25

POROUS MEDIA AND PACKED BEDS

25.1 INTRODUCTION

The flow of a fluid through porous media and/or a packed bed occurs frequently in chemical process applications. Some examples include flow through a fixed-bed catalytic reactor, flow through an adsorption tower, and flow through a filtration unit. An understanding of this type of flow is also important in the study of fluidization and some particle dynamics applications.(1)

This introductory section provides key definitions in this area and information on porous media flow regimes. The chapter concludes with six Illustrative Examples.

A porous medium consists of a solid phase with many void spaces that some refer to as pores. Examples include sponges, paper, sand, and concrete. As indicated above, *packed beds* (see Fig. 25.1) or porous materials are used in a number of engineering operations. These porous media are divided into two categories.

1. Impermeable media: solid media in which the pores are not interconnected, e.g., foamed polystyrene.
2. Permeable media: solid media in which the pores are interconnected, e.g., packed columns and catalytic reactors.

Before proceeding to the study of fluid flow through a porous medium (e.g., packed bed) the following key variables are defined.

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

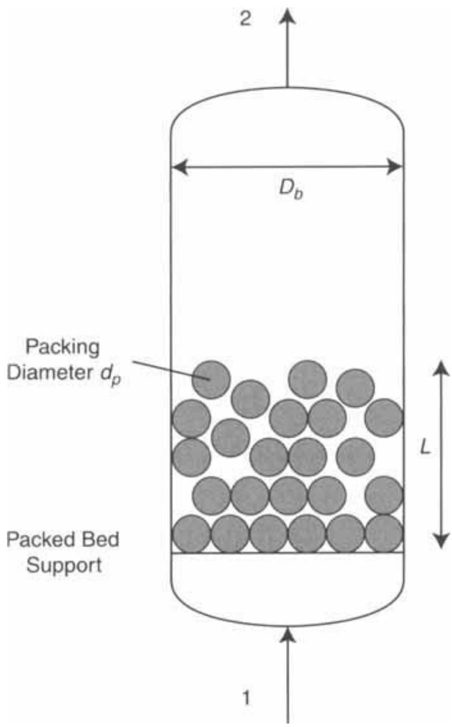

Figure 25.1 Packed bed.

25.2 DEFINITIONS

By definition, the *porosity* or *void fraction*, ε, is

$$\varepsilon = \frac{V_f}{V} = \frac{S_f}{S} \tag{25.1}$$

where V_f is the void volume (occupied by the fluid), V is the total system volume, S_f is the cross-sectional area of the void space, and S is the total cross-sectional area. Bed porosities are fractional numbers that range from approximately 0.4 to 0.6. *Solid fraction* is the fraction of the total bed volume occupied by solids and given by $1 - \varepsilon$. The *empty bed cross-section* (S) is

$$S = \frac{\pi D_b{}^2}{4} \tag{25.2}$$

where D_b is the bed diameter (see Fig. 25.2). The *actual bed cross-section* for flow (*open or void bed cross-section*), S_f, is

$$S_f = \frac{\pi D_b{}^2 \varepsilon}{4} \tag{25.3}$$

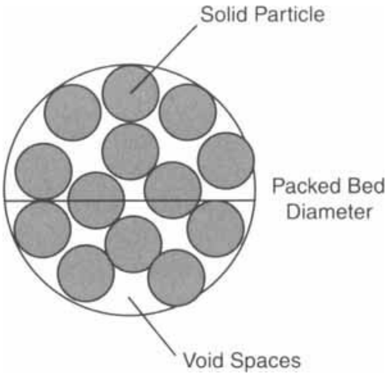

Figure 25.2 Packed bed cross-section.

The *ultimate* or *true density*, ρ_s, is the density of the solid material. The *bulk density*, ρ_B,

$$\rho_B = (1 - \varepsilon)\rho_s + \varepsilon\rho_f \tag{25.4}$$

where ρ_f is the fluid density. The *superficial velocity* (empty-tower velocity), v_s, is (with this subscript is referring to superficial, not solid)

$$v_s = \frac{q}{S} = \frac{4q}{\pi D_b{}^2} \tag{25.5}$$

The superficial velocity may be thought of as the average fluid velocity based on an empty cross-section. The *interstitial velocity*, v_I, is the actual velocity of fluid through pores

$$v_I = \frac{q}{S_v} = \frac{q}{\pi D_b{}^2 \varepsilon} = \frac{v_s}{\varepsilon} \tag{25.6}$$

The particle size, d_p, is expressed in different units depending on the size range involved. These units used are summarized in Table 25.1 with relation to particle size. *Standard screen mesh size* is often used to measure particles in the range of 3 inches to about 0.0015 inches. The testing sieves are standardized and have square openings.

