CHAPTER 9

Laboratory Tests

9.1 GENERAL

Laboratory testing, in situ testing, and geophysical testing are the options a geotechnical engineer has to obtain the soil information necessary for a geotechnical engineering project. There are advantages and drawbacks to each one of these options (see Table 6.1). Among the advantages of laboratory tests are that they lend themselves to theoretical analysis, that the boundary drainage conditions can be controlled, and that the boundary loading conditions can also be controlled. Some of the drawbacks are the small scale of such testing and the influence of disturbance on the results. Many laboratory tests are available, as shown in Figure 9.1. They are typically classified in the following main categories:

1. Tests for index properties (e.g., water content, unit weight, particle size, Atterberg limits)
2. Tests for deformation properties (e.g., consolidation, triaxial, simple shear, resonant column)
3. Tests for strength properties (e.g., direct shear, unconfined compression, triaxial, lab vane)
4. Tests for flow properties (e.g., constant head permeameter, falling head permeameter, erosion tests)

In each category, one can also distinguish between static tests and dynamic tests. The measurements made during the tests include normal stress, shear stress, normal strain, shear strain, displacements, water compression stress, water tension stress, and air stress. The tests for index properties were presented in Chapter 4. This chapter presents some of the laboratory tests that are often used in practice. They include the consolidation test, the direct shear test, the simple shear test, the unconfined compression test, the triaxial test, the resonant column test, the lab vane test, the constant head permeameter test, the falling head permeameter test, and the erosion function apparatus test.

9.2 MEASUREMENTS

9.2.1 Normal Stress or Pressure

In laboratory testing, measurements of normal stress (Figure 9.2) are most often made by measuring the force and dividing by the area, although normal stress measurements can also be made by using a pressure cell. The measurement of force is done in a number of ways. The simplest way is to add weights on a hanger, as in the classic consolidation test. A proving ring is another device to measure force. It is a stiff steel ring inserted between a jack and the sample; the proving ring is deformed like a spring and the decrease in diameter of the ring is measured using a dial gage. A dial gage is made of a stem with indentations that make a wheel rotate as the stem moves up and down; this wheel rotates a needle on a graduated dial. Dial gages are precise down to a few micrometers. A load cell is the most common way to measure load; it consists of a deformable piece of steel (S shape or cylindrical) instrumented with strain gages. The most common and inexpensive are foil strain gages made of very thin metal strips glued to a surface and connected with an electrical circuit. A change in length of the strain gage created by the deformation of the piece to which the strain gage is glued induces a change in voltage, which is recorded. The change in voltage is correlated with the change in strain of the piece to which the strain gage is glued and therefore to the change in stress and then the change in force. Measurements of normal stress or pressure can also be made by using pressure cells. Such cells are circular and have a metallic membrane that deforms when it is in contact with the stressed soil. The deflection of the membrane is measured with strain gages glued to that membrane and related to the pressure on the membrane. Alternatively, the pressure gage is filled with a fluid and the pressure in the fluid is measured through a diaphragm further away.

9.2.2 Shear Stress

The simplest way to measure shear stress is to measure the shear force and divide by the corresponding area. This is done in the direct shear test. Alternatively, the shear stress can be measured by a shear stress transducer, an example of which is shown in Figure 9.3. In this example, two thin posts are equipped with strain gages to quantify the bending of the posts when a shear force is applied to the top platen.

Geotechnical Engineering: Unsaturated and Saturated Soils Jean-Louis Briaud

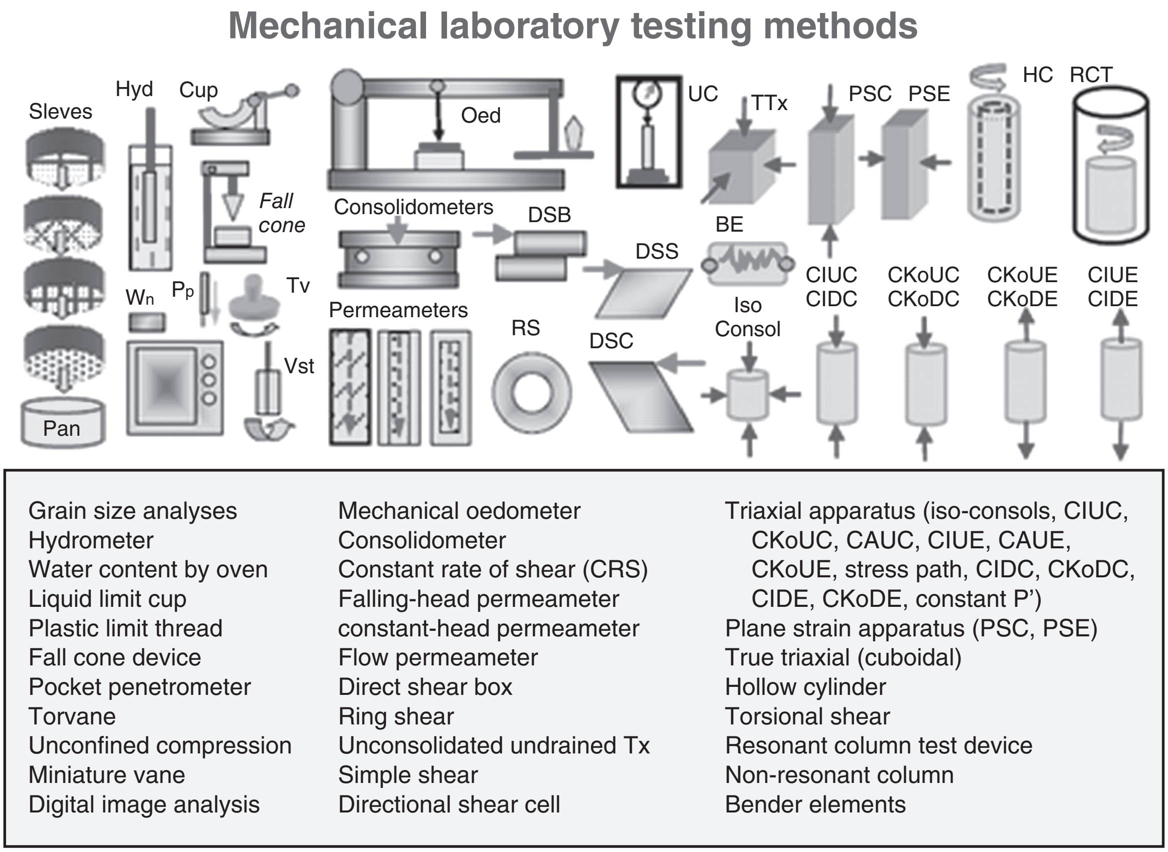


Figure 9.1 Laboratory tests. (From Mayne et al. 2009. Courtesy of Professor Paul Mayne, Georgia Institute of Technology, USA.)

Calibration of the transducer links the readings from the strain gages to the shear stress on the top platen.

9.2.3 Water Compression Stress

Water compression stress is also called *pore water pressure*. It can be measured through a manometer or through a pore-pressure transducer. A *manometer* or *standpipe* is simply a pipe connected to the point where the water compression stress is to be measured and open to the atmosphere at the other end. The pressure in the water makes the water rise in the manometer to the point of equilibrium. The water compression stress is then calculated as the vertical distance between the point of measurement and the water level in the manometer times the unit weight of water. A pore-pressure transducer measures the water pressure by letting that pressure deflect a membrane. A porous tip made of ceramic (Figure 9.4) is placed in contact with the soil where the water is in compression. This porous tip, which is saturated with de-aired water, allows water to come in but does not allow air to come in. This is called a *high air entry porous stone*. Behind the porous tip is a deformable body that responds to the pressure in the water. This could be a thin plate equipped with strain gages, although today it is most commonly a piezoelectric crystal. These crystals have the property of producing a voltage difference between the two sides of the crystal when they are subjected to deformations. So, by calibrating the crystal and measuring the voltage difference across the two sides, one can obtain the pressure.

9.2.4 Water Tension Stress

The tension stress in the water of a soil sample is generated by the suction potential. Suction has two components: matric suction and osmotic suction. Sometimes osmotic suction exists as a potential but is not realized as water tension (see Chapter 11 for more on this). Water tension and suction are usually measured in units of kPa, but sometimes water tension and suction are measured using the pF unit. A pF is defined as the decimal logarithm of the absolute value of the water tension stress or suction expressed in centimeters of water. For example, a water tension of −1000 kPa would correspond to −10,000 cm of water or 4 pF. Table 9.1 gives the equivalences for common values. The symbol *pF* reminds us of the chemical unit of pH which refers to the potential of hydrogen. The pF unit may be interpreted as the potential of flow because the tension in the water would create flow if water became available. Although the pF unit is not accepted

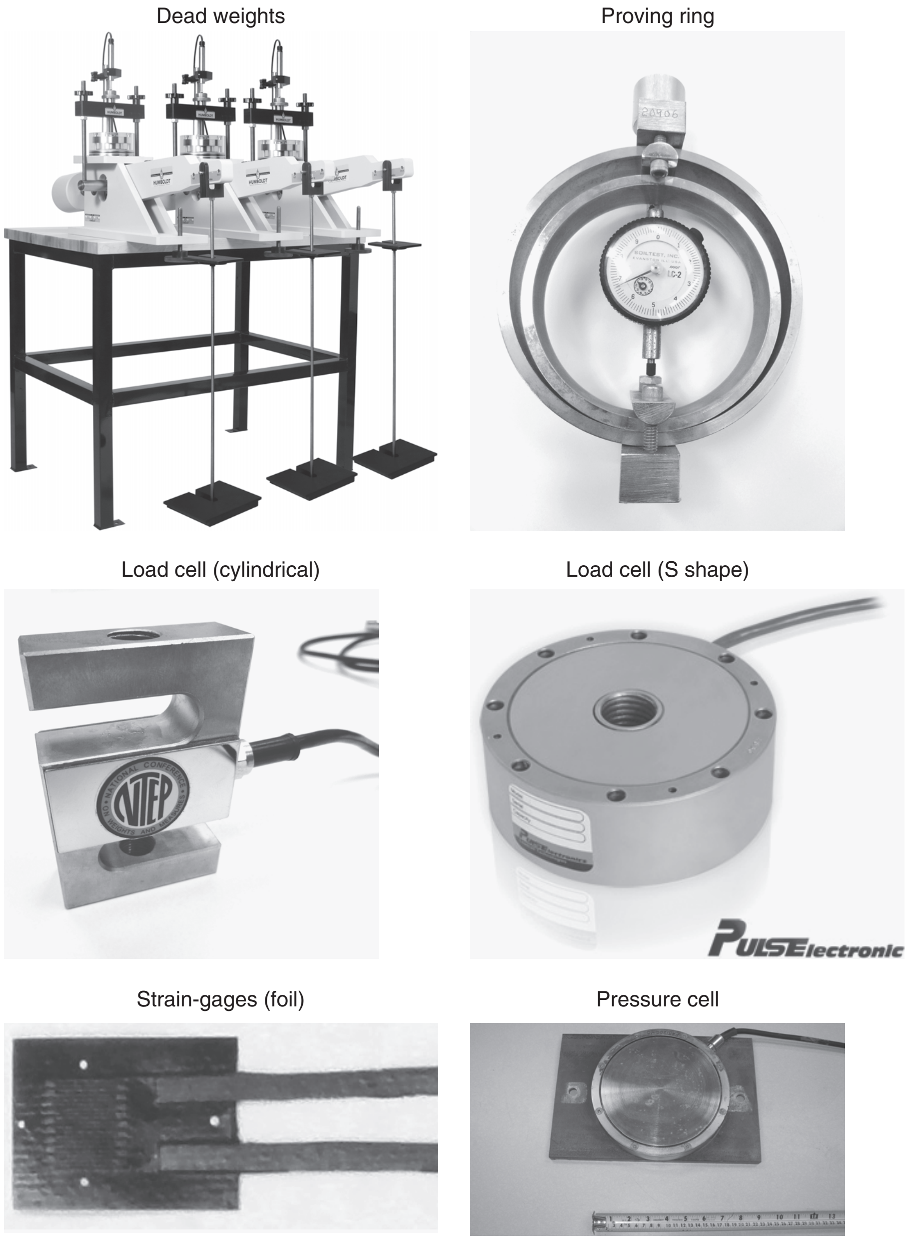


Figure 9.2 Devices to measure normal stress. (*a*) Dead weights. (*b*) Proving ring. (*c*) Load cell (S shape). (*d*) Load cell cylindrical. (*e*) Strain gages (foil). (*f*) Pressure cell. (*a*: Courtesy of Humboldt Mfg. Co.. *d*: Courtesy of Mediscale1.)

by the International Society for Soil Mechanics and Geotechnical Engineering, it conveys the message that using the log of the water tension is convenient.

Measuring water tension is not as easy as measuring water compression. As a result, there are many different measurement methods applicable to different ranges of water tension. These methods include filter paper, psychrometer, tensiometer, pressure plate apparatus, and salt solution equilibrium. Note that some devices or methods are geared toward measuring the natural water tension or suction in a sample, whereas others are geared toward forcing the sample to reach a chosen water tension. The second kind is often used to develop the soil water retention curve, also called the soil water characteristic curve, for a soil. Table 9.2 summarizes the range of application of these devices or methods.

Filter Paper Method

The filter paper method is the simplest of all. It consists of using a circular piece of filter paper (about 50 mm in diameter), weighing it dry, placing it either in contact with or above the soil sample, enclosing the filter paper and the sample in a sealed container until the filter paper comes

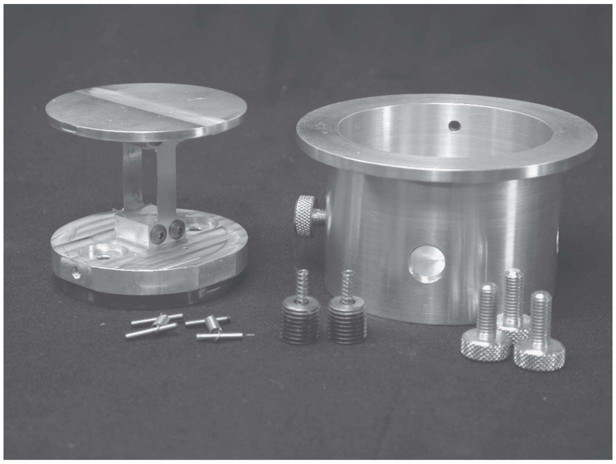


Figure 9.3 Shear stress sensor. (Courtesy of Department of Mechanical Engineering, University of Idaho, USA)

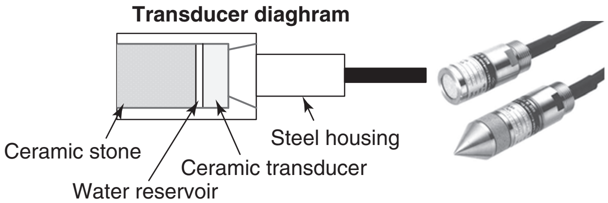


Figure 9.4 Water compression stress transducer or pore-pressure transducer. (*Right*: Courtesy of Bestech Australia provided, © 2010 Tokyo Sukki Kenkyujo Co., Ltd.)

into water tension equilibrium with the sample, retrieving the filter paper, and weighing it to obtain its water content (Bulut, Lytton, and Wray 2001). Because the soil sample is much larger than the filter paper, the water content of the

Table 9.1 Equivalency between kPa and pF Units

Water Tension in kPa	Water Tension in pF (log(cm of H_2O))
−10	2
−100	3
−1000	4
−10000	5
−100000	6
−1000000	7

sample remains unaffected by the amount of moisture drawn into the filter paper. The filter paper comes calibrated with a curve linking the filter paper moisture content and the water tension in the filter paper. Because the water tension is the same in the filter paper and the sample, the water tension of the sample is given in that fashion. Figure 9.5 shows the test in progress and a calibration curve. If the filter paper is in contact with the sample, the water drawn into the filter paper has the same chemistry as the water in the sample; therefore, the water tension due to osmotic suction is not distinguishable and only the water tension due to matric suction is measured. In contrast, if the filter paper is not in contact with the sample, then the water in the filter paper is pure water, while the water in the sample has its own chemistry. In this case the water tension due to osmotic suction is realized in addition to the matric suction and the water tension measured corresponds to the total suction. Note that the part of the procedure dealing with the weighing of the filter paper must be performed

Table 9.2 Methods to Measure Water Tension Stress or Suction

Device or Method	Water Tension or Suction	Range (kPa)	Natural or SWRC*	Time Required	Comments	ASTM
Filter paper	Total	Entire range	Natural	1 to 2 weeks	May measure matric suction if in good contact	D5298
Thermocouple psychrometer	Total	100 to 8000	Natural	1 to 2 hours	Constant environment required	E337
Chilled mirror psychrometer	Total	1000 to 8000	Natural	10 minutes	Scatter at suction values less than 1000 kPa	D6836
Tensiometer	Matric	0 to 90	Natural	10 minutes	Difficulties with cavitation and diffusion through ceramic cup	D3404
Pressure plate	Matric	0 to 1500	Natural or SWRC*	1 to 5 days	Difficulties with pressures higher than 1500 kPa	D6836
Salt solution	Total	Entire range	SWRC*	1 to 2 weeks	Mainly used for calibrating other devices	None

*Soil water retention curve; also called soil water characteristic curve.

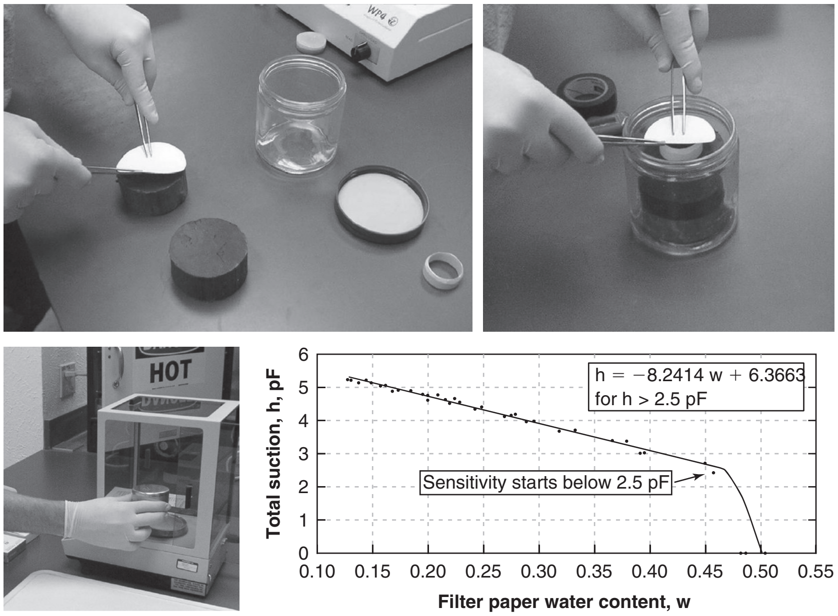


Figure 9.5 Filter paper method for water tension measurement. (a) Filter paper (matric suction only). (b) Filter paper (total suction). (c) High-precision scale with hood. (d) A calibration curve. (From Bulut et al., 2001. Courtesy of Dr. Bulut)

extremely carefully and quickly, as the weights involved are very small and the relative humidity of the air in the laboratory can influence the weight of the filter paper when it is transferred from the sample chamber to the scale chamber.

Thermocouple Psychrometers

Thermocouple psychrometers (*psykhros* means "cold" in Greek) can be used to give the total suction of a soil by measuring the relative humidity in the air phase of the soil pores or the region near the soil (Figure 9.6). They measure the total suction because the evaporation process creates pure water, while the water in the soil pores is not pure water. Hence, the osmotic suction is realized. Psychrometers give the relative humidity by measuring the difference in temperature between a nonevaporating surface and an evaporating surface. Imagine two thermometers, one with a dry bulb and the other with a wet bulb. The dry-bulb thermometer measures the ambient temperature, but the wet-bulb thermometer measures a temperature lower than ambient because the evaporation of the water on the bulb cools the bulb. The thermometers can be replaced by transistors in transistor psychrometers. If the air phase has a low relative humidity, the evaporation is faster, the cooling process is high, and the difference in temperature is larger. If the air phase has a high relative humidity, little evaporation takes place, the cooling process is limited, and the difference in temperature is smaller. The difference in temperature given by the two thermometers is related to the relative humidity, which in turn is related to the water tension or total suction. In the pores of a soil, there has to be a balance between the water tension in the air phase and in the water phase. See Chapter 11 for more details on these relationships. Because psychrometers work on the basis of precise temperature measurements, any exterior fluctuation in temperature will lead to poor precision. Therefore, psychrometers are not well suited for in situ measurements, because of the daily temperature cycle. It also takes a fair amount of time for equilibrium to be reached between the psychrometer and the air in the soil pores.

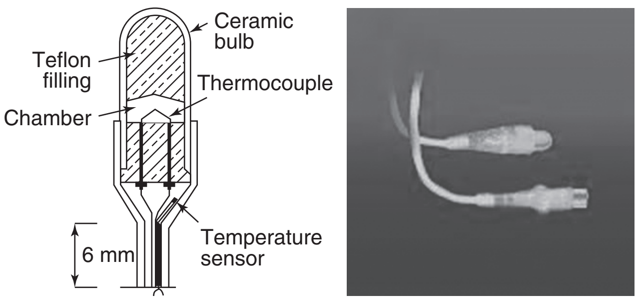


Figure 9.6 (a) Cross section of a thermocouple psychometer. (b) Thermocouple psychometer. (b: Courtesy of Wescor-Elitechgroup.)

Chilled Mirror Psychrometers

Chilled mirror psychrometers can be used to give the total suction of a soil (Figure 9.7). Much like the thermocouple psychrometers, they measure the relative humidity and then relate the relative humidity to the suction. The relative humidity in a chilled mirror psychrometer is obtained as follows: The soil is inserted into a small chamber that is sealed off from the outside air and has a mirror present. Facing the

Figure 9.7 Chilled mirror psychrometer. (Photo courtesy of Decagon Devices, Inc.)

mirror is a camera able to detect when dew forms on the mirror. The air in the chamber comes to relative humidity equilibrium with the air in the soil sample. Then the mirror is chilled down to the point where dew forms on the mirror and the temperature of the mirror at that point is recorded. The temperature of the soil is also recorded and the difference in temperature between the mirror at the dew point and the soil is related to the relative humidity in the soil. The suction is then obtained through its relationship with the relative humidity (see Chapter 11).

Tensiometers

Tensiometers can be used to measure the water tension or matric suction in a soil (Figure 9.8). A tensiometer consists of a high air entry porous ceramic tip (also called a ceramic cup) that is saturated with water and placed in good contact with the soil. In the tensiometer, the space behind the ceramic tip is filled with de-aired water and connected with a negative pressure measuring device. The stress slowly equalizes between the water tension in the tensiometer and the water tension in the soil pores. That tension is then measured either through a water-mercury manometer, a Bourdon-vacuum tube, or an electrical pressure transducer. The water tension that can be measured in a tensiometer is limited to approximately negative 90 kPa (2.95 pF) due to the possibility of water cavitation in the tensiometer above such a value.

Pressure Plate Apparatus (PPA)

The pressure plate apparatus (PPA) is a closed pressure chamber that can be used to increase the air pressure in the soil pores to the point where the air chases the water out of the pores (Figure 9.9). The sample is placed in the chamber on a high air entry ceramic disk. This disk, which is saturated with water, has the property of letting water go through but not air, up to a certain rated pressure, known as the air entry value of the disk. The air pressure is increased and the stress in the water is increased accordingly (decrease in tension). When the water tension becomes equal to zero, the water comes out and at that point, the air pressure is equal to the water tension. This technique is called the axis translation technique because it simply translates the origin of reference by applying an air pressure equal to the water tension (Figure 9.10).

The PPA can be used to determine the natural water tension or to generate a soil-water retention curve. If the soil sample is placed at its natural water content in the PPA, the air pressure that starts the water flow is the natural water tension. If the soil specimen starts as a saturated sample and the air pressure is increased in steps, each pressure step will drive

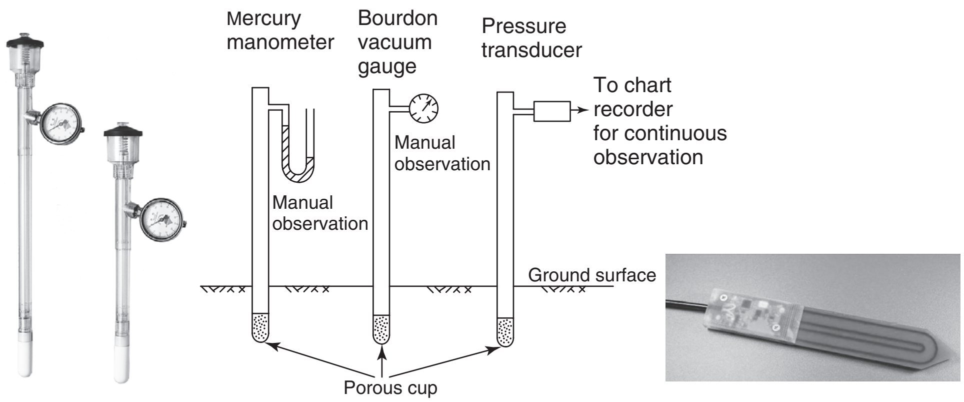


Figure 9.8 Tensiometers. (a) Tensiometer with pressure-vacuum gage. (b) Types of tensiometers. (c) Tensiometer with pressure transducer. (a: Courtesy of Envco Global. *c*: Courtesy of STEP Systems GmbH, www.stepsystems.de)

water out of the sample until equilibrium is reached, and this will give the water tension corresponding to the water content of the sample. This water content can be measured separately by stopping the test or inferred from the water loss read on the burette connected to the PPA. The air pressure is increased in steps and each step gives the water tension and the corresponding water content. The soil-water retention curve (SWRC) is thus obtained. The range of application of the PPA is from 0 to about 1500 kPa (4.17 pF).

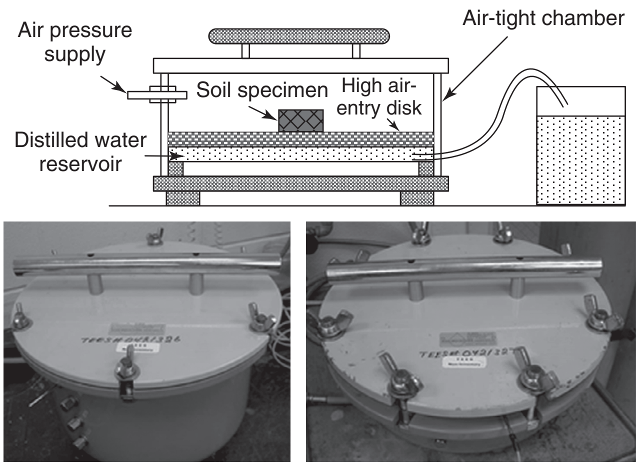


Figure 9.9 Pressure plate apparatus: (*a*) 500 kPa pressure plate. (*b*) 1500 kPa pressure plate.

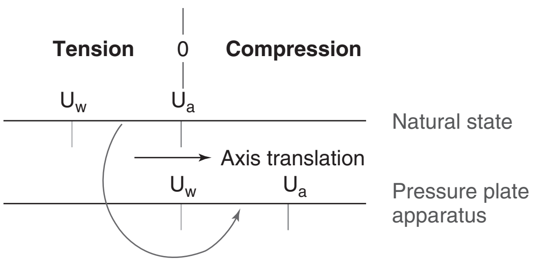


Figure 9.10 Axis translation for water tension determination.

Salt Solution Equilibrium (SSE)

Salt solution equilibrium (SSE) is a water tension measurement technique which relies on the fact that salt solutions have significant osmotic suction. As explained in Chapter 11, osmotic suction comes from the fact that water molecules are attracted to salt molecules: more salt, more attraction. A closed chamber with a salt solution at its lower part (Figure 9.11) will generate a certain relative humidity in the air above it. The higher the salt concentration is, the lower the relative humidity above the solution in the chamber will be. If a soil sample is suspended in the air above the salt solution, it will dry and the water tension in the soil sample will come to equilibrium with the ambient relative humidity. At equilibrium, the water tension is given by the relative humidity in the air of the chamber. This relative humidity depends on the salt concentration in the solution and can be calculated from it (see Chapter 11). This relationship depends on the type of salt, the molality, and the temperature. Table 9.3 gives the osmotic suction for different salts and

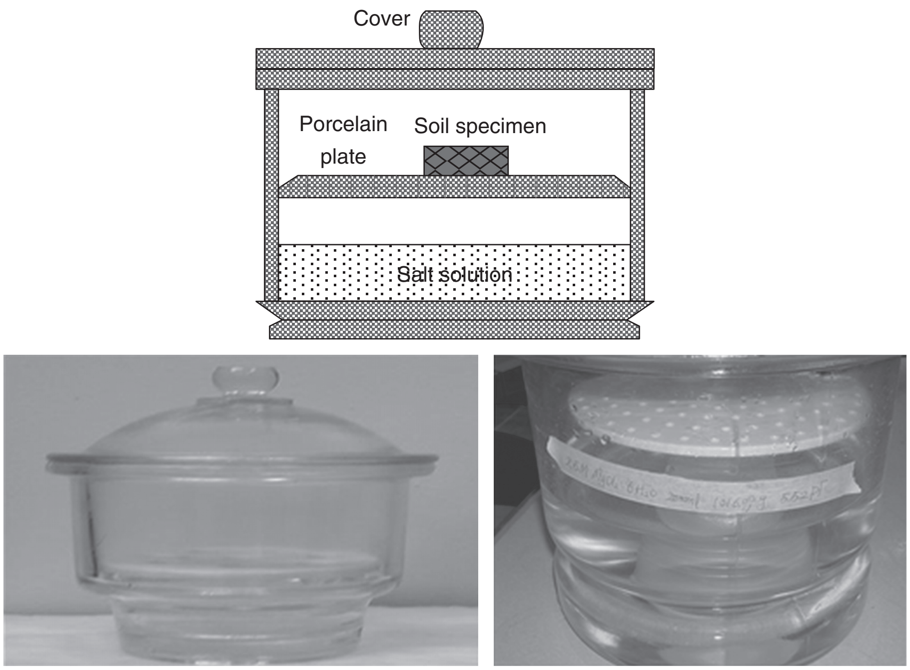


Figure 9.11 Salt solution equilibrium containers for water tension determination.

Table 9.3 Osmotic Suction in kPa of Some Salt Solutions at 25°C

	Osmotic Suction in kPa at 25°C						
Molality (mol/kg)	N_aCl	KCl	NH_4Cl	Na_2SO_4	$CaCl_2$	$Na_2S_2O_3$	$MgCl_2$
0.001	5	5	5	7	7	7	7
0.002	10	10	10	14	14	14	14
0.005	24	24	24	34	34	34	35
0.010	48	48	48	67	67	67	68
0.020	95	95	95	129	132	130	133
0.050	234	233	233	306	320	310	324
0.100	463	460	460	585	633	597	643
0.200	916	905	905	1115	1274	1148	1303
0.300	1370	1348	1348	1620	1946	1682	2000
0.400	1824	1789	1789	2108	2652	2206	2739
0.500	2283	2231	2231	2582	3396	2722	3523
0.600	2746	2674	2671	3045	4181	3234	4357
0.700	3214	3116	3113	3498	5008	3744	5244
0.800	3685	3562	3558	3944	5880	4254	6186
0.900	4159	4007	4002	4384	6799	4767	7187
1.000	4641	4452	4447	4820	7767	5285	8249
1.200	5616	5354	5343	N/A	N/A	N/A	N/A
1.400	6615	6261	6247	N/A	N/A	N/A	N/A
1.500	N/A	N/A	N/A	6998	13391	7994	14554
1.600	7631	7179	7155	N/A	N/A	N/A	N/A
1.800	8683	8104	8076	N/A	N/A	N/A	N/A
2.000	9757	9043	9003	9306	20457	11021	22682
2.500	12556	11440	11366	11901	29115	14489	32776

(After Bulut et al. 2001.)

different molalities. *Molality*, in this case, is the number of moles of salt per kilogram of water. Note that in most cases, molarity is different from molality because *molarity* is the number of moles per liter of solvent.

The range of application for the SSE technique is very wide, from 0 to close to 100,000 kPa (6 pF). It is also a very inexpensive and very reliable technique. Hence, it is used as a reference to calibrate many other techniques. The drawback is that it is quite time consuming: The time necessary for equilibrium to be reached between the water tension in the soil sample and the relative humidity in the surrounding air can be a couple of weeks.

9.2.5 Normal Strain

A normal strain ε is defined in one direction as the change in length Δz divided by the initial length z between two points. A normal strain is measured either by measuring a displacement and a length ($\Delta z/z$) or by using a strain gage (ε). Measurements of length are done with a ruler or a set of calipers (Figure 9.12). Displacements are measured with mechanical devices such as dial gages (Figure 9.12) or electrical devices such as LVDTs, DCDTs, and potentiometers.

A linear variable differential transformer (LVDT) (Figure 9.13) has three solenoid coils arranged like three side-by-side donuts. A small metallic rod is attached to the point where the displacement is to be measured and the solenoids are attached to an immobile reference point. The small rod passes through the center of the three solenoids without touching them. An alternating current through the center solenoid creates a voltage in the side solenoids. The movement of the metallic rod creates a change in voltage that is linearly proportional to the movement of the rod. The change in voltage is transformed into a displacement measurement through calibration. A direct current differential transformer (DCDT) is an LVDT in which the current passing through the solenoids is a direct current instead of an alternating current. A potentiometer or pot is a resistor with three terminals. Two are fixed and one moves between the two fixed terminals. By sliding the moving terminal, the resistance offered by the potentiometer varies and so does the voltage. The rod connected to the point where the movement

Figure 9.12 Mechanical devices to measure displacement: (a) Calipers. (b) Dial gage.

is to be measured is tied to the sliding terminal. The change in voltage induced by the movement of the rod is related to the movement through calibration.

Strain gages are of two main types: foil strain gages and vibrating wire strain gages. A foil gage is a thin sheet of metal (copper-nickel alloy is common) with a pattern (Figure 9.14) glued to the material that is deforming. Actually, a layer of flexible insulating material is first glued to the deforming material and then the foil gage is glued onto the insulator, so that the current passing through the gage only travels through the gage. When the material deforms, the foil length changes and so does its resistance. The voltage changes accordingly and the strain is related to the change in voltage through calibration. Vibrating wire strain gages consist of two small anchor blocks solidly connected to the material that is deforming. Between these two anchors is a high-tensile-strength wire brought taut to a chosen initial load. Around the wire is a cylinder that protects the wire and contains a permanent magnet and a plucking coil. When the wire is plucked, it vibrates at its natural frequency. If the material deforms, the end blocks move and the natural frequency of the vibrating wire changes. The change in natural frequency of the wire is related to the normal strain by theory and calibration.

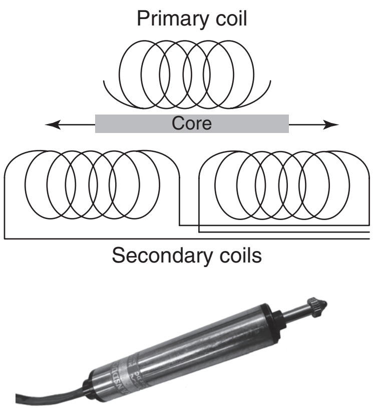


Figure 9.13 Linear variable differential transformers: (a) Principle. (b) Device.

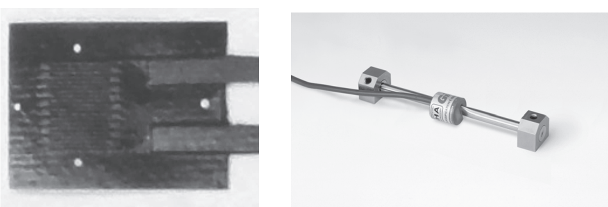


Figure 9.14 Foil and vibrating wire strain gages. (a) Foil strain gage. (b) Model 4000 Vibrating Wire strain gage. (*b*: Courtesy of Geokon, Inc.)

9.2.6 Shear Strain

A shear strain γ is defined for two perpendicular directions (x and y as shown in Figure 9.15). When the shear strain is small enough, the shear strain is equal to the change in angle γ expressed in radians between the two perpendicular directions due to the shearing process. Obtaining shear strain is most easily done by measuring the normal strain in two perpendicular directions (Figure 9.15). It can be shown (Chapter 10) that the shear strain in this case is given by:

$$\gamma_{xy} = \varepsilon_1 - \varepsilon_2 \tag{9.1}$$

9.2.7 Bender Elements

A *bender element* (Figure 9.16) is a small electromechanical device used to generate or sense bending waves. It is made of two thin piezoceramic plates glued together. Between the two plates and on the outside of the two plates are conducting surfaces. Because of the different polarizations of the two plates, when a voltage is driven through the plates, one shortens and the other lengthens; this forces the plates to shake in bending. If the small plates are buried in the soil,

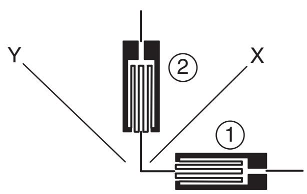


Figure 9.15 Getting shear strain from two normal strain gages.

