −2), (−1, −3)

17. (0.3, −1.4), (−1.1, −0.4)

18. (1.2, 2.1), (3.1, 2.4)

Functions



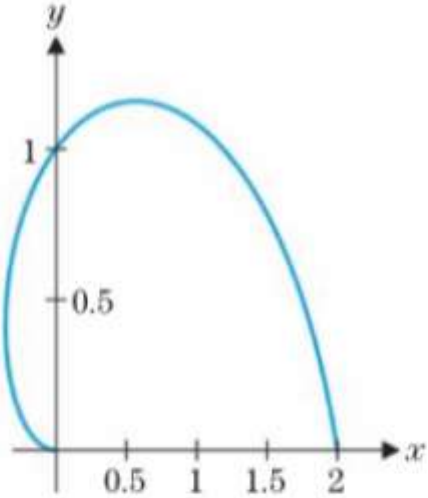
DEFINITION 1.3

A **function** f is a rule that assigns *exactly one* element y in a set B to each element x in a set A. In this case, we write $y = f(x)$.

We call the set A the **domain** of f. The set of all values $f(x)$ in B is called the **range** of f, written $\{y \mid y = f(x), \text{ for some } x \in A\}$. Unless explicitly stated otherwise, whenever a function f is given by a particular expression, the domain of f is the largest set of real numbers for which the expression is defined. We refer to x as the **independent variable** and to y as the **dependent variable.**

In exercises 35–38, use the vertical line test to determine whether the curve is the graph of a function.

38.



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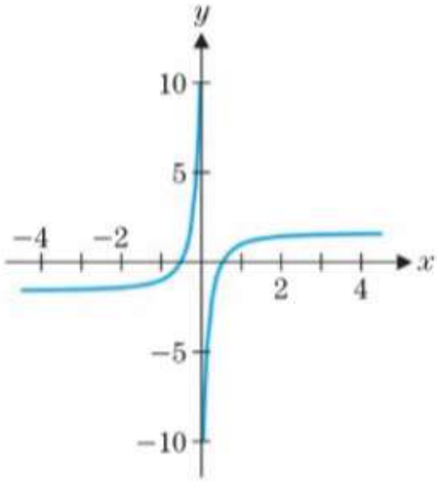

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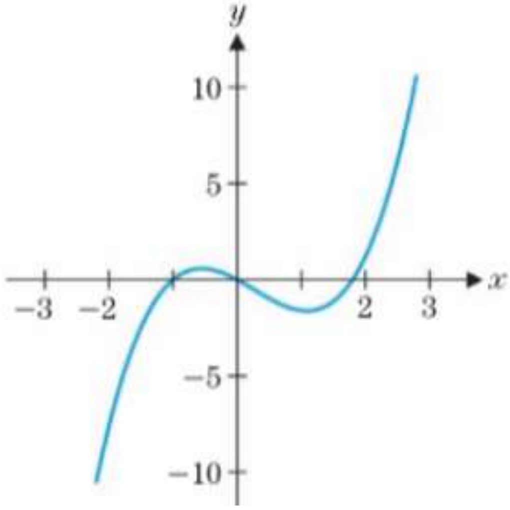

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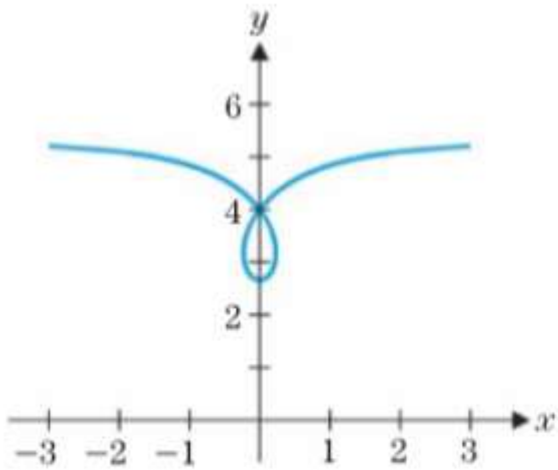


