TEMPERATURE FITTING METHOD FOR PREDICTING EQUIDISTANT VOLTAGE DROP OF ANODE ROD IN ALUMINUM REDUCTION CELL

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Key words: Temperature equation, Velocity, Aluminum reduction cell, Anode rod, Equidistant voltage drop

Abstract

In this paper, the temperature equation for an anode rod of aluminum reduction cell is derived based on principles of energy conservation and heat sources superposition. Electrical heat is used as the first inner heat source, while heat coming from surrounding media is used as the second inner heat source. All relevant parameters for representing inner current of anode rod are deduced completely. They are relative to rod's length, rod temperature, temperature and velocity of surrounding gas. A new test method and a series of curve fit equations for representing the state of equidistant voltage drop are presented. Large amount of on-line monitoring data on different anode rods show that the agreement between the curve which is obtained from curve fit equations and experimental curve of equidistant voltage drop is good, and the correlation coefficient is larger than 0.90.

Introduction

According to National Twelfth Five-Year Plan of China which focuses on energy, resources and ecological environment, the development of electrolytic aluminum industry has entered into the era of large scale production with reduction in energy consumption and emissions, intelligent production via better process control, online monitoring and cleaner process practices. In the search of optimum structure of aluminum reduction cell, the anodic current distribution has been drawing close attention because of its significant impact on current efficiency of electrolytic cell.

In recent years, considerable amount of work is carried out in the relevant fields such as the relation between efficiency and distribution of current^[11], the characteristic and design of anodic current distribution^[21], spectrum analysis of anodic current^[3,4], electro-thermal simulation and analysis of aluminum reduction cell^[5-11], the fundamental and applied research on on-line monitoring system for current distribution and level fluctuation of liquid aluminum^[12-16] and so on. The utilization of various simplifying assumptions results in deviations between the predictions of electro-thermal simulations and the measured values. So, real-time monitoring becomes the basic method for verifying and revising the simulation results. This monitoring method is also important for collecting data on the status analysis of aluminum reduction cell.

Presently, current distribution of anode rod is generally represented by the method of equidistant voltage drop measurements for which extensive preparations are needed. For an example, numerous connecting wires and test probes have to be used to ensure the contact between the interfaces of integrated control system for signal acquisition and conversion^[12-15]. In this study, a new perspective on temperature field equation of anode rod is put forward. The equation represents the combined effect of electro-thermal phenomena and convection heat transfer. The new parameters and curve fit equations are determined, a new method of acquiring an equivalent curve, which can perfectly reflect the state of equidistant voltage drop on anode rod, is derived by monitoring these parameters, determining their values and using curve fit equations.

1. Theory and selection of parameters to be measured

According to the theory on electro-thermal conversion, internal current of conductor can be related to surface temperatures of electrified conductor without taking the influence of surrounding media into consideration. In electrolytic production process of aluminum, anode rod is the major part between transverse busbar and anode carbon block and the environment of anode rod as shown in figure 1.



Figure1. Diagram of anode rod and its surrounding.

The presence of current is found to be the major factor determining the surface temperature of anode rod. When electric current flows through the anode rod from transverse busbar, first, the amperage of current has an impact on the electro-thermal conversion taking place in anode rods. Next, the temperature of surrounding media around the rods also affects surface temperatures of anode rod. Outside the cover of the cell, there is convective heat transfer taking place between the surrounding air around anode rods and the rods. While inside the cover of the cell, there is convective heat transfer between inner flue gas and anode rods. The relevant parameters for the amperage of internal electric current can be determined based on the temperature indicated by electro-thermal action. On the other hand, the relevant parameters can also be determined based on the temperature established by heat transfer with surrounding media.

In this study, the heat produced due to electric current passing through anode rod is used as the first inner heat source, while the heat transferred from surrounding media to the surface of anode rod is used as the second inner heat source. The principle of heat source superposition is applied in three-dimensional differential equation of heat transfer, which is based on the principle of energy conservation. The flow of heat into and out of the infinitesimal element of anode rod per unit time and the heat produced due to electric current flowing through anode rod are all taken into consideration.

