

## KEY TERMS AND DEFINITIONS

### 3.1 INTRODUCTION

This chapter is concerned with key terms and definitions in fluid flow. Since fluid flow is an important subject that finds wide application in engineering, the understanding of "fluid" flow jargon is therefore important to the practicing engineer. The handling and flow of either gases or liquids is much simpler, cheaper, and less troublesome than solids. Consequently, the engineer attempts to transport most quantities in the form of gases or liquids whenever possible. It is important to note that throughout this book, the word "fluid" will always be used to include both liquids and gases.

The mechanics of fluids are treated in most physics courses and form the basis of the subject of fluid flow and hydraulics. Key terms in these two topics that are of special interest to engineers are covered in this chapter. Fluid mechanics includes two topics: statics and dynamics. Fluid statics treats fluids at rest while fluid dynamics treats fluids in motion. The definition of key terms in this subject area is presented in Section 3.2.

### 3.1.1 Fluids

For the purpose of this text, a fluid may be defined as a substance that does not permanently resist distortion. An attempt to change the shape of a mass of fluid will result in layers of fluid sliding over one another until a new shape is attained. During the change in shape, shear stresses (forces parallel to a surface) will result,
the magnitude of which depends upon the viscosity (to be discussed shortly) of the fluid and the rate of sliding. However, when a final shape is reached, all shear stresses will have disappeared. Thus, a fluid at equilibrium is free from shear stresses. This definition applies for both liquids and gases.

### 3.2 DEFINITIONS

Standard key definitions, particularly as they apply to fluid flow, follow.

### 3.2.1 Temperature

Whether in a gaseous, liquid, or solid state, all molecules possess some degree of kinetic energy; that is, they are in constant motion-vibrating, rotating, or translating. The kinetic energies of individual molecules cannot be measured, but the combined effect of these energies in a very large number of molecules can. This measurable quantity is known as temperature; it is a macroscopic concept only and as such does not exist on the molecular level.

Temperature can be measured in many ways; the most common method makes use of the expansion of mercury (usually encased inside a glass capillary tube) with increasing temperature. (However, thermocouples or thermistors are more commonly employed in industry.) The two most commonly used temperature scales are the Celsius (or Centigrade) and Fahrenheit scales. The Celsius scale is based on the boiling and freezing points of water at 1 -atm pressure; to the former, a value of $100^{\circ} \mathrm{C}$ is assigned, and to the latter, a value of $0^{\circ} \mathrm{C}$. On the older Fahrenheit scale, these temperatures correspond to $212^{\circ} \mathrm{F}$ and $32^{\circ} \mathrm{F}$, respectively. Equations (3.1) and (3.2) show the conversion from one scale to the other:

$$
\begin{align*}
{ }^{\circ} \mathrm{F} & =1.8\left({ }^{\circ} \mathrm{C}\right)+32  \tag{3.1}\\
{ }^{\circ} \mathrm{C} & =\left({ }^{\circ} \mathrm{F}-32\right) / 1.8 \tag{3.2}
\end{align*}
$$

where ${ }^{\circ} \mathrm{F}=$ a temperature on the Fahrenheit scale and ${ }^{\circ} \mathrm{C}=$ a temperature on the Celsius scale.

Experiments with gases at low-to-moderate pressures (up to a few atmospheres) have shown that, if the pressure is kept constant, the volume of a gas and its temperature are linearly related (see Chapter 11-Charles' law) and that a decrease of $0.3663 \%$ or $(1 / 273)$ of the initial volume is experienced for every temperature drop of $1{ }^{\circ} \mathrm{C}$. These experiments were not extended to very low temperatures, but if the linear relationship were extrapolated, the volume of the gas would theoretically be zero at a temperature of approximately $-273^{\circ} \mathrm{C}$ or $-460^{\circ} \mathrm{F}$. This temperature has become known as absolute zero and is the basis for the definition of two absolute temperature scales. (An absolute scale is one that does not allow negative quantities.) These absolute temperature scales are the Kelvin (K) and Rankine ( ${ }^{\circ} \mathrm{R}$ ) scales; the former is defined by shifting the Celsius scale by $273^{\circ} \mathrm{C}$ so that 0 K is equal to $-273^{\circ} \mathrm{C}$. The Rankine scale is defined by shifting the Fahrenheit scale by $460^{\circ}$.

