Section 5

Motion

5.1 Making sense of equilibrium

The concept of equilibrium lies behind many types of engineering analyses and design.

5.1.1 Definitions

- *Formally* An object is in a state of equilibrium when the forces acting on it are such as to leave it in its state of rest or uniform motion in a straight line.
- *Practically* The most useful interpretation is that an object is in equilibrium when the forces acting on it are producing no tendency for the object to move.

Figure 5.1 shows the difference between equilibrium and non-equilibrium.

5.1.2 How is it used?

The concept of equilibrium is used to analyse engineering structures and components. By isolating a part of a structure (a joint or a member) which is in a state of equilibrium, this enables a 'free body diagram' to be drawn. This aids in the analysis of the stresses (and the resulting strains) in the structure. When co-planar forces acting at a point are in equilibrium, the vector diagram closes.

Engineers' Data Book, Fourth Edition. Clifford Matthews.

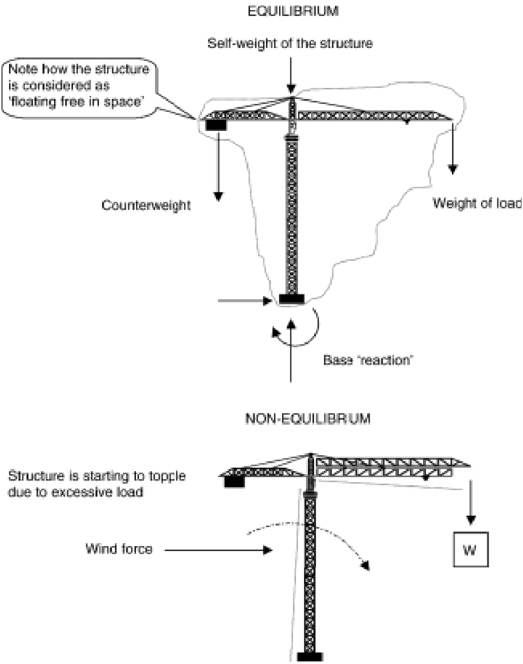


Figure 5.1

5.2 Motion equations

5.2.1 Uniformly accelerated motion

Bodies under uniformally accelerated motion follow the general equations given here.

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2 \qquad t = \text{time (s)}$$

$$a = \text{acceleration } (\text{m/s}^2)$$

$$s = \left(\frac{u+v}{2}\right)t \qquad s = \text{distance travelled (m)}$$

$$u = \text{initial velocity (m/s)}$$

$$v^2 = u^2 + 2as \qquad v = \text{final velocity (m/s)}$$

5.2.2 Angular motion

$\omega = \dfrac{2\pi N}{60}$	t = time (s)
$\omega_2 = \omega_1 + \alpha t$	θ = angle moved (rad)
$\theta = \left(\frac{\omega_1 + \omega_2}{2}\right) t$	α = angular acceleration (rad/s^2)
	N = angular speed (rev/min)
$\omega_2^2 = \omega_1^2 + 2\alpha s$	ω_1 = initial angular velocity (rad/s)
	ω_2 = final angular velocity (rad/s)
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	

5.2.3 General motion of a particle in a plane

$v = \mathrm{d}s/\mathrm{d}t$	s = distance
$a = \mathrm{d}v/\mathrm{d}t = \mathrm{d}^2 s/\mathrm{d}t^2$	t = time
$v = \int a\,\mathrm{d}t$	v = velocity
$s = \int v\,\mathrm{d}t$	a = acceleration

5.3 Newton's laws of motion

First law Everybody will remain at rest or continue in uniform motion in a straight line until acted upon by an external force.

Second law When an external force is applied to a body of constant mass it produces an acceleration which is directly proportional to the force. i.e. Force (F) = mass (m) $\times$ acceleration (a)

Third law Every action produces an equal and opposite reaction.

