Appendix F

Worked Example for Calculating the Performance of a Gas Turbine

F.1 THE REQUIREMENTS AND DATA GIVEN

A 12 MW gas turbine generator is required to operate at sea level with an ambient temperature T_1 of 20°C and a combustion temperature T_3 of 950°C. The following data apply.

Compressor	
Pressure ratio of the compressor r_{pc}	11.0
Compressor efficiency η_c	0.85
Ratio of specific heats for compression γ_c	1.4
Specific heat at constant pressure C_{pc}	1.005 kJ/kg°K
Ambient pressure	1.0 bar
Turbine	
Pressure ratio of the turbine r_{pt}, nominal	11.0
Turbine efficiency η_t	0.87
Combustion pressure drop $\Delta P_{23}/P_{23}$	0.04
Ratio of specific heats for expansion γ_t	1.33
Specific heat at constant pressure C_{pt}	1.147 kJ/kg°K
Heat rate	15.750 MJ/kWh
Losses	
Inlet ducting and silencer pressure drop ΔP_1	125 mm of water
Exhaust ducting pressure drop ΔP_4	50 mm of water
Gear box efficiency η_{gb} at full load	0.985 per unit
Generator energy conversion efficiency η_{gen}	0.985 per unit
Fuel	
LHV for hydrocarbon natural gas	37.50 MJ/kg
Fuel air ratio by mass	0.01 per unit

F.2 BASIC REQUIREMENTS

Assume constant specific heat $C_p = C_{pc}$, and $\gamma = \gamma_c$. Ignore the losses in the ducting, gear box and generator.

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Find the following:-

1. Ideal compressor outlet temperature T_2 in °K and °C.
2. Ideal turbine outlet temperature T_4 in °K and °C.
3. Ideal cycle efficiency η_i in per unit.
4. Compressor outlet temperature T_{2e} in °K and °C.
5. Turbine outlet temperature T_{4e} in °K and °C due to expansion efficiency η_t.
6. Practical cycle efficiency η_p per unit, with η_c and η_t included.
7. Find the pressure ratio $r_{p\,\max}$ that causes the maximum power to be delivered to the generator.

F.3 DETAILED REQUIREMENTS

Assume the specific heats are functions of temperature and take account of the pressure drops ΔP_1, ΔP_{23} and ΔP_4.

Find the following:-

8. Compressor outlet temperature T_{2ea} in °K and °C, due to compression efficiency η_c and the inlet pressure drop ΔP_1.
9. Turbine outlet temperature T_{4ea} in °K and °C, due to expansion efficiency η_t, the combustion pressure drop ΔP_{23}, and the outlet pressure drop ΔP_4.
10. The work done on the mass flow to produce the desired output power of 12 MW.
11. Theoretical thermal efficiency η_{pa} per unit, with all the losses included.
12. Overall thermal efficiency η_{pao} with all losses included.

F.4 BASIC SOLUTIONS

Step 1. From (2.14),

$$\delta = (1.0 - 1.4)/1.4 = -0.2857$$
$$(P_2/P_1)^\delta = 11.0^{-0.2857} = 0.50403$$

Therefore,

$$T_2 = T_1/0.50403 = (273.0 + 20.0)/0.50403 = 581.31^\circ\text{K or } 308.31^\circ\text{C}.$$

Step 2. From (2.15),

$$(P_3/P_4)^\delta = 11.0^{-0.2857} = 0.50403$$

Therefore,

$$T_4 = T_3 \times 0.50403 = (273.0 + 950.0) \times 0.50403 = 616.43^\circ\text{K or } 343.43^\circ\text{C}.$$

Step 3.

