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FLOW MECHANISMS

12.1 INTRODUCTION

When fluids move through a closed conduit of any cross-section, one of two different types of flow may occur. These two flow types are most easily visualized by referring to a classic experiment first performed by Osborne Reynolds in 1883. In Reynolds' experiment, a glass tube was connected to a reservoir of water in such a way that the velocity of the water flowing through the tube could be varied. A nozzle was inserted in the inlet end of the tube through which a fine stream of colored dye could be introduced.

Reynolds found that when the velocity of the water was low, the "thread" of dye color maintained itself throughout the tube. By locating the nozzle at different points in the cross-section, it was shown that there was no mixing of the dye with water and that the dye flowed in parallel, straight lines.

At high velocities, it was found that the "line" or "thread" of dye disappeared and the entire mass of flowing water was uniformly colored with the dye. In other words, the liquid, instead of flowing in an orderly manner parallel to the long axis of the tube, was now flowing in an erratic manner and so there was complete mixing.

These two forms of fluid motion are known as *laminar* or *viscous flow* (low velocity), and *turbulent flow* (high velocity). The velocity at which the flow changes from laminar to turbulent is defined as the *critical velocity*.[1]

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12.2 THE REYNOLDS NUMBER

Reynolds, in a later study of the conditions under which the two types of flow might occur, showed that the critical velocity depended on the diameter of the tube, the velocity of the fluid, its density, and its viscosity. Further, Reynolds showed that the term representing these four quantities could be combined in a manner that later came to be defined as the *Reynolds number.*

The Reynolds number, Re, is a *dimensionless* quantity, and can be shown to be the ratio of inertia to viscous forces in the fluid:

$$\mathrm{Re} = \frac{L\rho v}{\mu} = \frac{Lv}{\nu} \tag{12.1}$$

where L is a characteristic length, v is the average velocity, ρ is the fluid density, μ is the dynamic (or absolute) viscosity, and ν is the kinematic viscosity. In flow through round pipes and tubes, L is the diameter, D.

The Reynolds number provides information on flow behavior. It is particularly useful in scaling up bench-scale or pilot data to full-scale applications. Laminar flow is always encountered at a Reynolds number, Re, below 2100 in a circular duct, but it can persist up to higher Reynolds numbers in very smooth pipes. However, the flow is unstable and small disturbances may cause a transition to turbulent flow. Very slow flow (in circular ducts) for which Re is less than 1 is termed *creeping* or *Stokes* flow. Under ordinary conditions of flow (in circular ducts), the flow is turbulent at a Reynolds number above 4000. A transition region is observed between 2100 and 4000, where the type of flow may be either laminar or turbulent, and predictions are unreliable. The Reynolds numbers at which the fluid flow changes from laminar to transition or to turbulent are termed *critical Reynolds numbers.* In other geometries, different critical Re criteria exist.

Illustrative Example 12.1 The inlet flue to a furnace is at 200°F. It is piped through a 6.0-ft inside diameter duct at 25 ft/s. The furnace heats the gas to 1900°F. In order to maintain a velocity of 40 ft/s, what size duct would be required at the outlet of the furnace?

Solution Applying the continuity equation, the volumetric flowrate into the furnace is

$$q_1 = A_1 v_1$$

Since

$$A_1 = [\pi(6.0)^2]/4 = 28.3 \text{ ft}^2$$

then

$$q_1 = (28.3)(25) = 707.5 \text{ ft}^3/\text{s}$$

The volumetric flowrate out of the scrubber, using Charles' law, is

$$\begin{aligned} q_2 &= q_1(T_2/T_1) \quad (T \text{ in absolute units}) \\ &= (707.5)(2360/660) \\ &= 2530 \text{ ft}^3/\text{s} \end{aligned}$$

The cross-sectional area of outlet duct is given by

$$\begin{aligned} A_2 &= q_2/v_2 \\ &= 2530/40 \\ &= 63.25 \text{ ft}^2 \end{aligned}$$

The diameter of the duct is therefore

$$\begin{aligned} D &= (4A_2/\pi)^{0.5} \\ &= (4 \times 63.25/\pi)^{0.5} \\ &= 8.97 \text{ ft} = 108 \text{ in} \end{aligned}$$

These calculations become important and are necessary for determining the Reynolds number. The reader is left the exercise of calculating the Reynolds number. However, since the flowing fluid is a gas, one can be virtually certain that the flow will be turbulent.

