Chapter 4

Conservation Laws

INTRODUCTION

In order to better understand heat transfer, it is necessary to first understand the theory underlying this science. How can one predict at what temperature products will be emitted from cooled or heated effluent streams? At what temperature must a thermal device be operated? How much energy in the form of heat is given off during a combustion process? Is it economically feasible to recover that heat? Is the feed high enough in heating value, or must additional fuel be added to assist in the thermal process? If so, how much fuel must be added? The answers to these questions are rooted in the various theories of heat transfer, including thermodynamics, thermochemistry, and to a lesser degree, phase equilibrium and chemical reaction equilibrium.

One of the keys necessary to answer the above questions is often obtained via the application of one or more of the conservation laws. The contents of this chapter deal with these laws. The four topics covered are:

The Conservation Laws

The Conservation Law for Momentum

The Conservation Law for Mass

The Conservation Law for Energy

Obviously, at the heart of this chapter is topic (**4**) and, to a lesser extent, topic (**3**); however, some momentum considerations also come into play in part of this book.

Four important terms are defined below before proceeding to the conservation laws:

1. A *system* is any portion of the universe that is set aside for study.
2. Once a system has been chosen, the rest of the universe is referred to as the *surroundings*.
3. A *system* is described by specifying that it is in a certain state.
4. The *path*, or series of values certain variables assume in passing from one state to another, defines a process.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

THE CONSERVATION LAWS

Momentum, energy, and mass are all conserved. As such, each quantity obeys the conservation law below as applied within a system (see also Chapter 2):

$$\left\{\begin{matrix}\text{quantity}\\ \text{into}\\ \text{system}\end{matrix}\right\} - \left\{\begin{matrix}\text{quantity}\\ \text{out of}\\ \text{system}\end{matrix}\right\} + \left\{\begin{matrix}\text{quantity}\\ \text{generated}\\ \text{in system}\end{matrix}\right\} = \left\{\begin{matrix}\text{quantity}\\ \text{accumulated}\\ \text{in system}\end{matrix}\right\} \tag{4.1}$$

This equation may also be written on a *time* rate basis:

$$\left\{\begin{matrix}\text{rate of}\\ \text{quantity}\\ \text{into}\\ \text{system}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{quantity}\\ \text{out of}\\ \text{system}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of}\\ \text{quantity}\\ \text{generated}\\ \text{in system}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{quantity}\\ \text{accumulated}\\ \text{in system}\end{matrix}\right\} \tag{4.2}$$

The above conservation law may be applied at the macroscopic, microscopic, or molecular level.[1,2] Chapter 2 has illustrated the differences between these methods. As noted earlier, this text departs from the molecular approach even though this method has a great deal to commend it. Experience has indicated that the engineer possessing a working knowledge of the conservation laws is likely to obtain a more integrated and unified picture of heat transfer by applying the macroscopic approach. However, the microscopic approach receives some treatment in Chapters 7–9.

THE CONSERVATION LAW FOR MOMENTUM

The general conservation law for momentum on a rate basis is first applied to a volume element:

$$\underbrace{\left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{in by}\\ \text{convection}\end{matrix}\right\}}_{(1)} - \underbrace{\left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{out by}\\ \text{convection}\end{matrix}\right\}}_{(2)} + \underbrace{\left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{in by}\\ \text{molecular}\\ \text{diffusion}\end{matrix}\right\}}_{(3)} - \underbrace{\left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{out by}\\ \text{molecular}\\ \text{diffusion}\end{matrix}\right\}}_{(4)}$$

$$\downarrow \text{ bulk flow} \qquad\qquad \downarrow \text{ velocity gradients}$$

$$+ \underbrace{\left\{\begin{matrix}\text{external forces}\\ \text{exerted on fluid}\end{matrix}\right\}}_{(5)} = \underbrace{\left\{\begin{matrix}\text{rate of momentum}\\ \text{accumulation}\end{matrix}\right\}}_{(6)}$$

$$\downarrow \text{ surface forces and/or body forces} \qquad\qquad \downarrow \text{ inventory}$$

The above enumerates the rate of momentum and forces acting on a moving fluid in the volume element of concern at any time t. Each rate of momentum or force term in the above equation can be expressed in units of lb_f in order to maintain dimensional consistency. It is suggested that the reader refer to the literature for more information.[1,2]

The application of this conservation law finds extensive use in the field of fluid mechanics or fluid flow.[3] Applications in the field of heat transfer, as indicated above, are limited.

ILLUSTRATIVE EXAMPLE 4.1

A 10-cm-diameter horizontal line carries saturated steam at 420 m/s. Water is entrained by the steam at a rate of 0.15 kg/s. The line has a 90° bend. Calculate the force components in the horizontal and vertical directions required to hold the bend in place due to the entrained water.

