UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF
ZAYED

TEACHER EDITION

2018 - 2019

McGraw-Hill Education

Mathematics

8

General Stream

United Arab Emirates Edition

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Teacher Edition

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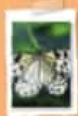
Geometry

Essential Question

HOW can you use different measurements to solve real-life problems?

Chapter 5
Triangles and the Pythagorean Theorem

In this chapter, you will use the Pythagorean Theorem to find side lengths of right triangles and distances on the coordinate plane.

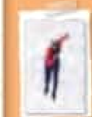


Chapter 6
Transformations

In this chapter, you will describe the effect of translations, reflections, rotations, and dilations on geometric figures.

Chapter 7
Congruence and Similarity

In this chapter, you will describe transformations that produce congruent and similar figures.



Chapter 8
Volume and Surface Area

In this chapter, you will find the volume and surface area of cones, cylinders, and spheres.

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Essential Question

At the end of this unit, students should be able to answer "How can you use different measurements to solve real-life problems?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

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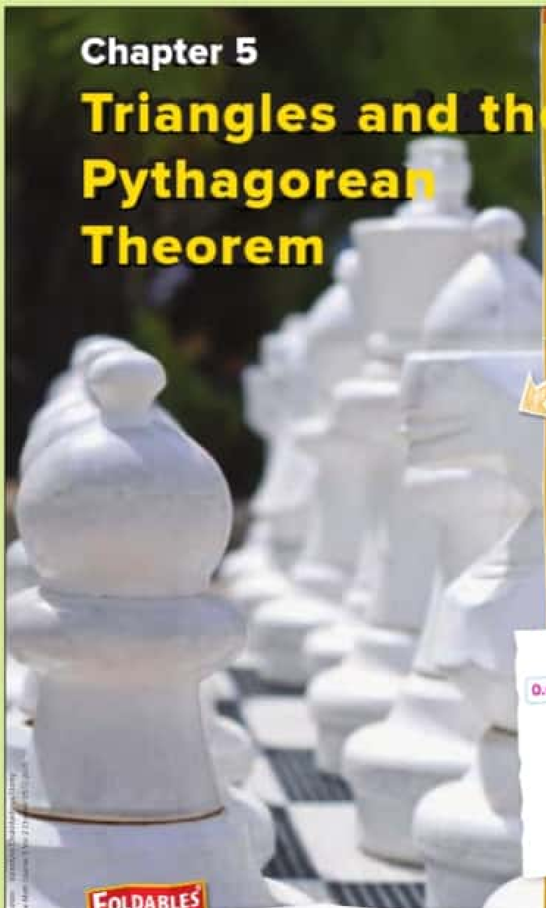
Chapter 5

Triangles and the Pythagorean Theorem



Chapter sourced from Glencoe Math Course 3 Vol 2 Chapter 05 © 2015

Geometry

Essential Question

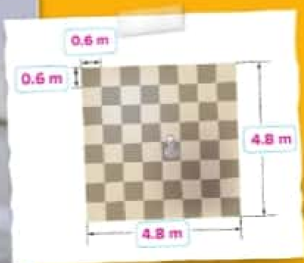
HOW can algebraic concepts be applied to geometry?

Mathematical Practices
1, 2, 3, 4, 5, 7, 8

Math in the Real World

Games At a park in Morro Bay, California, one of the world's largest chess boards is made of concrete and has an area of 23.04 square meters. The chess pieces used weigh between 8 and 14 kilograms each.

Label the dimensions of the chess board and one of the squares.



FOLDABLES Study Organizer

1 Cut out the Foldable from the end of the book.

2 Place your Foldable at the end of the chapter.

3 Use the Foldable throughout this chapter to help you learn about the Pythagorean Theorem.

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Focus narrowing the scope

This chapter focuses on content from the **Geometry** (G) domain.

Coherence connecting within and across grades

Previous	Now	Next
Students solved multi-step equations.	Students use algebraic concepts to find relationships between lines, angles, and triangles.	Students will perform transformations on geometric figures.

Rigor pursuing concepts, fluency, and applications

At the end of the chapter, students should be able to answer "HOW can algebraic concepts be applied to geometry?"

Launch the Chapter

Math in the Real World

Games The chessboard is in the shape of a square. Remind students that to find the length of one side of the square, they need to find the square root of the area of the square.

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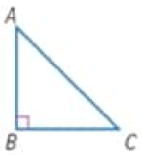
What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: The hypotenuse is the side opposite the right angle in a right triangle.

Example:



Ask:

- *Which side of the above triangle is the hypotenuse?* $\overline{AC}$

The Structure of Math

LA Have students read The Structure of Math section.

Ask:

- *What kinds of questions are asked in the diamonds in a flowchart?* **Sample answer: In a flowchart, the questions only have yes or no answers.**
- *What do the answers to the questions determine?* **Sample answer: They determine the path to follow.**
- *Where should all of the paths lead?* **the end oval**
- *What other uses can you think of for flowcharts?* **Sample answer: A flowchart can be used to clarify any process such as a computer program, the way a company produces goods, how to cook, how to write a paper in English class, and so on.**

What Tools Do You Need?

Vocabulary

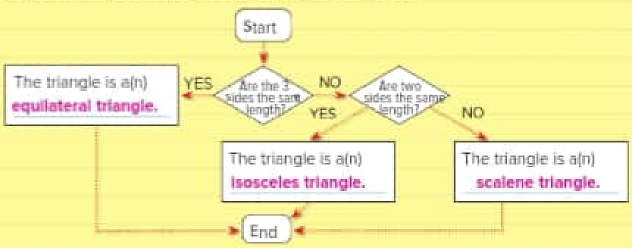
alternate exterior angles	hypotenuse	proof
alternate interior angles	inductive reasoning	Pythagorean Theorem
converse	informal proof	regular polygon
corresponding angles	interior angles	remote interior angles
deductive reasoning	legs	theorem
Distance Formula	paragraph proof	transversal
equiangular	parallel lines	triangle
exterior angles	perpendicular lines	two-column proof
formal proof	polygon	

Study Skill: The Structure of Math

Use a Flowchart A *flowchart* is like a map that tells you how to get from the beginning of a problem to the end.

Flowchart Symbols	
(diamond)	A diamond contains a question. You need to stop and make a decision.
(rectangle)	A rectangle tells you what to do.
(oval)	An oval indicates the beginning or end.

Complete the flowchart to classify triangles by their sides.



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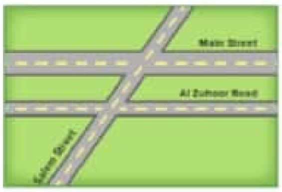
Inquiry Lab
Parallel Lines

Inquiry **WHAT are the angle relationships formed when a third line intersects two parallel lines?**

Mathematical Practices 1, 3, 5

A newspaper route has two parallel streets. The streets are cut by another street as shown in the figure below.



Hands-On Activity

Parallel lines have special angle relationships. You will examine those relationships in this activity.

Step 1 Use a protractor and angle relationships you have previously learned to find the measure of each numbered angle and record it in the table.

Angle	1	2	3	4	5	6	7	8
Measure	115°	65°	65°	115°	115°	65°	65°	115°

Step 2 Color the angles that have the same measure.
See students' work.

Step 3 Describe the position of the angles with the same measure.
Sample answer: Pairs of angles that have the same measurement are vertical angles and vertical angles are congruent.

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Focus narrowing the scope

Objective Examine angle relationships formed when parallel lines are cut by a transversal.

Coherence connecting within and across grades

Now
Students will examine angle relationships formed when parallel lines are cut by a transversal.

Next
Students will use angle relationships to find missing angles when parallel lines are cut by a transversal.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 370.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Materials: protractor

Hands-On Activity

AL BL LA **Roundrobin** Have students work in groups of 3–4 to find the measure of an angle either by use of a protractor or by angle relationships. After each student gives an angle measure, the rest of the group gives a thumbs-up or thumbs-down to denote agreement or disagreement. If there is disagreement, students work together to resolve it.
MP 1, 3, 5

Ask:
- *What do you notice about the position of the angles that have the same measure?* Sample answer: They are in the same location in relation to the top line and the bottom line. They are also opposite each other at the intersection of lines.

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2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–7	Exercises 8–10	Exercises 11, 12
Level 3			●
Level 2		●	
Level 1	●		

Investigate

BL EL **Pairs Present** Have students explain how they can determine all of the angle measures if they are given only one angle measure. Have them prepare a brief oral presentation to share with the class, using illustratio MP 1, 3, 5, 6, 7

Ask:

- *What do the obtuse angles have in common? the acute angles?* The obtuse angles have the same angle measure. The acute angles have the same measure.
- *What rule(s) can you generate to determine all of the missing angle measures when you are only given one angle measure?* Sample answer: Angles are either congruent or supplementary.

Create

Inquiry Students should be able to answer "WHAT are the angle relationships formed when a third line intersects two parallel lines?"

Investigate

MP **Use Math Tools** Work with a partner. If the measure of ∠1 in the figure at the right is 40°, determine the measure of each given angle without using a protractor. Then check your answers by measuring with a protractor.

1. ∠2 140°
2. ∠3 40°
3. ∠4 140°
4. ∠5 40°
5. ∠6 140°
6. ∠7 40°
7. ∠8 140°

Analyze and Reflect

Refer to the figure above.

8. What is the relationship between the two horizontal lines?
Sample answer: The lines appear to be parallel.

9. What is true about the measures of angles that are side by side?
Sample answer: Angles that are side by side are supplementary.

10. MP **Reason Inductively** Congruent angles are angles that have the same measure. Describe the position of the congruent angles.
Sample answer: Pairs of angles that appear to be congruent are in the same positions with respect to both horizontal lines.

Create

11. MP **Make a Conjecture** Draw a set of parallel lines cut by another line. Estimate the measures of the eight angles formed. Check your estimates by measuring each angle with a protractor. See students' work.

12. Inquiry WHAT are the angle relationships formed when a third line intersects two parallel lines?
Sample answer: Eight angles are formed. Some of them add up to 180° and some of them are congruent.

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Lesson 1
Lines

Vocabulary Start-Up

When two lines intersect in a plane and form right angles they are called **perpendicular lines**. Two lines are called **parallel lines** when they are in the same plane and do not intersect.

Complete the graphic organizer. **Sample answers are given.**

	Parallel Lines	Perpendicular Lines
Symbols	∥	⊥
Define it in your own words	two lines that never cross	two lines that cross and form 90° angles
Draw it		
Describe a real-world example of it	railroad tracks	an intersection where two streets cross

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

perpendicular lines
parallel lines
transversal
interior angles
exterior angles
alternate interior angles
alternate exterior angles
corresponding angles

Math Symbols

∥ is parallel to
⊥ is perpendicular to
$m\angle 1$ the measure of ∠1

Mathematical Practices
1, 3, 4

Real-World Link

A gymnastic event in the Summer Olympics involves the parallel bars. The women compete on uneven parallel bars and the men compete on the parallel bars like the one shown. Circle the parallel lines shown in the photo at the right. **See students' work.**

Mathematical Practices did you use?
Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

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Focus narrowing the scope

Objective Identify relationships of angles formed by two parallel lines cut by a transversal.

Coherence connecting within and across grades

Previous
Students examined angle relationships formed when parallel lines are cut by a transversal.

Now
Students will classify the angles formed and find missing angles when parallel lines are cut by a transversal.

Next
Students will explore the relationship among angles of a triangle.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 375.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have pairs complete the graphic organizer. Then have them share and revise their responses, if necessary, with another pair of students. Call on one pair to share their responses with the class. **MP 1, 2, 3, 4, 5, 6**

Alternate Strategy

AL **LA** Have students write *parallel lines* and *perpendicular lines* on the front of two index cards. On the back, have them draw an example and write the definition. Have them use these cards as a reference throughout the lesson. **MP 1, 2, 4, 5, 6**

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Classify angle pairs.

AL • *What does the term interior mean?* **inside**

• *What does the term exterior mean?* **outside**

OL • *Are ∠1 and ∠7 on the same side of the transversal or opposite sides?* **opposite sides**

• *Are the angles on the inside of the parallel lines or the outside?* **outside**

• *What type of angles are ∠1 and ∠7?* **alternate exterior angles**

BL • *What is true about alternate exterior angles?* **They are congruent if the lines are parallel.**

2. Classify angle pairs.

AL • *Look at the positions of ∠2 and ∠6 related to each of the lines. What do you notice?* **Sample answer: The angles are in the same position.**

OL • *Are the angles on the same side of the transversal or opposite sides?* **same side**

• *What type of angles are ∠2 and ∠6?* **corresponding**

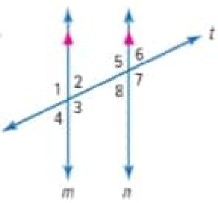
BL • *What is true about corresponding angles?* **They are congruent if the lines are parallel.**

Need Another Example?

Classify each pair of angles as *alternate interior, alternate exterior,* or *corresponding.*

∠3 and ∠7 **corresponding**

∠2 and ∠8 **alternate interior**



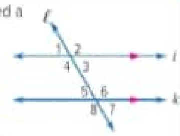
Key Concept

Transversals and Angles

A line that intersects two or more lines is called a **transversal** and eight angles are formed.

Interior angles lie inside the lines.
Examples: ∠3, ∠4, ∠5, ∠6

Exterior angles lie outside the lines.
Examples: ∠1, ∠2, ∠7, ∠8



Alternate interior angles are interior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal. **Examples:** m∠4 = m∠6, m∠3 = m∠5

Alternate exterior angles are exterior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal. **Examples:** m∠1 = m∠7, m∠2 = m∠8

Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal. When the lines are parallel, their measures are equal. **Examples:** m∠1 = m∠5, m∠2 = m∠6, m∠4 = m∠8, m∠3 = m∠7

Work Zone

Angles
Read m∠1 as the measure of angle 1.

Special notation is used to indicate perpendicular and parallel lines.

A red right angle symbol indicates that lines m and n are perpendicular.

Red arrowheads indicate that lines p and q are parallel.

Parallel and Perpendicular Lines
Read m ⊥ n as line m is perpendicular to line n.
Read p ∥ q as line p is parallel to line q.

Examples

Classify each pair of angles in the figure as *alternate interior, alternate exterior,* or *corresponding.*

1. ∠1 and ∠7

∠1 and ∠7 are exterior angles that lie on opposite sides of the transversal. They are alternate exterior angles.

2. ∠2 and ∠6

∠2 and ∠ 6 are in the same position on the two lines. They are corresponding angles.

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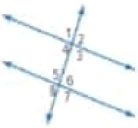
Get It? Do this problem to find out.

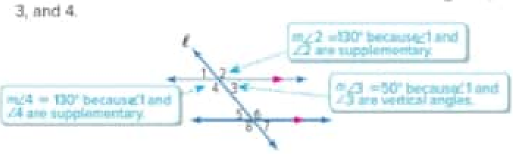
a. Classify the relationship between ∠4 and ∠6. Explain.



a. ∠4 and ∠6 are alternate interior angles because they lie inside the two lines but on opposite sides of the transversal.

Find Missing Angle Measures

When two parallel lines are cut by a transversal, special angle relationships exist. If you know the measure of one of the angles, you can find the measures of all of the angles. Suppose you know that $m\angle 1 = 50°$. You can use that to find the measures of angles 2, 3, and 4.



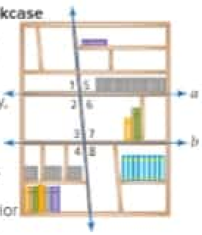
STOP and Reflect

In the figure, how do you know that m∠5 = 50°? Explain below.

Sample answer: The lines are parallel and ∠1 and ∠5 are corresponding angles, so they have equal measures.

Example

3. **A furniture designer built the bookcase shown. Line a is parallel to line b. If $m\angle 2 = 105°$, find $m\angle 6$ and $m\angle 3$. Justify your answer.**



Since ∠2 and ∠6 are supplementary, the sum of their measures is 180.
$m\angle 6 = 180° - 105°$ or 75°

Since ∠6 and ∠3 are interior angles that lie on opposite sides of the transversal, they are alternate interior angles. The measures of alternate interior angles are equal. $m\angle 3 = 75°$

Get It? Do this problem to find out.

b. Refer to the situation above. Find $m\angle 4$. Justify your answwe

b. 105°; Sample answer: ∠2 and ∠4 are corresponding angles, so their measures are equal.

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Example

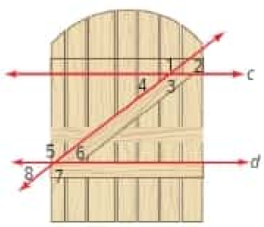
3. Find missing angle measures.

AL
- *How does ∠6 relate to ∠2?* ∠6 and ∠2 are supplementary.
- *If two angles are supplementary, what is the sum of their angle measures?* 180°
- *If I know one of the angle measures is 105°, how can I find the measure of the other angle?* Subtract 105° from 180°.

OL
- *What is the measure of ∠6?* 75°
- *How does ∠3 relate to ∠6?* They are alternate interior angles.
- *What is true about the measures of alternate interior angles?* The angle measures are the same if the lines are parallel.
- *What is the measure of ∠3?* 75°

BL
- *Explain how to find all of the missing angle measures in the diagram.* Sample answer: If $m\angle 2 = 105°$, then $m\angle 4 = 105°$, $m\angle 5 = 105°$, and $m\angle 7 = 105°$. The other four angles, ∠1, ∠6, ∠3, and ∠8 all have a measure of 75° using corresponding angles, vertical angles, alternate interior angles, and supplementary angles.

Need Another Example?
Mr. Mohammad installed the gate shown. Line c is parallel to line d. If $m\angle 4 = 40°$, find $m\angle 6$ and $m\angle 7$. Justify your answer. $m\angle 6 = 40°$ and $m\angle 7 = 140°$; Sample answer: ∠4 and ∠6 are alternate interior angles, so they are congruent. ∠6 and ∠7 are supplementary. Since $m\angle 6 = 40°$, $m\angle 7 = 140°$.



Example

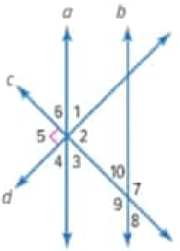
4. **Find missing angle measures.**

AL
- *If line q is perpendicular to line p, at what angle measure do they intersect?* **90°**
- *What kind of angle do ∠8, ∠7, and ∠6 form?* **a straight angle**
- *How many degrees are in a straight angle?* **180°**

OL
- *If $m\angle 1 = 40°$, what is $m\angle 6$? Explain.* **$m\angle 6 = 40°$; ∠1 and ∠6 are alternate exterior angles, so they have the same angle measure.**

BL
- *How can you use the measure of ∠6 to find $m\angle 7$?* **∠6, ∠7, and ∠8 form a straight angle. Since a straight angle measures 180°, I can solve $40 + 90 + m\angle 7 = 180$ to find $m\angle 7$.**

Need Another Example?
In the figure, line *a* is parallel to line *b*, and line *c* is perpendicular to line *d*. The measure of ∠7 is 125°. What is the measure of ∠4? **35°**



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

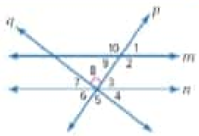
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Whole Group Discussion** Work as a large group to complete Exercises 1–4. Have students highlight in different colors which angles are congruent, which are complementary, and which are supplementary. MP 1, 5

BL LA **Round Table** Students take turns completing the steps for Exercises 2 and 3, checking the previous person's work as they progress through each problem. MP 1, 3

Example

4. **In the figure, line *m* is parallel to line *n*, and line *q* is perpendicular to line *p*. The measure of ∠1 is 40°. What is the measure of ∠7?**



Since ∠1 and ∠6 are alternate exterior angles, $m\angle 6 = 40°$.

Since ∠6, ∠7, and ∠8 form a straight line, the sum of their measures is 180.

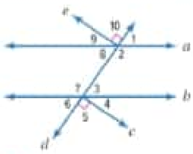
$40 + 90 + m\angle 7 = 180$

So, $m\angle 7$ is 50°.

Guided Practice

1. Refer to the porch stairs shown. Line *m* is parallel to line *n* and $m\angle 7$ is 35°. Find the measure of ∠1. Justify your answer. (Example 3)
145°; Sample answer: ∠7 and ∠5 are supplementary. So, $m\angle 5 = 180° - 35°$ or 145°. ∠5 and ∠1 are corresponding angles. Since corresponding angles are have the same measure, $m\angle 1 = 145°$.

Refer to the figure at the right. Line *a* is parallel to line *b* and $m\angle 2$ is 135. Find each given angle measure. Justify your answer. (Examples 1, 2, and 4)



2. $m\angle 9$ **45°; Sample answer: ∠2 and angles 9 and 10 are vertical angles. So, $m\angle 9 + m\angle 10 = 135°$. So, $m\angle 9 = 135 - 90°$ or 45°.**

3. $m\angle 7$ **135°; Sample answer: ∠2 and ∠7 are alternate interior angles. So, $m\angle 7 = 135°$.**

4. **Building on the Essential Question** How are the measures of the angles related when parallel lines are cut by a transversal?
Sample answer: The angles are either equal or supplementary.

Rate Yourself!
How confident are you about lines and angles? Check the box that applies.

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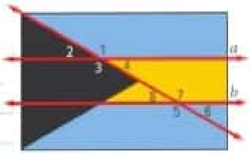
Name______________________ My Homework______________

Independent Practice

Classify each pair of angles as *alternate interior, alternate exterior, or corresponding.* (Examples 1 and 2)

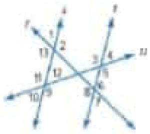
1. $\angle 2$ and $\angle 4$ **corresponding**

2. $\angle 4$ and $\angle 5$ **alternate exterior**



3. In the flag shown at the right, line a is parallel to line b. If $m\angle 1 = 150°$, find $m\angle 4$ and $m\angle 7$. Justify your answers. (Example 3) **$m\angle 4 = 30°$, $m\angle 7 = 150°$; Sample answer: $\angle 1$ and $\angle 7$ are corresponding angles so their measures are equal. $\angle 1$ and $\angle 4$ are supplementary. So, $m\angle 4 = 180° - 150°$ or $30°$.**

Refer to the figure at the right. Line s is parallel to line t. $m\angle 2$ is 110° and $m\angle 11$ is 137°. Find each given angle measure. Justify your answer. (Example 4)

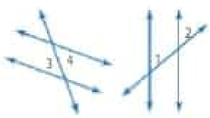


4. $m\angle 7$ **70°; Sample answer: $\angle 2$ and $\angle 6$ are corresponding angles, so they have the same measure. $\angle 6$ and $\angle 7$ are supplementary. So, $m\angle 7 = 180 - 110$ or $70°$.**

5. $m\angle 8$ **110°; Sample answer: $\angle 2$ and $\angle 8$ are alternate interior angles, so they have the same measure.**

6. $m\angle 3$ **137°; Sample answer: $\angle 11$ and $\angle 3$ are corresponding angles, so they have the same measure.**

7. The parallel lines at the right are cut by a transversal. Find the value of x.



a. Angles 1 and 2 are corresponding angles. $m\angle 1 = 45°$, and $m\angle 2 = (x + 25)°$. **20**

b. Angles 3 and 4 are alternate interior angles. $m\angle 3 = 2x°$, and $m\angle 4 = 80°$. **40**

8. Describe a method you could use to find the value of x in the figure at the right without using a protractor. **Sample answer: The two angles are supplementary. So, $x + 2x = 180°$; $x = 60$.**

Uncorrected first proof - for training purposes only

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–6, 13–19	7–9, 20–22	10–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 12, 21, 22
OL	On Level	1–5 odd, 7–9, 12, 21, 22
BL	Beyond Level	7–12, 21, 22

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MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	11, 12
4 Model with mathematics.	9, 20

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

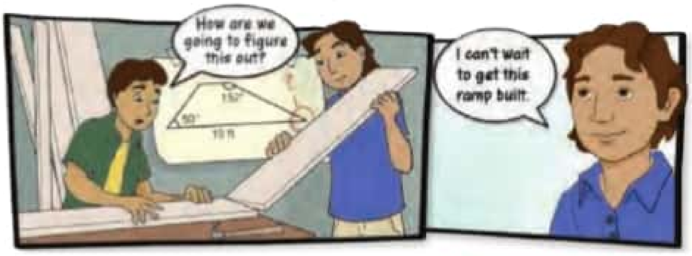
TICKET Out the Door

Ask students to describe the pairs of congruent angles in a set of parallel lines cut by a transversal.
See students' work.

Watch Out!

Common Error Students may make errors finding missing angle measures when there is more than one transversal. Encourage students to begin by locating the given angle measure and finding the angle measures that are adjacent and vertical to that angle. They can then identify the parallel lines and use the relationships between corresponding, alternate interior, and alternate exterior angles to find the remaining angle measures.

Model with Mathematics Refer to the graphic novel frame below for Exercises a–b.

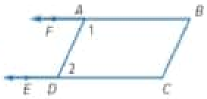


a. Describe a method you could use to find the missing angle. The top and bottom of the ramp are parallel. The slanted part of the ramp can be considered a transversal. You can use angle relationships of parallel lines to find the measure of the missing angle.

b. Use your method from part **a** to find the measure of the missing angle.
28°

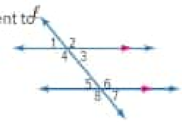
H.O.T. Problems Higher Order Thinking

10. **Persevere with Problems** Quadrilateral *ABCD* is a parallelogram. Make a conjecture about the relationship of $\angle 1$ and $\angle 2$. Justify your reasoning. $\angle 1$ and $\angle 2$ are supplementary. Sample answer: Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel, $m\angle 1 = m\angle ADE$ (alternate interior angles have the same measure). Since $\angle ADE$ and $\angle 2$ lie on the same line, they are supplementary, and $m\angle ADE + m\angle 2 = 180°$. Substitute $\angle 1$ for $\angle ADE$. Therefore, $m\angle 1 + m\angle 2 = 180°$.



11. **Reason Inductively** If two parallel lines are cut by a transversal, what relationship exists between interior angles that are on the same side of the transversal? They are supplementary.

12. **Reason Inductively** Suppose $m\angle 1 = x°$. Use an informal argument to write an expression for the measure of $\angle 6$ in the diagram at the right.
Sample answer: $\angle 1$ and $\angle 2$ are supplementary, so $m\angle 2 = 180° - x°$. $\angle 2$ and $\angle 6$ are corresponding angles. So $m\angle 6 = 180° - x°$.



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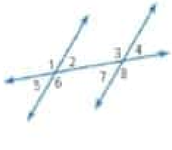
Name________________ My Homework________________

Extra Practice

Classify each pair of angles as *alternate interior, alternate exterior,* or *corresponding.*



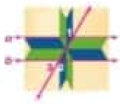
13. $\angle 3$ and $\angle 6$ alternate interior

$\angle 3$ and $\angle 6$ are interior angles that lie on opposite sides of the transversal. They are alternate interior angles

14. $\angle 1$ and $\angle 3$ **corresponding**

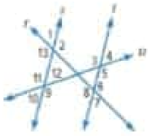
15. $\angle 2$ and $\angle 7$ **alternate interior**

16. In the quilt design at the right, line *a* is parallel to line *b*. If $m\angle 1 = 120°$, find $m\angle 2$ and $m\angle 3$.



Justify your answer. **$m\angle 2 = 120°$, $m\angle 3 = 60°$; Sample answer: $\angle 1$ and $\angle 2$ are alternate exterior angles, so they have the same measure. $\angle 2$ and $\angle 3$ are supplementary. So, $m\angle 3 = 180° - 120°$ or $60°$.**

Refer to the figure at the right. Line *s* is parallel to line *t*, $m\angle 2$ is 110° and $m\angle 11$ is 137°. Find each given angle measure. Justify your answer.



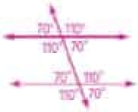
17. $m\angle 6$ **110°; Sample answer: $\angle 2$ and $\angle 6$ are corresponding angles, so they have the same measure.**

18. $m\angle 13$ **110°; Sample answer: $\angle 2$ and $\angle 13$ are vertical angles, so they have the same measure.**

19. $m\angle 4$ **43°; Sample answer: $\angle 11$ and $\angle 3$ are corresponding angles, so they have the same measure and $\angle 3$ and $\angle 4$ are supplementary. So, $m\angle 4 = 180 - 137$ or $43°$.**

20. **Model with Mathematics** Draw a pair of parallel lines cut by a transversal. Estimate the measure of one angle and label it. Without using a protractor, label all the other angles with their approximate measure.

Sample answer:



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Power Up! Test Practice

Exercises 21 and 22 prepare students for more rigorous thinking needed for assessment.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

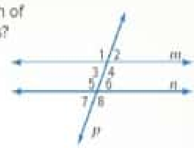
22.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly label all 7 of the angles.
1 point	Students correctly label 5–6 of the 7 angles.

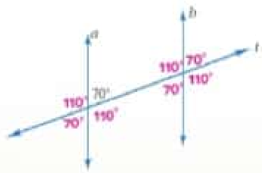
Power Up! Test Practice

21. Lines *m* and *n* are parallel and cut by the transversal *p*. Which of the following pairs of angles represent corresponding angles? Select all that apply.

- [x] ∠2 and ∠6
- [ ] ∠4 and ∠6
- [ ] ∠3 and ∠4
- [x] ∠1 and ∠5



22. Lines *a* and *b* are parallel and cut by the transversal *t*. Label each of the 7 unknown angles with the correct angle measure.

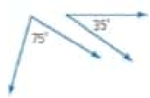


Spiral Review

23. A poster has a triangular image with a base that measure 10 centimeters, and a height that measures 20 centimeters. What is the area of the poster?
100 cm²

Classify each pair of angles as *complementary, supplementary,* or *neither*.

24. neither



25. supplementary

26. complementary

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Lesson 2

Geometric Proof

Real-World Link

Detectives A police detective uses analytical thinking to solve crimes. **Inductive reasoning** is the process of making a conjecture after observing several examples.

Unlike inductive reasoning, **deductive reasoning** uses facts, rules, definitions, or laws to make conjectures from given situations.

Complete the graphic organizer by matching each situation with the type of reasoning used.

Every time Abdalla watches his favorite team on television, the team loses. So, he decides to not watch the team play on TV.

In order to play sports, you need to have a B average. Faris has a B average, so he concludes he can play sports.

All triangles have 3 sides and 3 angles. Hala has a figure with 3 sides and 3 angles, so it must be a triangle.

After performing a science experiment, Ayoub concluded that only 80% of tomato seeds would grow into plants.

Deductive Reasoning

Inductive Reasoning

Ess enti l Q uestio

HOW can algebraic concepts be applied to geometry?

Vocabulary

inductive reasoning
deductive reasoning
proof
paragraph proof
informal proof
two-column proof
formal proof
theorem

Mathematical Practices
1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Write geometric proofs.

Coherence connecting within and across grades

Previous Students used the properties of mathematics to justify the steps to solve an equation.

Now Students use definitions, properties, and theorems to prove a hypothesis.

Next Students will prove the Pythagorean Theorem and its converse.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 383.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas fo r Use

You may w ish to launch the lesson using a whole group, smal group, thi nk-pair-share activity, or independent activity.

AL LA **Pairs Consult** Have pairs write *inductive reasoning* and *deductive reasoning* on two index cards. Have them write the attributes of each kind of reasoning on the back of each card. Have them use their cards to determine which type of reasoning was used for each situation in the graphic organize MP 1, 5, 6

Alternate Strategy

BL Have students provide their own examples of when they have used inductive and deductive reasoning to solve a problem in everyday lif MP 6

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Complete a paragraph proof.

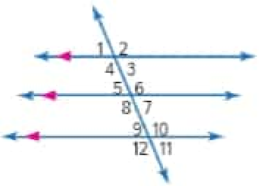
AL
- *What is a paragraph proof?* **a proof that is written in paragraph form**
- *What information do you know that is stated in the problem?* **$m\angle 1 = m\angle 4$**
- *What do you know about the relationship between $\angle 1$ and $\angle 2$ from the diagram?* **They are vertical angles.**
- *What do you know about the relationship between $\angle 3$ and $\angle 4$ from the diagram?* **They are vertical angles.**
- *What is true about the measures of vertical angles?* **They are equal.**

OL
- *If $m\angle 1 = m\angle 4$ and $m\angle 1 = m\angle 2$, what can you say about $m\angle 2$ and $m\angle 4$?* **$m\angle 2 = m\angle 4$**
- *If $m\angle 2 = m\angle 4$ and $m\angle 4 = m\angle 3$, what can you say about $m\angle 2$ and $m\angle 3$?* **$m\angle 2 = m\angle 3$**

BL
- *What definitions, properties, or relationships were used in this proof?* **substitution; vertical angles have equal measures**

Need Another Example?

Refer to the diagram. If $m\angle 1 = m\angle 5$, write a paragraph proof to show that $m\angle 1 = m\angle 11$.



$m\angle 1 = m\angle 9$ because they are corresponding angles. $m\angle 9 = m\angle 11$ because they are vertical angles. Since $m\angle 9 = m\angle 11$, then $m\angle 1 = m\angle 11$ by substitution.

Work Zone

Key Concept: The Proof Process

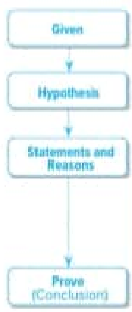
Step 1 List the given information, or what you know. If possible, draw a diagram to illustrate this information.

Step 2 State what is to be proven.

Step 3 Create a deductive argument by forming a logical chain of statements linking the given information to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, and theorems.

Step 5 State what it is you have proven.



A **proof** is a logical argument where each statement is justified by a reason. A **paragraph proof**, also called an **informal proof**, involves writing a paragraph to explain why a conjecture is true. In Example 1 below, you will use the algebraic property of substitution and the geometric relationship between vertical angles.

Proofs
Always end your proof with a statement that describes what you proved.

Example

1. The diamondback rattlesnake has a diamond pattern on its back. An enlargement of the skin is shown. If $m\angle 1 = m\angle 4$, write a paragraph proof to show that $m\angle 2 = m\angle 3$.

Given: $m\angle 1 = m\angle 4$

Prove: $m\angle 2 = m\angle 3$

Proof: $m\angle 1 = m\angle 2$ because they are vertical angles. Since $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 4$ by substitution. $m\angle 4 = m\angle 3$ because they are vertical angles. Since $m\angle 2 = m\angle 4$, then $m\angle 2 = m\angle 3$ also by substitution. Therefore, $m\angle 2 = m\angle 3$.

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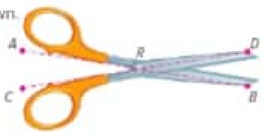
Got It? Do this problem to find out.

a. Refer to the diagram shown. $AR = CR$ and $DR = BR$. Write a paragraph proof to show that $AR + DR = CR + BR$.



Given: $AR =$ **CR** and $DR =$ **BR**

Prove: **$AR + DR$** $= CR + BR$.

Proof: You know that $AR = CR$ and $DR = BR$. $AR + DR = CR + DR$ by the **Addition** Property of Equality. So, $AR + DR = CR + BR$ by **substitution**.

Segment Notation
AR is read as the measure of line segment AR

Two-Column Proofs

A **two-column proof** or **formal proof** contains *statements* and *reasons* organized in two columns. Once a statement or conjecture has been proven, it is called a **theorem** and it can be used as a reason to justify statements in other proofs.

Example

2. **Write a two-column proof to show that if two angles are vertical angles, then they have the same measure.**

Given: lines m and n intersect; $\angle 1$ and $\angle 3$ are vertical angles

Prove: $m\angle 1 = m\angle 3$

	Statements	Reasons
a.	lines m and n intersect; $\angle 1$ and $\angle 3$ are vertical angles.	Given
b.	$\angle 1$ and $\angle 2$ are a linear pair and $\angle 3$ and $\angle 2$ are a linear pair.	Definition of linear pair
c.	$m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 2 = 180°$	Definition of supplementary angles
d.	$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	Substitution
e.	$m\angle 1 = m\angle 3$	Subtraction Property of Equality

Linear Pair
A linear pair of angles is a pair of adjacent angles formed by intersecting lines.

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Example

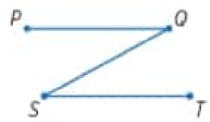
2. Complete a two-column proof.

AL
- *What information are you given?* **lines m and n intersect; $\angle 1$ and $\angle 3$ are vertical angles**
- *What is the relationship between $\angle 1$ and $\angle 2$?* **They are supplementary angles.**
- *What is the relationship between $\angle 2$ and $\angle 3$?* **They are supplementary angles.**
- *If two angles are supplementary, what is the sum of their angle measures?* **180°**

OL
- *If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 3 + m\angle 2 = 180°$, what new equation can be written using substitution?* **$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$**
- *What property allows you to subtract $\angle 2$ from each side?* **Subtraction Property of Equality**

BL
- *Do you prefer using a paragraph proof or a two-column proof? Explain.* **See students' preferences.**

Need Another Example?
Complete the two-column proof to show that if $PQ = QS$ and $QS = ST$, then $PQ = ST$.



Given: $PQ = QS$; $QS = ST$
Prove: $PQ = ST$

	Statements	Reasons
a.	$PQ = QS$ and $QS = ST$	Given
b.	$PQ = ST$	Substitution

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Guided Practice

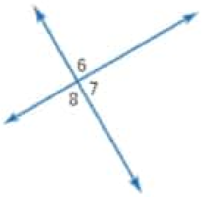
Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Roundrobin** Have students work in teams of three to complete Exercises 1 and 2. The first student should fill in the first blank and explain their answer, then move on to the second student, and so on. Have them trade their solutions with another team of students and discuss any differences. MP 1, 3, 5, 6, 7

BL LA **Pairs Consult** Have students work in pairs. Give them the following information and ask them to create a paragraph proof or a two-column proof. MP 1, 3, 5, 6, 7

In the figure, two lines intersect to form four angles. If ∠6 and ∠8 are supplementary angles, prove that ∠7 is a right angle. Have them trade their proofs with another pair of students and discuss any differences.



Get It? Do this problem to find out.

b. The statements for a two-column proof to show that if $m\angle Y = m\angle Z$, then $x = 100$ are given below. Complete the proof by providing the reasons.

Statements	Reasons
a. $m\angle Y = m\angle Z$, $m\angle Y = 2x - 90$, $m\angle Z = x + 10$	Given
b. $2x - 90 = x + 10$	Substitution
c. $x - 90 = 10$	Subtraction Property of Equality
d. $x = 100$	Addition Property of Equality

Guided Practice

Check

1. Use the figure to complete the paragraph proof. (Example 1)
Given: $m\angle 1 = m\angle 2$, ∠1 and ∠2 are supplementary.
Prove: ∠1 and ∠2 are right angles.

Proof: $m\angle 1 + m\angle 2 =$ **180°** since they are supplementary angles. Since $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 1 = 180°$ by **substitution**. Solving the equation gives $m\angle 1 =$ **90°**. Since $m\angle 1 = m\angle 2$, then $m\angle 2$ is also **90°**. Therefore ∠1 and ∠2 are right angles.

2. Refer to the figure above. Complete the two-column proof to show that if $EG = 3x - 1$, $ED = 2x + 4$, and $EG = ED$, then $x = 5$. (Example 2)

Statements	Reasons
a. $EG = 3x - 1$, $ED = 2x + 4$, $EG = ED$	Given
b. $3x - 1 = 2x + 4$	Substitution
c. $x - 1 = 4$	Subtraction Property of Equality
d. $x = 5$	Addition Property of Equality

3. **Building on the Essential Question** How is deductive reasoning used in algebra and geometry proofs?
Sample answer: You use facts, definitions, and properties in proofs.

Rate Yourself!
Are you ready to move on? Shade the section that applies.

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Name ______ My Homework ______

Independent Practice

1. In the figure at the right, two lines intersect to form four angles. If $m\angle 7 = 9x$ and $m\angle 8 = 11x$, complete the paragraph proof to show that $x = 9$. (Example 1)

Given: Two intersecting lines with $m\angle 7 = 9x$ and $m\angle 8 = 11x$

Prove: $x = 9$

Proof: $\angle 7$ and $\angle 8$ form a **straight** angle so they are **supplementary** angles. So, $m\angle 7 + m\angle 8 =$ **180°**, by the definition of supplementary angles. By substitution, **9x** $+ 11x = 180$. So, $x =$ **9** by the Division Property of Equality.

2. **Construct an Argument** Four towns lie on a straight road. Town B is midway between Town A and Town C. Town C is midway between Town B and Town D. Write a paragraph proof to show the distance from Town A to Town B is the same as the distance from Town C to Town D. (Example 1)

Given: B is the midpoint $\overline{AC}$ and C is the midpoint $\overline{BD}$.

Prove: $AB = CD$.

Proof: By the definition of midpoint, **AB** $= BC$ and **BC** $= CD$.

Therefore, $AB = CD$ by **substitution**.

3. **Construct an Argument** Complete the two-column proof to show that if $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 = m\angle 2$, then $\angle 1$ and $\angle 2$ are right angles. (Example 2)

Given: $\angle 1$ and $\angle 2$ are supplementary; $m\angle 1 = m\angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles

	Statements	Reasons
a.	$\angle 1$ and $\angle 2$ are supplementary; $m\angle 1 = m\angle 2$	**Given**
b.	$m\angle 1 + m\angle 2 = 180°$	**Definition of supplementary angles**
c.	$m\angle 1 + m\angle 1 = 180°$	**Substitution**
d.	$2(m\angle 1) = 180°$	**Simplify**
e.	$m\angle 1 = 90°$	**Division Property of Equality**
f.	$m\angle 2 = 90°$	$m\angle 1 = m\angle 2$ (Given)
g.	$\angle 1$ and $\angle 2$ are right angles.	**Definition of right angles**

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3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–3, 8–10	4, 11, 12	5–7
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 7, 11, 12
OL	On Level	1, 3–5, 7, 11, 12
BL	Beyond Level	4–7, 11, 12

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MATHEMATICAL PRACTICES

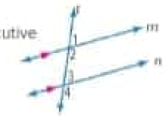
Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	6
2 Reason abstractly and quantitatively.	5
3 Construct viable arguments and critique the reasoning of others.	2–4, 7–10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door
Have students describe the differences between a paragraph proof and a two-column proof. See students' work.

4. **Construct an Argument** Complete the two-column proof to show that when two parallel lines are cut by a transversal, consecutive interior angles are supplementary.



Given: parallel lines m and n cut by transversal t
Prove: $\angle 2$ and $\angle 3$ are supplementary.

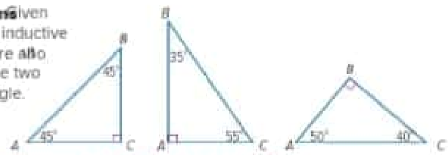
	Statements	Reasons
a.	Lines m and n are parallel and cut by transversal t	Given
b.	$\angle 1$ and $\angle 2$ form a straight angle.	Definition of straight angle
c.	$m\angle 1 + m\angle 2 = 180$	Definition of supplementary angles
d.	$m\angle 1 = m\angle 3$	Corresponding $\angle$s have equal measures
e.	$m\angle 3 + m\angle 2 = 180$	Substitution
f.	$\angle 2$ and $\angle 3$ are supplementary angles	Definition of supplementary angles

H.O.T. Problems Higher Order Thinking

5. **Reason Abstractly** Describe the theorem or definition you could use to find the measure of $\angle 2$.

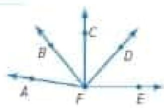
Sample answer: Vertical angles have the same measure.

6. **Persevere with Problems** Given the right triangles shown, use inductive reasoning to make a conjecture about the sum of the measures of the two acute angles of any right triangle.



Sample answer: The sum of the measures of the acute angles of a right triangle is 90°. So, the acute angles are complementary.

7. **Reason Inductively** In the diagram, $m\angle CFE = 90°$ and $m\angle AFB = m\angle CFD$. Which of the following conclusions does *not* have to be true? II



I $m\angle AFB + m\angle DFE = 90°$
II $\overline{BF}$ divides $\angle AFD$ in half
III $m\angle CFD = m\angle AFB$
IV $\angle CFE$ is a right angle.

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Name________________ My Homework________________

Extra Practice

8. **Construct an Argument** In the figure at the right, $\overline{AE} \cong \overline{DB}$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$. Complete the proof to show that $\overline{AC} \cong \overline{CB}$.

Given: $AE = DB$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$.

Prove: $AC = CB$

Proof: Since C is the midpoint of **$\overline{AE}$** and **$\overline{DB}$**, $AC = CE = \frac{1}{2}$ **AE** and $DC = CB = \frac{1}{2}$ **DB** by the definition of midpoint. We are given $AE = DB$. By the **Multiplication** Property of Equality, $\frac{1}{2}AE = \frac{1}{2}DB$. So, by **substitution**, $AC = CB$.

9. **Construct an Argument** Refer to the figure at the right. Complete the two-column proof to show if $m\angle 3 = 2x - 15$ and $m\angle 6 = x + 55$, then $x = 70$.

Given: $j \parallel k$, transversal ℓ; $m\angle 3 = 2x - 15$, $m\angle 6 = x + 55$

Prove: $x = 70$

Statements	Reasons
a. $j \parallel k$, transversal ℓ; $m\angle 3 = 2x - 15$, $m\angle 6 = x + 55$	**Given**
b. $m\angle 3 = m\angle 6$	**Alternate interior angles have the same measure.**
c. $2x - 15 = x + 55$	**Substitution**
d. $x - 15 = 55$	**Subtraction Property of Equality**
e. $x = 70$	**Addition Property of Equality**

10. **Construct an Argument** Refer to the figure at the right. Complete the two-column proof to show if $\angle ABE$ and $\angle DBC$ are right angles, then $m\angle ABD = m\angle EBC$.

Given: $\angle ABE$ and $\angle DBC$ are right angles.

Prove: $m\angle ABD = m\angle EBC$

Statements	Reasons
a. $\angle ABE$ and $\angle DBC$ are right angles.	**Given**
b. $m\angle ABE = 90$ and $m\angle DBC = 90$	**Definition of right angles**
c. $m\angle ABD + m\angle DBE = 90$ $m\angle DBE + m\angle EBC = 90$	**Angle addition**
d. $m\angle ABD + m\angle DBE = m\angle DBE + m\angle EBC$	**Substitution**
e. $m\angle ABD = m\angle EBC$	**Subtraction Property of Equality**

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Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for assessment.

11.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

12.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly identify all 4 steps in the proof.
1 point	Students correctly identify 3 of the 4 steps in the proof.

Power Up! Test Practice

11. In the diagram shown, $\overleftrightarrow{AE}$ intersects $\overleftrightarrow{DB}$ at C.

Determine if each of the following conclusions will always be true. Select yes or no.

a. $m\angle ACD = m\angle BCE$ ■ Yes ☐ No

b. $\angle ACD$ and $\angle ECD$ form a linear pair. ■ Yes ☐ No

c. $\angle DCE$ and $\angle ACB$ are vertical angles. ■ Yes ☐ No

d. $\angle ACB$ and $\angle BCE$ are complementary angles. ☐ Yes ■ No

12. Select the appropriate reason for each statement of the geometric proof below.

Substitution	Division Property of Equality	Vertical angles have equal measures.
Given	Alternate interior angles have equal measures.	Corresponding angles have equal measures.

Given: two parallel lines cut by a transversal, $m\angle 1 = 2x$, $m\angle 3 = 94$

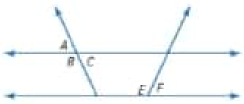
Prove: $x = 47$

Proof:

Statements	Reasons
a. $m\angle 1 = 2x$, $m\angle 3 = 94$	Given
b. $m\angle 1 = m\angle 3$	Corresponding angles have equal measures.
c. $2x = 94$	Substitution
d. $x = 47$	Division Property of Equality

Spiral Review

Refer to the diagram. Identify each pair of angles as *adjacent*, *vertical*, or *neither*.



13. $\angle A$ and $\angle B$ adjacent

14. $\angle A$ and $\angle C$ vertical

15. $\angle C$ and $\angle E$ neither

16. $\angle E$ and $\angle F$ adjacent

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Inquiry Lab
Triangles

WHAT is the relationship among the measures of the angles of a triangle?

Mathematical Practices 1, 3

Fahd has a metal bracket that is in the shape of an angle that attaches a bag to the frame of a bike. The angle of the bracket measures 35°. Fahd wonders if it will fit into the frame of the bike by the handlebars.

Hands-On Activity

Triangle means *three angles*. In this Activity you will explore how the three angles of a triangle are related.

Step 1 On a separate piece of paper, draw a triangle like the one shown below.

Step 2 Label the corners 1, 2, and 3. Then tear off each corner.

Step 3 Rearrange the torn pieces so that the corners all meet at one point. Label the torn pieces with 1, 2, and 3.

What does each torn corner represent?
an angle of the triangle

The point where these corners meet is the vertex of another angle. Classify this angle as *acute*, *right*, *obtuse*, or *straight*.
Explain. Straight; the angle forms a straight line.

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Focus narrowing the scope

Objective Explore the relationship among the angles of a triangle.

Coherence connecting within and across grades

Now
Students explore the relationship among the angles of a triangle.

Next
Students will find missing angle measures in triangles.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 388.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Think-Pair-Share** Have students work in pairs to complete the Activity. Give students a few minutes to individually read through each step in the activity and think about how they would complete each step. Students then gather in pairs to discuss their responses and progress through completing each step in the activity. Students can then present their responses to the questions in Step 3 to the class. MP 1, 3, 4, 6, 7

BL LA **Pairs Consult** Have students work in pairs to alter the activity so that an obtuse or right triangle is used instead of an equilateral triangle. Have them note whether the type of triangle affects the angle that the three corners form. MP 1, 3, 4, 6, 7, 8

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2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1, 2	Exercises 3, 4	Exercises 5, 6
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA **Rally Coach** Have students work in pairs to complete Exercises 1 and 2. The pair has one piece of paper and pencil. Student 1 completes the first exercise while Student 2 watches and listens, coaches and praises. Students switch roles for the second exercise. MP 1, 3, 4, 6, 8

Analyze and Reflect

BL LA **Pairs Check** Have students work in pairs to complete Exercises 3 and 4 and trade their answers with another pair of students to check their work. MP 1, 3, 5, 6, 7, 8

Create

Inquiry Students should be able to answer "WHAT is the relationship among the measures of the angles of a triangle?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Repeat Steps 1–3 of the Activity on the previous page for each of the following triangles. Draw or tape your results in the space provided.

See students' work.

1.

2.

Analyze and Reflect

3. MP **Reason Inductively** What is the sum of the measures of the angles for each of your triangles? 180°

Verify your conjecture below by measuring each angle using a protractor.

Exercise 1: $m\angle 1 + m\angle 2 + m\angle 3 =$ $65° + 90° + 25° = 180°$

Exercise 2: $m\angle 1 + m\angle 2 + m\angle 3 =$ $140° + 15° + 25° = 180°$

4. MP **Justify Conclusions** Refer to the bicycle problem on the previous page. Will the bracket fit exactly into Fahd's bike? Explain. No; $87° + 55° + 35° = 177°$, so the bracket will be too small.

Create

5. MP **Use Math Tools** Find a real-world example of a triangle. Measure the angles of the triangle. What is the sum of the measures of the angles? Does your answer support your findings in this Inquiry Lab? Explain. See students' work. Answers should be close to 180, but likely not exact. Reasons may include the triangle was too large to measure accurately or the lines weren't exactly straight.

6. Inquiry WHAT is the relationship among the measures of the angles of a triangle? The sum of the measures of a triangle is 180°.

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Geom

Geometry

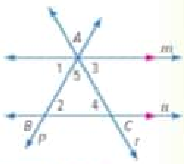
Lesson 3

Angles of Triangles

Real-World Link

STEM Eiman and Asma are building a bridge out of toothpicks for a science competition. Asma thinks the sides should be constructed using triangles. Use the activity to find the sum of the measures of the angles in a triangle.

Lines m and n are parallel. Lines p and r are transversals that intersect at point A.



1. What is true about the measures of $\angle 1$ and $\angle 2$? Explain.
   They are equal because they are alternate interior angles.

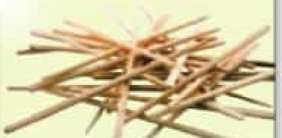
2. What is true about the measures $\angle 3$ and $\angle 4$? Explain.
   They are equal because they are alternate interior angles.

3. What kind of angle is formed by $\angle 1$, $\angle 5$, and $\angle 3$? Write an equation representing the relationship between the 3 angles.
   straight angle; $m\angle 1 + m\angle 5 + m\angle 3 = 180$

4. Use the information from Exercises 1, 2, and 3 to draw a conclusion about the sum of the measures of the angles of $\triangle ABC$. Explain your reasoning. **The sum of the measures of the angles in $\triangle ABC$ is 180°. Since $m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$, and $m\angle 1 + m\angle 5 + m\angle 3 = 180°$, by substitution, $m\angle 2 + m\angle 5 + m\angle 4 = 180°$.**

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning



Ess ential Q uestion

HOW can algebraic concepts be applied to geometry?

Vocabulary

triangle
interior angle
exterior angle
remote interior angles

Mathematical Practices
1, 2, 3, 4

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Focus narrowing the scope

Objective Find missing angle measures in triangles.

Coherence connecting within and across grades

Previous Students explored the relationship of angle measures in triangles.

Now Students will find missing angle measures in triangles.

Next Students will examine the relationship among angles of regular polygons.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 393.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Circle the Sage** Poll students to see who has a solid understanding of angles formed when parallel lines are cut by a transversal. Have those students (the sages) spread out around the room. Place remaining students in teams. Have students report to different sages, with no two team members going to the same sage, if possible. Have the sages lead the work for the Real-World Link. Then have students report back to their teams and discuss any differences in solution. MP 1, 2, 3, 4, 5, 6, 7

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

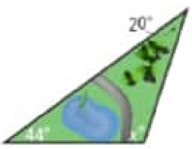
1. Find missing angle measures in triangles.

AL
- *What is the sum of the measures of the interior angles of a triangle?* **180°**
- *What are the known angle measures?* **55° and 90°**

OL
- *What equation could be used to determine the missing angle measure?* **$x° + 55° + 90° = 180°$**
- *What is the value of x?* **35**

BL
- *If the measure of one known angle in a triangle is 90°, what is the sum of the other two angle measures?* **90°**
- *What other equation could you use to solve for x in this example?* **$55° + x° = 90°$**

Need Another Example?
The city park is in the shape of a triangle. Find the value of *x*. **116**



2. Use ratios to find angle measures.

AL
- *What expression could be used to represent the measure of the first angle?* **x** *the second angle?* **4x** *the third angle?* **5x**

OL
- *What equation could be used to find the value of x?* **$x + 4x + 5x = 180$**
- *Why can we write $x + 4x + 5x$ as $10x$?* **They are like terms.**

BL
- *How can we check our work?* **Sample answer: Check that the measures 18°, 72°, and 90° are in the ratio 1:4:5.**

Need Another Example?
The measures of the angles of triangle *DEF* are in the ratio 1:2:3. What are the measures of the angles? **30°, 60°, and 90°**

Key Concept

Angle Sum of a Triangle

Words The sum of the measures of the interior angles of a triangle is 180°.

Model

Symbols $x + y + z = 180°$

A **triangle** is formed by three line segments that intersect only at their endpoints. A point where the segments intersect is a vertex. The angle formed by the segments that lies inside the triangle is an **interior angle**.

Work Zone

Example

1. Find the value of *x* in the Antigua and Barbuda flag.

$x + 55 + 90 = 180$		Write the equation.
$x + 145 = 180$		Simplify.
$-145 = -145$		Subtract.
$x = 35$		Simplify.

The value of *x* is 35.

Got It? Do this problem to find out.

a. In $\triangle XYZ$, if $m\angle X = 72°$ and $m\angle Y = 74°$, what is $m\angle Z$?

a. **34°**

Example

2. The measures of the angles of $\triangle ABC$ are in the ratio 1:4:5. What are the measures of the angles?

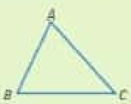
Let *x* represent the measure of angle *A*.

Then $4x$ and $5x$ represent angle *B* and angle *C*.

$x + 4x + 5x = 180$	Write the equation.
$10x = 180$	Collect like terms.
$x = 18$	Division Property of Equality

Since $x = 18$, $4x = 4(18)$ or 72, and $5x = 5(18)$ or 90. The measures of the angles are 18°, 72°, and 90°.

Segments
$\overline{AB}$ is read as segment AB. So the sides of the triangle below are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.



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Geometry

Got It? Do this problem to find out.

b. The measures of the angles of $\triangle LMN$ are in the ratio 2:4:6. What are the measures of the angles?

b. 30°, 60°, 90°

Exterior Angles of a Triangle

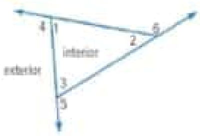
Key Concept

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Model

Symbols $m\angle A + m\angle B = m\angle 1$

In addition to its three interior angles, a triangle can have **exterior angle** formed by one side of the triangle and the extension of the adjacent side. Each exterior angle of the triangle has **remote interior angles** that are not adjacent to the exterior angle.



$\angle 4$ is an exterior angle of the triangle. Its two remote interior angles are $\angle 2$ and $\angle 3$.

$m\angle 4 = m\angle 2 + m\angle 3$

STOP and Reflect

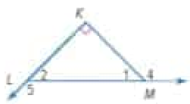
Measure ∠2, ∠3, and ∠4 to verify that $m\angle 2 + m\angle 3 = m\angle 4$. Repeat the process for exterior angles 5 and 6. What is true about $m\angle 5$ and $m\angle 6$?

$m\angle 5 = m\angle 1 + m\angle 2$

$m\angle 6 = m\angle 1 + m\angle 3$

Example

3. Suppose $m\angle 4 = 135°$. Find the measure of $\angle 2$.



Angle 4 is an exterior angle. Its two remote interior angles are $\angle 2$ and $\angle LKM$.

$m\angle 2 + m\angle LKM = m\angle 4$ Write the equation.

$x + 90° = 135°$ $m\angle 2 = x°$, $m\angle LKM = 90°$, $m\angle 4 = 135°$

$x = 45°$ Subtraction Property of Equality

So, the $m\angle 2 = 45°$.

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Example

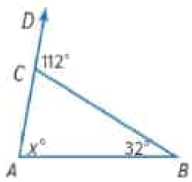
3. Use exterior angles to find a missing angle.

AL
- *What is the sum of the interior angle measures of a triangle?* 180°
- *What kind of angle is angle 4?* exterior
- *What are the two remote interior angles for angle 4?* $\angle 2$ and $\angle LKM$
- *What is the measure of $\angle LKM$?* 90°

OL
- *What is true about an exterior angle and its two remote interior angles?* The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.
- *What equation could be used to find the measure of $\angle 2$?* Sample answer: $m\angle 2 + 90 = 135$

BL
- *Explain another way to find the measure of $\angle 2$.* Sample answer: $\angle 4$ and $\angle 1$ form a straight line, which means that the sum of their angle measures is 180°. 135° + $m\angle 1 = 180°$, so $m\angle 1 = 45°$. The total degrees in a triangle are 180°. 45° + 90° + $m\angle 2 = 180°$, so $m\angle 2 = 45°$.

Need Another Example?
Find the value of x in the triangle. 80



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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students complete Exercise 1 in a team of four students, then complete Exercises 2 and 3 in pairs. Have them complete Exercises 4 and 5 on their own and then compare answers with their original team. MP 1, 3, 5, 6, 7

BL LA **Pairs Research** Have students research or look for a real-world example of a triangle formed by transversals crossing parallel lines (a map, for example). Have them explain how known angle measures can help to find unknown angle measures in the triangle. MP 1, 2, 3, 4, 5, 6, 7

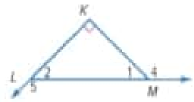
Watch Out!

Common Error When working with angle ratios, students may solve for x and leave out calculating the measures of the three angles. Remind students to multiply x by each of the coefficients in the original equation to find the measures of each of the three angles. Then have them find the sum of the angles to check their work.

c. 57°

Got it? Do this problem to find out.

c. Refer to the figure at the right. Suppose $m\angle 5 = 147°$. Find $m\angle 1$.



Guided Practice

1. Find the value of x in the triangle. (Example 1) 45

2. What is the value of x in the sail of the sailboat? (Example 1) 90

3. The measures of the angles of $\triangle LMN$ are in the ratio 1:2:5. What are the measures of the angles? (Example 2)
22.5°, 45°, 112.5°

4. Find the value of x in the triangle. (Example 3) 31

5. **Building on the Essential Question** How can you find the missing measure of an angle in a triangle if you know the measure of two of the interior angles?
Sample answer: If you know the measure of two of the interior angles, you can subtract the sum of those angles' measures from 180 to find the missing measure.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

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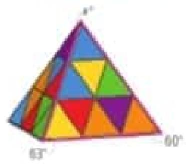
Name_______________ My Homework_______________

Independent Practice

1. The top of a kite is shown below. What is the value of x? (Example 1) **55**

2. A popular toy puzzle is shown below. What is the value of x? (Example 1) **57**



3. The measures of the angles $\triangle RST$ are in the ratio 2:4:9. What are the measures of the angles? (Example 2) **24°, 48°, 108°**

4. The measures of the angles $\triangle XYZ$ are in the ratio 3:3:6. What are the measures of the angles? (Example 2) **45°, 45°, 90°**

Find the value of x in each triangle. (Example 3)

5. **112**

6. **62**

7. **45**

8. In $\triangle ABC$ the measure of angle A is $2x$ − 3, the measure of angle B is $4x + 2$, and the measure of angle C is $2x$. What are the measures of the angles? **$m\angle A = 47°$, $m\angle B = 90°$, $m\angle C = 43°$**

9. **Reason Abstractly** What is the measure of the third angle of a triangle if one angle measures 25° and the second angle measures 50°? **105°**

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3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 16–26	8–12, 27–29	13–15
Level 3			•
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 14, 15, 28, 29
OL	On Level	1–7 odd, 8–12, 14, 15, 28, 29
BL	Beyond Level	8–15, 28, 29

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
2 Reason abstractly and quantitatively.	9
3 Construct viable arguments and critique the reasoning of others.	14, 15, 27

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Draw a triangle on the board and label the measures of two of the three angles. Have students describe the steps they would take to find the missing angle measure.
See students' work.

Find the measures of the angles in each triangle.

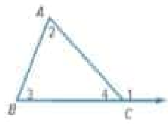
10. 120°, 30°, 30°

11. 90°, 60°, 30°

12. 53°, 55°, 72°

H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** Use the figure at the right to informally prove that an exterior angle of a triangle is equal to the sum of its two remote interior angles.



Given: $\triangle ABC$; $\angle 1$ is an exterior angle.

Prove: $m\angle 1 = m\angle 2 + m\angle 3$

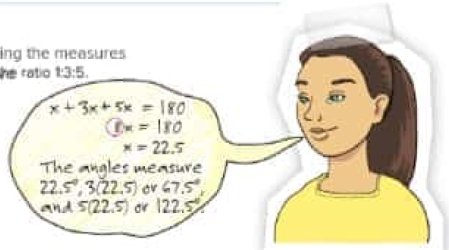
Proof: Sample answer: Since $\angle 1$ and $\angle 4$ form a straight angle, $m\angle 1 + m\angle 4 = 180°$. By the Subtraction Property of Equality, $\angle 1$= $180 - m\angle 4$. Since ABC is a triangle, $m\angle 2 + m\angle 3 + m\angle 4 = 180$. By the Subtraction Property of Equality, $m\angle 2 + m\angle 3 = 180 - m\angle 4$. So by substitution, $m\angle 2 + m\angle 3 = m\angle 1$.

14. **Find the Error** Nisreen is finding the measures of the angles in a triangle that hathe ratio 1:3:5. Circle her mistake and correct it.

$9x = 180$

$x = 20$

The angles measure 20°, 60°, and 100°.



15. **Justify Conclusions** Make a conjecture about the sum of the interior angles of a quadrilateral. Justify your reasoning.
Sample answer: The sum is 360°. Drawing the diagonal of a quadrilateral forms two triangles. So, the sum of the interior angles is 2(180°), or 360°.

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Name ______________________ My Homework ______________________

Extra Practice

Find the value of *x* in each triangle with the given angle measures.

16.

