Chapter 11

Radiation

INTRODUCTION

In addition to conduction and convection, heat can be transmitted by radiation. Conduction and convection both require the presence of molecules to "carry" or pass along energy. Unlike conduction or convection, radiation does not require the presence of any medium between the heat source and the heat sink since the thermal energy travels as electromagnetic waves. This radiant energy (thermal radiation) phenomena is emitted by every body having a temperature greater than absolute zero. Quantities of radiation emitted by a body are a function of both temperature and surface conditions, details of which will be presented later in this chapter. Applications of thermal radiation include industrial heating, drying, energy conversion, solar radiation, and combustion.

The amount of thermal radiation emitted is not always significant. Its importance in a heat transfer process depends on the quantity of heat being transferred simultaneously by the other aforementioned mechanisms. The reader should note that the thermal radiation of systems operating at or below room temperature is often negligible. In contrast, thermal radiation tends to be the principal mechanism for heat transfer for systems operating in excess of 1200°F. When systems operate between room temperature and 1200°F, the amount of heat transfer contributed by radiation depends on such variables as the convection film coefficient and the nature of the radiating surface.

Radiation heat transfer in an industrial boiler from the hot gases to most solid surfaces inside the combustion chamber is considerable. However, in most heat exchangers, the contribution of radiation as a heat transfer mechanism is usually minor.

In the heat transfer mechanisms of conduction and convection discussed in Chapters 7–10, the movement of energy in the form of heat takes place through a material medium—a fluid in the case of convection. Since a transfer medium is not required for this third mechanism, the energy is carried by electromagnetic radiation. Thus, a piece of steel plate heated in a furnace until it is glowing red and then placed several inches away from a cold piece of steel plate will cause the temperature of the cold steel to rise, even if the process takes place in an evacuated container.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

As noted above, radiation generally becomes important as a heat transfer mechanism only when the temperature of the source is very high. As will become clear later in the chapter, the energy transfer is approximately proportional to the fourth power of the *absolute* temperature of the radiating body. However, the driving force for conduction and convection is simply the temperature difference (driving force) between the source and the receptor; the actual temperatures have only a minor influence. For these two mechanisms, it does not matter whether the temperatures are 120°F and 60°F or 520°F and 460°F. Radiation, on the other hand, is strongly influenced by the temperature level; as the temperature increases, the extent of radiation as a heat transfer mechanism increases rapidly. It therefore follows that, at very low temperatures, conduction and convection are the major contributors to the total heat transfer; however, at very high temperatures, radiation is often the prime contributor.

ILLUSTRATIVE EXAMPLE 11.1

Qualitatively discuss radiation.

***SOLUTION*:**

1. Radiation is energy transported in the form of electromagnetic waves at the velocity of light.
2. In contrast to conduction and convection, radiation does not require matter for the transfer of energy; for example, radiation can be transferred through a vacuum.
3. Molecules and atoms emit electromagnetic waves when at high energy levels corresponding to high temperatures; this release of energy allows for the atom or molecule to return to a lower energy state. ■

Knowledge of the mechanism of heat transfer is essential in selecting an appropriate heat transfer equation, which is the first step in designing heaters, coolers, condensers, evaporators, and so on. In analyzing the performance of a thermal system, the engineer must therefore be able to identify the relevant heat transfer process (there are generally more than one). Only then can the system behavior be properly quantified. These mechanisms are discussed in the following illustrative example.

ILLUSTRATIVE EXAMPLE 11.2

Identify the pertinent heat transfer processes for the following systems.

1. A heat exchanger made of metal tubing with the fluid inside the tube hotter than the outside fluid.
2. An insulation blanket placed around a tank of Liquefied Nitrogen Gas (LNG).
3. Air flowing across a heated radiator.

4. The heat transfer process that determine the temperature of an asphalt pavement on a breezy summer's day.
5. A thermocouple junction is used to measure the temperature of a hot gas stream flowing through a channel by inserting the junction into the mainstream of the gas. Identify the heat transfer process associated with the junction surface.
6. With respect to (5), will the junction sense a temperature that is less than, equal to, or greater than the gas temperature?

***SOLUTION*:**

1. In scenario 1, metallic solids transfer heat by conduction. The metal tube wall conducts heat from the hot fluid, through the metal wall, to the cold fluid.
2. In scenario 2, solid insulators also transfer heat by conduction. The insulation blanket conducts heat from the warmer outside air to the colder metal of the tank. Because of the high thermal resistance of the blanket, the rate of transfer is slow.
3. In scenario 3, heat is transferred by natural convection. If the air currents are caused by an external force, the heat transfer from the radiator surface to the air is by forced convection.
4. Pathways for energy to and from asphalt paving for (4) include incident solar radiation (a large portion of which is absorbed by the asphalt). Asphalt emits heat by radiation to the surroundings and also loses heat by convection to the air.
5. Pathways for energy to and from the thermocouple with respect to scenario (5) include convection of heat from the hot gas to the thermocouple junction and radiation from the thermocouple junction to the wall.
6. Finally, in (6), since the thermocouple is losing heat by radiation, it will indicate a temperature lower than that of the actual gas temperature. ■

Several additional examples of heat transfer mechanisms by radiation have appeared in the literature. Badger and Banchero[(1)] provided the following explanation of radiation: "If radiation is passing through empty space, it is not transformed to heat or any other form of energy and it is not diverted from its path. If, however, matter appears in its path, it is only the absorbed energy that appears as heat, and this transformation is quantitative. For example, fused quartz transmits practically all the radiation which strikes it; a polished opaque surface or mirror will reflect most of the radiation impinging on it; a black surface will absorb most of the radiation received by it (as one can experience on a sunny day while wearing a black shirt) and will transform such absorbed energy quantitatively into heat." The relationship between the energy transmitted, reflected, and absorbed is discussed in the next section; see also Equation (11.7). Bennett and Meyers[(2)] provide an additional example involving the operation of a steam "radiator."

