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COMPRESSIBLE AND SONIC FLOW

15.1 INTRODUCTION

Compressibility refers to a condition where the volume or density of a fluid varies with the pressure. In fluid flow applications, it is a consideration only when vapors/gases are involved; liquids can safely be considered incompressible in these calculations. When the pressure drop in a flowing gas system is less than (on the order of) 10–20% of the absolute pressure in the system, satisfactory engineering accuracy is obtained in pressure drop calculations by assuming the fluid incompressible at conditions corresponding to the average pressure in the system. For larger pressure drops, compressibility effects can become important. The compressible flow of a fluid is further complicated by the fact that the fluid density is dependent on temperature as well as on pressure. In such systems, temperature may vary in accordance with thermodynamic principles[1] (see Chapters 8 and 11 for more details).

Although this chapter is primarily concerned with sonic flow, it also addresses the general topic of compressible flow. The presentation that follows first examines compressible flow, which in turn is followed by sonic flow, which in turn is followed by key pressure drop equations that may be employed in engineering flow calculations for this topic.

15.2 COMPRESSIBLE FLOW

As noted above, flowing fluids are typically considered compressible when the density varies by more than 10–20% during a particular application. In practice,

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compressible flows are normally limited to gases, supercritical fluids, and multiphase flows containing gases; flowing liquids are normally considered incompressible. In industrial applications, one-dimensional gas flow through nozzles or orifices and in pipelines are the most important applications of compressible flow. Multidimensional external flows are of interest mainly in aerodynamic applications, a topic beyond the scope of this text.[2]

In addition to the factors discussed above, compressible flow calculations are further complicated by other system parameter variations. For example, for a given pipe diameter and mass flow rate, the friction factor depends upon the viscosity, which, in turn, depends upon temperature. This problem does not exist for isothermal flow but can be important during adiabatic operation. However, in adiabatic compressible flow, Reynolds numbers are usually high indicating turbulent flow and any variation of the friction factor due to temperature variations along the pipe length is small. Thus, the friction factor may be assumed constant.

The first step in a compressible–incompressible flow analysis is to classify the flow. One has to specify either steady or unsteady flow as well as whether the flow is compressible or incompressible. Steady and unsteady flow refers to variations with time, while incompressible and compressible flow refers to density variations. If the density is constant, or its variation is very small (most liquids, and gases with a Mach number less than 0.3), the flow is deemed *incompressible*. The Mach number is discussed in the next section.

Illustrative Example 15.1 The inlet air density to an expanding nozzle is 0.071 lb/ft^3. The outlet density is 0.0049 lb/ft^3. Is the flow compressible?

Solution Since the density of the fluid changes during its residence in the nozzle, the flow can be considered compressible.

15.3 SONIC FLOW

The Mach number, *Ma*, is a dimensionless number defined as the ratio of fluid velocity to the speed of sound in the fluid, i.e.,

$$Ma = \frac{v}{c} \tag{15.1}$$

where v is the average velocity of the fluid and c is the speed of sound. If the Mach number is less than or equal to 0.3, compressibility effects may usually be neglected, and one may safely assume incompressible flow.

The speed of sound in (selected) common liquids is given in Table 15.1. The speed of sound, c, in an ideal gas may be calculated from

$$c = \sqrt{\frac{kRT}{\text{MW}}} \tag{15.2}$$

Table 15.1 Speed of Sound in Various Liquids

Liquid	Sound Velocity (m/s)
Acetone	1174
Benzene	1298
Ethanol	1144
Ethylene glycol	1644
Methanol	1103
Water	1498

where k (see Table 15.2) is the ratio of C_p/C_v, R the universal gas constant, T the absolute temperature and MW the molecular weight of the fluid. The derivation of this from basic principles is available in the literature. Note that the k values in Table 15.2 are approximate for 1 atm and 25°C; a decrease in temperature or an increase in pressure will generally result in higher values.[3]

For air, Equation (15.2) simplifies to

Table 15.2 Values of *k*

Gas	k
C_2H_6	1.2
CO_2, SO_2, H_2O, H_2S, NH_3, Cl_2, CH_4, C_2H_2, C_2H_4	1.3
Air, H_2, O_2, N_2, CO, NO, HCl	1.4
Monatomic gases	1.67

$$c = 20\sqrt{T(\mathrm{K})};\ \ \mathrm{m/s} \tag{15.3}$$

and

$$c = 20\sqrt{T(^\circ\mathrm{R})};\ \ \mathrm{ft/s} \tag{15.4}$$

Illustrative Example 15.2 Nitrogen gas at 20°C and 1 atm flows in a duct at a velocity of 82 m/s. Is it reasonable to neglect compressibility effects? Assume $k = 1.4$ for nitrogen.

