21

ACADEMIC APPLICATIONS

The illustrative examples provided in this chapter pertain to academic applications. Those readers desiring more technical and industry-oriented calculations should bypass these examples and proceed directly to the next chapter. There are 18 Illustrative Examples in this chapter; several of the earlier examples are qualitative in nature.

Illustrative Example 21.1 Qualitatively explain pipe schedule number.

Solution The wall thickness of a pipe is specified by a schedule number, which is a function of the internal pressure and allowable stress. The describing equation is:

$$\text{Schedule number} = 1000\ P/S$$

where P = internal working pressure ($\text{lb}_\text{f}/\text{in}^2$ gauge)
S = allowable stress ($\text{lb}_\text{f}/\text{in}^2$)

There are *10* schedule numbers in use and these range from *10* to *160*; the thickness of the pipe wall increases with the schedule number. Schedule 40 is the most commonly used pipe thickness for normal temperature and pressure applications.

Illustrative Example 21.2 Selecting the appropriate pipe diameter to handle a particular liquid flow application is a function of many varieties. Provide information on suggested pipe diameters for various flow ranges.[1]

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

Table 21.1 Nominal pipe diameters for liquid flows

Capacity (gpm)	Nominal Pipe Diameter (in.)
0–15	1
15–70	2
70–150	3
150–250	4

Solution Typical pipe diameters for various liquid flow capacities are given in Table 21.1.

Illustrative Example 21.3 Provide information on how one would estimate the minimum pipe thickness required for a particular application.

Solution In calculating the minimum thickness of pipe wall (t) required for any specific pressure, temperature, and corrosive condition, the following formula may be employed for estimation purposes

$$t = \frac{PD_o}{2S} + C$$

where t = minimum pipe wall thickness allowable on inspection, in.
P = maximum internal service pressure, lb/in.^2
D_o = outside diameter of the pipe, in.
S = allowable bursting stress in the pipe material, lb/in.^2
C = allowance for threading, mechanical strength and corrosion.

Appropriate values for S and C are available in the literature.[2]

Illustrative Example 21.4 Provide a qualitative discussion of pipes and tubing.

Solution The manufacturing of various classes of pipe uses many materials including ceramic, metal, and plastic. Pipes and tubing are used as conduits for transporting liquids and gases. In the past, piping materials have included wood and lead. However, current materials include ceramic, metal, and plastic. Metal pipes are commonly made from unfinished black (lacquer) or galvanized steel, brass, and ductile iron. Other sources of metal pipes include copper, which is popular for plumbing systems. Plastic tubing is widely used for its light weight, chemical resistance, non-corrosive properties, and ease of making connections. Raw materials include polyvinyl chloride (PVC), polyvinyl dichloride (CPVC), polyethylene (PE), polybutylene (PB), and acrylonitrile butadiene styrene (ABS). Ceramic pipes are usually used for low pressure applications such as gravity flow or drainage.

Commonly used steel pipe ratings are Schedule 40 (standard) and Schedule 80 (extra strong). In most cases in the U.S., Schedule 40 piping is used for heating applications, while Schedule 80 is employed for high pressure applications or cases where higher than normal corrosion rates are expected.

Illustrative Example 21.5 Briefly describe the following four classes of fittings.

1. Fittings that extend or terminate pipe runs
2. Fittings that change a pipe's direction
3. Fittings that connect two or more pipes
4. Fittings that change pipe size

