

## Chapter 5

---

# Life Cycle Costing

---

EDWARD POHL, Ph.D.  
HEATHER NACHTMANN, Ph.D.

The last 10 percent of performance generates one-third of the cost and two-thirds of the problems.

—Norman R. Augustine

### 5.1 INTRODUCTION TO LIFE CYCLE COSTING

In Chapter 1, a system is defined as “an integrated set of elements that accomplishes a defined objective. System elements include products (hardware, software, firmware), processes, people, information, techniques, facilities, services, and other support elements” [1]. As part of a system, these integrated elements have a system life cycle consisting of seven stages: conceptualization, design, development, production, deployment, operation, and retirement of the system. This system life cycle was described in Chapter 3. Throughout each of these seven stages, various levels of life cycle costs occur, including development, production, support, and disposal costs.

*Life cycle costing* (LCC) should be used for solution design (Chapter 11) and is required for systems decision making (see Chapter 12). LCC is used by a systems engineering team to estimate whether the new system or the proposed system modifications will meet functional requirements at a reasonable total cost over the duration of its anticipated life. When successfully employed and managed, life cycle

costing is also a tool for systems decision making. The Society of Cost Estimating and Analysis [2] defines a life cycle cost estimate in the following way:

Life cycle cost estimate is an estimate that covers all of the cost projected for a system's life cycle, and which aids in the selection of a cost-effective total system design, by comparing costs of various trade-offs among design and support factors to determine their impact of total system acquisition and ownership costs.

The concept map in Figure 5.1 provides a pictorial overview of life cycle costing. Life cycle costing centers around the development of a system cost estimate, which in conjunction with the schedules is used by a program manager to manage the system's acquisition, operation, or disposal. System design and operational concepts drive the key cost parameters, which in turn identify the data required for developing a system cost estimate. As part of a systems engineering team, cost analysts rely on historical data, subject matter experts (SME), system schedules, and budget quantities to provide data to use life cycle costing techniques.

System risk depends on the life cycle stage of the system, and this risk affects the key cost parameters that drive the system cost estimate. Cost estimating is a critical activity for successful public and private organizations. Cost estimates are required to (a) develop a budget to obtain a new system, (b) prepare a bid on a project proposal, (c) negotiate a price for a system, and (d) provide a baseline from which to track and manage actual costs. Major cost estimating errors can dramatically impact the credibility and long-term viability of the organization.

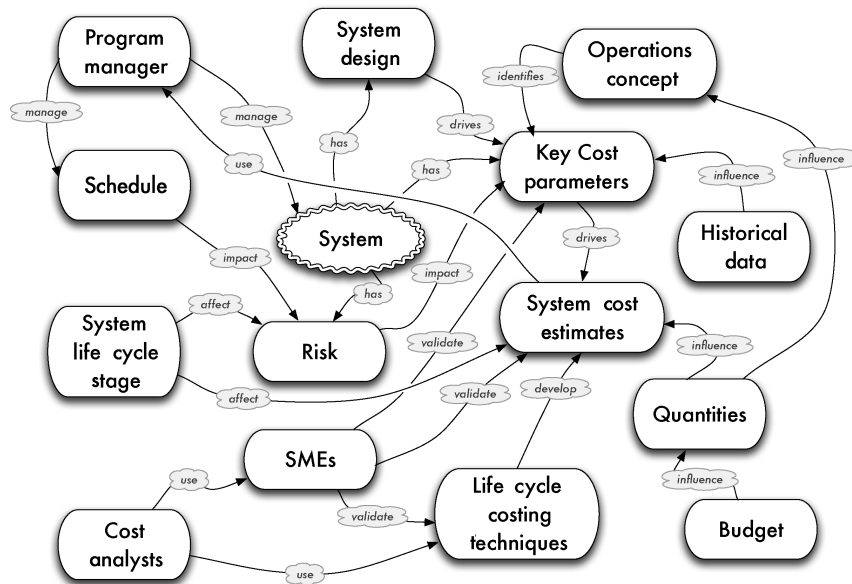


Figure 5.1 Concept map for life cycle costing.

Selection of the most appropriate LCC technique depends on the amount and type of available data and the perceived system risks. As each stage of the life cycle progresses, additional information concerning system requirements, system design and system performance becomes available, and some dimensions of uncertainty are resolved. Therefore, which LCC technique is most useful depends on the stage of the system life cycle. Recommendations of LCC techniques by life cycle stage are provided in Table 5.1 along with suggested references for each technique.

The Association for the Advancement of Cost Engineering (AACE) International prescribes a cost estimation classification system that can be generalized for applying estimate classification principles to system cost estimates [3]. Under this classification system, the level of system definition is the primary characteristic for classifying cost estimates. Other secondary characteristics shown in Table 5.2 (adapted from reference [4]) include the end usage of the estimate, estimating methodology, expected accuracy range, and effort to prepare the estimate.

Estimates are grouped into classes ranging from Class 1 to Class 5. Class 5 estimates are the least precise because they are based on the lowest level of system definition, while Class 1 estimates are the most precise because they are closest to full system definition and maturity. Successive estimates are prepared as the level of system definition increases until a final system cost estimate is developed.

The *level of systems definition* provides ranges of typical completion percentages for systems within each class, which can provide information about the maturity and extent of available input data. The *end usage* describes the typical use of cost estimates generated at that level of system definition, such that Class 5 estimates are generally used only for low-level screening or feasibility analysis. The *methodology* contains the typical estimating methods that are employed to generate each class of estimate. The less knowledge we have of the system, the more appropriate it is to provide a cost estimate range instead of a single number.

The *expected accuracy range* indicates the degree to which the final cost outcome for a given system is expected to vary from the estimated cost. The values in this column do not represent percentages as generally given for expected accuracy but instead represent an index value relative to a best range index value of 1. For example, if a given industry expects a Class 1 accuracy range of +15 to -10, then a Class 5 estimate with a relative index value of 10 would have an accuracy range of +150 to -100 percent. The final characteristic, *preparation effort* provides an indication of the cost, time, and other resources required to prepare a given estimate, which is again a relative index value.

## 5.2 INTRODUCTION TO COST ESTIMATING TECHNIQUES

Once definitions have been determined for all the cost elements forecasts to occur during a system's life cycle, the systems engineering team begins cost estimating—defined by Stewart [5] as “the process of predicting or forecasting the cost of a work activity or work output.” This process is divided into 12 major steps discussed in more detail in the *Cost Estimator's Reference Manual* [5].

TABLE 5.1 LCC Techniques by Life Cycle Stage

| LCC Techniques                     | Life Cycle Stages              |                                   |  |  |                              |                              |                            |
|------------------------------------|--------------------------------|-----------------------------------|--|--|------------------------------|------------------------------|----------------------------|
|                                    | Concept                        | Design                            | Development                              | Production                               | Deployment                   | Operation                    | Retirement                 |
| Expert judgment                    |                                | Estimate by analogy               | Estimate by analogy                      | Estimate by analogy                      |                              |                              |                            |
| Cost estimating relationship       | Prepare initial cost estimates | Refine cost estimates             |  | Create production estimates              |                              |                              |                            |
| ship [5]                           |                                |                                   |  |  |                              |                              |                            |
| Activity-based costing [6]         |                                |                                   |  | Provides indirect product costs          |                              | Use for operational trades   |                            |
| Learning curves [7, 8]             |                                |                                   | Provides development and test unit costs | Provide direct labor production costs    |                              |                              |                            |
| Break-even analysis [9]            |                                |                                   | Use in design trades                     | Provides production quantities           |                              | Use for operational trades   |                            |
| Uncertainty and risk analysis [10] | Use with CER estimates         | Use with analogy or CER estimates | Affects development cost                 | Affects direct and indirect product cost | Affects deployment schedules | Affects O&S cost projections |                            |
| Replacement analysis [4]           |                                |                                   |  |  |                              |                              | Determines retirement date |

TABLE 5.2 AACE Cost Estimate Classification Matrix [4]

| Primary Characteristic |   | Secondary Characteristic        |                                |  |   |
|------------------------|---|---------------------------------|--------------------------------|--|---|
| Estimate class         | Level of system definition expressed as percentage of complete definition | End usage                       | Methodology                    | Expected accuracy range                                      | Preparation effort  |
| Class 5                | 0% to 2%  | Typical purpose of estimate     | Typical estimating method      | Typical $\pm$ range relative to best index of 1 <sup>a</sup> | Typical degree of effort relative to least cost index of 1 <sup>b</sup> |
| Class 4                | 1% to 15%   | Screening or feasibility        | Stochastic or judgmental       | 4 to 20  | 1   |
| Class 3                | 10% to 40%  | Concept study or feasibility    | Primarily stochastic           | 3 to 12  | 2 to 4  |
| Class 2                | 30% to 70%  | Budget authorization or control | Mixed but primarily stochastic | 2 to 6   | 3 to 10   |
| Class 1                | 50% to 100%   | Control or bid tender           | Primarily deterministic        | 1 to 3   | 5 to 20   |
|                        |   | Check estimate or bid tender    | Deterministic                  | 1  | 10 to 100   |

<sup>a</sup>If the range index value of "1" represents +10 – 5%, then an index value of 10 represents +100 – 50%.

<sup>b</sup>If the cost index value of "1" represents 0.005% of project costs, then an index value of 100 represents 0.5%.

This manual is an excellent source for developing life cycle cost estimates. The book contains extensive discussion and numerous examples of how to develop a detailed life cycle cost estimate. The 12 steps are:

1. Developing the work breakdown structure
2. Scheduling the work elements
3. Retrieving and organizing historical data
4. Developing and using cost estimating relationships
5. Developing and using production learning curves
6. Identifying skill categories, skill levels, and labor rates
7. Developing labor hour and material estimates
8. Developing overhead and administrative costs
9. Applying inflation and escalation (cost growth) factors
10. Computing the total estimated costs
11. Analyzing and adjusting the estimate
12. Publishing and presenting the estimate to management/customer

While all of these steps are important, the earlier steps are critical since they define the scope of the system, the appropriate historical data, and the appropriate cost models to use. In addition, the identification of technology maturity for each cost element is critical since many cost studies cite technology immaturity as the major source of cost estimating errors [11].

The use of these steps is dependent upon the phase of the life cycle, which impacts the level of detail required for the cost estimate. Many of these steps are by-products of a properly executed systems engineering and management process. In the project planning phase (see Chapter 13, Solution Implementation), a work breakdown structure for the systems is established. The low-level activities are then scheduled to develop a preliminary schedule.

Once all the activities have been identified and scheduled, the next task is to estimate their costs. The best approach is to estimate the cost of the activities based on past experience. Specifically, one would like to be able to estimate the costs associated with the activities using historical cost and schedule data. Finding these data and organizing them into a useful format is one of the most difficult and can be the most time-consuming step in the process. Once the data are found and organized, the analysts must ensure that it is complete and accurate. Part of this accuracy check is to make sure that the data are “normalized.” This is accomplished to ensure that the analyst is not making an “apples to oranges” comparison.

One form of data normalization is to use the proper inflationary/deflationary indices on estimates associated with future costs (Step 9). Once the data have been normalized, it is then used to develop statistical relationships between physical and performance elements of the system and cost. Steps 4 and 5 are used to establish baseline cost estimating relationships and adjust the costs based on the quantities purchased. Steps 6, 7, and 8 are used when a detailed “engineering” level estimate

(Class 2 or Class 3 estimate) is being performed on a system. This is a very time-consuming task, and these steps are necessary if one wants to build a “bottom-up” estimate by consolidating individual estimates of each of the work activities into a total project cost estimate. Like the earlier techniques in Steps 4 and 5, these steps are even more dependent on collecting detailed historical information on activities and their associated costs.

Finally, Steps 11 and 12 are necessary elements to cost estimating. Step 11 provides the analyst the opportunity to revise and update the estimate as more information becomes available about the system. Specifically, this may be an opportunity for the analysts to revise or adjust the estimate based on the maturity of the technology [11]. Additionally, it provides the analyst the opportunity to assess the risk associated with the estimate. The analyst can account for the data uncertainty quantitatively by performing a Monte Carlo analysis on the estimate and creating a distribution for the systems life cycle cost in a similar fashion as is done for value measures in Chapter 12. Step 12 is one of the most important steps, it does not matter how good an estimate is if an analyst cannot convince the management that they have done a thorough and complete cost analysis.

All assumptions should be clearly articulated in a manner that provides insight on the data sources and key assumptions used. The foundation for a cost estimate is the basic list of ground rules and assumptions associated with that estimate. Specifically, all assumptions, such as data sources, inflation rates, quantities procured, amount of testing, spares provisioning, and so on, should be clearly documented up front in order to avoid confusion and the appearance of an inaccurate or misleading cost estimate.