Solution Calculate the speed of sound in N_2 at 20°C. See Equation (15.2).

$$c = \sqrt{\frac{kRT}{M}} = \sqrt{\frac{(1.4)(8314.4)(293)}{28}}$$
$$= 349\,\mathrm{m/s}$$

The Mach number can then be calculated from Equation (15.1).

$$Ma = \frac{v}{c} = \frac{82}{349} = 0.235$$

Since $0.235 < 0.3$, compressibility effects may be neglected.

Equation (15.2) indicates that the square of the velocity of sound is proportional to the absolute temperature of the ideal gas. Thus, the velocity of sound may be viewed as being proportional to the internal kinetic energy of the gas. Since the kinetic energy of a flowing gas is proportional to v^2, the ratio v^2/c^2 provides a measure of the ratio of the kinetic energy to the internal energy. The velocity of sound in air at room temperature is approximately 1100 ft/s. Thus, for a velocity of 220 ft/s, and noting that v/c is defined as the Mach number, Ma is 0.2 and $(Ma)^2$ is only 0.04. This indicates that kinetic energy effects do not become important until somewhat higher Mach numbers are achieved.[3]

Most often, the Mach number is calculated using the speed of sound evaluated at the local pressure and temperature. When $Ma = 1$, the flow is *critical* or *sonic*, and the velocity equals the local speed of sound. For subsonic flow, $Ma < 1$, while supersonic flow has $Ma > 1$. A potential error is to assume that compressibility effects are always negligible when the Mach number is small. Proper assessment of whether compressibility is important should be based on relative density changes, not on Mach number alone.[2] However, the Mach number is usually employed in engineering calculations.

Equations developed earlier for incompressible fluids are applicable to compressible fluids—in a general sense. However, these same equations may often be applied to compressible fluids if the *fractional change* in pressure is not large. For example, compressibility effects may not be important if there is a change in pressure from 14.7 to 15.7 psia, but could be very important if the change is from 0.1 to 1.0 psia.[3]

A detailed treatment of sonic flow though a variety of process units is provided in Perry's Handbook.[1] Included in the treatment are defining equations for:

1. Flow through a frictionless nozzle
2. *Adiabatic flow with friction in a duct of constant cross-section*
3. Compressible flow with friction loss
4. Convergent/divergent nozzles

Illustrative Example 15.3 Propane ($k = C_p/C_v = 1.3$) at 17°C and 0.35 MPa is flowing in a tube (inside diameter of 1 in) at an average velocity of 43 m/s. Determine the speed of sound in the propane. Is the propane flow compressible? Why or why not? Is the propane flow laminar or turbulent?

Solution The speed of sound in propane is first calculated from Equation (15.2).

$$c = \sqrt{\frac{kRT}{M}} = \sqrt{\frac{1.3(83.14)(290)}{44}}$$
$$= 267\ \mathrm{m/s}$$

The Mach number is therefore [see Eq. (15.1)]

$$Ma = \frac{v}{c} = \frac{43}{267} = 0.161$$

Since $0.161 < 0.3$, the flow is incompressible.

Determine the Reynolds number. First calculate the density from the ideal gas law

$$\rho = 6.39\,\text{kg/m}^3$$

The viscosity is estimated from Fig. B.2 in the Appendix,

$$\mu = 8 \times 10^{-3}\text{cP} = 8 \times 10^{-6}\,\text{m}^2/\text{s}$$

Therefore,

$$\text{Re} = \frac{43(0.0254)(6.39)}{8 \times 10^{-6}} = 872{,}000$$

Since Re is 872,000 and >4000, the flow is turbulent.

15.4 PRESSURE DROP EQUATIONS

Two equations for pressure drop are presented in this section—one for laminar flow and one for turbulent flow. Both can be employed for most real-world applications involving compressible flow.

