Pre Algebra A Vocabulary
Chronological List

Term	Definition
SUM	sum – the answer to an addition problem Ex. 4 + 5 = 9…The **sum** is 9.
DIFFERENCE	difference – the answer to a subtraction problem Ex. 8 – 2 = 6…The **difference** is 6.
PRODUCT	product – the answer to a multiplication problem Ex. 3 × 4 = 12…The **product** is 12.
QUOTIENT	quotient – the answer to a division problem Ex. 18 ÷ 6 = 3…The **quotient** is 3.
ESTIMATE	estimate (noun/verb) – an answer that is close to the real answer; to quickly perform an approximation Ex. 28.7 + 42.25 $\approx$ 30 + 40 = 70…The **estimate** is 70.
COMPATIBLE NUMBERS	compatible numbers – numbers that can be divided evenly; useful in estimating quotients Ex. 27.2 ÷ 4.14 $\approx$ 28 ÷ 4 = 7…28 and 4 are **compatible #s**.
PROPER FRACTION	proper fraction – a fraction that represents a positive number that has a value less than 1 (denominator is larger than numerator) Ex. $\frac{3}{4}$ is a **proper fraction**.

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Term	Definition
IMPROPER FRACTION	improper fraction – a fraction that represents a positive number that has a value more than 1 (numerator is larger than denominator) Ex. $\frac{7}{6}$ is an **improper fraction**.
EQUIVALENT FRACTION	equivalent fraction – a fraction that has the same value as a given fraction Ex. $\frac{2}{3}$ and $\frac{4}{6}$ are **equivalent fractions**.
SIMPLEST FORM	simplest form (of a fraction) – an equivalent fraction for which the only common factor of the numerator and denominator is 1 Ex. The **simplest form** of the fraction $\frac{20}{30}$ is $\frac{2}{3}$.
MIXED NUMBER	mixed number – the sum of a whole number and a fraction Ex. $3\frac{1}{4}$ is a **mixed number**.
RECIPROCAL	reciprocal – a number that can be multiplied by another number to make 1 (numerator and denominator are switched) Ex. $\frac{8}{3}$ is the **reciprocal** of $\frac{3}{8}$ because $\frac{8}{3} \times \frac{3}{8} = 1$
PERCENT	percent – a ratio that compares a number to 100 Ex. 25% is a **percent** that represents $\frac{25}{100}$ ($\frac{1}{4}$).
SEQUENCE	sequence – a set of numbers that follow a pattern Ex. 4, 6, 8, 10, 12…is a **sequence** of numbers.

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Term	Definition
ARITHMETIC SEQUENCE	arithmetic sequence – a sequence where each term is found by adding or subtracting the exact same number to the previous term Ex. 4, 6, 8, 10, 12…is an **arithmetic sequence** (add 2)
GEOMTRIC SEQUENCE	geometric sequence – a sequence where each term is found by multiplying or dividing by the exact same number to the previous term Ex. 2, 6, 18, 54, 162…is a **geometric sequence** (multiply 3)
GROUPING SYMBOLS	Ex. (parenthesis), [brackets], {braces}, long division bar
ORDER OF OPERATIONS	order of operations – the procedure to follow when simplifying a numerical expression 1 – Grouping symbols 2 – Exponents 3 – Multiplication and Division (from left to right) 4 – Addition and Subtraction (from left to right)
NUMERICAL EXPRESSION	numerical expression – a mathematical phrase that contains numbers and operation symbols Ex. $14 + 8 \div 4 - 21$
VARIABLE EXPRESSION	variable expression – a mathematical phrase that contains variables, numbers, and operation symbols Ex. $45 - (x + 3y)$

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Term	Definition
EVALUATE	evaluate – to replace variables with numbers and then simplify an expression Ex. To **evaluate** 4x + 10 when x = 3, replace "x" with 3 and simplify: 4(3) + 10 = 12 + 10 = 22
ABSOLUTE VALUE	absolute value – the distance a number is from zero on the number line Ex. |-3| = 3; "The **absolute value** of -3 is 3."
OPPOSITES	opposites – pairs of numbers that have the same absolute value Ex. 4 and -4 are **opposites** because they are 4 units from 0.
INTEGERS	integers – the set of numbers that includes whole numbers and their opposites
X-AXIS	x-axis – the horizontal number line that, together with the y-axis, establishes the coordinate plane
Y-AXIS	y-axis – the vertical number line that, together with the x-axis, establishes the coordinate plane
COORDINATE PLANE	coordinate plane – plane formed by two number lines (the horizontal x-axis and the vertical y-axis) intersecting at their zero points
QUADRANT	quadrant – one of four sections on the coordinate plane formed by the intersection of the x-axis and the y-axis

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Term	Definition
ORDERED PAIR	ordered pair – a pair of numbers that gives the location of a point in the coordinate plane. Also referred to as the “coordinates” of a point Ex. The **ordered pair** (3, 2) describes the location that is found by moving 3 units to the right of zero on the x-axis and then 2 units up from the x-axis.
ORIGIN	origin – the intersection of the x-axis and the y-axis on the coordinate plane Ex. The **origin** is described by the ordered pair (0,0).
X-COORDINATE	x-coordinate – the number that indicates the position of a point to the left or right of the y-axis Ex. The 4 in (4,3) is the **x-coordinate**, and tells you to move 4 places to the right of the y-axis
Y-COORDINATE	y-coordinate – the number that indicates the position of a point above or below the x-axis Ex. The 3 in (4,3) is the **y-coordinate**, and tells you to move 3 places above the x-axis
EQUATION	equation – a mathematical sentence that uses an equals (=) sign to indicate that the side to the left of the equals sign has the same value as the side to the right of the equals sign Ex. The **equation** $x + 4 = 10$ has a solution of $x = 6$.
INVERSE OPERATION	inverse operations – operations that undo each other Ex. Addition and subtraction are **inverse operations**. Multiplication and division are also **inverse operations**.

Pre Algebra A Vocabulary
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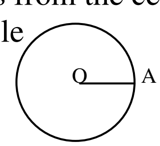
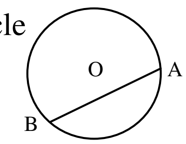
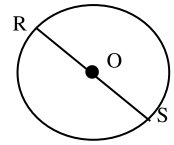
Term	Definition
INEQUALITY	inequality – a mathematical sentence that uses a symbol ($<, >, \leq, \geq, \neq$) to indicate that the left and right sides of the sentence hold values that are different Ex. The **inequality** $x > 8$ has an infinite number of solutions.
PERIMETER	perimeter – the distance around the outside of a figure Ex. The **perimeter** of a rectangle whose length is 18 feet and width is 5 feet is: 18+5+18+5 = 46 feet.
CIRCUMFERENCE	circumference – the distance around a circle Ex. The **circumference** of a circle whose radius is 4 inches is: $2\pi(4) = 8\pi$ inches or approximately 25.12 inches.
AREA	area – the number of square units inside a 2-dimensional figure Ex. The **area** of a rectangle whose length is 18 feet and width is 5 feet is: $18\times5 = 90$ square feet.
VOLUME	volume – the number of cubic units inside a 3-dimensional figure Ex. The **volume** of a rectangular prism whose length is 10 feet, width is 4 feet, and height is 2 feet is: $10\times4\times2 = 80$ cubic feet.
RADIUS	radius – a line segment that runs from the center of the circle to somewhere on the circle Ex. $\overline{OA}$ is a radius 

Pre Algebra A Vocabulary
Chronological List

Term	Definition
CHORD	chord – a line segment that runs from somewhere on the circle to another place on the circle Ex. $\overline{AB}$ is a chord 
DIAMETER	diameter – a chord that passes through the center of the circle Ex. $\overline{RS}$ is a diameter 
CENTRAL TENDENCY	central tendency – an attempt to find the "average" or "central value" of a given set of data. Ex. In statistics, there are three main measures of **central tendency**: MEAN, MEDIAN, MODE
MEAN	mean – the sum of the data items divided by the number of data items Ex. The **mean** of (16, 10, 13, 11, 10) is 60/5 = 12.
MEDIAN	median – the middle data item found after sorting the data items in ascending order; could be the mean of two middle numbers if the data set has an even number of items Ex. The **median** of (10, 10, 11, 13, 16) is 11.
MODE	mode – the data item that occurs most often; there could be no mode, one mode, or multiple modes Ex. The **mode** of (10, 10, 11, 13, 16) is 10.