DEFINITION 1.4

A **polynomial** is any function that can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

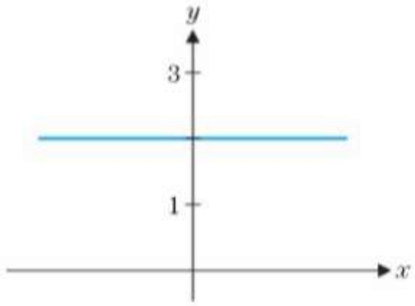
where $a_0, a_1, a_2, . a_n$ are real numbers (the **coefficients** of the polynomial) with $a_n \neq 0$ and $n \geq 0$ is an integer (the **degree** of the polynomial).

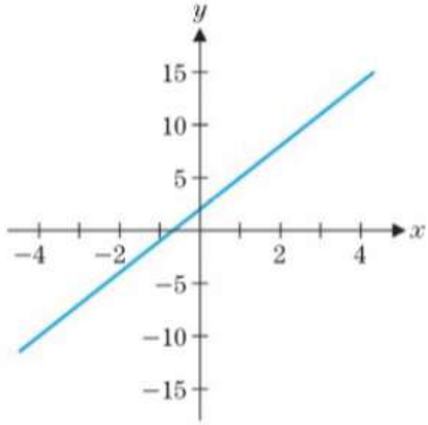
The following are all examples of polynomials:

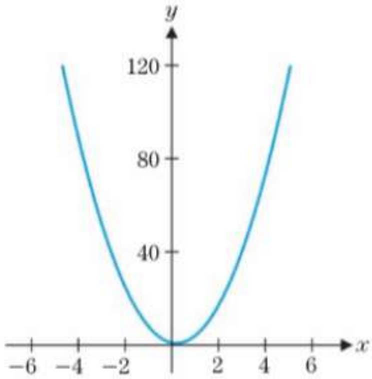
$f(x) = 2$ (polynomial of degree 0 or **constant**),
$f(x) = 3x + 2$ (polynomial of degree 1 or **linear** polynomial),
$f(x) = 5x^2 - 2x + 2/3$ (polynomial of degree 2 or **quadratic** polynomial),
$f(x) = x^3 - 2x + 1$ (polynomial of degree 3 or **cubic** polynomial),
$f(x) = -6x^4 + 12x^2 - 3x + 13$ (polynomial of degree 4 or **quartic** polynomial),

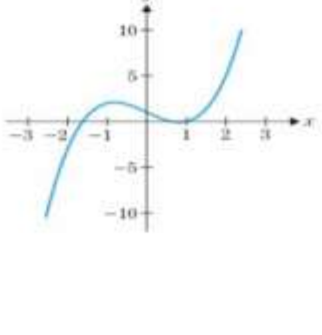
and

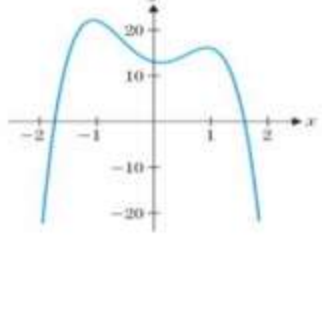
$f(x) = 2x^5 + 6x^4 - 8x^2 + x - 3$ (polynomial of degree 5 or **quintic** polynomial).

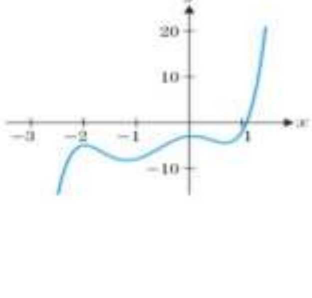












DEFINITION 1.5

Any function that can be written in the form

$$f(x) = \frac{p(x)}{q(x)},$$

where p and q are polynomials, is called a **rational** function.

EXAMPLE 1.18 A Sample Rational Function

Find the domain of the function

$$f(x) = \frac{x^2 + 7x - 11}{x^2 - 4}.$$

EXAMPLE 1.19 Finding the Domain of a Function Involving a Square Root or a Cube Root

Find the domains of $f(x) = \sqrt{x^2 - 4}$ and $g(x) = \sqrt[3]{x^2 - 4}$.

