## 19

## FLOW MEASUREMENT

### 19.1 INTRODUCTION

Measurement of a flowing fluid can be difficult since it requires that the mass or volume of material be quantified as it moves through a pipe or conduit. Problems may arise due to the complexity of the dynamics of flow. Further, flow measurements draw on a host of physical parameters that are also often difficult to quantify.

This chapter serves to review standard industrial methods that are employed to measure fluid flow rates. Information provided can include the velocity or the amount of fluid that passes through a given cross-section of a pipe or conduit per unit time. Local velocity variations across the cross-section or short-time fluctuations (e.g., turbulence) are not considered. These concerns can be important, particularly the former. For example, in air pollution applications, it is often necessary to traverse a stack to obtain local velocity variations with position.

Hydrodynamic methods are primarily used by industry in the measuring the flow of fluids. These methods include the use of the following equipment:

## 1. Pitot tube

2. Venturi meter
3. Orifice meter

Other approaches include weighing, direct displacement, and dilution. Weighing involves mass or gravitational approaches which, as one might suppose, cannot be
used for gases. Direct displacement can be applied to liquids and is based on a displacement of either a moving part of the unit or the moving fluid. Dilution methods involve adding a second fluid of a known rate to the stream of fluid to be measured and determining the concentration of this second fluid at some displaced point. In addition, there is the vane anemometer that is in effect a windmill consisting of a number of light blades mounted on radial arms attached to a common spindle rotating in two jeweled bearings; when placed parallel to a moving gas stream, the forces on the blades cause the spindle to rotate at a rate depending mainly on the gas velocity. An extension of this unit is the hot-wire anemometer that essentially consists of a fine, electrically-heated wire exposed to the gas stream in which the velocity is being measured; the velocity of the gas determines the cooling effect upon the wire, which in turn affects the electrical resistance. Finally, the rotameter is the most widely used form of area meter that is essentially a vertical tapered glass tube inserted into a pipe line by means of special end connections and containing a float that moves up and down as the flow increases or decreases; graduations are etched onto the side of the tube to indicate the rate of flow.

This chapter will primarily key on the five hydrodynamic methods listed above, introducing the subject with pressure measurement and the general topic of manometry.

### 19.2 MANOMETRY AND PRESSURE MEASUREMENTS

Pressure is usually measured by allowing it to act across some area and opposing it with some type of force (e.g., gravity, compressed spring, electrical, and so on). If the force is gravity, the device is usually a manometer.

A very common device to measure pressure is the Bourdon-tube pressure gauge. It is a reliable and inexpensive direct displacement device. It is made of a stiff metal tube bent in a circular shape. One end is fixed and the other is free to deflect when pressurized. This deflection is measured by a linkage attached to a calibrated dial (see Fig. 19.1). Bourdon gauges are available with an accuracy of $\pm 0.1 \%$ of the full scale.

Other pressure gauges measure the pressure by the displacement of the sensing element electrically. Among the common methods are capacitance, resistive and inductive. However, the interest in this section is primarily with the manometer.

Consider the open manometer shown in Fig. 19.2. $P_{1}$ is unknown and $P_{a}$ is the known atmospheric pressure. The heights $z_{a}, z_{1}$, and $z_{2}$ are also known. Applying Bernoulli's hydrostatic equation at points 1 and 2, and again at points a and 2 yields:

$$
\begin{align*}
& P_{1}-P_{2}=-\rho_{1} \frac{g}{g_{c}}\left(z_{1}-z_{2}\right)  \tag{19.1}\\
& P_{2}-P_{a}=-\rho_{2} \frac{g}{g_{c}}\left(z_{2}-z_{a}\right) \tag{19.2}
\end{align*}
$$



Figure 19.1 Bourdon-tube pressure gauge.


Figure 19.2 Open manometer.

