الحجم : شرائح واقراص وحلقات

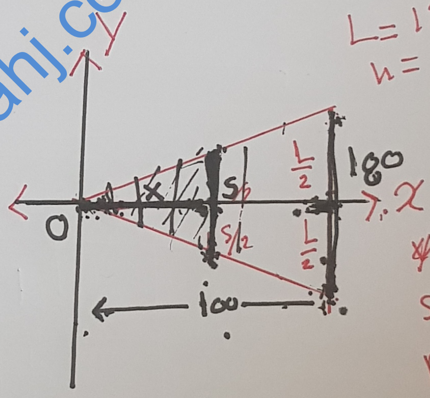
الشرائح : يتم تقطيع المجسم إلى شرائح ومن ثم ايجاد مساحة الشريحة وتطبيق القاعدة

$$V = \int_a^b A(x)\,dx$$

حيث $A(x)$ مساحة المقطع العرضي (الشريحة)

$L = 180$

$h = 100$



$$\frac{x}{100} = \frac{\frac{s}{2}}{\frac{L}{2}}$$

$$S = \frac{Lx}{h}$$

$$\frac{x}{100} = \frac{\frac{s}{2}}{90}$$

$$\boxed{\frac{x}{100} = \frac{s}{180}} \Rightarrow s = \frac{x(180)}{100} = 1.8x$$

$$A(x) = s^2 = (1.8x)^2.$$

$$V = \int_{0}^{100} (1.8x)^2 \, dx$$

$$= (1.8)^2 \left. \frac{x^3}{3} \right]_{0}^{100} = (1.8)^2 \frac{(100)^3}{3}$$

$\frac{1}{3} b.h$

$$= 1080000 \, m^3$$

اوجد حجم المجسم $\frac{1}{430}$ مساحة المقطع

$\boxed{A(x) = 10e^{0.01x}}$, $0 \le x \le 10$

$$V = \int_{0}^{10} 10e^{0.01x} \, dx = \cancel{10.05}$$

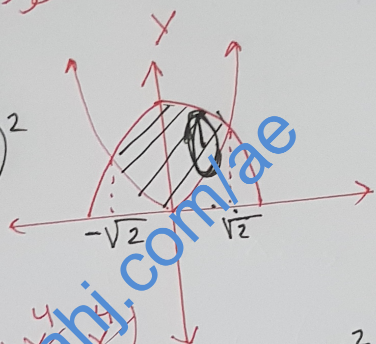
505.025

وحدة مكعبة

$\frac{18}{430}$] $Y = 4 - x^2 \,,\; Y = x^2$ 1

حول

a) حول المحور x b) حول $y=4$



$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (4-x^2)^2 - (x^2)^2$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (16 - 8x^2 + \cancel{x^4} - \cancel{x^4})$$

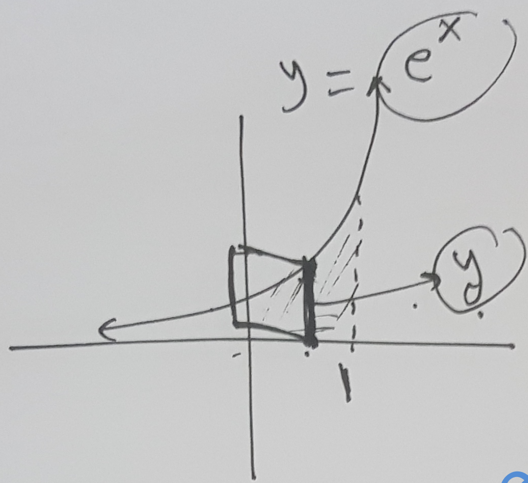
$$4 - x^2 = x^2$$

$$4 = 2x^2$$

$$x = \pm\sqrt{2}$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (16 - 8x^2)\,dx$$

$$= \pi \left[16x - \frac{8x^3}{3} \right]_{-\sqrt{2}}^{\sqrt{2}} = \frac{64\pi\sqrt{2}}{3}$$