$x + 75 + 75 = 180$

$x + 150 = 180$

$x = 30$

30

17.

100

18.

65

19. 70°, 60°, $x°$ 50

20. $x°$, 60°, 25° 95

21. $x°$, 35°, 25° 120

22. The measures of the angles $\triangle DEF$ are in the ratio 2:4:4. What are the measures of the angles?

36°, 72°, 72°

23. The measures of the angles $\triangle XYZ$ are in the ratio 4:5:6. What are the measures of the angles?

48°, 60°, 72°

Copy and Solve **Find the value of *x* in each triangle. Show your work on a separate sheet of paper.**

24.

75

25.

70

26.

125

27. **Reason Inductively** Apply what you know about angles and lines to find the values of *x* and *y* in the figure at the right.

$x =$ 25 $y =$ 50

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Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

28.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly place all values to complete the model and find the value of x.
1 point	Students correctly place all values to complete the model OR find the value of x.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

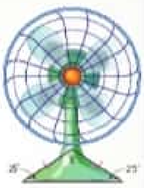
Power Up! Test Practice

28. When viewed from the front, the base of an upright fan has a triangular face with the angle measures shown. Select the correct values to complete the model that could be used to find the value of x.

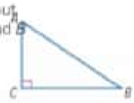
x + 2 · 25 = 180

What is the value of x? 130

x	65
2	90
25	180



29. Which of the following statements are always true about the relationship between the measures of angles A and B of the right triangle shown? Select all that apply.



- ☐ The y are equivalent.
- ☐ The y are supplementary.
- ■ The y are acute.
- ■ They are complementary.

Spiral Review

30. The street maintenance vehicles for the city of Badr cannot safely make turns less than 70°. Should the proposed site of the new maintenance garage at the northeast corner of Park and Main be approved? Explain.

Yes; the two corners at the intersection have measures of 108° and 72°. Therefore it is within the safety limit.

31. $\angle A$ and $\angle B$ are complementary, and the measure of $\angle A$ is 39°. What is the measure of $\angle B$? 51°

Solve each equation.

32. $x + 72 + 63 + 120 = 360$ **105**

33. $90 + 90 + (2x + 4) + (3x - 29) = 360$ **41**

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Lesson 4

Polygons and Angles

Vocabulary Start-Up

A **polygon** is a simple closed figure formed by three or more line segments. The segments intersect only at their endpoints.

A map of the United States is shown. List the states that are in the shape of a polygon. Then list the number of segments that form the polygon.
Sample answers are given. Some students may interpret some state borders as straight segments.

State	Number of Segments
New Mexico	8
Utah	6
Colorado	4
North Dakota	4
Wyoming	4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

polygon
equiangular
regular polygon

Mathematical Practices
1, 3, 4

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Focus narrowing the scope

Objective Find the sum of the angle measures of a polygon and the measure of one interior angle of a regular polygon.

Coherence connecting within and across grades

Previous Students used properties of triangles to find missing angle measures.

Now Students find the measures of angles in polygons.

Next Students use angle measures to show congruence and similarity of figures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 401.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Team Consult** Organize students into three teams. Assign each team a two-dimensional figure to find in a picture: 5-sided, 6-sided, or 8-sided. You can provide the picture or they can research a picture using the Internet. Tell them that the figures should not have any curved sides. Have them present their pictures with the figures outlined to the class. MP 7

Alternate Strategy

BL Ask students to give a few examples of states that are not polygons. Then have them justify their responses. MP 3, 6

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the sum of the interior angle measures of a polygon.

AL
- *How many years are in a decade?* 10
- *Decagon contains the same root word as decade. How many sides are in a decagon?* 10
- *Draw a decagon, then draw all of the diagonals from one vertex. How many triangles did you create?* 8
- *What is the sum of the angle measures for each triangle?* 180°
- *What is the sum of the interior angle measures of eight triangles? Explain how you found the sum.* 1,440°; Multiply 180° by 8.
- *What is the sum of the angle measures of a decagon?* 1,440°

OL
- *What equation can be used to find the sum of the interior angles measures of a polygon with n sides?* $S = (n - 2)180$
- *What equation can be used to find the sum of the interior angle measures of a decagon?* $S = (10 - 2)180$

BL
- *In the equation to find the sum of the measures of the interior angles of a polygon, what does n − 2 represent?* the number of triangles you get when you draw all of the diagonals from one vertex
- *Why does the equation use n − 2 instead of just n?* The number of triangles that a polygon can be separated into is not equal to the number of sides, n. It is equal to 2 less than the number of sides.

Need Another Example?
Find the sum of the measures of the interior angles of a 13-gon. 1,980°

Key Concept

Interior Angle Sum of a Polygon

Words The sum of the measures of the interior angles of a polygon is $(n - 2)180$, where n represents the number of sides.

Symbols $S = (n - 2)180$

Work Zone

You can use the sum of the angle measures of a triangle to find the sum of the interior angle measures of various polygons. A polygon that is equilateral (all sides are the same length) **equiangular** (all angles have the same measure) is called **regular polygon**.

Number of Sides	Sketch of Figure	Number of Triangles	Sum of Angle Measures
3		1	$1(180°) = 180°$
4		2	$2(180°) = 360°$
5		3	$3(180°) = 540°$
6		4	$4(180°) = 720°$

Everyday Use
Deca: a prefix meaning ten, as in decade
Math Use
Decagon: a polygon with ten sides

Example

1. Find the sum of the measures of the interior angles of a decagon.

$S = (n - 2)\ 180$ Write an equation.

$S = (10 - 2)\ 180$ A decagon has 10 sides. Replace n with 10.

$S = (8)180$ or 1,440 Simplify.

The sum of the measures of the interior angles of a decagon is 1,440°.

Got It? Do these problems to find out.

Find the sum of the interior angle measures of each polygon.

a. hexagon **b.** octagon **c.** 15-gon

a. 720°
b. 1,080°
c. 2,340°

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Example

2. Each chamber of a bee honeycomb is a regular hexagon. Find the measure of an interior angle of a regular hexagon.

Step 1 Find the sum of the measures of the angles.

$S = (n - 2)180$ Write an equation.

$S = (6 - 2)180$ Replace n with 6.

$S = (4)180$ or 720 Simplify.

The sum of the measures of the interior angles is 720°.

Step 2 Divide 720 by 6, the number of interior angles, to find the measure of one interior angle. So, the measure of one interior angle of a regular hexagon is 720 ÷ 6 or 120°.

Got It? Do these problems to find out.

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

d. octagon e. heptagon f. 20-gon

d. 135°
e. 128.6°
f. 162°

Exterior Angles of a Polygon

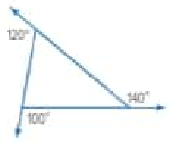
Key Concept

Words In a polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

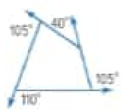
Model

Symbols $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

Regardless of the number of sides in a polygon, the sum of the exterior angle measures is equal to 360°.



$120 + 100 + 140 = 360°$



$105 + 110 + 105 + 40 = 360°$

STOP and Reflect

Draw another quadrilateral and a pentagon. Extend the sides to show the exterior angles. Then find the sum of each figure's exterior angle measures.

See students' work for drawings. 360°; 360°

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Example

2. Find the measure of one interior angle of a regular polygon.

AL • *What is a regular polygon?* a polygon with equal side lengths and equal angle measures

• *How many sides are in a hexagon?* 6

OL • *What equation can be used to find the sum of the interior angle measures of a hexagon?* $S = (6 - 2)180$

• *What is the sum of the interior angle measures of a hexagon?* 720°

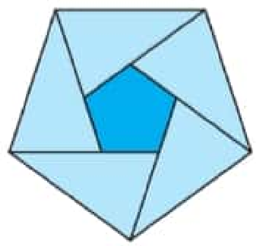
• *How can you find the measure of one of the interior angles of a regular hexagon?* Divide 720 by 6.

BL • *Write an equation that can be used to find the measure of one angle M of a regular polygon with n sides.*

$M = \frac{(n - 2) \cdot 180}{n}$

Need Another Example?

A designer is creating a new logo for a bank. The logo consists of a regular pentagon surrounded by isosceles triangles. Find the measure of an interior angle of a regular pentagon. 108°



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Example

3. Find the measure of one exterior angle in a regular polygon.

AL • *What is the sum of the measures of the exterior angles of any polygon?* **360°**

• *How many exterior angles are in a hexagon?* **6**

• *How would you find the measure of one exterior angle of a regular hexagon?* **Find 360 ÷ 6.**

OL • *What is the measure of each exterior angle?* **60°**

• *Write an equation to find the measure of one exterior angle m in a regular polygon with n sides.* $m = \frac{360}{n}$

BL • *Can you use this method to find the measure of an exterior angle of a polygon that is not regular? Explain.* **No, the exterior angles of a polygon that is not regular are not all the same.**

Need Another Example?
Find the measure of an exterior angle in a regular 30-gon. **12°**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Check** Have students work in pairs. Give each pair an index card. On the card, they should list the names of polygons and corresponding number of sides from triangle to decagon. Have students refer to their cards when completing Exercises 1–5. Then have them compare their answers with another pair of students. MP 7

BL LA **Pairs Consult** Have students read the information about tessellations in Exercises 8 and 9. Have students work in pairs to research M.C. Escher and tessellations. They should write a paragraph about the attributes of polygon tessellations. Give them sheets of colored paper or allow them to work on a computer to create their own tessellations. Display the tessellations throughout the room. MP 6

Example

3. Find the measure of an exterior angle in a regular hexagon.

Let x represent the measure of each exterior angle.

$6x = 360$ Write an equation. A hexagon has 6 exterior angles.

$x = 60$ Division Property of Equality

So, each exterior angle of a regular hexagon measures 60°.

Got it? Do these problems to find out.

Find the measure of an exterior angle of each regular polygon.

g. triangle h. quadrilateral i. octagon

g. **120°**

h. **90°**

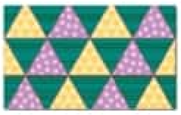
i. **45°**

Guided Practice

Find the sum of the interior angle measures of each polygon. (Example 1)

1. quadrilateral **360°**
2. nonagon **1,260°**
3. 12-gon **1,800°**

4. The quilt pattern shown is made of repeating equilateral triangles. What is the measure of one interior angle of an equilateral triangle? (Example 2)



60°

5. Find the measure of an exterior angle of a regular pentagon. (Example 3) **72°**

6. **Building on the Essential Question** How can I find the sum of the interior angle measures of a polygon?
Sample answer: Subtract 2 from the number of sides of the polygon and then multiply by 180.

Rate Yourself!

☐ I understand how to find the sum of the interior angle measures of a polygon.
Great! You're ready to move on!

☐ I still have some questions about the angles of polygons.

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Name ______ My Homework ______

Independent Practice

Find the sum of the interior angle measures of each polygon. (Example 1)

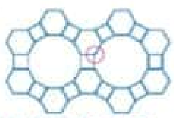
1. pentagon 540°
2. 11-gon 1,620°
3. 13-gon 1,980°

4. The soccer ball at the right consists of repeating regular pentagons and hexagons. Find the measure of one interior angle of a pentagon. (Example 2) 108°

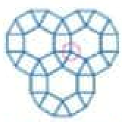
Find the measure of an exterior angle of each regular polygon. (Example 3)

5. decagon 36°
6. 20-gon 18°
7. 15-gon 24°

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. For each tessellation, find the measure of each angle at the circled vertex. Then find the sum of the angles.

8. 

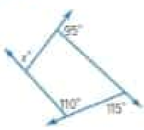
90°, 120°, 150°; 360°

9. 

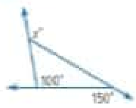
60°, 90°, 90°, 120°; 360°

Find the value of x in each figure.

10. 80



11. 130



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3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used f... additional reinforcement or as a second-day assignment.

Levels of Complexity

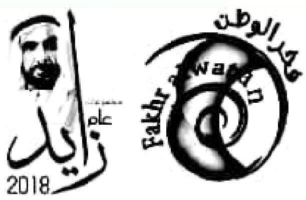
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 16–25	8–12, 26–29	13–15
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 14, 15, 28, 29
OL	On Level	1–7 odd, 8–12, 14, 15, 28, 29
BL	Beyond Level	8–15, 28, 29



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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
3 Construct viable arguments and critique the reasoning of others.	14, 15, 27
4 Model with mathematics.	12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain h ow to find the sum of the interior angles of a polygon wh en they know the number of sides of the polygon. See students' work.

12. **Model with Mathematics** Refer to the graphic novel frame below. Find the measures of the two missing angles using properties of quadrilaterals and parallel lines. 130° and 28°

H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** How many sides does a regular polygon have if the measure of an interior angle is 160°? Justify your answer.

18; $\frac{(n-2)180}{n} = 160$
$(n-2)180 = 160n$ Multiplication Property of Equality
$180n - 360 = 160n$ Distributive Property
$20n = 360$ Properties of Equality
$n = 18$ Division Property of Equality

14. **Reason Inductively** If the number of sides of a polygon increases by 1, what happens to the sum of the measures of the interior angles?
It increases by 180°.

15. **Reason Inductively** Jamal drew a regular polygon and measured one of its interior angles. Explain why it is impossible for his angle measure to be 145°.
Regular decagons have equal angles measuring 144° and regular 11-sided polygons have angles measuring 147.27°. 145° is between these two values so it cannot be the interior angle measure of a regular polygon.

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Name ______ My Homework ______

Extra Practice

Find the sum of the interior angle measures of each polygon.

16. heptagon 900°

$S = (n - 2)180$
$S = (7 - 2)180$
$S = 5 \cdot 180$
$S = 900$

17. 14-gon **2,160°**

18. 24-gon **3,960°**

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

19. nonagon **140°**

20. decagon **144°**

21. 19-gon **161.1°**

22. 16-gon **157.5°**

Find the measure of an exterior angle of each regular polygon.

23. nonagon **40°**

24. 12-gon **30°**

25. 18-gon **20°**

26. The surface of the dome of Spaceship Earth in Orlando consists of repeating equilateral triangles as shown. Find the measure of each angle in each outlined triangle. Then make a conjecture about the interior angle measures in equilateral triangles of different sizes.

The measure of each angle in each outlined triangle is 60°. If a triangle is equilateral, the measure of each angle will be 60° regardless of the size of the triangle.

27. **Justify Conclusions** What is the sum of the interior angles of nonregular hexagons? Explain your reasoning to a classmate.

Sample answer: The sum of the interior angles will still be 720° because even though the figures are not regular, they are still hexagons.

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Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

28.

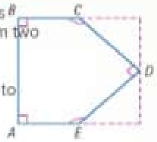
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the model and find the measure of angle *AED*.
1 point	Students correctly complete the model OR find the measure of angle *AED*.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly answer all four parts of the question.
1 point	Students correctly answer three of the four parts of the question.

Power Up! Test Practice

28. After the first two folds of an origami paper design, the paper is shaped like a square with two isosceles triangles removed from two adjacent corners.



Angles *AED* and *BCD* are congruent. Select the correct values to complete the model below to find the measure of angle *AED*.

x	2	3	45	90
180	360	540	720	

2 · x + 3 · 90 = 540

What is m∠AED? 135°

29. Fill in each box to make each statement true.

a. The sum of the interior angle measures of a quadrilateral is 360.

b. The sum of the interior angle measures of a hexagon is 720.

c. The sum of the interior angle measures of an octagon is 1,080.

d. The sum of the interior angle measures of a 11-gon is 1,620.

Spiral Review

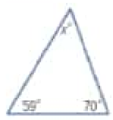
Classify each pair of angles as *complementary, supplementary,* or *neither*.

30. angle 1: 35° complementary
angle 2: 55°

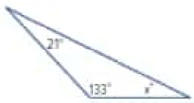
31. angle 1: 62° neither
angle 1: 108°

Find the value of *x* in each triangle.

32. 51



33. 26



34. 27

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Problem-Solving Investigation

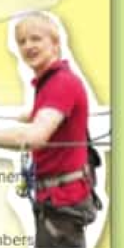
Look for a Pattern

Mathematical Practices 1, 3, 8

Case #1 Spider Web

An activity in a low ropes course creates the inside of a spider web using string. The group members form a polygon. The strings stretch from each person to every nonadjacent member of the figure. Saeed's group has 20 members.

How many strings will Saeed hold in the web?



1 **Understand** *What are the facts?*

- There are 20 group members that form a polygon.
- A string stretches from each person to every nonadjacent group mem

2 **Plan** *What is your strategy to solve this problem?*

Drawing a 20-gon would be difficult. Begin with a group of four members and look for a pattern. Then make a table to find the pattern.

3 **Solve** *How can you apply the strategy?*

Draw figures using four, five, and six members. Draw the diagonals from one member to show number of strings. Some figures are drawn for you.

Number of Members	4	5	6	7	8	9
Number of Strings	1	2	3	4	5	6

How many strings will Saeed hold? 17 strings

4 **Check** *Does the answer make sense?*

Draw a 20-gon and count the number of diagonals from one vertex.

Analyze the Strategy

Identify Repeated Reasoning How would the pattern change if Saeed was looking for the total number of strings from every person in the web?

Sample answer: If n represented the number of people, they would need $\frac{n(n-3)}{2}$ strings.

Uncorrected first proof - for training purposes only

Focus narrowing the scope

Objective Solve problems by using the *look for a pattern* strategy. This lesson emphasizes **Mathematical Practice 8** Identify Repeated Reasoning.

Look for a Pattern Looking for a pattern is a good strategy for solving a variety of problems. When working with patterns, it is sometimes helpful to organize information in a table.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the look for a pattern strategy to analyze the relationship between the side lengths of a right triangle.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 407.

Engage Explore Explain Elaborate Evaluate

1 Launch the Lesson

The problems on pages 405 and 406 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 405 walks students through the solution, while the problem on page 406 asks students to come up with their own solutions.

Case #1 Spider Web

BL Have students extend the problem by answering the following question.

Ask:

- *Write an expression that can be used to find the number of strings Chen will hold for n members.* $n - 3$

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Case #2 Follow the Bouncing Ball

AL LA **Rally Coach** Have students work in pairs to solve the problem. Have Student A complete the first step, speaking out loud, while Student B listens carefully, coaches, and praises. Next, have Student B complete the second step while Student A listens carefully, coaches, and praises. Partners take turns until they have solved the problem. MP 1, 2, 3, 4, 5, 6, 7, 8

BL LA **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1, 2, 3, 4, 5, 6, 7, 8

Ask:

- *How could you solve this problem a different way?* Sample answer: The height of each bounce is $\frac{2}{3}$ the height of the previous bounce. I could draw a diagram to show the height of each bounce after the third bounce.

Need Another Example?

All new showerheads are required to restrict their water flow. Determine how long it will take Hidaya to use 18 liters of water.

Number of Minutes	1	2	3	4
Number of Liters	$2\frac{1}{2}$	5	$7\frac{1}{2}$	10

Each minute, she uses $2\frac{1}{2}$ liters of water. She will use 18 liters of water between the 7th and 8th minute.

Case #2 Follow the Bouncing Ball

A ball was dropped from a height of 27 centimeters. After the first, second, and third bounces, the heights were 18 centimeters, 12 centimeters, and 8 centimeters, respectively.

After which bounce will the height of the ball be less than 3 centimeters?

1 Understand

Read the problem. What are you being asked to find?

I need to find after which bounce will the height of ball be less than 3 centimeters.

Underline key words and values. What information do you know?

The ball is dropped from 27 centimeters. The first bounce is 18 centimeters high, the second bounce 12 centimeters high, and the third bounce is 8 centimeters high.

2 Plan

Choose a problem-solving strategy.

I will use the look for a pattern strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

+1 +1

Bounce	0	1	2	3	4	5	6
Height (cm)	27	18	12	8	$5\frac{1}{3}$	$3\frac{5}{9}$	$2\frac{10}{27}$

$\times\frac{2}{3}$ $\times\frac{2}{3}$

So, the height of each bounce $\frac{2}{3}$ of the previous bounce and will be less than 3 centimeters after the sixth bounce.

4 Check

Use information from the problem to check your answer.

Start with the sixth bounce height and work backward using inverse operations.

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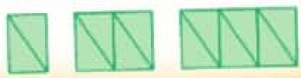
Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Geometry

Right triangles are arranged as shown. The sum of the measures of the angles in the first figure is 360°.

What is the sum of the measures of the angles in the fifth figure?

1,800°



Case #4 Seating

A theater has 12 seats in the first row, 17 seats in the second row, 22 seats in the third row, and so on.

How many seats are in the eighth row? the *n*th row?

47 seats; $(5n + 7)$ seats

Case #5 Mental Math Tricks

Study the pattern.

1 × 1 = 1
11 × 11 = 121
111 × 111 = 12,321
1111 × 1111 = 1,234,321

Without doing the multiplication, what is the answer to 1,111,111 × 1,111,111?

1,234,567,654,321

Use any strategy!

Case #6 Time

Hareb and his friends are going out to bowl, eat dinner, and see a movie. The movie starts at 8:10 P.M. and they want to arrive 20 minutes before it starts. They will bowl for one hour and dinner will take 1 hour and 15 minutes. Travel time is 20 minutes to the bowling alley, 45 minutes to the restaurant, and 10 minutes to the theater.

At what time should they plan to leave Hareb's house?

4:20 P.M.

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2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	3, 4	5	6
Level 3			●
Level 2		●	
Level 1	●		

AL LA **Roundrobin** Have students work in pairs to extend the pattern in Case 4 to find the number of seats in the fourth row, fifth row, sixth row, and so on, until they reach the eighth row. If they are struggling with the number of seats in the *n*th row, provide them with the beginning of the expression, ___$n + 7$. Have them find the coefficient of *n*. MP 1, 2, 3, 4, 5, 6, 7, 8

BL LA **Trade-a-Problem** Have students create their own real-world problem with a pattern. Then have them trade their problems with each other, solve, and compare solutions. If the solutions do not agree, students should work together to find the errors. MP 1, 2, 3, 4, 5, 6, 7, 8

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Mid-Chapter Check

If students have trouble with Exercises 1–10, they may need help with the following concepts.

Concept	Exercise(s)
parallel lines and transversals (Lesson 1)	1, 3–8, 10
polygons and angles (Lesson 4)	2, 9

Vocabulary Activity

LA **Numbered Heads Together** Have students work in groups of 4 to complete Exercise 1. Each student is assigned a number from 1 to 4. Each student is also assigned one of the angle relationships, such as corresponding angles. Students are responsible to ensure that each group member understands the meaning of their angle relationship. Students should ask each other for clarification and assistance, as needed. Call on one numbered student to define their angle relationship for the class. Then have the groups complete Exercise 2. MP 1, 4, 6, 7

Alternate Strategies

AL Have students use highlighters or colored pencils to identify examples of angle relationships shown on the diagram in Exercise 1.

BL Have students write equations that represent the angle relationships shown on the diagram in Exercise 1.

Mid-Chapter Check

Vocabulary Check

1. Name a pair of angles for each of the following. (Lesson 1) **Sample answers: 1a–1d.**
   a. corresponding angles **2 and 6**
   b. alternate interior angles **3 and 5**
   c. vertical angles **4 and 2**
   d. alternate exterior angles **1 and 7**

2. List two attributes of regular polygons. (Lesson 4)
   all of the sides are the same length
   all of the angles have the same measure

Skills Check and Problem Solving

Refer to the figure at the right. Find the missing measure of each angle. (Lessons 1 and 3)

3. $m\angle 1 =$ **80°**
4. $m\angle 2 =$ **57°**
5. $m\angle 3 =$ **57°**
6. $m\angle 4 =$ **43°**
7. $m\angle 5 =$ **137°**
8. $m\angle 6 =$ **123°**

9. A building is in the shape of a regular polygon with five sides. What is the measure of one of the interior angles of the building? (Lesson 4) **108°**

10. MP **Use Math Tools** In the figure, line a is parallel to line b. Which of the following are equal to the measure of $\angle T$? (Lesson 1) **I, III, IV**
    I the supplement of $\angle S$
    II the complement of $\angle X$
    III the angle adjacent to $\angle Z$
    IV angle corresponding to $\angle R$

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Geom

Inquiry Lab
Right Triangle Relationships

Inquiry **WHAT is the relationship among the sides of a right triangle?**

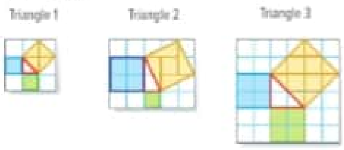
Mathematical Practices 1, 3, 4

Three square tents at the funfair are situated as shown below.
The backs of the orange tent and the green tent form a right angle. The back of the blue tent finishes the triangle.

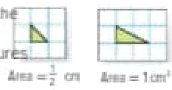
Hands-On Activity

Using grid paper can help to investigate the relationship between the sides of a right triangle.

Step 1 In each figure below, the sides of three squares form a right triangle.



Step 2 Find the area of each square that is attached to the triangle. Record your results in the table below. The first one is already done for you. Use the figures at the right to help find the area of partial grids.



Triangle	Area of Green Square	Area of Blue Square	Area of Yellow Square
1	1	1	2
2	1	4	5
3	4	4	8

What relationship exists among the areas of the three squares bordering each triangle? The sum of the areas of the two smaller squares is equal to the area of the larger square.

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Focus narrowing the scope

Objective Model the relationship among the sides of a right triangle.

Coherence connecting within and across grades

Now Students will model the relationship among the sides of a right triangle.

Next Students will use the Pythagorean Theorem and its converse to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 410.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Three-Step Interview** Have pairs complete the activty. Up on completion, have Student 1 interview Student 2, using the table and and question in Step 2 as interview questions. Then Student 1 asks Student 2 any clarification questions about the relationship and how Student 2 can verify the relationship using the values they generated in the table. MP 1, 3, 4, 6, 7, 8

EL LA **Pairs Consult** Have students work with a partner to translate their verbal response to the question in Step 2 into an equation. Have them use the letters *a*, *b*, and *c* to represent the lengths of the sides of the triangle, letting *c* represent the length of the longest side. MP 1, 2, 4, 6, 7

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1, 2	Exercises 3	Exercises 4, 5
Level 3			•
Level 2		•	
Level 1	•		

Investigate

AL LA **Pairs Discussion** Have students work in pairs. Have each student complete Exercise 1 on their own. Then have them trade their triangles and complete their partner's triangle for Exercise 2. Then have them compare solutions and discuss any differences. MP 1, 3, 4, 6

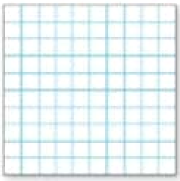
Create

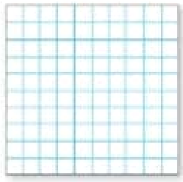
BL Have students verify their conjecture in Exercise 4 by using grid paper to draw the triangle. Then have them make a conjecture about the length of the longest side of a triangle if the lengths of the two shorter sides are double that of the sides in Exercise 4. MP 1, 3, 4, 5, 6, 7

Inquiry Students should be able to answer "WHAT is the relationship among the sides of a right triangle?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Draw a right triangle different than those of the Activity on grid paper. Find the area of each square that is attached to the triangle. See students' work.

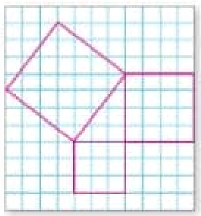
1. 

2. 

1.	2.
Area of square 1= ______	Area of square 1= ______
Area of square 2= ______	Area of square 2= ______
Area of square 3= ______	Area of square 3= ______

Analyze and Reflect

3. MP **Model with Mathematics** On the grid paper shown, draw a right triangle so the two shorter sides are 3 units and 4 units long. Draw squares attached to each side of the triangle.



What is the area of each square?
9, 16, and 25 square units

What is the length of each side?
3, 4, and 5 units

Create

4. MP **Reason Inductively** Make a conjecture about the length of the longest side of a right triangle if the lengths of the two shorter sides are 6 centimeters and 8 centimeters.
The length of the longest side would be 10 cm.

5. Inquiry WHAT is the relationship among the sides of a right triangle?
The sum of the squares of the two smallest sides is equal to the square of the largest side.

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Lesson 5

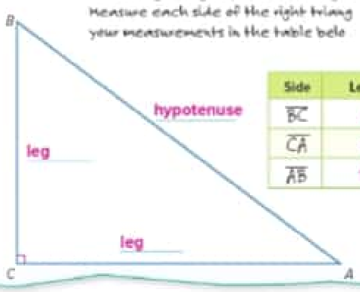
The Pythagorean Theorem

Vocabulary Start-Up

A right triangle is a triangle with one right angle. **legs** are the sides that form the right angle. The **hypotenuse** is the side opposite the right angle. It is the longest side of the triangle.

Complete the graphic organizer. Label the legs and the hypotenuse.

Draw a right angle symbol on the right angle.
Measure each side of the right triangle and write your measurements in the table below.



Side	Length (cm)
$\overline{BC}$	6 cm
$\overline{CA}$	8 cm
$\overline{AB}$	10 cm

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

legs
hypotenuse
Pythagorean Theorem
converse

Mathematical Practices
1, 3, 4, 5

Real-World Link

When viewed from the side, the shape of some wooden waterskiing ramps is a right triangle. Suppose the height of a ramp is 90 centimeters and the length of the base of the ramp is 120 centimeters. How long do you think the ramp will be? Explain your reasoning.

Accept all reasonable answers; the length of the ramp is 150 cm.

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

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Focus narrowing the scope

Objective Use the Pythagorean Theorem.

Coherence connecting within and across grades

Previous Students used a model to explore the relationship among the sides of a right triangle.

Now Students will use the Pythagorean Theorem and its converse to solve problems.

Next Students will explore proofs of the Pythagorean Theorem.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 415.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Pairs Discussion Ask students to draw a rectangle on a piece of paper. Then have them draw the diagonal. Ask the questions below, then have students work in pairs to complete the Vocabulary Start-Up on the student page. MP 1, 2, 3, 4, 5, 6

Ask:

- *When a diagonal is drawn, what is formed?* two triangles
- *How would you classify the triangles?* right triangles
- *Is the diagonal longer or shorter than the sides of the rectangle?* longer

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

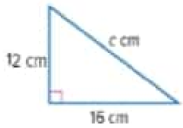
Example

1. Find a missing length in a right triangle.

- AL • *Do you need to find the length of one of the legs or the hypotenuse?* **hypotenuse**
  - *What are the lengths of the legs?* **9 cm and 12 cm**
- OL • *What equation do we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
  - *What value would you substitute for a in the equation?* **12**
  - *What value would you substitute for b in the equation?* **9**
- BL • *Why can the value of c not be negative?* **Length must be positive, so you need to use the positive square root.**
  - *How do we know that our answer is reasonable?* **Sample answer: The hypotenuse is the longest side of a right triangle. Since 15 > 12 > 9, the answer is reasonable.**

Need Another Example?

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.



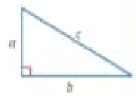
$12^2 + 16^2 = c^2$; 20 cm

Key Concept

Pythagorean Theorem

Words In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Model



Symbols $a^2 + b^2 = c^2$

Work Zone

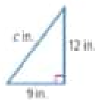
The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for *any* right triangle.

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the other two sides.

Examples

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$12^2 + 9^2 = c^2$	Replace a with 12 and b with 9.
$144 + 81 = c^2$	Evaluate 12^2 and 9^2.
$225 = c^2$	Add 81 and 144.
$\pm\sqrt{225} = c$	Definition of square root
$c = 15$ or -15	Simplify.

The equation has two solutions, 15 and −15. However, the length of a side must be positive. So, the hypotenuse is 15 centimeters long.

Check: $a^2 + b^2 = c^2$

$12^2 + 9^2 \stackrel{?}{=} 15^2$

$144 + 81 \stackrel{?}{=} 225$

$225 = 225$ ✓

Right Angle
The symbol ⌐ indicates an angle with a measure of 90°.

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2.

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$8^2 + b^2 = 24^2$ — Replace a with 8 and c with 24.

$64 + b^2 = 576$ — Evaluate 8^2 and 24^2

$64 - 64 + b^2 = 576 - 64$ — Subtract 64 from each side.

$b^2 = 512$ — Simplify.

$b = \pm\sqrt{512}$ — Definition of square root

$b \approx 22.6$ or -22.6 — Use a calculator.

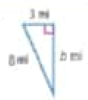
The length of side b is about 22.6 meters.

Check for Reasonableness The hypotenuse is always the longest side in a right triangle. Since 22.6 is less than 24, the answer is reasonable.

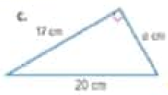
Got It? Do these problems to find out.

a.

b.



c.



a. $18^2 + 24^2 = c^2$; 30 dm

b. $3^2 + b^2 = 8^2$; 7.4 km

c. $a^2 + 17^2 = 20^2$; 10.5 cm

Converse of Pythagorean Theorem — Key Concept

If the sides of a triangle have lengths a, b, and c units such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

If you reverse the parts of the Pythagorean Theorem, you have formed its **converse**.

Statement: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

The converse of the Pythagorean Theorem is also true.

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Example

2. Find a missing length in a right triangle.

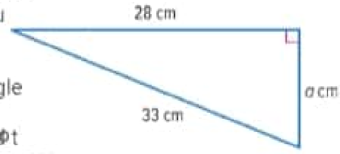
AL
- *Do you need to determine the length of one of the legs or the hypotenuse?* one of the legs
- *What is the length of the known leg?* 8 m
- *What is the length of the hypotenuse?* 24 m

OL
- *What equation do we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- *What value would you substitute for a in the equation?* 8
- *What value would you substitute for c in the equation?* 24

BL
- *Why do we not consider the negative square root, −22.6?* The side length of a triangle cannot be negative.
- *How do we know if our answer is reasonable?* Sample answer: The hypotenuse is the longest side of a right triangle. Since 22.6 is less than 24, the answer is reasonable.

Need Another Example?

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.



$a^2 + 28^2 = 33^2$; 17.5 cm

Watch Out!

Common Error Students may think that they always substitute the known side lengths for a and b and solve for c in the Pythagorean Theorem. Remind them that c is always the hypotenuse of the triangle, so if the hypotenuse is one of the known sides, they will need to substitute the known lengths for a and c, and then solve for b.

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Example

3. Use the converse of the Pythagorean Theorem.

AL • *What are the lengths of the sides of the triangle?* 5 cm, 12 cm, and 13 cm

• *Which side length is the longest?* 13 cm

OL • *If this is a right triangle, which side would be the hypotenuse?* 13 cm

• *What equation do we use to determine if the triangle is a right triangle?* $a^2 + b^2 = c^2$

• *What value would you substitute for a in the equation?* 5 *b?* 12 *c?* 13

BL • *Sets of numbers that work in the Pythagorean Theorem are called Pythagorean Triples. Can you think of other values for a, b, and c that are Pythagorean Triples?* Sample answer: 3, 4, and 5

Need Another Example?
The measures of three sides of a triangle are 24 centimeters, 7 centimeters, and 25 centimeters. Determine whether the triangle is a right triangle. yes; $7^2 + 24^2 = 25^2$

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL BL LA **Team-Pair-Solo** Have students work in a team of 4 (making sure there are some approaching level students grouped with beyond level students) to complete Exercises 1 and 3. Then have them work in pairs to complete Exercises 2 and 4. Finally, have them work individually to complete Exercise 5. Upon completion, have them regroup with their original team to compare answers and resolve any differences.

MP 1, 2, 3, 4, 6

STOP and Reflect

State three measures that could be the side measures of a right triangle. Justify your answer below.

Sample answer: 3, 4, 5; $3^2 + 4^2 = 5^2$

Example

3. **The measures of three sides of a triangle are 5 centimeters, 12 centimeters, and 13 centimeters. Determine whether the triangle is a right triangle.**

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 12^2 \stackrel{?}{=} 13^2$ $a = 5, b = 12, c = 13$

$25 + 144 \stackrel{?}{=} 169$ Evaluate 5^2, 12^2, and 13^2.

$169 = 169$ ✓ Simplify.

The triangle is a right triangle.

Got It? Do these problems to find out.

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

d. 36 km, 48 km, 60 km **e.** 4 m, 7 m, 5 m

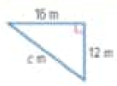
d. yes; $36^2 + 48^2 = 60^2$

e. no; $4^2 + 5^2 \neq 7^2$

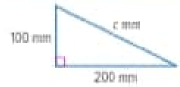
Guided Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. $12^2 + 16^2 = c^2$; 20 m



2. $100^2 + 200^2 = c^2$; 223.6 mm



Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

3. 5 cm, 10 cm, 12 cm no; $5^2 + 10^2 \neq 12^2$

4. 9 m, 40 m, 41 m yes; $9^2 + 40^2 = 41^2$

5. **Building on the Essential Question** What is the relationship among the legs and the hypotenuse of a right triangle?

The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Rate Yourself!

How confident are you about using the Pythagorean Theorem? Check the box that applies.

FOLDABLES Time to update your Foldable!

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Name ____________________ My Homework ____________________

Independent Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. $5^2 + 12^2 = c^2$; 13 cm.

2. $a^2 + 51^2 = 60^2$; 31.6 m

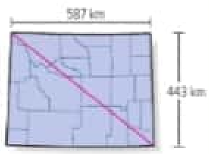
3. $8^2 + b^2 = 18^2$; 16.1 m

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

4. 28 m, 195 m, 197 m
yes; $28^2 + 195^2 = 197^2$

5. 30 cm, 122 cm, 125 cm
no; $30^2 + 122^2 \neq 125^2$

6. Calculate the length of the diagonal of the rectangle.
about 735 km



Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

7. $a = 48$ m; $b = 55$ m
$48^2 + 55^2 = c^2$; 73 m

8. $a = 23$ cm; $b = 18$ cm
$23^2 + 18^2 = c^2$; 29.2 cm

9. $b = 5.1$ m; $c = 12.3$ m
$a^2 + 5.1^2 = 12.3^2$; 11.2 m

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–6, 15–22	7–10, 23–25	11–14
Level 3			•
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 12–14, 24, 25
OL	On Level	1–5 odd, 6–10, 12–14, 24, 25
BL	Beyond Level	6–14, 24, 25

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
3 Construct viable arguments and critique the reasoning of others.	12–14, 23
5 Use appropriate tools strategically.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students determin e whether a triangle with side lengths of 1 centimeter, 3 centimeters, and 3 centimeters is a right triangle. Have them justify their respon**no**se.
Sample answer: $1^2 + 3^2 \neq 3^2$.

Watch Out!

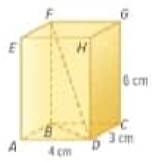
Find the Error In Exercise 12, Amani identified the hypotenuse as one of the triangle's legs. Ask students to label the two legs of the right triangle a and b and the hypotenuse c. Point out that it is often helpful to write the formula before replacing variables with numbers.

10. **Use Math Tools**The whole numbers 3, 4, and 5 are called Pythagorean triples because they satisfy the Pythagorean Theorem. Complete the graphic organizer shown to list 4 additional sets of Pythagorean triples.

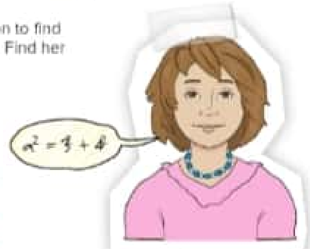
Pythagorean Triples		
3	4	5
6	8	10
9	12	15
5	12	13
8	15	17

H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** the figure$\overline{BD}$ is the diagonal of the base a$\overline{FD}$ is the diagonal of the figure. Fin$\overline{BD}$ to the nearest tenth.
7.8 cm

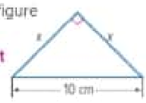


12. **Find the Error**Amani is writing an equation to find the length of the third side of the right triangle. Find her mistake and correct it.



She used the sides given as legs, when one is a hypotenuse; $x^2 + 3^2 = 4^2$.

13. **Justify Conclusions**What does the value of x have to be for the figure to be classified as a right isosceles triangle? Justify your reasoning.



about 7.1 cm; Sample answer: The Pythagorean Theorem states that $c^2 = a^2 + b^2$. Since both legs are x inches, $c^2 = 2x^2$. When you replace c with 10 and simplify, $x \approx 7.1$.

14. **Justify Conclusions**The hypotenuse of a right triangle is 23 centimeters long. Find possible measures for the legs of the triangle. Round to the nearest hundredth. Justify your answer.
Sample answer: 15 cm and 17.44 cm; $23^2 = 529$ and $15^2 + 17.44^2 = 529.1536$. So, $23^2 \approx 15^2 + 17.44^2$.

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Name________________________ My Homework

Extra Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

15. $27^2 + 36^2 = c^2$; 45 meter

$27^2 + 36^2 = c^2$

$729 + 1{,}296 = c^2$

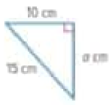
$2{,}025 = c^2$

$\pm\sqrt{2{,}025} = c$

$\pm 45 = c$

Since length cannot be negative, the length of side c is 45 meters.

16. $a^2 + 10^2 = 15^2$; 11.2 cm



17. $30^2 + b^2 = 80^2$; 74.2 mm

Copy and Solve **Determine whether each triangle is a right triangle. Justify your answer. Show your work on a separate piece of paper.**

18. 24 m, 143 m, 145 m
yes; $24^2 + 143^2 = 145^2$

19. 135 cm, 140 cm, 175 cm
no; $135^2 + 140^2 \neq 175^2$

20. 56 m, 65 m, 16 m
no; $56^2 + 16^2 \neq 65^2$

21. 44 cm, 70 cm, 55 cm
no; $44^2 + 55^2 \neq 70^2$

22. A triangle is formed by three towns, as shown on the map. Is this triangle a right triangle? Explain.
no; $12^2 + 24^2 \neq 29^2$

23. **Construct an Argument** Explain to a classmate why you can use two sides of a right triangle to find the third side. Sample answer: If you know the lengths of two sides of a right triangle, you can substitute the values in the Pythagorean Theorem and find the missing side.

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Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed when taking assessment.

24.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3, MP4, MP5
Scoring Rubric	
2 points	Students correctly model the situation, find the height of the ladder and explain their response.
1 point	Students correctly model the situation OR find the height of the ladder and explain their response.

25.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2, MP6
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

24. The base of a 3.90-meter ladder stands 1.50 meters from a house. Sketch a drawing to model this situation.

How many meters up the side of the house does the ladder reach? Explain how drawing the picture helped you solve the problem.

3.60 m; Sample answer: Drawing and labeling the picture helps you to see how to apply the Pythagorean Theorem to solve the problem.

25. Which of the following lengths represent the sides of a right triangle? Select all that apply.

- ☐ 9 cm, 12 cm, 16 cm
- ■ 8 cm, 15 cm, 17 cm
- ☐ 10 cm, 24 cm, 28 cm
- ■ 6 cm, 8 cm, 10 cm

Spiral Review

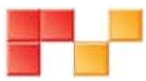
Simplify each expression.

26. $10^2 + 14^2 =$ **296**

27. $16^2 + 2^2 =$ **260**

28. $20^2 - 17^2 =$ **111**

29. The area of each square is 16 square units. Find the perimeter of the figure shown.



64 units

Find each square root. Round to the nearest tenth.

30. $\sqrt{200} \approx$ **14.1**

31. $\sqrt{45} \approx$ **6.7**

32. $\sqrt{126} \approx$ **11.2**

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Inquiry Lab
Proofs About the Pythagorean Theorem

Inquiry **HOW can you prove the Pythagorean Theorem and its converse?**

Mathematical Practices 1, 3, 7

The Pythagorean Theorem is named after a famous Greek mathematician Pythagoras who lived around 500 B.C. The properties of the theorem, however, were known by the ancient Egyptians, Babylonians, and Chinese. The following geometric proof is similar to a visual proof shown in a Chinese document written between 500 B.C. and 200 B.C.

Hands-On Activity 1

Step 1 Draw and cut out 8 copies of a right triangle. Label each pair of legs a and b, and each hypotenuse c.

Step 2 On a separate piece of paper arrange four of the triangles in a square as shown. Trace the figure formed by the hypotenuses.

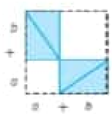
The length of each side of the large square is $a + b$, so the area of the large square is $(a + b)^2$.

Is the figure formed by the hypotenuses a square? Explain.

yes; Sample answer: The sides of the figure have the same measure c and all of the angles measure 90°.

Write an expression for the area of the inside square. c^2

Step 3 On the same paper, arrange the remaining triangles as shown. Draw the two figures shown by the dashed lines.



The length of each side of the large square is $a + b$, so the area of the large square is $(a + b)^2$.

Are the two figures represented by dashed line squares? Explain.

yes; Sample answer: The sides of each figure have the same measure and all of the angles measure 90°.

Write an expression for the area of the small square.

Write an expression for the area of the large square.

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Focus narrowing the scope

Objective Prove the Pythagorean Theorem and its converse.

Coherence connecting within and across grades

Now Students will use models and diagrams to prove the Pythagorean Thoerem.

Next Students will use the Pythagorean Theorem and its converse to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 420.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

AL BL LA **Teammates Consult** Have students work in small groups to complete the activity. Group one or more Approaching Level students and one or more Beyond Level students in the same groups, if possible. Have one student read each set of directions aloud, checking to see if the team understands what procedures to follow. Have students who understand the procedures check the work of those who have trouble or need clarification. For each step, have the student who is reading the directions pause and ask if students have questions. If no one on the team is able to answer a specific question, the team may ask you for help and assistance.

MP 1, 4, 5

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

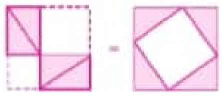
	Exercises 1–3	Exercises 4–6	Exercises 7, 8
Level 3			•
Level 2		•	
Level 1	•		

Investigate

AL BL LA **Teammates Consult** Continue the groupings from Activity 1. After researching the Egyptian technique in Exercise 1, have students work in their small groups to demonstrate the procedure. Give groups ropes of different lengths with knots already tied. Have students trace their triangle on paper or on the board. MP 1, 4, 5

BL LA **Teammates Consult** Using the same activity above, give the groups ropes of different lengths that do not have any knots. Have students tie equally spaced knots and trace their triangle on paper or on the board. MP 1, 4, 5

Step 4 Since the area of each of the two composite figures you created is $(a+b)^2$, the areas are equal. Use the space provided to draw each figure from Step 2 and Step 3. Place an equal sign between the two figures to show the two areas are equal.



Step 5 Remove the triangles from each side. Use the space provided to draw the remaining figures.

What property justifies removing the triangles from each side of the equation? **Subtraction Property of Equality**

Write an algebraic equation that represents the relationship between the figures shown in Step 5. $a^2 + b^2 = c^2$

Summarize the relationship among the sides of a right triangle measuring *a* units, *b* units, and *c* units.

Sample answer: The sum of the squares of the two smaller sides is equal to the square of the largest side.

Investigate

Work with a partner.

1. A legend states that the ancient Egyptians could create a right triangle by using a knotted rope. Research this on the Internet. Describe their technique in the space provided and draw a diagram to illustrate the technique.

Sample answer: The Egyptians tied 12 knots in a rope that were equally spaced. They then laid it out so one side had 3 units, another side had 4 units and the third side had 5 units. This made a right triangle.

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Hands-On Activity 2

The converse of the Pythagorean Theorem states if a triangle has side lengths, a, b, and c units such that $a^2 b^2 = c^2$, then the triangle is a right triangle. In this Activity, you will prove the converse of the Pythagorean Theorem by using a two-column proof.

Given: $\triangle ABC$ such that $a^2 + b^2 = c^2$.

Prove: $\triangle ABC$ is a right triangle.

Complete the proof with the correct reasons justifying each statement.

	Statements	Reasons
a.	Draw a right triangle DEF so that $\overline{DE}$ is a units long and $\overline{DF}$ is b units long. Label $\overline{FE}$ as d.	
b.	Write an equation that describes the relationship between the side lengths of $\triangle DEF$. State the theorem that allows you to make that statement.	**$a^2 + b^2 = d^2$; Pythagorean Theorem**
c.	$a^2 + b^2 = c^2$	Given
d.	If $a^2 + b^2 = c^2$ and $a^2 + b^2 = d^2$, then $d^2 = c^2$.	**Substitute c^2 for $a^2 + b^2$ in the second equation.**
e.	If $d^2 = c^2$, then $d = c$.	**Definition of square root**
f.	If $d = c$, then $FE = AB$.	**Definition of equal segments**
g.	If $AC = FD$, $CB = DE$, and $AB = FE$, the two triangles are the same shape and size.	If three sides of a triangle are the same length as the corresponding sides of another triangle, the triangles are the same shape and size.
h.	$m\angle C = m\angle D$	Corresponding parts of the triangles with the same size and shape have the same measures.
i.	$\angle C$ is a right angle.	**Definition of right angle**
j.	$\triangle ABC$ is a right triangle	**Definition of right triangle**

So, if a triangle has side lengths, a, b, and c units such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

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Hands-On Activity 2

AL LA Think-Pair-Share Have students work with a partner to complete the activity. For each step, give students about 10–20 seconds to think through how they would provide a reason for each step. Then have them share their responses with their partner, asking for support and clarification if needed from their partner, or you. You may wish to provide students with numerical measurements for the side lengths of the triangle instead of variable measurements, if students are struggling with the algebraic manipulation.
MP 1, 2, 3, 4, 5, 6, 7

BL LA Popcorn Share Have students work in small groups to complete Activity 2. Have each student be responsible for providing the reason for each step. Have students take turns providing the reasons. When it is their turn to provide a reason, have that student stand up, verbally give the reason, and then explain the reason in their own words. MP 1, 2, 3, 4, 5, 6, 7

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Investigate

AL **LA** **Group-Solo** Have students work as a small group to complete Exercise 2, ensuring that each group member understands how to determine whether the triangle is a right triangle. Then have students complete Exercise 3 individually. Upon completion, have them share their responses with their group to check their work. **MP** 1, 2, 3, 4, 5, 6, 7

Ask:

- *What equation can you use to determine if the triangle is a right triangle?* $a^2 + b^2 = c^2$
- *In Exercise 1, what length on the triangle must be substituted for c? Why?* 6; Sample answer: because it is the longest side of the triangle

Analyze and Reflect

AL **BL** **LA** **Value Line** Students place themselves on a pretend line, using the number 10 to represent that they understand the Pythagorean Theorem and its converse completely and the number 1 to represent that they do not understand or have a lot of questions. Have students pair up with someone from the other side of the line to complete Exercises 4–6. **MP** 1, 2, 4, 5, 6, 7

Create

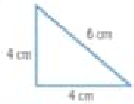
AL **LA** **Think-Pair-Share** Have students work in pairs. Give students several minutes to think through their responses to Exercises 7 and 8. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. **MP** 1, 2, 4, 5, 6

Inquiry Students should be able to answer "HOW can you prove the Pythagorean Theorem and its converse?" Check for student understanding and provide guidance, if needed.

Investigate

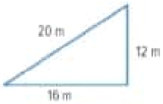
Work with a partner. Determine whether the following figures are right triangles. Justify your answer.

2.



no; $4^2 + 4^2 \neq 6^2$

3.



yes; $12^2 + 16^2 = 20^2$

Analyze and Reflect

MP Justify Conclusions **Work with a partner. The whole numbers 3, 4, and 5 are called *Pythagorean Triples* because they satisfy the Pythagorean Theorem. Determine if each of the following is a Pythagorean Triple. Explain your reasoning.**

4. 7, 24, 25
yes; Sample answer: $7^2 + 24^2 = 25^2$

5. 15, 20, 25
yes; Sample answer: $15^2 + 20^2 = 25^2$

6. 9, 12, 16
no; Sample answer: $9^2 + 12^2 = 15^2$, not 16^2

Create

7. **MP Identify Structure** In the Activity in the "Right Triangle Relationships" Inquiry Lab, you examined the relationship between the sides of a right triangle. Compare the process used in that Activity with the one you completed when you did the "Proofs About the Pythagorean Theorum" Inquiry Lab. What kind of reasoning was used in each Activity?

Sample answer: In the first Activity I used measurement to demonstrate the Pythagorean Theorem so that was inductive reasoning. In this Activity, I used properties of mathematics to prove the Pythagorean Theorem so that was deductive reasoning.

8. Inquiry HOW can you prove the Pythagorean Theorem and its converse?

Sample answer: You can use a physical model and the properties of mathematics to construct proofs of the Pythagorean Theorem and its converse.

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Lesson 6

Use the Pythagorean Theorem

Real-World Link

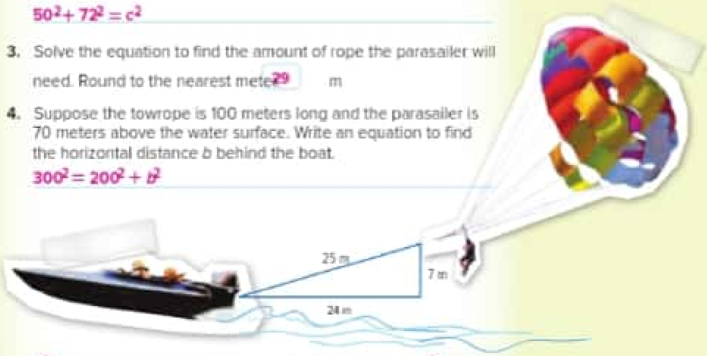
Parasailing In parasailing, a towrope is used to attach a parasailer to a boat. Refer to the diagram below for Exercises 1–4.

1. What type of triangle is formed by the horizontal distance, the vertical height, and the length of the towrope? Explain.
   right triangle; Since the sum of the squares of two sides is equal to the square of the third side, the triangle is a right triangle: $24^2 + 72^2 = 75^2$.

2. Suppose the wind picks up and the parasailer rises to 17 m and remains 24 m behind the boat. Write an equation that will help you find how much towrope *c* the parasailer will need.
   $50^2 + 72^2 = c^2$

3. Solve the equation to find the amount of rope the parasailer will need. Round to the nearest meter. **29** m

4. Suppose the towrope is 100 meters long and the parasailer is 70 meters above the water surface. Write an equation to find the horizontal distance *b* behind the boat.
   $300^2 = 200^2 + b^2$



Essential Question

HOW can algebraic concepts be applied to geometry?

Mathematical Practices 1, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Solve problems using the Pythagorean Theorem.

Coherence connecting within and across grades

Previous Students used models and diagrams to prove the Pythagorean Theorem.

Now Students will use the Pythagorean Theorem to solve problems.

Next Students will find the distance between two points on a coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 427.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Think-Pair-Solo Have students work in pairs. Give them about 20 seconds to think through their response to Exercise 1 individually, then have them share their response with a partner, taking care to make sure they justify their response. Then have them work together to complete Exercises 2 and 3. Have them work individually to complete Exercise 4. MP 1, 2, 3, 4, 6

Alternate Strategy

AL Provide students with copies of a blank Pythagorean Theorem equation, ____^{2} + ____^{2} = ____^{2} to use as they complete the exercises.

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2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Solve a right triangle.

- AL • *How can you tell that the triangle is a right triangle?* There is a right angle symbol.
- OL • *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- BL • *Why do we not use the negative square root?* Sample answer: The length of a ladder cannot be negative.

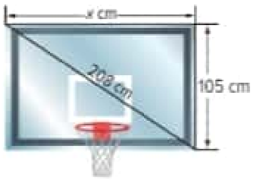
Need Another Example?
Write an equation that can be used to find the length of the boat ramp. Then solve. Round to the nearest tenth.

$4.2^2 + 25^2 = c^2$; 25.4 m

2. Solve a right triangle.

- AL • *Do you need to find the length of one of the legs or the hypotenuse?* one of the legs
- OL • *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- BL • *Derive a different equation that you can use to solve for the leg of a right triangle, when the other two sides are known.* Sample answer: When b is the unknown, you must subtract a^2 from both sides to get b^2 by itself; $b^2 = c^2 - a^2$.

Need Another Example?
Write an equation that can be used to find the length of the backboard. Then solve. Round to the nearest tenth.



$105^2 + x^2 = 208.5^2$; 180 cm

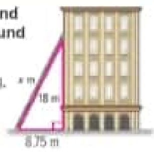
Work Zone

Solve a Right Triangle

The Pythagorean Theorem can be used to solve a variety of problems. It is helpful to use a diagram to determine what part of the right triangle is unknown.

Examples

1. Write an equation that can be used to find the length of the ladder. Then solve. Round to the nearest tenth.



Notice that the distance from the building, the building itself, and the ladder form a right triangle. Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8.75^2 + 18^2 = c^2$	Replace a with 8.75 and b with 18.
$76.5625 + 324 = c^2$	Evaluate 8.75^2 and 18^2.
$400.5625 = c^2$	Add 76.5625 and 324.
$\pm\sqrt{400.5625} = c$	Definition of square root
$\pm 20.0 \approx c$	Use a calculator.

Since length cannot be negative, the ladder is about 20 meters long.

2. Write an equation that can be used to find the height of the plane. Then solve. Round to the nearest tenth.

The distance between the planes is the hypotenuse of a right triangle. Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$10^2 + b^2 = 12^2$	Replace a with 10 and c with 12.
$100 + b^2 = 144$	Evaluate 10^2 and 12^2.
$b^2 = 44$	Subtraction Property of Equality
$b = \pm\sqrt{44}$	Definition of square root.
$b \approx \pm 6.6$	Use a calculator.

Since length cannot be negative, the height of the plane is about 6.6 kilometers.

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Got It? Do this problem to find out.

a. Mr. Khalid wants to build a new banister for the staircase shown. If the *rise* of the stairs of a building is 1.5 meters and the *run* is 3.6 meters, what will be the length of the new banister?

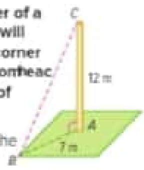
a. 3.9 m

The Pythagorean Theorem in Three-Dimensions

You can use the Pythagorean Theorem to find missing measures in three-dimensional figures.

Example

3. A 12 meter flagpole is placed in the center of a square area. To stabilize the pole, a wire will stretch from the top of the pole to each corner of the square. The flagpole is 7 meters from each corner of the square. What is the length of each wire? Round to the nearest tenth.



Draw right triangle *ABC*. You want to find the length of each wire or *BC*. This is the hypotenuse of a right triangle, so use the Pythagorean Theorem.

$AB^2 + AC^2 = BC^2$	Pythagorean Theorem
$7^2 + 12^2 = BC^2$	Replace AB with 7 and AC with 12.
$49 + 144 = BC^2$	Evaluate 7^2 and 12^2.
$193 = BC^2$	Simplify.
$\pm\sqrt{193} = BC$	Definition of square root.
$\pm 13.9 \approx BC$	Use a calculator.

Since length cannot be negative, the length of the wire is about 13.9 m.

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Example

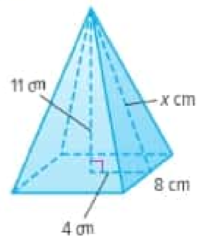
3. Use the Pythagorean Theorem in three dimensions.

AL
- *What do the wire and the pole form?* the hypotenuse and one leg of a right triangle
- *What could be used as the second leg of the right triangle?* the length from the pole to one corner of the square

OL
- *Do you need to find the length of one of the legs or the hypotenuse?* hypotenuse
- *What are the lengths of the legs?* 7 m and 12 m
- *Use AB and AC to denote the legs of the triangle. Use BC to denote the hypotenuse. What equation can be used to find the length of the wire?* $AB^2 + AC^2 = BC^2$
- *What value would you substitute for AB? AC?* 7; 12
- *Explain why we don't consider the negative square root to be a solution.* The length of the wire cannot be negative.

BL
- *Suppose your friend solved this problem and came up with an answer of approximately 9.7 meters. How would you know that this answer is incorrect without performing the calculations?* Sample answer: The length of the wire must be the longest side because it is the hypotenuse. Since 9.7 < 12, 9.7 cannot represent the hypotenuse. So, this answer is incorrect.

Need Another Example?
The *slant height* of a pyramid is the height of each lateral face. What is the slant height of the pyramid shown? Round to the nearest tenth. 11.7 cm



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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–4, making sure that every member understands. Call on a specific number from one team to present the team's solution to the class. Ask each student clarifying questions to ensure understanding. MP 1, 2, 4, 5, 6

Ask:

- *How did you know how to set up the equation?* See students' work.
- *What steps did you take to solve the equation?* See students' work.
- *How can you determine if your answer is reasonable?* See students' work.

BL **LA** **Gallery Walk** Have students work in pairs to create a real-world problem in which the Pythagorean Theorem must be used to solve the problem. In their problem, they should create a drawing. Then have them post the problems around the room. Pairs of students should walk around the room and select a problem, not their own, and return to their seats to solve it. After every pair has solved their problem, have pairs of students re-post the solutions around the room. The original pair locates their problem and determines if the solution is correct. MP 1, 2, 3, 4, 5, 6

Got It? Do this problem to find out.

b. The top part of a circus tent is in the shape of a cone. The tent has a radius of 16 meters. The distance from the top of the tent to the edge is 20 meters. How tall is the top part of the tent? Round to the nearest whole number.

b. 12 m

Guided Practice

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary. (Examples 1 and 2)

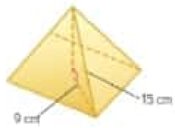
1. What is the height of the tent?

$3^2 + h^2 = 5^2$; 4 m

2. How high is the wheelchair ramp?

$30^2 + h^2 = 36.1^2$; 2.7 m

3. Nisreen made a model of a pyramid like the one shown for history class. What is the height of the model? (Example 3) 12 cm



4. **Building on the Essential Question** How do you solve a right triangle?
Sample answer: You need to determine what measurements represent the legs and the hypotenuse, and appropriately use the Pythagorean Theorem.

Rate Yourself!

☐ I understand how to apply the Pythagorean Theorem.

Great! You're ready to move on!

☐ I still have questions about how to apply the Pythagorean Theorem.

FOLDABLES Time to update your Foldable!

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Name ______________________ My Homework ______________________

Independent Practice

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. How far up the tree is the cat?

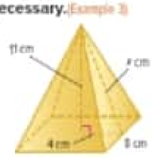
$1.5^2 + h^2 = 3.7^2$; 3.4 m

2. How deep is the water?

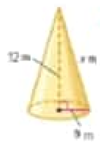
$6^2 + x^2 = 15^2$; 13.7 m

Find the missing measure in each figure below. Round to the nearest tenth if necessary. (Example 3)

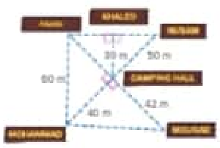
3. 11.7 cm



4. 15 m



5. Refer to the map of the Scout's Camp at the right. Round your answers to the nearest tenth.



a. How far is it from Khaled's cabin to Husam's cabin? 40 m

b. A camper in Fahd's cabin wants to visit a friend in Mohammad's cabin. How much farther is it if he walks to the Camping Hall first? 24.7 m

6. **Justify Conclusions** Ibrahim is buying a 165-centimeter-long fishing rod for his father. He wants to put it in a box so that his dad will not be able to guess what is in the box. The box he wants to use is 120 centimeters long and 120 centimeters wide. Will the pole fit in the box? Justify your reasoning.

yes; Sample answer: The corner of the box is a right angle. Find the length of the diagonal using the Pythagorean Theorem: $120^2 + 120^2 = 28,800$, $\sqrt{28}$ ≈ 170. Since the fishing rod is 165 centimeters long, it will fit diagonally in the box.

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Engage Explore **Explain** **Elaborate** **Evaluate**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 12, 13	5–7, 14–19	8–11
Level 3			•
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 11, 18, 19
OL	On Level	1–5 odd, 6–9, 11, 18, 19
BL	Beyond Level	5–11, 18, 19

Watch Out!

Common Error Students may substitute the side lengths of the triangle for any variable and then solve. Remind them that the variables *a* and *b* must be legs of the triangle and *c* must be the hypotenuse.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10, 16, 17
3 Construct viable arguments and critique the reasoning of others.	6, 9
4 Model with mathematics.	8, 11
7 Look for and make use of structure.	7

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

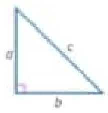
Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to write how they think using the Pythagorean Theorem will connect with the next lesson about finding the distance between two points on the coordinate plane. Use the writing prompt below. See students' work.

- The Pythagorean Theorem will help me find the distance between two points on the coordinate plane because ...

7. **Identify Structure** How do you use the Pythagorean Theorem?