It is assumed that anode rod is isotropic and it is a continuous medium. Based on the principle of energy conservation, if the heat of formation of inner heat source per unit volume per unit time is dF_v , the added value of thermodynamics energy per unit time can be obtained by the total heat flux passing through infinitesimal element per unit time and dF_v . The coefficient of heat conduction of aluminum rod, k_{Al} , is taken as a constant. ρ_{Al} and c_{Al} represent density and specific heat of aluminum, respectively. Then, at any time t, three-dimensional differential equation of heat transfer at a point, with Cartesian coordinate (x, y, z), on aluminum rods which have inner heat source, can be written as:

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + f(x, y, z, t)$$
(1)

where *u* represents the temperature of anode rod, whereas, a^2 and f(x, y, z, t) can be further expressed as:

$$a^2 = \frac{k_{Al}}{\rho_{Al}c_{Al}} \tag{2}$$

$$f(x, y, z, t) = \frac{F_v}{\rho_{Al} c_{Al}}$$
(3)

For the inner heat source, r is the electrical resistance coefficient of aluminum rod, S is the cross-sectional area of aluminum rod. $2\delta_1$ and $2\delta_2$ are the length and the width of this area, respectively. k_1 is the heat transfer coefficient between surrounding gas and the rod surface. So, according to equivalence of work and heat, heat produced can be expressed as follows when electric current i passes through per unit length of aluminum rod:

$$F_1(x, y, z, t) = \frac{0.24i^2 r}{S^2} = \frac{0.24i^2 r}{(2\delta_1 \cdot 2\delta_2)^2}$$
(4)

For the second inner heat source, if an aluminum rod with temperature u(x, y, z, t) is put into a gas medium with temperature $u_1(x, y, z, t)$, according to Newtonian law, heat transfer, Q, between aluminum rod and gas medium is directly proportional to the temperature difference between them as shown below:

$$dQ = k_1(u - u_1)dSdt \tag{5}$$

The heat transfer between the surface of aluminum rod and surrounding media can be deemed as passive source. The inflow heat, produced heat and required heat for temperature change in a section of rod, with coordinates from (x, y, z) to $(x + \Delta x, y + \Delta y, z + \Delta z)$ and a time from t to $t + \Delta t$, are taken as dQ_1 , dQ_2 and dQ_3 , respectively. Based on the principle of energy conservation, the relation between them is given as: $dQ_3 = dQ_1 + dQ_2$. If $\Delta x, \Delta y, \Delta z \rightarrow 0$, the following relationship is obtained:

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{1}{\rho_{Al} c_{Al}} \frac{k_1 (\delta_1 + \delta_2)}{\delta_1 \delta_2} (u - u_1)$$
(6)

Only considering the heat transfer between aluminum rod and surrounding gas medium without taking the heat transfer by electric current passing through the rod into account, the comparison of the equations (1) and (6) gives:

$$F_2(x, y, z, t) = -\frac{k_1(\delta_1 + \delta_2)}{\delta_1 \delta_2} (u - u_1)$$
(7)

So, the total heat source can be written as:

$$F_v = F_1 + F_2 \tag{8}$$

Further, based on the principle of heat source superposition and substituting equation (8) into the equation (1), the heat balance equation for the anode rod (9) is given as:

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{0.03i^2 r - 2\delta_1 \delta_2 k_1 (\delta_1 + \delta_2)(u - u_1)}{2\delta_1^2 \delta_2^2}$$
(9)

In modeling electro-thermal systems, the selection of initial-boundary value condition is, in general, the key for solving the governing equations established according to Laplace and Poisson equations. The differential equation of the temperature field of anode rod in aluminum reduction cell is derived and the relevant parameters reflecting inner electric current of anode rod are shown in this paper. According to the knowledge of the authors, no such equation exists in existing literature. It is clear from the equation (9) that the temperature of any point in anode rod, with coordinates (x, y, z), is affected not only by the conductor's physical properties such as k_{Al} , ρ_{Al} , c_{Al} , δ_1 , δ_2 , but also by the temperature of surrounding medium u_1 and the surface heat transfer coefficient k_1 . The surface heat transfer coefficient includes both convection and radiation. However, it should be noted that the materials exchanging heat with anode rod are both gaseous and can be assumed transparent. Therefore, the temperature and velocity of the gases affecting the surface heat transfer coefficient are the factors needed to be considered.

