

# Chapter 12

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## Condensation and Boiling

### INTRODUCTION

It should be noted that phase-change processes involve changes (sometimes significantly) in density, viscosity, heat capacity, and thermal conductivity of the fluid in question. The heat transfer process and the applicable heat transfer coefficients for boiling and condensation is more involved and complicated than that for a single-phase process. It is therefore not surprising that most real-world applications involving boiling and condensation require the use of empirical correlations.

The transfer of heat, which accompanies a change of phase, is often characterized by high rates. Heat fluxes as high as 50 million Btu/h · ft<sup>2</sup> have been obtained in some boiling systems. This mechanism of transferring heat has become important in rocket technology and nuclear-reactor design where large quantities of heat are usually produced in confined spaces. Although condensation rates have not reached a similar magnitude, heat transfer coefficients for condensation as high as 20,000 Btu/h · ft<sup>2</sup> · °F have been reported in the literature.<sup>(1)</sup> Due to the somewhat complex nature of these two phenomena, simple pragmatic calculations and numerical details are provided later in this chapter.

This chapter addresses phenomena associated with the change in phase of a fluid. The processes almost always occur at a solid–liquid interface and are referred to as *boiling* and *condensation*. The change from liquid to vapor due to boiling occurs because of heat transfer from the solid surface; alternatively, condensation of vapor to liquid occurs due to heat transfer to the solid surface.

Phase changes of substances can only occur if heat transfer is involved in the process. The phase change processes that arise include:

1. Boiling (or evaporation)
2. Condensation
3. Melting (or thawing)
4. Freezing (or fusion)
5. Sublimation.

The corresponding heat of transformation arising during these processes are:

1. Enthalpy of vaporization (or condensation)
2. Enthalpy of fusion (or melting)
3. Enthalpy of sublimation.

These common phase change operations are listed in Table 12.1. Other phase changes besides these are possible. During any phase change, it is usual (but not necessary) to have heat transfer without an accompanying change in temperature.

The applications of phase change involving heat transfer are numerous and include utility units where water is boiled, evaporators in refrigeration systems where a refrigerant may be either vaporized or boiled, or both, and condensers that are used to cool vapors to liquids. For example, in a power cycle, pressurized liquid is converted to vapor in a *boiler*. After expansion in a turbine, the vapor is restored to its liquid state in a *condenser*; it is then pumped to the boiler to repeat the cycle. Evaporators, in which the boiling process occurs, and condensers are also essential components in vapor-compression refrigeration cycles. Thus, the practicing engineer needs to be familiar with phase change processes.

Applications involving the solidification or melting of materials are also important. Typical examples include the making of ice, freezing of foods, freeze-drying processes, solidification and melting of metals, and so on. The freezing of food and other biological matter usually involves the removal of energy in the form of both sensible heat (enthalpy) and latent heat of freezing. A large part of biological matter is liquid water, which has a latent enthalpy of freezing,  $h_{sf}$ , of approximately 335 kJ/kg (144 Btu/lb or 80 cal/g). When meat is frozen from room temperature, it is typically placed in a freezer at  $-30^{\circ}\text{C}$ , which is considerably lower than the freezing point. The sensible heat to cool any liquids from the initial temperature to the freezing point is first removed, followed by the latent heat,  $h_{sf}$ , to accomplish the actual freezing. Once frozen, the substance is often cooled further by removing some sensible heat of the solid.

Dimensionless parameters arise in boiling and condensation. The Nusselt and Prandtl numbers, presented earlier in convection analyses appear once again. The new dimensionless parameters are the Jakob number, Ja, the Bond (no relation to James) number, Bo, and the condensation number, Co. The Jakob number is the ratio of the maximum sensible energy absorbed by the liquid (vapor) to the latent energy absorbed by the liquid (vapor). However, in many applications, the sensible

**Table 12.1** Phase Change Operations

Process	Description
Solidification	Change from liquid to solid
Melting	Change from solid to liquid
Boiling	Change from liquid to vapor
Condensation	Change from vapor to liquid
Sublimation	Change from solid to vapor

energy is much less than the latent energy and  $Ja$  has a small numerical value. The Bond number is the ratio of the buoyancy force to the surface tension force and finds application in some nucleate boiling equations. The condensation number, as one might suppose, is employed in condensation calculations.

### ILLUSTRATIVE EXAMPLE 12.1

It was noted earlier in the section that “other phase changes besides these are possible” (see Table 12.1). Provide an example of a solid-to-solid phase change.

**SOLUTION:** Iron–carbon undergoes a solid-to-solid phase change at a temperature of  $727^{\circ}\text{C}$ . ■

The objectives of this chapter are to develop an understanding of the physical conditions associated with boiling and condensation and to provide a basis for performing related heat transfer calculations. The remaining chapter contents are highlighted with four sections:

- Condensation Fundamentals
- Condensation Principles
- Boiling Fundamentals
- Boiling Principles

## CONDENSATION FUNDAMENTALS

Although several earlier chapters dealt with situations in which the fluid medium remained in a single phase, a significant number of real-world engineering applications involve a phase change that occurs simultaneously with the heat transfer process. As discussed earlier, the process of condensation of a vapor is usually accomplished by allowing it to come into contact with a surface where the temperature is maintained at a value lower than the saturation temperature of the vapor for the pressure at which it exists. The removal of thermal energy from the vapor causes it to lose its latent heat of vaporization and, hence, to condense onto the surface.

The appearance of the liquid phase on the cooling surface, either in the form of individual drops or in the form of a continuous film, offers resistance to the removal of heat from the vapor. In most applications, the condensate is removed by the action of gravity. As one would expect, the rate of removal of condensate (and the rate of heat removal from the vapor) is greater for vertical surfaces than for horizontal surfaces. Most condensing equipment consists of an assembly of tubes around which the vapor to be condensed is allowed to flow. The cool temperature of the outer tube surface is maintained by circulating a colder medium, often water, through the inside of the tube.

There are primarily three types of condensation processes:

1. *Surface Condensation.* This type of condensation occurs when vapor is in contact with a cool surface. This process is common in industrial applications and is discussed below.
2. *Homogeneous Condensation.* Homogeneous condensation occurs when the vapor condenses out as droplets in the gas phase.
3. *Direct Contact Condensation.* This process occurs when vapor is in contact with a cold liquid.

Surface condensation may occur in one of two modes depending upon the conditions of the surface.

1. *Film Condensation.* When the surface is clean and uncontaminated, the condensed vapor forms a liquid film that covers the entire condensing surface; this film contributes an additional resistance to heat transfer.
2. *Dropwise Condensation.* When the surface is coated with a substance that inhibits wetting, the condensed vapor forms drops in cracks and cavities on the surface. The drops often grow and coalesce and “drop” from the surface available for heat transfer. Thus, the additional resistance to heat transfer can be either reduced or eliminated. Up to 90% of the surface can be covered by drops. This topic receives a superficial treatment later in the chapter, with film condensation receiving the bulk of the analysis.

