CHAPTER 77

Demand Forecasting and Planning

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Our goal in this chapter is to provide a number of possible demand models, and illustrate their use in operational decision making. We cover a number of demand models, ranging from the constant demand model to a model of demand as a distribution to use of information over time to generate adaptive estimates of demand. In each case we provide numerical examples to illustrate use of the technique. For additional technical details see Additional Reading at the end of this chapter.

1. CONSTANT DEMAND FORECAST AND ITS USE IN PLANNING

One of the simplest models of demand is to use an estimate of the average demand. This average demand, assuming a constant rate each period, can then be used to understand the effect of production costs or transport costs on inventory levels. Such models are appropriate when we deal with products in situations with predictable demand, that is, low forecast error. In particular, we will focus on the

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impact of setup costs and their interaction with holding costs. Our focus in this section is on discussing use of a constant demand rate model in an economic order quantity (EOQ) model. This requires us to define some of the costs that are important in understanding the impact of alternative decisions in an inventory system.

1.1. Inventory Costs

There are three main inventory costs we focus on:

1. *Holding or carrying costs:* These costs, expressed as a cost per unit of inventory per unit of time, model the cost associated with storage facilities, handling, insurance, pilferage, obsolescence, opportunity cost of capital, and so on.
2. *Fixed ordering or setup costs:* These costs are incurred each time an order is placed and model the costs to prepare the purchase or production order, costs associated with equipment setups, or costs associated with transportation (a single delivery truck).
3. *Shortage costs:* These are costs associated with loss of demand or penalties associated with delays.

1.2. An Inventory Policy

The context is the following: An inventory manager faces a constant demand rate that has to be satisfied from inventory. Ordering costs entail a fixed cost per order. Inventory held in the system incurs a holding cost. An *inventory policy* determines the quantity and frequency of orders. Each inventory policy has an associated cost to the organization. The two components of an inventory policy are:

- *Review period:* how often inventory levels are monitored and orders placed
- Order quantity: fixed or variable order quantity

We illustrate this concept through the use of an example.

1.2.1. Example: Choosing an Inventory Policy

Consider the problem of managing the inventory of Xerox paper in a warehouse. Although demands from retailers may fluctuate a bit in their demand, your aggregate demand for the item is fairly constant at 100,000 cases for the year. Due to your volume, your supplier has agreed to provide you an everyday low price of $55.00 a case. You calculate that it will cost about $4.00 per case per year to hold each case. Costs associated with each order and delivery charges by your supplier yield a fixed ordering cost of $75.00.

Consider the following inventory policy:

1. Place an order for Q units.
2. When inventory reaches zero, place another order for Q units and repeat.
3. Assume lead time is zero.

Which inventory policy do you recommend?

- Review daily resulting in daily shipments of about 385 cases each.
- Review weekly, resulting in order sizes of about 1923 cases each.
- Place an order each month at about 8333 cases each.
- Place only two orders per year at about 50000 cases each.

1.2.2. Evaluating the Cost of an Inventory Policy

To calculate costs of each policy, let:

d = yearly demand in units
h = holding costs in dollars per unit per year
s = setup or ordering costs in dollars
Q = quantity of each order
Q = uniquely defines this simple inventory policy

$$\text{Total cost (AHO)} = \text{annual carrying costs} + \text{annual ordering costs}$$

$$\text{AHO} = \frac{Q}{2}h + \frac{d}{Q}s$$

So:

d = 100,000 units/yr
h = 4 dollars/unit-yr
s = 75 dollars

$$1.\ Q = 385,\ \text{AHO} = \frac{385}{2}4 + \frac{100{,}000}{385}75 = 20{,}250$$

$$2.\ Q = 1{,}923,\ \text{AHO} = \frac{1{,}923}{2}4 + \frac{100{,}000}{1{,}923}75 = 7{,}746$$

$$3.\ Q = 8{,}333,\ \text{AHO} = \frac{8{,}333}{2}4 + \frac{100{,}000}{8{,}333}75 = 17{,}566$$

$$4.\ Q = 50{,}000,\ \text{AHO} = \frac{50{,}000}{2}4 + \frac{100{,}000}{50{,}000}75 = 100{,}150$$

From these policies the lowest cost is to order weekly with Q = 1,923.

