

Chapter 14

Introduction to Heat Exchangers

INTRODUCTION

Heat exchangers are defined as equipment that effect the transfer of thermal energy in the form of heat from one fluid to another. The simplest exchangers involve the direct mixing of hot and cold fluids. Most industrial exchangers are those in which the fluids are separated by a wall. The latter type, referred to by some as a *recuperator*, can range from a simple plane wall between two flowing fluids to more complex configurations involving multiple passes, fins, or baffles. Conductive and convective heat transfer principles are required to describe and design these units; radiation effects are generally neglected.

Heat exchangers for the chemical, petrochemical, petroleum, paper, and power industries encompass a wide variety of designs that are available from many manufacturers. Equipment design practice first requires the selection of safe operable equipment. The selection and design process must also seek a cost-effective balance between initial (capital) installation costs, operating costs, and maintenance costs.

The proper application of heat exchange principles can significantly minimize both the initial cost of a plant and the daily operating and/or utility costs. Each heat exchange application may be accomplished by the use of many types of heat exchange equipment. To perform these applications, their design and materials of construction must be suitable for the desired operating conditions; the selection of materials of construction is primarily influenced by the operating temperature and the corrosive nature of the fluids being handled.

A more detailed description of heat exchanger equipment is warranted before proceeding to the development of the heat exchanger equation. The presentation in this chapter keys not only on the various heat exchanger equipment (and their classification) but also on the log mean temperature difference driving force, ΔT_{lm} , and the overall heat transfer coefficient, U . The development of both ΔT_{lm}

and U ultimately leads to the classic heat exchanger equation. Topics addressed in this chapter include:

- Energy relationships
- Heat exchange equipment classification
- The log mean temperature difference (LMTD) driving force
- Overall heat transfer coefficients
- The heat transfer equation

ENERGY RELATIONSHIPS

The flow of heat from a hot fluid to a cooler fluid through a solid wall is a situation often encountered in engineering equipment; examples of such equipment are the aforementioned heat exchangers, condensers, evaporators, boilers, and economizers. The heat absorbed by the cool fluid or given up by the hot fluid may be sensible heat, causing a temperature change in the fluid, or it may be latent heat, causing a phase change such as vaporization or condensation. In a typical waste heat boiler, for example, the hot flue gas gives up heat to water through thin metal tube walls separating the two fluids. As the flue gas loses heat, its temperature drops. As the water gains heat, its temperature quickly reaches the boiling point where it continues to absorb heat with no further temperature rise as it changes into steam. The rate of heat transfer between the two streams, assuming no heat loss due to the surroundings, may be calculated by the enthalpy change of either fluid:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c(h_{c1} - h_{c2}) \quad (14.1)$$

where \dot{Q} is the rate of heat flow (Btu/h), \dot{m}_h is the mass flow rate of hot fluid (lb/h), \dot{m}_c is the mass flow rate of cold fluid (lb/h), h_{h1} is the enthalpy of entering hot fluid (Btu/lb), h_{h2} is the enthalpy of exiting hot fluid (Btu/lb), h_{c1} is the enthalpy of entering cold fluid (Btu/lb), and h_{c2} is the enthalpy of exiting cold fluid (Btu/lb).

Equation (14.1) is applicable to the heat exchange between two fluids whether a phase change is involved or not. In the above waste heat boiler example, the enthalpy change of the flue gas is calculated from its sensible temperature change:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c c_{ph}(T_{h1} - T_{h2}) \quad (14.2)$$

where c_{ph} is the heat capacity of the hot fluid (Btu/lb · °F), T_{h1} is the temperature of the entering hot fluid (°F), and T_{h2} is the temperature of the exiting hot fluid (°F). The enthalpy change of the water, on the other hand, involves a small amount of sensible heat to bring the water to its boiling point plus a considerable amount (usually) of latent heat to vaporize the water. Assuming all of the water is vaporized and no superheating of the steam occurs, the enthalpy change is

$$\dot{Q} = \dot{m}_c(h_{c2} - h_{c1}) = \dot{m}_c c_{pc}(t_{c2} - t_{c1}) + \dot{m}_c \Delta h_{\text{vap}} \quad (14.3)$$

where c_{pc} is the heat capacity of the cold fluid (Btu/lb · °F), t_{c1} is the temperature of the entering cold fluid (°F), t_{c2} is the temperature of the exiting cold fluid (°F), and Δh_{vap} is

the heat of vaporization of the cold fluid (Btu/lb). Note that, wherever possible, lower case t and upper case T will be employed to represent the cooler fluid temperature and hotter fluid temperature, respectively; in addition, lowercase c /uppercase C and lowercase h /uppercase H will be employed to represent the cold and hot fluid, respectively.

ILLUSTRATIVE EXAMPLE 14.1

If 20,000 scfm (32°F, 1 atm) of an air stream is heated from 200°F to 2000°F, calculate the heat transfer rate required to bring about this change in temperature. Use the following enthalpy and average heat capacity data:

$$\begin{aligned}H_{200^\circ\text{F}} &= 1170 \text{ Btu/lbmol} \\H_{2000^\circ\text{F}} &= 14,970 \text{ Btu/lbmol} \\ \bar{C}_{p,\text{av}} &= 7.53 \text{ Btu/lbmol} \cdot ^\circ\text{F} \text{ (over the } 200\text{--}2000^\circ\text{F range)}\end{aligned}$$

SOLUTION: Convert the flow rate of air to a molar flow rate, \dot{n} :

$$\dot{n} = \frac{\text{scfm}}{359} = \frac{20,000}{359} = 55.7 \text{ lbmol/min}$$

Note that at standard conditions of 32°F and 1 atm, 1.0 lbmol of an ideal gas occupies 359 ft³ or 22.4 (see also Chapter 3).