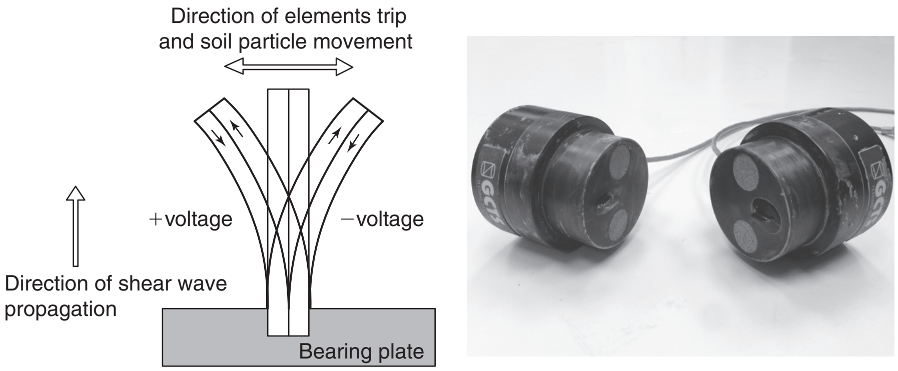


Figure 9.16 Bender elements: (a) Principle. (b) Device.

the repeated lateral motion of the plates generates a wave that propagates in shear through the soil. This is the wave generation function of a bender element. At the other end of the sample, a similar bender element is also buried in the soil and acts as a receiver. This receiver senses the arrival of the shear wave because that wave forces the two plates to move sideways. This bending movement shortens one and lengthens the other; this alternating tension and compression creates an electrical signal that can be measured. When the bender element generates a shear wave, the wave travels through the soil and reaches the bender element, which detects its arrival. Knowing the length of travel (sample length) and the time necessary for the wave to propagate from the generating bender element to the receiving bender element, one can calculate the shear-wave velocity v_s. Theory on shear-wave propagation in an elastic body tells us that the shear modulus G of the soil from measurement of shear-wave velocity v_S is given by:

$$G = \rho\ (v_S)^2 \tag{9.2}$$

where ρ is the mass density of the soil sample. Note that the shear modulus measured in this fashion is associated with very small shear strains.

9.3 COMPACTION TEST: DRY UNIT WEIGHT

9.3.1 Saturated Soils

Most of the time, the soil in a compaction test is unsaturated.

9.3.2 Unsaturated Soils

The compaction test dates back to the work of Ralph Proctor, an American civil engineer, in the early 1930s. Today, the test is actually two tests: the Standard Proctor Compaction Test (SPCT; ASTM D698) and the Modified Proctor Compaction Test, (MPCT; ASTM D1557). Proctor developed the SPCT, but in the late 1950s, as compaction machines became much bigger than in the 1930s, the MPCT was developed to better correspond to the higher energy generated by the larger roller compactors. In both cases, the result of the test is the dry unit weight γ_d vs. water content w curve (Figure 9.17).

The first step in the SPCT is to take a soil sample, dry it, break the clumps of soil down to individual particles (e.g., with a mortar and rubber-tip pestle), and measure its weight W_d. Then, calculate the weight of water W_w that must be added to the dry soil sample to reach a chosen water content w:

$$W_w = w\ W_s \tag{9.3}$$

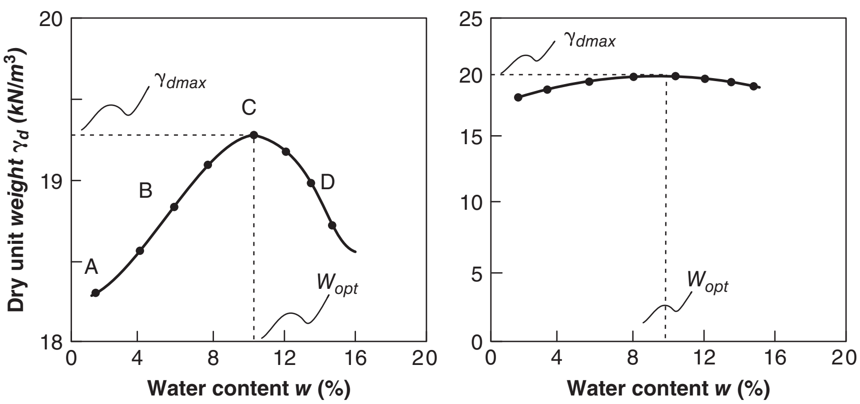


Figure 9.17 Compaction curve.

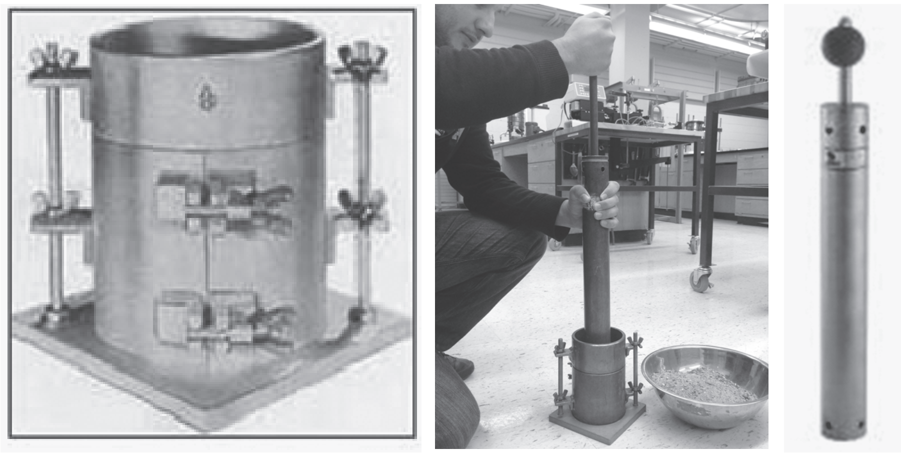


Figure 9.18 Compaction equipment and test: (a) Compaction mold. (b) Compaction test. (c) Compaction hammer. (a and c: Courtesy of Forney LP, Hermitage, PA.)

Add the water to the soil and mix thoroughly. Weigh the empty compaction mold to be used for the test. Using the prepared soil mixture, place a first layer in the compaction mold (Figure 9.18) and compact that layer of loose soil by dropping a standard compaction hammer a standard number of times. The blows should be distributed evenly across the soil layer to reach uniform compaction. Repeat this process for all layers and aim for the last layer to coincide with the top of the mold. Two mold sizes are used; Table 9.4 gives the detailed requirements. At the end, weigh the mold plus soil and calculate the soil weight W_t. The dry unit weight is obtained by:

$$\gamma_d = \frac{W_t}{V_t(1-w)} \tag{9.4}$$

where γ_d is the dry unit weight, W_t is the total weight of the soil sample in the mold, V_t is the total volume of the sample, and w is the water content of the sample. The combination of γ_d and w gives one point on the compaction curve. By repeating the SPCT for different water contents, the compaction curve is described point by point (Figure 9.17). Note that this curve has a well-defined bell shape because the vertical scale is concentrated around the range of values within which the dry unit weight varies. If the same curve is plotted at the full scale of the unit weight, the curve still has a bell shape but shows that the dry unit weight is not very sensitive to the water content.

The reason for this bell curve is that at point A on Figure 9.17 the soil is relatively dry and it is difficult for a given compaction energy to bring the particles closer together. At point B the water content is such that water tension exists between the particles and hinders the effectiveness of the compaction process. At point C, the water tension loses its effect and the primary role of the water is to lubricate the contacts between particles, thereby allowing the given compaction effort to reach a low void ratio and a high dry density. At point D the soil is nearing saturation and the added water simply increases the volume of the voids, which negates the benefit of the compaction.

The compaction curve is bounded on the right side by the saturation line for a degree of saturation equal to 1. Indeed, the relationship between the dry unit weight γ_d and the water content w is a function of the degree of saturation:

$$\gamma_d = \frac{SG_s\gamma_w}{S+G_s w} \tag{9.5}$$

This relationship can be demonstrated as follows:

$$\gamma_d = \frac{W_s}{V_t} = \frac{\gamma_s V_s}{V_v + V_s} = \frac{G_s\gamma_w}{1+\dfrac{V_v}{V_s}} = \frac{SG_s\gamma_w}{S+\dfrac{V_w}{V_v}\dfrac{V_v}{V_s}}$$
$$= \frac{SG_s\gamma_w}{S+\dfrac{V_w}{W_w}\dfrac{W_w}{W_s}\dfrac{W_s}{V_s}} = \frac{SG_s\gamma_w}{S+G_s w} \tag{9.6}$$

Equation 9.6 shows that the relationship between the dry unit weight and the water content for a given degree of saturation S is a hyperbola. This hyperbola is called the *saturation line* and corresponds to S (Figure 9.19). The

Table 9.4 Compaction Requirements for Standard Proctor Compaction Test

102 mm diameter 116 mm high mold	152 mm diameter 116 mm high mold
3 soil layers 25 blows per soil layer Hammer weight 24.5 N Hammer drop height 305 mm Volume 9.43×10^{-4} m^3 Total energy 600 kN.m/m^3	3 soil layers 56 blows per soil layer Hammer weight 24.5 N Hammer drop height 305 mm Volume 21.2×10^{-4} m^3 Total energy 600 kN.m/m^3

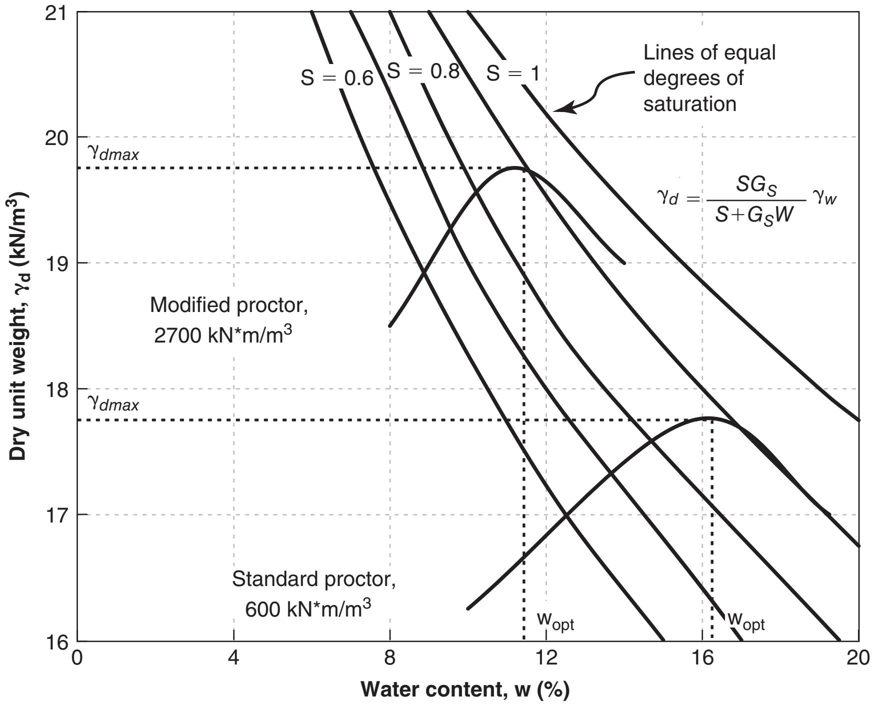


Figure 9.19 Compaction curve for Standard and Modified Proctor Compaction Tests.

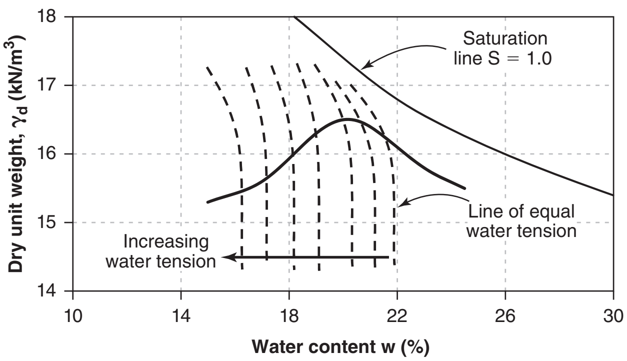


Figure 9.20 Compaction test and water tension lines.

Table 9.5 Compaction Requirements for Modified Proctor Compaction Test

102 mm diameter 116 mm high mold	152 mm diameter 116 mm high mold
5 soil layers 25 blows per soil layer Hammer weight 44.5 N Hammer drop height 457 mm Volume 9.43×10^{-4} m^3 Total energy 2700 kN.m/m^3	5 soil layers 56 blows per soil layer Hammer weight 44.5 N Hammer drop height 457 mm Volume 21.2×10^{-4} m^3 Total energy 2700 kN.m/m^3

saturation line for S = 1 is a bounding envelope for all compaction curves for that soil, called the *zero air void line*. It is also possible to draw the lines of equal water tension on the same graph as the compaction curve, as shown in Figure 9.20.

In 1958, a second compaction test, the Modified Proctor Compaction Test (MPCT), was developed as an ASTM standard. A higher compaction standard was necessary to better correspond to the larger and heavier compaction equipment, such as large vibratory compactors and heavier steam rollers. The MPCT is very similar to the SPCT except for the different requirements listed in Table 9.5. The data reduction is the same and the result is also the γ_d vs. w curve. The difference is that, due to the higher compaction effort (2700 kN.m/m^3 compared to 600 kN.m/m^3), the curve for the MPCT is located higher than the curve for the SPCT (Figure 9.19).

The peak of the curve has the coordinates γ_{dmax} and w_{opt}, called the *maximum dry density* and the *optimum water content* respectively. The specifications for field applications usually require that the water content be within ±x% of the optimum water content and that the dry density be at least y% of the maximum dry density. Then these requirements are checked by field testing at the compaction site (see Chapter 7 on in situ testing).

Note that the dry unit weight is used on the vertical axis of the compaction curve and not the total unit weight. The reason is best explained through the example of Figure 9.21. Both soil A and soil B have a total unit weight of 20 kN/m^3,

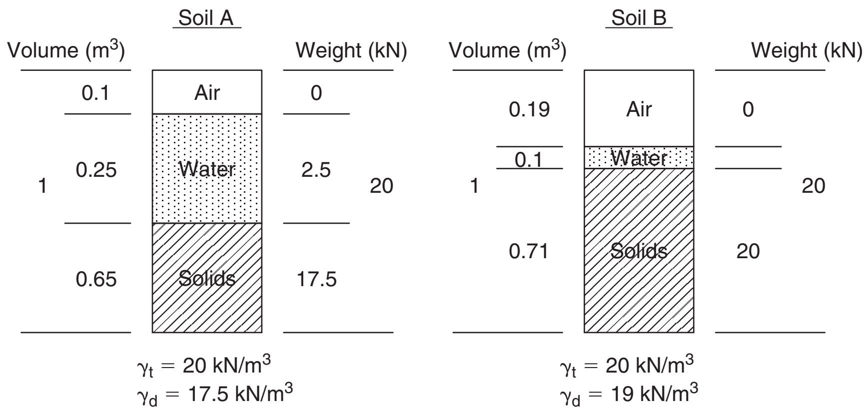


Figure 9.21 Three-phase diagram showing the usefulness of dry unit weight.

yet soil A has a dry unit weight of 17.5 kN/m^3 whereas soil B has a total and dry unit weight of 19 kN/m^3. Soil B has more solid constituents per unit volume and is therefore more compact. The selection of soil B over soil A can be made on the basis of the dry unit weight (19 vs. 17.5) but not on the basis of the total unit weight (20 vs. 20).

9.4 COMPACTION TEST: SOIL MODULUS

9.4.1 Saturated Soils

Most of the time, the soil in a compaction test is unsaturated.

9.4.2 Unsaturated Soils

The compaction test described in section 9.3 yields the dry unit weight γ_d vs. water content w curve. The soil modulus also plays a very important role in the field of compaction. Indeed, one of the major goals of compaction is to minimize deformation, so a sufficiently high modulus should be reached for compaction to be adequate. A modulus E vs. water content w curve can be generated in parallel with the γ_d vs. w curve by using a device called the BCD (Figure 9.22). It consists of a 150 mm diameter thin and flexible steel plate at the bottom of a rod with handles—a kind of scientific cane. Strain gages are mounted on the back of the plate to record the bending that takes place during the loading test. When the operator leans on the handle, the load on the plate increases and the plate bends. If the soil is soft (low modulus), the plate bends a lot. If the soil is hard (high modulus), the plate does not bend much. The amount of bending is recorded by the strain gages and is correlated to the modulus of the soil below.

This test, called the *BCD test* (Briaud et al. 2006), consists of the following steps. First, the BCD plate is placed on top of the sample in the 152 mm diameter compaction mold (Figure 9.22). The operator then leans on the handles of the BCD and the vertical load increases. When the load goes through 223 N, a load sensor triggers the reading of the strain gages. The device averages the strain gage values, uses the internal calibration equation linking the strains to the modulus, and displays the modulus E. This gives one point on the modulus vs. water content curve. By repeating this test for different water contents when the SPCT or MPCT is performed, a complete E vs. w curve can be obtained (Figure 9.23).

The modulus obtained with the BCD corresponds to a reload modulus, to a mean stress level averaging about 50 kPa

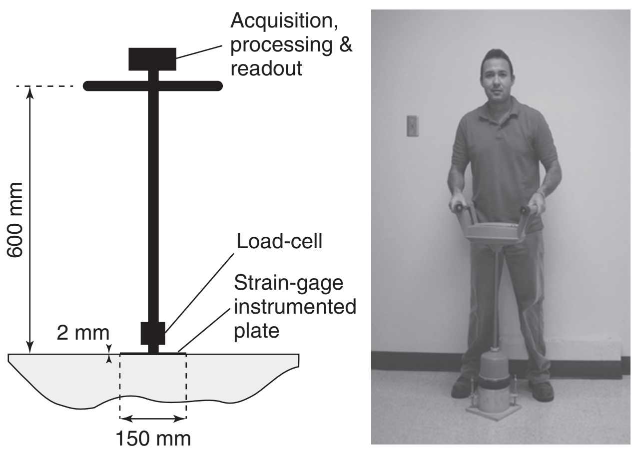


Figure 9.22 BCD apparatus to get soil modulus during a Proctor compaction test: (a) BCD principle. (b) BCD on Proctor mold.

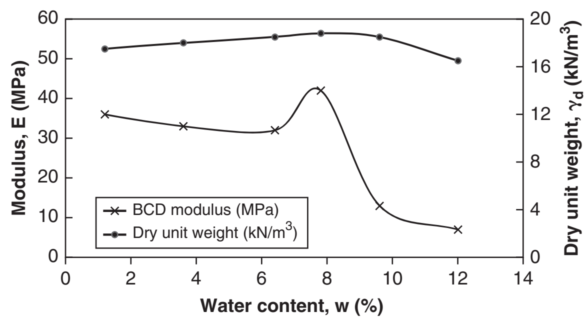


Figure 9.23 Compaction equipment and test.

within the zone of influence, to a strain level averaging 10^{-3} within the zone of influence, and to a time of loading averaging about 2 s.

9.5 CONSOLIDATION TEST

9.5.1 Saturated Soils

The consolidation test dates back to the early 1900s, and it may be appropriate to attribute its early development to Terzaghi, around 1925, with Cassagrande and Taylor making significant contributions as well. The consolidation test (ASTM D2435) is used mostly for determining the compressibility of saturated fine-grained soils. It consists of placing a disk of soil approximately 25 mm high and 75 mm in diameter in a steel ring of the same diameter and applying a vertical load on the sample while recording the decrease in thickness of the sample (Figure 9.24). Filter stones are placed at the top and bottom of the sample to allow the water squeezed out of the sample to drain at both ends. There are several loading procedures: incremental loading, constant rate of strain, and constant gradient.

The *incremental loading procedure* is the most popular and consists of placing a load on the sample for 24 hours while recording the decrease in sample thickness. The load creates a constant total normal stress σ on the surface of the sample. When σ is applied, the water stress u_w goes up because the water has difficulty escaping from the small soil pores quickly enough (Figure 9.25). It takes some time for the water stress u_w to decrease and come back to its original value. This decrease in u_w is associated with a corresponding

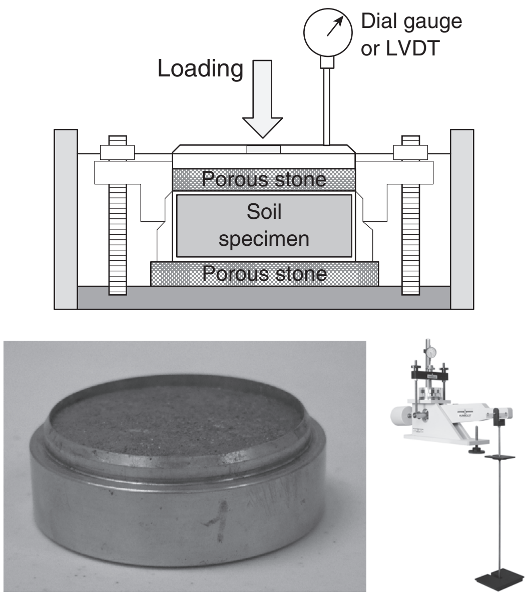


Figure 9.24 Consolidation test and equipment: (a) Principle. (b) Sample in ring. (c) Complete setup. (b: Courtesy of Lev Buchko, P.E. // Timely Engineering Soil Tests, LLC. c: Courtesy of Humboldt Mfg. Co.)

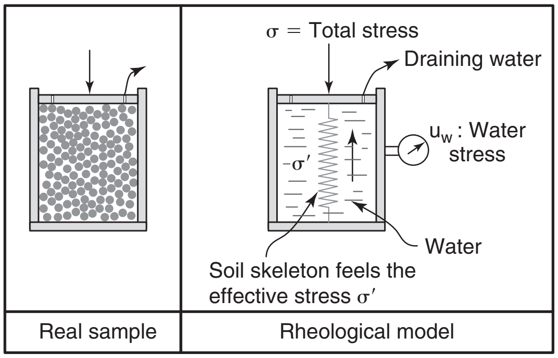


Figure 9.25 Consolidation model.

increase in effective stress ($\sigma' = \sigma - u_w$ in this case, because the soil is saturated) and a settlement of the soil; this is the process of consolidation (Figure 9.26).

The 24-hour loading step is considered to be sufficient in general for the water stress u_w to decrease back to zero. Therefore, it is assumed that at the end of each 24-hour loading step, the water stress is back to zero and the total normal stress σ is equal to the effective normal stress σ'. The loads and associated pressures are applied in a sequence where the load is doubled each time. A typical sequence is 12, 25, 50, 100, 200 kPa for σ.

The last point at the end of the 24-hour loading step curve (displacement vs. time, Fig. 9.26) gives one point (vertical effective stress σ' and vertical strain ε) on the consolidation test stress-strain curve (stress vs. strain, Figure 9.27*a*). The upward curvature of this stress-strain curve and the lack of maximum stress or failure stress or strength is due to the steel ring that confines the soil sample. The more load that is applied to the sample, the more the steel ring contributes to the resistance. Note that this curve is often presented as void ratio e versus the decimal logarithm of the effective stress $\log \sigma'$ (Figure 9.27*b*). The compression index C_c is defined as the slope of the linear portion of the $e - \log \sigma'$ curve past the initial rounded part of that curve (Figure 9.27). As such, C_c is:

$$C_c = \frac{\Delta e}{\Delta \log \sigma}, \tag{9.7}$$

During each 24-hour loading step, the decrease in sample height ΔH is recorded as a function of time t to be able to develop the ΔH vs. t curve. The vertical strain ε is obtained by dividing the change in height ΔH by the original height H_o of the sample. Figure 9.28 shows the ε vs. t curve for three loading steps.

The coefficient of consolidation c_v can be obtained from the ε vs. t curve of each load step through the formula:

$$c_v = T\frac{H^2}{t} \tag{9.8}$$

where T is the time factor, H the drainage length, and t the time elapsed. The drainage length H is equal to the height

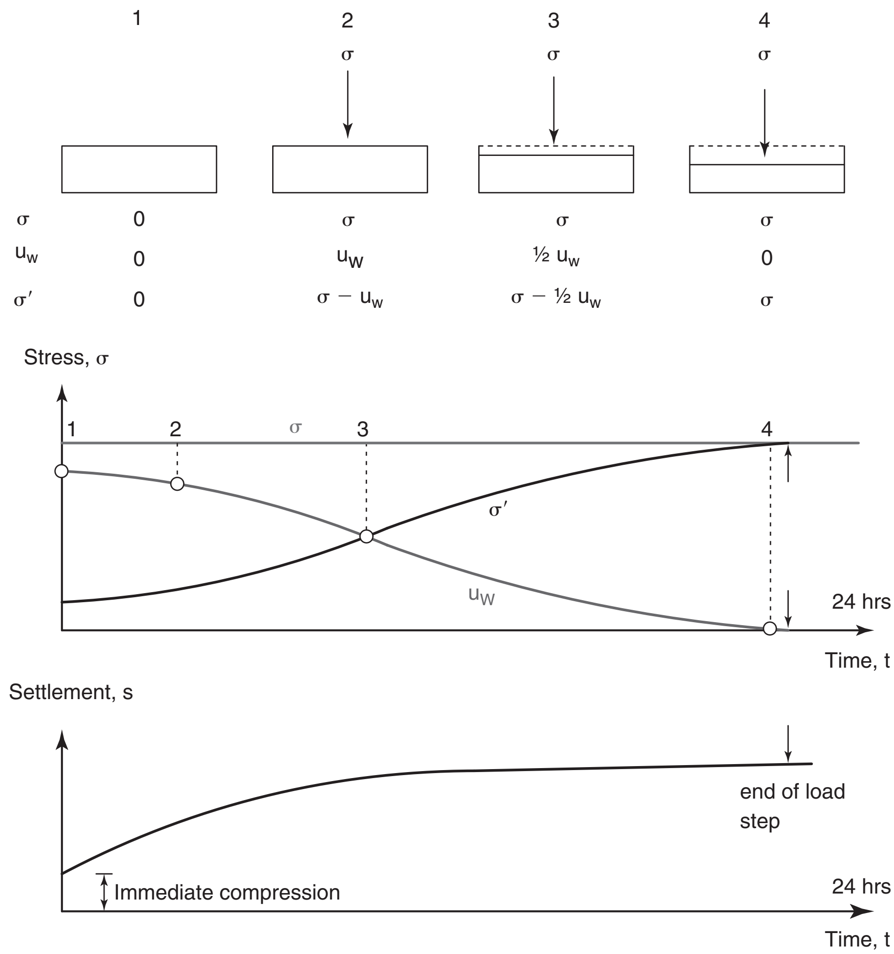


Figure 9.26 Consolidation process.

H_o of the sample if there is drainage only on one side of the sample (top only or bottom only) and equal to half the height of the sample, $H_o/2$, if there is drainage on the top and bottom of the sample. The time factor T comes from the solution of the governing differential equation for the one-dimensional consolidation theory (see Chapters 11 and 14 for more on consolidation theory). This time factor is linked to the average percent consolidation U, defined as:

$$U = \frac{s(t)}{s_{\max}} = 1 - \frac{u_e}{u_{e\,\max}} \tag{9.9}$$

where $s(t)$ is the settlement at time t, $s_{\max}$ is the settlement at a time equal to infinity, u_e is the excess water stress or pore pressure at time t, and $u_{e\,\max}$ is the maximum excess water stress. The theoretical curve linking the average percent consolidation U to the time factor T is shown in Figure 9.29. This curve describes the normalized displacement vs. time curve for the sample according to the one-dimensional consolidation theory. It represents a normalized version of the settlement vs. time curve under a given load.

A value of c_v can be obtained for each load step by choosing a value of the percent consolidation U (50% or 90%, for example) and finding the corresponding time on the ε vs. t curve. Two methods are available to do this: the log time method developed by Cassagrande (1938) and the square root of time method developed by Taylor (1948). The log time (Cassagrande) method requires that ε_0 and ε_{100} be found on the ε vs. t curve (Figure 9.30). Note that ε_0 is not necessarily zero, as ε_0 refers to zero percent consolidation, not zero deformation. This is a subtle distinction, as the first part of the deformation may be elastic in nature and does not correspond to water being expelled from the pores (consolidation). Cassagrande proposed the following way to find ε_0 (Figure 9.30): Plot the ε vs. t curve as ε vs. log t; choose a point near the beginning of the ε vs. log t curve with coordinates ε_1 and log t_1; find the point with coordinates ε_2 and log t_2 = log 4 t_1; calculate the difference $\varepsilon_1 - \varepsilon_2$; and find ε_0 as:

$$\varepsilon_0 = \varepsilon_1 - (\varepsilon_2 - \varepsilon_1) = 2\,\varepsilon_1 - \varepsilon_2 \tag{9.10}$$

The basis for this technique is that, according to the theory, the beginning of the ε vs. t curve is a parabola, so that the

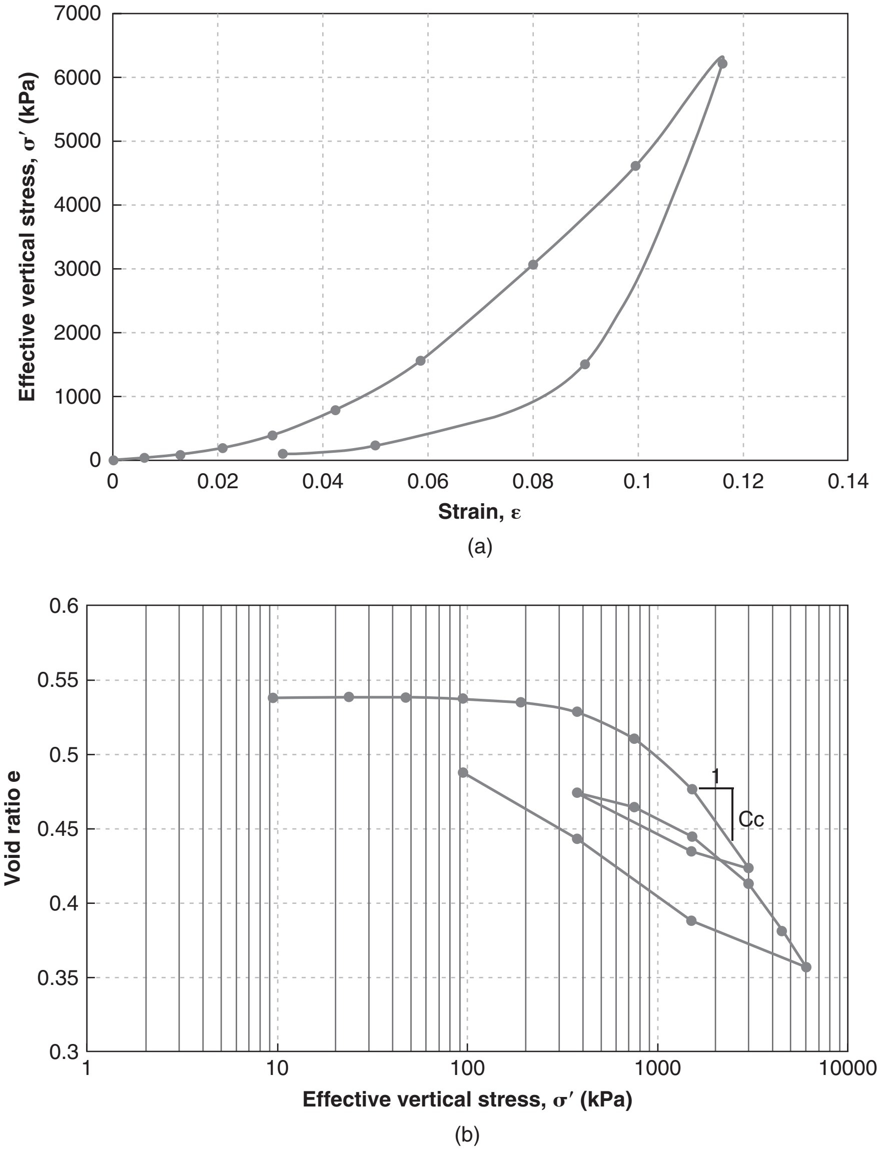


Figure 9.27 Consolidation test results: (*a*) Stress-strain curve. (*b*) e log σ' curve

beginning of the parabola satisfies equation 9.10. Once ε_0 is known, ε_{100} is found at the intersection of the two straight lines drawn on the ε vs. log t curve as shown in Figure 9.30. Then ε_{50} is read on the curve halfway between ε_0 and ε_{100}. The time t_{50} is read as the time corresponding to ε_{50} on the curve. Once t_{50} is obtained, Eq. 9.8 is used to calculate c_v. All other quantities are known, including $T_{50} = 0.197$, and the drainage length as described previously.

The square root of time (Taylor) method consists of plotting the ε vs. t curve as ε vs. $\sqrt{t}$ curve (Figure 9.31). Then a straight line is fitted to the early part of the curve (AB on Figure 9.31). A straight line with a slope equal to 1/1.15 times the slope of the first line is then drawn through point A (AC on Figure 9.31). The intersection of line AC with the ε vs. $\sqrt{t}$ curve corresponds to $\sqrt{t_{90}}$. Once t_{90} is known, Eq. 9.8 is used to calculate c_v. All other quantities are known, including $T_{90} = 0.848$, and the drainage length as described previously.

The preconsolidation pressure σ'_p is another important soil parameter that can be obtained from the consolidation test. It is the effective vertical stress before which the deformation of the soil is small and after which the deformation of the soil increases more rapidly. It can be thought of as a vertical yield stress, although failure does not necessarily happen at σ'_p. This effective stress corresponds to the highest long-term effective stress that the soil has been subjected to. The following procedure is recommended to obtain σ'_p from the consolidation test (Figure 9.32). Choose the point of highest curvature on the ε vs. log σ' curve (Point A on Figure 9.32); then draw a horizontal line through that point and a line tangent to the curve at that point. Then draw the bisectrice of the angle formed by these two lines. Draw the straight line

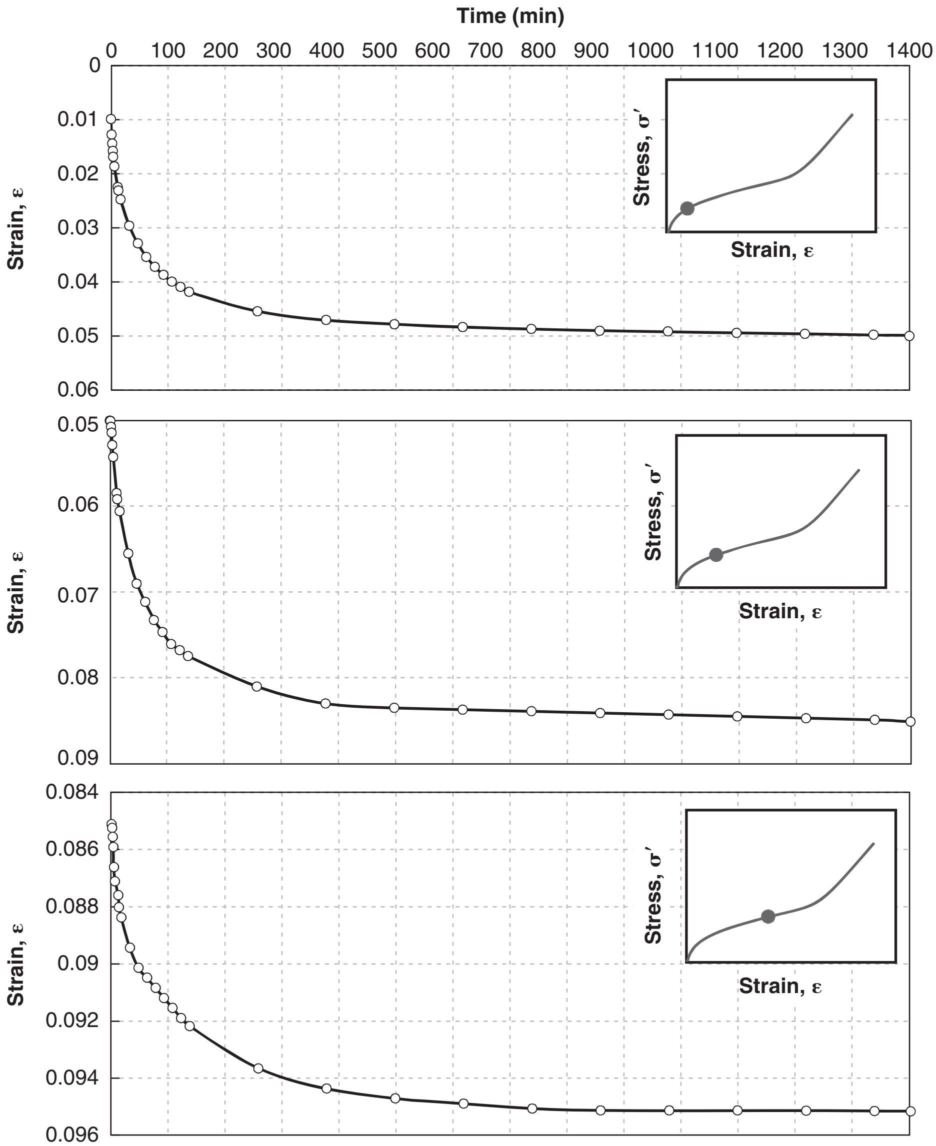


Figure 9.28 Vertical strain vs. time for three consolidation test loading steps.

that best fits the portion of the ε vs. log σ' curve past the σ'_p value. The intersection between this best-fit straight line and the bisectrice is a point that defines the preconsolidation pressure σ'_p (Figure 9.32).

The *constant rate of strain procedure* consists of the same procedure as the incremental loading procedure but with the following differences. The water is allowed to drain from the top of the sample but not from the bottom of the sample, where the water stress is measured. The sample is then deformed at a constant rate of displacement with time. This rate is chosen in such a way that the increase in water stress Δu_w at the bottom of the sample is kept at 5 to 10% of the vertical stress σ applied on the sample.

The *constant gradient procedure* consists of the same procedure as the constant rate of strain procedure but with the following differences. When the load is applied, a water stress (pore pressure) Δu_w develops throughout the sample. Soon the excess water stress at the top of the sample decreases to zero, because drainage is allowed but the bottom water stress remains close to Δu_w because the sample is not allowed to drain at the bottom. This creates a gradient between the top and bottom of the sample. This gradient is maintained constant as the load on the sample is slowly increased. However, at the end of each loading step, the water stress is allowed to dissipate to obtain an equilibrium compression of the soil.

