Section 9

Basic Fluid Mechanics and Aerodynamics

9.1 Basic properties

9.1.1 Basic relationships

Fluids are divided into (a) liquids, which are virtually incompressible and (b) gases, which are compressible. A fluid consists of a collection of molecules in constant motion. A liquid adopts the shape of the vessel containing it, while a gas expands to fill any container in which it is placed. Some basic fluid relationships are given in Table 9.1.

Table 9.1 Basic fluid relationships

Density (ρ)	Mass per unit volume. Units kg/m^3 (lb/in^3)
Specific gravity (s)	Ratio of density to that of water i.e. $s = \rho/\rho_{water}$
Specific volume (v)	Reciprocal of density i.e. $s = 1/\rho$. Units m^3/kg (in^3/lb)
Dynamic viscosity (μ)	A force per unit area or shear stress of a fluid. Units Ns/m^2 (lbf.s/ft^2)
Kinematic viscosity (ν)	A ratio of dynamic viscosity to density i.e. $\nu = \mu/\rho$. Units m^2/s (ft^2/s)

9.1.2 Perfect gas

A perfect, or 'ideal', gas is one which follows Boyles/Charles law $pv = RT$ where

p = pressure of the gas
v = specific volume
T = absolute temperature
R = the universal gas constant

Although no actual gases follow this law totally, the behaviour of most gases at temperatures well above their liquification temperature will approximate to it and so they can be *considered* as a perfect gas.

Engineers' Data Book, Fourth Edition. Clifford Matthews.

9.1.3 Changes of state

When a perfect gas changes state its behaviour approximates to

$$pv^n = \text{Constant}$$

where n is known as the polytropic exponent.

Figure 9.1 shows the four main changes of state relevant to aeronautics; isothermal, adiabatic, polytropic, and isobaric.

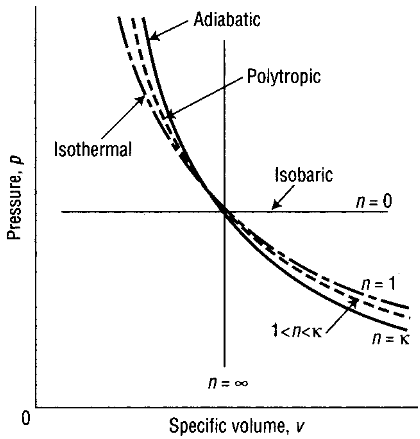


Figure 9.1

9.1.4 Compressibility

The extent to which a fluid can be compressed in volume is expressed using the compressibility coefficient β.

$$\beta = \frac{\Delta v/v}{\Delta p} = \frac{1}{K}$$

where

Δv = change in volume
v = initial volume
Δp = change in pressure
K = bulk modulus

Also

$$K = \rho \frac{\Delta p}{\Delta \rho} = \rho \frac{\mathrm{d}p}{\mathrm{d}\rho}$$

and

$$a = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}} = \sqrt{\frac{K}{\rho}}$$

where

a = the velocity of propagation of a pressure wave in the fluid.

9.1.5 Fluid statics

Fluid statics is the study of fluids that are at rest (i.e. not flowing) relative to the vessel containing them. Pressure has four important characteristics:

- pressure applied to a fluid in a closed vessel (such as a hydraulic ram) is transmitted to all parts of the closed vessel at the same value (Pascal's law);
- magnitude of pressure force acting at any point in a static fluid is the same, irrespective of direction;
- pressure force always acts perpendicular to the boundary containing it;
- the pressure 'inside' a liquid increases in proportion to its depth.

Other important static pressure statements are:

- absolute pressure = gauge pressure + atmospheric pressure;
- pressure (p) at depth (h) in a liquid is given by $p = \rho g h$;
- a general equation for a fluid at rest is

$$p\mathrm{d}A - \left(p + \frac{\mathrm{d}p}{\mathrm{d}z} \cdot \mathrm{d}z\right)\mathrm{d}A - \rho g \,\mathrm{d}A\,\mathrm{d}z = 0$$

This relates to an infinitesimal vertical cylinder of fluid.

9.2 Flow equations

Flow of a fluid may be one dimensional (1-D), two dimensional (2-D), or three dimensional (3D), depending on the way in which the flow is constrained.

9.2.1 One-dimensional flow

One-dimensional flow has a single-direction coordinate x and a velocity in the direction of v. Flow in a pipe or tube is generally considered one-dimensional. The equations for 1-D flow are derived by considering flow along a straight-stream tube (see Fig. 9.2). Table 9.2 shows the principles, and their resulting equations.

