Computer Aided Process and Product Engineering
Luis Puigianer and Georges Heyen

3 Facing Uncertainty in Demand by Cost-effective Manufacturing Flexibility

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3.1 Introduction

This chapter deals with a case of flexible production planning for a multiproduct plant to optimize expected proceeds from product sales when facing uncertainty in the demands for existing and emerging new products over the planning period. The manufacturing capacities of the plant (that is, the nominal production rates) for the existing and new products are fixed by its design and these are not subject to adaptations by making changes to the plant. Hence, the flexibility refers exclusively to the planning problem and it is not coupled with a plant redesign.

As the inherent uncertainty in customers' demand forecasts is hard to defeat by a company, the industry's specific capabilities with respect to responding rapidly to new and changing orders must be improved. New technologies are required, including tools that can swiftly convert customer orders into actual production and delivery actions. On the production side, this may require new planning technologies or new types of equipment that are, for example, dedicated to product families, rather than to individual products. Many companies need to use medium term planning in their product development and manufacturing processes in order to sustain the reliability of supply and the responsiveness to changing customer requirements.

Flexibility is often referred to in operations and manufacturing research as the solution for dealing with swift changes in customer demands and requests for in-time delivery (Bengtsson 2001). The concept has received even more attention with the upcoming of e-business in the chemical industry. The actual meaning, interpretation and consequences of "operating flexibility" are, however, not instantly clear for a particular case or company (Berry and Cooper 1999). A number of uncertainties may induce organisations to seek more flexible manufacturing systems. Common sources of uncertainties are depicted in Fig. 3.1.

Computer Aided Process and Product Engineering. Edited by Luis Puigjaner and Georges Heyen

ISBN: 3-527-30804-0

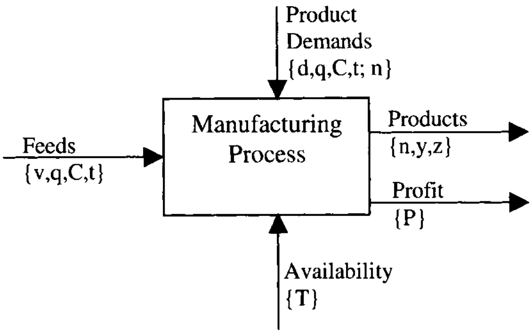

Figure 3.1 Types of uncertainties

On the input side, manufacturing systems have to deal with suppliers' reliability with respect to feed stock supply, involving quantities (*v*), quality (*q*), cost (*C*), and with uncertainties in time (*t*). Secondly, process inherent uncertainties exist, concerning equipment availability (*T*), and modeling uncertainties. On the product demand side, the same types of uncertainties can be found for each product, involving demand (*d*), quality (*q*), and cost (*C*), and time (*t*). For the products a distinction is made regarding two different sales conditions. At the beginning of the planning period some sales contracts can be secured, under which the amounts (*y*) that can be manufactured and sold. The excess manufacturing capacity of the plant can be used to make the amounts (*z*), which will capture market opportunities during the planning period. It is noticed that the number of products (*n*) can change over time. This change reflects a trend towards diversification in many production markets. To achieve this diversification and to cope with shorter product life span, it seems preferable for manufacturing systems to have *flexible* resources.

Extensive research has been done into the flexibility of (chemical) processes that are subject to uncertainties on the input side and with respect to the availability of the processing equipment, possibly influencing the feasible operating region of the plant (Bansal et al. 1998; Swaney and Grossmann 1985). Less research has been done, however, into flexibility that is characterized by the possibility to cope with changes in demand or product mix. The right way to respond to change is always system specific, and dependent on the system's flexibility. Many approaches for dealing with uncertainties exist (Corrêa 1994). As this study concerns product mix variations and demand variations, the monitoring and forecasting technique was selected. The uncertainty aspect is modeled by means of a stochastic approach.

In the development of a planning technique its applicability requires careful consideration. Firstly, the technique should be compatible with the work processes and the associated level of technical competence. Among others, this requires that the input and the output can be well understood and interpreted by those who will use it. Secondly, the cost of using the technique (time and money wise) should remain low. It would be very helpful to use input data that can be obtained without excessive efforts, while the results of the planning can be easily (re)produced with small computational effort.

3.2 The Production Planning Problem

A case study was developed based on experiences at a company that makes various types of food additives in a multiproduct batch plant. In this plant several groups of products are produced on a number of reactors. At the beginning of every new production period of one year, planning management agrees with the customers about the amount and price of products that will be produced to meet customer demands in the coming period. These agreements between company and customers are laid down in annual contracts. The demand for products that could be sold in these annual contracts is in general very large and the total capacity of the plant could have been sold out. However, planning management has strong indications that the demand for a new and very profitable product will increase during the coming production period and it could be very attractive to keep some of the capacity free for this newcomer on the market.

Not only that, but also for the current products it could be quite profitable to not sell all capacity beforehand, since the price for which the products can be sold during the production period is in general significantly higher than before by contract price. Unfortunately, the demand for the products during the production period cannot be assured. Planning management would like to establish in the production planning how much of the current products they should sell in annual contracts and how much capacity they should leave open for every individual current product and for the new one in such a way that the final profit achieved at the end of the production period is as high as possible. The plant production capacity acts as a restriction on the total amount that can be produced.

The next sections will introduce a simple probabilistic model for the product demands as well as a manufacturing capacity constraint (Section 3.3). The realised product sales are related to the corresponding profit over the planning period (Section 3.4). In order to optimize the manufacturing performance, two objective functions are chosen that take into account the distributive nature of the demands and product sales (Section 3.5). The first objective is the expected value of the final profit over the planning period. The second objective is a measure for the robustness of the planning; it involves maximization of the first quartile of the profit. The outcome of the modeling is a multiobjective, piecewise linear optimization problem (Section 3.6). Due to the discontinuities the problem is solved by means of a direct search method, the Nelder and Mead algorithm. The multiobjective problem is turned into two single objective problems. The solutions to these problems define the full range between maximum expected profit (with a high risk) and the robust profit (for a low risk scenario). This approach allows a production manager to take a preferred position between these two extremes. The result is a production planning and the associated profit.

Each step in the model development is illustrated by its application to the case study of the food additives plant, taking base case values for model parameters. To be able to make a good evaluation of the risks, the sensitivity of the profit and the optimal planning are studied for small changes from the nominal model parameters

(Section 3.7). Finally, the implementation aspects of the proposed planning method are discussed (Section 3.8).

3.3
Mathematical Description of the Planning Problem

To solve this production planning problem we need to formulate it in a more formal way. Assume that the current product portfolio consists of n products that can be produced on several exchangeable units. The decision about on which specific unit a product will be made is established in the production schedule and is considered to be outside the scope of the production planning. In the production planning, the planners take the overall production capacity into consideration without allocating products to specific units.

In the production planning the following decisions variables should be established:

- the amount of the current products sold in annual contracts in ton per year:

  $y_i,\ i \in \{1, 2, \ldots, n\}$;

- the capacity left open for the current products and for the new one in ton per year:

  $x_i,\ i \in \{1, 2, \ldots, n, n+1\}$.

