CHAPTER 17

Shallow Foundations

17.1 DEFINITIONS

Shallow foundations (Figure 17.1) are those placed close to the ground surface, typically at a depth less than one times the width of the foundation. A 1 m thick, 3 m by 3 m foundation under a column, placed at a depth of 1.5 m, would be a shallow foundation called a *spread footing*. Spread footings can be square, circular, or very long compared to their width, in which case they are called *strip footings*. A 3 m thick, 40 m by 40 m square foundation, placed at a depth of 10 m, would be considered a particular type of shallow foundation called a *mat foundation*. A 0.1 m thick, 15 m by 15 m foundation stiffened with 1 m deep beams 3 m apart in both directions would be a shallow foundation called a *stiffened slab on grade*.

17.2 CASE HISTORY

This case history illustrates the behavior of shallow foundations. Five tests of spread footings were performed at the National Geotechnical Experimentation Site at Texas A&M University. The soil at the site is a medium-dense, fairly uniform, silty fine silica sand with the following average properties near the footings and within the top 5 meters: mean grain size $D_{50} = 0.2$ mm, SPT (standard penetration test) blow count 18 blows per 0.3 m, CPT (cone penetrometer test) point resistance 6 MPa, PMT (pressuremeter test) limit pressure 800 kPa, PMT modulus 8.5 MPa, DMT (dilatometer test) modulus 30 MPa, borehole shear test friction angle 32^o, estimated total unit weight 15.5 kN/m^3, and cross hole shear wave velocity 240 m/s. The water table is 4.9 m deep. Additional data can be found in Briaud and Gibbens (1999; 1994). Geologically, the top layer of sand is a flood plain deposit of Pleistocene age about 3 m thick with a high fine content. The next layer of sand is a river channel deposit of Pleistocene age about 3 m thick, clean and uniform. The third layer is a mixed unit with an increasing amount of clay seams and gravel layers; it is also of Pleistocene age and was deposited by a stream of fluctuating energy. Below these 200,000-year-old sand layers and about 10 m below the ground surface is the 45-million-year-old Eocene bedrock; this bedrock is a dark gray clay shale that was deposited in a series of marine transgressions and regressions. Erosion of the Eocene marine clay took place before the Pleistocene river sediments were deposited.

The test setup is shown in Figure 17.2. The 5 footings were square with a side dimension equal to 1 m, 1.5 m, 2.5 m, 3 m, and 3 m. They were embedded 0.75 m into the sand and were 1.2 m thick. They were loaded in load step increments, each one lasting 30 minutes, while settlement was recorded every minute during the load step. All footings were pushed downward until the settlement reached 0.15 m. Figure 17.3 shows an example of the load settlement curve obtained for the 3 m by 3 m north footing, as well as the log of the settlement vs. the log of time for several load steps. The pressure vs. settlement curves for all footings are shown in Figure 17.4. These curves were normalized by dividing the pressure by the limit pressure of the pressuremeter and the settlement by the width of the footing. Figure 17.4 indicates that this normalization makes the footing size disappear: The p/p_L vs. s/B curve becomes a property of the soil, much like a stress-strain curve. Tell tales and inclinometers were placed below and on the side of the footing, respectively. They indicated the depth to which the soil was compressed and the lateral movement of the soil during the load application. Figure 17.5 shows the soil movement as a function of depth for four of the footings and the lateral movement for the 3 m north footing. The data show that most of the settlement and lateral movement occurs within one footing width below the footing.

17.3 DEFINITIONS AND DESIGN STRATEGY

The most important considerations in foundation design are to ensure:

1. The safety of the foundation against soil failure (ultimate limit state)

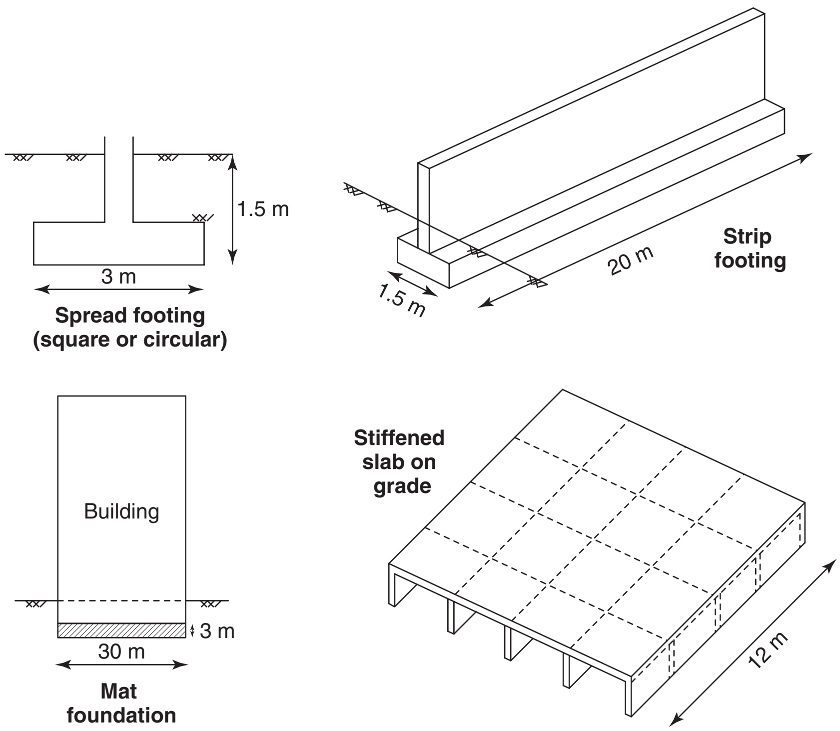

Figure 17.1 Types of shallow foundations.

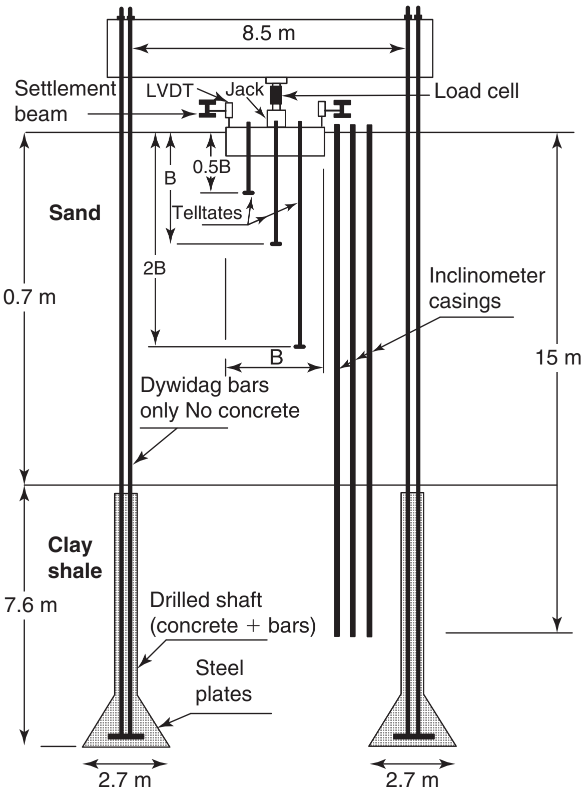

Figure 17.2 Load test setup: (a) Load settlement curve with 30-minute load steps. (b) Settlement time curves for each load step.

2. The functionality of the foundation and the structure above by minimizing the foundation movement and distortion (serviceability limit state)
3. The safety of the foundation against structural failure

Item 3 is handled primarily by the structural engineer and is not covered in this book. Items 1 and 2 in the preceding list are primarily geotechnical engineering considerations involving soil shear strength and the soil increase and decrease in volume when loaded. They are the topic of this chapter for shallow foundations and of Chapter 18 for deep foundations.

The geotechnical design of a shallow foundation consists of estimating the size and depth of the foundation. The depth is chosen on the basis of several factors, including profile of soil strength and compressibility, depth of the zone that shrinks and swells, depth of frost penetration, groundwater level, and ease of construction. The size is typically chosen once the depth is chosen.

No foundation can be designed to ensure zero probability of failure. This is because any calculation is associated with some uncertainty; because the engineering profession's knowledge, while having made great strides, is still incomplete in many respects; because human beings are not error free; because budgets are limited; and because the engineer designs the bridge or building for conditions that do not include extremely unlikely events, such as a big airplane hitting the bridge at the same time as an earthquake, a hurricane,

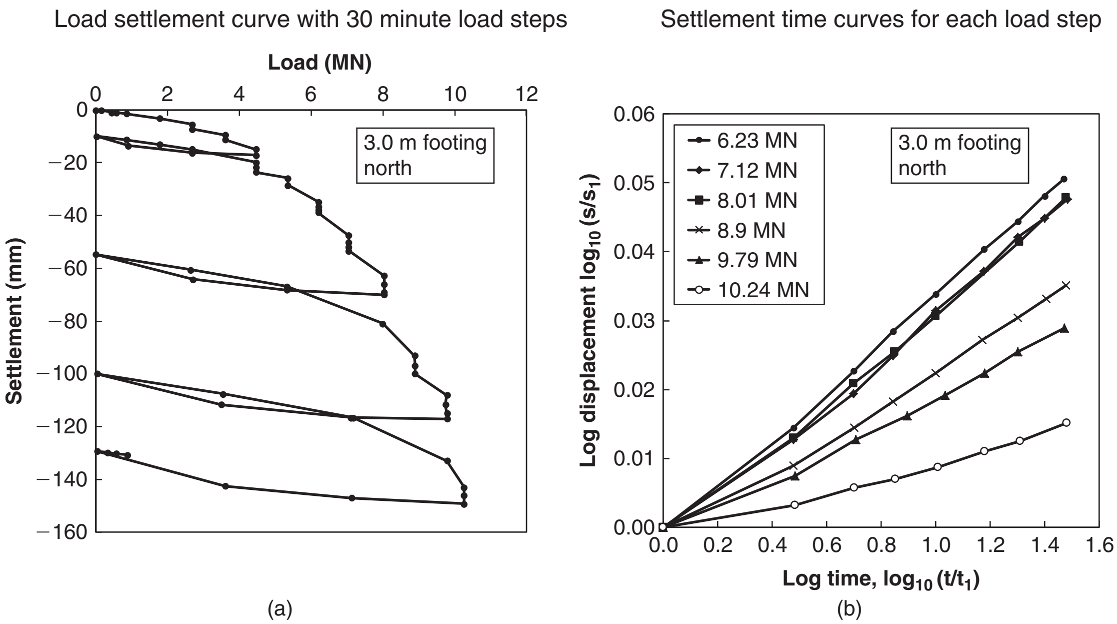

Figure 17.3 Result for the 3 m by 3 m north footing: (a) Pressure-settlement curve. (b) Normalized curves.

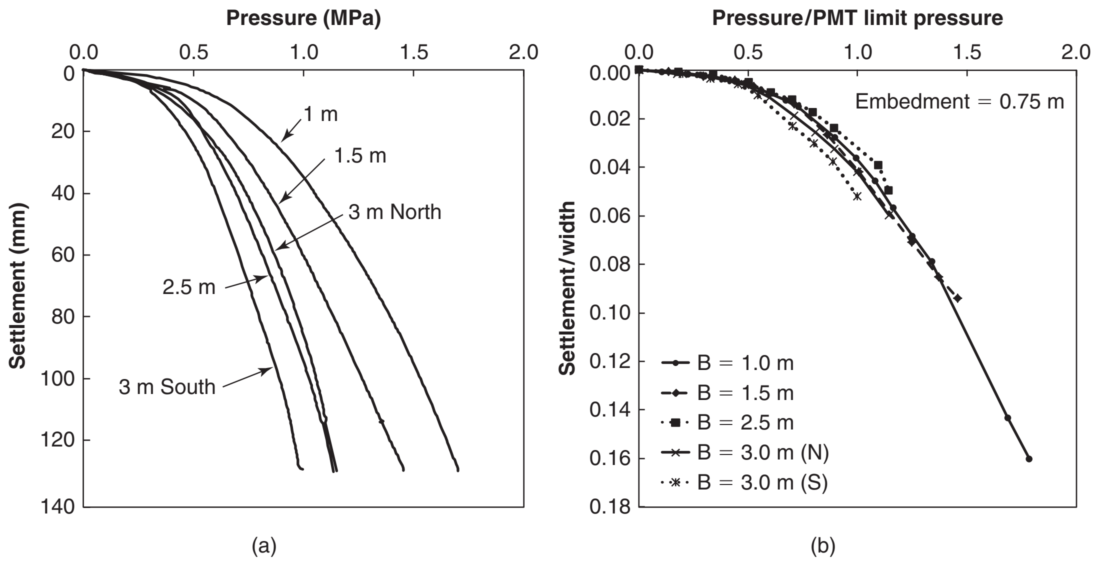

Figure 17.4 Pressure vs. settlement curve for all footings and normalized curves: (a) Pressure-settlement curve. (b) Normalized curves.

and a 500-year-flood during rush hour. The engineer and the public must accept a certain level of probability of failure. This acceptable level of probability of failure is tied to the number of deaths that the public accepts on a daily basis (fatalities) and to the amount of money that it can afford to spend (economy). In geotechnical engineering and in structural engineering, this acceptable probability of failure is typically less than 1 chance in 1000 (10^{-3}).

Design procedures have been developed to calculate a foundation size that meets these low probabilities of failure. These procedures involve:

1. Selecting the design issues (limit states)
2. Selecting load factors and resistance factors that are consistent with the low target probability of not meeting the design criterion
3. Determining the minimum size of the foundation that satisfies the low probability of not meeting the design criterion

For example, let's go back in time to the year 1100 and design the foundation of the Tower of Pisa, but with today's knowledge. The load is calculated to be 150,000 kN. The uncertainty about this load is small because the

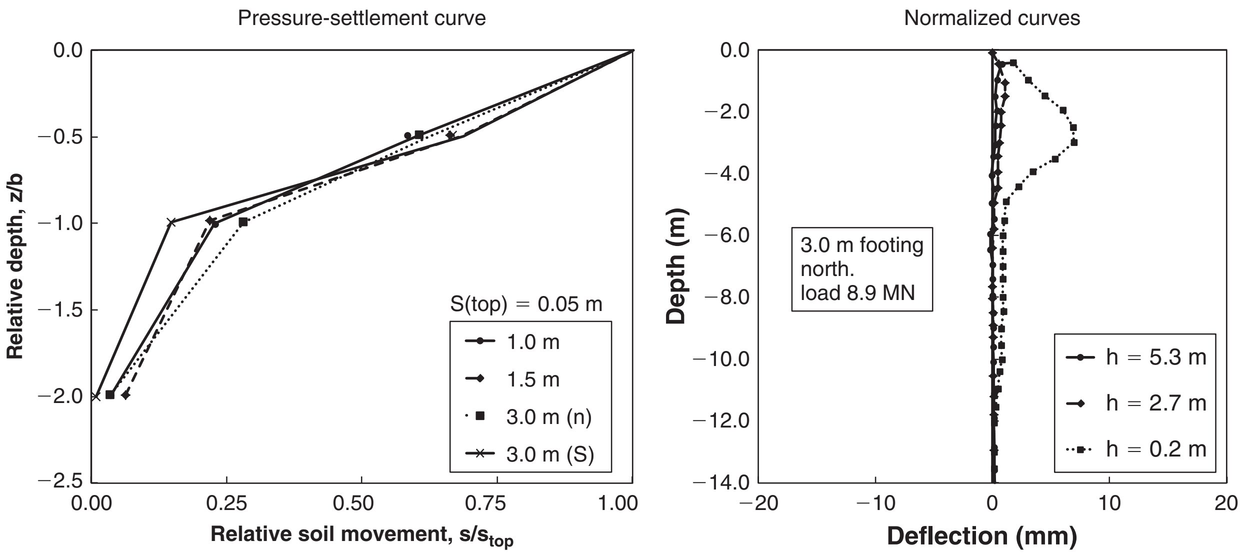

Figure 17.5 Vertical and horizontal movement vs. depth.

dimensions of the structure are on the plans. Nevertheless, a load factor of 1.2 is used to obtain the factored load of $1.2 \times 150{,}000 = 180{,}000$ kN, which lowers the probability of exceeding the load. The resistance is the ultimate bearing pressure of the soil below the tower. It is calculated as 6 times the undrained shear strength s_u of the soil within the depth of influence of the foundation (section 17.6.1). From the borings, in situ tests, and laboratory tests, a value of 80 kPa is selected for s_u. This leads to an ultimate bearing pressure of 480 kPa. The uncertainty associated with the undrained shear strength and the calculation model is not negligible, so a resistance factor of 0.6 is selected. The factored resistance is $0.6 \times 480 = 288$ kPa, which lowers the probability of not having the necessary resistance. The load factor 1.2 and the resistance factor 0.6 are based on the probability distribution of the load and of the resistance, and on ensuring that the probability that the difference between the factored load and the factored resistance is negative (failure) is less than approximately 10^{-3}. The difference between the load and the resistance is called the *limit state function*. We decide to place the 15 m diameter Tower of Pisa on a circular mat foundation 1 m thick with a diameter B. Now the ultimate limit state equation is written as:

$$1.2 \times 150000 < 0.6 \times 480 \times \pi \, B^2/4 \qquad (17.1)$$

which leads to $B > 28$ m. The actual, as-built foundation was less than 15 m in diameter and the soil below the foundation failed. The design should also include other considerations such as the serviceability limit state, but this simple example illustrates the design process and the concept of load and resistance factors.

More specifically, the design process proceeds as follows:

1. Decide on the foundation depth.
2. Make a reasonable estimate of the foundation size.
3. Calculate the ultimate bearing pressure of the foundation, p_u.
4. Check if the ultimate bearing pressure satisfies the safety criterion under the given load (ultimate limit state).
5. Repeat steps 1 through 3 until the safety criterion is satisfied and obtain the safe foundation pressure p_s, which is the unfactored load divided by the foundation area.
6. Under the safe foundation pressure p_s, check that the foundation satisfies the serviceability limit state by calculating the movement of the foundation and ensuring that it is less than the allowable movement.
7. If the calculated movement is larger than the acceptable movement s_a, increase the foundation size and/or the foundation stiffness and repeat step 6.
8. If the movement is acceptable, the design is complete, as the pressure applied is safe and allows only acceptable movement.

In addition to the preceding steps concerning soil strength and compressibility, the foundation must be well designed structurally. For example, one must ensure that the column will not punch through the spread footing, or that the mat foundation will not bend excessively. The structural aspect of foundation design is not covered in this book.

Shallow foundations are typically less expensive than deep foundations. Therefore, it is economically prudent, in most cases, to start with a shallow foundation solution. Only if it is shown to be insufficient or inappropriate should the design proceed with deep foundations.

17.4 LIMIT STATES, LOAD AND RESISTANCE FACTORS, AND FACTOR OF SAFETY

Limit states are the loading situations and the associated equations that are considered during the design of a foundation. They must be satisfied to yield a proper design. There are two major limit states: the ultimate limit state and the service limit state. In foundation engineering, *ultimate limit state* involves calculations of ultimate capacity using primarily the shear strength of the soil. Satisfying the ultimate limit state ensures that the foundation will meet a chosen level of safety against failure. The *service limit state* involves calculations of movements using deformation parameters. Satisfying the service limit state ensures that the foundation will meet a chosen degree of confidence against excessive movement or distortion of the structure.

The ultimate limit state refers to satisfaction of equations ensuring that the foundation will function far enough away from failure of the soil. This requires the choice of load factors γ and resistance factors φ that will achieve the chosen level of probability of success. These equations are of the form:

$$\gamma L < \varphi R \tag{17.2}$$

where γ is the load factor, L is the load, φ is the resistance factor, and R is the resistance. The resistance here is meant to be the ultimate resistance of the soil. In the case of complex loading and multiple resistances, Eq. 17.2 becomes:

$$\sum \gamma_i L_i < \sum \varphi_i R_i \tag{17.3}$$

where γ_i is the load factors, L_i is the loads, φ_i is the resistance factors, and R_i is the resistances. The load factors and the resistance factors make it possible to address separately the uncertainties associated with each load and each resistance. The term $\sum \gamma_i L_i$ also makes it possible to select the most appropriate combination(s) of loads that the soil has to resist. An example of an ultimate limit state equation is:

$$1.25\ DL + 1.75\ LL < 0.5\ R_u \tag{17.4}$$

where DL is the dead load and permanent live load on the foundation, LL is the nonpermanent live load on the foundation, and R_u is the ultimate resistance of the foundation from the soil point of view. Typical load factors for ultimate limit state are shown in Table 17.1; typical resistance factors for ultimate limit state are shown in Table 17.2. Note that there are two choices for the resistance side. The first one consists of applying a factor φ to the resistance (resistance factor); the second one consists of applying factors to the individual material properties such as the components of the shear strength (material factors). The Eurocode gives designers the choice to use either of the approaches (not both), whereas the AASHTO specifications only use the resistance factors. The selection of the soil parameter is a very important step. The AASHTO specifications tend to use mean values of the parameters, whereas the Eurocode uses "cautious estimates" of the soil parameters. This affects the selection of the resistance and material factors.

These factors γ and φ are developed by using the following procedure:

1. The unbiased estimates or best estimates or true values or measured values of the ultimate resistance and the load are R_m and L_m. The nominal values or design

Table 17.1 Typical Load Factors for Ultimate Limit State

Type of Loading	Load Factor γ (AASHTO) For bridges	Load Factor γ (ASCE 7) For buildings	Load Factor γ_E (Eurocode 7) For buildings
Dead load and permanent live load	1.25	1.2	1.35
Other live load	1.75	1.6	1.5
Extreme events (earthquake, hurricane, etc.)	1	1	1

Table 17.2 Typical Resistance Factors for Ultimate Limit State and Shallow Foundations

Type of Soil Testing	Resistance Factor φ (AASHTO)	(Eurocode 7) Material Factor $\gamma_M = 1/\varphi$	(Eurocode 7) Resistance Factor $\gamma_R = 1/\varphi$
Many high-quality tests	0.5 to 0.6	1.25–1.4 (may be reduced for extreme events)	1.1 to 1.7 (footings) 1.1 to 1.6 (piles) (may be reduced for extreme events)
Ordinary quantity and quality of tests	0.4 to 0.5		
Extreme events (earthquake, hurricane, etc.)	1		

values or predicted values of the resistance and the load are R_p and L_p.

2. Obtain the probability distribution of the load L_m and of the ultimate resistance R_m. Note that L_m and R_m are probabilistic. Each follows a certain distribution (for example, lognormal) with specified means (μ_{Rm} and μ_{Lm}) and standard deviation (σ_{Rm} and σ_{Lm}).
3. Write the likelihood function as $g = R_m - L_m$. Because R_m and L_m are random, g is also random.
4. Compute, using reliability software such as FERUM (2001):
   a. the probability $P(g \leq 0)$
   b. the corresponding value of the generalized reliability index β
   c. the coordinates of the failure point (R_m^*, L_m^*)
5. Choose a target reliability index β_{target}, usually 2.33 for redundant systems and 3 for nonredundant systems.
6. Compare the β from step 4 with the β_{target} from step 5. If the β from step 4 is equal to the β_{target} from step 5, then the central resistance factor $\overline{\varphi}$ and the central load factor $\overline{\gamma}$ can be calculated as:

$$\overline{\varphi} = R_m^*/\mu_{Rm}$$

$$\overline{\gamma} = \mathrm{L}_m^*/\mu_{\mathrm{Lm}}$$

7. Otherwise, increase or decrease μ_{Rm} and repeat steps 1 through 5.
8. Calculate the nominal resistance factor φ and the nominal load factor γ as follows:

$$\varphi = \overline{\varphi}\frac{\mu_{Rm}}{\mu_{Rp}}$$

$$\gamma = \overline{\gamma}\frac{\mu_{Lm}}{\mu_{Lp}}$$

For normal distributions, the reliability index β is the inverse of the coefficient of variation and tells us how many standard deviations the mean of $R_m - L_m$ is from the zero origin. For more complex distributions, this definition does not hold true. Typical β values are 2.33 for redundant systems and 3 for nonredundant systems. These β values correspond to probabilities of failure equal to $10^{-2}(\beta = 2.33)$ and $10^{-3}(\beta = 3.0)$.

The service limit state involves calculations of movements using deformation parameters. Satisfying the service limit state ensures that the foundation will meet a chosen degree of confidence against excessive movement or distortion of the structure. The equations have the same format as the ultimate limit state equations. The load factors are applied to the loads to be considered for movement calculations and the resistance factors are applied to the predicted movement or the soil deformation parameters. Typically, however, the load factors and resistance factors are taken as equal to 1. The nonpermanent live loads are not included in the loads considered for calculating settlements that take a long time to develop, such as consolidation settlements in saturated clays.

For example, the service limit state in terms of loads for a spread footing can be written as follows:

$$\gamma_1 DL + \gamma_2 LL \leq \varphi \frac{s_{all}BE}{I(1-\nu^2)} \tag{17.5}$$

where s_{all} is the allowable settlement of the foundation, B is the width of the spread footing, E is the modulus of the soil below the footing, I is a shape factor, and ν is the Poisson's ratio of the soil. The term $\frac{s_{all}BE}{I(1-\nu^2)}$ on the right-hand side of Eq. 17.5 is the load that generates the allowable settlement of the footing on an elastic half space; it is the resistance of the system at the service limit state. As mentioned earlier, the load factors and the resistance factors are usually taken as equal to 1. Furthermore, if the settlement will take place over a long period of time, the live load is not included in the settlement calculations except for the permanent live load.

Before the development of the load and resistance factor design (LRFD) approach, also called limit state design (LSD), the working stress design (WSD), also called the allowable stress design (ASD), approach was used. WSD consists of applying a global factor of safety against the ultimate bearing capacity of the soil in order to obtain the safe load. The equation is:

$$L < R/F \tag{17.6}$$

where L is the applied load to be safely carried, R is the ultimate resistance, and F is the global factor of safety. The factor of safety varies depending on the type of design (shallow foundation, deep foundation, slope stability, retaining wall) and is typically between 1.5 and 3 (Table 17.3). For the ultimate bearing pressure under a shallow foundation obtained by calculations, it is 3. The settlement is calculated using the dead loads and permanent live load without applying any factors.

One is always tempted, when comparing the WSD and LRFD approaches, to compare the global factor of safety with the ratio of the load factor divided by the resistance factor. Indeed, from Eqs. 17.2 and 17.6 comes $F = \gamma/\varphi$.

Table 17.3 Typical Global Factors of Safety against Soil Failure

Type of Geotechnical Application	Global Factor of Safety F
Shallow foundations	2.5 to 3
Deep foundations	2 to 2.5
Retaining wall	1.5 to 2
Slope stability	1.3 to 1.5

Using this expression and the extreme values of the load factors (dead load) and resistance factors gives a global factor of safety ranging from 1.3 to 4.2. This is a larger range than the values in Table 17.3 and shows that not all geotechnical methods give the same degree of precision on the predicted resistance. The LRFD approach takes this factor clearly into account.

One very important issue is how the geotechnical design parameters are selected from the borings, tests results, and soundings resulting from the site investigation. For example, the issue is to know which value to select from an undrained shear strength profile or a blow count profile or a cone penetrometer point resistance profile. This value is called the *characteristic value*, and its selection obviously will have a major impact on the uncertainty associated with predictions of the resistance. The Eurocode 7 defines the characteristic value as "a cautious estimate of the value affecting the occurrence of the limit state." So, in this case the selection is tied to the limit state itself.

Design methods can be classified into three categories: design by theory, design by empiricism, and design by analogy. Design methods by theory rely on theoretical derivations for recommending the design equations. Design methods by empiricism rely on experimental data and correlations for recommending the design equations. Design methods by analogy rely on the close analogy between the mode of deformation in the soil test and under the foundation. Generally speaking, the best methods include—and accumulate the advantages of—all three, by using a close analogy, experimental data, and a solid theoretical background.

17.5 GENERAL BEHAVIOR

In a load test on a shallow foundation (say, a 3 m by 3 m spread footing), the load on the foundation is increased in steps (jacking against an anchored frame or accumulating dead weight) and the corresponding downward movement is recorded. The load settlement curve is plotted and usually shows a relatively linear part at lower loads (elastic behavior), followed by a curved part, followed by a part where the movement accelerates faster than the load (Figure 17.6).

Load tests on silts and clays often plunge; load tests on sands and gravel rarely do, with the load increasing steadily with more deflection (Figure 17.6). The reason for the difference is that the fine-grained soils tend to shear in an undrained mode during a load test that may last a few hours, whereas coarse-grained soils likely shear in a drained mode. The undrained shear strength of a clay does not vary much with the stress and confinement level (s_u = constant), so when the load on the footing increases, the shear strength does not increase and the failure is clearly defined. The drained shear strength of a sand depends on the stress and confinement level ($s = \sigma' \tan\varphi'$); thus, when the load increases, so does the stress level and therefore the shear strength. Hence, the ultimate resistance of the sand increases as more load is applied and the failure is ill

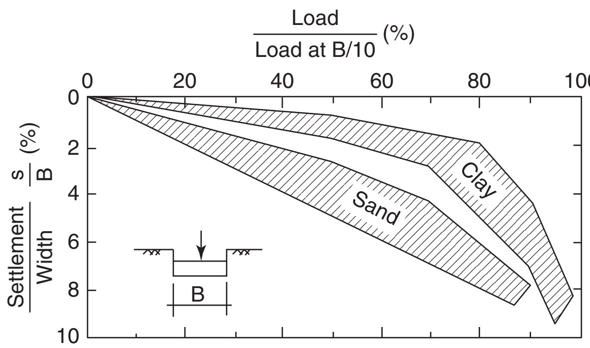

Figure 17.6 Typical shape of load test results on shallow foundations.

defined. In such a case, the ultimate load can be defined as the load corresponding to a movement equal to one-tenth of the foundation width. The true ultimate resistance of a footing on sand or gravel does exist, but at much larger displacements. These displacements are on the order of the width of the footing, as can be shown by the cone penetrometer test.

One important part of shallow foundation behavior is the movement of the foundation under sustained load, because most foundations are loaded with a static load for the life of the structure, which may be several decades or more. During the load test, the load can be maintained for a period of time and the movement can be observed as a function of time during that period.

17.6 ULTIMATE BEARING CAPACITY

The ultimate bearing capacity p_u is one of the critical values to be estimated when designing a shallow foundation. It is defined as the highest pressure the soil can resist. As explained in section 17.5, p_u corresponds to a plunging load in fine-grained soils, but to a load at large displacement (such as one-tenth of the footing width or B/10) in coarse-grained soils, because of the shape of the load settlement curve. Thus, the ultimate bearing capacity tends to control the design of shallow foundations on clay, whereas settlement tends to control the design of shallow foundations on sand. The value of p_u can come from an empirical formula (pressuremeter test, cone penetrometer test, or standard penetration test), from a formula based on theory (general bearing capacity equation), or from a load test. Load tests on shallow foundations are rare.

17.6.1 Direct Strength Equations

Direct strength equations rely on the average value of the strength of the soil within the depth of influence of the foundation below the foundation level. They are generally of the form:

$$p_u = k\,s + \gamma D \tag{17.7}$$

where k is the bearing capacity factor, γ is the effective unit weight of the soil, D is the embedment depth, and s is a measure of the soil strength averaged over the depth of influence. This depth of influence is typically taken as one foundation width below the foundation level for a uniform soil. The case of layered soils is addressed in section 17.6.3.

The first direct strength equation was proposed by Skempton (1951); it addresses the problem of the undrained ultimate bearing capacity of a shallow foundation on a fine-grained soil. The equation makes use of the average undrained shear strength s_u within the depth of influence below the footing. The theoretical background for this equation is rooted in the information presented in section 11.4.2. The equation is:

$$p_u = N_c s_u + \gamma D \tag{17.8}$$

where N_c is the bearing capacity factor (Figure 17.7) proposed by Skempton after calibration against field data, γ is the total unit weight of the soil above the foundation depth, and D is the depth of embedment. Note that N_c is higher for square footings than for strip footings. The reason is that the square footing can develop a relatively larger failure surface, because the failure surface can develop in four directions, whereas the failure surface for the strip footing is confined to only two directions. The N_c values for the square footing and the strip footing are related by:

$$N_{c(square)} = 1.2 N_{c(strip)} \tag{17.9}$$

Note also that N_c gradually increases with the relative depth of embedment, due to the gradual increase in the length of the failure surface with embedment. The values of N_c peak at:

$$N_{c(square)\,\max} = 9 \quad and \quad N_{c(strip)\,\max} = 7.5 \tag{17.10}$$

The second direct strength equation was proposed by Menard (1963a; 1963b); it addresses the problem of the ultimate bearing capacity of any soil in which the pressuremeter test can be performed. The theoretical background of this equation is rooted in the solution to the expansion of a cylindrical cavity. The equation is:

$$p_u = k_p p_L^* + \gamma D \tag{17.11}$$

where k_p is the pressuremeter bearing capacity factor, γ is the total unit weight of the soil above the footing depth, D is the depth of embedment, and p_L* is the net limit pressure equal to the PMT limit pressure p_L minus the horizontal total stress at rest σ_{oh}:

$$p_L^* = p_L - \sigma_{oh} \tag{17.12}$$

The PMT bearing capacity factor k_p is given in two steps (Frank, 1999, 2013, Norme Francaise AFNOR P94-261), first a soil classification step (Table 17.4) and then an equation for each soil category (Eqs. 17.13 to 17.18).

