# 11
# Fault Calculations and Stability Studies

## 11.1 INTRODUCTION

When a short circuit occurs in a power supply, larger than normal currents are caused to flow into the short circuit. The magnitude of the short-circuit current is determined by the impedance of AC systems, or the resistance of DC systems, that exists between the short circuit and the sources of voltage. That impedance or the resistance will be called the 'source impedance' in the discussions that follow. In DC systems the source impedance is often the series addition of the supply cable resistance, the rectifier or thyristor internal resistance and any other resistance that may be connected in the circuit. The calculation of the short-circuit current in a DC circuit is therefore a reasonably simple process once the resistance data are known.

For AC systems the calculation of the short-circuit current is more complicated, particularly when generators and motors are both present in the system. The simplest calculations occur when the source of voltage can be assumed to be of constant magnitude during the fault duration. In AC systems the source impedance will be the addition of the cable impedance, busbar impedance, transformer internal impedance, the appropriate internal impedance of the generator, the appropriate internal impedance of the motors in system and the impedance of the overhead transmission lines.

The sub-sections that now follow will begin with the simplest situations and end with the more complicated.

## 11.2 CONSTANT VOLTAGE SOURCE – HIGH VOLTAGE

A constant voltage source is one in which the voltage that drives the short-circuit current maintains a constant magnitude before, during and after the fault occurs. This is usually considered to be the case when the source power capacity is very much greater than the normal power rating of the circuit in which the fault has occurred. An example of such a situation is shown in Figure 11.1 for an onshore, high voltage transmission network.

The cables and busbars connecting the transformers to the switchboards are very short in comparison with the length of the transmission lines and the transformer reactances and so their impedances may be ignored. Consider the fault being applied to the busbars of the T4 switchboard. The fault circuit for the switching configuration shown is through T2 and T4.

The simple series circuit for this configuration is shown in Figure 11.2.

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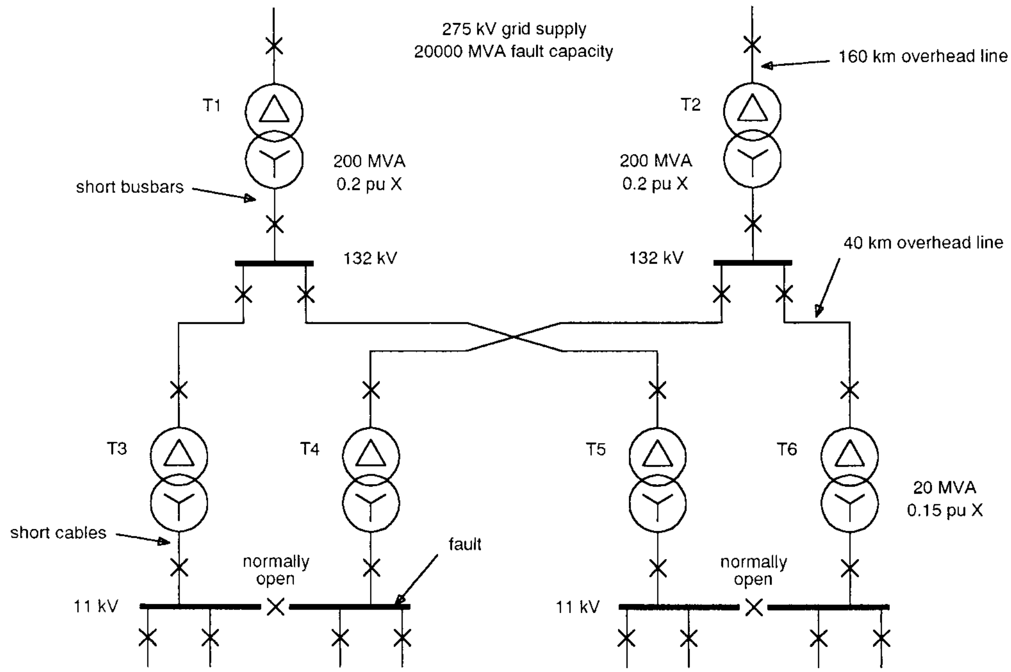


**Figure 11.1** One-line diagram of faulted high voltage system.

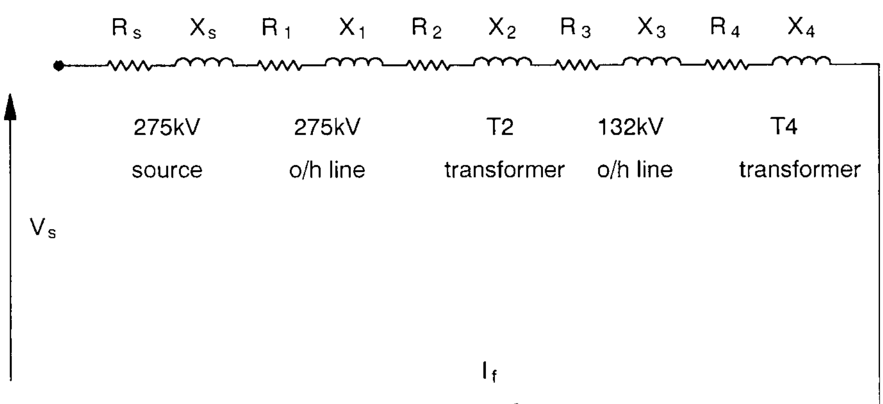


**Figure 11.2** Equivalent circuit of faulted high voltage system.

The base MVA rating chosen for this system is 100 MVA. The impedance data is given in Table 11.1.

Hence the total series per unit impedance is $R = 0.069$ pu, $X = 1.021$ pu. The short-circuit current is therefore:

$$I_f = \frac{V}{R + jX} = \frac{1.0}{0.069 + jl.02.1} = 0.9775 \text{ pu}$$

Therefore the fault MVA is $0.9775 \times 100 = 97.75$ MVA.

**Table 11.1.** Impedance data values

| Item | R (ohms) | X (ohms) | R (pu) | X (pu) |
|---|---|---|---|---|
| | | | at 100 MVA | |
| Source fault impedance | | | | 0.005 |
| 275 KV O/H line | 5.47 | 53.91 | 0.0072 | 0.0713 |
| T2 | – | – | 0.0010 | 0.10 |
| 132 KV O/H Line | 6.235 | 16.495 | 0.0358 | 0.0947 |
| T4 | – | – | 0.025 | 0.75 |
| TOTAL | | | 0.069 | 1.021 |

## Observations:

- It can be seen that for most of the circuit items their X-to-R ratio is more than 10. Hence their resistance may be neglected for fault calculations but this only applies to high voltage systems, e.g. above 3300 volts. The X-to-R ratio of LV components is usually low, e.g. between 1 and 3.
- For different switching configurations the equivalent circuit will be different, and so appropriate additional calculations must be made to find the worst-case situation.

## 11.3 CONSTANT VOLTAGE SOURCE – LOW VOLTAGE

Consider a LV motor control centre fed from a HV/LV transformer as shown in Figure 11.3.

In this case the cables and busbars are not ignored, as will be demonstrated in the calculations. The base MVA is assumed to be 100 MVA in this case, and the equivalent circuit is given in Figure 11.4.

The impedance data is given in Table 11.2 from which it may be seen that the total series per unit impedance is $R = 0.6092$ pu and $X = 3.9614$ pu.

The short-circuit current is therefore:

$$I_f = \frac{V}{R + jX} = \frac{1.0}{0.6092 + j3.9614} = 0.038 + j0.247 \text{ pu.}$$

## Observations:

a) It can be seen that the X-to-R ratio for the LV items is less than 10 and that the total impedance has an X-to-R ratio of 6.5. Since R is relatively large it cannot be ignored in the LV circuits.

b) When designing a new installation in the early stages, it is acceptable to ignore the impedance of the LV busbars and cables. However, as the design becomes more defined it may occur that,

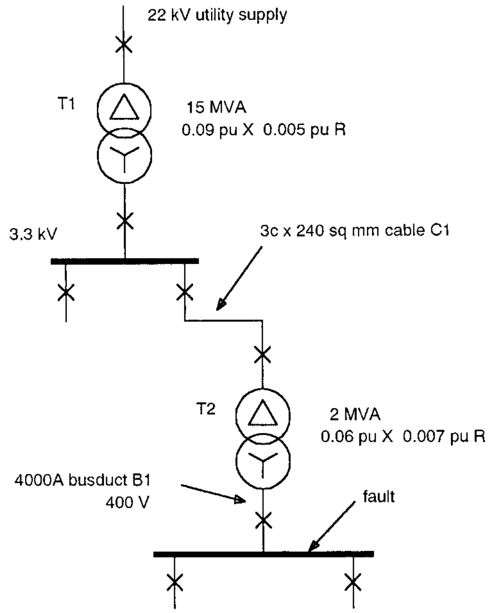


**Figure 11.3** One-line diagram of faulted low voltage system.

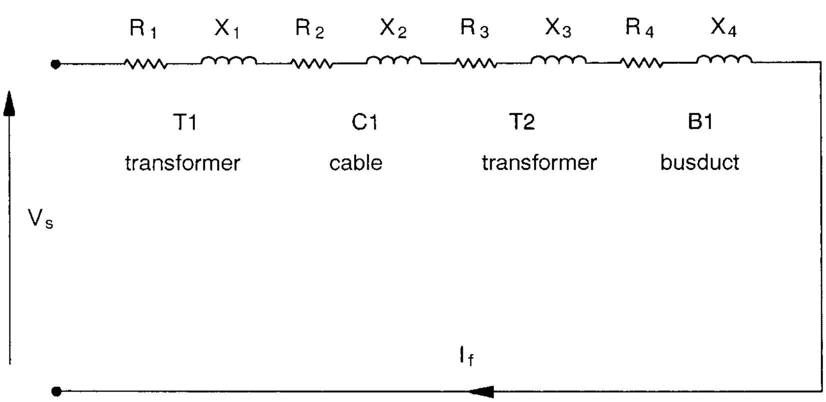


**Figure 11.4** Equivalent circuit of faulted low voltage system.

**Table 11.2.** Impedance data values

| Item | R1 | X1 | R2 | X2 | R3 | X3 | R4 | X4 |
|---|---|---|---|---|---|---|---|---|
| T1 | | 0.0333*pu | 0.6*pu | | | | | |
| 3c-240 $mm^2$ | | | 0.01976 | 0.0146 | | | | |
| Cable C1 | | | 0.1815*pu | 0.1343*pu | | | | |
| T2 | | | | | 0.35*pu | 3*pu | | |
| Busbars | | | | | | | 0.000086 | 0.00044 |
| B1 | | | | | | | 0.0444*pu | 0.2273*pu |

*per unit at 100 MVA base.

for economic or technical reasons, a choice of MCC ratings may become critically dependent on fault ratings. In such a situation, the resistances and reactances of the LV components should be used in the fault calculations.

c) When designing modifications or uprated systems it is essential that both the resistances and reactances of the LV components are used in the fault calculations, otherwise the existing equipment might be exposed to fault currents higher than expected and a dangerous situation could ensue. This is particularly the case when the HV system is being uprated, e.g. by adding more generators or transformers, and it is too easy to ignore the effects on the LV part of the system. Uprated system design can be more difficult in practice than new system design.

d) Fault currents can be contributed by LV generators and LV motors and so care must be taken to allow for this possibility. This subject will be discussed in detail in later pages.

## 11.4 NON-CONSTANT VOLTAGE SOURCES – ALL VOLTAGE LEVELS

So far it has been assumed that the source impedance and the source voltage remain constant during a fault situation. This is the case for power systems that do not contain rotating machines, i.e. generators and motors. Motors, and especially generators, exhibit peculiar reactance and voltage characteristics during fault situations and these are generally grouped into three types:-

- Sub-transient effects.
- Transient effects.
- Steady state (or synchronous) effects.

For installations that have self-contained power generation plants, e.g. offshore platforms and onshore gathering stations, proper allowance must be made for the presence of generators and motors, especially at the generator switchboard. This subject is a complicated one and so it is now necessary to give due attention to the design and dynamic characteristics of firstly the generators and secondly the motors.

A synchronous generator (and a synchronous motor) can be represented by many inductances and reactances to account for transformer-type induction, rotational induction, mutual coupling between windings, leakage and self-induction, magnetising and excitation induction and the effects of the pole-face damper windings. Extremely complex equivalent circuits have been developed for synchronous machines, see References 1 and 2 as examples.

For most hand calculations in power systems, only three of the generator reactances are of particular interest:

- The sub-transient reactance $X''_d$.
- The transient reactance $X'_d$.
- The synchronous reactance $X_{sd}$.

The suffix '$d$' relates to the 'direct axis' values, i.e. those that can be represented along the pole axis of the excitation winding. Occasionally, the 'quadrature axis' reactances are encountered and these are denoted by the suffix '$q$'. See Chapter 3 for a further discussion of the '$d$' and '$q$' axis parameters.