الشريحة

عبارة عن مربعات

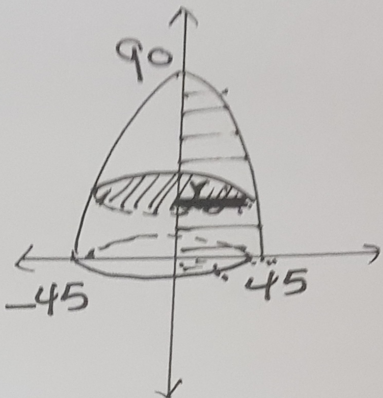
أصغر عمودية على محور x

$$V = \int_0^1 A(x)\,dx$$

$$V = \int_0^1 (e^x)^2\,dx$$

$$= \int_0^1 e^{2x}\,dx = \frac{1}{2}e^{2x}$$

$$Y = \frac{-2}{45}x^2 + 90$$



$$r = x$$

$$\frac{2}{45}x^2 = 90 - Y$$

$$x^2 = \frac{45}{2}(90 - Y)$$

$$\boxed{x = \sqrt{\frac{45}{2}(90 - y)}} = r(y)$$

$$A(y) = \pi\left(\frac{45}{2}(90 - y)\right)$$

$$V = \int_0^{90} \pi\left(2025 - \frac{45}{2}y\right) dy$$

$$\approx 91,125\pi \ cm^3$$

286.277

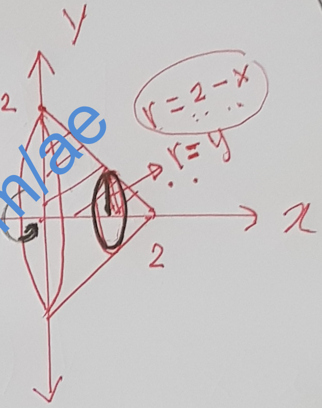
$\frac{17}{430}$] $Y=0\ ,\ Y=2-x\ ,\ x=0$

$Y=3$ (b) x محور (a حول

$A(x) = \pi r^2$

$= \pi (2-x)^2$

$V = \pi \int_0^2 \underbrace{(2-x)^2}_{r^2} dx$



الحل :

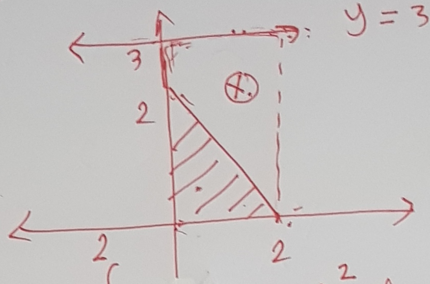
a) $V = \pi \int_0^2 (f(x))^2 \, dx$

$V = \pi \int_0^2 (2-x)^2 \, dx = \pi \left[\frac{(2-x)^3}{-3} \right]_0^2$

$= \frac{-\pi}{3} [-8] = \frac{8\pi}{3}$ وحدة الحجم

$V = \int_a^b A(x) \, dx \rightarrow$ مساحة الدائرة

b



$$V = \pi \int_0^2 (3-0)^2 - \pi \int_0^2 (3-(2-x))^2 \, dx$$

$$V = \pi \int_0^2 9 \, dx - \pi \int_0^2 (1+x)^2 \, dx$$

$$= 18\pi - \pi \frac{(1+x)^3}{3} \Big]_0^2$$

$$= 18\pi - \pi\left(9 - \frac{1}{3}\right)$$

$$= 18\pi - \frac{26}{3}\pi$$

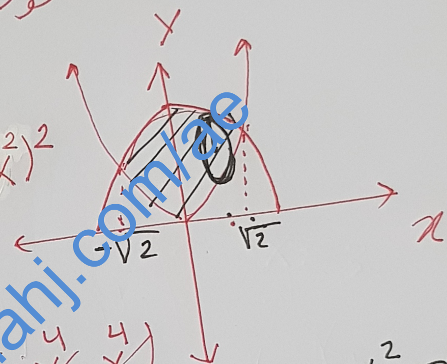
$$= \frac{54\pi - 26\pi}{3} = \boxed{\frac{28}{3}\pi}$$

$\frac{18}{430}$] $Y = 4 - x^2 , \quad Y = x^2$ 1

حول

(a المحور x

(b حول $y=4$



$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - x^2)^2 - (x^2)^2$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (16 - 8x^2 + \cancel{x^4} - \cancel{x^4})$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (16 - 8x^2)\, dx$$

$$= \pi \left[16x - \frac{8x^3}{3} \right]_{-\sqrt{2}}^{\sqrt{2}} = \frac{64\pi\sqrt{2}}{3}$$

$4 - x^2 = x^2$

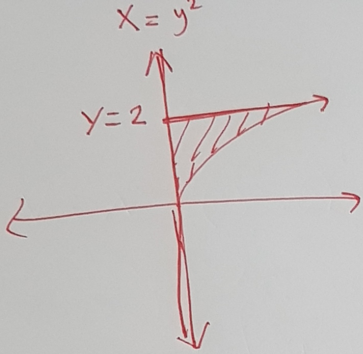
$4 = 2x^2$

$x = \pm\sqrt{2}$

(b) $V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - x^2)^2 - (x^2)^2 \, dx$