The information that is needed to make these decisions consists of the following parameters:

- the profit that can be made with the production of one ton of a certain product, depending on the retail price and on the production costs, divided into:
  - the profit made on the current products sold in annual contracts in dollars per ton:

    $\sigma_i,\ i \in \{1, 2, \ldots, n\}$;

  - the profit made on sold amounts of the current products and of the new one during the production period in dollars per ton:

    $\varrho_i,\ i \in \{1, 2, \ldots, n, n+1\}$;

- the total production time available in hours per year: T;
- the production time needed to make the products in hours per ton:

  $\tau_i,\ i \in \{1, 2, \ldots, n, n+1\}$;

- the demand for the current products that can be sold in the annual contracts:

  $\delta_i,\ i \in \{1, 2, \ldots, n\}$;

Since the total amount of product made during the production period cannot exceed the total available production time, the decision variables are restricted by:

$$T = \sum_{i=1}^{n} \tau_i y_i + \sum_{i=1}^{n+1} \tau_i x_i \tag{1}$$

The assumption is made that the planners have enough and correct information to make a good estimation of the values of these parameters. Therefore, these parameters are assumed to be the deterministic factors in the planning problem.

- the demand for the current products and for the new one during the production period in ton per year: d_i, $i \in \{1, 2, \ldots, n, n+1\}$.

The uncertainty of the demand during the production period is quite large and therefore these factors are assumed to have a stochastic nature. The assumption is made that the planners have enough information to indicate the minimum and maximum demand that can be expected and the mode of the demand, that is, the demand for which the probability density function is maximized.

The demand for the products will therefore be modeled by a triangular distribution with the probability density function given in Eq. (2). This triangular form (see Fig. 3.2) corresponds with the shape used in fuzzy modeling.

$$f(d_i) = \begin{cases} \dfrac{2(d_i - \alpha_i)}{(\beta_i - \alpha_i)(\gamma_i - \alpha_i)} & \alpha_i \le d_i \le \beta_i \\ \dfrac{2(\gamma_i - d_i)}{(\gamma_i - \beta_i)(\gamma_i - \alpha_i)} & \beta_i < d_i \le \gamma_i, \ i \in \{1, 2, \ldots, n, n+1\} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

in which α_i is the minimum, β_i the mode and γ_i the maximum of the demand d_i for a certain product i.

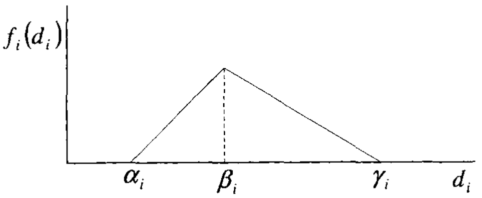

Figure 3.2 The triangular probability density function of the demand

The expected demand for product i will then be:

$E(d_i) = \dfrac{\alpha_i + \beta_i + \gamma_i}{3}$, $\in \{1, 2, \ldots, n, n+1\}$.

The probability distribution of the demand d_i reads:

$$F(d_i) = \begin{cases} 0 & d_i \le \alpha_i \\ \dfrac{(d_i - \alpha_i)^2}{(\beta_i - \alpha_i)(\gamma_i - \alpha_i)} & \alpha_i < d_i \le \beta_i \\ 1 - \dfrac{(\gamma_i - d_i)^2}{(\gamma_i - \beta)(\gamma_i - \alpha_i)} & \beta_i < d_i \le \gamma_i \\ 1 & d_i > \gamma_i \end{cases}, \ i \in \{1, 2, \ldots, n, n+1\}. \tag{3}$$

In Section 3.4 the mathematical description of the planning problem will be continued, but the generic problem will first be applied to a case study in a plant where various types of food additives were made.

3.3.1
Case Study in a Food Additives Plant

In a multiproduct multipurpose batch plant different food additives are produced on two reactors. The present portfolio consists of two product groups A and B. Having strong indications about a growing demand for a new product C, operations management wants to reevaluate the current product portfolio and the production planning for the coming year.

The product groups A and B are manufactured on two exchangeable reactors. Planning management has estimated the production times based on the current *annual operation plan*. The total amount of available operating time for the reactors is determined by the available time in a year minus 15 % down and changeover time, resulting in 4625 hours for reactor 1 and 4390 hours for reactor 2. Together this results in a total production time for the coming year of $T = 9015$ hours.

Table 3.1 shows the estimated values for all parameters in the planning problem. From the figures it is clear that the total production capacity could have been sold out in the annual contracts, since the demand for the product groups A and B is high enough. The new product C however is expected to be very profitable and it would very likely be an unwise decision to sell out the total production capacity. Unfortunately the demand for the new product C is not very certain.

Table 3.1 Estimated values for the planning parameters

	Product group A	**Product group B**	**Product C**
Production time in hours per ton	$\tau_A = 0.24$	$\tau_B = 0.47$	$\tau_C = 1.4$
Contract profit in \$ per ton	$\sigma_A = 1478$	$\sigma_B = 897$	–
Demand for contracts in ton per year	$\delta_A = 20\,000$	$\delta_B = 11\,000$	–
Profit in production period in \$ per ton	$\varrho_A = 1534$	$\varrho_B = 953$	$\varrho_C = 3350$
Minimum demand in ton per year	$\alpha_A = 16\,040$	$\alpha_B = 8350$	$\alpha_C = 0$
Mode of demand in ton per year	$\beta_A = 17\,550$	$\beta_B = 8900$	$\beta_C = 850$
Maximum demand in ton per year	$\gamma_A = 19\,900$	$\gamma_B = 9150$	$\gamma_C = 1600$

This case study will be continued in Section 3.4.1.

3.4
Modeling the Profit of the Production Planning

The criterion on which planning will be assessed is the total profit that is achieved after the production period when the production is executed in accordance with the production planning. For that, not only is the expected profit important, but also the certainty that this profit will be achieved should be taken into account in the final decision. If a small deviation of the expected demand results in a much lower profit than expected, it could be safer to choose a more robust planning with a lower, but more certain profit.

Let z_i, $i \in \{1, 2, \ldots n, n+1\}$ be the sold amount of products when the production period is finished. Together with the products that are sold before the production period in annual contracts, the final profit that will be made in this period equals:

$$P(y_1, \ldots, y_n, z_1, \ldots, z_n, z_{n+1}) = \sum_{i=1}^{n} \sigma_i y_i + \sum_{i=1}^{n+1} \rho_i z_i \tag{4}$$

The amount of products sold during the production period will depend on the demand for these products and the available production time.

If the demand is lower than the amount that can be produced in the available production time, then the total demand can be satisfied. However, if the demand is larger than the available capacity then only that amount of product can be made and sold. Therefore, the total amount of product sold during the production period will equal $z_i = \min(d_i, x_i)$, $i \in \{1, 2, \ldots n, n+1\}$. The same holds for the amounts sold in annual contracts. The overall profit will then be

$$P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i \min(\delta_i, y_i) + \sum_{i=1}^{n+1} \rho_i \min(d_i, x_i). \tag{5}$$

For fixed values of the decision variables, the maximum and minimum total profit that can be achieved depends on the planned amounts of the products on the production planning, and on the maximum, respectively minimum demand for the products:

$$\begin{aligned} \max P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) &= \sum_{i=1}^{n} \sigma_i \min(y_i, \delta_i) + \sum_{i=1}^{n+1} \rho_i \min(x_i, \gamma_i) \\ \min P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) &= \sum_{i=1}^{n} \sigma_i \min(y_i, \delta_i) + \sum_{i=1}^{n+1} \rho_i \min(x_i, \alpha_i). \end{aligned} \tag{6}$$

For fixed values of the decision variables the *probability density of the final profit* can now be derived from the probability density of the demand for the products during the production period, under the assumption that the demands for these products are mutually independent.

