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STABILITY OF ALUMINUM CELLS - A NEW APPROACH

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Small periodic disturbances are superposed to the equilibrium shape of the interface between cryolite and aluminum, and a linear analysis of their properties is proposed. Perturbations of electric quantities are first calculated, and then perturbations of velocities and pressure in the two liquids follow from 3D Navier-Stokes equations with a linear drag. The result is a characteristic equation which relates the parameters of the disturbance (wave length, rate of amplification) to the parameters of the cell (current densities, thicknesses of liquid layers, friction coefficients, equilibrium velocities...).

It is found that two different kinds of instabilities may develop. One is the classical Kelvin-Helmholtz instability of sheared interfaces slightly modified by MHD effects ; it may generate small scale disturbances (20 cm). The other is a new instability, essentially electromagnetic in nature, able to generate large scale waves (1.5 m) propagating in the direction of horizontal current.

INTRODUCTION

The interest in the instabilities of aluminum reduction cells and in appropriate techniques for preventing their development became more important during the past few years. The main reason is undoubtedly the perspectives of development of new cells with electric current up to 200 K Amp or more, and it is remarkable that such an increase of interest coïncides with the 100th anniversary of the invention of smelting techniques by Hall and Héroult.

Two main papers |1,2| were concerned with analytical models of these instabilities in simplified conditions. They both neglect viscosity or any other damping effect. Urata's model |1| is quite close to real cells, but the different mechanisms (influence of shear at the interface, of finite size of the two rectangular liquid layers, of the horizontal electric current, etc) are studied in different sections and do not belong to the same and unique theory. Sneyd's analysis |2|, the most recent paper on the subject, apparently ignores Urata's pioneering work, and is limited to an extremely idealized model without any horizontal electric current, an ingredient reputed to be at the root of the phenomenon. For this reason these two papers do not take into account the exact force field acting on disturbances of the interface and do not start with equations complete enough to allow a full description of the real instabilities. This certainly explains the differencies between predictions from Urata's theory and experiments on a real pot |3|.

The purposes of this new paper on the subject is i) to propose some progress in the analysis of the key physical mechanisms of these instabilities, ii) to introduce a better theoretical basis for modelling the phenomena and iii) to provide some relation between the parameters of the cell (geometry, electric current, magnetic field...) and the properties of disturbances of the interface (wave length, rate of amplification, phase velocity...). This new analysis takes into account the horizontal current in the aluminum as well as the mean velocities of the two superposed liquids and the influence of turbulence. It is however still a first step in the sense that the finite size of the cell and the variations along the interface of unperturbed quantities (velocities, electric current, magnetic field...) are not taken into account. The friction on the electrodes and the vertical transport of momentum in each liquid layer are modelled with the linear assumption introduced by Moreau and Evans |4|, which was recently confirmed by a laboratory experiment |5|.

THE DISTURBANCE OF ELECTRIC QUANTITIES

In the equilibrium state the curvature of the interface is supposed to be negligible. According to the general linear theory of hydrodynamic stability (see Chandrasekhar |6|) a disturbance of that interface by the form

$$z_o = A\, e^{st + i\, \underline{k}.\underline{r}} \tag{1}$$

is introduced, characterized by a wave vector $\underline{k}$, the two components of which (k_x and k_y) are real numbers. The increment s has to be determined as a function of $\underline{k}$ and of the parameters of the cell. The stability criterion is obviously Re(s) < 0.

The disturbance of the electric potential due to (1) comes from the Laplace equation $\Delta\phi=0$, together with the boundary conditions at the electrodes which are assumed to be sufficiently conductive so that

$$\phi(z=h) = \phi(z=-H) = 0, \tag{2}$$

and at the interface where the total potential and the normal component of the electric current density are continuous

$$\Phi_+(z_o) + \phi_+(z_o) = \Phi_-(z_o) + \phi_-(z_o) \tag{3}$$

$$\sigma_+ \left(\frac{\partial\phi_+}{\partial n}\right)_{z_o} = \sigma_- \left(\frac{\partial\phi_-}{\partial n}\right)_{z_o} \tag{4}$$

(Subscripts + or − refer to the side z>0 or z<0 of the interface, capital letters to unperturbed quantities, and small letters to their disturbances.)

