2

Gas Turbine Driven Generators

2.1 CLASSIFICATION OF GAS TURBINE ENGINES

For an individual generator that is rated above 1000 kW, and is to be used in the oil industry, it is usual practice to use a gas turbine as the driving machine (also called the prime mover). Below 1000 kW a diesel engine is normally preferred, usually because it is an emergency generator running on diesel oil fuel.

Gas turbines can be classified in several ways, common forms are:-

- Aero-derivative gas turbines.
- Light industrial gas turbines.
- Heavy industrial gas turbines.

2.1.1 Aero-derivative Gas Turbines

Aircraft engines are used as 'gas generators', i.e. as a source of hot, high velocity gas. This gas is then directed into a power turbine, which is placed close up to the exhaust of the gas generator. The power turbine drives the generator. The benefits of this arrangement are:-

- Easy maintenance since the gas generator can be removed as a single, simple module. This can be achieved very quickly when compared with other systems.
- High power-to-weight ratio, which is very beneficial in an offshore situation.
- Can be easily designed for single lift modular installations.
- Easy to operate.
- They use the minimum of floor area.

The main disadvantages are:-

- Relatively high costs of maintenance due to short running times between overhauls.
- Fuel economy is usually lower than other types of gas turbines.
- The gas generators are expensive to replace.

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Aero-derivative generators are available in single unit form for power outputs from about 8 MW up to about 25 MW. These outputs fall conveniently into the typical power outputs required in the oil and gas production industry, such as those on offshore platforms.

2.1.2 Light Industrial Gas Turbines

Some manufacturers utilize certain of the advantages of the aero-derivative machines, i.e. high power-to-weight ratio and easy maintenance. The high power-to-weight ratios are achieved by running the machines with high combustion and exhaust temperatures and by operating the primary air compressors at reasonably high compression ratios i.e. above 7. A minimum of metal is used and so a more frequent maintenance programme is needed. Easier maintenance is achieved by designing the combustion chambers, the gas generator and compressor turbine section to be easily removable as a single modular type of unit. The ratings of machines in this category are limited to about 10 MW.

2.1.3 Heavy Industrial Gas Turbines

Heavy industrial gas turbines are usually to be found in refineries, chemical plants and power utilities. They are chosen mainly because of their long and reliable running times between major maintenance overhauls. They are also capable of burning most types of liquid and gaseous fuel, even the heavier crude oils. They also tend to tolerate a higher level of impurities in the fuels. Heavy industrial machines are unsuitable for offshore applications because:-

- Their poor power-to-weight ratio means that the structures supporting them would need to be much larger and stronger.
- Maintenance shutdown time is usually much longer and is inconvenient because the machine must be disassembled into many separate components. A modular concept is not possible in the design of these heavy industrial machines.
- The thermodynamic performance is usually poorer than that of the light and medium machines. This is partly due to the need for low compression ratios in the compressor.

They do, however, lend themselves to various methods of heat energy recovery e.g. exhaust heat exchangers, recuperators on the inlet air.

Figures 2.1 and 2.2 show the relative costs and weights for these types of machines.

2.1.4 Single and Two-shaft Gas Turbines

There are basically two gas turbine driving methods, known as ‘single-shaft’ and ‘two (or twin) shaft’ drives. In a single-shaft gas turbine, all the rotating elements share a common shaft. The common elements are the air compressor, the compressor turbine and the power turbine. The power turbine drives the generator.

In some gas turbines, the compressor turbine and the power turbine are an integral component. This tends to be the case with heavy-duty machines.

The basic arrangement is shown in Figure 2.3.

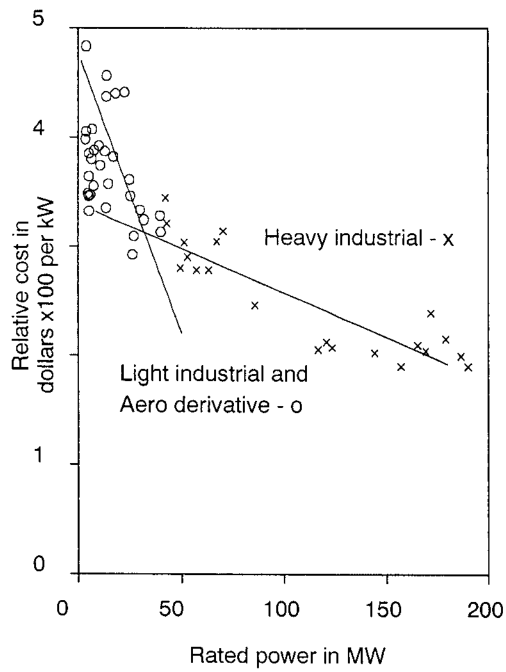

Figure 2.1 Relative cost of gas turbo-generators versus power rating.

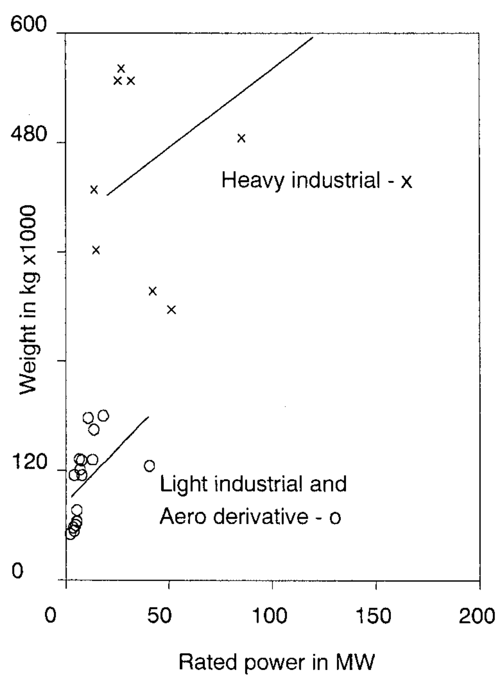

Figure 2.2 Weight of gas turbo-generators versus power rating.

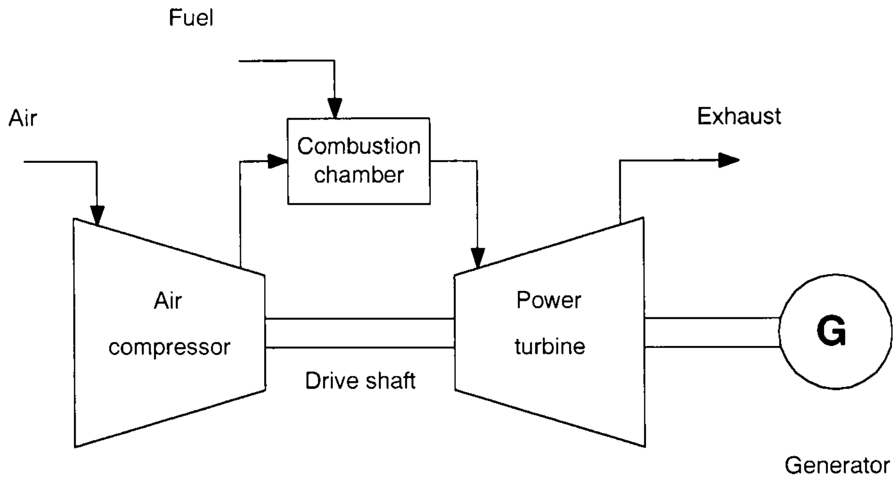

Figure 2.3 Single-shaft gas turbine.

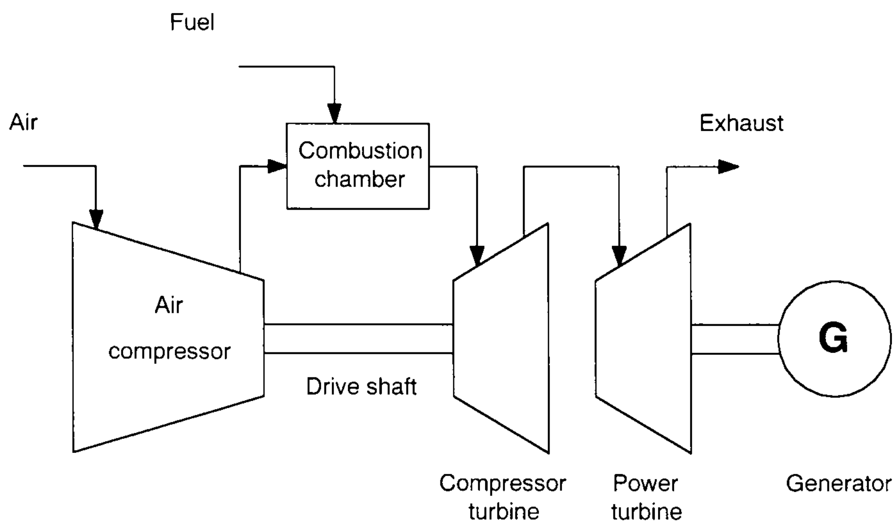

Figure 2.4 Two-shaft gas turbine.

In a two-shaft gas turbine the compressor is driven by a high pressure turbine called the compressor turbine, and the generator is driven separately by a low pressure turbine called the power turbine.

The basic arrangement is shown in Figure 2.4.

Two-shaft systems are generally those which use aero-derivative engines as 'gas generators', i.e. they produce hot, high velocity, high pressure gas which is directed into the power turbine. Some light industrial gas turbines have been designed for either type of drive. This is achieved by fitting a removable coupling shaft between the two turbines. Some points to consider with regard to the two types of driver are:-

a) High speed of rotation tends to improve the compressor and turbine efficiency. Hence, with two separate shafts, the best thermodynamic performance from both turbines and the compressor is obtainable.
b) Using aero-derivative machines means that a simple ‘add on’ power turbine can be fitted in the exhaust streams of the aero engine. This enables many manufacturers to design a simple power turbine and to use a particular aero engine.
c) Two-shaft machines are often criticised as electrical generators because of their slower response to power demands in comparison with the single-shaft machines. This can be a problem when a two-shaft machine may have to operate in synchronism with other single-shaft machines or steam turbine generators. Sometimes the slower response may affect the power system performance during the starting period of large motors. A power system computerised stability study should be carried out to investigate these types of problem.

Some of the recent aero engines could be called ‘three-shaft’ arrangements because within the gas generator there are two compressor turbines and two compressors.

2.1.5 Fuel for Gas Turbines

The fuels usually consumed in gas turbines are either in liquid or dry gas forms and, in most cases, are hydrocarbons. In special cases non-hydrocarbon fuels may be used, but the machines may then need to be specially modified to handle the combustion temperatures and the chemical composition of the fuel and its combustion products.

Gas turbine internal components such as blades, vanes, combustors, seals and fuel gas valves are sensitive to corrosive components present in the fuel or its combustion products such as carbon dioxide, sulphur, sodium or alkali contaminants, see also sub-section 2.2.5.

The fuel can generally be divided into several classifications:-

- Low heating value gas.
- Natural gas.
- High heating value gas.
- Distillate oils.
- Crude oil.
- Residual oil.

2.2 ENERGY OBTAINED FROM A GAS TURBINE

A gas turbine functions as a heat engine using the thermodynamic Joule cycle, as explained in many textbooks, see for example References 1 to 5. Most gas turbines used in the oil industry use the

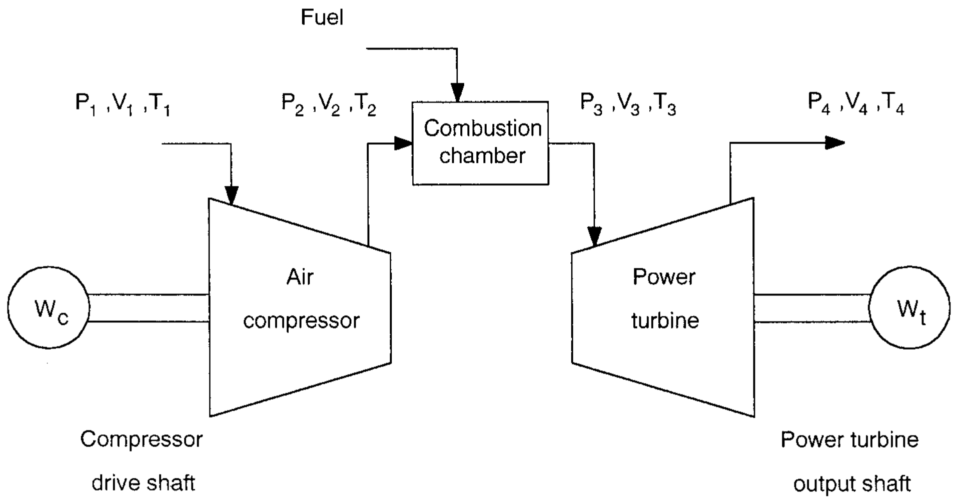

Figure 2.5 Gas turbine thermodynamic cycle. Simple-cycle gas turbine.