Equation (3.3) shows this relationship for both absolute temperatures:

$$
\begin{align*}
\mathrm{K} & ={ }^{\circ} \mathrm{C}+273 \\
{ }^{\circ} \mathrm{R} & ={ }^{\circ} \mathrm{F}+460 \tag{3.3}
\end{align*}
$$

### 3.2.2 Pressure

There are a number of different methods used to express a pressure term or measurement. Some of them are based on a force per unit area (e.g., pound-force per square inch, dyne, and so on) and others are based on fluid height (e.g., inches of water, millimeters of mercury, etc.). Pressure units based on fluid height are convenient when the pressure is indicated by a difference between two levels of a liquid. Standard barometric (or atmospheric) pressure is 1 atm and is equivalent to 14.7 psi, 33.91 ft of water, and 29.92 inches of mercury.

Gauge pressure is the pressure relative to the surrounding (or atmospheric) pressure and it is related to the absolute pressure by the following equation:

$$
\begin{equation*}
P=P_{a}+P_{g} \tag{3.4}
\end{equation*}
$$

where $P$ is the absolute pressure ( psia ), $P_{a}$ is the atmospheric pressure ( psi ) and $P_{g}$ is the gauge pressure. The absolute pressure scale is absolute in the same sense that the absolute temperature scale is absolute; i.e., a pressure of zero psia is the lowest possible pressure theoretically achievable-a perfect vacuum.

In stationary fluids subjected to a gravitational field, the hydrostatic pressure difference between two locations A and B is defined as

$$
\begin{equation*}
P_{\mathrm{A}}-P_{\mathrm{B}}=-\int_{z_{\mathrm{A}}}^{z_{\mathrm{B}}} \rho g \mathrm{~d} z \tag{3.5}
\end{equation*}
$$

where $z$ is a vertical upwards direction, $g$ is the gravitational acceleration, and $\rho$ is the fluid density. This equation will be revisited in Chapter 10.

Expressed in various units, the standard atmosphere is equal to 1.00 atmosphere (atm), 33.91 feet of water ( $\mathrm{ft} \mathrm{H}_{2} \mathrm{O}$ ), 14.7 pound-force per square inch absolute ( psia ), 2116 pound-force per square foot ( psfa ), 29.92 inches of mercury (in Hg ), 760.0 millimeters of mercury ( mm Hg ), and $1.013 \times 10^{5}$ Newtons per square meter $\left(\mathrm{N} / \mathrm{m}^{2}\right)$. The pressure term will be reviewed again in several later chapters.

Vapor pressure, usually denoted $p^{\prime}$, is an important property of liquids and, to a much lesser extent, of solids. If a liquid is allowed to evaporate in a confined space, the pressure in the vapor space increases as the amount of vapor increases. If there is sufficient liquid present, a point is eventually reached at which the pressure in the vapor space is exactly equal to the pressure exerted by the liquid at its own surface. At this point, a dynamic equilibrium exists in which vaporization and condensation take place at equal rates and the pressure in the vapor space remains
constant. The pressure exerted at equilibrium is called the vapor pressure of the liquid. The magnitude of this pressure for a given liquid depends on the temperature, but not on the amount of liquid present. Solids, like liquids, also exert a vapor pressure. Evaporation of solids (called sublimation) is noticeable only for those with appreciable vapor pressures.