In general for a linear combination $w = ax + by$, where the stochastic variables x and y are independent, the density function of w reads (Papoulis 1965):

$$f_W(w) = \frac{1}{|ab|} \int_{-\infty}^{\infty} f_x\left(\frac{w - y_1}{a}\right) f_y\left(\frac{y_1}{b}\right) \mathrm{d}y_1 \quad \text{with } y_1 = by. \tag{7}$$

The final profit was defined by

$$P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i \min(\delta_i, y_i) + \sum_{i=1}^{n+1} \varrho_i \min(d_i, x_i).$$

Let p_i, $i \in \{1, 2, \ldots, n, n+1\}$ be the profit made by selling the amount z_i of product i during the production period, then $p_i = \varrho_i z_i$, with a minimum of 0 and a maximum of $\varrho_i x_i$.

Applying the general proposition (Eq. (1)) on the final profit P, the probability density function of P reads:

$$f_P(p) = \frac{1}{\prod_{i=1}^{n+1} \rho_i} \int_0^{\rho_1 x_1} \int_0^{\rho_2 x_2} \cdots \int_0^{\rho_n x_n} \prod_{i=1}^{n+1} f_{z_i}\left(\frac{p_i}{\rho_i}\right) \mathrm{d}p_n \ldots \mathrm{d}p_2 \, \mathrm{d}p_1$$
$$\text{with } p_{n+1} = p - \prod_{i=1}^{n} p_i - \sum_{i=1}^{n} \sigma_i y_i. \tag{8}$$

If the actual demand d_i, $i \in \{1, 2, \ldots, n, n+1\}$ is smaller than the planned capacity x_i then the sold amount of product z_i will equal the demand d_i. In that case, the probability density of z_i will follow the probability density of the demand d_i. However, if the actual demand d_i is larger than the planned capacity x_i then only the amount $z_i = x_i$ will be produced and sold. The probability that this will happen is the probability of a demand larger than the planned capacity, that is, $d_i \geq x_i$ (see Fig. 3.3).

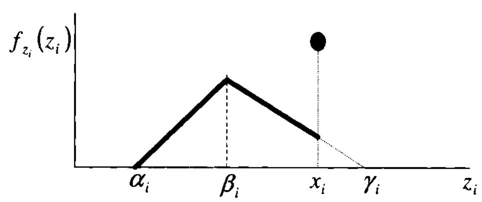

Figure 3.3 The probability density function of the sold amount of product

By this observation, the probability density $f_{Z_i}(z_i)$, $i \in \{1, 2, \ldots, n, n+1\}$ of the sold amount of product z_i satisfies:

$$f_{z_i}(z_i) = \begin{cases} f_{D_i}(z_i) & 0 < z_i < x_i \\ 1 - F_{D_i}(z_i) & z_i = x_i \\ 0 & z_i > x_i \end{cases}, \; i \in \{1, 2, \ldots, n, n+1\} \tag{9}$$

Unfortunately, by the local discontinuities in the probability density function of the final profit, the integrals cannot be solved analytically. The derived theoretical results will now be applied on the case study described in Section 3.3.1.

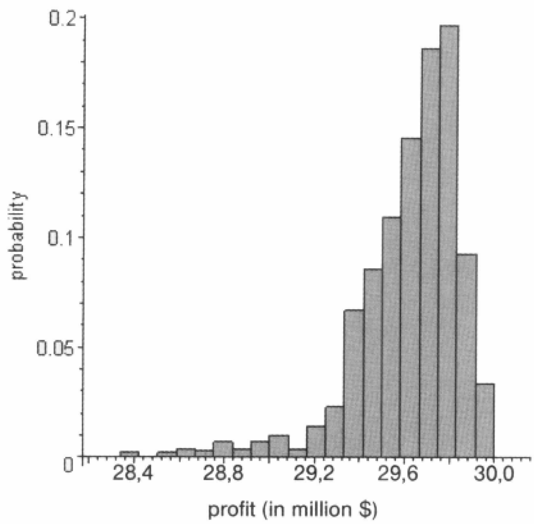

Figure 3.4 Simulation of the final profit for one possible production planning

3.4.1
Modeling the Profit for the Food Additives Plant

In the aforementioned case study the production planning should be made for two current product groups A and B and one new product C. For reasons of comprehensibility, the assumption is made that no products were sold before the production period, that is $y_A = y_B = 0$.

The total profit that can be made will now depend on the demand for the products during the production period and on the planned amounts of the different products, that is, $P(x_A, x_B, x_C) = 1534 \min(d_A, x_A) + 953 \min(d_B, x_B) + 3350 \min(d_C, x_C)$,

under the restriction that the total production time will not be exceeded, $0.24\, x_A + 0.47\, x_B + 1.4\, x_C = 9015$.

The probability density function of the profit satisfies:

$$f_P(p) = \frac{1}{1534 \cdot 953 \cdot 3350} \cdot \int_0^{1534 x_A} \int_0^{953 x_B} f_{Z_A}\left(\frac{p_A}{1534}\right) \cdot f_{Z_B}\left(\frac{p_B}{953}\right) \cdot f_{Z_C}\left(\frac{p - p_A - p_B}{3350}\right) \mathrm{d}p_B\, \mathrm{d}p_A \tag{10}$$

Although this probability density cannot be solved analytically, it can be simulated for fixed values of x_A, x_B, x_C by randomly picking a certain demand for the products A, B and C from their individual probability density functions.

Figure 3.4 shows a probability histogram of the simulated profit for a production planning with $x_A = 17\,000$, $x_B = 9588$, $x_C = 950$ ton per year. The sample size taken is 1000. The unequal distribution in the left tail is caused by the sample size and would not be present in the theoretical distribution. This histogram shows a very skewed distribution to the left. This skewness is caused by the discontinuities in the function $P(x_A, x_B, x_C)$. In Section 3.5.1 this case study will be continued.

3.5 Modeling the Objective Functions

The quality of a certain production planning will be assessed on the expected value of the profit that can be achieved with the planning. The expected value of the final profit satisfies:

$$E_P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i \min(\delta_i, y_i) + \sum_{i=1}^{n+1} \rho_i E_{Z_i}(x_i) \tag{11}$$

with $y_i \leq \delta_i, \ i \in \{1, 2, \ldots, n\}$

From the probability density function $f_{Z_i}(z_i)$, $i \in \{1, 2, ..., n, n+1\}$ of the amount of sold products, the expected value of the amount z_i can be determined by:

$$E_{Z_i}(x_i) = \int_0^{x_i} z_i f_{d_i}(z_i)\, dz_i + x_i\big(1 - F_{d_i}(x_i)\big), \ i \in \{1, 2, \ldots, n, n+1\} \tag{12}$$

There are four possibilities for the planned amount x_i in comparison to the expected demand d_i, $i \in \{1, 2, ..., n, n+1\}$. Remember that d_i was expected to lie between α_i and γ_i with a mode β_i. Elaboration of Eq. (12) yields:

1. If $x_i \leq \alpha_i$ then $E_{Z_i}(x_i) = x_i$.
2. If $\alpha_i \leq x_i \leq \beta_i$ then

$$E_{Z_i}(x_i) = \frac{1}{(\beta_i - \alpha_i)(\gamma_i - \alpha_i)} \left[-\frac{1}{3}x_i^3 + \alpha_i x_i^2 + (\beta_i\gamma_i - \alpha_i\beta_i - \alpha_i\gamma_i)x_i + \frac{1}{3}\alpha_i \right].$$

3. If $\beta_i < x_i \leq \gamma_i$ then

$$\begin{aligned} E_{Z_i}(x_i) &= \frac{1}{(\beta_i - \alpha_i)(\gamma_i - \alpha_i)} \left[\frac{2}{3}\beta_i^3 - \alpha_i\beta_i^2 + \frac{1}{3}\alpha_i^3 \right] \\ &+ \frac{1}{(\gamma_i - \beta_i)(\gamma_i - \alpha_i)} \left[\frac{1}{3}x_i^3 - \gamma_i x_i^2 + \gamma_i^2 x_i + \frac{2}{3}\beta_i^3 - \gamma_i\beta_i^2 \right] \end{aligned}$$

4. If $x_i > \gamma_i$ then $E_{Z_i}(x_i) = \dfrac{\alpha + \beta + \gamma}{3}$.

Unfortunately, the expected value of the profit that can be achieved with a certain production planning, does not guarantee that this profit will be achieved in reality.