Clay and silt—strip footing:

$$k_p = 0.8 + \left(0.2 + 0.02\frac{D}{B}\right)\left(1 - e^{-1.3\frac{D}{B}}\right) \tag{17.13}$$

Clay and silt—square footing:

$$k_p = 0.8 + \left(0.3 + 0.02\frac{D}{B}\right)\left(1 - e^{-1.5\frac{D}{B}}\right) \tag{17.14}$$

Clay and silt—rectangular:

$$k_{p(B/L)} = k_{p(B/L=0)}\left(1 - \frac{B}{L}\right) + k_{p(B/L=1)}\frac{B}{L} \tag{17.15}$$

Sand and gravel—strip footing:

$$k_p = 1 + \left(0.3 + 0.05\frac{D}{B}\right)\left(1 - e^{-2\frac{D}{B}}\right) \tag{17.16}$$

Sand and gravel—square footing:

$$k_p = 1 + \left(0.22 + 0.18\frac{D}{B}\right)\left(1 - e^{-5\frac{D}{B}}\right) \tag{17.17}$$

Sand and gravel—rectangular:

$$k_{p(B/L)} = k_{p(B/L=0)}\left(1 - \frac{B}{L}\right) + k_{p(B/L=1)}\frac{B}{L} \tag{17.18}$$

where B and L are the width and length of the footing respectively, and D is the depth of embedment. These rules are primarily based on load tests with 1 m by 1 m square footings. As can be seen, the k_p factor varies within a typical

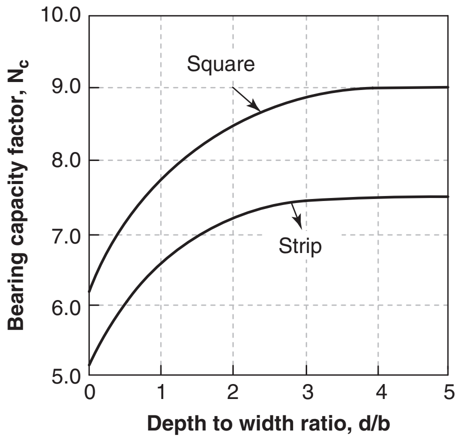

Figure 17.7 Skempton chart for N_c. (Skempton 1951)

Table 17.4 Soil Classification for the PMT and CPT Foundation Rules (After Frank, 2013)

Soil Type	Strength	PMT $p_L *$ (MPa)	CPT q_c (MPa)	SPT N(bpf)	Shear Strength s_u (kPa)
Clay, Silt	Very soft to soft	<0.4	<1		<75
	Firm	0.4 to 1.2	1 to 2.5		75 to 150
	Stiff	1.2 to 2	2.5 to 4		150 to 300
	Very stiff	>2	>4		>300
Sand, Gravel	Very loose	<0.2	<1.5	<3	
	Loose	0.2 to 0.5	1.5 to 4	3 to 8	
	Medium dense	0.5 to 1	4 to 10	8 to 25	
	Dense	1 to 2	10 to 20	25 to 42	
	Very dense	>2	>20	42 to 58	

After Norme Francaise AFNOR P94-261 as presented in Frank, 2013

range of 0.9 to 1 for clay and 1.2 to 1.4 for sands. The following simpler rule seems conservative in most cases:

$$p_u = k_p p_L \quad with \quad k_p = 0.9 \ for\ clay \quad and$$
$$k_p = 1.2\, for\ sand \qquad (17.19)$$

Interestingly, it can be shown that the horizontal resistance of a soil is the major component of the vertical resistance. Referring to Figure 17.8, consider a circular footing with a diameter D founded on the ground surface. The soil is a saturated clay layer with a thickness equal to $2D$. According to Eq. 17.8, the vertical ultimate bearing capacity p_u of that footing is $6.2\, s_u$. To calculate how much of p_u comes from the horizontal soil resistance, let's remove that lateral support. In this case the footing sits on top of a large sample loaded in an unconfined compression test; thus, the ultimate bearing capacity that it generates is equal to $2s_u$. The difference between the two diagrams on Figure 17.8 gives the contribution of the horizontal strength of the soil to the vertical ultimate bearing pressure: $4.2s_u$. Therefore, 68% of the vertical ultimate bearing pressure is due to horizontal resistance. For sand, the percent contribution of the horizontal resistance is even larger, as the unconfined compression resistance of a sand is very small.

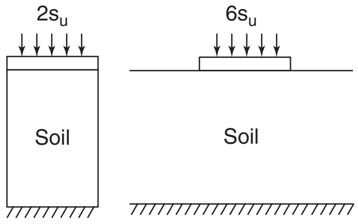

Figure 17.8 Lateral support as main contributor to vertical capacity. (Baguelin et al. 1978)

The third direct strength equation makes use of the cone penetrometer point resistance q_c; it addresses the problem of the ultimate bearing capacity of any soil into which the cone penetrometer can be pushed. The theoretical background of this equation is rooted in the solution to the expansion of a spherical cavity. There is one equation for clays and another one for sands. For clays, the equation is based on the work of Tand et al. (1986):

$$p_u = k_c q_c + \gamma D \qquad (17.20)$$

where k_c is the cone penetrometer bearing capacity factor (Figure 17.9), q_c is the average point resistance within one footing width below the footing, γ is the total unit weight of the soil above the footing, and D is the depth of embedment. All in all, it appears that a k_c value of 0.35 is a reasonable estimate for shallow foundations on clay. In sand, a value of

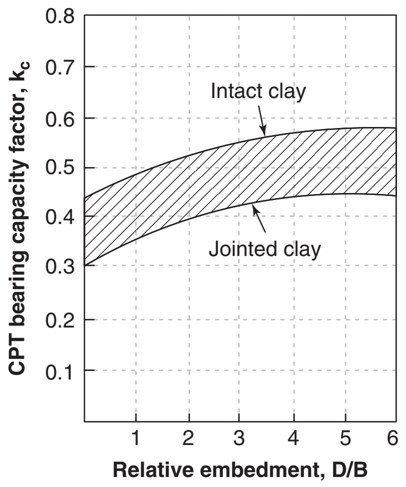

Figure 17.9 Chart for the CPT bearing capacity factor k_c. (After Tand et al. 1986)

k_c equal to 0.23 has been proposed by Briaud and Gibbens (1999). So, in summary:

$$\text{Clays} \quad p_u = 0.40q_c + \gamma D \tag{17.21}$$

$$\text{Sands} \quad p_u = 0.20q_c + \gamma D \tag{17.22}$$

The Norme Francaise AFNOR P94-261 as presented in Frank (2013) gives the following recommendations for k_c:

Clay and silt—strip footing:

$$k_c = 0.27 + \left(0.07 + 0.007\frac{D}{B}\right)\left(1 - e^{-1.3\frac{D}{B}}\right) \tag{17.23}$$

Clay and silt—square footing:

$$k_c = 0.27 + \left(0.1 + 0.007\frac{D}{B}\right)\left(1 - e^{-1.5\frac{D}{B}}\right) \tag{17.24}$$

Sand and gravle—strip footing:

$$k_c = 0.09 + \left(0.04 + 0.006\frac{D}{B}\right)\left(1 - e^{-2\frac{D}{B}}\right) \tag{17.25}$$

Sand and gravel—square footing:

$$k_c = 0.09 + \left(0.03 + 0.02\frac{D}{B}\right)\left(1 - e^{-5\frac{D}{B}}\right) \tag{17.26}$$

where B and L are the width and length of the footing respectively, and D_e is the depth of embedment. For the case of a rectangular footing, Eq. 17.15 is used. As can be seen, the k_c factor recommended by AFNOR varies within a typical range of 0.30 to 0.35 for clay and 0.10 to 0.14 for sands.

The fourth direct strength method makes use of the SPT blow count N; it addresses the problem of the ultimate bearing capacity of any soil in which the standard penetration test can be performed. There is one equation for sands and another one for clays. The form of the equation is:

$$p_u = k_N N\, p_a + \gamma D \tag{17.27}$$

where k_N is the SPT bearing capacity factor, N is the average blow count within one footing width below the footing, p_a is the atmospheric pressure used for normalization, γ is the total unit weight of the soil above the footing, and D is the depth of embedment. For sands, the k_N value is based on the work of Briaud and Gibbens (1999) and for clay the k_N value is back-calculated using Eq. 17.8 and the correlation between the blow count and the undrained shear strength. Note that calculating p_u based on the SPT blow count is probably the least accurate of all direct methods. So, in summary:

$$\text{Sands} \quad p_u = 0.60N\, p_a + \gamma D \tag{17.28}$$

$$\text{Clays} \quad p_u = 0.35N\, p_a + \gamma D \tag{17.29}$$

17.6.2 Terzaghi's Ultimate Bearing Capacity Equation

This equation is called the *general bearing capacity equation*. The assumptions made in deriving this equation are that the soil has no water, that it has a constant friction angle and cohesion c, and that it has a constant unit weight. As such, it corresponds to a soil strength profile that increases linearly with depth (Figure 17.10). If the soil strength profile does not meet this requirement, this equation should not be used, as it will give erroneous values of p_u.

The Terzaghi equation also assumes that a failure mechanism develops with a shear plane under the foundation (Figure 17.11) and that the soil mass is pushed sideways to allow for the foundation penetration. This was not observed in the large footing tests by Briaud and Gibbens (Figure 17.5).

The general bearing capacity equation for a strip footing is:

$$p_u = c'N_c + \frac{1}{2}\gamma BN_\gamma + \gamma DN_q \tag{17.30}$$

where p_u is the ultimate bearing capacity of the soil; c' is the effective stress cohesion intercept; N_c, N_γ, and N_q are bearing capacity factors function of the effective stress friction angle φ'; γ is the effective unit weight; B is the width of the foundation; and D is the depth of embedment of the foundation. The assumption of constant φ' and constant γ implies that the shear strength profile increases linearly with depth. If this matches the soil strength profile observed at the site, the equation is applicable. However, most field situations do not exhibit such simple linear profiles. In this case, the empirical equations give a more representative estimate of p_u. Note that the general bearing capacity equation is to be used with effective stress parameters (c', φ') and

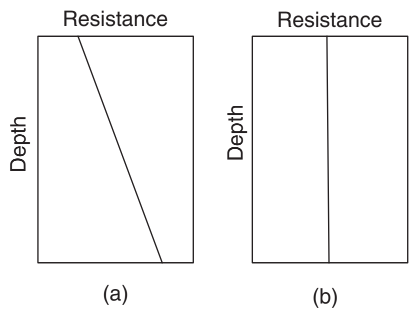

Figure 17.10 Soil strength profiles.

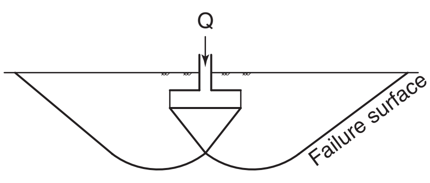

Figure 17.11 Bearing capacity failure mechanism.

drained conditions. It gives the long-term capacity of fine-grained soils and the short- and long-term capacity of coarse-grained soils. The undrained ultimate bearing capacity of fine-grained soils is given by Eq. 17.8.

The following derivation is an illustration of how the bearing capacity factors N_c, N_γ, and N_q can be obtained. The footing is a strip footing, which ensures a plane strain condition. The step-by-step procedure explained in section 11.4.1 is followed to obtain the failure load.

1. The failure mechanism of Figure 17.11 is assumed.
2. The free-body diagram of the wedge below the footing is drawn (Figure 17.12) and the reasoning is carried out on half of the wedge because of symmetry (OAB in Figure 17.12). The angle of the side of the wedge with the vertical is the angle of the failure plane. It is considered to be $45 + \varphi'/2$ because that is the angle of the failure plane in a triaxial test (see section 9.12.1) and with a passive pressure type of failure (see Chapter 21). All external forces are shown; they include the ultimate load Q_u at the soil-foundation interface (Q_u(kN/m) $= p_u \times$ B), the weight W of the half wedge, the cohesion force C along the face AB, and the passive earth pressure force P_p (also on face AB).
3. Vertical equilibrium of forces is the fundamental equation used. Note that the forces are in force per unit length, as this is a plane strain problem:

$$\frac{Q_u}{2} = P_p \cos\left(45 - \frac{\varphi'}{2}\right) + C\cos\left(45 - \frac{\varphi'}{2}\right) - W \tag{17.31}$$

where φ' is the soil friction angle.
Referring to Figure 17.12, the weight W of the half wedge is:

$$W = \frac{1}{2}\gamma\frac{B}{2}H = \frac{1}{4}\gamma B\frac{B}{2}\tan\left(45 + \frac{\varphi'}{2}\right) = \frac{1}{8}\gamma B^2\tan\left(45 + \frac{\varphi'}{2}\right) \tag{17.32}$$

where γ is the unit weight of the soil.

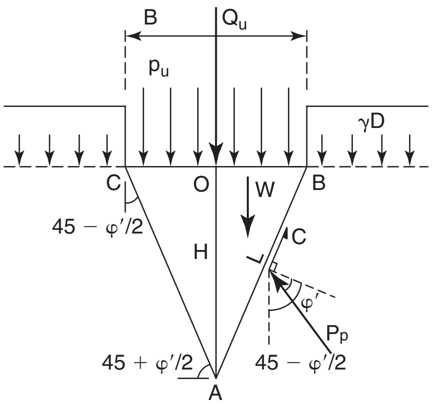

Figure 17.12 Free-body diagram of soil wedge in bearing capacity failure.

The cohesion force is:

$$C = c'L = c'\frac{B}{2\sin\left(45 - \frac{\varphi'}{2}\right)} \tag{17.33}$$

where c' is the soil cohesion intercept.
The passive resistance P_p is given by an equation presented in Chapter 21:

$$P_p = \frac{1}{2}K_p\gamma H^2 + 2c'H\sqrt{K_p} + \gamma DHK_p \tag{17.34}$$

where K_p is the passive earth pressure coefficient (see Chapter 21). This coefficient depends on φ'. Regrouping Eqs. 17.31 to 17.34 gives:

$$p_u = \frac{Q_u/2}{B/2} = c'\left(1 + 2\sqrt{K_p}\cos\left(45 - \frac{\varphi'}{2}\right)\right)\tan\left(45 + \frac{\varphi'}{2}\right) + \frac{1}{2}\gamma B\left(\frac{K_p\cos\left(45 - \frac{\varphi'}{2}\right)}{2\tan\left(45 - \frac{\varphi'}{2}\right)} - \frac{1}{2}\right)\tan\left(45 + \frac{\varphi'}{2}\right) + \gamma DK_p\tan\left(45 + \frac{\varphi'}{2}\right) \tag{17.35}$$

This can be rewritten as:

$$p_u = c'N_c + \frac{1}{2}\gamma BN_\gamma + \gamma DN_q \tag{17.36}$$

and the expressions of the bearing capacity factors N_c, N_γ, and N_q become clear. In Eq. 17.36, p_u is the ultimate bearing pressure the soil can resist, c' is the effective stress cohesion, γ is the soil effective unit weight, B is the foundation width, D is the depth of embedment, and N_c, N_γ, and N_q are the bearing capacity factors.

4. Note that the constitutive equation is buried in Eq. 17.34, which makes use of the shear strength equation of the soil. This is discussed in Chapter 21. The problem now is to obtain the expression of K_p as a function of φ'. Taking the expression that comes from Chapter 21 is not appropriate, because the assumptions for the retaining walls dealt with in Chapter 21 are not applicable to the extreme inclination of the "retaining wall" associated with plane AB in Figure 17.12 and Figure 17.13. In Chapter 21, a plane

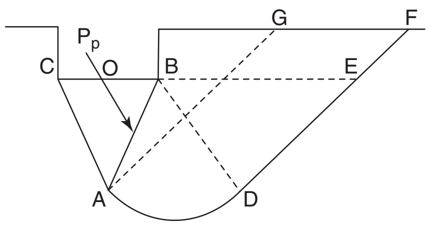

Figure 17.13 Evaluation of passive resistance.

is assumed as a failure surface (line AG in Figure 17.13), whereas a different shape failure surface is assumed for the bearing capacity failure (line ADE in Figure 17.13). Different assumptions have been made for line ADEF, and each one leads to a different set of bearing capacity factors N_c, N_γ, and N_q. Some assume a circle for line AD, some assume a log spiral, some assume that line DF stops at E, some go all the way to F, and some use a wedge ABC that is not a triangle.

5. The solution originally proposed by Terzaghi (1943) was decomposed into three superposed states:

a. State I, soil with cohesion and friction but no weight and no surcharge
b. State II, soil with friction and surcharge but no weight and no cohesion
c. State III, soil with weight and friction but no surcharge and no cohesion

Then each State is solved with separate failure envelopes and the solutions for each State are added in superposition of all States, to end up with Eq. 17.36. Although such a superposition principle is not theoretically correct in plasticity (or any other nonlinear) theory, the error appears to be small.

Many different bearing capacity factors have been proposed by various authors. All in all, the N_c, N_γ, and N_q factors most commonly used are those shown in Figure 17.14. They come from the work of Reissner (1924) for N_c and N_q and from the work of Meyerhof (1955) for N_γ.

The general bearing capacity equation requires that the soil be rigid enough to push the whole soil wedge from the footing to the ground surface. This may be the case when the soil is very dense or very stiff, but not when it is loose or soft. This also requires a very large amount of movement. To alleviate this limitation, Terzaghi and Peck (1963) recommended a correction that consists of reducing the value of the friction angle to $0.67\varphi'$ for loose and soft soils.

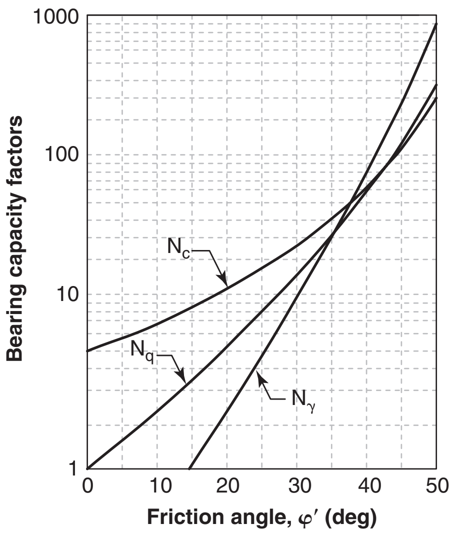

Figure 17.14 Bearing capacity factors.

Recall that one of the assumptions for the development of Eq. 17.36 is that the soil has no water. If the groundwater level (GWL) is within the depth of influence of the footing (1B below the footing), the unit weight in Eq. 17.36 should be the effective unit weight:

If the soil is below the GWL $\quad \gamma_{eff} = \gamma_t - \gamma_w \quad (17.37)$

If the soil is above the GWL $\quad \gamma_{eff} = \gamma_t \quad (17.38)$

For example, if the GWL is at the level of the foundation, the γ value for the third term in Eq. 17.36 should be γ_t, because that term refers to the soil above the foundation level, but the γ value for the second term in Eq. 17.36 should be $\gamma_t - \gamma_w$ because it refers to the soil below the foundation level.

17.6.3 Layered Soils

The previous two subsections dealt with relatively uniform soils. If the strength profile indicates that a layered system is involved in the responses to the foundation loading, modifications to the equations are necessary. The following simple examples show how this can be done for a strip footing.

Hard clay over soft clay. The first step is always to find a reasonable failure mechanism. Referring to Figure 17.15, it seems reasonable to assume that if the thickness H of the hard layer is large enough, the ultimate bearing pressure will be the one of the hard layer, $p_{u\,(hard)}$. If the thickness of the hard layer is negligible, then the ultimate bearing pressure will be $p_{u\,(soft)}$. If the thickness of the hard layer is intermediate, then the foundation will punch through the hard layer into the soft layer. This is very similar to punching through the ice layer when you walk across a frozen lake, if the ice is not thick enough.

Vertical equilibrium of forces for the failing mass (ABCD in Figure 17.15*a*) gives:

$$p_u B + \gamma_{(hard)} HB = 2F + p_{u(soft)} B = 2s_{u(hard)} H + (N_{c(soft)} s_{u(soft)} + \gamma_{(hard)} H) B \quad (17.39)$$

Or

$$p_u = N_{c(soft)} s_{u(soft)} + 2s_{u(hard)} \frac{H}{B} \quad (17.40)$$

where p_u is the ultimate bearing pressure of the foundation, N_c is the bearing capacity factor from Figure 17.7 for a depth of embedment of H/B, $s_{u\,(soft)}$ and $s_{u\,(hard)}$ are the undrained shear strength of the soft layer and hard layer respectively, $\gamma_{(hard)}$ is the unit weight of the hard layer, H is the thickness of the hard layer, and B is the width of the footing. Note that in Eq. 17.40 all forces are in kN/m, because they are calculated per unit length of footing perpendicular to the page. The p_u values for both layers taken independently are:

$$p_{u(soft)} = N_{c(soft)} s_{u(soft)} + \gamma_{(hard)} H \quad (17.41)$$

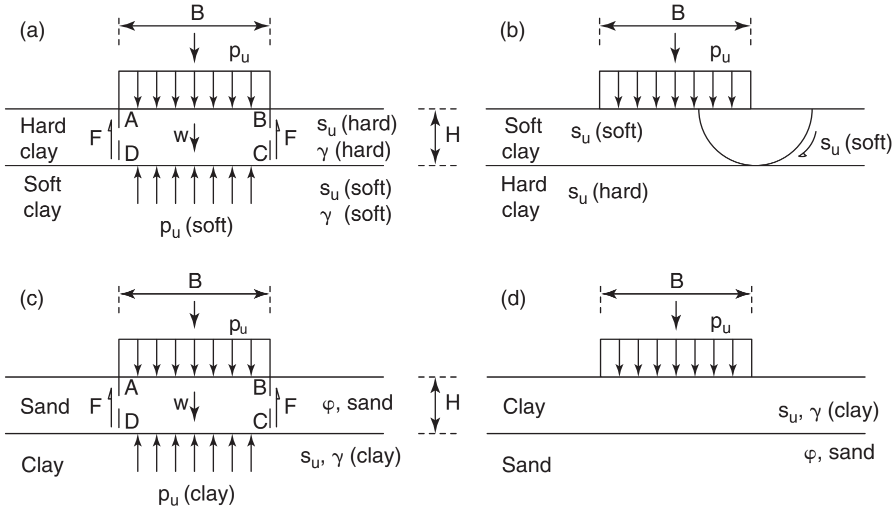

Figure 17.15 Layered systems.

$$p_{u(hard)} = N_{c(hard)} s_{u(hard)} \tag{17.42}$$

Then the critical height ratio, H_c/B, where the failure changes from a punching failure of the layered system to failure in the hard layer alone, can be found by writing that at that point the value of p_u for the layered system is equal to the p_u value for the hard layer:

$$N_{c(hard)} s_{u(hard)} = N_{c(soft)} s_{u(soft)} + 2s_{u(hard)} \frac{H}{B} \tag{17.43}$$

Note that a distinction must be made between $N_{c(hard)}$ and $N_{c(soft)}$ because of the different depth of embedment for the foundation on top of the hard layer and the foundation on top of the soft layer. Because the top of the hard layer is at the ground surface, $N_{c(hard)}$ is equal to 5.14. Then the expression for H_c/B is:

$$\frac{H_c}{B} = \frac{5.14 s_{u(hard)} - N_{c(soft)} s_{u(soft)}}{2 s_{u(hard)}} = 2.57 - \frac{N_{c(soft)}}{2} \frac{s_{u(soft)}}{s_{u(hard)}} \tag{17.44}$$

Because $N_{c(soft)}$ depends on H/B, Eq. 17.44 has to be solved by iteration. Figure 17.16 illustrates the variation of p_u with an increase in H/B. As can be inferred from Eq. 17.44, the critical depth H_c varies from about 2 for significant strength contrast between the two layers to about 1 when the strength contrast is not very significant.

Soft clay over hard clay. In this case, the failure mechanism is different from the one for the hard clay over the soft clay. If the soft clay layer is thick enough, the failure will occur in the soft clay and p_u is equal to $p_{u(soft)}$. If the thickness of the soft layer is negligible, then it should be removed and p_u is equal to $p_{u(hard)}$. If the thickness of the soft layer is intermediate, then the failure mechanism is that the soft layer squeezes out

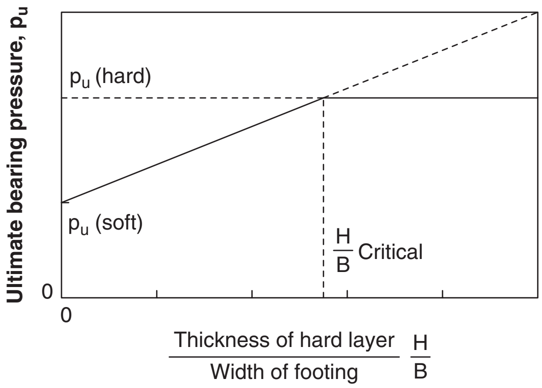

Figure 17.16 Ultimate bearing capacity for a layered system.

on the side of the footing. More scientifically put, a local failure occurs in the soft layer as shown in Figure 17.15*b*. Therefore, for a soft layer over a hard layer, the ultimate bearing pressure is always $p_{u(soft)}$.

Sand over clay. If the sand is very loose and the clay is very hard, a local failure in the sand layer can occur. Most of the time, in the case of a hard layer over a soft layer, the punching mechanism is likely to apply. If the thickness H of the sand layer is large enough, the ultimate bearing pressure will be the one of the sand layer, $p_{u(sand)}$. If the thickness of the sand layer is negligible, then the ultimate bearing pressure will be $p_{u(clay)}$. If the thickness of the sand layer is intermediate, then the foundation will punch through the hard layer into the soft layer. The force F in this case is equal to the horizontal force P_p times the coefficient of friction $\tan\varphi'$. The horizontal force P_p is the resultant force corresponding to the passive earth pressure distribution on the vertical plane BC (Figure 17.15c). Indeed, this plane is pushed sideways into the soil and generates the passive earth pressure at ultimate load. This force P_p is given by:

$$P_p = \frac{1}{2} K_{p(sand)} \gamma_{(sand)} H^2 \tag{17.45}$$

where $K_{p(sand)}$ is the coefficient of passive earth pressure for the sand. From Chapter 21 we get:

$$K_{p(sand)} = \frac{1+\sin\varphi'}{1-\sin\varphi'} \tag{17.46}$$

Vertical equilibrium of forces for the failing mass (ABCD in Figure 17.15*c*) gives:

$$\begin{aligned} p_u B + \gamma_{(sand)} HB &= 2F + p_{u(clay)} B \\ &= 2\frac{1}{2} K_{p(sand)} \gamma_{(sand)} H^2 \tan\varphi' \\ &\quad + (N_c s_{u(clay)} + \gamma_{(clay)} H) B \end{aligned} \tag{17.47}$$

Therefore, the ultimate bearing pressure is:

$$p_u = K_{p(sand)} \gamma_{(sand)} \frac{H^2}{B} \tan\varphi' + N_c s_{u(clay)} \tag{17.48}$$

where p_u is the ultimate bearing pressure of the foundation, $K_{p(sand)}$ is the coefficient of passive earth pressure for the sand, $\gamma_{(sand)}$ and $\gamma_{(clay)}$ are the unit weight of the sand and of the clay respectively, H is the thickness of the sand layer, B is the width of the foundation, φ' is the friction angle of the sand, N_c is the bearing capacity factor from Figure 17.7 for a depth of embedment of H/B, and $s_{u(clay)}$ is the undrained shear strength of the clay. Then the critical height ratio, H_c/B, where the failure changes from a punching failure of the layered system to failure in the sand layer, can be found by writing that at that point the value of p_u for the layered system is equal to the $p_{u(sand)}$ value for the sand layer, which is given by an equation of the form of Eq. 17.7.

Other combinations of layered systems should be addressed by considering the most likely failure mechanism and using the procedure outlined in section 11.4.1 to obtain p_u. If several failure mechanisms are possible, p_u should be calculated for each one and the minimum value should be retained, because the soil will fail at the lowest failure load encountered.

17.6.4 Special Loading

Most of the solutions for ultimate bearing pressure presented so far have been for simple cases. However, shallow foundations can be more complex (Figure 17.17) including:

1. Influence of the foundation shapes (rectangular, square, circular, strip), i_s
2. Influence of the depth of embedment, i_d
3. Influence of the load eccentricity, i_e
4. Influence of the load inclination, i_i
5. Influence of a nearby slope, i_β

An increase in the depth of embedment tends to increase the ultimate bearing pressure p_u, while the eccentricity, the inclination, and the slope presence tend to decrease p_u. In each case, an influence factor must be added in front of the equation for the base case. Such factors have been proposed for the pressuremeter method, the cone penetrometer method, and the general bearing capacity method. The influence factors for the cone penetrometer method are the same as the ones for the pressuremeter method.

Pressuremeter method. These factors are recommended by Frank (1999) and Norme Francaise AFNOR P94-261 (2013) and are as follows. Note that the influence of the foundation shape and of the depth of embedment are already included in the formulas for the bearing capacity factor k_p and k_c (Eqs. 17.13 to 17.18 and Eqs. 17.23 to 17.26). If the load applied to a $B \times L$ footing has an eccentricity e_B along the width B and e_L along the length L, the influence of the eccentricity is taken into account by using a rule attributed to Meyerhof. This rule consists of reducing the footing size as follows:

$$B' = B - 2e_B \quad and \quad L' = L - 2e_L \tag{17.49}$$

Then the design rules are applied to the reduced-size $B' \times L'$ footing, but the final recommendation is a $B \times L$ footing.

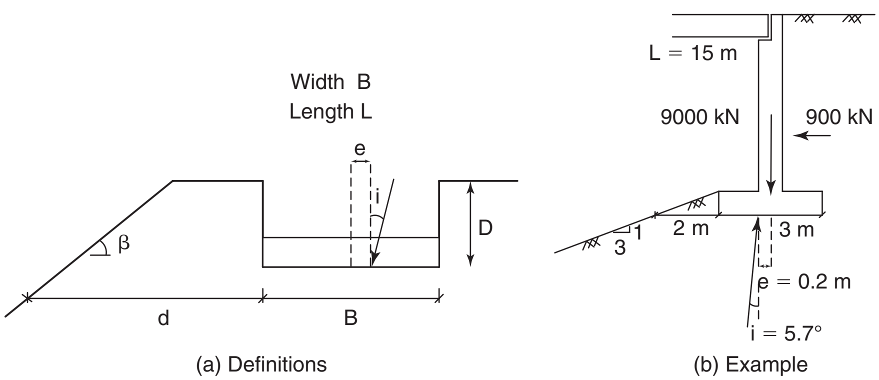

Figure 17.17 Complex loading cases for a shallow foundation.

If a footing is subjected to a centered inclined load making an angle α with the vertical, the influence factor i_i is given by Figure 17.18. Note that in Figure 17.18, the upper curve is for fine-grained soils, whereas the three lower curves are for coarse-grained soils and for three different relative depths of embedment D/B.

If a footing is located close to a slope and subjected to a centered vertical load, the presence of the slope reduces the ultimate bearing pressure. The influence factor i_β is given by Figure 17.19 as a function of d/B where d is the horizontal distance between the front edge of the bottom of the footing to the slope and B is the footing width. Each curve on Figure 17.19 corresponds to a slope angle β. Note that this figure corresponds to zero embedment depth. A simplified straight line relationship is also shown on Figure 17.19.

It is common practice to multiply the influence factors when several conditions are present at the same time.

General bearing capacity method. Several recommendations have been made for the influence factors to apply to the general bearing capacity equation. They take into account the foundation shape, the load eccentricity, the load inclination, and the presence of a nearby slope. They can be found in many manuals, including the Canadian foundation manual, the NAVFAC manual, the AASHTO bridge specifications, the API RP2A manual, the Norme Francaise AFNOR as presented by Frank (1999), and many others.

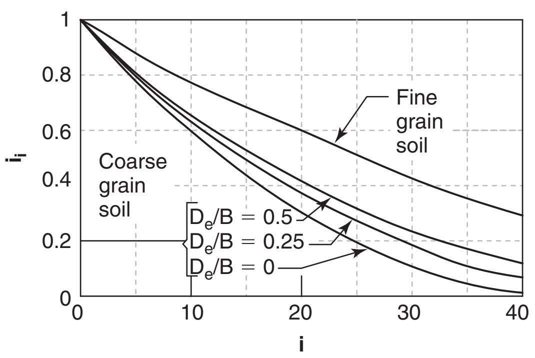

Figure 17.18 Influence of inclination. (After Frank, 1999)

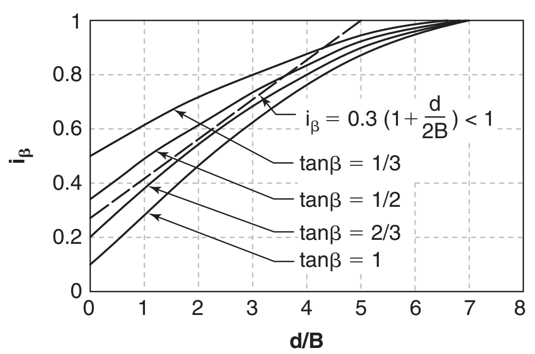

Figure 17.19 Influence of nearby slope. (After Frank, 1999)

The recommendations vary, but a review of these factors leads to the factors shown in Table 17.5, which represent reasonable averages. Note that there is a different factor for each of the three terms in Eq. 17.36. The subscript c is used for the cN_c term, the subscript γ is used for the term $0.5\gamma BN\gamma$, and the subscript q is used for the term γDN_q. Thus, the general formula is:

$$p_u = i_{cs} i_{ce} i_{ci} i_{c\beta} cN_c + i_{\gamma s} i_{\gamma e} i_{\gamma i} i_{\gamma\beta} \frac{1}{2}\gamma BN_\gamma + i_{qs} i_{qe} i_{qi} i_{q\beta} \gamma DN_q \qquad (17.50)$$

17.6.5 Ultimate Bearing Capacity of Unsaturated Soils

Unsaturated soils and saturated soils with water in tension generally have higher ultimate bearing capacity p_u than the same soils with water in compression. Indeed, the water tension increases the effective stress and therefore the shear strength, which affects the value of p_u.

In the case of the direct equations, nothing changes because the change in strength is directly taken into account because the test itself takes the increase in strength into account. The PMT limit pressure, the CPT point resistance, the SPT blow count, and the undrained shear strength all reflect the impact of water tension on these soil parameters. Therefore, if one is using a direct method such as Eqs. 17.8, 17.11, 17.21, 17.22. 17.28, or 17.29, there is no need to change anything in the approach to be taken. Nevertheless, one must be aware of the fact that if the strength test is performed when the soil is very dry (high water tension), as is often the case in the summer, the predicted value of p_u will be high. If the soil loses that water tension in the winter, then the value of p_u will become much smaller. It is very possible for the water tension to vary significantly from one season to the next down to a depth of 3 m below the surface. Because shallow foundations are often placed within that depth, it is desirable to test the soil when it is in its wet state. If this is not possible, experience should be used from prior comparisons between summer and winter strength to reduce the strength accordingly before computing p_u.

