Impedance

The impedance **Z** of a resistance **R** in series with a reactance **X** is:
$\mathbf{Z = R + jX}$
Rectangular and polar forms of impedance **Z**:
$\mathbf{Z = R + jX = (R^2 + X^2)^{½}\angle tan^{-1}(X / R) = |Z|\angle\phi = |Z|cos\phi + j|Z|sin\phi}$
Addition of impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$:
$\mathbf{Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2) = (R_1 + R_2) + j(X_1 + X_2)}$
Subtraction of impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$:
$\mathbf{Z_1 - Z_2 = (R_1 + jX_1) - (R_2 + jX_2) = (R_1 - R_2) + j(X_1 - X_2)}$
Multiplication of impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$:
$\mathbf{Z_1 * Z_2 = |Z_1|\angle\phi_1 * |Z_2|\angle\phi_2 = (|Z_1| * |Z_2|)\angle(\phi_1 + \phi_2)}$
Division of impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$:
$\mathbf{Z_1 / Z_2 = |Z_1|\angle\phi_1 / |Z_2|\angle\phi_2 = (|Z_1| / |Z_2|)\angle(\phi_1 - \phi_2)}$
In summary:
- use the rectangular form for addition and subtraction,
- use the polar form for multiplication and division.

Admittance

An impedance **Z** comprising a resistance **R** in series with a reactance **X** can be converted to an admittance **Y** comprising a conductance **G** in parallel with a susceptance **B**:
$\mathbf{Y = Z^{-1} = 1 / (R + jX) = (R - jX) / (R^2 + X^2) = R / (R^2 + X^2) - jX / (R^2 + X^2) = G - jB}$
$\mathbf{G = R / (R^2 + X^2) = R / |Z|^2}$
$\mathbf{B = X / (R^2 + X^2) = X / |Z|^2}$
Using the polar form of impedance **Z**:
$\mathbf{Y = 1 / |Z|\angle\phi = |Z|^{-1}\angle{-\phi} = |Y|\angle{-\phi} = |Y|cos\phi - j|Y|sin\phi}$
Conversely, an admittance **Y** comprising a conductance **G** in parallel with a susceptance **B** can be converted to an impedance **Z** comprising a resistance **R** in series with a reactance **X**:
$\mathbf{Z = Y^{-1} = 1 / (G - jB) = (G + jB) / (G^2 + B^2) = G / (G^2 + B^2) + jB / (G^2 + B^2) = R + jX}$
$\mathbf{R = G / (G^2 + B^2) = G / |Y|^2}$
$\mathbf{X = B / (G^2 + B^2) = B / |Y|^2}$
Using the polar form of admittance **Y**:
$\mathbf{Z = 1 / |Y|\angle{-\phi} = |Y|^{-1}\angle\phi = |Z|\angle\phi = |Z|cos\phi + j|Z|sin\phi}$
The total impedance $\mathbf{Z_S}$ of impedances $\mathbf{Z_1}$, $\mathbf{Z_2}$, $\mathbf{Z_3}$,... connected in series is:
$\mathbf{Z_S = Z_1 + Z_1 + Z_1 +...}$
The total admittance $\mathbf{Y_P}$ of admittances $\mathbf{Y_1}$, $\mathbf{Y_2}$, $\mathbf{Y_3}$,... connected in parallel is:
$\mathbf{Y_P = Y_1 + Y_1 + Y_1 +...}$
In summary:
- use impedances when operating on series circuits,
- use admittances when operating on parallel circuits.

Reactance

Inductive Reactance

The inductive reactance $\mathbf{X_L}$ of an inductance **L** at angular frequency **ω** and frequency **f** is:
$\mathbf{X_L = \omega L = 2\pi fL}$
For a sinusoidal current **i** of amplitude **I** and angular frequency **ω**:
$\mathbf{i = I\ sin\omega t}$
If sinusoidal current **i** is passed through an inductance **L**, the voltage **e** across the inductance is:
$\mathbf{e = L\ di/dt = \omega LI\ cos\omega t = X_L I\ cos\omega t}$
The current through an inductance lags the voltage across it by 90°.

Capacitive Reactance

The capacitive reactance $\mathbf{X_C}$ of a capacitance **C** at angular frequency **ω** and frequency **f** is:
$\mathbf{X_C = 1 / \omega C = 1 / 2\pi fC}$
For a sinusoidal voltage **v** of amplitude **V** and angular frequency **ω**:
$\mathbf{v = V\ sin\omega t}$
If sinusoidal voltage **v** is applied across a capacitance **C**, the current **i** through the capacitance is:
$\mathbf{i = C\ dv/dt = \omega CV\ cos\omega t = V\ cos\omega t / X_C}$
The current through a capacitance leads the voltage across it by 90°.

Resonance

Series Resonance

A series circuit comprising an inductance **L**, a resistance **R** and a capacitance **C** has an impedance $\mathbf{Z_S}$ of:

$\mathbf{Z_S = R + j(X_L - X_C)}$

where $\mathbf{X_L = \omega L}$ and $\mathbf{X_C = 1 / \omega C}$

At resonance, the imaginary part of $\mathbf{Z_S}$ is zero:

$\mathbf{X_C = X_L}$

$\mathbf{Z_{Sr} = R}$

$\mathbf{\omega_r = (1 / LC)^{1/2} = 2\pi f_r}$

Parallel resonance

A parallel circuit comprising an inductance **L** with a series resistance **R**, connected in parallel with a capacitance **C**, has an admittance $\mathbf{Y_P}$ of:

$\mathbf{Y_P = 1 / (R + jX_L) + 1 / (- jX_C) = (R / (R^2 + X_L^2)) - j(X_L / (R^2 + X_L^2) - 1 / X_C)}$

where $\mathbf{X_L = \omega L}$ and $\mathbf{X_C = 1 / \omega C}$

At resonance, the imaginary part of $\mathbf{Y_P}$ is zero:

$\mathbf{X_C = (R^2 + X_L^2) / X_L = X_L + R^2 / X_L = X_L(1 + R^2 / X_L^2)}$

$\mathbf{Z_{Pr} = Y_{Pr}^{-1} = (R^2 + X_L^2) / R = X_L X_C / R = L / CR}$

$\mathbf{\omega_r = (1 / LC - R^2 / L^2)^{1/2} = 2\pi f_r}$

Note that for the same values of **L**, **R** and **C**, the parallel resonance frequency is lower than the series resonance frequency, but if the ratio **R / L** is small then the parallel resonance frequency is close to the series resonance frequency.

