26

FLUIDIZATION

26.1 INTRODUCTION

Fluidization is the process in which fine solid particles are transformed into a fluid-like state through contact with either a gas or liquid, or both. Fluidization is normally carried out in a vessel filled with solids. The fluid is introduced through the bottom of the vessel and forced up through the bed. At a low flow rate, the fluid (liquid or gas) moves through the void spaces between the stationary and solid particles and the bed is referred to as *fixed*. (This topic is treated first in the development that follows.) As the flow rate increases, the particles begin to vibrate and move about slightly, resulting in the onset of an *expanded bed*. When the flow of fluid reaches a certain velocity, the solid particles become suspended because the upward frictional force between the particle and the fluid balances the gravity force associated with the weight of the particle. This point is termed *minimum fluidization* or *incipient fluidization* and the velocity at this point is defined as the *minimum* or *incipient fluidization velocity*. Beyond this stage, the bed enters the fluidization state where bubbles of fluid rise through the solid particles, thereby producing a circulatory and/or mixing pattern.[1]

From a force balance perspective, as the flow rate upward through a packed bed is increased, a point is reached at which the frictional drag and buoyant force is enough to overcome the downward force exerted on the bed by gravity. Although the bed is supported at the bottom by a screen, it is free to expand upward, as it will if the velocity is increased above the aforementioned minimum fluidization velocity. At this point, the particles are no longer supported by the screen, but rather are suspended

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in the fluid in equilibrium and act and behave as the fluid. The bed is then said to be *fluidized*. From a momentum or force balance perspective, the sum of the drag, buoyancy, and gravity forces must be equal to zero.

The terminal settling velocity can be evaluated for the case of flow past one bed particle. By superimposition, this case is equivalent to that of the terminal velocity that a particle would attain flowing through a fluid. Once again, a force balance can be applied and empirical data used to evaluate a friction coefficient (see Chapter 23 for more details).

At intermediate velocities between the minimum fluidization velocity and the terminal velocity, the bed is expanded above the volume that it would occupy at the minimum value. Note also that above the minimum fluidization velocity, the pressure drop stays essentially constant.

One of the novel characteristics of fluidized beds is the uniformity of temperature found throughout the system. Essentially constant conditions are known to exist in both the horizontal and vertical directions in both short and long beds. This homogeneity is due to the turbulent motion and rapid circulation rate of the solid particles within the fluid stream described above. In effect, excellent fluid-particle contact results. Temperature variations can occur in some beds in regions where quantities of relatively hot or cold particles are present but these effects can generally be neglected. Consequently, fluidized beds find wide application in industry, e.g., oil cracking, zinc coating, coal combustion, gas desulfurization, heat exchangers, plastics cooling and fine powder granulation.

26.2 FIXED BEDS[2]

The friction factor f for a "fixed" packed bed is defined as:

$$\frac{\Delta P}{\frac{1}{2}\rho v_s^2} = \left(\frac{L}{d_p}\right)4f \qquad (26.1)$$

in which d_p is the particle diameter (defined presently) and v_s is the superficial velocity defined in the previous chapter as the average linear velocity that the fluid would have in the column if no packing were present. The term L is the length of the packed column. The friction factor for laminar flow and that for turbulent flow can now be estimated separately.

For laminar flow in circular tubes of radius R, it was shown that

$$v = \frac{\Delta P R^2}{8\mu L} \qquad (26.2)$$

Now imagine that a packed bed is just a tube with a very complicated cross-sectional area with hydraulic radius r_h. The average flow velocity in the cross section available for flow is then

$$v = \frac{\Delta P r_h{}^2}{2\mu L}; \quad r_h = \frac{\text{cross-section available for flow}}{\text{wetted perimeter}} \tag{26.3}$$

The hydraulic radius may be expressed in terms of the void fraction ε and the wetted surface "a" per unit volume of bed in the following way:

$$\begin{aligned} r_h &= \frac{\text{cross-section available for flow}}{\text{wetted perimeter}} \\ &= \frac{\text{volume available for flow}}{\text{total wetted surface}} \\ &= \frac{\left(\dfrac{\text{volume of voids}}{\text{volume of bed}}\right)}{\left(\dfrac{\text{wetted surface}}{\text{volume of bed}}\right)} = \frac{\varepsilon}{a} \end{aligned} \tag{26.4}$$

The quantity "a" is related to the "specific surface" a_v (total particle surface/volume of the particles) by

$$a = a_v(1 - \varepsilon) \tag{26.5}$$

The quantity a_v is in turn used to define the mean particle diameter d_p:

$$d_p = \varepsilon / a_v \tag{26.6}$$

This definition is chosen because, for spheres, Equation (26.6) reduces to just d_p as the diameter of sphere. Finally, note that the average value of the velocity in the interstices, v_I, is not of general interest to the engineer but rather the aforementioned superficial velocity v_s; these two velocities are related by

$$v_s = v_I \varepsilon \tag{26.7}$$

If the above definitions are combined with the modified Hagen–Poiseuille equation, the superficial velocity can be expressed as

$$\begin{aligned} v_s &= \frac{\Delta P r_h^2}{2\mu L} \\ &= \frac{\Delta P \varepsilon^2}{2\mu L a^2} \\ &= \frac{\Delta P \varepsilon^2}{2\mu L a_v{}^2(1-\varepsilon)^2} \\ &= \frac{\Delta P d_p{}^2}{2L(36\mu)} \frac{\varepsilon^2}{(1-\varepsilon)^2} \end{aligned}$$

or finally

$$v_s = \frac{\Delta P}{L}\frac{d_p{}^2}{2(36\mu)}\frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{26.8}$$

In laminar flow, the assumption of mean hydraulic radius frequently gives a throughput velocity too large for a given pressure gradient. Because of this assumption, one would expect that the right side of Equation (26.8) should be somewhat smaller. A second assumption implicitly made in the foregoing development is that the path of the fluid flowing through the bed is of length L, i.e., it is the same as the length of the packed column. Actually, of course, the fluid traverses a very tortuous path, the length of which may be approximately twice as long as the length L. Here, again, one would expect that the right side of Equation (26.8) should be somewhat diminished.

