مراجعة صيغ التكامل

أولاً: أجد كلاً من التكاملات

① $\int_1^4 \sqrt{x}(2+\sqrt{x})^2\,dx = \int_1^4 \sqrt{x}(4+4\sqrt{x}+x)\,dx$

$= \int_1^4 x^{\frac{1}{2}}(4+4x^{\frac{1}{2}}+x)\,dx = \int_1^4 (4x^{\frac{1}{2}}+4x+x^{\frac{3}{2}})\,dx$

$= \left[4(\frac{2}{3})x^{\frac{3}{2}}+2x^2+\frac{2}{5}x^{\frac{5}{2}}\right]_1^4 = \left[\frac{8}{3}x^{\frac{3}{2}}+2x^2+\frac{2}{5}x^{\frac{5}{2}}\right]_1^4$

$= (\frac{992}{15}) - (\frac{76}{15}) = \boxed{\frac{916}{15}}$

② $\int (e^{3x} - \frac{5}{x} - e^2)\,dx$

ثابت

$= \frac{1}{3}e^{3x} - 5\ln|x| - e^2x + c$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{a}{x}\,dx = a\ln|x| + c$

③ $\int \frac{5x}{1+x^2}\,dx = 5\int \frac{x}{1+x^2}\,dx \rightarrow \int \frac{f'(x)}{f(x)}\,dx$

$= \frac{5}{2}\int \frac{2x}{1+x^2}\,dx$

$= \frac{5}{2}\ln|1+x^2| + c$

(2)

ثانياً

أوجد

ثابت

1) $\int \left(3\sin\left(\frac{\pi x}{2}\right) - \cos(3x) + \tan\frac{\pi}{4}\right)dx$

$= \int \left(3\sin\left(\frac{\pi}{2}x\right) - \cos(3x) + 1\right)dx$

$= -3\,\frac{\cos\left(\frac{\pi}{2}x\right)}{\frac{\pi}{2}} - \frac{\sin(3x)}{3} + x + c$

$= \frac{-6}{\pi}\cos\left(\frac{\pi}{2}x\right) - \frac{1}{3}\sin(3x) + x + c$

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2) $\int \left(\frac{4}{\cos^2 2x} - 4\csc^2\frac{x}{2} + \cancel{\sec^2 x\,\cos^2 x}\right)dX$

$= \int \left(4\sec^2(2x) - 4\csc^2\left(\tfrac{1}{2}x\right) + 1\right)dx$

$= \frac{4\tan(2x)}{2} + \frac{4\cot\left(\frac{1}{2}x\right)}{\frac{1}{2}} + x + c$

$= 2\tan(2x) + 8\cot\left(\tfrac{1}{2}x\right) + x + c$

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3) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

الزاوية ليست على صورة ax

إذاً فهي تركيب $\leftarrow u$

$= \int \frac{\cos(u)}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,du$

$= 2\int \cos(u)\,du = 2\sin u + c$

$= 2\sin(\sqrt{x}) + c$

$\sqrt{x} = u$

$\frac{1}{2\sqrt{x}}\,dx = du$

$dx = 2\sqrt{x}\,du$

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4) $\int \frac{1}{x^2}\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)dx$

$= \int \cancel{\frac{1}{x^2}}\sec(u)\tan(u)\cdot -\cancel{x^2}\,du$

$= -\sec u + c = -\sec\left(\frac{1}{x}\right) + c$

$\frac{1}{x} = u$

$\frac{-1}{x^2}\,dx = du$

$dx = -x^2\,du$

5 $\int \cos x \csc^2(\sin x)\, dx$

ثالثاً أصعب التكاملات

① مقدار ليس درجة أسه لسطح والمقام

* اذا كان واحد ← ln

* اذا كان الفرق الكبرى واحد tan⁻¹

(3) الشروط

$$\int \frac{\text{مشتقة ما داخل القوس}}{1+(\ )^2}\,dx$$

مربع كامل

① $\int \frac{4x}{9+9x^2}\,dx$

$4\int \frac{x}{9+9x^2}\,dx$

$\frac{4}{18}\int \frac{18x}{9+9x^2}\,dx$

$= \frac{2}{9}\ln|9+9x^2| + c$

فرق الدرجات الكبرى واحد

② $\int \frac{4x}{1+x^4}\,dx$

مربع كامل

$= \frac{4}{2}\int \frac{2x}{1+(x^2)^2}\,dx = 2\int \frac{(2x)}{1+(x^2)^2}\,dx$

$= 2\tan^{-1}(x^2) + c$

فرق الدرجات الكبرى واحد tan⁻¹

③ $\int \frac{4x}{9+9x^4}\,dx$

$= 4\int \frac{x}{9(1+x^4)}\,dx = \frac{4}{9}\int \frac{x}{1+(x^2)^2}\,dx$

$= \frac{\cancel{4}^{2}}{9}\left(\cancel{\frac{1}{2}}\right)\int \frac{2x}{1+(x^2)^2}\,dx$