1.2.3. Economic Order Quantity (EOQ) Model

Consider the following inventory model:
Place an order for Q units.
When inventory reaches zero, place another order for Q units and repeat.

d = yearly demand in units.
h = carrying or holding costs in dollars per unit per year.
s = setup or ordering costs in dollars.

$$Q^* = \text{economic order quantity (EOQ)}$$

$$Q^* = \text{EOQ} = \sqrt{\frac{2ds}{h}}$$

minimizes the total cost function:

$$\text{AHO} = \frac{Q^*}{2}h + \frac{d}{Q^*}s$$

For:

d = 100,000 units/yr
h = 4 dollars/unit-yr
s = 75 dollars

$$Q^* = \sqrt{\frac{2(100{,}000)(75)}{4}} = 1{,}936$$

$$\text{AHO} = \frac{1{,}936}{2}4 + \frac{100{,}000}{1{,}936}75 = 7{,}746$$

$$\text{Total cost} = \text{AHO} + (cD)$$

1.3. A Service Application—Training Airline Flight Attendants

You manage the training department for airline attendants for a major airline. You have to organize and plan the training sessions for all new hires. Each week personnel sends you a list of the new hires that you must schedule into the next available training session—they are on the payroll and basically idle until they can be trained and put into service. The yearly demand for new attendants is fairly constant at 1200, and personnel is always interviewing and hiring to keep up with the demand. The new hires, on average, cost the company about \$40,000 per year in salary and benefits.

You wonder if there is a way to think about the costs in the system and better manage them. First you realize that personnel and training should coordinate better. Why should personnel put someone on the payroll before they can be scheduled for training? After all, a new hire won't argue about a week here or there on a start date. This seems like an easy thing to accomplish, but it certainly puts the burden on you to schedule the sessions and keep personnel abreast of your plans. Next you wonder how to best plan the training sessions—how often should you hold sessions and how many trainees should you put in each class. You gather the following data: The cost of the teachers, the conference rooms, and so on combine for a fixed cost of \$10,000 per training session.

How should you plan the sessions to minimize total cost? Note that given the data, we see that $s = 10{,}000$, $d = 1{,}200$ and $h = 40{,}000$. Thus, the optimal training batch size is

$$\sqrt{\frac{2 \times 10{,}000 \times 1{,}200}{40{,}000}}$$

which is rounded up to 25 per batch. This corresponds to training approximately once a week. The cost associated with this decision

$$\frac{10{,}000 \times 1{,}200}{25} + \frac{40{,}000 \times 25}{2} = 980{,}000$$

1.4. Sensitivity Analysis of the EOQ Model

While the EOQ model provides quick estimates of optimal behavior, it requires knowledge of the costs described earlier. It also requires us to have a good estimate of demand rate. However, the benefit of the model is that it is robust to variation in the assumptions. This is indicated as follows:

*1.4.1. Impact of Using a Q Other Than Q**

Suppose, in practice, the actual Q used in a situation is different from Q^*. We will examine the impact of this difference is Q on average annual costs AHO(Q). We define the average annual costs as

$$AHO(Q) = \frac{sd}{Q} + \frac{hQ}{2}$$

and

$$Q^* = \sqrt{\frac{2sd}{h}}$$

(In this analysis, we thus drop the term cd from both expressions.) Note that

$$\frac{\mathrm{AHO}(Q)}{\mathrm{AHO}(Q^*)} = \frac{1}{2}\left\{\frac{Q}{Q^*} + \frac{Q^*}{Q}\right\}$$

The equation above is obtained by substituting the optimal Q^* in the cost expression for AHO(Q^*)

Note that if Q varies between $Q^*/2$ and $2Q^*$, the impact on average annual cost is less than 25%. Similarly, if Q varies between $0.8Q^*$ and $1.25Q^*$, the average annual costs increase by no more than 2.5%.

These results suggest that the EOQ model is robust, that is, variations in decisions result in small variation in their cost impact as long as we are around the optimal EOQ model.

2. DEMAND MODELED AS HAVING A DISTRIBUTION

In the earlier section, we modeled demand as a constant rate. Often, however, demand is not very predictable but has a significant amount of randomness. To understand the effect of demand forecast error, we first focus on problems where decisions regarding inventory are made once for an entire period. Examples of products that might require inventory decisions that cover demand over a single period include

- Newspapers for which the period refers to one day
- Fruits/flowers for which the period refers to one week

- Fashion products for which the period refers to a season (three months)
- Hotel room reservations for which the period refers to one day
- Airline reservations for which we refer to a particular flight

To illustrate the basic ideas, we start with a numerical example.