Liquid condensate provides resistance to heat transfer between the vapor and the surface. This resistance naturally increases as the thickness of the condensate later increases. For design purposes, it is usually desirable to have the condensation occur on surfaces that discourage the formation of thick liquid layers (e.g., short vertical surfaces or horizontal cylinders, or tube bundles, through which a coolant liquid flows). Dropwise condensation has a lower thermal resistance (and therefore a higher heat transfer rate) than filmwise condensation. In industrial practice, surface coatings that inhibit wetting are often used. Examples of such coatings include Teflon, silicones, waxes, and fatty acids. The drawback is that these coatings lose their effectiveness over time, which results in the condensation mode eventually changing from dropwise to filmwise. Obviously, all other things being equal, dropwise condensation is preferred to film condensation. In fact, steps are often put in place to induce this “dropwise” effect on heat transfer surfaces.

Finally, it should be noted that the local heat transfer coefficient varies along a flat surface and along the length of a vertical tube and around the perimeter of a horizontal tube (i.e., the local coefficient of heat transfer for a vapor condensing in a horizontal tube is a function of position, just like a vertical tube). As one would suppose, the highest coefficient is located at the top of a tube where the condensate film is thinnest. The condensing temperature in the application of the equations to be presented in the next section involving film coefficients is taken as the temperature at the vapor–liquid interface.

## Phase Equilibrium<sup>(2)</sup>

The *degrees of freedom*,  $F$ , or the *variance* of a system is defined as the *smallest number* of independent variables (such as pressure, temperature, concentration) that must be specified in order to completely define (the remaining variables of) the system. The significance of the degrees of freedom of a system may be drawn from the following examples. In order to specify the density of gaseous (vapor) steam, it is necessary to state both the temperature and pressure to which this density corresponds. For example, the density of steam has a particular value at 150°C and 1 atm pressure. A statement of this density at 150°C without mention of pressure does not clearly define the state of the steam, for at 150°C the steam may exist at many other possible pressures. Similarly, mention of the pressure without the temperature leaves ambiguity. Therefore, for the complete description of the state of the steam, two variables must be given, and this phase, when present alone in a system, possesses two degrees of freedom, or the system is said to be *bivariant*. When liquid water and steam exist in equilibrium, however, the temperature and the densities of the phases are determined only by the pressure, and a statement of some arbitrary value of the latter is sufficient to define all the other variables. The same applies to the choice of temperature as the independent variable. At each arbitrarily chosen temperature (within the range of existence of the two phases), equilibrium is possible only at a given pressure, and once again the system is defined in terms of one variable. Under these conditions, the system possesses only one degree of freedom or it is *monovariant*.

There is a definite relation in a system between the number of degrees of freedom, the number of components, and the number of phases present. This relationship was first established by J. Willard Gibbs in 1876.<sup>(3)</sup> This relation, known as the *Gibbs Phase Rule*, is a principle of the widest generality. It is one of the most often used rules in thermodynamic analyses, particularly in the representation of equilibrium conditions existing in heterogeneous systems.

To arrive at a mathematical description of the phase rule, consider a system of  $C$  components in which  $P$  phases are present. The problem is to determine the total number of variables upon which such a system depends. First, the state of the system will depend upon the pressure and the temperature. Again, in order to define the composition of each phase, it is necessary to specify the concentration of  $C - 1$  constituents of the phase, the concentration of the remaining component being determined by difference. Since there are  $P$  phases, the total number of concentration variables will be  $P(C - 1)$ , and these along with the temperature and pressure constitute a total of  $[P(C - 1) + 2]$  variables.

Recall from algebra that when a system possesses  $n$  independent variables,  $n$  independent equations are necessary in order to solve for the value of each of these. Similarly, in order to define the  $[P(C - 1) + 2]$  variables of a system, this number of equations relating these variables would have to be available. The next question is then: How many equations involving these variables can possibly be written from the conditions describing the system? To answer this, recourse must be had to thermodynamics. Equilibrium thermodynamic principles makes it possible to write *one*

equation among the variables for each constituent distributed between any two phases.<sup>(2)</sup> When  $P$  phases are present,  $P - 1$  equations are available for each constituent, and for  $C$  constituents a total of  $C(P - 1)$  equations arise. If this number of equations is equal to the number of variables, the system is completely defined. However, this will generally not be the case, and the number of variables will exceed the number of equations by  $F$ , where

$$\begin{aligned} F &= \text{number of variables} - \text{number of equations} \\ &= [P(C - 1) + 2] - [C(P - 1)] \\ &= C - P + 2 \end{aligned} \quad (12.1)$$

Equation (12.1) is the celebrated Gibbs Phase Rule (GPR). The  $F$  term is the number of degrees of freedom of a system and provides the number of variables whose values must be specified (arbitrarily) before the state of the system can be completely and unambiguously characterized. According to the phase rule, the number of degrees of freedom of a system whose pressure and temperature are specified is therefore given by the difference between the number of components and the number of phases present (i.e., by  $C - P$ ).

It is assumed in the above derivation that each component is present in every phase. If a component is missing from a particular phase, however, the number of concentration variables is decreased by one. But, at the same time, the number of possible equations is also decreased by one. Hence the value of  $C - P$ , and therefore  $F$ , remains the same whether each constituent is present in every phase or not. This effectively means that the phase rule is not restricted by this assumption and is generally valid under all conditions of distribution provided that equilibrium exists in the system.

The simplest case of GPR is one in which only a single component in a single phase is present, as with ice or steam. When more than one component and/or phase is present in a system, the number of degrees of freedom correspondingly increases in accordance with Equation (12.1). Most (but not all) of the applications in this chapter involve one-component one-phase systems.

### ILLUSTRATIVE EXAMPLE 12.2

Calculate the number of degrees of freedom for a one-component, one-phase system.

**SOLUTION:** Refer to Equation (12.1)

$$F = C - P + 2$$

Since  $C = 1$  and  $P = 1$ ,

$$\begin{aligned} F &= 1 - 1 + 2 \\ &= 2 \end{aligned}$$

Thus, two independent variables must be specified to completely define the system. ■

From a thermodynamic point-of-view, condensation of a condensable vapor in a condensable vapor–noncondensable vapor mixture can be induced by either increasing the pressure or decreasing the temperature, or both. Condensation most often occurs when a vapor mixture contacts a surface at a temperature lower than the saturation (dew point) temperature of the mixture. The dew point<sup>(2)</sup> of a vapor mixture is the temperature at which the vapor pressure exerted by the condensable component(s) is equal to the(ir) partial pressure in the vapor. For example, consider an air–water mixture at 75°F that is 80% saturated with water (or has a relative humidity, RH, of 80%):

$$\% \text{ Sat} = \% \text{ RH} = \frac{p_{\text{H}_2\text{O}}}{p'}(100) \quad (12.2)$$

where  $p_{\text{H}_2\text{O}}$  = partial pressure of water  
 $p'$  = vapor pressure of water = 0.43 psia at 75°F (see Steam Tables, Appendix, Table C.1)

For this condition,  $p_{\text{H}_2\text{O}} = (0.8)(0.43) = 0.344$  psia.

Pure vapors condense (or vaporize) at their vapor pressure at a given temperature. For example, water at 212°F and 1 atm will vaporize to steam (or steam will condense to water). A vapor or a mixture of vapors in a non-condensable gas is more difficult to analyze. A typical example is steam (water) in air or a high molecular weight organic in air.

There are two key vapor–liquid mixtures of interest to the practicing engineer: air–water and steam–water (liquid). Information on the former is available on a psychrometric chart while steam tables provide information on the later. A discussion on both follows.