In this chapter, we will highlight a few of the key tools and techniques that are necessary to develop a reasonable preliminary cost estimate. Specifically, we will focus on developing and using cost estimating relationships and learning or cost progress curves. The details associated with developing a comprehensive detailed estimate are extensive and cannot be given justice within a single textbook chapter. Interested readers are referred to Stewart et al. [5] and Ostwald [7].

Once the estimate is developed and approved, it can be used to develop a budget, create a bid on a project or proposal, establish the price, or to form a baseline from which to track actual costs. A second benefit is that it can be used as a tool for cost analysis for future estimates in the organization.

### 5.2.1 Types of Costs

First, although most people think of “costs” in terms of dollars, cost can refer to any term that represents resources to an organization—for example, hours, man years, facilities space, and so on. These measures are an important factor in making meaningful tradeoff decisions that affect the organization whose system is being studied. These resource measures can often be converted to dollars and usually are in order to provide senior management a unifying measure they can easily assess.

There are a variety of costs associated with developing new systems or modifying existing ones. These costs vary based on where in the life cycle they occur

and the type of system being developed, constructed, or acquired. These cost classifications are useful in identifying the various sources of cost as well as the effect those sources have on the system life cycle cost. Costs are partitioned into four classes [5]: acquisition, fixed and variable, recurring and nonrecurring, direct and indirect.

*Acquisition Cost.* The total cost to procure, install, and put into operation a system, a product, or a specific piece of infrastructure (e.g., building, bridge, tunnel, transportation system). These are the costs associated with planning, designing, engineering, testing, manufacturing, and deploying/installing a system or process.

*Fixed and Variable Costs.* Fixed costs are those costs that remain constant independent of the quantity or phase of the life cycle being addressed in the estimate. Typical fixed costs include such items as research, lease rentals, depreciation, taxes, insurance, and security. Variable costs are those costs that change as a function of the number of systems or system output. Variable costs increase or decrease as the amount of product or service output from a system increases or decreases. Typical variable costs include direct labor, direct material, direct power, and the like. In other words, any cost that can be readily allocated to each unit of system output can be considered a variable cost.

*Recurring and Nonrecurring Costs.* Recurring costs are costs that repeat with every unit of product or every time a system process takes place. Like variable costs, they are a function of the quantity of items output. Nonrecurring costs are those costs that occur only once in the life cycle of a system. For example, the costs associated with design, engineering, testing, and other nonproduction activities would be normally classified as nonrecurring when developing a system cost estimate, because they are not anticipated to repeat once they occur.

*Direct and Indirect Costs.* Direct costs are those costs that are associated with a specific system, end item, product, process, or service. Direct costs can be further subdivided into direct labor, direct material, and direct expenses. Labor costs are those costs associated with the labor used directly on an item. Direct material costs are those costs associated with the bills of material purchased for the manufacture of the item and direct expense may be subcontracted work for part of the system or product. Indirect costs are those costs that cannot be assigned to a specific product or process; are usually pooled into an overhead account, which is applied to direct costs as a burden. Examples of indirect costs may include items like security, accounting and finance labor, janitorial services, executive management, training, and other activities and costs that are not directly related to the specific product or process but are an integral part of the organization that is responsible for the product, process, or service. Activity-based costing is a life cycle costing technique that can provide more accurate indirect cost analysis based on the premise that indirect costs should be allocated according to important functional activities



that are performed during the system life cycle [6]. Indirect costs associated with these activities are identified and grouped into multiple cost pools based on similar cost drivers. Resulting system life cycle costs are thus based on a more detailed analysis than traditional indirect costing.

### 5.3 COST ESTIMATION TECHNIQUES

As mentioned earlier, a variety of tools and techniques are available to assist in developing the life cycle cost estimate for a system [12]. In order to begin, an analyst must have a very good understanding of the system operations concept, the system functions, the system hardware (and software), the technology maturity, the quantities desired, and the system life cycle. All of these are developed in the course of applying the SDP in each life cycle stage. This information is necessary in order to develop a credible initial cost estimate.

In this section, we explore several techniques for developing life cycle cost estimates. First, we discuss the use of expert judgment and its role in establishing initial estimates. This technique is useful in developing initial estimates for comparison of alternatives early on the concept exploration phase. Second, we explore the use of cost estimating relationships as a vehicle to estimating the cost of a system, product, or process. These are used to provide more refined estimates of specific alternatives when selecting between alternatives and are often used to develop initial cost baselines for a system. Finally, we will discuss the use of learning curves in a production cost estimate. This tool is used to analyze and account for the effect quantity has on the cost of an item. This tool is often used in conjunction with cost estimating relationships (CERs) to build a life cycle cost estimate.

#### 5.3.1 Estimating by Analogy Using Expert Judgment

Engineers are often asked to develop cost estimates for products and services that are in the system concept stage. The engineers may have nothing more than a preliminary set of functions and requirements and a rough description of a feasible system concept. Given this very preliminary information, the program manager, the cost analyst, the systems engineer, and the engineering design team are often asked to develop a life cycle cost estimate for the proposed system in order to obtain approval and funding for the system. Given the scarcity of information at this stage, many program managers and cost analysts will rely on their own experience and/or the experience of other stakeholders and experts to construct an initial cost estimate. The use of expert judgment to construct an initial estimate for a system is not uncommon and is often used for Class 4 and 5 estimates. This underscores yet another reason why a good deal of time and effort is dedicated to stakeholder analysis in the Problem Definition phase of the SDP.

Often, technological advances create the requirements and/or market opportunities for new systems. When this occurs, the existing system can serve as a reference point from which a baseline cost estimate for a new system may be constructed.

If historical cost and engineering data are available for a similar system, then that system may serve as a useful baseline from which modifications can be made based upon the complexity of the advances in technology and the increase in requirements for system performance.

Many times, an expert will be consulted to describe the increase in complexity by focusing on a single system element (e.g., the new processor is three times as complex). The cost analyst translates this into a cost factor by referencing past experience for example, “The last time we changed processors, it was two times as complex and it increased cost by 20% over the earlier generation.” A possible first-order estimate may be to take the baseline cost, say \$10,000, and create a cost factor based on the information elicited from the experts.

- Past cost factor:  $2 \times \text{complexity} = 20\%$  increase
- Current estimate:  $3 \times \text{complexity}$  may increase cost 30%

These factors will be based on personal experience and historical precedent. In this example, the underlying assumption that the expert is making is that there is a linear relationship between the cost factor and the complexity factor. Given this assumption, a baseline estimate for the new technology might be ( $\$10,000 \times 1.3 = \$13,000$ ). Estimating by analogy can be accomplished at the meta system level as well when the new system has proposed characteristics in common with existing systems. For example, the cost of unmanned aeronautical vehicles (UAVs) could initially be estimated by drawing analogies between missiles and UAVs, because UAVs and missiles use similar technologies. By making appropriate adjustments for size, speed, payload, and other performance parameters, one could make an initial life cycle cost estimate based on historical missile data.

A major disadvantage associated with estimating by analogy is the significant dependency on the judgment of the expert and existing historical data. The credibility of the estimate rests largely upon the expertise and experience of the person constructing the analogy. Estimating by analogy requires significantly less effort and therefore is not as costly in terms of time and level of effort as other methods. Therefore, it is often used as a check of the more detailed estimates that are constructed as the system description evolves.

### 5.3.2 Parametric Estimation Using Cost Estimating Relationships

Parametric estimates are used to create estimates of life cycle costs using statistical analysis techniques. The use of parametric cost estimates was first introduced in the late 1950s by the RAND Corporation for use in predicting costs of military systems. These techniques rely heavily on historical data. In general, parametric cost estimates are preferred to expert judgment techniques. However, if sufficient historical data are not available or the product and its associated technology have changed significantly, then constructing a parametric cost estimate may not be possible.

The level at which parametric cost estimating is accomplished is largely dependent on the system life cycle stage. Parametric cost estimation is frequently used during early stages in the life cycle before detailed design information is available. Parametric techniques can also be constructed and/or revised using detailed design and production information. Because they are designed to forecast costs into the future, they are often used to estimate operation and support costs as well.

The end goal of this statistical approach is to develop a cost estimating relationship (mathematical relationship between one or more system physical and performance parameters and the total system cost estimate). For example, the cost of a satellite may be estimated as a function of weight, power, and orbit. The cost of a house may be estimated by forming a relationship between cost and the square footage, location, and number of levels in a house.

As mentioned previously, when constructing a system life cycle cost estimate, one should use the baseline work breakdown structure (WBS) for the system. This will help ensure that all the necessary elements of the system have been appropriately accounted for in the cost estimate. As an example, a three-level WBS for the air-vehicle portion of a missile system has been adapted from Mil-Hdbk-881a WBS Missiles [13] and is presented in Table 5.3. The missile system has many more level 2 components. For example, at WBS Level 2, one must also consider the costs of the command and launch components, the systems engineering and

**TABLE 5.3 Levels of Cost Estimating [13]**

| Level 1        | Level 2  | Level 3  |
|----------------|--|--|
| Missile system | Air vehicle  | Propulsion system<br>Payload<br>Airframe<br>Guidance and control<br>Fuzing<br>Integration and assembly |
|                | Command and launch components<br>Systems engineering and program management<br>System test and evaluation<br>Training<br>Data<br>Support equipment<br>Site activation<br>Storage facilities<br>Initial spares<br>Operating and support costs<br>Retirement costs |  |

program management costs, the system test and evaluation costs, training costs, data costs, support equipment costs, site activation costs, facilities costs, initial spares costs, operational and support costs, and retirement costs. Each of these Level 2 elements can be further broken down into Level 3 WBS elements as has been done for the air vehicle.

A parametric cost estimating relationship can be developed at any of the three levels of the WBS depending on the technological maturity of the system components, available engineering and cost data, and amount of time available to create the estimate. In general, the further along in the life cycle, the more engineering data available and the lower the WBS level (higher level number) from which an estimate can be constructed.

In the next section, we present the basic approach for constructing a cost estimating relationship and provide guidance on how it can be used to develop a system estimate. We will use our simplified missile air vehicle system as an example.

***Common Cost Estimating Relationship Forms*** A cost estimating relationship (CER) is a mathematical function that relates a specific cost category to one or more system variables. These variables must have some logical relationship to the system cost. One must be sure that the data used to develop the CER is relevant to the system and its specific technology. We will discuss the four basic forms for cost estimating relationships.

*Linear CER with Fixed and Variable Cost.* Many WBS elements can be modeled reasonably well by a simple linear relationship,  $Y = aX$ . For example, facility cost can be modeled as a function of area. Personnel costs can be modeled by multiplying labor rates by personnel hours. It is also possible to have a linear relationship that includes a fixed cost, denoted by  $b$ . For example, suppose the cost of the facility also includes the cost of the land purchase. Then it would have a fixed cost associated with the land purchase and a variable cost that is dependent on the size of the facility built on the land. The relationship is given by  $Y = aX + b$ . Both of these basic forms are shown below in Figure 5.2.

*Power CER with Fixed and Variable Cost.* Some systems may not have a linear relationship between cost and the selected estimating parameter. An economy of scale effect may occur; for example, as the size of a house increases, there will be a point at which the cost/ft<sup>2</sup> decreases. Similarly, one can also encounter situations where there are diseconomies of scale. For example, large gemstones often have higher costs per unit size than smaller gemstones. Figure 5.3 illustrates the various shapes that a power CER can take as well as the functional form of the various cost estimating relationships.

*Exponential CER with Fixed and Variable Cost.* Another functional form that is often used is the exponential cost estimating relationship. In this form, it is assumed

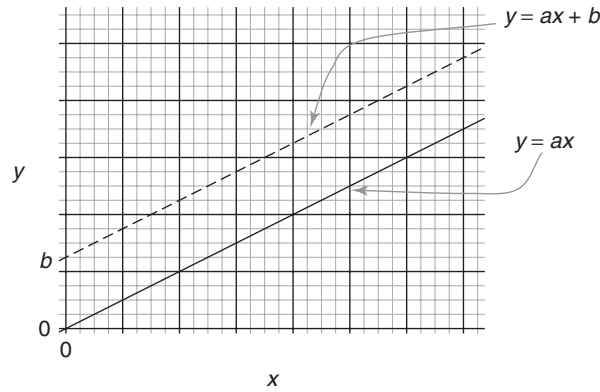


Figure 5.2 Linear CERs.

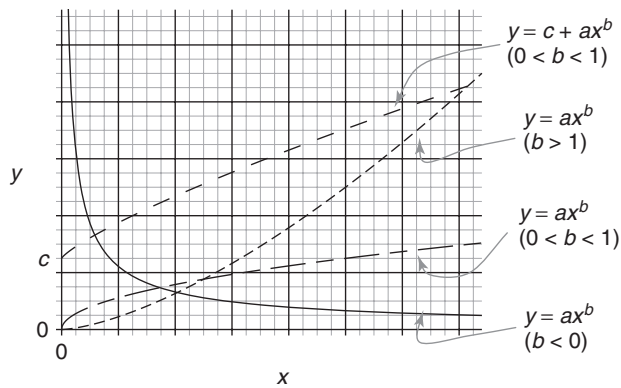


Figure 5.3 Power CERs.

that the relationship between the independent variable and cost is such that a unit change in the independent variable causes a relatively constant percentage change in cost. Figure 5.4 illustrates the shape for an exponential CER for a variety of functional forms.