Write the Pythagorean Theorem.
$a^2 + b^2 = c^2$

How do you find c when a and b are known?	How do you find a when c and b are known?
See students' work.	See students' work.

H.O.T. Problems Higher Order Thinking

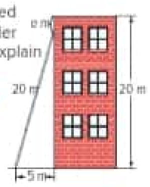
8. **Model with Mathematics** Write a real-world problem that can be solved by using the Pythagorean Theorem. Then explain how to solve the problem.
Sample answer: Nasser leaves his house. He walks 2 kilometers north, and then turns and walks 3 kilometers west. How far is Nasser from his house? Using the Pythagorean Theorem, $c^2 = 2^2 + 3^2$. Solving for c, Nasser is about 3.6 kilometers from his house.

9. **Which One Doesn't Belong?** Each set of numbers represents the side measures of a triangle. Identify the set that does not belong with the other three. Explain your reasoning.

3-4-5	12-35-37	3-5-7	6-8-10

3–5–7; $3^2 + 5^2 \neq 7^2$

10. **Persevere with Problems** Suppose a ladder 20 meters long is placed against a vertical wall 20 meters high. How far would the top of the ladder move down the wall by pulling out the bottom of the ladder 5 meters? Explain your reasoning.



about 0.6 m; By solving $20^2 = x^2 + 5^2$, you find that the ladder reaches approximately 19.4 m up the wall. Therefore, the top of the ladder would move down 20 m − 19.4 m or 0.6 m. by pulling out the bottom of the ladder 5 meters.

11. **Model with Mathematics** Write and solve a real-world problem that involves using the Pythagorean Theorem or its converse.
See students' work.

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Name ______________________ My Homework ______________________

Extra Practice

12. Write an equation to find how far the bird is from the boy. Then solve the equation. Round to the nearest tenth.

$70^2 + 20^2 = x^2$; 72.8 m

$a = 70, b = 20$, and $c = x$

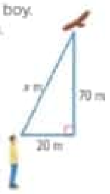
$70^2 + 20^2 = x^2$

$4{,}900 + 400 = x^2$

$5{,}300 = x^2$

$\sqrt{5{,}300} = x^2$

$72.8 \approx x$

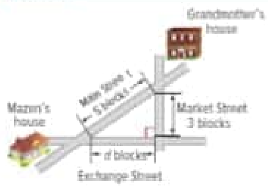


13. A hat is in the shape of a cone with dimensions shown.

Find the height of the hat. Round to the nearest tenth. **22.5 centimeters**

14. Mazen wants to go from his house to his grandmother's house. How much distance is saved if he takes Main Street instead of Market and Exchange?

2 blocks

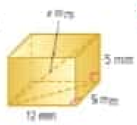


15. Suppose three towns form a right triangle. What is the distance between the two towns that form the hypotenuse?

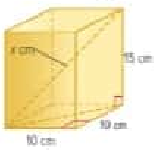
about 105 km

Persevere with Problems Find the missing measure in each figure below. Round to the nearest tenth if necessary.

16. **13.9 mm**



17. **20.6 cm**



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Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed when taking assessment.

18. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP7
Scoring Rubric	
2 points	Students correctly label the diagram and find the perimeter.
1 point	Students correctly label the diagram OR find the perimeter.

19. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

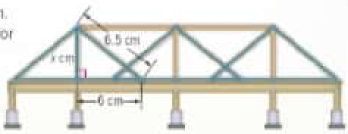
18. Suhaila designed a decorative glass window in the shape of a kite. Select the correct measures to label the dimensions of the window.

12 cm	42 cm
31 cm	45 cm
36 cm	60 cm
39 cm	

What is the perimeter of the window?

168 cm

19. Ayoub is building the model bridge shown. How long must he cut the piece of wood for one of the vertical support beams, represented by x?



2.5 cm

Spiral Review

20. Determine whether a triangle with sides 20 centimeters, 48 centimeters, and 52 centimeters long is a right triangle. Justify your answer.

yes; $20^2 + 48^2 = 52^2$

Estimate each of the following to the nearest whole number. Justify your reasoning.

21. $\sqrt{39} \approx$ 6

$6^2 = 36$ and $7^2 = 49$, since 39 is closer to 36 than 49, $\sqrt{39} \approx 6$.

22. $-\sqrt{146} \approx$ −12

$-(12^2) = -144$ and $-(13^2) = -169$. Since −146 is closer to −144 than −169, $-\sqrt{146} \approx -12$.

23. $\sqrt[3]{30} \approx$ 3

$3^3 = 27$ and $4^3 = 64$. Since 30 is closer to 27 than 64, $\sqrt[3]{30} \approx 3$.

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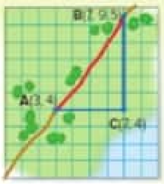
Lesson 7

Distance on the Coordinate Plane

Real-World Link

Mountain Biking Saeed was biking on a trail. A map of the trail is shown. His brother timed his ride from point *A* to point *B*.



1. What do the blue and red lines on the graph represent?
   The blue lines represent the horizontal and vertical distances between the two points. The red line represents the actual distance between the two points.
2. What type of triangle is formed by the lines?
   a right triangle
3. How can you find the length $\overline{AC}$ and $\overline{BC}$ without counting the number of units?
   Sample answer: subtract the *x*-coordinates and subtract the *y*-coordinates
4. What are the lengths of the two blue lines?
   $AC =$ 4 units $BC =$ 5.5 units
5. Write an equation using the Pythagorean Theorem that you can use to find the length $\overline{AB}$.
   $4^2 + 5.5^2 = c^2$

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

Distance Formula

Mathematical Practices

1, 3, 4, 5

Race ya!

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Find the distance between two points on the coordinate plane.

Coherence connecting within and across grades

Previous	Now	Next
Students used the Pythagorean Theorem to solve problems.	Students will find the distance between two points on a coordinate plane.	Students will apply distance on the coordinate plane to transformations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 435.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

ELL **Numbered Heads Together** Have students work in groups of 3–4 to complete Exercises 1–5. Assign each student a number. Each student is responsible to ask for help or support and for ensuring that their teammates understand each exercise. Call on one numbered student to explain the group's responses to the class. MP 1, 4, 5, 6

Alternate Strategy

AL Students may benefit from a quick review of how the Pythagorean Theorem can be used to find the length of a missing side of a right triangle.

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

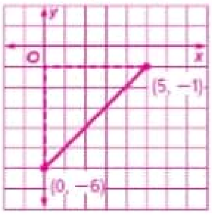
Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find distance on the coordinate plane.

- AL • *A right triangle can be formed using these two points and a third point (3, –5). On the drawn triangle, which side represents the distance that we need to determine?* **the hypotenuse, c**
  - *What is the length of the horizontal leg?* **4 units**
  - *What is the length of the vertical leg?* **5 units**
- OL • *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
  - *What value will we use for a? b?* **4 units; 5 units**
  - *Could we have used 5 units for a and 4 units for b?* **yes; The order in which you add does not matter.**
  - *Why do we not use the negative square root?* **Distance cannot be negative.**
- BL • *Could you draw a different triangle and still determine the same distance? Explain.* **yes; You could draw a horizontal line to the right of (3, 0) and a vertical line up from (7, –5).**

Need Another Example?
Graph the ordered pairs (0, –6) and (5, –1). Then find the distance c between the points. Round to the nearest tenth. **7.1 units**



Work Zone

Distance
To find the distance between two points on the coordinate plane, graph the points then draw a right triangle with c as the hypotenuse.

a. 3.2 units

Find Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

Example

1. Graph the ordered pairs (3, 0) and (7, –5). Then find the distance c between the two points. Round to the nearest tenth.

$a^2 + b^2 = c^2$ — Pythagorean Theorem

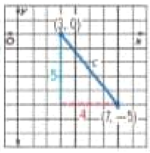
$4^2 + 5^2 = c^2$ — Replace a with 4 and b with 5.

$41 = c^2$ — $4^2 + 5^2 = 16 + 25$ or 41

$\pm\sqrt{41} = \sqrt{c^2}$ — Definition of square root

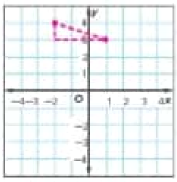
$\pm 6.4 \approx c$ — Use a calculator.

The points are about 6.4 units apart.



Get it? Do this problem to find out.

a. (1, 3), (–2, 4)

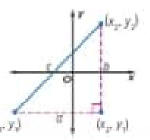


Key Concept

Distance Formula

Symbols The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Model



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You can also use the **Distance Formula** to find the distance between two points on the coordinate plane. You can use the model from the Key Concept box to see how the Distance Formula is based on the Pythagorean Theorem as shown below.

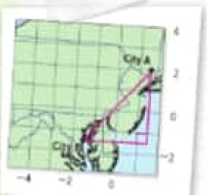
$c^2 = a^2 + b^2$ — Pythagorean Theorem

$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ — Substitute. The length of side a is $(x_2 - x_1)$ and the length of side b is $(y_2 - y_1)$.

$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Definition of square root

Example

2. **On the map, each unit represents 72 kilometers. City A is located at (1.5, 2) and City B is located at (−1.5, −1.5). What is the approximate distance between City A and City B?**



Method 1

Use the Pythagorean Theorem

Let c represent the distance between City A and City B. Then $a = 3$ and $b = 3.5$.

$a^2 + b^2 = c^2$

$3^2 + 3.5^2 = c^2$

$21.25 = c^2$

$\pm\sqrt{21.25} = \sqrt{c^2}$

$\pm 4.6 \approx c$

Method 2

Use the Distance Formula

Let $(x_1, y_1) = (1.5, 2)$ and $(x_2, y_2) = (-1.5, -1.5)$.

$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$c = \sqrt{(-1.5 - 1.5)^2 + (-1.5 - 2)^2}$

$c = \sqrt{(-3)^2 + (-3.5)^2}$

$c = \sqrt{9 + 12.25}$

$c = \sqrt{21.25}$

$c \approx \pm 4.6$

Since each map unit equals 72 kilometers, the distance between the cities is 4.6 · 72 or about 331 kilometers.

Got It? Do this problem to find out.

b. K Field is located at (2.5, 3.5) and L Field at (1.5, 4.5) on a map. If each map unit is 0.16 kilometers, about how far apart are the fields?

b. 0.224 kilometers

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Example

2. Use the Distance Formula to find actual distances.

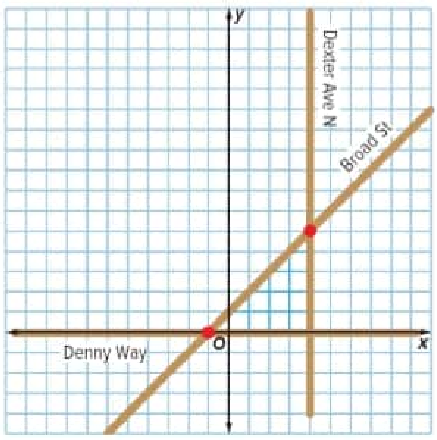
AL
- *What ordered pair represents City A?* **(1.5, 2)**
- *What ordered pair represents City B?* **(−1.5, −1.5)**

OL
- *If you make a right triangle to find the distance, what would be the length of each side?* **3 units and 3.5 units**
- *If you were to use the Distance Formula, what point would you use for (x_1, y_1)?* **(1.5, 2)**
- *What point would you use for (x_2, y_2)?* **(−1.5, −1.5)**
- *Once you find the map distance, what do you need to do to find the actual distance?* **Multiply 4.6 by 45.**

BL
- *Does it matter whether we use the point (1.5, 2) for (x_1, y_1) or for (x_2, y_2)? Explain.* **no; Sample answer: The distance between the two points will be the same, regardless of which point you consider, the first or second point.**

Need Another Example?

Fahd lives in Seattle, Washington. One unit on this map is 0.08 mile. Find the approximate distance he drives between Broad Street at Denny Way (−1, 0) and Broad Street at Dexter Avenue North (4, 5). **0.57 mi**



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Example

3. Use the Distance Formula.

AL • *What point would you use for* (x_1, y_1)? **(5, −4)**

• *What point would you use for* (x_2, y_2)? **(−3, −2)**

OL • *When you substitute the values in the formula, what expression is inside the radical sign?* $(-3-5)^2 + [-2-(-4)]^2$

• *How can you simplify this expression?* $(-8)^2 + 2^2$, or 68

BL • *Why do we not consider the negative square root?* **Distance is never negative.**

• *Suppose you forgot the Distance Formula. How could you determine the distance between these points?* **Sample answer: Use the Pythagorean Theorem.**

Need Another Example?
Use the Distance Formula to find the distance between $G(-3, -2)$ and $H(-6, 5)$. Round to the nearest tenth, if necessary. **7.6 units**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work in a group of 3–4 students to complete Exercise 1, ensuring that every team member understands. Then have them work in pairs to complete Exercise 2. Individually, they should complete Exercise 3 and then share their responses with their partner. MP 1, 4, 5, 6

BL LA **Pairs Present** Have students work with a partner to prepare a brief oral presentation about how the Distance Formula is derived from the Pythagorean Theorem. Their presentation should include illustrations. Have them present to the class. MP 1, 4, 5, 6

STOP and Reflect

Explain below how to find the length of a non-vertical and a non-horizontal segment whose endpoints are (x_1, y_1) and (x_2, y_2).

Sample answer: substitute the values for x_2 and x_1 and y_2 and y_1 into the Distance Formula. Then simplify.

Example

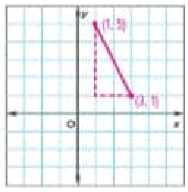
3. Use the Distance Formula to find the distance between $X(5, -4)$ and $Y(-3, -2)$. Round to the nearest tenth if necessary.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$XY = \sqrt{(-3-5)^2 + [-2-(-4)]^2}$	$(x_1, y_1) = (5, -4)$, $(x_2, y_2) = (-3, -2)$
$XY = \sqrt{(-8)^2 + 2^2}$	Simplify.
$XY = \sqrt{64 + 4}$	Evaluate $(-8)^2$ and 2^2.
$XY = \sqrt{68}$	Add 64 and 4.
$XY \approx \pm 8.2$	Simplify.

So, the distance between points X and Y is about 8.2 units.

Guided Practice

1. Graph the ordered pairs (1, 5) and (3, 1). Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)
4.5 units



2. On a park map, the ranger station is located at (2.5, 3.5) and the nature center is located at (0.5, 4). Each unit in the map is equal to 0.8 kilometers. What is the approximate distance between the ranger station and the nature center? (Examples 2 and 3) **1.6 kilometers**

3. **Building on the Essential Question** How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane?
Sample answer: After you plot the points, draw a right triangle. Use the Pythagorean Theorem to find the length of the hypotenuse which is the distance between the two points.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

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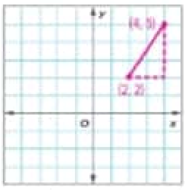
Name ______________________ My Homework ______________________

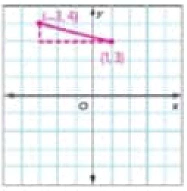
Independent Practice

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)

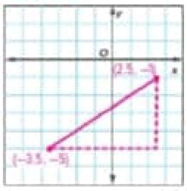
1. (4, 5), (2, 2) **3.6 units**



2. (−3, 4), (1, 3) **4.1 units**



(2.5, −1), (−3.5, −5) **7.2 units**



4. A ferry sets sail from an island located at (4, 12) on a map. Its destination is Ferry Landing *B* at (6, 2). How far will the ferry travel if each unit on the grid is 0.5 kilometer? (Example 2) **about 5.1 km**

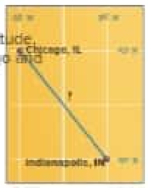
Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary. (Example 3)

C(−5, −3), *D*(−4, −2) **1.4 units**

6. *Y*(3.5, 1), *Z*(4, 2.5) **7.6 units**

7. $K\left(8\frac{1}{2}, 12\right), L\left(-6\frac{3}{4}, 7\frac{1}{2}\right)$ **15.9 units**

8. Chicago, Illinois, has a longitude of 88°W and a latitude of 42°N. Indianapolis, Indiana, is located at 86°W and 40°N. At this longitude/latitude, each degree is about 85 kilometers. Find the distance between Chicago and Indianapolis. **about 240 km**



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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 13–19	8, 9, 20, 21	10–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 10, 12, 20, 21
OL	On Level	1–7 odd, 8–10, 12, 20, 21
BL	Beyond Level	8–12, 20, 21

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	12
3 Construct viable arguments and critique the reasoning of others.	9
5 Use appropriate tools strategically.	10, 16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

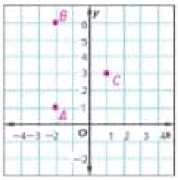
TICKET Out the Door

Have students explain how their knowledge of the Pythagorean Theorem helped them to find the distance between two points on a coordinate plane.
See students' work.

Watch Out!

Common Error When the Distance Formula calculations are done mentally, it is easy to incorrectly subtract a negative number from another number or replace a variable with an incorrect value. Encourage students to designate which values are x_1, x_2, y_1, and y_2, write the formula and all the steps, and then evaluate the formula.

9. **Multiple Representations** Points $A(-2, 1)$, $B(-2, 6)$, and $C(1, 3)$ are the vertices of a triangle.



   a. **Graphs** Graph the points $A(-2, 1)$, $B(-2, 6)$ and $C(1, 3)$.

   b. **Words** Explain how to find the length of segment BC.
   Sample answer: Use the Distance Formula and the points (−2, 6) and (1, 3).

   c. **Numbers** Find the length of each side of $\triangle ABC$. Round to the nearest tenth if necessary.
   $AC \approx$ 3.6 units
   $AB =$ 5 units
   $BC \approx$ 4.2 units

   d. **Numbers** What is the perimeter of $\triangle ABC$? Use the values from part c.
   perimeter ≈ 12.8 units

H.O.T. Problems Higher Order Thinking

10. **Use Math Tools** Kayla needs to find the distance between the points $A(-2.4, 3.7)$ and $B(4.5, -1.4)$. Suggest a tool she could use to find the length. Then find the length. Explain your reasoning.
Sample answer: Calculator; it will be most helpful when squaring and finding the square root involving decimals; about 8.6 units.

11. **Persevere with Problems** Apply what you have learned about distance on the coordinate plane to write the coordinates of two possible endpoints of a line segment that is neither horizontal nor vertical and has a length of 5 units.
Sample answer: (1, 2) and (4, 6)

12. **Reason Inductively** Compare the steps to find the distance between two points on the coordinate plane by first using the Pythagorean Theorem and then using the Distance Formula.
Sample answer: To use the Pythagorean Theorem, connect the points to form a right triangle. Then use the Pythagorean Theorem to find the length of the hypotenuse. To use the Distance Formula, replace (x_1, y_1) and (x_2, y_2) in the formula with the coordinates of the two endpoints and simplify.

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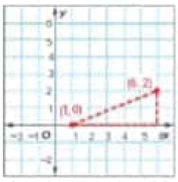
Name ______________ My Homework ______________

Extra Practice

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

13. (6, 2), (1, 0) 5.4 units



$a = 2, b = 5$

$a^2 + b^2 = c^2$

$2^2 + 5^2 = c^2$

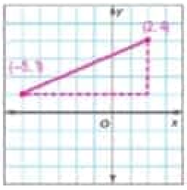
$4 + 25 = c^2$

$29 = c^2$

$\pm\sqrt{29} = \sqrt{c^2}$

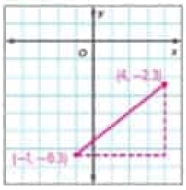
$\pm 5.4 \approx c$

14. (−5, 1), (2, 4) **7.6 units**



15. (4, −2.3), (−1, −6.3)

6.4 units



16. **Use Math Tools** On a map, Al Jabar is located at (3, 2.5), and Dhaman is located at (8.5, 14.5). Each unit on the map equals 26.4 kilometers. What is the approximate distance between the cities?

about 348.5 km

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary.

17. $W(1, 7)$, $X(-2, -4)$

11.4 units

18. $G(-6.25, 5)$, $H(-3.75, 2)$

3.9 units

19. $P\left(-9\frac{1}{4}, -7\frac{1}{2}\right)$, $Q(-4, 5)$

13.6 units

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Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed when taking assessment.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4, MP5
Scoring Rubric	
2 points	Students correctly plot and connect the points and also find the shortest combined distance.
1 point	Students correctly plot and connect the points, but fail to find the shortest combined distance OR students incorrectly plot points but draw lines and find a distance based on the incorrect points.

21.

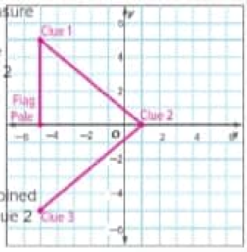
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2, MP5
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

20. Mr. Mansour is using a coordinate plane to design a treasure hunt for his students. The hunt begins at the flagpole. The first clue is hidden 5 units north of the flagpole. The second clue is located 6 units east of the flagpole. Clue 2 says that Clue 3 is located 5 units south of the flagpole.

Plot the locations of the flagpole and the 3 clues on the coordinate grid and show the path students will follow with straight lines.

Each unit represents 15 meters. What is the shortest combined distance along the path from the flagpole to Clue 1 to Clue 2 to Clue 3? Round to the nearest foot if necessary.



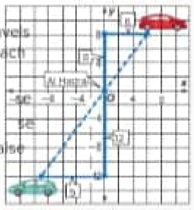
310 meters

21. Two cars leave a house in Al Hadra. The first car travels 8 kilometers north and then 6 kilometers east. The second car travels 12 kilometers south and then 9 kilometers west. Determine if each statement is true or false.

a. The first car is 10 kilometers from Al Hnd ■ True □ Fal

b. The second car is 15 kilometers from Al Hadra. ■ True □ Fal

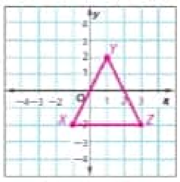
c. The cars are 35 kilometers apart. □ True ■ False



Spiral Review

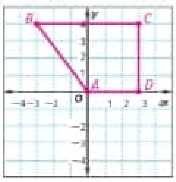
Graph each set of points on the coordinate plane. Then connect the points and identify the figure drawn. 6.G.3

22. $X(-1, -2)$, $Y(1, 2)$, $Z(3, -2)$



isosceles triangle

23. $A(0, 0)$, $B(-3, 4)$, $C(3, 4)$, $D(3, 0)$



right trapezoid

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA **Circle the Sage** Poll the class to see which students have knowledge about the Pythagorean Theorem. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Exercises 1–6. When the exercises are complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. MP 1, 2, 3, 4, 5, 6, 7

Ask:

- *How do right triangles help in reading a map and finding distances?* **Sample answer: Right triangles help find distances that are not easily distinguishable.**

BL LA **Pairs Discussion** Have students work in pairs to extend the activity by answering the following question. MP 1, 4, 5, 6, 7

Ask:

- *Describe the locations of Marathon, Cudjoe Key, and Islamorada using ordered pairs. Why is Cudjoe Key closer to Marathon than Islamorada?* **Sample answer: There are about 17.68 miles between Cudjoe Key and Marathon and about 20.16 miles between Islamorada and Marathon.**

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Agents with backgrounds in computer science, geography, communication, foreign languages, or world history often have a greater chance of being hired by travel agencies. These backgrounds show that the agents have an interest in travel and culture, which clients find appealing.

Time to Get Away!

Use the map to solve each problem. Round to the nearest tenth if necessary.

1. What is the approximate distance between Key Largo and Islamorada? **23 kilometers**
2. Draw and label a right triangle to find the distance between Plantation Key and Islamorada. Then find the approximate distance. 2.5 km, 2.5 km; **5.6 kilometers**
3. Describe the ordered pairs that represent Layton and Plantation Key. Then find the approximate distance between Layton and Plantation Key. **Layton: (7,–2); Plantation Key: (11, 1); 20 kilometers**
4. To the nearest 0.5 unit, name the ordered pairs that represent Key West and Cudjoe Key. Then use the ordered pairs to estimate the distance between the keys. **Sample answer: Key West (0, –7.5); Cudjoe Key (–4, –5); 26 kilometers**
5. What is the approximate distance between Key West and Layton? **Sample answer: 71.5 kilometers**
6. What is the approximate distance between Tavernier and Big Pine Key? **Sample answer: 62.5 kilometers**

Career Project

It's time to update your career portfolio! Go online and research a career as a travel agent.

Describe three things that you learned about being a travel agent that you did not know.

List several jobs that are created by the travel and tourism industry.

-
-
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Chapter Review

Vocabulary Check

Fill in the blank with the correct vocabulary term.

1. A line that intersects two or more lines is called a **transversal**.
2. **Corresponding angles** are those angles that are in the same position on the two lines in relation to the transversal.
3. **Deductive reasoning** uses facts, rules, definitions, or laws to make conjectures from given situations.
4. A statement or conjecture that has been proven and can be used as a reason to justify statements in other proofs is called a **theorem**
5. The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse of *any* right triangle.
6. Interior angles that lie on opposite sides of the transversal are called **alternate interior angles**.
7. Two lines that are in the same plane and do not intersect are called **parallel lines**
8. **Inductive reasoning** is the process making a conjecture after observing several examples.
9. The **hypotenuse** is the side opposite the right angle in a right triangle.
10. A polygon that is equilateral (all sides the same length) and equiangular (all angles have the same measure) is called a **regular polygon**

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Vocabulary Check

LA **Rally Coach** Have students work in pairs to complete the Vocabulary Check. Have Student A complete the first exercise, speaking out loud, while Student B listens carefully, coaches, and praises. Next have Student B complete the second exercise while Student A listens carefully, coaches, and praises. Partners take turns until they have completed the Vocabulary Check. MP 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- alternate interior angles (Lesson 1)
- corresponding angles (Lesson 1)
- deductive reasoning (Lesson 2)
- hypotenuse (Lesson 3)
- inductive reasoning (Lesson 2)
- parallel lines (Lesson 1)
- Pythagorean Theorem (Lesson 3)
- regular polygon (Lesson 4)
- theorem (Lesson 3)
- transversal (Lesson 1)

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Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How are the measures of angles related when parallel lines are cut by a transversal? (p. 374)
- How is deductive reasoning used in algebra and geometry proofs? (p. 382)
- How can you find the missing measure of an angle in a triangle if you know the measure of two of the interior angles? (p. 392)
- How can I find the sum of the interior angle measure of a polygon? (p. 400)
- What is the relationship among the legs and the hypotenuse of a right triangle? (p. 414)
- How do you solve a right triangle? (p. 426)
- How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane? (p. 434)

Ideas for Use

Think-Pair-Share Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 6

Track Your Progress

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

Reflect

Answering the Essential Question

Use what you learned about triangles and the Pythagorean Theorem to complete the graphic organizer. List three ways you used algebra in this chapter. Draw a model to represent each way.

Sample answers are given. See students' work for models.

Essential Question

HOW can algebraic concepts be applied to geometry?

Find the missing value in a triangle.	Find the measure of an exterior angle in a regular hexagon.	Use the Pythagorean Theorem to find the length of the missing side.

Answer the Essential Question. HOW can algebraic concepts be applied to geometry?

See students' work.

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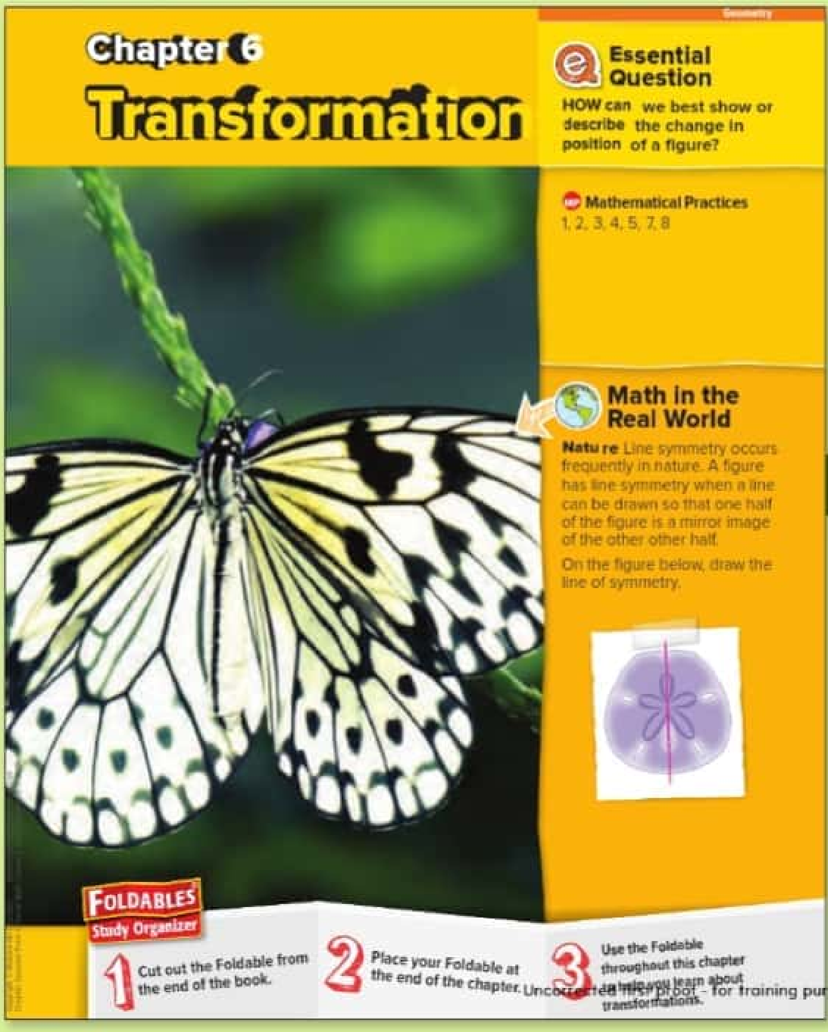

Chapter 6

Transformation

Essential Question

HOW can we best show or describe the change in position of a figure?

Mathematical Practices
1, 2, 3, 4, 5, 7, 8

Math in the Real World

Natu re Line symmetry occurs frequently in nature. A figure has line symmetry when a line can be drawn so that one half of the figure is a mirror image of the other other half.

On the figure below, draw the line of symmetry.

FOLDABLES Study Organizer

1. Cut out the Foldable from the end of the book.
2. Place your Foldable at the end of the chapter.
3. Use the Foldable throughout this chapter to help you learn about transformations.

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Focus narrowing the scope

This chapter focuses on content from the **Geometry** (G) domain.

Coherence connecting within and across grades

Previous Students used the Pythagorean Theorem.

Now Students study the effects of various types of transformations.

Next Students will explore congruence and similarity of figures.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Nature Remind students that a line of symmetry will always go through the center of an object.

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Are You Ready

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Graph on the coordinate plane.
2	Add integers.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3
Two vertices of a rectangle are $A(-2, 4)$ and $B(3, 4)$. The height of the rectangle is 3 units. Graph the rectangle and label the other two vertices. See Answer Appendix.

Exercises 4–11
Find $12 + (-4)$. 8

Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.

Are You Ready?

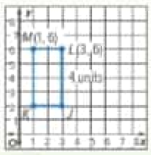
Try the Quick Check below.

Quick Review

Review

Example 1

Two vertices of a rectangle are $J(3, 2)$ and $K(1, 2)$. The length of the rectangle is 4 units. Graph the rectangle and label the other two vertices.



Example 2

Find $2 + (-6)$.

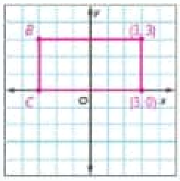
$2 + (-6) = -4$

$|2| - |-6| = -4$
The sum is negative because $|-6| > |2|$.

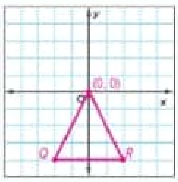
Quick Check

Coordinate Plane Graph each figure and label the missing vertices. Sample answers: 1 and 2

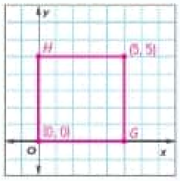
1. rectangle with vertices: $B(-3, 3)$, $C(-3, 0)$; side length: 6 units



2. triangle with vertices: $Q(-2, -4)$, $R(2, -4)$; height: 4 units



3. square with vertices: $G(5, 0)$, $H(0, 5)$; side lengths: 5 units



Integers Add.

4. $-5 + 3 =$ -2
5. $7 + (-9) =$ -2
6. $-4 + (-9) =$ -13
7. $-2 + 8 =$ 6
8. $-8 + (-6) =$ -14
9. $0 + (-6) =$ -6
10. $-8 + 2 =$ -6
11. $3 + (-1) =$ 2

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9 10 11

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Hands-On Activity 2

AL **LA** **Paired Heads Together** Have students work with a partner to complete the activity. Each student is assigned a number. Provide students with tracing paper, a protractor, and a ruler. Have them follow the steps using the given tools. Then have them complete the questions. Upon completion of the activity, call on one numbered student to share their responses with the class. **MP** 1, 5, 6, 7

BL **LA** **Pairs Discussion** Provide students with tracing paper, a protractor, and a ruler. Have them draw an angle that is not a right angle and label the points as in the directions. Have them follow the steps using the given tools. Then have them work in pairs to check each other's work and answer the questions. **MP** 1, 5, 6, 7

Hands-On Activity 3

AL **LA** **Solo to Groups** Provide students with tracing paper. Have them work individually to follow the steps, making sure to rotate in the correct direction. Then have students work in small groups to discuss the questions. **MP** 1, 5, 6, 7

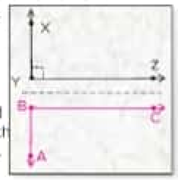
BL **LA** **Pairs Consult** Have students work in pairs to complete the activity and the questions. Then have them answer the following extension question. **MP** 1, 5, 6, 7

Ask:

- *The orientation of a figure is determined by the order in which the vertices are named. If you walk around the vertices W, Y, Z, and X in the original figure, you walk clockwise. What happens if you walk around the vertices W, Y, Z, and X after the turn? What does this tell you about the figure's orientation after the turn?* **Sample answer: You still walk clockwise. So, orientation is preserved after the turn.**

Hands-On Activity 2

Step 1 Draw right angle *XYZ* on a piece of tracing paper. Place a dashed line on the paper as shown.



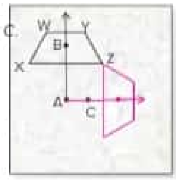
Step 2 Fold the paper along the dashed line. Trace the angle onto the folded portion of the paper. Unfold and label the angle *ABC* so that *A* matches up with *X*, *B* matches up with *Y*, and *C* matches up with *Z*. Tape the paper to your book.

Use a protractor to find the measure of ∠*XYZ* and ∠*ABC*. Did the measure of the angle change after the flip? **90°; 90°; no**

Use a centimeter ruler to measure the shortest distance from *X* and *A* to the dashed line. Repeat for *Y* and *B* and for *Z* and *C*. What do you notice?
See students' work; Sample answer: The distance from the original image to the dotted line is the same as the distance from the image to the dotted line.

Hands-On Activity 3

Step 1 Place a piece of tracing paper over the trapezoid shown. Copy the trapezoid. Draw points *A*, *B*, and *C*. Draw $\overline{AB}$.



Step 2 Place the eraser end of your pencil on *A*. Turn the tracing paper until $\overline{AB}$ passes through *C*. Tape the paper to your book.

Did the shape of the trapezoid change when you moved it? If yes, describe the change. **no**

Did the size of the trapezoid change when you moved it? If yes, describe the change. **no**

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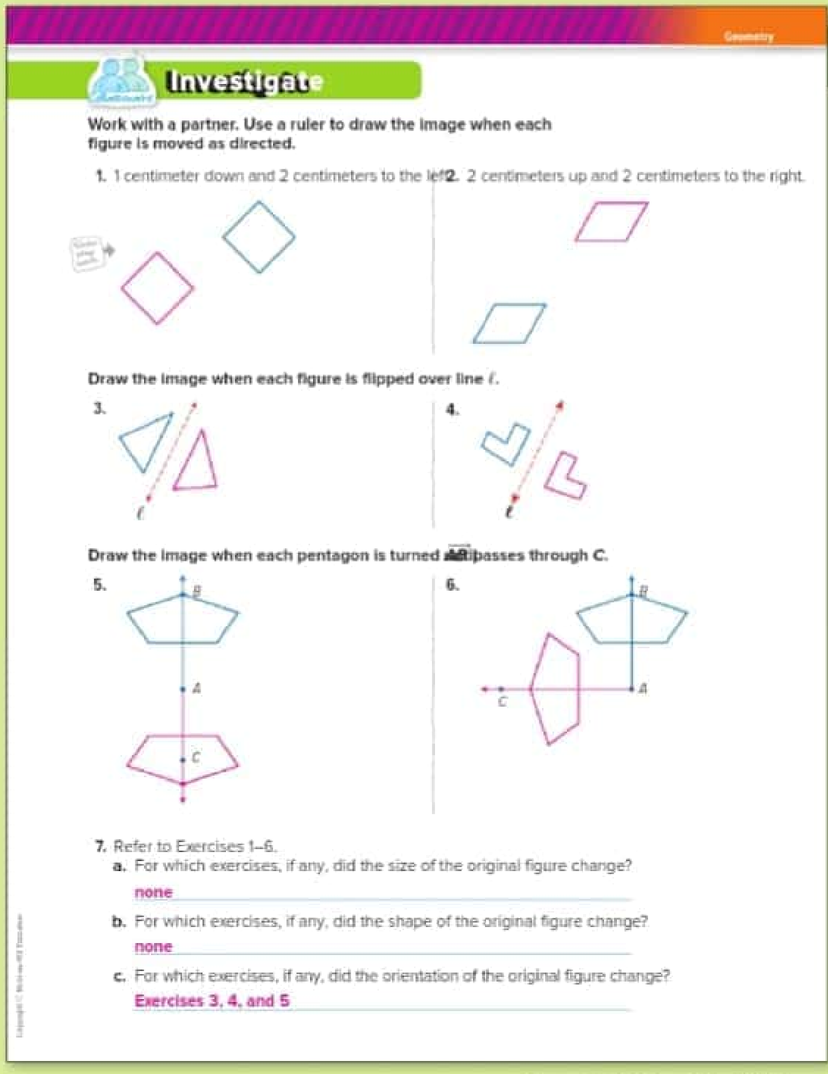

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–7	Exercises 8–13	Exercises 14, 15
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA **Group-Solo-Partner** Ask a volunteer to lead Exercise 1 in front of the class while students watch and listen carefully. Then have students complete Exercise 2 on their own. Continue this process for Exercises 3–6. Then have students discuss Exercise 7 with a partner. MP 1, 5, 6, 7

Ask:

- *For Exercises 3 and 4, what tool could we use in place of the dotted line to help draw the new image?* a mirror

BL LA **Roundrobin** Have students work in pairs. Have Student 1 complete Exercises 1, 3, and 5 while Student 2 completes Exercises 2, 4, and 6. Then have students exchange solutions and check each other's work. Have them discuss and respond to Exercise 7. MP 1, 5, 6, 7

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Analyze and Reflect

AL **LA** **Pairs Discussion** Have students use tracing paper to complete Exercises 8 and 9 as they did in the activities. Then have them work with a partner to complete Exercises 10–12. MP 1, 5, 6, 7

BL **LA** **Category Sort** Have students draw a pair of figures in which either a slide, flip, or turn has taken place. Then have students place all of their drawings in the center of the room. Have students pop up out of their seats to randomly select one drawing and classify it as a slide, flip, or turn. Have them place the drawing into a pile labeled either *Slide*, *Flip*, or *Turn*. MP 1, 5, 6, 7

Create

BL **LA** **Pairs Present** Have pairs research online or look indoors and outdoors for real-world examples of slides, flips, and turns. Have them explain to the class what they discovered and how they determined the type of transformation. MP 1, 4, 5, 6, 7

Inquiry Students should be able to answer "WHAT are some rigid motions of the plane?" Check for student understanding and provide guidance, if needed.

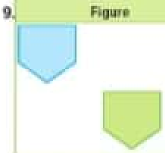
Analyze and Reflect

For each pair of figures, describe a movement or movements that will place the blue figure on top of the green figure. 8–9. Sample answers are given

8.

Figure	Movement(s)
	$\frac{1}{3}$ turn clockwise

9.

Figure	Movement(s)
	slide to the right and down

10. Refer to Activity 1 and Exercises 1 and 2. Circle the word that best describes the movement of the figures: flip (slide) turn

11. Refer to Activity 2 and Exercises 3 and 4. Circle the word that best describes the movement of the figures: (flip) slide turn

12. Refer to Activity 3 and Exercises 5 and 6. Measure one side of the original figures. Then measure that same side after the turn. Did the length of the side change after you turned it? If yes, describe the change.
no

13. **Justify Conclusions** Activity 3 $\overline{WY}$ and $\overline{XZ}$ are parallel. Were the segments still parallel after the turn? Would they still be parallel after a slide? flip? Explain.
yes; yes; yes; Sample answer: Since the size of the figure does not change in any of the movements, the distance between the two lines is the same, so the lines remain parallel.

Create

14. **Reason Inductively** Slides, flips, and turns are called *rigid motions of the plane*. Based on the Activities, describe two characteristics of a rigid motion of the plane. Sample answer: The shape and the size of a figure do not change in a rigid motion of the plane.

15. Inquiry WHAT are some rigid motions of the plane? Slides, flips, and turns are some rigid motions of the plane.

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Lesson 1
Translations

Vocabulary Start-Up

A **transformation** is an operation that maps an original geometric figure, the **preimage** onto a new figure called the **image**. A **translation** slides a figure from one position to another without turning it.

Scan the lesson and complete the graphic organizer.
Sample answers are given.

Translation

Define in Your Own Words	List 3 Characteristics
a slide without turning or flipping	Shape stays the same Size stays the same Faces the same way

Draw an Example

Draw a Nonexample

Essential Question

HOW can we best show or describe the change in position of a figure?

Vocabulary

transformation
preimage
image
translation
congruent

Math Symbols

$(x, y) \rightarrow (x + a, y + b)$
A' is read *A* prime

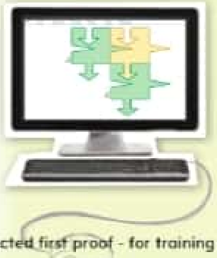
Mathematical Practices
1, 2, 3, 4, 8

Real-World Link

Amani created the design at the right on her computer.

1. Describe the motion involved in moving the design from *A* to *A'*.
over 1 space and down 1 space

2. Compare the size, shape, and orientation of the design piece in its original position to that of the piece in its new position.
They are the same.



Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Graph translations on the coordinate plane.

Coherence connecting within and across grades

Previous Students identified the properties of translations.

Now Students will graph translations on the coordinate plane.

Next Students will graph reflections on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 457.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Think-Pair-Share Give students about one minute to think through their responses to the graphic organizer on the student page. Then have them share their responses with a partner. MP 1, 5, 6

Alternate Strategy

AL LA Have students describe what a *translation* is in everyday life, such as the translation of a word from Spanish to English. Then have them explain the meaning of the term *translation* in mathematics. MP 1, 5, 6

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

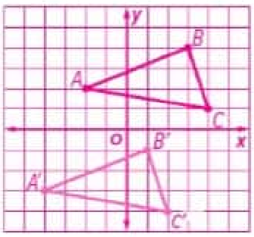
1. Translate a figure in the coordinate plane.

AL
- *How do we graph coordinate J?* **Start at the origin and move 3 units left and then 4 units up.** *coordinate K?* **Start at the origin and move 1 unit right and 3 units up.** *coordinate L?* **Start at the origin, move 4 units left, then 1 unit up.**

OL
- *If point J(−3, 4) is moved two units right and 5 units down, what are the coordinates of point J'?* **(−1, −1)**
- *If point K(1, 3) is moved two units right and 5 units down, what are the coordinates of point K'?* **(3, −2)**
- *If point L(−4, 1) is moved two units right and 5 units down, what are the coordinates of point L'?* **(−2, −4)**

BL
- *Was the orientation of the image changed?* **no**
- *Are the two images congruent?* **yes**

Need Another Example?
Graph △*ABC* with vertices *A*(2, 2), *B*(3, 4), and *C*(4, 1). Then graph the image of △*ABC* after a translation 2 units left and 5 units down. Write the coordinates of its vertices. **A'(−4, −3), B'(1, −1), C'(2, −4)**

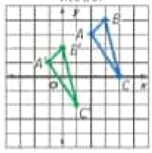


Key Concept

Translations in the Coordinate Plane

Words When a figure is translated, the *x*-coordinate of the preimage changes by the value of the horizontal translation *a*. The *y*-coordinate of the preimage changes by the vertical translation *b*.

Model



Symbols $(x, y) \rightarrow (x + a, y + b)$

Work Zone

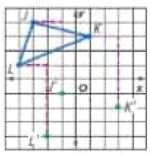
Prime Symbols
Use prime symbols for vertices in a transformed image.
A → A'
B → B'
B → B'
A' is read A prime.

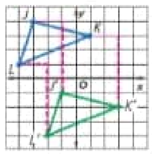
When translating a figure, every point of the preimage is moved the same distance and in the same direction. The image and the preimage are congruent. **Congruent** figures have the same shape and same size. So, line segments in the preimage have the same length as line segments in the image. Angles in the preimage have the same measure as angles in the image.

Example

1. Graph △*JKL* with vertices *J*(−3, 4), *K*(1, 3), and *L*(−4, 1). Then graph the image of △*JKL* after a translation 2 units right and 5 units down. Write the coordinates of its vertices.

Move each vertex of the triangle 2 units right and 5 units down. Use prime symbols for the vertices of the image.



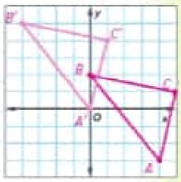


From the graph, the coordinates of the vertices of the image are *J'*(−1, −1), *K'*(3, −2), and *L'*(−2, −4).

Got it? Do this problem to find out.

a. Graph △*ABC* with vertices *A*(4, −3), *B*(0, 2) and *C*(5, 1). Then graph its image after a translation of 4 units left and 3 units up. Write the coordinates of the image.

a. **A'(0, 0), B'(−4, 5) and C'(1, 4)**



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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Paired Heads Together Have a student volunteer to lead a class discussion on how to translate the figure in Exercise 1. Then have students complete Exercise 2 with a partner, ensuring that each partner understands how to translate a figure. Have a student-volunteer draw the image on the board. If students are ready, have them complete Exercises 3–5 on their own. If not, have them work with a partner, while you walk around the room and monitor their progress. **MP 1, 5, 6, 7**

BL LA Trade-a-Problem Have students draw an image on a coordinate grid. Then have them write instructions in coordinate form, such as translation notation, for how to translate the image. Have students trade graph paper and translate each other's images using the translation notations only. **MP 1, 5, 6, 7**

Got it? Do this problem to find out.

c. Refer to the figure in Example 3. If point A was at (1, 5), use translation notation to describe the translation from point A to point B.

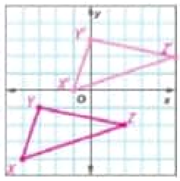
c. $(x + 1, y - 4)$

Guided Practice

Graph $\triangle XYZ$ with vertices $X(4, -4)$, $Y(-3, -1)$, and $Z(2, -2)$. Then graph the image of $\triangle XYZ$ after each translation, and write the coordinates of its vertices. (Example 1)

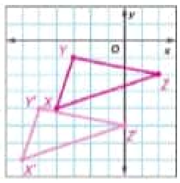
1. 3 units right and 4 units up

$X'(-1, 0)$, $Y'(0, 3)$, $Z'(5, 2)$



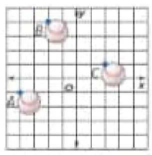
2. 2 units left and 3 units down

$X'(-6, -7)$, $Y'(-5, -4)$, $Z'(0, -5)$



3. The ball at the right was filmed using stop-motion animation so it appears to be thrown in the air. Use translation notation to describe the translation from point A to point B. (Example 3)

$(x + 2, y + 5)$



4. Quadrilateral $DEFG$ has vertices at $D(1, 0)$, $E(-2)$, $F(2, 4)$, and $G(6, -3)$. Find the vertices of $D'E'F'G'$ after a translation of 4 units right and 5 units down. (Example 2)

Sample answer: $D'(5, -5)$, $E'(2, -7)$, $F'(6, -1)$, and $G'(10, -8)$

5. **Building on the Essential Question** How are figures translated on the coordinate plane?

Sample answer: They are slid up or down and right or left.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

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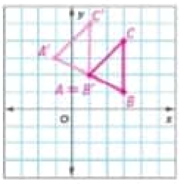
Name ____________________ My Homework ____________________

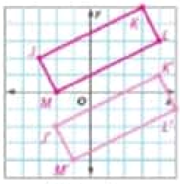
Independent Practice

Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices. (Example 1)

1. △*ABC* with vertices *A*(1, 2), *B*(3, 1), and *C*(3, 4) translated 2 units left and 1 unit up
   A′(−1, 3), B′(1, 2), C′(1, 5)



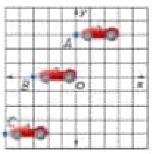
2. rectangle *JKLM* with vertices *J*(−3, 2), *K*(3, 5), *L*(4, 3), and *M*(2, 0) translated 1 unit right and 4 units down
   J′(−2, −2), K′(4, 1), L′(5, −1), M′(−1, −4)



Triangle *PQR* has vertices *P*(0, 0), *Q*(5, −2), and *R*(−3, 6). Find the vertices of *P′Q′R′* after each translation. (Example 2)

3. 6 units right and 5 units up **P′(6, 5), Q′(11, 3), R′(3, 11)**
4. 8 units left and 1 unit down **P′(−8, −1), Q′(−3, −3), R′(−11, 5)**

Use the image of the race car at the right. (Example 3)



5. Use translation notation to describe the translation from point *A* to point *B*. **(x − 3, y − 3)**
6. Use translation notation to describe the translation from point *B* to point *C*. **(x − 2, y − 4)**

7. Quadrilateral *KLMN* has vertices *K*(2, −2), *L*(1, 1), *M*(0, 4), and *N*(−3, 5). It is first translated by (*x* + 2, *y* − 1) and then translated by (*x* − 3, *y* + 4). When a figure is translated twice, a double prime symbol is used. Find the coordinates of quadrilateral *K″L″M″N″* after both translations.
   K″(−3, 1), L″(0, 4), M″(−1, 7), N″(−4, 8)

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

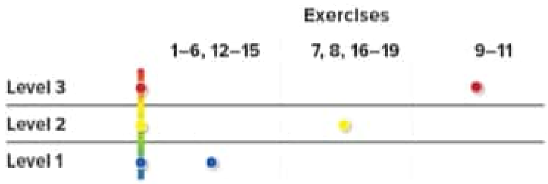
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 18, 19
OL	On Level	1–5 odd, 7–9, 11, 18, 19
BL	Beyond Level	7–11, 18, 19

Watch Out!

Common Error Suggest to students that they graph the original figures in one color and the translated images in another color to avoid confusion.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	9, 11
4 Model with mathematics.	8
8 Look for and express regularity in repeated reasoning.	17

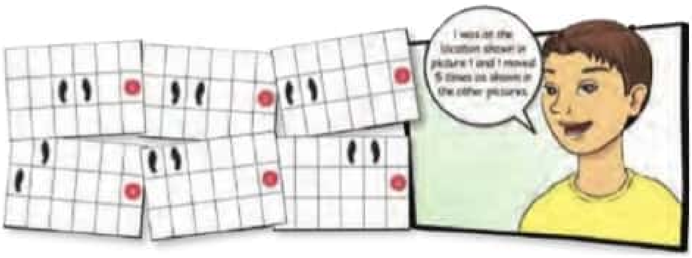
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students answer the following question: If point $P(-3, 2)$ is translated 3 units right and 2 units down, what are the coordinates of P'? **(0, 0)**

8. **Model with Mathematics** Refer to the graphic novel frame below. List the five steps the girl took and identify any transformations used in the movements. **Sample answer: right crosses over left; left crosses behind right; right forward one step; left forward one step; both hop three to the right; Steps and hops are translations.**



H.O.T. Problems Higher Order Thinking

9. **Reason Inductively** A figure is translated by $(x - 5, y + 7)$, then by $(x + 5, y - 7)$. Without graphing, what is the final position of the figure? Explain your reasoning to a classmate. **the same as the original position of the figure; Sample answer: Since −5 and 5 are opposites, and −7 and 7 are opposites, the translations cancel each other out.**

10. **Persevere with Problems** What are the coordinates of the point (x, y) after being translated m units left and n units up? $(x - m, y + n)$

11. **Reason Inductively** Determine whether each of the following statements is *always*, *sometimes*, or *never* true. Justify your reasoning.

a. A translation preserves orientation. **always; Sample answer: Each point moves the same distance and in the same direction.**

b. A preimage and its translated image are the same size, but not the same shape. **never; Sample answer: A preimage and image in a translation will always have the same size and shape.**

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Name ______________________ My Homework ______________________

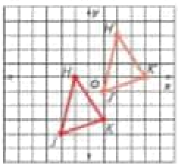
Extra Practice

Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

12. △*HJK* with vertices *H*(1, 0), *J*(−2, −4) and *K*(1, −3) translated 3 units right and 3 units up

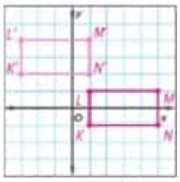
$H'(2, 3), J'(1, -1), K'(4, 0)$

Graph each point and connect them to form a triangle. Then move each point 3 units to the right and then 3 units up. Connect them to form △H'J'K'.



13. Rectangle *KLMN* with vertices *K*(1, −1), *L*(1, 1), *M*(5, 1), and *N*(5, −1) translated 4 units left and 3 units up

$K'(-3, 2), L'(-3, 4), M'(1, 4), N'(1, 2)$



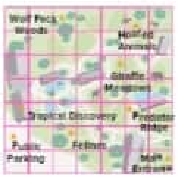
Quadrilateral *ABCD* has vertices *A*(−5, −1), *B*(−3, 0), *C*(2, −2), and *D*(0, −6). Find the vertices of *A'B'C'D'* after each translation.

14. 4 units up **$A'(-5, 3), B'(-3, 4), C'(2, 2), D'(0, -2)$**

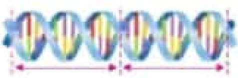
15. 2 units right and 2 units down **$A'(-3, -3), B'(-1, -2), C'(4, -4), D'(2, -8)$**

16. Ismail is in Colorado exploring part of the Denver Zoo as shown. He starts at the Felines exhibit and travels 3 units to the right and 5 units up. At which exhibit is Ismail located? If the Felines exhibit is located at (3, 1), what are the coordinates of Ismail's new location?

Hoofed Animals; (6, 6)



17. **Identify Repeated Reasoning** diagram of a DNA double helix is shown below. Look for a pattern. On the diagram indicate where this pattern repeats or is translated. Find how many translations of the original pattern are shown in the diagram. 1



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Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed for assessment.

18.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the figures and list the vertices.
1 point	Students correctly graph the figures but fail to list the vertices OR students correctly graph one figure and list the vertices.

19.

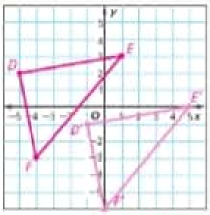
This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

18. Graph triangle *DEF* with vertices *D*(−5, 2), *E*(1, 3), and *F*(−4, −3). Then graph the image of the triangle after it is translated 4 units right and 3 units down.

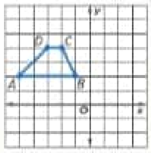
What are the vertices of triangle *D′E′F′*?

D′(−1, −1), *E′*(5, 0), *F′*(0, −6)



19. Trapezoid *ABCD* is shown on the coordinate plan. Suppose the trapezoid is translated 3 units right and 2 units up. Which of the following are vertices of the translated figure? Select all that apply.

- [x] *A′*(−2, 4)
- [ ] *B′*(1, −1)
- [ ] *C′*(0, 7)
- [x] *D′*(0, 6)



Spiral Review

Find each sum.

20. −5 + 12 = 7

21. 23 + (−3) = 20

22. −36 + (−42) = −78

23. 256 + (−82) = 174

24. −121 + (−119) = −240

25. −452 + 97 = −355

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Lesson 2

Reflections

Real-World Link

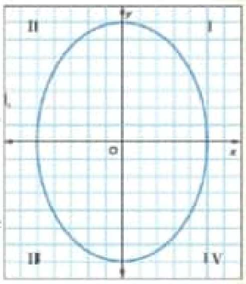
Art Pysanky is the ancient Ukrainian art of egg decorating. Many artists use flips and line symmetry to create their designs. Use the activity to create your own pysanky desi**See students' work.**

The template shown represents the front view of an egg. The template has been divided into four sections.

Step 1 To create your egg, draw a design in Quadrant II.

Step 2 To complete Quadrant I, draw the mirror image over the *y*-axis.

Step 3 Repeat Steps 2 and 3 to fill in Quadrants III and IV. You can create a new design or you can draw the mirror image over the *x*-axis.



Add color to your design by using colored pencils or markers to complete the design.

1. *Line symmetry* is when a figure can be folded so one side is the mirror image of the other side. Does your pysanky have line symmetry? Explain. **yes; Sample answer: The design was drawn as the mirror image over the *y*-axis.**

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Essential Question

HOW can you best show or describe the change in position of a figure?

Vocabulary

reflection
line of reflection

Math Symbols

$(x, y) \to (x, -y)$
$(x, y) \to (-x, y)$

MP **Mathematical Practices**
1, 3, 4, 7

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Focus narrowing the scope

Objective Graph reflections on the coordinate plane.

Coherence connecting within and across grades

Previous Students identified the properties of reflections.

Now Students will graph reflections on the coordinate plane.

Next Students will graph rotations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 465.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Have students work with a partner to complete the activity. Give students about one minute to think through how their design would look in Quadrants I, III, and IV. Then have them discuss their responses with a partner. Call on several pairs of students to share their drawings with the class. MP 1, 5, 6, 7

Alternate Strategy

AL Have students fold their paper in half to verify that their design has line symmetry.

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2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Reflect a figure over the *x*-axis.

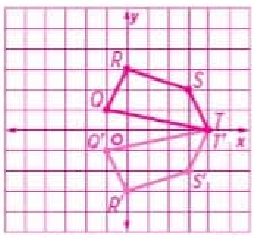
AL
- *What line in the coordinate plane are we using as the mirror?* **x-axis**
- *When reflecting a point over the x-axis, which coordinate remains the same?* **x-coordinate** *Which coordinate changes?* **y-coordinate**

OL
- *If point A is 2 units above the x-axis, where will it be after the reflection?* **2 units below the x-axis**
- *What algebraic notation explains the effect of this reflection?* **$(x, y) \rightarrow (x, -y)$**
- *What operation does the algebraic notation tell you to perform on the y-coordinate?* **Multiply by −1.**

BL
- *Is the orientation of the triangle preserved? Explain.* **no; Sample answer: After the triangle is reflected, the orientation of the points are reversed from counter clockwise to clockwise.**
- *Are the figures congruent?* **yes**

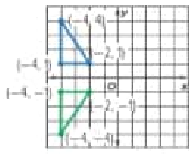
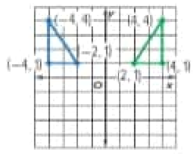
Need Another Example?

Quadrilateral *QRST* has vertices *Q*(−1, 1), *R*(0, 3), *S*(3, 2), and *T*(4, 0). Graph the figure and its reflected image over the *x*-axis. Then find the coordinates of the vertices of the reflected image. **Q′(−1, −1), R′(0, −3), S′(3, −2), T′(4, 0)**



Key Concept

Reflections in the Coordinate Plane

	Over the *x*-axis	Over the *y*-axis
Words	To reflect a figure over the *x*-axis, multiply the *y*-coordinates by −1.	To reflect a figure over the *y*-axis, multiply the *x*-coordinates by −1.
Symbols	$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$
Models		

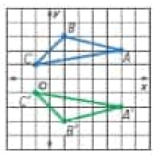
Work Zone

A **reflection** is a mirror image of the original figure. It is the result of a transformation of a figure over a line called a **line of reflection**. In a reflection, each point of the preimage and its image are the same distance from the line of reflection. So, in a reflection, the image is congruent to the preimage.

Examples

1. Triangle *ABC* has vertices *A*(5, 2), *B*(1, 3), and *C*(−1, 1). Graph the figure and its reflected image over the *x*-axis. Then find the coordinates of the vertices of the reflected image.

The *x*-axis is the line of reflection. So, plot each vertex of *A′B′C′* the same distance from the *x*-axis as its corresponding vertex on *ABC*.



Point *A* is 2 units above the *x*-axis, . . .

. . . so point *A′* is plotted 2 units below the *x*-axis.

The coordinates are *A′*(5, −2), *B′*(1, −3), and *C′*(−1, −1).

Check

Check the coordinates of the image by multiplying the *y*-coordinates by −1.

$(x, y) \rightarrow (x, -y)$
$(5, 2) \rightarrow (5, -2)$
$(1, 3) \rightarrow (1, -3)$
$(-1, 1) \rightarrow (-1, -1)$ ✓

Uncorrected first proof - for training purposes only

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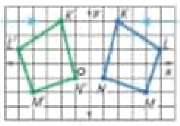
2. **Quadrilateral $KLMN$ has vertices $K(2, 3)$, $L(5, 1)$, M[illegible], and $N(1, -1)$. Graph the figure and its reflection over the y-axis. Then find the coordinates of the vertices of the reflected image.**

The y-axis is the line of reflection. So, plot each vertex of $K'L'M'N'$ the same distance from the y-axis as its corresponding vertex on $KLMN$.

Point K' is 2 units to the left of the y-axis.

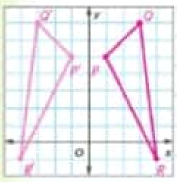


Point K is 2 units to the right of the y-axis.

The coordinates are $K'(-2, 3)$, $L'(-5, 1)$, $M'(-4, -2)$, and $N'(-1, -1)$.

Got it? Do this problem to find out.

a. Triangle PQR has vertices $P(1, 5)$, $Q(3, 7)$, and R[illegible]. Graph the figure and its reflection over the y-axis. Then find the coordinates of the reflected image.



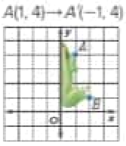
a. $P'(-1, 5)$, $Q'(-3, 7)$, $R'(-4, -1)$

Example

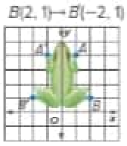
3. **The figure below is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the figure and its image on the coordinate plane.**

Point A is located at $(1, 4)$. Point B is located at $(2, 1)$. Since the figure is being reflected over the y-axis, multiply the x-coordinates by -1.

$A(1, 4) \rightarrow A'(-1, 4)$



$B(2, 1) \rightarrow B'(-2, 1)$



STOP and Reflect

Explain below how the x- and y-coordinates of an image relate to the x- and y-coordinates of the preimage after a reflection over the y-axis.

The x-coordinates are opposites and the y-coordinates are the same.

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Examples

2. Reflect a figure over the y-axis.

AL
- *What line in the coordinate plane are we using as the mirror?* y-axis
- *When reflecting a point over the y-axis, which coordinate remains the same?* y-coordinate *Which coordinate changes?* x-coordinate

OL
- *If point K is 2 units to the right of the y-axis, where will it be after the reflection?* 2 units to the left of the y-axis
- *What algebraic notation explains the effect of this reflection?* $(x, y) \rightarrow (-x, y)$

BL
- *What is another way to determine the coordinates of the reflected points?* Sample answer: Determine each point's location in respect to the origin. Point K is 2 units to the right of the origin and 3 units up. After the reflection, it will be 2 units to the left of the origin and 3 units up. Repeat this process for each point.

Need Another Example?

Triangle XYZ has vertices $X(1, 2)$, $Y(2, 1)$, and Z[illegible]. Graph the figure and its reflected image over the y-axis. Then find the coordinates of the vertices of the reflected image. See Answer Appendix.

3. Use a reflection to sketch the figure.

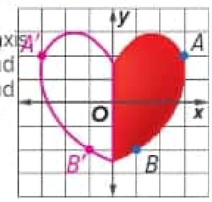
AL
- *Is the image of the frog symmetrical? Explain.* Yes, the left side is a reflection of the right side.

OL
- *How do you determine the new coordinates of a point that is reflected across the y-axis?* Multiply the x-coordinates by -1.

