CHAPTER 14

Deformation Properties

Two of the major reasons for soil to deform are mechanical loading (settlement or rebound) and environmental changes (water content, temperature, chemistry). Deformations due to loading are governed by an appropriate stress-strain curve of the soil, and deformations due to environmental changes by corresponding constitutive relationships. For shrinking and swelling, for example, those relationships would be the water content-strain curve or the water tension-strain curve.

The choice of an appropriate laboratory test or in situ test and appropriate parameters to solve a particular deformation problem in the field is not easy. One of the important concepts in making that choice is to favor a laboratory test or an in situ test that duplicates the deformation condition of the soil in the field at the element level or the global level. For example, if the problem is a wide embankment over a comparatively thin layer of compressible soil, the consolidation test makes sense and is often used. Although the rigid vertical boundaries in the consolidation test do not exist in the field, the confinement created by the friction between the thin clay layer and the stronger top and bottom layers minimize the lateral movement, just as the consolidation ring does. The remaining difference leads to the need for a correction factor in the calculations. As another example, if the problem is a foundation over a deep deposit, the lateral squeezing of the soil is well represented by the horizontal deformation around the pressuremeter; thus, the pressuremeter is well suited to predicting the settlement in such a case. This approach also requires correction factors to compensate for the lack of complete correspondence.

There are several ways to quantify the deformation characteristics of a soil. One of the simplest is through a modulus of deformation.

14.1 MODULUS OF DEFORMATION: GENERAL

The shape of the stress-strain curve for a soil is typically nonlinear and depends on a number of factors. The early part of that curve can be approximated by a straight line, where the theory of linear elasticity becomes very useful. The slope of that line is related to the modulus of elasticity E (Young's modulus) and to Poisson's ratio ν. E is called *Young's modulus* after Thomas Young, a British physician and physicist who made his contribution around the turn of the 1800s. Poisson's ratio is named after Simeon Poisson, a French mathematician and physicist who lived around the turn of the 1800s and had Lagrange and Laplace as his doctoral advisors at the École Polytechnique in Paris.

Elasticity refers to the ability of a material to regain its original shape when deformed by load. That is not the case with soils, as they experience irrecoverable (plastic) deformations even at low stresses. *Linear elasticity* refers to the fact that the stress-strain curve is linear. That also is not the case with soils, as they exhibit nonlinear behavior very early in the stress-strain curve. Nevertheless, a modulus can be calculated from a soil stress-strain curve by using the secant line from the origin to the point considered on the stress-strain curve (*first load modulus*) or the slope of an unload-reload cycle loop (*cyclic modulus*). Note that the slope of the line calculated as the stress increment divided by the strain increment is not generally the modulus. This is true only if the loading is unconfined, as in an unconfined compression test. The theory of elasticity is presented in section 12.1.1. In the general case, the modulus E is given by applying the elasticity equations (Eqs. 12.1 to 12.3). In the case of a triaxial test (Figure 14.1) for example, the modulus is given by:

$$E = \frac{\sigma_1 - 2\nu\sigma_3}{\varepsilon_1} \tag{14.1}$$

where σ_1 and σ_3 are the major and minor principal stresses, ν is Poisson's ratio, and ε_1 is the major principal strain. As can be seen, the slope s is equal to E only if σ_3 is zero (unconfined compression test).

$$E = \frac{\sigma_1}{\varepsilon_1} \quad \textit{only for unconfined compression test} \tag{14.2}$$

The Poisson's ratio ν is also obtained by applying the elasticity equations (Eqs. 12.1 to 12.3). In the case of the

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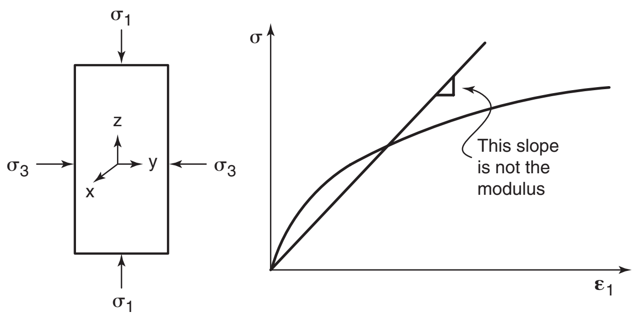

Figure 14.1 Case of a triaxial test.

triaxial test, and if the radial strain ε_3 measurements are made (rare), then:

$$\nu = \frac{-\varepsilon_3\sigma_1 + \varepsilon_1\sigma_3}{\varepsilon_1\sigma_1 + \varepsilon_1\sigma_3 - 2\varepsilon_3\sigma_3} \tag{14.3}$$

If σ_3 is zero (unconfined compression test), the Poisson's ratio is given by.

$$\nu = -\frac{\varepsilon_3}{\varepsilon_1} \quad \textit{only for unconfined compression test} \tag{14.4}$$

The minus sign indicates that when ε_1 is in compression, ε_3 is in tension and the Poisson's ratio is positive.

The stress-strain curve of a soil, and therefore the soil modulus, is influenced by state factors and by loading factors. The state factors include the soil density, the soil structure, the soil water content, the soil stress history, and any cementation between the particles. The loading factors include the stress level, the strain level, the strain rate, the number of cycles, and the drainage conditions. The modulus typically increases when the density increases, when the water content decreases, when the soil has been prestressed by overburden or desiccation, when cementation increases, when the mean stress level increases, when the strain level decreases, when the strain rate increases, when the number of cycles decreases, and when better drainage takes place.

14.2 MODULUS: WHICH ONE?

Because soils do not exhibit a linear stress-strain curve, many moduli can be defined from triaxial test results, for example. In section 14.1, it was pointed out that the slope of the stress-strain curve is not the modulus of the soil. However, the slope of that curve is related to the modulus, and it is convenient to associate the slope of the stress-strain curve with a modulus. This gives a simple image tied to the modulus value; note, however, that in the figures the slope is never labeled as modulus E, but rather as slope S. Referring to Figure 14.2, if the slope is drawn from the origin to a point on the curve (O to A in Figure 14.2), the secant slope S_s is obtained and the secant modulus E_s is calculated from it. One would use such a modulus for predicting the movement due to the first application of a load, as in the case of a spread footing. If the slope is drawn as the tangent to the point considered on the stress-strain curve, then the tangent slope S_t is obtained and the tangent modulus E_t is calculated from it. One would use such a modulus to calculate the incremental movement due to an incremental load, as in the case of the movement due to one more story in a high-rise building. If the slope is drawn as the line joining points A and B in Figure 14.2, then the unloading slope S_u is obtained and the unloading modulus E_u is calculated from it. One would use such a modulus when calculating the heave at the bottom of an excavation or the rebound of a pavement after the loading by a truck tire (*resilient modulus*). If the slope is drawn from point B to point D in Figure 14.2, then the reloading slope S_r is obtained and the reload modulus E_r is calculated from it. One would use this modulus to calculate the movement at the bottom of an excavation if the excavated soil or a building of equal weight is placed back in the excavation, or to calculate the movement of the pavement under reloading by the same truck tire. If the slope is drawn from point B to point C in Figure 14.2, then the cyclic slope S_c is obtained and the cyclic modulus E_c is calculated from it. One would use such a modulus, and its evolution as a function of the number of cycles, to calculate the movement of a pile foundation subjected to repeated wave loading.

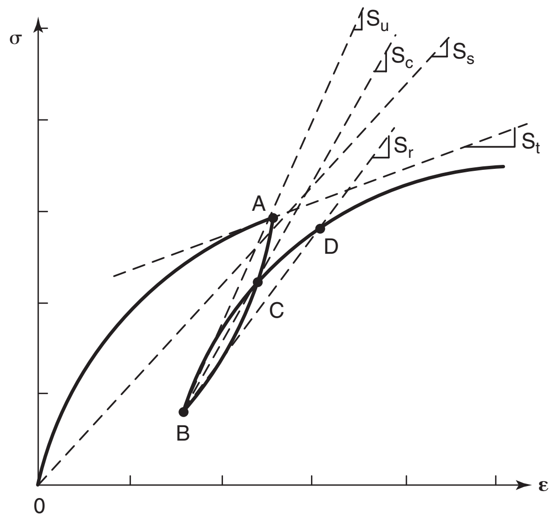

Figure 14.2 Definition of soil modulus.

Regardless of which modulus is defined and considered, the state of the soil at any given time will affect that modulus. Section 14.3 describes some of the main state parameters influencing soil moduli.

14.3 MODULUS: INFLUENCE OF STATE FACTORS

The state factors include particle packing and organization, water content, past stress history, and cementation.

How closely packed are the particles? If the particles are closely packed, the modulus tends to be high. This is measured by the dry density (ratio of the weight of solids over the total volume of the wet sample) of the soil, for example; it can also be measured by the porosity (ratio of the volume of voids over the total volume of the wet sample).

How are the particles organized? This factor refers to the structure of the soil. For example, a coarse-grained soil can have a loose or dense structure; a fine-grained soil can have a dispersed or flocculated structure. Note that two soil samples can have the same dry density yet different structures and therefore different soil moduli. This is why taking a disturbed sample of a coarse-grained soil in the field and reconstituting it to the same dry density and water content in the laboratory can lead to differing laboratory and field moduli.

What is the water content? This parameter has a major impact because at low water contents the water binds the particles (especially for fine-grained soils) and increases the effective stress between the particles through the water tension (suction) phenomenon. Therefore, in this case low water contents lead to high soil moduli. This is why a clay shrinks and becomes very stiff when it dries. At the same time, at very low water contents the compaction of coarse-grained soils is not as efficient as it is at higher water contents, because the lubrication effect of water is not present. Therefore, in this case very low water contents lead to low moduli. As the water content increases, water lubrication increases the effect of compaction and the modulus increases as well. However, if the water content rises beyond an optimum value, the water occupies more and more room and gets to the point where it pushes the particles apart, thereby increasing compressibility and reducing the modulus.

What has the soil been subjected to in the past? This is referred to as the *stress history factor*. If the soil has been prestressed in the past, it is called *overconsolidated*. This prestressing could come, for example, from a glacier that was 100 meters thick 10,000 years ago and has now totally melted. Prestressing can also come from the drying and wetting cycles of the seasons in semiarid parts of the world. If the soil has not been prestressed in the past—in other words, if today's stress is the highest stress ever experienced by the soil—and if the soil is at equilibrium under this stress, the soil is normally consolidated. An overconsolidated (OC) soil will generally have higher moduli than the same normally consolidated (NC) soil, because the OC soil is on the reload part of the stress-strain curve whereas the NC soil is on the first loading part. Some soils are still in the process of consolidating under their own weight. These so-called *underconsolidated* soils are those such as the clays deposited offshore of the Mississippi Delta, where the deposition rate is faster than the rate that would allow the pore water pressures induced by deposition to dissipate. These clays have very low moduli.

What about cementation? *Cementation* refers to the "glue" that may exist at the contacts between particles. As discussed earlier, low water contents in fine-grained soils can generate water tension strong enough to simulate a significant "glue effect" between particles. This effect is temporary, as an increase in water content will destroy it. Another glue effect is due to the chemical cementation that can develop at the contacts. This cementation can be due to the deposition of calcium at the particle-to-particle contacts, for example. Such cementation leads to a significant increase in modulus.

These are some of the most important factors related to the state of the soil and influencing its modulus. Section 14.4 discusses the factors associated with the loading process.

14.4 MODULUS: INFLUENCE OF LOADING FACTOR

In this section it is assumed that the state factors for the soil considered are fixed (unchanging). In other words, the discussion of each of the following factors can be prefaced by saying "all other factors being equal." Also, in this section the secant modulus is used.

What is the mean stress level in the soil? The loading process induces stresses in the soil. These stresses can be shear stresses or normal stresses or a combination of both. At any given point and at any given time in a soil mass, there is a set of three principal normal stresses. The mean of these three stresses has a significant influence on the soil modulus, called the *confinement effect*. Figure 14.3*a* shows two sample stress-strain curves at two different confinement levels. As common sense would indicate, the higher the confinement is, the higher the soil modulus will be. A common model for quantifying the influence of confinement on the soil modulus is given in Figure 14.3*a* and is usually attributed to the work of Kondner. According to this model, the modulus is proportional to a power law of the confinement stress. The modulus E_0 is the modulus obtained when the confinement stress is equal to the atmospheric pressure p_a. A common value for the power exponent a in Figure 14.3*a* is 0.5.

What is the strain level in the soil? The loading process induces strains in the soil mass. Because soils are nonlinear materials, the secant modulus depends on the mean strain level in the zone of influence. In most cases the secant modulus will decrease as the strain level increases, because the stress-strain curve has a downward curvature. Note that an exception to this downward curvature occurs when the results of a consolidation test are plotted as a stress-strain curve on arithmetic scales for both axes. In this case the stress-strain curve exhibits an upward curvature, because the increase in confinement brought about by the steel ring is more influential than the decrease in modulus due to the increase in strain in the soil. In the triaxial test, the stress-strain curve can be fitted with a hyperbola up to the peak value; the associated model for this modulus is shown on Figure 14.3*b*. This hyperbolic model is usually attributed to the work of Duncan. In this model (Figure 14.3*b*), E_0 is the initial tangent modulus, also equal to the secant modulus for a strain of zero. The parameter

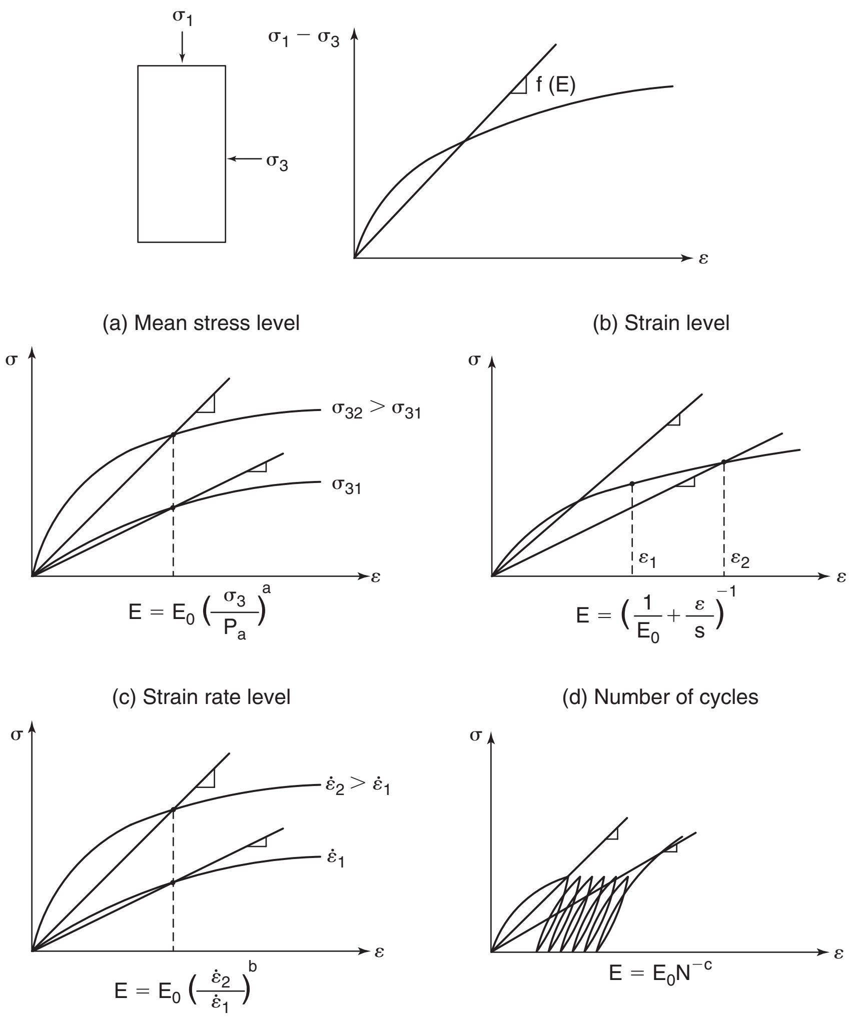

Figure 14.3 Loading factors for soil moduli.

s is the asymptotic value of the stress for a strain equal to infinity. In that sense it is related to the strength of the soil.

What is the strain rate in the soil? Soils, like many other materials, are viscous. This means that the faster a soil is loaded, the stiffer it is and therefore the higher the modulus is. In some instances, though, the reverse behavior is observed. Figure 14.3*c* shows an example of two stress-strain curves obtained by loading the soil at two drastically different strain rates. *Strain rate* is defined as the strain accumulated per unit of time. The modulus usually varies as a straight line on a log-log plot of modulus vs. strain rate. The slope of that line is the exponent b in Figure 14.3*c*. In clays, common values of this exponent vary from 0.02 for stiff clays to 0.1 for very soft clays. In sands, common values of b vary from 0.01 to 0.03. The modulus E_0 is the modulus obtained at a reference strain rate. Much of the work on this model has been done at Texas A&M University.

What is the number of cycles experienced by the soil? If the loading process is repeated a number of times, the number of cycles applied will influence the soil modulus. Again referring to the secant modulus, the larger the number of cycles, the smaller the modulus becomes. This is consistent with the accumulation of movement with an increasing number of cycles. The model used to describe this phenomenon is shown in Figure 14.3*d*. The exponent c in the model is negative and varies significantly. The most common values are on the order of -0.1 to -0.3. Much of the work on this model has also been done at Texas A&M University.

Is there time for the water to drain during the loading process? Two extreme cases can occur: drained or undrained loading. The undrained case may occur if the drainage valve is closed during a laboratory test or if the test is run sufficiently quickly in the field. The time required to maintain an undrained behavior or to ensure that complete drainage takes place depends mainly on the soil type. For example, a 10-minute test in a high-plasticity clay is probably undrained, whereas a 10-minute test in a clean sand is probably drained. The Poisson's ratio is sensitive to whether or not drainage takes place. For example, if no drainage takes place during loading in a clay, it is common to assume a Poisson's ratio equal to 0.5 (no volume change). In contrast, if complete drainage takes place (excess pore pressures are kept equal to zero), then a Poisson's ratio value of 0.35 may be reasonable. The difference between the two calculated moduli is the difference between the undrained modulus and the drained modulus. Note that the shear modulus remains theoretically constant when the drainage varies, because the effective shear stress is equal to the total shear stress. Note also that the Poisson's ratio can be larger than 0.5 if the soil dilates during shear associated with compression.

14.5 MODULUS: DIFFERENCES BETWEEN FIELDS OF APPLICATION

The modulus is useful in many fields of geotechnical engineering, but the modulus required for one field may be significantly different from the modulus for another field.

In the case of shallow foundations, the mean stress level applied under the foundation is often between 100 and 200 kPa. The normal strain level in the vertical direction is about 0.01 or less and is typically associated with a movement of about 25 mm. The rate of loading is extremely slow because that strain occurs first at the construction rate, and then the load is sustained over many years. The number of cycles is one unless cycles due to seasonal variations or other cyclic loading (such as compressor foundations or wind loads) are included. Example values of the modulus in this case are 10,000 to 20,000 kPa.

In the case of deep foundations, the mean stress level varies because the side friction on the piles occurs over a range of depth, whereas the point resistance occurs at a relatively large depth. The strain level at the pile point is usually smaller than in shallow foundations because a percentage of the load dissipates in friction before getting to the pile point. The strain rate is similar to the case of shallow foundations, with rates associated with months of construction and years of sustained loads. Some of the highest strain rates occur in the case of earthquake or wave loading. Cycles can be a major issue for earthquake loading of buildings and bridges or for wave loading of offshore structures. Because deep foundations are used in very different types of soils and for very different types of loading, the moduli vary over a much wider range of values than do the moduli for shallow foundations.

In the case of slope stability and retaining structures, movements are associated with the deformation of the soil mass essentially under its own weight. Therefore, the stress level corresponds to gravity-induced stresses. The strains are usually very small and the strain rate is again associated with the rate during initial construction and then the long-term deformation rate during the life of the slope or of the retaining structure. Cycles may occur due to earthquakes or other cyclic phenomena. For properly designed slopes and retaining structures, the moduli tend to be higher than in foundation engineering because the strain levels tend to be smaller.

In the case of pavements, the mean stress level in the subgrade is relatively low. The pressure applied to the pavement is on the order of 200 kPa for car tires, 500 kPa for truck tires, and 1700 kPa for airplane tires. However, the vertical stress at the top of the subgrade under a properly designed pavement may be only one-tenth of the tire pressure applied at the surface of the pavement. The strain level is very low because the purpose of the pavement is to limit long-term movements so that they do not exceed a few tenths of a millimeter. Typical strain levels are 0.001 or less at the top of the subgrade. The rate of loading is very high and associated with the passing of a traveling vehicle. The loading time is on the order of milliseconds for a car traveling at 100 km/h, but is measured in hours for an airplane parked at the gate. The number of cycles is tied to the number of vehicles traveling on the pavement during the life of the pavement. This number varies drastically from less than a million vehicle cycles for small roads to tens of millions for busy interstates. Typical modulus values for the subgrade range from 20,000 kPa to 150,000 kPa.

