# **Humidification and Drying**

# INTRODUCTION

In many unit operations, it is necessary to perform calculations involving the properties of mixtures of air and water vapor. Such calculations often require knowledge of:

- 1 the amount of water vapor carried by air under various conditions,
- 2 the thermal properties of such mixtures, and
- **3** the changes in enthalpy content and moisture content as air containing some moisture is brought into contact with water or wet solids, and other similar processes.

This chapter will review the properties of mixtures of air and water vapor, the mechanisms of humidification and drying processes, and the equipment in which these processes are carried out. The remainder of this chapter will focus on three topics:

- **1** Psychrometry and the Psychrometric Chart (including some key definitions)
- 2 Humidification
- 3 Drying

# PSYCHROMETRY AND THE PSYCHROMETRIC CHART

Some key (and important) terms are introduced before proceeding to the general subject of psychrometry. In the discussion of the physical properties of mixtures of air and water vapor, certain terms need to be defined. The definitions of several key terms follow.<sup>(1)</sup>

1 The *humidity*, or *absolute humidity*,  $Y_A$ , is defined as the mass of water (designated with subscript *A*) carried by one unit of mass of bone dry air (BDA). The *molal humidity*,  $Y'_A$ , can also be defined, but most of the literature on

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humidification and drying is in terms of  $Y_A$ . The two quantities are related by:

$$Y_A = \frac{18}{29} Y'_A \tag{13.1}$$

The humidity at 1.0 atm is then related to the mole fraction,  $y_A$ , and the partial pressure,  $p_A$ , by:

$$Y_A = \frac{18}{29} \left( \frac{y_A}{1.0 - y_A} \right) = \frac{18}{29} \left( \frac{p_A}{1.0 - p_A} \right)$$
(13.2)

2 Air is said to be saturated with water vapor at a given temperature and pressure when it contains the maximum amount of water vapor possible at that condition. This is achieved when the air is in equilibrium with liquid water. The *saturation humidity*,  $Y_{As}$ , is the value of  $Y_A$  corresponding to a partial pressure,  $p_A$ , equal to the vapor pressure of water,  $p'_A$ , at the given temperature. The *relative humidity*,  $Y_R$ , is defined by:

$$Y_R = 100 \left(\frac{p_A}{p'_A}\right) \tag{13.3}$$

3 The percent humidity, or percent saturation, PS, is given by:

$$PS = 100 \left(\frac{Y_A}{Y_{As}}\right) = \frac{p_A}{p'_A} \left(\frac{1.0 - p'_A}{1.0 - p_A}\right) \times 100$$
(13.4)

where  $p'_A$  and  $p_A$  must be expressed in atmospheres.

**4** The *humid heat capacity*,  $C_{pH}$ , is defined as the energy required to raise the temperature of 1 lb of the carrier gas (air, *B*) and its accompanying vapor (water, *A*) by 1°F. Thus, if  $C_{pB}$  and  $C_{pA}$  are the heat capacities of the carrier gas and vapor, respectively,  $C_{pH}$  is given by:

$$C_{pH} = C_{pB} + Y_A C_{pA} \tag{13.5}$$

For the air-water system, the above equation becomes:

$$C_{pH} = 0.24 + 0.46Y_A; \quad [Btu/{}^{\circ}F \cdot lb BDA]$$
(13.6)

Note that heat capacities on a mass basis are designated with a *capital C* in this field. The molal humid heat is based on moles of BDA.

5 The *humid volume*,  $V_H$ , is defined as the volume, in ft<sup>3</sup>, of "moist" gas per unit mass of bone dry gas where

$$V_H = V_R + Y_A V_A; \quad [ft^3/lb BDA]$$
(13.7)

The terms  $Y_R$  and  $V_A$  represent the volume, in ft<sup>3</sup>, of BDA/lb BDA and volume of A/lb A, respectively.

**6** The enthalpy of moist air,  $H_y$ , at  $T_y$  is defined by Equation (13.8) in units consistent with the present discussion:

$$H_{v} = C_{pH}(T_{v} - T_{o}) + \lambda_{o}Y_{A}; \quad [Btu/lb BDA]$$
(13.8)

The term  $\lambda_o$  is the heat of vaporization at some reference temperature,  $T_o$ , which is normally assumed to be either 0°F or 32°F. This effectively represents the sensible enthalpy of the air–water mixture relative to  $T_o$  plus the enthalpy of vaporization at  $T_o$ .

- 7 The *adiabatic saturation temperature*,  $T_{as}$ , of moist air is the temperature that air reaches when it is saturated adiabatically (i.e., at constant enthalpy).
- 8 The *wet-bulb temperature*,  $T_{wb}$ , is the temperature attained by a small reservoir of water in contact with a large amount of air flowing past it. Normally, one can assume that  $T_{wb}$  and  $T_{as}$  are the same for the water–air system at temperatures low enough to form a dilute gas-phase solution. For other liquid–vapor mixtures, the adiabatic saturation and wet-bulb temperatures are normally not equal.
- **9** The *dew point temperature*,  $T_{dp}$ , is the temperature at which a given sample of moist air becomes saturated as it is cooled at constant pressure and absolute humidity. Thus, the dew-point pressure is the (total) pressure to which moist air must be compressed at constant temperature and humidity to bring it to saturation. As an example, suppose the air in a room at 70°F is at 50% relative humidity. From steam tables (see Appendix), the vapor pressure of water at 70°F is 0.3631 psi, which means that the air at 50% humidity holds water vapor with a partial pressure of (0.50)(0.3631) or 0.1812 psi. If the temperature is dropped at constant pressure to the point where 0.1812 psi equals the water vapor pressure (around 52°F), the air becomes saturated with water and any further drop in temperature will cause condensation. The dew point of this air mixture is then 52°F. Obviously, if the air were already saturated at 70°F (i.e., 100% relative humidity), then the dew point would also be 70°F.

A summary of the above definitions is provided in Table 13.1.

One vapor-liquid phase equilibrium example containing air-water raw data is the *psychrometric* or *humidity chart*. A humidity chart is used to determine the properties of moist air and calculate the moisture content in air. The ordinate of the chart is the absolute humidity, Y (with the subscript A dropped for convenience), which was defined earlier as the mass of water vapor per mass of bone dry air. (Note that some charts base the ordinate on moles instead of mass). The previously defined concepts and definitions are normally presented graphically on the aforementioned psychrometric or humidity chart (see Figs 13.1 and 13.2). There are also charts that apply to other single noncondensible gases and single condensable components at a fixed pressure (usually 1 atm).

Curves showing the relative humidity (ratio of the partial pressure of the water vapor in the air to the vapor pressure of water at the system temperature) of humid air also appear on the charts. The curve for 100% relative humidity is also referred

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Term		Definition	Units
1.	Absolute humidity	Vapor content of a gas	lb vapor/lb noncondensible gas (BDA)
2.	Molal humidity	Vapor content of a gas	moles vapor/mole noncondensible gas
3.	Relative saturation or relative humidity	Ratio of partial pressure of vapor to partial pressure of vapor at saturation	atm/atm, or mole fraction/ mole fraction, often expressed as a percent
4.	Percent saturation or percent humidity	Ratio of concentration of vapor to the concentration of vapor at saturation with concentrations expressed as mole ratios	mole ratio/mole ratio, often expressed as a percent
5.	Humid volume	Volume occupied by 1 lbmol of dry gas plus its associated vapor	ft <sup>3</sup> /lbmole of BDA
6.	Humid heat capacity	Energy required to raise the temperature of 1 lbmol of bone dry gas plus its associated vapor 1°F	Btu/lbmol of BDA $\cdot{}^\circ F$
7.	Adiabatic- saturation temperature	Temperature that would be attained if the gas were saturated in an adiabatic process	°F
8.	Wet-bulb temperature	Steady-state temperature attained by a wet-bulb thermometer under standardized conditions	°F
9.	Dew-point temperature	Temperature at which vapor begins to condense when the gas phase is cooled at constant pressure	°F

 Table 13.1
 Definitions of Psychrometric Terms

to as the *saturation curve*. The abscissa of the humidity chart is the air temperature, also known as the dry-bulb temperature,  $T_{db}$ . The *wet-bulb* temperature, which has also already been defined (see Table 13.1), is another measure of the humidity. As described earlier, it is the temperature at which a thermometer stabilizes when it has a wet wick wrapped around the bulb. As water evaporates from the wick to the ambient air, the bulb is cooled; the rate of cooling depends on how humid the air is. No evaporation occurs if the air is saturated with water; hence,  $T_{wb}$  and  $T_{db}$  are the same and the lower the humidity, the greater the difference between these two temperatures. On the psychrometric chart, constant wet-bulb temperature lines are straight with negative slopes. The value of  $T_{wb}$  corresponds to the value of the abscissa at the point of intersection of this line with the saturation curve.

The *humid volume*, also defined earlier, is the volume of wet air per mass of BDA and is linearly related to the humidity. This quantity is used as an alternate ordinate in Figure 13.2. Note the straight parallel lines labeled with units of cubic feet. The *humid enthalpy* is the enthalpy of the moist air on a bone-dry basis. Since enthalpy is a measure of the energy content of the mixture, the enthalpy for saturated air can be read from the chart by extending the approximate wet-bulb temperature line upwards to the diagonal scale labeled *enthalpy of saturation*.







Figure 13.2 Psychrometric chart—high temperatures. Barometric pressure, 29.92 in Hg.

A common experimental method for determining the humidity of air is to determine the wet-bulb and dry-bulb temperatures simultaneously. The operation described above can be accomplished by rapidly passing a stream of air over two thermometers, the bulb of one which is dry. The bulb of the other is kept wet by means of a cloth sack either dipped in water or supplied with water. In a sling psychrometer, the two thermometers are fastened in a metal frame that may be whirled about a handle. The psychrometer is whirled for some seconds and the reading of the wet-bulb thermometer is observed as quickly as possible. The operation is repeated until successive readings of the wet-bulb thermometer show that it has reached its minimum temperature. Thus, a psychrometer consists mainly of two thermometers for wet- and dry-bulb readings. The wet-bulb temperature is the equilibrium temperature obtained by the aforementioned action of an unsaturated vapor-gas mixture flowing past a wetted wick completely covering the bulb of the thermometer. At this equilibrium temperature, the sensible heat given to the water from the air is balanced by the loss of heat from the water by evaporation. An expression for the absolute humidity Y can be obtained from both a heat and material balance in terms of the dry-bulb temperature,  $T_{db}$ , and the wet-bulb temperature,  $T_{wb}$ :

$$Y = Y_w - \frac{h}{k\lambda_w}(T_{db} - T_{wb})$$
(13.9)

where  $Y_w$  = the saturated absolute humidity at the wet-bulb temperature

 $\lambda_w$  = the latent heat of vaporization at  $T_{wb}$ 

h = the gas film coefficient for heat transfer by conduction and convection

k = the mass transfer coefficient

The ratio of h to k (English units) has been experimentally determined to equal 0.236 for an air-water system. It is assumed that radiation effects are negligible in Equation (13.9).