$\doteq 64\pi\sqrt{3}/3$

$\boxed{19}$ $y = \sqrt{x}$, $y = 2$, $x = 0$



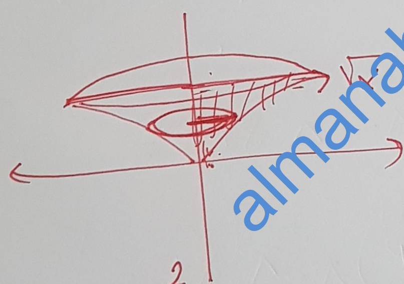
a) حول محور Y

* التحديد المنطقة التي سوف تدور

* $x = y^2$

$A = \pi (y^2)^2$

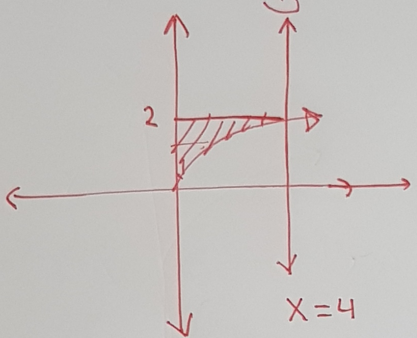
$A = \pi y^4$



$$V = \pi \int_0^2 y^4 \, dy = \pi \frac{y^5}{5} \Big]_0^2$$

$$= \frac{32\pi}{5} \text{ unit cub}$$

b) حول $X=4$



هناك يوجد فجوة في المجسم الناتج

$$V = \pi \int_0^2 (4-0)^2\,dy - \pi \int_0^2 (4-y^2)^2\,dy$$

$$= 32\pi - \pi \int_0^2 (16 - 8y^2 + y^4)\,dy$$

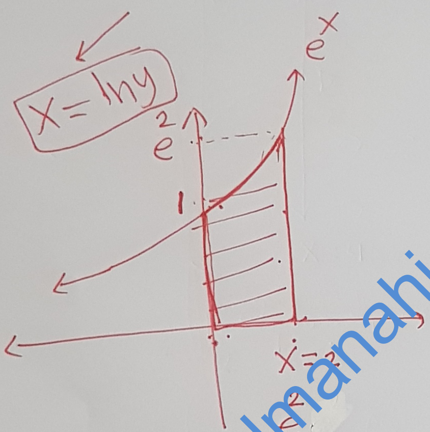
$$= 32\pi - \pi \left[16y - \frac{8y^3}{3} + \frac{y^5}{5}\right]_0^2$$

$$= \frac{224}{15}\pi$$

(21)

$Y = e^{x} \quad , X = 0 , \quad x = 2 \quad , Y = 0$

(a حول محور y

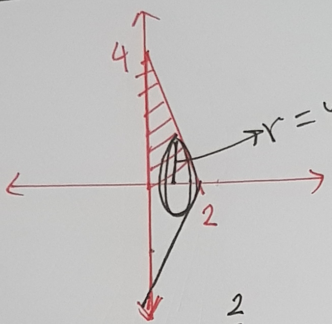


$$V = \pi \int_0^{e^2} (2-0)^2 \, dy - \pi \int_1^{e^2} (\ln y)^2 \, dy$$

$$= 4\pi e^2 - \left[y(\ln y)^2 - 2y \ln y + 2y \right]_1^{e^2}$$

$$= 2\pi (e^2 + 1)$$

25 $Y = 4 - 2X \quad , \quad X = 0 \quad , \quad Y = 0$



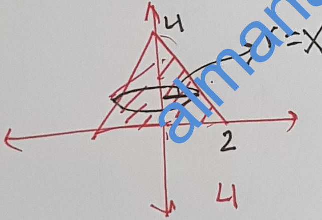
a) حول المحور X

المقاطع العرضية دوائر

لا يوجد مخروم

$$V = \pi \int_0^2 (4-2x)^2 \, dx = \frac{32\pi}{3}$$

b) حول المحور Y

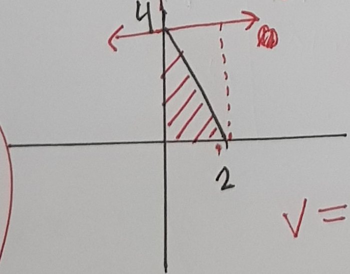


$r = x \Rightarrow 2X = 4 - Y$

$$\boxed{X = 2 - \frac{Y}{2}}$$

$$V = \pi \int_0^2 \left(2 - \frac{y}{2}\right)^2 dy = \frac{16\pi}{3}$$

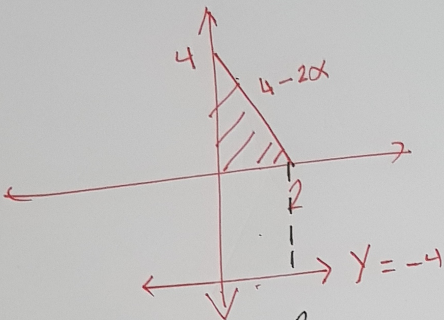
(c) حول $Y=4$ (يوجد مخروم)