Table 25.1 Measurement units of particle sizes

Particle Size	Measurement Units
Coarse particles	Inches or centimeters
Fine particles	Standard screen mesh size
Very fine particles	Micrometers or nanometers
Ultrafine particles	Specific surface area, or surface area per unit mass

A common group of screens is the Tyler standard screen scales, listed in Table 25.2. As noted in Chapter 23, a common method of specifying large particle sizes is to designate the screen mesh that has an aperture corresponding to the particle diameter. Since various screen scales are in use, confusion may result unless the screen scale involved is specified. The screen mesh generally refers to the number of screen openings per unit of length or area. The aperture for a given mesh will depend on the wire size employed. An expanded version, the Tyler Standard Screen Scales in Table 23.2, including English units, is provided in Table 25.2.

The clear space between the individual wires of the screen is termed the screen aperture. As indicated above, the term *mesh* is applied to the number of apertures per linear inch; for example, a 10-mesh screen will have 10 openings per inch, and the aperture will be 0.1 in. minus the diameter of the wire.

The *particle specific surface area*, a_p, is the surface area per unit volume of the packed bed particles. For a spherical particle of diameter d_p,

$$a_p = \frac{S_p}{V_p} = \frac{\pi d_p^{\,2}}{(\pi/6)d_p^{\,3}} = \frac{6}{d_p} \tag{25.7}$$

The *bed specific surface area* (a_b) is

$$a_b = a_p(1-\varepsilon) = \frac{6(1-\varepsilon)}{d_p} \tag{25.8}$$

The *effective diameter* ($d_{p,e}$) of non-spherical particles is the diameter of a sphere that has the same specific surface area, a_p, as that of a non-spherical particle. In effect, it is the surface area of a sphere having a volume equal to that of the particle, divided by the surface area of the particle. If all the particles are not the same size, one should employ the *mean effective diameter*, $\overline{d_{p,e}}$, where

$$\overline{d_{p,e}} = \frac{1.0}{\sum (w_i)(d_{p,e})_i} \tag{25.9}$$

and w_i is the mass fraction in size of particle i. The *hydraulic diameter* provides a measure of the space in which the fluid flows, either through the particle pores or the spaces between particles. By definition, the hydraulic diameter, D_h, is given by

$$D_h = 4r_h = \frac{\text{volume open to flow}}{\text{total wetted surface}} = \frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right)d_p \tag{25.10}$$

where r_h is the hydraulic radius.

Table 25.2 Tyler standard screen scale

Mesh	Inches	Clear Opening, mm	Wire Diameter, in
2.5	0.312	7.925	0.088
3	0.263	6.680	0.070
3.5	0.221	5.613	0.065
4	0.185	4.699	0.065
5	0.156	3.962	0.065
6	0.131	3.327	0.036
7	0.110	2.794	0.0328
8	0.093	2.362	0.032
9	0.078	1.981	0.033
10	0.065	1.651	0.035
12	0.055	1.397	0.028
14	0.046	1.168	0.025
16	0.039	0.991	0.0235
20	0.0328	0.833	0.0172
24	0.0276	0.701	0.0141
28	0.0232	0.589	0.0125
32	0.0195	0.495	0.0118
35	0.0164	0.417	0.0122
42	0.0138	0.351	0.0100
48	0.0116	0.295	0.0092
60	0.0097	0.246	0.0070
65	0.0082	0.208	0.0072
80	0.0069	0.174	0.0056
100	0.0058	0.147	0.0042
115	0.0059	0.124	0.0038
150	0.0041	0.104	0.0026
170	0.0035	0.088	0.0024
200	0.0029	0.074	0.0021
230	0.0024	0.061	0.0016
270	0.0024	0.061	0.0016
325	0.0017	0.043	0.0014
400	0.0015	0.038	0.0010

25.3 FLOW REGIMES

The Reynolds number (Re) is based on the actual (interstitial) velocity and the hydraulic diameter:

$$\mathrm{Re} = \frac{d_h v_I \rho_f}{\mu_f} \tag{25.11}$$

Substituting for v_I and d_h leads to

$$\mathrm{Re} = \frac{2}{3} \frac{d_p v_s \rho_f}{(1-\varepsilon)\mu_f} \tag{25.12}$$

Ergun(2) defined a porous medium Reynolds number as $\mathrm{Re}_p = 1.5\,\mathrm{Re}$, so that

$$\mathrm{Re}_p = \frac{d_p v_s \rho_f}{(1-\varepsilon)\mu_f} \tag{25.13}$$

Flow regimes in porous media may be laminar, transition, or turbulent, based on the porous medium Reynolds number. For laminar flow, $\mathrm{Re}_p < 10$. For transition flow, $10 < \mathrm{Re}_p < 1000$ and for turbulent flow, $\mathrm{Re}_p > 1000$. The effect of media *roughness* is less significant than the other variables but may become more important in the highly turbulent region. Generally, for flow in the laminar and early turbulent region, roughness has little effect on pressure drop and should not be included in most correlations for porous media.(3)