According to Moreau and Evans (4) the unperturbed electric potential at the interface is taken as

$$\Phi(z_o) = \frac{J_o}{\sigma}\, z_o \tag{5}$$

in the two liquids (J_o is the uniform vertical component of the current density). This does not imply that the electric current in the aluminum is neglected, since it is a second order term in z_o^2.

The solution is :

$$\phi = A\, e^{st + i\, \underline{k}.\underline{r}}.f(z) \tag{6}$$

with

$$f_+(z) = J_o \frac{\sigma_- - \sigma_+}{\sigma_+} \cdot \frac{chkH.shk(z-h)}{\sigma_- shkh.chkH + \sigma_+ shkH.chkh}$$

$$f_-(z) = J_o \frac{\sigma_- - \sigma_+}{\sigma_-} \cdot \frac{chkh.shk(z+H)}{\sigma_- shkh.chkH + \sigma_+ shkH.chkh} \tag{7}$$

Expressions of the disturbances for the current density directly follow :

$$\underline{j} = -\,\sigma\, A\, e^{st+i\underline{k}.\underline{r}} \cdot \left[if(z)\, \underline{k} + f'(z)\, \underline{\lambda} \right] \tag{8}$$

$\underline{\lambda}$ being the unit vector in the z direction. Thus the magnetic field induced by such an electric current is :

$$\underline{b} = \mu\sigma\, A\, e^{st+i\underline{k}.\underline{r}}\, F(z) \cdot \frac{\underline{k}}{k} \times \underline{\lambda}$$

with :

$$F_+(z) = -\, iJ_o \frac{\sigma_- - \sigma_+}{\sigma_-} \cdot \frac{chkH.chk(z-h)}{\sigma_- shkh.chkH + \sigma_+ shkH.chkh}$$

$$F_-(z) = -\, iJ_o \frac{\sigma_- - \sigma_+}{\sigma -} \cdot \frac{chkh.chk(z+H)}{\sigma_- shkh.chkH + \sigma_+ shkH.chkh} \tag{10}$$

THE DISTURBANCE OF THE FORCES

Lorentz forces per unit volume may be written so:

$$\left[\underline{J}_H + J_v\,\underline{\lambda} + \underline{j}_H + j_v\,\underline{\lambda}\right] \times \left[\underline{B}_H + B_v\,\underline{\lambda} + b\left(\frac{\underline{k}}{k} \times \lambda\right)\right]$$

There are five linear vector products distinguishable in such a linear stability analysis :

$$\begin{aligned}
\underline{f}_1 &= J_v\,\underline{\lambda} \times b\,\left(\frac{\underline{k}}{k} \times \underline{\lambda}\right) = J_v\, b\,\frac{\underline{k}}{k} \\
\underline{f}_2 &= J_v\,\underline{\lambda} \times \underline{B}_H \\
\underline{f}_3 &= \underline{j}_H \times B_v\,\underline{\lambda} \\
\underline{f}_4 &= \underline{j}_H \times \underline{B}_H \\
\underline{f}_5 &= \underline{J}_H \times b\,\left(\frac{\underline{k}}{k} \times \underline{\lambda}\right)
\end{aligned} \tag{11}$$

It may first be noticed that three of them have no influence at all on the disturbance, specifically those related to the undisturbed magnetic field

$\underline{B}_H + Bv\,\underline{\lambda}$. This result is not obvious. It comes i) from the fact that the combination of $\underline{f}_2$ and $\underline{f}_4$ which intervenes in the detailed treatment of the motion equations

$$-\frac{\partial}{\partial z}(\nabla_H.\underline{f}_2) + \Delta_H(\underline{f}_4.\underline{\lambda}) = -i\sigma\Delta\phi\ \underline{\lambda}.(\underline{k} \times \underline{B}_H)$$

is necessarily zero because $\Delta\phi = 0$, and ii) from the fact that $\underline{f}_3$ only intervenes through the scalar product $\underline{k}.\underline{f}_3$ which is identically zero. Therefore any uniform external magnetic field has no influence on periodic disturbances such as (1). This is in agreement with Urata's conclusion |3| that the distribution of the magnetic field rather than its absolute intensity may have some influence.