### 3.2.3 Density

At a given temperature and pressure, a fluid possesses density, $\rho$, which is measured as mass per unit volume. The density of a fluid depends on both temperature and pressure; if a fluid is not affected by changes in pressure, it is said to be incompressible, and most liquids are incompressible. The density of a liquid can, however, change if there are extreme changes in temperature, and not appreciably affected by moderate changes in pressure. In the case of gases, the density may be affected appreciably by both temperature and pressure. Gases subjected to small changes in pressure and temperature vary so little in density that they can be considered incompressible and the change in density can be neglected without serious error. Density, specific gravity, and other similar properties have the same significance for fluids as for solids.

### 3.2.4 Viscosity

Viscosity, $\mu$, is an important fluid property that provides a measure of the resistance to flow. The viscosity is frequently referred to as the absolute or dynamic viscosity. The principal reason for the difference in the flow characteristics of water and of molasses is that molasses has a much higher viscosity than water. Note also that the viscosity of a liquid decreases with increasing temperature, while the viscosity of a gas increases with increasing temperature.

One set of units of viscosity in SI units is $\mathrm{g} /(\mathrm{cm} \cdot \mathrm{s})$, which is defined as a poise (P). Since this numerical unit is somewhat high for many engineering applications, viscosities are frequently reported in centipoises (cP) where one poise is 100 centipoises. In English or engineering units, the dimensions of viscosity are in lb/ $\mathrm{ft} \cdot \mathrm{s}$. To convert from poises to this unit, one may simply multiply by ( $30.48 /$ 453.6 ) or ( 0.0672 ); to convert from centipoises, multiply by $6.72 \times 10^{-4}$. To convert centipoises to $\mathrm{lb} / \mathrm{ft} \cdot \mathrm{hr}$, multiply by 2.42 .

Kinematic viscosity, $\nu$, is the absolute viscosity divided by the density ( $\mu / \rho$ ) and has the dimensions of (volume)/length • time. The corresponding unit to the poise is the stoke, having the SI dimensions of $\mathrm{cm}^{2} / \mathrm{s}$. The specific viscosity is the ratio of the viscosity to the viscosity of a standard fluid expressed in the same units and measured at the same temperature and pressure. Although all real fluids possess viscosity, an ideal fluid is a hypothetical fluid that has a viscosity of zero and possesses no resistance to shear.

The viscosity is a fluid property listed in many engineering books, including Perry's Handbook. ${ }^{(1)}$ Data are given as tables, charts, or nomographs. Figures B. 1 and B. 2 (see Appendix) are two nomographs that can be used to obtain the absolute
(or dynamic) viscosity of liquids and gases, respectively. ${ }^{(2,3)}$ In addition, the kinematic viscosities of some common liquids and gases at a temperature of $20^{\circ} \mathrm{C}$ are listed ${ }^{(2,3)}$ in Tables A. 2 and A.3, respectively (see Appendix).

Illustrative Example 3.1 To illustrate the use of nomograph, calculate the dynamic viscosity of a $98 \%$ sulfuric acid solution at $45^{\circ} \mathrm{C}$.

Solution From Fig. B. 1 in the Appendix, the coordinates of $98 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ are given as $\mathrm{X}=7.0$ and $\mathrm{Y}=24.8$ (number 97). Locate these coordinates on the grid and call it point A . From $45^{\circ} \mathrm{C}$, draw a straight line through point A and extend it to cut the viscosity axis. The intersection occurs at approximately 12 centipoise (cP). Therefore,

$$
\mu=12 \mathrm{cP}=0.12 \mathrm{P}=0.12 \mathrm{~g} / \mathrm{cm} \cdot \mathrm{~s}
$$

### 3.2.5 Surface Tension: Capillary Rise

A liquid forms an interface with another fluid. At the surface, the molecules are more densely packed than those within the fluid. This results in surface tension effects and interfacial phenomena. The surface tension coefficient, $\sigma$, is the force per unit length of the circumference of the interface, or the energy per unit area of the interface area. The surface tension for water is listed in Table A. 4 (see Appendix).

