Chapter 10

Free Convection

INTRODUCTION

Convective effects, previously described as *forced convection*, are due to the bulk motion of the fluid. The bulk motion is caused by external forces, such as that provided by pumps, fans, compressors, etc., and is essentially independent of "thermal" effects. *Free convection* is another effect that occasionally develops and was briefly discussed in the previous chapter. This effect is almost always attributed to buoyant forces that arise due to density differences within a system. It is treated analytically as another external force term in the momentum equation. The momentum (velocity) and energy (temperature) effects are therefore interdependent; consequently, both equations must be solved simultaneously. This treatment is beyond the scope of this text but is available in the literature.[1]

Consider a heated body in an unbounded medium. In natural convection, the velocity is zero (no-slip boundary condition) at the heated body. The velocity increases rapidly in a thin boundary adjacent to the body, and ultimately approaches zero when significantly displaced from the body. In reality, both natural convection and forced convection effects occur simultaneously so that one may be required to determine which is predominant. Both may therefore be required to be included in some analyses, even though one is often tempted to attach less significance to free convection effects. However, this temptation should be resisted since free convection occasionally plays the more important role in the design and/or performance of some heated systems.

As noted above, free convection fluid motion arises due to buoyant forces. Buoyancy arises due to the combined presence of a fluid density gradient and a body force that is proportional to density. The body force is usually gravity. Density gradients arise due to the presence of a temperature gradient. Furthermore, the density of gases and liquids depends on temperature, generally decreasing with increasing temperature, i.e., $(\partial\rho/\partial T)_P < 0$.

There are both industrial and environmental applications. Free convection influences industrial heat transfer from and within pipes. It is also important in transferring heat from heaters or radiators to ambient air and in removing heat from the coil of a

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

refrigeration unit to the surrounding air. It is also relevant to environmental sciences and engineering where it gives rise to both atmospheric and oceanic motion, a topic treated in the last section of this chapter.

In addition to this Introductory section, this chapter addresses the following three topics:

Key dimensionless numbers

Describing equations

Environmental applications[(2)]

KEY DIMENSIONLESS NUMBERS

If a solid surface at temperature, T_s, is in contact with a gas or liquid at temperature, T_∞, the fluid moves solely as a result of density variations in natural convection. It is the fluid motion that causes the so-called natural convection. The nature of the buoyant force is characterized by the coefficient of volumetric expansion, β. For an ideal gas, β is given by

$$\beta = \frac{1}{T} \tag{10.1}$$

where T is the *absolute* temperature; this is an important term in natural convection theory and applications.

Semi-theoretical equations for natural convection use the following key dimensionless numbers, some of which have been discussed earlier:

$$\text{Gr} = \text{Grashof number} = \frac{L^3 g\beta\Delta T}{\nu^2} = \frac{L^3\rho^3 g\beta\Delta T}{\mu^2} \tag{10.2}$$

$$\text{Nu} = \text{Nusselt number} = hL/k \tag{10.3}$$

$$\text{Ra} = \text{Rayleigh number} = (\text{Gr})(\text{Pr}) = \frac{L^3 g\beta\Delta T}{\nu\alpha} = \frac{L^3 g\beta\Delta T\rho^2 c_p}{\mu k} \tag{10.4}$$

$$\text{Pr} = \text{Prandtl number} = \frac{\nu}{\alpha} = \frac{c_p\mu}{k} \tag{10.5}$$

where ν = kinematic viscosity

μ = absolute viscosity

α = thermal diffusivity $= k/\rho c_p$

ρ = fluid density

c_p = fluid heat capacity

k = thermal conductivity

L = characteristic length of system

ΔT = temperature difference between the surface and the fluid $= |T_s - T_\infty|$

The above Rayleigh number is used to classify natural convection as either laminar or turbulent:

$$\text{Ra} < 10^9 \quad \text{laminar free convection} \tag{10.6}$$

$$\text{Ra} > 10^9 \quad \text{turbulent free convection} \tag{10.7}$$

In the previous chapter on forced convection, the effects of natural convection were neglected, a valid assumption in many applications characterized by moderate-to-high-velocity fluids. Free convection may be significant with low-velocity fluids. A measure of the influence of each convection effect is provided by the ratio

$$\frac{\text{Gr}}{\text{Re}^2} = \frac{\text{buoyancy force}}{\text{inertia force}} = \frac{\rho g \beta L(\Delta T)}{V^2};\ V = \text{fluid velocity} \tag{10.8}$$