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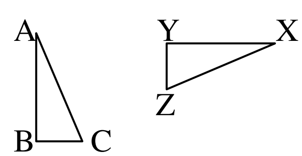
Term	Definition
RANGE	range – the difference between the highest and the lowest data item Ex. The **range** of (10, 10, 11, 13, 16) is 16 – 10 = 6.
OUTLIER	outlier – a data item that is much higher or much lower than all the other data items Ex. The **outlier** of (2, 3, 5, 5, 6, 7, 10, 45) is 45.
RATIO	ratio – a comparison of two quantities by division Ex. The **ratio** of students to faculty members at a given university is 16:1 (also 16 to 1…or 16/1).
RATE	rate – a ratio that compares quantities measured in different units Ex. The student's typing **rate** was 200 words per 6 minutes.
UNIT RATE	unit rate – a rate that has a denominator of 1 Ex. The **unit rate** describing his speed was 14 meters per second.
PROPORTION	proportion – a statement (equation) showing two ratios to be equal Ex. $\frac{3}{4} = \frac{9}{12}$ is a **proportion**.
CONGRUENT FIGURES	congruent figures – figures that have the same size AND same shape Ex. ΔABC ≅ ΔXYZ 

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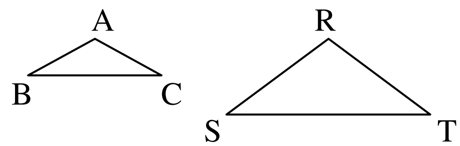
Chronological List

Term	Definition
SIMILAR FIGURES	similar figures – figures that have the same shape BUT different size. The corresponding sides of similar figures are proportional in length. Ex. ΔABC ~ ΔRST 
SCALE DRAWING	scale drawing – an enlarged or reduced drawing that is similar to an actual object or place
SCALE	scale – the ratio of the model distance to the actual distance Ex. The **scale** on the map is 1 in. to 25 mi.
OUTCOMES	outcomes – possible results of action
EVENT	event – an outcome or a group of outcomes
COMPLEMENT of an event	complement (of an event) – the opposite of the event
PROBABILITY	probability – a ratio that explains the likelihood of an event
THEORETICAL PROBABILITY	theoretical probability – the ratio of the number of favorable outcomes to the number of possible outcomes (based on what is expected to occur). Ex. The **theoretical probability** of rolling a four on the 6-sided die is $\frac{1}{6}$. 

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Term	Definition
EXPERIMENTAL PROBABILITY	experimental probability – the ratio of the number of times an event occurs to the number of times an experiment is done (based on real experimental data). Ex. If you actually roll the die 25 times and it lands on the four 5 times, the **experimental probability** is $\frac{5}{25}$ or $\frac{1}{5}$. 
DISTRIBUTIVE PROPERTY	distributive property – a way to simplify an expression that contains a single term being multiplied by a group of terms. ***For any numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$ Ex. 5(2x + 3) = 10x + 15
TERM	term – a number, a variable, or product of a number and a variable(s) Ex. There are 3 **terms** in the expression: 4x + y + 2
CONSTANT	Constant – a term with no variable part (i.e. a number)
COEFFICIENT	Coefficient – a number that multiplies a variable Ex. For the term 8x, the 8 is the **coefficient**.
LIKE TERMS	like terms – terms with the same variable part (including exponent)…like terms can be combined using the distributive property in reverse Ex. 4x + 6x = (4 + 6)x = 10x

Pre Algebra A Vocabulary
Chronological List

Term	Definition
DISCOUNT	discount – the amount by which a price is decreased Ex. If shoes marked at $56 have a **discount** of $10, the new price is now $46.
MARKUP	Markup – the amount by which a price is increased Ex. If the jacket was purchased at $25 from the manufacturer, and a $50 **markup** is applied, the new price is $75.

Solving Equations

Solving Two Step Equations

You will be able to solve a two-step equation for the value of an unknown variable. 8.8.C, A.5.A

Example A

$$2x + 5 = 17$$

x is being multiplied by 2. Then 5 is being added.
Work backward performing the inverse operations:

$$-5 \quad -5$$

Subtract 5 on both sides.

$$\frac{2x}{2} = \frac{12}{2}$$

Since x is being multiplied by 2, divide both sides by 2.

$$x = 6$$

Example B

$$-8 = \frac{y}{2} - 3$$

y is being divided by 2. Then 3 is being subtracted.
Work backward performing the inverse operations:

$$+3 \quad +3$$

Add 3 to both sides.

$$(2) - 5 = \frac{y}{2}(2)$$

Since y is being divided by 2, multiply both sides by 2.

$$-10 = y$$

Example C

$$\frac{2}{3}n - \frac{1}{2} = \frac{5}{2}$$

n is being multiplied by $\frac{2}{3}$. Then $\frac{1}{2}$ is being subtracted.
Work backward performing the inverse operations:

$$+\frac{1}{2} + \frac{1}{2}$$

Add $\frac{1}{2}$ to both sides.

$$\left(\frac{3}{2}\right)\frac{2}{3}n = 3\left(\frac{3}{2}\right)$$

Since n is being multiplied by $\frac{2}{3}$, multiply by the reciprocal.

$$n = \frac{9}{2}$$

Vocabulary

Equation = a mathematical statement which shows that two expressions are equal

Inverse Operations = opposite operations that undo each other. Addition and subtraction are inverse operations. Multiplication and Division are inverse operations.

Independent Practice.

1. $5r+2=17$
2. $25=-2w-3$
3. $-7=4y+9$
4. $-3f+19=4$
5. $-22=-x-12$
6. $\frac{y}{3}-8=1$
7. $\frac{y}{3}$ - 8 = 1
8. $\frac{2}{3}h-\frac{1}{4}=\frac{1}{3}$
9. $\frac{-2}{5}=\frac{-1}{4}m+\frac{3}{5}$
10. $12.5=2g-3.5$
11. $6.3=2x-4.5$
12. $-6=\frac{y}{5}=4$
13. $\frac{7}{9}=2n+\frac{1}{9}$
14. $-9y-4.2=13.8$
15. $-1=\frac{b}{4}-7$
16. $0.6x+1.5=4$

Solving Equations by Combining Like Terms

You will be able to solve an equation by combining like terms.

8.8.C, A.5.A

Example A

$2x + 6 + 3x = 21$ — Since 2x and 3x are like terms, they can be combined.

$5x + 6 = 21$ — x is being multiplied by 5. Then 6 is being added. Work backward.

$\quad - 6 \quad - 6$ — Subtract 6 from both sides.

$\frac{5x}{5} = \frac{15}{5}$ — Since x is being multiplied by 5, divide both sides by 5.

$x = 3$

Example B

$-4n + 2 - 6n = 82$ — Since -4n and -6n are like terms, they can be combined.

$-10n + 2 = 82$ — n is being multiplied by -10. Then 2 is being added. Work backward.

$\quad - 2 \quad - 2$ — Subtract 2 from both sides.

$\frac{-10n}{-10} = \frac{80}{-10}$ — Since n is being multiplied by -10, divide both sides by -10.

$n = -8$

Example C

$$\frac{m}{3} - 8 + 7 = 4$$ Since -8 and 7 are like terms, they can be combined.

$$\frac{m}{3} - 1 = 21$$ m is being divided by 3. Then 1 is being subtracted. Work backward.

$$+1 \quad +1$$ Add 1 to both sides.

$$(3)\frac{m}{3} = 22(3)$$ Since m is being divided by 3, multiply both sides by 3.

$$m = 66$$

Vocabulary

Like Terms = terms whose variables and exponents are the same. In other words, terms that are "like" each other.

Combine Like Terms = a mathematical process in which like terms are added or subtracted in order to simplify the expression or equation.

Independent Practice

Solve each equation.