In the case of the general bearing capacity equation, it is important to understand the role of each of the three terms. The first term, $c'N_c$, refers to the contribution made by the effective stress cohesion of the soil along the failure plane. The second term, $0.5\gamma BN\gamma$, refers to the contribution made by the friction along the failure plane due to the effective stress below the foundation but without a surcharge. The third term, γDN_q, refers to the contribution made by the friction along the failure plane due to the presence of the surcharge γD. It is relatively common practice to calculate the bearing capacity of soils with water tension (unsaturated or saturated) by increasing the cohesion c′ to include the apparent cohesion $c_{app} = \alpha\, u_w \tan\varphi$ in the value of c'. Then the equation is:

$$p_u = (c' - \alpha u_w \tan\varphi')N_c + \frac{1}{2}\gamma BN_\gamma + \gamma DN_q \qquad (17.51)$$

Table 17.5 Influence Factors for the General Bearing Capacity Equation

	i_c for cN_c term	i_γ for $0.5\gamma BN_\gamma$ term	i_q for γDN_q term
Shape*	1 + 0.2 (B/L)	1 − 0.3(B/L)	1
Eccentricity	Meyerhof rule	Meyerhof rule	Meyerhof rule
Inclination**	$(1-\alpha/90)^2$	$(1-\tan\alpha)^{2.5}$	$(1-\tan\alpha)^{1.5}$
Nearby slope***	0.3(1 + (d/2B))	0.3(1 + (d/2B))	0.3(1 + (d/2B))

*B is the footing width and L is the footing length
**α is the angle of inclination of the load
***For slope angles between 2 to 1 and 3 to 1, d is the horizontal distance from the footing edge to the slope, B is the footing width

This practice does not recognize the fact that the apparent cohesion is due to an increase in effective stress through the water tension and not to an increase in "glue" between the grains. It appears more appropriate to include this increase in effective stress in the second term. The expression 0.5γB represents the effective stress σ'_{ov} for a "no water" condition at a depth of 0.5B below the foundation level in the case of no surcharge. This expression should be replaced by the effective stress at that same location but after consideration of the water tension. The bearing capacity for soils with water tension (unsaturated or saturated) would then be:

$$p_u = c'N_c + \frac{1}{2}(\gamma B - \alpha u_w)N_\gamma + \gamma DN_q \qquad (17.52)$$

Unfortunately, there are no known large-scale footing tests in which water tension was measured during a load test so as to provide verification for either approach. In any case, it is recommended that the direct method equations be used rather than the general bearing capacity equation, because the former methods are not restricted by the shape of the soil strength profile and have been extensively calibrated against footing load tests, particularly the PMT and CPT methods.

17.7 LOAD SETTLEMENT CURVE APPROACH

The design of a shallow foundation, much like the design of a deep foundation, is split into two steps. One addresses the ultimate bearing capacity, the other the movement at working loads. The load settlement curve (LSC) method (Jeanjean 1995; Briaud 2007) is used to predict the entire load settlement curve of the shallow foundation, rather than being limited to predicting only two points on that curve. It was developed in part after testing five large-scale footings (Figures 17.3 and 17.4). During these tests, inclinometer casings placed vertically at the edge of the footings gave the lateral deformation of the soil below the footings (Figure 17.5). These lateral deformation profiles never indicated that a plane of failure was developing as assumed in Figure 17.11. Instead, it showed that a "barreling" effect was progressively increasing in the same shape as the one created by the pressuremeter test. This is why the PMT curve was chosen as the curve to use and transform it into the footing load settlement curve. So, the LSC method is a way to transform the pressuremeter curve into the load settlement curve for a footing (Figure 17.20). During these large-scale tests, it was also observed that the normalized curve, plotted as pressure on the footing divided by the soil strength (PMT limit pressure) versus the settlement divided by the footing width, was independent of the footing size and essentially a unique property of the soil (Figure 17.4).

The transformation of the PMT curve into the footing curve is based on two equations as follows:

$$\frac{s}{B} = 0.24\frac{\Delta R}{R_o} \qquad (17.53)$$

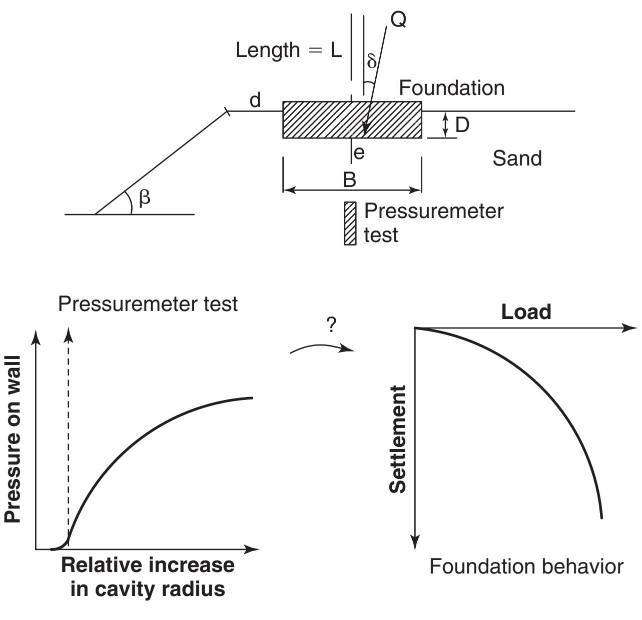

Figure 17.20 The load settlement curve (LSC) method. (Briaud 2007.)

$$p_f = f_{L/B} f_e f_\delta f_{\beta,d} \Gamma p_p \tag{17.54}$$

where s is the footing settlement, B is the footing width, R_o is the initial radius of the pressuremeter cavity, ΔR is the increase in cavity radius, p_f is the footing pressure corresponding to the settlement s, p_p is the pressuremeter pressure corresponding to $\Delta R/R_o$, and $f_{L/B}$, f_e, f_δ, and $f_{\beta,d}$ are the factors to include the influence of the footing shape, the load eccentricity, the load inclination, and the presence of a slope.

Equation 17.53 serves as a strain compatibility equation because it matches the strains at the ultimate values, which are $s/B = 0.1$ for the footing (a typical reference) and $\Delta R/R_o$ equal to 0.414 for the PMT (corresponding to the definition of the limit pressure). The value of 0.24 in Eq. 17.53 is the ratio of 0.1/0.414. In Eq. 17.54, Γ is a function of s/B (or 0.24 $\Delta R/R_o$), which represents the ratio between the footing pressure p_f and the PMT pressure p_p for the reference case of a centered vertical load on flat ground. Figure 17.21 shows the data from many sites used to generate the average Γ function and the design Γ function of Figure 17.22. The design Γ function is one standard deviation below the mean Γ function with respect to the data shown on Figure 17.21 and is recommended for design. The precision of the method can be gauged by the scatter on Figure 17.21.

The equations for the influence factors came mostly from numerical simulations (Hossain 1996; Briaud 2007):

$$\text{Shape} \quad f_{L/B} = 0.8 + 0.2\frac{B}{L} \tag{17.55}$$

$$\text{Load eccentricity} \quad f_e = 1 - 0.33\frac{e}{B} \quad \text{for the center} \tag{17.56}$$

$$\text{Load eccentricity} \quad f_e = 1 - \left(\frac{e}{B}\right)^{0.5} \quad \text{for the edge} \tag{17.57}$$

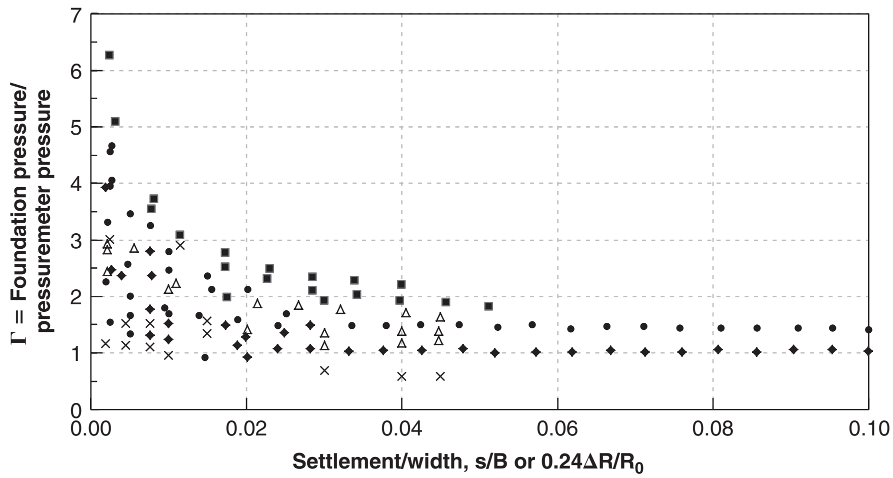

Figure 17.21 Data accumulated to generate the Γ function.

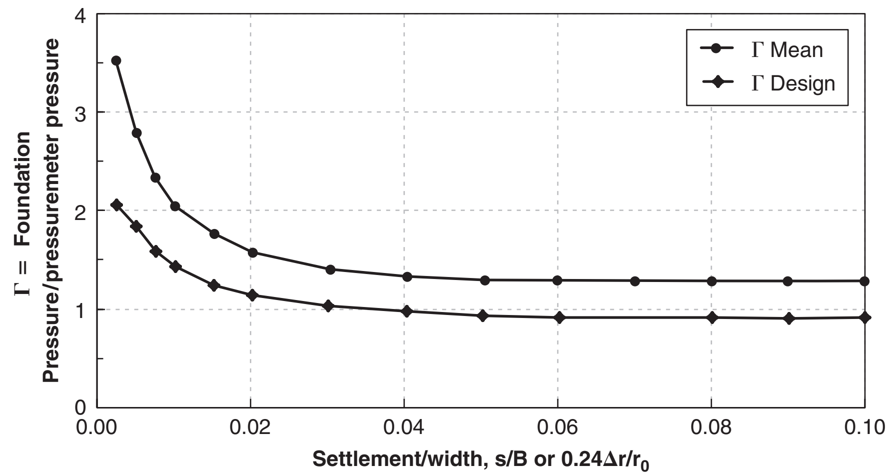

Figure 17.22 The Γ function for the load settlement curve method.

Load inclination $f_i = 1 - \left(\dfrac{i}{90}\right)^2$ for the center (17.58)

Load inclination $f_i = 1 - \left(\dfrac{i}{360}\right)^{0.5}$ for the edge (17.59)

Near a slope $f_{\beta,d} = 0.8\left(1 + \dfrac{d}{B}\right)^{0.1}$ for a 3 to 1 slope (17.60)

Near a slope $f_{\beta,d} = 0.7\left(1 + \dfrac{d}{B}\right)^{0.15}$ for a 2 to 1 slope (17.61)

where B and L are the footing width and length respectively, e is the load eccentricity, i is the inclination angle of the load, and d is the horizontal distance from the edge of the footing to the slope surface (Figure 17.17). The influence factor for the influence of a nearby slope is given for two common highway slopes: a 3 to 1 slope has a β angle with the horizontal of 18.4 degrees, and a 2 to 1 slope has a β angle with the horizontal of 26.6 degrees.

During the large footing tests discussed in section 17.2, the load was held for 30 minutes at each load level (Figure 17.3) and the settlement s was recorded as a function of time t. Figure 17.3 shows the relationship between the log of settlement and the log of time for each load step. The settlement $s(t)$ is normalized by the settlement value at the beginning of that load step $s(t_1)$ and the time t is normalized by a time t_1 equal to one minute. As can be seen from Figure 17.3, the relationship is linear in the log space; therefore, the model is a power model with an exponent n equal to the slope of the line in the log space:

$$\frac{s(t)}{s(t_1)} = \left(\frac{t}{t_1}\right)^n \tag{17.62}$$

The exponent n can be measured in a pressuremeter test where the pressure is held at an appropriate pressure level while the relative increase in radius $\Delta R/R_o$ is recorded as a function of time t. Equation 17.62 is then applied to the PMT data and n is back-calculated. The n value tends to be between 0.01 and 0.03 for sands and between 0.02 to 0.05 for stiff to hard clays.

The step-by-step procedure for the load settlement curve method is as follows:

1. Perform preboring pressuremeter tests within the zone of influence of the footing.
2. Plot the PMT curves as pressure p_p on the cavity wall versus relative increase in cavity radius $\Delta R/R_o$ for each test. Extend the straight-line part of the PMT curve to zero pressure and shift the vertical axis to the value of $\Delta R/R_o$ where that straight line intersects the horizontal axis; re-zero that axis. This is done to correct the origin for the initial expansion of the pressuremeter to allow it to come into contact with the borehole wall.
3. Develop the mean pressuremeter curve of all the PMT curves within the depth of influence of the footing. To do so, choose a value of $\Delta R/R_o$ and average the corresponding pressures of all the PMT curves; in doing so, give more weight to the shallower PMT curves, which will have more influence on the settlement than the deep PMT curves (Briaud 2007).
4. Transform the PMT curve point by point into the footing curve by using Eqs. 17.53 and 17.54.
5. Generate the short-term load settlement curve for the footing from the normalized curve.
6. Generate the long-term load settlement curve by multiplying all settlement values by the factor $(t/t_1)^n$ where t is the design life of the structure, t_1 is 1 hr, and n is the time exponent obtained from PMT tests or set equal to 0.03 as the default value.

Figure 17.23 is an example of the LSC method.

17.8 SETTLEMENT

17.8.1 General Behavior

Once the ultimate bearing capacity has been calculated and once the dimensions of the footing have been established such that the ultimate limit state (safety criterion) is satisfied, the settlement under the foundation pressure is calculated. This is the *service limit state*. Typically in this case, the load factors and resistance factors are taken as equal to 1. The nonpermanent live loads are not included in the loads considered for calculating settlements that take a long time to develop, such as consolidation settlements in saturated clays. The settlement of a structure is often decomposed into an elastic component (elastic settlement), then a time-delayed component associated with water stress dissipation (consolidation), then a time-delayed component associated with the slow movement of particles as a function of time (creep settlement). The settlement of a structure can also be decomposed into the settlement induced by the deviatoric stress tensor (shearing) and by the spherical stress tensor (compression). In cases where the settlement is concentrated in a thin (relative to the width of the foundation) layer, the settlement due to the spherical part of the tensor dominates. This would be the case of a wide embankment on top of a thin layer of soft clay. If, in contrast, the soil layer is deep (relative to the width of the foundation), the settlement due to the deviatoric tensor dominates. This would be the case of a tall building on top of a mat foundation underlain by a deep deposit of very stiff clay.

The pressure distribution under a shallow foundation depends on the flexibility of the foundation (Figure 17.24). For flexible foundations, the pressure is constant but the settlement is not. The settlement at the center $s_{\text{flex (center)}}$ is

Problem: A bridge abutment rests on a shallow foundation 15 m long and 3 m wide. The foundation is subjected to a vertical and centered load equal to 9000 kN. The lateral earth pressure generates a load of 900 kN on the back of the abutment. The resultant of the two forces has an eccentricity equal to 0.2 m. The soil is a sand characterized by the average pressuremeter curve shown.

Solution Load Settlement Curve

$f_{L/B} = 0.8 + 0.2\text{x}3/15 = 0.84$
$f_e = 1\text{-}0.33\text{x}0.2/3 = 0.978$
$f_\delta = 1\text{-} (\text{Arctan}(900/9000) / 90)^2$
$= 0.996$
$f_{\beta,\,d} = 0.8\text{x}(1+2/3)^{0.1} = 0.842$
$f = f_{L/B} f_e f_\delta f_{\beta,d} = 0.689$

$\Delta R/R_0$	P_p (kN/m²)	s/B	s (mm)	Γ	f	P_f (kN/m²)	Q (MN)
0.006	75	0.00144	4.32	2.25	0.689	116.3	5.23
0.012	120	0.00288	8.64	2.00	0.689	165.4	7.44
0.024	220	0.00576	17.28	1.60	0.689	242.5	10.91
0.032	300	0.00768	23.04	1.50	0.689	310.0	13.95
0.055	450	0.0132	39.6	1.30	0.689	403.1	18.14
0.10	650	0.0240	72.0	1.10	0.689	492.6	22.17
0.15	775	0.036	108	1.00	0.689	534.0	24.03
0.20	850	0.048	144.0	0.95	0.689	556.4	25.04

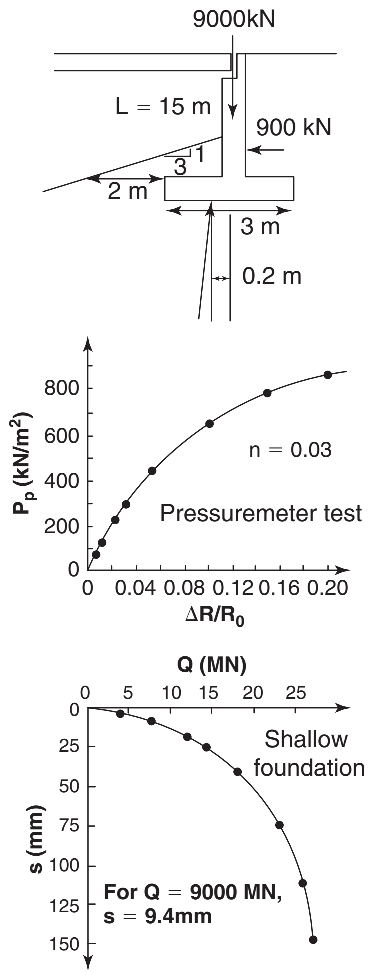

Figure 17.23 Example of the load settlement curve method.

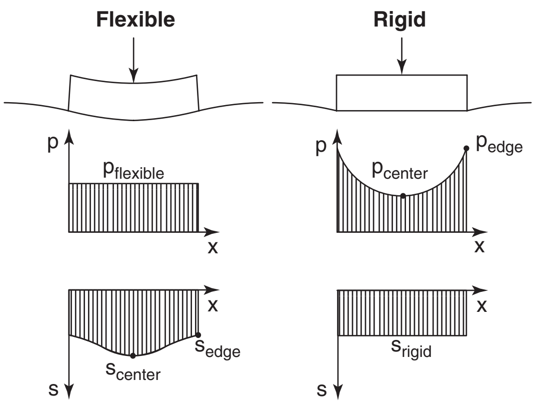

Figure 17.24 Settlement and pressure distribution below footings.

larger than the settlement at the edge $s_{flex(\text{edge})}$. For rigid foundations, the settlement s_{rigid} is constant but the pressure is not—at least initially. The following is an approximate relationship between the settlements:

$$s_{flex\ (center)} \simeq 2 s_{flex\ (edge)} \simeq 1.33 s_{rigid} \qquad (17.63)$$

In other words, the settlement at the center of a flexible footing is about twice as large as the settlement at the edge of a flexible footing, and the settlement of a rigid footing is about the average of the center and the edge of a flexible footing. These observations are based on the theory of elasticity. Also in elasticity, the pressure near the edge of a rigid footing is very large and the pressure in the center of that footing is much smaller (Figure 17.24); in fact, it is about one-half the mean pressure. As will be discussed in section 17.8.7, the soil tends to develop a constant pressure under the foundation in the long term even if the foundation is very rigid.

There are a number of methods for performing settlement calculations:

1. Elasticity approach
2. Load settlement curve method (see section 17.7)

3. Chart approach
4. General layered soil approach
5. Consolidation settlement approach

17.8.2 Elasticity Approach for Homogeneous Soils

Soils are not elastic, as they do not recover all the deformation they experience when strained. Soils are not linear either, as their stress-strain curve are not a straight line. Nevertheless, if a foundation is loaded with a certain load Q and experiences a settlement s as a result, there is always a modulus E that, when combined with Q, can give the right s value. The elasticity equations also have a significant advantage in that they are simple to use. They have a drawback in that they require a very sensible and often very difficult choice of the soil modulus. The best way to obtain the modulus is to run a test that closely reproduces what the soil will be subjected to under the structure. At the preliminary design stage, one may wish to use estimated values as presented in Chapter 14.

The equation for the elastic settlement s of a shallow foundation is:

$$s = I(1-\nu^2)\frac{pB}{E} \tag{17.64}$$

where I is an influence factor for any deviation from a footing on the ground surface subjected to a centered vertical load; ν is Poisson's ratio, usually taken as 0.35 for drained conditions and 0.5 for undrained conditions; p is the average pressure at the foundation level; B is the width of the foundation; and E is the soil modulus of deformation. The factor I can be written as:

$$I = I_s I_e I_h \tag{17.65}$$

where I_s is the factor for the influence of the shape of the footing, I_e is the factor for the influence of the embedment depth, and I_h is the factor for the presence of a hard layer at depth. Table 17.6 gives the values of I_s and shows that the strip footing settles a lot more than the square footing.

Table 17.6 Values of the Elastic Influence Factor I_s for Foundation Shape

Shape	Length-to-Width Ratio L/B	Influence Factor for Shape, I_s.		
		Rigid	Flexible	
			Center	Corner
Circular	1	0.79	1	0.64
	1	0.88	1.12	0.56
	1.5	1.07	1.36	0.68
Rectangular	2	1.21	1.53	0.77
	3	1.42	1.78	0.89
	5	1.7	2.1	1.05
	10	2.1	2.54	1.27

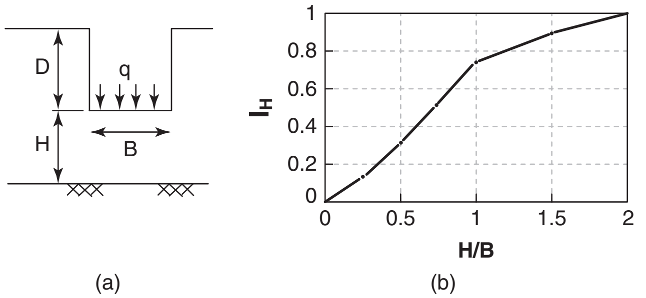

Figure 17.25 Influence factor for hard layer within depth of influence: (a) Hard layer at depth H. (b) Influence factor I_h. (After Christian and Carrier 1978)

This is due to a much larger depth of influence for the strip footing compared to the square footing. The factor I_e reduces settlement compared to a surface footing, because of the beneficial effect of having more mass to deform for a deeper footing than a shallower footing. The factor I_e can be estimated for footings with a relative depth of embedment (D/B) less than 1 (shallow foundations) by:

$$I_e = 1 - 0.1\frac{D}{B} \tag{17.66}$$

The maximum reduction for larger values of D/B is 15% $(I_e = 0.85)$. The factor I_h is a reduction factor when there is a hard layer within the depth of influence of the footing. Figure 17.25 gives the values of I_h when it is assumed that beyond 2B the hard layer has no reduction influence on the settlement and that the hard layer is incompressible.

The previous method assumes that the soil has a modulus which is constant with depth. If the soil has a modulus profile that increases linearly with depth (Figure 17.26), a correction factor I_G can be used. The equation for the soil modulus profile is:

$$E = E_o + E_1\left(\frac{z}{B}\right) \tag{17.67}$$

where E is the soil modulus at a depth z, E_o is the soil modulus at the ground surface, and E_1 is the rate of increase of the soil modulus as a function of the normalized depth (z/B). The influence factor I_G takes the modulus profile into account and is defined as:

$$I_G = \frac{s_1}{s_o} \tag{17.68}$$

where s_1 is the settlement calculated using E from Eq. 17.67 and s_o is the settlement calculated from Eq. 17.64 using a constant modulus E_o with depth ($E_1 = 0$). Figure 17.26 shows the influence factor I_G as a function of the ratio E_1/E_o.

17.8.3 Elasticity Approach for Layered Soils

Another way to use elasticity to solve a settlement problem is to decompose the depth of influence z_i into several soil layers H_i thick and calculate the compression ΔH_i of each

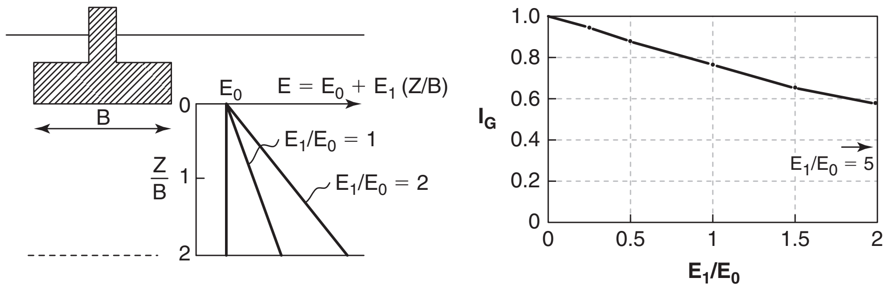

Figure 17.26 Influence of modulus increase vs. depth (Gibson soil).

layer. The vertical strain in each layer is ε_i and is related to the increase in stress $\Delta\sigma_i$ in the middle of that layer. The settlement s is expressed as:

$$s = \sum_{i=1}^{n} \Delta H_i = \sum_{i=1}^{n} \varepsilon_i H_i = \sum_{i=1}^{n} \frac{\Delta\sigma_i}{E_i} H_i \tag{17.69}$$

where s is settlement, n is the number of layers within the depth of influence, i refers to the ith layer, H_i is the thickness of the ith layer, ΔH_i is the compression of the ith layer, ε_i is the mean vertical strain of the ith layer, and $\Delta\sigma_i$ is the increase in stress in the center of the ith layer. Equation 17.69 assumes that the relationship between ε_i and $\Delta\sigma_i$ is given by:

$$\varepsilon_i = \frac{\Delta\sigma_i}{E_i} \tag{17.70}$$

This relationship ignores the influence of confinement on the strain and therefore is an approximation. This assumption is conservative, as taking the confinement into account would reduce the strain. How to obtain the magnitude of $\Delta\sigma_i$ in the middle of each layer is discussed in section 17.8.7.

Schmertmann (1970; 1978) used this approach and proposed a method to calculate the settlement s of footings on sand:

$$s = C_1 C_2 \Delta p \sum_{i=1}^{n} \frac{I_{zi}}{E_i} H_i \tag{17.71}$$

where C_1 takes into account the beneficial effect of the embedment, C_2 takes into account the increase in settlement with time, Δp is the net bearing pressure expressed as the difference between the footing pressure p (load over area) minus σ'_{ov} (the vertical effective stress in the soil at the level of the foundation near the footing), I_{zi} is called the strain influence factor, E_i is the soil modulus, and H_i is the thickness of the ith layer. The coefficient C_1 is:

$$C_1 = 1 - 0.5\frac{\sigma'_{ov}}{\Delta p} \geq 0.5 \tag{17.72}$$

where σ'_{ov} is the vertical effective stress in the soil at the level of the foundation near the footing, and Δp is the net increase in pressure expressed as the difference between the footing pressure p (load over area) minus σ'_{ov}. The coefficient C_2 is:

$$C_2 = 1 + 0.2\log\left(\frac{t\ (years)}{0.1}\right) \tag{17.73}$$

where t is the time in years.

The strain influence factor I_{zi} is such that $I_{zi} \times \Delta p$ represents $\Delta\sigma_i$ in Eq. 17.69. It is shown in Figure 17.27. In that figure, I_z increases first and then decreases. The peak value

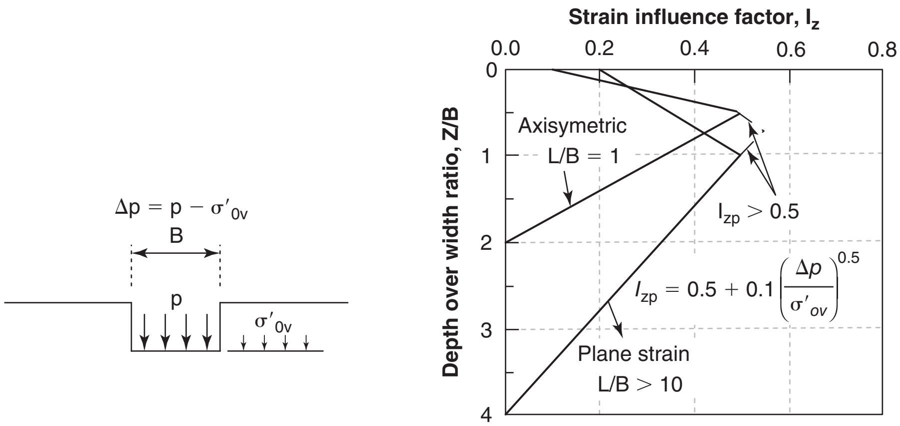

Figure 17.27 Strain influence factor. (After Schmertmann 1970)

Table 17.7 Conversion from CPT to SPT Values for Sands

Soil	q_c (kPa)/N (bpf)
Silts, sandy silts, slightly cohesive silt-sand	200
Clean, fine to medium sands and slightly silty sands	350
Coarse sands and sands with little gravel	500
Sandy gravel and gravel	600

of I_z is called I_{zp}. It is shown as 0.5 on Figure 17.27 but in fact it is given by:

$$I_{zp} = 0.5 + 0.1\left(\frac{\Delta p}{\sigma'_{Izp}}\right)^{0.5} \qquad (17.74)$$

where σ'_{Izp} is the vertical effective stress at the location of I_{zp}. The soil modulus E_i is recommended by Schmertmann as follows:

For circular or square footings $\quad E = 2.5q_c \quad$ (17.75)

For strip footings (L/B > 10) $\quad E = 3.5q_c \quad$ (17.76)

where q_c is the CPT point resistance. Schmertmann adds the conversion values of Table 17.7 between q_c and N.

17.8.4 Chart Approach

The chart approach consists of simplifying the problem sufficiently so that the calculations are minimized and a chart can be read for the answer. Such a chart approach was developed by Terzaghi and Peck (1963) for footings on sand (Figure 17.28). This chart is only for footings on sands, and it gives the pressure that satisfies both the ultimate bearing pressure criterion and the settlement criterion of 25 mm. This chart was developed before LRFD was developed and as such

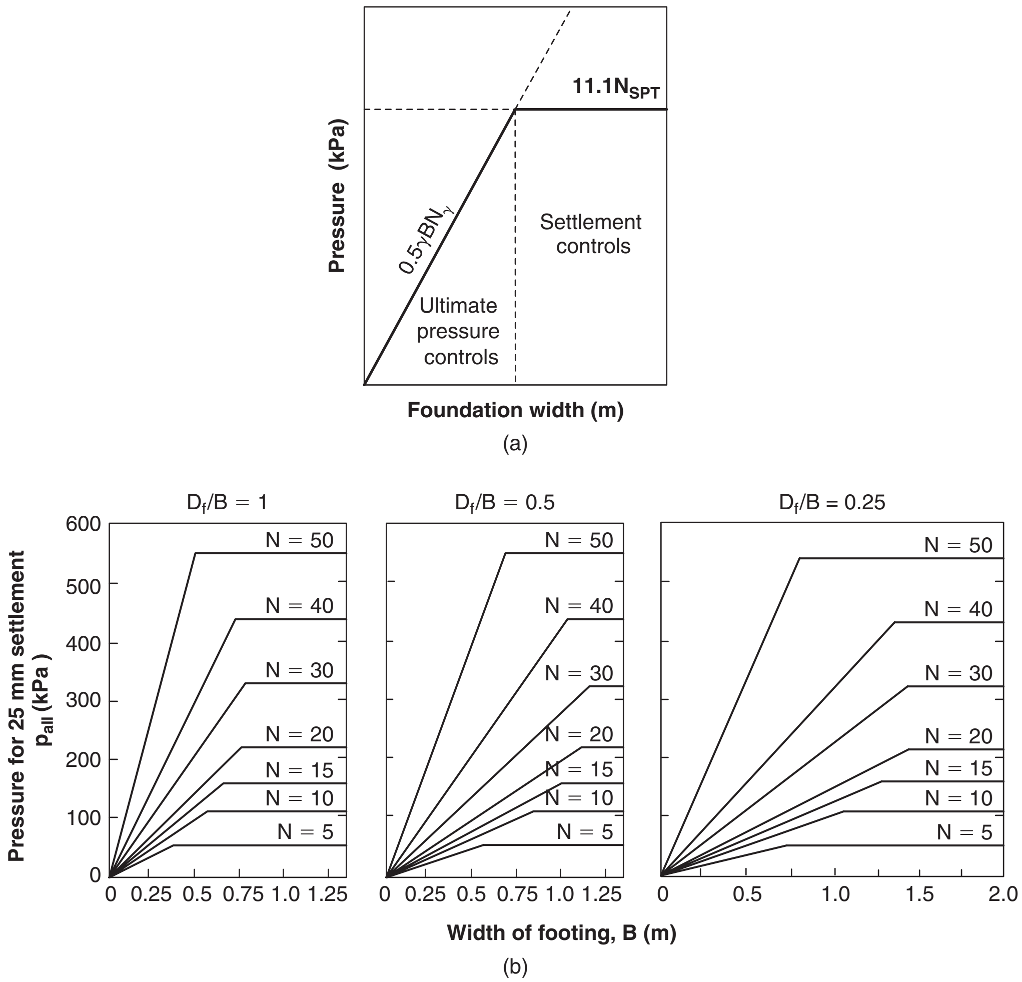

Figure 17.28 Chart for pressure leading to 25 mm settlement of footings on sand. (Terzaghi and Peck 1963)

is based on the following. The safe pressure criterion ensures that a reasonable factor of safety is applied to the ultimate bearing pressure:

$$p_{safe} = \frac{p_u}{F} \tag{17.77}$$

where p_{safe} is the safe bearing pressure, p_u is the ultimate bearing pressure, and F is the factor of safety. The allowable pressure criterion ensures that the settlement will be less than 25 mm in this case:

$$p_{allowable} = p \; for \; 25 \text{ mm } settlement \tag{17.78}$$

The chart of Figure 17.28 gives the minimum of p_{safe} and $p_{allowable}$. The first part of the design curves on the chart increases linearly with the width B of the footing for the following reason. For small values of B, it turns out that the ultimate bearing pressure criterion controls the design, and since there is no cohesion for sands, it is expressed as:

$$p_u = \frac{1}{2}\gamma B N_\gamma \tag{17.79}$$

As a result, p_{safe} increases linearly with B. The influence of the depth of embedment D is included by having several charts for different relative depths of embedment D/B. For the settlement s of the footing, Terzaghi and Peck found that s was proportional to the SPT blow count as follows:

$$p_{allowable}(\text{kPa}) \; for \; 25 \text{ mm } settlement = 11.1N(blows/0.30 \text{ m}) \tag{17.80}$$

This indicates that $p_{allowable}$ is not a function of B and therefore it shows up as a horizontal line on Figure 17.28. As a result, the ultimate pressure criterion controls for small footings and the settlement criterion controls for larger footings. If Eq. 17.80 is extended to other settlement values, and assuming linear behavior, the equation becomes:

$$s \text{ (mm)} = 2.3\frac{p(\text{kPa})}{N(\text{bpf})} \tag{17.81}$$

17.8.5 General Approach

The general approach to calculating the settlement of a structure is valid in all cases and proceeds as follows:

1. Determine the depth of influence z_i.
2. Divide that depth into an appropriate number n of layers (4 is a minimum), each layer being H_i thick.
3. Calculate the vertical effective stress σ'_{ovi} in the middle of each layer i before any load is applied.
4. Calculate the increase in stress $\Delta\sigma_{\text{vi}}$ in the middle of each layer i due to load.
5. Calculate the vertical effective stress $\sigma'_{\text{ovi}} + \Delta\sigma_{\text{vi}}$ in the middle of each layer i long after loading.
6. Obtain the vertical strain ε_{bi} before any load is applied, corresponding to the stress σ'_{ovi}.
7. Obtain the vertical strain ε_{ai} long after the load application corresponding to the stress $\sigma'_{ovi} + \Delta\sigma_{vi}$.
8. Calculate the compression ΔH_i of each layer i as:

$$\Delta H_i = (\varepsilon_{ai} - \varepsilon_{bi})H_i \tag{17.82}$$

9. Calculate the settlement ΔH as:

$$\Delta H = \sum_{i=1}^{n} \Delta H_i = \sum_{i=1}^{n} (\varepsilon_{ai} - \varepsilon_{bi})H_i \tag{17.83}$$

This general approach requires some other steps, which are addressed in the next sections. These steps are where one determines the zone of influence z_i (step 1), finds the increase in stress $\Delta\sigma_v$ (step 4), and obtains the strains ε_{bi} and ε_{ai} given the stresses σ'_{ov} and $\sigma'_{ov} + \Delta\sigma_v$ (steps 6 and 7).