This dimensionless number is represented by LT by the author,[(2)] so that

$$\frac{\text{Gr}}{\text{Re}^2} = LT \tag{10.9}$$

For $LT > 1.0$, free convection is important. The regimes of these convection effects are:

$$\text{Free convection predominates, i.e., } LT \gg 1.0 \text{ or Gr} \gg \text{Re}^2 \tag{10.10}$$

$$\text{Forced convection predominates, i.e., } LT \ll 1.0 \text{ or Gr} \ll \text{Re}^2 \tag{10.11}$$

Both effects contribute; mixed free and forced convection, i.e.,

$$LT \approx 1.0 \text{or Gr} \approx \text{Re}^2 \tag{10.12}$$

Combining these three convection regimes with the two flow regimes—laminar and turbulent—produces six subregimes of potential interest.

ILLUSTRATIVE EXAMPLE 10.1

The Grashof and Reynolds numbers for a system involved in a heat transfer process are approximately 100 and 50, respectively. Can free convection effects be neglected.

***SOLUTION*:** Employ Equation (10.9).

$$LT = \frac{\text{Gr}}{\text{Re}^2}$$

Substituting

$$LT = \frac{100}{50^2}$$

$$= 0.04$$

Since $LT \ll 1.0$, free convection effects can be neglected. ■

DESCRIBING EQUATIONS

Free convection is an important consideration in the calculation of heat transfer rates from pipes, transmission lines, electronic devices, and electric baseboards. The average Nusselt number, $\overline{\text{Nu}}$ and the Rayleigh number can be related through the following semi-theoretical correlation:

$$\overline{\text{Nu}} = \bar{h}L/k = c\,\text{Ra}^m \tag{10.13}$$

with all fluid properties evaluated at the film temperature, T_f,

$$T_f = (T_S + T_\infty)/2 \tag{10.14}$$

Generally, but with some exceptions,

$$\begin{aligned} m &= \tfrac{1}{4} \quad \text{for laminar free convection} \\ m &= \tfrac{1}{3} \quad \text{for turbulent free convection} \end{aligned} \tag{10.15}$$

As one might expect, the characteristic length, L, depends on the geometry. For a vertical plate, L is the plate height and for a horizontal plate, the plate length. For a horizontal cylinder, L is the diameter, D, and for a horizontal disk, L is given by: $L = 0.9D$. The constants C and m to be used with Equation (10.13) are listed in Table 10.1 for several geometries and a wide range of Rayleigh numbers.

Another correlation that can be used to calculate the heat transfer coefficient for natural convection from spheres is Churchill's equation,

$$\overline{\text{Nu}} = 2 + \frac{0.589\,\text{Ra}^{0.25}}{\left[1 + \left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} \tag{10.16}$$

Table 10.1 Coefficients for Equation (10.13)

Geometry	Gr Pr = Ra	c	m
Vertical planes and cylinders	10^4–10^9	0.59	0.25
	10^9–10^{13}	0.10	0.3333
Horizontal cylinders	0–10^{-5}	0.4	0
	10^{-5}–10^4	0.85	0.188
	10^4–10^9	0.53	0.25
	10^9–10^{12}	0.13	0.3333
Spheres	0–10^{12}	0.60	0.25
Upper surface of horizontal heated plates; plate is hotter than surroundings ($T_S > T_\infty$) or lower surface of horizontal cooled plates ($T_S < T_\infty$)	2×10^4–8×10^6	0.54	0.25
	8×10^6–10^{11}	0.15	0.3333
Lower surface of horizontal heated plates ($T_S > T_\infty$) or upper surface of horizontal cooled ($T_S < T_\infty$) plates	10^5–10^{11}	0.58	0.2

The Rayleigh number, Ra, and the Nusselt number, Nu, in Equation (10.16) are based on the diameter of the sphere. Churchill's equation is valid for $\mathrm{Pr} \geq 0.7$ and $\mathrm{Ra} \leq 10^{11}$.

There are also *simplified* correlations for natural (or free) convection in air at 1 atm. The correlations are dimensional and are based on the following SI units: $h =$ heat transfer coefficient, $\mathrm{W/m^2 \cdot K}$; $\Delta T = T_S - T_\infty$, °C; $T_s =$ surface temperature, °C; $T_\infty =$ surroundings temperature, °C; $L =$ vertical or horizontal dimension, m and $D =$ diameter, m. These correlations are presented in Table 10.2.

