CHAPTER 90

Discounted Cash Flow Methods

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1. DEFINING ALTERNATIVE CAPITAL EXPENDITURE PROPOSALS

Expenditures are made with the anticipation that more will be gained in benefits from expenditures than they will cost. If those benefits were to accrue over more than one year, then they would be called *capital expenditures*. Buildings, machinery, equipment, software, or research and development would be typical capital expenditures. When evaluating an expenditure proposal, the primary question is whether or not the money invested will generate a worthwhile annual flow of benefits.

To determine the economic feasibility of a proposed expenditure, it is necessary to define and relate four variables:

1. The economic life of the proposed alternative
2. The amount and pattern of the expenditures and the time period over which they will occur
3. The amount and pattern of the benefits and the time period over which they will occur
4. The interest rate that will appropriately represent the capital structure of the firm and the risk involved

The recognition of risk is especially important because all estimates and actual outcomes are subject to variability.

1.1. Determining the Economic Life of an Asset

The economic life of a capital expenditure is the number of years that this asset will make a positive economic contribution to the firm. The equipment used for a project will be retired when management observes (1) unsatisfactory functional characteristics, such as wear and deterioration; (2) unsatisfactory economic characteristics: it costs more than it earns; (3) termination of need: no one wants to pay for its output anymore; or (4) obsolescence due to changes in policy regulations or technology. The economic life is estimated by considering when the preceding conditions will occur. Such estimations can be facilitated by using data from historical service records of comparable assets, either within the firm or from other sources required to keep accurate records. Utilities or government agencies will often make such data available. Economic service lives can vary significantly. Telecommunications and computer equipment can be obsolete in 18 months, while aircraft and power generators can have life cycles of 30 years. Estimation of economic service lives can be complicated by the potential of legislative or regulatory action terminating or significantly modifying the use of an asset. Finally, economic service lives are not the same as tax lives, which are established for tax strategies, not operational plans.

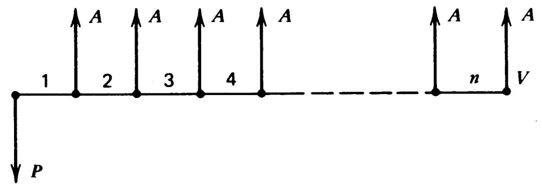
1.2. Developing Cash Flow Profiles

The cash flow profile should include all cash items anticipated to flow into or out of a project. Each cash flow item must be identified specifically according to the time at which this flow occurs. The major items included in an engineering economy study are:

1. First cost, P, is the sum of the costs of engineering, construction, purchasing, installation, and so on, to bring the asset into service. This cost is considered to occur at the installation of the project, $t = 0$. However, if the project requires a number of years to construct, then the major expenditures to be made each year during construction should be identified at the time the cash flow occurs.
2. Salvage, V, is the net sum realized from the disposal of the project or asset at the termination of its useful economic life.
3. Income (revenues), other benefits, and expenses are identified according to their type of flow over time.
   - **(a)** Periodic cash flow items occur at specific times, such as an overhaul of an engine every three years.
   - **(b)** Uniform series revenues or expenses are equal periodic amounts such as property taxes, leasing costs, interest on debt, etc.
   - **(c)** Continuous flow of revenues or expenses occur continuously and uniformly over the life of the project, such as the savings realized from a new assembly technique or the revenue stream from evenly divided alphabetical billing.

1.2.1. Traditional Engineering Economy Cash Flow Profiles

To ensure that all positive and negative cash flow items are included in the analysis and to better visualize these cash flows, it is useful to construct a *cash flow diagram* or *table.* A generalized illustration of a cash flow diagram for n periods of time is shown below. In the diagram, P is the present value of the first cost; A is the value of an annuity, a uniform series of equal cash savings occurring at the end of each period; and V is the salvage value received for the asset upon the termination of the project.



Convention in engineering economy studies dictates that all discrete cash flows are considered to occur at the end of the period, which is normally at the end of each year. However, the period may be quarterly, monthly, or even daily.

To describe a specific cash flow, consider the following discrete items, with the cash flow developed using a tabular format.

End of Year	Receipts	Disbursements	F_{jt}
0	\$ 0	\$−6000	\$6000
1	2000	−500	1500
2	3500	−1000	2500
3	5000	−2000	3000
4	5000	−1000	4000

In this table F_{jt} = net cash flow for investment j at time t. If $F_{jt} < 0$, then F_{jt} represents a net cash disbursement of expense. If $F_{jt} > 0$, then F_{jt} represents a net gain or revenue.

1.2.2. Computer Spreadsheet Cash Flow Profiles

Spreadsheets, such as Microsoft Excel, have largely replaced the traditional tabular cash flow profiles and calculators for setting up and solving cash flow problems. The spreadsheet format has become the business standard for collecting, analyzing, and displaying data. Entering the previous data into an Excel format would result in the following table.

	A	B	C	D
1	End of	Receipts	Disbursements	Fjt
2	Year	\$	\$	\$
3	0	—	(6,000)	(6,000)
4	1	2,000	(500)	1,500
5	2	3,500	(1,000)	2,500
6	3	5,000	(2,000)	3,000
7	4	5,000	(1,000)	4,000

The calculation of F_{j0} would be accomplished as before, using the Excel commands = B3 + C3. Then the remaining F_{jt} values would be found by duplicating the formula for all subsequent F_{jt} cells.

The principal value of spreadsheets, other than their universal acceptance and use over the past decade, is the ability to display efficiently and use individual estimates of costs and benefits for each period over the life of the project, reflecting anticipated variability rather than a uniform average value or an approximate mathematical series. This will be discussed as each discounted cash flow method is presented. The principal disadvantage of a spreadsheet is that computational errors are hidden, even though an ''audit tool'' exists. However, the problem of quality assurance is not a new one, and the engineer must always be alert to the prevention and elimination of errors.

1.3. Selecting the Interest Rate

When analyzing the feasibility of any investment, two principles must be observed. First, the value of a sum of money is a function of the time span between the base point of reference and the date an expenditure or the receipt of revenue will occur. For example, the value of $100 one year from today is not the same as that of $100 today. Consequently, money has a time value, which is measured by the interest rate. Second, because of this time value of money, all comparisons of receipts and disbursements must be made at a common specific point of time. Compound interest factors provide the means of shifting cash flows to this common time.

1.3.1. Weighted Average Cost of Capital

The minimum interest rate a company should earn on its invested capital is determined by the capital structure of the firm. The firm must earn an adequate return both to support the long-term debt and to compensate the stockholders adequately for their equity investment. This minimum interest rate, C_C, is calculated using the capital asset pricing model and is often referred to as the weighted cost of capital.

$$C_C = C_D + C_E$$

where C_C = weighted cost of capital, %
C_D = weighted cost of long-term debt, %
C_E = weighted cost of equity, %

The weighted cost of long-term debt, C_D, can be determined by

$$C_D = k_D \times W_{LTD}$$

where k_D is the return necessary to support debt, stated as a percentage and W_{LTD} is the percentage of the firm's capital structure represented by long-term debt. The values of k_D and W_{LTD} are determined by examining the financial statements of the firm. The value of k_D would represent the average interest rate charged on the firm's long-term obligations.

$$W_{\mathrm{LTD}} = \frac{\text{long-term debt}}{\text{long-term debt} + \text{equity}} = \frac{\mathrm{LTD}}{\mathrm{LTD} + E}$$

The weighted cost of equity, C_E, can be determined by

$$C_E = k_E \times W_E$$

where k_E is the return necessary to support the firm's equity stated as a percentage and W_E is the percentage of the firm's capital structure represented by equity. As would be expected, W_E is the complement of W_{LTD} and would be calculated by

$$W_E = \frac{\text{equity}}{\text{long-term debt} + \text{equity}} = \frac{E}{\mathrm{LTD} + E}$$

The before-tax rate of return required to support the firm's equity structure would reflect both the risk attributable to equity instruments and the volatility associated with the firm's stock price relative to the equity market average. The rate of return, k_E, required to support the equity portion of the firm's capital structure is

$$k_E = \frac{(\text{risk-free rate}) + (\text{risk premium})}{1 - \text{tax rate}}$$

$$\mathrm{k}_E = \frac{r^* + \beta(R_m - r^*)}{1 - t}$$

where r^* = risk-free rate of return
R_m = risk attributable to the general equity market
t = effective tax rate
β = systematic risk of a stock due to underlying movements in security prices

The value of the risk-free rate of return, r^*, can be estimated as being equal to the interest rate paid on U.S. Treasury Bills or other guaranteed savings instruments, approximately 6% in mid-2000. The interest rate equivalent to the general equity market, R_m, was found to be approximately 9% by Fisher and Lorie (1968). Over the past 30 years this figure has risen to 11%. The effective tax rate, t, can be calculated from data obtained from the annual consolidated statement of earnings in the firm's annual report. Values for the systematic risk of a stock due to underlying movements in security prices, β, can be obtained for the majority of firms traded on major stock exchanges from references such as *Value Line Investment Survey.*

As an example of determining the minimum rate of return that must be earned to maintain the capital structure of the firm, consider the following calculation for Johnson & Johnson using the data contained in its*1999 Annual Report.* The data are in $million.

$$t = \frac{\text{taxes paid}}{\text{profit before taxes}} = \frac{\$\ 1{,}586}{\$\ 5{,}753} = 27.6\%$$

$$k_E = \frac{r^* + \beta(R_M - r^*)}{1 - t} = \frac{6 + 0.8(11 - 6)}{1 - 0.276} = 13.8\%$$

$$W_E = \frac{E}{\text{LTD} + E} = \frac{\$16{,}231}{\$18{,}689} = 0.8675$$

Cost of equity

$$C_E = W_E \times k_E = .8675 \text{ x } 13.8 = 11.97\%$$

$$W_D = \frac{\text{LTD}}{\text{LTD} + E} = \frac{\$\ 2{,}476}{\$18{,}689} = 0.1325$$

Cost of long-term debt

$$C_D = W_D \times k_D = 0.1325 \times 6.42 = 0.85\%$$

Weighted cost of capital

$$C_C = C_D + C_E = 11.97\% + 0.85\% = 12.82\%$$

1.3.2. Factors Impacting the Selection of an Interest Rate

The preceding calculation will determine the *minimum* interest rate a company should earn on its invested capital to preserve its capital structure. Any lesser rate would erode the firm's reserves. Most companies use a "hurdle rate" of approximately two times the weighted cost of capital. Since research has shown that the average project returns approximately half of the original estimate, using a higher rate of return is a good policy. However, a firm may adopt differing interest rates in order to establish strategic investment guidelines for different types of business or functions within the business. For example, an energy company might require quite different interest rate hurdles between marketing and exploration and production, or a technology firm could encourage research and development expenditures by using a rate in a feasibility study which would be a fraction of that applied to a "cash cow" operation. For example, consider projects for:

1. *Safety, quality, or legal requirement:* A specific rate of return is usually not required. The immediate need for the project or the lack of an alternative solution may preclude an earnings test for such an expenditure.
2. *Increased profit:*
   - **(a)** *Cost reduction:* Projects with a rate of return of at least 25% would qualify for consideration.
   - **(b)** *Existing product:* Projects to increase the production capacity, flexibility, or delivery speed for an existing product would be considered if the return were at least 25%.
   - **(c)** *New product line:* Because of the greater risk of demand, technology, and life cycle, the rate of return should be at least 50% over the projected life of the project.
3. *Country risk:* When operating offshore, the stability of a country's government, labor force, and infrastructure operations can vary the risk to the firm. The hurdle rate should be varied accordingly.

The interest rate used to evaluate capital investment alternatives should be greater than the weighted cost of capital. However, the specific rate to be used in capital budgeting and project feasibility analysis must be a management decision, depending on the type of activity, the risk, and the opportunity costs.