17.8.6 Zone of Influence

The zone of influence z_i below a loaded area can be defined in one of two ways:

1. The depth at which the stress increase in the soil $\Delta\sigma_v$ has decreased to 10% of the stress increase p at the foundation level. This depth is called $z_{0.1\sigma}$.
2. The depth at which the downward movement of the soil becomes equal to 10% of the downward movement at the surface. This depth of influence is called $z_{0.1\text{s}}$.

Although the stress-based definition is the most commonly used in geotechnical engineering, the movement-based definition seems more reasonable because it ensures that 90% of the settlement is being calculated. Multiplying the answer by 1.11 will then give the full value of settlement.

The value of $z_{0.1\sigma}$ is typically taken as 2 times the width B of the footing for square and circular footings, and as 4 times the width B of the footing for long strip footings. These values are based on the elastic analysis of a uniform soil. Interpolation based on the ratio of width over length (B/L) is used for rectangular footings:

$$\frac{z_{0.1\sigma}}{B} = 4 - 2\left(\frac{B}{L}\right) \tag{17.84}$$

The value of $z_{0.1\text{s}}$ is the same as the value of $z_{0.1\sigma}$ if the soil modulus is constant with depth. If the soil modulus increases with depth, the value of $\text{z}_{0.1\sigma}$ does not change, but that of $z_{0.1\text{s}}$ does. The increase in modulus with depth is characterized by Eq. 17.67. Figure 17.29 shows the variation of $z_{0.1\text{s}}$ for a strip footing and for various values of the increase in modulus with depth characterized by E_1/E_o. For very small values of E_1/E_o (constant modulus with depth), the value of 4B is confirmed, but for high values of $\text{E}_1/\text{E}_\text{o}$ the zone of influence based on settlement criterion $z_{0.1\sigma}$ is much smaller than $z_{0.1\text{s}}$. It decreases from 4B to 1B and reaches 1B for a modulus which is zero at the surface and increases linearly with depth. This phenomenon is explained as follows. When the soil is uniform and the soil modulus is constant with depth, the zone of influence is relatively deep (4B). At the other extreme,

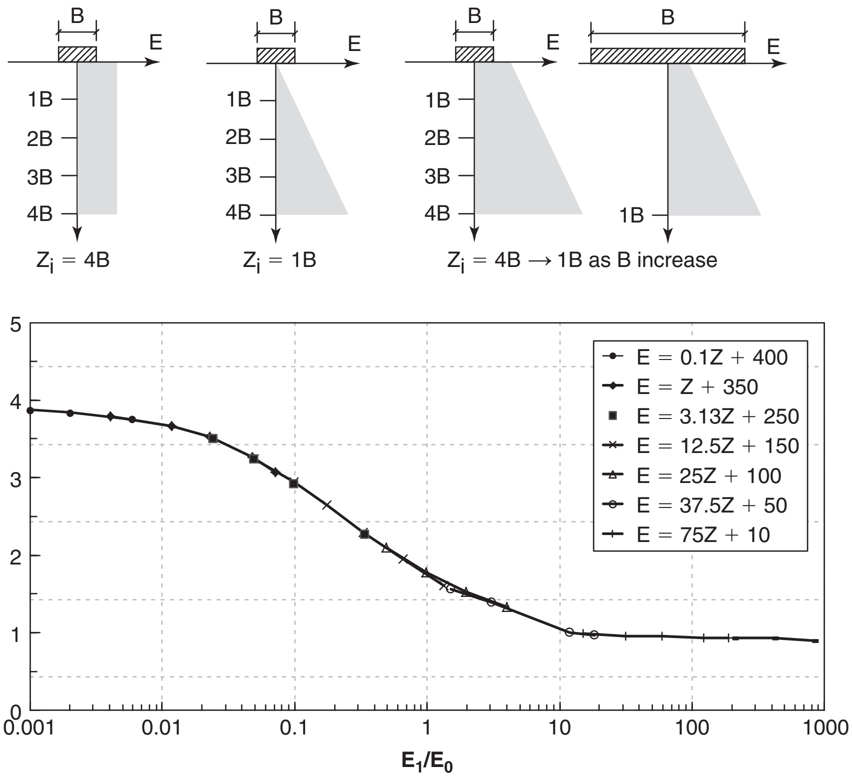

Figure 17.29 Zone of influence based on settlement criterion. (Briaud et al. 2007)

when the soil modulus increases with depth from a value of zero at the surface, the deeper layers are stiffer and do not compress as much as the shallower layers, which are softer. As a result, 90% of the settlement takes place within a much shallower depth and $z_{0.1s}$ is only 1B. For intermediate modulus profiles, the depth of influence depends on B and varies from 4B for small B values to 1B for large B values.

The procedure for finding the zone of influence below a foundation based on the settlement criterion using Figure 17.29 is as follows:

1. Fit the soil modulus profile with a straight line.
2. Determine the ratio E_1/E_o.
3. Obtain the depth of influence $z_{0.1s}$ from Figure 17.29 knowing the footing width B.
4. Calculate the settlement within that depth.
5. Multiply the answer by 1.11 to obtain the total settlement.

This approach has not been verified at full scale and is based solely on numerical simulations.

17.8.7 Stress Increase with Depth

2 to 1 method: One simple way to calculate the increase in stress below a foundation is the 2 to 1 method. This method consists of spreading the load with depth, as shown in Figure 17.30. The foundation is B wide and L long and is subjected to a load Q. At a depth z, the area over which the load is applied is increased by $z/2$ on both sides and becomes

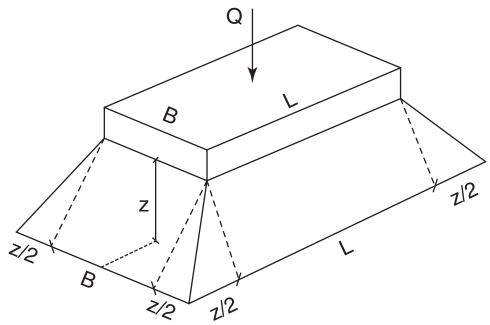

Figure 17.30 2 to 1 method for stress increase calculations.

$B+z$ and $L+z$. The average increase in stress at depth z is given by:

For a rectangular foundation $$\Delta\sigma_v = \frac{Q}{(B+z)(L+z)} \tag{17.85}$$

If the foundation is circular, then the diameter is increased by $z/2$ all around and the average increase in stress is given by:

For a circular foundation $$\Delta\sigma_v = \frac{4Q}{\pi(D+z)^2} \tag{17.86}$$

If the foundation is infinitely long (strip footing or embankment, for example), the load is defined as a line load (kN/m).

Furthermore, the load cannot spread in the direction of the length L, so the expression becomes:

$$\text{For a strip foundation} \quad \Delta\sigma_v = \frac{Q}{B+z} \tag{17.87}$$

Note that this method aims at estimating the average increase in vertical stress under the foundation. In elasticity, the increase in stress at the edge of a rigid foundation is different from the increase in stress at the center.

Bulbs of pressure: A more precise way to obtain the increase in stress at depth is the bulb of pressure chart (Figure 17.31). This chart gives the increase in stress below a square foundation and below a strip foundation for a uniform elastic soil. By using this chart, you can get the increase in stress at any location in the soil mass in the vicinity of the foundation. It is particularly useful for obtaining the increase in stress at the edge and at the center of the footing because this difference can affect the distortion of the foundation. Note that Figure 17.31 is for a flexible foundation where the pressure is uniform at the foundation level. Although foundations can be rigid, using the flexible solution in all cases is recommended for the following reason. Full-scale measurements (Focht, Khan, and Gemeinhardt 1978) indicate that the initially uneven pressure distribution under relatively rigid foundations redistributes itself and becomes close to the constant pressure under a flexible footing. This is attributed to the inability of the soil to sustain a large stress gradient for a long period of time. Therefore, the long-term settlement

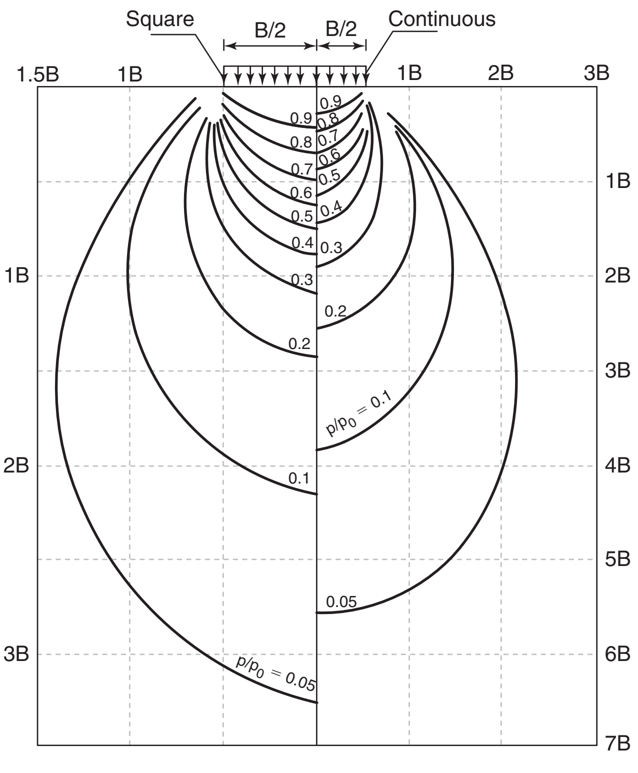

Figure 17.31 Bulbs of pressure based on Boussinesq elastic solution for a flexible foundation.

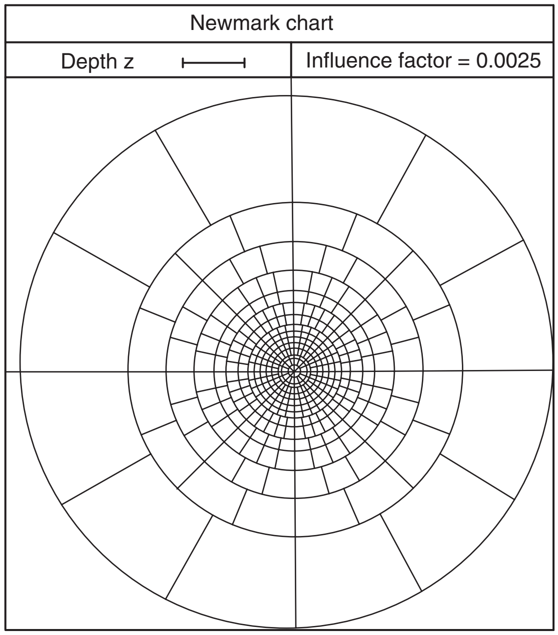

Figure 17.32 Newmark's chart.

of a foundation should be calculated on the basis of the stress distribution below a flexible distribution, regardless of whether the foundation is actually rigid or flexible.

Newmark's chart: The bulbs of pressure method gives the increase in stress under a square foundation or a strip foundation. If the foundation is more complicated, one possible solution is Newmark's chart (Figure 17.32). This chart is also for a uniform elastic soil and gives the increase in stress at any location below a foundation of any shape.

The best way to use the Newmark's chart is to make a transparency of the chart. If you do not have a transparency, then the drawing of the foundation has to be made on the original Newmark's chart, making it hard to reuse that chart. The transparency allows you many uses.

The procedure to use Newmark's chart is as follows:

1. Choose the depth z at which the stress increase $\Delta\sigma_v$ is required. Set the scale on the Newmark's chart (AB on Figure 17.32) equal to z. This gives the scale to be used for step 2.
2. Draw the foundation to the scale determined by $z = \text{AB}$.
3. Choose the point C in plan view on the foundation drawing under which $\Delta\sigma_v$ is required.
4. Overlay the transparent Newmark's chart on the foundation drawing such that point C of the foundation drawing is at the center of the Newmark's chart.
5. Count the number n of squares or fields of the Newmark's chart covered by the foundation drawing.
6. Calculate the increase in stress $\Delta\sigma_v$ as:

$$\Delta\sigma_v = n\, I\, p \tag{17.88}$$

where n is the number of squares, I is the influence factor of the Newmark's chart indicated on the chart, and p is the mean foundation pressure.

All of the solutions described in this subsection are limited to a uniform soil. If a layered soil such as a pavement is involved, or if the modulus is not constant with depth, the finite element method may be a good solution for finding the increase in stress with depth.

17.8.8 Choosing a Stress-Strain Curve and Setting Up the Calculations

Possibly the most difficult step in settlement calculations is to select the best and most applicable stress-strain curve to link the stress increment to the strain increment. This step requires a lot of thought and engineering judgment based on experience. Sections 17.8.2 and 17.8.3 described the elasticity approach, in which the stress-strain curve is a straight line and the modulus E is used to define the slope. Choosing such a modulus is a very difficult task; the content of Chapter 14 can help in that respect. The best and most applicable stress-strain curve is usually the one that most closely duplicates what the soil is being subjected to in the field, before and during the construction and then during the life of the structure. This includes the stress path, the strain path, the weather, the soil profile, the load level, and many more factors.

In general, if the structure is wide compared to the thickness of the compressing soil layer, then a test such as the consolidation tests will duplicate the soil deformation process quite closely. This would be true in the case of a wide embankment on a relatively thin, compressible layer, for example, because in this instance the friction between the embankment and the soil generates a natural resistance against lateral expansion of the soil much like the steel ring in the consolidation test. In contrast, if the soil deposit is deep compared to the width of the structure, then a test such as the pressuremeter test or the triaxial test duplicates the soil deformation process quite closely. Indeed, in this instance the soil below the foundation tends to barrel out in the same fashion as the soil around the pressuremeter or in the triaxial test.

Once the stress-strain curve is chosen, the strains corresponding to the stresses can be determined. The strains before and long after the loading (ε_{bi} and ε_{ai}) are obtained from the curve for the corresponding stresses σ'_{ov} and $\sigma'_{ov} + \Delta\sigma_v$. Note that although in the field the strain ε_{bi} corresponding to σ'_{ov} was zero, it is unlikely to be zero when reading the stress-strain curve. This is attributed to possible disturbance and stress relief upon extrusion. The calculations are then set up in the form of a spreadsheet, as shown in Table 17.8, where i is the layer number, H_i is the thickness of layer i, σ'_{ovi} is the vertical effective stress in the middle of layer I before loading, $\Delta\sigma_{vi}$ is the increase in stress in the middle of layer i due to loading, ε_{bi} is the vertical strain corresponding to σ'_{ovi}, ε_{ai} is the vertical strain corresponding to $\sigma'_{ovi} + \Delta\sigma_{vi}$, and ΔH_i is the compression of layer i. The sum of the last column in the table corresponds to Eq. 17.83 and represents the settlement.

Table 17.8 Calculation of Settlement by the General Approach

i	H_i (m)	σ'_{ovi} (kPa)	$\Delta\sigma_{vi}$ (kPa)	$\sigma'_{ovi} + \Delta\sigma_{vi}$ (kPa)	ε_{bi}	ε_{ai}	ΔH_i
1							
2							
3							
4							

17.8.9 Consolidation Settlement: Magnitude

As pointed out in section 17.8.8, the consolidation test is well suited to predicting the settlement of structures when most of the settlement is due to vertical compression and very little is due to lateral deformation. This limited horizontal movement can be created by the friction on the top and the bottom of a thin layer under a wide load. In this case the consolidation test curve can be used as the stress-strain curve in the general method, and the strains can be obtained by reading the curve for the corresponding stresses (Figure 17.33). It is recommended that you read the curve directly rather than reconstructing an undisturbed curve. This should be an incentive for obtaining quality samples, as disturbance is likely to increase the settlement prediction. Note that the consolidation curve is made of points corresponding to equilibrium points at the end of the 24-hour test period under each load step. The settlement is then calculated as:

$$\Delta H = H_o \frac{(e_{before} - e_{after})}{1 + e_o} = H_o(\varepsilon_{after} - \varepsilon_{before}) \quad (17.89)$$

Or, when the compressing zone is divided into several layers:

$$\Delta H = \sum_{i=1}^{n} H_{oi} \frac{(e_{i\ before} - e_{i\ after})}{1 + e_{oi}} = \sum_{i=1}^{n} H_{oi}(\varepsilon_{i\ after} - \varepsilon_{i\ before}) \quad (17.90)$$

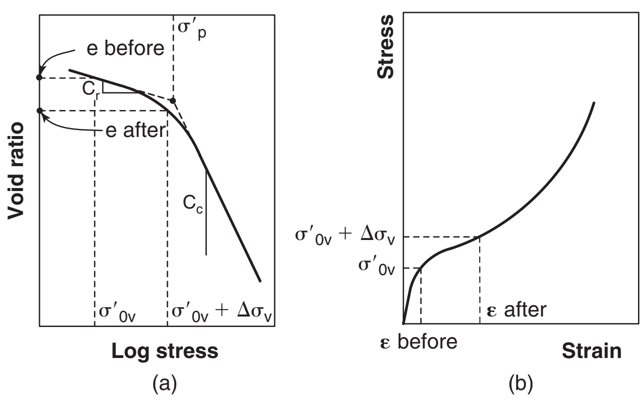

Figure 17.33 Obtaining the strains from the stresses for settlement calculations.

where ΔH is the settlement, H_{oi} is the thickness of each layer, $e_{ibefore}$ and e_{iafter} are the void ratios read on the consolidation curve (Figure 17.33) at σ'_{ov} and $\sigma'_{ov} + \Delta\sigma_v$ respectively, e_{oi} is the initial void ratio in each layer, and ε_{iafter} and $\varepsilon_{ibefore}$ are the strains read on the consolidation curve at $\sigma'_{ov} + \Delta\sigma_v$ and σ'_{ov} respectively.

If consolidation curves are not available, then the following equations can be used but the precision of the predictions will be affected. These equations correspond to the bilinear shape of the void ratio or vertical strain versus log of vertical effective stress (Figure 17.33a). Beyond the preconsolidation pressure σ'_p, the slope of the curve is the compression index C_c, defined as:

$$C_c = \frac{\Delta e}{\Delta \log \sigma'} = \frac{e_1 - e_2}{\log\left(\dfrac{\sigma'_1}{\sigma'_2}\right)} \tag{17.91}$$

Rough estimates of C_c can be obtained by correlation with index properties (see section 14.12). Before the preconsolidation pressure σ'_p, the slope of the curve is the recompression index C_r. It is defined as:

$$C_r = \frac{\Delta e}{\Delta \log \sigma'} = \frac{e_1 - e_2}{\log\left(\dfrac{\sigma'_1}{\sigma'_2}\right)} \tag{17.92}$$

The choice of the right equation for calculating the settlement is based on the relative magnitude of the effective stress before loading σ'_{ov} and the effective stress long after loading $\sigma'_{ov} + \Delta\sigma_v$, compared with the preconsolidation pressure σ'_p (see section 14.11). Normally consolidated (NC) soils have a vertical effective stress σ'_{ov} equal to the preconsolidation pressure σ'_{p}, and overconsolidated (OC) soils have a vertical effective stress σ'_{ov} smaller than the preconsolidation pressure σ'_p.

NC soils :

$$\Delta H = \frac{H_o}{1 + e_o} C_c \log\left(\frac{\sigma'_{ov} + \Delta\sigma_v}{\sigma'_{ov}}\right) \tag{17.93}$$

OC soils and $\sigma'_{ov} + \Delta\sigma_v < \sigma'_p$

$$\Delta H = \frac{H_o}{1 + e_o} C_r \log\left(\frac{\sigma'_{ov} + \Delta\sigma_v}{\sigma'_{ov}}\right) \tag{17.94}$$

OC soils and $\sigma'_{ov} + \Delta\sigma_v > \sigma'_p$

$$\Delta H = \frac{H_o}{1 + e_o}\left(C_r \log\left(\frac{\sigma'_p}{\sigma'_{ov}}\right) + C_c \log\left(\frac{\sigma'_{ov} + \Delta\sigma_v}{\sigma'_p}\right)\right) \tag{17.95}$$

For Eq. 17.93, the curve is simply a single straight line in the e-log σ' set of axes, so there is only one term. For Eq. 17.94, the curve is bilinear, but the stresses σ'_{ov} and $\sigma'_{\mathrm{ov}} + \Delta\sigma_v$ are both on the recompression part of the curve. In Eq. 17.95, the stresses σ'_{ov} and $\sigma'_{ov} + \Delta\sigma_v$ straddle the preconsolidation pressure; therefore, both C_c and C_r are involved and the equation has two terms. The first term represents the recompression from σ'_{ov} to σ'_p, the second term represents the virgin compression from σ'_p to $\sigma'_{ov} + \Delta\sigma_v$.

17.8.10 Consolidation Settlement: Time Rate

The time rate of settlement can be estimated by using the consolidation theory solution described in section 11.4.6. The time required for a given percentage of the settlement to take place is given by:

$$t_U = T_U \frac{H^2}{c_v} \tag{17.96}$$

where t_U is the time required for U% of the settlement to take place, T_U is the time factor (which comes from the theoretical solution and is obtained from Figure 17.34), H is the drainage length, and c_v is the coefficient of consolidation for the soil obtained from a consolidation test (see section 9.5.1). On Figure 17.34, curve C_1 represents the most common case. The parameter U is the average percent consolidation, which is a function of the time t and is defined as:

$$U(t) = \frac{\Delta H(t)}{\Delta H_{\max}} \tag{17.97}$$

where $\Delta H(t)$ is the settlement after a time t and $\Delta H_{\max}$ is the maximum settlement at time equal to infinity. $\Delta H_{\max}$ is the settlement obtained from section 17.8.9. The drainage length (Figure 17.35) depends on the ability of the upper layer and the lower layer to drain the water away. If both the top and bottom layers are free draining (two-way drainage), then the drainage length H is equal to one-half the layer thickness H_o. This is because the furthest that a water molecule has to travel to get out of the compressing layer is one-half of the layer thickness. If only one of the two layers, top or bottom, is free draining (one-way drainage), then the drainage length H is equal to the layer thickness H_o. This is because the furthest that a water molecule has to travel to get out of the compressing layer is the layer thickness. Then, the complete settlement vs. time curve ($\Delta H(t)$ vs. t_U, Figure 17.36) can be created by using the combination of Eq. 17.96 and Eq. 17.97.

17.8.11 Creep Settlement

The consolidation settlement is associated with the dissipation of excess water stress by drainage of the soil mass. When the excess water stress has dissipated, the settlement may continue to occur due to creep in the soil. The creep settlement is attributed to the slow movement of particles with respect to each other with no change in water stress. This creep settlement can occur in saturated soils as well as in unsaturated soils. It is often slow and small, but can be significant in soft soils and soils with high organic content.

One can use consolidation test data to estimate the creep settlement. During each load step in a consolidation test,

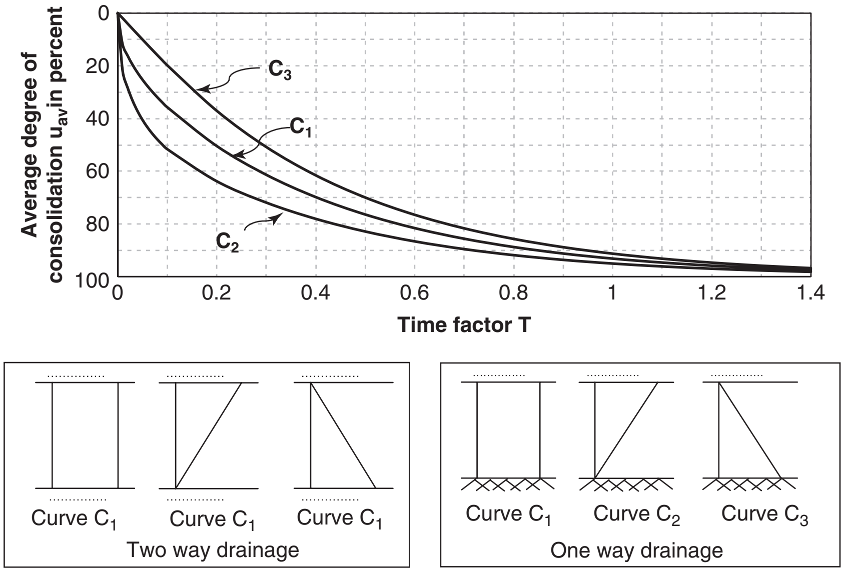

Figure 17.34 Time factor and percent consolidation curve.

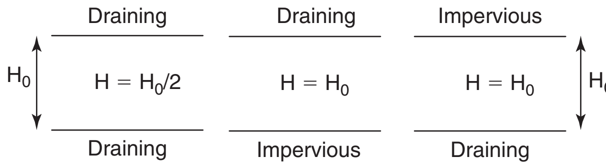

Figure 17.35 Drainage length.

the sample compression is recorded as a function of time (see section 14.12). The slope of the tail end of that curve corresponds to the creep settlement; because at that point on the curve, the excess water stress has dissipated. That slope C_α is called the *secondary compression index* and is defined as:

$$C_\alpha = \frac{\Delta e}{\Delta \log t} \tag{17.98}$$

where Δe is the change in void ratio between the start time t_{start} and the end time t_{end} and $\Delta \log\ t$ is the change in the log base 10 of the time. Then, the creep settlement is estimated as:

$$\Delta H = \frac{H_o}{1+e_o} C_\alpha \log\left(\frac{t_{end}}{t_{start}}\right) \tag{17.99}$$

where ΔH is the creep settlement, H_o is the layer thickness, e_o is the initial void ratio, C_α is the secondary compression index, t_{start} is the start time, and t_{end} is the end time.

To estimate the creep settlement, pressuremeter test data can be used, as discussed in section 17.7. The equation in this case is:

$$\frac{s(t_{end})}{s(t_{start})} = \left(\frac{t_{end}}{t_{start}}\right)^n \tag{17.100}$$

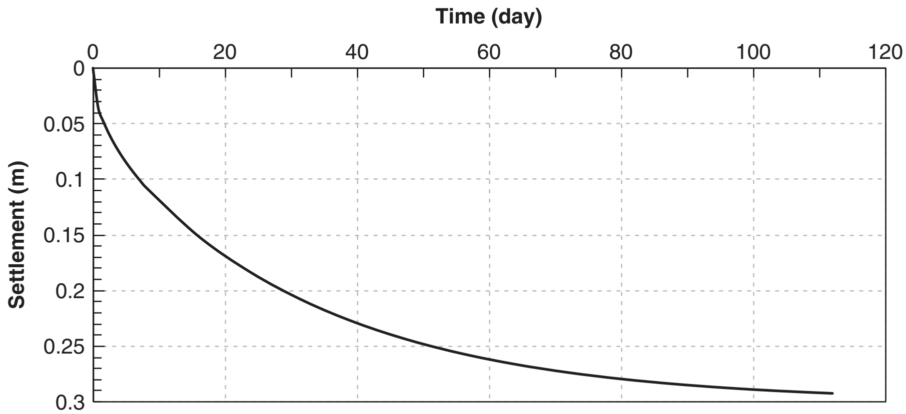

Figure 17.36 Settlement vs. time curve.

Table 17.9 Range of Possible Values for Footing Pressures

Category	Types of soils	Presumed allowable bearing value
Coarse-grained soils	Dense gravel or dense sand and gravel	>600 kN/m^2
	Medium-dense gravel, or medium-dense sand and gravel	200 to 600 kN/m^2
	Loose gravel, or loose sand and gravel	<200 kN/m^2
	Dense sand	>300 kN/m^2
	Medium-dense sand	100 to 300 kN/m^2
	Loose sand	<100 kN/m^2
Fine-grained soils	Very stiff and hard clays	300 to 600 kN/m^2
	Stiff clays	150 to 300 kN/m^2
	Firm clay	75 to 150 kN/m^2
	Soft clays and silts	<75 kN/m^2
	Very soft clay	Not applicable

where $s(t_{end})$ and $s(t_{start})$ are the settlements at the times t_{end} and t_{start} respectively and n is the rate effect exponent. This exponent can be measured on a site-specific basis by performing the pressuremeter tests discussed in section 17.7. The typical range of values for n is 0.01 and 0.03 for sands and 0.02 to 0.05 for stiff to hard clays.

17.8.12 Bearing Pressure Values

The allowable pressure on a shallow foundation should always be calculated according to proper design procedures. It is useful to have an idea of what to expect as a range of possible pressure for various soils. Any pressure significantly outside of those ranges should be checked very carefully. Table 17.9 gives estimates of these ranges.

17.9 SHRINK-SWELL MOVEMENT

17.9.1 Water Content or Water Tension vs. Strain Curve

Most soils swell and shrink when they get wet and dry. Some soils are particularly prone to such movements, which must be taken into account in the foundation design. Such soils can be identified in a number of ways. One index is the *plasticity index*: the difference between the liquid limit and the plastic limit. The higher the plasticity index, the more prone to large shrink-swell movements the soil is. The reason is that a high plasticity index is indicative of a higher content of very small clay particles, and very small clay particles tend to absorb more water than larger clay particles. Another index is the *shrink-swell index*: the range of water content over which the soil freely shrinks and swells. The higher the shrink-swell index, the more prone to large shrink-swell movement the soil is. The reason is simply that the volume change is related to the water content and that a larger variation in water content leads to a larger change in volume.

The basic behavior of a soil with regard to water content changes can be shown by performing two simple tests: a free swell test and a free shrink test (see sections 9.6 and 9.7). For a shrink test, a disk of soil is placed on a table and allowed to dry; its weight and dimensions are recorded as a function of time. At the end of the test, the sample is oven dried and the dry weight is obtained. The test data give the water content versus volume change of the sample. For the swell test, a disk of soil is placed in a consolidometer ring, submerged, and allowed to swell. The thickness of the sample and the weight of the sample are kept constant so that the water content versus volume change curve can be plotted. The free shrink curve and the free swell curve are joined on the same graph to give the shrink-swell curve for the sample. The maximum water content that the soil can reach is the swell limit. As the soil dries, the soil shrinks along the water content vs. volume change curve, which is a straight line until the shrinkage limit is reached (Figure 17.37).

During shrinkage, the soil particles come closer and closer together until they can no longer get any closer; at that point, called the *shrinkage limit*, any further loss of water will no longer represent a loss of volume. In first approximation, it

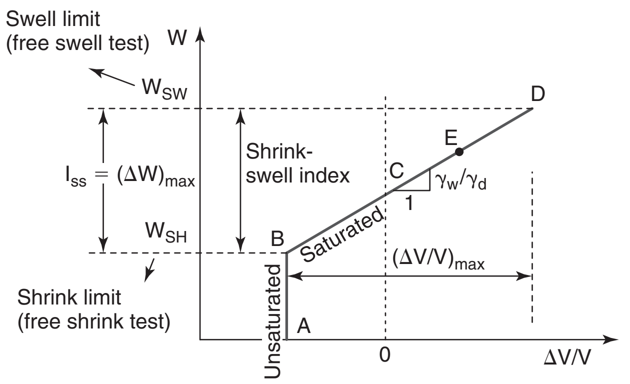

Figure 17.37 Water content vs. relative volume change.

can be said that above the shrinkage limit, the soil is saturated, and below the shrinkage limit, the soil is no longer saturated. The slope of the water content vs. volume change line is called the shrink-swell modulus E_w. Simple weight-volume relationships (see section 14.16) give E_w as:

$$E_w = \frac{\gamma_w}{\gamma_d} \tag{17.101}$$

where γ_w, and γ_d are the unit weight of water and the dry unit weight of the soil respectively.

17.9.2 Shrink-Swell Movement Calculation Methods

Several different types of methods are available to predict the shrink-swell movement of a soil: the potential vertical rise (PVR) method (McDowell 1956), the suction method, and the water content method. The PVR method consists of obtaining samples at the site, measuring the water content and the Atterberg limits, and using charts based on observations to calculate the maximum possible vertical movement. This movement corresponds to the case where the ground surface would be inundated for a very long time. This movement depends on the water content of the soil at the time of sampling. Therefore, a sample taken during the summer months will lead to a large predicted PVR and one taken during the winter months will lead to a small predicted PVR. The PVR only gives an indication of the swelling potential, not the shrinkage potential.

The suction method (Lytton 1994) relates the settlement to the log of the water tension. Lytton includes the settlement due to the change in mechanical stress in addition to the movement due to the change in water tension and proposes the following equation:

$$s = \sum_{i=1}^{n} f_i H_i \left(-\gamma_{hi} \log \frac{h_{fi}}{h_{ii}} - \gamma_{\sigma i} \log \frac{\sigma_{fi}}{\sigma_{ii}} - \gamma_{\pi i} \log \frac{\pi_{fi}}{\pi_{ii}} \right) \tag{17.102}$$

where i is the layer number, f_{i} is the crack fabric factor to convert the volumetric strain into vertical strain, $\mathrm{H_i}$ is the layer thickness, γ_{hi} is the matrix suction compression index for layer i, h_{fi} and h_{ii} are the final and initial values of the matric suction in layer i respectively, $\gamma_{\sigma i}$ is the mean principal stress compression index for layer i, σ_{fi} and σ_{ii} are the final and initial values of the mean principal stress in layer i respectively, $\gamma_{\pi i}$ is the osmotic suction compression index for layer i, and $\pi_{f\mathrm{i}}$ and π_{ii} are the final and initial values of the osmotic suction in layer i respectively. The three compression indices are obtained from correlations with index soil properties on samples from layer i, or from testing samples, and the boundary values of the matric and osmotic suction are based on experience. For the crack fabric factor f, Lytton recommends $f = 0.5$ when the soil is drying and $f = 0.8$ when the soil is wetting.