BL
- *What is another method you could use to plot A'?* A is 1 unit to the right and 4 units up from the origin. So, A' is 1 unit to the left and 4 units up from the origin.

Need Another Example?

The figure is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the figure and its image on the coordinate plane.



$A'(-3, 2)$, $B'(-1, -2)$

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Team-Pair-Solo Have students work in a four-person team to complete Exercise 1, then with a partner to complete Exercise 2. Teams and partners should ensure that each person understands how to reflect a figure. Have students individually think through their response to Exercise 3, then discuss their response with their partner or group. MP 1, 5, 6, 7

BL LA Trade-a-Problem Have students draw their own figure in one quadrant of the coordinate grid and plot at least 4 points. Have students trade the grid and reflect each other's figure over the *x*- or *y*-axis. Then have them work together to write the algebraic notations that explain the effect of the reflections. MP 1, 5, 6, 7

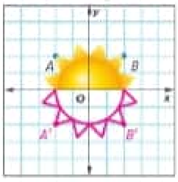
Watch Out!

Common Error In Exercise 1, students may reflect the image over the *y*-axis. Remind students that when reflecting over the *x*-axis, the *y*-coordinates are multiplied by –1, not the *x*-coordinates.

A'(–2, –2),
b. B'(2, –2)

Got it? Do this problem to find out.

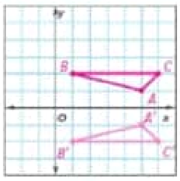
b. The figure at the right is reflected over the *x*-axis. Find the coordinates of point *A* and point *B*. Then sketch the image on the coordinate plane.



Guided Practice

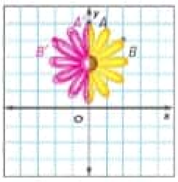
1. Graph △*ABC* with vertices *A*(5, 1), *B*(1, 2), and *C*(6, 2) and its reflection over the *x*-axis. Then find the coordinates of the image. (Examples 1 and 2)

A(5, –1), *B*(1, –2), *C*(6, –2)



2. The figure is reflected over the *y*-axis. Find the coordinates of point *A* and point *B*. Then sketch the image on the coordinate plane. (Example 3)

A'(0, 5), *B*'(–2, 4)



3. **Building on the Essential Question** How can you determine the coordinates of a figure after a reflection over either axis?

Sample answer: If you reflect over the *x*-axis, keep the *x*-coordinate and take the opposite of the *y*-coordinate. If you reflect over the *y*-axis, take the opposite of the *x*-coordinate and keep the *y*-coordinate.

Rate Yourself!

How well do you understand reflections? Circle the image that applies.

FOLDABLES Time to update your Foldable!

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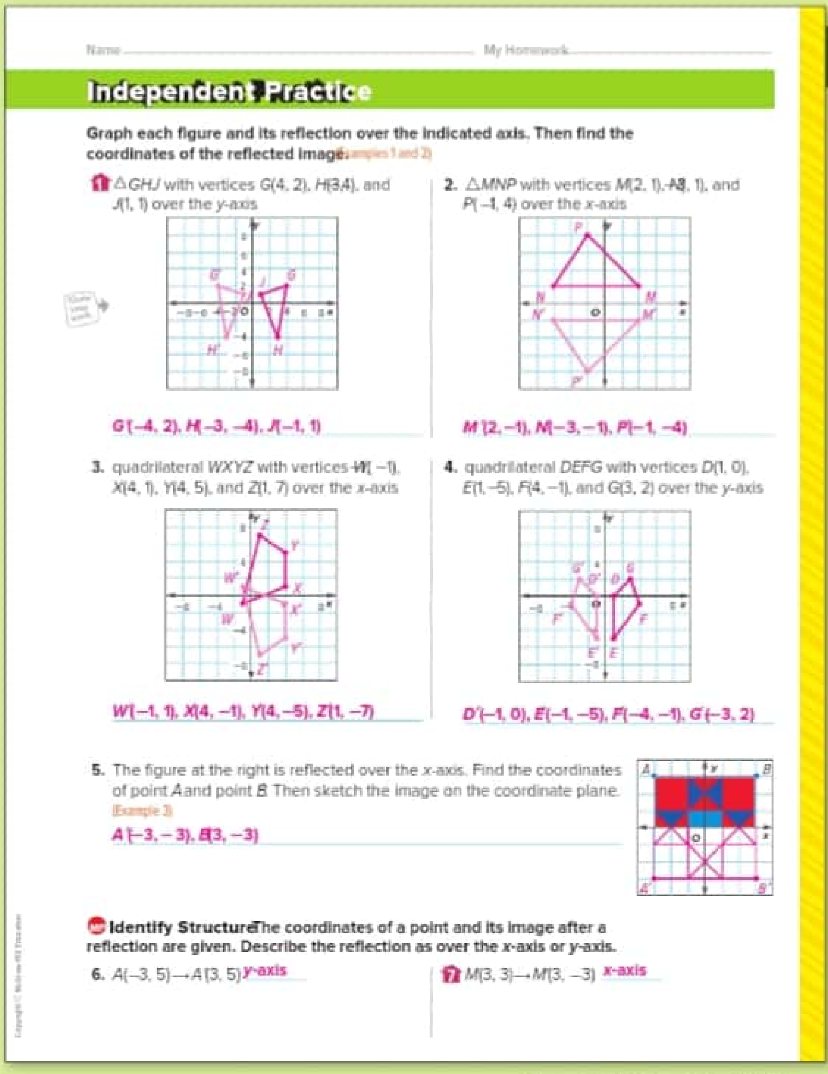
Name ______ My Homework ______

Independent Practice

Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image. (Examples 1 and 2)

1. △*GHJ* with vertices *G*(4, 2), *H*(3, 4), and *J*(1, 1) over the *y*-axis

   G′(−4, 2), H′(−3, −4), J′(−1, 1)

2. △*MNP* with vertices *M*(2, 1), *N*(−3, 1), and *P*(−1, 4) over the *x*-axis

   M′(2, −1), N′(−3, −1), P′(−1, −4)

3. quadrilateral *WXYZ* with vertices *W*(−1, −1), *X*(4, 1), *Y*(4, 5), and *Z*(1, 7) over the *x*-axis

   W′(−1, 1), X′(4, −1), Y′(4, −5), Z′(1, −7)

4. quadrilateral *DEFG* with vertices *D*(1, 0), *E*(1, −5), *F*(4, −1), and *G*(3, 2) over the *y*-axis

   D′(−1, 0), E′(−1, −5), F′(−4, −1), G′(−3, 2)

5. The figure at the right is reflected over the *x*-axis. Find the coordinates of point *A* and point *B*. Then sketch the image on the coordinate plane. (Example 3)

   A′(−3, −3), B′(3, −3)

Identify Structure The coordinates of a point and its image after a reflection are given. Describe the reflection as over the *x*-axis or *y*-axis.

6. *A*(−3, 5) → *A*′(3, 5) y-axis

7. *M*(3, 3) → *M*′(3, −3) x-axis

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–5, 13–17	6, 7, 18–21	8–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 8, 10–12, 20, 21
OL	On Level	1–5 odd, 6–8, 10–12, 20, 21
BL	Beyond Level	6–12, 20, 21

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9
3 Construct viable arguments and critique the reasoning of others.	8, 10, 11, 12
7 Look for and make use of structure.	6, 7, 18, 19

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

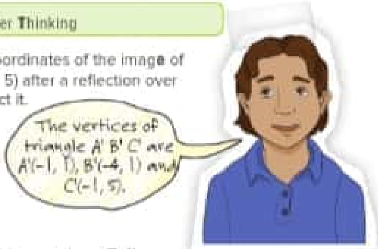
TICKET Out the Door

Ask students to compare translating a figure and reflecting a figure, and to write how previous lessons helped prepare them for concepts introduced in this lesson. Have them use the writing prompts below. **See students' work.**

- Reflecting a figure is similar to translating a figure because...
- Reflecting a figure is different from translating a figure because...
- What I learned about translations helped me to understand reflections in that...

H.O.T. Problems Higher Order Thinking

8. **Find the Error** Mazen is finding the coordinates of the image of triangle with vertices $A(1, 1)$, $B(4, 1)$ and $C(1, 5)$ after a reflection over the x-axis. Describe his mistake and correct it.



Mazen reflected the triangle over the y-axis. The coordinates should be $A'(1, -1)$, $B'(4, -1)$ and $C'(1, -5)$.

9. **Persevere with Problems** Triangle JKL has vertices $J(-7, 4)$, $K(7, 1)$, and $L(2, -2)$. Without graphing, find the new coordinates of the vertices of the triangle after a reflection first over the x-axis and then over the y-axis. **$J''(7, -4)$, $K''(-7, -1)$, $L''(-2, 2)$**

10. **Reason Inductively** Suppose you reflect a triangle in Quadrant I over the y-axis, then translate the image 2 units left and 3 units down. Is there a single transformation that maps the preimage onto the image? Explain your reasoning. **no; Sample answer: If the vertices of $\triangle ABC$ are $A(1, 2)$, $B(3, 4)$, and $C(1, 4)$, then the vertices of the final image are $A'(-3, -1)$, $B'(-5, 1)$, and $C'(-3, 1)$.**

11. **Reason Inductively** Suppose you reflect a nonregular figure over the x-axis and then reflect it over the y-axis. Is there a single transformation using reflections or translations that maps the preimage onto the image? Explain your reasoning.
no; Sample answer: If the vertices of $\triangle ABC$ are $A(0, 0)$, $B(2, 2)$, and $C(0, 4)$, then the vertices of the final image are $A'(0, 0)$, $B'(-2, -2)$, and $C'(0, -4)$.

12. **Which One Doesn't Belong?** Triangle ABC has vertices $A(1, 2)$, $B(1, 5)$, and $C(4, 2)$ and undergoes a transformation. Circle the set of vertices that does not belong. Explain your reasoning.

$A'(1, -1)$, $B'(1, 2)$, $C'(4, -1)$	$A'(5, 2)$, $B'(5, 5)$, $C'(8, 2)$
$A'(1, -2)$, $B'(1, -5)$, $C'(4, -2)$ (circled)	$A'(3, 3)$, $B'(3, 6)$, $C'(6, 3)$

Sample answer: This set is a reflection over the x-axis of $\triangle ABC$. The other sets are translations.

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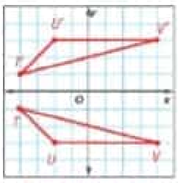
Name ______________________ My Homework ______________________

Extra Practice

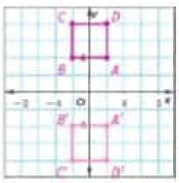
Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image.

13. △*TUV* with vertices *T*(6, −1), *U*(−2, −3), and *V*(4, −3) over the *x*-axis



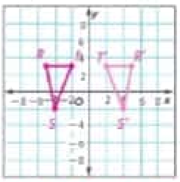
T′(−4, 1), U′(−2, 3), V′(4, 3)

14. square *ABCD* with vertices *A*(2, 4), *B*(, 4), *C*(−2, 8), and *D*(2, 8) over the *x*-axis



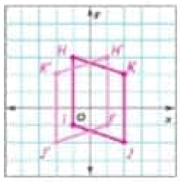
A′(2, −4), B′(−2, −4), C′(−2, −8), D′(2, −8)

15. △*RST* with vertices *R*(5, 3), *S*(−4, −2), and *T*(−2, 3) over the *y*-axis



R′(5, 3), S′(4, −2), T′(2, 3)

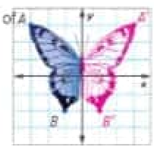
16. parallelogram *HIJK* with vertices *H*(3), *I*(−1, −1), *J*(2, −2), and *K*(2, 2) over the *y*-axis



H′(1, 3), I′(1, −1), J′(−2, −2), K′(−2, 2)

17. The figure at the right is reflected over the *y*-axis. Find the coordinates of point *A* and point *B*. Then sketch the image on the coordinate plane.

A′(3, 3), B′(1, −2)



Identify Structure The coordinates of a point and its image after a reflection are given. Describe the reflection as over the *x*-axis or *y*-axis.

18. *X*(−1, −4) → *X*′(−1, 4) *x*-axis

19. *W*(−4, 0) → *W*′(4, 0) *y*-axis

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Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the figure and list the vertices.
1 point	Students correctly graph the figure OR list the vertices.

21.

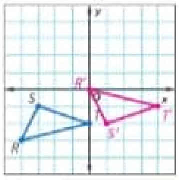
This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP5
Scoring Rubric	
1 point	Students correctly answer both parts of the question.

Power Up! Test Practice

20. Graph the image of triangle *RST* after it is reflected over the *x*-axis then translated 4 units to the right and 3 units down.

What are the vertices of triangle *R'S'T'*?

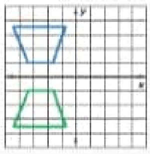
$R'(0, 0), S'(1, -2), T'(4, -1)$



21. The figure shown at the right was transformed from Quadrant II to Quadrant III.

Fill in each box to make a true statement to describe the transformation.

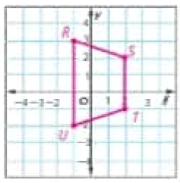
The figure was **reflected** over the ***x*-axis**.



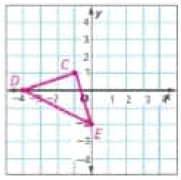
Spiral Review

Graph and label each figure on the coordinate plane.

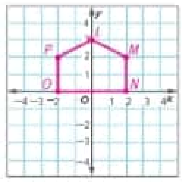
22. trapezoid *RSTU* with vertices $R(-1, 3)$, $S(2, 2)$, $T(2, -1)$, and $U(-1, -2)$



23. $\triangle CDE$ with vertices $C(1, 1)$, $D(-4, 0)$, and $E(0, -2)$



24. pentagon *LMNOP* with vertices $L(0, 3)$, $M(2, 2)$, $N(2, 0)$, $O(-2, 0)$, and $P(-2, 2)$



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Mid-Chapter Check

If students have trouble with Exercises 1–5, they may need help with the following concepts.

Concept	Exercise(s)
translations (Lesson 1)	1, 3, 5
reflections (Lesson 2)	2, 4, 5

Vocabulary Activity

EL Think-Pair-Share Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 6

Alternate Strategies

AL Provide students with the coordinates of the vertices of a triangle. Have students work with a partner to perform a reflection of the triangle over the *x*- or *y*-axis. Then have students discuss the role of the line of reflection in their transformation.

BL Have students create a graphic organizer of their own choosing to show the similarities and differences of translations and reflections.

Mid-Chapter Check

Vocabulary Check

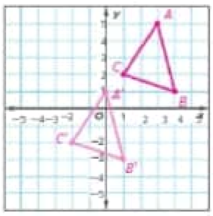
1. **Be Precise** Define *transformation* using the words *preimage* and *image*. (Lesson 1)
   Sample answer: A transformation maps a figure, the preimage, onto a new figure called the image.

2. Describe the role of the line of reflection in a transformation. (Lesson 2)
   Sample answer: The line of reflection is the fixed line over which a figure is reflected.

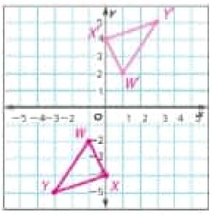
Skills Check and Problem Solving

Graph each triangle with the given vertices. Then graph the image after the given transformation and write the coordinates of the image's vertices. (Lessons 1 and 2)

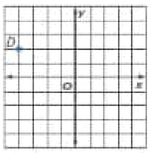
3. $\triangle ABC$ with vertices $A(3, 5)$, $B(4, 1)$, and $C(1, 2)$; translation of 3 units left and 4 units down
   $A'(0, 1)$, $B'(1, -3)$, $C'(-2, -2)$



4. $\triangle WXY$ with vertices $W(1, -2)$, $X(0, -4)$, and $Y(-3, -5)$; reflection over the *x*-axis followed by a reflection over the *y*-axis
   $W'(1, 2)$, $X'(0, 4)$, $Y'(3, 5)$



5. **Persevere with Problems** Point *D* is translated 5 units right and 2 units down, then reflected over the *y*-axis. Write an algebraic representation to represent the final location of point *D*. (Lessons 1 and 2)
   $(x, y) \rightarrow (-x - 5, y - 2)$



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Inquiry Lab
Rotational Symmetry

Inquiry **HOW can you identify rotational symmetry?**

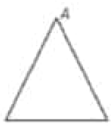
MP Mathematical Practices 1, 3

Many products have logos so people can easily identify them. If you turn the first aid logo below 180°, will the logo look the same as the original figure?

Hands-On Activity

A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position.

Step 1 Copy the outline of the equilateral triangle onto a piece of tracing paper. Label one vertex *A*.



Step 2 Place the tracing paper over the outline in Step 1. Put your pencil point at the center of the figure to hold the tracing paper in place. Turn the tracing paper clockwise from its original position until the two figures match. Draw and label the new figure in the space provided.

Step 3 Continue turning the tracing paper until the logo is back to its original position. Does the figure have rotational symmetry? Explain.

Yes; Sample answer: the figure was turned less than 360° about its center and still looked like the original.

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Focus narrowing the scope

Objective Identify rotational symmetry.

Coherence connecting within and across grades

Now Students will identify properties of rotational symmetry.

Next Students will graph rotations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 474.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Teammates Consult** Have students work in small teams. Give each team tracing paper. Ask a volunteer to lead the activity, showing how to rotate the paper without letting it slide. Then have them discuss the question in Step 3. Have one team share their responses with the class to initiate a class discussion about what it means for a figure to have rotational symmetry. MP 1, 3, 5, 6, 7

BL LA **Pairs Consult** After completing the activity, have students work with a partner to draw and color a pattern inside the logo that will change its rotational symmetry to be 180°. MP 1, 5, 6, 7

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises

	1–3	4–6	7, 8
Level 3			•
Level 2		•	
Level 1	•		

Investigate

AL LA **Team-Pair-Solo** Give students tracing paper and have them trace the outline of each figure. As they did in the activity, have them use a pencil to hold the paper in place and rotate the paper until it matches with the original picture. Have students work in small teams to complete Exercise 1. Then have them work with a partner to complete Exercise 2, and have them work on their own to complete Exercise 3. Have them rejoin their original teams to discuss and compare response s. MP 1, 5, 6, 7

Create

BL Have students design and color their own personal logo with a degree of rotational symmetry. Pass their designs around the room and have students determine the degree of rotationa l symmetry for each other's work. MP 1, 5, 6, 7

Inquiry Students should be able to answer "HOW can you identify rotational symmetry?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Determine whether the figure has rotational symmetry. Write *yes* or *no*.

1. 
yes

2. 
no

3. 
yes

Analyze and Reflect

4. MP **Reason Inductively** The degree measure of an angle through which the figure is rotated is called the **angle of rotation**. Find the first angle of rotation of the equilateral triangle by dividing 360° by the total number of times the figures matched. 90°

5. List the other angles of rotation of the equilateral triangle by adding the measure of the first angle of rotation to the previous angle measure. Stop when you reach 360°. 180°, 270°

6. What is the angle of rotation of each figure in Exercises 1–3? Write *no* if there is no rotational symmetry.
Exercise 1: 180° Exercise 2: no Exercise 3: 60°

Create

7. MP **Model with Mathematics** Draw two figures, one that has rotational symmetry and one that does not. See students' work.

8. Inquiry HOW can you identify rotational symmetry?
Sample answer: You can identify rotational symmetry by turning the figure less than 360° and determining if the figure looks the same as the original.

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Lesson 3

Rotations

Real-World Link

Prizes Majed is spinning the prize wheel shown below.

1. A spin can be *clockwise* or *counterclockwise*. Define these two words in your own words.

   clockwise **rotating to the right**

   counterclockwise **rotating to the left**

2. If the section labeled 8 on the left part of the wheel spins 90° clockwise, where will it land? **at the top**

3. If one of the sections labeled 4 makes three complete turns counterclockwise, how many degrees will it have traveled? **1,080** °

4. Are there any points on the wheel that stay fixed, or do not move, when the wheel spins? If so, what are the points?
   yes; the center

5. Does the center of the wheel change if the wheel is spun counterclockwise as opposed to clockwise? **no**

6. Does the distance from the center to the edge change as it spins? Explain.
   no; Sample answer: The distance from the center to the edge is the radius of the circle. The size of the circle does not change as it spins so the radius does not change.

Essential Question

HOW can we best show or describe the change in position of a figure?

Vocabulary

rotation
center of rotation

Math Symbols

$(x, y) \rightarrow (y, -x)$
$(x, y) \rightarrow (-x, -y)$
$(x, y) \rightarrow (-y, x)$

Mathematical Practices
1, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Graph rotations on the coordinate plane.

Coherence connecting within and across grades

Previous Students identified the properties of rotational symmetry.

Now Students will graph rotations on the coordinate plane.

Next Students will graph dilations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 479.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Cooperative Play** Show a similar spinner at the front of the class or use an online spinner. Give students time to play with the spinner so that they can visualize the movements made in the activity. Then have them complete the activity with a partner. MP 1, 4, 5, 7

Alternate Strategy

AL LA Have students use a clock face to visualize the meanings of *clockwise* and *counterclockwise*. MP 1, 5

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Rotate a figure about a point.

AL • *About what point are we rotating?* **about vertex L**

OL • *Describe vertex M in comparison to vertex L.* **Vertex M is 3 units up from vertex L.**

• *After the rotation, how far away from vertex L will vertex M' be?* **Vertex M' will be 3 units down from vertex L.**

• *Describe vertex N in comparison to vertex L.* **Vertex N is 3 units up and 3 units to the right from vertex L.**

• *After the rotation, how far away from vertex L will vertex N' be?* **Vertex N' will be 3 units down and 3 units to the left from vertex L.**

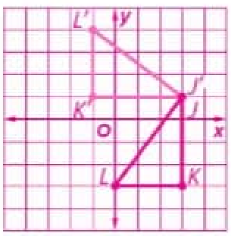
BL • *After the rotation, where will vertex L' be in comparison to vertex L? Explain.* **Vertex L' will be in the same location as vertex L. Vertex L is the center of rotation.**

• *Are the two figures congruent?* **yes**

Need Another Example?

Triangle *JKL* has vertices *J*(3, 1), *K*(3, −3), and *L*(0, −3). Graph the figure and its image after a clockwise rotation of 90° about vertex *J*. Then give the coordinates of the vertices for △*J'K'L'*.

***J'*(3, 1), *K'*(−1, 1), *L'*(−1, 4)**



Work Zone

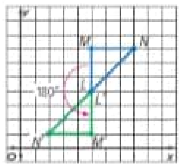
Rotate a Figure About a Point

A **rotation** is a transformation in which a figure is rotated, or turned, about a fixed point. The **center of rotation** is the fixed point. A rotation does not change the size or shape of the figure. So, the preimage and the image are congruent.

Example

1. Triangle *LMN* with vertices *L*(5, 4), *M*(5, 7), and *N*(8, 7) represents a desk in Ibrahim's bedroom. He wants to rotate the desk counterclockwise 180° about vertex *L*. Graph the figure and its image. Then give the coordinates of the vertices for △*L'M'N'*.

Step 1 Graph the original triangle.



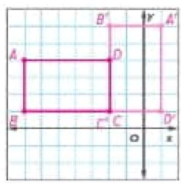
Step 2 Graph the rotated image. Use a protractor to measure an angle of 180° with *M* as one point on the ray and *L* as the vertex. Mark off a point the same length as $\overline{ML}$. Label this point *M'* as shown.

Step 3 Repeat Step 2 for point *N*. Since *L* is the point at which △*LMN* is rotated, *L'* will be in the same position as *L*.

So, the coordinates of the vertices of △*L'M'N'* are *L'*(5, 4), *M'*(5, 1), and *N'*(2, 1).

Got it? Do this problem to find out.

a. Rectangle *ABCD* with vertices *A*(−7, 4), *B*(−7, 1), *C*(−2, 1), and *D*(−2, 4) represents the bed in Ibrahim's room. Graph the figure and its image after a clockwise rotation of 90° about vertex *C*. Then give the coordinates of the vertices for rectangle *A'B'C'D'*.



a. ***A'*(1, 6), *B'*(−2, 6), *C'*(−2, 1), *D'*(1, 1)**

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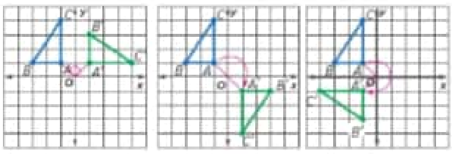
Rotations About the Origin

Key Concept

Words A rotation is a transformation around a fixed point. Each point of the original figure and its image are the same distance from the center of rotation.

Models The rotations shown are clockwise rotations about the origin.

90° Rotation **180° Rotation** **270° Rotation**



Symbols

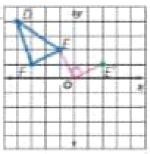
$(x, y) \rightarrow (y, -x)$ $(x, y) \rightarrow (-x, -y)$ $(x, y) \rightarrow (-y, x)$

Figures can also be rotated about the origin.

Example

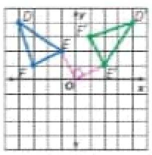
2. Triangle *DEF* has vertices *D*(−4, 4), *E*(−1, 2), and *F*(−3, 1). Graph the figure and its image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices of △*D'E'F'*.

Step 1 Graph △*DEF* on a coordinate plane.



Step 2 Sketch segment $\overline{EO}$ connecting point *E* to the origin. Sketch another segment, $\overline{E'O}$, so that the angle between point *E*, *O*, and *E'* measures 90° and the segment is the same length as $\overline{EO}$.

Step 3 Repeat Step 2 for points *D* and *F*. Then connect the vertices to form △*D'E'F'*.



So, the coordinates of the vertices of △*D'E'F'* are *D'*(4, 4), *E'*(2, 1), and *F'*(1, 3).

Check

Check the coordinates of the image.

$(x, y) \rightarrow (y, -x)$

$(-4, 4) \rightarrow (4, 4)$

$(-1, 2) \rightarrow (2, 1)$

$(-3, 1) \rightarrow (1, 3)$ ✓

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Example

1. Rotate a figure about the origin.

AL • *About what point are we rotating?* **about the origin**

OL • *Describe the location of point E in reference to the origin.* **It is 1 unit to the left of and 2 units above the origin.**

• *Describe the location of point E' in reference to the origin.* **It is 2 units to the right of and 1 unit above the origin.**

• *Using this as a guide, what will be the location of point F' in reference to the origin? point D'?* **Point F' will be 1 unit to the right and 3 units above the origin. Point D' will be 4 units to the right and 4 units above the origin.**

BL • *Are the two figures congruent?* **yes**

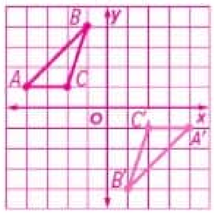
• *After a 180° rotation clockwise about the origin, what does the point (x, y) become?* **$(x, y) \rightarrow (-x, -y)$**

• *After a 270° rotation clockwise about the origin, what does the point (x, y) become?* **$(x, y) \rightarrow (-y, x)$**

• *After a 360° rotation clockwise about the origin, what does the point (x, y) become?* **$(x, y) \rightarrow (x, y)$**

Need Another Example?

Triangle *ABC* has vertices *A*(−4, 1), *B*(−1, 4), and *C*(−2, 1). Graph the figure and its image after a counterclockwise rotation of 180° about the origin. Then give the coordinates of the vertices for △*A'B'C'*. ***A'*(4, −1), *B'*(1, −4), *C'*(2, −1)**



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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

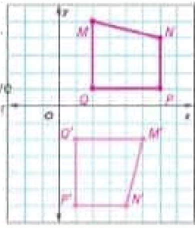
AL LA Groups-Pairs-Solo If students are having trouble understanding how to rotate images, you may wish to complete Exercise 1 together as a whole group. You may also wish to give the students tracing paper to help them see how the image is rotated. Have them trace the original triangle and then press their pencil down on the point of rotation (vertex *X* for Exercise 1, the origin for Exercise 2) to rotate the paper. Then have students work with a partner to complete Exercise 2. Have them work individually to complete Exercise 3. Then have them rejoin the whole group to discuss answers and compare solutions. MP 1, 5, 7

BL LA Pairs Discussion For Exercises 1 and 2, have students predict the coordinates of the image after the rotation without graphing. Then have them compare the coordinates after graphing to see if their prediction was correct. MP 1, 3, 5, 6, 7

b. $M'(5, -2)$, $N'(4, -6)$, $P'(1, -6)$, $Q'(1, -2)$

Got it? Do this problem to find out.

b. Quadrilateral *MNPQ* has vertices *M*(2, 5), *N*(6, 4), *P*(6, 1), and *Q*(2, 1). Graph the figure and its image after a counterclockwise rotation of 270° about the origin. Then give the coordinates of the vertices for quadrilateral *M'N'P'Q'*.

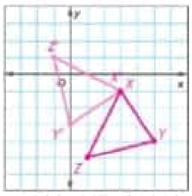


Guided Practice

Triangle *XYZ* has vertices *X*(3, −1), *Y*(5, −4), and *Z*(1, −5). Graph *XYZ* and its image after each rotation. Then give the coordinates of the vertices for △*X'Y'Z'*. (Examples 1 and 2)

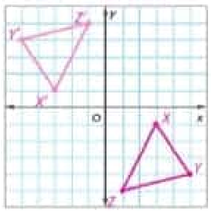
1. 270° counterclockwise about vertex *X*

$X'(3, -1)$, $Y'(0, -3)$, $Z'(-1, 1)$



2. 180° clockwise about the origin

$X'(-3, 1)$, $Y'(-5, 4)$, $Z'(-1, 5)$



3. **Building on the Essential Question** What is the difference between rotating a figure about a given point that is a vertex and rotating the same figure about the origin if the rotation is less than 360°?

Sample answer: If you rotate the figure about one of the vertices, that point stays the same. If you rotate the same figure about the origin, all of the points are different unless one of the vertices is the origin.

Rate Yourself!

How confident are you about rotations? Check the box that applies.

FOLDABLES Time to update your Foldable!

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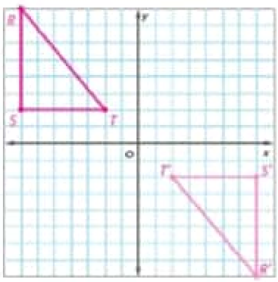
Name ______________________ My Homework ______________________

Independent Practice

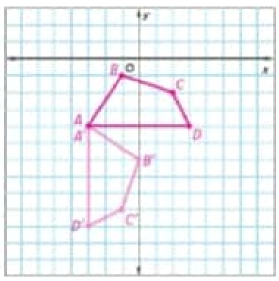
1. Triangle *RST* represents the placement of Fawzia's tricycle in the driveway and has vertices *R*(–7, 8), *S*(–7, 2), and *T*(–2, 2). Graph the figure and its rotated image after a clockwise rotation of 180° about the origin. Then give the coordinates of the vertices for triangle *R'S'T'*. (Example 2)

R'(7, –8), S'(7, –2), T'(2, –2)

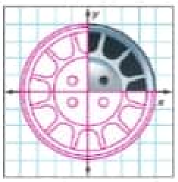


2. Quadrilateral *ABCD* has vertices at *A*(–3, –4), *B*(–1, –1), *C*(2, –2), and *D*(3, –4). Graph quadrilateral *ABCD* and its image after a 90° clockwise rotation about vertex *A*. Then give the coordinates of the vertices of the image. (Example 1)

A'(–3, –4), B'(0, –6), C'(–1, –9), D'(–3, –10)

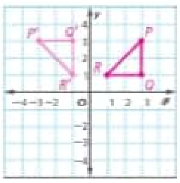


3. **Model with Mathematics** A partial hubcap is shown. Copy and complete the figure so that the completed hubcap has rotational symmetry of 90°, 180°, and 270°.



4. The right isosceles triangle *PQR* has vertices *P*(3, 3), *Q*(3, 1), and *R*(*x*, *y*) and is rotated 90° counterclockwise about the origin. Find the missing vertex of the triangle. Then graph the triangle and its image. **Sample answer:**

R(*x*, *y*) = *R*(**1** , **1**)



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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

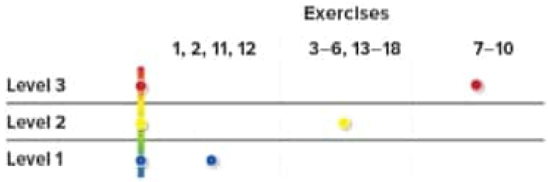
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 9, 10, 17, 18
OL	On Level	1, 3–6, 9, 10, 17, 18
BL	Beyond Level	3–10, 17, 18

Watch Out!

Common Error Watch for students who rotate figures about a vertex rather than about the origin. Remind students to first determine the center of rotation.

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MP MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7, 8
2 Reason abstractly and quantitatively.	10
4 Model with mathematics.	3, 6, 9
7 Look for and make use of structure.	13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

5. Which capital letters in ISOSCELES produce the same letter after being rotated 180° in the plane of the page? **I, S, and O**

H.O.T. Problems Higher Order Thinking

6. **Persevere with Problems** Triangle ABC has vertices $A(0, 4)$, $B(0, 2)$, and $C(2, 0)$. The triangle is reflected over the x-axis. Then the image is rotated 180° counterclockwise about the origin. What are the coordinates of the final image?
$A''(0, 4)$, $B''(0, -2)$, $C''(-2, 0)$

7. **Persevere with Problems** Triangle QRS is translated 7 units right, then rotated 90° clockwise about the origin. The vertices of triangle $Q''R''S''$ are $Q''(6, -1)$, $R''(0, -1)$, and $S''(0, -7)$. Find the coordinates of $\triangle QRS$.
$Q(-6, 6)$, $R(-6, 0)$, $S(0, 0)$

8. **Model with Mathematics** A triangle is rotated 90° clockwise about the origin. Then the image is rotated 270° clockwise about the origin.
   a. Complete the algebraic representation to explain the effect of the series of transformations performed.
   $$(x, y) \rightarrow (y, -x) \rightarrow (x, y)$$
   b. Based on your answer to part a, what can you conclude about a rotation of 90° followed by a rotation of 270°? **They are the same as a rotation of 360°.**

9. **Reason Inductively** Will a geometric figure and its rotated image *always*, *sometimes*, or *never* have the same perimeter? Explain your reasoning.
always; Sample answer: The figure and its image have the same size and shape. Since the corresponding lengths are equal, the perimeters are the same.

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Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students respond to the following question: If point $T(4, -3)$ is rotated 90° counterclockwise about the origin, what are the coordinates of T'? **$T'(3, 4)$**

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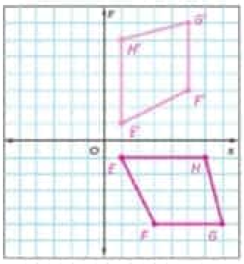
Name ______________________ My Homework ______________________

Extra Practice

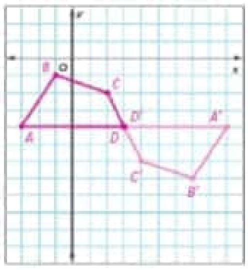
10. Quadrilateral *EFGH* has vertices *E*(1, 1), *F*(3, −5), *G*(7, −5), and *H*(6, −1). Graph the figure and its rotated image after a counterclockwise rotation of 90° about the origin. Then give the coordinates of the vertices for quadrilateral *E′F′G′H′*.

E′(1, 1), *F′*(5, 3), *G′*(5, 7), *H′*(1, 6)



11. Quadrilateral *ABCD* has vertices at *A*(−3, −4), *B*(−1, −1), *C*(2, −2), and *D*(3, −4). Graph quadrilateral *ABCD* and its image after a 180° counterclockwise rotation about vertex *D*. Then give the coordinates of the vertices of the image.

A′(9, −4), *B′*(7, −7), *C′*(4, −6), *D′*(3, −4)



12. **Identify Structure** Identify each transformation as a *translation*, *reflection*, or *rotation*.

a.

reflection

b.

translation

c.

rotation

Copy and Solve Triangle *MNP* has vertices *M*(1, 4), *N*(3, 1), and *P*(5, 3). Find the vertices of *M′N′P′* after each rotation about the origin. Show your work on a separate piece of paper.

13. 90° clockwise
M′(4, −1),
N′(1, −3),
P′(3, −5)

14. 180° clockwise
M′(−1, −4),
N′(−3, −1),
P′(−5, −3)

15. 90° counterclockwise
M′(−4, 1),
N′(−1, 3),
P′(−3, 5)

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Power Up! Test Practice

Exercises 17 and 18 prepare students for more rigorous thinking needed for assessment.

17. This test item requires students to reason abstractly and quantitatively when problem solving.

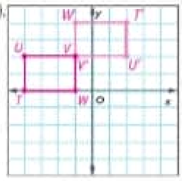
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

18. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly draw the figure and its rotation and list the coordinates.
1 point	Students correctly draw the figure and its rotation but fail to list the coordinates OR students correctly draw one figure and list the coordinates OR students correctly list the coordinates but fail to draw the figures.

Power Up! Test Practice

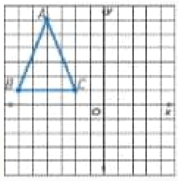
16. On a floor plan, *TUVW* with vertices *T*(−4, 0), *U*(−4, 2), *V*(−1, 2), and *W*(−1, 0) represents the location of Hiyam's bed in her bedroom. Hiyam would like to rotate her bed 180° clockwise about point *V* to see if she likes the new placement. Draw the bed and the rotated image on the coordinate plane.



What are the coordinates of the corners of the rotated bed?

T'(2, 4), *U'*(2, 2), *V'*(−1, 2), *W'*(−1, 4)

17. Triangle *ABC* is rotated 90° counterclockwise about the origin. Determine if each statement is true or false.



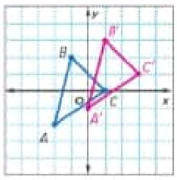
a. The image of point *A* is *A'*(−6, 4). ☐ True ■ False

b. The image of point *B* is *B'*(−1, −6). ■ True ☐ False

c. The image of point *C* is *C'*(−1, −2). ■ True ☐ False

Spiral Review

18. Use the graph of △*ABC* shown at the right.



a. What are the coordinates of △*A'B'C'* when △*ABC* is reflected over the *x*-axis? *A'*(−2, 2), *B'*(−1, −2), *C'*(1, 0)

b. Graph the image of △*ABC* after it is translated 2 units right and 1 unit up.

19. Triangle *FGH* has vertices *F*(3, 7), *G*(−1, 5), and *H*(−2, 2). Find the vertices of its image after a translation of 4 units right and 2 units down followed by a reflection over the *y*-axis.

F'(−1, 5), *G'*(−3, 3), *H'*(−2, 0)

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Inquiry Lab
Dilations

Inquiry WHAT are the results of a dilation of a triangle?

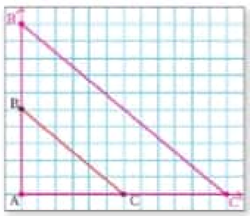
Mathematical Practices 1, 3, 5

One way to create murals on a wall is to use a drawing grid method. Artists draw a grid on the artwork to be copied and draw a similar grid on the wall. By transferring sections of the artwork, the mural is the same shape as the artwork, but a different size.

Hands-On Activity 1

In this Activity, you will enlarge $\triangle ABC$ by a *scale factor* of 2 using grid paper. Point A will be the center point for the enlargement.

Step 1 On the grid shown below, $\overrightarrow{AB}$ is drawn to the edge of the grid. Draw $\overrightarrow{AC}$ in the same way.



Step 2 Draw point B' on $\overrightarrow{AB}$ so that $AB' = 2(AB)$. Draw point C' on $\overrightarrow{AC}$ so that $AC' = 2(AC)$.

Step 3 Draw $\overline{B'C'}$ to complete $\triangle AB'C'$.

What is the ratio of the length $\overline{AB}$ to the length of $\overline{AB'}$? $\frac{5}{10}$ or $\frac{1}{2}$

What is the ratio of the length $\overline{AC}$ to the length of $\overline{AC'}$? $\frac{6}{12}$ or $\frac{1}{2}$

What is the ratio of the length $\overline{BC}$ to the length of $\overline{B'C'}$? $\frac{1}{2}$

What do you notice about the ratios of corresponding sides?

Is $\triangle ABC$ similar to $\triangle AB'C'$? They are equal; yes.

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Focus narrowing the scope

Objective Identify dilations.

Coherence connecting within and across grades

Now Students will measure angles and sides to generalize the properties of a dilation.

Next Students will graph dilations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 485.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

AL BL LA **Circle the Sage** Poll the class to see which students have some knowledge of dilations. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Activity 1. When the activity is complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. MP 1, 5, 6, 7

Ask:

- *How does triangle ABC compare to triangle AB'C'?* Sample answer: They are the same shape but different sizes.

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Hands-On Activity 2

AL LA **Circle the Sage** Select new sages to lead the activity based on their understanding of Activity 1. Repeat the same process, assigning new teams. MP 1, 3, 5, 6, 7

Ask:

- *Compare the scale factor in Activity 2 to the scale factor in Activity 1. How does the change in scale factor affect the dilation?* **Sample answer: The scale factor in Activity 1 was 2. The scale factor in Activity 2 is $\frac{1}{2}$. In Activity 1, the dilation was larger than the original triangle. In Activity 2, the dilation is smaller than the original triangle.**

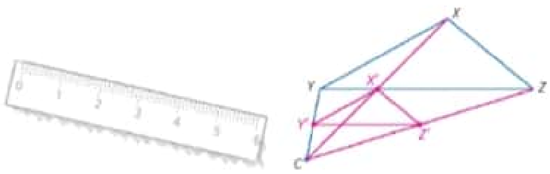
BL LA **Rally Table** Assign students to pairs. Students take turns completing the tasks in Activity 2. Have them discuss their responses to the questions in Step 4, with each student listening carefully to the other's reasoning. Have them ask for clarification or assistance, if needed. MP 1, 5, 6, 7

Hands-On Activity 2

In Activity 1, you used a dilation to transform $\triangle ABC$ by a scale factor of 2. **dilation** is a transformation that enlarges or reduces a figure by a scale factor relative to a center point. That point is called **center of dilation**.

In this Activity, you will draw the image of $\triangle XYZ$ after a dilation with a scale factor of $\frac{1}{2}$. Point C will be the center of dilation.

Step 1 Triangle XYZ is shown below. Point C is the center of dilation. Using a ruler, draw line segments connecting C to each of the vertices of the triangle. $\overline{CY}$ is done for you.



Step 2 Measure $\overline{CY}$. Draw point Y' on $\overline{CY}$ so that $CY' = \frac{1}{2}(CY)$.

Step 3 Repeat Step 2 for the two remaining sides. Draw point X' on $\overline{CX}$ so that $CX' = \frac{1}{2}(CX)$ and point Z' on $\overline{CZ}$ so that $CZ' = \frac{1}{2}(CZ)$.

Step 4 Draw $\triangle X'Y'Z'$.

Is $\triangle X'Y'Z'$ the same shape as $\triangle XYZ$? **yes**

Measure and compare the corresponding lengths on the original and new triangles. Describe the relationship between these measurements. **Sample answer: the measures of the original triangle's side lengths are 2 times the new triangle's side lengths.**

Measure and compare the corresponding angles on the original and new triangles. Describe the relationship between these measurements. **The measures of the corresponding angles on the original and new triangles are the same.**

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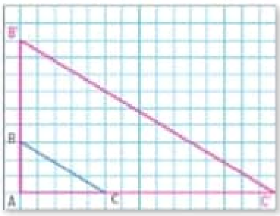
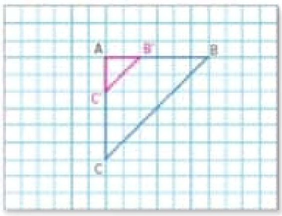
Investigate

Work with a partner. Draw the image after a dilation with the given scale factor. Point *A* is the center of dilation.

1. scale factor: 3

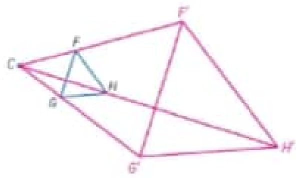


2. scale factor: $\frac{1}{3}$

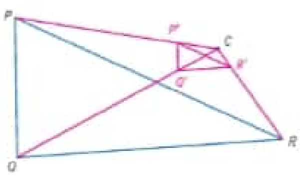


Work with a partner. Use a ruler to draw the image after a dilation with the given scale factor. Point *C* is the center of dilation.

3. scale factor: 3



4. scale factor: $\frac{1}{5}$



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4	5–10	11, 12
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA **Rally Coach** Have students work in pairs. While Student A works through Exercise 1, Student B watches, listens, coaches, and praises. Then partners trade roles for Exercise 2. Continue for Exercises 3 and 4. MP 1, 5, 6, 7

Ask:

- *What happens to the image when it is dilated by a scale factor that is a whole number greater than 1?* Sample answer: The new image is bigger.
- *What happens to the image when it is dilated by a scale factor that is a fraction between 0 and 1?* Sample answer: The new image is smaller.

BL LA **Trade-a-Problem** Students draw an image with 4 or 5 sides on a grid paper and choose a scale factor for the dilation. Students trade drawings and perform the dilation on their partner's drawing. Have students compare drawings and discuss any differences in solutions. MP 1, 5, 6, 7

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Analyze and Reflect

AL LA **Think-Pair-Share** Give students time to complete Exercises 5–10 on their own. Then have them share answers with a partner and resolve any differences. MP 1, 5, 6, 7

Ask:

- *How are ratios written?* In fraction form

BL LA **Pairs Discussion** Give students time to complete Exercises 5–10 in pairs. Then have them trade answers with another pair of students and resolve any differences. MP 1, 5, 6, 7

Ask:

- *For Exercise 6, explain how you can complete the table without measuring.* Sample answer: Because the dilation had a scale factor of 3, I can multiply my answers in Exercise 5 by 3 to find the new side lengths.

Create

BL Discuss in a small group how to answer Exercises 11 and 12. Assign one group member as the leader. The leader facilitates the discussion and makes sure that every group member understands. MP 1, 3, 5, 6, 7

Inquiry Students should be able to answer "WHAT are the results of a dilation of a triangle?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Sample answers: 5, 6, 10

MP **Use Math Tools** For each figure from Exercise 3, measure the given side lengths in millimeters. Complete the table.

5.

Figure	Side Lengths (mm)		
$\triangle FGH$	FG	GH	HF
	13	12	14

6.

Figure	Side Lengths (mm)		
$\triangle F'G'H'$	F'G'	G'H'	H'F'
	39	36	42

7. What is the ratio of side FG to side $F'G'$? $\frac{1}{3}$

8. What is the ratio of side GH to side $G'H'$? $\frac{1}{3}$

9. What is the ratio of side HF to side $H'F'$? $\frac{1}{3}$

10. Measure the angles of $\triangle FGH$ and $\triangle F'G'H'$ in Exercise 3 using a protractor.

Angle Measure (°)		
$\angle F$	$\angle G$	$\angle H$
54	66	60
$\angle F'$	$\angle G'$	$\angle H'$
54	66	60

Describe the relationship between the corresponding angles.

The corresponding angles have the same measure.

Create

11. MP **Reason Inductively** Based on the Activities and Exercises, write a conjecture about the effects of a dilation on the sides and angles of a triangle.

Sample answer: After a dilation, the new triangle's angles have the same measure to the original triangle's angles. The ratios of the corresponding sides are equal to the scale factor.

12. Inquiry WHAT are the results of a dilation of a triangle?

When a triangle is dilated, the resulting triangle has the same shape, but is a different size.

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Lesson 4

Dilations

Vocabulary Start-Up

A dilation uses a scale factor to enlarge or reduce a figure.

Scan the lesson and complete the graphic organizer.
Sample answers are given.

Dilation

Things that are like this	Characteristics	Things I've already learned I can use for this
enlargement reduction scale drawing	different size and same shape as original	ratio scale factor graphing

Essential Question

HOW can we best show or describe the change in position of a figure?

Math Symbols

$(x, y) \rightarrow (kx, ky)$

Mathematical Practices
1, 3, 4

Real-World Link

Photography Nada wants to insert a photo of her cat on her blog. The current size of the photo is 480 pixels by 640 pixels.

1. Suppose she wants to reduce the photo to 120 pixels by 160 pixels. Compare and contrast the original photo and the reduction. **The dimensions of the original photo are 4 times the dimensions of the new photo.**
2. What is the scale factor from the original to the reduction? $\frac{1}{4}$

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Use scale factors to graph dilations.

Coherence connecting within and across grades

Previous Students generalized the properties of a shape and its dilation.

Now Students will graph dilations using scale factors.

Next Students will model the effect of a dilation on perimeter and area.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 491.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Circle the Sage** Poll students on their knowledge of dilations and scale factors. Choose 4 to 5 students to be the sages and have them spread out around the room. Have the remaining students surround the sages while the sages provide explanations and answer questions. After students return to their seats, call on students to repeat what they learned from their sages. MP 1, 3, 5, 6, 7

Alternate Strategy

AL Have students graph line segment *AB* using *A*(1, 3) and *B*(3, 1). Then have them double the coordinates and draw a line segment using the new coordinates. Have them measure both segments and make a conjecture about the length of a line segment if the coordinates were tripled. MP 1, 3, 5, 6, 7

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find coordinates after an enlargement.

AL
- *Since the scale factor is 4, will the dilation result in an enlargement or a reduction? Explain.* enlargement; 4 > 1
- *Why didn't the coordinates for vertex A change after the dilation was applied?* The coordinates were 0, and 0 multiplied by any number remains 0.

OL
- *What do you need to do to the x- and y-coordinates of each vertex to find the coordinates of the dilated figure?* Multiply each coordinate by the scale factor, 4.
- *Using a scale factor of 4, what does the point (x, y) become after the scale factor is applied?* $(x, y) \rightarrow (4x, 4y)$

BL
- *Could a dilation ever be represented by the notation* $(x, y) \rightarrow \left(\frac{1}{4}x, 4y\right)$*? Explain.* no; Each of the x- and y-coordinates must be multiplied by the same scale factor.

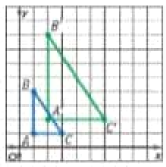
Need Another Example?
A triangle has vertices D(1, 2), E(0, 4), and F(1, −1). Find the coordinates of the triangle after a dilation with a scale factor of 3. D′(3, 6), E′(0, 12), F′(3, −3)

Work Zone

Key Concept: Dilations in the Coordinate Plane

Words A dilation with a scale factor of *k* will be:
- an enlargement, or an image larger than the original, if $k > 1$,
- a reduction, or an image smaller than the original, if $0 < k < 1$,
- the same as the original figure if $k = 1$.

Model



When the center of dilation in the coordinate plane is the origin, each coordinate of the preimage is multiplied by the scale factor *k* to find the coordinates of the image.

Symbols $(x, y) \rightarrow (kx, ky)$

The preimage and the image are the same shape but not necessarily the same size since the figure is enlarged or reduced by a scale factor.

Example

1. A triangle has vertices A(0, 0), B(8, 0), and C(3, −2). Find the coordinates of the triangle after a dilation with a scale factor of 4.

The dilation is $(x, y) \rightarrow (4x, 4y)$. Multiply the coordinates of each vertex by 4.

$A(0, 0) \rightarrow (4 \cdot 0,\ 4 \cdot 0) \rightarrow (0, 0)$
$B(8, 0) \rightarrow (4 \cdot 8,\ 4 \cdot 0) \rightarrow (32, 0)$
$C(3, -2) \rightarrow [4 \cdot 3,\ 4 \cdot (-2)] \rightarrow (12, -8)$

So, the coordinates after the dilation are A′(0, 0), B′(32, 0), and C′(12, −8).

Got it? Do this problem to find out.

a. A figure has vertices W(−2, 4), X(1, 4), Y(3, −1), and Z(−3, −1). Find the coordinates of the figure after a dilation with a scale factor of 2.

a. W′(−4, 8), X′(2, 8), Y′(6, −2), Z′(−6, −2)

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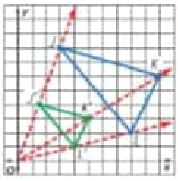
Example

2. A figure has vertices $J(3, 8)$, $K(10, 6)$, and $L(8, 2)$. Graph the figure and the image of the figure after a dilation with a scale factor of $\frac{1}{2}$.

The dilation is $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$. Multiply the coordinates of each vertex by $\frac{1}{2}$. Then graph both figures on the coordinate plane.



$J(3, 8) \rightarrow \left(\frac{1}{2} \cdot 3, \frac{1}{2} \cdot 8\right) \rightarrow J'\left(\frac{3}{2}, 4\right)$

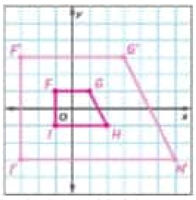
$K(10, 6) \rightarrow \left(\frac{1}{2} \cdot 10, \frac{1}{2} \cdot 6\right) \rightarrow K'(5, 3)$

$L(8, 2) \rightarrow \left(\frac{1}{2} \cdot 8, \frac{1}{2} \cdot 2\right) \rightarrow L'(4, 1)$

Check Draw lines throught the origin and each of the vertices of the original figure. The vertices of the dilation should lie on those same lines. ✓

Got it? Do these problems to find out.

b. A figure has vertices $F(-1, 1)$, $G(1, 1)$, $H(2, -1)$, and $I(-1, -1)$. Graph the figure and the image of the figure after a dilation with a scale factor of 3.



STOP and Reflect

Explain below how you can determine if a dilation is a reduction or an enlargement based on the scale factor.

If the scale factor is greater than 1, the dilation is an enlargement. If the scale factor is between 0 and 1, the dilation is a reduction.

Example

3. Through a microscope, the image of a grain of sand with a 0.25-millimeter diameter appears to have a diameter of 11.25 millimeters. What is the scale factor of the dilation?

Write a ratio comparing the diameters of the two images.

$\frac{\text{diameter in dilation}}{\text{diameter in original}} = \frac{11.25}{0.25}$

$= 45$

So, the scale factor of the dilation is 45.

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Examples

2. Find coordinates after a reduction.

AL • *Since the scale factor is $\frac{1}{2}$, will the dilation result in an enlargement or a reduction? Explain.* reduction; The scale factor is less than one.

OL • *What do you need to do to the x- and y-coordinates of each vertex to find the coordinates of the dilated figure?* Multiply each coordinate by the scale factor, $\frac{1}{2}$.

• *How are the figures alike? How are they different?* Sample answer: They have the same shape. The dilation is smaller than the original figure because the dilation was a reduction.

BL • *If your friend wrote the dilation as $(x, y) \rightarrow \left(\frac{x}{2}, \frac{y}{2}\right)$, would that be correct or incorrect? Explain.* correct; Multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Need Another Example?
A figure has vertices $H(8, 4)$, $J(6, 4)$, $K(6, -4)$, and $L(-8, -4)$. Graph the figure and the image of the figure after a dilation with a scale factor of $\frac{3}{4}$. See Answer Appendix.

3. Find the scale factor of a dilation.

AL • *What is the diameter of the grain of sand in the dilation?* 11.25 mm

• *What is the diameter of the original grain of sand?* 0.25 mm

OL • *Find the ratio of the diameter in the dilation to the diameter in the original.* $\frac{11.25}{0.25}$ or $\frac{45}{1}$

BL • *If your friend said the scale factor was 0.45, how could you use the context of the problem to show your friend that the scale factor could not be 0.45?* Sample answer: The image is larger than the grain, so the dilation is an enlargement. The scale factor must be greater than 1.

Need Another Example?
The pupil of Omar's eye is 6 millimeters in diameter. His doctor uses medicine to dilate his pupils so that they are 9 millimeters in diameter. What is the scale factor of the dilation? $\frac{3}{2}$

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Guided Practice

Formative A ssessment Use these exercises to assess students' un derstanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL EL Complete Exercise 1 together as a class. Ask a student to volunteer to write the directions for each step on the board and the remaining students copy the notes on the side of the page in their texts. Direct them to follow the same steps for Exercises 2 and MP 1, 5, 6, 7

BL EL **Teammates Consult** Teams discuss Exercise 1 with Student 1 leading the discussion. When everyone on the team has contributed to the discussion and any differences have been resolved, team members silently write their own answers without further discussion. Repeat the process for Exercise 2 with Student 2 leading the discussi MP 1, 5, 6, 7

c. $2\frac{1}{2}$ or 2.5

Got it? Do this problem to find out.

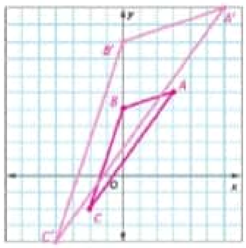
c. Rahd wants to enlarge a 7- by 12-centimeter photo to a $17\frac{1}{2}$- by 30-centimeter photo. What is the scale factor of the dilation?

Guided Practice

Find the coordinates of the vertices of each figure after a dilation with the given scale factor *k*. Then graph the original image and the dilation. (Examples 1 and 2)

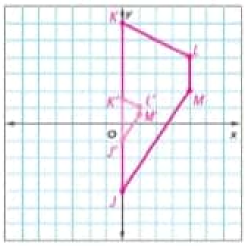
1. $A(3, 5)$, $B(0, 4)$, $C(2, -2)$; $k = 2$

$A'(6, 10)$, $B'(0, 8)$, $C'(-4, -4)$



2. $J(0, -4)$, $K(0, 6)$, $L(4, 4)$, $M(4, 2)$; $k = \frac{1}{4}$

$J'(0, -1)$, $K'\left(0, 1\frac{1}{2}\right)$, $L'(1, 1)$, $M'\left(1, \frac{1}{2}\right)$



3. STEM Mrs. Hidaya's pupils are creating a Web page for their school's Intranet site. They need to reduce a scanned photograph so it is 720 pixels by 320 pixels. If the scanned photograph is 1,080 pixels by 480 pixels, what is the scale factor of the dilation? (Example 3) $\frac{2}{3}$

4. **Building on the Essential Question** How are dilations similar to scale drawings?

Sample answer: Both represent enlargements or reductions of other figures. Both use a scale factor to determine the size of the dilation or scale drawing.

Rate Yourself!

☐ I understand how to dilate a figure.

Great! You're ready to move on!

☐ I still have some questions about how to dilate a figure.

FOLDABLES Time to update your Foldable!

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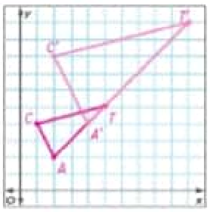
Name ______ My Homework ______

Independent Practice

Find the coordinates of the vertices of each figure after a dilation with the given scale factor *k*. Then graph the original image and the dilation.
(Examples 1 and 2)

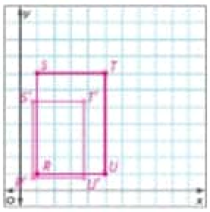
1. $C(1, 4)$, $A(2, 2)$, $T(5, 5)$; $k = 2$

$C'(2, 8)$, $A'(4, 4)$, $T'(10, 10)$



2. $R(1, 1)$, $S(1, 7)$, $T(5, 7)$, $U(5, 1)$; $k = \frac{3}{4}$

$R'\left(\frac{3}{4}, \frac{3}{4}\right)$, $S'\left(\frac{3}{4}, 5\frac{1}{4}\right)$, $T'\left(3\frac{3}{4}, 5\frac{1}{4}\right)$, $U'\left(3\frac{3}{4}, \frac{3}{4}\right)$



3. A graphic designer created a logo on $2\frac{1}{4}$ by $27\frac{1}{2}$-centimeter paper. In order to be placed on a business card, the logo needs to be $\frac{1}{4}$ centimeters by $5\frac{1}{2}$ centimeters. What is the scale factor of the dilation? (Example 3)

$\frac{1}{5}$

4. Faleh wants to build a regulation-size pool table that is 275 centimeters in length. The plans he ordered are 45 by 90 centimeters. What is the scale factor of the dilation he must use to build the regulation pool table? (Example 3)

3

5. A triangle has vertices $A(2, 3)$, $B(0, 0)$, and $C(1, 1)$.

a. Find the coordinates of the triangle if it is reflected over the *x*-axis, then dilated by a scale factor of 3.

$A'(-6, -9)$, $B'(0, 0)$, $C'(3, -3)$

b. Find the coordinates if the original triangle is dilated by a scale factor of 3, then reflected over the *x*-axis.

$A'(-6, -9)$, $B'(0, 0)$, $C'(3, -3)$

c. Are the two transformations commutative? Explain.

Yes; Sample answer: since the coordinates of the answers to Exercises a and b are the same, the order in which you perform them does not matter.

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–4, 10–12	Exercises 5, 6, 13–15	Exercises 7–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 14, 15
OL	On Level	1, 3, 5–7, 14, 15
BL	Beyond Level	5–9, 14, 15

Watch Out!

Common Error When finding the coordinates of the dilation, students might mistakenly multiply only the *x*-coordinates of the vertices by the scale factor. Remind students that in a dilation, both the *x*- and *y*-coordinates of each vertex must be multiplied by the scale factor.

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MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	8, 9
3 Construct viable arguments and critique the reasoning of others.	7, 13
4 Model with mathematics.	6

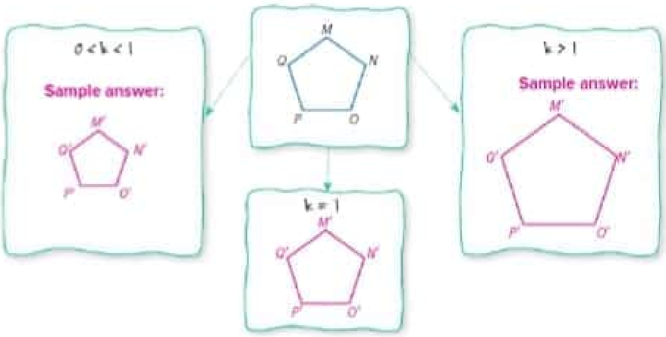
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to answer the following question on a piece of paper. Students should hand the paper to you as they leave the room. If the scale factor of a dilation is $\frac{3}{2}$, is the dilation an enlargement or a reduction? Explain.
enlargement; The scale factor is greater than one.

6. **Model with Mathematics** In each part of the graphic organizer, sketch an image of pentagon $MNOPQ$ after a dilation within the given parameters.



H.O.T. Problems Higher Order Thinking

7. **Make a Conjecture** A figure has a vertex at the point (4, −6). The figure is dilated with the center at the origin with a scale factor of 5. The resulting image is then dilated with a scale factor of $\frac{3}{5}$.

   a. What are the coordinates of the vertex in the final image? **(12, −18)**

   b. How do they compare with those of the original image?
   The final coordinates are three times the original coordinates.

   c. Can you predict the scale factor of a compound dilation? Explain.
   Sample answer: Yes; multiply the scale factors of each dilation to find the scale factor of the final dilation.

8. **Persevere with Problems** The coordinates of two triangles are shown in the table. Is $\triangle WXY$ a dilation of $\triangle ABC$? Explain.
No; Sample answer: both coordinates of all the points must be multiplied by the same scale factor. The x-coordinates are multiplied by 4, but the y-coordinates are only multiplied by 2.

WXY		ABC	
W	(a, b)	A	$(4a, 2b)$
X	(a, c)	B	$(4a, 2c)$
Y	(d, b)	C	$(4d, 2b)$

9. **Persevere with Problems** The algebraic representation of a dilation is $(x, y) \rightarrow \left(\frac{1}{a}x, \frac{1}{a}y\right)$. If the dilation is an enlargement, give three possible values of a. **Sample answer: $a = \frac{1}{3}$, $a = \frac{1}{5}$, $a = \frac{1}{2}$**

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Name ______________________ My Homework ______________________

Extra Practice

Find the coordinates of the vertices of each figure after a dilation with the given scale factor *k*. Then graph the original image and the dilation.

10. $R(5, 5)$, $S(5, 10)$, $T(10, 10)$, $U(10, 5)$; $k = \frac{2}{5}$

$R'(2, 2)$, $S'(2, 4)$, $T'(4, 4)$, $U'(4, 2)$

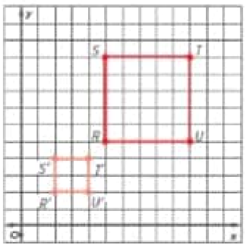
Multiply each coordinate in each ordered pair by scale factor. Then graph the two figures.