Calculate the heat transfer rate: \dot{Q} , using enthalpy data:

$$\dot{Q} = \dot{n}\Delta H = 55.7(14,970 - 1170) = 7.69 \times 10^5 \text{ Btu/min}$$

Also, calculate the heat transfer rate using the average heat capacity data:

$$\dot{Q} = \dot{n}\bar{C}_{p,\text{av}}\Delta T = 55.7(7.53)(2000 - 200) = 7.55 \times 10^5 \text{ Btu/min}$$

Both approaches provide near identical results. ■

ILLUSTRATIVE EXAMPLE 14.2

Given the mass flow rate of a gas stream and its heat capacity, determine the required heat rate to change the gas stream from one temperature to another. Data are provided below:

$$\begin{aligned}\dot{n} &= 1200 \text{ lbmol/min} \\ \bar{C}_{p,\text{av}} &= 0.26 \text{ Btu/lbmol} \cdot ^\circ\text{F} \text{ (over the } 200\text{--}2000^\circ\text{F range)} \\ T_1 &= 200^\circ\text{F} \\ T_2 &= 1200^\circ\text{F}\end{aligned}$$

SOLUTION: Write the equation describing the required heat rate, \dot{Q} , and solve:

$$\dot{Q} = \dot{n}\bar{C}_{p,\text{av}}\Delta T = (1200)(0.26)(1200 - 200) = 3.12 \times 10^5 \text{ Btu/min} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 14.3

Obtain the heat transfer rate in an exchanger if equal similar liquid flow rates are used. The hot fluid is cooled from 84°C to 72°C while the cold fluid is initially at 25°C.

SOLUTION: The final temperature of the cold liquid can be found from direct application of Equation (14.2):

$$\begin{aligned}\dot{m}_c c_{pc}(t_{c,2} - t_{c,1}) &= \dot{m}_h c_{ph}(T_{h,2} - T_{h,1}) \\ \dot{m}_c c_{pc}(t_{c,2} - 25) &= \dot{m}_h c_{ph}(84 - 72)\end{aligned}$$

Since

$$\begin{aligned}\dot{m}_c &= \dot{m}_h \\ c_{ph} &= c_{pc}\end{aligned}$$

one can conclude that the temperature *change* for both liquids is equal. This yields

$$t_{c,2} = 37^\circ\text{F}$$

■

HEAT EXCHANGE EQUIPMENT CLASSIFICATION

There is a near infinite variety of heat exchange equipment. These can vary from a simple electric heater in the home to a giant boiler in a utility power plant. A limited number of heat transfer devices likely to be encountered by the practicing engineer have been selected for description in this Part. Most of the units transfer heat from one fluid to another fluid, with the heat passing through a solid interface such as a tube wall that separates the two (or more) fluids. The size, shape, and material employed to separate the two fluids is of course important. Another problem in a heat exchanger is the method of confining one or both of the two fluids involved in the heat transfer process.

As noted above, heat exchangers are devices used to transfer heat from a hot fluid to a cold fluid. They are classified by their functions, as shown in the Table 14.1. In a general sense, heat exchangers are classified into three broad types:

1. Recuperators or through-the-wall non-storing exchangers (e.g., double-pipe heat exchangers, shell-and-tube heat exchangers)
2. Direct contact non-storing exchangers
3. Regenerators, accumulators, or heat storage exchangers

Through-the-wall non-storing exchangers can be further classified into:

1. Double pipe heat exchangers
2. Shell-and-tube heat exchangers
3. Cross-flow exchangers

Exchangers (1) and (2) receive treatment in the next two chapters.

Table 14.1 Heat Exchanger Equipment

Equipment	Function
Chiller	Cools a fluid to a temperature below that obtainable if only water were used as a coolant. It often uses a refrigerant such as ammonia or freon.
Condenser	Condenses a vapor or mixture of vapors, either alone or in the presence of a non-condensable gas.
Cooler	Cools liquids or gases by means of water.
Exchanger	Performs a double function: (1) heats a cold fluid and (2) cools a hot fluid. Little or none of the transferred heat is normally lost.
Final condenser	Condenses the vapors to a final storage temperature of approximately 100°F. It uses water cooling, which means the transferred heat is often lost to the process.
Forced-circulation reboiler	A pump is used to force liquid through the reboiler (see reboiler below).
Heater	Imparts sensible heat to a liquid or a gas by means of condensing steam or some other hot fluid (e.g., Dowtherm).
Partial condenser	Condenses vapors at a temperature high enough to provide a temperature difference sufficient to preheat a cold stream of process fluid. This saves heat and eliminates the need for providing a separate preheater.
Reboiler	Connected to the bottom of a fractionating tower, it provides the reboil heat necessary for distillation. The heating medium may be either steam (usually) or a hot process fluid.
Steam generator	Generates steam for use elsewhere in the plant by using available high-level heat, e.g., from tar or a heavy oil.
Superheater	Heats a vapor above its saturation temperature.
Thermosiphon reboiler	Natural circulation of the boiling medium is obtained by maintaining sufficient liquid head to provide for circulation (see reboiler).
Vaporizer	A heater which vaporizes all or part of the liquid.
Waste-heat boiler	Produces steam; similar to a steam generator, except that the heating medium is a hot gas or liquid produced in a chemical reaction.

The flow in the double pipe heat exchanger⁽¹⁾ may be countercurrent or parallel (co-current). In countercurrent flow, the fluid in the pipe flows in a direction opposite to the fluid in the annulus. In parallel/co-current flow, the two fluids flow in the same direction. The variations of fluid temperature within the heat exchanger depend on whether the flow is parallel (co-current) or countercurrent. The definitions below are employed in the development to follow:

t_1 = temperature of fluid entering the inside tube

t_2 = temperature of fluid exiting the inside tube

T_1 = temperature of fluid entering the annulus (space between the two tubes)

T_2 = temperature of fluid exiting the annulus.

The difference between the temperature of the tube side fluid and that of the annulus side is the temperature difference driving force, ΔT .

In a parallel heat exchanger, both hot and cold fluids enter on the same side and flow through the exchanger in the same direction. The *temperature approach* is defined as the temperature difference driving force at the heat exchanger entrance, ΔT_1 or $(T_1 - t_1)$. This driving force drops as the streams approach the exit of the exchanger. At the exit, the temperature difference driving force is ΔT_2 or $(T_2 - t_2)$. The heat exchanger is more effective at the entrance than at the exit because of the higher driving force.

In a countercurrent flow exchanger, the two fluids exchanging heat flow in opposite directions. The temperature approach at the tube entrance end, ΔT_1 or $(T_1 - t_2)$, and at the annulus entrance end, ΔT_2 or $(T_2 - t_1)$ are usually roughly the same. The thermal driving force is normally relatively constant over the length of the exchanger.

