CHAPTER 3

Soil Components and Weight-Volume Parameters

3.1 PARTICLES, LIQUID, AND GAS

Soils are made of particles, gas (most often air), and fluid (most often water). Particles are also called *grains*. The space between the particles makes up the voids sometimes also called *pores*. If the voids are completely filled with air, the soil is called *dry*. If the voids are completely filled with water, the soil is called *saturated*. If the soil is filled partly with air and partly with water, the soil is called *unsaturated*. Figure 3.1 shows a soil sample and its graphical representation (the three-phase diagram discussed later in this chapter).

Note that in some cases, there is a subtle distinction between saying that a soil is dry and saying that a soil has no water. If a small sample of wet soft clay is left in the sun or in a low-humidity laboratory, it will become "dry" after a while and at the same time much stronger than when it was wet. This "dry" clay still has a tiny bit of water firmly bound between the particles. This water is in tension and sucks the particles together through a phenomenon called *suction* (explained in Chapter 10 on effective stress). This suction is responsible for the increase in strength of the clay. If the dried clay is ground into individual particles and placed in an oven at $100°C$, then it will have no water and no strength. Thus, it becomes important to make a distinction between dried and no water; for example, a dried clay is a hard block of soil whereas a clay with no water may simply be a dry powder.
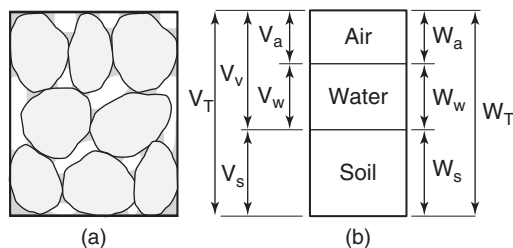
3.2 PARTICLE SIZE, SHAPE, AND COLOR

Depending on their size, soil particles are called gravel size particles, sand size particles, silt size particles, or clay size particles. Gravel, sand, and the coarser silt particles are typically made of quartz and are more rounded in shape. They can be seen with the naked eye or a simple microscope. Clay and the finer silt particles are too small to be seen with the naked eye; they are visible only with the use of electron microscopy or X-ray diffractometry. Figure 3.2 shows photos of soil particles.

Ranges of particle sizes are defined as:

Gravel-size particles: 20 mm to 4.75 mm
Sand-size particles: 4.75 mm to 0.075 mm
Silt-size particles: 0.075 mm to 0.002 mm
Clay-size particles: less than 0.002 mm

These ranges indicate a huge difference in size between a sand-size particle and a clay-size particle. For example, if the clay particle were a postage stamp, the sand particle would be a very large airplane. Soil particle sizes are so dramatically different that showing them on a natural scale is not very helpful (Figure 3.3); instead, a logarithmic scale is used which allows the very small particle to appear on the scale as well as the very large ones. Figure 3.4 shows such a scale and summarizes the main differences between soil particles.

There is also a big difference in shape between the gravel- and sand-size particles on the one hand and the silt- and clay-size particles on the other. Gravel, sand, and the larger silt particles tend to be rounded, whereas clays and the smaller silt size particles tend to be rodlike or platelike. This is because minerals such as quartz, which form the larger particles, are much more stable and resistant to weathering than the minerals, such as kaolinite (baby powder), that form the smaller particles. The surface of sand and gravel particles can present various degrees of roughness. At one end of the spectrum are the angular particles (freshly broken from the parent rock, for example) and at the other are the smooth,

Figure 3.1 Three-phase diagram representation.

Figure 3.2 Examples of cobbles, gravel-, sand-, silt-, and clay-size particles.
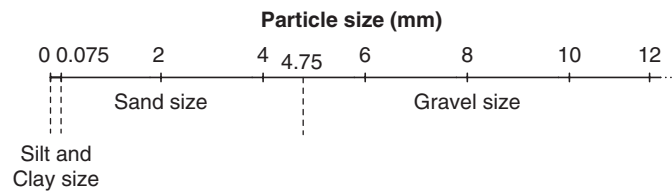
Figure 3.3 Particle sizes on a natural scale.

Particle size	Clay size		Silt size	Sand size		Gravel size	
Particle size in mm on a log scale	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
Log of particle size on an arithmatic scale	-4	-3	-2	-1	0	1	2
			$\log(0.002)=-2.7$	$\log(0.075)=-1.125$		$\log(4.75)=0.677$	
Seeing the particle	Electron microscope		Microscope	Hand lens		Naked eye	
Shape	Plate like-rod like			Sphere like			
Minerals	Smectite-montmorillonite-bentonite-illite-kaolinite			Mostly quartz-some felspar and mica			
Forces involved	Electostatic-electromagnetic-van der waals-intermolecular-gravity			Gravity			
Structure	Flocculated-dispersed			Loose-dense			
Other factors	Cation exchange capacity-suction			Surface roughness-suction			

Figure 3.4 Particle sizes on logarithmic scale and some characteristics of each size.

rounded particles (eroded by water over a long period of time, for example). Clays and silts are typically much smoother to the touch than sands and gravels.

Soil particles are grey, tan, brown, or reddish. The brown or reddish color may come from the presence of iron. The wetter the soil is, the darker the color will be; this may help in determining the location of the groundwater level when retrieving samples from a boring. A darker color may also indicate the presence of organic matter, although a foul smell is another and possibly better indicator.

3.3   COMPOSITION OF GRAVEL, SAND, AND SILT PARTICLES

Soil particles are made of mineral or organic matter. Mineral matter is inert matter such as silica, whereas organic matter is of biological origin (basically, anything that lives or has lived). Organic particles include leaves, plants, grasses, fibers, tree trunks, shells, and fossils. Most soil particles are made of minerals, which have a crystalline structure. The most common mineral is silica; indeed, silica makes up 95% of the Earth's crust. Minerals are to particles what bricks are to houses: they are the building blocks of the particle. The most stable minerals are framework minerals, which are resistant to erosion and weathering, and form the larger particles (gravel and sand). The least stable minerals are the sheet minerals which make up the clay particles. The most common constituent mineral in gravel, sand, and the coarser silt particles is quartz (SiO_2), but feldspar ($KAlSi_3O_8$), and mica (SiO_2, Al_2O_3) are also encountered. The behavior of gravel particles, sand particles, and the coarser silt particles is determined by the weight of the particle and associated friction. Other phenomena, such as electromagnetic and intermolecular forces, do exist, but in these coarser particles their effects are negligible compared to the weight. However, this is not the case for extremely small particles, such as clay particles or the finer silt particles.

3.4   COMPOSITION OF CLAY AND SILT PARTICLES

Note that silt particles are listed in the title of this section and the last section. The reason is that silt particles straddle the properties of coarse-grained particles and clay particles. Three major minerals make up clay particles. In decreasing order of size, they are kaolinite, illite, and smectite (Mitchell and Soga 2005). Montmorillonite and bentonite are subgroups of the smectite minerals. These minerals are composed of elementary sheets, which are the silica sheet (SiO_2), the gibbsite sheet ($Al_2(OH)_6$), and the brucite sheet ($Mg_3(OH)_6$).

The mineral kaolinite ($Al_2Si_2O_5(OH)_4$) is made of a stack of a silica sheet and a gibbsite sheet. Kaolinite makes up the larger clay particles with length on the order of 1000 nanometers (Figure 3.5), a thickness of about 100 nanometers, and a specific surface (particle surface per unit mass) of 10 m^2/g. Kaolinite is commonly used in baby powder. Smectite ($Al_2Si_4O_{10}(OH)_2$ and x interlayers of H_2O) is made of a gibbsite sheet sandwiched between two silica sheets. Smectite makes up the smaller clay particles with length on the order of 100 nanometers (Figure 3.5), a thickness on the order of 1 nanometer, and a specific surface (particle surface per unit mass) of 800 m^2/g. This remarkably high specific surface allows the smectite particle to absorb a significant amount of water between the elementary sheets. This leads to extreme swelling and shrinking potential for these clays
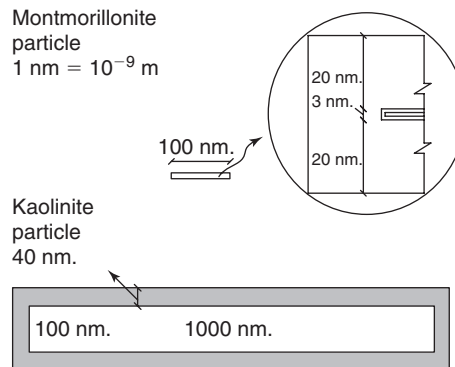

Figure 3.5   Approximate dimensions of montmorillonite and kaolinite particles.

(Figure 3.6). Montmorillonite and bentonite are subgroups of the smectite mineral group. Bentonite is sold commercially for drilling mud applications because it can form a nearly impervious cake on the wall of the borehole and keep groundwater from penetrating the borehole (see Chapter 6 on site investigation). The mineral illite has properties intermediate between those of kaolinite and smectite.