$$V = \pi \int_0^2 (4)^2 \, dx - \pi \int_0^2 [4-(4-2x)]^2$$

$64\pi/3$

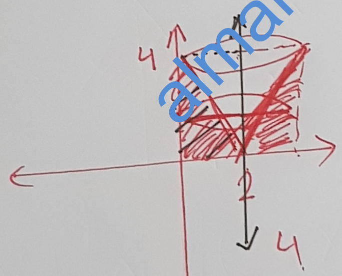
d) حول $y = -4$



$$V = \pi \int_0^2 (8 - 2x)^2 - \pi \int_0^2 (0 - -4)^2 \, dx$$

$$\frac{128\pi}{3}$$

e) حول $x = 2$

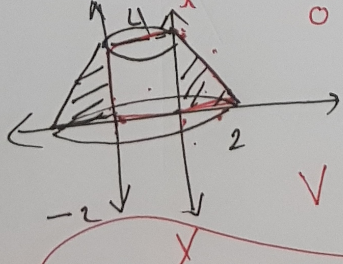


$y = 2, \quad x = 2 - \frac{y}{2}$

$$V = \pi \int_0^4 (2 - 0)^2 - \int_0^4 \left[2 - \left(2 - \frac{y}{2}\right)\right]^2 dy$$

$$32\pi/2$$

f) حول $x = -2$



$$V = \pi \int_0^4 \left(2 - \frac{y}{2} + 2\right)^2 dy - \pi \int_0^4 (0 - -2)^2$$

$$256\pi/2$$

① اقراص وحلقات — الحجوم — 12/4/2020

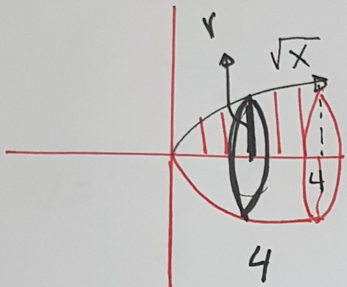
مثال

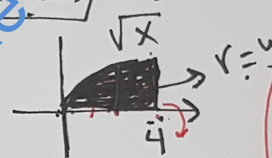
$Y=\sqrt{x}$, $[0,4]$

الدوران حول محور x

$\boxed{r=\sqrt{x}=y}$

$\boxed{A(x)}=\pi x$

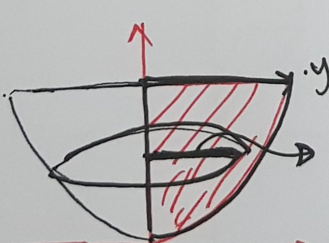




$$V=\pi\int_0^4(\sqrt{x})^2\,dx=\underline{\underline{8\pi}}$$

$Y=\frac{1}{4}x^2$, $X=0$, $Y=1$

① $\boxed{\text{حول محور } Y}$

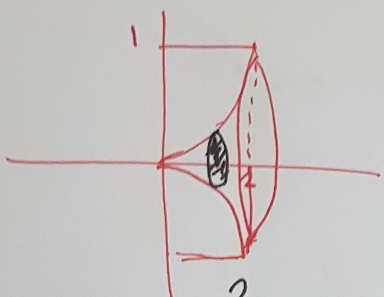


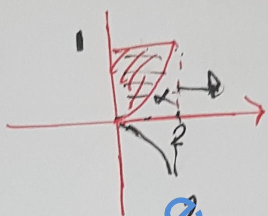
$\longrightarrow r=x \Rightarrow \boxed{x=\sqrt{4y}}$

$$V=\pi\int_0^1(\sqrt{4y})^2\,dy$$

$$\pi\int_0^1 4y\,dy=2\pi$$

حول محور x

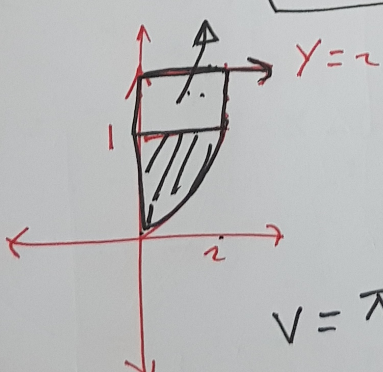




$$V = \pi \int_0^2 \pi (1)^2 \, dx - \pi \int_0^2 \left(\frac{1}{4}x^2\right)^2 dx$$

$$= \frac{8}{5}\pi$$

* $\boxed{\text{حول المستقيم } Y=2}$



$$V = \pi \int_0^2 \left(2 - \frac{1}{4}x^2\right)^2 dx - \pi \int_0^2 (2-1)^2 dx$$

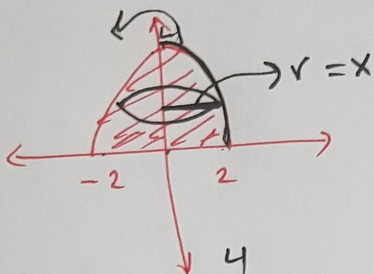
$$\frac{56}{15}\pi$$

3

$y = 4 - x^2 \quad , \; y = 0$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