Experimental measurements indicate that the above theoretical formula can be improved if the 2 in the denominator on the right-hand side is changed to a value somewhere between 4 and 5. Analysis of a great deal of data has led to the value 25/6, which is accepted here. Insertion of that value into Equation (26.8) then gives

$$v_s = \frac{\Delta P}{L}\frac{d_p{}^2}{150\mu}\frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{26.9}$$

which some have defined as the Blake–Kozeny equation. This result is generally good for void fractions less than 0.5 and is valid only in the laminar region where the particle Reynolds number is given by $d_pG_s/\mu(1-\varepsilon) < 10$; $G_s = \rho v_s$.[3] Note that the Blake–Kozeny equation corresponds to a bed friction factor of

$$f = \left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right]\frac{75}{d_pG_s/\mu} \tag{26.10}$$

Exactly the same treatment can be repeated for highly turbulent flow in packed columns. One begins with the expression for the friction-factor definition for flow in a circular tube. This time, however, note that for highly turbulent flow in tubes with any appreciable roughness, the friction factor becomes a function of the roughness only. Assuming that all packed beds have similar roughness characteristics, a unique friction factor f_0 may be used for turbulent flow. This leads to the following results if the some procedure as before is applied:

$$\frac{\Delta P}{L} = \frac{1}{D}\frac{\rho v_s{}^2}{2}4f_0 = 6f_0\frac{1}{d_p}\frac{\rho v_s{}^2}{2}\left(\frac{1-\varepsilon}{\varepsilon^3}\right) \tag{26.11}$$

Experimental data indicate that $6f_0 = 3.50$. Hence Equation (26.11) becomes

$$\begin{aligned}\frac{\Delta P}{L} &= 3.50\frac{1}{d_p}\frac{\rho v_s{}^2}{2}\frac{1-\varepsilon}{\varepsilon^3} \\ &= 1.75\frac{\rho v_s{}^2}{d_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right) \end{aligned}\tag{26.12}$$

which some have defined as the Burke–Plummer equation and is valid for $(d_p G_s/\mu)(1 - \varepsilon) > 1000$. This result corresponds to a friction factor given by

$$f_0 = 0.875\frac{1-\varepsilon}{\varepsilon^3} \tag{26.13}$$

Note that this dependence on ε is different from that given for laminar flow.

When the Blake–Kozeny equation for laminar flow and the Burke–Plummer equation for turbulent flow are simply added together, the result is

$$\frac{\Delta P}{L} = \frac{150\mu v_s}{d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho v_s^2}{d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \tag{26.14}$$

This may be rewritten in terms of dimensionless groups (numbers):

$$\left(\frac{\Delta P\rho}{G_0^2}\right)\left(\frac{d_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\frac{1-\varepsilon}{d_p G_0/\mu} + 1.75 \tag{26.15}$$

This is the Ergun equation. It has been applied with success to gases by using the density of the gas at the arithmetic average of the end pressures. For large pressure drops, however, it seems more reasonable to use Equation (26.14) with the pressure gradient in differential form. Note that G_s is a constant through the bed whereas v_s changes through the bed for a compressible fluid. The d_p used in this equation is that defined in Equation (26.6).

Equation (26.14) may be written in the following form

$$\Delta P = 150\frac{v_s\mu}{g}\frac{(1-\varepsilon)^2}{\varepsilon^3}\left(\frac{L}{d_p^2}\right) + 1.75\frac{v_s^2}{g}\frac{(1-\varepsilon)}{\varepsilon^3}\left(\frac{L}{d_p}\right)\rho \tag{26.16}$$

Equation (26.15) may also be written in a similar form. Other terms have been used to represent ΔP, including

$$\Delta P = \Delta P_f = h_f' \tag{26.17}$$

where the subscript f is a reminder that the pressure drop term represents friction due to the flowing fluid. Thus,

$$h_f' = 150\frac{v_s\mu}{g\rho}\frac{(1-\varepsilon)^2}{\varepsilon^3}\left(\frac{L}{d_p^2}\right) + 1.75\frac{v_s^2}{g}\frac{(1-\varepsilon)}{\varepsilon^3}\left(\frac{L}{d_p}\right) \tag{26.16}$$

The reader should note that the pressure drop term in Equation (26.16) has units of height of flowing fluid, e.g., in H_2O. This may be converted into units of force per unit area (e.g., psf), by applying the hydrostatic pressure equation

$$\Delta P = \frac{\rho g h}{g_c} \tag{26.18}$$

This equation can then be employed to rewrite Equation (26.16) in the following form:

$$\Delta P = \frac{v_s \mu}{g_c} \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{L}{d_p^2} + 1.75 \frac{v_s^2 \rho}{g_c} \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{L}{d_p}\right) \tag{26.19}$$

The units of ΔP are then those of pressure (i.e., force per unit area).

Illustrative Example 26.1 Comment on the relationship between the Ergun equation and the Burke–Plummer and Blake–Kozeny equations.

Solution Note that for high rates of flow, the first term on the right-hand side drops out and the equation reduces to the Burke–Plummer equation. At low rates of flow, the second term on the right-hand side drops out and the Blake–Kozeny equation is obtained. It should be emphasized that the Ergun equation is but one of many that have been proposed for describing the pressure drop across packed columns.

26.3 PERMEABILITY

In porous medium applications involving laminar flow, the Carmen–Kozeny equation is rewritten as

$$h_f' = 150 \frac{v_s \mu}{g \rho} \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{L}{d_p^2} = \frac{1}{k} \frac{v_s \mu L}{g \rho} \tag{26.20}$$

where k is the permeability of the medium. The permeability may then be written as

$$k = \frac{1}{150} \frac{\varepsilon^3}{(1-\varepsilon)^2} d_p^2 \tag{26.21}$$

The permeability may be expressed in units of darcies, where 1 darcy $= 0.99 \times 10^{-12}\ \text{m}^2 = 1.06 \times 10^{-11}\ \text{ft}^2$.

Illustrative Example 26.2 A water softener unit consists of a large diameter tank of height $h = 0.25$ ft. The bottom of the tank is connected to a vertical ion-exchange pipe of length $L = 1$ ft and a diameter D of 2 inches. The ion exchange resin particle diameter is 0.05 in. $= 0.00417$ ft, and the bed porosity is 0.25. The water has an absolute viscosity of 6.76×10^{-4} lb/ft · s and a density of 62.4 lb/ft^3. Calculate the water flow rate and the superficial velocity (see Fig. 26.1).

Solution First determine the total fluid height, $h_L = h_f'$

$$h_L = h_f' = z_1 - z_2 = 1.25\ \text{ft of water}$$

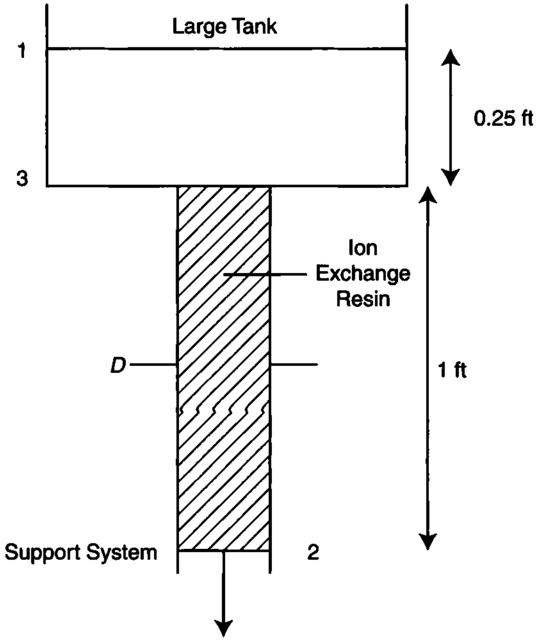

Figure 26.1 Ion-exchange softener.