2.1. Example—The Fashion Store

The Fashion Store sells fashion items. The store has to order these items many months in advance of the fashion season in order to get a good price on the items. Each unit costs Fashion $100. These units are sold to customers at a price of $160 per unit. Items not sold during the season can be sold to the outlet store at $75 per unit. If the store runs out of an item during the season, it has to obtain the item from alternative sources and the cost including air freight to Fashion is $190 per unit.

Fashion wants help in choosing the initial order quantity to minimize costs to run the store.

Historical data from comparable items over the last few years have generated 100 demand observations as follows:

86	94	90	86	82	84	91	76	85	83	92	82	89	88	79	83	83	85	89	90
73	84	86	90	90	92	83	91	85	85	82	81	81	76	81	81	78	85	84	82
88	86	85	88	86	89	87	84	83	79	90	87	83	87	82	81	85	84	87	89
82	80	92	85	88	85	83	87	84	84	86	80	87	80	89	79	83	80	86	87
81	93	91	89	80	86	87	86	88	84	81	84	84	82	77	93	94	97	87	75

A frequency table of these points and the corresponding cumulative probabilities for each demand value are as follows:

Demand (D)	Frequency	P(demand) P(D)	Cum. Prob. P(Demand ≤ D)
73	1	0.01	0.01
75	1	0.01	0.02
76	2	0.02	0.04
77	1	0.01	0.05
78	1	0.01	0.06
79	3	0.03	0.09
80	5	0.05	0.14
81	7	0.07	0.21
82	7	0.07	0.28
83	8	0.08	0.36
84	10	0.10	0.46
85	10	0.10	0.56
86	9	0.09	0.65
87	8	0.08	0.73
88	5	0.05	0.78
89	6	0.06	0.84
90	5	0.05	0.89
91	3	0.03	0.92
92	3	0.03	0.95
93	2	0.02	0.97
94	2	0.02	0.99
97	1	0.01	1.00
	100	1.0	

The sample mean is 85 units and the standard deviation is 4.43 units.

2.2. The Single-Period Inventory Model

Given the problem described earlier, it turns out that the optimal decision is described using two cost parameters:

1. C_e = cost per unit of an excess item at the end of period
2. C_s = cost per unit of an item short

The optimal service level defined as the probability that all demand is satisfied in a period immediately from primary stock is described as

$$\text{ser}^* = \frac{C_s}{C_s + C_e}$$

From our example we have:

$$C_e = 25$$
$$C_s = 90$$

So,

$$\text{ser}^* = \frac{C_s}{C_s + C_e} = \frac{90}{90 + 25} = 0.782$$

From frequency table we get for a service level of 0.782,

$$r^* \approx 89$$

2.2.1. Suppose Demand Follows a Normal Distribution

Example 1: Fashion Store. If we approximate demand by a normal distribution with mean $m = 85$ and standard deviation $\sigma = 4.43$, we get

$$\text{ser}^* = \frac{C_s}{C_s + C_e} = \frac{90}{90 + 25} = 0.782$$

so

$$r^* = 85 + Z_{0.782}(4.43) = 85 + 0.76(4.43) = 88.45 \approx 89$$

Example 2: Fashion Store. Approximate by Normal with $m = 85$ and $\sigma = 4.43$

Suppose the Fashion Store wants to provide a service level of 90%. What level of inventory is required?

$$r_{0.90} = 85 + Z_{.90}(4.43) = 85 + 1.29(4.43) = 90.7 \approx 91$$

Therefore, if Fashion has an inventory of 91 items with probability 0.90 all the demand in a period is satisfied from primary stock.

3. UNCERTAIN DEMAND OVER MULTIPLE PERIODS

In this section we return to making decisions over continuous time. Demands each period are modeled as following a distribution. We first consider the case where we can only make decisions once each period. The costs consist of the costs of holding inventory each period and the costs of shortage each period. We include the case where there may be a fixed lead time L for the supplier to deliver an order. The basic idea is that in the presence of uncertainty and lead time for delivery, orders have to be placed well in advance of inventory running out in order to guarantee a high level of availability.

3.1. Impact of Lead Time

If we consider the problem in the earlier sections with constant demand and just add in a lead time L between order placement and delivery, note that orders should be placed when the inventory level hits dL. This level is called the reorder level. The order size remains Q as calculated by the EOQ model.