14.6 MODULUS, MODULUS OF SUBGRADE REACTION, AND STIFFNESS

The modulus of deformation E was defined in Figure 14.1. It is measured in units of force per unit area (kN/m^2). The stiffness K is defined here as the ratio of the force Q applied on a boundary through a loading area divided by the displacement s experienced by the loaded area. It is notated in units of force per unit length (kN/m). The loaded area is typically a plate, which can be square or circular. There is a relationship between the modulus E and the stiffness K. For the case of a circular plate having a diameter B, the elastic settlement s of the plate is given by:

$$s = I_1 \frac{Q}{EB} \tag{14.5}$$

Where I_1 is a constant. Therefore, the relationship between K and E is:

$$K = \frac{EB}{I_1} \tag{14.6}$$

This relationship shows that, if the modulus is a soil property, *the stiffness is not a soil property*, because it depends on the size of the loaded area. Therefore, for an elastic material, the stiffness measured with one test will be different from the stiffness measured with another test if the loading areas are different. Yet, for the same elastic material, the modulus obtained from both tests would be the same. In that sense, the stiffness is not as convenient as the modulus, so the use of the modulus is preferred.

Similar considerations apply to the modulus of subgrade reaction k. The *modulus of subgrade reaction* is defined here as the ratio of the pressure p applied to the boundary through a loading area divided by the displacement s experienced by the loaded area. It is noted in units of force per unit volume (kN/m^3). The loaded area can be a footing (coefficient of vertical subgrade reaction) or a horizontally loaded pile (coefficient of horizontal subgrade reaction). There is a relationship between the modulus E and the coefficient of subgrade reaction k. Eq. 14.5 can be rewritten as:

$$s = I_2 \frac{pB}{E} \tag{14.7}$$

Therefore, the relationship between k and E is:

$$k = \frac{E}{I_2 B} \tag{14.8}$$

where B is the footing width or the pile width or diameter. This relationship shows that, if the modulus is a soil property, *the coefficient of subgrade reaction is not a soil property*, because it depends on the size of the loaded area. Therefore, if a coefficient of subgrade reaction k is derived from load tests on a footing or a pile of a certain dimension, the value of k cannot be used directly for other footing or pile sizes. Indeed, in this case careful considerations of size and scale must be addressed. The modulus is not affected by this problem. In that sense, the coefficient of subgrade reaction is not as convenient as the modulus, so the use of the modulus is preferred.

14.7 COMMON VALUES OF YOUNG'S MODULUS AND POISSON'S RATIO

Considering all those factors, it is clear that the modulus of a soil is not a unique number. Therefore, when one says that

Table 14.1 Range of Quoted Modulus and Poisson's Ratio Values for Clays

Clay	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges	Undrained Strength (kPa)
Very soft clay	2.5–15	0.4	<12
	2–15	0.35–0.45	
	3	0.4–0.5 undrained	
	1–3	0.1–0.3 unsaturated	
	2–4	0.2–0.3 sandy clay	
	0.5–5		
Soft clay	2–25	0.15–0.25	12–25
	1.8–3.5	0.4–0.5 undrained	
	5–20	0.1–0.3 unsaturated	
		0.2–0.3 sandy clay	
Medium or firm clay	15–50	0.3	25–50
	15–50	0.3–0.35	
	7	0.2–0.5	
	5–10	0.4–0.5 undrained	
	20–50	0.1–0.3 unsaturated	
		0.2–0.3 sandy clay	
Stiff clay	15–50	0.1–0.3	50–100
	2.5–5	0.4–0.5 undrained	
	8–19	0.1–0.3 unsaturated	
	4.2–8	0.2–0.3 sandy clay	
	50–100		
Very stiff clay	50–100	0.4–0.5 undrained	100–200
		0.1–0.3 unsaturated	
		0.2–0.3 sandy clay	
Hard clay	50–100	0.25	200–400
	14	0.4–0.5 undrained	
	8–19	0.1–0.3 unsaturated	
	6–14	0.2–0.3 sandy clay	
Very hard clay	100–200	0.4–0.5 undrained	>400
		0.1–0.3 unsaturated	
		0.2–0.3 sandy clay	

Table 14.2 Range of Quoted Modulus and Poisson's Ratio Values for Silts

Silt	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges
Silt	2–20	0.3–0.35
	3–10	0.3–0.35
	2–19	0.3–0.35
	2–20	
	2–20	
Soft silt	2–5	
	0.5–3	
	4–8	
Firm silt	5–20	

the modulus of a soil is 10,000 kPa, for example, the very next question should be: What are the conditions associated with this number? It is also clear that the best way to obtain an appropriate modulus for a soil is to measure it directly with a test that reproduces the situation that the soil will undergo during the deformation process. Hence, tests like the pressuremeter test, the triaxial test, and the consolidation test are among the best for such a measurement. These tests are not always available or even within the budget of small projects. The next best way to obtain an appropriate modulus is to use correlations to other results or tests, such as the undrained shear strength s_u or the standard penetration test blow count N. The last resort for estimating a modulus is to use tables that give ranges of typically encountered values. Tables 14.1 through 14.7 are a collection of ranges quoted in various publications for the values of soil and rock moduli. It is not always clear from these publications

Table 14.3 Range of Quoted Modulus and Poisson's Ratio Values for Sands

Sand	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges	SPT Blow Count N (bpf)
Very loose			<4
Loose	8–12 (fine)	As low as 0.1	4–10
	10–28	0.25 (fine)	
	10–25	0.2–0.36	
	10–30	0.2	
	15	0.2–0.35	
	10–21	0.35–0.4	
	20–80	0.2–0.4	
	10–29		
Medium or compact	12–20 (fine)	0.25 (fine)	10–30
	30–50	0.3–0.35	
	50–150	0.25–0.4	
	29–48		
Dense	20–30 (fine)	0.25 (fine)	30–50
	50–80	0.3–0.4	
	35–70	0.3–0.45	
	50–81	0.3	
	80	0.3–0.36	
	52–83	0.3–0.4	
	49–78	0.25–0.3	
	48–77	Up to 1	
Very dense			>50

Table 14.4 Range of Quoted Modulus and Poisson's Ratio Values for Gravels

Gravel	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges	SPT Blow Count N (bpf)
Loose	50–150	0.2–0.35	4–10
	50–150	0.2	
	100		
	29–77		
	30–80		
Medium or compact	80–100		10–30
Dense	100–200	0.3–0.4	30–50
	100–200	0.3	
	150		
	102–204		
	96–192		

Table 14.5 Range of Quoted Modulus and Poisson's Ratio Values for Other Soils

Other Soils	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges
Peat	0.1–0.3 (pure)	
	0.4–1	
	0.8–2 (some clay)	
Loess	14–60	–0.3
	15–60	0.1–0.3
	14–58	
Glacial till	10–150 (loose)	
	150–720 (dense)	
	500–1440 (Very dense)	
Clay shale	100–200	0.25–0.33
	150–5000	
	10000–40000 (intact)	

Table 14.6 Range of Quoted Modulus and Poisson's Ratio Values for Some Rocks

Rocks	Modulus E (MPa) Quoted ranges	Poisson's Ratio ν Quoted ranges
Dolomite	110000–121000	0.3
Gneiss	83000–118000	0.15–0.2
Granite	73000–86000	0.23–0.27
	31000–57000	0.15–0.24
	7000–14000 (partially decomposed)	
Limestone	87000–108000	0.27–0.3
	21000–48000	0.16–0.23
Marble	87000–108000	0.27–0.3
Mica schist	79000–101000	0.15–0.2
Quartzite	82000–97000	0.12–0.15
Rock salt	35000	0.25
Slate	79000–112000	0.15–0.2
Sandstone	38000–76000	0.25–0.33
Coal	10000–20000	

what conditions are associated with these modulus values (stress level, strain level, rate of loading, number of cycles, undrained or drained). Nevertheless, they offer some guidance for the overall range of possible values. Among the sources of these ranges of values are Lambe and Whitman (1979), Hunt (1986), USACE (1990), Bowles (1996), AASHTO (2007), and FHWA (2010). In summary, common values of soil moduli vary between 1 MPa and 150 MPa; Poisson's ratio is about 0.3 to 0.35 for drained behavior and unsaturated soils and close to 0.5 for undrained behavior of saturated soils. Poisson's ratio can be higher than 0.5 for dilatant soils because the volume can increase during compression.

Table 14.7 Range of Quoted Modulus and Poisson's Ratio Values for Other Materials

Other Material	Modulus E (MPa) Quoted ranges	Poisson's Ratio v Quoted ranges
Steel	1220000	
Steel	200000	0.28–0.29
		0.33
Aluminum	55000–76000	0.34–0.36
Concrete	20000–40000	0.15
Wood	11000–14500	
Glass	65000	
Plastic (polyethylene)	13000	
Ice	7000	0.36
Water	2200 (bulk modulus)	
Air	0.1 (bulk modulus)	

14.8 CORRELATIONS WITH OTHER TESTS

Correlations have been developed between soil modulus and the results of soil tests, mostly in situ tests. The correlations are presented in Tables 14.8 through 14.10. Among the sources for these correlations are Bowles (1996), FHWA (2010), Briaud and Miran (1992a, 1992b), Briaud (1992), and Mayne (2007a, 2007b). Note that most of these correlations lead to a modulus that would be associated with foundation settlements at working loads. There is one exception to this statement for the relationship between the soil modulus of clays and the undrained shear strength. In this case the modulus refers to the elastic immediate settlement, a higher modulus than would be used for the long-term settlement. Figure 14.4 shows a more detailed relationship between the undrained modulus E_u and the undrained shear strength s_u as a function of the overconsolidation ratio (OCR). Recall that the OCR is the ratio between the effective preconsolidation pressure σ'_p and the vertical effective stress at rest σ'_{ov}. The pressure σ'_p is found on the stress-strain curve of the consolidation test around the maximum curvature of the semilog plot (see Figure 9.32).

14.9 MODULUS: A COMPREHENSIVE MODEL

To acknowledge the influence of the various loading factors on the modulus, it is useful to regroup them into one single model. The model should include the influence of:

- Stress confinement level
- Strain level
- Rate of loading
- Number of cycles

First, the influence of the confinement level on the modulus is quantified. For this, the initial tangent modulus E_i is selected to isolate the influence of the strain level, which is to be included separately. This confinement is due to the mean normal stress σ_M:

$$\sigma_M = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{14.9}$$

where σ_1, σ_2, and σ_3 are the principal stresses. The influence of the confinement is quantified through the Kondner model:

$$E_{\sigma i} = E_{\text{ai}}\left(\frac{\sigma_M}{p_a}\right)^m \tag{14.10}$$

where $E_{\sigma i}$ is the initial tangent modulus, E_{ai} is the value of E_i for a confinement equal to the atmospheric pressure, σ_M is the mean confining stress, p_a is the atmospheric pressure used as a reference, and m is the stress level exponent. Then the

Table 14.8 Correlations between Soil Modulus and Soil Test Results for Sands and Gravels

Soil Types	Correlation
Silts, sandy silts, slightly cohesive mixtures	$E(kPa) = 400\ N(bpf)^*$
Clean fine to medium sands and slightly silty sands	$E(kPa) = 700\ N(bpf)^*$
Coarse sand and sand with little gravel	$E(kPa) = 1000\ N(bpf)^*$
Sandy gravels and gravels	$E(kPa) = 1200\ N(bpf)^*$
	$E(kPa) = 7000\ (N(bpf))^{0.5}$
Sand (normally consolidated)	$E(kPa) = (15000 \text{ to } 22000)\ \log_e(N(bpf))$
	$E(kPa) = 500\ (N(bpf)^* + 15)$
Sand (saturated)	$E(kPa) = 250\ (N(bpf)^* + 15)$
Sand (overconsolidated)	$E(kPa) = 40000 + 1050\ N(bpf)^*$
Gravelly sand	$E(kPa) = 1200\ (N(bpf)^* + 6)$
	$E = 2\ q_c^{**}$
Sandy soils (normally consolidated)	$E = (2.5 \text{ to } 3.5)\ q_c^{**}$ recent < 100 yrs
	$E = (3.5 \text{ to } 6)\ q_c^{**}$ old > 3000 yrs
Sand (normally consolidated)	$E = (1 + Dr^2)\ q_c$ Dr is relative density as ratio
Sand (overconsolidated)	$E = (6 \text{ to } 10)\ q_c^{**}$
Sand: $q_c < 5$ MPa	$E = 2\ q_c^{**}$
Sand: $q_c > 10$ MPa	$E = 1.5\ q_c^{**}$

*SPT blow count N in bpf, blows per 0.3 m
**CPT point resistance in units of pressure

Table 14.9 Correlations between Soil Modulus and Soil Test Results for Clays and Silts

Soil Types	Correlation
Normally consolidated sensitive clay	$E = (200 \text{ to } 500)\ s_u^{***}$ for immediate undrained settlement
Normally consolidated insensitive and lightly overconsolidated clay	$E = (750 \text{ to } 1200)\ s_u^{***}$ for immediate undrained settlement
Heavily overconsolidated clay	$E = (1500 \text{ to } 2000)\ s_u^{***}$ for immediate undrained settlement
Clays of low plasticity (CL)	$M^* = (1 \text{ to } 2.5)\ q_c^{**}$ for $q_c > 2$ MPa $M^* = (2 \text{ to } 5)\ q_c^{**}$ for $0.7 < q_c < 2$ MPa $M^* = (3 \text{ to } 8)\ q_c^{**}$ for $q_c < 0.7$ MPa
Silts of low plasticity (ML)	$M^* = (3 \text{ to } 6)\ q_c^{**}$ for $q_c > 2$ MPa $M^* = (1 \text{ to } 3)\ q_c^{**}$ for $q_c < 2$ MPa
High-plasticity silts and clays (MH, CH)	$M^* = (2 \text{ to } 6)\ q_c^{**}$ for $q_c < 2$ MPa
Organic silt (OL)	$M^* = (2 \text{ to } 8)\ q_c^{**}$ for $q_c < 1.2$ MPa
Peat and organic clay (Pt, OH, $q_c < 0.7$ MPa)	$M^* = (1.5 \text{ to } 4)\ q_c^{**}$ for $50 < w < 100$ $M^* = (1 \text{ to } 1.5)\ q_c^{**}$ for $100 < w < 200$ $M^* = (0.4 \text{ to } 1)\ q_c^{**}$ for $w > 200$
Chalk	$M^* = (2 \text{ to } 4)\ q_c^{**}$ for $2 < q_c < 3$ MPa

*M is the constrained modulus: $M = E(1-v)/((1+v)(1-2v))$
**CPT point resistance in units of pressure
***Undrained shear strength

Table 14.10 Correlations between Pressuremeter Modulus and Other Data

(a) Correlations for Sand
Column A = number in table × row B

B A	E_0 (kPa)	E_R (kPa)	p^*_L (kPa)	q_c (kPa)	f_s (kPa)	N (bl/30 cm)
E_0 (kPa)	1	0.125	8	1.15	57.5	383
E_R (kPa)	8	1	64	6.25	312.5	2174
p^*_L (kPa)	0.125	0.0156	1	0.11	5.5	47.9
q_c (kPa)	0.87	0.16	9	1	50	479
f_s (kPa)	0.0174	0.0032	0.182	0.02	1	9.58
N (bl/30 cm)	0.0026	0.00046	0.021	0.0021	0.104	1

(b) Correlations for Clay
Column A = number in table × row B

B A	E_0 (kPa)	E_R (kPa)	p^*_L (kPa)	q_c (kPa)	f_s (kPa)	s_u (kPa)	N (bl/30 cm)
E_0 (kPa)	1	0.278	14	2.5	56	100	667
E_R (kPa)	3.6	1	50	13	260	300	2000
p^*_L (kPa)	0.071	0.02	1	0.2	4	7.5	50
q_c (kPa)	0.40	0.077	5	1	20	27	180
f_s (kPa)	0.079	0.0038	0.25	0.05	1	1.6	10.7
s_u (kPa)	0.010	0.0033	0.133	0.037	0.625	1	6.7
N (bl/30 cm)	0.0015	0.0005	0.02	0.0056	0.091	0.14	1

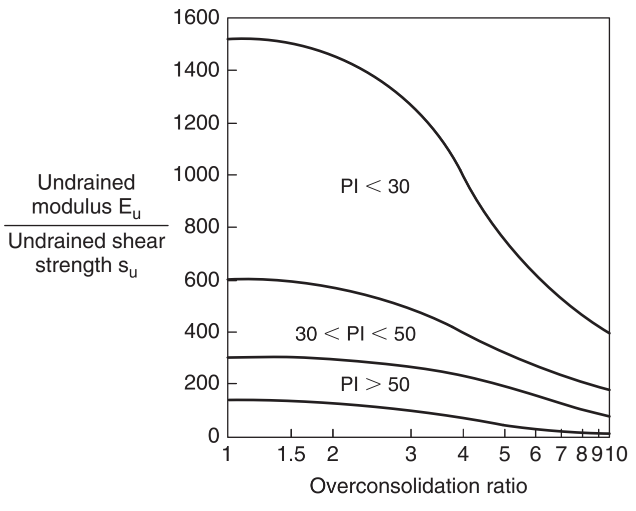

Figure 14.4 Modulus of clays correlated to undrained shear strength (After Duncan and Buchignani, 1976)

influence of the strain level is included by using the Duncan hyperbolic model. This model states that the stress-strain curve is well described by a hyperbola:

$$\sigma = \frac{\varepsilon}{a + b\varepsilon} \tag{14.11}$$

Note that when ε goes to zero, the ratio σ/ε goes to $1/a$, therefore $1/a$ is associated with the initial tangent modulus E_i. Also, when ε goes to infinity, σ goes to $1/b$, which represents the ultimate strength of the soil σ_{ult}. Equation 14.11 can be rewritten in terms of modulus variation as:

$$E_{\sigma\varepsilon} = \left(\frac{1}{E_{\sigma i}} + \frac{\varepsilon}{\sigma_{ult}}\right)^{-1} \tag{14.12}$$

Now we can include the rate effect on the modulus. This is done by using a rate effect exponent model (Briaud and Garland 1985) that quantifies the modulus increase when the time of loading t decreases or the strain rate $\dot{\varepsilon}$ increases:

$$E_t = E_{t0}\left(\frac{t}{t_o}\right)^{-n} = E_{t0}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o}\right)^{n} \quad (14.13)$$

where t_o is a reference time of loading, $\dot{\varepsilon}_o$ is a reference strain rate, and n is the rate effect exponent for the soil. Figure 14.5 shows a correlation between the exponent n and the undrained shear strength s_u. The best-fit equation is:

$$n = 0.12(s_{u\,ref}(\text{kPa}))^{-0.22} \quad (14.14)$$

The number of loading cycles N is included by using a power law (Briaud 1992):

$$E_N = E_1 N^{-p} \quad (14.15)$$

where E_N and E_1 are the secant moduli to the top of the Nth cycle and the first cycle respectively and p is the cyclic degradation exponent. By combining all effects, the general model becomes:

$$E_{\sigma\varepsilon tN} = \left(\frac{1}{E_{ai}\left(\frac{\sigma_M}{p_a}\right)^m} + \frac{\varepsilon}{\sigma_{ult}}\right)^{-1}\left(\frac{t}{t_o}\right)^{-n} N^{-p} \quad (14.16)$$

Values around 0.5 are common for the stress level exponent m and for drained or unsaturated conditions ($S < 0.8$). The value of m becomes very low and even zero for the undrained behavior of fine-grained soils. In clays, common values of the rate effect exponent n vary from 0.02 for stiff clays to 0.1 for very soft clays. In sands, common values of n vary from 0.01 to 0.03. The cyclic degradation exponent p varies widely depending on how close to failure the soil is loaded. At working loads, this exponent is generally less than 0.1. The time t_o that serves as a reference for strain rate effect can correspond to the typical length of a soil test and may be taken as 10 minutes. The time t can vary from 75 years for the typical design life of a bridge to 10 milliseconds for a car impact on a guardrail post or the passage of a vehicle on a pavement. The number of cycles can vary from 1 for a building, to about 1000 for hurricane wave loading on an offshore platform, and to millions for a pavement. As can be seen in Eq. 14.16, the modulus E_{ai} is very important. It is the reference modulus for all other calculations and represents the initial tangent modulus (zero strain) at a stress level corresponding to atmospheric pressure (100 kPa), at a loading time of possibly 10 minutes, and for the first loading (one cycle).