Surface tension causes a contact angle to appear when a liquid interface is in contact with a solid surface, as shown in Fig. 3.1. If the contact angle $\theta$ is $<90^{\circ}$, the liquid is termed wetting. If $\theta>90^{\circ}$, it is a nonwetting liquid. Surface tension causes a fluid interface to rise (or fall) in a capillary tube. The capillary rise is obtained by equating the vertical component of the surface tension force, $F_{\sigma}$, to the weight of the liquid of height $h, F_{g}$ (see Fig. 3.2). These two forces are shown


Figure 3.1 Surface tension figure.


Figure 3.2 Capillary rise in a circular tube.
in the following equations:

$$
\begin{align*}
& F_{\sigma}=2 \pi R \sigma \cos \theta  \tag{3.6}\\
& F_{g}=\rho g \pi R^{2} h \tag{3.7}
\end{align*}
$$

Equating the above two forces gives:

$$
\begin{align*}
2 \pi r \sigma \cos \theta & =\rho g \pi R^{2} h \\
h & =\frac{2 \sigma \cos \theta}{\rho g R} \tag{3.8}
\end{align*}
$$

where $\sigma$ is the surface tension $(\mathrm{N} / \mathrm{m}), \theta$ the contact angle, $\rho$ the liquid density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$, $g$ is the acceleration due to gravity ( $9.807 \mathrm{~m} / \mathrm{s}^{2}$ ), and $R$ is the tube radius (m).

For a droplet, the pressure is higher on the inside than on the outside. The pressure increase in the interior of the liquid droplet is balanced by the surface tension force. By applying a force balance on the interior of a spherical droplet, see Fig. 3.3, one can obtain the force due to the pressure increase, $F_{p}$, which equals the surface tension force on the ring, $F_{\sigma}$ (see Eqs. 3.9 and 3.10). This force balance neglects the weight of the liquid in the droplet

$$
\begin{align*}
& F_{p}=\pi r^{2} \Delta P  \tag{3.9}\\
& F_{\sigma}=2 \pi r \sigma \tag{3.10}
\end{align*}
$$



Figure 3.3 Surface tension in a spherical droplet.

Equating the two forces gives,

$$
\begin{equation*}
\pi r^{2} \Delta P=2 \pi r \sigma \tag{3.11}
\end{equation*}
$$

The pressure increase is therefore,

$$
\begin{equation*}
\Delta P=\frac{2 \sigma}{r} \tag{3.12}
\end{equation*}
$$

where $\Delta P$ is the pressure increase ( Pa or psi ) and $r$ is the droplet radius ( m or ft ).
Illustrative Example 3.2 A capillary tube is inserted into a liquid. Determine the rise, $h$, of the liquid interface inside the capillary tube. Data are provided below.

Liquid-gas system is water-air
Temperature is $30^{\circ} \mathrm{C}$ and pressure is 1 atm
Capillary tube diameter $=8 \mathrm{~mm}=0.008 \mathrm{~m}$
Water density $=1000 \mathrm{~kg} / \mathrm{m}^{3}$
Contact angle, $\theta=0^{\circ}$
Solution The height equation is first written

$$
\begin{equation*}
h=\frac{2 \sigma \cos \theta}{\rho g R} \tag{3.8}
\end{equation*}
$$

The surface tension of water (see Table A. 4 in the Appendix) at $30^{\circ} \mathrm{C}$ is

$$
\sigma=0.0712 \mathrm{~N} / \mathrm{m}=0.0712 \mathrm{~kg} / \mathrm{s}^{2}
$$

The height is therefore

$$
\begin{aligned}
h & =\frac{(2)(0.0712) \cos 0^{\circ}}{(1000)(9.807)(0.004)} \\
& =0.00363 \mathrm{~m}=3.63 \mathrm{~mm}
\end{aligned}
$$

Note that for most industrial applications involving pipes, the diameters are large enough that any capillary rise may be neglected.