Solution

1. Couplings extend a run by connecting two lengths of pipe. They are available in all standard pipe sizes and nearly all varieties of pipe. They are called reducing couplings if they are connecting differently-sized pipe.
   Caps and plugs end a run of pipe by closing it off with a watertight seal.
2. Elbows change the direction of pipes. The most commonly used are 90° and 45° elbows, but they are also available in other sizes. They are identified by their angle but they are ordinarily referred to by number only. An elbow may be female at both ends, or in the case of a street elbow, may be male on one end and female on the other.
3. Tees offer the most varieties of any type of fitting. Tees are fittings in the shape of a "T" where the top of the "T" is the continuous pipe run, and the vertical section is a branch connected to it. They may be reducing tees, where the branch and/or one end of the through section is a smaller diameter than the inlet. They may have side-inlets, which allow for a fourth pipe to join them. These may be left- or right-handed depending on which side the inlet enters.
4. Reducers can be couplings, tees, or elbows, where one end is smaller than the other. This reduces the pipe's diameter between the inlet and outlet. In the case of fittings that connect more than two pipes, one of the outlets is of a smaller diameter, (not counting side-inlets, which are always smaller). Some reduce pipe only one size; others can reduce several sizes. Both ends are female.
   Bushings serve the same purpose as reducers except that they have one male and one female end. In steel pipe, they are threaded inside and out; instead of screwing directly onto pipe threads, they screw into a coupling and pipe is threaded into them. They are virtually invisible once installed. In PVC and copper, they are not threaded but work the same way.
   Expanders serve the opposite purpose of reducers. They increase the pipes diameter between inlet and outlet.

Illustrative Example 21.6 List the typical uses and applications for each of the following six valves, plus their advantages and disadvantages.

1. Gate
2. Globe
3. Ball
4. Butterfly
5. Pinch
6. Plug

Solution

1. Gate valve

   Recommended uses:
   1. Fully open/closed, non-throttling
   2. Infrequent operation
   3. Minimal fluid trapping in line

   Applications: Oil, gas, air, slurries, heavy liquids, steam, noncondensing gases, and corrosive liquids

   Advantages:
   1. High capacity
   2. Tight shutoff
   3. Low cost
   4. Little resistance to flow

   Disadvantages:
   1. Poor control
   2. Cavitate at low pressure drops
   3. Cannot be used for throttling
2. Globe valve

   Recommended uses:
   1. Throttling service/flow regulation
   2. Frequent operation

   Applications: Liquids, vapors, gases, corrosive substances, slurries

   Advantages:
   1. Efficient throttling
   2. Accurate flow control
   3. Available in multiple ports

   Disadvantages:
   1. High pressure drop
   2. More expensive than other valves

3. Ball valve
   Recommended uses:
   1. Fully open/closed, limited-throttling
   2. Higher temperature fluids

   Applications: Most liquids, high temperatures, slurries
   Advantages:
   1. Low cost
   2. High capacity
   3. Low leakage and maintenance
   4. Tight sealing with low torque

   Disadvantages:
   1. Poor throttling characteristics
   2. Prone to cavitation
4. Butterfly valve
   Recommended uses:
   1. Fully open/closed or throttling services
   2. Frequent operation
   3. Minimal fluid trapping in line

   Applications: Liquids, gases, slurries, liquids with suspended solids
   Advantages:
   1. Low cost and maintenance
   2. High capacity
   3. Good flow control
   4. Low pressure drop

   Disadvantages:
   1. High torque required for control
   2. Prone to cavitation at lower flows
5. Pinch valve
   Recommended uses:
   1. Fully open/closed, or throttling services
   2. Abrasives and corrosives

   Applications: Medical, pharmaceutical, wastewater, slurries, pulp, powder and pellets
   Advantages:
   1. Streamlined flow
   2. High coefficient of flow

   Disadvantages:
   1. Limited materials
   2. Low shut-off capabilities
   3. Low pressure limits

6. Plug valve

   Recommended uses:

   1. Fully open/closed, non-throttling
   2. Maintain flow

   Applications: Sewage, sludge, and wastewater

   Advantages:

   1. Easy operation
   2. Medium to high flow
   3. Good shut off

   Disadvantages:

   1. Low cleanliness
   2. Inability to handle slurry

Illustrative Example 21.7 If water at 70°F is flowing through a 3/8 in schedule 40 brass pipe at a volumetric flow rate of 2.0 gpm, calculate the Reynolds number. Also determine whether the flow is in the laminar or turbulent region.