The average heat transfer coefficient can be calculated once the Nusselt number has been determined. Rearranging Equation (10.13) gives

$$\bar{h} = \overline{\mathrm{Nu}}\, k/L \tag{10.17}$$

The heat transfer rate, $\dot{Q}$, is given by the standard heat transfer equation:

$$\dot{Q} = \bar{h}A(T_S - T_\infty) \tag{10.18}$$

If the air is at a pressure other than 1 atm, the following correction may be applied to the reference value at 1 atm:

$$h = h_{\mathrm{ref}}(P \text{ in atmospheres})^n \tag{10.19}$$

where

$$\begin{aligned} n &= \tfrac{1}{2} \text{ for laminar cases } (\mathrm{Ra} < 10^9) \\ n &= \tfrac{2}{3} \text{ for turbulent cases } (\mathrm{Ra} > 10^9) \end{aligned} \tag{10.20}$$

Table 10.2 Free Convection Equation in Air

Geometry	$10^4 < \mathrm{Gr\,Pr} = \mathrm{Ra} < 10^9$ Laminar	$\mathrm{Gr\,Pr} = \mathrm{Ra} > 10^9$ Turbulent
Vertical planes and cylinders	$h = 1.42\ (\Delta T/L)^{0.25}$	$h = 0.95\ (\Delta T)^{0.333}$
Horizontal cylinders	$h = 1.32\ (\Delta T/D)^{0.25}$	$h = 1.24\ (\Delta T)^{0.333}$
Upper surface of horizontal heated plates; plate is hotter than surroundings ($T_S > T_\infty$) or lower surface of horizontal cooled plates ($T_S > T_\infty$)	$h = 1.32\ (\Delta T/L)^{0.25}$	$h = 1.43\ (\Delta T/L)^{0.333}$
Lower surface of horizontal heated plates or upper surface of horizontal cooled plates ($T_S < T_\infty$) plates	$h = 0.61\ (\Delta T/L^2)^{0.2}$	

ILLUSTRATIVE EXAMPLE 10.2

The heat flux rate incident on a vertical flat plate at 110°C is 800 W/m^2. The plate is 2 m wide and 3.5 m high and is well insulated on the back side. The ambient air temperature is 30°C. All the incident radiation (800 W/m^2) on the plate is absorbed and dissipated by free convection to the ambient air at 30°C. Determine the Grashof and Rayleigh numbers.

***SOLUTION*:** Obtain ν, k, and Pr from the Appendix for air at the film temperature, T_f.

$$\begin{aligned} T_f &= (T_S + T_\infty)/2 \\ &= (110 + 30)/2 \\ &= 70^\circ\text{C} = 343\,\text{K} \end{aligned}$$

$$\begin{aligned} \nu &= 2.0 \times 10^{-5}\ \text{m}^2/\text{s} \\ k &= 0.029\ \text{W/m}\cdot\text{K} \\ \text{Pr} &= 0.7 \end{aligned}$$

Calculate the coefficient of expansion β from Equation (10.1).

$$\begin{aligned} \beta &= \frac{1}{343} \\ &= 0.0029\,\text{K}^{-1} \end{aligned}$$

Calculate the Grashof and Rayleigh numbers:

$$\text{Gr} = g\beta\Delta T L^3/\nu^2 \tag{10.2}$$

$$= \frac{(9.807)(0.0029)(80)(3.5)^3}{(2.0 \times 10^{-5})^2}$$

$$= 2.44 \times 10^{11}$$

$$\text{Ra} = (\text{Gr})(\text{Pr}) \tag{10.4}$$

$$= (2.44 \times 10^{11})(0.7)$$

$$= 1.71 \times 10^{11}$$

■

ILLUSTRATIVE EXAMPLE 10.3

Refer to Illustrative Example 10.2. Determine the type of natural convection (flow regime).

***SOLUTION*:** Determine the free convection flow type. Refer to Table 10.2.

$$\text{Since Ra} > 10^9, \text{ the convection flow catagory is turbulent.}$$

■

ILLUSTRATIVE EXAMPLE 10.4

Refer to Illustrative Example 10.2. Determine the average heat transfer coefficient.