## Psychrometric Chart

A vapor–liquid phase equilibrium example involving raw data is the psychrometric or humidity chart.<sup>(2)</sup> A humidity chart is used to determine the properties of moist air and to calculate moisture content in air. The ordinate of the chart is the absolute humidity  $\mathcal{H}$ , which is defined as the mass of water vapor per mass of bone-dry air. (Some charts base the ordinate on moles instead of mass.) Based on this definition, Equation (12.3) gives  $\mathcal{H}$  in terms of moles and also in terms of partial pressure:

$$\mathcal{H} = \frac{18n_{\text{H}_2\text{O}}}{29(n_T - n_{\text{H}_2\text{O}})} = \frac{18p_{\text{H}_2\text{O}}}{29(P - p_{\text{H}_2\text{O}})} \quad (12.3)$$

where  $n_{\text{H}_2\text{O}}$  = number of moles of water vapor  
 $n_T$  = total number of moles in gas  
 $p_{\text{H}_2\text{O}}$  = partial pressure of water vapor  
 $P$  = total system pressure

Curves showing the *relative humidity* (ratio of the mass of the water vapor in the air to the maximum mass of water vapor that the air could hold at that temperature, i.e.,

if the air were saturated) of humid air also appear on the charts. The curve for 100% relative humidity is also referred to as the *saturation curve*. The abscissa of the humidity chart is air temperature, also known as the *dry-bulb* temperature ( $T_{DB}$ ). The *wet-bulb* temperature ( $T_{WB}$ ) is another measure of humidity; it is the temperature at which a thermometer with a wet wick wrapped around the bulb stabilizes. As water evaporates from the wick to the ambient air, the bulb is cooled; the rate of cooling depends on how humid the air is. No evaporation occurs if the air is saturated with water; hence,  $T_{WB}$  and  $T_{DB}$  are the same. The lower the humidity, the greater the difference between these two temperatures. On the psychrometric chart, constant wet-bulb temperature lines are straight with negative slopes. The value of  $T_{WB}$  corresponds to the value of the abscissa at the point of intersection of this line with the saturation curve.

Given the dry bulb and wet bulb temperatures, the relative humidity (along with any other quantity on the chart) may be determined by finding the point of intersection between the dry bulb abscissa and wet bulb ordinate. The point of intersection describes all humidity properties of the system.

## Steam Tables

The steam tables comprise a tabular representation of the thermodynamic properties of water. These tables are divided into three separate categories:

1. The *saturated*-steam tables provide the value of the enthalpy, specific volume, and entropy of saturated steam and saturated water as functions of pressures and/or temperatures (condensation or boiling points). Changes in these extensive properties during the evaporation of 1 lb of the saturated liquid are also tabulated. The tables normally extend from 32 to 705°F, the temperature range where saturated liquid and vapor can coexist.
2. The *superheat* tables list the same properties in the superheated-vapor region. Degrees superheat or number of degrees above the boiling point (at the pressure in question) are also listed.
3. The Mollier chart (enthalpy–entropy diagram) for water is so frequently used in engineering practice that it also deserves mention. This diagram is useful since the *entropy function stays constant during any reversible adiabatic expansion or compression.*<sup>(2)</sup>

The steam tables are located in the Appendix.

### ILLUSTRATIVE EXAMPLE 12.3

A large *enclosed* main contains steam at 500 psia and 233°F superheat. Find the temperature and pressure in the main after half the steam has condensed.

**SOLUTION:** From the saturated steam tables in the Appendix, the condensation temperature is 467°F at 500 psia. Therefore, the steam temperature is  $467 + 233 = 700^\circ\text{F}$ . The



corresponding initial specific volume is approximately  $1.30 \text{ ft}^3/\text{lb}$ . After condensation, only half of the steam vapor remains. Therefore, the final specific volume will be<sup>(4)</sup>  $(1.30)/0.5 = 2.60 \text{ ft}^3/\text{lb}$ . Since two phases exist at equilibrium, Gibbs phase rule indicates that

$$F = (C - P) + 2 = (1 - 2) + 2 = 1$$

Employing the saturated tables, one notes (by interpolation)<sup>(4)</sup> that  $T = 370^\circ\text{F}$  and  $P = 175 \text{ psia}$  as the final state. The specific volume of the liquid is approximately  $0.018 \text{ ft}^3/\text{lb}$ . ■

### ILLUSTRATIVE EXAMPLE 12.4

Refer to the previous illustrative example. How much heat is removed from the system during this process?

**SOLUTION:** This is a constant volume system since the main is enclosed. From the first law<sup>(2)</sup>

$$Q = \Delta U = U_2 - U_1; U = \text{internal energy}$$

From the steam tables<sup>(4)</sup>

$$U_1 = 1237.1 \text{ Btu/lb}$$

$$U_{2,g} = 1112.2 \text{ Btu/lb}$$

$$U_{2,l} = 343.15 \text{ Btu/lb}$$

Substituting,

$$\begin{aligned} Q &= 0.5(1112.2 + 343.15) - (1)(1237.1) \\ &= 727.7 - 1237.1 \\ &= -509.4 \text{ Btu/lb (heat is removed)} \end{aligned}$$

Regarding solid–vapor equilibrium, a solid, like a liquid, has a definite vapor pressure at each temperature (some solids cannot exist in contact with vapor). Examples are helium below the critical temperature and various forms of ice; this pressure may be extremely small but it is nevertheless finite. The vapor pressure of a solid increases with temperature and the variation can be represented by a curve similar to that for a liquid; it is generally called a *sublimation curve*, with the term *sublimation* used to indicate the direct conversion of solid to vapor without the intervention of liquid. The change from solid to vapor is accompanied, like the analogous change from liquid to vapor, by an absorption of heat; this is referred to as the latent enthalpy of sublimation.

## CONDENSATION PRINCIPLES

As noted earlier, condensation occurs when the temperature of a vapor is reduced below its saturation temperature and results from contact between the vapor and a cooler surface. The latent enthalpy of the vapor is released, heat is transferred to the surface, and the condensate forms.

As noted in the Introduction to this chapter, there are three basic types of condensation: *surface condensation*, *homogeneous condensation*, and *direct contact condensation*. The corresponding modes are: *film condensation* and *dropwise condensation*. Although it is desirable to achieve dropwise condensation in real-world applications, it is often difficult to maintain this condition. Although convection coefficients for film condensation are smaller than those for the dropwise case, condenser design calculations may be and usually are based on the assumption of film condensation. It is for this reason that the paragraphs to follow in this section focus on film condensation.

Filmwise condensation may be laminar or turbulent, depending on the Reynolds number of the condensate. The condensate Reynolds number is defined as

$$\text{Re} = \frac{4\rho_L v_{mL} \delta}{\mu_L} = \frac{4\dot{m}'}{\mu_L} = \frac{4\dot{m}}{W\mu_L} \quad (12.4)$$

where  $\dot{m}'$  is the mass flow rate of condensate per unit width of the surface;  $\dot{m}$ , the mass flow rate of condensate;  $v_{mL}$ , the mean velocity of the condensate liquid film;  $W$ , the width of the condensing surface;  $\delta$ , the average thickness of the condensed liquid film;  $\rho_L$ , the density of the liquid condensate; and  $\mu_L$ , the absolute viscosity of the liquid condensate.