As mentioned earlier, the relative humidity,  $Y_R$ , is the ratio of the partial pressure of the water vapor,  $p_A$ , in the mixture to the vapor pressure of liquid water,  $p'_A$ , at the system temperature. This was expressed in Equation (13.3) on a percentage basis.

In addition, the dew point is the temperature to which a vapor–gas mixture must be cooled at constant total pressure and humidity to become saturated. This can be determined by calculating  $p_A$  in the formula for  $Y_R$  [Equation (13.3)], and looking up the temperature that corresponds to this vapor pressure of water.

In an adiabatic "saturator", evaporation cools the water to  $T_{as}$ , the adiabatic saturation temperature. If the contact time between the air and water is long enough, the air becomes saturated and leaves the apparatus at the steady-state temperature  $T_{as}$ . The following equation can be derived from an enthalpy balance, where the subscript *as* refers to conditions at  $T_{as}$ :

$$T - T_{as} = \frac{\lambda_o(Y_{as} - Y)}{C_{pH}} \tag{13.10}$$

For the air-water system at ordinary temperature ranges, the heat capacity may be assumed as constant and one may employ Equation (13.6) for  $C_{pH}$ .

Equations (13.9) and (13.10) for absolute humidity can be compared in the following form:

$$Y_w - Y = \frac{h}{k\lambda_w}(T - T_w);$$
 for wet-dry bulb data (13.11)

$$Y_{as} - Y = \frac{C_{pH}}{\lambda_s}(T - T_{as});$$
 for adiabatic saturation data (13.12)

where *h* and *k* again represent the heat and mass transfer coefficients, respectively. Both equations would be identical if  $C_{pH} = h/k$ . This is almost always true for the air–water system. Hence, the adiabatic cooling line on the psychrometric chart may be used for wet-bulb problems under ordinary conditions.

Figures 13.1 and 13.2 contain the psychrometric charts from thermodynamic properties drawn from the literature. A General Electric chart is provided in Figure 13.3.<sup>(2)</sup>

The following are some helpful points on the use of psychrometric charts:

- 1 Heating or cooling at temperatures above the dew point (temperature at which the vapor begins to condense) corresponds to a horizontal movement on the chart. As long as no condensation occurs, the absolute humidity remains constant.
- **2** If the air is cooled, the system follows the appropriate horizontal line to the left until it reaches the saturation curve and follows it thereafter.
- **3** In problems involving use of the humidity chart, it is convenient to choose the mass of air on a dry basis since the chart uses this basis.

#### **ILLUSTRATIVE EXAMPLE 13.1**

For air with a dry- and wet-bulb temperature of 80°F and 65°F, respectively, calculate the following:

- 1 Relative humidity
- 2 Moisture content
- 3 Dew point
- 4 Total heat
- 5 Specific volume
- 6 Vapor pressure

**SOLUTION:** The solution is obtained by carefully reading the charts in Figures 13.1 and 13.2. Answers are provided below.

- 1 Relative humidity = 45%
- 2 Moisture content = 0.151 lb/lb BDA
- 3 Dew point =  $56^{\circ}$ F
- 4 Total heat = 30.0 Btu/lb BDA

- 5 Specific volume =  $13.81 \text{ ft}^3/\text{lb}$  BDA
- **6** Vapor pressure = 0.228 psi

#### **ILLUSTRATIVE EXAMPLE 13.2**

Qualitatively explain the effect of the wet-bulb temperature on how a sling psychrometer works.

**SOLUTION:** As described in this section, the wet-bulb temperature  $(T_{wb})$  is the temperature at which a thermometer with a wet wick wrapped around the bulb stabilizes. As water evaporates from the wick to the ambient air, the bulb is cooled; the rate of cooling depends on how humid the air is. No evaporation occurs if the air is saturated with water; hence, the wet-bulb temperature and the dry-bulb temperature are the same when the air has a 100% relative humidity.

#### **ILLUSTRATIVE EXAMPLE 13.3**

Determine the key properties for humid air at a dry-bulb temperature of  $160^{\circ}$ F and a wet-bulb temperature of  $100^{\circ}$ F.

**SOLUTION:** Refer to the psychrometric chart in Figure 13.1. If the air were to be cooled until moisture just begins to condense, the dew point would be reached. This is represented by a horizontal line at constant absolute humidity intersecting the saturation curve at a dew point of approximately  $87.5^{\circ}$ F.

The relative humidity is approximately 14% (interpolating between the 10 and 15% relative humidity lines). The absolute humidity is the horizontal line extended to the right; it intersects the ordinate at a humidity of 0.0285 lb  $H_2O/lb$  BDA.

The humid volume is approximately  $16.3 \text{ ft}^3$  moist air/lb BDA (interpolating between 16 and 17 ft<sup>3</sup> moist air volume).

The enthalpy for saturated air at a  $T_{wb}$  of 100°F is 71.8 Btu/lb BDA. For the unsaturated air, the enthalpy deviation is -1.0 Btu/lb BDA; therefore, the actual enthalpy for the moist air at a  $T_{wb}$  of 100°F and a  $T_{db}$  of 160°F is 70.8 Btu/lb dry air.

#### **ILLUSTRATIVE EXAMPLE 13.4**

A stream of moist air is cooled and humidified adiabatically from  $T_{db}$  of 100°F and  $T_{wb}$  of 70°F to a  $T_{db}$  of 80°F. How much more moisture is added per pound of BDA?

**SOLUTION:** Once again, refer to the psychrometric chart in Figure 13.1. Adiabatic cooling follows the wet-bulb temperature line upwards (toward the saturation curve). The difference in the final and initial humidities is the moisture added:

State	Y	
Initial	0.0090	
Final	0.0133	



**Figure 13.3** General Electric psychrometric chart.<sup>(2)</sup> (a) Sensible heating and cooling of air are represented on the psychrometric chart by a straight horizontal line between the dry-bulb temperature limits of the process. These processes are distinguished by a change in dry-bulb temperature, relative humidity, wet-bulb temperature, total heat, specific volume, and by no change in moisture content, dew-point temperature, and vapor pressure of the air. (b) Cooling and dehumidifying of air are represented on the psychrometric chart by a straight line drawn between the initial condition of the air and the point on the 100 per cent line corresponding to the temperature of the cooling surface. This applies only when the surface temperature is below the initial dew point. The final condition of the air will depend on the total heat extracted from the air. This process is distinguished by a change in all properties of the air. (c) Humidifying of air, with no temperature changes, is represented by a straight vertical line along the dry-bulb temperature line of the air between the moisture content limits of the process. This process is distinguished by an increase in relative humidity, wet-bulb temperature, total heat, specific volume, moisture content, dew-point temperature and vapor pressure of the air. (d) Mixing of air at one condition with air at some other condition is represented by a straight line drawn between the points representing the two air conditions. The condition of the resultant mixture will fall on this line at a point determined by the relative masses (lever rule ) of air being mixed. (e) Evaporative cooling of air, by bringing it in contact with water at a temperature equal to the wet-bulb temperature of the air, is represented by a straight line drawn along the wet-bulb

Therefore

$$\Delta Y = 0.0133 - 0.0090 = 0.0043 = 4.3 \times 10^{-3} \text{ lb H}_2\text{O/lb} \text{ BDA}$$

# **ILLUSTRATIVE EXAMPLE 13.5**

A gas is discharged at  $140^{\circ}$ F from an HCl absorber. If 9000 lb/h (MW = 30) of gas enters the absorber essentially dry (negligible water) at  $560^{\circ}$ F, calculate the moisture content, the mass flow rate, and the volumetric flow rate of the discharge gas. The discharge gas from the absorber may safely be assumed to be saturated with water vapor.

SOLUTION: From Figure 13.1, the discharge humidity of the gas is approximately

$$Y_{\text{out}} = 0.0814 \,\text{lb}\,\text{H}_2\text{O}/\text{lb}\,\text{BDA}$$

This represents the moisture content of the gas at outlet conditions. If the gas is assumed to have the properties of air (i.e., BDA), the discharge water vapor rate is:

$$\dot{m}_{\rm H_2O} = (0.0814)(9000)$$
  
= 733 lb/h

The total flow rate leaving the absorber is:

$$\dot{m}_T = 733 + 9000$$
  
= 9733 lb/h

The volumetric (or molar) flow rate can only be calculated if the molecular weight of the gas is known. The average molecular weight of the discharge gas must first be calculated from the mole fraction of the gas (g) and water vapor (w):

$$y_g = \frac{9000/30}{(9000/30) + (733/18)} = 0.88$$
$$y_w = \frac{733/18}{(733/18) + (9000/30)} = 0.12$$
$$\overline{MW} = (0.88)(30) + (0.12)(18) = 28.6$$

**Figure 13.3** (*Continued*) temperature line of the air between the limits of the process. In this process, the total heat of the air remains unchanged because the sensible heat extracted from the air is returned as latent heat by an increase of moisture content. This process is distinguished by a change in dry bulb temperature, relative humidity, specific volume, moisture content, dew-point temperature, vapor pressure, and by no change in wet-bulb temperature. (f) Chemical drying of air is represented by a straight line along the wet-bulb temperature between the limits of the process (AB) only in case the drying is purely by adsorption (the drying agent does not dissolve in the water extracted from the air) and only in case the drying agent does not dissolve in the heat of vaporization liberated when the water is condensed on the surface of the adsorbent. In case an appreciable amount of this heat is retained by the adsorbent, the process takes place on a line below the wet-bulb temperature (AB'). If the drying agent is soluble in water (such as calcium chloride) the drying process is above (AB'') or below (AB') the wet-bulb temperature, depending on whether heat is liberated or absorbed when the agent is dissolved in water.

The ideal gas law is employed to calculate the volumetric flow rate,

$$Pq = \frac{\dot{m}_T}{(\overline{MW})} RT$$

$$q = \left(\frac{9733}{28.6}\right) \frac{(0.73)(460 + 140)}{1.0}$$

$$q = 1.49 \times 10^5 \, \text{ft}^3/\text{h}$$

#### **ILLUSTRATIVE EXAMPLE 13.6**

A dehumidification unit in a large factory draws in moist air at 500 lb/h. Moist air enters the unit at a temperature of 82.5°F and a wet-bulb temperature of 75°F. The exit stream of the unit is 60.8°F and saturated with water. Determine the amount of water (in lb/h) condensed in the dehumidifier.