***SOLUTION*:** Calculate the average Nusselt number. From Table 10.1, $c = 0.1$ and $m = \frac{1}{3}$. Therefore,

$$\overline{\mathrm{Nu}} = c\,\mathrm{Ra}^m \tag{10.13}$$

Substituting,

$$= 0.1(1.71 \times 10^{11})^{1/3}$$
$$= 555.0$$

Calculate the average heat transfer coefficient employing Equation (10.13):

$$\bar{h} = (\overline{\mathrm{Nu}})(k)/(L)$$
$$= (555.1)(0.029)/3.5$$
$$= 4.6\,\mathrm{W/m^2 \cdot K}$$

■

ILLUSTRATIVE EXAMPLE 10.5

Comment on the calculations performed in the previous example.

***SOLUTION*:** The approximate correlations for natural air convection provided in Table 10.2 could simplify the calculations of the heat transfer rate. This is left as an exercise for the reader. [Hint: the answer in approximately $4.0\,\mathrm{W/m^2 \cdot K}$] ■

ILLUSTRATIVE EXAMPLE 10.6 (adapted from Holman[3])

Calculate the air heat transfer film coefficient for a horizontal 6 inch diameter pipe whose surface temperature is 200°F in a room containing air at 70°F.

***SOLUTION*:**

$$T_{\mathrm{av}} = (200 + 70)/2 = 135^\circ\mathrm{F}$$

The fluid volume density is obtained directly from the ideal gas law:

$$v_f = RT/P = (0.73)(135 + 460)/(1.0) = 434\ \mathrm{ft^3/lbmol}$$

The mass density of the fluid is

$$\rho_f(\mathrm{air},\ \mathrm{MW} = 29) = 29/434 = 0.0668\,\mathrm{lb/ft^3}$$

At 70°F (from the Appendix)

$$\mu = 0.019\,\mathrm{cP} = 1.28 \times 10^{-5}\,\mathrm{lb/\,ft \cdot s}$$
$$k = 0.016\,\mathrm{Btu/h \cdot\ ft \cdot {}^\circ F}$$

Calculate the Grashof number; employ Equation (10.2). Since

$$\beta = 1/T_{av} = 1/(135 + 460) = 0.0017\,^{\circ}\mathrm{R}^{-1}$$

$$\mathrm{Gr} = \frac{D^3\rho^2 g\beta\Delta T}{\mu^2} = \frac{(0.5)^3(0.0668)^2(32.174)(0.017)(660 - 530)}{(1.28 \times 10^{-5})^2} = 24.2 \times 10^7$$

The Prandtl number may also be calculated. At 135°F, $c_p = 0.25$ Btu/lb · °F. Substituting into Equation (10.5),

$$\mathrm{Pr} = \frac{c_p\mu}{k} = \frac{(0.25)(1.28 \times 10^{-5})}{0.016/3600} = 0.72$$

Note that for air, Pr is generally taken as 0.7 (see earlier examples). The following term is calculated:

$$\log_{10}[(\mathrm{Gr})(\mathrm{Pr})] = 8.24$$

From Holman,[3] at this value

$$\log_{10}(\mathrm{Nu}) \cong 1.5$$

so that

$$\mathrm{Nu} = 31.6$$

Since

$$\mathrm{Nu} = \frac{hD}{k} \tag{10.13}$$

$$h = \mathrm{Nu}\left(\frac{k}{D}\right) = 31.6\left(\frac{0.016}{0.5}\right) = 1.01\ \mathrm{Btu/h\cdot ft\cdot {}^{\circ}F}$$ ■

ILLUSTRATIVE EXAMPLE 10.7

Calculate the free convection heat transfer coefficient for a plate 6 ft high and 8 ft wide at 120°F that is exposed to nitrogen at 60°F.

***SOLUTION*:** The mean film temperature is

$$T_f = (120 + 60)/2 = 90^{\circ}\mathrm{F} = 550^{\circ}\mathrm{R}$$

From the Appendix,

$$\rho = 0.0713\ \mathrm{lb/ft^3}$$
$$k = 0.01514\ \mathrm{Btu/h\cdot ft\cdot {}^{\circ}F}$$
$$\nu = 16.82 \times 10^{-5}\ \mathrm{ft^2/s}$$
$$\mathrm{Pr} = 0.713$$

In addition,

$$\beta = 1/T = 1/550 = 1.818 \times 10^{-3}\,^{\circ}\mathrm{R}^{-1}$$

Therefore

$$\text{Gr} = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2} \tag{10.2}$$