Characteristic wavelengths of radiation are provided in Table 11.1. Note that light received from the Sun passes through the Earth's atmosphere which absorbs some of the energy and thus affects the quality of visible light as it is received. The units of wavelength may be expressed in meters (m), centimeters (cm), micrometers (μm), or Angstroms ($1.0\ \text{Å} = 10\ \mu$m), with the centimeter being the unit of choice.[(3)] The speed of electromagnetic radiation is approximately 3×10^8 m/s in a vacuum. This

Table 11.1 Characteristic Wavelengths[3]

Type of radiation	$\lambda \times 10^8$
Gamma rays	0.01–0.15 cm
X-rays	0.06–1000 cm
Ultraviolet	100–35,000 cm
Visible	3500–7800 cm
Infrared	7800–4,000,000 cm
Radio	0.01–0.15 cm

velocity (c) is given by the product of the wavelength (λ) and the frequency (ν) of the radiation, that is,

$$c = \lambda\nu; \quad \text{consistent units} \tag{11.1}$$

As noted earlier, the energy emitted from a "hot" surface is in the form of electromagnetic waves. One of the types of electromagnetic waves is thermal radiation. Thermal radiation is defined as electromagnetic waves falling within the following range:

$$0.1\ \mu\text{m} < \lambda < 100\ \mu\text{m}; \quad 1.0\ \mu\text{m} = 10^{-6}\ \text{m} = 10^{-4}\ \text{cm}$$

However, most of this energy is in the interval from 0.1 to 10 μm. The visible range of thermal radiation lies within the narrow range of 0.4 μm (violet) $< \lambda <$ 0.8 μm (red).

The remainder of the chapter consists of five additional sections:

Energy and Intensity

Radiant Exchange

Kirchoff's Law

Emissivity Factors

View Factors

An additional 15 illustrative examples complement the presentation.

ENERGY AND INTENSITY

A body at a given temperature will emit radiation over a range of wavelengths, not a single wavelength. Information is available on the *intensity* of the radiant energy I (Btu/h·ft^2·μm) as a function of the wavelength, λ (μm). In addition, at any given temperature, a wavelength exists at which the amount of energy given off is a maximum. For the same body at a lower temperature, the maximum intensity of radiation is obviously less; however, it is also significant that the wavelength at which the maximum intensity exists is higher in value.

Since the $I - \lambda$ curve for a single temperature depicts the amount of energy emitted at a given wavelength, the sum of all the energy radiated by the body at all its wavelengths is simply the area under a plot of I vs. λ. This quantity of radiant energy (of all wavelengths) emitted by a body per unit area and time is defined as the *total emissive power* E (Btu/h·ft^2). Given the intensity of the radiation at any wavelength, I, one may calculate the total emissive power, E, from

$$E = \int_0^\infty I\, d\lambda \tag{11.2}$$

ILLUSTRATIVE EXAMPLE 11.3

The intensity of radiation as a function of wavelength ($\lambda = \mu$m) is specified as

$$I = 40e^{-\lambda^2}; \quad \text{Btu/h} \cdot \text{ft}^2 \cdot \mu\text{m}$$

Calculate the total emissive power.

***SOLUTION*:** Apply Equation (11.2):

$$E = \int_0^\infty I\, d\lambda$$

Substitute,

$$E = \int_0^\infty 40e^{-\lambda^2}\, d\lambda = 40\int_0^\infty e^{-\lambda^2}\, d\lambda$$

The above integral is calculated as follows.[(4)] Set

$$\lambda^2 = x; \quad \lambda = x^{1/2}$$

so that

$$2\lambda\, d\lambda = dx$$

$$d\lambda = \frac{dx}{2\lambda}$$

$$= \tfrac{1}{2}x^{-1/2}\, dx$$

Insertion into the above integral gives

$$E = \frac{40}{2}\int_0^\infty e^{-x}x^{-1/2}\, dx$$

This integral is the gamma function of $\frac{1}{2}$, that is, $\Gamma(\frac{1}{2})$. Since

$$\Gamma(\tfrac{1}{2}) = \pi^{1/2} = \sqrt{\pi}$$

$$E = 20\sqrt{\pi}$$

$$= 35.5 \text{ Btu/h} \cdot \text{ft}^2$$

■

Maxwell Planck was the first to fit the I vs. λ relationship to equation form, as

$$I_\lambda = \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1} \tag{11.3}$$

where I_λ = intensity of emission (Btu/h · ft^2 · μm) at λ
λ = wavelength (μm)
C_1 = 1.16×10^8 (dimensionless)
C_2 = 25,740 (dimensionless)
T = temperature of the body (°R)

It was later shown that the product of the wavelength of the maximum value of the intensity of emission and the absolute temperature is a constant. This is referred to as Wien's displacement law,

$$\lambda T = 2884\ \mu\text{m} \cdot {}^\circ\text{R} \approx 5200\ \mu\text{m} \cdot \text{K} \tag{11.4}$$

One can derive Equation (11.4) from (11.3) as follows. Since $dI_\lambda/d\lambda = 0$ at the maximum value of the intensity

$$\frac{dI_\lambda}{d\lambda} = d\left(\frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1}\right)\Big/ d\lambda = 0 \tag{11.5}$$

After differentiation,

$$(-5C_1\lambda^{-6})(e^{C_2/\lambda T} - 1) + C_1\lambda^{-5}(e^{C_2/\lambda T})/(e^{C_2/\lambda T} - 1)^2 = 0$$

which can be reduced to

$$(-5C_1\lambda^{-6})(e^{C_2/\lambda T} - 1) + C_1\lambda^{-5}(e^{C_2/\lambda T})\left(\frac{C_2}{\lambda^2 T}\right) = 0$$

This ultimately simplifies to

$$(-5 + \frac{C_2}{\lambda T})e^{C_2/\lambda T} + 5 = 0;\ C_2 = 25{,}740 \tag{11.6}$$

The reader is left with the exercise of showing that the first term equals -5 when $\lambda T = 2884$.

Atmospheric data indicates that the maximum intensity, I, of the Sun is experienced around 0.25 μm wavelength. This accounts for the predominance of blue in the visible spectrum and the high ultraviolet content of the Sun's rays.

ILLUSTRATIVE EXAMPLE 11.4

Estimate the Sun's temperature. Employ equation (11.4).

***SOLUTION*:** Assuming a wavelength of 0.25 μm, substitute into Equation (11.4):

$$\lambda T = 2884\ \mu\text{m} \cdot {}^\circ\text{R},\ \lambda = 0.25\ \mu\text{m}$$
$$T = 2884\ \mu\text{m} \cdot {}^\circ\text{R}/0.25\ \mu\text{m}$$
$$T = 11{,}500^\circ\text{R} \approx 11{,}000^\circ\text{F}$$

■

RADIANT EXCHANGE

The conservation law of energy indicates that any radiant energy incident on a body will partially absorb, reflect, or transmit stored energy. An energy balance around a receiving body on which the total incident energy is assumed to be unity gives

$$\alpha + \rho + \tau = 1 \tag{11.7}$$

where the absorptivity α is the fraction absorbed, the reflectivity ρ is the fraction reflected, and the transmissivity τ is the fraction transmitted. It should be noted that the majority of engineering applications involve opaque substances having transmissivities approaching zero (i.e., $\tau = 0$). This topic receives additional treatment below in a later paragraph.