Solution From Table A.5 in the Appendix,

D_i (3/8 inch pipe) = 0.493 in = 0.0411 ft
S_i (3/8 inch pipe) = 0.00133 ft^2

From Table A.4 in the Appendix,

μ (at 70°F) = 0.982 cP = 6.598×10^{-4} lb/ft · s
$\rho = 62.4$ lb/ft^3

The volumetric flow rate is converted to ft^3/s by noting that 1 gpm = 0.00228 ft^3/s.

$$\begin{aligned} q &= (2.0)(0.00228)\ \text{ft}^3/\text{s} \\ &= 0.00456\ \text{ft}^3/\text{s} \end{aligned}$$

The velocity of the fluid is calculated by dividing the volumetric flow rate by the cross-sectional area of the 3/8 inch pipe:

$$\begin{aligned} v &= \frac{q}{S} \\ &= \frac{0.00456}{0.00133} \\ &= 3.43\ \text{ft/s} \end{aligned}$$

The Reynolds number is then calculated by employing Equation (12.1).

$$\begin{aligned}\mathrm{Re} &= \frac{Dv\rho}{\mu}\\ &= \frac{(0.0411)(3.43)(62.4)}{0.0006598}\\ &= 13{,}330\end{aligned}$$

Since the Reynolds number is above 2100, the flow of the water is in the turbulent region.

Illustrative Example 21.8 Part of a Fluid Flow Unit Operations experiment at Manhattan College requires the following calculation. Determine the Reynolds numbers for water flows of 6.0 gpm and 1.5 gpm through 3/8 in and 1/2 in schedule 40 pipes.

Solution For water,

$$\rho = 62.4\ \mathrm{lb/ft^3}$$

and

$$\mu = 6.72 \times 10^{-4}\ \mathrm{lb/ft \cdot s}$$

For 3/8 inch and 1/2 inch schedule 40 pipe, the outside and inside diameters can again be found in the Appendix and provided below.

Pipe size	D_o (in)	D_i (in)	D_o (ft)	D_i (ft)
3/8 in.	0.675	0.493	0.0563	0.041
1/2 in.	0.84	0.622	0.07	0.0518

Use the following equation to calculate the average velocity:

$$\begin{aligned}v = q/S &= (\mathrm{gpm})\left(\frac{35.3}{264}\right)\left(\frac{1}{60}\right)\left(\frac{(4)(144)}{\pi D_i^2}\right)\\ &= (0.409)(\mathrm{gpm})/(D_i)^2\end{aligned}$$

where $v = \mathrm{ft/s}$
$q = \mathrm{gal/min}$
$D_i = \mathrm{inches}$

For a 3/8 in pipe with an inside diameter of 0.493 inches, the results are

q	v
1.5	2.52
6.0	10.1

For a 1/2 in. pipe with an inside diameter of 0.622 inches, the results are

q	v
1.5	1.58
6.0	6.33

Use Equation (12.1) for the calculation of the Reynolds number

$$\mathrm{Re} = Dv\rho/\mu$$

and the properties of water at room temperature provided above. The following values result:

For a 3/8 in. pipe and 1.5 gpm, Re = 9594
For a 3/8 in. pipe and 6 gpm, Re = 38,452
For a 1/2 in. pipe and 1.5 gpm, Re = 7600
For a 1/2 in. pipe and 6 gpm, Re = 30,447

This indicates that all of the flows are turbulent although for smaller pipes the result could approach the "intermediate" regime.

Illustrative Example 21.9 If water is flowing in an upward 15 ft vertical pipe in a 3/8 in. schedule 40 brass pipe, calculate the frictional loss using the Bernoulli equation if the pressure drop of the flowing fluid from bottom to top is 4.5 $\mathrm{lb_f/ft^2}$.

Solution The Bernoulli equation is employed. The velocity head is neglected since the water is flowing through a straight vertical pipe and the velocity can be assumed not to change. From Equation (17.11), with location 1 at the bottom of the pipe

$$\frac{P_2 - P_1}{\rho} + \frac{g}{g_c}(z_2 - z_1) + \cancelto{0}{\frac{v_2^2 - v_1^2}{2g_c}} = h_f$$

Substituting the information provided,

$$\begin{aligned} h_f &= (-4.5)/(62.4) + (32.2/32.2)(15 - 0) \\ &= -0.0721 + 15 \\ &= 14.9\ \mathrm{ft \cdot lb_f/lb} \end{aligned}$$

The frictional loss of the vertical pipe is therefore 14.9 $\mathrm{ft \cdot lb_f/lb}$. Note that the contribution of the "pressure" loss is negligible in comparison to potential or hydrostatic head.