The quadrature axis reactances are those that can be represented on an axis at right angles to the pole or direct axis. These reactances do not normally appear in the hand calculation of fault currents.

When generators are being considered it is usually necessary to know the form and magnitude of fault currents when a fault occurs close to the main terminals of the generators. Several aspects of the fault current are of interest:

- The peak value of the fault current during the first cycle of instantaneous current. This value determines the 'peak asymmetrical' duty of the switchgear connected to the generator. This value is determined by the sub-transient reactance.
- The rms value of the symmetrical component of the fault current during the first cycle. This is the first result obtained from the calculation process and from this is then calculated, or estimated, the peak value mentioned above (due to the phenomenon called 'current doubling'). This value is determined by the sub-transient reactance.
- The rms value of the symmetrical component of the fault current several cycles after the fault occurs. This value determines the 'symmetrical breaking' duty of the switchgear connected to the generator. This value is determined by the transient reactance
- Occasionally a critical situation occurs in which the alternating fault current does not reach a zero value, or becomes negative, until several cycles have passed, see sub-section 7.2.10 and Figure 7.1. This is very important because the basis of interrupting fault current in a circuit breaker is highly dependent on current zeros and crossing points occurring naturally in the circuit. When a current zero occurs, the arc-gap has a short time to become de-ionised and the dielectric strength of the insulating medium in the gap to be restored. While arcing occurs, these two processes cannot take place and energy is released in the arc. If this process is overly delayed then too much energy will be released in the arc and damage due to overheating can occur.
- The switchboard must be specified to withstand this peculiar situation and it is the task of the engineer to investigate the possibility of it taking place, see sub-section 7.2.11. The controlling factor that determines whether or not it takes place is the X-to-R ratio of the source impedance of the generator and its connecting components (cables, busbars and transformers) up to the switchboard. If X is very much larger than R then the phenomenon described may occur. The time constant $T_a$ of the generator influences the time taken for a zero-crossing to occur.

## 11.5 CALCULATION OF FAULT CURRENT DUE TO FAULTS AT THE TERMINALS OF A GENERATOR

### 11.5.1 Pre-Fault or Initial Conditions

Since the peak value of the fault current reduces in time due to the effects of the sub-transient and transient reactances, it is necessary to establish a driving voltage suitable for each part of the process and calculation. The concept used is one which assigns an emf 'behind' an appropriate impedance of, in the case of generators, an appropriate reactance.

This is shown diagrammatically in Figure 11.5.

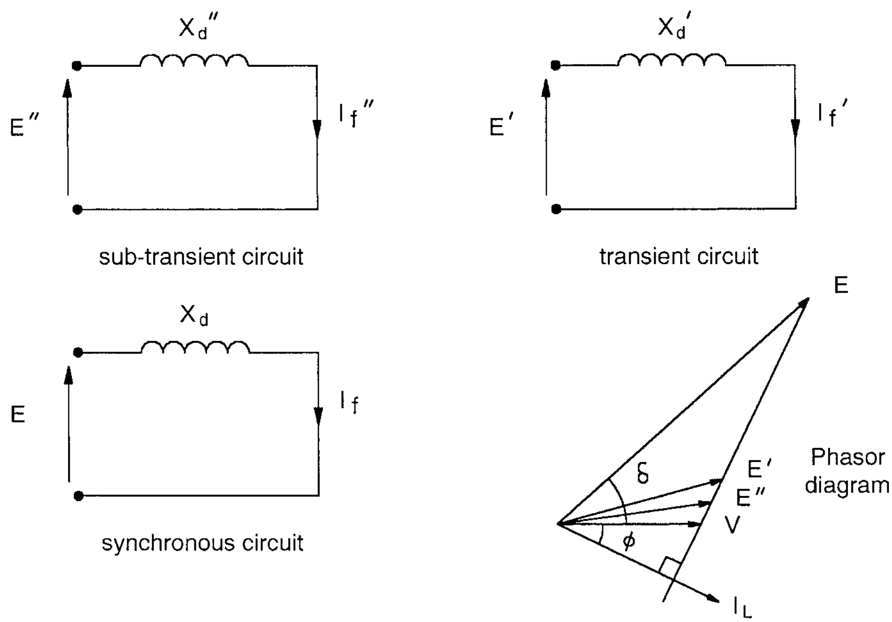


**Figure 11.5** Equivalent circuits and the phasor diagram of a faulted synchronous generator.

$E''$ denotes the sub-transient emf behind the sub-transient reactance $X_d''$. Used to calculate the initial peak asymmetrical and symmetrical fault currents.

$E'$ denotes the transient emf behind the transient reactance $X_d'$. Used to calculate the fault-breaking currents several cycles after the fault occurs.

$E$ shows the synchronous emf behind the synchronous reactance $X_{sd}$. Used for calculating the steady state fault current, which will then be fully symmetrical, since all the sub-transient and transient effects will have decayed to zero. The emf E will be the ceiling voltage of the exciter since the AVR will have seen a severe depression in terminal voltage and will have forced the exciter to give its maximum possible output. See also sub-sections 7.2.8 and 12.2.2.1.

The next step is to determine each of the emfs $E''$, $E'$ and $E$ that apply to the circuit before the fault occurs. In order to do this it is necessary to know the pre-fault load conditions of the generator. It is usually the case to assume that the generator is running at its rated output just before the fault occurs.

The phasor diagram for full load conditions is shown in Figure 11.5.

Where Ø is the power factor angle, $V$ is the terminal voltage $= 1.0$ pu and $I_L$ is the terminal rated current.

The same method that is described for transformers in sub-section 6.3 is used to find $E''$, $E'$ and $E$. Simply replace $X_{se}$ in the equations by $X_d''$, $X_d'$ or $X_{sd}$ as appropriate and assume $R$ and $R_{se}$ to be equal to zero. Now that the driving voltage has been calculated, it is a simple matter to calculate the symmetrical fault currents.

## 11.5.2 Calculation of Fault Current – rms Symmetrical Values

From sub-section 11.5.1 the emf $E$ ($E''$, $E'$ or $E$) and appropriate reactance $X$ ($X''_d$, $X'_d$ or $X_{sd}$) are known. Hence the symmetrical fault current $I_f$ may be easily calculated:

$$I_f = \frac{E}{X} \text{ per unit}$$

For example:

A 6600 V, 4.13 MVA generator has $X''_d = 15.5\%$, $X'_d = 23.5\%$ and $X_{sd} = 205\%$

At full load with a power factor of 0.8 lagging the corresponding emfs are therefore:

$$E'' = 1.1 \text{ pu}, E' = 1.156 \text{ pu and E } = 2.77 \text{ pu}$$

The rms fault currents are therefore:

$$I''_f = \frac{1.1}{0.155} = 7.097 \text{ pu (2564 amps)}$$

$$I'_f = \frac{1.156}{0235} = 4.919 \text{ pu (1776 amps)}$$

$$I_f = \frac{2.77}{2.05} = 1.351 \text{ pu (488 amps)}$$

A typical oil industry power system can be approximated as shown in Figure 11.6. The majority of oil industry systems are of the radial distribution type, with feeders radiating away from a

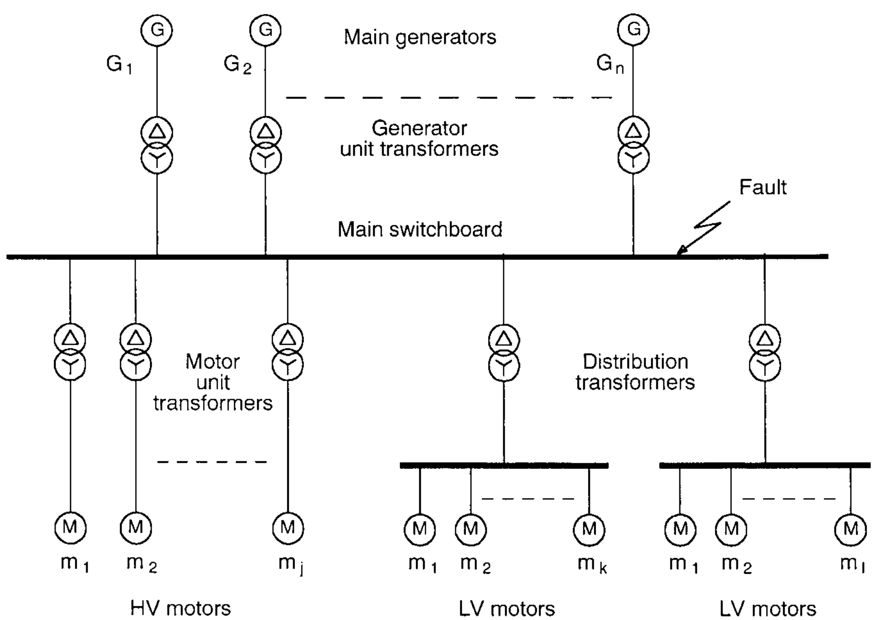


**Figure 11.6** One-line diagram of an equivalent power system that has its own dedicated generators.

centralised main switchboard. Mesh or looped systems such as those found in utility or countrywide networks are rarely used. Occasionally a simple form of a ‘ring-main’ may be used between adjacent plants to improve power equipment utilisation and availability. Radial systems have the benefit that the calculation of load flows and fault currents are relatively easy to carry out by hand or with the use of a simple digital computer program.

Estimating load flows and fault currents are two of the earliest tasks that are necessary to undertake when designing a new plant. Such estimates are carried out so that budget costs and physical dimensions can be established at an early stage of a project.

The sub-transient rms and peak fault currents are needed so that the worst-case maximum fault making duty of the main switchgear can be assessed. The decaying components of the fault current are also of interest in assessing the fault breaking duty of the switchgear at the times that correspond to the circuit breaker clearing times e.g. 0.08 to 0.2 seconds. The long-term steady state fault current is of little concern, unless the system is fed from a utility grid instead of close-up generators. The long-term decrement of the generator current that feeds into a major fault is mainly of interest in setting the protective relays in the generator circuit breaker.

The following discussion and worked example for an LNG plant show how to carry out simple but reasonably accurate estimates of the sub-transient fault current and its decay in the first few cycles. Following is a discussion on how to assess the fault breaking current.

If the total generating capacity exceeds about 120 MVA then the generators should be connected to the main switchboard through unit transformers. The main switchboard voltage should be about 33 kV. Each of the various groups of generators, transformers and motors can be represented by a single equivalent unit, using the methods given below.

### *11.5.2.1 The load*

A preliminary estimate of the load is $S_l$, which may be assumed to consist of a certain amount of motor load and some static load. The motor load can be assumed to be connected to the main high voltage switchboard and at lower voltage switchboards. Let the high voltage motor load be $S_{hm}$ and the lower voltage motor load be $S_{lm}$. Assume the static load $S_{ls}$ to be connected to the lower voltage switchboards. Typical power factors for these loads are 0.87, 0.85 and 0.97 respectively. The total active and reactive power estimates are,

$$P_{\text{load}} = 0.87S_{hm} + 0.85S_{lm} + 0.97S_{ls}$$

$$Q_{\text{load}} = (0.493S_{hm} + 0.527S_{lm} + 0.243S_{ls})\,1.015$$

$$S_{\text{load}} = P_{\text{load}} + jQ_{\text{load}}$$

Where the factor 1.015 is an allowance for the $I^2X$ reactive power losses in the transformers.

### *11.5.2.2 Generators and their transformers*

For a new plant it may be assumed that all the generators that are connected to the main switchboard have the same rating and parameters, i.e. identical machines. Similarly their transformers may be assumed to be identical. The total capacity of the generators $S_{\text{gen}}$ must be greater than the load $S_{\text{load}}$.

Therefore,

$$S_{\text{gen}} = \sum_{i=1}^{n} S_{gi} = K_g S_{\text{load}}$$

Where $S_{gi}$ is the rating of the $i_{th}$ generator and $K_g$ is a marginal factor $> 1.0$.

The generator unit transformers have a total capacity $S_{tg}$ slightly higher than $S_{\text{gen}}$,

$$S_{tg} = K_{tg} S_{\text{gen}}$$

Where $K_{tg}$ is a marginal factor $> 1.0$.

Assume the leakage reactance $X_{tg}$ of each generator unit transformer to be 0.08 per unit, and ignore the resistance.

#### *11.5.2.3 High voltage motors and their transformers*

The high voltage motor unit transformers have a total capacity $S_{tm}$ slightly higher than that of the motors $S_{hm}$,

$$S_{tm} = K_{tm} S_{hm}$$

Where $K_{tm}$ is a marginal factor $> 1.0$.

Assume the leakage reactance of each motor unit transformer to be 0.06 per unit, and ignore the resistance.