*Logarithm CERs.* Finally, one other common form that may be useful for describing the relationship between cost and a particular independent variable is the logarithm CER. Figure 5.5 illustrates the shape for a logarithm CER for a variety of functional forms.

**Constructing a CER** To construct a cost estimating relationship we need a sufficient amount of data to fit the curve. What is sufficient is a judgmental decision

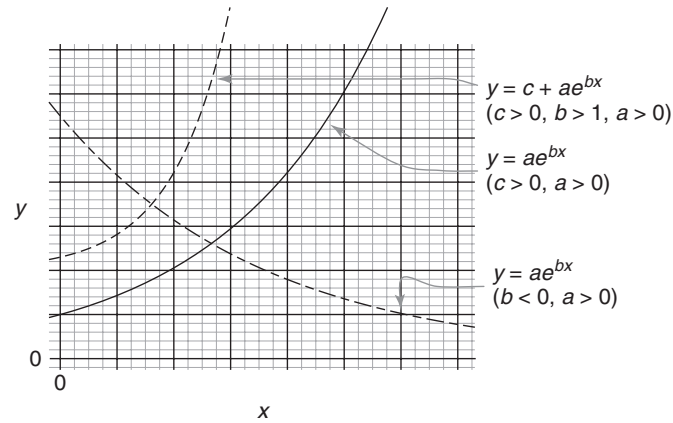


Figure 5.4 Exponential CERs.

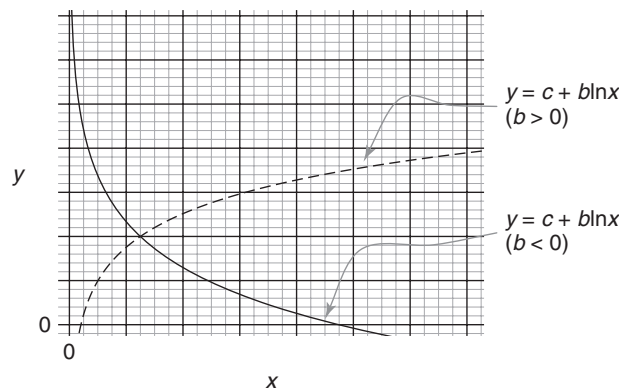


Figure 5.5 Logarithm CERs.

and is usually dependent on what is available. For most of these models, a minimum of three or four data points would be sufficient to construct a CER. A CER constructed from so few points is likely to have a significant amount of error. Ordinarily, linear regression is used to construct the cost estimating relationship. We can use linear regression on all of the functional forms discussed previously by performing a transformation on the data. Table 5.4, adapted from Stewart et al. [5], shows the relationship between the various CERs and their associated transformations.

Linear regression is used to estimate the parameters for the CERs once the data has been appropriately transformed. Linear regression is a statistical method used to fit a straight line through a set of data points. The goal is to determine the

**TABLE 5.4 Linear Transformations for CERs**

|  | Linear       | Power                     | Exponential          | Logarithmic       |
|--|--------------|---------------------------|----------------------|-------------------|
| Equation form desired                  | $Y = a + bX$ | $Y = ax^b$                | $Y = ae^{bX}$        | $Y = a + b \ln X$ |
| Linear equation form                   | $Y = a + bX$ | $\ln Y = \ln a + b \ln X$ | $\ln Y = \ln a + bX$ | $Y = a + b \ln X$ |
| Required data transform                | $X, Y$       | $\ln X, \ln Y$            | $X, \ln Y$           | $\ln X, Y$        |
| Regression coefficient obtained        | $a, b$       | $\ln a, b$                | $\ln a, b$           | $a, b$            |
| Coefficient reverse transform required | None         | $\exp(\ln a), b$          | $\exp(\ln a), b$     | None              |
| Final coefficient                      | $a, b$       | $a, b$                    | $a, b$               | $a, b$            |

coefficient values for the parameters  $a$  and  $b$  of the linear equation. The parameters are determined by using the following formulas:

$$b = \frac{\sum_{i=1}^n x_i y_i - \left[ \frac{\sum_{i=1}^n x_i}{n} \right] \sum_{i=1}^n y_i}{\sum_{i=1}^n x_i^2 - \left[ \frac{\sum_{i=1}^n x_i}{n} \right] \sum_{i=1}^n x_i} \tag{5.1}$$

$$a = \frac{\sum_{i=1}^n y_i}{n} - b \left[ \frac{\sum_{i=1}^n x_i}{n} \right] \tag{5.2}$$

Most of the time, especially when we have a reasonable size data set, a statistical analysis package such as Minitab [14], JMP [15], or even Excel will be used to perform the regression analysis on the cost data.

**TABLE 5.5 Labor Hours and Costs for Highway Construction**

| X<br>Labor hours | Y<br>Cost [\$] |
|------------------|----------------|
| 940.87           | 252.87         |
| 5814.28          | 4708.28        |
| 302.31           | 137.31         |
| 292.44           | 303.44         |
| 149.46           | 149.46         |
| 2698.94          | 1385.94        |
| 680.64           | 362.64         |
| 1078.32          | 364.32         |
| 6961.21          | 5269.21        |
| 4174.96          | 1192.96        |
| 1277.78          | 813.78         |
| 1493.08          | 957.08         |
| 4731.84          | 2342.84        |

**Example.** Suppose we have collected the following data on labor hours and construction costs for highways in Table 5.5. We would like to establish a cost estimating relationship between labor hours and cost. Analyze the data using a linear model and a power model.

We will fit the data to a simple linear model. The first thing we should do is to construct a scatter plot of the data such as that shown in Figure 5.6. We can estimate the parameters for a line that minimizes the squared error between the line and the actual data points. If we summarize the data, we get the following:

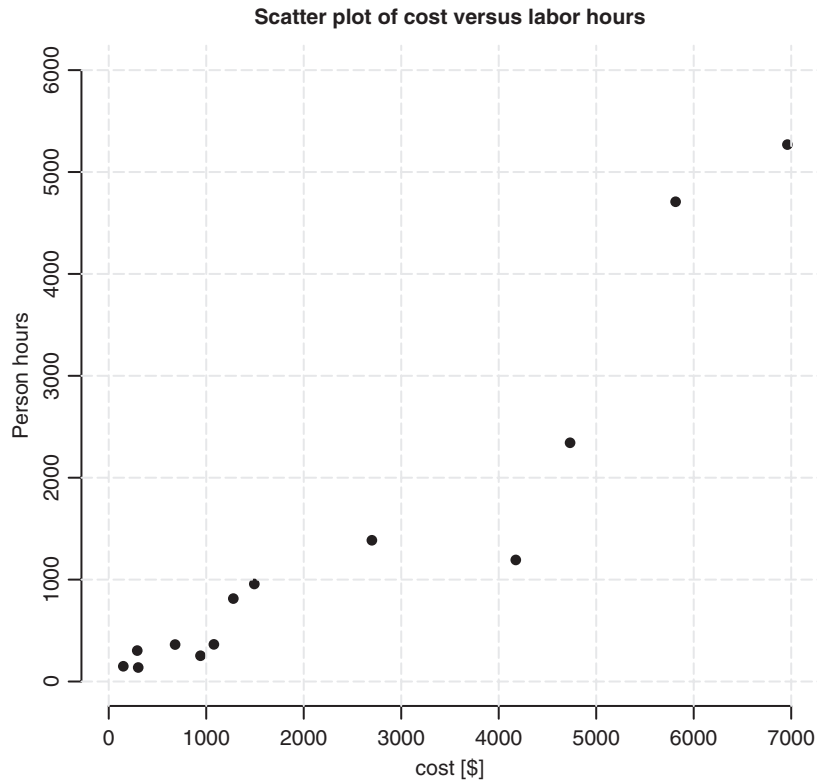
$$\sum_{i=1}^{13} x_i = 30596.13$$

$$\sum_{i=1}^{13} y_i = 18240.13$$

$$\sum_{i=1}^{13} x_i y_i = 87361422.71$$

$$\sum_{i=1}^{13} x_i^2 = 135941716.08$$





**Figure 5.6** Scatter plot of the data.

Using the summary data we can calculate the following coefficients for the linear relationship:

$$b = \frac{87361422.71 - \left(\frac{30596.13}{13}\right) 18240.13}{135941716.08 - \left(\frac{30596.13}{13}\right) 30596.13} = 0.695$$

$$a = \frac{1824.13}{13} - 0.695 \left(\frac{30596.13}{13}\right) = -232.61$$

If we enter the same data set into Minitab we obtain the following output:

The regression equation is  
 Cost = -233 + 0.695 Labor Hours

| Predictor   | Coef    | SE Coef | T     | P     |
|-------------|---------|---------|-------|-------|
| Constant    | -232.6  | 253.8   | -0.92 | 0.379 |
| Labor hours | 0.69499 | 0.07849 | 8.85  | 0.000 |

S = 627.579 R-Sq = 87.7\% R-Sq(adj) = 86.6\%

Analysis of variance

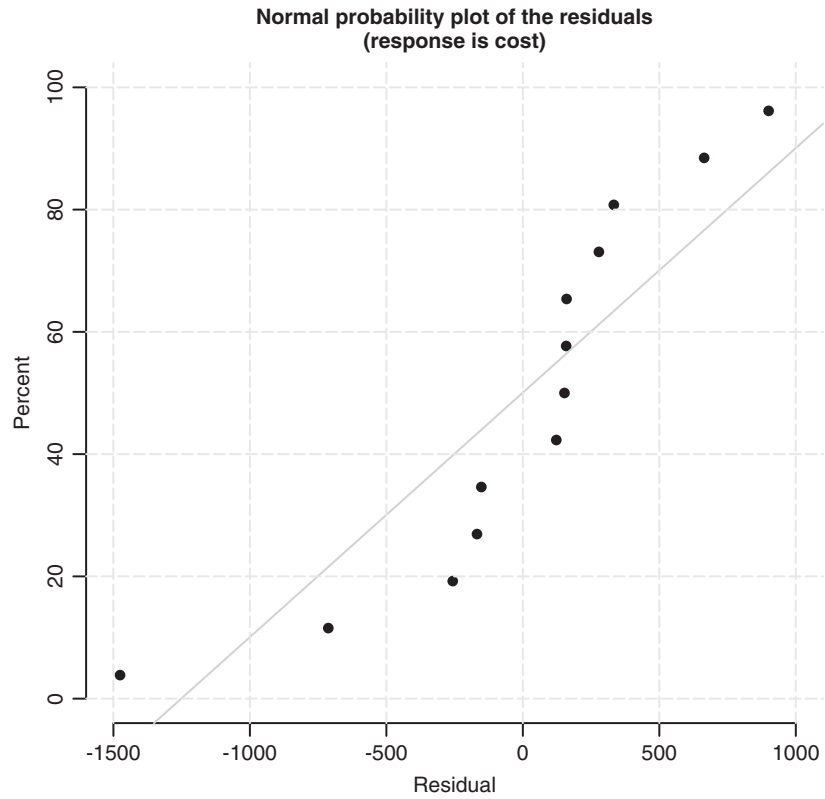
| Source         | DF | SS       | MS       | F     | P     |
|----------------|----|----------|----------|-------|-------|
| Regression     | 1  | 30880140 | 30880140 | 78.40 | 0.000 |
| Residual error | 11 | 4332404  | 393855   |       |       |
| Total          | 12 | 35212544 |          |       |       |

Examining the output, we see that the model is significant and that it accounts for approximately 87% of the total variation in the data. We note that the intercept term has a  $p$ -value of 0.379 and therefore could be eliminated from the model. As part of the analysis, one needs to check the underlying assumptions associated with the basic regression model. The underlying assumption is that the errors are normally distributed, with a mean of zero and a constant variance. If we examine the normal probability plot (Figure 5.7) and the associated residual plot (Figure 5.8), we see that our underlying assumptions may not be valid. We see that the residual data does not fall along a straight line and therefore is probably not normally distributed. Second, it appears that the variance is not constant. For larger values of man hours the variance increases.