1. $6a+5a=-11$
2. $-6n-2n=16$
3. $-3+6-3x=-18$
4. $x+11+8x=29$
5. $0=-5n-2n$
6. $-10=-14v+14v$
7. $43=8.3m+13.2m$
8. $a-2+3=-2$
9. $4x+6+3=17$
10. $-10p+9p=12$
11. $5x+8-5x=8$
12. $\frac{3}{4}x-1+\frac{1}{2}x=11$
13. $\frac{6}{7}y+\frac{2}{5}+\frac{3}{5}+\frac{1}{7}y=10$
14. $7q+4-3q-7+5q=15$
15. $6.25x-\frac{1}{4}x+2.25+7.3x=0$
16. $0.5y+5-5y+7+4.5y=0.25$

- Equation
- Expression
- Variable
- Like terms
- Inverse operations

Solving Equations Using the Distributive Property

You will solve an equation using the distributive property.

Example A

$4(y - 2) = 7$ Distribute 4 to the expression in parenthesis.

$4y - 8 = 7$
$\quad + 8 \quad + 8$

y is being multiplied by 4. Then 8 is being subtracted. Work backward. Add 8 to both sides.

$\frac{4y}{4} = \frac{15}{4}$ Since y is being multiplied by 4, divide both sides by 4.

$y = 3.75 \text{ or } \frac{15}{4}$

Example B

$11 = -(c + 9)$ Distribute -1 to the expression in parenthesis.

$11 = -c - 9$
$+ 9 \quad\quad + 9$

c is being multiplied by -1. Then 9 is being subtracted. Work backward. Add 9 to both sides.

$\frac{20}{-1} = \frac{-c}{-1}$ Since c is being multiplied by -1, divided both sides by -1.

$-20 = c$

Example C

$-3(x - 1) + 8x = -9$	Distribute -3 to the expression in parenthesis.
$-3x + 1 + 8x = -9$	Since -3x and 8x are like terms, they can be combined.
$5x + 1 = -9$ $\quad -1 \quad -1$	x is being multiplied by 5. Then 1 is being added. Work backward. Subtract 1 from both sides.
$\frac{5x}{5} = \frac{-10}{5}$	Since x is being multiplied by 5, divide both sides by 5.
$x = -2$	

Vocabulary

Distributive Property = the mathematical law which states that $a(b + c) = ab + ac$.

Independent Practice

Solve each equation.

1. $5(3x+10)=215$
2. $6(5y-2)=18$
3. $68=4(3b-1)$
4. $-7(x-3.5)=0$
5. $-185=5(3x-4)$
6. $-\frac{1}{3}(x+6)=21$
7. $22=2(4h-9)$
8. $-6(2w+4)=-84$
9. $5(-10-6f)=-290$
10. $42=7(6-7x)$
11. $4(y-5)=-15$
12. $\frac{2}{3}(3x-12)=10$
13. $5(x+6)-3=37$
14. $5(1+4m)-2m=-13$
15. $-(n-8)+10=-2$
16. $8=8v-4(v+8)$
17. $10(1+3b)+15b=-20$
18. $-5-8(1+7n)=-8$
19. $\frac{2}{3}(6x+9)-x-2=-17$
20. $-7.2+2(2.5x-4)=12$

- Equation
- Expression
- Solve
- Simplify
- Inverse operations
- Variable
- Like Terms

Solving Equations with a Variable on Both Sides

You will use distribution to solve equations with variables on both sides.

Example A

$$9b + 1 = 2b + 29$$
$$-2b \qquad -2b$$

To collect the variables on one side, subtract 2b from both sides.

$$7b + 1 = 29$$
$$-1 \quad -1$$

b is being multiplied by 7. Then 1 is being added. Work backward. Subtract 1 from both sides.

$$7b = 28$$

Since b is being multiplied by 7, divide both sides by 7.

.

$$-2(n + 3) = 4n - 3$$

Distribute -2 to the expression in parenthesis.

$$-2n - 6 = 4n - 3$$
$$-4n \qquad -4n$$

To collect the variables on one side, subtract 4n from both sides.

$$-6n - 6 = -3$$
$$+6 \quad +6$$

n is being multiplied by -6. Then 6 is being subtracted. Work backward. Add 6 to both sides.

$$\frac{-6n}{-6} = \frac{3}{-6}$$

Since n is being multiplied by -6, divide both sides by -6.

$$n = -.5 \text{ or } -\frac{1}{2}$$

Example C

$-2x + 8 - 3x = -5x - 2$

$-2x + 8 - 3x = -5x - 2$ Since -2x and -3x are like terms, they can be combined.

$-5x + 8 = -5x - 2$

$+5x \qquad +5x$ To collect the variables on one side, add 5x to both sides.

$8 = \qquad -2$ Since this is a false statement (8 is not equal to -2), the answer is NO SOLUTION. When the answer is a true statement (for example, 3 = 3), the answer is INFINITE SOLUTIONS.

Independent Practice.

Solve each equation.

1. $5x-17=4x+36$
2. $36+19c=24c+6$
3. $-3y+8=2y-2$
4. $4+6p=-8p+32$
5. $-2a+6=30-5a$
6. $6x-7=4x+1$
7. $6y-8=1+9y$
8. $-14g-8=-10g+40$
9. $5x+6=5x-10$
10. $6p+2=-3p-1$
11. $-3x+9=9-3x$
12. $10x=2x-16$
13. $6y=-1+6y$
14. $-5m+2+4m=-2m+11$
15. $2(3p+5)+p=13-2p+15$
16. $-3y-10=4(y+2)+2y$
17. $2(3b-4)=8b-11$
18. $-6(2x+1)=-3x+7-9x$
19. $8s-10=27-(3s-7)$
20. $3b+12=3(b-6)+4$

Solving Rational Equations

You will learn how to solve an equation that involves fractions.

Example A

$\frac{2}{n} = \frac{5}{n-3}$ Cross multiply to eliminate the denominators. (Rewrite in distribution form)

$5(n) = 2(n-3)$ Distribute the 5 and the 2 to the expressions in parenthesis.

$5n = 2n - 6$

$-2n \quad -2n$ To collect variables on one side, subtract 2n from both sides.

$\frac{3n}{3} = \frac{-6}{3}$ Since n is being multiplied by 3, divide both sides by 3.

$n = -2$

Example B

$\frac{12}{y-2} = 8$ Cross multiply to eliminate the denominators. (Rewrite in distribution form)

$8(y-2) = 12(1)$ Distribute the 8 and the 12 to the expressions in parenthesis.

$8y - 16 = 12$

$+16 + 16$ y is being multiplied by 8. Then 16 is being subtracted. Work backward. Add 16 to both sides.

$\frac{7y}{7} = \frac{28}{7}$ Since y is being multiplied by 7, divide both sides by 7.

$y = 4$

Example C

$$\frac{4}{d+3} = \frac{6}{d-1}$$

Cross multiply to eliminate the denominators.
(Rewrite in distribution form)

$$6(d + 3) = 4(d - 1)$$

Distribute the 6 and the 4 to the expressions in parenthesis.

$$\begin{array}{r} 6d + 18 = 4d - 4 \\ -4d \qquad\quad -4d \\ \hline \end{array}$$

To collect the variables on one side, subtract 4d from both sides.

$$\begin{array}{r} 2d + 18 = -4 \\ -18 \quad -18 \\ \hline \end{array}$$

d is being multiplied by 2. Then 18 is being added. Work backward.
Subtract 18 from both sides.

$$\frac{2d}{2} = \frac{-22}{2}$$

Since d is being multiplied by 2, divide both sides by 2.

$$d = -11$$

Vocabulary

Rational Equation = an equation in which one or more of the terms is a fractional one.

Independent Practice

Solve the following rational equations using cross products.

1. $\frac{3}{c} = \frac{4}{c-3}$
2. $\frac{1}{x-1} = 3$
3. $\frac{2}{r} = \frac{2}{2-r}$
4. $\frac{5}{x+3} = \frac{2}{x}$
5. $\frac{-4}{x-1} = \frac{2}{x}$
6. $\frac{3}{c+2} = \frac{2}{c+2}$
7. $4 = \frac{8}{x+2}$
8. $\frac{2}{j+4} = \frac{4}{j-1}$
9. $\frac{5}{y-3} = \frac{-8}{y-4}$
10. $\frac{-2}{-b+5} = \frac{1}{b-2}$
11. $\frac{2}{-x-5} = \frac{3}{-2x-3}$
12. $\frac{6}{x+1} = \frac{-3}{3-x}$
13. $\frac{-2}{x+5} = \frac{-1}{2-x}$
14. $\frac{-4}{1+x} = \frac{-3}{5-3x}$

- Rational Numbers
- Cross Products
- Distributive Property

Solving Literal Equations

You will learn how to solve for any specified variable in any given formula.