***SOLUTION*:** A line diagram of the system is provided in Figure 4.1. Select the fluid in the bend as the system and apply the conservation law for mass (see next Section):

$$\dot{m}_1 = \dot{m}_2$$

Since the density and cross-sectional area are constant,

$$V_1 = V_2$$

where $\dot{m}_1, \dot{m}_2$ = mass flowrate at 1 and 2, respectively
V_1, V_2 = velocity at 1 and 2, respectively

A linear momentum ($\dot{M}$) balance in the horizontal direction provides the force applied by the channel wall on the fluid in the x-direction, F_x:

$$F_x g_c = \dot{M}_{x,\text{out}} - \dot{M}_{x,\text{in}}$$
$$= \frac{d}{dt}(mV)_{x,\text{out}} - \frac{d}{dt}(mV)_{x,\text{in}}$$

Note that the above equation assumes that the pressure drop across the bend is negligible.

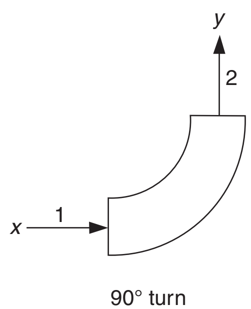

Figure 4.1 Diagram for Illustrative Example 4.1.

Since $V_{x,\text{out}} = 0$ and $dm/dt = \dot{m}$,

$$F_x g_c = 0 - \dot{m}V_{x,\text{in}} = -\frac{(0.15)(420)}{(1)} = -63\,\text{N} = -14.1\,\text{lb}_\text{f}$$

The x-direction supporting force acting on the 90° elbow is 14.1 lb_f acting toward the left.

A linear momentum balance in the vertical direction results in

$$\begin{aligned} F_y g_c &= \dot{M}_{y,\text{out}} - \dot{M}_{y,\text{in}} \\ &= \dot{m}V_{y,\text{out}} - \dot{m}V_{y,\text{in}} \\ &= \dot{m}V_2 - 0 = (0.15)(420) = 63\,\text{N} = 14.1\,\text{lb}_\text{f} \end{aligned}$$

The y-direction supporting force on the 90° elbow is 14.1 lb_f acting upwards. ■

ILLUSTRATIVE EXAMPLE 4.2

Refer to Illustrative Example 4.1. Calculate the magnitude and direction of the resultant force.

***SOLUTION*:** The resultant supporting force is given by:

$$F_\text{res} = \sqrt{F_x^2 + F_y^2}$$

Substituting,

$$F_\text{res} = \sqrt{(-63)^2 + (63)^2} = 89.1\,\text{N} = 19.1\,\text{lb}_\text{f}$$

The direction is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{63}{-63} = -1$$

$$\theta = 135°$$

where θ is the angle between the positive x axis and the direction of the force. The counter-clockwise rotation of the direction from the x axis is defined as positive.

The supporting force is therefore 19.1 lb_f acting in the "northwest" direction. ■

THE CONSERVATION LAW FOR MASS

The *conservation law* for mass can be applied to any process or system. The general form of this law is given as:

$$\text{mass in} - \text{mass out} + \text{mass generated} = \text{mass accumulated} \qquad (4.4)$$

or, on a time rate basis, by

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of mass}\\ \text{generated}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of mass}\\ \text{accumulated}\end{matrix}\right\} \tag{4.5}$$

In heat transfer-related processes, it is often necessary to obtain quantitative relationships by writing mass balances on the various elements in a system. This equation may be applied either to the total mass involved or to a particular species on either a mole or mass basis. This law can be applied to steady-state or unsteady-state (transient) processes and to batch or continuous systems. As noted earlier, in order to isolate a system for study, it is separated from the surroundings by a boundary or envelope. This boundary may be real (e.g., the walls of a heat exchanger) or imaginary. Mass crossing the boundary and entering the system is part of the *mass in* term in Equation 4.5, while that crossing the boundary and leaving the system is part of the *mass out* term. Equation 4.5 may be written for any compound whose quantity is not changed by chemical reaction and for any chemical element whether or not it has participated in a chemical reaction. It may be written for a heat exchanger, one piece of equipment, around several pieces of equipment, or around an entire process. It may be used to calculate an unknown quantity directly, to check the validity of experimental data, or to express one or more of the independent relationships among the unknown quantities in a particular problem situation.

A *steady-state* process is one in which there is no change in conditions (pressure, temperature, composition, etc.) or rates of flow with time at any given point in the system. The accumulation term in Equation 4.5 is then zero. (If there is no chemical or nuclear reaction, the generation term is also zero.) All other processes are *unsteady state*.

In a *batch* process, a given quantity of feed is placed in a container or heat exchanger and a change in temperature can be made to occur. At the end of the process, the container or exchanger holds the heated or cooled product(s). In a *continuous* process, heated or cooled feed is continuously introduced into a heat exchanger, a piece of equipment, or several pieces in series, and products are continuously removed from one or more locations. A continuous process may or may not be steady-state. A coal-fired power plant, for example, operates continuously. However, because of the wide variation in power demand between peak and slack periods, there is an equally wide variation in the rate at which the coal is fired. For this reason, power plant problems may require the use of average data over long periods of time. However, most, but not all, heat transfer operations are assumed to be steady-state and continuous.

As indicated previously, Equation 4.5 may be applied to the total mass of each stream (referred to as an *overall* or *total material balance*) or to the individual component(s) of the stream (referred to as a *componential* or *component material balance*). The primary task in preparing a material balance in heat transfer calculations is often to develop the quantitative relationships among the streams.