'simple-cycle' version of the Joule cycle. The main components of the gas turbine are shown in Figure 2.5.

The thermodynamic relationships used to describe the operation of the gas turbine are the pressure (P) versus volume (V) characteristic in Figure 2.6 and the temperature (T) versus entropy (S) characteristic in Figure 2.7. These figures also show the effect of practical inefficiencies that occur both in the air compressor and the turbine.

Air is drawn into the compressor at atmospheric pressure P_1 (in practice slightly lower due to the inlet silencer, filter and ducting) and atmospheric temperature T_1, and compressed adiabatically to a higher pressure P_2 to reduce its volume to V_2 and raise its temperature to T_2. The adiabatic compression is given by the following equations; see standard textbooks e.g. References 1 to 5.

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} = \text{constant} \tag{2.1}$$

$$P_2V_2{}^{\gamma} = P_1V_1{}^{\gamma} = \text{constant} \tag{2.2}$$

The work done in the compressor per kg of fluid U_c is,

$$U_c = \frac{\gamma}{\gamma - 1}(P_2V_2 - P_1V_1) \tag{2.3}$$

The following standard relationships apply,

$$P_1V_1 = RT_1 \tag{2.4}$$

$$P_2V_2 = RT_2 \tag{2.5}$$

$$C_p - C_v = R \tag{2.6}$$

$$\frac{C_p}{C_v} = \gamma \tag{2.7}$$

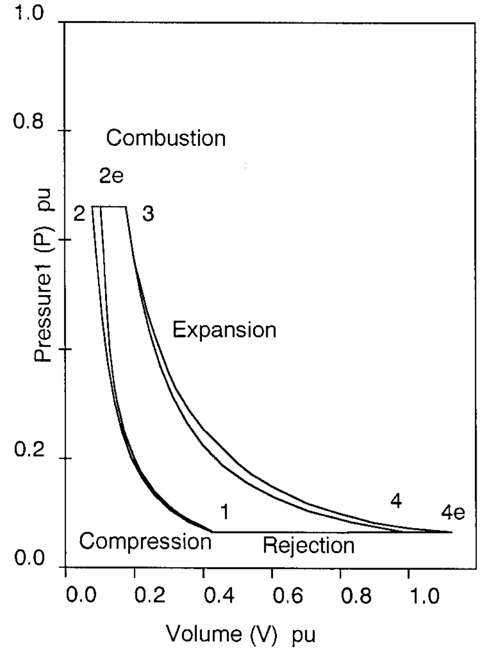

Figure 2.6 Pressure versus volume in the thermodynamic cycle.

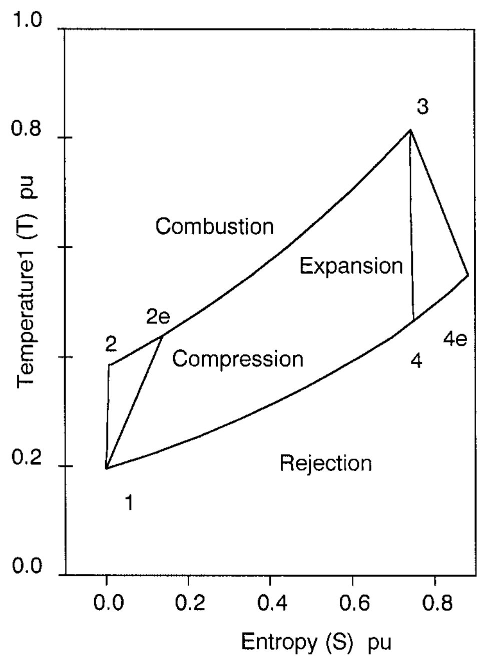

Figure 2.7 Temperature versus entropy in the thermodynamic cycle.

Where, C_p is the specific heat of the air at constant pressure, kcal/kg K $\simeq$ 1.005
C_v is the specific heat of the air at constant volume, kcal/kg K $\simeq$ 0.718
R is the particular gas constant for air, kJ/kg K $\simeq$ 0.287
γ is the ratio of specific heats $\simeq$ 1.4

From (2.3) and (2.7),

$$\frac{\gamma}{\gamma - 1} = \frac{C_p}{R} \tag{2.8}$$

Substitute (2.4, 2.5 and 2.8) into (2.1),

$$U_c = C_p(T_2 - T_1) \text{ kJ/kg} \tag{2.9}$$

The air leaving the compressor at pressure P_2 passes into the combustion chamber where its temperature is raised to T_3, at constant pressure.

The hot air–fuel mixture burns and the gaseous products of combustion pass into the turbine where the pressure falls to the atmospheric pressure $P_4 = P_1$ (in practice slightly higher due to the resistance or 'back pressure' of the exhaust silencer and ducting). The exhaust gas temperature is T_4 and is lower than the combustion temperature T_3. (The ducting systems should be arranged so that the exhaust gas is discharged at a point far enough away from the inlet ducting entrance that no interaction occurs i.e. T_4 does not influence T_1.)

The turbine expansion process can be described by similar equations to (2.1) through (2.7), with T_3 replacing T_2 and T_4 replacing T_1. Hence the work done by the turbine (U_t) is,

$$U_t = C_p(T_3 - T_4) \text{ kJ/kg} \tag{2.10}$$

The heat supplied by the fuel is C_p $(T_3 - T_2)$.

In a conventional gas turbine the turbine supplies power to drive its compressor and so the power available to drive a generator is the net power available from the turbine. Neglecting inefficiencies in the compressor and the turbine, the work done on the generator at the coupling of the gas turbine is U_{out},

$$U_{\text{out}} = U_t - U_c = C_p(T_3 - T_4 - T_2 + T_1) \text{ kJ/kg} \tag{2.11}$$

The ideal cycle efficiency η_i of the gas turbine is:

$$\begin{aligned}\eta_i &= \frac{C_p(T_3 - T_4 - T_2 + T_1)}{C_p(T_3 - T_2)} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2}\right) \\ &= 1 - \frac{\text{Rejection temperature difference}}{\text{Combustion temperature difference}}\end{aligned} \tag{2.12}$$

From (2.1), raise to the power γ,

$$\left(\frac{P_2 V_2}{T_2}\right)^{\gamma} = \left(\frac{P_1 V_1}{T_1}\right)^{\gamma} \tag{2.13}$$

From here onwards the following substitutions will be used in order to keep the presentation of the equations in a simpler format.

$$\beta = \frac{\gamma - 1}{\gamma},\ \beta_c = \frac{\gamma_c - 1}{\gamma_c},\ \beta_t = \frac{\gamma_t - 1}{\gamma_t}$$
$$\delta = \frac{1 - \gamma}{\gamma},\ \delta_c = \frac{1 - \gamma_c}{\gamma_c},\ \delta_t = \frac{1 - \gamma_t}{\gamma_t}$$

Where subscript 'c' refers to the compressor and 't' to the turbine, the absence of a subscript means a general case.

Divide (2.2) by (2.13) to obtain an expression for the compressor,

$$\left(\frac{P_2}{P_1}\right)^{\delta} = \frac{T_1}{T_2} \tag{2.14}$$

Similarly for the turbine,

$$\left(\frac{P_3}{P_4}\right)^{\delta} = \frac{T_4}{T_3} \tag{2.15}$$

It is of interest to determine the work done on the generator in terms of the ambient temperature T_1 and the combustion temperature T_3.

From (2.14),

$$T_2 = T_1 {r_p}^{\beta}$$

And from (2.15),

$$T_4 = T_3 {r_p}^{\delta}$$

Therefore (2.11) becomes,

$$\begin{aligned} U_{\text{out}} &= C_p(T_3 - T_3{r_p}^{\delta} - T_1{r_p}^{\beta} + T_1) \\ &= C_p T_3(1 - {r_p}^{\delta}) - C_p T_1({r_p}^{\beta} - 1) \end{aligned} \tag{2.16}$$

The ideal cycle efficiency η_i can also be expressed in terms of T_1 and T_3.

$$\eta_i = 1 - \left(\frac{T_3{r_p}^{\delta} - T_1}{T_3 - T_1{r_p}^{\beta}}\right) \tag{2.17}$$

The specific heat C_p is assumed to be constant and equal for both compression and expansion. In practice these assumptions are not valid because the specific heat C_p is a function of temperature. The average temperature in the turbine is about twice that in the compressor. Also the products of combustion i.e. water vapour and carbon dioxide, slightly increase the specific heat of air–gas mixture in the turbine. Figures 2.8 and 2.9 show the spread of values for the pressure ratio and exhaust temperature for a range of gas turbines from 1 MW to approximately 75 MW.

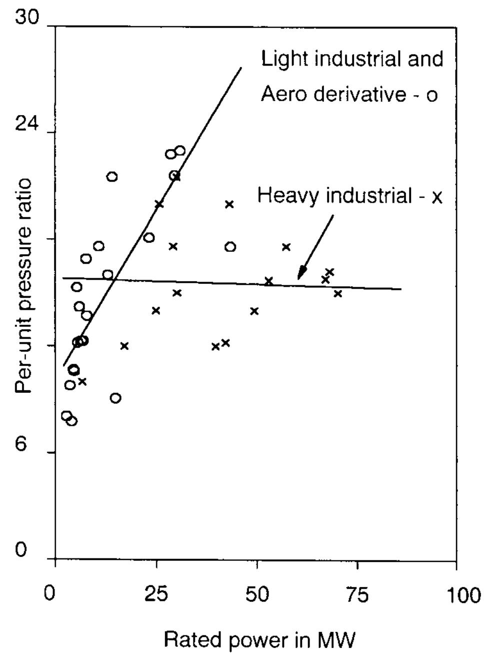

Figure 2.8 Per-unit pressure ratio versus power rating.

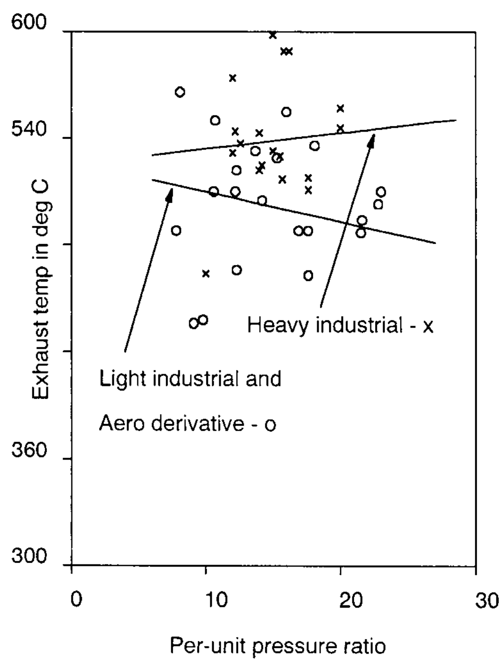

Figure 2.9 Exhaust temperature versus pressure ratio.

2.2.1 Effect of an Inefficient Compressor and Turbine

Frictional losses in the compressor raise the output temperature. Similarly the losses in the turbine raise the exhaust temperature. These losses are quantified by modifying the temperatures T_2 and T_4 to account for their increases.

The compression ratio (P_2/P_1) of the compressor is usually given by the manufacturer and therefore the temperature of the air leaving the compressor is easily found from (2.13). If the efficiency of compression η_c is known e.g. 90% and that of the turbine η_t is known e.g. 85% then a better estimate of the output energy can be calculated. In this situation T_2 becomes T_{2e} and T_4 becomes T_{4e}, as follows:-

$$T_{2e} = \frac{T_2}{\eta_c} + \left(1 - \frac{1}{\eta_c}\right) T_1 \quad \text{and} \quad T_{4e} = T_4 \eta_t + (1 - \eta_t) T_3 \tag{2.18}$$

These would be the temperatures measurable in practice. In (2.14) and (2.15) the pressure ratios are theoretically equal, and in practice nearly equal, hence:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = {r_p}^{\beta} \tag{2.19}$$

Where r_p is the pressure ratio $\dfrac{P_2}{P_1}$ or $\dfrac{P_3}{P_4}$

In practice the temperatures T_1 and T_3 are known from the manufacturer or from measuring instruments installed on the machine. The pressure ratio r_p is also known. The ratio of specific heats is also known or can be taken as 1.4 for air. If the compressor and turbine efficiencies are taken into account then the practical cycle efficiency η_p of the gas turbine can be expressed as:

$$\eta_p = \frac{T_3(1 - {r_p}^{\delta})\eta_c \eta_t - T_1({r_p}^{\beta} - 1)}{T_3 \eta_c - T_1(r_p - 1 + \eta_c)} \tag{2.20}$$

which has a similar form to (2.17) for comparison.