Orientation is an important variable in special cases. However, variations in orientation do not occur with random packing as encountered in most industrial practices. Oriented beds have been occasionally used in some absorbers and for other specialized applications where the packing is stacked by hand rather than dumped into the vessel; however, real world applications of this case are rare.(3)

Illustrative Example 25.1 Calculate the effective particle diameter for a set of packing with the following given characteristics (obtained during a packed column experiment at Manhattan College):

Weight of packing:	5.86 g
Total volume:	21 mL
Packing volume:	0.2 mL
Number of packing particles:	99
Average surface area:	2.18 mm^2 (from 20 randomly drawn packing)

Solution The volume of a single particle is calculated by the weight of the packing by the number of particles, n (assumed to be 100 for the purposes of calculation)

$$V_p = \frac{V}{n}$$

$$V_p = 0.2\,\text{mL}/100 = 0.2\,\text{cm}^3/100$$

$$V_p = 0.002\,\text{cm}^2 = 2.0\,\text{mm}^2$$

The specific surface area, a_p, is calculated by dividing the average surface area of the particle by the volume of a single particle. See Equation (25.7).

$$a_p = \frac{S_p}{V_p}$$

$$a_p = \frac{2.18\,\text{mm}^2}{2.0\,\text{mm}^3}$$

$$a_p = 1.09\,(\text{mm})^{-1}$$

The effective particle diameter, d_p, is also calculated by employing the same equation.

$$d_p = \frac{6}{a_p}$$

$$d_p = \frac{6}{1.09\,\text{mm}}$$

$$d_p = 5.50\,\text{mm}$$

Illustrative Example 25.2 For the packing in Illustrative Example 25.1, calculate the Reynolds number if a fluid is flowing through a column containing the same packing with an interstitial velocity of 10 cm/s. The fluid density and viscosity are $0.235\,\text{g/cm}^3$ and 0.02 cP.

Solution Employ the Reynolds number defined in Equation (25.11)

$$\text{Re} = \frac{D_h \rho v_I}{\mu}$$

while noting that

$$0.02\,\text{cP} = 2.0 \times 10^{-4}\,\text{g/cm}\cdot\text{s}$$

Substituting gives

$$\begin{aligned}\text{Re} &= (5.50/10)(0.235)(10)/2.0 \times 10^{-4}\\ &= 6463\end{aligned}$$

This illustrates that at a velocity of 10 m/s, the Reynolds number is 646,300 and the flow of the fluid would be in the turbulent region.

Illustrative Example 25.3 Air ($\rho = 1.2\,\text{kg/m}^3$, $\mu = 1.25 \times 10^{-5}\,\text{kg/m}\cdot\text{s}$) flows across a packed bed. The bed porosity is 0.4 and the superficial velocity of the air is 0.1 m/s. The packing consists of solid cylindrical particles of diameter equal to 1.5 cm and height of 2.5 cm (see Fig. 25.3). Calculate the particle specific surface and the effective diameter of the particle.

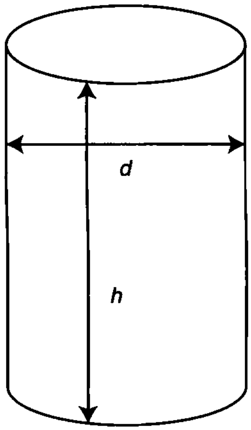

Figure 25.3 Cylindrical packing.

Solution Calculate the volume, V_p, of the cylindrical particle

$$V_p = \frac{\pi d_p{}^2 h}{4} = \frac{\pi(1.5)^2(2.5)}{4}$$
$$= 4.418\,\text{cm}^3$$

Calculate the cylindrical particle surface area, S_p

$$S_p = \pi d_p h + 2\left(\frac{\pi d_p^2}{4}\right) = \pi(1.5)(2.5) + \frac{\pi(1.5)^2}{2}$$
$$= 15.32\,\text{cm}^2$$

Calculate the particle specific surface, a_p

$$a_p = \frac{S_p}{V_p} = \frac{15.32}{4.418}$$
$$= 3.467\,\text{cm}^{-1}$$

Calculate the effective particle diameter, $d_{p,e}$

$$d_{p,e} = \frac{6}{3.467} = 1.73\,\text{cm}$$

Illustrative Example 25.4 An absorber bed packing consists of cube-shaped particles of edge length equal to $\frac{3}{4}$ inch. Calculate the particle specific surface and the effective particle diameter.