Four important processing concepts are *bypass*, *recycle*, *purge*, and *makeup* (see Figure 4.2). With *bypass*, part of the inlet stream is diverted around the heat exchanger or equipment to rejoin the (main) stream after the unit. This stream effectively moves

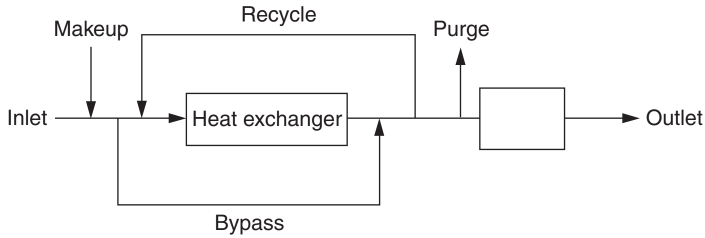

Figure 4.2 Recycle, bypass, and purge.

in parallel with the stream passing through the equipment. In *recycle*, part of the heated or cooled product stream is sent back to mix with the feed stream. If a small quantity of nonreactive material is present in the feed to a process that includes recycle, it may be necessary to remove the nonreactive material in a *purge* stream to prevent it from building up to above a maximum tolerable value. This can also occur in a process without recycle; if a nonreactive material is added in the feed and not totally removed in the products, it will accumulate until purged. The purging process is sometimes referred to as *blowdown. Makeup*, as its name implies, involves adding or making up part of a stream that has been removed from a process. Makeup may be thought of as the opposite of purge and/or blowdown.

ILLUSTRATIVE EXAMPLE 4.3

Fuel is fed into a boiler at a rate of 10,000 lb/h in the presence of 20,000 lb/h of air. Due to the low heating value of the fuel, 2000 lb/h of methane is added to assist in the combustion of the fuel. At what rate (lb/h) do the product gases exit the incinerator?

***SOLUTION*:** Apply the conservation law for mass to the boiler. Assume steady-state conditions to apply:

$$\text{Rate of mass in } (\dot{m}_{\text{in}}) = \text{rate of mass out } (\dot{m}_{\text{out}}) \tag{4.5}$$

Substituting,

$$\begin{aligned} \dot{m}_{\text{in}} &= (10{,}000 + 20{,}000 + 2000) \\ &= 32{,}000 \text{ lb/h} \end{aligned}$$

Therefore,

$$\dot{m}_{\text{out}} = 32{,}000 \text{ lb/h}$$

■

ILLUSTRATIVE EXAMPLE 4.4

A proposed heat exchanger design requires that a packed column and a spray tower are to be used in series for the removal of HCl from a gas feed stream. The spray tower is operating at an efficiency of 65% and the packed column at an efficiency of 98%. Calculate the mass flow rate of

HCl leaving the spray tower, the mass flow rate of HCl entering the packed tower, and the overall fractional efficiency of the removal system if 76.0 lb of HCl enters the system every hour.

***SOLUTION*:** By definition, the efficiency, E, is given by

$$E = (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})/\dot{m}_{\text{in}}$$
$$\dot{m}_{\text{out}} = (1 - E)(\dot{m}_{\text{in}}) \tag{4.6}$$

For the spray tower:

$$\dot{m}_{\text{out}} = (1 - 0.65)(76.0)$$
$$= 26.6 \text{ lb/h HCl}$$

Note that the mass flow rate of HCl leaving the spray tower equals the mass flow rate of HCl entering the packed column.

For the packed column:

$$\dot{m}_{\text{out}} = (1 - 0.98)(26.6)$$
$$= 0.532 \text{ lb/h HCl}$$

The overall fractional efficiency is therefore

$$E = (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})/\dot{m}_{\text{in}}$$
$$= (76.0 - 0.532)/76.0$$
$$= 0.993 = 99.3\%$$

■

ILLUSTRATIVE EXAMPLE 4.5

Consider the flow diagram in Figure 4.3 for a wastewater treatment system. The following flowrate data are given:

$$\dot{m}_1 = 1000 \text{ lb/min}$$
$$\dot{m}_2 = 1000 \text{ lb/min}$$
$$\dot{m}_4 = 200 \text{ lb/min}$$

Calculate the amount of water lost by evaporation in the operation, $\dot{m}$.

***SOLUTION*:** Apply a material balance around the treatment system to determine the value of $\dot{m}_5$. The value of $\dot{m}_5$ is given by:

$$\dot{m}_4 + \dot{m}_5 = \dot{m}_3$$
$$\dot{m}_4 + \dot{m}_5 = \dot{m}_1 + \dot{m}_2$$
$$200 + \dot{m}_5 = 1000 + 1000$$
$$\dot{m}_5 = 1800 \text{ lb/min}$$