5.3.1 Comparisons: rotational and translational motion

Translation		*Rotation*	
Linear displacement from a datum	x	Angular displacement	θ
Linear velocity	v	Angular velocity	ω
Linear acceleration	$a = \mathrm{d}v/\mathrm{d}t$	Angular acceleration	$\alpha = \mathrm{d}\omega/\mathrm{d}t$
Kinetic energy	$\mathrm{KE} = mv^2/2$	Kinetic energy	$\mathrm{KE} = I\omega^2/2$
Momentum	mv	Momentum	$I\omega$
Newton's second law	$\mathrm{F} = \mathrm{md}_2\mathrm{x}/\mathrm{d}t^2$	Newton's second law	$M = \mathrm{d}_2\theta/\mathrm{d}t^2$

5.4 Simple harmonic motion (SHM)

A particle moves with SHM when it has constant angular velocity (ω). The projected displacement, velocity, and acceleration of a point P on the x.y axes are a sinusoidal function of time (t).

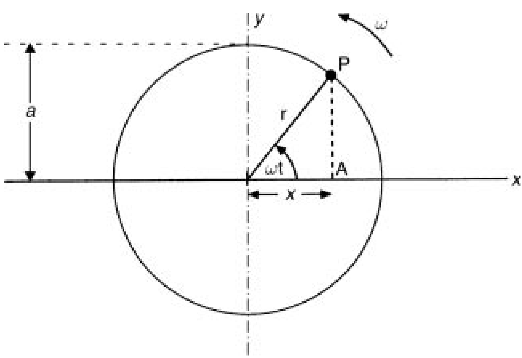


Figure 5.2 Simple harmonic motion

Angular velocity $(\omega) = 2\pi N/60$ where N is in rev/min

Periodic time $(T) = 2\pi/\omega$

Velocity (v) of point A on the x axis is

$v = \mathrm{d}s/\mathrm{d}t = \omega r \sin \omega t$

Acceleration $(a) = \mathrm{d}_2 s/\mathrm{d}t^2 = \mathrm{d}v/\mathrm{d}t = -\omega^2 r \cos \omega t$

5.5 Understanding acceleration

The dangerous thing about acceleration is that it represents a *rate of change* of speed or velocity. When this rate of change is high it puts high stresses on engineering components, causing them to deform and break. In the neat world of physical science, objects in a vacuum experience a constant acceleration (g) due to gravity of 9.81m/s^2 – so if you drop a hammer and a feather they will reach the ground at the same time.

Unfortunately, you won't find many engineering products made of hammers and feathers locked inside vacuum chambers. In practice, the components of engineering machines experience acceleration many times the force of gravity so they have to be designed to resist the forces that result. Remember that these forces can be caused as a result of either linear or angular accelerations and that there is a correspondence between the two as shown on the following page.

Linear acceleration	*Angular acceleration*
$a = \frac{v-u}{t} \mathrm{m/s^2}$	$\alpha = \frac{\omega_2-\omega_1}{t} \mathrm{rad/s^2}$

5.5.1 Design hint

When analysing (or designing) any machine or mechanism think about linear and angular accelerations *first* – they are always important.

5.6 Dynamic balancing

Virtually all rotating machines (pumps, shafts, turbines, gear-sets, generators, etc.) are subject to dynamic balancing during manufacture. The objective is to maintain the operating vibration of the machine within manageable limits.

Dynamic balancing normally involves two measurement/correction planes and involves the calculation of vector quantities. The component is mounted in a balancing rig which rotates it at or near its operating speed, and both senses and records out-of-balance forces and phase angle in two planes. Balance weights are then added (or removed) to bring the imbalance forces to an acceptable level.

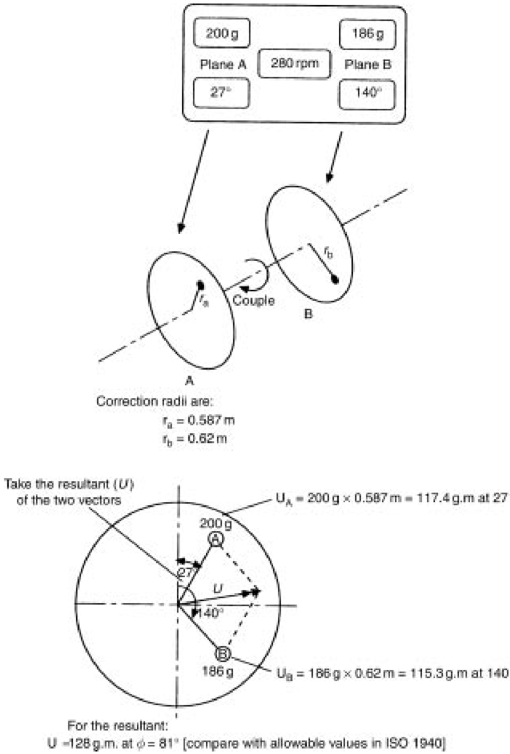


Figure 5.3

5.6.1 Balancing standard: ISO 1940/1: 2003

The standard ISO 1940/1: 2003 (identical to BS 6861: Part 1: 1987): Balance quality requirements of rigid rotors is widely used. It sets acceptable imbalance limits for various types of rotating equipment. It specifies various (G) grades. A similar approach is used by the standard ISO 10816-1.