$$r_p^\delta = 11.0^{-0.2857} = 0.50403$$

and

$$r_p^\beta = 11.0^{+0.2857} = 1.984$$

Therefore, from (2.17),

$$\eta_i = 1.0 - \frac{(273.0 + 950.0) \times 0.50403 - (273.0 + 20.0)}{(273.0 + 950.0) - ((273.0 + 20.0) \times 1.984)}$$

$$= 1.0 - \frac{323.43}{641.69} = 0.496 \text{ per unit}$$

Step 4. From (2.18),

$$T_{2e} = \frac{581.31}{0.85} + \left(1.0 - \frac{1.0}{0.85}\right) \times 293.0$$

$$= 632.18^{\circ}\text{K or } 359.18^{\circ}\text{C}.$$

Step 5. Also from (2.18),

$$T_{4e} = 616.43 \times 0.87 + (1.0 - 0.87) \times 1223.0$$

$$= 695.28^{\circ}\text{K or } 422.28^{\circ}\text{C}.$$

Step 6. From (2.20),

$$\eta_p = \frac{1223.0(1.0 - 0.50403) \times 0.85 \times 0.87 - 293.0(1.984 - 1.0)}{1223.0 \times 0.85 - 293.0(1.984 - 1.0 + 0.85)}$$

$$= \frac{160.25}{502.188} = 0.319 \text{ per unit}$$

Step 7. From (2.27),

Let

$$d = \frac{1.4}{2(1.0 - 1.4)} = -1.75$$

$$r_{\text{p max}} = (293.0/(1223.0 \times 0.85 \times 0.87))^{d}$$

$$= 7.187 \text{ per unit}$$

F.5 DETAILED SOLUTIONS

Step 8. Initially convert the pressure drops into the SI system of measurement units of 'bar'.

$$\Delta P_1 = 125.0/10200.0 = 0.01226 \text{ bar}$$

And

$$\Delta P_4 = 50.0/10200.0 = 0.0049 \text{ bar}$$

The combustion pressure drop in 'bar' is,

$$\Delta P_4 = r_{pt} \times P_4 \times 0.04 = 11.0 \times 1.0 \times 0.04 = 0.44 \text{ bar}$$

Step 9. The relationship between 'γ' over the range of 1.33 to 1.4 and 'C_p' over the range of 1.005 and 1.147 respectively, is approximately a straight-line law of the form '$y = a + bx$'. Hence by using these pairs of points, $a = 1.895425$ and $b = -0.49296$.

Therefore,

$$\gamma = 1.895425 - 0.49296\ C_p$$

Step 10. The pressure ratio is not affected by the change in inlet pressure to the compressor. The outlet temperature will remain constant at $T_2 = T_{2e} = 632.18°\text{K}$ or 359.18°C.

Step 11. The outlet pressure of the compressor will be,

$$P_2 + \Delta P_2 = r_p(P_1 + \Delta P_1) = 11.0 \times (1.0 - 0.01226)$$
$$= 10.8651 \text{ bar}$$

The inlet pressure to the turbine will be,

$$P_3{}' = P_2 + \Delta P_2 - \Delta P_{23} = 10.8651 - 0.44 = 10.4251 \text{ bar}$$

The outlet pressure of the turbine will be,

$$P_4{}' = P_4 + \Delta P_4 = 1.0 + 0.0049 = 1.0049 \text{ bar}$$

Hence the pressure ratio of the turbine is,

$$r_{pt} = \frac{P_3{}'}{P_4{}'} = \frac{10.4251}{1.0049} = 10.3743$$

The specific heats C_{pc} and C_{pt} are functions of the temperature within the compressor and turbine respectively. A reasonable approximation is to use the average of T_1 and T_{2e} for the compressor, call this T_{12e}, and the average of T_3 and T_{4e} for the turbine, call this T_{34e}. The variation of C_p with temperature is given in Table 2.1 as a cubic equation for three fuel-to-air ratios, zero, 0.01 and 0.02 per unit by mass. The value of 0.01 is appropriate for this example. At the same time the ratio of specific heats γ_c and γ_t are functions of the specific heat at constant pressure. Simple linear functions can be used to estimate the appropriate value of γ for a given C_p, as follows,

$$\gamma_c = a_c + b_c C_{pc} \quad \text{and} \quad \gamma_t = a_t + b_t C_{pt},$$

where

$$a_c = a_t = 1.895425 \quad \text{and} \quad b_c = b_t = -0.49296$$

An iterative procedure is necessary in order to stabilise the values of C_{pc}, γ_c and T_{2e} for the compressor and C_{pt}, γ_t and T_{4e} for the turbine. The conditions for the compressor need to be calculated before those of the turbine.