$R(5, 5) \rightarrow \left(\frac{2}{5} \cdot 5, \frac{2}{5} \cdot 5\right) \rightarrow R'(2, 2)$

$S(5, 10) \rightarrow \left(\frac{2}{5} \cdot 5, \frac{2}{5} \cdot 10\right) \rightarrow S'(2, 4)$

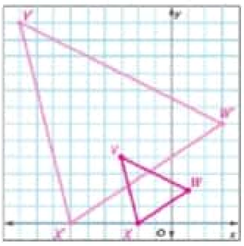
$T(10, 10) \rightarrow \left(\frac{2}{5} \cdot 10, \frac{2}{5} \cdot 10\right) \rightarrow T'(4, 4)$

$U(10, 5) \rightarrow \left(\frac{2}{5} \cdot 10, \frac{2}{5} \cdot 5\right) \rightarrow U'(4, 2)$



11. $V(-3, 4)$, $X(-2, 0)$, $W(1, 2)$; $k = 3$

$V'(-9, 12)$, $X'(-6, 0)$, $W'(3, 6)$



12. To place a picture in his class newsletter, Faris must reduce the picture by a scale factor of 0.3. Find the dimensions of the reduced picture if the original is 15 centimeters wide and 10 centimeters high.

4.5 cm by 3 cm

13. **Multiple Representations** Triangle *XYZ* has vertices $X(0, 0)$, $Y(3, 1)$ and $Z(2, 3)$.

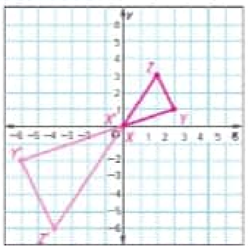
a. **Numbers** Find the coordinates of the image $\triangle X'Y'Z'$ after a dilation with a scale factor of -2.

$X'(0, 0)$, $Y'(-6, -2)$, $Z'(-4, -6)$

b. **Algebra** Graph $\triangle XYZ$ and the image on the coordinate plane.

c. **Words** Describe the locations of $\triangle XYZ$ and $\triangle X'Y'Z'$ using transformations.

$\triangle X'Y'Z'$ is the image of $\triangle XYZ$ after a dilation of 2 and a rotation of 180° about the origin.



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Power Up! Test Practice

Exercises 14 and 15 prepare students for more rigorous thinking needed for assessment.

14.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly graph the figure, find the scale factor, indicate it is an enlargement and explain.
1 point	Students correctly graph the figure, find the scale factor and/or indicate it is an enlargement but fail to explain OR students incorrectly graph the figure, but answer the rest of the question according to the incorrect figure.

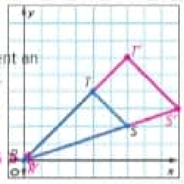
15.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

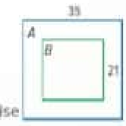
14. Triangle *RST* is dilated so that the image of point *T* is *T'*(6, 6). Draw triangle *R'S'T'*.

What is the scale factor of the dilation? Does the dilation represent an enlargement or a reduction? Explain how you found your answer.



1.5; enlargement; Sample answer: Compare point T(4, 4) and point T'(6, 6). To transform point T(4, 4) to T'(6, 6), you multiply by each coordinate by 1.5. So the scale factor is 1.5. Since 1.5 > 1 the dilation represents an enlargement.

15. Squares *A* and *B* are related by a dilation. Determine if each statement is true or false.



a. The scale factor from figure *A* to figure *B* is $\frac{3}{5}$. ■ True ☐ False

b. The scale factor from figure *B* to figure *A* is $\frac{5}{3}$. ■ True ☐ False

c. The dilation from figure *A* to figure *B* is an enlargement. ☐ True ■ False

Spiral Review

16. A model airplane is built with a wing span of 45 centimeters. The actual wing span of the airplane is 27 meters. Find the scale.

1 cm = 60 cm

Find the scale factor for each scale.

17. 15 cm = 3 m $\frac{1}{20}$

18. 4 cm = 2.5 mm $\frac{16}{1}$

19. 500 cm = 45 m $\frac{1}{9}$

20. On a map of the UAE, the scale is 1 cm = 50 km. Using the scale, complete the table showing the distance between cities.

Cities	Map	Actual
Ruwais to Al Ain	7.5 cm	**375 km**
Dubai to Abu Dhabi	**3 cm**	150 km

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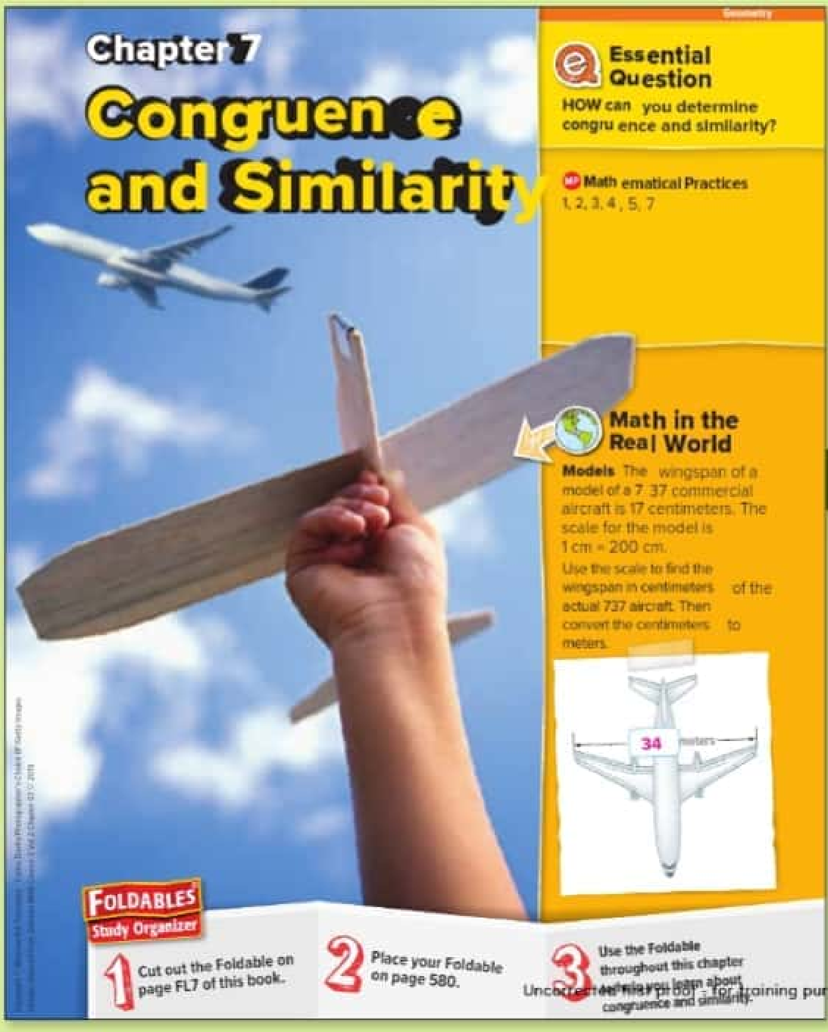

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Focus narrowing the scope

This chapter focuses on content from **Geometry**.

Coherence connecting within and across grades

Previous Students graphed translations, reflections, rotations, and dilations.

Now Students determine congruence and similarity of figures.

Next Students will find the volume and surface area of three-dimensional figures.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Models Remind students that to find the wingspan in centimeters, they need to multiply 17 by 200. To convert to meters they need to divide the product by 100.

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Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

COMMON CORE REVIEW	
Example	**Skill**
1	Solve proportions
2	Find slope

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–6

Solve $\frac{a}{12} = \frac{3}{8}$. 4.5

Exercises 7–8

Find the slope of the line that passes through (4, 7) and (−2, 3). $\frac{2}{3}$

Track Your Progress

Prior to beginning this chapter, have your students rate their current knowledge. At the end of the chapter, you will be reminded to have your students rate their knowledge again. They should see that their knowledge of the key ideas has increased.

Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Solve $\frac{w}{12} = \frac{5}{6}$.

$\frac{w}{12} = \frac{5}{6}$ Write the proportion.

$6 \times w = 12 \times 5$ Find cross products.

$6w = 60$ Simplify.

$w = 10$ Division Property of Equality

Example 2

Find the slope of the line that passes through (3, 8) and (−1, 0).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{0 - 8}{-1 - 3}$ $(x_1, y_1) = (3, 8)$; $(x_2, y_2) = (-1, 0)$

$m = \frac{-8}{-4}$ or 2 Simplify.

Quick Check

Proportions Solve each proportion.

1. $\frac{x}{15} = \frac{7}{30}$ 3.5
2. $\frac{4}{9} = \frac{14}{y}$ 31.5
3. $\frac{12}{z} = \frac{30}{37}$ 14.8
4. $\frac{8}{15} = \frac{m}{21}$ 11.2
5. $\frac{n}{5} = \frac{18}{45}$ 2
6. $\frac{3}{7} = \frac{21}{p}$ 49

Find Slope Find the slope of the line that passes through each pair of points.

7. (−1, 1), (−3, 7) −3
8. (2, 0), (0, 2) −1
9. (−6, −1), (−3, 4) $\frac{5}{3}$

How Did You Do? Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

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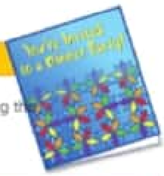
Inquiry Lab
Composition of Transformations

HOW does a combination of transformations differ from a single transformation? How are they the same?

Mathematical Practices 1, 3

Graphic artists often use several transformations to create designs. When a transformation is applied to a figure and then another transformation is applied to the image, the result is called **composition of transformations**.



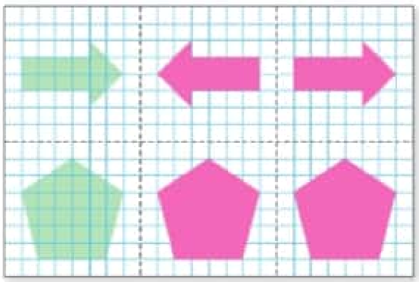
Hands-On Activity 1

Step 1 Fold the page in your book vertically into three sections along the dotted lines.

Step 2 Draw the reflection of the arrow over the fold in the middle section.

Step 3 Draw a reflection of the 2nd arrow over the fold in the right-hand section.

Step 4 Repeat Steps 2 and 3 with the pentagon.



How are the original figures and the final figures related?

Sample answer: The arrows are the same and the pentagons are the same.

Would the final images be the same as the original figure if the second reflection was reflected over the horizontal line? Explain.

Sample answer: The arrows would be the same. The pentagon would be the same shape and the same size, but it would be upside down.

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Focus narrowing the scope

Objective Draw compositions of translations, reflections, and rotations.

Coherence connecting within and across grades

Now
Students will draw compositions of translations, reflections, and rotations.

Next
Students will differentiate between transformations that preserve congruence and those that do not.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 507.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

AL LA **Pairs Discussion** Have students work with a partner to complete the activity. If students have trouble visualizing what each figure would look like reflected over a horizontal line, have them make a horizontal fold and draw the reflection. This way they can easily compare the reflection over a horizontal line and a reflection over a vertical MP 1, 4, 5

BL LA **Pairs Consult** Have students consult with a partner to draw the reflections without using the paper. They should discover how to use the grid lines to count out where each corner and line will lie after the reflection. MP 1, 4, 5

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Hands-On Activity 2

AL **LA** **Teammates Consult** Have students work in small teams to complete the activity. You may wish to have them draw their figure on a separate sheet of paper first. Tell them to make it something simple that can be redrawn many times without being distorted. For Exercise 3, you may need to remind students about dilations that were previously taught in Chapter 6. MP 1, 4, 5

BL **LA** **Pairs Consult** Have students follow Steps 1 and 2 as written. Before moving on to Step 3, have students dilate the figure by a scale factor of 2, then continue following Steps 3 and 4. MP 1, 4, 5

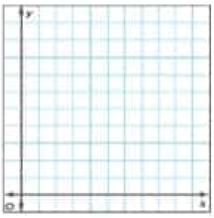
Ask:

- *Prior to the dilation, was congruence of the original figure preserved after the translation?* **yes**
- *After the dilation, was congruence of the original figure preserved? Explain.* **no; Sample answer: The figure after the dilation is larger than the original figure.**
- *If the reflection occurred prior to the dilation, would congruence of the original figure be preserved after the reflection and before the dilation?* **yes**
- *If the reflection occurred prior to the dilation, would orientation of the original figure be preserved after the reflection and before the dilation?* **Orientation is not preserved for reflections; see students' work.**

Hands-On Activity 2

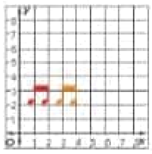
In this Activity, you will use a translation and a reflection to create a decorative border.

Step 1 Draw a figure on the coordinate plane shown, close to the origin.

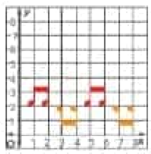


See students' work.

Step 2 On the coordinate plane in Step 1, translate your figure. Lightly draw the image since it will not be in its final location. In this example, the red figure is translated 2 units to the right.



Step 3 On the coordinate plane in Step 1, reflect the drawn image across a horizontal line. This will be the final location so you can draw this in your book. In this example, the image is reflected across the line $y = 2$.



Step 4 Repeat the process to create your border.

How are the size and shape of the original figure related to the size and shape of the images?

Sample answer: They are the same.

Suppose you wanted your border to run up the side of the page instead of across the bottom of the page. Describe what transformations you might use to do this. **Sample answer: You could use a vertical reflection followed by a translation up the page.**

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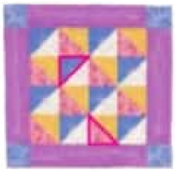
Investigate

Work with a partner. Describe the transformations combined to create the outlined patterns shown in Exercises 1–4.

1. 

a translation and a reflection

2. 

a translation and a rotation

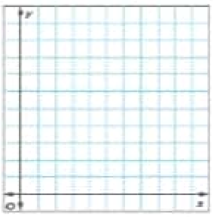
3. 

Sample answer: a translation and a reflection

4. 

a rotation and a translation

5. Draw a figure on the coordinate plane shown. Use a reflection and a rotation to create a logo for a company. See students' work.



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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–5	6, 7	8, 9
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA **Teammates Consult** Have students work in small groups to try out different combinations of transformations until they determine what results in the given figure. Then have students work in pairs or small groups to complete Exercises 1–5. Provide them with graph paper and tracing paper. Allow them to use any method they choose to manipulate the drawing to determine the transformation. MP 1, 4, 5

EL LA **Pairs Discussion** Have students determine as many combinations of transformations as possible. MP

Ask:

- *When there are two transformations not on the coordinate plane, does the order of the transformation matter?* Sample answer: no; A reflection then a translation will result in the same figure as a translation and then a reflection.

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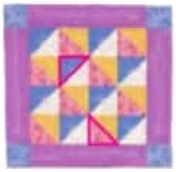
Investigate

Work with a partner. Describe the transformations combined to create the outlined patterns shown in Exercises 1–4.

1. 

a translation and a reflection

2. 

a translation and a rotation

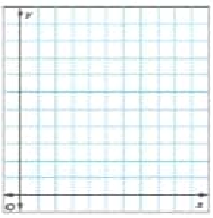
3. 

Sample answer: a translation and a reflection

4. 

a rotation and a translation

5. Draw a figure on the coordinate plane shown. Use a reflection and a rotation to create a logo for a company. See students' work.



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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

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	1–5	6, 7	8, 9
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Level 1	•		

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Ask:

- *When there are two transformations not on the coordinate plane, does the order of the transformation matter?* Sample answer: no; A reflection then a translation will result in the same figure as a translation and then a reflection.

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Analyze and Reflect

AL **LA** **Roundrobin** Have students work in small teams. The first student draws the line reflected across the *y*-axis. The second student draws the reflection of this image across the *x*-axis. The third student determines what single kind of transformation the image is compared to the pre-image. The fourth student, if available, verifies that the single transformation of the pre-image would result in the same final figure. **MP** 1, 3, 5

BL **LA** **Pairs Extend** Have students work in pairs to create their own composition of transformations that result in a different kind of single transformation. **MP** 1, 4, 5

Create

BL **LA** **Pairs Check** Have students draw two examples of each composition of transformations that prove their conjectures. Have them share their conjectures and drawings with a partner. Each partner should check and verify each other's work. **MP** 1, 3, 4

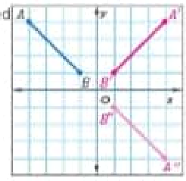
Inquiry Students should be able to answer "HOW does a combination of transformations differ from a single transformation? How are they the same?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

In some cases, a composition of transformations is the same as a single transformation. Draw the composition of transformations described. Then identify the single transformation that would produce the same image as each composition.

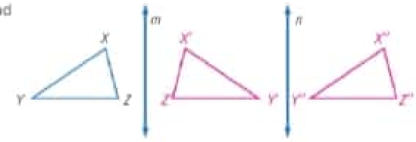
6. $\overline{AB}$ is reflected across the *y*-axis, then reflected across the *x*-axis.

a 180° rotation about the origin



7. $\triangle XYZ$ is reflected across line *m* and then reflected across line *n*.

a translation



Create

8. **MP** **Make a Conjecture** The transformations in the Activities and Exercises have been translations, reflections, and rotations which preserve distance. Make a conjecture about the position, size, and shape of a figure if a composition of transformations included a dilation.

Sample answer: The original figure would change position and change size. The shape of the figure would not change.

9. **Inquiry** HOW does a combination of transformations differ from a single transformation? How are they the same?

Sample answer: A combination of transformations includes more than one transformation, so many times the image could not be obtained by a single transformation. They are the same because regardless of the number of transformations, the image is the same shape as the preimage.

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Determine if figures are congruent by using transformations.

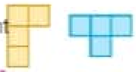
- AL • *Assuming the triangles are congruent, which vertex in triangle ABC matches up with vertex X?* **vertex C**
- OL • *What do you need to do to triangle ABC so that the vertices are in the same position as the vertices in triangle ZYX?* **Reflect triangle *ABC* over a vertical line and then translate the triangle.**
- BL • *Explain why following this series of transformations demonstrates the figures are congruent.* **Sample answer: By reflecting and translating, I am able to obtain triangle *XYZ* without changing the size or shape of triangle *ABC*.**

Need Another Example?
Determine if the two figures are congruent by using transformations. Explain your reasoning. **congruent; A rotation followed by a translation maps figure A onto figure B.**

2. Determine if figures are congruent by using transformations.

- OL • *What transformation should we do first? Explain.* **Sample answer: reflection; The square that is farthest left on the pre-image is farthest right on the image.**
- BL • *Are there any other transformations we can use to make the figures congruent? Explain.* **Sample answer: no; There are no transformations that will match the figures up exactly.**

Need Another Example?
Determine if the two figures are congruent by using transformations. Explain your reasoning. **not congruent; No transformations will match the figures up exactly.**



Work Zone

Transformations

Translations, reflections and rotations are called isometries. In an isometry the distance between two points in an image is the same as the distance in the preimage.

iso / metry
↓ ↓
same distance

Identify Congruence

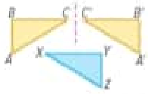
On the previous page, you matched Figure *A* to Figure *B* by a translation and a reflection. Two figures are congruent if the second can be obtained from the first by a series of rotations, reflections, and/or translations.

Examples

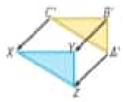
Determine if the two figures are congruent by using transformations. Explain your reasoning.

1.

Step 1 Reflect $\triangle ABC$ over a vertical line. Label the vertices of the image A', B', and C'.

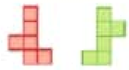


Step 2 Translate $\triangle A'B'C'$ until all sides and angles match $\triangle XYZ$.

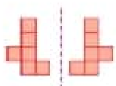


So, the two triangles are congruent because a reflection followed by a translation will map $\triangle ABC$ onto $\triangle ZYX$.

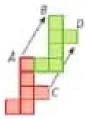
2.



Reflect the red figure over a vertical line.



Even if the reflected figure is translated up and over, it will not match the green figure exactly. The two figures are not congruent.



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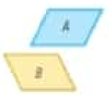
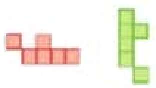
Got it? Do these problems to find out.

a.



b.



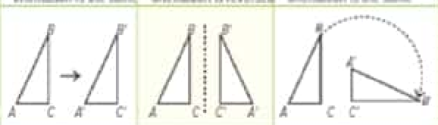
a. congruent; A reflection followed by a translation maps figure A onto figure B.

b. not congruent; No transformations will match the two figures up exactly.

Determine the Transformations

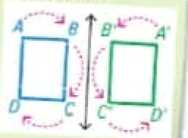
If you have two congruent figures, you can determine the transformation, or series of transformations, that maps one figure onto the other by analyzing the orientation or relative position of the figures.

Translation	Reflection	Rotation
• length is the same • orientation is the same	• length is the same • orientation is reversed	• length is the same • orientation is the same



Orientation

The order in which the vertices of a figure are named determines the figure's orientation. In the reflection shown, the vertices of the preimage are named in a clockwise direction, but the vertices of the image are named in a counterclockwise direction. The orientation has been reversed.



Example

3. **Eiman created the logo shown. What transformations did she use if the letter "d" is the preimage and the letter "p" is the image? Are the two figures congruent?**

Step 1 Start with the preimage. Rotate the letter "d" 180° about point A.

Step 2 Translate the new image up.

Eiman used a rotation and translation to create the logo. The letters are congruent because images produced by a rotation and translation have the same shape and size.

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Example

3. Determine transformations.

AL • *Looking at the given table, which transformations affect the orientation of a figure?* reflections

OL • *Is the letter "d" reflected or rotated to create the letter "p"?* rotated

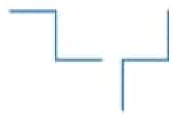
• *After the letter "d" is rotated, what has to happen in order to get the exact location of the letter "p" in the logo?* It needs to be translated up.

BL • *Is there another set of translations that results in the same figure?* Sample answer: The "d" could be reflected over a vertical line, then reflected over a horizontal line, then translated up.

• *Which set of translations is more efficient? Explain.* Sample answer: A rotation and translation only takes two transformations, so it is more efficient.

Need Another Example?

The pattern below appears along the edge of a plate. What transformations could be used if the first figure is the preimage and the second is the image? Sample answer: a rotation followed by a translation



Watch Out!

Common Error Students may have difficulty visualizing the transformations. For Example 3, they may wish to trace the shape of the "d" and then try out different transformations until they find what works.

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Guided Practice

Formative A ssessment Use these exercises to assess students' un derstanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Pairs Discussion Give pairs of students tracing paper. Have them trace both figures for Exercise 1 and cut them out. Then have them use the cut-outs to try different transformations, keeping a record of which transformations they tried, until they determine if the figures are congruent. Have them write down which set of transformations worked. Then have them use the same strategy for Exercises 2 and 3. MP 1, 4, 5

BL LA Trade-a-Problem Have students draw a preimage and an image. The figures may or may not be congruent. Have them trade drawings with a partner and determine if their partner's preimage and image are congruent. If they are congruent, have students explain what transformation they used to map the preimage onto the ima MP 1, 3, 4, 5

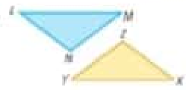
c. Sample answer: a vertical reflection followed by a translation; yes, images produced by a reflection and a translation are congruent.

Got it? Do this problem to find out.

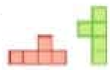
c. Wha t transformations could be used if the lette r "W" is the preimage and the letter "M" is th e image in the logo shown? Are the two figu res congruent? Explain.

Guided Practice

Determine if the two figures are congruent by using transformations. Explain your reasoning. (Examples 1 and 2)

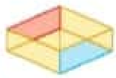
1. 

The two triangles are congruent because a rotation followed by a translation will map $\triangle LMN$ onto $\triangle XYZ$.

2. 

The two figures are congruent because a clockwise rotation of 90° followed by a translation maps the red figure onto the green figure.

3. The Boyd Box Company uses the logo shown. What transformations could be used if the red trapezoid is the preimage and the blue trapezoid is the image? Are the two figures congruent? Explain. (Example 3)



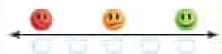
Sample answer: a rotation followed by a translation; they are congruent because an image produced by a rotaion and a translation have the same size and shape.

4. **Building on the Essential Question** Why do translations, reflections, and rotations create congruent images?

Sample answer: Rotations, reflections, and translations do not change size or shape. Since congruent figures have the same size and shape, the three transformations create congruent images.

Rate Yourself!

How confident are you about the relationship between congruence and transformations? Check the box that applies.



FOLDABLES Time to update your foldable!

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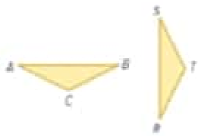
Independent Practice

Determine if the two figures are congruent by using transformations. Explain your reasoning. (Examples 1 and 2)

1. 

not congruent; No sequence of transformations maps *RSTU* onto *WXYZ* exactly.

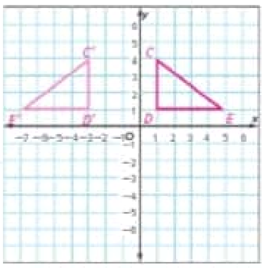
2. 

congruent; A counterclockwise rotation of 90° followed by a translation maps △*ABC* onto △*RST*.

3. Najla purchased some custom printed stationery with her initials. What transformations could be used if the letter "Z" is the preimage and the letter "N" is the image in the design shown? Are the two figures congruent? Explain. (Example 3)

Sample answer: 90° clockwise rotation followed by a translation; they are congruent because an image produced by a rotation and a translation have the same size and shape.

4. **Multiple Representations** One way to identify congruent triangles is to prove their matching sides have the same measure. Triangle *CDE* has vertices at (1, 4), (1, 1), and (5, 1).



a. **Graphs** Graph △*CDE*.

b. **Numbers** Find the lengths of the sides of △*CDE*.
DC = 3 units, *DE* = 4 units, *CE* = 5 units

c. **Geometry** Reflect △*CDE* over the *y*-axis, then translate it 2 units left. Label the vertices of the image *C'D'E'*. Write the coordinates of △*C'D'E'* below.
C'(−3, 4), *D'*(−3, 1), *E'*(−7, 1)

d. **Numbers** Find the lengths of the sides of △*C'D'E'*.
D'C' = 3 units, *D'E'* = 4 units, *C'E'* = 5 units

e. **Words** Are the two triangles congruent? Justify your response.
yes; Sample answer: The matching sides are the same length.

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Engage Explore **Explain** **Elaborate** **Evaluate**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	1–3, 10–13	4, 5, 14–16	6–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 6, 9, 15, 16
OL	On Level	1, 3–6, 9, 15, 16
BL	Beyond Level	4–9, 15, 16

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7, 8
3 Construct viable arguments and critique the reasoning of others.	4, 9
4 Model with mathematics.	6, 14

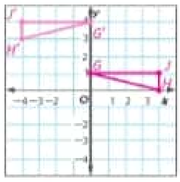
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Write the coordinates for triangle PQR on the board: $P(3, 4)$, $Q(1, 2)$, and $R(0, -1)$. Have students choose two transformations to perform on the triangle and have them give the coordinates of the image. Then have them describe whether the two figures are congruent. **See students' work.**

5. Graph $\triangle GHJ$ with vertices at $G(0, 1)$, $H(4, 0)$, and $J(4, 1)$. Then graph the image of the triangle after a translation of 3 units up followed by a reflection over the y-axis. Find the lengths of each side of the preimage and the image. Then determine if the two figures are congruent.



4 units, 1 unit, $\sqrt{17}$ units; 4 units, 1 unit, $\sqrt{17}$ units; yes

H.O.T. Problems Higher Order Thinking

6. **Model with Mathematics** Create a design in the space at the right, using a series of transformations that produce congruent figures. Exchange designs with a classmate and determine what transformations were used to create their design.
See students' work.

7. **Persevere with Problems** Triangle $A'B'C'$ has vertices $A'(4, 5)$, $B'(-1, 4)$, and $C'(-2, 0)$. Triangle ABC was rotated 90° in a clockwise direction about the origin, translated 2 units up, and reflected over the y-axis. What were the coordinates of the vertices of triangle ABC?
$A(-3, 4)$, $B(-2, 1)$, $C(2, 2)$

8. **Persevere with Problems** Line segment XY has endpoints at $X(3, 1)$ and $Y(-2, 0)$. Its image after a series of transformations has endpoints at $X'(0, 1)$ and $Y'(5, 0)$. Find the series of transformations that maps $\overline{XY}$ onto $\overline{X'Y'}$. Then find the exact length of both segments.
Sample answer: a reflection over the y-axis followed by a translation of 3 units to the right; $\sqrt{26}$

9. **Justify Conclusions** A line segment has endpoints at (a, b) and (c, d). Determine whether the following statements are *true* or *false*. Justify your reasoning.

a. The line segment with endpoints at $(a + x, b)$ and $(c + x, d)$ is congruent to the original segment. **true; Sample answer: The segment was translated x units to the right.**

b. The line segment with endpoints $\left(\frac{2}{3}a, \frac{2}{3}b\right)$ and $\left(\frac{2}{3}c, \frac{2}{3}d\right)$ is congruent to the original segment. **false; Sample answer: The segment was dilated by a scale factor of $\frac{2}{3}$.**

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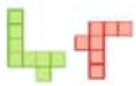
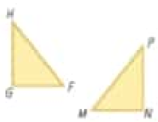
Extra Practice

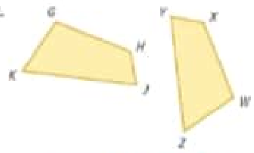
Determine if the two figures are congruent by using transformations. Explain your reasoning.

10. 

The two figures are not congruent because no sequence of transformations will map the green figure onto the red figure exactly.

11. 

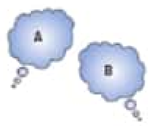
congruent; A reflection followed by a translation maps $\triangle FGH$ onto $\triangle MNP$.

12. 

congruent; A reflection followed by a clockwise rotation of 90° followed by a translation maps *GHJK* onto *WXYZ*.

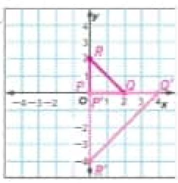
13. Ismail is illustrating a graphic novel for a friend. He is using the two thought bubbles shown. What transformations did he use if Figure A is the preimage and Figure B is the image?

Sample answer: a reflection followed by a translation



14. **Model with Mathematics** Graph $\triangle PQR$ with vertices at $P(0, 0)$, $Q(2, 0)$, and $R(0, 2)$. Then graph the image of the triangle after a reflection over the x-axis followed by a dilation with a scale factor of 2. Find the lengths of each side of the preimage and the image. Then determine if the two figures are congruent.

2 units, 2 units, $\sqrt{8}$ units; 4 units, 4 units, $\sqrt{32}$ units; no



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Power Up! Test Practice

Exercises 15 and 16 prepare students for more rigorous thinking.

15.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP3
Scoring Rubric	
1 point	Students correctly answer the question.

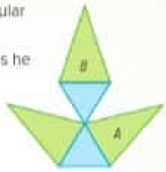
16.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly label the vertices.

Power Up! Test Practice

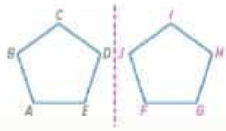
15. Usama is creating a mosaic for art class. He started by using triangular tiles as shown.

Triangles *A* and *B* are congruent. Describe possible transformations he could have used if triangle *A* is the preimage and triangle *B* is the image?



Sample answer: Rotate triangle *A* counterclockwise, then translate it up.

16. Pentagon *ABCDE* is reflected across the line shown and then rotated 72° clockwise about its center to create congruent pentagon *FGHIJ*. Label the vertices of *FGHIJ* in the correct positions on the image.

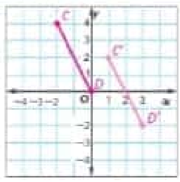


Spiral Review

Graph each figure with the given vertices and its image after the indicated transformation. Then give the coordinates of the final image.

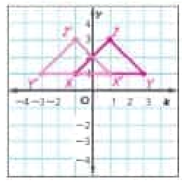
17. $\overline{CD}$: $C(-2, 4)$, $D(0, 0)$; translation of 3 units right and 2 units down

C′(1, 2), D′(3, −2)



18. $\triangle XYZ$: $X(-1, 1)$, $Y(3, 1)$, $Z(1, 3)$; reflection over the *y*-axis

X′(1, 1), Y′(−3, 1), Z′(−1, 3)



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Inquiry Lab
Investigate Congruent Triangles

inquiry **WHICH three pairs of corresponding parts can be used to show that two triangles are congruent?**

MP Mathematical Practices 1, 3

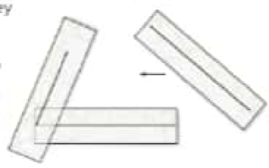
While driving past a bridge with his family, Ayman noticed that the bridge truss was made up of congruent triangles.

Hands-On Activity 1

In this Activity you will investigate whether it is possible to show that two triangles are congruent without showing that all six pairs of corresponding parts are congruent.

Step 1 Copy the sides of the triangle shown onto a piece of patty paper and cut them out.

Step 2 Arrange and tape the pieces together so that they form a triangle.



Is the triangle you formed congruent to the original triangle? Explain. Yes; Sample answer: When placed on top of each other, the corresponding sides and angles are congruent.

Rotate the triangle you formed 180°. Is the triangle congruent to the original triangle? Explain. Yes; Sample answer: The rotation of the triangle does not change the shape or size of a triangle.

Try to form another triangle with the given sides. Is it congruent to the original triangle? See students' work; yes.

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Focus narrowing the scope

Objective Determine which three pairs of corresponding parts can be used to show that two triangles are congruent.

Coherence connecting within and across grades

Now Students will generalize the properties of congruence.

Next Students will find missing measures in congruent figures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 519.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: tracing paper

Hands-On Activity 1

AL LA **Paired Heads Together** Have students work in pairs to check for congruence by placing the triangle they make over the original triangle. MP 1, 5

EL LA **Pairs Consult** Have students work with a partner to answer the following extension question. MP 1, 3

Ask:
- *Does the order in which you tape the sides of the triangle together matter? Explain.* Sample answer: no; The lengths of the sides of the triangle determine its shape and size, so it does not matter how you put it together.

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Hands-On Activity 2

AL LA **Pairs to Groups** Have students complete the activity with a partner, making sure their copies match closely to the original and their lines are straight. Then have them join with another pair to complete Exercises Steps 1 an MP 2, 5

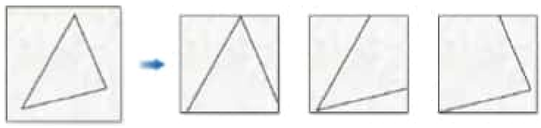
BL LA **Pairs Consult** Have students make a conjecture about congruence of triangles using angles before completing the activity. Give them time to draw examples to prove their conjecture. Then have them complete the activity with a partner. When completed, have them refer back to their conjecture to see if they were corre MP 1, 3, 5

Ask:

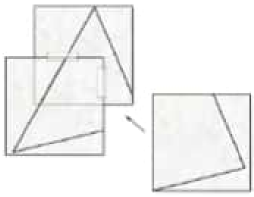
- *Was your conjecture correct? Why or why n* See students' work.
- *Can you draw a new example that proves or disproves your conjecture?* See students' work.

Hands-On Activity 2

Step 1 Draw a triangle on a piece of patty paper. Copy each angle of the triangle onto a separate piece of patty paper. Extend each side of each angle to the edge of the patty paper.



Step 2 Arrange and tape the pieces together so that they form a triangle.



Is the triangle you formed congruent to the original triangle? Explain.

No; Sample answer: The triangle has the same shape as the original triangle, but it is larger than the original, so it is not congruent to the original triangle.

Try to form another triangle with the given angles. Is it congruent to the original triangle? **See students' work; Most students' triangles will be similar, but not congruent to the original triangle.**

A *counterexample* disproves a statement by showing an example of when the statement is not true. Based on this activity, is the following statement true? If not, provide a counterexample.

If the angles of one triangle are congruent to the angles of another triangle, the two triangles are congruent.

no; See students' work.

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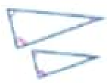
Investigate

1. Draw a triangle on a piece of tracing paper. Copy two sides of the triangle and the angle between them onto separate pieces of tracing paper and cut them out. Arrange and tape pieces together so that the two sides are joined to form the rays of the angle. Connect the two rays to form a triangle.
   a. Is the triangle you formed congruent to the original triangle? Explain. **Yes; Sample answer: When placed on top of each other, the corresponding sides and angles are congruent.**
   b. Try to form another triangle with the given sides and angle. Is it congruent to the original triangle? **See students' work; yes.**

2. Determine if two triangles with the following congruent parts are congruent. If not, draw a counterexample.

Various Parts	Congruent?	Counterexample
3 angles	No	
2 sides	No	See students' work.
2 angles and 1 side	Yes	
2 angles and the side between the 2 angles	Yes	
2 angles	No	See students' work.
3 sides	Yes	

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Engage Explore Explain Elaborate Evaluate

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1, 2	3–5	6, 7
Level 3			•
Level 2		•	
Level 1	•		

Investigate

AL LA **Team-Pair-Solo** Have students work in teams of 3 to complete Exercise 2, ensuring that each team member understands what congruent triangles look like. Then have the teams complete the second and third row in the table. Have them agree upon one response and one counterexample (if needed) to share with the class. MP 1, 3, 5

BL LA **Pairs Consult** Have pairs of students discuss and explain why certain parts of a triangle can be used to determine congruent triangles and other parts cannot. MP 1, 3

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Analyze and Reflect

AL LA Think-Pair-Share Give students one minute to think through their responses to Exercises 3–5. Then arrange students in pairs and have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 3

BL LA Pairs Consult After completing Exercises 3–5 on their own, have students explain to a partner why each exercise does or does not show congruence. MP 1, 3

Create

BL LA Trade-a-Problem Have students write their own case, then trade with a partner. Have them read each other's case and decide if it can be used to show congruence. Have them verify their conjecture with drawings. MP 1, 3, 4

Inquiry Students should be able to answer "WHICH three pairs of corresponding parts can be used to show that two triangles are congruent?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

3. Based on Activity 1, can three pairs of congruent sides be used to show that two triangles are congruent? yes

4. Based on Activity 2, can three pairs of congruent angles be used to show that two triangles are congruent? no

5. Based on Exercise 1, can two pairs of congruent sides and the pair of congruent angles between them be used to show that two triangles are congruent? yes

Create

6. MP **Make a Conjecture** Use patty paper to investigate the relationship between two triangles with the given information. Make a conjecture about whether each of these cases can be used to show that two triangles are congruent.

Case 1 two pairs of congruent sides and a pair of congruent angles not between them Sample answer: To make two congruent triangles, you cannot use two pairs of congruent sides and a pair of angles that are not included.

Case 2 two pairs of congruent angles and the pair of congruent sides between them Sample answer: You can use two pairs of congruent angles and the pair of congruent sides not between them to make congruent triangles.

Case 3 two pairs of congruent angles and a pair of congruent sides not between them Sample answer: You can use two pairs of congruent angles and the pair of congruent sides not between them to make congruent triangles.

7. Inquiry WHICH three pairs of corresponding parts can be used to show that two triangles are congruent?
Sample answer: Three pairs of congruent sides, two pairs of congruent sides with the pair of congruent angles between them, two pairs of congruent angles with any pair of congruent sides.

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Lesson 2

Congruence

Real-World Link

Crafts Buthaina is creating a quilt using the geometric pattern shown. She wants to make sure that all of the triangles in the pattern are the same shape and size.

1. What would Buthaina need to do to show the two triangles are congruent?
   Measure the sides and angles of each triangle and compare them.

2. Complete the lists of the parts △ABC and △DEF. Then draw lines between the corresponding parts of each triangle.

   $\overline{CB}$ $\overline{CA}$ $\overline{BA}$ ∠BAC ∠ABC ∠C

   $\overline{FD}$ $\overline{ED}$ $\overline{FE}$ ∠DEF ∠F ∠EDF

3. Suppose you cut out the two triangles and laid one on top of the other so the parts of the same measures were matched up. What is true about the triangles?
   They are congruent.

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

corresponding parts

Math Symbols

≅ is congruent to

Mathematical Practices

1, 2, 3, 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Write congruence statements for congruent figures.

Coherence connecting within and across grades

Previous Students generalized the properties of congruence.

Now Students will determine if two figures are congruent and find missing measures of congruent figures.

Next Students will use transformations to determine if two figures are congruent.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 525.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Paired Heads Together Have students work with a partner to trace the triangles on separate pieces of paper and label the sides and angles on the inside of the triangles. Have one student place the triangles on top of each other and match up segments and angles while the other student confirms the matched parts in Exercises 2. MP 1, 4, 5

Alternate Strategies

AL Have students use a protractor and a ruler to verify that the triangles are congruent. MP 1, 5, 7

BL Have students explain why three letters are necessary to name some of the angles, while only a single letter is needed to name others. MP 1, 3, 7

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2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

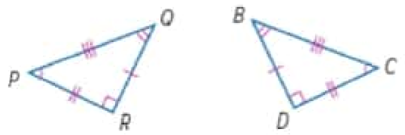
1. Write congruence statements.

AL
- *What symbol is used to indicate congruent angles?* **arcs**
- *What symbol is used to indicate congruent segments?* **tick marks**

OL
- *Which angles are congruent?* $\angle G \cong \angle J; \angle I \cong \angle L; \angle H \cong \angle K$
- *Which sides are congruent?* $\overline{GH} \cong \overline{JK}; \overline{HI} \cong \overline{KL}; \overline{GI} \cong \overline{JL}$

BL
- *How do we know, based on the markings, that the triangles are congruent?* **Sample answer: All three sides are congruent; therefore, the triangles must be congruent.**

Need Another Example?
Write congruence statements comparing the corresponding parts in the congruent triangles shown. $\angle P \cong \angle C$, $\angle Q \cong \angle B$, $\angle R \cong \angle D$; $\overline{RQ} \cong \overline{DB}$; $\overline{RP} \cong \overline{DC}$; $\overline{PQ} \cong \overline{CB}$



Watch Out!

Common Error Remind students that they need to look at the congruency marks when writing a congruency statement and not just list parts in alphabetical order.

Key Concept

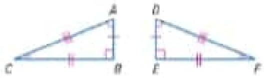
Work Zone

Congruence
To indicate that sides are congruent, an equal number of tick marks is drawn on the corresponding sides. To show that angles are congruent, an equal number of arcs is drawn on the corresponding angles.

Corresponding Parts of Congruent Figures

Words If two figures are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

Model

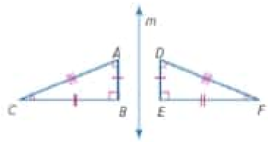


Symbols $\triangle ABC \cong \triangle DEF$

Congruent Angles: $\angle A \cong \angle D$; $\angle B \cong \angle E$; $\angle C \cong \angle F$

Congruent Sides: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$; $\overline{CA} \cong \overline{FD}$

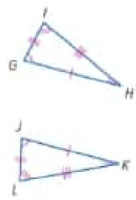
In the figure below, the two triangles are congruent because $\triangle DEF$ is the image of $\triangle ABC$ reflected over line m. The notation $\triangle ABC \cong \triangle DEF$ is read *triangle ABC is congruent to triangle DEF*.



The parts of congruent figures that *match* or correspond, are called **corresponding parts**.

Example

1. Write congruence statements comparing the corresponding parts in the congruent triangles shown.



Use the matching arcs and tick marks to identify the corresponding parts.

Corresponding angles:
$\angle J \cong \angle G$, $\angle L \cong \angle I$, $\angle K \cong \angle H$

Corresponding sides:
$\overline{JK} \cong \overline{GH}$, $\overline{KL} \cong \overline{HI}$, $\overline{LJ} \cong \overline{IG}$

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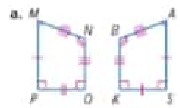
Got it? Do this problem to find out.

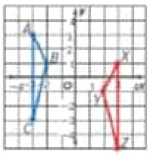
a.



a. $\angle A \cong \angle M$, $\angle B \cong \angle N$, $\angle K \cong \angle O$, $\angle S \cong \angle P$
$\overline{AB} \cong \overline{MN}$, $\overline{BK} \cong \overline{NO}$, $\overline{KS} \cong \overline{OP}$, $\overline{SA} \cong \overline{PM}$

Example

2. Triangle *ABC* is congruent to *XYZ*. Write congruence statements comparing the corresponding parts. Then determine which transformations map △*ABC* onto △*XYZ*.



Step 1 Analyze the figures to determine which angles and sides of the figures correspond.

Corresponding angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$
Corresponding sides: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{CA} \cong \overline{ZX}$

Step 2 Determine any changes in the orientation of the triangles. The orientation is reversed so at least one of the transformations is a reflection. If you reflect △*ABC* over the *y*-axis and then translate it down 2 units, it coincides with △*XYZ*.

The transformations that map △*ABC* onto △*XYZ* consist of a reflection over the *y*-axis followed by a translation of 2 units down.

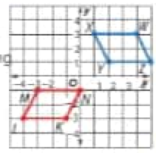
STOP and Reflect

When writing congruence statements, why is it important to match up corresponding points in the statement?

Sample answer: You can determine which points correspond by using the congruence statements.

Got it? Do this problem to find out.

b. Parallelogram *WXYZ* is congruent to parallelogram *KLMN*. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map parallelogram *WXYZ* onto parallelogram *KLMN*.



b. $\angle W \cong \angle K$, $\angle X \cong \angle L$, $\angle Y \cong \angle M$, $\angle Z \cong \angle N$
$\overline{WX} \cong \overline{KL}$, $\overline{XY} \cong \overline{LM}$, $\overline{YZ} \cong \overline{MN}$, $\overline{ZW} \cong \overline{NK}$;
Sample answer: If you reflect *KLMN* over the *x*-axis and then translate it to the right 5 units, it coincides with *WXYZ*.

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Example

2. Write congruence statements and determine transformations.

AL
- *Is the orientation of the triangles the same?* no
- *Write congruence statements to show which angles are corresponding.* $\angle A \cong \angle X$; $\angle B \cong \angle Y$; $\angle C \cong \angle Z$

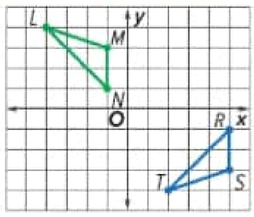
OL
- *How can you determine which sides are corresponding?* Find the lengths of the segments. The segments that are congruent are the corresponding sides.
- *Write congruence statements to show which sides are corresponding.* $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{CA} \cong \overline{ZX}$
- *Since the orientation is different, which transformation could have occurred?* rotation or reflection and translation

BL
- *Without looking at the graph of the triangles, how can you name the corresponding parts?* Sample answer: the order of letters in the name of △*ABC* corresponds with the letters in the name of △*XYZ*. Because *A* is written first, it means it corresponds with *X*, which is also written first.

Need Another Example?

Triangle *RST* is congruent to △*NML*. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) maps △*RST* onto △*NML*.

$\angle R \cong \angle N$, $\angle S \cong \angle M$, $\angle T \cong \angle L$; $\overline{RS} \cong \overline{NM}$; $\overline{RT} \cong \overline{NL}$; $\overline{ST} \cong \overline{ML}$;
Sample answer: If you reflect △*RST* over the *x*-axis and then translate it to the left 6 units, it coincides with △*NML*.



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Example

3. Determine missing measures.

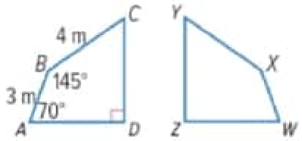
AL • *How can we determine the missing angle measure of a triangle?* Add the known angle measures and subtract from 180.

• *What is true about angles CBE and FDG?* They are corresponding angles of congruent figures.

OL • *What is true about corresponding angles of congruent figures?* They are congruent.

BL • *If $m\angle BCE = 90°$, explain how we can use $\triangle BCE$ to determine the missing angle measure of $\triangle DFG$.* $\triangle BCE$ and $\triangle DFG$ are congruent; therefore, $m\angle FDG$ is congruent to $m\angle CBE$, which is 40°.

Need Another Example?
In the figure below, quadrilateral *ABCD* is congruent to quadrilateral *WXYZ*. What is the measure of $\angle X$? 145°



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Three Stay, One Stray** Have students work in four-person teams to complete Exercise 1, making sure that all team members understand and can explain their solution. Then have one student from each team stand and stray to a new team. The new teams discuss and compare answers. Then have the new teams complete Exercise 2. Follow the same process until Exercise 3 is complete. MP 1, 3, 6

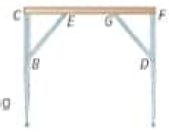
Find Missing Measures

You can use properties of congruent figures to find the missing measures of angles and sides in a figure.

Congruence
Congruent angles have the same measure and congruent sides have equal length.

Example

3. Badria is using a brace to support a tabletop. In the figure, $\triangle BCE \cong \triangle DFG$. If $m\angle CEB = 50°$, what is the measure of $\angle FGD$?



Since $\angle CEB$ and $\angle FGD$ are corresponding parts in congruent figures, they are congruent. So $\angle FGD$ measures 50°.

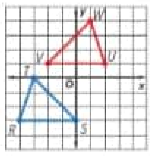
Got it? Do this problem to find out.

c. In the figure shown above, the length of $\overline{CE}$ is 0.6 meters. What is the length of $\overline{FG}$?

c. 0.6 meters

Guided Practice

1. Triangle *RST* is congruent to $\triangle UVW$. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map $\triangle RST$ onto $\triangle UVW$. (Examples 1 and 2)



$\angle R \cong \angle U$, $\angle S \cong \angle V$, $\angle T \cong \angle W$; $\overline{RS} \cong \overline{UV}$, $\overline{ST} \cong \overline{VW}$, $\overline{TR} \cong \overline{WU}$

Sample answer: If you translate $\triangle RST$ up 4 units then 2 units right, then reflect it over the *y*-axis, it coincides with $\triangle UVW$.

2. In the table design shown in Example 3, suppose $BE = 45$ centimeters. What is *DG*? (Example 3)

45 cm

3. **Building on the Essential Question** How can the coordinate plane help you determine that corresponding sides are congruent? Sample answer: You can find the lengths of corresponding sides by using the coordinates of the vertices and the Distance Formula.

Rate Yourself!
How confident are you about congruence? Check the box that applies.

FOLDABLES Time to update your Foldable!

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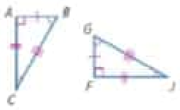
Name ______ My Homework ______

Independent Practice

Write congruence statements comparing the corresponding parts in each set of congruent figures. Example 1

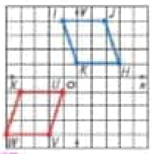
1. 

$\angle N \cong \angle S$, $\angle M \cong \angle T$, $\angle O \cong \angle V$
$\overline{ON} \cong \overline{VS}$, $\overline{NM} \cong \overline{ST}$, $\overline{MO} \cong \overline{TV}$

2. 

$\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle J$
$\overline{AB} \cong \overline{FG}$, $\overline{BC} \cong \overline{GJ}$, $\overline{CA} \cong \overline{JF}$

3. Parallelograms *UVWX* and *HJIK* are congruent. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map parallelogram *UVWX* onto parallelogram *HJIK*. Example 2



$\angle U \cong \angle H$, $\angle V \cong \angle J$, $\angle W \cong \angle I$, $\angle X \cong \angle K$
$\overline{UV} \cong \overline{HJ}$, $\overline{VW} \cong \overline{JI}$, $\overline{WX} \cong \overline{IK}$, $\overline{XU} \cong \overline{KH}$
Sample answer: If you reflect parallelogram *UVWX* over the *x*-axis, then translate it 4 units to the right, it coincides with parallelogram *HJIK*.

4. In the umbrella shown at the right, $\triangle JLK \cong \triangle NLM$. Example 3

a. If $m\angle JKL = 66°$, then $m\angle NML =$ **66°**.
b. If $MN = 35$ centimeters, then $KJ =$ **35 cm**.

5. **Reason Abstractly** In the figure, $\triangle ABC \cong \triangle EBD$.
a. On the figure, draw arc and tic marks to identify the corresponding parts.
b. Find the value of *x*.
6

6. In the figure at the right, $\triangle EFG \cong \triangle LMN$. Find the value of *x*. Then describe the transformations that map $\triangle EFG$ onto $\triangle LMN$.
10; Sample answer: a rotation followed by a translation

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Engage Explore **Explain** **Elaborate** **Evaluate**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 11–14	5–7, 15–17	8–10
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 8, 10, 16, 17
OL	On Level	1, 3, 5–8, 10, 16, 17
BL	Beyond Level	5–10, 16, 17

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9
2 Reason abstractly and quantitatively.	5, 15
3 Construct viable arguments and critique the reasoning of others.	7, 8
4 Model with mathematics.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write what must be true about the corresponding angles and corresponding sides for two polygons to be congruent. See students' work.

7. **Make a Conjecture** Hexagon *ABCDEF* has six congruent sides.

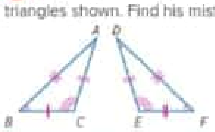
a. Draw $\overline{CA}$, $\overline{CF}$, and $\overline{CE}$.

b. How many triangles were formed? 4

c. Make a conjecture about which triangles are congruent. Test your conjecture by measuring the sides and angles of the triangles.
$\triangle ABC \cong \triangle CDE$ and $\triangle CAF \cong \triangle CEF$. See students' work for measurements; corresponding parts should be congruent.

H.O.T. Problems Higher Order Thinking

8. **Find the Error** Bilal is making a congruence statement for the congruent triangles shown. Find his mistake and correct it.



Bilal did not state the order correctly in the congruence statement. He should have said $\triangle ABC$ is congruent to $\triangle DFE$.

9. **Persevere with Problems** Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.

a. If two figures are congruent, their perimeters are equal.
true; Sample answer: If the figures are congruent, the corresponding sides have equal length. Therefore, the sum of the lengths of the sides will be equal.

b. If two figures have the same perimeter, they are congruent.
false; Sample answer: Triangle *ABC* has a perimeter of 24 centimeters. Square *MNOP* has a perimeter of 24 centimeters. They have the same perimeter but because they are different shapes, they are not congruent.

10. **Model with Mathematics** Write and solve a real-world problem that involves using the properties of congruent figures to find a missing measure. See students' work.

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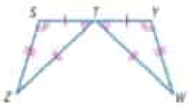
Name ______ My Homework ______

Extra Practice

Write congruence statements comparing the corresponding parts in each set of congruent figures.

11.



Use the matching arcs and tick marks to identify the corresponding parts.

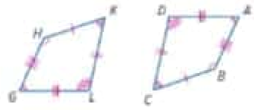
Corresponding angles:

$\angle S \cong \angle Y$, $\angle STZ \cong \angle YTW$, $\angle Z \cong \angle W$

Corresponding sides:

$\overline{SZ} \cong \overline{YW}$, $\overline{ZT} \cong \overline{WT}$, $\overline{TS} \cong \overline{TY}$

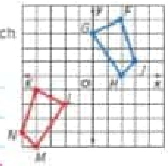
12.



$\angle G \cong \angle A$, $\angle H \cong \angle B$, $\angle K \cong \angle C$, $\angle L \cong \angle D$

$\overline{GH} \cong \overline{AB}$, $\overline{HK} \cong \overline{BC}$, $\overline{KL} \cong \overline{CD}$, $\overline{LG} \cong \overline{DA}$

13. Quadrilaterals *KLMN* and *FGHJ* are congruent. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map quadrilateral *KLMN* onto quadrilateral *FGHJ*.



$\angle K \cong \angle F$, $\angle L \cong \angle G$, $\angle M \cong \angle H$, $\angle N \cong \angle J$

$\overline{KL} \cong \overline{FG}$, $\overline{LM} \cong \overline{GH}$, $\overline{MN} \cong \overline{HJ}$, $\overline{NK} \cong \overline{JF}$

Sample answer: If you reflect quadrilateral *KLMN* over the *y*-axis, then translate it 5 units up and 2 units to the left, it coincides with quadrilateral *FGHJ*.

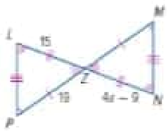
14. In the quilt design shown, $\triangle ABC \cong \triangle ADE$. What is the measure of $\angle BCA$?

59°

15. **Reason Abstractly** In the figure $\triangle LZP \cong \triangle NZM$.

a. On the figure, draw arc and tic marks to identify the corresponding parts.

b. Find the value of x. 6



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Power Up! Test Practice

Exercises 16 and 17 prepare students for more rigorous thinking.

16. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly answer each part of the question.
1 point	Students correctly answer four or five of the six parts.

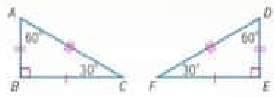
17. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

16. The triangles shown are congruent.

Complete the congruence statements to compare the corresponding parts.



a. $\angle A \cong$ $\angle D$ b. $\angle B \cong$ $\angle E$

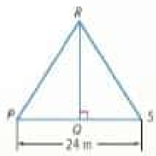
c. $\angle C \cong$ $\angle F$ d. $\overline{AB} \cong$ $\overline{DE}$

e. $\overline{BC} \cong$ $\overline{EF}$ f. $\overline{AC} \cong$ $\overline{DF}$

$\angle A$	$\angle D$	$\overline{AB}$	$\overline{DE}$
$\angle B$	$\angle E$	$\overline{AC}$	$\overline{DF}$
$\angle C$	$\angle F$	$\overline{BC}$	$\overline{EF}$

17. In the figure $\triangle PQR \cong \triangle SQR$. Which of the following represent a congruence statement for the corresponding parts? Select all that apply.

- [ ] $\angle RQP \cong \angle QSR$
- [ ] $\overline{PQ} \cong \overline{RQ}$
- [x] $\overline{RP} \cong \overline{RS}$
- [x] $\angle SRQ \cong \angle PRQ$

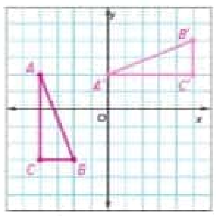


Spiral Review

Graph each figure with the given vertices and its image after the indicated transformations. Then give the coordinates of the final image.

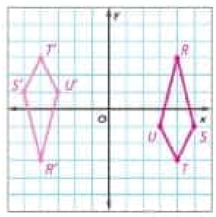
18. $\triangle ABC$: $A(-4, 2)$, $B(-2, -3)$, $C(-4, -3)$; 90° counterclockwise rotation about A followed by a translation of 4 units to the right

$A'(0, 2)$, $B'(5, 4)$, $C'(5, 2)$



19. quadrilateral $RSTU$: $R(4, 3)$, $S(5, -1)$, $T(4, -3)$, $U(3, -1)$; reflection over the x-axis followed by a reflection over the y-axis

$R'(-4, -3)$, $S'(-5, 1)$, $T'(-4, 3)$, $U'(-3, 1)$



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Problem-Solving Investigation

Draw a Diagram

Mathematical Practices 1, 3, 4

Case #1 Hammer Time

Hasan wants to make shelves to store his game system and other electronics in his room. He will make brackets in the shape of right triangles to hold the shelves. Since it is a right triangle, one of the angles measure 90°.

What is the relationship of the other two angles in a right triangle?

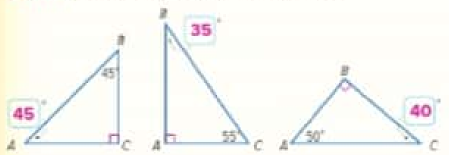
1 **Understand** *What are the facts?*

The bracket is in the shape of a right triangle, so one of the angles measures 90°.

2 **Plan** *What is your strategy to solve this problem?*

Draw several right triangles, measure each angle, and look for a pattern.

3 **Solve** *How can you apply the strategy?*



It appears that the sum of the measures of the acute angles of a right triangle is 90°. So, the acute angles are complementary.

4 **Check** *Does the answer make sense?*

You can try several more examples to see whether your conjecture appears to be true. But at this point, it is just a conjecture, not an actual proof.

Analyze the Strategy

Justify Conclusions Inductive reasoning is the process of making a conjecture after observing several examples. Did Hasan use inductive reasoning? Explain. Yes; Sample answer: Hasan observed that the acute angles of different right triangles were complementary. He decided that the acute angles of all right triangles are complementary.

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Focus narrowing the scope

Objective Solve problems by drawing a diagram.

This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Draw a Diagram Drawing a diagram is a good strategy for solving spatial and geometric problems. Students may find it helpful to list the information from a problem to make the diagram.

Coherence connecting within and across grades

Now Students apply the content standard to solve non-routine problems.

Next Students will use the draw a diagram strategy to determine congruence and similarity.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 531.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on these pages are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on this page walks students through the solution, while the problem on the following page asks students to come up with their own solutions.

Case #1 Hammer Time

BL Have students extend the problem by having them answer the question below.

Ask:

- *What is the greatest whole-number angle measure a right triangle can have for one of its two non-right angles?* 89

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Mid-Chapter Check

If students have trouble with Exercises 1–7, they may need help with the following concepts.

Concept	Exercise(s)
congruence and transformations (Lesson 1)	1, 3–5
congruence (Lesson 2)	2, 6, 7

Vocabulary Activity

LA **Think-Pair-Share** Give students about one minute to individually think through their responses to Exercises 1 and 2. Have students work in pairs to complete Exercises 1 and 2. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 6

Alternate Strategies

AL Have students define congruent in their own words before completing Exercise 1.

BL Have students explain the similarities and differences between congruent and similar shapes to a partner.

Mid-Chapter Check

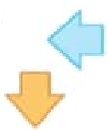
Vocabulary Check

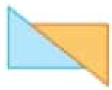
1. What transformations can be used to show two figures are congruent? (Lesson 1)
   translations, reflections, and rotations

2. List two attributes of two congruent polygons. (Lesson 2)
   Sample answer: corresponding sides are the same length and corresponding angles have the same measure.

Skills Check and Problem Solving

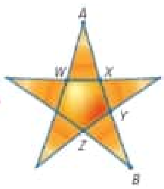
Determine if the two figures are congruent by using transformations. Explain your reasoning. (Lesson 1)

3. 
   congruent; A rotation followed by a translation maps the blue figure onto the gold figure.

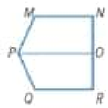
4. 
   congruent; A rotation followed by a translation maps the blue figure onto the gold figure.

5. 
   not congruent; No transformation will map one onto the other.

6. Husam is creating the logo shown using a pentagon and five congruent triangles. Triangle *WAX* is congruent to triangle *YBZ*. Describe the transformations that map △*WAX* onto △*YBZ*. If *WX* measures 5 centimeters, what is the length of *YZ*? (Lesson 2)
   Sample answer: a rotation about *X* followed by a translation; 5 cm.



7. MP **Persevere with Problems** Trapezoid *MNOP* is congruent to trapezoid *QROP*. Which transformation maps *MNOP* onto *QROP*? (Lesson 2)
   reflection



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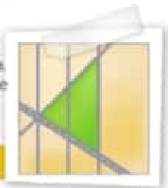
Inquiry Lab
Similar Triangles

Inquiry **HOW are two triangles related if they have the same shape but different sizes?**

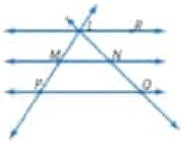
MP Mathematical Practices 1, 3

While flying in an airplane, Houriyya looked out the window and saw roads and a field like the one shown. She wondered if there was a relationship between the two triangles she saw.



Hands-On Activity

To determine if there is a relationship between the two triangles, use the diagram shown.



$\overleftrightarrow{LR} \parallel \overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$
$\overleftrightarrow{LP}$ and $\overleftrightarrow{LQ}$ are transversals.

Step 1 Measure and record the lengths of the line segments in millimeters and angles in degrees in the table.

△LPQ		△LMN	
$LP =$ 18 mm	$m\angle L = 78°$	$LM =$ 9 mm	$m\angle L = 78°$
$LQ =$ 21 mm	$m\angle P = 58°$	$LN =$ 10.5 mm	$m\angle M = 58°$
$PQ =$ 25 mm	$m\angle Q = 44°$	$MN =$ 12.5 mm	$m\angle N = 44°$

What do you notice about the measure of the corresponding angles of the triangles? The measures are equal.

Step 2 Express the lengths of the corresponding sides of the triangles as ratios.

$\frac{LP}{LM} = \frac{18}{9}$ or 2 $\quad \frac{LQ}{LN} = \frac{21}{10.5}$ or 2 $\quad \frac{PQ}{MN} = \frac{25}{12.5}$ or 2

What do you notice about the ratios of the corresponding sides of the triangles? They are equal.