If the two equations are added, one obtains:

$$
\begin{equation*}
P_{1}-P_{a}=-\left[\rho_{1} \frac{g}{g_{c}}\left(z_{1}-z_{2}\right)-\rho_{2} \frac{g}{g_{c}}\left(z_{2}-z_{a}\right)\right] \tag{19.3}
\end{equation*}
$$

The reader should refer to Chapter 10 for more details on manometry.
Illustrative Example 19.1 Find the air pressure $\left(P_{1}\right)$ in the oil tank pictured in Fig. 19.3, given the heights and densities of the fluids in the manometer. The oil has a specific gravity of 0.8 , the specific gravity of mercury is 13.6 , and the density of air is $1.2 \mathrm{~kg} / \mathrm{m}^{3}$. Employ SI units.

Solution Apply the manometer equation between points 1 and 2:

$$
P_{1}+\rho_{1} \frac{g}{g_{c}} z_{1}=P_{2}+\rho_{2} \frac{g}{g_{c}} z_{2}
$$

Since $z_{1}=z_{2}$ and $\rho_{1}=\rho_{2}$

$$
P_{1}=P_{2}
$$



Figure 19.3 Air pressure in a tank.

Also apply the manometer equation between points 2 and 3:

$$
P_{2}+\rho_{2} \frac{g}{g_{c}} z_{2}=P_{3}+\rho_{3} \frac{g}{g_{c}} z_{3}
$$

Since $\rho_{2}=\rho_{3}=\rho_{\text {oil }}$ and $z_{3}-z_{2}=0.4 \mathrm{~m}$ of oil

$$
\begin{aligned}
P_{2} & =P_{3}+\rho_{\mathrm{oil}} \frac{g}{g_{c}}\left(z_{3}-z_{2}\right) \\
& =P_{3}+(0.8)(1000)(9.807)(0.4) \\
& =P_{3}+3138
\end{aligned}
$$

Consider points 4 and 5:

$$
\begin{aligned}
P_{3} & =P_{4}=P_{5}+\rho_{\mathrm{air}} \frac{g}{g_{c}}\left(z_{5}-z_{4}\right) \\
& =P_{5}-(1.2)(9.807)(1) \\
& =P_{5}-11.77
\end{aligned}
$$

Consider points 6 and 7:

$$
\begin{aligned}
P_{5} & =P_{6}=P_{7}+\rho_{\mathrm{Hg}} \frac{g}{g_{c}}\left(z_{7}-z_{6}\right) \\
& =P_{7}+(13,600)(9.807)(0.8) \\
& =P_{7}+106,700
\end{aligned}
$$

Since $P_{7}=0$ (gauge basis),

$$
P_{5}=P_{6}=106,700 \mathrm{Pag}
$$

Back substitute to obtain the desired pressures.

$$
\begin{aligned}
& P_{3}=P_{4}=P_{5}-11.77=106,700-11.77=106,688 \mathrm{Pag}=2.05 \mathrm{~atm} \\
& P_{2}=P_{1}=106,688+3138=109,826 \mathrm{Pag}=211,151 \mathrm{~Pa}=2.08 \mathrm{~atm}
\end{aligned}
$$

Note:

1. The contribution of the gas section of the manometer to the answer is negligible.
2. Manometers that are open to the atmosphere are gauge-pressure devices.
3. The cross-sectional area of the manometer tube does not affect the calculation.

Liquid depths in tanks are commonly measured by the scheme shown in Fig. 19.4. Compressed air (or nitrogen) bubbles slowly through a dip tube in the liquid. The flow of the air is so slow that it may be considered static. The tank is vented to the atmosphere. The gauge pressure reading at the top of the dip tube is then primarily due to the liquid depth in the tank.


Figure 19.4 Liquid depth measurement.