THE LOG MEAN TEMPERATURE DIFFERENCE (LMTD) DRIVING FORCE

When heat is exchanged between a surface and a fluid, or between two fluids flowing through a heat exchanger, the local temperature driving force, ΔT , usually varies along the flow path. This effect is treated through the log mean temperature difference approach, as discussed below.

The concept of a log-mean temperature is first developed before proceeding to the log-mean temperature difference driving force. Consider an absolute temperature profile $T(x)$ that is continuous and where T_1 is the absolute temperature at some point x_1 with T_2 at x_2 . By definition, the mean value of the reciprocal temperatures ($1/T$) between x_1 and x_2 is given by

$$\begin{aligned} \left(\overline{\frac{1}{T}}\right) &= \int_{T_1}^{T_2} \frac{1}{T} dT \bigg/ \int_{T_1}^{T_2} dT = \left(\frac{1}{T_2 - T_1}\right) \int_{T_1}^{T_2} \frac{1}{T} dT \\ &= \frac{\ln(T_2/T_1)}{T_2 - T_1} \end{aligned} \quad (14.4)$$

If the average of the reciprocal is a reasonable approximation to the reciprocal of the average, i.e.,

$$\left(\overline{\frac{1}{T}}\right) \approx \frac{1}{\overline{T}} \quad (14.5)$$

then one can combine the above two equations to give

$$\frac{1}{\overline{T}} \approx \frac{\ln(T_2/T_1)}{T_2 - T_1}$$

or

$$\bar{T} \approx \frac{T_2 - T_1}{\ln(T_2/T_1)} \quad (14.6)$$

The right-hand side of Equation (14.6) is *defined* to be the log-mean temperature between T_1 and T_2 , that is,

$$T_{\text{lm}} = \frac{T_2 - T_1}{\ln(T_2/T_1)} \quad (14.7)$$

Note once again that absolute temperatures must be employed.

Assuming steady-state operation and constant properties independent of the temperature, an overall energy balance is first applied to a fluid in a conduit (e.g., a pipe with inlet and exit temperatures t_1 and t_2 , respectively), and heated by a source at temperature T_s (see also Figure 14.1):

$$\dot{Q} = \dot{m}c_p(t_2 - t_1) = \dot{m}c_p[(T_s - t_1) - (T_s - t_2)] = \dot{m}c_p(\Delta T_1 - \Delta T_2) \quad (14.8)$$

where ΔT_1 , or $(T_s - t_1)$, is the temperature difference driving force (also termed the *approach*) at the fluid entrance, and ΔT_2 , or $(T_s - t_2)$, is the temperature driving force, or approach, at the fluid exit. The differential balance is

$$d\dot{Q} = \dot{m}c_p dt = \dot{m}c_p d(\Delta T) \quad (14.9)$$

One may also apply an energy balance to a differential fluid element of cross-sectional area, A , and a thickness of dx . Set $x = 0$ at the pipe entrance and $x = L$ at the pipe exit. The pipe diameter is D , so

$$A = \frac{\pi D^2}{4} \quad (14.10)$$

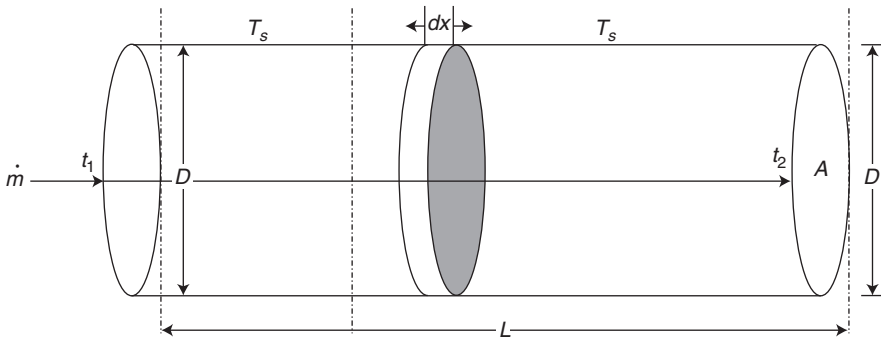


Figure 14.1 Energy balance of a differential element of a pipe.

The outer surface differential area of the element is $dA = \pi D(dx)$. One may also apply the energy balance across the walls of the unit

$$d\dot{Q} = h(T_s - t)(\pi D)(dx) = \dot{m}c_p dt \quad (14.11)$$

where h is the heat transfer coefficient of the fluid defined earlier in Chapter 9. Separation of variables in Equation (14.11) yields

$$\frac{dt}{T_s - t} = \frac{h}{\dot{m}c_p}(\pi D)(dx) \quad (14.12)$$

Integrating from $x = 0$, where $t = t_1$, to $x = L$, where $t = t_2$, yields

$$\int_{t_1}^{t_2} \frac{dt}{T_s - t} = \frac{h}{\dot{m}c_p}(\pi D) \int_0^L (dx) \quad (14.13)$$

$$\ln\left(\frac{T_s - t_1}{T_s - t_2}\right) = \frac{h}{\dot{m}c_p}(\pi D)L \quad (14.14)$$

Letting $\Delta T_1 = T_s - t_1$ and $\Delta T_2 = T_s - t_2$ leads to

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{h}{\dot{m}c_p}(\pi D)L \quad (14.15)$$

Thus,

$$\dot{m}c_p = \frac{h(\pi D)L}{\ln(\Delta T_1/\Delta T_2)} \quad (14.16)$$

Combining Equations (14.4) and (14.16) gives

$$\dot{Q} = h(\pi D)L \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = hA\Delta T_{\text{lm}} \quad (14.17)$$

where $\Delta T_{\text{lm}} = \log$ mean temperature difference, or log mean temperature approach, and is defined as

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \quad (14.18)$$

For the special case of $\Delta T_1 = \Delta T_2$,

$$\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2 \quad (14.19)$$

Heat transfer calculations using ΔT_{lm} are convenient when terminal temperatures are known. If the temperature of the fluid leaving the tube or exchanger is not known, the procedure may require a trial-and-error calculation.