Advantages of the consolidation test include its relative simplicity and its yield of the response of a soil sample to one-dimensional confined compression. A drawback is that the confinement provided by the steel ring around the sample

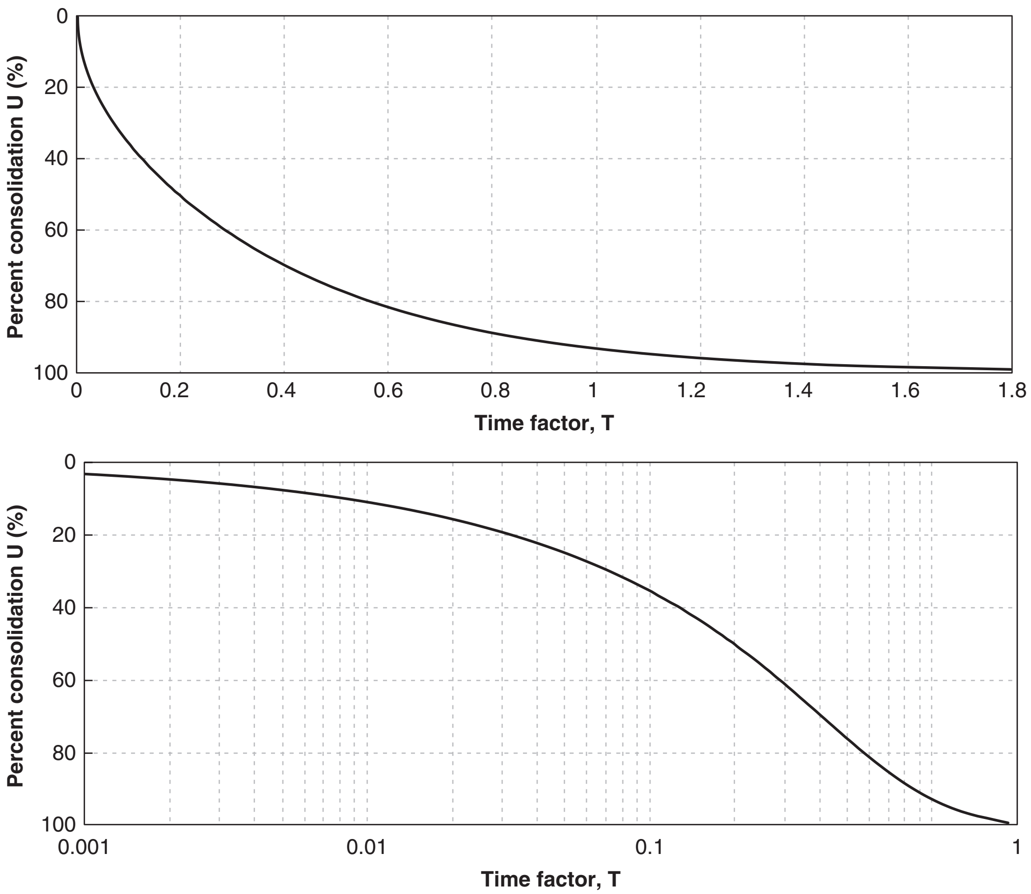


Figure 9.29 Percent consolidation vs. time factor.

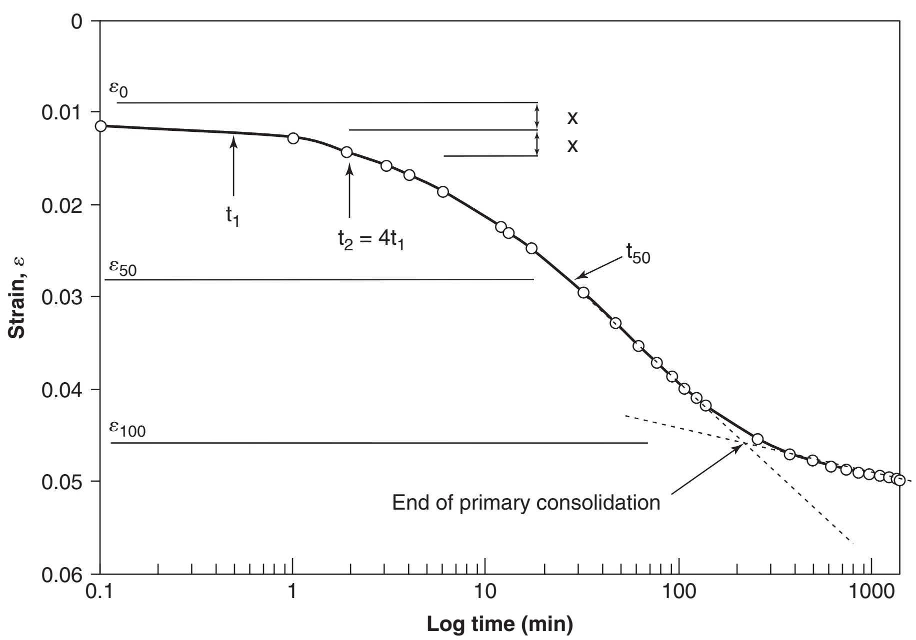


Figure 9.30 Log time method to obtain the coefficient of consolidation. (After Cassagrande 1938.)

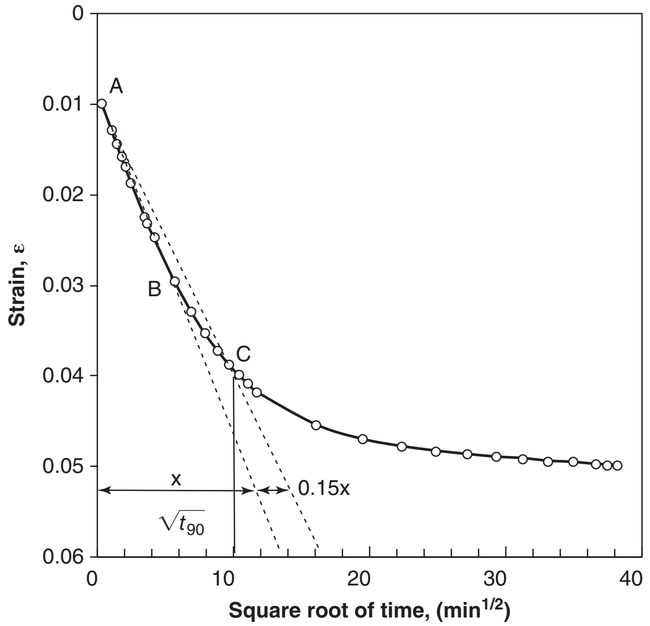


Figure 9.31 Square root of time method to obtain the coefficient of consolidation. (After Taylor 1948.)

prevents lateral deformations and may not represent the true deformation of the soil in the field.

9.5.2 Unsaturated Soils

If the soil is unsaturated, the test procedures are unchanged. However, the water is in tension initially, when the sample is placed in the consolidometer. The increase in vertical stress on the sample as the test proceeds may create enough of an increase in water stress that it goes from tension to compression. If the soil is saturated, it is implicitly assumed that at the end of each 24-hour loading step in the loading step procedure, the water stress is zero; that way the effective stress on the sample can be calculated for each step. In the case of unsaturated soils, it becomes more difficult to calculate the effective stress on the sample. The following expression can be used if the air stress is zero (see Chapter 10):

$$\sigma' = \sigma - \alpha \, u_w \tag{9.11}$$

where σ' is the effective stress, σ the total stress, α the water area ratio coefficient, and u_w the water tension stress. The coefficient α can be estimated as the degree of saturation S, but the error can be $\pm 40\%$ of the correct value. A better estimate consists of using the air entry value, as shown in Chapter 10. Either way, obtaining σ' requires that the water tension u_w be measured during the test. Most of the time, a soil in the saturated state with the water in compression is more compressible than the same soil in the unsaturated state with the water in tension. One exception is collapsible soils; with such materials, an unsaturated soil can experience significant and sudden compression when inundated (see section 9.8).

9.6 SWELL TEST

9.6.1 Saturated Soils

When soils absorb water, they may swell; some soils swell more than others. This is why it is important in many cases to measure how much swelling takes place when a soil has access to water. Consider a sample of dry, clean gravel in a container: When you add water to it, the water will fill the voids, but when the voids are full, no more water will be absorbed by the gravel. Clean gravel does not swell during wetting. Now consider a dry piece of montmorillonite clay

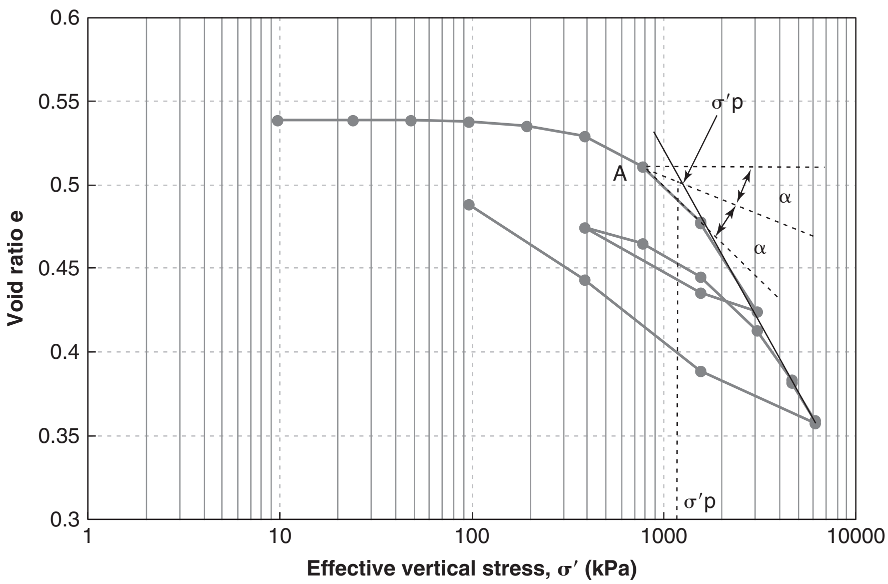


Figure 9.32 Method to determine the preconsolidation pressure from the consolidation test.

with a high dry density and place it at the bottom of a glass of water. The first thing that you will see through the wall is tiny explosions at the surface of the clay sample. The reason is that the water is drawn into the voids, but these voids are full of air that cannot escape because the water is coming in. This pressurizes the voids. The pressure increases until it overcomes the tensile strength of the dry clay, and a series of mini explosions is created. After a while the air finds a way to escape and the water enters the voids. The amount of swelling then depends on what the soil particles are made of. Montmorillonite minerals have a tremendous ability to attract water, so the swelling can be very significant for such clays and the sample may more than double in height. Swelling soils have very fine, highly plastic clay particles and are relatively dense. If they are located in regions where the water content of the soil varies significantly from one season to the next, they can create a lot of damage to structures, particularly light ones like houses, as they swell or shrink unevenly and distort those structures.

If the water in the voids is in compression (below the groundwater level), then no swelling will take place. If the water in the voids is in tension (above the groundwater level), then more water will be attracted into the voids. Thus, the swell test is more useful for soils above the groundwater level. These soils may be saturated or unsaturated. The procedure for the swell test is the same for both saturated soils and unsaturated soils and is described in section 9.6.2.

9.6.2 Unsaturated Soils

The *swell test* (Figure 9.33; ASTM 4546) consists of placing a soil sample in a snug-fitting cylindrical container (consolidometer ring), inundating the soil by placing it in a bath of water, and measuring the vertical swell movement (vertical strain) as a function of time (Figure 9.34). The vertical strain is the change in height of the sample divided by the initial height of the sample. Water access to the sample is provided by porous disks placed at the top and bottom of the sample. The swelling can take days or even weeks. If the top of the sample is not subjected to any vertical load, the test is called a *free swell test* (path AB on Figure 9.35, path CD on Figure 9.36). If a vertical load is applied, the

Figure 9.33 Swell test equipment.

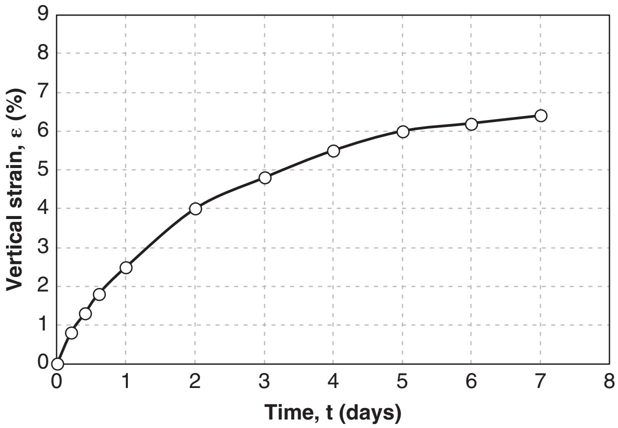


Figure 9.34 Swell test results: Vertical strain vs. time.

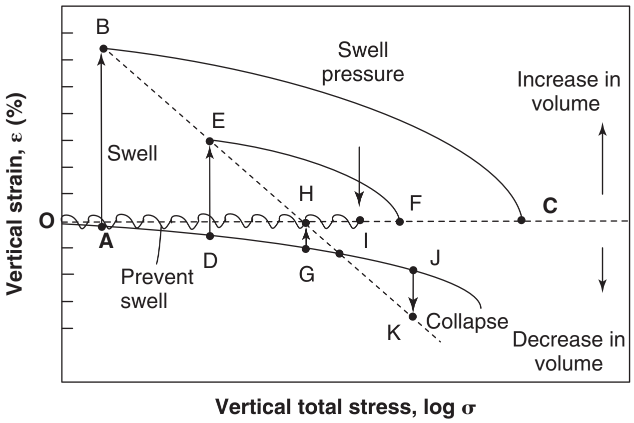


Figure 9.35 Shrink-swell test results: Vertical strain vs. vertical total stress.

test is simply called a swell test (path DE on Figure 9.35, path CE on Figure 9.36). Note that after swelling, a regular consolidation test can be performed on the sample (path BC and EF on Figure 9.35).

The free swell test gives the *swell limit*, which is the water content of the sample at the end of the free swell test (point B on Figure 9.35). The swell limit represents an upper limit of the water content that the soil can reach in the undisturbed state. When a vertical load is applied, it is usually applied before water is added on top of the sample and swelling starts. The magnitude of the load influences the swelling. It is often advantageous to apply a vertical stress on the sample equal to the stress that the soil will experience in the field (under the planned structure, for example). Sometimes the pressure is high enough that no swelling can take place, and settlement takes place instead.

Another way to run the swell test is to add the water first so that swelling can start and to increase the vertical stress on the sample gradually to prevent any swelling (path OI on Figure 9.35). During this test, the volume of the sample is maintained constant and equal to its initial volume. When

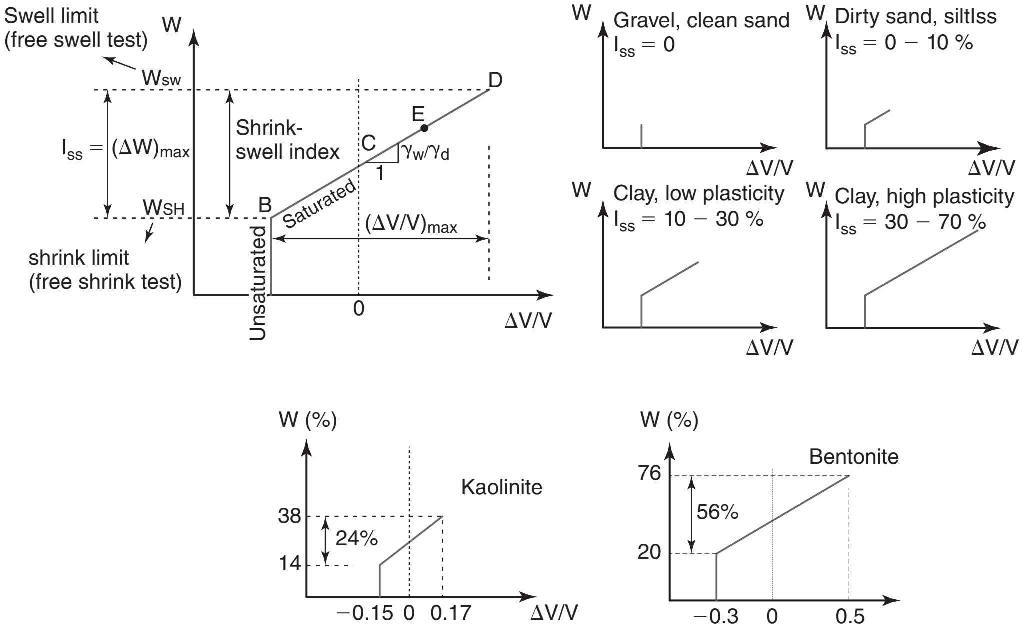


Figure 9.36 Shrink-swell test results: Water content vs. relative change in volume. (*a*) Idealized behavior. (*b*) Typical ranges. (*c*) Low-plasticity clay example.

the vertical stress reaches an equilibrium value, that stress is called the *swelling pressure*. Swelling pressures can reach 1000 kPa or higher for high-plasticity clays.

9.7 SHRINK TEST

9.7.1 Saturated Soils

The shrink test (Figure 9.37) consists of trimming a sample of soil into a cylindrical shape, measuring its dimensions, and recording its weight. The initial volume V_o and the initial weight W_o are recorded. Then the sample is left to dry while the dimensions and the weight are measured as a function of time. This gives the volume V(t) and weight W(t). When the sample is air-dried, it is placed in the oven to obtain the oven dry weight W_s. The average water content w of the sample at any time during the test is $(W(t) - Ws)/Ws$. The results of the test consist of a plot of the relative change in volume $(V(t) - V_0)/V_0 = \Delta V/V_0$ as a function of time t (Figure 9.38), and the water content w as a function of the relative change in volume $\Delta V/V_0$ (Figure 9.36). The undisturbed sample shrinkage limit w_{SH} is the water content corresponding to the point where the sample first stops decreasing in volume (point B on Figure 9.36(a)).

Figure 9.37 Free shrink test for shrinkage limit of undisturbed sample.

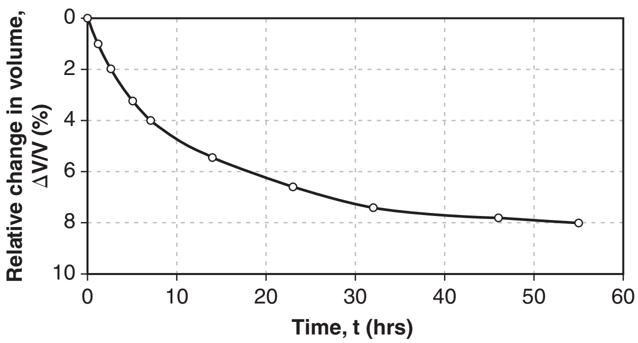


Figure 9.38 Free shrink test result: Relative change in volume vs. time.

As in the case of the swell test, the shrink test can be performed without any vertical load applied (free shrink test) or with vertical load applied (shrink test). The free shrink test is much more common.

9.7.2 Unsaturated Soils

The shrink test applies equally to saturated soils and unsaturated soils. In both cases the water is in tension throughout the test. The soil may start as a saturated soil, but, as it dries, it goes through the air entry value u_{we}, at which point it becomes unsaturated. The shape of the relative volume change vs. time curve for the free shrink test (Figure 9.38) is similar to the shape of the relative volume change vs. time curve for the free swell test (Figure 9.34). During the free shrink test, the weight of the sample is measured as a function of time, so it is possible to plot the water content as a function of relative volume change (BCD on Figure 9.36*a*). This curve indicates where the undisturbed shrink limit w_{sh} occurs. Note that the undisturbed shrink limit is more obvious for low-plasticity soils than for high-plasticity soils. The undisturbed shrink limit is different from the Atterberg shrink limit, which is obtained on a remolded sample.

9.8 COLLAPSE TEST

9.8.1 Saturated Soils

Consider a natural sample of dry silt with a low dry density and a reasonable strength. Place it in a steel ring and place some weight on top of the sample. In the dry state, the sample has no problem carrying the load without much deformation. Now add water on top of the sample: You will likely see a significant amount of compression take place due to collapse of the soil skeleton. What happens is that the small amount of water tension that exists at the contacts between the silt particles is lost when the water enters the voids and the loose structure of the silt collapses. It is important to check if a soil is collapsible; you can imagine the distress associated with any structure built on such soils if a significant amount of water permeates below the foundation.

Collapsible soils consist of loose, dry, low-density materials (say less than 16 kN/m^3) that decrease in volume (collapse and compact) with the addition of water. These soils are often found in arid regions, specifically in areas of wind-blown silty sediments (loess), young alluvial fans, and debris flow sediments. Soil collapse occurs within soils above the groundwater level. The process of saturation weakens or eliminates the clay bonds holding the soil grains together through water tension.

9.8.2 Unsaturated Soils

The collapse test (ASTM D5333) is the same for saturated and unsaturated soils. It is performed with the sample confined in a consolidometer ring. Typically, it consists of loading the soil sample to a vertical stress equal to the vertical total stress that the soil will experience at a chosen depth, recording the vertical strain vs. time curve (consolidation test), and then (once the compression is complete) inundating the sample while continuing to record the vertical strain vs. time curve. Once the collapse is completed, the consolidation test can be resumed by increasing the vertical stress. A sample vertical strain vs. vertical stress curve is shown in Figure 9.39.

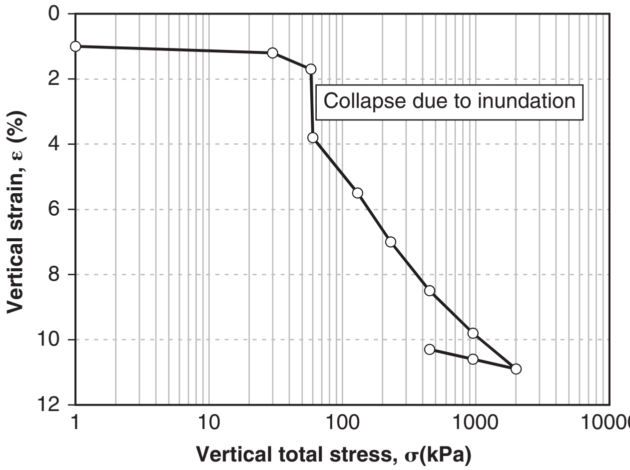


Figure 9.39 Collapse test: Vertical strain vs. vertical stress.

9.9 DIRECT SHEAR TEST

9.9.1 Saturated Soils

The direct shear test (ASTM D3080) is a simple test used to obtain the shear strength of a soil. A disk of soil is placed in a steel cylinder split horizontally at mid height (Figure 9.40). The cylinder is made of two rings stacked on top of each other. One filter stone is placed on top and one at the bottom of the sample so that the water can drain from the sample during the test. A vertical load is applied to the top of the sample and maintained constant during the test. This vertical load creates a total normal stress σ. Then the soil sample is sheared horizontally by pushing on the bottom ring while holding the top ring. This forces a shear plane to develop around the mid height of the sample. During the shearing process, the shear force is measured with a load cell, the horizontal displacement with an LVDT or dial gage, and the vertical displacement with an LVDT or dial gage. The result of a direct shear test is a shear stress vs. horizontal displacement curve and, if the vertical movement is also measured, a vertical movement vs. horizontal movement curve (Figure 9.41).

During the first part of the direct shear test, the soil sample is allowed to consolidate under the vertical stress applied, if such a stress is applied. The consolidation is monitored by recording the vertical movement of the sample as a function of time. When the settlement stops or becomes very small, it is assumed that the water stress has returned to zero and the shearing part of the test can start. During the second part of the test, the sample is sheared and shearing takes place along a thin horizontal band at mid height of the sample near the junction between the two steel rings. The shear stress versus horizontal movement curve is obtained point by point. The shear strength is the maximum shear stress on the shear stress versus horizontal movement curve. This shear strength is the undrained shear strength if the shearing part of the test is run quickly enough that water does not have time to drain; it is the drained shear strength if the test is run slowly enough that

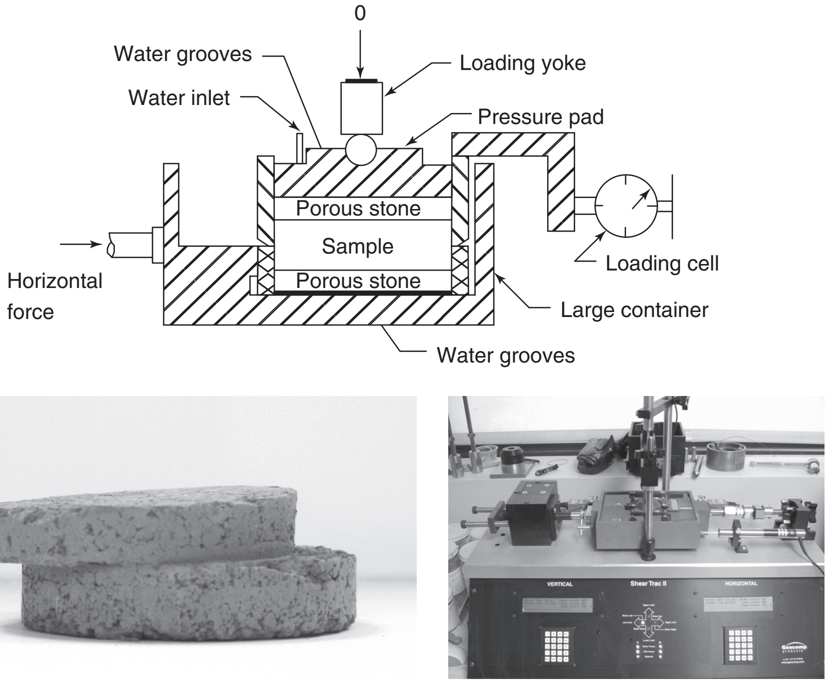


Figure 9.40 Direct shear test and equipment. (*a*) Principle. (*b*) Sample. (*c*) Complete setup. (*b*: Courtesy of Lev Buchko, P.E. // Timely Engineering Soil Tests, LLC.)

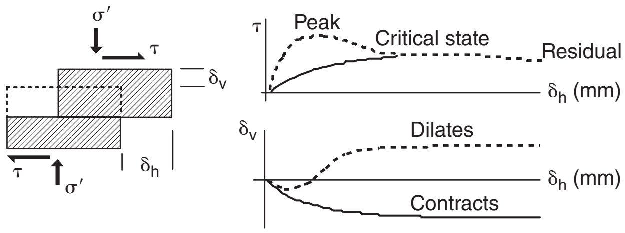


Figure 9.41 Direct shear test results stress-displacement curve.

the water stress remains zero. It is best also to measure the pore pressure or water stress, but that is not common with this simple test.

The shear strength measured in an undrained direct shear test is the undrained shear strength s_u. This undrained shear strength corresponds to the effective stress σ' generated at the end of the consolidation phase. This undrained shear strength also corresponds to the stress path followed in a direct shear test.

The shear strength measured in a drained direct shear test provides one point on the shear strength envelope. This envelope links the shear strength to the effective stress σ' normal to the plane of failure. As described in Chapter 15 on shear strength, the envelope is represented by the following equation:

$$s' = c + \sigma' \tan \phi' \tag{9.12}$$

This equation has two soil parameters: the effective stress cohesion c′ and the effective stress friction angle ϕ'. Because the drained direct shear test gives only one point on the envelope, it is necessary to run at least two direct shear tests to obtain c′ and ϕ' for a given soil (Figure 9.42).

When soils are subjected to shearing, they can increase in volume (dilate), decrease in volume (contract), or not change volume. If a soil dilates during shear, the shear strength

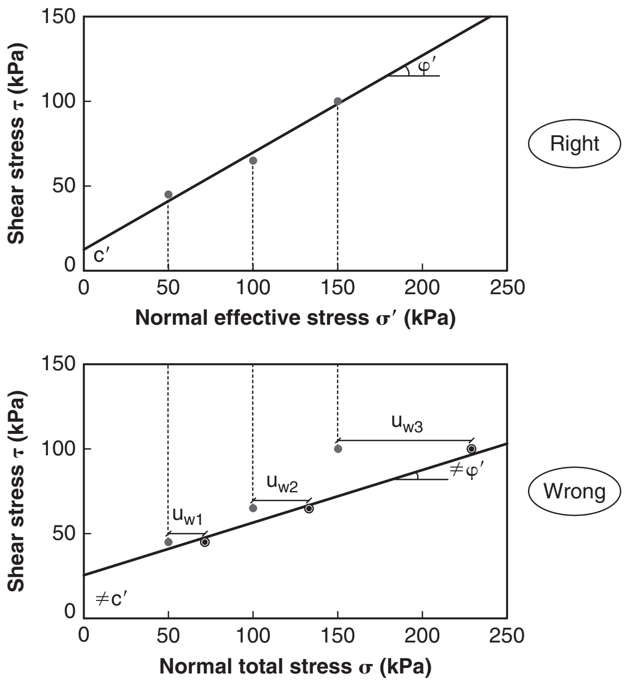


Figure 9.42 Example of direct shear test strength results for saturated soils.

increases compared to a soil that does not change in volume. The increase in shear strength is reflected by the dilation angle ψ (see Chapter 15 for more details). The dilation angle ψ can be estimated from a direct shear test as the slope of the curve linking the vertical movement z to the horizontal movement x. Because this curve is rarely a straight line, the equation is written in an incremental fashion.

$$\tan\psi = \Delta z/\Delta x \tag{9.13}$$

Advantages of the direct shear test include that it is easy to perform and gives a shear strength of the soil. A drawback of the direct shear test is that it cannot give the shear strain of the soil as it is sheared, because the thickness of the shearing zone is not known.

9.9.2 Unsaturated Soils

If the soil is unsaturated, or if the soil is saturated but the water in the voids is in tension (e.g., above the groundwater level), then the direct shear test requires measurement of the water tension stress (suction) to obtain the effective stress shear strength parameters c′ and φ'. Indeed, although the test procedure is the same for a soil with water in compression and for a soil with water in tension, the assumption that the water stress is zero when the test is performed slowly is not valid when the water is in tension. The reason is that if the water is in compression at the beginning of the direct shear test, the water compression stress is very small compared to the general stress level; in contrast, if the water is in tension, the water tension stress can be very large when the degree of saturation is low. The water tension stress u_w can be measured by any one of the methods described in section 9.2.4, but it is most often done with a tensiometer during the shear test. Once the water tension stress is known, the effective stress (assuming the air stress u_a is zero) is calculated as:

$$\sigma' = \sigma - \alpha\, u_w \tag{9.14}$$

where σ' is the effective stress, σ the total stress, α the water tension coefficient, and u_w the water tension stress. The coefficient α can be estimated as the degree of saturation S, but the error can be as large as ±40% of the correct value. A better estimate can be obtained by using the correlation to the air entry value u_{we} as shown in Chapter 10. It is assumed here that the air stress remains zero during the test.

The results are then plotted as shear strength vs. effective normal stress, as shown in Figure 9.43. If the results of direct shear tests on soils where the water is in tension are plotted as shear strength vs. total stress, the cohesion intercept will be much larger, as it includes the effect of the water tension on the soil strength (Figure 9.43). The apparent cohesion c_{app} is equal to:

$$c_{app} = -\alpha\, u_w \tag{9.15}$$

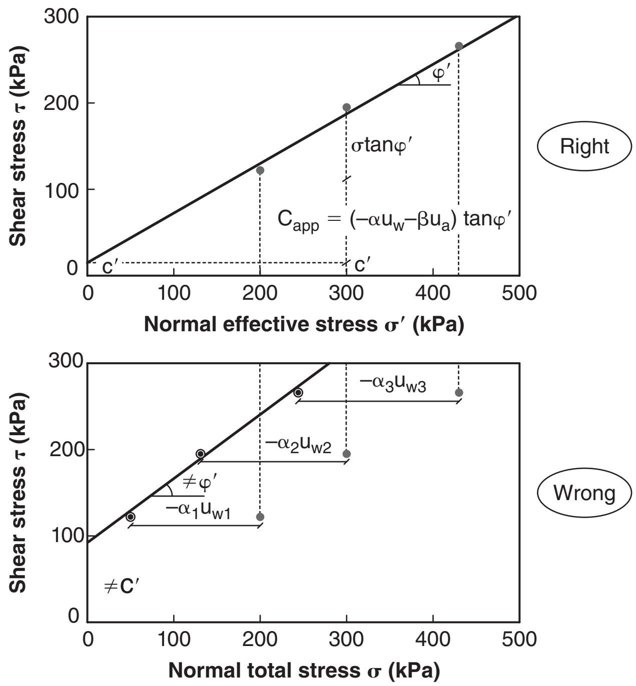


Figure 9.43 Example of direct shear test strength results for unsaturated soil.

However, c_{app} is not a constant for a given soil, because u_w depends on the water content of the sample. The apparent cohesion is called apparent rather than true cohesion because it is due to the effective stress created by the water tension and because it disappears if the soil is inundated (water tension goes to zero). In contrast, the parameter c′ is a characteristic of the soil that is constant and independent of the water content.

9.10 SIMPLE SHEAR TEST

9.10.1 Saturated Soils

The simple shear test (ASTM D6528) can be traced back to the mid 1960s with a publication by Bjerrum and Landva (1966). A disk of soil is placed in a flexible membrane with a porous stone on the top and on the bottom of the disk (Figure 9.44). A vertical load is applied to the top of the sample and maintained constant during the test. This vertical

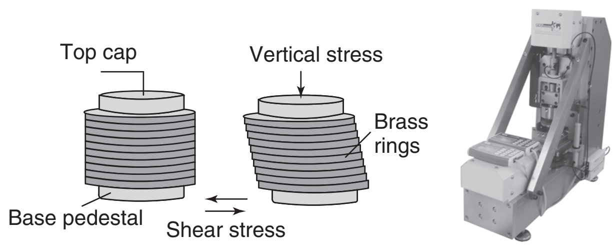


Figure 9.44 Simple shear test equipment: (a) Principle. (b) Complete setup. (b: Courtesy of GDS Instruments.)

load creates a total normal stress σ. Then the soil is sheared by holding one of the two platens and pushing the other one horizontally. The major difference between the direct shear test and the simple shear test is that in the direct shear test, the shearing takes place along a predetermined thin band of soil near the middle of the sample. In the simple shear test, the shearing takes places over the entire height of the sample. Therefore, the shearing strain γ can be measured in the simple shear test as:

$$\gamma = \Delta x/h_o \tag{9.16}$$

where Δx is the difference in horizontal movement between the top and the bottom of the sample and h_o is the initial height of the sample. The shear stress τ is measured as the shear force divided by the cross-sectional area of the sample. Thus, the simple shear test gives the shear stress-shear strain curve for the sample and therefore a shear modulus G.

During the first part of the simple shear test, the soil sample is allowed to consolidate (through drainage) under the vertical stress applied if such a stress is applied. The consolidation is monitored by recording the vertical movement of the sample as a function of time. When the settlement stops or becomes very small, it is assumed that the water stress has returned to zero and the shearing part of the test can start. During the second part of the test, the sample is sheared. The shear stress vs. shear strain curve is obtained point by point (Figure 9.45). The shear strength τ_f is the maximum shear stress on the shear stress vs. shear strain curve. This shear strength is the undrained shear strength if the shearing part of the test is run without allowing water to drain out of the sample; it is the drained shear strength if the test is run slowly enough that the water stress remains zero or if the water stress (pore pressure) is measured.

The shear strength τ_f is obtained in the same fashion as for the direct shear test, including the shear strength parameters c' and ϕ'. The shear modulus G is the slope of the τ vs. γ curve. Because the curve is typically nonlinear, G varies with γ and a G vs. γ curve can be generated. Therefore, an advantage of the simple shear test is that it can give the shear modulus G as a function of shear strain, in addition to the shear strength of the soil sample.

When soils are subjected to shearing, they can increase in volume (dilate), decrease in volume (contract), or not change volume. If a soil dilates during shear, the shear strength increases compared to a soil that does not change in volume. If a soil contracts during shear, the shear strength decreases compared to a soil with no change in volume. The increase or decrease in shear strength is reflected by the dilation angle ψ (see Chapter 15 on shear strength for more details). The dilatancy angle ψ can be estimated from a simple shear test as the slope of the curve linking the change in vertical movement Δz to the change in horizontal movement Δx (Eq. 9.13).

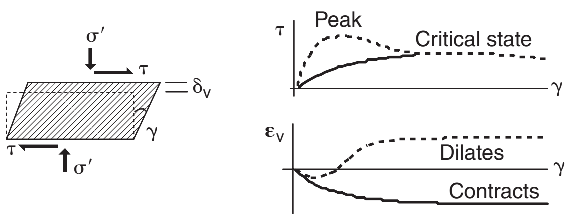


Figure 9.45 Simple shear test results.

9.10.2 Unsaturated Soils

If the soil is unsaturated, or if it is saturated but the water is in tension, the testing procedure is unchanged except for measurement of the water stress.The tensile stress in the water will typically require the use of a different measuring device, such as a tensiometer. The data reduction requires calculation of the effective stress, as discussed for the direct shear test.

9.11 UNCONFINED COMPRESSION TEST

9.11.1 Saturated Soils

The unconfined compression test (ASTM D2166) (Figure 9.46) is one of the simplest tests to perform if the soil can stand up under its own weight. In this test, the sample is a cylinder with a diameter d and a height h equal to about 2 times the diameter. The ratio h/d is about 2, to ensure that the oblique shear plane that typically develops during failure can propagate through the entire sample without intersecting the top or bottom platen. The sample remains unconfined during the test; therefore, the minor principal stress σ_3 is zero. A vertical load is applied to the sample by pushing up on the bottom platen at a constant rate of displacement while holding the top platen in a fixed position. The vertical total stress σ is calculated by dividing the vertical load by the cross-sectional area of the sample. Because it is assumed that there is no shear between the top of the sample and the bottom of the top platen, that stress is the major principal stress σ_1. The sample compresses and the vertical displacement Δh is measured with an LVDT or a dial gage. Knowing the initial height h of the sample, the vertical strain ε can be obtained as $\varepsilon = \Delta h/h$. The result of an unconfined compression test

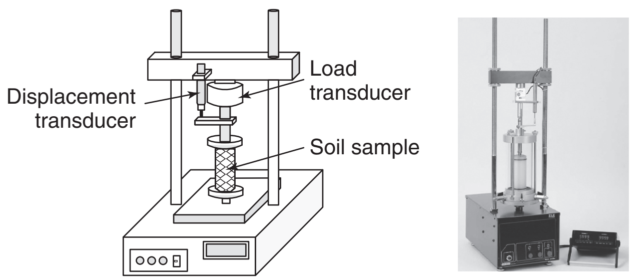


Figure 9.46 Unconfined compression test equipment: (a) Principle. (b) Complete setup. (*a*: After Ian Smith. *b*: Courtesy of ELE International.)