Reactive Loads and Power Factor

Resistance and Series Reactance

The impedance **Z** of a reactive load comprising resistance **R** and series reactance **X** is:

$\mathbf{Z = R + jX = |Z|\angle\phi}$

Converting to the equivalent admittance **Y**:

$\mathbf{Y = 1 / Z = 1 / (R + jX) = (R - jX) / (R^2 + X^2) = R / |Z|^2 - jX / |Z|^2}$

When a voltage **V** (taken as reference) is applied across the reactive load **Z**, the current **I** is:

$\mathbf{I = VY = V(R / |Z|^2 - jX / |Z|^2) = VR / |Z|^2 - jVX / |Z|^2 = I_P - jI_Q}$

The active current $\mathbf{I_P}$ and the reactive current $\mathbf{I_Q}$ are:

$\mathbf{I_P = VR / |Z|^2 = |I|\cos\phi}$

$\mathbf{I_Q = VX / |Z|^2 = |I|\sin\phi}$

The apparent power **S**, active power **P** and reactive power **Q** are:

$\mathbf{S = V|I| = V^2 / |Z| = |I|^2|Z|}$

$\mathbf{P = VI_P = I_P^2|Z|^2 / R = V^2R / |Z|^2 = |I|^2R}$

$\mathbf{Q = VI_Q = I_Q^2|Z|^2 / X = V^2X / |Z|^2 = |I|^2X}$

The power factor **cosϕ** and reactive factor **sinϕ** are:

$\mathbf{\cos\phi = I_P / |I| = P / S = R / |Z|}$

$\mathbf{\sin\phi = I_Q / |I| = Q / S = X / |Z|}$

Resistance and Shunt Reactance

The impedance **Z** of a reactive load comprising resistance **R** and shunt reactance **X** is found from:

$\mathbf{1 / Z = 1 / R + 1 / jX}$

Converting to the equivalent admittance **Y** comprising conductance **G** and shunt susceptance **B**:

$\mathbf{Y = 1 / Z = 1 / R - j / X = G - jB = |Y|\angle{-\phi}}$

When a voltage **V** (taken as reference) is applied across the reactive load **Y**, the current **I** is:

$\mathbf{I = VY = V(G - jB) = VG - jVB = I_P - jI_Q}$

The active current $\mathbf{I_P}$ and the reactive current $\mathbf{I_Q}$ are:

$\mathbf{I_P = VG = V / R = |I|\cos\phi}$

$\mathbf{I_Q = VB = V / X = |I|\sin\phi}$

The apparent power **S**, active power **P** and reactive power **Q** are:

$\mathbf{S = V|I| = |I|^2 / |Y| = V^2|Y|}$

$\mathbf{P = VI_P = I_P^2 / G = |I|^2G / |Y|^2 = V^2G}$

$\mathbf{Q = VI_Q = I_Q^2 / B = |I|^2B / |Y|^2 = V^2B}$

The power factor **cosϕ** and reactive factor **sinϕ** are:

$\mathbf{\cos\phi = I_P / |I| = P / S = G / |Y|}$

$\mathbf{\sin\phi = I_Q / |I| = Q / S = B / |Y|}$

Complex Power

When a voltage **V** causes a current **I** to flow through a reactive load **Z**, the complex power **S** is:
S = VI* where **I*** is the conjugate of the complex current **I**.

Inductive Load

$Z = R + jX_L$

$I = I_P - jI_Q$

$\cos\phi = R / |Z|$ (lagging)

$I^* = I_P + jI_Q$

$S = P + jQ$

An inductive load is a sink of lagging VArs (a source of leading VArs).

Capacitive Load

$Z = R - jX_C$

$I = I_P + jI_Q$

$\cos\phi = R / |Z|$ (leading)

$I^* = I_P - jI_Q$

$S = P - jQ$

A capacitive load is a source of lagging VArs (a sink of leading VArs).

Three Phase Power

For a balanced **star** connected load with line voltage V_{line} and line current I_{line}:

$V_{star} = V_{line} / \sqrt{3}$

$I_{star} = I_{line}$

$Z_{star} = V_{star} / I_{star} = V_{line} / \sqrt{3}I_{line}$

$S_{star} = 3V_{star}I_{star} = \sqrt{3}V_{line}I_{line} = V_{line}^2 / Z_{star} = 3I_{line}^2Z_{star}$

For a balanced **delta** connected load with line voltage V_{line} and line current I_{line}:

$V_{delta} = V_{line}$

$I_{delta} = I_{line} / \sqrt{3}$

$Z_{delta} = V_{delta} / I_{delta} = \sqrt{3}V_{line} / I_{line}$

$S_{delta} = 3V_{delta}I_{delta} = \sqrt{3}V_{line}I_{line} = 3V_{line}^2 / Z_{delta} = I_{line}^2Z_{delta}$

The apparent power **S**, active power **P** and reactive power **Q** are related by:

$S^2 = P^2 + Q^2$

$P = S\cos\phi$

$Q = S\sin\phi$

where **cosϕ** is the power factor and **sinϕ** is the reactive factor

Note that for equivalence between balanced star and delta connected loads:

$Z_{delta} = 3Z_{star}$

Per-unit System

For each system parameter, per-unit value is equal to the actual value divided by a base value:

$E_{pu} = E / E_{base}$

$I_{pu} = I / I_{base}$

$Z_{pu} = Z / Z_{base}$

Select rated values as base values, usually rated power in MVA and rated phase voltage in kV:

$S_{base} = S_{rated} = \sqrt{3}E_{line}I_{line}$

$E_{base} = E_{phase} = E_{line} / \sqrt{3}$

The base values for line current in kA and per-phase star impedance in Ohms/phase are:

$I_{base} = S_{base} / 3E_{base}$ $(= S_{base} / \sqrt{3}E_{line})$

$Z_{base} = E_{base} / I_{base} = 3E_{base}^2 / S_{base}$ $(= E_{line}^2 / S_{base})$

Note that selecting the base values for any two of S_{base}, E_{base}, I_{base} or Z_{base} fixes the base values of all four. Note also that Ohm's Law is satisfied by each of the sets of actual, base and per-unit values for voltage, current and impedance.