#### *11.5.2.4 Lower voltage distribution transformers*

Assume that the lower voltage switchboards are each fed by two transformers and that the bus-section circuit breaker is normally open. In this configuration each transformer carries half the load on its switchboard. Therefore the total capacity of the distribution transformers $S_{td}$ is at least twice that of the load,

$$S_{td} = 2K_{td}(S_{lm} + S_{ls})$$

Where $K_{td}$ is a marginal factor to account for future increase in load, assume $K_{td}$ to be 1.3.

Assume the leakage reactance $X_{td}$ of each transformer to be 0.055 per unit, and ignore the resistance.

#### *11.5.2.5 Equivalent transformer*

Suppose a main switchboard feeds load through transformers of different ratings and impedances. For the purpose of estimating fault current at an early stage in a project it is reasonable to combine all the distribution transformers into one equivalent transformer. The equivalent rating $S_{te}$ of all the transformers is simply the arithmetic sum of their individual ratings $S_{ti}$.

$$S_{te} = \sum_{i=1}^{n} S_{ti}$$

The equivalent impedance $Z_{te}$ of the transformers may be found from,

$$Z_{te} = \frac{S_{te}}{\displaystyle\sum_{i=1}^{n} \frac{S_{ti}}{Z_{ti}}}$$

#### *11.5.2.6 Worked example*

Three transformers feed a load from a main switchboard. Their ratings and impedances are,

$$\begin{aligned}
\text{Transformer No. 1} \qquad S_{ti} &= 10\ \text{MVA} \\
Z_{ti} &= 0.008 + j0.09\ \text{pu} \\
\text{Transformer No. 2} \qquad S_{t2} &= 15\ \text{MVA} \\
Z_{t2} &= 0.009 + j0.1\ \text{pu} \\
\text{Transformer No. 3} \qquad S_{t3} &= 25\ \text{MVA} \\
Z_{t3} &= 0.01 + j0.12\ \text{pu} \\
\text{The total capacity} \qquad S_{te} &= 10.0 + 15.0 + 25.0 \\
&= 50.0\ \text{MVA}
\end{aligned}$$

$$\sum_{i=1}^{n} \frac{S_{ti}}{Z_{ti}} = \frac{10.0}{0.008 + j0.09} + \frac{15.0}{0.009 + j0 + 1} + \frac{25.0}{0.01 + j0.12}$$

$$= 40.432 - j465.93$$

$$Z_{te} = \frac{50.0}{40.432 - j465.93} = 0.0092 + j0.1065\ \text{pu}$$

## 11.6 CALCULATE THE SUB-TRANSIENT SYMMETRICAL RMS FAULT CURRENT CONTRIBUTIONS

The method adopted below is based upon the principles set out in IEC60363 and IEC60909, both of which describe how to calculate sub-transient and transient fault currents, and are well suited to oil industry power systems. The method will use the per-unit system of parameters and variables. Choose the base MVA to be $S_{\text{base}}$.

It is customary to assume that all the generators are operating and that they are heavily loaded. In which case the emf $E''_g$ behind the sub-transient reactance $X''_d$ is about 5 to 10% above the rated terminal voltage, hence assume $E''_g$ is 1.1 pu. This emf drives the fault current around the circuit. In IEC60909 the elevation in driving emf, or voltage, is given in Table I as 'factor c' and discussed in Clause 6 therein.

The contribution of fault current $I''_g$ from the generators is,

$$
\begin{aligned}
I''_g &= \frac{E''_g}{\left(\dfrac{X''_d}{S_{\text{gen}}} + \dfrac{X_{tg}}{S_{tg}}\right) S_{\text{base}}} \text{ pu} \\
&= \frac{1.1}{\left(\dfrac{X''_d}{S_{\text{gen}}} + \dfrac{0.08}{K_{tg} S_{\text{gen}}}\right) S_{\text{base}}} \\
I''_g &= \frac{1.1 S_{\text{gen}}}{\left(X''_d + \dfrac{0.08}{K_{tg}}\right) S_{\text{base}}} \qquad (11.1)
\end{aligned}
$$

The contribution from the high voltage motors is found as follows.

It may be assumed that the average ratio of starting current to rated current $(I_s/I_n)$ of the motor is,

$$\frac{I_s}{I_n} = 6.0 \text{ pu for high voltage motors}$$

Consequently the sub-transient impedance $Z''_{hm}$ of the motors is,

$$Z''_{hm} = \frac{1.0}{6.0} = 0.167 \text{ pu (at } S_{hm})$$

For typical high voltage motors the starting power factor is between 0.15 and 0.2 lagging, hence assume 0.2. The sub-transient impedance becomes,

$$Z''_{hm} = 0.033 + j0.164 \text{ pu}$$

The equivalent impedance $Z_{tm}$ of the motor unit transformers is 0.06 pu at a total capacity of $S_{tm}$.

$$Z_{tm} = 0.0 + j0.06 \text{ pu}$$

The emf $E''_{hm}$ behind the motor sub-transient impedance is the air-gap emf and will in practice be slightly less than 1.0 pu, hence it is reasonable and conservative to assume it to be 1.0 pu.

The contribution of fault current $I''_{hm}$ from the main switchboard motors is

$$
\begin{aligned}
I''_{hm} &= \frac{E''_{hm}}{\left(\dfrac{Z''_{hm}}{S_{hm}} + \dfrac{Z_{tm}}{K_{tm} S_{hm}}\right) S_{\text{base}}} \text{ pu} \\
&= \frac{1.0 S_{hm}}{\left(0.033 + j0.164 + \dfrac{j0.06}{K_{tm}}\right) S_{\text{base}}} \qquad (11.2)
\end{aligned}
$$

The contribution from the lower voltage motors is found as follows.

The average ratio of starting current to rated current ($I_s/I_n$) for the lower voltage motors may be assumed to be,

$$\frac{I_s}{I_n} = 6.5 \text{ pu for lower voltage motors}$$

Their sub-transient impedance $Z''_{lm}$ is

$$Z''_{lm} = \frac{1.0}{6.5} = 0.153 \text{ pu(at } S_{lm})$$

Typical lower voltage motors have a starting power factor of between 0.25 and 0.35 lagging, hence assume 0.35.

The sub-transient impedance $Z''_{lm}$ of the motors becomes,

$$Z''_{lm} = 0.054 + j0.143 \text{ pu}$$

The equivalent impedance $Z_{td}$ of the distribution transformers can be found by the method in sub-section 11.5.2.5 or taken as,

$$Z_{td} = 0.0 + j0.055 \text{ pu(at } S_{td})$$

Again assume that the air-gap emf $E''_{lm}$ is 1.0 pu.

The contribution $I''_{lm}$ from the lower voltage motors is,

$$\begin{aligned} I''_{lm} &= \frac{E''_{lm}}{\left(\dfrac{Z''_{lm}}{S_{lm}} + \dfrac{Z_{td}}{2K_{td}(S_{lm} + S_{ls})}\right) S_{\text{base}}} \\ &= \frac{1.0 S_{lm}}{\left(0.054 + j0.143 + \dfrac{j0.06}{K_{td2}}\right) S_{\text{base}}} \end{aligned} \tag{11.3}$$

Where

$$K_{td2} = 2K_{td}\left(1.0 + \frac{S_{ls}}{S_{lm}}\right)$$

The total sub-transient symmetrical rms fault current $I''_{frms}$ is,

$$I''_{frms} = I''_g + I''_{hm} + I''_{lm} \tag{11.4}$$

### 11.6.1 Calculate the Sub-Transient Peak Fault Current Contributions

Many power system networks can be reduced to a simple series-connected circuit containing a resistance $R$ and an inductance $L$, for the purpose of calculating the transient fault current. Furthermore a

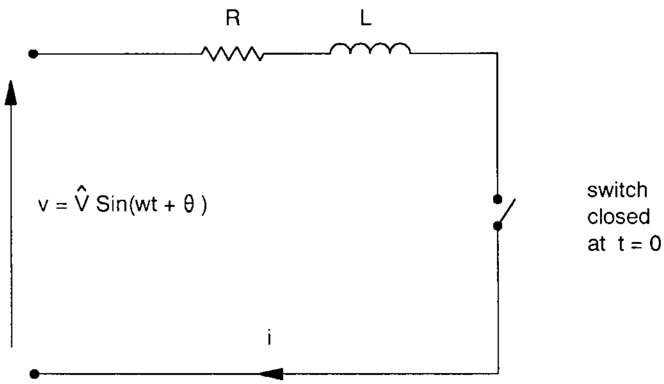


**Figure 11.7** Instantaneous current response in a series-connected R-L circuit that is fed by a sinusoidal voltage.

single-phase AC circuit can be used to represent a three-phase circuit in which a line-to-line-to-line short circuit occurs.

Figure 11.7 shows the single-phase circuit, which is supplied by a sinusoidal voltage $v$.

The differential equation for the current $i$ that responds to the applied voltage $v$ is,

$$\mathrm{Ri} + L\frac{di}{dt} = v = \hat{V}\sin(\omega t + \theta)$$

Where $\omega$ = the angular frequency in rad/sec
$\theta$ = the angular displacement of $v$ at $t = 0$
$t$ = the time in seconds
$\hat{V}$ = peak value of $V$ the rms applied voltage, i.e. $\sqrt{2}V$.

The complete solution of this equation can be found by several methods e.g. Laplace transforms, method of undetermined coefficients, see Reference 3. The solution for $i$ is,

$$i = \frac{\hat{V}}{Z}\left(-e^{\frac{-Rt}{L}}\sin(\theta - \phi) + \sin(\omega t + (\theta - \phi))\right) \tag{11.5}$$

where

$$Z = \sqrt{(R^2 + \omega^2 L^2)}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{X}{R}\right)$$

and

$$X = \omega L \text{ the inductive reactance.}$$

The exponential term has its maximum positive value when $\theta - \phi$ equals $-\pi/2$ radians. Therefore the maximum value occurs when $\theta = \phi - \pi/2$.

The oscillating term reaches its first maximum value when wt $+\theta-\phi$ equals $+\pi/2$, or,

$$t_{m1} = \frac{1}{\omega}\left(\frac{\pi}{2} - \theta + \phi\right) \text{ seconds}$$

There are two important cases to consider. Firstly when the resistance is much smaller than the inductive reactance and secondly when it is much greater, Tables 11.3 and 11.4 show the peak maximum and minimum values of the instantaneous current when the maximum value of V is 1.414 per unit,

Z = 1.0 per unit, and the ratio of X to R has different values over a wide range.

#### *11.6.1.1 Resistance smaller than inductive reactance*

This case often represents a circuit in which a circuit breaker or a contactor is subject to its most onerous duty, because 'current doubling' occurs.

The angle $\phi$ approaches $\pi/2$, and $\theta$ approach zero. The first maximum value then occurs at,

$$t_{\mathrm{m1x}} = \frac{1}{\omega}\left(\frac{\pi}{2} - 0 + \frac{\pi}{2}\right) = \frac{\pi}{\omega} \text{ seconds}$$

Which is half the periodic time of the sinusoidal forcing function $v$.

**Table 11.3.** Values of maximum and minimum currents for a 50 Hz power system

| X-to-R ratio | $I_1$ (pu) | $I_2$ (pu) | $I_3$ (pu) | $I_4$ (pu) | $I_5$ (pu) | $I_6$ (pu) |
|---|---|---|---|---|---|---|
| 500.0000 | 2.8196 | −.0177 | 2.8020 | −.0351 | 2.7847 | −.0523 |
| 100.0000 | 2.7845 | −.0864 | 2.7011 | −.1672 | 2.6227 | −.2432 |
| 50.0000 | 2.7418 | −.1680 | 2.5850 | −.3151 | 2.4467 | −.4449 |
| 30.0000 | 2.6863 | −.2697 | 2.4459 | −.4859 | 2.2509 | −.6612 |
| 25.0000 | 2.6593 | −.3176 | 2.3825 | −.5612 | 2.1672 | −.7507 |
| 20.0000 | 2.6196 | −.3861 | 2.2944 | −.6631 | 2.0570 | −.8655 |
| 16.0000 | 2.5714 | −.4661 | 2.1952 | −.7737 | 1.9413 | −.9816 |
| 14.0000 | 2.5378 | −.5197 | 2.1310 | −.8428 | 1.8716 | −1.0493 |
| 12.0000 | 2.4943 | −.5868 | 2.0533 | −.9237 | 1.7926 | −1.1235 |
| 10.0000 | 2.4355 | −.6729 | 1.9581 | −1.0182 | 1.7041 | −1.2028 |
| 8.0000 | 2.3520 | −.7864 | 1.8406 | −1.1275 | 1.6084 | −1.2834 |
| 6.0000 | 2.2246 | −.9402 | 1.6971 | −1.2474 | 1.5133 | −1.3556 |
| 5.0000 | 2.1327 | −1.0368 | 1.6172 | −1.3064 | 1.4719 | −1.3835 |
| 4.0000 | 2.0105 | −1.1475 | 1.5369 | −1.3585 | 1.4397 | −1.4026 |
| 3.0000 | 1.8444 | −1.2667 | 1.4664 | −1.3959 | 1.4206 | −1.4120 |
| 2.0000 | 1.6267 | −1.3710 | 1.4232 | −1.4123 | 1.4146 | −1.4141 |
| 1.0000 | 1.4341 | −1.4134 | 1.4143 | −1.4142 | 1.4142 | −1.4142 |
| .5000 | 1.4143 | −1.4142 | 1.4142 | −1.4142 | 1.4142 | −1.4142 |
| .2000 | 1.4142 | −1.4142 | 1.4142 | −1.4142 | 1.4142 | −1.4142 |
| .1000 | 1.4142 | −1.4142 | 1.4142 | −1.4142 | 1.4142 | −1.4142 |