2.2.1.1 Worked example

A light industrial gas turbine operates at an ambient temperature T_1 of 25°C and the combustion temperature T_3 is 950°C. The pressure ratio r_p is 10.

If the overall efficiency is 32% find the efficiency of the compressor assuming the turbine efficiency to be 86%.

From (2.20),

$$T_1 = 273 + 25 = 298^\circ\text{K}$$

$$T_3 = 273 + 950 = 1223^\circ\text{K}$$

$${r_p}^{\delta} = 10^{-0.2857} = 0.51796 \quad \text{and} \quad {r_p}^{\beta} = 10^{+0.2857} = 1.93063$$

Therefore,

$$\eta_p = 0.32 = \frac{1223(1.0 - 0.51796)\eta_c(0.86) - 298(1.93063 - 1.0)}{1223\eta_c - 298(1.93063 - 1.0 + \eta_c)}$$

Transposing for η_c results in $\eta_c = 0.894$. Hence the compressor efficiency would be 89.4%.

2.2.2 Maximum Work Done on the Generator

If the temperatures T_{2e} and T_{4e} are used in (2.11) to compensate for the efficiencies of the compressor and turbine, then it is possible to determine the maximum power output that can be obtained as a function of the pressure ratio r_p.

The revised turbine work done U_{te} is,

$$U_{te} = C_p(T_3 - T_4)\eta_t \text{ kJ/kg} \tag{2.21}$$

The revised compressor work done U_{ce} is,

$$U_{ce} = C_p(T_2 - T_1)\frac{1}{\eta_c} \text{ kJ/kg} \tag{2.22}$$

The revised heat input from the fuel U_{fe} is,

$$U_{fe} = C_p(T_3 - T_{2e}) \text{ kJ/kg} \tag{2.23}$$

where,

$$T_{2e} = T_1\left(\frac{{r_p}^{\beta} - 1 + \eta_c}{\eta_c}\right)$$

From (2.19),

$$T_4 = T_3{r_p}^{\delta} \tag{2.24}$$

and

$$T_2 = T_1{r_p}^{\beta} \tag{2.25}$$

Substituting for T_2, T_{2e} and T_4 gives the resulting output work done U_{oute} to be,

$$U_{\text{oute}} = U_{te} - U_{ce} = C_p(T_3 - T_3{r_p}^{\delta})\eta_t - C_p\left(\frac{T_1{r_p}^{\beta} - T_1}{\eta_c}\right)$$

$$= C_p\left[T_3(1 - r^{\delta})\eta t - \frac{T_1}{\eta_c}({r_p}^{\beta} - 1)\right]\text{kJ/kg} \tag{2.26}$$

To find the maximum value of U_{oute} differentiate U_{oute} with respect to γ_p and equate the result to zero. The optimum value of γ_p to give the maximum value of U_{oute} is,

$$r_{p\max} = \left(\frac{T_1}{T_3\eta_c\eta_t}\right)^{d} \tag{2.27}$$

Where

$$d = \frac{1}{2\delta}$$

which when substituted in (2.26) gives the maximum work done U_{outemax}.

2.2.2.1 Worked example

Find $r_{p\max}$ for the worked example in sub-section 2.2.1.1.

Given that,

$$T_1 = 298\ K, T_3 = 1223^{\circ}\text{C},$$
$$r = 1.4,\ \eta_t = 0.86 \text{ and } \eta_c = 0.894$$
$$d = \frac{\gamma}{2(1-\gamma)} = \frac{1.4}{2(1.0 - 1.4)} = -1.75$$
$$r_{p\max} = \left[\frac{298}{1223(0.894)(0.86)}\right]^{-1.75}$$
$$= 0.3169^{-1.75} = 7.4$$

2.2.3 Variation of Specific Heat

As mentioned in sub-section 2.2 the specific heat C_p changes with temperature. From Reference 4, Figure 4.4, an approximate cubic equation can be used to describe C_p in the range of temperature 300 K to 1300 K when the fuel-to-air ratio by mass is 0.01, and for the air alone for compression, as shown in Table 2.1. The specific heat for the compressor can be denoted as C_{pc} and for the turbine C_{pt}. The appropriate values of C_{pc} and C_{pt} can be found iteratively from the cubic expression and equations (2.24) and (2.25). At each iteration the average of T_1 and T_2 can be used to recalculate C_{pc}, and T_3 and T_4 to recalculate C_{pt}. The initial value of γ can be taken as 1.4 in both cases, and C_v can be assumed constant at 0.24/1.4 = 0.171 kcal/kg K. The pressure ratio is constant. Having found suitable values of C_{pc} and C_{pt} it is now possible to revise the equations for thermal efficiency η_{pa} and output energy U_{outea}, where the suffix 'a' is added to note the inclusion of variations in specific heat C_p.

Table 2.1. Specific heat C_p as a cubic function of absolute temperature K in the range 373 K to 1273 K $C_p = a + bT + cT^2 + dT^3$

Fuel-air ratio	Cubic equation constants			
	$a \times 10^0$	$b \times 10^{-4}$	$c \times 10^{-7}$	$d \times 10^{-10}$
0.0	0.99653	−1.6117	+5.4984	−2.4164
0.01	1.0011	−1.4117	+5.4973	−2.4691
0.02	1.0057	−1.2117	+5.4962	−2.5218

The energy equations for the compressor and turbine become,

$$U_{cea} = C_{pc}(T_2 - T_1)\left(\frac{1}{\eta_c}\right) \text{ kJ/kg} \tag{2.28}$$

and

$$U_{tea} = C_{pt}(T_3 - T_4)\left(\frac{1}{\eta_t}\right) \text{ kJ/kg} \tag{2.29}$$

Also assume that the specific heat C_{pf} of the fuel–air mixture is the value corresponding to the average value of T_2 and T_3, see Reference 4, sub-section 4.7.1, (2.23).

Hence the fuel energy equation becomes, from (2.23),

$$U_{fea} = C_{pf}(T_3 - T_{2ea}) \text{ kJ/kg} \tag{2.30}$$

Where

$$T_{2ea} = \frac{T_1({r_p}^{\beta_c} - 1 + \eta_c)}{\eta_c} \tag{2.31}$$

Where r_c and r_t apply to the compressor and turbine and are found from C_{pc}, C_{pt} and C_v.

The work done on the generator is now,

$$U_{\text{outea}} = C_{pt}\ T_3(1 - {r_p}^{\delta_t})\eta_t - \frac{C_{pc}T_1}{\eta_c}({r_p}^{\beta_t} - 1) \tag{2.32}$$

and

$$T_{4ea} = T_3(\eta_t {r_p}^{\delta_c} + 1 - \eta_t)$$

From U_{fea} and U_{outea} the thermal efficiency η_{pa} can be found as,

$$\eta_{pa} = \frac{U_{\text{outea}}}{U_{fea}} \tag{2.33}$$

2.2.4 Effect of Ducting Pressure Drop and Combustion Chamber Pressure Drop

Practical gas turbines are fitted with inlet and exhaust silencing and ducting systems to enable the incoming air to be taken from a convenient source and the outgoing gas to be discharged to a second convenient location. These systems can be long enough to create significant pressure drops at the inlet and outlet of the gas turbine itself. The inlet system reduces the pressure at the entry to the compressor, by an amount $\Delta P_{1.}$ The exhaust system increases the pressure at the exit of the power turbine, by an amount ΔP_4.

Between the outlet of the compressor and the inlet to the turbine there is a small pressure drop caused by the presence of the combustion chamber and the throttling effect of its casing. Let this pressure drop be ΔP_{23}.

The effects of ΔP_1, ΔP_{23} and ΔP_4 can be found by modifying their corresponding pressure ratios, r_{pc} for the compressor and r_{pt} for the turbine, and using the binomial theorem to simplify the results. ΔP_{23} and ΔP_4 apply to the turbine pressure ratio.

After a gas turbine has been operating for a long time the inlet filter pressure drop may become high enough to indicate that the filter needs cleaning. The drop in pressure across silencers will remain almost constant; the effect of ingress of particles or development of soot can be neglected.

The pressure ratio terms in (2.31) and (2.32) are of the general form,

$$y + \Delta y = \left(\frac{x + \Delta x}{w + \Delta w}\right)^n \tag{2.34}$$

and,

$$y = \left(\frac{x}{w}\right)^n \tag{2.35}$$

which upon expanding becomes,

$$yw^n + nyw^{n-1}\Delta w + w^n\Delta y = x^n + nx^{n-1}\Delta x \tag{2.36}$$

Where the second and higher orders of Δ are neglected. If the initial values are deducted then the expression relating the small changes becomes,

$$nyw^{n-1}\Delta w + w^n\Delta y = nx^{n-1}\Delta x \tag{2.37}$$

Hence the change in y becomes,

$$\Delta y = \frac{nx^{n-1}}{w^n}\Delta x - \frac{ny}{w}\Delta w \tag{2.38}$$

For the compressor it is assumed that the inlet pressure is increased by ΔP_1. The pressure ratio remains unchanged and so the change in output pressure is,

$$\Delta P_2 = r_p \Delta P_1$$

Since the pressure ratio is unchanged the output temperature will be unchanged at T_2.

The heat from the fuel is a function of T_2 and therefore it will also be unchanged.

For the turbine there are three pressure drops to consider. One for the compressor discharge ΔP_2, one for the practical throttling effect in the combustion chamber ΔP_{23} and one for the turbine exhaust pressure due to ducting ΔP_4. The two pressure drops at the inlet to the turbine can be combined as,

$$\Delta P_{223} = \Delta P_2 + \Delta P_{23} \tag{2.39}$$

In (2.34) Δx is ΔP_{223} and Δ_w is ΔP_4. Hence their effect on the turbine pressure ratio is $\Delta \mathrm{r}_{pt}{}^{nt}$,

$$\Delta r_{pt}{}^{nt} = \frac{n_t P_3{}^{nt-1}}{P_4{}^{nt}}\Delta P_{223} - \frac{n_t r_{pt}{}^{nt}}{P_4}\Delta P_4 \tag{2.40}$$

The turbine energy changes from U_{tea} to $U_{tea} + \Delta U_{tea}$. Substitute (2.40) into (2.29),

$$U_{tea} + \Delta U_{tea} = C_{pt} T_3 (1 - (r_{pt} + \Delta r_{pt})^{nt})\eta_t$$

$$= C_{pt} T_3 \eta_t \left[1 - \left(r_{pt}{}^{n} + \frac{n_t P_3{}^{nt-1}}{P_4{}^{nt}} \Delta P_{223} - \frac{n_t r_{pt}{}^{nt}}{P_4} \Delta P_4 \right) \right]$$

from which,

$$\Delta U_{tea} = +n_t \eta_t C_{pt} T_3 r_{pt}{}^{nt-1} \left[\frac{r_{pt} \Delta P_4 - r_{pt} \Delta P_1 - \Delta P_{23}}{P_4} \right] \tag{2.41}$$

The change in efficiency η_{pa} in (2.33) is,

$$\eta_{pa} + \Delta \eta_{pa} = \frac{U_{tea} + \Delta U_{tea} - U_{cea} - \Delta U_{cea}}{U_{fea} + \Delta U_{fea}} \tag{2.42}$$

from which, by substituting for ΔU_{tea}, $\Delta U_{cea} = 0.0$ and $\Delta U_{fea} = 0.0$ and deducting the initial conditions gives,

$$\Delta \eta_{pa} = \frac{\Delta U_{tea}}{U_{fea}} \tag{2.43}$$

The change in work done on the generator

$$\Delta U_{\text{outea}} = \Delta U_{tea} \text{ kJ/kg} \tag{2.44}$$

Note that in the above analysis the signs of the practical changes are,

ΔP_1 is negative

ΔP_{23} is negative

and ΔP_4 is positive

The pressure drops ΔP_1 and ΔP_4 are dependent upon the layout of the gas turbine generator, the dimensions of the ducting systems and the specification of silencers and filters. ΔP_{23} is fixed by the design of the combustion system and cannot be changed by external factors such as ducting systems.