14.10 INITIAL TANGENT MODULUS G_o OR G_{max}

This modulus is typically referred to as G_o or G_{max}. This is because the shear modulus G is more convenient than the Young's modulus E. Indeed, the shear modulus G does not require knowledge of the Poisson's ratio, whereas E does. The subscript "o" or "max" refers to the fact that it is the modulus at the origin and also the maximum shear modulus value one can expect for the soil. Several expressions for G_{max} have been formulated. Hardin and Drnevich (1972) and Hardin (1978) proposed, for all soil types:

$$\frac{G_{max}}{p_a} = \frac{625}{0.3 + 0.7e^2}(OCR)^k\left(\frac{\sigma'_M}{p_a}\right)^n \quad (14.17)$$

where p_a is the atmospheric pressure, e is the void ratio, OCR is the overconsolidation ratio, σ'_M is the mean effective normal stress, and k and n are exponents. Then Jamiolkowski et al. (1991) proposed:

$$\frac{G_{max}}{p_a} = \frac{625}{e^{1.3}}(OCR)^k\left(\frac{\sigma'_M}{p_a}\right)^n \quad (14.18)$$

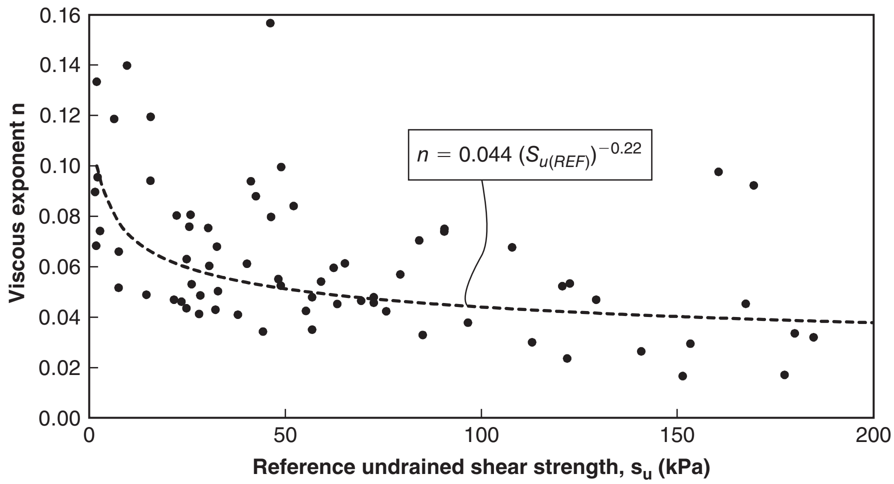

Figure 14.5 Rate effect exponent for clays.

The exponent n is usually taken equal to 0.5, and k is given in Table 14.11.

For sands, Seed and Idriss (1970) proposed:

$$\frac{G_{\max}}{p_a} = 22.4 K_{2,\max} \left(\frac{\sigma'_M}{p_a}\right)^{0.5} \tag{14.19}$$

where p_a is the atmospheric pressure and $K_{2,\max}$ is a modulus number given in Table 14.12 for sands. For gravel, $K_{2,\max}$ is higher than for sands, ranging between 80 and 180.

For fine-grained soils, Kramer (1996) suggests relating $G_{\max}$ to the OCR and the undrained shear strength s_u measured in a CU triaxial test (Table 14.13).

The value of $G_{\max}$ has also been correlated to the results of in situ tests, in particular the SPT and the CPT. Ohta and Goto (1976) and Seed et al. (1986) proposed, for sands:

$$\frac{G_{\max}}{p_a} = 447 N^{0.33} \left(\frac{\sigma'_M}{p_a}\right)^{0.5} \tag{14.20}$$

where N is the SPT blow count corrected for 60% of maximum energy and corrected to 100 kPa of pressure (see Chapter 7). Rix and Stokoe (1991) proposed a correlation for quartz sands with the CPT point resistance q_c (Chapter 7):

$$\frac{G_{\max}}{p_a} = 290 \left(\frac{q_c}{p_a}\right)^{0.25} \left(\frac{\sigma'_M}{p_a}\right)^{0.375} \tag{14.21}$$

Table 14.11 Overconsolidation Exponent *k*

Plasticity Index	Value of k
0	0.00
20	0.18
40	0.30
60	0.41
80	0.48
100	0.50

(After Hardin and Drnevich 1972; Kramer 1996)

Table 14.12 Values of $K_{2,\max}$

Void ratio	$K_{2,\max}$	Relative density (%)	$K_{2,\max}$
0.4	70	30	34
0.5	60	40	40
0.6	51	45	43
0.7	44	60	52
0.8	39	75	59
0.9	34	90	70

(After Seed and Idriss 1970; Kramer 1996.)

Table 14.13 Values of $G_{\max}/s_u$

	Overconsolidation ratio, OCR		
Plasticity index	1	2	5
15–20	1100	900	600
20–25	700	600	500
035–45	450	380	300

(After Kramer 1996.)

Table 14.14 Common Values of $G_{\max}$ for Different Soils Based on Shear Wave Velocity

Type of Soil	Small-Strain Shear Wave Velocity, v_s (m/s)	Initial Shear Modulus, $G_{\max}$ (MPa)
Soft clay	40–90	3–14
Firm clay	65–140	7–36
Loose sand	125–270	29–144
Dense sand and gravel	270–400	72–360

For clay, Mayne and Rix (1993) proposed:

$$\frac{G_{\max}}{p_a} = 100 \left(\frac{q_c}{p_a}\right)^{0.695} e^{-1.13} \tag{14.22}$$

In the end, however, the best way to obtain $G_{\max}$ is through testing. In the field, the tests that can be used are the cross hole test and the SASW, as described in Chapter 7. Some common values are shown in Table 14.14. In the laboratory, the best test is the resonant column test (Chapter 9), yet sample disturbance may lead to lower values of $G_{\max}$ compared to the field values. The field values come from testing a large, undisturbed mass of soil through wave propagation, whereas disturbance has a much more pronounced effect on the small scale of the lab test. The contrary is likely true for weathered rocks, where the sample is likely to be much stiffer than the rock mass.

14.11 REDUCTION OF $G_{\max}$ WITH STRAIN: THE $G/G_{\max}$ CURVE

As discussed earlier (section 14.9, and section 9.13.1 in Chapter 9), the modulus decreases with an increase in strain. As a result, the $G_{\max}$ value, which corresponds to zero strain, is the highest shear modulus attainable. When the strain

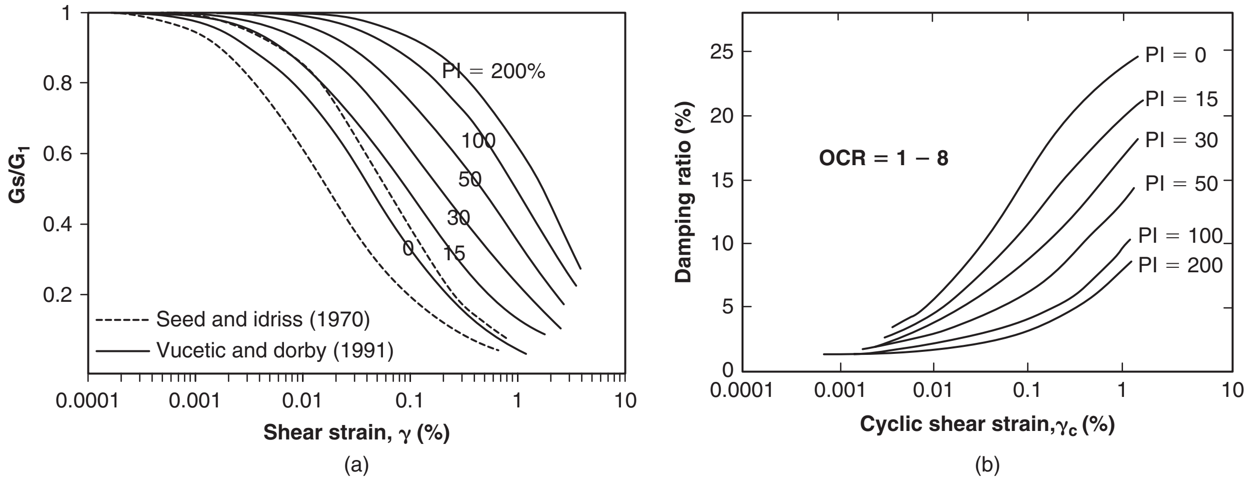

Figure 14.6 Degradation of the shear modulus and damping ratio with shear strain: (a) Modulus. (b) Damping ratio. (After Vucetic and Dobry 1996)

increases, G decreases and the ratio G/G_{max} becomes less than 1. The G/G_{max} ratio is plotted against the shear strain γ in a semilog plot: G/G_{max} on a natural scale and γ on a log scale. Figure 14.6 shows such a plot, proposed by Vucetic and Dobry (1991), indicating the influence of the plasticity index PI on the G/G_{max} curve. As can be seen from Figure 14.6, the high PI clays maintain the G_{max} value over a larger range of strain than the low PI clays. Kramer (1996) cautiously suggests that the $PI = 0$ curve could also be used for sands.

Parallel to this variation is the evolution of the damping ratio as defined in section 9.13.1. The damping ratio quantifies the loss of energy in the deformation process. During cyclic loading, the loop of the shear stress vs. shear strain curve widens, and it takes more and more energy to deform the soil. This leads to an increase in the damping ratio, as shown in Figure 14.6.

Mayne (2007a; 2007b) summarizes the data of several authors using the following equation, which is based on the mobilized strength rather than the strain as the variable influencing the reduction in shear modulus:

$$\frac{G}{G_{max}} = 1 - \left(\frac{\tau}{\tau_{max}}\right)^g \tag{14.23}$$

where τ is the applied shear stress, τ_{max} is the maximum shear stress or shear strength of the soil, and g is the reduction exponent (which has a range of 0.2 to 0.4 for common soils). Figure 14.7 shows plots of Eq. 14.23 with data for various soils.

Again, testing is the best way to obtain the G/G_{max} degradation curve, but no single test can be used to give the entire curve. The problem is that the strain range associated with the G/G_{max} degradation curve goes from shear strains of 10^{-6} to 10^{-1} or 10^{-4} percent to 10^{1} percent. To cover this wide range of strains, several tests are used, as shown in Figure 14.8.

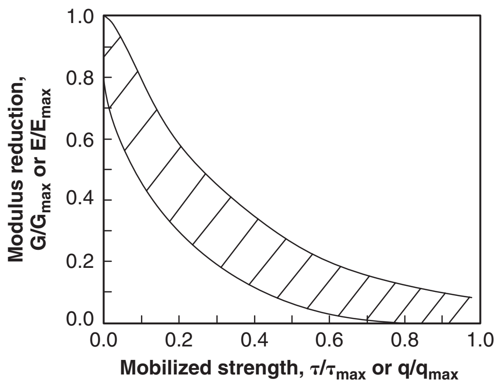

Figure 14.7 Degradation of the shear modulus with mobilized stress. (After Mayne 2007a; 2007b)

14.12 PRECONSOLIDATION PRESSURE AND OVERCONSOLIDATION RATIO FROM CONSOLIDATION TEST

One of the oldest tests in geotechnical engineering is the consolidation test (see Chapters 9 and 11). This test yields a stress-strain curve and a set of strain-time curves from which soil properties can be obtained regarding the magnitude of deformation and the time rate of deformation. The preconsolidation pressure σ'_p at a depth z is the effective normal stress found from the effective stress vs. strain consolidation curve on a sample from depth z by the technique described in section 9.5 and Figure 9.32. The vertical effective stress at depth z where the sample was taken is σ'_{ov}. Soils can be classified according to whether σ'_{ov} is larger than, smaller than, or equal to σ'_p. Three categories are identified:

- Normally consolidated soils: $\sigma'_{ov} = \sigma'_p$
- Overconsolidated soils: $\sigma'_{ov} < \sigma'_p$
- Underconsolidated soils: $\sigma'_{ov} > \sigma'_p$

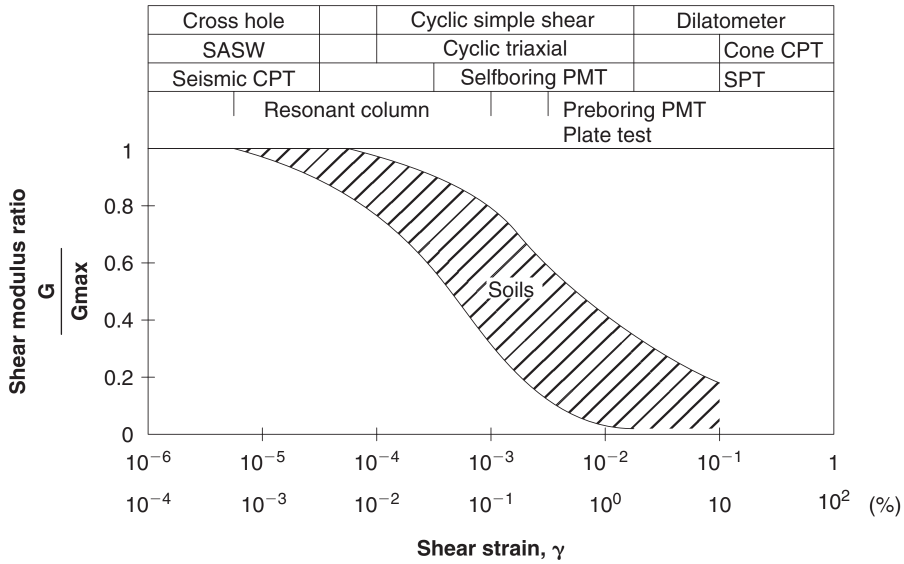

Figure 14.8 Range of strains covered by various soil tests to generate the G/G_{max} vs. γ curve.

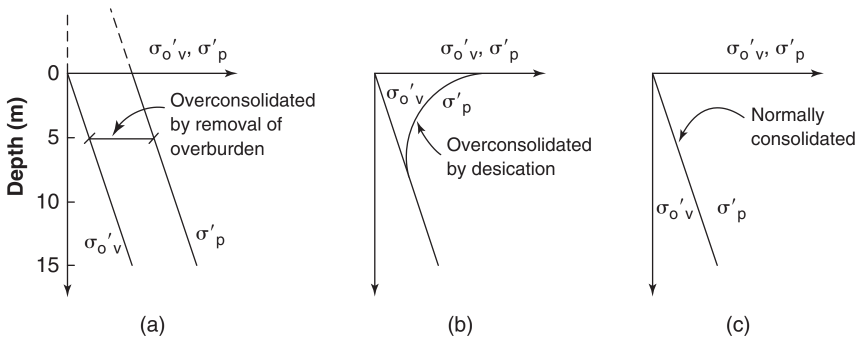

Figure 14.9 Overconsolidation profiles.

The pressure σ'_p is very important for many reasons; it essentially plays the role of a yield stress for soils when the consolidation test applies to the soil problem being studied. For example, the settlement of a structure or embankment will likely be small if the pressure under the foundation on an overconsolidated soil is kept below σ'_p. This pressure can be due to a number of phenomena, including overburden removal and desiccation. Indeed, if a soil exists for a long time with a high overburden pressure and then this overburden pressure is significantly reduced, the soil will keep the "memory" of the high overburden pressure and will not exhibit large deformation until the high overburden pressure is surpassed during the soil loading process. For example, if a 100 m thick glacier was covering an area 10,000 years ago but has completely melted today, the soil below the glacier will have a preconsolidation pressure equal to the pressure of the large glacier (100 m $\times$ 9 kN/m^3 = 900 kPa). One would expect small settlements for pressures less than 900 kPa but larger settlements for pressures above 900 kPa. This stress relief would be felt throughout the depth of the deposit, and the profile of preconsolidation pressure would look like the one in Figure 14.9*a*. If, in contrast, the soil had not been subjected to erosion, but had been subjected to a series of wetting and drying cycles, it would exhibit an overconsolidation profile, as shown in Figure 14.9*b*. Indeed, repeated wetting and drying cycles induce and release a significant effective stress that prestresses the soil. This effect is typically localized near the ground surface where the wetting and drying cycles are prevalent. If the soil had not been subjected to any significant erosion, loading, wetting, or desiccation, the current vertical effective stress would be the same as the preconsolidation pressure and the soil would be normally consolidated (Figure 14.9*c*). Underconsolidated soils are soils that are still consolidating. This is the case, for example, with the soils in the delta of the Mississippi River, where the sediments drained from the plains of the USA through erosion are loading the bottom of the Gulf of Mexico faster than the soft clays can consolidate.

The overconsolidation ratio (OCR) is the ratio of the preconsolidation pressure σ'_p over the vertical effective stress σ'_{ov}:

$$OCR = \frac{\sigma'_p}{\sigma'_{ov}} \quad (14.24)$$

The OCR varies from 1 for normally consolidated soils up to 5 for heavily overconsolidated soils, with values between 1.5 and 2.5 being relatively common.

14.13 COMPRESSION INDEX, RECOMPRESSION INDEX, AND SECONDARY COMPRESSION INDEX FROM CONSOLIDATION TEST

The *compression index* C_c is defined as the slope of the linear portion of the e–logσ' curve after the initial curved part (Figure 14.10):

$$C_c = \frac{\Delta e}{\Delta \log \sigma}, \quad (14.25)$$

The value of C_c for common saturated soils varies between 0.2 and 1. Terzaghi and Peck (1967) gave the following empirical equation:

$$C_c = 0.009(LL - 10) \quad (14.26)$$

where LL is the liquid limit expressed as a percentage. This equation has a reliability range of ±30% and should not be used for clays with a sensitivity greater than 4, or a liquid limit greater than 100, or a large percentage of organic matter (Holtz et al. 2011). The compression index increases with the initial void ratio and the water content for saturated soils. Among other correlations for saturated clays are:

$$C_c = 1.15(e_o - 0.35) \quad (14.27)$$

$$C_c = w \quad (14.28)$$

Table 14.15 Compression Index C_c Values for Saturated Soils

Soil	Compression Index C_c
Peat	10–15
Organic clays	2–8
Sensitive clays	1–4
High-plasticity clays	0.5–0.9
Low-plasticity clays	0.15–1.2

where e_o is the initial void ratio and w is the natural water content expressed as a ratio rather than a percentage. The compression index does increase with the initial void ratio and the water content for saturated soils. Table 14.15 gives some ranges of observed values of C_c for various soils.

The *recompression index* C_r is the slope of the unload-reload loop performed during a consolidation test. The problem is that the value of C_r depends on both the point at which the unloading is started and the extent of the unloading. This is why it is important, during the test, to reproduce the loading path experienced by the soil. Typically, the higher the stress at which the unloading starts, the higher the C_r; conversely, the larger the unloading stress, the smaller the C_r is.

The secondary compression index C_α (Figure 14.11) is associated with the void ratio vs. log time curve. This curve is the one obtained during each load step in the conventional consolidation test. Toward the end of the consolidation process, once the pore pressures have dissipated, the void ratio vs. log time curve tends toward a straight line. At that point, the soil deformation is called *secondary consolidation* or *creep*. The slope of the later part of the e versus. log t curve

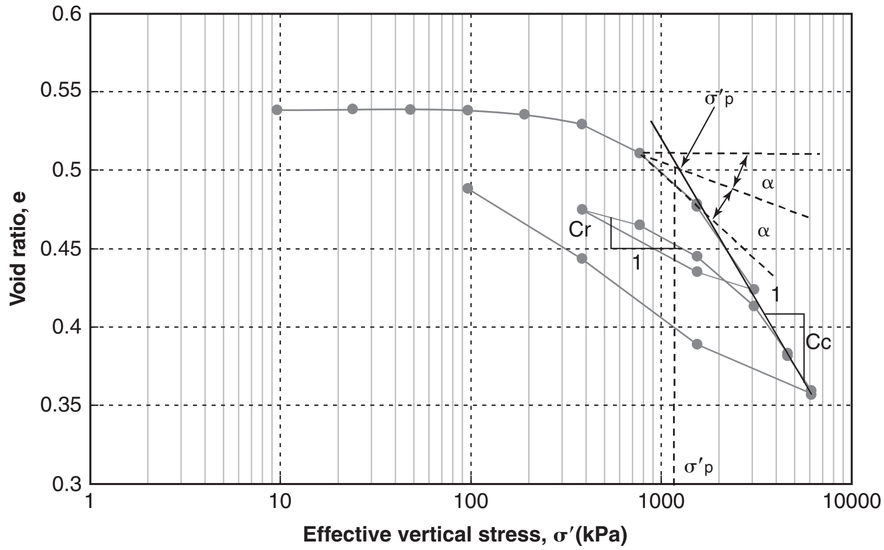

Figure 14.10 Definition of σ'_p, the compression index C_c, and the recompression index C_r.