THEOREM 1.3

A polynomial of degree n has *at most* n distinct zeros.

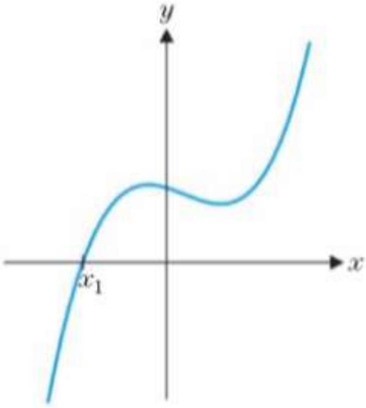


FIGURE 1.23a
One zero

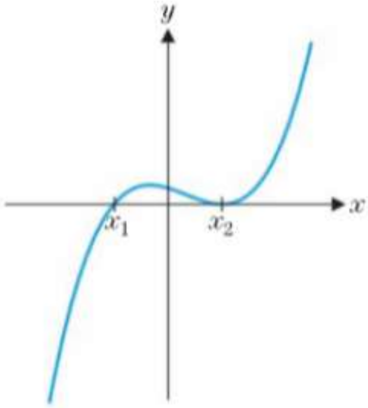


FIGURE 1.23b
Two zeros

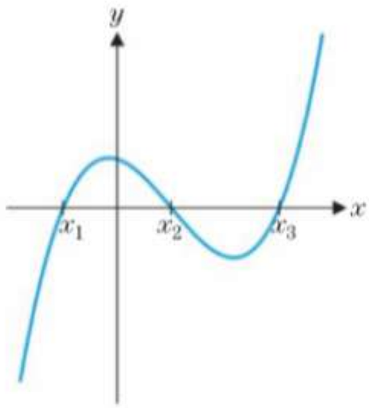


FIGURE 1.23c
Three zeros

THEOREM 1.4 (Factor Theorem)

For any polynomial function f, $f(a) = 0$ if and only if $(x - a)$ is a factor of $f(x)$.

In exercises 39–42, identify the given function as polynomial, rational, both or neither.

39. $f(x) = x^3 - 4x + 1$

40. $f(x) = \dfrac{x^3 + 4x - 1}{x^4 - 1}$

41. $f(x) = \dfrac{x^2 + 2x - 1}{x + 1}$

42. $f(x) = \sqrt{x^2 + 1}$

In exercises 43–48, find the domain of the function.

43. $f(x) = \sqrt{x+2}$

44. $f(x) = \sqrt[3]{x-1}$

45. $f(x) = \dfrac{\sqrt{x^2 - x - 6}}{x - 5}$

46. $f(x) = \dfrac{\sqrt{x^2 - 4}}{\sqrt{9 - x^2}}$

47. $f(x) = \dfrac{4}{x^2 - 1}$

48. $f(x) = \dfrac{4x}{x^2 + 2x - 6}$

In exercises 65–72, factor and/or use the quadratic formula to find all zeros of the given function.

65. $f(x) = x^2 - 4x + 3$

66. $f(x) = x^2 + x - 12$

67. $f(x) = x^2 - 4x + 2$

68. $f(x) = 2x^2 + 4x - 1$

69. $f(x) = x^3 - 3x^2 + 2x$

70. $f(x) = x^3 - 2x^2 - x + 2$

In exercises 73 and 74, find all points of intersection.

73. $y = x^2 + 2x + 3$ and $y = x + 5$

74. $y = x^2 + 4x - 2$ and $y = 2x^2 + x - 6$

In exercises 59–64, find all intercepts of the given graph.

59. $y = x^2 - 2x - 8$

60. $y = x^2 + 4x + 4$

61. $y = x^3 - 8$

62. $y = x^3 - 3x^2 + 3x - 1$

63. $y = \dfrac{x^2 - 4}{x + 1}$

64. $y = \dfrac{2x - 1}{x^2 - 4}$