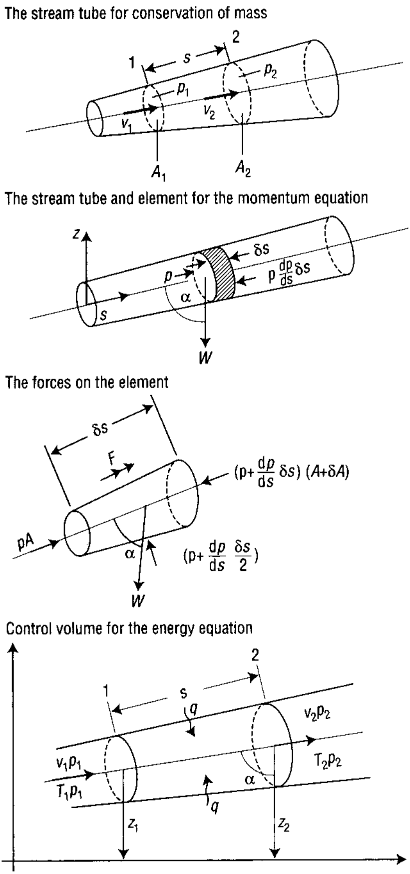


Figure 9.2

Table 9.2 Fluid principles

Law	*Basis*	*Resulting equations*
Conservation of mass	Matter (in a stream tube or anywhere else) cannot be created or destroyed.	$\rho vA = \text{constant}$
Conservation of momentum	The rate of change of momentum in a given direction = algebraic sum of the forces acting in that direction (Newton's second law of motion).	$\int \sqrt{\frac{dp}{\rho}} + 1/2\, v^2 + gz = \text{constant}$ This is Bernoulli's equation.
Conservation of energy	Energy, heat, and work are convertible into each other and are in balance in a steadily operating system.	$c_p T + \frac{v^2}{2} = \text{constant}$ for an adiabatic (no heat transferred) flow system.
Equation of state	Perfect gas state: $p/\rho T = R$ and the first law of thermodynamics	$p = k\rho^{\gamma}$ k = constant y = ratio of specific heat c_p/c_v

9.2.2 Two-dimensional flow

Two-dimensional flow (as in the space between two parallel flat plates) is that in which all velocities are parallel to a given plane. Either rectangular (x, y) or polar (r, θ) coordinates may be used to describe the characteristics of 2-D flow. Figure 9.3 and Table 9.3 show the fundamental equations.

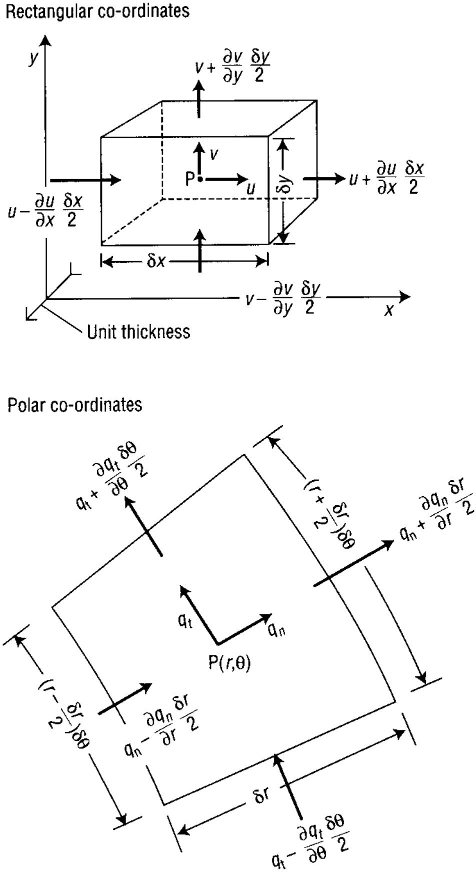

Rectangular co-ordinates
y
$v + \frac{\partial v}{\partial y}\frac{\delta y}{2}$
v
P
u
δy
$u + \frac{\partial u}{\partial x}\frac{\delta x}{2}$
$u - \frac{\partial u}{\partial x}\frac{\delta x}{2}$
δx
$v - \frac{\partial v}{\partial y}\frac{\delta y}{2}$
x
Unit thickness
Polar co-ordinates
$q_t + \frac{\partial q_t}{\partial \theta}\frac{\delta \theta}{2}$
$(r + \frac{\delta r}{2})\delta\theta$
$q_n + \frac{\partial q_n}{\partial r}\frac{\delta r}{2}$
q_t
q_n
$P(r,\theta)$
$(r - \frac{\delta r}{2})\delta\theta$
$q_n - \frac{\partial q_n}{\partial r}\frac{\delta r}{2}$
δr
$q_t - \frac{\partial q_t}{\partial \theta}\frac{\delta \theta}{2}$

Figure 9.3

Table 9.3 Two-dimensional flow: fundamental equations

Basis	*The equation*	*Explanation*
Laplace's equation	$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}$ or $\nabla^2\phi=\nabla^2\psi=0$ where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$	A flow described by a unique velocity potential is irrational.
Equation of motion in 2-D	$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\left(Y-\frac{\partial p}{\partial x}\right)$ $\frac{\partial v}{\partial t}+u\frac{\partial y}{\partial x}+v\frac{\partial v}{\partial t}=\frac{1}{\rho}\left(Y-\frac{\partial p}{\partial y}\right)$	The principle of force = mass × acceleration (Newton's law of motion) applies to fluids and fluid particles.
Equation of continuity in 2-D (incompressible flow)	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ or in polar $\frac{q_n}{r}+\frac{\partial q_n}{\partial r}+\frac{1}{r}\frac{\partial q_t}{\partial\theta}=0$	If fluid velocity increases in the x direction, it must decrease in the y direction (see Fig. 9.3)
Equation of vorticity	$\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=\varsigma$ or, in polar $\varsigma=\frac{q_t}{r}+\frac{\partial q_t}{\partial r}-\frac{1}{r}\frac{\partial q_n}{\partial\theta}$	A rotating or spinning element of fluid can be investigated by assuming it is a solid. (See Fig. 9.4)
Stream function ψ (incompressible flow)	Velocity at a point is given by $u=\frac{\partial\psi}{\partial y}$ $v=-\frac{\partial\psi}{\partial x}$	ψ is the stream function. Lines of constant ψ give the flow pattern of a fluid stream (See Fig. 9.5)
Velocity potential φ (irrotational 2-D flow)	Velocity at a point is given by $u=\frac{\partial\phi}{\partial x}$ $v=-\frac{\partial\phi}{\partial y}$	φ is defined as $\phi=\int_{op} q\cos\beta\, ds$ (See Fig. 9.6)