Assume turbulent flow to calculate the superficial velocity, v_s. Employ a revised version of the Burke–Plummer equation [Equation (26.11)].

$$v_s = \sqrt{\frac{gh_f'}{1.75}\frac{\varepsilon^3}{1-\varepsilon}\frac{d_p}{L}} = \sqrt{\frac{(32.174)(1.25)}{(1.75)}\frac{(0.25)^3}{(1-0.25)}\frac{(0.00417)}{(1.0)}}$$
$$= 0.0446\,\text{ft/s}$$

Check the turbulent flow assumption

$$\text{Re}_p = \frac{d_p v_s \rho}{(1-\varepsilon)\mu} = \frac{(0.00417)(0.0446)(62.4)}{(1-0.25)6.76\times 10^{-4}}$$
$$= 22.9 < 1000$$

Since the Reynolds number is low, the calculation is not valid. Assume laminar flow and use a modified form of the Blake–Kozeny equation [see Equation (26.9)].

$$v_s = \frac{\rho g h_L}{150\mu}\frac{\varepsilon^3}{(1-\varepsilon)^2}\frac{d_p{}^2}{L} = \frac{(62.4)(32.174)(1.25)}{150(6.76\times 10^{-4})}\frac{(0.25)^3}{(1-0.25)^2}\frac{(0.00417)^2}{1}$$
$$= 0.0119\,\text{ft/s}$$

Once again, check the porous medium Reynolds number

$$\mathrm{Re}_p = \frac{d_p v_s \rho}{(1-\varepsilon)\mu} = \frac{(0.00417)(0.0119)(62.4)}{(1-0.25)6.76\times 10^{-4}} = 6.11 < 10$$

The flow is therefore laminar.

Calculate the empty cross-sectional area, S

$$S = \frac{\pi D^2}{4} = \frac{\pi(0.167)^2}{4} = 0.0218\,\text{ft}^2$$

The volumetric flow rate of water, q, is then

$$q = v_s S = (0.0119)(0.0218) = 0.000252\,\text{ft}^3/\text{s}$$

Illustrative Example 26.3 Refer to Illustrative Example 26.2. Calculate the pressure drop due to friction and the pressure drop across the resin bed.

Solution Calculate the packed bed permeability, k, using Equation (26.21).

$$k = \frac{1}{150}\frac{\varepsilon^3}{(1-\varepsilon)^2}d_p{}^2 = \frac{1}{150}\frac{(0.25)^3}{(1-0.25)^2}(0.00417)^2$$
$$= 3253 \text{ Darcies}$$

The friction pressure drop across the resin bed, ΔP_{fr}, may also be calculated noting that $h_L = h_f'$.

$$\Delta P_{fr} = \frac{\rho_f g h_f'}{g_c} = 62.4(1.25) = 78.0\,\text{psf}$$

Finally, calculate the pressure drop across the resin bed by applying Bernoulli's equation across the resin bed (between points 2 and 3):

$$\frac{P_3}{\rho} + \frac{v_3{}^2}{2g_c} + \frac{g}{g_c}z_3 = \frac{P_2}{\rho} + \frac{v_2{}^2}{2g_c} + \frac{g}{g_c}z_2 + h_f - h_s$$

Noting that $v_3 = v_2$ and $h_s = 0$,

$$\frac{\Delta P}{\rho} = \frac{P_3 - P_2}{\rho} = (z_2 - z_3)\frac{g}{g_c} + h_f' = -1 + 1.25 = 0.25\,\text{ft of liquid}$$

$$\Delta P = \rho\left(\frac{\Delta P}{\rho}\right) = 62.4(0.25) = 15.6\,\text{psf} = 0.108\,\text{psi}$$

The total pressure drop represents both the friction drop and the height of the fluid (water).

26.4 MINIMUM FLUIDIZATION VELOCITY

Figure 26.2 is a photograph of the fluidization experimental unit at Manhattan College. Figure 26.3 shows the kinds of contact between solids and a fluid, starting from a packed bed and ending with "pneumatic" transport. At a low fluid velocity, one observes a fixed bed configuration of height L_m, a term that is referred to as the slumped bed height. As the velocity increases, fluidization starts, and this is termed the *onset of fluidization*. The superficial velocity (that velocity which would occur if the actual flow rate passed through an empty vessel) of the fluid at the onset of fluidization is noted again as the *minimum fluidization velocity*, v_{mf}, and the bed height is L_{mf}. As the fluid velocity increases beyond v_{mf}, the bed expands and the bed void volume increases. At low fluidization velocities (fluid velocity > v_{mf}), the operation is termed *dense phase fluidization*.

The onset of fluidization (or *minimum fluidization* condition) in a packed bed occurs when drag forces due to friction by the upward moving gas equal the

Figure 26.2 Fluidization experiment.

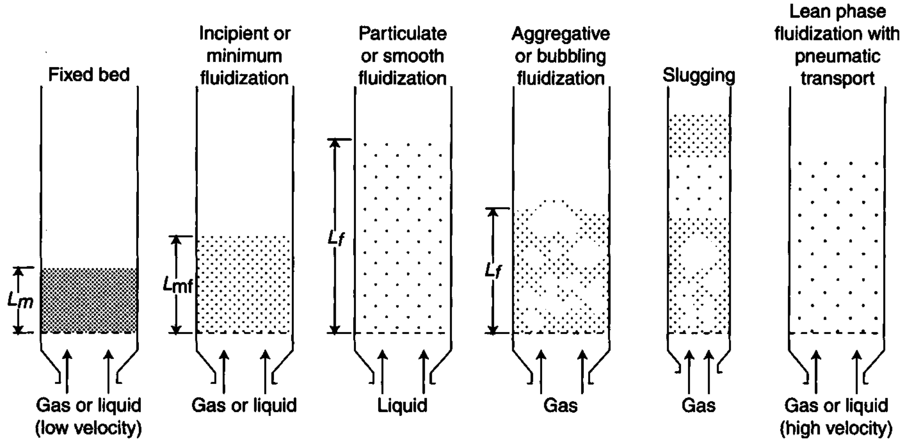

Figure 26.3 Types of particle-fluid contact in a bed.