**Table 11.4.** Values of time corresponding to the currents in Table 11.3

| X-to-R ratio | $T_1$ (sec) | $T_2$ (sec) | $T_3$ (sec) | $T_4$ (sec) | $T_5$ (sec) | $T_6$ (sec) |
|---|---|---|---|---|---|---|
| 500.0000 | .010000 | .02002 | .03000 | .04002 | .0500 | .0600 |
| 100.0000 | .010000 | .02006 | .03000 | .04006 | .0500 | .0601 |
| 50.0000 | .010000 | .02012 | .03002 | .04012 | .0500 | .0601 |
| 30.0000 | .010020 | .02020 | .03002 | .04018 | .0500 | .0602 |
| 25.0000 | .010020 | .02022 | .03004 | .04020 | .0501 | .0602 |
| 20.0000 | .010040 | .02028 | .03006 | .04024 | .0501 | .0602 |
| 16.0000 | .010040 | .02034 | .03008 | .04028 | .0501 | .0603 |
| 14.0000 | .010060 | .02038 | .03012 | .04032 | .0502 | .0603 |
| 12.0000 | .010080 | .02042 | .03014 | .04036 | .0502 | .0603 |
| 10.0000 | .010140 | .02048 | .03020 | .04040 | .0503 | .0604 |
| 8.0000 | .010220 | .02058 | .03028 | .04048 | .0503 | .0604 |
| 6.0000 | .010300 | .02070 | .03042 | .04058 | .0505 | .0605 |
| 5.0000 | .010440 | .02080 | .03054 | .04068 | .0506 | .0606 |
| 4.0000 | .010000 | .02092 | .03072 | .04082 | .0508 | .0608 |
| 3.0000 | .010700 | .02114 | .03098 | .04104 | .0510 | .0610 |
| 2.0000 | .011240 | .02152 | .03146 | .04148 | .0515 | .0615 |
| 1.0000 | .012460 | .02250 | .03250 | .04250 | .0525 | .0625 |
| 0.5000 | .013520 | .02352 | .03352 | .04352 | .0535 | .0635 |
| 0.2000 | .014380 | .02438 | .03438 | .04438 | .0544 | .0644 |
| 0.1000 | .014680 | .02468 | .03468 | .04468 | .0547 | .0647 |

Notes:

a) $I_1$, $I_3$ and $I_5$ are the maximum or upper peak values at times $T_1$, $T_3$ and $T_5$.

b) $I_2$, $I_4$ and $I_6$ are the minimum or lower peak values at times $T_2$, $T_4$ and $T_6$.

c) If the supply frequency is 60 Hz then multiply the times by the ratio of 50:60.

The worst case is where the resistance is zero. The current $i$ response is,

$$\begin{aligned} i &= \frac{\hat{V}}{\omega L}\left(-\sin\left(\theta - \frac{\pi}{2}\right) + \sin\omega t\left(+\theta - \frac{\pi}{2}\right)\right) \\ &= \frac{\hat{V}}{\omega L}(\cos\theta - \cos(\omega t + \theta)) \end{aligned}$$

With $\theta = 0$ this current becomes

$$i = \frac{\hat{V}}{\omega L}(1 - \cos\omega t)$$

The sinusoidal term in the brackets oscillates between zero and +2.0. This term is called the 'doubling factor' when the time is $t = \pi/\omega$ has the value of 2.0.

The bad cases occur when the resistance is small, see Figure 11.8.

The response is then,

$$i = \frac{\hat{V}}{Z}\left(e^{\frac{-Rt}{L}} - \cos\omega t\right) \tag{11.6}$$

The term in brackets is again called the 'doubling factor' but it is now less than 2.0 when $t = \pi/\omega$. Table H.1b shows the doubling factor for different ratios of X to R.

Note: The doubling factor is sometimes combined with $\sqrt{2}$ when $V$ is given as the root-mean-square value. In which case the doubling factor has a maximum value of 2.8284 and a minimum value of 1.4142.

#### *11.6.1.2 Resistance larger than inductive reactance*

This case represents the least onerous duty for the switchgear. The angle $\phi$ becomes small as the resistance increases. The worst-case switching angle $\theta$ approaches zero. The conditions that produce a minimum or a maximum can be found by differentiating i in equation (11.5) with respect to the time t and equating the result to zero. This yields the following conditions,

$$\frac{+Re^{\frac{-Rt}{L}}}{L} = \frac{-\omega\cos(\omega t + \theta - \phi)}{\sin(\theta - \phi)} \tag{11.7}$$

When $R >> L$, $e^{\frac{-Rt}{L}}$ approaches zero for $t$ in the range of one or two periods.

The angle $\phi$ approaches zero.

Transposing equation (11.7) for the cosine term gives,

$$\cos(\omega t + \theta) = -\frac{R}{\omega L}\Delta \sin\theta$$

Where $\Delta$ is the small value of $e^{\frac{-Rt}{L}}$, which approaches zero.

The right-hand side approaches zero as $\Delta$ becomes very small. Therefore the left-hand side becomes,

$$\cos(\omega t + \theta) = 0$$

Now since $\theta$ also approaches zero $\cos\omega t$ equals zero for the first time when $\omega t = \pi/2$.

If the above conditions are substituted into (11.5) the current becomes,

$$i = \frac{\hat{V}}{Z}\sin(\omega t + (0 - 0)) = \frac{\hat{V}}{R}\sin\omega t$$

Which is in phase with $V$ as can be expected. Note, the switching angle $\theta$ need not be zero when the inductance is negligible, see Figure 11.9.

#### *11.6.1.3 The doubling factor*

The conditions given by equation (11.7) apply to all combinations of resistance and inductance, and the switching angle $\theta$. Equation (11.7) can be used with little error for cases where the resistance

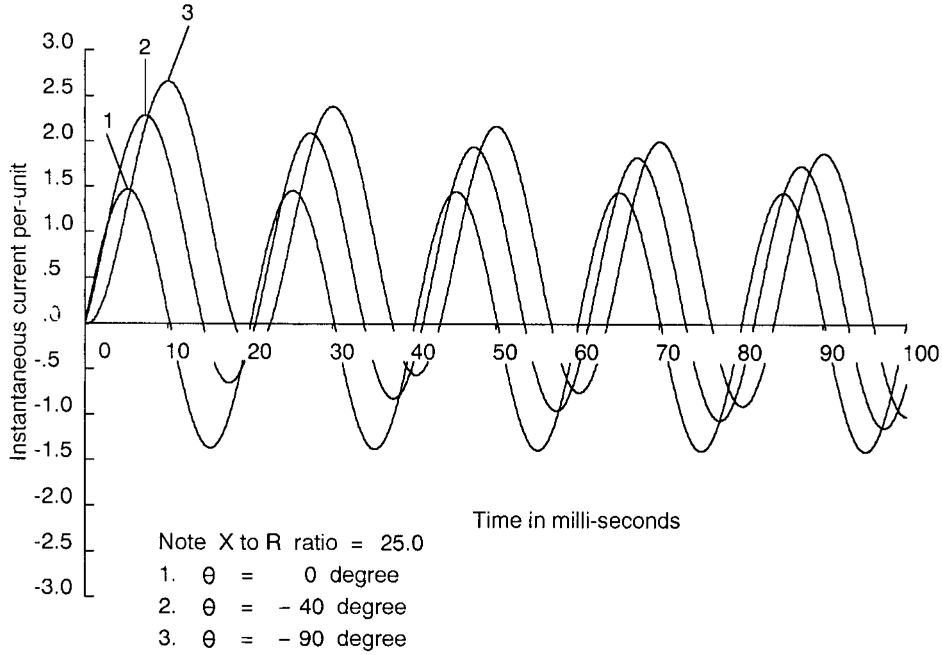


**Figure 11.8** Short-circuit current waveform of a series connected R-L circuit that is fed by a sinusoidal voltage. The X-to-R ratio of the circuit is 25 pu. The responses are for three values of the switching angle $\theta$.

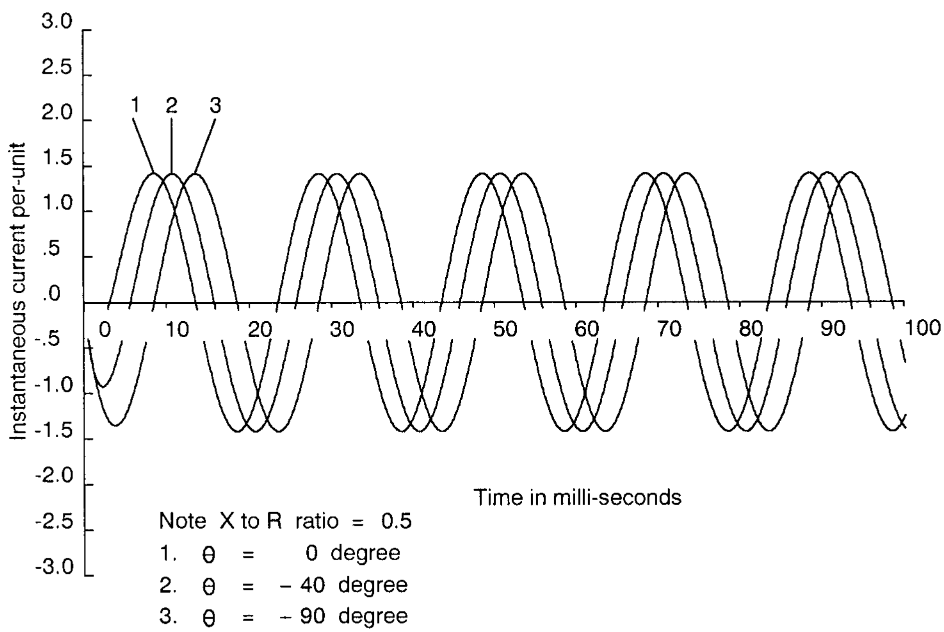


**Figure 11.9** Short-circuit current waveform of a series connected R-L circuit that is fed by a sinusoidal voltage. The X-to-R ratio of the circuit is 0.5 pu. The responses are for three values of the switching angle $\theta$.

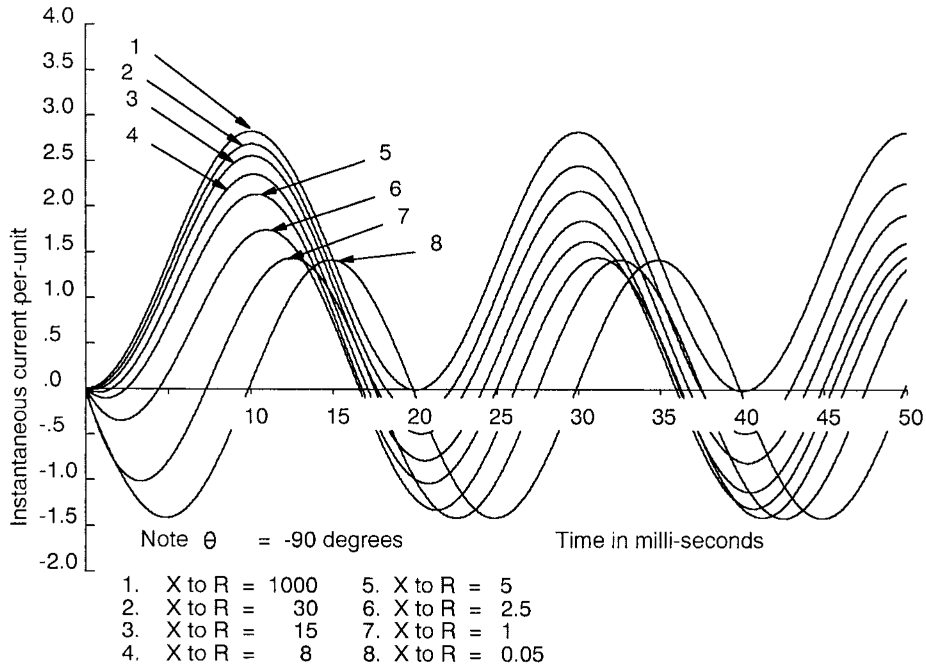


**Figure 11.10** Short-circuit current waveform of a series-connected R-L circuit that is fed by a sinusoidal voltage. The switching angle $\theta$ is $-90$ degrees, which represents the worst case. The responses are for eight values of the circuit X-to-R ratio.

is small i.e. X-to-R ratios greater than about 5.0. Hence substituting for $\omega t = \pi$ gives the doubling factor (DF) as,

$$\mathrm{DF} \cong e^{\frac{-R\pi}{x}} + 1.0$$

For intermediate X-to-R ratios i.e. 0.1 to 5.0, the equality in (11.7) must be satisfied, which is best achieved by iteration for a solution in the vacinity of $\omega t = 3\pi/4$, e.g. by Newton's method, see Reference 4.