Transformers

The primary and secondary MVA ratings of a transformer are equal, but the voltages and currents in the primary (subscript $_1$) and the secondary (subscript $_2$) are usually different:

$\sqrt{3}\mathbf{E_{1line}I_{1line} = S = \sqrt{3}E_{2line}I_{2line}}$

Converting to base (per-phase star) values:

$\mathbf{3E_{1base}I_{1base} = S_{base} = 3E_{2base}I_{2base}}$

$\mathbf{E_{1base} / E_{2base} = I_{2base} / I_{1base}}$

$\mathbf{Z_{1base} / Z_{2base} = (E_{1base} / E_{2base})^2}$

The impedance $\mathbf{Z_{21pu}}$ referred to the primary side, equivalent to an impedance $\mathbf{Z_{2pu}}$ on the secondary side, is:

$\mathbf{Z_{21pu} = Z_{2pu}(E_{1base} / E_{2base})^2}$

The impedance $\mathbf{Z_{12pu}}$ referred to the secondary side, equivalent to an impedance $\mathbf{Z_{1pu}}$ on the primary side, is:

$\mathbf{Z_{12pu} = Z_{1pu}(E_{2base} / E_{1base})^2}$

Note that per-unit and percentage values are related by:

$\mathbf{Z_{pu} = Z_{\%} / 100}$

Symmetrical Components

In any three phase system, the line currents $\mathbf{I_a}$, $\mathbf{I_b}$ and $\mathbf{I_c}$ may be expressed as the phasor sum of:
- a set of balanced positive phase sequence currents $\mathbf{I_{a1}}$, $\mathbf{I_{b1}}$ and $\mathbf{I_{c1}}$ (phase sequence a-b-c),
- a set of balanced negative phase sequence currents $\mathbf{I_{a2}}$, $\mathbf{I_{b2}}$ and $\mathbf{I_{c2}}$ (phase sequence a-c-b),
- a set of identical zero phase sequence currents $\mathbf{I_{a0}}$, $\mathbf{I_{b0}}$ and $\mathbf{I_{c0}}$ (cophasal, no phase sequence).

The positive, negative and zero sequence currents are calculated from the line currents using:

$\mathbf{I_{a1} = (I_a + hI_b + h^2I_c) / 3}$

$\mathbf{I_{a2} = (I_a + h^2I_b + hI_c) / 3}$

$\mathbf{I_{a0} = (I_a + I_b + I_c) / 3}$

The positive, negative and zero sequence currents are combined to give the line currents using:

$\mathbf{I_a = I_{a1} + I_{a2} + I_{a0}}$

$\mathbf{I_b = I_{b1} + I_{b2} + I_{b0} = h^2I_{a1} + hI_{a2} + I_{a0}}$

$\mathbf{I_c = I_{c1} + I_{c2} + I_{c0} = hI_{a1} + h^2I_{a2} + I_{a0}}$

The residual current $\mathbf{I_r}$ is equal to the total zero sequence current:

$\mathbf{I_r = I_{a0} + I_{b0} + I_{c0} = 3I_{a0} = I_a + I_b + I_c = I_e}$

which is measured using three current transformers with parallel connected secondaries.

$\mathbf{I_e}$ is the earth fault current of the system.

Similarly, for phase-to-earth voltages $\mathbf{V_{ae}}$, $\mathbf{V_{be}}$ and $\mathbf{V_{ce}}$, the residual voltage $\mathbf{V_r}$ is equal to the total zero sequence voltage:

$\mathbf{V_r = V_{a0} + V_{b0} + V_{c0} = 3V_{a0} = V_{ae} + V_{be} + V_{ce} = 3V_{ne}}$

which is measured using an earthed-star / open-delta connected voltage transformer.

$\mathbf{V_{ne}}$ is the neutral displacement voltage of the system.

The h-operator

The **h**-operator ($1\angle 120°$) is the complex cube root of unity:

$\mathbf{h = -1/2 + j\sqrt{3}/2 = 1\angle 120° = 1\angle -240°}$

$\mathbf{h^2 = -1/2 - j\sqrt{3}/2 = 1\angle 240° = 1\angle -120°}$

Some useful properties of **h** are:

$\mathbf{1 + h + h^2 = 0}$

$\mathbf{h + h^2 = -1 = 1\angle 180°}$

$\mathbf{h - h^2 = j\sqrt{3} = \sqrt{3}\angle 90°}$

$\mathbf{h^2 - h = -j\sqrt{3} = \sqrt{3}\angle -90°}$

Fault Calculations

The different types of short-circuit fault which occur on a power system are:
- single phase to earth,
- double phase,
- double phase to earth,
- three phase,
- three phase to earth.

For each type of short-circuit fault occurring on an unloaded system:
- the first column states the phase voltage and line current conditions at the fault,
- the second column states the phase 'a' sequence current and voltage conditions at the fault,
- the third column provides formulae for the phase 'a' sequence currents at the fault,
- the fourth column provides formulae for the fault current and the resulting line currents.

By convention, the faulted phases are selected for fault symmetry with respect to reference phase 'a'.

$\mathbf{I_f}$ = fault current
$\mathbf{I_e}$ = earth fault current
$\mathbf{E_a}$ = normal phase voltage at the fault location
$\mathbf{Z_1}$ = positive phase sequence network impedance to the fault
$\mathbf{Z_2}$ = negative phase sequence network impedance to the fault
$\mathbf{Z_0}$ = zero phase sequence network impedance to the fault

Single phase to earth - fault from phase 'a' to earth:

$V_a = 0$ $I_b = I_c = 0$ $I_f = I_a = I_e$	$I_{a1} = I_{a2} = I_{a0} = I_a / 3$ $V_{a1} + V_{a2} + V_{a0} = 0$	$I_{a1} = E_a / (Z_1 + Z_2 + Z_0)$ $I_{a2} = I_{a1}$ $I_{a0} = I_{a1}$	$I_f = 3I_{a0} = 3E_a / (Z_1 + Z_2 + Z_0) = I_e$ $I_a = I_f = 3E_a / (Z_1 + Z_2 + Z_0)$