gravity force of the particles minus the buoyancy force on the particles. This can be represented in equation form as

$$F_D = W_{\text{net}} = \left(W - F_{\text{bouy}}\right)_{\text{particle}} \tag{26.22}$$

where W is the weight of the particle. The drag force, F_D, exerted by the upward gas is a product of the friction pressure drop in gas flow across the bed and the bed cross-section area. From Bernoulli's equation, the total pressure drop ΔP is given by

$$\Delta P = \Delta P_{fr} + \rho_f \frac{g}{g_c} L_{\text{mf}}$$

or

$$\Delta P_{fr} = \Delta P - \rho_f \frac{g}{g_c} L_{\text{mf}} \tag{26.23}$$

with the latter term representing the fluid head. Therefore,

$$F_D = \Delta P_{fr} S_b = \left(\Delta P - \rho_f \frac{g}{g_c} L_{\text{mf}}\right) S_b \tag{26.24}$$

The net gravity force, W_{net}, is due to the gravity of the solid particles and to the fluid buoyancy:

$$W_{\text{net}} = S_b L_{\text{mf}} \left((1 - \varepsilon_{\text{mf}})(\rho_s - \rho_f) \frac{g}{g_c}\right) \tag{26.25}$$

Combining Equations (26.24) and (26.25), one obtains the condition for minimum fluidization:

$$\Delta P_{fr} = \rho_f \frac{g}{g_c} h_f' = (1 - \varepsilon_{\text{mf}})(\rho_s - \rho_f) \frac{g}{g_c} L_{\text{mf}} \tag{26.26}$$

or

$$h_f' = (1 - \varepsilon_{mf})\left(\frac{\rho_s - \rho_f}{\rho_f}\right)L_{mf} \tag{26.27}$$

The equations for minimum fluidization are similar to those presented for a fixed bed, i.e., Equations (26.16)–(26.19). For laminar flow conditions ($Re_p < 10$), the Blake–Kozeny equation is used to express h_f' in terms of the superficial gas velocity at minimum fluidization, v_{mf}, and other fluid and bed properties. This equation is obtained from Equations (26.9) and (26.27)

$$h_f' = (1 - \varepsilon_{mf})\left(\frac{\rho_s - \rho_f}{\rho_f}\right)L_{mf} = 150\frac{v_{mf}\mu_f}{\rho_f g}\frac{(1-\varepsilon_{mf})^2}{\varepsilon_{mf}^3}\frac{L_{mf}}{d_p^2} \tag{26.28}$$

Rearranging, one obtains the minimum fluidization velocity:

$$v_{mf} = \frac{1}{150}\left(\frac{\varepsilon_{mf}^3}{1-\varepsilon_{mf}}\right)\frac{g(\rho_s - \rho_f)d_p^2}{\mu_f}(Re_p < 10) \tag{26.29}$$

with (once again)

$$Re_p = \frac{d_p v_{mf}\rho}{\mu_f(1-\varepsilon_{mf})} \tag{26.30}$$

The Burke–Plummer equation is used to express the head loss, h_f'. For turbulent flow conditions ($Re_p > 1000$). For this condition, the result is:

$$v_{mf} = \sqrt{\frac{1}{1.75}\varepsilon_{mf}^3\left(\frac{\rho_s - \rho_f}{\rho_f}\right)gd_p} \tag{26.31}$$

In the absence of ε_{mf} data, the above equations can be approximated as

$$v_{mf} = \frac{1}{1650}\frac{g(\rho_s - \rho_f)d_p^2}{\mu_f} \quad Re_p < 10 \tag{26.32}$$

$$v_{mf} = \sqrt{\frac{1}{24.5}\left(\frac{\rho_s - \rho_f}{\rho_f}\right)gd_p} \quad Re_p > 1000 \tag{26.33}$$

where Re_p is the particle Reynolds number at minimum fluidization and is equal to:

$$Re_p = \frac{d_p v_{mf}\rho}{\mu_f} \tag{26.34}$$

Illustrative Example 26.4 Air is used to fluidize a bed of spherical particles. The particles are 200 mesh uniform spheres; bed diameter, D_b, is 0.2 m; ultimate solids density, ρ_s, is 2200 kg/m^3; voidage at minimum fluidization, ε_{mf}, equals 0.45; bed length (height) at minimum fluidization, L_{mf}, is 0.3 m; and air properties are $\rho_f = 1.2$ kg/m^3 and $\mu_f = 1.89 \times 10^{-5}$ kg/m-s.

Calculate the minimum fluidization mass flow rate of air and the pressure drop of air across the bed at minimum fluidization.

Solution Obtain the diameter of a 200 mesh particle from Table 23.2.

$$d_p = 74\,\mu\text{m} = 7.4 \times 10^{-5}\,\text{m}$$

Assume turbulent flow to apply and calculate v_{mf} from Equation (26.31).

$$v_{mf} = \sqrt{\frac{(0.45)^3}{1.75}\left(\frac{2200 - 1.2}{1.2}\right)(9.807)(7.4 \times 10^{-5})} = 0.263\,\text{m/s}$$

Check the flow regime. Employ Equation (26.30):

$$\text{Re}_p = \frac{v_{mf} d_p}{v_f(1 - \varepsilon_{mf})} = \frac{(0.263)(7.4 \times 10^{-5})}{(1.89 \times 10^{-5})(1 - 0.45)} = 1.87 < 1000$$

Therefore, turbulent flow conditions do not apply.

Assume laminar flow, with $\rho_s - \rho_f = \rho_s$, and employ Equation (26.29).

$$v_{mf} = \frac{1}{150}\frac{(1 - 0.45)}{(0.45)^3}\frac{9.807(2200)(7.4 \times 10^{-5})^2}{1.89 \times 10^{-5}} = 0.25\,\text{m/s}$$

Once again, check the flow regime

$$\text{Re}_p = \frac{v_{mf} d_p}{\mu_f(1 - \varepsilon_{mf})} = \frac{(0.25)(7.4 \times 10^{-5})}{(1.89 \times 10^{-5})(1 - 0.45)}$$
$$= 1.79 < 10$$

The flow is indeed laminar.