Illustrative Example 21.10 A centrifugal pump is needed to transport water from sea level to 10,000 feet above sea level. Atmospheric pressure at 10,000 feet is 10.2 psi. Using a mass flow rate of 50 lb/s and an efficiency of no less than 65%, determine the actual pump work in $\text{ft} \cdot \text{lb}_f/\text{s}$ and hp. Neglect frictional losses.

Solution Using Equations (13.4) or (17.11) and neglecting kinetic energy effects and frictional losses, one may write

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2g_c} + \Delta z\left(\frac{g}{g_c}\right) - h_s + h_f = 0$$

$$h_s = \frac{P_2 - P_1}{\rho} + \cancelto{0}{\frac{v_2^2 - v_1^2}{2g_c}} + (z_2 - z_1)\frac{g}{g_c} + \cancelto{0}{h_f/\eta_p}$$

where P_2 = pressure at 10,000 ft ($z_2 = 10{,}000$ ft)
P_1 = atmospheric pressure at sea level conditions ($z_1 = 0$ ft)
ρ = density of water
η_p = frictional efficency of pump

Plugging in for these values yields

$$\begin{aligned} h_s &= \frac{(10.2 - 14.7)(144)}{62.4} + 10{,}000 - 0 \\ &= -10 + 10{,}000 \\ &= 9990 \text{ ft} \cdot \text{lb}_f/\text{lb} \\ &= (9990)(50) \\ &= 499{,}500 \text{ ft} \cdot \text{lb}_f/\text{s} \\ &= 499{,}500/550 \\ &= 908 \text{ hp} \end{aligned}$$

The above value represents the work delivered by the pump to the fluid (water). The actual pump work is calculated by dividing the above terms by the frictional efficiency. For example,

$$\begin{aligned} W_p &= 908/0.65 \\ &= 1397 \text{ hp} \end{aligned}$$

Finally the reader should note that there is a credit of "pressure" energy since the discharge pressure is lower than the input (inlet) pressure.

Illustrative Example 21.11 What is the maximum water velocity allowed in a pipe length of 150 ft, given a required pressure drop of no more than $5 \text{ lb}_f/\text{in}^2$.

Solution First a calculation of the Reynolds number is required. However, the velocity is not known, so a trial-and-error method will be employed using the

Table 21.2 Trial-and-error calculations for Illustrative Example 21.11

v (ft/s)	Re	f	ΔP (lb/ft^2)	ΔP (lb/in^2)
3	5060	0.00956	965	7.00
2	3370	0.0107	480	3.33
1	1690	0.0130	146	1.01
1.5	2530	0.0116	292	2.0

following three equations:

$$\mathrm{Re} = \frac{Dv\rho}{\mu} \tag{12.1}$$

$$\Delta P = \frac{4f\rho vL}{2D_i g_c} \tag{14.3}$$

$$f = 0.0014 + \frac{0.125}{\mathrm{Re}^{0.32}}$$

The trial-and-error calculations are provided in the Table 21.2.

As can be seen from Table 21.2, a maximum velocity between 2 and 3 ft/s will allow for a pressure drop of no more than 5.0 lb$_f$/in^2 (psi). The exact maximum allowable velocity can be shown to be 2.53 ft/s with a corresponding Re and ΔP of 4257 and 5.00 psi, respectively.

Illustrative Example 21.12 Refer to Illustrative Example 21.4. If the pipe contains two globe valves and one straight through tee, what is the friction loss?