Cations are positive ions that are attracted to the surface of clay particles. Silicium (Si^{4+}) is a very common cation in soils. Because Si^{4+} has a high valence, a negative charge will be generated if it is replaced by another cation such as Al^{3+} or Mg^{2+} or Na^+. This cation exchange is called *isomorphous substitution* because the exchange cation has the same shape (*isomorphous* means "same shape" in Greek), allowing it to fit in the crystalline lattice, but a lower valence. This substitution will occur if an exchange cation is available when a Si^{4+} cation is not. The cation exchange capacity or CEC is a measure of how many cations a clay particle can catch; it is measured in milliequivalents per unit mass (meq/100 g). The milliequivalent is a unit of amount of substance and is related to the mole, the SI unit used to quantify the amount of substance. Kaolinite has a smaller CEC ($\sim$5 meq/100 g) than montmorillonite ($\sim$80 meq/100 g). As a result of isomorphous substitution, the surface of clay particles is negatively charged except at the ends of the particles, where positive charges may appear due to broken bonds. In this case, clay particles can be thought of as little magnets that attract or repel each other. The negative and sometimes positive electrical charges on the surface of clay particles influence the way the structure of the clay mass develops (flocculated or dispersed).

The water next to the clay particle surface is made of molecules that can be thought of as electrical dipoles (H^+ and OH^-). The H^+ end of the dipole is attracted to the negative charges on the clay particle surface and the water molecule adheres to the surface. Cations such as Na^+ may also be present in the water and will be attracted to the surface in an effort to neutralize the negative charge. The sodium adsorption ratio or SAR gives an indication of how
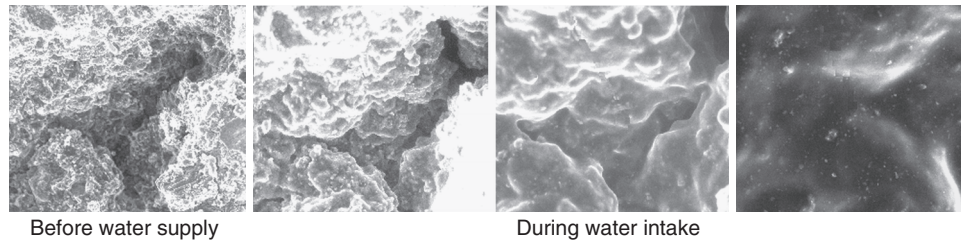
Before water supply   During water intake

Figure 3.6 Absorption of water in bentonite. (Courtesy of Komine and Ogata, 2004.)
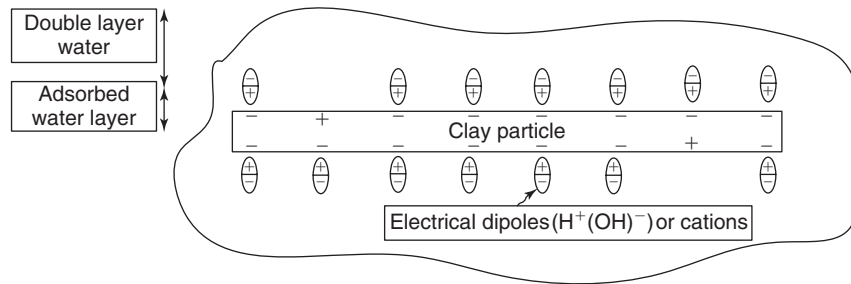

Figure 3.7 The electrical double layer of clay particles.

much sodium is available around the particles. It is defined as:

$$SAR = \frac{[Na^+]}{\left(\frac{[Ca^{2+}] + [Mg^{2+}]}{2}\right)^{0.5}} \qquad (3.1)$$

where the value within brackets [] is the concentration of cations in meq/liter. This layer of bound water is called the *electrical double layer* (Figure 3.7) and its thickness is on the order of 1 to 50 nm, with the higher values found in very active clay particles such as montmorillonite and bentonite. The layer of water most closely bound to the particle surface within the electrical double layer is called the *adsorbed water layer* (Figure 3.7).

The attraction between clay particles is attributed to the Van der Waals forces that overcome the repulsion between two negatively charged particles. Van der Waals forces are intermolecular forces that give water its tensile strength, for example. The other important source of cohesion in a clay is the attraction between water and silica, which sucks the particles together. This phenomenon, called *suction*, and is discussed in Chapter 10 on effective stress.

3.5 PARTICLE BEHAVIOR

Gravels and sands are called coarse-grained soils, while silts and clays are called fine-grained soils. The weight of soil particles varies tremendously; for example, a gravel-size particle is about 10 billion times heavier than a clay-size particle. Coarse-grained soil particles tend to behave according to

their weight. In contrast, the behavior of fine-grained, clay-size particles is significantly influenced by the electrostatic and electromagnetic forces that exist at the particle surface. These forces create attraction and repulsion much like small magnets would do. They give clays their consistency, which you might wish to think of as stickiness. The behavior of silt-size particles is intermediate between that of gravel and sand on the one hand and that of clay on the other.

In addition to the weight of the particle and the electrostatic/electromagnetic forces affecting the particles, water can strongly influence the behavior of an assembly of particles (Figure 3.8). First, the water can create buoyancy if the particle is below the groundwater level. This buoyancy reduces the effective weight of the particle (like when you go into
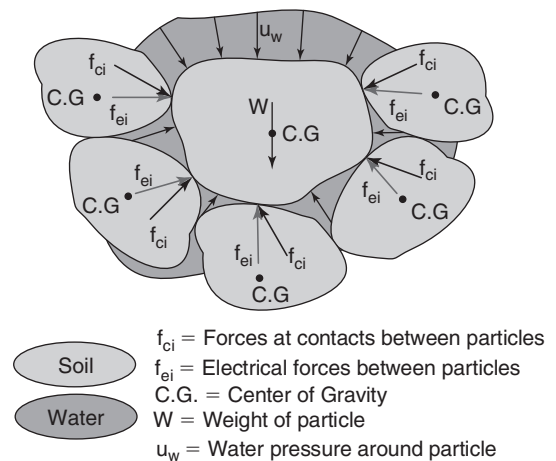

f_{ci} = Forces at contacts between particles
f_{ei} = Electrical forces between particles
C.G. = Center of Gravity
W = Weight of particle
u_w = Water pressure around particle

Figure 3.8 Forces acting on a soil particle.

a swimming pool) and therefore reduces the friction that it can generate when rubbing against other particles. Second, even above the groundwater level water is still present in the voids because of two fundamental phenomena: the attraction between water and the clay minerals (e.g., water is attracted to silica, which leads to capillary suction) and the attraction between water and salt (osmosis). Both phenomena allow the water to stay in the voids, go into tension, and suck the particles together. This "glue" between particles influences the behavior of the particles, contributes to soil plasticity (stickiness), and is responsible for the strength of a dry clay. This topic is developed in Chapter 10 on effective stress.

3.6   SOIL STRUCTURE

The *structure* of a soil refers to the arrangement of the soil grains. Loose or dense structures are found in coarse-grained soils, whereas flocculated and dispersed structures exist in fine-grained soils.

A loose soil structure is similar to the arrangement of the spheres shown in Figure 3.9. In this case, the contacts between particles are mostly at 90 and 180 degrees on the rosette of contacts. Shearing the mass would lead to a loss of volume of the mass, as the particles will tend to move toward a more stable arrangement. This soil would be called *contractive*. Such loose structures are found, for example, when the soil settles under water in a very low-energy environment and without vibration. This can be the case with hydraulic fills. A dense soil structure is similar to the arrangement of the spheres shown in Figure 3.10. In this case, the contacts between particles are mostly at 45 and 135 degrees on the rosette of contacts. Shearing the mass would lead to an increase in volume of the mass, as each particle will tend
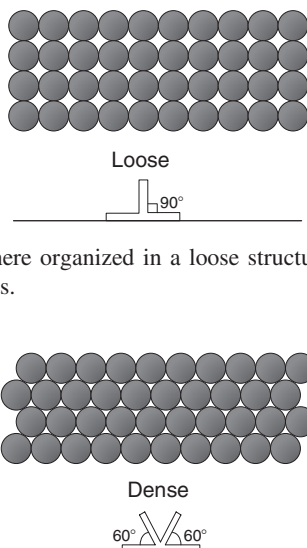

Figure 3.9 Sphere organized in a loose structure and associated rosette of contacts.
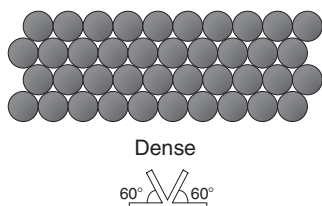

Figure 3.10 Sphere organized in a dense structure and associated rosette of contacts.

to ride on top of the next one. This soil would be called *dilatant*. Such dense structures are found in compacted soils for dams or pavements that are densified during placement by a combination of pressure and vibration.