Example A

Solve for r.

$d = rt$ To solve the equation for r, work backward.

$\frac{d}{t} = \frac{rt}{t}$ Since r is being multiplied by t, divide both sides by t.

$\frac{d}{t} = r$

Example B

Solve for a.

$a - b = 7$ To solve the equation for a, work backward.

$+b \quad +b$ Since a is being subtracted by b, add b to both sides.

$a = b + 7$

Example C

Solve for h.

$A = \frac{bh}{2}$ To solve for h, cross multiply to eliminate the denominator.

$1(bh) = 2(A)$ Distribute the 1 and the 2 to the expressions in parenthesis.

$\frac{bh}{b} = \frac{2A}{b}$ Since h is being multiplied by b, divide both sides by b.

$h = \frac{2A}{b}$

Vocabulary

Literal Equation = an equation made up of mostly letters or variables.

Independent Practice

Solve the following equations.

1. $5 = x + y$ Solve for x
2. $w = x + 5$ Solve for x
3. $a + b = 3$ Solve for a
4. $a + b = 3$ Solve for b
5. $p + t = q$ Solve for p
6. $a^2 + b^2 = c^2$ Solve for c^2
7. $A = lw$ Solve for w
8. $A = \prod r^2$ Solve for $\prod$
9. $d = rt$ Solve for t
10. $r = \frac{m}{2p}$ Solve for m
11. $\frac{c}{d} = \prod$ Solve for c
12. $\frac{c}{d} = \prod$ Solve for d

- Formula
- Multi-variable
- Literal equations

Absolute Value Equations and Inequalities

Absolute Value Definition - The absolute value of *x*, is defined as…

$$|\boldsymbol{x}| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$$

where ***x*** is called the "argument"

Steps for Solving *Linear* Absolute Value *Equations*: $i.e.\quad |ax+b| = c$

1. Isolate the absolute value.

2. Identify what the isolated absolute value is set equal to…
   a. If the absolute value is set **equal to zero**, remove absolute value symbols & solve the equation to get **one solution**.

   b. If the absolute value is set **equal to a negative** number, there is **no solution.**

   c. If the absolute value is set **equal to a positive** number, set the argument (*expression within the absolute value*) equal to the number **and** set it equal to the opposite of the number, using an 'or' statement in between the two equations. Then solve each equation separately to get **two solutions**.

Examples:

a. $|3x+12|+7=7$

$|3x+12|=0$

Because this equals **0**, there is **ONE** solution.

$3x+12=0$

$3x=-12$

$\boldsymbol{x=-4}$

b. $|3x-7|+7=2$

$|3x-7|=-5$

Because this equals a **negative** number, there is **NO** solution.

No Solution

c. $|3x-7|+7=9$

$|3x-7|=2$

Because this equals a **positive** number there are **TWO** sltns.

$3x-7=2$ or $3x-7=-2$

$3x=9$ or $3x=5$

$\boldsymbol{x=3}$ or $\boldsymbol{x=\frac{5}{3}}$

d. $|x+5| = |2x-1|$ → **Set up two Equations**

$x+5=+(2x-1)$ or $x+5=-(2x-1)$

$\boldsymbol{x=6}$ or $x+5=-2x+1 \rightarrow 3x=-4 \rightarrow \boldsymbol{x=-\frac{4}{3}}$

<u>Steps for Solving *Linear* Absolute Value *Inequalities*:</u> $i.e \quad |ax+b| \le c$

1. Isolate the absolute value.

2. Identify what the absolute value inequality is set "equal" to…

"Zero"

   a. If the absolute value is **less than zero**, there is **no solution.**

   b. If the absolute value is **less than or equal to zero**, there is **one solution**. Just set the argument equal to zero and solve.

   c. If the absolute value is **greater than or equal to zero**, the solution is **all real numbers.**

   d. If the absolute value is **greater than zero**, the solution is all real numbers **except** for the value which makes it equal to zero. This will be written as a **union**.

"Negative" Number

   e. If the absolute value is **less than** <u>or</u> **less than or equal to a negative number,** there is **no solution.** The absolute value of something will *never* be less than or equal to a negative number.

   f. If the absolute value is **greater than** <u>or</u> **greater than or equal to a negative number,** the solution is **all real numbers.** The absolute value of something will *always* be greater than a negative number.

"Positive" Number

   g. If the absolute value is **less than** <u>or</u> **less than or equal to a positive number**, the problem can be approached two ways. Either way, the solution will be written as an **intersection**.
      i. Place the argument in a 3-part inequality (compound) between the opposite of the number and the number, then solve.
      ii. Set the argument less than the number **and** greater than the opposite of the number using an "and" statement in between the two inequalities.

   h. If the absolute value is **greater than** <u>or</u> **greater than or equal to a positive number**, set the argument less than the opposite of the number **and** greater than the number using an 'or' statement in between the two inequalities. Then solve each inequality, writing the solution as a **union** of the two solutions.

3. Graph the answer on a number line <u>and</u> write the answer in interval notation.

Examples:

a. $|x-4| \ge 0$

All Real Numbers

b. $|2x-1|+4 < 4$

$|2x-1| < 0$

No Solution

c. $-3+|x+1| \le -3$

$|x+1| \le 0$

Set $x+1=0$

So $\boldsymbol{x=-1}$

d. $|3x+4|+5 \leq 3$

$|3x+4| \leq -2$

No Solution

e. $2|x-1|-4 \geq 2$

$2|x-1| \geq 6$

$|x-1| \geq 3$

$x-1 \geq 3 \quad OR \quad x-1 \leq -3$

$x \geq 4 \quad OR \quad x \leq -2$

$(-\infty, -\mathbf{2}] \cup [\mathbf{4}, \infty)$

f. $|x-6|+6 \geq -4$

$|x-6| \geq -10$

All Real Numbers

g. $|2-x| < 8$

$2-x < 8 \quad OR \quad 2-x > -8$

$-x < -6 \quad OR \quad -x > -10$

$x > 6 \quad OR \quad x < 10$

$(\mathbf{6}, \mathbf{10})$

h. $3|4x-1| \leq 9$

$|4x-1| \leq 3$

i. $|x+6| > 0$

Set $\quad x+6 \neq 0$

So $\quad x \neq -6$

$(-\infty, -\mathbf{6}) \cup (-\mathbf{6}, \infty)$

Problem "h" can be solved using two different approaches.

Option 1 – Split in to two different Inequalities joined by an "AND" statement (Intersection)

$3|4x-1| \leq 9$

$|4x-1| \leq 3$

$4x-1 \leq 3 \quad AND \quad 4x-1 \geq -3$

$x \leq 1 \quad AND \quad x \geq -\frac{1}{2}$

$\left[-\frac{1}{2}, 1\right]$

Option 2 – Write as a compound inequality (Intersection)

$3|4x-1| \leq 9$

$|4x-1| \leq 3$

$-3 \leq 4x-1 \leq 3$ *(add 1)*

$-2 \leq 4x \leq 4$ *(divide by 4)*

$-\frac{1}{2} \leq x \leq 1$

$\left[-\frac{1}{2}, 1\right]$

Steps for Solving *NON- Linear* Absolute Value *Equations*:

Follow the same steps as outlined for the linear absolute value equations, but all answers must be plugged back in to the original equation to verify whether they are valid or not (i.e. ***"Check your answers."***) Occasionally, "extraneous" solutions can be introduced that are not correct and they must be excluded from the final answer.

Examples:

a. $|x^2 + 1| = 5$

2 Equations

$x^2 + 1 = 5$ or $x^2 + 1 = -5$

$x^2 = 4$ or $x^2 = -6$

$\sqrt{x^2} = \sqrt{4}$ or $\sqrt{x^2} = \sqrt{-6}$

$\boldsymbol{x = \pm 2}$ or $x = imaginary!$

Check your answers!

Check: $x = 2$	Check: $x = -2$
$\|(2)^2 + 1\| = 5$	$\|(-2)^2 + 1\| = 5$
$\|5\| = 5$	$\|5\| = 5$
$5 = 5$ ✓	$5 = 5$ ✓
$\boldsymbol{x = 2}$ **Works!**	$\boldsymbol{x = -2}$ **Works!**

b. $|x^2 + 5x + 4| = 0$

Only 1 Equation

$x^2 + 5x + 4 = 0$

$(x + 1)(x + 4) = 0$

$x + 1 = 0$ and $x + 4 = 0$

$\boldsymbol{x = -1}$ and $\boldsymbol{x = -4}$

Check your answers!