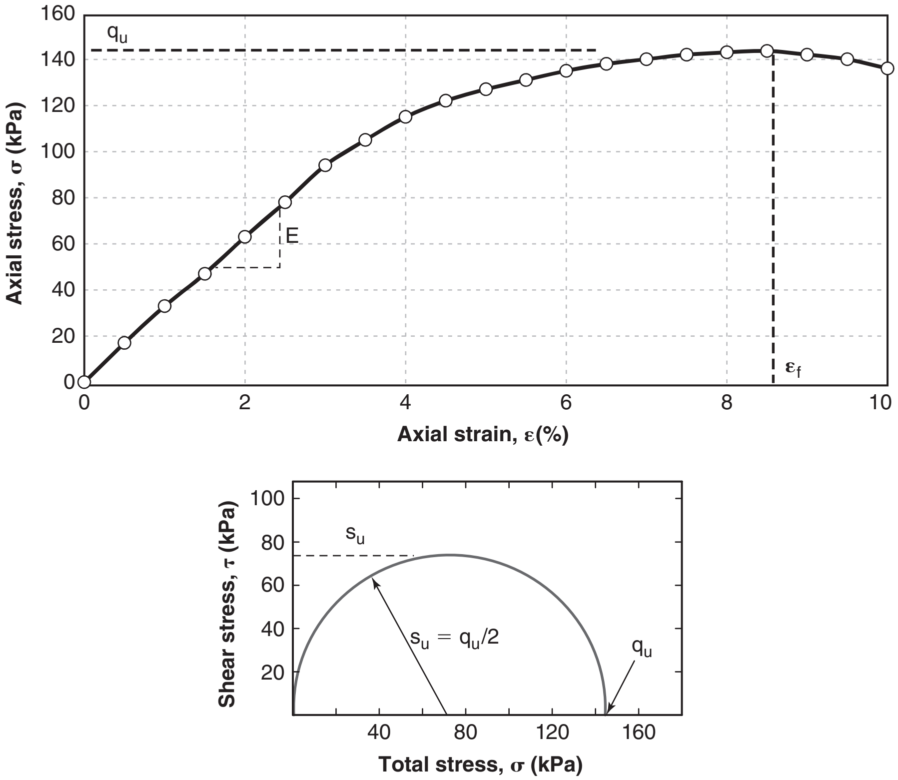


Figure 9.47 Unconfined compression test results.

is a complete total stress σ vs. strain ε curve for the soil sample under zero lateral confinement (Figure 9.47). The maximum stress on the curve is the unconfined compression strength q_u. Because the test is rather rapid, the shearing process is considered to be undrained for fine-grained soils. The undrained shear strength s_u is equal to $q_u/2$, as shown in Chapter 15.

$$s_u = q_u/2 \tag{9.17}$$

An unconfined compression modulus of deformation E can also be obtained from this test as:

$$E = \sigma_1/\varepsilon \tag{9.18}$$

Because the curve is often nonlinear, several moduli can be obtained depending on the chosen strain level. Advantages of the unconfined compression test are its simplicity and the fact that it gives both an undrained shear strength and a modulus of deformation for fine-grained soils.

9.11.2 Unsaturated Soils

If the soil is unsaturated, the test procedure is unchanged. Because the water stress is not measured in this test, there is also no difference in measurement and data reduction. One interesting observation is that the water tension can be estimated from the unconfined compression strength q_u. Indeed, the shear strength equation for unsaturated soils when the air stress u_a is assumed to be zero is:

$$s = c' + (\sigma - \alpha\, u_w)\ \tan\varphi' \tag{9.19}$$

In the unconfined compression test, the horizontal total stress is zero, therefore:

$$\sigma_h = 0 = \sigma'_h + \alpha\, u_w \text{ and therefore } \sigma'_h = -\alpha\, u_w \tag{9.20}$$

Meanwhile, the vertical total stress at failure is equal to q_u; therefore:

$$\sigma_v = q_u = \sigma'_v + \alpha\, u_w \text{ and therefore } \sigma'_v = q_u - \alpha\, u_w \tag{9.21}$$

The shear strength s is given by the point of tangency between the effective stress Mohr circle and the shear strength envelope (Figure 9.48). Triangle ACD on Figure 9.48 is such that:

$$\frac{CD}{AD} = \sin\varphi' = \frac{0.5((q_u - \alpha u_w) - (-\alpha u_w))}{0.5((q_u - \alpha u_w) + (-\alpha u_w)) + \dfrac{c'}{\tan\varphi'}} \tag{9.22}$$

Which leads to

$$u_w = \frac{0.5 q_u(\sin\varphi' - 1) + c'\cos\varphi'}{\alpha\sin\varphi'} \tag{9.23}$$

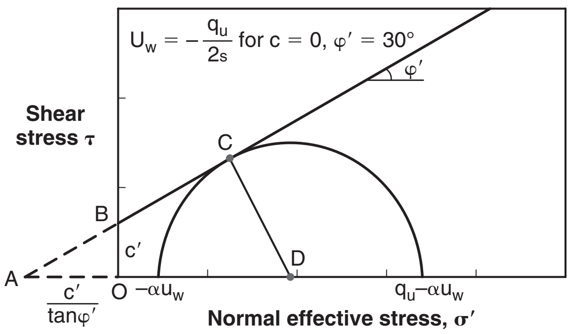


Figure 9.48 Water tension and unconfined compression strength relationship.

Eq. 9.23 gives the water tension at failure in the unconfined compression test. If it is further assumed that $c' = 0, \varphi' = 30^\circ$, and $\alpha = S$, then Eq. 9.23 becomes:

$$u_w = -\frac{q_u}{2S} \tag{9.24}$$

9.12 TRIAXIAL TEST

9.12.1 Saturated Soils

The triaxial test (ASTM D5311) (Figure 9.49) is similar to the unconfined compression test except that a chosen confining pressure is applied to the sample before compression takes place. The sample has a height equal to about two times the diameter to ensure that the oblique shear plane that typically develops during failure in compression can propagate through the entire sample without intersecting the top or bottom platen. Typical diameters range from 30 to 75 mm. First, porous disks (also called filter stones) are placed at the top and bottom of the sample. Then the sample is fit in an impervious rubber membrane and set on the pedestal of the triaxial cell. The top platen is placed, and the top of the triaxial cell is brought down to cover the sample. The shaft of the piston is lowered in contact with the top platen on one side and connected to the load cell or proving ring on the other. The cell is filled with liquid (water or oil) and the confining pressure is applied. Sometimes the cell is not filled with liquid and only air pressure is used. The triaxial cell is placed in a frame and the load is applied by moving the bottom of the frame upward and at a constant rate of displacement against the stationary top of the frame. The movement of the sample is typically obtained by measuring the movement of the shaft applying the load with respect to the triaxial cell. For more advanced testing, the movement measurements are taken between two rings directly tied to the sample. Pore-pressure measurements are an option and are typically made by placing a saturated porous stone at the base of the sample and measuring the pressure in the water through a pressure transducer tied to the base platen. Measuring the change in volume of the sample is also an option.

There are many different types of triaxial tests because of the possible combinations related to drainage and type and sequence of stress applications. However, nearly all triaxial tests start with a consolidation phase followed by a shearing phase. The consolidation phase is designed to bring the sample to a desired state of stress that is often intended to match the stress conditions that the sample would face in the field under the project conditions. During the consolidation phase, the cell pressure is increased to a chosen value of the confining pressure. This pressure confines the sample hydrostatically and represents the minor principal stress σ_3. During this phase of consolidation, drainage may or may not be allowed. If drainage is not allowed, the word "unconsolidated" is used in describing the triaxial test and the letter U is used in the acronym. If drainage is allowed and the water stress (pore pressure) generated by the application of σ_3 is allowed to dissipate back to zero, the word "consolidated" is used to describe the test and the letter C is used in the acronym.

During the shearing phase of the test, the vertical load Q on the sample is increased gradually and the stress in the

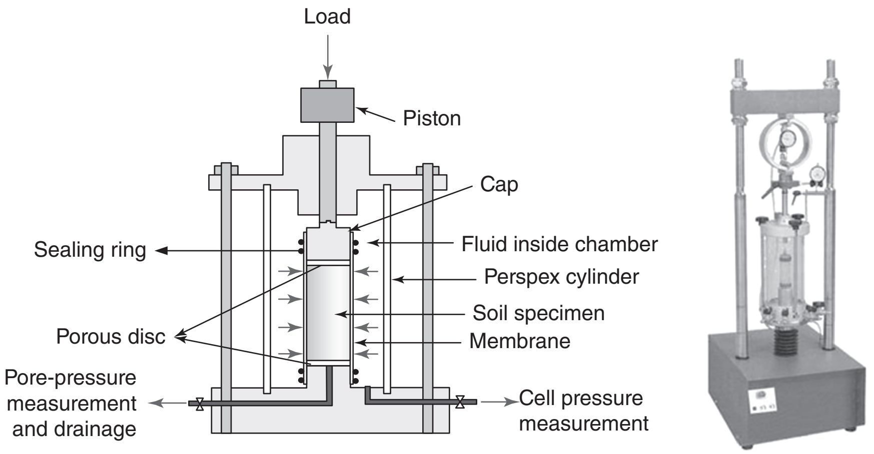


Figure 9.49 Triaxial test equipment: (a) Principle. (b) Equipment. (b: Courtesy of Geotechnical Testing Equipment Ltd., UK.)

vertical direction increases. This stress is the major principal stress σ_1:

$$\sigma_1 = \sigma_3 + Q/A \tag{9.25}$$

where σ_3 is the confining pressure, Q is the vertical load and A is the cross section of the sample. If drainage is not allowed during the shearing phase, the word "undrained" and the letter U are used. If drainage is allowed and the excess water stress (pore pressure) is kept equal to zero (very slow loading), then the word "drained" and the letter D are used. So, in the end, the following triaxial tests are possible:

1. UU test: unconsolidated undrained test
2. CU test: consolidated undrained test
3. CD test: consolidated drained test

A UD test is not possible, because allowing drainage during the shearing phase would also allow some consolidation under σ_3. UU tests are commonly performed to obtain the undrained shear strength, particularly in offshore studies where recompressing the sample to the high bottom pressures is important; UU tests are also simpler and faster than the other two. CD tests are quite time consuming, as loading must be slow enough not to generate water stresses (pore pressures), but they are simple to run. CU tests with water stress (pore pressure) measurements are faster to run, but require more sophisticated equipment because water stress (pore pressure) must be measured. Both CD tests and CU tests with water stress measurements are used to obtain the effective stress shear strength parameters c′ and ϕ'.

The result of a triaxial test is a stress-strain curve that typically links the deviator stress $(\sigma_1 - \sigma_3)$ to the vertical strain $(\varepsilon = \Delta h/h)$ where h is the initial height of the sample and Δh is the change in height of the sample. Figure 9.50 shows some results for two categories of soils: overconsolidated or dense soils on the one hand and normally consolidated or loose soils on the other. The first category exhibits a clear peak stress (maximum strength), followed by strain softening to reach a residual strength. The second category exhibits strain hardening, with the strength being reached at larger strain.

The peak stress value on this curve is the failure deviator stress $(\sigma_{1f} - \sigma_3)$. This failure stress, along with information on the water stress, is used to obtain the effective stress shear strength parameters c′ and ϕ'. This process requires use of the Mohr circle (see Figure 9.51 and Chapter 15). A *Mohr circle* is a circle in the shear stress vs. normal stress set of axes that describes the state of stress at a point when the principal stresses reduce from 3 stresses to 2 stresses. This is the case in the triaxial test where σ_1' and σ_3' are different and σ_3' is equal to σ_2'. The points corresponding to the principal stresses σ_1' and σ_3' plot on the horizontal axis because they exist on planes with zero shear stress. The circle representing the state of stress in the triaxial sample at failure is drawn (Figure 9.51). Because the failure envelope is described by two parameters c′ and φ' (Eq. 9.12), a minimum of two triaxial tests at two different confining pressures (σ_3) must be performed to obtain the effective stress cohesion intercept c′ and the effective stress friction angle φ'. Figure 9.52 shows the difference between the Mohr circles in the effective stress set of axes and in the total stress set of axes.

A modulus of deformation E can also be obtained from the stress-strain curve as follows:

$$E = (\sigma_1 - 2\nu\sigma_3)/\varepsilon \tag{9.26}$$

where E is the total stress modulus of deformation of the soil, σ_1 and σ_3 are the major and minor principal total stresses respectively, ν is Poisson's ratio, and ε is the vertical strain. Note that because the stress-strain curve is rarely linear, many different moduli can be obtained depending on the strain level among other factors. The modulus defined in terms of effective stress is typically more useful and more fundamentally rooted:

$$E' = (\sigma_1' - 2\nu\sigma_3')/\varepsilon \tag{9.27}$$

where E′ is the effective stress modulus of deformation of the soil, and σ_1' and σ_3' are the major and minor principal effective stresses respectively.

The stress path describes the evolution of certain stresses during the test. Specifically, it tracks the path described by the points with p, q stress coordinates where p and q are defined as follows:

$$p = \frac{\sigma_1 + \sigma_3}{2} \quad \text{or} \quad p = \frac{\sigma_v + \sigma_h}{2} \tag{9.28}$$

$$q = \frac{\sigma_1 - \sigma_3}{2} \quad \text{or} \quad q = \frac{\sigma_v - \sigma_h}{2} \tag{9.29}$$

where σ_v and σ_v are the vertical and horizontal total stresses in a triaxial test, for example. The most useful stress paths are plotted in terms of effective stresses (p′ and q′):

$$p' = \frac{\sigma_1' + \sigma_3'}{2} \quad \text{or} \quad p' = \frac{\sigma_v' + \sigma_h'}{2} \tag{9.30}$$

$$q' = \frac{\sigma_1' - \sigma_3'}{2} = q \quad \text{or} \quad q' = \frac{\sigma_v' - \sigma_h'}{2} = q \tag{9.31}$$

where σ_v' and σ_v' are the vertical and horizontal total stresses in a triaxial test, for example. Examples of effective stress paths are shown in Figure 9.53 for different types of tests. In any lab test, it is most desirable to match the effective stress path followed by the soil in the field during the project construction and the project life.

9.12.2 Unsaturated Soils

If the soil is unsaturated, or if it is saturated and the water in the voids is in tension, the test procedure does not change, but the water and air stress measurements change. The water stress can be measured with a tensiometer and the air stress with a pressure transducer.

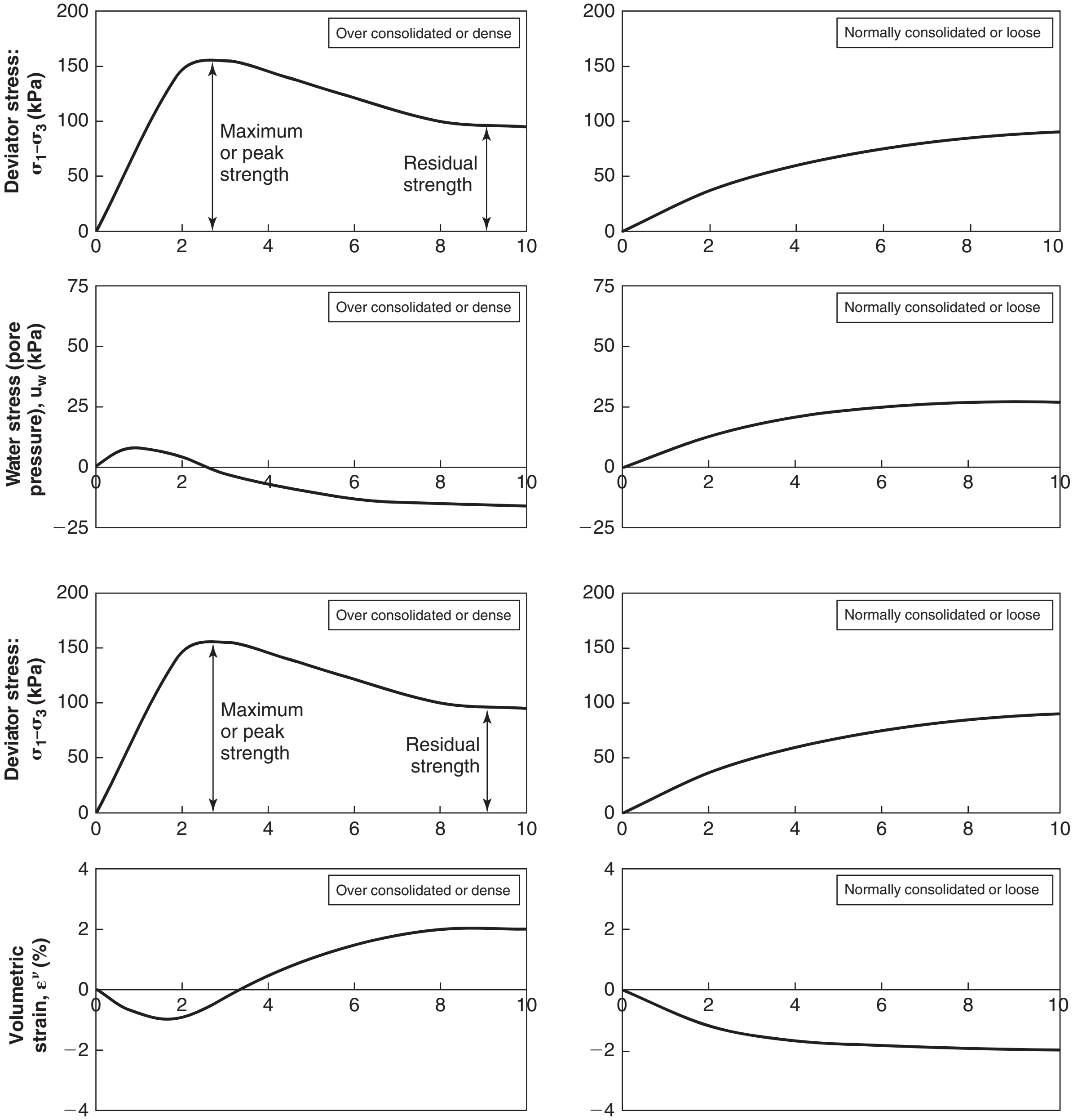


Figure 9.50 Triaxial test results (example stress-strain curves): (a) Consolidation, undrained test. (b) Consolidation, drained test.

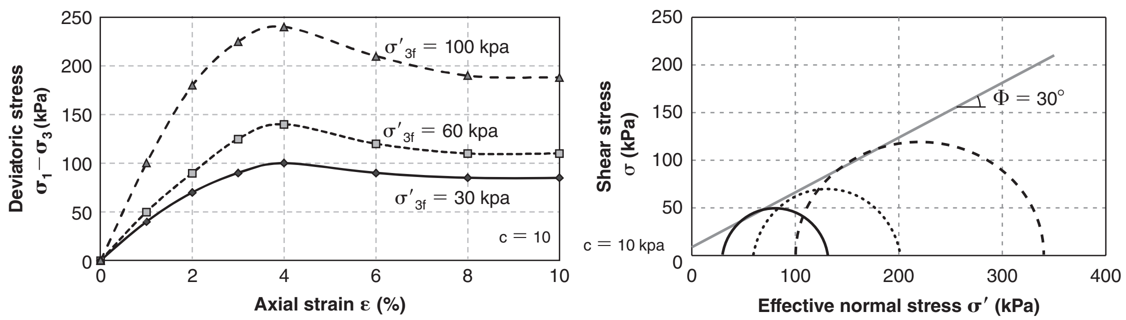


Figure 9.51 Triaxial test results: Examples of Mohr circles and strength envelope for saturated soils.

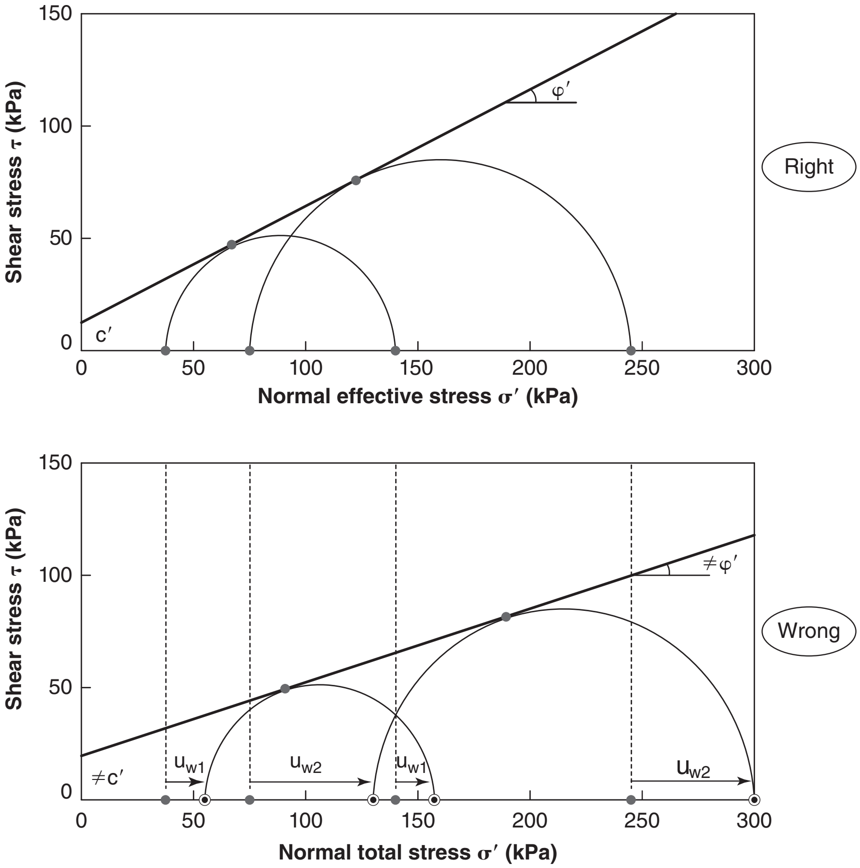


Figure 9.52 Triaxial test results: Mohr circles and strength envelope for saturated soils.

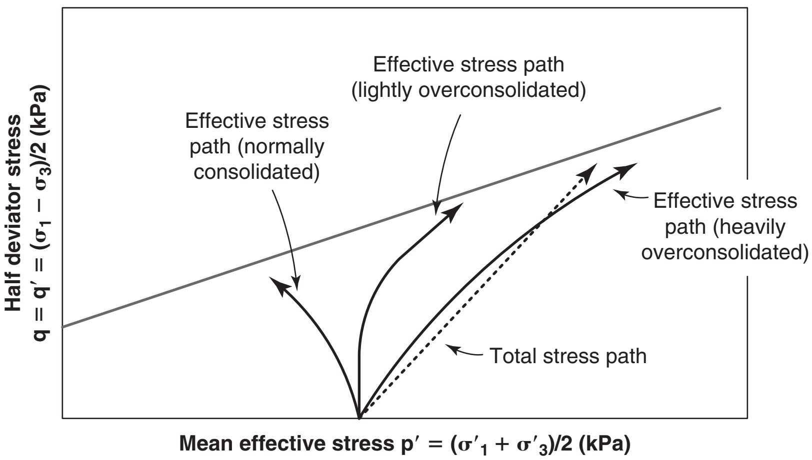


Figure 9.53 Triaxial test results: Stress paths.

The meaning of the tests that were described for saturated soils changes as well:

1. UU test: unconsolidated undrained test. For unsaturated soils, UU means that both the air and water are prevented from draining from the beginning to the end of the test. The air stress increases as the air compresses and the water stress increases (decrease in the absolute value of the water tension).
2. CU test: consolidated undrained test. For unsaturated soils, both air and water are allowed to drain during the consolidation phase. During the shearing phase, both are prevented from draining, so both pressures must be measured. Typically, the air stress and the water stress

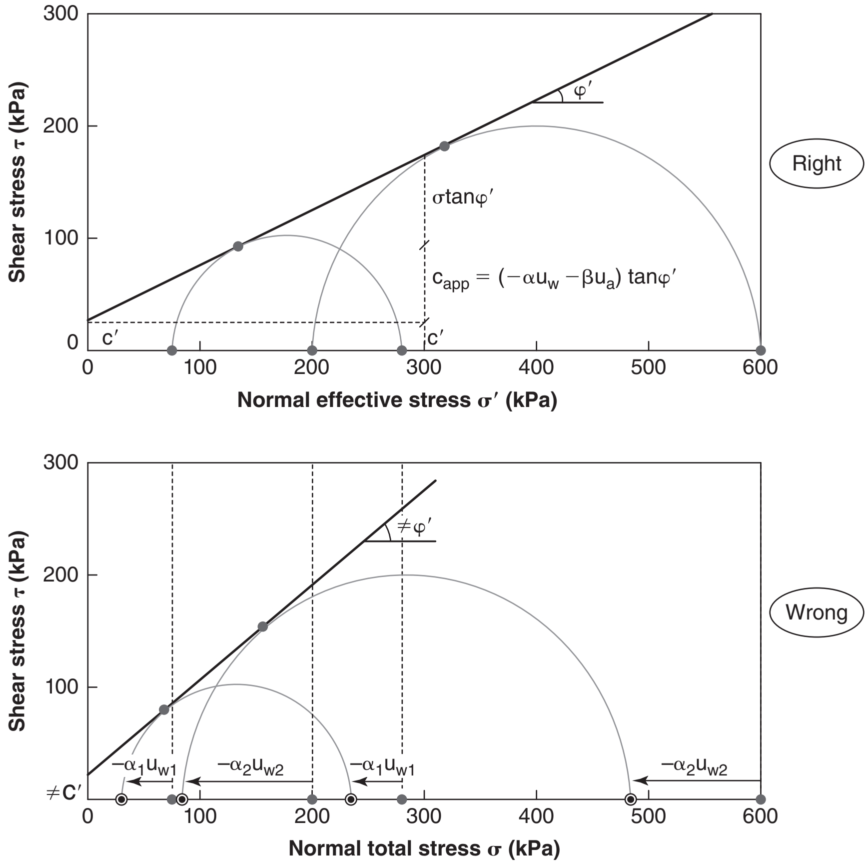


Figure 9.54 Triaxial test results: Mohr circles and strength envelope for unsaturated soils.

increase (decrease in water tension) during the shearing phase because the soil volume decreases (except for dilatant soils).

3. CD test: consolidated drained test. Both the air and the water are permitted to drain. The water tension can therefore be held constant throughout the test. The strain rate must be sufficiently slow to allow for flow of water from the soil through the high air entry disk.
4. CWC test: constant water content test. For unsaturated soils, it is also possible to conduct a test where the air can drain but not the water. Air drains much faster than water, so a judiciously chosen strain rate can achieve this condition.

The data reduction changes as well. The effective stress must be calculated according to the following formula (instead of $\sigma' = \sigma - u_w$):

$$\sigma' = \sigma - \alpha\, u_w - \beta\, u_a \tag{9.32}$$

where σ' is the normal effective stress, σ the normal total stress, α the water area ratio parameter, u_w the water stress, β the air area ratio parameter, and u_a the air stress. This difference will affect the location of the Mohr circle on the shear stress τ vs. effective normal stress σ' graph. If instead the results are plotted in the shear stress τ vs. total normal stress σ graph, then the effective stress shear strength parameters c' and φ' cannot be obtained. The cohesion intercept c' in the shear stress τ vs. total normal stress σ graph is much larger than c', as it includes the effect of the water tension on the soil strength (Figure 9.54). The apparent cohesion c_{app} is equal to:

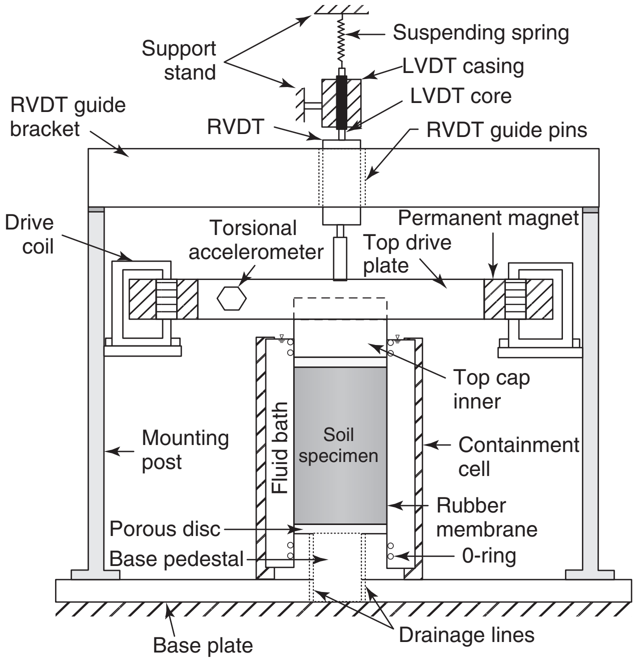
$$c_{app} = -\alpha\, u_w - \beta\, u_a \tag{9.33}$$

This cohesion is called apparent cohesion rather than true cohesion because it is due to water tension and because it disappears if the soil is inundated (water and air stresses go to zero).

9.13 RESONANT COLUMN TEST

9.13.1 Saturated Soils

The resonant column test (ASTM D4015) is used to determine the dynamic small strain properties of a soil. Such results are applied in earthquake engineering and machine vibration, for example. A cylinder of soil with a height-to-diameter ratio of about 2 is placed in a cell where a confining pressure can be applied. The base of the sample is fixed to the bottom platen, which does not move. The top of the sample is mounted with a top platen having a mass m and able to generate



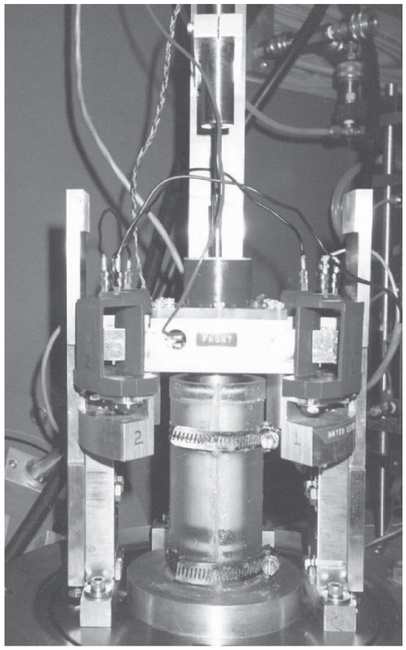


Figure 9.55 Resonant column test: (*a*) Principle. (*b*) Equipment. (*b*: Courtesy of Geotechnical Research Lab, Dept. of Civil Engineering, University of British Columbia.)

cyclic torsion (Figure 9.55). The test consists of applying a sinusoidal torque $T(\omega)$ to the top of the sample. This torque is generated through an electromagnetic drive system that controls the angular frequency ω of the sinusoidal torque application. The response of the sample is monitored by measuring (through LVDTs, for example) the rotation of the top of the sample. The water stress (pore pressure) is sometimes also measured during this test.

In a first step, a confining pressure is applied to the sample. Then the top of the sample is subjected to a chosen torque. The torque applied gives the shear stress τ imposed on the sample and the rotation θ is used to obtain the shear strain γ of the sample. The response is presented in term of loops linking τ to γ. The frequency of the sinusoidal torque is increased gradually while recording the strain in the sample. Resonance occurs when the frequency of the soil vibrations matches the frequency of the torque application (Figure 9.56). This frequency is ω_n. At that point the sample rotation reaches its maximum value.

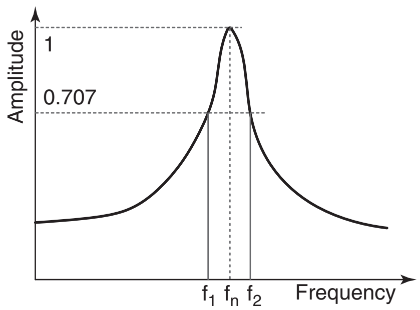


Figure 9.56 Rotation amplitude vs. frequency of induced vibration.

The data are used as follows to obtain the soil shear modulus G when the sample is fixed at the bottom and free at the top where the torque is applied. The mass polar moment of inertia of the sample J_s is:

$$J_s = M_s d_s^2/8 \tag{9.34}$$

where M_s is the sample mass and d_s is the sample diameter. The mass polar moment of inertia of the mass on top of the sample J_m is:

$$J_m = M_m d_m^2/8 \tag{9.35}$$

where M_m is the mass of the mass on top of the sample and d_m is the diameter of that mass. By using fundamental and constitutive equations, it can be shown that:

$$\frac{J_s}{J_m} = \frac{\omega_n L}{v_s} \tan\left(\frac{\omega_n L}{v_s}\right) \quad \text{or} \quad \frac{J_s}{J_m} = \frac{2\pi f_n L}{v_s} \tan\left(\frac{2\pi f_n L}{v_s}\right) \tag{9.36}$$

where J_s and J_m are the polar moments of inertia of the sample and of the mass on top of the sample respectively, ω_n is the resonant angular frequency, v_s is the shear-wave velocity in the sample, L is the length of the sample, and f_n is the natural frequency of the soil.

In Eq. 9.36, J_s, J_m, and L are known, f_n is measured in the test, and v_s can be back-calculated. Then the shear modulus is obtained from:

$$G = \rho v_s^2 \tag{9.37}$$

For the case where there is no mass at the top, $J_m = 0$, then $2\pi f_n L/v_s = \pi/2$, and then

$$G = \rho\, v_s^2 = 16\, \rho\, f_n^2 L^2 \tag{9.38}$$

Eq. 9.38 gives the shear modulus G for a given shear strain amplitude γ.

There are several ways to obtain the damping ratio, and each way has its own advantages and limitations. One way is to stop the excitation and let the sample vibration die out while recording the sample rotation as a function of time. This is called the *logarithmic decrement method*. The damping ratio *D* is defined as the ratio of the damping coefficient to the critical damping coefficient. The critical damping is the minimum amount of damping that results in the sample returning to its original position without oscillation. The damping ratio can be obtained from the decay curve (Figure 9.57) as follows. The amplitude of the first cycle is x_1 and the amplitude of the n^{th} cycle is x_n, which is smaller than x_1. It can be shown that:

$$\frac{Ln\, x_1 - Ln\, x_n}{n-1} = \frac{2\pi\, D}{\sqrt{1-D^2}} \tag{9.39}$$

In Eq. 9.39, all quantities are known except for D, the damping ratio. The damping obtained by this method includes the damping of the device, which must be accounted for separately. This method also requires stopping the test, and the strain level decreases during the vibration decay. Another way to obtain the damping ratio is to use the half-power bandwidth method. This method makes use of the amplitude vs. frequency plot (Figure 9.57) obtained during steady-state torsional vibration of the sample:

$$D = (f_2 - f_1)/2f_n \tag{9.40}$$

where f_2, f_1, and f_n are defined in Figure 9.57. This method is best applied when the system is linear.

These curves can also be obtained from direct measurements of the shear stress and the shear strain. The maximum shear stress τ generated during the cycles is calculated as

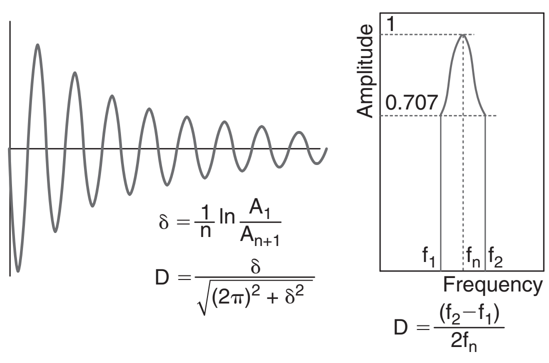


Figure 9.57 Method to obtain damping ratio from resonant column test: (a) Logarithmic decrement. (b) Half-power bandwidth.

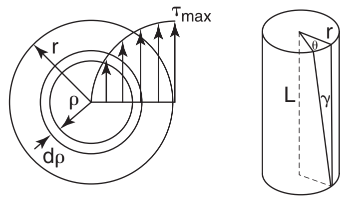


Figure 9.58 Shear stress and shear strain in a resonant column torsion test: (*a*) Shear stress. (*b*) Shear strain.

an average of the shear stress generated on the sample cross section. This shear stress is zero at the center of the sample ($\tau_{center} = 0$) and maximum at the edge (τ_{edge}) (Figure 9.58). The mean shear stress τ is related to the maximum torque T as follows:

$$\tau = 2T/\pi r_e^3 \tag{9.41}$$

where r_e is the equivalent radius, which can be anywhere from 0.6r to 0.8r where r is the radius of the sample. The maximum shear strain during the cycle exists at the edge of the sample (γ_{edge}), while the shear strain is zero along the axis of the cylindrical sample ($\gamma_{center} = 0$) (Figure 9.58). The mean shear strain in the sample is usually taken as.

$$\gamma = r_e \theta / L \tag{9.42}$$

where again r_e is the equivalent radius, often taken as 0.8r where r is the radius of the sample.

A typical τ vs. γ curve is shown in Figure 9.59. The shear modulus G is calculated as the slope of the line joining the two extremities of the loop. Alternatively, this curve can be generated by calculating the shear strain first, obtaining the shear modulus by the resonant frequency method, and then calculating the shear stress as $G\gamma$. The damping ratio D is defined from the curve as the ratio of the energy necessary to perform one cycle of torsion to the elastic energy expanded to load the sample to the peak of the cycle (Figure 9.59):

$$D = A_c / 4\pi A_e \tag{9.43}$$

where D is the damping ratio, A_c is the area inside the cycle, and A_e is the area inside the triangle shown in Figure 9.59.