Similarly (for tank 2),

$$\dot{m}_6 = \dot{m}_2$$
$$\dot{m}_6 = 1000 \text{ lb/min}$$

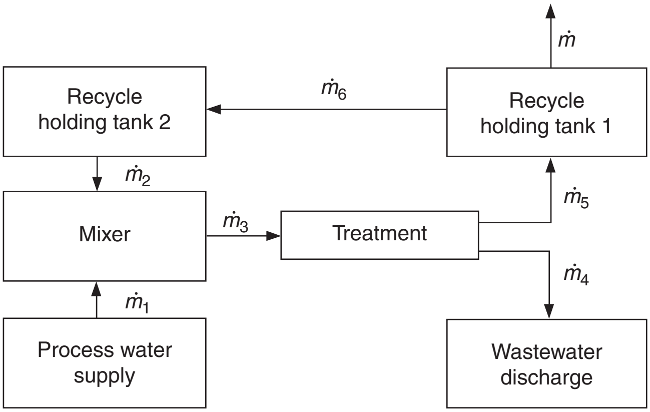

Figure 4.3 Flow diagram for Illustrative Example 4.5.

Thus (for tank 1),

$$\dot{m}_5 + \dot{m} = \dot{m}_6$$
$$1800 - \dot{m} = 1000$$
$$\dot{m} = 800\,\text{lb/min}$$

Therefore, 800 lb of water per minute is lost in the operation. ■

ILLUSTRATIVE EXAMPLE 4.6

Consider the system shown in Figure 4.4. The following volumetric flowrate and phosphate concentration (volume basis) data have been provided by the plant manager. Are the data correct and/or consistent?

$$\begin{aligned} q_1 &= 1000 \text{ gal/day} & C_1 &= 4 \text{ ppm} \\ q_2 &= 1000 \text{ gal/day} & C_2 &= 0 \text{ ppm} \\ q_3 &= 2000 \text{ gal/day} & C_3 &= 2 \text{ ppm} \\ q_4 &= 200 \text{ gal/day} & C_4 &= 20 \text{ ppm} \\ q_5 &= 1800 \text{ gal/day} & C_5 &= 0 \text{ ppm} \\ q_6 &= 1000 \text{ gal/day} & C_6 &= 0 \text{ ppm} \end{aligned}$$

***SOLUTION*:** A componential balance around the mixer (in lb/day) gives (with the conversion factor for water of 120,000 gal/10^6 lb)

$$C_1q_1 + C_2q_2 = C_3q_3$$
$$(4)(1000/120{,}000) + (0)(1000/120{,}000) = (2)(2000/120{,}000)$$
$$4000 = 4000 \qquad \text{OK}$$

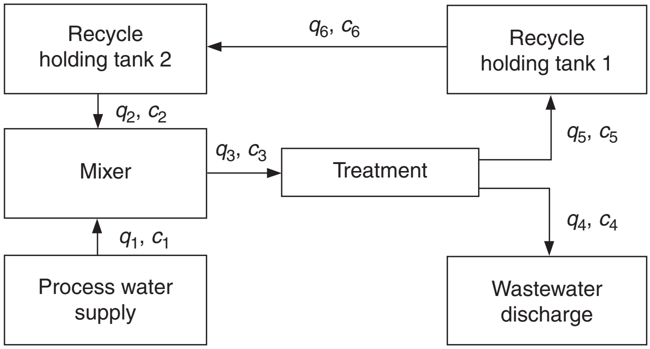

Figure 4.4 Flow diagram for Illustrative Example 4.6.

A balance around the treatment tank gives

$$C_3 q_3 = C_4 q_4 + C_5 q_5$$
$$(2)(2000/120{,}000) = (20)(200/120{,}000) + (0)(1800/120{,}000)$$
$$4000 = 4000 \qquad \text{OK}$$

A balance around holding tank 1 gives

$$C_5 q_5 = C_6 q_6$$
$$(0)(1800) = (0)(1000)$$
$$0 = 0 \qquad \text{OK}$$

A balance around holding tank 2 gives

$$C_2 q_2 = C_6 q_6$$
$$(0)(1000) = (0)(1000)$$
$$0 = 0 \qquad \text{OK}$$

The data appear to be consistent. ■

THE CONSERVATION LAW FOR ENERGY

From the early recognition of energy, man has studied its effects upon objects and its transfer from object to object. This field of study is an integral part of heat transfer. Before proceeding to the first law of thermodynamics, often referred to as the conservation law for energy, certain important terms are defined as follows:

1. *Isothermal* is constant temperature.
2. *Isobaric* is constant pressure.
3. *Isochoric* is constant volume.
4. *Adiabatic* specifies no transfer of heat to or from system(s).

The first law of thermodynamics specifies that energy is conserved. In effect, this law states that energy is neither created nor destroyed. Thus, the change in energy of a system is exactly equal to the negative of the change in the surroundings. For a system of constant mass (a closed system), the only way the system and surroundings may interchange energy is by work and heat. Work and heat are defined as energy in transit. They are not properties and cannot be stored in a system. Two common forms of work are expansion and electrical. Heat is energy in transit because of a temperature difference. This heat transfer may take place by conduction, convection, or radiation, topics to be discussed in Part II.