The mass flow rate of air is

$$\dot{m} = \frac{\pi D^2}{4} v_{mf} \rho_f = \frac{\pi (0.2)^2}{4}(0.25)(1.2) = 9.40^{-3}\,\text{kg/s}$$

Calculate the gas pressure drop across the bed. Use Equation (26.28).

$$\Delta P_{fr} = (1 - 0.45)(2200)(9.807)(0.3) = 3560\,\text{Pa} = 0.0351\,\text{atm}$$

Both Equations (26.16) and (26.19) may be rewritten in a slightly different form and viewed as contributing terms to a more general equation. An equation covering the entire range of flow rates but for *various shaped* particles can be obtained by assuming that the laminar and turbulent effects are additive. This result is also referred to as the Ergun equation.[3] Thus,

$$\frac{\Delta P}{L}=\frac{150 v_0 \mu}{g_c {\phi_s}^2 {d_p}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+\frac{1.75 \rho {v_0}^2}{g_c \phi_s d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \tag{26.35}$$

where ϕ_s is the sphericity or shape factor of the fluidized particles. Typical values for the sphericity of typical fluidized particles are in the 0.75–1.0 range. In lieu of any information on ϕ_s, one should employ a value of 1.0, typical for spheres, cubes and cylinders ($L = d$).

Another approach that may be employed to calculate the minimum fluidization velocity is to employ Happel's equation.[4] Happel's equation is given by:

$$\frac{v_{\mathrm{mf}}}{v_t}=\frac{3-4.5(1-\varepsilon_{\mathrm{mf}})^{1/3}+4.5(1-\varepsilon_{\mathrm{mf}})^{5/3}-3(1-\varepsilon_{\mathrm{mf}})^2}{3+2(1-\varepsilon_{\mathrm{mf}})^{5/3}} \tag{26.36}$$

where v_{mf} is the minimum fluidization velocity, v_t the terminal velocity, v, and $\varepsilon_{\mathrm{mf}}$ the bed porosity at minimum fluidization.

Illustrative Example 26.5 Determine the pressure drop of 60°F air flowing through a 3-inch diameter 10-ft packed bed with 0.24-inch protruded packing made of 316 stainless steel. The superficial velocity is 4.65 ft/s. The protruded packing has a fraction void volume, effective particle diameter and surface area per unit packing of 0.89, 0.0078.5 ft, and 3305 ft^{-1}, respectively.

Solution Use the Ergun equation

$$\Delta P=\left[\left(\frac{150 \overline{V}_0 \mu_g}{g_c(\Phi_s D_p)^2}\right)\left(\frac{(1-\varepsilon)^2}{\varepsilon^3}\right)+\left(\frac{1.75 \rho_g {\overline{V}_0}^2}{g_c(\Phi_s D_p)}\right)\left(\frac{(1-\varepsilon)^2}{\varepsilon^3}\right)\right]L \tag{26.35}$$

For air at 60°F, Appendix A.9, indicates

$$\mu = 1.3\times 10^{-5}\,\mathrm{lb/ft\cdot s}$$

$$\rho = 0.067\,\mathrm{lb/ft^3}$$

Plugging in values from the problem statement, one obtains

$$\begin{aligned}\Delta P &= \left[\left(\frac{(150)(4.65)(1.3E^{-5})}{(32.2)(0.007815)^2}\right)\left(\frac{(1-0.89)^2}{(0.89)^3}\right)\right.\\ &\quad \left.+\left(\frac{(1.75)(0.67)(4.65)^2}{32.2(0.007815)}\right)\left(\frac{(1-0.89)^2}{(0.89)^3}\right)\right]10\\ &= 10.25\,\mathrm{lb/ft^2}\end{aligned}$$

26.5 BED HEIGHT, PRESSURE DROP AND POROSITY

The above development is now extended above and beyond the state of minimum fluidization. As described above, when a fluid moves upward in a packed bed of solid particles, it exerts an upward drag force. Minimum fluidization occurs at a point when the drag force equals the net gravity force. By increasing the fluid velocity above minimum fluidization, the bed expands, the porosity increases, and the pressure drop remains the same.

The fluidized bed height, L_f, at any voidage, ε, can be found from the minimum fluidization conditions (L_{mf} and $\varepsilon_{\mathrm{mf}}$), or from the bed height at zero porosity, L_0, that is,

$$\begin{aligned} m &= \rho_s S_b L_0 = \rho_s S_b L_{\mathrm{mf}}(1-\varepsilon_{\mathrm{mf}}) \\ &= \rho_s S_b L_f(1-\varepsilon) \end{aligned}$$

so that

$$L_f = L_{\mathrm{mf}}\frac{1-\varepsilon_{\mathrm{mf}}}{1-\varepsilon} = \frac{L_0}{1-\varepsilon} \tag{26.37}$$

The pressure drop at minimum fluidization remains constant at any fluidization height, L_f, [see Equation (26.17)] so that

$$\Delta P_{fr} = \frac{g}{g_c}(\rho_s-\rho_f)L_{\mathrm{mf}}(1-\varepsilon_{\mathrm{mf}}) = \frac{g}{g_c}(\rho_s-\rho_f)L_f(1-\varepsilon) \tag{26.38}$$

The effect of bed pressure drop on the superficial velocity is now briefly discussed. Initially, the bed pressure drop increases rapidly with a slight increase in velocity. Then, the pressure drop begins to level off. This point, as defined earlier, is called incipient fluidization. Beyond this point, the pressure drop remains fairly constant as the superficial velocity increases. This is the fluidized region.

The variation of porosity (and hence bed height) with the superficial velocity is calculated from the Blake–Kozeny equation, assuming laminar flow and $\rho_f \ll \rho_s$.

$$v_s = \frac{d_p{}^2 g}{150\mu_f}\frac{\varepsilon^3}{1-\varepsilon}(\rho_s-\rho_f) \quad \text{for } \varepsilon < 0.8 \tag{26.39}$$

It should be noted that the bed density is a function of superficial velocity. As the velocity increases, the bed density decreases. This occurs because the volume of the expanding fluidized bed increases while the mass of the bed remains constant. Therefore, the bed density decreases. An increase in gas velocity causes a greater force on the particles and thus drives them further apart. The increased distance between the particles causes an increase in bed volume. After the point of incipient fluidization, the decrease in density is more dramatic as the bed volume increases rapidly during fluidization. The bed porosity is also a function of superficial velocity.

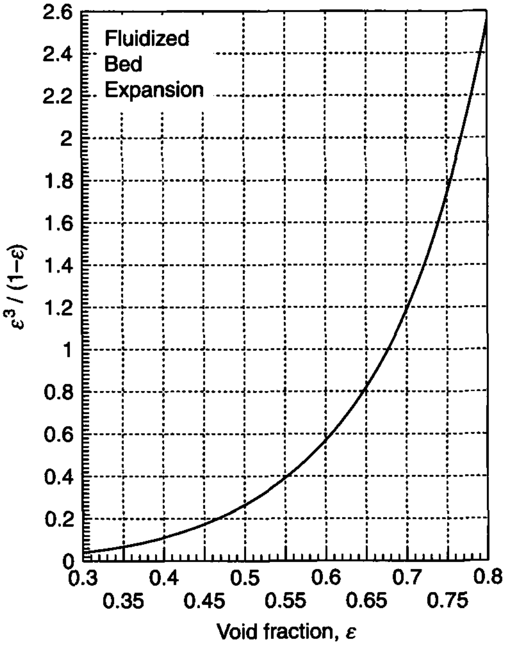

Figure 26.4 Expanded bed porosity.