Laminar condensation occurs when  $\text{Re} < 1800$ , while turbulent condensation occurs when the  $\text{Re} > 1800$ . Since the flow may not occur in a circular conduit, the hydraulic diameter,  $D_h$ , must be used. The hydraulic diameter,  $D_h$  (as defined earlier), is:

$$D_h = \frac{4A_C}{P_W} = \frac{4\dot{m}}{\rho_L v_{mL} P_W} \quad (12.5)$$

where  $A_C$  is the area of the conduit and  $P_W$  is the wetted perimeter. Assuming that the vapor is at its saturation temperature,  $T_{\text{sat}}$ , and letting  $T_S$  represent the surface temperature, the rate of heat transfer,  $\dot{Q}$ , can be related to the rate of condensation,  $\dot{m}'$ , through an energy balance, as shown in the following equations:

$$\dot{Q} = hA(T_{\text{sat}} - T_S) = \dot{m}h_{\text{vap}} \quad (12.6)$$

with  $A$  representing the heat transfer area. Rearranging Equation (12.6) leads to

$$\dot{m}' = \frac{\dot{m}}{P_W} = \frac{hA(T_{\text{sat}} - T_S)}{P_W h_{\text{vap}}} \quad (12.7)$$

Substituting Equation (12.7) for  $\dot{m}'$  in the condensate Reynolds number definition in Equation (12.4) gives

$$\text{Re} = \frac{4\dot{m}'}{\mu_L} = \frac{4hA(T_{\text{sat}} - T_S)}{P_W h_{\text{vap}} \mu_L} \quad (12.8)$$

The values of  $A$ ,  $P_W$  and the ratio of  $A/P_W$  for several geometries are summarized in Table 12.2.

**Table 12.2** Condensation Geometries

Geometry	Wetted perimeter, $P_W$	Heat transfer area, $A$	$A/P_W$
Vertical plate ( $W \times Z$ )	$W$	$WZ$	$Z$
Vertical tube	$\pi D$	$\pi DZ$	$Z$
Horizontal tube	$Z$	$\pi DZ$	$\pi D$
Horizontal tube bank $n_1$ rows $\times$ $n_2$ columns	$n_2 Z$	$n_1 n_2 \pi DZ$	$\pi n_1 D$

Note:  $W$  = width;  $Z$  = height;  $D$  = diameter.

The following correlations are available for calculating the heat transfer coefficient during condensation.

1. For laminar film condensation on a vertical plate or vertical cylinder:

$$\text{Co} = 1.874 \text{Re}_f^{-1/3} \quad (12.9)$$

where Co is the average condensation number and is given by

$$\text{Co} = \frac{\bar{h}}{k} \left( \frac{v_L^2}{g(1 - [\rho_v/\rho_L])} \right)^{1/3}; \quad g = \text{acceleration due to gravity} \quad (12.10)$$

This equation can also be written in the dimensional form:

$$\bar{h} = 1.13 \left[ \frac{\rho_L(\rho_L - \rho_v)gh_{\text{vap}}k_L^3}{(A_h/P_W)\mu_L(T_{\text{sat}} - T_S)} \right]^{0.25} \quad (12.11)$$

where  $k_L$  is the thermal conductivity of the liquid and  $\rho_L$  is the density of the liquid.

2. For laminar film condensation on horizontal tubes of diameter  $D$ , the Nusselt equation is:

$$\text{Co} = 1.514 \text{Re}_f^{-1/3} \quad (12.12)$$

In dimensional form:

$$\bar{h} = 0.725 \left[ \frac{\rho_L(\rho_L - \rho_v)gh_{\text{vap}}k_L^3}{D\mu_L(T_{\text{sat}} - T_S)} \right]^{0.25} \quad (12.13)$$

3. For turbulent film condensation, the Kirkbride equation is:

$$\text{Co} = 0.0077 \text{Re}_f^{0.4} \quad (12.14)$$

with Co once again given by Equation (12.10).

A superheated vapor occurs when the temperature of the vapor is higher than that of the boiling point at the corresponding pressure. When a superheated vapor enters a condenser, the sensible heat of superheat and the latent heat of condensation must be

transferred through the cooling surface. The condensation mechanism is therefore somewhat different if the condensing vapor is superheated rather than saturated. Experimental results have shown that, in most cases, the effect of superheat may be ignored and the equations for saturated vapors may be used with negligible error. It should be noted that

$$T_{\text{sat}} - T = \Delta T \quad (12.15)$$

is still the temperature difference driving force, and that the actual superheated vapor temperature does not enter into the calculations.

For mixed vapors (i.e., if the vapor contains two or more volatile components), the condensation temperature is no longer constant at a given pressure unless the mixture is azeotropic. If the cooling surface temperature is low, the vapor may condense and the condensate may be assumed to be the same as that of the original vapor. If the cooling surface temperature is straddled by the condensation temperatures of the components, part of the vapor will condense and some vapor must be vented from the condenser.

Regarding dropwise condensation, heat transfer coefficients are an order of magnitude larger than those for film condensation. If other thermal resistances in the system are significantly larger than that due to condensation, the condensation resistance may be neglected. The following expressions are recommended for estimation purposes for dropwise condensation:

$$\bar{h} = 51,000 + 2000T_{\text{sat}} \quad 22^\circ\text{C} < T_{\text{sat}} < 100^\circ\text{C} \quad (12.16)$$

$$\bar{h} = 255,000 \quad 100^\circ\text{C} < T_{\text{sat}} \quad (12.17)$$

where the heat transfer coefficient  $h$  has units of  $\text{W}/\text{m}^2 \cdot \text{K}$ .

### ILLUSTRATIVE EXAMPLE 12.5

A vertical rectangular plate is 0.2 m wide and 0.4 m high. One surface of the plate is exposed to saturated steam at atmospheric pressure. The plate surface is maintained at  $98^\circ\text{C}$ . Filmwise condensation may be assumed. Calculate the type of condensation (flow type) and the condensation heat transfer coefficient. Assume steady-state, constant properties, an isotropic homogeneous surface, and that the properties of the water condensate may be evaluated at a film temperature of  $99^\circ\text{C}$ . The thermal conductivity of water may be assumed equal to  $0.68 \text{ W}/\text{m} \cdot \text{K}$ .

**SOLUTION:** Obtain the latent heat of steam,  $h_{\text{vap}}$ , at the saturation temperature. From the steam tables (see Appendix), at  $100^\circ\text{C}$ ,

$$h_{\text{vap}} = 970.3 \text{ Btu}/\text{lb} = 2.255 \times 10^6 \text{ J}/\text{kg}$$

Obtain the density of steam at the mean film temperature. From the same steam tables,

$$\rho_v = 0.577 \text{ kg}/\text{m}^3 \text{ at } 99^\circ\text{C}$$

Also, obtain the properties of liquid water condensate ( $\rho_L$ ,  $\mu_L$ ,  $k_L$ ) at the mean film temperature. From the Appendix, at 99°C (some interpolation required),

$$\begin{aligned}\rho_L &= 960 \text{ kg/m}^3 \\ \mu_L &= 2.82 \times 10^{-4} \text{ kg/m} \cdot \text{s} \\ k_L &= 0.68 \text{ W/m}^2 \cdot \text{K}\end{aligned}$$