Finer balance grades are used for precision assemblies such as instruments and gyroscopes. The principles are the same.

5.7 Vibration

Vibration is a subset of the subject of dynamics. It has particular relevance to both structures and machinery in the way that they respond to applied disturbances.

5.7.1 General model

The most common model of vibration is a concentrated spring-mounted mass which is subject to a disturbing force and retarding force.

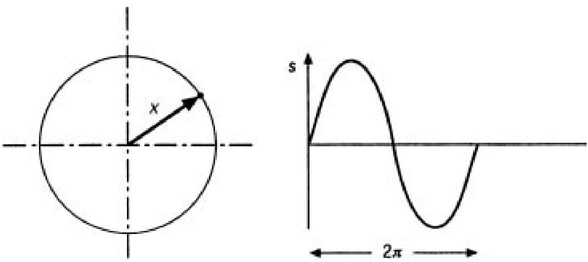


Figure 5.4

The motion is represented graphically as shown by the projection of the rotating vector x. Relevant quantities are

$$\text{frequency (Hz)} = \sqrt{\frac{k}{m}}/2\pi$$

The ideal case represents simple harmonic motion with the waveform being sinusoidal. Hence the motion follows the general pattern:

$$\text{Vibration displacement (amplitude)} = s$$
$$\text{Vibration velocity} = v = \mathrm{d}s/\mathrm{d}t$$
$$\text{Vibration acceleration} = a = \mathrm{d}v/\mathrm{d}t$$

5.8 Machine vibration

There are two types of vibration relevant to rotating machines

- Bearing *housing* vibration. This is assumed to be sinusoidal. It normally uses the velocity (V_{rms}) parameter.
- *Shaft* vibration. This is generally not sinusoidal. It normally uses displacement (s) as the measured parameter.

5.8.1 Bearing housing vibration

Relevant points are:

- It only measures vibration at the 'surface'.
- It excludes torsional vibration.
- V_{rms} is normally measured across the frequency range and then distilled down to a single value.

$$\text{i.e. } V_{rms} = \sqrt{\frac{1}{2}\left(\sum \text{amplitudes} \times \text{angular frequences}\right)}$$

- It is covered in the German standard VDI 2056: Criteria for assessing mechanical vibration of machines and BS 7854: 1995: Mechanical vibration.

5.8.2 Acceptance levels

Technical standards, and manufacturers' practices, differ in their acceptance levels. General 'rule of thumb' acceptance levels are shown in Figures 5.5 and 5.6.

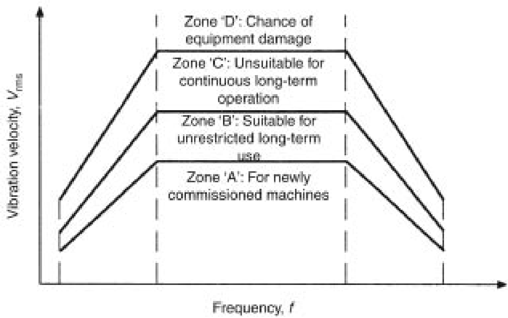
Machine	V_{rms}(mm/s)
Precision components and machines – gas turbines, etc.	1.12
Helical and epicyclic gearboxes	1.8
Spur-gearboxes, turbines	2.8
General service pumps	4.5
Long-shaft pumps	4.5–7.1
Diesel engines	7.1
Reciprocating large machines	7.1–11.2

Figure 5.5

Typical balance grades: from International Standard ISO 1940–1

Balance grade	Types of rotor (general examples)
G 1	Grinding machines, tape-recording equipment
G2.5	Turbines, compressors, electric armatures
G6.3	Pump impellers fans, gears, machine tools
G 16	Cardan shafts, agricultural machinery
G 40	Car wheels, engine crankshafts
G 100	Complete engines for cars and trucks