**SOLUTION:** This problem is meant to serve as an example of a basic material balance. As with all material balance problems, it is first recommended that a picture be drawn as follows:



In order to solve this problem, one must make use of the aforementioned *relative humidity* ( $Y_R$ ). As noted earlier, the relative humidity is the ratio of the partial pressure of water in the gas phase to the vapor pressure of water at temperature *T*. Since the outlet stream is saturated with water, its relative humidity is unity. The relative humidity of the inlet stream, however, can be determined via Figure 13.1. Locating the appropriate intersection of *T* and  $T_{wb}$  for the inlet conditions,

$$Y_R = 0.73 = 73\%$$

Note that,

$$Y_R = \frac{y_W P}{p'_W}$$

At the inlet temperature, the vapor pressure of water is approximately 0.0373 atm (see steam tables in the Appendix). Therefore, at the inlet:

$$y_w = \frac{Y_R p'_w}{P} = \frac{(0.73)(0.0373)}{(1)} = 0.027$$

Also note  $y_{BDA} = 1 - y_w$ . At the exit conditions, the vapor pressure of water is approximately 0.0179 atm. Therefore, at the exit:

$$y_w = \frac{Y_R p'_w}{P}$$
  
=  $\frac{(1)(0.0179)}{(1)}$   
= 0.0179

The total material balance equation for the system may be written

$$500 = \dot{m}_W + \dot{m}_E$$

Similarly, a componential balance for water may be written around the dehumidifier, as

$$(0.027)(500) = (\dot{m}_W) + (0.0179)(\dot{m}_E)$$

The details of solving the above two equations simultaneously is left as an exercise for the reader. The final result is,

$$\dot{m}_W = 4.6 \,\mathrm{lb/h}$$

#### HUMIDIFICATION

The evaporation of water into air for the purpose of increasing the air humidity is known as *humidification*. Closely allied to this is the evaporation of water into air for the purpose of cooling the water. *Dehumidification* consists of condensing water from air to decrease the air humidity. All these processes are of considerable industrial importance and involve the contacting of air and water accompanied by heat and mass transfer. These processes are discussed in this section.

In terms of industrial applications of humidification, it is often necessary to employ air at a known temperature and a known humidity. This can be accomplished by bringing the air into contact with water under such conditions that a desired humidity is reached. This mixing process can occur in any of the gas-liquid contacting devices discussed early in this Part.

If conditions in a humidifier are such that the air reaches complete saturation, the humidity is fixed. However, if the equipment is operated in such a manner that the exit air is not saturated, then process conditions are somewhat indeterminate. Many of these applicable calculations can be obtained directly from the psychrometric chart provided in the previous section, or from an equivalent chart.

Direct contact of a condensable vapor-noncondensable gas mixture with a liquid can produce any of several results, including humidifying of the gas or cooling of the liquid. The direction of liquid transfer (either humidification or dehumidification) depends on the difference in humidity of the bulk of the gas and that at the liquid surface. If the liquid is normally a pure substance, no concentration gradient exists within it and the resistance to mass transfer lies entirely within the gas. Since evaporation and condensation of vapor simultaneously involves a latent enthalpy of vaporization or condensation, there will always be a transfer of latent heat in the direction of mass transfer. The temperature differences existing within the system additionally control the direction of any sensible heat transfer that may occur. Furthermore, since temperature gradients may reside within the liquid, within the gas, or within both, the sensible heat transfer resistance may include effects in either or both phases. The effects of latent and sensible heat transfer may be simultaneously considered in terms of the enthalpy changes which occur.

The operation of an adiabatic humidifier normally involves make-up water entering the unit at the adiabatic saturation temperature. Under these conditions, the temperature of the water in the system is assumed constant at the adiabatic saturation temperature, and both the air temperature and humidity water remain constant. In addition, all the heat required to vaporize the water is supplied from the sensible heat of the air.

When water is present in air, it is possible to extract the water by cooling the air– water mixture below the mixture's dew point temperature. The dew point was defined earlier as the temperature at which the air can no longer absorb more water (i.e., it is the 100% relative humidity point) and if the temperature is then reduced further, water is forced out of the air–water mixture as condensation (dew) is formed. This process is described as *dehumidification*. The amount of water that is removed from a mixture as a result of cooling can be determined by drawing a line on a psychrometric chart (see Fig. 13.1) from the mixture's initial conditions (e.g., dry-bulb temperature and relative humidity), horizontally to the left (i.e., the cooling direction) until the 100% relative humidity line is encountered.

The dehumidification process can also be accomplished by bringing moist air into contact with a spray of water—the temperature of which is lower than the dew point of the entering air. An example is passing the air through sprays. Furthermore, dehumidification of air may also be accomplished by passing a cold fluid through the inside of (finned) tubes arranged in banks through which the air is blown. The outside surface of the metal tubes must be below the dew point of the air so that water will condense out of the air.<sup>(3)</sup>

*Cooling towers* also find application in industry. The same operation that is used to humidify air may also be used to cool water. There are many cases in practice in which warm water is discharged from condensers or other equipment and where the value of this water is such that it is more economical to cool it and reuse it than to discard it. Water shortages and thermal pollution have made the cooling tower a vital part of many plants in the chemical process industry. Cooling towers are normally

employed for this purpose and they may be destined to have an increasingly important role in almost all phases of industry. Modern (newer) power-generating stations remain under construction or in the planning stage, and both water shortages and thermal pollution are serious problems that must be dealt with.

The cooling of water in a cooling tower is accomplished by bringing the water into contact with unsaturated air under such conditions that the air is humidified and the water is brought approximately to the wet-bulb temperature. This method is applicable only in those cases where the wet-bulb temperature of the air is below the desired temperature of the exit water.<sup>(3)</sup>

Qualitatively speaking, water is cooled in cooling towers by the exchange of sensible heat, latent heat, and water vapor with a stream of relatively cool dry air. The basic relationships developed for dehumidifiers also apply to cooling towers although the transfer is in the opposite direction since the unit acts as a humidifier rather than as a dehumidifier of air.<sup>(4)</sup>

Brown and Associates<sup>(5)</sup> have provided empirical correlations from the literature<sup>(6,7)</sup> for estimating (roughly) sizes and capacities of conventional towers.

# Equipment

Any of the absorption contact equipment discussed in Chapter 10 are applicable to humidification, dehumidification, and water-cooling applications. Packed towers and plate towers are particularly effective but there are other types of specialized units. These are briefly discussed below.

As mentioned above, warm water flows down cooling towers countercurrent to rising unsaturated air. Water is cooled by furnishing part of the latent heat required to vaporize some of the water into the air stream. The air is thus humidified as it rises. The calculation of the tower height for the vaporization process can be accomplished by the stepwise procedure detailed for distillation in Chapter 9. The operation of a cooling tower is indicated diagrammatically in Figure 13.4. As described above, the hot water at  $T_{I2}$  is introduced at the top of the tower and leaves the bottom at  $T_{I1}$ . The air flows countercurrent to the water. It enters at the bottom (point 1) and leaves the top (point 2). The temperature of the air-water interface tends to approach the adiabatic saturation or wet-bulb temperature of the air. At the top of the tower, heat is being transferred from the inlet (hot) water to the air, since the temperature of the water is higher than that of the interface, and the interface temperature is usually higher than that of the air. This sensible heat removed from the water appears as sensible and latent heat of the air-water mixture. At the bottom of the tower, the temperature of the water and of the interface may both be lower than that of the air, with sensible heat being transferred both from the liquid and air to the interface, resulting in the vaporization of the water. Water may thus be cooled by air at a higher temperature, provided that a humidity difference driving force (which produces evaporation) is maintained.<sup>(5)</sup>

Cooling towers were originally primarily constructed of redwood, a material which is very durable when in continuous contact with water; however, there are also



Figure 13.4 Adiabatic humidification.

moisture resistant polymer composite materials that are now available. The internal packing is usually in the form of horizontal wooden slats. The void volume is usually greater than 90%, leading to a pressure drop that is extremely low. The air–water interfacial surface includes not only the liquid films that wet the slats but also the surface of droplets, which settle as a "rain" from each tier of slats to the next.

*Natural-circulation* towers consist of two types: atmospheric and natural-draft. In *atmospheric* towers, air circulation is dependent solely on the prevailing winds, which essentially produces a crossflow of the air and water. The towers are generally long with narrow horizontal cross-sectional areas. This leads to adequate penetration of the air into the central portions of the unit. Louvers on the sides of the tower help reduce the losses of water entrained in the gas stream. *Natural-draft* towers provide a free convection effect. This ensures air movement even in calm weather; it is similar to a stack or a chimney. Large cross-sectional areas are usually required in order to maintain a low air velocity and, consequently, a low pressure drop. (Both *natural-circulation* and *natural-draft* towers must be relatively tall.) A pump is required for the water, but there are no fans or the accompanying power cost associated with moving the air.

The emphasis in recent years has been to employ *mechanical-draft* equipment. These towers may be of the forced-draft type where the air is blown into the tower by a fan at the bottom, or of the induced-draft type where the air is drawn upward by a fan at the top. Since the forced-draft tower ensures the recirculation of the hot, humid discharged air, the effectiveness of the tower is somewhat compromised. However, the induced-draft tower discharges the air at a higher velocity and can lead to a more uniform air distribution in the packing. Based on these considerations, this unit is often preferred despite the fact that the fan power is higher since the air density is lower. Water is usually distributed over the packing by weirs or spray nozzles. Spray eliminators at, or near, the top of the unit can reduce water carryover. All these units incur losses defined as blowdown. In addition, make-up water is required for evaporation and entrainment losses as well as the blowdown.

*Spray columns* (see Chapter 10) are either forced or induced-draft towers without any internal packing. Contact relies entirely on the inlet water sprays at the top to provide interfacial surface for mass transfer. Low gas pressure drop is normal for this unit.

*Spray chambers* are essentially horizontal spray columns. They too are frequently used for adiabatic humidification–cooling operations. Dehumidification is possible by cooling the water prior to spraying or by inserting refrigerating coils in the side spray chamber. Generally, three banks of sprays in series will bring the gas to substantial equilibrium with the incoming spray liquid.

*Spray ponds* are occasionally used for water cooling where a close approach to the air wet-bulb temperature is not required. These units essentially contain fountains from which the water is sprayed vertically upward into the air and allowed to settle by gravity into a collection basin. They are obviously subject to water loss due to any prevailing winds.

*Cooling ponds* or reservoirs are used for removing a relatively small amount of heat from water over a small temperature range. The pond required may simply be estimated from any relationship that provides the rate of evaporation of water into still air. The heat is then calculated from the latent enthalpy (of vaporization) of water. As one might expect, any wind will increase the capacity of a pond. The cooling capacity can be further increased if a system of spray nozzles can be installed above the surface of a cooling pond.

Other methods of dehumidification can also include adsorption, such as those methods discussed in Chapter 11. Adsorbents employed include silica gel or alumina (commonly referred to as *desiccants*). In addition, treating/washing the gas with liquids can also reduce the water content of the gas.