Therefore only the elementary forces $\underline{f}_1$ and $\underline{f}_5$, representing the interaction between the undisturbed current $\underline{J}_H + J_v\,\underline{\lambda}$ and the disturbance of the magnetic field $\underline{b}$, are relevant. The force $\underline{f}_1$ is easy to interpret. It represents the well known pinch effect. Near a bulge of the interface, it is directed towards the center of the bulge where extra-current is passing. The effect acts on the two fluids but with different intensities. It can be predicted that an extra-pressure δp appears at the center of the bulge in order to balance the pinching forces, with different values δp_+ and δp_- in the two liquids. The sign of the difference $\delta p_+ + \delta p_-$ controls the stabilizing, or destabilizing, character of this "differential pinch effect".

The force $\underline{f}_5$ is vertical and is confined within the aluminum layer where $\underline{J}_H$ is non zero. Being proportional to b, which is not in phase with the deformation of the interface, it tends to elevate the aluminum on one side of a bulge and to lower it on the other side, thus provoking some propagation in the direction of $\underline{J}_H$ as well as some torque on each half wave length of the interface. One may suppose that such a torque is a source of vorticity and is destabilizing.

Expressions of the relevant forces are :

$$\underline{f}_{1+} = i\mu J_o^2 \frac{\mathrm{ch}\ k(z-h)}{\mathrm{sh}\ kh} A\ e^{st + i\underline{k}.\underline{r}} . \frac{\underline{k}}{k} \qquad (12)$$

$$\underline{f}_{1-} = i\mu J_o (J_o + J_1 \frac{z}{H}) \frac{\mathrm{ch}\ k(z+H)}{\mathrm{th}\ kh.\mathrm{ch}\ kH} . A\ e^{st+i\underline{k}.\underline{r}} \qquad (13)$$

$$\underline{f}_{5-} = i\mu J_o (\underline{J}_H . \frac{\underline{k}}{k}) \frac{\mathrm{ch}\ k(z+H)}{\mathrm{th}\ kh.\mathrm{ch}\ kH} . A\ e^{st+i\underline{k}.\underline{r}} . \underline{\lambda} \qquad (14)$$

In (13) the expression of the vertical current density in the aluminum is written $J_o + J_1 \frac{z}{H}$ whereas in cryolite (eq. 12) it is just J_o, as in Moreau and Evans |4|.

MOTION EQUATIONS

Writing $\underline{u}_H + w\underline{\lambda}$ the local disturbance of velocity in each fluid, the linearized motion equations are :

$$\nabla_H.\underline{u}_H + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \underline{u}_H}{\partial t} + (\underline{U}_H.\nabla_H)\underline{u}_H = -\frac{1}{\rho}\nabla_H p + \frac{1}{\rho}(\underline{f}_1 + \underline{f}_2 + \underline{f}_3) - \kappa \underline{u}_H$$

$$\frac{\partial w}{\partial t} + (\underline{U}_H.\nabla_H)\ w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho}(\underline{f}_4 + \underline{f}_5) - \kappa w \qquad (15)$$

As in |4| the symbol $\underline{U}_H$ stands for the local value of the mean horizontal velocity, and κ is the friction coefficient. They may have different values in the two liquids, and $\underline{U}_H$ should vary with position. However in this first step of the theory the variation of $\underline{U}_H$ in the horizontal directions is not taken into account. Notice that the velocity field associated with the disturbance is three-dimensional. This is necessary to express the kinematic conditions at the interface :

$$w_+(o) = (\frac{dz_o}{dt})_+ = (s + i\ \underline{k}.\underline{U}_+)z_o$$

$$w_-(o) = (\frac{dz_o}{dt})_- = (s + i\ \underline{k}.\underline{U}_-)z_o \qquad (16)$$

The other boundary conditions on this velocity are :

$$w_+(z = h) = w_-(z = -H) = 0 \qquad (17)$$

Let this perturbation also be written as :

$$\underline{u}_H = \underline{U}(z)\ e^{st+i\ \underline{k}.\underline{r}}$$
$$w = W(z)\ e^{st+i\ \underline{k}.\underline{r}}$$
$$p = P(z)\ e^{st+i\ \underline{k}.\underline{r}}$$
$$\underline{f}_i = F_i(z)\ e^{st+i\underline{k}.\underline{r}} \qquad (18)$$

Equations (15) give the following system of ordinary differential equations :

$$\underline{ik}\cdot\underline{U}_H + W' = 0$$

$$(s + \kappa + \underline{ik}\cdot\underline{U}_H)\ \underline{U} = -\frac{1}{\rho}\underline{kP} + \frac{1}{\rho}(\underline{F}_1+\underline{F}_2+\underline{F}_3)$$

$$(z + \kappa + \underline{ik}\cdot\underline{U}_H)\ W = -\frac{1}{\rho}P' + \frac{1}{\rho}(\underline{F}_4+\underline{F}_5) \qquad (19)$$

Its general solution satisfying the boundary conditions (16) and (17) is straightforward.