Illustrative Example 3.3 At $30^{\circ} \mathrm{C}$, what diameter glass tube is necessary to keep the capillary height change of water less than one millimeter? Assume negligible angle of contact.

Solution For air-water-glass, assume the contact angle $\theta=0$, noting that $\cos \left(0^{\circ}\right)=1$. Obtain the properties of water from Table A. 2 in the Appendix.

$$
\begin{aligned}
\rho & =996 \mathrm{~kg} / \mathrm{m}^{3} \\
\sigma & =0.071 \mathrm{~N} / \mathrm{m} \text { (surface tension) }
\end{aligned}
$$

Use the capillary rise Equation (3.8) to calculate the tube radius

$$
\begin{aligned}
& h=\frac{2 \sigma \cos \theta}{\rho g R} \\
& R=\frac{2 \sigma \cos \theta}{\rho g h}=\frac{2(0.071)(1)}{(996)(9.807)(0.001)}=0.0145 \mathrm{~m}=14.5 \mathrm{~mm} \\
& D=29 \mathrm{~mm}
\end{aligned}
$$

If the tube diameter is greater than 29 mm , then the capillary rise will be less than 1 mm .

### 3.2.6 Newton's Law

The relationship between force mass, velocity, and acceleration may be expressed by Newton's second law with force equaling the time rate of change of momentum, $\dot{M}$.

$$
\begin{equation*}
F=\frac{1}{g_{c}} \frac{\mathrm{~d}(m v)}{\mathrm{d} t}=\frac{\mathrm{d} M}{\mathrm{~d} t}=\dot{M} \tag{3.13}
\end{equation*}
$$

If the mass is constant,

$$
\begin{equation*}
F=\frac{m a}{g_{c}} \tag{3.14}
\end{equation*}
$$

where $a=$ acceleration or $\mathrm{d} v / \mathrm{d} t$.

In the English engineering system of units, the pound-force $\left(\mathrm{lb}_{\mathrm{f}}\right)$ is defined as that force which accelerates 1 pound-mass (lb) $32.174 \mathrm{ft} / \mathrm{s}$. Newton's law must therefore include a dimensional conversion constant for consistency. This constant, $g_{c}$, is $32.174\left(\mathrm{lb} / \mathrm{lb}_{\mathrm{f}}\right)\left(\mathrm{ft} / \mathrm{s}^{2}\right)$. When employing SI units, the value of $g_{c}$ becomes unity and has no dimensions associated with it, i.e., $g_{c}=1.0$ (see previous chapter for more details). Thus, the $g_{c}$ term is normally retained in equations involving force where English units are employed. The SI unit of force is the Newton (N), which simply expresses force $F$ as the product of mass $m$ and acceleration $a$ (see Equation 3.14 once again). The Newton is defined as the force, when applied to a mass of 1 kg , produces an acceleration of $1 \mathrm{~m} / \mathrm{s}^{2}$; the term $g_{c}$ is not retained in this (and similar) equations when SI units are employed.

The term $g_{c}$ is carried in most of the force and force-related terms and equations presented in this and the following chapters. Although both sets of units are employed in the Illustrative Examples and Problems, the reader should note that despite statements to the contrary by academics and theorists, English units are almost exclusively employed by industry in the US.

As described earlier, pressure is a force per unit area. The conversion of force per unit area ( $S$ ) to a height of fluid follows from Newton's law, i.e.,

$$
\begin{equation*}
P=\frac{F}{S}=m \frac{g}{g_{c}} / S \tag{3.15}
\end{equation*}
$$

and

$$
\begin{equation*}
m=\rho S h \tag{3.16}
\end{equation*}
$$

Thus, a vertical column of a given fluid under the influence of gravity exerts a pressure at its base that is directly proportional to its height so that pressure may also be expressed as the equivalent height of a fluid column. The pressure to which a fluid height corresponds may be determined from the density of the fluid and the local acceleration of gravity.