Obtain the ratio,  $A/P_W$  for a vertical plate from Table 12.2:

$$\begin{aligned}Z &= \text{height} = 0.4 \text{ m} \\ P_W &= W = 0.2 \text{ m} \\ A/P_W &= Z = 0.4 \text{ m}\end{aligned}$$

Generate an expression for the condensation number, Co, in terms of the average heat transfer coefficient using Equation (12.10):

$$\begin{aligned}v_L &= \frac{\mu_L}{\rho_L} = \frac{2.82 \times 10^{-4}}{960} = 2.94 \times 10^{-7} \text{ m}^2/\text{s} \\ \text{Co} &= \frac{\bar{h}}{k} \left( \frac{v_L^2}{g(1 - \rho_v/\rho_L)} \right)^{1/3} = \frac{\bar{h}}{0.68} \left( \frac{(2.94 \times 10^{-7})^2}{9.807(1 - 0.577/960)} \right)^{1/3} \\ &= 3.038 \times 10^{-5} \bar{h}\end{aligned}$$

Also, generate the Reynolds number in terms of the average heat transfer coefficient using Equation (12.8), noting that  $A/P_W = 0.4$ :

$$\text{Re} = \frac{4\bar{h}(0.4)(100 - 98)}{(2.82 \times 10^{-4})(2.255 \times 10^6)} = 5.03 \times 10^{-3} \bar{h}$$

Assume a flow type (laminar or turbulent) and determine Co in terms of  $\text{Re}_L$ . First, assume turbulent condensation on a vertical plate so that Equation (12.14) applies:

$$\text{Co} = 0.0077 \text{Re}^{0.4}$$

Obtain a value for  $\bar{h}$  by substituting into the equations above:

$$\begin{aligned}3.038 \times 10^{-5} \bar{h} &= (0.0077)(5.03 \times 10^{-3})^{0.4} (\bar{h})^{0.4} \\ \bar{h} &= 298 \text{ W/m}^2 \cdot \text{K}\end{aligned}$$

Check the Reynolds number to verify the assumption of flow type:

$$\text{Re} = (5.03 \times 10^{-3}) \bar{h} = (5.03 \times 10^{-3})(298) = 1.50 < 1800$$

The assumption of turbulent condensation is not valid. Therefore, assume laminar flow and employ the expression for Co in terms of Re in Equation (12.9):

$$\text{Co} = 1.874 \text{Re}^{-1/3}$$

Once again, obtain a value for  $\bar{h}$  by substituting into the equation above

$$\begin{aligned} 3.038 \times 10^{-5} \bar{h} &= (1.874)(5.03 \times 10^{-3})^{-1/3} (\bar{h})^{-1/3} \\ \bar{h} &= 14,700 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

Check the Reynolds number to verify the assumption of flow type:

$$\text{Re} = (5.03 \times 10^{-3}) \bar{h} = (5.03 \times 10^{-3})(14,700) = 73.9 < 1800$$

The assumption of laminar flow is valid. ■

### ILLUSTRATIVE EXAMPLE 12.6

Refer to Illustrative Example 12.5. Calculate

1. the mass flow rate of condensate,  $\dot{m}'$
2. the heat transfer rate  $\dot{Q}$ .

**SOLUTION:** First calculate  $\dot{m}'$ :

$$\begin{aligned} \text{Re} &= \frac{4\dot{m}'}{\mu_L} \\ \dot{m}' &= \frac{\text{Re}\mu_L}{4} \end{aligned} \tag{12.4}$$

Substituting,

$$\begin{aligned} \dot{m}' &= \frac{(73.9)(2.82 \times 10^{-4})}{4} \\ &= 0.0052 \text{ kg/m} \cdot \text{s} \end{aligned}$$

Calculate  $\dot{m}$  employing Equation (12.7):

$$\begin{aligned} \dot{m} &= P_w \dot{m}' = W \dot{m}' \\ &= (0.2)(0.0052) \\ &= 0.00104 \text{ kg/s} \end{aligned}$$

Calculate Co using the result from the previous illustrative example:

$$\text{Co} = (3.0377 \times 10^{-5}) \bar{h} = (3.0377 \times 10^{-5})(14,700) = 0.4465$$

Finally, calculate  $\dot{Q}$ :

$$\begin{aligned}\dot{Q} &= hA(T_{\text{sat}} - T_S) \\ &= (14,700)(0.2)(0.4)(100 - 98) \\ &= 2350 \text{ W} \\ &= 2.35 \text{ kW}\end{aligned}$$

### ILLUSTRATIVE EXAMPLE 12.7

A horizontal 4-inch OD tube is surrounded by saturated steam at 2.0 psia. The tube is maintained at 64°F. What is the average heat-transfer coefficient? Assume laminar flow.

**SOLUTION:** Assuming laminar flow, the average heat-transfer coefficient is given by Equation (12.13). The liquid properties are evaluated at the mean film temperature,  $T_f = (T_{\text{sat}} + T)/2 = (126 + 64)/2 = 95^\circ\text{F}$ . The approximate values (at 100°F) of key properties are:

$$\begin{aligned}h_{\text{vap}} &= 1022 \text{ Btu/lb (at } T_{\text{sat}}) \\ \rho_v &= 0.00576 \text{ lb/ft}^3 \text{ (at } T_{\text{sat}}) \\ \rho_L &= 62.03 \text{ lb/ft}^3 \\ k_L &= 0.364 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \\ \mu_L &= 4.26 \times 10^{-4} \text{ lb/ft} \cdot \text{s}\end{aligned}$$

Solving,

$$\begin{aligned}\bar{h} &= 0.725 \left[ \frac{\rho_L(\rho_L - \rho_v)gh_{\text{vap}}k_L^3}{\mu_L D(T_{\text{sat}} - T_S)} \right]^{1/4} \\ &= 0.725 \left[ \frac{(62.03)(62.03 - 0.00576)(32.2)(1022)(0.364)^3}{(4.26 \times 10^{-4})(4/12)(126 - 64)/(3600)} \right]^{1/4} \\ &= 911.6 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}\end{aligned} \quad (12.13)$$

## BOILING FUNDAMENTALS

The process of converting a liquid into a vapor is also of importance to practicing engineers. The production of steam for electrical power generation is a prime example. Many other processes, particularly in the refining of petroleum and the manufacture of chemicals, require the vaporization of a liquid.

Boiling is the opposite of condensation. Boiling occurs when a liquid at its saturation temperature,  $T_{\text{sat}}$ , is in contact with a solid surface at a temperature,  $T_S$ , which is above  $T_{\text{sat}}$ . The excess temperature,  $\Delta T_e$ , is defined as:

$$\Delta T_e = T_S - T_{\text{sat}} \quad (12.18)$$

The  $\Delta T_e$  driving force causes heat to flow from the surface into the liquid, which results in the formation of vapor bubbles that move up through the liquid. Because of bubble formation, the surface tension of the liquid has an impact on the rate of heat transfer.

Boiling may be classified as *pool boiling* or *forced convection boiling*. In pool boiling, the liquid forms a “pool” in a container while submerged surfaces supply the heat. The liquid motion is induced by the formation of bubbles as well as density variations. In forced convection, the liquid motion is induced by external means (e.g., pumping of liquids through a heated tube).

Another common classification for types of boiling is based on the relationship of the liquid temperature to its saturated temperature. Boiling is *subcooled* (also known as *local*) when the liquid temperature is below the saturation temperature (i.e.,  $T < T_{\text{sat}}$ ). When the liquid is at the saturation temperature, the boiling is *saturated* boiling.