'Acceptance criteria': from International Standard ISO 10816–1



Typical 'boundary limits': from International Standard ISO 10816–1

<table>
<tr><th>V_{rms}</th><th>Class I</th><th>Class II</th><th>Class III</th><th>Class IV</th></tr>
<tr><td>0.71</td><td>A</td><td rowspan="2">A</td><td rowspan="3">A</td><td rowspan="4">A</td></tr>
<tr><td>1.12</td><td rowspan="2">B</td></tr>
<tr><td>1.8</td><td rowspan="2">B</td></tr>
<tr><td>2.8</td><td rowspan="2">C</td><td rowspan="2">B</td></tr>
<tr><td>4.5</td><td rowspan="2">C</td><td rowspan="2">B</td></tr>
<tr><td>7.1</td><td rowspan="3">D</td><td rowspan="2">C</td></tr>
<tr><td>11.2</td><td rowspan="2">D</td><td rowspan="2">C</td></tr>
<tr><td>18</td><td>D</td></tr>
</table>

<u>Class suitability</u>

Class I	Machines < 15kW
Class II	< 300kW
Class III	Large machines with rigid foundations
Class IV	Large machines with 'soft' foundations

(Note how wide these classes are)

Figure 5.6

5.9 Machinery noise

5.9.1 Principles

Noise is most easily thought of as air-borne pressure pulses set up by a vibrating surface source. It is measured by an instrument which detects these pressure changes in the air and then relates this measured sound pressure to an accepted zero level. Because a machine produces a mixture of frequencies (termed *broad-band* noise), there is no single noise measurement that will fully describe a noise emission. In practice, two methods used are:

- The '*overall noise*' level. This is often used as a colloquial term for what is properly described as the *A-weighted sound pressure level*. It incorporates multiple frequencies, and weights them according to a formula which results in the best approximation of the loudness of the noise. This is displayed as a single instrument reading expressed as decibels dB(A).
- *Frequency band* sound pressure level. This involves measuring the sound pressure level in a number of frequency bands. These are arranged in either octave or one-third octave bands in terms of their mid-band frequency. The range of frequencies of interest in measuring machinery noise is from about 30 Hz to 10 000 Hz. Note that frequency band sound pressure levels are also expressed in decibels (dB).

The decibel scale itself is a logarithmic scale – a sound pressure level in dB being defined as:

$$\text{dB} = 10\log_{10}(p_1/p_0)^2$$

where

p_1 = measured sound pressure
p_0 = a reference zero pressure level

Noise tests on rotating machines are carried out by defining a 'reference surface' and then positioning microphones at locations 1 m from it.

5.9.2 Typical levels

Approximate 'rule of thumb' noise levels are given in Table 5.1.

A normal 'specification' level is 90–95 dB (A) at 1 m from operating equipment. Noisier equipment needs an acoustic

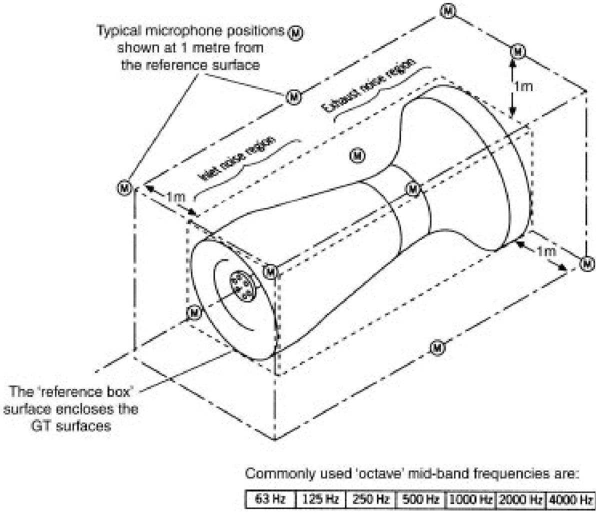


Figure 5.7

Table 5.1

Machine/environment	*dB*(A)
A whisper	20
Office noise	50
Noisy factory	90
Large diesel engine	97
Turbocompressor/gas turbine	98

enclosure. Humans can continue to hear increasing sound levels up to about 120 dB. Above this causes serious discomfort and long-term damage.