Based on the derived equation above and the following two points, the parameters related to the equidistant voltage drop (ε) and the inner current of anode rod (ψ) are ultimately determined to be temperatures of anode rods, temperatures of surrounding gases, and velocities of surrounding gases. First, the physical properties, the shape and size of every anode rod in one aluminum reduction cell are the same. Second, the principle for position selection of measured points is the same. However, with the consumption and replacement of anode carbon used in the production of electrolytic aluminum, the anode height changes constantly. Altough the measured points of different rods in one cell are at the same level, which is the same absolute position along the length of the rod (the height of measured point which is shown with white dashed line in Figure 2), their relative positions (the height between measured points and yoke which is marked with white short dash-dotted line in Figure 2) are different. So, a parameter which is related to the height of the rod and represented with l is used in the calculations. Besides, it should be pointed out that the parameter u_1 and k_1 in equation (9) have different values at different positions. They represent the temperature of surrounding gases and the surface heat transfer coefficient between anode rods and surrounding gases, respectively. Furthermore, they represent the temperature of surrounding air and surface heat transfer coefficient between anode rods and surrounding gas inside the cell cover whereas they represent the temperature of flue gas and surface heat transfer coefficient between anode rods and flue gas outside of the cell cover.

From the above analysis, it is proposed that the function representing the inner current of anode rods can be expressed as a function of a number of parameters which can be measured experimentally at the same absolute positions:

$$\psi = f(u_{rod}, u_{air}, u_{flue-gas}, v_{air}, v_{flue-gas}, l)$$
(10)

where u_{rod} is the temperature of anode rod, u_{air} and v_{air} are respectively the temperature and the velocity of air around the anode rod outside the cell cover, $u_{flue-gas}$ and $v_{flue-gas}$ are respectively the temperature and the velocity of flue gas around the anode rod inside the cell cover.



Figure 2. Measured position of anode rod.

2. Industrial measured parameters and the fitting curve equivalent to the curve of equidistant voltage drop

2.1 Specification on industrial measured parameters

During the industrial tests to measure the parameters such as u_{rod} , u_{air} , $u_{flue-gas}$, v_{air} , $v_{flue-gas}$, l, attention should be paid to the following points.

First point of concern is the selection of the anode rod temperature measurement positions. The electric current passes through transverse busbar to anode rod and then from voke to anode carbon. The internal electrical current distribution of the rod is inhomogeneous in two positions: close to transverse busbar and yoke. So, it is decided that the measurement points should be selected where the internal electric current distribution is uniform. The points should be far away from contact points of the rod and transverse busbar as well as contact points of the rod and yoke. Because the temperature of anode rods in flue gas inside the cover of the cell is difficult to be measured and it is greatly influenced by the flow of surrounding flue gas, the measurement points should be selected outside the cover of the cell but as close to the cover as possible. In other words, it is necessary that the absolute positions of measurement points of different rods are the same during the actual measurements.

Second point of concern is duration of the measurement time of anode rod temperature. Model HFM-215N heat flow meter is used for the temperature measurements. To ensure that the surface temperature can accurately reflect the internal temperature, it is necessary to have adequate contact time between temperature transducer and the surface of the rod. Based on a large number of experimental data, it is proved that infrared thermometer used for the surface temperature measurement of anode rod is efficient and the measurement results confirm this conclusion.

Third point is related to the measurement of other parameters. In the plants, the influence of two parameters (u_{air} and v_{air}) of air around the rods located at both ends of the cell is important, while their influence on the ten middle rods of the cell is relatively minor. So, parameters u_{air} and v_{air} are neglected in practical measurements in order to reduce measurement work. Model TH-880F smoke dust sampled-data control system controlled by microcomputer is used for the measurement of the other two parameters ($u_{flue-gas}$ and $v_{flue-gas}$) for the flue gas.