Due to the skewed density function, for most choices of the production planning, the median of the profit will be higher than the expected value of the profit. This means that with a probability of more than 50 % the real profit will be higher than the expected value. And with a probability of less than 50 % the real profit will be lower than the expected value. As a consequence, the average deviation below the expected profit will be larger than the average deviation above the expected profit.

As a measure for the robustness of the planning the first quartile $Q_P(y_i, x_i)$ of the profit is chosen. The probability that the real profit will be lower than this first quartile equals 25 %. Under the assumption that the demands for the different products are mutually independent, the first quartile of the total profit will be a linear combination of the first quartiles of the sold amounts of products z_i, $i \in \{1, 2, \ldots, n, n+1\}$ and can be written as:

$$Q_P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i y_i + \sum_{i=1}^{n+1} \rho_i Q_{Z_i}(x_i) \tag{13}$$

with $y_i \leq \delta_i,\ i \in \{1, 2, \ldots, n\}$

The first quartile of the sold product, $Q_{Z_i}(x_i)$, will equal the first quartile of the demand d_i if the planned amount x_i is larger than this demand, otherwise it will equal x_i:

$$Q_{Z_i}(x_i) = \min\left(Q_{D_i}(d_i), x_i\right),\ i \in \{1, 2, \ldots, n, n+1\} \tag{14}$$

As long as the first quartile $Q_{D_i}(d_i)$ is smaller than the mode β_i of the demand, i.e., $F_{D_i}(\beta_i) = \dfrac{(\beta_i - \alpha_i)^2}{(\beta_i - \alpha_i)\,(\gamma_i - \alpha_i)} \leq 0.25 \Leftrightarrow \beta_i \leq 0.75\ \alpha_i + 0.25\ \gamma_i$, the first quartile $Q_{D_i}(d_i)$ is the

solution of $F_{D_i}(d_i) = \dfrac{(d_i - \alpha_i)^2}{(\beta_i - \alpha_i)\,(\gamma_i - \alpha_i)} = 0.25$ for d_i, i.e.,

$$Q_{D_i}(d_i) = \alpha_i + \sqrt{0.25(\beta_i - \alpha_i)(\gamma_i - \alpha_i)}. \tag{15}$$

If the first quartile $Q_{D_i}(d_i)$ is larger than the mode β_i of the demand, i.e., $\beta_i > 0.75\ \alpha_i + 0.25\ \gamma_i$, then

$$Q_{D_i}(d_i) = \gamma_i + \sqrt{0.75(\gamma_i - \beta_i)(\gamma_i - \alpha_i)}. \tag{16}$$

3.5.1 Modeling the Objective Functions of the Food Additives Plant

For the case study described in Section 3.4.1, the objective functions can now be modeled.

The expected value of the final profit satisfies:

$$E_P(x_A, x_B, x_C) = 1534 E_{Z_A}(x_A) + 953 E_{Z_B}(x_B) + 3350 E_{Z_C}(x_C) \tag{17}$$

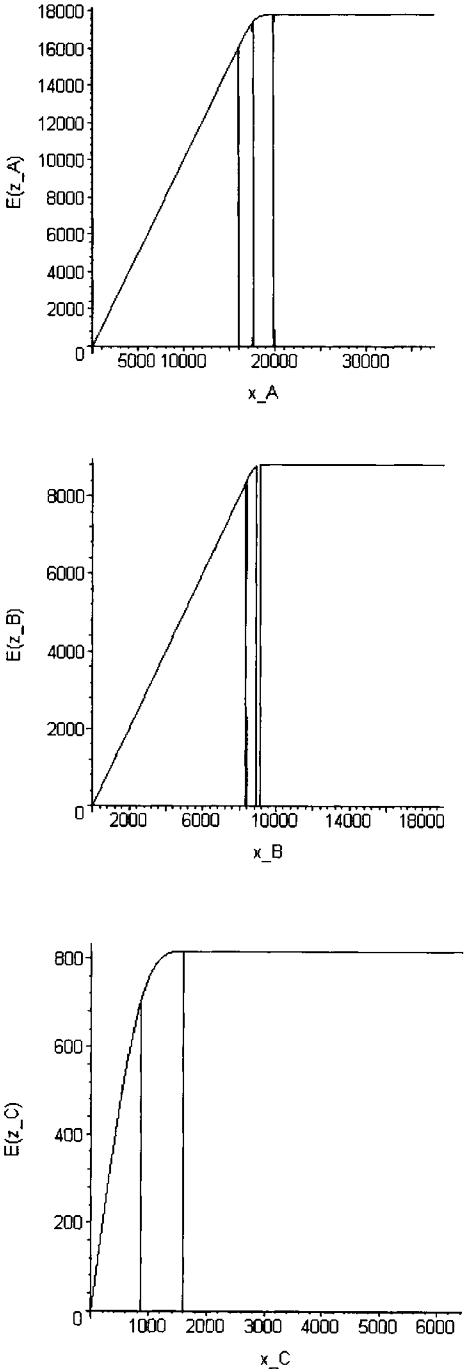

Figure 3.5 The expected amount of sold products A, B and C for a certain production planning

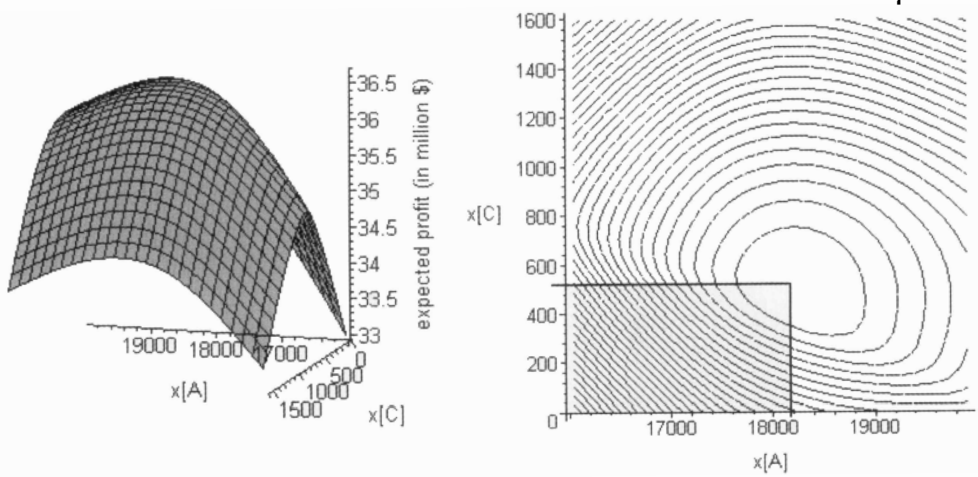

Figure 3.6 The expected profit for different choices of the production planning

Figure 3.5 shows the functions of $E_{Z_A}(x_A)$, $E_{Z_B}(x_B)$ and $E_{Z_C}(x_C)$, respectively. The vertical lines indicate the different parts of the piecewise functions, that is, $x_i < \mathrm{a}_i$, $\mathrm{a}_i \leq x_i \leq \mathrm{b}_i$, $\beta_i < x_i \leq \gamma_i$, $x_i > \gamma_i$, $i \in \{A, B, C\}$.