2. USING INTEREST FACTORS TO FIND EQUIVALENT MONETARY AMOUNTS

Equivalence is equating monetary values occurring at different points in time. Comparisons of financial alternatives must be made among equivalent units. The Interest rate provides the mechanism for converting a cash flow at one specific time into an equivalent cash flow at another time. This conversion is facilitated by using either the Excel spreadsheet functions in Table 1 or the interest tables at the end of the chapter.

2.1. Simple Interest

The simple interest payment each year, i_n, is found by multiplying the interest rate, i, times the invested capital, or principal, P. Thus, $i_n = Pi$. After any n time periods, the accumulated value of money owed under simple interest, F_n, would be

$$F_n = P(1 + i_n)$$

For example, \$100 invested now at 9% simple interest for eight years would yield

$$F_8 = \$100[1 + 0.09(8)] = \$172$$

Simple interest forgoes the money earned by annual compounding and is rarely used in engineering economy analyses.

2.2. Compound Interest

The interest payment each year, or each period, is found by multiplying the interest rate by the accumulated value of money, both principal and interest.

TABLE 1 Compound Interest Factors: Discrete Cash Flow, Discrete Compounding

To Find	Given	Name of Factor	Algebraic	Format: Functional	Format: Excel
F	P	Compound amount factor (single payment)	$(1 + i)^n$	$(F/P, i\%, N)$	FV (rate, nper, pmt, pv, type)
P	F	Present worth factor (single payment)	$(1 + i)^{-n}$	$(P/F, i\%, N)$	PV(rate, nper, pmt, fv, type)
F	A	Compound amount factor (uniform series)	$\dfrac{(1 + i)^n - 1}{i}$	$(F/A, i\%, N)$	FV(rate, nper, pmt, pv, type)
A	F	Sinking fund factor	$\dfrac{i}{(1 + i)^n - 1}$	$(A/F, i\%, N)$	PMT(rate, nper, pv, fv, type)
A	P	Capital recovery factor	$\dfrac{i(1 + i)^n}{(1 + i)^n - 1}$	$(A/P, i\%, N)$	PMT(rate, nper, pv, fv, type)
P	A	Present worth factor (uniform series)	$\dfrac{(1 + i)^n - 1}{i(1 + i)^n}$	$(P/A, i\%, N)$	PV(ratte, nper, pmt, fv, type)
A	G	Arithmetic gradient conversion factor to uniform series	$\dfrac{(1 + i)^n - (1 + ni)}{(1 + i)^n - 1}$	$(A/G, i\%, N)$	
P	G	Arithmetic gradient conversion factor to present value	$\dfrac{1 - (1 + ni)(1 + i)^{-n}}{i^2}$	$(P/G, i\%, N)$	

End of Period (EOP)	Accumulated BOP Value of Amount Owed (1)	Interest for Period (2)	Amount Owed or Value Accumulated End of Period (3) = (1) + (2)
0	0		P
1	P	Pi	$P + Pi = P(1 + i)^2$
2	$P(1 + i)^1$	$[P(1 + i)^1]i$	$P(1 + i)^1 + P(1 + i) = P(1 + i)^2$
3	$P(1 + i)^2$	$[P(1 + i)^2]i$	$P(1 + i)^2 + P(1 + i)^2\, i = P(1 + i)^3$
. . .	. . .	. . .	. . .

Consequently, the value for an amount P invested for n periods at i rate of interest would be

$$F_n = P(1 + i)^n$$

For example, \$100 invested now at 9% compound interest for eight years would yield

$$F_8 = \$100(1 + 0.09)^8 = 100(1.9926) = \$199.26$$

2.3. Nominal and Effective Interest Rates

For many financial feasibility studies it is appropriate to consider interest periods of one year. However, financial agreements may call for interest to be compounded or paid more frequently, say quarterly, monthly, or even daily. Interest rates associated with more frequent compounding, say of quarterly interest periods, are usually stated as "8% compounded quarterly."

The nominal interest rate, r, is expressed as an annual rate, without considering the impact of any compounding per period during the year. It is obtained by multiplying the periodic interest rate, i, by the number of periods per year, m.

$$r = im$$

For example, the nominal annual rate would be 8% with a 2% quarterly rate.

The effective annual interest rate is the true or actual annual interest rate, taking into account the monetary gain obtained by compounding the invested capital each period during the year. The effective interest rate per year, i_a, is

$$i_a = (1 + i)^m - 1 \text{ when } m < \infty$$

For example when the effective interest rate is 2% per month, the nominal interest rate per year is

$$r = (0.02)(12) = 24\%$$

and the effective interest rate per year is

$$i_a = (1 + 0.02)^{12} = 26.8\%$$

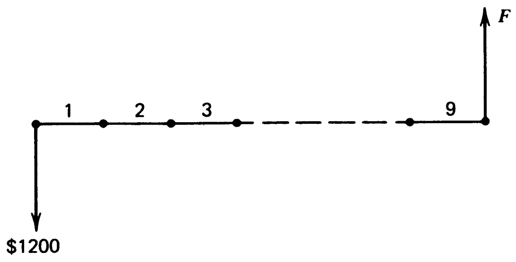
2.4. Compound Interest Factors: Discrete Cash Flow, Discrete Compounding

The compound interest factors described in this section are used for discrete cash flows compounded discretely at the end of each interest period. All of these factors can be found in Table 1, including algebraic and functional formats and the Excel functions. The numerical values for each factor for selected interest rates can be found in the tables at the end of this chapter. Complete tables can be found in the Additional Reading at the end of the chapter. The notation used in this chapter is

i = effective interest rate.
n = number of compounding periods.
A = end-of-period cash flows (or equivalent end-of-period vales) in a uniform series continuing for a specified number of periods (the letter A implies annual or an annuity).
F = future sum of money (the letter F implies future or equivalent future value).
P = present sum of money (the letter P implies present or equivalent present value).

2.4.1. *Compound Amount Factor (Single Payment)*

This factor finds the equivalent future worth, F, of a present investment, P, held for n periods at a rate of i interest. For example, what is the value in nine years of $1200 invested now at 10% interest?



Algebraic format

$$\begin{aligned} F &= P(1 + i)^n \\ &= \$1200(1 + 0.10)^9 \\ &= \$1200(2.3579) \\ F &= \$2829 \end{aligned}$$

Functional format (note that throughout this chapter only the functional format will be used)

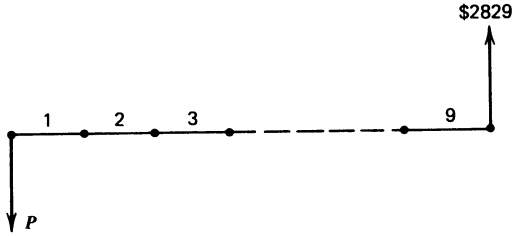
$$\begin{aligned} F &= P(F/P, i\%, N) \\ &= P(F/P, 10\%, 9) \\ &= \$1200(2.3579) \\ F &= \$2{,}829 \end{aligned}$$

Excel format
Highlight the location where you want the solution. Click on the f_x button on the Standard Tool Bar. Follow the Excel instructions to enter the appropriate variables.

F = FV (rate, nper, pmt, pv, type)
= FV (0.10, 9, 0, A1, 0)
pmt = no intervening payments
pv = A1, location of the present value payment of $1,200 in the spreadsheet. Note that from the cash flow diagram, pv should be negative if F is to be positive.
Type = 0 for end of the period payments. Excel default. 0 for beginning of the period payments
F = $2830

2.4.2. *Present Worth Factor (Single Payment)*

This factor finds the equivalent present value, P, of a single future cash flow, F, occurring at n periods in the future when the interest rate is $i\%$ per period. Note that this factor is the reciprocal of the compound amount factor (single payment). For example, what amount would you have to invest now to yield $2829 in nine years if the interest rate were 10%?



$$\begin{aligned} P &= F(P/F, 10\%, 9) \\ &= \$2829\ (0.4241) \\ &= \$1200 \end{aligned}$$

Functional format

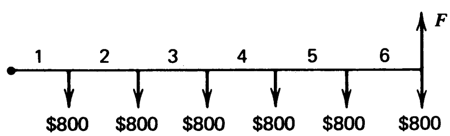
$$\begin{aligned} P &= F(P/F, \text{i}\%, N) \\ &= F(P/F, 10\%, 9) \\ &= \$2839(0.4241) \\ &= \$1200. \end{aligned}$$

Excel format

$$\begin{aligned} P &= \text{PV(rate, nper, pmt, pv, type)} \\ &= \text{FV}(0.10, 9, 0, \$1200, 0) \\ &= \$(1200) \end{aligned}$$

2.4.3. *Compound Amount Factor (Uniform Series)*

This factor finds the equivalent future value, F, of the accumulation of a uniform series of equal annual payments, A, occurring over n periods at i rate of interest per period. For example, what would be the future worth of an annual year-end cash flow of \$800 for six years at 12% interest per year?



$$\begin{aligned} \mathbf{F} &= \mathbf{A(F/A, 12\%, 6)} \\ &= \mathbf{\$800\ (12.2997)} \\ &= \mathbf{\$9840} \end{aligned}$$

Functional format

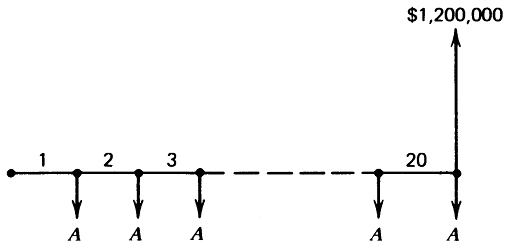
$$\begin{aligned} F &= A(F/A, \text{i}\%, N) \\ &= A(F/A, 12\%, 6) \\ &= \$800(8.1152) \\ &= \$6492 \end{aligned}$$

Excel format

$$\begin{aligned} F &= \text{FV(rate, nper, pmt, pv, type)} \\ &= \text{FV}(0.10, 6, -800, 0, 0) \\ &= \$6492 \end{aligned}$$

2.4.4. *Sinking Fund Factor*

This factor determines how much must be deposited each period in a uniform series, A, occurring over n periods at i rate of interest per period to yield *a specified* future sum, F. For example, if a \$1.2 million bond issue is to be retired at the end of 20 years, how much must be deposited annually into a sinking fund at 10% interest per year?



$$
\begin{aligned}
A &= F(A/F, 7\%, 20) \\
&= \$1{,}200{,}000\ (0.0244) \\
&= \$29{,}280
\end{aligned}
$$

Functional format

$$
\begin{aligned}
A &= F(A/F, \text{i}\%, N) \\
&= F(A/F, 10\%, 20) \\
&= \$1{,}200{,}000(0.01746) \\
&= \$20{,}952
\end{aligned}
$$

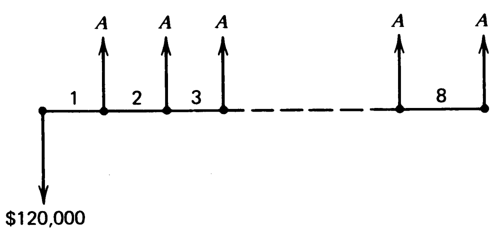
Excel format

$$
\begin{aligned}
A &= \text{PMT(rate, nper, pv, fv, type)} \\
&= \text{PMT}(0.10, 20, 0, \$1{,}200{,}000, 0) \\
&= \$20{,}952
\end{aligned}
$$

This factor was historically used to find the required annual payments that must be made into a "sinking fund" to retire a bond issue by a particular date.