15.4.1 Isothermal Flow

For laminar flow of gases in pipes and other conduits, the pressure drop from P_1 to P_2 may be estimated from Equation (15.5) for laminar flow conditions.

$$P_1^2 - P_2^2 = \frac{8\mu RTG}{g_c MD}\left[\frac{8L}{D} + \frac{\text{Re}}{3}\ln\left(\frac{P_1}{P_2}\right)\right] \tag{15.5}$$

where Re = Reynolds number,
μ = gas viscosity,
T = absolute temperature,
G = mass velocity flux,
M = gas molecular weight,
D = pipe/conduit diameter,
L = pipe/conduit length.

Equation (15.5) may be used for engineering purposes provided that the Mach number is below 0.5 (i.e., $Ma < 0.5$).

Finally, one should note that if the flow rate is unknown (this is often the desired quantity), a trial-and-error solution is involved since the friction factor depends on the velocity. However, the value of f does not vary significantly over a very wide range of Reynolds numbers and the solution is therefore not sensitive to the value of f. Consequently, an (initial) assumption of an average value of 0.004 for f is satisfactory. This will yield a value of the velocity for which the Reynolds number can be calculated and the corresponding value of f determined. An iterative calculation using these new and updated values of f will provide an acceptable answer.

$$P_1^2 - P_2^2 = \frac{4fLG^2RT}{g_cD(\mathrm{MW})}\left(1 + \frac{2D}{fL}\ln\frac{P_1}{P_2}\right) \tag{15.6}$$

In ducts of appreciable length, the second term in the parentheses can be assumed negligible unless the pressure drop is very large. When this term is omitted, Equation (15.6) becomes

$$P_1 - P_2 = \frac{2fLG^2}{2g_c\rho_{\mathrm{av}}D} \tag{15.7}$$

where ρ_{avg} is the density at the average pressure of $(P_1 + P_2)/2$ for the mass rate of flow of the system. This equation may also be written as

$$\dot{m} = \frac{\pi}{8}\sqrt{\frac{(P_1^2 - P_2^2)g_cD^5(\mathrm{MW})}{fLRT}} \tag{15.8}$$

It should be noted that in most applications, the flow can more appropriately be described as adiabatic rather than truly isothermal.

The equation for adiabatic flow is based on the condition that the flow arises from the adiabatic expansion of the gas through a frictionless nozzle leading from an inlet source where the velocity is negligible. Such a system is frequently encountered in practice. For this system, the describing equation is given by

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{(k-1)/k}$$

$$\frac{P_0}{P_1} = \left[1 + \frac{G^2}{2g_c}\left(\frac{k-1}{k}\right)\frac{RT_1}{(\mathrm{MW})P_1^2}\right]^{k/(k-1)} \tag{15.9}$$

Illustrative Example 15.5 Verify Equation (15.8); assume Equation (15.6), with the second term neglected, to be correct.

Solution Start with Equation (15.6). Neglecting the second term,

$$P_1^2 - P_2^2 = \frac{4fLG^2RT}{g_cD(\mathrm{MW})}$$

Note that

$$G = \frac{\dot{m}}{(\pi/4)D^2}$$

Substituting and rearranging

$$P_1^2 - P_2^2 = \frac{4fL(\dot{m})^2 RT(16)}{g_c D(D^2)^2(\pi)^2(\mathrm{MW})}$$

Solving for $\dot{m}$

$$(\dot{m})^2 = \frac{(P_1^2 - P_2^2)g_c D^5 \pi^2(\mathrm{MW})}{64fLRT}$$

$$\dot{m} = \frac{\pi}{8}\sqrt{\frac{(P_1^2 - P_2^2)g_c D^5(\mathrm{MW})}{fLRT}}$$

Illustrative Example 15.6 Calculate the pressure drop accompanying the flow of natural gas (which may be assumed to be methane) at 70°F through a horizontal steel pipe 12 inches in diameter and 3 miles long. The gas enters the pipe at 75 psig and at a rate of 236 scfs (14.7 psia, 60°F). The viscosity of methane at 70°F is 0.011 centipoise and a friction factor of 0.008 may be assumed.