The energy balance makes use of the conservation law to account for all the energy in a chemical process, or in any other process for that matter. After a system is defined, the energy balance considers the energy entering the system across the boundary, the energy leaving the system across the boundary, and the accumulation of energy within the system. This may be written in a simplified equation form as

$$\text{Energy in} - \text{Energy out} = \text{Energy accumulation} \tag{4.7}$$

This expression has the same form as the general law of conservation of mass as well as the conservation law for momentum. It may also be written on a time rate basis.

All forms of energy must be included in an energy balance. In many processes, certain energy forms remain constant and changes in them may be neglected. However, these forms should be recognized and understood before their magnitude and constancy can be determined. Some forms of energy are easily recognized in everyday life: the energy of a moving object, the energy given off by a fire, and the energy content of a container of hot water. Other forms of energy are less easily recognized. However, the five key energy terms are kinetic, potential, internal, heat, and work. These are briefly described below.

1. *Kinetic energy* (also defined in Chapter 3). The energy of a moving object is called "kinetic energy." A baseball thrown by a pitcher possesses a definite kinetic energy as it travels toward the catcher. A pound of flowing fluid possesses kinetic energy as it travels through a duct.
2. *Potential energy* (also defined in Chapter 3). The energy possessed by a mass by virtue of its position in the Earth's gravitational field is called "potential energy." A boulder lying at the top of a cliff possesses potential energy with reference to the bottom of the cliff. If the boulder is pushed off the cliff, its potential energy is transformed into kinetic energy as it falls. Similarly, a mass of fluid in a flowing system possesses a potential energy because of its height above an arbitrary reference level.
3. *Internal energy*. The component molecules of a substance are constantly moving within the substance. This motion imparts "internal energy" to the material. The molecules may rotate, vibrate, or migrate within the substance. The addition of heat to a material increases its molecular activity and, hence, its internal energy. The temperature of a material is a measure of its internal energy.

4. *Heat*. As noted above, when energy if transferred between a system and its surroundings, it is transferred either as work or as heat. Thus, heat is energy in transit. This type of energy transfer occurs whenever a hot body is brought into contact with a cold body. Energy flows as heat from the hot body to the cold body until the temperature difference is dissipated—that is, until thermal equilibrium is established. For this reason, heat may be considered as energy being transferred due to a temperature difference.
5. *Work*. Work is also energy in transit. Work is done whenever a force acts through a distance.

The first law of thermodynamics may be stated formally—as opposed to equation form—in many ways. One of these is as follows: although energy assumes many forms, the total quantity of energy is constant, and when energy disappears in one form, it must appear simultaneously in other forms.

As noted earlier, application of the conservation law for energy gives rise to the first law of thermodynamics. This law, in steady-state equation form for batch and flow processes, is presented below.

For *batch* processes:

$$\Delta U = Q + W \tag{4.8}$$

For *flow* processes:

$$\Delta H = Q + W_s \tag{4.9}$$

where potential, kinetic, and other energy effects have been neglected and

Q = energy in the form of heat transferred across the boundaries of the system
W = energy in the form of work transferred across the boundaries of the system
W_s = energy in the form of mechanical work transferred across the boundaries of the system
U = internal energy of the system
H = enthalpy of the system (defined below)
ΔU, ΔH = changes in the internal energy and enthalpy, respectively, during the process.

Contrary to an earlier convention, both Q and W (or W_s) are considered positive if added/transferred *to* the system. Also note that for a flow process.

$$\Delta H = Q \tag{4.10}$$

if $W_s = 0$.

The internal energy and enthalpy in Equations (4.8) and (4.9), as well as the other equations in this section, may be on a *mass* basis (i.e., for 1 g or 1 lb of material), on a *mole* basis (i.e., for 1 gmol or 1 lbmol of material), or represent the total internal energy and enthalpy of the entire system. As long as these equations are dimensionally consistent, it makes no difference. For the sake of clarity, the same convention that is used for heat capacities will be employed throughout this text—uppercase letters (e.g., H, U, C_P) represent properties on a mole basis, while lowercase letters (e.g., h, u, c_p) represent properties on a mass basis. Properties for the entire system will rarely be used and therefore require no special symbols.

Perhaps the most important thermodynamic function that the engineer works with is the aforementioned *enthalpy*. The enthalpy is defined by:

$$H = U + PV \tag{4.11}$$

where P = pressure of the system

V = volume of the system

ILLUSTRATIVE EXAMPLE 4.7

A lake is located at the top of a mountain. A power plant has been constructed at the bottom of the mountain. The potential energy of the water traveling downhill can be used to spin turbines and generate electricity. This is the operating mode in the daytime during peak electrical demand. At night, when demand is reduced, the water is pumped back up the mountain. The operation is shown in Figure 4.5.

Using the method of power "production" described above, determine how much power (Watts) is generated by the lake located at an elevation of 3000 ft above the power plant. The flowrate of water is 500,000 gpm. The turbine efficiency is 30%. Neglect friction effects.

Note: This programmed-instructional problem is a modified and edited version (with permission) of an illustrative example prepared by Marie Gillman, a graduate mechanical engineering student at Manhattan College.