Fourth point is related to facilitate the demonstration of the dimension l which denotes two parameters, l_{rod} and $l_1 \cdot l_{rod}$ represents the distance between measured point and the top of anode rod and l_1 represents the distance between the lower edge of transverse busbar and the upper edge of horizontal cover.

2.2 Illustrations of equivalence between the curve fitted to ψ and the curve of equidistant voltage drop

According to equation (10) and assuming that the same parameters also influence the equidistant voltage drop (ε) and the inner current of anode rod (ψ), the curve fit for ψ is carried out by making use of large amount of industrial data, which are u_{rod} , $u_{flue-gas}$, $v_{flue-gas}$, l_{rod} and l_1 , measured at each rod. Detailed data fit procedure is as follows. u_{rod} , $u_{flue-gas}$, $v_{flue-gas}$, are plotted separately as a function of rod number and the ε is obtained from the measured data of equidistant voltage drop also as a function of anode rod number (figure 3). From this figure, it can be observed that the trends of

 u_{rod} and ε are relatively similar and the difference between them might be explained with the influence of the surrounding media. So, the influence of the four parameters, which are $u_{flue-gas}$, $v_{flue-gas}$, l_{rod} and l_1 , can be considered as the relevant parameters affecting u_{rod} . The fit is carried out using the Equations (11) to (15) repeatedly until the agreement of the trends between the curve fitted to ψ and the curve ε becomes better and better. Excel and Origin software are used for fitting the data, and a new curve is obtained after number of trials. The constants of the fit equations are determined. The correlation coefficients of the two curves, ψ and ε , are larger than 0.90, which shows that the fit equations have certain applicability.

Below, a set of industrial data is used as an example to compare and explain the equivalence of the two curves. The industrial measurements are carried out in a 400kA prebaked anode aluminum reduction cell with 24 group anode rods. The measured cell is located around the middle of the line. The number of current input points on the cell is six and there are six input points along the length of the cell. Electric current, cell voltage, temperature of electrolytic reduction, the height of electrolyte and liquid aluminum in the process of measuring are 429kA, 3.87V, 963°C and 20cm/29cm, respectively.

The data for u_{rod} , $u_{flue-gas}$, $v_{flue-gas}$, l_{rod} are listed in table I and the parameter l_1 is measured as 383mm. The value of equidistant voltage drop of anode rod, ε , is obtained from the on-line monitoring and recording system which is made by Northeastern University. The value of ε per anode rod, shown in the sixth column of table I, is the average value of data sampled for 10 minutes and the detailed operation steps are given in reference [13]. The value of ψ , obtained from the curve fit equations (11) - (15) shown in Section 2.3 below, is listed in the last column of table I.

Table I. Industrial Measurements (columns 1 to 5) and Calculation (column 6) Results

No.	u _{nod} ∕°C	u _{gas} ∕°C	<i>V</i> _{gas} /(m/s)	l _{rod} /mm	€ /mV	Ψ
1	148.3	62.8	1.3	1149	378.2	139.1
2	176.1	85.4	1.42	1000	351.7	206.4
3	191.9	89.1	1.62	1000	362.2	215.6
4	164.4	119.2	1.64	1130	315.7	183.0
5	167.6	125.2	1.73	1102	315.9	191.1
6	117.2	126.7	1.78	1092	231.2	139.1
7	150.3	134.9	1.99	995	297.0	179.6
8	163.7	169.7	1.99	1105	328.0	202.4
9	185.1	165.5	2.09	1005	383.5	235.9
10	164.6	169.3	2.09	970	366.7	222.1
11	153.0	180.9	2.28	997	334.1	193.1
12	119.8	171.5	2.28	1118	288.3	136.0

In figure 3, five curves are for the measured variables (anode rod and flue gas temperatures, flue gas velocity, distance between

measured point and the top of the anode rod, equidistant voltage drop on anode rods) as a function of anode rod number) and the sixth curve is the curve fit for ψ .