Figure 11.10 shows 'worst-case' responses of $i$ for different values of the ratio X to R.

## 11.7 APPLICATION OF THE DOUBLING FACTOR TO FAULT CURRENT $I''_{frms}$ FOUND IN 11.6

Now returning to the rms equations for $I''_g$, $I''_{hm}$ and $I''_{lm}$ in sub-section 11.6 it can be seen that each of these currents can have different X-to-R ratios and will therefore decay at different rates. The peak fault current is,

$$I''_{fpk} = \sqrt{2}(DF_g I''_g + DF_{hm} I''_{hm} + DF_{lm} I''_{lm})$$

Where the doubling factors $DF_g$, $DF_{hm}$ and $DF_{lm}$ are evaluated from the X-to-R ratios of each component using equation (11.5) or their nearest ratio given in Table 11.3 as $I_1$ (pu) or in Table H.1b.

### 11.7.1 Worked Example

An LNG plant has an estimated load of 90 MW and is supplied by five 34.0 MVA generators. The main switchboard operates at 33 kV, and supplies DOL induction motors with unit transformers totalling 35 MW. There is a group of 10 DOL motors operating at 6.6 kV the total kW of which is 25 MW. There is a total of 5 MW of low voltage motors operating at 400 V. A large group of high voltage motors operate from variable speed power electronic rectifier-inverters. These consume a total of 23 MW , and can be regarded as static loads in that they do not contribute fault currents to the main switchboard. There is a miscellaneous static load at 400 V totalling 2 MW.

Each generator has a sub-transient reactance of 0.13 pu.

Each generator unit transformer is rated at 42.5 MVA and has a reactance of 0.08 pu.

The marginal factors for MVA ratings are,

- $K_{tg} = 1.25$
- $K_{tm} = 1.10$
- $K_{td} = 1.3$

The operating power factors of the loads are,

- 0.87 for high voltage motors
- 0.85 for low voltage motors
- 0.86 for high voltage static loads (motors)
- 0.97 for low voltage static loads.

The MVA values for these loads are,

$$S_{hm1} = 35.0/0.87 = 40.23 \text{ MVA}$$
$$S_{hm2} = 25.0/0.87 = 28.74 \text{ MVA}$$
$$S_{lm} = 5.0/0.85 = 5.88 \text{ MVA}$$
$$S_{hs} = 23.0/0.86 = 26.74 \text{ MVA}$$
$$S_{ls} = 2.0/0.97 = 2.06 \text{ MVA}$$

The summations of active and reactive powers are,

$$P_{hm1} = 35.0 \text{ MW},\ P_{hm2} = 25.0 \text{ MVA}$$
$$P_{lm} = 5.0 \text{ MW},\ P_{hs} = 23.0 \text{ MVA}$$
$$P_{ls} = 2.0$$

Hence

$$
\begin{aligned}
P_{\text{load}} &= P_{hm1} + P_{hm2} + P_{lm} + P_{hs} + P_{ls} \\
&= 90.0 \text{ MW} \\
Q_{hm1} &= 19.84 \text{ MVA}_r,\ Q_{hm2} = 14.17 \text{ MVA}_r \\
Q_{lm} &= 3.10 \text{ MVA}_r,\ Q_{hs} = 13.65 \text{ MVA}_r \\
Q_{ls} &= 0.5 \text{ MVA}_r \\
Q_{\text{load}} &= (Q_{hm1} + Q_{hm2} + Q_{lm} + Q_{hs} + Q_{ls})\, 1.015 \\
&= 52.03 \\
S_{\text{load}} &= \sqrt{}(P_{\text{load}}{}^2 + Q_{\text{load}}{}^2) = 103.96 \text{ MVA}
\end{aligned}
$$

The operating power factor $\text{PF}_{\text{load}}$ is,

$$\text{PF}_{\text{load}} = \frac{P_{\text{load}}}{S_{\text{load}}} = \frac{90.0}{103.96} = 0.8657 \text{ lagging}$$

The generator MVA is $S_{\text{gen}}$ which equals 37.5.

Choose the base MVA to be $S_{\text{base}} = 100$.

Assume all five generators are operating when the three-phase zero impedance fault occurs.

Calculate the rms symmetrical fault currents for the generators and each type of load.

a) The generators and unit transformers

$$I''_g = \frac{5 \times 1.1 \times 34.0}{j\left(0.13 + \dfrac{0.08}{1.25}\right) 100.0} = 0.0 - j9.639 \text{ pu}$$

- The high voltage motors and unit transformers.
  These consist of two groups $S_{hm1}$ and $S_{hm2}$, let their total be $S_{hm}$.

$$
\begin{aligned}
I''_{hm} &= \frac{1.0 \times (40.23 + 28.74)}{\left(0.033 + j0.164 + \dfrac{j0.06}{1.10}\right) 100.0} \\
&= \frac{0.6897}{0.033 + j0.164 + j0.0545} \\
&= 14.1184(0.033 - j0.2185) \\
&= 0.4659 - j3.0849 \text{ pu}
\end{aligned}
$$

- The high voltage variable speed drive motors.
  These can be ignored as sources of sub-transient current.

- The low voltage motors and distribution transformers.

  There is one group of low voltage motors connected to various switchboards. Their total MVA is $S_{lm}$. The transformer ratings also require the value of the total static MVA which is $S_{ls}$.

$$
\begin{aligned}
K_{td2} &= 2K_{td}\left(1.0 + \frac{S_{ls}}{S_{lm}}\right)\\
&= 2 \times 1.3\left(1.0 + \frac{2.06}{5.88}\right)\\
&= 3.511\\
I''_{lm} &= \frac{1.0 \times 5.88}{\left(0.054 + j0.143 + \dfrac{j0.055}{3.511}\right)100.0}\\
&= \frac{0.0588}{0.054 + j0.143 + j0.0157}\\
&= 2.0924(0.054 - j0.1587)\\
&= 0.1130 - j0.3321 \text{ pu}
\end{aligned}
$$

- The total rms symmetrical sub-transient fault current.

The total rms fault current $I''_{frms}$ is,

$$
\begin{aligned}
I''_{frms} &= I''_g + I''_{hm} + I''_{lm}\\
&= 0.0 - j9.639\\
&\quad + 0.4659 - j3.0849\\
&\quad + 0.1130 - j0.3321\\
&= 0.5789 - j13.056 \text{ pu}
\end{aligned}
$$

The base current $I_{\text{base}}$ is,

$$
\begin{aligned}
I_{\text{base}} &= \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{100 \times 10^6}{\sqrt{3} \times 33{,}000}\\
&= 1749.6 \text{ amps}
\end{aligned}
$$

Hence the total fault current in rms amps is,

$$
\begin{aligned}
I''_{frms} &= 1749.6(0.5789 - j13.056)\\
&= 1012.8 - j22842.7 \text{ amps}
\end{aligned}
$$

The magnitude is,

$$|I''_{frms}| = 22{,}865 \text{ amps}$$

Find the peak sub-transient fault current.

The X-to-R ratios of the three symmetrical currents can be found from their real and imaginary parts, as shown in the table below:-

| Current | Imaginary part | Real part | X-to-R ratio |
|---|---|---|---|
| $I''_g$ | 9.639 | 0 | infinity |
| $I''_{hm}$ | 3.0849 | 0.4659 | 6.6214 |
| $I''_{lm}$ | 0.3321 | 0.1130 | 2.9389 |

The three ‘doubling factors’ are 2.0, 1.622 and 1.343 per unit. The magnitudes of the three rms currents in amps are 16,864, 5458 and 614 respectively. Multiply each of these currents by 1.414 × doubling factor,

$$I''_{gpk} = 2.828 \times 16{,}864 = 47{,}691 \text{ amps}$$

$$I''_{hmpk} = 2.294 \times 5458 = 12,518 \text{ amps}$$

$$I''_{lmpk} = 1.899 \times 614 = 1166 \text{ amps}$$

The total of these currents is the peak asymmetrical sub-transient fault current $I''_{pk}$ which is 61,375 amps. This is a conservative summation because it assumes that the three peaks occur at the same time. The fault making duty of the main switchboard must be greater than this value of current, i.e., choose a duty of at least 70,000 amps.

### 11.7.2 Breaking Duty Current

Modern switchboard circuit breakers are often able to clear a major fault current within 120 milliseconds, which is typically five or six cycles of the fundamental current. When these circuit breakers are used with generators, and switchboards that are fed by generators located only a short distance away, the decay of the sub-transient current merges with the decay of the transient current. Even at 120 milliseconds the current may have a substantial value. There are several ways of assessing the breaking duty current,

- Use the rigorous equations for a salient pole generator,

$$i_{fa} = \hat{V}\left[\frac{1}{X_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{\frac{-t}{T'd}} + \left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{\frac{-t}{T''d}}\right]\cos(\omega t + \theta) - \hat{V}\left[\frac{(X''_d + X''_q)}{2X''_d X''_q}e^{\frac{-t}{Ta}}\right]$$
$$\cos\theta - \hat{V}\left[\frac{X''_q - X''_d}{2X''_d X''_q}\right]e^{\frac{-t}{Ta}}\cos(2\omega t + \theta) \quad (11.8)$$

  See sub-sections 3.4 and 7.2.7 for an explanation of the variables and parameters.

- Use the above equation but ignore the sub-transient terms, thereby leaving,

$$i_{fa} = \hat{V}\left[\frac{1}{X_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{\frac{-t}{T'd}}\right]\cos(\omega t + \theta) \quad (11.9)$$

- Use equation (11.5) and substitute suitable values for R and L the transient parameters of the generator.

- Use a set of multiplying factors to modify the precalculated value of the rms symmetrical sub-transient current $I''_f$. Apply the factor at the given fault clearance time. (This factor functions in a manner similar to the 'doubling factor' described in sub-sections 11.6 and 11.6.1.3.) Suitable values of the factor are given in clause 12.2.1.3 of IEC60909, equation (47) and Figure 16 therein.

Whichever method is used it is not usually necessary to include the contribution of fault current from induction motors, because such current will have decayed to almost zero at the fault clearance time. If there are large motors connected to the main switchboard then their contribution will be similar to a generator and should be included, see sub-section 7.2.7 and Reference 3 therein, and sub-section 11.8.5.

## 11.8 COMPUTER PROGRAMS FOR CALCULATING FAULT CURRENTS

Now that computers have become so widely available in both the office and in the home, it is relatively easy to program the calculations described in the previous sub-sections. Radial system equations are particularly easy to compute.

As a project moves into the detail design phase it acquires more precise data for all aspects of the work. It is then possible to calculate the fault currents more accurately. However, it should be noted that the tolerances on most of the data are seldom better than plus or minus 15%, and so increasing the quantity of data will not necessarily improve the results significantly. During the detail design phase the power system tends to be modified and additional switchboards added. It is then necessary to calculate the fault currents at least at the busbars of each switchboard, and this can become a laborious task if hand calculations are attempted.

There are many commercially available computer programs for calculating fault currents. Some programs include other features such as load flow, harmonic penetration, transient stability, motor starting and volt-drop calculations, since these features tend to use the same database. Usually a program that calculates fault currents will have several special features for different types of faults e.g.,

- Radial and meshed networks.
- Three-phase zero impedance fault.
- Three-phase non-zero impedance fault.
- Single-phase faults.
- Line-to-line faults.
- Line-to-line-to-ground faults.

These features are calculated with the aid of symmetrical component theory, see Reference 5 to 8. Apart from the simplest situations the solutions are too complicated and time consuming to attempt by hand.

### 11.8.1 Calculation of Fault Current – RMS and Peak Asymmetrical Values

For most LV and all HV generators it is often acceptable to ignore the armature resistance as far as calculating the magnitude of 'first-cycle' fault currents is concerned. It is usual to assume that the

X-to-R ratio of generators is high, e.g. between 20 (for LV generators) and 100 (for HV generators). However, the value of armature resistance is of most importance when considering the downstream circuit-breaker fault clearance capabilities. This aspect is described in sub-sections 7.2.7 and 7.2.11. The calculation of current magnitudes may be carried out in several ways depending upon the amount and accuracy of the data available.

### 11.8.2 Simplest Case

Assume that only $X''_d$ is given, and that this figure is only accurate to about ±15% accuracy. Hence, assume that the X-to-R ratio is infinity; this means that full current doubling will occur (the doubling factor from Table H.1b is 2.848).