Illustrative Example 12.2 A liquid with a viscosity of 0.78 cP and a density of 1.50 g/cm^3 flows through a 1-inch diameter tube at 20 cm/s. Calculate the Reynolds number. Is the flow laminar or turbulent?

Solution By definition, the Reynolds number (Re) is equal to:

$$\text{Re} = L\rho v/\mu \tag{12.1}$$

where ρ = fluid density
v = fluid velocity
L = characteristic length, usually the conduit diameter D
μ = fluid viscosity

Since

$$1 \text{ cP} = 10^{-2} \text{ g/(cm} \cdot \text{s)}$$

$$\mu = 0.78 \times 10^{-2} \text{ g/(cm} \cdot \text{s)}$$

$$1 \text{ in} = 2.54 \text{ cm}$$

$$\begin{aligned} \text{Re} &= (1.50)(20)(2.54)/(0.78 \times 10^{-2}) \\ &= 9770 \end{aligned}$$

The flow is therefore turbulent. Once again, the value of the Reynolds number indicates the nature of the fluid flow in a duct or pipe and generally:

$\mathrm{Re} < 2100$; flow is streamline (laminar or viscous)
$\mathrm{Re} > 4000$; flow is turbulent
$2100 \leq \mathrm{Re} \leq 4000$; transition region

Illustrative Example 12.3 Given the physical properties and velocity of a gas stream flowing through a circular duct, determine the Reynolds number of the gas stream. The velocity through the duct is 3.8 m/s, the duct diameter is 0.45 m, the gas viscosity 1.73×10^{-5} kg/m · s, and the gas density is 1.2 kg/m^3.

Solution Substitution into Equation (12.1) gives

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{(0.45)(3.8)(1.2)}{1.73 \times 10^{-5}} = 118{,}600$$

The flow is in the turbulent regime and for most engineering applications, one can assume turbulent (high Reynolds number) flow for gases. The reader should also note that the Reynolds number appears in many semi-empirical and empirical equations that involve fluid flow, heat transfer, and mass transfer applications. For flow in non-circular conduits, some other appropriate length (termed the *hydraulic diameter*) replaces the diameter in Re. This is discussed later in the next chapter.

12.3 STRAIN RATE, SHEAR RATE, AND VELOCITY PROFILE

When a fluid flows past a stationary solid wall, the fluid adheres to the wall at the interface between the solid and fluid. This condition is referred to as "no slip." Therefore, the local velocity, v, of the fluid at the interface is zero. At some distance, y, normal to and displaced from the wall, the velocity of the fluid is finite. Therefore, there is a velocity variation from point to point in the flowing fluid. This causes a velocity field, in which the velocity is a function of the normal distance from the wall, that is, $v = f(y)$. If $y = 0$ at the wall, $v = 0$, and v increases with y. The rate of change of velocity with respect to distance is defined the velocity gradient,

$$\frac{\mathrm{d}v}{\mathrm{d}y} = \frac{\Delta v}{\Delta y} \tag{12.2}$$

This velocity derivative (or gradient) is also referred to as the shear rate, time rate of shear, or rate of deformation. See Chapter 5 for more details.

Illustrative Example 12.4 The local velocity v (ft/s or fps) near a wall varies with the normal distance, y (ft), from a stationary wall according to the equation

$$V = 20y - y^2$$

Generate an equation describing the shear rate.

Solution Check the consistency of the given profile at the wall where $y = 0$:

$$v = 20y - y^2 = 20(0) - (0)^2 = 0$$

The profile is consistent with respect to the boundary condition.

Generate the equation of the strain rate (see Eq. (12.2)) using the above velocity profile:

$$\frac{dv}{dy} = 20 - 2y$$

The units of the strain rate are s^{-1}. Strain rate is important in the classification of real fluids. The relationships between shear stress and strain rate are presented in diagrams called *rheograms*.

12.4 VELOCITY PROFILE AND AVERAGE VELOCITY

The velocity profile for either laminar and turbulent flow is provided in Fig. 12.1. In laminar flow, the velocity profile approaches a true parabola slightly pointed in the middle and tangent to the walls of the pipe. The average velocity over the whole cross-section (volumetric flow rate divided by the cross-sectional area) is 0.5 times the maximum velocity. This fact will be derived in the next chapter. In turbulent