DISPERSION EQUATION AND MAIN RESULTS

As in classical theory |6| of sheared interfaces, the condition of continuity of the pressure at the interface yields this supplementary condition

$$P_+(o) + \underline{P}(o) = -A\left[g(\rho_- - \rho_+) + Tk^2\right] \qquad (20)$$

where g is gravity and T surface tension. The expression of $P_+(o)$ and $P_-(o)$ directly follow from the solution of (19), (16) and (17), and may be substituted into (20), giving the dispersion equation :

$$\frac{\rho_-\beta_-(\beta_-+\kappa_-)}{th\ kH} + \frac{\rho_+\beta_+(\beta_++\kappa_+)}{th\ kh} = -\ gk(\rho_- - \rho_+)\ )-Tk^3$$

$$+\ \frac{\mu J_o^2}{thkh}\left(\frac{kH}{sh\ 2kH} - \frac{kh}{sh\ 2kh}\right)$$

$$-\ \frac{\mu J_o J_1}{4\ thkh}\left[\frac{2kH}{sh\ 2kH} + \frac{thkH}{kH}\ (1-2i\ kH\cos\theta\ \right] \qquad (21)$$

The symbols β_+ and β_- stand for $s + \underline{ik}\cdot\underline{U}_+$ and $s + \underline{ik}\cdot\underline{U}_-$, and θ is the angle between $\underline{k}$ and $\underline{J}_H$.

The most unstable wave vectors are those oriented in the same direction as $\underline{J}_H$. Then in the following cos θ is supposed to be 1 to focus on the most unstable disturbance.

When J_o and J_1 are zero this equation is nothing but the classical dispersion equation for a sheared interface (with particular assumption on the friction in each liquid layer). The two first terms at the right hand side exhibit the stabilizing influence of gravity (because $\rho_- > \rho_+$) and surface tension (efficient for k large). The destabilizing mechanism responsible for Kelvin-Helmholtz instability lies at the left hand side of (21) in terms involving $|U_+ - U_-|$. This classical results are only slightly changed by our special assumption on friction law. They bring to the curve (a) on Fig. 1 which shows typical curves of neutral stability (such that Re(s) = 0) for different conditions. It is essentially characterized by two asymptotic branches, one controlled by gravity (small wave numbers), the other by surface tension (large wave numbers), and by a minimum which corresponds to critical conditions.

When J_o and J_1 are taken into account the results may change dramatically. To enlighten the physics of the phenomenon it is interesting to consider first the case of liquid layers of infinite depth. Then the differential pinch effect is identically zero. Against this the term in J_oJ_1 overthrows the behaviour at small wave numbers since it is predominant (as k^{-2}) upon the gravity term (as k^{-1}) and since the asymptotic branch to the neutral curve is always negative. This means that a second critical value of the wave number exists k_{**}, below which stability is impossible. Of course, the higher the electric current, or the smaller the friction coefficient, the larger this critical wave number k.

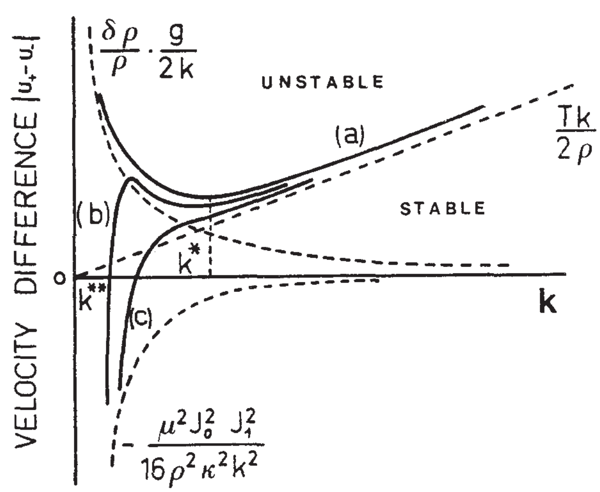

Fig. 1 - General pattern of curves of neutral stability and their asymptotic behaviour, when liquid thicknesses are infinite. (a) : ordinary Kelvin-Helmholtz neutral curve when J_o = 0. (b) : Typical curves in the presence of electric current.