Substituting,

$$\text{Gr} = \frac{(32.2\ \text{ft/s}^2)(1.818 \times 10^{-3}\ {}^\circ\text{R}^{-1})[(120 - 40)^\circ\text{R}](6\ \text{ft})^3}{(16.82 \times 10^{-5})^2\ \text{ft}^4/\text{s}^2} = 3.576 \times 10^{10}$$

In addition, from Equation (10.4),

$$\text{Ra} = (\text{Gr})(\text{Pr}) = (3.576 \times 10^{10})(0.713) = 2.549 \times 10^{10}$$

The flow is therefore turbulent (see Table 10.2). Equation (10.13) applies, with appropriate constants from Table 10.1, to give

$$\frac{\bar{h}L}{k} = (0.10)(\text{Ra})^{1/3}$$

$$\bar{h} = \frac{0.01514\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}}{6\ \text{ft}}(0.10)(2.549 \times 10^{10})^{1/3}$$

$$= 0.743\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 10.8

Calculate the heat loss in the previous example.

***SOLUTION*:** Apply Equation (10.18).

$$\dot{Q} = \bar{h}A(T_S - T_\infty)$$

$$= \left(0.743\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}\right)[(6 \times 8)\ \text{ft}^2][(120 - 60)^\circ\text{F}]$$

$$= 2140\ \text{Btu/h}$$

■

ILLUSTRATIVE EXAMPLE 10.9

Calculate the heat transfer from a 100-W light bulb at 113°C to 31°C ambient air. Approximate the bulb as a 120-mm-diameter sphere.

***SOLUTION*:** For this example,

$$T_f = (T_S + T_\infty)/2 = 72^\circ\text{C}$$

From the Appendix,

$$\nu = (22.38 \times 10^{-5})(0.0929) = 2.079 \times 10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.70$$
$$k = (0.01735)(1.729) = 0.0300\ \text{W/m} \cdot \text{K}$$
$$\beta = 1/T = 1/345 = 2.899 \times 10^{-3}\ \text{K}^{-1}$$

Employ the characteristic length as the diameter of the sphere, D, in Equation (10.2).

$$\begin{aligned}\text{Gr} &= \frac{g\beta(T_S - T_\infty)D^3}{\nu^2} \\ &= \frac{(9.80\ \text{m/s}^2)(2.899 \times 10^{-3}\ \text{K}^{-1})(113 - 31)(\text{K})(0.060\ \text{m})^3}{(2.079 \times 10^{-5})^2\ \text{m}^4/\text{s}^2} \\ &= 1.16 \times 10^6\end{aligned}$$

Apply Equation (10.13) with constants drawn from Table 10.1, i.e.,

$$\begin{aligned}\frac{\overline{h}D}{k} &= (0.60)(\text{Ra})^{1/4} \\ \overline{h} &= \frac{0.0300\ \text{W/m} \cdot \text{K}}{0.060\ \text{m}}(0.60)[(1.16 \times 10^6)(0.7)]^{1/4} \\ &= 9.01\ \text{W/m}^2 \cdot \text{K}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.10

Refer to Illustrative Example 10.9. Calculate the heat transfer lost by free convection from the light bulb.

***SOLUTION*:** Once again, apply Equation (10.18)

$$\begin{aligned}\dot{Q} &= \overline{h}A(T_S - T_\infty) \\ &= \left(9.01\ \text{W/m}^2 \cdot \text{K}\right)\pi(0.060\ \text{m})^2(82\ \text{K}) \\ &= 8.36\ \text{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.11

With reference to Illustrative Examples 10.9–10.10, what percentage of the energy is lost by free convection.

***SOLUTION*:**

$$\text{Energy lost} = \frac{8.36}{100}(100\%) = 8.36\% = 0.0836$$

■

ILLUSTRATIVE EXAMPLE 10.12

How else could Illustrative Examples 10.9–10.11 have been solved.

***SOLUTION*:** Employ Equation (10.16) rather than Equation (10.13). This is left as an exercise for the reader ■

ENVIRONMENTAL APPLICATIONS[2]

Two applications that involve the environment comprise the concluding section of this chapter. The first is concerned with lapse rates and the other with plume rise.