2.4.5. Capital Recovery Factor

This factor finds an annuity, or uniform series of payments, over n periods at $i\%$ interest per period that is equivalent to a present value, P. For example, what savings in annual manufacturing costs over an eight-year period would justify the purchase of a $120,000 machine if a firm's minimum attractive rate of return (MARR) were 20%?



$$
\begin{aligned}
A &= P(A/P, 25\%, 8) \\
&= \$120{,}000\ (0.3004) \\
&= \$36{,}048
\end{aligned}
$$

Functional format

$$\begin{aligned} A &= P(A/P, \text{i}\%, N) \\ &= P(A/P, 20\%, 8) \\ &= \$120{,}000(0.2606) \\ &= \$31{,}272 \end{aligned}$$

Excel format

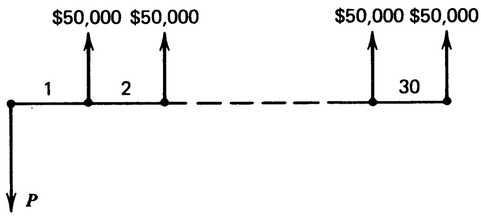
$$\begin{aligned} A &= \text{PMT(rate, nper, pv, fv, type)} \\ &= \text{PMT}(0.20, 8, \$120{,}000, 0) \\ &= \$31{,}273 \end{aligned}$$

This problem of finding the revenue that must be generated each period to justify a capital expenditure is one of the most common facing the engineer. The analyst should be reminded that there are two elements of the capital recovery factor. The first is the recovery of the $120,000 original investment, and the second is the necessity of earning 20% on the capital invested over the life of the project. In this case,

$$\begin{aligned} 8 \text{ payments} \times \$31{,}273 &= \$250{,}184 \text{ cash flow} \\ &= \underline{(\$120{,}000)} \text{ return of original investment} \\ &= \$130{,}184 \text{ interest paid on invested capital} \end{aligned}$$

2.4.6. *Present Worth Factor (Uniform Series)*

This factor finds the equivalent present value, P, of a series of end-of-period payments, a, for n periods at $i\%$ interest per period. For example, a donor has offered to give the Hospital Authority a new wing for treatment of allergies. However, since the operation and maintenance of the existing facility requires all of the funds available under the authority's taxing limits, how much additional endowment would be required to provide $50,000/year over the 30-year economic life of the structure. The endowment is expected to earn 10% on its invested capital.



$$\begin{aligned} P &= \$50{,}000\ (P/A, 7\%, 30) \\ &= \$50{,}000\ (12{,}409) \\ &= \$620{,}450 \end{aligned}$$

Functional format

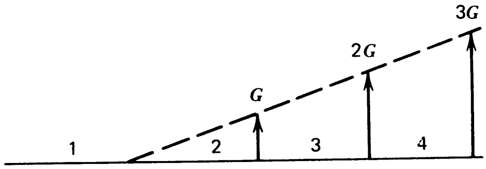
$$\begin{aligned} P &= A(P/A, i\%, N) \\ &= \$50{,}000(P/A, 10\%, 30) \\ &= \$50{,}000(9.4269) \\ &= \$471{,}345 \end{aligned}$$

Excel format

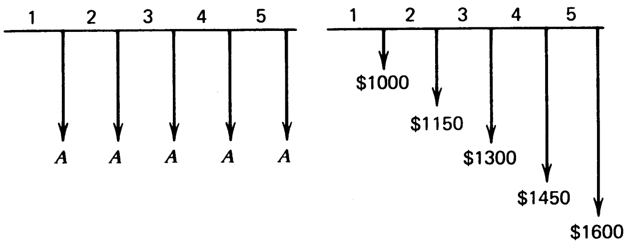
$$\begin{aligned} P &= \text{PV(rate, nper, pmt, fv, type)} \\ &= \text{PV}(0.10, 30, -50000, 0, 0) \\ &= \$471{,}345 \end{aligned}$$

2.4.7. *Arithmetic Gradient Conversion Factor (to a Uniform Series)*

Many times annual payments do not occur in equal amounts. Inflation causes annual increases in operating costs, and maintenance costs often increase with the age of the equipment. If a series of payments increases by an equal amount or gradient, G, each year, then a special compound interest factor can be used to reduce the gradient series to an equivalent equal-payment series. The following illustration shows a four-period gradient series that increases by G each period.



The arithmetic gradient conversion factor (to uniform series) is used when it is necessary to convert a gradient series into a uniform series of equal payments. For example, what would be the equal annual series, A, that would have the same net present value (i.e., be equivalent) at 20% interest per year to a five-year gradient series that started at \$1000 for the first year and increased \$150 every year thereafter?



$$\begin{aligned} A &= A_g + G(A/G, 20\%, 5) \\ &= \$1000 + \$150\,(1.6405) \\ &= \$1246 \end{aligned}$$

Functional format

$$\begin{aligned} A &= A_g + G(A/G, i\%, N) \\ &= A_g + G(A/G, 20\%, 5) \\ &= \$1000 + \$150(1.6405) \\ &= \$1246 \end{aligned}$$

where A_g is the uniform base value of the gradient series. In the previous four-period gradient series illustration, $A_g = 0$.

Excel format

It is necessary to solve this problem in two steps

1. Net present value of the series of cash flows:

$$\begin{aligned} \text{NPV} &= (\text{rate, value 1, value 2, value 3, value 4, value 5}) \\ &= (0.20, 1000, 1150, 1300, 1450, 1600) \\ &= \$3762 \end{aligned}$$

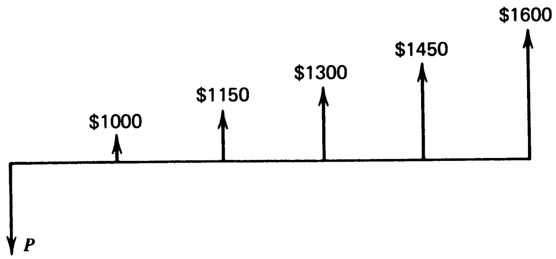
2. Equal annual cash flows equivalent to the series of cash flows:

$$\begin{aligned} A &= \text{PMT (rate, nper, pv, fv, type)} \\ &= \text{PMT (0.20, 5, 3726, 0, 0)} \\ &= \$1246 \end{aligned}$$

Excel Spreadsheet can calculate equivalent values from a series of cash flows regardless of whether they follow a particular gradient series, such as in this example, or are just best estimates of the cash flows for each period. This is a powerful feature eclipsing the requirement of uniform cash flow series necessary in traditional engineering economy models.

2.4.8. Arithmetic Gradient Conversion Factor (to a Present Value)

This factor converts a series of cash amounts increasing by a gradient value, G, each period to an equivalent present value at $i\%$ interest per period. For example a machine will require \$1000 maintenance in the first year of its five-year operating life. Further, the cost of maintenance will increase by \$150 each year. What is the present worth of this series of maintenance costs if the firm's minimum attractive rate of return (MARR) is 20%?



$$\begin{aligned} P &= A(P/A, 20\%, 5) + G(P/G, 20\%, 5) \\ &= \$1000\ (2.9906) + \$150\ (4.9061) \\ &= \$3727 \end{aligned}$$

Functional format

$$\begin{aligned} P &= A(P/A, i\%, N) + G(P/G, i\%, N) \\ &= A(P/A, 20\%, 5) + \$150(P/G, 20\%, 5) \\ &= \$1000(2.9906) + \$150(4.9061) \\ &= \$3727 \end{aligned}$$

Excel format—referring to the previous example

$$\begin{aligned} \text{NPV} &= \text{(rate, value 1, value 2, value 3, value 4, value 5)} \\ &= (0.20, 1000, 1150, 1300, 1450, 1600) \\ &= \$3726 \end{aligned}$$

2.5. Compound Interest Factors: Discrete Cash Flow, Continuous Compounding

Monetary institutions and industrial firms alike strive to keep their funds working at all times. Techniques of cash management, such as electronic funds transfer, provide the potential to put to work cash receipts immediately. This has shortened the compounding periods to the point where the use of continuous compounding is the most appropriate cash flow model. In the concept of discrete cash flows with continuous compounding, it is assumed that the cash flows occur once per year but that compounding is continuous throughout the year. Thus, if

$$\begin{aligned} r &= \text{nominal interest rate per year} \\ M &= \text{number of compounding periods per year} \\ N &= \text{number of years} \end{aligned}$$

then at the end of one year one unit of principal will equal

$$\left[1 + \left(\frac{r}{n}\right)\right]^{M} \tag{1}$$

Letting $k = M/r$, Eq. (1) becomes

$$\left[1 + \frac{1}{k}\right]^{rk} = \left[\left(1 + \frac{1}{k}\right]^{k}\right]^{r} \tag{2}$$

The limit of $(1 + 1/k)^k$ as k approaches infinity is e. Thus, Eq. (2) can be written as e^r, and the single-payment continuous compounding amount factor at $r\%$ nominal annual interest rate for N years is e^{rN}. Also, since e^{rN} (for continuous compounding) corresponds to $(1 + i)^N$ for discrete compounding,

$$e^r = 1 + i$$

or

$$i = e^r - 1$$

By the use of this relationship, the compound interest factors for discrete cash flows compound continuously shown in Table 1 can be derived from the discrete compounding factors in Table 2.

TABLE 2 Compound Interest Factors: Discrete Cash Flow, Continuous Compounding

To Find	Given	Name of Factor	Format: Algebraic	Format: Functional
F	P	Continuous compounding Compound amount factor (single payment)	e^{rn}	$(F/P, r\%, N)$
P	F	Continuous compounding Present worth factor (single payment)	e^{-rn}	$(P/F, r\%, N)$
F	A	Continuous compounding Compound amount factor (uniform series)	$\frac{e^{rn} - 1}{e^r - 1}$	$(F/A, r\%, N)$
A	F	Continuous compounding Sinking fund factor	$\frac{e^r - 1}{e^{rn} - 1}$	$(A/F, r\%, N)$
A	P	Continuous compounding Capital recovery factor	$\frac{e^{rn}(e^r - 1)}{e^{rn} - 1}$	$(A/P, r\%, N)$
P	A	Continuous compounding Present worth factor (uniform series)	$\frac{e^{rn} - 1}{e^{rn}(e^r - 1)}$	$(P/A, r\%, N)$
A	G	Continuous compounding Arithmetic gradient conversion factor (to uniform series)	$\frac{1}{e^r - 1} - \frac{n}{e^{rn} - 1}$	$(A/G, r\%, N)$
P	G	Continuous compounding Arithmetic gradient conversion factor (to present value)	$\frac{e^{rn} - 1 - n(e^r - 1)}{e^{rn}(e^r - 1)^2}$	$(P/G, r\%, N)$
P	A_1, c	Continuous compounding Geometric gradient Conversion fctor (to present value)	$\frac{1 - e^{(C-r)n}}{e^r - e^c}$	$(P/A, r\%, c\%, N)$ $r \neq c$
F	A_1, c	Continuous compounding Geometric gradient Conversion factor (to uniform series)	$\frac{e^{rn} - e^{cn}}{e^r - e^c}$	$(F/A, r\%, c\%, N)$ $r \neq c$

2.5.1. Continuous Compounding Compound Amount Factor (Single Payment)

This factor is used to find the equivalent future worth, F, of a present value, P, when the interest is continuously compounded at the nominal annual rate of $r\%$. For example, consider the problem of finding the future worth in six years of \$5,000 invested now at 9% nominal interest rate compounded continuously.

$$F = Pe^{rn}$$

Functional format

$$\begin{aligned} F &= P(F/P, r\%, N) \\ &= P(F/P, 9\%, 6) \\ &= \$5000(1.7160) \\ &= \$8580 \end{aligned}$$

Note that the only difference between continuous compounding and discrete compounding in finding equivalent values of F, P, A, and G is the interest factor used (r, the nominal annual interest rate). Consequently, to solve discrete cash flow continuous compounding problems, use the same procedures illustrated for discrete compounding with the functional format.