When an ordinary body emits radiation to another body, it will have some of the emitted energy returned to itself by reflection. Equation (11.7) assumes that none of the emitted energy is returned; this is equivalent to assuming that bodies having zero transmissivity also have zero reflectivity. This introduces the concept of a perfect "black body" for which $\alpha = 1$. Not all substances radiate energy at the same rate at a given temperature. The theoretical substance to which most radiation discussions refer to is called a "black body." This is defined as a body that radiates the maximum possible amount of energy at a given temperature. Much of the development to follow is based on this concept.

To summarize, when radiation strikes the surface of a semi-transparent material such as a glass plate or a layer of water, three types of interactive effects occur. Some of the incident radiation is reflected off the surface, some of it is absorbed within the material, the remainder is transmitted through the material. Examining the three fates of the incident radiation, one can see that α and ρ depend on inherent properties of the material; it is for this reason that they are referred to as *surface* properties. The transmissivity, τ, on the other hand, depends on the amount of the material in question; it is therefore referred to as a *volumetric* property.

It is appropriate to examine a few common surfaces. An opaque surface, the most commonly encountered surface type, has $\tau \approx 0$. Because of this, Equation (11.7) becomes:

$$\alpha + \rho = 1 \tag{11.8}$$

or

$$\rho = 1 - \alpha \tag{11.9}$$

Equation (11.9) may also be applied to gases; this may be counterintuitive since most gases are invisible. With respect to the reflectivity term, ρ, surfaces may have either *specular* reflection, in which the angle of incidence of the radiation is equal to the angle of reflection, or *diffuse* reflection, in which the reflected radiation scatters in all directions. In addition, a *gray* surface is one for which the absorptivity is the same as the emissivity, ε, at the temperature of the radiation source. For this case,

$$\alpha = \varepsilon \tag{11.10}$$

$$\rho = 1 - \varepsilon \tag{11.11}$$

Reflectivity and transmissivity are characteristics experienced in the everyday world. Polished metallic surfaces have high reflectivities and granular surfaces have low reflectivities. Reflection from a surface depends greatly on the characteristics of the surface. If a surface is very smooth, the angles of incidence and reflection are essentially the same. However, most surfaces encountered in engineering practice are sufficiently rough so that some reflection occurs in all directions. Finally, one may state that a system in thermal equilibrium has its absorptivity equal to the emissivity.

KIRCHOFF'S LAW

Consider a body of given size and shape placed within a hollow sphere of constant temperature, and assume that the air has been evacuated. After *thermal equilibrium* has been reached, the temperature of the body and that of the enclosure (the sphere) will be the same, inferring that the body is absorbing and radiating heat at equal rates. Let the total intensity of radiation falling on the body be I (now with units of Btu/h·ft^2), the fraction absorbed α_1, and the total emissive power E_1 (Btu/h·ft^2). Kirchhoff noted that the energy emitted by a body of surface A_1 at thermal equilibrium is equal to that received, so that:

$$E_1 A_1 = I \alpha_1 A_1 \tag{11.12}$$

or simply,

$$E_1 = I \alpha_1 \tag{11.13}$$

If the body is replaced by another of identical shape, then:

$$E_2 = I \alpha_2 \tag{11.14}$$

If a third body that is a black body is introduced, then:

$$E_b = I \tag{11.15}$$

Since the absorptivity, α, of a black body is 1.0, one may write:

$$I = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = E_b \tag{11.16}$$

Thus, at thermal equilibrium, the ratio of the total emissive power to the absorptivity for all bodies is the same. This is referred to as Kirchhoff's law. Since $\alpha = \varepsilon$, the above equation may also be written

$$\frac{E_1}{E_b} = \alpha_1 = \varepsilon_1 \quad \text{and}$$

$$\frac{E_2}{E_b} = \alpha_2 = \varepsilon_2 \tag{11.17}$$

where the ratio of the actual emissive power to the black-body emissive power is defined as the aforementioned emissivity, ε. Values of ε for various bodies and surfaces are given in Table 11.2.

If a *black* body radiates energy, the total radiation may be determined from Planck's law:

$$I = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \tag{11.3}$$

Integration over the entire spectrum at a particular temperature yields:

$$E_b = \int_0^{\infty} \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}\, d\lambda \tag{11.18}$$

The evaluation of the previous integral can be shown to be:

$$E_b = 0.173 \times 10^{-8} T^4 = \sigma T^4; \quad T \equiv {}^\circ\mathrm{R} \tag{11.19}$$

Thus, the total radiation from a perfect black body is proportional to the fourth power of the absolute temperature of the body. This is also referred to as the Stefan–Boltzmann law. The constant 0.173×10^{-8} Btu/h·ft^2·°R$^{(2)}$ is known as the Stefan–Boltzmann constant, usually designated by σ. Its counterpart in SI units is 5.669×10^{-8} W/m^2·K^4. However, note that this equation was derived for a perfect black body.

If the body is non-black, the emissivity is given by

$$E = E_b \varepsilon \tag{11.16}$$

Substituting Equation (11.16) into Equation (11.19) gives

$$E = \varepsilon \sigma T^4 \tag{11.20}$$

or, since $E = \dot{Q}/A$,

$$\frac{\dot{Q}}{A} = \varepsilon \sigma T^4 \tag{11.21}$$

Thus, when the law is applied to a real surface with emissivity, ε, the total emissive power of a real body is given by Equation (11.21). Typical values for the emissivity were provided in Table 11.2.

Table 11.2 Total Emissivity of Various Sources

Surface	T, °F	Emissivity, ε
Aluminum		
Anodized		0.76
Highly polished plate, 98.3% pure	440–1070	0.039–0.057
Commercial sheet	212	0.09
Heavily oxidized	299–940	0.20–0.31
Al-surfaced roofing	100	0.216
Brass		
Highly polished		
73.2% Cu, 26.7% Zn	476–674	0.028–0.031
62.4% Cu, 36.8% Zn, 0.4% Pb, 0.3% Al	494–710	0.033–0.037
82.9% Cu, 17.0% Zn	530	0.030
Hard-rolled, polished, but with the direction of polishing visible	70	0.038
Dull plate	120–660	0.22
Chromium		
Polished	100–2000	0.08–0.38
Copper		
Polished	242	0.028
Plate, heated a long time, and covered with thick oxide layer	77	0.78
Gold		
Pure, highly polished	440–1160	0.018–0.035
Iron and steel (not including stainless)		
Steel, polished	212	0.066
Iron, polished	800–1800	0.14–0.38
Cast iron, newly turned	72	0.44
Turned and heated	1620–1810	0.60–0.70
Milled steel	450–1950	0.20–0.32
Oxidized surfaces		
Iron plate, pickled, then rusted red	68	0.61
Iron, dark gray surface	212	0.31
Rough ingot iron	1700–2040	0.87–0.95
Sheet steel with strong, rough oxide layer	75	0.80
Lead		
Unoxidized, 99.96% pure	260–440	0.057–0.075
Gray oxidized	75	0.28
Oxidized at 300°F	390	0.63
Magnesium		
Magnesium oxide	530–1520	0.055–0.20