ILLUSTRATIVE EXAMPLE 14.4

Steam at 1 atm outside a pipe is heating a fluid inside a pipe whose temperature may be assumed uniform at 25°C. Calculate the LMTD.

SOLUTION: At 1 atm, the steam temperature is 100°C. For this case, Equation (14.19) applies and

$$\Delta T_{lm} = 100 - 25 = 75^\circ\text{C} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 14.5

Repeat the calculation in the previous example if the fluid is heated from 25°C and 80°C.

SOLUTION: For this case, Equation (14.18) applies with

$$\Delta T_1 = 100 - 25 = 75^\circ\text{C}$$

$$\Delta T_2 = 100 - 80 = 20^\circ\text{C}$$

Therefore,

$$\Delta T_{lm} = \frac{75 - 20}{\ln(75/20)} = 43.9^\circ\text{C} \quad \blacksquare$$

Temperature Profiles

Consider a heated surface, as represented in Figure 14.2, where temperatures are plotted against distance along the surface. The temperature drop at the left-hand side of the figure is much greater than that at the right-hand side. Heat is being transferred more rapidly at the left-hand end since the hot fluid and the cold fluid enter at the same end of the unit and flow parallel to each other. This arrangement is known as *parallel flow* or *co-current flow*.

The alternative method is to feed the hot fluid at one end of the unit and the cold fluid at the other, allowing the fluids to pass by each other in opposite directions. Such an arrangement is called *counterflow* or *counter-current flow*. The temperature gradients for this case are provided in Figure 14.3. In this case of counter-current operation, the temperature drop along the length of the unit is relatively more constant than in parallel flow.

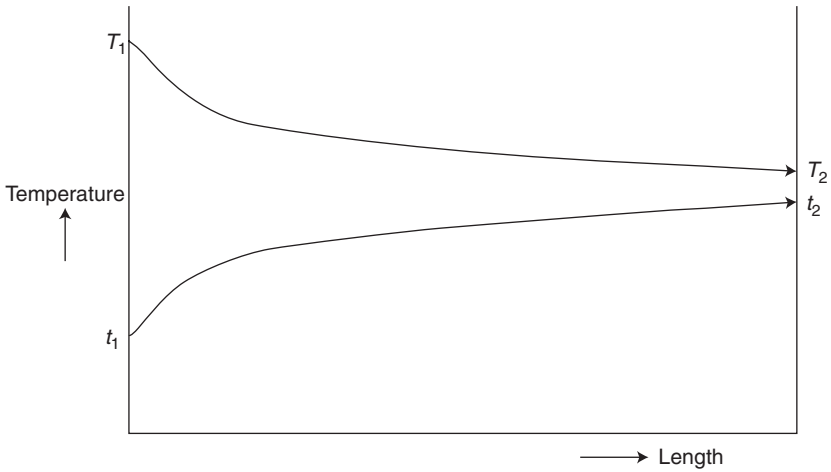


Figure 14.2 Heat transfer with parallel flow.

ILLUSTRATIVE EXAMPLE 14.6

A liquid-to-liquid *counterflow* heat exchanger is used to heat a cold fluid from 120°F to 310°F. Assuming that the hot fluid enters at 500°F and leaves at 400°F, calculate the log mean temperature difference (driving force) for the heat exchanger.

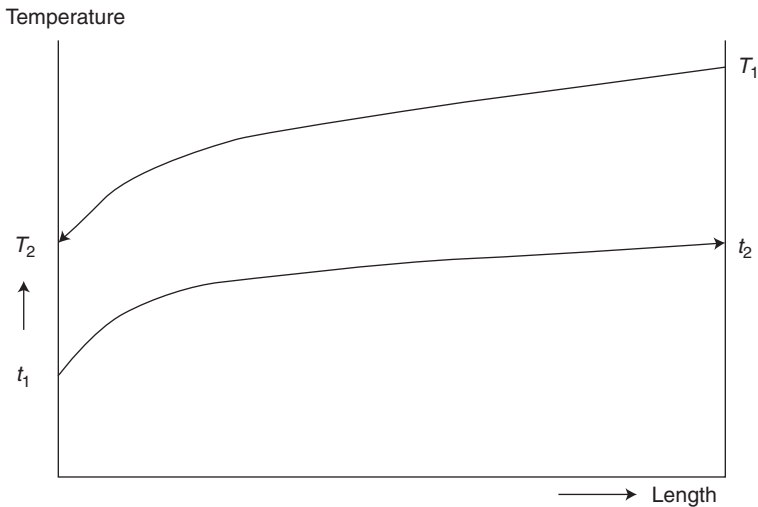


Figure 14.3 Heat transfer with counterflow.

SOLUTION: For this application

$$\Delta T_1 = T_1 - t_2 = 500 - 310 = 190^\circ\text{F}$$

$$\Delta T_2 = T_2 - t_1 = 400 - 120 = 280^\circ\text{F}$$

Therefore,

$$\Delta T_{\text{lm}} = \frac{190 - 280}{\ln(190/280)} = 232^\circ\text{F}$$

ILLUSTRATIVE EXAMPLE 14.7

A heavy hydrocarbon oil with heat capacity, $c_p = 0.55$ Btu/lb \cdot $^\circ\text{F}$, is being cooled from $T_1 = 210^\circ\text{F}$ to $T_2 = 170^\circ\text{F}$. The oil flows inside a tube at a rate of 8000 lb/h and the tube surface temperature is maintained at 60°F . Calculate the heat transfer rate and the LMTD in $^\circ\text{F}$.

SOLUTION: Calculate the heat transferred from the heavy oil:

$$\dot{Q} = \dot{m}c_p\Delta T = (8000)(0.55)(170 - 210) = -176,000 \text{ Btu/h}$$

The negative sign is a reminder that heat is lost by the oil. The thermal driving forces, or the temperature approaches, at the pipe entrance and exit are

$$\Delta T_1 = T_1 - t = 210 - 60 = 150^\circ\text{F}$$

$$\Delta T_2 = T_2 - t = 170 - 60 = 110^\circ\text{F}$$

The LMTD is therefore

$$\Delta T_{\text{lm}} = \frac{150 - 110}{\ln(150/110)} = 129^\circ\text{F}$$

Note once again that the use of ΔT_{lm} is valid for heating or cooling. ■

ILLUSTRATIVE EXAMPLE 14.8

An oil cooler is to operate with an inlet of 138°F and outlet of 103°F , and the coolant entering at 88°F is to be allowed to rise to 98°F . What is the LMTD for this unit, if it is considered as

1. counter-current flow unit, and
2. a parallel flow unit.