Lapse Rates

The concept behind the so-called *lapse rate* that has found application in environmental science and engineering can be best demonstrated with the following example. Consider the situation where a fluid is enclosed by two horizontal plates of different temperature ($T_1 \neq T_2$), see Figure 10.1. In case (a), the temperature of the lower plate exceeds that of the upper plate ($T_2 > T_1$) and the density decreases in the (downward) direction of the gravitational force. If the temperature difference exceeds a particular value, conditions are termed *unstable* and buoyancy forces become important. In Figure 10.1(a), the gravitational force of the cooler and denser fluid near the top plate exceeds that acting on the lighter hot fluid near the bottom plate and the circulation pattern as shown on the right-hand side of Figure 10.1(a) will exist. The cooler heavier fluid will descend, being warmed in the process, while the lighter hot fluid will rise, cooling as it moves. However, for case (b) where $T_1 > T_2$, the density no longer decreases in the direction of the gravitational force. Conditions are now

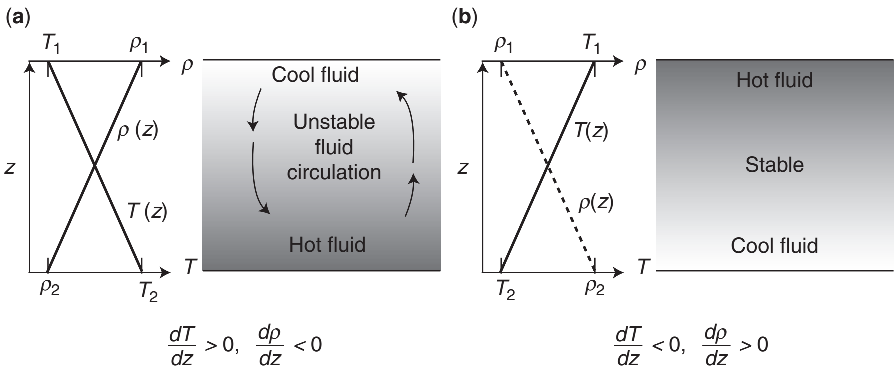

Figure 10.1 A fluid contained between two horizontal plates at different temperatures. (**a**) Unstable temperature gradient. (**b**) Stable temperature gradient.

reversed and are defined as *stable* since there is no bulk fluid motion. In case (*a*), heat transfer occurs from the bottom to the top surface by free convection; for case (*b*), any heat transfer (from top to bottom) occurs by conduction. These two conditions are similar to that experienced in the environment, particularly with regard to the atmosphere. The extension of the above development is now applied to the atmosphere but could just as easily be applied to oceanographic systems.[(2)]

Apart from the mechanical interference of the steady flow of air caused by buildings and other obstacles, the most important factor that determines the degree of turbulence, and hence how fast diffusion in the lower air occurs, is the variation of temperature with height above the ground (i.e., the aforementioned "lapse rate"). Air is a good insulator. Therefore, heat transfer in the atmosphere is caused by radiative heating or by mixing due to turbulence. If there is no mixing, an air parcel rises adiabatically (no heat transfer) in the atmosphere.

The Earth's atmosphere is normally treated as a perfect gas mixture. If a moving air parcel is chosen as a control volume, it will contain a fixed number of molecules. The volume of such an air parcel must be inversely proportional to the density. The perfect gas law is

$$PV \propto RT$$

or

$$P \propto \rho RT \tag{10.21}$$

The air pressure at a fixed point is caused by the weight of the air above that point. The pressure is highest at the Earth's surface and decreases with altitude. This is a hydrostatic pressure distribution with the change in pressure proportional to the change in height.[(3)]

$$dP = -\rho g \, dz \tag{10.22}$$

Since air is compressible, the density is also a decreasing function of height. An air parcel must have the same pressure as the surrounding air and so, as it rises, its pressure decreases. As the pressure drops, the parcel must expand adiabatically. The work done in the adiabatic expansion ($P\,dV$) comes from the thermal energy of the air parcel.[(4)] As the parcel expands, the internal energy then decreases and the temperature decreases.

If a parcel of air is treated as a perfect gas rising in a hydrostatic pressure distribution, the rate of cooling produced by the adiabatic expansion can be calculated. The rate of expansion with altitude is fixed by the vertical pressure variation described in Equation (10.22). Near the Earth's surface, a rising air parcel's temperature normally decreases by 0.98°C with every 100-m increase in altitude.