The explanation of the strange behavior of boiling systems lies in the fact that boiling heat transfer occurs by several different mechanisms and the mechanism is often more important in determining the heat rate than the temperature-difference driving force. Consider, for example, the heating of water in an open pot. Heat is initially transferred within the water by natural convection. As the heat rate is further increased, the surface temperature at the base of the pot increases to and above 212°F. Bubbles begin to form and then rise in columns from the heating surface creating a condition favorable to heat transfer. As the temperature increases further, more sites become available until the liquid can no longer reach the heating surface at a sufficient rate to form the required amount of vapor. This ends the *nucleate boiling* stage (to be discussed later). At this point, the mechanism changes to *film boiling* since the heating surface is now covered with a film of vapor. The temperature of the base of the pot rises (even though the heat rate is constant). Vaporization takes place at the liquid–vapor interface and the vapors disengage from the film in irregularly-shaped bubbles at random locations.

Although the phenomena of superheating occurs in most boiling systems, the temperature of the boiling liquid, measured some distance from the heated surface, is higher than the temperature of the vapor above the liquid (which is at the saturation temperature). The liquid superheat adjacent to the heated surface may be as high as 25 to 50°F. Most of the temperature change occurs in a narrow thin film from the surface. This superheat occurs because the internal pressure in the vapor bubble is higher due to surface tension effects.

The formation, growth, and release of bubbles is an extremely rapid sequence of events. The rapid growth and departure of vapor bubbles causes turbulence in the liquid, especially in the aforementioned zone of superheat near the heated surface. This turbulence assists the transport of heat from the heated surface to the liquid evaporating at the bubble surface. The rapid growth of bubbles and the turbulence in the liquid complement each other, resulting in high heat transfer coefficients.

As the superheat of a boiling liquid is further increased, the concentration of active centers on the heating surface increases and the heat rate correspondingly increases. The mass rate of the vapor rising from the surface must be equal to the mass rate of liquid proceeding toward the surface if steady-state conditions prevail. As the boiling



rate increases, the rate of liquid influx must increase since the area available for flow decreases with the increasing number of bubble columns; in addition, the liquid velocity must also increase. At this limiting condition, the liquid flow toward the heated surface cannot increase and the surface becomes largely blanketed with vapor. If the heat rate to the surface is held constant, the surface temperature will rise to a high value at which point heat is transmitted to the fluid by the mechanism of film boiling. Film boiling occurs when the superheat is sufficiently high to keep the heated surface completely blanketed with vapor. Heat may then be transmitted through the gas film by conduction, convection, and radiation.

Finally, the *Leidenfrost* phenomenon is one of the many complex properties of boiling. If a liquid is dropped on a surface that has a temperature significantly higher than the liquid's boiling point, the liquid will skitter across the surface and evaporate at a slower rate than expected. This is due to an instantaneous layer of vapor, suspending the liquid, which acts as insulation between the hot surface and the drop of liquid. At temperatures closer to boiling, the liquid will evaporate at a much faster rate.

Interestingly, there is a unique relationship between a liquid's normal boiling point and critical point. The describing equation is given by:

$$\frac{T_b}{T_c} \approx 0.6 \quad (12.19)$$

The relationship is demonstrated in Table 12.3.

Boiling point elevation is another interesting phenomenon. When a solute (e.g., NaOH) is dissolved in water, the vapor pressure of the aqueous solution is less than that of water at the same temperature. Thus, at a given pressure, the solution boils at a higher temperature than water. This increase is termed the boiling point elevation (BPE) of the solution. Dilute solutions of organic compounds exhibit small (negligible) BPEs. For aqueous solutions of inorganic compounds, the BPE may be as

**Table 12.3** Ratio of Boiling Point to Critical Temperature

Substance	$T_b$ , K	$T_c$ , K	$T_b/T_c$ Ratio
Nitrogen	77.3	126.0	0.61
Oxygen	90.1	154.3	0.58
Carbon dioxide	194.6	304.2	0.64
Ammonia	239.7	405.5	0.59
Water	373.2	647	0.58
Ethane	185	305	0.61
<i>n</i> -Pentane	309	470	0.66
Benzene	353	562	0.63
Carbon tetrachloride	350	556	0.63
Ethyl alcohol	351	516	0.68
Acetic acid	391	594	0.66

high as 150°F. This BPE represents a loss in the thermal driving force and has to be accounted for in some calculations.

### ILLUSTRATIVE EXAMPLE 12.8

The rate of heat flow per unit area of heat transfer surface to a boiling liquid

- (a) increases with the temperature of the heating medium.
- (b) decreases with the temperature of the heating medium.
- (c) increases with the temperature of the boiling liquid.
- (d) decreases with the temperature of the boiling liquid.
- (e) may increase or decrease with the temperature of the heating medium.
- (f) None of the above.

Which statement is correct?

**SOLUTION:** The answer is (e) due to the aforementioned “bubble” effect. ■

## BOILING PRINCIPLES

It has been found that the heat transfer coefficient,  $h$ , in boiling systems depends on the excess temperature,  $\Delta T_e$ . For free convection boiling ( $\Delta T_e < 5^\circ\text{C}$ ):

$$h \propto (\Delta T_e)^n \quad (12.20)$$

where  $n = \frac{1}{4}$  laminar free convection conditions  
 $= \frac{1}{3}$  turbulent free convection conditions

The boiling heat flux,  $\dot{Q}'_s$ , is calculated as:

$$\dot{Q}'_s = h(T_S - T_{\text{sat}}) \quad (12.21)$$

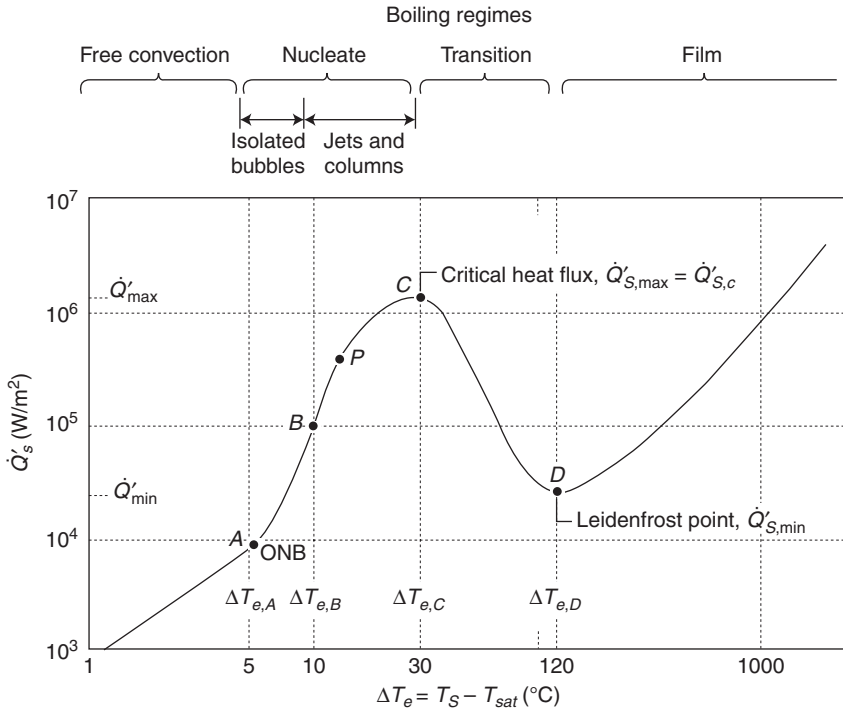
and

$$\Delta T_e = T_S - T_{\text{sat}} \quad (12.22)$$

The dimensionless number of importance in boiling heat transfer is the liquid Jakob number,  $\text{Ja}_L$ . It is defined as

$$\text{Ja}_L = \frac{c_p \Delta T_e}{h_{\text{vap}}} \quad (12.23)$$

The mechanism of pool boiling heat transfer depends on the excess temperature,  $\Delta T_e$ , where, once again,  $\Delta T_e$  is the difference between surface temperature and the fluid saturation temperature. A plot of the boiling heat transfer flux,  $\dot{Q}'_s$ , versus the excess temperature, is shown in Figure 12.1. When  $\Delta T_e > 5^\circ\text{C}$ , bubbles start to