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Focus narrowing the scope

Objective Investigate properties of similar triangles.

Coherence connecting within and across grades

Now
Students will measure angles and sides and set up ratios to identify properties of similar shapes.

Next
Students will generalize the properties of similar shapes.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on the following page.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The Activity is intended to be used as a whole-group activity.

Materials: rulers, protractors

Hands-On Activity

AL LA Have students work in pairs to complete the activity. Have one student be the measurer and one be the recorder for the first triangle, then have them trade roles for the second triangle. MP 1, 2, 5

Ask:

- *Two triangles are formed by the parallel lines and transversals. Name them.* △LMN and △LPQ
- *Which angles in the triangles are corresponding angles?* ∠LMN and ∠LPQ; ∠LNM and ∠LQP

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1	Exercises 2–4	Exercises 5, 6
Level 3			•
Level 2		•	
Level 1	•		

Investigate

AL BL LA **Think-Pair-Share** Give students several minutes to draw and find a solution for Exercise 1. Have them share responses with a partner. MP 1, 4, 5

Analyze and Reflect

AL LA **Think-Pair-Share** Assemble students in pairs to complete Exercises 2–4, referring back to Exercise 1. MP 1, 5

Create

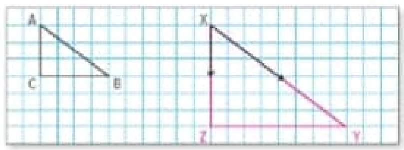
BL LA **Rally Coach** For Exercises 5 and 6, have students work with a partner. Partner A talks through Exercise 5 while Partner B coaches, listens, and praises. Partners switch roles for Exercise 6. MP 1, 3

Inquiry Students should be able to answer "HOW are two triangles related if they have the same shape but different sizes?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner.

1. MP **Model with Mathematics** Triangle *ABC* is a right triangle with $m\angle A = 53°$. On the grid, draw and label a different right triangle, *XYZ*, using the given angle *X*, which also measures 53°. **Sample answer:**



What do you notice about the shape of the triangles? **Sample answer: They appear to be the same shape.**

Analyze and Reflect

For Exercises 2–4, refer to the triangles in Exercise 1.

2. What is the measure of ∠B? the measure of the angle that corresponds to ∠B in △XYZ? **37°; 37°**

3. Express the lengths of the corresponding sides of the triangles as ratios. **Sample answer:**

$$\frac{AC}{XZ} = \frac{3}{6} \text{ or } \frac{1}{2} \qquad \frac{CB}{ZY} = \frac{4}{8} \text{ or } \frac{1}{2} \qquad \frac{AB}{XY} = \frac{5}{10} \text{ or } \frac{1}{2}$$

4. What do you notice about the ratios? **They are equal.**

Create

5. MP **Reason Inductively** The two triangles in the Activity and in Exercise 1 are called *similar triangles*. Based on your discoveries, make a conjecture about the properties of similar triangles.
Sample answer: Corresponding angles in two similar triangles have the same measures, and the ratios of the corresponding sides are equal.

6. Inquiry HOW are two triangles related if they have the same shape but different sizes?
Sample answer: The triangles are similar.

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Lesson 3

Similarity and Transformations

Vocabulary Start-Up

Recall that a dilation changes the size of a figure by a scale factor, but does not change the shape of the figure. Since the size is changed, the image and the preimage are not congruent.

Complete the graphic organizer. Consider each word on the Rating Scale and place a check in the appropriate column next to the word. If you do not know the meaning of a word, find the meaning in the glossary or on the Internet.
See students' work. Sample answers for definitions are given.

Word	Know it well	Have seen or heard it	No clue	What it means
dilation				an enlargement or reduction of a figure
scale factor				the ratio of the length of a side on a scale model to the same side of the original object
similar figures				two figures that are the same shape but may have different sizes

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

similar

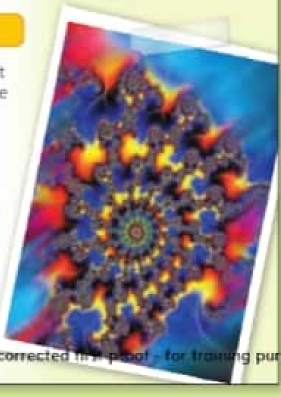
Mathematical Practices

1, 3, 4, 7

Real-World Link

A *fractal* is a geometric image that can be divided into parts that are smaller copies of the whole. The photo at the right is an example of a fractal.

1. Circle two different size parts of the figure that are smaller copies of the whole. See students' work.



Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

1 Persevere with Problems
2 Reason Abstractly
3 Construct an Argument
4 Model with Mathematics
5 Use Math Tools
6 Attend to Precision
7 Make Use of Structure
8 Use Repeated Reasoning

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Focus narrowing the scope

Objective Use transformations to create similar figures.

Coherence connecting within and across grades

Previous Students used transformations to determine congruence.

Now Students use properties of transformations to determine similarity.

Next Students will find missing measures in similar figures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 539.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Find the Fib** Have students work in teams. Each student writes down two facts and one fib about one of the following: dilation, scale factor, or similar figures. The job of the team is to identify the fib. MP 1, 3

Alternate Strategy

AL Have students list the transformations that produce congruent figures. Then discuss the effects dilation has on a figure. Remind students that dilation with a scale factor other than 1 is the only transformation that results in a figure that is not congruent to the original figure. MP 1, 7

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–2. Identify similarity.

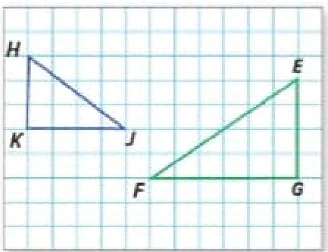
AL
- *Assuming the figures are similar in Example 1, which vertex in triangle DEF corresponds with vertex G in triangle GHI?* **D**
- *How would you translate △DEF so that D maps onto G?* **Translate the triangle 5 units right and 2 units down.**

OL
- *In Example 1, how would you find EF and HI?* **use the Pythagorean Theorem**
- *Are the ratios of the corresponding sides equal?* **yes**

BL
- *In Example 1, is △GHI a result of a dilation of △DEF? Explain.* **yes; Sample answer: The lengths of the sides of △GHI are two times the lengths of the sides of △DEF.**

Need Another Example?

Determine if the two triangles are similar using transformations. **no; Sample answer: The ratios of the side lengths are not equal for all of the sides. $\frac{HK}{EG} = \frac{3}{4}$, while $\frac{KJ}{GF} = \frac{2}{3}$.**



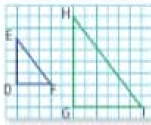
Work Zone

Identify Similarity

Two figures are **similar** if the second can be obtained from the first by a sequence of transformations and dilations.

Examples

1. Determine if the two triangles are similar by using transformations.



Since the orientation of the figures is the same, one of the transformations might be a translation.

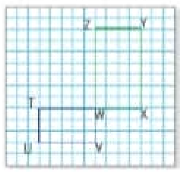
Step 1 Translate △*DEF* down 2 units and 5 units to the right so *D* maps onto *G*.

Step 2 Write ratios comparing the lengths of each side.

$\frac{HG}{ED} = \frac{8}{4}$ or $\frac{2}{1}$ $\quad \frac{GI}{DF} = \frac{6}{3}$ or $\frac{2}{1}$ $\quad \frac{IH}{FE} = \frac{10}{5}$ or $\frac{2}{1}$

Since the ratios are equal, △*HGI* is the dilated image of △*EDF*. So, the two triangles are similar because a translation and a dilation maps △*EDF* onto △*HGI*.

2. Determine if the two rectangles are similar by using transformations.



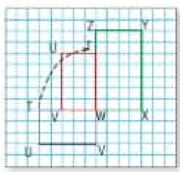
The orientation of the figures is the same, so one of the transformations might be a rotation.

Step 1 Rotate rectangle *VWTU* 90° clockwise about *W* so that it is oriented the same way as rectangle *WXYZ*.

Step 2 Write ratios comparing the lengths of each side.

$\frac{WT}{XY} = \frac{5}{7}$ $\quad \frac{TU}{YZ} = \frac{3}{4}$

$\frac{UV}{ZW} = \frac{5}{7}$ $\quad \frac{VW}{WX} = \frac{3}{4}$



The ratios are not equal. So, the two rectangles are not similar since a dilation did not occur.

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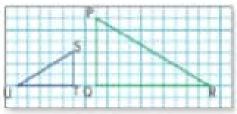
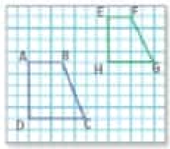
Got it? Do these problems to find out.

a.



b.



a. yes; Sample answer: A reflection and a dilation with a scale factor of 2 maps $\triangle STU$ onto $\triangle PQR$.

b. no; Sample answer: The ratio of the side lengths are not equal for all of the sides; $\frac{AB}{EF} = \frac{3}{2}$ while $\frac{AD}{EH} = \frac{5}{4}$.

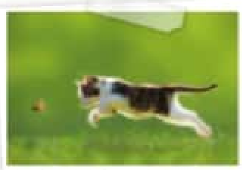
Use the Scale Factor

Similar figures have the same shape, but may have different sizes. The sizes of the two figures are related to the scale factor of the dilation.

If the scale factor of the dilation is ...	then the dilated figure is ...
between 0 and 1	smaller than the original
equal to 1	the same size as the original
greater than 1	larger than the original

Example

3. **Hamad enlarges the phot shown by a scale factor of for his webpage. He then enlarges the webpage phot by a scale factor of 1.5 to print. If the original photo is 5 centimeters by 7.5 centimeters, what are the dimensions of the print? Are the enlarged photos similar to the original?**



Multiply each dimension of the original photo by 2 to find the dimensions of the webpage photo.

5 cm × 2 = 10 cm   7.5 cm × 2 = 15 cm

So, the webpage photo will be 10 centimeters by 15 centimeters. Multiply the dimensions of that photo by 1.5 to find the dimensions of the print.

10 cm × 1.5 = 15 cm   15 cm × 1.5 = 22.5 cm

The printed photo will be 15 centimeters by 22.5 centimeters. All three photos are similar since each enlargement was the result of a dilation.

STOP and Reflect

List below at least two topics in mathematics that use a scale factor.

Sample answer: dilations, scale drawings

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Example

3. Use scale factor.

AL
- *What do you need to find?* the dimensions of the printed photo and if it is similar to the original photo
- *How will you know if the two photos are similar?* If one is the result of one or more dilations of the other, they are similar.
- *What are the dimensions of the original photo?* 5 cm × 7.5 cm
- *What are the dimensions of the webpage photo?* 10 cm × 15 cm
- *What photo is enlarged to make the print photo?* the webpage photo
- *What will you multiply the dimensions by to find the dimensions of the print photo?* 1.5
- *What are the dimensions of the print photo?* 15 cm × 22.5 cm

OL
- *How would you find the dimensions of the photo on the webpage?* Since the scale factor is 2, multiply each dimension of the original photo by 2.
- *How would you find the dimensions of the photo Ken will print?* Since the scale factor is 1.5, multiply each dimension of the webpage photo by 1.5.

BL
- *How can you find the dimensions of the printed photo without finding the dimensions of the webpage photo?* Sample answer: Multiply the original photo dimensions by 2 • 1.5 or 3.

Need Another Example?

A baker is reducing an 20 cm × 25 cm photo to place the image on a cake. He reduces it by a scale factor of 0.8. Then decides the image is still too large, and reduces it by a scale factor of 0.9. What are the dimensions of the final image? Is the reduced image similar to the original? 14.4 cm × 18 cm

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Guided Practice

Formative A ssessment Use these exercises to assess students' un derstanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Pairs Check Have students work in pairs to complete Exercises 1–3. One partner solves the problem while the other coaches. Students switch roles for each Exercise. After all the Exercises are completed, pairs check their solutions with another pair and discuss any differences. MP 1, 5

BL LA Pairs Consult Students work in pairs to find the area and perimeter of the figures in Exercises 1 and 2. Then each pair makes a conjecture about how the area and perimeter of similar figures are related. Pairs share their conjectures with another pair of students and discuss any differences. Have students apply their area conjecture in Exercise 3. MP 1, 3

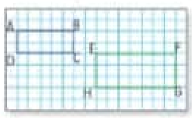
c. 108 cm by 135 cm; yes

Got it? Do this problem to find out.

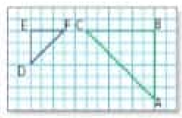
c. An art show offers different size prints of the same painting. The o riginal print measures 24 centimeters by 30 centimeters. A pri nter enlarges the original by a scale factor of 1.5, and then enla rges the second image by a scale factor of 3. What are the dim ensions of the largest print? Are both of the enlarged prints sim ilar to the original?

Guided Practice

Determine if the two figures are similar by using transformations. Explain your reasoning. (Examples 1 and 2)

1. 

no; Sample answer: The ratios of the side lengths are not equal for all of the sides; $\frac{EH}{AD} = \frac{3}{2}$ while $\frac{EF}{AB} = \frac{7}{5}$.

2. 

yes; Sample answer: A reflection and a dilation with a scale factor $\frac{1}{2}$ maps $\triangle CBA$ onto $\triangle FED$.

3. A T-shirt iron-on measures 5 cm by 2.5 cm. It is enlarged by a scale factor of 3 for the back of the shirt. The second iron-on is enlarged by a scale factor of 2 for the front of the shirt. What are the dimensions of the largest iron-on? Are both of the enlarged iron-ons similar to the original? (Example 3) 30 cm by 15 cm

4. **Building on the Essential Question** What is the difference between using transformations to create similar figures versus using transformations to create congruent figures?

Sample answer: A dilation with a scale factor that is not equal to one is always used to create similar figures while it is never used to create congruent figures.

Rate Yourself!

How confident are you about similar figures? Shade the ring on the target.

FOLDABLES Time to update your Foldable!

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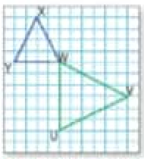
Name ________ My Homework ________

Independent Practice

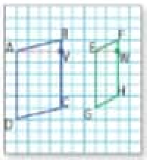
Determine if the two figures are similar by using transformations. Explain your reasoning. (Examples 1 and 2)

1.



yes; Sample answer: A rotation, a translation of 4 units down, and a dilation with a scale factor of $\frac{3}{2}$ maps onto

2.



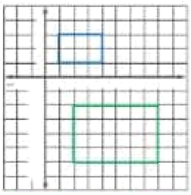
no; Sample answer:
$\frac{6}{5}$ and $\frac{4}{2}$, $\frac{6}{5}$ $\frac{4}{2}$

3.

(Example 3) 13.5 cm by 22.5 cm

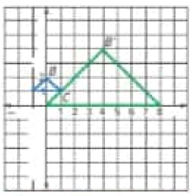
Persevere with Problems Each preimage and image are similar. Describe a sequence of transformations that maps the preimage onto the image.

4.



Sample answer: reflection over the -axis followed by a dilation with a scale factor of 2

5.



Sample answer: translation of 1 unit to the right and 1 unit down followed by a dilation with a scale factor of 4

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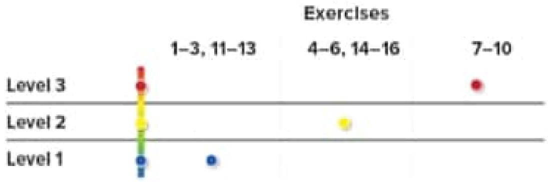
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 8–10, 15, 16
OL	On Level	1, 3, 4–6, 8–10, 15, 16
BL	Beyond Level	4–10, 15, 16

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	4, 5, 7
3 Construct viable arguments and critique the reasoning of others.	9
4 Model with mathematics	8, 10, 14
7 Look for and make use of structure.	6

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Tell students that a figure is dilated by a scale factor of *a* and then the image is dilated by a scale factor of *b*. Have them determine whether the result is the same if the figure is first dilated by the scale factor *b* and then by the scale factor *a*. Have them explain their reasoning. **Yes; multiplication is commutative.**

6. **Identify Structure** Use the graphic organizer to compare and contrast similar and congruent figures. **Sample answers are given.**

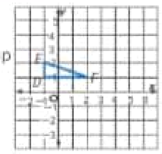
	Similar Figures	Congruent Figures
Side Measures	usually different	always the same
Angle Measures	always the same	always the same
Transformations Used	always a dilation, may use rotation, reflection, or translation	always use rotation, reflection, or translation, never use a dilation

H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** Using at least one dilation, describe a series of transformations where the image is congruent to the preimage.

See students' work; product of dilation(s) should equal 1.

8. **Model with Mathematics** The image of △*DEF* after two transformations has vertices at *D*′(3, 3), *E*′(6, 3) and *F*′(3, −6). If the two triangles are similar, determine what two transformations map △*DEF* onto △*D*′*E*′*F*′.

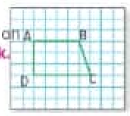


Sample answer: 90° clockwise rotation about the origin followed by a dilation with a scale factor of 3

9. **Construct an Argument** True or *false*. If a dilation is in a composition of transformations, the order in which you perform the composition does not matter. Explain your reasoning.

false; Sample answer: If you perform the dilation after a translation, the translation is multiplied by the same scale factor.

10. **Model with Mathematics** Trapezoid *ABCD* is shown at the right. Perform a series of transformations on the trapezoid and draw the image on the coordinate plane. List the transformations used below. **See students' work.**



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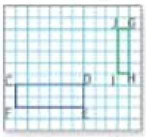
Name ____________________ My Homework ____________________

Extra Practice

Determine if the two figures are similar by using transformations. Explain your reasoning.

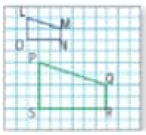
11.



no; The ratios of the side lengths are not equal.

Find the ratios of the side lengths. $\frac{CD}{GH} = \frac{6}{4}$ and $\frac{DE}{JG} = \frac{2}{1}$; $\frac{6}{4} \neq \frac{2}{1}$, so the two figures are not similar.

12.

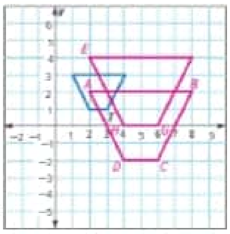


yes; Sample answer: A translation and a dilation with a scale factor of 2 maps trapezoid ______ onto trapezoid ______.

13. Reham is making three different sizes of blankets from the same material. The first measures 1 meter by 0.6 meters. She wants to enlarge it by a scale factor of 2 to make the second blanket. Then she will enlarge the second one by a scale factor of 1.5 to make the third blanket. What are the dimensions of the third blanket? Are all of the blankets similar?

3 m x 1.8 m

14. **Model with Mathematics** In the figure shown, trapezoid *RSTU* has vertices *R*(1, 3), *S*(4, 3), *T*(3, 1), and *U*(2, 1).



a. Draw the image of *RSTU* after a translation of 2 units down followed by a dilation with a scale factor of 2. Label the vertices *ABCD*.

b. Draw the image of *RSTU* after a dilation with a scale factor of 2, followed by a translation of 2 units down. Label the vertices *EFGH*.

c. Which figures are similar? Which figures are congruent?

______ and ______ are similar; ______ and ______ are similar; ______ and ______ are congruent

d. Are *ABCD* and *EFGH* in the same location? If they are not, what transformation would map *ABCD* onto *EFGH*?

no; a translation of 2 units up

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Power Up! Test Practice

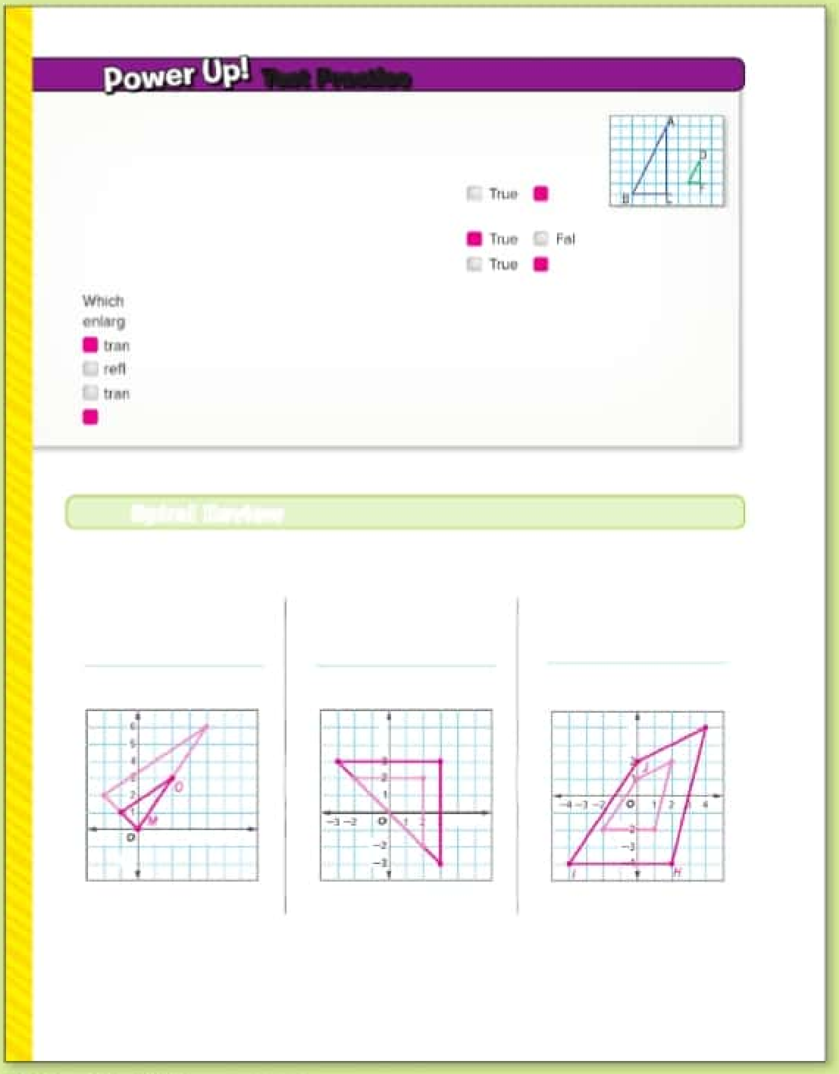
Exercises 15 and 16 prepare students for more rigorous thinking.

15.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

16.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



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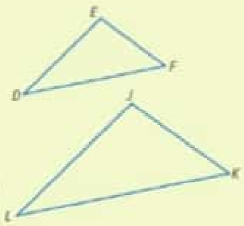
Lesson 4

Properties of Similar Polygons

Real-World Link

Photos Salma is printing pictures at a photo kiosk in the store. She can choose between 4 6 prints or 5× 7 prints. Are the side lengths of the two prints proportional? Explain. **No; $\frac{4}{5} \neq \frac{6}{7}$**

Follow the steps to discover how the triangles are related.



1. Using a centimeter ruler, measure the sides of the two triangles. Then, use a protractor to measure the angles. Write the results in the table. **Sample answers are given.**

Figure	Side Length (cm)			Angle Measure (°)		
△EFD	DE	EF	FD	∠D	∠E	∠F
	2.7	2	3.6	33	100	47
△LJK	LJ	JK	KL	∠L	∠J	∠K
	4.1	3	5.4	33	100	47

2. Are the side lengths proportional? Explain.
yes; $\frac{DE}{LJ} = \frac{EF}{JK} = \frac{FD}{KL} = \frac{2}{3}$

3. What do you notice about the angles of the two triangles?
The corresponding angles of the triangles have the same measure.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

similar polygons
scale factor

Math Symbols

~ is similar to

Mathematical Practices
1, 2, 3, 4

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Focus narrowing the scope

Objective Identify similar polygons and find missing measures of similar polygons.

Coherence connecting within and across grades

Previous Students used transformations to determine if two figures were congruent or similar.

Now Students use proportional reasoning to find missing measures of similar figures.

Next Students will use similar triangles to find missing measures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 547.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Check** Place students in pairs to complete the activity and Exercises 1–3. Then have pairs check results with other pairs and resolve any discrepancies. **MP 1, 2, 3, 5**

Alternate Strategy

AL **LA** In Exercise 3, give students a writing prompt such as, "The ratios of corresponding sides are..." instead of having them say the sides of triangle *LJK* are longer. **MP 1, 6, 7**

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Describe attributes of similar polygons.

AL • *How do you know that the corresponding angles are congruent?* **Sample answer: All of the angles are right angles, so they are all congruent.**

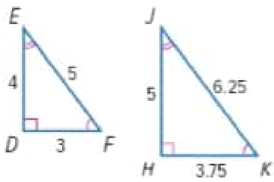
OL • *What is the ratio of the length $\overline{JK}$ to $\overline{NP}$?* $\frac{3}{6}$ **or** $\frac{1}{2}$

• *What is the ratio of the length $\overline{JH}$ to $\overline{NM}$?* $\frac{7}{10}$

• *Are the ratios of the corresponding sides equal?* **no**

BL • *If corresponding angles are congruent, does that always mean that the ratios of corresponding sides will be proportional? Explain.* **no; Sample answer: If the corresponding angles are congruent, the shapes may not be the same and so not necessarily have proportional side lengths.**

Need Another Example?
Determine whether triangle *DEF* is similar to triangle *HJK*. Explain. **Yes; the corresponding angles are congruent and** $\frac{4}{5} = \frac{5}{6.25} = \frac{3}{3.75}$



Key Concept **Similar Polygons**

Work Zone

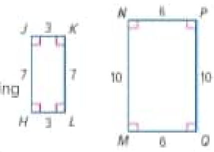
Words	If two polygons are similar, then • their corresponding angles are congruent and • the measures of their corresponding sides are proportional.
Model	 $\triangle ABC \sim \triangle XYZ$
Symbols	$\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

Polygons that have the same shape are called **similar polygons**. In the Key Concept box, triangle *ABC* is similar to triangle *XYZ*. This is written as $\triangle ABC \sim \triangle XYZ$. The parts of similar figures that "match" are called corresponding parts.

Common Error
Do not assume that two rectangles are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.

Example

1. Determine whether rectangle *HJKL* is similar to rectangle *MNPQ*. Explain.



First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

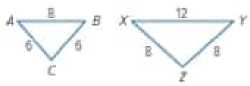
Next, check to see if corresponding sides are proportional.

$$\frac{HJ}{MN} = \frac{7}{10} \quad \frac{JK}{NP} = \frac{3}{6} \text{ or } \frac{1}{2} \quad \frac{KL}{PQ} = \frac{7}{10} \quad \frac{LH}{QM} = \frac{3}{6} \text{ or } \frac{1}{2}$$

Since $\frac{7}{10}$ and $\frac{1}{2}$ are not equivalent, the rectangles are *not* similar.

Got it? Do this problem to find out.

a. Determine whether $\triangle ABC$ is similar to $\triangle XYZ$. Explain.



a. **No; The corresponding angles are not congruent and** $\frac{8}{12} \neq \frac{6}{8}$

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Find Missing Measures

Scale factor is the ratio of the lengths of two corresponding sides of two similar polygons. You can use the scale factor of similar figures to find missing measures.

Example

2. **Quadrilateral *WXYZ* is similar to quadrilateral *ABCD*.**

a. Describe the transformations that map *WXYZ* onto *ABCD*.

Since the figures are similar, they are not the same size. Choose two corresponding sides and determine what transformations will map one onto the other. A translation followed by a dilation will map onto .

b. Find the missing measure.

Method 1

Find the scale factor from quadrilateral to quadrilateral .

scale factor: $\frac{15}{10}$ or $\frac{3}{2}$

So, a length on polygon $\frac{3}{2}$ times as long as the corresponding length on polygon . Let represent the measure of .

$\frac{3}{2}(12)$ Write the equation.

18 Multiply.

Method 2

Set up a proportion to find the missing measure.

Write the proportion.

12 $\frac{15}{10}$. 12, 15, 10

10 12 15 Find the cross products.

10 180 Simplify.

18 Division Property of Equality

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Example

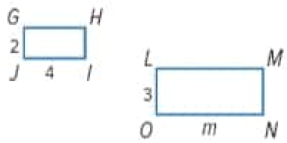
2. **Determine missing measures.**

AL
- *Are the figures similar or congruent? How do you know?* similar; They are the same shape, but not the same size.
- *What angle corresponds to angle B? How do you know?* angle *X*; They have the same number of arcs.

OL
- *How would you determine the scale factor?* Sample answer: Determine the ratio of the lengths of two corresponding sides of two similar polygons.
- *How can you use the scale factor to find the missing length?* Multiply the length of $\overline{BC}$ by the scale factor, $\frac{3}{2}$.

BL
- *Compare and contrast Method 1 and Method 2.* Sample answer: In Method 1, I need to first determine the scale factor, then use it to determine the missing length. In Method 2, I do not need to determine the scale factor, but I need to set up a proportion using two sets of corresponding sides.

Need Another Example?
Rectangle *LMNO* is similar to rectangle *GHIJ*.



a. Describe the transformations that map *GHIJ* onto *LMNO*. Since the figures are similar, they are not the same size. So, the transformation is a translation followed by a dilation.

b. Find the missing measure. 6

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Three-Step Interview** Have students work in pairs to complete Exercises 1–5. Student 1 interviews Student 2 with questions about method preferences, which angles and sides correspond, and how to check your work. While Student 2 explains the answer, Student 1 carefully listens and asks clarifying questions. Students trade roles for successive exercises. MP 1, 3

BL LA **Class Coaches** Students complete Exercises 1–5 individually. Check their work for accuracy. Students should then walk around the room and help those who are struggling, explaining the process in a different way than was explained in previous exercises. MP 1, 3

Watch Out!

Common Error Students may not remember how to determine which angles and sides correspond in the similar figures. Remind students that the order of the letters in the names of the figures tells us the corresponding parts. For example, because the letter *J* is given first in rectangle *JKLM*, it corresponds with *R*, the first letter given in rectangle *RSTU*.

---

b. 19.5

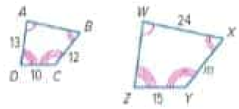
c. 16

Got it? Do these problems to find out.

Find each missing measure.

b. *WZ*

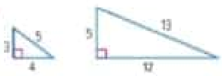
c. *AB*



Guided Practice

Determine whether each pair of polygons is similar. Explain.

1.



No; $\frac{5}{3} \neq \frac{13}{5}$

2.

Yes; the corresponding angles are congruent and $\frac{8}{6} = \frac{8}{6} = \frac{10}{7.5} = \frac{18}{13.5}$

3. The two triangles are similar.

a. Determine the transformations that map one figure onto the other.

Sample answer: rotation and dilation

b. Find the missing side measures. $GH = 12$; $KL = 4.5$

4. The two triangles are similar.

a. Determine the transformations that map one figure onto the other.

rotation, translation, and dilation

b. Find the missing side measure. 6

5. **Building on the Essential Question** How does the scale factor of a dilation relate to the ratio of two of the corresponding sides of the preimage and the image?

The scale factor and the ratio are equal.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

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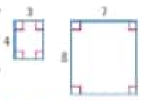
Name ______ My Homework ______

Independent Practice

Determine whether each pair of polygons is similar. Explain.

1.



No; The corresponding angles are congruent, but $\frac{3}{7} \neq \frac{4}{8}$.

2.

Yes; The corresponding angles are congruent and $\frac{3}{5} = \frac{3}{5}$.

Each pair of polygons is similar. Determine the transformations that map one figure onto the other. Then find the missing side measures.

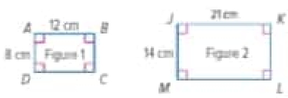
3.

translation and dilation; 4.5

4.

translation and dilation; 20

5. **Persevere with Problems** The figures at the right are similar.



a. Find the area of both figures.
Figure 1: 96 cm²; Figure 2: 294 cm²

b. Compare the scale factor of the side lengths and the ratio of the areas.
Sample answer: The scale factor of the side lengths is $\frac{14}{8}$ or $\frac{7}{4}$. The ratio of the areas is $\frac{49}{16}$. The ratio of the areas is the scale factor of the side lengths squared.

6. **STEM** The scale factor from the model of a human inner ear to the actual ear is 55:2. If one of the bones of the model is 8.25 centimeters long, how long is the actual bone in a human ear? 0.3 cm

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	1–4, 12–16	5–7, 17, 18	8–11
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9–11, 17, 18
OL	On Level	1, 3, 5–7, 9–11, 17, 18
BL	Beyond Level	5–11, 17, 18

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	5, 8
3 Construct viable arguments and critique the reasoning of others.	9, 10
4 Model with mathematics.	7, 11, 15

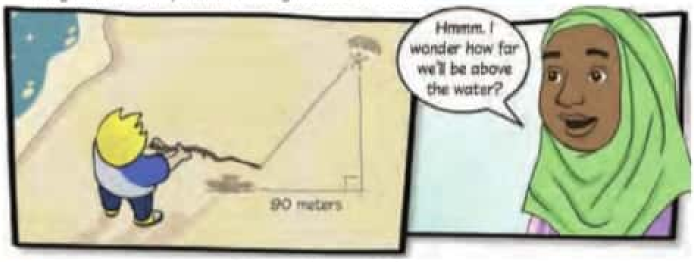
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students find the missing height given two similar rectangles, one with length 10 meters and height 12 meters, the other rectangle with length 15 meters. 18 m

7. **Model with Mathematics** Refer to the graphic novel frame below. The brochure says that the rope is 150 meters long. Use the properties of similar triangles to find the parasailer's height above the water. 120 m

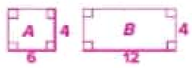


H.O.T. Problems Higher Order Thinking

8. **Persevere with Problems** Suppose two rectangles are similar with a scale factor of 2. What is the ratio of their areas? Explain. 1:4 or 4:1; Choose two rectangles with a scale factor of 2, 2 by 4 and 4 by 8. Compare the areas, 8 and 32.

Justify Conclusions Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, provide a counterexample.

9. All rectangles are similar.
false; Sample answer: In rectangles, all corresponding angles are congruent but not all sides are proportional. Rectangle *A* is not similar to Rectangle *B*, since $\frac{4}{4} \neq \frac{1}{2}$.



10. All squares are similar.
true; Sample answer: Since all four angles in a square are right angles, all corresponding angles between squares are congruent. In addition, all sides in a square are congruent. Therefore, all four ratios of corresponding sides are equal.

11. **Model with Mathematics** Draw two similar polygons in the space provided. Include the measures of the sides on your drawing, and identify the scale factor. See students' work.

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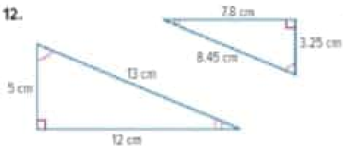
Name ______ My Homework ______

Extra Practice

Determine whether each pair of polygons is similar. Explain.

12.

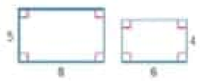


As indicated by the arc marks, corresponding angles are congruent. Check to see if the corresponding sides are proportional.

$\frac{3.25}{5} = \frac{8.45}{13} = \frac{7.8}{12}$

The sides are proportional so the triangles are similar.

13.



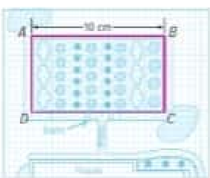
No; the corresponding angles are congruent, but $\frac{5}{4} \neq \frac{8}{6}$

14. The two figures are similar. Determine the transformations that map one figure onto the other. Then find the missing side length.

translation and dilation; 16.25

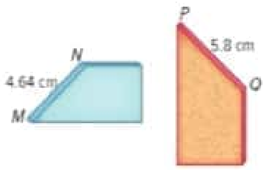
15. **Model with Mathematics** Manal wants to build a fence around the rectangular garden in her backyard. In the scale drawing, the perimeter of the garden is 34 centimeters. If the actual length $\overline{AB}$ is 6 meters, how many meters of fencing will she need?

20.4 m



16. Abdulaziz is making a mosaic using different pieces of tile. The tiles shown at the right are similar. If the perimeter of the larger tile is 23 centimeters, what is the perimeter of the smaller tile?

18.4 cm



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Power Up! Test Practice

Exercises 17 and 18 prepare students for more rigorous thinking.

17.

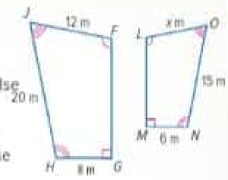
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answers each part of the question.

18.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly place both values.

Power Up! Test Practice

17. Quadrilateral *FGHJ* was transformed to create similar quadrilateral *LMNO*.



Determine if each statement is true ~~else~~.

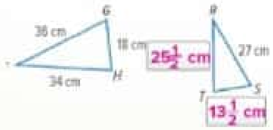
a. *FGHJ* was reflected and dilated to create *LMNO*. ■ True □ False

b. The scale factor of the dilation is $\frac{3}{4}$. ■ True □ False

c. The value of x is 16. □ True ■ False

18. Triangle *FGH* is similar to triangle *RST*. Select the correct values to label the missing side lengths of triangle *RST*.

$13\frac{1}{2}$ cm
$14\frac{5}{7}$ cm
$22\frac{2}{3}$ cm
24 cm
$25\frac{1}{2}$ cm



$25\frac{1}{2}$ cm

$13\frac{1}{2}$ cm

Spiral Review

Find the scale factor for each scale drawing.

19. 6 cm = 1.44 m $\frac{1}{24}$

20. 20 cm = 10 m $\frac{1}{50}$

21. 15 cm = 0.3 m $\frac{1}{2}$

22. 8 cm = 2.5 mm $\frac{32}{1}$

23. 2 cm = 0.5 km $\frac{1}{25,000}$

24. 5 m = 5 km $\frac{1}{1,000}$

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Lesson 5

Similar Triangles and Indirect Measurement

Vocabulary Start-Up

Indirect measurement allows you to use properties of similar polygons to find distances or lengths that are difficult to measure directly.

Complete the graphic organizer. List three real-world examples in the Venn diagram for each method of measurement.
Sample answers are given.

Direct Measurement	Indirect Measurement
globe's circumference	Earth's circumference
long jump distance; circumference of a volleyball; length of your bedroom wall	a single bacteria; height of a roller coaster; diameter of the moon

Write the name of an object that could be measured by either method. a person's height

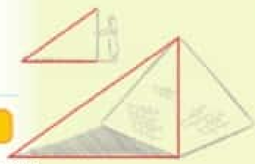
Essential Question

HOW can you determine congruence and similarity?

Vocabulary

indirect measurement

Mathematical Practices
1, 3, 4, 7



Real-World Link

Shadows Legend says that Thales, the first Greek mathematician, was the first to determine the height of the pyramids by examining the shadows made by the Sun.

1. What appears to be true about the corresponding angles in the two triangles? They are equal.
2. If the corresponding sides are proportional, what could you conclude about the triangles? They are similar.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Solve problems involving similar triangles.

Coherence connecting within and across grades

Previous	Now	Next
Students used properties of similar figures to find missing measurements.	Students establish facts about angle-angle similarity and use proportions to solve problems involving indirect measurement.	Students will solve problems involving slope triangles.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Pairs Discussion Give pairs of students 6 straws that have been cut into different lengths and have them make different-sized triangles. Students should measure all 3 angles in each triangle, add the angle measures for each triangle, and record their measurements in a table. They should notice that the three angles always add up to 180°. MP 1, 5

Alternate Strategy

AL Hand out a paper with triangles drawn and 2 of the 3 angles measurements given. Have students measure the third angle in each triangle and record the measurements in a table. Have them find the sum of the measures and determine a rule. MP 1, 3, 5, 6, 7

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2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

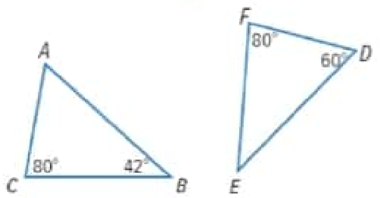
1. Determine similarity.

AL
- *Which angles have the same measure?* angle A and angle E
- *What is the sum of the angle measures in a triangle?* 180°

OL
- *How would you determine the measure of angle G?* Sample answer: The sum of the angle measures of a triangle is 180°. Subtract the known angle measures from 180 to find the missing angle measure.
- *Are at least two of the angle pairs congruent?* yes

BL
- *Do we need to determine the measures of both angle G and angle B? Explain.* Sample answer: No, we only need to know the measure of one angle. If angle G is congruent to angle C on the other triangle, then we know the triangles are similar.

Need Another Example?
Determine whether the triangles are similar. If so, write a similarity statement. The triangles are not similar.



Key Concept: Angle-Angle (AA) Similarity

Work Zone

Words If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Symbols If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.

Model

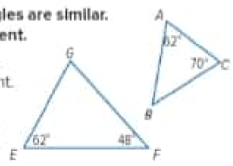
In the figure below $\angle X \cong \angle P$ and $\angle Y \cong \angle Q$. If you extend the sides of each figure to form a triangle, you can see the two triangles are similar. So, triangle similarity can be proven by showing two pairs of corresponding angles are congruent.

STOP and Reflect
What do you know about the third pair of angles in the triangle?
They are congruent.

Example

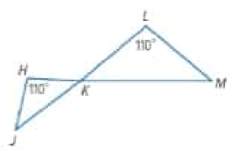
1. Determine whether the triangles are similar. If so, write a similarity statement.

Angle A and $\angle E$ have the same measure, so they are congruent. Since $180 - 62 - 48 = 70$, $\angle G$ measures 70°. Two angles of $\triangle EFG$ are congruent to two angles of $\triangle ABC$, so $\triangle ABC \sim \triangle EFG$.



Got it? Do this problem to find out.

a.



a. $\angle JKH \cong \angle MKL$ and $\angle L \cong \angle H$ so $\triangle JKH \sim \triangle MKL$

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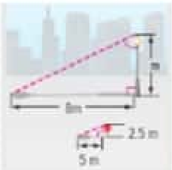
Use Indirect Measurement

One type of indirect measurement is *shadow reckoning*. Two objects and their shadows form two sides of right triangles. In shadow problems, you can assume that the angles formed by the Sun's rays with two objects at the same location are congruent. Since two pairs of corresponding angles are congruent, the two right triangles are similar. You can also use similar triangles that do not involve shadows to find missing measures.

Examples

2. **A road sign 2.5 meters high casts a 5-meter shadow. How tall is a street light that casts an 8-meter shadow at the same time? Let *h* represent the height of the street light.**



Shadow		Height
road sign →	$\frac{5}{8} = \frac{2.5}{h}$	← road sign
street light →		← street light

$5h = 8 \cdot 2.5$ Find the cross products.

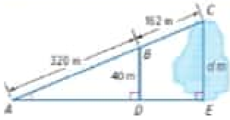
$5h = 20$ Multiply.

$\frac{5h}{5} = \frac{20}{5}$ Divide each side by 5.

$h = 4$

The street light is 4 meters tall.

3. **In the figure at the right, triangle *DBA* is similar to triangle *ECA*. Rami wants to know the distance across the lake.**



$\frac{AB}{AC} = \frac{BD}{CE}$ $\overline{AB}$ corresponds to $\overline{AC}$ and $\overline{BD}$ corresponds to $\overline{CE}$.

$\frac{320}{482} = \frac{40}{d}$ Replace *AB* with 320, *AC* with 482, and *BD* with 40.

$320d = 482 \cdot 40$ Find the cross products.

$\frac{320d}{320} = \frac{19{,}280}{320}$ Multiply. Then divide each side by 320.

$d = 60.25$

The distance across the lake is 60.25 meters.

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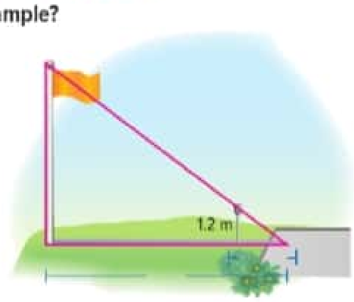
Examples

2. Use indirect measurement.

- AL • *What is the unknown?* the height of the street light
- OL • *What proportion could be used to find the height of the street light?* $\frac{5}{8} = \frac{2.5}{h}$
- BL • *Is there another proportion you can use to solve the problem? Explain.* yes; $\frac{5}{2.5} = \frac{8}{h}$; Sample answer: It yields the same cross products.

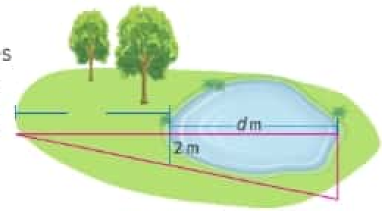
Need Another Example?
How tall is the flagpole? 9.6 m



3. Use indirect measurement.

- AL • *What are the names of the sides on the larger triangle?* $\overline{AE}$, $\overline{AC}$, $\overline{CE}$
- OL • *How can we find the length $\overline{AC}$?* Add the lengths of $\overline{AB}$ and $\overline{BC}$.
- BL • *Is there another proportion you can use to solve the problem? Explain.* yes; $\frac{320}{40} = \frac{482}{d}$; Sample answer: It yields the same cross products.

Need Another Example?
The two triangles in the figure are similar. Find the distance across the lake. 15 m



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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Rally Coach Have students work in pairs to complete Exercises 1–5. Student 1 completes Exercise 1, talking aloud through each step in the process, while Student 2 listens and encourages. Student 2 asks any clarifying questions. Students trade roles for each exercise. Some possible clarifying questions for Exercise 1 are below. MP 1, 3, 5

Ask:

- *Only one angle measure in each triangle is given. How did you determine another pair of angle measures?* **Angle *YWX* and angle *VWU* are vertical angles, so they are congruent.**
- *Why do you only need to know if two pairs of angles are congruent to determine that all three angle pairs are congruent in a triangle?* **The sum of the angle measures in a triangle is 180°. By knowing two of the angle measures, I can subtract from 180° to find the third angle measure.**

BL LA Trade-a-Problem Have students draw a pair of triangles that may or may not be similar. Have them predict whether they think the pair of triangles is similar. Then have them trade problems and use angle-angle similarity to determine if the triangles are similar. Have them discuss and resolve any differences. MP 1, 3, 5

Watch Out!

Common Error When the triangles do not mirror each other, as in Exercise 4, students often write incorrect proportions. Suggest that students trace and turn one of the triangles, making sure that the angle arcs and the vertices have the same orientation.

b. 8.2 m

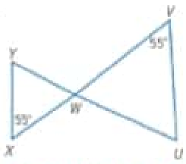
Got it? Do this problem to find out.

b. At the same time a 2-meter street sign casts a 3-meter shadow, a nearby telephone pole casts a 12.3-meter shadow. How tall is the telephone pole?

Guided Practice

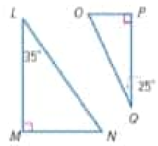
Determine whether the triangles are similar. If so, write a similarity statement. (Example 1)

1.



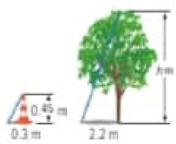
$\angle YWX \cong \angle UWV$ and $\angle V \cong \angle X$, so $\triangle YWX \sim \triangle UWV$

2.

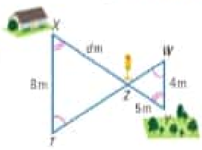


The triangles are not similar.

3. How tall is the tree? (Example 2) 3.3 m



4. Find the distance from the house to the street light. (Example 3) 10 m



5. **Building on the Essential Question** How do similar triangles make it easier to measure very tall objects? Sample answer: Two objects and their shadows form similar right triangles. If you can measure the height of one of the objects, you can use a proportion to find the measure of the other object.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES ? NO

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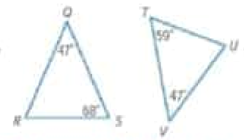
Name ______________________ My Homework

Independent Practice

Determine whether the triangles are similar. If so, write a similarity statement. (Example 1)

1.

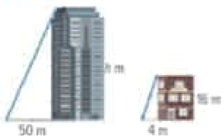


The triangles are not similar.

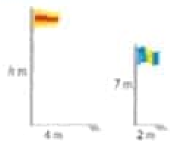
2.

$\angle BAC \cong \angle DAF$ and $\angle ABC \cong \angle D$, so $\triangle BAC \sim \triangle DAF$

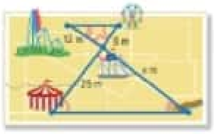
3. How tall is the building? (Example 2) **200 m**



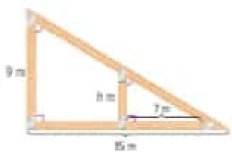
4. How tall is the taller flagpole? (Example 2) **14 m**



5. How far is it from the log ride to the pirate ship? (Example 3) **37.5 m**



6. Find the height of the brace. (Example 3) **4.2 m**



7. **Reason Abstractly** A giant ferris wheel is 136 meters tall. If the ferris wheel casts a 34-meter shadow, write and solve a proportion to find the height of a nearby street light that casts a $1\frac{1}{2}$-meter shadow.

$\frac{136}{34} = \frac{h}{1.5}$; 6 meters tall

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Engage Explore **Explain** **Elaborate** **Evaluate**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–6, 13–16	7, 17–20	8–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 19, 20
OL	On Level	1–5 odd, 7–9, 11, 12, 19, 20
BL	Beyond Level	7–12, 19, 20

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
2 Reason abstractly and quantitatively.	7
3 Construct viable arguments and critique the reasoning of others.	8, 12
4 Model with mathematics.	9, 18
7 Look for and make use of structure.	11

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

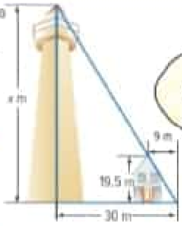
TICKET Out the Door

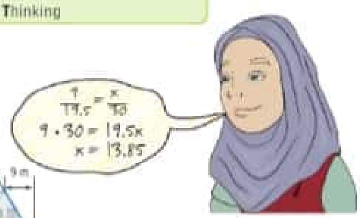
Ask students to explain how the previous lesson about similar triangles helped them to learn about angle-angle similarity. Give them the following writing prompt. **See students' work.**

- What I learned about similar triangles helped me to learn about angle-angle similarity because...

H.O.T. Problems Higher Order Thinking

8. **Find the Error** Metha is finding the height of the lighthouse shown in the diagram. Find her mistake and correct it.





Metha set up the proportion incorrectly.
$\frac{9}{30} = \frac{19.5}{x}$
$9x = 30 \cdot 19.5$
$x = 65$

9. **Model with Mathematics** On a separate sheet of paper, draw two different triangles so that each one contains both of the angles shown. Then verify that they are similar by determining which transformation will map one onto the other.

See students' work.

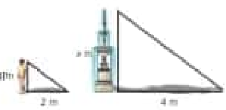
10. **Persevere with Problems** You cut a circular hole $\frac{1}{2}$-centimeter in diameter in a piece of cardboard. With the cardboard 60 centimeters from your face, the Moon fits exactly into the hole. The Moon is about 390,000 kilometers from Earth. Is the Moon's diameter more than 2,500 kilometers? Justify your reasoning.

Yes; $\frac{0.5}{60} = \frac{x}{390{,}000}$; $x = 3{,}250$ km; $3{,}250 > 2{,}500$

11. **Identify Structure** What measures must be known in order to calculate the height of tall objects using shadow reckoning?

Sample answer: The length of the tall object's shadow, the length of the shadow of a nearby object with a height that is directly measurable, and the height of the nearby object.

12. **Reason Inductively** Ali wants to estimate the height of a fountain in a local park. Ali's height and both shadow lengths are shown in the diagram. Is an estimate of 5 meters reasonable for the fountain's height? Explain your reasoning. **No; sample answer: Since Ali's height is less than his shadow's length, the statue's height must be less than its shadow's length.**



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Name ____________ My Homework ____________

Extra Practice

13. What is the height of the tree? 90 m

Homework Help

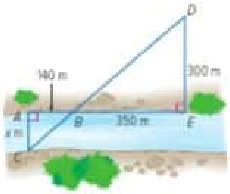
The triangles are similar. Write and solve a proportion.

$\frac{63}{42} = \frac{x}{60}$

$63 \cdot 60 = 42x$

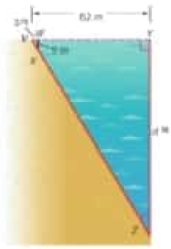
$90 = x$

14. Find the distance across the river. 120 m



15. About how long is the log that goes across the creeks? 6 m

16. How deep is the water 62 meters from the shore? $103\frac{1}{3}$ m

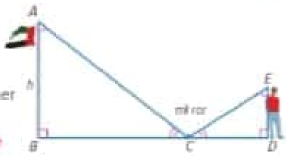


17. In the diagram shown at the right, $\triangle ABC \sim \triangle EDC$.

a. Write a proportion that could be used to solve for the height h of the flag pole. $\frac{h}{ED} = \frac{BC}{DC}$

b. What information would you need to know in order to solve this proportion?

The distance from the mirror to the person, the distance from the mirror to the base of the flag, the height of the person's eyes.



18. **Model with Mathematics** A 72 centimeter-tall child casts a shadow that is 48 centimeters long. At the same time, a nearby building casts a 16-meter-long shadow. Write and solve a proportion to find the height of the building.

$\frac{h}{72} = \frac{16}{48}$; 24 m

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فخر الوطن Fakhr alwatan

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Power Up! Test Practice

Exercises 19 and 20 prepare students for more rigorous thinking.

19.

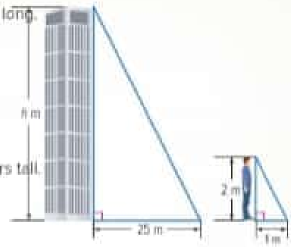
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Student finds proportion and solves.
1 point	Student finds a proportion but fails to solve it OR student finds the height but fails to set up a correct proportion OR student has an error in the proportion and finds the height based on the error.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly label the diagram and find the height of the pole.
1 point	Students label the diagram but fail to find the height OR students find the height but fail to label the diagram OR students have an error in labeling and find the height based upon the error.

Power Up! Test Practice

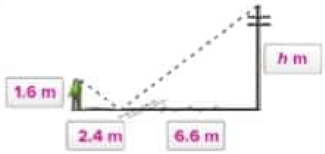
19. Omar is 2 meters tall and casts a shadow 1 meter long. At the same time, a nearby tower casts a shadow that is 25 meters long.



Write a proportion Omar can use to find the height of the tower. Sample answer: $\frac{2\text{ m}}{1\text{ m}} = \frac{h\text{ m}}{25\text{ m}}$

Using the proportion, the tower is 50 meters tall.

20. Eissa is 1.6 meters tall and is using similar triangles and a mirror to find the height of a telephone pole. The horizontal distance between Eissa and the telephone pole is 9 meters. He places the mirror on the ground 2.4 meters from himself so that he can see the top of the pole in the mirror's reflection as shown in the figure below.



1.6 m	2.4 m	4.0 m	4.5 m
6.6 m	9 m	h m	

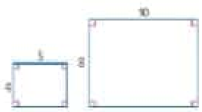
Select values to label the diagram with the correct dimensions.

What is the height of the telephone pole? 4.4 m

Spiral Review

Determine whether each pair of polygons is similar. Explain.

21.



Yes; the corresponding angles are congruent and $\frac{5}{10} = \frac{4}{8}$

22.

No; the corresponding angles are not congruent and $\frac{3}{5} \neq \frac{5}{9}$.

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Geometry

Lesson 6
Slope and Similar Triangles

Real-World Link

Physics In an experiment using a coiled spring toy, Faris and Faleh determined they needed to raise one side of a 5-unit board 2 units for the toy to move.

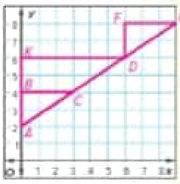
1. Find the slope of the board. (*Hint:* Use the Pythagorean Theorem to find how far the end of the board is from the books.)

Essential Question

HOW can you determine congruence and similarity?

Mathematical Practices
1, 2, 3, 4

Work with a partner. Use the graph to discover how slope triangles are related.



1. Draw the triangle formed by *A*(0, 2), *B*(0, 4), and *C*(3, 4). What kind of triangle did you draw?
   right triangle
2. Draw the triangle formed by *D*(6, 6), *F*(6, 8), and *G*(9, 8). How is △*DFG* related to △*ABC*?
   They are congruent.
3. Draw the triangle formed by *A*(0,2), *K*(0, 6), and *D*(6, 6). How is △*AKD* related to △*ABC*?
   They are similar.
4. What is true about the hypotenuses of the three triangles in Steps 1, 2, and 3?
   They all fall on the same line.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Relate the slope of a line to similar triangles.

Coherence connecting within and across grades

Previous	Now	Next
Students found the rate of change (slope) between two points.	Students use similar right triangles to develop a further understanding of slope.	Students will apply properties of similar figures to perimeter and area.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Value Line** Have students complete Exercises 1–4 individually. Then have them stand in a line across the room. Let one side of the room represent a solid understanding of slope and similar triangles. Let the other side of the room represent a lot of questions. Then pair students from opposite sides to discuss the exercises. MP 1, 3

Alternate Strategies

BL Have students determine the ratio of the rise to run for each triangle to verify that the triangles are similar. MP 1, 6, 7

BL **LA** Have students write an argument for why the slope of a line is the same for any two points on the line. Their argument should include drawings and definitions. Have them present to the class. MP 1, 3, 5, 6, 7, 8

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Compare slope with similar triangles.

AL
- *What line segment length represents the rise for triangle ABC? the run?* $\overline{AC}$; $\overline{BC}$
- *What line segment length represents the rise for triangle BDE? the run?* $\overline{BE}$; $\overline{DE}$

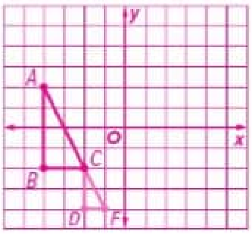
OL
- *What is true about corresponding sides of similar triangles?* Their lengths are proportional.
- *What is the ratio of the rise to the run for each triangle?* 2
- *How does this compare to the slope?* It is the same value.

BL
- *Give the coordinates of another right triangle that has the same ratio of the rise to the run as these triangles.* Sample answer: (1, 0), (4, 0), (4, 6)
- *How do you know that the coordinates of your new triangle has the same ratio?* Sample answer: The rise is 6 and the run is 3. The ratio of the rise to the run is 2.

Need Another Examples?

Graph $\triangle ABC$ with vertices $A(-4, 2)$, $B(-4, -2)$, and $C(-2, -2)$, and $\triangle CDF$ with vertices $C(-2, -2)$, $D(-2, -4)$, and $F(-1, -4)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

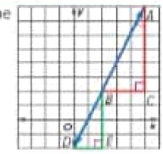
$\frac{AB}{BC} = \frac{CD}{DF}$; $\frac{2}{1}$



Work Zone

Similar Triangles and the Coordinate Pla

In the figure shown, $\triangle ABC$ and $\triangle BDE$ are slope triangles. Slope triangles are similar.



$\angle BAC \cong \angle DBE$ Given

$\angle ACB \cong \angle BED$ Given

$\triangle ABC \sim \triangle BDE$ Angle-Angle Similarity

You can use the properties of similar triangles to show the ratios of the rise to the run for each right triangle are equal.

Example

1. Write a proportion comparing the rise to the run for each of the similar slope triangles shown above. Then find the numeric value.

$\frac{AC}{BE} = \frac{BC}{DE}$ Corresponding sides of similar triangles are proportional.

$AC \cdot DE = BE \cdot BC$ Find the cross products.

$\frac{AC \cdot DE}{BC \cdot DE} = \frac{BE \cdot BC}{BC \cdot DE}$ Division Property of Equality

$\frac{AC}{BC} = \frac{BE}{DE}$ Simplify.

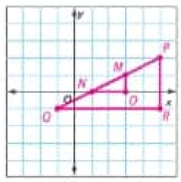
$\frac{6}{3} = \frac{4}{2}$ $AC = 6, BC = 3, BE = 4, DE = 2$

So, $\frac{AC}{BC} = \frac{BE}{DE}$ or $\frac{6}{3} = \frac{4}{2}$

Got it? **Do this problem to find out.**

a. Graph $\triangle MNO$ with vertices $M(3, 1)$, $N(1, 0)$, and $O(3, 0)$, and $\triangle PQR$ with vertices $P(5, 2)$, $Q(-1, -1)$, and $R(5, -1)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

a. $\frac{MO}{NO} = \frac{PR}{QR}$ or $\frac{1}{2}$



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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

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Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Compare slope with similar triangles.

AL
- *What line segment length represents the rise for triangle ABC? the run?* **$\overline{AC}$; $\overline{BC}$**
- *What line segment length represents the rise for triangle BDE? the run?* **$\overline{BE}$; $\overline{DE}$**

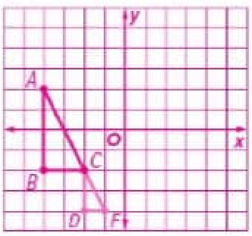
OL
- *What is true about corresponding sides of similar triangles?* **Their lengths are proportional.**
- *What is the ratio of the rise to the run for each triangle?* **2**
- *How does this compare to the slope?* **It is the same value.**

BL
- *Give the coordinates of another right triangle that has the same ratio of the rise to the run as these triangles.* **Sample answer: (1, 0), (4, 0), (4, 6)**
- *How do you know that the coordinates of your new triangle has the same ratio?* **Sample answer: The rise is 6 and the run is 3. The ratio of the rise to the run is 2.**

Need Another Examples?

Graph △*ABC* with vertices *A*(−4, 2), *B*(−4, −2), and *C*(−2, −2), and △*CDF* with vertices *C*(−2, −2), *D*(−2, −4), and *F*(−1, −4). Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

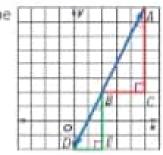
$\frac{AB}{BC} = \frac{CD}{DF}; \frac{2}{1}$



Work Zone

Similar Triangles and the Coordinate Pla

In the figure shown, △*ABC* and △*BDE* are slope triangles. Slope triangles are similar.



$\angle BAC \cong \angle DBE$	Given
$\angle ACB \cong \angle BED$	Given
$\triangle ABC \sim \triangle BDE$	Angle-Angle Similarity

You can use the properties of similar triangles to show the ratios of the rise to the run for each right triangle are equal.

Example

1. Write a proportion comparing the rise to the run for each of the similar slope triangles shown above. Then find the numeric value.

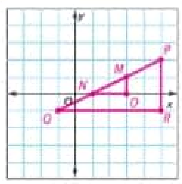
$\frac{AC}{BE} = \frac{BC}{DE}$	Corresponding sides of similar triangles are proportional.
$AC \cdot DE = BE \cdot BC$	Find the cross products.
$\frac{AC \cdot DE}{BC \cdot DE} = \frac{BE \cdot BC}{BC \cdot DE}$	Division Property of Equality
$\frac{AC}{BC} = \frac{BE}{DE}$	Simplify.
$\frac{6}{3} = \frac{4}{2}$	$AC = 6, BC = 3, BE = 4, DE = 2$

So, $\frac{AC}{BC} = \frac{BE}{DE}$ or $\frac{6}{3} = \frac{4}{2}$

Got it? **Do this problem to find out.**

a. Graph △*MNO* with vertices *M*(3, 1), *N*(1, 0), and *O*(3, 0), and △*PQR* with vertices *P*(5, 2), *Q*(1, −1), and *R*(5, −1). Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

a. $\frac{MO}{NO} = \frac{PR}{QR}$ or $\frac{1}{2}$



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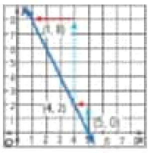
Similar Triangles and Slope

Key Concept

Words The ratio of the rise to the run of two slope triangles formed by a line is equal to the slope of the line.

Example



Larger Triangle

$\frac{\text{rise}}{\text{run}} = \frac{6}{-3} = -2$

Smaller Triangle

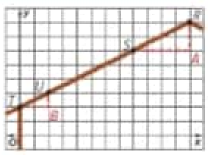
$\frac{\text{rise}}{\text{run}} = \frac{2}{-1} = -2$

$\text{slope} = \frac{-2}{1} = -2$

The ratios of the rise to the run of the two similar slope triangles in Example 1 are the same as the slope of the line. Since the ratios are equal, the slope m of a line is the same between any two distinct points on a non-vertical line in the coordinate plane.

Example

2. The pitch of a roof refers to the slope of the roof line. Choose two points on the roof and find the pitch of the roof shown. Then verify that the pitch is the same by choosing a different set of points.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{8 - 6}{12 - 8}$ Use the points S and R. $(x_1, y_1) = (8, 6)$ and $(x_2, y_2) = (12, 8)$

$m = \frac{2}{4} = \frac{1}{2}$ Simplify.