2.5.2. Continuous Compounding Geometric Conversion Factor (to Present Value)

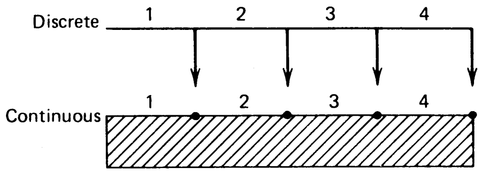
When conditions such as inflation cause a gradient that increases at a fixed percent per period, c, then geometric conversion factors should be used. For example, find the present worth of a series of costs that increase 10% per year from the initial first-year cost of \$1000 for five years when the firm's MARR is 15%.

Functional format

$$\begin{aligned} P &= A(P/A, r\%, c\%, N) \\ &= A(P/A, 15\%, 10\%, 5) \\ &= \$1000\ (3.9037) \\ &= \$3904 \end{aligned}$$

2.6. Compound Interest Factors: Continuous Uniform Cash Flows, Continuous Compounding

Many cash flows that engineers must consider can be assumed to occur continuously, such as accounts receivable, the cost savings resulting from productivity improvements, or the costs of carrying inventories. The following cash flow diagrams serve to illustrate the differences between discrete and continuous cash flows.



Again, as with discrete cash flows, to solve for one variable given another, it is only necessary to select the proper interest factor for continuous cash flow, continuous compounding. A listing of these factors can be found in Table 3.

As an example, consider the equivalent future worth of a uniform series of continuous cash flows totaling \$2000 per year for 10 years compounded continuously at 15% nominal annual rate of interest.

$$\begin{aligned} F &= \overline{\mathbf{A}}(F/\overline{\mathbf{A}}, r\%, N) \\ &= \$2000\ (F/\overline{\mathbf{A}}, 15\%, 10) \\ &= \$2000\ (23.2113) \\ &= \$46{,}423 \end{aligned}$$

TABLE 3 Compound Interest Factors: Continuous, Uniform Cash Flow, Continuous Compounding

To Find	Given	Name of Factor	Format: Algebraic	Format: Functional
F	$\bar{A}$	Continuous compounding Compound amount factor (continuous, uniform payments)	$\frac{e^{rn} - 1}{r}$	$(F/\bar{A}, r\%, N)$
$\bar{A}$	F	Continuous compounding Sinking fund factor (continuous, uniform payments)	$\frac{r}{e^{rn} - 1}$	$(\bar{A}/F, r\%, N)$
$\bar{A}$	P	Continuous compounding Capital recovery factor (continuous, uniform payments)	$\frac{re^{rn}}{e^{rn} - 1}$	$(\bar{A}/P, r\%, N)$
P	$\bar{A}$	Continuous compounding Present worth factor (continuous, uniform payments)	$\frac{e^{rn} - 1}{re^{rn}}$	$(P/\bar{A}, r\%, N)$

3. COMPARING ALTERNATIVES

In comparing alternatives to meet a need or an objective, plans (1) should provide the same quality and quantity (or level) of service and (2) should provide that service over the same period of time. Competing plans should be alternative ways to accomplish the same end. Any differences in expected revenue or other benefits must be credited to the plan providing the additional services. The analysis is concerned only with the *differences* in the cash flows between the alternatives.

3.1. Present Worth

The present worth method compares all of a project's estimated expenditures to all of its estimated revenues and other benefits at a reference time called the "present" ($t = 0$). For a particular interest rate, if the present value of the revenues and other benefits exceeds the present value of the expenses, the project is considered acceptable. The present worth of alternative j with cash flows that last of n periods of time at $i\%$ interest per period is

$$\text{PW } (i)_j = \sum_{t=1}^{N} F_{jt} \ (P/F, i\%, t)$$

If two or more alternatives are being compared, the alternative with the greatest present worth or net present value is recommended. To compare alternatives fairly using the present worth method, it is necessary that all alternatives have a common retirement date.

For example, two pieces of equipment are being considered by a hospital to perform a particular service. Brand A will cost \$30,000 and will have an annual operating and maintenance cost of \$5000 over its eight-year economic life with a salvage value of \$3000. Brand B will cost \$15,000, will have an annual operating and maintenance cost of \$8000 over the first three years and \$10,000 over the last three years of its economic life, and will have a negligible salvage value. Which brand of equipment would you recommend, using the present worth comparison with an interest rate of 10%/ year?

Cash Flow Item	Brand A	Brand B
First cost	\$(30,000)	\$(15,000)
Operating and maintenance		
A: = \$(5,000) $(P/A,10\%,6)$		
= \$(5,000) (4.3553)	(21,776)	
B: = \$(8,000)$(P/A,10\%,3)$		
= \$(8,000)(2.4869)	(19,895)	

Cash Flow Item	Brand A	Brand B
= \$(10,000)[(P/A,10%,3)(P/F,10%,3)]		
= \$(10,000)[(2.4869)(0.7531)]	\$(18,684)	
Salvage value		
A: = \$3,000(P/F,10%,6)		
= \$3,000(0.5645)	1,694	
B:		0
Present worth	\$(50,082)	\$(53,579)

Brand A would be the recommended alternative since the present worth of its total cost is smaller or its present worth is greater.

3.2. Annual Worth

The annual worth method converts all cash flows to an equivalent uniform series of equal annual payments. As in the present worth method, if the annual worth of the revenues is greater than the annual worth of the costs for the specified interest rate, then the project is acceptable. The annual worth of alternative j and i percent rate of interest per period, which lasts for n periods, is

$$\text{AW}(i)_j = \text{PW}(i)_j(A/P,\, i\%,\, N)$$

It is usually necessary to calculate the present worth of all cash flows, PW(i), first since these cash flows are rarely a uniform series that can be summed directly to find AW(i).

If two or more alternatives are being compared, the alternative with the greatest annual worth (cash receipts are positive and disbursements are negative) is the recommended alternative.

If you must compare alternatives with differing economic lives, the annual worth method is preferred when the ‘‘repeatability assumption’’ is valid for the analysis. (See Section 4 for a detailed discussion of the comparison of alternatives with unequal service lives.) If this assumption is valid, then the annual worth at the time of renewal of the asset is exactly the same as before. Therefore, you are actually comparing the annual worth of two infinite series.

Using the preceding example, the following calculations are made.

Cash Flow Item	Brand A	Brand B
First cost		
A: = \$(30,000) (P/A, 10%, 6)		
= \$(30,000)(0.2296)	\$ (6,888)	
A: = \$(15,000) (P/A, 10%, 6)		
= \$(15,000) (0.2296)		\$ (3,444)
Operating and maintenance		
A: =	\$ (5,000)	
B: = \$(8,000)(P/A, 10%, 3) (A/P, 10%, 6)		
= \$(8,000)(2.4869)(0.2296)		\$ (4,568)
= \$(10,000)[(P/A, 10%, 3)(P/F, 10%, 3) (A/P, 10%, 6)]		
= \$(10,000)[(2.4869)(0.7531)(0.2296)]		\$ (4,290)
Salvage value		
A: = \$3,000(A/F, 10%, 6)		
= \$3,000(0.1296)	389	
B:		0
Annual worth	\$(11,499)	\$(12,302)

Brand A is the recommended alternative since it has the greatest annual worth.
Note that either of these methods of comparison recommends the same alternative because they are dealing with equivalent monetary amounts.

A common method of finding the annual worth of an alternative is

$$\text{AW}(i)_j = R_j - \text{FC}(A/P,\, i\%,\, N) - (\text{O\&M}) + V(A/F,\, i\%,\, N)$$

where R = revenues per year (uniform series)
FC = first cost
V = salvage value
O&M = operating and maintenance cost (uniform series)

This is simply an equation form of the preceding tabular format.

3.3. Future Worth

The future worth (FW) method is comparable to the present worth method except that the comparison between the project's estimated expenditures and benefits occurs at a reference time called the "future" ($t = F$). As in present worth analysis, in future worth analysis a project is acceptable at a particular interest rate if the future value of the revenues and other benefits exceeds the future value of the expenses. Likewise, the preferred alternative, given equal future benefits, would be the alternative with the lowest future costs.

For example, if the estimated future worth of a stream of revenues and other benefits from proposed materials handling equipment at the end of 10 years is $1,200,000, should the new equipment be purchased? The firm's MARR before taxes is 25%. The initial cost would be $125,000, and the annual maintenance cost would be $750/year for the 10-year life.

$$
\begin{aligned}
\text{FW}(25\%) &= \$125{,}000\ (F/P,\ 25\%,\ 10) + \$750\ (F/A,\ 25\%,\ 10) \\
&= \$125{,}000\ (9.313) + \$750\ (33.253) \\
&= \$1{,}164{,}125 + \$25{,}940 \\
&= \$1{,}189{,}065
\end{aligned}
$$

Since the future worth of the benefits exceed the future worth of the costs, the purchase of the equipment would be justified.

3.4. Rate of Return

The rate of return method finds the interest rate that equates the cash flows of receipts and disbursements. That is an alternative's rate of return is the interest rate at which the present worth of the cash flows is equal to 0.

$$0 = \sum_{t=0}^{N} F_{jt}\,(1 + i)^{-k}$$

Thus, for alternative j the rate of return is the break-even interest rate between incomes and expenses.

For example, what rate of return would be earned from a $64,000 investment in a testing device if the savings were to be $16,000/year for 8 years?

Functional format

$$-\$64{,}000 = \$16{,}000\ (P/A,\ i\%,\ 8)$$

Solving for (P/A) and interpolating from the interest tables

$$i = 18.62\%$$

Excel format
IRR calculates the rate of return for a series of cash flows. Assume the data were located in an Excel spreadsheet with −64,000 in cell A1 and 16,000 in each of cells A2 through A9. Going to the function button and then to the financial micros,

$$i = \text{IRR(values, guess)}$$

Enter the location of the data array, A1:A9, in the value command box. The "Formula Result" then indicates the internal rate of return,

$$i = 0.1862$$

A more complex example would consider the investment yielding the four-year stream of cash flows illustrated in the discussion on cash flow profiles. What rate of return would equate the −$6000 disbursement at $t = 0$ with the positive cash flows of \$1500, \$2500, \$3000, and \$4000 at the end of years 1, 2, 3, and 4, respectively?

$$0 = -5000 + \$1500\,(P/F, i\%, 1) + \$2500\,(P/F, i\%, 2) + \$3000\,(P/F, i\%, 3) + \$4000\,(P/F, i\%, 4)$$

Functional format
The rate of return for this set of cash flows is approximately 34%/year and would be determined using an iterative trial-and-error solution method, guessing values of i until a zero solution is obtained.

Excel format
From an array of values of cash flow on the spreadsheet arranged from $n = 0$ to $n = 4$, enter A1: a5 into the values box.

$$i = \text{IRR(values, guess)}$$
$$i = 33.99\% \text{ by the formula}$$

The Excel solution is far faster and less error prone than an iterative method in the functional format.

3.5. Payback Period

The payback period method determines the length of time required to recover the initial investment, or first cost, at a zero rate of interest. The payback period (PP) for alternative j is

$$PP_j = \frac{\text{first cost of the project}}{\text{uniform } \textit{net} \text{ benefits per period}}$$
$$PP_j = \frac{FC_j}{R - D}$$

where R equals the equivalent uniform benefits per period and D equals the equivalent uniform costs per period. For example, find the payback period for a \$10,000 investment that will return net uniform benefits of \$1,250/year.

$$PP = \frac{\$\ 10{,}000}{\$\ 1{,}250} = 8 \text{ years}$$

The alternative with the shortest payback period would be the preferred alternative.

The payback period method is an approximate measure of preference. First, it does not consider the timing of cash flows prior to payback, ignoring the time value of money. It weighs cash flows 10 years from now the same as cash flows occurring today. Second, it ignores the duration of the cash flows. Cash flows after the payback period, such as major overhauls, are not included in the calculation. These weaknesses in the payback period method render it less desirable than the other measures of merit presented in this section.