However the depths of aluminum and cryolite are finite. Taking this into account changes again the behaviour, since a new asymptotic branch, strictly positive and varying as k^{-3}, is now predominant at very small k. The neutral curve may now have two minima. The one located at value k_* corresponds to the most unstable mode in the presence of the shear $|U_+ - U_-|$; it is only slightly modified by the electromagnetic effects which are almost negligible at large wave numbers. For typical cells of today this minimum k_* corresponds to wave lengths of the order of 15 to 20 cm. The other minimum located at values k_{**}, much smaller than k_*, corresponds to some purely electromagnetic instability. For cells of today the corresponding wave length is of the order of 1.5 to 2 m.

TABLE 1 : Numerical values used in calculations of curves of Fig. 2 and 3

h	0.05 m
H	0.15 m
Lx	7.00 m
Ly	2.50 ù
J_o	10^4 A.m^{-2}
μ	1.26×10^{-6} kg.A^{-2}.s^{-2}
σ_-	3.3×10^6 A.V^{-1}.m^{-1}
σ_+	200 A.V^{-1}.m^{-1}
ρ_+	2088 Kg.m^{-3}
ρ_-	2270 Kg.m^{-3}
T	0.50 Kg.s^{-2}

It is straightforward to solve numerically the dispersion equation (21) and to examine in detail how each parameter acts on the stability criterion. To give an idea of the main tendencies a few curves of neutral stability have been plotted on Fig. 2 using typical values of the cell parameters given in TABLE 1. These numerical results clearly confirm the behaviour just discussed above. They show the negligible influence of electric parameters and friction on the critical wave number $k_* (\simeq 55\ m^{-1})$ as well as on the critical shear V_* ($\simeq 0.12$ m.s^{-1}). In addition they put in evidence the drastic influence of these quantities for small wave numbers. It appears that a sufficient level of turbulence, providing friction coefficient $\kappa > 0.1\ s^{-1}$, may completly stabilize the electromagnetic effects. On the contrary, when $\kappa < 0.055\ s^{-1}$ the minimum at k_{**} becomes negative, what means that disturbances with k of the order of 4 m^{-1} should be unstable even without any mean motion of the two liquids.

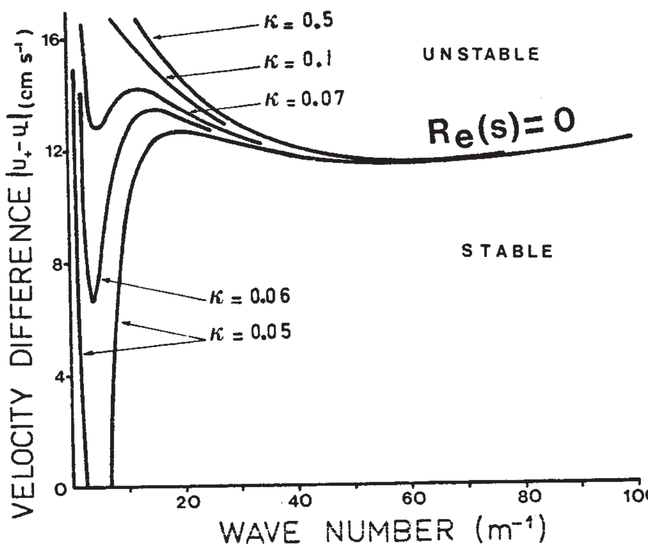

Fig. 2 - Curves of neutral stability deduced from equation (21) for values given in Table 1, when $J_1/J_o = 4$, for different values of $\kappa_+ = \kappa_- = \kappa$.