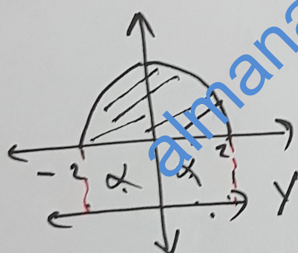


$\boxed{1}$ حول المحور y

حول $\boxed{x = 0}$

$$V = \pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy = 8\pi$$

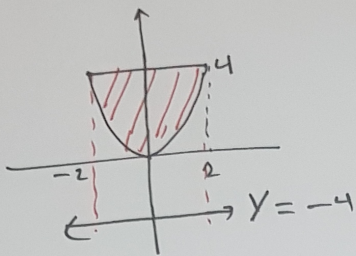
$\boxed{2}$ حول $y = -3$



$$V = \pi \int_{-2}^{2} \left(4 - x^2 - (-3)\right)^2 dx$$

$$- \pi \int_{-2}^{2} (0 - -3)^2 dx$$

$$= \frac{1472\pi}{15}$$



$x^2 = 4$

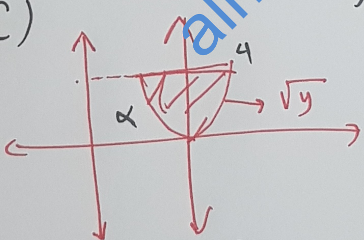
$x = \pm 2$

(d) $y = -4$ حول

$$V = \pi \int_{-2}^{2} (4 - -4)^2 \, dx - \pi \int_{-2}^{2} (x^2 - -4)^2 \, dx$$

$$\pi \int_{-2}^{2} 64 \, dx - \pi \int_{-2}^{2} (x^2 + 4)^2 \, dx$$

(f) $x = -4$ حول



$y = x^2$

$x = \pm \sqrt{y}$

$x = -4$

$$V = \pi \int_{0}^{4} (\sqrt{y} - -4)^2 \, dy - \pi \int_{0}^{4} (-\sqrt{y} - -4)^2 \, dy$$

$$\pi \int_{0}^{4} (\sqrt{y} + 4)^2 \, dy - \pi \int_{0}^{4} (4 - \sqrt{y})^2 \, dy$$

13/4

الثاني عشر

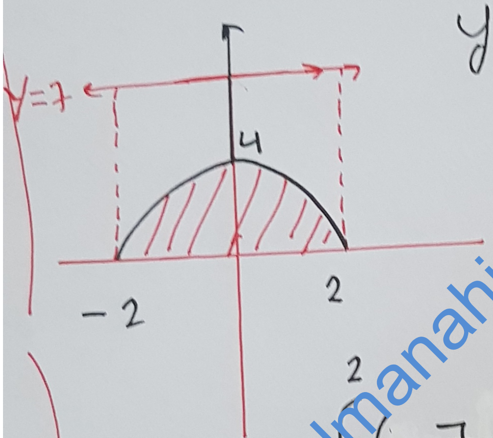
الحجوم / الأقراص

$\boxed{1}$

R : المنطقة المحددة بـ

$y = 4 - x^2 \ , \ y = 0$

$\boxed{1}$ حول $y = 7$



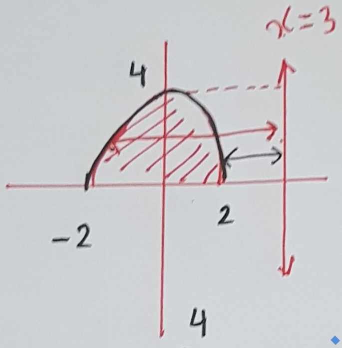
$$V = \pi \int_{-2}^{2} (7-0)^2 \, dx - \pi \int_{-2}^{2} ((7-(4-x^2))^2 \, dx$$

$$= \pi \int_{-2}^{2} 49 \, dx - \pi \int_{-2}^{2} (3+x^2)^2 \, dx$$

$$= \frac{576}{5} \pi$$

(2)

$\boxed{2}$ حول المستقيم $x = 3$



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \pm\sqrt{4-x}$

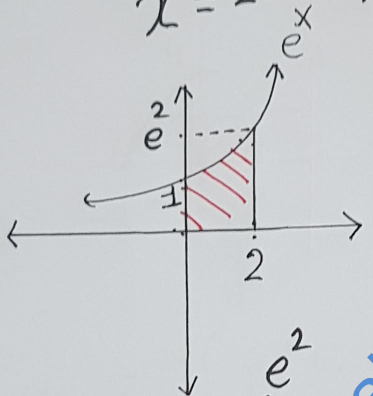
$$V = \pi \int_0^4 \left[\left(3 + \sqrt{4-y}\right)\right]^2 - \pi \int_0^4 \left(3 - \sqrt{4-y}\right)^2 dy$$