SOLUTION:

For the counter-current flow case,

$$\Delta T_1 = T_1 - t_2 = 138 - 98 = 40^\circ\text{F}$$

$$\Delta T_2 = T_2 - t_1 = 103 - 88 = 15^\circ\text{F}$$

Therefore,

$$\Delta T_{\text{lm}} = \frac{40 - 15}{\ln(40/15)} = 25.5^\circ\text{F}$$

For the parallel flow case,

$$\Delta T_1 = T_1 - t_1 = 138 - 88 = 50^\circ\text{F}$$

$$\Delta T_2 = T_2 - t_2 = 103 - 98 = 5^\circ\text{F}$$

Therefore,

$$\Delta T_{\text{lm}} = \frac{50 - 5}{\ln(50/5)} = 19.5^\circ\text{F} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 14.9

Comment on the results of the previous example.

SOLUTION: Note that the counter-current flow unit clearly provides the greater driving force for heat transfer. \blacksquare

OVERALL HEAT TRANSFER COEFFICIENTS

In order to design heat transfer equipment and calculate the required energy, one must know more than just the heat transfer rate calculated by the enthalpy (energy) balances described previously. The rate at which heat can travel from the hot fluid at T_H , through the tube walls, into the cold fluid at t_C , must also be considered in the calculation of certain design variables (e.g., the contact area). The slower this rate is, for given hot and cold fluid flow rates, the more contact area is required. The rate of heat transfer through a unit of contact area was referred to earlier as the *heat flux* and, at any point along the area or the tube length, is given by

$$\frac{d\dot{Q}}{dA} = U(T_H - t_C) \quad (14.20)$$

where $d\dot{Q}/dA$ is the local heat flux ($\text{Btu}/\text{h} \cdot \text{ft}^2$) and U is defined as the local *overall heat transfer coefficient* ($\text{Btu}/\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$), a term that provides a measure (inversely) of the resistance to heat transfer.

The use of the above overall heat transfer coefficient (U) is a simple, yet powerful concept. In most applications, it combines both conduction and convection effects, although heat transfer by radiation can also be included. In actual practice, it is not uncommon for vendors to provide a numerical value for U . For example, a typical value for U for estimating heat losses from an incinerator is approximately $0.1 \text{ Btu}/\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$. Methods for calculating the overall heat transfer coefficient are presented later in this section.

With reference to Equation (14.20), the temperatures T_H and t_C are actually local average values. As described in the previous section, when a fluid is being heated or cooled, the temperature will vary throughout the cross-section of the stream. If the fluid is being heated, its temperature will be highest at the tube wall and will decrease with increasing distance from the tube wall. The average temperature across the stream cross-section is therefore t_C ; i.e., the temperature that would be achieved if the fluid at this cross-section was suddenly mixed to a uniform temperature. If the fluid is being cooled, on the other hand, its temperature will be lowest at the tube wall and will increase with increasing distance from the wall.

In order to apply Equation (14.20) to an entire heat exchanger, the equation must be integrated. This cannot be done unless the geometry of the exchanger is first defined. For simplicity, one of the simplest geometries will be assumed here—the double pipe heat exchanger to be discussed in the next chapter. This device consists of two parallel concentric pipes. The outer surface of the outer pipe is well insulated so that no heat exchange with the surroundings may be assumed. One of the fluids flows through the center pipe and the other flows through the annular channel (known as the annulus) between the pipes. The fluid flows may be either co-current where the two fluids flow in the same directions, or countercurrent where the flows are in the opposite directions; however, the countercurrent arrangement is more efficient and is more commonly used. For this heat exchanger, integration of Equation (14.20) along the exchanger area or length, and applying several simplifying assumptions, yields:

$$\dot{Q} = UA\Delta T \quad (14.21)$$

The above equation was previously applied to heat transfer across a plane wall in Chapter 9 and it was shown that

$$\dot{Q} = \frac{T_H - T_C}{\frac{1}{h_i A} + \frac{\Delta x}{kA} + \frac{1}{h_o A}} \quad (14.22)$$

where the h 's represent the individual heat transfer coefficients, also discussed in Chapter 9. Since $\dot{Q} = UA\Delta T$

$$U = \frac{1}{\frac{1}{h_i} + \frac{\Delta x}{k} + \frac{1}{h_o}} \quad (14.23)$$

Since A is a constant,

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{\frac{1}{h_i A} + \frac{\Delta x}{kA} + \frac{1}{h_o A}} \quad (14.24)$$

For a tubular unit, $\dot{Q} = UA\Delta T$ still applies, but A is now a variable. Equation (14.24) is rewritten for this geometry as

$$\frac{\dot{Q}}{\Delta T} = \frac{1}{\frac{1}{h_i A_i} + \frac{\Delta D}{2kA_{lm}} + \frac{1}{h_o A_o}}; \Delta D = D_o - D_i = 2(r_o - r_i) \quad (14.25)$$

where the subscripts i and o refer to the inside and outside of the tube, respectively. In addition,

$$A_{lm} = \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{\pi L \Delta D}{\ln(D_o/D_i)} \quad (14.26)$$

and $L =$ tube length (ft). Thus,

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi Lk} + \frac{1}{h_o A_o}} \quad (14.27)$$

Equation (14.27) may again be written as

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{R_i + R_w + R_o} \quad (14.28)$$

The term U in Equation (14.27) may be based on the inner area (A_i) or the outer area (A_o) so that

$$\frac{\dot{Q}}{\Delta T} = U_i A_i = U_o A_o = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi Lk} + \frac{1}{h_o A_o}} \quad (14.29)$$

Dividing by either A_i or A_o yields an expression for U_i and U_o :