The water content method (Briaud et al. 2003) makes use of the water content vs. volume change curve recorded in a free shrink test to calculate the amplitude of the vertical movement. The equation used is:

$$s = \sum_{1}^{n} f_i \frac{\Delta w_i}{E_{wi}} H_i \tag{17.103}$$

where s is the vertical movement of the ground surface, n is the number of layers making up the depth of the active zone involved in the shrink-swell movement, f_i is the factor used to transform the volumetric strain into the vertical strain (0.33, according to Briaud et al. 2003), H_i is the thickness of the i th layer, Δw_i is the change of water content in the i th layer during the calculation period (expressed as a ratio, not a percentage), and E_{wi} is the shrink-swell modulus of the soil in the i th layer as given by Eq. 17.101 or measured in a shrink test or a swell test.

The number of layers involved in the calculations is given by the depth of the active zone, which is often obtained from local experience on water content profiles observed over several years. These profiles typically show large variations in water content near the surface, and a decrease in variation with depth down to a depth where the variation is negligible; that depth is the depth of the active zone. Typical values range between 3 and 5 m. The value of E_{wi} is obtained from measurements on samples (shrink test) from layer i or from using Eq. 17.101.

The value of Δw_i should not be taken as the difference between the swell limit and the shrink limit for the soil. This would assume that, during the life of the structure, the soil will shrink to the shrink limit and swell to the swell limit. This is extremely conservative and very unlikely. Instead, Δw is obtained from local experience as the amplitude of the water content variation at the chosen depth (middle of H_i) read on the water content profiles collected in an area over time. Briaud et al. (2003) collected more than 8000 water content measurements over a period of time and as a function of depth. They obtained values of the variation of water content over several seasons and found the amplitude of Δw for four cities in Texas (Figure 17.38). The samples came from right outside of the foundation as well as from under the foundation. The Δw values ranged from 0.05 to 0.08 for the samples outside of the foundation imprint. The Δw values for the samples directly under the foundation were lower:

$$\Delta w_{under} = 0.7 \Delta w_{outside} \tag{17.104}$$

17.9.3 Step-by-Step Procedure

Calculating the shrink-swell movement of a soil proceeds much like calculating the settlement of a building. The parallel is drawn in the following step-by-step procedure (Figure 17.39):

1. Determine the depth of the active zone H (the zone within which the movement takes place over time). This

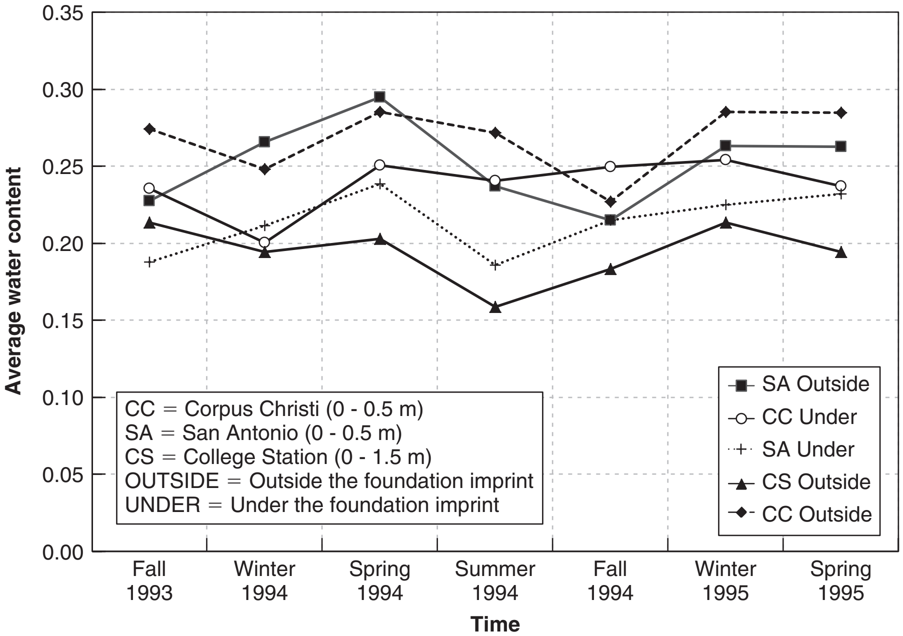

Figure 17.38 Water content variation for three cities in Texas. (Briaud et al. 2003)

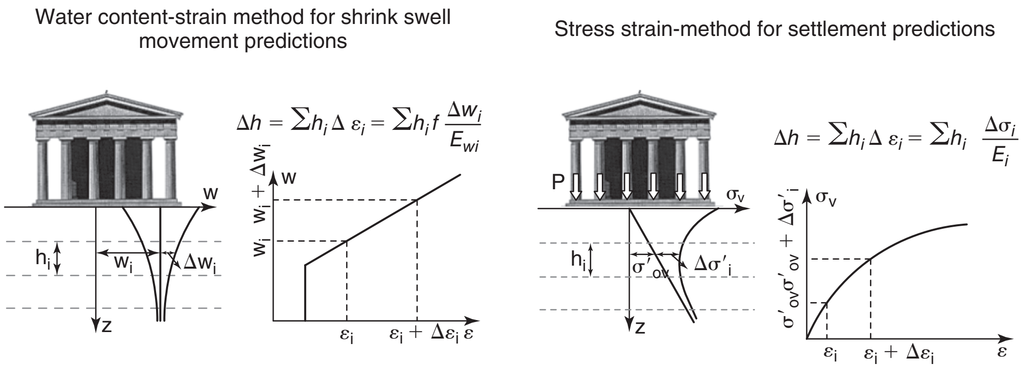

Figure 17.39 Parallel between shrink-swell and settlement methods.

is parallel to the zone of influence for the settlement case. The active zone is usually estimated from local experience or from water content or water tension profiles gathered over many years. Typical values vary between 3 and 5 m.

2. Decompose that zone into an appropriate number n of soil layers. This is the same step as in the settlement procedure.
3. Estimate the initial water content w_i in the center of each layer. The water content and associated water tension play the role of the effective stress in the settlement procedure.
4. Estimate the final water content w_f in the center of each layer. Again, the water content and associated water tension play the role of the effective stress in the settlement procedure. The final water content can be obtained from the soil boring files accumulated over time by a consulting company in a given geological area. This is what was done for Figure 17.38. This step is parallel to obtaining the increase in stress with depth by the elasticity method for the settlement case. Indeed, the water content in the shrink-swell calculations plays the role of the stress in the settlement calculations.
5. Obtain the relationship between the water content and the vertical strain by performing simple tests like the free shrink test or the free swell test. This relationship plays the role of the stress-strain curve in the settlement calculations.
6. Using the water content vs. vertical strain curve, obtain the strains ε_i and ε_f corresponding to the initial and final water content w_i and w_f. This is the same step as in the settlement calculations, but using water content instead of stress.

7. Calculate the shrink or swell movement using:

$$s = \sum_{1}^{n} f_i \frac{w_f - w_i}{E_{wi}} H_i \qquad (17.105)$$

where s is the vertical movement of the ground surface, n is the number of layers making up the depth of the active zone involved in the shrink-swell movement, f_i is the factor used to transform the volumetric strain into the vertical strain (0.33, according to Briaud et al. 2003), H_i is the thickness of the ith layer, w_f and w_i are the final water and initial contents in the ith layer during the calculation period (expressed as a ratio, not a percentage), and E_{wi} is the shrink-swell modulus of the soil in the ith layer as given by Eq. 17.101 or measured in a shrink test or a swell test. Equation 17.105 for shrink-swell movement is the same as Eq. 17.83 for settlement calculations.

This procedure uses the water content as the main variable. The water content can be replaced by the water tension or suction in this procedure when using the suction-based shrink-swell movement method.

17.9.4 Case History

Four footings were placed at a site near Dallas, Texas, where the soil is a CL-CH (Figure 17.40). The soil below footings W1 and W2 was water injected, whereas the soil below footings RF1 and RF2 was left intact. The soil properties are shown in Figure 17.40; the groundwater level was about 4.5 m deep. The footings were constructed at the ground surface and were 2 m by 2 m by 0.6 m thick. The movement

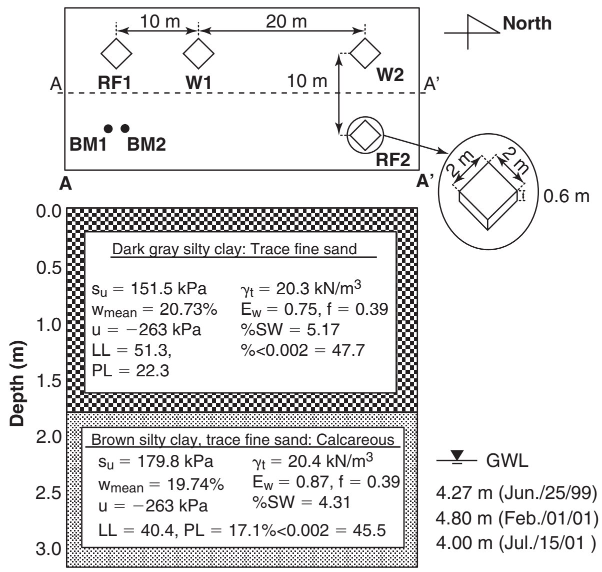

Figure 17.40 Footings and soil stratigraphy.

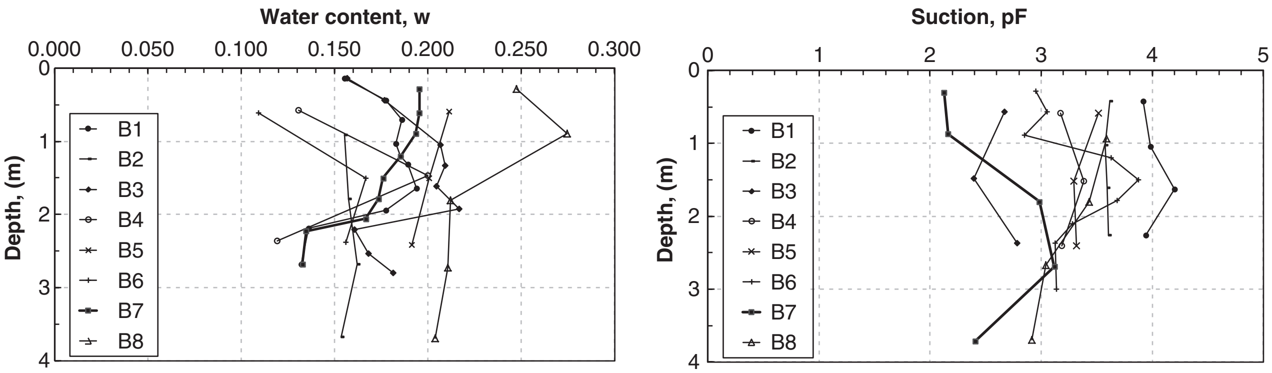

Figure 17.41 Water content and water tension variation over two years.

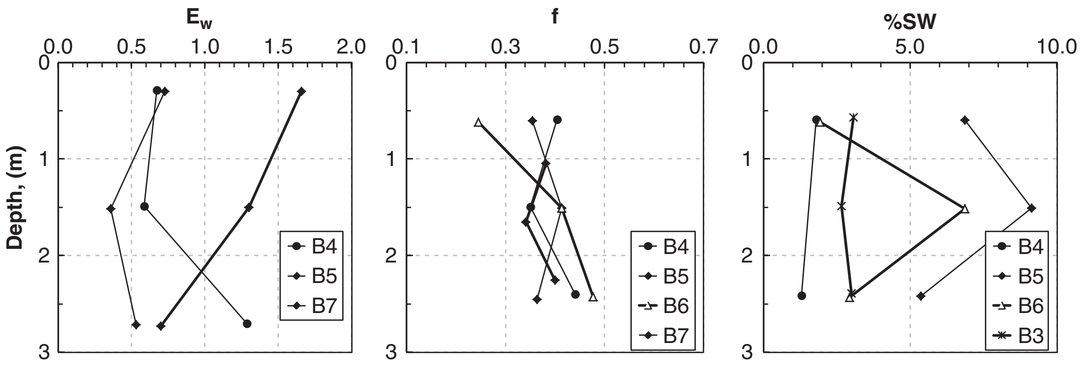

Figure 17.42 Soil properties.

of the footings was measured every month for 2 years and borings were drilled every 3 months. The samples were tested and gave the water content and water tension profiles shown in Figure 17.41. Additional properties, including the shrink-swell modulus E_w, the f factor to convert volumetric strain to vertical strain, and the maximum percent swell (%SW), are shown in Figure 17.42. The recorded movement of the footings is shown by Figure 17.43, and the temperature and rainfall variation during the same two years is shown in Figure 17.44. These data indicate the following:

1. The amplitude of movement of the footings on the water-injected soil is the same as the amplitude of the movement of the footings on the intact soil.
2. The footings on the water-injected soil swelled less and shrank more than the footings on the intact soil.
3. The movement was very small during the first year when the rainfall was very evenly distributed (Figure 17.44). During the second year, a three-month drought followed by three months of heavy rainfall created a lot of movement amplitude.

17.10 FOUNDATIONS ON SHRINK-SWELL SOILS

17.10.1 Types of Foundations on Shrink-Swell Soils

Predicting the vertical movement of the ground surface is useful but not a direct input to the design of foundations on shrink-swell soils. The problem with shrink-swell soils is that the soil shrinks and swells more at the edges of the building than under the center of the building. This tends to distort the building and damage it if the distortion exceeds the building's ability to deform. The best foundation systems are those that minimize building distortion even when the soil movement is very uneven. Foundations that have been used include (Figure 17.45):

1. Stiffened slab on grade for smaller structures (1 to 3 stories). These slabs consist of a thin (~0.1 m thick) slab on grade connected to deep beams (say, 1 to 1.2 m deep, 0.3 m wide, placed in both directions with a 3 to 5 m spacing center to center). This solution, sometimes called *waffle slab*, is typically economical (~\$100/m^2 in 2010) and very satisfactory if the slab is stiff enough.

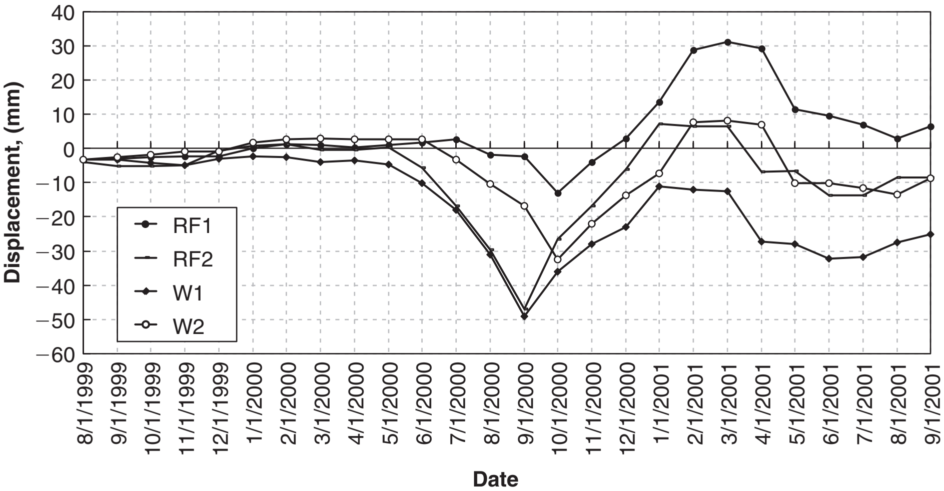

Figure 17.43 Observed movement over two years.

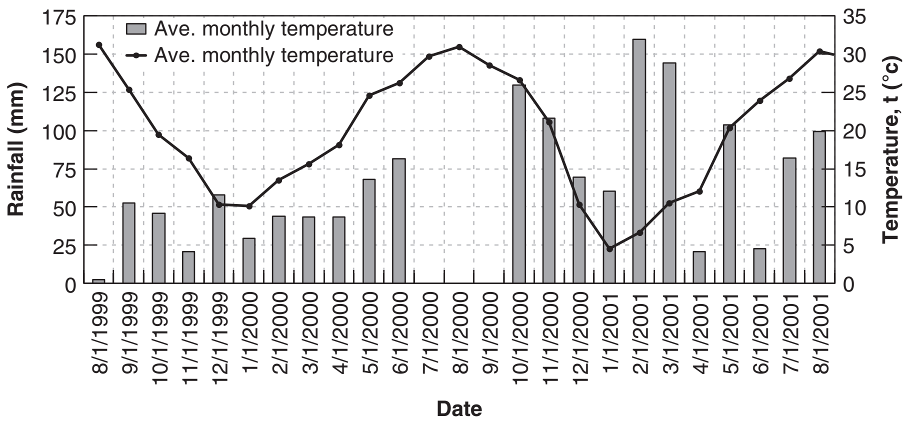

Figure 17.44 Rainfall and air temperature over the two-year period.

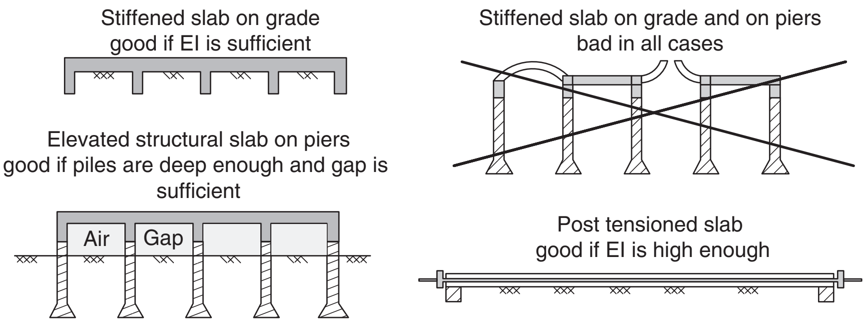

Figure 17.45 Types of foundations used for light buildings on shrink-swell soils.

2. Elevated structural slab for larger structures (3 to 15 stories). This system consists of a structural slab (a slab that can sustain the dead load and live load in free span) connected to piles in such a way that there is a sufficient gap (say, 0.3 m) between the ground surface and the bottom of the beams stiffening the slab. This solution is more expensive ($\sim\$200/\text{m}^2$ in 2010) but very satisfactory provided the piles go deep enough below the zone of movement.
3. Anchored slab on grade. This system consists of a slab on grade and on piles. There is no gap between the ground surface and the slab that rests on it. This is a very undesirable system because when the soil swells, the bored piles prevent the slab from moving up, so the slab deforms and can break under the swell pressure. Alternatively, if the soil shrinks under the slab, the slab on grade becomes unsupported and breaks because it is not designed to carry the load in free span.
4. Posttensioned slabs are typically flat slabs that are posttensioned to keep the concrete in compression during bending. They are satisfactory systems provided they are stiff enough to minimize distortion. For equal stiffness, they are not as economical as a stiffened slab on grade. Thin posttensioned slabs on grade are undesirable for buildings because they are overly flexible. Although they minimize the cracking of the slab, they do not prevent distortion of the superstructure. Thin posttensioned slabs are a very good solution, however, for playing surfaces such as tennis or basketball courts on shrink-swell soils, because they minimize cracking.

17.10.2 Design Method for Stiffened Slabs on Grade

A stiffened slab on grade has deep beams (e.g., 1 m deep, 0.3 m thick), spaced relatively closely (e.g., 4 m) in both directions. These beams stiffen the slab, which is sometimes called a *waffle slab*. The stiffening limits the amount of distortion that the superstructure is subjected to in case of soil movement. In the summer, when the soil shrinks around the periphery of the structure and a gap develops between the ground surface and the edge of the foundation, the edges of the slab do not drop significantly, because of the rigidity of the slab and beams; this prevents excessive distortion of the superstructure. In the winter, when the soil swells around the periphery of the structure and lifts the foundation, the center of the foundation does not sag significantly, again because of the rigidity of the slab and beams; this prevents excessive deformations of the superstructure. The design of the foundation is therefore controlled by these two conditions, sometimes called *edge drop* and *edge lift*. The critical design parameters for these two conditions are the *cantilever edge distance* for the edge drop condition and the *free span distance* for the edge lift condition. These parameters depend on several factors, including weather, vegetation, soil shrink-swell sensitivity, soil stiffness, and slab stiffness. Several design procedures have been suggested over the years, including:

1. BRAB method (Building Research Advisory Board 1968)
2. PTI method (Post Tensioning Institute 2004)
3. WRI method (Wire Reinforcing Institute 1981)
4. Australian method (Australian Standard (AS) 2870, 1996)
5. TAMU-Slab method (Briaud et al. 2010)

The TAMU-Slab method is based on the use of charts. The details of the research work on which the method is based can be found in Abdelmalak (2007) and Briaud et al. (2010). The design parameters necessary to size the beams and their spacing are the maximum bending moment $M_{\max}$, the maximum shear force $V_{\max}$, and the maximum deflection $\Delta_{\max}$ of the slab (Figure 17.46).

In the TAMU-Slab method, these quantities are linked to the equations of $M_{\max}$, $V_{\max}$, and $\Delta_{\max}$ for an equivalent cantilever beam with a length L_{eqv}. These equations are applied to the design of the beams for the stiffened slab by using modification factors:

$$M_{\max} = \frac{1}{2} q L_{eqv}{}^2 \tag{17.106}$$

$$V_{\max} = F_v q L_{eqv} \tag{17.107}$$

$$\Delta_{\max} = \frac{q L_{eqv}^4}{F_{\Delta\max} EI} \tag{17.108}$$

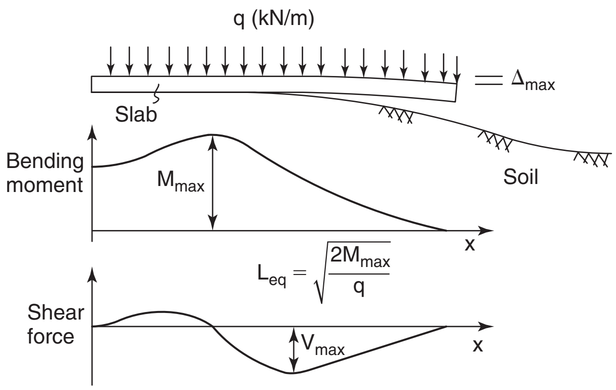

Figure 17.46 Half slab with deflection, bending moment, and shear.

where q (kN/m) is the distributed load on the cantilever beam, F_v is the modification factor for the shear force, $F_{\Delta\,\max}$ is the modification factor for the deflection, and $EI(\text{kN.m}^2)$ is the bending stiffness of the beam product of the modulus $E(\text{kN/m}^2)$ by the moment of inertia $I\,(\text{m}^4)$. For a true cantilever beam, the equivalent cantilever length L_{eqv} would simply be the length of the cantilever beam, the maximum shear modification factor F_V would be 1, and the maximum deflection factor $F_{\Delta\,\max}$ would be 8. For the stiffened slab on grade, these factors were obtained from numerical simulations. The equivalent length and the modification factors are given by charts for various slab thicknesses.

The three most important factors affecting the final choice of the beam depth are the weather, the soil, and the slab stiffness. The weather and the soil were found (Abdelmalak 2007) to be best characterized by the soil and weather index I_{SW}. This soil and weather index can be defined on the basis of the water tension or the water content:

$$I_{SW(Water\ Tension)} = I_{SS}\,H\;\Delta U_{edge} \tag{17.109}$$

$$I_{SW(Water\ Content)} = \mathrm{H}\;\Delta w_{edge} = 0.5 I_{SW(Water\ Tension)} \tag{17.110}$$

where $I_{SW(Water\ Tension)}$ and $I_{SW(Water\ Content)}$ are the soil and weather indices on the basis of the water tension and the water content respectively; I_{SS} is the shrink-swell index, a soil parameter equal to the difference between the swell limit and the shrink limit; H is the depth of the active zone in meters; ΔU_{edge} is the change in the $\log_{10}$ of the water tension in kPa at the edge of the foundation over the period considered for the design; and Δw_{edge} is the change in water content expressed as a ratio at the edge of the foundation over the period considered for the design. The change in log10 of water tension is:

$$\Delta U_{edge} = \log_{10}\frac{u_w(\mathit{final\ at\ edge})}{u_w(\mathit{initial\ at\ edge})} \tag{17.111}$$

Based on testing of a number of clays, the relationship between $I_{SW(Water\ tension)}$ and $I_{SW(Water\ content)}$ was found to be:

$$I_{SW(Water\ Content)} = 0.5 I_{SW(Water\ Tension)}$$

because

$$\Delta w_{edge} = 0.5 I_{SS}\,\Delta U_{edge} \tag{17.112}$$

A few cities in the United States where the shrink-swell soil problem is acute were selected and the weather over the past 20 years was simulated to obtain estimates of the change in $\log_{10}$ of water tension (kPa), ΔU_{edge}. Simulations were performed for the free field and the edge of the foundation. It was found that the value at the edge was about one-half the value in the free field. The results are shown in Table 17.10.

The slab stiffness was represented by the slab equivalent depth d_{eq}, which represents the thickness of a flat slab having the same moment of inertia as the moment of inertia of a stiffened slab with a beam depth equal to D, a beam width equal to b, and a beam spacing equal to S. The slab equivalent depth can be calculated by:

$$S\,{d_{eq}}^3 = b\,D^3 \tag{17.113}$$

Step-by-step procedure for the water content method

1. Obtain the dimensions of the slab $B \times L$ and the loading pressure on the slab w (kPa).
2. Estimate the depth H of the active zone. This is best based on local practice and experience. In Texas, H is typically considered to be between 3 and 5 m.
3. Estimate the change in water content Δw_{edge} at the edge of the foundation. This is also best estimated from local practice and experience. Note that Δw_{edge} was found to be equal to one-half of the change in water content $\Delta w_{free\ \text{field}}$ in the free field. The borings

Table 17.10 Change in $\log_{10}$ of Water Tension in kPa for Six Cities in the USA

	College Station, TX	San Antonio, TX	Austin, TX	Dallas, TX	Houston, TX	Denver, CO
$\Delta U_{\text{free field}}$	0.788	1.392	0.866	1.295	1.283	1.374
ΔU_{edge}	0.394	0.696	0.433	0.648	0.642	0.687

(Abdelmalak 2007)

accumulated over time by a local company may come in very handy in estimating $\Delta w_{free\,\text{field}}$ and therefore Δw_{edge}.

4. Choose a beam spacing s and a beam width b. Typical numbers for s are 3 to 5 m and for b are equal to 0.3 m.
5. Make a first assumption as to the beam depth D (say, 1 m).
6. Calculate the thickness of the equivalent slab d_{eq} by using Eq. 17.113 and the soil and weather index $I_{SW\,(water\ content)}$ by using Eq. 17.110.
7. Use the design charts of Figures 17.47 to 17.49 for the edge drop case (summer shrinkage) and Figure 17.50 to 17.52 for the edge lift case (winter swelling), and obtain the equivalent length L_{eq}, the shear factor F_v, and the deflection factor $F_{\Delta\,\max}$.
8. Use the distributed load on the beam q(kN/m) (q = $w \times$ s) to calculate the maximum bending moment $M_{\max}$, maximum shear $V_{\max}$, and maximum deflection $\Delta_{\max}$ according to Eqs. 17.106 to 17.108.
9. Calculate the ratio $0.5L/\Delta_{\max}$ and $L_{eq}/\Delta_{\max}$. Ratios larger than 500 typically lead to acceptable distortions. If this criterion is not reached, repeat steps 5 to 9 with a larger beam depth D.
10. If the deflection criterion of step 9 is met, use the maximum bending moment and maximum shear to design the beam reinforcement and the slab.

Step-by-step procedure for the water tension method

The steps for this method are the same as the steps for the water content method except for the following. In step 3, the change in $\log_{10}$ of the water tension in kPa is needed instead

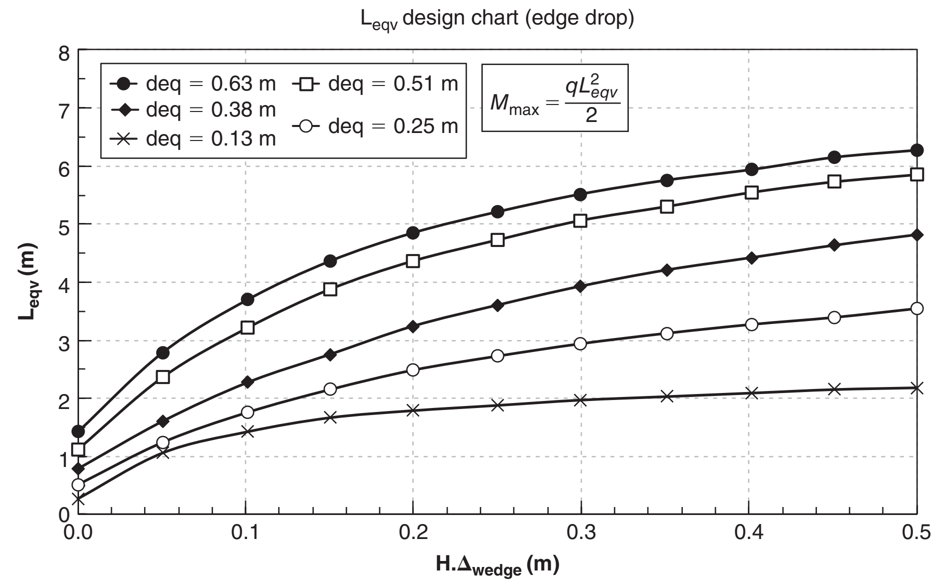

Figure 17.47 Equivalent cantilever length—water content method—edge drop case.

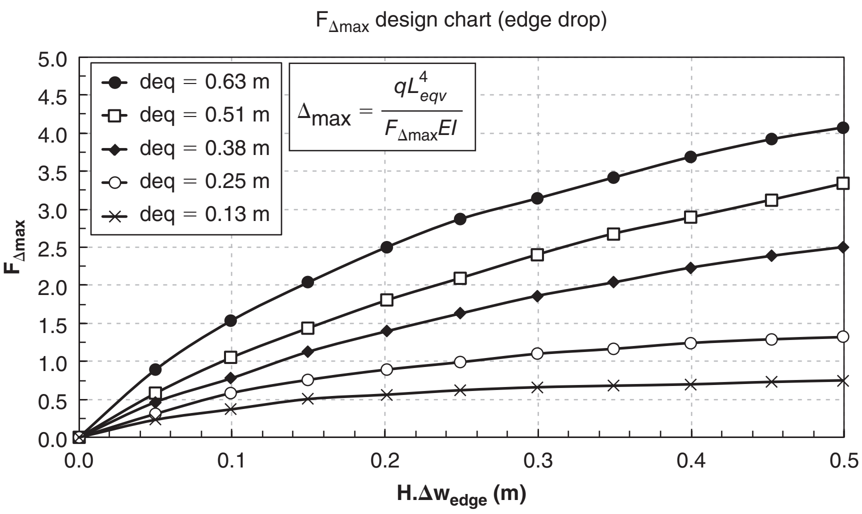

Figure 17.48 Maximum deflection factor—water content method—edge drop case.

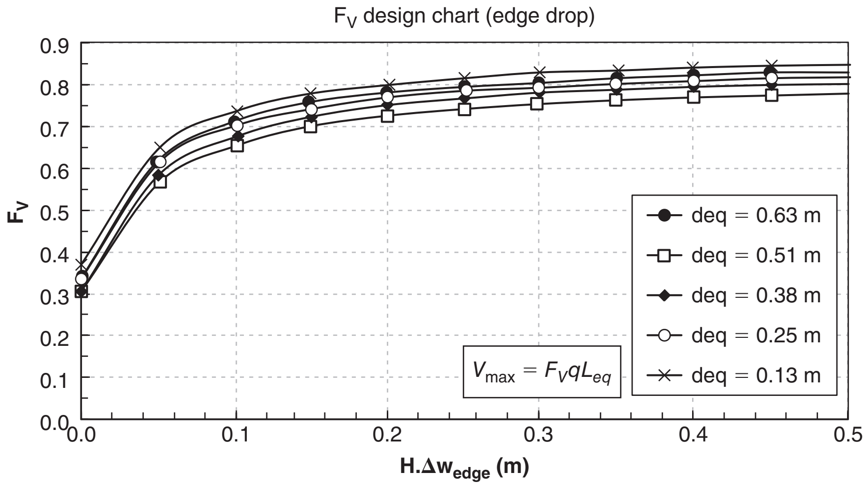

Figure 17.49 Maximum shear factor—water content method—edge drop case.

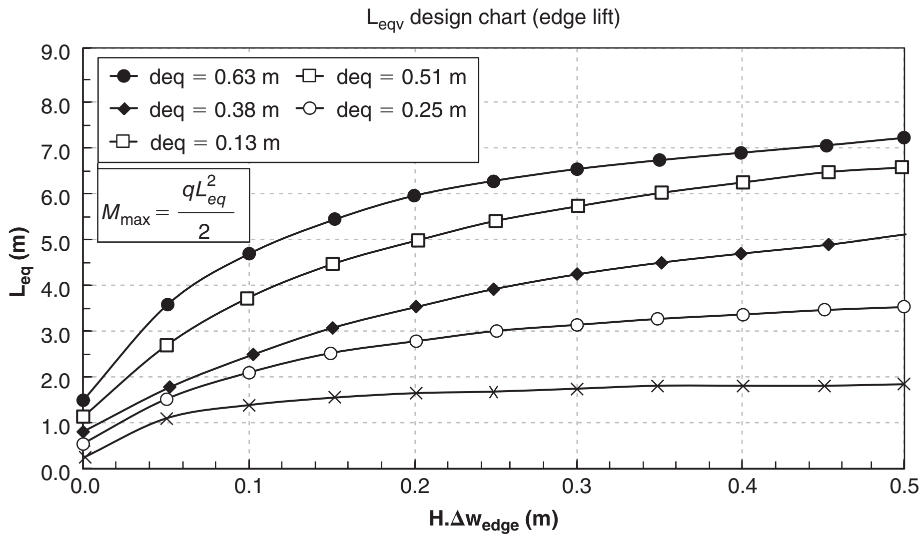

Figure 17.50 Equivalent cantilever length—water content method—edge lift case.