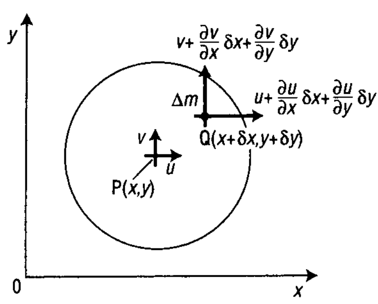


Figure 9.4

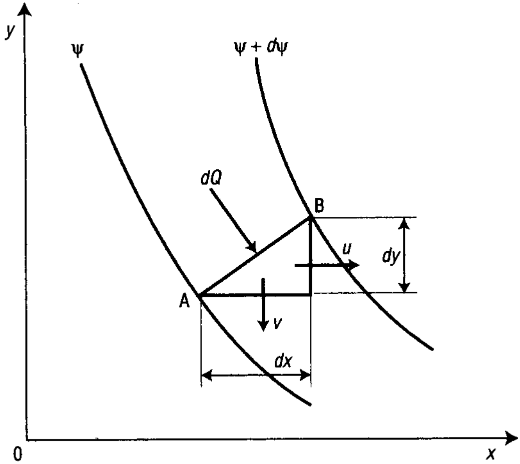


Figure 9.5

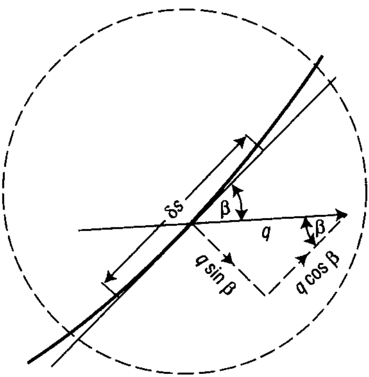


Figure 9.6

9.2.3 The navier–stokes equations

The Navier–Stokes equations are written as

$$\left.\begin{aligned}
\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) &= \rho X-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\
\underbrace{\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)}_{\text{Inertia term}} &= \underbrace{\rho Y}_{\text{Body Force Term}}\underbrace{-\frac{\partial p}{\partial y}}_{\text{Pressure term}}+\underbrace{\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)}_{\text{Viscous term}}
\end{aligned}\right\}$$

9.2.4 Sources and sinks

A '*source*' is an arrangement in which a volume of fluid (+ q) flows out evenly from an origin toward the periphery of an (imaginary) circle around it. If q is negative, such a point is termed a *sink* (see Fig. 9.7). If a source and sink of equal strength have their extremities infinitesimally close to each other, while increasing the strength, this is termed a '*doublet*'.

9.3 Flow regimes

9.3.1 General descriptions

Flow regimes can generally be described as follows (see Fig. 9.8):

- *Steady flow* Flow parameters at any point do not vary with time (even though they may differ between points).
- *Unsteady flow* Flow parameters at any point vary with time.
- *Laminar flow* Flow that is generally considered smooth, i.e. not broken up by eddies.
- *Turbulent flow* Non-smooth flow in which any small disturbance is magnified, causing eddies and turbulence.
- *Transition flow* The condition lying between laminar and turbulent flow regimes.

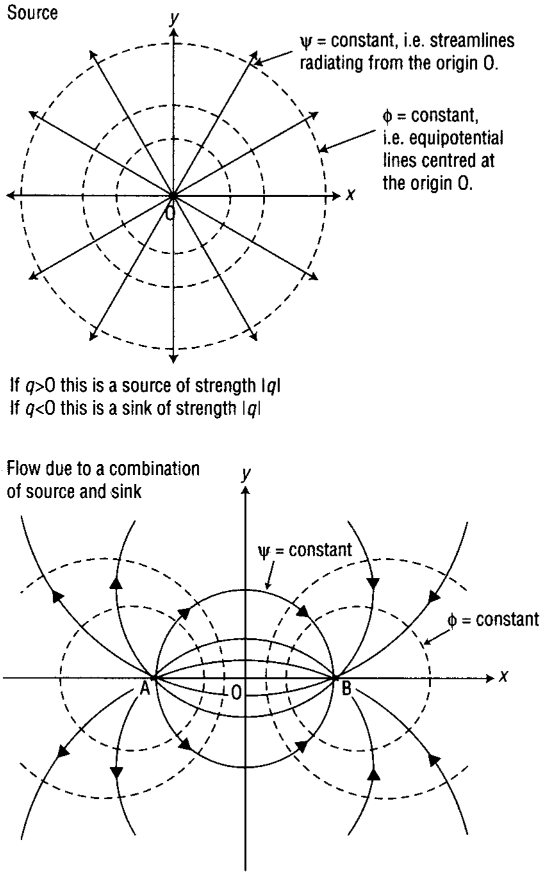

Source
y
x
O
ψ = constant, i.e. streamlines radiating from the origin O.
ϕ = constant, i.e. equipotential lines centred at the origin O.
If q>0 this is a source of strength |q|
If q<0 this is a sink of strength |q|
Flow due to a combination of source and sink
y
x
A
O
B
ψ = constant
ϕ = constant