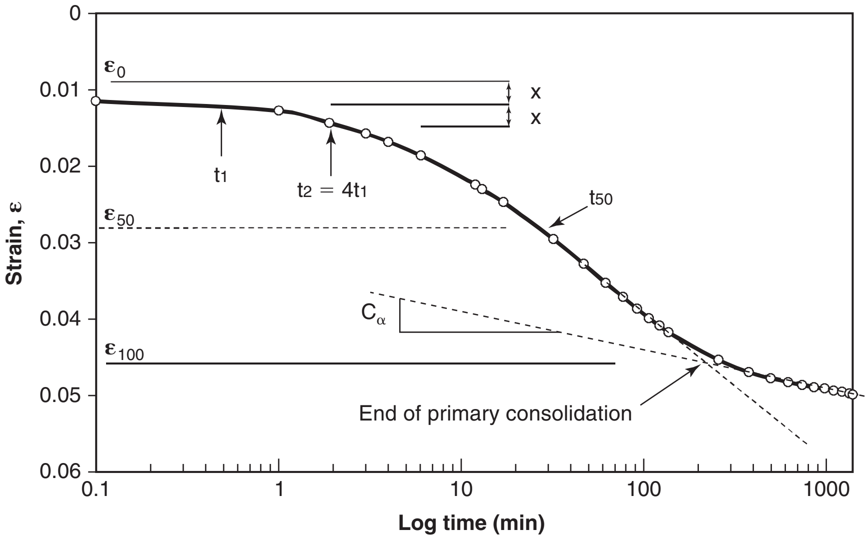

Figure 14.11 Definition of the secondary compression index C_α.

is the secondary compression index C_α:

$$C_\alpha = \frac{\Delta e}{\Delta \log t} \tag{14.29}$$

C_α tends to increase with an increase in organic content in the soil and decrease with an increase in sand content in fine-grained soils. Terzaghi et al. (1996) state that the secondary compression index C_α is linked to the compression index C_c and propose Table 14.16.

Note that a modulus E can be obtained from the consolidation test. By definition, the constrained modulus M is the ratio between the vertical stress σ_z and the vertical strain ε_z when any lateral expansion is restrained. The constrained modulus M is related to the Young's modulus E by:

$$M = \frac{\sigma_z}{\varepsilon_z} = E\frac{1-\nu}{(1+\nu)(1-2\nu)} \tag{14.30}$$

Figure 14.12 shows consolidation curves plotted on axes with natural scale and associated constrained modulus M. Note that as the strain increases, the value of M increases, because the steel ring constraining the soil is playing an increasing role.

Table 14.16 Ratio of Secondary Compression Index C_α over Compression Index C_c for Various Soils

Soil	C_α/C_c
Granular soils	0.02 ± 0.01
Inorganic clays and silts	0.04 ± 0.01
Organic clays and silts	0.05 ± 0.01
Peat	0.06 ± 0.01

(After Terzaghi et al. 1996)

14.14 TIME EFFECT FROM CONSOLIDATION TEST

The rate of deformation can be quantified by the coefficient of consolidation c_v. As demonstrated in Chapter 11 (section 11.4.6), the time rate of settlement can be predicted by the following equation, provided the assumptions associated with that derivation are satisfied:

$$t = \frac{TH^2}{c_v} \tag{14.31}$$

where t is the time required for $U\%$ of the settlement to take place, T is the unitless time factor corresponding to $U\%$ of settlement (Figure 14.13), H is the drainage length (m), and c_v is the coefficient of consolidation (m^2/s). The drainage length H is equal to the thickness H_o of the consolidating layer if drainage can take place on only one side (top or bottom) of the layer and equal to half the thickness of the layer $H_o/2$ if drainage can take place on both sides of the layer (top and bottom) (Figure 14.14). The assumptions required for this equation to be applicable are:

a. The soil is saturated with water.
b. The water is incompressible.
c. The soil skeleton is linear elastic (linear stress-strain relation).
d. The soil particles are incompressible.
e. Darcy's law governs the flow of water through the soil.
f. The water drains through one or both of the horizontal boundaries.
g. The flow is in the vertical direction only.

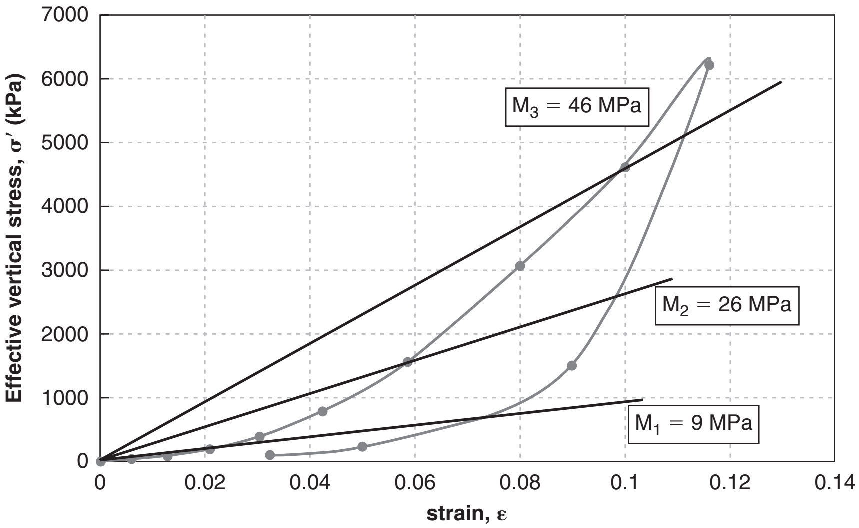

Figure 14.12 Stress-strain curve from consolidation test.

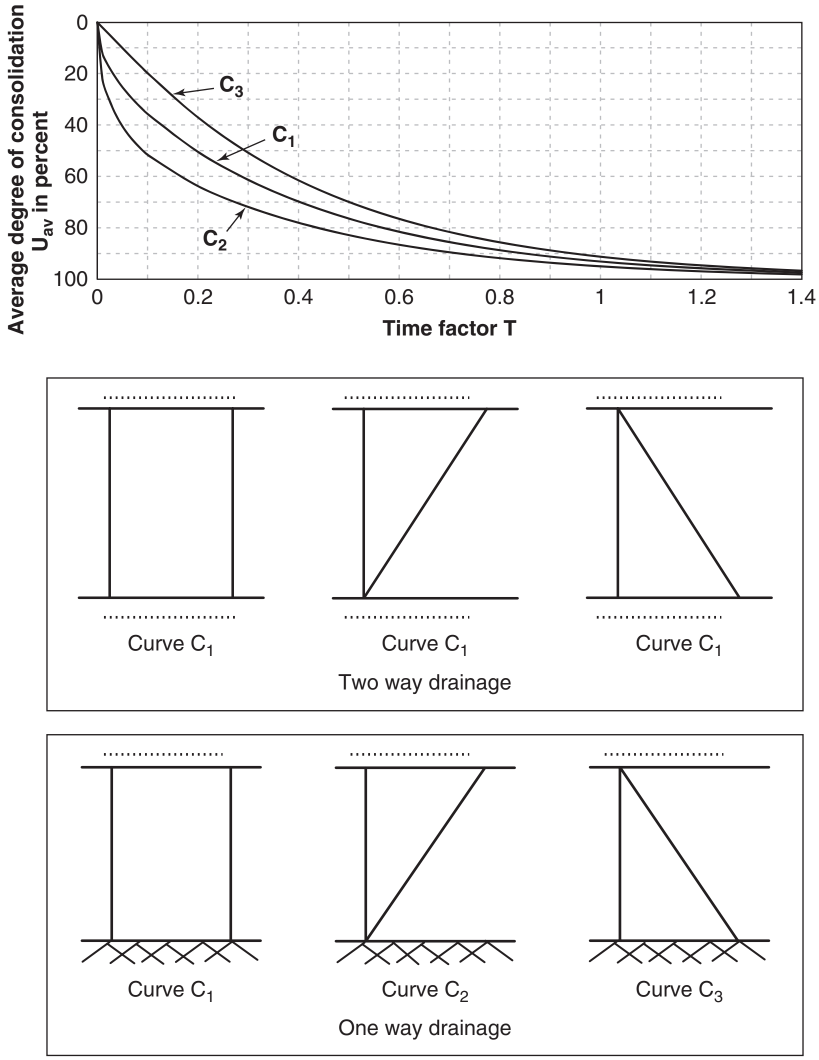

Figure 14.13 Average degree of consolidation U_{av} vs. time factor T on natural scale for different stress increase profiles.

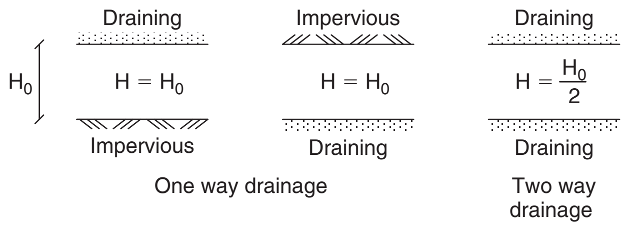

Figure 14.14 Drainage length.

h. The increase in stress $\Delta\sigma$ in the layer due to the embankment is constant within the layer.
i. The excess water stress u_{we} increases by $\Delta\sigma$ when the embankment is placed.
j. No lateral soil movement takes place.

The coefficient of consolidation c_v can be obtained from the consolidation test, as explained in Chapter 9 (section 9.5.1). Typical values of c_v for fine-grained soils range from 10^{-3} m^2/day to 10^{-1} m^2/day.

14.15 MODULUS, TIME EFFECT, AND CYCLIC EFFECT FROM PRESSUREMETER TEST

One of the tools with which one can measure a soil modulus in situ is the pressuremeter (PMT). There are three types of pressuremeters: the preboring pressuremeter, the self-boring pressuremeter, and the push-in or cone pressuremeter. With a preboring PMT, a borehole is drilled first; then the drilling tool is removed and the PMT probe is inserted in the open hole. With a self-boring PMT, the probe is equipped with its own drilling equipment, and bores itself into the soil to avoid decompression of the soil due to preboring. With a push-in PMT, the probe is pushed into the soil and full displacement takes place during the insertion, as in the cone penetrometer test. This section refers to tests done with the preboring pressuremeter, which is by far the most common of the three.

The stress tensor in the soil can be decomposed into the spherical component (confinement) and the deviatoric component (shearing). The consolidation test mostly generates an increase in confinement, and the spherical part of the stress tensor dominates the deformation process measured in that test. The pressuremeter test, in contrast, mostly generates an increase in shear around the probe, while the mean confining stress remains relatively unchanged. Thus, for any deformation process in which shearing dominates, the PMT is a good candidate test.

A PMT curve is shown in Figure 14.15. The pressure p_y is the yield pressure, and it represents an important threshold of pressure. If the soil is loaded at pressures below this value, the creep deformation will be small. Such creep deformation will increase gradually and significantly beyond p_y. The modulus E_o is the modulus obtained from the first loading part of the PMT curve below p_y (see Chapter 7, section 7.3). This modulus corresponds to about 1% strain and to pressure levels below p_y associated with ordinary foundation work. It is measured in minutes and with the first cycle of loading. Thus, it is a relatively low modulus that tends to give reasonable to conservative values of settlement when used in elastic equations. Typical values and correlations for the PMT modulus are presented in Chapter 7 (see Tables 7.3 to 7.5).

The influence of time on the modulus can be measured if a PMT is run by holding the pressure constant while measuring the relative increase in radius (strain) as a function of time (Figure 14.16). The model described in Eq. 14.13 is used and the exponent n is back-calculated directly from the measurements taken during the test. This is done by plotting the modulus E_o as a function of time on a log-log plot and

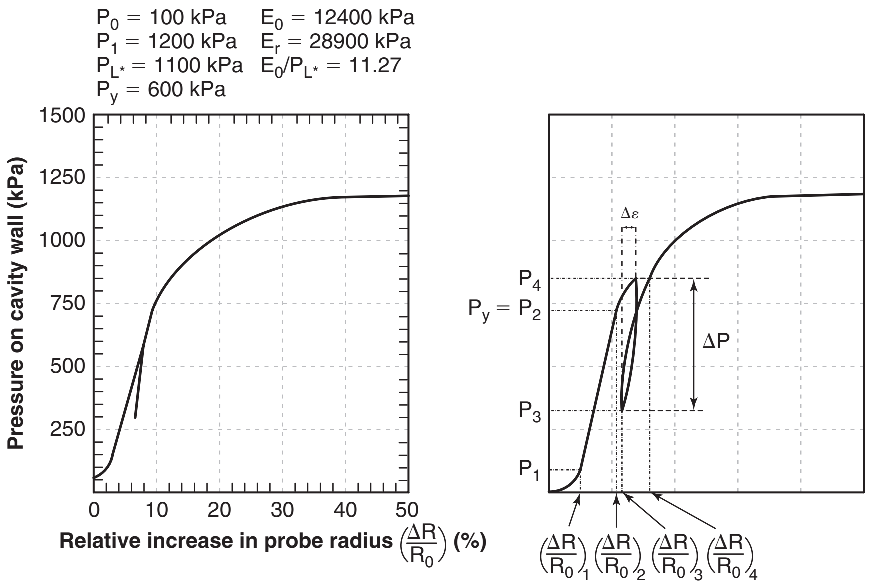

Figure 14.15 Pressuremeter curve and modulus.

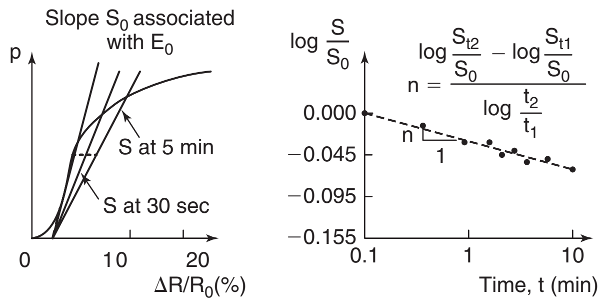

Figure 14.16 PMT modulus as a function of time.

obtaining the slope of that line (Figure 14.16). That way a modulus corresponding to the building settlement after 50 years can be calculated. For example, if E_o is 30 MPa as measured in the PMT in 1 minute; and if n is 0.03, also as measured in the PMT, then the modulus to use for the settlement at 50 years would be:

$$E_{o(50\ \text{years})} = E_{o(1\ \text{min})}\left(\frac{50 \times 365 \times 24 \times 60}{1}\right)^{-0.03} = 0.6E_{o(1\ \text{min})} \quad (14.32)$$

and the settlement at 50 years would be 1.67 times larger than the settlement at 1 minute.

The unload-reload modulus E_r comes from an unload-reload loop performed around the value of p_y. The value of E_r depends on the amplitude of the unload-reload cycle ($\Delta p = p_4 - p_3$; Figure 14.15). The larger Δp is, the smaller E_r will be. The strain amplitude $\Delta\varepsilon$ corresponding to the pressure amplitude Δp can be controlled and the unload-reload modulus can be associated with that strain amplitude as a way to obtain a modulus as a function of strain:

$$E_{\sigma\varepsilon} = \left(\frac{1}{E_{\sigma i}} + \frac{\Delta\varepsilon}{p_L}\right)^{-1} \quad (14.33)$$

$$\Delta\varepsilon = \left(\frac{\Delta R}{R_0}\right)_4 - \left(\frac{\Delta R}{R_0}\right)_3 \quad (14.34)$$

where $E_{\sigma\varepsilon}$ is a modulus corresponding to the strain amplitude $\Delta\varepsilon$, $E_{\sigma i}$ would be obtained from a very small unload-reload loop at p_4, $\Delta\varepsilon$ is given by Eq. 14.34 and Figure 14.15, and p_L is the PMT limit pressure. In practice, the unload-reload loop is performed by unloading the pressure around the p_y value (p_4; Figure 14.15) down to a pressure equal to about one-half of that value ($\Delta p = p_4/2$). The reload modulus E_r obtained in this fashion and used with elastic equations seems to give reasonable to optimistic values of settlement.

A PMT modulus can also be obtained as a function of the number of cycles by repeating the unload-reload loop (Figure 14.17). As can be seen, the secant modulus E_{sN} to the top of the cycle will decrease as the number of cycles increases, but the cyclic modulus E_{cN} obtained from the slope of the unload-reload loop will increase—at least at low stress-to-strength ratios. In both cases, the cyclic exponent can be obtained from the evolution of the modulus as a function of cycles. It remains important to think whether or not the loading process around the pressuremeter (stress path) is analogous to the loading process in the geotechnical problem at hand.

$$E_{sN} = E_{s1}N^{-n_c} \quad (14.35)$$

$$E_{cN} = E_{c1}N^{n_c} \quad (14.36)$$

14.16 RESILIENT MODULUS FOR PAVEMENTS

The *resilient modulus* is used in pavement engineering to quantify the deformation characteristic of the various layers involved in the response of a pavement to the cyclic loading of traffic. It is measured in a triaxial test and is defined as the ratio of the applied cyclic stress to the recoverable strain after many cycles of repeated loading (Figure 14.18):

$$M_R = \frac{\Delta\sigma_c}{\Delta\varepsilon_c} \quad (14.37)$$

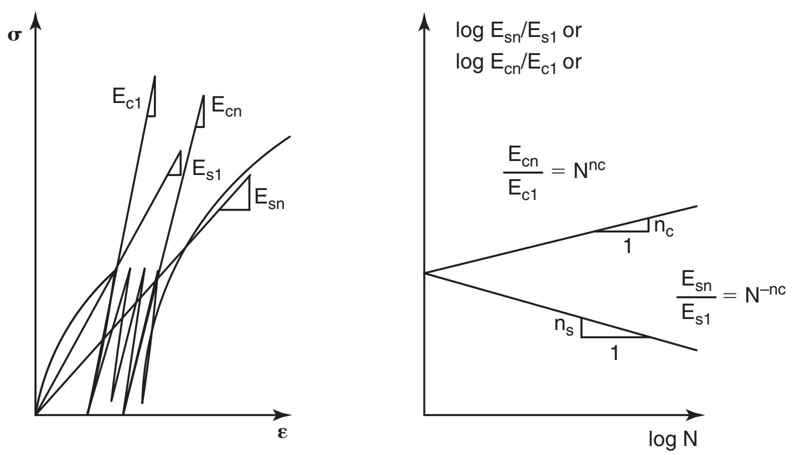

Figure 14.17 PMT modulus as a function of number of cycles.

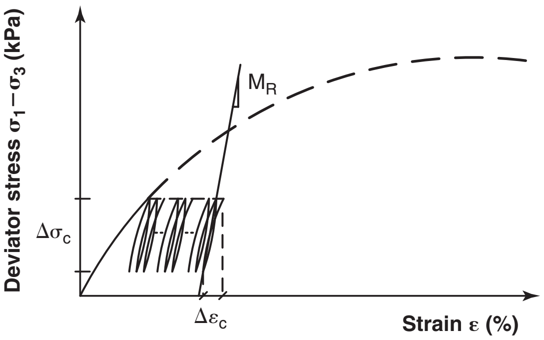

Figure 14.18 Resilient modulus test result.

where $\Delta\sigma_c$ is the amplitude of the deviatoric cyclic stress $(\sigma_1 - \sigma_3)$ over which the cycles are performed (Figure 14.18) and $\Delta\varepsilon_c$ is the corresponding strain amplitude after many cycles. The number of cycles should be sufficient for the value of M_R to reach a constant value for several cycles in a row. Generally, the value of M_R will increase (stiffening) as the number of cycles increases, at least if the peak stress is much lower than the strength of the soil. However, if the peak cyclic stress (top of the cycle) is too close to the strength of the soil sample, the value of M_R will decrease as the number of cycles increases, and the sample may fail at a cyclic strength value less than the static strength value.

The confinement stress and the amplitude of the deviator stress should be chosen to match the expected values during the pavement loading. The confinement stress is fairly small for pavements, as the layers are not very thick and the depth is shallow. Typical confinement for pavement layers varies from 30 to 200 kPa. The amplitude of the deviator stress also depends on the depth below the rolling surface, and may be 50 to 200 kPa for cars, 100 to 500 kPa for trucks, and 350 to 1500 kPa for airplanes. Table 14.17 gives some range and typical values of the resilient modulus and Table 14.18 gives some ranges and typical values of the Poisson's ratio for soils in pavement layers.