Figure 3.6 shows the three-dimensional plot and the contour plot of $E_p(x_A, x_B, x_C)$ with the restriction that the total production time is filled, but not exceeded, that is,

$$x_B = \frac{9015 - 0.24x_A - 1.4x_C}{0.47} \tag{18}$$

The figures show that there is one production planning that leads to a maximum value for the expected profit $E_P(x_A, x_B, x_C)$. A rough estimation can already be made from the contour plot that in this production planning around 18,200 tons will be planned of product A, around 500 tons will be planned for product C, which will leave capacity for about 8400 tons of product B.

The second objective function is the first quartile of the total profit, that is,

$$Q_P(x_A, x_B, x_C) = 1534 \min(17247, x_A) + 953 \min(8682, x_B) + 3350 \min(583, x_C). \tag{19}$$

Figure 3.7 shows the three-dimensional plot and the contour plot of $Q_P(x_A, x_B, x_C)$ again with the restriction that the total production time is filled, but not exceeded (Eq. (18)).

Also these figures show that there is one production planning that leads to a maximum value for the first quartile $Q_P(x_A, x_B, x_C)$ of the final profit. Again a rough estimation can be made from the contour plot. In this production planning around 17,200 tons will be planned of product A, around 600 tons will be planned for product C, which will leave capacity for about 8600 tons of product B.

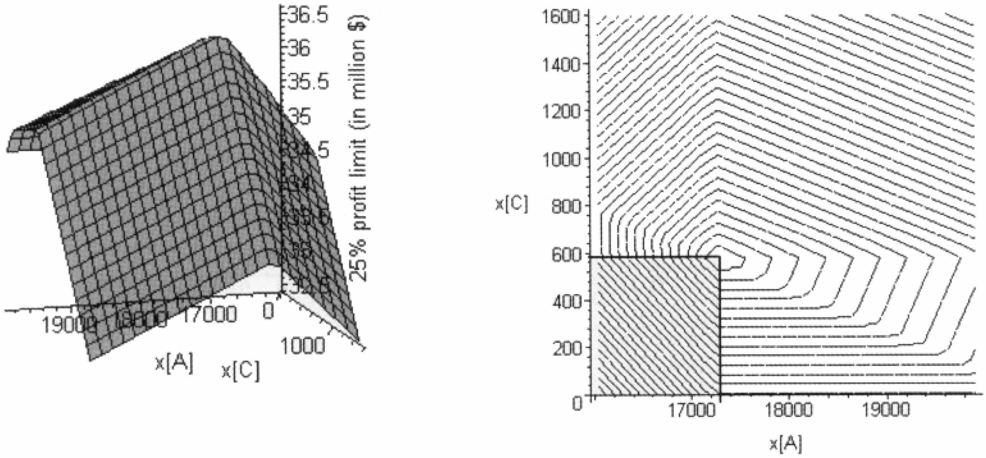

Figure 3.7 The first quartile of the profit for different choices of the production planning

3.6 Solving the Optimization Problem

The production planning problem is translated into a multicriteria piecewise linear optimization problem. The problem, however, will not be solved as a multicriteria problem, since the objectives are the extremes of one scale, from an uncertain but high profit to a more certain but low profit. For planning management willing to run a higher risk for a higher expected profit, the most profitable planning, corresponding with the maximum expected profit, may be the right choice. For planning management not willing to run any risk the most robust planning, that is, the one with the highest first quartile will be a more certain choice, although the expected profit will be much lower in that case. For every nuance of profitableness at a certain risk, a production planning in between those two extremes can be found.

The optimization problem is as follows:

determine $y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}$ for which

$$E_P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i y_i + \sum_{i=1}^{n+1} \rho_i E_{Z_i}(x_i)$$

or

$$Q_P(y_1, \ldots, y_n, x_1, \ldots, x_n, x_{n+1}) = \sum_{i=1}^{n} \sigma_i y_i + \sum_{i=1} \rho_i Q_{Z_i}(x_i)$$

is maximized, subject to

$$T = \sum_{i=1}^{n} \tau_i y_i + \sum_{i=1}^{n+1} \tau_i x_i .$$

Due to the piecewise character of the objective functions, common gradient methods for optimization cannot be used. Therefore a choice is made to use a direct-search method, the simplex method of Nelder and Mead (Nelder 1965). The Nelder-Mead algorithm is mentioned in many textbooks, but very seldom explained in detail. That is why a short description of the working method is given here (Fig. 3.8).

If there are n decision variables in the optimization problem, the Nelder-Mead algorithm will start by choosing $n + 1$ points arbitrarily. For reason of simplicity, assume that there are two decision variables. Then there will be 3 starting points (P1, P2, P3), which together form a triangle. This triangle is called the *simplex*. If the objective function is to be maximised then the point with the smallest value (P1) is reflected into the middle of the opposite side, in the supposition that this will lead to a better value for the objective function.

There are four different possibilities on how the search will continue.

1. If the objective value of the new point (P4) lies between the best and the worst values of the other points (P2, P3), then P4 is accepted as a new starting point and the new simplex is (P2, P3, P4) (Fig. 3.8a).
2. If the objective value of the new point (P4) is better than all others then the point is even further drawn out, twice as far from the reflecting point as P4. Therefore, P5 will form the new simplex with P2 and P3 (Fig. 3.8b).
3. If the objective value of the new point (P4) is worse than all others but better than the original (P1), then a new point P6 is evaluated half as far from the reflecting point as P4 (Fig. 3.8c). Again there are two possibilities:
   a. If P6 is worse than all others then the whole simplex is decreased by half towards the best point in the simplex. The new simplex is then (P2, P3', P6') (Fig. 3.8d).
   b. Otherwise, the new simplex is (P2, P3, P6).
4. If the objective value of the new point (P4) is worse than P1 then a new point P7 is defined halfway between the reflecting point and P1 itself. The new simplex will then be (P2, P3, P7) (Fig. 3.8e).

The new simplex is now used as starting point and the same procedure is performed until the best point and the second best point differ less than a fixed value ε. For both objective functions the Nelder-Mead algorithm could be used to find the production planning with the highest expected profit and the production planning with the highest first quartile, that is, the most robust planning. The planners will be provided with information about the robustness and the expected profit of different possibilities of the planning, on which they can base their final choice.

In the next paragraph, the Nelder-Mead algorithm will be applied on the objective functions in the case study.