In a *dispersed* structure, the particle arrangement is like a deck of cards (Figure 3.11). Such structures tend to be very stable and exhibit high stiffness. However, they have little strength against shearing that takes place in the direction of the "cards." The stacks of particles can, however, be organized in different ways within a single soil, and that will influence the overall behavior. In a *flocculated* structure, the particle arrangement looks like a card castle (Figure 3.11). Such structures tend to be unstable and can easily collapse. When a flocculated clay derives the strength of its particle contact from salt bonding, a quick clay may be formed. These quick clays (such as found in Norway and Canada) may liquefy if the salt is leached from the contacts by exposure to fresh water and/or if an event triggers the breaking of the bonds. The Risa event in Norway was a remarkable quick-clay landslide in which the clay literally turned into liquid—to the extent that houses floated down the hill. (This landslide was videotaped by an amateur, and the movie can be obtained by contacting the Norwegian Geotechnical Institute (NGI) in Oslo, Norway.) Most natural clays exhibit a mixture of dispersed and flocculated structures. Examples of clay and sand structures are shown in Figure 3.12 (Terzaghi et al. 1996).

Composite structures are associated with mixtures of coarse particles and fine particles. In a matrix structure, the fine particles are predominant and the coarse particles do not touch each other. In a void bound structure, the coarse particles touch each other and are bound together by the fine particles, which effectively act as a glue.

3.7   THREE-PHASE DIAGRAM

The three-phase diagram is a graphical representation of the soil components. Figure 3.1(a) shows a soil sample in its natural state, with particles, gas (most often air), and liquid (most often water) all mixed together. All the air can be regrouped into one volume V_a, and all the water can be regrouped into one volume V_w. The sum of V_a and V_w is the volume of voids V_v. Once all the air is in V_a and all the water is in V_w, then what is left are the particles regrouped into one volume V_s. This particle volume has no voids, because they have been sucked out into V_a and V_w; therefore, the volume V_s is a solid piece of rock with no voids. The unit weight of this solid piece of rock made with the particulate material has a unit weight called the *unit weight of solids*, γ_s, a ratio of the weight of solids W_s over the volume of solids V_s. The unit weight of solids varies depending on the mineral or organic matter of the particles, but for mineral matter it is in the range of 25.5 to 27 kN/m^3 and for organic matter it is 9 to 13 kN/m^3. The specific gravity of solids, Gs, is the ratio
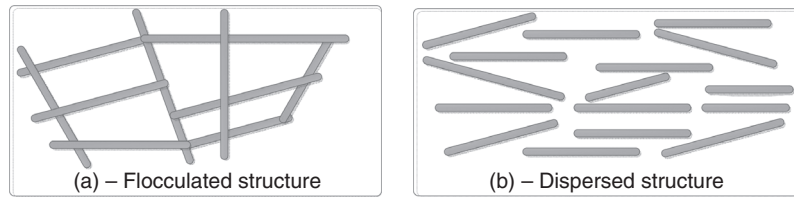
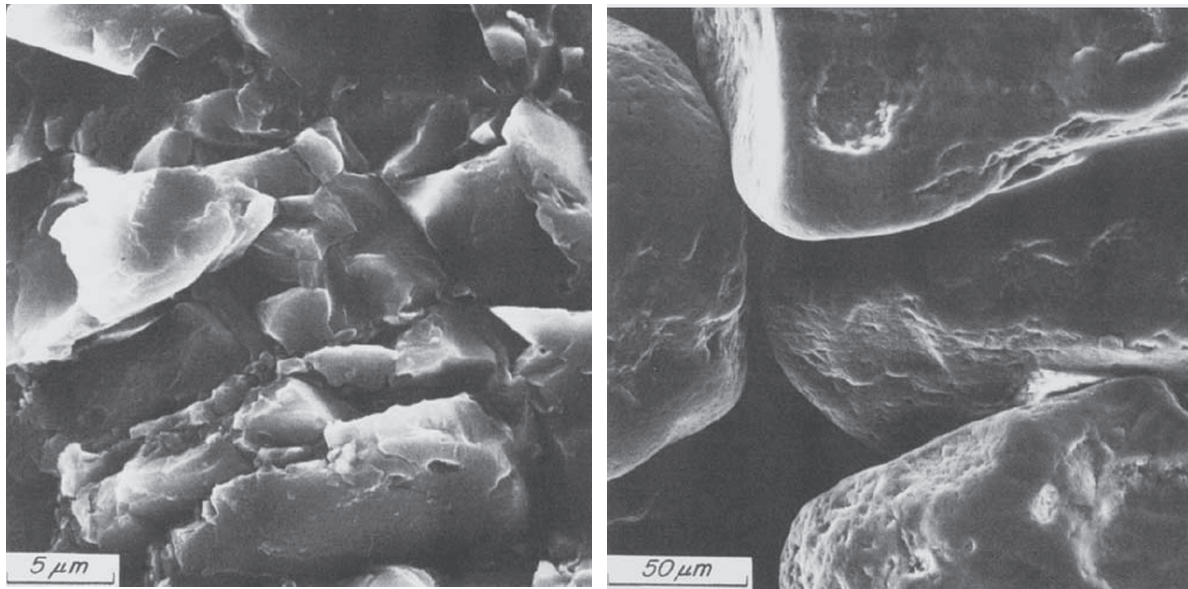**Figure 3.11**   Flocculated and dispersed clay structures.

Figure 3.12   Example of clay and sand structure. (From Terzaghi et al. 1996. This material is reproduced with permission of John Wiley & Sons.)

between the unit weight of solids γ_s and the unit weight of water γ_w.

3.8   WEIGHT-VOLUME PARAMETERS

Some of the most important parameters describing the volume and weight of soils are the unit weight, the water content, the void ratio, the porosity, the degree of saturation, the specific gravity of solids, and the density index (Table 3.1).

The *natural unit weight* (total weight of soil W_t over total volume of soil V_t) is the unit weight of the soil as it is found in its natural environment. The natural unit weight is also called *total unit weight* or simply *unit weight*. Numbers between 17 and 22 kN/m^3 are common. The *dry unit weight* is the unit weight of the dry soil (weight of solids W_s over total volume of soil V_t). Numbers between 14 and 18 kN/m^3 are common. The *saturated unit weight* is the ratio of the weight of the soil when the voids are full of water or liquid over the total volume of the soil. The weight of the saturated soil is the weight of solids plus the weight of water necessary to fill the voids ($W_s + V_v \gamma_w$). Numbers between 18 and 22 kN/m^3 are common. The *submerged unit weight* is the difference between the saturated unit weight and the

unit weight of water. The *effective unit weight* is equal to the total unit weight for a point in the soil mass above the groundwater level and equal to the submerged unit weight for a point below the groundwater level. The unit weight of solids γ_s is the unit weight of the particle itself. It is the ratio of the weight of solids W_s over the volume of solids V_s. The unit weight of solids varies depending on the composition of the particles (mineral or organic matter), but for mineral matter it is in the range of 25.5 to 27 kN/m^3 and for organic matter it is in the range of 9 to 13 kN/m^3. The specific gravity of solids G_s is the ratio between the unit weight of solids γ_s and the unit weight of water γ_w.

The water content w, also called *gravimetric water content*, is the ratio of the weight of water W_w over the weight of solids W_s or weight of dry soil. Although the water content is a ratio and should be used as such in most formulas, it is most often quoted as a percentage. Numbers around 10 to 40% are common, but the water content can be 0 for a dry soil and can reach 400%, as in the Mexico City silt, or even 2000% for some peaty soils (soil near swamps, made up mostly of grass and plants). Indeed, there is no theoretical upper limit to the water content. The gravimetric water content is the water content measure most widely used in geotechnical

Table 3.1   **Weight-Volume Parameters and Typical Values**

Parameter	Symbol	Definition	Typical range	Observation
Total unit weight	γ_t	W_t/V_t	17–22 kN/m^3	Unit weight of soil in natural state
Dry unit weight	γ_d	W_s/V_t	14–18 kN/m^3	Unit weight of dry soil
Maximum dry unit weight	γ_{dmax}	W_{smax}/V_t	15–19 kN/m^3	Densest state
Minimum dry unit weight	γ_{dmin}	W_{smin}/V_t	13–17 kN/m^3	Loosest state
Unit weight of solids	γ_s	W_s/V_s	25.5–27 kN/m^3 for mineral 9–13 kN/m^3 for organic	Unit weight of particles
Specific gravity of solids	G_s	γ_s/γ_w	2.6–2.7 for mineral 0.9–1.3 for organic	Dimensionless
Saturated unit weight	γ_{sat}	$(W_s + V_v\gamma_w)/V_t$	18–22 kN/m^3	Voids are full of water
Submerged unit weight	γ_{sub}	$\gamma_{sat}-\gamma_w$	8–12 kN/m^3	Buoyancy force accounted for
Effective unit weight	γ_{eff}	γ_t if above GWL $\gamma_{sat}-\gamma_w$ if below GWL	See γ_t and γ_{sub}	
Unit weight of water	γ_w	W_w/V_w	9.81 kN/m^3	
Water content (gravimetric)	w	W_w/W_s	10–40%	0–∞ theoretical range
Volumetric water content	θ_w	V_w/V_t	5–30%	0–1 theoretical range
Degree of saturation	S	V_w/V_v	50–100%	0–100% theoretical range
Porosity	n	V_v/V_t	25–50%	0–100% theoretical range
Void ratio	e	V_v/V_s	0.4–1	0–∞ theoretical range
Maximum void ratio	e_{max}	V_{vmax}/V_s	0.6–1.2	Loosest state
Minimum void ratio	e_{min}	V_{vmin}/V_s	0.3–0.9	Densest state
Density Index	I_d or D_r	$(e_{max}-e)/(e_{max}-e_{min})$	20–90%	0–100% theoretical range

engineering. Sometimes for unsaturated soils, the volumetric water content θ_w is used; θ_w is defined as the ratio of the volume of water V_w over the total volume V_t. Numbers between 5 and 30% are common; θ_w is zero for a dry soil and approaches 100% for extremely wet soils such as peat. The degree of saturation S is the volume of water V_w over the volume of voids V_v. Although the degree of saturation is a ratio and should be used as such in most formulas, it is most often quoted as a percentage. Numbers from 0 to 100% are found, although most soils below the groundwater level and some distance above it are saturated or nearly saturated. In many cases soils near the surface are unsaturated.