Check: $x = -1$	Check: $x = -4$
$\|(-1)^2 + 5(-1) + 4\| = 0$	$\|(-4)^2 + 5(-4) + 4\| = 0$
$\|1 - 5 + 4\| = 0$	$\|16 - 20 + 4\| = 0$
$\|0\| = 0 \rightarrow 0 = 0$ ✓	$\|0\| = 0 \rightarrow 0 = 0$ ✓
$\boldsymbol{x = -1}$ **Works!**	$\boldsymbol{x = -4}$ **Works!**

c. $|x + 3| = x^2 - 4x - 3$

2 Equations

$x + 3 = x^2 - 4x - 3$ or $x + 3 = -(x^2 - 4x - 3)$

$x^2 - 5x - 6 = 0$ or $x + 3 = -x^2 + 4x + 3$

$(x - 6)(x + 1) = 0$ or $x^2 - 3x = 0$

$x - 6 = 0$ and $x + 1 = 0$ or $x(x - 3) = 0$

$\boldsymbol{x = 6}$ and $\boldsymbol{x = -1}$ or $\boldsymbol{x = 0}$ and $\boldsymbol{x = 3}$

Check your answers!

Plugging each of the 4 answers into original equation results in …

$x = -1 \rightarrow 2 = 2$ ✓

$x = 6 \rightarrow 9 = 9$ ✓

$x = 0 \rightarrow 3 \neq -3$

$x = 3 \rightarrow 6 \neq -6$

So, the only answers to the problem are $\boldsymbol{x = -1}$ and $\boldsymbol{x = 6}$. *($x = 0$ and $x = 3$ are extraneous).*

RATIOS AND PROPORTIONS

Ratios

A ratio is a comparison of two quantities that have the *same units*. You can express a ratio in any one of the following ways:

$\frac{18}{5}$ 18:5 18 to 5

Example #1: If one store has 360 items and another store has 100 of the same items, express the ratio of the items.

$\frac{360}{100}$ **or** 360:100 **or** 360 to 100

Ratios are usually written in lowest terms; therefore, the above example would reduce in this way: $\frac{360 \div 20}{100 \div 20}$ (What is the largest number you can divide both values by?)

$\frac{18}{5}$

Example #2: John earns $350 a week. His take-home pay, however, is $295. What is the ratio of his gross pay to his take-home pay.

$\frac{350}{295} = \frac{70}{59}$

Rates

A rate is a comparison of two quantities that have *different units*. Rates are usually expressed in the fractional form.

Example: Francine paid $16 for her 12-month subscription to *Better Homes and Gardens* magazine. Express as a rate.

$\frac{\$16.00}{12 \text{ magazines}} = \frac{\$4.00}{3 \text{ magazines}}$

If Francine wants to know how much she pays for each (1) magazine, she can divide $4 by 3 magazines. This will give her the price per magazine (also called the **unit rate**).

$\frac{\$4.00}{3} = \$1.33\text{/magazine}$

Proportions

A proportion is a statement that two ratios or rates are equal. It can be given as a sentence in words, but most often a proportion is an algebraic equation.

The arithmetic equation $\frac{3}{5} = \frac{21}{35}$ is a proportion because its cross products are equal.

$3 \times 35 = \mathbf{105}$ and $5 \times 21 = \mathbf{105}$

Proportions are solved by using this cross-product rule.

Example #1: $\frac{4}{9} = \frac{x}{36}$

$4 \times 36 = 9x$

$144 = 9x$

$\frac{144}{9} = x$

$16 = x$

Example #2: $\frac{72}{1.5} = \frac{12}{x}$

$72x = 1.5 \times 12$

$72x = 18$

$x = \frac{18}{72}$

$x = .25$ or ¼

Applied Proportion Problems

Many problems can be solved by setting up a **direct proportion** (an increase in one quantity leads to a proportional increase in the other quantity) or by setting up **equivalent rates**.

Example: In one day you earn $75 for 8 hours of work. If you work 37.5 hours for the week, what will your weekly pay be?

$\frac{8 \text{ hours}}{37.5 \text{ hours}} = \frac{\$75}{x}$

$8x = 75 \times 37.5$

$8x = 2812.5$

$x = \frac{2812.5}{8}$

$x = \$351.56$

or

$\frac{8 \text{ hours}}{\$75} = \frac{37.5 \text{ hours}}{x}$

$8x = 75 \times 37.5$

$8x = 2812.5$

$x = \frac{2812.5}{8}$

$x = \$351.56$

RATIOS AND PROPORTIONS
PRACTICE SHEET

A. Write each ratio as a fraction in lowest terms.

1. 2 to 4
2. $\frac{15}{20}$
3. 6:18
4. 21:15
5. $\frac{12}{18}$
6. 3 to 12
7. 7: 4
8. $\frac{18}{12}$
9. 20:16
10. 15 to 36
11. 35:7
12. $\frac{8}{28}$
13. 24 to 96
14. 9:27
15. $\frac{11}{88}$

B. Write each of the following rates as a unit rate.

1. $\frac{\text{3 Tbsp}}{\text{2 tsp}}$
2. $\frac{\text{135 pitches}}{\text{45 strikes}}$
3. $\frac{\text{128 miles}}{\text{4 hours}}$
4. $\frac{\text{2250 pencils}}{\text{18 boxes}}$
5. $\frac{\$450}{\text{18 shares}}$
6. $\frac{\text{2500 meters}}{\text{15 seconds}}$
7. $\frac{\$5{,}082}{\text{475 sq.yds.}}$
8. $\frac{\text{750 gallons}}{\text{14 minutes}}$

C. Solve each proportion and give the answer in simplest form.

1. 6 : 8 = n : 12
2. $\frac{2}{7} = \frac{8}{n}$
3. $\frac{n}{6} = \frac{11}{3}$
4. 4 : n = 6 : 9

5. $\frac{3}{n} = \frac{2}{5}$

6. $\frac{0.4}{1.5} = \frac{12}{n}$

7. 2 ½ : 3 ½ = n : 2

8. 1: 2 = n : 9

9. 4 to 8 = 15 to n

10. 18 : n = 3 :11

11. $\frac{5}{6} = \frac{n}{30}$

12. $\frac{12}{40} = \frac{n}{25}$

13. 8 :19 = 14:n

14. $\frac{10}{n} = \frac{2}{1.7}$

15. 24 : ¼ = n : ⅓

16. 44 to 121 = n to 11

D. Solve by using a proportion. Round answers to the nearest hundredth if necessary.

1. You jog 3.6 miles in 30 minutes. At that rate, how long will it take you to jog 4.8 miles?

2. You earn $33 in 8 hours. At that rate, how much would you earn in 5 hours?

3. An airplane flies 105 miles in ½ hour. How far can it fly in 1 ¼ hours at the same rate of speed?

4. What is the cost of six filters if eight filters cost $39.92?

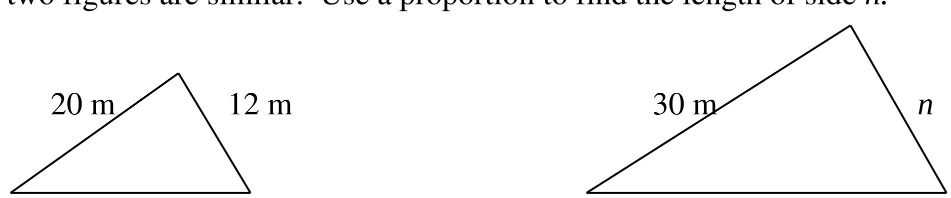
5. If one gallon of paint covers 825 sq. ft., how much paint is needed to cover 2640 sq. ft.?

6. A map scale designates 1” = 50 miles. If the distance between two towns on the map is 2.75 inches, how many miles must you drive to go from the first town to the second?

7. Bob is taking his son to look at colleges. The first college they plan to visit is 150 miles from their home. In the first hour they drive at a rate of 60 mph. If they want to reach their destination in 2 ½ hours, what speed must they average for the remainder of their trip?