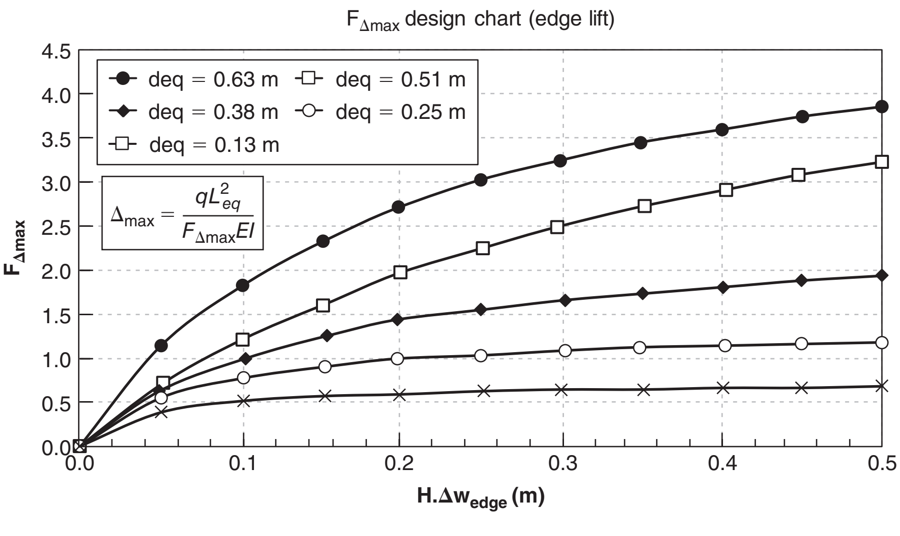

Figure 17.51 Maximum deflection factor—water content method—edge lift case.

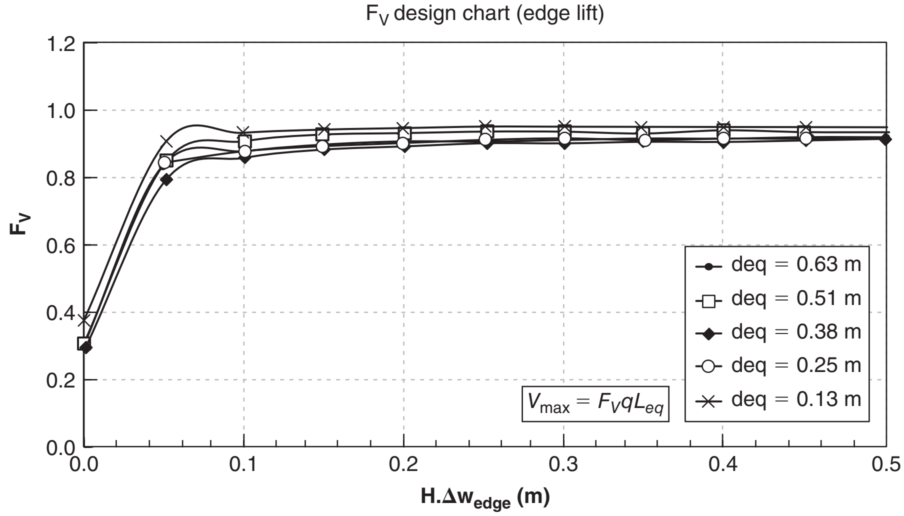

Figure 17.52 Maximum shear factor—water content method—edge lift case.

of the change in water content. In step 6, the soil and weather index $I_{SW(water\ tension)}$ is used instead of $I_{SW(water\ content)}$ based on Eq. 17.112. In step 7, the same charts are used after converting the $I_{SW(water\ tension)}$ into $I_{SW(water\ content)}$ based on Eq. 17.112.

17.11 TOLERABLE MOVEMENTS

Tolerable movements depend on the structure that is being built. Some embankments can tolerate 1 m of settlement as long as the pavement is built after the settlement takes place. Some sensitive facilities can tolerate only a few millimeters of settlement. Very tall buildings can settle anywhere from a few millimeters to 200 or 300 mm. One problem with a large total settlement is the connection to utilities outside the building, because typically the building settles or moves with respect to its surroundings. The total settlement s_t is one issue, but the differential settlement s_d is even more important in many cases. The differential settlement is rarely calculated, however, as there are rarely enough borings to make a settlement calculation at each building column or at each bridge pier. The practice is to calculate s_t and to assume that s_d is 3/4 of s_t:

$$s_d = 0.75 s_t \tag{17.114}$$

Bridges can tolerate a lot of differential movement, as documented in the study by Moulton et al. (1985). *Simply supported bridges* are bridges where each span is made of beams resting on top of the piers; the beams are not connected from one span to the next. For a *continuous bridge*, the beams continuously span several piers from one end of the bridge to the other. Simply supported bridges are easier and faster to construct, but continuous bridges make better use of the material and are thus lighter and therefore can be cheaper. Moulton surveyed more than 400 bridges and found that the supports (abutments and piers) moved vertically an average of 94 mm and horizontally an average of 68 mm. He found that bridges supported on shallow foundations had the same average movement as the ones founded on piles. From damage inspection, he concluded that total vertical and horizontal movements of up to 50 mm were tolerable. The *longitudinal distortion* is the ratio of the differential movement s_d between adjacent piers over the span length L. Moulton et al. (1985) and Barker et al. (1991) made recommendations for the limits of longitudinal angular distortion. In the end, it appears reasonable to accept 0.004 for continuous bridges and 0.008 for simply supported bridges. Simply supported bridges can sustain more differential movement than continuous bridges.

$$\text{For simply supported bridges} \quad \frac{L}{s_d} \geq 125 \tag{17.115}$$

$$\text{For continuous bridges} \quad \frac{L}{s_d} \geq 250 \tag{17.116}$$

where L is the span length and s_d is the differential vertical movement between adjacent piers. The amount of movement and distortion that buildings can tolerate has been studied by many researchers, including Skempton and MacDonald (1956), Polshin and Tokar (1957), Wahls (1994), and Zhang and Ng (2007). The tolerable amount of movement and distortion also depends on the level of damage that can be tolerated by the building, including the appearance and the function. It varies with many factors, such as the type, size, function, and properties of the structure; the soil type and properties; the method and time of construction; the type and stiffness of the foundation; and the rate and uniformity of the settlement. Zhang and Ng (2007) collected data for 380 buildings; the results are shown in Table 17.11. Many codes include tolerable values as well. All in all, it appears that, for buildings, vertical movements of 50 mm are generally tolerable and that

Table 17.11 Allowable Vertical Displacement and Angular Distortion for Buildings

Building category	Allowable value (FS = 1.5) Vertical displacement (mm)	Allowable value (FS = 1.5) Span length over differential movement	Allowable value (95% percentile) Vertical displacement (mm)	Allowable value (95% percentile) Span length over differential movement
Foundation type:				
Shallow foundation	145	245	49	833
Deep foundation	71	770	42	3333
Structural type:				
Frame buildings	92	323	29	1000
Load-bearing wall	60	417	22	1111
Soil type:				
Clay	113	333	44	1666
Sand	57		27	
Building use:				
Factory	141	263	53	526
Office	81	435	47	833

(Zhang and Ng 2007)

larger movements can be tolerated if they occur uniformly. It is common practice, however, to design buildings for 25 mm settlement. A span length over differential movement ratio of 500 also seems good guidance in most cases:

$$\text{For buildings} \quad \frac{L}{s_d} \geq 500 \qquad (17.117)$$

where L is the span length and s_d is the differential vertical movement between adjacent columns.

17.12 LARGE MAT FOUNDATIONS

17.12.1 General Principles

Mat foundations, also called *raft foundations*, are shallow foundations. A large mat may be used as the foundation for a tall building in an area where the soil strength does not increase significantly with depth. The design strategy is to place the foundation at a depth such that the weight of the excavated soil is nearly equal to the weight of the building. For this reason it is often called a *floating foundation*. The unit weight of soil (about 16 to 20 kN/m^3) is much larger than the unit weight of a building (about 2.5 to 5 kN/m^3). A story is about 3 m high; therefore, using a ratio of unit weight of soil over unit weight of building equal to 5, when a mat foundation is placed at a depth of 12 m, the weight of soil removed is equal to a 60 m high building with 20 stories (16 out of the ground). Placing such a building on such a foundation would lead to a postconstruction stress on the soil equal to the one in the soil prior to construction. Therefore, there would likely be very little problem with ultimate capacity and settlement. More precisely, the soil movement would be reduced to the unloading and reloading, which would take place during excavation of the soil and construction of the building. If a building taller than 20 stories were built, the ultimate capacity and settlement would have to be considered under the excess load beyond that of 20 stories. These are the basic geotechnical governing principles for the design of large mat foundations for tall buildings.

The design of the mat itself is controlled by the bending that it will undergo. The column loads represent point loads on the mat that must be transferred to the soil without punching or excessive bending. This requires an amount of concrete reinforcement dictated by the interaction between the stiff mat and the softer soil. The analysis can proceed in one of two ways: beam on elastic foundation approach or finite element approach. In the beam on elastic foundation approach, the stiffness of the soil comes from a stress-strain curve obtained, for example, from a consolidation test, and the stiffness of the mat is given by its bending stiffness value EI, where E is the modulus of concrete and I is the moment of inertia of the section. Because the stiffness of the soil is dependent on the strain experienced by the soil, and because the soil strain also depends on the mat stiffness, an iteration process develops where a run is made with the mat stiffness and a chosen soil stiffness; then the results are used to calculate the new soil strain and the next value of the soil stiffness. This process is repeated until the assumed soil stiffness and the calculated soil stiffness are within an acceptable tolerance. At that point, the bending moments in the mat are used to choose the amount of reinforcement necessary. With the more sophisticated FEM approach, the interaction is taken into account directly and the output gives the pressure distribution under the mat, the mat settlement profile, and the mat bending moment and shear. These mats often end up being of a uniform thickness equal to about 3 m. Pouring such mats is a large operation requiring many concrete trucks lined up one after the other; the concrete sets while developing very high temperatures (up to 80° C or more) due to the heat of hydration.

17.12.2 Example of Settlement Calculations

A large building weighing 400 MN is to be built on a deep deposit of very stiff clay (Figure 17.53). The building is to be placed at the bottom of a 15 m deep excavation, which will correspond to 4 levels of parking garages and 1 level for a mall. The height of the building is 180 m or 60 stories in addition to the 15 m of embedment. The footprint is a 30 m × 30 m square and the building will be founded on a thick mat foundation. The soil has a total unit weight of 20 kN/m^3 and the groundwater level is deeper than the zone of influence of the foundation. Pressuremeter tests gave a profile of first load modulus E_o and reload modulus E_r, as shown on Figure 17.53, with the equations:

$$\text{First load PMT modulus } E_o \text{ (MPa)} = 10 + 0.5z\text{(m)} \quad (17.118)$$

$$\text{Reload PMT modulus } E_r \text{ (MPa)} = 50 + 2z\text{(m)} \quad (17.119)$$

The strength has been deemed sufficient not to create problems of ultimate bearing capacity, but the settlement must be estimated.

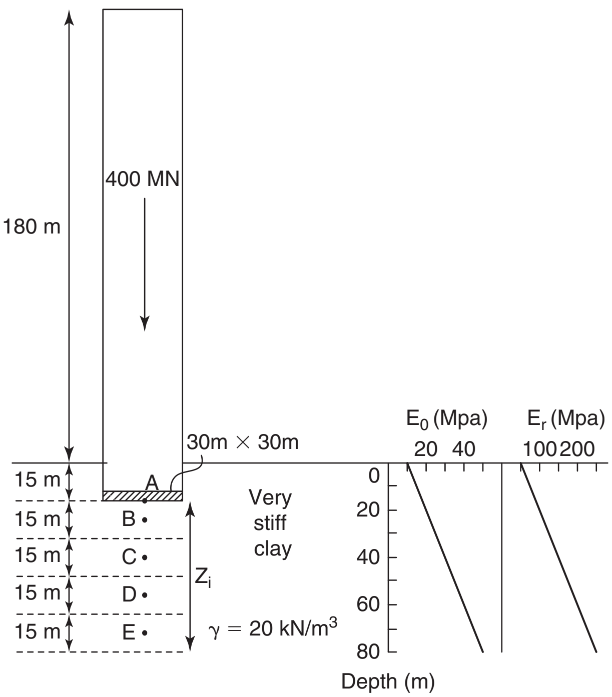

Figure 17.53 High-rise settlement: Problem definition.

It is first necessary to understand clearly what the soil will undergo at any depth below the building (Figure 17.54). Before any excavation, the vertical stress at a point in the soil is σ_{ov}. The 15 m deep excavation creates a decrease in stress equal to $\Delta\sigma_{exc}$ below the excavation level, such that after the excavation the stress has decreased to $\sigma_{ov} - \Delta\sigma_{exc}$. The construction of the building creates an increase in stress equal to $\Delta\sigma_{\text{bldg}}$ such that after the building is completed, the stress in the soil is $\sigma_{ov} - \Delta\sigma_{exc} + \Delta\sigma_{bldg}$. When the building is constructed starting at the bottom of the excavation, the soil first follows a reloading curve until the weight of the building becomes equal to the weight of soil removed. In other words, the recompression settlement S_{rel} should be calculated under the stress increment $\Delta\sigma_{exc}$ while using the PMT reload modulus E_r. Then, as construction continues, the soil is loaded in the virgin behavior and the settlement beyond the recompression settlement (sometimes called the *net settlement*, S_{net}) should be calculated under the stress increment $\Delta\sigma_{bldg} - \Delta\sigma_{exc}$ (sometimes called the *net increase in stress*) while using the PMT first load modulus E_o. This process is illustrated in Figure 17.54.

The steps outlined in section 17.8.5 are followed and are presented in Table 17.12.

1. The zone of influence z_i is taken as 2B because the building imprint is square: $z_i = 60$ m.
2. The zone of influence in this case is decomposed into 4 layers (Figure 17.53), each 15 m thick. This is column 1 in Table 17.12.
3. We calculate the initial stress σ_{ov} at the center of each layer. For example, the center of layer 1 is at a depth of 22.5 m and therefore the vertical stress at the center is $\sigma_{ov} = 22.5 \times 20 = 450$ kPa. This is column 2 in Table 17.12.
4. The first load modulus E_o is calculated in the center of each layer. For example, the modulus in the middle of layer 1 is $E_o = 10 + 0.5 \times 22.5 = 21.25$ MPa. This is column 3 in Table 17.12.
5. The reload modulus E_r is calculated in the center of each layer. For example, the modulus in the middle of layer 1 is $E_r = 50 + 2 \times 22.5 = 95$ MPa. This is column 4 in Table 17.12.
6. We calculate the decrease in stress $\Delta\sigma_{exc}$ in the middle of each layer due to the excavation. The total pressure decrease at the bottom of the excavation is $p_{exc} = 15 \times 20 = 300$ kPa. The bulb of pressure method is used to obtain the decrease in stress in the middle of each layer. For example, the decrease in stress $\Delta\sigma_{exc}$ in the middle of layer 1 is 0.85 $p_{exc} = 255$ kPa according to the bulb of pressure shown in Figure 17.31. This is column 5 in Table 17.12.
7. We calculate the increase in stress $\Delta\sigma_{bldg}$ in the middle of each layer due to the construction of the building. The total pressure increase at the foundation level is the weight of the building divided by the foundation area or $p_{bldg} = 400000/30 \times 30 = 444$ kPa. The bulb

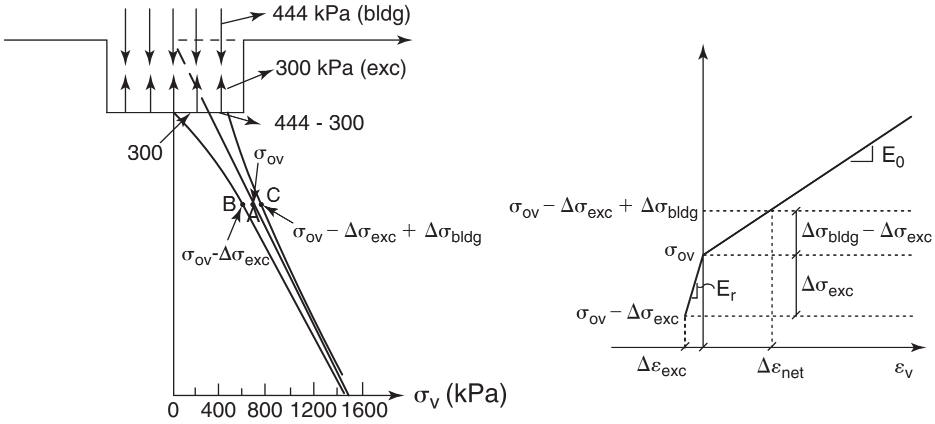

Figure 17.54 High-rise settlement: Stresses.

Table 17.12 High-Rise Settlement Calculations

1	2	3	4	5	6	7	8	9	10
H (m)	σ_{ov} (kPa)	E_o (kPa)	E_r (kPa)	$\Delta\sigma_{exc}$ (kPa)	$\Delta\sigma_{bldg}$ (kPa)	$\Delta\varepsilon_{exc}$	$\Delta\varepsilon_{net}$	ΔH_{rel} (mm)	ΔH_{net} (mm)
15	450	21250	95000	255	378	0.00268	0.00579	40	87
15	750	28750	125000	150	222	0.00120	0.00250	18	38
15	1050	36250	155000	75	111	0.00048	0.00099	7	15
15	1350	43750	185000	45	67	0.00024	0.00050	4	8
								69	148

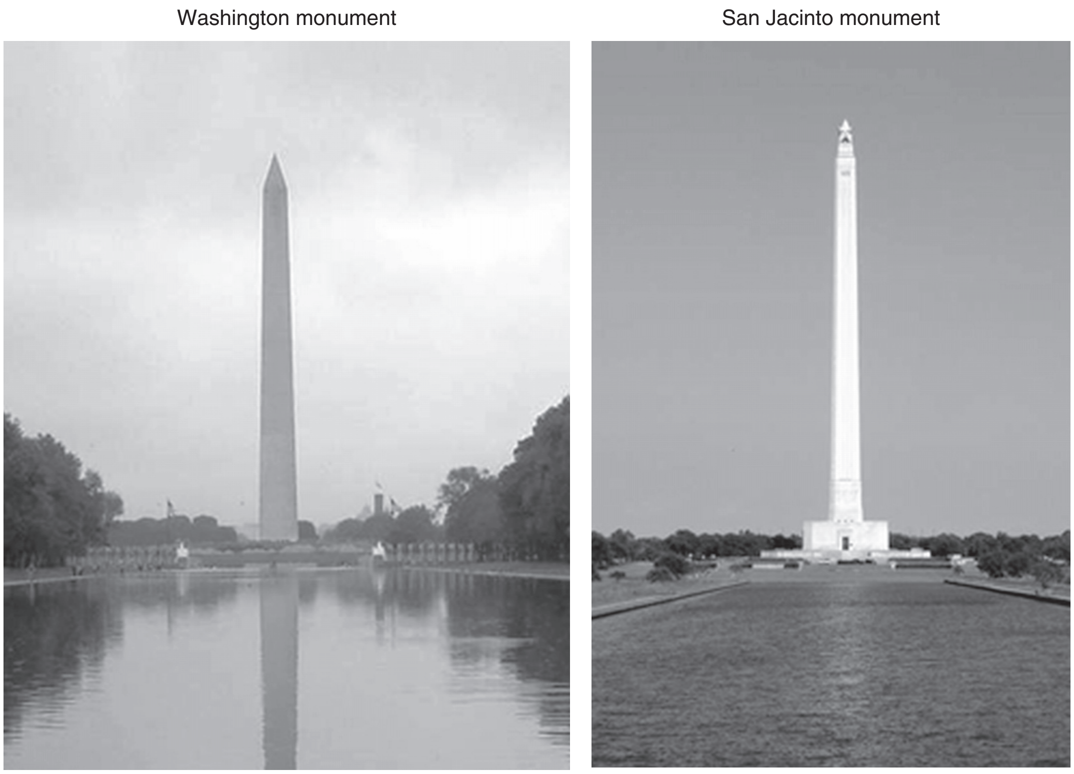

Figure 17.55 Washington Monument and San Jacinto Monument on large mats.

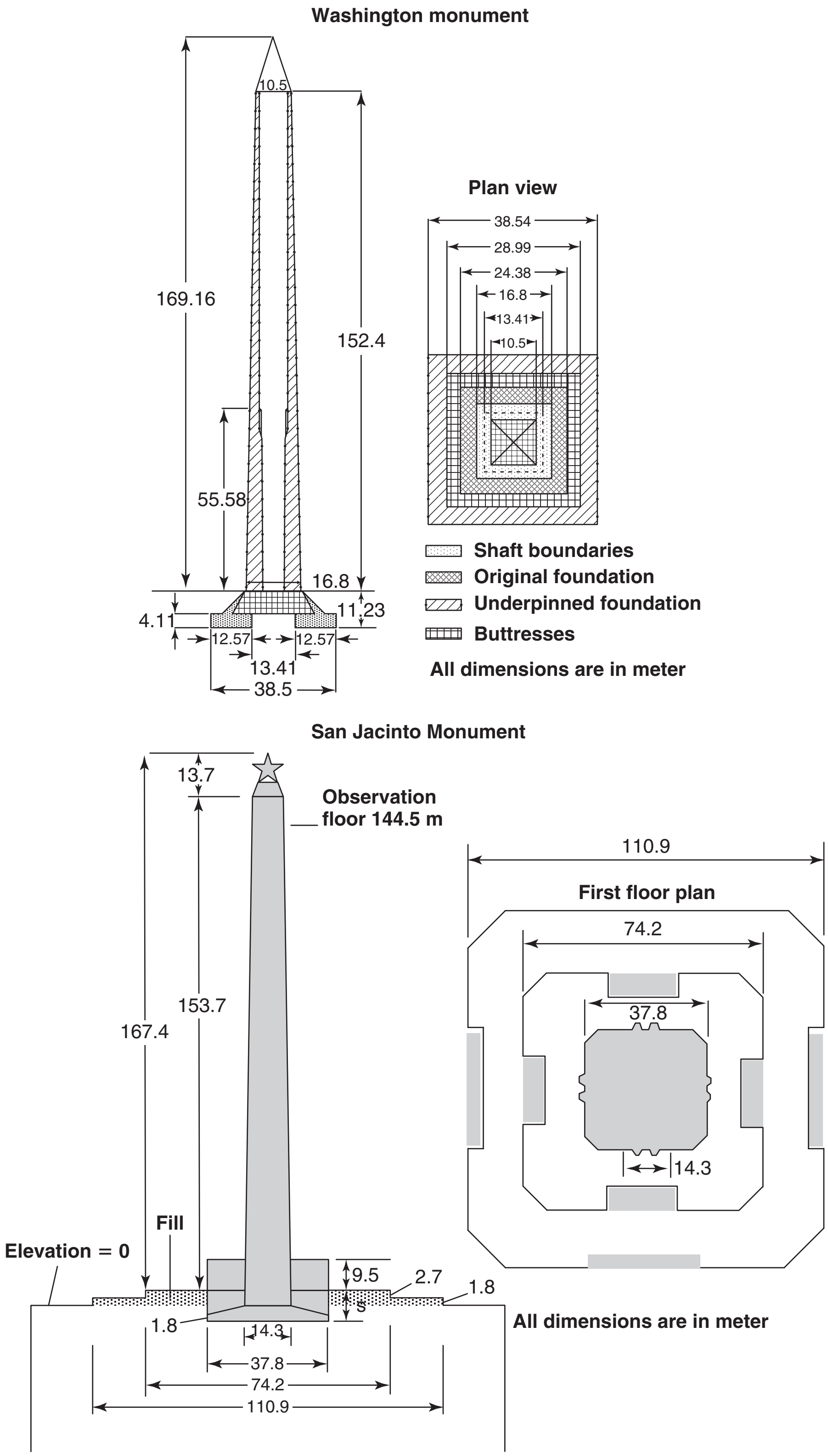

Figure 17.56 Washington Monument and San Jacinto Monument dimensions.

of pressure method is used to obtain the increase in stress in the middle of each layer. For example, the increase in stress $\Delta\sigma_{bldg}$ in the middle of layer 1 is $0.85 p_{exc} = 378$ kPa according to the bulb of pressure shown in Figure 17.31. This is column 6 in Table 17.12.

8. The weight of the soil excavated is equal to 20 × 30 × 30 × 15 = 270,000 kN. Because the building weighs 400,000 kN, the weight of soil excavated represents 67.5% of the total weight of the building. When the building has reached a weight of 270,000 kN, we assume that the soil will have been recompressed to its initial position. We calculate the strain increment $\Delta\varepsilon_{\text{rel}}$ in the middle of each layer due to the reloading by the building construction from zero load to a load equal to the weight of the excavated soil as:

$$\Delta\varepsilon_{rel} = \frac{\Delta\sigma_{exc}}{E_r} \tag{17.120}$$

This is column 7 in Table 17.12. Equation 17.120 neglects the influence of the stress confinement on the strain (conservative).

9. The increase in stress $\Delta\sigma_{bldg}$ in the middle of each layer corresponds to the weight of the entire building. Because the strain increment corresponding to $\Delta\sigma_{exc}$ has already been calculated (Eq. 17.120), we now need to calculate the strain increment $\Delta\varepsilon_{net}$ due to $(\Delta\sigma_{bldg} - \Delta\sigma_{exc})$. This is done as follows:

$$\Delta\varepsilon_{net} = \frac{\Delta\sigma_{bldg} - \Delta\sigma_{exc}}{E_o} \tag{17.121}$$

10. Then the compression ΔH_{rel} of each layer due to $\Delta\varepsilon_{rel}$ is calculated as:

$$\Delta H_{rel} = H\Delta\varepsilon_{rel} \tag{17.122}$$

and the compression ΔH_{net} of each layer due to $\Delta\varepsilon_{net}$ is calculated as:

$$\Delta H_{net} = H\Delta\varepsilon_{net} \tag{17.123}$$

This corresponds to columns 9 and 10 in Table 17.12.

11. Finally, the settlement is calculated by adding the compression of the four layers. The settlement S_{rel} due to the reloading of the soil under the part of the weight of the building equal to the weight of soil excavated is:

$$S_{rel} = \sum_{i=1}^{4} H_i \Delta\varepsilon_{rel\ i} = \sum_{i=1}^{4} \Delta H_{rel\ i} = 69 \text{ mm} \tag{17.124}$$

Then the settlement S_{net} due to the weight of the building in excess of the weight of the excavated soil is:

$$S_{net} = \sum_{i=1}^{4} H_i \Delta\varepsilon_{net\ i} = \sum_{i=1}^{4} \Delta H_{net\ i} = 148 \text{ mm} \tag{17.125}$$

The total settlement S_{tot} of the building is:

$$S_{tot} = S_{rel} + S_{net} = 217 \text{ mm} \tag{17.126}$$

17.12.3 Two Case Histories

Two large mat case histories are presented here: the Washington Monument (Briaud et al. 2009) and the San Jacinto Monument (Briaud et al. 2007) (Figure 17.55). The Washington Monument (Washington, DC) was completed in 1884 and stands at 169.16 m tall above ground (Figure 17.56). In a first phase, it was built to a height of 55.6 m on a square mat 24.38 m by 24.38 m when construction stopped with a calculated settlement of 1.33 m. In a second phase, the mat was underpinned and extended to a square ring mat 38.54 m by 38.54 m on the outside and 13.41 m by 13.41 m on the inside. The monument experienced an additional measured settlement of 0.12 m while construction was completed. Additional data are shown in Table 17.13. The mat rests on an 8.3 m thick layer of sand and gravel with a blow count averaging 100 bpf underlain by a 11.7 m thick layer of very stiff clay with an average undrained shear strength of 100 kPa. Below the very stiff clay is the bedrock. The settlement during phase one was calculated, on the basis of available consolidation tests, to be 1.33 m (Briaud et al. 2009), whereas the settlement during phase two was only 0.17 m (Figure 17.57). The reason the settlement was so large during the first phase is that the bottom of the first mat foundation was shallow and rested

Table 17.13 Data for the Washington Monument and the San Jacinto Monument

Washington Monument	San Jacinto Monument
Total weight = 608 MN	Total weight = 313 MN
Weight of foundation = 184 MN	Weight of foundation = 133 MN
Pressure at foundation level = 465 kPa	Pressure at foundation level = 224 kPa
Net pressure = 252 kPa	Net pressure = 141 kPa
Calculated total settlement = 1.50 m	Calculated total settlement = 0.61 m
Measured settlement after underpinning = 0.17 m	Measured settlement after mat placed = 0.33 m

on a very compressible soft clay layer. The underpinning brought the foundation down to the strong sand and gravel layer. More details can be found in Briaud et al. (2009).

The San Jacinto Monument (Houston, Texas, USA) was completed in 1936 and stands at 171.9 m tall above ground; it is the tallest free standing column in the world. It rests on a square mat foundation 37.8 m by 37.8 m. Some of the data regarding weight and pressure are shown in Table 17.13. The mat rests on a deep deposit of very stiff clay with an average undrained shear strength equal to 100 kPa. The CPT point resistance is 1000 kPa at the ground surface and increases to 3000 kPa at 10 m depth. The pressuremeter limit pressure is 800 kPa at the ground surface, increasing to 3000 kPa at a depth of 40 m. The PMT first load modulus is 15 MPa at the ground surface, increasing to 60 MPa at a depth of 40 m. The PMT reload modulus is 2.1 times larger than the first load modulus on the average. The PMT viscous exponent n averages 0.045. The settlement was calculated, on the basis of available consolidation tests, to be 0.61 m (Briaud et al. 2007). The settlement measured after the mat was poured reached 0.33 m (Figure 17.58). More details can be found in Briaud et al. (2007).

These two tall, columnar structures on large mats settled significantly, yet both are as straight as possible, with no lean detectable to the naked eye. If heterogeneity had been an issue, these structures would likely have tilted. However, at the scale of a 38 m by 38 m mat, the soil is much more homogeneous than at the scale of a cone penetrometer, for example. This shows that heterogeneity is scale dependent and that tall structures can stand much larger settlement than might be thought. The weight of these simple structures is shown in Table 17.13. By comparison, the Eiffel Tower in Paris, France, weighs 94 MN; the Tower of Pisa in Pisa, Italy, weighs 142 MN; each tower of the World Trade Center in New York weighed 4500 MN; and the Burj Khalifa in Dubai weighs about 5000 MN.

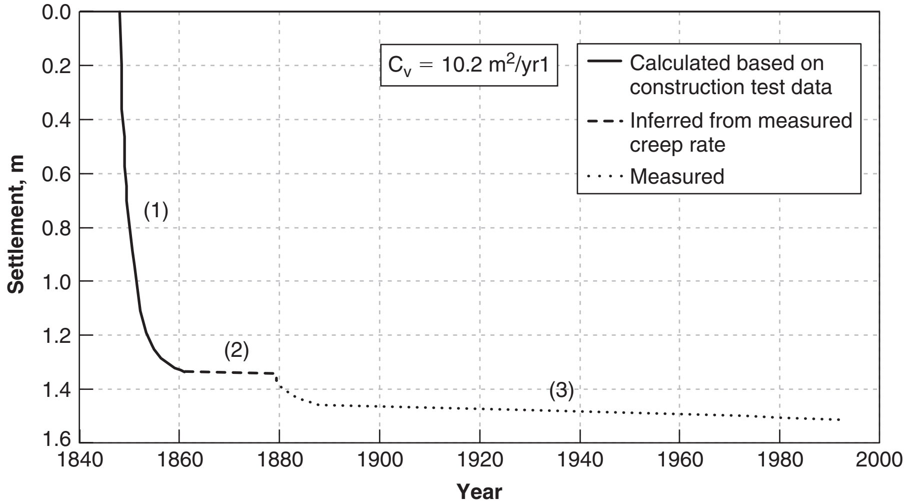

Figure 17.57 Settlement of the Washington Monument.

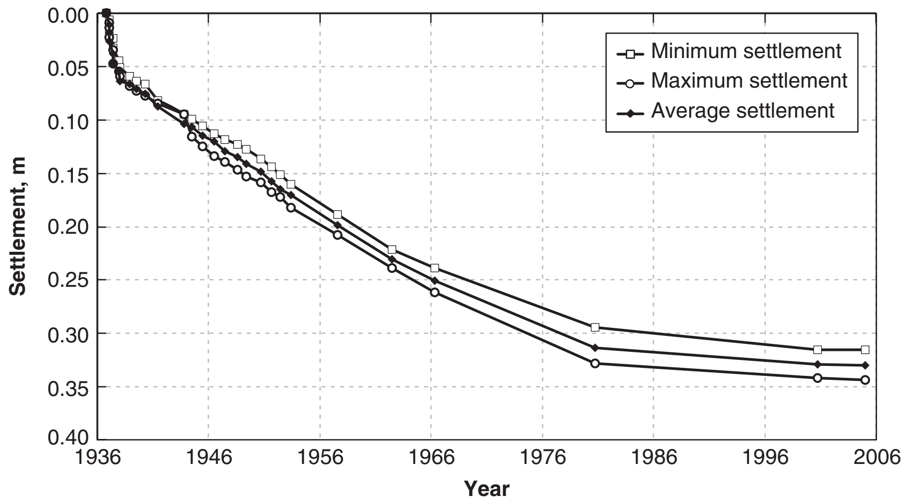

Figure 17.58 Settlement of the San Jacinto Monument.

PROBLEMS

17.1 If a shallow foundation test is performed on clay, there is a clear plunging load. If a shallow foundation test is performed on sand, the load continues to increase and a clear plunging load is not obvious. Explain why. How is the ultimate load defined for the load test on sand?

17.2 Calculate the ultimate bearing pressure for the footings and the soil described in section 17.2 by all applicable methods listed in section 17.6. Additional soil data can be obtained from Briaud and Gibbens (1999). If you had to give one answer what would you choose to do?