Payback period is still widely used for comparing alternatives. Its use precludes the necessity of specifying an interest rate or performing interest rate calculations. However, its widest use is as a surrogate measure of risk. The faster the cash is returned from the project, the less the likelihood of unforeseen risks harming the outcome.

3.6. Benefit–Cost Analysis

The benefit–cost method is often utilized to determine the feasibility of public sector expenditures. The benefit–cost criterion for the jth alternative, B/C_j, can be expressed as

$$B/C_j = \frac{\sum_{t=1}^{Nt} B_{jt}\,(1 + i)^{-t}}{\sum_{t=1}^{tN} C_{jt}\,(1 + i)^{-t}}$$

where B_{jt} equals the public benefits accruing to alternative project j during year t and C_{jt} equals the governmental costs associated with the alternative project j during year t. A project is deemed to be acceptable if $B/C_j \geq 1.0$, that is, if the project's benefits equal or exceed its costs.

The calculation of the benefit–cost ratio is not different in principle from the other methods of comparing alternatives. However, determining the monetary value of the costs and benefits associated with projects in the public sector is difficult. This difficulty arises because of the often subjective nature of the costs and benefits and because they often occur far into the future. A detailed discussion of the problems of defining public costs and benefits and of their use in calculating the benefit–cost ratio can be found in other sources, such as Smith (1987) or White et al. (1998).

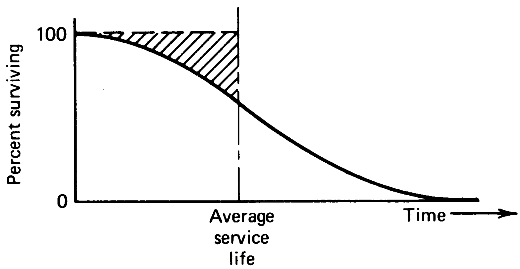
An example of the use of the benefit–cost method is as follows: A government bridge project requires an initial investment of $10 million and operation and maintenance costs of $250,000/year for the 20-year life of the project. The annual user benefits of $1,950,000/year are estimated to arise from savings in travel distance and time. At an annual interest rate of 7%, the benefit–cost ratio is

$$
\begin{aligned}
B/C &= \frac{\$1{,}950{,}000\ (P/A,7\%,20)}{\$10{,}000{,}000 + \$250{,}000\ (P/A,7\%,20)} \\
&= \frac{\$1{,}950{,}000\ (10.5940)}{\$10{,}000{,}000 + \$250{,}000\ (10.5940)} \\
&= \frac{\$20{,}658{,}300}{\$10{,}000{,}000 + \$2{,}648{,}500} \\
&= 1.63
\end{aligned}
$$

Thus, the project benefits would exceed its costs.

4. NONUNIFORM SERVICE LIVES

Engineers are often required to purchase or install a number of items at one time in order to provide a service or manufacture a product. The group of such items is called a vintage group. For example, you may install 2000 telephone poles or 50 automobiles. Assuming that all these items must be in working order in order to provide the required level of service, whenever an item fails, it must be replaced. Thus, if 10 poles are destroyed or fail during the first year of service, they must be replace in order to maintain the desired level of service. The result of having to replace items to maintain the desired level of service means that the true cost will always be higher than the estimated by conventional interest factors. Therefore, the error is always one of *underestimation*. The following figure illustrates the effect of assuming the rectangular distribution as opposed to a survival distribution, representing the true failure rate of the items in question. The shaded portion of the distribution indicates the items that must be replaced to maintain a given level of service.



It is the cost of these replacement items that provides the error of underestimation. See Smith (1987) for details on how to correct for this error in group properties that is due to nonuniform service lives.

5. PERPETUITIES AND CAPITALIZED COSTS

In some major public works projects, such as dams, locks, or bridges, the life of the investment is considered to be infinite. In the case of an infinite life asset, the amount needed to construct or acquire that asset initially plus the amount to provide for the perpetual maintenance and replacement of that asset is referred to as the capitalized cost. A ‘‘perpetuity’’ is a uniform series of payments that continues indefinitely, such as one would find from the conversion of a capitalized cost to an annuity.

For example, what would be the capitalized cost of an irrigation dam at a 5% rate of interest per year if the initial construction cost were $5 million/year for three years and there was a $50,000 maintenance cost/year forever after the completion of the construction?

$$
\begin{aligned}
P &= \$5{,}000{,}000\ (P/A,\ 5\%,\ 3) + \$50{,}000\ (P/A,\ 5\%,\ \infty)\ (P/F,\ 5\%,\ 3) \\
&= \$5{,}000{,}000\ (2.7233) + \$50{,}000\ (20.00)(0.8638) \\
&= \$13{,}616{,}500 + \$863{,}800 \\
&= \$14{,}480{,}300
\end{aligned}
$$

Note that in this example the present worth factor of the annuity is the reciprocal of the interest rate.

$$(P/A,\ i\%,\ \infty) = 1/i$$

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APPENDIX
Interest Tables: Selected Rates

CONTINUOUS COMPOUNDING INTEREST FACTORS
Interest Rate 9%

	Single Payment		Uniform Series				Arithmetic Gradient		
N	Compound Amount Factor Find *F* Given *P* *F/P, i, N*	Present Worth Factor Find *P* Given *F* *P/F, i, N*	Capital Recovery Factor Find *A* Given *P* *A/P, i, N*	Present Worth Factor Find *P* Given *A* *P/A, i, N*	Compound Amount Factor Find *F* Given *A* *F/A, i, N*	Sinking Fund Factor Find *A* Given *F* *A/F, i, N*	Uniform Series Factor Find *A* Given *G* *A/G, i, N*	Present Worth Factor Find *P* Given *G* *P/G, i, N*	*N*
1	1.0942	0.9139	1.0942	0.9139	1.0000	1.0000	0.0000	0.0000	1
2	1.1972	0.8353	0.5717	1.7492	2.0942	0.4775	0.4775	0.8353	2
3	1.3100	0.7634	0.3980	2.5126	3.2914	0.3038	0.9401	2.3620	3
4	1.4333	0.6977	0.3115	3.2103	4.6014	0.2173	1.3878	4.4551	4
5	1.5683	0.6376	0.2599	3.8479	6.0347	0.1657	1.8206	7.0056	5
6	1.7160	0.5827	0.2257	4.4306	7.6030	0.1315	2.2388	9.9193	6
7	1.8776	0.5326	0.2015	4.9632	9.3190	0.1073	2.6424	13.1149	7
8	2.0544	0.4868	0.1835	5.4500	11.1966	0.0893	3.0316	16.5221	8
9	2.2479	0.4449	0.1696	5.8948	13.2510	0.0755	3.4065	20.0810	9
10	2.4596	0.4066	0.1587	6.3014	15.4990	0.0645	3.7674	23.7401	10
11	2.6912	0.3716	0.1499	6.6730	17.9586	0.0557	4.1145	27.4559	11
12	2.9447	0.3396	0.1426	7.0126	20.6498	0.0484	4.4479	31.1914	12
13	3.2220	0.3104	0.1366	7.3229	23.5945	0.0424	4.7680	34.9158	13
14	3.5254	0.2837	0.1315	7.6066	26.8165	0.0373	5.0750	38.6033	14
15	3.8574	0.2592	0.1271	7.8658	30.3419	0.0330	5.3691	42.2327	15
16	4.2207	0.2369	0.1234	8.1028	34.1993	0.0292	5.6507	45.7866	16
17	4.6182	0.2165	0.1202	8.3193	38.4200	0.0260	5.9201	49.2512	17
18	5.0531	0.1979	0.1174	8.5172	43.0382	0.0232	6.1776	52.6155	18
19	5.5290	0.1809	0.1150	8.6981	48.0913	0.0208	6.4234	55.8711	19
20	6.0496	0.1653	0.1128	8.8634	53.6202	0.0186	6.6579	59.0117	20
21	6.6194	0.1511	0.1109	9.0144	59.6699	0.0168	6.8815	62.0332	21
22	7.2427	0.1381	0.1093	9.1525	66.2893	0.0151	7.0945	64.9326	22
23	7.9248	0.1262	0.1078	9.2787	73.5320	0.0136	7.2972	67.7087	23
24	8.6711	0.1153	0.1065	9.3940	81.4568	0.0123	7.4900	70.3612	24
25	9.4877	0.1054	0.1053	9.4994	90.1280	0.0111	7.6732	72.8908	25
26	10.3812	0.0963	0.1042	9.5957	99.6157	0.0100	7.8471	75.2990	26
27	11.3589	0.0880	0.1033	9.6838	109.9969	0.0091	8.0122	77.5879	27
28	12.4286	0.0805	0.1024	9.7642	121.3558	0.0082	8.1686	79.7603	28
29	13.5991	0.0735	0.1016	9.8378	133.7844	0.0075	8.3168	81.8193	29
30	14.8797	0.0672	0.1010	9.9050	147.3835	0.0068	8.4572	83.7683	30
31	16.2810	0.0614	0.1003	9.9664	162.2632	0.0062	8.5899	85.6109	31
32	17.8143	0.0561	0.0998	10.0225	178.5442	0.0056	8.7155	87.3511	32
33	19.4919	0.0513	0.0993	10.0738	196.3585	0.0051	8.8340	88.9928	33
34	21.3276	0.0469	0.0988	10.1207	215.8504	0.0046	8.9460	90.5401	34
35	23.3361	0.0429	0.0984	10.1636	237.1780	0.0042	9.0516	91.9970	35
36	25.5337	0.0392	0.0980	10.2027	260.5140	0.0038	9.1512	93.3678	36
37	27.9383	0.0358	0.0977	10.2385	286.0477	0.0035	9.2451	94.6563	37
38	30.5694	0.0327	0.0974	10.2712	313.9861	0.0032	9.3335	95.8667	38
39	33.4483	0.0299	0.0971	10.3011	344.5555	0.0029	9.4167	97.0028	39
40	36.5982	0.0273	0.0968	10.3285	378.0038	0.0026	9.4950	98.0684	40
41	40.0448	0.0250	0.0966	10.3534	414.6020	0.0024	9.5685	99.0673	41
42	43.8160	0.0228	0.0964	10.3763	454.6469	0.0022	9.6377	100.0030	42
43	47.9424	0.0209	0.0962	10.3971	498.4629	0.0020	9.7026	100.8791	43
44	52.4573	0.0191	0.0960	10.4162	546.4053	0.0018	9.7635	101.6988	44
45	57.3975	0.0174	0.0958	10.4336	598.8626	0.0017	9.8207	102.4654	45
46	62.8028	0.0159	0.0957	10.4495	656.2601	0.0015	9.8743	103.1819	46
47	68.7172	0.0146	0.0956	10.4641	719.0629	0.0014	9.9245	103.8513	47
48	75.1886	0.0133	0.0954	10.4774	787.7801	0.0013	9.9716	104.4764	48
49	82.2695	0.0122	0.0953	10.4895	862.9687	0.0012	10.0157	105.0598	49
50	90.0171	0.0111	0.0952	10.5006	945.2382	0.0011	10.0569	105.6042	50