An interesting issue is the fact that in the absence of demand forecast error, lead time has no impact on costs. This is seen by the fact that while the order trigger times are affected by lead time

L, the physical inventory levels and the rate of order placements are unaffected by L. Since these two parameters affect costs in this model, costs are unaffected by lead time.

3.2. Lead Time and Demand Uncertainty

In the presence of lead time and demand uncertainty, the reorder level requires us to decide how much of the demand should be satisfied from stock. This factor can be expressed as *service level.* We express service level as the *probability* of being in stock. This probability is a number that the customers (generating the demand) can use to do their own planning. This probability will depend (intuitively) on the industry and on the extent of competition faced by the company.

If demand follows a normal distribution, then the reorder level requires us to merely generate a Z value that corresponds to the desired cumulative probability.

Then set the reorder level as

$$r = (dL) + (Z\sigma\sqrt{L})$$

where σ refers to the standard deviation of demand per unit time and d is the mean demand per unit time.

3.2.1. Example Problem

The Reliable Hardware Store sells electric pumps. The supplier lead time is one week. Each pump costs Reliable \$100. Annual inventory carrying cost is 25% of the investment. If Reliable stocks out, it will fill demand by buying the required pumps elsewhere at an additional cost of \$20, that is, Reliable's cost would be \$120. Reliable's planned probability of in stock is 91.2%.

Demand recorded for the last 100 weeks shows the following data:

86	94	90	86	82	84	91	76	85	83	92	82	89	88	79	83	83	85	89	90
73	84	86	90	90	92	83	91	85	85	82	81	81	76	81	81	78	85	84	82
88	86	85	88	86	89	87	84	83	79	90	87	83	87	82	81	85	84	87	89
82	80	92	85	88	85	83	87	84	84	86	80	87	80	89	79	83	80	86	87
81	93	91	89	80	86	87	86	88	84	81	84	84	82	77	93	94	97	87	75

The mean demand is 85 units and the standard deviation is 4.43 units. The same data can be plotted as a histogram or as a line graph using a spreadsheet.

Note that when Reliable reorders, there should be sufficient inventory on hand to cover demand during the lead time of one week.

3.3. A (Q, r) Policy

We now formalize our multiperiod inventory system. By convention, some parameters are usually given in annual terms (the EOQ parameters), while other parameters are stated in smaller period terms (the lead time parameters).

s = ordering cost
d = average annual demand
h = annual holding cost per item
L = fixed lead time in periods
ser = planned service level
Z_{ser} = the Z value that generates the required in stock probability
m = mean demand during a period
σ = standard deviation of demand during a period

Then we first calculate:

$$Q = \sqrt{\frac{2ds}{h}}$$

And then the reorder point:

$$r = (mL) + (Z_{\text{ser}}\ \sigma\sqrt{L})$$

$$\text{Average physical inventory level} = \frac{Q}{2} + r - (mL)$$

3.3.1. Another Example Problem

Thus, for the problem faced by Steco, a retailer of titanium rods. Weekly demand for these rods follows a normal distribution with a mean of 100 units and a standard deviation of 5 units per week. These rods cost \$5 each and the cost of holding a rod in inventory for a year is 20% of its cost. The cost to Steco to place an order for replenishment is \$25/order. Delivery lead time is one week. Management wants a less than 6% probability of stocking out. Assume 50 weeks per year.

Develop a (Q, r) policy for Steco.

Answer:

Ordering cost $= s =$ \$25/order
Holding cost $= h =$ \$1/unit/year
Annual demand $= d = 100 \times 50 = 5000$ units/year

$$Q = \sqrt{\frac{2sd}{h}} = 500$$

$$\begin{aligned} m &= 100, \\ \sigma &= 5, \\ L &= 1, \\ \text{ser} &= 0.94, \text{ and} \\ Z_{0.94} &= 1.56 \text{ (from the normal table)} \\ r &= 108 \end{aligned}$$

Thus, Steco should reorder Q units whenever the pipeline inventory level falls to below r units.

Steco's average inventory level $= Q/2 + r - (mL) = 258$ units

4. DEMAND AS A MIXTURE OF DISTRIBUTIONS

We now consider an alternative representation of demand that will permit us to incorporate information collected to lower demand uncertainty. The role of the decision maker is to use the best available assessment of the weights to be assigned to each demand model in order to model the demand at that point in time. We do this by representing demand as a mixture of a number of possible demand models. This in effect means that the model of demand will change over time, reflecting the link between information and its role in reducing demand uncertainty. We illustrate this idea using an example problem.