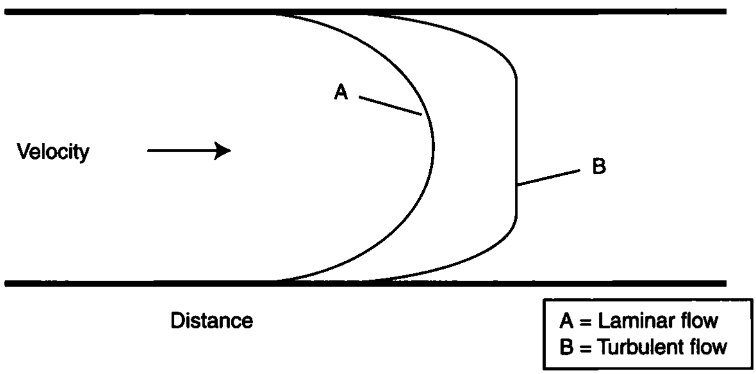

Figure 12.1 Velocity profile.

flow, the profile approaches a flattened parabola and the average velocity is usually approximately 0.8 times the maximum.

Because of its viscosity, a real fluid in contact with a nonmoving wall will have a velocity of zero at the wall. Similarly, a fluid in contact with a wall moving at a velocity, v, will move at the same velocity. This earlier described "no-slip" condition of real fluids flowing in a duct results in a fluid velocity at the wall of zero.

To calculate the volumetric flow rate, q, of the fluid passing through a perpendicular surface, S, one must integrate the product of the component of the velocity that is normal to the area and the area, over the whole cross-section area of the duct, i.e.,

$$q = \int_S v \, \mathrm{d}S \tag{12.3}$$

In accordance with the definition of average values, the average velocity of the fluid passing through the surface, S, is then given by

$$\bar{v} = v_{\mathrm{av}} = \frac{\int_S v \, \mathrm{d}S}{\int_S \mathrm{d}S} = \frac{q}{S} \tag{12.4}$$

Illustrative Example 12.5 A liquid has a specific gravity (SG) of 0.96 and an absolute viscosity of 9 cP. The liquid flows through a long circular tube of radius, $R = 3$ cm. The liquid has the following linear distribution of the axial velocity, v, (the velocity in the direction of the flow):

$$v\,(\mathrm{m/s}) = 6 - 200r$$

where r is the radial position (in meters) measured from the tube centerline. A total of 20 m^3 of liquid passes through the tube.

Calculate the average velocity of the fluid and the volumetric flow rate. Also, calculate the time for a specified volume (or mass) of fluid to pass through a section of the duct.

Solution Write the equation for q in a differential equation form in terms of r, the radial coordinate. By definition,

$$\mathrm{d}q = v \, \mathrm{d}S$$

and in cylindrical coordinates,

$$\mathrm{d}S = 2\pi r \, \mathrm{d}r$$

Therefore,

$$dq = 2\pi r v\,dr = 2\pi r(6 - 200r)\,dr = 2\pi(6r - 200r^2)\,dr$$

Integrate the above equation between the limits of $r = 0$ and $r = R$

$$q = 2\pi\int_0^R (6r - 200r^2)\,dr = 2\pi\left(3R^2 - \frac{200}{3}R^3\right)$$

Calculate the volumetric flow rate. Set $R = 3$ cm $= 0.03$ m.

$$q = 2\pi\left[3(0.03)^2 - \frac{200}{3}(0.03)^3\right] = 0.00565\,\mathrm{m^3/s}$$

Calculate the mass flow rate, $\dot{m}$

$$\dot{m} = \rho q = (\mathrm{SG})(1000\,\mathrm{kg/m^3})q$$
$$\dot{m} = 0.96(1000)(0.00565) = 5.42\,\mathrm{kg/s}$$

Calculate the average velocity. Since

$$S = \pi R^2$$

and (see Eq. (12.4))

$$v_{\mathrm{av}} = \frac{q}{S} = \frac{2\pi\left[3(R)^2 - \frac{200}{3}(R)^3\right]}{\pi R^2}$$
$$= 2\left[3 - \left(\frac{200}{3}\right)R\right]$$

By setting $R = 3$ cm $= 0.03$ m
$v_{\mathrm{av}} = 2.0\,\mathrm{m/s}$

Illustrative Example 12.6 Refer to Illustrative Example 12.5. Calculate the time to pass 20 $\mathrm{m^3}$ of the liquid through the cross-section of the pipe.