The porosity n is the ratio of the volume of voids V_v over the total volume V_t. Although the porosity is a ratio and should be used as such in most formulas, it is most often quoted as a percentage. Numbers in the range of 25 to 50% are common, and the porosity is always between 0 and 100%. The void ratio e is the ratio of the volume of voids V_v over the volume of solids V_s. It is most often quoted as a number. Numbers from 0.4 to 1 are common. Although the theoretical limits of the void ratio are 0 and infinity, the practical limits for a given soil are the minimum void ratio

e_{min} and the maximum void ratio e_{max}. The minimum void ratio corresponds to the densest state of a given soil, and the maximum void ratio corresponds to the loosest state for a given soil. Both e_{min} and e_{max} are particularly useful in the case of coarse-grained soils and lead to the definition of the density index I_d (also designated as D_r), which is quoted as a percentage and expresses the density of a coarse-grained soil as a percentage between the two extreme states of density ($I_d = (e_{max} - e)/(e_{max} - e_{min})$). Also associated with the densest and loosest states are the maximum and minimum dry densities γ_{dmax} and γ_{dmin}. Note that γ_{dmax} corresponds to e_{min} and that γ_{dmin} corresponds to e_{max}. The density index can be expressed in terms of γ_d, γ_{dmax}, and γ_{dmin} as

$$I_d = \frac{\gamma_{d\,max}}{\gamma_d}\left(\frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right) \tag{3.2}$$

3.9 MEASUREMENT OF THE WEIGHT-VOLUME PARAMETERS

To obtain the natural or total unit weight of a soil, the sample is trimmed into a simple geometrical shape, the dimensions

are measured to obtain the volume, the weight is measured, and the weight over volume is calculated (ASTM 2005b [ASTM D2937]). This test is possible only if the sample can keep a geometric shape long enough for the measurements to be made. If this is not possible, as in the case of a dry sand or gravel for example, then the unit weight is typically obtained by correlation with other measurements such as the blow count during a standard penetration test (SPT) (ASTM 2005a [ASTM D1586]). The water content is obtained by taking a small piece of the sample and measuring its wet weight (W_t), drying it in an oven at $100°C$ for 24 hours, and obtaining its dry weight W_s, and then calculating the water content $w = (W_t - W_s)/W_s$ (ASTM 2005c [ASTM D4959]). These two measurements, natural unit weight and water content, are the two most common measurements on a soil sample.

Unless the sample is dry or saturated, a third input parameter is necessary to obtain all the weight-volume parameters for a soil. This parameter is often the specific gravity of solids G_s. If it is known that the soil particles are mineral and not organic, then a reasonable assumption can be made for G_s, such as $G_s = 2.65$. If the composition is not known, or if a more precise value for G_s is needed, then G_s is determined by the specific gravity test (ASTM 2005d [ASTM D854]). This test consists of drying the soil in an oven, pulverizing it by grinding, placing the ground-up material in a container, and filling the container with water up to a chosen level. The container with water plus soil is weighed. Then the container is emptied, cleaned, and filled up to the same chosen level with water only and weighed. The weight of the container with water plus soil minus the weight of the container with water only gives the weight of the buoyant soil. The buoyancy force is the difference between the weight of the buoyant soil and the dry soil. The ratio of the dry weight over the buoyancy force is the specific gravity of the solids.

If the unit weight of the soil, its water content, and the specific gravity of solids are known, all other weight-volume parameters can be obtained by calculations (see section 3.10), including the dry unit weight, the saturated unit weight, the submerged unit weight, the effective unit weight, the degree of saturation, the porosity, and the void ratio.

Finding the density index of a coarse-grained soil requires two special tests in addition to the determination of the natural dry unit weight γ_d: one test to obtain the maximum dry unit weight γ_{dmax} (ASTM D4253) and one test to obtain the minimum dry unit weight γ_{dmin} (ASTM D4254). The maximum dry unit weight is obtained by pouring the dry sand or dry gravel into a container of known volume, placing a standard weight on top of the sample surface, and vibrating the soil and the container for a standard time. During the vibrations, the soil volume decreases and reaches equilibrium at the maximum dry unit weight. Measurements of weight and volume at that time allow one to calculate the maximum dry unit weight. The minimum dry unit weight is obtained by very gently pouring a dry sand or gravel sample into a container of known volume, measuring the weight, and calculating the dry unit weight. Once γ_d, γ_{dmax}, and γ_{dmin} are known, the density index I_d (or D_r) is calculated according to equation 3.2.

3.10 SOLVING A WEIGHT-VOLUME PROBLEM

Geotechnical engineers often encounter problems where some information related to the weight or volume of a soil is known but different weight-volume properties are required. The best way to solve such problems is to follow these steps:

1. Draw a three-phase diagram and indicate the known quantities. If the soil is dry or saturated, then only a two-phase diagram is necessary.
2. If no quantity is given (for example, you are given a unit weight but not a weight or a volume), assume a volume of solid of 1 m³.
3. Using the information in the specific problem case, complete the weight and volume values for the different phases. If some information is missing, make reasonable assumptions (e.g., the unit weight of solids.) Also realize that the unit weight of water is known (9.81 kN/m³).
4. Complete the calculations to derive the weight-volume parameters required.

The assumptions made in step 2 have no impact on the answers as long as the answers are in the form of ratios (unit weight, void ratio, porosity, degree of saturation); if a different volume of solids were assumed, the final answer would be the same. Although the step-by-step procedure described here is foolproof, it might be faster in some cases to use the relationships existing between weight-volume parameters. Table 3.2 shows some of these.

Table 3.2 Useful Relationships between Weight-Volume Parameters

$n = e/(1 + e)$

$e = n/(1 - n)$

$e = (\gamma_s - \gamma_d)/\gamma_d$

$Se = G_s w$

$W_s = W_t/(1 + w)$

$\gamma_t = \gamma_d (1 + w)$

$\gamma_t = \gamma_w (G_s(1 - n) + Sn)$

$\gamma_t = \gamma_w (G_s + Se)/(1 + e)$

$\gamma_d = \gamma_w G_s(1 - n)$

PROBLEMS

3.1 A sample of clay is brought back from the field, extruded from the Shelby tube, and trimmed to the following dimensions: height = 150 mm, diameter = 75 mm. It weighs 13.2 N. The water content has been determined to be 25% and the soil does not exhibit any signs of the presence of organic matter (e.g., the soil is not very dark and does not smell foul). Find the following parameters for the clay:
   a. Natural unit weight
   b. Degree of saturation
   c. Porosity
   d. Void ratio
   e. Dry unit weight
   f. Saturated unit weight

3.2 a. The sample from problem 3.1 shrinks by 10% when it dries. What is the difference between the dry unit weight and the unit weight of the dry soil?
   b. The sample from problem 3.1 is placed under water and has swollen by 15% when it reaches its swell limit. What is the difference between the saturated unit weight and the unit weight of the soil at the swell limit?

3.3 A farmer wants to buy a 10 kg bag of fertilizer (organic soil). He has the choice between two merchants. Merchant A sells the 10 kg bag for $10 and the bag indicates that the fertilizer is completely dry. Merchant B sells the 10 kg bag for $8 and the bag indicates that the fertilizer has a water content equal to 20%. If the farmer wishes to buy the least expensive solid constituents, which merchant should he buy from? Show your calculations.

3.4 An airport runway is being extended into a bay and requires a 10 m high embankment above the bottom of the bay. Calculations indicate that, once constructed, the long-term settlement of the soil beneath the embankment will be about 1 m. The sand used to build the embankment is taken from a pit where the sand has a relative density of 40%. The maximum void ratio is 0.7; the minimum void ratio is 0.4. Once compacted in the embankment, the sand will have a relative density of 90%. What height of sand must be obtained from the borrow pit so that, a long time after completion, the embankment will be 10 m above the initial position of the bottom of the bay before construction started?