***SOLUTION*:** First, convert height and flowrate to SI units in order to solve for the power in Watts:

$$(3000\,\text{ft})(0.3048\,\text{m/ft}) = 914.4\,\text{m}$$

$$(500{,}000\,\text{gal/min})(0.00378\,\text{m}^3/\text{gal}) = 1890\,\text{m}^3/\text{min}$$

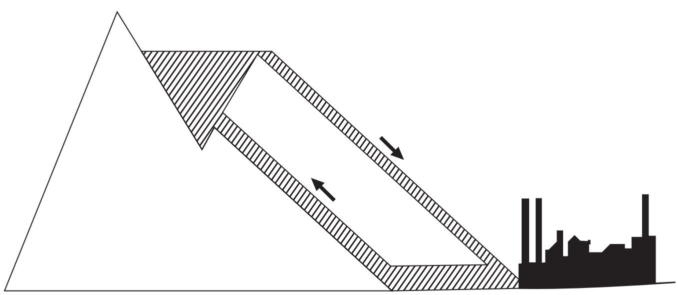

Figure 4.5 Schematic for Illustrative Example 4.7.

The mass flow rate of the water in kilograms/second is

$$\frac{(1890\,\text{m}^3/\text{min})(1000\,\text{kg/m}^3)}{60\,\text{s/min}} = 31{,}500\,\text{kg/s}$$

The loss in potential energy, ΔPE, of the water flow is given by

$$\Delta\text{PE} = \frac{mg\Delta z}{g_c}$$

Substituting yields

$$\begin{aligned}\Delta\text{PE} &= (31{,}500\,\text{kg/s})(9.8\,\text{m/s}^2)(914.4\,\text{m})\\ &= 2.82 \times 10^8\,\text{kg}\cdot\text{m/s}^3\\ &= 2.82 \times 10^8\,\text{N/s}\\ &= 282\text{ MW}\end{aligned}$$

Note that $g_c = 1$ in the SI system of units.

Assuming that the potential energy decrease is entirely converted to energy input to the turbine, the actual power output AP is

$$\text{AP} = (0.30)(282) = 84.6\,\text{MW}$$

This is enough power for a small town. No pollutants or greenhouse gases are generated because no fossil fuel is required. The initial construction expense could be quite high, but the long-term cost of producing electricity might be economical. ■

The flow of heat from a hot fluid to a cooler fluid through a solid wall is a situation often encountered in engineering equipment; examples of such equipment (to be discussed in Part III) are heat exchangers, condensers, evaporators, boilers, and economizers. The heat absorbed by the cool fluid or given up by the hot fluid may be sensible heat, causing a temperature change in the fluid, or it may be latent heat, causing a phase change such as vaporization or condensation. In a typical heat exchanger (for example, a waste heat boiler) hot flue gas gives up heat to water through thin metal tube walls separating the two fluids. As the flue gas loses heat, its temperature drops. As the water gains heat, its temperature quickly reaches the boiling point where it continues to absorb heat with no further temperature rise as it changes into steam. The rate of heat transfer between the two streams, assuming no heat loss due to the surroundings, may be calculated by the enthalpy change of either fluid:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c(h_{c1} - h_{c2}) \tag{4.12}$$

where $\dot{Q}$ is the rate of heat flow (Btu/h); $\dot{m}_h$, mass flow rate of hot fluid (lb/h); $\dot{m}_c$, mass flow rate of hot fluid (lb/h); h_{h1}, enthalpy of entering hot fluid (Btu/lb); h_{h2}, enthalpy of exiting hot fluid (Btu/lb); h_{c1}, enthalpy of entering cold fluid (Btu/lb); h_{c2}, enthalpy of exiting cold fluid (Btu/lb). The reader should also note that upper and lower case letters are employed in the text to represent the hot fluid, i.e., H and h respectively; both C and c are employed for the cold fluid.

Equation (4.12) is applicable to the heat exchange between two fluids whether a phase change is involved or not. In the aforementioned waste heat boiler example, the enthalpy change of the flue gas is calculated from its sensible temperature change:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_h c_{ph}(T_{h1} - T_{h2}) \tag{4.13}$$

where c_{ph} is the heat capacity of the hot fluid (Btu/lb·°F); T_{h1}, temperature of the entering hot fluid (°F); and T_{h2}, temperature of the existing hot fluid (°F). The enthalpy change of the water, on the other hand, involves a small amount of sensible heat to bring the water to its boiling point plus a considerable amount of latent heat to vaporize the water. Assuming all of the water is vaporized and no superheating of the steam occurs, the enthalpy change is

$$\dot{Q} = \dot{m}_c(h_{c2} - h_{c1}) = \dot{m}_c c_{pc}(T_{c2} - T_{c1}) + \dot{m}_c \Delta h_{\text{vap}} \tag{4.14}$$

where c_{pc} is the heat capacity of the cold liquid water (Btu/lb·°F), T_{c1}, temperature of the entering liquid water (°F), T_{c2}, temperature of the exiting steam (°F), and Δh_{vap}, heat of vaporization at T_{c2} of the water (Btu/lb).(4)