Table 14.17 Default MR Values for Unbound Granular and Subgrade Materials at Unsoaked Optimum Moisture Content and Density Conditions

USCS Soil Class	Range of resilient modulus (MPa)	Typical value (MPa)
GW	2725–2898	2829
GP	2449–2760	2622
GM	2277–2898	2656
GC	1656–2587	2070
GW-GM	2449–2794	2656
GP-GM	2070–2760	2484
GW-GC	1932–2760	2380
GP-GC	1932–2691	2346
SW	1932–2587	2208
SP	1656–2277	1932
SM	1932–2587	2208
SC	1483–1932	1656
SW-SM	1656–2277	1932
SP-SM	1656–2277	1932
SW-SC	1483–2070	1759
SP-SC	1483–2070	1759
ML	1173–1759	1380
CL	931–1656	1173
MH	480–1207	793
CH	345–931	480

(After FHWA 2006.)

Table 14.18 Typical Poisson's Ratio Values for Geomaterials in Pavements

Material Description	Poisson's ratio ν Range	ν Typical
Clay (saturated)	0.4–0.5	0.45
Clay (unsaturated)	0.1–0.3	0.2
Sandy clay	0.2–0.3	0.25
Silt	0.3–0.35	0.325
Dense sand	0.2–0.4	0.3
Coarse-grained sand	0.15	0.15
Fine-grained sand	0.25	0.25
Bedrock	0.1–0.4	0.25

(After FHWA 2006.)

14.17 UNSATURATED SOILS: EFFECT OF DRYING AND WETTING ON THE MODULUS

When a soil dries, it becomes stiffer, because the water tension that develops acts as glue by increasing the effective stress between particles; thus, the soil skeleton becomes stiffer. This has been studied particularly for pavement geotechnics, where the compaction curve helps to document the influence of the water content on the dry density and on the modulus of a soil. Figure 14.19 shows the variation of the soil modulus as a function of the water content. This modulus curve was obtained with the BCD (see Chapter 9, section 9.4). As can be seen, the modulus decreases drastically on the wet side of optimum, but the variation is not as severe on the dry side of optimum. Note that this variation of modulus with water content is associated with a remolded soil that was prepared by compaction at each water content. It does not represent the increase in modulus as the soil dries.

Some models have been proposed to document the increase in modulus with an increase in water tension and a decrease

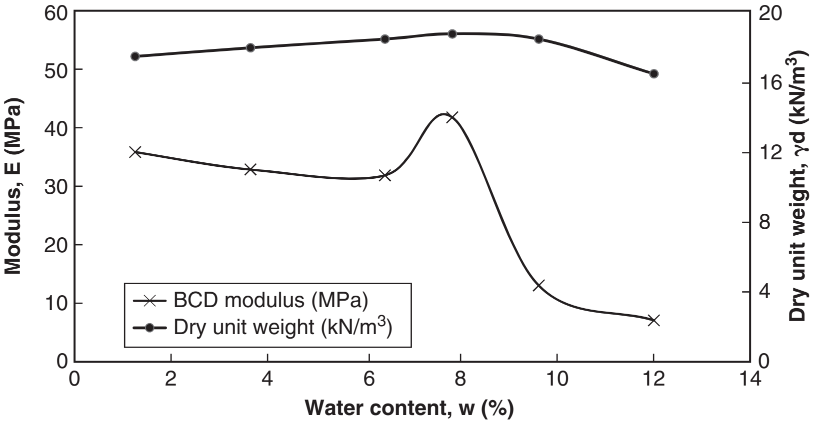

Figure 14.19 Variation of soil modulus measured in the Proctor Compaction Test.

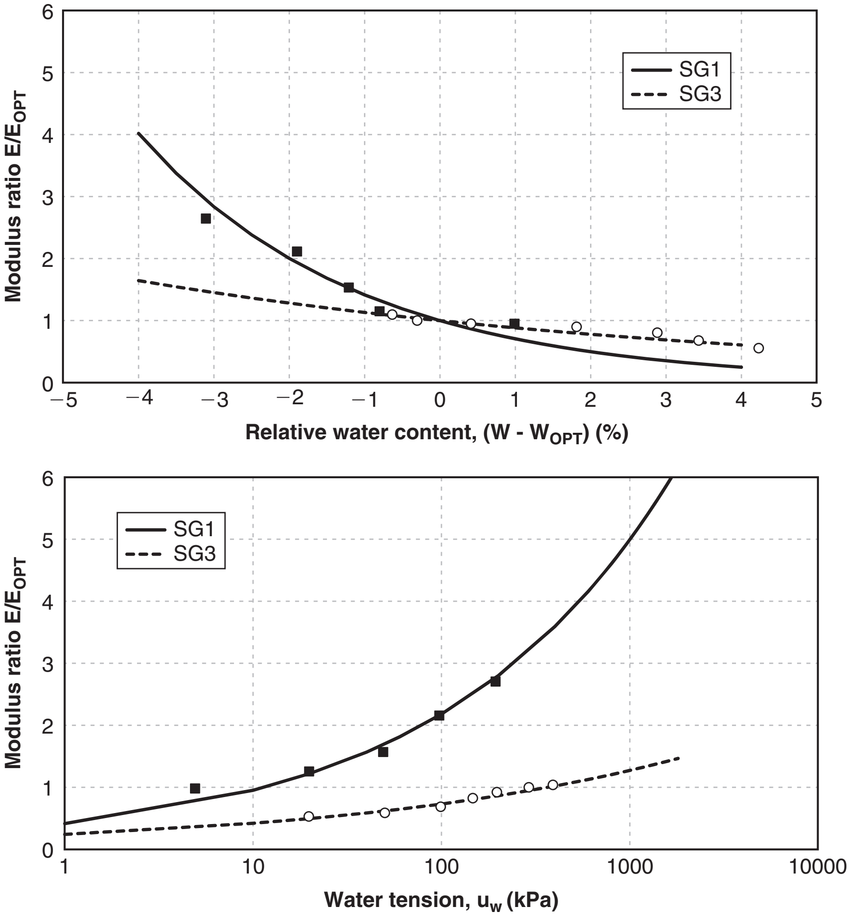

Figure 14.20 Soil modulus as a function of water content and water tension. (After Kim et al. 2006)

in water content. They use the optimum water content w_{opt} as the reference:

$$\frac{E}{E_{opt}} = 10^{k_1(w-w_{opt})} \tag{14.38}$$

where E and E_{opt} are the modulus at the water content w and w_{opt} (in percent) respectively and k_1 is a dimensionless constant. Kim et al. (2006) measured values of k_1 ranging from −0.05 to −0.15 and present the relation shown in Figure 14.20. In terms of water stress or suction:

$$\frac{E}{E_{opt}} = k_2\left(\frac{u_w}{u_{w\ opt}}\right)^{k_3} \tag{14.39}$$

where E and E_{opt} are the modulus at the water tension u_w and the water tension $u_{w\ opt}$ corresponding to the optimum water content respectively, and k_2 and k_3 are dimensionless constants. Kim et al. (2006) measured values of k_2 close to 1 and values of k_3 ranging from 0.25 to 0.35 (Figure 14.20).

14.18 SHRINK-SWELL DEFORMATION BEHAVIOR, SHRINK-SWELL MODULUS

Now let's talk about deformations due to changes in water content or water tension in soils that are not collapsible soils. When the unstressed soil has access to as much water as it can absorb, the soil swells and the water content ends up reaching an equilibrium value called the *free swell limit* w_{SW}. It is not possible for the water content w to be higher than this value under natural conditions. As the soil dries, it shrinks and decreases in volume. This happens because the loss of water brings the particles closer together, as the water no longer occupies that space. During this shrinking process, the soil remains saturated until it reaches the shrinkage limit w_{SH}. Air then enters the voids, the water content continues to decrease, and the soil stops changing volume, or at least the volume change is drastically decreased.

The soil remains saturated from the swell limit to the shrink limit, but the water tension gradually increases. At the shrink limit, the water tension reaches the air entry value u_{wae}, air first enters the soil pores, the soil starts to lose saturation, and the volume stops decreasing (see Chapter 4, Figure 4.15). You can think of this shrinking process as the soil particles coming closer and closer together. At the shrink limit, they touch each other and therefore the volume change stops—but the soil can still lose water if air comes into the voids to replace the water. For high-plasticity clays, this is not quite true: the soil continues to decrease in volume past the shrink limit, but at a much slower rate (Figure 4.15). This is because water is bound to the surface of the particle to various degrees and the transition is not as clearly defined as in the case of a sand or a gravel.

It turns out that the slope of the water content w vs. the relative decrease in volume ΔV/V (volumetric strain) between the swell limit and the shrinkage limit is well approximated by a straight line (Figure 14.21). During the seasons, the soil within a few meters of the ground surface shrinks and swells along this straight line. The double slope line linking the water content to the volumetric strain is to shrink-swell soils what the stress-strain curve is to compressible soils: a constitutive law. In this analogy, the water content plays the role of the stress. Note that the water content could be replaced by the log of the water tension to define the same type of curve. The slope of the straight line from the shrink limit to the swell limit is called the *shrink-swell modulus* E_{SS}, by analogy with the loading problem.

$$E_{SS} = \frac{\Delta w}{\Delta(\Delta V/V)} \qquad (14.40)$$

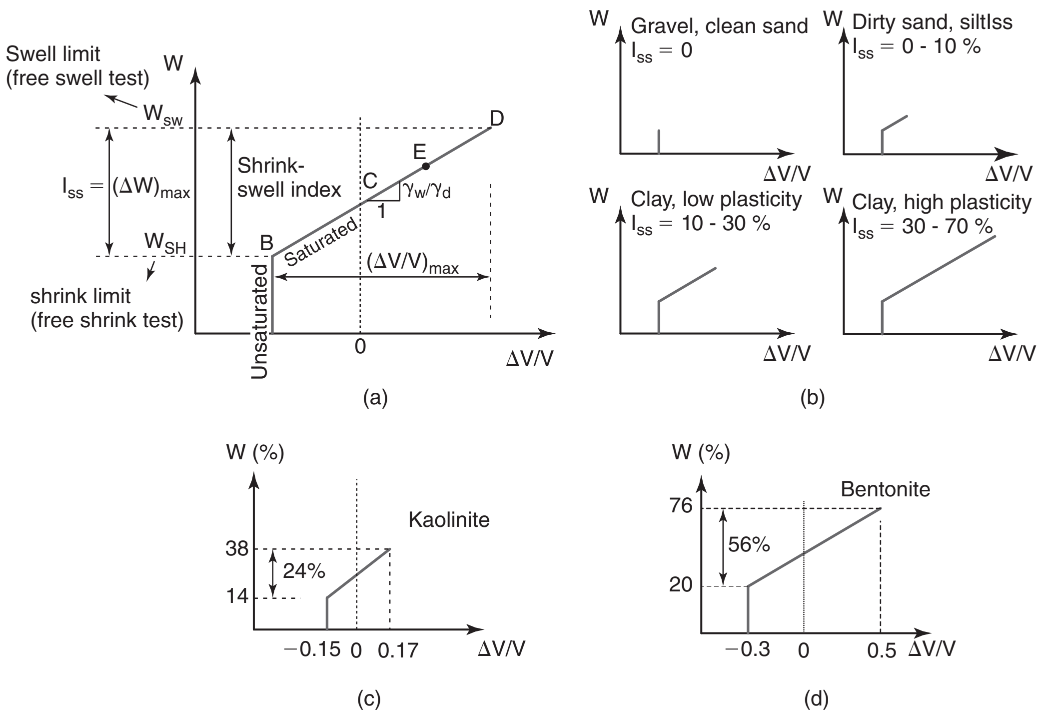

Figure 14.21 Water content versus relative change in volume: *(a)* Idealized behavior. *(b)* Typical ranges. *(c)* Low-plasticity clay, example. *(d)* High-plasticity clay, example.

Recall that the soil remains saturated along the straight line. Therefore, the change in soil volume ΔV along that line is also the change in volume of water ΔV_w and thus:

$$E_{SS} = \frac{\Delta w}{\Delta(\Delta V/V)} = \frac{\Delta W_w}{W_s} \times \frac{V_t}{\Delta V_w} = \frac{\gamma_w}{\gamma_d} \qquad (14.41)$$

Equation 14.41 shows that the shrink-swell modulus E_{SS} is equal to the ratio of the unit weight of water over the dry unit weight of the soil. Unlike the Young's modulus E, the shrink-swell modulus is a constant for a given soil and does not vary much from soil to soil, with values in the range of 0.5 to 1 and an average of 0.7. The shrink-swell modulus E_{SS} can be obtained in the laboratory by performing a free shrink test (see Chapter 9, section 9.6) or a free swell test (section 9.7). The much simpler free shrink test gives the shrink limit, but the free swell test is necessary to obtain the swell limit. The shrink-swell modulus E_{SS} is independent of the stress level. What happens is that if a vertical stress is applied, it will decrease the amount of swelling $\Delta V/V$ and the associated change in water content Δw, but it will not change the ratio that is E_{SS}. The vertical stress can reach a value large enough to prevent swelling altogether; that vertical stress is called the *swelling pressure* p_{SW} of the soil. By the same token, the swell limit depends on the vertical stress and the swell limit w_{SW} refers to the free swell limit, the one obtained when there is no pressure on top of the soil.

If you dug a hole in the soil under your feet, you would likely encounter three zones (Figure 14.22). The first zone would be unsaturated (degree of saturation less than 1). This zone corresponds to water contents below the shrink limit in Figure 14.21; therefore, this zone can swell all the way from the shrink limit to the swell limit if enough water becomes available, but this zone has very little shrinkage potential. The weather or a leaking pipe, for example, could affect the water content and water tension in that zone and create significant swelling. The water tension in this zone is quite high and above the air entry value, as the soil is unsaturated. The thickness of the first zone can vary drastically, from nearly nonexistent in wet regions to hundreds of meters in arid and desert regions.

If you kept on digging, you would encounter a second zone, a zone of saturated soil called the *capillary zone*. There would be no standing water in the hole because you would be above the groundwater level and the water would be held in the soil by water tension. The soil in this zone is at a natural water content somewhere between the swell limit and the shrink limit. Therefore, it can shrink or swell according to any change in water regime. The weather, for example, could affect the water content and water tension in that zone and create significant movements. The thickness of this second zone can also vary drastically, from nearly nonexistent near the coast or near rivers where the groundwater level is high to tens of meters of thickness. The thickness of this zone increases when the soil particles become finer; indeed, the height above the groundwater level to which the soil can draw water up and saturate the zone depends on the size of the soil particles (see Chapter 10, section 10.17.1).

If you kept on digging, you would encounter a third zone, a zone of saturated soil where, given enough time, water would collect in the hole and stand at a constant level called the *groundwater level* (GWL). The water content in this zone is at the swell limit, because the soil has access to all the water it can absorb and has been in this condition for a long time. Note that this swell limit is less than the free swell limit and depends on the vertical stress at rest applied at that depth. In this zone, the soil cannot swell, because it is at the swell limit, but it can shrink should water migrate upward (during a drought, for example). This would be associated with a lowering of the groundwater level.

One good way to tell if a soil is sensitive to shrink-swell deformation is to measure the water content difference between the swell limit and the shrinkage limit. This difference in percent is called the *shrink-swell index* I_{SS}. Shrink-swell

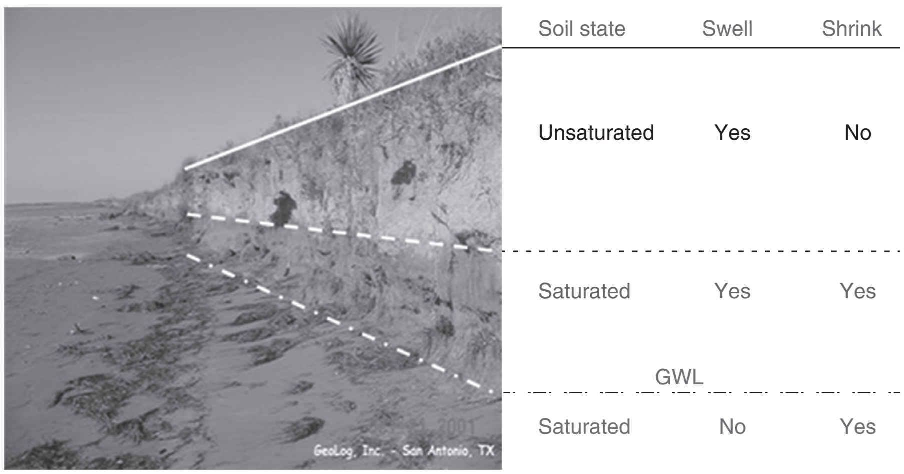

Figure 14.22 The three soil zones. (Courtesy of Art Koenig)

Table 14.19 Shrink-Swell Potential and Various Soil Parameters

Shrink-Swell Potential	Potential Volume Change[1]	Shrink-Swell Index[2]	Plasticity Index[3]	Percent Passing #200[4]	Swell Pressure(kPa)[5]	Slope of SWRC[6]
Very low	<5	<15	<10	<10	<50	>–5
Low	5–10	15–30	10–20	10–30	50–150	–7.5 to –5
Medium	10–20	30–45	20–30	30–60	150–250	–10 to –7.5
High	20–30	45–60	30–40	60–95	250–1000	–17 to –10
Very high	>30	>60	>40	>95	>1000	<–17

[1]Potential volume change ΔV/V in percent from dry to swell limit in free swell test.
[2]Difference between swell limit and shrink limit in percent.
[3]Difference between the liquid limit and the plastic limit in percent.
[4]Percent passing sieve number 200 with opening equal to 0.075 mm.
[5]Lowest pressure necessary to prevent swelling of an inundated sample.
[6]Slope of the soil water retention curve: $\Delta w = C_w \Delta(\log_{10}|u_w|)$ where Δw is the change in water content expressed as a percent and u_w is the water tension in kPa or any other unit, and C_w is the slope of the SWRC; it is negative because when the water content increases, the absolute value of the water tension decreases.

indices lower than 0.2 indicate a soil without much shrink-swell deformation potential. Shrink-swell indices with values above 0.6 indicate a soil with very high shrink-swell deformation potential. Other indices to evaluate the shrink-swell potential of a clay include the plasticity index, the percent passing sieve #200, the swelling pressure, and the slope of the soil water retention curve. Table 14.19 gives some guidance on the shrink-swell potential of clays.

It is the movements in these three zones that accumulate to create the shrink-swell movement of the ground surface. Shrink-swell deformations take place when two conditions exist: (1) there is a water content change between the shrinkage limit and the swell limit, and (2) the soil is sensitive to such water content changes. The natural water content cannot be higher than the free swell limit, by definition of the free swell limit. If the water content changes but remains below the shrink limit, there is either no change or very little change in soil volume, again by definition of the shrink limit. Because the soil is saturated between the shrink limit and the swell limit, most of the volume change of a soil takes place when the soil is saturated. The volume change is given by:

$$\frac{\Delta V}{V} = \frac{\Delta w}{E_{SS}} \tag{14.42}$$

Often, one is not interested in the volume change but in the change in height. Free shrink tests indicate that the shrinkage is about the same in all directions and therefore:

$$\frac{\Delta H}{H} = \frac{\Delta V}{3V} = \frac{1}{3}\frac{\Delta w}{E_{SS}} \tag{14.43}$$

Water content and water tension are tied together through the soil water retention curve. The main part of the SWRC leads to a linear relationship between the water content w and the decimal log of the water tension u_w (see Chapter 9, Figure 9.63):

$$\Delta w = C_w \Delta(\log_{10} u_w) \tag{14.44}$$

Therefore,

$$\frac{\Delta H}{H} = \frac{\Delta V}{3V} = \frac{1}{3}\left(\frac{C_w \Delta\left(\log_{10} u_w\right)}{E_{SS}}\right) \tag{14.45}$$

14.19 COLLAPSE DEFORMATION BEHAVIOR

Collapsible soils consist of loose, dry, low-density materials (say, less than 16 kN/m^3) that decrease in volume (collapse and compact) under the addition of water. These soils are often found in arid regions, specifically in areas of wind-blown silty sediments (loess), young alluvial fans, and debris flow sediments. Soil collapse can occur in these soils when they are above the groundwater level. The process of saturation weakens or eliminates the clay bonds holding the soil grains together through water tension.

Here are some indicators to help recognize if a soil is collapsible (USACE 1990):

1. Liquid limit below 45
2. Plasticity index below 25
3. Dry unit weight between 10 and 17 kN/m^3
4. Porosity between 40 and 60%

Table 14.20 Guidelines to Determine Collapsible Soils

Criterion	Source
$\gamma_d(\text{kN}/m^3) < \dfrac{25.5}{1+0.026\text{LL}(\%)}$	Gibbs and Bara, 1962
or	
$e_o > \dfrac{2.6LL(\%)}{100}$	
$\dfrac{\frac{w_o}{S_o} - PL(\%)}{PI(\%)} > 0.85$	Feda, 1966

γ_d = dry unit weight, LL = liquid limit, e_o = natural void ratio, w_o = natural water content, S_o = natural degree of saturation, PL = plastic limit, PI = plasticity index

Table 14.20 gives additional criteria to determine if a soil can be collapsible or not.