(*Continued*)

Table 11.2 *Continued*

Surface	T, °F	Emissivity, ε
Molybdenum		
Filament	1340–4700	0.096–0.202
Massive, polished	212	0.071
Monel metal		
Oxidized at 1110°F	390–1110	0.41–0.46
Nickel		
Polished	212	0.072
Nickel oxide	1200–2290	0.59–0.86
Nickel alloys		
Copper–nickel, polished	212	0.059
Nichrome wire, bright	120–1830	0.65–0.79
Nichrome wire, oxidized	120–930	0.95–0.98
Platinum		
Polished plate, pure	440–1160	0.054–0.104
Silver		
Polished, pure	440–1160	0.020–0.032
Polished	100–700	0.022–0.031
Stainless steels		
Polished	212	0.074
Type 301; B	450–1725	0.54–0.63
Tin		
Bright tinned iron	76	0.043–0.064
Tungsten		
Filament	6000	0.39
Zinc		
Galvanized, fairly bright	82	0.23
Alumina; effect of mean grain size (μm)		
10		0.18–0.30
50		0.28–0.39
100		0.40–0.50
Asbestos		
Board	74	0.96
Brick		
Red, rough, but no gross irregularities	70	0.93
Fireclay	1832	0.75

(*Continued*)

Table 11.2 *Continued*

Surface	T, °F	Emissivity, ε
Carbon		
T-Carbon 0.9% ash, started with an emissivity of 0.72 at 260°F but on heating changed to given values	260–1160	0.79–0.81
Filament	1900–2560	0.526
Rough plate	212–608	0.77
Lampblack, rough deposit	212–932	0.78–0.84
Concrete tiles	1832	0.63
Enamel		
White fused, on iron	66	0.90
Glass		
Smooth	72	0.94
Pyrex, lead and soda	500–1000	0.85–0.95
Rubber		
Hard, glossy plate	74	0.94
Water	32–212	0.95–0.963

ILLUSTRATIVE EXAMPLE 11.5

Estimate the increase in heat transferred by radiation of a black body at 1500°F relative to one at 1000°F.

***SOLUTION*:** Convert to *absolute* temperatures:

$$T_1 = 1500^\circ\mathrm{F} = 1960^\circ\mathrm{R}$$
$$T_2 = 1000^\circ\mathrm{F} = 1460^\circ\mathrm{R}$$

The ratio of the quantity/rate of heat transferred from Equation (11.19) is:

$$\frac{T_1^4}{T_2^4} = \frac{1960^\circ\mathrm{R}^4}{1460^\circ\mathrm{R}^4} \approx 3.25$$

This represents a 225% increase in heat transfer. ■

Now consider the energy transferred *between* two *black* bodies. Assume the energy transferred from the hotter body and the colder body is E_H and E_C, respectively. All of the energy that each body receives is absorbed since they are black bodies. Then, the net exchange between the two bodies maintained at two constant temperatures, T_H and T_C, is therefore

$$\frac{\dot{Q}}{A} = E_H - E_C = \sigma(T_H^4 - T_C^4) \tag{11.22}$$

$$= 0.173\left[\left(\frac{T_H}{100}\right)^4 - \left(\frac{T_C}{100}\right)^4\right] \tag{11.23}$$

ILLUSTRATIVE EXAMPLE 11.6

Two large walls are required to be maintained at constant temperatures of 800°F and 1200°F. Assuming the walls are black bodies, how much heat must be removed from the colder wall to maintain a steady-state, constant temperature?

***SOLUTION*:** For this application,

$$T_1 = 1200°\text{F} = 1660°\text{R} \qquad T_2 = 800°\text{F} = 1260°\text{R}$$

Apply Equation (11.23) and substitute:

$$\frac{\dot{Q}}{A} = 0.173\left[\left(\frac{1660}{100}\right)^4 - \left(\frac{1260}{100}\right)^4\right]$$

$$= (0.173)(75{,}900 - 25{,}200)$$

$$= 8770\ \text{Btu/h} \cdot \text{ft}^2$$

Note once again that the above solution applies to black bodies. ■

EMISSIVITY FACTORS

If the two large walls in Illustrative Example 11.6 are not black bodies, and instead (each) have an emissivity ε, then the net interchange of radiant energy is given by

$$\frac{\dot{Q}}{A} = \varepsilon\sigma(T_H^4 - T_C^4) \tag{11.24}$$

This equation can be "verified" as follows.[2] If the two planes discussed above are not black bodies and have *different* emissivities, the net exchange of energy will be different. Some of the energy emitted from the first body will be absorbed and the remainder radiated back to the other source. For two parallel bodies of infinite size, the radiation of each body can be accounted for. If the energy emitted from the first body is E_H with emissivity ε_H, the second body will absorb $E_H\varepsilon_C$ and reflect $1-\varepsilon_C$ of it, i.e., $E_H(1-\varepsilon_C)$. The first body will then receive $E_H(1-\varepsilon_C)\varepsilon_H$ and again radiate to the cold body, but in the amount $E_H(1-\varepsilon_C)(1-\varepsilon_H)$. The exchanges for the two bodies are therefore:

Hot body
Radiated: E_H
Reflected back: $E_H(1-\varepsilon_C)$
Radiated: $E_H(1-\varepsilon_C)(1-\varepsilon_H)$
Reflected back: $E_H(1-\varepsilon_C)(1-\varepsilon_H)(1-\varepsilon_C)$
etc.

Cold body
Radiated: E_C
Reflected back: $E_C(1-\varepsilon_H)$
Radiated: $E_C(1-\varepsilon_H)(1-\varepsilon_C)$
Reflected back: $E_C(1-\varepsilon_H)(1-\varepsilon_C)(1-\varepsilon_H)$
etc.