ILLUSTRATIVE EXAMPLE 4.8

If 111.4 lbmol/min of an air stream is heated from 200°F to 600°F, calculate the heat transfer rate required to bring about this change in temperature. Use the following enthalpy and average heat capacity data:

$$H_{200^\circ\text{F}} = 1170\ \text{Btu/lbmol}$$
$$H_{600^\circ\text{F}} = 4010\ \text{Btu/lbmol}$$

***SOLUTION*:** Calculate the heat transfer rate, $\dot{Q}$, using enthalpy data:

$$\dot{Q} = \dot{n}\Delta H = 111.4(4010 - 1170) = 3.16 \times 10^5\ \text{Btu/min}$$

The heat transfer rate could also be calculated using the average heat capacity data. This is addressed in the next example. ■

ILLUSTRATIVE EXAMPLE 4.9

Given the mass flow rate of a fluid and its heat capacity, determine the required heat rate to change the fluid from one temperature to another. Data are provided below:

$$\dot{n} = 600\ \text{lbmol/min}$$
$$\bar{C}_{P,\text{AV}} = 0.271\ \text{Btu/lbmol}\cdot{}^\circ\text{F (over the 200–600°F range)}$$
$$T_1 = 200^\circ\text{F}$$
$$T_2 = 600^\circ\text{F}$$

***SOLUTION*:** Write the equation describing the required heat rate, $\dot{Q}$, and solve:

$$\dot{Q} = \dot{n}\bar{C}_{P,\text{AV}}\Delta T = (600)(0.271)(600 - 200) = 65{,}000\ \text{Btu/min}$$ ■

ILLUSTRATIVE EXAMPLE 4.10

Obtain the heat transfer rate in an exchanger if equal mass liquid flow rates are used. The hot fluid is cooled from 94°C to 82°C while the cold fluid is initially at 20°C.

***SOLUTION*:** The final temperature of the cold liquid can be found from direct application of Equation (4.12):

$$\dot{m}_c c_{pc}(T_{c,2} - T_{c,1}) = \dot{m}_h c_{ph}(T_{h,2} - T_{h,1})$$
$$\dot{m}_c c_{pc}(T_{c,2} - 20) = \dot{m}_h c_{ph}(94 - 82)$$

Since

$$\dot{m}_c = \dot{m}_h$$
$$c_{ph} = c_{pc}$$

one can conclude that the temperature change for both liquids are equal. This yields

$$T_{c,2} = 32^\circ\text{F}$$

This and the two previous examples will be revisited in Chapter 14. ■

ILLUSTRATIVE EXAMPLE 4.11

As a gas flows through a cooler, 5.5 MW of heat is transferred from the gas. The average heat capacity of the gas is 1090 J/(kg · °C), the gas mass flow rate, $\dot{m}$, is 9 kg/s and the gas inlet temperature, T_1, is 650°C. For this example, kinetic and potential energy effects are again neglected. Furthermore, there is no shaft work. Determine the gas outlet temperature.

***SOLUTION*:** Since there are no kinetic, potential, or shaft work effects in this flow process, Equation (4.13) applies

$$\dot{Q} = \Delta H$$

where

$$\dot{Q} = \Delta H = \dot{m}\bar{c}_p\,\Delta T = \dot{m}\bar{c}_p(T_2 - T_1)$$

Solving for the gas outlet temperature, T_2,

$$T_2 = \frac{\dot{Q}}{\dot{m}\bar{c}_p} + T_1 = \frac{-5.5 \times 10^{-6}}{9(1090)} + 650 = 89^\circ\text{C}$$

Note that the sign of $\dot{Q}$ is negative since the heat is transferred out from the gas. ■

ILLUSTRATIVE EXAMPLE 4.12

A heat pump takes in 3500 gpm of water at a temperature of 38°F and discharges it back to the lake at 36.2°F. How many Btu are removed from the water per day [C_P for H_2O = 75.4 J/(gmol · °C), $\rho = 62.4\ \text{lb/ft}^3$]?

***SOLUTION*:** The following equation is employed to calculate the heat load:

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) \tag{4.13}$$

where

$$\begin{aligned}\dot{m} &= \frac{(3500\ \text{gal/min})(62.4\ \text{lb/ft}^3)(1440\ \text{min/day})}{7.48\ \text{gal/ft}^3} \\ &= 4.20 \times 10^7\ \text{lb/day}\end{aligned}$$

The heat capacity is converted into consistent units and placed on a mass bases as follows:

$$c_p = \frac{[75.4\ \text{J/(gmol} \cdot {}^\circ\text{C)}](454\ \text{g/lb})}{(1054\ \text{J/Btu})(18\ \text{g/gmol})(1.8^\circ\text{F}/^\circ\text{C})} = 1.00\ \text{Btu/(lb} \cdot {}^\circ\text{F)}$$

Therefore,

$$\begin{aligned}\dot{Q} &= (4.20 \times 10^7\ \text{lb/day})[1.0\ \text{Btu/(lb} \cdot {}^\circ\text{F)}](38 - 36.2^\circ\text{F}) \\ &= 1.36 \times 10^8\ \text{Btu/day}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 4.13

Determine the percentage of a river stream's flow available to an industry for cooling in order that the river temperature does not increase more than 10°F. Fifty percent of the industrial withdrawal is lost by evaporation and the industrial water returned to the river is 60°F warmer than the river.