### 19.3 PITOT TUBE

Bernoulli's equation provides the basis to analyze some devices for fluid flow measurement. A common device is the Pitot tube shown in Fig. 19.5. It essentially consists of one tube with an opening normal to the direction of flow and a second tube in which the opening is parallel to the flow. It measures both the static pressure (through the side holes at station 2) and the stagnation, or impact, pressure (through the hole in the front at station 1). Applying Bernoulli's equation between stations 1 and 2 , one obtains (after neglecting frictional effects):

$$
\begin{equation*}
P_{1}+\frac{\rho v_{1}^{2}}{2 g_{c}}+\rho \frac{g}{g_{c}} z_{1}=P_{2}+\frac{\rho v_{2}^{2}}{2 g_{c}}+\rho \frac{g}{g_{c}} z_{2} \tag{19.4}
\end{equation*}
$$

Since

$$
\begin{aligned}
& v_{1}=0 \text { (stagnation) } \\
& z_{1}=z_{2} \text { (horizontal) } \\
& v_{2}=v=\text { fluid velocity }
\end{aligned}
$$

$$
\begin{equation*}
v=\sqrt{\frac{2\left(P_{1}-P_{2}\right) g_{c}}{\rho}} \tag{19.5}
\end{equation*}
$$

This is the Pitot tube formula.
The pressure difference $\left(P_{1}-P_{2}\right)$ is often measured by connecting the ends of the Pitot tube to a manometer. The manometer liquid (density $\rho_{M}$ ) develops a differential height, $h$, due to the flowing fluid. Applying Bernoulli's equation at the manometer


Figure 19.5 Pitot tube for velocity measurement.
(as presented in Fig. 19.5 yields):

$$
P_{3}=P_{4}
$$

and

$$
\begin{equation*}
P_{5}+\rho \frac{g}{g_{c}} h=P_{6}+\rho_{M} \frac{g}{g_{c}} h \tag{19.6}
\end{equation*}
$$

In addition

$$
\begin{equation*}
P_{5}-P_{6}=P_{1}-P_{2}=\frac{g}{g_{c}} h\left(\rho_{M}-\rho\right) \tag{19.7}
\end{equation*}
$$

This is a modified form of Equation (19.3). Substituting Equation (19.7) into the Pitot tube formula, Equation (19.5) gives

$$
\begin{equation*}
v=\sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho}} \tag{19.8}
\end{equation*}
$$

This equation has also been written as

$$
\begin{equation*}
v=C \sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho}} \tag{19.9}
\end{equation*}
$$

The term $C$ is included to account for the assumption of negligible frictional effects. However, for most Pitot tubes, $C$ is approximately unity.

Illustrative Example 19.2 A Pitot tube is located at the center line of a horizontal 12 -inch ID pipe transporting dry air at $70^{\circ} \mathrm{F}$ and at atmospheric pressure. The horizontal deflection on a U-tube (inclined 10 inch horizontal to 1 inch vertical and connected to the impact and static openings) shows 2 inch of water. Calculate the actual velocity of air at the point where the reading is taken and the average velocity through this cross-section if the average velocity is $81.5 \%$ of the maximum velocity.

Solution The density of the gas at the point of reading is approximately

$$
\rho=0.075 \mathrm{lb} / \mathrm{ft}^{3}
$$

Owing to the 10 -to-1 inclination, the actual difference in levels is only 0.2 inch of water.

The velocity is calculated directly from Equation (19.8):

$$
\begin{aligned}
v & =\sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho}} ; \rho_{M}-\rho \approx \rho_{M} \\
& =\sqrt{\frac{(2)(32.2)(0.2 / 12)(62.4)}{0.075}} \\
& =29.9 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

This represents the velocity at the point where the reading was taken; that is, the centerline of the pipe. Thus,

$$
v_{\max }=29.9 \mathrm{ft} / \mathrm{s}
$$

Since the flowing fluid is air at a high velocity, the flow has a high probability of being turbulent. For this condition, assume (see Chapter 14)

$$
\bar{v} / v_{\max }=0.815
$$

so that

$$
\begin{aligned}
\bar{v} & =(0.815)(29.9) \\
& =24.4 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

Illustrative Example 19.3 Refer to Illustrative Example 19.3. Calculate the mass flow rate of the air.