For a non-black body, ε is not unity and must be included in Equation (11.19), that is,

$$E = 0.173\varepsilon\left(\frac{T}{100}\right)^4 \tag{11.25}$$

When Equation (11.25) is applied to the above infinite series analysis, one can show that Equation (11.26) results:

$$E = \frac{\dot{Q}}{A} = \frac{\sigma}{\left[\left(\frac{1}{\varepsilon_H}\right) + \left(\frac{1}{\varepsilon_C}\right) - 1\right]}(T_H^4 - T_C^4) \tag{11.26}$$

ILLUSTRATIVE EXAMPLE 11.7

If the two bodies from Illustrative Example 11.6 have emissivities of 0.5 and 0.75, respectively, what is the net energy exchange (per unit area)? Assume that the temperatures remain constant at 1660°R and 1260°R, and the two bodies are of infinite size.

***SOLUTION*:** Apply Equation (11.26) and substitute:

$$\frac{\dot{Q}}{A} = \frac{0.173}{\left[\left(\frac{1}{0.5}\right) + \left(\frac{1}{0.75}\right) - 1\right]}\left[(16.6)^4 - (12.6)^4\right]$$
$$= 3760\,\text{Btu/h}\cdot\text{ft}^2$$

■

ILLUSTRATIVE EXAMPLE 11.8

Compare and discuss the results of the last two illustrative examples.

***SOLUTION*:** The black bodies had an energy exchange of 8770 Btu/h·ft^2, while the non-black bodies had an exchange of 3760 $\text{Btu/h}\cdot\text{ft}^2$. The percent difference (relative to the black body calculation) is:

$$\begin{aligned}\%\ \text{difference} &= \left(\frac{8770 - 3760}{8770}\right)100 \\ &= 57.1\%\end{aligned}$$

■

The radiation between a sphere and an enclosed sphere of radii R_H and R_C, respectively, may be treated in a manner similar to that provided above. The radiation emitted initially by the inner sphere is $E_H A_H$, all of which falls on A_C. Of this total, however, $(1-\varepsilon_C)E_H A_H$ is reflected back to the hot body. If this analysis is similarly extended as before, the radiant exchange will again be represented by an infinite series whose solution may be shown to give

$$\begin{aligned}E_H = \frac{\dot{Q}}{A_H} &= \frac{\sigma_H(T_H^4 - T_C^4)}{\left[\dfrac{1}{\varepsilon_H} + \left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C} - 1\right)\right]} \\ &= \frac{\sigma_H(T_H^4 - T_C^4)}{\left[\dfrac{1}{\varepsilon_H} + \left(\dfrac{R_H}{R_C}\right)^2\left(\dfrac{1}{\varepsilon_C} - 1\right)\right]} \qquad (11.27)\end{aligned}$$

A similar relation applies for infinitely long concentric cylinders except that A_H/A_C is replaced R_H/R_C, not R_H^2/R_C^2.

In general, an emissivity correction factor, F_ε, is introduced to account for the exchange of energy between different surfaces of different emissivities. The describing equation takes the form

$$E_H = \frac{\dot{Q}}{A_H} = F_\varepsilon \sigma (T_H^4 - T_C^4) \qquad (11.28)$$

Values of F_ε for the interchange between surfaces are provided in Table 11.3 for the three cases already considered, i.e., (a), (b), (c), plus three additional cases (d), (e), (f).

Table 11.3 Values of F_ε

Condition	F_ε
a Surface A_H small compared with the totally enclosing surface A_C	ε_H
b Surfaces A_C and A_H of infinite parallel planes or surface A_H of a completely enclosed body is small compared with A_H	$\dfrac{1}{\left(\dfrac{1}{\varepsilon_H}+\dfrac{1}{\varepsilon_C}\right)-1}$
c Concentric spheres or infinite concentric cylinders with surfaces A_H and A_C	$\dfrac{1}{\dfrac{1}{\varepsilon_H}+\left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C}-1\right)}$
d Surfaces A_H and A_C of parallel disks, squares, 2:1 rectangles, long rectangles (see Figures 11.2 and 11.4 later)	$\varepsilon_H\varepsilon_C$
e Surface A_H and A_C of perpendicular rectangles having a common side (see Figure 11.3 later)	$\varepsilon_H\varepsilon_C$
f Surface A_H and parallel rectangular surface A_C with one corner of rectangle above A_H	$\varepsilon_H\varepsilon_C$

ILLUSTRATIVE EXAMPLE 11.9

Calculate the radiation from a 2-inch IPS cast iron pipe (assume polished) carrying steam at 300°F and passing through the center of a 1 ft × 1 ft galvanized zinc duct at 75°F and whose outside is insulated.

***SOLUTION*:** Base the calculation on 1 ft of pipe/duct. For a 2-inch pipe $A_H = 0.622\ \text{ft}^2$ of external surface per foot of pipe (see Table 6.2 in Part One). The emissivity of oxidized steel from Table 11.2 is $\varepsilon_H = 0.44$. The surface of the duct is $A_C = 4(1)(1) = 4.0\ \text{ft}^2$, and for galvanized zinc, $\varepsilon_C = 0.23$. Assume condition (c) in Table 11.3 applies with the physical representation of the duct replaced by a cylinder of the same area. Therefore,

$$F_\varepsilon = \frac{1}{\left[\dfrac{1}{\varepsilon_H}+\left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C}-1\right)\right]} = \frac{1}{\left[\dfrac{1}{0.44}+\left(\dfrac{0.622}{4.0}\right)\left(\dfrac{1}{0.23}-1\right)\right]} = 0.358$$

Apply Equation (11.28):

$$\dot{Q}_H = \dot{Q} = F_\varepsilon A\sigma(T_H^4 - T_C^4)$$

Substituting,

$$\begin{aligned}\dot{Q} &= (0.358)(0.622)(0.173\times10^{-8})(760^4 - 535^4)\\ &= 97.85\ \text{Btu/h}\cdot\text{ft}\end{aligned}$$

■

Finally, when a heat source is small compared to its enclosure, it is customary to assume that some of the heat radiated from the source is reflected back to it. Such is often the case on the loss of heat from a pipe to surrounding air. For these applications, it is convenient to represent the net radiation heat transfer in the same form employed for convection, i.e.,

$$\dot{Q} = h_r A(T_H - T_C) \tag{11.29}$$

where h_r is the *effective* radiation heat transfer coefficient.
When $T_H - T_C$ is less than 120°C (120 K or 216°R), one may calculate the radiation heat transfer coefficient using

$$h_r = 4\varepsilon\sigma T_{\text{av}}^3 \tag{11.30}$$

where $T_{\text{av}} = (T_H + T_C)/2$.

ILLUSTRATIVE EXAMPLE 11.10

The outside temperature of a 10 ft^2 hot insulated pipe is 140°F and the surrounding atmosphere is 60°F. The heat loss by free convection and radiation is 13,020 Btu/h, and the combined coefficient of heat transfer is estimated to be 2.10 Btu/h · ft^2 · °F. How much of the heat loss is due to radiation? Assume the pipe emissivity is approximately 0.9.