Solution The time, t, to pass the liquid is given by

$$t = \frac{V}{q} = \frac{20}{0.00565} = 3540\,\mathrm{s} = 59\ \mathrm{min}$$

As noted in Chapter 8, applying the conservation law of energy mandates that all forms of energy entering the system equal that of those leaving. See Equation (8.14) in Chapter 8. Expressing all terms in consistent units, e.g., energy per unit mass of fluid flowing, resulted in the total energy balance equation (rewritten below)

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g_c} + \frac{g}{g_c} z_1 + E_1 + Q + W_s = \frac{P_2}{\rho} + \frac{v_2^2}{2g_c} + \frac{g}{g_c} z_2 + E_2 \tag{12.5}$$

An important point needs to be made before leaving this subject. By definition, the kinetic energy of a small parcel of fluid with local velocity, v, is $v^2/2g_c$ (ft · lb_f/lb). If the local velocities at all points in the cross-section were uniform, v_{av} would be equal to v, and the kinetic energy term can be retained as written. Ordinarily there is a velocity gradient across the passage; this introduces an error, the magnitude of which depends on the nature of the velocity profile and the shape of the cross-section. For the usual case where the velocity is approximately uniform (i.e., turbulent flow), the error is not serious, and since the error tends to cancel because of the appearance of kinetic terms on each side of any energy balance equation, it is customary to ignore the effect of velocity gradients. When the error cannot be ignored, the introduction of a correction factor that is used to multiply the v^2/g_c term is needed. The term α, called the kinetic energy correction factor, is employed, where

$$\alpha = \frac{\int_S v^3 \, dS}{v_{av}{}^3 S} \tag{12.6}$$

For most engineering applications, the flow is turbulent and α may be assumed to be unity. Where the velocity distribution is parabolic, as in laminar flow, it can be shown that the exact value of α is 2. For transition state flow, $1 \leq \alpha \leq 2$.[1]

Illustrative Example 12.7 Given 1000 scfm (28.3 scmm) of gas flowing in a circular duct with a 1.2 ft (0.366 m) diameter at 300°F and 1 atm, calculate the average velocity, and the Reynolds number. Standard conditions are 60°F and 1.0 atm. The viscosity of the gas at 300°F is 2.2×10^{-5} kg/m · s and its molecular weight is 33.

Solution Calculate the actual volumetric flow rate, q, using the ideal gas law (see Chapter 11)

$$q = q_s\left(\frac{T}{T_s}\right)\left(\frac{P_s}{P}\right) = 1000\frac{(300+460)}{(60+460)}\left(\frac{1}{1}\right)$$

$$= 1461.5 \text{ acfm} = 41.36 \text{ acmm}$$

The cross-sectional area of the duct, S, is

$$S = \frac{\pi D^2}{4} = \frac{\pi(1.2)^2}{4} = 1.131 \text{ ft}^2 = 0.105 \text{ m}^2$$

The velocity, v, is

$$\begin{aligned} v &= \frac{q}{S} = \frac{1461.5}{1.131} \\ &= 1292.2 \text{ ft/min} \\ &= 21.5 \text{ ft/s} \\ &= 6.55 \text{ m/s} \end{aligned}$$

Note that the average velocity was simply calculated by dividing the actual volumetric gas flow rate by the cross-sectional area through which the gas flows. The velocity calculated here is therefore the bulk, or average, velocity. Note that the same calculational procedure may be employed to calculate the average velocity in any process unit, including chemical reactors, pipes, stacks, etc.

Calculate the gas density from the ideal gas law:

$$\rho = \frac{P(\text{MW})}{RT} = \frac{(1)(33)}{(0.7302)(760)} = 0.0595 \text{ lb/ft}^3 = 0.952 \text{ kg/m}^3$$

The mass flow rate is calculated as follows:

$$\dot{m} = \rho v S = (0.0595)(21.5)(0.105) = 0.134 \text{ lb/s} = 0.656 \text{ kg/s}$$

Calculate the Reynolds number

$$\text{Re} = \frac{Dv\rho}{\mu} = \frac{(0.366)(6.55)(0.952)}{2.2 \times 10^{-5}} = 103{,}670$$

As noted above, the velocity calculated is an average value. Plug flow, characterized by a uniform velocity distribution, is often assumed. In actual operation, the following velocity profiles might develop (see Fig. 12.1):

1. Parabolic—laminar flow.
2. "Flattened" parabola—turbulent flow, wherein velocities are low (often near zero at the perimeter/walls) and high (often near 20% above the average velocity) at the center.
3. Random distribution—following a bend, valve, or disturbance.

These profiles are discussed further in the next two chapters.

REFERENCE

1. W. Badger and J. Banchero, "Introduction to Chemical Engineering," McGraw-Hill, New York, 1955.

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