# **Describing Equations**

For cooling or humidification, the operating line lies below the equilibrium line. As noted earlier, an operating line refers to the actual vapor–liquid relationship of a key component, in contrast to the true equilibrium relationship. There is, therefore, a minimum air rate that can be used to accomplish a specified water-cooling duty. In dehumidification, cool water is used to reduce both the humidity and the temperature of the air introduced at the bottom of the tower. The operating line for this case lies above the equilibrium line and there exists a minimum amount of water that can be used to dehumidifiers and cooling towers apply to dehumidifiers, although the transfer occurs in the opposite direction; in effect, the tower is a dehumidifier rather than a humidifier of air.

#### **344** Chapter 13 Humidification and Drying

An adiabatic, gas-liquid, continuous, countercurrent contact system may now be examined. The diagram in Figure 13.4 shows such a system and the corresponding notation of Equations 13.13 and 13.14. For any process involving humidification, a moisture mass balance and an enthalpy balance can be written. The mass balance is:

$$L_2 + G_1 Y_1 = L_1 + G_2 Y_2 \tag{13.13}$$

In addition, the enthalpy balance is:

$$L_2 H_{L2} + G_1 H_1 + \dot{Q} = L_1 H_{L1} + G_2 H_2 \tag{13.14}$$

where  $\dot{Q}$  is the heat transferred to the units ( $\dot{Q} = 0$  if operated adiabatically).

One can ultimately show that for the differential element dZ the equation describing the rate of sensible heat transfer on the liquid side (assuming *L* and *G* are relatively constant) takes the form:

$$LC_L dT_L = h_L a(T_L - T_i) dZ; \quad C_L = C_{PB}$$
 (13.15)

Equation (13.16) applies on the gas side:

$$GC_G dT = h_G a(T_i - T) dZ; \quad C_G = C_{PA}$$
 (13.16)

The rate of mass transfer can be written as:

$$G dY = k_L a(Y_i - Y) dZ$$
(13.17)

where *a* is the interfacial surface area per unit volume and the subscript *i* refers to interfacial values.

If the interfacial values are constant through column height Z, one may integrate Equations (13.16) and (13.17) to give:

$$\ln\left(\frac{T_i - T_1}{T_i - T_2}\right) = \frac{h_G aZ}{GC_G}$$
(13.18)

$$\ln\left(\frac{Y_i - Y_1}{Y_i - Y_2}\right) = \frac{k_L a Z}{G}$$
(13.19)

In the above derivations, it was assumed that  $T_i$  and  $Y_i$  are constant throughout length Z; strictly speaking, this is not necessarily true. However, by taking an average  $T_i$  (assumed to be the average liquid temperature of the unit) and corresponding average  $Y_i$  for the length of the column, satisfactory answers can be obtained once heat and mass transfer coefficient values are provided.<sup>(8,9)</sup>

## **ILLUSTRATIVE EXAMPLE 13.7**

A flue gas (MW = 30, dry basis) is being discharged from a scrubber at  $180^{\circ}F$  (dry-bulb) and  $125^{\circ}F$  (wet-bulb). The gas flow rate on a dry basis is 10,000 lb/h. The absolute humidity at the

dry-bulb temperature of  $180^{\circ}F$  and wet-bulb temperature of  $125^{\circ}F$  is 0.0805 lb H<sub>2</sub>O/lb dry gas. What is the mass flow rate of the wet gas?<sup>(8)</sup>

**SOLUTION:** Calculate the flow rate of water (w) in the air and the total (T) flow rate:

$$\dot{m}_w = (0.0805)(10,000) = 805 \text{ lb/h}$$
  
 $\dot{m}_T = 10,000 + 805 = 10,805 \text{ lb/h}$ 

#### **ILLUSTRATIVE EXAMPLE 13.8**

Refer to Illustrative Example 13.7. What is the molar and volumetric flow rate of the wet gas?

**SOLUTION:** To determine the actual volumetric flow rate, first calculate the molar rate of water and dry gas (dg):

$$\dot{n}_{dg} = \frac{10,000}{30} = 333.3 \text{ lbmol/h}$$
  
 $\dot{n}_w = \frac{805}{18} = 44.7 \text{ lbmol/h}$ 

Calculate the mole fraction of water vapor:

$$y_w = \frac{44.7}{44.7 + 333.3} = 0.12$$

Calculate the average molecular weight of the mixture:

$$\overline{MW} = (1 - 0.12)(30) + (0.12)(18) = 28.6 \,\text{lb/lbmol}$$

Determine the molar flow rate of the wet gas (*wg*):

$$\dot{n}_{wg} = \frac{10,805}{28.6} = 378 \,\text{lbmol/h}$$

Calculate the volumetric flow rate of the wet gas by employing the ideal gas law.

$$q_{wg} = \frac{\dot{n}_{wg}RT}{P} = \frac{(378)(0.73)(460 + 180)}{1.0} = 1.77 \times 10^5 \,\text{ft}^3/\text{h}$$

## **ILLUSTRATIVE EXAMPLE 13.9**

Calculate the height of a packed tower operated in a countercurrent mode that is required to cool a utility water discharge from 90°F to 80°F. Ambient air at a wet-bulb and dry-bulb temperature of 47°F and 56°F, respectively, is to be employed. The water and gas rates have been set at 500 and 1000 lb/h  $\cdot$  ft<sup>2</sup>, respectively. Based on a review of the literature, one may assume that:

$$h_G a = (7.5 \times 10^{-3})(G')^{0.7}(L')^{0.6}$$

where  $h_G a$  = heat transfer coefficient, Btu/h · ft<sup>3</sup> · °F G' = air mass flux, lb/h · ft<sup>2</sup> L' = water mass flux, lb/h · ft<sup>2</sup>

Since the air at discharge conditions is essentially saturated, assume the humid heat capacity to be 0.245 Btu/lb BDA  $\cdot$  °F. Also, assume the adiabatic saturation temperature to be 68°F.

SOLUTION: First calculate the heat transfer coefficient:

$$h_G a = (7.5 \times 10^{-3})(500)^{0.7}(1000)^{0.6}$$
  
 $h_G a = 36.68 \operatorname{Btu/h} \cdot \operatorname{ft}^3 \cdot^{\circ} \mathrm{F}$ 

Use Equation (13.18) to solve for Z:

$$\ln\left(\frac{T_i - T_1}{T_i - T_2}\right) = \frac{h_G aZ}{GC_G}$$

In order to do this, the unknown values in the above equation must be determined; these are  $T_1$ ,  $T_2$ ,  $T_i$ , and  $C_G$ .  $T_1$  is the dry-bulb temperature of the air stream, which is given in the problem statement as 56°F. The heat capacity of air may be assumed equal to approximately 0.245 Btu/lb BDA · °F.  $T_i$  is to be assumed constant throughout the height, Z, of the column. This value is therefore assumed to be the average liquid temperature of the unit. This is determined via:

$$T_i = \frac{90 + 80}{2} = 85^{\circ} \text{F}$$

The value of  $T_2$  is determined in the following manner. The heat duty  $\dot{Q}'$  of the packed column is calculated using the information given for the water stream:

$$\dot{Q}' = L'C_{PL}\Delta T_w$$
  
= (500 lb/h · ft<sup>2</sup>)(1.0 Btu/lb · °F)(90 - 80)°F  
= 5000 Btu/h · ft<sup>2</sup>

Next, the duty of the water stream must be equal to the heat duty of the gas stream. Therefore:

$$-\dot{Q}' = G' C_{PG} \Delta T_G$$
  
-5000 Btu/h · ft<sup>2</sup> = (1000 lb/h · ft<sup>2</sup>)(0.245 Btu/lb · °F)(56 - T<sub>2</sub>)°F  
T<sub>2</sub> = 76.4°F

Substituting the appropriate values in Equation (13.18) yields the height of the packed column, which is calculated below:

$$\ln\left(\frac{T_i - T_1}{T_i - T_2}\right) = \frac{h_G aZ}{GC_G}$$
$$Z = \left[\frac{(1000)(0.245)}{36.68}\right] \ln\left(\frac{85 - 56}{85 - 76.4}\right)$$
$$= 8.1 \text{ ft}$$

# DRYING

In the drying process, a liquid (usually water) is separated from a wet solid by use of a hot dry gas (usually air). The drying of solids to remove moisture involves the simultaneous processes of heat and mass transfer. Heat is transferred from the gas to the solid (and liquid) in order to evaporate the liquid contained in the solid. Mass is transferred as either a liquid or vapor within the solid and then as a vapor from the surface of the solid. Additional details on this process are provided later in this section.

The energy required to vaporize the liquid in a solid is almost always furnished by a hot inert carrier gas that enters the drier. In some driers, the solid may be in contact with heated metal surfaces where the required heat of vaporization flows to the solid by conduction. In vacuum drying (where there is essentially no carrier gas), the heat of vaporization is furnished by conduction or radiation; here, the capacity of the drier is largely influenced by the heat-transfer surface available within the dryer.

The curve provided in Figure 13.5 is obtained when a substance saturated with water is dried. During the drying process, a thin film of water exists on the surface of the material where water is supplied from the solid fast enough to keep the surface entirely wet. As this water is evaporated, water from the interior of the sample rises to the surface essentially by capillary action with the solid temperature approximately given by the wet-bulb temperature of the air. After drying has proceeded for some time, the surface film begins to disappear and the rate of drying decreases. This critical moisture content leads to dry patches on the surface, and—as noted later in Figure 13.6—the drying rate begins to fall. This corresponds to the curved portion of the graph in Figure 13.5. Ultimately, water ceases to evaporate and a final *equilibrium moisture content* (denoted by the dashed line) is achieved.

The curve in Figure 13.5 can also be plotted as shown in Figure 13.6. In this plot, the horizontal portion corresponds to the constant rate period. The value of the



Figure 13.5 Typical drying process: moisture content vs time.



Figure 13.6 Typical drying process: drying rate vs moisture content.

mass rate of water evaporated per unit time, R [mass/time · area], corresponding to the horizontal portion is defined as the *critical drying rate*,  $R_c$ , while the value of X(mass H<sub>2</sub>O evaporated/mass solid) corresponding to  $R_c$  is called the *critical moisture content*,  $X_c$ . Note that the drying rate may reach zero before the solid is completely dry. The resulting equilibrium moisture content,  $X_e$ , for the solid is a function of the drying rate. The reader is reminded that the term X is a concentration term based on a *water free basis*.

An equation for the total time of drying can be derived. First note that:

$$R = \frac{dm}{A \, dt}; \quad dm = RA \, dt \tag{13.20}$$

The term *m* is the mass of water, *A* is the drying surface, and *dt* is the time of drying. In addition,

$$dm = 2sA\rho_s \, dX \tag{13.21}$$

where *s* is specified as one-half of the thickness of the material being dried,  $\rho_s$  is the density of the dry material and *X* is the moisture content on a water-free basis. Combining Equations (13.20) and (13.21), while eliminating *dm* and solving for *R*, gives:

$$R = \frac{\left[-2sA\rho_s\right]dX}{A\,dt} = -2s\rho_s\frac{dX}{dt} \tag{13.22}$$

The total drying time,  $t_t$ , is then found by integration,

$$\int_{0}^{t_{t}} dt = t_{t} = 2s\rho_{s} \int_{X_{2}}^{X_{1}} \frac{dX}{R}$$
(13.23)

where  $X_1$  and  $X_2$  are the initial and final moisture contents, respectively.