The previous discussion identifies how G, D, and γ can be obtained for a given amount of torque applied at the top of the sample. This torque can then be increased to create a larger shear strain in the sample. The test is repeated and a new set of values of G, D, and γ are obtained. Point by point, the G vs. γ curve and the D vs. γ curve are described (Figure 9.60). The G vs. γ curve and the D vs. γ curve are the two results of a resonant column test. The strain that can be tested with this test typically ranges from 10^{-6} to 10^{-3}.

9.13.2 Unsaturated Soils

If the soil is unsaturated, or if the soil is saturated but the water is in tension, neither the test procedure nor the data

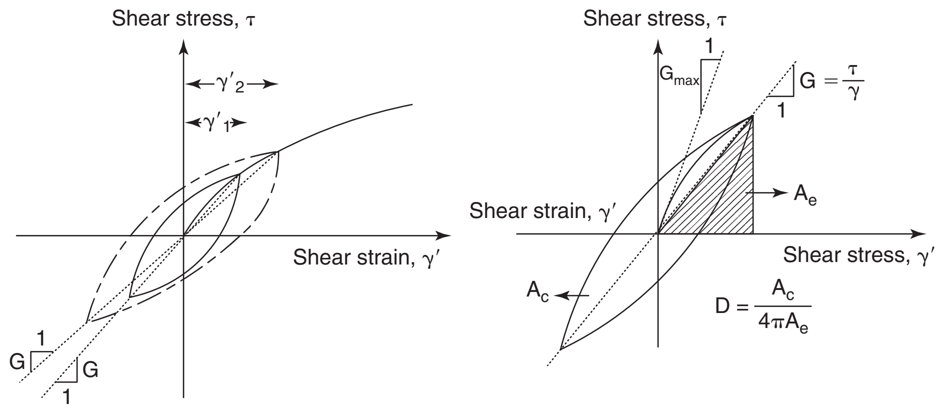


Figure 9.59 Shear stress-strain loops in resonant column test and damping ratio calculation: (*a*) Evolution of stress-strain loop. (*b*) Calculation of shear modulus and damping ratio.

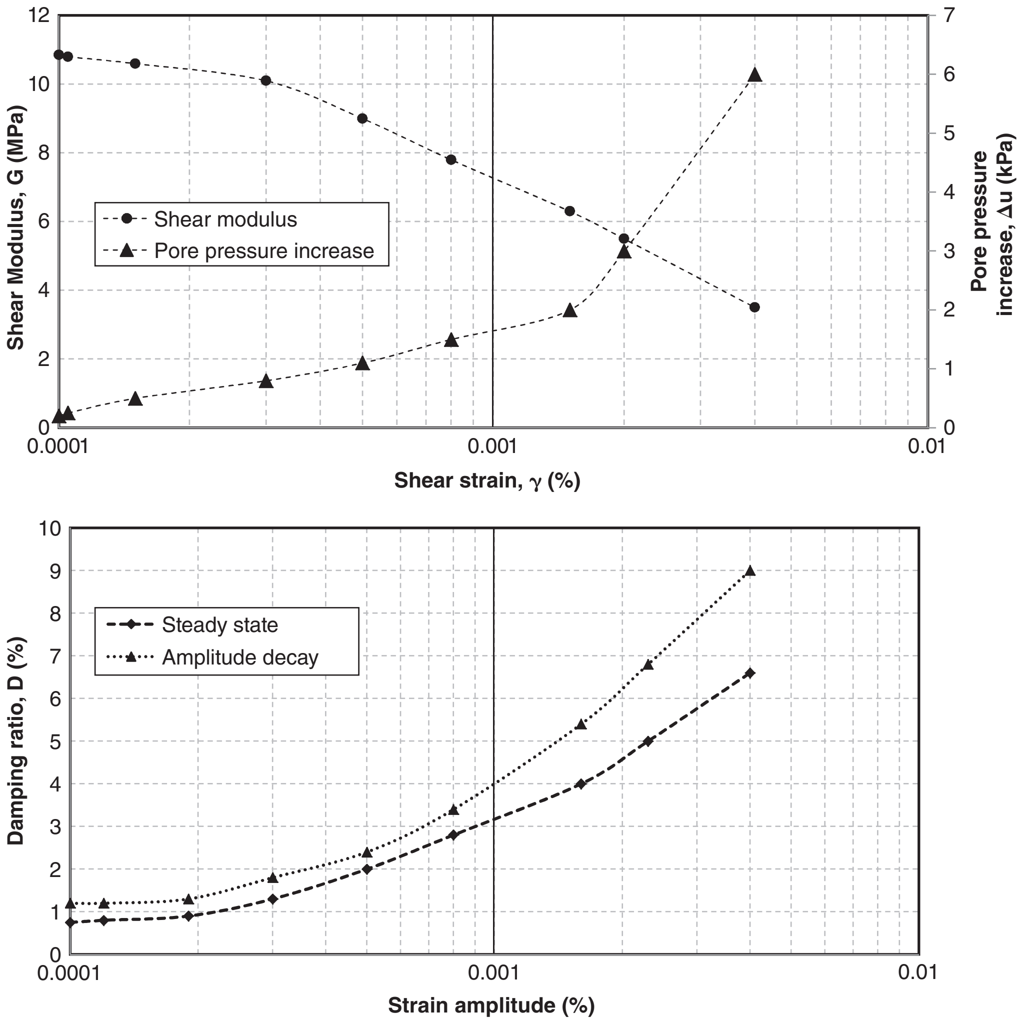


Figure 9.60 Shear modulus vs. shear strain and damping coefficient vs. shear strain: (*a*) Shearing modulus. (*b*) Damping ratio.

reduction changes. Indeed, the water stress is rarely measured during the resonant column test.

9.14 LAB VANE TEST

9.14.1 Saturated Soils

The lab vane test or VST (Figure 9.61) is used to determine the undrained shear strength of fine-grained soils (clays and silts). It can be performed either in the field with a field vane (ASTM D2573), or on the sample with a mini vane or a hand vane (ASTM D4648; Figure 9.61). The lab vane is made of two perpendicular blades, each having a 2-to-1 height-to-width ratio. The width of the blades varies from 12 to 25 mm; the larger vanes are used in softer soils. The vane is pushed perpendicularly into the end of a sample until the tops of the blades are one blade height below the surface of the sample. Then the vane is rotated at a slow rate (less than 1 degree per minute) while the testers measure the torque developed and the rotation angle (Figure 9.62). The peak value of the torque is recorded as T_{max}. The blade is then rotated at least 10 times rapidly and a new maximum torque value, T_{res}, is measured.

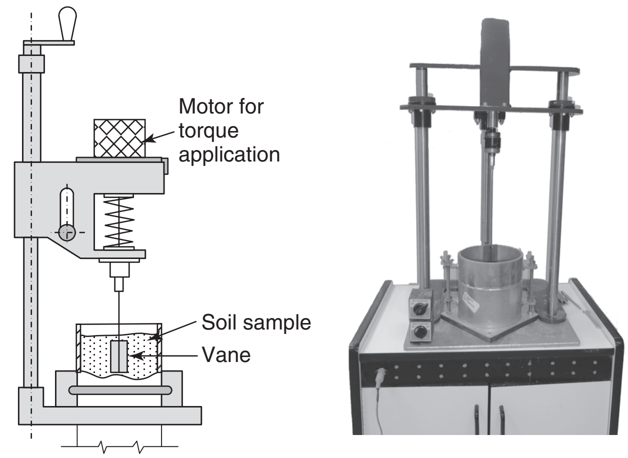


Figure 9.61 Lab vane test equipment: (*a*) Principle. (b) Equipment. (*a*: Adapted from BS 1377-7: 1990.)

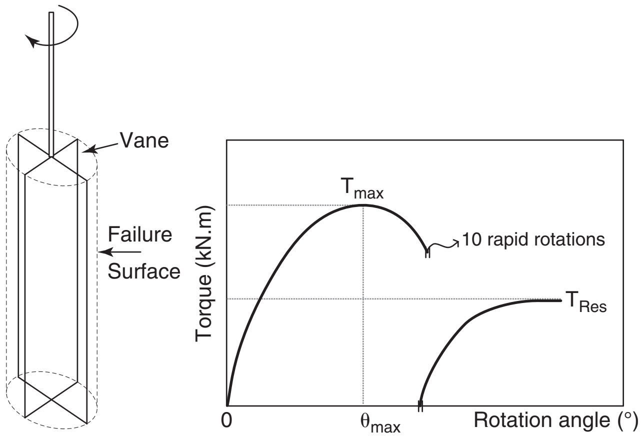


Figure 9.62 Lab vane test results.

The VST is used in saturated fine-grained soils to obtain the undrained shear strength s_u. The reason is that these soils have a low permeability and do not allow appreciable drainage during a test, which typically lasts less than 10 minutes. Therefore, for these saturated fine-grained soils, it is reasonable to assume that the undrained shear strength s_u is the parameter being measured. For a rectangular vane, the following equation gives s_u from T_{max}:

$$T_{max} = \pi s_u D^2 \left(\frac{H}{2} + \frac{D}{6} \right) \tag{9.44}$$

where D is the diameter of the vane and H is the height of the vane. Proof of this equation is shown in the solution to problem 7.4. The residual undrained shear strength s_{ur} is obtained from the same formula using T_{res}:

$$T_{res} = \pi s_{ur} D^2 \left(\frac{H}{2} + \frac{D}{6} \right) \tag{9.45}$$

The VST can be performed in coarse-grained soils, but no useful result can be obtained. These soils drain fast enough that one would not measure the undrained shear strength, but instead the drained or partially drained shear strength. Back-calculating the shear strength parameters from this test would require knowledge of the normal effective stress on the plane of failure in addition to T_{max}. This is not measured during the VST. The VST has the advantages of being fast, simple, economical, and useful for obtaining the undrained shear strength of fine-grained soils. Its drawbacks include that it is limited to fine-grained soils.

9.14.2 Unsaturated Soils

If the soil is unsaturated, or if the soil is saturated but the water is in tension, neither the test procedure nor the data reduction changes. Water stress is not measured during the vane test.

9.15 SOIL WATER RETENTION CURVE (SOIL WATER CHARACTERISTIC CURVE) TEST

9.15.1 Saturated Soils

The soil water retention curve (SWRC), also known as the soil water characteristic curve, is a property of the soil much like the shear strength parameters (Figure 9.63). It is a plot of the water content of the soil as a function of the water tension stress (suction) in the soil pores. It depends on many factors, including the particle size distribution, pore size distribution, soil structure, and soil texture.

During the drying process from a saturated state, the water tension in the soil will increase until it becomes large enough to force air into the soil pores. This water tension value is called the *air entry value* u_{wae}. Beyond the air entry value,

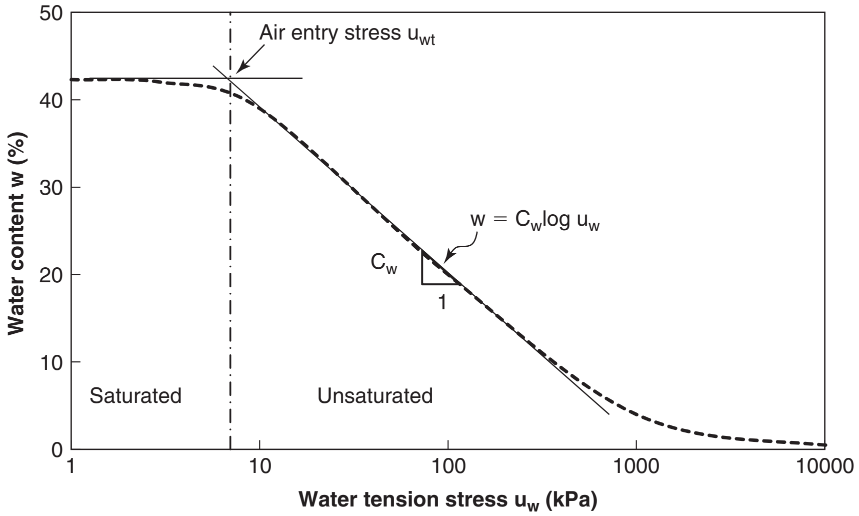


Figure 9.63 Soil water retention curve.

the decrease in water content is well approximated by a linear relationship between the water content and the log of the water tension, which can be written as:

$$dw = C_w d(\log_{10} u_w) \tag{9.46}$$

where w is in percent, u_w is in kPa (positive), and C_w is the slope of the SWRC.

The gravimetric water content is the most commonly used water content definition in geotechnical engineering, but for the SWRC the volumetric water content often is used. These are defined in the following equations:

$$\text{Gravimetric water content}: w = \mathrm{W}_w/\mathrm{W}_\mathrm{s} \tag{9.47}$$

$$\text{Volumetric water content}: \theta_w = \mathrm{V}_w/\mathrm{V} \tag{9.48}$$

where W_w and V_w are the weight and volume of water respectively, W_s is the weight of solids, and V is the total volume. Example SWRCs are presented in Figure 9.64. It stands to reason that different soils will have different SWRCs: A sand will not retain water the same way a clay would. Imagine that you insert a straw into a sand; it would not take much sucking to get the water out of the sand. Now imagine that your straw is inserted into a clay; in this instance it would take a lot of sucking to get a little bit of water out. The suction or water tension that you would have to exert through the straw would be much higher for the clay than for the sand. This phenomenon is what the SWRC characterizes.

Soils under the groundwater level (GWL) are generally saturated and the water is in compression. Soils above the GWL can be saturated or unsaturated, but in both cases the water is in tension (suction). The SWRC is a property of a

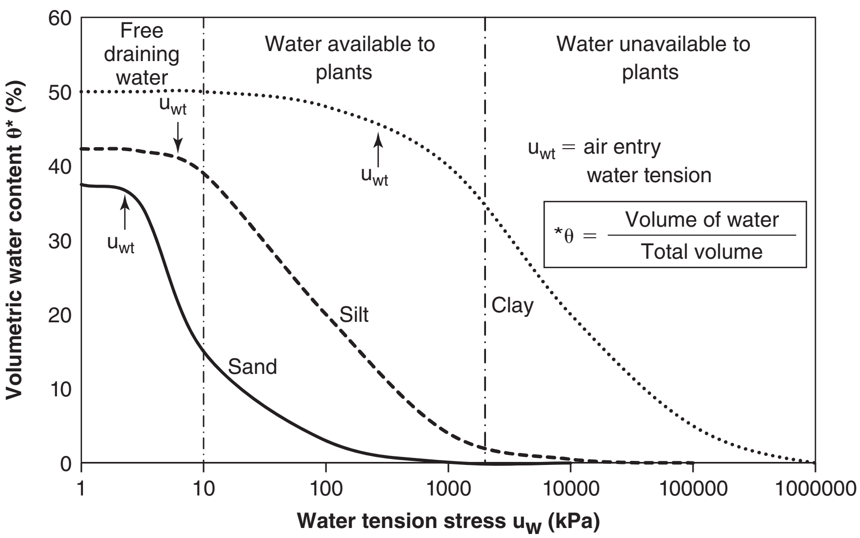


Figure 9.64 Example of soil water retention curve.

soil where the water is in tension. As such, the SWRC for a saturated soil refers to the case where the soil is saturated above the GWL by capillary action and other electrochemically based phenomena such as the affinity between water and clay minerals.

If a saturated soil sample is placed on a table top and is strong enough to stand by itself, it is likely held together by water tension unless it has some cementation (effective stress cohesion). As the soil dries, it initially shrinks while remaining saturated. The water tension increases and at a given water tension stress (suction), air enters the pores. The water tension at this point is called the *air entry value* (u_{we}). From this point on during the drying process, the soil is unsaturated. The procedure to determine the SWRC is the same below (saturated) and above (unsaturated) the air entry value. This procedure is detailed in section 9.15.2 related to unsaturated soils.

9.15.2 Unsaturated Soils

There are essentially two methods for obtaining the SWRC (ASTM D6836). The first consists of taking a saturated soil sample and measuring the water tension and the water content of the sample as a function of time as it dries up. The water content measurement was described in section 3.9; the water tension measurement was described in section 9.2.4 and summarized in Table 9.2. In this case, the two most common methods to measure water tension for the SWRC are the filter paper method and the chilled mirror psychrometer. For lower values of water tension, the hanging column method can also be used (ASTM D6836). As a guide, and for tests performed in an air-conditioned laboratory environment where the relative humidity is around 50%, a 25 mm high, 75 mm diameter sample is likely to become air-dry in about 24 hours. In these circumstances, a water content and water tension measurement every 1 to 2 hours is suitable to get a good description of the SWRC.

The second method of obtaining the SWRC is to use a saturated soil sample and force the sample to come to equilibrium at a selected series of water tension (suction), while measuring the water content for each one of those water tension values. The pressure plate apparatus can be used in this case (Figure 9.9); it makes use of the axis translation technique (Figure 9.10) and increases the air pressure to push the water out of the soil pores. The air pressure is equal to the water tension in the sample when the water starts moving out of the pores. The water content of the soil sample is measured when the water stops flowing. Such measurements are made at increasingly higher air pressures so as to describe the complete SWRC.

Yet another way is to use the salt solution equilibrium technique, in which "identical" samples are placed in different salt solution chambers (Figure 9.11) and left in the chamber until the water tension in the sample comes into equilibrium with the relative humidity created by the salt solution at the bottom of the chamber; reaching this equilibrium may take 1 or 2 weeks. The salt concentration in each chamber is different and is chosen to create a series of values of the relative humidity and therefore water tension, which gives a good description of the SWRC. After equilibrium is reached, the soil water content is measured in each chamber and the SWRC can be plotted.

The SWRC describes the fact that the water tension increases when the water content decreases but recognizes that this relationship is not the same when the soil is drying as when it is wetting; this is called the *hysteresis* in the SWRC. Figure 9.65 shows the difference between the drying curve

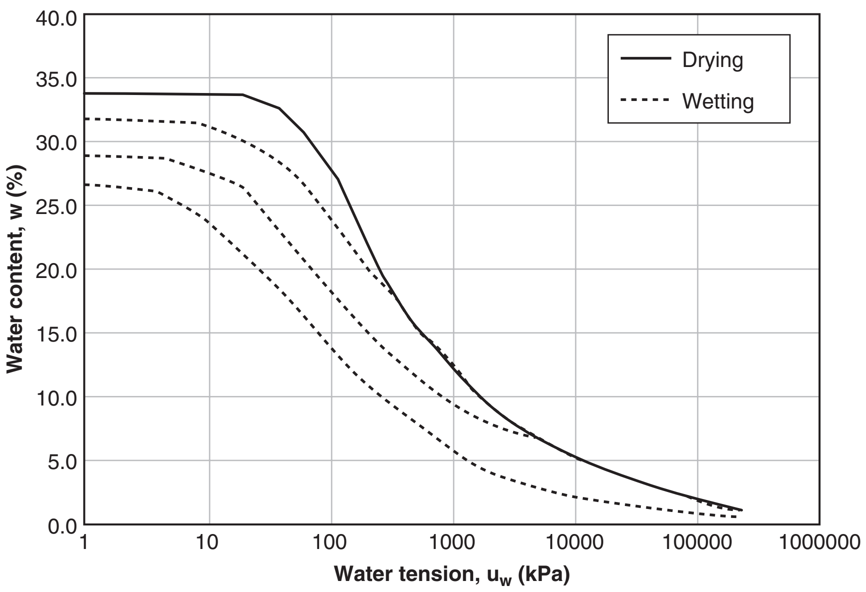


Figure 9.65 Drying and wetting hysteresis loop in the SWRC.

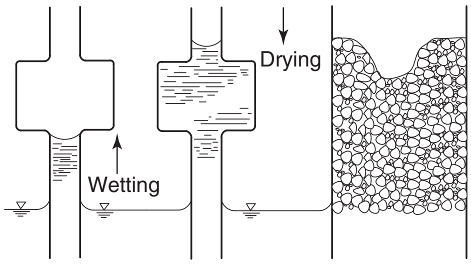


Figure 9.66 Geometrical explanation of drying and wetting hysteresis in the SWRC.

and the wetting curve. It is likely that this difference decreases as the number of drying and wetting cycles between the same values increases. The hysteresis effect may be attributed to several causes: the geometric nonuniformity of the individual pores, the pore fluid contact angle, entrapped air, and swelling, shrinking, or aging. The geometric nonuniformity of the pores can be explained as follows (Figure 9.66). When the soil is drying, the water level in the conduits formed by the voids between particles can drop down through a larger void cross section, as shown in Figure 9.66. However, if the soil is wetting, there is a limit to how large a cross section the water can move up, as the capillary force is limited. As a result, the loss of water is larger during drying than the gain of water during wetting and thus the wetting curve is below the drying curve (Figure 9.65). Several stages are identifiable in the drying or wetting process, as shown in Figure 9.67. During drying, at first the soil is saturated (S = 1) until the air entry value of the water tension u_{wae} is reached; then a linear semilog relationship exists between the water content and the water tension; and then the soil reaches a residual stage ($S = S_r$) where the water no longer forms continuous conduits in the pores, but rather exists only at the contacts between particles. The effective degree of saturation S_e is defined for a given degree of saturation S as:

$$S_e = \frac{S - S_r}{1 - S_r} \tag{9.49}$$

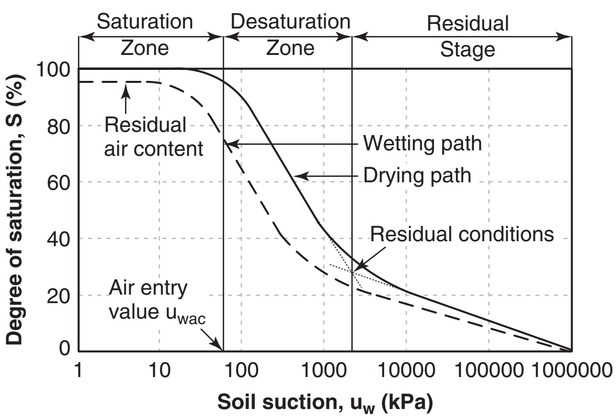


Figure 9.67 Various stages in the SWRC.

During the wetting process, a similar progression takes place in reverse and after the saturation phase, where again there is a linear semilog relationship between water content and water tension. The soil reaches a residual air content when the air is occluded and cannot be chased out of the voids through normal means.

Various empirical models have been proposed to describe the SWRC. Among the most common are:

Brooks and Corey (1964) $$S_e = \begin{pmatrix} 1 \; if \; u_w \le u_{wae} \\ \left(\dfrac{u_w}{u_{wae}}\right)^{-\lambda} \; if \; u_w \le u_{wae} \end{pmatrix} \tag{9.50}$$

van Genuchten (1980) $$S_e = \left(\frac{1}{1 + (\alpha u_w)^n}\right)^m \quad \text{with } m = 1 - 1/n \tag{9.51}$$

Fredlund and Xing (1994) $$\theta = C(u_w)\theta_s \left(\frac{1}{Ln(e + (u_w/a)^n}\right)^m \tag{9.52}$$

where S_e is the effective degree of saturation; u_w is the water tension (kPa); u_{wae} is the air entry value of the water tension (kPa); λ is a fitting parameter mostly influenced by the pore size distribution of the soil; α, n, and m are fitting parameters; θ is the volumetric water content (volume of water over total volume); θ_s is the volumetric water content at saturation; C(u_w) is a correction factor that forces the model through a prescribed water tension value of 10^6 kPa at zero water content; a is a fitting parameter; and e is the logarithmic constant ($Ln\, e = 1$). More details on these models can be found in Lu and Likos (2004). ARA-ERES (2000) proposed a set of SWRCs (Figure 9.68) predicted on the basis of D_{60} in mm, the particle size for which 60% by weight is finer, and an index called the wPI. The wPI is defined as the product of the percent passing sieve number 200 as a decimal (ratio not percentage) and the plasticity index as a percent.

9.16 CONSTANT HEAD PERMEAMETER TEST

9.16.1 Saturated Soils

The constant head permeameter (CHP) (ASTM D2434; Figure 9.69) is used to obtain the coefficient of hydraulic conductivity k of saturated coarse-grained soils. The soil sample is placed in a cylinder about 75 mm in diameter and 150 mm high, with one filter stone at the top and another at the bottom. The top of the sample is connected by tubing to a container in which the water level is kept constant through an overflow regulator. The bottom of the sample is connected to another container in which the water level is also kept constant. The bottom container is kept lower than the top container and the flow Q (m^3/s) out of the bottom container is measured. The measurement simply consists of weighing the amount of water collected in the overflow container

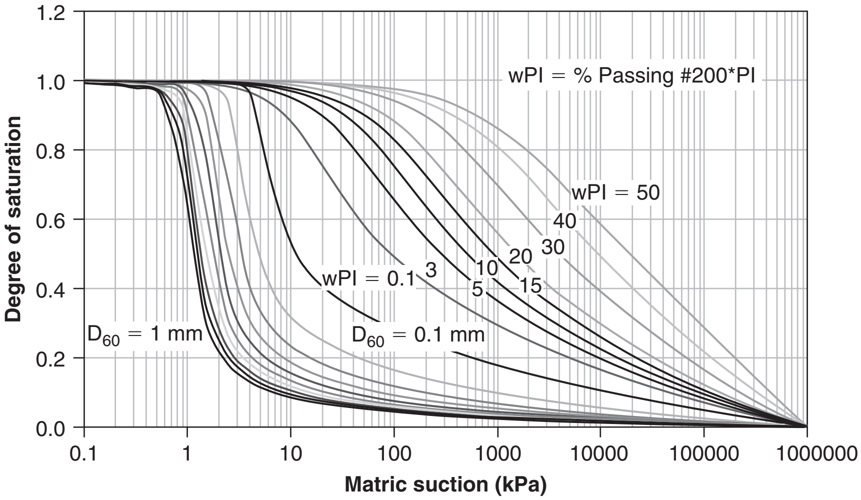


Figure 9.68 SWRC as a function of percent passing #200 and plasticity index. (Courtesy of NCHRP.)

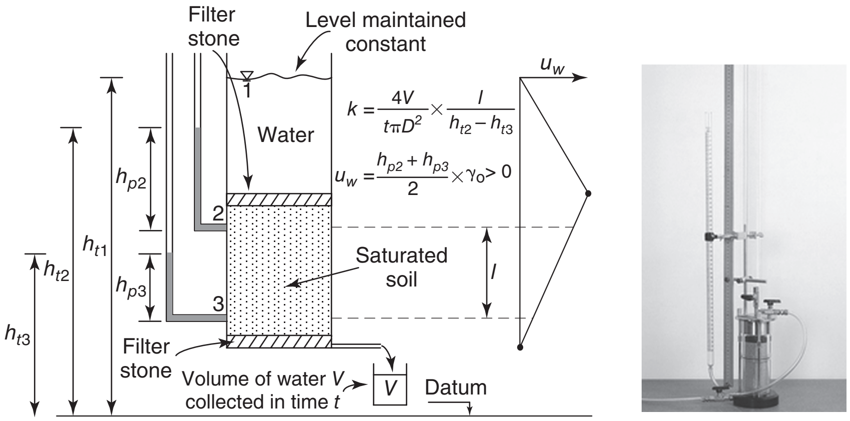


Figure 9.69 Constant head permeameter equipment: (*a*) Principle. (*b*) Equipment. (b: Courtesy of ELE International.)

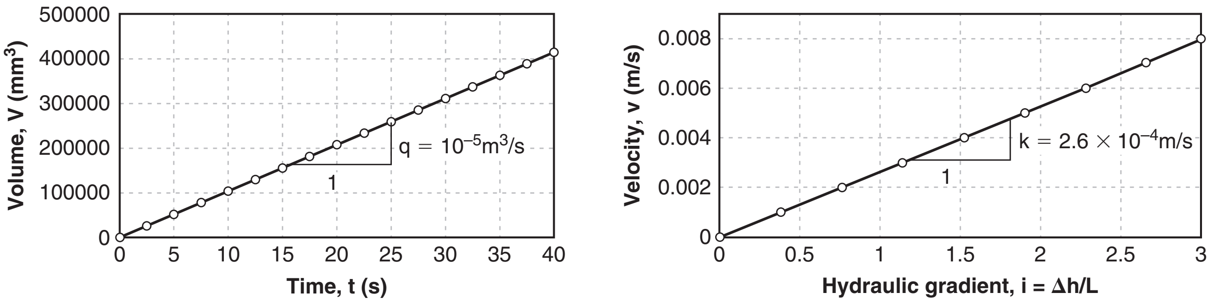


Figure 9.70 Constant head permeameter test results.

during a corresponding time. Typical results are shown in Figure 9.70. Often manometer tubes are connected to the side of the sample container at two points to give the water stress (pressure) at those two locations. (See Chapter 13 on flow through soils for an explanation of the following equations and parameters.) Darcy's law gives:

$$v = \mathrm{k}\,\mathrm{i} \tag{9.53}$$

where v is the discharge velocity through the sample, k is the hydraulic conductivity, and i is the hydraulic gradient. The hydraulic gradient in this case is given by:

$$i = h/l \tag{9.54}$$

where h is the loss of total head through the flow distance l (Figure 9.69). Also, conservation of mass gives:

$$Q = v\,A \tag{9.55}$$

where Q is the flow out of the sample, v is the discharge velocity, and A is the cross-sectional area of the sample. Note that A is the total cross-sectional area of the sample, not just the area of the pores through which the water is flowing. As a result, v is not the actual speed of the water molecules flowing through the pores (*seepage velocity*) but rather an equivalent speed called the *discharge velocity*. Combining equations 9.53 through 9.55 then gives the value of k:

$$k = Q\,l/h\,A \tag{9.56}$$

where Q is the discharge (m^3/s), l is the flow length between 2 points in the sample, h is the loss of total head between the same 2 points, and A is sample cross-sectional area. The discharge Q is the volume V collected in a time t divided by t. The cross-sectional area A is $\pi D^2/4$ where D is the sample diameter. The loss of total head h is $h_{t2} - h_{t3}$, as shown in Figure 9.69. Therefore, the hydraulic conductivity k is:

$$k = \frac{4V}{t\pi D^2}\frac{l}{h_{t2} - h_{t3}} \tag{9.57}$$

The advantage of the constant head permeameter test is that it is a very simple test to run; the drawback is that it is limited to measuring the hydraulic conductivity k of coarse-grained soils at a small scale. The k values typically measured with this test range from 10^{-1} to 10^{-6} m/s.

9.16.2 Unsaturated Soils

If the soil is unsaturated, things are quite different. The first thing to realize is that the hydraulic conductivity of an unsaturated soil is less than that of a saturated soil: Water goes through an unsaturated soil more slowly than through a saturated soil. The reason is that air is in the way of the flow, and the water is attracted to the walls of the tiny conduits formed by the particles. Of course, one must remember that in the equation giving the water velocity v (m/s) from the flow discharge Q (m^3/s) (Eq. 9.55), A is the total cross-sectional area of the sample, not the actual water flow area. Because the flow area is significantly reduced in the case of unsaturated flow, the actual water velocity is quite a bit higher than the velocity given by Eq. 9.55.

The steady-state permeameter test for unsaturated soils consists of the same equipment except for two differences: (1) The measurements of water compression are changed to measurements of water tension, and (2) a tube is connected to the center of the sample to control the air pressure in the sample. The measurement of the water tension is made at two locations, using tensiometers or other appropriate devices. Figure 9.71 shows the diagram for an unsaturated steady-state permeability test. The water level is maintained on the upstream side (point 1) and the water starts flowing. It arrives at the high air entry disk. This disk lets the water go through but not the air; that is a property of that disk. Then the water goes through the soil voids. One would think that it would flood the voids as it is attracted by the water tension (suction) in the water phase. But the air is in the way, and it has no way to escape because there is another high air entry disk at the other end of the sample. So the water is forced to flow through the continuous water phase around the air phase. The water tension is larger at point 3 than at point 2 (Figure 9.71) because the water loses energy as it drives through the soil. A friction force arises between the water molecules and the soil particles as the water drags through the voids. This force is called the *seepage force*. As a result of this force, there is an associated loss of pressure between points 2 and 3. Because the pressure at point 2 is negative (water tension), the pressure at point 3 is even more negative ($h_{p3} < h_{p2} < 0$).

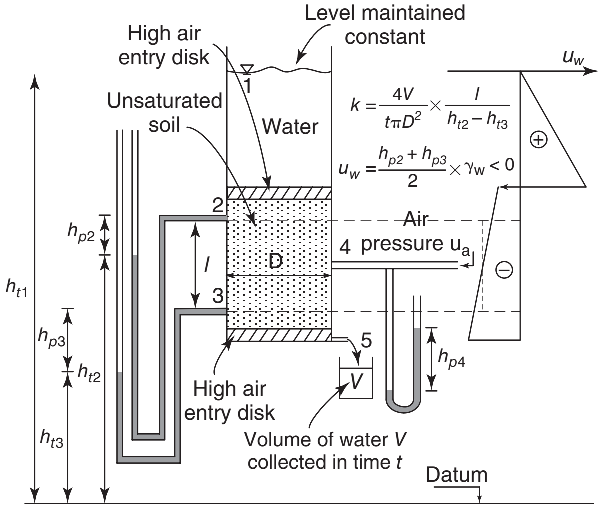


Figure 9.71 Constant head permeameter test for unsaturated soils.

The hydraulic conductivity depends on the water tension (Figure 9.72). As the water tension increases, the amount of water in the soil decreases, and it becomes harder and harder for the water to percolate through the soil: There is less room for the water to flow and a higher attraction between the water and the soil particles. The effect of the water tension on the hydraulic conductivity can be documented in this test by changing the air pressure through port 4. Applying an air pressure u_a different from zero changes the water tension u_w. This allows one to run the permeability test at different water tensions and establish the relationship between hydraulic conductivity and water tension.

From the calculations point of view, the hydraulic conductivity k is obtained as:

$$k = \frac{4V}{t\pi D^2}\frac{l}{(h_{t2} - h_{t3})} \tag{9.58}$$

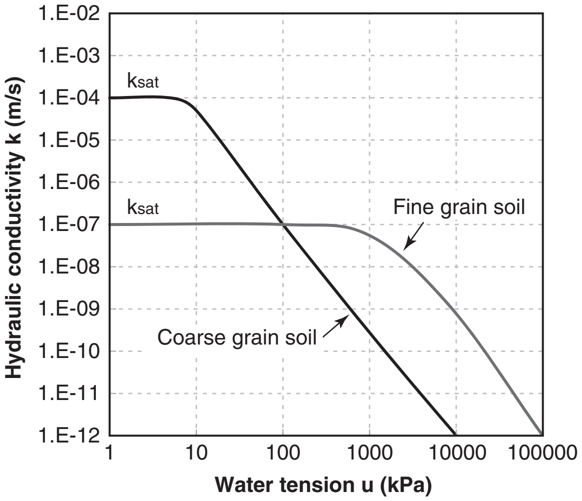


Figure 9.72 Constant head permeameter test results for unsaturated soils.

where V is the volume of water collected in a time t, D is the sample diameter, and l is the distance between the two points where the total heads h_{t2} and h_{t3} are measured. Note that Eq. 9.58 is the same as Eq. 9.57. The difference is that h_{t2} and h_{t3} are different, because the soil is unsaturated. The average water tension u_w associated with the hydraulic conductivity k of Eq. 9.58 is:

$$u_w = \frac{h_{p2} + h_{p3}}{2} < 0 \qquad (9.59)$$

The combination (k, u_w) gives the coordinates of one point on the hydraulic conductivity vs. water tension curve. By testing the soil at different water content (water tension), one can get the complete curve (Figure 9.72).

Note that the chemistry of the water makes a difference when running a permeability test. If the water that seeps through the soil has a much different salt chemistry than the sample water, the osmotic suction could be activated and lead to a different water tension in the sample than if the water seeping through the sample had the same chemistry than the water in the sample.

9.17 FALLING HEAD PERMEAMETER TEST FOR SATURATED SOILS

The falling head permeameter (FHP) (Figure 9.73) is used to obtain the hydraulic conductivity k of saturated fine-grained soils. The soil sample is placed in a cylinder about 75 mm in diameter and 150 mm high with one filter stone at the top and another at the bottom. The top of the sample is connected to a tube with a much smaller diameter (say, 10 mm) filled with water. Unlike the constant head permeameter, the water level in this tube goes down with time. The bottom of the sample is connected to a container where the water level is kept constant through an overflow. The measurements consist of recording the time on one hand and the drop in height in the small tube on the other (Figure 9.74). Darcy's law gives:

$$v = k\,i \qquad (9.60)$$

where v is the discharge velocity through the sample, k is the hydraulic conductivity, and i is the hydraulic gradient. The hydraulic gradient in this case is given by:

$$i = h/l \qquad (9.61)$$

where h is the loss of total head through the flow distance l (Figure 9.73). Also, conservation of mass gives:

$$Q = v\,A = k\,i\,A = k\,A\,h/l \qquad (9.62)$$

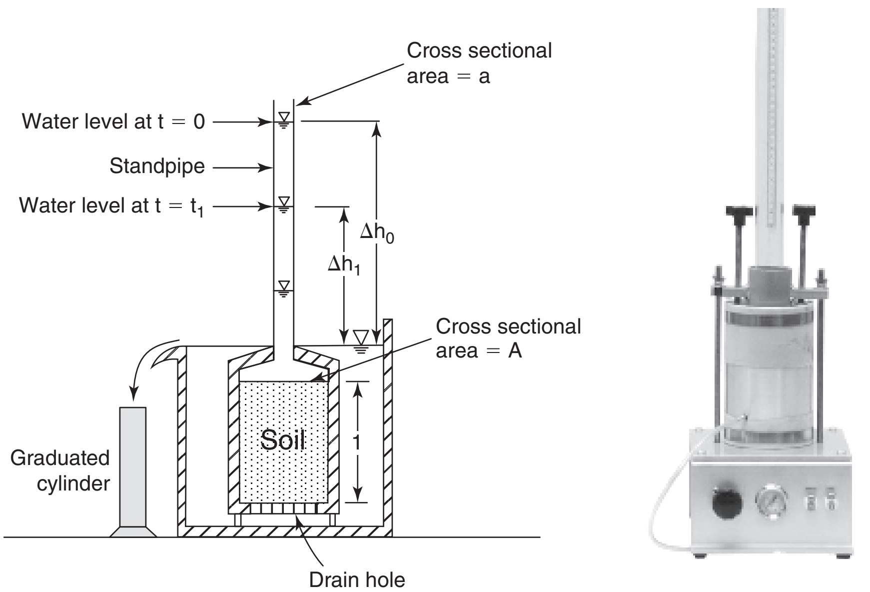


Figure 9.73 Falling head permeameter equipment: (a) Principle. (b) Equipment. (a: After FHWA. b: Courtesy of Gilson Company, Inc.)