Note: This problem is a modified and edited version (with permission) of an illustrative example prepared by Ms. Marie Gillman, a graduate mechanical engineering student at Manhattan College.

***SOLUTION*:** Draw a flow diagram representing the process as shown in Figure 4.6. Express the volumetric flow lost by evaporation from the process in terms of that entering the process:

$$q_{\text{lost}} = 0.5q_{\text{in}}$$

Express the process outlet temperature and the maximum river temperature in terms of the upstream temperature:

$$\begin{aligned}T_{\text{out}} &= T_{\text{up}} + 60^\circ\text{F} \\ T_{\text{mix}} &= T_{\text{up}} + 10^\circ\text{F}\end{aligned}$$

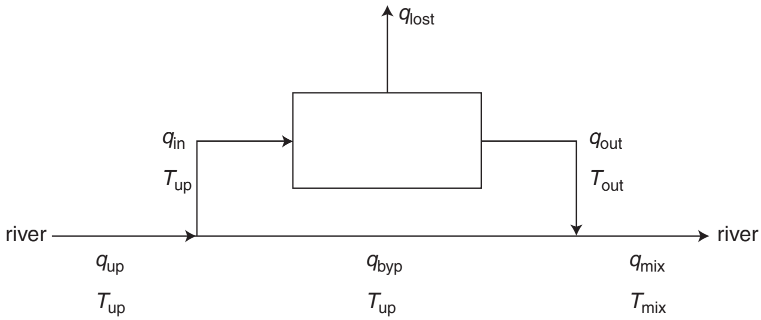

Figure 4.6 Flow diagram for Illustrative Example 4.13.

Using the conservation law for mass, express the process outlet volumetric flow in terms of the process inlet flow. Also express the flow bypassing the process in terms of the upstream and process inlet flows:

$$q_{\text{out}} = 0.5q_{\text{in}}$$
$$q_{\text{byp}} = q_{\text{up}} - q_{\text{in}}$$
$$q_{\text{mix}} = q_{\text{up}} - 0.5q_{\text{in}}$$

The flow diagram with the expressions developed above are shown in Figure 4.7.

Noting that the enthalpy of any stream can be represented by $qc_p\rho(T - T_{\text{ref}})$. An energy balance around the downstream mixing point leads to

$$(q_{\text{up}} - q_{\text{in}})c_p\rho(T_{\text{up}} - T_{\text{ref}}) + 0.5q_{\text{in}}c_p\rho(T_{\text{up}} + 60 - T_{\text{ref}})$$
$$= (q_{\text{up}} - 0.5q_{\text{in}})c_p\rho(T_{\text{up}} + 10 - T_{\text{ref}})$$

Note that T_{ref} is arbitrary and indirectly defines a basis for the enthalpy. Setting $T_{\text{ref}} = 0$ and assuming that density and heat capacity are constant yields

$$(q_{\text{up}} - q_{\text{in}})T_{\text{up}} + 0.5q_{\text{in}}(T_{\text{up}} + 60) = (q_{\text{up}} - 0.5q_{\text{in}})(T_{\text{up}} + 10)$$

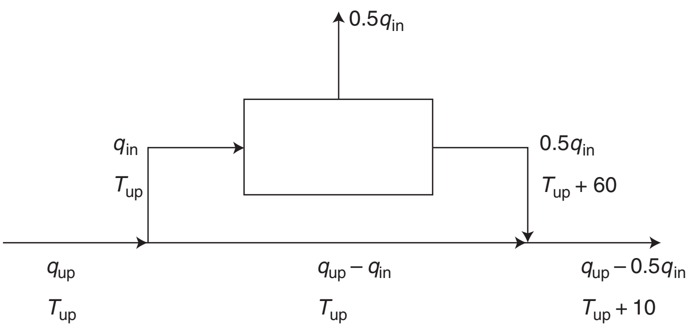

Figure 4.7 Flow diagram after applying mass balances.

The equation may now be solved for the inlet volumetric flow to the process in terms of the upstream flow:

$$
\begin{aligned}
q_{\mathrm{up}}T_{\mathrm{up}} - q_{\mathrm{in}}T_{\mathrm{up}} + 0.5q_{\mathrm{in}}T_{\mathrm{up}} + 30q_{\mathrm{in}} \\
= q_{\mathrm{up}}T_{\mathrm{up}} + 10q_{\mathrm{up}} - 0.5q_{\mathrm{in}}T_{\mathrm{up}} - 5q_{\mathrm{in}}
\end{aligned}
$$

Canceling terms produces

$$
\begin{aligned}
35q_{\mathrm{in}} &= 10q_{\mathrm{up}} \\
q_{\mathrm{in}} &= 0.286q_{\mathrm{up}}
\end{aligned}
$$

Therefore, 28.6% of the original flow, q_{up}, is available for cooling.

Note that the problem can also be solved by setting $T_{\mathrm{ref}} = T_{\mathrm{up}}$. Since for this condition, $T_{\mathrm{ref}} - T_{\mathrm{up}} = 0$, the solution to the problem is greatly simplified. ■

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