Once it is recognized that a soil may be collapsible using these criteria, the extent of the collapse and its severity can be gauged by the scale proposed by Jennings and Knight (1975). Their scale is based on the collapse potential index CP:

$$CP = \frac{e_o - e_c}{1 + e_o} \times 100 \tag{14.46}$$

where e_o is the void ratio of the soil at its natural water content under 200 kPa of vertical pressure in the consolidation test before wetting, and e_c is the void ratio after soaking under 200 kPa of vertical pressure. Table 14.21 gives a severity scale for the collapse.

The best way to determine the amount of collapse that may occur is to perform a consolidation test and simulate what would happen to the soil in the field. For this, the sample at its natural water content is placed in the consolidometer, the sample height h is recorded, and the vertical pressure p is increased in steps. For each step, the change in height Δh of the sample is recorded every 30 minutes and the curve of stress p vs. strain $\Delta h/h$ is plotted (Figure 14.23). Each pressure is kept on the sample until the rate of strain is less than 0.1%/hour. When the vertical pressure p reaches the pressure p_f anticipated in the field (under the foundation, for example) and at the end of that load step, the sample is inundated and the readings of strain continue during the collapse as a function of time. The end of the collapse step is when the strain has become less than 0.1%/hour. The next pressure step is applied, and so on, until the curve is completed (Figure 14.23). For collapse pressures less than p_f, a line is drawn from A to C on the curve and used to estimate the collapse strain for intermediate values of p_f.

Table 14.21 Severity of Collapsible Soils Scale

CP, Collapse Potential in Percent	Severity
0–1	Negligible
1–5	Moderate trouble
5–10	Trouble
10–20	Severe trouble
> 20	Very severe trouble

(After Jennings and Knight 1975.)

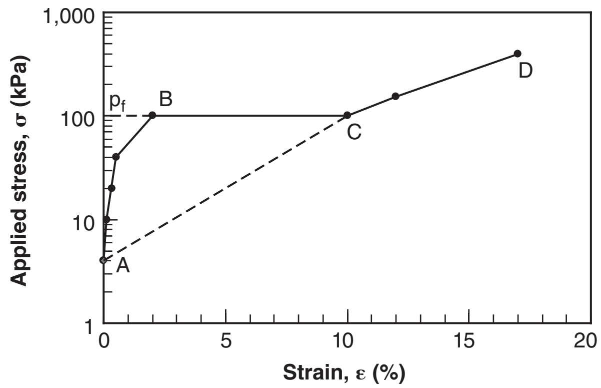

Figure 14.23 Stress-strain curve for a collapsible soil.

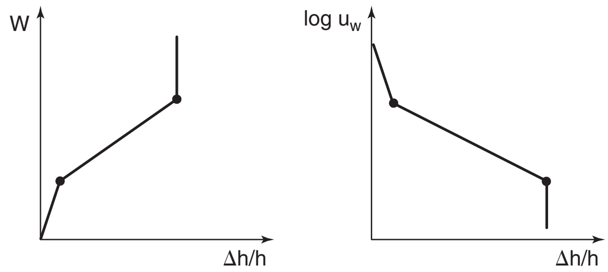

Figure 14.24 Water content and water tension vs. collapse strain.

In this consolidation test, the sample is inundated and the collapse occurs under the extreme situation in which the sample can absorb all the water it is able to absorb. In reality, there could be a limit to the amount of water available to the soil (the rainstorm stops, for example) and the collapse might be partial. A model to predict the collapse strain under partial wetting links the water content to the strain or the water tension to the strain (Figure 14.24) (Pereira and Fredlund 2000).

PROBLEMS

14.1 Consider the stress-strain curve from a triaxial test shown in Figure 14.1s:
   a. Why is $\varepsilon_r < 0$ when $\varepsilon_z > 0$?
   b. Calculate Poisson's ratio.
   c. Calculate the soil modulus between 0 and A.
   d. Calculate the ratio between this soil modulus and the modulus of concrete.

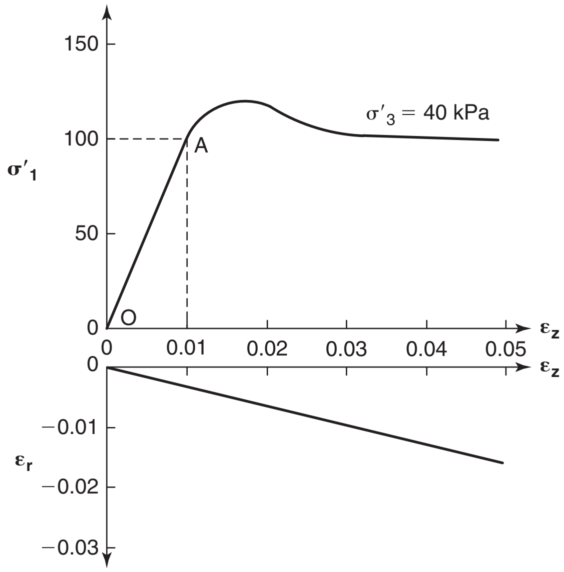

Figure 14.1s Triaxial test results.

14.2 Given that Figure 14.2s is the result of an unconfined compression test, calculate the secant modulus (OA), the unload modulus (AB), the unload-reload modulus (BC), the reload modulus (BD), and the tangent modulus at A. Which is the smallest modulus? Which is the largest modulus? Which is the right modulus?

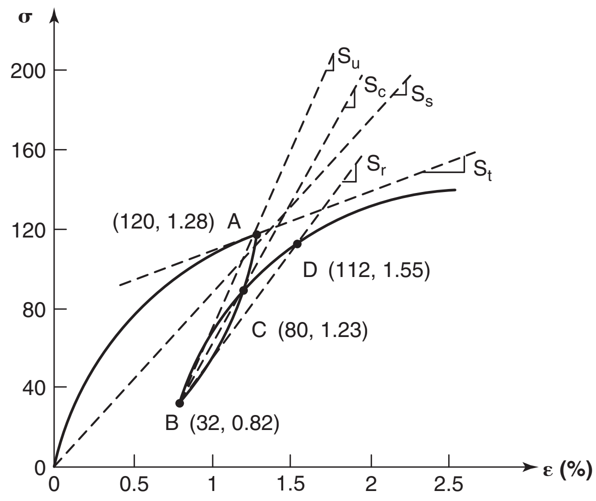

Figure 14.2s Modulus values.

14.3 Explain the difference between the modulus, the stiffness, and the modulus of subgrade reaction. Comment on which one is a true soil property and why.

14.4 Equation 14.12 gives the secant modulus for any confinement level, any strain level, any time of loading, and any number of cycles. *If* E_{ai} is equal to 1000 MPa, n is 0.5, σ_{ult} is 100 kPa, t_o is 1 minute, m is 0.03, and p is 0.1:

a. Plot the initial tangent modulus E_i as a function of the confinement level σ_M for the reference loading time t_o and for monotonic loading ($N = 1$).

b. Plot the secant modulus E_s as a function of the strain level ε for a confinement of 50 kPa, for the reference loading time t_o, and for monotonic loading ($N = 1$).

c. Plot the secant modulus E_s as a function of the time of loading t for a confinement stress of 50 kPa, a strain of 0.5%, and for monotonic loading ($N = 1$).

d. Plot the secant modulus E_s as a function of the number of cycles N for a confinement of 50 kPa, an initial strain of 0.5%, and the reference time t_o.

$$E_{\sigma\varepsilon tN} = \left(\frac{1}{E_{ai}\left(\dfrac{\sigma_M}{p_a}\right)^n} + \frac{\varepsilon}{\sigma_{ult}} \right)^{-1} \left(\frac{t}{t_o}\right)^{-m} N^{-p} \tag{14.12}$$

14.5 A soil sample has a void ratio e = 0.6, an OCR = 2, a PI = 20%, and a shear strength of 40 kPa at a confining pressure of 70 kPa. Use an equation similar to Eq. 14.12 for the shear modulus G and prepare two plots of G/G_{max} versus γ. The first one is G/G_{max} versus γ on natural scales and the second one is G/G_{max} on the vertical natural scale and γ on the horizontal decimal log scale. What other influencing factors are missing from this classical G-γ curve?

14.6 Given the log of vertical effective stress vs. vertical strain curve (Figure 14.3s), find the preconsolidation pressure σ'_p and calculate the compression index C_c.

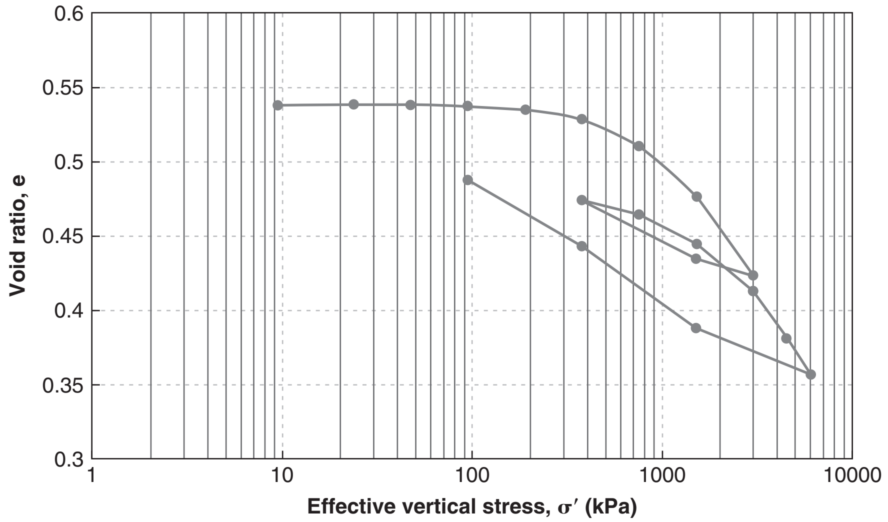

Figure 14.3s Strain vs. log of stress consolidation curve.

14.7 Given a straight-line relationship (Figure 14.4s) between the vertical effective stress and the vertical strain ($\sigma' = \sigma'_{ov} + 40{,}000\ \varepsilon$), a vertical effective stress at rest of 100 kPa, and an initial void ratio e_o of 1, draw the log of vertical stress vs. void ratio curve, and find the preconsolidation pressure σ'_p and the compression index C_c from that curve. Discuss.

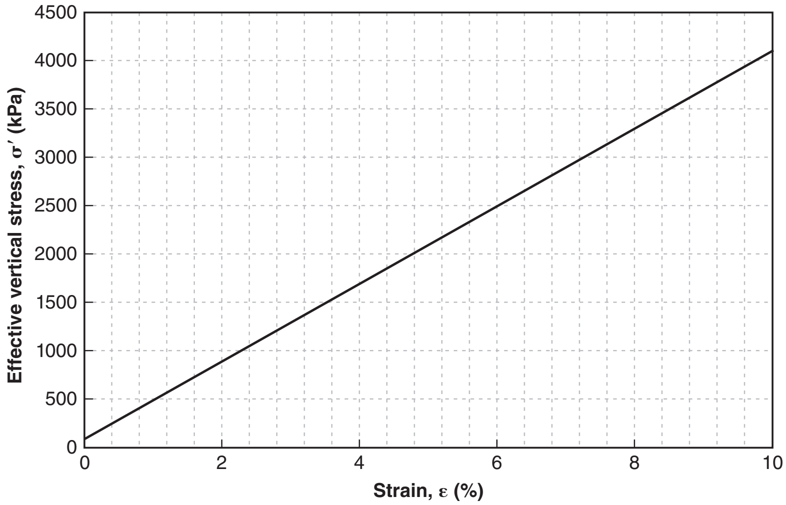

Figure 14.4s Effective vertical stress vs. strain.

14.8 Given the three vertical strain versus time curves of Figure 14.5s from a consolidation test with drainage top and bottom, and the original height of the sample of 14.2 mm, calculate the coefficient of consolidation c_v by the t_{50} method and by the log time method.

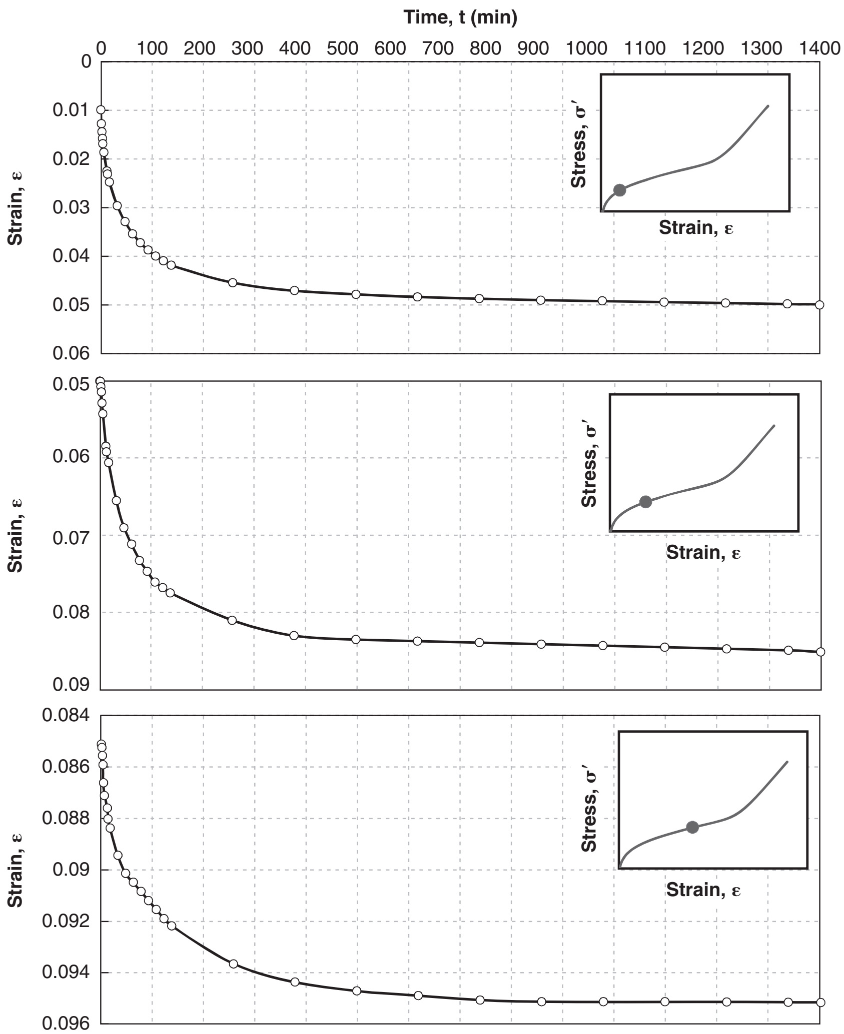

Figure 14.5s Strain vs. time consolidation curves.

14.9 Given the strain versus. time curve of Figure 14.6s, and knowing that the initial void ratio e_o is 0.7, calculate the secondary compression index C_α.

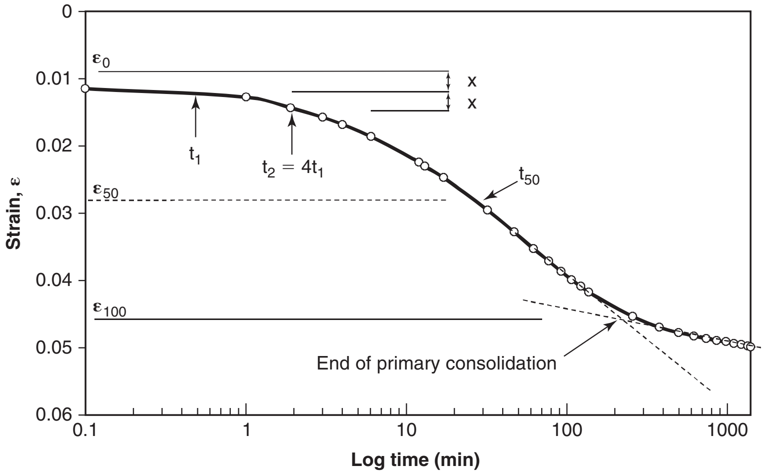

Figure 14.6s Strain vs. log time consolidation curve.

14.10 Devise a pressuremeter test procedure that allows you to measure as many parameters as possible for equation 14.12.

14.11 Give the range of shrink-swell modulus that can be expected for soils. Use that range and the range of shrink-swell indices in Table 14.19 to give the range of expected relative volume change in shrink-swell soils.

14.12 Which of the following two soils is the most likely to collapse upon wetting?

a. Silt with a dry unit weight of 14 kN/m^3, a liquid limit of 40%, a plastic limit of 20%, a porosity of 50%, and a natural water content of 10%

b. Clay with a dry unit weight of 16 kN/m^3, a liquid limit of 55, a plastic limit of 20, a porosity of 35%, and natural water content of 20%

Problems and Solutions

Problem 14.1

Consider the stress-strain curve from a triaxial test shown in Figure 14.1s:

a. Why is $\varepsilon_r < 0$ when $\varepsilon_z > 0$?
b. Calculate Poisson's ratio.
c. Calculate the soil modulus between 0 and A.
d. Calculate the ratio between this soil modulus and the modulus of concrete.

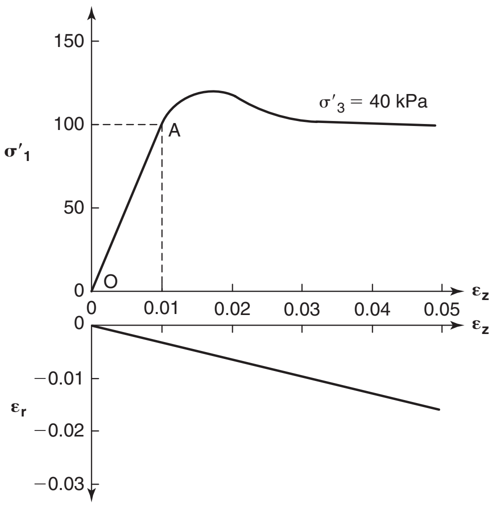

Figure 14.1s Triaxial test results.

Solution 14.1

The Poisson's ratio is obtained from the elasticity equations (Eq. 14.3). In the principal directions, the Poisson's ratio is given by:

$$\nu = \frac{-\varepsilon_3\sigma_1 + \varepsilon_1\sigma_3}{\varepsilon_1\sigma_1 + \varepsilon_1\sigma_3 - 2\varepsilon_3\sigma_3} \tag{14.1s}$$

Considering the geometry of the triaxial test:

$$\varepsilon_z = \varepsilon_1 \quad \textit{and} \quad \varepsilon_r = \varepsilon_3 \tag{14.2s}$$

At point A, the values of the strains are:

$$\varepsilon_z = \varepsilon_1 = 0.01 \quad \textit{and} \quad \varepsilon_r = \varepsilon_3 = -0.004 \tag{14.3s}$$

At point A, the values of the stresses are:

$$\sigma_z = \sigma_1 = 100\ \text{kPa} \quad \textit{and} \quad \sigma_r = \sigma_3 = 40\ \text{kPa} \tag{14.4s}$$

Then the Poisson's ratio is calculated as:

$$\nu = \frac{-(-0.004) \times 100 + 0.01 \times 40}{0.01 \times 100 + 0.01 \times 40 - 2 \times (-0.004) \times 40} = 0.47 \tag{14.5s}$$

Note that ν is different from the ratio of $-\varepsilon_r/\varepsilon_z$, which would be 0.4. The soil modulus E (Eq. 14.1) is given by:

$$E = \frac{\sigma_1 - 2\nu\sigma_3}{\varepsilon_1} = \frac{100 - 2 \times 0.47 \times 40}{0.01} = 6240\ \text{kPa} \tag{14.6s}$$

Note that E is different from the ratio σ_1/σ_3, which would be 10,000 kPa. The ratio between this soil modulus and the modulus of concrete (20,000 MPa) is:

$$\frac{E_{Concrete}}{E_{Soil}} = \frac{20000 \times 10^3}{6240} = 3205 \tag{14.7s}$$

Problem 14.2

Given that Figure 14.2s is the result of an unconfined compression test, calculate the secant modulus (OA), the unload modulus (AB), the unload-reload modulus (BC), the reload modulus (BD), and the tangent modulus at A. Which is the smallest modulus? Which is the largest modulus? Which is the right modulus?