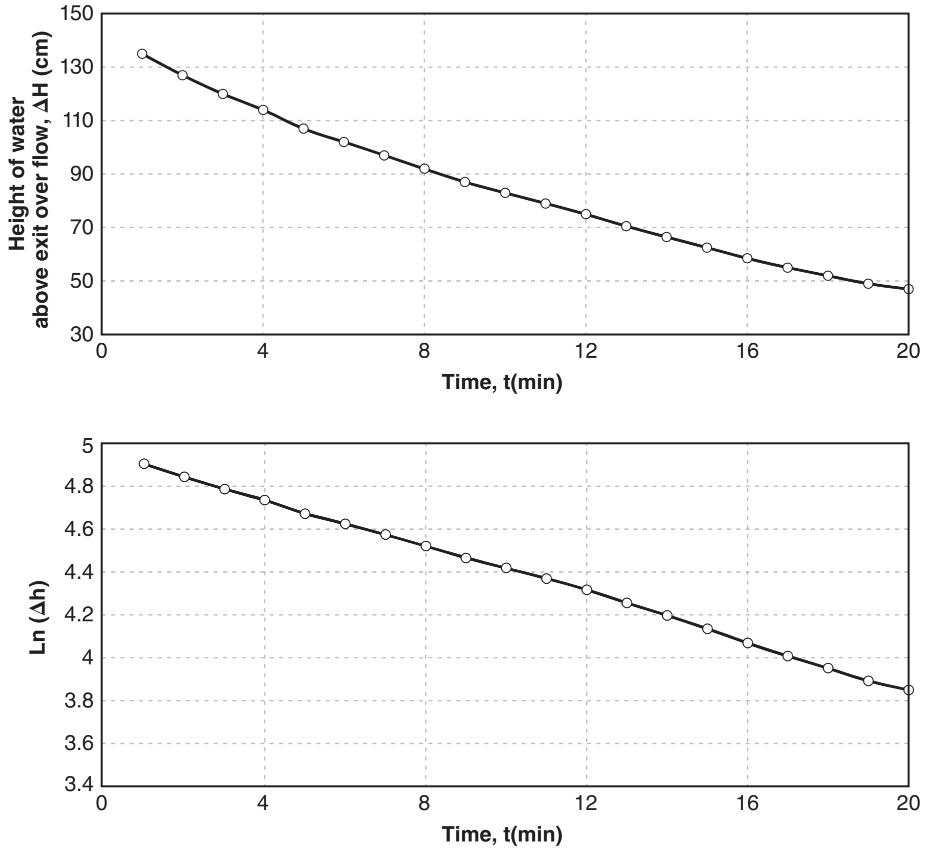


Figure 9.74 Falling head permeameter test results.

where Q is the flow out of the sample, v is the discharge velocity, and A is the cross-sectional area of the sample. The flow Q through the sample is also given by:

$$Q = -a\, dh/dt \tag{9.63}$$

where a is the cross-sectional area of the small tube, and dh is the drop in water level in the small tube during the time *dt*. The minus sign is necessary because *dh* is negative (the water level drops) while all other quantities are positive. Regrouping equations 9.58 and 9.59 gives:

$$dt = \frac{al}{Ak}\left(-\frac{dh}{h}\right) \tag{9.64}$$

After integration between the times 0 and t_1 corresponding to the losses of total head equal to Δh_0 and Δh_1 (Figure 9.73), the hydraulic conductivity is given by:

$$k = \frac{al}{At_1} Ln \frac{\Delta h_0}{\Delta h_1} = 2.3 \frac{al}{At_1} \log \frac{\Delta h_0}{\Delta h_1} \tag{9.65}$$

The advantage of the falling head permeameter test is that it is very simple to run; however, it is limited to measuring the hydraulic conductivity *k* of fine-grained soils at a small scale. The *k* values typically measured with this test range from 10^{-7} to 10^{-11} m/s.

9.18 WETTING FRONT TEST FOR UNSATURATED SOILS

The wetting front test is used to measure the hydraulic conductivity of unsaturated soils as a function of the water tension or water content. In this test the water progresses through the soil and saturates (wets) the soil as it goes. Three methods exist: the instantaneous profile method, the capillary rise method, and the wetting front method. The instantaneous profile method is described here. The other methods are described in Li, Zhang, and Fredlund (2009).

The test setup is shown in Figure 9.75. The water enters the sample from the left and wets the sample progressively toward the right. The air is chased away in front of the wetting front and escapes through the filter stone at the right end of the sample. Tensiometers or psychrometers are placed at regular intervals along the sample to measure the water tension u_w. During the test, the water content of the sample increases progressively while the water tension decreases accordingly. Because the hydraulic conductivity *k* depends on the water tension u_w, during this single test the hydraulic conductivity varies significantly. The measurements of water tension give the water content through the SWRC and also the velocity of the water as a function of time. The result of this test consists of a curve linking the hydraulic conductivity to the water tension or water content.

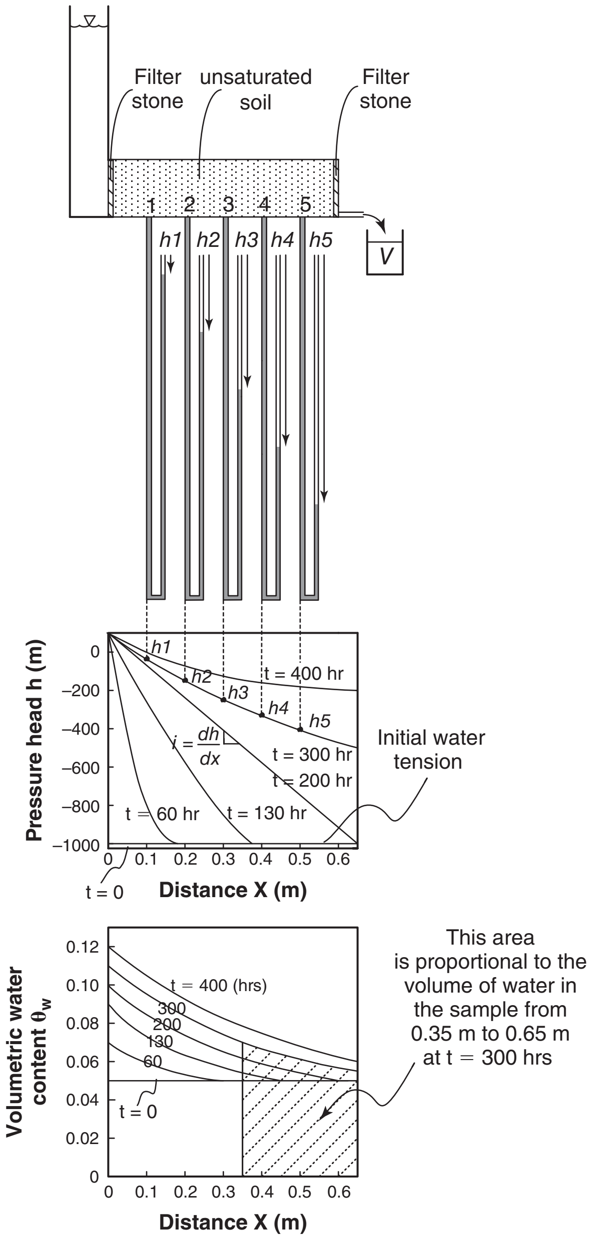


Figure 9.75 Wetting front test for unsaturated soils.

The data reduction proceeds as follows. The hydraulic gradient i varies as a function of time t and is given by the slope of the total head h_t versus distance x. Because the elevation head is constant, it is also the slope of the pressure head h_p versus distance x (Figure 9.75):

$$i_{(t)} = \frac{dh_t}{dx} = \frac{dh_p}{dx} \tag{9.66}$$

Note that the pressure head h_p is related to the water tension u_w and the unit weight of water γ_w as follows:

$$u_w = \gamma_w h_p \tag{9.67}$$

The volumetric water content θ_w may be obtained in the sample at any time and at any location by using the measured water tension at that time and at that location and the SWRC. In the center part of the curve, the relationship between θ_w and u_w can be approximated by:

$$\theta_w = C_w \log u_w + a \quad \text{or} \quad d\theta_w = C_w\, d(\log u_w) \tag{9.68}$$

where C_w is the slope of the θ_w vs. log u_w curve. Then the volume of water V_w in the sample between a given point j and the end of the sample is given by:

$$V_w = \int_{x_j}^{l} \theta_{w(x)} A dx \tag{9.69}$$

The velocity v_w of the water passing point j during an interval of time dt is given by:

$$v_w = \frac{dV_w}{Adt} \tag{9.70}$$

and the corresponding average hydraulic gradient is:

$$i_{ave} = \frac{1}{2}\left(i_{(t)} + i_{(t+dt)}\right) \tag{9.71}$$

Then the hydraulic conductivity $k(u_w)$ is found as the ratio of the velocity and the hydraulic gradient. The value of k depends on the water tension u_w; by using different corresponding pairs of pressure head vs. distance curves and volumetric water content vs. distance curves, the graph of $k(u_w)$ vs u_w can be generated with a single test.

9.19 AIR PERMEABILITY TEST FOR UNSATURATED SOILS

For unsaturated soils, it is sometimes necessary to measure the hydraulic conductivity of the soil to air flow k_a. One way to measure k_a is to use the apparatus shown in Figure 9.76. Air is supplied at a constant pressure p to the bottom of the unsaturated soil sample. The air flows through the soil and comes out at the top of the sample. The volume of air V_a collected during a time t is measured through a U-shaped, graduated burette. The air at the top of the sample is kept at

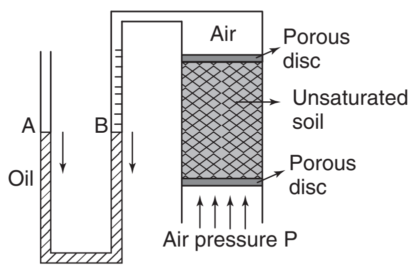


Figure 9.76 Measuring the hydraulic conductivity of air through an unsaturated soil.

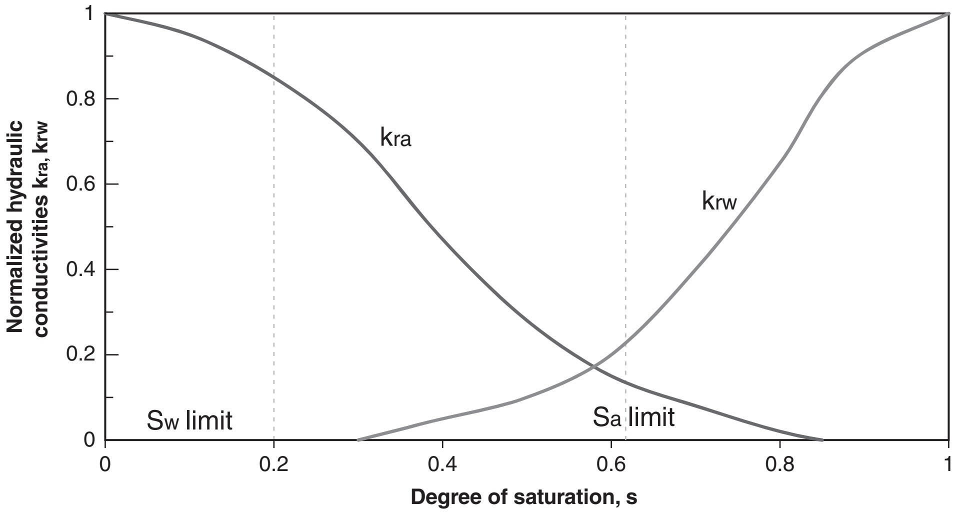


Figure 9.77 Relative hydraulic conductivity of water and air as a function of degree of saturation.

atmospheric pressure by adjusting the height of the burette so that the level of the oil remains the same on both sides (points A and B on Figure 9.76). Because point A is connected to the atmosphere, the pressure at B is also atmospheric.

The data reduction consists of the following. Darcy's law seems to describe the flow of air through soil reasonably well (Fredlund and Rahardjo 1993, p. 119). Therefore:

$$v_a = k_a i_a \tag{9.72}$$

where v_a is the air flow velocity, k_a is the hydraulic conductivity of air, and i_a is the hydraulic gradient for the air flow. Furthermore, the hydraulic gradient for the air is:

$$i_a = \frac{dh_a}{dx} = \frac{d\left(\dfrac{u_a}{\gamma_a}\right)}{dx} = \frac{1}{\gamma_a}\frac{du_a}{dx} \tag{9.73}$$

where h_a is the total head for air, x is the flow distance along the soil sample, u_a is the air pressure, and γ_a is the unit weight of air (0.0118 kN/m^3 at 20C). Note that the change in elevation head for air is typically negligible compared to the change in pressure head for air. This is why the change in total head is taken to be equal to the change in pressure head. The air pressure at the bottom of the sample is p and is maintained at 0 at the top of the sample, which has a length L. Therefore, the hydraulic conductivity of the air through the soil sample is given by:

$$k_a = \frac{\gamma_a V_a L}{Apt} \tag{9.74}$$

where k_a is the hydraulic conductivity of air, γ_a is the unit weight of air (0.0118 kN/m^3 at 20C), V_a is the volume of air flowing through the sample during a time t, L is the sample length, and p is the air pressure applied at the bottom of the sample.

The value of k_a depends on how dry the sample is as measured by the water content or water tension. The test described previously can be performed at different values of the water content or water tension by simply letting the sample dry and repeating the test at different water contents. The drier the sample is, the higher the value of k_a will be for a given soil. When the soil is dry, the value of k_a is maximum and equal to $k_{a(dry)}$. This trend is contrary to the trend for the hydraulic conductivity of water k_w. Indeed, k_w decreases when the soil gets drier; it is maximum when the soil is saturated and equal to $k_{w(sat)}$. Both hydraulic conductivities are often presented as normalized values as follows:

$$k_w = k_{rw} k_{w(sat)} \tag{9.75}$$

$$k_a = k_{ra} k_{a(dry)} \tag{9.76}$$

Figure 9.77 shows an example of the combined variation of both normalized hydraulic conductivity values k_{rw} and k_{ra} as a function of the degree of saturation S. Note that there is a limiting degree of saturation S_w (0.3 on Figure 9.77) where the water is no longer mobile and at the same time a limiting degree of saturation S_a (0.85 on Figure 9.77) where the air is no longer mobile.

9.20 EROSION TEST

9.20.1 Saturated Soils

The erosion function apparatus (EFA) test was developed in the early 1990s to measure the erodibility of soils and soft rocks (Figure 9.78; Briaud 2008). The principle is to go to the site where erosion is being investigated, collect samples within the depth of concern, bring them back to the laboratory and test them in the EFA. The 75 mm outside diameter sampling tube is placed through the bottom of the conduit where water flows at a constant velocity (Figure 9.78).

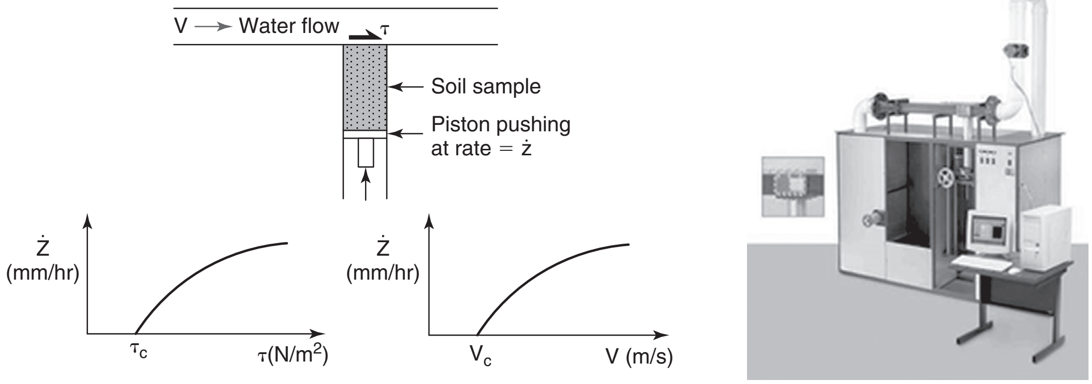


Figure 9.78 Erosion function apparatus test equipment: (*a*) Principle. (*b*) Equipment.

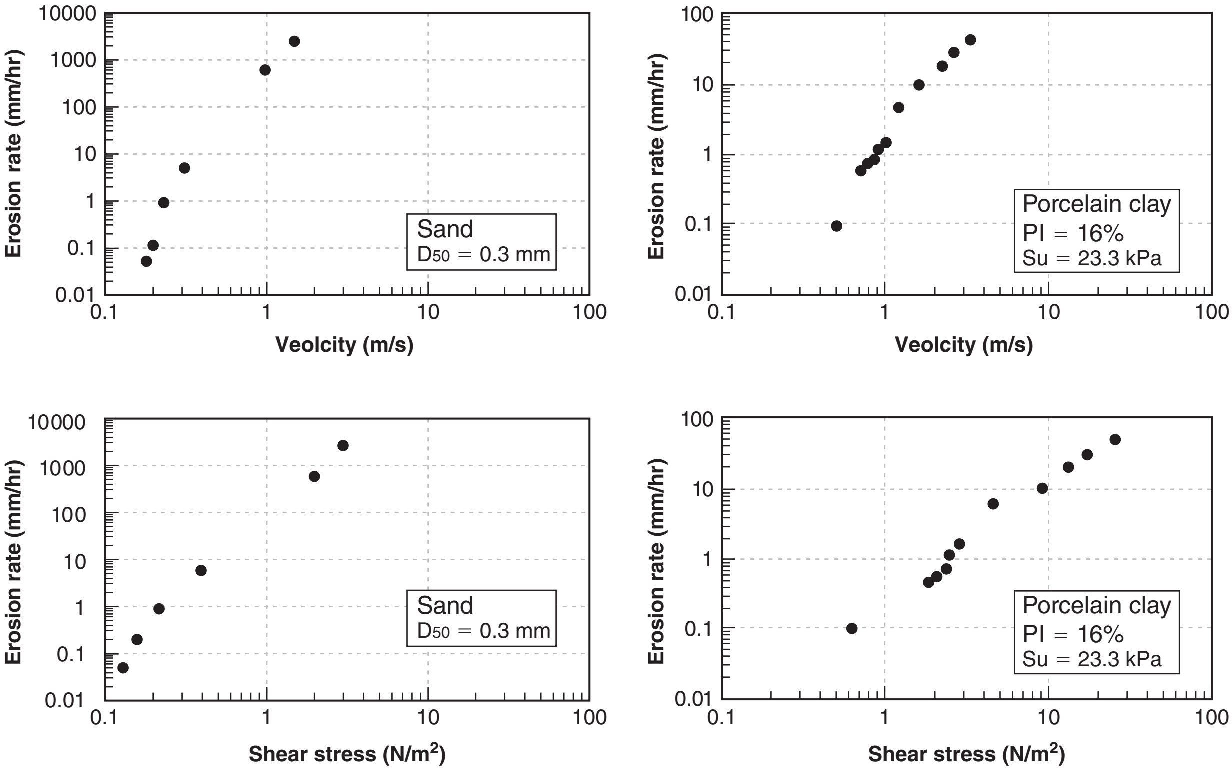


Figure 9.79 EFA test results: (*a*, *c*) A sand. (*b*, *d*) A clay.

The soil or rock is pushed by a piston out of the sampling tube only as fast as it is eroded by the water flowing over it.

The test result consists of the erosion rate $\dot{z}$ vs. shear stress τ curve and erosion rate $\dot{z}$ vs. mean flow velocity V curve (Figure 9.79). For each flow velocity V, the erosion rate $\dot{z}$ (mm/hr) is simply obtained by dividing the length h of sample eroded by the time t required to do so:

$$\dot{z} = \frac{h}{t} \tag{9.77}$$

The velocity V is obtained by measuring the flow Q and dividing by the flow area A. The shear stress τ is obtained by using the Moody Chart (Figure 9.80; Moody 1944) for pipe flows:

$$\tau = \frac{1}{8} f \rho V^2 \tag{9.78}$$

where τ is the shear stress on the wall of the pipe, f is the friction factor obtained from the Moody Chart (Figure 9.80), ρ is the mass density of water (1000 kg/m^3), and V is the mean flow velocity in the pipe. The friction factor f is a function of the pipe Reynolds Number R_e and the pipe roughness ε/D. The Reynolds Number is VD/ν where D is the pipe diameter and ν is the kinematic viscosity of water (10^{-6} m^2/s at 20°C).

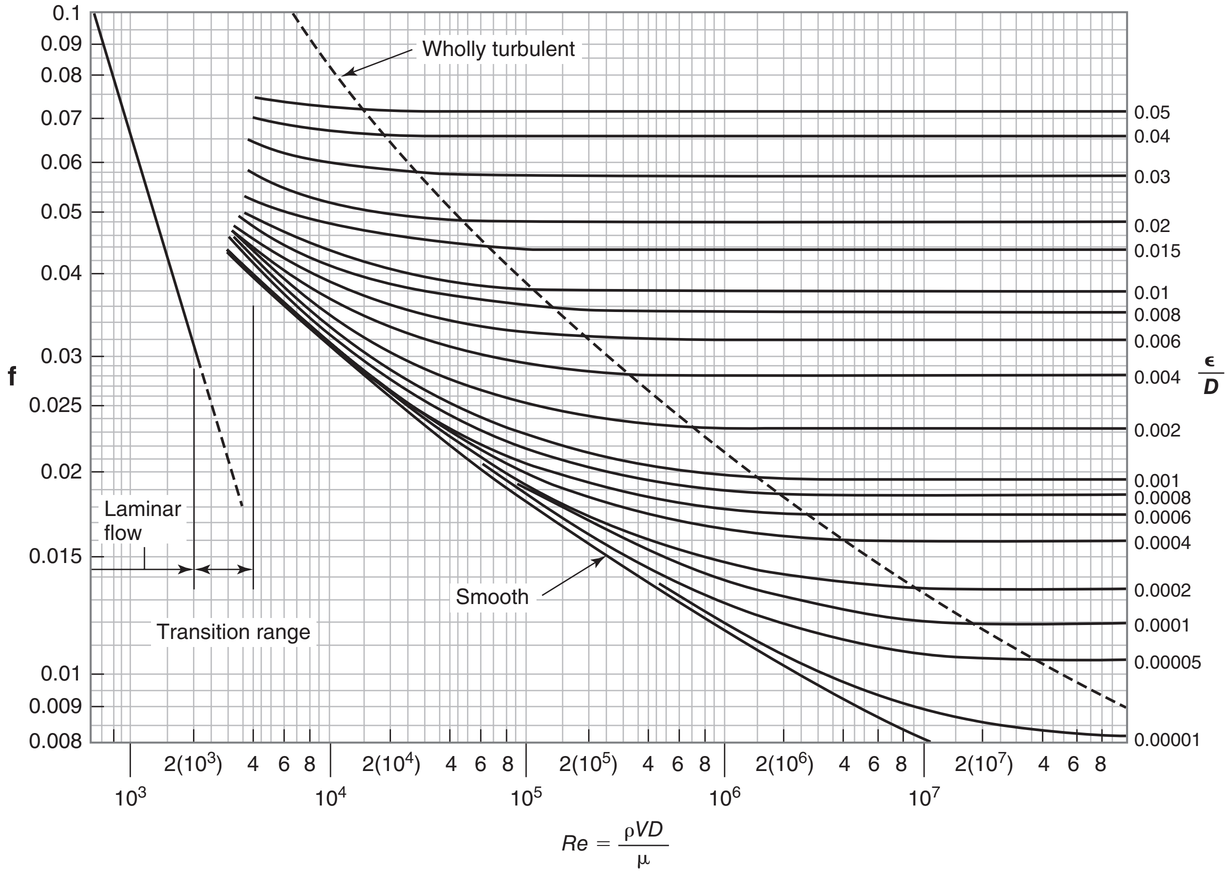


Figure 9.80 Moody Chart. (After Munson et al. 2012.)

Because the pipe in the EFA has a rectangular cross section, D is taken as the hydraulic diameter D = 4A/P where A is the cross-sectional flow area, P is the wetted perimeter, and the factor 4 is used to ensure that the hydraulic diameter is equal to the diameter for a circular pipe. For a rectangular cross-section pipe:

$$D = \frac{2ab}{a+b} \tag{9.79}$$

where a and b are the dimensions of the sides of the rectangle. The relative roughness ε/D is the ratio of the average height ε of the sample roughness over the pipe diameter D. The average height of the sample roughness ε is taken equal to $0.5D_{50}$ where D_{50} is the mean grain size for the soil. The factor 0.5 is used because it is assumed that the top half of the particle protrudes into the flow while the bottom half is buried in the soil mass. For fine-grained soils, the roughness is taken as one-half of the depth of the asperities on the sample surface.

For fine-grained and coarse-grained soils, ASTM standard thin-wall steel tube samples are favored. If such samples cannot be obtained (e.g., with coarse-grained soils), split spoon SPT samples are obtained and the coarse-grained soil is reconstituted in the thin-wall steel tube. Fortunately, in the case of erosion of uncemented coarse-grained soils, soil disturbance does not affect the results significantly. For erosion of rocks—if it is representative of the rock erosion process to test a 75 mm diameter rock sample—the rock core is placed in the thin-wall steel tube and tested in the EFA. Example erosion functions are shown in Figure 9.79 for a fine sand and for a low-plasticity clay. Note that for the same average velocity of 1 m/s in the EFA test conduit, the rate of erosion for the sand is about 1000 times faster than for the clay. This indicates that the rate of erosion can be very different for different soils.

Other devices have also been developed to evaluate how resistant earth materials are to water flow. These include the rotating cylinder to measure the erosion properties of stiff soils (e.g., Chapuis and Gatien 1986), the jet erosion test to evaluate the erodibility of soils (e.g., Hanson 1991), and the hole erosion test to measure the erosion properties of stiff soils (e.g., Wan and Fell 2004).

9.20.2 Unsaturated Soils

If the soil is unsaturated, the EFA test procedure and the data reduction are unchanged, except that a decision must be made on whether to let water stand on top of the sample for an extended period of time before starting the water flow. The decision should be based on which option best represents the field conditions. If the unsaturated state of the sample was created by drying, then the water should be left standing on top of the sample until saturation is recreated before testing. In ephemeral streams, for example, the soil dries in the summer, and a fine-grained soil may crack, thereby generating a thin crust of soil. This thin crust will be washed away as soon as significant water flow occurs. In this case it is necessary not to soak the sample before erosion testing. Note that in all cases, as the test progresses, the sample is likely to become nearly saturated.

PROBLEMS

9.1 What are the four main categories of laboratory tests? Give three examples for each category.

9.2 What device would you use to measure the following quantities? In each case explain the basic principle of the device.
  a. Force
  b. Pressure
  c. Shear stress
  d. Water compression stress

9.3 What devices and techniques would you use to measure the water tension stress (suction)? In each case explain the basic principle of the device.

9.4 What devices would you use to measure the following quantities? In each case explain the basic principle of the device.
  a. Displacement
  b. Normal strain
  c. Shear strain

9.5 Table 9.1s shows the results of a Standard Proctor Compaction Test for the same soil tested at different water contents. Calculate the dry unit weight and water content for each sample, plot a graph of dry unit weight versus water content, determine the maximum dry unit weight and optimum water content, and plot the saturation lines for S = 100%, S = 90%, and S = 80%.

Table 9.1s Results of a Standard Proctor Compaction Test

Volume of mold (m^3)	Unit weight (kN/m^3)	Weight wet soil + container (N)	Weight dry soil + container (N)	Weight of water (N)	Weight of container (N)	Weight of dry soil (N)
0.000943	18.48	0.194	0.182	0.012	0.048	0.134
0.000943	19.26	0.146	0.136	0.010	0.052	0.084
0.000943	19.80	0.204	0.184	0.020	0.047	0.137
0.000943	20.12	0.14	0.128	0.012	0.052	0.076
0.000943	20.21	0.273	0.238	0.035	0.054	0.184

9.6 A Modified Proctor Compaction Test is performed on a sample of silty sand in a 152 mm diameter mold. The maximum dry unit weight is 19.6 kN/m^3 and the optimum water content is 11%. If the specific gravity of solids is 2.65, draw the three-phase diagram of the sample in the mold and calculate all volumes and weight for that sample. What is the degree of saturation?

9.7 Referring to Figure 9.23, explain the following:
  a. Why is the dry unit weight vs. water content curve relatively flat compared to the modulus vs. water content curve?
  b. Why does the modulus vs. water content curve go downward as the water content increases from 1% to 6%?
  c. Why does the modulus vs. water content curve drop so significantly when the water content goes from 8% to 10%?

9.8 For the consolidation test, what is the difference between the incremental loading procedure and the constant rate of strain procedure?

9.9 For the consolidation test curve shown in Figure 9.27, calculate the compression index C_c and the recompression index C_r.

9.10 For the consolidation test curves shown in Figure 9.30 and Figure 9.31, calculate the coefficient of consolidation c_v according to the log time method and according to the square root of time method. Compare and comment.

9.11 For the consolidation curve shown in Figure 9.27, determine the preconsolidation effective stress.

9.12 Regarding deformation laboratory tests, discuss the differences between tests on saturated soils and tests on unsaturated soils (or, more precisely, tests on soils where the water is in compression and tests on soils where the water is in tension).

9.13 A direct shear test is performed on a sample of saturated clay. The sample is 25 mm high and 75 mm in diameter. The test cell is inundated such that the water stress is hydrostatic at the beginning of the test.

a. How would you run the test so as to measure the undrained shear strength of the clay?
b. How would you run the test so as to obtain the drained shear strength parameters for the clay?

9.14 A direct shear test is performed on a sample of dry sand. The sample is 50 mm in diameter and 25 mm high and is subjected to a vertical force of 100 N. At failure, the shear force applied is 60 N, the horizontal movement is 3 mm, and the vertical movement is 0.5 mm. Calculate the shear strength of the sand and the friction angle, and estimate the dilation angle.

9.15 A direct shear test is performed on a sample of saturated clay. The test is a quick test such that water does not have time to drain during the test. The vertical load on the sample induces a total normal stress of 50 kPa and at failure the shear force induces a shear stress of 100 kPa.

a. Calculate the undrained shear strength of the clay.
b. How is it possible for this clay to have such high shear strength, considering the low normal stress?

9.16 Two direct shear tests are performed on a sample of saturated clay. The tests are slow tests such that the water stress (pore pressure) remains equal to zero.

$$\text{Test 1: N} = 300\text{ N, T} = 250\text{ N, A} = 0.01\text{ m}^2\text{, S} = 100\%\text{, u}_w = 0\text{ kPa}$$

$$\text{Test 2: N} = 600\text{ N, T} = 400\text{ N, A} = 0.01\text{ m}^2\text{, S} = 100\%\text{, u}_w = 0\text{ kPa}$$

where N is the normal force, T is the shear force, A is the sample cross-sectional area, S is the degree of saturation, and u_w is the water stress. Calculate the effective stress cohesion and effective stress friction angle of the clay.

9.17 For strength laboratory tests, discuss the differences between tests on saturated soils and tests on unsaturated soils (or, more precisely, tests on soils where the water is in compression and tests on soils where the water is in tension).

9.18 Assume the conditions as in problem 9.16, but this time the soil is unsaturated and the readings are as follows:

$$\text{Test 1: N} = 600\text{ N, T} = 1900\text{ N, A} = 0.01\text{ m}^2\text{, S} = 60\%\text{, u}_w = -400\text{ kPa}$$

$$\text{Test 2: N} = 200\text{ N, T} = 900\text{ N, A} = 0.01\text{ m}^2\text{, S} = 40\%\text{, u}_w = -300\text{ kPa}$$

where N is the normal force, T is the shear force, A is the sample cross-sectional area, S is the degree of saturation, and u_w is the water tension stress. Calculate the effective stress cohesion and effective stress friction angle of the clay.

9.19 What are the differences between the direct shear test and the simple shear test? Explain your answers.

9.20 A simple shear test is performed on a sample of silt. The sample is 50 mm in diameter and 20 mm high. When the shear force applied is 200 N, the horizontal displacement of the top of the sample is 0.2 mm. Calculate the shear stress, the shear strain, and the shear modulus of the sample at that point on the stress-strain curve.

9.21 An unconfined compression (UC) test on a sample of clay gives the stress-strain curve shown in Figure 9.47. Calculate the undrained shear strength and the UC modulus for this sample. What geotechnical problem do you think this undrained shear strength and this modulus could be used for?

9.22 What are the two main phases in running a triaxial test? With respect to drainage during each one of these two phases, what are the different types of tests that can be run? For each type of test, what parameters can you obtain from the data?

9.23 A triaxial test is performed on a sample that is 50 mm in diameter and 100 mm high. The confining pressure is 30 kPa and at failure the vertical load on the sample is 118 N. Is the vertical total stress on the sample at failure expressed in N/m^2 equal to $\dfrac{118N}{\pi(25.10^{-3})^2}$? If yes, explain your answer. If not, what is it?

9.24 A triaxial test with water stress (pore-pressure) measurements is performed on a sample of saturated silty crushed rockfill and gives the results shown in Figure 9.50. The total confining stress is 35 kPa.

a. Calculate the total stress secant modulus E and the effective stress secant modulus E′ for a vertical strain equal to 0.2%, 0.5%, 1%, 2%, 3%, 4%, and 5%. Then plot the curve giving soil modulus both as a function of strain and as a function of log strain.
b. At failure, the vertical effective stress is 100 kPa. Calculate the effective stress friction angle of the sand if the effective stress cohesion c is zero.

9.25 Two CU triaxial tests are performed on a sample of saturated, overconsolidated, high-plasticity clay. At failure, the results are as follows:

$$\text{Test 1}: \sigma_3 = 30\text{ kPa, Q} = 0.45\text{ kN, A} = 0.01\text{ m}^2\text{, u} = 10\text{ kPa}$$

$$\text{Test 2}: \sigma_3 = 60\text{ kPa, Q} = 0.70\text{ kN, A} = 0.01\text{ m}^2\text{, u} = 20\text{ kPa}$$

where σ_3 is the total confinement stress, Q is the vertical load on the sample, A is the sample cross section, and u is the water stress (pore pressure).

a. Calculate σ_3, σ_1, σ_3', and σ_1' at failure.
b. Draw the Mohr circle at failure in the τ vs. σ' set of axes.
c. Draw the failure envelope and find the effective stress strength parameters c′ and φ'.

9.26 Two CU triaxial tests are performed on a sample of unsaturated clay. At failure, the results are as follows:

$$\text{Test 1}: \sigma_3 = 20\text{ kPa}, \sigma_1 = 190\text{ kPa, S} = 60\%, \text{u}_w = -100\text{ kPa}$$

$$\text{Test 2}: \sigma_3 = 60\text{ kPa}, \sigma_1 = 450\text{ kPa, S} = 50\%, \text{u}_w = -300\text{ kPa}$$

where σ_3 is the total confinement stress, σ_1 is the total vertical stress at failure, S is the degree of saturation, and u_w is the water stress.

a. Calculate σ_3, σ_1, σ_3', and σ_1' at failure.
b. Draw the Mohr circle at failure in the τ vs. σ' set of axes.
c. Draw the failure envelope and find the effective stress strength parameters c′ and φ'.

9.27 What is the stress path, and what shape does it typically have for the triaxial test?

9.28 A lab vane test is performed on a silty clay. At failure, the maximum torque is 5.7 N.m. The vane is 50 mm high and 25 mm in diameter. Calculate the undrained shear strength of the silty clay. The vane is rotated 10 times rapidly and the torque on the tenth revolution is measured to be 3.5 N.m. Calculate the residual undrained shear strength of the silty clay.

9.29 A silty sand is subjected to a constant head permeameter test. The flow collected at the downstream end is 221 mm^3/s; the sample is 75 mm in diameter and 100 mm high. The difference between the water level in the upstream overflow and the downstream overflow is 0.5 m. Calculate the hydraulic conductivity k of the silty sand.

9.30 A clay sample is tested in a falling head permeameter. The sample is 75 mm in diameter and 100 mm high. The small tube is 3 mm in diameter. The difference in height between the water level in the small tube above the sample and the downstream overflow is measured as a function of time. At time $t = 0$, the difference is 1.1 m and at time $t = 1$hr, the difference is 1.05 m. Calculate the hydraulic conductivity k of the clay.

9.31 This problem refers to Figure 9.69. A sample of unsaturated silt is tested in a constant head permeameter and the following parameters are measured: $D = 75$ mm, $l = 150$ mm, $V = 10$ cm^3, $t = 1$ hour, $h_{p2} = -100$ mm, $h_{p3} = -200$ mm, $h_{p4} = 0$ mm. Calculate the unsaturated hydraulic conductivity k of the silt and the water tension u_w corresponding to that value.

9.32 A 0.65 m long, 75 mm diameter sample of unsaturated clay is tested in a wetting front permeameter. Initially the water tension in the sample is −1000 kPa. The results are shown in Figure 9.75. Use the results to develop the hydraulic conductivity k vs. water tension u_w curve for this clay.