***SOLUTION*:** For this example,

$$T_H = 140 + 460 = 600°\text{R} \qquad T_C = 60 + 460 = 520°\text{R}$$

Apply Equation (11.21) and substitute:

$$\dot{Q} = \dot{Q}_{\text{rad}} = (0.9)(10)(0.173)\left[\left(\frac{600}{100}\right)^4 - \left(\frac{520}{100}\right)^4\right] = 880\,\text{Btu/h}$$

■

ILLUSTRATIVE EXAMPLE 11.11

With reference to Illustrative Example 11.10, calculate the radiation heat transfer coefficient, h_r.

***SOLUTION*:** Apply Equation (11.29):

$$h_r = \frac{\dot{Q}}{A(T_H - T_C)}$$

Substituting,

$$h_r = \frac{880}{10(600 - 520)} = 1.10\,\text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}$$

■

ILLUSTRATIVE EXAMPLE 11.12

A small oxidized horizontal metal tube is placed in a very large furnace enclosure with firebrick walls. The metal tube has an outside diameter of 1 inch, a length of 2 ft, a surface emissivity of 0.6, and its surface is maintained at 600°F. The hot air in the furnace is at 1500°F and the furnace brick walls are at 1350°F. The convection heat transfer coefficient for the horizontal tube is 2.8 Btu/h · ft^2 · °F. Calculate

1. The convective, radiative, and total heat transferred to the metal tube;
2. The percent of total heat transferred by radiation;
3. The radiation heat transfer coefficient;
4. Is it appropriate to use the approximate equation presented in Equation (11.30) for (3)?

***SOLUTION*:**

1. Calculate the tube area:

$$A = \pi DL = \pi(0.0833)(2) = 0.524\ \text{ft}^2$$

The convective heat transfer (from air to metal tube) is therefore

$$\dot{Q}_{\text{conv}} = hA(T_{\text{air}} - T_{\text{tube}}) = (2.8)(0.524)(1500 - 600) = 1320\ \text{Btu/h}$$

Calculate the radiation heat transfer (from wall to metal tube). Apply Equation (11.24):

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A(T_{\text{air}}^4 - T_{\text{tube}}^4);\ T_{\text{air}} = 1350°\text{F},\ T_{\text{tube}} = 600°\text{F}\\ &= (0.6)(0.1713 \times 10^{-8})(0.524)(1810^4 - 1060^4)\\ &= 5100\ \text{Btu/h}\end{aligned}$$

Determine the total heat transfer:

$$\dot{Q} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} = 1320 + 5100 = 6420\ \text{Btu/h}$$

2. The radiation percent contribution to the total heat transfer rate is

$$(5100/6420) \times 100\% = 79.4\% = 80\%$$

3. Determine the radiation heat transfer coefficient. Apply Equation (11.29):

$$\begin{aligned}h_r &= \frac{\dot{Q}}{A(T_{\text{wall}} - T_{\text{tube}})}\\ &= \frac{5100}{(0.524)(750)} = 13.1\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}\end{aligned}$$

4. Finally, since $|T_{\text{tube}} - T_{\text{wall}}| = 1350 - 600 = 750°\text{F} > 200°\text{F}$, the use of the approximation Equation (11.30), $h_r = 4\varepsilon\sigma T_{\text{av}}^3$, is not appropriate. ■

ILLUSTRATIVE EXAMPLE 11.13

The filament of a light bulb is at a temperature of 900°C and emits 5 W of heat toward the glass bulb. The interior of a light bulb can be considered a vacuum and the temperature of the glass bulb is 150°C. Ignoring heat transfer to the room and assuming the emissivity of the filament is 1.0, calculate the surface area of the filament in cm^2.

***SOLUTION*:** Write the equation for radiation heat transfer, see Equation (11.24),

$$\dot{Q} = \varepsilon\sigma A(T_1^4 - T_2^4)$$

Solve for the unknown, surface A,

$$A = \frac{\dot{Q}}{\varepsilon\sigma(T_1^4 - T_2^4)}$$

Substitute known values and compute A:

$$\begin{aligned} A &= \frac{\dot{Q}}{\varepsilon\sigma(T_1^4 - T_2^4)} \\ &= \frac{5.0}{\{(1)(5.669 \times 10^{-8})[(273 + 900)^4 - (273 + 150)^4]\}} \\ &= 4.74 \times 10^{-5}\ \text{m}^2 \\ &= 0.47\ \text{cm}^2 \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 11.14

A system consists of an uninsulated steam pipe made of anodized aluminum with a diameter $D = 0.06$ m and a length $L = 100$ m. The surface temperature is $T_1 = 127$°C and the surface emissivity of anodized aluminum is $\varepsilon = 0.76$. The pipe is in a large room with a wall temperature $T_2 = 20$°C. The air in the room is at a temperature $T_3 = 22$°C. The pipe convective heat transfer coefficient is $h = 15\ \text{W/m}^2 \cdot \text{K}$.

Estimate the emissive power, the total heat transfer by convection and radiation, and the radiation heat transfer coefficient. Assume steady-state operation, constant properties, and a room surface area much larger than the pipe surface area.

***SOLUTION*:** Calculate the emissive energy of the pipe surface assuming it is a blackbody:

$$T_1 = 127°\text{C} = 400\ \text{K}$$

$$E_b = \sigma T_1^4 = (5.669 \times 10^{-8})(400)^4 = 1451\ \text{W/m}^2$$

Employ Equation (11.17). Calculate the emissive power from the surface of the pipe:

$$E = \varepsilon E_b = (0.76)(1451) = 1103\ \text{W/m}^2$$

The total heat transfer, $\dot{Q}$, from the pipe to the air and walls is

$$\dot{Q} = \dot{Q}_c + \dot{Q}_r$$

With reference to the convection equation,

$$\dot{Q}_c = hA(T_1 - T_3)$$

Calculate the surface area of the pipe:

$$A = \pi(0.06)(100) = 18.85\,\mathrm{m}^2$$

The convective heat transfer to the air is therefore

$$\dot{Q}_c = (15)(18.85)(127 - 22) = 29{,}700\,\mathrm{W} = 29.7\,\mathrm{kW}$$

With reference to the radiation heat transfer rate, $\dot{Q}_r$, apply Equation (11.24):

$$\begin{aligned}\dot{Q}_r &= \varepsilon\sigma A(T_1^4 - T_2^4)\\ &= (0.76)(5.669 \times 10^{-8})(18.85)(400^4 - 293^4)\\ &= 14{,}800\,\mathrm{W} = 14.8\,\mathrm{kW}\end{aligned}$$