$= \frac{2}{9}\tan^{-1}(x^2) + c$

1 $\displaystyle\int \frac{3x}{\sqrt{1-x^2}}\,dx$

① درجة البسط ودرجة ما تحت الجذر

* اذا كان الفرق واحد (التعويض)

* = = الفرق اكبر من واحد

$\sin^{-1}$

تحقق ثلاث شروط

نستخدم التعويض

$$= \int \frac{3\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-2\cancel{x}}$$

$$= -\frac{3}{2}\int u^{-\frac{1}{2}}\,du = \cancel{-\frac{3}{2}\cdot(2)}\,u^{\frac{1}{2}}+c$$

$$1-x^2=u$$
$$-2x\,dx=du$$
$$dx=\frac{du}{-2x}$$

$$= -3\sqrt{u}+c = -3\sqrt{1-x^2}+c$$

2 $\displaystyle\int \frac{3x}{\sqrt{16-x^4}}\,dx$

فرق الدرجات اكبر من واحد

$\sin^{-1}$

$$= 3\int \frac{x}{\sqrt{16\cdot\left(1-\frac{x^4}{16}\right)}}\,dx = \frac{3}{4}\int \frac{x}{\sqrt{1-\left(\frac{x^2}{4}\right)^2}}\,dx$$

$\frac{1}{4}x^2$

$$= \frac{3}{\cancel{4}_{2}}\cancel{(2)}\int \frac{\frac{1}{2}x}{\sqrt{1-\left(\frac{x^2}{4}\right)^2}}\,dx = \frac{3}{2}\sin^{-1}\left(\frac{x^2}{4}\right)+C$$

3 $\displaystyle\int \frac{5}{\sqrt{4-x^2}}\,dx$

فرق الدرجات اكبر من واحد

$\sin^{-1}$

$$5\int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx = \frac{5}{2}(2)\int \frac{\frac{1}{2}\,1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx$$

$\frac{1}{2}x \rightarrow \frac{1}{2}$

$$= 5\sin^{-1}\left(\frac{x}{2}\right)+C$$

أصعب (تمارين)

$$\int \frac{3x}{1+5x^2}\,dx$$

$$\int \frac{7x}{\sqrt{4-4x^4}}\,dx$$

$$\int \frac{9}{25+x^2}\,dx$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

فكرة إكمال المربع الكامل

$\boxed{1}$ $x^2 + 6x + 5$ مقدار ثلاثي

جذر الأول

نفس الإشارة

نصف العدد

$$= (x+3)^2 - 9 + 5$$

مربع

$$= (x+3)^2 - 4$$

$\boxed{2}$ $5 - 4x - x^2$

معامل $x^2 = -1$ قبل إجراء مربع الكامل

$$-(-5 + 4x + x^2)$$

الترتيب

$$-[x^2 + 4x - 5]$$

$$-[(x+2)^2 - 4 - 5]$$

مربع

$$-[(x+2)^2 - 9]$$

$$-(x+2)^2 + 9$$

$$9 - (x+2)^2$$

أوجد التكاملات التالية

1 $$\int \frac{x+2}{\underbrace{5-4x-x^2}_{-4-2x}}\,dx$$

فرق الدرجات واحد

$\ln$

$$\int \frac{f'(x)}{f(x)}\,dx$$

$$= \frac{-1}{2}\int \frac{-2x-4}{5-4x-x^2}\,dx$$

$$= \frac{-1}{2}\ln|5-4x-x^2| + c$$

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2 $$\int \frac{5}{5+4x+x^2}\,dx$$