Figure 9.7

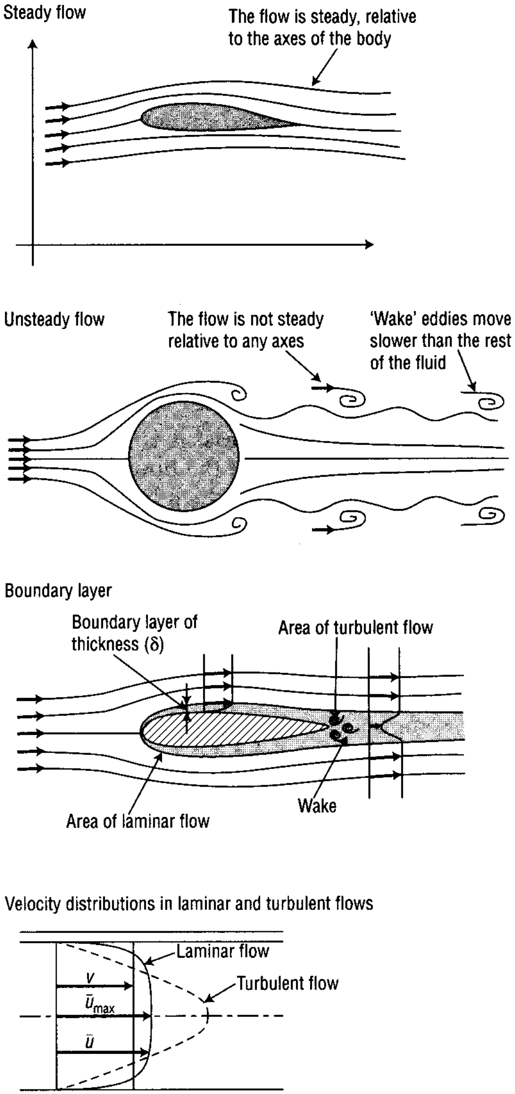

Steady flow
The flow is steady, relative to the axes of the body
Unsteady flow
The flow is not steady relative to any axes
'Wake' eddies move slower than the rest of the fluid
Boundary layer
Boundary layer of thickness (δ)
Area of turbulent flow
Area of laminar flow
Wake
Velocity distributions in laminar and turbulent flows
Laminar flow
Turbulent flow
V
Ū max
ū

Figure 9.8

9.3.2 Reynolds number

Reynolds number is a dimensionless quantity that determines the nature of flow of fluid over a surface.

$$\text{Renolds number (Re)} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

where

ρ = density
μ = dynamic viscosity
ν = kinematic viscosity
V = velocity
D = effective diameter

- Low Reynolds numbers (below about 2000) result in laminar flow.
- High Reynolds numbers (above about 2300) result in turbulent flow.
- Values of *Re* for $2000 < Re < 2300$ are generally considered to result in transition flow. Exact flow regimes are difficult to predict in this region.

9.4 Boundary layers

9.4.1 Definitions

- *The boundary layer* is the region near a surface or wall where the movement of the fluid flow is governed by frictional resistance.
- *The main flow* is the region outside the boundary layer which is not influenced by frictional resistance and can be assumed to be 'ideal' fluid flow.
- *Boundary layer thickness*: it is convention to assume that the edge of the boundary layer lies at a point in the flow which has a velocity equal to 99 per cent of the local mainstream velocity.

9.4.2 Some boundary layer equations

Figure 9.9 shows boundary layer velocity profiles for dimensional and non-dimensional cases. The non-dimensional case is used to allow comparison between boundary layer profiles of different thickness.

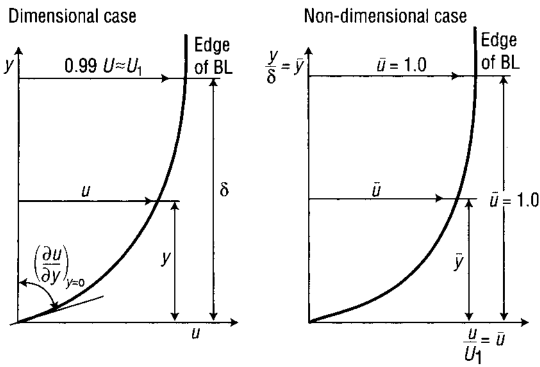


Figure 9.9

u = velocity parallel to the surface
y = perpendicular distance from the surface
δ = boundary layer thickness
U_1 = mainstrean velocity
$\bar{u}$ = velocity parameters u/U_1

Boundary layer equations of turbulent flow:

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{u}}{\partial y}\right)=-\frac{\partial \bar{p}}{\partial x}+\frac{\partial \tau}{\partial y}$$

$$\tau=\mu\frac{\partial \bar{u}}{\partial y}-\overline{\rho u' v'}$$

$$\frac{\partial \bar{p}}{\partial x}=0$$

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0$$

9.5 Isentropic flow

For flow in a smooth pipe with no abrupt changes of section:

continuity equation	$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$
equation of momentum conservation	$-dp\,A=(A\rho u)du$
isentropic relationship	$p=c\rho^k$
sonic velocity	$a^2=\frac{dp}{d\rho}$

These lead to an equation being derived on the basis of mass continuity i.e.

$$\frac{d\rho}{\rho}=-M^2\frac{du}{u}$$

or

$$M^2=-\frac{d\rho}{\rho}\Big/\frac{du}{u}$$

Table 9.4 shows equations relating to convergent and convergent–divergent nozzle flow.

Table 9.4 Isentropic flows

Pipe flows		$\frac{-dp}{\rho}\Big/\frac{du}{u}=M^2$
Convergent nozzle flows	Flow velocity	$u=\sqrt{2\left(\frac{k}{k-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1-\frac{\rho^{\frac{k-1}{k}}}{p_0}\right]}$
	Flow rate	$m=\rho uA$
Convergent–divergent nozzle flow	Area ratio	$\frac{A}{A^*}=\frac{\left(\frac{2}{k+1}\right)^{\frac{1}{(k-1)}}\left(\frac{p_0}{p}\right)^{1/k}}{\sqrt{\frac{k+1}{k-1}\left[1-\frac{p_0^{\frac{(1-k)}{k}}}{p}\right]}}$

9.6 Compressible one-dimensional flow

Basic equations for 1-D compressible flow are given below.