9.33 This problem refers to Figure 9.76. A sample of unsaturated clayey sand has a degree of saturation of 40%. The sample length is 150 mm and the sample diameter is 75 mm. It is tested in a permeameter to determine the hydraulic conductivity of air through the sample. The air pressure at the base of the sample is 10 kPa and the volume of air collected at the top of the sample in one hour of testing is 10^{-3} m^3. The top of the sample is kept at atmospheric pressure. Calculate the air hydraulic conductivity of the sample k_a and the air stress u_a associated with this hydraulic conductivity value.

9.34 A 1.8 m tall human being drinks one liter of water. Three hours later, this person goes to the bathroom and eliminates the liter of water. Is this case a constant head permeameter or a falling head permeameter? Calculate the hydraulic conductivity of the human body. Make reasonable assumptions when necessary.

9.35 A sample of fine sand is tested in the EFA. The mean diameter of the grains is $D_{50} = 1$ mm. When the velocity is set at 1 m/s, the piston below the sample of sand has to be raised at a rate of 16.7 mm/minute. The cross section of the conduit where the water is flowing is rectangular, with a width of 100 mm and a height of 50 mm. Calculate the shear stress at the interface between the water and the sand for the 1 m/s velocity.

9.36 A sample of low-plasticity clay is tested in the EFA. The surface of the clay sample is considered smooth. When the velocity is set at 3 m/s, the piston below the sample of sand has to be raised at a rate of 1 mm every 3 minutes. The cross section of the conduit where the water is flowing is rectangular, with a width of 100 mm and a height of 50 mm. Calculate the shear stress at the interface between the water and the sand for the 3 m/s velocity.

Problems and Solutions

Problem 9.1

What are the four main categories of laboratory tests? Give three examples for each category.

Solution 9.1

1. Tests for index properties
   a. Water content
   b. Unit weight
   c. Particle size
2. Tests for deformation properties
   a. Consolidation
   b. Triaxial
   c. Simple shear
3. Tests for strength properties
   a. Direct shear
   b. Unconfined compression
   c. Lab vane
4. Tests for flow properties
   a. Constant head permeameter
   b. Falling head permeameter
   c. Erosion tests

Problem 9.2

What device would you use to measure the following quantities? In each case explain the basic principle of the device.

a. Force
b. Pressure
c. Shear stress
d. Water compression stress

Solution 9.2

a. Force

A load cell is the most common way to measure load; it consists of a deformable piece of steel (S shape or cylindrical) instrumented with strain gages.

b. Pressure

Pressure cells are used to measure pressure. They are circular and have a metallic membrane that deforms when it is in contact with the stressed soil. The bending of the membrane is measured with strain gages glued to that membrane and the strains are related to the pressure on the membrane.

c. Shear stress

The simplest way to measure shear stress is to measure the shear force and divide by the corresponding area. This is the case with the direct shear test. Alternatively, shear stress can be measured by a shear stress transducer.

d. Water compression stress

A manometer can be used to measure water compression stress. A manometer or standpipe is simply a pipe connected to the point where the water compression stress must be measured and open to the atmosphere at the other end. The pressure in the water makes the water rise in the manometer to the point of equilibrium. The water compression stress is then calculated as the vertical distance between the point of measurement and the water level in the manometer times the unit weight of water.

A pore-pressure transducer can be also used to measure water pressure. The pore-pressure transducer measures the water pressure by letting that pressure deflect a membrane. A porous tip made of ceramic (Figure 9.4) is placed in contact with the soil where the water is in compression. This porous tip, which is saturated with de-aired water, allows water to come in but does not allow air to come in.

Problem 9.3

What devices and techniques would you use to measure the water tension stress (suction)? In each case explain the basic principle of the device.

Solution 9.3

Filter paper test

The filter paper test consists of using a circular piece of filter paper (about 50 mm in diameter), weighing it dry, placing it either in contact with or above the soil sample, enclosing the filter paper and the sample in a sealed container until the filter paper comes into water tension equilibrium with the sample, retrieving the filter paper, and weighing it to obtain its water content. Because the soil sample is much larger than the filter paper, the water content of the sample remains unaffected by the amount of moisture drawn into the filter paper. The filter paper comes calibrated with a curve linking the filter paper moisture content and the water tension in the filter paper. Because the water tension is the same in the filter paper and the sample, the water tension of the sample is determined in that fashion. The filter paper method, however, can measure matric suction only or matric suction plus osmotic suction, depending on whether or not the filter paper is in contact with the sample.

Thermocouple psychrometer

Psychrometers give the relative humidity by measuring the difference in temperature between a nonevaporating surface and an evaporating surface. Psychrometers measure the total suction because the evaporation process creates pure water, whereas the water in the soil pores is not pure.

Tensiometer

A tensiometer consists of a high air entry porous ceramic tip saturated with water and placed in good contact with the soil. In the tensiometer, the space behind the ceramic tip is filled with de-aired water and connected to a negative pressure measuring device. The stress slowly equalizes between the water tension in the tensiometer and the water tension in the soil pores. Then that tension is measured either through a water-mercury manometer, a Bourdon-vacuum tube, or an electrical pressure transducer. The water tension that can be measured in a tensiometer is limited to approximately negative 90 kPa (2.95 pF) due to the possibility of water cavitation in the tensiometer above such a value.

Pressure plate

A pressure plate is a closed pressure chamber that can be used to increase the air pressure in the soil pores to the point where the air drives the water out of the pores. The sample is placed in the chamber on a high air entry ceramic disk. This disk, which is saturated with water, has the property of letting water but not air go through up to a certain rated pressure (the air entry value of the disk). The air pressure is increased and the stress in the water is increased accordingly (decrease in tension). When the water tension becomes equal to zero, the water comes out; at that point, the air pressure is equal to the water tension. This technique is called the axis translation technique because it simply translates the origin of reference by applying an air pressure equal to the water tension.

Salt solution

Salt solution equilibrium is a water tension measurement that relies on the fact that salt solutions have significant osmotic suction. A closed chamber with a salt solution at its lower part will generate a certain relative humidity in the air above it. The higher the salt concentration is, the lower the relative humidity above the solution in the chamber will be. If a soil sample is suspended in the air above the salt solution, it will dry and the water tension in the soil sample will come to equilibrium with the ambient relative humidity. At equilibrium, the water tension is given by the relative humidity in the air of the chamber. This relative humidity depends on the salt concentration in the solution and can be calculated from it. This relationship depends on the type of salt, its molality, and the temperature.

Problem 9.4

What devices would you use to measure the following quantities? In each case explain the basic principle of the device.

a. Displacement
b. Normal strain
c. Shear strain

Solution 9.4

a. Displacement.

Displacement can be measured with a linear variable differential transformer. An LVDT has three solenoid coils, arranged like three side-by-side donuts. A metallic rod attached to the point where the displacement is to be measured passes through the center of the three solenoids without touching them. An alternating current through the center solenoid creates a voltage in the side solenoids. The movement of the metallic rod creates a change in voltage which is linearly proportional to the movement of the rod. The change in voltage is transformed into a displacement measurement through calibration.

b. Normal strain.

Normal strain can be measured using a foil strain gage. A foil gage is a thin sheet of metal (copper-nickel alloy is common) with a pattern glued to the material that will deform. Actually, a layer of insulating flexible material is first glued to the deforming material and then the foil gage is glued on the insulator so that the current passing through the gage only travels through the gage. When the material deforms, the foil length changes and so does its resistance. The voltage changes accordingly and the strain is related to the change in voltage through calibration.

c. Shear strain.

Shear strain can be measured using the same strain gage described for normal strain. Shear strain γ is defined for two perpendicular directions. When the shear strain is small enough, the shear strain is equal to the change in angle γ, expressed in radians between the two perpendicular directions due to the shearing process. Shear strain is most easily obtained by measuring the normal strain in two perpendicular directions. It can be shown that the shear strain measured in the x and y direction is given by: $\gamma_{xy} = \varepsilon_1 - \varepsilon_2$.

Problem 9.5

Table 9.1s shows the results of a Standard Proctor Compaction Test for the same soil tested at different water contents. Calculate the dry unit weight and water content for each sample, plot a graph of dry unit weight versus water content, determine the maximum dry unit weight and optimum water content, and plot the saturation lines for S = 100%, S = 90%, and S = 80%.

Table 9.1s Results of a Standard Proctor Compaction Test

Volume of mold (m^3)	Unit weight (kN/m^3)	Weight wet soil + container (N)	Weight dry soil + container (N)	Weight of water (N)	Weight of container (N)	Weight of dry soil (N)
0.000943	18.48	0.194	0.182	0.012	0.048	0.134
0.000943	19.26	0.146	0.136	0.010	0.052	0.084
0.000943	19.80	0.204	0.184	0.020	0.047	0.137
0.000943	20.12	0.14	0.128	0.012	0.052	0.076
0.000943	20.21	0.273	0.238	0.035	0.054	0.184

Solution 9.5

Assuming that Gs = 2.65, the dry unit weight and the water content for each sample are calculated in Table 9.2s.

Table 9.2s Results of a Standard Proctor Compaction Test

Volume of mold (m^3)	Unit weight (kN/m^3)	Weight wet soil + container (N)	Weight dry soil + container (N)	Weight of water (N)	Weight of container (N)	Weight of dry soil (N)	Water content (%)	Dry unit weight (kN/m^3)
0.000943	18.48	0.194	0.182	0.012	0.048	0.134	8.96	16.96
0.000943	19.26	0.146	0.136	0.01	0.052	0.084	11.90	17.21
0.000943	19.80	0.204	0.184	0.02	0.047	0.137	14.60	17.28
0.000943	20.12	0.14	0.128	0.012	0.052	0.076	15.79	17.37
0.000943	20.21	0.273	0.238	0.035	0.054	0.184	19.02	16.98

The plot of dry unit weight function of water content, and the saturation lines for S = 100%, S = 90%, and S = 80%, are shown in Figure 9.1s.

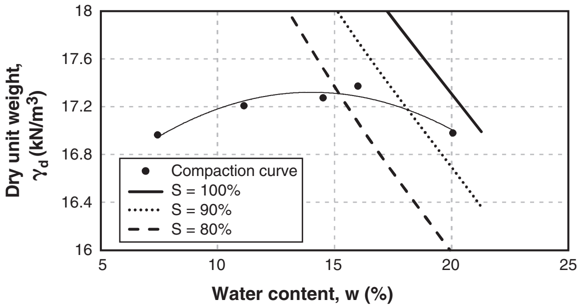


Figure 9.1s Plot of dry unit weight versus water content.

Problem 9.6

A Modified Proctor Compaction Test is performed on a sample of silty sand in a 152 mm diameter mold. The maximum dry unit weight is 19.6 kN/m^3 and the optimum water content is 11%. If the specific gravity of solids is 2.65, draw the three-phase diagram of the sample in the mold and calculate all volumes and weight for that sample. What is the degree of saturation?

Solution 9.6

Given that $\gamma_d = 19.6$ kN/m^3, $w_{opt} = 11\%$, $G_s = 2.65$, and D = 152 mm, we know the volume of the mold is 21.2×10^{-4} m^3 (Table 9.5).

The three-phase diagram of the sample is shown in Figure 9.2s.

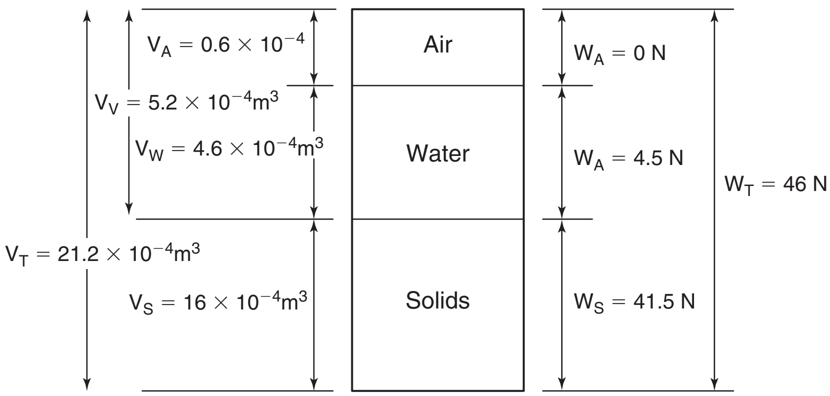


Figure 9.2s Three-phase diagram.

$$\gamma_d = \frac{w_s}{V_T} \rightarrow w_s = \gamma_d \times V_T = 19.6 \times 21.2 \times 10^{-4} = 4.15 \times 10^{-2}\ \text{kN} = 41.5\ \text{N}$$

$$V_s = \frac{w_s}{G_s \gamma_w} = \frac{41.5 \times 10^{-3}}{2.65 \times 9.81} = 1.6 \times 10^{-3}\ \text{m}^3$$

$$\gamma_d = \frac{\gamma_T}{1 + w} \rightarrow \gamma_T = \gamma_d \times (1 + w) = 19.6 \times (1 + 0.11) = 21.7\ \text{kN/m}^3$$

$$\gamma_T = \frac{W_T}{V_T} \rightarrow W_T = \gamma_T \times V_T = 21.7 \times 21.2 \times 10^{-4} = 46 \times 10^{-3}\ \text{kN} = 46\ \text{N}$$

$$\text{W}_\text{T} = \text{W}_\text{s} + \text{W}_w \rightarrow \text{W}_w = 46 - 41.5 = 4.5\ \text{N}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{4.5 \times 10^{-3}}{9.8} = 4.6 \times 10^{-4}\ \text{m}^3$$

$$\text{V}_\text{T} = \text{V}_\text{A} + \text{V}_\text{W} + \text{V}_\text{S} \rightarrow \text{V}_\text{A} = 0.6 \times 10^{-4}\ \text{m}^3$$

$$\text{V}_v = \text{V}_\text{A} + \text{V}_w = 6.2 \times 10^{-4}\ \text{m}^3$$

Degree of saturation:

$$S = \frac{V_w}{V_v} = \frac{4.6 \times 10^{-4}}{5.2 \times 10^{-4}} \times 100 = 88.5\%$$

Problem 9.7

Referring to Figure 9.23, explain the following:

a. Why is the dry unit weight vs. water content curve relatively flat compared to the modulus vs. water content curve?
b. Why does the modulus vs. water content curve go downward as the water content increases from 1% to 6%?
c. Why does the modulus vs. water content curve drop so significantly when the water content goes from 8% to 10%?

Solution 9.7

a. The dry unit weight vs. water content curve is relatively flat compared to the modulus vs. water content curve because the dry unit weight is much less sensitive to the water content than the modulus. The amount of dry particles in a given volume does not change much with increasing water content. However, the modulus will change much more dramatically because the effective stress and the skeleton structure of the soil change much more dramatically with the water content.
b. Increasing the water content from 1% to 6% decreases the tension stress in the water (suction), which weakens the soil; thus the BCD plate bends more, leading to lower E moduli. Beyond that point, the added water lubricates the particles and allows them to achieve a more compact arrangement for the given compaction effort. The soil becomes stiffer and the modulus becomes larger.
c. Near the optimum water content, the soil is approaching saturation. Increasing the water content beyond the optimum water content (8%) simply increases the size of the voids by filling them with water; thus, the soil becomes softer and the modulus drops significantly.

Problem 9.8

For the consolidation test, what is the difference between the incremental loading procedure and the constant rate of strain procedure?

Solution 9.8

The incremental loading procedure consists of placing a load on the sample for 24 hours while recording the decrease in sample thickness. Also, water is allowed to drain from both the top and the bottom of the sample. In the constant rate of strain procedure, the sample is deformed at a constant rate of displacement with time. Also, water is allowed to drain only from the top of the sample, while the tester ensures that the pore pressure does not rise above a set limit at the bottom of the sample.

Problem 9.9

For the consolidation test curve shown in Figure 9.27, calculate the compression index C_c and the recompression index C_r.

Solution 9.9

The compression index C_c and the recompression index C_r (Figure 9.3s) are calculated as follows:

$$C_c = \frac{\Delta e}{\log\left(\frac{\sigma_2'}{\sigma_1'}\right)} = \frac{0.48 - 0.36}{\log\left(\frac{6100}{1500}\right)} = 0.192$$

$$C_r = \frac{\Delta e}{\log\left(\frac{\sigma_2'}{\sigma_1'}\right)} = \frac{0.47 - 0.42}{\log\left(\frac{3000}{750}\right)} = 0.0823$$

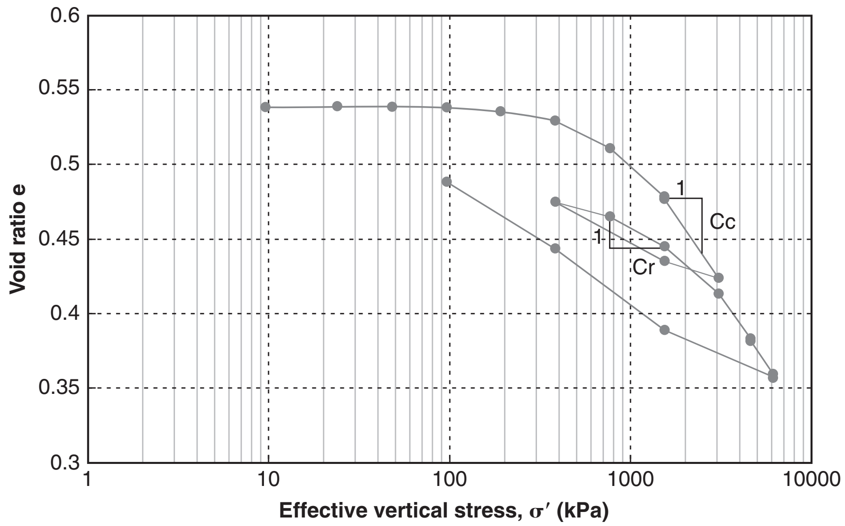


Figure 9.3s Consolidation test results.

Problem 9.10

For the consolidation test curves shown in Figure 9.30 and Figure 9.31, calculate the coefficient of consolidation c_v according to the log time method and according to the square root of time method. Compare and comment.

Solution 9.10

The coefficient of consolidation according to the log time method is calculated using Figure 9.4s. From the curve:

$$\mathrm{x} = 0.0145 - 0.0115 = 0.003$$

$$\varepsilon_0 = 0.012 - 0.002 = 0.009$$

$$\varepsilon_{100} = 0.046$$

$$\varepsilon_{50} = (\varepsilon_0 + \varepsilon_{100})/2 = (0.009 + 0.046)/2 = 0.0275$$

The time corresponding to 50% of consolidation $t_{50} = 30$ min $= 1800$ sec.

Assuming that during the consolidation test the sample is double-drained, the coefficient of consolidation is calculated as follows:

$$c_v = T_{50}\left(\frac{H_{dr}^2}{t_{50}}\right) = 0.197\left(\frac{0.0125^2}{1800}\right) = 1.59 \times 10^{-8}\ \mathrm{m^2/s}$$

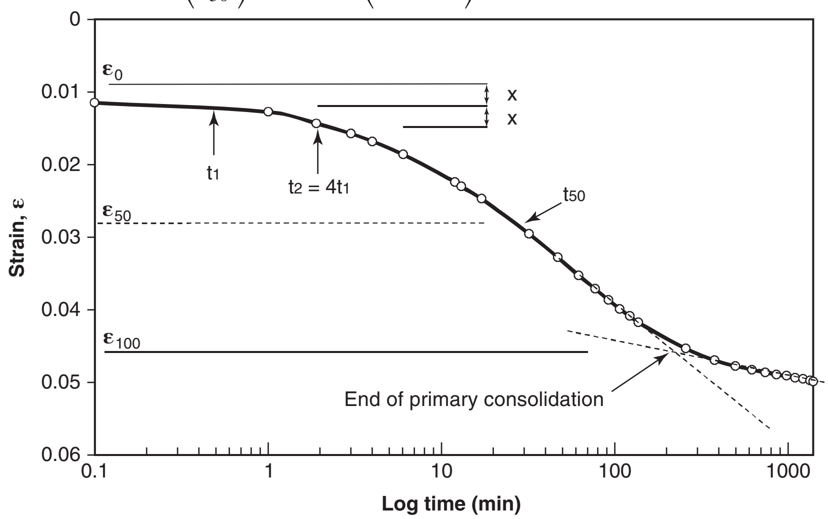


Figure 9.4s Log time method.

The coefficient of consolidation according to the square root of time method is calculated using Figure 9.5s. From the curve:

$$\sqrt{t_{90}} = 10 \Rightarrow t_{90} = 100\,\text{min} = 6000\,\text{sec}$$

Assuming that during the consolidation test the sample is double-drained, the coefficient of consolidation is calculated as follows:

$$c_v = T_{90}\left(\frac{H_{dr}^2}{t_{90}}\right) = 0.848\left(\frac{0.0125^2}{6000}\right) = 2.21 \times 10^{-8}\,\text{m}^2/\text{s}$$

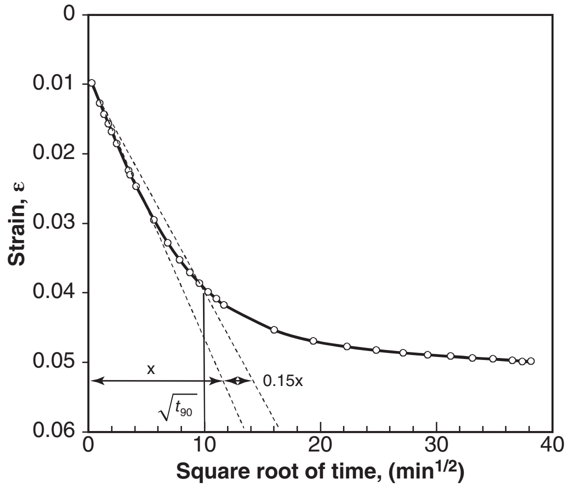


Figure 9.5s Square root of time method.

Problem 9.11

For the consolidation curve shown in Figure 9.27, determine the preconsolidation effective stress.

Solution 9.11

From the consolidation curve (Figure 9.6s) and using the Cassagrande construction, the preconsolidation effective stress is $\sigma'_p = 1020$ kPa.

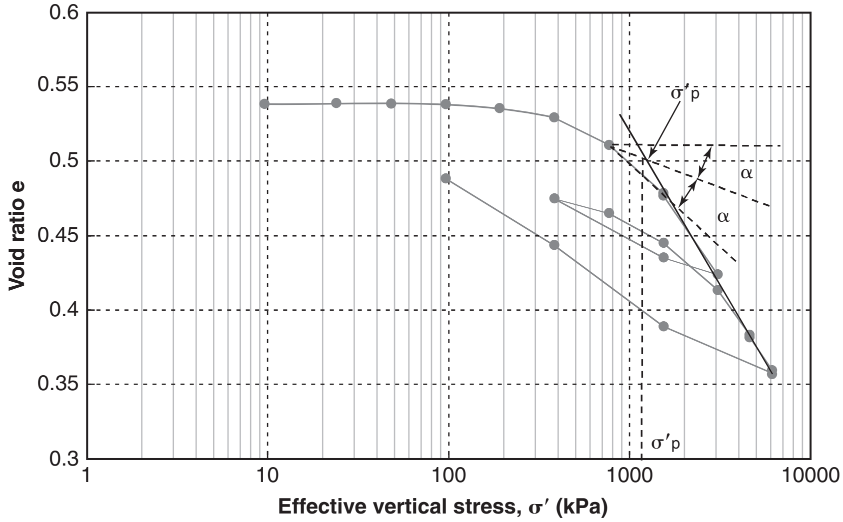


Figure 9.6s Consolidation test curve.

Problem 9.12

Regarding deformation laboratory tests, discuss the differences between tests on saturated soils and tests on unsaturated soils (or, more precisely, tests on soils where the water is in compression and tests on soils where the water is in tension).

Solution 9.12

For deformation laboratory tests on saturated and unsaturated soils, the test procedures are generally the same except for the measurement of water and air stress. For unsaturated soils, the effective stress on the sample should be calculated using the expressions:

$$\sigma' = \sigma - \alpha\, u_w \;\text{ or }\; \sigma' = \sigma - \alpha u_w - \beta\, u_a$$

if the air pressure is not zero. In these expressions, σ' is the effective stress, σ is the total stress, α is the water area coefficient, u_w is the water tension stress, β is the air area coefficient, and u_a is the air stress. The water tension stress u_w can be measured by several methods; most often this is done with a tensiometer during the test. The coefficient α can also be estimated as the degree of saturation S. Typically, saturated soils with the water in compression are more compressible than the same soils in the unsaturated state with the water in tension.

Problem 9.13

A direct shear test is performed on a sample of saturated clay. The sample is 25 mm high and 75 mm in diameter. The test cell is inundated such that the water stress is hydrostatic at the beginning of the test.

a. How would you run the test so as to measure the undrained shear strength of the clay?
b. How would you run the test so as to obtain the drained shear strength parameters for the clay?

Solution 9.13

a. The test has to be run quickly enough that the water does not have time to drain.
b. The test has to be run slowly enough that the water stress remains zero.

Problem 9.14

A direct shear test is performed on a sample of dry sand. The sample is 50 mm in diameter and 25 mm high and is subjected to a vertical force of 100 N. At failure, the shear force applied is 60 N, the horizontal movement is 3 mm, and the vertical movement is 0.5 mm. Calculate the shear strength of the sand and the friction angle, and estimate the dilation angle.

Solution 9.14

a. Shear strength of the sand:

$$A = \pi r^2 = 1.96 \times 10^3 \text{ m}^2$$

$$c = \frac{F_s}{A} = \frac{60\ N}{1.96 \times 10^{-3} \text{ m}^2} = 30600 \text{ Pa} = 30.6 \text{ kPa}$$

b. Friction angle of the sand:

$$\varphi = \tan^{-1}\left(\frac{F_s}{F_n}\right) = \tan^{-1}\left(\frac{60}{100}\right) = 31^\circ$$

c. Dilation angle of the sand (Figure 9.7s):

$$\psi = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.5}{3}\right) = 9.5^\circ$$

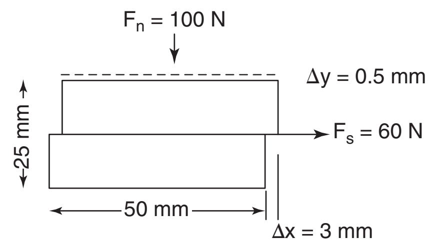


Figure 9.7s Direct shear test.

Problem 9.15

A direct shear test is performed on a sample of saturated clay. The test is a quick test such that water does not have time to drain during the test. The vertical load on the sample induces a total normal stress of 50 kPa and at failure the shear force induces a shear stress of 100 kPa.

a. Calculate the undrained shear strength of the clay.
b. How is it possible for this clay to have such high shear strength, considering the low normal stress?

Solution 9.15

a. The undrained shear strength of the saturated clay is associated with the shear force measured at failure. Therefore, the undrained shear strength is 100 kPa.
b. The shear strength equation is:

$$s = c + \sigma' \tan \varphi$$

We know that at failure the shear stress is 100 kPa, and we also know that the total normal stress is 50 kPa:

$$100 = c + (50 - \alpha u_w) \tan \varphi$$

The explanation is that either there is a lot of cohesion (cementation, for example), or there is a significant amount of tension in the water (highly negative u_w). The latter is more likely.

Problem 9.16

Two direct shear tests are performed on a sample of saturated clay. The tests are slow tests such that the water stress (pore pressure) remains equal to zero.

$$\text{Test 1: N} = 300\text{ N, T} = 250\text{ N, A} = 0.01\text{ m}^2\text{, S} = 100\%\text{, u}_w = 0\text{ kPa}$$

$$\text{Test 2: N} = 600\text{ N, T} = 400\text{ N, A} = 0.01\text{ m}^2\text{, S} = 100\%\text{, u}_w = 0\text{ kPa}$$

where N is the normal force, T is the shear force, A is the sample cross-sectional area, S is the degree of saturation, and u_w is the water stress. Calculate the effective stress cohesion and effective stress friction angle of the clay.

Solution 9.16

The effective stress cohesion and the effective stress friction angle can be calculated from the Mohr circle (Table 9.3s).

$$\sigma = \frac{N}{A} \quad \tau = \frac{T}{A} \quad \sigma' = \sigma - u_w = \sigma$$

Table 9.3s Calculation of Normal and Shear Stress for the Direct Shear Test

Test	Normal Force (N)	Shear Force (N)	Area (m^2)	S (%)	u_w (kPa)	σ' (kPa)	τ (kPa)
1	300	250	0.01	100	0	30	25
2	600	400	0.01	100	0	60	40

Figure 9.8s shows the Mohr-Coulomb envelope for the two sample tests. Because the pore water pressure remains zero, the intersection of the envelope with the y_{axis} represents the effective cohesion of the soil (c′) and the slope of the envelope represents the effective friction angle of the soil (φ'). The plot shows a c′ and a φ' of 10 kPa and 26.6 degrees, respectively. The friction angle can also be calculated as:

$$\varphi' = \tan^{-1}\left(\frac{\Delta\tau}{\Delta\sigma'}\right) = \tan^{-1}\left(\frac{\tau_2 - \tau_1}{\sigma'_2 - \sigma'_1}\right) = \tan^{-1}\left(\frac{40 - 25}{60 - 30}\right) = 26.6^{\circ}$$

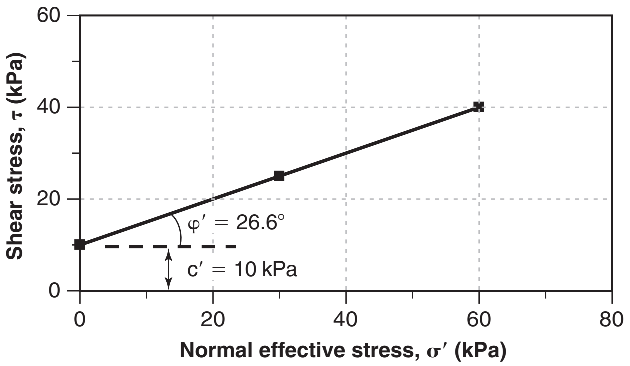


Figure 9.8s Shear stress and effective stress diagram.

Problem 9.17

For strength laboratory tests, discuss the differences between tests on saturated soils and tests on unsaturated soils (or, more precisely, tests on soils where the water is in compression and tests on soils where the water is in tension).

Solution 9.17

1. *Direct shear test*: When the soil is saturated (soil sample is inundated) there is no suction in the soil and the sample can be tested at low or high shearing rate. The high rate provides the undrained parameter of the soil and the low rate provides the drained parameters of the soil. If the soil is unsaturated, or if the soil is saturated but the water in the voids is in tension, then the direct shear test requires measurement of the water tension stress (suction) to obtain the effective stress shear strength parameters c′ and φ'.
2. *Simple shear test*: If the soil is unsaturated, or if it is saturated but the water is in tension, the testing procedure is the same as for the saturated case except for the measurement of the water stress.
3. *Unconfined compression test*: The test procedure and data reduction are the same for both saturated and unsaturated soils. The water stress is not measured in this test; however, it can be determined from the equations as shown in the text.
4. *Triaxial test*: If the soil is unsaturated, or if it is saturated and the water in the voids is in tension, the test procedure does not change from the saturated case; however, the water and air stress measurements change depending on whether the test is drained or undrained.
5. *Resonant column test*: The test procedure and data reduction are the same for both saturated and unsaturated soils. The water stress is rarely measured in this test.
6. *Lab vane test*: the test procedure and data reduction are the same for both saturated and unsaturated soils. The water stress is not measured in this test.

Problem 9.18

Assume the same conditions as in problem 9.16 but this time the soil is unsaturated and the readings are as follows:

$$\text{Test 1: N} = 600\text{ N, T} = 1900\text{ N, A} = 0.01\text{ m}^2\text{, S} = 60\%\text{, u}_w = -400\text{ kPa}$$

$$\text{Test 2: N} = 200\text{ N, T} = 900\text{ N,}\quad\text{A} = 0.01\text{ m}^2\text{, S} = 40\%\text{, u}_w = -300\text{ kPa}$$

where N is the normal force, T is the shear force, A is the sample cross-sectional area, S is the degree of saturation, and u_w is the water tension stress. Calculate the effective stress cohesion and effective stress friction angle of the clay.

Solution 9.18

For Test 1 (at failure):

$$\sigma = \frac{N}{A} = \frac{600}{0.01} = 60000\text{ Pa} = 60\text{ kPa}$$

$$\sigma' = \sigma - \alpha u_w = \sigma - S \cdot u_w = 60 - 0.6 \times (-400) = 300\text{ kPa}$$

$$\tau = \frac{T}{A} = \frac{1900}{0.01} = 190000\text{ Pa} = 190\text{ kPa}$$

For Test 2 (at failure):

$$\sigma = \frac{N}{A} = \frac{200}{0.01} = 20000\ \text{Pa} = 20\ \text{kPa}$$
$$\sigma' = \sigma - \alpha u_w = \sigma - S \cdot u_w = 20 - 0.4 \times (-300) = 140\ \text{kPa}$$
$$\tau = \frac{T}{A} = \frac{900}{0.01} = 90000\ \text{Pa} = 90\ \text{kPa}$$

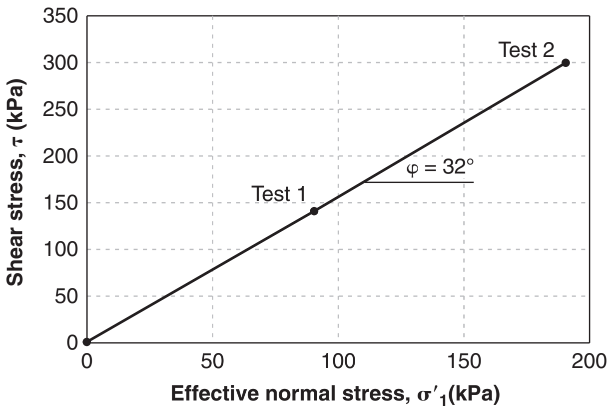


Figure 9.9s Failure envelope for the direct shear test on unsaturated soil.

The effective stress cohesion c' and effective stress friction angle φ' can be calculated by using the plotted curve of effective normal stress vs. shear stress:

$$\tan \phi' = \frac{\tau_1 - \tau_2}{\sigma_1' - \sigma_2'} = \frac{190 - 90}{300 - 140} = 0.625$$

Therefore, the effective friction angle φ' is 32°.

The effective stress cohesion can be calculated using the following equation:

$$\tau = \sigma' \cdot \tan \phi' + c'$$
$$90 = 140 \times \tan 32^\circ + c'$$

Therefore, the effective stress cohesion c' is 2.5 kPa.

Problem 9.19

What are the differences between the direct shear test and the simple shear test? Explain your answers.

Solution 9.19

In the direct shear test, the shearing takes place along a predetermined thin band of soil near the middle of the sample, whereas in the simple shear test the shearing takes places over the entire height of the sample. The shear strength can be obtained from both tests (including the strength parameters c′ and φ'). However, the simple shear test has the advantage of giving the shear modulus G as a function of shear strain in addition to the shear strength of the soil sample.

Problem 9.20

A simple shear test is performed on a sample of silt. The sample is 50 mm in diameter and 20 mm high. When the shear force applied is 200 N, the horizontal displacement of the top of the sample is 0.2 mm. Calculate the shear stress, the shear strain, and the shear modulus of the sample at that point on the stress-strain curve.

Solution 9.20

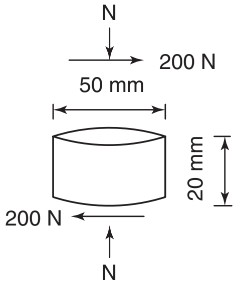


Figure 9.10s Silt sample.

Shear strain is:

$$\gamma = \frac{\Delta x}{h} = \frac{0.2\ \text{mm}}{20\ \text{mm}} = 0.01 = 1\%$$

Shear stress is:

$$\tau = \frac{F}{A} = \frac{200}{\frac{\pi}{4} \times (50 \times 10^{-3})^2} = 1.02 \times 10^5\ \text{Pa} = 102\ \text{kPa}$$

Assuming that τ and γ follow a linear relationship, then the shear modulus of the sample at that point is calculated:

$$G = \frac{\tau}{\gamma} = \frac{102\ \text{kPa}}{0.01} = 10.2\ \text{MPa}$$

Problem 9.21

An unconfined compression (UC) test on a sample of clay gives the stress-strain curve shown in Figure 9.11s. Calculate the undrained shear strength and the UC modulus for this sample. What geotechnical problem do you think this undrained shear strength and this modulus could be used for?

Solution 9.21

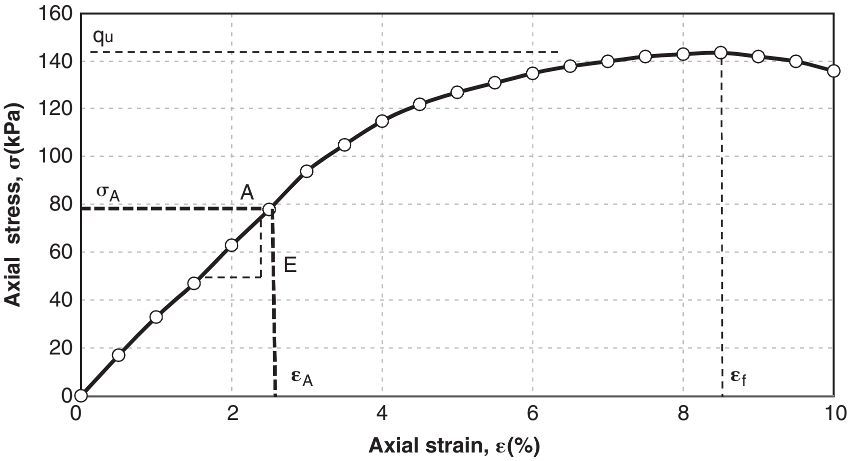


Figure 9.11s Stress-strain curve for unconfined compression test on clay.