تكملة المربع الكامل

فرق الدرجات اكبر من واحد

$\tan^{-1}$

$$= 5\int \frac{1}{1+(x+2)^2}\,dx$$

مربع كامل

$$= 5\tan^{-1}(x+2) + c$$

$5+4x+x^2$

x^2+4x+5

$(x+2)^2-4+5$

$(x+2)^2+1$

$1+(x+2)^2$

درجة ثانية

$$\boxed{3}\ \int \frac{3x^2+8x}{\sqrt{5-4x^2-x^3}}\,dx$$

درجة ثالثة

قرب الدرجات واحد

التعويض

$$= \int \frac{\cancel{(3x^2+8x)}}{\sqrt{u}} \cdot \frac{du}{-\cancel{(8x+3x^2)}}$$

$$= -\int u^{-\frac{1}{2}}\,du = -\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -2\sqrt{5-4x^2-x^3} + C$$

$5-4x^2-x^3 = u$

$-8x-3x^2\,dx = du$

$$dx = \frac{du}{-8x-3x^2}$$

$$= \frac{du}{-(8x+3x^2)}$$

درجة صفرية

$$\boxed{4}\ \int \frac{5}{\sqrt{3-2x-x^2}}\,dx$$

درجة ثانية

فرق الدرجات أكبر من واحد

$\sin^{-1}$

تكملة المربع كامل

$$= 5\int \frac{1}{\sqrt{4-(x+1)^2}}\,dx$$

$$= \frac{5}{2}\int \frac{1}{\sqrt{1-\frac{(x+1)^2}{4}}}\,dx$$

$$= \frac{5}{2}\int \frac{1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx$$

$$= \cancel{\frac{5}{2}}(2)\int \frac{\left(\frac{1}{2}\right)\ 1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx$$

$$= 5\sin^{-1}\left(\frac{x+1}{2}\right) + C$$

$3-2x-x^2$

$-[x^2+2x-3]$

$-[(x+1)^2-1-3]$

$-[(x+1)^2-4]$

$-(x+1)^2+4$

$$\boxed{4-(x+1)^2}$$

$$\left(\frac{x+1}{2}\right) = \frac{x}{2}+\frac{1}{2}$$

$$\left(\frac{1}{2}x+\frac{1}{2}\right)' = \frac{1}{2}$$

$\boxed{5}$ $\displaystyle\int \frac{e^{2x}}{\sqrt{4-e^{2x}}}\,dx$

$4 - e^{2x} = u$

$0 - 2e^{2x}\,dx = du$

$dx = \dfrac{du}{-2e^{2x}}$

$$= \int \frac{\cancel{e^{2x}}}{\sqrt{u}} \cdot \frac{du}{\cancel{-2e^{2x}}}$$

$$= -\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$= \cancel{-\frac{1}{2}} \cdot \frac{u^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} + c$$

$$= -\sqrt{4-e^{2x}} + c$$

$\boxed{6}$ $\displaystyle\int \frac{e^{x}}{\sqrt{4-e^{2x}}}\,dx$

$$= \frac{1}{2}\int \frac{e^{x}}{\sqrt{1-\frac{e^{2x}}{4}}}\,dx$$

$$= \frac{1}{2}(2)\int \frac{(\frac{1}{2})e^{x}}{\sqrt{1-(\frac{e^{x}}{2})^{2}}}\,dx$$

المشتقة $(\frac{1}{2}e^{x})' = \frac{1}{2}e^{x}$

$$= \sin^{-1}\left(\frac{e^{x}}{2}\right) + c$$

أمثلة مقادير الأسس النسبية في المقام

① $\int \frac{4}{x^{\frac{1}{3}}\left(1+x^{\frac{2}{3}}\right)}\,dx$

$$\boxed{\int \frac{4}{x^{\frac{1}{3}}\left(1+x^{\frac{2}{3}}\right)}\,dx}$$

$1+x^{\frac{2}{3}}=u$

$\frac{2}{3}x^{-\frac{1}{3}}\,dx=du$

$dx=\frac{3}{2}\,\frac{du}{x^{-\frac{1}{3}}}$

$dx=\frac{3}{2}x^{\frac{1}{3}}\,du$

$$4\int \frac{1}{\cancel{x^{\frac{1}{3}}}\cdot u}\cdot\frac{3}{2}\cancel{x^{\frac{1}{3}}}\,du$$

$$4\left(\frac{3}{2}\right)\int \frac{1}{u}\,du=6\ln|u|+c$$

$$=6\ln\left|1+x^{\frac{2}{3}}\right|+C$$

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② $\int_1^4 \frac{3}{\sqrt{x}+x}\,dx=\int_1^4 \frac{3}{\sqrt{x}\,(1+\sqrt{x})}\,dx$

