

# 23

## **Term Projects (2): Applied Mathematics**

23.1 Simplified Procedure for Solving Differential Equations

23.2 The Weighted Sum Method of Analysis

## Term Project 23.1

### Simplified Procedure for Solving Differential Equations

There are essentially two classes of differential equations that the chemical engineer occasionally encounters: ordinary and partial. Generally speaking, an equation containing one or more derivatives is called a *differential equation*. If these are ordinary derivative, the equation is said to be an *ordinary differential equation* (ODE); i.e., it contains only one independent variable, and as a consequence, is referred to as a total or ordinary derivative. Partial differential equations contain several independent variables, and as a consequence, are referred to as partial derivative, and the equation is termed a *partial differential equation* (PDE).

An ordinary differential equation, which is of the first degree in the dependent variable and all its derivatives, is termed a *linear equation*. Correspondingly, an equation in which the dependent variable or any of its derivatives is of the second or higher degree, is termed a *nonlinear equation*. However, the order of an equation is the order of the highest derivative contained therein.

For many chemical engineering applications, the dependent variable is defined in terms of more than one independent variable. Variation with respect to the independent parameters is possible and the derivatives describing these independent variations are defined as the aforementioned partial derivatives and the differential equations involving partial derivatives are known as partial differential equations. The three most often encountered PDEs are the *elliptic, parabolic, and hyperbolic* equations.

Providing specific details on the various methods of solving both ODEs and PDEs are beyond the scope of both this term project and this book. However, it may be possible to develop simple procedures to solve either or both of these two classes of differential equations. With this possibility in mind, you have been requested to prepare a sample; e.g., cookbook; method of solving:

1. ordinary differential equations, and
2. partial differential equations.

Comment: One very simple approach to solving differential equations is to guess a solution [1]. See also Illustrative Problem 2 in Part II Chapter 2. See also material available in the literature [2,3].

## Term Project 23.2

### The Weighted Sum Method of Analysis

The Weighted Sum Method (WSM) is a semi-quantitative method for screening and ranking process and design options. This method can and has been applied previously in quantifying the important criteria that affect equipment selection [4], energy resources [5], waste management at a particular facility [6], and pollution prevention [7], etc. This method involves three steps.

1. Determine what the important criteria are in terms of the program goals and constraints, and the other corporate goals and constraints. Example criteria as applied to pollution prevention are listed below.
  - a. reduction in waste quantity
  - b. reduction in waste hazard (e.g., toxicity, flammability, reactivity)
  - c. reduction in waste treatment/disposal costs
  - d. reduction in raw material costs
  - e. reduction in liability and insurance costs
  - f. previous successful use within the company
  - g. previous successful use in industry
  - h. not detrimental to product quality
  - i. low capital cost
  - j. low operating and maintenance costs
  - k. short implementation period with minimal disruption of plant operations
  - l. improved public relations
  - m. reduced workman's compensation
  - n. improved employee morale
  - o. reductions or elimination of liability
  - p. reduction or elimination of regulatory concerns.

A weight factor for each criteria is assigned. This is defined as the *weight of the criteria*. The weights (on a scale of 0 to 10, for example) are determined for each of the criteria in relation to their importance. For example, if reduction in waste treatment and disposal costs are very important, while previous successful use within the company is of minor importance, then the reduction in waste costs is given

- a high weight factor of 9 or 10, and the previous use within the company is given a low weight factor of either 1 or 2. Criteria that are not important are either not included, or are given a weight factor of 0.
2. Each criteria is then rated for its effect on the various options to be investigated. These ratings are defined as *effectiveness factors*. Again, a scale of 0 to 10 can be used (0 for low and 10 for high).
  3. Finally, the *effectiveness factor* (step 2) for a particular criterion is multiplied by the *weight factor of the criterion* (step 1). An option's *overall rating* is the sum of the *effectiveness factor* and the *weight factor criterion*.

The option(s) with the best overall rating(s) is (are) then selected for further technical and/or economic feasibility analyses.

Apply the Weighted Sum Method to a process/design/project of your choice. Comment on the results.

## References

1. Personal Notes, L. Theodore, East Williston, NY, 1969.
2. R. Ketter and S. Prawler, *Modern Methods of Engineering Computation*, McGraw-Hill, New York City, NY 1969.
3. L. Theodore, *Transport Phenomena for Engineers*, Theodore Tutorials, East Williston, NY, originally published by International Textbook CO., Scranton, PA, 1971.
4. A.J. Buonicore, J. Reynolds, and L. Theodore, *Control Technology for Fine Particulate Emissions*, Argonne National Laboratory, Chicago, IL, 1979.
5. K. Skipka and L. Theodore, *U.S. Energy Resources: Past, Present, and Future Management*, CRC Press/Taylor & Francis Group, Boca Raton, FL, 2014.
6. J. Santoleri, J. Reynolds, and L. Theodore, *Introductions to Hazardous Waste Incineration*, 2<sup>nd</sup> edition, John Wiley & Sons, Hoboken, NJ, 2004.
7. R. Dupont, L. Theodore, and K. Ganesan, *Pollution Prevention: The Waste Management Option for the 21<sup>st</sup> Century*, CRC Press/ Taylor & Francis Group, Boca Raton, FL, 2000.