CONTINUOUS COMPOUNDING
CONTINUOUS FLOW FACTORS
Interest Rate 15%

N	Capital Recovery Factor Find $\bar{A}$ Given *P* *A*/*P*, *i*, *N*	Present Worth Factor Find *P* Given $\bar{A}$ *P*/*A*, *i*, *N*	Compound Amount Factor Find *F* Given $\bar{A}$ *F*/*A*, *i*, *N*	Sinking Fund Factor Find $\bar{A}$ Given *F* *A*/*F*, *i*, *N*	*N*
1	1.0769	0.9286	1.0789	0.9269	1
2	0.5787	1.7279	2.3324	0.4287	2
3	0.4139	2.4158	3.7887	0.2639	3
4	0.3325	3.0079	5.4808	0.1825	4
5	0.2843	3.5176	7.4467	0.1343	5
6	0.2528	3.9562	9.7307	0.1028	6
7	0.2307	4.3337	12.3843	0.0807	7
8	0.2147	4.6587	15.4674	0.0647	8
9	0.2025	4.9384	19.0495	0.0525	9
10	0.1931	5.1791	23.2113	0.0431	10
11	0.1857	5.3863	28.0465	0.0357	11
12	0.1797	5.5647	33.6643	0.0297	12
13	0.1749	5.7182	40.1913	0.0249	13
14	0.1709	5.8503	47.7745	0.0209	14
15	0.1677	5.9640	56.5849	0.0177	15
16	0.1650	6.0619	66.8212	0.0150	16
17	0.1627	6.1461	78.7140	0.0127	17
18	0.1608	6.2186	92.5315	0.0108	18
19	0.1592	6.2810	108.5852	0.0092	19
20	0.1579	6.3348	127.2369	0.0079	20
21	0.1567	6.3810	148.9071	0.0067	21
22	0.1557	6.4208	174.0843	0.0057	22
23	0.1549	6.4550	203.3359	0.0049	23
24	0.1542	6.4845	237.3216	0.0042	24
25	0.1536	6.5099	276.8072	0.0036	25
26	0.1531	6.5317	322.6830	0.0031	26
27	0.1527	6.5505	375.9830	0.0027	27
28	0.1523	6.5667	437.9089	0.0023	28
29	0.1520	6.5806	509.8564	0.0020	29
30	0.1517	6.5926	593.4475	0.0017	30
31	0.1514	6.6029	690.5666	0.0014	31
32	0.1512	6.6118	803.4028	0.0012	32
33	0.1511	6.6194	934.4998	0.0011	33
34	0.1509	6.6260	1086.8127	0.0009	34
35	0.1508	6.6317	1263.7751	0.0008	35
36	0.1507	6.6366	1469.3761	0.0007	36
37	0.1506	6.6408	1708.2504	0.0006	37
38	0.1505	6.6444	1985.7827	0.0005	38
39	0.1504	6.6475	2308.2292	0.0004	39
40	0.1504	6.6501	2682.8586	0.0004	40
41	0.1503	6.6524	3118.1159	0.0003	41
42	0.1503	6.6544	3623.8127	0.0003	42
43	0.1502	6.6561	4211.3486	0.0002	43
44	0.1502	6.6576	4893.9679	0.0002	44
45	0.1502	6.6589	5687.0584	0.0002	45
46	0.1502	6.6599	6608.4981	0.0002	46
47	0.1501	6.6609	7679.0583	0.0001	47
48	0.1501	6.6617	8922.8718	0.0001	48
49	0.1501	6.6624	10367.9769	0.0001	49
50	0.1501	6.6630	12046.9494	0.0001	50

DISCRETE COMPOUND INTEREST FACTORS
Interest Rate 10%

	Single Payment		Uniform Series				Arithmetic Gradient		
N	Compound Amount Factor Find *F* Given *P* *F*/*P*, *i*, *N*	Present Worth Factor Find *P* Given *F* *P*/*F*, *i*, *N*	Capital Recovery Factor Find *A* Given *P* *A*/*P*, *i*, *N*	Present Worth Factor Find *P* Given *A* *P*/*A*, *i*, *N*	Compound Amount Factor Find *F* Given *A* *F*/*A*, *i*, *N*	Sinking Fund Factor Find *A* Given *F* *A*/*F*, *i*, *N*	Uniform Series Factor Find *A* Given *G* *A*/*G*, *i*, *N*	Present Worth Factor Find *P* Given *G* *P*/*G*, *i*, *N*	*N*
1	1.1000	0.9091	1.1000	0.909090	1.0000	1.0000	0.0000	0.0000	1
2	1.2100	0.8264	0.5762	1.735537	2.1000	0.47619	0.47619	0.8264	2
3	1.3310	0.7513	0.4021	2.486852	3.3100	0.3021	0.93656	2.3291	3
4	1.4641	0.6830	0.3155	3.1699	4.6410	0.21547	1.38117	4.3781	4
5	1.6105	0.6209	0.2638	3.790787	6.1051	0.1638	1.81013	6.8618	5
6	1.7716	0.5645	0.2296	4.355261	7.7156	0.12961	2.22356	9.6842	6
7	1.9487	0.5132	0.2054	4.868419	9.4872	0.10541	2.62162	12.7631	7
8	2.1436	0.4665	0.1874	5.334926	11.4359	0.08744	3.00448	16.0287	8
9	2.3579	0.4241	0.1736	5.7590	13.5795	0.07364	3.37235	19.4215	9
10	2.5937	0.3855	0.1627	6.144567	15.9374	0.06275	3.72546	22.8913	10
11	2.8531	0.3505	0.1540	6.495061	18.5312	0.0540	4.06405	26.3963	11
12	3.1384	0.3186	0.1468	6.813692	21.3843	0.04676	4.3884	29.9012	12
13	3.4523	0.2897	0.1408	7.103356	24.5227	0.04078	4.69879	33.3772	13
14	3.7975	0.2633	0.1357	7.366687	27.9750	0.03575	4.99553	36.8005	14
15	4.1772	0.2394	0.1315	7.60608	31.7725	0.03147	5.2789	40.1520	15
16	4.5950	0.2176	0.1278	7.823709	35.9497	0.02782	5.54934	43.4164	16
17	5.0545	0.1978	0.1247	8.021553	40.5447	0.02466	5.8071	46.5819	17
18	5.5599	0.1799	0.1219	8.2014	45.5992	0.02193	6.05256	49.63954	18
19	6.1159	0.1635	0.1195	8.36492	51.1591	0.01955	6.2861	52.58268	19
20	6.7275	0.1486	0.1175	8.513564	57.2750	0.01746	6.50808	55.40691	20
21	7.4002	0.1351	0.1156	8.648694	64.0025	0.01562	6.71888	58.10952	21
22	8.1403	0.1228	0.1140	8.77154	71.4027	0.0140	6.9189	60.6893	22
23	8.9543	0.1117	0.1126	8.883218	79.5430	0.01257	7.1085	63.14621	23
24	9.8497	0.1015	0.1113	8.984744	88.4973	0.0113	7.28805	65.4813	24
25	10.8347	0.0923	0.1102	9.0770	98.3471	0.01017	7.4580	67.6964	25
26	11.9182	0.0839	0.1092	9.160945	109.18177	0.00916	7.61865	69.7940	26
27	13.1100	0.0763	0.1083	9.237223	121.09994	0.00826	7.77044	71.77726	27
28	14.4210	0.0693	0.1075	9.306567	134.20994	0.00745	7.9137	73.64953	28
29	15.8631	0.0630	0.1067	9.369606	148.63093	0.00673	8.04886	75.41463	29
30	17.4494	0.0573	0.1061	9.4269	164.4940	0.00608	8.17632	77.07658	30
31	19.1943	0.0521	0.1055	9.479013	181.94342	0.0055	8.29617	78.63954	31
32	21.1138	0.0474	0.1050	9.526376	201.13777	0.0050	8.4091	80.10777	32
33	23.2252	0.0431	0.1045	9.569432	222.25154	0.0045	8.5152	81.48559	33
34	25.5477	0.0391	0.1041	9.608575	245.4767	0.00407	8.61494	82.77729	34
35	28.1024	0.0356	0.1037	9.644158	271.02437	0.00369	8.7086	83.98715	35
36	30.9127	0.0323	0.1033	9.676508	299.1268	0.0033	8.7965	85.11938	36
37	34.0039	0.0294	0.1030	9.705917	330.0395	0.0030	8.87892	86.17808	37
38	37.4043	0.0267	0.1027	9.732651	364.0434	0.00275	8.95617	87.16727	38
39	41.1448	0.0243	0.1025	9.7570	401.4478	0.00249	9.02852	88.09083	39
40	45.2593	0.0221	0.1023	9.779051	442.5926	0.00226	9.09623	88.9525	40
41	49.7852	0.0201	0.1020	9.7991	487.8518	0.0020	9.15958	89.7560	41
42	54.7637	0.0183	0.1019	9.817397	537.6370	0.00186	9.2188	90.50466	42
43	60.2401	0.0166	0.1017	9.8340	592.4007	0.00169	9.27414	91.20187	43
44	66.2641	0.0151	0.1015	9.849089	652.6408	0.00153	9.32582	91.85079	44
45	72.8905	0.0137	0.1014	9.862808	718.9048	0.00139	9.3740	92.45443	45
46	80.1795	0.0125	0.1013	9.87528	791.7953	0.00126	9.4190	93.01567	46
47	88.1975	0.0113	0.1011	9.886618	871.9749	0.00115	9.4610	93.53723	47
48	97.0172	0.0103	0.1010	9.896926	960.1723	0.0010	9.5001	94.02168	48
49	106.7190	0.0094	0.1009	9.906296	1057.1896	0.00095	9.53651	94.47146	49
50	117.3909	0.0085	0.1009	9.914814	1163.9085	0.00086	9.57041	94.88887	50