Given that the underlying assumptions are not met for the basic linear regression model, one should consider some other type of cost estimating relationship. Let us consider a simple power cost estimating relationship. In this case, we need to transform our data according to Table 5.6, which contains the original data and the transformed data that we will use to fit a linear regression model. Note that

**TABLE 5.6 Transformed Data**

| X       | Y       | ln(X) | ln(Y) | (ln(X)) <sup>2</sup> | (ln(X))(ln(Y)) |
|---------|---------|-------|-------|----------------------|----------------|
| 940.87  | 252.87  | 6.85  | 5.53  | 46.88                | 37.88          |
| 5814.28 | 4708.28 | 8.67  | 8.46  | 75.14                | 73.31          |
| 302.31  | 137.31  | 5.71  | 4.92  | 32.62                | 28.11          |
| 292.44  | 303.44  | 5.68  | 5.72  | 32.24                | 32.45          |
| 149.46  | 149.46  | 5.01  | 5.01  | 25.07                | 25.07          |
| 2698.94 | 1385.94 | 7.90  | 7.23  | 62.42                | 57.15          |
| 680.64  | 362.64  | 6.52  | 5.89  | 42.55                | 38.44          |
| 1078.32 | 364.32  | 6.98  | 5.90  | 48.76                | 41.19          |
| 6961.21 | 5269.21 | 8.85  | 8.57  | 78.29                | 75.83          |
| 4174.96 | 1192.96 | 8.34  | 7.08  | 69.5                 | 59.06          |
| 1277.78 | 813.78  | 7.15  | 6.70  | 51.16                | 47.94          |
| 1493.08 | 957.08  | 7.31  | 6.86  | 53.42                | 50.17          |
| 4731.84 | 2342.84 | 8.46  | 7.75  | 71.60                | 65.65          |



**Figure 5.7** Normal probability plot.

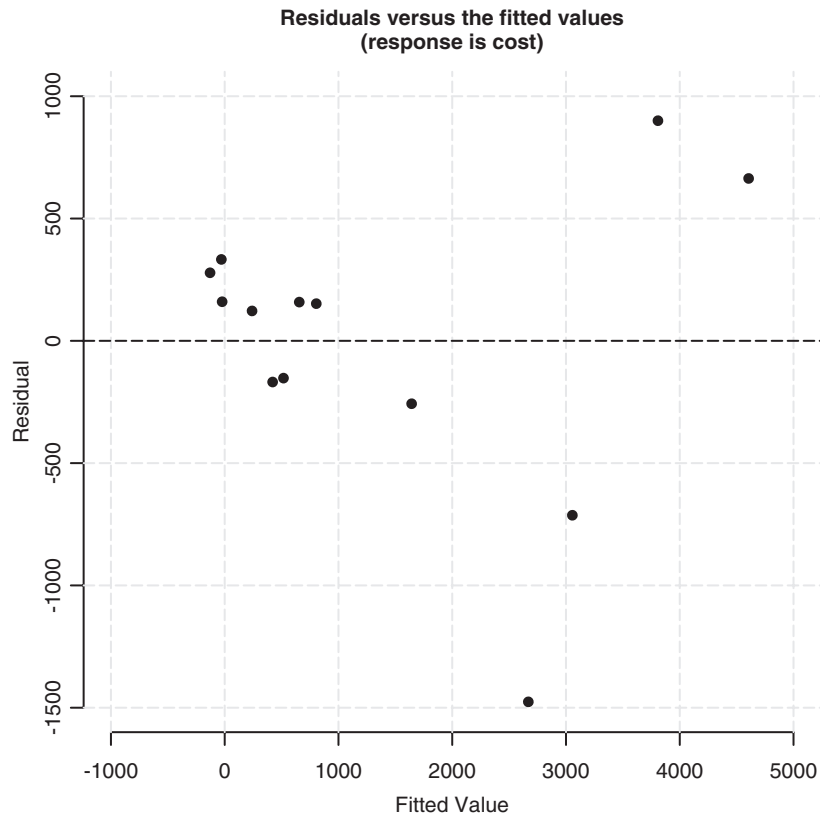
we perform our transformation by taking the natural logarithm of the cost and the natural logarithm of the labor hours. Using this transformed data set, we can calculate our coefficients for our transformed linear model.

$$\sum_{i=1}^{13} \ln x_i = 93.43$$

$$\sum_{i=1}^{13} \ln y_i = 85.64$$

$$\sum_{i=1}^{13} (\ln x_i) \ln y_i = 632.25$$

$$\sum_{i=1}^{13} (\ln x_i)^2 = 689.66$$



**Figure 5.8** Residual plot.

Using the summary data, we can calculate the coefficients for the transformed linear relationship.

$$b = \frac{632.25 - \left(\frac{93.43}{13}\right) 85.64}{689.66 - \left(\frac{93.43}{13}\right) 93.43} = 0.9209$$

$$a = \frac{85.64}{13} - 0.921 \left(\frac{93.43}{13}\right) = -0.0313$$

If we enter the same data set into Minitab we obtain the following output:

The regression equation is  
 $\ln Y = -0.031 + 0.921 \ln X$

| Predictor | Coef    | SE Coef | T     | P     |
|-----------|---------|---------|-------|-------|
| Constant  | -0.0313 | 0.7613  | -0.04 | 0.968 |
| LnX       | 0.9209  | 0.1045  | 8.81  | 0.000 |

S = 0.446321 R-Sq = 87.6\% R-Sq(adj) = 86.5\%

Analysis of variance

| Source         | DF | SS     | MS     | F     | P     |
|----------------|----|--------|--------|-------|-------|
| Regression     | 1  | 15.461 | 15.461 | 77.62 | 0.000 |
| Residual Error | 11 | 2.191  | 0.199  |       |       |
| Total          | 12 | 17.653 |        |       |       |

Examining the output, we see that the model is significant and that it accounts for approximately 87% of the total variation in the data. Again, we note that the intercept term has a *p*-value of 0.968 and therefore could be eliminated from the model at any level of significance less than or equal to  $\alpha = 0.968$ . Once again we must check the underlying assumptions associated with the basic regression model. The underlying assumption is that the errors are normally distributed, with a mean of zero and a constant variance. If we examine the normal probability plot and the associated residual plot (Figures 5.9 and 5.10) we see that our underlying assumptions appear to be valid. The residuals appear to be normal and have a constant variance with a mean of zero. Now we need to take the inverse transform to put our cost estimating relationship in its standard power form.

$$Y = e^{-0.031} X^{0.921}$$

$$Y = 0.97X^{0.921}$$

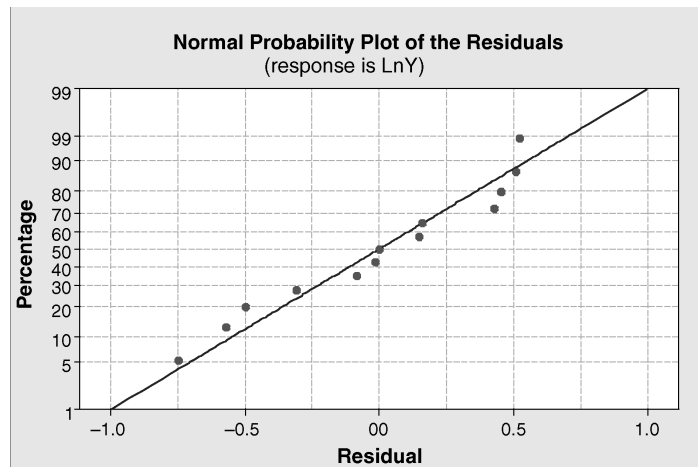


Figure 5.9 Normal probability plot.

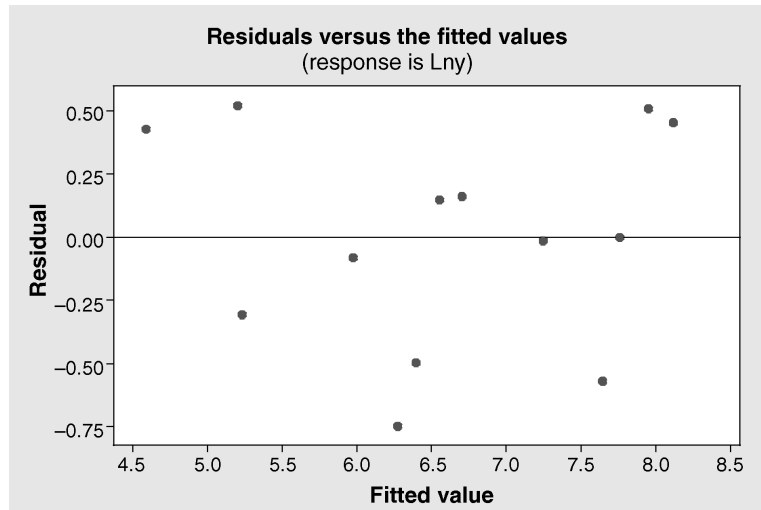


Figure 5.10 Residual plot.

**Example.** In this section, we provide several examples of hypothetical CERs that could be used to assemble a cost estimate for the air-vehicle component of the missile system [16, 17] described in the WBS given in Table 5.3. To estimate the unit production cost of the air-vehicle component, we sum the first unit costs for the propulsion system, the guidance and control system, the airframe, the payload, and the associated integration and assembly. Suppose the system that we are trying to estimate the first unit production cost for has the following engineering characteristics:

- Requires 15,000 lb of thrust
- Requires a 26-GHz guidance and control computer
- Has a 6-inch aperture on the antenna, operating in the narrow band
- Airframe weight of 300 lb
- Payload weight of 100 lb
- System uses electro-optics
- System checkout requires seven different test procedures

Suppose the following CERs have been developed using data from 20 different missile programs during the last 20 years.

**Propulsion CER.** The following CER was constructed using the propulsion costs from 15 of the 20 missile programs. Five of the programs were excluded because the technology used in those programs was not relevant for the missile system

currently being estimated. The CER for the propulsion system is given by

$$\begin{aligned}\text{Mfg \$ (FY 2000)} &= (\text{Thrust (lb)})e^{-0.1(\text{Yr}-1980)} \\ \text{Mfg \$ (FY 2000)} &= (15,000)e^{-0.1(2000-1980)} = 1114.10\end{aligned}$$

The manufacturing cost in dollars for the propulsion system is a function of thrust as well as the age of the motor technology (current year minus 1980).

**Guidance and Control CER.** The guidance and control CER was constructed using data from the three most recent missile programs. This technology has evolved rapidly and it is distinct from many of the early systems. Therefore, the cost analysts chose to use the reduced data set to come up with the following CER:

$$\begin{aligned}\text{Mfg \$ K (FY 2000)} &= 7.43(\text{GHz})^{0.45} \\ &\quad \times (\text{Aper (inches)})^{0.35}e^{0.7(\text{Wide/Narrow})} \\ \text{Mfg \$ K (FY 2000)} &= 7.43(26)^{0.45}(6)^{0.35}e^{0.7(1)} = 121.36\end{aligned}$$

The manufacturing cost in thousands of dollars for the guidance and control system is a function of the operating rate of the computer, the diameter of the antenna for the seeker, and whether or not the system operates over a wide band (0) or narrow band (1).

**Airframe CER.** Suppose the following CER was constructed using the airframe cost data from the 20 missile programs. The CER for the airframe is given by

$$\begin{aligned}\text{Mfg \$ (FY 2000)} &= 5.575(\text{Wt.lb})^{0.85} \\ \text{Mfg \$ (FY 2000)} &= 5.575(300)^{0.85} = 710.88\end{aligned}$$

Thus, the manufacturing costs in dollars for the airframe can be estimated if the analyst knows or has an estimate of the weight of the airframe.

**Fuzing System.** The following CER was established using data from five of the previous missile programs. The proximity fuse in the system being estimated is technologically similar to only five of the previous development efforts.

$$\begin{aligned}\text{Mfg \$ (FY 2000)} &= 15(\text{payloadwt.lb})e^{0.3(\text{EO/RF})} \\ \text{Mfg \$ (FY 2000)} &= 15(100)e^{0.3(0)} = 150\end{aligned}$$

The manufacturing cost for the fuzing system in dollars is a function of the weight of the payload and the type of technology used. The term EO/RF is equal to 0 if it uses electro-optic technology and 1 if it uses radio-frequency technology.

*Payload.* The payload CER is given by the following relationship in dollars:

$$\text{Mfg \$ (FY 2000)} = 150(\text{payloadwt.lb})$$

$$\text{Mfg \$ (FY 2000)} = 150(100) = 15000$$

**Integration and Assembly.** This represents the costs in dollars associated with integrating all of the air-vehicle components, testing them as they are integrated, and performing final checkout once the air vehicle has been assembled. Letting  $n$  represent the number of system test procedures, we have the following.

$$\text{Mfg \$ (FY 2000)} = 1.25 \left( \sum \text{Hardware costs} \right) e^{-(n)}$$

$$\begin{aligned} \text{Mfg \$ (FY 2000)} &= 1.25 (15,000 + 150 + 710.88 + 121.36 + 1114.10) \\ &\quad \times e^{-(7)} \end{aligned}$$

$$\text{Mfg \$ (FY 2000)} = 157.68$$

**Air-Vehicle Cost.** Using this information, the first unit cost of the air-vehicle system is constructed below:

$$\begin{aligned} \text{Mfg \$ (FY 2000)} &= 15,000 + 150 + 710.88 + 121.36 \\ &\quad + 1114.1 + 157.68 \end{aligned}$$

$$\text{Mfg \$ (FY 2000)} = 17254.02$$

This cost is in fiscal year 2000 dollars and it must be inflated to current year dollars (2006) using the methods discussed in Section 5.4.2. Once the cost has been inflated, the initial unit cost can be used to calculate the total cost for a purchase of 1000 missiles using an appropriate learning curve as discussed in the next section.