During the *constant rate period*, R is constant and may be set equal to  $R_c$ . One may therefore integrate Equation (13.23) to give:

$$t_c = \left(\frac{2s\rho_s}{R_c}\right)(X_1 - X_2) \tag{13.24}$$

In the *falling rate period*, an assumption can be made that there is a linear relationship, that is, y = mx + b (as shown in Fig. 13.6), between falling rate, *R*, and moisture content, *X*. This effectively assumes that the curve approximates a straight line from the *critical moisture content* to the origin during the falling rate period. Thus:

$$R = mX + b \tag{13.25}$$

In addition:

$$dR = m \, dX; \quad dR/m = dX \tag{13.26}$$

For this falling rate period, one may replace dX in Equation (13.23) by Equation (13.26) to give:

$$t_f = 2s\rho_s \int_{X_o}^{X_c} \frac{dX}{R} = \frac{2s\rho_s}{m} \int_{R_o}^{R_c} \frac{dR}{R}$$
(13.27)

where  $X_o$  refers to the *final moisture content*. Upon integration, the time of the falling rate period,  $t_f$ , becomes:

$$t_f = \frac{2s\rho_s}{m}\ln\left(\frac{R_c}{R_o}\right) \tag{13.28}$$

If the *y*-intercept in Figure 13.6 is assumed to be zero, i.e.,  $R_o = X_o = 0$ , then

$$m = \frac{R_c - R_o}{X_c - X_o} = \frac{R_c}{X_c}$$
(13.29)

Substituting this value of *m* into Equation (13.28) gives:

$$t_f = 2s\rho_s\left(\frac{X_c}{R_c}\right)\ln\left(\frac{R_c}{R_o}\right) \tag{13.30}$$

The total time for drying is the sum of the times for the constant rate period and the falling rate period, i.e.,

$$t_t = t_c + t_f = \frac{2s\rho_s}{R_c} \left[ (X_1 - X_2) + X_c \ln\left(\frac{R_c}{R_o}\right) \right]$$
(13.31)

If both rate periods are present,  $X_2 = X_c$  so that the above is given by

$$t_t = \frac{2s\rho_s}{R_c} \left[ (X_1 - X_c) + X_c \ln\left(\frac{R_c}{R_o}\right) \right]$$
(13.32)

Another term that has received attention in this field is the *free moisture content*, *F*. It is defined as the difference between the total moisture content *X* and the equilibrium moisture content  $X_e$  (the content of water in the solid that cannot be removed by the air), expressed as mass of water per mass of dry solid:

$$F = X - X_e \tag{13.33}$$

The free moisture content is a function of the same variables as the equilibrium moisture content. As one might surmise, F is of major interest in drying calculations.

It should be noted that drying is one method of separating a liquid from a solid. Technically, it is the aforementioned vaporization process in which the heat rate and mass transfer rates control equipment design. In most dryers, heat is transferred by convection from a gaseous drying medium to the surface of the wet solid. (In some designs, radiation from the walls of the dryer to the wet material supplements convection.) This heat vaporizes the liquid, which is usually water. The vapor that is thus formed must diffuse into the gas phase. In so doing, it passes through the same gas-phase convective resistance through which the heat passed. However, the transfer of heat is in the opposite direction to the transfer of mass. Depending on operating conditions, a small portion of the heat can act as sensible heat to raise the temperature of the wet solid.

In some respects, drying is identical to humidification in that water is provided from a pure liquid while in drying it comes from liquid dispersed in a solid. Thus, drying requires that a new resistance be considered. This resistance can be thought of opposing the movement of the liquid through the solid to the gas-solid interface. Although the next two subsections will key on rotary dryers and spray dryers, the significance of the above resistance becomes apparent if one analyzes typical drying data obtained in a batch dryer.

Summarizing, one notes that the rate at which the liquid is vaporized is constant at first. This constant rate holds until the moisture content of the wet stock reaches a critical value, at which point the rate begins to decrease. It continues to decrease thereafter, ultimately falling to or approaching zero when the moisture content has been reduced to an equilibrium value that is the lowest value that it can reach with the drying conditions employed.<sup>(4)</sup>

#### **ILLUSTRATIVE EXAMPLE 13.10**

A granular solid weighing 5000 lb (bone dry) is to be dried under constant drying conditions from a moisture content of 0.20 lb/lb to a final moisture content of 0.02 lb/lb. The material

Moisture content (lb/lb)	Drying rate, $R (lb/ft^2-h)$	
0.300	0.35	
0.200	0.35	
0.140	0.35	
0.114	0.30	
0.096	0.265	
0.056	0.180	
0.042	0.150	
0.026	0.110	
0.016	0.075	

 Table 13.2
 Drying Rate Data, Illustrative Example 13.10

has an effective area of  $0.25 \text{ ft}^2/\text{lb}$ . The drying rates shown in Table 13.2 were previously obtained under the same conditions. Calculate the time required for drying.

**SOLUTION:** Based on the problem statement,  $A = (0.25 \text{ ft}^2/\text{lb}) (5000 \text{ lb}) = 1250 \text{ ft}^2$ , m = 5000 lb, and the critical moisture content, from Table 13.2, is approximately 0.14 lb/lb. For the constant rate period,

$$t_c = \frac{2s\rho_s}{R_c}(X_1 - X_2); \quad X_1 = 0.20; \quad X_2 = X_c = 0.14$$
 (13.24)

with

$$2s\rho_s = \frac{m}{A}$$

This may be rewritten as

$$t_c = \frac{m}{AR_c} (X_1 - X_2)$$

Substituting,

$$t_c = \frac{5000}{(1250)(0.35)}(0.20 - 0.14) = 0.69 \,\mathrm{h}$$

The rate decreases following this constant rate period. This is treated in the next illustrative example.

#### **ILLUSTRATIVE EXAMPLE 13.11**

Refer to the previous example. If the falling rate period is approximately described by

$$R = 2.17X + 0.047$$

calculate the total drying time.

*SOLUTION:* An integration is required for the falling time calculation. Apply Equation (13.30):

$$t_f = 2s\rho_s\left(\frac{X_c}{R_c}\right)\ln\left(\frac{R_c}{R_o}\right)$$

This may be rewritten as

$$t_f = \frac{m}{A} \left( \frac{X_c}{R_c} \right) \ln \left( \frac{R_c}{R_o} \right)$$

Substituting,

$$t_f = \left(\frac{5000}{1250}\right) \left(\frac{0.14}{0.35}\right) \ln\left(\frac{0.35}{2.17(0.02) + 0.047}\right) = 2.17 \,\mathrm{h}$$

The required total time is then

$$t_t = t_c + t_f = 0.69 + 2.17 = 2.86 \,\mathrm{h}$$

## **Rotary Dryers**

The rotary dryer is a popular device suitable for the drying of free-flowing materials that can be tumbled without concern for breaking. Moist solid is continuously fed into one end of a rotating cylinder with the simultaneous introduction of heated air. The cylinder is installed at a slight angle and with internal lifting flights so that the solid is showered through the hot air as it traverses the dryer. A bench scale unit, located in the Unit Operations Laboratory at Manhattan College, is provided in Figure 13.7. In the lab's experiment, a liquid (water) is separated from a wet solid (cornmeal) by use of a hot dry gas (air). The rotary dryer is operated in the cocurrent mode. The hot air is cooled as it is humidified; at the same time, the solid is heated and dried by contact with the hot air.

A typical industrial rotary dryer consists of a cylinder, rotated upon suitable bearings and usually slightly inclined to the horizontal. The length of the cylinder may range from four to more than ten feet. Solids fed into one end of the cylinder progress through it by virtue of rotation, head effect, the slope of the cylinder, and discharge as finished product at the other end. Gases flowing through the cylinder may retard or increase the rate of solids flow, depending upon whether the gas flow is countercurrent or cocurrent with respect to solid flow.

Rotary dryers have been classified as direct, indirect-direct, indirect, and special types. These terms refer to the method of heat transfer: *direct* when heat is added to or removed from the solids by direct exchange between flowing gas and solids, and *indirect* when the heating medium is separated from physical contact with the solids by a metal wall or tube.

Rotating equipment is applicable to batch or continuous processing solids, which are relatively free-flowing and granular when discharged as product. Materials that are not completely free-flowing in their feed condition are handled in a special



Figure 13.7 Rotary dryer: Manhattan College's Unit Operations Laboratory.

manner—either by recycling a portion of the final product and premixing with the feed in an external mixer to form a uniform granular feed to the process, or by maintaining a bed of free-flowing product in the cylinder at the feed end and, in essence, performing a premixing in the cylinder itself.

The method of feeding rotating equipment depends upon material characteristics and the location and type of upstream processing equipment. When the feed comes from above, a chute extending into the cylinder is usually employed. For sealing purposes or if gravity feed is not convenient, a screw feeder is normally used. On cocurrent direct-heat units, cold-water jacketing of the feed chute or conveyer may be desirable if it is contacted by the inlet hot gas stream. This will prevent overheating of the metal wall with resultant scaling or overheating of heat sensitive feed materials. One method of feeding direct cocurrent drying equipment utilizes dryer exhaust gases to convey, mix, and pre-dry wet feed. The latter is added to the exhaust gases from the dryer at a high velocity. The wet feed, mixed with dust entrained from the dryer, separates from the exhaust gases in a cyclone (typically) and usually drops into the feed end of the cylinder. This technique combines pneumatic and rotary drying. The dust entrained in the exit-gas stream is customarily removed in the cyclone collector(s). This dust may be returned into the process or separately collected. For expensive materials or extremely fine particles, bag collectors (a baghouse) may follow a cyclone collector, assuming fabric temperature stability is not limiting and there are assurances of no temperature excursions.

Rotating equipment, with the exception of brick-lined vessels operated above ambient temperatures, are usually insulated to reduce heat losses. Other exceptions are direct-heat units of bare metal construction operating at high temperatures where heat losses from the shell are necessary to prevent overheating of the metal. Insulation is the rule with cocurrent direct-heat units. It is not unusual for product cooling or condensation on the shell to occur in the last 10 to 50 percent of the cylinder length if it is not sufficiently insulated.