3.5 A shrink test is performed on a sample of clay. At time zero, the sample is 25 mm high, 75 mm in diameter, weighs 2.2 N, and is saturated. The sample is left on a laboratory table; this laboratory is at 20°C and 50% relative humidity. The sample dries and shrinks. It is weighed and the dimensions are measured with digital calipers as a function of time. At the end of the test, the sample is placed in the oven to obtain its dry weight, which comes out to be 1.8 N. The results of the test are shown in the following table.

Time (hr)	0	1	2	3	5	8	12	24
Height (mm)	25	24.932	24.662	24.490	24.315	24.138	23.958	23.958
Diameter (mm)	75	74.497	73.987	73.470	72.946	72.414	71.874	71.874
Weight (N)	2.200	2.160	2.115	2.079	2.034	1.989	1.944	1.872

Plot the curve of water content versus relative decrease in volume. Comment on the shape of that curve.

3.6 A 2.2 N sample of clay is 25 mm high and 75 mm in diameter and has a water content of 22.2% (same sample as in problem 3.5). It is placed in a stainless steel ring has the same dimensions as the sample, so the sample cannot expand laterally. The sample is inundated and allowed to swell vertically for several weeks until it reaches equilibrium. The height and time measurements are shown in the following table. At the end of this free swell test, the sample is taken out of the steel ring and weighed; it weighs 2.40 N. Plot the relative increase in volume of the sample versus time and calculate the swell limit for this clay. The *swell limit* is the water content at which the soil can no longer absorb any additional water.

Time (hr)	0	100	200	300	400	500
Height (mm)	25	28	29.5	30.25	30.75	31

3.7 A silty sand is compacted in a mold. The volume of the mold is 9.46×10^{-4} m^3. The weight of compacted soil in the mold is 18.9 N and the water content is 8%. Assume that G_s is 2.65 and calculate the dry unit weight and the degree of saturation.

V = 9.46 x 10^{-4} m^3	V$_v$ = 2.69 x 10^{-4} m^3	V$_A$= (9.46–1.43–6.74) x 10^{-4} = 1.29 x 10^{-4} m^3	Air	W$_A$ = 0	W = 18.91 N
		V$_w$ = 1.4/9810 = 1.43 x 10^{-4} m^3	Water	W$_w$ = 0.08 x 17.51 = 1.40	
	V$_s$ = 17.51/9810/2.65 = 6.74 x 10^{-4} m^3		Soil	W$_s$ = 18.91/1.08 = 17.51 N	

Figure 3.10s   Three-phase diagram.

3.8 A consolidation test is performed on a sample of soft clay that is 25 mm high and 50 mm in diameter. The test consists of placing a disk of soil in a steel ring and applying load on the sample in a series of steps. The steps last 24 hours and measurements of vertical compression are obtained at the end of each step. The following table shows the time, load, and compression results of the test. Calculate the pressure and vertical strain for the sample at the end of each load step and plot the curve that links the pressure to the vertical strain (stress-strain curve). Why does this curve indicate an apparently surprising result, in that the more load applied to the sample, the stiffer the sample becomes? Can the sample fail?

Time (days)	0	1	2	3	4	5	6
Load (N)	0	294	589	1177	2453	4906	9812
Stress (kN/m^2)	0	149.73	299.98	599.45	1249.31	2498.61	4997.21
Height (mm)	25	24.62	24.25	23.62	22.87	21.47	19.7
Displacement (mm)	0	0.38	0.75	1.38	2.13	3.53	5.3
Strain	0	0.0152	0.03	0.0552	0.0852	0.1412	0.212

3.9 A silt has a unit weight of 20 kN/m^3 and a water content of 26%. What is the specific gravity of the particles?

3.10 A 5 m high embankment is made of a sand that has a void ratio of 0.55 and the following boundary void ratios: $e_{max} = 0.6$, $e_{min} = 0.4$. The embankment is subjected to an earthquake that creates a settlement of 0.32 m due to vibration without a change in lateral dimensions. Calculate the void ratio and the relative density of the sand after the earthquake.

3.11 Find the relationship between the dry unit weight, the void ratio, and the specific gravity of solids for a soil.

3.12 Demonstrate that $S \cdot e = G_s \cdot w$.

Problems and Solutions

Problem 3.1

A sample of clay is brought back from the field, extruded from the Shelby tube, and trimmed to the following dimensions: height = 150 mm, diameter = 75 mm. It weighs 13.2 N. The water content has been determined to be 25% and the soil does not exhibit any signs of the presence of organic matter (e.g., the soil is not very dark and does not smell foul). Find the following parameters for the clay:

  a. Natural unit weight
  b. Degree of saturation
  c. Porosity
  d. Void ratio
  e. Dry unit weight
  f. Saturated unit weight

Solution 3.1

$$\text{The volume of the sample is: } V_t = \frac{\pi D^2}{4} \times h = \frac{\pi \times 0.075^2}{4} \times 0.15 = 6.62 \times 10^{-4} \text{ m}^3$$

$$\text{The weight of the solid is: } W_s = \frac{W_t}{1+w} = \frac{13.2 \times 10^{-3}}{1.25} = 10.6 \times 10^{-3} \text{ kN}$$

$$\text{The weight of the water is: } W_w = W_t - W_s = 0.0132 - 0.0106 = 2.64 \times 10^{-3} \text{ kN.}$$

Assuming that the unit weight of the solids is $Y_s = 27$ kN/m^3, the volume of solid is:

$$V_s = \frac{W_s}{\gamma_s} = \frac{0.01056}{27} = 3.91 \times 10^{-4} \text{ m}^3$$

$$\text{The volume of water is: } V_w = \frac{W_w}{\gamma_w} = \frac{0.00264}{9.81} = 2.69 \times 10^{-4} \text{ m}^3$$

$$\text{The volume of air is: } V_a = V_t - V_w - V_s = 6.62 \times 10^{-4} - 3.91 \times 10^{-4} - 2.69 \times 10^{-4} = 2.48 \times 10^{-6} \text{ m}^3.$$

Based on these results, the three-phase diagram of this sample is shown in Figure 3.1s.
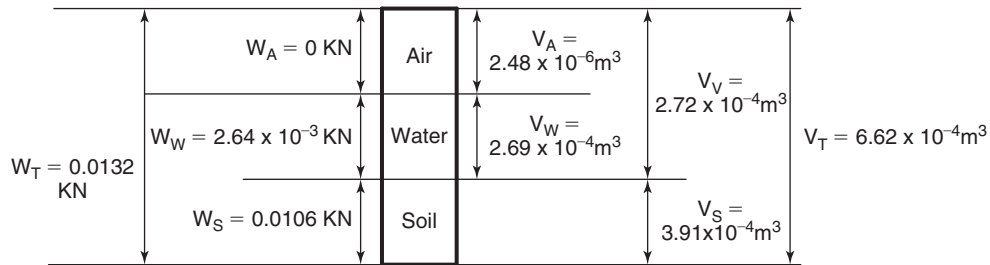

$W_A = 0$ KN	Air	$V_A = 2.48 \times 10^{-6}$ m^3		
$W_W = 2.64 \times 10^{-3}$ KN	Water	$V_W = 2.69 \times 10^{-4}$ m^3	$V_V = 2.72 \times 10^{-4}$ m^3	$V_T = 6.62 \times 10^{-4}$ m^3
$W_S = 0.0106$ KN	Soil		$V_S = 3.91 \times 10^{-4}$ m^3	

$W_T = 0.0132$ KN

Figure 3.1s   Three-phase diagram.

a. Natural unit weight

$$\gamma_t = \frac{W_T}{V} = \frac{13.2}{\dfrac{\pi \times 0.075^2}{4} \times 0.15} = 19.92 \times 10^3 \ \frac{N}{m^3} = 19.92 \ \frac{kN}{m^3}$$

b. Degree of saturation

$$S = \frac{V_w}{V_V} = \frac{2.69 \times 10^{-4}}{2.72 \times 10^{-4}} = 0.991 \times 100\% = 99.1\%$$

c. Porosity

$$n = \frac{V_V}{V_T} = \frac{2.72 \times 10^{-4}}{6.627 \times 10^{-4}} = 0.409 \times 100\% = 40.9\%$$

d. Void ratio

$$e = \frac{V_V}{V_s} = \frac{2.72 \times 10^{-4}}{3.91 \times 10^{-4}} = 0.694$$

e. Dry unit weight

$$\gamma_d = \frac{\gamma_t}{1+w} = \frac{19.92}{1+0.25} = 15.94 \ \frac{kN}{m^3}$$

f. Saturated unit weight

$$\gamma_{sat} = \frac{W_s + (V_V \times \gamma_w)}{V} = \frac{0.0106 + (2.72 \times 10^{-4} \times 9.81)}{6.627 \times 10^{-4}} = 19.95 \ \frac{kN}{m^3}$$

Problem 3.2

a. The sample from problem 3.1 shrinks by 10% when it dries. What is the difference between the dry unit weight and the unit weight of the dry soil?

b. The sample from problem 3.1 is placed under water and has swollen by 15% when it reaches its swell limit. What is the difference between the saturated unit weight and the unit weight of the soil at the swell limit?