### 5.3.3 Learning Curves

Learning curves are an essential tool for adequately modeling the costs associated with the development and manufacture of large quantities of systems [8]. Many studies have shown that performance improves the more times a task is performed (supporting the old adage that practice makes perfect!). This “learning” effect was first noticed by Wright [18] when he analyzed aircraft data in the 1930s. Empirical evidence from a variety of other manufacturing industries has shown that human performance improves by some constant amount each time the production quantity is doubled [19]. This concept is especially applicable to labor-intensive products. Each time the production quantity is doubled, the labor requirements necessary to create a unit decrease to a fixed percentage of their previous value. This percentage is referred to as the *learning rate*.



Learning effects typically produce a cost and time savings of 5% to 30% each time the production quantity is doubled [10]. By convention, the 10% to 30% labor savings equates to a 90% to 70% learning rate. This learning rate is influenced by a variety of factors, including the amount of preproduction planning, the maturity of the design of the system being manufactured, training of the production force, the complexity of the manufacturing process, and the length of the production runs. Figure 5.11 shows a plot of a 90% learning rate and a 70% learning rate for a task that initially would take 100 hours. As evidenced by the plot, a 70% learning rate results in significant improvement of unit task times over a 90% curve. Delionback [5] defines learning rates by industry sector. For example:

- Aerospace—85%
- Repetitive electronics manufacturing—90% to 95%
- Repetitive machining—90% to 95%
- Construction operations—70% to 90%.

**Unit Learning Curve Formula** The mathematical formula used to describe the learning effect shown in Figure 5.11 is given by

$$T_X = T_1 X^r \quad (5.3)$$

where  $T_X$  = the cost or time required to build the  $X$ th unit,  $T_1$  = the cost or time required to build the initial unit,  $X$  = the number of units to be built, and  $r$  = negative numerical factor that is derived from the learning rate and is given by

$$r = \frac{\ln(\text{learning rate})}{\ln(2)} \quad (5.4)$$

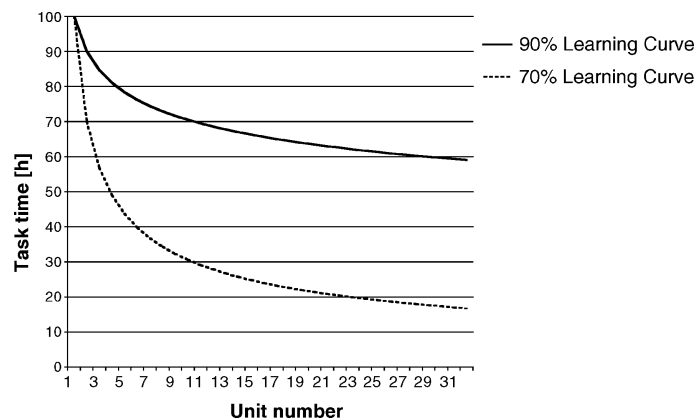


Figure 5.11 Plot of learning curves.

**TABLE 5.7 Factors for Various Learning Rates**

| Learning Rate (%) | $r$     |
|-------------------|---------|
| 100               | 0       |
| 95                | -0.074  |
| 90                | -0.152  |
| 85                | -0.2354 |
| 80                | -0.322  |
| 75                | -0.415  |
| 70                | -0.515  |

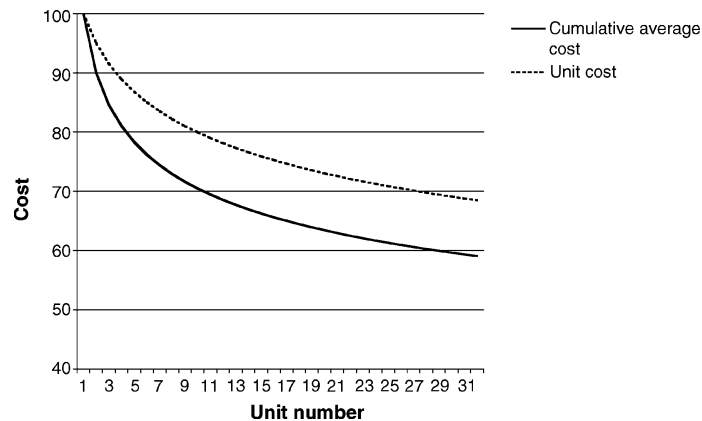
Typical values for  $r$  are given in Table 5.7. For example, with a learning rate of 95%, the resulting factor is  $r = \ln(0.95)/\ln(2) = 0.074$ .

The total time required for all units of production run of size  $N$  is

$$\text{total time} = T_1 \sum_{X=1}^N X^r \quad (5.5)$$

Examining the above equation, and using the appropriate factor for a 90% learning rate, we can calculate the unit cost for the first eight items. Assuming an initial cost of \$100, Table 5.8 provides the unit cost for the first eight items as well as the cumulative average cost per unit required to build  $X$  units. Figure 5.12 plots the unit cost curve and the cumulative average cost curve for a 90% learning rate for 32 units.

Note that the cumulative average curve is above the unit cost curve. When using data constructed with a learning curve, the analyst must be careful to note whether



**Figure 5.12** A 90% learning curve for cumulative average cost and unit cost.

**TABLE 5.8 Unit Cost and Cumulative Average Cost**

| Total Units Produced | Cost to Produce (X <sup>th</sup> ) unit | Cumulative Cost | Cumulative Average Cost |
|----------------------|---|-----------------|-------------------------|
| 1                    | 100                                     | 100             | 100                     |
| 2                    | 90                                      | 190             | 95                      |
| 3                    | 84.6                                    | 274.6           | 91.53                   |
| 4                    | 81                                      | 355.6           | 88.9                    |
| 5                    | 78.3                                    | 433.9           | 86.78                   |
| 6                    | 76.2                                    | 510.1           | 85.02                   |
| 7                    | 74.4                                    | 584.5           | 83.5                    |
| 8                    | 72.9                                    | 657.4           | 82.175                  |

they are using cumulative average data or unit cost data. It is easy to derive one from the other, but it is imperative to know what type of data one is working with to calculate the total system cost correctly.

**Example 1.** Suppose it takes 40 minutes to assemble the fins for a rocket motor the first time and takes 36 minutes the second time it is attempted. How long will it take to assemble the eighth unit?

First, that task is said to have a 90% learning rate because the cost of the second unit is 90% of the cost of the first. If we double the output again, from two to four units, then we would expect the fourth unit to be assembled in  $(36 \text{ minutes}) \times (0.9) = 32.4$  minutes. If we double again from four to eight units, the task time to assemble the eighth fin assembly would be  $(32.4) \times (0.9) = 29.16$  minutes.

**Example 2.** Suppose we wish to identify the assembly time for 25th unit, assuming a 90% learning rate.

First, we need to define  $r$ :

$$r = \frac{\ln(0.9)}{\ln(2)} = -0.152$$

Given  $r$ , we can determine the assembly time for the 25th unit as follows:

$$T_X = T_1 X^r$$

$$T_{25} = (40 \text{ min}) (25)^{-0.152}$$

$$T_{25} = 24.52 \text{ min}$$

Figure 5.13 plots a 90% learning curve for the fin assembly.

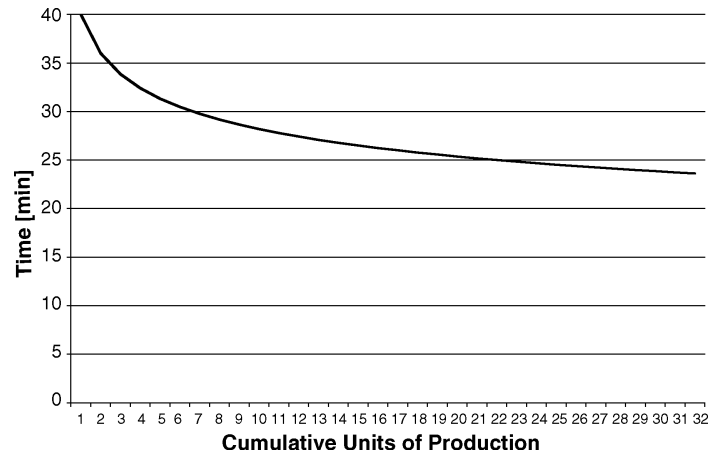


Figure 5.13 A 90% learning curve for fin assembly.

**Example 3.** It is common for organizations to define a standard of performance based on the 100th or 1000th unit. Suppose your organization sets a target assembly time for the 100th unit of 50 h. Suppose that your company has historically operated at an 80% learning rate, what is the expected assembly time of the first unit?

$$T_X = T_1 X^r$$

$$T_X x^{-r} = T_1$$

$$T_1 = 50(100)^{-(-0.322)}$$

$$T_1 = 220.20 \text{ h}$$

**Composite Learning Curves** Frequently, a new system will be constructed using a variety of processes, each of which may have its own learning rate. A single composite learning rate can be constructed that characterizes the learning rate for the entire system using the rates of the individual processes. One approach used to do this [5] weights each process in proportion to its individual dollar or time value. The composite learning curve is given by

$$r_c = \sum_p \left( \frac{T_p}{T} \right) r_p \quad (5.6)$$

where  $r_c$  = composite learning rate,  $r_p$  = learning rate for process  $p$ ,  $T_p$  = time or cost for process  $p$ , and  $T$  = total time or cost for the system.

**Example 4.** Suppose our rocket has a final assembly cost of \$50,000 and the final assembly task has a historic learning rate of 70%. Suppose that the rocket motor construction has a total cost of \$100,000 and that it has a historic learning rate of 80%. Finally, the guidance section has total cost of \$200,000 and a historic learning rate of 90%. Calculate the composite learning rate for the rocket?

$$r_c = \left[ \frac{50000}{350000} \right] (70\%) + \left[ \frac{100000}{350000} \right] (80\%) + \left[ \frac{200000}{350000} \right] (90\%)$$

$$= 84.29\%$$

**Cumulative Average Formula** The formula for calculating the approximate cumulative average cost or cumulative average number of labor hours required to produce  $X$  units is given by

$$T_c \approx \frac{T_1}{X(1+r)} [(X+0.5)^{(1+r)} - (0.5)^{(1+r)}] \quad (5.7)$$

This formula is accurate to within 5% when the quantity is greater than 10.

**Example 5.** Using the cumulative average formula, compute the cumulative average cost for eight units, assuming an initial cost of \$100 and a 90% learning rate.

$$T_c \approx \frac{100}{8(1-0.152)} [(8.5)^{(1-0.152)} - (0.5)^{(1-0.152)}]$$

$$\approx 82.31$$

Note that this value is very close to the actual cost found in Table 5.8.

**Constructing a Learning Curve from Historical Data** The previous formulas are all dependent upon having a value for the learning rate. The learning rate can be derived for specific tasks in a specific organization by using historical cost and performance data. The basic data requirements for constructing a learning rate for an activity include the dates of labor expenditure, or cumulative task hours, and associated completed units. The learning rate is found by comparing the total hours expended at the end of a given date and the corresponding number of units completed.

By taking the natural logarithm of both sides of the learning curve function discussed in Section 5.3.3, one can construct a linear equation which can be used to find the learning rate.

$$T_x = T_1 X^r \quad (5.8)$$

$$\ln(T_x) = \ln(T_1) + r \ln(X) \quad (5.9)$$

The intercept for this linear equation is  $\ln(T_1)$  and the slope of the line is given by  $r$ . Given  $r$ , the learning rate can be found using the following relation:

$$\text{learning rate}\% = 100(2^r) \quad (5.10)$$

This is best illustrated through an example.