4.1. A Mixture Model of Demand

Consider a retailer (ASSORT) who sells women's dresses. Analysis of the historical data has indicated that at the end of the season, some dresses follow a demand whose distribution is uniform between 1 and 5. Other (in-fashion) dresses follow a demand whose distribution is between 4 and 8 units. About eight months in advance, when the order is placed, the best estimate is that demand for a particular dress will be either of these two distributions with a 50% probability.

These dresses are bought for \$100 and sell for \$200. If ASSORT runs out of dresses, the future profit impact is estimated to be \$200. Dresses that are held through the fashion season incur a holding cost of \$20. Those dresses that do not sell during the season are sold to an outlet store for \$40.

Under this system, what will be ASSORT's order size and associated expected profit if orders have to be placed eight months in advance?

Answer: Given these costs, note that ASSORT will estimate the optimal service level to be

$$\text{Service level} = \frac{r + g - c}{r + h + g - s}$$

where $r = 200$, $c = 100$, $h = 20$, $s = 40$, $g = 200$. Thus, the optimal service level is 78.9%.

The demand distribution at this point in time is

Demand	Probability
1	0.1
2	0.1
3	0.1
4	0.2
5	0.2
6	0.1
7	0.1
8	0.1

With this demand distribution, the inventory of dresses purchased is 6 units. Associated with this inventory purchased, ASSORT's expected profit is as follows:

$$
\begin{aligned}
&(-100*6) + \text{(purchase costs)} \\
&((0.1*200*1) + (0.1*200*2) + (0.1*200*3) + (0.2*200*4) + (0.2*200*5) + \\
&(0.1*200*6) + (0.1*200*6) + (0.1*200*6))+ && \text{(expected revenue)} \\
&((0.1*20*5) + (0.1*20*4) + (0.1*20*3)+ \\
&(0.2*20*2) + (0.2*20*1)) + && \text{(expected salvage} - \text{Holding Costs)} \\
&((-0.1*200*1) + (-0.1*200*2)) && \text{(expected penalty costs)} \\
&= 216
\end{aligned}
$$

The associated manufacturer revenue is $100 * 6 = \$600$.

4.2. Impact of Information on the Demand Model

The retailer has heard of a new scheme called quick response (QR). Under this scheme, the manufacturer has to receive the order only four months in advance. This enables the retailer to collect data regarding sales of similar products. These similar product sales enable the retailer to further refine the demand distribution estimates. What will be the impact of QR on the retailer?

Answer: Under QR, the retailer observes a draw from the demand distribution. Depending on the value of this observed demand, ASSORT will adjust demand estimates as follows:

Demand	Probability
$1 \leq d1 \leq 3$	$P(1) = 1$, $P(2) = 0$
$4 \leq d1 \leq 5$	$P(1)=1/2$, $P(2) = 1/2$
$6 \leq d1 \leq 8$	$P(1) = 0$, $P(2) = 1$

Thus, the observed demand changes the weights we would place on each of the two demand distributions.

As before, we can then derive the optimal inventory policy and associated expected profit for the retailer as follows:

Demand ($d1$)	Probability	Inventory	Expected Profit
$1 \leq d1 \leq 3$	0.3	4	144
$4 \leq d1 \leq 5$	0.4	6	216
$6 \leq d1 \leq 8$	0.3	7	444
			Total expected profit 262.8

Thus, the retailer's profit increases from 216 to 262.8, an increase of 22%.

What is the expected quantity purchased from the manufacturer?

$$(0.3 * 4) + (0.4 * 6) + (0.3 * 7) = 5.7$$

Thus, manufacturer revenues decreases to \$570, a drop of 5%.

4.3. Quick Response—Service Commitment

Suppose the retailer commits to a service level of 100% in return for the manufacturer providing QR. What will be the impact on the system?

Answer: We will have to change the inventory purchased after observing demand and thus get the following results:

Demand ($d1$)	Probability	Inventory	Expected Profit
$1 \le d1 \le 3$	0.3	5	140
$4 \le d1 \le 5$	0.4	8	170
$6 \le d1 \le 8$	0.3	8	440
		Total expected profit	242.0

The associated manufacturer revenue is

$$100\,[(0.3 * 5) + (0.4 * 8) + (0.4 * 8)] = 710$$

Thus, the manufacturer and the retailer are better off than under the old system.