Euler's equation of motion in the steady state along a streamline

$$\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}s}+\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{2}u^2\right)=0$$

or

$$\int\frac{\mathrm{d}p}{\rho}+\frac{1}{2}u^2=\text{constant}$$

so

$$\frac{k}{k-1}RT+\frac{1}{2}u^2=\text{constant}$$

$$\frac{p_o}{p}=\left(\frac{T_o}{T}\right)^{k/(k-1)}=\left(1+\frac{k-1}{2}M^2\right)^{k/(k-1)}$$

where $T_o=$ total temperature

9.7 Normal shock waves

9.7.1 One-dimensional flow

A shock wave is a pressure front that travels at speed through a gas. Shock waves cause an increase in pressure, temperature, density and entropy and a decrease in normal velocity.

Equations of state and equations of conservation applied to a unit area of shock wave give (see Fig. 9.10).

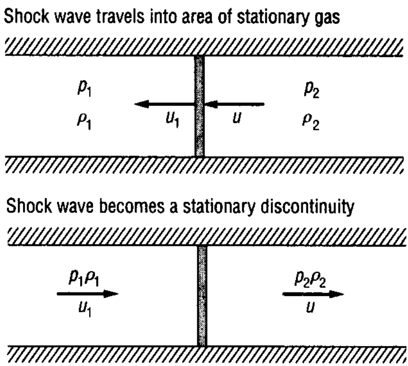


Figure 9.10

State $\quad p_1/\rho_1 T_1 - 1 = p_2/\rho_2 T_2$

Mass flow $\quad \dot{m} = \rho_1 u_1 = \rho_2 u_2$

Momentum $\quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

Energy $\quad c_p T_1 + \dfrac{u_1^2}{2} = c_p T_2 + \dfrac{u_2^2}{2} = c_p T_0$

Pressure and density relationships across the shock are given by the Rankine–Hugoniot equations

$$\frac{p_2}{p_1} = \frac{\dfrac{\gamma+1}{\gamma-1}\dfrac{\rho_2}{\rho_1} - 1}{\dfrac{\gamma+1}{\gamma-1} - \dfrac{\rho_2}{\rho_1}}$$

$$\frac{p_2}{p_1} = \frac{\dfrac{\gamma+1}{\gamma-1}\dfrac{p_2}{p_1} + 1}{\dfrac{\gamma+1}{\gamma-1} + \dfrac{p_2}{p_1}}$$

Static pressure ratio across the shock is given by

$$\frac{p_2}{p_1} = \frac{2\gamma M_2^2 - (\gamma - 1)}{\gamma + 1}$$

Temperature ratio across the shock is given by

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \Big/ \frac{\rho_2}{\rho_1}$$

$$\frac{T_2}{T_1} = \left(\frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}\right)\left(\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}\right)$$

Velocity ratio across the shock is given by

From continuity

$$u_2/u_1 = \rho_1/\rho_2$$

so

$$\frac{u_2}{u_1} = \frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}$$

In axisymmetric flow the variables are independent of θ so the continuity equation can be expressed as

$$\frac{1}{R^2}\frac{\partial(R^2 q_R)}{\partial R} + \frac{1}{R\sin\varphi}\frac{\partial(\sin\varphi\, q_\varphi)}{\partial\varphi} = 0$$

Similarly in terms of stream function ψ

$$q_R = \frac{1}{R^2\sin\varphi}\frac{\partial\psi}{\partial\varphi}$$

$$q_\varphi = -\frac{1}{R\sin\varphi}\frac{\partial\psi}{\partial R}$$

9.7.2 The pitot tube equation

An important criterion is the Rayleigh supersonic pitot tube equation (see Fig. 9.11).

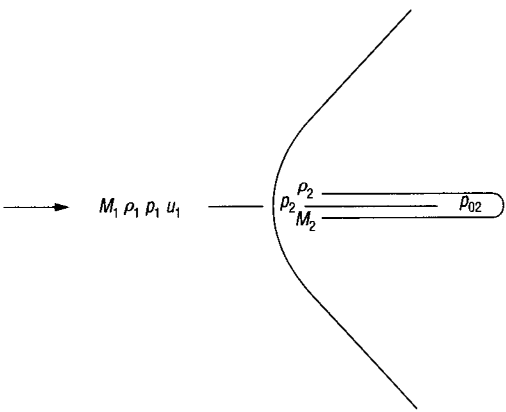


Figure 9.11

$$\text{Pressure ratio } \frac{p_{02}}{p_1} = \frac{\left[\frac{\gamma+1}{2}M_1^2\right]^{\gamma/(\gamma-1)}}{\left[\frac{2\gamma M_1^2-(\gamma-1)}{\gamma+1}\right]^{1/(\gamma-1)}}$$

9.8 Axisymmetric flows

Axisymmetric potential flows occur when bodies such as cones and spheres are aligned into a fluid flow. Figure 9.12 shows the layout of spherical coordinates used to analyse these types of flow.

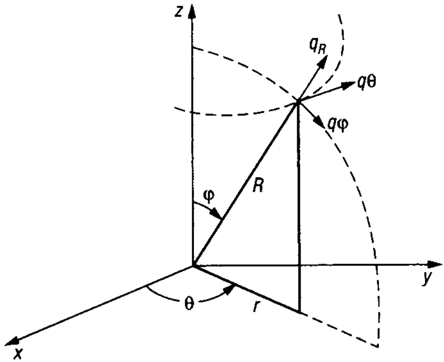


Figure 9.12

Relationships between the velocity components and potential are given by

$$q_R = \frac{\partial \phi}{\partial R} \quad q_\theta = \frac{1}{R \sin \varphi} \frac{\partial \phi}{\partial \theta} \quad q_\varphi = \frac{1}{R} \frac{\partial \phi}{\partial \varphi}$$

9.9 Drag coefficients

Figures 9.13(a) and (b) show drag types and ‘rule of thumb’ coefficient values.