**Example 6.** Suppose the data in Table 5.9 is pulled from the company accounting system.

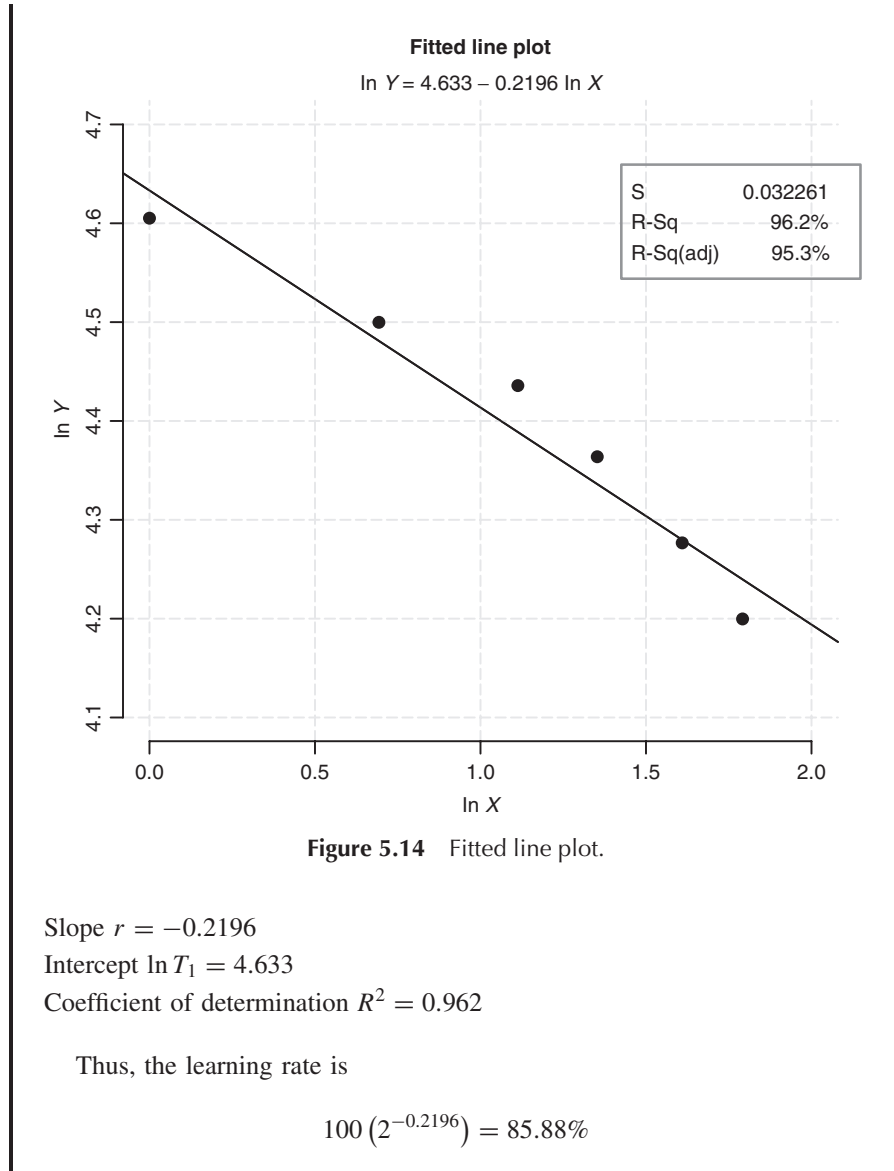
Transforming the data by taking the natural logarithm of the cumulative units and associated cumulative average hours yields Table 5.10. Figure 5.14 is a plot of the transformed data. Performing linear regression on the transformed data yields the following values for the slope and intercept of the linear equation.

**TABLE 5.9 Accounting System Data**

| Week | Cumulative Hours Expended | Cumulative Units Complete | Cumulative Average Hours |
|------|---------------------------|---------------------------|--------------------------|
| 1    | 040                       | 0                         |                          |
| 2    | 100                       | 1                         | 100/1 = 100              |
| 3    | 180                       | 2                         | 180/2 = 90               |
| 4    | 250                       | 3                         | 250/3 = 83.33            |
| 5    | 310                       | 4                         | 310/4 = 77.5             |
| 6    | 360                       | 5                         | 360/5 = 72               |
| 7    | 400                       | 6                         | 400/6 = 66.67            |

**TABLE 5.10 Natural Logarithm of Cumulative Units Completed and Cumulative Average Hours**

| Cumulative Units Completed $X$ | $\ln X$  | Cumulative Average Hours $TX$ | $\ln TX$ |
|--------------------------------|----------|-------------------------------|----------|
| 1                              | 0        | 100                           | 4.60517  |
| 2                              | 0.693147 | 90                            | 4.49981  |
| 3                              | 1.098612 | 83.33                         | 4.42281  |
| 4                              | 1.386294 | 77.5                          | 4.35028  |
| 5                              | 1.609437 | 72                            | 4.27667  |
| 6                              | 1.791759 | 66.67                         | 4.19975  |



#### 5.4 SYSTEM COST FOR SYSTEMS DECISION MAKING

In the previous section, we described how the system costs could be calculated using expert judgment, cost estimating relationships, and learning curves. Typically, this gives us a cost estimate in different year dollars. This section provides an overview of life cycle costing techniques that support the selection of economically feasible

systems. A discussion of the time value of money is provided along with time equivalence formulas and a discussion of inflation impacts. These techniques are demonstrated in a system selection example, and a brief overview of additional life cycle costing techniques for system decision making is provided. In Chapter 12, we tie our development of life cycle cost estimates with system value so that a tradeoff decision can be made which takes into account all the candidate design solutions for a systems decision problem.

#### 5.4.1 Time Value of Money

Life cycle costs must be identified in terms of timing as well as amount, because a dollar today is not the same as a dollar five years from now because of inflation and other economic effects. Two costs at different points in time are equivalent if they are equal to each other at some point in time for a given interest rate. For example, at a 10% annual interest rate, \$500 today is equivalent to \$605 two years from now. To compare costs occurring over the duration of a system's life cycle, we need to convert annual and future life cycle costs into their equivalent in present time. In addition, we may want to convert costs incurred today (present time) to future costs or analyze present and future costs as equal annual payments, depending on the specifics of the situation. Well-established equivalence formulas for converting costs in terms of time are shown in Table 5.11 along with an example for each conversion. These formulas are presented using the notation below:

- $i$  = Effective interest rate per period
- $N$  = Number of compounding periods
- $P$  = Present life cycle cost
- $F$  = Future life cycle cost
- $A$  = Annual life cycle cost

#### 5.4.2 Inflation

It is possible that the data available to support system life cycle costing is collected from different years. In these cases, it is necessary to convert this cost data from actual dollars (actual dollars at the point of time in which they occurred) into constant dollars (adjusted dollars representing purchasing power at some base point in time). This conversion is referred to as an inflation adjustment where inflation refers to rising prices measured against a standard level of purchasing power. Annual inflation rates vary across different types of goods and services and over time. Life cycle cost analysts estimate future costs. Since this requires making economic assumptions about the future, inflation is one of the key economic assumptions the analysts need to make.

The consumer price index (CPI) is a measure of the average change in prices over time of goods and services purchased by households [4]. The CPI is commonly used in the conversion of actual to constant dollars. Table 5.12 provides the end of



**TABLE 5.11** Equivalence Table [9]

| Conversion              | Formula   | Example   |
|-------------------------|---|---|
| Find $F$ when given $P$ | $F = P(1 + i)^N$  | A firm borrows \$1000 for 10 years. How much must it repay in a lump sum at the end of the 10 year? $i = 5\%$<br>$F = 1000(1 + 0.05)^{10} = \$1628.89$  |
| Find $P$ when given $F$ | $P = F \left[ \frac{1}{(1 + i)^N} \right]$              | A company desires to have \$1,000 seven years from now. What amount is needed now to provide for it? ( $i = 5\%$ )<br>$P = 1000 \left[ \frac{1}{(1 + 0.05)^7} \right] = \$710.68$   |
| Find $F$ when given $A$ | $F = A \left[ \frac{(1 + i)^N - 1}{i} \right]$          | If eight annual deposits of \$1000 each are placed in an account, how much money has accumulated immediately after the last deposit? ( $i = 5\%$ )<br>$F = 1000 \left[ \frac{(1 + 0.05)^8 - 1}{(0.05)} \right] = \$9549.11$ |
| Find $A$ when given $F$ | $A = F \left[ \frac{i}{(1 + i)^N - 1} \right]$          | How much should be deposited each year in an account in order to accumulate \$5000 at the time of the sixth annual deposit? ( $i = 5\%$ ),<br>$A = 5000 \left[ \frac{0.05}{(1 + 0.05)^6 - 1} \right] = \$735.09$            |
| Find $P$ when given $A$ | $P = A \left[ \frac{(1 + i)^N - 1}{i(1 + i)^N} \right]$ | How much should be deposited in a fund to provide for 10 annual withdrawals of \$500 each? ( $i = 5\%$ ),<br>$P = 500 \left[ \frac{(1 + 0.05)^{10} - 1}{0.05(1 + 0.05)^{10}} \right] = \$3860.87$                           |
| Find $A$ when given $P$ | $A = P \left[ \frac{i(i + 1)^N}{(1 + i)^N - 1} \right]$ | What is the size of 10 equal payments to repay a loan of \$10,000? $i = 5\%$ ,<br>$A = 10,000 \left[ \frac{0.05(1 + 0.05)^{10}}{(1 + 0.05)^{10} - 1} \right] = \$1295.05$   |

**TABLE 5.12 Consumer Price Index and Yearly Inflation Rate for 2000–2006**

| Year ( <i>i</i> ) | CPI (EOY) | Inflation rate % ( $f_i$ ) |
|-------------------|-----------|----------------------------|
| 2000              | 174       | —                          |
| 2001              | 176.7     | 1.55                       |
| 2002              | 180.9     | 2.38                       |
| 2003              | 184.3     | 1.88                       |
| 2004              | 190.3     | 3.26                       |
| 2005              | 196.8     | 3.42                       |
| 2006              | 201.8     | 2.54                       |

year (EOY) CPI for years 2000 through 2006 and the corresponding year-by-year change rates as provided by the U.S. Bureau of Labor Statistics [4].

Constant dollars ( $C\$$ ) at any time  $n$  of purchasing power as of any base time  $k$  can be converted to actual dollars ( $A\$$ ) at any time  $n$  by using the equivalence formula for finding  $F$  given  $P$ .

$$A\$ = C\$_n^{(k)}(1 + f)^{n-k} \quad (5.11)$$

where  $f$  = average inflation rate per period over the  $n - k$  periods.

**Example.**  $C\$10,000$  incurred in 2002 can be converted to 2005 dollars as follows:

$$f = \frac{f_{2003} + f_{2004} + f_{2005}}{3} = \frac{0.0188 + 0.0326 + 0.0342}{3} = 2.85\%$$

$$A\$ = \$10,000_{2005}^{(2002)} (1 + 0.0285)^{2005-2002}$$

$$A\$ = \$10,879.59$$

**Example.** Let us revisit the air-vehicle unit cost estimate developed in Section 5.3.2. Remember that the unit cost for the production of the first air vehicle was estimated to be \$138,492.58 in base year 2000. We can convert this to 2006 dollars as follows:

$$f = \frac{f_{2001} + f_{2002} + f_{2003} + f_{2004} + f_{2005} + f_{2006}}{6}$$

$$f = \frac{0.015 + 0.0238 + 0.0188 + 0.0326 + 0.0342 + 0.0254}{6} = 2.51\%$$

$$A\$ = \$138,492.58 (1 + 0.0251)^{2006-2000} = \$160,702.9$$

### 5.4.3 Net Present Value

A system decision based on life cycle costs should take both time value of money and inflation into account. To conduct this analysis when selecting among multiple systems, the following conditions must exist [9]:

- Candidate system solutions must be mutually exclusive where the choice of one excludes the choice of any other.
- All systems must be considered over the same length of time. If the systems have different expected total life cycles, the time period should equal the lowest common multiples of their lives or the length of time the selected system will be used.

**Example.** An automated assembly system is being considered to assist in the production of our rockets. The initial purchase and installation cost is assumed to be \$300,000. The life of the system is assumed to be seven years with annual operating and maintenance costs of \$65,000. It is expected that an annual increase of \$100,000 in revenue will be obtained from increased production, and the system can be salvaged for \$175,000 at the end of its seven-year life. Our minimum attractive rate of return (interest rate) is 7%, and an inflation rate of 3% is assumed. Given these cost parameters, compute the net present value of the assembly system to determine if the system should be purchased. The first step is to adjust the annual net cash flows for inflation as shown in Table 5.13.