The direct-heat rotary dryer is usually equipped with flights (discussed above) on the interior for lifting and showering the solids through the gas stream during passage through the cylinder. These flights are usually offset every 2-6 feet to ensure more continuous and uniform curtains of solids in the gas. The shape of the flights depends upon the handling characteristics of the solids. For free-flowing materials, a radial flight with a 90-degree lip is employed. For sticky materials, a flat radial flight without any lip is used. When materials change their characteristics during drying, the flight design is usually changed along the dryer length. Many standard dryer designs employ flat flights with no lips in the first one-third of the dryer measured from the feed end, flights with 45-degree lips in the middle one-third, and flights with 90-degree lips in the final one-third of the cylinder. Spiral flights are usually provided in the first few feet at the feed end to accelerate forward flow from under the feed chute or the conveyer to prevent leakage over the feed-end retainer. When cocurrent gas-solid flow is used, flights may be left out of the final few feet at the exit end to reduce entrainment of dry product in the exit gas. Showering of wet feed at the feed end of a countercurrent dryer will, on the other hand, frequently serve as an effective means for scrubbing dry entrained solids from the gas stream before it leaves the cylinder. Some dryers are provided with sawtooth flights to obtain uniform showering while others use lengths of chain attached to the underside of the flights to scrape over and knock the walls of the cylinder, thereby removing sticky solids that might normally adhere to it. Solids sticking on flights and walls are usually removed more efficiently by external shell knockers. In dryers of large cross-section, internal elements or partitions are sometimes used to increase the effectiveness of material distribution and reduce dusting and impact grinding. Use of internal members increases the difficulty of cleaning and maintenance unless sufficient free area is left between partitions for the easy access of an individual. Some examples of the more common flight arrangements are shown in Figure 13.8.

Countercurrent flow of gas and solids gives greater heat-transfer efficiency with a given inlet gas temperature. However, cocurrent flow is used more frequently to dry



Figure 13.8 Rotary dryer flight arrangements.

heat-sensitive materials at higher inlet gas temperatures because of the rapid cooling of the gas during the initial evaporation of surface moisture.

A major design variable of the rotary dryer is the drying rate. Since the solid (cornmeal in the Manhattan College experiment) loses moisture while the air stream is gaining moisture, the drying rate can be calculated in either of two ways. Data taken from a dryer can provide the drying rate based either on the moisture lost by the solid and/or the humidity gained by the air stream between the inlet and exit.<sup>(10)</sup>

As noted above, the mechanism of drying involves the transfer of heat by convection for vaporizing the material. Mass is transferred as a result of a moisture concentration gradient at a rate dependent on the characteristics of the solid.

The dynamic equilibrium prevailing between the rate of heat transfer to the material and rate of vapor removal from the surface during the constant drying rate period also provides a means for calculating the heat and mass transfer coefficients. The drying rate, dm/dt can be measured by the two ways presented above, i.e., based on the loss of water by the solids or on the increase in water content of the air stream. Dry-bulb temperatures and saturated (wet-bulb) temperatures at each end of the dryer are normally employed to calculate the log mean temperature difference. This enables one to determine the heat transfer coefficient, h, based on the air and water data as follows:

$$\dot{m} = \left(\frac{dm}{dt}\right) = \frac{hV(T - T_s)_{mean}}{\lambda}$$
(13.34)

with

$$(\dot{m}_{water})_{solid} = (\dot{m}_{water})_{air}$$

The above terms are defined as follows:

- $\dot{m} = dm/dt = drying$  rate (lb water evaporated/h)
- h = heat transfer coefficient based on dryer volume (Btu/h · ft<sup>2</sup> · °F)
- $V = dryer volume (ft^3)$
- T = temperature of drying medium, dry-bulb temperature (°F), determined at inlet and outlet conditions
- $T_s$  = surface temperature of solid (°F), wet-bulb temperature, determined at inlet and outlet conditions
- $\lambda =$  latent heat of vaporization (Btu/lb)

Similarly, humidities can be obtained at the inlet and outlet (including saturated humidities) and the log mean humidity difference is used to calculate the mass transfer coefficient, *k*, based on either the air and water data as follows since  $(\dot{m}_{water})_{solid} = (\dot{m}_{water})_{air} = \dot{m}$ :

$$\dot{m} = \frac{dm}{dt} = kV(Y_{as} - Y)_{mean}$$
(13.35)

where  $k = \text{mass transfer coefficient based on dryer volume (lb/h · ft<sup>3</sup> · <math>\Delta Y$ )

- Y = absolute humidity of drying medium at dry-bulb temperature (lb water/lb BDA), determined at inlet and outlet conditions
- $Y_{as}$  = absolute saturated humidity of surface at wet-bulb temperature (lb water/lb BDA), determined at inlet and outlet conditions

For comparison purposes, heat transfer coefficients can also be calculated from an empirical equation. Friedman and Marshall<sup>(11)</sup> expressed the heat transfer coefficient as an empirical equation in terms of air mass flux, G (lb/ft<sup>2</sup> · s):

$$h = \frac{\alpha G^m}{D} \tag{13.36}$$

where  $\alpha$  and *m* are empirical constants and *D* is the diameter of the dryer in feet. The value of *m* varies from 0.16–0.67 depending on the individual unit. The value of  $\alpha$  may be approximated by

$$\alpha = \frac{0.05(N_f - 1)}{0.786} \tag{13.37}$$

where  $N_f$  is the number of flights in the dryer.

Another important parameter in a dryer operation is the holdup  $\Phi$ , which refers to the fraction of dryer volume occupied by the solid at any instant. The describing equation is:

$$\Phi = \frac{\theta L_s}{\rho_s Z} \tag{13.38}$$

where  $\Phi = \text{holdup fraction (solid volume/dryer volume)}$ 

 $\theta$  = retention time of solids passing through dryer (h)

 $L_s$  = rate of flow of solids per dryer cross-sectional area (lb/h · ft<sup>2</sup>)

Z =length of dryer (ft)

 $\rho_s$  = density of solid material (lb/ft<sup>3</sup>)

#### **ILLUSTRATIVE EXAMPLE 13.12**

A rotary dryer is operating at  $320^{\circ}$ F with a gas flow rate of  $600 \text{ scfm} (60^{\circ}\text{F})$ . The dryer is 4 ft in diameter and 16 ft long. The minimum residence time required for operation is 0.4 min. Calculate (1) the actual gas flow rate, (2) the volume required for minimum residence time and, (3) the maximum residence time.

#### SOLUTION:

1 The actual gas flow rate is calculated by Charles' law:

$$q_a = q_s \left(\frac{T_a}{T_s}\right)$$

where  $T_a$  and  $T_s$  are in absolute units

$$q_a = (600 \text{ scfm}) \left[ \frac{320 + 460}{60 + 460} \right]$$
  
= 900 acfm

2 The volume required for minimum residence time is

$$V_m = q_a t_{\min}$$
  
= (900 acfm)(0.4 min)  
= 360 ft<sup>3</sup>

**3** The maximum residence time,  $t_{max}$ , takes into account the actual volume,  $V_a$ , in the equation:

$$t_{\max} = \frac{V_a}{q_a}$$

where

$$V_a = (\pi/4)D^2L$$
  
= (0.785)(4)<sup>2</sup>(16)  
= 201 ft<sup>3</sup>  
$$t_{max} = 210/900$$
  
= 0.223 min = 13.4 s

The minimum residence time required is therefore not satisfied. Note that the gas residence time, calculated by dividing the volume of the dryer by the gas flow rate, is only an approximate value since fully developed flow does not necessarily exist. Thus, a residence time distribution exists with real systems.

#### **ILLUSTRATIVE EXAMPLE 13.13**

An engineer involved in the analysis of a rotary dryer unit suspects that the dryer is operating below the desired residence time of 1 hour. The available operating data are

Dryer length = 25 ftDryer diameter = 10 ftSlope of dryer = 0.01 ft/ft of length

The present dryer rotation velocity is 0.9 rpm.

- 1 Is this velocity providing the necessary residence time?
- 2 What maximum kiln rotation velocity would meet the desired residence time?

The following equation may be assumed to apply:

$$t = \frac{0.19L}{NDS}$$

where t = retention time (min)

L = dryer length (ft) N = dryer rotational velocity (rpm)

D = dryer diameter (ft)

S = dryer slope (ft/ft of length)

SOLUTION: Calculate the residence time using the equation provided.

$$t = \frac{(0.19)(25)}{(0.9)(10)(0.01)}$$
  
= 53 min

Therefore, the necessary residence time of 60 min is not being met. Set the residence time of this equation to 60 min to find the minimum dryer rotation rate. Rearrange this equation and solve for N:

$$60 \min = \frac{(0.19)(25)}{(N)(10)(0.01)}$$
$$N = \frac{(0.19)(25)}{(60)(10)(0.01)}$$
$$= 0.8 \text{ rpm}$$

Therefore, a maximum rotational velocity of 0.8 rpm is required to achieve the desired residence time of 1 hour in this rotary dryer.

*Note*: The lead coefficient of 0.19 in the above equation has been estimated from limited experimental data. Other values appear in the literature. Therefore, the residence time calculated through the use of this coefficient is, at the very best, a rough estimate.

## **ILLUSTRATIVE EXAMPLE 13.14**

The following data was obtained during a rotary dryer experiment at Manhattan College:

Barometric pressure – 29.92 in Hg	Inlet air temperature (dry-bulb) – 74°F
Air flow rate $-210$ ft/min	Inlet air temperature (wet-bulb) – $61^{\circ}F$
Mass of feed – 29.8 g	Exit air temperature (dry-bulb) – $90^{\circ}F$
Feed time – 5.11 min	Exit air temperature (wet-bulb) – $67^{\circ}F$
Dryer diameter – 4.0 in	Inlet solids temperature - 182°F
Dryer length – 18.75 in	Exit solids temperature $-123^{\circ}F$

Calculate the drying rate, dm/dt.

**SOLUTION:** The drying rate equals the mass flow rate of BDA,  $\dot{m}$  (BDA) times the change in humidity (*Y*) of the air from inlet to outlet (lb H<sub>2</sub>O/lb BDA). As a first step, calculate the area available for air flow:

cross-sectional area of dryer = 
$$(\pi/4)(D^2)$$
  
=  $(0.785)(4/12)^2$   
=  $0.0872$  ft<sup>2</sup>

The density of moist air at these conditions may be assumed to be  $0.0739 \text{ lb/ft}^3$ . Thus,

$$\dot{m}_{tot} = (210 \,\text{ft/min})(60 \,\text{min/h})(0.0872 \,\text{ft}^2)(0.0739 \,\text{lb/ft}^3)$$
  
= 81.20 lb/h

This represents the mass of air plus water in air in the feed. To convert to a dry air basis, the following relation is used:

$$\frac{\text{lb}(\text{H}_2\text{O})}{\text{lb}(\text{BDA})} = \frac{\dot{m}_w}{\dot{m}_{tot} - \dot{m}_w}$$

The lb ( $H_2O$ )/lb (BDA) is known from the wet-bulb temperature of the inlet air. It is merely the humidity, *Y*, which is 0.0087. Therefore,

$$Y = \frac{\dot{m}_w}{\dot{m}_{tot} - \dot{m}_w}$$
$$0.0087 \frac{\text{lb}(\text{H}_2\text{O})}{\text{lb}(\text{BDA})} = \frac{\dot{m}_w}{81.20 - \dot{m}_w}$$
$$\dot{m}_w = 0.70 \frac{\text{lb}(\text{H}_2\text{O})}{h}$$

Therefore, by difference,  $\dot{m}_{air} = 80.50 \text{ lb/h}$ .