$$= 64\pi$$

$\boxed{3}$

$\boxed{21}$ $y = e^x \;,\; x = 0$

$x = 2 \;,\; y = 0$ $\quad \boxed{x = \ln y}$



حول محور Y

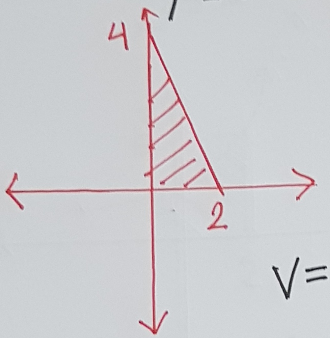
$$V = \pi \int_0^{e^2} (2-0)^2 \, dy - \pi \int_1^{e^2} (\ln y)^2 \, dy$$

$$= 2\pi(e^2+1)$$

$\boxed{4}$

$\boxed{25}$ $y = 4 - 2x \ , \ x = 0$

$y = 0$



$\boxed{1}$ حول محور x

$$V = \pi \int_0^2 (4-2x)^2 \, dx$$

$$= \frac{32\pi}{3}$$

$\boxed{2}$ حول المحور y

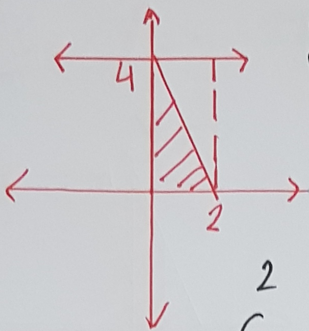
$$x = 2 - \frac{y}{2}$$

$$V = \pi \int_0^4 \left(2 - \frac{y}{2}\right)^2 dy$$

$$= \frac{16\pi}{3}$$

$\boxed{5}$

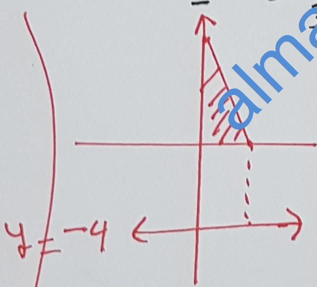
$\boxed{3}$ حول $y=4$



$$V = \pi \int_0^2 (4)^2\,dx - \pi \int_0^2 \left[4-(4-2x)\right]^2 dx$$

$$= \frac{64\pi}{3}$$

$\boxed{4}$ حول $y=-4$

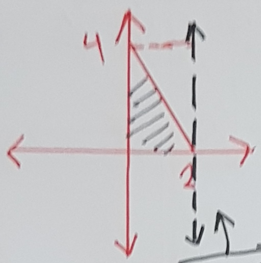


$$V = \pi \int_0^2 \left[(4-2x)-(-4)\right]^2 - \pi \int_0^2 (0--4)^2\,dx$$

$$= \frac{128\pi}{3}$$

$\boxed{6}$

* حول $x=2$



$X = 2 - \frac{y}{2}$

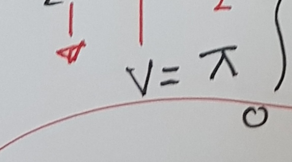
$\boxed{x=2}$

$$V = \pi \int_0^4 (2-0)^2\,dy - \pi \int_0^4 \left[\left(2-\left(2-\frac{y}{2}\right)\right]^2 dy$$

$$= \pi \int_0^4 4\,dy - \pi \int_0^4 \left(\frac{y}{2}\right)^2 dy$$

$$= \frac{32\pi}{2}$$

* حول $x=-2$

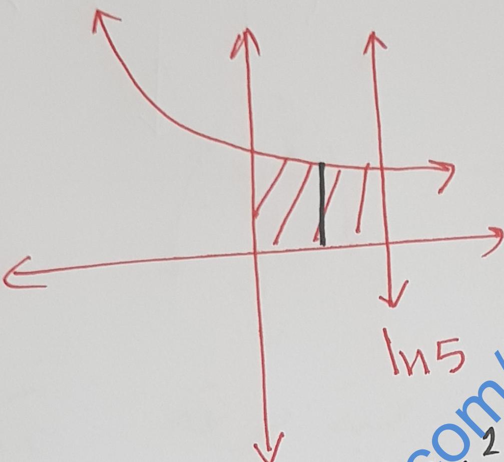


$$V = \pi \int_0^4 \left(2-\frac{y}{2}+2\right)^2 dy - \pi \int_0^4 (0 - -2)^2\,dy$$

$$= 256\pi/2$$

$\boxed{43}$ $y = e^{-2x}$ $y = 0$, $x = 0$

$x = \ln 5$



(a) $A(x) = \left(e^{-2x}\right)^2 = e^{-4x}$

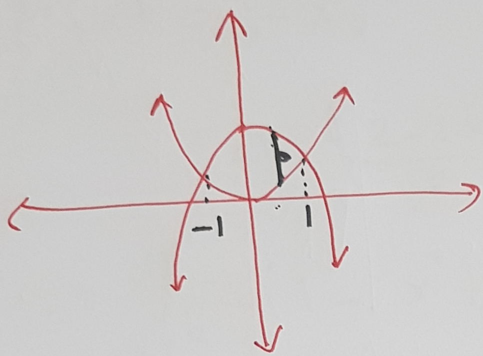
$$V = \int_0^{\ln 5} e^{-4x}\,dx = 0.2496$$