17.3 Calculate the ultimate bearing pressure (edge failure) for the mat of the San Jacinto Monument (section 17.12.2) by all applicable methods listed in section 17.6. If you had to give one answer, what would you choose to do?

17.4 Redo the example of Figure 17.23 but using the mean curve instead of the design curve for the Γ function.

17.5 Calculate the increase in stress under the center of the circular footing (Figure 17.1s) as a function of depth by all the methods presented in section 17.8.7. Show the profile of effective stress before construction and after construction. At what depth is $\Delta\sigma'(z)$ equal to 1/10 of $\Delta\sigma'(z = 0)$?

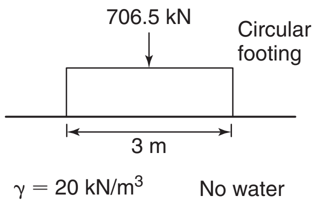

Figure 17.1s Circular footing.

17.6 Calculate the settlement of the footing shown in Figure 17.2s. If only 10 mm of settlement can be tolerated by the structure, what is the size of the footing required to carry the same load?

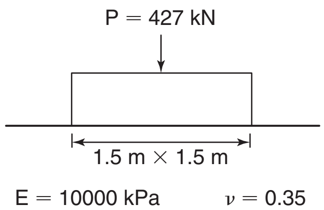

Figure 17.2s Square footing.

17.7 A square foundation is 3 m × 3 m and rests on a deep layer of sand at a depth of 1.5 m. The soil modulus at the ground surface is 10 MPa and increases linearly to 50 MPa at a depth of 10 m. What load can the footing carry if the allowable settlement is 25 mm?

17.8 Using the Schmertmann method, simplify the equation giving the settlement of a footing at the surface of a sand deposit when the soil is uniform with a constant value of E. Compare that equation to the elasticity equation.

17.9 A column load of 4000 kN is to be supported by a square spread footing on a medium-dense sand. Recommend the size and the embedment of the footing after addressing the issue of bearing capacity and settlement of the footing (25 mm is tolerable). Soil properties: N = 30 blows/ft, $q_c = 8$ MPa, $f_c = 70$ kPa, $p_L = 1500$ kPa, $E_o = 12$ MPa, $\gamma = 20$ kN/m^3. If you need additional properties, assume reasonable values.

17.10 A column load of 2000 kN is to be supported by a square spread footing on a very stiff clay. Recommend the size of the footing after addressing the issue of bearing capacity and settlement of the footing (25 mm is tolerable). Soil properties: $s_u = 100$ kPa, $q_c = 1.5$ MPa, $f_c = 70$ kPa, $p_L = 500$ kPa, $E_o = 7.5$ MPa, $C_c = 0.3$, $c_v = 10^{-4}$ cm$^2/s$, $\gamma =$ 18 kN/m^3. If you need additional properties, assume reasonable values.

17.11 In 1955, an oil tank 10 m high and 38 m in diameter is built as shown in Figure 17.3s.

a. Calculate the settlement of the center of this tank (point C on Figure 17.3s) using the data from Figure 17.4s. Assume that the stress increase in the middle of the compressible layer is equal to the pressure under the tank because the layer is thin.

In 1975, this tank is removed; a year later, a new tank 15 m high and 76 m in diameter is built. The edge of the new tank goes through the center of the old tank.

b. Calculate the settlement of the edge of the new tank away from the old tank (point B on Figure 17.3s) using the data from Figure 17.4s. Assume that the stress increase at the edge of the new tank in the middle of the compressible layer is equal to one half of the pressure under the new tank.

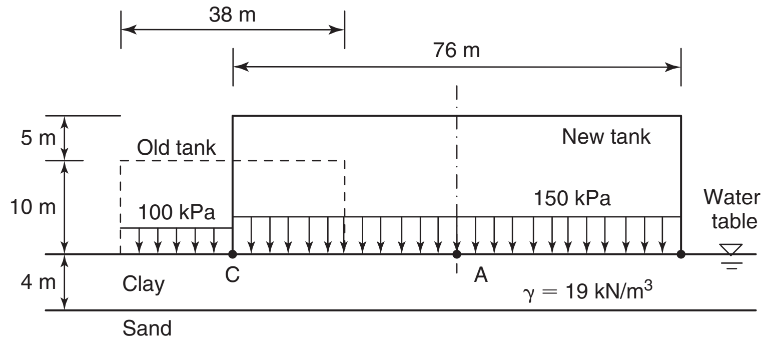

Figure 17.3s Old and new oil tanks.

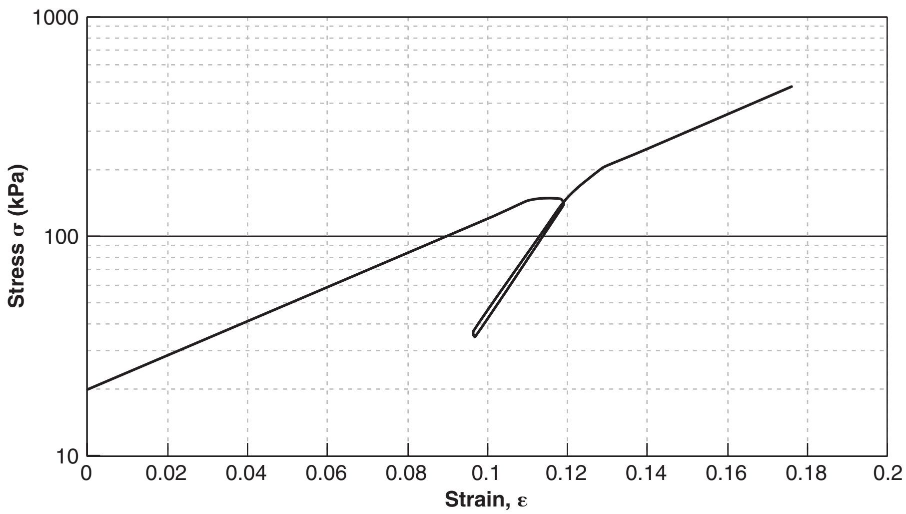

Figure 17.4s Stress-strain curve for oil tank problem.

c. Calculate the settlement of the edge of the new tank that passes over the center of the old tank (point C on Figure 17.3s) using the data from Figure 17.4s. Make the same assumption as in b.

d. Do you see any problem with the difference in settlement between C and B for the new tank?

17.12 Use the shrink-swell case history from section 17.9.4 to calculate the footing movements and compare your results with the measured movements.

17.13 The high-rise building shown in Figure 17.53 is subjected to a hurricane wind of 200 km/h. This wind creates a pressure of 3 kPa on the flat side of the building. Calculate the pressure diagram under the foundation.

17.14 The high-rise building shown in Figure 17.53 is placed on a stiff clay with the following properties: compression index C_c equal to 0.4, recompression index C_r equal to 0.1, initial void ratio e_o equal to 0.5, and total unit weight equal to 20 kN/m^3. The soil is lightly overconsolidated by overburden removal and has a preconsolidation pressure σ'_p 150 kPa higher than the effective stress σ'_{ov}. The groundwater level is at the ground surface. Calculate the settlement of the building. How would you estimate the time required for the settlement to take place if c_v were known?

17.15 The annual drying and wetting condition of a site is shown in Figure 17.5s. Calculate the shrink and swell displacement at the center and at the edge of the building, and then calculate the differential movement between the two points. Hint: Use

$$\varepsilon = \frac{\Delta H_i}{H_i} = f\frac{\Delta w_i}{E_{wi}} \qquad \therefore \varepsilon = 0.33\frac{\Delta w_i}{(\gamma_w/\gamma_d)} \text{ and } w_i = 25\%, \qquad \gamma_d = 14\ \text{kN/m}^3, \gamma_w = 10\ \text{kN/m}^3$$

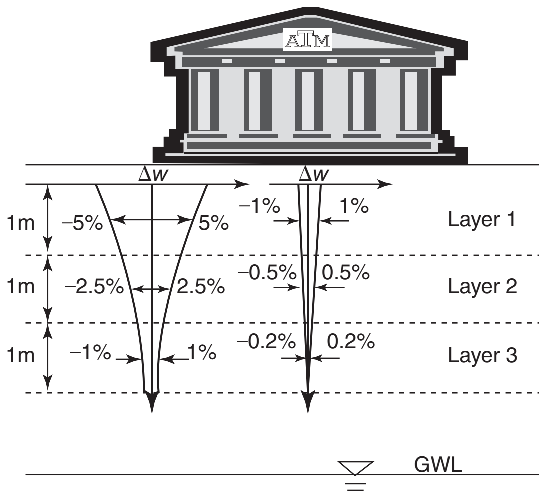

Figure 17.5s Annual drying and wetting condition.

17.16 A stiffened slab on grade for a two-story house is to be designed. The slab and site data are given as follows: slab dimensions 20 m by 20 m, beam spacing $s = 3.0$ m (for both directions), beam width $b = 0.3$ m, slab load $w = 10$ kPa, depth of movement zone $H = 3.0$ m, soil surface water content change $\Delta w_o = 20\%$. Recommend a beam depth that will minimize the distortion of the slab for the edge drop case to more than $L/\Delta = 500$.

17.17 Calculate the settlement of the San Jacinto Monument using the pressuremeter data given in section 17.12.3.

Problems and Solutions

Problem 17.1

If a shallow foundation test is performed on clay, there is a clear plunging load. If a shallow foundation test is performed on sand, the load continues to increase and a clear plunging load is not obvious. Explain why. How is the ultimate load defined for the load test on sand?

Solution 17.1

The reason a shallow foundation test performed on sand shows no clear plunging load is that fine-grained soils tend to shear in an undrained mode during a load test, whereas coarse-grained soils likely shear in a drained mode. The undrained shear strength of a clay does not vary much with the stress and confinement level (s_u = constant), so when the load on the footing increases, the shear strength does not increase and the failure is clearly defined. The drained shear strength of a sand depends on the stress and the confinement level ($s = \sigma' \tan \varphi'$), so when the load increases the shear strength also increases. Therefore, the failure for the sand is ill defined, and no obvious plunging load is observed. The ultimate load on the sand can be defined as the load corresponding to a movement equal to one-tenth of the foundation width.

Problem 17.2

Calculate the ultimate bearing pressure for the footings and the soil described in section 17.2 by all applicable methods listed in section 17.6. Additional soil data can be obtained from Briaud and Gibbens (1999). If you had to give one answer what would you choose to do?

Solution 17.2

Figure 17.6s shows the illustration of the soil profile.

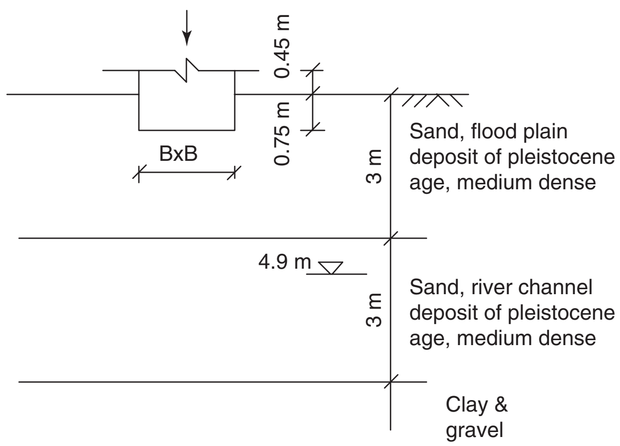

Figure 17.6s Footing and soil profile.

The methods include Skempton method, PMT method, CPT method, SPT method, and Terzaghi's general bearing capacity equation (GBE) method. The Skempton method is applicable only to fine-grained soil, so it is not discussed in this solution. The simple versions of the PMT method, CPT method, SPT method, and GBE method are used to solve the bearing capacity for the 1 m × 1 m footing, 1.5 m × 1.5 m footing, 2.5 m × 2.5 m footing, and 3 m × 3 m footing respectively. All footings are embedded 0.75 m. The soil unit weight is 15.5 kN/m^3. The following soil properties are selected within the zone of influence of the footings from the soil profiles presented in Figures 17.7s, 17.8s, 17.9s: the PMT limit pressure is 800 kPa, the CPT point resistance is 6000 kPa, and the SPT blow count is 18 bpf. Since the footing width does not appear in the ultimate bearing pressure p_u equations, and since the embedment depth is the same for all footings, then the value of p_u will be the same for all footings.

PMT method:

$$p_u = k_p p_L + \gamma D \tag{17.1s}$$

where k_p is the pressuremeter bearing capacity factor, p_L is the pressuremeter limit pressure, γ is the total unit weight of the soil above the footing depth, and D is the embedment of the footing.

Based on Equation 17.19, k_p is 1.2 and p_u is:

$$p_u = 1.2 \times 800 + 15.5 \times 0.75 = 972 \text{ kPa}$$

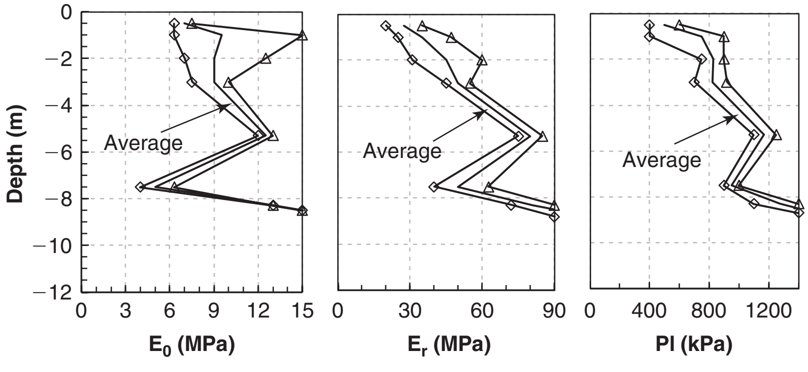

Figure 17.7s PMT profile.

CPT method:

$$p_u = k_c q_c + \gamma D \tag{17.2s}$$

where k_c is the cone penetrometer bearing capacity factor, q_c is the average point resistance within one footing width below the footing, γ is the total unit weight of the soil above the footing depth, and D is the embedment of the footing. Based on Equation 17.22, k_c is 0.2 and p_u is:

$$p_u = 0.2 \times 6000 + 15.5 \times 0.75 = 1212 \text{ kPa}$$

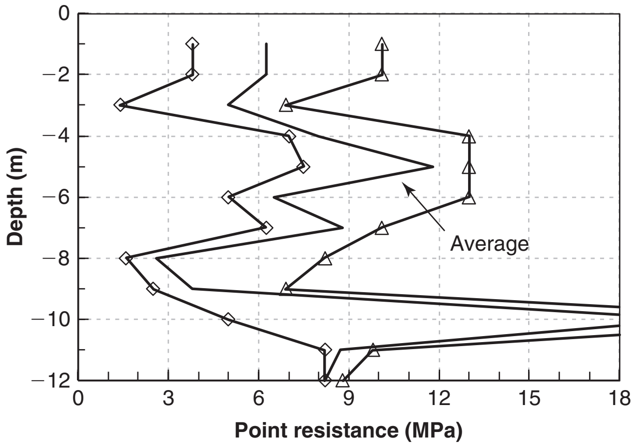

Figure 17.8s CPT profile.

SPT method:

$$p_u = k_N N p_a + \gamma D \tag{17.3s}$$

where k_N is the SPT bearing capacity factor, N is the average blow counts within one footing width below the footing, p_a is the atmospheric pressure, γ is the total unit weight of the soil above the footing depth, and D is the embedment of the footing. Based on Equation 17.28, k_N is 0.6 and p_u is:

$$p_u = 0.6 \times 18 \times 101.3 + 15.5 \times 0.75 = 1106 \text{ kPa}$$

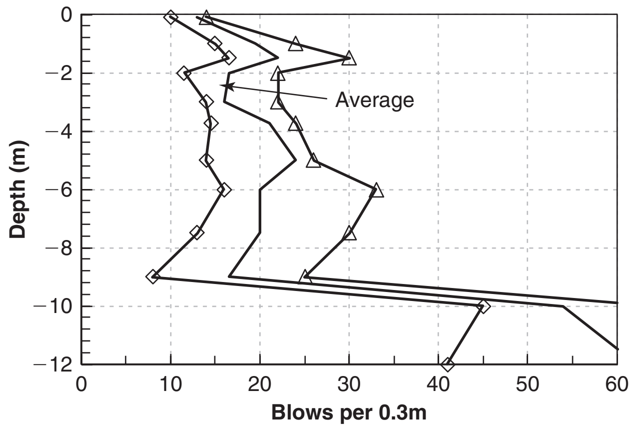

Figure 17.9s SPT profile.

Terzaghi's GBE:

This method is based on the following equation:

$$p_u = c' N_c + \frac{1}{2}\gamma_1 B N_r + \gamma_2 D N_q \tag{17.4s}$$

where c' is the effective stress cohesion; N_c, N_r, and N_q are bearing capacity factors, γ_1 is the average effective unit weight of soil within the one footing width below the foundation, B is the width of the foundation, γ_2 is the effective unit weight of the soil above the foundation, and D is the depth of embedment of the foundation.

For this case, $N = 18$ bl/ft, $c' = 0$, $\phi' = 32.5^\circ$ (Figure 15.12), $N_c = 39$, $N_r = 23$, and $N_q = 23$ (Figure 17.14):

$$p_u = 0 \times 39 + \frac{1}{2} \times 15.5 \times B \times 23 + 15.5 \times 0.75 \times 23 = 178.25B + 267.38$$

$$1 \times 1 \text{ m footing} : p_u = 446 \text{ kPa}$$
$$1.5 \times 1.5 \text{ m footing} : p_u = 535 \text{ kPa}$$
$$2.5 \times 2.5 \text{ m footing} : p_u = 713 \text{ kPa}$$
$$3 \times 3 \text{ m footing} : p_u = 802 \text{ kPa}$$

Based on this analysis, I would choose the average p_u value from the PMT method, CPT method, and SPT method as the ultimate bearing pressure of the footings. I would not use the general bearing capacity equation predictions because the soil profile does not correspond to the assumption made to derive that equation (linear strength increase with depth)

$$p_u = \frac{972 + 1212 + 1106}{3} = 1097 \text{ kPa}$$

Note that these footings were load tested (Briaud, Gibbens, 1999) individually and gave ultimate bearing pressures (pressure at one tenth of the footing width) equal to:

$$1 \times 1 \text{ m footing} : p_u = 1500 \text{ kPa}$$
$$1.5 \times 1.5 \text{ m footing} : p_u = 1500 \text{ kPa}$$
$$2.5 \times 2.5 \text{ m footing} : p_u = 1300 \text{ kPa}$$
$$3 \times 3 \text{ m footing (North)} : p_u = 1250 \text{ kPa}$$
$$3 \times 3 \text{ m footing (South)} : p_u = 1500 \text{ kPa}$$

Problem 17.3

Calculate the ultimate bearing pressure (edge failure) for the mat of the San Jacinto Monument (section 17.12.2) by all applicable methods listed in section 17.6. If you had to give one answer, what would you choose to do?

Solution 17.3

Skempton:

$$p_u = N_c s_u + \gamma D$$

$D = 9.1$ m (Figure 17.56), and $B = 37.8$ m, so the ratio $D/B = 0.24$.

From the Skempton chart in Figure 17.7, $N_c = 6.7$. The undrained shear strength below the monument is given as $S_u = 100$ kPa. Assuming a unit weight of 19 kN/m^3:

$$p_u = 100 \times 6.7 + 19 \times 9.1 = 843 \text{ kPa}$$

PMT Method (for clay):

$$p_u = k_p p_L^* + \gamma D$$

$D = 9.1$ m (Figure 17.56), $B = L = 37.8$ m, so the ratio $B/L = 1$, and $D/B = 0.24$:

$$p_u = 0.9 \times 800 + 19 \times 9.1 = 893 \text{ kPa}$$

CPT Method (for clay):

$$p_u = 0.40 q_c + \gamma D$$

Because the profile of q_c increases from 1000 kPa at the surface to 3000 kPa at 10 m depth, and because there is 9.1 m embedment, it appears reasonable to select 3000 kPa as the cautious design value for q_c:

$$p_u = 0.4 \times 3000 + 19 \times 9.1 = 1373 \text{ kPa}$$

There are no SPT data, c', or φ' to use with the remaining methods.

If I had to choose one answer, I would choose 900 kPa as a conservative yet substantiated value (supported by two methods). The CPT method seems a bit optimistic in this case.

Problem 17.4

Redo the example of Figure 17.23 but using the mean curve instead of the design curve for the Γ function.

Solution 17.4

The f factor does not change when we use the mean curve instead of the design curve:

$\Delta R/R_o$	p_p (kN/m²)	s/B	s (mm)	Γ_{Mean}	f	p_f (kN/m²)	Q(kN)
0	0	0	0	0	0.689	0	0
0.006	75	0.00144	4.32	3.6	0.689	186.0	8370
0.012	120	0.00288	8.64	3.1	0.689	256.3	11533
0.024	220	0.00576	17.28	2.75	0.689	416.8	18756
0.032	300	0.00768	23.04	2.25	0.689	465.1	20929
0.055	450	0.0132	39.6	1.9	0.689	589.1	26509
0.1	650	0.024	72	1.5	0.689	671.8	30231
0.15	775	0.036	108	1.35	0.689	720.9	32440
0.20	850	0.048	144.0	1.3	0.689	761.3	34258

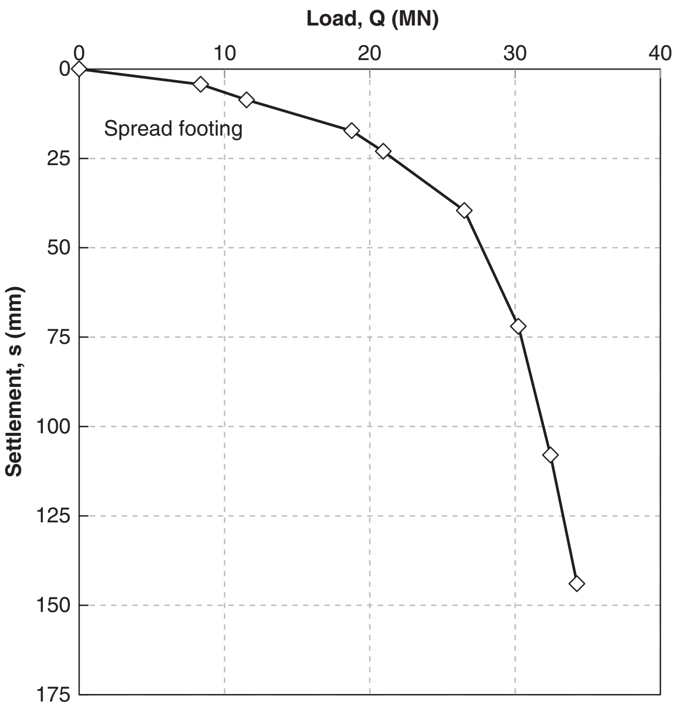

Figure 17.10s Load settlement curve.

Problem 17.5

Calculate the increase in stress under the center of the circular footing (Figure 17.1s) as a function of depth by all the methods presented in section 17.8.7. Show the profile of effective stress before construction and after construction. At what depth is $\Delta\sigma'(z)$ equal to 1/10 of $\Delta\sigma'(z = 0)$?

Solution 17.5

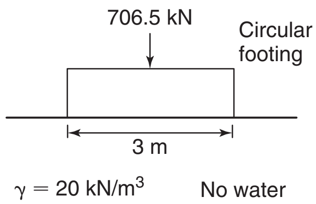

Figure 17.1s Circular footing.

2 to 1 method:

The average pressure under the footing is:

$$p_{ave} = \frac{Q}{\pi B^2/4} = \frac{706.5}{\pi (3)^2/4} = 100 \text{ kPa}$$

The increase in stress for a circular footing is:

$$\sigma_z = \frac{Q}{\pi (B + z)^2/4}$$

Depth (m)	$\Delta\sigma'_z$ (kN/m^2)	Before construction (kN/m^2)	After construction (kN/m^2)
0	100	0.0	100
1	56.2	20.0	76.2
2	36.0	40.0	76.0
3	25.0	60.0	85.0
4	18.4	80.0	98.4
5	14.1	100.0	114.1
6	11.1	120.0	131.1
7	9.0	140.0	149.0
8	7.4	160.0	167.4
9	6.2	180.0	186.2
10	5.3	200.0	205.3

Bulbs of pressure (using Figure 17.31):

Depth (m)	Depth/diameter	Factor (from bulbs of pressure)	$\Delta\sigma'_z$ (kN/m^2)	Before construction (kN/m^2)	After construction (kN/m^2)
0	0.00	1	100	0.0	100
1	0.33	0.8	80	20.0	100
2	0.67	0.57	57	40.0	97
3	1.00	0.35	35	60.0	95
4	1.33	0.23	23	80.0	103
5	1.67	0.17	17	100.0	117
6	2.00	0.12	12	120.0	132
7	2.33	0.09	9	140.0	149
8	2.67	0.07	7	160.0	167
9	3.00	0.06	6	180.0	186
10	3.33	0.04	5	200.0	204

Newmark's chart:

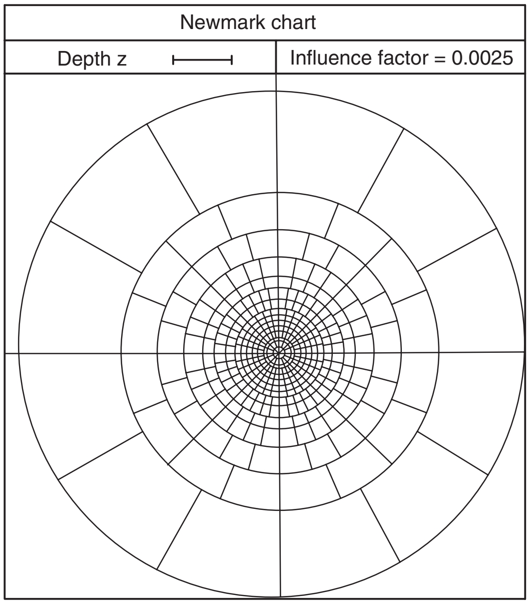

Figure 17.11s Newmark's chart.

Depth (m)	Number of squares	Factor	$\Delta\sigma'_z$ (kN/m^2)	Before construction (kN/m^2)	After construction (kN/m^2)
0		1.0	100	0.0	100
1	316	0.79	79	20.0	99
2	200	0.5	50	40.0	90
3	104	0.26	26	60.0	86
4	72	0.18	18	80.0	98
5	40	0.1	10	100.0	110
6	32	0.08	8	120.0	128
7	30	0.075	7.5	140.0	147.5
8	24	0.06	6	160.0	166.0
9	18	0.045	4.5	180.0	184.5
10	10	0.025	2.5	200.0	202.5

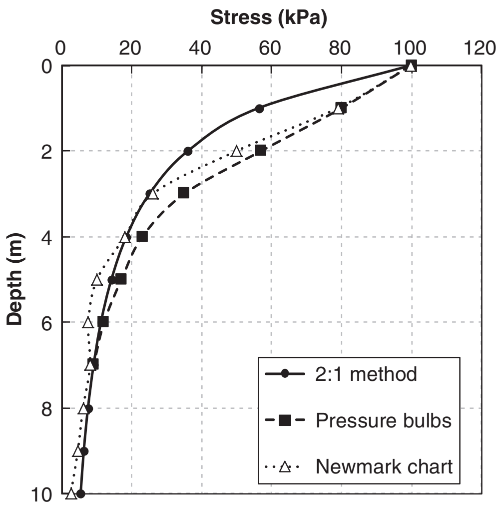

Figure 17.12s Difference between the three methods.

Problem 17.6

Calculate the settlement of the footing shown in Figure 17.2s. If only 10 mm of settlement can be tolerated by the structure, what is the size of the footing required to carry the same load?

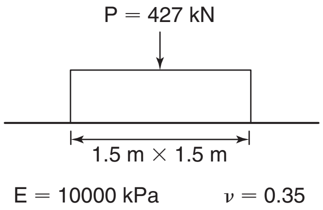

Figure 17.2s Square footing.

Solution 17.6

The settlement equation for an elastic soil gives (rigid square foundation: I = 0.88):

$$s = pB\frac{(1-\upsilon^2)}{E}I = \frac{427}{1.5 \times 1.5} \times 1.5 \times \frac{(1-0.35^2)}{10000} \times 0.88 = 0.022 \text{ m}$$

Footing size

$$s = pB\frac{(1-\upsilon^2)}{E}I = \frac{Q}{B^2}B\frac{(1-\upsilon^2)}{E}I$$

$$B = \frac{Q}{s}\frac{(1-\upsilon^2)}{E}I = \frac{427}{0.01}\frac{(1-0.35^2)}{10000}0.88 = 3.3 \text{ m}$$

The footing size should be 3.3 m by 3.3 m.

Problem 17.7

A square foundation is 3 m × 3 m and rests on a deep layer of sand at a depth of 1.5 m. The soil modulus at the ground surface is 10 MPa and increases linearly to 50 MPa at a depth of 10 m. What load can the footing carry if the allowable settlement is 25 mm?

Solution 17.7

$$s_o = pB\frac{(1-v^2)}{E_o} \times I$$

where:
s_o : reference settlement for uniform soil with E_o modulus
p : Pressure = ?
B : width of foundation = 3m
V : Poisson's ratio = 0.35
E_o : modulus of elasticity at the bottom of the foundation:

$$10 + \frac{1.5}{10}(50-10) = 16 \text{ MPa}$$

I : influence factor for square footing = 0.88

$$s_o = p \times 3 \times \frac{(1-0.35^2)}{16000} \times 0.88 = 1.448 \times 10^{-4} p$$

The settlement of the footing in the case of the increasing modulus with depth is s_1, such that:

$$I_G = \frac{s_1}{s_o}$$

where I_G is read on Figure 17.26 for the corresponding value of E_1/E_o, which is:

$$E = E_o + E_1\left(\frac{z}{B}\right) \quad or \quad \frac{E_1}{E_o} = \left(\frac{E}{E_o} - 1\right)\frac{B}{z}$$

We know that at a depth z equal to 8.5 m below the footing, the modulus E is 50 MPa, which gives an E_1/E_o ratio of:

$$\frac{E_1}{E_o} = \left(\frac{50}{16} - 1\right)\frac{3}{8.5} = 0.75$$

Figure 17.26 gives $I_G = 0.80$, so the settlement expression becomes:

$$s_1 = I_G \times pB\frac{(1-\nu^2)}{E_o} \times I = 0.80 \times p \times 3 \times \frac{(1-0.35^2)}{16000} \times 0.88 = 1.158 \times 10^{-4}p$$

Because s_1 must be 25 mm, then $p = 0.025/1.158 \times 10^{-4} = 216$ kPa.
The allowable footing load is then:

$$Q_{all} = 216 \times 3 \times 3 = 1944 \text{ kN}$$

Problem 17.8

Using the Schmertmann method, simplify the equation giving the settlement of a footing at the surface of a sand deposit when the soil is uniform with a constant value of E. Compare that equation to the elasticity equation.

Solution 17.8

The Schmertmann equation is:

$$s = C_1C_2p\Sigma\frac{I_{zi}}{E_i}H_i$$

The footing is placed on the ground surface, so:

$$\sigma'_{ov} = 0 \Rightarrow C_1 = 1 - 0.5\frac{\sigma'_{ov}}{\Delta p} = 1$$

Let's assume that the settlement occurs in 0.1 years, so:

$$C_2 = 1 + 0.2\log\left(\frac{0.1}{0.1}\right) = 1$$

E is a constant, so the final equation is:

$$s = \frac{p}{E}\Sigma H_i I_{zi}$$

The quantity $\Sigma H_i I_{zi}$ is the area under the strain influence factor curve on Figure 17.27. This area depends on the maximum value of I_{zp}, which is:

$$I_{zp} = 0.5 + 0.1\left(\frac{\Delta p}{\sigma'_{Izp}}\right)^{0.5}$$

A reasonable range for I_{zp} may be found when the ratio $\left(\frac{\Delta p}{\sigma'_{Izp}}\right)$ varies between 2 and 20 or a corresponding range for I_{zp} between 0.6 and 0.9 with an average of 0.75. For the value of 0.75, the area under the I_z diagram is:

$$0.5(0.1 + 0.75) \times 0.5B + 0.5 \times 0.75 \times 1.5B = 0.785B$$

and the final Schmertmann equation becomes:

$$s = 0.785\frac{pB}{E}$$

The elasticity equation is:

$$s = I(1-\nu^2)\frac{pB}{E} = 0.88 \times (1-0.35^2)\frac{pB}{E} = 0.77\frac{pB}{E}$$

Problem 17.9

A column load of 4000 kN is to be supported by a square spread footing on a medium-dense sand. Recommend the size and the embedment of the footing after addressing the issue of bearing capacity and settlement of the footing (25 mm is tolerable). Soil properties: $N = 30$ blows/ft, $q_c = 8$ MPa, $f_c = 70$ kPa, $p_L = 1500$ kPa, $E_o = 12$ MPa, $\gamma = 20$ kN/m^3. If you need additional properties, assume reasonable values.

Solution 17.9

Let's assume that D = 0.5 m.

Ultimate bearing capacity: SPT

$$p_u\ (\text{kN/m}^2) = 60N + \gamma D = 60 \times 30 + 20 \times 0.5 = 1810\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{1810}{3} = 603\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{603}} = \sqrt{\frac{4000}{603}} = 2.58\ \text{m}$$

Ultimate bearing capacity: CPT

$$p_u\ (\text{kN/m}^2) = 0.2q_c + \gamma D = 0.2 \times 8000 + 20 \times 0.5 = 1610\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{1610}{3} = 537\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{537}} = \sqrt{\frac{4000}{537}} = 2.73\ \text{m}$$

Ultimate bearing capacity: PMT

$$p_u\ (\text{kN/m}^2) = 1.2p_L + \gamma D = 1.2 \times 1500 + 20 \times 0.5 = 1810\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{1810}{3} = 603\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{603}} = \sqrt{\frac{4000}{603}} = 2.58\ \text{m}$$

Settlement (25 mm tolerable)

$$s = pB(1-\nu^2)\frac{I}{E}$$

I = shape factor (0.88 for square footing)

p = mean pressure under the foundation

B = foundation *w*idth

E = elasticity modulus of the soil ($E = 2E_0$(sand) from Briaud (1992))

ν = Poisson's ratio

$$25 \times 10^{-3} = \frac{4000}{B^2} \times B(1-0.35^2)\frac{0.88}{2 \times 12000}$$

$$B = 5.15\ \text{m}$$

So the recommended foundation size is 5.2 m × 5.2 m and the settlement criterion controls the design.