Solution Since the area is $0.785 \mathrm{ft}^{2}$

$$
\begin{aligned}
q & =(24.4)(0.785)(60) \\
& =1150 \mathrm{ft}^{3} \mathrm{~min}
\end{aligned}
$$

And,

$$
\begin{aligned}
\dot{m} & =(1150)(0.075)(60) \\
& =5175 \mathrm{lb} / \mathrm{hr}
\end{aligned}
$$

Illustrative Example 19.4 Water ( $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}, \mu=0.001 \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}$ ) flows in a circular pipe. The pipe is a 3 inch schedule 40 steel pipe. A Pitot tube is used to measure the water velocity. The liquid in the manometer is mercury ( $\rho_{\mathrm{Hg}}=13,600 \mathrm{~kg} / \mathrm{m}^{3}$ ). The manometer height, $h\left(z_{6}-z_{4}\right.$ in Fig. 19.5), is 7 cm . Determine the water velocity (in $\mathrm{m} / \mathrm{s}$ and fps ), volumetric flow rate (in $\mathrm{m}^{3} / \mathrm{s}$ and gpm ), and flow regime.

Solution Calculate the water velocity from the Pitot tube equation. Note

$$
h=0.07 \mathrm{~m}
$$

and

$$
\left(\rho_{\mathrm{M}}-\rho\right) / \rho=12.6
$$

Employing Equation (19.8),

$$
v=\sqrt{(2)(9.807)(0.07)(12.6)}=4.2 \mathrm{~m} / \mathrm{s}
$$

Obtain the pipe inside diameter. Use Table A. 5 in the Appendix. For a 3 inch schedule 40 pipe:

$$
\begin{aligned}
& \mathrm{OD}=3.5 \mathrm{in} \\
& \text { Wall thickness }=0.216 \mathrm{in} \\
& \mathrm{ID}=3.068 \mathrm{in}=0.0779 \mathrm{~m} \\
& \text { Pipe weight }=7.58 \mathrm{lb} / \mathrm{ft}
\end{aligned}
$$

Calculate volumetric flow rate

$$
\begin{aligned}
& S=\frac{\pi}{4}(0.0779)^{2}=0.00477 \mathrm{~m}^{2} \\
& q=v S=(4.2)(0.00477)=0.02 \mathrm{~m}^{3} / \mathrm{s}=317 \mathrm{gpm}
\end{aligned}
$$

The flow is turbulent since

$$
\operatorname{Re}=\frac{1000(4.2)(0.0779)}{0.001}=327,180
$$

Note that a Pitot tube measures the local velocity at only one point. To obtain the average velocity over the cross-section, it is necessary to read the velocity at a number of specific locations in the cross-section of the pipe. Also note that when the Pitot tube is used for measuring low-pressure gases, the pressure difference reading is usually extremely small, and can lead to large errors.

### 19.4 VENTURI METER

The Venturi meter is also a device for measuring a fluid flow rate. As shown in Fig. 19.6, it consists of three sections: a converging section to accelerate the flow, a short cylindrical section (called the throat), and a diverging section to increase the cross-sectional area to its original (upstream) value. There is a change in pressure between the upstream (point 1) and the throat (point 2). This pressure difference is measured (often with a manometer). The Venturi meter can determine the volumetric flow rate from either the pressure difference ( $P_{1}-P_{2}$ ) or the manometer head ( $h$ ). The development of pertinent equations is presented below.


Figure 19.6 Venturi meter.