Once we have our adjusted annual net cash flows, the net present value (NPV) can be computed using the equivalence formulas provided in Table 5.11. The system will be selected as economically justified if the

**TABLE 5.13 Inflation-Adjusted Annual Net Cash Flows**

| EOY | Cash Outflows [\$] | Cash Inflows [\$] | Net cash Flow [\$] | Inflation Conversion Factor | Cash Flow in Actual [\$] |
|-----|--------------------|-------------------|--------------------|-----------------------------|--------------------------|
| 0   | -300,000.00        |                   | -300,000.00        | $(1 + 0.03)^0$              | -300,000.00              |
| 1   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^1$              | 36,050.00                |
| 2   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^2$              | 37,132.00                |
| 3   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^3$              | 38,245.00                |
| 4   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^4$              | 39,393.00                |
| 5   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^5$              | 40,576.00                |
| 6   | -65,000.00         | 100,000.00        | 35,000.00          | $(1 + 0.03)^6$              | 41,794.00                |
| 7   | -65,000.00         | 275,000.00        | 210,000.00         | $(1 + 0.03)^7$              | 258,279.00               |

NPV is greater than zero.

$$\begin{aligned} \text{NPV} &= -\$300,000 + \$36,050 \left[ \frac{1}{(1+0.07)^1} \right] + \$37,132 \left[ \frac{1}{(1+0.07)^2} \right] \\ &\quad + \$38,245 \left[ \frac{1}{(1+0.07)^3} \right] + \$39,393 \left[ \frac{1}{(1+0.07)^4} \right] \\ &\quad + \$40,576 \left[ \frac{1}{(1+0.07)^5} \right] + \$41,794 \left[ \frac{1}{(1+0.07)^6} \right] \\ &\quad + \$258,279 \left[ \frac{1}{(1+0.07)^7} \right] \end{aligned}$$

$$\text{NPV} = \$45,018.57$$

The resulting NPV of the assembly system is \$45,018.57, and therefore we will recommend that the company approve the system for purchase and implementation.

#### 5.4.4 Breakeven Analysis and Replacement Analysis

Two additional system selection techniques warrant discussion in this chapter due to their applicability to life cycle costing: breakeven analysis and replacement analysis [7]. In a breakeven analysis, the system output quantity required to earn a zero profit (breakeven) is determined as a function of the sales per output unit, variable cost per output unit, and total fixed cost. Once the required breakeven output quantity is determined, a judgment as to whether or not this level of output is reasonable determines if the system should be selected. Replacement analysis is generally performed using the equivalence formulas presented in Table 5.11. The primary decision is whether an existing system (defender) should be retired from use, continued in service, or replaced with a new system (challenger). Accurate replacement analysis is very important in the retirement stage of the life cycle.

#### 5.5 RISK AND UNCERTAINTY IN COST ESTIMATION

As mentioned throughout this chapter, there is often considerable uncertainty associated with cost estimates, especially early in a systems life cycle. Cost risk has been defined in Chapter 3 as “the probability of exceeding the development, production, or operating budget in whole or in part.” These probabilities are a function of the amount of uncertainty present in the cost estimate. All estimates have some level of uncertainty in them. Table 5.2 illustrates the relative accuracy for the different classes of cost estimates.

When presenting cost estimates to management, the analyst should attempt to quantify the uncertainty associated with the estimate. For example, when a linear regression model is used to develop a CER, the analyst can compute the two-sided confidence limits on the coefficients of the model. The confidence limits can be used to express a measure of uncertainty for the cost estimating relationship. If an analyst is using expert judgment and creates a cost estimate by analogy, the analyst can attempt to capture the level of certainty from the expert or could use the uncertainty expressed in Table 5.2 for Class 1 or Class 2 estimates.

For example, in the analogy estimate provided in Section 5.3.1, the expert stated that the technology was three times more complex than the previous generation of the system. The expert may really believe that it is most likely three times more complex than the previous technology but no less than two times more complex and is no more than 4.5 times as complex as the previous generation of technology. This information on the uncertainty associated with the expert's quantification of complexity can then bound the estimate:

$$\text{Cost}_{\min} = (1.2) \times \$10,000 = \$12,000$$

$$\text{Cost}_{\text{likely}} = (1.3) \times \$10,000 = \$13,000$$

$$\text{Cost}_{\max} = (1.45) \times \$10,000 = \$14,500$$

### 5.5.1 Monte Carlo Simulation Analysis

Monte Carlo analysis is a useful tool for quantifying the uncertainty in a cost estimate. The Monte Carlo process creates a probability distribution for the cost estimate by rolling up all forms of uncertainty into a single distribution that represents the potential system costs. Once this distribution has been constructed, the analyst can provide management with meaningful insight about the probability that the cost exceeds a certain threshold, or that a schedule is longer than a specific target time. In Chapter 12 we will illustrate another use of Monte Carlo simulation analysis as a means of conducting sensitivity analysis on how uncertainty present in measure scores affects the total value for a system. Kerzner [10] provides five steps for conducting a Monte Carlo analysis for cost and schedule models. These steps are:

1. Identify the appropriate WBS level for modeling; the level will be a function of the stage in the system life cycle. In general, as the system definition matures, lower-level WBS elements can be modeled.
2. Construct an initial estimate for the cost or duration for each of the WBS elements in the model.
3. Identify those WBS elements that contain significant levels of uncertainty. Not all elements will have uncertainty associated with them. For example, if part of your system under study has off-the-shelf components and you have firm-fixed price quotes for the material, then there would be no uncertainty with the costs for those elements for the WBS.

4. Quantify the uncertainty for each of the WBS elements with an appropriate probability distribution.
5. Aggregate all of the lower-level WBS probability distributions into a single WBS Level 1 estimate by using a Monte Carlo simulation. This step will yield a cumulative probability distribution for the system cost. This distribution can be used to quantify the cost risk as well as identify the cost drivers in the system estimate.

Kerzner [10] emphasizes that caution should be taken when using Monte Carlo analysis. Like all models, the results are only as good as the data used to construct the model, the old adage that “garbage in, yields garbage out” applies to these situations. The specific distribution used to model the uncertainty in WBS elements depends on the information known about each estimate. Many cost analysts default to the use of a triangle probability distribution to express uncertainty. The choice of a triangle distribution is often a matter of convenience rather than the result of analysis. The probability distribution selected should fit some historical cost data for the WBS element being modeled. The triangle distribution will often be used for early life cycle estimates where minimal information is available (lower and upper bounds) and an expert is used to estimate the likeliest cost. When only the bounds on a WBS element are able to be reasonably estimated, a uniform distribution is frequently used in a Monte Carlo simulation to allow all values between the bounds to occur with equal likelihood. As the system definition matures and relevant cost data become available, other distributions should be considered and the cost estimate updated.

**Example.** Suppose you have been tasked to provide a cost estimate for the software nonrecurring costs for a Department of Defense (DoD) satellite system. The following CERs have been developed to estimate the cost of the ground control software, the system support software, and the mission embedded flight software during the conceptual design phase of the life cycle.

**Application and system software:** Person-months =  $4.3(\text{EKSLOC})^{1.2}(1.4)^{\text{DoD}}$

**Support software:** Person-months =  $5.3(\text{EKSLOC})^{0.95}$

**Space mission embedded flight software:** Person-months =  $8.7(\text{EKSLOC})^{1.6}(1.6)^{\text{DoD}}$

The DoD parameter equals 1 if it is a DoD satellite, and 0 otherwise. EKSLOC is a measure of the size of the software coding effort. It is the estimated size measured in thousands of source lines of code, but the engineers still need to estimate these sizes for their project.

Suppose that it is early in the design process and the engineers are uncertain about how big the coding effort is. The cost analyst has chosen to use a triangle distribution to estimate the EKSLOC parameter. The analysts ask the expert to provide a most likely line of code estimate,  $m$ , which is the mode, and two other estimates, a pessimistic size estimate,  $b$  and an optimistic size estimate,  $a$ . The

estimates of  $a$  and  $b$  should be selected such that the expert believes that the actual size of the source lines of code will never be less (greater) than  $a$  ( $b$ ). These become the lower and upper bound estimates for the triangular distribution. Law and Kelton [20] provide computational formulas for a variety of continuous and discrete distributions. The expected value and variance of the triangle distribution are calculated as follows:

$$\text{Expected value} = \frac{a + m + b}{3} \quad (5.12)$$

$$\text{Variance} = \frac{a^2 + m^2 + b^2 - am - ab - mb}{18} \quad (5.13)$$

Suppose our expert defines the following values for EKSLOC for each of the software components.

Using these values and the associated CERs a Monte Carlo analysis is performed using Oracle® Crystal Ball [21] software designed for use with Excel. The probability density function (PDF) for the embedded flight software is shown in Figure 5.15.

The PDF for the estimated labor hours is given in Figure 5.16 below for 10,000 simulation runs. Finally, suppose that management is uncertain about the labor cost for software engineers. Management believes the labor cost is distributed as a normal random variable with a mean of \$20 per hour and standard deviation of \$5. The CDF for the total software development cost is given below. This estimate assumes that engineers work 36 hours in a week on coding and that there are 4 weeks in a month. Figure 5.17 shows the PDF for the software development costs. The primary observation to take away from Figure 5.17 is the spread in possible software development costs due to the uncertainty assumptions imposed on the WBS elements when the Monte Carlo simulation was constructed. For this example, while it is more likely that the actual software development costs will clump around \$12 million, it is possible for them to be up to four times as much or as little as one-tenth as much because of this uncertainty. In the former case, the project could be threatened; in the latter, the project would continue well within budget. What is the probability of these values occurring?

Once we have the PDF for the cost of development, we can construct the cumulative distribution function (CDF) to answer this question. Applications such as Crystal Ball accomplish this task easily. Figure 5.18 contains the CDF for

**TABLE 5.14 EKSLOC Values by Software Component**

| Software type   | Optimistic<br>Size Estimate<br>(KSLOC) | Most Likely<br>Size Estimate<br>(KSLOC) | Pessimistic<br>Size Estimate<br>(KSLOC) | Mean<br>(KSLOC) | Variance<br>(KSLOC) |
|-----------------|--|---|---|-----------------|---------------------|
| Application     | 5                                      | 10                                      | 35                                      | 16.67           | 43.05               |
| Support         | 70                                     | 150                                     | 300                                     | 173.33          | 2272.22             |
| Embedded flight | 7                                      | 25                                      | 80                                      | 37.33           | 241.06              |

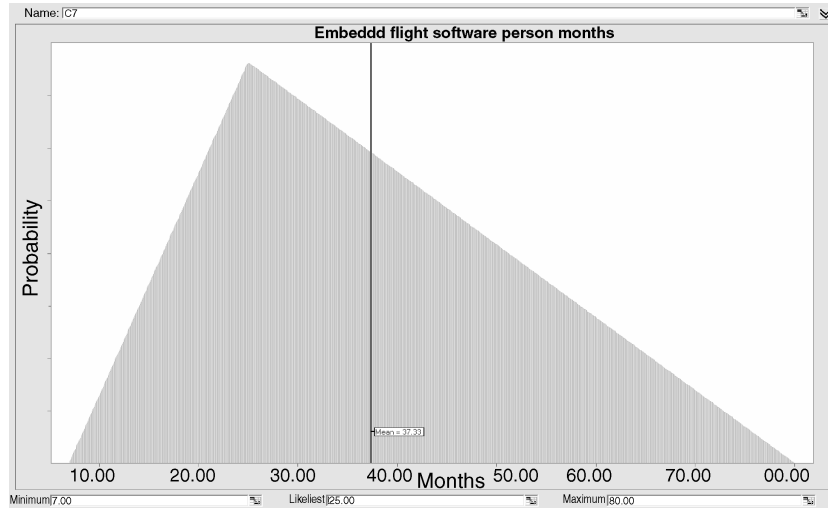


Figure 5.15 Triangular distribution for embedded flight software.

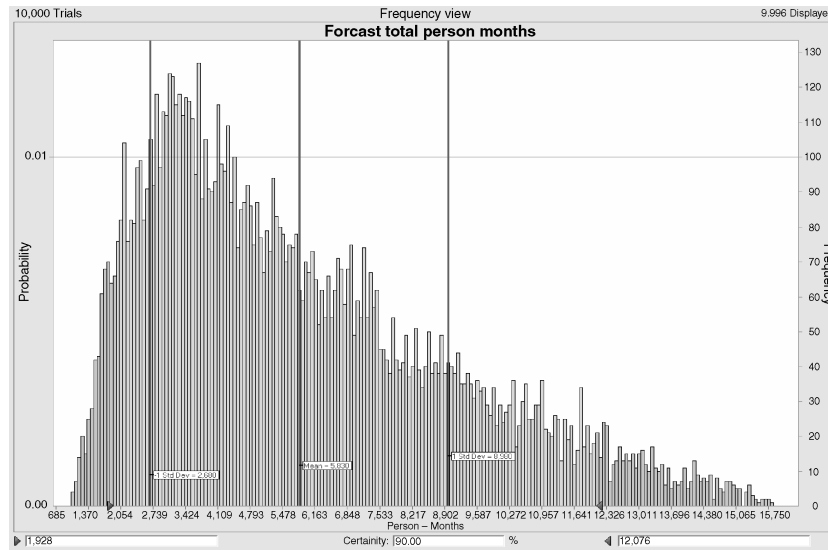


Figure 5.16 PDF for software person months.

software development costs. Using the CDF, we can make probability statements related to the software development cost. For example, we can state that there is a 50% probability that the software development costs will be less than \$15.96 million; similarly, there is a 20% probability that the software development costs will exceed \$26.59 million. This information is useful to senior-level management as they assess the cost risks associated with your program.