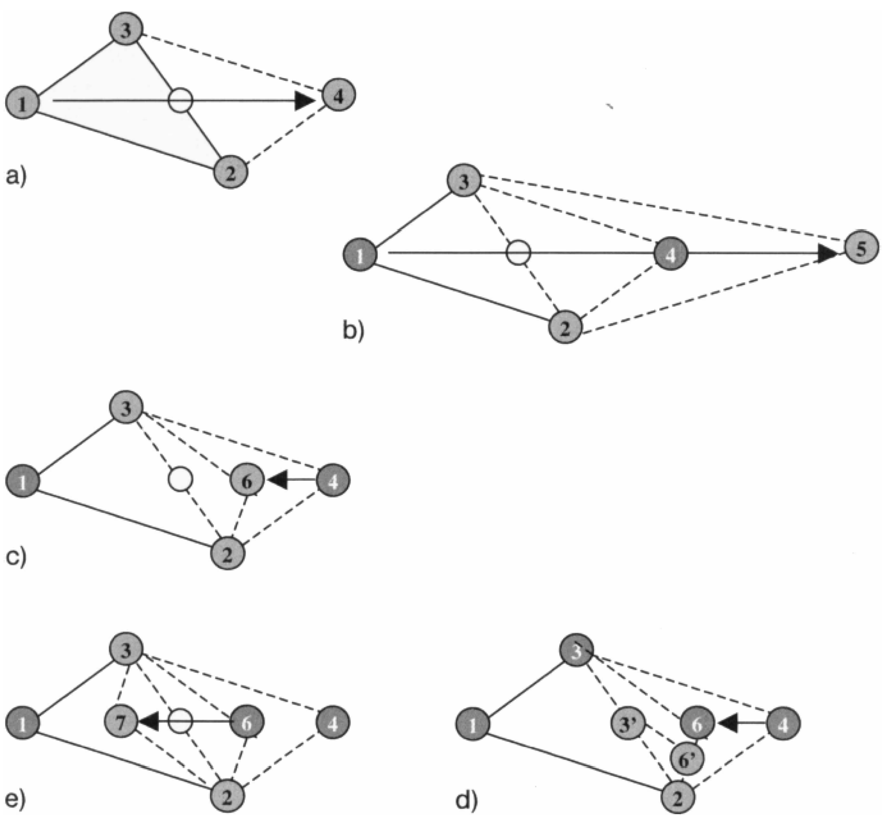

Figure 3.8 Nelder-Mead algorithm

3.6.1 Solving the Optimization Problem in the Case Study

For the case study the production planning problem is translated into the following optimization problem:

Determine x_A, x_B, x_C *for which*

$$E_P\,(x_A, x_B, x_C) = 1534\ E_{Z_A}\,(x_A) + 953\ E_{Z_B}\,(x_B) + 3350\ E_{Z_C}\,(x_C)$$

or

$$Q_P\,(x_A, x_B, x_C) = 1534\ \min(17247, x_A) + 953\ \min(8682, x_B) + 3350\ \min(583, x_C)$$

is maximized, subject to

$$x_B = \frac{9015 - 0.24\ x_A - 1.4\ x_C}{0.47}.$$

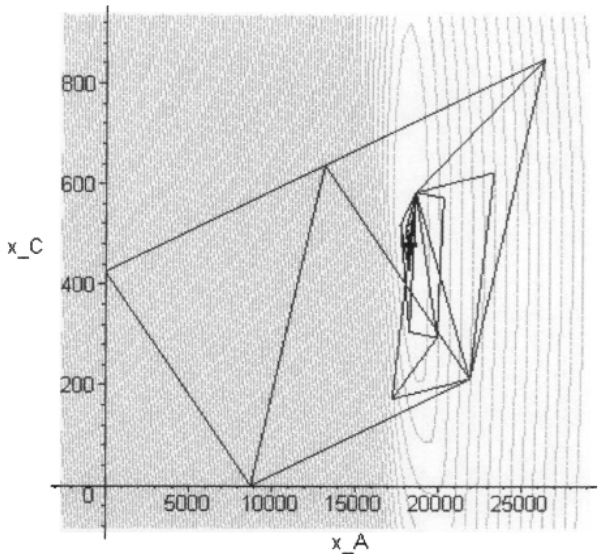

Figure 3.9 The Nelder-Mead algorithm to determine the maximum expected profit

Figure 3.9 shows for the decision variables x_A and x_C, the contour plot of the expected profit $E_P(x_A, x_B, x_C)$ with the simplices resulting from the Nelder-Mead algorithm.

The optimal planning found is the planning in which 18,221 tons for product A, 8445 tons for product B and 481 tons for product C are planned (Table 3.2). The expected profit for this planning equals: $E_P(18\,221, 8445, 481) = 3.67 \cdot 10^7$ dollars. The first quartile for this planning, that is, the robustness of the planning, equals $Q_P\,(18\,221, 8445, 481) = 3.61 \cdot 10^7$ dollars.

The most robust planning, that is, the one with the highest first quartile, is found by applying the Nelder-Mead algorithm to:

$$Q_P\,(x_A, x_B, x_C) = 1534 \min(17247, x_A) + 953 \min(8682, x_B) + 3350 \min(583, x_C)$$

$$\text{subject to } x_B = \frac{9015 - 0.24\,x_A - 1.4\,x_C}{0.47}.$$

The company itself should now decide if they will speculate on a higher expected profit or if they will prefer a lower but more certain profit. The diagram in Fig. 3.10 below can serve as an informative tool for the decision making.

Table 3.2 Results of the optimization

Optimization	$E_p(x_A, x_B, x_C)$ (million dollars)	$Q_p(x_A, x_B, x_C)$ (million dollars)	x_A (ton)	x_B (ton)	x_C (ton)
Max. expected profit	36.7	36.1	18 221	8445	481
Max. robustness	36.3	36.6	17 248	8647	579

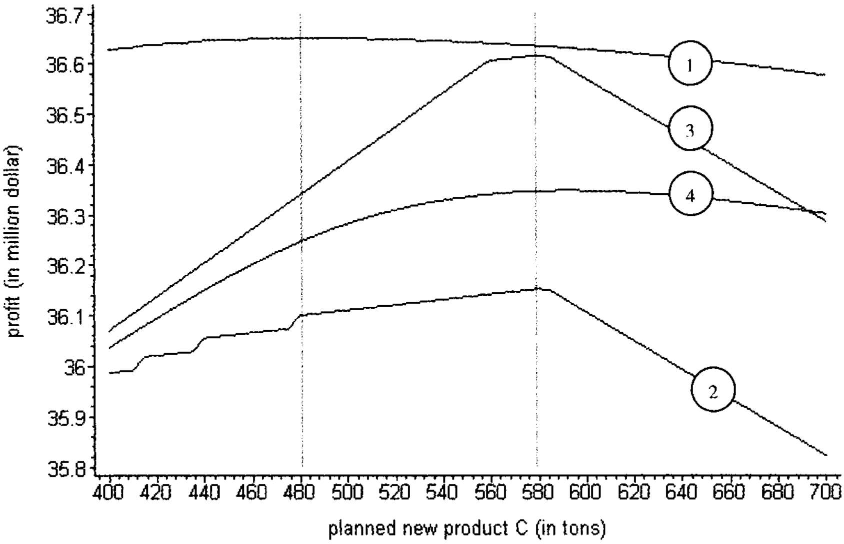

Figure 3.10 The profit levels for planned amounts of product C

In the planning with the maximum expected profit 481 tons were planned for product C. In the planning with the maximum 25 % limit of the profit 579 tons were planned for product C. Figure 3.10 shows for the interesting region for the planned amounts of product C, that is, between 400 and 700 tons, four different profit levels:

- Profit level 1 is the maximum expected profit that can be achieved with the planned amount of product C, assuming that the remaining capacity is optimally divided over the product groups A and B.
- Profit level 2 is the 25 % profit limit if the planning with the maximum expected profit (see profit level 1) is implemented.
- Profit level 3 is the maximum 25 % profit limit that can be achieved with the planned amount of product C, assuming that the remaining capacity is optimally divided over the product groups A and B.
- Profit level 4 is the expected profit if the planning with the maximum 25 % profit limit (see profit level 3) is implemented.

Assume that the company, based on the information from Fig. 3.10, decides to plan 579 tons of product C. This seems to be a profitable but not too risky choice. Compared to the planning with for example $x_C = 481$, the maximum expected profit is a bit lower, but all other profit levels are very high. It will now depend on the choice for the planned amounts of products in the groups A and B, if they can expect a higher profit with more uncertainty or a lower profit with less uncertainty. However, the 25 % profit limit gives no information about how the profit is distributed below this limit. To have more information on how low the profit could be, Fig. 3.11 shows, for 579 tons planned for product C, the probability distribution of the profit for different amounts planned for product groups A and B.