2.2.4.1 Typical values of pressure drop losses

A newly installed gas turbine generator can be taken to have the typical losses given in Table 2.2.

Table 2.2. Typical pressure drop losses in gas turbine

Inlet or exhaust	Pressure drop		% change in	
	Bar	Inches of water	Power output	Heat rate
Inlet	0.01245	5.0	−2.00	+0.75
Exhaust	0.006227	2.5	−0.50	+0.40

2.2.5 Heat Rate and Fuel Consumption

The heat rate is the ratio of heat given up by the fuel, in terms of its lower calorific or heating value (LHV), to the power available at the gas turbine coupling to its generator. It has the SI units of kJ/kWh. The lower heating value of typical fuels is given in Table 2.3.

The heat rate for a particular gas turbine will be given by its manufacturer at ISO conditions, and at various ambient temperatures. The typical variation of heat rate and power output, in relation to their ISO values, are shown in Figure 2.10. For a definition of ISO conditions see sub-section 2.3.2.

Table 2.3. Lower heating values of fuels

Fuel	Lower heating value (LHV) MJ/m^3 for gases MJ/kg for liquids	Btu/ft^3 for gases Btu/lb for liquids
GASES		
Natural gas	35.40 to 39.12	950 to 1050
Methane	33.94	911
Ethane	60.77	1,631
Propane	87.67	2,353
Butane	115.54	3,101
Hydrogen	10.17	273
Hydrogen sulphide	23.14	621
LIQUIDS		
Diesel oil	45.36	19,500
Kerosene	41.87	18,000
Distillate	44.89	19,300
Crude oil	44.66	19,200

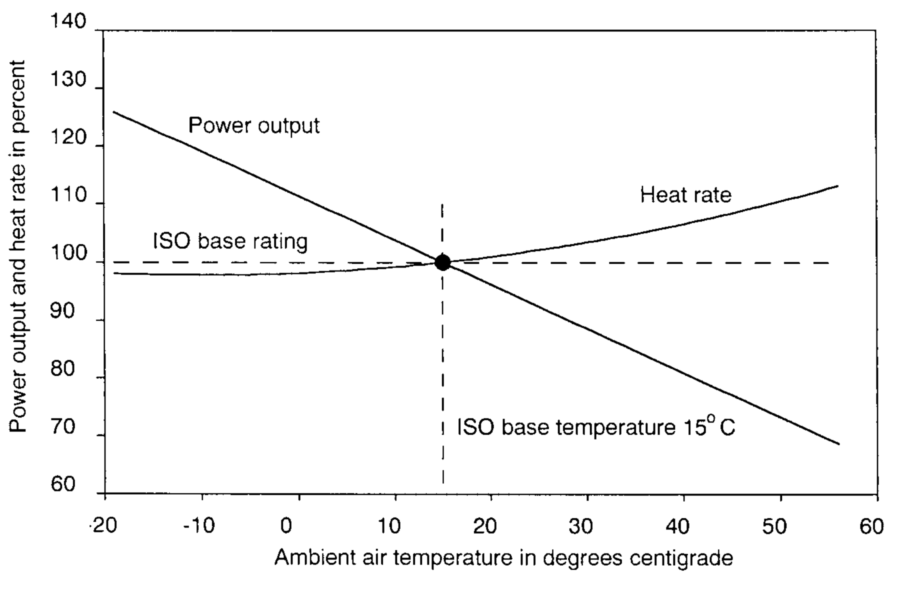

Figure 2.10 Power output and heat rate versus ambient air temperature.

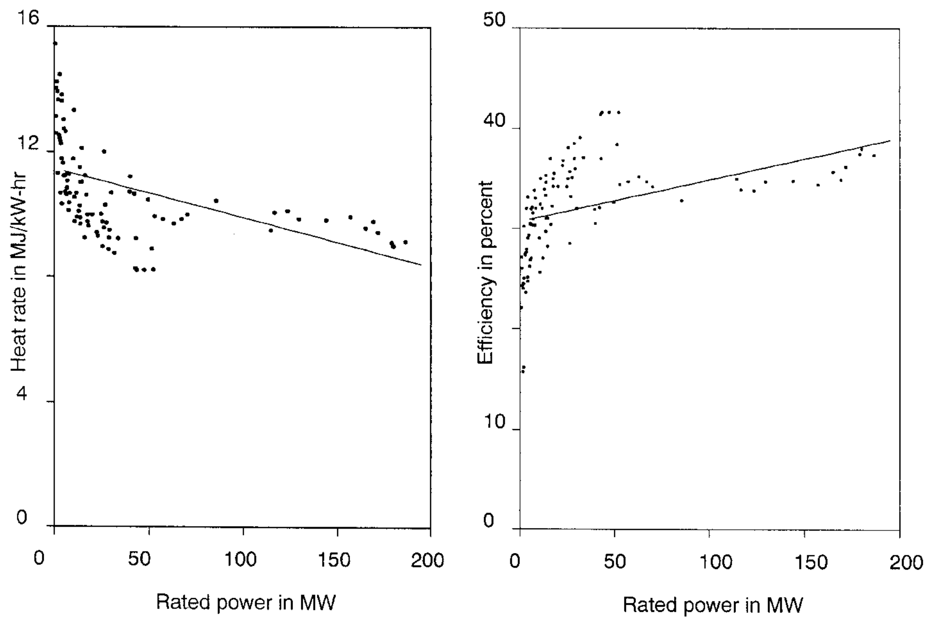

Figure 2.11 Heat rate and efficiency versus power rating.

The reduction in output power is typically 0.5 to 0.8%/°C.

The fuel consumption can be calculated approximately from,

$$\text{Fuel consumption} = \frac{\text{Power output} \times \text{Heat rate}}{\text{Fuel LHV}} \ \text{m}^3/\text{h (or kg/h)}$$

For situations where there is a mixture of gases it is advisable to consult the manufacturer of the gas turbine, since he will have a data bank containing all kinds of fuel compositions and heating values.

The heat rate and overall thermal efficiencies for typical modern gas turbines in the range of ISO power ratings 1 MW to 200 MW are shown in Figure 2.11. The data were derived from Reference 6.

2.3 POWER OUTPUT FROM A GAS TURBINE

In sub-section 2.2 the performance of a gas turbine was determined as the energy obtainable at the output shaft coupling. The energy equations are based on a unit of mass flow, 1.0 kg/s, of the fluid passing through the gas turbine i.e. from the air intake to the exhaust aperture.

The mass flow through the turbine is about 1% higher than that through the compressor because of the presence of the burnt fuel. Hence the mass flow rate (m) to produce the output power is,

$$m = \frac{\text{Output power to the generator}}{\text{Output energy per unit mass}} \quad \text{kg/s}$$

$$= \frac{W_{\text{out}}}{U_{\text{out}}} \qquad \left(\frac{\text{kW kg}}{\text{kj}} = \text{kg/s}\right)$$

Therefore it is a simple matter to predetermine the required output power and divide this by the specific energy available to the generator. The result is then the mass flow rate.

2.3.1 Mechanical and Electrical Power Losses

The power and specific energy available to drive the generator determined in the previous sub-section are those at the output shaft of the gas turbine. In most situations in the oil industry, where these machines seldom are rated above 40 MW, a speed-reducing gearbox is placed between the turbine and the generator. The generators are usually 4-pole machines that operate at 1500 or 1800 rev/min. The power loss in a typical gearbox is about 1.5% of the rated output power. Let the gearbox efficiency be η_{gb}.

The efficiency (η_{gen}) of electromechanical conversion in the generator can be defined as,

$$\eta_{gen} = \frac{\text{Power output at the terminals}}{\text{Power input to the shaft coupling}} \quad \text{pu}$$

Most rotating electrical machines above about 500 kW have efficiencies above 95%, which increases to about 98% for large machines in the hundreds of megawatts range. Their losses are due to windage between the rotor and the stator, friction in the bearings and seals, iron and copper electrical losses.

In some situations, such as ‘packaged’ gas turbine generators, all the necessary auxiliary electrical power consumers are supplied from the terminals of the generator through a transformer and a small motor control centre (or switchboard). These auxiliaries include lubricating oil pumps, fuel pumps, filter drive motors, cooling fans, purging air fans, local lighting, and sump heaters. Some of these operate continuously while others are intermittent. A rule-of-thumb estimate of the consumed power of these auxiliaries is between 1% and 5% of the rated power of the generator.

Care needs to be taken when referring to the efficiency of a gas-turbine generator set. See the worked example in Appendix F. The power system engineer is concerned with the power output from the terminals of the generator that is obtainable from the fuel consumed. Hence he considers the practical efficiency η_{pa} of the gas turbine, and the conversion efficiency through the gearbox η_{gb} and generator η_{gen}. Hence the Overall Thermal Efficiency η_{pao} would be:-

$$\eta_{pao} = \eta_{pa} \times \eta_g \times \eta_{gen}$$

2.3.2 Factors to be Considered at the Design Stage of a Power Plant

The electrical engineer should take full account of the site location and environmental conditions that a gas turbine generator will need to endure. These conditions can seriously effect the electrical power output that will be achievable from the machine. The starting point when considering the possible output is the ISO rating. This is the declared rating of the machine for the following conditions:-

- Sea level elevation.
- 15°C (59°F) ambient temperature.
- Basic engine, no losses for inlet or exhaust systems, no losses for gearbox and mechanical transmission.
- Clean engine, as delivered from the factory.

The gas turbine manufacturer provides a standardised mechanical output power versus ambient temperature characteristic, e.g., Figure 2.10. (Some manufacturers also give the electrical output power versus ambient temperature characteristic. Therefore care must be exercised to be sure exactly which data are to be given and used.)

The following derating factors should be used in the estimation of the continuous site rating for the complete machine:

- ISO to a higher site ambient temperature, typically 0.5 to 0.8% per °C.
- Altitude, usually not necessary for most oil industry plants since they are near sea level.
- Dirty engine losses and the ageing of the gas turbine, assume 5%.
- Fuel composition and heating value losses, discuss with the manufacturer.
- Silencer, filter and ducting losses, assume 2 to 5%.
- Gearbox loss, typically 1 to 2%.
- Generator electromechanical inefficiency, typically 2 to 4%.
- Auxiliary loads connected to the generator, typically 1 to 5%.

2.3.2.1 Dirty engine losses

Consideration should be given to the fact that engines become contaminated with the combustion deposits, the lubrication oil becomes less efficient, blades erode and lose their thermodynamic efficiency and air filters become less efficient due to the presence of filtered particles. These effects combine to reduce the output of the machine. A rule-of-thumb figure for derating a gas turbine for dirty engine operation is 5%. This depends upon the type of fuel, the type of engine, the environment and how long the engine operates between clean-up maintenance periods. Individual manufacturers can advise suitable data for their engines operating in particular conditions. Dirty engine conditions should be considered, otherwise embarrassment will follow later once the machine is in regular service.

2.3.2.2 Fuel composition and heating value losses

The chemical composition and quality of the fuel will to some extent influence the power output. However, it is usually the case that more or less fuel has to be supplied by the fuel control valve for a given throughput of combustion air. Hence it is usually possible to obtain the declared normal rating from the machine, but attention has to be given to the supply of the fuel. In extreme cases the profile of the fuel control valve may require modification so that adequate feedback control is maintained over the full range of power output. The appropriate derating factor is usually 100%, i.e. no derating.

2.3.2.3 Silencer, filter and ducting losses

The amount of silencing and filtering of the inlet combustion air depends upon the site environment and the operational considerations.

Site environmental conditions may be particularly bad, e.g. deserts where sand storms are frequent; offshore where rain storms are frequent and long lasting. The more filtering that is required, the more will be the pressure lost across the filters, both during clean and dirty operation. This pressure drop causes a loss of power output from the machine.

The amount of inlet and exhaust noise silencing will depend upon, the location of machine with respect to people in say offices or control rooms, how many machines will be in one group since this affects the maintenance staff and total noise level permitted by international or national standards. The effects of a silencer are similar to a filter since the silencing elements cause a pressure drop.