**Figure 12.1** Boiling curve. (Adapted from Incropera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981.)

form at the onset of nucleate boiling (ONB). As noted earlier, the bubbles rise and are dissipated in the liquid; the resulting stirring causes an increase in the heat flux. This regime was termed *nucleate* boiling. The heat transfer coefficient in this regime varies with  $\Delta T_e$  as follows:

$$h \propto (\Delta T_e)^n \quad (12.20)$$

where  $n$  varies between 3 or 4. As  $\Delta T_e$  increases, the rate of bubble formation increases. This causes  $h$  to increase, but at higher rates of bubble formation some bubbles cannot diffuse quickly enough and form a blanket around the heating element. This blanket increases the heat transfer resistance and slows down the increase in both  $h$  and the heat flux. These two factors balance out at approximately  $\Delta T_e = 30^\circ\text{C}$  where the flux (introduced below) reaches its maximum. The peak flux is termed the *critical heat flux*,  $\dot{Q}'_{S,c}$ . For boiling water,  $\dot{Q}'_{S,c}$  is greater than  $1 \text{ MW/m}^2$ . As  $\Delta T_e$  increases still further, the gas film formed by the coalescing bubbles around the element prevents the liquid from coming in and causes a further increase in the heat transfer resistance. This mechanism is defined as *film* boiling.

For the approximate  $30^\circ\text{C} < \Delta T_e < 120^\circ\text{C}$  range, the mechanism becomes *unstable film* boiling, also known as *transition film* boiling, or *partial film* boiling. The heat transfer coefficient decreases with increasing  $\Delta T_e$  until a stable value is reached. At these high surface temperatures,  $T_s$ , thermal radiation contributes significantly to the heat transfer. This contribution causes the heat flux to increase again. The minimum in the curve is referred to as the *Leidenfrost point* (see previous section) and the heat flux is given by  $\dot{Q}'_{S,\min}$ .

At the point of peak (or critical) flux,  $\dot{Q}'_{S,c}$ , a small increase in  $\Delta T_e$  causes  $\dot{Q}'_S$  to decrease. However, with the lower heat flux, the energy from the heating surface cannot be completely dissipated, causing a further increase in  $\Delta T_e$ . Eventually, the fluid temperature can exceed the melting point of the heating medium. The critical heat flux (at point C on Figure 12.1) is therefore also termed the *boiling crisis point* or the *burnout point*.

Boiling equipment should be operated in the nucleate boiling region, close to the critical flux, but never exceeding it. To calculate the boiling heat flux in the nucleate boiling regime, one may use the Rohsenow equation:<sup>(5)</sup>

$$\dot{Q}'_s = \mu_L h_{\text{vap}} \sqrt{\frac{g(\rho_L - \rho_v)}{\sigma}} \left( \frac{\text{Ja}_L}{C_{sf} \text{Pr}_L^n} \right)^3 \quad (12.24)$$

where  $\sigma$  is the liquid surface tension (lb<sub>f</sub>/ft or N/m),  $\mu_L$  is the dynamic viscosity of the liquid (lb/ft · h or kg/m · s),  $C_{sf}$  is the correlation coefficient that depends on the surface–fluid combination,  $n$  is the Prandtl number exponent that depends on the surface–fluid combination and  $\text{Ja}_L$  is the liquid Jakob number, as presented in Equation (12.23). Numerical values of  $C_{sf}$  and  $n$  can be obtained from Table 12.4. Note that  $n = 1$  for water and 1.7 for other liquids.

The critical heat flux (or burnout flux) is calculated from the Zuber equation:<sup>(6,7)</sup>

$$\dot{Q}'_{S,c} = \frac{\pi}{24} h_{\text{vap}} \rho_v \left( 1 + \frac{\rho_v}{\rho_L} \right)^{1/2} \left( \frac{\sigma g (\rho_L - \rho_v)}{\rho_v^2} \right)^{1/4} \quad (12.25)$$

If  $\rho_v \ll \rho_L$ , the Zuber equation simplifies to:

$$\dot{Q}'_{S,\max} = \dot{Q}'_{S,c} = \frac{\pi}{24} h_{\text{vap}} (\sigma g \rho_L / \rho_v^2)^{1/4} \quad (12.26)$$

At the so-called *Leidenfrost point*, the heating surface is covered by a stable vapor blanket and the heat flux reaches its minimum condition,  $\dot{Q}'_{S,\min}$ . Zuber developed the following relationship to calculate the minimum heat flux from a large horizontal plate:

$$\dot{Q}'_{S,\min} = 0.09 h_{\text{vap}} \rho_v \left( \frac{\sigma g (\rho_L - \rho_v)}{(\rho_L + \rho_v)^2} \right)^{1/4} \quad (12.27)$$

**Table 12.4** Values of  $C_{sf}$  for Various Fluid–Surface Combinations to be Used with the Rohsenow Equation

Fluid–surface combination	$C_{sf}$	$n$
Water–copper		
Scored	0.0068	1.0
Polished	0.0130	1.0
Water–stainless steel		
Chemically etched	0.0130	1.0
Mechanically polished	0.0130	1.0
Ground and polished	0.0060	1.0
Water–brass	0.0060	1.0
Water–nickel	0.0060	1.0
<i>n</i> -Pentane–copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene–chromium	0.0101	1.7
Ethyl alcohol–chromium	0.0027	1.7

If  $\rho_v \ll \rho_L$ , the Zuber minimum flux equation simplifies to:

$$\dot{Q}'_{S, \min} = 0.09 h_{\text{vap}} \rho_v \left( \frac{\sigma g}{\rho_L} \right)^{1/4} \quad (12.28)$$

Water is the most common fluid used in boiling applications. To facilitate calculations involving water, approximate heat transfer relations have been developed to calculate the surface flux,  $\dot{Q}'_S$ , for the boiling of water during both pool boiling and forced convection. The surface flux,  $\dot{Q}'_S$ , is given by:

$$\dot{Q}'_S = h(T_s - T_w) \quad (12.29)$$

For forced convection boiling inside vertical tubes, the heat transfer coefficient is approximated by the dimensional relation:

$$h = 2.54(\Delta T_e)^n e^{(P/1.55)} \quad (12.30)$$

and for *pool* boiling:

$$h = C(\Delta T_e)^n (P/P_a)^{0.4} \quad (12.31)$$

where  $h$  is the heat transfer coefficient from the surface to the boiling water,  $P$  is the absolute pressure in mega pascals (MPa), and  $P_a$  is the ambient absolute pressure (e.g., 1 atm = 0.1013 MPa). The coefficients  $C$  and  $n$  are constants that depend on the orientation of the surface and the magnitude of the surface heat flux,  $\dot{Q}'_S$ . Numerical values of  $C$  and  $n$  are given in Table 12.5.