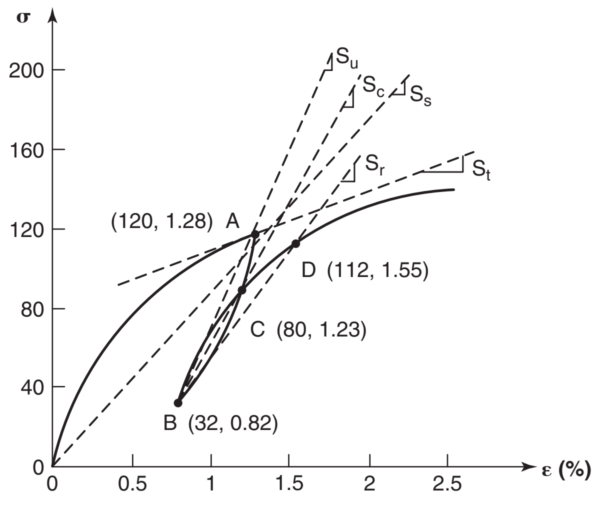

Figure 14.2s Modulus values.

Solution 14.2

Because the test is an unconfined compression test, each modulus can be calculated directly from the slopes in Figure 14.2s. From the secant slope OA (S_s), the secant modulus is:

$$E_s = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{120 - 0}{0.0128 - 0} = 9375\text{ kPa} \tag{14.8s}$$

From the unload slope AB (S_u), the unload modulus is:

$$E_u = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{120 - 32}{0.0128 - 0.0082} = 19130\text{ kPa} \tag{14.9s}$$

From the cyclic slope BC (S_c), the unload-reload modulus is:

$$E_c = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{80 - 32}{0.0123 - 0.0082} = 10707\text{ kPa} \tag{14.10s}$$

From the reloading slope BD (S_r), the reload modulus is:

$$E_r = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{112 - 32}{0.0155 - 0.0082} = 10960\text{ kPa} \tag{14.11s}$$

From the tangent slope (S_t), the tangent modulus is:

$$E_t = \frac{\sigma}{\varepsilon} = \frac{167 - 80}{0.03 - 0} = 2900\text{ kPa} \tag{14.12s}$$

- The smallest modulus is the tangent modulus.
- The largest modulus is the unload modulus.
- There is no “right” modulus. Each slope or modulus is used for different situations or applications.

Problem 14.3

Explain the difference between the modulus, the stiffness, and the modulus of subgrade reaction. Comment on which one is a true soil property and why.

Solution 14.3

The modulus of deformation (kN/m^2) is defined by the equations of elasticity and as the slope of the line of a stress-strain curve of a material in the case of an unconfined compression test. Stiffness (kN/m) is the ratio of a force Q applied on a boundary through a loading area divided by the displacement s experienced by the loaded area (square or circular shape). The modulus of subgrade reaction (kN/m^3) is the ratio of pressure p applied to the boundary through a loading area divided by the displacement s experienced by the loaded area. Only the modulus of deformation is a true soil property, because stiffness and modulus of subgrade reaction depend on the size of the loaded area. The results of stiffness and modulus of subgrade reaction in one test will be different from the results of other tests with different areas. The modulus of deformation for the same material is not affected by the size of the loaded area.

Problem 14.4

Equation 14.12 gives the secant modulus for any confinement level, any strain level, any time of loading, and any number of cycles. If E_{ai} is equal to 1000 MPa, n is 0.5, σ_{ult} is 100 kPa, t_o is 1 minute, m is 0.03, and p is 0.1:

a. Plot the initial tangent modulus E_i as a function of the confinement level σ_M for the reference loading time t_o and for monotonic loading ($N = 1$).
b. Plot the secant modulus E_s as a function of the strain level ε for a confinement of 50 kPa, for the reference loading time t_o, and for monotonic loading ($N = 1$).
c. Plot the secant modulus E_s as a function of the time of loading t for a confinement stress of 50 kPa, a strain of 0.5%, and for monotonic loading ($N = 1$).

d. Plot the secant modulus E_s as a function of the number of cycles N for a confinement of 50 kPa, an initial strain of 0.5%, and the reference time t_o.

$$E_{\sigma\varepsilon tN} = \left(\frac{1}{E_{ai}\left(\frac{\sigma_M}{p_a}\right)^n} + \frac{\varepsilon}{\sigma_{ult}} \right)^{-1} \left(\frac{t}{t_o}\right)^{-m} N^{-p} \tag{14.12}$$

Solution 14.4

a. For the initial tangent modulus, for the reference time t_o, and for monotonic loading ($N = 1$), the general equation becomes as shown in Eq. 14.13s, and the results are plotted in Figure 14.7s.

$$E_{\sigma\varepsilon tN}(\text{MPa}) = E_{ai}\left(\frac{\sigma_M}{p_a}\right)^n = 1000\left(\frac{\sigma_M\ (\text{kPa})}{100}\right)^{0.5} \tag{14.13s}$$

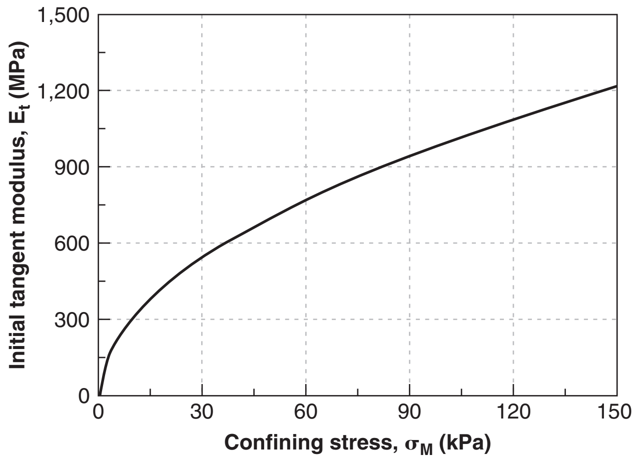

Figure 14.7s Initial tangent modulus vs confining stress.

b. For the secant modulus E_s as a function of the strain level ε for a confinement of 50 kPa, for the reference loading time t_o, and for monotonic loading ($N = 1$), the equation becomes as shown in Eq. 14.14s and the results are plotted in Figure 14.8s:

$$E_{\sigma\varepsilon tN}(\text{MPa}) = \left(\frac{1}{E_{ai}\left(\frac{\sigma_M}{p_a}\right)^n} + \frac{\varepsilon}{\sigma_{ult}} \right)^{-1} = (1.414 \times 10^{-3} + 10\varepsilon)^{-1} \tag{14.14s}$$

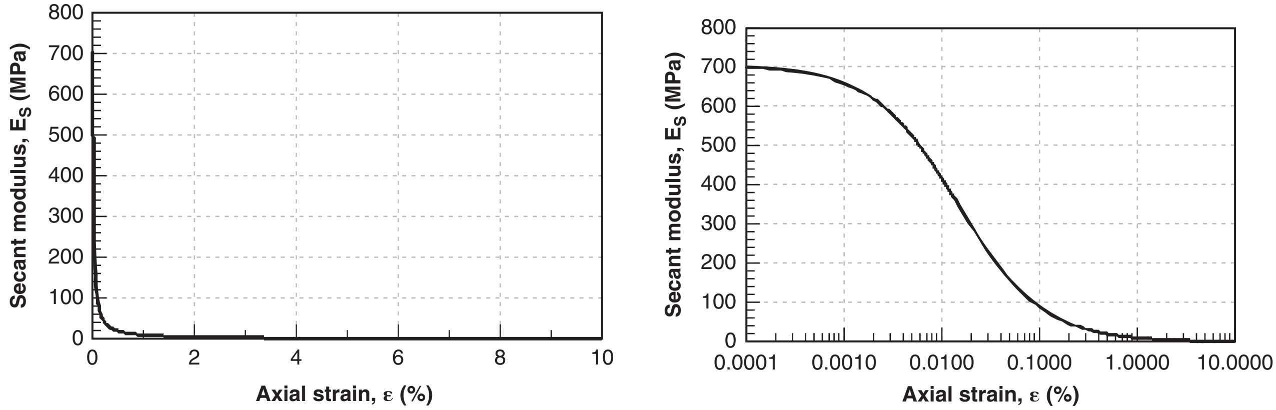

Figure 14.8s Secant modulus vs. axial strain.

c. For the secant modulus E_s as a function of the time of loading t for a confinement stress of 50 kPa, a strain of 0.5%, and for monotonic loading ($N = 1$), the equation becomes as shown in Eq. 14.15s and the results are plotted in Figure 14.9s:

$$E_{\sigma\varepsilon tN}(\text{MPa}) = \left(\frac{1}{E_{ai}\left(\frac{\sigma_M}{p_a}\right)^n} + \frac{\varepsilon}{\sigma_{ult}} \right)^{-1} \left(\frac{t}{t_o}\right)^{-m} = 19.45 \times (t\,(\text{min}))^{-0.03} \tag{14.15s}$$

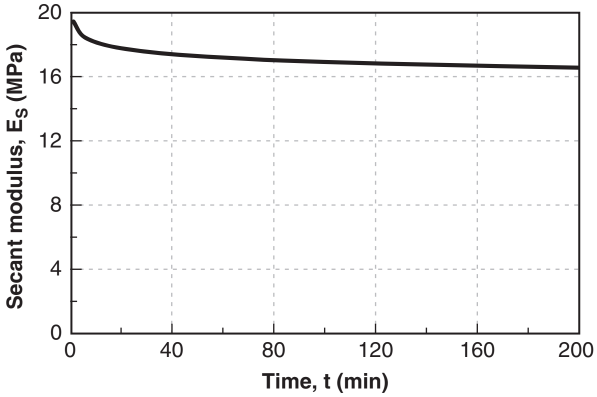

Figure 14.9s Secant modulus vs time

d. For the secant modulus E_s as a function of the number of cycles N for a confinement of 50 kPa, an initial strain of 0.5%, and the reference time t_o, the equation becomes as shown in Eq. 14.16s and the results are plotted in Figure 14.10s:

$$E_{\sigma\varepsilon tN} = \left(\frac{1}{E_{ai}\left(\frac{\sigma_M}{p_a}\right)^n} + \frac{\varepsilon}{\sigma_{ult}} \right)^{-1} N^{-p} = 19.45 \times N^{-0.1} \tag{14.16s}$$

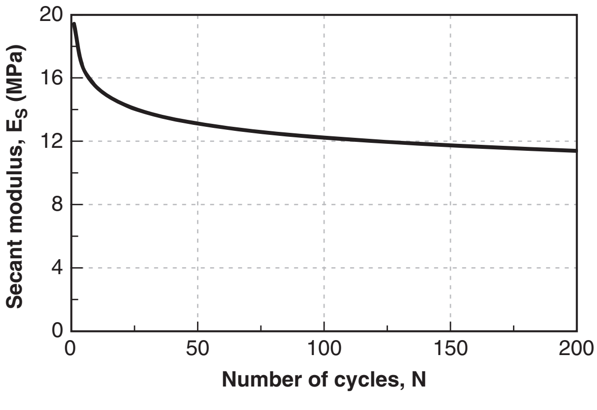

Figure 14.10s Secant modulus vs. number of cycles.

Problem 14.5

A soil sample has a void ratio $e = 0.6$, an OCR $= 2$, a $PI = 20\%$, and a shear strength of 40 kPa at a confining pressure of 70 kPa. Use an equation similar to Eq. 14.12 for the shear modulus G and prepare two plots of $G/G_{\max}$ versus γ. The first one is $G/G_{\max}$ versus γ on natural scales and the second one is G/Gmax on the vertical natural scale and γ on the horizontal decimal log scale. What other influencing factors are missing from this classical G-γ curve?

Solution 14.5

We select an equation of the form

$$G = \left(\frac{1}{G_{\max}} + \frac{\gamma}{s}\right)^{-1} \quad (14.17s)$$

We are given $e = 0.6$, $OCR = 2$, $PI = 20\%$, $p_a = 101.325$ kPa, s = 40 kPa, and $\sigma_M = 70$ kPa. The equation proposed by Jamiolkowski (1991) can be used to estimate the maximum shear modulus ($G_{\max}$). The overconsolidation exponent k is 0.18 for $PI = 20\%$ (Table 14.11).

$$\frac{G_{\max}}{P_a} = \frac{625}{e^{1.3}}(OCR)^k\left(\frac{\sigma'_M}{P_a}\right)^{0.5} \quad (14.18s)$$

$$G_{\max} = 101.325\left(\frac{625}{(0.6)^{1.3}}(2)^{0.18}\left(\frac{70}{101.325}\right)^{0.5}\right) = 115844 \text{ kPa} \quad (14.19s)$$

Then the equation for $G/G_{\max}$ is:

$$\frac{G}{G_{\max}} = \frac{1}{G_{\max}}\left(\frac{1}{G_{\max}} + \frac{\gamma}{s}\right)^{-1} = \frac{1}{115844}\left(\frac{1}{115844} + \frac{\gamma}{40}\right)^{-1} \quad (14.20s)$$

The plots of $G/G_{\max}$ versus γ are shown in Figure 14.11s. This plot includes the effect of strain level and confinement level, but not rate effect or the influence of cycles.

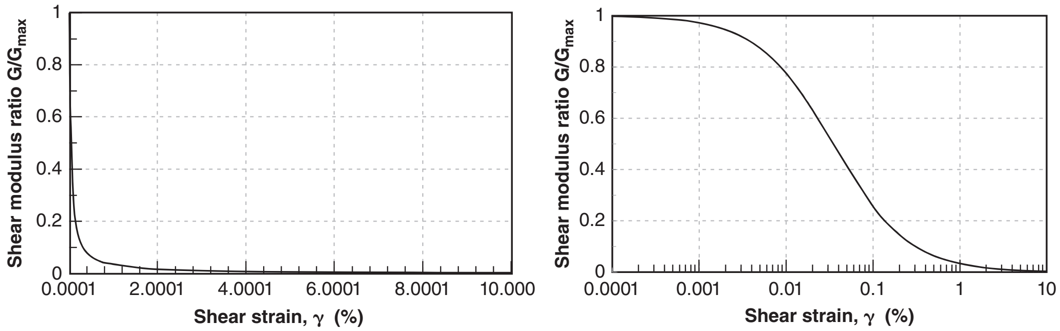

Figure 14.11s $G/G_{\max}$ vs. shear strain.

Problem 14.6

Given the log of vertical effective stress vs. vertical strain curve (Figure 14.3s), find the preconsolidation pressure σ'_p and calculate the compression index C_c.

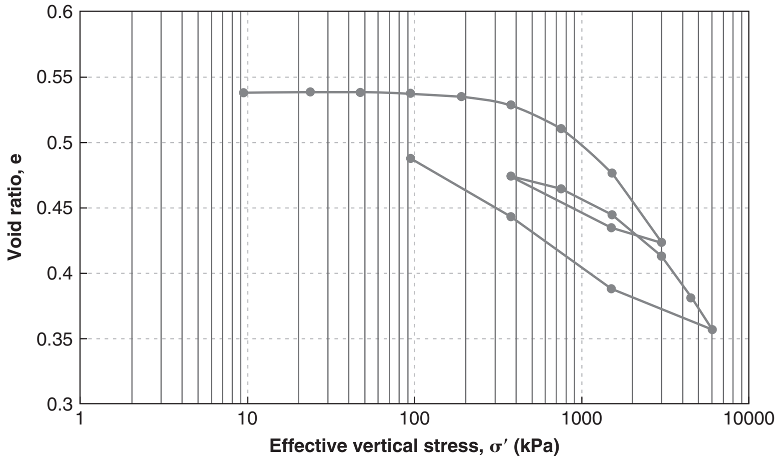

Figure 14.3s Strain vs. log of stress consolidation curve.

Solution 14.6

- Based on Cassagrande's method shown in Figure 14.10:

$$\sigma'_p = 1200(\text{kPa})$$

- The compression index C_c is:

$$C_c = \frac{\Delta e}{\log \sigma_1 - \log \sigma_2} = \frac{0.52 - 0.30}{\log\left(\dfrac{10000}{1000}\right)} = 0.22 \tag{14.21s}$$

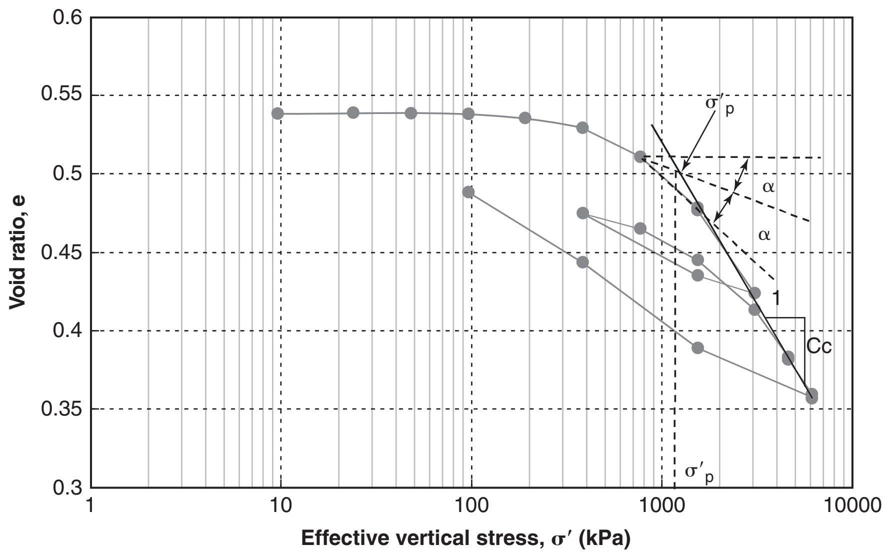

Figure 14.12s Void ratio vs. effective vertical stress.

Problem 14.7

Given a straight-line relationship (Figure 14.4s) between the vertical effective stress and the vertical strain ($\sigma' = \sigma'_{ov} + 40{,}000\,\varepsilon$), a vertical effective stress at rest of 100 kPa, and an initial void ratio e_o of 1, draw the log of vertical stress versus void ratio curve, and find the preconsolidation pressure σ'_p and the compression index C_c from that curve. Discuss.

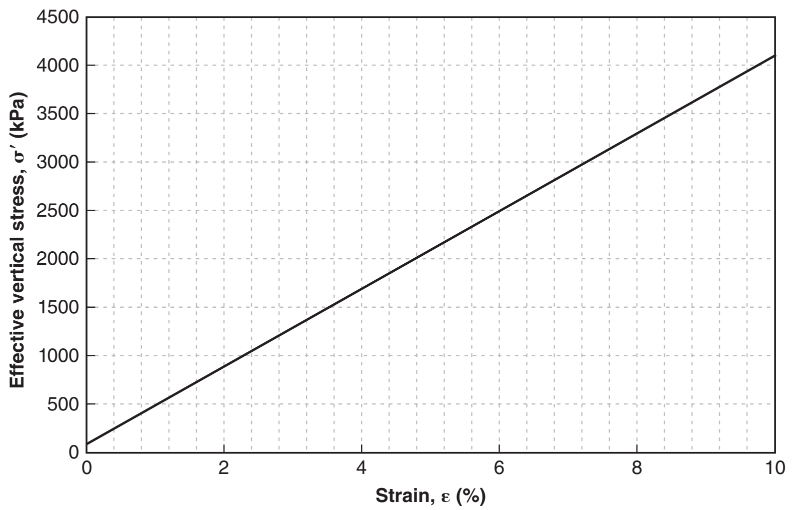

Figure 14.4s Effective vertical stress vs. strain.

Solution 14.7

The relationship between the strain and the void ratio is $\varepsilon = (e - e_o)/(1 + e_o)$. By substituting in the stress-strain equation, we get:

$$\sigma' = \sigma'_{ov} + 40{,}000\left(\frac{e - e_o}{1 + e_o}\right) = 100 + 20{,}000(e - 1) \tag{14.22s}$$

If we plot this equation as $\log \sigma'$ versus e, we get Figure 14.13s. Using this curve and Cassagrande construction, we find a preconsolidation pressure of the order of 2000 kPa (Figure 14.13s). Then we can calculate the compression index as:

$$C_c = \frac{\Delta e}{\log \sigma_1 - \log \sigma_2} = \frac{0.965 - 0.80}{\log\left(\dfrac{4000}{2000}\right)} = 0.548$$

As can be seen from Figure 14.13s, a preconsolidation pressure can be found for a soil that obviously does not have one (Figure 14.14s), as it is linear. The distortion created by the semilog plot creates the apparent preconsolidation pressure in this case.