Where are the retailer's increases in expected profit coming from?

Answer: Consider the service level in the old system and the new QR system with a 100% service level.

Demand ($d1$)	Probability	Old System Service Level	QR System 100% Service Level
$1 \le d1 \le 3$	0.3	100%	100%
$4 \le d1 \le 5$	0.4	80%	100%
$6 \le d1 \le 8$	0.3	60%	100%

By choosing the assortment of dresses closer to the season, ASSORT faces a lower forecast error. This enables the retailer to have fewer stockouts, the manufacturer to have higher revenue, and the customer to have a higher service level. All of this is accomplished without a decrease in retailer profits.

Note that providing a 100% service level in the old system would have decreased retailer pro?ts to \$ 170 because of the increased holding and salvage related costs.

Thus, QR allows the customer service level to be increased without decreasing retailer profits. This example thus illustrates the benefit of using a mixture of demands to represent demand for a product. As information is collected, the weights associated with the possible distributions can be adjusted, thus providing a convenient way to incorporate information into the forecasting process. It also shows the benefit of members of a channel working together to permit information to be incorporated into a decision-making process.

5. USING AN EXPONENTIAL SMOOTHING FORECASTING MODEL

We now illustrate the role of another classic demand-forecasting model—the exponential smoothing model. The exponential smoothing model works as follows: Given a demand forecast from previous periods and an observation this period, and a parameter α, the exponential smoothing model is that

$$\text{Demand forecast} = (\alpha \text{ observed demand}) + ((1 - \alpha) \text{ previous forecast})$$

We now illustrate the role of the exponential smoothing model in a supply chain. The model will allow us to understand how a supply chain reacts to partial information in adjusting its decisions. We start with a two-level supply chain involving a retailer and a manufacturer with a lead time L. The system operates as follows:

1. Retailer receives deliveries and then faces demand. Thus, the retailer has either too much or too little inventory at the end of the period.
2. Retailer updates demand forecast based on observed demand. This forecast follows an exponential smoothing model with parameter α.

Thus,

$$\text{Demand forecast} = (\alpha \text{ observed demand}) + [(1 - \alpha) \text{ previous forecast}]$$

3. Given a total lead time of L, the base stock level is set as $(L + 1)$ demand forecast
4. Retailer places an order to bring pipeline inventory level up to the base stock level.
5. Orders placed by the retailer reach the manufacturer after an information lead time.
6. The manufacturer receives deliveries and then faces demand. Thus, the manufacturer has either too much or too little inventory at the end of the period. Manufacturer shipments reach the retailer after a delivery lead time.
7. Manufacturer updates demand forecast based on observed demand. This forecast follows an exponential smoothing model with parameter α. Thus,

$$\text{Demand forecast} = (\alpha \text{ observed demand}) + [(1 - \alpha) \text{ previous forecast}]$$

8. Given a total lead time of L, the base stock level is set as $(L + 1)$ demand forecast
9. Manufacturer places an order to bring pipeline inventory level up to the base stock level.
10. Manufacturer orders are filled after a production lead time.

5.1. Sample Calculations for a Demand Forecasting Model

We illustrate the model in the earlier section for a four-level supply chain consisting of a retailer, a wholesaler, a distributor, and a manufacturer. There is a 4-period lead time between successive levels, consisting of 2 periods to transmit an order upstream and 2 periods to deliver downstream between successive levels. Manufacturing lead time is 2 periods. Demand is 4 units at the retailer for the first 10 periods and goes to 8 units from then on. In this section, we summarize the steps in generating values that reflect orders placed at each of the four stages of the beergame in response to demand changes. We will consider the case where each level uses $\alpha = 0.2$. The steps in calculating the order size are outlined below. We will assume that each level uses a base stock level = $(L + 1 + 2) \times$ demand forecast. The parameter L refers to the lead time for order delivery. The value 2 refers to 2 periods of safety stock (assumed).
Retailer—period 11:

1. Observe demand of 8 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 8) + [(1 - 0.2) \times 4] = 4.8$$

3. Base stock level = $(4 + 1 + 2) \times 4.8 = 33.6$.
4. Pipeline inventory level after satisfying period 11 demand = $28 - 8 = 20$
5. Order placed in period 11 = $33.6 - 20 = 13.6$.