**Table 12.5** Constants for Use with the Approximate Pool Boiling Heat Transfer Coefficient Correlation for Water

Surface orientation	Surface flux, $\dot{Q}'_s$ , kW/m <sup>2</sup>	$C$	$n$
Horizontal	$\dot{Q}'_s < 15.8$	1040	$\frac{1}{3}$
	$15.8 < \dot{Q}'_s < 236$	5.56	3
Vertical	$\dot{Q}'_s < 3.15$	539	7
	$3.15 < \dot{Q}'_s < 63.1$	7.95	3

Since the surface flux is often unknown, the calculation of  $h$  involves trial-and-error. A typical procedure is to:

1. assume the range of the surface flux,  $\dot{Q}'_s$
2. read  $C$  and  $n$  from Table 12.5
3. calculate  $h$  using Equation (12.30) or Equation (12.31)
4. calculate  $\dot{Q}'_s$  from Equation (12.29), and
5. determine if the assumption made in (1) is valid.

For heat transfer coefficients involving boiling water, the effect of pressure on the heat transfer coefficient may be estimated from

$$h = h_{1 \text{ atm}} \left( \frac{P}{P_a} \right)^{0.4} \quad (12.32)$$

This pressure correction is recommended for other liquids if another correlation is not available.

### ILLUSTRATIVE EXAMPLE 12.9

Boiling water at 100°C and 1 atm is supplied with heat from a surface at 102°C. The heat flux is 9800 W/m<sup>2</sup>. If the surface temperature is raised to 103°C, determine the new heat flux.

At 1 atm, the saturation temperature of the water is 100°C. The liquid water properties (SI units) are: density = 957.9 kg/m<sup>3</sup>, heat capacity = 4217 J/kg · K, latent heat of vaporization = 2.26 × 10<sup>6</sup> J/kg. Assume steady-state operation, constant properties, and laminar conditions.

**SOLUTION:** Calculate the heat transfer coefficient at 102°C using Equation (12.21)

$$h = \frac{\dot{Q}'_s}{T_S - T_L} = \frac{\dot{Q}'_s}{T_S - T_{\text{sat}}} = \frac{9800}{102 - 100} = 4900 \text{ W/m}^2 \cdot \text{K}$$

The new heat transfer coefficient may now be calculated, see Equation (12.20). Since  $h \propto (\Delta T_e)^n$  and  $n = 0.25$ , for laminar flow;

$$h_2 = h_1 \left( \frac{\Delta T_{e1}}{\Delta T_{e2}} \right)^{1/4} = 4900 \left( \frac{3}{2} \right)^{1/4} = 5421 \text{ W/m}^2 \cdot \text{K}$$

The new heat flux is:

$$\dot{Q}'_s = h(T_S - T_L) = 5421(103 - 100) = 16,260 \text{ W/m}^2 \quad \blacksquare$$

### ILLUSTRATIVE EXAMPLE 12.10

With respect to Illustrative Example 12.9, determine the mechanism of boiling heat transfer. Is the earlier assumed boiling mechanism still valid?

**SOLUTION:** Calculate the original excess temperature:

$$\Delta T_e = T_S - T_{\text{sat}} = 102 - 100 = 2^\circ\text{C}$$

Calculate the new excess temperature (for  $T_S = 103^\circ\text{C}$ ).

$$\Delta T_e = T_S - T_{\text{sat}} = 103 - 100 = 3^\circ\text{C}$$

The assumption of the free convection mechanism is valid since  $\Delta T_e < 5^\circ\text{C}$ . \blacksquare

### ILLUSTRATIVE EXAMPLE 12.11

Refer to Illustrative Example 12.9 and calculate the liquid Jakob number.

**SOLUTION:** The liquid Jakob number may be calculated from Equation (12.22):

$$\text{Ja}_L = \frac{c_p \Delta T_e}{h_{\text{vap}}} = \frac{4127(3)}{2.26 \times 10^6} = 0.00548 \quad \blacksquare$$

### ILLUSTRATIVE EXAMPLE 12.12

An electric heater 0.3 m in diameter at  $106^\circ\text{C}$  is used to heat water. The water is at 1 atm and  $100^\circ\text{C}$ . Determine

1. the boiling regime, and
2. The heat transfer coefficient.

Assume steady-state operation and constant properties.

**SOLUTION:** Calculate the excess temperature,  $\Delta T_e$ ,

$$\Delta T_e = T_S - T_{\text{sat}} = 106 - 100 = 6^\circ\text{C}$$

See Figure 12.1. Since  $\Delta T_e$  is between  $5^\circ\text{C}$  and  $30^\circ\text{C}$ , nucleate boiling applies.

Assume a range for the surface flux,  $\dot{Q}'_S$ , and obtain the corresponding values of  $C$  and  $n$ . Assuming that  $15.8 < \dot{Q}'_S < 236 \text{ kW/m}^2$ , then, from Table 12.5,  $C = 5.56$  and  $n = 3$ . For pool, rather than forced convection, the water boiling heat transfer coefficient is given by Equation (12.31):

$$\begin{aligned} h &= C(\Delta T_e)^n (P/P_a)^{0.4} \\ &= 5.56(6)^3 (1)^{0.4} \\ &= 1200 \text{ W/m}^2 \end{aligned}$$

Calculate the surface flux using Equation (12.29) and check the assumption made above:

$$\begin{aligned} \dot{Q}'_S &= h\Delta T_e = (1200)(6) \\ &= 7200 \text{ W/m}^2 \\ &= 7.2 \text{ kW/m}^2 \end{aligned}$$

Since  $\dot{Q}'_S < 15.8$ , the assumption above is *not* correct. Therefore, assume a new range for the surface flux (e.g., that  $\dot{Q}'_S < 15.8 \text{ kW/m}^2$ ). From Table 12.5,  $C = 1040$  and  $n = \frac{1}{3}$ .

Once again, calculate  $h$  and  $\dot{Q}'_S$ :

$$\begin{aligned} h &= 1040(6)^{1/3} (1)^{0.4} = 1890 \text{ W/m}^2 \cdot \text{K} \\ \dot{Q}'_S &= h(\Delta T_e) \\ &= (1890)(6) \\ &= 11,340 \text{ W/m}^2 \\ &= 11.34 \text{ kW/m}^2 \end{aligned}$$

The second assumption is valid. ■

### ILLUSTRATIVE EXAMPLE 12.13

Refer to the previous example. Calculate the rate of heat transfer.

**SOLUTION:** To calculate the heat transfer rate,  $\dot{Q}_S$ ,

$$\dot{Q}_S = \dot{Q}'_S A = \dot{Q}'_S \left( \frac{\pi D^2}{4} \right) = (11,340) \left[ \frac{\pi (0.3)^2}{4} \right] = 801 \text{ W}$$
■



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