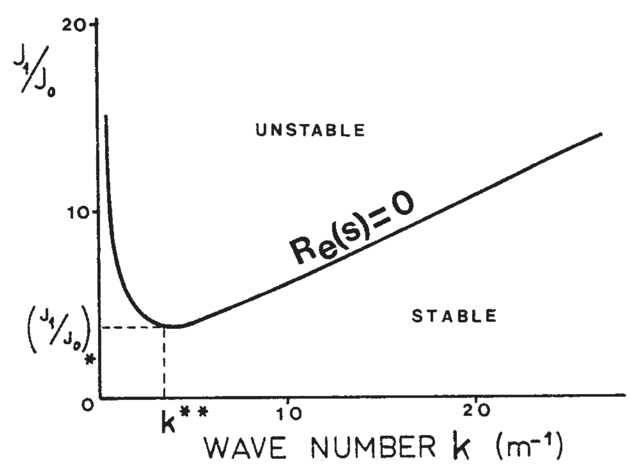

Fig. 3 - Curve of neutral stability without any mean motion deduced from equation (23) for values given in Table 1, when $\kappa_+ = \kappa_- = 0.06\ s^{-1}$

A new hydrodynamic instability is therefore demonstrated, almost independent of the mean motion in the two liquids. It is interesting to study this phenomenon in itself, assuming that $U_+ = U_- = 0$ (and $\cos\theta = 1$). The dispersion equation then becomes :

$$a\,s^2 + bs + c - id = 0 \qquad (22)$$

with the following definition of the coefficients :

$$a = \frac{\rho_-}{\mathrm{th}\,kH} + \frac{\rho_+}{\mathrm{th}\,kh} \qquad c = gk(\rho_- + \rho_+) + TK^3$$

$$b = \frac{\rho_-\,\kappa_+}{\mathrm{th}\,kH} + \frac{\rho_+\,\kappa_+}{\mathrm{th}\,kh} + \frac{\mu J_o^2}{\mathrm{th}\,kh}\left(\frac{kh}{\mathrm{th}\,kh} - \frac{kh}{\mathrm{sh}\,2kH}\right)$$

$$d = \frac{\mu J_o J_1}{2}\cdot\frac{\mathrm{th}\,kH}{\mathrm{th}\,kh} + \frac{\mu J_o J_1}{4\mathrm{th}kh}\left(\frac{2kH}{\mathrm{sh}\,2kH} + \frac{\mathrm{th}\,kH}{kH}\right)$$

The equation of the curves of neutral stability

$$ad^2 - cb^2 = 0$$

as been studied in the typical conditions of Table 1, with $\kappa_+ = \kappa_- = 0.06\ s^{-1}$. Fig. 3 shows that values $J_1/J_o \simeq 4$ are sufficient to destabilize wave numbers of the order of $4\ m^{-1}$ with such a small friction. The horizontal current in the aluminum J_1 may be estimated from J_o with relation

$$\frac{J_1}{J_o} \simeq \frac{L_y}{H}$$

because of continuity of electric current. Values of J_1/J_o of the order of 10 are quite plausible. One is therefore inclined to consider that the electromagnetic instability just analysed could be the root of the large scale oscillations observed in some cells.

CONCLUDING REMARKS

A new formulation of the problem of instabilities in the Hall Héroult cells is proposed. It is based on full Navier-Stokes equations and on an explicit calculation of the electromagnetic forces associated with the disturbance. The analysis follows the classical technique of the linear theory of hydrodynamic stability, except that a linear assumption is used for the friction on the electrodes in each fluid, previously introduced for modelling the mean flows |4| and verified in a laboratory experiment |5|.

At this stage the theory is only developped at a first level, ignoring the horizontal variations of undisturbed quantities (magnetic field, electric current, velocities) as well as the finite size of the cell. Therefore the results are still limited to the general behaviour and improvements are necessary before applying them to a particular pot. Nevertheless some predictions are in agreement with reported observations |3| :

- An electromagnetic instability may develop if the horizontal current density in the aluminum liquid is large enough.
- It may generate large scale waves (1.5 to 2 m) propagating in the direction of horizontal current.
- Small scale waves (15 to 20 cm) driven by the shear of the interface may be superposed.

In the frame of this linear analysis each unstable mode is independent of the others. Non-linear theories, which are now tractable, should be necessary to analyse the interactions between modes, and to predict the evolution in time of the instability. In this context it is remarkable that increasing the turbulence level in the two liquids (or increasing the friction coefficients) may stabilize the disturbances. This suggests the existence of some mechanism of saturation, and inclines to develop non-linear models able to distinguish conditions of saturation from conditions of developing chaos.

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