Solution 3.2

a. Shrinking case

The volume of the sample is 6.627×10^{-4} m^3.

The volume of the sample after the 10% reduction due to shrinkage is $V_{T\ (Shrink)} = 6.627 \times 10^{-4}$ m$^3 \times 0.90 = 5.96 \times 10^{-4}$ m^3.

$$\gamma_{dried\ soil} = \frac{W_S}{V} = \frac{10.6}{5.96 \times 10^{-4}\ \text{m}^3} = 17.78 \times 10^3\ \frac{N}{\text{m}^3} = 17.78\ \frac{\text{kN}}{\text{m}^3}$$

Based on these results, the unit weight of the dry soil is higher than the dry unit weight and the difference is $17.78 - 15.94 = 1.84\ \frac{\text{kN}}{\text{m}^3}$.

b. Swelling case

The volume of the sample after the 15% volume increase due to swelling is 6.627×10^{-4} m$^3 \times 1.15 = 7.62 \times 10^{-4}$ m^3. It is assumed that during the swelling process the soil becomes completely saturated. Therefore, the volume of air in the original sample is replaced by a volume of water.

The increase in weight of the sample is equal to the weight of water corresponding to an increase in volume of water equal to $(7.62 - 6.627) \times 10^{-4}$ m^3 plus the *volume* of water necessary to fill the air voids in the original sample.

Additional weight of water: $(7.62 - 6.627) \times 10^{-4} \times 9.81 + 2.48 \times 10^{-6} \times 9.81 = 9.98 \times 10^{-4}$ kN $= 0.998$ N

$$\gamma_{swollen\ soil} = \frac{W_T}{V} = \frac{13.2 + 0.998}{7.62 \times 10^{-4}\ \text{m}^3} = 18.63 \times 10^3\ \frac{N}{\text{m}^3} = 18.63\ \frac{\text{kN}}{\text{m}^3}$$

Based on these results, the unit weight of the swollen soil at the swell limit is lower than the unit weight of the saturated soil in the previous problem and the difference is: $19.95 - 18.63 = 1.32\ \frac{\text{kN}}{\text{m}^3}$.

Problem 3.3

A farmer wants to buy a 10 kg bag of fertilizer (organic soil). He has the choice between two merchants. Merchant A sells the 10 kg bag for $10 and the bag indicates that the fertilizer is completely dry. Merchant B sells the 10 kg bag for $8 and the bag indicates that the fertilizer has a water content equal to 20%. If the farmer wishes to buy the least expensive solid constituents, which merchant should he buy from? Show your calculations.

Solution 3.3

Case 1

Merchant A (fertilizer in completely dry condition). The three-phase diagram for the fertilizer from merchant A is shown in Figure 3.2s.
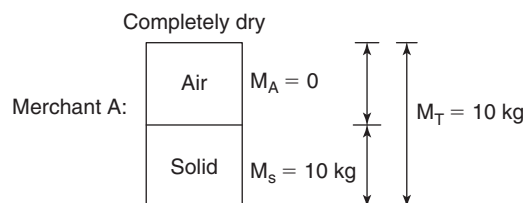

Figure 3.2s   Three-phase diagram for the fertilizer from merchant A.

The unit price for the solid constituents of merchant A is $\frac{10\$}{10\ \text{kg}} = 1\$/\text{kg}$

Case 2

Merchant B (fertilizer with water content = 20%). The three-phase diagram for the fertilizer from merchant B is shown in Figure 3.3s.
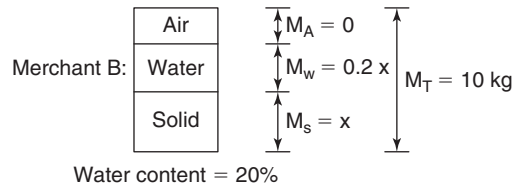

Water content = 20%

Figure 3.3s   Three-phase diagram for the fertilizer from merchant B.

Assume that the mass of the solids is x; then the mass of the water is $0.2x$. The total mass of the fertilizer bag is $1.2x$, which is equal to 10 kg. So the mass of the solids can be obtained from the following equation: $1.2x = 10$.

$$\text{So } x = 8.33 \text{ kg}$$

The unit price for the solid constituents in merchant B's bag is: $\frac{8\$}{8.33 \text{ kg}} = 0.96\$/\text{kg}$
So, the farmer should buy the fertilizer from merchant B.

Problem 3.4

An airport runway is being extended into a bay and requires a 10 m high embankment above the bottom of the bay. Calculations indicate that, once constructed, the long-term settlement of the soil beneath the embankment will be about 1 m. The sand used to build the embankment is taken from a pit where the sand has a relative density of 40%. The maximum void ratio is 0.7; the minimum void ratio is 0.4. Once compacted in the embankment, the sand will have a relative density of 90%. What height of sand must be obtained from the borrow pit so that, a long time after completion, the embankment will be 10 m above the initial position of the bottom of the bay before construction started?
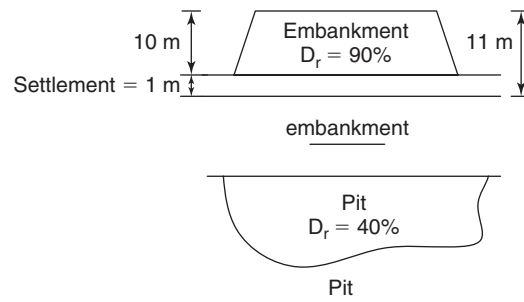
Solution 3.4

Figure 3.4s   Illustration of embankment and pit.

The void ratio of the soil in the pit is obtained with the equation:

$$D_r = \frac{e_{max} - e_{40}}{e_{max} - e_{min}} = \frac{0.7 - e_{40}}{0.7 - 0.4} = 0.4$$

Therefore, $e_{40} = 0.58$. For the soil after compaction:

$$D_r = \frac{e_{max} - e_{90}}{e_{max} - e_{min}} = \frac{0.7 - e_{90}}{0.7 - 0.4} = 0.9$$

Therefore, $e_{90} = 0.43$

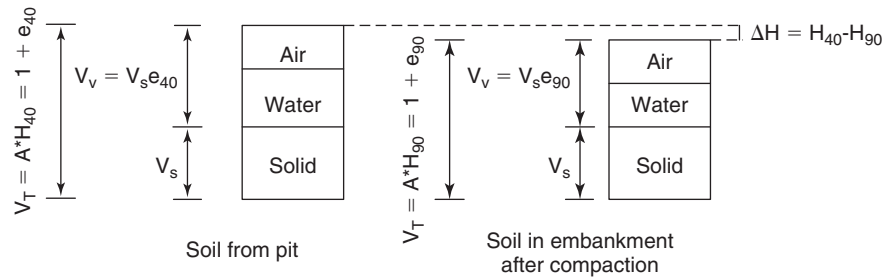The three-phase diagram for the soil in both conditions is shown in Figure 3.5s.

Figure 3.5s   Three-phase diagram for the soil in the two conditions.

Based on the three-phase diagram in Figure 3.5s, we can write:

$$\frac{H_{40}}{H_{90}} = \frac{V_s(1+e_{40})}{V_s(1+e_{90})}$$

Knowing that the long-term settlement of the soil in the bay beneath the embankment will be 1 m, the total height of soil necessary is 11 m. We have to calculate the height of soil H_{40} that should be taken from the pit with a 40% relative density such that when compacted to 90% relative density, the height H_{90} will equal 11 m.

$$\frac{H_{40}}{H_{90}} = \frac{H_{40}}{11} = \frac{V_s(1+e_{40})}{V_s(1+e_{90})} = \frac{1+0.58}{1+0.43} \qquad \text{or} \qquad H_{40} = 12.15 \text{ m}$$

The height of the soil that must be taken from the pit is 12.15 m.

Problem 3.5

A shrink test is performed on a sample of clay. At time zero, the sample is 25 mm high, 75 mm in diameter, weighs 2.2 N, and is saturated. The sample is left on a laboratory table; this laboratory is at 20°C and 50% relative humidity. The sample dries and shrinks. It is weighed and the dimensions are measured with digital calipers as a function of time. At the end of the test, the sample is placed in the oven to obtain its dry weight, which comes out to be 1.8 N. The results of the test are shown in the following table.

Time (hr)	0	1	2	3	5	8	12	24
Height (mm)	25	24.932	24.662	24.490	24.315	24.138	23.958	23.958
Diameter (mm)	75	74.497	73.987	73.470	72.946	72.414	71.874	71.874
Weight (N)	2.200	2.160	2.115	2.079	2.034	1.989	1.944	1.872

Plot the curve of water content versus decrease in volume. Comment on the shape of that curve.