The vertical temperature gradient in the atmosphere (the amount the temperature changes with altitude, dT/dz) is defined as the *lapse rate*. The dry *adiabatic lapse rate*

(DALR) is the temperature change for a rising parcel of dry air. The dry adiabatic lapse rate is approximately $-1^{\circ}\mathrm{C}/100\,\mathrm{m}$ or $dT/dz = -10^{-2\circ}\mathrm{C/m}$ or $-5.4^{\circ}\mathrm{F}/1000\,\mathrm{ft}$. Strongly stable lapse rates are commonly referred to as *inversions*; $dT/dz > 0$. The strong stability inhibits mixing. Normally, these conditions of strong stability only extend for several hundred meters vertically. The vertical extent of the inversion is referred to as the inversion height. Thus, a positive rate is particularly important in air pollution episodes because it limits vertical motion (i.e., the inversion traps the pollutants between the ground and the inversion layer).

Ground-level inversions inhibit the downward mixing of pollutants emitted from automobiles, smoke stacks, etc. This increases the ground-level concentrations of pollutants. At night the ground reradiates the solar energy that it received during the day. On a clear night with low wind speeds, the air near the ground is cooled and forms a ground-level inversion. By morning, the inversion depth may be 200–300 m with a 5–10° temperature difference from bottom to top. Clouds cut down the amount of heat radiated by the ground because they reflect the radiation back to the ground. Higher wind speeds tend to cause more mixing and spread the cooling effect over a larger vertical segment of the atmosphere, thus decreasing the change in lapse rate during the night.(4)

Since temperature inversions arise because of solar radiation, the effects of nocturnal radiation often results in the formation of frost. When the Sun is down, some thermal radiation is still received by the Earth's surface from space, but the amount is small. Consequently, there is a net loss of radiation energy from the ground at night. If the air is relatively still, the surface temperature may drop below 32°F. Thus, frost can form, even though the air temperature is above freezing. This frost is often avoided with the presence of a slight breeze or by cloud cover.

Plume Rise

Smoke from a stack will usually rise above the top of the stack for a certain distance (see Figure 10.2). The distance that the plume rises above the stack is called the *plume rise*. It is actually calculated as the distance to the imaginary centerline of the plume rather than to the upper or lower edge of the plume. Plume rise, normally denoted Δh, depends on the stack's physical characteristics and on the effluent's (stack gas) characteristics. For example, the effluent characteristic of stack gas temperature T_s in relation to the surrounding air temperature T_a is more important than the stack characteristic of height. The difference in temperature between the stack gas and ambient air determines plume density, and this density affects plume rise. Therefore, smoke from a short stack could climb just as high as smoke from a taller stack.

Stack characteristics are used to determine momentum, and effluent characteristics are used to determine buoyancy. The *momentum* of the effluent is initially provided by the stack. It is determined by the speed of the effluent as it exits the stack. As momentum carries the effluent out of the stack, atmospheric conditions begin to affect the plume.

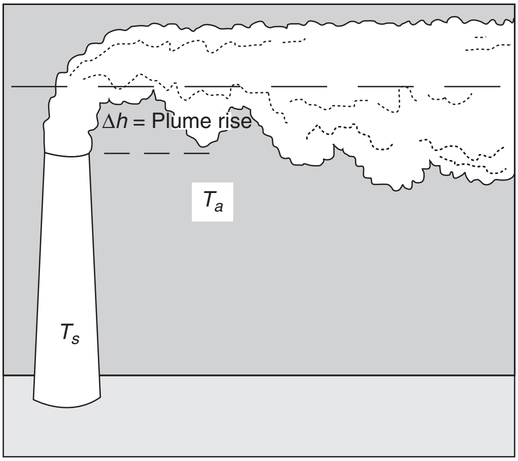

Figure 10.2 Plume rise.

The condition of the atmosphere, including the winds and temperature profile along the path of the plume, will primarily determine the plume's rise. As the plume rises from the stack, the wind speed across the stack top begins to tilt (or bend) the plume. Wind speed usually increases with distance above the Earth's surface. As the plume continues upward, stronger winds tilt the plume even farther. This process continues until the plume may appear to be horizontal to the ground. The point where the plume appears level may be a considerable distance downwind from the stack.

Plume buoyancy is a function of temperature. When the effluent's temperature, T_s, is warmer than the atmosphere's temperature, T_a, the plume will be less dense than the surrounding air. In this case, the density difference between the plume and air will cause the plume to rise. The greater the temperature difference, ΔT, the more buoyant the plume. As long as the temperature of the pollutant remains warmer than the atmosphere, the plume will continue to rise. The distance downwind where the pollutant cools to atmospheric temperature may also be quite displaced from its original release point.