Refer to Fig. 19.6. From the conservation law of mass $\dot{m}_{1}=\dot{m}_{2}$ at steady state. If the fluid is assumed incompressible, then:

$$
\begin{aligned}
& \rho_{1}=\rho_{2}=\rho \\
& q_{1}=q_{2}
\end{aligned}
$$

so that

$$
\begin{equation*}
\frac{\pi D_{1}^{2}}{4} v_{1}=\frac{\pi D_{2}^{2}}{4} v_{2} \tag{19.10}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{1}=\left(\frac{D_{2}^{2}}{D_{1}^{2}}\right) v_{2} \tag{19.11}
\end{equation*}
$$

Applying Bernoulli's equation between points 1 and 2, and assuming no frictional losses (see Eq. 19.4)

$$
\begin{equation*}
P_{1}+\frac{\rho v_{1}^{2}}{2 g_{c}}+\rho \frac{g}{g_{c}} z_{1}=P_{2}+\frac{\rho v_{2}^{2}}{2 g_{c}}+\rho \frac{g}{g_{c}} z_{2} \tag{19.12}
\end{equation*}
$$

For a horizontal Venturi meter, $z_{1}=z_{2}$. Therefore, the above equation simplifies to:

$$
\begin{equation*}
P_{1}+\frac{\rho v_{1}^{2}}{2 g_{c}}=P_{2}+\frac{\rho v_{2}^{2}}{2 g_{c}} \tag{19.13}
\end{equation*}
$$

Rearranging Equation (19.13) and substituting for $v_{1}$ from Equation (19.11) leads to:

$$
\begin{equation*}
v_{2}=\sqrt{\frac{2 g_{c}\left(P_{1}-P_{2}\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}} \tag{19.14}
\end{equation*}
$$

Substituting the manometer equation for ( $P_{1}-P_{2}$ ), for example see Equation (19.7), into Equation (19.14) yields

$$
\begin{equation*}
v_{2}=\sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}} \tag{19.15}
\end{equation*}
$$

where once again $\rho_{M}$ is the manometer fluid density, and $\rho$ the flowing fluid density. The volumetric flow rate, $q$, is

$$
\begin{equation*}
q=\left(\frac{\pi D_{2}^{2}}{4}\right) v_{2} \tag{19.16}
\end{equation*}
$$

Equation (19.15) is often referred to as the Venturi formula. It applies to frictionless flow. To account for the small friction loss between points (1) and (2), a Venturi discharge coefficient, $C_{\mathrm{v}}$, is introduced in the above equation, that is,

$$
\begin{equation*}
v_{2}=C_{\mathrm{v}} \sqrt{\frac{2 g_{\mathrm{c}}\left(P_{1}-P_{2}\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}}=C_{\mathrm{v}} \sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}} \tag{19.17}
\end{equation*}
$$

For well-designed Venturi meters, $C_{\mathrm{v}}$, is approximately 0.96 .
There is a permanent pressure loss, $\Delta P_{L}$, in the Venturi of about $10 \%$ of $\left(P_{1}-P_{2}\right)$. This means that $90 \%$ of the $\left(P_{1}-P_{2}\right)$ is recovered in the divergent section of the Venturi. This pressure loss causes an overall loss of energy. The power requirement to operate a Venturi meter (or the power loss) is calculated from the volumetric flow rate and the pressure loss, that is,

$$
\begin{equation*}
\dot{W}_{L}=q\left(\Delta P_{L}\right) \tag{19.18}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta P_{L}=0.1\left(P_{1}-P_{2}\right) \tag{19.19}
\end{equation*}
$$

Illustrative Example 19.5 A Venturi meter has gasoline flowing through it. The upstream diameter, $D_{1}$, is 0.06 m and the throat diameter, $D_{2}$, is 0.02 m . The manometer fluid is mercury, with a height difference, $h$, of 35 mm Hg . Gasoline properties are $\rho=680 \mathrm{~kg} / \mathrm{m}^{3}$ and $\mu=2.9 \times 10^{-4} \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}$. The density of mercury is $13,600 \mathrm{~kg} / \mathrm{m}^{3}$. Find the flow rate of gasoline in $\mathrm{m}^{3} / \mathrm{s}$ and gpm . If $P_{1}$, the upstream pressure is 1 atm , what is the pressure, $P_{2}$, at the throat of the Venturi meter? If the pressure loss is $10 \%$ of $\left(P_{1}-P_{2}\right)$, calculate the power loss.