With offshore platforms it is not always practical to locate the main generators in a good place regarding the position and routing of the inlet and exhaust ducting. Long runs of ducting are sometimes unavoidable. It is then necessary to allow a derating factor for the pressure drop that will occur. The manufacturer should be consulted for advice on this aspect. For a typical offshore or onshore situation with a reasonable degree of silencing a rule-of-thumb derating factor would be 98%. In a poor location assume 95%.

2.4 STARTING METHODS FOR GAS TURBINES

Gas turbines are usually started by a DC motor or an air motor. Either system is available for most turbines up to about 20 MW. Occasionally AC motors are used. Beyond 20 MW, when heavy industrial machines tend to be used, it becomes more practical to use air motors or even diesel engine starters. DC motors require a powerful battery system. The DC motor and battery systems tend to be more reliable and less space consuming, which is important for offshore systems. Air motors require air receivers and compressors. The compressors require AC motors or diesel engines. Air start and diesel start systems are more popular for onshore plants especially remote plants, e.g. in the desert. This is partly due to the fact that batteries tend to suffer from poor maintenance in hot, dry locations. Air systems require regular maintenance and must be kept fully charged in readiness for a quick start. Air system receivers can become very large if more than three successive starting attempts are required. More starts can probably be obtained by a battery system that occupies the same physical space.

Occasionally process gas can be used instead of air to drive the air/gas starter motor. This eliminates the need for receivers and compressors. However, there should always be a reliable source of gas available. The exhaust gas from the starter motor should be safely discharged e.g. into a ventilating pipeline.

2.5 SPEED GOVERNING OF GAS TURBINES

2.5.1 Open-loop Speed-torque Characteristic

The ungoverned or open-loop speed-torque characteristic of a gas turbine has a very steep negative slope and is unsuitable for regulating the power output of the generator. The open-loop characteristic is explicitly determined by the thermodynamic design of the gas turbine, together with the mechanical inertial and frictional characteristics of the rotating masses. Without closed-loop feedback control action the initial decline in speed in response to an increase in shaft torque would be mainly determined by the shaft inertia. Let T, ω and P be the torque, speed and shaft power respectively in per-unit terms. The expression relating these variables is,

$$P = T\omega \tag{2.45}$$

The open-loop speed-torque function may be expressed as,

$$\omega = f(T) \tag{2.46}$$

which may be represented by a simple linear function,

$$\omega = \omega_o - kT \tag{2.47}$$

where k is a positive number in the order of 1.0 pu equal to the open-loop slope, and ω_o is the shaft speed at no-load.

Reference 7 discusses the slope k in Chapter 2, Section 2.3.1.

Assume that the turbine is designed to deliver unit torque at unit speed, therefore,

$$1.0 = \omega_o - k(1.0) = \omega_o - k \tag{2.48}$$

From which $\omega_o = 1 + k$ and so (2.47) becomes,

$$\omega = 1 + k - kT \text{ or } T = \frac{1 + k - \omega}{k} \tag{2.49}$$

The speed can now be related to the shaft power rather than the torque,

$$P = \left(\frac{1 + k - \omega}{k}\right)\omega \tag{2.50}$$

Or in the form of a quadratic equation,

$$0 = \omega^2 - (1 + k)\omega + kP \tag{2.51}$$

The two roots of which are,

$$\omega_{1,2} = \frac{1 + k}{2} \pm \left(\frac{(1 + k)^2 - 4kP}{2}\right)^{1/2} \tag{2.52}$$

The positive root applies to the stable operating region, whilst the negative root applies to the unstable region after stalling occurs.

For example assume $k = 1.5$. Table 2.4 shows the values of the two roots for an increase in shaft power.

Table 2.4. Open-loop steady state speed-power characteristic of a gas turbine ($k = 1.5$)

Shaft power P (per unit)	Shaft speed ω (per unit)	
	Positive root	Negative root
0.0	2.5	0.0
0.5	2.151	0.349
0.75	1.911	0.589
1.00	1.500	1.000
1.04	1.250	1.250
1.04 + (unstable)		

Table 2.5. Open-loop steady state speed-power characteristic of a gas turbine ($k = 0.1$)

Shaft power P (per unit)	Shaft speed ω (per unit)	
	Positive root	Negative root
0.0	1.10	0.0
0.5	1.0525	0.0475
0.75	1.027	0.073
1.00	1.000	0.100
1.04	0.9955	0.1045
1.50	0.9405	0.1550
2.00	0.8700	0.2300
3.00	0.6000	0.5000
3.025	0.5500	0.0

At $P = 1.0$ the torque corresponding to the positive root is $T = 0.667$ pu, whilst that for the negative root is $T = 1.00$ pu. Hence the torque at full-load power is less than unity (due to the speed being higher than unity). The above example illustrates the impractical nature of the open-loop speed-torque and speed-power characteristics.

Suppose the design of the engine could be substantially improved such that k could be reduced to say 0.1 (approaching a value for a typical closed-loop feedback controlled system). Table 2.5 shows comparable results to those given in Table 2.4.

It can be seen that unit power is obtained at unit speed in the stable region, and that the stalling point is at a power much greater than unity. The above illustrates more desirable open-loop speed-torque and speed-power characteristics. Unfortunately reducing k to values between say 0.01 and 0.1 by thermodynamic design is not practical. Consequently a closed-loop feedback control system is necessary. Figure 2.12 shows the open-loop speed-power responses for different values of k. The transient response of the gas turbine just after a disturbance in the shaft power is of interest when underfrequency protective relays are to be used to protect the power system from overloading, see sub-section 12.2.10.

2.5.2 Closed-loop Speed-power Characteristic

All prime-moves used for driving electrical generators are equipped with closed-loop speed governors. Their main purpose is to reduce the variation in shaft speed to a small amount over the full range of shaft power. Deviations in speed are measured and amplified. The amplified signal is used to operate the fuel value in such a manner as to reduce the deviation in speed. It may be assumed that a linear relationship exists between the amplified signal received at the value and the shaft power created by the fuel passed through the valve orifice. The fuel valve may be regarded as a regulating device for power available at the shaft. It may therefore be assumed that the output of the valve is the shaft power P, whilst its inputs are a reference power P_{ref} and the amplified speed error P_e.

Therefore,

$$P = P_{\text{ref}} - P_e \tag{2.53}$$

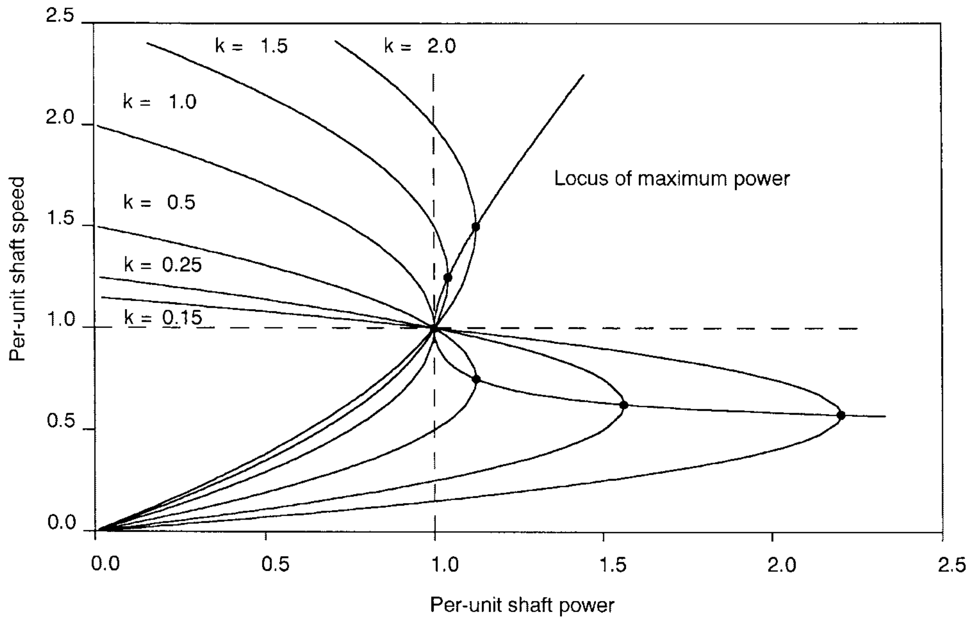

Figure 2.12 Open-loop speed regulation of a gas turbine.

where

$$P_e = F(\omega_o - \omega) \tag{2.54}$$

and ω_o is the nominal shaft speed, and F is the feedback gain.

Hence the closed-loop control system for steady state conditions may be described by the forward transfer function of (2.52), using the positive root, and the feedback transfer function of (2.54). In order to establish suitable relationships between k and F it is necessary to consider small changes in the variables and by so doing linearise the equations using a two-term Taylor's series. Transpose and square the positive root of (2.52).

$$\left(\omega - \frac{1+k}{2}\right)^2 = \frac{1}{4}\left((1+k)^2 - 4\,kP\right) \tag{2.55}$$

Let ω be increased by $\Delta\omega$ as the power P is increased by ΔP.

Equation (2.55) becomes,

$$2\omega\Delta\omega - \Delta\omega(1+k) + \omega^2 - \omega(1+k) + \left(\frac{1+k}{4}\right)^2$$
$$= \left(\frac{1+k}{4}\right)^2 - kP - k\Delta P \tag{2.56}$$

Subtract the predisturbance state,

$$\frac{\Delta\omega}{\Delta P} = \frac{-k}{4(2\omega - 1 - k)} \tag{2.57}$$

In (2.53) and (2.54) let ω be increased by $\Delta\omega$ and P by ΔP, and subtract the predisturbance state,

Hence, $$\Delta P_e = F\Delta\omega$$

or $$\frac{\Delta\omega}{\Delta P_e} = \frac{1}{F} \tag{2.58}$$

and $$\Delta P = \Delta P_{\text{ref}} - \Delta P_e \tag{2.59}$$

A change in the demand for shaft power ΔP_d may be added to the summing point of ΔP_{ref} and ΔP_e, and ΔP_{ref} assumed to be zero. Hence the overall closed-loop transfer function gain G_c at the speed ω is found to be,

$$G_c = \frac{\Delta\omega}{\Delta P_d} = \frac{\text{Forward gain}}{1 + (\text{Forward gain})(\text{Feedback gain})}$$

$$= \frac{\dfrac{-k}{4(2\omega - 1 - k)}}{1 - \dfrac{kF}{4(2\omega - 1 - k)}}$$

$$= \frac{k}{kF - 4(2\omega - 1 - k)} \tag{2.60}$$

For typical power system applications the transfer function gain has the per-unit value of 0.04, and the operating shaft speed ω is within a small range centred around the rated speed. The rated speed corresponds to the nominal frequency of the power system. Hence the term $4(2\omega - 1 - k)$ may be neglected since k is typically in the range of 1.0 to 2.0.

The transfer function simplifies to become,

$$G_c = \frac{1}{F} \quad \text{where F is typically 25 per unit} \tag{2.61}$$

The transfer function gain is also called the 'droop' characteristic of the gas turbine.

2.5.3 Governing Systems for Gas Turbines

The following discussions outline the important principles behind the governing of gas turbines. In all power systems the requirement is that the steady state speed deviation, and hence frequency, is kept small for incremental changes in power demand, even if these power increments are quite large – 20%, for example.

There are two main methods used for speed governing gas turbines,

a) Droop governing.
b) Isochronous governing.

Droop governing requires a steady state error in speed to create the necessary feedback control of the fuel value. ‘Droop’ means that a fall in shaft speed (and hence generator electrical frequency) will occur as load is increased. It is customary that a droop of about 4% should occur when 100% load is applied. Droop governing provides the simplest method of sharing load between a group of generators connected to the same power system.

In control theory terminology this action is called ‘proportional control’. This method of governing is the one most commonly used in power systems because it provides a reasonably accurate load sharing capability between groups of generators.

Isochronous governing causes the steady state speed error to become zero, thereby producing a constant speed at the shaft and a constant frequency for the power system. Isochronous governing is also a form of ‘integral control’. This method is best suited to a power system that is supplied by one generator. This type of power system has very limited application. However, there are situations where one isochronously governed generator can operate in parallel with one or more droop-governed generators. The droop-governed generators will each have a fixed amount of power assigned to them for the particular system frequency. This is achieved by adjusting their set points. As the demand on the whole system changes, positively or negatively, the isochronously governed generator will take up or reject these changes, and the steady state frequency will remain constant. This hybrid type of load sharing is seldom used in the oil industry.