Forces that act on a fluid can be classified as either body forces or surface forces. Body forces are distributed throughout the material, e.g., gravitational, centrifugal, and electromagnetic forces. Body forces therefore act on the bulk of the object from a distance and are proportional to its mass; the most common examples are the aforementioned gravitational and electromagnetic forces. Surface forces are forces that act on the surface of a material. Surface forces are exerted on the surface of the object by other objects in contact with it; they generally increase with increasing contact area. Stress is a force per unit area. If the force is parallel to the surface, the force per unit area is called shear stress. When the force is perpendicular (normal) to a surface, the force per unit area is called normal stress or pressure.

For a stationary (static, non-moving) fluid, the sum of all forces acting on the fluid ( $\sum F$ ) is zero. Newton's second law simplifies to

$$
\begin{equation*}
\sum F=0 \tag{3.17}
\end{equation*}
$$

When there are two opposing forces, for example, a gravity force and a pressure force, $P$, (acting on a surface) is then

$$
\begin{aligned}
& F_{\text {pres }}=F_{\mathrm{grav}} \\
& F_{\text {pres }}=P(S) \\
& F_{\text {grav }}=m\left(g / g_{c}\right)
\end{aligned}
$$

Equating the two forces gives the result described in Equation (3.15)

$$
\begin{equation*}
m\left(g / g_{c}\right)=P S \tag{3.18}
\end{equation*}
$$

Illustrative Example 3.4 Given a force $F=10 \mathrm{lb}_{\mathrm{f}}$, acting on a surface of area $S=$ $2 \mathrm{ft}^{2}$, at an angle $\theta=30^{\circ}$ to the normal of the surface. Determine the magnitude of the normal and parallel force components, the shear stress, and the pressure.

Solution When a force acts at an angle to a surface, the component of the force parallel to that surface is $F \cos \theta$. Noting that $\cos \left(30^{\circ}\right)=0.866$.

$$
\begin{aligned}
F_{\mathrm{para}} & =F \cos \theta=10 \cos \left(30^{\circ}\right) \\
& =8.66 \mathrm{lb}_{\mathrm{f}}
\end{aligned}
$$

The normal (perpendicular) component of the force is $F \sin \theta$, noting that $\sin \left(30^{\circ}\right)=$ 0.500 .

$$
\begin{aligned}
F_{\text {norn }} & =F \sin \theta=10 \sin \left(30^{\circ}\right) \\
& =5 \mathrm{lb}_{\mathrm{f}}
\end{aligned}
$$

The shear stress, $\tau$, is defined as

$$
\begin{aligned}
\tau & =\frac{F_{\mathrm{para}}}{S}=\frac{8.66}{2} \\
& =4.33 \mathrm{psf}
\end{aligned}
$$

Likewise, the pressure, $P$, is defined as

$$
\begin{aligned}
P & =\frac{F_{\text {norm }}}{S}=\frac{5}{2} \\
& =2.50 \mathrm{psf}
\end{aligned}
$$

### 3.2.7 Kinetic Energy

Consider a body of mass, $m$, that is acted upon by a force, $F$. If the mass is displaced a distance, $\mathrm{d} L$, during a differential interval of time, $\mathrm{d} t$, the energy expended is given by

$$
\begin{equation*}
\mathrm{d} E_{k}=m \frac{a}{g_{c}} \mathrm{~d} L \tag{3.19}
\end{equation*}
$$

Since the acceleration is given by $a=\mathrm{d} v / \mathrm{d} t$,

$$
\begin{equation*}
\mathrm{d} E_{k}=\frac{m}{g_{c}} \frac{\mathrm{~d} v}{\mathrm{~d} t} \mathrm{~d} L=\frac{m}{g_{c}} \frac{\mathrm{~d} L}{\mathrm{~d} t} \mathrm{~d} v \tag{3.20}
\end{equation*}
$$