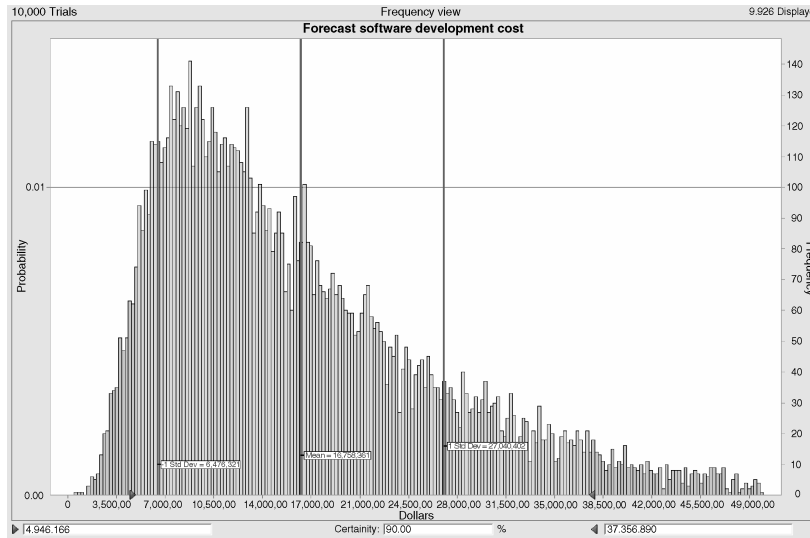


Figure 5.17 PDF for software development cost.

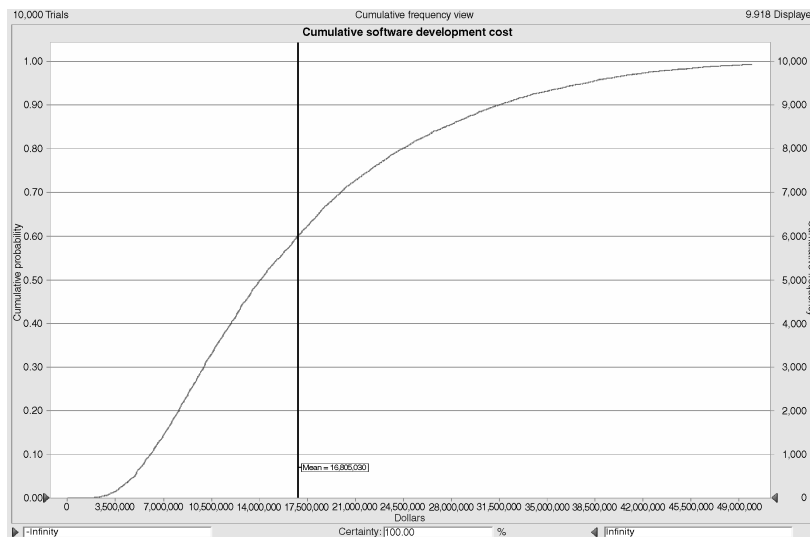


Figure 5.18 CDF for software development cost.

### 5.5.2 Sensitivity Analysis

The results can then be analyzed to identify those elements that are the most significant cost drivers. In our example, it is relatively easy because our total cost is only a function of three cost elements and one other factor. But realistic cost estimates may have on the order of 25 to 100 cost elements/factors, and choosing

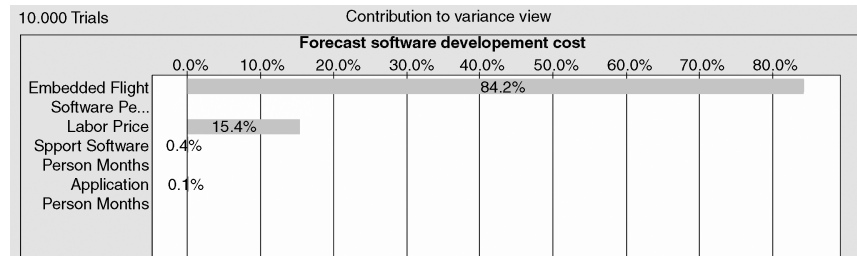


Figure 5.19 Sensitivity chart.

the cost drivers from this set is not so easy. Fortunately, Crystal Ball provides a tornado diagram (named for the shape) that analyzes the relative contribution of each of the uncertain components to the overall cost and variance for the system estimate. Figure 5.19 shows the sensitivity output for this example. Examining the chart in Figure 5.19, we find that the uncertainty associated with “embedded flight software person months” is the main contributor to the variability in the cost estimate, followed by the uncertainty in labor rate for software engineers. The cost analysis should consider spending more time getting a better estimate for the “embedded flight software person months,” since reductions in the uncertainty associated with this WBS element will have the greatest impact on reducing the variability in the total cost estimate seen in Figure 5.17.

## 5.6 SUMMARY

Estimating the cost of a system is an essential component of the systems decision process. Since many decisions are based on cost, it is imperative that the estimates constructed be as accurate as possible. This chapter has introduced some of the fundamental concepts and techniques necessary for developing a meaningful cost estimate of a system. Life cycle cost estimating, like many things in the systems engineering process, is an iterative task. Costs estimates should undergo updates as the system is defined and as the system moves through the various phases of the life cycle. In this chapter, we have discussed the evolution of the cost estimating process. A variety of techniques have been introduced including estimating by analogy, parametric cost estimates, learning curve analysis, and Monte Carlo analysis. Systems engineers and engineering managers must know how to perform and interpret life cycle cost analysis.

## 5.7 EXERCISES

- 5.1. Describe how the system life cycle affects the life cycle costing process.
- 5.2. Think of three factors that influence the accuracy of a life cycle cost estimate and explain how each of the factors influences cost estimation accuracy.

- 5.3.** Why is it important to consider the system uncertainty when developing a life cycle cost estimate?
- 5.4.** To help manage production of a company's missile systems, the purchase of a production scheduling software is being considered. The software is expected to have a useful life of four years and can be purchased and installed for \$150,000. Annual staff training and upgrades are expected to cost \$40,000. The system engineers anticipate an operating cost savings of \$75,000 due to an improved production schedule. It is predicted that the software will have no value at the end of its useful life. The company has a minimum attractive rate of return (interest rate) of 10% and uses an estimated annual inflation rate of 3%.
- (a) Use net present value to determine if the software should be purchased.
- (b) Suppose the company increases their minimum attractive rate of return to 15%, should they still purchase the software?
- 5.5.** Open the daily newspaper to the local real estate section. Using the data available to you on the real estate pages of the newspaper, develop a CER for the cost of housing in a particular community. Using your CER, predict the cost of a particular house in another community and compare it to the list price. Explain the reasoning for any differences. Is this good practice?
- 5.6.** Open the daily news paper to the automotive section and collect some cost data on at least 10 new automobiles. You will also need to collect some performance data and technical specifications for the vehicles. Using the information, construct a CER for the cost of a vehicle. Using your CER, predict the cost of another vehicle using its specifications and compare it to its list price. Explain the reason for any difference between the CER and the list price for the vehicle.
- 5.7.** Suppose we have collected the following cost and performance data on rocket motors for missile propulsion systems. Establish a cost estimating relationship between cost and thrust,. Establish a CER between cost and length. Finally, establish a CER that utilizes both thrust and length.

| Cost<br>(\$) | Thrust<br>(pounds) | Length<br>(inches) |
|--------------|--------------------|--------------------|
| 33,003       | 1,000              | 36                 |
| 34,712       | 1,200              | 40                 |
| 27,253       | 500                | 24                 |
| 39,767       | 2,000              | 42                 |
| 51,093       | 5,000              | 62                 |
| 62,073       | 10,000             | 98                 |
| 36,818       | 1,500              | 40                 |
| 79,466       | 25,000             | 124                |
| 68,628       | 15,000             | 78                 |

- 5.8.** One of Best Buys new Geek Squad members changed his first motherboard on a Thursday morning in 50 minutes. You know that the learning rate for this kind of task is 70%. If the chief Geek has him change 7 additional motherboards on Thursday morning, how long will it take him to change the last one?
- 5.9.** Suppose we wish to identify the assembly time of the 100th unit for a disc brake system. Assume we know that the assembly time for the 1st unit is 120 minutes. Assuming a learning rate of 80% compute the assembly time for the 100th unit.
- 5.10.** The automotive union has established a standard of performance for assembling the door panel based on the 100th unit. Suppose the standard is such that the target assembly time for the 100th unit is 150 minutes. Suppose the negotiated learning rate is 75%, what is the expected assembly time of the first unit?
- 5.11.** Suppose you have collected the following data on time required to perform an overhaul on an aircraft system from the aircraft depot information system. Determine the learning rate for the overhaul process.

| Week | Cumulative hours | Cumulative units completed |
|------|------------------|----------------------------|
| 1    | 120              | 0                          |
| 2    | 200              | 1                          |
| 3    | 310              | 2                          |
| 4    | 380              | 3                          |
| 5    | 440              | 4                          |
| 6    | 500              | 5                          |

- 5.12.** Consider problem nine again. Assume that you are uncertain of the actual learning rate, and suppose you believe it is most likely 80%, it would have a minimum value of 70% and a maximum value of 95%. Using a triangle distribution to represent your uncertainty, conduct a Monte Carlo analysis on the expected time of assembly for the 100th unit. Compute the probability that the assembly time of the 100th unit is longer than 90 minutes.
- 5.13.** In problem 5.4, assume that you are uncertain about the interest rate as well as the inflation rate. Suppose the most likely value for the interest rate is 10% and the min value is 8% and the max value is 11%. Similarly, suppose the most likely value for the inflation rate is 3% and the min value is 1% while the max value is 6%. Assuming that both parameters can be modeled using a triangle distribution, conduct a Monte Carlo analysis to determine the distribution for the NPV. Based on the levels of uncertainty associated with the interest rate and inflation rate, would you purchase the software?

## REFERENCES

1. International Committee for Systems Engineering (INCOSE). Retrieved 2007 March, from [www.incose.org](http://www.incose.org).
2. Society of Cost Estimating and Analysis. 2007. Glossary. Retrieved 2007 April, from [http://www.sceaonline.org/prof\\_dev/glossary-l.cfm](http://www.sceaonline.org/prof_dev/glossary-l.cfm).
3. AACE, Inc. *AACE International Recommended Practice*. No. 17R-97. Cost Estimation Classification System. 1997.
4. U.S. Bureau of Labor Statistics, Division of Consumer Prices and Price Indexes. Retrieved April 2007, from <http://www.bls.gov/CPI>.
5. Stewart, R, Wyskida, R, Johannes, J. *Cost Estimator's Reference Manual*, 2nd ed. New York: John Wiley & Sons, 1995.
6. Canada, J, Sullivan, W, White, J, Kulonda, D. *Capital Investment Analysis for Engineering and Management*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 2005.
7. Ostwald, P. *Cost Estimating*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1992.
8. Lee, D. *The Cost Analyst's Companion*. McLean, VA: Logistics Management Institute, 1997.
9. Park, C. *Fundamentals of Engineering Economics*. Upper Saddle River, NJ: Pearson-Prentice-Hall, 2004.
10. Kerzner, H. *Project Management*, 9th ed. Hoboken, NJ: John Wiley & Sons, 2006.
11. GAO-07-406SP. Defense acquisitions: Assessment of selected weapon systems, United States Government Accountability Office. Retrived from <http://www.gao.gov/new.items/d07406sp.pdf>.
12. Fabrycky, W, Blanchard, B. *Life Cycle Cost and Economic Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
13. Mil-Hdbk-881a. *Department of Defense Handbook*. Work Breakdown Structures for Defense Material Items, 30 July 2005. Retrived from <http://dcarc.pae.osd.mil/881handbook/881A.pdf>.
14. Minitab Statistical Software. Available at <http://www.minitab.com>. Accessed 25 April 2010.
15. JMP Statistical Software. Available at <http://www.jmp.com>. Accessed 25 April 2010.
16. Brown, C, Horak, J, Waller, W, Lopez, B. *Users Manual for TBMD Missile Cost Model*. TR-9609-01. Santa Barbara, CA: Technomics, 1997.
17. Shafer, W, Golberg, M, Om, N, Robinson, M. *Strategic System Costs: Cost Estimating Relationships and Cost Progress Curves*. IDA Paper P-2702, November 1993.
18. Wright, T. Factors, affecting the cost of airplanes. *Journal of Aeronautical Sciences*, 1936; 3 (February), 122–128.
19. Thuesen, G, Fabrycky, W. *Engineering Economy*, 7th ed. Englewood Cliffs, NJ: Prentice-Hall, 1989.
20. Law, A, Kelton, W. *Simulation Modeling and Analysis*, 3rd ed. Boston, MA: McGraw-Hill, 2000.
21. ORACLE® Crystal Ball. 2005. Denver, CO: Decisioneering, Inc. Available at <http://www.decisioneering.com>; Internet.