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(D_o/D_i)}{2\pi Lk} + \frac{A_i}{h_o A_o}} \quad (14.30)$$

$$\begin{aligned} U_o &= \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o \ln(D_o/D_i)}{2\pi Lk} + \frac{1}{h_o}} \\ &= \frac{1}{\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o}} \end{aligned} \quad (14.31)$$

Numerical values of U can range from as low as 2 Btu/h · ft² · °F (10 W/m² · K) for gas-to-gas heat exchangers to as high as 250 Btu/h · ft² · °F for water-to-water units. Additional values are provided in the literature.⁽¹⁾

Fouling Factors

During heat exchange operation with liquids and/or gases, a “dirt” film gradually builds up on the exchanger surface(s). This deposit may be rust, boiler scale, silt, coke, or any number of things. Its effect, which is referred to as *fouling*, is to increase the thermal resistance, R , which results in decreased performance. The nature of the rate of deposit is generally difficult to predict *a priori*. Therefore, only the performance of clean exchangers is usually guaranteed. The fouling resistance is often obtained from field, pilot, or lab data, or from experience.⁽²⁾ This unknown factor enters into every design.

The scale of fouling is dependent on the fluids, their temperature, velocity, and to a certain extent, the nature of the heat transfer surface and its chemical composition. Due to the unknown nature of the assumptions, these fouling factors can markedly affect the design of heat transfer equipment.

The effect (resistance) of fouling, R_f , can be obtained from the following equation:

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \quad (14.32)$$

where U_{dirty} is the overall heat transfer coefficient for design (with fouling) and U_{clean} is the overall heat transfer coefficient for the clean condition (without fouling). Thus, in general, Equation (14.28) can be rewritten as

$$UA = \frac{1}{R_i + R_{fi} + R_w + R_{fo} + R_o} = \frac{1}{R_t} \quad (14.33)$$

where R_t = total resistance.

If U is based on the outside area of the tube, A_o ,

$$U_o = \frac{1}{\frac{1}{h_i} \left(\frac{A_o}{A_i} \right) + R_{fi} \left(\frac{A_o}{A_i} \right) + R_w + R_{fo} + \frac{1}{h_o}} \quad (14.34)$$

where R_w is once again the wall resistance and given by (see also Equation (14.31))

$$R_w = \frac{A_o \ln(D_o/D_i)}{2\pi Lk} \quad (14.35)$$

The two fouling factors, R_{fi} and R_{fo} , are sometimes combined and treated as one resistance, R_f . For this condition,

$$U_o = \frac{1}{\frac{1}{h_i} \left(\frac{A_o}{A_i} \right) + \frac{A_o \ln(D_o/D_i)}{2\pi Lk} + R_f + \frac{1}{h_o}} \quad (14.36)$$

Table 14.2 Fouling Coefficients (English Units)

Fluid	$R_f, \text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}/\text{Btu}$
Alcohol vapors	0.0005
Boiler feedwater	0.001
Fuel oil	0.005
Industrial air	0.002
Quench oil	0.004
Refrigerating liquids	0.001
Seawater	0.0008
Steam, non oil-bearing	0.0005

Typical values of the fouling coefficient, h_f , are shown in Table 14.2. Additional values are available.⁽¹⁾ Note that

$$h_f = R_f^{-1} \quad (14.37)$$

One of the main contributors to fouling is corrosion. Therefore, corrosion protection is important and the selection of equipment materials with reasonable corrosion rates must be balanced against initial and replacement costs. Materials chosen for corrosion resistance in a given process service may include carbon steels, stainless steels, aluminum, copper alloys, nickel alloys, graphite, glass, and other non-metallics. Corrosion effects can be reduced without costly materials by means of special design and fabrication techniques; among these are limitations on flow velocities and use of impingement baffles.

As noted above, fouling factors are usually determined experimentally by generating data for both the dirty and clean unit. This topic will be revisited in the next chapter.

The Controlling Resistance

The reader should note that the numerical value of the overall heat transfer coefficient may be primarily governed by only one of the individual coefficients (and, therefore, only one of the resistances). That coefficient is then referred to as the *controlling* resistance. As one might expect, the resistance due to conduction (the wall) is often negligible relative to the convective resistances.⁽²⁾

ILLUSTRATIVE EXAMPLE 14.10

A rectangular plane window glass panel is mounted on a house. The glass is 0.125 inch thick and has a surface area of 1.0 m^2 ; its thermal conductivity, k_2 , is $1.4 \text{ W}/\text{m} \cdot \text{K}$. The inside house temperature, T_1 , and the outside air temperature, t_4 , are 25°C and -14°C , respectively. The heat transfer coefficient inside the room, h_1 , is $11.0 \text{ W}/\text{m}^2 \cdot \text{K}$ and the heat transfer coefficient

from the window to the surrounding cold air, h_3 , is $9.0 \text{ W/m}^2 \cdot \text{K}$. Calculate the overall heat transfer coefficient in $\text{W/m}^2 \cdot \text{K}$.

SOLUTION: Calculate the internal convection resistance, R_1 :

$$R_1 = \frac{1}{h_1 A} = \frac{1}{(11)(1)} = 0.0909 \text{ K/W}$$

Calculate the conduction resistance through the glass panel:

$$L_2 = \Delta x = 0.125 \text{ inch} \left(\frac{0.0254 \text{ m}}{\text{inch}} \right) = 0.00318 \text{ m}$$

$$R_2 = \frac{\Delta x}{k_2 A} = \frac{0.00318}{(1.4)(1)} = 0.00227 \text{ K/W}$$

Also calculate the outside convection resistance:

$$R_3 = \frac{1}{h_3 A} = \frac{1}{(9)(1)} = 0.111 \text{ K/W}$$

The total thermal resistance is therefore, from Equation (14.33),

$$R_t = 0.0909 + 0.00227 + 0.111 = 0.204 \text{ K/W}$$

The overall heat transfer coefficient may now be calculated:

$$U = \frac{1}{AR_t} \quad (14.33)$$

Substituting,

$$U = \frac{1}{(1)(0.204)} = 4.9 \text{ W/m}^2 \cdot \text{K}$$

■

ILLUSTRATIVE EXAMPLE 14.11

A heat exchanger wall consists of a copper plate 0.049 inch thick. If the two surface film coefficients are 208 and $10.8 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$, respectively, calculate the overall heat transfer coefficient in $\text{Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$.