DISCRETE COMPOUND INTEREST FACTORS
Interest Rate 12%

	Single Payment		Uniform Series				Arithmetic Gradient		
N	Compound Amount Factor Find *F* Given *P* *F/P, i, N*	Present Worth Factor Find *P* Given *F* *P/F, i, N*	Capital Recovery Factor Find *A* Given *P* *A/P, i, N*	Present Worth Factor Find *P* Given *A* *P/A, i, N*	Compound Amount Factor Find *F* Given *A* *F/A, i, N*	Sinking Fund Factor Find *A* Given *F* *A/F, i, N*	Uniform Series Factor Find *A* Given *G* *A/G, i, N*	Present Worth Factor Find *P* Given *G* *P/G, i, N*	N
1	1.1200	0.8929	1.1200	0.89286	1.0000	1.0000	0.0000	0.0000	1
2	1.2544	0.7972	0.5917	1.69005	2.1200	0.4717	0.4717	0.7972	2
3	1.4049	0.7118	0.4163	2.40183	3.3744	0.29635	0.92461	2.2208	3
4	1.5735	0.6355	0.3292	3.03735	4.7793	0.20923	1.35885	4.1273	4
5	1.7623	0.5674	0.2774	3.60478	6.3528	0.15741	1.77459	6.3970	5
6	1.9738	0.5066	0.2432	4.11141	8.1152	0.12323	2.1720	8.9302	6
7	2.2107	0.4523	0.2191	4.56376	10.0890	0.09912	2.55147	11.6443	7
8	2.4760	0.4039	0.2013	4.96764	12.2997	0.0813	2.91314	14.4714	8
9	2.7731	0.3606	0.1877	5.3282	14.7757	0.06768	3.25742	17.3563	9
10	3.1058	0.3220	0.1770	5.65022	17.5487	0.05698	3.58465	20.2541	10
11	3.4785	0.2875	0.1684	5.9377	20.6546	0.0484	3.89525	23.1288	11
12	3.8960	0.2567	0.1614	6.19437	24.1331	0.04144	4.18965	25.9523	12
13	4.3635	0.2292	0.1557	6.42355	28.0291	0.03568	4.4683	28.7024	13
14	4.8871	0.2046	0.1509	6.62817	32.3926	0.03087	4.73169	31.3624	14
15	5.4736	0.1827	0.1468	6.81086	37.2797	0.02682	4.9803	33.9202	15
16	6.1304	0.1631	0.1434	6.97399	42.7533	0.02339	5.21466	36.3670	16
17	6.8660	0.1456	0.1405	7.11963	48.8837	0.02046	5.4353	38.6973	17
18	7.6900	0.1300	0.1379	7.24967	55.7497	0.01794	5.64274	40.9080	18
19	8.6128	0.1161	0.1358	7.36578	63.4397	0.01576	5.83752	42.9979	19
20	9.6463	0.1037	0.1339	7.46944	72.0524	0.01388	6.0202	44.96757	20
21	10.8038	0.0926	0.1322	7.5620	81.6987	0.01224	6.19132	46.81876	21
22	12.1003	0.0826	0.1308	7.64465	92.5026	0.0108	6.35141	48.55425	22
23	13.5523	0.0738	0.1296	7.71843	104.6029	0.00956	6.5010	50.17759	23
24	15.1786	0.0659	0.1285	7.78432	118.1552	0.00846	6.64064	51.69288	24
25	17.0001	0.0588	0.1275	7.8431	133.3339	0.0075	6.7708	53.10464	25
26	19.0401	0.0525	0.1267	7.89566	150.33393	0.00665	6.8921	54.4177	26
27	21.3249	0.0469	0.1259	7.94255	169.3740	0.0059	7.00491	55.63689	27
28	23.8839	0.0419	0.1252	7.98442	190.69889	0.00524	7.10976	56.76736	28
29	26.7499	0.0374	0.1247	8.02181	214.58275	0.00466	7.20712	57.81409	29
30	29.9599	0.0334	0.1241	8.05518	241.3327	0.00414	7.29742	58.78205	30
31	33.5551	0.0298	0.1237	8.0850	271.29261	0.00369	7.3811	59.6761	31
32	37.5817	0.0266	0.1233	8.11159	304.84772	0.0033	7.45858	60.5010	32
33	42.0915	0.0238	0.1229	8.13535	342.42945	0.00292	7.53025	61.26122	33
34	47.1425	0.0212	0.1226	8.15656	384.5210	0.0026	7.59649	61.96123	34
35	52.7996	0.0189	0.1223	8.1755	431.6635	0.00232	7.65765	62.60517	35
36	59.1356	0.0169	0.1221	8.19241	484.46312	0.00206	7.71409	63.1970	36
37	66.2318	0.0151	0.1218	8.20751	543.59869	0.0018	7.76613	63.74058	37
38	74.1797	0.0135	0.1216	8.2210	609.83053	0.00164	7.81406	64.23936	38
39	83.0812	0.0120	0.1215	8.2330	684.0102	0.00146	7.85819	64.69675	39
40	93.0510	0.0107	0.1213	8.24378	767.09142	0.0013	7.89879	65.11587	40
41	104.2171	0.0096	0.1212	8.25337	860.14239	0.0012	7.93611	65.4997	41
42	116.7231	0.0086	0.1210	8.26194	964.3595	0.0010	7.9704	65.85095	42
43	130.7299	0.0076	0.1209	8.2696	1081.0826	0.00092	8.00188	66.17222	43
44	146.4175	0.0068	0.1208	8.27642	1211.8125	0.00083	8.03076	66.4659	44
45	163.9876	0.0061	0.1207	8.28252	1358.2300	0.00074	8.0572	66.73421	45
46	183.6661	0.0054	0.1207	8.2880	1522.2176	0.000	8.0815	66.97922	46
47	205.7061	0.0049	0.1206	8.29282	1705.8838	0.000	8.1037	67.20284	47
48	230.3908	0.0043	0.1205	8.29716	1911.5898	0.0005	8.12408	67.40684	48
49	258.0377	0.0039	0.1205	8.3010	2141.9806	0.00047	8.1427	67.59286	49
50	289.0022	0.0035	0.1204	8.3045	2400.0182	0.00042	8.15972	67.76241	50

DISCRETE COMPOUND INTEREST FACTORS
Interest Rate 15%

	Single Payment		Uniform Series				Arithmetic Gradient		
N	Compound Amount Factor Find *F* Given *P* *F/P, i, N*	Present Worth Factor Find *P* Given *F* *P/F, i, N*	Capital Recovery Factor Find *A* Given *P* *A/P, i, N*	Present Worth Factor Find *P* Given *A* *P/A, i, N*	Compound Amount Factor Find *F* Given *A* *F/A, i, N*	Sinking Fund Factor Find *A* Given *F* *A/F, i, N*	Uniform Series Factor Find *A* Given *G* *A/G, i, N*	Present Worth Factor Find *P* Given *G* *P/G, i, N*	N
1	1.1500	0.8696	1.1500	0.869565	1.0000	1.0000	0.0000	0.0000	1
2	1.3225	0.7561	0.6151	1.625709	2.1500	0.46512	0.46512	0.7561	2
3	1.5209	0.6575	0.4380	2.283225	3.4725	0.2880	0.90713	2.0712	3
4	1.7490	0.5718	0.3503	2.8550	4.9934	0.20027	1.32626	3.7864	4
5	2.0114	0.4972	0.2983	3.352155	6.7424	0.14832	1.72281	5.7751	5
6	2.3131	0.4323	0.2642	3.784483	8.7537	0.11424	2.09719	7.9368	6
7	2.6600	0.3759	0.2404	4.16042	11.0668	0.09036	2.44985	10.1924	7
8	3.0590	0.3269	0.2229	4.487322	13.7268	0.07285	2.78133	12.4807	8
9	3.5179	0.2843	0.2096	4.7716	16.7858	0.05957	3.09223	14.7548	9
10	4.0456	0.2472	0.1993	5.018769	20.3037	0.04925	3.3832	16.9795	10
11	4.6524	0.2149	0.1911	5.233712	24.3493	0.0411	3.65494	19.1289	11
12	5.3503	0.1869	0.1845	5.420619	29.0017	0.03448	3.9082	21.1849	12
13	6.1528	0.1625	0.1791	5.583147	34.3519	0.02911	4.14376	23.1352	13
14	7.0757	0.1413	0.1747	5.724476	40.5047	0.02469	4.36241	24.9725	14
15	8.1371	0.1229	0.1710	5.84737	47.5804	0.0210	4.5650	26.6930	15
16	9.3576	0.1069	0.1679	5.954235	55.7175	0.01795	4.75225	28.2960	16
17	10.7613	0.0929	0.1654	6.047161	65.0751	0.01537	4.92509	29.7828	17
18	12.3755	0.0808	0.1632	6.1280	75.8364	0.01319	5.08431	31.15649	18
19	14.2318	0.0703	0.1613	6.198231	88.2118	0.01134	5.23073	32.42127	19
20	16.3665	0.0611	0.1598	6.259331	102.4436	0.00976	5.36514	33.58217	20
21	18.8215	0.0531	0.1584	6.312462	118.81012	0.00842	5.48832	34.64479	21
22	21.6447	0.0462	0.1573	6.358663	137.6316	0.0073	5.6010	35.6150	22
23	24.8915	0.0402	0.1563	6.398837	159.2764	0.00628	5.7040	36.49884	23
24	28.6252	0.0349	0.1554	6.433771	184.1678	0.00543	5.79789	37.30232	24
25	32.9190	0.0304	0.1547	6.4641	212.7930	0.0047	5.8834	38.03139	25
26	37.8568	0.0264	0.1541	6.490564	245.7120	0.00407	5.96123	38.6918	26
27	43.5353	0.0230	0.1535	6.513534	283.56877	0.00353	6.0319	39.2890	27
28	50.0656	0.0200	0.1531	6.533508	327.10408	0.00306	6.0960	39.82828	28
29	57.5755	0.0174	0.1527	6.550877	377.16969	0.00265	6.15408	40.3146	29
30	66.2118	0.0151	0.1523	6.5660	434.7451	0.0023	6.20663	40.75259	30
31	76.1435	0.0131	0.1520	6.579113	500.95692	0.002	6.25412	41.14658	31
32	87.5651	0.0114	0.1517	6.590533	577.10046	0.0017	6.2970	41.50060	32
33	100.6998	0.0099	0.1515	6.600463	664.66552	0.0015	6.33567	41.81838	33
34	115.8048	0.0086	0.1513	6.609099	765.36535	0.00131	6.37051	42.10334	34
35	133.1755	0.0075	0.1511	6.616607	881.17016	0.00113	6.40187	42.35864	35
36	153.1519	0.0065	0.1510	6.623137	1014.3457	0.0010	6.43006	42.58717	36
37	176.1246	0.0057	0.1509	6.628815	1167.4975	0.0009	6.45539	42.79157	37
38	202.5433	0.0049	0.1507	6.633752	1343.6222	0.00074	6.47812	42.97425	38
39	232.9248	0.0043	0.1506	6.6380	1546.1655	0.00065	6.49851	43.13739	39
40	267.8635	0.0037	0.1506	6.641778	1779.0903	0.00056	6.51678	43.2830	40
41	308.0431	0.0032	0.1505	6.6450	2046.9539	0.0005	6.53313	43.4128	41
42	354.2495	0.0028	0.1504	6.647848	2354.9969	0.00042	6.54777	43.52858	42
43	407.3870	0.0025	0.1504	6.6503	2709.2465	0.00037	6.56086	43.63168	43
44	468.4950	0.0021	0.1503	6.652437	3116.6334	0.00032	6.57255	43.72346	44
45	538.7693	0.0019	0.1503	6.654293	3585.1285	0.00028	6.5830	43.80513	45
46	619.5847	0.0016	0.1502	6.655907	4123.8977	0.00024	6.5923	43.87776	46
47	712.5224	0.0014	0.1502	6.65731	4743.4824	0.00021	6.6006	43.94232	47
48	819.4007	0.0012	0.1502	6.658531	5456.0047	0.0002	6.6080	43.99967	48
49	942.3108	0.0011	0.1502	6.659592	6275.4055	0.00016	6.61461	44.05061	49
50	1083.6574	0.0009	0.1501	6.660515	7217.7163	0.00014	6.62048	44.09583	50