Solution 3.5

The water content and decrease in volume at each reading is given by the following equations:

$$w\% = \frac{(W_{Total} - W_{dry})}{W_{dry}} \times 100$$

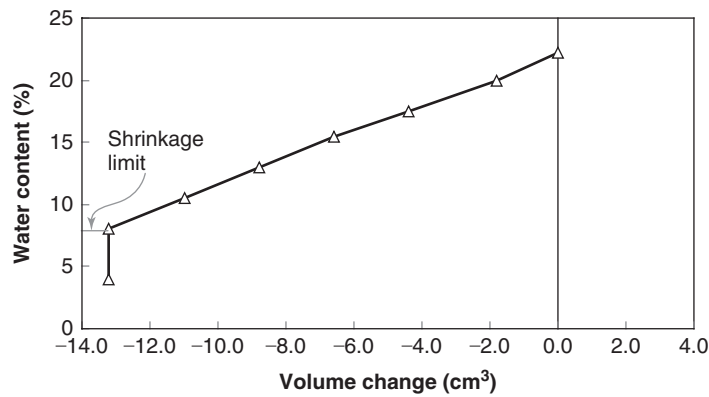$$\Delta V = \frac{\pi D_o{}^2}{4}H_o - \frac{\pi D^2}{4}H$$

Figure 3.6s   Water content versus decrease in volume.

We can make two observations:

1. The curve is almost linear for most of the test. This indicates that within that range of water content, the relative change in volume of the soil is linearly proportional to the change in water content.
2. At a water content of 8%, further drying does not lead to further reduction in volume. The soil has reached its shrinkage limit, which is 8% in this case. Note that this shrinkage limit is the shrinkage limit of the undisturbed soil, not the shrinkage limit of the Atterberg limit that would be obtained from a remolded sample.

Problem 3.6

A 2.2 N sample of clay is 25 mm high and 75 mm in diameter and has a water content of 22.2% (same sample as in problem 3.5). It is placed in a stainless steel ring has the same dimensions as the sample, so the sample cannot expand laterally. The sample is inundated and allowed to swell vertically for several weeks until it reaches equilibrium. The height and time measurements are shown in the following table. At the end of this free swell test, the sample is taken out of the steel ring and weighed; it weighs 2.40 N. Plot the relative increase in volume of the sample versus time and calculate the swell limit for this clay. The *swell limit* is the water content at which the soil can no longer absorb any additional water.

Time (hr)	0	100	200	300	400	500
Height (mm)	25	28	29.5	30.25	30.75	31

Solution 3.6

The volume, relative volume, and relative increase in volume are calculated in the following table:

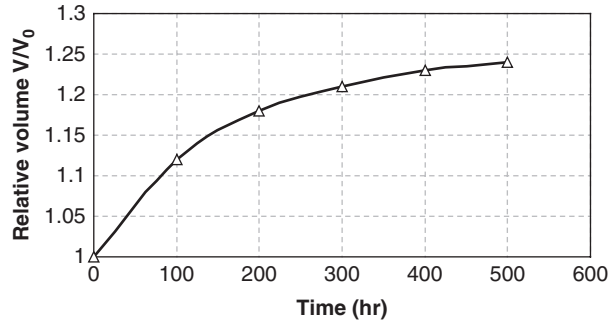| Time (hr)	0	100	200	300	400	500
Height (mm)	25	28	29.5	30.25	30.75	31
Volume (cm^3)	110.45	123.70	130.33	133.64	135.85	136.95
Relative volume V/V_0	1	1.12	1.18	1.21	1.23	1.24
Relative increase in volume $\Delta V/V_0$	0	0.12	0.18	0.21	0.23	0.24
$t/(\Delta V/V_0)$		833	1111	1428	1739	2083

Figure 3.7s   Relative volume variation with time.

As can be seen in Figure 3.7s, the sample did not reach the swell limit because at the end of the test its volume is still increasing slightly. One way to solve this issue is to use the hyperbolic extension method. To do so, we assume that the curve in the figure is a hyperbola with an equation:

$$\frac{\Delta V}{V} = \frac{t}{a + bt}$$

To determine the constants a and b, we write:

$$\frac{t}{(\Delta V/V)} = a + bt$$

Then we plot $\frac{t}{(\Delta V/V_0)}$ versus t and fit a straight line through the data as shown in Figure 3.8s.
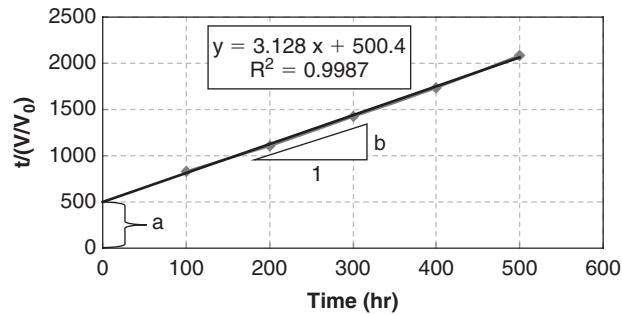

Figure 3.8s   Graph showing the parameters a and b.

According to this graph, $a = 500.4$ hr and $b = 3.13$. The extended swell test curve is shown in Figure 3.9s.
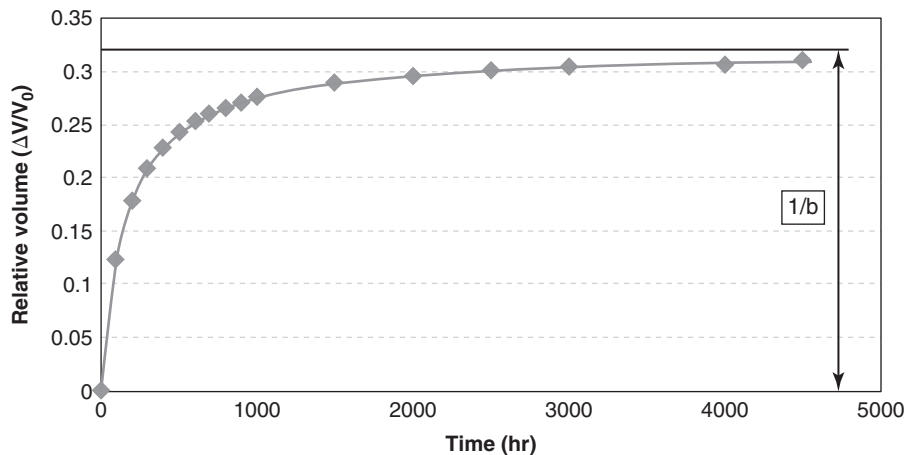

Figure 3.9s   Extended swell test curve.

When t goes to infinity, $\Delta V/V$ goes to $1/b$.

$$\lim_{t \to \infty} \left(\frac{\Delta V}{V} \right) = \lim_{t \to \infty} \left(\frac{t}{a + bt} \right) = \frac{1}{b}$$

So, the limit value of $\Delta V/V = 1/b = 0.319$. The asymptotic volume of the sample at the swell limit is:

$$\frac{V_{\text{swell limit}}}{V_{\text{initial}}} = 1.319 \quad or$$

$$V_{\text{swell limit}} = 1.319 \times 2.5 \times \frac{7.5^2}{4} \times \pi = 145.67 \times 10^{-6} \text{ m}^3$$

The volume of solids in the sample is:

$$W_{\text{dry}} = 1.8 \text{ N}$$

$$W_w = 2.2 - 1.8 = 0.4 \text{ N}$$

$$V_w = \frac{0.4}{9810} \times 10^6 = 40.77 \times 10^{-6} \text{ m}^3$$

$$V_s = \overbrace{25 \times \frac{75^2}{4} \times \pi \times \frac{1}{1000}}^{\text{Total volume}} - 40.77 = 69.67 \times 10^{-6} \text{ m}^3$$

At the swell limit, the water content of the sample is:

$$V_w = V_t - V_s = 145.67 - 69.67 = 76 \times 10^{-6} \text{ m}^3$$

$$W_w = 76 \times 10^{-6} \times 9810 = 0.745 \text{ N}$$

$$w\% = \frac{W_w}{W_s} \times 100 = \frac{0.745}{1.8} \times 100 = 41.3\%$$

Problem 3.7

A silty sand is compacted in a mold. The volume of the mold is 9.46×10^{-4} m³. The weight of compacted soil in the mold is 18.9 N and the water content is 8%. Assume that G_s is 2.65 and calculate the dry unit weight and the degree of saturation.