Take the $X''_d$ figure and deduct 15% of its value. Calculate $I_f$ using the method of sub-section 11.5.2. This will give a safe estimate of the situation.

### 11.8.3 The Circuit X-to-R Ratio is Known

The method of sub-section 11.8.2 may be used, but an allowance for fault current decrement needs to be made (because the X-to-R ratio is known). Table H.1b gives the appropriate 'doubling factor' for the situation at one-quarter of a cycle for a known X-to-R ratio.

If, for example, the X-to-R ratio happened to be 25 for the numerical example in sub-section 11.8.2 then the 'doubling factor' would be 2.663 instead of 2.848.

### 11.8.4 Detailed Generator Data is Available

A more exact result may be obtained by using equation (7.2). However, all the necessary data must be available, e.g. $X''_d$, $X'_d$, $X_d$, $R_a$, $T''_d$, $T'_d$, $T_a$. It is also advisable to consider the worst-case situation where the reactances take their low tolerance values.

In this method the rms value of the asymmetrical fault current is calculated from the symmetrical rms value and the DC offset value by using the following equation:

$$\text{rms value of asymmetrical fault current} = \sqrt{\begin{array}{c}\text{(rms value of symmetrical fault current during the}\\ \text{first half-cycle)}^2 + \text{(DC offset current)}^2\end{array}}$$

Note: This equation is based on the theory used for calculating the rms value of waveforms that contain harmonic components.

The peak asymmetrical value may be found directly from (7.2) when $t = 0.005$ sec (for 50 Hz systems) or 0.00417 sec (for 60 Hz systems).

### 11.8.5 Motor Contribution to Fault Currents

During a fault condition, the load side of the power system can contribute currents to the fault. The origin of such contribution is motors, which can be either induction or synchronous machines.

Induction motors react as sub-transient generators during the fault. The magnitude of the sub-transient current is normally taken as the starting current or, more specifically, determined by the air-gap emf and the sub-transient impedance of the induction motor. (It is worth noting that some literature treats the rotor of an induction motor as a transient impedance rather than a sub-transient impedance. The difference is not critical but it should be recognised, see Reference 14 and 15.) Since the induction motor has no external excitation system to create flux, then during a disturbance the flux in the machine is that which is 'trapped' in it. This trapped flux decays at a rate determined by the sub-transient impedance of the machine. Hence, induction motors contribute fault current only for a very short time and, consequently, the importance of this contribution is in the fault-making duty of switchgear.

Synchronous motors behave in the same way as synchronous generators during the fault, the only difference being the pre-fault condition of the motor. The emf $E''$ is usually just less than unity, e.g., 0.95 pu.

Since the synchronous motor has an external source of excitation power it can maintain flux for a longer time during a fault. The rotor pole face construction and the field circuit help to maintain the air-gap flux and generated emf. The decay of flux during the fault is determined for the most part by the transient impedance of the synchronous motor.

The sub-transient impedance determines the initial decay, i.e. in the first cycle or so. Therefore the emfs $E''$ and $E'$, together with the reactances $X''_d$ and $X'_d$, need to be used for calculating the fault currents. In a similar way to induction motors, the synchronous motors will contribute to fault-making duty requirements. However, they will also contribute towards the fault-breaking duty because of the transient effects.

All these considerations apply to HV motors, particularly if they are fed directly from the main generator switchboard. LV motors can often be grouped together and considered as one large equivalent motor. It is sometimes possible to ignore the contributions from LV motors because their circuits often have a low X-to-R ratio, which causes the motor contribution to decay very fast. Also, the connected cables, busbars and transformers in the circuit will tend to attenuate the motor fault contribution.

LV motors can occasionally be ignored when HV switchboard faults are being calculated but this will depend upon circumstances, e.g. the number of intermediate voltages exist in the system, whether there are many small motors or a few large motors, the average route length of motor and transformer feeder cables. On offshore platforms it is advisable to seriously consider the LV network. LV motor control centres will be influenced by their motor loads, and the effect of motor contribution will mainly be determined by the fuse, contractor and circuit breaker configurations.

Induction motors can be represented by the 2-axis theory, by using the derivations for synchronous machines but deleting the field winding. In this case some of the reactances become zero, and the field resistance is infinity. Hence, the derived reactances $X''_d$, $X''_q$, etc. and the various time constants $T''_d$, $T''_{do}$ etc. can be redefined for the induction motor.

## 11.9 THE USE OF REACTORS

Reactors are inductance coils and the name 'reactor' is used to imply their use for limiting fault current. Current limiting is often achieved by adding reactance into part of the power system. Reactors perform this function economically.

When power systems grow in size and complexity it often happens that the fault levels in some parts of the existing system become too high for the equipment. Reactors can be inserted to maintain the fault levels below the equipment limits. The most common application is in the feeders to switchgear.

In the oil industry it is often found necessary to increase the number of generators on an existing system. Sometimes this causes fault level problems at the generator switchboard. Rather than replace the switchboard it may be possible to insert one or more reactors. Several solutions are possible:-

- Insert a reactor in series with the new generator.
- Insert a reactor in series with each existing and the new generator, see Figure 11.11.
- Insert a reactor between sections of the main busbars, see Figures 11.12, 11.13 and 11.4.

The preferred solution depends on how much fault level reduction is necessary. If the change in fault level is greater than about 20% the value of the reactance may become too large and cause voltage regulation problems under normal operating conditions. A high reactance inserted into the circuit between generators may cause hunting and stability problems.

Figures 11.11 to 11.14 show different methods of installing fault limiting reactors into a power system. Figure 11.11 shows the simplest method in which one reactor is connected in series with each main generator. This is also the least expensive because no additional switchgear is required. However, it may not be the best technical solution because the value of reactance for each reactor tends to be higher than other options, and this could lead to stability problems. Also the terminal voltage of each generator under normal conditions will need to be kept slightly higher than before due to the reactive volt-drop in the reactor. This may require some modifications to the AVR set-point circuits.

The reactor systems shown in Figures 11.12 and 11.13 are very similar, one being a star-connected system and the other a delta-connected system. The star system has the advantage of

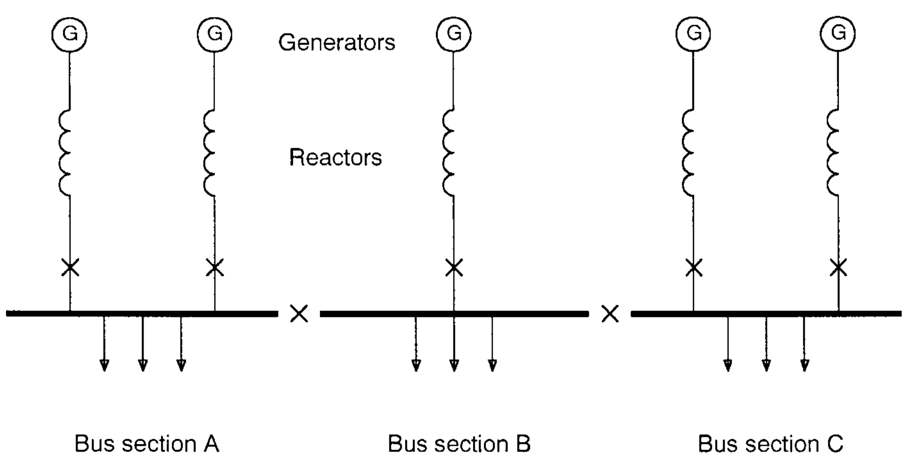


**Figure 11.11** One-line diagram of a simple reactor system for reducing the fault level at the switchgear in the system.

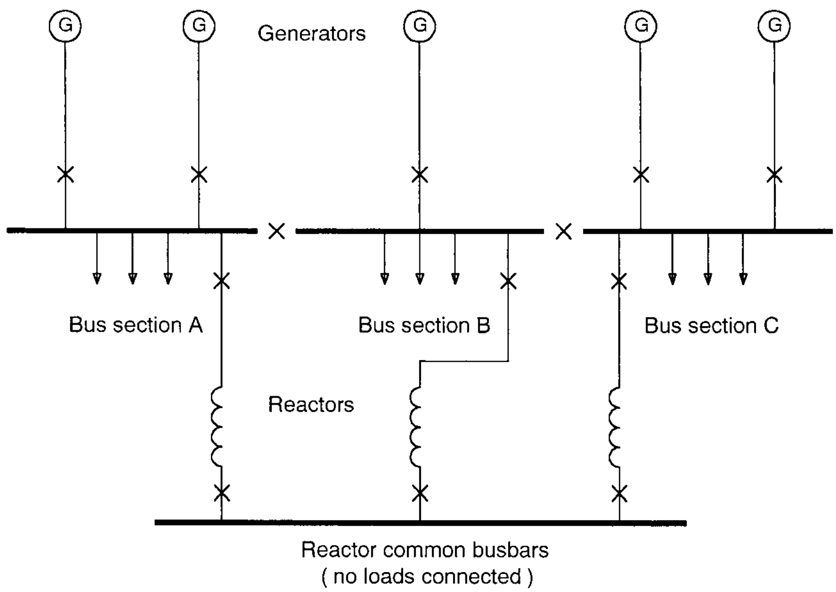


**Figure 11.12** One-line diagram of a star-connected reactor system for reducing the fault level at the switchgear in the system.

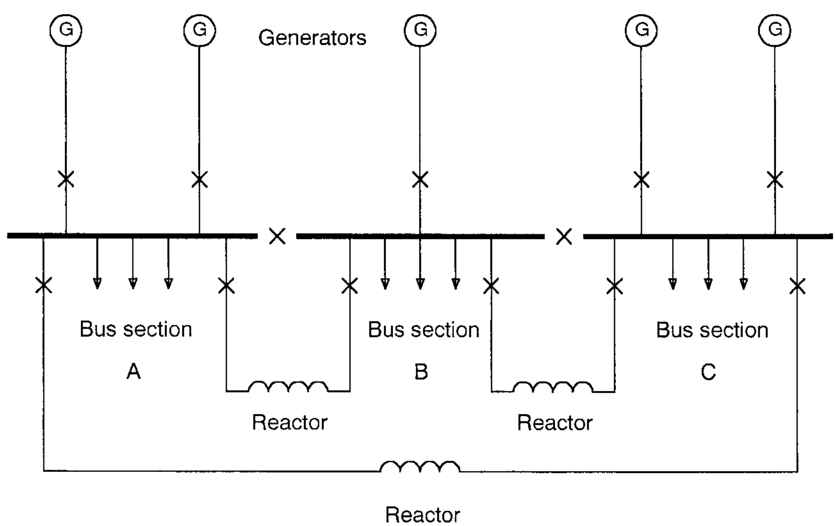


**Figure 11.13** One-line diagram of a delta-connected reactor system for reducing the fault level at the switchgear in the system.

only using three circuit breakers in the existing switchgear, whereas the delta system needs six. This economises the modification of the existing switchgear in terms of cost and space, however, the star system requires a new but small switchboard for the common connections. This new switchboard could be fitted with load break switches instead of circuit breakers, with protection being given by the circuit breaker in the existing switchgear.

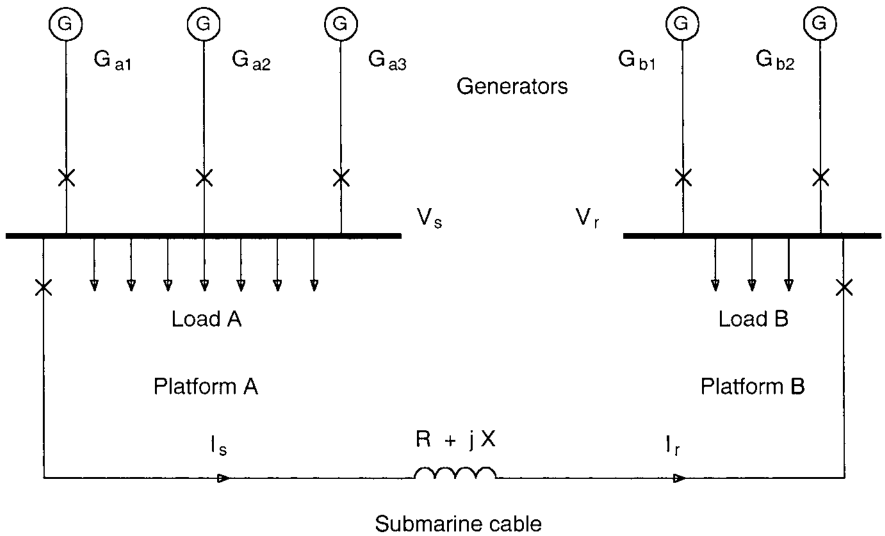


**Figure 11.14** One-line diagram of a two-platform power system.

The star and delta configurations use reactors that have lower reactances than the simple method of Figure 11.11. This will give rise to better stability in both the steady state and the transient state. In addition the AVR set-point circuits should not need to be modified.