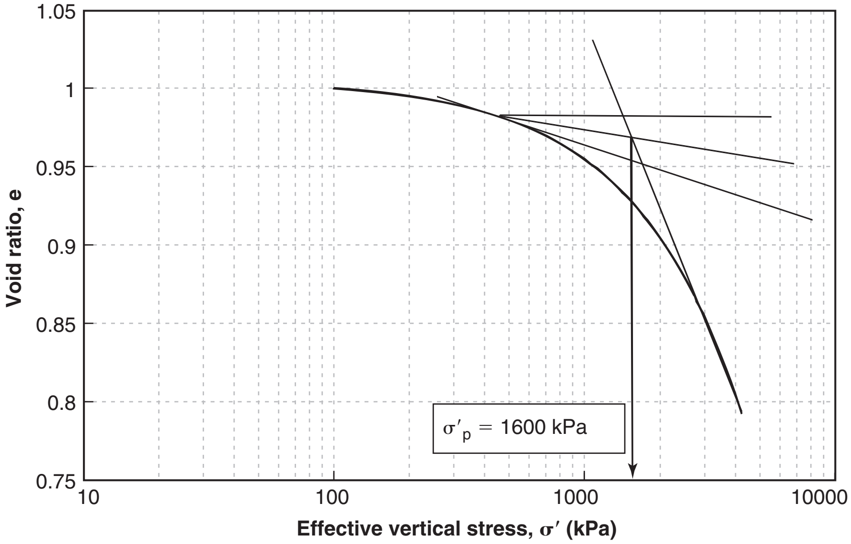

Figure 14.13s Preconsolidation pressure.

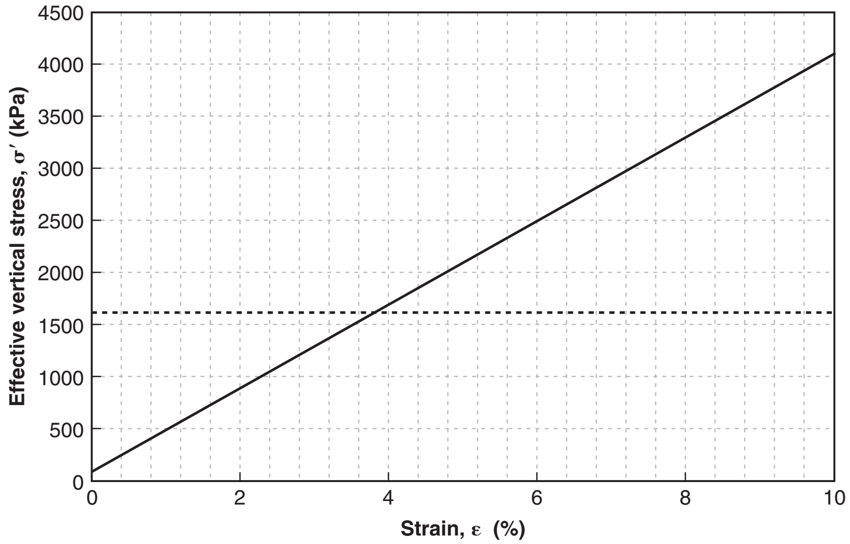

Figure 14.14s Effective vertical stress vs. strain.

Problem 14.8

Given the three vertical strain versus time curves of Figure 14.5s from a consolidation test with drainage top and bottom, and the original height of the sample of 14.2 mm, calculate the coefficient of consolidation c_v by the t_{50} method and by the log time method.

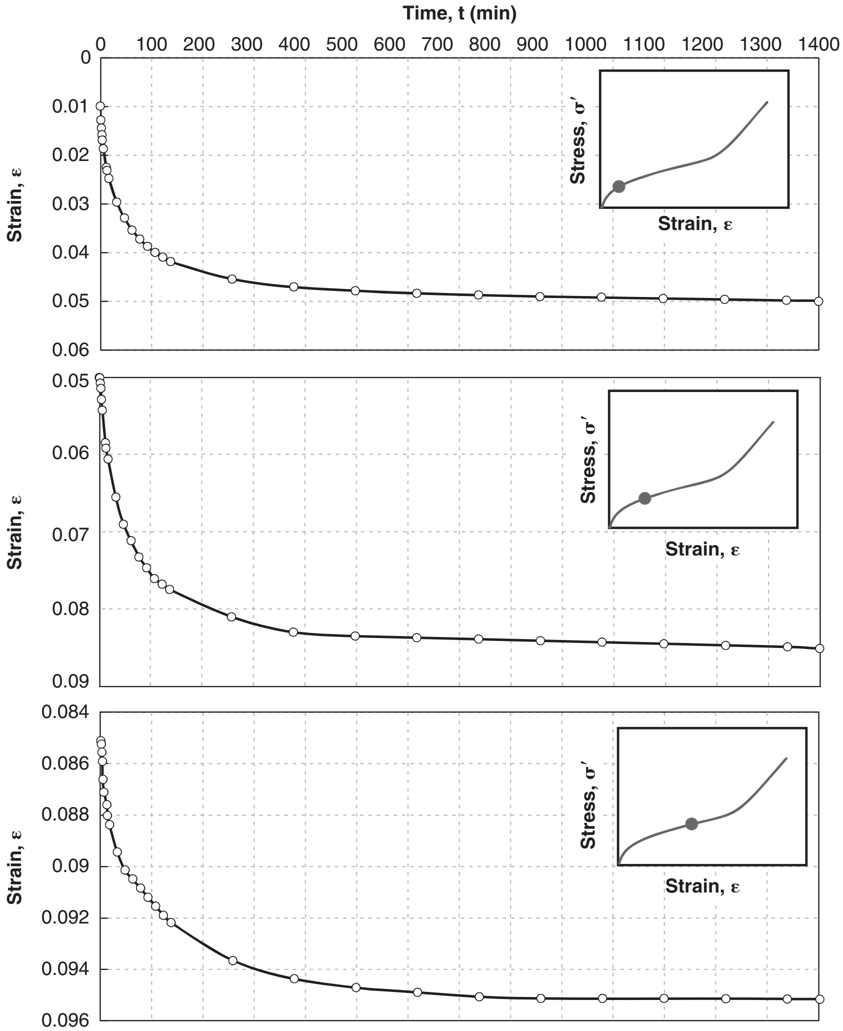

Figure 14.5s Strain vs. time consolidation curves.

Solution 14.8

a. $\sqrt{t}$ Method
   1. Plot the sample height H versus $\sqrt{t}$
   2. Draw the tangent to the initial part of the curve
   3. Choose a point M at an arbitrary but convenient $\sqrt{t_1}$ value and a height H_1 on that tangent
   4. Plot a point N with coordinates $\sqrt{t_2} = 1.15\sqrt{t_1}$ and H_1
   5. Connect N to the start of the curve
   6. The intersection with the curve gives $\sqrt{t_{90}}$
   7. Calculate C_v from the equation:

$$C_v = \frac{0.848\, d^2}{t_{90}} \tag{14.23s}$$

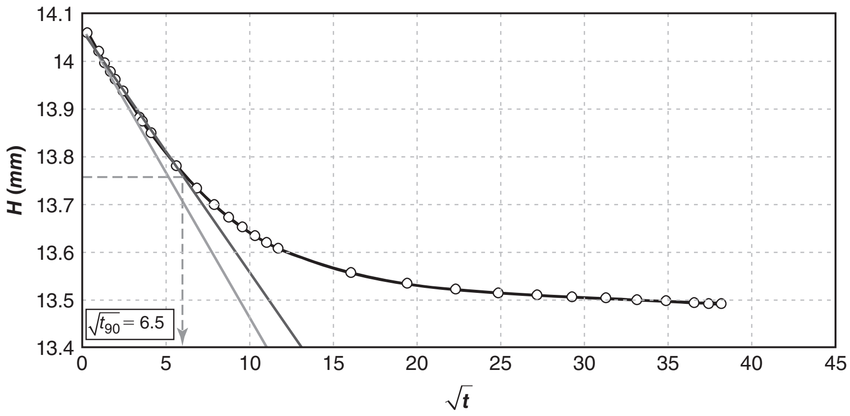

Figure 14.15s Square root of time method.

$$C_{v1} = \frac{0.848d^2}{t_{90}} = \frac{0.848^*(14.2/2)^2}{6.5^2} = 1.0118(\text{mm}^2/\text{min})$$

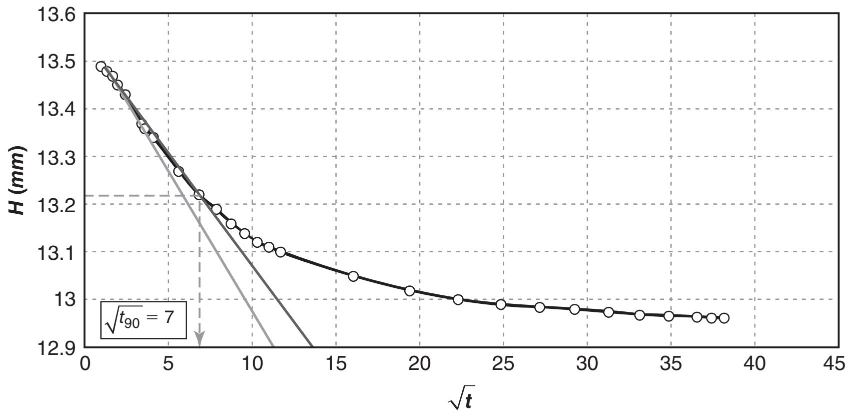

Figure 14.16s Square root of time method.

$$C_{v2} = \frac{0.848d^2}{t_{90}} = \frac{0.848^*(14.2/2)^2}{7^2} = 0.8724(\text{mm}^2/\text{min})$$

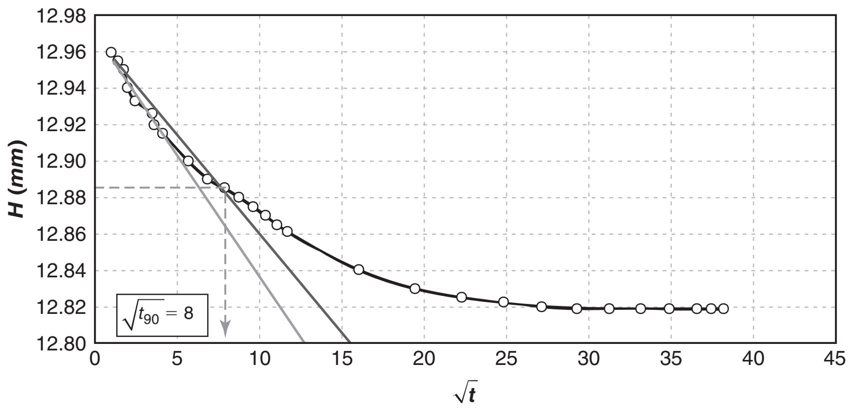

Figure 14.17s Square root of time method.

$$C_{v1} = \frac{0.848d^2}{t_{90}} = \frac{0.848 * (14.2/2)^2}{8^2} = 0.6679(\text{mm}^2/\text{min})$$

b. Log time method (Figure 14.18s)
1. Plot H vs. log time
2. Find H_{100} and H_0 (as shown in Figure 14.18s), then calculate H_{50} from the equation:

$$H_{50} = \frac{H_0 + H_{100}}{2} \quad (14.24s)$$

3. Find t_{50} from the plot
4. Calculate C_v as follows:

$$C_v = \frac{0.197 d^2}{t_{50}} \quad (14.25s)$$

$$H_0 = 14.08(\text{ mm})$$

$$H_{100} = 13.55(\text{mm})$$

$$H_{50} = 13.815(\text{mm}) \rightarrow t_{50} = 20\,\text{min}$$

$$C_v = \frac{0.197 d^2}{t_{50}} = \frac{0.197(14.2/2)^2}{20} = 0.50\ (\text{mm}^2/\text{min})$$

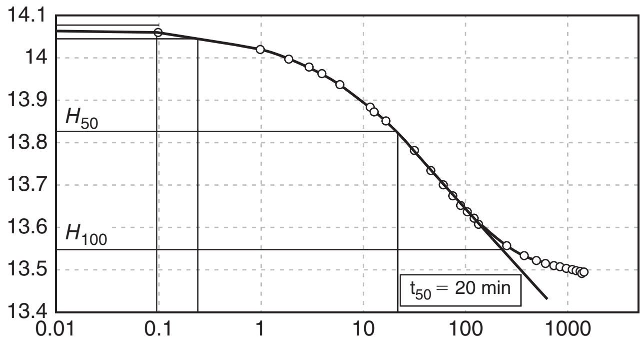

Figure 14.18s Log time method.

Problem 14.9

Given the strain versus time curve of Figure 14.6s, and knowing that the initial void ratio e_o is 0.7, calculate the secondary compression index C_α.

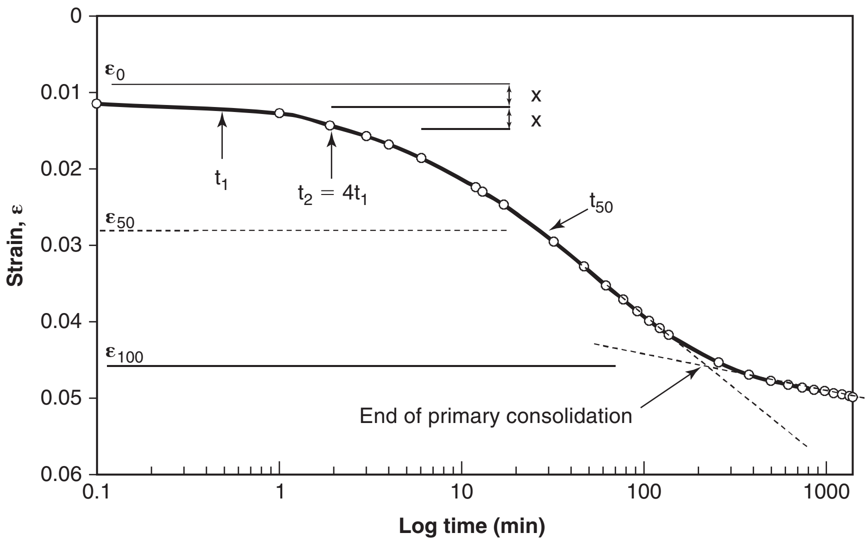

Figure 14.6s Strain vs. log time consolidation curve.

Solution 14.9

$$C_\alpha = \frac{\Delta e}{\Delta \log t} \tag{14.26s}$$

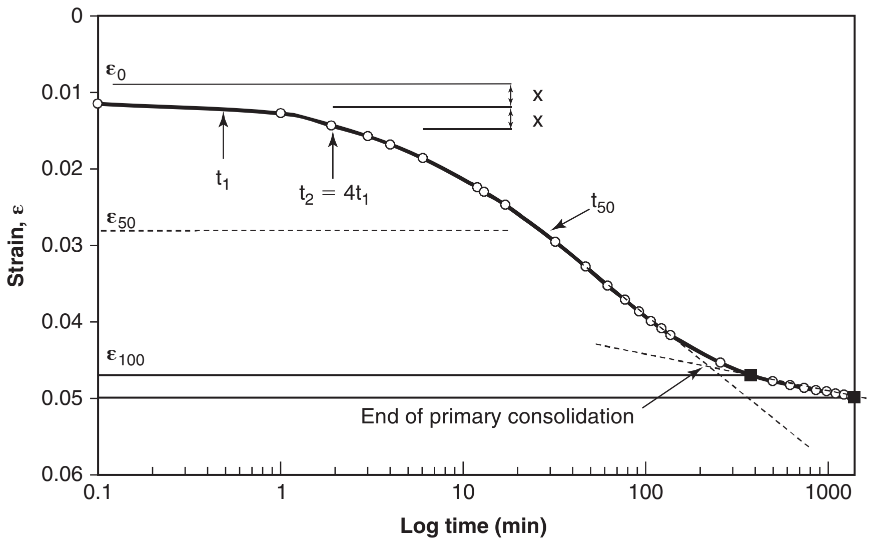

Figure 14.19s Strain vs. log time consolidation curve.

From Figure 14.19s:

$$\begin{aligned} \varepsilon_1 &= 0.04696 \\ t_1 &= 377(\text{min}) \\ \varepsilon_2 &= 0.0499 \\ t_2 &= 1400(\text{min}) \end{aligned}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0} \rightarrow \Delta H = \varepsilon_1 H_0 - \varepsilon_2 H_0 \rightarrow \frac{\Delta H}{H_0} = \Delta\varepsilon \tag{14.27s}$$

$$\Delta\varepsilon = \frac{\Delta e}{1+e_0} \tag{14.28s}$$

$$(0.0499 - 0.04696) = \frac{\Delta e}{1+0.7} \rightarrow \Delta e = 0.004998$$

Recalling Eq. 14.26s:

$$C_\alpha = \frac{0.004998}{\log(1400) - \log(377)} = 0.008772 \qquad (\text{in } 1/\log(\text{minutes}))$$

Problem 14.10

Devise a pressuremeter test procedure that allows you to measure as many parameters as possible for equation 14.12.

Solution 14.10

Strain level influence: A PMT can be run by performing unload-reload cycles around a chosen mean borehole pressure. The cycles would be of increasing amplitude to vary the associated strain, but always around the same mean pressure so that the mean stress level would not change and the influence of the strain amplitude would be isolated. This would quantify the influence of the strain level.

Stress level influence: A PMT can be run by performing unload-reload cycles with the same strain amplitude but at different stress levels. The loops of the cycles would have the same amplitude of $\Delta(\Delta R/R_o)$, but would take place at increasingly higher pressure over limit pressure ratio. This would isolate the influence of the stress level. One must be cautious here and realize that because the pressuremeter test is primarily a shear test, the influence of the stress level would not be the influence of the confinement level.

Time influence: A PMT can be run by holding a chosen pressure p and recording the relative increase in probe radius $\Delta R/R_o$ as a function of time *t*. This $\Delta R/R_o$ vs. t curve will give information on the time dependency of the soil deformation.

Cycle influence: A PMT can be run by performing cycles between two chosen pressure levels. The evolution of the relative increase in radius $\Delta R/R_o$ with the number of cycles *N* will give a quantification of the sensitivity of the soil to cyclic loading.

It is very important, when running these kinds of PMTs, to keep in mind the analogy or difference between the stress path and deformation process around the pressuremeter and in the geotechnical project. The closer the analogy, the more useful the information.

Problem 14.11

Give the range of shrink-swell modulus that can be expected for soils. Use that range and the range of shrink-swell indices in Table 14.19 to give the range of expected relative volume change in shrink-swell soils.

Solution 14.11

The shrink-swell modulus (E_{ss}) is a constant for a given soil and does not vary much from soil to soil, with values in the range of 0.5 to 1. Table 14.1s shows the expected range of relative volume change, $\Delta(\Delta V/V)$, for the corresponding range of shrink-swell indices (I_{ss}):

$$\Delta\left(\frac{\Delta V}{V}\right) = \frac{\Delta w}{E_{ss}} = \frac{I_{ss}}{E_{ss}} \tag{14.29s}$$

Table 14.1s Relative Change in Volume of a Soil in Percent for Various Values of Shrink-Swell Modulus and Shrink-Swell Index

		Shrink-swell index				
		0–15	15–30	30–45	45–60	>60
Shrink-swell modulus	**0.5**	0–30	30–60	60–90	90–120	>120
	0.6	0–25	25–50	50–75	75–100	>100
	0.7	0–21.4	21.4–42.9	42.9–64.3	64.3–85.7	>85.7
	0.8	0–18.8	18.8–37.5	37.5–56.3	56.3–75	>75
	0.9	0–16.7	16.7–33.3	33.3–50	50–66.7	>66.7
	1.0	0–15	15–30	30–45	45–60	>60

Problem 14.12

Which of the following two soils is the most likely to collapse upon wetting?

a. Silt with a dry unit weight of 14 kN/m^3, a liquid limit of 40%, a plastic limit of 20%, a porosity of 50%, and a natural water content of 10%
b. Clay with a dry unit weight of 16 kN/m^3, a liquid limit of 55, a plastic limit of 20, a porosity of 35%, and natural water content of 20%

Solution 14.12

Use the USACE (1990) indicators to determine if a soil is likely to collapse.

a. Silt:
   1. $LL = 40\%$ (less than 45%)
   2. $PI = 40\% - 20\% = 20\%$ (less than 25%)

3. $\gamma_d = 14\ \text{kN/m}^3$ (between 10 and 17)
4. $n = 50\%$ (between 40% and 60%)

b. Clay
1. $LL = 55\%$ (not less than 45%)
2. $PI = 55\%–20\% = 25\%$ (equals 25%)
3. $\gamma_d = 16\ \text{kN/m}^3$ (between 10 and 17)
4. $n = 35\%$ (not between 40% and 60%)

According to these guidelines, the silt is more likely to collapse than the clay.