Knowing the wet-bulb temperature at the outlet gives *Y* at the outlet:

$$Y = 0.0095 \, \text{lb} \, (\text{H}_2\text{O})/\text{lb} \, (\text{BDA})$$

The drying rate, dm/dt, can now be solved for in the following manner:

$$\dot{m}(\text{BDA}) = \frac{dm}{dt} = \left[ (0.0095 - 0.0087) \frac{\text{lb} (\text{H}_2\text{O})}{\text{lb} (\text{air})} \right] 80.50 \frac{\text{lb} (\text{air})}{h}$$
$$= 0.0644 \frac{\text{lb} (\text{H}_2\text{O})}{h}$$

# **ILLUSTRATIVE EXAMPLE 13.15**

Refer to Illustrative Example 13.14. Calculate the log mean temperature difference  $(T - T_s)_{lm}$ .

**SOLUTION:** Since rotary dryers are cocurrent devices, the describing equation for the logarithmic mean temperature difference is given by:

$$(T - T_s)_{lm} = \frac{(T - T_s)_{exit} - (T - T_s)_{inlet}}{\ln\left[\frac{(T - T_s)_{exit}}{(T - T_s)_{inlet}}\right]}$$

Since this is a measure of the temperature difference driving force down the length of the dryer, the inlet and exit solids temperatures are used for *T*. The corresponding air inlet and exit air temperatures are given by

$$T_s$$
 (exit) = 90°F  
 $T_s$  (inlet) = 74°F

Thus,

$$(T - T_s)_{lm} = \frac{(123 - 90)^\circ F - (182 - 74)^\circ F}{\ln\left[\frac{(123 - 90)^\circ F}{(182 - 74)^\circ F}\right]}$$
$$(T - T_s)_{lm} = 63.258^\circ F$$

# **ILLUSTRATIVE EXAMPLE 13.16**

Refer to Illustrative Example 13.14. Calculate the heat transfer coefficient, *h*.

SOLUTION: The dryer volume and the latent heat of vaporization must first be found:

$$V = (\text{length})(\text{cross-sectional area})$$
$$= (\pi/4)(D^2)(L)$$
$$= 0.1363 \text{ ft}^3$$

The heat of vaporization  $\lambda$  is approximately 910 Btu/lb. Thus, the heat transfer coefficient can be calculated from Equation (13.35):

$$h = \frac{dm}{dt} \left[ \frac{\lambda}{(T - T_s)_{lm} V} \right]; \quad \frac{dm}{dt} = \dot{m}(\text{BDA})$$

Substituting:

$$h = 0.0644 \left[ \frac{910}{(66.28)(0.1363)} \right]$$
  
= 6.487 Btu/ft<sup>2</sup> · h · °F

#### **ILLUSTRATIVE EXAMPLE 13.17**

Refer to Illustrative Example 13.14. Determine the mass transfer coefficient, k.

**SOLUTION:** The drying rate remains the same. Calculations are now required for  $(Y_{as} - Y)_{lm}$  and then *k*.

The calculation of the log mean humidity difference  $(Y_{as} - Y)_{lm}$  is as follows:

$$(Y_{as} - Y)_{lm} = \frac{(Y_{as} - Y)_{exit} - (Y_{as} - Y)_{inlet}}{\ln\left[\frac{(Y_{as} - Y)_{exit}}{(Y_{as} - Y)_{inlet}}\right]}$$

As given, *Y* (inlet) = 0.0087 lb (H<sub>2</sub>O)/lb (BDA) and *Y* (exit) = 0.0095 lb (H<sub>2</sub>O)/lb (BDA). Since this is the humidity difference along the length of the drum,  $Y_{as}$  is taken at the saturation temperatures listed. Therefore,  $Y_{as}$  (inlet) = 0.0115 lb (H<sub>2</sub>O)/lb (BDA) and  $Y_{as}$  (exit) = 0.0144 lb (H<sub>2</sub>O)/lb (BDA).

Substituting into the above equation gives

$$(Y_{as} - Y)_{lm} = \frac{(0.0115 - 0.0095) \, \text{lb/lb} - (0.0144 - 0.0087) \, \text{lb/lb}}{\ln\left[\frac{(0.0115 - 0.0095)}{(0.0144 - 0.0087)} \, \text{lb/lb}\right]}$$
$$= \frac{0.002 - 0.0057}{\ln\left[\frac{0.002}{0.0057}\right]}$$
$$= 0.00353 \, \text{lb/lb} \text{ (BDA)}$$

The calculation of the mass transfer coefficient, k, is determined by:

$$k = \left(\frac{dm}{dt}\right) \left[\frac{1}{V(Y_{as} - Y)_{lm}}\right]$$
(13.35)  
= (0.0644 lb/h)  $\left[\frac{1}{(0.1363 \text{ ft}^3)(0.00353 \text{ lb/lb})}\right]$   
= 133.8 lb(BDA)/h · ft<sup>3</sup>

#### Spray Dryers

Spray dryers (SD) are often utilized in the chemical processing industry to obtain a dry product in a granular or powder form. Various spray-dried products include coffee, detergents, and instant beverages. Spray drying is a drying technique that involves the drying of a solid in solution via atomization of the solution. The atomized drops are then contacted with a hot air stream. The dry product is normally collected by a cyclone (or another particulate control device) at or near the bottom of the unit. Spray dryers may be operated cocurrently or countercurrently. The spray dryer in the Unit Operations Laboratory of Manhattan College is shown in Figure 13.9.



Figure 13.9 Spray dryer: Manhattan College's Unit Operations Laboratory.

The method of operation of the spray dryer is relatively simple, requiring only two major equipment items—a spray dryer similar to those used in the chemical food-processing and mineral preparation industries and a cyclone or fabric filter (baghouse) or electrostatic precipitator (ESP) to collect the fly ash and entrained solids. In the spray dryer, the solution (or slurry) is atomized into the incoming hot gas stream to increase the liquid–gas interface and to promote the mass transfer of the gas to the slurry droplets, where it is absorbed.

The slurry solution may be mechanically atomized by either a rotary atomizer or by spray nozzles and injected into the gas stream. The action of the rotary atomizer results in smaller droplet size and size distribution, and is less subject to plugging and wear than the spray nozzle; however, it is higher in cost. This atomizer propels the droplets radially outward and perpendicular to the gas flow. The droplets decelerate rapidly owing (in part) to the drag forces of the downward moving gas and eventually attain the velocity of the gas. The radial distance between the atomizer and the dryer wall must be sufficient to allow for adequate drying of the largest droplets. The length to diameter (L/D) ratio of the cylindrical section of the SD is typically 0.8:1. In a SD with a dual-fluid pneumatic nozzle, atomization is in the direction of the gas flow and the L/D ratio is typically 2:1. Optimum SD performance is achieved through the proper choice of L/D, droplet size, and residence time.

Spray dryers have several advantages over other types of dryers. The drying time is very short, permitting drying of highly heat sensitive materials and the creation of a solid or a hollow spherical product. Some products, such as food and detergents, require certain appearances, consistencies, and bulk densities; other drying methods may fail to produce these valuable properties. Another desirable characteristic of spray dryers is their ability to produce a dry product ready for packaging from a feed solution. This may greatly simplify the manufacturing process.

Drying a solid means removing small amounts of liquid from a solid material to reduce the liquid content of the solid to an acceptably low value. For example, a solid dissolved in a solution can be dispersed into a stream of hot gas (usually air) in the form of fine droplets. Moisture vaporizes from the droplets, leaving residual particles of dry solid behind. The inlet dry gas gains the moisture that evaporates from the droplets.

A dry solids material balance requires the subtraction of the water from the product in performing the balance. This is accomplished by determining the moisture content of the feed and the product. After determining the amount of water in the feed and in the product and subtracting the water content for both the feed and the product, one may compare the dry feed and the dry product to determine any experimental error.

As atomized droplets are contacted with the hot, dry air in the drying chamber, the water evaporates and enters the air. The evaporation rate is determined from the following water mass balance equation (employing SI units in this development):

$$\frac{dm}{dt}(\text{water}) = (\dot{m}_1)(X_1) - (\dot{m}_2)(X_2)$$
(13.39)

where  $\frac{dm}{dt}$  (water) =  $\dot{m}_w$  = evaporation rate (kg H<sub>2</sub>O/s)  $\dot{m}_1$  = dry solids feed rate (kg dry feed/s)  $X_1$  = feed moisture content (kg H<sub>2</sub>O/kg dry feed)  $\dot{m}_2$  = dry solids product rate (kg dry feed/s)  $X_2$  = product moisture content (kg H<sub>2</sub>O/kg dry feed)

The moisture gain by the drying gas may be similarly calculated by:

$$\dot{m}_g = \frac{dm}{dt}(\text{gas}) = (\dot{m}_2)(Y_2) - (\dot{m}_1)(Y_1)$$
 (13.40)

where  $\frac{dm}{dt}(\text{gas}) = \dot{m}_g$  = moisture gain by drying gas (kg H<sub>2</sub>O/s)  $\dot{m}_2$  = dry air mass flow from dryer (kg dry air/s)  $Y_2 = air humidity at dryer outlet (kg H_2O/kg dry air)$ 

 $\dot{m}_1 = dry air mass flow to dryer (kg dry air/s)$ 

 $Y_1 = \text{air humidity at dryer inlet (kg H_2O/kg dry air)}$ 

The air humidities in Equation (13.40) can be determined from a psychrometric chart. The moisture gain of the drying gas should agree within 10% of the evaporation rate provided in Equation (13.39).

The average drop diameter,  $d_p$ , of the atomized feed solution may be estimated from<sup>(9)</sup>:

$$\frac{d_p}{r} = 0.4 \left(\frac{\Gamma}{\rho_s n r^2}\right)^{0.6} \left(\frac{\mu}{\Gamma}\right)^{0.2} \left(\frac{\sigma \rho_s L_w}{\Gamma^2}\right)^{0.1}$$
(13.41)

where  $d_p$  = average drop diameter (ft)

 $\Gamma = \text{spray mass velocity (lb/ft}^2 \cdot \min)$ 

 $\rho_s$  = density of the solution (lb/ft<sup>3</sup>)

n = atomizer disk speed (rev/min)

r = radius of the centrifugal disk (ft)

 $\mu = \text{viscosity of the solution (lb/ft · min)}$ 

 $\sigma = \text{surface tension (lb/min<sup>2</sup>)}$ 

 $L_w$  = wetted disk periphery (ft)

In order to completely dry the solution, it is customary to base the drying calculations on the *maximum drop diameter*,  $d_{p,max}$ . The maximum drop diameter is often assumed to be three times the average drop diameter, i.e.,

$$d_{p,\max} = 3d_p \tag{13.42}$$

The mass flow rate of the feed is necessary to determine the surface area available for heat transfer. It is often assumed that the drops are dried in one second; this approximation is fairly reasonable. The dimension of a drop also needs to be determined. This may be obtained from the equation for a sphere. It is also necessary to determine the number of drops. The *number of drops* is simply the volume of the feed divided by the volume of one drop. The surface area available for heat transfer is then obtained by multiplying the surface area of one drop by the total number of drops. These calculations are usually performed using the maximum drop diameter.