SOLUTION: Since information on the internal and external area is not given, neglect the effect of area. Equation (14.24) may be applied:

$$U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2}}$$

Substituting,

$$U = \frac{1}{\frac{1}{208} + \frac{(0.049/12)}{220} + \frac{1}{10.8}} = 10.26 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

ILLUSTRATIVE EXAMPLE 14.12

A coolant flows through a steel pipe (inside diameter of 0.05 m, outside diameter of 0.06 m) at a velocity of 0.25 m/s. What is the overall heat transfer coefficient for the system based on the pipe's outside surface area if the inside and outside coefficients are $2000 \text{ W/m}^2 \cdot \text{K}$ and $8.25 \text{ W/m}^2 \cdot \text{K}$, respectively. Assume the "resistance" term for the steel, that is,

$$R = \frac{D_o \ln(D_o/D_i)}{2k}$$

is $1.33 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$.

SOLUTION: For this example, Equation (14.31) applies. Therefore,

$$\begin{aligned} U_o &= \frac{1}{\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o}} \\ &= \frac{1}{\frac{0.06}{(2000)(0.05)} + 1.33 \times 10^{-4} + \frac{1}{8.25}} \\ &= \frac{1}{0.0006 + 1.33 \times 10^{-4} + 0.121} = 8.21 \text{ W/m}^2 \cdot ^\circ\text{K} \end{aligned}$$

ILLUSTRATIVE EXAMPLE 14.13

With reference to the previous example, determine the controlling resistance.

SOLUTION: Obviously, the controlling resistance is located outside (external to) the pipe ($1/h_o = 0.121$). ■

ILLUSTRATIVE EXAMPLE 14.14

Steam at 247°F is flowing through a pipe exposed to air but covered with 1.5 inch thick insulation. The following data is provided:

1. Pipe diameter, inside = 0.825 inch
2. Pipe diameter, outside = 1.05 inch
3. Surrounding air temperature = 60°F

4. Thermal conductivity, pipe = 26 Btu/h · ft · °F
5. Thermal conductivity, insulation = 0.037 Btu/h · ft · °F
6. Steam film coefficient = 800 Btu/h · ft² · °F
7. Air film coefficient = 2.5 Btu/h · ft² · °F

Calculate the overall heat transfer coefficient based on the inside area of the pipe.

SOLUTION: Obviously, cylindrical coordinates are employed again. Assume a basis 1.0 ft of pipe length. The inside (*i*) and outside (*o*) areas of the pipe (*P*) plus the insulation (*I*) areas are

$$\begin{aligned}A_i &= 0.2157 \text{ ft}^2 \\A_o &= 0.2750 \text{ ft}^2 \\A_I &= 1.060 \text{ ft}^2\end{aligned}$$

The log mean area for the steel pipe (*P*) and insulation (*I*) are, therefore

$$\begin{aligned}A_{P\text{lm}} &= \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{0.2750 - 0.2157}{\ln(0.2750/0.2157)} = 0.245 \text{ ft}^2 \\A_{I\text{lm}} &= \frac{A_I - A_o}{\ln(A_I/A_o)} = \frac{1.060 - 0.2750}{\ln(1.060/0.2750)} = 0.582 \text{ ft}^2\end{aligned}$$

A slightly modified form of Equation (14.25) and (14.30) is employed to calculate the overall heat coefficient based on the inside area:

$$\begin{aligned}U_i &= \frac{1}{A_i \sum R} = \frac{1}{A_i(R_i + R_P + R_o + R_I)} = \frac{1}{A_i \left(\frac{1}{h_i A_i} + \frac{r_o - r_i}{k_P A_{P\text{lm}}} + \frac{1}{h_o A_I} + \frac{r_1 - r_o}{k_I A_{I\text{lm}}} \right)} \\&= \frac{1}{0.2157 \left(\frac{1}{(800)(0.2157)} + \frac{(0.525 - 0.412)/12}{(26)(0.245)} + \frac{1}{(2.5)(1.060)} + \frac{(2.025 - 0.525)/12}{(0.037)(0.582)} \right)} \\&= \frac{1}{0.2157(0.005795 + 0.001478 + 0.37736 + 5.80477)} \\&= 0.749 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}\end{aligned}$$

Obviously, the major resistance to heat transfer is the insulation. ■

ILLUSTRATIVE EXAMPLE 14.15

Calculate the rate of heat transfer in the previous example.

SOLUTION: The above result can be extended to calculate the heat transfer rate.

$$A_i = \pi D_i L = \pi(0.825/12)(1) = 0.2157 \text{ ft}^2 \text{ (see previous example)}$$

$$\dot{Q} = U_i A_i (T_s - t_a) = 0.7492(0.2157)(247 - 60) = 30.2 \text{ Btu/h}$$

ILLUSTRATIVE EXAMPLE 14.16

The following information is provided for a heat exchanger:

$$h_w(\text{water}) = 200 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

$$h_o(\text{oil}) = 50 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

$$h_f(\text{fouling}) = 1000 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

$$\Delta T_{\text{lm}} = 90^\circ\text{F}$$

$$A = 15.0 \text{ ft}^2$$

Calculate the heat transfer rate for the exchanger if the wall resistance can be neglected.