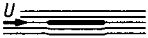
Shape	*Pressure drag* D_P *(%)*	*Friction drag* D_f *(%)*
	0	100
	≈ 10	≈ 90
	≈ 90	≈ 10
	100	0

Figure 9.13(a)

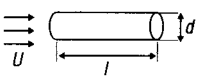
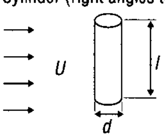
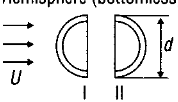
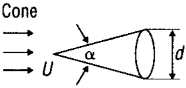
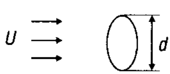
Shape	*Dimensional ratio*	*Datum area, A*	*Approximate drag coefficient,* C_D
Cylinder (flow direction)	$l/d = 1$		0.91
	2		0.85
	4	$\frac{\pi}{4} d^2$	0.87
	7		0.99
Cylinder (right angles to flow)	$l/d = 1$		0.63
	2		0.68
	5		0.74
	10	dl	0.82
	40		0.98
	∞		1.20
Hemisphere (bottomless)	I	$\frac{\pi}{4} d^2$	0.34
	II		1.33
Cone	$a = 60°$	$\frac{\pi}{4} d^2$	0.51
	$a = 30°$		0.34
		$\frac{\pi}{4} d^2$	1.2

Figure 9.13(b)

9.10 General airfoil theory

When an airfoil is located in an airstream, the flow divides at the leading edge – the stagnation point. The camber of the airfoil section means that the air passing over the top surface has further to travel to reach the trailing edge than that travelling along the lower surface. In accordance with Bernoulli's equation the higher velocity along the upper airfoil surface results in a lower pressure, producing a lift force. The net result of the velocity differences produces an effect equivalent to that of a parallel air stream and a rotational velocity ('vortex'), see Figures 9.14 and 9.15.

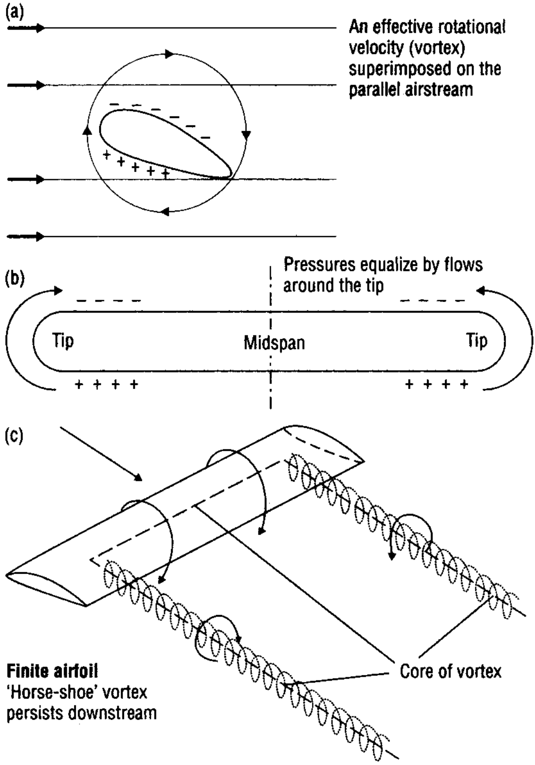


Figure 9.14

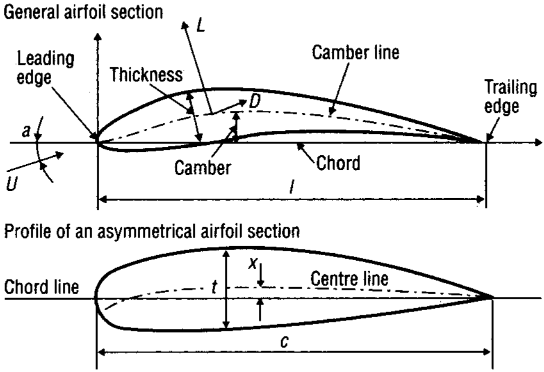


Figure 9.15

For the case of a theoretical finite airfoil section, the pressure on the upper and lower surface, tries to equalize by flowing around the tips. This rotation persists downstream of the wing resulting in a long 'U'-shaped vortex (see Fig. 9.14). The generation of these vortices needs the input of a continuous supply of energy, the net result being to increase the drag of the wing, by the addition of so-called '*induced*' *drag*.

9.11 Airfoil coefficients

Lift, drag, and moment (L, D, M) acting on an aircraft wing are expressed by the equations:

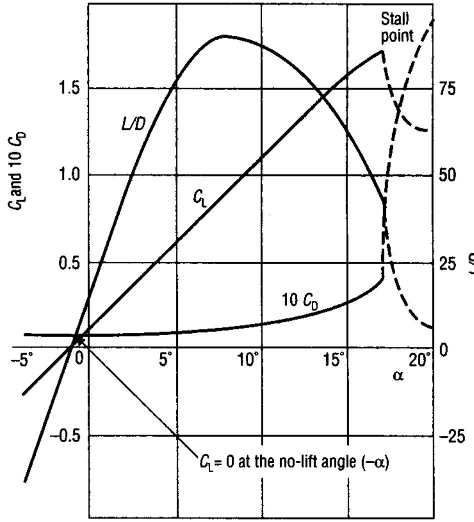
$$\text{Lift } (L) \text{ per unit width} = C_L l^2 \frac{\rho U^2}{2}$$

$$\text{Drag } (D) \text{ per unit width} = C_D l^2 \frac{\rho U^2}{2}$$

$$\text{Moment } (M) \text{ about leading edge (LE) or } 1/4 \text{ chord} = C_M l^2 \frac{\rho U^2}{2} \text{ per unit width.}$$

The lift, drag, and moment coefficients are C_L, C_D, and C_M respectively. Figure 9.16 shows typical values plotted against the angle of attack, or incidence α. The value of C_D is small so a value of 10 C_D is often used for the characteristic curve. C_L rises towards