## 11.9.1 Worked Example

Consider the systems shown in Figures 11.11 to 11.13. Assume that all the generators are rated at 20 MVA, with sub-transient reactances of 0.15 pu and the main switchboard operates at 11 kV. The symmetrical making current duty of the switchboard is 30,000 amps. Ignore motor contribution in this example. Calculate the per-unit reactance $X_r$ required for each reactor in the three different systems.

a) Case A: Simple system

The 1 pu current of each generator is,

$$I_1 = \frac{S}{\sqrt{3}V} = \frac{20{,}000{,}000}{\sqrt{3} \times 11{,}000} = 1049.8 \text{ amps}$$

The short-circuit current available from each generator is,

$$I_{fg} = \frac{E''I_1}{VX''_d} = \frac{1.1 \times 1049.8}{1.0 \times 0.15} = 7698.2 \text{ amps}$$

Without reactors and with five generators operating the total fault current is

$$I_{fa} = 5 \times 7698.2 = 38{,}491 \text{ amps}$$

which exceeds the duty of the switchgear by 8491 amps.

In order to reduce the total current to 30,000 amps, each generator needs to contribute 6000 amps. Therefore the following condition must be satisfied,

$$I_{fg} = \frac{E''I_1}{V(X''_d + X_r)} = \frac{1.1 \times 1049.8}{1.0 \times (0.15 + X_r)} = 6000 \text{ amps}$$

Transposing gives,

$$X_r = \frac{1.1 \times 1049.8}{1.0 \times 6000} - 0.15 = 0.0425 \text{ pu}$$

b) Case B: Star-connected reactors

From Figure 11.12 it can be seen that the left-hand side and right-hand pairs of generators contribute the same amount of fault current, because the system is symmetrical. The combined impedance of a pair of generators is $X''_d/2$. The combined impedance of two pairs of generator and their shared reactors is,

$$Z_{\text{pairs}} = \frac{X''_d}{4} + \frac{X_r}{2}$$

The fault current contributed by the four outer generators is,

$$I_{fg4} = \frac{E''I_1}{VZ_{\text{pairs}}}$$

The contribution from the centre generator is 7698.2 as found in a).

The total fault current is again 30,000 amps and is given by,

$$I_{f5} = \frac{E''I_1}{V}\left[\frac{1}{Z_{\text{pairs}} + X_r}\right] + 7698.2 = 30000.0$$

$$= \frac{1.1 \times 1049.8}{1.0}\left[\frac{1}{\frac{0.15}{4} + \frac{X_r}{2} + X_r}\right] + 7698.2 \text{ amps}$$

Transposing gives,

$$X_r = 0.00952 \text{ pu}$$

c) Case C: Delta-connected reactors

This case is similar to the star case of b) and the equivalent delta reactance values are simply three times those of the star reactance values.

Hence, $X_r = 0.02856$ pu.

It is interesting to note that in this case there will be no current flowing in the reactor that couples the two outer busbars. However, this reactor cannot be omitted because it serves its purpose when faults occur at the outer switchboards.

d) Comparison of cases

As a rough estimate it may be assumed that the cost of a reactor is directly proportional to its current rating and its value of reactance.

**Table 11.5.** Companion of reactor configurations

| Case | No. of reactors | Reactance (pu) | Current rating (amps) | Product | Cost factor |
|---|---|---|---|---|---|
| | $N$ | $X_r$ | $I$ | $IX_r$ | $NIX_r$ |
| A | 5 | 0.0425 | 1049.8 | 44.62 | 223.10 |
| B | 3 | 0.00952 | 2099.6 | 19.99 | 59.97 |
| C | 3 | 0.02856 | 1049.8 | 29.98 | 89.95 |

Table 11.5 compares the cases in terms of cost, but without the cost of the extra switchgear being included. The cost of a circuit breaker would be in the same order of magnitude as its associated reactor.

When the cost of the switchgear is taken into account, cases B and C become closer in cost, and possibly either is more expensive than case A. Case A does not require extra switchgear.

Occasionally it is desirable to interconnect isolated power generating stations, e.g. offshore platforms or desert gathering stations. Although this often seems a good idea when considering improved power availability and minimising redundancy and spare generators, it frequently causes difficult fault level problems. However, these problems can sometimes be solved by using reactors or transformers in the interconnecting cables or overhead lines. Figure 11.14 shows an interconnection of two offshore platforms.

Even when the reactors are inserted, it may be necessary to impose operational restrictions on the system configuration, e.g. it may not be permissible to have all the generators connected when the interconnector is in service. This aspect may be overcome to some extent by introducing a system of electrical or mechanical interlocks.

Reactors are usually a solution to progressive problems. They should not be designed into a new system.

Reactors may be iron-cored or air-cored. Iron-cored units are preferred but care has to be taken in their design so that they do not become saturated when fault currents pass through them. If the fault current exceeds about three times their rated current then air-cored units become more economically attractive.

They may be of dry-type or liquid-immersed construction, the latter tending to be most common because:-

- They are more suitable for outdoor locations.
- They have a high factor of safety with regard to internal flashover.
- They have a tank, which tends to retain all magnetic fluxes inside the unit. This is important when the location of the reactor is being considered. The radiation of the flux can cause eddy current heating in adjacent steelwork and magnetic interference with other nearby electrical and electronic circuits.
- They have high thermal capacity and can therefore absorb the fault current heat more efficiently.
- The manufacturer can use standard tank and cooling designs that would normally be used for transformers.

## 11.10 SOME COMMENTS ON THE APPLICATION OF IEC60363 AND IEC 60909

IEC60363 was first available in 1972 and IEC60909 in 1988. IEC60363 was issued for evaluating the short circuits in power systems that are used onboard ships. It covers both the transient and sub-transient fault situations. AC power systems on modern large ships have certain similarities to those in oil industry, marine and onshore installations, e.g.

- Independent from other sources of power, i.e. 'island' operation.
- Generators connected directly to the main busbars.
- The main busbars supply induction motors that have relatively high ratings.
- Short cable routes and therefore minimal attenuation of fault currents.
- Significant contribution of sub-transient fault current from induction motor consumers.

IEC60636 is presented in two parts, the first for AC systems and the second for DC systems. The first part gives formulae and tables for calculating the steady state and dynamic fault currents at generators, near to generators and remote from generators. It takes account of the external impedance, beyond the generator terminals, that alters the values of the various time constants that are frequently used in short-circuit calculations. (This aspect is sometimes overlooked when dynamic calculations are being carried out.) The publication also uses only parameters and data that are readily available from manufacturers or databases, which is very convenient. The decrements in the fault currents are also described and illustrated by worked examples. Motor contribution to fault currents is also described and illustrated. The publication briefly addresses the effect of the generators being fully or highly loaded before the fault occurs. In recent years this subject has become more significant in the selection of equipment, relatively small variations due to loading should be considered.

IEC60909 is also presented in two parts but does not cater for DC power systems. It addresses in detail balanced and unbalanced faults near to and far away from a generator. The aspect of a loaded generator is catered for by using a factor 'c' to multiply the rated voltage $U_n$ of the generator, see clauses 6 and 11.4, Table I therein. A second factor $K_G$ is also introduced to modify the sub-transient impedance of the generator, as a function of the load power factor. Appendix A of the publication gives numerical examples.

## 11.11 STABILITY STUDIES

So far the power system has been designed to meet the steady state load distribution requirements and the steady state and transient fault currents that could occur under the worst conditions. Most power systems in the oil industry have their own generators. Consequently, the transient performance of the system and its generators is of great concern when relatively large disturbances are applied, e.g. starting large motors, switching out loaded feeders, recovery from fault clearance.

The analysis and study of the dynamic behaviour of the power system is part of what is generally called 'Stability Studies'.

The stability of a power system can be studied in several ways but, generally speaking, only two ways are important, i.e. steady state and transient stability. The results of these studies usually cause only minor changes to the system that was originally proposed provided that the system had been well thought out initially.

Typical changes would be transformer and generator reactances, limiting the maximum size of the largest motors, providing special starters for large motors (e.g. Korndorfer method), provision of special interlocks or inhibits on the switchgear. Occasionally, however, it is necessary to extend the existing power system, e.g. extra load, more generators, adding an unusually large motor, or to interconnect systems using long-distance cables or overhead lines. When this happens it is essential to carry out a stability study to ensure that the existing equipment still performs satisfactorily and that any new equipment is compatible in all respects.

### 11.11.1 Steady State Stability

Steady state stability relates to the ability of the synchronous source (generators) to transfer power to the synchronous sink (motors and/or other generators). This may be explained by simplifying the synchronous power system as a transmission link (cable or overhead line) of reactance X and zero resistance, a synchronous source (generator at the sending end of the link) and a synchronous sink (load at the receiving end).

The source has an internal emf $E_S$ and the sink has an internal emf $E_R$,

Where phasor

$$\hat{E}_S = |E_S|\angle\delta^\circ$$

and

$$\hat{E}_{R,} = |E_R|\angle 0^\circ \quad \text{(reference phasor)}$$

The current flowing between $E_S$ and $E_R$ is:

$$\hat{I} = |I|\angle{-\text{Ø}} = \frac{\hat{E}_S - \hat{E}_R}{X}$$

Since the reactance X consumes no power, the receiving end power must equal the sending end power. (If the end voltages are not in steady state synchronism then the system is regarded as being unstable.)

Hence:-

$$\begin{aligned}
\text{Power transferred (P)} &= \text{Real part of } \hat{E}_R\hat{I} \text{ or } \hat{E}_S\hat{I} \\
&= \text{Real}\left\{\frac{(E_R\angle 0^\circ)(E_S\angle\delta^\circ - E_R\angle 0^\circ)}{X}\right\} \\
&= \text{Real}\left\{\frac{E_R(E_S\cos\delta + jE_S\sin\delta) - {E_R}^2}{jX}\right\} \\
&= \text{Real}\left\{\frac{-jE_RE_S\cos\delta + E_RE_S\sin\delta + j{E_R}^2}{X}\right\}
\end{aligned}$$

Therefore,

$$P = \frac{E_RE_S\sin\delta}{X} \tag{11.10}$$

A more detailed treatment of this aspect is given in sub-section 3.5, however, (11.10) will be used to illustrate the stability problem.

#### *11.11.1.1 Steady state stability of a generator or motor*

Equation (11.10) applies to any simple form of synchronous source and sink where $E_R$ and $E_S$ and the voltages at either side of the linking reactance $X$. $\delta$ is the phase angle between $E_R$ and $E_S$. For the generator case, $E_R$ and $E_S$ may be replaced by $V$ and $E_g$ and $X$ by $X_{sg}$. (For the synchronous motor case $E_R$ and $E_S$ may be replaced by $E_m$ and $V$ and $X$ by $X_{sm}$.)

Hence, for the generator:

$$P = \frac{VE_g}{X_{sg}} \sin \delta_g$$

Now $V$ is usually kept close to the system rated voltage, i.e. 1.0 pu $\pm$ 0.05 pu and $X_{gs}$, the synchronous reactance of the generator may be assumed constant i.e. typically 1.8 pu to 2.9 pu (depending on the generator rating).

$E_q$ is the internal emf produced by the field winding on the rotor. Hence, for any given value of power P supplied by the generator there will be a wide range of $E_g$ and rotor angle $\delta_g$ values.

**Example:**

Let $\quad V = 1.0$ pu, $X_{sg} = 2.5$ pu and $P = 1.0$ pu (full load).

$$P = 1.0 = \frac{1.0 E_g \sin \delta_g}{2.5}$$

$$\sin \delta_g = \frac{2.5}{E_g} \leq 1.0$$

It can be seen that the larger the value of $E_g$ the smaller will be the value of $\delta_g$.

For full-load normal operation $\delta_g$ is about 50 degrees, which would require $E_g$ to be 3.264 pu.

Suppose $E_g$ is reduced to 2.51 pu, then $\delta_g$ would be 85 degrees.

If $E_g$ is reduced again, to 2.5 pu, then $\delta_g$ would be 90 degrees.

If $E_g$ is reduced below 2.5 pu then there is not a value of $\delta_g$ to satisfy the equation and this means that the power cannot be transferred if $\delta_g$ is caused to exceed 90 degrees. The generator rotor can no longer be kept in synchronism with the terminal voltage to which it is connected. $\delta_g$ can be caused to exceed 90 degrees by either reducing the field excitation, as described above, or by allowing more power to be applied to the generator from its prime-mover, e.g. gas turbine. This can happen at any level of power loading on the generator (above zero power). When the rotor angle $\delta_g$ exceeds 90 degrees, and the generator rotor pulls out of synchronism, the condition is unstable which means the limit of steady state stability has been exceeded.