Solution 3.7

The volume of the sample is:
$$V_t = 9.46 \times 10^{-4} \text{ m}^3$$

The weight of the sample is:
$$W_t = 18.9 \text{ N}$$

The weight of the solids is:
$$W_s = \frac{W_t}{1 + \omega} = \frac{18.9}{1.08} = 17.5 \text{ N}$$

The weight of the water is:
$$W_w = W_t - W_s = 18.9 - 17.5 = 1.40 \text{ N}$$

Assuming that the density of solids is $G_s = 2.65$, the volume of solids is:

$$V_s = \frac{W_s}{G_s \times \gamma_w} = \frac{17.5}{2.65 \times 9810} = 6.74 \times 10^{-4} \text{ m}^3$$

The volume of water is:

$$V_w = \frac{W_w}{\gamma_w} = \frac{1.40}{9810} = 1.43 \times 10^{-4} \text{ m}^3$$

The volume of air is: $V_a = V_t - V_w - V_s = (9.46 - 6.74 - 1.42) \times 10^{-4} = 1.29 \times 10^{-4} \text{m}^3$

	$V_v = 2.69 \times 10^{-4}$ m³	$V_A = (9.46-1.43-6.74) \times 10^{-4} = 1.29 \times 10^{-4}$ m³	Air	$W_A = 0$	
$V = 9.46 \times 10^{-4}$ m³		$V_w = 1.4/9810 = 1.43 \times 10^{-4}$ m³	Water	$W_w = 0.08 \times 17.51$ = 1.40	$W = 18.91$ N
	$V_s = 17.51/9810/2.65 = 6.74 \times 10^{-4}$ m³		Soil	$W_s = 18.91/1.08$ = 17.51 N	

Figure 3.10s Three-phase diagram.

The degree of saturation can be calculated from the following formula:

$$S = \frac{V_w}{V_v} = \frac{1.43 \times 10^{-4}}{(1.29 + 1.43) \times 10^{-4}} \times 100 = 53\%$$

or

$$w.G_s = S.e$$

$$0.08 \times 2.65 = S \times \frac{1.29 + 1.43}{6.74}$$

$$S = 53\%$$

The dry unit weight of the sample is:

$$\gamma_d = \frac{W_s}{V_t} = \frac{17.5}{9.46 \times 10^{-4}} = 18.5 \ \frac{\text{kN}}{\text{m}^3}$$

Problem 3.8

A consolidation test is performed on a sample of soft clay that is 25 mm high and 50 mm in diameter. The test consists of placing a disk of soil in a steel ring and applying load on the sample in a series of steps. The steps last 24 hours and measurements of vertical compression are obtained at the end of each step. The following table shows the time, load, and compression results of the test. Calculate the pressure and vertical strain for the sample at the end of each load step and plot the curve that links the pressure to the vertical strain (stress-strain curve). Why does this curve indicate an apparently surprising result, in that the more load applied to the sample, the stiffer the sample becomes? Can the sample fail?

Time (days)	0	1	2	3	4	5	6
Load (N)	0	294	589	1177	2453	4906	9812
Stress (kN/m²)	0	149.73	299.98	599.45	1249.31	2498.61	4997.21
Height (mm)	25	24.62	24.25	23.62	22.87	21.47	19.7
Displacement (mm)	0	0.38	0.75	1.38	2.13	3.53	5.3
Strain	0	0.0152	0.03	0.0552	0.0852	0.1412	0.212

Solution 3.8
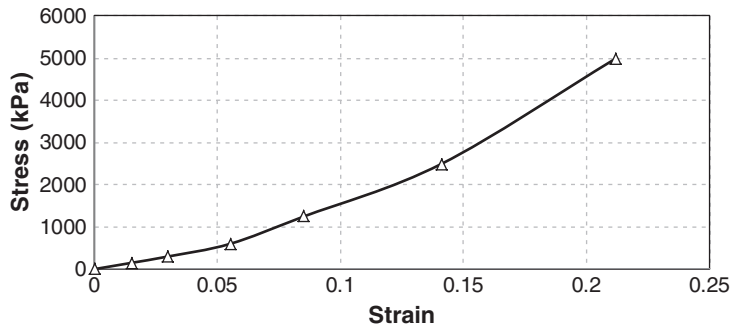
Figure 3.11s   Stress-strain curve.

   The stress-strain curve indicates that the soil become stiffer as the stress increases. Indeed, the ratio of stress increment to strain increment becomes increasingly larger. The reason is that as the stress increases, the influence of the steel ring becomes more important in providing confinement to the sample. With increased confinement, the sample becomes stiffer. The sample cannot fail unless the steel ring fails.

Problem 3.9

A silt has a unit weight of 20 kN/m^3 and a water content of 26%. What is the specific gravity of the particles?

Solution 3.9

The information given in this problem is not sufficient to solve for specific gravity; thus, the exact answer is that it is not possible to solve this problem. However, if we make one assumption, then it becomes possible. For example, assume that the sample is saturated.

$$w = 0.26, Y = 20 \text{ kN/m}^3, Y_w = 10 \text{ kN/m}^3, S = 1$$

$$\gamma = \frac{G_s \, \gamma_w (1 + w)}{1 + e}$$

$$S.e = G_s.w \quad \text{and} \quad S = 1 \quad \text{therefore} \quad e = G_s.w$$

$$\gamma = \frac{G_s \, \gamma_w (1 + w)}{1 + G_s.w} \rightarrow 20 = \frac{G_s \times 10(1 + 0.26)}{1 + G_s \, .0.26} \rightarrow G_s = 2.7$$

Problem 3.10

A 5 m high embankment is made of a sand that has a void ratio of 0.55 and the following boundary void ratios: $e_{\max} = 0.6, e_{\min} = 0.4$. The embankment is subjected to an earthquake that creates a settlement of 0.32 m due to vibration without a change in lateral dimensions. Calculate the void ratio and the relative density of the sand after the earthquake.

Solution 3.10

Let's assume a reference volume of solids equal to 1 m^3:

$$e = \frac{V_v}{V_s} = \frac{V_v}{1} = V_v.$$

   The total height of the embankment, H, is proportional to $1 + e$, so that

$$\frac{H \text{(before earthquake)}}{H \text{(after earthquake)}} = \frac{5}{5 - 0.32} = \frac{1 + 0.55}{1 + e \text{(after earthquake)}}$$

$$\text{Therefore} \quad e \text{(after earthquake)} = \frac{(5 - 0.32)}{5} \times (1 + 0.55) - 1 = 0.45$$

The relative density is

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100(\%) = \frac{0.6 - 0.45}{0.6 - 0.4} \times 100 = 75\%$$

Problem 3.11

Find the relationship between the dry unit weight, the void ratio, and the specific gravity of solids for a soil.
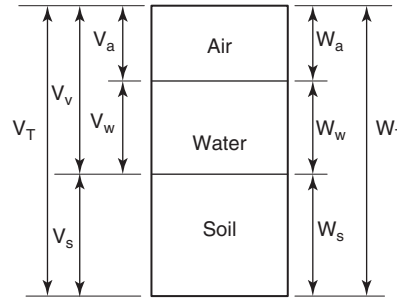
Solution 3.11

Figure 3.12s   Three-phase diagram.

Definition:

$$S = V_w/V_v, e = V_v/V_s, G_s = W_s/(V_s \cdot \gamma_w),$$
$$w = W_w/W_s, \gamma_t = W_T/V_T, \text{ and } \gamma_d = W_s/V_T$$

Using the definition of S and e, the volume of water and air can be rewritten as:

$$V_a = V_v - V_w = e \cdot V_s - S \cdot e \cdot V_s = (1 - S) \cdot e \cdot V_s$$
$$V_w = S \cdot e \cdot V_s$$

Using the relationships and definitions, the dry unit weight is:

$$\gamma_d = \frac{W_s}{V_T} = \frac{W_s}{V_s + V_w + V_a} = \frac{G_s \cdot \gamma_w \cdot V_s}{V_s + V_w + V_a}$$
$$= \frac{G_s \cdot \gamma_w \cdot V_s}{V_s + S \cdot e \cdot V_s + (1 - S) \cdot e \cdot V_s} = \frac{G_s \cdot \gamma_w \cdot V_s}{V_s \cdot (1 + e)} = \frac{G_s \cdot \gamma_w}{1 + e}$$

Problem 3.12

Demonstrate that $S \cdot e = G_s \cdot w$.

Solution 3.12

Definition:

$$S = V_w/V_v, e = V_v/V_s, G_s = W_s/(V_s \cdot \gamma_w), \text{ and } w = W_w/W_s$$
$$G_s \cdot w = \frac{W_s}{V_s \cdot \gamma_w} \cdot \frac{W_w}{W_s} = \frac{W_w}{V_s \cdot \gamma_w} = \frac{V_w \cdot \gamma_w}{V_s \cdot \gamma_w} = \frac{V_w}{V_s}$$
$$S \cdot e = \frac{V_w}{V_v} \cdot \frac{V_v}{V_s} = \frac{V_w}{V_s}$$
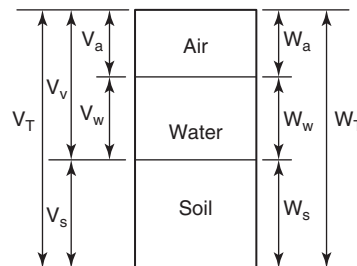$$S \cdot e = G_s \cdot w$$

Figure 3.13s   Three-phase diagram.