Problem 17.10

A column load of 2000 kN is to be supported by a square spread footing on a very stiff clay. Recommend the size of the footing after addressing the issue of bearing capacity and settlement of the footing (25 mm is tolerable). Soil properties: $s_u = 100$ kPa, $q_c = 1.5$ MPa, $f_c = 70$ kPa, $p_L = 500$ kPa, $E_o = 7.5$ MPa, $C_c = 0.3$, $c_v = 10^{-4}\text{cm}^2/\text{s}$, $\gamma = 18\ \text{kN/m}^3$. If you need additional properties, assume reasonable values.

Solution 17.10

Let's assume that $D = 0.5$ m.

Ultimate bearing capacity: CPT

$$p_u\ (\text{kN/m}^2) = 0.4q_c + \gamma D = 0.4 \times 1500 + 18 \times 0.5 = 609\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{609}{3} = 203\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{203}} = \sqrt{\frac{2000}{203}} = 3.14\ \text{m}$$

Ultimate bearing capacity: Pressuremeter

$$p_u\ (\text{kN/m}^2) = 0.9p_L + \gamma D = 0.9 \times 500 + 18 \times 0.5 = 459\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{459}{3} = 153\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{153}} = \sqrt{\frac{2000}{153}} = 3.62\ \text{m}$$

Ultimate bearing capacity: Undrained shear strength

$$p_u\ (\text{kN/m}^2) = N_c S_u + \gamma D = 6.3 \times 100 + 18 \times 0.5 = 639\ \text{kPa}$$

$$p_{safe}\ (\text{kN/m}^2) = \frac{639}{3} = 213\ \text{kPa} = \frac{Q}{B^2}$$

$$B = \sqrt{\frac{Q}{213}} = \sqrt{\frac{2000}{213}} = 3.06\ \text{m}$$

Settlement (25 mm tolerable)

$$s = pB(1 - \nu^2)\frac{I}{E}$$

I = shape factor (0.88 for square footing)
p = mean pressure under the foundation
B = foundation *w*idth
E = elasticity modulus of the soil ($E = E_0$(clay) from Briaud (1992))
ν = Poisson's ratio

$$25 \times 10^{-3} = \frac{2000}{B^2} \times B(1 - 0.35^2)\frac{0.88}{7500}$$

$$B = 8.24\ \text{m}$$

So the recommended foundation size is 8.3 m × 8.3 m

Problem 17.11

In 1955, an oil tank 10 m high and 38m in diameter is built as shown in Figure 17.3s.

a. Calculate the settlement of the center of this tank (point C on Figure 17.3s) using the data from Figure 17.4s. Assume that the stress increase in the middle of the compressible layer is equal to the pressure under the tank because the layer is thin.

   In 1975, this tank is removed; a year later, a new tank 15 m high and 76 m in diameter is built. The edge of the new tank goes through the center of the old tank.

b. Calculate the settlement of the edge of the new tank away from the old tank (point B on Figure 17.3s) using the data from Figure 17.4s. Assume that the stress increase at the edge of the new tank in the middle of the compressible layer is equal to one half of the pressure under the new tank.
c. Calculate the settlement of the edge of the new tank that passes over the center of the old tank (point C on Figure 17.3s) using the data from Figure 17.4s. Make the same assumption as in b.
d. Do you see any problem with the difference in settlement between C and B for the new tank?

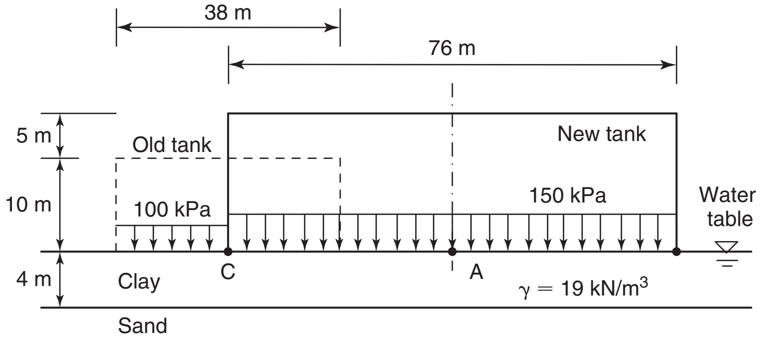

Figure 17.3s Old and new oil tanks.

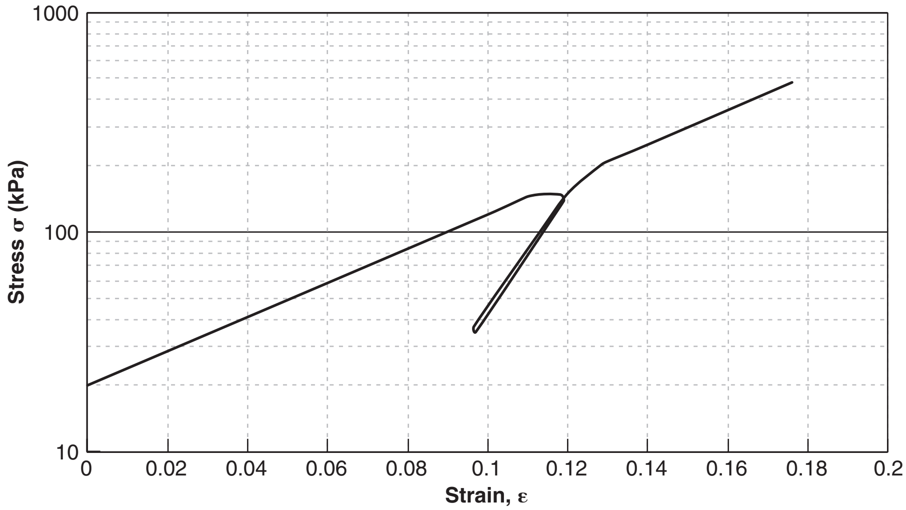

Figure 17.4s Stress-strain curve for oil tank problem.

Solution 17.11

$$\Delta H = \frac{\Delta e}{1+e_o} H = \frac{C_c H}{1+e_o} \log \frac{\sigma'_{ov} + \Delta\sigma'}{\sigma'_{ov}} = \Delta\varepsilon H$$

a. Settlement at point C for old tank

$$\sigma'_{ov} = 19 \times 2 - 9.81 \times 2 = 19.62 \text{ kPa}$$

$$\Delta\sigma = 100 \text{ kPa}$$

$$\sigma'_{ov} + \Delta\sigma = 119.62 \text{ kPa}$$

From Figure 17.4s, $\varepsilon_{119.6\text{kPa}} = 0.1$, $\varepsilon_{19.6\text{kPa}} = 0$, $\Delta\varepsilon = 0.1 - 0 = 0.1$

$$\Delta H = \Delta\varepsilon H = 0.1 \times 4 \text{ m} = 0.4 \text{ m}$$

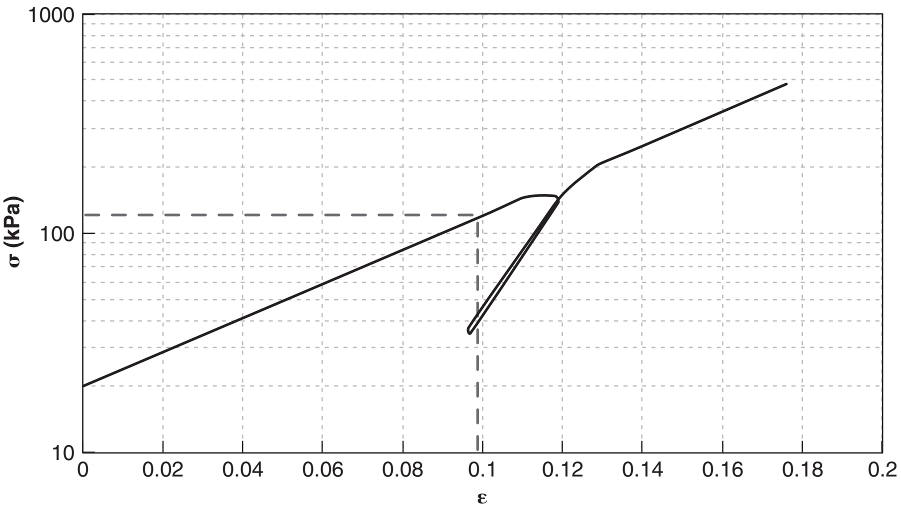

Figure 17.13s Settlement at point C for old tank.

b. Settlement at point B for new tank

$$\sigma'_{ov} = 19 \times 2 - 9.81 \times 2 = 19.62 \text{ kPa}$$
$$\Delta\sigma = 150 \times 1/2 = 75 \text{ kPa}$$
$$\sigma'_{ov} + \Delta\sigma = 94.62 \text{ kPa}$$

From Figure 17.4s, $\varepsilon_{94.6\text{kPa}} = 0.088$, $\varepsilon_{19.6\text{kPa}} = 0$, $\Delta\varepsilon = 0.088 - 0 = 0.088$

$$\Delta H = \Delta\varepsilon H = 0.088 \times 4 \text{ m} = 0.352 \text{ m}$$

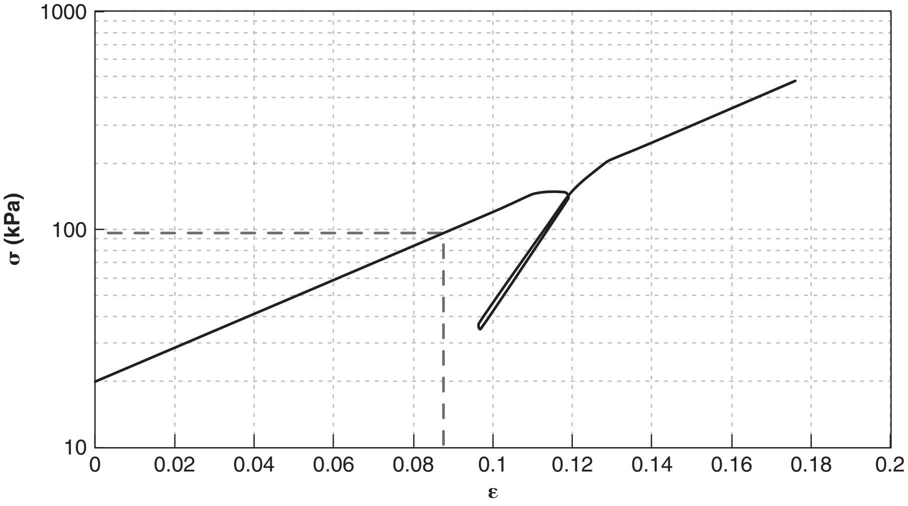

Figure 17.14s Settlement at point B for new tank.

c. Settlement at point C for new tank

$$\sigma'_{ov} = 19 \times 2 - 9.81 \times 2 = 19.62 \text{ kPa}$$
$$\Delta\sigma = 150 \times 1/2 = 75 \text{ kPa}$$
$$\sigma'_{ov} + \Delta\sigma = 94.62 \text{ kPa}$$

In 1955, the preconsolidation pressure was 119.6 kPa. Therefore:

$$\sigma'_{ov} + \Delta\sigma = 94.62 \text{ kPa} < \sigma'_c = 119.6 \text{ kPa} \quad \therefore \quad \textit{Overconsolidated clay}$$

From the rebound curve in Figure 17.4s:

$$\varepsilon_{94.6\text{kPa}} = 0.114, \varepsilon_{19.6\text{kPa}} = 0.086, \Delta\varepsilon = 0.114 - 0.086 = 0.028$$
$$\Delta H = \Delta\varepsilon H = 0.028 \times 4 \text{ m} = 0.112 \text{ m}$$

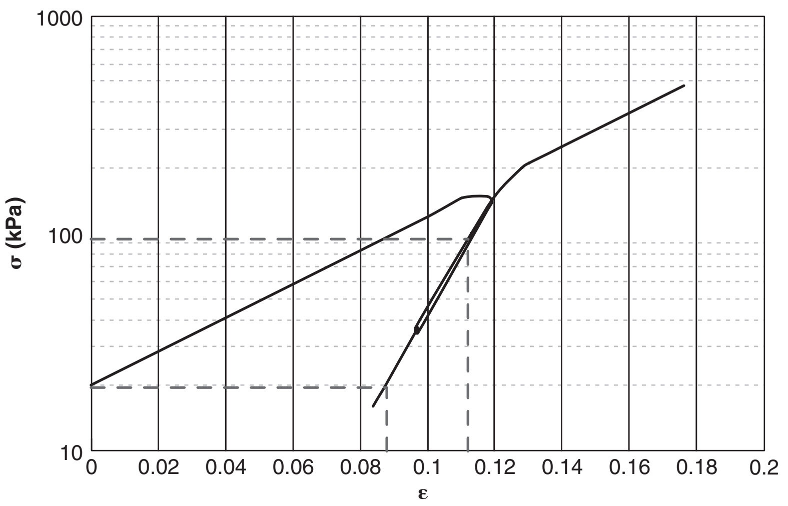

Figure 17.15s Settlement at point C for new tank.

d. The differential settlement between point B and point C:

$$\Delta H_{B-C} = \Delta H_B - \Delta H_C = 0.352 \text{ m} - 0.112 \text{ m} = 0.24 \text{ m}$$

This differential settlement between C and B is significant and will cause bending of the foundation and the new oil tank. This bending may create a problem with the sliding roof often used in such oil tanks.

Problem 17.12

Use the shrink-swell case history from section 17.9.4 to calculate the footing movements and compare your results with the measured movements.

Solution 17.12

The settlement time history was developed using the data presented in section 17.9.4. The change in water content was computed from the boring information and used to estimate the settlement. The analysis was conducted using a representative layer of 0.5 m for the dark gray silty clay and for the brown silty clay. The total settlement was estimated by the contribution of each layer. The results are presented in Table 17.2s and Table 17.3s. The average estimated settlement was compared with the average measured settlement from the four footings, as shown in the figure. The water content method seems to yield a reasonable prediction of the movement of the foundation:

$$s = \sum_{1}^{n} f_i \frac{w_f - w_i}{E_{wi}} H_i$$

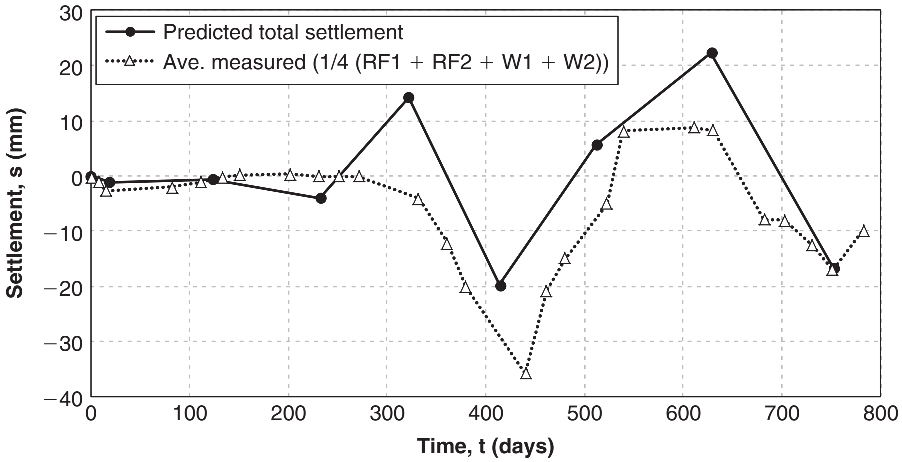

Figure 17.16s Measured vs. predicted settlement.

Table 17.2s Settlement of the Dark Gray Silty Clay

Boring	Boring Date	Elapsed time (days)	w_i(%)	Δw_i(%)	f_i	E_{wi}	$\Delta\varepsilon_i$	s_1(m)
B1	6/24/1999	0	0.19	—	0.33	0.752	0.0000	0.0
B2	7/13/1999	19	0.185	−0.005	0.33	0.752	−0.0026	−1.1
B3	10/25/1999	123	0.17	−0.02	0.33	0.752	−0.0104	−4.4
B4	2/11/2000	232	0.165	−0.005	0.33	0.752	−0.0026	−1.1
B5	5/11/2000	322	0.2	0.035	0.33	0.752	0.0182	7.7
B6	8/11/2000	414	0.14	−0.06	0.33	0.752	−0.0311	−13.2
B7	11/17/2000	512	0.17	0.03	0.33	0.752	0.0156	6.6
B8	3/13/2001	628	0.22	0.05	0.33	0.752	0.0259	11
B9	7/15/2001	752	0.17	−0.05	0.33	0.752	−0.0259	−13.0

Table 17.3s Settlement of Brown Silty Clay

Boring	Boring Date	Elapsed time (days)	w_i(%)	Δw_i(%)	f_i	E_{wi}	$\Delta\varepsilon_i$	s_2(m)	s_{total}(m)
B1	6/24/1999	0	0.15	—	0.33	0.869	0.0000	0.0	0.0
B2	7/13/1999	19	0.15	0	0.33	0.869	0.0000	0.0	−1.1
B3	10/25/1999	123	0.17	0.02	0.33	0.869	0.0090	3.8	−0.6
B4	2/11/2000	232	0.155	−0.015	0.33	0.869	−0.0067	−2.9	−4.0
B5	5/11/2000	322	0.19	0.035	0.33	0.869	0.0157	6.7	14.3
B6	8/11/2000	414	0.155	−0.035	0.33	0.869	−0.0157	−6.7	−19.8
B7	11/17/2000	512	0.15	−0.005	0.33	0.869	−0.0022	−0.9	5.7
B8	3/13/2001	628	0.21	0.06	0.33	0.869	0.0269	11.4	22.3
B9	7/15/2001	752	0.18	−0.03	0.33	0.869	−0.0135	−5.7	−16.7

Problem 17.13

The high-rise building shown in Figure 17.53 is subjected to a hurricane wind of 200 km/h. This wind creates a pressure of 3 kPa on the flat side of the building. Calculate the pressure diagram under the foundation.

Solution 17.13

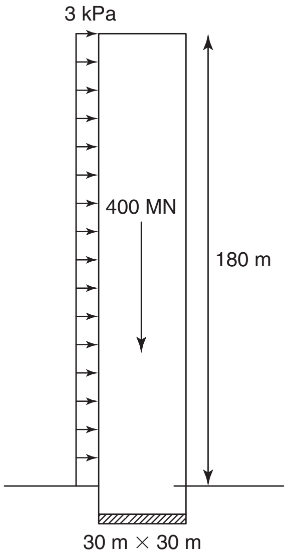

Figure 17.17s High-rise building.

The average pressure is:

$$p_{ave} = \frac{400000}{30 \times 30} = 444 \text{ kPa}$$

The wind creates a horizontal force equal to:

$$H = pA = 3 \times 30 \times 180 = 16200 \text{ kN}$$

The point of application of that force is at a height of 90 m above the ground surface. Therefore, the moment applied on the foundation is:

$$M = Hb = 16200 \times 90 = 1458000 \text{ kN} \cdot \text{m}$$

This moment will create a trapezoidal pressure distribution under the foundation such that the high pressure will be $p_{\max}$ and the low pressure $p_{\min}$. The high pressure $p_{\max}$ is such that:

$$M = \frac{1}{2}(p_{\max} - p_{ave})B \times \frac{B}{2} \times \frac{2}{3} \times \frac{B}{2} \times 2 = \frac{B^3}{6}(p_{\max} - p_{ave})$$

But the average pressure is equal to the vertical load V divided by the foundation area A:

$$p_{ave} = \frac{V}{B^2}$$

and the eccentricity e is given by:

$$M = Ve$$

In the end,

$$p_{\max} = p_{ave}\left(1 + \frac{6e}{B}\right)$$

and then:

$$p_{\min} = p_{ave}\left(1 - \frac{6e}{B}\right)$$

Numerically:

$$p_{\max} = 444\left(1 + \frac{6(1458000/400000}{30}\right) = 768 \text{ kPa}$$

$$p_{\min} = 444\left(1 - \frac{6(1458000/400000}{30}\right) = 120 \text{ kPa}$$

The p_{max} value would have to be checked against the ultimate bearing capacity of the soil.

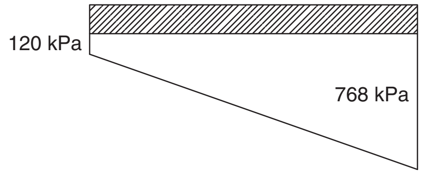

Figure 17.18s Stresses under foundation.

Problem 17.14

The high-rise building shown in Figure 17.53 is placed on a stiff clay with the following properties: compression index C_c equal to 0.4, recompression index C_r equal to 0.1, initial void ratio e_o equal to 0.5, and total unit weight equal to 20 kN/m^3. The soil is lightly overconsolidated by overburden removal and has a preconsolidation pressure σ'_p 150 kPa higher than the effective stress σ'_{ov}. The groundwater level is at the ground surface. Calculate the settlement of the building. How would you estimate the time required for the settlement to take place if c_v was known?

Solution 17.14

The solution is the same as the one presented in Table 17.12 except that the consolidation test is used instead of the pressuremeter test. For the consolidation test, the equations change for the evaluation of the settlement due to the building pressure. The following equations cover all the possible cases at various depths:

$$\text{If } \sigma'_{ov} + \Delta\sigma_{bldg} < \sigma'_p \quad \text{use} \quad \Delta H = \frac{H_o}{1+e_o} C_r \log\left(\frac{\sigma'_{ov} + \Delta\sigma_{bldg}}{\sigma'_{ov}}\right)$$

$$\text{If } \sigma'_{ov} + \Delta\sigma_{bldg} > \sigma'_p \quad \text{use} \quad \Delta H = \frac{H_o}{1+e_o}\left(C_r \log\left(\frac{\sigma'_p}{\sigma'_{ov}}\right) + C_c \log\left(\frac{\sigma'_{ov} + \Delta\sigma_{bldg}}{\sigma'_p}\right)\right)$$

Note that from the statement of the problem:

$$\sigma'_p = \sigma'_{ov} + 150 \text{ kPa}$$

It is assumed that the excavation and subsequent construction are done under undrained conditions. Hence, the effective stress after excavation and therefore at the beginning of construction is the same as the effective stress before excavation begins. This assumption states that during the undrained behavior, the change in total stress due to excavation is the same as the change in water stress. The following table shows the calculations for each of the four layers of Figure 17.19s.

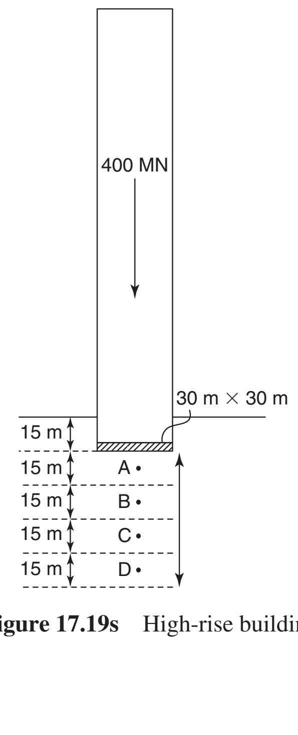

Figure 17.19s High-rise building.

Table 17.4s Calculations for the consolidation settlement of the highrise

1	2	3	4	5	6	7
Point	H (m)	Depth below ground surface (m)	Depth below foundation level (m)	σ_{ov}(kPa)	σ'_{ov}(kPa)	$\Delta\sigma_{bldg}$(kPa)
A	15	22.5	7.5	450	225	378
B	15	37.5	22.5	750	375	222
C	15	52.5	37.5	1050	525	111
D	15	67.5	52.5	1350	675	67

8	9	10	11	12	13	14
$\sigma'_{ov} + \Delta\sigma_{bldg}$(kPa)	σ'_p(kPa)	C_c	C_r	ΔH_R(mm)	ΔH_V(mm)	ΔH_T(mm)
603	375	0.4	0.1	221	825	1046
597	525	0.4	0.1	146	223	369
636	675	0.4	0.1	83	0	83
742	825	0.4	0.1	41	0	41

Summing column 14 gives a total settlement of 1539 mm. This is obviously not tolerable for such a building.

The time rate of settlement can be estimated by using the consolidation theory solution described in section 11.4.6. The time required for a given percentage of the settlement to take place is given by:

$$t_U = T_U \frac{H^2}{c_v}$$

where t_U is the time required for $U\%$ of the settlement to take place, T_U is the time factor (which comes from the theoretical solution and is obtained from Figure 17.34), H is the drainage length, and c_v is the coefficient of consolidation for the soil obtained from a consolidation test (see section 9.5.1). The parameter U is the average percent consolidation, which is a function of the time t and is defined as:

$$U(t) = \frac{\Delta H(t)}{\Delta H_{max}}$$

where $\Delta H(t)$ is the settlement after a time t and ΔH_{max} is the maximum settlement at time equal to infinity. ΔH_{max} is the settlement obtained from previous calculations.

The major question in this case is to find the drainage length H. This is done by carefully analyzing the stratigraphy to estimate the thickness of the compressing layer between two draining layers. Identifying the presence of sand seams in a clay deposit becomes very important in this case.

Problem 17.15

The annual drying and wetting condition of a site is shown in Figure 17.5s. Calculate the shrink and swell displacement at the center and at the edge of the building, and then calculate the differential movement between the two points. Hint: Use

$$\varepsilon_i = \frac{\Delta H_i}{H_i} = f\frac{\Delta w_i}{E_{wi}} \qquad \therefore \varepsilon_i = 0.33\frac{\Delta w_i}{(\gamma_w/\gamma_d)}$$

and

$$w_i = 25\%, \qquad \gamma_d = 14\ \text{kN/m}^3, \gamma_w = 10\ \text{kN/m}^3$$

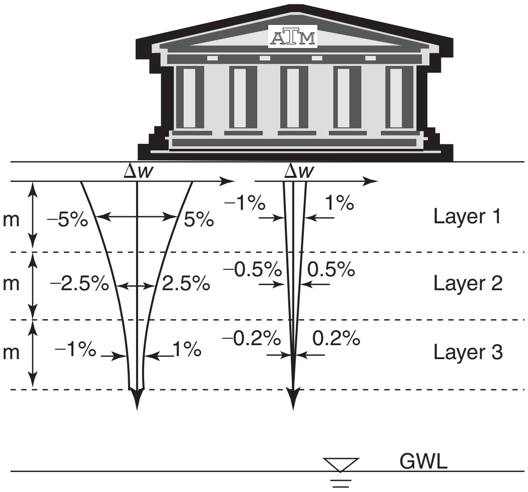

Figure 17.5s Annual drying and wetting condition.

Solution 17.15

$$\varepsilon = 0.33\frac{\Delta w_i}{(\gamma_w/\gamma_d)} = 0.33\frac{\Delta w_i}{10/14} = 0.462\Delta w_i$$

$$\varepsilon_i = \frac{\Delta H_i}{H_i} \qquad \therefore \Delta H_i = \varepsilon_i H_i$$

a. Center

Layer	w_i	Δw_i	ε_i	ΔH_i (mm)
1	0.25	±0.01	±0.00462	±4.62
2	0.25	±0.005	±0.00231	±2.31
3	0.25	±0.002	±0.00092	±0.92

The total displacement at the center of the foundation:

- Shrinking: −7.85 mm
- Swelling: +7.85 mm

b. Edge

Layer	w_i	Δw_i	ε_i	ΔH_i (mm)
1	0.25	±0.05	±0.0231	±23.1
2	0.25	±0.025	±0.01155	±11.55
3	0.25	±0.01	±0.00462	±4.62

The total displacement at the edge of the foundation:

- Shrinking: −39.27 mm
- Swelling: +39.27 mm

Problem 17.16

A stiffened slab on grade for a two-story house is to be designed. The slab and site data are given as follows: slab dimensions 20 m by 20 m, beam spacing s = 3.0 m (for both directions), beam width $b = 0.3$ m, slab load $w = 10$ kPa, depth of movement zone $H = 3.0$ m, soil surface water content change $\Delta w_o = 20\%$. Recommend a beam depth that will minimize the distortion of the slab for the edge drop case to more than $L/\Delta = 500$.

Solution 17.16

The design process advances by trial and error in the sense that the beam depth is assumed and then the resulting deflection is calculated and checked against the distortion criterion. If the deflection criterion is not met, a larger beam depth is assumed. Let's assume a beam depth of 1.2 m. The calculations then proceed with the soil-weather index I_{s-w} calculations:

$$\Delta w_{edge} = 0.5\ \Delta w_0 = 0.5 \times 0.2 = 0.1 \text{ or } 10\%$$

$$\mathrm{I}_{s-w} = \Delta w_{edge} \times H = 0.1 \times 3 = 0.3 \text{ m}$$

and then the slab bending stiffness:

$$\mathrm{EI} = \mathrm{E\ bh}^3/12 = 2 \times 10^7 \times 0.3 \times 1.2^3/12 = 8.64 \times 10^5 \text{ kN.m}^2$$

which leads to the equivalent slab thickness:

$$b\ h^3/12 = s\ d_{eq}{}^3/12$$

$$d_{eq} = h(b/s)^{1/3} = 1.2\ (0.3/3)^{1/3} = 0.56 \text{ m}$$

The values of the design parameters are read on the water content charts for the edge drop case:

$$L_{\mathrm{eq}} = 5.3 \text{ m for maximum moment}$$

$$L_{\mathrm{gap}} = 3.6 \text{ m for information}$$

$$F_{\Delta\max} = 2.9 \text{ for maximum deflection}$$

$$F_v = 0.8 \text{ for maximum shear}$$

The maximum bending moment is calculated as:

$$q = 10 \times 3 = 30 \text{ kN/m line load on each beam}$$

$$M_{\max} = 0.5\ \mathrm{q}L_{eq}{}^2 = 0.5 \times 30 \times 5.3^2 = 421.3 \text{ kN.m}$$

The maximum deflection is calculated as:

$$\Delta_{\max} = q\ L_{eq}{}^4/F_{\Delta\max}EI = 30 \times 5.34/2.9 \times 8.64 \times 10^5$$

$$\Delta_{\max} = 9.5 \times 10^{-3}\text{m}$$

The maximum shear force is calculated as:

$$V_{\max} = F_v q\ L_{eq} = 0.8 \times 30 \times 5.3 = 127.2 \text{ kN}$$

This results in a distortion of:

$$0.5\mathrm{L}/\Delta_{\max} = 10/9.5 \times 10^{-3} = 1050$$

$$L_{\mathrm{eq}}/\Delta_{\max} = 5.3/9.5 \times 10^{-3} = 558$$

Note that this example is an extreme case, as a Δw_0 of 20% corresponds to extreme weather conditions and a distributed pressure of 10 kPa is quite high for a house. This is why the beam depth is significant.

Problem 17.17

Calculate the settlement of the San Jacinto Monument using the pressuremeter data given in section 17.12.3.

Solution 17.17

The steps outlined in section 17.8.5 are used to solve this problem. The calculations are shown in Table 17.5s. The bulbs of pressure were used to obtain the change in stress at depth. The unit weight of the soil is 18 kN/m^3, the size of the foundation is 37.8 m × 37.8 m, the excavation depth is 4.5 m, the excavation pressure is −83 kPa, the contact pressure between the foundation and the soil is 224 kPa, and the following equations were assumed, using the values of the moduli given in section 17.12.3:

$$E_o(\text{MPa}) = 15 + 1.125\ \text{z(m)} = 15 + 1.125 \times 38\ \text{(z/B)} = E_o + E_1(z/B)$$

$$E_R(\text{MPa}) = 31.5 + 2.362\ \text{z(m)} = 31.5 + 2.362 \times 38\text{(z/B)} = E_{ro} + E_{r1}(z/B)$$

The release of pressure due to excavating 4.5 m of soil is:

$$\Delta p_{exc} = 4.5 \times 18 = 81\ \text{kPa}$$

The contact pressure under the building is:

$$\Delta p_{bldg} = 313000/(37.8 \times 37.8) = 219\ \text{kPa}$$

The total settlement of the monument is the sum of the last two columns or:

$$S_{total} = 41.5 + 114.1 = 155.6\ \text{mm}$$

Table 17.5s Calculations of the San Jacinto Monument settlement

H (m)	Depth to center of layer (m)	σ_{ov} (kPa)	E_o (kPa)	E_r (kPa)	Pressure factor	$\Delta\sigma_{exc}$ (kPa)	$\Delta\sigma_{bldg}$ (kPa)	$\Delta\varepsilon_{exc}$	$\Delta\varepsilon_{net}$	ΔH_{rel} (mm)	ΔH_{net} (mm)
19	14	252	30750	64575	0.88	71	193	0.001100	0.003970	20.9	75.4
19	33	594	52125	109462	0.50	40	110	0.000365	0.001343	16.9	25.5
19	52	936	73500	154350	0.25	20	55	0.000139	0.000476	2.6	9.0
19	71	1278	94875	199237	0.15	12	33	0.000060	0.000221	1.1	4.2
										$\Sigma = 41.5$	$\Sigma = 114.1$

Also:

- Elastic settlement (Equation 17.64)—Using an average modulus of 30 MPa and Poisson's ratio of 0.35:

$$s = I(1 - \nu^2)\frac{pB}{E} = 0.88(1 - 0.35^2)\frac{141 \times 38}{30} = 138\ \text{mm}$$

- Long-term settlement (Equation 17.100)—Where $s(t_o) = 138$ mm, $t_o = 5$ min, $t = 70$ years, and $n = 0.045$:

$$\frac{s(t)}{s(t_o)} = \left(\frac{t}{t_o}\right)^n$$

$$\frac{s(t)}{138} = \left(\frac{70 \times 365 \times 24 \times 60}{5}\right)^{0.045}$$

$$s(t) = 281.1\ \text{mm}$$

- Settlement using linear increase of modulus with depth (Equation 17.68)—Using the elastic settlement equation with the elastic modulus at the surface:

$$s_o = I(1 - v^2)\frac{pB}{E_o} = 0.88(1 - 0.35^2)\frac{141 \times 38}{15} = 276 \text{ mm}$$

From Figure 17.26, I_G is 0.5 (using $E_1/E_{\mathrm{o}} = 2.85$):

$$\frac{E_1}{E_o} = \frac{1.125 \times 38}{15} = 2.85$$

From Equation 17.68, the settlement is:

$$I_G = \frac{s_1}{s_o}$$

$$s_1 = I_G s_o = 0.5(276) = 138 \text{ mm}$$