Solution Calculate the velocity of gasoline at the throat using the following data

$$
\begin{aligned}
h & =0.035 \mathrm{~m} \\
D_{2} / D_{1} & =1 / 3 \\
\rho_{M} / \rho & =13,600 / 680=20
\end{aligned}
$$

Assume $C_{\mathrm{v}}=1.0$ and substitute into Equation (19.15)

$$
\begin{aligned}
v_{2} & =\sqrt{\frac{2(9.807)(0.035)(20-1)}{1-(1 / 3)^{4}}} \\
& =3.63 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Calculate the volumetric flow rate.

$$
\begin{aligned}
q & =\frac{\pi(0.02)^{2}}{4}(3.63) \\
& =0.00114 \mathrm{~m}^{3} / \mathrm{s} \\
& =18.1 \mathrm{gpm}
\end{aligned}
$$

Calculate $P_{2}$ from the manometer equation and the corresponding pressure loss, $\Delta P_{L}$.

$$
\begin{aligned}
P_{2} & =101,325-(9.807)(0.035)(13,600-680) \\
& =96,900 \mathrm{~Pa} \\
\Delta P & =P_{1}-P_{2} \\
& =101,325-96,900=4425 \mathrm{~Pa}
\end{aligned}
$$

For a $10 \%$ loss,

$$
\begin{aligned}
\Delta P_{L} & =0.1(4425) \\
& =442.5 \mathrm{~Pa}
\end{aligned}
$$

Calculate the power loss:

$$
\begin{aligned}
\dot{W}_{L} & =(0.00114)(442.5) \\
& =0.5 \mathrm{~W}=6.71 \times 10^{-4} \mathrm{hp}
\end{aligned}
$$

Illustrative Example 19.6 Refer to Illustrative Example 19.6. If gasoline has a vapor pressure of $50,000 \mathrm{~Pa}$, what flow rate will cause cavitation to occur?

Solution Set $P_{2}=p^{\prime}=50,000 \mathrm{~Pa}$ and use Equation (19.14):

$$
\begin{aligned}
v_{2} & =\sqrt{\frac{2 g_{c}\left(P_{1}-P_{2}\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}}=\sqrt{\frac{2(101,325-50,000)}{680\left[1-1^{4} /(1 / 3)^{4}\right]}} \\
& =12.36 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Also note that

$$
\begin{aligned}
q & =\frac{\pi 0.02^{2}}{4} 12.36 \\
& =0.0388 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

### 19.5 ORIFICE METER

Another device used for flow measurement is the orifice meter (see Fig. 19.7). The pressure difference is measured (often with a manometer) between the upstream (point 1) and the orifice (point 2). Although it operates on the same principles as a Venturi meter, orifice plates can be easily changed to accommodate a wide range of flow rates.

The orifice can be employed to determine either the volumetric flow rate from the pressure difference, ( $P_{1}-P_{2}$ ), or from the manometer head, $h$. For a horizontal orifice meter, the velocity equation is the same as the Venturi meter, that is,

$$
\begin{equation*}
v_{2}=C_{0} \sqrt{\frac{2 g_{c}\left(P_{1}-P_{2}\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}}=C_{\mathrm{o}} \sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho\left[1-\left(D_{2}^{4} / D_{1}^{4}\right)\right]}} \tag{19.20}
\end{equation*}
$$

The volumetric flow rate is once again:

$$
\begin{equation*}
q=\left(\frac{\pi D_{2}^{2}}{4}\right) v_{2} \tag{19.21}
\end{equation*}
$$

The orifice meter is simpler in construction and less expensive than a Venturi meter, and occupies less space. However, it has a lower pressure recovery (around 70\%). The discharge coefficient, $C_{0}$, for drilled-plate orifices is shown in Fig. 19.8 where $C_{0}$ is a function of $D_{2} / D_{1}$ and the Reynolds number at the throat, $\mathrm{Re}_{2}$. At $\mathrm{Re}_{2}$ values greater than 20,000 , the discharge coefficient, $C_{\mathrm{o}}$, is approximately constant at $0.61-0.62$.


