$$V = \int_a^b 2\pi\, x\, f(x)\, dx$$

x : نصف القطر = r

- تمثل البعد عن محور الدوران

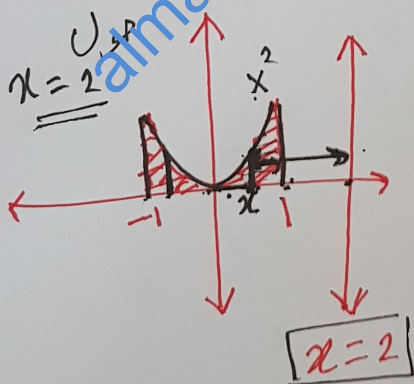
$f(x)$: الارتفاع = h

ارتفاع الاسطوانة المرسومة

(ارسم الخط موازي لمحور الدوران)

dx : السماكة

1] حول $x = 2$



$\boxed{r}$: $2 - x$

h = ارتفاع الاسطوانة هو $f(x)$

$\boxed{h = x^2}$

$$V = 2\pi \int_{-1}^{1} \underbrace{(2-x)}_{r}\underbrace{(x^2)}_{f(x)} . \, dx$$

$$= 2\pi \int_{-1}^{1} (2x^2 - x^3) \, dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1}$$

$$= 2\pi \left[\left(\frac{2}{3} - \frac{1}{4}\right) - \left(\frac{-2}{3} - \frac{1}{4}\right) \right]$$

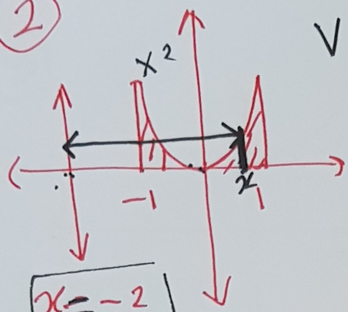
$$= 2\pi \left[\frac{5}{12} + \frac{2}{3} + \frac{1}{4} \right]$$

$$= 2\pi \left[\frac{5+8+3}{12} \right]$$

$$= 2\pi \left(\frac{16}{12} \right)$$

$$= \frac{8\pi}{3}$$

②



$\boxed{x=-2}$

محور الدوران

$$V = 2\pi \int_{-1}^{1} (\underbrace{x+2}_{r})(x^2)\,dx$$

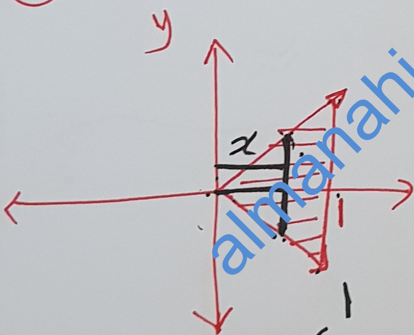
$r = x+2$

$= \frac{8\pi}{3}$

$h = x^2$

③ $y = x$ $y = -x$ $x = 1$

$\boxed{\text{حول المحور } y}$



$r = x$ ✓

$h = 2x$ ✓

$$V = 2\pi \int_{0}^{1} x\,(2x)\,dx$$

$$= 2\pi \int_{0}^{1} 2x^2\,dx = \frac{4\pi}{3}$$ ✓

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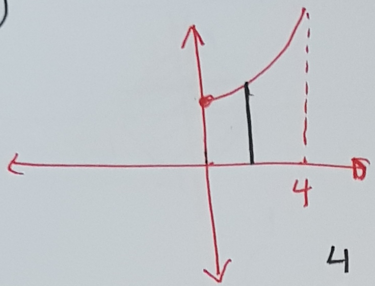
حول محور الصادات