The pitch of the roof is $\frac{1}{2}$. Verify that the pitch is the same using two other points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{2 - 3}{0 - 2}$ Use the points U and T. $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (0, 2)$

$m = \frac{-1}{-2} = \frac{1}{2}$ Simplify. The pitch is the same.

STOP and Reflect

Is the statement $\triangle RAS \sim \triangle UBT$ true? Explain below.

yes; Sample answer: The two triangles are slope triangles so they are similar.

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Example

2. Similar triangles and slope.

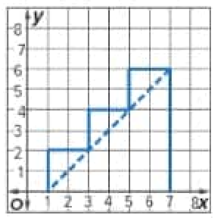
AL
- *What are the coordinates of each point?* R(12, 8), U(2, 3), S(8, 6), T(0, 2)
- *What is the slope formula?* $m = \frac{y_2 - y_1}{x_2 - x_1}$

OL
- *What is the slope between points S and R?* $\frac{1}{2}$
- *What is the slope between points U and T?* $\frac{1}{2}$
- *If you choose any other set of points on the line represented by the roof, will the slope be the same? Explain.* yes; Sample answer: The line is a straight line with a constant slope, or rate of change.

BL
- *Are the two triangles similar? Explain.* yes; Sample answer: They are slope triangles and slope triangles are similar.
- *If the pitch of the roof is $\frac{1}{2}$, give the horizontal run of the roof if the vertical rise is 3 meters.* 6 meters

Need Another Example?

Choose two points along the stairs and find the slope of the stairs. Then verify that the slope is the same by choosing a different set of points. $m = 1$; The other slope should equal 1.



Watch Out!

Common Error Students may flip the order of the y-coordinates and x-coordinates when substituting values into the slope formula. Have students choose which ordered pair will be used for (x_1, y_1) values first and label them. Then have students substitute those values into the formula.

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

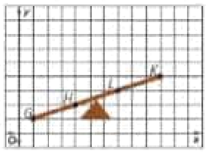
AL LA Rally Coach Have students work in pairs. For Exercise 1, Student A completes the problem while Student B watches, listens, coaches, and praises. For Exercise 2, students trade roles. MP 1, 3

BL LA Gallery Walk Have pairs of students write their own real-world problem, similar to Exercise 2, involving the slope of an object in a real-world setting. Then have them draw a diagram representing the object using graph paper. Post diagrams around the room. Have students walk around the room to find the slope of each object. Have them verify the slope by choosing another set of points. MP 1, 4, 5

b. $m = \frac{1}{3}$; See students' work for other slope. The other slope should equal $\frac{1}{3}$.

Got it? Do this problem to find out.

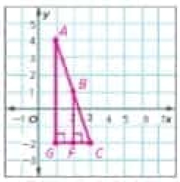
b. The plans for a teeter-totter are shown at the right. Using points G and L, find the slope of the teeter-totter. Then verify that the slope is the same at a different location by choosing a different set of points.



Guided Practice

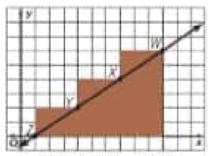
1. Graph △ACG with vertices A(1, 4), C(3, 2), and G(1, –2), and △BCF with vertices B(2, 1), C(3, –2), and F(2, –2). Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value. (Example 1)

$\frac{GA}{GC} = \frac{FB}{FC}$ or $-\frac{3}{1}$ or -3



2. The plans for a set of stairs are shown below. Using points X and Z, find the slope of the line down the stairs. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)

$m = \frac{2}{3}$; See students' work for other slope. The other slope should equal $\frac{2}{3}$.



3. **Building on the Essential Question** How is the slope of a line related to the similar slope triangles formed by the line?

Sample answer: The ratio of the vertical leg to the horizontal leg of each similar slope triangle formed by the line is equivalent to the absolute value of the slope.

Rate Yourself!

How confident are you about slope and similar triangles? Check the box that applies.

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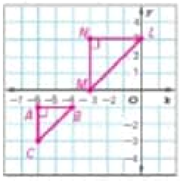
Name ________ My Homework ________

Independent Practice

Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value. (Example 1)

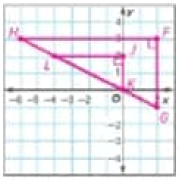
1. △ABC with vertices A(6, −1), B(−4, −1), and C(−6, −3); △NLM with vertices N(3, 3), L(0, 3), and M(−3, 0)

$\frac{AC}{AB} = \frac{NM}{NL}$, or $\frac{1}{1}$



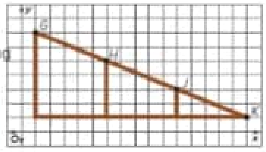
2. △FGH with vertices F(2, 3), G(2, −1), and H(−6, 3); △JKL with vertices J(0, 2), K(0, 0), and L(−4, 2)

$\frac{GF}{FH} = \frac{KJ}{JL}$, or $-\frac{1}{2}$



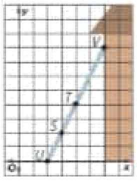
3. The plans for a skateboard ramp are shown. Use two points to find the slope of the ramp. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)

$m = -\frac{2}{5}$; See students' work for other slope. The other slope should equal $\frac{2}{5}$.



4. A ladder is leaning up against the side of a house. Use two points to find the slope of the ladder. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)

$m = 2$; See students' work for other slope. The other slope should equal 2.



5. **Reason Abstractly** Triangle XYZ has vertices X(0, 0), Y(10, 0), and Z(0, 6). Triangle MYP has vertices M(5, 0), Y(10, 0), and P(x, y). Find the missing coordinates for P if △MYP ~ △XYZ.

P(5, 3)

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 10, 11	5, 6, 12–17	7–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 16, 17
OL	On Level	1, 3, 5–7, 9, 16, 17
BL	Beyond Level	5–9, 16, 17

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	8
2 Reason abstractly and quantitatively.	5
3 Construct viable arguments and critique the reasoning of others.	9
4 Model with mathematics.	6, 7, 12, 13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students graph and connect the following coordinates: (0, 1), (5, 4), and (10, 7). Ask students to find the slope. Then, have students use the slope and the coordinates given to draw a congruent right triangle on the grid. Finally, have students create a triangle that is similar to the two triangles they just drew. **slope $= \frac{3}{5}$; See students' work for similar triangles.**

6. **Model with Mathematics** Refer to the graphic novel frame below. On the beach, a cable is attached to the pier. The line formed by the cable has a slope of $\frac{3}{5}$. Is the triangle formed by the pier, the beach, and the cable similar to the triangle formed by the boat, the parasailer, and the rope? Explain. **No; the slope of the triangle formed by the boat, parasailer and rope is $\frac{400}{300}$ or $\frac{4}{3}$. Since the slopes are not the same, the triangles are not similar.**

H.O.T. Problems Higher Order Thinking

7. **Model with Mathematics** On a separate piece of grid paper, draw the graph of a line with a positive slope. Draw two slope triangles formed by the line. Demonstrate that the simplified ratio of the rise to the run of each triangle is equivalent to the slope. **See students' work.**

8. **Persevere with Problems** The slope of a line is 3.5. Find two possible measurements for the legs of similar slope triangles. Explain your reasoning. **Sample answer: 3.5 and 1; 7 and 2; Since the slope is the ratio of the two legs of a slope triangle, look for values that simplify to |−3.5|. $\frac{3.5}{1}$ and $\frac{7}{2}$ both simplify to 3.5.**

9. **Reason Inductively** Triangle *JKL* has vertices *J*(0, 0), *K*(1, 0), and *L*(1, 2). Determine if each triangle is similar to and/or a slope triangle with $\triangle JKL$.
   a. $\triangle ABC$: *A*(1, 2), *B*(1, 6), *C*(3, 6) **similar triangle, slope triangle**
   b. $\triangle MNP$: *M*(3, 1), *N*(6, 1), *P*(6, 7) **similar triangle**
   c. $\triangle RST$: *R*(1, 2), *S*(4, 2), *T*(4, 5) **neither**
   d. $\triangle WXY$: *W*(0, 0), *X*(1, −2), *Y*(0, −2) **similar triangle, slope triangle**

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Name ______________________ My Homework ______________________

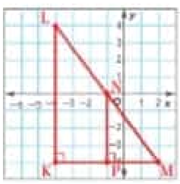
Extra Practice

Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

10. $\triangle LKM$ with vertices $L(-4, 4)$, $K(-4, -4)$, and $M(2, -4)$; $\triangle NPM$ with vertices $N(1, 0)$, and $P(-1, -4)$

$\frac{LK}{KM} = \frac{NP}{PM}$ or $-\frac{4}{3}$

Graph and label each triangle.



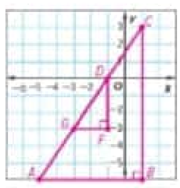
write the proportion using the side labels.

$\frac{LK}{KM} = \frac{NP}{PM}$

$-\frac{8}{6} = -\frac{4}{3}$

11. $\triangle ABC$ with vertices $A(-5, -6)$, $B(1, -6)$, and $C(1, 3)$; $\triangle GFD$ with vertices $G(-3, -3)$, $F(-1, -3)$, and $D(-1, 0)$

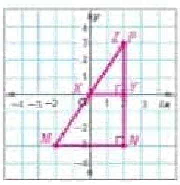
$\frac{CB}{BA} = \frac{DF}{FG}$ or $\frac{3}{2}$



Model with Mathematics Use a graph to find the missing coordinates for point Z if $\triangle MNP \sim \triangle XYZ$.

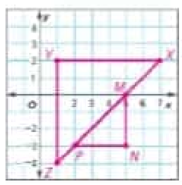
12. $M(-2, -3)$, $N(2, -3)$, $P(2, 3)$, $X(0, 0)$, $Y(2, 0)$

$Z(2, 3)$



13. $M(5, 0)$, $N(5, -3)$, $P(2, -3)$, $X(7, 2)$, $Y(1, 2)$

$Z(1, -4)$



Copy and Solve Find the missing coordinates for point D if $\triangle ABC \sim \triangle DEF$. Show your work on a separate sheet of paper.

14. $A(-1, 3)$, $B(1, 3)$, $C(1, 6)$, $E(-4, -7)$, $F(-4, -1)$ $D(-8, -7)$

15. $A(1, 11)$, $B(1, 6)$, $C(3, 6)$, $E(1, 1)$, $F(5, 1)$ $D(1, 11)$

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Power Up! Test Practice

Exercises 16 and 17 prepare students for more rigorous thinking.

16. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly graph the triangles, the line formed by the triangles, find the slope and explain their response.
1 point	Students correctly graph the triangles and the line, and find the slope but fail to explain OR students correctly graph the triangles and find the slope but fail to graph the line and may or may not explain OR students incorrectly graph and base their written responses on their errors, using proper mathematical reasoning.

17. This test item requires students to reason abstractly and quantitatively when problem solving.

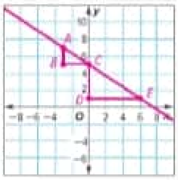
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

16. Triangle *ABC* with vertices *A*(−3, 7), *B*(−3, 5), and *C*(0, 5), and triangle *CDE* with vertices *C*(0, 5), *D*(0, 1), and *E*(6, 1) are slope triangles.

Draw the triangles and the line that they represent on the coordinate plane.

Find the slope of the line. Then describe the relationship between the slope triangles and the slope of the line.



$-\frac{2}{3}$; **Sample answer: The legs of the slope triangles represent the rise and run of the slope of the line.**

17. The statements below refer to any non-vertical line in the coordinate plane. Determine if each statement is true or false.

a. All of the slope triangles on the line are similar. ■ True ☐ False

b. The slope is the same between any two distinct points on the line. ■ True ☐ False

c. In the slope triangles, the ratios of the rise to the run are equal to the absolute value of the slope. ■ True ☐ False

Spiral Review

Find the slope of the line that passes through each pair of points.

18. (2, 2), (−2, −2) **1**

19. (5, −4), (9, −4) **0**

20. (4, 3), (−1, 6) $-\frac{3}{5}$

21. (3, 3), (3, 5) **undefined**

22. (0, 0), (3, −6) **−2**

23. (−8, −15), (−2, −5) $\frac{5}{3}$

24. (−3, 5), (3, 6) $\frac{1}{6}$

25. (0.2, 0.7), (1.7, 1.2) $\frac{1}{3}$

26. (−5, 0), (3, −2) $-\frac{1}{4}$

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the perimeter of similar figures.

- AL • *Which rectangle will have a greater perimeter, the original or the new? Explain.* **the new rectangle; The length of the new rectangle is greater than the length of the original rectangle.**
- OL • *Will the scale factor be less than or greater than 1? Explain how you know.* **Sample answer: greater than 1; Because the new length is greater than the original, the dilation is larger, which means the scale factor must be greater than 1.**
- BL • *How can you use the scale factor and the perimeter of the smaller rectangle to determine the perimeter of the larger rectangle?* **Multiply the perimeter of the smaller rectangle by the scale factor.**
  • *How can you check your answer for accuracy?* **Sample answer: The ratio of the perimeters is $\frac{28}{24}$, which simplifies to $\frac{7}{6}$, which is the correct scale factor.**

Need Another Example?
Two rectangles are similar. One has a length of 10 centimeters and a perimeter of 36 centimeters. The other rectangle has a length of 7.5 centimeters. What is the perimeter of this rectangle? **27 cm.**

Watch Out!

Common Error When setting up the scale factor, students may not be sure which number to put in the numerator and which number to put in the denominator. Have them decide first if the new figure is going to be bigger or smaller than the original. Remind them that if it will be bigger, the scale factor must be greater than 1.

Key Concept

Perimeter and Area of Similar Figures

Work Zone

Perimeter

Words If figure B is similar to figure A by a scale factor, then the perimeter of B is equal to the perimeter of A times the scale factor.

Symbols perimeter of figure B = perimeter of figure A • scale factor

Area

Words If figure B is similar to figure A by a scale factor, then the area of B is equal to the area of A times the square of the scale factor.

Symbols area of figure B = area of figure A • (scale factor)^2

Models

In similar figures, the perimeters are related by the scale factor, k. What about area? The area of one similar figure is equal to the area of the other similar figure times the *square* of the scale factor, or k^2.

Example

1. Two rectangles are similar. One has a length of 6 centimeters and a perimeter of 24 centimeters. The other has a length of 7 centimeters. What is the perimeter of this rectangle?

The scale factor is $\frac{7}{6}$. The perimeter of the original is 24 centimeters.

$x = 24\left(\frac{7}{6}\right)$ Multiply by the scale factor.

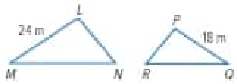
$x = \frac{\overset{4}{24}}{1}\left(\frac{7}{\underset{1}{6}}\right)$ Divide out common factors.

$x = 28$ Simplify.

So, the perimeter of the new rectangle is 28 centimeters.

Got it? **Do this problem to find out.**

a. Triangle LMN is similar to triangle PQR. If the perimeter of $\triangle LMN$ is 64 meters, what is the perimeter of $\triangle PQR$?



a. **48 m**

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Rally Coach Students work in pairs to complete Exercises 1–5. Partner A completes Exercise 1 while Partner B watches, listens, coaches, and praises. Then the partners trade roles for Exercise 2. Partners continue taking turns until all problems are completed. Partners are responsible for each one understanding. Have them ask each other for support, if needed. MP 1, 3

BL LA Trade-a-Problem Students complete Exercises 1–5 individually. Have them trade papers with another student to check their work for accuracy. Have them research how similar figures are used in the real world. Then have them write their own real-world problem, similar to Exercises 3 or 4. Have them trade with each other to solve each other's problem. MP 1, 3, 6

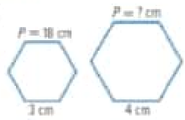
Got it? Do this problem to find out.

c. Noura is painting a mural on her bedroom wall. The image she is reproducing is 4.8 centimeters by 7.2 centimeters. If the dimensions of the mural are 10 times the dimensions of the image, find the area of the mural in square centimeters.

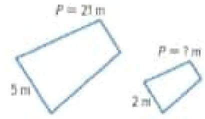
c. 3,456 cm²

Guided Practice

For each pair of similar figures, find the perimeter of the second figure. (Example 1)

1. 

24 cm

2. 

8.4 m

3. Hidaya is enlarging a digital photograph on her computer. The original photograph is 5 centimeters by 7 centimeters. If she enlarges the dimensions 1.5 times, what will be the perimeter and area of the new image? (Examples 2 and 3)

36 cm; 78.75 cm²

4. Mahmoud is flying a kite that is made up of three similar rectangles. The sides of the three rectangles are in the ratio 1:2:3. If the area of the smallest rectangle is 72 square centimeters, what are the areas of the other two rectangles? (Example 3) 288 cm²; 648 cm²

5. **Building on the Essential Question** If you know two figures are similar and you are given the area of both figures, how can you determine the scale factor of the similarity?

Sample answer: Write the ratio of the two areas and then take the square root of the ratio.

Rate Yourself!

☐ I understand how to find the perimeter and area of similar figures.

Great! You're ready to move on!

☐ I still have some questions about the perimeter and area of similar figures.

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Name ____________________ My Homework

Independent Practice

For each pair of similar figures, find the perimeter of the second figure. (Example 1)

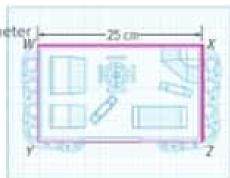
1. 

57 mm

2. 

14.55 cm

3. A city is planning to build a skate park. An architect designed the area shown at the right. In the plan, the perimeter of the park is 80 centimeters. If the actual length $\overline{WX}$ is 50 meters, what will be the perimeter of the actual skate park? (Example 2) 160 m



4. A child's desk is made so that the dimensions are two-thirds the dimensions of a full-size adult desk. Suppose the top of the full-size desk measures 135 centimeters long by 90 centimeters wide. What is the perimeter and area of the top of the child's desk? (Examples 2 and 3)

300 cm; 5,400 cm^2

5. Mansour is constructing a miniature putting green in his backyard. He wants it to be similar to a putting green at the local golf course, but one third the dimensions. The area of the putting green at the golf course is 378 square meters. What will be the area of the putting green Mansour constructs?

42 m^2

6. Nasser is making a model version of his neighborhood that uses model trains. The ratio of the model train to the actual train is 1:64. His neighborhood covers an area of 18,432 square meters. What will be the area of the model neighborhood?

4.5 m^2

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ENGAGE EXPLORE **EXPLAIN ELABORATE EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 11–15	**5–7, 16–18**	**8–10**
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 10, 17, 18
OL	On Level	1, 3, 5–7, 9, 10, 17, 18
BL	Beyond Level	5–10, 17, 18

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	8, 16
3 Construct viable arguments and critique the reasoning of others.	9
5 Use appropriate tools strategically	10
7 Look for and make use of structure.	7

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write how previous lessons involving similar figures helped them with the material in this lesson by using the following prompts. See students' work.

- In previous lessons, I learned...
- The previous lessons helped me understand what I learned in this lesson by...

7. **Identify Structure** Complete the graphic organizer to compare how the scale factor affects the side lengths, perimeter, and area of similar rectangles.

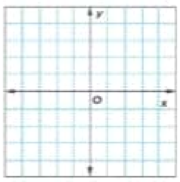
If the scale factor is...	Multiply the ...			
	Length by	Width by	Perimeter by	Area by
2	2	2	2	4
4	4	4	4	16
0.5	0.5	0.5	0.5	0.25
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{9}$
k	k	k	k	k^2

H.O.T. Problems Higher Order Thinking

8. **Persevere with Problems** Two circles have circumferences π and 3π. What is the ratio of the area of the circles? the diameters? the radii?
1:9; 1:3; 1:3

9. **Justify Conclusions** A company wants to reduce the dimensions of its logo from 15 centimeters by 10 centimeters to 7.5 centimeters by 5 centimeters to use on business cards. Yousif thinks that the new logo is $\frac{1}{4}$ the size of the original logo. Saeed thinks that it's $\frac{1}{2}$ of the original size. Explain their thinking to a classmate.
Yousif is thinking of size in terms of area and Saeed is thinking of size in terms of perimeter.

10. **Use Math Tools** Use the coordinate plane to draw a rectangle. Dilate the rectangle and draw the dilation. Then determine the perimeter and area of each rectangle to model the effect of the dilation.
See students' work.



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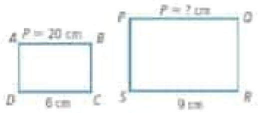
Name ____________________ My Homework ____________________

Extra Practice

For each pair of similar figures, find the unknown perimeter.

11.

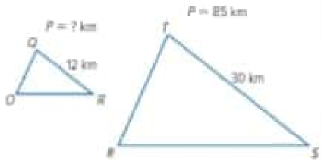


The scale factor is $\frac{3}{2}$. Multiply the perimeter of ABCD by $\frac{3}{2}$.

$P = 20 \cdot \frac{3}{2}$ or 30

30 cm

12.



34 km

13. For your dinner party, you make a map to your house on a 3-centimeter-wide by 5-centimeter-long index card. What will be the perimeter and area of your map if you use a copier to enlarge it so it is 8 centimeters long?

25.6 cm; 38.4 cm^2

14. A company wants to reduce the dimensions of its logo by one fourth to use on business cards. If the area of the original logo is 16 square centimeters, what is the area of the logo that will be used on the business cards?

1 cm^2

15. Two picture frames are similar. The ratio of the perimeters of the two pieces is 3:5. If the area of the smaller frame is 108 square centimeters, what is the area of the larger frame? 300 cm^2

16. **Persevere with Problems** Mr. Ayoub is enlarging a logo for printing on the back of a T-shirt. He wants to enlarge a logo that is 3 centimeters by 5 centimeters so that the dimensions are 3 times larger than the original. How many times as large as the original logo will the area of the printing be?

9 times larger

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Power Up! Test Practice

Exercises 17 and 18 prepare students for more rigorous thinking.

17.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2
Scoring Rubric	
1 point	Students correctly answer each part of the question.

18.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP3
Scoring Rubric	
1 point	Students correctly complete each statement.

Power Up! Test Practice

17. A photograph is enlarged to 3 times the size of the original. Fill in the boxes to complete each statement.

The area of the enlargement is **9** times the original area.

The perimeter of the enlargement is **3** times the original perimeter.

18. A smaller version of this school banner is being made to appear on the front of the students' homework agenda books.

The perimeter of the smaller version of the flag is 2 meters. Select the correct values to complete each statement.

1	2	4	8	16
$\frac{1}{64}$	$\frac{1}{8}$	$\frac{15}{8}$	$\frac{15}{64}$	

a. The perimeter of the full size flag is **16** meters.

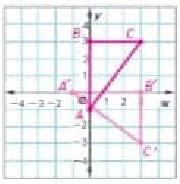
b. The scale factor of the reduction is $\frac{1}{8}$.

c. The area of the smaller version of the flag is $\frac{15}{64}$ square meters.

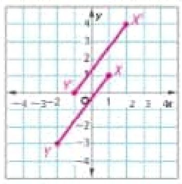
Spiral Review

Graph each figure with the given vertices and its image after the indicated transformation.

19. $\triangle ABC$: $A(0, -1)$, $B(0, 3)$, $C(3, 3)$
90° clockwise rotation about the origin



20. $\overline{XY}$: $X(1, 1)$, $Y(-2, -3)$
translation of 1 unit right and 3 units up



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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA **Think-Pair-Write** Have students work in pairs to write answers to the following questions related to Exercise 3.
MP 1, 3, 6

Ask:

- *How is a scale factor of $\frac{1}{25}$ different from a scale factor of 25?* **Sample answer: A scale factor of $\frac{1}{25}$ is a reduction and a scale factor of 25 is an enlargement.**
- *The scale factor $\frac{1}{2}$ is used to make a toy car from the model. What are the side view measurements of the toy car?* **9 cm and 5 cm**

BL LA **Pairs Discussion** Have students work in pairs to extend the activity by answering the following questions.
MP 1, 3, 6

Ask:

- *How would you find the dimensions of an actual figure based on a scale model or scale drawing?* **Sample answer: use the scale factor**
- *What are some other careers where scale models or scale drawings are used?* **See students' response.**

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

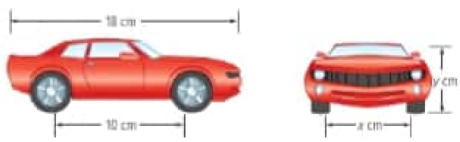
Career Facts

Because of the concern about fuel efficiency and protecting the environment, the car design field has experienced major job growth in the past few years. This is causing large demand for designers with background knowledge in environmental sciences.

Drive Yourself to Success

Use the information on the drawing to solve each problem.

1. What transformation maps the drawing to the actual car? **dilation**
2. Are the views of the drawing of the car similar to views of the actual car? Explain. **yes; Sample answer: The drawing is the same shape as the actual car but a different size.**
3. If the scale factor is $\frac{1}{25}$, find the following:
   a. the length of the actual car **450 cm**
   b. the distance from the front wheel to the rear wheel of the actual car **250 cm**
4. If the actual height of the car is 150 centimeters, what is y? **6 cm**
5. If $x = 7$ centimeters, what is the actual distance between the tires on the car? **175 cm**



Career Project

It's time to update your career portfolio! Describe the features that you, as a car designer, would include in a new car design. Determine whether these features already exist in cars today.

List several challenges associated with this career.

-
-
-
-
-

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Chapter Review

Vocabulary Check

Reconstruct the vocabulary word and definition from the letters under the grid. The letters for each column are scrambled directly under that column.

	S	I	M	I	L	A	R		
P	O	L	Y	G	O	N	S	:	
			T	W	O				
	P	O	L	Y	G	O	N	S	
	W	I	T	H		T	H	E	
S	A	M	E		S	H	A	P	E
			T						
	A	L	M	I	G	O	S		
	P	I	T	W	S	H	R		
	O	O	E	H	O	T	H	S	
S	S	I	L	G	O	A	A	P	
P	W	M	Y	Y	L	N	N	E	E

Complete each sentence using vocabulary from the chapter.

1. Two figures are congruent if one can be obtained from the other by a series of rotations, reflections, or translations.
2. Indirect measurement uses properties of similar polygons to find distances or lengths that are difficult to measure directly.
3. The parts of congruent figures that match are called corresponding parts.
4. Two figures are similar if one can be obtained from the other by a series of transformations and dilations.
5. When a transformation is applied to a figure and then another transformation is applied to the image, the result is called a composition of transformations.

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Vocabulary Check

LA **One Stray** Assign students to 3- to 4- person learning teams. Each member is assigned a number from 1 to 4. Each team completes the Vocabulary Check, making sure every team member understands the terms and their definitions. Call on a specific number to stand and move to a different group. The student who moved compares his or her answers with the new team. Then have the students return to their original teams and discuss similarities and differences. MP 1, 3

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- compostition of transformations (Lesson 1)
- congruent (Lesson 1)
- corresponding parts (Lesson 2)
- indirect measurement (Lesson 5)
- similar (Lesson 3)
- similar polygons (Lesson 3)

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Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of similar and congruent figures.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 3

Got It?

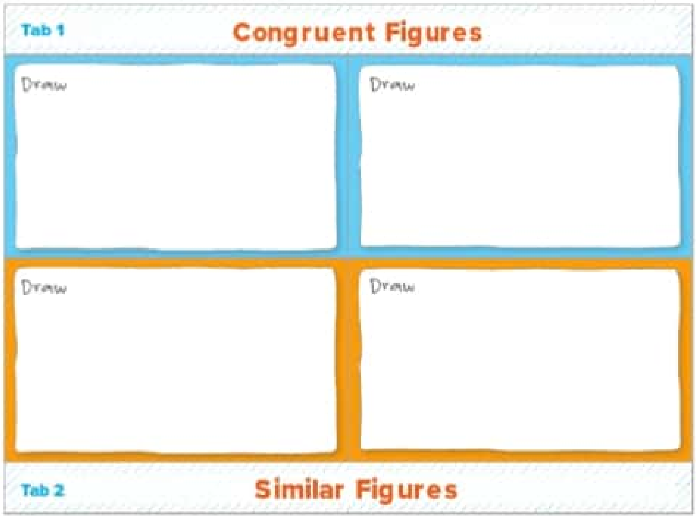
If students have trouble with Exercises 1–4, they may need help with the following concept(s).

Concept	Exercise(s)
congruence and transformations (Lesson 1)	1, 3
similarity and transformations (Lesson 3)	2, 4

Key Concept Check

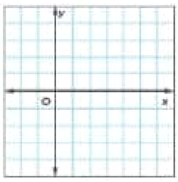
Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Triangle *ABC* has vertices $A(0, 0)$, $B(2, 4)$, $C(6, 0)$. Match each image with the description of its transformation.



1. $A'(0, 0)$, $B'(2, -4)$, $C'(6, 0)$ — c
2. $A'(0, 0)$, $B'(1, 2)$, $C'(3, 0)$ — a
3. $A'(0, 0)$, $B'(4, -2)$, $C'(0, -6)$ — b
4. $A'(2, -6)$, $B'(6, 2)$, $C'(14, -6)$ — d

a. similar; a dilation with a scale factor of $\frac{1}{2}$

b. congruent; a 90° clockwise rotation about the origin

c. congruent; a reflection over the *x*-axis

d. similar; a translation of $(x + 1, y - 3)$ followed by a dilation with a scale factor of 2

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Focus narrowing the scope

This chapter focuses on content from **Geometry**.

Coherence connecting within and across grades

Previous	Now	Next
Students studied properties of triangles, transformations, congruency, and similarity.	Students find the volume and surface area of cylinders and cones.	Students will construct and interpret scatter plots, and analyze data.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Ice Skating Area formulas are used to find the number of square units needed to cover a surface. Students can use the formulas $A = \pi r^2$ and $A = \ell w$ to find the area of the skating rink. Some students may find the area of each semicircle and add them. Other students may find the total area by using the fact that the total area of the semicircles is equal to the area of one circle with the same radius.

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Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

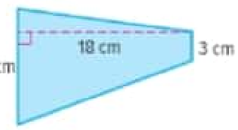
REVIEW	
Example	**Skill**
1	Area of a Polygon
2	Simplifying Expressions

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–6

Find the area of the trapezoid.
$135\ cm^2$

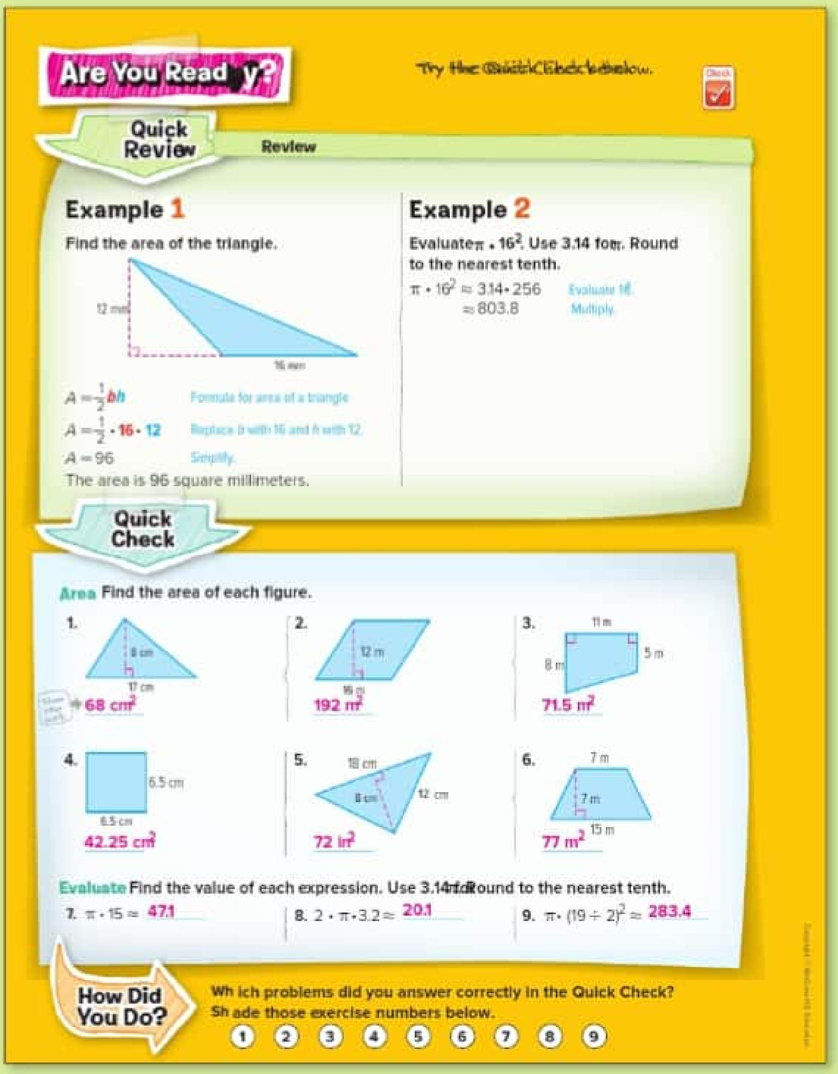


Exercises 7–9

Find the value of $\pi \cdot (17 \div 2)^2$. Use 3.14 for π. Round to the nearest tenth. 226.9

Track Your Progress

Prior to beginning this chapter, have your students rate their current knowledge. At the end of the chapter, have your students rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Are You Ready?

Try the Quick Check below.

Quick Review — Review

Example 1

Find the area of the triangle.

$A = \frac{1}{2}bh$ — Formula for area of a triangle

$A = \frac{1}{2} \cdot 16 \cdot 12$ — Replace b with 16 and h with 12.

$A = 96$ — Simplify.

The area is 96 square millimeters.

Example 2

Evaluate $\pi \cdot 16^2$. Use 3.14 for π. Round to the nearest tenth.

$\pi \cdot 16^2 \approx 3.14 \cdot 256$ — Evaluate 16^2.

≈ 803.8 — Multiply.

Quick Check

Area Find the area of each figure.

1. $68\ cm^2$
2. $192\ m^2$
3. $71.5\ m^2$
4. $42.25\ cm^2$
5. $72\ in^2$
6. $77\ m^2$

Evaluate Find the value of each expression. Use 3.14 for π. Round to the nearest tenth.

7. $\pi \cdot 15 \approx$ 47.1
8. $2 \cdot \pi \cdot 3.2 \approx$ 20.1
9. $\pi \cdot (19 \div 2)^2 \approx$ 283.4

How Did You Do? Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9

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Inquiry Lab

Three-Dimensional Figures

Inquiry HOW are some three-dimensional figures related to circles?

Mathematical Practices 1, 3

Bilal is training on an obstacle course. One of the activities in the course is the tunnel, an open tube through which he runs.

Hands-On Activity

A three-dimensional figure with faces that are polygons is called a **polyhedron**. There are three-dimensional figures that are *not* polyhedrons. Some examples of these figures are *cylinders, cones,* and *spheres.*

Step 1 For each figure, list three real-world items that represent the figure. **Sample answers are given.**

Cylinder	Cone	Sphere
		
can	ice cream cone	basketball
oatmeal container	funnel	Earth
science beaker	orange traffic cone	bubble

Step 2 Just as a rectangular prism and a pyramid have bases, a cylinder and a cone have bases as well. What is the shape of the base of a cylinder? **a circle** a cone? **a circle**

Step 3 Interesting shapes can occur when you find the cross section of a figure that is not a polyhedron. Describe the shape of the figure resulting from a horizontal cross section of each of the following.

circle **circle** **circle**

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Focus narrowing the scope

Objective Determine how some three-dimensional figures are related to circles.

Coherence connecting within and across grades

Now
Students explore the relationship of three-dimensional figures and circles using cross sections.

Next
Students will use a formula to find the volume of cylinders.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Team-Pair-Solo** Have students work in a team of four to complete Step 1 of the Activity. After Step 1 is complete, have students break into pairs to complete Step 2. Have students complete Step 3 individually. Give them time to discuss their answers within their original teams. MP 1, 4, 5

BL LA **Pairs Consult** Have students work in pairs to alter Step 3 so that the cross sections of the cylinder, cone, and sphere are slanted. Ask students to describe the shape of the cross section. Then have them to describe the shape of a vertical cross section of each figure. MP 1, 4, 5

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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–6	Exercises 7–8
Level 3		•
Level 2		
Level 1	•	

Investigate

AL LA **Think-Pair-Draw** Have students work in pairs to complete Exercises 1–6. Give them a few minutes to think about their responses to each exercise. Have the students draw their answers and discuss any differences and make any necessary changes. MP 1, 4, 5

Create

BL **Pair Discussion** Have students complete Exercise 7 individually. Have them provide drawings or models of any examples or counterexamples. Then have them share their responses with a partner. Have them justify their examples or counterexamples. MP 1, 3, 4, 5

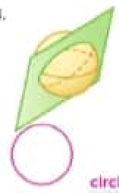
Inquiry Students should be able to answer "HOW are some three-dimensional figures related to circles?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Draw and describe the shape resulting from each cross section.

1. 
oval

2. 
triangle

3. 
circle

4. 
a parabola or part of an oval

5. 
circle

6. 
rectangle

Create

7. MP **Use a Counterexample** True or *false*: The cross section of a cylinder, a cone, and a sphere will *always* be a circle or an oval. If false, provide a counterexample.
false; Sample answer: The vertical cross section of a cylinder is a rectangle.

8. Inquiry HOW are some three-dimensional figures related to circles?
Sample answer: Cylinders and cones have circles as their base. A sphere is like a three-dimensional circle.

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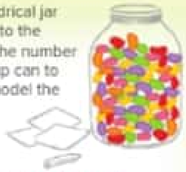
Lesson 1

Volume of Cylinders

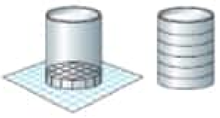
Real-World Link

Jelly Beans Muna's teacher filled a cylindrical jar with jelly beans. She is awarding a prize to the student who most accurately estimates the number of jelly beans in the jar. Muna used a soup can to model the jar and centimeter cubes to model the jelly beans.



Work with a partner. Sample answers are given.

1. Set a soup can on a piece of grid paper. Trace the area around the base as shown.



About how many centimeter cubes would fit at the bottom of the container? Remember to include partial cubes in your total. about 18

2. Suppose each layer is 1 centimeter high. How many layers would it take to fill the cylinder? 6

3. **Be Precise** Write a formula that allows you to find the volume of the container. $V = \pi r^2 h$

Essential Question

WHY are formulas important in math and science?

Vocabulary

volume
cylinder
composite solids

Mathematical Practices
1, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Find the volume of cylinders.

Coherence connecting within and across grades

Previous Students determined how some three-dimensional figures are related to circles.

Now Students solve problems involving the volume of cylinders.

Next Students will solve problems involving the volume of cones.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Pairs Prediction Have students work in pairs. Have students guess how many jelly beans they think will fill the can. Then have them complete the activity. Have them refer back to their guess to see how close they were.
MP 1, 4, 5, 6

Alternate Strategies

AL Have students make a list of everyday objects that are cylinders, such as cans, water glasses, thermoses, and so on.
MP 1, 5

BL Have students share their formulas from Exercise 3. Have them brainstorm if there are any different ways to accurately express the formula for the volume of a cylinder. MP 1, 2, 7

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the volume of a cylinder.

- AL • *What is the height of the cylinder?* **8.3 cm**
  • *What is the radius of the base?* **5 cm**
- OL • *In the formula $V = \pi r^2h$, what do we replace r with? h?* **Replace r with 5 and h with 8.3.**
  • *In the formula $V = Bh$, what does B represent?* **the area of the circular base, πr^2**
- BL • *How might our calculation of the volume of the cylinder change if we use 3.14 for π?* **Sample answer: The rounded answer would be different by a few tenths.**

Need Another Example?
Find the volume of the cylinder.
Round to the nearest tenth. **855.3 cm³**

2. Find the volume of a cylinder.

- AL • *What is the height of the cylinder?* **20 cm.**
  • *What is the radius of the base?* **8 cm.**
- OL • *How would you find the radius of the base?* **Divide the diameter by 2.**
  • *How would you find the volume of the cylinder?* **Multiply the area of the base times the height.**
- BL • *How do you know that the answer is reasonable?* **Sample answer: Use estimation. $\pi(8)^2(20) \approx 3(60)(20)$, or 3,600, so our answer is reasonable.**

Need Another Example?
Find the volume of a cylinder with a diameter of 12 meters and a height of 4 meters. Round to the nearest tenth. **452.4 m³**

Key Concept

Volume of a Cylinder

Words The volume V of a cylinder with radius r is the area of the base B times the height h.

Symbols $V = Bh$, where $B = \pi r^2$ or $V = \pi r^2h$

Model

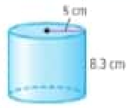
Work Zone

STOP and Reflect
What formula do you use to find the area of the base of a cylinder?
$A = \pi r^2$

Volume is the measure of the space occupied by a solid. Volume is measured in cubic units. A **cylinder** is a three-dimensional figure with two parallel congruent circular bases connected by a curved surface. The area of the base of a cylinder tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder.

Examples

1. Find the volume of the cylinder. Round to the nearest tenth.



$V = \pi r^2h$ Volume of a cylinder

$V = \pi(5)^2(8.3)$ Replace r with 5 and h with 8.3.

Use a calculator.

2nd [π] × 5 x² × 8.3 ENTER 651.8804756

The volume is about 651.9 cubic centimeters.

Circles
Recall that the radius is half the diameter.

2. Find the volume of a cylinder with a diameter of 16 centimeters and a height of 20 centimeters. Round to the nearest tenth.

$V = \pi r^2h$ Volume of a cylinder

$V = \pi(8)^2(20)$ The diameter is 16 so the radius is 8. Replace h with 20.

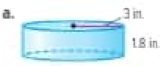
$V \approx 4,021.2$ Use a calculator.

The volume is about 4,021.2 cubic centimeters.

Got it? Do these problems to find out.

Find the volume of each cylinder. Round to the nearest tenth.

a.



b. diameter: 12 mm
height: 5 mm

a. 50.9 in³

b. 565.5 mm³

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Example

3. A metal paperweight is in the shape of a cylinder. The paperweight has a height of 1.5 centimeters and a diameter of 2 centimeters. How much does the paperweight weigh if 1 cubic centimeters weighs 50 grams? Round to the nearest tenth.

First find the volume of the paperweight.

$V = \pi r^2 h$ — Volume of a cylinder

$V = \pi(1)^2 1.5$ — Replace with 1 and h with 1.5.

$V \approx 4.7$ — Simplify.

To find the weight of the paperweight, multiply the volume by 50.

4.7(50) = 235

So, the weight of the paperweight is about 235 grams.

Got it? Do this problem to find out.

c. The Faris family uses a container shaped like a cylinder to recycle aluminum cans. It has a height of 130 centimeters and diameter of 50 centimeters. The container is full. How much do its contents weigh if the average weight of aluminum cans is 0.037 g per cubic centimeter? Round to the nearest tenth of a kilogra

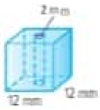
c. 9.4 kg

Volume of a Composite Solid

Objects made up of more than one type of solid are **composite solids**. To find the volume of a composite solid, decompose the figure into solids whose volumes you know how to find.

Example

4. Badria uses cube-shaped beads to make jewelry. Each bead has a circular hole through the middle. Find the volume of each bead.



The bead is made of one rectangular prism and one cylinder. Find the volume of each solid. Then subtract to find the volume of the bead.

Rectangular Prism

$V = Bh$

$V = (12 \cdot 12)12$ or 1,728

Cylinder

$V = Bh$

$V = (\pi \cdot 1^2)12$ or 37.7

The volume of the bead is 1,728 − 37.7 or 1,690.3 cubic millimeters.

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Examples

3. Find the volume of a cylinder to solve a real-world problem.

AL
- *What is the height of the cylinder?* 1.5 cm.
- *What is the radius of the base?* 1 cm.

OL
- *What does the shape of the paperweight tell you about which formula you can use?* It is cylindrical in shape. Use the formula $V = \pi r^2 h$ (or $V = Bh$) to find the volume.
- *What is the volume of the paperweight?* 4.7 cm^3
- *How can you find the weight of the paperweight?* Multiply the volume by 50.

BL
- *About what fraction of a kilogram is the weight of the paperweight?* about $\frac{1}{4}$ kg

Need Another Example?

A cylindrical container of ice cream is 10 centimeters in diameter and 12 centimeters tall. How long will it take for all of the ice cream in the container to melt if it melts at a rate of 2.1 cubic centimeters every minute? Round to the nearest tenth. 448.8 min or 7.5 h

4. Find the volume of a composite figure.

AL
- *What is the height of the cylinder?* 12 mm
- *What is the radius of the base of the cylinder?* 1 mm

OL
- *What is the volume of the cylinder?* 37.7 mm^3
- *What is the volume of the cube?* 1,728 mm^3
- *How can you find the volume of the bead?* Subtract the volume of the cylinder from the volume of the cube.

BL
- *About what percent of a whole cube has been drilled out to make the cylindrical hole?* about 2%

Need Another Example?

A roll of paper towels has the dimensions shown. Find the volume of the roll. Round to the nearest tenth. 3,271.2 cm^3

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

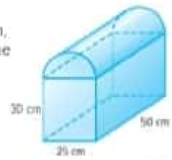
AL LA Paired Heads Together Have students work with a partner to complete Exercises 1 and 2. Have them use the following template to find the volume of each cylinder: *volume = area of base × height of cylinder*. If they are still struggling, have them replace *area of base* in the template with $\pi \cdot$ *radius* $\cdot$ *radius*. MP 1, 5, 7

BL LA Trade-a-Problem Have students draw a composite figure similar to Exercise 3 and write a real-world problem that involves finding the volume. Have them trade figures and problems with another student to solve each other's problem. Have them discuss responses and compare solutions. MP 3, 4, 5

d. 49,771.8 cm³

Got it? Do this problem to find out.

d. The Service Club is building models of storage chests, like the one shown, to donate to a charity. Find the volume of the chest to the nearest tenth.



Guided Practice

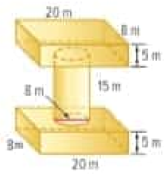
Find the volume of each cylinder. Round to the nearest tenth. (Examples 1 and 2)

1. 56.5 cm³

2. 402.1 cm³

diameter: 8 cm.
height: 8 cm.

3. A platform like the one shown was built to hold a sculpture for an art exhibit. What is the volume of the figure? (Example 4)
2,354.0 m³



4. A scented candle is in the shape of a cylinder. The radius is 4 centimeters and the height is 12 centimeters. Find the mass of the wax needed to make the candle if 1 cubic centimeter of wax has a mass of 3.5 grams. Round to the nearest tenth. (Example 3)
2,111.2 g

5. **Building on the Essential Question** How is the formula for the volume of a cylinder similar to the formula for the volume of a rectangular prism?
Sample answer: In both, the volume equals the area of the base times the height.

Rate Yourself!

How confident are you about volume of cylinders? Check the box that applies.

FOLDABLES Time to update your Foldable!

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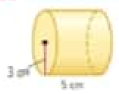
Name ________________ My Homework ________________

Independent Practice

Find the volume of each cylinder. Round to the nearest tenth. (Examples 1 and 2)

1. 141.4 cm³



2. 103.4 m³
diameter 4.5 m
height 6.5 m

3. Asma's parents have a cylindrical oak tree stump that has a diameter of 1.3 meters and a height of 2 meters. How much does the stump weigh if the average weight of oak is 946 kilograms per cubic meter? Round to the nearest tenth. (Example 3) 2,139.8 kg

4. An unused roll of paper towels is shown. What is the volume of the unused roll? (Example 4) 2,580.3 cm³

5. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.

a. Find the volume of the bag and candle. Round to the nearest tenth.
bag: 2,400 cm³; candle: 369.5 cm³

b. How much packing material is needed to fill the empty space in the bag after the candle is placed in the bag? 2,030.5 cm³

c. There are 70 teachers in the school. If each package of packing material contains 11,000 cubic centimeters of material, how many packages do they need to buy to fill all of the gift bags? 13 packages

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–4, 10–14	Exercises 5, 6, 15–19	Exercises 7–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 8, 9, 18, 19
OL	On Level	1, 3, 5, 6, 8, 9, 18, 19
BL	Beyond Level	5–9, 18, 19

Watch Out!

Common Error Students may multiply the radius by 2 instead of squaring it when using a formula to find the volume of a cylinder. Point out that the 2 is an exponent. Then remind students what each part of the formula shows: πr^2 is the area of the base, and h is the height.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7
2 Reason abstractly and quantitatively.	9
3 Construct viable arguments and critique the reasoning of others.	17
4 Model with mathematics.	5, 8
5 Use appropriate tools strategically.	6

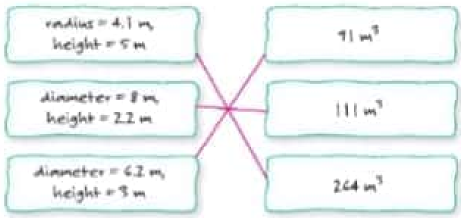
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

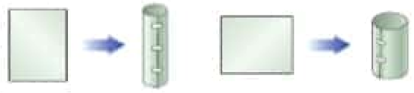
Pose the following problem to students: A cylindrical bale of hay has a height of 2 meters and a diameter of 2 meters. What is its approximate volume to the nearest cubic meter? about 6 m^3

6. **Use Math Tools** Match each cylinder with its approximate volume.



H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** Two equally-sized sheets of construction paper are rolled; one along the length and the other along the width, as shown. Which cylinder has the greater volume? Explain.



Sample answer: The shorter cylinder, because the radius is larger and that is the squared value in the formula.

8. **Model with Mathematics** Draw and label a cylinder that has a larger radius but less volume than the cylinder shown below.

9. **Reason Abstractly** Find the ratios of the volume of cylinder A to cylinder B.

a. Cylinder A has the same radius but twice the height of cylinder B.
2:1

b. Cylinder A has the same height but twice the radius of cylinder B.
4:1

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Name ____________ My Homework ____________

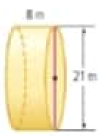
Extra Practice

Copy and Solve For Exercises 10–27, show your work and answers on a separate piece of paper.

Find the volume of each cylinder. Round to the nearest tenth.

10. 

2,261.9 mm³



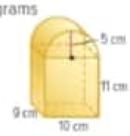
2,770.9 m³

12. 

167.1 cm³

13. Ahmed has a container of flour in the shape of a cylinder. The container has a diameter of 10 centimeters and a height of 8 centimeters. If the container is full, how much will the flour weigh if the average weight of flour is 0.23 grams per cubic centimeter? Round to the nearest tenth. 144.5 grams

14. Amna wants to make a box like the one shown. What is the volume of the box? Round to the nearest tenth. 1,343.4 cm³



15. Cylinder A has a radius of 4 centimeters and a height of 2 centimeters. Cylinder B has a radius of 2 centimeters. What is the height of Cylinder B to the nearest centimeter if both cylinders have the same volume? 8 cm

16. Which will hold more cake batter, the rectangular pan or two round pans? Explain your reasoning to a classmate. See margin.

17. **Multiple Representations** The dimensions for four cylinders are shown in the table. 17a–b, d. See margin.

a. **Symbols** Write an equation to find the volume of each cylinder.

b. **Words** Compare the dimensions of Cylinder A with the dimensions of Cylinders B, C, and D.

c. **Numbers** Complete the table.

d. **Words** Explain how changing the dimensions of a cylinder affects the cylinder's volume.

	Radius (cm)	Height (cm)	Volume (cm³)
Cylinder A	1	1	3.14 cm³
Cylinder B	1	2	6.28 cm³
Cylinder C	2	1	12.57 cm³
Cylinder D	2	2	25.13 cm³

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Additional Answers

16. rectangular pan; Sample answer: The rectangular pan's volume is 234 cm³ and the total volume of the two round pans is about 201 cm³.

17a. $V = \pi(1)^2(1)$; $V = \pi(1)^2(2)$; $V = \pi(2)^2(1)$; $V = \pi(2)^2(2)$

17b. The height of Cylinder B is twice the height of Cylinder A. The radius of Cylinder C is twice the radius of Cylinder A. The radius and height of Cylinder D are twice the radius and height of Cylinder A.

17d. When the radius is doubled, the volume is four times the original volume. When the height is doubled, the volume is twice the original volume. When the radius and height are doubled, the volume is eight times the original volume.

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Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking.

18.

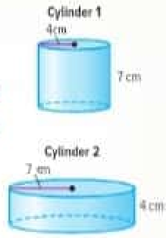
This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3, MP6
Scoring Rubric	
2 points	Students correctly answer the question, explain their response, and find the volume of each cylinder.
1 point	Students correctly answer the question and find the volume of each cylinder, but fail to explain their response OR students correctly answer and explain and may or may not find the volume of one cylinder OR students correctly find the volume of each cylinder, but fail to answer the question and explain their response.

19.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP6
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

18. Without doing any calculations, do you think Cylinder 1 and Cylinder 2 will have the same volume? Explain your reasoning.



no; Sample answer: Even though the dimension of both cylinders are 4 cm and 7 cm, 4 cm is the radius of Cylinder 1 and 7 cm is the radius of Cylinder 2. Since this radius is squared in the volume formula, the volume of Cylinder 2 will be greater than the volume of Cylinder 1.

Fill in each box to complete the following statements.

To the nearest tenth, the volume of Cylinder 1 is $351.9\ cm^3$

To the nearest tenth, the volume of Cylinder 2 is $615.8\ cm^3$

19. The oatmeal container shown has a diameter of 3 centimeters and a height of 9 centimeters. Which of the following statements are true? Select all that apply.

- ☐ The area of each base is exactly 9π square centimeters.
- ■ The volume of the container is exactly 20.25π cubic centimeters.
- ■ The volume of the container to the nearest tenth is about 63.6 cubic centimeters.

Spiral Review

Find the area of each circle. Round to the nearest tenth.

20. 

$176.7\ cm^2$

21. 

$201.1\ cm^2$

22. 

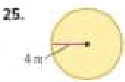
$254.5\ cm^2$

23. 

$28.3\ cm^2$

24. 

$30.2\ cm^2$

25. 

$50.3\ m^2$

Find the volume of each prism.

26. 

$36\ m^3$

27. 

$539\ m^3$

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

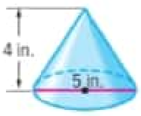
1. Find the volume of a cone.

AL
- *What is the height of the cone?* 6 cm.
- *What is the radius of the base?* 3 cm.

OL
- *How is the volume of this cone related to the volume of a cylinder with the same base area and height?* The volume of the cone is $\frac{1}{3}$ the volume of the cylinder.
- *In the formula $V = \frac{1}{3}\pi r^2h$, what does πr^2 represent?* the area of the circular base

BL
- *What would be the first step to solve the problem if you were given that the diameter is 6 centimeters?* Sample answer: Divide by 2 to find the radius.

Need Another Example?
Find the volume of the cone. Round to the nearest tenth. 26.2 cm³



2. Find the volume of a cone.

AL
- *What is the radius of the base of the cup?* 4 cm
- *What is the height of the paper cup?* 10 cm

OL
- *How are the formulas $V = \frac{1}{3}Bh$ and $V = \frac{1}{3}\pi r^2h$ for a cone related?* Sample answer: For a cone, they are the same formula because the base is a circle.

BL
- *Why is the label for the solution given in cubic centimeters?* Sample answer: We are determining volume, which is a three-dimensional property; therefore, the answer is in three dimensions, or cubic units.

Need Another Example?
A cone-shaped vase has a height of 15 centimeters and a diameter of 8 centimeters. What is the volume of the vase? Round to the nearest tenth. 251.3 cm³

Key Concept

Volume of a Cone

Words The volume V of a cone with radius r is one third the area of the base B times the height h.

Model

Symbol $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$

Work Zone

A **cone** is a three-dimensional figure with one circular base connected by a curved surface to a single vertex.

Example

1. Find the volume of the cone. Round to the nearest tenth.

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 6$ $r = 3, h = 6$

$V \approx 56.5$ Simplify.

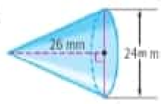
The volume is about 56.5 cubic centimeters.

Got it? Do these problems to find out.

Find the volume of each cone. Round to the nearest tenth.

a.

b.



a. 29.3 m³

b. 3,920.7 mm³

Example

2. A cone-shaped paper cup is filled with water. The height of the cup is 10 centimeters and the diameter is 8 centimeters. What is the volume of the paper cup? Round to the nearest tenth.

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 10$ $r = 4, h = 10$

$V \approx 167.6$ Simplify.

The volume of the paper cup is about 167.6 cubic centimeters.

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Got it? Do this problem to find out.

c. Sumayya is filling six identical cones for her competition. Each cone has a radius of 3.6 centimeters and a height of 21 centimeters. What is the total volume of the cones? Round to the nearest tenth.

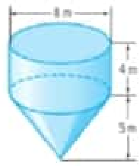
c. 284.9 cm^3

Volume of Composite Solids

When a composite solid includes cylinders and cones, you can find the volume by decomposing it into solids whose volumes you know how to find.

Example

3. Find the volume of the solid. Round to the nearest tenth.



Step 1 Find the volume of the cylinder.

$V = \pi r^2 h$ Volume of a cylinder

$V = \pi \cdot 4^2 \cdot 4$ $r = 4, h = 4$

$V = \pi \cdot 16 \cdot 4$ Simplify.

$V \approx 201.1$ Simplify.

Step 2 Find the volume of the cone.

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 5$ $r = 4, h = 5$

$V = \frac{1}{3} \cdot \pi \cdot 16 \cdot 5$ Simplify.

$V \approx 83.8$ Simplify.

So, the volume of the solid is about 201.1 + 83.8 or 284.9 cubic meters.

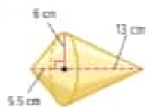
STOP and Reflect

Hareb and Hamad are simplifying $\pi \cdot 5^2$. Hareb rounds π to 3.14 and Hamad uses the π key on her calculator. Which student's calculation is closer to the exact value? Explain below.

Hamad's calculation; Sample answer: Since π is a nonterminating decimal, the product of π and 5^2 is also nonterminating. Hamad's calculation is closer to the exact value because his approximation of π contained more decimal places.

Got it? Do this problem to find out.

d. Find the volume of the solid.



d. 697.4 cm^3

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Example

3. Find the volume of a composite solid.

AL
- *Look at the figure. Into what solids can it be broken up?* a cylinder and a cone
- *What is the formula for the volume of a cylinder?* $V = \pi r^2 h$
- *What is the formula for the volume of a cone?* $V = \frac{1}{3}\pi r^2 h$

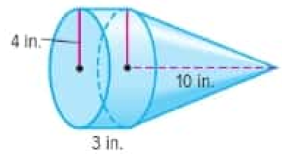
OL
- *What is the height of the cylinder?* 4 m *the cone?* 5 m
- *What is the radius of the base of the cylinder?* 4 m *the cone?* 4 m
- *After we find the volume of the cylinder and the cone, what is the final step to finding the volume of the composite solid?* Add the volumes of the cylinder and cone.

BL
- *Why is the volume of the cone not $\frac{1}{3}$ the volume of the cylinder?* Sample answer: Although the cylinder and cone have congruent base areas, they do not have the same height.
- *If you used 3.14 for π, how would your final answer for the volume of the composite solid change?* The total volume would be 284.7 cubic meters, rounded to the nearest tenth.

Need Another Example?

Find the volume of the solid. Round to the nearest tenth.

318.3 in^3



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Guided Practice

Formative A ssessment Use these exercises to assess students' un derstanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Team-Pair-Solo Have students complete Exercises 1 and 2 in a four-person team. Then have them complete Exercises 3 and 4 in pairs. If they are ready, have them complete Exercises 5 and 6 on their own and compare answers with their original team. Then have them discuss in their teams the answer for Exercise 7 and write down an agreed-upon answer. MP 1, 2, 4, 5

BL LA Pairs Present Have students work in pairs to complete Exercise 7 and prepare a brief oral presentation for the connection between the formulas for the volume of a cylinder and a cone having congruent base areas and heights. In their presentation, they should use algebraic manipulation as well as concrete examples and diagrams or illustrations. Have them present to the class, while the rest of the class listens carefully, and asks any clarifying questions. MP 1, 2, 4, 5

Guided Practice

Find the volume of each cone. Round to the nearest tenth. (Examples 1 and 2)

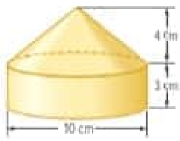
1. 2,668.3 m^3

2. 1,055.6 mm^3

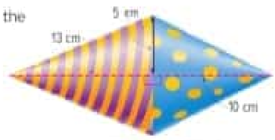
3. height: 9 m
   diameter: 10 m 235.6 m^3

4. height: 120 millimeters
   radius: 45 millimeters 254,469.0 mm^3

5. Find the volume of the solid at the right. Round to the nearest tenth. (Example 3) 340.3 cm^3



6. Find the volume of the pair of cones shown. Round to the nearest tenth. (Example 3) 602.1 cm^3



7. **Building on the Essential Question** What would have a greater effect on the volume of a cone: doubling its radius or doubling its height? Explain.
   Sample answer: Depending on the length of the radius and the height, generally doubling the radius has more effect since it is squared in the formula.

Rate Yourself!

How confident are yo u about volume of cones? Sha de the section that applies.

FOLDABLES Time to update your foldable!

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Name ______ My Homework ______

Independent Practice

Go online for Step-by-Step Solutions

Find the volume of each cone. Round to the nearest tenth. (Example 1)

1. 4,720.8 mm³

2. 4.8 m³

3. height: 8.4 meters
diameter: 3.5 meters 26.9 m³

4. height: 3.9 meters
radius: 1.7 meters 11.8 m³

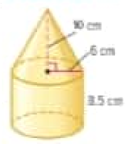
5. A cone like the one at the right is going to be filled with candy. What is the volume of the cone? Round to the nearest tenth. (Example 2)
102.6 cm³

6. Mr. Ibrahim is building a storage shed in a conical shape. The base of the shed is 4 meters in diameter and the height of the shed is 3.8 meters. What is the volume of the shed? Round to the nearest tenth. (Example 2) 15.9 m³

Find the volume of each solid. Round to the nearest tenth. (Example 3)

7. 1,608.5 cm³

8. 1,338.3 cm³



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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–8, 15–22	9–11, 23–29	12–14
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options			
AL	Approaching Level	1–9, 11, 12, 14, 28, 29
OL	On Level	1–7 odd, 9–12, 14, 28, 29
BL	Beyond Level	9–14, 28, 29

Watch Out!

Common Error Students may mistakenly use the diameter as the radius in the formula for the volume of a cone. Remind students that the radius of the base, not the diameter, is used to find the volume.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13, 25–27
2 Reason abstractly and quantitatively.	10
3 Construct viable arguments and critique the reasoning of others.	12, 14

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how the previous lesson on finding the volume of a cylinder helped them with this lesson. Use the writing prompts below. **See students' work.**

- In the previous lesson, I learned...
- In this lesson, I learned...
- What I learned in the previous lesson helped me in this lesson because...

Watch Out!

Common Error In Exercise 12, Faleh mistakenly uses the diameter as the radius in the formula for the volume of a cone. Remind students that the radius of the base, not the diameter, is used to find the volume.

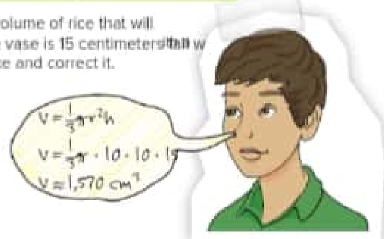
A cylinder has a radius of 5 centimeters and a height of 12 centimeters. What would the height of a cone need to be if it has the same volume and radius? Round to the nearest centimeter. **36 cm**

10. **Reason Abstractly** Alyssa is making cone-shaped ice cubes by using a mold. The radius of the mold is 1.5 centimeters and the height is 2 centimeters. If one cubic centimeter is about 1 gram, how many grams will ten ice cubes weigh? Round to the nearest tenth. **47.1 g**

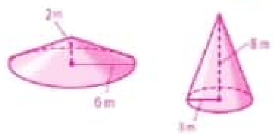
11. The volume of a cone with a 30-millimeter radius is 9,420 cubic millimeters. What is the height of the cone to the nearest millimeter? **10 mm**

H.O.T. Problems Higher Order Thinking

12. **Find the Error** Faleh is finding the volume of rice that will fill a cone-shaped decorative vase. The vase is 15 centimeters tall with a 10-centimeter diameter. Find her mistake and correct it.
Faleh used the incorrect radius; 392.5 cm³



13. **Persevere with Problems** Draw and label two cones with different dimensions but the same volume. **Sample answer:**



14. **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never true*. Explain your reasoning.