$x=(\sqrt{x})^2$

$1+\sqrt{x}=u$

$\frac{1}{2\sqrt{x}}\,dx=du$

$dx=2\sqrt{x}\,du$

حدود التكامل الجديدة

$x=1\rightarrow u=2$

$x=4\rightarrow u=3$

$$=3\int_2^3 \frac{1}{\cancel{\sqrt{x}}\,(u)}\,2\cdot\cancel{\sqrt{x}}\,du$$

$$=6\int_2^3 \frac{1}{u}\,du=6\Big[\ln|u|\Big]_2^3$$

$$=6\left[\ln(3)-\ln(2)\right]=6\ln\left(\frac{3}{2}\right)$$

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③ $\int \frac{1}{x^{\frac{1}{4}}+x}\,dx=\int \frac{1}{x^{\frac{1}{4}}\left(1+x^{\frac{3}{4}}\right)}\,dx$

$1+x^{\frac{3}{4}}=u$

$\frac{3}{4}x^{-\frac{1}{4}}\,dx=du$

$dx=\frac{4}{3}\,\frac{du}{x^{-\frac{1}{4}}}$

$dx=\frac{4}{3}x^{\frac{1}{4}}\,du$

$$=\int \frac{1}{\cancel{x^{\frac{1}{4}}}\cdot u}\cdot\frac{4}{3}\cancel{x^{\frac{1}{4}}}\,du$$

$$=\frac{4}{3}\int \frac{1}{u}\,du=\frac{4}{3}\ln|u|+c$$

$$=\frac{4}{3}\ln\left|1+x^{\frac{3}{4}}\right|+C$$

أوجد

① $\int e^{2\ln x}\,dx$

$= \int \cancel{e}^{\cancel{\ln} x^2}\,dx = \int x^2\,dx$

$= \frac{x^3}{3} + C$

خواص

$\cancel{e}^{\cancel{\ln} x} = x$

$\ln x^n = n \ln x$

$\ln^n(x) = (\ln x)^n$

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② $\int \frac{\ln x^2}{x}\,dx$ الأس زوجي $= \int \frac{2\ln|x|}{x}\,dx$ مطلق

$2\int \frac{\ln(x)}{x}\,dx$

$2\int \frac{u}{\cancel{x}}\cdot\cancel{x}\,du$

$= \cancel{2}\cdot\frac{u^2}{\cancel{2}} + c = \ln^2 x + c$

$\ln x = u$

$\frac{1}{x}\,dx = du$

$dx = x\,du$

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③ $\int \frac{\ln^2 x}{x}\,dx$ مرة $= \int \frac{(\ln(x))^2}{x}\,dx$

$= \int \frac{u^2}{\cancel{x}}\cdot\cancel{x}\,du$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}\ln^3 x + C$