Double phase - fault from phase 'b' to phase 'c':

$V_b = V_c$ $I_a = 0$ $I_f = I_b = - I_c$	$I_{a1} + I_{a2} = 0$ $I_{a0} = 0$ $V_{a1} = V_{a2}$	$I_{a1} = E_a / (Z_1 + Z_2)$ $I_{a2} = - I_{a1}$ $I_{a0} = 0$	$I_f = - j\sqrt{3}I_{a1} = - j\sqrt{3}E_a / (Z_1 + Z_2)$ $I_b = I_f = - j\sqrt{3}E_a / (Z_1 + Z_2)$ $I_c = - I_f = j\sqrt{3}E_a / (Z_1 + Z_2)$

Double phase to earth - fault from phase 'b' to phase 'c' to earth:

$V_b = V_c = 0$ $I_a = 0$ $I_f = I_b + I_c = I_e$	$I_{a1} + I_{a2} + I_{a0} = 0$ $V_{a1} = V_{a2} = V_{a0}$	$I_{a1} = E_a / Z_{net}$ $I_{a2} = - I_{a1}Z_0 / (Z_2 + Z_0)$ $I_{a0} = - I_{a1}Z_2 / (Z_2 + Z_0)$	$I_f = 3I_{a0} = - 3E_aZ_2 / \Sigma_{zz} = I_e$ $I_b = I_f / 2 - j\sqrt{3}E_a(Z_2 / 2 + Z_0) / \Sigma_{zz}$ $I_c = I_f / 2 + j\sqrt{3}E_a(Z_2 / 2 + Z_0) / \Sigma_{zz}$

$\mathbf{Z_{net} = Z_1 + Z_2Z_0 / (Z_2 + Z_0)}$ and $\mathbf{\Sigma_{zz} = Z_1Z_2 + Z_2Z_0 + Z_0Z_1 = (Z_2 + Z_0)Z_{net}}$

Three phase (and three phase to earth) - fault from phase 'a' to phase 'b' to phase 'c' (to earth):

$V_a = V_b = V_c\ (= 0)$ $I_a + I_b + I_c = 0\ (= I_e)$ $I_f = I_a = hI_b = h^2I_c$	$V_{a0} = V_a\ (= 0)$ $V_{a1} = V_{a2} = 0$	$I_{a1} = E_a / Z_1$ $I_{a2} = 0$ $I_{a0} = 0$	$I_f = I_{a1} = E_a / Z_1 = I_a$ $I_b = E_b / Z_1$ $I_c = E_c / Z_1$

Note that the single phase fault current is greater than the three phase fault current if $\mathbf{Z_0}$ is less than $\mathbf{(2Z_1 - Z_2)}$.
The values of $\mathbf{Z_1}$, $\mathbf{Z_2}$ and $\mathbf{Z_0}$ are each determined from the respective positive, negative and zero sequence impedance networks by network reduction to a single impedance.
Note that if the system is earthed through an impedance $\mathbf{Z_n}$ (carrying current $\mathbf{3I_0}$) then an impedance $\mathbf{3Z_n}$ (carrying current $\mathbf{I_0}$) must be included in the zero sequence impedance network.

Three Phase Fault Level

The symmetrical three phase short-circuit current $\mathbf{I_{sc}}$ of a power system with no-load line and phase voltages $\mathbf{E_{line}}$ and $\mathbf{E_{phase}}$ and source impedance $\mathbf{Z_S}$ per-phase star is:
$\mathbf{I_{sc} = E_{phase} / |Z_S| = E_{line} / \sqrt{3}|Z_S|}$
The three phase fault level $\mathbf{S_{sc}}$ of the power system is:
$\mathbf{S_{sc} = 3I_{sc}^2|Z_S| = 3E_{phase}I_{sc} = 3E_{phase}^2 / |Z_S| = E_{line}^2 / |Z_S|}$
Note that if the $\mathbf{X / R}$ ratio of the source impedance $\mathbf{Z_S}$ (comprising resistance $\mathbf{R_S}$ and reactance $\mathbf{X_S}$) is sufficiently large, $\mathbf{|Z_S| \approx X_S}$.

Power Factor Correction

If an inductive load with an active power demand **P** has an uncorrected power factor of $\cos\phi_1$ lagging, and is required to have a corrected power factor of $\cos\phi_2$ lagging, the uncorrected and corrected reactive power demands, Q_1 and Q_2, are:

$Q_1 = P \tan\phi_1$

$Q_2 = P \tan\phi_2$

where $\tan\phi_n = (1 / \cos^2\phi_n - 1)^{½}$

The leading (capacitive) reactive power demand Q_C which must be connected across the load is:

$Q_C = Q_1 - Q_2 = P (\tan\phi_1 - \tan\phi_2)$

The uncorrected and corrected apparent power demands, S_1 and S_2, are related by:

$S_1\cos\phi_1 = P = S_2\cos\phi_2$

Comparing corrected and uncorrected load currents and apparent power demands:

$I_2 / I_1 = S_2 / S_1 = \cos\phi_1 / \cos\phi_2$

If the load is required to have a corrected power factor of unity, Q_2 is zero and:

$Q_C = Q_1 = P \tan\phi_1$

$I_2 / I_1 = S_2 / S_1 = \cos\phi_1 = P / S_1$

Shunt Capacitors

For star-connected shunt capacitors each of capacitance C_{star} on a three phase system of line voltage V_{line} and frequency **f**, the leading reactive power demand Q_{Cstar} and the leading reactive line current I_{line} are:

$Q_{Cstar} = V_{line}^2 / X_{Cstar} = 2\pi f C_{star} V_{line}^2$

$I_{line} = Q_{Cstar} / \surd 3 V_{line} = V_{line} / \surd 3 X_{Cstar}$

$C_{star} = Q_{Cstar} / 2\pi f V_{line}^2$

For delta-connected shunt capacitors each of capacitance C_{delta} on a three phase system of line voltage V_{line} and frequency **f**, the leading reactive power demand Q_{Cdelta} and the leading reactive line current I_{line} are:

$Q_{Cdelta} = 3V_{line}^2 / X_{Cdelta} = 6\pi f C_{delta} V_{line}^2$

$I_{line} = Q_{Cdelta} / \surd 3 V_{line} = \surd 3 V_{line} / X_{Cdelta}$

$C_{delta} = Q_{Cdelta} / 6\pi f V_{line}^2$

Note that for the same leading reactive power Q_C:

$X_{Cdelta} = 3X_{Cstar}$

$C_{delta} = C_{star} / 3$

Reactors

Shunt Reactors

For star-connected shunt reactors each of inductance L_{star} on a three phase system of line voltage V_{line} and frequency **f**, the lagging reactive power demand Q_{Lstar} and the lagging reactive line current I_{line} are:

$Q_{Lstar} = V_{line}^2 / X_{Lstar} = V_{line}^2 / 2\pi f L_{star}$

$I_{line} = Q_{Lstar} / \surd 3 V_{line} = V_{line} / \surd 3 X_{Lstar}$

$L_{star} = V_{line}^2 / 2\pi f Q_{Lstar}$

For delta-connected shunt reactors each of inductance L_{delta} on a three phase system of line voltage V_{line} and frequency **f**, the lagging reactive power demand Q_{Ldelta} and the lagging reactive line current I_{line} are:

$Q_{Ldelta} = 3V_{line}^2 / X_{Ldelta} = 3V_{line}^2 / 2\pi f L_{delta}$

$I_{line} = Q_{Ldelta} / \surd 3 V_{line} = \surd 3 V_{line} / X_{Ldelta}$

$L_{delta} = 3V_{line}^2 / 2\pi f Q_{Ldelta}$

Note that for the same lagging reactive power Q_L:

$X_{Ldelta} = 3X_{Lstar}$

$L_{delta} = 3L_{star}$

Series Reactors

For series line reactors each of inductance L_{series} carrying line current I_{line} on a three phase system of frequency **f**, the voltage drop V_{drop} across each line reactor and the total lagging reactive power demand $Q_{Lseries}$ of the set of three line reactors are:

$V_{drop} = I_{line} X_{Lseries} = 2\pi f L_{series} I_{line}$

$Q_{Lseries} = 3V_{drop}^2 / X_{Lseries} = 3V_{drop} I_{line} = 3I_{line}^2 X_{Lseries} = 6\pi f L_{series} I_{line}^2$

$L_{series} = Q_{Lseries} / 6\pi f I_{line}^2$

Note that the apparent power rating S_{rating} of the set of three line reactors is based on the line voltage V_{line} and not the voltage drop V_{drop}:

$S_{rating} = \surd 3 V_{line} I_{line}$

Harmonic Resonance

If a node in a power system operating at frequency **f** has a inductive source reactance $\mathbf{X_L}$ per phase and has power factor correction with a capacitive reactance $\mathbf{X_C}$ per phase, the source inductance **L** and the correction capacitance **C** are:

$\mathbf{L = X_L / \omega}$

$\mathbf{C = 1 / \omega X_C}$

where $\omega = 2\pi f$

The series resonance angular frequency ω_r of an inductance **L** with a capacitance **C** is:

$\mathbf{\omega_r = (1 / LC)^{1/2} = \omega(X_C / X_L)^{1/2}}$

The three phase fault level $\mathbf{S_{sc}}$ at the node for no-load phase voltage **E** and source impedance **Z** per-phase star is:

$\mathbf{S_{sc} = 3E^2 / |Z| = 3E^2 / |R + jX_L|}$

If the ratio $\mathbf{X_L / R}$ of the source impedance **Z** is sufficiently large, $|Z| \approx X_L$ so that:

$\mathbf{S_{sc} \approx 3E^2 / X_L}$

The reactive power rating $\mathbf{Q_C}$ of the power factor correction capacitors for a capacitive reactance $\mathbf{X_C}$ per phase at phase voltage **E** is:

$\mathbf{Q_C = 3E^2 / X_C}$

The harmonic number $\mathbf{f_r / f}$ of the series resonance of $\mathbf{X_L}$ with $\mathbf{X_C}$ is:

$\mathbf{f_r / f = \omega_r / \omega = (X_C / X_L)^{1/2} \approx (S_{sc} / Q_C)^{1/2}}$

Note that the ratio $\mathbf{X_L / X_C}$ which results in a harmonic number $\mathbf{f_r / f}$ is:

$\mathbf{X_L / X_C = 1 / (f_r / f)^2}$

so for $\mathbf{f_r / f}$ to be equal to the geometric mean of the third and fifth harmonics:

$\mathbf{f_r / f = \sqrt{15} = 3.873}$

$\mathbf{X_L / X_C = 1 / 15 = 0.067}$

Dielectric Dissipation Factor

If an alternating voltage **V** of frequency **f** is applied across an insulation system comprising capacitance **C** and equivalent series loss resistance $\mathbf{R_S}$, then the voltage $\mathbf{V_R}$ across $\mathbf{R_S}$ and the voltage $\mathbf{V_C}$ across **C** due to the resulting current **I** are:

$\mathbf{V_R = IR_S}$

$\mathbf{V_C = IX_C}$

$\mathbf{V = (V_R^2 + V_C^2)^{1/2}}$

The dielectric dissipation factor of the insulation system is the tangent of the dielectric loss angle δ between $\mathbf{V_C}$ and **V**:

$\mathbf{\tan\delta = V_R / V_C = R_S / X_C = 2\pi fCR_S}$

$\mathbf{R_S = X_C\tan\delta = \tan\delta / 2\pi fC}$

Note that an increase in the dielectric losses of a insulation system (from an increase in the series loss resistance $\mathbf{R_S}$) results in an increase in $\mathbf{\tan\delta}$. Note also that $\mathbf{\tan\delta}$ increases with frequency.

The dielectric power loss **P** is related to the capacitive reactive power $\mathbf{Q_C}$ by:

$\mathbf{P = I^2R_S = I^2X_C\tan\delta = Q_C\tan\delta}$

The power factor of the insulation system is the cosine of the phase angle ϕ between $\mathbf{V_R}$ and **V**:

$\mathbf{\cos\phi = V_R / V}$

so that δ and ϕ are related by:

$\mathbf{\delta + \phi = 90°}$

$\mathbf{\tan\delta}$ and $\mathbf{\cos\phi}$ are related by:

$\mathbf{\tan\delta = 1 / \tan\phi = \cos\phi / \sin\phi = \cos\phi / (1 - \cos^2\phi)^{1/2}}$

so that when $\mathbf{\cos\phi}$ is close to zero, $\mathbf{\tan\delta \approx \cos\phi}$

Note that the series loss resistance $\mathbf{R_S}$ is not related to the shunt leakage resistance of the insulation system (which is measured using direct current).