From Fig. 3.11 it is clear that the more product A is planned for, the more profit can be expected, but the more uncertainty exists whether this profit will be achieved. For instance, if the company decides to plan 18,000 tons for products from product group A, then the maximum profit they can achieve is about US \$37.5 million and there is a probability of 60% that this profit will not be achieved. There even is a probability of 25% that the profit will be lower than US \$36.3 million.

3.7
Sensitivity Analysis of the Optimization

The planners settle several parameters on which the determination of the optimal planning is based. Some deviation from the expected values will lead, after completion of the production period, to profit results that differ from what was expected. To be able to make a good evaluation of the risks, the sensitivity of the profit and the optimal planning will be studied for small deviations from the expected values of the following parameters in the objective functions:

1. the profit ϱ_i, $i \in \{1, 2, ..., n, n+1\}$ made on the sold amounts of the current products and the new one during the production period;
2. the minimum α_i, the mode β_i and the maximum γ_i of the demand d_i, $i \in \{1, 2, ..., n, n+1\}$ of all products during the production period.

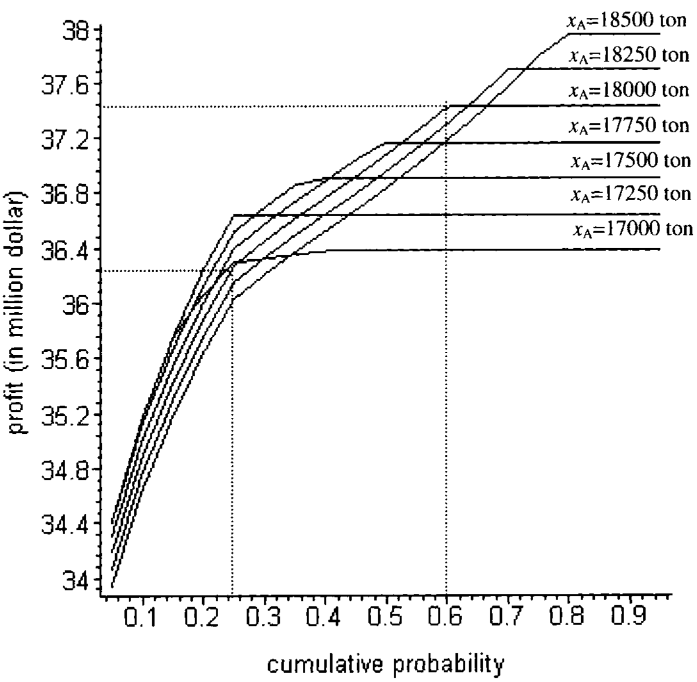

Figure 3.11 Probability distributions of the profit for different amounts of product A

Also the sensitivity of the solution to the parameters in the production time constraint will be studied:

3. the total production time T available per year;
4. the production time τ_i, $i \in \{1, 2, \ldots, n, n+1\}$ that is needed to make one ton of product.

Let $(\underline{y}_I^{OPT}, \underline{x}_I^{OPT})$ be the optimal planning with respect to the maximum expected profit. Let $E_P^{OPT} = E_P\,(\underline{y}_I^{OPT}, \underline{x}_I^{OPT})$ be the maximum value of the expected profit.

The effect on the optimal value E_P^{OPT} of a small change in for example the parameter ϱ_1 is now given by $\dfrac{\partial E_P^{OPT}}{\partial \varrho_1}$, the absolute sensitivity coefficient of ϱ_1. The same can be done for all other parameters and for both objective functions.

The stepwise character of the objective functions is in this case for the differentiation not a problem, since the discontinuities of the function are only found in the decision variables, not in the parameters. For the parameters, the objective functions are continuous and therefore differentiable.

The size of the absolute sensitivity coefficients depends on the scale on which the parameters are measured. To make them comparable they are scaled by $\dfrac{\partial E_P^{OPT}}{\partial \varrho_1} \cdot \dfrac{\varrho_1}{E_P^{OPT}}$, which form the relative sensitivity coefficients, for example, for the parameter ϱ_1.

The other uncertain parameters influence the only constraint in the problem: $\sum\limits_{i=1}^{n} \tau_i y_i + \sum\limits_{i=1}^{n+1} \tau_i x_i = T$. Changes in the parameters τ_i or in T will cause the optimal planning to be no longer feasible. In practice a decrease in comparison to the expected values of τ_i, $i \in \{1, 2, \ldots, n, n+1\}$ or an increase in comparison to the expected value of T will not cause any problem. The planned amounts of products can still be made and it will be possible to make an even higher amount of product than was planned, although it is not guaranteed that this extra amount can be sold as well. Information is needed to determine for which product the extra production time should be used to make as much profit as possible.

An increase, however, of the values of τ_i, $i \in \{1, 2, \ldots, n, n+1\}$ or a decrease of the overall production time T will cause the optimal planning to be unachievable, and information is needed at the expense of which product the reduction of production time should be found to keep the profit as high as possible.

In general Lagrange multipliers can be used to investigate the influence of changes in the right hand side of the constraint parameters, but due to the piecewise character of the objective functions the Lagrange multiplier cannot analytically be calculated for an arbitrary T. The sensitivity analysis will be illustrated on the basis of the case study from Section 3.6.1.

3.7.1
Sensitivity Analysis in the Case Study

Assume that the company, based on the information given in Figs. 3.10 and 3.11, has decided to plan 18,000 tons for product group A, 8265 tons for product group B and 579 tons of product C. The expected profit for this planning equals US $36.6 million and there is a probability of 25 % that the real profit is lower than US $36.3 million.

To study the sensitivity of the results for small changes in the profit parameters ϱ_A, ϱ_B, ϱ_C and the demand parameters α_A, β_A, γ_A, α_B, β_B, γ_B, α_C, β_C, γ_C, the relative sensitivity coefficients are calculated in the neighbourhood of the expected values of these parameters, presented in Table 3.3.

Table 3.3 Sensitivity of the profit for different planning parameters

Parameter	Expected value	Expected profit		25 % Probability limit	
		Absolute sensitivity	Relative sensitivity	Absolute sensitivity	Relative sensitivity
ϱ_A	1534 $ per ton	17,578	0.74	17,247	0.73
ϱ_B	953 $ per ton	8265	0.22	8265	0.22
ϱ_C	3350 $ per ton	531	0.05	579	0.05
α_A	16,040 ton	411	0.18	0	0
β_A	17,550 ton	347	0.17	0	0
γ_A	19,900 ton	166	0.09	0	0
α_B	8350 ton	0	0	0	0
β_B	8900 ton	0	0	0	0
γ_B	9150 ton	0	0	0	0
α_C	0 ton	539	0	0	0
β_C	850 ton	188	0.00	0	0
γ_C	1600 ton	100	0.00	0	0

The expected profit and the first quartile of the profit are most sensitive for the profit that can be made with the current product A, due to the high amounts of this product planned to be made and sold, according to expectations. Furthermore, the profit is sensitive to changes with respect to the profit that can be made with one ton of product B, and for changes in the demand of product A. Small changes in the other parameters have hardly any influence on the profit that can be made with the chosen planning.

A change in the total production time will also influence the profit that could be achieved with the implementation of the chosen planning. Figure 3.12 shows the change in expected profit, respectively 25 % profit limit, if the increase or decrease in production time T is totally covered by an increase, i.e., a decrease in planned amounts of product group A, B or product C.