$\ln x = u$

$\frac{1}{x}\,dx = du$

$dx = x\,du$

$$\int_{-2}^{-1} \frac{\ln(x^2)}{x}\,dx$$

$$= \int_{-2}^{-1} \frac{2\ln|x|}{x}\,dx$$

$$= \int_{-2}^{-1} \frac{2\ln(-x)}{x}\,dx$$

$$= 2\int_{-2}^{-1} \frac{\ln(-x)}{x}\,dx$$

$$= -2\int_{-2}^{-1} \frac{-1}{x}\ln(-x)\,dx$$

$$= -2\left[\ln(-x)\right]_{-2}^{-1}$$

$$= -2\left[\cancel{\ln(1)} - \ln(2)\right] = 2\ln 2$$

$$\int_{-2}^{-1} \frac{\ln(x^2)}{x}\, dx$$

تكامل محدود

x^2 ← أس زوجي

$$= \int_{-2}^{-1} \frac{2\ln|x|}{x}\, dx$$

اعادة تعريف المطلق

$$= \int_{-2}^{-1} \frac{2\ln(-x)}{x}\, dx$$

$\ln(-x) = u$

$\frac{-1}{x}\, dx = du$

$dx = -x\, du$

$x = -2 \to \ln(2)$

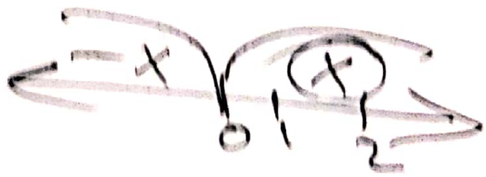
$x = -1 \to \ln(1) = 0$

$$= \int_{\ln(2)}^{0} \frac{2u}{\cancel{x}} \cdot (\cancel{-x})\, du$$

$$= -2\int_{\ln 2}^{0} u\, du = -2\left[\frac{u^2}{2}\right]_{\ln 2}^{0}$$

$$= -2\left[0 - \frac{(\ln 2)^2}{2}\right] = \ln^2(2) = 0.48$$

$$\int_{1}^{2} \frac{\ln x^2}{x}\, dx$$



$$= \int_{1}^{2} \frac{2\ln|x|}{x}\, dx$$

$\ln(x) = u$

$\frac{1}{x}\, dx = du$

$dx = x\, du$

$x = 1 \to u = 0$

$x = 2 \to \ln 2$

$$= \int_{1}^{2} \frac{2\ln(x)}{x}\, dx$$

$$= \int_{0}^{\ln(2)} \frac{2u}{\cancel{x}}\, (\cancel{x}\, d(x)) = \left[u^2\right]_{0}^{\ln 2}$$

$$= (\ln 2)^2 - (0)^2 = \boxed{0.48}$$

الخاصية

1

$$\ln x^2 = 2\ln(x)$$

تكامل غير المحدود

غير محدود

$$\int \frac{\ln x^2}{x}\,dx$$

$$\int 2\,\frac{\ln(x)}{x}\,dx$$

نفرض $u = \ln x$

2

في التكامل المحدود $\int_a^b \frac{\ln x^n}{x}\,dx$

n

زوجي — فردي

$n\ln|x|$ — $n\ln(x)$

$$\int_3^4 x\sqrt{x-3}\,dx = 2.4$$

الحل

$$x - 3 = u$$
$$dx = du$$
$$x = 3 \rightarrow u = 0$$
$$x = 4 \rightarrow u = 1$$
$$x = (u+3)$$

$$= \int_0^1 x\sqrt{u}\,.\,du$$

$$= \int_0^1 (u+3)\,.\,u^{\frac{1}{2}}\,du$$

$$= \int_0^1 \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\right) du = \left[\frac{2}{5}u^{\frac{5}{2}} + 3\left(\frac{2}{3}\right)u^{\frac{3}{2}}\right]_0^1$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}}\right]_0^1$$

$$= \left(\frac{12}{5}\right) - (0) = \boxed{2.4}$$

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$$\int x\cdot\sqrt{x-3}\;dx$$

$x-3=u$

$1dx=du$

$x-3=u$

$\boxed{x=(u+3)}$

$$=\int \underline{\underline{x}}\sqrt{u}\cdot du$$

$$=\int (u+3)\cdot u^{\frac{1}{2}}\,du$$

توزيع الضرب على الجمع

$$=\int (u^{\frac{3}{2}}+3u^{\frac{1}{2}})\,du$$

$$=\frac{2}{5}u^{\frac{5}{2}}+\cancel{3}\left(\frac{2}{\cancel{3}}\right)u^{\frac{3}{2}}+c$$

$$=\frac{2}{5}(x-3)^{\frac{5}{2}}+2(x-3)^{\frac{3}{2}}+c$$

حاوية

$$\int_1^3 e^{2\ln x}\,dx$$

$\boxed{\ln(x)^n=n\ln x}$

$\boxed{e^{\ln x}=x}$

$$\int_1^3 \cancel{e}^{\cancel{\ln}(x)^2}\,dx$$

$$\int_1^3 x^2\,dx=\left[\frac{x^3}{3}\right]_1^3$$

$$=(9)-\left(\frac{1}{3}\right)$$

$$=\boxed{\frac{26}{3}}$$

مراجعة

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$\sqrt{x} = u$

$\frac{1}{2\sqrt{x}}\,dx = du$

$dx = 2\sqrt{x}\,du$

$$= \int \frac{\sin(u)}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,du$$

$$= 2\int \sin(u)\,du$$

$$= -2\cos(u) + C$$

$$= -2\cos(\sqrt{x}) + C$$

$$\int \frac{1+x}{1+x^2}\,dx \Rightarrow \boxed{\frac{a \mp b}{c} = \frac{a}{c} \mp \frac{b}{c}}$$

$$= \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$$

$$= \int \frac{1}{1+(x)^2}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= \tan^{-1}(x) + \frac{1}{2}\ln(1+x^2) + C$$

$$= \tan^{-1}(x) + \ln\sqrt{1+x^2} + C$$