Figure 3. Diagram on the curve of measured parameters, the curve of equidistant voltage drop and fitting curve

The trend of ε and ψ is quite similar as can be seen in figure 3. Using the data of curve ε and ψ given in table I, the correlation coefficients of the two curves are determined as 0.62. If the terminal measured points of the cell are excluded, the correlation coefficient of both curves becomes 0.99. However, the variations of curve ε and ψ at terminal measurement points, respectively at duct end and tube end, have an obvious difference because of the influence of surrounding air. Moreover, because the operations such as liquid aluminum tapping and residue salvaging also affect the fluctuation of the curves ε and ψ observed at terminal measurement points due to t_{air} and v_{air} can be ignored for practical purposes.

The analysis on the electric current distribution state of anode rods represented by the curve of equidistant voltage drop can also be found in the literature [13]. Since the agreement of the trend between curves ε and ψ is found to be good, the method described in this paper can replace the method of equidistant voltage drop for analyzing electric current distribution of anode rods used in aluminum reduction.

2.3 Curve fit equations

It is important that the above parameters are measured repetitiously not only in many more aluminum reduction cells but also in one cell with the same performance conditions to ensure the accuracy of fitting curve and the universality of equivalence between fitting curve and the curve of equidistant voltage drop on anode rods. The curve fit equations are as follows:

$$\psi = \frac{6A^{C_1 + C_2}}{B} \tag{11}$$

A, B, C_1 , C_2 in equation (11) are expressed in the following equations respectively.

$$A = \frac{\exp\left[\frac{3}{16}u_{rod} + \frac{1}{20}u_{flue-gas} + \frac{1}{12}\left(\bar{u} + u_{max}\right)\right]}{100}$$
(12)

$$B = (0.75l_{rod} + 1.65l_1)^{1.65} + (0.25l_{rod})^{1.8}$$
(13)

$$C_1 = 1.2\left(\nu + \nu_{\rm m \ i}\right) e^{x} \left(\frac{\nu_{\rm m \ a \ x} - \nu}{\overline{\nu}}\right)$$
(14)

$$C_2 = 72(0.0 \, \ln_{rod})^{\frac{\nu_{\max} + \nu}{6}} + 2(1.2u)^{\frac{\nu_{\min} + \nu}{12}}$$
(15)

where u, \overline{u} , u_{max} are the value, average value and maximum value of the temperature in flue gas around each rod, respectively. v, \overline{v} , v_{max} , v_{min} are the value, average and maximum velocity of flue gas around each rod, respectively.

The principle of equivalence between the two curves is similar with the operation principle of thermo-electric couplers. It is assumed that the anode rod and the flue gas current are in contact with good conductor of heat, and they constitute a closed circuit. So, there is temperature gradient only away from the rod due to the influence of surrounding gases. Electrical current can exist in this circuit. Then, Seebeck electromotive force is generated in the distance, which is called Seebeck effect. Therefore, the dimension ψ can be described as the thermo-electromotive force represented by the temperature difference of anode rod per unit distance in measuring range. This is the reason why there is higher inosculation between the fitting curve of ψ and the curve ε of equidistant voltage drop. The index of numerator in equation (11) is equivalent to the conduction impact of thermo-electric couplers principle, where C_1 (Equation 14) represents the impact of convection and C_2 (Equation 15) represents the impact of radiation. The denominator in equation (11) is equivalent to the effective distance of thermo-electric couplers, in which electromotive force is generated.

3. Conclusions

First, the differential equation (9) on temperature of anode rod in aluminum reduction cell is derived, and it is clearly pointed out that the relevant factors for reflecting internal current of anode rod are $u_{rod}, u_{gas}, v_{gas}$ and l based on the temperature of the rod.

Second, a mass of industrial test results show that the five parameters, including u_{rod} , $u_{flue-gas}$, $v_{flue-gas}$, l_{rod} and l_1 , can accurately represent the behavior of equidistant voltage drop on anode rods in aluminum reduction cell based on the curve fit ψ determined during this research. The correlation coefficient of curve ε and ψ is larger than 0.90.

Third, the method described in this paper can reduce the experimental work which involves drilling in anode rods, probe localization, connecting and protecting of signal cable.

Fourth, the state of curve ψ is identical with the state of the curve ε . According to reference [13], the method presented in this paper can also represent the trend of electric current distribution on anode rods in aluminum reduction cell.

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