(b) $A(x) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{e^{-2x}}{2}\right)^2$

$= \frac{\pi}{8} e^{-4x}$

$$V = \int_0^{\ln 5} \frac{\pi}{8} e^{-4x}\,dx \approx 0.980$$

$\boxed{41}$ $y = x^2 \;,\; y = 2 - x^2$



$2 - x^2 = x^2$

$2 = 2x^2$

$x = \pm 1$

$(2 - 2x^2)$

(a) مقاطع عرضية مربعة

$A(x) = (2 - x^2 - x^2)^2 = (2 - 2x^2)^2$

$= 4(1 - x^2)^2$

$= 4(1 - 2x^2 + x^4)$

$$V = \int_{-1}^{1} 4(1 - 2x^2 + x^4)\, dx$$

$$= \frac{64}{15}$$

(b) انصاف دوائر

$$A(x) = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi\left(\frac{2-2x^2}{2}\right)^2 = \frac{1}{2}\pi(1-x^2)^2$$

$$V = \int_{-1}^{1} \frac{1}{2}\pi(1-x^2)^2\,dx = \int_{-1}^{1} \frac{1}{2}\pi(1-2x^2+x^4)\,dx$$

$$= \pi\left[x - 2\frac{x^3}{3} + \frac{x}{5}\right]_0^1$$

$$= \pi\left[1 - \frac{2}{3} + \frac{1}{5}\right]$$

$$= \pi\left[\frac{1}{3} + \frac{1}{5}\right] = \pi\left(\frac{8}{15}\right)$$

$$= \frac{\pi}{15}$$

(C)

مثلثات متساوية الاضلاع

$$A(x) = \frac{1}{2}(2-2x^2)^2 \cdot \sin 60$$

$$= \frac{\sqrt{3}}{4}(2-2x^2)^2$$

$$= \frac{\sqrt{3}}{4}[2(1-x^2)]^2$$

$$= \sqrt{3}(1-x^2)^2$$

$$V = \int_{-1}^{1} \sqrt{3}(1-x^2)^2 = 2\sqrt{3}\int_0^1 (1-2x^2+x^4)\,dx$$

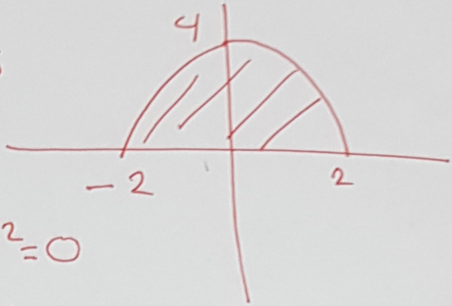
$$= 2\sqrt{3}\left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right]_0^1$$

$$= 2\sqrt{3}\left[1 - \frac{2}{3} + \frac{1}{5}\right]$$

$$= 2\sqrt{3}\left[\frac{1}{3} + \frac{1}{5}\right] = \frac{16\sqrt{3}}{15}$$

$y = 4 - x^2$

نقاط التقاطع مع محور السينات



$4 - x^2 = 0$

$x = \pm 2$

$$A = \int_{-2}^{2} (4 - x^2)\,dx$$

$$= 2 \int_{0}^{2} (4 - x^2)\,dx$$

$$= 2 \left[4x - \frac{x^3}{3} \right]_0^2$$

$$= 2 \left[\frac{8}{1} - \frac{8}{3} \right] = 2 \left(\frac{24 - 8}{3} \right)$$

$$= \frac{32}{3}$$

الاحجام بالاصداف الاسطوانية (6-3)

$$V = \int_a^b 2\pi x f(x)\, dx$$

x : نصف القطر $= r$

تمثل البعد عن محور الدوران

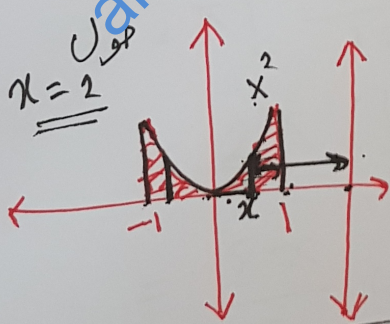
$f(x)$: الارتفاع $= h$

ارتفاع الاسطوانة المرسومة

اذا رسم الخط موازي لمحور الدوران

dx : السماكة

1] حول $\underline{\underline{x=2}}$



$\boxed{x=2}$

$\boxed{r}$: $2-x$

h = ارتفاع الاسطوانة هو $f(x)$

$\boxed{h = x^2}$

$\boxed{2}$

$$V = 2\pi \int_{-1}^{1} \underbrace{(2-x)}_{r} \underbrace{(x^2)}_{f(x)} \cdot dx$$