Noting that $v=\mathrm{d} L / \mathrm{d} t$, the above expression becomes:

$$
\begin{equation*}
\mathrm{d} E_{k}=m \frac{v}{g_{c}} \mathrm{~d} v \tag{3.21}
\end{equation*}
$$

If this equation is integrated from $v_{1}$ to $v_{2}$, the change in energy is

$$
\begin{equation*}
\Delta E_{k}=\frac{m}{g_{c}} \int_{v_{1}}^{v_{2}} v \mathrm{~d} v=\frac{m}{g_{c}}\left(\frac{v_{2}^{2}}{2}-\frac{v_{1}^{2}}{2}\right) \tag{3.22}
\end{equation*}
$$

or

$$
\begin{equation*}
\Delta E_{k}=\left(\frac{m v_{2}^{2}}{2 g_{c}}-\frac{m v_{1}^{2}}{2 g_{c}}\right)=\Delta\left(\frac{m v^{2}}{2 g_{c}}\right) \tag{3.23}
\end{equation*}
$$

The term above is defined as the change in kinetic energy.
The reader should note that for flow through conduits, the above kinetic energy term can be retained as written if the velocity profile is uniform; that is, the local velocities at all points in the cross-section are the same. Ordinarily, there is a velocity gradient across the passage; this introduces an error, the magnitude of which depends on the nature of the velocity profile and the shape of the cross section. For the usual case where the velocity is approximately uniform (e.g., turbulent flow) (see Chapter 14), the error is not serious, and since the error tends to cancel because of the appearance of kinetic terms on each side of any energy balance equation, it is customary to ignore the effect of velocity gradients. When the error cannot be ignored, the introduction of a correction factor, that is used to multiply the $v^{2} / g_{c}$ term, is needed. This is quantitatively treated in Chapter 8.

### 3.2.8 Potential Energy

A body of mass $m$ is raised vertically from an initial position $z_{1}$ to $z_{2}$. For this condition, an upward force at least equal to the weight of the body must be exerted on it, and this force must move through the distance $z_{2}-z_{1}$. Since the weight of the body is the force of gravity on it, the minimum force required is again given by Newton's law:

$$
\begin{equation*}
F=\frac{m a}{g_{c}}=m \frac{g}{g_{c}} \tag{3.24}
\end{equation*}
$$

where $g$ is the local acceleration of gravity. The minimum work required to raise the body is the product of this force and the change in vertical displacement, that is,

$$
\begin{equation*}
\Delta E_{\mathrm{PE}}=F\left(z_{2}-z_{1}\right)=m \frac{g}{g_{c}}\left(z_{2}-z_{1}\right)=\Delta\left(m \frac{g}{g_{c}} z\right) \tag{3.25}
\end{equation*}
$$

The term above is defined as the potential energy of the mass.

Illustrative Example 3.5 As part of a fluid flow course, a young environmental engineering major has been requested to determine the potential energy of water before it flows over a waterfall 10 meters in height above ground level conditions.

Solution The potential energy of water depends on two considerations:

1. the quantity of water, and
2. a reference height.

For the problem at hand, take as a basis 1 kilogram of water and assume the potential energy to be zero at ground level conditions. Apply Equation (3.25) based on the problem statement, set $z_{1}=0 \mathrm{~m}$ and $z_{2}=10 \mathrm{~m}$, so that

$$
\Delta z=10 \mathrm{~m}
$$

At ground level conditions,

$$
\mathrm{PE}_{1}=0
$$

Therefore

$$
\begin{aligned}
\Delta(\mathrm{PE}) & =\mathrm{PE}_{2}-\mathrm{PE}_{1}=\mathrm{PE}_{2} \\
\mathrm{PE}_{2} & =m\left(g / g_{c}\right) z_{2} \\
& =(1 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(10 \mathrm{~m}) \\
& =98 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2} \\
& =98 \mathrm{~J}
\end{aligned}
$$

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