Figure 3.12 shows clearly that a decrease in the total production time should never be covered at the expense of product group A, but should be found in a smaller amount of product group B or product C. On the other hand, when the production time is higher than expected, the extra time should be used to produce more of product C, although the differences are not so large. The robustness of the planning for

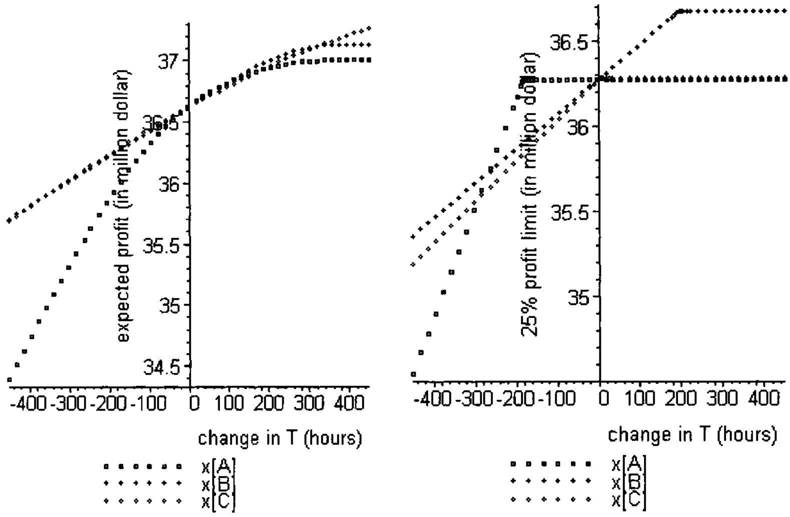

Figure 3.12 The changes in profit for smaller or large production time

a small decrease in total production time will not change if less of product A is made. However, a large decrease should go at the expense of the other products. An increase of the production time should as well be used to make more of the product group B or product C.

Figure 3.13 shows the change in expected profit, respectively 25 % profit limit, if the increase or decrease in the time t_A needed to produce one ton of product A is totally covered by an increase, i.e., a decrease in the planned amounts of product group A, B or product C.

Figure 3.13 shows that an increase of the production time needed to make one ton of product A can best be covered by making less of product group B or product C, although to keep the same robustness it is better to make less of product A. For a decrease, it is best to make more of product C. An increase or decrease of the production time needed to make one ton of product group B or of product C, gives more or less the same results as for product group A.

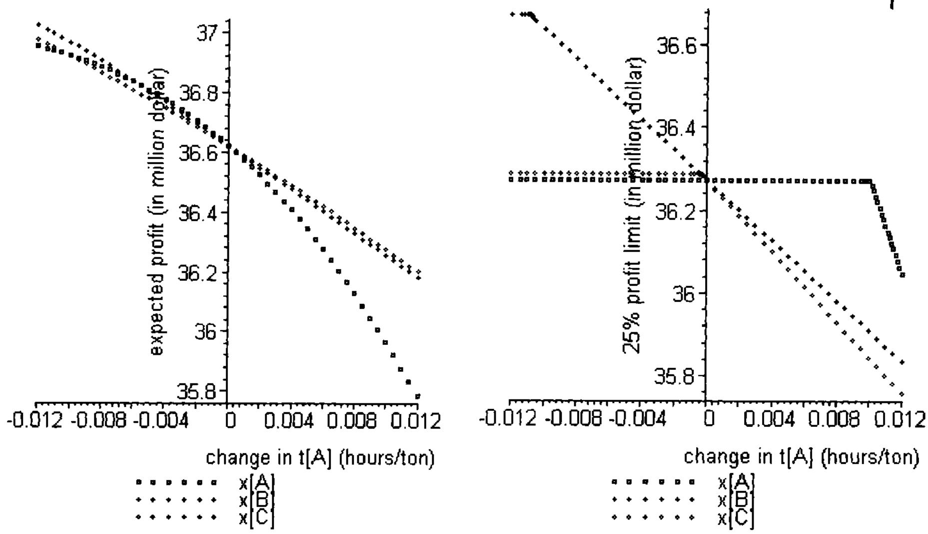

Figure 3.13 The changes in profit for a smaller or larger time/ton for product A

3.8 Implementation of the Optimization of the Production Planning

In the development of the method the focus was on the practical use for the planners in a multiproduct plant. The users of the method should not be bothered with the mathematical background of the method, which from their point of view can be considered as a black box. The emphasis should be on the information that should be acquired from them as an input for the method and on the results obtained from this information to be presented in a comprehensible and useful way. Knowledge of the transformation from the input into the output can increase the confidence in the results, but is not required to be able to use the planning method (see Fig. 3.14).

The *information from the planners*, needed as an input for the method, should consist of:

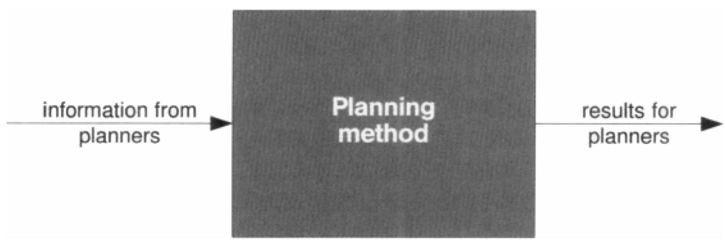

Figure 3.14 Implementation of the planning method

- the profits made on the current products sold in annual contracts in dollars per ton;
- the profits made on sold amounts of the current products and of new ones during the production period in dollars per ton;
- the total production time available in hours per year;
- the production times needed to make the products in hours per ton;
- the demand for the current products that can be sold in the annual contracts;
- the demand for the current products and for the new ones during the production period in ton per year described by:
  - the minimum demand in ton per year;
  - the mode of the demand in ton per year;
  - the maximum demand in ton per year.

Table 3.1 is an example of such input information.

The *results for the planners*, produced by the planning method, consist of:

- results of the optimization showing the optimal planning with respect to the maximum expected profit and the optimal planning with respect to the maximum 25 % profit limit (Table 3.2);
- the profit levels for planned amounts of new products showing the maximum expected profit and its corresponding 25 % profit limit and the maximum 25 % limit and its corresponding expected profit for different choices of free capacity for the new product(s). If more than one new product is taken into consideration, then the aggregate free capacity for these new products will be showed on the *x*-axis (Fig. 3.10).
- probability distributions of the profit for different planned amounts of the products showing the total probability distribution of a certain planning. For practical use, it should be easy to change the chosen amounts for all products to assess the effect of the changes on the profit distribution (Fig. 3.11).
- sensitivity of the profit for different planning parameters showing for a chosen planning which parameters are really influencing the profit, and by that require a good estimation of the expected value (Table 3.3);
- the changes in profit for smaller or larger production time (Fig. 3.12);
- the changes in profit for a smaller or larger time per ton for the products showing which adaptation to the planning should be made if the real values of production times differ from the expected ones (Fig. 3.13).

When the results are presented in such a way that the planners have full insight into the consequences of a chosen planning, the method will serve as a valuable decision support tool.

3.9
Conclusions and Final Remarks

A production planning method has been presented for a multiproduct manufacturing plant, which optimizes the profit under uncertainties in product demands. In the method these uncertainties are modeled by means of a simple triangular probability distribution, which is easy to specify. The optimization goal can be formulated as either a maximum expected profit or a robust profit (first quartile of the profit) to lower the risk. Due to discontinuities in the probabilistic distribution function of sold products a direct search optimization technique, Nelder-Mead, must be applied rather than a gradient-based optimization. The development and the application of the method have been highlighted by means of a case study taken from a food additives plant.

This method is considered practical because the required input data for the demand and process models and the profit function is easy to get by the users of the method, while the output information facilitates the interpretation of sensitivities of the optimized production planning in terms of common economic and product demand specification parameters. The method should be accessible to plant production management rather than to operations research specialized planning experts.

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