Solution The mass flow rate is

$$\dot{m} = (236/379)16$$
$$= 10\,\mathrm{lb/s}$$

The mass velocity flux is

$$G = 10/[(\pi/4)(1)^2]$$
$$= 12.7\,\mathrm{lb/ft^2 \cdot s}$$

Apply Equation (15.7) since the flow is turbulent (gas). Assume an average density based on inlet conditions

$$\rho = \frac{P(\mathrm{MW})}{RT}$$
$$= \frac{(89.7)(16)}{(10.73)(530)}$$
$$= 0.252\,\mathrm{lb/ft^3}$$

$$P_1 - P_2 = \frac{2fLG^2}{g_c \rho_{\mathrm{av}} D}$$
$$= \frac{2(0.008)(3)(5280)(12.7)^2}{(32.2)(0.252)(1)}$$
$$= 5036\,\mathrm{psf}$$

Rearranging

$$\begin{aligned} P_2 &= P_1 - 5036 \\ &= (89.7 \times 144) - 5036 \\ &= 12{,}917 - 5036 \\ &= 7881\,\text{psf} \\ &= 54.7\,\text{psia} \\ &= 40.0\,\text{psig} \end{aligned}$$

The pressure drop is approximately given by

$$\Delta P = 89.7 - 54.7 = 35.0\,\text{psia}$$

Strictly speaking, the calculation should be repeated with an updated value for the density at the average of the inlet and outlet pressure.

Illustrative Example 15.7 Refer of Illustrative Example 15.6. Calculate the Reynolds number for the system. Comment on the results.

Solution The calculation is based on inlet conditions. The Reynolds number is

$$\begin{aligned} \text{Re} &= \frac{DG}{\mu} \\ &= \frac{(1)(12.7)}{\mu} \end{aligned}$$

Convert cP to lb/ft · s.

$$\begin{aligned} \mu &= (0.011\,\text{cP})(6.72 \times 10^{-4}) \\ &= 7.39 \times 10^{-6}\,\text{lb/ft} \cdot \text{s} \end{aligned}$$

The Reynolds number is therefore

$$\begin{aligned} \text{Re} &= \frac{(1)(12.7)}{7.39 \times 10^{-6}} \\ &= 17.2 \times 10^{5} \end{aligned}$$

The value provided for the friction factor is reasonable for this Reynolds number.

Illustrative Example 15.8 Air at 2.7 atm and 15°C enters a horizontal 8.5 cm steel pipe that is 6.5 m long. The velocity at the entrance of the pipe is 30 m/s. What is the pressure drop across the line?

Solution Assume isothermal flow and apply Equation (15.7). From the ideal gas law

$$\rho = \left(\frac{29}{22.4}\right)\left(\frac{2.7}{1}\right)\left(\frac{273}{288}\right)$$
$$= 3.31\,\mathrm{kg/m^3}$$

The mass velocity is

$$G = v\rho = (30)(3.31)$$
$$= 99.3\,\mathrm{kg/m^2 \cdot s}$$

Assume (initially)

$$f = 0.004$$

Rearranging Equation (15.7)

$$P_1 - P_2 = \frac{2fLG^2}{g_c \rho D}$$
$$P_2 = P_1 - \frac{2fLG^2}{g_c \rho D}$$

Substituting, while noting 1 atm = $101{,}325\,\mathrm{kg/m \cdot s^2}$,

$$P_2 = 2.7 - \left[\frac{(2)(0.004)(65)(99.3)^2}{(3.31)(0.085)(101{,}325)}\right]$$
$$= 2.7 - 0.18$$
$$= 2.52\,\mathrm{atm}$$

The pressure drop is therefore

$$\Delta P = P_1 - P_2$$
$$= 0.18\,\mathrm{atm} = 2.65\,\mathrm{psi}$$

Illustrative Example 15.9 Refer to Illustrative Example 15.8. Is the assumption for the friction factor reasonable?

Solution From Table A.3 in the Appendix,

$$\mu = 0.0174\,\mathrm{cP} = 1.74 \times 10^{-5}\,\mathrm{kg/(m \cdot s)}$$

Calculate the Reynolds number,

$$\mathrm{Re} = \frac{DG}{\mu}$$

$$= \frac{(0.085)(99.3)}{1.74 \times 10^{-5}} = 485{,}000$$

Refer to Fig. 14.2. The assumption of $f = 0.004$ is reasonable.

REFERENCES

1. D. Green and R. Perry (editors), "Perry's Chemical Engineers' Handbook," 8th edition, McGraw-Hill, New York, 2008.
2. I. Farag, "Fluid Flow," A Theodore Tutorial, East Williston, NY, 1995.
3. C. Bennett and J. Myers, "Momentum, Heat, and Mass Transfer," McGraw-Hill, New York, 1962.

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