Solution One must now include the friction losses for each fitting in the calculation. Employ Equation (18.14).

$$h_f = \left(4f\frac{L}{D} + 2K_{f,\mathrm{globe}} + K_{f,\mathrm{tee}}\right)\frac{v^2}{2g_c}$$

where $K_{f,\mathrm{globe}} = 6.0$
$K_{f,\mathrm{tee}} = 0.4$

The first term represents the frictional loss associated with the pipe; this value was previously "calculated" to be 5.0 psia. Subtracting the above equation gives:

$$\begin{aligned} h_f &= (5.0)(144/62.4) + [(2)(6.0) + 0.4](2.53)^2/(2)(32.2) \\ &= 11.5 + 1.23 \\ &= 12.73\ \mathrm{ft\cdot lb_f/lb} \end{aligned}$$

Illustrative Example 21.13 A Pitot tube is inserted in a 55 mm diameter circular pipe to measure the flow velocity. The tube is inserted so that it points upstream into the flow and the pressure sensed by the probe is the stagnation pressure. The static pressure is measured at the same location in the flow using a wall pressure tap. The change in elevation between the tip of the Pitot tube and the wall pressure tap is negligible. The flowing fluid is soybean oil at 20°C and the fluid in the manometer tube is mercury. Refer to Fig. 21.1. Is the height, h, correct or should the manometer fluid be higher on the left side? If the magnitude of h is 40 mm, determine the flow speed. If it is assumed that the velocity is uniform across the cross-section of the 55 mm pipe, what is the mass flow rate of the fluid? What is the flow type (laminar or turbulent)? Assume $\rho_{\text{oil}} = 919\,\text{kg/m}^3$, $\mu_{\text{oil}} = 0.04\,\text{kg/m}\cdot\text{s}$ and $\rho_M = 13{,}600\,\text{kg/m}^3$.

Solution Note that point 2 is a stagnation point. Thus, $P_2 > P_1$, and the manometer fluid should be higher on the left side ($h < 0$).

Calculate the flow velocity, given that $h = 40$ mm of mercury. Use the Pitot tube equation. See Equation (19.9) and assume $C = 1.0$.

$$v = \sqrt{2gh\left(\frac{\rho_M}{\rho} - 1\right)} = \sqrt{2(9.804)(0.04)\left(\frac{13{,}600}{919}\right) - 1}$$
$$= 3.29\,\text{m/s}$$

Assuming a uniform velocity, calculate the mass flow rate

$$\dot{m} = \rho q = \rho v S = (919)(3.29)\pi(0.055)^2/4$$
$$= 7.18\,\text{kg/s}$$

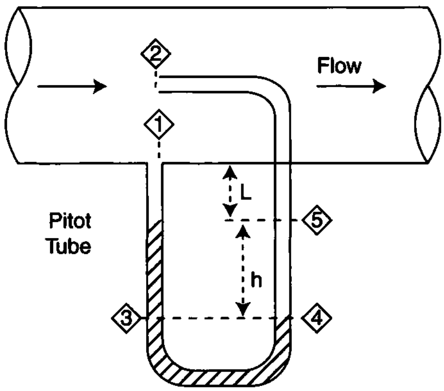

Figure 21.1 Pitot tube.

Calculate the Reynolds number

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{(0.055)(3.29)(919)}{0.04}$$
$$= 4160$$

The flow is therefore turbulent.

Illustrative Example 21.14 Given a 50 ft pipe with flowing water, determine the flow rate if there is an expansion from 3/8 inch to 1/2 inch and immediately back to 3/8 inch with an overall pressure loss no greater than 2 $\mathrm{lb_f/ft^2}$.

Solution The flow rate will need to be determined using trial-and-error. Use the equation:

$$h_f = \left(4f\frac{L}{D} + K_e + K_c\right)\frac{v^2}{2g_c} \tag{18.14}$$

with

$$K_e = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 = \left(1 - \frac{S_1}{S_2}\right)^2 \tag{18.8}$$

$$K_c = 0.4\left[1 - \left(\frac{D_2}{D_1}\right)^2\right] = 0.4\left(1 - \frac{S_2}{S_1}\right) \tag{18.10}$$

Since (see Table A.5 in the Appendix) $S_{3/8} = 0.00133\ \mathrm{ft^2}$ and $S_{1/2} = 0.00211\ \mathrm{ft^2}$.
A trial-and-error calculation is again required for the equation:

$$h_f = \left(4f\frac{50}{0.03125} + 0.06224 - 0.13334\right)\frac{v^2}{2(32.2)}$$

Results are provided in Table 21.3.