The total heat transfer rate is then

$$\dot{Q} = \dot{Q}_c + \dot{Q}_r = 29{,}700 + 14{,}800 = 44{,}500\,\mathrm{W} = 151{,}834\,\mathrm{Btu/h}$$

Since $(T_1 - T_2) = 107°\mathrm{C}$, which is less than $120°\mathrm{C}$, it is valid to use the approximation equation,

$$h_r = 4\varepsilon\sigma T_{\mathrm{av}}^3 \tag{11.30}$$

where

$$T_{\mathrm{av}} = (T_H + T_C)/2$$

in order to calculate the radiation heat transfer coefficient. First calculate T_{av}:

$$T_{\mathrm{av}} = (T_H + T_C)/2 = [(127 + 273) + (20 + 273)]/2 = 346.5\,\mathrm{K}$$

Substituting into Equation (11.30), and solving for h_r

$$h_r = 4\varepsilon\sigma T_{\mathrm{av}}^3 = 4(0.76)(5.669 \times 10^{-8})(346.5)^3 = 7.2\ \mathrm{W/m^2 \cdot K}$$ ■

ILLUSTRATIVE EXAMPLE 11.15

Refer to Illustrative Example 11.14. How much does radiation contribute to the total heat transfer?

***SOLUTION*:** Since the convection heat transfer coefficient is $15.0\,\mathrm{W/m^2 \cdot K}$ and the radiation heat transfer coefficient is $7.2\ \mathrm{W/m^2 \cdot K}$,

$$\%\ \text{heat transfer by radiation} = \frac{7.2}{15.0 + 7.2} \times 100 = 32.4\%$$

Therefore, radiation accounts for approximately one-third of the total heat transfer. ■

VIEW FACTORS

As indicated earlier, the amount of heat transfer between two surfaces depends on geometry *and* orientation of the two surfaces. Again, it is assumed that the intervening medium is non-participating. The previous analyses were concerned with sources that were situated so that every point on one surface could be "connected" with every surface on the second . . . in effect possessing a perfect view. This is very rarely the case in real-world engineering applications, particularly in the design of boilers and furnaces. Here, the receiving surface, such as a bank of tubes, is cylindrical and may partially obscure some of the surfaces from "viewing" the source. These systems are difficult to evaluate. The simplest cases are addressed below; however, many practical applications must resort to the use of empirical methods.

To introduce the subject of view factors, the reader should note that the flow of radiant heat is analogous to the flow of light (i.e., one may follow the path of radiant heat as one may follow the path of light). If any object is placed between a hot and cold body, a light from the hot body would cast a shadow on the cold body and prevent it from receiving all the light leaving the hot one.

As noted above, the computation of actual problems involving "viewing" difficulty is beyond the scope of this book. One simple problem may illustrate the nature of these complications. Contemplate two black bodies, with surfaces consisting of parallel planes of finite size and separated by a finite distance (see Figure 11.1). Then, a small differential unit of surface *dA* can see the colder body only through solid angel β, and any radiation emitted by it through solid angles γ and γ^1 will fall elsewhere. To evaluate the heat lost by radiation from body *H*, one must integrate the loss from element *dA* over the whole surface of *A*.

The previously mentioned problem is sufficiently complicated. However, it can become more complicated. If the colder body is not a black body, then it will reflect some of the energy imparted upon it. Some of the reflected energy will return to the hot body. Since the hot body is black, it will absorb the reflected energy, tending to raise its temperature. One can envision even more complicated scenarios.

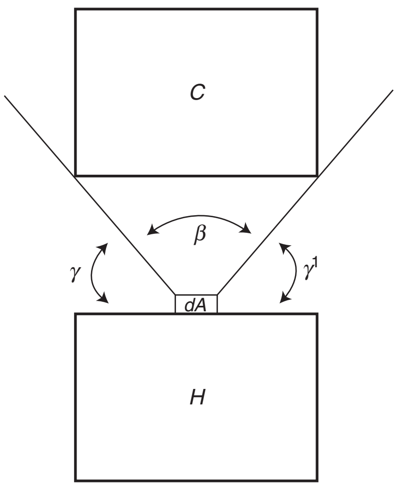

Figure 11.1 View factor illustration.

Table 11.4 Values of F_v

Condition	F_v
a Surface A_H small compared with the totally enclosing surface A_C	1.0
b Surfaces A_H and A_C of infinite parallel planes or surface A_H of a completely enclosed body is small compared with A_H	1.0
c Concentric spheres or infinite concentric cylinders with surfaces A_H and A_C	1.0
d Surfaces A_H and A_C of parallel disks, squares, 2:1 rectangles, long rectangles	Figure 11.2
e Surfaces A_H or A_C of perpendicular rectangles having a common side	Figure 11.3
f Coaxial parallel disks	Figure 11.4

In order to include the effect of "viewing", Equation (11.28) is expanded to

$$\frac{\dot{Q}}{A_H} = F_v F_\varepsilon \sigma (T_H^4 - T_C^4) \tag{11.31}$$

with the inclusion of view factor, F_v. View factors for the cases considered in Table 11.3 are presented in Table 11.4 in conjunction with Figures 11.2–11.3. View factors for parallel disks are provided in Figure 11.4. Note that $F_{i,j}$ represents the fraction of energy leaving surface i that strikes surface j. Other information and applications involving view factors are available in the literature.[5–7]

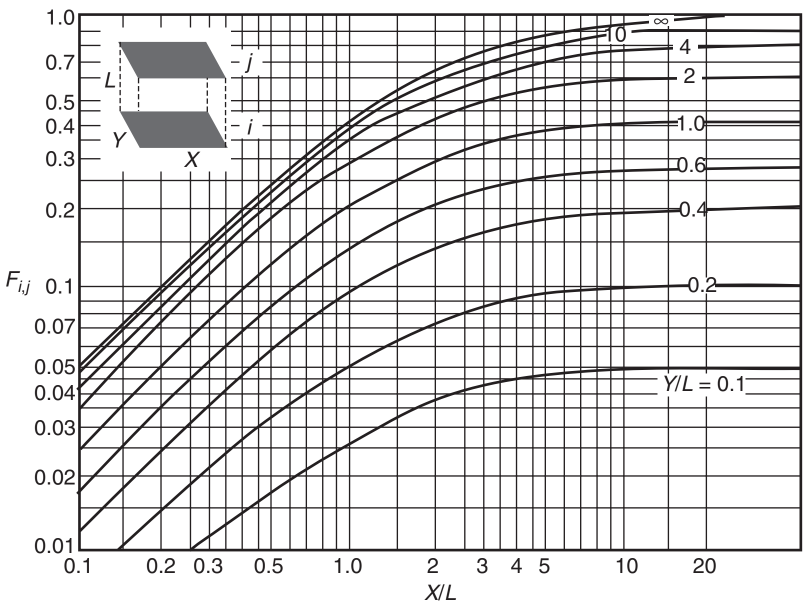

Figure 11.2 View factors for aligned parallel rectangles.[7]