The bed porosity also increases with an increase in superficial velocity. (Once again, the porosity is a measure of the empty space existing between the particles in the bed.) Initially, the porosity increases gradually; however, after incipient fluidization, the particles are rapidly forced further apart and the porosity increases at a greater rate.

The term,

$$\frac{\varepsilon^3}{1-\varepsilon}$$

appears often in packed bed and fluidized bed equations. To simplify the calculations, it is plotted versus the void fraction, ε, in Fig. 26.4. This plot may be useful to some in estimating the expanded bed porosity and height without trial-and-error.

26.6 FLUIDIZATION MODES

There are two modes of fluidization. When the fluid and solid densities are not too different, or the particles are very small, and therefore, the velocity of the flow is low, the bed is fluidized evenly with each particle moving individually through the bed. This is called smooth fluidization, and is typical of liquid–solid systems. If the fluid and solid densities are significantly different, or the particles are large, the velocity of the flow must be relatively high. In this case, fluidization is uneven, and the fluid passes through the bed mainly in large bubbles. These bubbles burst at the surface, spraying solid particles above the bed. Here, the bed has many of the characteristics of a liquid with the fluid phase acting as a gas bubbling through it. This is called bubbling (or aggregative) fluidization; it is typical of gas–solid systems and is due to the large density difference between the solid and gas. The approximate criterion to estimate the transition from bubbling to smooth fluidization is expressed in terms of the dimensionless Froude number at minimum fluidization. This is expressed in terms of the minimum fluidization velocity, v_{mf}, the particle diameter, d_p, and the acceleration due to gravity, g, as:

$$Fr = \frac{v_{mf}^2}{g d_p} \tag{26.40}$$

If $Fr < 0.13$, the fluidization mode is smooth; if $Fr > 0.13$, then the fluidization is bubbling.

Illustrative Example 26.6 A bed of 35 mesh pulverized coal is to be fluidized with a liquid oil. The bed diameter is 4 ft. At minimum fluidization, the bed height is 8 ft and its void fraction in 0.38. The coal particle density is 84 lb/ft^3. The liquid oil properties are: density = 55 lb/ft^3 and dynamic viscosity = 15 cP. What is the pressure drop required for fluidization?

Solution Obtain the particle diameter from Table 23.2.

$$35\,\text{mesh};\quad d_p = 0.417\,\text{mm} = 0.0164\,\text{in} = 0.00137\,\text{ft}$$

Calculate the pressure drop from Equation (26.15):

$$\Delta P = \Delta P_{fr} - \rho_f \frac{g}{g_c} L_{mf}$$

Note that since the fluidizing fluid is a liquid, with a specific gravity comparable to the specific gravity of the solid particles, the fluid gravity term may not be neglected. In this case, the overall pressure drop, ΔP, is not the same as the friction pressure drop. ΔP_{fr} can be obtained from Equation (26.38). Therefore,

$$\begin{aligned}\Delta P &= \frac{g}{g_c}(\rho_s - \rho_f)L_{mf}(1 - \varepsilon_{mf}) - \rho_f \frac{g}{g_c} L_{mf}\\ &= (1 - 0.38)(84 - 55)(8) - (55)(8) = 583.7\,\text{psf}\end{aligned}$$

Illustrative Example 26.7 Refer to Illustrative Example 26.6. If the bed is fluidized such that the bed height is 10 ft, calculate the volumetric flow rate of oil (in gpm).

Solution Calculate the bed voidage employing Equation (26.37).

$$L_f = L_{\mathrm{mf}} \frac{1 - \varepsilon_{\mathrm{mf}}}{1 - \varepsilon}$$

$$10 = 8 \frac{1 - 0.38}{1 - \varepsilon}$$

$$\varepsilon = 0.504$$

Calculate the superficial velocity of the oil, assuming laminar flow [see Equation (26.29)].

$$v_s = \frac{1}{150} \frac{d_p{}^2 g}{\mu_f} \frac{\varepsilon^3}{1 - \varepsilon} (\rho_s - \rho_f)$$

$$= \frac{1}{150} \frac{(0.00137)^2 (32.174)}{(3.13 \times 10^{-4})} \frac{0.504^3}{1 - 0.504} (84 - 55)$$

$$= 9.6 \times 10^{-3} \text{ ft/s}$$

Calculate the volumetric flow rate.

$$q = \frac{\pi D^2}{4} v_s = \frac{\pi (4)^2}{4} 9.6 \times 10^{-3} = 0.121 \text{ ft}^3/\text{s}$$

Check on the laminar flow assumption:

$$\mathrm{Re}_p = \frac{d_p v_s \rho_f}{\mu_f (1 - \varepsilon)} = \frac{(0.00137)(9.6 \times 10^{-3})(55)}{(0.01)(1 - 0.504)} = 0.145 < 10$$

The flow is therefore laminar.

Illustrative Example 26.8 Refer to Illustrative Example 25.6. Calculate the following:

1. The porous medium friction factor.
2. The Reynolds number.
3. The absolute pressure of the inlet gas.
4. The permeability of the catalyst bed in darcies.

Solution Obtain the porous medium friction factor using the Burke–Plummer equation. Since the flow is turbulent, Equation (26.6) applies and

$$f_{\mathrm{PM}} = 1.75$$

The head loss, h_f, is [see Equation (26.7)]

$$h_f' = 1.75\frac{v_s^2}{g}\frac{(1-\varepsilon)}{\varepsilon^3}\frac{L}{d_p} = 1.75\frac{2^2}{32.174}\frac{0.6}{(0.4)^3}\frac{50}{0.0833} = 1224.3\,\text{ft of propane}$$

Check on the assumption of neglecting the dynamic head (kinetic effects)

$$\frac{v_I^2}{2g} = \frac{5^2}{2(32.174)} = 9.71 \ll 1224.3\,\text{ft}$$

The assumption is justified.

Write Bernoulli's equation between the entrance and gas exit. Neglect the dynamic head.

$$P_1 - P_2 = \rho_f\frac{g}{g_c}[(z_2 - z_1) + h_f]$$

$$\begin{aligned} P_1 &= P_2 + \rho_f\frac{g}{g_c}[(z_2 - z_1) + h_f] \\ &= 4320 + 0.0128[(1)(-50) + 1224.3] = 4335\,\text{psf} \\ &= 30.10\,\text{psi} = 2.048\,\text{atm} \end{aligned}$$

The permeability of the medium, k, is defined only for laminar flow. Since the flow is turbulent, k cannot be calculated.