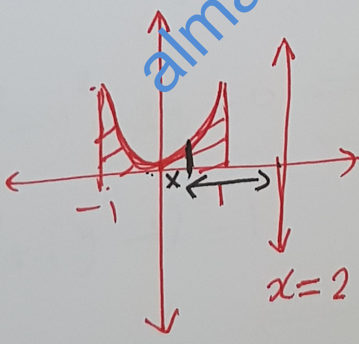
$r = x$

$h = f(x) = \sqrt{x^2+1}$

$$V = 2\pi \int_0^4 x\sqrt{x^2+1}\,dx$$

$$\approx 144.707$$

6



$r = 2 - x$

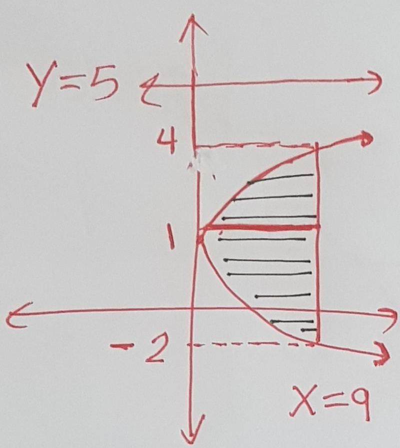
$h = f(x) = x^2$

$$V = 2\pi \int_{-1}^{1} (2-x)(x^2)\,dx$$

$$= \frac{8\pi}{3}$$

$$x = (y-1)^2 \quad , \quad x = 9$$

حول $y = 5$



اجد نقاط التقاطع

$$9 = (y-1)^2$$

$$\pm 3 = y - 1$$

$$y - 1 = 3$$

$$\boxed{y = 4}$$

$$y - 1 = -3$$

$$\boxed{y = -2}$$

$$r = 5 - y$$

$$h = 9 - (y-1)^2$$

$$v = 2\pi \int_{-2}^{4} (5-y)\left(9-(y-1)^2\right) dy$$

$$= 288\pi$$

اوجد طول القوس

$$S=\int_a^b \sqrt{1+(f'(x))^2}\,dx$$

مثال

$$f(x)=2x+1 \quad , [0,2]$$

$$f'(x)=2 \Rightarrow (f'(x))^2=4$$

$$1+(f'(x))^2=5$$

$$S=\int_0^2 \sqrt{5}\,dx=(2-0)\sqrt{5}=2\sqrt{5}$$

مثال :

$$f(x)=\sqrt{1-x^2} \quad , [-1,1]$$

$$f'(x)=\frac{-2x}{2\sqrt{1-x^2}} \Rightarrow (f'(x))^2=\frac{x^2}{1-x^2}$$

$1+(f')^2 = \frac{1}{1} + \frac{x^2}{1-x^2}$

$= \frac{1-x^2+x^2}{1-x^2} = \frac{1}{1-x^2}$

$S = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x \Big]_{-1}^{1} = \pi$

مثال :

$Y = 4x^{3/2}+1 \quad , [1,2]$

$Y' = 6x^{1/2} \Rightarrow (Y')^2 = 36x$

$S = \int_{1}^{2} \sqrt{1+36x}\, dx = \frac{2}{3}(1+36x)^{3/2} \Big]_{1}^{2}$

$\simeq 7.38$

$$y = \frac{1}{4}\left(e^{2x} + e^{-2x}\right), \quad [0, 1]$$

$$y' = \frac{1}{4}\left(2e^{2x} - 2e^{-2x}\right)$$

$$y' = \frac{1}{2}\left(e^{2x} - e^{-2x}\right)$$

$$(y')^2 = \frac{1}{4}\left(e^{2x} - e^{-2x}\right)^2$$

$$(y')^2 + 1 = 1 + \frac{1}{4}\left(e^{2x} - e^{-2x}\right)^2$$

$$= \frac{1}{4}\left(4 + e^{4x} - 2e^{2x} \cdot e^{-2x} + e^{-4x}\right)$$

$$= \frac{1}{4}\left[e^{4x} + e^{-4x} + 2\right]$$

$$= \frac{1}{4}\left(e^{2x} + e^{-2x}\right)^2$$

$$S = \int_0^1 \sqrt{1 + (f'(x))^2}$$

$$= \int_0^1 \frac{1}{2}\left(e^{2x} + e^{-2x}\right) dx$$

$$\approx \boxed{1.81}$$

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الحل

$$Y=\frac{1}{4}x^2-\frac{1}{2}\ln x \quad ,[1,2]$$

$$y'=\frac{1}{2}x-\frac{1}{2x}=\frac{1}{2}\left(x-\frac{1}{x}\right)$$

$$1+(y')^2=1+\frac{1}{4}\left(x-\frac{1}{x}\right)^2$$

$$=\frac{1}{4}\left(4+\left(x-\frac{1}{x}\right)^2\right)$$

$$=\frac{1}{4}\left(4+x^2-2+\frac{1}{x^2}\right)$$

$$=\frac{1}{4}\left(x^2+2+\frac{1}{x^2}\right)$$

$$=\left[\frac{1}{2}\left(x+\frac{1}{x}\right)\right]^2$$

$$S=\frac{1}{2}\int_1^2\left(x+\frac{1}{x}\right)dx$$

$$=\frac{1}{2}\left(\frac{x^2}{2}+\ln x\right)\Big]_1^2=1.0965$$