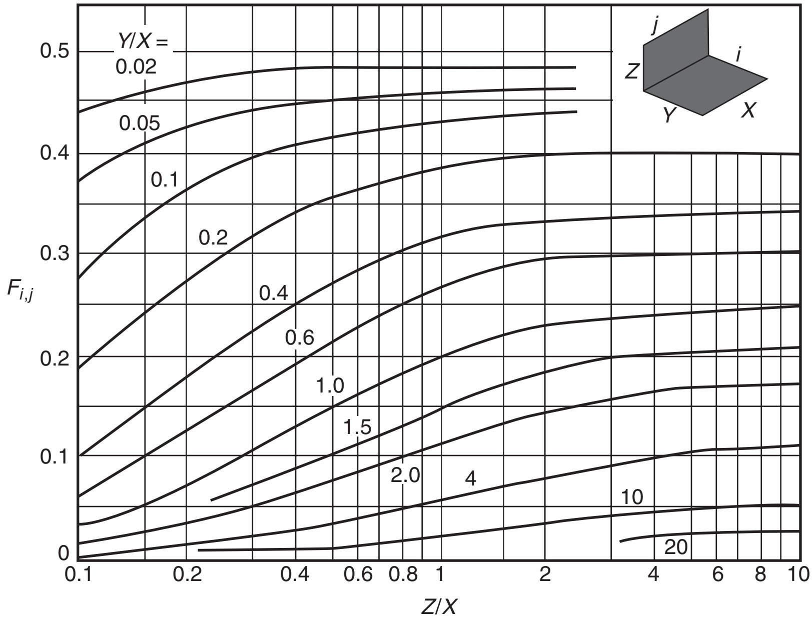

Figure 11.3 View factors for perpendicular rectangles with a common edge.[7]

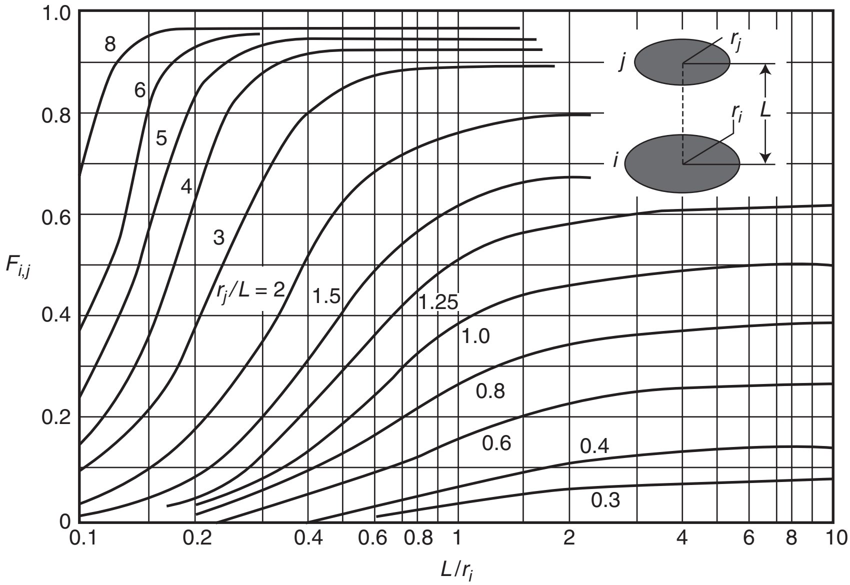

Figure 11.4 View factor for coaxial parallel disks.[7]

ILLUSTRATIVE EXAMPLE 11.16

Refer to Illustrative Example 11.9. Calculate the heat transfer rate if $F_v = 1.0$.

***SOLUTION*:** Apply Equation (11.31):

$$\dot{Q} = F_v F_\varepsilon \sigma A(T_H^4 - T_C^4)$$

Since $F_v = 1.0$, the solution remains unchanged:

$$\dot{Q} = 97.85\,\text{Btu/h} \cdot \text{ft}$$ ■

ILLUSTRATIVE EXAMPLE 11.17

Two parallel rectangular *black* plates 0.5 m by 2.0 m are spaced 1.0 m apart. One plate is maintained at 1000°C and the other at 2000°C. What is the net radiant heat exchange between the two plates?

***SOLUTION*:** Figure 11.2 is to be employed. For this application, $L = 1.0$, $X = 0.5$ and $Y = 2.0$. Therefore,

$$\frac{Y}{L} = \frac{2.0}{1.0} = 2.0$$

and

$$\frac{X}{L} = \frac{0.5}{1.0} = 0.5$$

From Figure 11.2,

$$F_v \cong 0.18$$

The net heat transfer exchange rate is calculated employing Equation (11.31), noting that $F_\varepsilon = 1.0$ and $\sigma = 5.669 \times 10^{-8}$ (when employing temperatures in K):

$$\begin{aligned}\dot{Q} &= F_v F_\varepsilon \sigma A(T_H^4 - T_C^4)\\ &= (0.18)(1.0)(5.669 \times 10^{-8})(0.5)(2.0)(2273^4 - 1273^4)\\ &= 245{,}600\ \text{kW} = 8.37 \times 10^8\ \text{Btu/h}\end{aligned}$$ ■

REFERENCES

1. W. Badger and J. Banchero, *Introduction to Chemical Engineering*, McGraw-Hill, New York City, NY, 1955.
2. C. Bennett and J. Meyers, *Momentum, Heat and Mass Transfer*, McGraw-Hill, New York City, NY, 1962.
3. D. Green and R. Perry (editors), *Perry's Chemical Engineers' Handbook*, 8th edition, McGraw-Hill, New York City, NY, 2008.
4. L. Theodore: personal notes, East Williston, NY, 2010.
5. J. Holman, *Heat Transfer*, McGraw-Hill, New York City, NY, 1981.
6. I. Farag and J. Reynolds, *Heat Transfer*, A Theodore Tutorial, Theodore Tutorials, East Williston, NY, 1994.
7. F. Incropera and D. DeWitt, *Fundamentals of Heat Transfer*, John Wiley and Sons, Hoboken, NJ, 2002.