#### *11.11.1.2 Steady state stability of an interconnected power system*

As an example, consider two offshore platforms, each with its own generators and loads, operating in synchronism through an interconnecting power cable of reactance $X$ (as shown in Figure 11.14. Assume the resistance of the interconnecting cable is zero.

In this situation it is desirable to keep both platform voltages close to their rated values, i.e. 1.0 pu $\pm$ 0.05. A particular operating condition requires one of the generators on platform B to be out of service for maintenance but the load still needs to be supplied.

This is achieved by operating an extra generator on platform A and transferring the surplus power from A to B through the interconnecting cable X.

The value of X depends upon the route length and the maximum amount of power that is ever likely to be continuously transferred under normal conditions (for example, it may be decided to size the cable to handle the rated power output of one generator on one of the platforms).

The equation for the power transferred would be:

$$P = \frac{V_s \cdot V_R}{X} \sin \delta_c$$

Where $V_s$ is the sending end voltage on Platform A
$V_R$ is the receiving end voltage on Platform B
$\delta_c$ is the load angle across the cable reactance X.

A typical situation could be that the cable reactance X would be 0.2 pu, and 1.0 pu of its power capability is being transferred. With $V_S$ and $V_R$ each about 1.0 pu then the load angle would be about 11.5 degrees. This represents a ‘tight’ coupling between the two platforms since the load angle is small and considerable margin exists before the 90 degree limit of steady state stability is exceeded.

In order to even approach 90 degrees, considerable current would have to flow in the cable (four to five times full-load power in this example). Therefore, a ‘tightly’ coupled system is unlikely to become unstable in the steady state for normal and near-normal situations.

Problems can arise when a long cable or overhead line is rated for a relatively small amount of power transfer, because its impedance will be relatively large. In this situation, the load angle will be large and a small disturbance could bring about instability. Such a system may be described as being ‘loosely’ coupled.

### 11.11.2 Transient Stability

This is a more complex subject since it is closely related to the dynamic behaviour of the generators, prime-movers, motors, loads and the control systems used with these machines. The static elements in an interconnected power system also have considerable effect on the transient responses of the machines in the system.

In an interconnected power system there will be two or more synchronous machines (or groups of machines). These machines will be coupled through their own internal reactances and through additional reactances (or impedances) due to the presence of cables, overhead lines and transformers. The system will be assumed to be stable in the steady state.

In order to change the operating conditions of the system there must be a change in the load (or loads). This may be due to starting a motor, switching in or out a cable or overhead line, changing the load on a motor or changing a static load. When a load change occurs, the relative position of the generator rotors will change, i.e. $\delta_g$ of each generator will change. This angular change of rotor position will be accompanied by an oscillatory movement of the rotors as they reposition themselves. The amplitude and duration of the oscillatory motion is mainly determined by the mechanical inertia and the damping characteristics of the generators and their prime-movers.

The inertia and damping characteristics can be represented by an accelerating power term and a frictional or damping power term in a simplified second-order differential equation for each generator. Also in the equation is a term for the electrical power generated. The right-hand side of the equation represents the mechanical power that is applied to the shaft of the generator.

Each generator prime-mover unit can be thought to be rather like a mechanical spring/mass/damper dynamic system. Once disturbed in any way, the mass will oscillate and eventually settle at a new position. The static characteristic of the spring is analogous to the electrical power generated and sent out from the generator. The inertia term includes all the rotating masses of the generator, its prime-mover and a gearbox that may be used. The damping term consists of two parts; firstly the damping due to eddy current induction in the rotor electrical circuits and, secondly, the damping due to the friction, windage and governor action at the prime-mover.

The subject of electromagnetic damping within synchronous machines is a complicated one and some of the earliest analytical work was recorded in the 1920s e.g. References 9 to 11 using mechanical analogues. A later mechanical analogue was made by Westinghouse Electrical Corporation, Reference 7, Chapter 13, based on that given in Reference 9. A comprehensive summary of the historical developments made in this subject, and automatic voltage regulation, from 1926 to 1973 can be found in Reference 12.

A typical set of system equations will now be described in their simpler form. There are many variations on the general theme, depending upon the results being sought. The analysis of fast-acting transients to match field tests would require very detailed modelling of all the dynamic components of the machinery in the system. The starting of motors or the loss of generation would not require such a detailed representation since the transients of interest take longer to manifest themselves, i.e. 20 seconds, instead of 1 second, are required to pass in order to reach a conclusion.

#### *11.11.2.1 The equation of motion of one generator*

The transient power balance equation of an individual generator prime-mover set may be written as:-

$$P_a + P_{fw} + P_{em} + P_{\text{elec}} = P_{\text{mech}}$$

Where: $P_a$ = accelerating power for the polar moment of inertia.

$P_{fw}$ = friction and windage power.

$P_{em}$ = electromagnetic damping power.

$P_{\text{elec}}$ = electrical power delivered from the generator terminals.

$P_{\text{mech}}$ = mechanical power received by the generator at its coupling.

and $$P_a = 2\pi M \frac{d_f}{d_t}$$

$$P_{fw} = F_{fw} \cdot f$$

$$P_{em} = f_{em}(X''_d, X''_q, R_d, R_q, X_f, R_f, [f_o - f])$$

$$P_{\text{elec}} = f_{\text{elec}}(V, E, \sin\delta_c, X_{dg}, X_q, X'_d, X'_q, R_a)$$

$$P_{\text{mech}} = G_{pm(p)}\left[P_{\text{ref}} + A\left(\frac{f - f_o}{f_o}\right)\right]$$

Where:
$M$ = polar moment of inertia of the generator and its prime-mover.
$f$ = generator shaft speed (i.e. frequency).
$f_o$ = reference frequency of the system, e.g. 50 Hz or 60 Hz.
$F_{f\omega}$ = friction and windage coefficient.
$V$ = terminal voltage of the generator.
$E$ = $f_e(I_f)$ = internal emf of the generator as created by the field current $I_f$.
$\delta$ = rotor angle between the terminal voltage and the rotor direct axis.
$X_d$ = direct axis synchronous reactance.
$X_{qg}$ = quadrature axis synchronous reactance.
$X'_d$ = direct axis transient reactance.
$X'_q$ = quadrature axis transient reactance.
$X''_d$ = direct axis sub-transient reactance.
$X''_q$ = quadrature axis sub-transient reactance.
$X_{fg}$ = rotor field leakage reactance.
$R_d$ = direct axis rotor damper bar resistance.
$R_q$ = quadrature axis rotor damper bar resistance.
$R_f$ = rotor field circuit resistance.
$R_a$ = stator resistance.
$G_{mp}(p)$ = transfer function for the dynamics of the prime mover.
$(p)$ = general differential operator $\dfrac{\mathrm{d}()}{\mathrm{d}t}$
$P_{\text{ref}}$ = power set-point of the prime mover.
$A$ = governor droop setting.
$f_e$, $f_{em}$ and $f_{\text{elec}}$ are functions of the variables shown.

In some situations, the rate of change of shaft frequency is equal to the second rate of change of rotor angle, e.g. when the system frequency remains almost constant or changes slowly. Hence:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{1}{2\pi}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2}$$

#### *11.11.2.2 Multi-generator situations*

The equations of sub-section 11.11.2.1 can be applied to all the generators in an interconnected system. At steady state stable conditions all the generator shaft frequencies $f$ must be equal. During disturbed conditions, the average frequency of rotation of each generator shaft will be equal, otherwise unstable operation will exist (i.e. averaged over several cycles of the alternating current delivered from the generators). The elements that connect all the generators in the equations are the electrical power terms $P_{elec}$ ($P_{mech}$ will change due to the governor action sensing the change of shaft speed). The $P_{elec}$ terms are connected and balanced through algebraic equations that represent the power balance and exchange that occurs in the static electrical interconnecting network, e.g. cables, overhead lines, transformers, loads.

Hence the simultaneous solution of the generator prime-mover equations also requires the simultaneous solution of the algebraic power transfer equations of the electrical network. Digital computers must be used for the accurate solution of these complex equations. Manual solution is almost impossible, even for relatively simple situations. An excellent treatment of these complex equations for multi-machine systems is given in Reference 13, which lends itself to being reasonably easy to program in a digital computer. The reference also compares the benefits and disadvantages obtained when the mathematical modelling of the generators becomes very detailed.

#### *11.11.2.3 Limit of transient stability*

In the same way that steady state stability was assessed by concentrating on the variations of the rotor angle $\delta_g$, so also is the limit of transient stability assessed. However, the situation is not so exact. The transient variation of $\delta_g$ for any one machine can exceed 90 degrees, and even reach 120 degrees, before unstable operation occurs. The limit of transient stability can therefore exceed 90 degrees and is influenced by several factors:

- The inertia constant ($H$) of the machines.
- Effectiveness of the electromagnetic rotor damping.
- The pre-disturbance operating conditions and how close they are to the rated conditions.
- The amplitude of the disturbance.
- The time function of the disturbance, e.g. step function such as a fault, slowly changing function such as a motor start.
- The 'tightness' or 'looseness' of the interconnections in the system (see sub-section 11.11.1.2).
- The time constants and gains of the control systems used in the automatic voltage regulators, governors and prime-movers.
- The non-linear limits imposed on the control systems, e.g. constraints on excitation current, valve limits on fuel valves.
- The dynamic characteristics of motor loads.
- The mixture ratio of dynamic to static loads.
- Operating power factors before the disturbance is applied.

#### *11.11.2.4 Applications*

In the oil, gas and petro-chemical industries, the need for stability studies is primarily due to the fact that most plants have their own power generation facilities which are occasionally interconnected between themselves or with a large public utility. In either case, the stable performance of the system is of great importance, otherwise unwarranted shutdowns can occur with a resulting loss of production.

Stability studies will help to minimise these possibilities. When planning a stability study the main aspects that are usually included are:-

- Application of major faults on the electrical network.
- Sudden loss of a generator, e.g. due to an unexpected failure.
- Starting large induction motors direct-on-line.
- Reduced voltage methods for the starting of motors.
- Tripping large motors.
- Switching in or out interconnecting cables or overhead lines.

The performance is assessed in terms of the following:-

- Voltage recovery throughout the system.
- Frequency recovery throughout the system.
- Synchronous operation is maintained.
- Motors recover to their normal operation.
- No prolonged overloads occur.
- Generators share load changes properly.
- Hunting oscillations do not develop.
- Transient oscillations die away within a few seconds after a sudden disturbance is applied.

#### *11.11.2.5 Depth of study – preliminary stage*

A stability study should be seriously considered necessary at an early stage of a project so that the basic configuration of the power system network may be established with confidence. This is especially applicable to remote or self-contained power plants which have a large number of motors, e.g. an offshore platform.

At the early stage it is acceptable to use typical data for particular plant items and a number of simplifications are justified:

- Use typical data for generators, motors, gas turbines, pumps and compressors. This can be obtained as ‘budget’ data when screening vendors and manufacturers for suitable machinery.
- Neglect high voltage cable impedances unless the route distances are long.
- Use simplified block models for the turbine and generator control systems.
- Represent all the low voltage motors on a typical motor control centre by one, two or perhaps three equivalent motors to cover the kilowatt range. A typical selection would be 20 kW and 100 kW. The equivalent motor would have the electrical parameters, inertia constant and pump characteristic of the typical machine, but would have the rating of the total of all the motors in the group.
- All low voltage motors would be assumed to be driving centrifugal machinery.
- Separate out any special case low voltage motors, e.g. extra large motors, large motors driving reciprocating machinery.

- Include typical transformers by their per-unit reactance. Neglect their resistance.
- Include all high voltage motors and their driven machinery.

The results of the preliminary study will enable potential problem areas to be seen ahead of the detail design stage. The results will have been obtained at a minimum cost.

#### *11.11.2.6 Depth of study – detail design stage*

As the detail design work develops, the data available for the network and individual plant items become more precisely defined. Particular manufacturers may have been selected, the cable routes and lengths fixed. The network configuration becomes more definite and the turbine and generator control systems can be precisely identified. Hence, the detail to which the network can be represented may be increased with confidence.

The preliminary studies can be re-run with a revised network and new data, and additional operational options can be considered.

#### *11.11.2.7 Theoretical basis of a computer program*

The programs used for this type of study are based on the mathematical theory of electrical machines known in various forms as:

- two-axis theory.
- $d$-$q$ axis theory.
- generalised theory of machines.

The theory has been developed by many researchers over the last 70 years, e.g. H R Park, E Kimbark, C Concordia, B Adkins, G Shackshaft, G Kron, A Rankin.

The synchronous generators and motors are represented by their sub-transient, transient and synchronous reactances and time constants in both the 'd' and the 'q' axes, hence saliency is accounted for.

The control systems for the governors and automatic voltage regulators can be chosen from standard IEEE forms or can be built up separately to any degree of detail necessary.

A two-axis model is often used for the induction motors but the two axis parameters are usually created within the program from the customary impedances that are given in per-unit form.

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## FURTHER READING

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