Table 21.3 Friction loss calculation for Illustrative Example 21.14

v	Re	f	h_f
10	14083.64	0.007279	72.22989
8	11266.91	0.007714	48.99488
6	8450.186	0.008323	29.73801
4	5633.457	0.009282	14.74203
2	2816.729	0.01124	4.463628
0.5	704.1821	0.016734	0.415472

The velocity for this piping system is estimated to be 1.93 ft/s by linear interpolation.

Illustrative Example 21.15 Refer to Illustrative Example 21.14. How would the velocity differ if an orifice meter were used instead?

Solution If using an orifice meter instead of the expansion and contraction, the following equation would be used:

$$v_2 = C_o\sqrt{\frac{2gh(\rho_M - \rho)}{\rho[1 - (D_2/D_1)^4]}} \tag{19.17}$$

Obviously, the calculation cannot be performed since the diameter of the orifice has not been specified. One would expect the velocity to be lower since the orifice offers more resistance to flow.

Illustrative Example 21.16 Water flows at a velocity of 0.02 m/s in a concrete pipe (diameter, $D_p = 1.5$ m; length, $L_p = 20$ m; roughness, $k_p = 0.003$ m). This prototype is to be modeled in a lab using a 1/30th scale pipe (diameter, D_m; length, L_m; roughness, k_m). The fluid in the model is castor oil ($\rho_m = 961.3\ \text{kg/m}^3$, $\mu_m = 0.0721$ kg/m · s). Determine the model dimensions, the velocity of castor oil (v_m), and, if the pressure drop in the model (ΔP_m) is measured to be 100 kPa, what is the pressure drop (ΔP_p) in the prototype?

Solution Refer to Chapter 3. Achieve geometric similarity using a circular pipe of 1/30 the scale:

$$\frac{D_m}{D_p} = \frac{D_m}{1.5} = \frac{1}{30}$$

$$D_m = 0.05\ \text{m} = 5\ \text{cm}$$

Achieve dynamic similarity using the dimensionless ratios k/D and L/D, plus the Reynolds number:

$$k_m = k_p\frac{D_m}{D_p} = 0.0001\ \text{m}$$

$$L_m = L_p\frac{D_m}{D_p} = 0.667\ \text{m}$$

$$\text{Re} = \frac{\rho_m v_m D_m}{\mu_m} = \frac{\rho_p v_p D_p}{\mu_p}$$

For water, $\rho_p = 1000\,\text{kg/m}^3$ and $\mu_p = 0.001\,\text{kg/m}\cdot\text{s}$. Therefore,

$$\frac{(961.3)v_m(0.05)}{0.0721} = \frac{(1000)(0.02)(1.5)}{0.001}$$

$$v_m = 45\,\text{m/s}$$

Finally, calculate ΔP in the prototype noting that $\text{Eu}_m = \text{Eu}_p$. See Illustrative Example (2.2).

$$\frac{\Delta P_m}{0.5\rho_m v_m{}^2} = \frac{\Delta P_p}{0.5\rho_p v_p{}^2}$$

$$\frac{10^5}{0.5(961.3)(45)^2} = \frac{\Delta P_p}{0.5(1000)(0.02)^2}$$

$$\Delta P_p = 0.0206\,\text{Pa}$$

Illustrative Example 21.17 Air at 75°F and 1 atm (kinematic viscosity = 0.00016 ft^2/s) flows at a rate of 4800 cfm in an inclined (1 ft × 2 ft) commercial steel rectangular duct (Fig. 21.2). The duct is 1000 ft long and is inclined upward at 5° to the horizontal. Neglect "minor losses" and assume fully developed flow.