Step 12. Find the compressor conditions

The starting conditions for iterating the compressor variables are,

$$C_{pc} = 1.005$$
$$\gamma_c = 1.895425 - 0.49296 \times 1.005 = 1.4$$
$$T_1 = 293.0^\circ\text{K}$$
$$T_{2e} = 632.18^\circ\text{K}, \text{ found from Step 4} \quad \text{[step 12.1]}$$

The average value of T_1 and T_{2e} is 462.59°K. From the cubic expression in Table 2.1 for a fuel-to-air ratio of zero, the revised value of C_{pc} is,

$$\begin{aligned} C_{pcn} &= 0.99653 - 1.6117 \times 10^{-4} \times 462.59 \\ &\quad + 5.4984 \times 10^{-7} \times 462.59^2 - 2.4164 \times 10^{-10} \times 462.59^3 \\ &= 0.99653 - 0.074557 + 0.117662 - 0.023921 = 1.015718 \end{aligned}$$

The new value of γ_c is $1.895425 - 0.49296 \times 1.015718 = 1.3947$. Now recalculate T_{2e},

$$T_{2e} = \frac{293.0 \times (11.0 - 1.0 + 0.85)}{0.85} = 627.78^\circ\text{K}$$

The new average value of T_1 and T_{2e} is 460.39°K.

Step 13. Recycle.

Repeat this iterative process from [step 12.1] until the variables settle at their stable values. These eventually become,

$$C_{pc} = C_{pcn} = 1.01531$$
$$\gamma_c = \gamma_{cn} = 1.394917$$
$$T_{2e} = 627.934^\circ\text{K or } 354.934^\circ\text{C}$$

Step 14. Find the turbine conditions.

The starting conditions for iterating the turbine variables are,

$$C_{pt} = 1.005$$
$$\gamma_t = 1.895425 - 0.49296 \times 1.147 = 1.33$$
$$T_1 = 293.0^\circ\text{K}$$
$$T_4 = r_{pt}{}^{0.2481} = 10.3743^{0.2481} = 684.46^\circ\text{K} \quad \text{[step 14.1]}$$
$$T_{4e} = 684.46 \times \eta_t + (1 - \eta_t) \times 1223.0 = 754.47^\circ\text{K}$$

The average value of T_3 and T_{4e} is 988.734°K. From the cubic expression in Table 2.1 for a fuel-to-air ratio of 0.01, the revised value of C_{pt} is,

$$\begin{aligned} C_{ptn} &= 1.0011 - 1.4117 \times 10^{-4} \times 988.734 \\ &\quad + 5.4973 \times 10^{-7} \times 988.734^2 - 2.4691 \times 10^{-10} \times 988.734^3 \\ &= 1.160278 \end{aligned}$$

The new value of γ_t is $1.895425 - 0.49296 \times 1.160278 = 1.32345$, and $T_4 = 690.436$°K. Now recalculate T_{4e},

$$T_{4e} = 690.436 \times 0.87 + (1.0 - 0.87) \times 1223.0 = 759.67^\circ\text{K}$$

The new average value of T_3 and T_{4e} is 991.334°K.

Step 15. Recycle.

Repeat this iterative process from [step 14.1] until the variables settle at their stable values. These eventually become,

$$\begin{aligned} C_{pt} &= C_{ptn} = 1.16088 \\ \gamma_t &= \gamma_{tn} = 1.323156 \\ T_{4e} &= 991.455^\circ\text{K or } 718.455^\circ\text{C} \end{aligned}$$

Step 16. The work done on the gearbox input shaft, from (2.32) is found as follows,

$$\delta_t = \frac{1 - \gamma_t}{\gamma_t} = \frac{1.0 - 1.323156}{1.323156} = -0.24423$$

$$\begin{aligned} U_{\text{outea}} &= 1.16088 \times 1223.0 \times (1.0 - 10.3743^{-0.24423}) \times 0.87 \\ &= 537.592 - 340.062 = 197.53 \text{ kJ/kg} \end{aligned}$$

Step 17. Include the gearbox and generator losses.