Accurate power sharing and constant speed control can be obtained by using a specially designed controller. This controller incorporates load measurement of each generator, measurement of common system frequency and a sub-system to reduce the power mismatches of each generator to zero. The controller regularly or even continuously trims the speed set points of each gas turbine to maintain zero mismatches. A slowly operating integrator can be superimposed onto these set points to adjust them simultaneously so that the frequency is kept constant. This is a form of ‘proportional-integral’ control. See also Chapter 16 for a further discussion of these subjects.

The basic control system of most gas turbine generator systems is shown in Figure 2.13.

Where ω = shaft speed
ω_{ref} = reference speed
P_e = electrical power at the generator shaft
P_m = mechanical output power of the gas turbine
P_a = accelerating power
P_f = friction and windage power

2.5.4 Load Sharing between Droop-governed Gas Turbines

Consider a number of generators connected to the same busbars. For the purpose of generality it will be assumed that each of the generators has a different power rating, and that each governor has a different droop. The droop characteristic for the i_{th} gas turbine is,

$$f = f_{zi} - \frac{D_i P_i f_o}{G_i} \tag{2.62}$$

Where f_o = the nominal system frequency in Hz
f = the actual system frequency in Hz

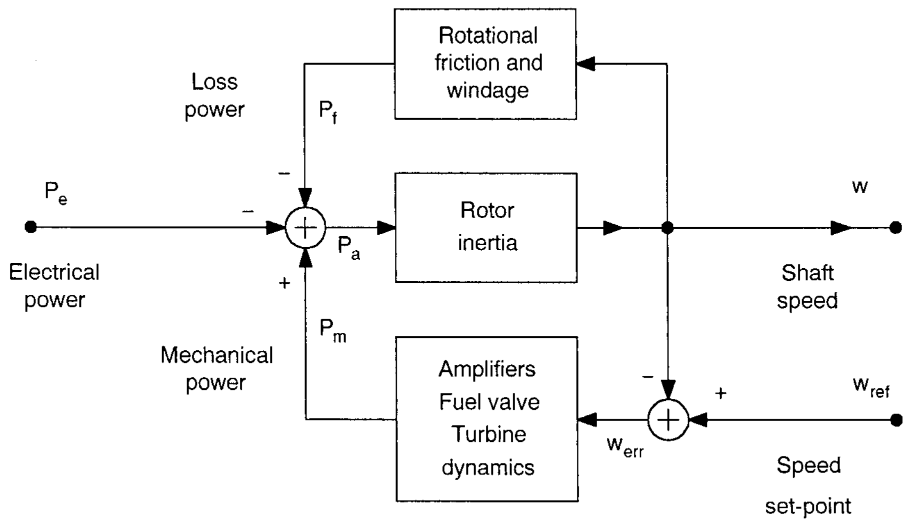

Figure 2.13 Basic control system block diagram of a gas turbine. The diagram represents the main elements of the equation of motion.

f_{zi} = frequency set-point of the i_{th} governor in Hz

D_i = governor droop in per unit of the i_{th} governor

P_i = electrical load of the i_{th} generator in kW

G_i = electrical power rating of the i_{th} generator in kW

Transpose (2.62) to find P_i,

$$P_i = (f_{zi} - f)\frac{G_i}{D_i f_o} \tag{2.63}$$

The total power demand P for n generators is,

$$P = \sum_{i=1}^{i=n} P_i \tag{2.64}$$

$$= \frac{1}{f_o}\sum_{i=1}^{i=n}\frac{f_{zi}G_i}{D_i} - \frac{f}{f_o}\sum_{i=1}^{i=n}\frac{G_i}{D_I} \tag{2.65}$$

Which is of the simple form,

$$P = a - bf \tag{2.66}$$

where

$$a = \frac{1}{f_o}\sum_{i=1}^{i=n}\frac{f_{zi}G_i}{D_i} \text{ is a constant} \tag{2.67}$$

and

$$b = \frac{1}{f_o} \sum_{i=1}^{i=n} \frac{G_i}{D_i} \text{ is also a constant} \quad (2.68)$$

(2.65) and (2.66) represent the overall droop characteristic of the power system.

The application of (2.63) and (2.64) can be demonstrated graphically for a system in which two generators are sharing a common load.

Consider two gas turbine generators, called Gen.1 and Gen.2, of the same size are sharing a common load. Assume Gen.1 takes 60% and Gen.2 the remaining 40%. Let the system frequency be 60 Hz at full load and the droop of each machine be 4%.

The speed (frequency) versus load sharing situations can be shown graphically as in Figure 2.14 where point 'A' is the initial situation.

Now, supposing it is necessary to equalise the load shared by the two machines, then one or both of the speed settings will need to be adjusted depending upon the final common speed (frequency) required by the machines. It can be seen that unless the speed settings are changed, the load taken by each machine cannot change. There are several methods by which this may be done, by changing the speed setting of Gen.1 or Gen.2 or both.

Method 1. Change the speed setting of Gen.1 only:

The droop characteristic line 1A-A must be lowered to the new position ID-D so that it crosses the line 2A-D of Gen.2 at point 'D' for 50% sharing of load. Thus the speed

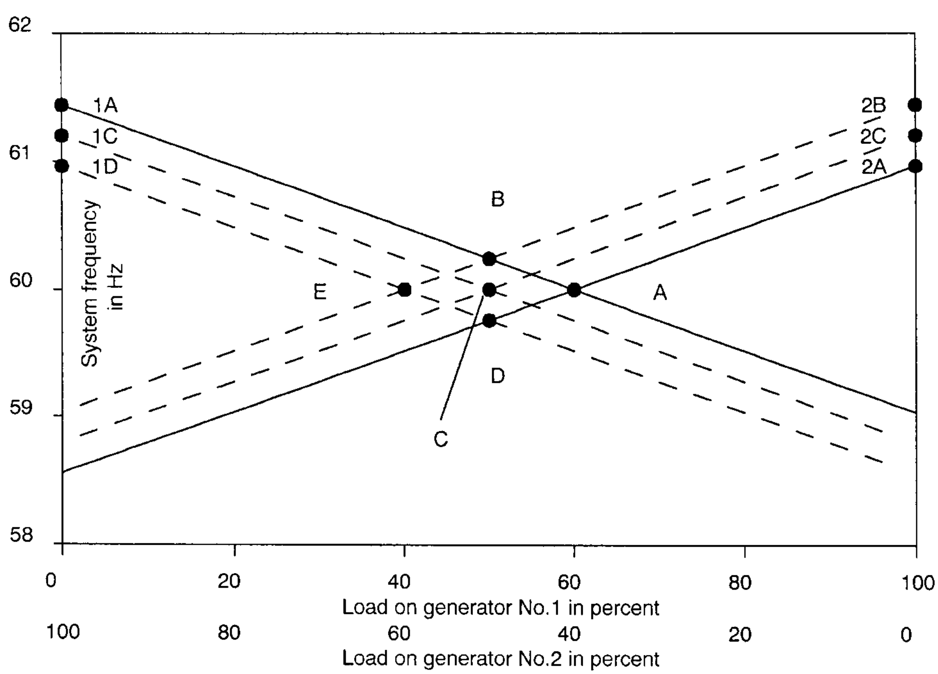

Figure 2.14 Frequency droop governing and load sharing of two gas turbines.

setting must be reduced from 102.4% (61.44 Hz) to 101.6% (60.96 Hz) i.e. the same as that of Gen.2. The common new frequency will be at point 'D' as 99.6% (59.76 Hz).

Method 2. Change the speed setting of Gen.2 only:

The droop characteristic line 2A-A must be raised to the new position 2B-B so that it crosses the line 1A-B of Gen.1 at point 'B' for 50% sharing of load. Thus the speed setting must be raised from 101.6% (60.96 Hz) to 102.4% (61.44 Hz) i.e. the same as that of Gen.1. The new frequency will be at point 'B' as 100.4% (60.24 Hz).

Method 3. Change the speed setting of Gen.1 and Gen.2.

In order to recover the frequency to 100% (60 Hz) both speed settings will need to be changed.

Gen.1 speed setting will be reduced to 102% (61.2 Hz).

Gen.2 speed setting will be raised to 102%.

The operating point will be 'C'.

The droop lines will be 1C-C and 2C-C.

2.5.4.1 Worked example

Three generators have different ratings and operate in a power system that has a nominal frequency of 60 Hz. Each generator is partially loaded and the total load is 25 MW.

a) Find the loading of each generator and the system frequency if the total load increases to 40.5 MW, whilst their set points remain unchanged.
b) Also find the changes required for the set points that will cause the system frequency to be restored to 60 Hz. The initial loads on each generator and their droop values are, shown in Table 2.6.
c) Find the changes in the set points that will enable the generators to be equally loaded at the new total load, with the system frequency found in a).
d) Find the additional changes in the set points that will enable the frequency to be recovered to 60 Hz.

Step 1. Find the initial set points f_{zi} before the load is increased. Transpose (2.62) to find f_{zi}

$$f_{zi} = f + \frac{D_i P_i f_o}{G_i} \tag{2.69}$$

For generator No. 1,

$$f_{z1} = 60.0 + \frac{0.03 \times 60.0 \times 10.0}{20} = 60.9 \text{ Hz}$$

Table 2.6. Data and initial conditions of three generators

Generator rating (MW)	Initial loading (MW)	Drop in per unit
20	10	0.03
15	10	0.04
10	5	0.05

Similarly for generators Nos. 2 and 3,

$$f_{z2} = 61.6 \text{ Hz and } f_{z3} = 61.5 \text{ Hz}$$

Step 2. The common system frequency after the load increases is found from (2.66), (2.67) and (2.68).

$$a = \frac{1}{60.0}\left(\frac{60.9 \times 20.0}{0.03} + \frac{61.6 \times 15.0}{0.04} + \frac{61.5 \times 10.0}{0.05}\right) = 1266.67$$

$$b = \frac{1}{60}\left(\frac{20.0}{0.03} + \frac{15.0}{0.04} + \frac{10.0}{0.05}\right) = 20.6945$$

$$f = \frac{a - P}{b} = \frac{1266.67 - 40.5}{20.6945}$$
$$= 59.25101 \text{ Hz}$$

Step 3. Find the new load on each generator

$$P_1 = (f_{z1} - f)\frac{G_1}{D_1 f_o} = (60.9 - 59.25101)\frac{20.0}{0.03 \times 60.0}$$
$$= 18.3221 \text{ MW } (91.61\%)$$

Similarly for generators Nos. 2 and 3,

$$P_2 = 14.6819 \text{ MW } (97.88\%) \text{ and } P_3 = 7.4966 \text{ MW } (74.97\%)$$

Note,

$$P_{\text{new}} = P_1 + P_2 + P_3 = 18.3221 + 14.6819 + 7.4966$$
$$= 40.5 \text{ MW as required.}$$

Step 4. Find the new set points that will recover the frequency to 60 Hz.

If a change ΔP_i in P_i is added to the (2.69) then the change in the set point will be,

$$\Delta f_{zi} = \frac{D_i \Delta P_i f_o}{G_I} \text{ (or } 60.0 - f)$$

For generator No. 1,

$$\Delta f_{z1} = \frac{0.03 \times (18.3221 - 10.0)60.0}{20} = 0.74899$$

And so the new set-point is $f_{z1} + \Delta f_{z1} = 61.6489$ Hz

Similarly for generators Nos. 2 and 3

$$f_{z2} + \Delta f_{z2} = 62.3491 \text{ Hz, and } f_{z3} + \Delta f_{z3} = 62.2489 \text{ Hz}$$

Step 5. Find the set points that will enable the generators to be equally loaded.

For generator No. 1, the ratio K_1 of its new load to its rating is,

$$\frac{P_1 + \Delta P_1}{G_1} = K_1$$

Similarly for generators Nos. 2 and 3,

$$\frac{P_2 + \Delta P_2}{G_2} = K_2 \text{ and } \frac{P_3 + \Delta P_3}{G_3} = K_3$$

For the generators to be equally loaded $K_1 = K_2 = K_3 =$K.