From Figure 9.11s, we can determine that the maximum axial stress q_u is 144 kPa. The axial strain at failure is 8.5%. The undrained shear strength is half of the maximum axial stress: $s_u = \frac{q_u}{2} = \frac{144}{2}\text{kPa} = 72$ kPa. UC modulus can be obtained from the straight portion of the stress-strain curve for this sample. The following equation shows the result of UC modulus.

$$E = \frac{\sigma_A}{\varepsilon_A} = \frac{78\ \text{kPa}}{2.5\%} = 3120\ \text{kPa} = 3.12\ \text{MPa}$$

The results from the unconfined compression test can be used to estimate the short-term bearing capacity of fine-grained soils for foundation, estimate the short-term stability of slopes, and determine the stress-strain characteristics of a soil under fast (undrained) loading conditions.

Problem 9.22

What are the two main phases in running a triaxial test? With respect to drainage during each one of these two phases, what are the different types of tests that can be run? For each type of test, what parameters can you obtain from the data?

Solution 9.22

The two main phases in running a triaxial test are the consolidation phase and the shearing phase.

With respect to drainage during the two phases of the triaxial test, there are three different types of tests: UU test (unconsolidated undrained test), CU test (consolidated undrained test), and CD test (consolidated drained test).

From a UU test, we can obtain an undrained shear strength, the strain at failure, and undrained deformation moduli at different strains.

From a CU test with water stress measurement, we can obtain effective stress shear strength parameters: friction angle φ' and cohesion c$'$, and consolidated undrained deformation moduli at different strains.

From a CD test, we can obtain effective stress shear strength parameters: friction angle φ' and cohesion c$'$, and drained deformation moduli at different strains.

Problem 9.23

A triaxial test is performed on a sample that is 50 mm in diameter and 100 mm high. The confining pressure is 30 kPa and at failure the vertical load on the sample is 118 N. Is the vertical total stress on the sample at failure expressed in N/m^2 equal to $\frac{118\,\text{N}}{\pi(25.10^{-3})^2}$? If yes, explain your answer. If not, what is it?

Solution 9.23

The vertical total stress on the sample at failure expressed in N/m^2 is NOT equal to $\frac{118\ \text{N}}{\pi(25\times10^{-3})^2\ \text{m}^2}$. The confining pressure also contributes to the vertical total stress. Therefore, the vertical total stress on the sample at failure is equal to 30 kPa $+ \frac{0.118\ \text{kN}}{\pi(25\times10^{-3})^2\ \text{m}^2} = 90$ kPa. This answer does not consider the change in sample cross section during loading.

Problem 9.24

A triaxial test with water stress (pore-pressure) measurements is performed on a sample of saturated silty crushed rockfill and gives the results shown in Figure 9.50. The total confining stress is 35 kPa.

a. Calculate the total stress secant modulus E and the effective stress secant modulus E$'$ for a vertical strain equal to 0.2%, 0.5%, 1%, 2%, 3%, 4%, and 5%. Then plot the curve giving soil modulus both as a function of strain and as function of log strain.
b. At failure, the vertical effective stress is 100 kPa. Calculate the effective stress friction angle of the sand if the effective stress cohesion c is zero.

Solution 9.24

In the following calculation, Poisson's ratio is assumed to be 0.5 for the undrained modulus and 0.35 for the effective stress modulus. Figure 9.12s shows the soil sample in a triaxial test.

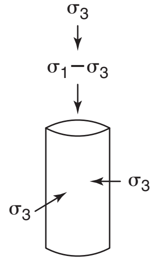


Figure 9.12s Illustration of soil sample in triaxial test.

From Figure 9.13s, we can determine that the deviator stresses $\sigma_1 - \sigma_3$ at the axial strains of 0.2%, 0.5%, 1%, 2%, 3%, 4% and 5% are 14 kPa, 40 kPa, 75 kPa, 145 kPa, 152 kPa, 142 kPa, and 131 kPa respectively. Given that $\sigma_3 = 35$ kPa, we can get both σ_1 and σ_3 for all the strain levels. The total stress secant modulus E can be obtained.

$$E = (\sigma_1 - 2\nu\sigma_3)/\varepsilon$$

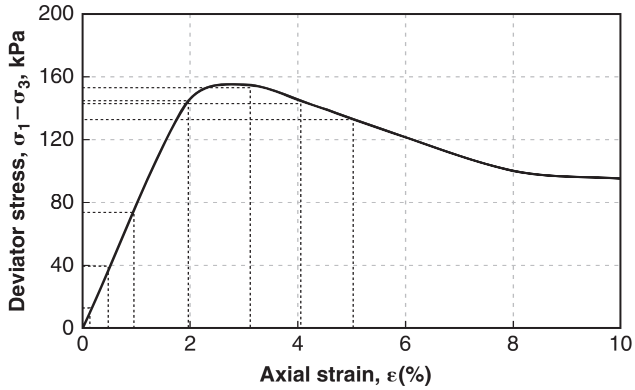


Figure 9.13s Deviator stress curve related to axial strain.

The results are shown in Table 9.4s. In Figure 9.14s, the water pressures can be read as 2 kPa, 5 kPa, 9 kPa, 11 kPa, 8 kPa, 4 kPa, and −2 kPa when the axial strain is 0.2%, 0.5%, 1%, 2%, 3%, 4%, and 5% respectively.

Table 9.4s Results of Calculation of Soil Modulus

ε	σ_3 (kPa)	$\sigma_1 - \sigma_3$ (kPa)	σ_1 (kPa)	E (MPa)	u (kPa)	σ_3' (kPa)	σ_1' (kPa)	E′(MPa)
0.2%	35	14	49	7.0	2	33	47	11.9
0.5%	35	40	75	8.0	5	30	70	9.8
1%	35	75	110	7.5	9	26	101	8.3
2%	35	145	180	7.2	11	24	169	7.6
3%	35	152	187	5.1	8	27	179	5.3
4%	35	142	177	3.5	4	31	173	3.8
5%	35	131	166	2.6	−2	37	168	2.8

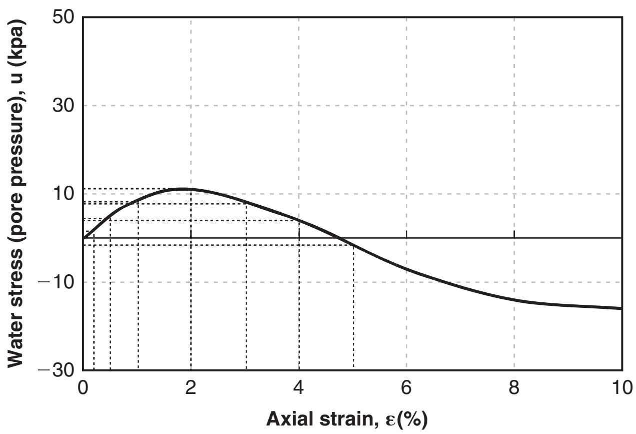


Figure 9.14s Water stress curve related to axial strain.

The effective stress can be calculated by using the total stress minus the water stress for this saturated soil. The effective stress secant modulus E' can then be obtained.

$$E' = (\sigma_1' - 2\nu'\sigma_3')/\varepsilon$$

The results are shown in Table 9.4s. Figure 9.15s shows the curves of total soil modulus and effective soil modulus as a function of strain, while Figure 9.16s shows the curve of total soil modulus and effective soil modulus as a function of logarithm of strain.

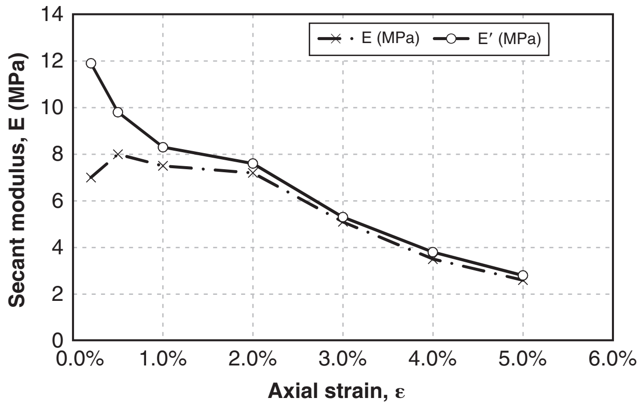


Figure 9.15s Curves of total soil modulus and effective soil modulus as a function of strain.

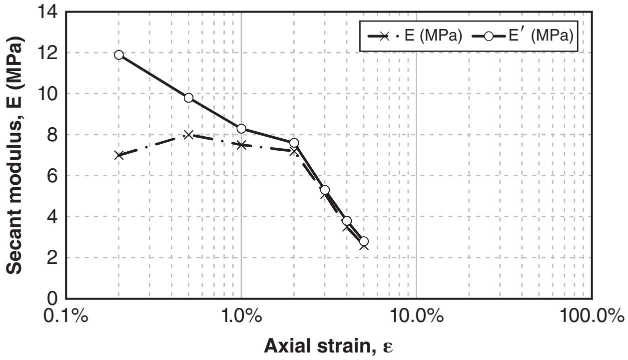


Figure 9.16s Curves of total soil modulus and effective soil modulus as a function of logarithm of strain.

Assuming that the effective cohesion is 0, the effective stress friction angle can be obtained from the Mohr circle using the effective principal stresses at failure (Figure 9.17s): $\varphi' = 47.5^\circ$

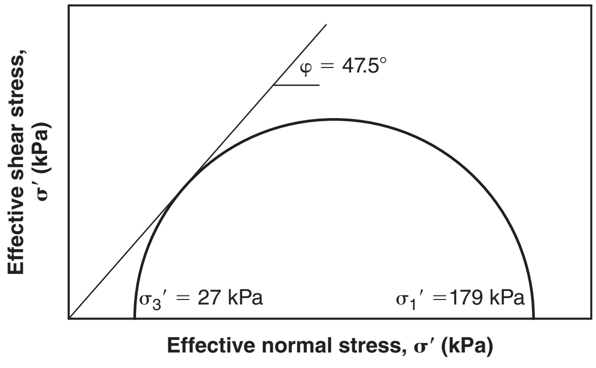


Figure 9.17s Mohr circle and friction angle.

Problem 9.25

Two CU triaxial tests are performed on a sample of saturated, overconsolidated, high-plasticity clay. At failure, the results are as follows:

$$\text{Test 1}: \sigma_3 = 30\text{ kPa, Q} = 0.45\text{ kN, A} = 0.01\text{ m}^2\text{, u} = 10\text{ kPa}$$

$$\text{Test 2}: \sigma_3 = 60\text{ kPa, Q} = 0.70\text{ kN, A} = 0.01\text{ m}^2\text{, u} = 20\text{ kPa}$$

where σ_3 is the total confinement stress, Q is the vertical load on the sample, A is the sample cross section, and u is the water stress (pore pressure).

a. Calculate σ_3, σ_1, σ_3', and σ_1' at failure.
b. Draw the Mohr circle at failure in the τ vs. σ' set of axes.
c. Draw the failure envelope and find the effective stress strength parameters c$'$ and φ'.

Solution 9.25

a. At failure, the stresses σ_3, σ_1, σ_3', and σ_1' are calculated as follows:

$$\sigma_1 = \sigma_3 + \frac{Q}{A} = 30 + \frac{0.45}{0.01} = 75 \text{ kPa}$$
$$\sigma_3 = 30 \text{ kPa}$$
$$\sigma_1' = \sigma_1 - \alpha u = 75 - 1 * 10 = 65 \text{ kPa}$$
$$\sigma_3' = \sigma_3 - \alpha u = 30 - 1 * 10 = 20 \text{ kPa}$$

Test 2:

$$\sigma_1 = \sigma_3 + \frac{Q}{A} = 60 + \frac{.70}{.01} = 130 \text{ kPa}$$
$$\sigma_3 = 60 \text{ kPa}$$
$$\sigma_1' = \sigma_1 - \alpha u = 130 - 1 * 20 = 110 \text{ kPa}$$
$$\sigma_3' = \sigma_3 - \alpha u = 60 - 1 * 20 = 40 \text{ kPa}$$

b. The Mohr circle at failure is shown in Figure 9.18s.

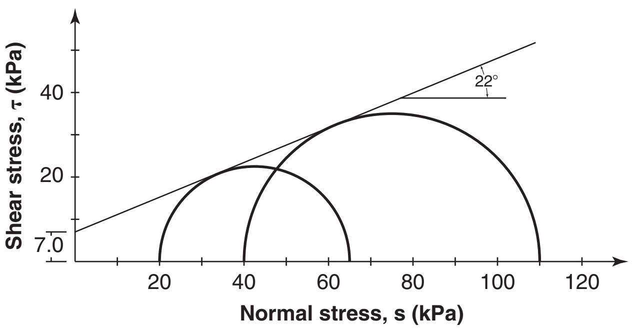


Figure 9.18s Mohr circle.

c. From the Mohr circle (Figure 9.18s), the cohesion c$' = 7$ kPa and the friction angle $\varphi' = 22^\circ$.

Problem 9.26

Two CU triaxial tests are performed on a sample of unsaturated clay. At failure, the results are as follows:

$$\text{Test 1: } \sigma_3 = 20 \text{ kPa}, \sigma_1 = 190 \text{ kPa}, \text{S} = 60\%, \text{u}_w = -100 \text{ kPa}$$
$$\text{Test 2: } \sigma_3 = 60 \text{ kPa}, \sigma_1 = 450 \text{ kPa}, \text{S} = 50\%, \text{u}_w = -300 \text{ kPa}$$

where σ_3 is the total confinement stress, σ_1 is the total vertical stress at failure, S is the degree of saturation, and u_w is the water stress.

a. Calculate σ_3, σ_1, σ_3', and σ_1' at failure.
b. Draw the Mohr circle at failure in the τ vs. σ' set of axes.
c. Draw the failure envelope and find the effective stress strength parameters c$'$ and φ'.

Solution 9.26

a. Calculate σ_3, σ_1, σ_3', and σ_1' at failure.
For Test 1 (at failure):

$$\sigma_3 = 20\ \text{kPa}$$
$$\sigma_3' = \sigma_3 - \alpha u_w = \sigma_3 - S \cdot u_w = 20 - 0.6 \times (-100) = 80\ \text{kPa}$$
$$\sigma_1 = 190\ \text{kPa}$$
$$\sigma_1' = \sigma_1 - \alpha u_w = \sigma_1 - S \cdot u_w = 190 - 0.6 \times (-100) = 250\ \text{kPa}$$

For Test 2 (at failure):

$$\sigma_3 = 60\ \text{kPa}$$
$$\sigma_3' = \sigma_3 - \alpha u_w = \sigma_3 - S \cdot u_w = 60 - 0.5 \times (-300) = 210\ \text{kPa}$$
$$\sigma_1 = 490\ \text{kPa}$$
$$\sigma_1' = \sigma_1 - \alpha u_w = \sigma_1 - S \cdot u_w = 450 - 0.5 \times (-300) = 600\ \text{kPa}$$

b & c. Figure 9.19s shows the Mohr circle and failure envelope for the test. From the Mohr circle and failure envelope, we can get the value of c′ and φ' as 10 kPa and 27° respectively.

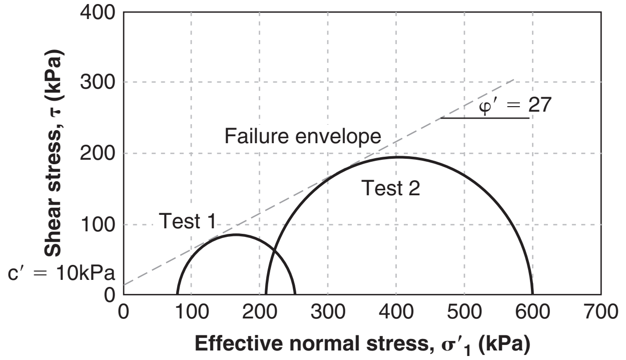


Figure 9.19s Mohr circle and failure envelope for the test.

Problem 9.27

What is the stress path and what shape does it typically have for the triaxial test?

Solution 9.27 (Figure 9.20s)

The stress path describes the evolution of certain stresses during the test. Specifically, it tracks the path described by the points with p, q stress coordinates where p and q are defined as follows:

Total stress

$$p = (\sigma_1 + \sigma_3)/2$$
$$q = (\sigma_1 - \sigma_3)/2$$

Effective stress

$$p' = (\sigma_1' + \sigma_3')/2$$
$$q' = (\sigma_1' - \sigma_3')/2$$

The most useful stress paths are plotted in terms of effective stresses (p′ and q′).

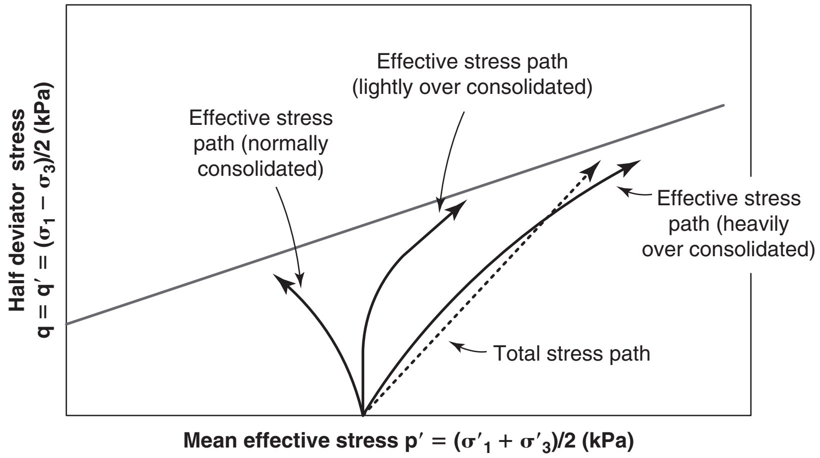


Figure 9.20s Stress paths.

Problem 9.28

A lab vane test is performed on a silty clay. At failure, the maximum torque is 5.7 N.m. The vane is 50 mm high and 25 mm in diameter. Calculate the undrained shear strength of the silty clay. The vane is rotated 10 times rapidly and the torque on the tenth revolution is measured to be 3.5 N.m. Calculate the residual undrained shear strength of the silty clay.

Solution 9.28

$$T = \pi s_u D^2 \left(\frac{H}{2} + \frac{D}{6}\right) \Rightarrow s_u = \frac{T}{\pi D^2 \left(\frac{H}{2} + \frac{D}{6}\right)}$$

$$T_{Max} = 5.7 \text{ N.m}$$

$$T_{res} = 3.5 \text{ N.m}$$

$$H = 0.05 \text{ m}$$

$$D = 0.025 \text{ m}$$

$$s_u = \frac{5.7}{\pi 0.025^2 \left(\frac{0.05}{2} + \frac{0.025}{6}\right)} = 99531 \text{ N/m}^2 \approx 100 \text{ kPa}$$

$$s_{ur} = \frac{3.5}{\pi 0.025^2 \left(\frac{0.05}{2} + \frac{0.025}{6}\right)} = 61115 \text{ N/m}^2 \approx 61 \text{ kPa}$$

Problem 9.29

A silty sand is subjected to a constant head permeameter test. The flow collected at the downstream end is 221 mm^3/s; the sample is 75 mm in diameter and 100 mm high. The difference between the water level in the upstream overflow and the downstream overflow is 0.5 m. Calculate the hydraulic conductivity k of the silty sand.

Solution 9.29

Given in the problem statement:

$$\text{Q} = 221 \text{ mm}^3/\text{s}$$

$$\text{A} = \pi \text{d}^2/4 = \pi (75 \text{ mm})^2/4 = 4417.8 \text{ mm}^2$$

$$\text{i} = \text{h/l} = 0.5 \text{ m}/0.1 \text{ m} = 5$$

Hydraulic conductivity:

$$k = (\mathrm{Q\,l})/(\mathrm{A\,h}) = 221 \times 0.1/(4417.8 \times 0.5) = 0.01\ \mathrm{mm/s}$$
$$k = 0.01\ \mathrm{mm/s} = 1.0 \times 10^{-5}\ \mathrm{m/s}$$

Problem 9.30

A clay sample is tested in a falling head permeameter. The sample is 75 mm in diameter and 100 mm high. The small tube is 3 mm in diameter. The difference in height between the water level in the small tube above the sample and the downstream overflow is measured as a function of time. At time $t = 0$, the difference is 1.1 m and at time $t = 1$ hr, the difference is 1.05 m. Calculate the hydraulic conductivity k of the clay.

Solution 9.30

Given in the problem statement:

Inner tube diameter, d = 3 mm
Sample diameter, D = 75 mm
Sample height = 100 mm

From test results:

$$\Delta t = 60\ \mathrm{min} = 3600\ \mathrm{s}$$
$$h_0 = 1100\ \mathrm{mm}$$
$$h_1 = 1050\ \mathrm{mm}$$

Calculations:

$$a = \pi \mathrm{d}^2/4 = \pi(3)^2/4 = 7.06\ \mathrm{mm}^2$$
$$A = \pi \mathrm{D}^2/4 = \pi(75)^2/4 = 4417.7\ \mathrm{mm}^2$$

Hydraulic conductivity:

$$k = 2.3\frac{al}{At}\log\frac{h_0}{h_1} = 2.3\frac{7.06 \times 100}{4417.7 \times 60 \times 60}\log\frac{1100}{1050} = 2.06 \times 10^{-6}\ \mathrm{mm/s} = 2.06 \times 10^{-9}\ \mathrm{m/s}$$

Problem 9.31

This problem refers to Figure 9.71. A sample of unsaturated silt is tested in a constant head permeameter and the following parameters are measured: $D = 75$ mm, l = 150 mm, V = 10 cm^3, $t = 1$ hour, $h_{p2} = -100$ mm, $h_{p3} = -200$ mm, $h_{p4} = 0$ mm. Calculate the unsaturated hydraulic conductivity k of the silt and the water tension u_w corresponding to that value.

Solution 9.31

$$k = \frac{4V}{t\pi D^2}\frac{1}{(h_{t2} - h_{t3})}$$
$$h_{t2} - h_{t3} = |h_{p3}| + l - |h_{p2}| = 200 + 150 - 100 = 250\ \mathrm{mm}$$
$$k = \frac{4 \times 10 \times 10^3\ \mathrm{mm}^3}{60 \times 60 \times \pi \times 75^2\ \mathrm{mm}^2}\left(\frac{150}{250}\right)$$
$$= 3.77 \times 10^{-4}\frac{\mathrm{mm}}{s} = 3.77 \times 10^{-7}\frac{\mathrm{m}}{s}$$
$$u_w = \frac{h_{p2} + h_{p3}}{2}\gamma_w$$
$$u_w = \left(\frac{-100 + (-200)}{2}\right)10^{-3} \times 9.81 = -1.47\ \mathrm{kPa}$$

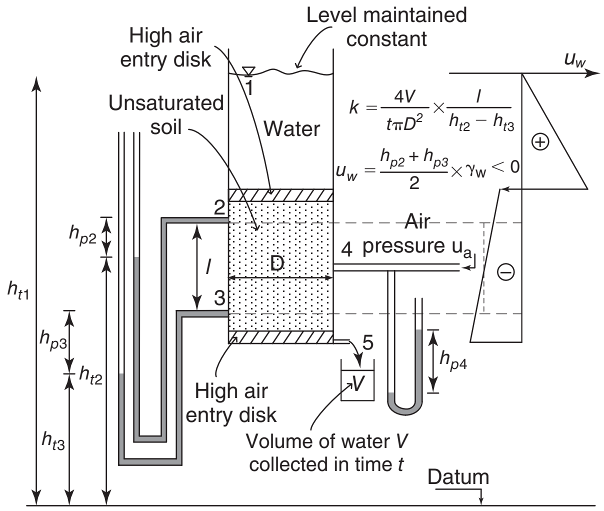


Figure 9.21s Constant head permeameter test for unsaturated soils.

Problem 9.32

This problem refers to Figure 9.75. A 0.65 m long 75 mm diameter sample of unsaturated clay is tested in a wetting front permeameter. Initially the water tension in the sample is −1000 kPa. The results are shown in Figure 9.22s. Use the results to develop the hydraulic conductivity k vs. water tension u_w curve for this clay.

Solution 9.32 (Figure 9.22s)

The hydraulic conductivity k is the ratio of water velocity v_w to the hydraulic gradient i, $k = v_w/\mathrm{i}$. To develop the hydraulic conductivity k vs. water tension u_w curve, v_w and i are calculated from the plot on Figure 9.22s as follows.

The water velocity v_w is equal to the volume of water $d\mathrm{V}_w$ passing through a point in a soil sample with a cross section A in a time interval dt:

$$v_w = \frac{d\mathrm{V}_w}{dt} \times \frac{1}{A}$$

The volume of water that passes through the soil sample in a time interval $dt = t_2 - t_1$ is:

$$dV_w = V_{w,t_2} - V_{w,t_1}$$

where $V_{w,\mathrm{ti}}$ is the volume of water present in the soil sample at time $\mathrm{t_i}$ between the distance x_J and the end of the sample. This volume can be calculated from the volumetric water content vs. distance curve and is equal to the area below the curve at a given time multiplied by the sample cross section A:

$$\mathrm{V}_w \text{ (between point J and end of sample)} = \int_{x_j}^{L} \theta_w(x) A dx$$

where $\theta_w = V_w/V$ is the volumetric water content and can be derived from the water tension as follows:

$$\theta_w = C_w \log|u_w| + a$$

where $u_w = \gamma_w \mathrm{h}$ is the water tension in the soil and can be calculated from the pressure head vs. distance graph.

Let's call "a" the difference in area between two curves corresponding to two different times on the θ_w vs. distance diagrams. Then $d\mathrm{V}_w$ is given as:

$$\mathrm{dV}_w = a \times A$$

The water velocity is then:

$$v_w = \frac{d\mathrm{V}_w}{dt} \times \frac{1}{A} = \frac{a \times A}{\Delta t} \times \frac{1}{A} = \frac{a}{\Delta t}$$

For each time interval, the hydraulic gradient is simply calculated from the pressure head vs. distance graph as follows:

$$\mathrm{i}_{\mathrm{avg}} = \frac{\Delta h}{\Delta x} = \frac{h_{x=0.6} - h_{x=0.0}}{0.60 - 0}$$

The hydraulic conductivity is then calculated as follows:

$$k = \frac{v_w}{i_{avg}} = \frac{a}{\Delta t} \times \frac{1}{i_{avg}}$$

Because the pressure head is variable through the section, the water tension u_w can be calculated as an average value in the soil sample for a given time from the pressure head vs. distance graph.

The following is an example of the calculations leading to one point on the k vs. u_w graph. This point is the one corresponding to the time interval $t = 300$ hr to 400 hr.

1. Calculation of average hydraulic gradient:

$$\mathrm{i}_{\mathrm{t=400}} = \frac{\Delta h}{\Delta x} = \frac{h_{x=0.60} - h_{x=0.0}}{0.60 - 0} = \frac{-190 - 0}{0.6 - 0} = -316$$

$$\mathrm{i}_{\mathrm{t=300}} = \frac{\Delta h}{\Delta x} = \frac{h_{x=0.6} - h_{x=0.0}}{0.6 - 0} = \frac{-420 - 0}{0.6 - 0} = -700$$

$$\mathrm{i}_{\mathrm{avg}} = \frac{\mathrm{i}_{\mathrm{t=400}} + \mathrm{i}_{\mathrm{t=300}}}{2} = -508$$

2. Calculation of "a":

The area S_1 below the $t = 400$ curve (assuming it is trapezoidal) is equal to:

$$S_1 = \frac{(0.12 + 0.062) \times (0.6 - 0)}{2} = 0.0546$$

The area S_1 below the $t = 300$ curve (assuming it is trapezoidal) is equal to:

$$S_2 = \frac{(0.11 + 0.056) \times (0.6 - 0)}{2} = 0.0498$$

$$\mathrm{a} = S_1 - S_2 = 4.8 \times 10^{-3}$$

3. Calculation of water velocity:

$$v_w = \frac{a \times L}{\Delta t} = \frac{4.8 \times 10^{-3} \times 0.6}{(400 - 300) \times 3600} = 8 \times 10^{-9}\ \mathrm{m/s}$$

4. The hydraulic conductivity k is equal to:

$$k = \frac{v_w}{i_{avg}} = \frac{8 \times 10^{-9}}{508} = 1.6 \times 10^{-11}\ \mathrm{m/s}$$

5. The average water tension corresponding to the hydraulic conductivity calculated in step 4 is:

$$u_{w,\mathrm{avg}} = \sum_{i=1}^{7} \frac{\gamma_w \mathrm{h_i}}{7} = \frac{9.81}{7} \times (0 - 20 - 60 - 100 - 150 - 170 - 190) = 967\ \mathrm{kPa}$$

More refined calculations can be done by using the actual pressure head vs. distance curve rather than a straight line and curved areas rather than trapezoidal shape assumptions for the volumetric water content vs. distance diagram.

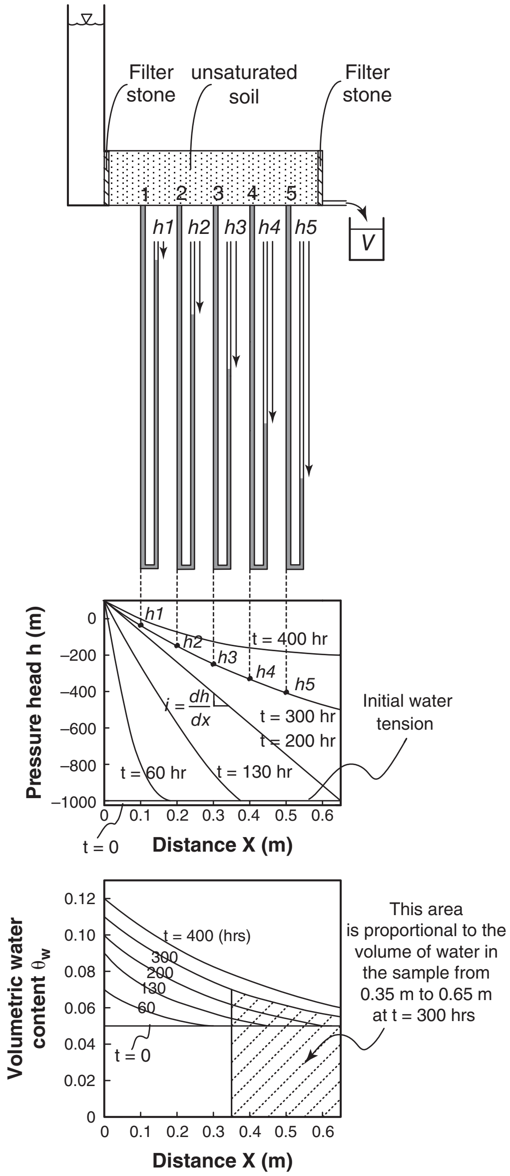


Figure 9.22s Wetting front test for unsaturated soils.

The hydraulic conductivity k vs. the water tension curve is shown in Figure 9.23s.

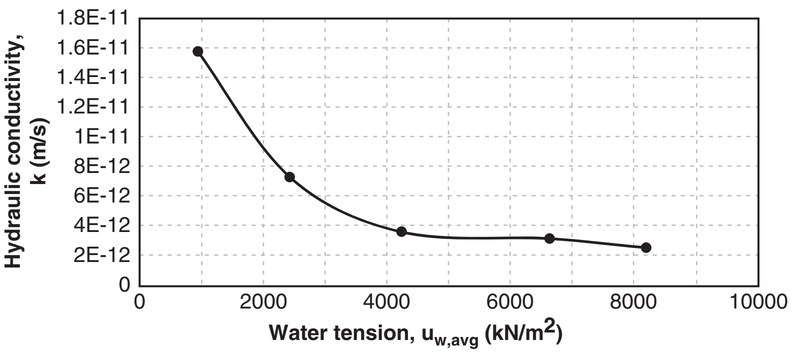


Figure 9.23s Hydraulic conductivity vs. water tension.

Problem 9.33

This problem refers to Figure 9.76. A sample of unsaturated clayey sand has a degree of saturation of 40%. The sample length is 150 mm and the sample diameter is 75 mm. It is tested in a permeameter to determine the hydraulic conductivity of air through the sample. The air pressure at the base of the sample is 10 kPa and the volume of air collected at the top of the sample in one hour of testing is 10^{-3} m^3. The top of the sample is kept at atmospheric pressure. Calculate the air hydraulic conductivity of the sample k_a and the air stress u_a associated with this hydraulic conductivity value.

Solution 9.33

Data:

$$\begin{aligned}
\gamma_{\mathrm{a}} &= 0.0118\ \mathrm{kN/m^3}\ \text{(assumed at } 20^{\circ}\mathrm{C)} \\
\mathrm{V_a} &= 10^{-3}\ \mathrm{m^3} \\
\mathrm{L} &= 0.15\ \mathrm{m} \\
\mathrm{D} &= 0.075\ \mathrm{m} \\
\mathrm{p_a} &= 10\ \mathrm{kPa} \\
\mathrm{t} &= 1\mathrm{hr} = 3600\ \ \mathrm{sec}
\end{aligned}$$

a. The permeability of the soil to air can be calculated as:

$$k_a = \frac{\gamma_a V_a L}{Apt}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.075)^2}{4} = 0.004418\ \mathrm{m}^2$$

$$k_a = \frac{\gamma_a V_a L}{Apt} = \frac{(0.0118\ \mathrm{kN/m^3}) \times (1 \times 10^{-3}\ \mathrm{m^3}) \times (0.15\ \mathrm{m})}{(0.0044818\ \mathrm{m^2}) \times (10\ \mathrm{kN/m^2}) \times (3600\ \ \mathrm{sec}.)} = 1.1 \times 10^{-8}\ \mathrm{m/\,sec}\,.$$

b. The air stress associated with the permeability k_a can be calculated as:

The air stress at the top of the sample is zero. The air stress at the bottom of the sample is 10 kPa. Thus, the average air stress in the sample associated with the measure of air hydraulic conductivity is $u_a = 0.5\ (0 + 10) = 5$ kPa.

Problem 9.34

A 1.8 m tall human being drinks one liter of water. Three hours later, this person goes to the bathroom and eliminates the liter of water. Is this case a constant head permeameter or a falling head permeameter? Calculate the hydraulic conductivity of the human body. Make reasonable assumptions when necessary.

Solution 9.34

This is a case of a falling head permeameter because the water level goes down with time. The equivalent hydraulic conductivity of the human body can be estimated using the following equation:

$$k = 2.3\frac{al}{At}\log\frac{h_0}{h_1}$$

Assumptions:

$$A = a;\ l = 0.21\ \mathrm{m};\ t = 2\ \mathrm{hrs};\ h_o = 1.3\ \mathrm{m};\ h_1 = 1\ \mathrm{m}$$

$$k = 2.3\frac{0.21}{2 \times 60 \times 60}\log\frac{1.3}{1} = 7.64 \times 10^{-6}\ \mathrm{m/s}$$

Problem 9.35

A sample of fine sand is tested in the EFA. The mean diameter of the grains is $D_{50} = 1$ mm. When the velocity is set at 1 m/s, the piston below the sample of sand has to be raised at a rate of 16.7 mm/minute. The cross section of the conduit where the water is flowing is rectangular, with a width of 100 mm and a height of 50 mm. Calculate the shear stress at the interface between the water and the sand for the 1 m/s velocity.

Solution 9.35

$$\tau = \frac{1}{8}\rho f v^2$$

Given in the problem statement are:

Flow velocity: $v = 1.0\ \mathrm{m/s}$
Mass density of water: $\rho = 1000\ \mathrm{kg/m^3}$
Mean grain size: $D_{50} = 1$ mm
Dimensions of test section = 50 mm by 100 mm
Hydraulic diameter, $D = 2ab/(a + b) = 2\ (50 \times 100)/(50 + 100) = 66.7$ mm = 0.0667 m
Viscosity of water: $\upsilon = 1.12 \times 10^{-6}\ \mathrm{m^2/s}$
Calculate friction factor, f, from Moody Chart
f is a function of roughness, ε, and Reynolds Number, Re
Roughness: $\varepsilon = D_{50}/2 = 1\ \mathrm{mm}/2 = 0.5$ mm
Reynolds Number: $Re = Dv/\upsilon = 0.0667 \times 1/1.12 \times 10^{-6} = 59553$
Friction factor read on Moody Chart: f = 0.032
Shear stress: $\tau = 1/8 \times 1000 \times 0.032 \times 1^2 = 4$ Pa

Problem 9.36

A sample of low-plasticity clay is tested in the EFA. The surface of the clay sample is considered smooth. When the velocity is set at 3 m/s, the piston below the sample of sand has to be raised at a rate of 1 mm every 3 minutes. The cross section of the conduit where the water is flowing is rectangular, with a width of 100 mm and a height of 50 mm. Calculate the shear stress at the interface between the water and the sand for the 3 m/s velocity.

Solution 9.36

$$\tau = \frac{1}{8}\rho f v^2$$

Given in problem statement:

Flow velocity: $v = 3.0\ \mathrm{m/s}$
Mass density: $\rho = 1000\ \mathrm{kg/m^3}$
Mean grain size: $D_{50} = 0$ mm—Smooth
Dimensions of test section = 50 mm by 100 mm
Hydraulic diameter, $D = 2ab/(a + b) = 2\ (50 \times 10)/(50 + 100) = 66.7$ mm = 0.0667 m
Viscosity of water: $\upsilon = 1.12 \times 10^{-6}\ \mathrm{m^2/s}$
Friction factor f is read on Moody Chart
f is a function of roughness, ε, and Reynolds Number, Re
Roughness is zero (smooth)
Reynolds Number: $Re = Dv/\upsilon = 0.0667 \times 3/1.12 \times 10^{-6} = 178660$
So, friction factor is read as f = 0.016
Shear stress: $\tau = 1/8 \times 1000 \times 0.016 \times 3^2 = 18$ Pa