$$= 2\pi \int_{-1}^{1} (2x^2 - x^3)\, dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1}$$

$$= 2\pi \left[\left(\frac{2}{3} - \frac{1}{4} \right) - \left(\frac{-2}{3} - \frac{1}{4} \right) \right]$$

$$= 2\pi \left[\frac{5}{12} + \frac{2}{3} + \frac{1}{4} \right]$$

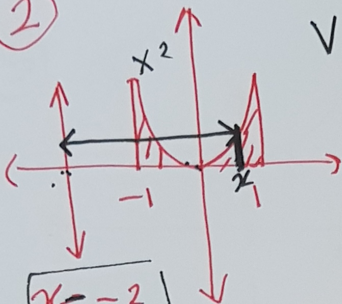
$$= 2\pi \left[\frac{5+8+3}{12} \right]$$

$$= 2\pi \left(\frac{16}{12} \right)$$

$$= \frac{8\pi}{3}$$

$\boxed{3}$

(2)



$\boxed{x = -2}$

محور الدوران

$$V = 2\pi \int_{-1}^{1} (\underbrace{x+2}_{r})(x^2)\, dx$$

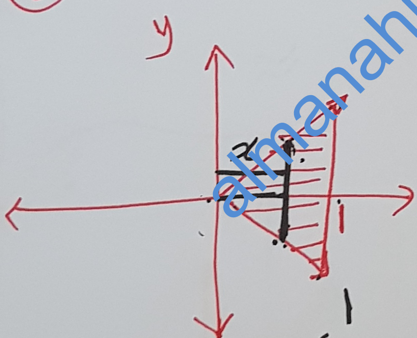
$$= \frac{8\pi}{3}$$

$r = x + 2$

$h = x^2$

(3) $y = x$ $\quad y = -x$ $\quad x = 1$

حول المحور y



$r = x$ ✓

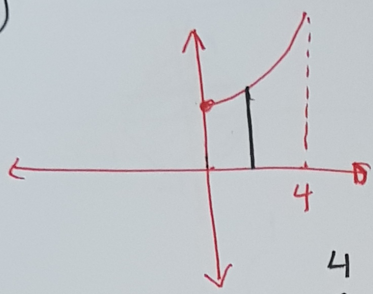
$h = 2x$ ✓

$$V = 2\pi \int_{0}^{1} x\,(2x)\, dx$$

$$= 2\pi \int_{0}^{1} 2x^2\, dx = \frac{4\pi}{3}$$ ✓

5

حول محور
الصادات



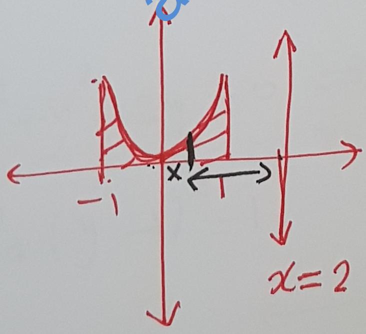
$r = x$

$h = f(x) = \sqrt{x^2+1}$

$$V = 2\pi \int_0^4 x\sqrt{x^2+1}\,dx$$

≈ 144707

6



$r = 2 - x$

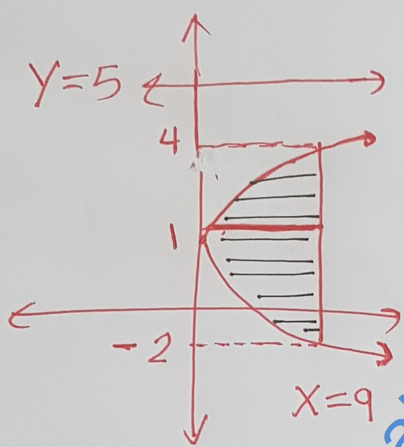
$h = f(x) = x^2$

$$V = 2\pi \int_{-1}^{1} (2-x)(x^2)\,d$$

$$= \frac{8\pi}{3}$$

$$x = (y-1)^2 \ , \ x = 9$$

$Y = 5$ حول



أجد نقاط التقاطع

$$9 = (y-1)^2$$

$$\pm 3 = y - 1$$

$$y - 1 = 3$$

$$\boxed{y = 4}$$

$$y - 1 = -3$$

$$\boxed{y = -2}$$

$$r = 5 - y$$

$$h = 9 - (y-1)^2$$

$$v = 2\pi \int_{-2}^{4} (5-y)(9-(y-1)^2)\,dy$$

$$= 288\pi$$