In addition the ratio of the total load to the total of the generator ratings must be the same as for each generator,

Hence,

$$K = \frac{P_1 + \Delta P_1 + P_2 + \Delta P_2 + P_3 + \Delta P_3}{G_1 + G_2 + G_3}$$

$$K = \frac{40.5}{20 + 15 + 10} = 0.9$$

Therefore since

$$\frac{P_1 + \Delta P_1}{G_1} = 0.9$$

$$\Delta P_1 = (0.9 \times 20) - 10.0 = 8.00 \text{ MW}$$

so that

$$P_1 + \Delta P_1 = 18.00 \text{ MW } (90\%)$$

and

$$\Delta P_2 = (0.9 \times 15) - 10.0 = 3.5 \text{ MW}$$

so that

$$P_2 + \Delta P_2 = 10.0 + 3.5 = 13.5 \text{ MW } (90\%)$$

and

$$\Delta P_3 = (0.9 \times 10) - 5.0 = 4.0 \text{ MW}$$

so that

$$P_3 + \Delta P_3 = 5.0 + 4.0 = 9.0 \text{ MW } (90\%)$$

Step 6. Find the new set points.

From (2.62), for generator No. 1, using the original frequency of 59.25101 Hz, found in Step 2,

$$f_{z1} = 59.25101 + \frac{0.03 \times 18.00 \times 60.0}{20.0}$$
$$= 60.871 \text{ Hz}$$

Similarly for generators Nos. 2 and 3,

$$f_{z2} = 61.411 \text{ Hz and } f_{z3} = 61.951 \text{ Hz}$$

Step 7. Find the new set points that will recover the frequency to 60 Hz whilst maintaining equally loaded generators.

Let the desired frequency of 60 Hz be denoted at f_d. In order to reach this frequency all the set points need to be increased by the difference between f_d and f, which is,

$$\Delta f = f_d - f = 60.0 - 59.25101 = 0.749 \text{ Hz}$$

Therefore,

$$f_{z1} = 60.871 + 0.749 = 61.62 \text{ Hz}$$
$$f_{z2} = 61.411 + 0.749 = 62.16 \text{ Hz}$$

and

$$f_{z3} = 61.951 + 0.749 = 62.70 \text{ Hz}$$

Check that f has now the correct value, by using (2.62),

$$\begin{aligned} f &= f_{z1} - \frac{D_1 P_1 f_o}{G_1} = 61.62 - \frac{0.03 \times 18.0 \times 60}{20} \\ &= 61.62 - 1.62 = 60.0 \text{ Hz} \\ f &= f_{z2} - \frac{D_2 P_2 f_o}{G_2} = 62.16 - \frac{0.04 \times 13.5 \times 60}{15} \\ &= 62.16 - 2.16 = 60.0 \text{ Hz} \end{aligned}$$

and,

$$\begin{aligned} f &= f_{z3} - \frac{D_3 P_3 f_o}{G_3} = 62.70 - \frac{0.05 \times 9.0 \times 60}{10} \\ &= 62.70 - 2.70 = 60.0 \text{ Hz} \end{aligned}$$

2.5.5 Load Sharing Controllers

The above worked example illustrates the combination of droop governing with an overall isochronous control function. In a practical control scheme the following variables can be easily measured by suitable transducers,

f = the system frequency.
P_i = the electrical power at the terminals of the generator (the generator losses and gearbox losses can be ignored).
f_{zi} = the governor set point within the controller that drives the fuel valve. A suitable potentiometer can be used to derive the signal.

The constants D_i, G_i and f_o can be incorporated into the controller as potentiometer adjustments, or in a program if a programmable computing type of controller is used.

The control action can be made continuous or intermittent, i.e. control signals dispatched at regular intervals.

2.5.5.1 Simulation of gas turbine generators

As described in sub-section 2.1.4 there are two main methods of transferring power from the gas turbine to the generator, i.e., single-shaft and two-shaft driving systems. Established practice has a preference for single-shaft machines for generator duty, but only where the ratings are available. There is a reluctance to have both types on a common self-contained power system, such as those used with offshore platforms or isolated land-based plants. It is generally considered that a single-shaft machine has a superior speed performance when sudden changes in electrical power occur. The deviation in shaft speed and frequency are lower and the recovery time is faster. In a two-shaft machine there is a finite delay caused by the fact that the compressor responds before the power turbine can respond.

The block diagrams for these two driving arrangements are different, the two-shaft arrangement being slightly more complicated. Figure 2.13 can be rearranged as Figure 2.15 to show the reference speed signal on the left-hand side as the main input to the system. The main output of interest is the shaft speed. The rotational friction and windage block can be ignored since its influence on the performance of the control system is very small. The complexity of these diagrams depends upon what data are available from the manufacturer and the nature of the study being performed. The diagrams from manufacturers sometimes show features, which are not usually needed for stability studies, for example overspeed safety loops. Therefore some reasonable simplification is usually acceptable.

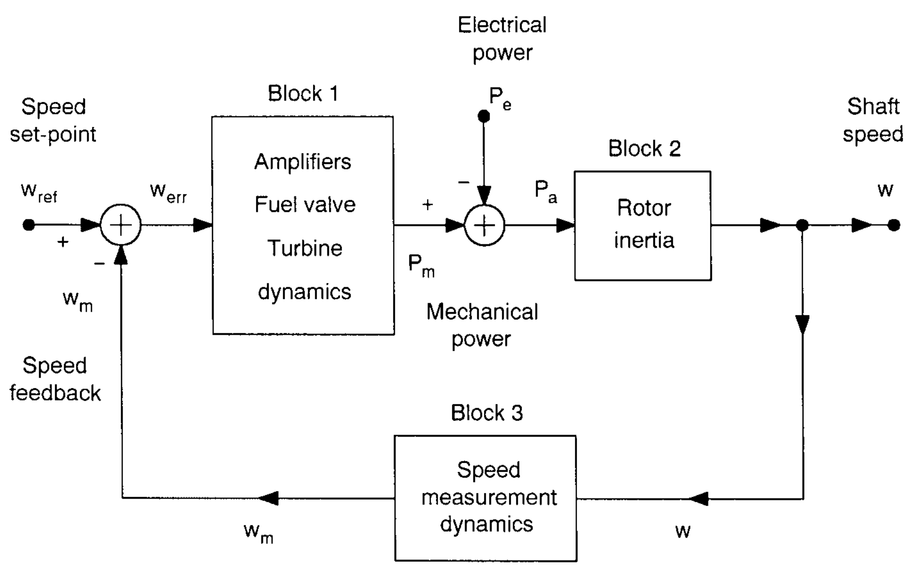

Figure 2.15 Simplified equation of motion of a gas turbine.

Block 1 in Figure 2.15 contains most of the main control and turbine functions, such as,

a) Governor gain.
b) Governor lead and lag compensating dynamics.
c) Derivative damping term for the speed signal.
d) Fuel valve gain, limits and dynamic terms.
e) Combustion system lag dynamic term.
f) Combustion system limits.
g) Power turbine dynamics.
h) Compressor dynamics.
i) Compressor protection system.
j) Turbine temperature measurement dynamics and limit or reference level.

The functions h), i) and j) are used when a two-shaft drive system needs to be simulated. When applied they usually require a special signal selection block to be incorporated just before the fuel valve or governor. The purpose of this signal selector is to automatically choose the lowest value or its two input signals, so that the least fuel is passed to the combustion system. This contributes to the slower response of a two-shaft machine.

The data supplied by the manufacturer is often given in physical units such as, the position of the fuel valve in angular degrees, shaft speed in revolutions per minute, power output in kilowatts, combustion temperature in degrees Kelvin. In most power system computer programs these data need to be converted into a compatible per-unit form. This can be a little difficult to achieve and a source of numerical errors, which can lead to incorrect results from the program. Manufacturers may also provide a per-unit form of the block diagram, if requested to do so. The time constants used in these diagrams vary significantly from one type and rating of gas turbine to another. It is difficult to generalise their values. The rotor inertia of the turbine should include the inertia of the gearbox and the rotor of the generator. The speed measurement block usually contains the governing lead and lag compensation time constants. These time constants and the derivative damping gain have a strong influence on the speed response to a change in electrical power, and should therefore be chosen or calculated carefully.

2.6 MATHEMATICAL MODELLING OF GAS TURBINE SPEED GOVERNING SYSTEMS

2.6.1 Modern Practice

Control systems used for the speed governing of gas turbines have become highly involved in electronic circuitry. Electromechanical fuel value control has largely replaced methods based on hydraulic control. The reliability of electronic and electrical devices has improved to such a level that they are generally preferred to hydraulic and mechanical devices, where their use is appropriate.

Most computer programs used for dynamic studies of power systems are capable of representing control systems and machinery dynamics to a reasonably high level of detail. Manufacturers of

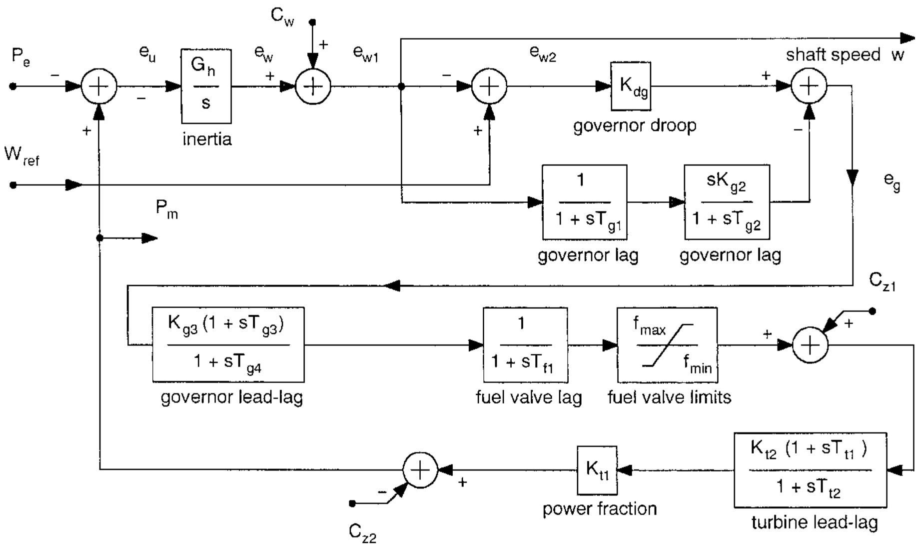

Figure 2.16 Control system for the speed governing of a single-shaft gas turbine.

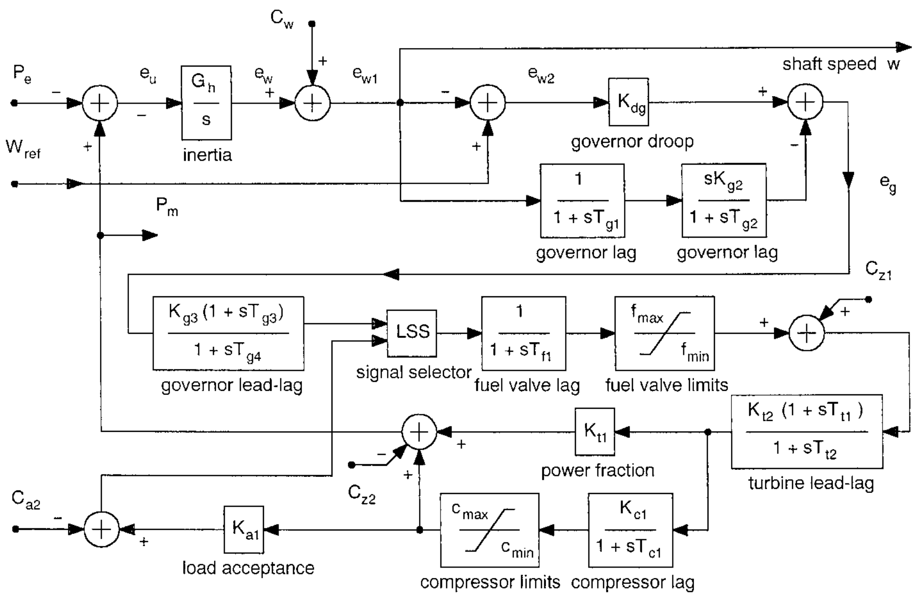

Figure 2.17 Control system for the speed governing of a two-shaft gas turbine.

gas turbines are normally able to provide detailed mathematical models of the machines and their control systems.

The modelling of the complete gas turbine including its control system and its interaction with the driven generator can be divided into several main functions. See Figures 2.16 and 2.17. Figure 2.16 represents a single-shaft gas turbine whilst Figure 2.17 represents a two-shaft machine.

The main functions are:-

- Summation of electrical and mechanical power.
- Acceleration of the rotating mass.
- Speed error sensing circuit to compare the shaft speed with a set or reference value.
- A power amplifier to amplify the error signal and to provide sufficient power to supply the fuel valve actuator.
- Fuel value limits and dynamics.
- Division of power between the power turbine and the compressor turbine.