Buoyancy is taken out of the plume by the same mechanism that tilts the plume over—the wind. The faster the wind speed, the faster this mixing with outside air takes place. This mixing is called *entrainment*. Strong wind will "rob" the plume of its buoyancy rapidly and, on windy days, the plume will not climb significantly above the stack.

Many individuals have studied plume rise over the years. The most popular plume rise formulas in use are those of Briggs.[(5)] Most plume rise equations are used on plumes with temperatures greater than the ambient air temperature. The equations proposed by Briggs are presented below in Illustrative Example 10.13. Plume rise formulas determine the imaginary *centerline* of the plume; the centerline is located where the *greatest concentration* of pollutant occurs at a given downward distance. Finally, plume rise is a linear measurement, usually expressed in feet or meters.

ILLUSTRATIVE EXAMPLE 10.13[6,7]

If a waste source emits a gas with a buoyancy flux of 50 m^4/s^3, and the wind speed averages 4 m/s, find the plume rise at a distance of 750 m downward from a stack that is 50 m high under unstable atmospheric conditions. Several plume rise equations are available. Use the equation proposed by Briggs.

Briggs[5] used the following equations to calculate the plume rise:

$$\Delta h = 1.6F^{1/3}u^{-1}x^{2/3}; \quad x < x_f \tag{10.23}$$

$$= 1.6F^{1/3}u^{-1}x_f^{2/3}; \quad \text{if } x \geq x_f$$

$$x^* = 14F^{5/8}; \quad \text{when } F < 55\,\text{m}^4/\text{s}^3$$

$$= 34F^{2/5}; \quad \text{when } F \geq 55\,\text{m}^4/\text{s}^3$$

$$x_f = 3.5x^*$$

where Δh = plume rise, m
F = buoyancy flux, $m^4/s^3 = 3.7 \times 10^{-5}\dot{Q}_H$
u = wind speed, m/s
x^* = downward distance, m
x_f = distance of transition from first stage of rise to the second stage of rise, m
$\dot{Q}_H$ = heat emission rate, kcal/s

If the term $\dot{Q}_H$ is not available, the term F may be estimated by

$$F = (g/\pi)q(T_S - T)/T_S \tag{10.24}$$

where g = gravity term 9.8 m/s^2
q = stack gas volumetric flowrate, m^3/s (actual conditions)
T_S, T = stack gas and ambient air temperature, K, respectively

***SOLUTION*:** Calculate x_f to determine which plume equation applies

$$x^* = 14F^{5/8}, \quad \text{since } F \text{ is less than } 55\,\text{m}^4/\text{s}^3 \tag{10.23}$$

$$= (14)(50)^{5/8}$$

$$= 161.43\,\text{m}$$

$$x_f = 3.5x^*$$

$$= (3.5)(161.43)$$

$$= 565.0\,\text{m}$$

The plume rise is therefore ($x > x_f$)

$$\Delta h = 1.6F^{1/3}u^{-1}x_f^{2/3}, \quad \text{since } x \geq x_f \tag{10.23}$$

$$= (1.6)(50)^{1/3}(4)^{-1}(565)^{2/3} = 101\,\text{m}$$

■

ILLUSTRATIVE EXAMPLE 10.14

Briefly discuss other plume rise equations.

SOLUTION: Many other plume rise equations may be found in the literature.[4,7,8] The Environmental Protection Agency (EPA) is mandated to use Brigg's equations to calculate plume rise. In past years, industry has often chosen to use the Holland or Davidson–Bryant equation. The Holland equation is[4,7,8]

$$\Delta h = D_s(v_s/u)[1.5 + 2.68 \times 10^{-3} P(\Delta T/T_s)/d_s] \tag{10.25}$$

where
D_s = inside stack diameter, m
v_s = stack exit velocity, m/s
u = wind speed, m/s
P = atmospheric pressure, mbar
T_s, T = stack gas and ambient temperature, respectively, K
$\Delta T = T_S - T$
Δh = plume rise, m

The Davidson–Bryant equation is[9]

$$\Delta h = D_s(v_s/u)^{1.4}\{1.0 + [(T_s + T)/T_s]\}$$

The reader should also note that the "plume rise" may be negative in some instances due to surrounding structures, topography, and so on. ■

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