Notation

The library uses the symbol font for some of the notation and formulae. If the symbols for the letters 'alpha beta delta' do not appear here [α β δ] then the symbol font needs to be installed before all notation and formulae will be displayed correctly.

C	capacitance	[farads, F]	**Q**	charge	[coulombs, C]
E	voltage source	[volts, V]	**q**	instantaneous **Q**	[coulombs, C]
e	instantaneous **E**	[volts, V]	**R**	resistance	[ohms, Ω]
G	conductance	[siemens, S]	**T**	time constant	[seconds, s]
I	current	[amps, A]	**t**	instantaneous time	[seconds, s]
i	instantaneous **I**	[amps, A]	**V**	voltage drop	[volts, V]
k	coefficient	[number]	**v**	instantaneous **V**	[volts, V]
L	inductance	[henrys, H]	**W**	energy	[joules, J]
M	mutual inductance	[henrys, H]	**Φ**	magnetic flux	[webers, Wb]
N	number of turns	[number]	**Ψ**	magnetic linkage	[webers, Wb]
P	power	[watts, W]	**ψ**	instantaneous **Ψ**	[webers, Wb]

Resistance

The resistance **R** of a circuit is equal to the applied direct voltage **E** divided by the resulting steady current **I**:
$\mathbf{R = E / I}$

Resistances in Series

When resistances $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, ... are connected in series, the total resistance $\mathbf{R_S}$ is:
$\mathbf{R_S = R_1 + R_2 + R_3 + ...}$

Voltage Division by Series Resistances

When a total voltage $\mathbf{E_S}$ is applied across series connected resistances $\mathbf{R_1}$ and $\mathbf{R_2}$, the current $\mathbf{I_S}$ which flows through the series circuit is:
$\mathbf{I_S = E_S / R_S = E_S / (R_1 + R_2)}$
The voltages $\mathbf{V_1}$ and $\mathbf{V_2}$ which appear across the respective resistances $\mathbf{R_1}$ and $\mathbf{R_2}$ are:
$\mathbf{V_1 = I_SR_1 = E_SR_1 / R_S = E_SR_1 / (R_1 + R_2)}$
$\mathbf{V_2 = I_SR_2 = E_SR_2 / R_S = E_SR_2 / (R_1 + R_2)}$
In general terms, for resistances $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, ... connected in series:
$\mathbf{I_S = E_S / R_S = E_S / (R_1 + R_2 + R_3 + ...)}$
$\mathbf{V_n = I_SR_n = E_SR_n / R_S = E_SR_n / (R_1 + R_2 + R_3 + ...)}$
Note that the highest voltage drop appears across the highest resistance.

Resistances in Parallel

When resistances $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, ... are connected in parallel, the total resistance $\mathbf{R_P}$ is:
$\mathbf{1 / R_P = 1 / R_1 + 1 / R_2 + 1 / R_3 + ...}$
Alternatively, when conductances $\mathbf{G_1}$, $\mathbf{G_2}$, $\mathbf{G_3}$, ... are connected in parallel, the total conductance $\mathbf{G_P}$ is:
$\mathbf{G_P = G_1 + G_2 + G_3 + ...}$
where $\mathbf{G_n = 1 / R_n}$
For two resistances $\mathbf{R_1}$ and $\mathbf{R_2}$ connected in parallel, the total resistance $\mathbf{R_P}$ is:
$\mathbf{R_P = R_1R_2 / (R_1 + R_2)}$
$\mathbf{R_P}$ = product / sum
The resistance $\mathbf{R_2}$ to be connected in parallel with resistance $\mathbf{R_1}$ to give a total resistance $\mathbf{R_P}$ is:
$\mathbf{R_2 = R_1R_P / (R_1 - R_P)}$
$\mathbf{R_2}$ = product / difference

Current Division by Parallel Resistances

When a total current $\mathbf{I_P}$ is passed through parallel connected resistances $\mathbf{R_1}$ and $\mathbf{R_2}$, the voltage $\mathbf{V_P}$ which appears across the parallel circuit is:
$\mathbf{V_P = I_PR_P = I_PR_1R_2 / (R_1 + R_2)}$
The currents $\mathbf{I_1}$ and $\mathbf{I_2}$ which pass through the respective resistances $\mathbf{R_1}$ and $\mathbf{R_2}$ are:
$\mathbf{I_1 = V_P / R_1 = I_PR_P / R_1 = I_PR_2 / (R_1 + R_2)}$
$\mathbf{I_2 = V_P / R_2 = I_PR_P / R_2 = I_PR_1 / (R_1 + R_2)}$
In general terms, for resistances $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, ... (with conductances $\mathbf{G_1}$, $\mathbf{G_2}$, $\mathbf{G_3}$, ...) connected in parallel:
$\mathbf{V_P = I_PR_P = I_P / G_P = I_P / (G_1 + G_2 + G_3 + ...)}$
$\mathbf{I_n = V_P / R_n = V_PG_n = I_PG_n / G_P = I_PG_n / (G_1 + G_2 + G_3 + ...)}$
where $\mathbf{G_n = 1 / R_n}$
Note that the highest current passes through the highest conductance (with the lowest resistance).