8. Four employees can wash 20 service vehicles in 5 hours. How long would it take 5 employees to wash the same number of vehicles?

9. These two figures are similar. Use a proportion to find the length of side *n*.



Percent Increase and Decrease

Construct Meaning

A packaging company located in Iceland exports the country's products and manufactures paper products for schools, homes, and businesses. If Iceland Packaging had 140 employees in 2000 and 175 employees in 2002, what was the percent increase of the number of employees?

Percent increase is a percent change that describes an increase in quantity.
Percent decrease is a percent change that describes a decrease in quantity.

To determine percent increase or percent decrease, first find the amount of change. In this case, subtract the number of employees in 2000 from the number in 2002 to determine the amount of change.

175 employees – 140 employees = 35 employees

Percent change may be percent increase or percent decrease.

A proportion may be used to find percent changes.

$$\frac{\text{percent change}}{100} = \frac{\text{amount of change}}{\text{original amount}}$$

Use the proportion to determine the percent increase in the number of employees at Iceland Packaging. Let n = percent increase.

Substitute the numbers. $\frac{n}{100} = \frac{35}{140}$

Use cross products. $140n = 100 \cdot 35$

Use inverse operations. $\frac{140n}{140} = \frac{3500}{140}$

$n = 25$

Write the percent. $\frac{n}{100} = \frac{25}{100} = 25\%$

The percent increase was 25%.

Use the proportion to find the number of tourists taking a shore excursion in Iceland during August. There was a 20% decrease in business from July, when 240 people took the tour. Let x represent the amount of change.

Substitute the numbers. $\frac{20}{100} = \frac{x}{240}$

Use cross products. $20 \cdot 240 = 100x$

Use inverse operations. $\frac{4800}{100} = \frac{100x}{100}$

$48 = x$ **There were 48 fewer tourists in August.**

To find the number of tourists in August, subtract the amount of change from the number of tourists in July.

$240 - 48 = 192$ THERE WERE 192 TOURISTS IN AUGUST.

Check Understanding

a. The number of customers that visited Perry's Bakery doubled in one year. The shop manager put a sign in the window that stated, "We now serve 200% more customers." Is the statement true or false? Explain.

b. Brooke decreased the amount of time she spent practicing the piano from one hour to 45 minutes. Mentally determine the percent decrease.

c. If Iceland Packaging hired 12 more employees, which is a 20% increase in the number of employees, what was the original number of employees?

Practice

Determine the percent increase or decrease.

1. Original price *$40*
Sale price *$28*

2. Original weight *102 lb*
New weight *127.5 lb*

3. Original number of students *215*
Current number of students *258*

Determine the new amount.

4. $45 is increased by 15%

5. 120 is decreased by 40%

6. 5 lb is increased by 112%

Determine the original amount.

7. Amount of decrease *$10*
Percent change *10%*

8. Amount of increase *540*
Percent change *80%*

9. Sale price *$35*
Discount *30%*

10. The population of Monterey Heights decreased from 16,750 to 15,745 in one year. Determine the percent decrease.

11. Mr. Divatz's car has decreased in value by 20% of its original price of $13,500. How much is Mr. Divatz's car worth?

12. Because of the success of the local rugby team, the number of people registered in the fan club increased by 200 people, which is 80% of the original number of fans. How many people are now registered in the fan club?

13. Mr. Bakke recorded the time it took the students to run around the perimeter of the school at the beginning of the year and at the end of the year. Between Gina and Andrea, who had the greater percent of improvement? What was that student's percent of improvement?

Name	Fall Quarter	Spring Quarter
Gina	1 min 30 sec	1 min 15 sec
Andrea	1 min 40 sec	1 min 25 sec

14. Todd bought a pack of basketball cards that contained a rookie card of a player who later became famous. He paid $0.03 for the rookie card, and fifteen years later, the card was worth $1200. Determine the percent increase of the card's value.

WORD PROBLEMS

Word Translations

There is nothing more important in mathematics than to be able to translate English to math and math to English. Vocabulary and notation are very important to understanding and communicating in mathematics. Without knowing what words mean, we'll certainly have trouble answering questions. The research is clear, there is no more single important factor that affects students comprehension than vocabulary.

Listed below are examples of statements translated to algebra. It's very important that you are familiar with these expressions and their translations so you won't later confuse algebraic difficulties with vocabulary deficiencies. – The following should be taught explicitly. Students need to memorize them!

STATEMENT	ALGEBRA
twice as much as a number	2x
two less than a number	x – 2
five more than an unknown	x + 5
three more than twice a number	2x + 3
a number decreased by 6	x – 6
ten decreased by a number	10 – x
Tom's age 4 years from now	x + 4
Tom's age ten years ago	x – 10
number of cents in x quarters	25x
number of cents in 2x dimes	10(2x)
number of cents in x + 3 nickels	5(x + 3)
separate 15 into 2 parts	x, 15 – x
distance traveled in x hrs at 50 mph	50x
two consecutive integers	x, x + 1
two consecutive odd integers	x, x + 2
sum of a number and 30	x + 30
product of a number and 5	5x
quotient of a number and 7	x ÷ 7
four times as much	4x
two less than 3 times a number	3x - 2

By familiarizing yourself with these expressions, you'll look forward to solving word problems. We have already identified and used strategies for solving linear equations in one variable. In word problems, all we do that is different is make our own equations. Piece of cake, don't you think?

The easiest and best way to learn vocabulary is to read your textbook. How you do on standardized tests will often be determined by your understanding of math vocabulary. College entrance exams, the ACT and SAT, use correct terminology so you best get used to it. Where your teacher might ask you to solve an equation, on a standardized test you will be asked to find the solution set. You need to know they mean the same thing.

Word translations. Using letters suggested in the problem, write an equation or expression for each of the following statements.

1. John is four years older than Frank, the sum of their ages is 36.

2. Bob has fives times as much money as John and together they have $60.00.

3. The second angle is thirty degrees more than the first.

4. The sum of the interior angles of a triangle is 180˚, The second angle of a triangle is 45˚ more than the first and the third angle is twice the first.

5. The area of a triangle is half the base times the height.

6. The perimeter of a rectangle is the sum of twice the length and twice the width.

7. Ted is four years older than three times Mary's age.

8. Mark earns a base salary of $400 per week plus a 6% commission on all his sales.

9. The cell phone bill has a base fee of $30 per month plus twenty cents per minute.

10. The circumference of a circle is equal to the diameter multiplied by π.

Sec. 2 Problem Solving

Now that you know how to translate English to math, it's time to use our knowledge of solving equations with our knowledge of translating English to mathematics. During your first year of algebra, you will learn how to set up different types of problems including, uniform motion, age, coin, mixture, geometry, number and investment. Like everything else in life, the more you do, the more comfortable and confident you will become. These learned formats should give you an idea how to set up and solve problems that you have not encountered in class.

Probably the most important thing to remember is most of us have to read a word problem 4, 5 or 6 times just to get all the information we need to solve the problem and make an equation that describes the relationship.

If there is any one trick to make your work easier, it is to write the smallest quantity as x and the other unknown in terms of x.

Study the word translations!

I can not stress enough how important it is for you to give your self a chance to be successful by reading and rereading the word problem in order to get the needed information.

Generally speaking, if you only read the problem once or twice, you won't get the information you need to setup and solve the problem.

Let's look at some word problems and see how to set them up. Remember, after we identify what we are looking for, determine the smallest value and call it x. The other unknowns will be described in terms of x.

Algorithm for Problem Solving

1. Read the problem through to determine the type of problem
2. Reread the problem to identify what you are looking for and label
3. Reread, Let x be the smallest quantity you are looking for.
4. Reread the problem again and label the other quantities in terms of x
5. Reread the problem to make an equation, use some fact or relationship involving your variables

WEIGHTED MEANS AND MEANS AS WEIGHTED SUMS

In the Speeds Problem we saw that there is more than one kind of "average." In this handout, we will explore this topic further.

The ordinary mean is sometimes called the "*arithmetic mean*" to distinguish it from other types of means.

Note on pronunciation: When "arithmetic" is used as an adjective (as in "arithmetic mean"), it is pronounced "air-rith-MAT-ic" or "air-rith-MET-ic" -- i.e., accent on the third syllable. (Analogy: "geometric").