The losses between the gearbox input shaft and the electrical terminals of the generator U_{losses} are,

$$U_{\text{losses}} = (0.015 + (1.0 - 0.985)) \times 12.0 = 0.36 \text{ MW}$$

Hence the input to the gearbox is $12.0 + 0.36 = 12.36$ MW. From sub-section 2.3 the mass flow of the air–fuel mixture 'm' is,

$$\begin{aligned} m &= \frac{W_{\text{out}}}{U_{\text{outea}}} = \frac{12.36 \times 1000.0}{197.53} \\ &= 62.573 \text{ kg/sec} = 225263 \text{ kg/hour} \end{aligned}$$

Step 18. Find the theoretical efficiency η_{pa}.

From (2.20) the theoretical efficiency η_{pa} can be found by using the appropriate pressure ratios and ratios of the specific heats.

Let

$$T_{4a} = T_3(1 - {r_{pt}}^{\delta t})\eta_c\ \eta_t$$
$$T_{1a} = T_1({r_{pt}}^{\beta t} - 1)$$
$$T_{3a} = T_3\ \eta_c$$

and

$$T_{2a} = T_1({r_{pc}}^{\beta t} - 1 + \eta_c)$$

then

$$\eta_{pa} = \frac{T_{4a} - T_{1a}}{T_{3a} - T_{2a}}$$

therefore,

$$T_{4a} = 1223.0 \times (1.0 - 10.3743^{-0.24423}) \times 0.85 \times 0.87 = 393.627^{\circ}\text{K}$$
$$\beta_c = \frac{\gamma_c - 1}{\gamma_c} = \frac{1.394917 - 1.0}{1.394917} = +0.28311$$
$$T_{1a} = 293.0 \times (11.0^{+0.28311} - 1.0) = 284.694^{\circ}\text{K}$$
$$T_{3a} = 1223.0 \times 0.85 = 1039.55^{\circ}\text{K}$$
$$T_{1a} = 293.0 \times (1.971652 - 1.0 + 0.85) = 533.744^{\circ}\text{K}$$
$$\eta_{pa} = \frac{393.627 - 284.694}{1039.55 - 533.744} = 0.2154 \text{ per unit}$$

Step 19. Find the overall thermal efficiency η_{pao}.

From (2.33) and allowing for the losses in the gearbox and generator, the overall thermal efficiency η_{pao} can be found as follows.

$$\eta_{pao} = \frac{U_{\text{oute}}}{U_{\text{fea}}}\ \eta_{gb}\ \eta_{\text{gen}}$$

The value of C_{pf} can be taken as the average value of T_3 and T_{2e}, call this T_{23},

$$T_{23} = \frac{1223.0 + 627.934}{2} = 925.467^{\circ}\text{K}$$

Substitute T_{23} in the cubic expression for a fuel–air ratio of 0.01 in Table 2.1 to find the appropriate value of C_{pf},

$$C_{pf} = 1.0011 - 1.4117 \times 10^{-4} \times 925.467$$
$$+ 5.4973 \times 10^{-7} \times 925.467^2 - 2.4691 \times 10^{-10} \times 925.467^3 = 1.14558$$
$$U_{\text{fea}} = 1.14558 \times (1223.0 - 627.934) = 681.695 \text{ kJ/kg}$$
$$\eta_{pa} = \frac{U_{\text{outea}}}{U_{\text{fea}}} = \frac{197.530}{681.695} = 0.28976 \text{ per unit}$$
$$\eta_{pao} = 0.28976\ \eta_{gb}\ \eta_{\text{gen}}$$
$$= 0.28976 \times 0.985 \times 0.985 = 0.28114 \text{ per unit}$$