Illustrative Example 26.9 What is the minimum pressure drop in an activated carbon bed (0.5 m in depth, particle diameters of 0.001 m, bed porosity of 0.25) for turbulent flow of water through the bed?

Solution At turbulent flow, Re is >1000. For minimum pressure drop, set

$$\text{Re} = 1000$$

Therefore

$$1000 = \frac{d_p v_s \rho}{\mu(1-\varepsilon)}$$

$$v_s = \frac{(1000)(\mu)}{d_p \rho}(1-\varepsilon)$$

Assume for water at room temperature (see Table A.4 in the Appendix): $\rho = 1000$ kg/m^3 and $\mu = 1\times10^{-3}$ kg/m · s. Therefore

$$\begin{aligned} v_s &= \frac{(1000)(1\times10^{-3}\,\text{kg/m}\cdot\text{s})(1-0.25)}{(0.001\,\text{m})(1000\,\text{kg/m}^3)} \\ &= 0.75\,\text{m/s} \end{aligned}$$

Then for turbulent flow [Equation (26.12)],

$$\Delta P = \frac{1.75 \rho L v_s^2 (1-\varepsilon)}{g_c \phi_s d_p \varepsilon^3}$$

$$= \frac{(1.75)(1000\,\mathrm{kg/m^3})(0.5\,\mathrm{m})(0.75\,\mathrm{m/s})^2(0.75)}{(\mathrm{kg\cdot m/N\cdot s})(1.0)(0.001\,\mathrm{m})(0.25)^3}$$

$$= 2.36 \times 10^7\,\mathrm{Pa}$$

Illustrative Example 26.10 A bed of 200 mesh particles is fluidized with air at 20°C. The bed has a diameter $D = 0.2$ m. The bed height and porosity at minimum fluidization are 0.3 and 0.45, respectively. The bed is operated with a superficial air velocity of 0.05 m/s. Determine the zero porosity bed height, the air pressure drop, the operating bed height and porosity, and the bed mass.

Solution The particle diameter, d_p, is again obtained from Table 23.2.

$$d_p = 74\,\mu\mathrm{m}$$

Calculate L_0 (the zero porosity bed height) from Equation (26.29).

$$L_0 = L_{\mathrm{mf}}(1-\varepsilon_{\mathrm{mf}}) = 0.3(1-0.45) = 0.165\,\mathrm{m}$$

Calculate the minimum fluidization velocity, v_{mf}, assuming laminar flow. Use Equation (26.9):

$$v_{\mathrm{mf}} = \frac{1}{150}\left(\frac{\varepsilon_{\mathrm{mf}}^3}{1-\varepsilon_{\mathrm{mf}}}\right)\frac{g(\rho_s-\rho_f)d_p^2}{\mu_f}$$

$$= \frac{1}{150}\left(\frac{(0.45)^3}{1-0.45}\right)\frac{9.807(2200-1.2)(7.4\times10^{-5})^2}{1.89\times10^{-5}} = 0.0069\,\mathrm{m/s}$$

The terminal falling velocity of the particle was calculated as 0.35 m/s in Illustrative Example 26.3. Calculate the porosity of the expanded bed from Equation (26.9).

$$0.35 = \left(\frac{1}{150}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right)\left(\frac{9.807(2200-1.2)(7.4\times10^{-5})^2}{1.89\times10^{-5}}\right)$$

$$\varepsilon = 0.91$$

Calculate the expanded bed height L and the bed inventory m.

$$L_f = \frac{L_0}{1-\varepsilon} = \frac{0.165}{1-0.91} = 1.833\,\mathrm{m}$$

$$m = \rho_s \pi d_b^2 L_0 = (2200)(\pi)(0.2)^2(0.165) = 45.6\,\mathrm{kg}$$

Illustrative Example 26.11 Refer to Illustrative Example 26.9. Specify whether the fluidization mode is smooth or bubbling.

Solution Determine the fluidization mode [see Equation (26.40)]:

$$Fr_{\mathrm{mf}} = \frac{v_{\mathrm{mf}}^2}{gd_p} = \frac{(0.0069)^2}{9.807(7.4 \times 10^{-5})} = 0.066 < 0.13$$

The fluidization is smooth.

26.7 FLUIDIZATION EXPERIMENT DATA AND CALCULATIONS

One of the experiments conducted in the Chemical Engineering Laboratory at Manhattan College is concerned with fluidization. Students perform the experiment and later submit a report. In addition to theory, experimental procedure, discussion of results, etc., the report contains sample calculations. The following is an (edited) example of those calculations that cover a wide range of fluidization principles and applications.

The fluidization experiment consists of two parts. The first part, which is examined here, determines the particle characteristics of sand in the fluidized bed. There are four parts to determine the sand characteristics. First, the bulk density of the particles is calculated, then the particle density, particle size distribution, and finally the "hydraulic" particle diameter.

In order to calculate the bulk density, the mass and volume of the particles need to be measured. A sample of the glass particles was placed in a 1-L graduated cylinder. The weight and volume of the particles in the cylinder were determined as follows:

$$m_{\mathrm{cyl}} = 623\ \mathrm{g}$$

$$m_{\mathrm{cyl+sand}} = 877\ \mathrm{g}$$

$$m_{\mathrm{sand}} = m_{\mathrm{cyl+sand}} - m_{\mathrm{cyl}} = 877\ \mathrm{g} - 623\ \mathrm{g}$$

$$= 254\ \mathrm{g}$$

The volume occupied by the particles in the cylinder was 170 ml. The bulk density of any substance or particle is given by:

$$\rho_B = \frac{m_{\mathrm{glass}}}{V_{\mathrm{glass}}}$$

Substituting the above gives

$$\rho_B = \frac{254\ \mathrm{g}}{170\ \mathrm{mL}} = 1.494\ \mathrm{g/mL} = 1494\ \mathrm{kg/m^3}$$

The area S of the bed has a width of 24 in and a depth of 2 in. Therefore,

$$S = (W)(D) = (24)(2)$$

$$= 48\ \mathrm{in}^2$$

$$= 0.0310\ \mathrm{m}^2$$

The bulk density of the particles is used to determine the total mass, m, of the sand beads in the bed of static height L:

$$m = (S)(L)(\rho_B)$$

The static bed height is obtained by taking an average of three height measurements at different points in the bed

$$L_1 = 24.09\,\text{in}$$
$$L_2 = 23.62\,\text{in}$$
$$L_3 = 25.98\,\text{in}$$
$$L = \frac{L_1 + L_2 + L_3}{3} = \frac{24.09 + 23.62 + 25.98}{3}$$
$$= 24.56\,\text{in} = 0.624\,\text{m}$$