Characteristics for an asymmetrical 'infinite-span 2D airfoil'



Characteristic curves of a practical wing

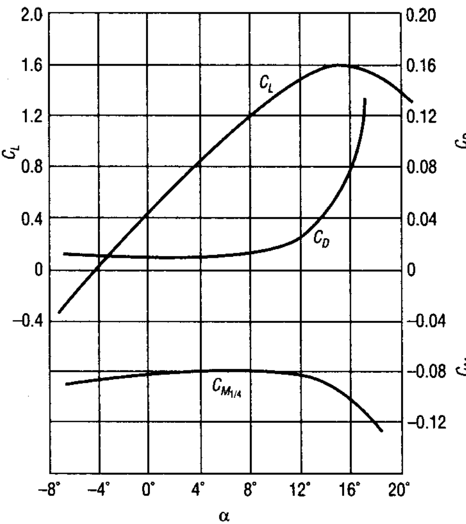


Figure 9.16

stall point and then falls off dramatically, as the wing enters the stalled condition. C_D rises gradually, increasing dramatically after the stall point. Other general relationships are outlined below.

- As a rule of thumb, a Reynolds number of $Re \cong 10^6$ is considered a general flight condition.
- Maximum C_L increases steadily for Reynolds numbers between 10^5 and 10^7.
- *CD* decreases rapidly up to Reynolds numbers of approximately 10^6, beyond which the rate of change reduces.
- Thickness and camber both affect the maximum C_L that can be achieved. As a general rule, C_L increases with thickness and then reduces again as the airfoil becomes even thicker. C_L generally increases as camber increases. The minimum C_D achievable increases fairly steadily with section thickness.

9.12 Pressure distributions

The pressure distribution across an airfoil section varies with the angle of attack α. Figure 9.17 shows the effect as α increases, and the notation used. The pressure coefficient C_p reduces towards the trailing edge.

9.13 Aerodynamic centre

The aerodynamic centre (AC) is defined as the point in the section about which the pitching moment coefficient (C_M) is constant i.e. does not vary with lift coefficient (C_L). Its theoretical positions are indicated in Table 9.5.

Using common approximations, the following equations can be derived

$$\frac{x_{AC}}{c} = \frac{9}{c} - \frac{\mathrm{d}}{\mathrm{d}C_L}(C_{Ma})$$

where

Table 9.5 Position of aerodynamic centre

Condition	*Theoretical position of the AC*
$\alpha < 10$ degrees	At approx $^1/_4$ chord somewhere near the chord line
Section with high aspect ratio	At 50% chord
Flat or curved plate: inviscid, incompressible flow	At approx $^1/_4$ chord

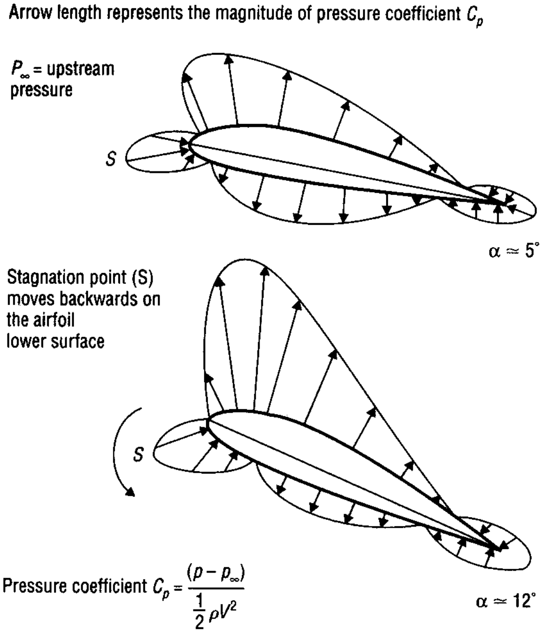


Figure 9.17

C_{Ma} = pitching moment coefficient at distance a back from LE.
x_{AC} = position of AC back from LE.
c = chord length.

9.14 Centre of pressure

The centre of pressure (CP) is defined as the point in the section about which there is no pitching moment, i.e. the aerodynamic forces on the entire section can be represented by a lift and drag force acting at this point. The CP does not have to lie within the airfoil profile and can change location, depending on the magnitude of the lift coefficient C_L. The CP is conventionally shown at distance k_{CP} back from the section leading edge (see Fig. 9.18).

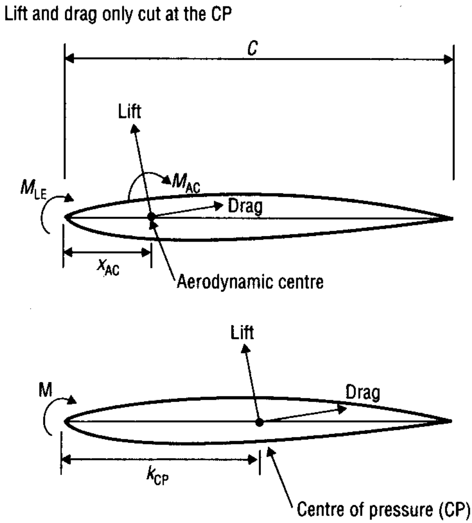


Figure 9.18

Using the principle of moments the following expression can be derived for k_{CP}

$$k_{CP} = \frac{x_{AC}}{c} - \frac{C_{M_{AC}}}{C_L \cos\alpha + C_D \sin\alpha}$$

Assuming that $\cos\alpha \cong 1$ and $C_D \sin\alpha \cong 0$ gives

$$k_{CP} \cong \frac{x_{AC}}{c} - \frac{C_{M_{AC}}}{C_L}$$

9.15 Supersonic conditions

As an aircraft is accelerated to approach supersonic, speed the equations of motion that describe the flow change in character. In order to predict the behaviour of airfoil sections in upper subsonic and supersonic regions, compressible flow equations are required.