Solution If the particle side length is L, then

$$V_p = L^3$$

and the surface area is

$$S_p = 6L^2$$

The specific packing (particle) surface area is defined as [see Eq. (25.7)]:

$$a_p = \frac{S}{V}$$

Substituting,

$$a_p = \frac{6L^2}{L^3} = \frac{6}{L} = \frac{6}{3/4} = 8\,\text{in}^2/\text{in}^3$$

The effective particle diameter, $d_{p,e}$, is calculated by equating a_p and $a_{p,e}$. Since

$$a_p = \frac{6}{L}$$

and

$$a_{p,e} = \frac{6}{d_{p,e}}$$

it is clear that the effective particle diameter is equal to L and

$$d_{p,e} = \frac{3}{4}\,\text{in} = 0.75\,\text{in}$$

Illustrative Example 25.5 A catalyst tower, 50 ft high and 20 ft in diameter, is packed with one-inch diameter spheres (ultimate density = 77.28 lb/ft^3). Gas enters at the top of the vertical bed at a temperature of 500°F, flows downward, and leaves at the same temperature. The absolute pressure at the bottom of the catalyst bed is 4320 psf. The bed porosity is 0.4. The gas has average properties similar to propane and the time of contact between the gas and the catalyst is 10 s. Assuming incompressible flow, calculate the interstitial velocity, the superficial velocity, the gas flow rate, the bulk density of the bed, and the particle and bed specific surface area.

Solution Calculate the density of propane assuming the ideal gas law to apply.

$$M = 44.1 \quad T = 500\,\text{deg F} = 960\,\text{deg R}$$

$$\rho = \frac{PM}{RT} = \frac{4320(44.1)}{(10.73)(144)(960)}$$
$$= 0.0128\,\text{lb/ft}^3$$

Also, estimate the viscosity of propane from the gas viscosity monogram (see Fig. B.2 in the Appendix) at 500°F

$$\mu \cong 1.33 \times 10^{-4}\,\text{P} = 8.54 \times 10^{-7}\,\text{lb/ft} \cdot \text{s}$$
$$= 0.0133\,\text{cP}$$

Calculate the bed volume.

$$V = \frac{\pi D^2}{4} L = \frac{\pi (20)^2 (50)}{4}$$
$$= 15{,}708\,\text{ft}^3$$

Use the contact time, θ, to calculate the volumetric flow rate.

$$q = \frac{V_f}{\theta} = \frac{V\varepsilon}{\theta} = \frac{(15{,}708)(0.4)}{10}$$
$$= 628.3\,\text{ft}^3/\text{s}$$

Calculate the superficial velocity, or empty tower velocity.

$$v_s = \frac{4q}{\pi D^2} = \frac{4(628.3)}{\pi (20)^2}$$
$$= 2.0\,\text{ft/s}$$

The interstitial velocity is therefore [see Eq. (25.6)]:

$$v_I = \frac{v_s}{\varepsilon} = \frac{2.0}{0.4}$$
$$= 5.0\,\text{ft/s}$$

Calculate the bulk density from Equation (25.4).

$$\rho_B = (1 - \varepsilon)\rho_s + \varepsilon\rho_f$$

Neglecting the latter term,

$$\rho_B = (0.6)(77.28)$$
$$= 46.4\,\text{lb/ft}^3$$

Finally, calculate the particle specific surface area, a_p, and bed specific surface a_b from Equations (25.7) and (25.8).

$$a_p = \frac{6}{d_p} = \frac{6}{0.0833}$$
$$= 72\,\text{ft}^{-1}$$
$$a_b = a_p(1 - \varepsilon) = (72)(0.6) = 43.2\,\text{ft}^{-1}$$

Illustrative Example 25.6 Refer to Illustrative Example 25.5. Calculate the hydraulic radius and hydraulic diameter.

Solution The hydraulic radius and diameter may now be calculated from the results of Illustrative Example 25.5. See Equation (25.10)

$$D_h = \frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right)d_p = \frac{2}{3}(0.0833)\frac{0.4}{0.6}$$
$$= 0.037\,\text{ft}$$

By definition

$$r_h = \frac{D_h}{4} = \frac{0.037}{4}$$
$$= 0.00926\,\text{ft}$$

REFERENCES

1. C. Bennett and J. Myers, "Momentum, Heat, and Mass Transfer," McGraw-Hill, New York, 1962.
2. S. Ergun, "Fluid Flow Through Packed Columns," CEP, New York, 48:89, 1952.
3. G. Brown & Associates, "Unit Operations," John Wiley & Sons, Hoboken, NJ, 1950.

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