Similarly, for period 12, the calculations are as follows:

1. Observe demand of 8 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 8) + [(1 - 0.2) \times 4.8] = 5.44$$

3. Base Stock Level = $(4 + 1 + 2) \times 5.44 = 38.08$.
4. Pipeline inventory level after satisfying period 11 demand = $33.6 - 8 = 25.6$.
5. Order placed in period 11 = $38.08 - 25.6 = 12.48$.

We now consider the orders placed by the wholesaler in period 13.
Wholesaler—period 13:

1. Observe demand of 13.6 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 13.6) + [(1 - 0.2) \times 4] = 5.92$$

3. Base stock level = $(4 + 1 + 2) \times 5.92 = 41.44$.
4. Pipeline inventory level after satisfying period 13 demand = $28 - 13.6 = 14.4$
5. Order placed in period 13 = $41.44 - 14.4 = 27.04$.

Similarly, for period 14, the calculations are as follows:

1. Observe demand of 12.48 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 12.48) + [(1 - 0.2) \times 5.92] = 7.232$$

3. Base stock level = $(4 + 1 + 2) \times 7.232 = 50.624$.
4. Pipeline inventory level after satisfying period 14 demand = $41.44 - 12.48 = 28.96$.
5. Order placed in period 14 = $50.624 - 28.96 = 21.664$.

We now consider the orders placed by the distributor in period 15.
Distributor—period 15:

1. Observe demand of 27.04 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 27.04) + [(1 - 0.2) \times 4] = 8.608$$

3. Base stock level = $(4 + 1 + 2) \times 8.608 = 60.256$.
4. Pipeline inventory level after satisfying period 15 demand = $28 - 27.04 = 0.96$.
5. Order placed in period 15 = $60.256 - 0.96 = 59.296$.

Similarly for period 16, the calculations are as follows:

1. Observe demand of 21.664 units.
2. Update demand Forecast as follows:

$$\text{Demand forecast} = (0.2 \times 21.664) + [(1 - 0.2) \times 8.608] = 11.2192$$

3. Base stock level = $(4 + 1 + 2) \times 11.2192 = 78.5344$.
4. Pipeline inventory level after satisfying period 16 demand = $60.256 - 21.664 = 38.592$.
5. Order placed in period 16 = $78.5344 - 38.592 = 39.9424$.

We now consider the orders placed by the factory in period 17.

Factory—period 17:

1. Observe demand of 59.296 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 59.296) + [(1 - 0.2) \times 4] = 15.0592$$

3. Base stock level = $(2 + 1 + 2) \times 15.0592 = 75.296$.
4. Pipeline inventory level after satisfying period 17 demand = $20 - 59.296 = -39.296$.
5. Order placed in period 17 = $75.296 - (-39.296) = 114.592$.

Similarly, for period 18, the calculations are as follows:

1. Observe demand of 39.9424 units.
2. Update demand forecast as follows:

$$\text{Demand forecast} = (0.2 \times 39.9424) + [(1 - 0.2) \times 15.0592] = 20.0358$$

3. Base stock level = $(2 + 1 + 2) \times 20.0358 = 100.179$.
4. Pipeline inventory level after satisfying period 16 demand = $75.296 - 39.9424 = 35.3536$.
5. Order placed in period 16 = $100.179 - 35.3536 = 64.8256$.

Note that the effect of the information and delivery lags between the stages is to increase a customer demand increase from 4 to 8 units in period 11 to a wholesale order change from 4 to 27.04 in period 13 to a distributor order increase from 4 to 59.296 in period 15 and a factory increase in its brewed cases from 4 to 114.592. Each decision reflects a rational choice given the parameters. This increase in variance or orders as we go up a supply chain is referred to as the bullwhip effect. Demand updating is only one possible reason for the bullwhip effect.

6. SUMMARY

In this chapter we have provided a quick review of four possible approaches to forecast demand and its use in planning. The constant demand model allows for a quick analysis of the effect of ordering costs in a system. The models of demand as a distribution permit details of lead time and demand uncertainty to be included. The modeling of demands as a mixture of distributions enables us to consider the role of information acquired over time. Finally, the exponential smoothing model shows how demand forecast updating can create large swings upstream in a supply chain.

ADDITIONAL READING

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