Front View of Orifice Plate

Figure 19.7 Orifice meter.


Figure 19.8 Discharge coefficients for drilled plate orifices. $D_{2}$ is the orifice diameter, and $D_{1}$ is the pipe diameter. The abscissa is the Reynolds number based on the orifice conditions.

An approximate equation that can be used to estimate the pressure recovery in an orifice meter is ${ }^{(1)}$ :

$$
\begin{equation*}
\text { Percentage Pressure Recovery } \Delta P_{\mathrm{rec}}=14 \frac{D_{2}}{D_{1}}+80\left(\frac{D_{2}}{D_{1}}\right)^{2} \tag{19.22}
\end{equation*}
$$

Since

$$
\text { Percentage Pressure Loss }=100-\Delta P_{\text {rec }}
$$

the pressure loss in the orifice meter is:

$$
\begin{equation*}
\Delta P_{L}=\left(1-\frac{\text { Percentage Pressure Recovery }}{100}\right)\left(P_{1}-P_{2}\right) \tag{19.23}
\end{equation*}
$$

The power loss (or power consumption) due to the orifice meter is (in consistent units):

$$
\begin{equation*}
\dot{W}_{L}=q \Delta P_{L} \tag{19.24}
\end{equation*}
$$

Illustrative Example 19.7 An orifice meter equipped with flange taps is installed to measure the flow rate of air in a circular duct of diameter, $D_{1}=0.25 \mathrm{~m}$. The orifice diameter, $D_{2}=0.19 \mathrm{~m}$. The air is flowing at a rate of $1.0 \mathrm{~m}^{3} / \mathrm{s}$ at 1 atm . Under these conditions, the air density, $\rho$, is $1.23 \mathrm{~kg} / \mathrm{m}^{3}$ and the absolute viscosity, $\mu$, is $1.8 \times 10^{-5} \mathrm{~kg} /(\mathrm{m} \cdot \mathrm{s})$. Water is used as the manometer fluid. Calculate the pressure drop if the orifice discharge coefficient is 1.0 , the actual pressure drop and the manometer head.

Solution Calculate the velocity through the orifice.

$$
\begin{aligned}
v_{2} & =\frac{4(1)}{\pi(0.19)^{2}} \\
& =35.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Assuming $C_{o}=1$, calculate $\Delta P$ using Equation (19.20):

$$
\begin{aligned}
\Delta P & =\frac{(1.23)(35.3)^{2}\left[1-(0.19 / 0.25)^{4}\right]}{2} \\
& =510 \mathrm{~Pa}=52 \mathrm{~mm} \mathrm{H}
\end{aligned}
$$

The orifice Reynolds number is

$$
\operatorname{Re}=\frac{(1.23)(35.3)(0.19)}{1.8 \times 10^{-5}}=458,310
$$

And

$$
\frac{D_{2}}{D_{1}}=\frac{0.19}{0.25}=0.76
$$

The actual discharge coefficient, $C_{0}$, from Fig. 19.8 is

$$
C_{0}=0.62
$$

The actual pressure drop is calculated by once again rearranging Equation (19.20):

$$
\Delta P=\frac{510}{(0.62)^{2}}=1327 \mathrm{~Pa}=135 \mathrm{~mm} \mathrm{H}_{2} \mathrm{O}
$$

The percent pressure recovery and pressure loss may now be calculated.

$$
\begin{aligned}
\text { Percentage Pressure Recovery } & =14(0.76)+80(0.76)^{2}=56.8 \% \\
\text { Percentage Pressure Loss } & =43.2 \%
\end{aligned}
$$

The actual pressure loss, $\Delta P_{L}$, in the orifice meter is then

$$
\Delta P_{L}=0.432(1326.2)=573 \mathrm{~Pa}
$$

Illustrative Example 19.8 Refer to Illustrative Example 19.8. Calculate the actual power consumption of the orifice meter.