The heat load of the dryer is necessary to determine the heat transfer coefficient. The following equation is suggested when centrifugal disk atomizers are used<sup>(9)</sup>:

$$\dot{Q} = \frac{4.19(k_f) \left(R - \frac{r}{2}\right)^2 \Delta T}{d_{p,\max}^2 \rho_s} \sqrt{\frac{w_s \rho_t}{(r)(n)}}$$
(13.43)

where

 $\dot{Q}$  = heat load (Btu/h)

 $k_f$  = thermal conductivity at the film temperature (Btu/h · ft · °F)

R = radius of the drying chamber (ft)

r = radius of the disk (ft)

 $\Delta T$  = temperature driving force for heat transfer (°F)

 $d_{p,\max} = \max \operatorname{maximum} \operatorname{drop} \operatorname{diameter} (\operatorname{ft})$ 

 $\rho_s$  = density of the liquid solution (lb/ft<sup>3</sup>)

 $w_s = \text{feed flow rate (lb/h)}$ 

 $\rho_t = \text{feed density (lb/ft^3)}$ 

n = disk speed, rev/h

Interestingly, the driving force for heat transfer is the difference between the drying gas outlet temperature and outlet gas wet-bulb temperature. The wet-bulb temperature may be found on a psychrometric chart from the outlet temperature and relative humidity of the outlet gas. A film temperature is also necessary to determine the physical properties of the solution; it is calculated by averaging the outlet air temperature and the outlet wet-bulb temperature. An *experimental* heat transfer coefficient is then found by:

$$h_{\rm exp} = \frac{\dot{Q}}{A\Delta T} \tag{13.44}$$

where  $h_{exp} = experimental heat transfer coefficient (Btu/h · ft<sup>2</sup> · °F)$ 

 $\dot{Q}$  = heat load (Btu/h)

A = surface area available for heat transfer (ft<sup>2</sup>)

 $\Delta T$  = temperature driving force for heat transfer (°F)

This experimental heat transfer coefficient may be compared to a *literature* heat transfer coefficient. This coefficient is normally based on the assumption that the Nusselt number is equal to 2.0, i.e.,

$$\frac{hd_{p,\max}}{k_f} = 2.0$$
 (13.45)

Mass transfer coefficients may also be determined for a spray dryer. The experimental mass transfer coefficient is found by:

$$k_{\exp} = \frac{N_A}{\Delta p} \tag{13.46}$$

where  $k_{exp} = experimental mass transfer coefficient (lbmol/s \cdot ft<sup>2</sup> · atm)$ 

 $N_A = \text{molar flux (lbmol A/s \cdot ft^2)}$ 

 $\Delta p$  = partial pressure driving force for mass transfer (atm)

The molar flux represents the number of moles diffusing per time per surface area. It is obtained by dividing the mass flow rate of the feed by the total surface area available

for mass transfer. The driving force for mass transfer is the difference between the interface partial pressure and the bulk partial pressure. To determine the interfacial partial pressure, one must first determine the mole fraction of water in the dry air at the interface. It is customary to assume that the interface temperature is the outlet wet-bulb temperature. The bulk partial pressure may be found by determining the mole fraction of water in the outlet dry gas.

To determine the *literature* mass transfer coefficient, the Sherwood number is assumed equal to 2.0, which is valid for a relative velocity of the drops through air of zero. Although it is not completely accurate, it provides for a good approximation. For this assumption:

$$\frac{k_{lit}d_{p,\max}\Delta p_f \overline{MW}}{D_{AB}\rho} = 2.0 \tag{13.47}$$

where  $k_{\text{lit}} = \text{literature mass transfer coefficient (lbmol/s \cdot ft^2 \cdot atm)}$ 

 $\Delta p_f =$  pressure difference across the mass transfer path (atm)

 $\overline{MW}$  = average molecular weight across the mass transfer path (lb/lbmol)

 $D_{AB}$  = diffusivity of water vapor (A) in air (B) (ft<sup>2</sup>/s)

 $\rho$  = density of air at exit conditions (lb/ft<sup>3</sup>)

The diffusivity of water vapor in air,  $D_{AB}$ , is provided in the literature.<sup>(9)</sup> At 25°C and 1.0 atm, the diffusivity of water vapor in air is 0.202 cm<sup>2</sup>/s. However, the diffusivity should be estimated at the dryer outlet temperature, which is assumed to be the temperature inside the dryer.

A companion spray unit to a spray dryer is the spray tower. This unit can also be employed to cool a hot gas. In one spray tower design, the gas (quench) cooling is accomplished simply by spraying water at the top of the tower as the hot gases travel upwards through the tower. When the unvaporized water reaches the pump at the bottom of the tower, it is recirculated by pumping it back up to the top. Since about 10% of the water stream is usually vaporized during contact with the hot gases, makeup water must be constantly added. For efficient evaporation of the water, the gas velocity should be from 400–600 fpm and the entire cross section of the gas stream should be covered with a fine spray of water. This necessitates proper location, type, and number of spray nozzles to ensure adequate coverage of the gas-side flow area.

Cooling hot gases with a water spray is relatively simple and requires little space. These towers are easy to operate and, with automatic temperature controls, only that amount of water that is needed is used to maintain the desired temperature of the gases at the discharge. Their installation and operating costs are generally considered to be less than that of other cooling methods. Quench towers require careful design review for applications where the gases to be cooled contain a significant amount of acid.

Before spray towers can be sized, the water flow rate necessary to bring the hot gas to the desired temperature must first be calculated. This is accomplished by means of both componential and total energy and material balances. Componential balances, where the mass and enthalpy of each component entering and exiting the tower is accounted for, are necessary because material will be exchanged between the water and gas streams. Design techniques of spray towers are available in the literature.<sup>(9)</sup>

## **ILLUSTRATIVE EXAMPLE 13.18**

List some advantages of a spray dryer.

SOLUTION: Among the inherent advantages that the dryer enjoys are:

- 1 Low capital costs
- 2 Low pressure drop losses
- 3 Reduced auxiliary power
- 4 Reduced water consumption
- 5 Continuous, two-stage operation, from liquid feed to dry product.

## **ILLUSTRATIVE EXAMPLE 13.19**

Discuss the equipment that is available for collecting the dry product from a spray dryer.

**SOLUTION:** The choice of many individuals in the dryer field is the cyclone. However, fabric filters (baghouses) and electrostatic precipitators (ESPs) may also be employed. Baghouses have an inherent advantage over ESPs in that they have high collection efficiency and simplicity in design. Two disadvantages of using a fabric filter are that the fabric is somewhat sensitive to wetting, so that a margin above the saturation temperature must be kept for bag protection, and there is always the possibility of catastrophic bag failure due to a temperature excursion. However, it is the cyclone that is the preferred choice in the industry.<sup>(12,13)</sup> Details on these units are provided in Chapter 16.

#### **ILLUSTRATIVE EXAMPLE 13.20**

An air (*a*) stream at 90°F, 14.7 psia, with a relative humidity of 60% is flowing into a water-spray chamber (SC) where enough water (*w*) is added to reach saturation. The outlet temperature and pressure of the saturated stream are 80°F and 12 psia, respectively. After the spray chamber, the stream is heated and compressed (H/C) to 30 psia and has a final relative humidity of 30%. What is the mole ratio of the amount of water added (i.e., absorbed by the air stream) to the amount of incoming moist air in the spray chamber?

**SOLUTION:** Choose a basis of 100 lbmol of feed and draw a flow diagram of the process (see Fig. 13.10).

Using Dalton's law and steam table data, determine the composition of the moist air feed stream, and the stream leaving the compressor/heater unit. Water saturation pressures are



Figure 13.10 Spray chamber/compressor diagram.

obtained from Table C1 in Appendix C.1:

$$p_w = (Y_R)p'_w = 0.60(0.69813) = 0.4189 \text{ psia;} 90^\circ \text{F}$$
  
 $y_w = \frac{p_w}{P} = \frac{0.4189}{14.7} = 0.0285$   
 $y_a = 1 - 0.0285 = 0.9715$ 

For the stream exiting the spray chamber:

$$p_w = (Y_R)p'_w = 1.00(0.50693) = 0.50693 \text{ psia}$$
$$y_w = \frac{p_w}{P} = \frac{0.50693}{12.0} = 0.0422$$
$$y_a = 1 - 0.0422 = 0.9578$$

Since no water is added to or removed from the air stream in the compressor/heater unit, the inlet composition must remain the same as the stream exiting the spray chamber. Perform a material balance on the air solely around the spray chamber unit to find the amount of moist air leaving this unit. For the SC,

air in = air out  

$$0.9715(100) = 0.9578N_3$$
  
 $N_3 = 101.43$  lbmol

Perform a total material balance around the same unit to find the amount of water added:

$$100 + N_2 = N_3$$
  
 $100 + N_2 = 101.43$   
 $N_2 = 1.43$  lbmol of water added

Finally, determine the mole ratio of the amount of water added to the amount of incoming moist air:

$$ratio = \frac{N_2}{N_1} = \frac{1.43}{100} = 0.0143$$

#### **ILLUSTRATIVE EXAMPLE 13.21**

Refer to Illustrative Example 13.20. What is the dew point temperature of the moist air leaving the compressor/heater unit and what is the final temperature of the moist air in  $^{\circ}$ F?

**SOLUTION:** Using the steam tables in Appendix C.1, determine the dew point of the moist air leaving the compressor/heater unit. The composition of the inlet and outlet stream to H/C remain the same. Thus, at the outlet conditions,

$$p_w = y_w P = 0.0422(30) = 1.266$$
 psia

From the tables, the temperature at which the air stream would be saturated (dew point) is somewhere between  $100^{\circ}$ F (0.94924 psia) and  $110^{\circ}$ F (1.2750 psia). Using linear interpolation:

$$T = 109.7^{\circ} \mathrm{F} \approx 110^{\circ} \mathrm{F}$$

Determine the vapor pressure of the water in the air stream leaving the compressor/heater unit. As before,

$$p_w = y_w P = 0.0422(30) = 1.266 \, \text{psia}$$

and

$$p'_w = \frac{p_w}{Y_R} = \frac{1.266}{30} = 4.22 \, \text{psia}$$

## REFERENCES

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