Often the data to be used in a computer program are provided in actual physical units based on the SI or English thermodynamic systems of measurement. Most programs require the data in a per unit format. Care needs to be taken in converting the data into a suitable per unit format, especially the constants, scaling factors and controller limits. Figures 2.16 and 2.17 have therefore been drawn using per unit quantities.

2.6.1.1 Summation of electrical and mechanical power

The electrical power P_e input comes from the generator equations, which are usually presented in their two-axis form. This power is the power demand at the shaft coupling of the generator. This is derived from the transient or sub-transient equations of the generator, as described in sub-section 3.4. The choice depends upon the mathematical model used for the generator. For studies using practical data that are subject to tolerances of typically $\pm 15\%$, and often approximations, the differences in the results obtained from a sub-transient or a transient model are small enough to ignore.

The mechanical output power P_m is the net power produced by the turbines of the gas turbine. This is the total power converted to mechanical power less the amount consumed by the compressor. In some models factors are given that show the proportion of power consumed by the compressor to that delivered to the power output turbine, as shown in Figure 2.17. The sum of two factors equals unity.

2.6.1.2 Acceleration of the rotating mass

The rotating mass considered in this part of the model is the total of the masses that form parts of the power turbine, its couplings, the gearbox rotating elements and the rotor of the generator (complete with its attachments such as the main exciter). It is customary to convert all the rotating polar moments of inertia into their 'inertia constants' and to use their total value in the model. Usually the turbine manufacturer will be able to advise the total polar inertia of the turbine plus the generator. However, the units used may be given in for example, SI (kgm^2), TM (kgfm^2) or English (Ibft^2) units. The TM system of units is commonly used in Europe, especially in Germany although it is being superseded by the SI system. A discussion of this aspect can be found in Chapter 1, Table 22 of Reference 8. If the polar moment of inertia is given in TM units of kgfm^2 then the equivalent quantity in SI units is 0.25 kgm^2, due to a fundamental difference in the definition of the radius of gyration. A possible source of error by a factor of four could result from simply ignoring the subscript 'f' in kgfm^2 and assuming it is the same as kgm^2.

The 'inertia constant H' is a constant used in electrical engineering to relate the actual moment of inertia of mechanical rotating components to a base of electrical volt-amperes. It was developed specifically for use in solving differential equations that describe the transient speed changes of generator shafts. Subsequently it has been used more widely in motor dynamic analysis. Two early references to the definitions of inertia constants are a report by Evans in 1937 (Reference 9), and a paper by Wagner and Evans in 1928 (Reference 10). The inertia constant H is defined as the energy stored in the rotating mass divided by the volt-ampere rating of the generator (or motor), which gives.

$$H = \frac{\text{kilo-joules}}{\text{kVA}} \quad \text{or} \quad \frac{\text{kWsec}}{\text{kVA}}$$
$$= \frac{2J{\omega_o}^2}{Sp^2} \text{ seconds}$$

where J is the polar moment of inertia
ω_o is the synchronous speed
S is the VA rating of the machine
p is the number of poles of the machine

In English units,

$$H = \frac{0.231JN^2 \times 10^{-6}}{S} \text{ seconds}$$

with J in Lbft^2
N in revs/min
S in kVA

In SI units,

$$H = \frac{J\pi^2N^2 \times 10^{-3}}{1800S} \text{ seconds}$$

with J in kgm^2
N in revs/min
S in kVA

It should be noted that H is a function of the synchronous speed of the machine. If the speed should vary over a wide range then the variation of H with speed should be included in the mathematical simulation. For small excursions in speed about the synchronous speed, the error in using a constant value of H is negligible. This point is discussed in Reference 11.

2.6.1.3 Speed error sensing circuit

The output from the inertia block is the speed change e_ω due to integration of the mismatch in power between P_e and P_m.

The governor responds to the actual speed of the shaft and so the speed change needs to be added to the 1.0 pu base speed C_ω. The actual shaft speed is compared to the reference or set-point speed resulting in the error $e_{\omega 2}$.

2.6.1.4 Power amplifier

Power amplification is necessary in order to develop sufficient power to drive the fuel value open or closed. The amplifier incorporates,

- The droop constant K_{d1}.
- The lag term time constants T_{g1} and T_{g2} which are inherently present in the electronic circuits.
- The derivative damping gain K_{g2} which is often made adjustable.

2.6.1.5 Governor compensation

In order to improve the speed of response a lag-lead compensation circuit is employed in some governor control systems. It contains a gain term K_{g3}, a lag time constant T_{g4} and a lead time constant T_{g3}. If data are not available for these they may be assumed to be $K_{g3} = 1.0$ and $T_{g3} = T_{g4} = 0$.

2.6.1.6 Fuel valve mechanism lag

The fuel valve actuator and its mechanism may have sufficient inductance or inertia to introduce a perceptible lag in the valve stem response to its input signal. The equivalent time constant is T_{f1}.

2.6.1.7 Fuel valve limits

The fuel valve naturally has an upper and lower physical limit of the 'hard' type, i.e. a limit that is suddenly reached by the moving part. (A 'soft' limit is one in which the moving part reaches a region of increasing resistance before it eventually comes to rest. An electrical analogy would be magnetic saturation in an exciter, see sub-section 4.2.) The two hard limits are $f_{\min}$ and $f_{\max}$ where $f_{\min}$ is usually set at zero. Occasionally $f_{\min}$ has a negative value to artificially account for the no-load turbine power needed to drive the compressor. Hence at no load on the gas-turbine coupling the valve would be represented as having its position set to zero, whereas in practice it would open to about 15% of its travel.

Some fuel valves are driven by constant speed servomechanisms such as stepper motors. When they move the stem from one position to another the initial acceleration to constant speed is rapid, and likewise when the final position is reached. Feedback is applied in the valve controller to accurately relate the stem position to the magnitude of the control signal. Often this type of device is not modelled in computer programs, and so some form of approximation should be used to account for the lag in time between the receipt of the signal and the valve stem reaching its correct position. The constant speed motion of the valve actuator is also called 'slewing' and the 'slewing rate' is the measure of the rate of change of position during the constant speed motion.

An exponential approximation of slewing is now considered. Assume that the valve can move from its zero position to its 100% position in T_{100} seconds, at a constant rate, when a step input signal is applied at $t = 0$ seconds. Assume that an equivalent exponential lag term responds to the same step input over the same period of T_{100} seconds. Figure 2.18 shows the two responses referred to a common base of time. A good 'measure of fit' can be made by choosing the time constant T_{fa} such that the area represented by the lower part (A) equals that represented by the upper area (B). This is determined by equating these two areas. The areas are found by integration. Area (A) is found by

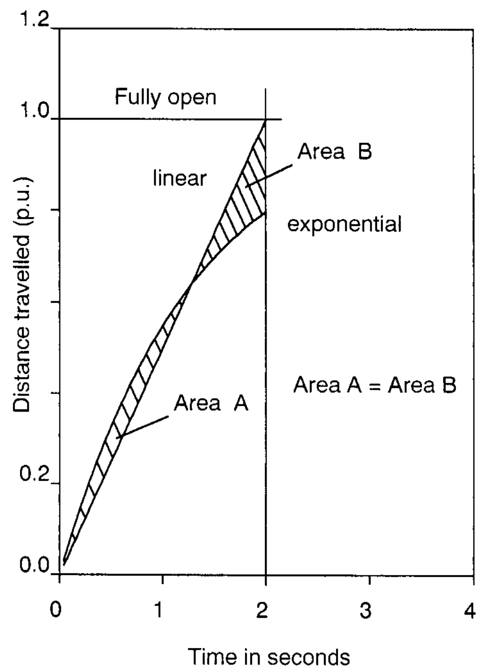

Figure 2.18 Simulation of slewing of the fuel valve by using an exponential approximation.

integrating between $t_1 = 0$ and $t_2 = T_e$, whilst area (B) is found by integrating between $t_1 = T_e$ and $t_2 = T_{100}$. If R is the slewing rate in per unit movement per second, then the solution for the best measure of fit is,

$$\frac{1}{2R} = (1 - e^{-f})T_{fa} \text{ seconds}$$

Where,

$$f = \frac{-1}{RT_{fa}}$$

Hence a unique value of T_{fa} can be found for each value of slewing rate R. The ratio of $1.0/T_{fa}$ to R that satisfies the above equation for all non-zero R is,

$$\frac{1}{RT_{fa}} = 1.5932, \text{ which may be rounded up to } 1.6$$

If for example the slewing rate is 3 per-unit travel/second then $1.0/R = 0.333$ and $T_{fa} = 0.333/1.6 = 0.208$ seconds.

If this approximation is made then an additional lag term should be inserted in the denominator of the 'fuel valve lag' block described in sub section 2.6.1.7 and the hard limits simply applied to the output of the block.

2.6.1.8 Combustion and turbine dynamics

After the fuel valve moves from one position to another the flow rate of the fuel delivered to the combustors changes, but a delay due to the inertia of the fuel occurs. The fuel enters the combustor and burns along its length at a finite burning rate. Completion of the combustion takes time and adds a further delay to the energy conversion process. A finite time is required for the burnt gas to pass through the power turbine and transfer part of its energy to the turbine. The 'turbine lead-lag' block approximates these conversion processes. The number of lead and lag terms varies from one gas turbine type to another.

In a single-shaft gas turbine the turbine lead-lag block represents the amount of energy or power that is convertible to mechanical power for accelerating the output shaft masses and to balance the electrical power demand.

In a two-shaft gas turbine the situation is slightly more complicated. Part of the convertible power is required to drive the separate compressor. The compressor has its own dynamic response and is shown as a parallel branch in Figure 2.17. This illustrates the fact that the resulting mechanical

Table 2.7. Typical data for simulating gas-turbine control systems

Parameter	Low	Values Typical	High
H note i)	1.2	1.5	2.0
G_h	0.25	0.33	0.42
C_w	1.0	1.0	1.0
K_{g1}	1.0	1.0	1.0
T_{g1}	0.05	0.01	0.015
K_{g2} note ii)	10.0	20.0	40.0
T_{g2}	0.02	0.04	0.15
K_{dg}	0.02	0.04	0.08
T_{g3}	0.25	0.50	0.75
T_{g4}	1.0	1.50	1.75
T_{f1}	0.01	0.02	0.05
f_{max}	1.2	1.35	1.5
f_{min}	−0.2	−0.15	0
K_{t1}	1.0	1.0	1.0
T_{t1}	0.3	0.6	0.9
T_{t2}	1.2	1.4	2.0
K_{t2}	0.4 (1.0)	0.5 (1.0)	06 (1.0)
K_{c1}	0.4 (0)	0.5 (0)	0.6 (0)
T_{c1}	Tt_2 (0)	Tt_2 (0)	Tt_2 (0)
C_{max}	1.1 (0)	1.2 (0)	1.3 (0)
C_{min}	0	0	0
K_{a1}	2.0 (0)	2.5 (0)	3.0 (0)
C_{a2}	0.38 (0)	0.4 (0)	0.42 (0)
C_{z1}	0.38 (0)	0.4 (0)	0.42 (0)
C_{z2}	0.48 (0)	0.5 (0)	0.52 (0)

Notes:

i) $G_h = \dfrac{1.0}{2H}$

ii) $K_{g1} x K_{g2} \simeq$ a constant value

iii) Data in brackets ( ) apply to the single-shaft mathematical model.

power has a part that is delayed when a disturbance occurs. It is generally considered that two-shaft gas turbines have a slower response characteristic to disturbances in electrical power, and that this gives rise to greater excursion in shaft speed. The delay due to the compressor being on a separate shaft accounts for this inferior performance.

With a two-shaft system the compressor is free to accelerate since it is not constrained by the heavy mass of the driven generator. In order to avoid excessive acceleration of the compressor a suitable signal is derived and passed through a safety control loop, often called the load schedule or acceleration control. The signal is compared with the output of the governor power amplifier and the least of these two signals is selected and sent to the fuel valve. The 'least signal selector' block carries out this comparison, as shown in Figure 2.17. Where the compressor loop is given with a slewing block, with upper and lower limits, the approximation of the slewing may be considered in the same manner as for the fuel valve actuator and its limits.

2.6.2 Typical Parameter Values for Speed Governing Systems

Table 2.7 shows typical per-unit values for the gains, limits and time constants used in the speed governing control systems for gas turbines having ratings up to approximately 25 MW.

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