Capacitance

When a voltage is applied to a circuit containing capacitance, current flows to accumulate charge in the capacitance:
$\mathbf{Q = \int idt = CV}$
Alternatively, by differentiation with respect to time:
$\mathbf{dq/dt = i = C\ dv/dt}$
Note that the rate of change of voltage has a polarity which opposes the flow of current.
The capacitance $\mathbf{C}$ of a circuit is equal to the charge divided by the voltage:
$\mathbf{C = Q / V = \int idt / V}$
Alternatively, the capacitance $\mathbf{C}$ of a circuit is equal to the charging current divided by the rate of change of voltage:
$\mathbf{C = i / dv/dt = dq/dt / dv/dt = dq/dv}$

Capacitances in Series

When capacitances $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, ... are connected in series, the total capacitance $\mathbf{C_S}$ is:
$\mathbf{1 / C_S = 1 / C_1 + 1 / C_2 + 1 / C_3 + ...}$
For two capacitances $\mathbf{C_1}$ and $\mathbf{C_2}$ connected in series, the total capacitance $\mathbf{C_S}$ is:
$\mathbf{C_S = C_1C_2 / (C_1 + C_2)}$
$\mathbf{C_S}$ = product / sum

Voltage Division by Series Capacitances

When a total voltage $\mathbf{E_S}$ is applied to series connected capacitances $\mathbf{C_1}$ and $\mathbf{C_2}$, the charge $\mathbf{Q_S}$ which accumulates in the series circuit is:
$\mathbf{Q_S = \int i_Sdt = E_SC_S = E_SC_1C_2 / (C_1 + C_2)}$
The voltages $\mathbf{V_1}$ and $\mathbf{V_2}$ which appear across the respective capacitances $\mathbf{C_1}$ and $\mathbf{C_2}$ are:
$\mathbf{V_1 = \int i_Sdt / C_1 = E_SC_S / C_1 = E_SC_2 / (C_1 + C_2)}$
$\mathbf{V_2 = \int i_Sdt / C_2 = E_SC_S / C_2 = E_SC_1 / (C_1 + C_2)}$
In general terms, for capacitances $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, ... connected in series:
$\mathbf{Q_S = \int i_Sdt = E_SC_S = E_S / (1 / C_S) = E_S / (1 / C_1 + 1 / C_2 + 1 / C_3 + ...)}$
$\mathbf{V_n = \int i_Sdt / C_n = E_SC_S / C_n = E_S / C_n(1 / C_S) = E_S / C_n(1 / C_1 + 1 / C_2 + 1 / C_3 + ...)}$
Note that the highest voltage appears across the lowest capacitance.

Capacitances in Parallel

When capacitances $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, ... are connected in parallel, the total capacitance $\mathbf{C_P}$ is:
$\mathbf{C_P = C_1 + C_2 + C_3 + ...}$

Charge Division by Parallel Capacitances

When a voltage $\mathbf{E_P}$ is applied to parallel connected capacitances $\mathbf{C_1}$ and $\mathbf{C_2}$, the charge $\mathbf{Q_P}$ which accumulates in the parallel circuit is:

$$Q_P = \int i_P dt = E_P C_P = E_P(C_1 + C_2)$$

The charges $\mathbf{Q_1}$ and $\mathbf{Q_2}$ which accumulate in the respective capacitances $\mathbf{C_1}$ and $\mathbf{C_2}$ are:

$$Q_1 = \int i_1 dt = E_P C_1 = Q_P C_1 / C_P = Q_P C_1 / (C_1 + C_2)$$

$$Q_2 = \int i_2 dt = E_P C_2 = Q_P C_2 / C_P = Q_P C_2 / (C_1 + C_2)$$

In general terms, for capacitances $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, ... connected in parallel:

$$Q_P = \int i_P dt = E_P C_P = E_P(C_1 + C_2 + C_3 + ...)$$

$$Q_n = \int i_n dt = E_P C_n = Q_P C_n / C_P = Q_P C_n / (C_1 + C_2 + C_3 + ...)$$

Note that the highest charge accumulates in the highest capacitance.

Inductance

When the current changes in a circuit containing inductance, the magnetic linkage changes and induces a voltage in the inductance:

$$d\psi/dt = e = L\ di/dt$$

Note that the induced voltage has a polarity which opposes the rate of change of current.
Alternatively, by integration with respect to time:

$$\Psi = \int e dt = LI$$

The inductance $\mathbf{L}$ of a circuit is equal to the induced voltage divided by the rate of change of current:

$$L = e / di/dt = d\psi/dt / di/dt = d\psi/di$$

Alternatively, the inductance $\mathbf{L}$ of a circuit is equal to the magnetic linkage divided by the current:

$$L = \Psi / I$$

Note that the magnetic linkage $\mathbf{\Psi}$ is equal to the product of the number of turns $\mathbf{N}$ and the magnetic flux $\mathbf{\Phi}$:

$$\Psi = N\Phi = LI$$

Mutual Inductance

The mutual inductance $\mathbf{M}$ of two coupled inductances $\mathbf{L_1}$ and $\mathbf{L_2}$ is equal to the mutually induced voltage in one inductance divided by the rate of change of current in the other inductance:

$$M = E_{2m} / (di_1/dt)$$

$$M = E_{1m} / (di_2/dt)$$

If the self induced voltages of the inductances $\mathbf{L_1}$ and $\mathbf{L_2}$ are respectively $\mathbf{E_{1s}}$ and $\mathbf{E_{2s}}$ for the same rates of change of the current that produced the mutually induced voltages $\mathbf{E_{1m}}$ and $\mathbf{E_{2m}}$, then:

$$M = (E_{2m} / E_{1s})L_1$$

$$M = (E_{1m} / E_{2s})L_2$$

Combining these two equations:

$$M = (E_{1m}E_{2m} / E_{1s}E_{2s})^{½} (L_1L_2)^{½} = k_M(L_1L_2)^{½}$$

where $\mathbf{k_M}$ is the mutual coupling coefficient of the two inductances $\mathbf{L_1}$ and $\mathbf{L_2}$.
If the coupling between the two inductances $\mathbf{L_1}$ and $\mathbf{L_2}$ is perfect, then the mutual inductance $\mathbf{M}$ is:

$$M = (L_1L_2)^{½}$$

Inductances in Series

When uncoupled inductances $\mathbf{L_1}$, $\mathbf{L_2}$, $\mathbf{L_3}$, ... are connected in series, the total inductance $\mathbf{L_S}$ is:

$$L_S = L_1 + L_2 + L_3 + ...$$

When two coupled inductances $\mathbf{L_1}$ and $\mathbf{L_2}$ with mutual inductance $\mathbf{M}$ are connected in series, the total inductance $\mathbf{L_S}$ is:

$$L_S = L_1 + L_2 \pm 2M$$

The plus or minus sign indicates that the coupling is either additive or subtractive, depending on the connection polarity.