9.15.1 Basic definitions

M $=$ Mach number
$M_\infty =$ free stream Mach number

M_c = critical Mach number, i.e. the value of M_∞ that results in flow of $M = 1$ at some location on the airfoil surface.

Figure 9.19 shows approximate forms of the pressure distribution on a two-dimensional airfoil around the critical region. Owing to the complex non-linear form of the equations of motion that describe high-speed flow, two popular simplifications are used: the *small perturbation* approximation and the so-called *exact* approximation.

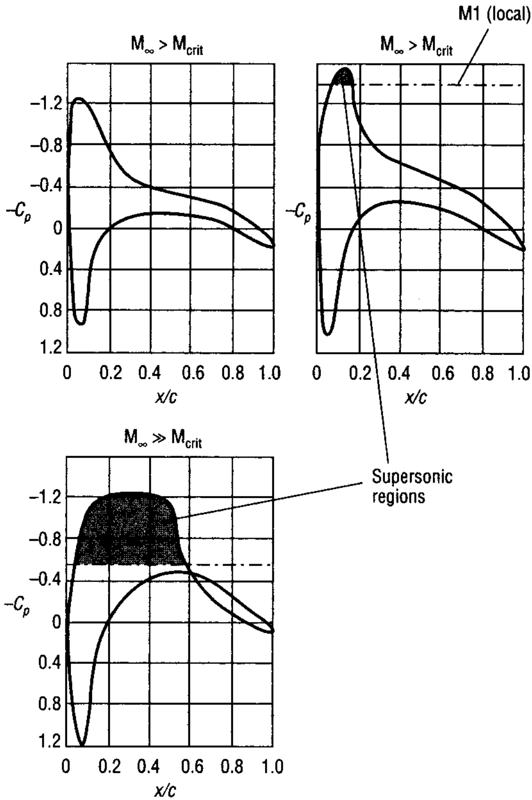


Figure 9.19

9.15.2 Supersonic effects on drag

In the supersonic region, induced drag (due to lift) increases in relation to the parameter $\sqrt{M^2-1}$ and is a function of the plan form geometry of the wing.

9.15.3 Supersonic effects on aerodynamic centre (AC)

Figure 9.20 shows the location of wing AC for several values of tip chord/root chord ratio (λ). These are empirically based results that can be used as a 'rule of thumb'.

9.16 Wing loading: semi-ellipse assumption

The simplest general loading condition assumption for symmetric flight is that of the semi-ellipse. The equivalent equations for lift, downwash, and induced drag become:

For lift

$$L = \rho \frac{V K_0 \pi s}{2}$$

replacing L by $C_L \, {}^1\!/\!_2 \rho V^2 S$ gives

$$K_0 = \frac{C_L V S}{\pi s}$$

For downwash velocity (w)

$$w = \frac{K_0}{4S} \text{ i.e. it is constant along the span.}$$

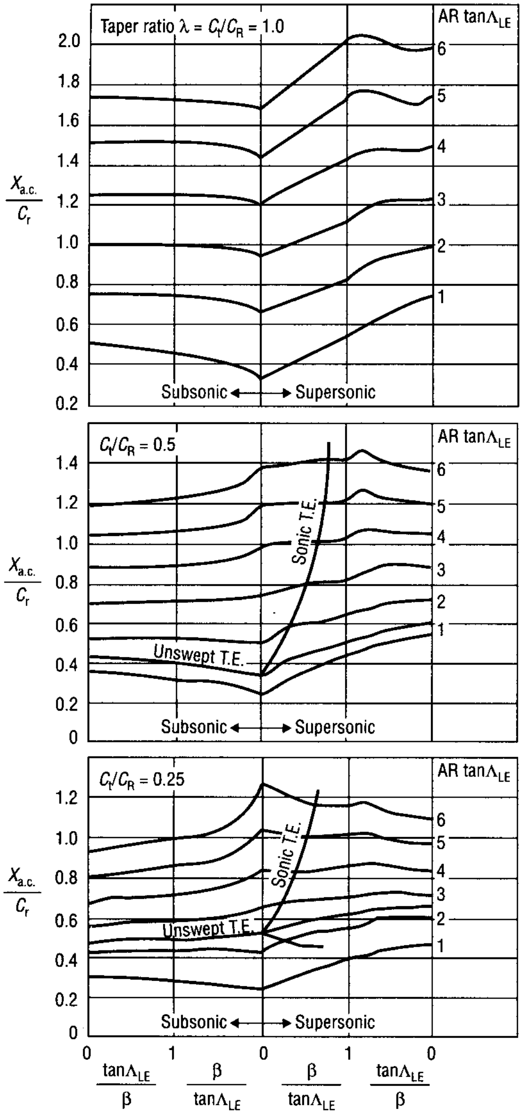
For induced drag (vortex)

$$C_{D_V} = \frac{C_L^2}{\pi AR}$$

where aspect ratio

$$(AR) = \frac{\text{span}^2}{\text{area}} = \frac{4s^2}{S}$$

Hence, C_{Dv} falls (theoretically) to zero as aspect ratio increases. At zero lift in symmetric flight, $C_{Dv} = 0$.



Taper ratio λ = C_t/C_R = 1.0
AR tanΛ$_{LE}$
6
5
4
3
2
1
$X_{a.c.}/C_r$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Subsonic ← → Supersonic
C_t/C_R = 0.5
AR tanΛ$_{LE}$
Sonic T.E.
Unswept T.E.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
C_t/C_R = 0.25
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1
0
1
0
tanΛ$_{LE}$/β
β/tanΛ$_{LE}$
β/tanΛ$_{LE}$
tanΛ$_{LE}$/β

Figure 9.20