SOLUTION:

Apply Equation (14.34) for a constant A ,

$$\frac{1}{U} = \frac{1}{h_w} + \frac{1}{h_o} + \frac{1}{h_f} \quad (14.34)$$

Substituting,

$$\begin{aligned} \frac{1}{U} &= \frac{1}{200} + \frac{1}{50} + \frac{1}{1000} \\ &= 0.005 + 0.02 + 0.001 \\ &= 0.026 \end{aligned}$$

Therefore,

$$U = 1.0/0.026 = 38.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

The heat transfer rate is

$$\begin{aligned} \dot{Q} &= UA\Delta T_{\text{lm}} \\ &= (38.46)(15.0)(90) \\ &= 51,920 \text{ Btu/h} \end{aligned} \quad (14.21)$$

Varying Overall Heat Transfer Coefficients

The calculation of ΔT_{lm} is valid for heating or cooling, i.e., whether ΔT_1 and ΔT_2 are both positive or both negative. It is not possible that ΔT_1 and ΔT_2 have opposite signs since it would constitute a violation of the Second Law of Thermodynamics. Finally, the LMTD should not be used when the overall heat transfer coefficient, U , changes appreciably through the unit. When U varies linearly with the temperature difference

over the entire heating surface, the rate of heat transfer may be estimated from the following equation:

$$\dot{Q} = A \left[\frac{U_2 \Delta T_1 - U_1 \Delta T_2}{\ln \left(\frac{U_2 \Delta T_1}{U_1 \Delta T_2} \right)} \right] = A(U \Delta T)_{\text{lm}} \quad (14.38)$$

where U_1 is the overall heat transfer coefficient at the fluid entrance side and U_2 is the overall heat transfer coefficient at the fluid exit side. Equation (14.38) requires the use of a log mean value of the product of U and ΔT . Note that the U at one end of the heater is multiplied by the temperature difference at the other end.

ILLUSTRATIVE EXAMPLE 14.17

A brine solution at 10°F in a food processing plant is heated by flowing through a heated pipe. The pipe surface is maintained at 80°F. The pipe surface area for heat transfer is 2.5 ft². The brine solution (with a density of 62.4 lb/ft³ and a heat capacity of 0.99 Btu/lb · °F) flows at a rate of 20 lb/min. The overall heat transfer coefficient varies linearly with the temperature approach, with values of 150 Btu/h · ft² · °F at the brine solution entrance (where the brine temperature is 10°F) and 140 Btu/h · ft² · °F at the brine solution exit. Determine the temperature approach at the brine inlet side, the exit temperature of the brine solution, and the rate of heat transfer, \dot{Q} .

SOLUTION: Set T as the surface temperature and t_1 and t_2 as the brine inlet and outlet temperatures, respectively. Calculate the temperature approach at the pipe entrance:

$$\Delta T_1 = T - t_1 = 80 - 10 = 70^\circ\text{F} = 70/1.8 = 38.9^\circ\text{C}$$

Note that ΔT_2 , which is equal to $(T - t_2)$, cannot be calculated since t_2 is not known. Apply an energy balance to the brine solution across the full length of the pipe:

$$\begin{aligned} \dot{Q} &= \dot{m}c_p(t_2 - t_1) = \dot{m}c_p(\Delta T_1 - \Delta T_2) \\ &= (1200)(0.99)(70 - \Delta T_2) = 1188(70 - \Delta T_2) \end{aligned}$$

The equation for the LMTD is

$$\Delta T_{\text{lm}} = (70 - \Delta T_2)/\ln(70/\Delta T_2)$$

Write the equation for the heat transfer rate. Note that U varies linearly with ΔT . See Equation (14.38):

$$\dot{Q} = A \left\{ \frac{U_2(\Delta T_1) - U_1(\Delta T_2)}{\ln \left(\frac{U_2 \Delta T_1}{U_1 \Delta T_2} \right)} \right\} = 2.5 \left\{ \frac{140(70) - 150(\Delta T_2)}{\ln \left(\frac{(140)(70)}{(150)(\Delta T_2)} \right)} \right\}$$

Combine the previous equations and eliminate \dot{Q} :

$$1188(70 - \Delta T_2) = 2.5 \left\{ \frac{(140)(70) - (150)(\Delta T_2)}{\ln\left(\frac{(140)(70)}{(150)(\Delta T_2)}\right)} \right\}$$

This equation is non-linear with one unknown (ΔT_2). Solve this equation by trial-and-error. Note that $0 \leq \Delta T_2 \leq 70^\circ\text{F}$. Solution gives $\Delta T_2 \simeq 51.6^\circ\text{F} \simeq 28.7^\circ\text{C}$.

Calculate the discharge temperature of the brine solution:

$$\begin{aligned} \Delta T_2 = 51.6 &= T - t_2 = 80 - t_2 \\ t_2 &= 80 - 51.6 = 28.4^\circ\text{F} \\ &= (28.4 - 32)/1.8 = -2^\circ\text{C} \end{aligned}$$

Finally, calculate the heat transfer rate, \dot{Q} . Using the earlier equation for \dot{Q} ,

$$\begin{aligned} \dot{Q} &= 1188(70 - 51.6) = 21,860 \text{ Btu/h} \\ &= 21,860/3.412 = 6407 \text{ W} \end{aligned}$$

THE HEAT TRANSFER EQUATION

The chapter concludes with a (very) short section entitled the heat transfer equation. By now the reader should have concluded that

$$\dot{Q} = UA\Delta T_{\text{lm}} \quad (14.21)$$

is that equation. It will find extensive use in the remainder of the text, particularly with the remaining chapters in this Part.

Summarizing,⁽³⁾ for a plane wall,

$$\begin{aligned} U &= \frac{1}{R_i + R_w + R_o} \\ &= \frac{1}{\frac{1}{h_i} + \frac{\Delta x}{k} + \frac{1}{h_o}} \end{aligned} \quad (14.23)$$

If the area is constant, this equation may be written as

$$UA = \frac{1}{\frac{1}{h_i A} + \frac{\Delta x}{kA} + \frac{1}{h_o A}} \quad (14.24)$$

For a tubular wall, Equation (14.24) remains the same, but which A should be employed? For inside/wall/outside resistances present, and employing the radius

rather than the diameter leads to:

$$U = \frac{1}{\frac{1}{h_i} + \frac{\Delta r}{k} + \frac{1}{h_o}} \quad (14.40)$$

In addition (and employing radii),

$$UA = \frac{1}{\frac{1}{h_i A_i} + \frac{\Delta r}{k A_{lm}} + \frac{1}{h_o A_o}} \quad (14.41)$$

with

$$A_{lm} = \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{2\pi L \Delta r}{\ln(r_o/r_i)} \quad (14.42)$$

Equation (14.41) may therefore be written as

$$UA = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \quad (14.43)$$

U may be based on (*i*) or (*o*). For $U_i A_i$,

$$U_i A_i = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \quad (14.44)$$

or

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o} \left(\frac{A_i}{A_o} \right)} \quad (14.45)$$

with a similar equation for U_o .

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