Calculate the equivalent diameter, the Reynolds number, the pressure drop in psf and psi, the ideal power requirement to move the air through the duct, and the percent of the total power due to friction. Also determine the brake (or actual) horsepower (bhp) of the blower if the efficiency is 60%.

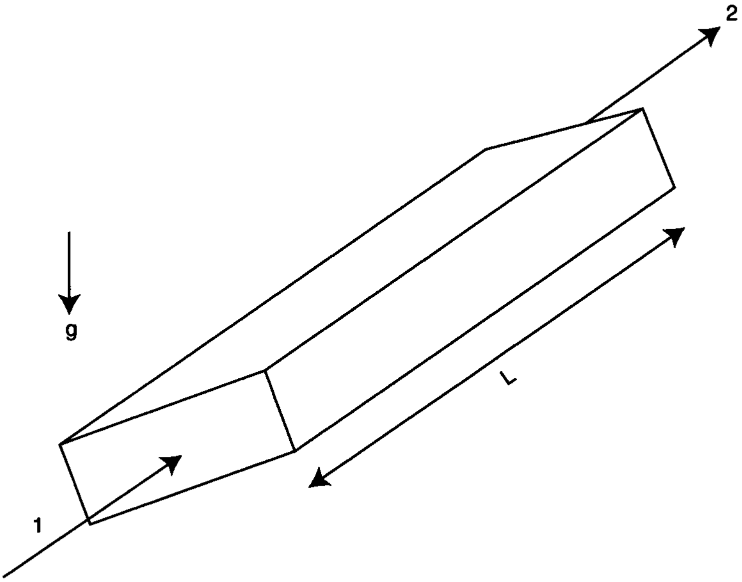

Figure 21.2 Inclined rectangular duct.

Solution Calculate the equivalent diameter.

$$D_{eq} = \frac{4S}{l_p} = \frac{4(1)(2)}{2(1+2)} = 1.333\,\text{ft}$$

Obtain the air density from the ideal gas law.

$$\rho = \frac{P(\text{MW})}{RT} = \frac{(1)(28.9)}{(0.7302)(460+75)} = 0.074\,\text{lb/ft}^3$$

Calculate flow velocity.

$$v = \frac{q}{S} = \frac{4800/60}{(1)(2)} = 40\,\text{ft/s}$$

The Reynolds number is:

$$\text{Re} = \frac{D_{eq}v}{\nu} = \frac{(1.333)(40)}{0.00016} = 3.33 \times 10^5$$

Therefore, the flow is in the turbulent regime.
From Table 14.1, $k = 0.00015$ ft. Thus,

$$k/D = 0.00015/1.333 = 0.000113$$

Obtain the Fanning friction factor from the Chart in Fig. 14.2

$$f = 0.00375$$

Calculate the head loss using the Hagen–Poiseuille (Darcy–Weisbach) equation (see Eq. (14.3)) expressing the friction/head loss in feet of flowing fluid rather than energy/mass.

$$\begin{aligned} h_f' &= 4f\frac{L}{D_{eq}}\frac{v^2}{2g} = 4(0.00375)\left(\frac{1000}{1.333}\right)\left(\frac{40^2}{2(32.174)}\right) \\ &= 280\,\text{ft of air} \end{aligned}$$

Apply Bernoulli's equation at the entrance and exit of the conduit. Note that the pipe is inclined, there is no shaft work, no (minor) head loss, and $v_1 = v_2$. Therefore,

$$\frac{P_1}{\rho}\frac{g_c}{g} = \frac{P_2}{\rho}\frac{g_c}{g} + h_f' + (z_2 - z_1)$$

$$P_1 - P_2 = \rho\frac{g}{g_c}(h_f' + z_2 - z_1)$$

Since

$$z_2 - z_1 = 1000\sin(5^\circ) = 87.2\,\text{ft}$$

ΔP can now be calculated

$$\Delta P = 0.074(1)(280 + 87.2) = 27.17\,\text{psf} = 0.19\,\text{psi}$$

The fluid power requirement is

$$\dot{W}_s = q\Delta P = 80(27.17) = 2173.6\,\frac{\text{ft}\cdot\text{lbf}}{\text{s}} = 3.95\,\text{hp}$$