DISCRETE COMPOUND INTEREST FACTORS
Interest Rate 20%

	Single Payment		Uniform Series				Arithmetic Gradient		
N	Compound Amount Factor Find *F* Given *P* *F/P, i, N*	Present Worth Factor Find *P* Given *F* *P/F, i, N*	Capital Recovery Factor Find *A* Given *P* *A/P, i, N*	Present Worth Factor Find *P* Given *A* *P/A, i, N*	Compound Amount Factor Find *F* Given *A* *F/A, i, N*	Sinking Fund Factor Find *A* Given *F* *A/F, i, N*	Uniform Series Factor Find *A* Given *G* *A/G, i, N*	Present Worth Factor Find *P* Given *G* *P/G, i, N*	N
1	1.2000	0.8333	1.2000	0.83333	1.0000	1.0000	0.0000	0.0000	1
2	1.4400	0.6944	0.6545	1.52778	2.2000	0.45455	0.45455	0.6944	2
3	1.7280	0.5787	0.4747	2.10648	3.6400	0.2747	0.87912	1.8519	3
4	2.0736	0.4823	0.3863	2.5887	5.3680	0.18629	1.27422	3.2986	4
5	2.4883	0.4019	0.3344	2.99061	7.4416	0.13438	1.64051	4.9061	5
6	2.9860	0.3349	0.3007	3.32551	9.9299	0.10071	1.97883	6.5806	6
7	3.5832	0.2791	0.2774	3.60459	12.9159	0.07742	2.29016	8.2551	7
8	4.2998	0.2326	0.2606	3.83716	16.4991	0.06061	2.57562	9.8831	8
9	5.1598	0.1938	0.2481	4.0310	20.7989	0.04808	2.83642	11.4335	9
10	6.1917	0.1615	0.2385	4.19247	25.9587	0.03852	3.07386	12.8871	10
11	7.4301	0.1346	0.2311	4.32706	32.1504	0.0311	3.28929	14.2330	11
12	8.9161	0.1122	0.2253	4.43922	39.5805	0.02526	3.4841	15.4667	12
13	10.6993	0.0935	0.2206	4.53268	48.4966	0.02062	3.6597	16.5883	13
14	12.8392	0.0779	0.2169	4.61057	59.1959	0.01689	3.81749	17.6008	14
15	15.4070	0.0649	0.2139	4.67547	72.0351	0.01388	3.9588	18.5095	15
16	18.4884	0.0541	0.2114	4.72956	87.4421	0.01144	4.08511	19.3208	16
17	22.1861	0.0451	0.2094	4.77463	105.9306	0.00944	4.19759	20.0419	17
18	26.6233	0.0376	0.2078	4.8122	128.1167	0.00781	4.29752	20.68048	18
19	31.9480	0.0313	0.2065	4.8435	154.7400	0.00646	4.38607	21.2439	19
20	38.3376	0.0261	0.2054	4.86958	186.6880	0.00536	4.46435	21.73949	20
21	46.0051	0.0217	0.2044	4.89132	225.0256	0.00444	4.53339	22.17423	21
22	55.2061	0.0181	0.2037	4.90943	271.0307	0.0037	4.5941	22.5546	22
23	66.2474	0.0151	0.2031	4.92453	326.2369	0.00307	4.6475	22.88671	23
24	79.4968	0.0126	0.2025	4.9371	392.4842	0.00255	4.69426	23.1760	24
25	95.3962	0.0105	0.2021	4.9476	471.9811	0.00212	4.7352	23.42761	25
26	114.4755	0.0087	0.2018	4.95632	567.3773	0.00176	4.77088	23.6460	26
27	137.3706	0.0073	0.2015	4.9636	681.8528	0.00147	4.8020	23.83527	27
28	164.8447	0.0061	0.2012	4.96967	819.2233	0.00122	4.8291	23.99906	28
29	197.8136	0.0051	0.2010	4.97472	984.0680	0.0010	4.85265	24.14061	29
30	237.3763	0.0042	0.2008	4.9789	1181.8816	0.00085	4.87308	24.26277	30
31	284.8516	0.0035	0.2007	4.98245	1419.2579	0.0007	4.89079	24.36809	31
32	341.8219	0.0029	0.2006	4.98537	1704.1095	0.0006	4.9061	24.45878	32
33	410.1863	0.0024	0.2005	4.98781	2045.9314	0.00049	4.91935	24.5368	33
34	492.2235	0.0020	0.2004	4.98984	2456.1176	0.00041	4.93079	24.60384	34
35	590.6682	0.0017	0.2003	4.99154	2948.3411	0.00034	4.94064	24.6614	35
36	708.8019	0.0014	0.2003	4.99295	3539.0094	0.0003	4.94914	24.71078	36
37	850.5622	0.0012	0.2002	4.99412	4247.8112	0.0002	4.95645	24.7531	37
38	1020.6747	0.0010	0.2002	4.9951	5098.3735	0.0002	4.96273	24.78936	38
39	1224.8096	0.0008	0.2002	4.9959	6119.0482	0.00016	4.96813	24.82038	39
40	1469.7716	0.0007	0.2001	4.9966	7343.8578	0.00014	4.97277	24.8469	40
41	1763.7259	0.0006	0.2001	4.9972	8813.6294	0.0001	4.97674	24.8696	41
42	2116.4711	0.0005	0.2001	4.99764	10577.3553	0.0001	4.98015	24.88897	42
43	2539.7653	0.0004	0.2001	4.9980	12693.8263	0.0001	4.98306	24.9055	43
44	3047.7183	0.0003	0.2001	4.99836	15233.5916	0.0001	4.98556	24.91961	44
45	3657.2620	0.0003	0.2001	4.99863	18281.3099	0.0001	4.9877	24.93164	45
46	4388.7144	0.0002	0.2000	4.99886	21938.5719	0.0000	4.9895	24.94189	46
47	5266.4573	0.0002	0.2000	4.99905	26327.2863	0.0000	4.9911	24.95063	47
48	6319.7487	0.0002	0.2000	4.99921	31593.7436	0.0000	4.9924	24.95807	48
49	7583.6985	0.0001	0.2000	4.99934	37913.4923	0.0000	4.99354	24.9644	49
50	9100.4382	0.0001	0.2000	4.99945	45497.1908	0.0000	4.99451	24.96978	50

GEOMETRIC SERIES FACTORS: DISCRETE COMPOUNDING
FUTURE WORTH FACTOR F/A
Interest Rate 15%

N	$C = 4$	$C = 5$	$C = 6$	$C = 8$	$C = 10$	$C = 12$	$C = 20$	N
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	4	4	4	4	4	4	4	3
4	5	5	6	6	6	6	7	4
5	7	8	8	8	8	9	10	5
6	10	10	10	11	11	12	14	6
7	13	13	13	14	15	16	20	7
8	16	17	17	18	19	21	27	8
9	20	21	21	23	25	27	37	9
10	25	26	27	29	31	34	49	10
11	30	31	33	36	39	43	64	11
12	37	38	40	44	48	53	83	12
13	44	46	48	53	59	66	108	13
14	53	56	58	65	73	82	139	14
15	63	67	70	79	88	100	178	15
16	75	80	84	95	107	122	227	16
17	90	95	100	113	129	149	288	17
18	106	112	119	136	156	181	365	18
19	125	133	142	162	187	219	460	19
20	148	157	168	193	224	264	579	20
21	174	185	198	229	268	318	728	21
22	204	218	234	271	319	381	912	22
23	240	256	275	321	380	457	1141	23
24	281	301	324	379	451	547	1425	24
25	329	353	380	447	535	653	1778	25
26	385	414	446	527	634	779	2214	26
27	450	484	523	620	750	928	2753	27
28	526	566	613	729	887	1104	3420	28
29	614	662	718	857	1047	1311	4244	29
30	716	774	840	1006	1234	1556	5261	30
31	836	903	982	1179	1454	1844	6516	31
32	974	1054	1147	1382	1711	2184	8064	32
33	1136	1230	1339	1619	2013	2584	9970	33
34	1323	1434	1563	1895	2366	3054	12319	34
35	1541	1672	1824	2217	2779	3608	15210	35
36	1795	1948	2127	2592	3262	4258	18769	36
37	2089	2269	2480	3029	3826	5023	23146	37
38	2432	2643	2891	3539	4486	5920	28527	38
39	2830	3077	3369	4132	5256	6974	35142	39
40	3293	3582	3924	4824	6156	8210	43270	40
41	3830	4169	4570	5629	7207	9660	53254	41
42	4455	4852	5322	6567	8434	11361	65513	42
43	5182	5645	6195	7658	9865	13354	80562	43
44	6026	6567	7211	8929	11536	15689	99031	44
45	7007	7639	8392	10407	13484	18425	121692	45
46	8147	8885	9765	12128	15756	21628	149489	46
47	9472	10332	11361	14130	18405	25377	183579	47
48	11011	12015	13216	16460	21494	29766	225377	48
49	12800	13970	15373	19171	25094	34900	276615	49
50	14879	16243	17880	22323	29289	40906	339415	50

GEOMETRIC SERIES FACTORS: DISCRETE COMPOUNDING
PRESENT VALUE FACTOR: P/A
Interest Rate 15%

N	$C = 4$	$C = 5$	$C = 6$	$C = 8$	$C = 10$	$C = 12$	$C = 20$	N
1	0.8607	0.8607	0.8607	0.8607	0.8607	0.8607	0.8607	1
2	1.6318	1.6395	1.6473	1.6632	1.6794	1.6960	1.7655	2
3	2.3225	2.3442	2.3663	2.4115	2.4582	2.5066	2.7168	3
4	2.9413	2.9818	3.0233	3.1092	3.1991	3.2932	3.7168	4
5	3.4956	3.5588	3.6238	3.7597	3.9037	4.0566	4.7680	5
6	3.9922	4.0808	4.1726	4.3662	4.5741	4.7974	5.8732	6
7	4.4370	4.5532	4.6742	4.9317	5.2117	5.5163	7.0351	7
8	4.8356	4.9806	5.1326	5.4590	5.8182	6.2140	8.2565	8
9	5.1926	5.3673	5.5515	5.9507	6.3952	6.8910	9.5405	9
10	5.5124	5.7173	5.9344	6.4091	6.9440	7.5481	10.8903	10
11	5.7989	6.0339	6.2844	6.8365	7.4660	8.1857	12.3094	11
12	6.0556	6.3204	6.6042	7.2350	7.9626	8.8045	13.8012	12
13	6.2855	6.5797	6.8965	7.6066	8.4350	9.4050	15.3695	13
14	6.4915	6.8142	7.1636	7.9530	8.8843	9.9877	17.0183	14
15	6.6760	7.0265	7.4078	8.2761	9.3117	10.5533	18.7515	15
16	6.8413	7.2185	7.6309	8.5773	9.7183	11.1021	20.5736	16
17	6.9894	7.3923	7.8348	8.8581	10.1050	11.6347	22.4892	17
18	7.1220	7.5495	8.0212	9.1199	10.4729	12.1515	24.5029	18
19	7.2409	7.6918	8.1915	9.3641	10.8229	12.6531	26.6199	19
20	7.3473	7.8206	8.3472	9.5917	11.1557	13.1399	28.8455	20
21	7.4427	7.9370	8.4895	9.8040	11.4724	13.6122	31.1851	21
22	7.5281	8.0424	8.6195	10.0019	11.7736	14.0706	33.6447	22
23	7.6046	8.1378	8.7383	10.1864	12.0601	14.5155	36.2304	23
24	7.6732	8.2241	8.8470	10.3584	12.3326	14.9472	38.9487	24
25	7.7346	8.3022	8.9462	10.5189	12.5918	15.3661	41.8064	25
26	7.7897	8.3728	9.0369	10.6684	12.8384	15.7727	44.8105	26
27	7.8389	8.4368	9.1198	10.8079	13.0730	16.1673	47.9687	27
28	7.8831	8.4946	9.1956	10.9379	13.2961	16.5502	51.2888	28
29	7.9227	8.5469	9.2649	11.0591	13.5084	16.9217	54.7792	29
30	7.9581	8.5943	9.3282	11.1722	13.7103	17.2823	58.4485	30
31	7.9898	8.6372	9.3860	11.2776	13.9023	17.6323	62.3059	31
32	8.0183	8.6759	9.4389	11.3759	14.0850	17.9719	66.3611	32
33	8.0438	8.7110	9.4872	11.4675	14.2588	18.3014	70.6242	33
34	8.0666	8.7428	9.5313	11.5529	14.4241	18.6212	75.1059	34
35	8.0870	8.7715	9.5717	11.6326	14.5813	18.9316	79.8174	35
36	8.1053	8.7975	9.6086	11.7069	14.7309	19.2328	84.7704	36
37	8.1218	8.8210	9.6423	11.7761	14.8732	19.5251	89.9774	37
38	8.1365	8.8423	9.6731	11.8407	15.0085	19.8088	95.4513	38
39	8.1496	8.8615	9.7013	11.9009	15.1372	20.0840	101.2059	39
40	8.1614	8.8789	9.7270	11.9570	15.2597	20.3512	107.2556	40
41	8.1720	8.8947	9.7505	12.0094	15.3762	20.6104	113.6154	41
42	8.1814	8.9090	9.7720	12.0582	15.4870	20.8620	120.3013	42
43	8.1899	8.9219	9.7916	12.1037	15.5924	21.1061	127.3300	43
44	8.1975	8.9336	9.8096	12.1461	15.6926	21.3431	134.7190	44
45	8.2043	8.9441	9.8260	12.1856	15.7880	21.5730	142.4869	45
46	8.2104	8.9537	9.8410	12.2225	15.8787	21.7961	150.6531	46
47	8.2159	8.9623	9.8547	12.2569	15.9650	22.0126	159.2380	47
48	8.2208	8.9702	9.8672	12.2890	16.0471	22.2228	168.2630	48
49	8.2252	8.9773	9.8787	12.3189	16.1252	22.4267	177.7507	49
50	8.2291	8.9837	9.8891	12.3468	16.1995	22.6246	187.7249	50