The mass is therefore

$$m = (0.0310\,\text{m}^2)(0.624\,\text{m})(1494\,\text{kg/m}^3)$$
$$= 28.9\,\text{kg}$$

The second necessary measurement in the experiment consists of finding the particle density. Approximately 75 g of sand particles were placed in a 100-ml volumetric flask. The mass of the glass is determined in the same way as it is found previously.

$$m_{\text{flask}} = 0.068\,\text{kg}$$
$$m_{\text{flask+sand}} = 0.143\,\text{kg}$$
$$m_{\text{sand}} = m_{\text{flask+sand}} - m_{\text{flask}} = 0.143\,\text{kg} - 0.068\,\text{kg}$$
$$= 0.075\,\text{kg}$$

Water is added to the flask—first up to $\frac{3}{4}$ full and later additional volume is added to the 100-ml mark. This reduces the void spaces between the particles. The volume of water added was 68.8 mL. Therefore, the volume of the glass particles is given by

$$V_{\text{sand}} = V_{\text{flask}} - V_{\text{water}} = 100\,\text{mL} - 68.8\,\text{mL}$$
$$= 31.2\,\text{mL} = 3.12 \times 10^{-5}\,\text{m}^3$$

The particle density is now determined

$$\rho_p = \frac{m_{\text{sand}}}{V_{\text{sand}}} = \frac{0.075\,\text{kg}}{3.12 \times 10^{-5}\,\text{m}^3}$$
$$= 2403.85\,\text{kg/m}^3$$

This particle density is used to calculate the height of the bed with no void spaces, L_0. This height is needed to determine the bed porosity. The following equation is used for this calculation

$$L_0 = \frac{m}{\rho_p S}$$

$$= \frac{(28.9\,\text{kg})}{(2403.85\,\text{kg/m}^3)(0.0310\,\text{m}^2)}$$

$$= 0.388\,\text{m}$$

The bed porosity is determined as follows:

$$\varepsilon = 1 - \frac{L_0}{L_{\text{static}}}$$

$$= 1 - \frac{0.388\,\text{m}}{0.624\,\text{m}}$$

$$= 0.378$$

The third part of the experiment consists of determining the particle size distribution. Here, approximately 1000 grams of sand were placed in the Tyler shaker (see Chapter 23 for more details) and the screen test was studied. Table 26.1 shows the screen numbers and sizes.

Two runs were performed in this part. After each run, the mass of the particles left in the trays were measured and an average was calculated as follows:

Run 1:

Tray No. 80

$$m_T = \sum m_{\text{tray+sand}} - m_{\text{tray}}$$

$$= (153\,\text{g} - 1.4989\,\text{g}) + (145\,\text{g} - 1.4981\,\text{g}) + (155\,\text{g} - 1.4753\,\text{g})$$
$$+ (145\,\text{g} - 1.5213\,\text{g}) + (13\,\text{g} - 1.4915\,\text{g})$$

$$= 603.5\,\text{g}$$

Table 26.1 Sieve trays screen number and sizes

No	Size, μm
50	297
60	250
80	177
100	149
120	125
140	105
170	88
200	74

The same calculation was made for the sand left on tray 80 on the second run:

Run 2:

Tray No. 80

$m_T = 451.8\,\text{g}$

The average mass for tray 80 is therefore:

$$m_{\text{ave}} = [m_T \text{ (Run 1)} + m_T \text{ (Run 2)}]/2$$
$$= 527.7\,\text{g}$$

From the data, the cumulative mass percent of material smaller than each of the screen sizes was calculated. Using tray No. 80 again, the following percentage was obtained:

Tray No. 80

$$\text{mass} > 177\,\mu\text{m} = \sum_{t=80}^{200} m_t; \quad t = \text{tray number}$$
$$= (373.02\,\text{g} + 527.7\,\text{g}) = 900.72\,\text{g}$$

The 373.02 g represents the mass on trays 100 through 200. The total average mass of glass for the two runs was determined as follows:

Run 1:

Sand mass = 986.3 g

Run 2:

Sand mass = 984.1 g

Average mass = (986.3 + 984.1)/2 = 985.2 g

The mass of glass smaller than 177 μm is calculated:

$$\text{Mass} < 177\,\mu\text{m} = \text{Average weight} - \text{Weight} > 177\,\mu\text{m}$$
$$= 985.2\,\text{g} - 900.72\,\text{g} = 84.5\,\text{g}$$

The cumulative percent smaller than 177 μm is therefore

$$\%\text{W} < 177\,\mu\text{m} = \frac{\text{Weight} < 177\,\mu\text{m}}{\text{Average weight}} \times 100 = \frac{84.5\,\text{g}}{985.2} \times 100$$
$$= 8.85\%$$

This same procedure was used for each screen size. The final values obtained were plotted in a log-probability paper so as to show the curve for the particles size distribution.

The last part of the characteristics section is to determine the particles' hydraulic diameter. A sample of the glass captured on the screen nearest to the 50% mass position on the size distribution curve was taken in order to perform the terminal velocity experiment. The sample was taken from the particles left in tray No. 60. Six runs were made for the terminal velocity. The time the particles took to travel 24 in was measured for each run and the velocity was determined from this values. For the first run, the particles took 14 s to travel the given distance. The velocity was calculated as follows;

$$v_t = \frac{d}{t} = \frac{(24\,\text{in})(0.0254\,\text{m/in})}{(14\,\text{s})} = 0.044\,\text{m/s}$$

Table 26.2 shows the terminal velocities measured for the six runs.

The average terminal velocity in water is therefore:

$$\begin{aligned} v_t &= \sum_{i=1}^{n} \frac{v_{ti}}{n} \\ &= \frac{(0.044) + (0.043) + (0.044) + (0.038) + (0.038) + (0.039)}{6} \\ &= 0.041\,\text{m/s} \end{aligned}$$

The hydraulic diameter and the terminal velocity were also determined. The procedure set forth in Chapter 23 was employed. The calculated diameter was

$$d_p = 3.05 \times 10^{-4}\,\text{m}$$

Using the calculated particle diameter, the terminal velocity of air can be obtained by substituting the properties of air instead of the ones for water in the appropriate Chapter 23 equations. The final result was:

$$v_{t,\text{air}} = 1.755\,\text{m/s}$$

Table 26.2 Terminal velocities for six runs

Run	Time (s)	v_t (m/s)
1	14	0.044
2	14.06	0.043
3	13.84	0.044
4	16.03	0.038
5	16.1	0.038
6	15.53	0.039

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