Note that

$$h_f{}' = 280\,\text{ft of air}$$

Illustrative Example 21.18 Water is drawn from a reservoir and pumped through an equivalent length of 2 miles of a horizontal, circular concrete duct of 10 inch ID. At the end of the duct, the flow is divided into a network consisting of a 4 inch and a 3 inch ID pipe. The 4 inch line has an equivalent length of 200 ft and rises to a point 50 ft above the surface of the water in the reservoir, where the flow discharges to the atmosphere. This flow must be maintained at a rate of 1000 gal/min. The 3 inch line discharges to the atmosphere at a point 700 ft from the junction at the level of the surface of the water in the reservoir. Outline how to calculate the horsepower input to the pump, which has an efficiency of 70%.

Solution A diagram of the system is shown in Fig. 21.3. The solution requires resolving five equations—three mechanical energy balances, one continuity and one pressure drop relationship that applies to the piping network. The three mechanical energy balance equations are

$$\frac{\Delta P_2}{\rho} + 50 + \left(\frac{4fLv^2}{2g_cD}\right)_2 = 0 \tag{1}$$

$$\frac{\Delta P_3}{\rho} + \left(\frac{4fLv^2}{2g_cD}\right)_3 = 0 \tag{2}$$

$$\frac{\Delta P_1}{\rho} + \left(\frac{4fLv^2}{2g_cD}\right)_3 = \eta_p W_p = h_s \tag{3}$$

The continuity equation,

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \tag{4}$$

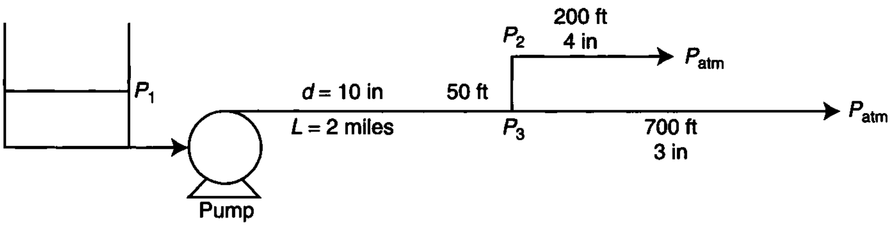

Figure 21.3 Application 21.18.

The pressure relationship is,

$$\Delta P_2 = \Delta P_3 \tag{5}$$

where

$$\Delta P_2 = P_2 - P_{atm}$$
$$\Delta P_3 = P_3 - P_{atm}$$
$$\Delta P = P_2 - P_1$$
$$P_1 = P_{atm}$$

There are five unknowns—P_2, P_3, v_2, v_3 and W_p. The outline of the solution follows.

1. Calculate v_2 from the continuity equation
2. Calculate ΔP_2 from Equation (1).
3. Set $\Delta P_3 = \Delta P_2$.
4. Calculate v_3 from Equation (2). This will require a trial-and-error procedure since both v_3 and Re_3 (and f_3) are not known.
5. Calculate v_1 from the continuity equation since both v_2 and v_3 are known.
6. Calculate $\dot{W}_s$ in Equation (3)

$$(z_2 - z_1) = 87.2\,\text{ft of air}$$

so that

$$h_f{}' + (z_2 - z_1) = 367.2\,\text{ft of air}$$

Therefore, the percent of the energy loss due to friction is

$$280/367.2 = 0.762 = 76.2\%$$

Finally,

$$\text{bhp} = \frac{\dot{W}_s}{E} = \frac{3.93}{0.6}$$
$$= 6.56\,\text{hp}$$

REFERENCES

1. J. Santoleri, J. Reynolds, and L. Theodore, "Introduction to Hazardous Waste Incineration," 2nd edition, John Wiley and Sons, Hoboken, NJ, 2000.
2. C. E. Lapple, "Fluid and Particle Dynamics," University of Delaware, Newark, Delaware, 1951.

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