Inductances in Parallel

When uncoupled inductances L_1, L_2, L_3, ... are connected in parallel, the total inductance L_P is:
$1 / L_P = 1 / L_1 + 1 / L_2 + 1 / L_3 + ...$

Time Constants

Capacitance and resistance

The time constant of a capacitance **C** and a resistance **R** is equal to **CR**, and represents the time to change the voltage on the capacitance from zero to **E** at a constant charging current **E / R** (which produces a rate of change of voltage **E / CR** across the capacitance).
Similarly, the time constant **CR** represents the time to change the charge on the capacitance from zero to **CE** at a constant charging current **E / R** (which produces a rate of change of voltage **E / CR** across the capacitance).
If a voltage **E** is applied to a series circuit comprising a discharged capacitance **C** and a resistance **R**, then after time **t** the current **i**, the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:
$i = (E / R)e^{-t / CR}$
$v_R = iR = Ee^{-t / CR}$
$v_C = E - v_R = E(1 - e^{-t / CR})$
$q_C = Cv_C = CE(1 - e^{-t / CR})$
If a capacitance **C** charged to voltage **V** is discharged through a resistance **R**, then after time **t** the current **i**, the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:
$i = (V / R)e^{-t / CR}$
$v_R = iR = Ve^{-t / CR}$
$v_C = v_R = Ve^{-t / CR}$
$q_C = Cv_C = CVe^{-t / CR}$

Inductance and resistance

The time constant of an inductance **L** and a resistance **R** is equal to **L / R**, and represents the time to change the current in the inductance from zero to **E / R** at a constant rate of change of current **E / L** (which produces an induced voltage **E** across the inductance).
If a voltage **E** is applied to a series circuit comprising an inductance **L** and a resistance **R**, then after time **t** the current **i**, the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:
$i = (E / R)(1 - e^{-tR / L})$
$v_R = iR = E(1 - e^{-tR / L})$
$v_L = E - v_R = Ee^{-tR / L}$
$\psi_L = Li = (LE / R)(1 - e^{-tR / L})$
If an inductance **L** carrying a current **I** is discharged through a resistance **R**, then after time **t** the current **i**, the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:
$i = Ie^{-tR / L}$
$v_R = iR = IRe^{-tR / L}$
$v_L = v_R = IRe^{-tR / L}$
$\psi_L = Li = LIe^{-tR / L}$

Rise Time and Fall Time

The rise time (or fall time) of a change is defined as the transition time between the 10% and 90% levels of the total change, so for an exponential rise (or fall) of time constant **T**, the rise time (or fall time) t_{10-90} is:
$t_{10-90} = (\ln 0.9 - \ln 0.1)T \approx 2.2T$
The half time of a change is defined as the transition time between the initial and 50% levels of the total change, so for an exponential change of time constant **T**, the half time t_{50} is :
$t_{50} = (\ln 1.0 - \ln 0.5)T \approx 0.69T$
Note that for an exponential change of time constant **T**:
- over time interval **T**, a rise changes by a factor $1 - e^{-1}$ (≈ 0.63) of the remaining change,
- over time interval **T**, a fall changes by a factor e^{-1} (≈ 0.37) of the remaining change,
- after time interval 3**T**, less than 5% of the total change remains,
- after time interval 5**T**, less than 1% of the total change remains.

Power

The power **P** dissipated by a resistance **R** carrying a current **I** with a voltage drop **V** is:
$P = V^2 / R = VI = I^2R$
Similarly, the power **P** dissipated by a conductance **G** carrying a current **I** with a voltage drop **V** is:
$P = V^2G = VI = I^2 / G$
The power **P** transferred by a capacitance **C** holding a changing voltage **V** with charge **Q** is:
$P = VI = CV(dv/dt) = Q(dv/dt) = Q(dq/dt) / C$
The power **P** transferred by an inductance **L** carrying a changing current **I** with magnetic linkage **Ψ** is:
$P = VI = LI(di/dt) = \Psi(di/dt) = \Psi(d\psi/dt) / L$

Energy

The energy **W** consumed over time **t** due to power **P** dissipated in a resistance **R** carrying a current **I** with a voltage drop **V** is:
$W = Pt = V^2t / R = VIt = I^2tR$
Similarly, the energy **W** consumed over time **t** due to power **P** dissipated in a conductance **G** carrying a current **I** with a voltage drop **V** is:
$W = Pt = V^2tG = VIt = I^2t / G$
The energy **W** stored in a capacitance **C** holding voltage **V** with charge **Q** is:
$W = CV^2 / 2 = QV / 2 = Q^2 / 2C$
The energy **W** stored in an inductance **L** carrying current **I** with magnetic linkage **Ψ** is:
$W = LI^2 / 2 = \Psi I / 2 = \Psi^2 / 2L$

Batteries

If a battery of open-circuit voltage E_B has a loaded voltage V_L when supplying load current I_L, the battery internal resistance R_B is:
$R_B = (E_B - V_L) / I_L$
The load voltage V_L and load current I_L for a load resistance R_L are:
$V_L = I_LR_L = E_B - I_LR_B = E_BR_L / (R_B + R_L)$
$I_L = V_L / R_L = (E_B - V_L) / R_B = E_B / (R_B + R_L)$
The battery short-circuit current I_{sc} is:
$I_{sc} = E_B / R_B = E_BI_L / (E_B - V_L)$

Voltmeter Multiplier

The resistance R_S to be connected in series with a voltmeter of full scale voltage V_V and full scale current drain I_V to increase the full scale voltage to **V** is:
$R_S = (V - V_V) / I_V$
The power **P** dissipated by the resistance R_S with voltage drop $(V - V_V)$ carrying current I_V is:
$P = (V - V_V)^2 / R_S = (V - V_V)I_V = I_V^2R_S$

Ammeter Shunt

The resistance R_P to be connected in parallel with an ammeter of full scale current I_A and full scale voltage drop V_A to increase the full scale current to **I** is:
$R_P = V_A / (I - I_A)$
The power **P** dissipated by the resistance R_P with voltage drop V_A carrying current $(I - I_A)$ is:
$P = V_A^2 / R_P = V_A(I - I_A) = (I - I_A)^2R_P$