The volume of a rectangular-based pyramid and a cone with the same height and equal areas of the base are equal.

always; Sample answer: The volumes are the same since both heights and the area of both bases are the same. Changing the shape of the base will not affect the volume.

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Name ________ My Homework ________

Extra Practice

Copy and Solve For Exercises 15–33, show your work and answers on a separate piece of paper.

Find the volume of each cone. Round to the nearest tenth.

15.


2,989.8 mm^3

16.


71.2 m^3

17.


398.2 m^3

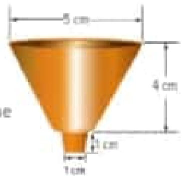
18.


603.2 m^3

19. height: 24 centimeters
diameter: 8 centimeters 402.1 cm^3

20. height: 9 centimeters
diameter: $7\frac{1}{2}$ centimeters 132.5 cm^3

21. Usama is using the funnel shown to fill a glass bottle with colored sand. Estimate the volume of the funnel.
Sample answer: 32.7 cm^3



22. Mount Rainier, a cone-shaped volcano in Washington, is about 4.4 kilometers tall and about 18 kilometers across its base. Find the volume of Mount Rainier to the nearest whole number.
about 373 km^3

23. The volume of a cone is 471.24 cubic centimeters and the height is 8 centimeters. What is the diameter? 15 cm

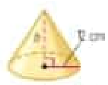
24. The volume of a cone is 593.46 cubic centimeters. The radius is 9 centimeters. Find the height of the cone to the nearest centimeter. 7 cm

Persevere with Problems Find the height of each cone. Round to the nearest tenth.

25.


Volume: 42.39 m^3

4.5 m

26.


Volume: 1,205.76 cm^3

8.0 cm

27.


Volume: 19.625 m^3

3.0 m

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Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking.

28.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly order each figure and find the corresponding volume.
1 point	Students correctly order three of the four figures and find the corresponding volume OR students correctly order each figure, but have calculation errors in finding the volume of one or two figures.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

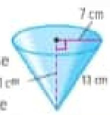
Power Up! Test Practice

28. Four cones have the dimensions shown below.

Sort the cones from least to greatest volume. Round to the nearest tenth.

	Figure	Volume (cm³)
Least	4	452.4
	3	513.1
	1	938.3
Greatest	2	1,102.7

29. Refer to the cone shown at the right. Determine if each statement is true or false.



a. The approximate area of the base is 153.9 square centimeters. ■ ueTr □ False

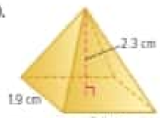
b. The approximate volume of the cone is 886.5 cubic centimeters. □ ueTr ■ False

c. The volume of a cylinder with the same height and radius would be 3 times the volume of the cone. ■ ueTr □ False

Spiral Review

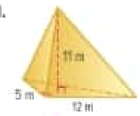
Find the volume of each pyramid. Round to the nearest tenth if necessary.

30.



3.5 cm³

31.



110 m³

32.

482.1 m³

33.

90 cm³

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Lesson 3

Volume of Spheres

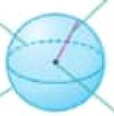
Vocabulary Start-Up

A **sphere** is a set of all points in space that are a given distance, known as the radius, from a given point, known as the center.

Complete the graphic organizer.

Describe the shape of the cross section. **circle**

Name this part of the sphere. **radius**



Name two examples of a sphere. **Sample answer: basketball, moon**

Name this part of the sphere. **center**

Essential Question

WHY are formulas important in math and science?

Vocabulary

sphere
hemisphere

Mathematical Practices
1, 3, 4

Real-World Link

Buthaina purchased a necklace that contained a round pearl with a diameter of 7.5 millimeters. What is the circumference of the largest circle around the outside of the pearl? Round to the nearest tenth. **23.6 mm**

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

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Focus narrowing the scope

Objective Find the volume of spheres.

Coherence connecting within and across grades

Previous Students solved problems involving the volume of cones.

Now Students solve problems involving the volume of spheres.

Next Students will use models and nets to find the surface area of cylinders.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Paired Heads Together Have pairs complete the graphic organizer. Have them come up with as many real-world examples of spheres as possible, or objects that are approximately spheres. MP 1, 4, 5

Alternate Strategies

AL Bring in a piece of fruit that is roughly spherical, such as an orange. Slice the fruit in half. Then have students identify the circular cross section, the radius, and the center. MP 1, 4, 5

BL Have students research whether or not Earth is an exact sphere. Have them share their results with the class. MP 1, 2

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the volume of a sphere.

AL
- *What formula do you use to find the volume of a sphere?* $V = \frac{4}{3}\pi r^3$
- *What is the radius of the sphere?* 6 mm

OL
- *What is the volume of the sphere, rounded to the nearest tenth?* 904.8 mm³
- *Why do we cube the radius when determining the volume?* Volume is measured in cubic units, so we need three dimensions.

BL
- *What is the exact volume of the sphere?* 288π
- *If the problem asks for the exact volume, why must we leave π in the answer?* Sample answer: π is an irrational number. When we multiply by it and round, we are not providing the exact answer.

Need Another Example?
Find the volume of the sphere.
Round to the nearest tenth. 113.1 cm³

2. Find the volume of a sphere.

AL
- *What is the diameter of the balloon? radius?* 8 m; 4 m

OL
- *Which operation do you perform first when using the volume formula?* Cube the radius.
- *What is the volume, rounded to the nearest tenth?* 268.1 m³

BL
- *What is the exact volume of the sphere?* $85\frac{1}{3}\pi$ m³

Need Another Example?
A regulation basketball used in men's basketball has a diameter of about 24 centimeters. What is the volume of a basketball? Round to the nearest tenth. 7,238.2 cm³

Key Concept

Volume of a Sphere

Words The volume V of a sphere is four thirds the product of π and the cube of the radius r.

Symbols $V = \frac{4}{3}\pi r^3$

Model

Work Zone

Exact and Approximate
Whenever you round or use 3.14 for π, you are finding the approximate value. An answer left in terms of π, such as $\frac{256}{3}\pi$, is an exact value.

You can use the formula for the volume of a sphere to solve mathematical and real-world problems.

Example

1. Find the volume of the sphere. Round to the nearest tenth.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 6^3$ Replace r with 6.

$V \approx 904.8$ Simplify. Use a calculator.

The volume of the sphere is about 904.8 cubic millimeters.

Got it? Do these problems to find out.

Find the volume of each sphere. Round to the nearest tenth.

a.

b.

a. 5,575.3 cm³

b. 523.6 mm³

Example

2. A spherical giant balloon has a diameter of about 8 meters. Find the volume of the spherical balloon. Round to the nearest tenth.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 4^3$ Replace r with 4.

$V \approx 268.1$ Simplify. Use a calculator.

The volume of the giant balloon is about 268.1 cubic meters.

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Got it? Do this problem to find out.

c. A dish contains a spherical scoop of vanilla ice cream with a radius of 3 centimeters. What is the volume of the ice cream?

c. About 113 cm^3

Example

3. A ball has a diameter of 10 centimeters. A pump can inflate the ball at a rate of 325 cubic centimeters per minute. How long will it take to inflate the ball? Round to the nearest tenth.

Find the volume of the ball. Then use a proportion.

$V = \frac{4}{3}\pi r^3$ — Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 5^3$ or 523.6 — Replace r with 5.

$\frac{325 \text{ cm}^3}{1 \text{ min}} = \frac{523.6 \text{ cm}^3}{x \text{ min}}$ — Write the proportion.

$325x = 523.6$ — Cross multiply.

$x = 1.6$ — Simplify.

So, it will take about 1.6 minutes to inflate the ball.

Got it? Do this problem to find out.

d. A snowball has a diameter of 6 centimeters. How long would it take the snowball to melt if it melts at a rate of 1.8 cubic centimeters per minute? Round to the nearest tenth.

d. 62.8 min

Volume of a Hemisphere

A circle separates a sphere into two congruent halves each called a **hemisphere**.

Hemisphere

The volume of a hemisphere is $\frac{1}{2}$ the volume of a sphere.

Example

4. Find the volume of the hemisphere. Round to the nearest tenth.

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ — Volume of a hemisphere

$V = \frac{1}{2}\left(\frac{4}{3} \cdot \pi \cdot 5^3\right)$ — Replace r with 5.

$V \approx 261.8$ — Simplify. Use a calculator.

The volume of the hemisphere is about 261.8 cubic centimeters.

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Examples

3. Solve a real-world problem involving the volume of a sphere.

AL
- *What is the radius of the ball?* 5 cm.
- *How is volume related to inflating a ball?* The amount of air it holds is its volume.

OL
- *What steps do we need to do to solve the problem?* We need to find the volume of the ball. Then we need to find how long it will take to inflate the ball.
- *What is the volume of the ball?* 523.6 cm^3.
- *How can you find the time it will take to inflate the ball?* Set up a proportion comparing the unit rate with the unknown amount of time to inflate the ball to its full volume.

BL
- *How would you find the time it will take to inflate the ball?* Multiply the volume by the rate.

Need Another Example?
A beach ball has a diameter of 12 centimeters. A pump can inflate the ball at a rate of 325 cubic centimeters per minute. How long will it take to inflate the ball? Round to the nearest tenth. 2.8 min

4. Find the volume of a hemisphere.

AL
- *What is a hemisphere?* One half of a sphere
- *Where might you have seen this word before?* Sample answer: Earth is divided into the Northern Hemisphere and the Southern Hemisphere.
- *What is the radius of the hemisphere in Example 4?* 5 cm

OL
- *How can you find the volume of a hemisphere?* Multiply the volume of the sphere by one half.
- *In the expression $\frac{1}{2}\left(\frac{4}{3} \cdot \pi \cdot 5^3\right)$, which operation do we perform first?* Find the cube of 5.

BL
- *Write a simplified formula that can be used to find the volume of a hemisphere.* Sample answer: $V = \frac{2}{3}\pi r^3$

Need Another Example?
Find the volume of a hemisphere with a radius of 9.5 meters. Round to the nearest tenth. 1,795.7 cm^3

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Pairs Consult Prior to beginning Exercises 1–5, have students work with a partner to create a booklet, noting the formulas for area of a circle, volume of a cylinder, cone, sphere, and hemisphere. For each formula, have them provide an example. Leave several blank pages to add formulas and notes for Lessons 4 and 5. Have students use the booklet to help complete Exercises 1–5 and to use as a reference for the rest of the chapter. MP 1, 4, 5, 7

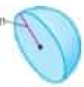
BL LA Pairs Present For Exercise 6, have students experiment using different-sized spheres and cylinders with the given dimensions of r and $2r$. Have them use actual numerical values for r and $2r$ and have them keep track of their work in a table. Then have them prepare a brief oral presentation and present their findings to the class. MP 1, 4, 5, 7

c. 2.1 cm^3

d. 1,526.8 m^3

Got it? Do these problems to find out.

e. 

f. 

Guided Practice

Find the volume of each sphere. Round to the nearest tenth. (Example 1)

1. 4,188.8 m^3

2. 1,150.3 km^3

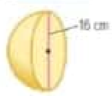
3. Sindiyya is blowing up spherical balloons for her brother's graduation party. One of the balloons has a radius of 7.5 centimeters Round to the nearest tenth.
(Examples 2 and 3)

   a. What is the volume of the balloon? 1,766.3 cm^3

   b. Suppose Sindiyya can inflate the balloon at a rate of 3,000 cubic centimeters per minute. How long will it take her to inflate the balloon? 0.6 min

Find the volume of each hemisphere. Round to the nearest tenth. (Example 4)

4. 1,072.3 cm^3



5. 883.6 mm^3

6. **Building on the Essential Question** True or *false*? The volume of a sphere is two-thirds the volume of a cylinder with the same radius r and height of $2r$. Explain your reasoning.
True; sample answer: The volume of the cylinder is $2\pi r^3$ units^3. $2\pi r^3 \cdot \frac{2}{3} = \frac{4}{3}\pi r^3$ or the volume of the sphere.

Rate Yourself!

How well do you understand volume of spheres? Circle the image that applies.

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Name _______________ My Homework _______________

Independent Practice

Find the volume of each sphere. Round to the nearest tenth. (Example 1)

1. 1,563.5 cm³

2. 904.8 m³

3. 2,144.7 mm³

4. 1,288.2 cm³

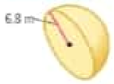
5. The radius of a ball is 4.7 centimeters. What is the volume of the basketball? Round to the nearest tenth. (Example 2) 434.9 cm³

6. Najat bought a game that contained a ball and 10 jacks. The ball had a radius of 2 centimeters. What is the volume of the ball? Round to the nearest tenth. (Example 2) 33.5 cm³

7. A spherical ball has a diameter of 8 centimeters. The ball has a slow leak in which the air escapes at the rate of 20 cubic centimeters per second. How long it would take the ball to deflate? Round to the nearest tenth. (Example 3)
107.2 s

Find the volume of each hemisphere. Round to the nearest tenth. (Example 4)

8. 658.5 m³



9. 1.5 mm³

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–9, 16–23	Exercises 10–12, 24–29	Exercises 13–15
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 14, 15, 28, 29
OL	On Level	1–9 odd, 10–12, 14, 15, 28, 29
BL	Beyond Level	10–15, 28, 29

Watch Out!

Common Error Students may forget to divide the volume by 2 when determining the volume of a hemisphere. Remind them that hemisphere means "half of a sphere" and that they can determine the volume of the whole sphere, then cut it in half.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10, 11, 13, 24
2 Reason abstractly and quantitatively.	14
3 Construct viable arguments and critique the reasoning of others.	15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students find the volume of a sphere with a diameter of 2 centimeters. Round to the nearest tenth. 4.2 cm³

Persevere with Problems Find the radius of each figure. Round to the nearest tenth.

10. sphere with a volume of 1,767.1 m^3

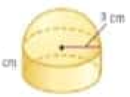
7.5 m

11. hemisphere with a volume of 2,712.3 cm^3

10.9 cm.

12. Find the volume of the composite solid shown. Round to the nearest tenth.

113.1 cm^3



H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** Three tennis balls are packaged in a box as shown below. The box is 12.1 centimeters long, 3.5 centimeters wide, and 3.5 centimeters tall. Each ball is 3.3 centimeters in diameter. What is the volume of the empty space in the box? 91.8 cm^3

14. **Reason Abstractly** A cylinder contains 150.8 cubic units of water. What is the minimum radius of a sphere that will hold the water? Round to the nearest tenth. 3.3 units

15. **Reason Inductively** Determine whether the following statement is *true* or *false*. Explain your reasoning.

Doubling a sphere's radius doubles its volume.

false; Sample answer: The radius is cubed when finding the volume of a sphere. When the radius is doubled, the volume is 2^3 or 8 times the original volume.

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Name__________ My Homework__________

Extra Practice

Copy and Solve For Exercises 16–36, show your work and answers on a separate piece of paper.

Find the volume of each figure. Round to the nearest tenth.

16. 

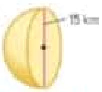
696.9 cm^3

17. 

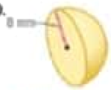
3,053.6 cm^3

18. 

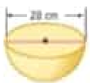
1,436.8 m^3

19. 

883.6 km^3

20. 

1,072.3 mm^3

21. 

5,747.0 cm^3

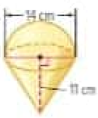
22. Ayesha is purchasing a ring that contains a 7.5 millimeter diameter round pearl. Find the volume of the pearl to the nearest te 220.9 mm^3

23. Amal is purchasing balloons for a party. Each spherical balloon is inflated with helium. How much helium is in the balloon if the balloon has a radius of 11 centimeters? Round to the nearest tenth. 5,575.3 cm^3

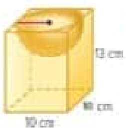
24. **Persevere with Problems** The volume of a ball is about 13.39 cubic centimeters. What is the diameter? Round to the nearest tenth.
2.9 cm

25. A golf ball has a diameter of 42.67 millimeters and a mass of 45.93 grams. What is the number of grams per cubic millimeter of the material used to make the golf ball? Round to the nearest ten-thousandth.
0.0011 g/mm^3

Find the volume of each composite solid. Round to the nearest tenth.

26. 

1,282.8 cm^3

27. 

1,038.2 cm^3

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Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking.

28. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP2, MP4
Scoring Rubric	
2 points	Students correctly model the equation and find the radius.
1 point	Students correctly model the equation OR find the radius.

29. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

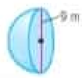
28. The volume of a golf ball is about 41.63 cm^3. Select the correct values to complete the formula below to find the radius of a golf ball.

2	6
3	9
4	41.63
	r

$41.63 = \frac{4}{3}\pi r^3$

To the nearest hundredth, what is the radius of the golf ball? 2.15 cm

29. Refer to the hemisphere shown. Fill in each box to make a complete statement. Round to the nearest tenth if necessary.



a. The radius of the hemisphere is 4.5 meters.

b. The volume of a sphere with a diameter of 9 meters is 381.7 cubic meters.

c. The volume of the hemisphere is 190.9 cubic meters.

Spiral Review

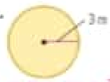
Find the circumference and area of each circle. Round to the nearest tenth.

30.


11.3 cm.; 10.2 cm^2

31.


25.1 mm; 50.3 mm^2

32.


18.8 m; 28.3 m^2

33.


19.5 m; 30.2 m^2

34. Find the area of a circle with a radius of 6 centimeters. Round to the nearest tenth. 113.1 cm^2

35. Find the area of a circle with a diameter of 13.1 centimeters. Round to the nearest tenth. 134.8 cm^2

36. An conical icicle has a volume of about 12 cubic centimeters. If the icicle has a height of 8 centimeters, what is the diameter of the icicle? 2.4 cm

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Mid-Chapter Check

If students have trouble with Exercises 1–8, they may need help with the following concepts.

Concept
volume of a cylinder (Lesson 1)
volume of a cone (Lesson 2)
volume of a sphere (Lesson 3)

Vocabulary Activity

LA Rally Coach Have students work in pairs to complete Exercises 1 and 2. Have Student 1 speak aloud their response to Exercise 1, while Student 2 listens, coaches, and encourages. Then have students trade roles for Exercise 2. MP 1, 2, 7

Alternate Strategies

AL LA If students are having trouble with Exercise 2, have them write the formulas for the volume of a cylinder and cone. Then have them replace *r* and *h* with numerical values, such as $r = 3$ centimeters and $h = 8$ centimeters in each formula. They should see that the difference between the two formulas is that the volume of a cone is one-third the volume of a cylinder, with the same base area and height. MP 1, 2, 7

BL LA Have students explain why the volume of a cone is *not* one-third the volume of a cylinder if the base area and height are not equivalent. They should use drawings or illustrations in their explanation. MP 1, 2, 4, 5, 7

Mid-Chapter Check

Vocabulary Check

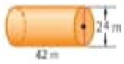
1. MP **Be Precise** Define *cylinder*. What are the symbols used to find the volume of a cylinder? (Lesson 1)
   A cylinder is a three-dimensional figure with two parallel congruent circular bases. Sample answer: The volume *V* of a cylinder with a radius *r* is the area of the base *B* times the height *h*, where $B = \pi r^2$.

Fill in the blank.

2. The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height. (Lesson 2)

Skills Check and Problem Solving

3. What is the volume of the cylinder shown at the right? Round to the nearest tenth. (Lesson 1) 19,000.4 m³



4. Find the height of a cone with a volume of 464,603 cubic centimeters and a diameter of 8 centimeters. (Lesson 2) 27.7 cm

Find the volume of each sphere. Round to the nearest tenth. (Lesson 3)

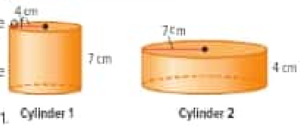
5. 24,429.0 m³

6. 6,044.0 cm³

7. 38,792.4 m³

8. MP **Reason Inductively** Refer to the cylinders shown. If a cone has a base and height congruent to Cylinder 1, which statement is true? (Lesson 2) IV

   I The volume of the cone is equal to the volume of Cylinder 1.
   II The volume of the cone is equal to the volume of Cylinder 2.
   III The cone has a greater volume than Cylinder 1.
   IV The cone has one third the volume of Cylinder 1.



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ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–4	Exercises 5	Exercises 6, 7
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL LA Paired Heads Together Have students work in pairs to complete Exercises 1–4. Each student is responsible to ask for help and support, if needed, from their partner or from the teacher if both students have the same question. Call on one of the students to share their response to each exercise. MP 1, 4, 5, 7

Analyze and Reflect

BL LA Pairs Consult Have students work with a partner to write the formula for the area of the curved surface of a cylinder. Have them justify their formula using drawings and illustrations. MP 1, 4, 5, 7

Create

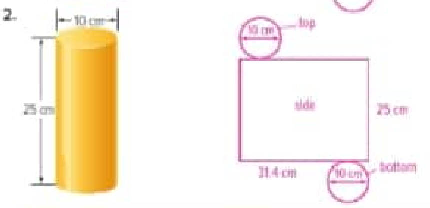
Inquiry Students should be able to answer "HOW can the surface area of a cylinder be determined?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Draw the net and label the parts of the cylinder and the measurements. Then complete the table to find the *total surface areas* for Exercises 1 and 2. Round to the nearest tenth.

1.

2.



	Area of top (πr^2)	Area of bottom (πr^2)	Curved Area	Total Surface Area
3.	78.5 cm^2	78.5 cm^2	628 cm^2	785 cm^2
4.	78.5 cm^2	78.5 cm^2	785 cm^2	942 cm^2

Analyze and Reflect

5. What is the total surface area of the container described at the beginning of the lesson? Round to the nearest tenth. 2,513.3 cm^2

Create

6. MP **Reason Inductively** Describe how to find the area of the curved surface of a cylinder. **Sample answer: Calculate the circumference of one circular base. This is the length of the rectangle of the curved surface. Multiply that value by the height of the cylinder.**

7. Inquiry How can the surface area of a cylinder be determined? **Sample answer: Calculate the area of one circular base then multiply this by 2 since there are two bases. Add the area of the curved side.**

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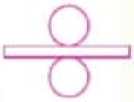
Lesson 4

Surface Area of Cylinders

Real-World Link

Bakery The Shiny Bright bakery is making a cake for Manal's wedding ceremony. The top tier of the cake will be in the shape of a cylinder with a height of 4 centimeters and a diameter of 14 centimeters.

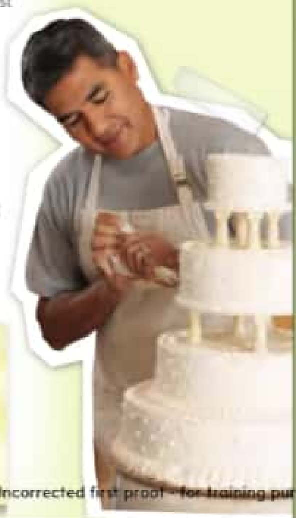
1. What are the shapes that make up the net of the cake? Sketch the net in the space provided.
   two circles and a rectangle



2. How is the length of the rectangle related to the circles that form the top and bottom of the cake?
   Sample answer: The length of the rectangle is the circumference of the circle. So, the length is

3. Find the area of each part of the cake. Round to the nearest whole number.
   Top: 154 cm² Bottom: 154 cm² Side: 176 cm²

4. Add the values from Exercise 3. What is the total surface area of the cake? 484 cm²



Essential Question

WHY are formulas important in math and science?

Vocabulary

lateral area
total surface area

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1 Persevere with Problems
2 Reason Abstractly
3 Construct an Argument
4 Model with Mathematics
5 Use Math Tools
6 Attend to Precision
7 Make Use of Structure
8 Use Repeated Reasoning

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Focus narrowing the scope

Objective Find the surface area of cylinders.

Coherence connecting within and across grades

Previous Students used nets to find the surface area of a cylinder.

Now Students solve problems involving the surface area of cylinders.

Next Students will solve problems involving the surface area of cones.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart below.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** Have students work in a small group to complete Exercises 1–4. Each student is responsible for leading the discussion for one of the exercises. Ask one student to share their group's responses with the class. MP 1, 3, 4, 5

Alternate Strategies

AL Have students create the net in Exercise 1 using a large piece of paper and have them label the area of each section. MP 1, 4, 5

BL Have students generate the formula for the surface area of a cylinder by studying the area of each section of its net. MP 1, 4, 7

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the surface area of a cylinder.

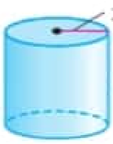
AL
- *What is the shape of the curved surface of a cylinder?* **rectangle**
- *What is the length of this rectangle?* **the circumference of the base**
- *What is the width of the rectangle?* **the height of the cylinder**

OL
- *In the formula $S.A. = 2\pi rh + 2\pi r^2$, what does $2\pi rh$ represent?* **the lateral area of the cylinder**
- *How can you find the lateral area of a cylinder?* **Determine the area of the curved surface by multiplying the circumference of the circular base by the height of the cylinder.**
- *How can you find the total surface area of a cylinder?* **Add the lateral surface area plus the area of the two circular bases, $2\pi r^2$**

BL
- *Why are the units for surface area square units and not cubic units?* **Sample answer: Area is a two-dimensional measurement. Volume is a three-dimensional measurement.**

Need Another Example?

Find the surface area of the cylinder. Round to the nearest tenth. **169.6 cm²**



Key Concept

Surface Area of a Cylinder

Work Zone

Lateral Area

Words The lateral area $L.A.$ of a cylinder with height h and radius r is the circumference of the base times the height.

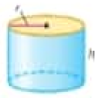
Symbols $L.A. = 2\pi rh$

Total Surface Area

Words The surface area $S.A.$ of a cylinder with height h and radius r is the lateral area plus the area of the two circular bases.

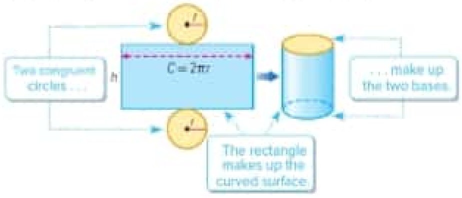
Symbols $S.A. = L.A. + 2\pi r^2$ or $S.A. = 2\pi rh + 2\pi r^2$

Model



area of base = πr^2

You can find the surface area of a cylinder using a net.



In the diagram above, the length of the rectangle is the same as the circumference of the circle, $2\pi r$. Also, the width of the rectangle is the same as the height of the cylinder.

The **lateral area** of a three-dimensional figure is the surface area of the figure, excluding the area of the base(s). So, the lateral area of a cylinder is the area of curved surface.

The **total surface area** of a three-dimensional figure is the sum of the areas of all its surfaces.

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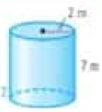
Example

1. **Find the surface area of the cylinder. Round to the nearest tenth.**



$S.A. = 2\pi rh + 2\pi r^2$ — Surface area of a cylinder

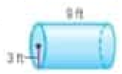
$S.A. = 2\pi(2)(7) + 2\pi(2)^2$ — Replace r with 2 and h with 7.

$S.A. \approx 113.1$ — Simplify.

The surface area is about 113.1 square meters.

Got it? **Do these problems to find out.**

Find the surface area of each cylinder. Round to the nearest tenth.

a. 

b. 

a. 226.2 m²

b. 747.7 cm²

Example

2. **A circular fence that is 2 meters high is to be built around the outside of a carousel. The distance from the center of the carousel to the edge of the fence will be 12 meters. What is the area of the fencing material that is needed to make the fence around the carousel?**

You need to find the lateral area. The radius of the circular fence is 12 meters. The height is 2 meters.

$L.A. = 2\pi rh$ — Lateral area of a cylinder

$L.A. = 2\pi(12)(2)$ — Replace r with 12 and h with 2.

$L.A. \approx 151$ — Simplify.

So, about 151 square meters of material is needed to make the fence.

Got it? **Do these problems to find out.**

c. Find the area of the label of a can of tuna with a radius of 5.1 centimeters and a height of 2.9 centimeters. Round to the nearest tenth.

d. Find the total surface area of a cylindrical candle with a diameter of 4 centimeters and a height of 8 centimeters. Round to the nearest tenth.

c. 92.9 cm²

d. 100.5 cm²

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Example

2. **Find the surface area of a cylinder.**

AL
- *What is the radius?* 12 m
- *What is the height of the fence?* 2 m

OL
- *How can you find the amount of material needed?* If the fence were a cylinder, find the area of the curved surface and not the bases. This is the lateral area.
- *What is the lateral area?* 151 m²

BL
- *If each square meter of fencing costs AED 2.35, what will be the total cost?* AED 1,033.53
- *Why do you not need to find the total surface area in this example?* The area of the fencing material will only include the lateral side, not the top or the bottom.

Need Another Example?

The Humaid family installed a cylindrical pool in their backyard. The pool is 3 meters deep. The distance from the center of the pool to the edge of the pool is 8 meters. How much material was needed to create the pool? Do not include the top surface. Round to the nearest tenth. 351.7 m²

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Team-Pair-Solo Before completing the Exercises, have students add the remaining formulas for surface area to their booklets they created in a previous lesson. Then have students work in four-person teams to complete Exercises 1 and 2. Then have them work in pairs to complete Exercises 3–5. Then have them work on their own to complete Exercises 6–8 and compare solutions with a partner. Have them discuss Exercise 9 in their original teams and write the final agreed-upon answer. MP 1, 4, 5, 7

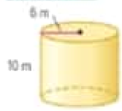
BL LA Find the Fib Have students work in pairs to write two facts and one fib about the surface area of a cylinder. For example, one fact could be that the surface area is the sum of the lateral area and the area of the two circular bases. One fib could be that the lateral area is the diameter of the circular base multiplied by the cylinder's height. Have students trade facts and fibs with another set of students. Each pair identifies the facts and fib and shares their responses with the original pair. MP 1, 3, 4, 5, 7

Guided Practice

Check

Find the total surface area of each cylinder. Round to the nearest tenth. (Example 1)

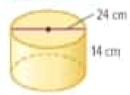
1. 603.2 m^2



2. 466.5 cm^2

3. 636.2 cm^2

4. 1960.4 cm^2

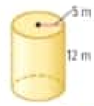


5. Find the total surface area of a water tank with a height of 10 meters and a diameter of 10 meters. Round to the nearest tenth. (Example 1) 471.2 m^2

Find the lateral area of each cylinder. Round to the nearest tenth. (Example 2)

6. 132.4 cm^2

7. 377.0 ft^2

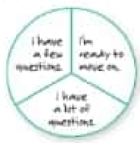


8. Find the area of the label of a cylindrical potato chip container with a radius of 7.5 cm and a height of 22 cm. Round to the nearest tenth. (Example 2)
1,036.2 cm^2

9. **Building on the Essential Question** How is a calculation affected if you round π to 3.14 or use the π key on your calculator? Explain.
Sample answer: Calculating with more decimal places produces an answer closer to the exact value.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

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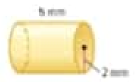
Name ________________ My Homework

Independent Practice

Find the total surface area of each cylinder. Round to the nearest tenth. (Example 1)

1. **88.0 mm²**



2. **1,215.8 m²**

3. **272.0 mm²**

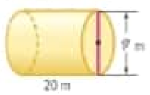
4. **1,120.0 cm²**

5. A cylindrical candle has a diameter of 4 centimeters and a height of 7 centimeters. To the nearest tenth, what is the total surface area of the candle? (Example 1) **113.1 cm²**

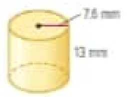
6. Find the total surface area of an unsharpened cylindrical pencil that has a radius of 0.5 centimeter and a height of 19 centimeters. Round to the nearest tenth. (Example 1) **61.3 cm²**

Find the lateral area of each cylinder. Round to the nearest tenth. (Example 2)

7. **1,068.1 m²**



8. **620.8 mm²**



9. Find the lateral area of a cylindrical copper pipe that has a diameter of 6.4 centimeters and a height of 12 centimeters. Round to the nearest tenth. (Example 2) **241.3 cm²**

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	1–9, 14–17	10, 18–24	11–13
Level 3			•
Level 2		•	
Level 1	•		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 12, 13, 23, 24
OL	On Level	1–9 odd, 10, 12, 13, 23, 24
BL	Beyond Level	10–13, 23, 24

Watch Out!

Common Error For certain exercises, students may substitute the given diameter measure for the radius when calculating surface areas. Remind students to use the radius of a circle. Point out that the radius is always half the diameter, so they can calculate the radius when given the diameter.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11, 22
3 Construct viable arguments and critique the reasoning of others.	12, 13
4 Model with mathematics.	10
5 Use appropriate tools strategically.	18–20

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Tell students that a cylinder has a diameter of 2 meters and a height of 1 meter. Have students find the cylinder's surface area to the nearest tenth. Have them write it on a piece of paper and hand it to you as they exit the class. 12.6 m^2

10. **Model with Mathematics** Refer to the graphic novel frame below.

a. What is the least amount of paper that will be needed to wrap one candle with no overlap? 263.76 cm^2

b. How many square centimeters of wrapping paper will be needed to wrap all 70 candles? About $18{,}463 \text{ cm}^2$ or 1.85 m^2

H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** If the height of a cylinder is doubled, will its surface area also double? Explain your reasoning.
No, the surface area of the side of the cylinder will double, but the area of the bases will not.

12. **Reason Inductively** Which has a greater surface area: a cylinder with radius 6 centimeters and height 3 centimeters or a cylinder with radius 3 centimeters and height 6 centimeters? Explain your reasoning.
A cylinder with radius 6 cm and height 3 cm has a greater surface area than a cylinder with height 6 cm and radius 3 cm; Sample answer: The first cylinder has a surface area of 339.3 cm^2 while the second cylinder has a surface area of 169.6 cm^2

13. **Reason Inductively** A baker is icing a cylindrical cake with radius r and height h. The baker will ice the top and sides of the cake. Write an equation giving the total area A that the baker will ice. Explain why your equation is not the same as the formula for the total surface area of a cylinder.
$A = 2\pi rh + \pi r^2$; Sample answer: The baker will not ice the bottom of the cake, so you only need to include the area of one of the bases in the equation.

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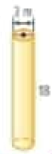
Name____________________ My Homework____________________

Extra Practice

Copy and Solve For Exercises 14–27, show your work and answers on a separate piece of paper.

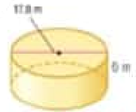
Find the lateral area and the total surface area of each cylinder. Round to the nearest tenth.

14. 

169.6 m^2; 183.8 m^2

15. 

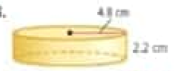
1,105.8 cm^2; 1,508.0 cm^2

16. 

335.5 m^2; 833.2 m^2

17. A lamp shade is in the shape of a cylinder with a height of 18 centimeters and a radius of 6.75 centimeters. Fabric will cover the lateral area of the lamp shade. Find the area of the fabric needed. Round to the nearest tenth. 763.4 cm^2

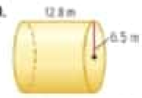
Use Math Tools Estimate the surface area of each cylinder.

18. 

Sample answer: $2 \cdot 3 \cdot 5^2 + 2 \cdot 3 \cdot 5 \cdot 2$ or 210 cm^2

19. 

Sample answer: $2 \cdot 3 \cdot 4^2 + 2 \cdot 3 \cdot 4 \cdot 4$ or 192 m^2

20. 

Sample answer: $2 \cdot 3 \cdot 7^2 + 2 \cdot 3 \cdot 7 \cdot 13$ or 840 m^2

21. The mail tube shown is made of cardboard and has plastic end caps. Approximately what percent of the surface area of the mail tube is cardboard? about 85.7%

22. **Persevere with Problems** A hot cocoa canister is a cylinder with a height of 24.5 centimeters and a diameter of 13 centimeters.

a. What is the lateral area of the hot cocoa canister to the nearest tenth? 1,000.6 cm^2

b. How does the lateral area change if the height is divided by 2?

It is divided by 2.

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Analyze and Reflect

AL EL **Roundrobin** Have students work in pairs to complete the table. Have each student be responsible for completing one column of the table for each exercise. As they complete their column, they should talk through their process and explain how they either found the measure of the central angle or the surface area. Have students trade roles for each exercise. MP 1, 2, 3, 6, 7

BL LA **Pairs Consult** Have students work in pairs to complete the table. Then have them use the proportion from Activity 1 to write an equation that gives the degree measure x of the central angle for the larger circle in terms of the circumference of each circle. Have them define variables for the circumference of each circle. For example, they may write $x = \frac{S \times 360}{L}$, where S represents the circumference of the smaller circle and L represents the circumference of the larger circle. MP 1, 2, 3, 6, 7

Create

BL LA **Think-Write-Pair** Have students review their previous activities to help them answer Exercises 7–9 independently. After they have completed their responses, have students work in pairs to discuss them. As a class, have them share any difference and make any necessary edits to their responses. MP 1, 2, 3, 8

Inquiry Students should be able to answer "HOW can the surface area of a cone be found?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner. Use the formula from Activity 2 to find the total surface area of each of the following cones given the radius of the base and the slant height. Round the measure of the central angle to the nearest whole number. Round the surface area to the nearest tenth.

	radius of base (r)	slant height (ℓ)	measure of central angle (°)	surface area ($\pi r\ell + \pi r^2$)
3.	2 m	5 m	144°	44.0 m²
4.	5 cm	15 cm	120°	314.2 cm²
5.	3 cm	20 cm	54°	216.8 cm²

6. Refer to Activity 1. What is the lateral area of the party hat that Hourlyya is covering with tissue paper? Round to the nearest tenth.

125.7 cm²

Create

7. MP **Make a Conjecture** Suppose the radius of the base of a cone is increased while the slant height stays the same. Make a conjecture about how the lateral surface area is affected.

Sample answer: The lateral surface area will increase by the same factor.

8. MP **Make a Conjecture** Suppose a cone's slant height is decreased. Make a conjecture about which is affected more: the base or the lateral area. Justify your response. the lateral area; Sample answer: In the formula, the slant height is only used in finding the lateral area. If the slant height were decreased, the lateral area would decrease while the base remains the same.

9. Inquiry HOW can the surface area of a cone be found?

Sample answer: The surface area of a cone can be found by multiplying pi times the radius times the slant height and adding the area of the base.

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ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

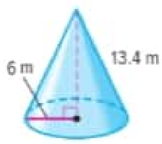
1. Find the lateral area of a cone.

AL
- *What is the lateral area of a cone?* the area of the curved surface
- *In the formula L.A. = $\pi r\ell$, what does r represent?* the radius of the base of the cone
- *In the formula L.A. = $\pi r\ell$, what does ℓ represent?* the slant height

OL
- *What formula is used to find the lateral area of a cone?* $L.A. = \pi r\ell$
- *What is the radius of the base?* 5 mm
- *What is the slant height?* 13 mm

BL
- *What is the exact lateral area of the cone in terms of π?* 65π

Need Another Example?
Find the lateral area of the cone. Round to the nearest tenth. 252.6 m²



Key Concept: Lateral Area of a Cone

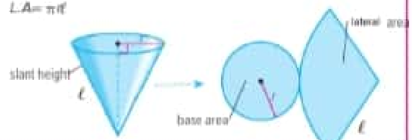
Work Zone

Lateral Area of a Cone
The lateral area of a cone is one-half the circumference of the base times the slant height.
$L.A. = \frac{1}{2}(2\pi r)\ell$
$L.A. = \pi r\ell$

Words The lateral area L.A. of a cone is π times the radius times the slant height ℓ.

Symbols $L.A. = \pi r\ell$

Model



Example

1. Find the lateral area of the cone. Round to the nearest tenth.

$L.A. = \pi r\ell$ Lateral area of a cone

$L.A. = \pi \cdot 5 \cdot 13$ Replace r with 5 and ℓ with 13.

$L.A. \approx 204.2$ Simplify.

The lateral area of the cone is about 204.2 square millimeters.

Got it? Do these problems to find out.

a. Find the lateral area of a cone with a radius of 4 centimeters and a slant height of 9.5 centimeters. Round to the nearest tenth.

b. Find the lateral area of a cone with a diameter of 16 centimeters and a slant height of 10 centimeters. Round to the nearest tenth.

a. 119.4 cm²

b. 251.3 cm²

Key Concept: Surface Area of a Cone

Words The surface area S.A. of a cone with slant height ℓ and radius r is the lateral area plus the area of the base.

Symbols $S.A. = L.A. + \pi r^2$ or $S.A. = \pi r\ell + \pi r^2$

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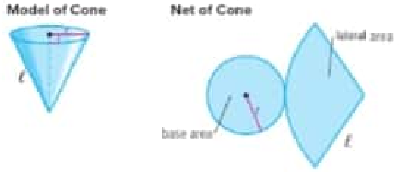
You can find the surface area of a cone using a net. The surface area of a cone is the sum of its lateral area and the area of its base.

Model of Cone **Net of Cone**



Example

2. **Find the surface area of the cone. Round to the nearest tenth.**

$S.A. = \pi r\ell + \pi r^2$ — Surface area of a cone

$S.A. = \pi \cdot 6 \cdot 6.2 + \pi \cdot 6^2$ — Replace r with 6 and ℓ with 6.2.

$S.A. \approx 230.0$ — Simplify.

The surface area of the cone is about 230.0 square centimeters.

Got it? **Do this problem to find out.**

c. Find the surface area of the cone. Round to the nearest tenth.

c. 346.4 mm²

Example

3. **A conical tent has a radius of 5 meters and a slant height of 12 meters. Find the lateral area of the tent. Round to the nearest tenth.**

$L.A. = \pi r\ell$ — Lateral area of a cone

$L.A. = \pi \cdot 5 \cdot 12$ — Replace r with 5 and ℓ with 12.

$L.A. \approx 188.5$ — Simplify.

The lateral area of the tent is about 188.5 square meters.

Got it? **Do this problem to find out.**

d. Rasheed bought candles that were in the shape of a cone. Each candle has a diameter of 8 centimeters and a slant height of 11 centimeters. Find the lateral area of one candle. Round to the

d. 138.2 cm²

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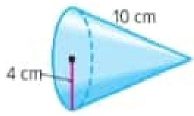
Examples

2. **Find the surface area of a cone.**

- AL • *How is finding the total surface area different from finding the lateral surface area of a cone?* The total surface area includes the area of the base.
  - *What is the shape of the base?* circle
  - *What formula is used to find the area of a circle?* $A = \pi r^2$
- OL • *What is the length of the radius?* 6 cm.
  - *What is the slant height?* 6.2 cm.
- BL • *What is the exact total surface area of the cone in terms of π?* 73.2π

Need Another Example?
Find the surface area of the cone. Round to the nearest tenth. 175.9 cm²



3. **Find the lateral area of a cone.**

- AL • *What do you need to find?* the lateral area of the tepee
  - *What formula will you use?* $L.A. = \pi r\ell$
- OL • *What is the slant height of the tent?* 12 m
  - *What is the radius of the base of the tent?* 5 m
- BL • *What is the exact lateral area of the tent in terms of π?* 60π

Need Another Example?
Ali is making conical hats for the school play. Each hat will have a slant height of 18 centimeters and a radius of 8 centimeters. How much fabric will be needed to cover the lateral surface of each hat? Round to the nearest tenth.
452.2 cm²

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

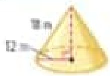
AL **LA** **Pairs Check** Have students work in pairs to complete Exercises 1–7. One partner decides which formula to use and the other partner solves the problem. Students switch roles for each exercise. After every two problems, pairs check their solutions with another pair and discuss and resolve any differences. MP 1, 2, 4, 5, 7

BL **LA** **Trade-a-Problem** Have students create their own problem, similar to Exercise 7. Challenge students to include finding the slant height as part of the solution. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 2, 4, 5, 7

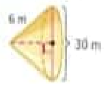
Guided Practice

Find the lateral area of each cone. Round to the nearest tenth. (Example 1)

1. 678.6 m²



2. 282.7 m²



3. 301.6 cm²

4. 230.9 m²

Find the surface area of each cone. Round to the nearest tenth. (Example 2)

5. 1,276.3 cm²

6. 122.5 m²

7. A local ice cream shop sells waffle cones dipped in chocolate. The waffle cone has a diameter 6.5 cm and a slant height of 15 cm. Find the lateral area of the waffle cone. Round to the nearest tenth. (Example 3)

153.1 cm²

8. **Building on the Essential Question** How does the volume of a three-dimensional figure differ from its surface area?

Volume is the amount a container holds. Surface area is the sum of the areas of the surfaces of the figure.

Rate Yourself!

☐ I understand the surface area of cones.

Great! You're ready to move on!

☐ I still have some questions about the surface area of cones.

FOLDABLES Time to update your Foldable!

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Name ______________ My Homework

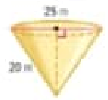
Independent Practice

Find the lateral area of each cone. Round to the nearest tenth. (Example 1)

1. 269.2 cm²

2. 1,979.2 cm²

3. 785.4 m²



Find the surface area of each cone. Round to the nearest tenth. (Example 2)

4. 2,082.9 cm²

5. 279.5 cm²

6. 505.8 m²

7. A snow cone has a diameter of 5 centimeters and a slant height of 12.7 centimeters. What is the lateral area of the snow cone? Round to the nearest tenth. (Example 3)
99.7 cm²

8. An active conical volcano has a radius of about 2.5 kilometers and slant height of about 9.6 kilometers. What is the lateral area of the volcano? Round to the nearest tenth. (Example 3) 75.4 km²

9. The lateral area of a cone with a diameter of 15 millimeters is about 333.5 square millimeters.
   a. Find the surface area of the cone. Round to the nearest tenth.
   510.2 mm²
   b. What is the slant height of the cone? Round to the nearest tenth.
   14.2 mm

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	1–8, 14–25	9, 10, 26–28	11–13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 13, 27, 28
OL	On Level	1–7 odd, 9–11, 13, 27, 28
BL	Beyond Level	9–13, 27, 28

Watch Out!

Common Error In certain exercises, students may confuse the slant height of the cone and the height of the cone. Have students sketch the cone and draw the slant height from the vertex of the cone to the base in a different color than the height of the cone.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	12
2 Reason abstractly and quantitatively.	26
3 Construct viable arguments and critique the reasoning of others.	11, 13
7 Look for and make use of structure.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

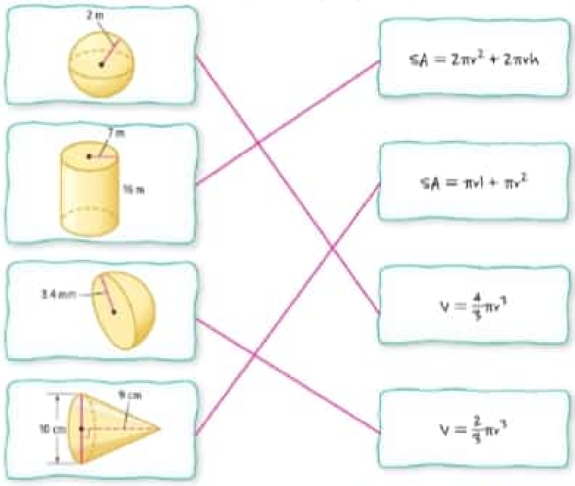
TICKET Out the Door

Have students write the formula for the lateral surface area of a cone and explain what each variable represents.
$L.A. = \pi r \ell$; Sample answer: r represents the radius and ℓ represents the slant height

Watch Out!

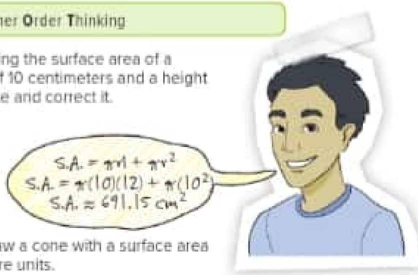
Common Error In Exercise 11, Salem used the diameter of the cone to find the surface area instead of the radius. Remind students to always check whether the diameter or radius is given. If the diameter is given, students must divide by 2 to find the radius.

10. **Identify Structure** Match the figure with its correct volume or surface area formula.



H.O.T. Problems Higher Order Thinking

11. **Find The Error** Enrique is finding the surface area of a cone. The cone has a diameter of 10 centimeters and a height of 12 centimeters. Find his mistake and correct it.



Enrique did not use the right radius. He did not divide the diameter by 2 to get the radius; 267.04 cm^2

12. **Persevere with Problems** Draw a cone with a surface area that is between 50 and 100 square units.

See students' work.

13. **Justify Conclusions** Which has a greater surface area: a square pyramid with a base of x units and a slant height of ℓ units or a cone with a diameter of x units and a slant height of ℓ units? Explain your reasoning.

Square pyramid; sample answer: The surface area of the pyramid is $x^2 + 2x\ell$. If you use $\pi \approx 3.14$, the surface area of the cone is $0.785x^2 + 1.57x\ell$. For all positive values of x and ℓ, the surface area of the pyramid is greater than the surface area of the cone.

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Name ______________________ My Homework ______________________

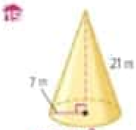
Extra Practice

Copy and Solve **For Exercises 14–35, show your work and answers on a separate piece of paper.**

Find the lateral area of each cone. Round to the nearest tenth.

14. 

424.1 cm²

15. 

461.8 m²

16. 

360.7 m²

17. 

113.1 cm²

18. 

73.5 mm²

19. 

62.8 m²

20. Find the lateral area of a cone with a radius of 3.5 millimeters and a slant height of 8 millimeters. Round to the nearest te 88.0 mm²

21. Find the lateral area of a cone with a radius of 9 centimeters and a slant height of 16 centimeters. Round to the nearest te 452.4 cm²

Find the surface area of each cone. Round to the nearest tenth.

22. 

241.4 cm²

23. 

188.5 m²

24. Find the surface area of a cone with a diameter of 20 millimeters and a slant height of 42 millimeters. Round to the nearest te 1,633.6 mm²

25. Find the surface area of a cone with a radius of 5.1 meters and a slant height of 17 meters. Round to the nearest te 354.1 m²

26. **Reason Abstractly** A conical hat has a radius of 7 centimeters and a height of 14 centimeters. Find the slant height of the hat. Then find the lateral area. Round to the nearest ten 15.7 cm.; 345.3 cm²

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Power Up! Test Practice

Exercises 27 and 28 prepare students for more rigorous thinking.

27. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

28. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

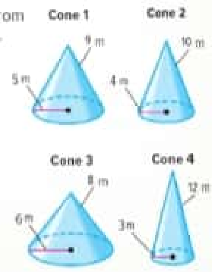
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly order each figure and find the corresponding lateral area.
1 point	Students correctly order each area but have errors in the lateral area of one or two figures OR students correctly order three figures and find the corresponding lateral area.

Power Up! Test Practice

27. A cone h as the radius and height shown. Which o f the following statements are true? Select all that apply.

- ■ The slant height of the cone is 13 cm.
- ■ The lateral area of the cone is about 204 square centimeters
- □ The total surface area of the cone is about 236 square centimeters.

28. Four cones have the dimensions shown. Sort the cones from least to greatest lateral areas. Round to the nearest tenth.



	Cone	Lateral Area (m^2)
Least	4	116.6
	2	135.3
	1	161.7
Greatest	3	180.5

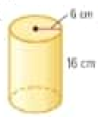
Spiral Review

Find the surface area of each cylinder. Round to the nearest tenth.

29. 150.8 m^2

30. 351.9 m^2

31. 829.4 cm^2



32. diameter: 10 meters
height: 24 meters
911.1 m^2

33. radius: 12 meters
height: 9 meters
1,583.4 m^2

34. Find the volume of a cylinder with a radius of 2 centimeters and a height of 25 centimeters. Round to the nearest te314.2 cm^3

35. Find the volume of a cone with a diameter of 16 meters and a height of 26 meters. Round the nearest ten1,742.5 m^3

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Engage Explore **Explain** Elaborate Evaluate

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the surface area of similar solids.

AL
- *What is the scale factor?* **3**
- *What is the surface area of the original prism?* **78 cm²**

OL
- *How can you find the surface area of the new prism?* **Multiply the surface area of the original prism by the square of the scale factor.**
- *Why do we multiply by the square of the scale factor and not just the scale factor?* **Sample answer: Surface area is two-dimensional. If I multiply by the scale factor, I am only changing one dimension. In order to change both dimensions, I have to multiply by the square of the scale factor.**

BL
- *What would be the surface area of a prism with dimensions that are 5 times the dimensions of the original prism?* **1,950 cm²**
- *What would be the surface area of a prism with dimensions that are half the dimensions of the original prisms?* **19.5 cm²**

Need Another Example?
The surface area of a rectangular prism is 90 square centimeters. What is the surface area of a similar prism with dimensions that are multiplied by a scale factor of 5?
2,250 cm²

Key Concept

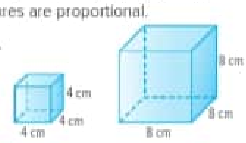
Surface Area of Similar Solids

If Solid X is similar to Solid Y by a scale factor, then the surface area of X is equal to the surface area of Y times the square of the scale factor.

Work Zone

Cubes are **similar solids** because they have the same shape and their corresponding linear measures are proportional.

The cubes at the right are similar. The ratio of their corresponding edge lengths is $\frac{8}{4}$ or 2. The scale factor is 2. How are their surface areas related?



S.A. of Small Cube
$S.A. = 6(4 \cdot 4)$
There are 6 faces.

S.A. of Large Cube
$S.A. = 6(2 \cdot 4)(2 \cdot 4)$
$= 2 \cdot 2(6)(4 \cdot 4)$
$= 2^2(6)(4 \cdot 4)$

To find the surface area of the large cube, multiply the surface area of the small cube by the *square* of the scale factor, or 2^2. This relationship is true for any similar solids.

Example

1. The surface area of a rectangular prism is 78 square centimeters. What is the surface area of a similar prism with dimensions that are 3 times as great as the dimensions of the original prism?

$S.A. = 78 \times 3^2$ Multiply by the square of the scale factor.
$S.A. = 78 \times 9$ Square 3.
$S.A. = 702$ cm² Simplify.

Got it? Do these problems to find out.

a. The surface area of a triangular prism is 34 square centimeters. What is the surface area of a similar prism with dimensions that are 3 times as great as the original prism?

b. A giant box has a surface area of 352 square meters. If the dimensions of a similar box are smaller than the giant box by a scale factor of $\frac{1}{48}$, what is its surface area?

a. 306 cm²
b. $\frac{11}{72}$ m² or 0.15 m²

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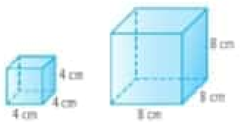
Volume of Similar Solids

Key Concept

If Solid X is similar to Solid Y by a scale factor, then the volume of X is equal to the volume of Y times the cube of the scale factor.

Refer to the cubes below.



Volume of Small Cube

$V = 4 \cdot 4 \cdot 4$

Volume of Large Cube

$V = (2 \cdot 4)(2 \cdot 4)(2 \cdot 4)$

$= 2 \cdot 2 \cdot 2(4 \cdot 4 \cdot 4)$

$= 2^3(4 \cdot 4 \cdot 4)$

The volumes of similar solids are related by the *cube* of the scale factor.

Example

2. A triangular prism has a volume of 432 cubic meters. If the dimensions of the prism are reduced to one third of the original dimensions, what is the volume of the new prism?

$V = 432 \times \left(\frac{1}{3}\right)^3$ Multiply by the cube of the scale factor.

$V = 432 \times \frac{1}{27}$ Cube $\frac{1}{3}$.

$V = 16\ m^3$ Simplify.

The volume of the new prism is 16 cubic meters.

Got it? Do these problems to find out.

c. A square pyramid has a volume of 512 cubic centimeters. What is the volume of a square pyramid with dimensions one-fourth of the original?

d. A cylinder has a volume of 432 cubic meters. What is the volume of a cylinder with dimensions one-third of the original?

c. $8\ cm^3$

d. $16\ m^3$

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Example

2. Find the volume of similar solids.

AL
- *What is the scale factor?* $\frac{1}{3}$
- *What is the volume of the original prism?* $432\ m^3$

OL
- *How can you find the volume of the new prism?* Multiply the volume of the original prism by the scale factor cubed.
- *Why do we multiply by the cube of the scale factor and not just the scale factor?* Sample answer: Volume is the amount that can fill a three-dimensional space. If I multiply by the scale factor, I am only changing one dimension. In order to change three dimensions, I have to multiply by the cube of the scale factor.

BL
- *Multiplying by the cube of $\frac{1}{3}$ is the same as dividing by which number? Explain.* Sample answer: Multiplying by the cube of $\frac{1}{3}$ is the same as dividing by 27 because the cube of $\frac{1}{3}$ is $\frac{1}{27}$ and multiplying by $\frac{1}{27}$ is the same as dividing by 27.

Need Another Example?

A triangular prism has a volume of 96 cubic meters. If the dimensions of the prism are reduced to one half its original dimensions, what is the volume of the new prism? $12\ m^3$

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Example

3. **Find the volume and surface area of similar solids.**

- AL • *What is the radius of the toy truck wheel?* **1.5 cm.** *the height?* **1 cm.**
- • *What is the scale factor?* **40**
- OL • *What is the volume of the toy truck wheel?* **7.065 cm³** *surface area?* **23.55 cm²**
- BL • *What will happen to the volume and surface area if the scale factor is 40?* **The volume is multiplied by a factor of 64,000 units and the surface area is multiplied by a factor of 1,600 units.**

Need Another Example?

A standard can of soup has the dimensions shown below. The radius and height of a large can of soup are 2 times the radius and height of a standard-sized can. Find the surface area and volume of the large can. Use 3.14 for π. Round to the nearest tenth. **1,081.7 cm²; 2,653.3 cm³**

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

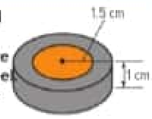
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Have students work in pairs. Give students several minutes to think through their response to Exercises 1–4. Have them share their response with their partner and discuss any differences. Finally, have the pairs share their responses with another pair of students. MP 1, 2, 7

BL LA **Trade-a-Problem** Have students work with a partner to write a real-world problem involving two similar solids. Have them trade problems with another pair of students to solve each other's problem. MP 1, 2, 7

Example

3. **The measurements for a toy truck wheel are shown at the right. The large truck wheel at the left has dimensions that are 40 times the dimensions of the toy wheel. Find the volume and surface area of the large wheel. Use 3.14 for π.**



Find the volume and surface area of the standard puck first.

$V = \pi r^2 h$ | $S.A. = 2(\pi r^2) + 2\pi r h$

$\approx (3.14)(1.5^2)(1)$ | $\approx 2(3.14)(1.5^2) + 2(3.14)(1.5)(1)$

$\approx 7.065 \text{ cm}^3$ | $\approx 14.13 + 9.42$

$\approx 23.55 \text{ cm}^2$

Find the volume and surface area of the giant puck using the computations for the standard puck and the scale factor.

$V = V(40^3)$ | $S.A. = S.A.(40^2)$

$= (7.065)(40^3)$ | $= (23.55)(40^2)$

$= 452{,}160 \text{ cm}^3$ | $= 37{,}680 \text{ cm}^2$

The large wheel has a volume of about 452,160 cubic centimeters and a surface area of about 37,680 square centimeters.

STOP and Reflect

What happens to the surface area of a cylinder if its radius and its height are doubled?

The surface area is quadrupled.

Guided Practice

1. The surface area of a rectangular prism is 35 square centimeters. What is the surface area of a similar solid with dimensions that have been enlarged by a scale factor of 7? (Example 1) **1,715 cm²**

2. The volume of a cylinder is about 425 cubic centimeters. What is the volume, to the nearest tenth, of a similar solid with dimensions that are smaller by a scale factor of $\frac{1}{3}$? (Example 2) **15.7 cm³**

3. A box with a sliding lid in Josh's art studio measures 16 centimeters by 15 centimeters by 6 centimeters. A second box used just for paintbrushes has a similar shape and is smaller by a scale factor of $\frac{1}{2}$. Find the volume and surface area of the second box. (Example 3) **180 cm³; 213 cm²**

4. **Building on the Essential Question** How is the volume of a prism affected when its dimensions are tripled?
The volume is 27 times greater.

Rate Yourself!

How confident are you about changes in dimensions? Check the box that applies.

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Name ________________ My Homework ________________

Independent Practice

1. The surface area of a rectangular prism is 95 square centimeters. What is the surface area of a similar prism with dimensions that are 4 times as great as the original prism? (Example 1) **1,520 cm²**

2. The surface area of a pyramid is 57.8 square centimeters. What is the surface area of a similar pyramid with dimensions that are 2 times as great as the original prism? (Example 1) **231.2 cm²**

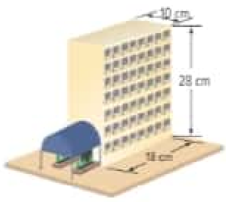
3. A cereal box has a surface area of 280 square centimeters. What is the surface area of a similar box that is larger by a scale factor of 1.4? (Example 1) **548.8 cm²**

4. A glass display box has a surface area of 378 square centimeters. How many square centimeters of glass are used to create a glass display box with dimensions that are one-half those of the original? (Example 1) **94.5 cm²**

5. A cone has a volume of 9,728 cubic millimeters. What is the volume of a similar cone with dimensions that are one-eighth the dimensions of the original? (Example 2) **19 mm³**

6. A triangular prism has a volume of 350 cubic meters. If the dimensions are tripled, what is the volume of the new prism? (Example 2) **9,450 m³**

7. The model of a new apartment building is shown. The architect plans for the building to be 360 times the dimensions of the model. What will be the volume and surface area of the new building, in cubic meters and square meters, when it is completed? (Example 3)
235,146 m³; 20,321 m²



ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises	1–8, 14–17	9, 18–25	10–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 24, 25
OL	On Level	1–11 odd, 12, 24, 25
BL	Beyond Level	9–12, 24, 25

Watch Out!

Common Error Students may make calculation errors when squaring and cubing scale factors that are fractional amounts. Remind students how to apply an exponent to a fraction: apply the exponent to the numerator and apply the exponent to the denominator.

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9, 10
3 Construct viable arguments and critique the reasoning of others.	11, 12, 19–23

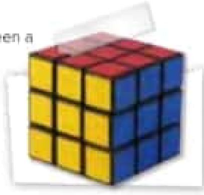
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how the scale factor between two similar solids is related to the ratio of the surface areas of the solids and to the ratio of the volumes of the solids. See students' work.

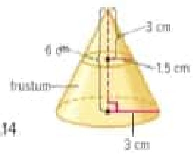
8. The world's largest cube puzzle is in Knoxville, Tennessee. It measures 180 centimeters on each side. The scale factor between a standard cube puzzle and the largest puzzle is $\frac{1}{24}$. Find the surface area and volume of the standard cube puzzle. (Example 3)
337.5 cm² or 421.875 cm³



9. **Persevere with Problems** Two spheres are similar in shape. The scale factor between the smaller sphere and the larger sphere is $\frac{3}{4}$. If the volume of the smaller sphere is 126.9 cubic meters, what is the volume of the larger sphere? 300.8 m³

H.O.T. Problems Higher Order Thinking

10. **Persevere with Problems** A *frustum* is the solid left after a cone is cut by a plane parallel to its base and the top cone is removed.



a. Is the smaller cone that is removed similar to the original cone? Justify your response. Yes, the ratios $\frac{3}{6}$ and $\frac{1.5}{3}$ are equal.

b. What is the volume of the smaller cone? the larger cone? Use 3.14 for π. 7.065 cm³; 56.52 cm³

c. What is the ratio of the volume of the smaller cone to the volume of the larger cone? 1:8

d. What is the volume of the frustum? 49.455 cm³

11. **Justify Conclusions** A cone has a volume of x cubic centimeters. If the dimensions of a second cone are one-sixth the original cone, what is the volume of the second cone? Explain your reasoning.
The volume of the first cone is x, so the first cone's volume multiplied by one-sixth cubed is the second cone's volume. The volume of the second cone is $\frac{1}{216}x$ cm³.

12. **Reason Inductively** Determine whether the following statement is *true* or *false*. Explain your reasoning.

All spheres are similar.

true; Sample answer: Spheres have only one measurement, the radius.

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