The most common way to think of the average (arithmetic mean) of numbers is to add them up and divide by the total number of summands:

e.g., the average of 1,1,2,3, 4,4,4 is (1+1+2+3+4+4+4)/7

But we could write this two other ways:

1. "Distributing" the denominator gives

$$(1/7)1 + (1/7)1 + (1/7)2 + (1/7)3 + (1/7)4 + (1/7)4 + (1/7)4.$$

Thus we have the mean as a sum of coefficients times the original numbers in the list. *Note that the sum of the coefficients is 1.*

2. Collecting like terms gives

$$(2x1 + 2 + 3 + 3x4)/7 = (2/7)1 + (1/7)2 + (1/7)3 + (3/7)4.$$

Now we have a sum of coefficients times the *distinct* values (not allowing repetitions) in the original list of numbers. The coefficient of a value is the *proportion* of that value in the original list of numbers. We still have the coefficients adding to 1, but they are no longer all the same. We now see the mean as a *weighted sum* of the *distinct values*, where each value is weighted according to its proportion in the total list of numbers. This perspective prompts two generalizations of the arithmetic mean.

A. **Weighted Means**

To form a *weighted mean* of numbers, we first multiply each number by a number ("weight") for that number, then add up all the weighted numbers, then divide by the sum of the weights. We often do this in computing course grades – e.g., weighting the final exam twice as much as a midterm exam. The ordinary (arithmetic) mean is a weighted mean with all weights equal to 1.

Another way to describe a *weighted mean* of a list of numbers is a sum of coefficients times the numbers, where the coefficients add up to 1. In this case, the coefficients are called the *weights*. (*Note the ambiguity in the use of "weight".)* If all the weights are the same, we get the ordinary arithmetic mean.

Why are these two descriptions equivalent?

Examples of weighted means:

1. The discussion above shows that the ordinary (arithmetic) mean can also be considered as a *weighted mean* of the *distinct values* being averaged, with the weight of a value being its proportion in the original list of numbers being averaged.

2. In part (b) of the Average Speeds Problem (Problem 2 in the handout "What Do You Mean by Average?"), the average speed can be written as a weighted mean:

$$\text{Average speed} = \frac{a_1 v_1 + a_2 v_2 + ... + a_n v_n}{a_1 + a_2 + ... + a_n} =$$

$$\frac{a_1}{a_1 + a_2 + ... + a_n} v_1 + \frac{a_2}{a_1 + a_2 + ... + a_n} v_2 + ... + \frac{a_n}{a_1 + a_2 + ... + a_n} v_n$$

$$= w_1 v_1 + w_2 v_2 + \ldots + w_n v_n,$$

where $w_i = \dfrac{a_i}{a_1 + a_2 + ... + a_n}$

Note that the sum of the w_i's is 1. Thus, the answer to part b in the Average Speeds Problem can be seen as *a weighted mean of the original speeds, with the weight of each speed being the fraction (proportion) of the total number of intervals that are traveled at that speed.*

3. Another place where weighted means are important is when the purpose of the study is to compare means of two groups, but the two groups are appreciably different in size. Consider for example, a study whose purpose is to compare the educational and workforce experiences of male and female electrical engineers. There are many fewer women in electrical engineering than men, so a simple random sample of all engineers in the population would include very few women, and therefore not give as good estimates for the women as the men. Instead, the researchers would use a *stratified* sample – they might, for example, sample 200 men and 200 women. But then if they want a *sample* "average" that estimates the average for *all* electrical engineers, including both men and women, they need to take a weighted average.

Exercise: Suppose that the total number of men in the *population* being studied (e.g., *all* electrical engineers) is N_M and the total number of women in the *population* is N_w. If 200 of each sex are sampled, what would be appropriate weights for calculating an average of some variable (e.g., salary; number of years in the profession) on which data are collected, if the intent is to estimate the average for the *entire population* of interest (e.g., all electrical engineers)?

Word Problems – Uniform Motion

Solving word problems is what kids in algebra live for. As there are different formats for solving different types of equations, there are different formats for solving different types of word problems.

You should keep in mind there are other methods for solving word problems than the ones I present.

To solve word problems involving uniform motion, we need to know that

DISTANCE = RATE X TIME

I will use a distance, rate, time chart, and solve the problems in terms of distance whenever possible. In that way I can avoid fractional equations.

From this perspective there are two types of uniform motion problems. Either

A. The distances are equal, or
B. The sum of the distances equal a number

TYPE A. If the distances are equal, one of two things must occur.

1. You go somewhere and return, or
2. You leave to go somewhere and someone else leaves later and catches up to you

In either case, the distances are equal. Mathematically we write **D1 = D2**

Type B. If someone did not catch up to you or if you did not go somewhere and come back, the distances are not equal. That means the sum of the distances must be equal to a number.

Mathematically, we write **D1 + D2 = #**

Let's see how all this works.

EXAMPLE
Two trains start from the same station at the same time and travel in opposite directions. One train travels at an average rate of 40 mph, the other at 65 mph. In how many hours will they be 315 miles apart?

First we'll make the d=rt chart. But we won't fill in the d.

	d	**=**	**r**	x	**t**
Train 1			40		x
Train 2			65		x

The reason we have an x in the time column is because they left at the same time and will be 315 at the same time. In other words, their times are equal.

Now, the big question. Are there distances equal? Since they do not meet the criteria in a TYPE A problem, the answer is no. That means the sum of the distances must be equal to a number.

$$D1 + D2 = \#$$
$$40x + 65 = 315$$
$$105x = 315$$
$$x = 3$$ It will take three hours.

EXAMPLE

Bob starts out in his car traveling 30 mph. Four hours later, Mr. Speedster starts out from the same point at 60 mph to overtake Bob. In how many hours will he catch him?

Making the d = rt chart

	d	=	**r**	x	**t**
Bob			30		x + 4
Mr. Speedster			60		x

Since Mr. Speedster traveled the least amount of time, we called that x. This is a TYPE A problem, the distances are equal.

$$\mathbf{D}_{Bob} = \mathbf{D}_{Speedster}$$

$$30(x + 4) = 60x$$
$$30x + 120 = 60x$$
$$120 = 30x$$
$$4 = x$$ It will take 4 hours to catch Bob.

Solve

1. Two trains start from the same station and run in opposite directions. One runs at an average rate of 40 miles per hour, and the other at 65 miles per hour. In how many hours will they be 315 miles apart?

First Train Starting point Second train

40 mph x hours
distance = rate x time

d = 40x

65 mph x hours
distance = rate x time

d = 65x

$40x + 65x = 315$ miles

Or you may like to set up a table like this:

Train	Time	Rate	Distance
First	X	40	40x
Second	X	65	65x

Then Complete. $40x + 65x = 315$

2. Two automobiles start from the same place and travel in opposite directions, one averaging 45 miles per hour and the other 30 miles per hour. In how many hours will they be 900 miles apart?

3. Two men, A and B, start toward each other at the same time from points 510 miles apart. If they travel 40 and 45 miles an hour respectively, in how many hours will they meet?

4. Jones and Brown start from two points, which are 375 miles apart and travel toward each other. The latter travels twice as fast as the former. They meet in 5 hours. Find the rates per hour.

5. A man rides out into the country at a uniform rate of 30 miles per hour. He rests 2 hours and then rides back at 20 miles per hour. He is gone 5 hours. How far did he go?

6. A motorboat starts out and travels 9 miles an hour. In 3 hours another motorboat traveling 18 miles an hour starts out to overtake the first one. In how many hours will the second boat overtake the first?

7. Mr. Williams starts out in his auto traveling 30 miles per hour. Four hours later Mr. Speedster starts out from the same point at 60 miles per hour to overtake Mr. Williams. In how many hours will he be overtaken?

   Hint: Remember that each will have traveled the same distance when they meet. X = Speedster's time.

   R X T = D

Williams	30	X + 4	30(x – 4)
Speedster	60	X	60X

   Therefore, $30(x + 4) = 60x$

8. A freight train is traveling 30 miles per hour. An automobile starts out from the same place 1 hour later and overtakes the train in 3 hours. What was the rate of the automobile?

9. C and D start from two points 480 miles apart and travel toward each other. They meet in 8 hours. If C travels 6 miles per hour faster than D, find their rates.

Word Problems - Mixture

One method of solving mixture problems is to do the problem in terms of what is being added.

That means if you have a problem involving a mixture of antifreeze and you are going to add water to it to dilute it, then do the problem in terms of water.