Solution Calculate the power consumption using Equation (19.18)

$$
\dot{W}_{L}=q\left(\Delta P_{L}\right)=(1.0)\left(\Delta P_{L}\right)=1.0(573)=573 \mathrm{~W}=0.77 \mathrm{hp}
$$

The reader should note that if the pressure drop or the manometer head is given, the calculation of the volumetric flow rate will usually involve trial-and-error.

Illustrative Example 19.9 Air at ambient condition is flowing at the rate of $0.50 \mathrm{lb} / \mathrm{s}$ in a 4-in ID pipe. What sized orifice would produce an orifice pressure drop of 10 in $\mathrm{H}_{2} \mathrm{O}$ ?

Solution This requires the simultaneous solution of Equations (19.6) and (19.7). At ambient conditions

$$
\rho=0.075 \mathrm{lb} / \mathrm{ft}^{3}
$$

The volumetric flowrate is therefore

$$
\begin{aligned}
q & =\frac{(0.5)}{(0.075)} \\
& =6.67 \mathrm{ft}^{3} / \mathrm{s}
\end{aligned}
$$

From Equation (19.16)

$$
v_{2}=q /\left(\pi D_{2}^{2} / 4\right)
$$

From Equation (19.17)

$$
v_{2}=C_{v} \sqrt{\frac{2 g h\left(\rho_{M}-\rho\right)}{\rho\left[1-\left(D_{2} / D_{1}\right)^{4}\right]}}
$$

Equating the above two equations, and solving by trial-and-error (assuming $C_{\mathrm{v}}=$ 0.61 ) gives $D_{2} \cong 2.825 \mathrm{in}$.


Figure 19.9 Constriction meter.

Is the assumption of $C_{\mathrm{v}}=0.61$ reasonable? This is left as an exercise for the reader. [Hint: Check to see if $\operatorname{Re}>20,000$ at the throat.]

Another way to measure the fluid flow rate in a pipe is to insert a long constriction (of smaller diameter than the pipe) inside the pipe and measure the pressure drop (or head loss) across the constriction (as shown in Fig. 19.9). The flow goes through a sudden contraction when it enters the constriction and through a sudden expansion as it exits the constriction. "Major" loss due to friction may be neglected. Only "minor" losses due to sudden expansion and sudden contraction are considered in the calculation of the flow rate. The calculation of the head loss follows the method outlined earlier for a sudden contraction and expansion.

The term flowmeter is sometimes used to designate any restricted opening or tube through which the rate of flow has been determined by calibration. For example, a 6 -inch pipe may be tapered down to 2 inches and then enlarged back to 6 inches. The pressure drop through this "opening" provides a measure of the rate of flow, but this relation should be determined by calibration.

### 19.6 SELECTION PROCESS

Several factors should be considered in selecting a flow measurement device. Engineering decisions on the selection process should consider the following:

1. Is the fluid phase a gas or liquid?
2. The range (or capacity) of the device
3. Accuracy
4. Desired readout
5. Fluid properties
6. Internal environment
7. External environment
8. Capital cost
9. Operating cost
10. Reliability

Considering the complexity and diversity of flow meter measuring devices and the wide range of flow conditions encountered in industrial applications, one should carefully compare the different options that are available before purchasing a device. It should be noted that it is possible that a number of devices may be suitable for a given application.

## REFERENCE

1. I. Farag, "Fluid Flow," A Theodore Tutorial, Theodore Tutorials, East Williston, NY, 1996.

NOTE: Additional problems are available for all readers at www.wiley.com. Follow links for this title.