An iodine problem that has you adding alcohol to dilute it should be done in terms of alcohol.

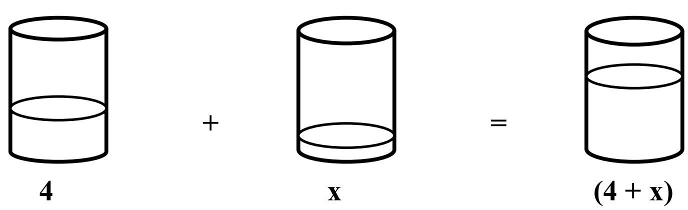
Water is water, salt is salt, if you don't get these right, it will be my fault. Just remember, when solving mixture problems, we **DON'T** start off with water and add salt to get salt water.

EXAMPLE
A pharmacist has 4 quarts of a 15% solution of iodine. How much alcohol must be added to reduce it to a 10% solution of iodine?

What's being added? Hopefully, you said alcohol. Therefore our equation will look like this:

ALCOHOL	+	**ALCOHOL**	=	**ALCOHOL**
4 qts +		x qts	=	(4 + x) qts

We start off with 4 quarts, add x quarts, then end up with (4 + x) quarts.



Notice that you started with 4 quarts and added x quarts on the left side of the equation and you ended up with (4 + x) on the other side. The parentheses indicate that it's one container, Neato!

Of the original 4 quarts, 15% is iodine. Since we are doing the problem in terms of what we are adding – alcohol, we must change 15% iodine solution to an 85% solution of alcohol. Let's write the equation.

$$.85(4) + x = .90(4x)$$

Where'd the .90 come from? Well, since I wanted to end up with a 10% solution of iodine, that meant it must be a 90% solution of alcohol.

Multiplying both sides of the equation by the common denominator – 100, we have

$$\begin{aligned} 85(4) + 100x &= 90(4 + x) \\ 340 + 100x &= 360 + 90x \\ 10x &= 20 \\ x &= 2 \end{aligned}$$

You would have to add 2 quarts of alcohol to reduce the mixture to 10% iodine.

EXAMPLE
How much water must be added to a 30 quarts of a 75% acid solution to reduce it to a 15% solution of acid?

I'm adding water, so we have

WATER + WATER = WATER

Starting off with 30 quarts and adding x quarts, we should end up with (30 + x) quarts

$$30 + x = (30 + x)$$

The problem describes the solution in terms of acid, we have set it up in terms of water. So, we have to change the percentages and put them in the problem.

$$.25(30) + x = .85(30 + x)$$

Again, we multiply both sides of the equation by 100 to get rid of the decimal point.

$$25(30) + 100x = 85(30 + x)$$
$$750 + 100x = 2550 + 85x$$
$$15x = 1800$$
$$x = 120 \text{ qts}$$

Remember to do the problem in terms of what's being added, then make sure your percents describe the solution in your equation.

I know, you love this stuff!

Solve

1. A grocer wishes to mix one-dollar coffee with 80-cent coffee to produce a mixture of 200 pounds to sell for 84 cents a pound. How many pounds of each kind should he use? Hint: Try making a table like this:

Kind	Number of Pounds	Value in Cents
$l coffee	N	100n
80-cent coffee	200 – n	80(200 – n)
84-cent coffee	200	200 84

Why do we use 200 – n to represent the number of pounds of 80-cent coffee?
Hence, 100n + 80 (200 – n) = 16,800
Complete the solution.

Solving Mixture Problems: The Bucket Method

Mixture problems occur in many different situations. For example, a store owner may wish to combine two goods in order to sell a new blend at a given price. A chemist may wish to obtain a solution of a desired strength by combining other solutions. In any case, mixture problems may all be solved by using the bucket method.

The key to the bucket method is setting up the buckets correctly. Generally, the buckets will be set up as follows:

what you start with	+	what you Add in	=	the end result or mixture

Each bucket must contain two values:

- An amount (liters, tons, pounds, ounces, grams, etc.)
- A type (usually either a percent or a price)

Once all of the buckets are "filled" with an amount and a type, an equation may be determined.

Example 1: How many pounds of coffee worth \$1.00 per pound must be mixed with 15 pounds of coffee worth \$1.60 per pound to obtain a blend worth \$1.20 per pound?

Solution: Let x= number of pounds of \$1.00 per pound coffee (this is what we are starting with, so it goes in the first bucket)

Amount	x	+		=	
Type	\$1.00				

Next, enter the values for the coffee that you are adding in (15 pounds of coffee worth \$1.60 per pound).

Amount	x	+	15	=	
Type	\$1.00		\$1.60		

And finally, enter the values for the desired mixture or blend (coffee worth $1.20 per pound).

Amount	x	+	15	=	
Type	$1.00		$1.60		$1.20

Notice that all of the buckets are not "filled." To get the missing value, think of the problem this way: if we started with 3 pounds of the $1.00 blend and added in 15 pounds of the $1.60 blend then we would have a total of 18 pounds. So the missing value is found by adding the first two amounts. Therefore, we have:

Amount	x	+	15	=	x+15
Type	$1.00		$1.60		$1.20

Now that our buckets are filled, we simply multiply the two values in each bucket, making sure that we keep the operation (the plus sign and the equal sign) between each product.

$$(1.00)(x) \quad + \quad (1.60)(15) \quad = \quad (1.20)(x+15)$$

$$\begin{array}{r} x+24=1.2x+18 \\ \underline{-x-18 \qquad -x-18} \\ 6=.2x \end{array}$$

$$\frac{6}{.2}=\frac{.2x}{.2}$$

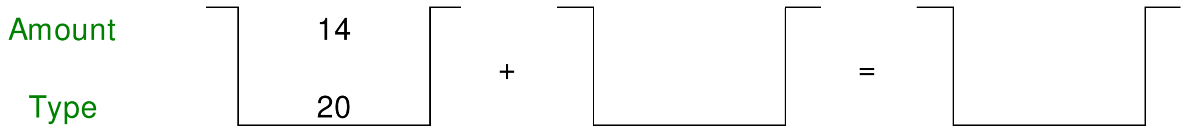
$$x=30$$

Thus, we should start with 30 pounds of the $1.00 blend coffee.■

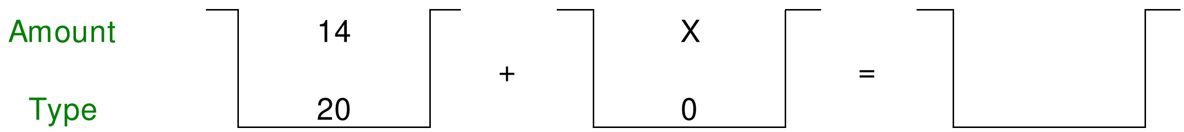
The next example is different from the first for a couple of reasons. First of all, it deals with a different type in our buckets, namely percents instead of prices. The second difference is that, at first glance, the problem does not look like it has enough given information (numbers) to fill the buckets up.

Example 2: How much water must be added to 14 oz of a 20% alcohol solution to obtain a 7% alcohol solution?

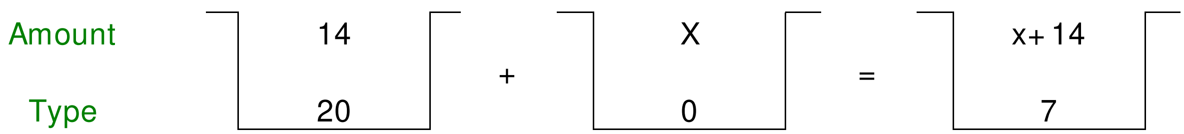
Solution: Let x= the number of ounces of water that we are adding in. Since we are starting off with 14 oz of a 20% alcohol solution, we have:



We are adding in x ounces of pure water. That means that the percentage of alcohol in the water is 0%.



We wish our end result to be 7% alcohol, so:



With our buckets filled we can now get:

$$(14)(20) + (0)(x) = (7)(x+14)$$

$$280 + 0 = 7x + 98$$

$$\begin{array}{rcl} 280 & = & 7x + 98 \\ -98 & & -98 \\ \hline 182 & = & 7x \end{array}$$

$$\frac{182}{7} = \frac{7x}{7}$$

$$x = 26$$

Thus, we should add in 26 oz of pure water.■